XV.

THE SHIELDING EFFECT OF PLANETARY MAGNETIC FIELDS.

(From the Dublin Institute for Advanced Studies.)

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1. INTRODUCTION. SUMMARY.

VERY much the same as in the case of Einstein's field-equations of gravitation in empty space, Maxwell's equations likewise admit of a term expressing that the *potentials* act also as *sources* of the field—the "cosmical term," as it is usually called. While in the case of gravitation anything but an *extremely* low order of magnitude of this term is excluded by observation, the widespread belief that the corresponding Maxwellian term must be of the same low order, neither rests on direct experimental evidence, nor are there strong theoretical grounds for it. As one of us has recently pointed out,[1] our present knowledge of the earth's magnetic field suggests for the constant in question (μ^2) a value, still moderately small, but after all about 10^{32}-times larger than the "cosmical" value. General field theories seldom fail to produce the term.[2] This fact ought not to be over-emphasized, for they may have been deceived (see the footnote), and at any rate they do not positively indicate the order of magnitude. But quite a strong argument *pro* (which we have never seen mentioned and which would be all-but-quenched, if the term were insignificantly small) is this: the two equations curl $A = H$ and curl $H = -\mu^2 A$ exclude an irrotational static magnetic field, and thus automatically *exclude the existence of an isolated magnetic pole* (the current version of Maxwell's theory has to exclude it explicitly).

Reviewing the situation we deem that, just as in the case of gravitation, the "μ-term" ought not to be regarded as a "new addition" to Maxwell's equations but as being virtually contained in them. We have to decide, not *whether* there is a μ-term or not, but *what upper limit observation sets to the constant* μ.

[1] E. Schrödinger, Proc. R.I.A. (A), 49, 135, 1943.

[2] H. Weyl, Raum-Zeit-Materie (Berlin, Springer, 1921), § 36; A. Einstein, Sitz. Ber. d Preuss. Akad., p. 137, 1923; E. Schrödinger, Proc. R.I.A. (A), 49, 55, 237, 275, 1943. In the *last* paper it is pointed out that, what had always been interpreted as the Maxwellian field, is in fact, very likely, the meson field. This is the possible *deception* alluded to above.

The limit is imposed by *large-scale static fields.* The observed structure of the magnetic field *on the surface* of the earth certainly excludes a value of the constant μ considerably larger than $(30{,}000 \text{ km.})^{-1}$; it possibly supports this figure, which, as far as we can see, would not be at variance with any other phenomenon.

The constant intervenes, of course, in computing from the known *surface-field* the field *surrounding* the earth, which is indeed much more strongly affected than the former—and so is its *shielding effect* towards charged particles, coming from outside (aurora-particles, cosmic rays). *This is the object of the present investigation.* We generalize, for the modified field, Störmer's results[3]—but not the refinements of Lemaître and Vallarta, which would be infinitely laborious and is not required in a first survey.

The constant μ enters in form of the product μa, i.e. of the ratio of the radius a of the celestial body and the length μ^{-1}. With the value of μ^{-1} mentioned above, $\mu a = 0{\cdot}2$ and roughly $= 20$ for the sun. That is why we carry the numerical computations as far as $\mu a = 20$.

We can hardly avoid reproducing the main trend of the whole theory of shielding, which we believe to have put into neat form. It proves handier to speak throughout not of "shielding" but of "escape." Since we disregard the bodily screening effect of the planet,[4] the orbits are, as it were, reversible. The minimum momentum which just enables a particle, coming from infinity, to *impinge* on a given point of the surface from a given direction, also just enables a particle of opposite charge to *escape* from there to infinity on the reversed orbit.

Specializing, for the purpose of illustration, in the μ-value quoted above $[\mu = (30{,}000 \text{ km.})^{-1}]$, our results are briefly these:—

$\mu a = 0{\cdot}2$ (earth): The minimum momentum decreases steadily towards the pole. At 50°, where the cosmic-ray latitude-effect stops, it is (for any direction of launching) by about 20 per cent. larger than with $\mu = 0$. The bearing on cosmic rays is hardly significant. But the *percentage increase* (as against the case $\mu = 0$) *increases* considerably at higher latitudes. Hence *there is* a mutual bearing of the precise value assumed for μ and any quantitative theory of the aurora;

$\mu a = 20$ (sun): The minimum momentum for escape is, of course, *considerably lowered* as against $\mu = 0$, though much less than we might anticipate from the exponential decrease of the field. The potential bearing on aurora theories is obvious. But, with the present scanty knowledge of the sun's magnetic field, it is useless.

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[3] For a general account of the Störmer theory cf. S. Chapman and J. Bartels, Geomagnetism, Vol. 2, p. 833, et seqq. (Oxford, Clarendon Press, 1940).

