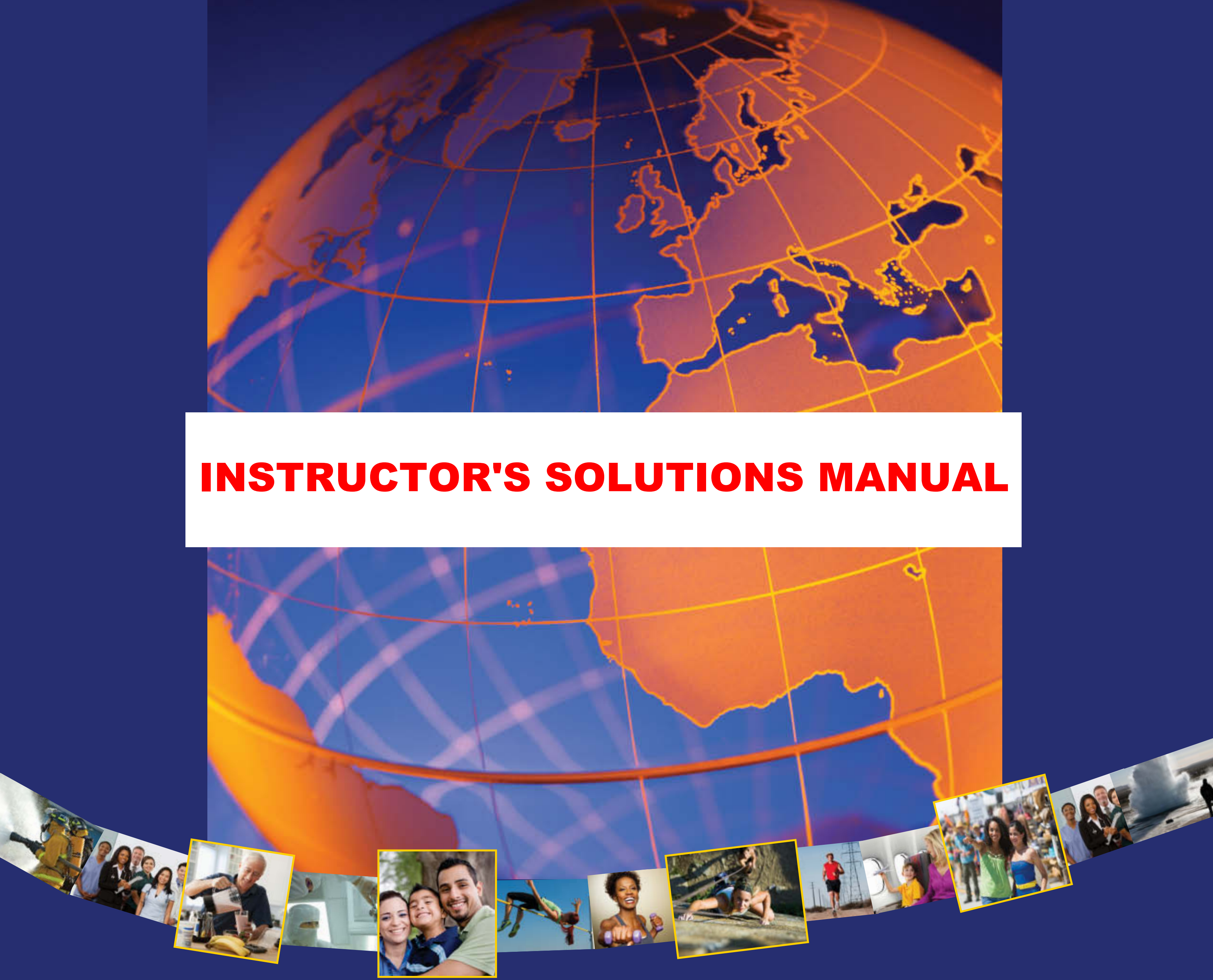
Michael J. Moran | Howard N. Shapiro | Daisie D. Boettner | Margaret B. Bailey



INSTRUCTOR'S SOLUTIONS MANUAL

FUNDAMENTALS OF ENGINEERING THERMODYNAMICS

Eighth Edition

WILEY

1.4 Perform the following unit conversions:

(a) $1\,\text{L}\left|\dfrac{0.0353\,\text{ft}^3}{1\,\text{L}}\right|\left|\dfrac{12\,\text{in.}}{1\,\text{ft}}\right|^3 = 61\,\text{in.}^3 \longleftarrow$

(b) $650\,\text{J}\left|\dfrac{1\,\text{kJ}}{10^3\,\text{J}}\right|\left|\dfrac{1\,\text{Btu}}{1.0551\,\text{kJ}}\right| = 0.616\,\text{Btu} \longleftarrow$

(c) $0.135\,\text{kW}\left|\dfrac{3413\,\text{Btu/h}}{1\,\text{kW}}\right|\left|\dfrac{1\,\text{h}}{3600\,\text{s}}\right|\left|\dfrac{778.17\,\text{ft}\cdot\text{lbf}}{1\,\text{Btu}}\right| = 99.596\,\dfrac{\text{ft}\cdot\text{lbf}}{\text{s}} \longleftarrow$

(d) $378\,\dfrac{\text{g}}{\text{s}}\left|\dfrac{1\,\text{kg}}{10^3\,\text{g}}\right|\left|\dfrac{1\,\text{lb}}{0.4536\,\text{kg}}\right|\left|\dfrac{60\,\text{s}}{1\,\text{min}}\right| = 50\,\dfrac{\text{lb}}{\text{min}} \longleftarrow$

(e) $304\,\text{kPa}\left|\dfrac{1\,\text{lbf/in.}^2}{6894.8\,\text{Pa}}\right|\left|\dfrac{10^3\,\text{Pa}}{1\,\text{kPa}}\right| = 44.09\,\dfrac{\text{lbf}}{\text{in.}^2} \longleftarrow$

(f) $55\,\dfrac{\text{m}^3}{\text{h}}\left|\dfrac{3.2808\,\text{ft}}{1\,\text{m}}\right|^3\left|\dfrac{1\,\text{h}}{3600\,\text{s}}\right| = 0.54\,\dfrac{\text{ft}^3}{\text{s}} \longleftarrow$

(g) $50\,\dfrac{\text{km}}{\text{h}}\left|\dfrac{10^3\,\text{m}}{1\,\text{km}}\right|\left|\dfrac{3.2808\,\text{ft}}{1\,\text{m}}\right|\left|\dfrac{1\,\text{h}}{3600\,\text{s}}\right| = 45.57\,\dfrac{\text{ft}}{\text{s}} \longleftarrow$

(h) $8896\,\text{N}\left|\dfrac{1\,\text{lbf}}{4.4482\,\text{N}}\right|\left|\dfrac{1\,\text{ton}}{2000\,\text{lbf}}\right| = 1\,\text{ton} \longleftarrow$

1.5 Perform the following unit conversions:

$$\text{(a)}\ 122\ \text{in.}^3 \left|\frac{1\ \text{cm}^3}{0.061024\ \text{in.}^3}\right|\left|\frac{1\ \text{m}}{10^2\ \text{cm}}\right|^3\left|\frac{1\ \text{L}}{10^{-3}\ \text{m}^3}\right| = 2\ \text{L} \leftarrow$$

$$\text{(b)}\ 778.17\ \text{ft}\cdot\text{lbf}\left|\frac{1\ \text{kJ}}{737.56\ \text{ft}\cdot\text{lbf}}\right| = 1.0551\ \text{kJ} \leftarrow$$

$$\text{(c)}\ 100\ \text{hp}\left|\frac{1\ \text{kW}}{1.341\ \text{hp}}\right| = 74.57\ \text{kW} \leftarrow$$

$$\text{(d)}\ 1000\ \frac{\text{lb}}{\text{h}}\left|\frac{1\ \text{h}}{3600\ \text{s}}\right|\left|\frac{1\ \text{kg}}{2.2046\ \text{lb}}\right| = 0.126\ \frac{\text{kg}}{\text{s}} \leftarrow$$

$$\text{(e)}\ 29.392\ \frac{\text{lbf}}{\text{in.}^2}\left|\frac{6894.8\ \text{Pa}}{1\ \text{lbf/in.}^2}\right|\left|\frac{1\ \text{N/m}^2}{1\ \text{Pa}}\right|\left|\frac{1\ \text{bar}}{10^5\ \text{N/m}^2}\right| = 2.027\ \text{bar} \leftarrow$$

$$\text{(f)}\ 2500\ \frac{\text{ft}^3}{\text{min}}\left|\frac{0.028317\ \text{m}^3}{1\ \text{ft}^3}\right|\left|\frac{1\ \text{min}}{60\ \text{s}}\right| = 1.18\ \frac{\text{m}^3}{\text{s}} \leftarrow$$

$$\text{(g)}\ 75\ \frac{\text{mile}}{\text{h}}\left|\frac{1.6093\ \text{km/h}}{1\ \text{mile/h}}\right| = 120.7\ \frac{\text{km}}{\text{h}} \leftarrow$$

$$\text{(h)}\ 1\ \text{ton}\left|\frac{2000\ \text{lbf}}{1\ \text{ton}}\right|\left|\frac{4.4482\ \text{N}}{1\ \text{lbf}}\right| = 8896\ \text{N} \leftarrow$$

1.6 Which of the following food items weighs approximately one newton?

a. a grain of rice
b. a small strawberry
c. a medium-sized apple
d. a large watermelon

1.7 A person whose mass is 150 lb weights 144.4 lbf. Determine (a) the *local* acceleration of gravity, in ft/s^2, and (b) the person's mass, in lb, and weight, in lbf, if g = 32.174 ft/s^2.

(a) $F_{\text{grav}} = mg \rightarrow$

$$g = \frac{F_{\text{grav}}}{m} = \frac{144.4\ \text{lbf}}{150\ \text{lb}}\left|\frac{32.174\ \text{lb}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right| = \underline{\mathbf{30.97\ ft/s^2}}$$

(b) Mass value remains the same. So

$$F_{\text{grav}} = mg = (150\ \text{lb})\left(32.174\frac{\text{ft}}{\text{s}^2}\right)\left|\frac{1\ \text{lbf}}{32.174\ \text{lb}\cdot\text{ft/s}^2}\right| = \underline{\mathbf{150\ lbf}}$$

1.8 The *Phoenix* with a mass of 350 kg was a spacecraft used for exploration of Mars. Determine the weight of the *Phoenix*, in N, (a) on the surface of Mars where the acceleration of gravity is 3.73 m/s^2 and (b) on Earth where the acceleration of gravity is 9.81 m/s^2.

KNOWN: *Phoenix* spacecraft has mass of 350 kg.

FIND: (a) Weight of *Phoenix* on Mars, in N, and (b) weight of *Phoenix* on Earth, in N.

SCHEMATIC AND GIVEN DATA:

m = 350 kg
g_{Mars} = 3.73 m/s^2
g_{Earth} = 9.81 m/s^2

ENGINEERING MODEL:
1. Acceleration of gravity is constant at the surface of both Mars and Earth.

ANALYSIS: Weight is the force of gravity. Applying Newton's second law using the mass of the *Phoenix* and the local acceleration of gravity

$$F = mg$$

(a) On Mars,

$$F = (350\text{ kg})\left(3.73\frac{\text{m}}{\text{s}^2}\right)\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right| = \underline{\mathbf{1305.5\ N}}$$

(b) On Earth,

$$F = (350\text{ kg})\left(9.81\frac{\text{m}}{\text{s}^2}\right)\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right| = \underline{\mathbf{3433.5\ N}}$$

Although the mass of the Phoenix is constant, the weight of the Phoenix is less on Mars than on Earth since the acceleration due to gravity is less on Mars than on Earth.

PROBLEM 1.9

Eq. 1.8 is used in both parts: $n = m/M$, where M is from Tables A-1.

(a) $m = Mn$, where $n = 20\,\text{kmol}$.

$$\left.\begin{aligned} \text{Air:} \quad & m = (28.97\,\text{kg/kmol})(20\,\text{kmol}) = 579.4\,\text{kg} \\ C: \quad & m = (12.01\,\text{kg/kmol})(20\,\text{kmol}) = 240.2\,\text{kg} \\ H_2O: \quad & m = (18.02\,\text{kg/kmol})(20\,\text{kmol}) = 360.4\,\text{kg} \\ CO_2: \quad & m = (44.01\,\text{kg/kmol})(20\,\text{kmol}) = 880.2\,\text{kg} \end{aligned}\right\} \leftarrow$$

(b) $n = m/M$, where $m = 50\,\text{lb}$.

$$\left.\begin{aligned} H_2: \quad & n = (50\,\text{lb})/(2.016\,\text{lb/lbmol}) = 24.802\,\text{lbmol} \\ N_2: \quad & n = (50\,\text{lb})/(28.01\,\text{lb/lbmol}) = 1.785\,\text{lbmol} \\ NH_3: \quad & n = (50\,\text{lb})/(17.03\,\text{lb/lbmol}) = 2.936\,\text{lbmol} \\ C_3H_8: \quad & n = (50\,\text{lb})/(44.09\,\text{lb/lbmol}) = 1.134\,\text{lbmol} \end{aligned}\right\} \leftarrow$$

PROBLEM 1.10

Using magnitudes,

$$|F| = m|a| \quad , |a| = 60g$$

$$= m(60g) = 60mg$$

$$= 60(50\,lb)(32.2\,\frac{ft}{s^2}) \left| \frac{1\,lbf}{32.2\,lb \cdot ft/s^2} \right| = 3000\,lbf$$

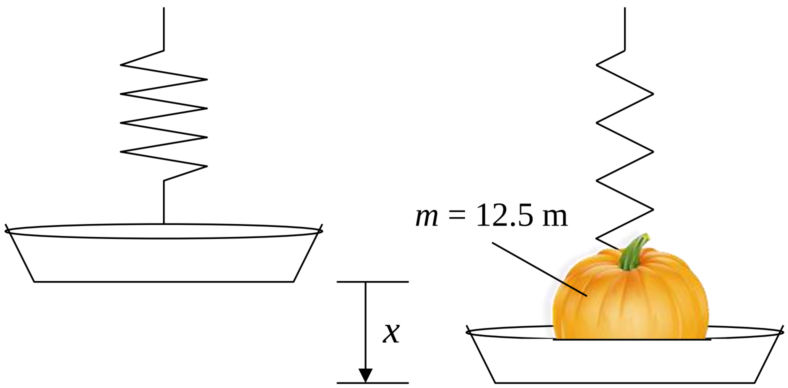
(rounded)

1.11 At the grocery store you place a pumpkin with a mass of 12.5 lb on the produce spring scale. The spring in the scale operates such that for each 4.7 lbf applied, the spring elongates one inch. If local acceleration of gravity is 32.2 ft/s^2, what distance, in inches, did the spring elongate?

KNOWN: Pumpkin placed on a spring scale causes the spring to elongate.

FIND: Distance spring elongated, in inches.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Spring constant is 4.7 lbf/in.
2. Local acceleration of gravity is 32.2 ft/s^2.

ANALYSIS:
The force applied to the spring to cause it to elongate can be expressed as the spring constant, *k*, times the elongation, *x*.

$$F = kx$$

The applied force is due to the weight of the pumpkin, which can be expressed as the mass (*m*) of the pumpkin times acceleration of gravity, (*g*).

$$F = Weight = mg = kx$$

Solving for elongation, *x*, substituting values for pumpkin mass, acceleration of gravity, and spring constant, and applying the appropriate conversion factor yield

$$x = \frac{mg}{k} = \frac{(12.5\text{ lb})\left(32.2\,\frac{\text{ft}}{\text{s}^2}\right)}{\left(4.7\,\frac{\text{lbf}}{\text{in.}}\right)}\left|\frac{1\text{ lbf}}{32.174\,\frac{\text{lb}\cdot\text{ft}}{\text{s}^2}}\right| = \underline{\mathbf{2.66\text{ in.}}}$$

PROBLEM 1.12

The spring is known to deflect 0.14 inch for every 1 lbf of applied force. Thus, we begin by determining the weight of the object (F_{grav}) using the deflection (Δx) given as 1.8 inches.

$$\Delta x = 1.8 \text{ inches} = (0.14\,\tfrac{\text{in}}{\text{lbf}})(F_{\text{grav}})$$

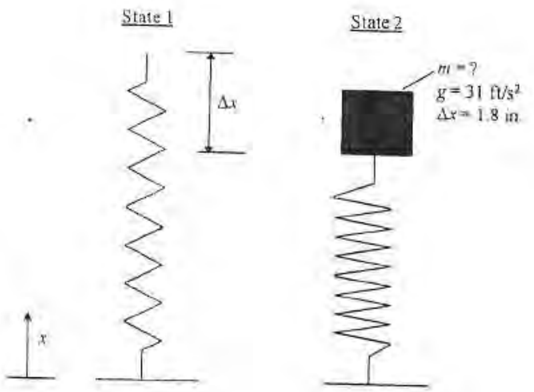
$$(F_{\text{grav}}) = \frac{1.8 \text{ inches}}{(0.14\,\tfrac{\text{in.}}{\text{lbf}})} = 12.86 \text{ lbf}$$

The mass can be solved from the expression $F_{grav} = \text{mg}$.

rounded

$$m = \frac{(F_{\text{grav}})}{g} = \frac{12.86\,\text{lbf}}{31\,\frac{\text{ft}}{\text{s}^2}} \left| \frac{32.2\,\text{ft}\cdot\text{lb}/\text{s}^2}{1\text{lbf}} \right| = 13.36\text{lb}$$

$m = 13.36$ lb



PROBLEM 1.13

Weight refers to the force of gravity: $F_{grav} = mg$.

Thus, when her mass is 120 lb and weight is 119 lbf, we have

$$g = \frac{F_{grav}}{m} = \frac{119\ \text{lbf}}{120\ \text{lb}} \left| \frac{32.174\ \text{lb}\cdot\text{ft/s}^2}{1\ \text{lbf}} \right| = 31.906\ \text{ft/s}^2 \longleftarrow$$

When her mass is 120 lb and $g = 32.05\ \text{ft/s}^2$, we have

$$F_{rav} = mg = (120\ \text{lb})(32.05\ \text{ft/s}^2) \left| \frac{1\ \text{lbf}}{32.174\ \text{lb}\cdot\text{ft/s}^2} \right|$$
$$= 119.54\ \text{lbf} \longleftarrow$$

COMMENT: Her mass remains constant, but weight depends on the local acceleration of gravity.

PROBLEM 1.14

The actual forces developed when birds and aircraft collide are difficult to determine precisely, but estimates can be calculated using average values of acceleration and force magnitudes, as follows:

The goose is accelerated from a very low velocity to 150 miles/h in 10^{-3} s. Thus the averge acceleration magnitude is

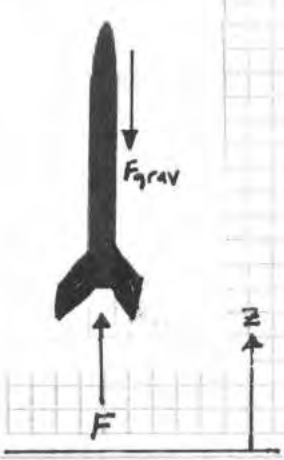
$$|a| = \left(\frac{150\ \text{miles/h} - 0}{10^{-3}\ \text{s}} \right) \left| \frac{1\ \text{h}}{3600\ \text{s}} \right| \left| \frac{5280\ \text{ft}}{1\ \text{mile}} \right| = 2.2 \times 10^5\ \frac{\text{ft}}{\text{s}^2}$$

The magnitude of the average force applied is

$$|F| = m|a| = (12\ \text{lb})\left(2.2\times10^5\ \frac{\text{ft}}{\text{s}^2}\right) \left| \frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2} \right| = 82{,}000\ \text{lbf} \longleftarrow$$

rounded

PROBLEM 1.15



$m = 4.5$ lb

$a = 3g$, where $g = 32.2\ \text{ft/s}^2$

$\Sigma F_z = ma$

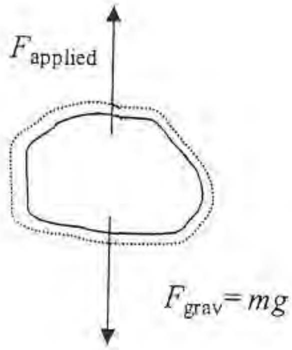
Neglecting air resistance,

$$F - F_{grav} = ma$$
$$\Rightarrow \quad F = ma + F_{grav}$$
$$= ma + mg \qquad = m(3g) + mg$$
$$= m(4g)$$
$$= (4.5\ \text{lb})(4 \times 32.2\ \text{ft/s}^2) \left| \frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2} \right| = 18\ \text{lbf} \longleftarrow$$

rounded

PROBLEM 1.16

The FBD of the object is as shown with an upward applied force of 10 lbf and the force downward due to gravity where $F_{grav} = mg$ and g is given as 32.2 ft/s^2. Summing forces yields the following equation that can be rearranged to solve for acceleration. It is assumed that up is positive.



$$F_{\text{applied}} = 10 \text{ lbf}$$

$$m = 50 \text{ lb}$$

$$g = 32.2 \frac{\text{ft}}{\text{s}^2}$$

$$a = ? \frac{\text{ft}}{\text{s}^2}$$

$$F_{\text{applied}} - F_{\text{grav}} = ma$$

$$F_{\text{grav}} = mg$$

$$a = \frac{F_{\text{applied}} - F_{\text{grav}}}{m} = \frac{F_{\text{applied}} - mg}{m} = \frac{F_{\text{applied}}}{m} - g$$

$$a = \frac{10 \text{ lbf}}{50 \text{ lb}} \left| \frac{32.2 \text{ ft} \cdot \text{lb}/\text{s}^2}{1 \text{lbf}} \right| - 32.2 \frac{\text{ft}}{\text{s}^2}$$

$$a = -25.8 \frac{\text{ft}}{\text{s}^2} \quad \text{downward}$$

1.17 A communications satellite weighs 4400 N on Earth where g = 9.81 m/s^2. What is the weight of the satellite, in N, as it orbits Earth where the acceleration of gravity is 0.224 m/s^2? Express each weight in lbf.

KNOWN: Weight of communications satellite on Earth.

FIND: Determine weight of the satellite, in N, as it orbits Earth where the acceleration of gravity is 0.224 m/s^2. Express the satellite weight, in lbf, on Earth and in orbit.

SCHEMATIC AND GIVEN DATA:

$W_{\text{Sat(Earth)}} = 4400$ N
$g_{\text{Earth}} = 9.81$ m/s^2
$g_{\text{orbit}} = 0.224$ m/s^2

ENGINEERING MODEL:
1. Gravitational acceleration on Earth is constant at 9.81 m/s^2.
2. Gravitational acceleration at orbital altitude is constant at 0.224 m/s^2.

ANALYSIS: Weight of the satellite is the force of gravity and varies with altitude. Mass of the satellite remains constant. Applying Newton's second law to solve for the mass of the satellite yields

$$W = mg \rightarrow m = W/g$$

On Earth,

$$m = W_{\text{Sat(Earth)}}/g_{\text{Earth}}$$

$$m = \frac{(4400\ \text{N})}{\left(9.81\dfrac{\text{m}}{\text{s}^2}\right)}\left|\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right| = 448.5\ \text{kg}$$

Solving for the satellite weight in orbit,

$$W_{\text{Sat(orbit)}} = mg_{\text{orbit}}$$

$$W_{\text{Sat(orbit)}} = (448.5\ \text{kg})\left(0.224\frac{\text{m}}{\text{s}^2}\right)\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right| = \underline{\mathbf{100.5\ N}}$$

Although the mass of the communications satellite is constant, the weight of the satellite is less at orbital altitude than on Earth since the acceleration due to gravity is less at orbital altitude than on Earth.

To determine the corresponding weights in lbf, apply the conversion factor, 1 lbf = 4.4482 N.

$$W_{\text{Sat(Earth)}} = (4400\text{ N})\left|\frac{1\text{ lbf}}{4.4482\text{ N}}\right| = \underline{\mathbf{989.2\ lbf}}$$

$$W_{\text{Sat(orbit)}} = (100.5\text{ N})\left|\frac{1\text{ lbf}}{4.4482\text{ N}}\right| = \underline{\mathbf{22.6\ lbf}}$$

PROBLEM 1.18

(a) Mexico City, $g = 9.779\ m/s^2$

$$F_{grav} = mg = (80\ kg)(9.779\ \tfrac{m}{s^2})\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|$$

$$= 782.32\ N$$

(b) Cape Town, $g = 9.796\ m/s^2$

$$F_{grav} = mg = (80\ kg)(9.796\ \tfrac{m}{s^2})\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|$$

$$= 783.68\ N$$

(c) Tokyo, $g = 9.798\ m/s^2$

$$F_{grav} = mg = (80\ kg)(9.798\ \tfrac{m}{s^2})\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|$$

$$= 783.84\ N$$

(d) Chicago, $g = 9.803\ m/s^2$

$$F_{grav} = mg = (80\ kg)(9.803\ \tfrac{m}{s^2})\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|$$

$$= 784.24\ N$$

(e) Copenhagen, $g = 9.815\ m/s^2$

$$F_{grav} = mg = (80\ kg)(9.815\ m/s^2)\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|$$

$$= 785.2\ N$$

PROBLEM 1.19

1. The weight of the tower itself is ignored.
2. Local acceleration of gravity is 32.1 ft/s^2.
3. $\rho_{water} = 62.4\ \text{lb/ft}^3$

The structure must exert a minimum force equivalent to the weight of the water, which can be expressed as the mass (m) of the water times acceleration of gravity, g.

$$F = Weight = mg$$

The mass of the water can be determined from its density times the volume the water occupies

$$m = \rho V = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)(1{,}000{,}000\ \text{gal})\left|\frac{0.13368\ \text{ft}^3}{1\ \text{gal}}\right| = 8{,}341{,}632\ \text{lb}$$

Substituting for mass and acceleration of gravity and applying the appropriate conversion factor yield

$$F = mg = (8{,}341{,}632\ \text{lb})\left(32.1\frac{\text{ft}}{\text{s}^2}\right)\left|\frac{1\ \text{lbf}}{32.174\frac{\text{lb}\cdot\text{ft}}{\text{s}^2}}\right| = \underline{\mathbf{8{,}322{,}446\ lbf}} \longleftarrow$$

PROBLEM 1.20

0.5 kmol NH_3, $V = 6\ m^3$

$g = 9.81\ m/s^2$

(a) $F_{grav} = m\,g$

Using Eq. 1.8, $m = n\,M = 0.5\ \text{kmol}\left(17.03\ \frac{\text{kg}}{\text{kmol}}\right) = 8.52\ \text{kg}$ ← Table A-1

$\therefore\ F_{grav} = (8.52\ \text{kg})(9.81\ \frac{m}{s^2})\left|\frac{1\ N}{1\ \text{kg}\cdot m/s^2}\right| = 83.58\ N \longleftarrow F_{grav}$

(b) $\bar{v} = \frac{V}{n} = \frac{6\ m^3}{0.5\ \text{kmol}} = 12\ \frac{m^3}{\text{kmol}}$, $v = \frac{V}{m} = \frac{6\ m^3}{8.52\ \text{kg}} = 0.704\ \frac{m^3}{\text{kg}} \longleftarrow \bar{v}, v$

1.21 A 2-lb sample of an unknown liquid occupies a volume of 62.6 in.^{3} For the liquid determine (a) the specific volume, in ft^3/lb, and (b) the density, in lb/ft^3.

KNOWN: Volume and mass of an unknown liquid sample.

FIND: Determine (a) the specific volume, in ft^3/lb, and (b) the density, in lb/ft^3.

SCHEMATIC AND GIVEN DATA:

m = 2 lb
V = 62.6 in.^{3}

ENGINEERING MODEL:
1. The liquid can be treated as continuous.

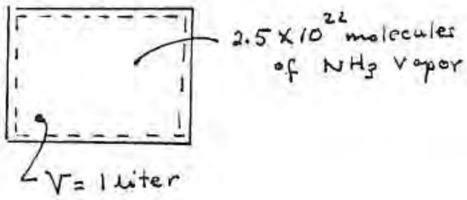
ANALYSIS:
(a) The specific volume is volume per unit mass and can be determined from the total volume and the mass of the liquid

$$v = \frac{V}{m} = \frac{62.6\ \text{in.}^3}{2\ \text{lb}}\left|\frac{1\ \text{ft}^3}{1728\ \text{in.}^3}\right| = \underline{\mathbf{0.0181\ ft^3/lb}}$$

(b) Density is the reciprocal of specific volume. Thus,

$$\rho = \frac{1}{v} = \frac{1}{0.0181\dfrac{\text{ft}^3}{\text{lb}}} = \underline{\mathbf{55.2\ lb/ft^3}}$$

PROBLEM 1.22



(a) From Sec. 1.5, the number of molecules in a gram mole (mol) is 6.022×10^{23} (Avogadro's number).

Thus $$n = \frac{2.5 \times 10^{22} \text{ molecules}}{6.022 \times 10^{23} \text{ molecules/mol}} = 4.15 \times 10^{-2} \text{ mols}$$

Converting to kmol,
$$n = 4.15 \times 10^{-2} \text{ mol} \left| \frac{1 \text{ kmol}}{10^3 \text{ mol}} \right| = 4.15 \times 10^{-5} \text{ kmol}$$

Using Eq. 1.8, $m = nM$, so
$$m = 4.15 \times 10^{-5} \text{ kmol} \left(\underbrace{17.03}_{\text{Table A-1}} \frac{\text{kg}}{\text{kmol}}\right) = 7.07 \times 10^{-4} \text{ kg}$$

Then
$$v = \frac{V}{m} = \frac{(1 \text{ liter})}{7.07 \times 10^{-4} \text{ kg}} \left| \frac{10^{-3} \text{ m}^3}{1 \text{ liter}} \right| = 1.41 \frac{\text{m}^3}{\text{kg}}$$

$$\bar{v} = \frac{V}{n} = \frac{10^{-3} \text{ m}^3}{4.15 \times 10^{-5} \text{ kmol}} = 24.1 \frac{\text{m}^3}{\text{kmol}}$$

1.23 The specific volume of 5 kg of water vapor at 1.5 MPa, 440°C is 0.2160 m^3/kg. Determine (a) the volume, in m^3, occupied by the water vapor, (b) the amount of water vapor present, in gram moles, and (c) the number of molecules.

KNOWN: Mass, pressure, temperature, and specific volume of water vapor.

FIND: Determine (a) the volume, in m^3, occupied by the water vapor, (b) the amount of water vapor present, in gram moles, and (c) the number of molecules.

SCHEMATIC AND GIVEN DATA:

m = 5 kg
p = 1.5 MPa
T = 440°C
v = 0.2160 m^3/kg

ENGINEERING MODEL:
1. The water vapor is a closed system.

ANALYSIS:
(a) The specific volume is volume per unit mass. Thus, the volume occupied by the water vapor can be determined by multiplying its mass by its specific volume.

$$V = mv = (5\,\text{kg})\left(0.2160\,\frac{\text{m}^3}{\text{kg}}\right) = \underline{\mathbf{1.08\ m^3}}$$

(b) Using molecular weight of water from Table A-1 and applying the appropriate relation to convert the water vapor mass to gram moles gives

$$n = \frac{m}{M} = \left(\frac{5\,\text{kg}}{18.02\,\frac{\text{kg}}{\text{kmol}}}\right)\left|\frac{1000\,\text{moles}}{1\,\text{kmol}}\right| = \underline{\mathbf{277.5\ moles}}$$

(c) Using Avogadro's number to determine the number of molecules yields

$$\#\,\text{Molecules} = \text{Avogadro's Number} \times \#\,\text{moles} = \left(6.022\times10^{23}\,\frac{\text{molecules}}{\text{mole}}\right)(277.5\,\text{moles})$$

$$\#\,\text{Molecules} = \underline{\mathbf{1.671\times10^{26}\ molecules}}$$

PROBLEM 1.24

$$p = A + \frac{B}{\mathcal{V}}$$

p: lbf/ft^2 ; $\mathcal{V}$: ft^3

By inspection of this equation, A has units of lbf/ft^2. ←

Rearranging,

$$B = \underbrace{[p-A]}_{\text{lbf/ft}^2}\ \underbrace{\mathcal{V}}_{\text{ft}^3} \Rightarrow$$

B has units of ft·lbf. ←

PROBLEM 1.25

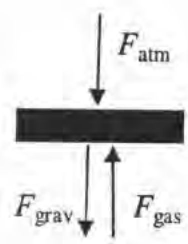
The FBD of the piston is as shown with a downward force due to the atmosphere (F_{atm}) where $F_{atm} = p_{atm} A_{piston}$ and A_{piston} is the cross sectional area of the piston. Another downward force is due to gravity (F_{grav}) where $F_{grav} = m_{piston} g$ and m_{piston} is the mass of the piston. The upward force (F_{gas}) is due to the pressure exerted on the bottom face of the piston by the substance where $F_{gas} = p_{gas} A_{piston}$ and p_{gas} is the pressure of the gas. Summing forces yields the following equation that can be rearranged to explore whether p_{gas} is constant. It is assumed that up is positive.

$$F_{gas} = F_{grav} + F_{atm}$$

$$F_{gas} = p_{gas} A_p \qquad F_{grav} = m_{piston}\, g \qquad F_{atm} = p_{atm} A_{piston}$$

$$p_{gas} A_{piston} = m_{piston}\, g + p_{atm} A_{piston}$$

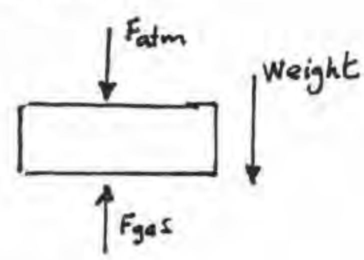
$$p_{gas} = \frac{m_{piston}\, g}{A_{piston}} + \frac{p_{atm} A_{piston}}{A_{piston}} = \frac{m_{piston}\, g}{A_{piston}} + p_{atm}$$



Since each of the four quantities on the right-side of the above equation is constant, it follows that the pressure acting on the bottom of the piston remains constant.
Volume change occurs as the gas is heated or cooled.

PROBLEM 1.26

Since the piston moves smoothly within the cylinder, the piston begins to rise when the force exerted by the gas exceeds the resisting force composed of the piston weight and the force exerted by the atmospheric pressure. That is,



$$F_{gas} \geq \text{Weight} + F_{atm}$$

$$p_{gas} A \geq mg + p_{atm} A$$

$$\Rightarrow \qquad p_{gas} \geq \frac{mg}{A} + p_{atm}$$

$$\geq \left[\frac{(50\,kg)(9.81\,m/s^2)}{0.01\,m^2}\right]\left|\frac{1\,bar}{10^5\,N/m^2}\right|\left|\frac{1\,N}{1\,kg\cdot m/s^2}\right| + 1\,bar$$

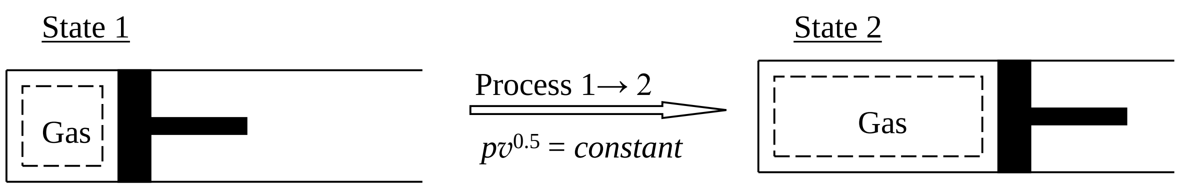
$$\geq 1.49\ bar$$

1.27 Three kg of gas in a piston-cylinder assembly undergo a process during which the relationship between pressure and specific volume is $pv^{0.5}$ = *constant*. The process begins with p_1 = 250 kPa and V_1 = 1.5 m^3 and ends with p_2 = 100 kPa. Determine the final specific volume, in m^3/kg. Plot the process on a graph of pressure versus specific volume.

KNOWN: A gas of known mass undergoes a process from a known initial state to a specified final pressure. The pressure-specific volume relationship for the process is given.

FIND: Determine the final specific volume and plot the process on a pressure versus specific volume graph.

SCHEMATIC AND GIVEN DATA:



m_1 = 3 kg
p_1 = 250 kPa
V_1 = 1.5 m^3

m_2 = 3 kg
p_2 = 100 kPa

ENGINEERING MODEL:
1. The gas is a closed system.
2. The system undergoes a polytropic process in which $pv^{0.5}$ = *constant*.

ANALYSIS:
The final specific volume, v_2, can be determined from the polytropic process equation

$$p_1 v_1^{0.5} = p_2 v_2^{0.5}$$

Solving for v_2 yields

$$v_2 = v_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{0.5}}$$

Specific volume at the initial state, v_1, can be determined by dividing the volume at the initial state, V_1, by the mass, m, of the system

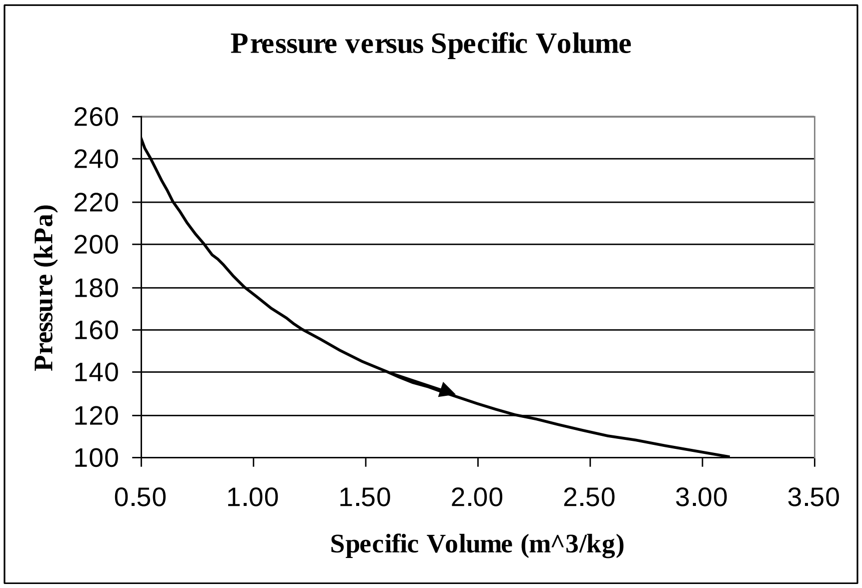
$$v_1 = \frac{V_1}{m} = \frac{1.5\,\text{m}^3}{3\,\text{kg}} = 0.5\ \text{m}^3/\text{kg}$$

Substituting values for pressures and specific volume yields

$$v_2 = \left(0.5\frac{m^3}{kg}\right)\left(\frac{250\,kPa}{100\,kPa}\right)^{\frac{1}{0.5}} = \underline{\mathbf{3.125\ m^3/kg}}$$

The volume of the system increased while pressure decreased during the process.

A plot of the process on a pressure versus specific volume graph is as follows:

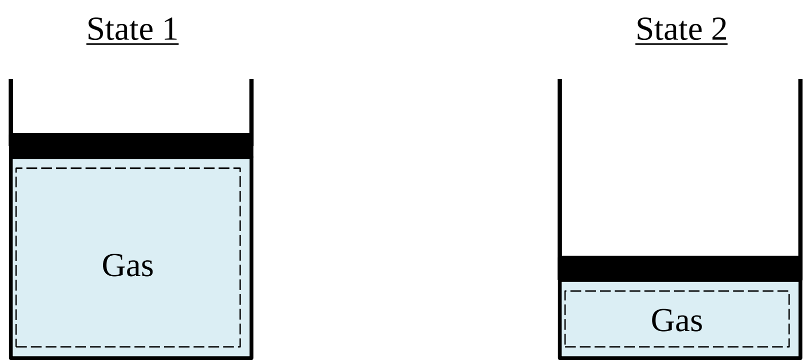


1.28 A closed system consisting of 4 lb of a gas undergoes a process during which the relation between pressure and volume is $pV^n = constant$. The process begins with p_1 = 15 lbf/in.^{2}, v_1 = 1.25 ft^3/lb and ends with p_2 = 53 lbf/in.^{2}, v_2 = 0.5 ft^3/lb. Determine (a) the volume, in ft^3, occupied by the gas at states 1 and 2 and (b) the value of n. (c) Sketch Process 1-2 on pressure-volume coordinates.

KNOWN: Gas undergoes a process from a known initial pressure and specific volume to a known final pressure and specific volume.

FIND: Determine (a) the volume, in ft^3, occupied by the gas at states 1 and 2 and (b) the value of n. (c) Sketch Process 1-2 on pressure-volume coordinates.

SCHEMATIC AND GIVEN DATA:



m = 4 lb
p_1 = 15 lbf/in.^{2}
v_1 = 1.25 ft^3/lb

p_2 = 53 lbf/in.^{2}
v_2 = 0.5 ft^3/lb

ENGINEERING MODEL:
1. The gas is a closed system.
2. The relation between pressure and volume is $pV^n = constant$ during process 1-2.