[4] Precisely to this point—and to the question of periodic orbits—does Lemaître's and Vallarta's improvement on Störmer refer.

§ 2. THE REDUCED HAMILTONIAN.

Using spatial coordinates with the line-element

$$ds^2 = g_{ik}\, dx_i\, dx_k \qquad (i, k = 1, 2, 3), \qquad (2, 1)$$

the well-known Hamiltonian of a particle with charge e and rest-mass m_0, moving in an electromagnetic field with the potentials V, A_k $(k = 1, 2, 3)$, reads as follows:

$$H = eV + m_0 c^2 \sqrt{1 + \frac{1}{m_0^2 c^2} g^{ik}\left(p_i - \frac{e}{c}A_i\right)\left(p_k - \frac{e}{c}A_k\right)}. \qquad (2, 2)$$

In ordinary vector notation the potentials of a static magnetic dipole **D** at the origin are[5]

$$\mathbf{A} = \text{curl}\left(\frac{\mathbf{D}e^{-\mu r}}{r}\right), \qquad V = 0. \qquad (2, 3)$$

From this, in polar coordinates, with **D** in the direction $\theta = 0$,

$$A_1 = A_2 = 0 \qquad A_3 = \frac{\sin^2\theta\, D f(r)}{r}, \qquad (2, 4)$$

where 1, 2, 3 refer to r, θ, ϕ respectively, D is the dipole-strength, f is short for

$$f(r) = (1 + \mu r)e^{-\mu r} \qquad (2, 5)$$

Hence (replacing the subscripts 1, 2, 3 by r, θ, ϕ for clarity):

$$H = \\ = m_0 c^2 \sqrt{1 + \frac{1}{m_0^2 c^2}\left[p_r^2 + \frac{1}{r^2} p_\theta^2 + \frac{1}{r^2 \sin^2\theta}\left(p_\phi - \frac{\sin\theta\, e D f}{c r}\right)^2\right]}. \qquad (2, 6)$$

Since ϕ is not contained, p_ϕ is a constant of the motion and the problem reduces to the two-dimensional motion in the meridional half-plane r, θ (with the *same* Hamiltonian). Moreover, H is a constant of the motion $\geqslant m_0 c^2$. Let m be *any* constant $\geqslant m_0$. Envisage the following function of the function H

$$F(H) = \frac{1}{2m}\left(\frac{H^2}{c^2} - m_0^2 c^2\right) = \frac{1}{2m}\, [\ldots] \qquad (2, 7)$$

[5] E. Schrödinger, Proc. R.I.A., 49, 137, 1943.

(the square bracket [. . .] being that of (2, 6)). If q is any one of the six variables r, θ, ϕ, p_r, p , p_ϕ,

$$\frac{\partial F}{\partial q} = F' \frac{\partial H}{\partial q} = \frac{H}{mc^2} \frac{\partial H}{\partial q}. \tag{2, 8}$$

Hence the equations of motion can be obtained from the (very much simpler) Hamiltonian $F(H)$, provided that we regard the constant m as depending on the particular motion and as linked with its energy constant H by

$$H = mc^2,$$

(which means, that m is the "relativistic mass" of the particle). In a word we have now formally reduced the problem to the *two-dimensional "non-relativistic" motion* with the Hamiltonian

$$\frac{1}{2m}\left[p_r^2 + \frac{1}{r^2} p_\theta^2 + \left(\frac{p_\phi}{r \sin\theta} - \frac{\sin\theta\, e D f}{r^2 c}\right)^2\right] = \frac{p^2}{2m}, \tag{2, 9}$$

which is of very familiar form. By putting it equal to $p^2/2m$, the constant p is obviously the total momentum of the (true) motion.

It is convenient to reduce the momenta by the factor[6] eD/c, and to put

$$\frac{c^2}{e^2 D^2}\left(p_r^2 + \frac{1}{r^2} p_\theta^2\right) = P'^2$$

$$\frac{c}{eD} p_\phi = h \tag{2, 10}$$

$$\frac{c^2}{e^2 D^2} p^2 = P^2.$$

Then (2, 9) reads

$$P'^2 + \left(\frac{h}{r \sin\theta} - \frac{\sin\theta f(r)}{r^2}\right)^2 = P^2. \tag{2, 11}$$

Just like P, we take P' non-negative. It is the momentum in the meridional plane or, speaking of the two-dimensional motion, the momentum. The quantity in brackets is the momentum ($m \times$ velocity) in the ϕ-direction. It provides, for the two-dimensional motion, a store of potential energy. But notice that the form of the latter still depends on the value of the integration constant h (= the canonical p_ϕ). Let

[6] We take $e > 0$, for convenience.

α be the angle between the three-dimensional velocity and the direction of increasing ϕ, the "magnetic West." Then

$$P' = P \sin \alpha \tag{2, 12}$$

and, from (2, 11),

$$\frac{h}{r \sin \theta} - \frac{\sin \theta f(r)}{r^2} = P \cos \alpha . \tag{2, 13}$$

The sign is not ambiguous, as you verify by forming, with the Hamiltonian (2, 9), the canonical equation which gives $\dot{\phi}$.

§ 3. The Potential Energy.

We have to investigate the restrictions on escape, possibly imposed by the "potential energy."

$$U(r, \theta) = \psi(r, \theta)^2, \tag{3, 1}$$

where

$$\psi(r, \theta) = \frac{h}{r \sin \theta} - \frac{\sin \theta f(r)}{r^2}. \tag{3, 2}$$

If $\mu = 0$, from (2, 5), $f = 1$. In this case the function U has been extensively studied and represented in diagrams[7]. Qualitatively it behaves alike for any μ. To examine the *radial* behaviour, form

$$r^3 \frac{\partial \psi}{\partial r} = - \frac{h}{\sin \theta} r + 2 \sin \theta (1 + \mu r + \tfrac{1}{2} \mu^2 r^2) e^{-\mu r}. \tag{3, 3}$$

For $h \leqslant 0$, we have $\psi < 0$ and $\partial \psi / \partial r > 0$. Thus U *de*creases permanently. *Hence there is no energetic restriction on escape unless h is positive.*

If it is, then from the fact that

$$\frac{\partial}{\partial r}\left(r^3 \frac{\partial \psi}{\partial r}\right) = - \frac{h}{\sin \theta} - \mu^3 \sin \theta \, r^2 e^{-\mu r} < 0, \tag{3, 4}$$

you easily infer, that ψ and u depend on r as qualitatively indicated in Fig. 1.

The relevant point is, that u passes *in every radial direction,* first through a zero-minimum, then through a maximum. The series of these maxima—call it the *rim*—has one minimum at $\theta = \frac{\pi}{2}$ — call it the *pass.*

[7] See e.g. S. Chapman and J. Bartels, l.c., page 837.

This is the point of easiest escape. If we disregard the screening effect of the body of the planet and also disregard a "point set of content zero" of exceptional initial values, then according to a famous theorem of Poincaré's, a particle launched anywhere inside the rim *will* sooner or later reach the *pass* and make its escape, provided it has sufficient energy.

In the *pass* $\partial\psi/\partial r = 0$ and $\theta = \frac{\pi}{2}$. Hence from (3, 2) and (3, 3)

$$\left.\begin{aligned} \psi_{pass} &= \frac{h}{r} - \frac{f(r)}{r^2} \\ h &= \frac{2}{r}\left(1 + \mu r + \tfrac{1}{2}\mu^2 r^2\right)e^{-\mu r} \end{aligned}\right\} \qquad (3, 5)$$

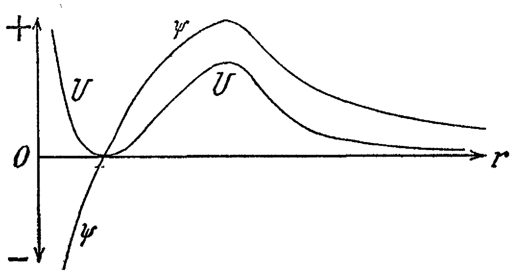

Fig. 1.

that is to say

$$\psi_{pass} = \frac{1}{r^2}\left(1 + \mu r + \mu^2 r^2\right)e^{-\mu r}. \qquad (3, 6)$$

The letter r shall henceforth refer to the pass.

§ 4. The Minimum Escape Momentum.

Now let the particle be launched at a distance a from the origin (a = radius of the planet), at a pole-distance $\theta = \lambda$ and under an angle α with the "magnetic West" (i.e. the direction of increasing ϕ). From (2, 13)

$$\frac{h}{a \sin\lambda} - \frac{\sin\lambda f(a)}{a^2} = P\cos\alpha\,. \qquad (4, 1)$$

We are out to find the smallest P which allows escape. To lower P, we have h at our free disposal. Now, for $\cos\alpha < 0$, h could be negative; but then we could, with impunity, lower P further by taking $h = 0$ instead, since we have seen that only with h positive does any restriction on escape arise. *Hence h must be positive anyhow.*

Therefore (3, 5) and (3, 6) obtain and, to permit escape, P must at least equal ψ_{pass}. But if P exceeded ψ_{pass} *by a finite amount,* you could in (4, 1) always change h in the direction that lowers P, irrespective of ψ_{pass} being thereby possibly raised. *So they must be equal.* From (3, 5), (3, 6), and (4, 1) that gives

$$\frac{\cos\alpha}{r^2}\left(1 + \mu r + \mu^2 r^2\right)e^{-\mu r} = \tag{4, 2}$$
$$= \frac{2}{a\, r \sin\lambda}\left(1 + \mu r + \tfrac{1}{2}\mu^2 r^2\right)e^{-\mu r} - \frac{\sin\lambda}{a^2}\left(1 + \mu a\right)e^{-\mu a} .$$