ANALYSIS:
(a) The specific volume is volume per unit mass. Thus, the volume occupied by the gas can be determined by multiplying its mass by its specific volume.

$$V = mv$$

For state 1

$$V_1 = mv_1 = (4\text{ lb})\left(1.25\frac{\text{ft}^3}{\text{lb}}\right) = \underline{\mathbf{5\ ft^3}}$$

For state 2

$$V_2 = mv_2 = (4\text{ lb})\left(0.5\frac{\text{ft}^3}{\text{lb}}\right) = \underline{\mathbf{2\ ft^3}}$$

(b) The value of n can be determined by substituting values into the relationship:

$$p_1(V_1)^n = constant = p_2(V_2)^n$$

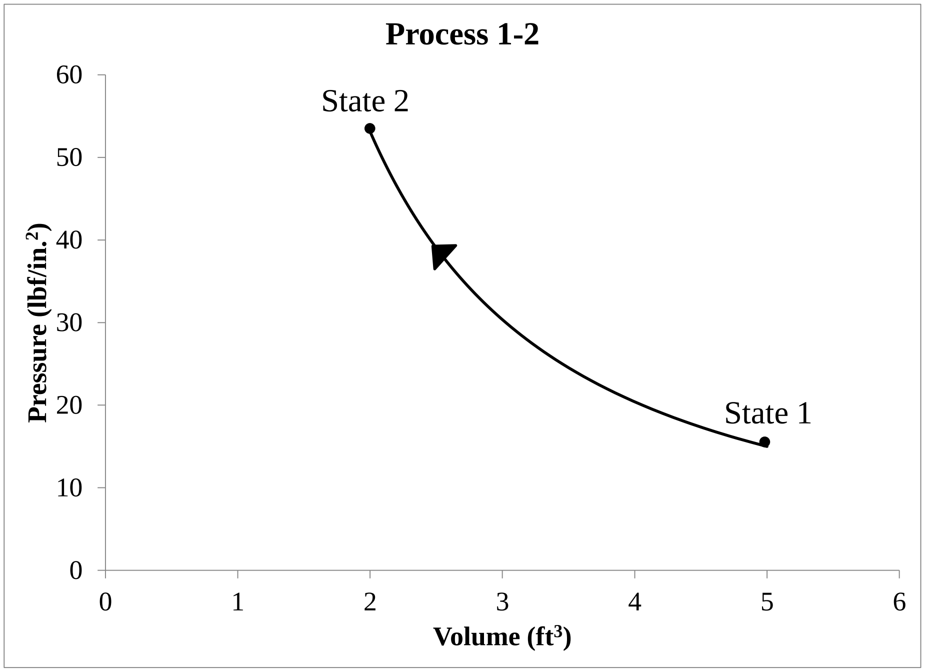
Solving for n

$$\frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^n$$

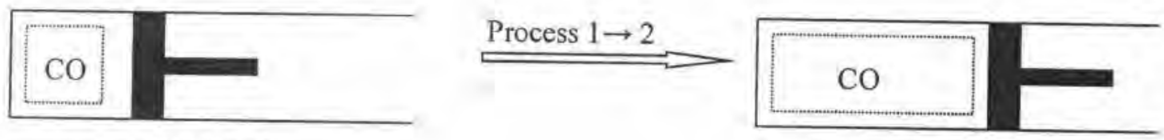
$$\ln\left(\frac{p_1}{p_2}\right) = n\ln\left(\frac{V_2}{V_1}\right)$$

$$n = \frac{\ln\left(\frac{p_1}{p_2}\right)}{\ln\left(\frac{V_2}{V_1}\right)} = \frac{\ln\left(\frac{15\ \text{lbf/in.}^2}{53\ \text{lbf/in.}^2}\right)}{\ln\left(\frac{2\ \text{ft}^3}{5\ \text{ft}^3}\right)} = \underline{\mathbf{1.38}}$$

(c) Process 1-2 is shown on pressure-volume coordinates below:



PROBLEM 1.29



$p_2 = 200$ lbf/in.^{2}
$V_2 = 2.0$ ft^3

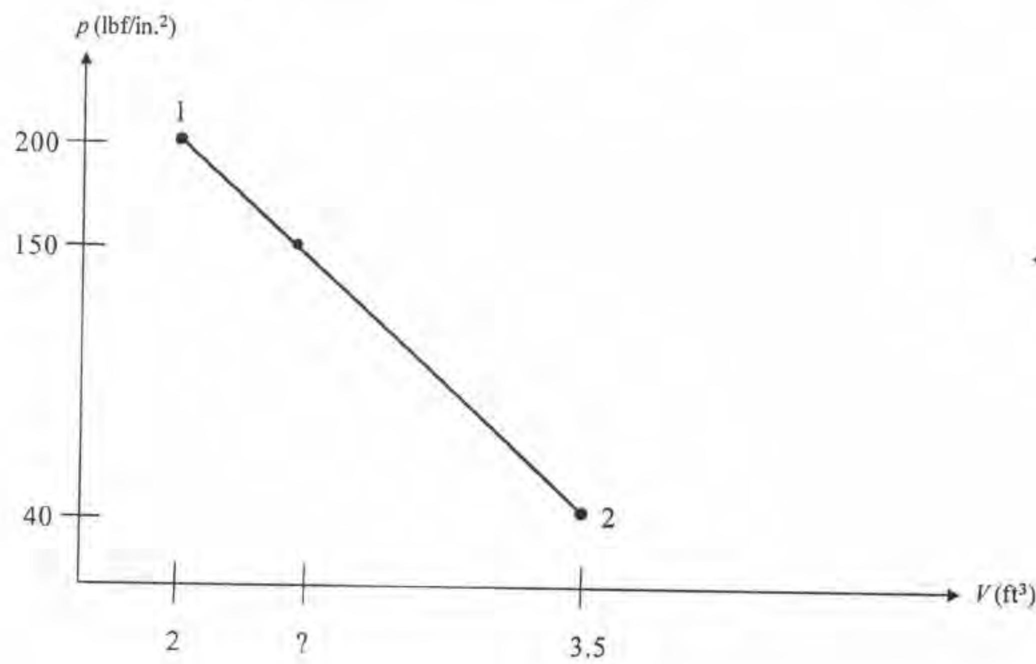
$p_1 = 40$ lbf/in.^{2}
$V_1 = 3.5$ ft^3

The pressure-volume relation is linear during the process. Therefore,

$$\frac{p-p_1}{V-V_1} = \frac{p_2-p_1}{V_2-V_1} \quad \textit{and} \quad V = \frac{p-p_1}{p_2-p_1}(V_2-V_1)+V_1$$

Using given data where $p = 150$ lbf/in.^{2}

$$V = \frac{(150-200)\frac{\text{lbf}}{\text{in.}^2}}{(40-200)\frac{\text{lbf}}{\text{in.}^2}}(3.5-2.0)\text{ft}^3 + 2.0\text{ ft}^3 = \frac{-50}{-160}(1.5)\text{ft}^3 + 2.0\text{ ft}^3 = 2.5\text{ ft}^3$$



PROBLEM 1.30

Since the piston moves smoothly within the cylinder, the force exerted by the gas equals the resisting force composed of the piston weight, shaft weight, force exerted by the atmospheric pressure, and the force acting on the shaft, F. That is, the sum of the forces acting vertically is zero, giving

$$F_{gas} = \begin{bmatrix}\text{Piston}\\ \text{Weight}\end{bmatrix} + \begin{bmatrix}\text{Shaft}\\ \text{Weight}\end{bmatrix} + F_{atm} + F$$

Solving

$$F = F_{gas} - \begin{bmatrix}\text{Piston}\\ \text{weight}\end{bmatrix} - \begin{bmatrix}\text{Shaft}\\ \text{weight}\end{bmatrix} - F_{atm} \qquad (*)$$

In this expression,

$F_{gas} = P_{gas} A_P$, where A_P is the piston face area:

$$A_P = \frac{\pi D^2}{4} = \frac{\pi (10 cm)^2}{4} = 78.54 cm^2$$

$$\therefore \quad F_{gas} = (3 bar) \left| \frac{10^5 N/m^2}{1 bar} \right| (78.54 cm^2) \left| \frac{1 m}{10^2 cm} \right|^2 = 2356.2 N$$

The pressure of the atmosphere acts only on the net area at the top of the piston — namely, the piston face area less the area occupied by the shaft. The force is then

$$F_{atm} = P_{atm}[A_P - A]$$

$$= (1 bar)(78.54 - 0.8) cm^2 \left| \frac{10^5 N/m^2}{1 bar} \right| \left| \frac{1 m}{10^2 cm} \right|^2 = 777.4 N$$

The total weight of the piston and shaft is

$$\text{Total weight} = (m_{piston} + m_{shaft}) g$$

$$= (25 kg)(9.81 \frac{m}{s^2}) \left| \frac{1 N}{1 kg \cdot m/s^2} \right| = 245.3 N$$

Collecting results, Eq. (*) gives

$$F = 2356.2 N - 245.3 N - 777.4 N$$

$$= 1333.5 N$$

1.31 A gas contained within a piston-cylinder assembly undergoes four processes in series:

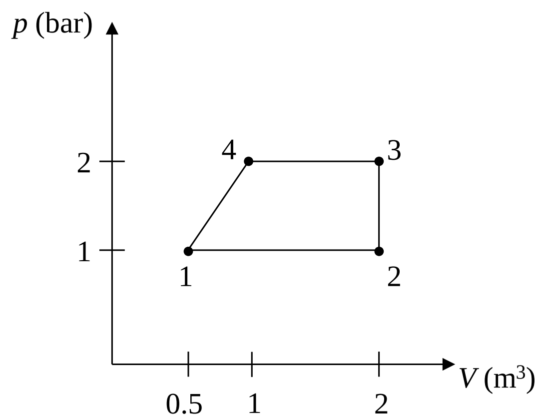
Process 1-2: Constant-pressure expansion at 1 bar from V_1 = 0.5 m^3 to V_2 = 2 m^3

Process 2-3: Constant volume to 2 bar

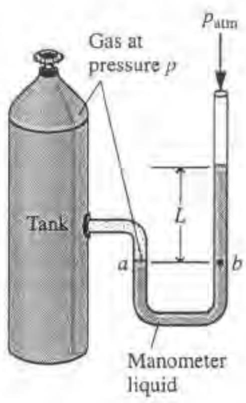
Process 3-4: Constant-pressure compression to 1 m^3

Process 4-1: Compression with $pV^{-1} = constant$

Sketch the process in series on a p-V diagram labeled with pressure and volume values at each numbered state.



PROBLEM 1.32



(a) We have $p_a = p_{gas}$ and $p_a = p_b$. p_b is evaluated using Eq. 1.11. Collecting results,

$$\Rightarrow \quad p_{gas} = p_{atm} + \rho_w g L$$

where $\rho_w = 997\ kg/m^3$ and $g = 9.81\ m/s^2$.

Solving for L

$$L = \frac{(p_{gas} - p_{atm})}{\rho_w g} = \frac{(1.5-1)\,bar \left|\frac{10^5 N/m^2}{1\,bar}\right|}{\left(997\frac{kg}{m^3}\right)\left(9.81\frac{m}{s^2}\right)\left|\frac{1\,N}{1\,kg\cdot m/s^2}\right|}$$

$$= 5.11\ m \longleftarrow$$

(b) Following the approach of part (a) with $\rho_m = 13.59\ \frac{g}{cm^3}$, $p_{atm} = 750\ mm\ Hg = 10^5\ N/m^2$ (see "For Example" on p. 16)

$$L = \frac{p_{gas} - p_{atm}}{\rho_m g} = \frac{(1.3-1) \times 10^5\ N/m^2}{\left(13.59\frac{g}{cm^3}\right)\left|\frac{1\,kg}{10^3\,g}\right|\left|\frac{10^2\,cm}{1\,m}\right|^3\left(9.81\frac{m}{s^2}\right)\left|\frac{1\,N}{1\,kg\cdot m/s^2}\right|}$$

$$= 0.225\,m\left|\frac{100\,cm}{1\,m}\right| = 22.5\ cm \longleftarrow$$

PROBLEM 1.33

Considering a manometer like that shown in Fig. 1.7 connected to the storage tank by a line filled with gas, we have $p_a = p_{gas}$ and $p_a = p_b$. p_b is evaluated using Eq. 1.11. Collecting results,

$$p_{gas} = p_{atm} + \rho g L$$

$$= 101\ kPa + \left(13.59\frac{g}{cm^3}\right)\left|\frac{1\,kg}{10^3\,g}\right|\left|\frac{10^2\,cm}{1\,m}\right|^3\left(9.81\frac{m}{s^2}\right)(1\,m)\left|\frac{1\,N}{1\,kg\cdot m/s^2}\right|\left|\frac{1\,kPa}{10^3\,N/m^2}\right|$$

$$= 101\ kPa + 133.3\ kPa$$

$$= 234.3\ kPa \longleftarrow$$

PROBLEM 1.34

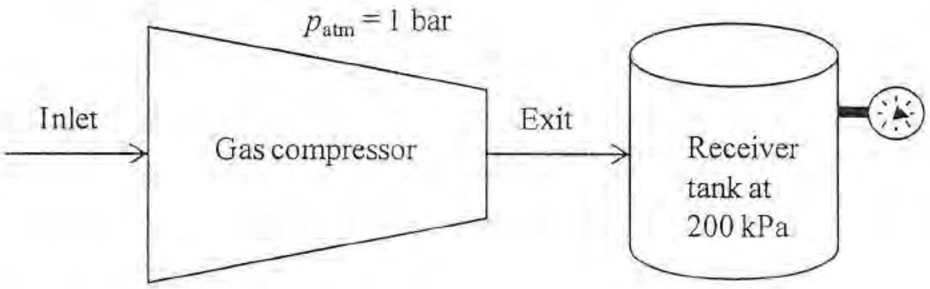


Fig. P1.34

Converting the local atmospheric pressure to kPa, we get $p_{atm} = 100$ kPa. Since the pressure in the tank, 200 kPa, is greater than the atmospheric pressure, the Bourdon reading is a *gage* pressure. Using the following relationship, $p_{gage} = p_{abs} - p_{atm}$ the Bourdon reading is 100 kPa.

PROBLEM 1.35

See Fig. P 1.35. Applying the principles discussed in Sec. 1.6.1, the atmospheric pressure is

$$p_{atm} = \rho_m g L \Rightarrow L = \frac{p_{atm}}{\rho_m g} = \frac{100 \times 10^3 \, N/m^2}{\left(13.59 \frac{g}{cm^3}\right)\left|\frac{1 kg}{10^3 g}\right|\left|\frac{10^2 cm}{1 m}\right|^3 \left(9.81 \frac{m}{s^2}\right)} \left|\frac{1 kg \cdot m/s^2}{1 N}\right|$$

$$= 0.75 \, m \left|\frac{10^3 \, mm}{1 \, m}\right|$$

$$= 750 \, mm \, Hg$$

Converting to in. Hg,

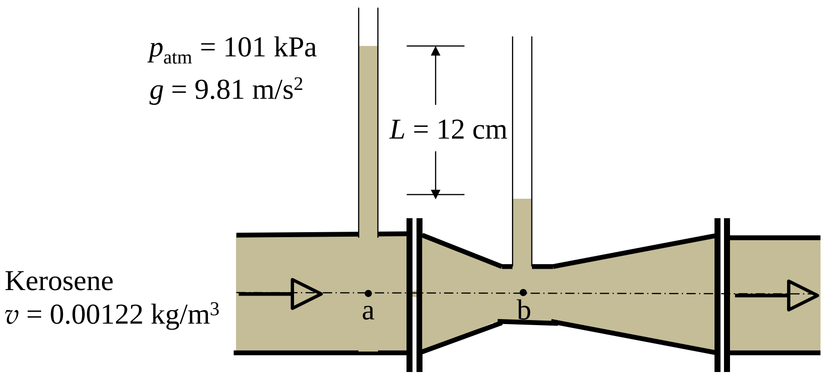
$$L = 750 \, mm \, Hg \left|\frac{1 \, cm}{10 \, mm}\right|\left|\frac{1 \, in.}{2.54 \, cm}\right| = 29.53 \, in. \, Hg$$

1.36 Liquid kerosene flows through a Venturi meter, as shown in Fig. P1.36. The pressure of the kerosene in the pipe supports columns of kerosene that differ in height by 12 cm. Determine the difference in pressure between points a and b, in kPa. Does the pressure increase or decrease as the kerosene flows from point a to point b as the pipe diameter decreases? The atmospheric pressure is 101 kPa, the specific volume of kerosene is 0.00122 m^3/kg, and the acceleration of gravity is $g = 9.81$ m/s^2.

KNOWN: Kerosene flows through a Venturi meter.

FIND: The pressure difference between points a and b, in kPa and whether pressure increases or decreases as the kerosene flows from point a to point b as the pipe diameter decreases.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The kerosene is incompressible.
2. Atmospheric pressure is exerted at the open end of the fluid columns.

ANALYSIS:
Equation 1.11 applies to both columns of fluid (a and b). Let h_b be the height of the fluid above point b. Then $h_b + L$ is the height of the fluid above point a. Applying Eq. 1.11 to each column yields

$$p_a = p_{atm} + \rho g(h_b + L) = p_{atm} + \rho g h_b + \rho g L$$

and

$$p_b = p_{atm} + \rho g h_b$$

Thus, the difference in pressure between point a and point b is

$$\Delta p = p_b - p_a = (p_{atm} + \rho g h_b) - (p_{atm} + \rho g h_b + \rho g L)$$

$$\Delta p = -\rho g L$$

Density of kerosene is the reciprocal of its specific volume

$$\rho = 1/v = 1/0.00122 \text{ m}^3/\text{kg} = 820 \text{ kg/m}^3$$

Solving for the difference in pressure yields

$$\Delta p = -\left(820 \frac{\text{kg}}{\text{m}^3}\right)\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(12 \text{ cm})\left|\frac{1 \text{ N}}{1 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}}\right|\left|\frac{1 \text{ m}}{100 \text{ cm}}\right|\left|\frac{1 \text{ kPa}}{1000 \frac{\text{N}}{\text{m}^2}}\right| = \underline{\mathbf{-0.965 \text{ kPa}}}$$

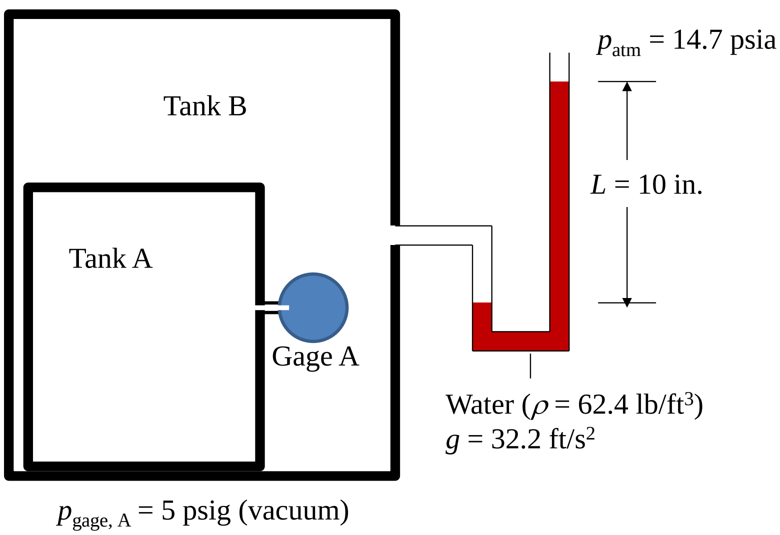
Since points a and b are at the same elevation in the flow, the difference in pressure is indicated by the difference in height between the two columns. **The negative sign indicates pressure decreases as the kerosene flows from point a to point b as the pipe diameter decreases.**

1.37 Figure P1.37 shows a tank within a tank, each containing air. Pressure gage A, which indicates pressure inside tank A, is located inside tank B and reads 5 psig (vacuum). The U-tube manometer connected to tank B contains water with a column length of 10 in. Using data on the diagram, determine the absolute pressure of the air inside tank B and inside tank A, both in psia. The atmospheric pressure surrounding tank B is 14.7 psia. The acceleration of gravity is g = 32.2 ft/s^2.

KNOWN: A tank within a tank, each containing air.

FIND: Absolute pressure of air in tank B and in tank A, both in psia.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The gas is a closed system.
2. Atmospheric pressure is exerted at the open end of the manometer.
3. The manometer fluid is water with a density of 62.4 lb/ft^3.

ANALYSIS:
(a) Applying Eq. 1.11

$$p_{gas,B} = p_{atm} + \rho g L$$

where p_{atm} is the local atmospheric pressure to tank B, ρ is the density of the manometer fluid (water), g is the acceleration due to gravity, and L is the column length of the manometer fluid. Substituting values

$$p_{gas,B} = 14.7\frac{\text{lbf}}{\text{in.}^2} + \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(32.2\frac{\text{ft}}{\text{s}^2}\right)(10\text{ in.})\left|\frac{1\text{ lbf}}{32.2\frac{\text{lbm}\cdot\text{ft}}{\text{s}^2}}\right|\left|\frac{1\text{ ft}^3}{1728\text{ in.}^3}\right| = \underline{\mathbf{15.1\ lbf/in.^2}}$$

Since the gage pressure of the air in tank A is a vacuum, Eq. 1.15 applies.

$$p(\text{vacuum}) = p_{\text{atm}}(\text{absolute}) - p(\text{absolute})$$

The pressure of the gas in tank B is the local atmospheric pressure to tank A. Solving for p (absolute) and substituting values yield

$$p(\text{absolute}) = p_{\text{atm}}(\text{absolute}) - p(\text{vacuum}) = 15.1 \text{ psia} - 5 \text{ psig} = \underline{\mathbf{10.1\ psia}}$$

PROBLEM 1.38

See Fig. P1.38.

The pressure acting on the vehicle at a depth $L = 1000\,ft$ is

$$p = p_{atm} + \rho g L$$

$$= 1\,atm + \left(62.4\,\frac{lb}{ft^3}\right)\left(32.2\,\frac{ft}{s^2}\right)(1000\,ft)\left|\frac{1\,atm}{14.696\,lbf/in^2}\right|\left|\frac{1\,ft^2}{144\,in^2}\right|\left|\frac{1\,lbf}{32.2\,lb\cdot ft/s^2}\right|$$

rounded

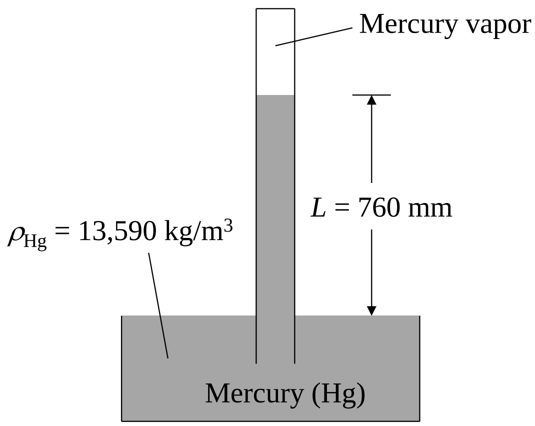
$$= 1\,atm + 29.49\,atm = 30.49\,atm$$

1.39 Show that a standard atmospheric pressure of 760 mmHg is equivalent to 101.3 kPa. The density of mercury is 13,590 kg/m^3 and g = 9.81 m/s^2.

KNOWN: Standard atmospheric pressure of 760 mmHg.

FIND: Show that 760 mmHg is equivalent to 101.3 kPa.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Local gravitational acceleration is 9.81 m/s^2.
2. Pressure of mercury vapor is much less than that of the atmosphere and can be neglected.

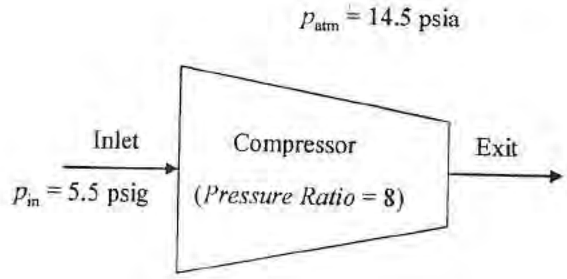
ANALYSIS:
Equation 1.12 applies.

$$p_{atm} = p_{vapor} + \rho_{Hg} g L = \rho_{Hg} g L$$

Neglecting the pressure of mercury vapor and applying appropriate conversion factors yield

$$p_{atm} = \left(13{,}590\,\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)(760\text{ mm})\left|\frac{1\text{ N}}{1\,\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right|\left|\frac{1\text{ m}}{1000\text{ mm}}\right|\left|\frac{1\text{ kPa}}{1000\,\frac{\text{N}}{\text{m}^2}}\right| = \underline{\mathbf{101.3\ kPa}}$$

PROBLEM 1.40



From the compressor pressure ratio, the exit pressure in psia can be determined from

$$\text{pressure ratio} = p_{exit}/p_{in} \quad \rightarrow \quad p_{exit} = p_{in}(\text{pressure ratio})$$

in psia

Inlet pressure must be expressed as absolute pressure to solve for exit pressure. Conversion from the inlet pressure gage reading to absolute pressure is determined from

$$p_{in}(\text{gage}) = p_{in}(\text{absolute}) - p_{atm}(\text{absolute})$$

Rearranging the equation to solve for p_{in}(absolute) and substituting values yield

$$p_{in}(\text{absolute}) = p_{in}(\text{gage}) + p_{atm}(\text{absolute}) = 5.5 \text{ psig} + 14.5 \text{ psia} = 20 \text{ psia}$$

Substituting absolute pressure at the inlet into the equation for exit pressure yields

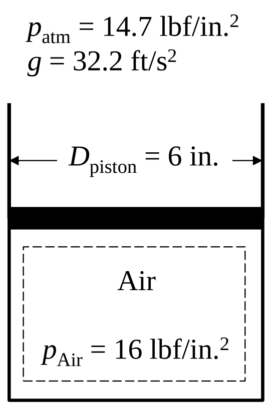
$$p_{exit} = (20 \text{ psia})(8) = \underline{\mathbf{160\ psia}}$$

1.41 As shown in Figure P1.41, air is contained in a vertical piston-cylinder assembly such that the piston is in static equilibrium. The atmosphere exerts a pressure of 14.7 lbf/in.^{2} on top of the 6-in.-diameter piston. The absolute pressure of the air inside the cylinder is 16 lbf/in.^{2} The local acceleration of gravity is g = 32.2 ft/s^2. Determine (a) the mass of the piston, in lb, and (b) the gage pressure of the air in the cylinder, in psig.

KNOWN: A piston-cylinder assembly contains air such that the piston is in static equilibrium.

FIND: (a) The mass of the piston, in lb, and (b) the gage pressure of the air in the cylinder, in psig.

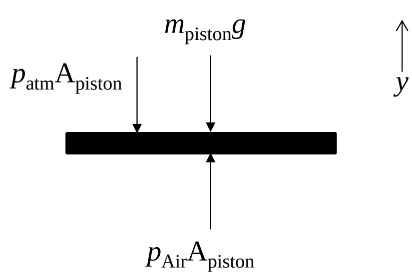
SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The air is a closed system.
2. The piston is in static equilibrium.
3. Atmospheric pressure is exerted on the top of the piston.
4. Local gravitational acceleration is 32.2 ft/s^2.

ANALYSIS:
(a) Draw a free body diagram indicating all forces acting on the piston. Taking upward as the positive y-direction, the sum of the forces acting on the piston in the y-direction must equal zero for static equilibrium of the piston.

Free Body Diagram



$$\uparrow \sum F_y = 0$$

$$p_{Air}A_{piston} - p_{atm}A_{piston} - m_{piston}g = 0$$

Solving for the mass of the piston,

$$m_{piston} = \frac{p_{Air}A_{piston} - p_{atm}A_{piston}}{g}$$

$$m_{\text{piston}} = \frac{\left(p_{\text{Air}} - p_{\text{atm}}\right)\text{A}_{\text{piston}}}{g}$$

The area of the piston is determined from the piston diameter

$$\text{A}_{\text{piston}} = \frac{\pi}{4}D^2 = \frac{\pi}{4}(6\,\text{in.})^2 = 28.3\ \text{in.}^2$$

Substituting values and solving for the mass of the piston,

$$m_{\text{piston}} = \frac{\left(16\,\frac{\text{lbf}}{\text{in.}^2} - 14.7\,\frac{\text{lbf}}{\text{in.}^2}\right)\left(28.3\,\text{in.}^2\right)}{32.2\,\frac{\text{ft}}{\text{s}^2}}\left|\frac{32.2\,\frac{\text{lb}\cdot\text{ft}}{\text{s}^2}}{1\,\text{lbf}}\right| = \underline{\mathbf{36.8\ lb}}$$

(b) Gage pressure of the air is given by Eq. 1.14

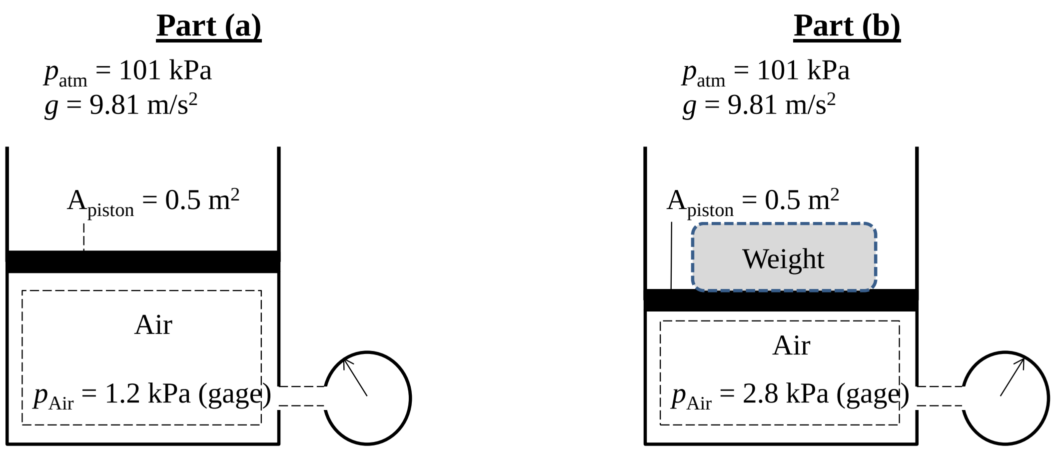
$p(\text{gage}) = p(\text{absolute}) - p_{\text{atm}}(\text{absolute}) = 16.0\ \text{psia} - 14.7\ \text{psia} =$ **1.3 psig**

1.42 Air is contained in a vertical piston-cylinder assembly such that the piston is in static equilibrium. The atmosphere exerts a pressure of 101 kPa on top of the 0.5-meter-diameter piston. The gage pressure of the air inside the cylinder is 1.2 kPa. The local acceleration of gravity is $g = 9.81$ m/s^2. Subsequently, a weight is placed on top of the piston causing the piston to fall until reaching a new static equilibrium position. At this position, the gage pressure of the air inside the cylinder is 2.8 kPa. Determine (a) the mass of the piston, in kg, and (b) the mass of the added weight, in kg.

KNOWN: A piston-cylinder assembly contains air such that the piston is in static equilibrium. Upon addition of a weight, the piston falls until reaching a new position of static equilibrium.

FIND: (a) The mass of the piston, in kg, and (b) the mass of the added weight, in kg.

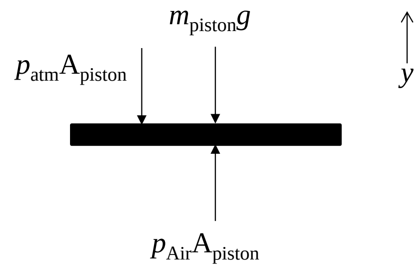
SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The air is a closed system.
2. The piston is in static equilibrium for both part (a) and part (b).
3. Atmospheric pressure is exerted on the top of the piston.
4. Local gravitational acceleration is 9.81 m/s^2.

ANALYSIS:
(a) Draw a free body diagram indicating all forces acting on the piston. Taking upward as the positive y-direction, the sum of the forces acting on the piston in the y-direction must equal zero for static equilibrium of the piston.

Free Body Diagram



$$\uparrow \sum F_y = 0$$

$$p_{\text{Air}}\text{A}_{\text{piston}} - p_{\text{atm}}\text{A}_{\text{piston}} - m_{\text{piston}}g = 0$$

Solving for the mass of the piston,

$$m_{\text{piston}} = \frac{p_{\text{Air}}\text{A}_{\text{piston}} - p_{\text{atm}}\text{A}_{\text{piston}}}{g}$$

$$m_{\text{piston}} = \frac{(p_{\text{Air}} - p_{\text{atm}})\text{A}_{\text{piston}}}{g}$$

From Eq. 1.14, the quantity in parenthesis is the gage pressure of the air in the cylinder. Rewriting the equation above

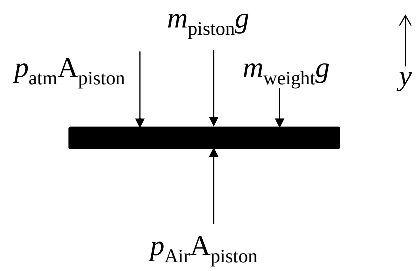
$$m_{\text{piston}} = \frac{p_{\text{Air(gage)}}\text{A}_{\text{piston}}}{g}$$

Substituting values and solving for the mass of the piston,

$$m_{\text{piston}} = \frac{(1.2\,\text{kPa})(0.5\,\text{m}^2)}{9.81\frac{\text{m}}{\text{s}^2}}\left|\frac{1000\frac{\text{N}}{\text{m}^2}}{1\,\text{kPa}}\right|\left|\frac{1\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}{1\,\text{N}}\right| = \mathbf{\underline{61.2\ kg}}$$

(b) Draw a second free body diagram indicating all forces acting on the piston including the newly added weight expressed as the product of its mass and gravitational acceleration. Taking upward as the positive *y*-direction, the sum of the forces acting on the piston in the *y*-direction must equal zero for static equilibrium of the piston.

Free Body Diagram



$$\uparrow\sum F_y = 0$$

$$p_{\text{Air}}\text{A}_{\text{piston}} - p_{\text{atm}}\text{A}_{\text{piston}} - m_{\text{piston}}g - m_{\text{weight}}g = 0$$

Solving for the mass of the weight,

$$m_{\text{weight}} = \frac{p_{\text{Air}}\text{A}_{\text{piston}} - p_{\text{atm}}\text{A}_{\text{piston}}}{g} - m_{\text{piston}}$$

$$m_{\text{weight}} = \frac{\left(p_{\text{Air}} - p_{\text{atm}}\right)\text{A}_{\text{piston}}}{g} - m_{\text{piston}}$$

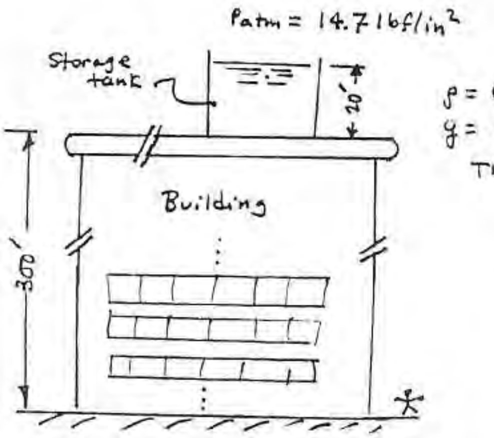
From Eq. 1.14, the quantity in parenthesis is the gage pressure of the air in the cylinder. Rewriting the equation above

$$m_{\text{weight}} = \frac{p_{\text{Air(gage)}}\text{A}_{\text{piston}}}{g} - m_{\text{piston}}$$

Substituting values and solving for the mass of the weight,

$$m_{\text{weight}} = \frac{(2.8\,\text{kPa})\left(0.5\,\text{m}^2\right)}{9.81\,\frac{\text{m}}{\text{s}^2}}\left|\frac{1000\,\frac{\text{N}}{\text{m}^2}}{1\,\text{kPa}}\right|\left|\frac{1\,\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}{1\,\text{N}}\right| - 61.2\,\text{kg} = \mathbf{\underline{81.5\,kg}}$$

PROBLEM 1.43



$\rho = 62.2\ lb/ft^3$

$g = 32.0\ ft/s^2$

The pressure at the bottom of the storage tank is

$$p = p_{atm} + \rho g L$$

$$= 14.7\frac{lbf}{in^2} + \left(62.2\frac{lb}{ft^3}\right)\left(32.0\frac{ft}{s^2}\right)(20\,ft)\left|\frac{1\,ft^2}{144\,in^2}\right|\left|\frac{1\,lbf}{32.2\,lb\,ft/s^2}\right|$$

rounded

$$= 14.7\frac{lbf}{in^2} + 8.6\frac{lbf}{in^2} = 23.3\frac{lbf}{in^2}$$

PROBLEM 1.44

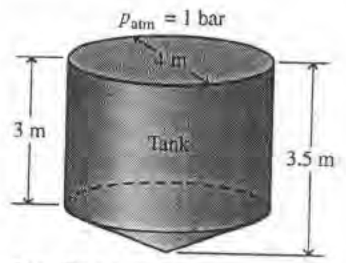


Fig. P1.44

(a) The depth at the center of the cistern is 3.5 m and the corresponding pressure at the center (p_c) in kPa is as follows

$$p_c = p_{atm} + \rho g h = 1\text{ bar}\left|\frac{100\text{ kPa}}{1\text{ bar}}\right| + (987.1\ \frac{\text{kg}}{\text{m}^3})(9.8\ \frac{\text{m}}{\text{s}^2})(3.5\text{ m})\left|\frac{1\text{ N}}{1\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right|\left|\frac{1\text{ kPa}}{10^3\ \frac{\text{N}}{\text{m}^2}}\right| =$$

$$p_c = 100\text{ kPa} + 33.9\text{ kPa} = 133.9\text{ kPa}$$

(b) The force acting on the bottom (F_{tot}) of the tank is the sum of the weight of the water plus the force of the atmosphere. The force of the atmosphere (F_{atm}) in kN is

$$F_{atm} = p_{atm}\pi\frac{D^2}{4} = 1\text{ bar}\left|\frac{10^5\ \frac{\text{N}}{\text{m}^2}}{1\text{ bar}}\right|\pi\frac{(4\text{m})^2}{4}\left|\frac{1\text{ kN}}{10^3\text{ N}}\right| = 12.6\times10^2\text{kN}$$

The weight of the water is given by

$$m_w g = \rho V g \qquad (1)$$

where ρ is the density of the water and g is the acceleration of gravity which were both given. The total volume of the water in the tank (V) is equal to the volume of a cylinder having a diameter, $D = 4$ m and a length, $L = 3$ m plus the volume of a cone having $D = 4$ m and a height, $H = 0.5$ m. Thus,

$$V = V_{cyl} + V_{cone} = \pi L\left(\frac{D^2}{4}\right) + \left(\frac{1}{3}\right)\pi H\left(\frac{D^2}{4}\right) = \pi\left(\frac{D^2}{4}\right)\left(L + \frac{H}{3}\right) = \pi\left(4\text{ m}^2\right)\left(3 + \frac{0.5}{3}\right)\text{m} = 39.8\text{ m}^3$$

Substituting values into Eq. (1)

$$\rho g V = 987.1\frac{\text{kg}}{\text{m}^3}\left(9.8\frac{\text{m}}{\text{s}^2}\right)\left(39.8\text{ m}^3\right)\left|\frac{1\text{ N}}{1\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right|\left|\frac{1\text{ kN}}{10^3\text{N}}\right| = 3.85\times10^2\text{ kN}$$

Finally, the total force acting on the bottom of the tank is

$$F_{tot} = \text{weight} + F_{atm} = 3.85\times10^2\text{kN} + 12.6\times10^2\text{kN} = 16.5\times10^2\text{kN}$$

PROBLEM 1.45

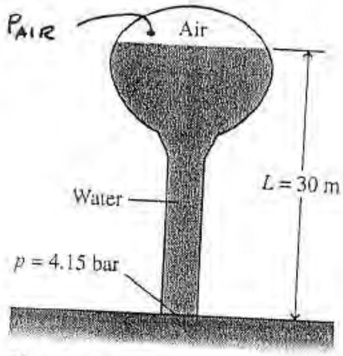


Fig. P1.45

Ignoring the vertical variation in pressure of the air trapped above water level,

$$p = P_{AIR} + \rho g L$$

pressure at the base

$$\Rightarrow \quad P_{AIR} = p - \rho g L$$

$$= 4.15\,\text{bar} - \left(10^3 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)(30\,m)\left|\frac{1N}{1\,kg \cdot m/s^2}\right|\left|\frac{1\,bar}{10^5 \frac{N}{m^2}}\right|$$

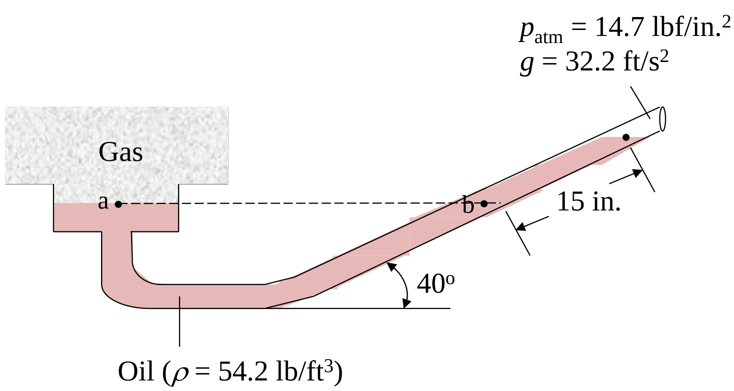
$$= 4.15\,\text{bar} - 2.94\,\text{bar} = 1.21\,\text{bar}$$

1.46 As shown in Figure P1.46, an inclined manometer is used to measure the pressure of the gas within the reservoir. (a) Using data on the figure, determine the gas pressure, in lbf/in.^{2} (b) Express the pressure as a gage or a vacuum pressure, as appropriate, in lbf/in.^{2} (c) What advantage does an inclined manometer have over the U-tube manometer shown in Figure 1.7?

KNOWN: A gas contained in a reservoir with inclined manometer attached.

FIND: (a) Pressure of gas within the reservoir, in lbf/in.^{2} (b) Pressure expressed as gage or vacuum pressure, as appropriate, in lbf/in.^{2} (c) Advantage of inclined manometer over the U-tube manometer.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The gas is a closed system.
2. Atmospheric pressure is exerted at the open end of the manometer.
3. The manometer fluid is oil with a density of 54.2 lb/ft^3.

ANALYSIS:
(a) Applying Eq. 1.11

$$p_{gas} = p_{atm} + \rho g L$$

where p_{atm} is the local atmospheric pressure, ρ is the density of the manometer fluid (oil), g is the acceleration due to gravity, and L is the vertical difference in liquid levels. Since level a is the same as level b, applying trigonometry to determine the vertical difference in liquid levels between level b and the liquid level at the free surface with the atmosphere yields

$$p_{gas} = p_{atm} + \rho g L(\sin 40^{\circ})$$

Substituting values

$$p_{gas} = 14.7\frac{\text{lbf}}{\text{in.}^2} + \left(54.2\frac{\text{lb}}{\text{ft}}\right)\left(32.2\frac{\text{ft}}{\text{s}^2}\right)(15\,\text{in.})(\sin 40°)\left|\frac{1\,\text{lbf}}{32.2\frac{\text{lbm}\cdot\text{ft}}{\text{s}^2}}\right|\left|\frac{1\,\text{ft}^3}{1728\,\text{in.}^3}\right| = \underline{\mathbf{15.0\ lbf/in.^2}}$$

(b) Since the pressure of the gas is greater than atmospheric pressure, gage pressure is given by Eq. 1.14

$$p(\text{gage}) = p(\text{absolute}) - p_{\text{atm}}(\text{absolute}) = 15.0\text{ psia} - 14.7\text{ psia} = \underline{\mathbf{0.3\ psig}}$$

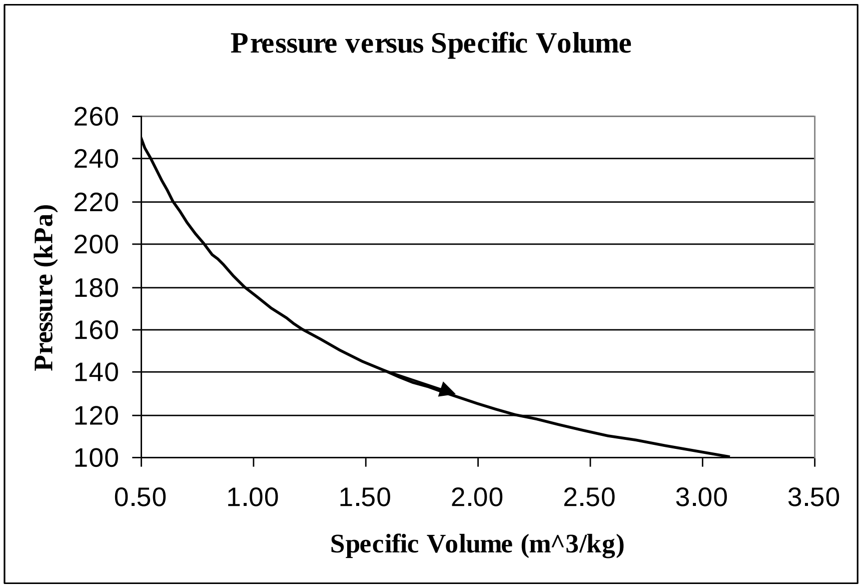
(c) The advantage of the inclined manometer is its easier readability since the surface of the liquid is wider than with a same diameter U-tube manometer. The scale on the inclined manometer is much more precise since more graduations are possible compared with the U-tube manometer.

Substituting values for pressures and specific volume yields

$$v_2 = \left(0.5\frac{\text{m}^3}{\text{kg}}\right)\left(\frac{250\,\text{kPa}}{100\,\text{kPa}}\right)^{\frac{1}{0.5}} = \underline{\mathbf{3.125\ m^3/kg}}$$

The volume of the system increased while pressure decreased during the process.

A plot of the process on a pressure versus specific volume graph is as follows:



PROBLEM 1.47

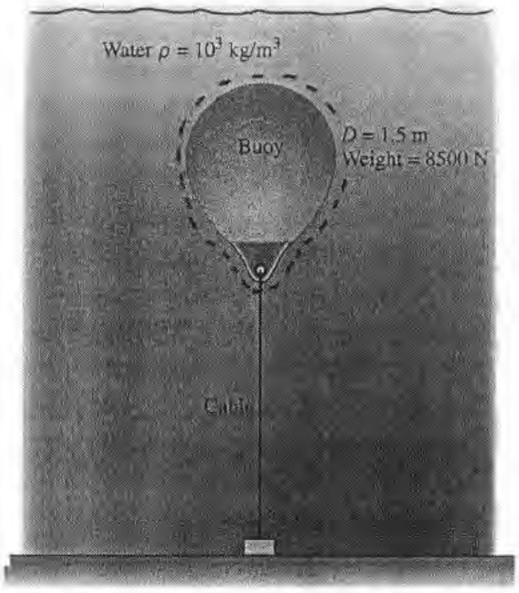


Fig. P1.47

The resultant pressure force acting on the system denoted by the dashed line is the buoyant force, F_B, acting vertically upward with a magnitude equal to the weight of the displaced water. See Sec. 1.6.2 for discussion.

Also acting on the system, vertically downward, is the weight of system and the force exerted by the cable.

In sum,

$$F_B = \text{Weight} + F_{CABLE}$$

$$\Rightarrow \quad F_{CABLE} = F_B - \text{Weight}$$

$$= (\rho V) g - \text{Weight}$$

$(\pi D^3/6)$

Calculating,

$$F_{CABLE} = \left(10^3 \frac{kg}{m^3}\right)\left(\frac{\pi(1.5\,m)^3}{6}\right)\left(9.81 \frac{m}{s^2}\right)\left|\frac{1\,N}{1\,kg \cdot m/s^2}\right| - 8500\,N$$

$$= (17336 - 8500)\,N = 8836\,N$$

PROBLEM 1.48

$\rho_o = 55\ lb/ft^3$

$\rho_w = 62\ lb/ft^3$

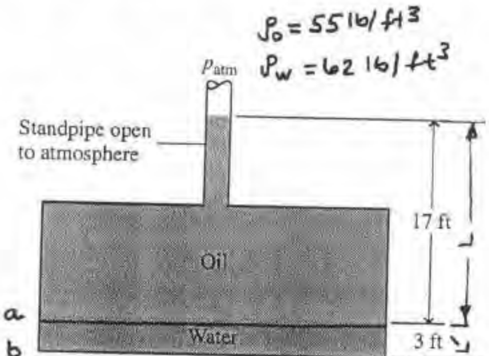


Fig. P1.48

With Eq. 1.11, the pressure at the oil-water interface is

$$p_a = p_{atm} + \rho_o g L$$

Expressed as a gage pressure, this is

$$[p_a - p_{atm}] = \rho_o g L$$

Calculating,

$$p_a(\text{gage}) = \left(55 \frac{lb}{ft^3}\right)\left(32.2 \frac{ft}{s^2}\right)(17\,ft)\left|\frac{1\,lbf}{32.2\,lb \cdot ft/s^2}\right|\left|\frac{1\,ft^2}{144\,in^2}\right|$$

(rounded)

$$= 6.49 \frac{lbf}{in^2} \ (\text{gage})$$

The pressure at the bottom of the tank is

$$p_b = p_a + \rho_w g L' = [p_{atm} + \rho_o g L] + \rho_w g L'$$

$$\Rightarrow p_b(\text{gage}) = \rho_o g L + \rho_w g L'$$

$$= 6.49 \frac{lbf}{in^2} + \left(62 \frac{lb}{ft^3}\right)\left(32.2 \frac{ft}{s^2}\right)(3\,ft)\left|\frac{1\,lbf}{32.2\,lb \cdot ft/s^2}\right|\left|\frac{1\,ft^2}{144\,in^2}\right|$$

$$= (6.49 + 1.29) \frac{lbf}{in^2} \ (\text{gage})$$

$$= 7.78 \frac{lbf}{in^2} \ (\text{gage})$$

PROBLEM 1.49

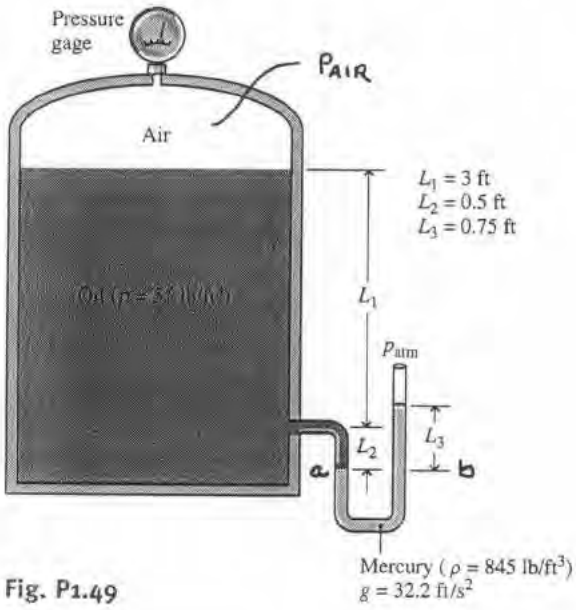


Fig. P1.49

Ignoring the vertical pressure variation of the air trapped above the oil, the gage reads

$$P_{gage} = P_{AIR} - P_{atm} \qquad (1)$$

We also have

$$P_a = P_{AIR} + \rho_o g (L_1 + L_2) \qquad (2)$$

and

$$P_b = P_{atm} + \rho_m g L_3 \qquad (3)$$

Then, since $P_a = P_b$, Eqs. (2) and (3) give

$$P_{AIR} + \rho_o g (L_1 + L_2) = P_{atm} + \rho_m g L_3$$

$$\Rightarrow \qquad P_{AIR} - P_{atm} = \rho_m g L_3 - \rho_o g (L_1 + L_2)$$

$$\Rightarrow \qquad P_{gage} = [\rho_m L_3 - \rho_o (L_1 + L_2)]\, g$$

Calculating,

$$P_{gage} = \left[\left(845 \frac{lb}{ft^3}\right)(0.75 ft) - \left(55 \frac{lb}{ft^3}\right)(3.5 ft)\right]\left(32.2 \frac{ft}{s^2}\right)\left|\frac{1\ lbf}{\underbrace{32.2\ lb \cdot ft/s^2}_{\text{rounded}}}\right|\left|\frac{1\ ft^2}{144\ in^2}\right|$$

$$= 3.06 \frac{lbf}{in^2} \text{ (gage)}$$

PROBLEM 1.50

First convert temperatures from °C to K by rearranging Eq. 1.17 to solve for temperature in K

$$T(^{\circ}C) = T(K) - 273.15 \quad \rightarrow \quad T(K) = T(^{\circ}C) + 273.15$$

For summer: T_{summer} (K) = 19.5°C + 273.15 = 292.65 K

For winter: T_{winter} (K) = –4.9°C + 273.15 = 268.25 K

Next apply Eq. 1.16 to solve for temperatures in °R

$$T(^{\circ}R) = 1.8T(K)$$

For summer: T_{summer} (°R) = (1.8)(292.65 K) = **526.77 °R**

For winter: T_{winter} (°R) = (1.8)(268.25 K) = **482.85°R**

Finally, apply Eq. 1.18 to solve for temperatures in °F

$$T(^{\circ}F) = T(^{\circ}R) - 459.67$$

For summer: T_{summer} (°F) = 526.77 °R – 459.67 = **67.10 °F**

For winter: T_{winter} (°F) = 482.85°R – 459.67 = **23.18 °F**

PROBLEM 1.51

Use the following equations to convert from °F to °C and then to K

$$T(^{\circ}F) = 1.8 \times T(^{\circ}C) + 32 \qquad (1.19)$$

$$\Rightarrow T(^{\circ}C) = \frac{T(^{\circ}F)}{1.8} - \frac{32}{1.8} = \frac{T(^{\circ}F)}{1.8} - 17.78$$

$$T(^{\circ}C) = T(K) - 273.15 \qquad (1.17)$$

$$\Rightarrow T(K) = T(^{\circ}C) + 273.15$$

(a) 86°F

$$T(^{\circ}C) = \frac{86}{1.8} - 17.78 = 30^{\circ}C$$

$$T(K) = 30 + 273.15 = 303.15\ K$$

(b) -22°F

$$T(^{\circ}C) = \frac{-22}{1.8} - 17.78 = -30^{\circ}C$$

$$T(K) = -30 + 273.15 = 243.15\ K$$

(c) 50°F

$$T(^{\circ}C) = \frac{50}{1.8} - 17.78 = 10^{\circ}C$$

$$T(K) = 10 + 273.15 = 283.15\ K$$

(d) -40°F

$$T(^{\circ}C) = \frac{-40}{1.8} - 17.78 = -40^{\circ}C$$

$$T(K) = -40 + 273.15 = 233.15\ K$$

(e) 32°F

$$T(^{\circ}C) = \frac{32}{1.8} - 17.78 = 0^{\circ}C$$

$$T(K) = 0 + 273.15 = 273.15\ K$$

(f) -459.67°F

$$T(^{\circ}C) = \frac{-459.67}{1.8} - 17.78 = -273.15^{\circ}C$$

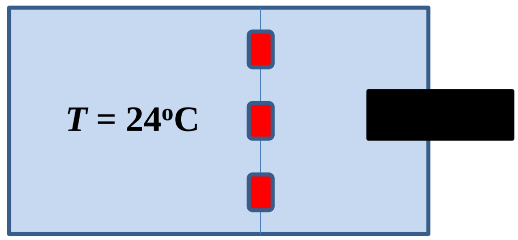
$$T(K) = -273.15 + 273.15 = 0\ K$$

1.52 Water in a swimming pool has a temperature of 24^oC. Express this temperature in K, ^oF, and ^oR.

KNOWN: Water is at a specified temperature in ^oC.

FIND: Equivalent temperature in K, ^oF, and ^oR.

SCHEMATIC AND GIVEN DATA:



ANALYSIS:
First convert temperature from ^oC to K by rearranging Eq. 1.17 to solve for temperature in K

$$T(^oC) = T(K) - 273.15 \quad \rightarrow \quad T(K) = T(^oC) + 273.15$$

$T_{water}\ (K) = 24^oC + 273.15 =$ **297.15 K**

Next apply Eq. 1.16 to solve for temperature in ^oR

$$T(^oR) = 1.8T(K)$$

$T_{water}\ (^oR) = (1.8)(297.15\ K) =$ **534.87 ^oR**

Finally, apply Eq. 1.18 to solve for temperatures in ^oF

$$T(^oF) = T(^oR) - 459.67$$

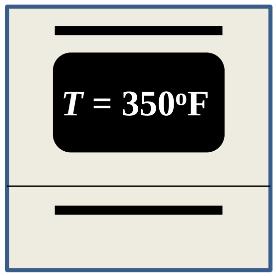
$T_{water}\ (^oF) = 534.87\ ^oR - 459.67 =$ **75.2^oF**

1.53 A cake recipe specifies an oven temperature of 350^{o}F. Express this temperature in ^{o}R, K, and ^{o}C.

KNOWN: Oven temperature is specified in ^{o}F.

FIND: Equivalent temperature in ^{o}R, K, and ^{o}C.

SCHEMATIC AND GIVEN DATA:



ANALYSIS:
First convert temperature from ^{o}F to ^{o}R using Eq. 1.18 to solve for temperature in ^{o}R

$$T(^oF) = T(^oR) - 459.67 \rightarrow T(^oR) = T(^oF) + 459.67$$

T_{oven} (^{o}R) = 350^{o}F + 459.67 = **809.67^{o}R**

Next apply Eq. 1.16 to solve for temperature in K

$$T(^oR) = 1.8T(K) \rightarrow T(K) = T(^oR)/1.8$$

T_{oven} (K) = 809.67^{o}R/1.8 = **449.82 K**

Finally, apply Eq. 1.17 to solve for temperature in ^{o}C

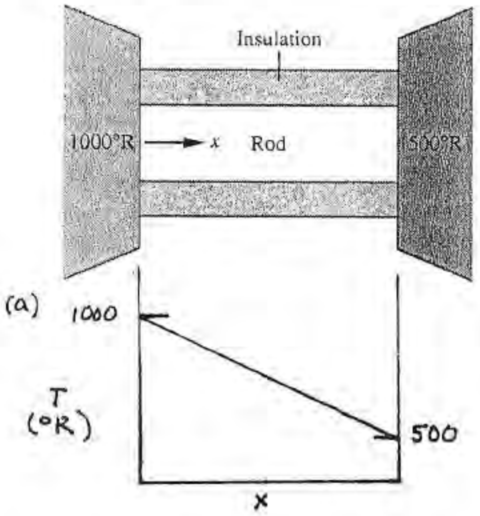
$$T(^oC) = T(K) - 273.15$$

T_{oven} (^{o}C) = 449.82 K – 273.15 = **176.67^{o}C**

PROBLEM 1.54

See Fig 1.14. At the steam point, for instance, 671.67 Rankine degrees correspond to 373.15 Kelvin degrees. The Kelvin is the larger unit.

PROBLEM 1.55



(b) Apply the test for equilibrium given in Sec. 1.3.4—namely, think of isolating the system and watching for changes in observable properties. In this instance, the rod is the system and the relevant observable is its temperature. If the rod is also insulated on its ends, its temperature will eventually become uniform throughout, indicating that the rod was not in equilibrium initially.

1.56 Left for independent study using the Internet.

1.57 Air temperature rises from a morning low of 42°F to an afternoon high of 70°F.
(a) Express these temperatures in °R, K, and °C.
(b) Determine the temperature *change* in °F, °R, K, and °C from morning low to afternoon high.
(c) What conclusion do you draw about temperature *change* for °F and °R scales?
(d) What conclusion do you draw about temperature *change* for °C and K scales?

KNOWN: Morning low temperature and afternoon high temperature, both in °F.

FIND: (a) Express these temperatures in °R, K, and °C, (b) temperature *change* in °F, °R, K, and °C from morning low to afternoon high, (c) conclusion about temperature *change* for °F and °R scales, (d) conclusion about temperature *change* for °C and K scales.

SCHEMATIC AND GIVEN DATA:

$T_{low} = 42^oF$
$T_{high} = 70^oF$

ANALYSIS:
(a) First convert temperatures from °F to °R using Eq. 1.18 to solve for temperatures in °R

$$T(^oF) = T(^oR) - 459.67 \rightarrow T(^oR) = T(^oF) + 459.67$$

$T_{low}\ (^oR) = 42\ ^oF + 459.67 =$ **501.67°R**

$T_{high}\ (^oR) = 70\ ^oF + 459.67 =$ **529.67°R**

Next apply Eq. 1.16 to solve for temperature in K

$$T(^oR) = 1.8T(K) \rightarrow T(K) = T(^oR)/1.8$$

$T_{low}\ (K) = 501.67^oR/1.8 =$ **278.71 K**

$T_{high}\ (K) = 529.67^oR/1.8 =$ **294.26 K**

Finally, apply Eq. 1.17 to solve for temperature in °C

$$T(^oC) = T(K) - 273.15$$

$T_{low}\ (^oC) = 278.71\ K - 273.15 =$ **5.56°C**

$T_{high}\ (^oC) = 294.26\ K - 273.15 =$ **21.11°C**

(b) Temperature change, ΔT, is $T_{high} - T_{low}$. Calculating the differences yields

$\Delta T(^oF) = 70^oF - 42^oF =$ **28°F**

$\Delta T(^oR) = 529.67^oR - 501.67^oR =$ **$\underline{28^oR}$**

$\Delta T(K) = 294.26\ K - 278.71\ K =$ **$\underline{15.55\ K}$**

$\Delta T(^oC) = 21.11\ ^oC - 5.56\ ^oC =$ **$\underline{15.55^oC}$**

(c) For ^oF and ^oR scales, the temperature *change* is the same since a Rankine degree and a Fahrenheit degree are the same temperature unit.

(d) For ^oC and K scales, the temperature *change* is the same since a Kelvin degree and a Celsius degree are the same temperature unit.

1.58 Left for independent study using the Internet.

PROBLEM 2.1

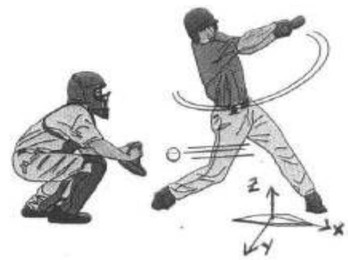


Fig. P2.1

$$KE = \frac{1}{2} m \underbrace{V^2}_{\text{relative to home plate}}$$

$$= \frac{1}{2}(0.3\,lb)\left\{ 44\frac{miles}{h}\left|\frac{1\,h}{3600\,s}\right|\left|\frac{5280\,ft}{1\,mile}\right|\right\}^2$$

$$= 2851.1\ \frac{lb\cdot ft}{s^2}\left|\frac{1\,lbf}{32.2\,lb\cdot ft/s^2}\right|\left|\frac{1\,Btu}{778\,ft\cdot lbf}\right|$$

rounded

$$KE = 0.114\ Btu$$

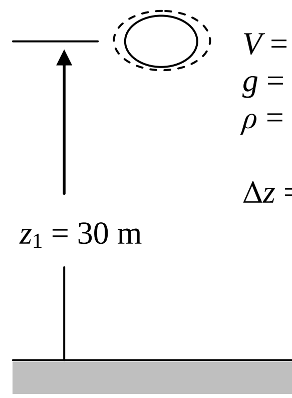
Problem 2.2

Determine the gravitational potential energy, in kJ, of 2 m^3 of liquid water at an elevation of 30 m above the surface of Earth. The acceleration of gravity is constant at 9.7 m/s^2 and the density of the water is uniform at 1000 kg/m^3. Determine the change in gravitational potential energy as the elevation decreased by 15 m.

KNOWN: The elevation of a known quantity of water is decreased from a given initial value by a given amount.

FIND: Determine the initial gravitational potential energy and the change in gravitational potential energy.

SCHEMATIC AND GIVEN DATA:



V = 2 m^3
g = 9.7 m/s^2
ρ = 1000 kg/m^3

Δz = - 15 m

ENGINERING MODEL:
(1) The water is a closed system. (2) The acceleration of gravity is constant. (3) The density of water is uniform.

ANALYSIS: The initial gravitational potential energy is

$$PE_1 = mgz_1 = (\rho \mathrm{V})gz_1$$

$$= \left(1000\,\frac{\mathrm{kg}}{\mathrm{m}^3}\right)(2\ \mathrm{m}^3)\left(9.7\,\frac{\mathrm{m}}{s^2}\right)(30\ \mathrm{m})\left|\frac{1\ \mathrm{N}}{1\ \mathrm{kg}\cdot\frac{\mathrm{m}}{\mathrm{s}^2}}\right|\left|\frac{1\ \mathrm{kJ}}{10^3\,\mathrm{N}\cdot\mathrm{m}}\right|$$

= 582 kJ

The change in potential energy is

$$\Delta PE = mg(z_2 - z_1) = mg\Delta z$$

$$= \left(1000\,\frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(9.7\,\frac{\mathrm{m}}{s^2}\right)(-15\ \mathrm{m})\left|\frac{1\ \mathrm{N}}{1\ \mathrm{kg}\cdot\frac{\mathrm{m}}{\mathrm{s}^2}}\right|\left|\frac{1\ \mathrm{kJ}}{10^3\,\mathrm{N}\cdot\mathrm{m}}\right|$$

= -291 kJ

PROBLEM 2.3

$\Delta KE = -500\ ft \cdot lbf$

$\Delta PE = +1500\ ft \cdot lbf$

$V_1 = 40\ ft/s$

$z_1 = 30\ ft$

$F_{grav} = mg = 100\ lbf$

$g = 32.2\ ft/s^2$

(a) $\Delta KE = \frac{1}{2} m \left[V_2^2 - V_1^2 \right]$, $m = \frac{F_{grav}}{g} = \frac{100\ lbf}{(32.2\ ft/s^2)} \left| \frac{32.2\ lb \cdot ft/s^2}{1\ lbf} \right| = 100\ lb$

Solving for V_2,

$$V_2 = \left[\frac{2\Delta KE}{m} + V_1^2 \right]^{\frac{1}{2}} = \left[\frac{2(-500\ ft \cdot lbf)}{100\ lb} \left| \frac{32.2\ lb \cdot ft/s^2}{1\ lbf} \right| + \left(40 \frac{ft}{s}\right)^2 \right]^{1/2}$$

$$= 35.75\ ft/s \longleftarrow$$

(b) $\Delta PE = \underbrace{mg}_{F_{grav}} (z_2 - z_1) \Rightarrow 1500\ ft \cdot lbf = 100\ lbf\,(z_2 - 30\ ft)$

Solving,

$$z_2 = 45\ ft \longleftarrow$$

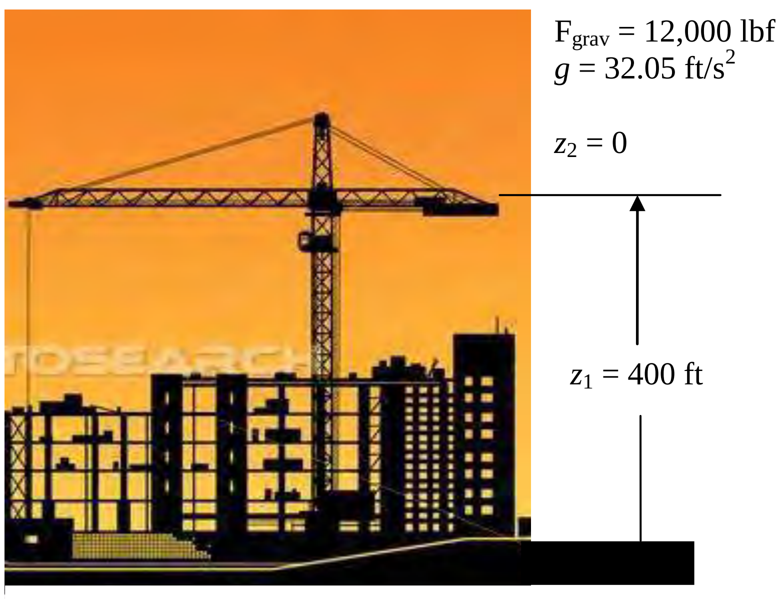
Problem 2.4

A construction crane weighing 12,000 lbf fell from a height of 400 ft to the street below during a severe storm. For g = 32.05 ft/s^2, determine mass, in lb, and the change in gravitational potential energy of the crane, in ft·lbf.

KNOWN: A crane of known weight falls from a known elevation to the street below.

FIND: Determine the change in gravitational potential energy of the crane.

SCHMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The crane is the closed system. (2) The acceleration of gravity is constant.

ANALYSIS:

To get the mass, note that $F_{grrav} = mg$. Thus

$$m = \frac{F_{grav}}{g} = \frac{12000 \text{ lbf}}{32.05 \text{ ft/s}^2}\left|\frac{32.174 \text{ lb·ft/s}^2}{1 \text{ lbf}}\right| = 12{,}046 \text{ lb} \longleftarrow$$

The change in gravitational potential energy is

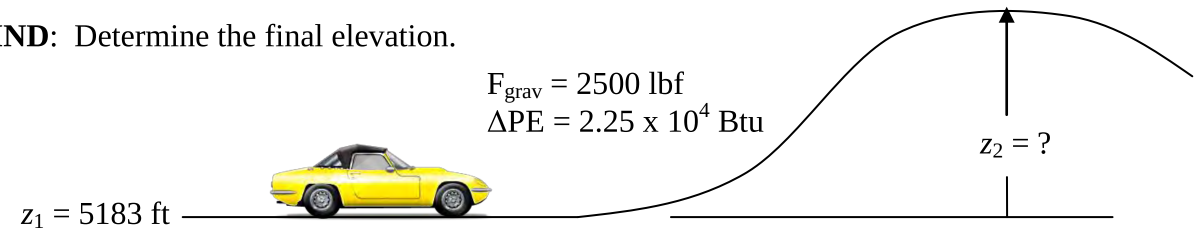
$$\Delta PE = mg(z_2 - z_1) = F_{grav}\Delta z = (12000 \text{ lbf})(-400 \text{ ft}) = -4.8 \text{ x } 10^6 \text{ ft} \cdot \text{lbf} \longleftarrow$$

Problem 2.5

A automobile weighing 2500-lbf increases its gravitational potential energy by 2.25 x 10^4 Btu in going from an elevation of 5,183 ft in Denver to the highest elevation on Trail Ridge road in the Rocky Mountains. What is the elevation at the high point of the road, in ft?

KNOWN: An automobile of known weight increases its gravitational potential energy by a given amount. The initial elevation is known.

FIND: Determine the final elevation.



ENGINEERING MODEL: (1) The automobile is the closed system. (2) The acceleration of gravity is constant.

ANALYSIS: The change in gravitational potential energy is: $\Delta PE = mg(z_2 - z_1)$. With $F_{grav} = mg$, we get

$$\Delta PE = F_{grav}(z_2 - z_1)$$

Solving for z_2

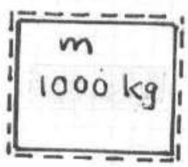
$$z_2 = \frac{\Delta PE}{F_{grav}} + z_1 = \frac{(2.25\ x\ 10^4\ \text{Btu})}{(2500\ \text{lbf})}\left|\frac{778\ \text{ft}\cdot\text{lbf}}{1\ \text{Btu}}\right| + 5183\ \text{ft} = 12{,}185\ \text{ft} \longleftarrow$$

PROBLEM 2.6

KNOWN: An object of known mass decelerates from a given initial velocity to a known final velocity.

FIND: Determine the change in kinetic energy of the object.

SCHEMATIC & GIVEN DATA:



$V_1 = 100$ m/s

$V_2 = 20$ m/s

ENGR. MODEL: The object is a closed system.

ANALYSIS: The change in kinetic energy is

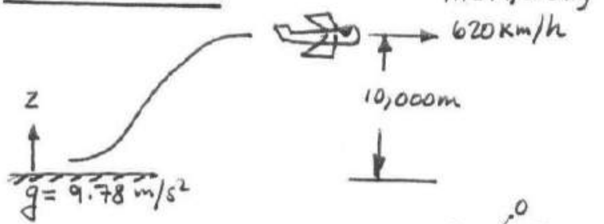
$$\Delta KE = \frac{1}{2} m \left[V_2^2 - V_1^2 \right]$$

Inserting known values and converting units

$$\Delta KE = \frac{1}{2}(1000 \text{ kg})\left[20^2 - 100^2\right]\frac{\text{m}^2}{\text{s}^2}\left|\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}}\right|$$

$$= -4800 \text{ kJ} \longleftarrow \Delta KE$$

PROBLEM 2.7



$$\Delta KE = \frac{1}{2} m \left[V_2^2 - \cancel{V_1^2}^{\,0} \right]$$

$$= \frac{1}{2} (14000\ \text{kg}) \left[620 \frac{\text{km}}{\text{h}} \left| \frac{1\ \text{h}}{3600\ \text{s}} \right| \left| \frac{1000\ \text{m}}{\text{km}} \right| \right]^2 \left| \frac{1\ \text{N}}{1\ \text{kg} \cdot \text{m/s}^2} \right| \left| \frac{1\ \text{kJ}}{10^3\ \text{N} \cdot \text{m}} \right|$$

$$= 207,623\ \text{kJ}$$

$$\Delta PE = mg\,(z_2 - \cancel{z_1}^{\,0})$$

$$= (14000\ \text{kg})(9.78\ \text{m/s}^2)(10,000\ \text{m}) \left| \frac{1\ \text{N}}{1\ \text{kg} \cdot \text{m/s}^2} \right| \left| \frac{1\ \text{kJ}}{10^3\ \text{N} \cdot \text{m}} \right|$$

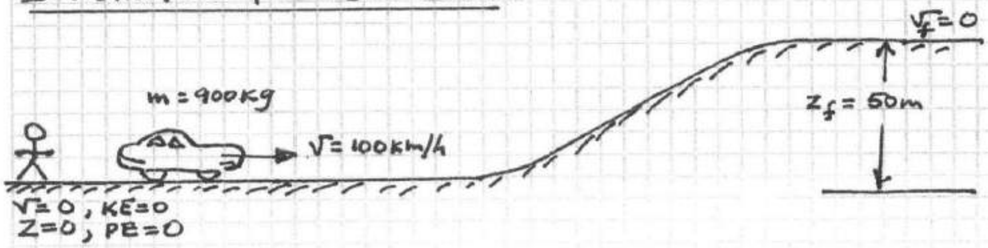
$$= 1,369,200\ \text{kJ}$$

PROBLEM 2.8

KNOWN: Data are provided for an automobile on the open road.

FIND: Determine the changes in kinetic energy and gravitational potential energy for the automobil, in kJ

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. As shown in the schematic, the automobile is the system.
2. The acceleration of gravity is constant, $g = 9.81 m/s^2$.
3. The datums for KE and PE are embedded in the road surface, where indicated by the stationary observer.

ANALYSIS:

The change in kinetic energy is

$$\Delta KE = \left(0 - \frac{1}{2} m \frac{V^2}{2}\right) = -\frac{1}{2}(900kg)\left(100 \frac{km}{h}\right)^2 \left|\frac{10^3 m}{1 km}\right|^2 \left|\frac{1h}{3600s}\right|^2 \left|\frac{1 N}{1 kg \cdot m/s^2}\right| \left|\frac{1 kJ}{10^3 N \cdot m}\right|$$

$$= -347.2 kJ \longleftarrow$$

The change in potential energy is

$$\Delta PE = \left[m g z_f - 0\right] = (900kg)\left(9.81 \frac{m}{s^2}\right)(50m) \left|\frac{1N}{1 kg \cdot m/s^2}\right| \left|\frac{1 kJ}{10^3 N \cdot m}\right|$$

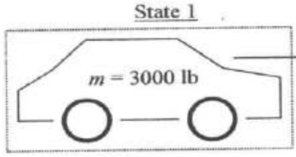
$$= +441.5 \ kJ \longleftarrow$$

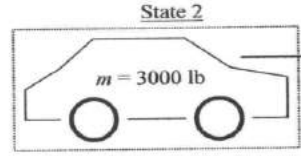
PROBLEM 2.9

KNOWN: Vehicle crumple zone absorbs energy during impact.

FIND: Change in kinetic energy, in Btu, of 3000-lb vehicle decelerating from 10 mph to 0 mph.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL:

1. The automobile is the system.
2. Crumple zone absorbs all kinetic energy of vehicle upon impact.
3. Weight of occupants can be neglected.

ANALYSIS:

Change in kinetic energy (ΔKE) is determined by

$$\Delta KE = \tfrac{1}{2}\, m(V_2^2 - V_1^2)$$

where m is mass of the vehicle and V is velocity of the vehicle relative to the roadway.

Substituting and applying appropriate conversion factors yield

$$\Delta KE = \frac{1}{2}(3000\ \text{lb})\left(\left(0\frac{\text{mi}}{\text{h}}\right)^2 - \left(10\frac{\text{mi}}{\text{h}}\right)^2\right)\left|\frac{(5280\ \text{ft})^2}{(\text{mi})^2}\right|\frac{1\ \text{lbf}}{32.2\dfrac{\text{lb}\cdot\text{ft}}{\text{s}^2}}\left|\frac{\text{h}^2}{(3600\ \text{s})^2}\right|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

$$\Delta KE = \underline{\mathbf{-12.9\ Btu}}$$

The negative sign indicates the energy of the moving vehicle is reduced and must be absorbed by the crumple zone to protect the occupants.

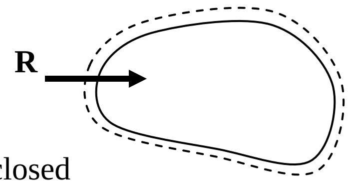
Problem 2.10

An object whose mass is 300 lb experiences changes in kinetic and potential energies owing to the action of a resultant force **R**. The work done on the object by the resultant force is 140 lbf. There are no other interactions between the object and its surroundings. It the object's elevation increases by 100 ft and its final velocity is 200 ft/s, what is the initial velocity, in ft/s? Let g = 32.2 ft/s^2.

KNOWN: An object of known mass experiences changes in kinetic and potential energy due to the action of a resultant force. The final velocity, the change in elevation, and the work done by the force are given.

FIND: Determine the final velocity.

SCHEMATIC AND GIVEN DATA:



m = 300 lb
V_2 = 200 ft/s
$z_2 - z_1$ = +100 ft
g = 32.2 ft/s^2

Work done by resultant force = 140 Btu

ENGINEEIRNG MODEL: (1) The object is a closed system. (2) The force of gravity acts on the object, and g = 32.2 ft/s^2. (3) The resultant force accounts for all interactions between the system and its surroundings.

ANALYSIS: By modeling assumption (3), the work of the resultant force must equal the sum of the changes in kinetic and gravitational potential energies. Thus, with Eq. 2.9

$$\text{Work} = \tfrac{1}{2}\, m(V_2^2 - V_1^2) + mg(z_2 - z_1)$$

Solving for V_1^2 and inserting values

$$V_1^2 = \frac{2[mg(z_2 - z_1) - \text{Work}]}{m} + V_2^2$$

First

$$mg(z_2 - z_1) = (300\text{ lb})(32.2\text{ ft/s}^2)(100\text{ ft})\left|\frac{1\text{ lbf}}{32.2\text{ lb·ft/s}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft·lbf}}\right| = 38.6\text{ Btu}$$

So

$$V_1^2 = \frac{2[38.6 - 140]\text{BTU}}{(300\text{ lb})}\left|\frac{778\text{ ft·lbf}}{1\text{ Btu}}\right|\left|\frac{32.2\text{ lb·ft/s}^2}{1\text{ lbf}}\right| + 200^2\text{ ft}^2/\text{s}^2 = 23065\text{ ft}^2/\text{s}^2$$

or

1 $$V_1 = 151.9\text{ ft/s}$$

---

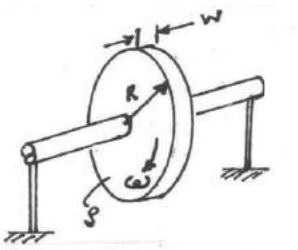
1. The increase in velocity reflects the increase in kinetic energy of the object as a result of energy transferred to it by the work of the resultant force. Carefully observe that in Eq. 2.9 the work of the resultant force acting *on* the body is positive.

PROBLEM 2.11

KNOWN: Data are provided for a disk-shaped flywheel.

FIND: (a) Obtain appropriate expressions for the moment of inertia and the kinetic energy. (b) Using given data, determine the kinetic energy and mass for a steel flywheel. (c) Using results from part (b), determine the radius and mass of an aluminum flywheel.

SCHEMATIC & GIVEN DATA:



Steel flywheel:
$\omega = 3000$ RPM
$R = 0.38$ m
$W = 0.025$ m

Aluminum flywheel:
$\omega = 3000$ RPM
$W = 0.025$ m

ENGR. MODEL: 1. The flywheel is the closed system. 2. Motion is relative to the flywheel support structure.

ANALYSIS:

(a) Evaluating the moment of inertia

$$I = \int_{vol} \rho r^2 dV$$

For the disk, $dV = (2\pi r dr)W$. Thus, since ρ is constant

$$I = \rho(2\pi)W\int_0^R r^3 dr = \rho\pi W R^4/2 \longleftarrow I$$

The kinetic energy is

$$KE = \int_{vol}\left(\tfrac{1}{2}\rho V^2\right)dV$$

and $V = r\omega$, so

$$KE = \int_0^R \left(\tfrac{1}{2}\rho r^2\omega^2\right)(2\pi r dr)W$$

$$= \tfrac{1}{2}\rho\omega^2(2\pi)W\int_0^R r^3 dr$$

$$= \tfrac{1}{2}\underbrace{\left(\rho\pi\frac{R^4}{2}W\right)}_{I}\omega^2$$

$$= \tfrac{1}{2}I\omega^2 \longleftarrow KE$$

(b) From Table A-19, the density of steel is $\rho = 8060$ kg/m³. Thus, the mass is

$$m = \rho V = \rho[W\cdot\pi R^2]$$
$$= \left(8060\ \tfrac{kg}{m^3}\right)\left[(0.025m)\cdot\pi\cdot(0.38m)^2\right] = 91.41\ kg \longleftarrow m$$

Using the result of part (a), $KE = \frac{1}{2}I\omega^2$, where

$$I = \pi\rho W\frac{R^4}{2} = \frac{\pi}{2}\left(8060\ \tfrac{kg}{m^3}\right)(0.025m)(0.38m)^4 = 6.6\ kg\cdot m^2$$

PROBLEM 2.11 (Contd.)