Putting

$$\mu a = x , \qquad \frac{r}{a} = u \tag{4, 3}$$

you get

$$\sin^2\lambda\,(1 + x)e^{-x} =$$
$$= \left[\frac{1 + u x}{u^2}\,(2u - \sin\lambda\cos\alpha) + x^2\,(u - \sin\lambda\cos\alpha)\right]e^{-ux} . \tag{4, 4}$$

This transcendental equation solves the problem. x is a given quantity. The equation has to be solved for u, which is "the distance of escape," expressed in the unit a, the radius of the planet. Thereafter the minimum momentum for escape is given by (3, 6), viz.

$$a^2 P = \frac{1 + u x + u^2 x^2}{u^2}\,e^{-ux} . \tag{4, 5}$$

The rest of this paper deals with the numerical evaluation of (4, 4) and (4, 5) in a number of cases, chosen for illustration.

But we must still remove an objection which may have occurred to the reader. What if the particle were launched *outside the rim*—then the altitude of the *pass* would be irrelevant? Well, it must not be forgotten, that, given the point and the direction of launching (a, λ, α), the position of the rim still depends on the momentum P we impart to the particle. It is not very difficult to show, that to make the rim contract so far that the point (a, λ) is outside already at the outset, would always require a larger P than the one we have determined.

§ 5. Numerical Evaluation.

From (2, 10) our P is the momentum times

$$\frac{c}{e\,D}$$

where D is the dipole moment. Our object is to study the influence which various assumptions about the basic constant μ have on the escape-momenta. Now it is not recommendable to make the comparison *for the same value of D*. For, given any information about the actual field at the surface, the value of D to comply with this information depends itself on the value adopted for μ.

We have therefore chosen, always to make the comparison *for the same equatorial surface field.*[8] Calling D' the "Gaussian value" of D i.e. the dipole moment which produces for $\mu = 0$ the same surface field on the equator, we have the relation[9]

$$\frac{D}{a^3}\left(1 + \mu a + \mu^2 a^2\right) e^{-\mu a} = \frac{D'}{a^3} \quad . \tag{5, 1}$$

With the notation (4, 3)

$$D' = D\,(1 + x + x^2)\, e^{-x}, \tag{5, 2}$$

so that we obtain from (4, 5) and the last relation (2, 10) the final formula for the minimum escape momentum p

$$\frac{a^2 c}{e D'} p = \frac{1}{u^2} \frac{1 + ux + u^2 x^2}{1 + x + x^2} e^{(1-u)x}, \tag{5, 3}$$

where u, as before, is determined by (4, 4) as a function of λ, a and $x\,(= \mu a)$.

It is customary and quite convenient to characterize a particle, instead of just by its momentum p, by the product

$$S = \frac{cp}{e}, \tag{5, 4}$$

which is called its *magnetic stiffness* and has, when the particle moves orthogonal to a magnetic field, the well-known meaning: field-strength times radius of curvature of the path.

The dimensionless factor on the right of (5, 3) thus gives S in the unit

$$\frac{D'}{a^2} = a \times H_{equat.} \tag{5, 5}$$

which characterizes the planetary field. Moreover, since e will nearly always be the electronic charge, it is convenient to think of (5, 4) and (5, 5) as expressed in "electron-volt."

The value of (5, 5) in this unit is

$$300\, a\, H_{equat.} = 5{\cdot}7 \times 10^{10} \text{ electron-volts}, \tag{5, 6}$$

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[8] This is the simplest to keep to in general. In the case of the earth the choice is irrelevant, the deviation of the surface-field from a classical dipole-field being certainly not larger than about 2 per cent.

[9] Cf. Proc. R.I.A. (A), 49, p. 137, 1943.

where a is in cm., $H'_{equat.}$ in Gauss and the numerical value refers to the earth. But let us recall, that S (though measured in electron-volt) is *not* in general *the energy.* The two coincide only in the limit for very fast particles, when the energy is a considerable multiple of the rest-energy.

In the following tables and graphs we omit the factor (5, 6), that is to say, *we tabulate and plot as "magnetic stiffness S" directly the dimensionless factor* (5, 3).[10] Only in Fig. 6 are the actual *energies* calculated for electrons and protons, the scale of ordinates is in electron-volt and refers, of course, to the earth.

In the same way we tabulate and plot the dimensionless quantity u as "escape distance,"[10] meaning really ua. The latter quantity can also be described as the radius of the *circular* orbit concentric with the equator in the equatorial plane. (In the two-dimensional motion the particle is in this case *at rest* on the "pass".)