Accordingly,

$$KE = \frac{1}{2} I \omega^2 = \frac{1}{2}(6.6\ kg \cdot m^2)\left(3000\ \frac{Rev}{min}\left|\frac{2\pi\ rad}{Rev}\right|\left|\frac{1\ min}{60\ s}\right|\right)^2\left|\frac{1\ N}{1\ kg \cdot m/s^2}\right|$$

$$= 32.57 \times 10^4\ N \cdot m \longleftarrow KE$$

(c) If ω, W, and KE are the same for the aluminum flywheel as for the steel flywheel

$$(KE)_{AL} = (KE)_{ST}$$

$$\left(\tfrac{1}{2} I \omega^2\right)_{AL} = \left(\tfrac{1}{2} I \omega^2\right)_{ST} \Rightarrow I_{AL} = I_{ST}$$

or

$$\left(\pi \rho W \frac{R^4}{2}\right)_{AL} = \left(\pi \rho W \frac{R^4}{2}\right)_{ST}$$

$$\Rightarrow (\rho R^4)_{AL} = (\rho R^4)_{ST}$$

$$R_{AL} = \left(\frac{\rho_{ST}}{\rho_{AL}}\right)^{1/4} R_{ST}$$

With ρ_{AL} from Table A-19, $\rho_{AL} = 2700\ kg/m^3$

$$R_{AL} = \left(\frac{8060}{2700}\right)^{\frac{1}{4}}(0.38\ m)$$

$$= 0.5\ m \longleftarrow R_{AL}$$

Then, the mass of the aluminum flywheel is

$$m = \rho V = \rho\left[w \pi R^2\right]$$

$$= \left(2700\ \frac{kg}{m^3}\right)(0.025\ m)(\pi)(0.5\ m)^2 = 53.01\ kg \longleftarrow m$$

PROBLEM 2.12

From Problem 2.11, $KE = \frac{1}{2} I \omega^2 \Rightarrow \omega = \left(\frac{2\,KE}{I}\right)^{1/2}$ where $I = 200\,lb \cdot ft^2$.

The change in potential energy of a 100 lb mass raised 30 ft is

$$\Delta PE = mg(z_2 - z_1) = (100\,lb)\left(32.2\,\frac{ft}{s^2}\right)(30\,ft)\left|\frac{1\,lbf}{32.2\,lb \cdot ft/s^2}\right| = 3000\,ft \cdot lbf.$$

Thus, with $KE = 3000\,ft \cdot lbf$,

$$\omega = \left(\frac{2\,(3000\,ft \cdot lbf)}{200\,lb \cdot ft^2}\left|\frac{32.2\,lb \cdot ft/s^2}{1\,lbf}\right|\right)^{1/2} = 31.08\,\frac{1}{s}$$

In terms of RPM

$$\omega = \left(31.08\,\frac{1}{s}\right)\left|\frac{1\,rev}{2\pi}\right|\left|\frac{60\,s}{1\,min}\right| = 297\,rev/min \longleftarrow$$

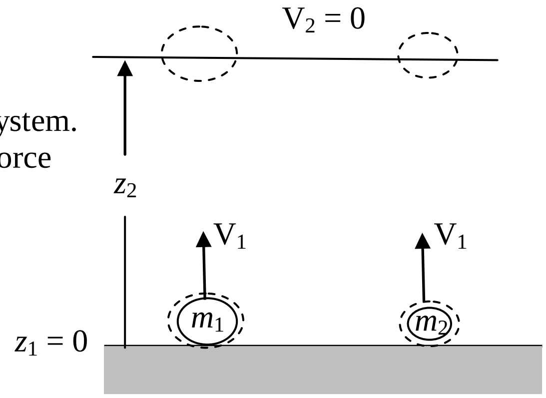
Problem 2.13

Two objects having different masses are propelled vertically from the surface of Earth, each with the same initial velocities. Assuming the objects are acted upon only by the force of gravity, show that they reach zero velocity at the same height.

KNOWN: Two objects are propelled upward from the surface of Earth with the same initial velocities and are acted upon only by the force of gravity.

FIND: Show that the reach zero velocity at the same height.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) Each object is a closed system. (2) The acceleration of gravity is constant. (3) The only force acting is the force of gravity.

ANALYSIS: For an object moving vertically under the influence of gravity only, Eq. 2.11 applies

$$\frac{1}{2}mg(V_2^2 - V_1^2) + mg(z_2 - z_1) = 0$$

For $V_2 = 0$ and $z_1 = 0$

$$-\frac{1}{2}mV_1^2 + mgz_2 = 0$$

Thus

$$z_2 = V_1^2/2g$$

Since the final height doesn't depend on mass, both objects will reach zero velocity at the same final height.

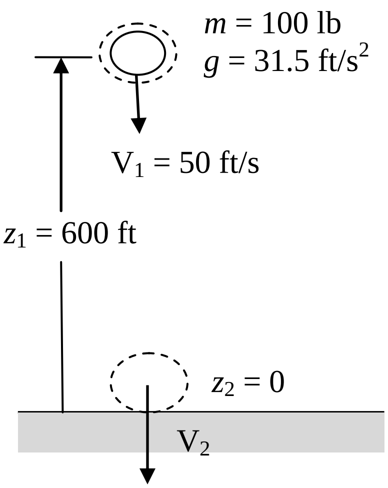
Problem 2.14

An object whose mass is 100 lb falls freely under the influence of gravity from an initial elevation of 600 ft above the surface of Earth. The initial velocity is downward with a magnitude of 50 ft/s. The effect of air resistance is negligible. Determine the velocity, in ft/s, of the object just before it strikes Earth. Assume g = 31.5 ft/s^2.

KNOWN: An object of known mass falls freely from a known elevation and with a given initial velocity. The only force acting is the force of gravity.

FIND: Determine the velocity of the object just before it strikes Earth.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The object is a closed system. (2) The acceleration of gravity is constant. (3) The only force acting on the object is the force of gravity.

ANALYSIS: Since the only force acting on the object is the force of gravity, Eq. 2.11 applies. Thus

① $$\frac{1}{2}\cancel{m}(V_2^2 - V_1^2) + \cancel{m}g(\cancelto{0}{z_2} - z_1) = 0$$

Solving for V_2

$$V_2 = \sqrt{V_1^2 + 2gz_1}$$

Inserting values

$$V_2 = \sqrt{50^2\,\frac{\text{ft}^2}{\text{s}^2} + 2\left(31.5\,\frac{\text{ft}}{\text{s}^2}\right)(600\text{ ft})} = 200.7\text{ ft/s}$$

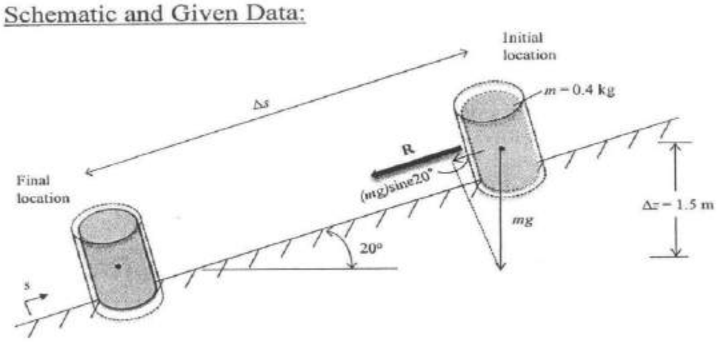
1. Note that the mass cancels out. Any object falling freely under the influence of gravity, with no effects of air resistance, would reach the same final velocity.

PROBLEM 2.15

Known: Can moves down a surface that is inclined relative to the horizontal. The can is acted upon by a constant force parallel to the incline and by the force of gravity.

Find: Can's change in kinetic energy, in J, and whether it is *increasing* or *decreasing*. If friction between the can and the inclined surface were significant, what effect would that have on the value of the change in kinetic energy?

Schematic and Given Data:



Engineering Model:
(1) The can is a closed system.
(2) The acceleration of gravity is constant.
(3) The applied force **R** is constant.
(4) Ignore friction between the can and inclined surface.

Analysis:

Begin with Eq. 2.6

$$\int_{s_1}^{s_2} \underline{F} \cdot d\underline{s} = \frac{1}{2} m\left(V_2^2 - V_1^2\right) = \Delta KE \qquad (1)$$

From the free body diagram in the schematic, the dot product can be expressed as

$$\underline{F} \cdot d\underline{s} = \left(\mathbf{R} + (mg)\text{sine}20^\circ\right)ds$$

Substituting into Eq. (1)

$$\int_{s_1}^{s_2} \underline{F} \cdot d\underline{s} = \int_{s_1}^{s_2} \left(\mathbf{R} + (mg)\text{sine}20^\circ\right)ds = \Delta KE \qquad (2)$$

Since $\Delta z = \Delta s$ sine 20°, Eq. (2) becomes

$$\int_{s_1}^{s_2} (\mathbf{R})ds + (mg)\Delta z = (\mathbf{R})\Delta s + (mg)\Delta z = \Delta KE \qquad (3)$$

Evaluate Δs

$$\Delta s = \frac{\Delta z}{\text{sine}\,20^\circ} = \frac{1.5\text{ m}}{0.342} = 4.39\text{ m}$$

PROBLEM 2.15 (Continued)

Subsituting all known and calculated data into Eq. (3)

$$\Delta KE = (0.05N)(4.39m)\left|\frac{1J}{1N\cdot m}\right| + (0.4kg)\left(9.8\frac{m}{s^2}\right)(1.5m)\left|\frac{1N}{1\frac{kg\cdot m}{s^2}}\right|\left|\frac{1J}{1N\cdot m}\right| =$$

#1

$$\Delta KE = 0.22\ J + 5.88\ J = 6.1\ J$$

Which corresponds to an *increase* in kinetic energy.

If friction were significant, the magnitude of the net force acting in the direction of motion would be less, and thus the kinetic energy change would be less than calculated.

1. Observe that in the absence of the force **R** the can is acted on only by gravity, and the can's change in kinetic energy is 5.88 J.

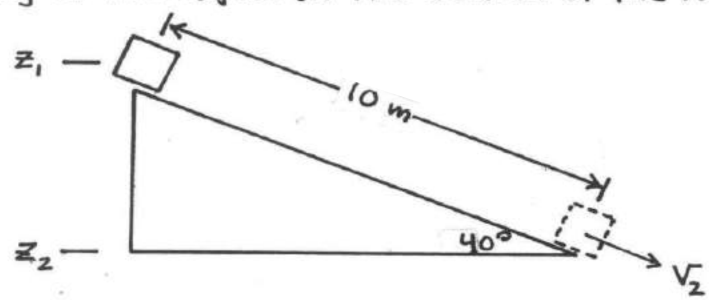
PROBLEM 2.16

KNOWN: Beginning from rest, an object of known mass slides down an inclined plane. The length of the ramp is given.

FIND: Determine the velocity of the object at the bottom of the ramp.

SCHEMATIC & GIVEN DATA:

$m = 200$ kg
$g = 9.81$ m/s²
$V_1 = 0$



ENGR. MODEL: (1) The mass is a closed system. (2) There is no friction between the mass and the ramp, and air resistance is negligible. (3) The acceleration of gravity is constant.

ANALYSIS: By assumption (2), the only force acting on the system is the force of gravity. Thus, Eq. 2.11 applies

① $$\frac{1}{2}\cancel{m}(V_2^2 - \cancel{V_1^2}) + \cancel{m}g(z_2 - z_1) = 0$$

Solving for V_2

$$V_2 = \sqrt{2g(z_1 - z_2)}$$

From trigonometric relationships

$$z_1 - z_2 = (10\text{ m})\sin 40°$$

Thus

$$V_2 = \sqrt{2(9.81\text{ m/s}^2)(10\text{ m})\sin 40°}$$

$$= 11.23\text{ m/s} \longleftarrow V_2$$

---

1. Even though the object travels along an inclined path, the vertical distance appears in this expression.

PROBLEM 2.17

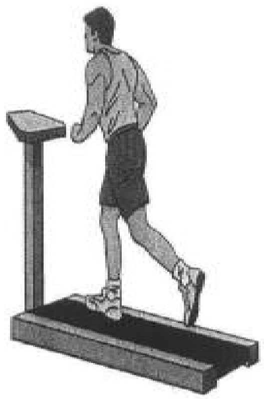


Fig. P2.17

⊙ Exercise value = 620 kcal

⊙ Calorie value, 1 cup of vanilla ice cream = 264 kcal
(Internet)

To break even calorie-wise, Jack may have

$$\frac{620\text{ kcal}}{264\text{ kcal/cup}} = 2.35 \text{ cups}$$

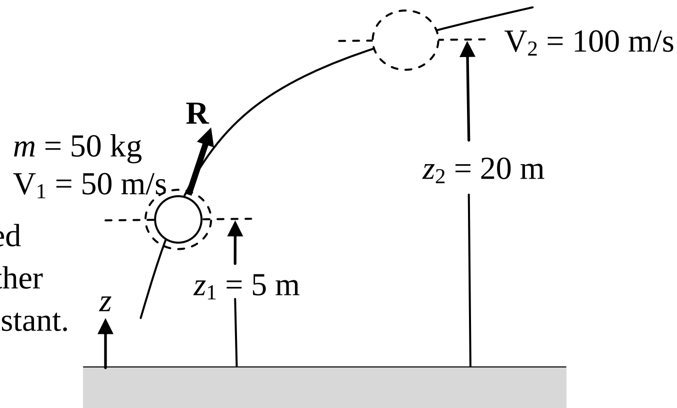
Problem 2.18

An object initially at an elevation of 5 m relative to Earth's surface and with a velocity of 50 m/s is acted on by an applied force **R** and moves along a path. Its final elevation is 20 m and its velocity is 100 m/s. The acceleration of gravity is 9.81 m/s^2. Determine the work done on the object by the applied force, in kJ.

KNOWN: An object moves along a path due to the action of an applied force. The elevation and velocities are known initially and finally.

FIND: Determine the work of the applied force.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The object is a closed system. (2) **R** is the only force acting on the object other than the force of gravity. (3) g = 9.81 m/s^2 and is constant.

ANALYSIS: To find the work of force **R** we use

$$\text{Work} = \int_1^2 \mathbf{R}\cdot d\mathbf{s} = \frac{1}{2}m(V_2^2 - V_1^2) + mg(z_2 - z_1)$$

Inserting values and converting units

$$\text{Work} = \left\{\frac{1}{2}(50\text{ kg})(100^2 - 50^2)\frac{\text{m}^2}{\text{s}^2} + (50\text{ kg})\left(9.81\frac{\text{m}}{\text{s}^2}\right)(20-5)\text{m}\right\}\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1\text{ kJ}}{10^3\text{N}\cdot\text{m}}\right|$$

$$= 187.5 + 7.36 = 194.9\text{ kJ}$$

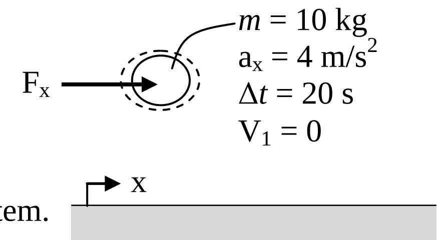
Problem 2.19

An object of mass 10 kg, initially at rest, experiences a constant horizontal acceleration of 4 m/s^2 due to the action of a resultant force applied for 20 s. Determine the total amount of energy transfer by work, in kJ.

KNOWN:
A system of known mass experiences a constant horizontal acceleration due to an applied force for a specified length of time.

FIND: Determine the amount of energy transfer by work.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The object is a closed system. (2) The horizontal acceleration is constant.

ANALYSIS: The work of the resultant force is determined using Eq. 2.6

$$\int_{x_1}^{x_2} F_x dx = \frac{1}{2} m(V_2^2 - \cancelto{0}{V_1^2})$$

To find V_2, use the fact that the acceleration is constant

$$A_x = \frac{dV}{dt} \quad \rightarrow \quad dV = a_x dt \quad \rightarrow \quad \int_{V_1}^{V_2} dV = \int_{t_1}^{t_2} a_x\, dt$$

or

$$(V_2 - \cancelto{0}{V_1}) = a_x(t_2 - t_1) = a_x \Delta t$$

Thus

$$V_2 = (4 \text{ m/s}^2)(20 \text{ s}) = 80 \text{ m/s}$$

Finally, the work of the resultant force is

$$\int_{x_1}^{x_2} F_x dx = \frac{1}{2} m V_2^2$$

$$= \frac{1}{2}(10 \text{ kg})(80^2)\frac{\text{m}^2}{\text{s}^2}\left|\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{N}\cdot\text{m}}\right| = 32 \text{ kJ} \longleftarrow$$

PROBLEM 2.20

KNOWN: Data are provided for an automobile moving at (a) 25 mph, (b) 70 mph.

FIND: For each case, determine the power, in hp, to overcome aerodynamic drag.

SCHEMATIC & GIVEN DATA:

$C_d = 0.28$, $A = 25\ ft^2$

Air
$\rho = 0.075\ lb/ft^3$

ENGINEERING MODEL:

1. The automobile is the system.

ANALYSIS:

The aerodynamic drag force is $F_d = C_d A \frac{1}{2} \rho V^2$. By Eq. 2.13, the required power is $\dot{W}_d = F_d \cdot V$, giving

$$\dot{W}_d = \left[C_d A \tfrac{1}{2} \rho V^2\right] V$$
$$= C_d A \tfrac{1}{2} \rho V^3 \qquad (1)$$

(a) $V = 25$ mph. Substituting into Eq. (1) and applying unit conversion factors,

$$\dot{W}_d = \frac{1}{2}(0.28)(25\,ft^2)\left(0.075\frac{lb}{ft^3}\right)\left[25\frac{mi}{h}\left|\frac{1\,h}{3600\,s}\right|\left|\frac{5280\,ft}{1\,mi}\right|\right]^3 \left|\frac{1\,lbf}{32.2\,lb\cdot ft/s^2}\right|\left|\frac{1\,hp}{550\,\frac{ft\cdot lbf}{s}}\right|$$

$$= 0.73\,hp \longleftarrow$$

(b) $V = 70$ mph. Substituting into Eq. (1) and applying the same unit conversions as in (a),

$$\dot{W}_d = \frac{1}{2}(0.28)(25)(0.075)\left[(70)\left|\frac{5280}{3600}\right|\right]^3\left|\frac{1}{32.2}\right|\left|\frac{1}{550}\right|$$

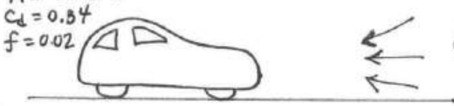
$$= 16.04\,hp \longleftarrow$$

PROBLEM 2.22

KNOWN: The drag force and the force associated with rolling resistance are known as functions of variables associated with a vehicle in motion.

FIND: (a) Determine the power required to overcome drag and rolling resistance when the vehicle is moving at 55 mi/h. (b) Plot the quantities of part (a) and their sum versus vehicle velocity ranging from 0 to 75 mi/h. Discuss the implication for vehicle fuel economy.

$\mathcal{W} = 3550$ lbf
$A = 23.3$ ft^2
$C_d = 0.34$
$f = 0.02$



$\rho = 0.08$ lb/ft^3

ENGR. MODEL: The vehicle is the system.

ANALYSIS: Applying Eq. 2.13, the power required to overcome drag is

$$\dot{W}_d = F_d \cdot V = \left(\tfrac{1}{2} C_d A \rho V^2\right) V = \tfrac{1}{2} C_d A \rho V^3$$

$$= \tfrac{1}{2}(0.34)(23.3\,ft^2)\left(0.08\,\frac{lb}{ft^3}\right)\left[V\left(\frac{mi}{h}\right)\left|\frac{5280\,ft}{mi}\right|\left|\frac{h}{3600\,s}\right|\right]^3\left|\frac{1\,lbf}{32.2\,lb\cdot ft/s^2}\right|\left|\frac{hp}{550\,ft\cdot lbf/s}\right|$$

$$= 5.65\times10^{-5}\,[V(mi/h)]^3\ hp \qquad (*)$$

The power required to overcome rolling resistance is

$$\dot{W}_r = F_r \cdot V = (f\,\mathcal{W})\,V = (0.02)(3550\,lbf)\left[V\left(\frac{mi}{h}\right)\left|\frac{5280\,ft}{mi}\right|\left|\frac{h}{3600\,s}\right|\right]\left|\frac{hp}{550\,ft\cdot lbf/s}\right|$$

$$= 0.189\,V\left(\frac{mi}{h}\right)\ hp \qquad (**)$$

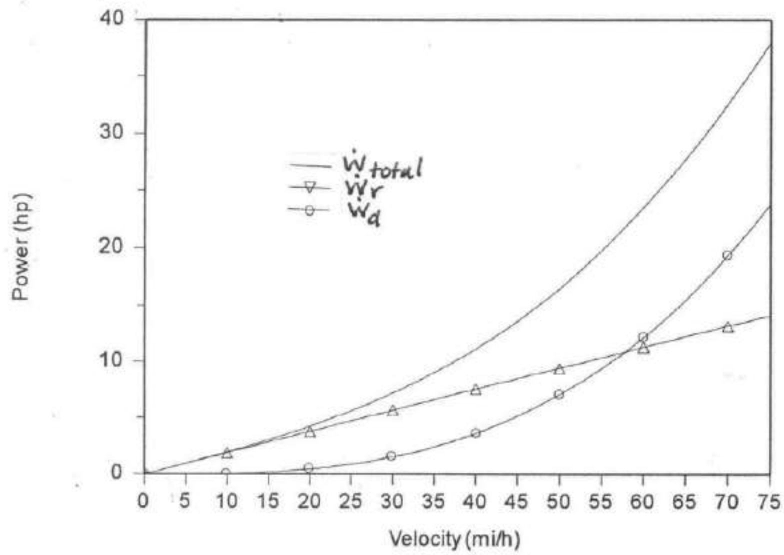
(a) When $V = 55$ mi/h, we have

$$\dot{W}_d = 5.65\times10^{-5}[55]^3 = 9.4\,hp$$
$$\dot{W}_r = 0.189[55] = 10.4\,hp$$

$\longleftarrow \dot{W}_d, \dot{W}_r$

PROBLEM 2.22 (Cont'd)

(b) Letting V range from 0 to 75 mi/h, the accompanying plots can be developed.



We see from the plots that up to about 58 mi/h, the power required to overcome rolling resistance is more significant than the power to overcome drag. At higher speeds, the drag effect becomes dominant because of the V^3 term in the expression for $\dot{W}_d$. Since the power to overcome these effects is developed by the engine from the fuel stored on board the vehicle, high-speed driving has an especially significant effect on fuel consumption.

Problem 2.23

The two major forces opposing the motion of a vehicle are the rolling resistance of the tires, F_r, and the aerodynamic drag force of the air flowing around the vehicle, F_d, given respectively by

$$F_r = f\mathscr{W}, \qquad\qquad F_d = C_d A(1/2)\rho V^2$$

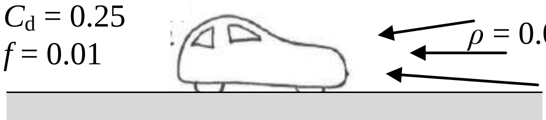
where f and C_d are constants known as rolling resistance and drag coefficient, respectively, $\mathscr{W}$ and A are the vehicle weight and projected frontal area, respectively, V is the vehicle velocity, , and ρ is the air density. For a popular gasoline hybrid car with $\mathscr{W}$ = 3040 lbf, A = 6.24 ft^2 and C_d = 0.25, when f = 0.02 and ρ= 0/08 lb/ft^3.
(a) determine the power required, in hp, to overcome rolling resistance and aerodynamic drag when V is 55 mph.
(b) plot versus vehicle velocity ranging from 0 to 90 mi/h (i) the power to overcome rolling resistance, (ii) the power to overcome aerodynamic drag, and (iii) the total power, all in hp. What implications for vehicle fuel economy can be deduced from the results of part (b)?

KNOWN: The drag force and the force associated with rolling resistance are known as functions of variables associated with a vehicle in motion.

FIND: (a) Determine the power required to overcome drag and rolling resistance when the vehicle is moving at 55 mi/h. (b) Plot the quantities of part (a) and their sum versus vehicle velocity ranging from 0 to 90 mi/h. Discuss implications for fuel economy.

SCHMATIC AND GIVEN DATA:

$\mathscr{W}$ = 3040 lbf
A = 6.24 ft^2
C_d = 0.25
f = 0.01



ρ = 0.08 lb/ft^3

ENGINEERING MODEL: The vehicle is the closed system.

ANALYSIS: Applying Eq. 2.13, the power, in hp, required to overcome aerodynamic drag is

$$\dot{W}_d = \mathbf{F}_d \cdot \mathbf{V} = \left(\frac{1}{2}C_d A\rho V^2\right)V = \frac{1}{2}C_d A\rho V^3$$

$$= \frac{1}{2}(0.25)(6.24\text{ft}^2)\left(0.08\ \frac{\text{lb}}{\text{ft}^3}\right)\left[V\frac{\text{mi}}{\text{h}}\left|\frac{5280\text{ ft}}{1\text{ mi}}\right|\left|\frac{1\text{ h}}{3600\text{ s}}\right|\right]^3\left|\frac{1\text{ lbf}}{32.2\text{ lb}\cdot\text{ft/s}^2}\right|\left|\frac{1\text{ hp}}{550\text{ ft}\cdot\text{lbf/s}}\right| \qquad (*)$$

$= 1.11 \times 10^{-5}$ [V^3] (where V is in mi/h)

The power, in hp, required to overcome rolling resistance is

Problem 2.23 (Continued) – Page 2

$$\dot{W}_r = \mathbf{F_r} \cdot \mathbf{V} = (f\,\mathscr{W})\mathrm{V}$$

$$= (0.01)(3040\text{ lbf})\left[\mathrm{V}\,\frac{\text{mi}}{\text{h}}\left|\frac{5280\text{ ft}}{1\text{ mi}}\right|\left|\frac{1\text{ h}}{3600\text{ s}}\right|\right]\left|\frac{1\text{ hp}}{550\text{ ft·lbf/s}}\right| = 0.0811\text{ V}\left(\frac{\text{mi}}{\text{h}}\right) \qquad (**)$$

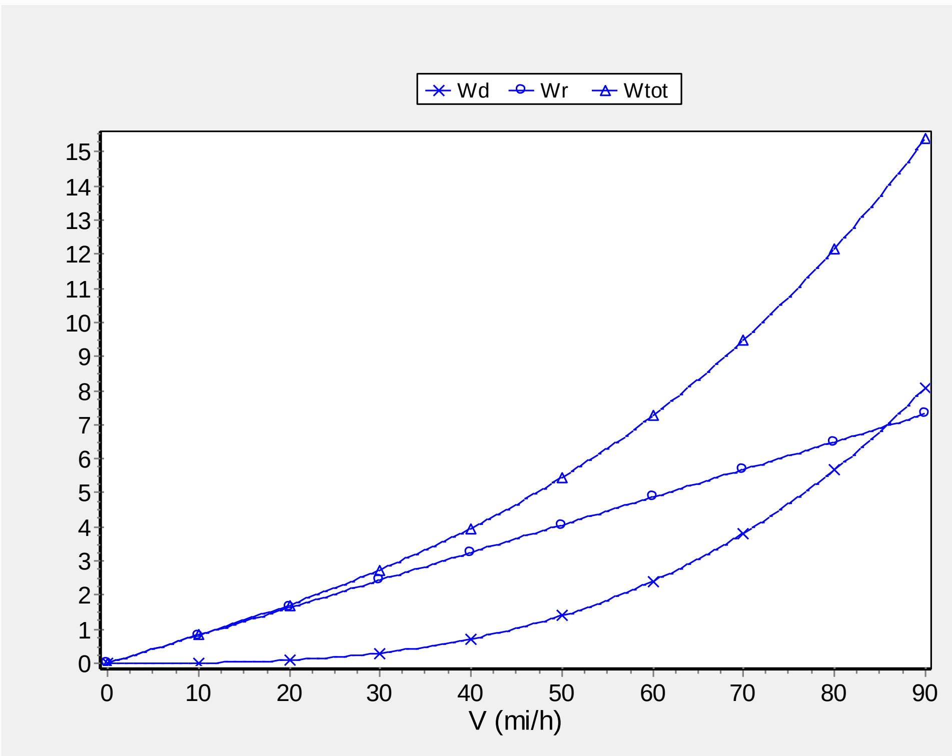
(a) When V = 55 mi/h, we get

$\dot{W}_d$= 1.847 hp

$\dot{W}_r$ = 4.461 hp

$\dot{W}_{total}$ = 6.308 hp

(b) The plots are developed by letting V vary from 0 to 90 mi/h:



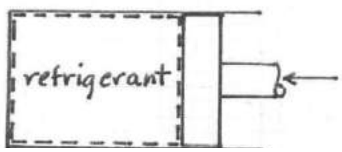
We see from the plots that up to about 87 mi/h, the power required to overcome rolling resistance exceeds the power to overcome aerodynamic drag. However, the total power required increases dramatically with velocity. The aerodynamic drag varies as the cube of velocity, so it increases rapidly and contributes much more significantly as speed increases. The total power required by the engine increases about 5-fold from 30 mi/h to 90mi/h. Since the power is developed by the engine from fuel stored on board the vehicle, high-speed driving has a significant negative effect on fuel consumption.

PROBLEM 2.24

KNOWN: Measured data for pressure versus volume during the compression of a refrigerant are given.

FIND: (a) Determine n for a fit of the data by pV^n = constant. (b) Use the result of part (a) to evaluate the work done during the compression. (c) Evaluate the work using graphical or numerical integration of the data, (d) compare and discuss parts (b) and (c).

SCHEMATIC & GIVEN DATA:



Data Point	p (lbf/in.^{2})	V (in.^{3})
1	112	13.0
2	131	11.0
3	157	9.0
4	197	7.0
5	270	5.0
6	424	3.0

ENGR. MODEL: 1. The refrigerant in the piston-cylinder assembly form a closed system. 2. The pressure values provided approximate the pressure at the piston face.

ANALYSIS:

(a) One approach to find n is to begin with pV^n= constant. Taking the log of both sides of this equation

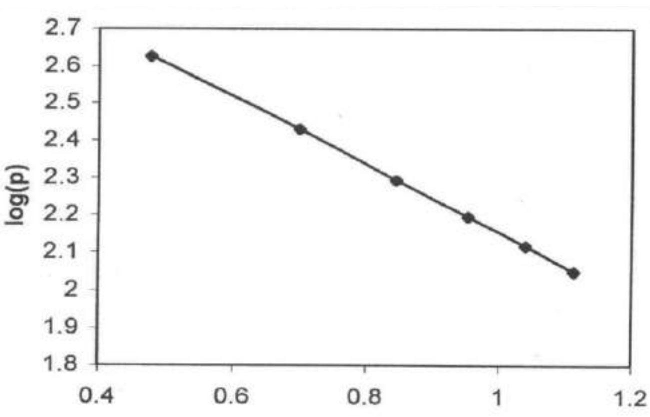
$$\log p + n \log V = \log c$$

or

$$\log p = (-n) \log V + \log c$$

Thus, $(-n)$ corresponds to the slope of a plot of log p vs. log V. Using

D a spreadsheet program to obtain the plot and the least squares best fit curve:



From the curve fit

$$(-n) = -0.90887$$

or

$$n = 0.90887 \leftarrow n$$

Thus

$$pV^{0.90887} = \text{constant}$$

PROBLEM 2.24 (Cont'd)

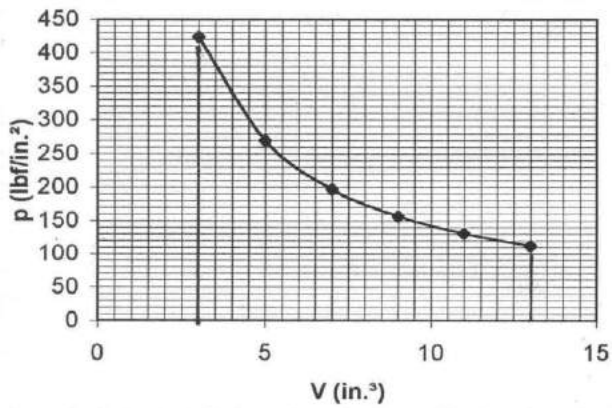
(b) Using the results of part (a) and the procedure of Example 2.1, the work is

$$W = \int_{V_1}^{V_2} p\,dV = \frac{p_2 V_2 - p_1 V_1}{(1-n)}$$

$$= \frac{(424\ \text{lbf/in}^2)(3.0\ \text{in}^3) - (112)(13.0)}{(1 - 0.90887)} \left|\frac{1\ \text{ft}}{12\ \text{in}}\right| \left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

$$= -0.2163\ \text{Btu} \quad W$$

② (c) A graphical evaluation of the work involves a plot of the tabulated data and a smooth curve drawn through the data points:



Each elemental rectangle in the plot contributes the following to the area under the curve:

$$\left(10\ \frac{\text{lbf}}{\text{in}^2}\right)(0.5\ \text{in}^3)\left|\frac{1\ \text{ft}}{12\ \text{in}}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right| = 5.356 \times 10^{-4}\ \text{Btu}$$

The number of retangles is approximately 401.1, thus

$$W \approx (401.1)(5.356 \times 10^{-4}) = -0.2148\ \text{Btu} \quad W$$

(d) The results obtained in parts (b) and (c) are in good agreement. Each should be considered a plausible estimate for the reasons presented in Sec. 2.2.4 in the discussion of actual expansion and compression processes.

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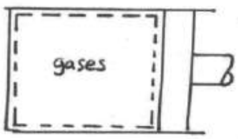
1. The software IT could be used to obtain the least squares curve fit by programming the equations for curve fitting. It is easier to use a spreadsheet program in this instance, however.
2. The only measured data are the tabulated data points, shown as filled circles. The smooth curve does not necessarily represent the actual pressure at the piston face for the corresponding volume.

PROBLEM 2.25

KNOWN: Measured pressure-volume data for an expansion of gases within the cylinder of an internal combustion engine are given.

FIND: (a) Determine n for a fit of the data by pV^n = constant. (b) Use the result of part (a) to evaluate the work done in the expansion. (c) Evaluate the work done using graphical or numerical integration of the data. (d) Compare and discuss parts (c), (d).

SCHEMATIC & GIVEN DATA:



Data Point	p (bar)	V (cm³)
1	15	300
2	12	361
3	9	459
4	6	644
5	4	903
6	2	1608

ENGR. MODEL: 1. As shown in the schematic, the gases within the piston-cylinder form the closed system. 2. The pressure values provided approximate the pressure at the piston face.

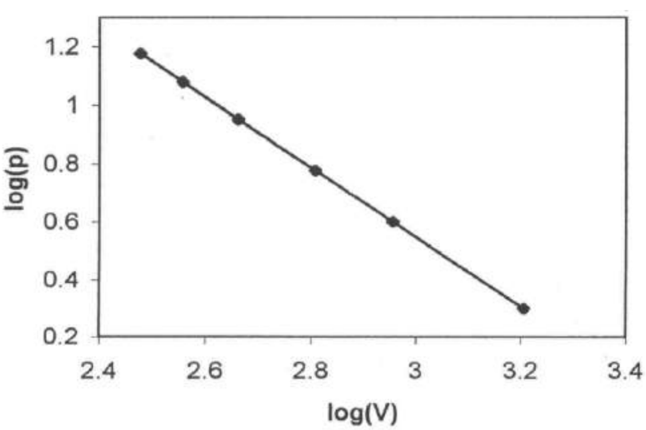
ANALYSIS: (a) One approach to find n is to begin with pV^n = constant. Taking the log of both sides of this equation

$$\log p + n \log V = \log c$$

or

$$\log p = (-n) \log V + \log c$$

Thus, (−n) corresponds to the slope of a plot of log p vs. log V. Using
① a spreadsheet program to obtain the plot and the least squares best fit curve:



From the curve fit

$(-n) = -1.1996$

or

$n = 1.1996$ ← n

Thus

$pV^{1.1996}$ = constant

PROBLEM 2.25 (Cont'd)

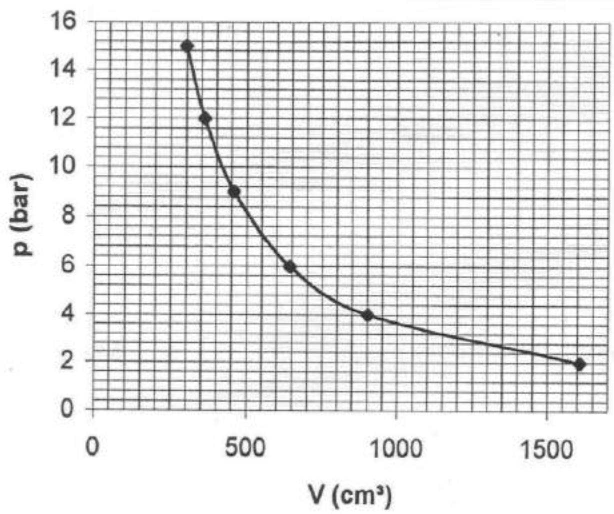
(b) Using the results of part (a) and the procedure of Example 2.1, the work is

$$W = \int_{V_1}^{V_2} p\,dV = \frac{p_2V_2 - p_1V_1}{(1-n)}$$

$$= \frac{(2\text{ bar})(1608\text{ cm}^3) - (15)(300)}{(1-1.1996)}\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ m}^3}{10^6\text{ cm}^3}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

$$= 0.643\text{ kJ} \qquad W$$

② (c) A graphical evaluation of the work involves a plot of the tabulated data and a smooth curve drawn through the data points:



Each elemental rectangle in the plot contributes the following to the area under the curve:

$$(.4\text{ bar})(50\text{ cm}^3)\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ m}^3}{10^6\text{ cm}^3}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| = 0.002\text{ kJ}$$

The number of rectangles is approximately 324, thus

$$W \approx (324)(0.002) = 0.648\text{ kJ} \qquad W$$

(d) The results obtained in parts (b) and (c) are in good agreement. Each should be considered a plausible estimate for the reasons presented in Sec. 2.2.4 in the discussion of actual expansion and compression processes.

1. The software IT could be used to obtain the least squares curve fit by programming the equations for curve fitting. It is easier to use a spreadsheet program in this instance, however.

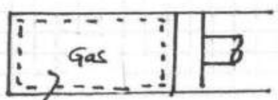
2. The only measured data are the tabulated data points, shown as filled circles. The smooth curve does not necessarily represent the actual pressure at the piston face for the corresponding volume.

PROBLEM 2.26

KNOWN: A gas in a piston-cylinder assembly undergoes a process during which pV^2 = constant. State data are provided.

FIND: Determine the final volume occupied by the gas, in m^3, and the work for the process, in kJ.

SCHEMATIC & GIVEN DATA:

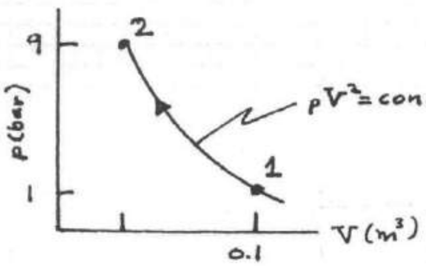


pV^2 = constant

$p_1 = 1$ bar

$V_1 = 0.1\ m^3$

$p_2 = 9$ bar



ENGINEERING MODEL:

1. The gas within the piston-cylinder is the closed system.
2. Volume change is the only work mode.
3. The process of the gas obeys pV^2 = constant.

ANALYSIS:

(a) We have pV^2 = constant. Thus, $p_1V_1^2$ = constant and $p_2V_2^2$ = constant.

$$\Rightarrow \quad p_2V_2^2 = p_1V_1^2 \quad \Rightarrow \quad V_2 = \left[\frac{p_1}{p_2}\right]^{1/2} V_1 = \underbrace{\left[\frac{0.1}{0.9}\right]^{1/2}}_{1/3} (0.1\ m^3) = 0.033\ m^3 \leftarrow$$

(b) Calling on Eq. 2.17,

$$W = \int_1^2 p\,dV = \frac{p_2V_2 - p_1V_1}{1-n} \quad \text{(See Example 2.1(a) for the integration.)}$$

from (a)

$$\therefore \quad W = \frac{p_2[V_1/3] - p_1V_1}{1-n} = \frac{V_1[p_2/3 - p_1]}{(-1)} = \frac{0.1\,m^3[3-1]\times 10^5\,N/m^2}{(-1)} \left| \frac{1\ kJ}{10^3\ N\cdot m} \right|$$

2

$$= -20\ kJ$$

Energy is transferred to the air by work in the compression process.

Problem 2.27

Carbon dioxide (CO_2) gas within a piston-cylinder assembly undergoes a process from a state where p_1 = 5 lbf/in.^{2}, V_1 = 2.5 ft^3 to a state where p_2 = 20 lbf/in.^{2}, V_2 = 0.5 ft^3. The relationship between pressure and volume during the process is given by $p = 23.75 – 7.5V$, where V is in ft^3 and p is in lbf/in.^{2} Determine the work for the process, in Btu.

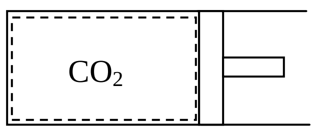
KNOWN: CO_2 gas within a piston-cylinder assembly undergoes a process where the *p*-*V* relation is given. The initial and final states are specified.

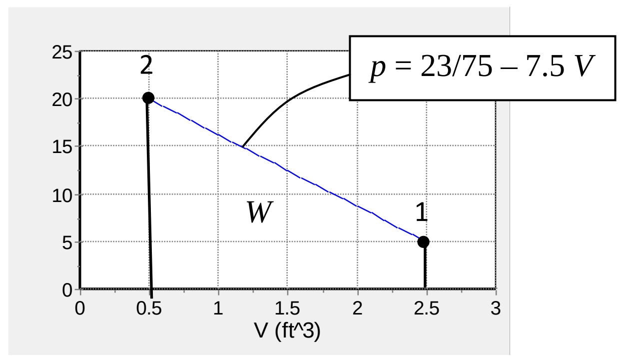
FIND: Determine the work for the process.

SCHEMATIC AND GIVEN DATA:

p_1 = 5 lbf/in.^{2}
V_1 = 2.5 ft^3

p_2 = 20 lbf/in.^{2}
V_2 = 0.5 ft^3





ENGINEERING MODEL: (1) The CO_2 is the closed system. (2) The *p*-*V* relation during the process is linear. (3) Volume change is the only work mode.

ANALYSIS: The given *p*-*V* relation can be used with Eq. 2.17 as follows:

$$W = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} [23.75 - 7.5V] dV = \left[23.75V - \frac{7.5V^2}{2}\right]_{V_1}^{V_2}$$

$$= 23.75[V_2 - V_1] - \frac{7.5}{2}[V_2^2 - V_1^2]$$

$$W = \left(23.75 \frac{\text{lbf}}{\text{in.}^2}\right)\left|\frac{144 \text{ in.}^2}{1 \text{ ft}^2}\right|[0.5 - 2.5]\text{ft}^3 - \left(\frac{7.5}{2}\frac{\text{lbf/in.}^2}{\text{ft}^3}\right)\left|\frac{144 \text{ in.}^2}{1 \text{ ft}^2}\right|[0.5^2 - 2.5^2](\text{ft}^3)^2$$

= -3600 ft·lbf

$= (-3600 \text{ ft} \cdot \text{lbf})\left|\frac{1 \text{ Btu}}{778 \text{ ft·lbf}}\right| = -4.63$ Btu (negative sign denotes energy transfer *in*.)

Alternative Solution

Since the *p*-*V* relation is linear, *W* can also be evaluated geometrically as the area under the process line:

$$W = p_{\text{ave}}(V_2 - V_1) = \left(\frac{p_1 + p_2}{2}\right)(V_2 - V_1) = \left(\frac{20 + 5}{2}\right)\frac{\text{lbf}}{\text{in}^2}\left|\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right|(0.5 - 2.5)\text{ft}^3\left|\frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}}\right|$$

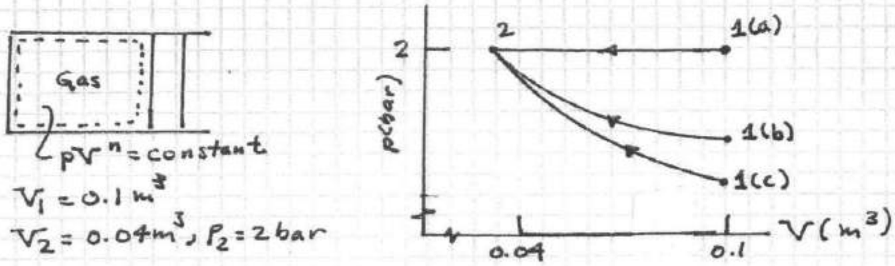
= -4.63 Btu

PROBLEM 2.28

KNOWN: A gas in a piston-cylinder assembly undergoes a compression process for which pV^n = constant. State data is provided.

FIND: For each of n = 0, n = 1, and n = 1.3, determine the initial pressure, in bar, and the work, in kJ.

SCHEMATIC & GIVEN DATA:



$V_1 = 0.1\ m^3$

$V_2 = 0.04\ m^3$, $p_2 = 2$ bar

ENGINEERING MODEL:

1. The gas within the piston-cylinder is the closed system.
2. Volume change is the only work mode.
3. The process of the gas obeys pV^n = constant, where (a) n = 0, (b) n = 1, (c) n = 1.3.

ANALYSIS:

(a) n = 0: Thus, pV^0 = constant ⇒ p = constant. So, p_1 = 2 bar

Using Eq. 2.17 with p = constant,

(1) $$W = \int_1^2 p\,dV = p[V_2 - V_1] = 2\text{bar}[0.04 - 0.1]\,m^3 \left|\frac{10^5 N/m^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3 N\cdot m}\right| = -12\text{ kJ}$$

(b) n = 1: Thus, pV = constant ⇒ $p_1V_1 = p_2V_2$ ⇒ $p_1 = p_2\left[\frac{V_2}{V_1}\right] = 2\text{bar}\left[\frac{0.04}{0.10}\right] = 0.8$ bar

$$W = \int_1^2 p\,dV = \int_1^2 \frac{c}{V}dV = \underbrace{c}_{= p_1V_1 = p_2V_2} \ln\frac{V_2}{V_1} = p_2V_2 \ln\frac{V_2}{V_1} = \left(2\times10^5 \frac{N}{m^2}\right)(0.04\,m^3)\left|\frac{1\text{kJ}}{10^3 N\cdot m}\right| \ln\frac{0.04}{0.10}$$

$$= -7.33\text{ kJ}$$

(c) n = 1.3: Thus, $p_1V_1^n = p_2V_2^n$, where n = 1.3. ⇒ $p_1 = p_2\left[\frac{V_2}{V_1}\right]^{1.3} = 2\text{bar}\left[\frac{0.04}{0.10}\right]^{1.3}$

$$= 0.608\text{ bar}$$

$$W = \int_1^2 p\,dV = \int_1^2 \frac{c}{V^n}dV = \frac{p_2V_2 - p_1V_1}{(1-n)}$$. See Example 2.1(a) for the integration.

$$\therefore W = \frac{(2\times10^5 N/m^2)(0.04m^3) - (0.608\times10^5 N/m^2)(0.1\,m^3)}{(1-1.3)}\left|\frac{1\text{kJ}}{10^3 N\,m}\right|$$

$$= -6.4\text{ kJ}$$

1. The negative sign for W denotes work done on the gas during compression.

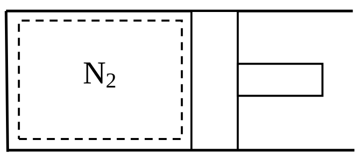
Problem 2.29

Nitrogen (N_2) gas within a piston-cylinder assembly undergoes a process from p_1 = 20 bar, V_1 = 0.5 m^3 to a state where V_2 = 2.75 m^3. The relationship between pressure and volume during the process is $pV^{1.35}$ = *constant*. For the N_2, determine (a) the pressure at state 2, in bar, and (b) the work, in kJ.

KNOWN: N_2 gas within a piston-cylinder assembly undergoes a process where the *p*-*V* relation is $pV^{1.35}$ = *constant*. Data are given at the initial and final states.

FIND: Determine the pressure at the final state and the work.

SCHEMATIC AND GIVEN DATA:



$pV^{1.35}$ = *constant*
p_1 = 20 bar, V_1 = 0.5 m^3
V_2 = 2.75 m^3

ENGINEERING MODEL: (1) The N_2 is the closed system. (2) The *p*-*v* relation is specified for the process. (3) Volume change is the only work mode.

ANALYSIS: (a) $p_1V_1^n = p_2V_2^n \quad \rightarrow \quad p_2 = p_1\left(\frac{V_1}{V_2}\right)^n$; n = 1.35. Thus

$$p_2 = (20\text{ bar})\left(\frac{0.5\text{ m}^3}{2.75\text{ m}^3}\right)^{1.35} = 2\text{ bar} \longleftarrow$$

(b) Since volume change is the only work mode, Eq. 2.17 applies. Following the procedure of part (a) of Example 2.1, we have

$$W = \frac{p_2V_2 - p_1V_1}{1-n} = \frac{(2\text{ bar})(2.75\text{ m}^3) - (20)(0.5)}{1-1.35}\left|\frac{10^5\text{N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{N}\cdot\text{m}}\right|$$

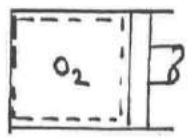
$$= 1285.7\text{ kJ} \longleftarrow$$

PROBLEM 2.30

KNOWN: O_2 gas within a piston-cylinder assembly undergoes an expansion where the p-V relation is $p = AV^{-1} + B$

FIND: Determine the initial and final pressures and the work.

SCHEMATIC & GIVEN DATA:



$p = AV^{-1} + B$

$A = 0.06\ \text{bar}\cdot\text{m}^3$

$B = 3.0\ \text{bar}$

$V_1 = 0.01\ \text{m}^3$, $V_2 = 0.03\ \text{m}^3$

ENGR. MODEL:

1. The O_2 is the closed system
2. The p-V relation during expansion is specified.
3. Volume change is the only work mode.

ANALYSIS:

(a) $p_1 = \left[(0.06\ \text{bar}\cdot\text{m}^3)/0.01\ \text{m}^3\right] + 3.0\ \text{bar}$

$\therefore\ p_1 = 9.0\ \text{bar}$

$p_2 = \left[(0.06\ \text{bar}\cdot\text{m}^3)/(0.03\ \text{m}^3)\right] + 3.0\ \text{bar}$

$\therefore\ p_2 = 5.0\ \text{bar}$ ⟵

(b) Since volume change is the work mode, Eq. 2.17 applies. That is,

$$W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2}\left[\frac{A}{V} + B\right]dV = A\ln\frac{V_2}{V_1} + B(V_2 - V_1)$$

$$= (0.06\ \text{bar}\cdot\text{m}^3)\ln\left(\frac{0.03\ \text{m}^3}{0.01\ \text{m}^3}\right) + (3.0\ \text{bar})[0.03 - 0.01]\ \text{m}^3$$

$$= [0.0659 + 0.06]\ \text{bar}\cdot\text{m}^3 \left|\frac{10^5\ \text{N/m}^2}{1\ \text{bar}}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|$$

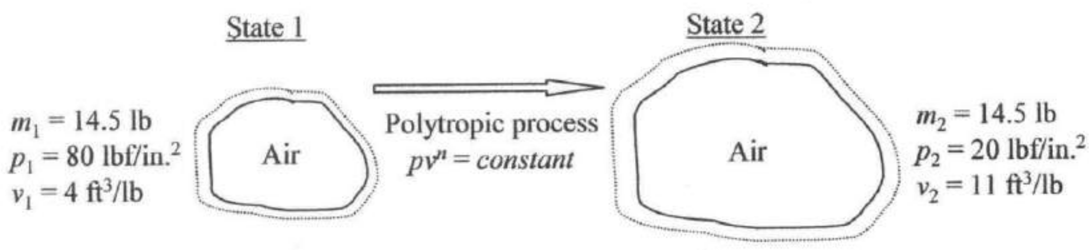
$$= 12.59\ \text{kJ}$$ ⟵

PROBLEM 2.31

KNOWN: Air undergoes a polytropic process between two specified states.

FIND: Determine the work.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The air is a closed system.
2. The system undergoes a polytropic process.

ANALYSIS:
The pressure-volume relationship for a polytropic process is

$$pV^n = constant$$
or
$$pv^n = constant$$

Thus

$$p_1 v_1^n = p_2 v_2^n$$

Solving for n yields

$$n = \frac{\ln\left(\frac{p_1}{p_2}\right)}{\ln\left(\frac{v_2}{v_1}\right)} = \frac{\ln\left(\dfrac{80\,\frac{\text{lbf}}{\text{in}^2}}{20\,\frac{\text{lbf}}{\text{in}^2}}\right)}{\ln\left(\dfrac{11\,\frac{\text{ft}^3}{\text{lb}}}{4\,\frac{\text{ft}^3}{\text{lb}}}\right)}$$

Problem 2.31 (Cont'd)

To solve for work

$$W = \int_{V_1}^{V_2} pdV = m\int_{v_1}^{v_2} pdv = m\int_{v_1}^{v_2} \frac{(constant)dv}{v^n}$$

$$W = m\frac{(constant)v_2^{1-n} - (constant)v_1^{1-n}}{1-n} = m\frac{(p_2v_2^n)v_2^{1-n} - (p_1v_1^n)v_1^{1-n}}{1-n} = m\frac{p_2v_2 - p_1v_1}{1-n}$$

$$W = (14.5\,\text{lb})\frac{\left(20\frac{\text{lbf}}{\text{in}^2}\right)\left(11\frac{\text{ft}^3}{\text{lb}}\right) - \left(80\frac{\text{lbf}}{\text{in}^2}\right)\left(4\frac{\text{ft}^3}{\text{lb}}\right)}{1-1.37}\left|\frac{144\,\text{in}^2}{1\,\text{ft}^2}\right|\left|\frac{1\,\text{Btu}}{778\,\text{ft}\cdot\text{lbf}}\right| = \underline{\mathbf{725.4\ Btu}}$$

The positive sign for work denotes energy transfer out of the system.

Problem 2.32

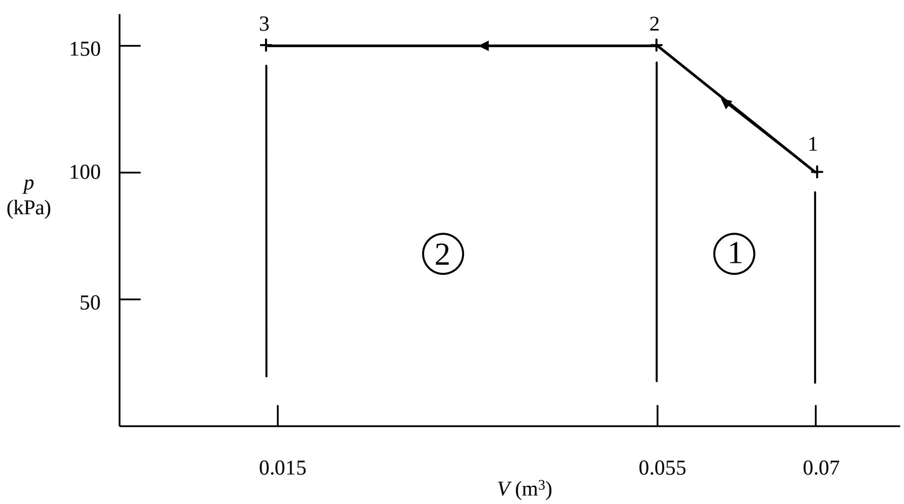
Air contained within a piston-cylinder assembly is slowly compressed. As shown in Fig P2.32, during this first process the pressure first varies linearly with volume and then remains constant. Determine the total work, in kJ.

KNOWN: Air within a piston-cylinder assembly undergoes two processes in series.

FIND: Determine the total work.

SCEMATIC AND GIVEN DATA:

ENGINEERING MODEL: (1) The air within the piston-cylinder assembly is the closed system. (2) The two-step *p*-*V* relation is specified graphically. (3) Volume change is the only work mode.



ANALYSIS: Since volume change is the work mode, Eq. 2.17 applies. Furthermore, the integral can be evaluated geometrically in terms of the total area under process lines:

$$W = \int_{V_1}^{V_2} p\,dV = \underbrace{p_{\text{ave}}(V_2 - V_1)}_{1} + \underbrace{p_2(V_3 - V_2)}_{2} = \left(\frac{p_1 + p_2}{2}\right)(V_2 - V_1) + p_2(V_3 - V_2)$$

$$= \left[\left(\frac{100+150}{2}\right)\text{kPa}(0.055 - 0.07)\text{m}^3 + (150)(0.015 - 0.055)\right]\left|\frac{10^3\,\text{N/m}^2}{1\text{ kPa}}\right|\left|\frac{1\text{ kJ}}{10^3\,\text{N}\cdot\text{m}}\right|$$

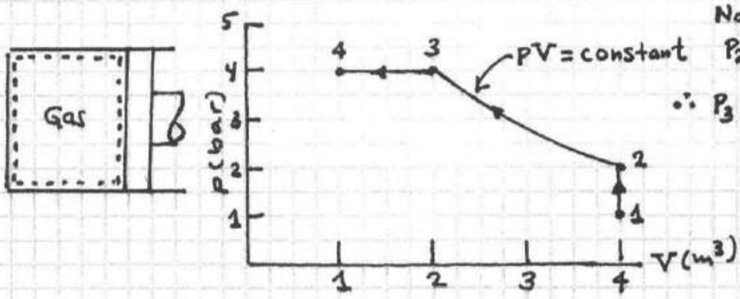
= (-1.875 kJ) + (-6 kJ) = -7.875 kJ (in)

PROBLEM 2.33

KNOWN: A gas contained within a piston-cylinder assembly undergoes three processes in series. State data are provided.

FIND: Sketch the processes in series on p-V coordinates and evaluate the work for each process, in kJ.

SCHEMATIC & GIVEN DATA:



Note: $P_2V_2 = P_3V_3$

$\therefore P_3 = P_2\left[\frac{V_2}{V_3}\right]$

$= 2\text{bar}\left[\frac{4}{2}\right]$

$= 4\text{bar}$

ENGINEERING MODEL:

1. The gas within the piston-cylinder is the closed system.
2. The gas experiences three processes, in series, as shown in the sketch.

ANALYSIS: The work is given by Eq. 2.17: $W = \int p\,dV$

Process 1-2: V is constant. Thus, the piston does not move, and $W_{12} = 0$.

Process 2-3: $W_{23} = \int_2^3 \frac{C}{V}\,dV = C \ln\frac{V_3}{V_2} = P_2V_2 \ln\frac{V_3}{V_2}$

$$= \left(2\times10^5\frac{N}{m^2}\right)(4\,m^3)\left|\frac{1\,kJ}{10^3\,N\cdot m}\right| \ln\left[\frac{2}{4}\right] = -554.5\,kJ$$

Process 3-4: $W_{34} = p[V_4 - V_3]$

$$= \left(4\times10^5\frac{N}{m^2}\right)(1-2)\,m^3\left|\frac{1\,kJ}{10^3\,N\cdot m}\right| = -400\,kJ$$

Note: Minus signs signify energy transfer by work to the gas.

Problem 2.34

Carbon monoxide gas (CO) contained within a piston-cylinder assembly undergoes three processes in series:

Process 1-2: Constant pressure expansion at 5 bar from V_1 = 0.2 m^3 to V_2 = 1 m^3.

Process 2-3: Constant volume cooling from state 2 to state 3 where p_3 = 1 bar.

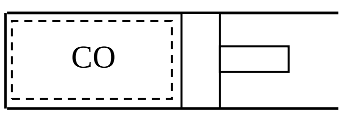
Process 3-1: Compression from state 3 to the initial state during which the pressure-volume relationship is pV = *constant*.

Sketch the processes in series on *p-V* coordinates and evaluate the work for each process, in kJ.

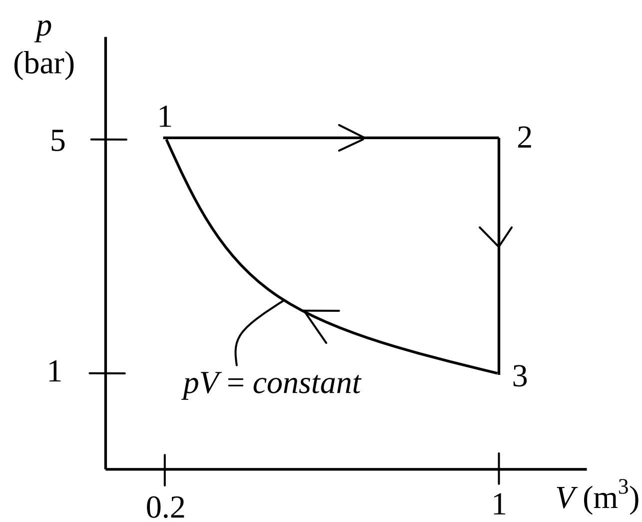
KNOWN: Carbon monoxide gas within a piston-cylinder assembly undergoes three processes in series.

FIND: Sketch the processes in series on a *p-V* diagram and evaluate the work for each process.

SCHEMATIC AND GIVEN DATA:



Process 1-2: Constant pressure expansion at 5 bar from V_1 = 0.2 m^3 to V_2 = 1 m^3.
Process 2-3: Constant volume cooling from state 2 to state 3 where p_3 = 1 bar.
Process 3-1: Compression from state 3 to the initial state during which the pressure-volume relationship is pV = *constant*.



ENGINEERING MODEL: (1) The gas is the closed system. (2) Volume change is the only work mode. (3) Each of the three processes is specified.

ANALYSIS: Since volume change is the only work mode, Eq. 2.17 applies.

Process 1-2: Constant pressure processes: $W_{12} = \int_{V_1}^{V_2} p dV = p_1(V_2 - V_1)$

$$W_{12} = (5\text{ bar})(1 - 0.2)\text{m}^3 \left|\frac{10^5\text{N/m}^2}{1\text{ bar}}\right| \left|\frac{1\text{ kJ}}{10^3\text{N}\cdot\text{m}}\right| = 400\text{ kJ (out)} \longleftarrow$$

Process 2-3: Constant volume (piston does not move). Thus $W_{23} = 0$ $\longleftarrow$

Problem 2.33 (Continued)

<u>Process 3-1</u>: For process 3-1, $pV = constant = p_1V_1$. Noting that $V_3 = V_2$, we get

$$W_{31} = \int_{V_3}^{V_1} pdV = \int_{V_3}^{V_1} \frac{C}{V} dV = C\ln\left(\frac{V_1}{V_3}\right) = (p_1V_1)\ln\left(\frac{V_1}{V_2}\right)$$

Inserting values and converting units

1 $$W_{31} = (5\text{ bar})(0.2\text{ m}^3)\ln\left(\frac{0.2\text{ m}^3}{1\text{ m}^3}\right)\left|\frac{10^5\text{N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{N·m}}\right| = -160.9\text{ kJ (in)}$$

---

1. The *net* work for the three process is

$W_{net} = W_{12} + W_{23} + W_{31} = (+400) + 0 + (-160.9) = 239.1$ kJ (*net* work is positive - out)

Problem 2.35

Air contained within a piston-cylinder assembly undergoes three processes in series:

Process 1-2: Compression during which the pressure-volume relationship is pV = *constant* from p_1 = 10 lbf/in.^{2}, V_1 = 4 ft^3 to p_2 = 50 lbf/in.^{2}

Process 2-3: Constant volume from state 2 to state 3 where p = 10 lbf/in.^{2}

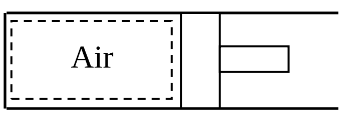
Process 3-1: Constant pressure expansion to the initial state.

Sketch the processes in series on a p-V diagram. Evaluate (a) the volume at state 2, in ft^3, and (b) the work for each process, in Btu.

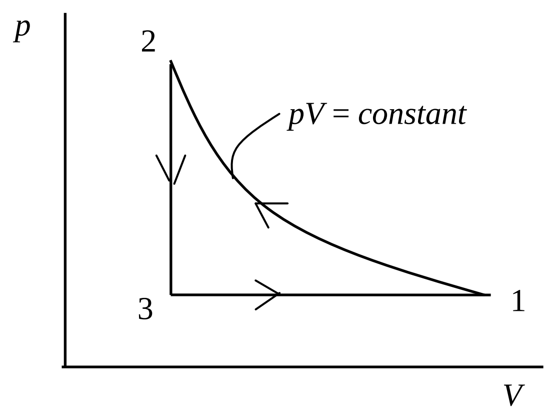
KNOWN: Air within a piston-cylinder assembly undergoes three processes in series.

FIND: Sketch the processes in series on a p-V diagram. Evaluate (a) the volume at state 2, and (b) the work for each process.

SCHEMATIC AND GIVEN DATA:



Process 1-2: Compression during which the pressure-volume relationship is pV = *constant* from p_1 = 10 lbf/in.^{2}, V_1 = 4 ft^3 to p_2 = 50 lbf/in.^{2}
Process 2-3: Constant volume from state 2 to state 3 where p = 10 lbf/in.^{2}
Process 3-1: Constant pressure expansion to the initial state.



ENGINEERING MODEL: (1) The gas is the closed system. (2) Volume change is the only work mode. (3) Each of the three processes is specified.

ANALYSIS: (a) For process 1-2; pV = *constant*. Thus $p_1V_1 = p_2V_2$, and

$$V_2 = \left(\frac{p_1}{p_2}\right)V_1 = \left(\frac{10\text{lbf/in.}^2}{50\text{lbf/in.}^2}\right)(4\text{ ft}^3) = 0.8\text{ ft}^3$$

(b) Since volume change is the only work mode, Eq. 2.17 applies.

<u>Process 1-2</u>: For process 1-2, $pV = constant = p_1V_1$. Thus

$$W_{12} = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} \frac{C}{V} dV = C\ln\left(\frac{V_2}{V_1}\right) = (p_1V_1)\ln\left(\frac{V_2}{V_1}\right)$$

Problem 2.35 (Continued)

Inserting values and converting units

$$W_{12} = \left(10\ \frac{\text{lbf}}{\text{in.}^2}\right)(4\ \text{ft}^3)\ \ln\left(\frac{0.8\ \text{ft}^3}{4\ \text{ft}^3}\right)\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right| = -11.92\ \text{Btu (in)} \longleftarrow$$

Process 2-3: Constant volume (piston does not move). Thus $W_{23} = 0$ ⟵

Process 3-1: Constant pressure processes ($p_3 = p_1$): $W_{31} = \int_{V_3}^{V_1} p dV = p_1(V_1 - V_3)$

Noting that $V_3 = V_2$

$$W_{31} = \left(10\ \frac{\text{lbf}}{\text{in}^2.}\right)(4 - 0.8)\text{ft}^3 \left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right| = 5.92\ \text{Btu (out)} \longleftarrow$$

---

1. The *net* work for the three process is

$W_{net} = W_{12} + W_{23} + W_{31}$ = (-11.92) + 0 + (5.92)) = - 6 kJ (*net* work is negative - in)

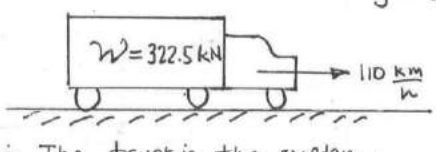
PROBLEM 2.21

KNOWN: The force associated with the rolling resistance of the tires of a truck is known as a function of the truck weight.

FIND: Determine the power required by the truck to overcome rolling resistance.

SCHEMATIC & GIVEN DATA:

$f = 0.0069$



ENGR. MODEL: The truck is the system.

ANALYSIS: Applying Eq. 2.13, the power required to overcome rolling resistance is

$$\dot{W}_r = F_r \cdot V = (f\mathcal{W})V$$

$$= (0.0069)\left(322.5\ kN\left|\frac{10^3 N}{1 kN}\right|\right)\left(110\ \frac{km}{h}\left|\frac{10^3 m}{1 km}\right|\left|\frac{1 h}{3600 s}\right|\right)\left|\frac{1 kJ}{10^3 N\cdot m}\right|\left|\frac{1 kW}{1 kJ/s}\right|$$

$$= 68\ kW \quad \dot{W}_r$$

PROBLEM 2.36

KNOWN: Operating data is provided for a belt sander.

FIND: Evaluate the power transmitted by the belt to the surface and work done in one minute of sanding.

SCHEMATIC & GIVEN DATA:

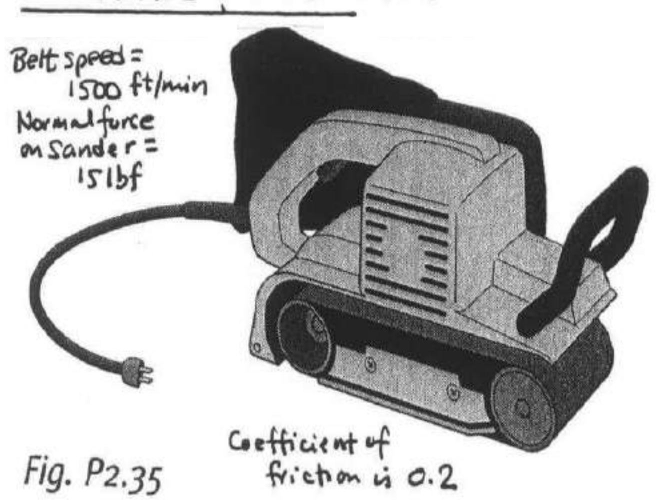


Fig. P2.35

ENGR. MODEL

1. The force exerted by the belt is related to the normal force, F_N, by the coefficient of friction:

$$F = \left(\begin{smallmatrix}\text{coeff. of}\\ \text{friction}\end{smallmatrix}\right) F_N = 0.2\, F_N$$

ANALYSIS: (a) Using Eq. 2.13, the power, $\dot{W}$, transmitted is

$$\dot{W} = F \cdot V = 0.2\, F_N\, V$$

or

$$\dot{W} = 0.2\,(15\ \text{lbf})\left(1500\ \frac{\text{ft}}{\text{min}}\right)\left|\frac{1\ \text{min}}{60\ \text{s}}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

$$= 0.096\ \text{Btu/s} \longleftarrow$$

or

$$\dot{W} = 0.096\ \frac{\text{Btu}}{\text{s}}\left|\frac{3600\ \text{s}}{\text{h}}\right|\left|\frac{1\ \text{hp}}{2545\ \text{Btu/h}}\right|$$

$$= 0.136\ \text{hp} \longleftarrow$$

(b) In one minute of sanding, the work done on the surface is

$$W = \left(0.096\ \frac{\text{Btu}}{\text{s}}\right)\left|\frac{60\ \text{s}}{\text{min}}\right|(1\ \text{min})$$

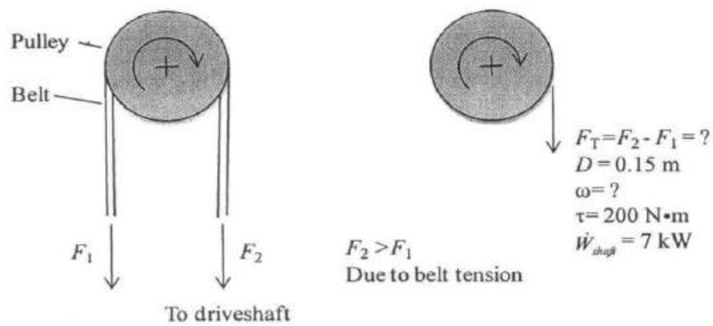
$$= 5.76\ \text{Btu} \longleftarrow$$

PROBLEM 2.37

Known: Pulley turns a belt rotating the driveshaft of a power plant pump with known torque and power transmitted.

Find: Determine the net force applied by the belt on the pulley, in kN, and the rotational speed of the driveshaft, in RPM.

Schematic and Given Data:



Engineering Model:
(1) The rotational speed of the pulley and drive shaft are assumed to be equal.
(2) Net tangential force (F_T) on the pulley is due to belt tension (see schematic).

Analysis:

The net force, in kN, applied by the belt on the pulley is calculated using the torque and the diameter of the pulley as follows

$$\tau = F_T\left(\frac{D}{2}\right) \text{ or } F_T = \frac{2\tau}{D} = \frac{2(200\text{N}\cdot\text{m})}{0.15\text{m}}\left|\frac{1\text{ kN}}{1000\text{N}}\right| = 2.67\text{ kN}$$

Using Eq. 2.20, the rotational speed of the driveshaft, in RPM, is determined using assumption 1, power transmitted, and torque as follows:

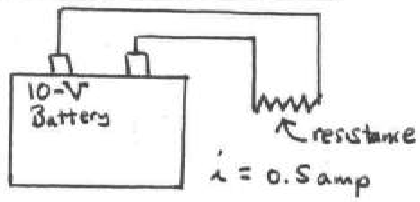
$$\dot{W}_{\text{shaft}} = \tau\omega \quad \text{or} \quad \omega = \frac{\dot{W}_{\text{shaft}}}{\tau} = \frac{7\text{kW}}{200\text{N}\cdot\text{m}}\left|\frac{1000\frac{\text{J}}{\text{s}}}{1\text{kW}}\right|\left|\frac{1\text{N}\cdot\text{m}}{1\text{J}}\right|\left|\frac{60\text{s}}{1\text{min}}\right|\left|\frac{\text{rev}}{2\pi\text{ radians}}\right| = 334.2\text{ RPM}$$

PROBLEM 2.37

KNOWN: Operating data are given for a 10-V battery providing current to a resistance.

FIND: Determine the resistance, in ohms, and the amount of energy transfer by work, in kJ.

SCHEMATIC & GIVEN DATA:



$i = 0.5\text{ amp}$

ANALYSIS:

$$\text{Resistance} = \frac{\text{Voltage}}{\text{Current}} = \frac{10\text{ volts}}{0.5\text{amp}}\left|\frac{1\text{ ohm}}{1\text{ volt/amp}}\right| = 20\text{ ohm} \longleftarrow$$

With Eq. 2.21 applied to the battery, which is discharging,

$$\dot{W} = (\text{voltage})(\text{current}) = (10\text{ volt})(0.5\text{amp})\left|\frac{1\text{ Watt/amp}}{1\text{ volt}}\right| = 5\text{ Watt}$$

Then, for 30 minutes of operation,

$$W = \int \underbrace{\dot{W}}_{\text{constant}} dt = (5\text{ watt})(30\text{min.})\left|\frac{60\text{s}}{1\text{min}}\right|\left|\frac{1\text{ J/s}}{1\text{ Watt}}\right|\left|\frac{1\text{kJ}}{10^3\text{J}}\right| = 9\text{ kJ} \longleftarrow$$

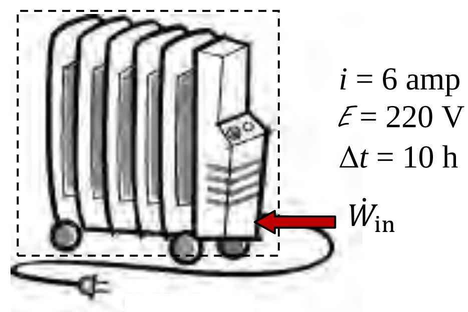
Problem 2.39

An electric heater draws a constant current of 6 amp, with an applied voltage of 220 V, for 24 h. Determine the instantaneous electric power provided to the heater, in kW, and the total amount of energy supplied to the heater by electrical work, in kW·h. If electric power is valued at \$0.08/kW·h, determine the cost of operation for one day.

KNOWN: An electric heater draws a constant current at a specified voltage for a given length of time. The cost of electricity is specified.

FIND: Determine the instantaneous power provided to the heater and the total amount of energy supplied by electrical work. Determine the cost of operation for one day.

SHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: The current and voltage are constant.

ANALYSIS: The constant power *input* to the heater is given by Eq. 2.21

$$\dot{W}_{\text{in}} = \mathcal{E} I = (220\text{ V})(6\text{ amp})\left|\frac{1\text{ W/amp}}{1\text{ V}}\right|\left|\frac{1\text{ kW}}{10^3\text{W}}\right| = 1.320\text{ kW} \longleftarrow$$

Thus, the total energy *input* is

$$W_{\text{in}} = \int_{t_1}^{t_2} \dot{W}_{\text{in}} dt = \dot{W}_{\text{in}}\Delta t = (1.320\text{ kW})(24\text{ h}) = 31.68\text{ kW}\cdot\text{h} \longleftarrow$$

Using the specified cost of electricity

Cost per day = (31.68 kW·h) (\$0.08/kW·h) = \$2.53 ⟵

PROBLEM 2.40

KNOWN: An expression for the power developed by an automobile engine in terms of torque and rotational speed is given.

FIND: For power, in hp, torque, in ft·lbf, and rotational speed, in RPM, evaluate the value and units of the constant appearing in the given expression.

ANALYSIS: The given expression is $\dot{W} = \mathcal{T}\omega / C$. When $\dot{W}$ is in hp, $\mathcal{T}$ is in ft·lbf, and ω is in RPM, by inspection the units of C are $\left[\frac{(\text{ft}\cdot\text{lbf})(\text{rev/min})}{\text{hp}}\right]$ ←

Beginning with $\dot{W} = \mathcal{T}\omega$, Eq. 2.20, and applying unit conversion factors for the product $\mathcal{T}\omega$, we get

$$\underbrace{\dot{W}}_{\text{in hp}} = \mathcal{T}(\text{ft}\cdot\text{lbf})\,\omega\left(\frac{\text{rev}}{\text{min}}\right)\left|\frac{2\pi\,\text{rad}}{1\,\text{rev}}\right|\left|\frac{1\,\text{min}}{60\,\text{s}}\right|\left|\frac{1\,\text{hp}}{550\,\text{ft}\cdot\text{lbf/s}}\right|$$

$$= \mathcal{T}(\text{ft}\cdot\text{lbf})\,\omega\left(\frac{\text{rev}}{\text{min}}\right)\left[\frac{1\,\text{hp}}{5252\,(\text{ft}\cdot\text{lbf})(\text{rev/min})}\right]$$

$$\therefore \quad \underbrace{\dot{W}}_{\text{in hp}} = \frac{\tau(\text{ft}\cdot\text{lbf})\;\omega\left(\frac{\text{rev}}{\text{min}}\right)}{C}$$

where

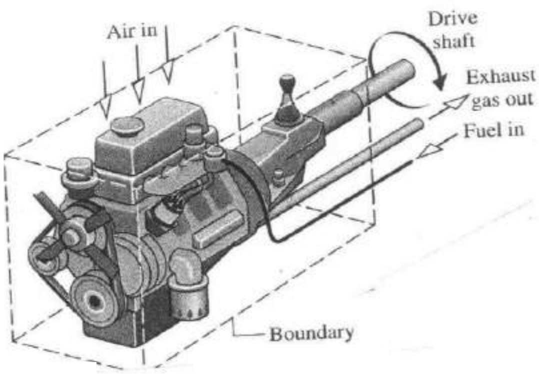
$$C = 5252\,\frac{(\text{ft}\cdot\text{lbf})(\text{rev/min})}{\text{hp}}$$ ←

PROBLEM 2.41

KNOWN: Operating data are provided for a V-6 automobile engine.

FIND: Determine the percentage of the developed power that is transferred to the driveshaft and discuss.

SCHEMATIC & GIVEN DATA:



Driveshaft:
① Rotational speed = 4700 RPM
② Torque = 248 ft·lbf

Engine develops 226 hp

ANALYSIS: Using Eq. 2.20, the power delivered to the drive shaft is

$$\dot{W} = \tau\omega$$
$$= (248\ \text{ft}\cdot\text{lbf})\left(4700\ \frac{\text{rev}}{\text{min}}\right)\left|\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right|\left|\frac{1\ \text{min}}{60\ \text{s}}\right|\left|\frac{1\ \text{hp}}{550\ \text{ft}\cdot\text{lbf/s}}\right|$$
$$= 221.9\ \text{hp}$$

The percentage of the power developed by the engine that is delivered to the driveshaft is

$$\% = \frac{221.9\ \text{hp}}{226\ \text{hp}} = 0.98 \quad (98\%)$$

Frictional and like effects account for the difference.

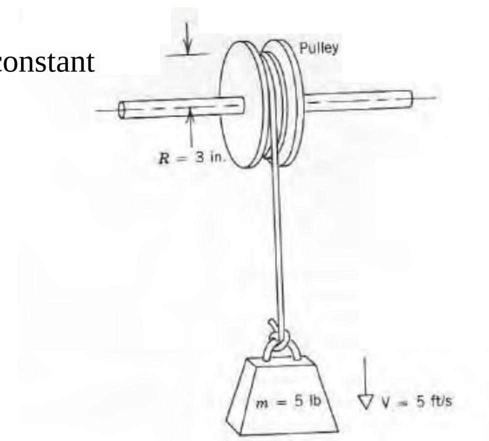
PROBLEM 2.42

Figure P2.42 shows an object whose mass is 5 lb attached to a rope wound around a pulley. The radius of the pulley is 3 in. If the mass falls at a constant velocity of 5 ft/s, determine the power transmitted to the pulley, in hp, and the rotational speed of the shaft, in revolutions per minute (RPM). The acceleration of gravity is 32.2 ft/ s^2.