The substantial task in the numerical evaluation is to find, given x, λ, a, the root u of equation (4, 4), which has then to be inserted in (5, 3). Though, as a rule, nothing but systematic trials, with subsequent interpolation, are of avail, a glance at the approximations obtainable for $x \ll 1$ and for $x \gg 1$ is useful.

For x *small* you get, by developing the exponentials in (4, 4), without difficulty

$$u = u_0 + \frac{\sin\lambda - \cos a}{4\sin\lambda\sqrt{1-\sin^3\lambda\cos a}}\, u_0^2 x^2 + \dots \qquad (5, 7)$$

and then from (5, 3), (with the notation (5, 4) and the omission of the factor (5, 5), as agreed upon)

$$S = S_0 + \tfrac{1}{2}\left(1 - \frac{1}{u_0^2} - \frac{(\sin\lambda - \cos a)}{u_0\sin\lambda\sqrt{1-\sin^3\lambda\cos a}}\right)x^2 + \dots, \qquad (5, 8)$$

where

$$u_0 = \frac{1+\sqrt{1-\sin^3\lambda\cos a}}{\sin^2\lambda}; \qquad S_0 = \frac{1}{u_0^2}$$

are the values for $x = \mu a = 0$, known from Störmer's work. The factors of x^2 prove to be *positive* in many cases (actually in all those dealt with below). Thus, strangely enough, there is an initial increase of stiffness and of escape-distance, but followed eventually, and usually

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[10] That is the meaning of the words "*reduced* stiffness," "*reduc*ed escape distance" in the explanation of the tables and graphs.

very soon, by a maximum and subsequent steady decrease. This complicated initial behaviour has an ill effect on the convergence of the power series, which are therefore as a rule of little value.

For x large you deduce from (4, 4)

$$(u - 1) = \log \left\{ \frac{(u + v)\,x}{\sin^2 \lambda \left(1 + \frac{1}{x}\right)} \left[1 + \frac{2u + v}{u^2 (u + v)} \left(\frac{u}{x} + \frac{1}{x^2} \right) \right] \right\}, \qquad (5, 9)$$

where for abbreviation

$$v = - \sin \lambda \cos \alpha \,. \qquad (5, 10)$$

Putting also

$$x' = \frac{x}{\sin^2 \lambda}\,, \qquad (5, 11)$$

you obtain, by carefully developing the logarithms,

$$u = 1 + \frac{\log (1 + v)\,x'}{x} + \frac{\log (1 + v)\,x' + 1}{(1 + v)\,x^2} - \frac{\frac{1}{2} (\log x')^2 + [(1 + v)^2 + \log (1 + v)] \log x'}{(1 + v)^2 x^3} \qquad (5, 12)$$

The order of terms corresponds roughly to their efficiency between about $x = 10$ and $x = 20$, where, generally speaking, the formula works well. Terms of order x^{-3} are neglected, but e.g. $\log x / x^3$ is included. The case of very small λ (neighbourhood of the pole) would need special attention. It is best to draw u numerically from (5, 12) and then to insert it in the following very good approximation of (5, 3)

$$S = \frac{\left(1 - \frac{1}{x^2}\right) \sin^2 \lambda}{1 + (u + v)\,x}\,, \qquad (5, 13)$$

in which only terms of *relative* order x^{-3} are neglected.

§ 6. Examples.

To study the escape-distance u and the required stiffness S for an extended range of the parameter $x = \mu a$ (viz., from $0 \rightarrow 20$) we picked out 4 cases: a particle launched from the *equator* or from Latitude 45°, either in the horizontal direction of *easiest* escape or vertically. (The "easiest escape" is *eastward* for a positive particle, it corresponds to "easiest arrival" *from the west*—for a positive particle.) The angles λ, a are:

Equator,	easiest:	$\lambda = \frac{\pi}{2}$	$a = \pi$
„	vertical:	$\lambda = \frac{\pi}{2}$	$a = \frac{\pi}{2}$
45°	easiest:	$\lambda = \frac{\pi}{4}$	$a = \pi$
„	vertical:	$\lambda = \frac{\pi}{4}$	$a = \frac{\pi}{2}$