KNOWN: An object attached to a rope wound around a pulley falls at a constant velocity.

FIND: Find the power transmitted to the pulley and the rotational speed.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) the object falls at a constant speed. (2) The acceleration of gravity is constant.

ANALYSIS: The power is obtained using Eq. 2.13

$$\dot{W} = \mathbf{F} \cdot \mathbf{V} = (mg)\mathrm{V}$$

$$= (5\text{ lb})\left(32.2\ \frac{\text{ft}}{\text{s}^2}\right)\left(5\ \frac{\text{ft}}{\text{s}}\right)\left|\frac{1\text{ lbf}}{32.2\text{ lb}\cdot\text{ft}/s^2}\right|$$

$$= 25\text{ ft}\cdot\text{lb/s}$$

Converting to horsepower

$$\dot{W} = \left(25\text{ ft}\cdot\frac{\text{lbf}}{\text{s}}\right)\left|\frac{1\text{ hp}}{550\text{ ft}\cdot\text{lbf/s}}\right| = 0.0455\text{ hp} \longleftarrow$$

The rotational speed of the pulley is related to the velocity of the object and the radius by V = Rω. Thus

$$\omega = \frac{\mathrm{V}}{\mathrm{R}} = \left(\frac{5\text{ ft/s}}{3/12\text{ ft}}\right)\left|\frac{1\text{ rev}}{2\pi}\right|\left|\frac{60\text{ s}}{1\text{ min}}\right| = 191\text{ rev/min} \longleftarrow$$

PROBLEM 2.43

KNOWN: A wire suspended vertically is stretched by an applied force.

FIND: Obtain an expression for the work done on the wire and evaluate the work for a given set of data.

SCHEMATIC & GIVEN DATA:

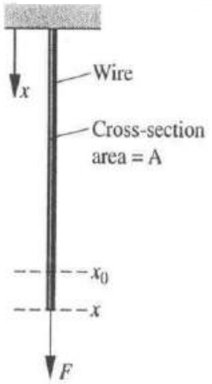


Fig. P2.39

normal stress, strain

$\sigma = C\epsilon$

$\epsilon = \frac{x - x_0}{x_0}$

C: Young's modulus

Data set:

$x_0 = 10$ ft

$x = 10.01$ ft

$C = 2.5 \times 10^7$ lbf/in^2

$A = 0.1$ in^2

ENGR. MODEL:

1. The wire is the closed system.
2. The moving boundary is the only work mode.
3. The change in area A is negligible.
4. The normal stress and thus the applied force varies linearly with strain.

ANALYSIS: (a) The work done on the wire is given by Eq. 2.18

$$W = -\int_{x_0}^{x} \sigma A\, dx$$

From the given stress-strain relation

$$\sigma = C\varepsilon = C\left(\frac{x - x_0}{x_0}\right)$$

Where C is a constant (Young's modulus). From this expression

$$d\varepsilon = \frac{dx}{x_0} \Rightarrow dx = x_0\, d\varepsilon$$

Substituting into the work expression

$$W = -\int_0^{\varepsilon} (C\varepsilon) A (x_0\, d\varepsilon) = -CAx_0 \int_0^{\varepsilon} \varepsilon\, d\varepsilon$$

Finally

$$W = -\frac{CAx_0\varepsilon^2}{2}$$ ← work expression

PROBLEM 2.43 (Continued)

(b) Substituting given data into the work expression

$$W = -\frac{\left(2.5\times10^{7}\,\frac{lbf}{in^2}\right)(0.1\,in^2)(10\,ft)\left[\frac{0.01}{10}\right]^2}{2} = -12.5\ ft\cdot lbf$$

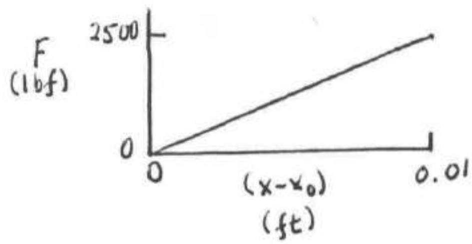
10^{-6}

The downward force varies with strain according to

$$F = \sigma A = C\epsilon A = AC\left[\frac{x - x_0}{x_0}\right]$$

When $x = x_0$, $F = 0$. When $x - x_0 = 0.01\,ft$,

$$F = (0.1\,in^2)\left(2.5\times10^{+7}\,\frac{lbf}{in^2}\right)\left(\frac{0.01\,ft}{10\,ft}\right) = 2500\ lbf$$

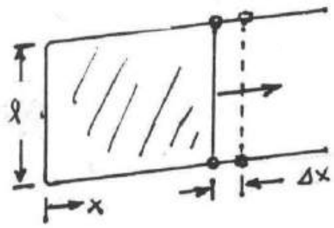


PROBLEM 2.44

KNOWN: A soap film on a wire frame is stretched.

FIND: Determine the work done.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The film is a closed system. (2) The moving boundary is the only work mode. (3) The surface tension is constant, acting on both sides of the film.

ANALYSIS: (a) The work is determined using Eq. 2.19

$$W = -\int_{A_1}^{A_2} \tau \, dA = -\int_{x_1}^{x_2} \tau \, 2\ell \, dx$$

For constant surface tension τ

$$W = -\tau \, 2\ell \, \Delta x$$

(b) If $\ell = 5 \text{ cm}$, $\Delta x = 0.5 \text{ cm}$, $\tau = 25 \times 10^{-5} \text{ N/cm}$,

$$W = -(25 \times 10^{-5} \text{ N/cm})(2)(5 \text{ cm})(0.5 \text{ cm}) \left| \frac{1 \text{ m}}{10^2 \text{ cm}} \right| \left| \frac{1 \text{ J}}{1 \text{ N·m}} \right| = -1.25 \times 10^{-5} \text{ J}$$

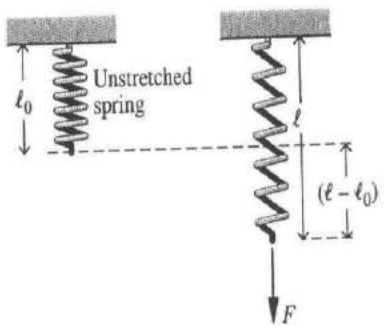
The negative sign denotes work done on the film. Note the small magnitude of the work required to stretch the film.

PROBLEM 2.45

KNOWN: Data is provided for a spring stretched by a force applied at its end.

FIND: Obtain an expression for the work done in streching the spring and evaluate the work using given data.

SCHEMATIC & GIVEN DATA:



$F = k(\ell - \ell_0)$

$\ell_0 = 3$ cm
$\ell_1 = 6$ cm
$\ell_2 = 10$ cm
$k = 10^4$ N/m

Fig. P2.41

ENGR. MODEL:

1. The spring is the closed system.
2. The moving boundary is the only work mode.
3. Hooke's law applies.

ANALYSIS: (a) The work done in stretching the spring is given by

$$W = -\int_1^2 F\,d\ell$$

Letting $x = \ell - \ell_0$, this becomes

$$W = -\int_1^2 kx\,dx = -k\left[\frac{x_2^2 - x_1^2}{2}\right]$$

$$= -\frac{k}{2}\left[(\ell_2 - \ell_0)^2 - (\ell_1 - \ell_0)^2\right]$$

(b) When $(\ell_1 - \ell_0) = 3$ cm and $(\ell_2 - \ell_0) = 7$ cm,

$$W = \frac{(-10^4 \text{ N/m})}{2}\left[(7\text{ cm})^2 - (3\text{ cm})^2\right]\left|\frac{1\text{ m}}{10^2\text{ cm}}\right|^2\left|\frac{1\text{ J}}{1\text{ N}\cdot\text{m}}\right|$$

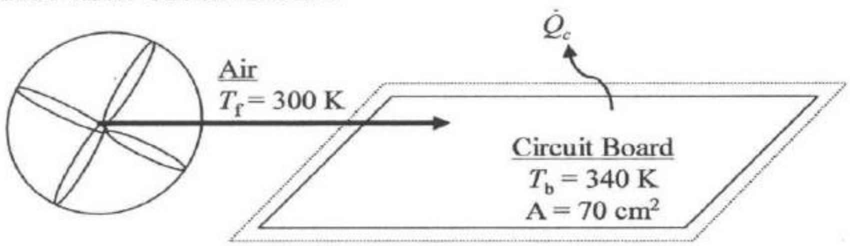
$$= -20 \text{ J}$$

PROBLEM 2.46

KNOWN: A fan forces air to flow over a circuit board to avoid overheating.

FIND: Largest and smallest heat transfer rates associated with this forced convection.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The circuit board is the system.
2. The system is at steady state.

ANALYSIS:
Newton's Law of Cooling is $\dot{Q}_c = hA(T_b - T_f)$, where $\dot{Q}_c$ is the rate of cooling heat transfer, h is the convection heat transfer coefficient, A is area of the surface, T_b is temperature of the surface, and T_f is temperature of the flowing fluid (air).

From Table 2.1 for forced convection using gases, the largest and smallest values for the convection heat transfer coefficient are

(Largest) h = 250 W/(m2·K)
(Smallest) h = 25 W/(m^2·K)

Substituting into Newton's Law of Cooling yields

$$\dot{Q}_c = \left(250 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}\right)(70\ \text{cm}^2)\left|\frac{\text{m}^2}{(100\ \text{cm})^2}\right|(340\ \text{K} - 300\ \text{K}) = \underline{\textbf{70 W}\ \text{(largest heat transfer rate)}}$$

and

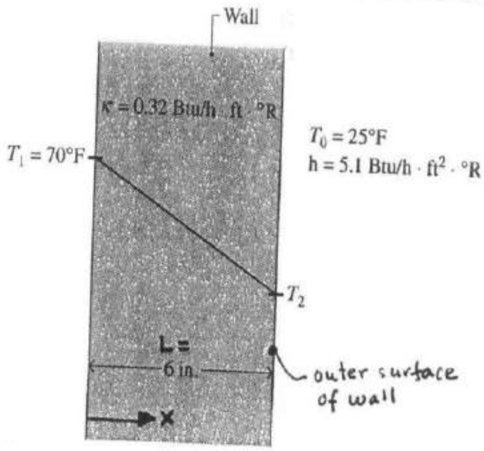
$$\dot{Q}_c = \left(25 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}\right)(70\ \text{cm}^2)\left|\frac{\text{m}^2}{(100\ \text{cm})^2}\right|(340\ \text{K} - 300\ \text{K}) = \underline{\textbf{7 W}\ \text{(smallest heat transfer rate)}}$$

PROBLEM 2.47

KNOWN: Data are provided for an exterior wall of a building at steady state.

FIND: Determine the outer surface temperature and the heat transfer rate.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The wall is at steady state.
2. The temperature varies linearly through the wall.
3. Heat transfer at the outer wall surface is by convection only.

ANALYSIS:

Using Eq. 2.31 together with assumption 2

$$\dot{Q}_x = -\kappa A \frac{dT}{dx}$$

$$= -\kappa A \left[\frac{T_2 - T_1}{L} \right] \qquad (*)$$

At steady state, the rate of heat transfer by conduction to the outer surface of the wall equals the rate of heat transfer by convection from the outer surface, where convection is given by Eq. 2.34: $\dot{Q}_c = hA[T_2 - T_0]$. Thus, at $x = L$

$$\dot{Q}_x = \dot{Q}_c$$

$$-\kappa A \left[\frac{T_2 - T_1}{L} \right] = hA[T_2 - T_0]$$

Solving

$$T_2 = \frac{hLT_0 + \kappa T_1}{hL + \kappa} = \frac{\left(5.1 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{R}}\right)(0.5\,\text{ft})(485^\circ\text{R}) + \left(0.32 \frac{\text{Btu}}{\text{h} \cdot \text{ft} \cdot {}^\circ\text{R}}\right)(530^\circ\text{R})}{\left(5.1 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{R}}\right)(0.5\,\text{ft}) + \left(0.32 \frac{\text{Btu}}{\text{h} \cdot \text{ft} \cdot {}^\circ\text{R}}\right)}$$

$$= 490^\circ\text{R} \; (30^\circ\text{F}) \longleftarrow$$

Then, using Eq. (*) we get

$$\frac{\dot{Q}_x}{A} = -\kappa \left[\frac{T_2 - T_1}{L} \right]$$

$$= -0.32 \frac{\text{Btu}}{\text{h} \cdot \text{ft} \cdot {}^\circ\text{R}} \left[\frac{490^\circ\text{R} - 530^\circ\text{R}}{0.5\,\text{ft}} \right] = 25.6 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2} \longleftarrow$$

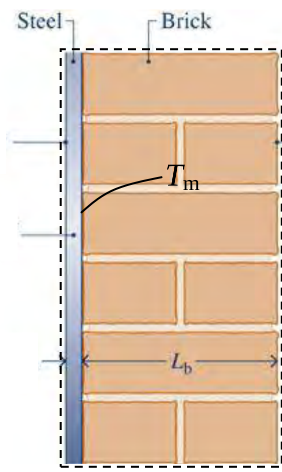
PROBLEM 2.48

As shown in Fig. P2.48, an oven wall consists of a 0.635-cm-thick layer of steel (κ_s = 15.1 W/m·K) and a layer of brick (κ_b = 0.72 W/m·K). At steady state, a temperature decrease of 0.7°C occurs over the steel layer. The inner temperature of the steel layer is 300 K. If the temperature of the outer surface of the brick must be no greater than 40°C, determine the thickness of brick, in cm, that ensures this limit is met. What is the rate of conduction, in kW per m^2 of wall surface area?

KNOWN: Steady-state data are provided for a composite wall formed from a steel layer and a brick layer.

FIND: Determine the minimum thickness of the brick layer to keep the outer surface temperature of the brick at or below a specified value.

SCHEMATIC AND GIVEN DATA:



T_i = 300 K

ΔT = -0.7°C

L_s = 0.635 cm

$T_o \leq 40$°C

κ_s = 15.1 W/m·K

κ_b = 0.72 W/m·K

ENGINEERING MODEL: (1) The wall is the system at steady state. (2) The temperature varies linearly through each layer.

ANALYSIS: Using Eq. 2.31 together with model assumption 2

$$\left(\frac{\dot{Q}}{A}\right)_{\text{steel}} = -\kappa_s\left[\frac{T_m - T_i}{L_s}\right] \quad \text{and} \quad \left(\frac{\dot{Q}}{A}\right)_{\text{brick}} = -\kappa_b\left[\frac{T_o - T_m}{L_b}\right]$$

where T_m denotes the temperature at the steel-brick interface.

At steady state, the rate of conduction *to* the interface through the steel must equal the rate of conduction *from* the interface through the brick: $(\dot{Q}/A)_{\text{steel}} = (\dot{Q}/A)_{\text{brick}}$. Thus

$$-\kappa_s\left[\frac{T_m - T_i}{L_s}\right] = -\kappa_b\left[\frac{T_o - T_m}{L_b}\right]$$

And solving for L_b we get

(300 – 0.7) = 299.3 °C

$$L_b = \frac{\kappa_b}{\kappa_s}\left[\frac{T_o - T_m}{\underline{T_m - T_i}}\right] L_s$$

= – 0.7 °C

PROBLEM 2.48 (Continued)

$$L_\text{b} = \left(\frac{0.72\ \text{W/m}\cdot\text{K}}{15.1\ \text{W/m}\cdot\text{K}}\right)\left[\frac{299.3 - T_\text{o}}{0.7}\right](0.635\ \text{cm})$$

Since $T_\text{o} \leq 40\ ^\text{o}\text{C}$

$$L_\text{b} \geq \left(\frac{0.72}{15.1}\right)\left[\frac{299.3 - 40}{0.7}\right](0.635\ \text{cm})$$

$$L_\text{b} \geq 11.22\ \text{cm} \longleftarrow$$

The rate of conduction is

$$\left(\frac{\dot{Q}}{\text{A}}\right)_\text{steel} = -\kappa_\text{s}\left[\frac{T_\text{m} - T_\text{i}}{L_\text{s}}\right] = -(15.1\ \text{W/m}\cdot\text{K})\left[\frac{299.3 - 300}{0.635\ \text{cm}}\right]\left|\frac{100\ \text{cm}}{1\ \text{m}}\right|\left|\frac{1\ \text{kW}}{10^3\text{W}}\right| = 1.665\ \text{kW/m}^2 \longleftarrow$$

or

$$\left(\frac{\dot{Q}}{\text{A}}\right)_\text{brick} = -\kappa_\text{s}\left[\frac{T_\text{o} - T_\text{m}}{L_\text{b}}\right] = -(0.72\ \text{W/m}\cdot\text{K})\left[\frac{40 - 299.3}{11.22\ \text{cm}}\right]\left|\frac{100\ \text{cm}}{1\ \text{m}}\right|\left|\frac{1\ \text{kW}}{10^3\text{W}}\right| = 1.664\ \text{kW/m}^2 \longleftarrow$$

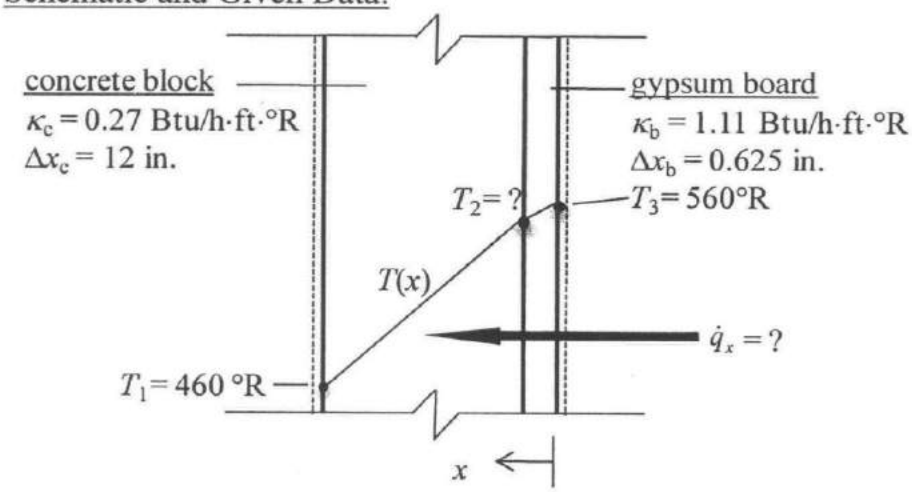
The slight difference is due to round-off.

PROBLEM 2.49

Known: Insulated composite plane wall consists of insulated concrete block and gypsum board with known surface temperatures. There is perfect contact at the interface between the two layers.

Find: Determine at steady state the instantaneous rate of heat transfer, in Btu/h per ft^2 of surface area, and the temperature, in °R, at the interface between the concrete block and gypsum board.

Schematic and Given Data:



Engineering Model:
(1) The wall is a closed system at steady state.
(2) The temperature distributions are linear in both layers.
(3) The two layers are in perfect thermal contact.
(4) The thermal conductivity in each layer is uniform.

Analysis:

Begin with Eq. 2.31

$$\dot{Q}_x = -\kappa A \frac{dT}{dx}$$

$$\dot{q}_x = \frac{\dot{Q}_x}{A} = -\kappa \frac{dT}{dx} = -\kappa_c \left.\frac{dT}{dx}\right|_{\text{concrete}} = -\kappa_b \left.\frac{dT}{dx}\right|_{\text{gypsum board}}$$

PROBLEM 2.49 (CONTINUED)

Using assumption (2)

$$\dot{q}_x = \kappa_c \frac{T_2 - T_1}{\Delta x_c} = \kappa_b \frac{T_3 - T_2}{\Delta x_b}$$

Use thermal resistance, $R = \Delta x/\kappa$, to simplify

$$R_c = \frac{\Delta x_c}{\kappa_c} \quad and \quad R_b = \frac{\Delta x_b}{\kappa_b}$$

$$\dot{q}_x = \frac{T_2 - T_1}{R_c} = \frac{T_3 - T_2}{R_b} \qquad (1)$$

Rearrange first part of Eq. (1) (2)

$$T_2 = T_1 + R_c \dot{q}_x$$

Substitute Eq. (2) into the second part of Eq. (1) and rearrange

#1 $$\dot{q}_x = \frac{T_3 - T_2}{R_b} = \frac{T_3 - (T_1 + R_c \dot{q}_x)}{R_b} = \frac{T_3 - T_1}{R_c + R_b} \qquad (3)$$

Solve for R values

$$R_c = \frac{\Delta x_c}{\kappa_E} = \frac{12 \text{ in.}}{0.27 \dfrac{\text{Btu}}{\text{h} \cdot \text{ft} \cdot {}^\circ\text{R}}} \left| \frac{1 \text{ ft}}{12 \text{ in.}} \right| = 3.7 \frac{\text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{R}}{\text{Btu}}$$

$$R_E = \frac{\Delta x_b}{\kappa_b} = \frac{0.625 \text{ in.}}{1.11 \dfrac{\text{Btu}}{\text{h} \cdot \text{ft} \cdot {}^\circ\text{R}}} \left| \frac{1 \text{ ft}}{12 \text{ in.}} \right| = 0.047 \frac{\text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{R}}{\text{Btu}}$$

Solve for $\dot{q}_x$ and T_2

$$\dot{q}_x = \frac{T_3 - T_1}{R_E + R_b} = \frac{560^\circ\text{R} - 460^\circ\text{R}}{(3.7 + 0.047) \dfrac{\text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{R}}{\text{Btu}}} = 26.69 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2} \longleftarrow$$

#2 $$T_2 = T_1 + R_c \dot{q}_x = 460^\circ\text{R} + \left(3.7 \frac{\text{h} \cdot \text{ft}^2 \cdot {}^\circ\text{R}}{\text{Btu}} \right) \left(26.69 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2} \right) = 558.8^\circ\text{R} \longleftarrow$$

---

1. Eq. (3) illustrates the analogy between heat conduction through a composite wall and electric current flow through a series of resistances. The temperature difference in the numerator is analogous to a voltage difference, and the value R_c and R_b are "thermal resistances" analogous to electrical resistances.
2. Due to the relatively low value for R_b, the temperature change across the gypsum board is minimal compared to the concrete block.

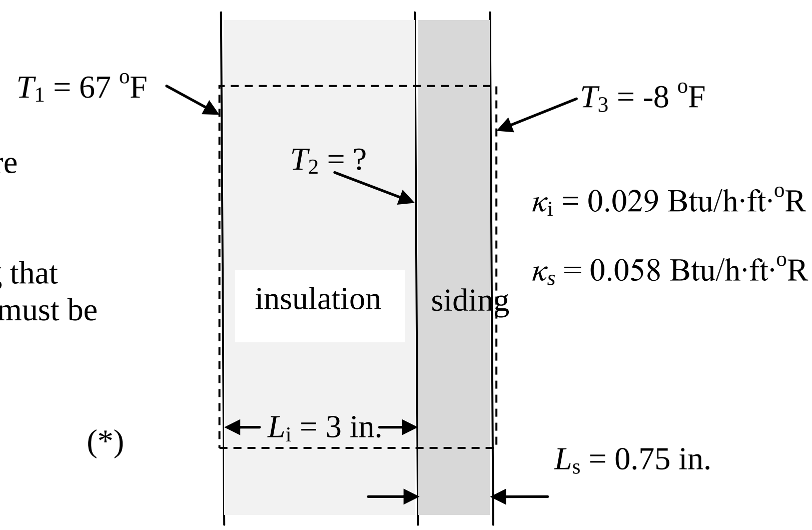
Problem 2.50

A composite plane wall consists of a 3-in.-thick layer of insulation (κ_i = 0.029 Btu/h·ft·°R) and a 0.75-in.-thick layer of siding (κ_s = 0.058 Btu/h·ft·°R). The inner temperature of the insulation is 67°F. The outer temperature of the siding is -8°F. Determine at steady state (a) the temperature at the interface of the two layers, in °F, and (b) the rate of heat transfer through the wall in Btu per ft^2 of surface area.

KNOWN: Energy transfer by conduction occurs through a composite wall consisting of two layers.

FIND: Determine the temperature at the interface between the two layers and the rate of heat transfer per unit area through the wall.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The wall is the system at steady state. (2) The temperature varies linearly through each layer.

ANALYSIS: With Eq. 2.17, and recognizing that at steady state the rates of energy conduction must be equal through each layer

$$\frac{\dot{Q}}{A} = -\kappa_i\left[\frac{T_2 - T_1}{L_i}\right] = -\kappa_s\left[\frac{T_3 - T_2}{L_s}\right] \qquad (*)$$

Solving for T_2

$$T_2 = \frac{\left(\frac{\kappa_i}{L_i}T_1 + \frac{\kappa_s}{L_s}T_3\right)}{(\kappa_i/L_i) + (\kappa_s/L_s)}$$

$$\frac{\kappa_i}{L_i} = \frac{0.029\ \text{Btu/h·ft·R}}{3\ \text{in.}}\left|\frac{12\ \text{in.}}{1\ \text{ft}}\right| = 0.116\ \text{Btu/h·°R} \qquad \frac{\kappa_s}{L_s} = \frac{0.058\ \text{Btu/h·ft·R}}{.75\ \text{in.}}\left|\frac{12}{1}\right| = 0.928\ \text{Btu/h·°R}$$

Thus

$$T_2 = \frac{(0.116)(527) + (0.928)(452)}{(0.116) + (0.928)} = 460.3\ \text{°R} = 0.33\ \text{°F}$$

Thus, using Eq. (*)

$$\frac{\dot{Q}}{A} = -\kappa_i\left[\frac{T_2 - T_1}{L_i}\right] = \left(-0.029\frac{\text{Btu}}{\text{h·ft·R}}\right)\left[\frac{(0.33-67)\text{R}}{\frac{3}{12}\text{ft}}\right] = 7.73\ \text{Btu/ft}^2$$

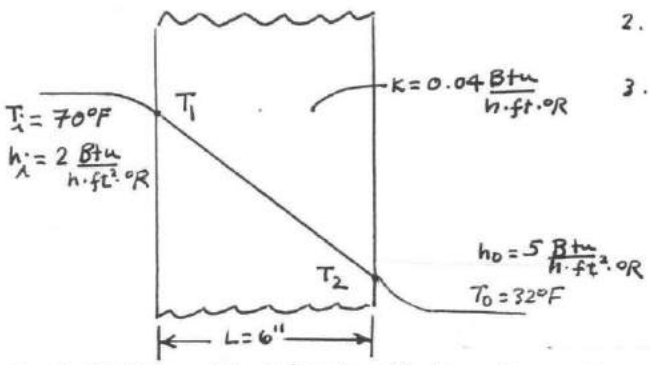
$$\frac{\dot{Q}}{A} = -\kappa_s\left[\frac{T_3 - T_2}{L_2}\right] = \left(-0.058\frac{\text{Btu}}{\text{h·ft·R}}\right)\left[\frac{(-8-0.33)\text{R}}{\frac{0.75}{12}\text{ft}}\right] = 7.73\ \text{Btu/ft}^2$$

PROBLEM 2.51

KNOWN: Steady-state data are provided for the outer wall of a house.

FIND: Determine the rate of heat transfer through the wall.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The wall is at steady state.
2. Temperature varies linearly through the wall.
3. At the inner and outer surfaces, convection is the only heat transfer mode.

ANALYSIS: At steady state, the rates of energy transfer to the wall by convection, through the wall by conduction, and from the wall by convection are all equal. That is,

$$\frac{\dot{Q}}{A} = \underbrace{h_i (T_i - T_1)}_{\text{convection to wall}} = \underbrace{-k\left[\frac{T_2 - T_1}{L}\right]}_{\text{conduction through wall}} = \underbrace{h_o [T_2 - T_o]}_{\text{convection from wall}} \qquad (*)$$

Solving the first and third of Eqs. (*),

$$T_1 = T_i - \frac{\dot{Q}/A}{h_i}, \quad T_2 = T_o + \frac{\dot{Q}/A}{h_o}$$

Substituting into the second of Eqs. (*) and simplifying

(1)
$$\frac{\dot{Q}}{A} = -k\left[\frac{\left(T_o + \frac{\dot{Q}/A}{h_o}\right) - \left(T_i - \frac{\dot{Q}/A}{h_i}\right)}{L}\right] = \frac{T_i - T_o}{\underbrace{\left(\frac{1}{h_i} + \frac{L}{k} + \frac{1}{h_o}\right)}_{\text{thermal "resistances"}}} \qquad (**)$$

Inserting values into Eq. (**)

$$\frac{\dot{Q}}{A} = \frac{[530°R - 492°R] \quad (Btu/h \cdot ft^2 \cdot °R)}{\left[\underbrace{\frac{1}{2}}_{0.5} + \underbrace{\frac{0.5}{0.04}}_{12.5} + \underbrace{\frac{1}{5}}_{0.2}\right]}$$

$$= 2.88 \ \frac{Btu/h}{ft^2} \longleftarrow$$

---

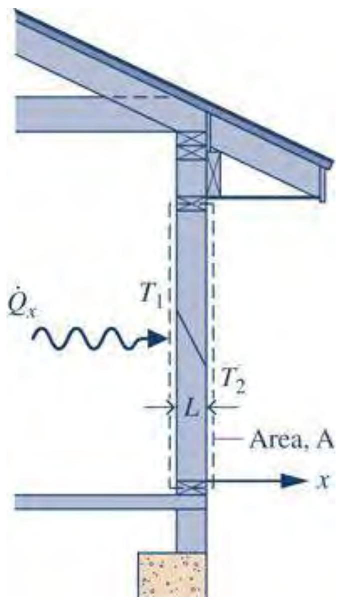
1. The form of Eq. (**) illustrates the analogy between heat transfer and electric current flow through resistances in series. The temperature difference in the numerator is analogous to the voltage difference, and the terms in the denominator account for "thermal resistances" analogous to electrical resistances. In this case, the insulated wall is the greater of the three resistances, as shown by the calculation below.

Problem 2.52

Complete the following exercise using heat transfer relations:

(a) Referring to Fig. 2.12, determine the rate of conduction heat transfer, in W, for κ = 0.07 W/m·K, A = 0.125 m^2, T_1 = 298 K, T_2 = 273 K.
(b) Referring to Fig. 2.14, determine the rate of convection heat transfer from the surface to the air, in W, for h = 10 W/m^2, A = 0.125 m^2, T_b = 305 K, T_f = 298 K.

(a) Referring to Fig. 2.12, determine the rate of conduction heat transfer, in W, for κ = 0.07 W/m·K, A = 0.125 m^2, T_1 = 298 K, T_2 = 273 K.

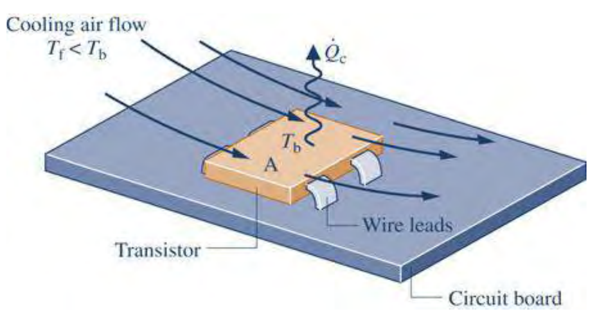


Using Eq. 2.31 and noting that the temperature varies linearly through the wall

$$\dot{Q}_x = -\kappa A\left[\frac{T_2 - T_1}{L}\right]$$

$$= -\left(0.07\,\frac{\text{W}}{\text{m·K}}\right)(0.125\ \text{m}^2)\left[\frac{(273-298)\text{K}}{(0.127\ \text{m})}\right] = 1.722\ \text{W}$$

(b) Referring to Fig. 2.14, determine the rate of convection heat transfer from the surface to the air, in W, for h = 10 W/m^2, A = 0.125 m^2, T_b = 305 K, T_f = 298 K.



Using Eq. 2.34

$$\dot{Q}_c = hA[T_b - T_f]$$

$$= \left(10\,\frac{\text{W}}{\text{m}^2}\right)(0.125\ \text{m}^2)[305 - 298]\text{K}$$

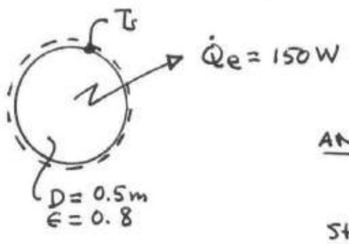
$$= 8.75\ \text{W}$$

PROBLEM 2.53

KNOWN: Steady-state operating data are provided for a spherical interplanetary probe.

FIND: Determine the surface temperature of the sphere, in K.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The probe is at steady state.
2. The probe emits but does not receive radiation.

ANALYSIS: In this case, Eq. 2.32 applies:

$\dot{Q}_e = \epsilon \sigma A T_s^4$, where $A = \pi D^2$ and σ is the Stefan-Boltzmann constant, $5.67 \times 10^{-8}\ W/m^2 \cdot K^4$. Solving for T_s

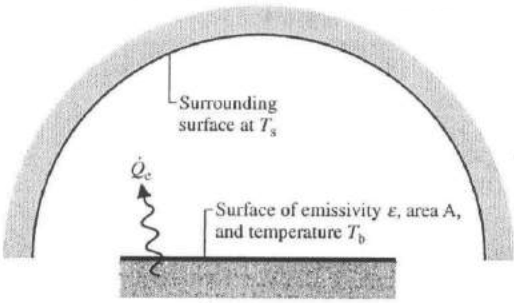
$$T_s = \left[\frac{\dot{Q}_e}{\epsilon \pi D^2 \sigma}\right]^{1/4} = \left[\frac{150 W}{0.8 \pi (0.5 m)^2 (5.67 \times 10^{-8} W/m^2 \cdot K)}\right]^{1/4} = 255 K \longleftarrow$$

PROBLEM 2.54

KNOWN: Data are provided for a body placed in a large, evacuated chamber.

FIND: Determine the rate at which radiation is emitted from the surface and the net rate at which radiation is exchanged between the body and chamber.

SCHEMATIC & GIVEN DATA:



$A = 0.5\ m^2$, $\epsilon = 0.8$, $T_b = 423K$, $T_s = 298K$

ENGR. MODEL:

1. The area of the enclosed surface is much less than that of the chamber walls.
2. The chamber is evacuated.

ANALYSIS:

(a) The rate radiation is emitted from the surface is given by Eq. 2.32, where σ is the Stefan-Boltzmann constant. That is

$$\dot{Q}_e = \epsilon \sigma A T_b^4 = 0.8(0.5m^2)\left(5.67\times10^{-8}\frac{W}{m^2\cdot K^4}\right)(423K)^4$$
$$= 726\ W$$

(b) The net rate at which radiation is transferred from the surface to the chamber walls is given by Eq. 2.33. That is

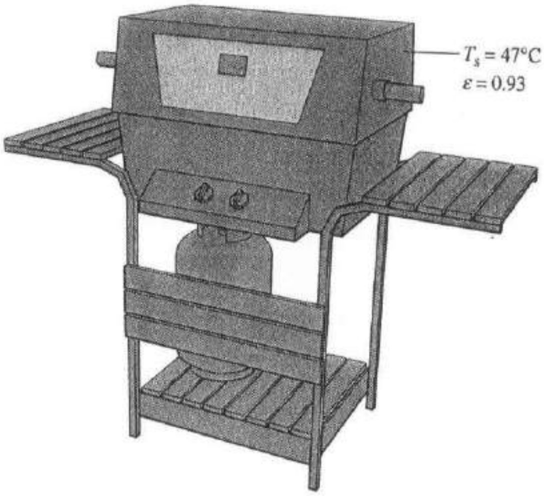
$$(\dot{Q}_e)_{net} = \epsilon\sigma A\left[T_b^4 - T_s^4\right] = 0.8(0.5m^2)\left(5.67\times10^{-8}\frac{W}{m^2\cdot K^4}\right)\left((423K)^4 - (298K)^4\right)$$
$$= 547\ W$$

PROBLEM 2.55

KNOWN: Data are provided for a grill hood.

FIND: Determine the net rate of heat transfer between the hood and the surroundings by convection and radiation, per unit area of hood surface.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. Radiative heat transfer between the hood and surroundings is modeled as an exchange between a surface at T_s and a much larger surrounding surface at T_0.

ANALYSIS:

Using Eq. 2.34, convection heat transfer is

$$\frac{\dot{Q}_c}{A} = h[T_s - T_0] = 10\frac{W}{m^2 K}[20K]\left|\frac{1 kW}{10^3 W}\right|$$

$$= 0.2\ kW/m^2 \longleftarrow$$

Using Eq. 2.33, radiative heat transfer is

$$\frac{\dot{Q}_e}{A} = \epsilon\sigma[T_s^4 - T_0^4]$$

$$= 0.93\left[5.67\times10^{-8}\frac{W}{m^2 \cdot K^4}\right]\left[(320K)^4 - (300K)^4\right]$$

$$= 125.8\frac{W}{m^2}\left|\frac{1 kW}{10^3 W}\right| = 0.13\ kW/m^2 \longleftarrow$$

$$\text{Total} = (\dot{Q}_c/A) + (\dot{Q}_e/A) = 0.33\ kW/m^2 \longleftarrow$$

Problem 2.56

Each line of the following table gives data for a process of a closed system. Each entry has the same energy units. Determine the missing entries.

Process	Q	W	E_1	E_2	ΔE
a	+50		-20		+70
b		+20		+50	+30
c		-60	+40	+60	
d	-40		+50		0
e	+50	+150		-80	

Process	Q	W	E_1	E_2	ΔE
a	+50	***-20***	-20	***+50***	+70
b	***+50***	+20	*+20*	+50	+30
c	***-40***	-60	+40	+60	***+20***
d	-90	***-90***	+50	***+50***	0
e	+50	+150	***+20***	-80	***-100***

$\Delta E = Q - W$

Process a:

$W = Q - \Delta E = +50 - (+70) = -20$ ←

$\Delta E = E_2 - E_1$

$E_2 = \Delta E + E_1 = +70 + (-20) = +50$ ←

Process b:

$Q = \Delta E + W = +30 + (+20) = +50$ ←

$\Delta E = E_2 - E_1$

$E_1 = E_2 - \Delta E = +50 - (+30) = +20$ ←

Process c:

$\Delta E = E_2 - E_1 = +60 - (+40) = +20$ ←

$Q = \Delta E + W = +20 + (-60) = -40$ ←

Process d:

$W = Q - \Delta E = (-90) - 0 = -90$ ←

$\Delta E = E_2 - E_1$

$E_2 = \Delta E + E_1 = 0 + 50 = +50$ ←

Process e:

$\Delta E = Q - W = +50 - (+150) = -100$ ←

$E_1 = E_2 - \Delta E = (-80) - (-100) = +20$ ←

PROBLEM 2.57

Each line of the following table gives data, in Btu, for a process of a closed system. Determine the missing table entries, in Btu.

Process	Q	W	E_1	E_2	ΔE
a	+40		+15		+15
b		+5	+7	+22	
c	-4	+10		-8	
d	-10		-10		+20
e	+3	-3	+8		

Process	Q	W	E_1	E_2	ΔE
a	+40	***-25***	+15	***+30***	+15
b	***+20***	+5	+7	+22	***+15***
c	-4	+10	***+6***	-8	***-14***
d	-10	***-30***	-10	***+10***	+20
e	+3	-3	+8	***+14***	***+6***

$\Delta E = Q - W$

<u>Process a</u>:

$W = Q - \Delta E = +40 - (+15) = -25$ Btu $\longleftarrow$

$\Delta E = E_2 - E_1$

$E_2 = \Delta E + E_1 = +15 + (+15) = +30$ Btu $\longleftarrow$

<u>Process b</u>:

$\Delta E = E_2 - E_1 = 22 - 7 = +15$ Btu $\longleftarrow$

$Q = \Delta E + W = +15 + 5 = +20$ Btu $\longleftarrow$

<u>Process c</u>:

$\Delta E = Q - W = (-4) - (10) = -14$ Btu $\longleftarrow$

$E_1 = E_2 - \Delta E = (-8) - (-14) = 6$ Btu $\longleftarrow$

<u>Process d</u>:

$W = Q - \Delta E = (-10) - (+20) = -30$ Btu $\longleftarrow$

$\Delta E = E_2 - E_1$

$E_2 = \Delta E + E_1 = +20 + (-10) = +10$ Btu $\longleftarrow$

<u>Process e</u>:

$\Delta E = Q - W = +3 - (-3) = +6$ Btu $\longleftarrow$

$E_2 = \Delta E + E_1 = (+6) + (+8) = +14$ Btu $\longleftarrow$

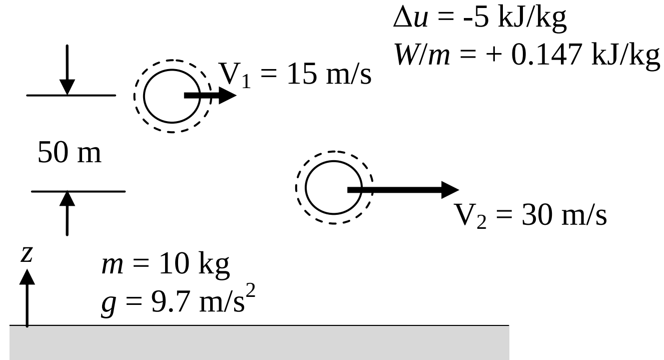
Problem 2.58

A closed system of mass of 10 kg undergoes a process during which there is energy transfer by work from the system of 0.147 kJ per kg, an elevation decrease of 50 m, and an increase in velocity from 15 m/s to 30 m/s. The specific internal energy decreases by 5 kJ/kg and the acceleration of gravity is constant at 9.7 m/s^2. Determine the heat transfer for the process, in kJ.