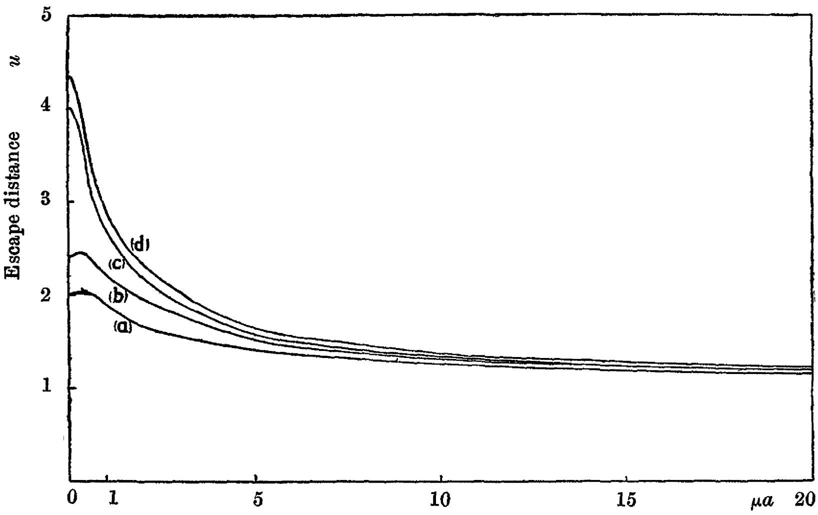

Fig. 2.—Escape distance (in the unit a) as a function of μa for the cases—
(a) vertical escape from magnetic equator; (c) vertical escape from 45° magnetic latitude;
(b) easiest „ „ „ „ ; (d) easiest „ „ „ „ „ .

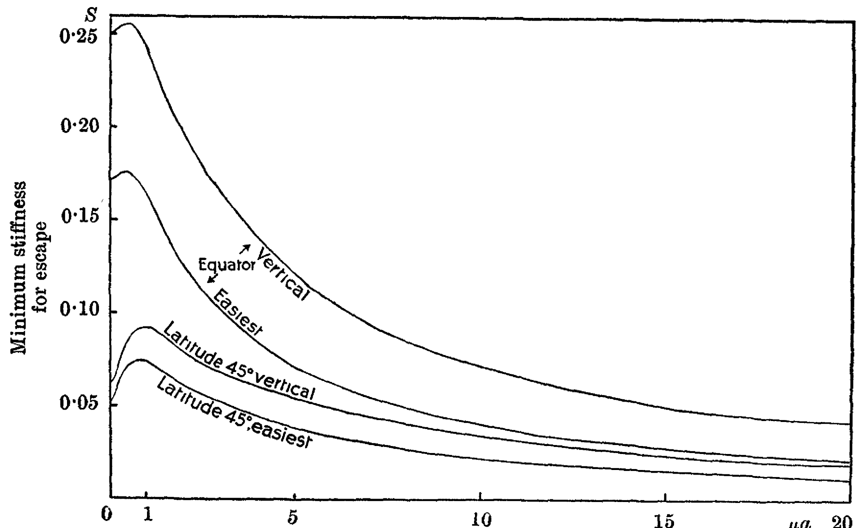

Fig. 3.—Reduced minimum magnetic stiffness as a function of μa for magnetic latitudes 0° and 45° and for vertical and easiest escape.

TABLE I.—Equator.

$x = \mu a$	Vertical				Easiest			
	u	S	$\frac{u}{u_0}$	S/S_0	u	S	$\frac{u}{u_0}$	S/S_0
0	2	0·25	1	1	2·414	0·1716	1	1
0 1	2·007	0·2510	1·003	1·004	2 428	0 1725	1·006	1·005
0·2	2·019	0·2529	1·009	1·011	2·449	0·1741	1·014	1·014
0·25	2·024	0·2537	1·012	1·015	2 455	0·1749	1·017	1·019
0·3	2 027	0·2544	1·013	1 018	2·457	0·1755	1·018	1·023
0·4	2 027	0·2554	1·013	1·021	2·448	0·1761	1·014	1·026
0 5	2·018	0 2553	1·009	1·021	2·424	0·1756	1·004	1·024
1	1·908	0·2418	0·954	0 967	2·220	0·1627	0·920	0·948
5	1 410	0·1206	0·705	0·482	1 513	0·0713	0 627	0·415
10	1·260	0·0726	0·630	0·291	1 317	0·0409	0·546	0·238
20	1·159	0 0433	0 579	0·173	1·190	0·0222	0·493	0·129

u = reduced escape distance

u_0 = ,, ,, ,, for $\mu = 0$

S = reduced stiffness

S_0 = ,, ,, for $\mu = 0$

TABLE II.—Latitude 45°.

$x = \mu a$	Vertical				Easiest			
	u	S	$\frac{u}{u_0}$	S/S_0	u	S	$\frac{u}{u_0}$	S/S_0
0	4	0·0625	1	1	4·327	0·0535	1	1
0·04	4·001	0·0631	1·0002	1·009	4·332	0·0540	1·001	1·008
0·1	3·987	0 0655	0·997	1·048	4·325	0·0560	0·9996	1·047
0·2	3·891	0·0712	0·973	1·139	4·226	0·0607	0·996	1·135
0·25	3·813	0·0742	0·953	1·188	4·140	0·0631	0·935	1·179
0·3	3·725	0·0771	0·931	1·233	4·040	0·0653	0·934	1·220
0·4	3 537	0·0820	0·884	1·313	3·828	0·0689	0·885	1·288
0·5	3·356	0·0858	0·839	1·373	3·625	0·0715	0·838	1·336
1	2·693	0·0925	0·673	1·481	2·890	0·0739	0·668	1·382
5	1·564	0·0550	0·391	0·880	1·636	0·0382	0·378	0·714
10	1·334	0·0345	0·333	0·552	1·376	0·0226	0·318	0·423
20	1 195	0·0201	0·299	0·321	1·220	0 0122	0·282	0·228

Tables I and II and Figs. 2 and 3 contain the results (but differently collected in the figures and in the tables, for practical reasons). Let us repeat, that the "absolute" values u, S are given in the unit a and $aH_{equ.}$ respectively, while the *relative* values u/u_0, S/S_0 are to show the percentage change as compared with the customary assumption $\mu = 0$.

The behaviour can be read off the figures and hardly needs a commentary. It is, on the whole, what was to be expected, except for one notable point. While the initial rising of the S-curves *above* the

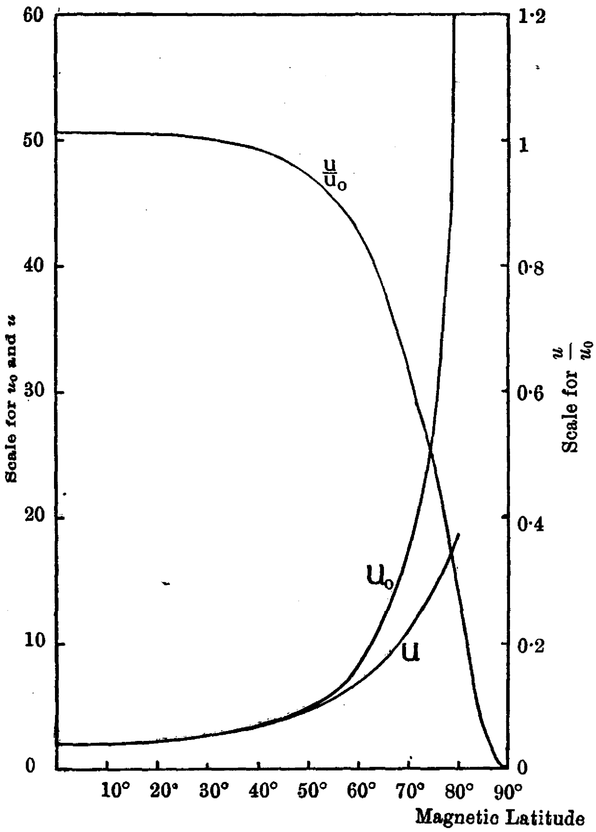

Fig. 4.—The relative escape distances for vertical escape, u_0 and u (for $\mu = 0$ and $\mu a = 0{\cdot}2$ respectively) as functions of the magnetic latitude $\left(\frac{\pi}{2} - \lambda\right)$.

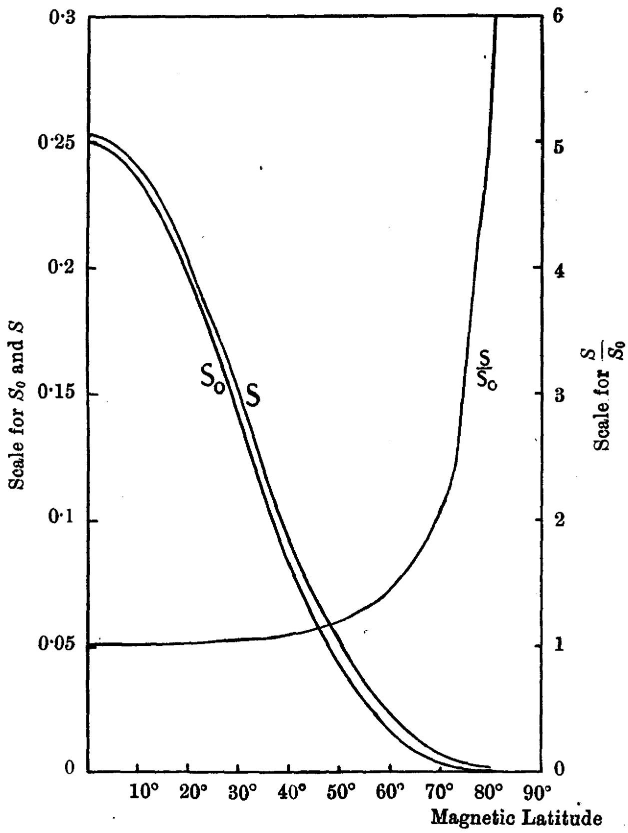

Fig. 5.—The magnetic stiffnesses for vertical escape, S_0 and S (for $\mu = 0$ and $\mu a = 0{\cdot}2$ respectively), as functions of the magnetic latitude $\left(\frac{\pi}{2} - \lambda\right)$.

starting value ($\mu = 0$, customary theory) is quite insignificant at the equator, it develops into a notable feature at the higher latitude.