KNOWN: Data are provided for a closed system undergoing a process involving work, heat transfer, change in elevation, and change in velocity.

FIND: Determine the heat transfer for the process.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The system is a closed system. (2) The acceleration of gravity is constant.

ANALYSIS:

$$\Delta U + \Delta PE + \Delta KE = Q - W \quad \rightarrow \quad Q = \Delta U + \Delta PE + \Delta KE - W$$

✓ $W = m\,[W/m] = 10\text{ kg}\,[-0.147\text{ kJ/kg}] = -1.47\text{ kJ}$

✓ $\Delta U = m\Delta u = 10\text{ kg}\,[-5\text{ kJ/kg}] = -50\text{ kJ}$

✓ $\Delta KE = \frac{m}{2}(\mathrm{V}_2^2 - \mathrm{V}_1^2) = \frac{10\text{ kg}}{2}\left[\left(30\ \frac{\text{m}}{\text{s}}\right)^2 - \left(15\ \frac{\text{m}}{\text{s}}\right)^2\right]\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1\text{ kJ}}{10^3\text{N}\cdot\text{m}}\right| = +3.38\text{ kJ}$

✓ $\Delta PE = mg(z_2 - z_1) = (10\text{ kg})\,(9.7\text{ m/s}^2)(-50\text{ m})\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1\text{ kJ}}{10^3\text{N}\cdot\text{m}}\right| = -4.85\text{ kJ}$

$Q = (-50) + (-4.85) + (3.38) - (-1.47) = -50\text{ kJ (out)}$ ←

PROBLEM 2.59

KNOWN: A gas contained in a piston-cylinder assembly undergoes a constant-pressure expansion while being slowly heated. State data are provided.

FIND: For the gas, evaluate work and heat transfer. For the piston, evaluate work and change in potential energy.

SCHEMATIC & GIVEN DATA:

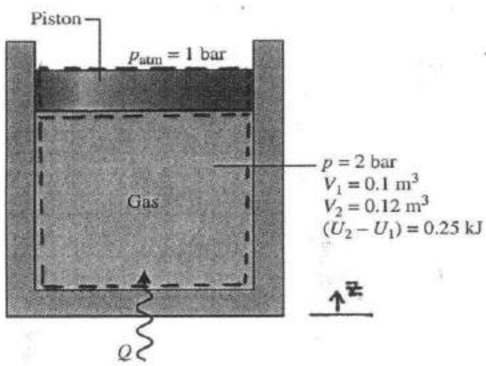


Fig. P2.56

ENGINEERING MODEL:

1. As shown in the schematic, two closed systems are considered: the gas and the piston.
2. The gas undergoes a constant-pressure process.
3. For the gas there is no change in potential energy (see Example 2.3) and no overall change in kinetic energy.
4. For the piston, there is no heat transfer. Also, there is no change in internal energy, no overall change in kinetic energy, and no friction.

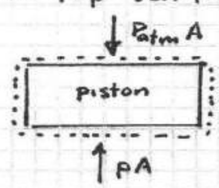
ANALYSIS: (a) Taking the gas as the system, the work is obtained from Eq. 2.17:

$$W = \int_1^2 p\,dV = p[V_2 - V_1] = \left(2\times10^5 \frac{N}{m^2}\right)(0.12 - 0.1)\,m^3 \left|\frac{1\,kJ}{10^3\,N\cdot m}\right| = 4\,kJ$$

Reducing an energy balance, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W \Rightarrow Q = W + \Delta U$

$$\Rightarrow Q = 4\,kJ + 0.25\,kJ = 4.25\,kJ$$

(b) Taking the piston as the system, an energy transfer by work occurs on the bottom surface from the gas. At the top surface the piston does work on the atmosphere:



(No friction)

$$W_{piston} = \int F\,dz = (p_{atm}A - pA)\Delta z = (p_{atm} - p)(A\Delta z)$$

$$= (p_{atm} - p)\Delta \underbrace{V}_{(gas)}$$

$$= (1-2)\left(10^5 \frac{N}{m^2}\right)(0.12 - 0.1)\,m^3 \left|\frac{1\,kJ}{10^3\,N\cdot m}\right|$$

$$= -2\,kJ$$

An energy balance for the piston reduces as follows:

$$[\cancel{\Delta U} + \cancel{\Delta KE} + \Delta PE]_{piston} = \cancel{Q_{piston}} - W_{piston}$$

(1)

$$\Rightarrow \Delta PE]_{piston} = -W_{piston}$$

$$= +2\,kJ$$

---

1. Overall energy "balance sheet" in terms of magnitudes:

Input: $Q = 4.25\,kJ$

Disposition of the energy input:

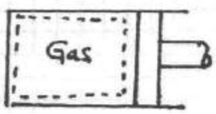
⊙ Stored as ΔU in the gas:	0.25 kJ
⊙ Stored as ΔPE in the piston:	2.00 kJ
⊙ Transfer by work to the atmosphere	2.00 kJ
	4.25 kJ

PROBLEM 2.60

KNOWN: A gas contained in a piston-cylinder assembly undergoes two processes, A and B, between the same end states. State data are provided.

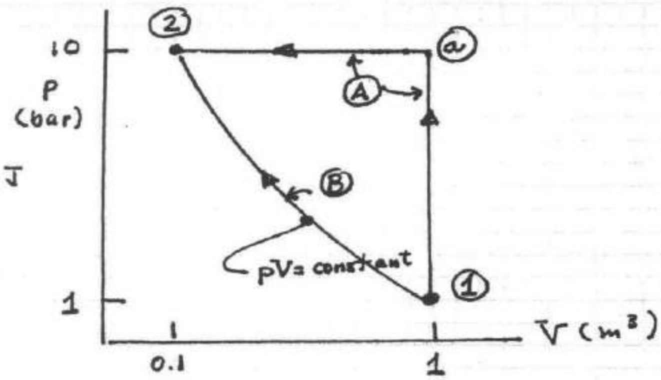
FIND: For each process, sketch it on p-V coordinates and evaluate the work and heat transfer, each in kJ.

SCHEMATIC & GIVEN DATA:



$p_1 = 1$ bar, $V_1 = 1\ m^3$, $U_1 = 400$ kJ

$p_2 = 10$ bar, $V_2 = 0.1\ m^3$, $U_2 = 450$ kJ



ENGINEERING MODEL:

1. The gas is the closed system.
2. Volume change is the only work mode.
3. Kinetic and potential energy effects are ignored.

ANALYSIS: Reducing an energy balance for the gas: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$

$$\Rightarrow Q = \Delta U + W = (450 - 400)\text{ kJ} + W$$

$$\Rightarrow Q = 50\text{ kJ} + W \qquad (1)$$

Eq. (1) applies to each of the processes, A and B.

Evaluating work from Eq. 2.17, $W = \int_1^2 p\,dV$.

Process A:

$$W_A = \cancel{W_{1a}}^{\,0} + W_{a2} = p_2[V_2 - V_1]$$

$$= \left(10 \times 10^5 \frac{N}{m^2}\right)(0.1 - 1)\,m^3 \left| \frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}} \right| = -900\text{ kJ}$$

Then, with Eq. (1)

$$Q_A = 50\text{ kJ} + (-900\text{ kJ}) = -850\text{ kJ}$$

Process B:

$$W_B = \int_1^2 p\,dV = \int_1^2 \frac{C}{V}\,dV = C \ln\frac{V_2}{V_1} = p_1 V_1 \ln\frac{V_2}{V_1}$$

$$= \left(10^5 \frac{N}{m^2}\right)(1\,m^3) \ln\left[\frac{0.1}{1.0}\right] \left| \frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}} \right| = -230.3\text{ kJ}$$

Then, with Eq. (1)

$$Q_B = 50\text{ kJ} + (-230.3\text{ kJ}) = -180.3\text{ kJ}$$

PROBLEM 2.61

KNOWN: A gas within a piston-cylinder assembly undergoes two different processes between the same end states.

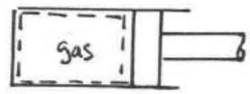
FIND: Sketch the processes on p-V coordinates. For each process evaluate W and Q.

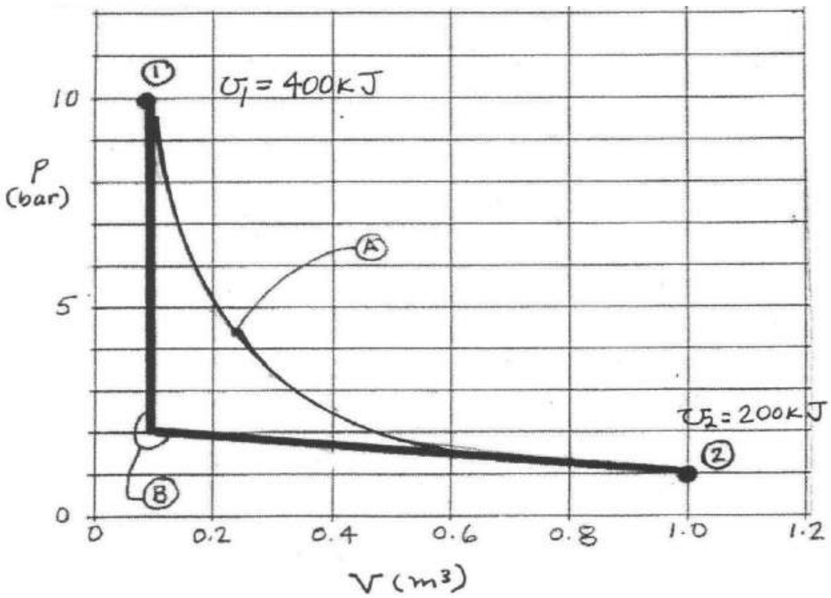
SCHEMATIC & GIVEN DATA:

Data: $p_1 = 10$ bar, $V_1 = 0.1\ \text{m}^3$, $U_1 = 400$ kJ and $p_2 = 1$ bar, $V_2 = 1.0\ \text{m}^3$, $U_2 = 200$ kJ:

Process A: Process from 1 to 2 during which the pressure-volume relation is $pV = constant$

Process B: Constant-volume process from state 1 to a pressure of 2 bar, followed by a linear pressure-volume process to state 2.





ENGR. MODEL:

1. The gas is the closed system.
2. Kinetic and potential energy effects can be ignored.
3. The p-V relation is specified for each process.
4. The moving boundary is the only work mode.

ANALYSIS: For Process A, $W_A = \int_1^2 p\,dV = \int_{V_1}^{V_2} \frac{C}{V}\,dV = \underbrace{C}_{= P_1V_1 = P_2V_2} \ln\frac{V_2}{V_1} = P_1V_1 \ln\frac{V_2}{V_1}$

Thus, $W_A = (10\text{ bar})(0.1\text{ m}^3)\ln\left[\frac{1.0\text{ m}^3}{0.1\text{ m}^3}\right]\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| = +230.26\text{ kJ}$ ← W_A

$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_A - W_A \Rightarrow Q_A = \Delta U + W_A = (200-400)\text{ kJ} + 230.26\text{ kJ} = +30.26\text{ kJ}$ ← Q_A

For Process B, the piston does not move for the first step (constant volume) and thus there is no work. The work can be evaluated geometrically for the second step, during which the p-V relation is linear.

$W_B = P_{ave}[V_2 - V_1] = \left[\frac{2\text{ bar} + 1\text{ bar}}{2}\right]\left[1.0\text{ m}^3 - 0.1\text{ m}^3\right]\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| = 135\text{ kJ}$ ← W_B

$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_B - W_B \Rightarrow Q_B = \Delta U + W$

Note: Since U is a property, ΔU is the same for each process

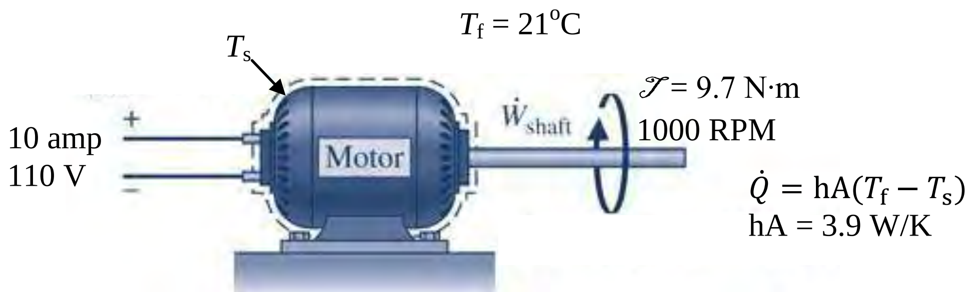
$Q_B = (200-400)\text{ kJ} + 135\text{ kJ} = -65\text{ kJ}$ ← Q_B

Problem 2.62

An electric motor draws a current of 10 amp with a voltage of 110 V, as shown in Fig. P2.62. The output shaft develops a torque of 9.7 N·m and a rotational speed of 1000 RPM. For operation at steady state, determine for the motor
(a) the electric power required, in kW.
(b) the power developed by the output shaft, in kW.
(c) the average surface temperature, T_s, in ^oC, if heat transfer occurs by convection to the surroundings at $T_f = 21^oC$.

KNOWN: Operating data are provided for an electric motor at steady state.

FIND: Determine (a) the electric power required, (b) the power developed by the output shaft, and (c) average the surface temperature.



ENGINEERING MODEL: (1) The motor is the closed system. (2) The system is at steady state.

ANALYSIS: (a) Using Eq. 2.21

$$\dot{W}_{electric} = \text{- (voltage) (current)} = \text{- (110 V)(10 amp)}\left|\frac{1\ \text{W/amp}}{1\ \text{V}}\right|\left|\frac{1\ \text{kW}}{10^3\text{W}}\right| = \text{-1.1 kW (in)} \longleftarrow$$

(b) Using Eq. 2.20

$$\dot{W}_{shaft} = \text{(torque) (angular velocity)}$$

$$= (9.7\ \text{N}\cdot\text{m})\left(1000\,\frac{\text{rev}}{\text{min}}\right)\left|\frac{2\pi\ \text{rad}}{\text{rev}}\right|\left|\frac{1\ \text{min}}{60\ \text{s}}\right|\left|\frac{1\ \text{kW}}{10^3\text{N·m/s}}\right| = \text{1.016 kW (out)} \longleftarrow$$

(c) To determine the surface temperature, first find the rate of energy transfer by heat using the energy balance

$$\overset{0}{\cancel{\frac{dE}{dt}}} = \dot{Q} - \dot{W} = \dot{Q} - (\dot{W}_{electric} + \dot{W}_{shaft})$$

$$\dot{Q} = (\dot{W}_{electric} + \dot{W}_{shaft}) = (\text{-1.1 kW}) + (\text{1.016 kW}) = \text{-0.084 kW}$$

The surface temperature of the motor is

Problem 2.62 (Continued)

$$T_s = \left(\dot{Q}/\text{hA}\right) + T_f = (\text{-}0.084\ \text{kW})/(3.9\ \text{W/K})\left|\frac{10^3\ \text{W}}{1\ \text{kW}}\right| + 294\ \text{K}$$

$$= 315.5\ \text{K} = 42.5\ ^{\circ}\text{C} \longleftarrow$$

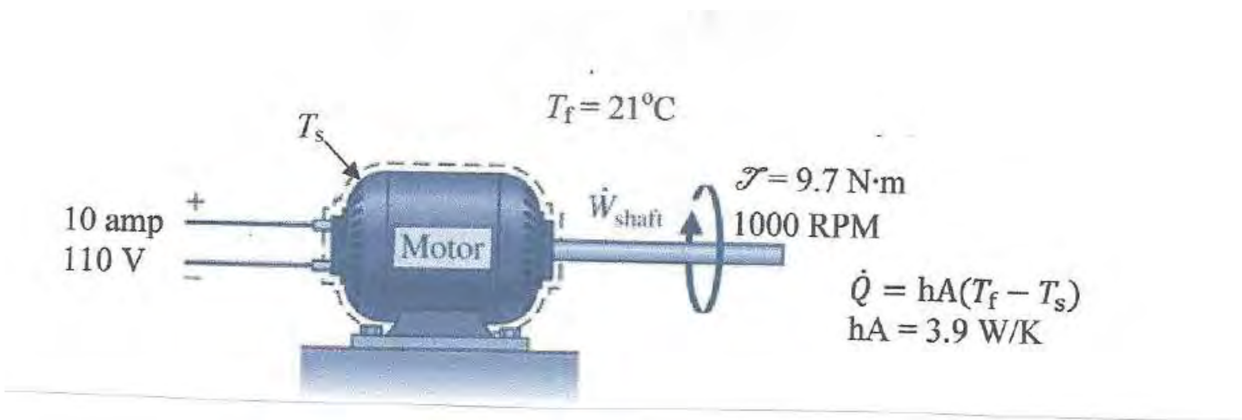


Fig. P2.62 – 8e

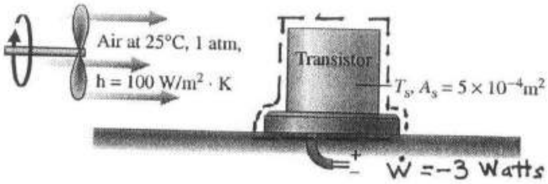
Pick-up motor graphic from Fig. E2.6 – 7e

PROBLEM 6.63

KNOWN: Steady-state data are provided for a transistor cooled convectively.

FIND: Determine for the transistor the heat transfer rate and the outer surface temperature.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The transistor is the closed system.
2. The system is at steady state.
3. No heat transfer occurs through the base of the transistor.

ANALYSIS: (a). An energy rate balance reads $\cancel{\frac{dE}{dt}}^{0} = \dot{Q} - \dot{W} \Rightarrow \dot{Q} = \dot{W} = -3$ Watts

(b) Since cooling occurs convectively, $\dot{Q} = -hA[T_s - T_{air}]$, where the minus sign is introduced because heat transfer is from the transistor and $T_s > T_{air}$. Solving

$$T_s = T_{air} - \left[\frac{\dot{Q}}{hA}\right] = 298K - \left[\frac{-3\ W}{\left(100 \frac{W}{m^2 \cdot K}\right)\left(5 \times 10^{-4} m^2\right)}\right] = 358K\ (85°C)$$

PROBLEM 2.64

KNOWN: Data are provided for a ridgid, closed tank fitted with a paddle wheel. The tank contains Refrigerant 22.

FIND: (a) Q and W for the refrigerant. (b) Evaluate the constant K in the heat transfer relation provided.

SCHEMATIC & GIVEN DATA:

$\dot{Q} = -Kt$ (K in kW/min, t in min), $\dot{W} = -0.1\,\text{kW}$

$u_1 = 232.92$ kJ/kg

$u_2 = 276.67$ kJ/kg

$P_1 = 0.9$ MPa, $P_2 = 1.2$ MPa $\quad \Delta t = 20$ min.

ENGR. MODEL:

1. The Refrigerant 22 is the closed system.
2. No overall changes in kinetic or potential energy occur.

ANALYSIS: (a)

$$W = \int_1^2 \dot{W}\,dt = -(0.1\,\text{kW})(20\,\text{min})\left|\frac{60\,\text{s}}{1\,\text{min}}\right|\left|\frac{1\,\text{kJ/s}}{1\,\text{kW}}\right| = -120\,\text{kJ}$$

Energy Balance:

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W \Rightarrow Q = \Delta U + W \Rightarrow Q = m(u_2 - u_1) + W$$

$$\therefore \; Q = 1\,\text{kg}\,(276.67 - 232.92) + (-120\,\text{kJ}) = -76.25\,\text{kJ}$$

(b)

$$Q = \int_1^2 \dot{Q}\,dt = \int_0^t -Kt\,dt = -\frac{Kt^2}{2} \Rightarrow$$

$$K = -\frac{2Q}{t^2} = -\frac{2(-76.25\,\text{kJ})}{(20\,\text{min})^2}\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right|\left|\frac{1\,\text{min}}{60\,\text{s}}\right|$$

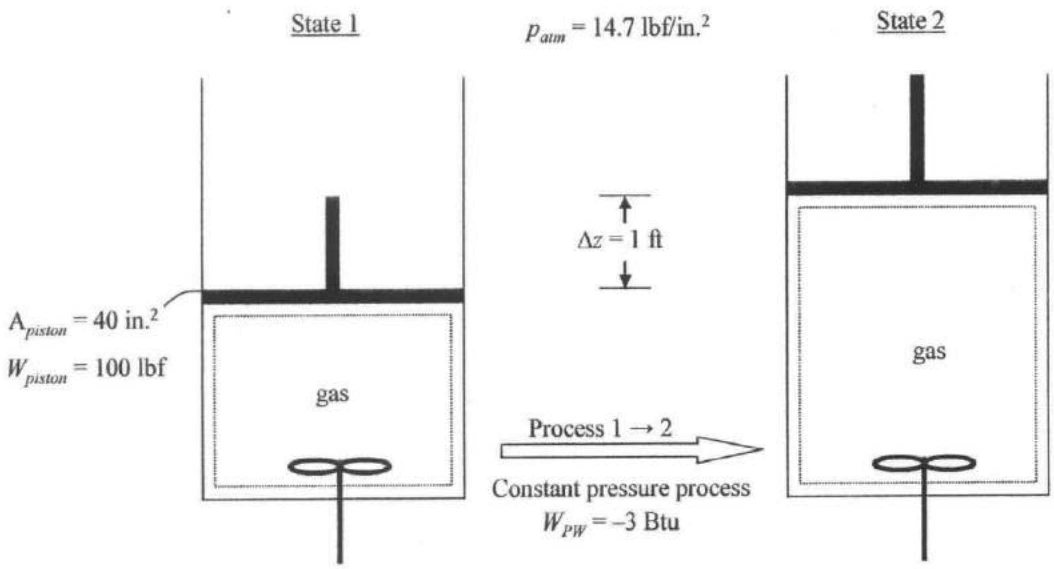
$$= 6.354 \times 10^{-3}\ \frac{\text{kW}}{\text{min}}$$

PROBLEM 2.65

KNOWN: A rotating shaft transfers energy to a gas contained in a vertical piston-cylinder assembly during a constant-pressure process.

FIND: Determine the change in internal energy of the gas.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The gas is the system.
2. The system undergoes an adiabatic process.
3. Kinetic and potential energy effects are negligible.
4. Thermal conduction and friction between the piston and cylinder can be neglected.

ANALYSIS:
The change in internal energy can be determined from the energy balance

$$\Delta \text{KE} + \Delta \text{PE} + \Delta U = Q - W$$

Neglecting changes in kinetic energy (ΔKE = 0) and potential energy (ΔPE = 0), assuming an adiabatic process (Q = 0), and solving for internal energy give

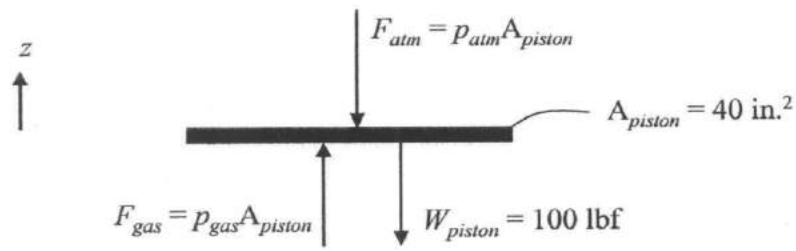
$$\Delta U = -W = -(W_{PW} + W_{exp})$$

PROBLEM 2.65 (Continued)

where W_{PW} = -3 Btu and W_{exp} is determined next:

$$W_{exp} = \int_{V_1}^{V_2} p_{gas} dV = \int_{x1}^{x2} p_{gas} \mathrm{A}_{piston} dx = \mathrm{A}_{piston} \int_{x1}^{x2} p_{gas} dx$$

To determine the gas pressure, draw a free body diagram for the piston. For equilibrium, sum of the forces in the z-direction equals zero.



$$\uparrow \Sigma F_z = 0$$

$$p_{gas}\mathrm{A}_{piston} - p_{atm}\mathrm{A}_{piston} - W_{piston} = 0$$

Solving for p_{gas} yields

$$p_{gas} = p_{atm} + \frac{W_{piston}}{\mathrm{A}_{piston}} = 14.7\frac{\text{lbf}}{\text{in.}^2} + \frac{100\text{ lbf}}{40\text{ in.}^2} = 17.2\text{ lbf/in.}^2$$

Since p_{gas} is constant, the expression for W_{exp} can be integrated as follows:

$$W_{exp} = p_{gas}\mathrm{A}_{piston}\int_{x1}^{x2} dx = p_{gas}\mathrm{A}_{piston}(x_2 - x_1)$$

Substitute the value for p_{gas} to solve for work of expansion.

$$W_{exp} = \left(17.2\frac{\text{lbf}}{\text{in.}^2}\right)\left(40\text{ in.}^2\right)\left(1\text{ ft}\right)\left|\frac{\text{Btu}}{778\text{ ft}\cdot\text{lbf}}\right| = 0.88\text{ Btu}$$

Substituting values for expansion work and shaft work yields change in internal energy.

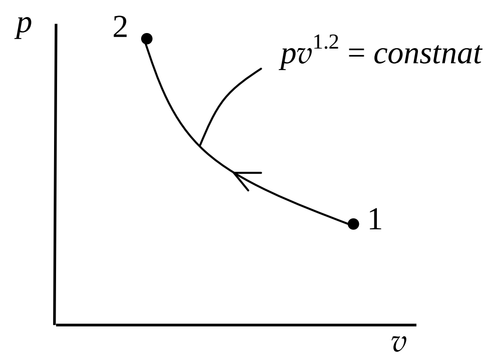
$$\Delta U = -W = -(W_{PW} + W_{exp}) = -[(0.88\text{ Btu}) - (-3\text{ Btu})] = \underline{\mathbf{2.12\ Btu}}$$

Problem 2.66

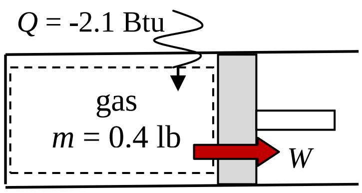
A gas undergoes a process in a piston-cylinder assembly during which the pressure-specific volume relation is $pv^{1.2}$ = *constant*. The mass of the gas is 0.4 lb and the following data are known: p_1 = 160 lbf/in.^{2}, V_1 = 1 ft^3, and p_2 = 390 lbf/in.^{2} During the process, heat transfer *from* the gas is 2.1 Btu. Kinetic and potential energy effects are negligible. Determine the change in specific internal energy of the gas, in Btu/lb.

KNOWN: A gas is compressed in a piston-cylinder assembly. The pressure-specific volume relation is specified.

FIND: Determine the change in specific internal energy.



SCHEMATIC AND GIVEN DATA:



p_1 = 160 lbf/in.^{2}
V_1 = 1 ft^3
p_2 = 390 lbf/in.^{2}

ENGINEERING MODEL: (1) The gas is a closed system. (2) The process follows $pv^{1.2}$ = *constant*. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: The change in specific internal energy will be found from an energy balance. First, determine the work. Since volume change is the only work mode, Eq. 2.17 applies:

$$W = \int_{V_1}^{V_2} p dV = = \int_{V_1}^{V_2} \frac{const}{V^{1.2}} dV = \frac{(p_2V_2 - p_1V_1)}{1-1.2}$$

Evaluating V_2

$$V_2 = \left(\frac{p_1}{p_2}\right)^{1/1.2} V_1 = \left(\frac{160 \text{ lbf/in.}^2}{390 \text{ lbf/in.}^2}\right)^{1/1.2} (1 \text{ ft}^3) = 0.4759 \text{ ft}^3$$

Thus

$$W = \left[\frac{(390 \text{ lbf/in.}^2)(0.4759 \text{ ft}^3) - (160)(1)}{1-1.2}\right] \left|\frac{144 \text{ in.}^2}{1 \text{ ft}^2}\right| \left|\frac{1 \text{ Btu}}{778 \text{ ft·lbf}}\right| = -23.69 \text{ Btu (in)}$$

Now, writing the energy balance: $\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = Q - W$

With $\Delta U = m\Delta u$

1 $$\Delta u = \frac{Q - W}{m} = \frac{(-2.1 \text{ Btu}) - (-23.69 \text{ Btu})}{0.4 \text{ lb}} = 54.0 \text{ Btu}$$

---

1. The amount of energy transfer in by work exceeds the amount of energy transfer out by heat, resulting in a net increase in internal energy.

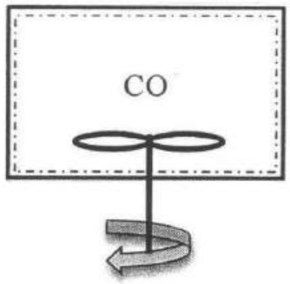
PROBLEM 2.67

Known: Carbon monoxide (CO) is contained in a rigid tank with a paddle wheel that transfers energy to the air at a constant rate of 14 W for 1 h. During the process, the specific internal energy of the carbon monoxide increases.

Find: Determine the specific volume at the final state, in m^3/kg; the energy transfer by work, in kJ; and the energy transfer by heat transfer, in kJ, with direction.

Schematic and Given Data:

$V = 1\ m^3$
$m = 4\ kg$
$\Delta u = 10\ kJ/kg$
$\dot{W} = -14\ W$
$\Delta t = 1\ h$



Engineering Model:
(1) The carbon monoxide within the tank is the closed system.
(2) The tank is rigid, therefore $V_1 = V_2$.
(3) The system experiences no change in potential and kinetic energy.

Analysis:

(a) The mass and volume remain constant in the process due to assumptions (1) and (2), therefore

$$v = \frac{V}{m} = \frac{1\ m^3}{4\ kg} = 0.25\frac{m^3}{kg}$$

(b) To evaluate W, in kJ, integrate the following

$$\int_0^{1\,h} \dot{W}dt = \int_0^{1\,h} (-14\ W)dt = (-14\ W)(1\ h)\left|\frac{3600\ s}{1\ h}\right|\left|\frac{1\ \frac{J}{s}}{1\ W}\right|\left|\frac{1\ kJ}{1000\ J}\right| = -50.4\ kJ$$

The minus sign for W indicates that energy is added to the system by work, as expected.

(c) To evaluate Q, in kJ, use the closed system energy balance

$$\Delta KE + \Delta PE + \Delta U = Q - W$$

$$Q = \Delta U + W = m\Delta u + W = (4\ kg)\left(10\ \frac{kJ}{kg}\right) + (-50.4\ kJ) = -10.4\ kJ$$

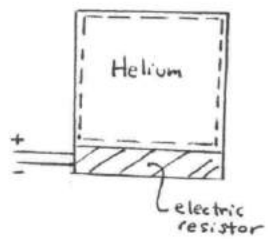
Energy is removed from the system through heat transfer.

PROBLEM 2.68

KNOWN: Data are provided for helium contained in a closed, rigid tank fitted with an electrical resistance.

FIND: Plot the change in energy of the helium, in kJ, for $t \geq 0$ and comment.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The helium is the system.
2. For the system, $\dot{W} = 0$.

ANALYSIS: An energy rate balance reads

$$\frac{dE}{dt} = \dot{Q} - \cancel{\dot{W}}^{\,0}$$

As the system receives energy by heat transfer from the resistor at a rate of 1 kW and loses energy by heat transfer to its surroundings at the rate of 5t W,

$$\dot{Q} = [1000 - \underbrace{5}_{W/min}\ \underbrace{t}_{min}]\ W$$

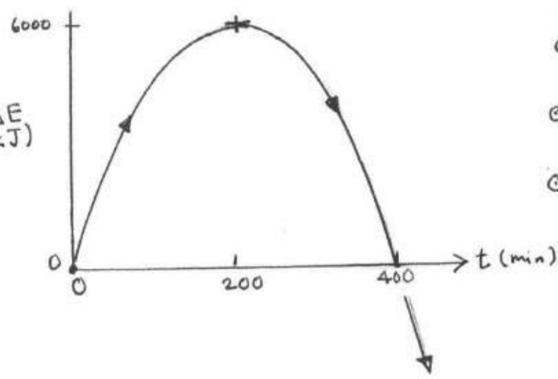
Thus,

$$\frac{dE}{dt} = 1000 - 5t$$

and

$$\Delta E = \int_0^t \frac{dE}{dt}dt = \int_0^t (1000-5t)dt = \left[1000t - \frac{5t^2}{2}\right] W\cdot min$$

$$= \left[1000t - \frac{5t^2}{2}\right] W\cdot min \left|\frac{1\ kJ/s}{10^3\ W}\right|\left|\frac{60 s}{min}\right| = [60t - 0.15\underbrace{t^2}_{t \text{ in min.}}]\ kJ$$



$\Delta E = E(t) - E(0) = E(t) - E_0$

- $0 \to 200$ min. Energy increases from the initial value at $t=0$: E_0.
- $200 \to 400$ min. Energy decreases to its initial value, E_0.
- 400 min $\to$. Energy decreases from its initial value, E_0.

Note that since any arbitrary value E_0 can be assigned to the energy of the system at $t=0$, no particular significance can be attached to the value of energy at the initial state or at any other state. Only changes in the energy of the system have significance.

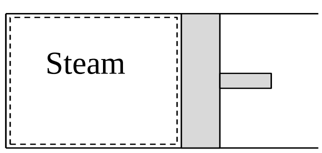
PROBLEM 2.69

Steam in a piston-cylinder assembly undergoes a polytropic process. Data for the initial and final states are given in the accompanying table. Kinetic and potential energy effects are negligible. For the process, determine the work and heat transfer, each in Btu per lb of steam.

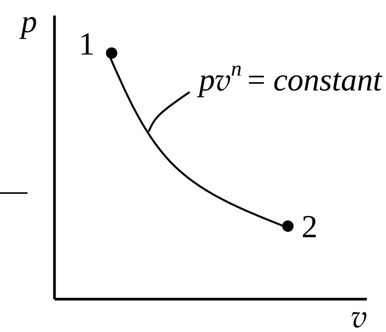
State	p (lbf/in.^{2})	v (ft^3/lb)	u (Btu/lb)
1	100	4.934	1136.2
2	40	11.04	1124.2

KNOWN: Steam undergoes a polytropic process in a piston-cylinder assembly. Data are known at the initial and final states.

FIND: Determine the work and heat transfer, each per unit mass of steam.



State	p (lbf/in.^{2})	v (ft^3/lb)	u (Btu/lb)
1	100	4.934	1136.2
2	40	11.04	1124.2



ENGINEERING MODEL: (1) The steam is a closed system. (2) The process is polytropic process, and volume change is the only work mode. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: Since the process is polytropic, Eq 2.17 applies for the work:

$$W/m = \int_{v_1}^{v_2} p\,dv = \int_{v_1}^{v_2} \frac{const}{v^n}\,dv = \frac{(p_2 v_2 - p_1 v_1)}{1-n}$$

The pressures and specific volumes are known at each state, but n is unknown. To find n, pv^n = *constant* , as follows:

$$p_1 v_1^n = p_2 v_2^n \quad \rightarrow \quad \frac{p_1}{p_2} = \left(\frac{v_2}{v_1}\right)^n \quad \rightarrow \quad n = \frac{\ln(p_1/p_2)}{\ln(v_2/v_1)} = \frac{\ln(100/40)}{\ln(11.04/4.934)} = 1.1377$$

Thus

$$W/m = \frac{(40\text{ lbf/in.}^2)(11.04\text{ ft}^3/\text{lb}) - (100)(4.934)}{1-1.1377}\left|\frac{144\text{ in.}^2}{1\text{ ft}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right| = 69.63\text{ Btu/lb (out)} \longleftarrow$$

The heat transfer is obtained using the energy balance.

PROBLEM 2.69 (CONTINUED)

$$\Delta U + \Delta KE + \Delta PE = Q - W \quad \rightarrow \quad Q = \Delta U + W$$

With $\Delta U = m\,\Delta u = m(u_2 - u_1)$

$$Q/m = (u_2 - u_1) + (W/m) = (1124.2 \; - 1136.2) \text{ Btu/lb} + (69.63 \text{ Btu/lb})$$

$$= \; 57.63 \text{ Btu/lb} \;\; \text{(in)}$$

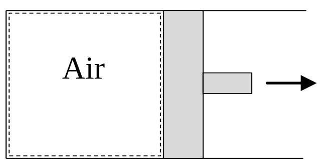
PROBLEM 2.70

Air expands adiabatically in a piston-cylinder assembly from an initial state where p_1 = 100 lbf/in.^{2}, v_1 = 3.704 ft^3/lb, and T_1 = 1000 ^{o}R, to a final state where p_2 = 50 lbf/in.^{2} The process is polytropic with n = 1.4. The change in specific internal energy, in Btu/lb, can be expressed in terms of temperature change as Δu = (0.171)(T_2 – T_1). Determine the final temperature, in ^{o}R. Kinetic and potential energy effects can be neglected.

KNOWN: Air undergoes a polytropic process with known n in a piston-cylinder assembly. Data are known at the initial and final states, and the change in specific internal energy is expressed as a function of temperature change.

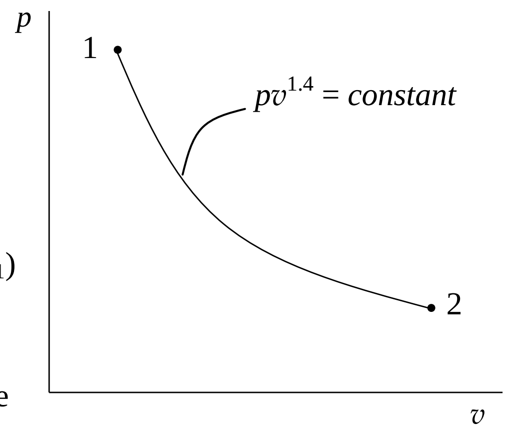
FIND: Determine the final temperature.

SCHEMATIC AND GIVEN DATA:



Q = 0

p_1 = 100 lbf/in.^{2}
v_1 = 3.704 ft^3/lb
T_1 = 1000 ^{o}R
p_2 = 50 lbf/in.^{2}
Δu = (0.171Btu/lb·^{o}R)(T_2 – T_1)



ENGINEERING MODEL: (1) The air is a closed system. (2) The process is polytropic with n = 1.4 and volume change is the only work mode. (3) The process is adiabatic: Q = 0. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: To find the final temperature, we will use the energy balance with the given expression for change in specific internal energy as a function of temperature change. First, determine the work using Eq. 2.17

$$W/m = \int_{v_1}^{v_2} p\,dv = \int_{v_1}^{v_2} \frac{const}{v^{1.4}}\,dv = \frac{(p_2v_2 - p_1v_1)}{1-1.4}$$

For the polytropic proess, $p_1v_1^{1.4} = p_2v_2^{1.4}$. Thus

$$v_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{1.4}} v_1 = \left(\frac{100\text{ lbf/in.}^2}{50\text{ lbf/in.}^2}\right)^{\frac{1}{1.4}} (3.704\text{ ft}^3\text{/lb}) = 6.077\text{ ft}^3\text{/lb}$$

So, the work is

$$W/m = \frac{(50\text{ lbf/in.}^2)(6.077\text{ ft}^3\text{/lb}) - (100\,)(3.704)}{1-1.4}\left|\frac{144\text{ in.}^2}{1\text{ ft}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft·lbf}}\right| = 30.794\text{ Btu/lb}$$

The energy balance is: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - W$. With $\Delta U = m(u_2 - u_1)$

PROBLEM 2.70 (CONTINUED)

$$(u_2 - u_1) = -W/m$$

Inserting values

$$(0.171 \text{ Btu/lb}\cdot{}^{o}\text{R})(T_2 - 1000\ {}^{o}\text{R}) = -(30.794 \text{ Btu/lb})$$

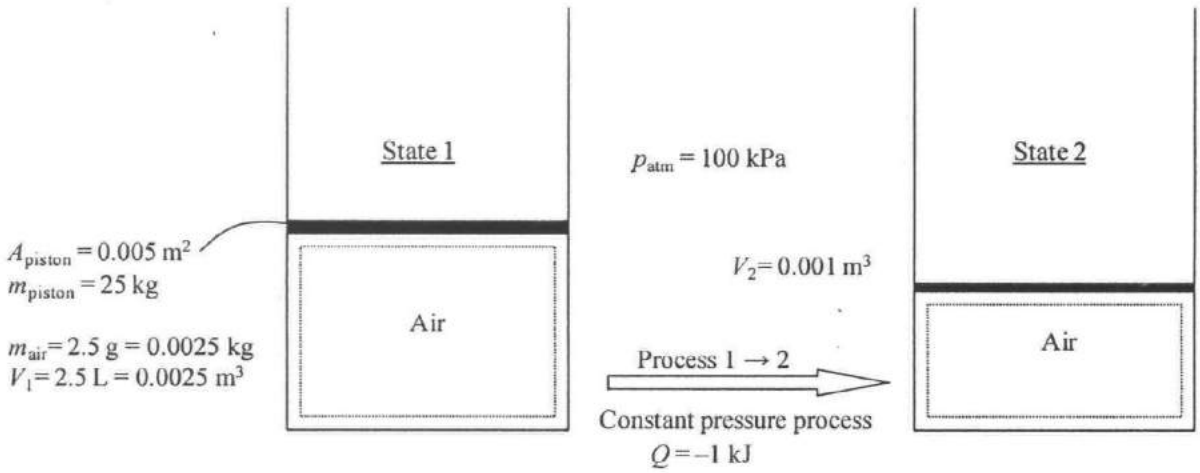
Solving; $T_2 = (-30.794)/(0.171) + 1000 = 819.9\ {}^{o}\text{R}$ ←

PROBLEM 2.71

Known: Vertical piston-cylinder assembly contains air of known mass. The initial and final volumes of air, and the heat transfer are specified.