This induced us, to examine in more detail the dependence on latitude and we chose the case of vertical launching and the parameter $x = \mu a = 0{\cdot}2$ (somewhat suggested[11] by geomagnetic data). The results are given in Figs. 4 and 5 and in Table III.

[11] Cf. Proc. R.I.A. (A), 49, p. 139, 1943.

TABLE III.—Magnetic stiffness and escape distance for vertical escape at different latitudes.

Magnetic Latitude	u_0	S_0	u	S	u/u_0	S/S_0
0°	2	0·25	2·019	0·253	1·0095	1·011
10°	2·062	0·235	2·080	0·239	1·009	1·015
20°	2·265	0·195	2·279	0·200	1·006	1·026
30°	2·667	0·141	2·668	0·148	1·0005	1·049
45°	4	0·062	3·891	0·071	0·973	1·139
60°	8	0·016	6·847	0·023	0·856	1·451
70°	17·097	0·0034	10·917	0·0074	0·639	2·164
80°	66·328	0·00023	18·735	0·0012	0·282	5·472

u_0 = reduced escape distance for $\mu = 0$

u = ,, ,, ,, ,, $\mu a = 0{\cdot}2$

S_0 = reduced stiffness for $\mu = 0$

S = ,, ,, ,, $\mu a = 0{\cdot}2$

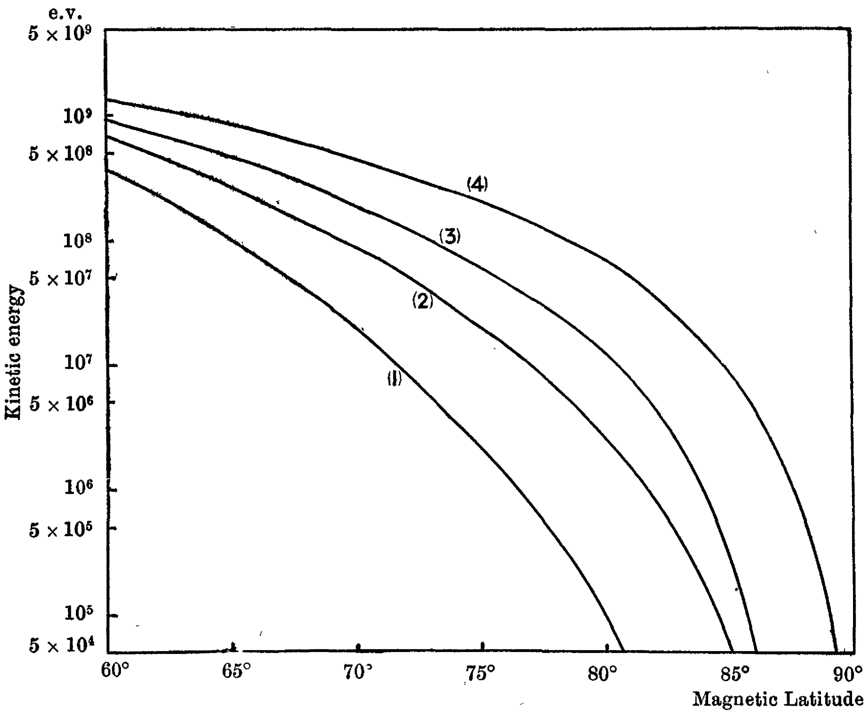

FIG. 6.—Minimum kinetic energy in electron-volts for the aurora region. Vertical launching (or arrival). The scale of ordinates is logarithmic.

(1) proton, $\mu = 0$; (3) electron, $\mu = 0$;
(2) ,, , $\mu a = 0{\cdot}2$; (4) ,, , $\mu a = 0{\cdot}2$.

It will be seen that this moderate value of x, which in the neighbourhood of the equator has no relevant consequences, increases the magnetic stiffness required by a particle to reach the earth in the zone of aurora, by a factor 2, 3, 4 . . . ., which actually increases to ∞ at the pole. At the same time the escape-distance u is steadily *reduced* in a roughly, reciprocal manner. (Of course, none the less, S goes monotonically to zero, u monotonically to infinity with increasing latitude.)

Thus aurora theories would be quantitatively affected, if μa were 0·2 (instead of being zero). We have plotted in Fig. 6 the *energies* of electrons and of protons, required to reach the earth within the auroral zone—both for $\mu = 0$ (curves (1) and (3)) and for $\mu a = 0\ 2$ (curves (2) and (4)). The scale of ordinates is logarithmic.

It will also be observed, that the range of energies which is noticeably affected is considerably lower than that of *cosmic ray particles,* so that *their* expected behaviour is not appreciably altered.