Find: Determine the change in specific internal energy of the air, in kJ/kg.

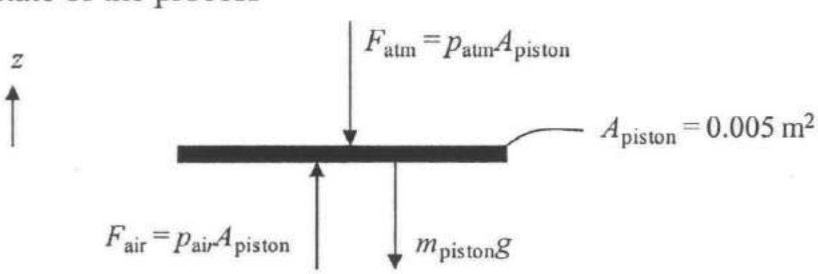
Schematic and Given Data:



Engineering Model:
(1) The air within the piston-cylinder assembly is the closed system.
(2) Kinetic and potential energy effects are negligible for the air.
(3) There is no friction between the piston and the cylinder wall.
(4) The process occurs slowly with no acceleration of the piston.
(5) The acceleration of gravity is constant.

Analysis:

Using a FBD with $\Sigma F_z = 0$, determine p_{air} which is the pressure exerted by the air at each state of the process



PROBLEM 2.71 (Continued)

$$p_{\text{air}} A_{\text{piston}} = p_{\text{atm}} A_{\text{piston}} + m_{\text{piston}} g$$

$$p_{\text{air}} = p_{\text{atm}} + \frac{m_{\text{piston}} g}{A_{\text{piston}}} = 100\text{kPa} + \frac{(25\text{kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)}{0.005\text{m}^2}\left|\frac{1\text{N}}{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right|\left|\frac{1\text{kPa}}{10^3\frac{\text{N}}{\text{m}^2}}\right|$$

$$p_{\text{air}} = (100 + 49)\text{kPa} = 149\text{ kPa}$$

Thus the pressure of the air remains constant.

To evaluate W, in kJ, begin with Eq. 2.17, noting that the pressure is constant, and integrate

$$W = \int_{V_1}^{V_2} p\,dV = p\int_{V_1}^{V_2} dV = p(V_2 - V_1) = 149\text{kPa}(0.001 - 0.0025)\text{m}^3\left|\frac{1000\frac{\text{N}}{\text{m}^2}}{1\text{kPa}}\right|\left|\frac{1\text{kJ}}{1000\text{N}\cdot\text{m}}\right| = -0.2235\text{ kJ}$$

Determine the change in specific internal energy, in kJ/kg, using the closed system energy balance

$$\Delta\text{KE} + \Delta\text{PE} + \Delta U = Q - W$$

$$\Delta U = Q - W = -1\text{ kJ} - (-0.2235\text{kJ}) = -0.7765\text{ kJ}$$

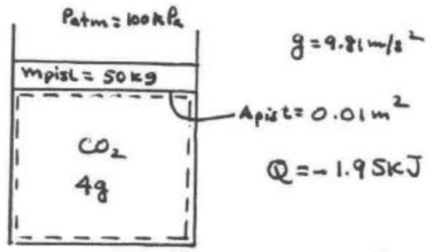
$$\Delta U = m(u_2 - u_1) \quad \text{or} \quad (u_2 - u_1) = \frac{\Delta U}{m} = \frac{-0.7765\text{ kJ}}{0.0025\text{ kg}} = -310.6\frac{\text{kJ}}{\text{kg}}$$

PROBLEM 2.72

KNOWN: Data are provided for CO_2 gas contained in a piston-cylinder assembly.

FIND: For the CO_2 determine the pressure and the final specific internal energy.

SCHEMATIC & GIVEN DATA:



$g = 9.81\ m/s^2$

$Q = -1.95\ kJ$

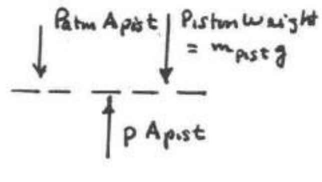
$V_1 = 0.005\ m^3$ $V_2 = 0.0025\ m^3$

$u_1 = 657\ \frac{kJ}{kg}$ $u_2 = ?$

ENGR. MODEL:

1. The CO_2 is the closed system.
2. The moving boundary is the only work mode.
3. As cooling occurs slowly there is no acceleration of the piston. The value of g remains constant.
4. Friction between the piston and cylinder can be ignored.
5. For the CO_2, $\Delta KE = 0$ and ΔPE can be ignored.

ANALYSIS: (a) Since there is no friction and the piston is not accelerated, the force exerted by the CO_2 in the cylinder on the bottom of the piston is equal to the weight of the piston plus the force exerted by the atmosphere on the top of the piston:



$$\Rightarrow \quad p\,A_{pist} = p_{atm} A_{pist} + m_{pist}\, g$$

$$p = p_{atm} + \frac{m_{pist}\, g}{A_{pist}}$$

$$= 100\ kPa + \frac{(50\ kg)(9.81\ m/s^2)}{0.01\ m^2}\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|\left|\frac{1\ kPa}{10^3\ N/m^2}\right|$$

$$= 149.1\ kPa\left|\frac{1\ bar}{10^2\ kPa}\right| = 1.491\ bar \quad \longleftarrow p$$

(b) The work can be evaluated using Eq. 2.17. Since the pressure remains constant

$$W = \int_1^2 p\, dV = p[V_2 - V_1] = 1.491\ bar\left|\frac{10^5\ N/m^2}{1\ bar}\right|\left[0.0025 - 0.005\right] m^3 \left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

$$= -0.373\ kJ$$

An energy balance for the CO_2 reads,

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$$

$$\Delta U = -1.95\ kJ - (-0.373\ kJ) = -1.577\ kJ$$

Then, with $\Delta U = m(u_2 - u_1)$,

$$u_2 = \frac{\Delta U}{m} + u_1$$

$$= \frac{-1.577\ kJ}{(4/1000)\ kg} + 657\ kJ/kg$$

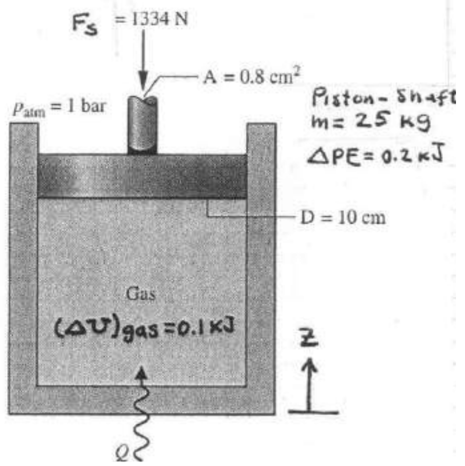
$$= 262.8\ \frac{kJ}{kg} \quad \longleftarrow u_2$$

PROBLEM 2.73

KNOWN: A gas contained in a piston-cylinder assembly is slowly heated. State data and operating data are provided.

FIND: Determine the work done by the shaft mounted on the top of the piston and work done in displacing the atmosphere, each in kJ. Also, determine the heat tranfer to the gas, in kJ, and develop an accounting of the heat transfer.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The closed system is the gas plus the piston and attached shaft.
2. There is no overall change in kinetic energy. For the piston-shaft, $\Delta U = 0$. For the gas, $\Delta PE = 0$.
3. $g = 9.81\ m/s^2$

ANALYSIS:

The work can be evaluated using $F\Delta z$, where Δz is the change in elevation of the piston-shaft found as follows:

$$\Delta PE = mg\,\Delta z$$

$$\Rightarrow \Delta z = \frac{\Delta PE}{mg} = \frac{0.2\ kJ}{(25 kg)(9.81 m/s^2)}\left|\frac{10^3 N\cdot m}{1 kJ}\right|\left|\frac{1 kg\cdot m/s^2}{1 N}\right|$$
$$= 0.82\ m$$

Thus, the work done by the shaft is

$$W_s = F_s\,\Delta z = (1334\ N)(0.82\ m)\left|\frac{1\ kJ}{10^3 N\cdot m}\right| = 1.094\ kJ$$

The work done in displacing the atmosphere is $W_{atm} = (P_{atm} A_{net})\Delta z$, where A_{net} is the net area: Area of piston face less area of the shaft. That is,

$$A_{net} = \left[\frac{\pi D^2}{4} - A\right] = \left[\frac{\pi (10 cm)^2}{4} - 0.8 cm^2\right] = 77.74\ cm^2.\ \text{Thus}$$

$$W_{atm} = \left(10^5 \frac{N}{m^2}\right)(77.74\ cm^2)\left|\frac{1 m}{100 cm}\right|^2\left|\frac{1\ kJ}{10^3 N\cdot m}\right|(0.82 m) = 0.637\ kJ$$

An energy balance for the system reads

$$\left\{[\cancel{\Delta KE} + \Delta PE + \cancel{\Delta U}]_{\text{piston-shaft}} + [\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U]_{gas}\right\} = Q - W$$

$$\Rightarrow Q = (\Delta PE)_{\text{piston-shaft}} + (\Delta U)_{gas} + W$$
$$= (0.2\ kJ) + (0.1\ kJ) + [1.094\ kJ + 0.637\ kJ] = 2.031\ kJ$$

ENERGY "balance sheet":

ENERGY IN:

$Q = 2.031\ kJ$

DISPOSITION OF THE ENERGY IN:

ENERGY STORED: $(\Delta U)_{gas}$	0.10 kJ	(4.92%)
ENERGY STORED: $(\Delta PE)_{\text{piston-shaft}}$	0.20 kJ	(9.85%)
ENERGY OUT BY WORK: ✓ SHAFT	1.094 kJ	(53.87%)
✓ ATM	0.637 kJ	(31.36%)
	2.031 kJ	

PROBLEM 2.74

KNOWN: A system undergoes a power cycle consisting of four processes in series.

FIND: Complete the table of energy values provided for the cycle and evaluate the thermal efficiency.

SCHEMATIC & GIVEN DATA:

Process	ΔE	Q	W
1-2	-1200	0	
2-3		800	
3-4		-200	-200
4-1	400		400

ANALYSIS:

(a) Process 1-2: $\Delta E = Q_{12} - W_{12} \Rightarrow W_{12} = Q_{12} - \Delta E = 0 - (-1200) = +1200$

Process 3-4: $\Delta E = Q_{34} - W_{34} \Rightarrow \Delta E = (-200) - (-200) = 0$

Process 4-1: $\Delta E = Q_{41} - W_{41} \Rightarrow Q_{41} = \Delta E + W_{41} = 400 + 600 = 1000$

Process 2-3:

Method 1: use $\Sigma(\Delta E) = 0$ to get

$$(-1200) + (E_3 - E_2) + (0) + (400) = 0$$

$$\Rightarrow (E_3 - E_2) = 800$$

Then, $\Delta E = Q_{23} - W_{23} \Rightarrow W_{23} = Q_{23} - \Delta E = 800 - 800 = 0$

Method 2: Use $\Sigma(Q) = \Sigma(W)$ to get

$$\underbrace{0 + 800 + (-200) + (1000)}_{1600} = 1200 + W_{23} + (-200) + (600)$$

$$= 1600 + W_{23} \Rightarrow W_{23} = 0$$

Then $(E_3 - E_2) = Q_{23} - \cancel{W_{23}} = 800$

(b) Thermal efficiency: $\eta = \dfrac{W_{cycle}}{Q_{in}}$

$$W_{cycle} = (+1200) + (0) + (-200) + (600) = 1600$$

$$Q_{in} = (+800) + (+1000) = 1800$$

$$\therefore \quad \eta = \frac{1600}{1800} = 0.889 \; (88.9\%)$$

PROBLEM 2.75

KNOWN: A system undergoes a power cycle consisting of four processes in series.

FIND: Complete the table of energy values provided for the cycle and evaluate the thermal efficiency.

SCHEMATIC & GIVEN DATA:

Process	ΔU	ΔKE	ΔPE	ΔE	Q	W
1–2	950	50	0	+1000 (c)	1000	0 (e)
2–3	−500 (a)	0	50	−450	0 (f)	450
3–4	−650	+50 (b)	0	−600	−600 (g)	0
4–1	200	−100	−50	+50 (d)	0	−50 (h)

ANALYSIS:

(a) For a cycle, the overall changes in U, KE, PE and E are zero:

$$\Sigma(\Delta U) = 950 + (a) - 650 + 200 = 0 \Rightarrow (a) = -500.$$

$$\Sigma(\Delta KE) = 50 + 0 + (b) - 100 = 0 \Rightarrow (b) = +50$$

$$\Sigma(\Delta PE) = 0 + 50 + 0 - 50 = 0 \checkmark$$

$$\Sigma(\Delta E) = (c) - 450 - 600 + (d) = 0 \Rightarrow (c) + (d) = 1050$$

For Process 1-2, $\Delta E = \Delta U + \Delta KE + \Delta PE$

$$= 950 + 50 + 0 = \underset{(c)}{1000}$$

So, (d) = +50

Also, for Process 1-2,

$$\Delta E = Q - W \Rightarrow W = \Delta E - Q$$
$$= 1000 - 1000 = \underset{(e)}{0}$$

For Process 2-3,

$$\Delta E = Q - W \Rightarrow Q = \Delta E + W$$
$$= -450 + 450 = \underset{(f)}{0}$$

For Process 3-4,

$$\Delta E = Q - W \Rightarrow Q = \Delta E + W$$
$$= -600 + 0 = -600 (g)$$

For Process 4-1,

$$\Delta E = Q - W \Rightarrow W = Q - \Delta E$$
$$= 0 - 50 = -50 (h)$$

(b) For any power cycle,

$$\eta = \frac{W_{cycle}}{Q_{in}}$$

Here, $W_{cycle} = 0 + 450 + 0 - 50 = 400$

$Q_{in} = 1000$

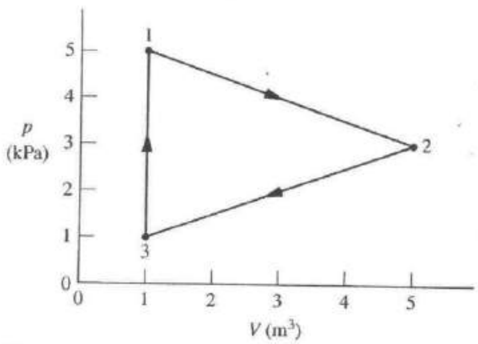
$$\Rightarrow \eta = \frac{400}{1000} = 0.40 \quad (40\%)$$

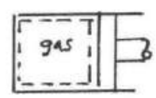
PROBLEM 2.76

KNOWN: Data are provided for a power cycle executed by a gas in a piston-cylinder assembly.

FIND: For each process evaluate W. Find Q for processes 1-2, 2-3. Evaluate the thermal efficiency.

SCHEMATIC & GIVEN DATA:





$U_2 - U_1 = 15\,kJ$
$Q_{31} = 10\,kJ$

Fig. P2.73

ENGR. MODEL

1. The gas is the closed system.
2. Volume change is the only work mode.
3. For each process, $\Delta KE = \Delta PE = 0$

ANALYSIS: (a) The work can be evaluated from Eq. 2.17. For Process 3-1, the piston does not move (volume is constant). Thus, $W_{31} = 0$. ← W_{31}

For Processes 1-2 and 2-3, the work can be evaluated geometrically. That is,

$$W_{12} = P_{ave}[V_2 - V_1] = \left(\frac{P_1+P_2}{2}\right)(V_2 - V_1) = \left[\left(\frac{5+3}{2}\right)kPa\right]\left[5-1\right]m^3 \left|\frac{10^3\,N/m^2}{1\,kPa}\right|\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|$$
$$= 16\,kJ$$ ← W_{12}

$$W_{23} = P_{ave}[V_3 - V_2] = \left(\frac{P_3+P_2}{2}\right)(V_3 - V_2) = \left[\left(\frac{3+1}{2}\right)kPa\right]\left[1-5\right]m^3 \left|\frac{10^3\,N/m^2}{1\,kPa}\right|\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|$$
$$= -8\,kJ$$ ← W_{23}

(b) Q_{31} is given. For Process 1-2, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{12} - W_{12}$

$\Rightarrow Q_{12} = \Delta U + W_{12} = 15\,kJ + 16\,kJ = 31\,kJ$ ← Q_{12}

For Process 2-3, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{23} - W_{23}$

$\Rightarrow Q_{23} = (U_3 - U_2) + W_{23}$

To find $(U_3 - U_2)$, note that since internal energy is a property

$$(U_2 - U_1) + (U_3 - U_2) + (U_1 - U_3) = 0$$

$$\Rightarrow (U_3 - U_2) = -\underbrace{(U_2 - U_1)}_{15\,kJ} - (U_1 - U_3)$$

Energy balance for Process 3-1:
$(U_1 - U_3) = Q_{31} - \cancel{W_{31}}^{\,0}$
$= 10\,kJ$

$\therefore \quad (U_3 - U_2) = -15\,kJ - 10\,kJ = -25\,kJ$

① $\quad \therefore Q_{23} = -25\,kJ + (-8\,kJ) = -33\,kJ$ ← Q_{23}

(c) For any power cycle, the thermal efficiency is $\eta = \dfrac{W_{cycle}}{Q_{in}}$

Here, $W_{cycle} = W_{12} + W_{23} + W_{31} = 16 - 8 + 0 = 8\,kJ$
$Q_{in} = Q_{12} + Q_{31} = 31 + 10 = 41\,kJ$

$$\therefore \eta = \frac{8\,kJ}{41\,kJ} = 0.195\ (19.5\%)$$ ← η

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1. Also, note that for any cycle, $W_{cycle} = Q_{cycle}$ (Eq. 2.40). Thus
$W_{12} + W_{23} + W_{31} = Q_{12} + Q_{23} + Q_{31} \Rightarrow Q_{23} = W_{12} + W_{23} + W_{31} - Q_{12} - Q_{31}$, or
$Q_{23} = 16 + (-8) + 0 - 31 - 10 = -33\,kJ$.

PROBLEM 2.77

KNOWN: A gas within a piston–cylinder assembly undergoes a thermodynamic cycle consisting of three processes in series.

FIND: Determine Q_{12}, Q_{31}, U_3. Determine if the cycle can be a power cycle.

SCHEMATIC & GIVEN DATA:

Process 1-2: Compression with pV = *constant*, W_{12} = -104 kJ, U_1 = 512 kJ, U_2 = 690 kJ
Process 2-3: W_{23} = 0, Q_{23} = -150 kJ
Process 3-1: W_{31} = +50 kJ

ENGR. MODEL:

1. The gas is the closed system.
2. Volume change is the only work mode.
3. For each process, $\Delta KE = \Delta PE = 0$.

ANALYSIS: (a) Process 1-2, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{12} - W_{12} \Rightarrow$

$$Q_{12} = [U_2 - U_1] + W_{12} = (690 - 512)\text{kJ} + (-104\text{kJ}) = +74\text{kJ} \quad \longleftarrow Q_{12}$$

For any cycle, $W_{cycle} = Q_{cycle}$ (Eq. 2.40). Thus

$$W_{12} + W_{23} + W_{31} = Q_{12} + Q_{23} + Q_{31}$$

$$\Rightarrow Q_{31} = W_{12} + W_{23} + W_{31} - Q_{12} - Q_{23}$$

$$= (-104) + 0 + 50 - 74 - (-150) = +22\text{ kJ} \quad \longleftarrow Q_{31}$$

Process 3-1: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{31} - W_{31} \Rightarrow U_1 - U_3 = Q_{31} - W_{31}$

(1) $\Rightarrow U_3 = U_1 - Q_{31} + W_{31} = 512 - 22 + 50 = 540\text{kJ} \quad \longleftarrow U_3$

(b) A power cycle is one for which $W_{cycle} > 0$. For the current cycle,

$$W_{cycle} = W_{12} + W_{23} + W_{31}$$

$$= +(-104) + (0) + (50) = -54\text{kJ}$$

*

NO. This cycle cannot be a power cycle. ⟵

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1. As checks on these calculations, note that

Process 2-3: $(U_3 - U_2) = Q_{23} - W_{23}$

$$= (-150\text{kJ}) - 0 \Rightarrow U_3 - U_2 = -150\text{kJ}$$

$$\Rightarrow U_3 = 690 - 150 = 540\text{kJ}$$

Since U is a property,

$$(U_2 - U_1) + (U_3 - U_2) + (U_1 - U_3) = 0$$

$$(690 - 512) + (U_3 - 690) + (512 - U_3) = 0$$

$$540 \qquad 540$$

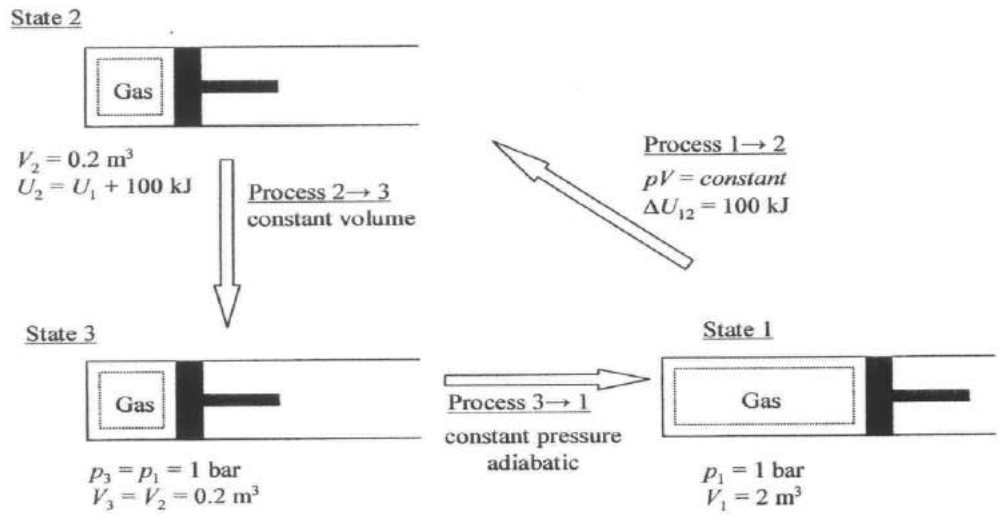
$$178 + (-150) + (-28) = 0 \checkmark$$

PROBLEM 2.78

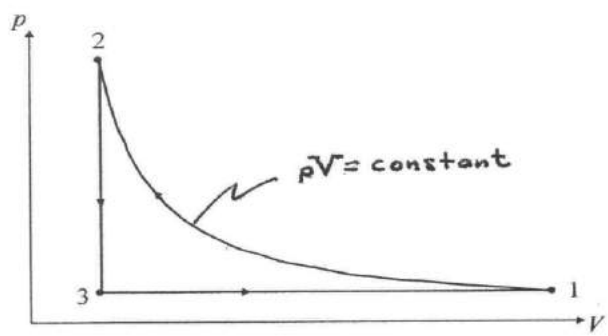
KNOWN: A closed system undergoes a cycle consisting of three processes.

FIND: Determine the net work of the cycle and the heat transfer for process 2-3. Identify whether the cycle is a power cycle or a refrigeration cycle and explain.

SCHEMATIC AND GIVEN DATA:



p-V Diagram



ENGINEERING MODEL:
1. The gas is a closed system.
2. Kinetic and potential energy effects are negligible
3. Process 1-2 is polytropic in which $pV = constant$.
4. Process 2-3 is constant volume.
5. Process 3-1 is constant pressure and adiabatic.

ANALYSIS:
Cycle work is the sum of work associated with each process in the cycle

$$W_{cycle} = W_{12} + W_{23} + W_{31}$$

Process 1-2 is a polytropic process with $pV = constant$. Therefore, $p = constant/V$.

$$W_{12} = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{(\text{constant})dV}{V} = (\text{constant}) \int_{V_1}^{V_2} \frac{dV}{V} = (\text{constant}) \ln\left(\frac{V_2}{V_1}\right) = p_1 V_1 \ln\left(\frac{V_2}{V_1}\right)$$

$$W_{12} = (1\text{ bar})(2\text{ m}^3) \left| \frac{10^5 \frac{\text{N}}{\text{m}^2}}{1\text{ bar}} \right| \left| \frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}} \right| \ln\left(\frac{0.2\text{ m}^3}{2\text{ m}^3}\right) = -460.5\text{ kJ}$$

Process 2-3 is constant volume; thus $W_{23} = \int_{V_2}^{V_3} p\,dV = 0\text{ kJ}$

Process 3-1 is constant pressure; thus $W_{31} = \int_{V_3}^{V_1} p\,dV = p_1(V_1 - V_3)$

$$W_{31} = (1\text{ bar})(2\text{ m}^3 - 0.2\text{ m}^3) \left| \frac{10^5 \frac{\text{N}}{\text{m}^2}}{1\text{ bar}} \right| \left| \frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}} \right| = 180\text{ kJ}$$

Substituting the work associated with each process yields the cycle work

$$W_{cycle} = (-460.5\text{ kJ}) + 0\text{ kJ} + 180\text{ kJ} = \underline{\mathbf{-280.5\text{ kJ}}}$$

Since the net cycle work is into the cycle (negative), the cycle is a <u>refrigeration cycle</u>.

For process 2-3, an energy balance is

$$\Delta\text{KE}_{23} + \Delta\text{PE}_{23} + (U_3 - U_2) = Q_{23} - W_{23}$$

Neglecting changes in kinetic energy ($\Delta\text{KE}_{23} = 0$) and potential energy ($\Delta\text{PE}_{23} = 0$), substituting $W_{23} = 0$ (determined above), and solving for Q_{23} yield

$$Q_{23} = U_3 - U_2$$

For the cycle,

$$(U_2 - U_1) + (U_3 - U_2) + (U_1 - U_3) = 0$$

Solving for $(U_3 - U_2)$ yields

$$(U_3 - U_2) = -(U_2 - U_1) - (U_1 - U_3)$$

From the problem statement, $(U_2 - U_1) = 100\text{ kJ}$

For process 3-1, an energy balance is

$$\Delta\text{KE}_{31} + \Delta\text{PE}_{31} + (U_1 - U_3) = Q_{31} - W_{31}$$

Problem 2.78 (Continued) – Page 2

Neglecting changes in kinetic energy ($\Delta KE_{31} = 0$) and potential energy ($\Delta PE_{31} = 0$), substituting $Q_{31} = 0$ since process 3-1 is adiabatic, and solving for $(U_1 - U_3)$ give

$$(U_1 - U_3) = -W_{31} = -180 \text{ kJ}$$

Substituting for changes in internal energy gives

$$(U_3 - U_2) = -(100 \text{ kJ}) - (-180 \text{ kJ}) = 80 \text{ kJ}$$

Solving for Q_{23}

1 $$Q_{23} = U_3 - U_2 = \underline{\mathbf{80\ kJ}}$$

The heat transfer is positive during process 2-3, denoting energy transfer by heat into the gas during this process.

---

1. ***As an alternative solution***, for the overall cycle, $Q_{cycle} = Q_{12} + Q_{23} + Q_{31} = W_{cycle}$

Thus, $Q_{23} = W_{cycle} - Q_{12} - Q_{31}$

For process 1-2, an energy balance is

$$\Delta KE_{12} + \Delta PE_{12} + \Delta U_{12} = Q_{12} - W_{12}$$

Neglecting changes in kinetic energy ($\Delta KE_{12} = 0$) and potential energy ($\Delta PE_{12} = 0$) and solving for heat transfer give

$$Q_{12} = \Delta U_{12} + W_{12} = 100 \text{ kJ} + (-460.5 \text{ kJ}) = -360.5 \text{ kJ}$$

The heat transfer is negative during process 1-2, denoting energy transfer by heat from the gas during this process.

Since process 3-1 is adiabatic, $Q_{31} = 0$ kJ.

Substituting values for W_{cycle} and heat transfer associated with each process yields

$$Q_{23} = (-280.5 \text{ kJ}) - (-360.5 \text{ kJ}) - 0 \text{ kJ} = \underline{\mathbf{80\ kJ}}$$

The heat transfer is positive during process 2-3, denoting energy transfer by heat into the gas during this process.

Problem 2.79

A gas undergoes a cycle in a piston-cylinder assembly consisting of the following three processes:

Process 1-2: Constant pressure, $p = 1.4$ bar, $V_1 = 0.028\ \text{m}^3$, $W_{12} = 10.5$ kJ

Process 2-3: Compression with $pV = constant$, $U_3 = U_2$

Process 3-1: Constant volume, $U_1 - U_3 = -26.4$ kJ

There are no significant changes in kinetic or potential energy.

(a) Sketch the cycle on a p-V diagram.
(b) Calculate the net work for the cycle, in kJ.
(c) Calculate the heat transfer for process 1-2, in kJ

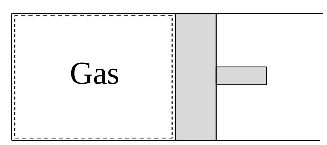
KNOWN: A gas undergoes a cycle consisting of three processes.

FIND: Sketch the cycle on a p-V diagram and determine the net work for the cycle and the heat transfer for process 1-2.

SCHEMATIC AND GIVEN DATA:
Process 1-2: Constant pressure, $p = 1.4$ bar, $V_1 = 0.028\ \text{m}^3$, $W_{12} = 10.5$ kJ

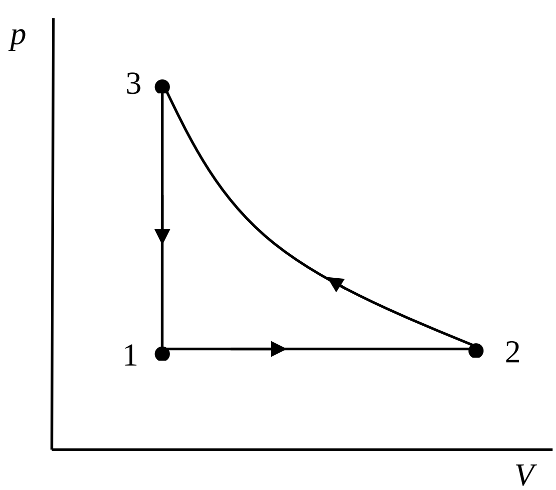
Process 2-3: Compression with $pV = constant$, $U_3 = U_2$

Process 3-1: Constant volume, $U_1 - U_3 = -26.4$ kJ



ENGINEERING MODEL: (1) The gas is a closed system. (2) Kinetic and potential energy effects are negligible. (3) The compression from state 2 to 3 is a polytropic process.

ANALYSIS: (a) Since $W_{12} > 0$, the process is an expansion. Thus



Problem 2.79 (Continued)

(b) The net work for the cycle is $W_{cycle} = W_{12} + W_{23} + \cancelto{0}{W_{31}}$. $W_{12} = 10.5$ kJ, so we need W_{23}.

$$W_{23} = \int_{V_2}^{V_3} p\,dV = \int_{V_2}^{V_3} \frac{const}{V}\,dV = (p_2V_2)\ln\left(\frac{V_3}{V_2}\right) = (p_2V_2)\ln\left(\frac{V_1}{V_2}\right) \qquad (*)$$

where $V_3 = V_1$ has been incorporated. But, we still need to evaluate V_2. For Process 1-2 at constant pressure

$$W_{12} = \int_{V_1}^{V_2} p\,dV = p(V_2 - V_1)$$

or

$$V_2 = \frac{W_{12}}{p} + V_1 = \frac{(10.5\text{ kJ})}{(1.4\text{ bar})}\left|\frac{10^3\text{N}\cdot\text{m}}{1\text{ kJ}}\right|\left|\frac{1\text{ bar}}{10^5\text{N/m}^2}\right| + 0.028\text{ m}^3 = 0.103\text{ m}^3$$

Thus, with Eq. (*)

$$W_{23} = (1.4\text{ bar})(0.103\text{ m}^3)\ln\left(\frac{0.028}{0.103}\right)\left|\frac{10^5\text{N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{N}\cdot\text{m}}\right| = -18.78\text{ kJ}$$

Thus

$$W_{cycle} = 10.5\text{ kJ} + (-18.78\text{ kJ}) + 0 = -8.28\text{ kJ} \longleftarrow$$

(c) To get Q_{12}, we apply the energy balance to process 1-2: $\cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} + (U_2 - U_1) = Q_{12} - W_{12}$

With $U_2 = U_3$,

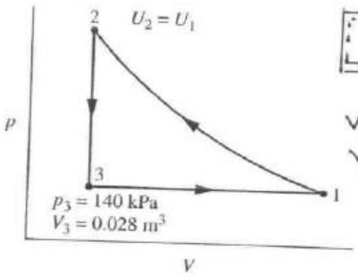
$$Q_{12} = (U_3 - U_1) + W_{12} = (+26.4\text{ kJ}) + (10.5\text{ kJ}) = 36.9\text{ kJ} \longleftarrow$$

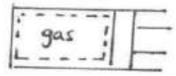
PROBLEM 2.80

KNOWN: A gas within a piston-cylinder assembly undergoes a thermodynamic cycle consisting of three processes in series.

FIND: Determine V_1, W_{12}, and Q_{12}. Determine if the cycle can be power cycle or a refrigeration cycle.

SCHEMATIC & GIVEN DATA:





$W_{31} = 10.5$ kJ
$W_{cycle} = -8.3$ kJ

ENGR. MODEL:

1. The gas is the closed system.
2. Volume change is the only work mode.
3. For each process, $\Delta KE = \Delta PE = 0$

ANALYSIS:

(a) To find V_1, note that

$$W_{31} = \int_3^1 p\,dV = \underset{140\,kPa}{p}\,[V_1 - V_3] \Rightarrow V_1 = V_3 + \frac{W_{31}}{p}$$

or

$$V_1 = 0.028\,m^3 + \frac{10.5\,kJ}{140\,kPa}\left|\frac{10^3\,N\cdot m}{1\,kJ}\right|\left|\frac{1\,kPa}{10^3\,N/m^2}\right| = 0.103\,m^3 \quad \xleftarrow{V_1}$$

(b) To find W_{12}, write $W_{cycle} = W_{12} + W_{23} + W_{31}$. Since the only work mode is volume change, the work is given by Eq. 2.17. Since the piston does not move in process 2-3 (volume is constant), $W_{23} = 0$. Thus

$$W_{12} = W_{cycle} - \cancel{W_{23}}^{\,0} - W_{31}$$
$$= -8.3\,kJ - 10.5\,kJ = -18.8\,kJ \quad \xleftarrow{W_{12}}$$

A power cycle is one for which $W_{cycle} > 0$. Here, we have $W_{cycle} = -8.3$ kJ. So, the cycle cannot be a power cycle, but it can be a refrigeration (or heat pump) cycle. $\xleftarrow{\text{cycle type}}$

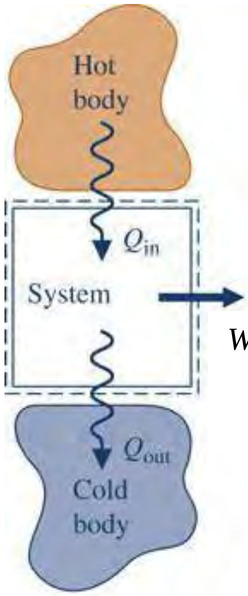
To find Q_{12}, write an energy balance: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{12} - W_{12}$

$$\Rightarrow Q_{12} = \underbrace{\cancel{\Delta U}^{\,0}}_{U_2 = U_1} + W_{12}$$

$$\Rightarrow Q_{12} = -18.8\,kJ \quad \xleftarrow{Q_{12}}$$

Problem 2.81

The net work of a power cycle operating as in Fig. 2.17a is 10,000 kJ, and the thermal efficiency is 0.4. Determine the heat transfers Q_{in} and Q_{out}, each in kJ.



W_{cycle} = 10,000 kJ

$\eta = 0.4$

$$\eta = \frac{W_{\mathrm{cycle}}}{Q_{\mathrm{in}}} \quad \rightarrow \quad Q_{\mathrm{in}} = \frac{W_{\mathrm{cycle}}}{\eta}$$

Q_{in} = (10,000 kJ) / (0.4) = 25,000 kJ ←

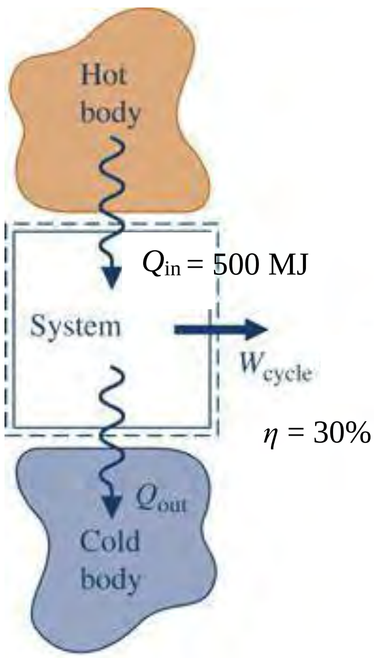
$W_{cycle} = Q_{cycle} = Q_{in} - Q_{out}$

Thus

$Q_{out} = Q_{in} – W_{cycle}$ = 25,000 – 10,000 = 15,000 kJ ←

Problem 2.82

For a power cycle operating as shown in Fig. 2.17a, the energy transfer by heat into the cycle, Q_{in}, is 500 MJ. What is the net work developed, in MJ, if the cycle thermal efficiency is 30%? What is the value of Q_{out}, in MJ?



$$\eta = \frac{W_{cycle}}{Q_{in}}$$

$W_{cycle} = \eta Q_{in} = (0.3)(500 \text{ MJ}) = 150 \text{ MJ}$ ←

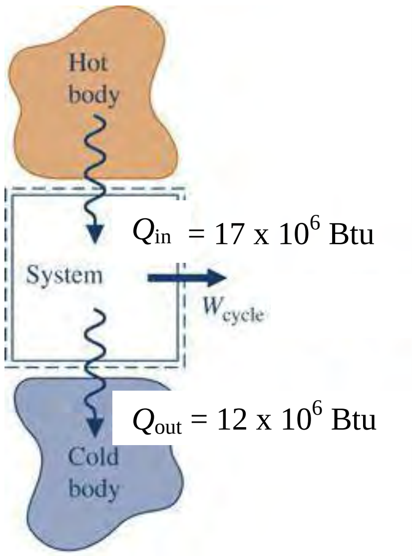
$W_{cycle} = Q_{cycle} = Q_{in} - Q_{out}$

Thus

$Q_{out} = Q_{in} - w_{cycle} = 500 \text{ MJ} - 150 \text{ MJ} = 350 \text{ MJ}$ ←

Problem 2.83

For a power cycle operating as in fig. 2.17a, Q_{in} = 17 x 10^6 Btu and Q_{out} = 12 x 10^6 Btu. Determine W_{cycle}, in Btu, and η.



$W_{cycle} = Q_{cycle} = Q_{in} - Q_{out}$

$= (17 \text{ x } 10^6) - (12 \text{ x } 10^6) = 5 \text{ x } 10^6$ Btu ←

$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{5 \text{ x } 10^6 \text{Btu}}{17 \text{ x } 10^6 \text{Btu}} = 0.294\ (29.4\%) \leftarrow$$

Alternatively

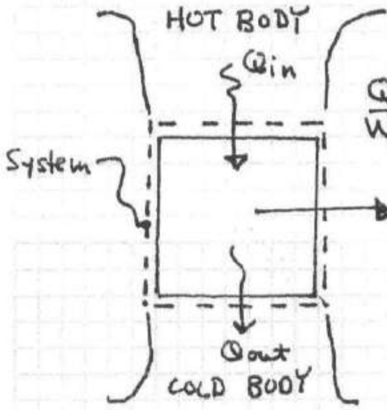
$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{12 \text{ x } 10^6}{17 \text{ x } 10^6} = 0.294$$

PROBLEM 2.84

KNOWN: Operating data are provided for a system undergoing a power cycle.

FIND: Determine the thermal efficiency.

SCHEMATIC & GIVEN DATA:



$$\frac{Q_{in}}{W_{cycle}} = 10^4 \frac{\text{Btu}}{\text{kW}\cdot\text{h}}$$

ENGINEERING MODEL:

1. The system undergoes a power cycle.
2. Energy transfers are positive in the direction of arrows on the schematic.

ANALYSIS: The thermal efficiency is $\eta = \dfrac{W_{cycle}}{Q_{in}}$.

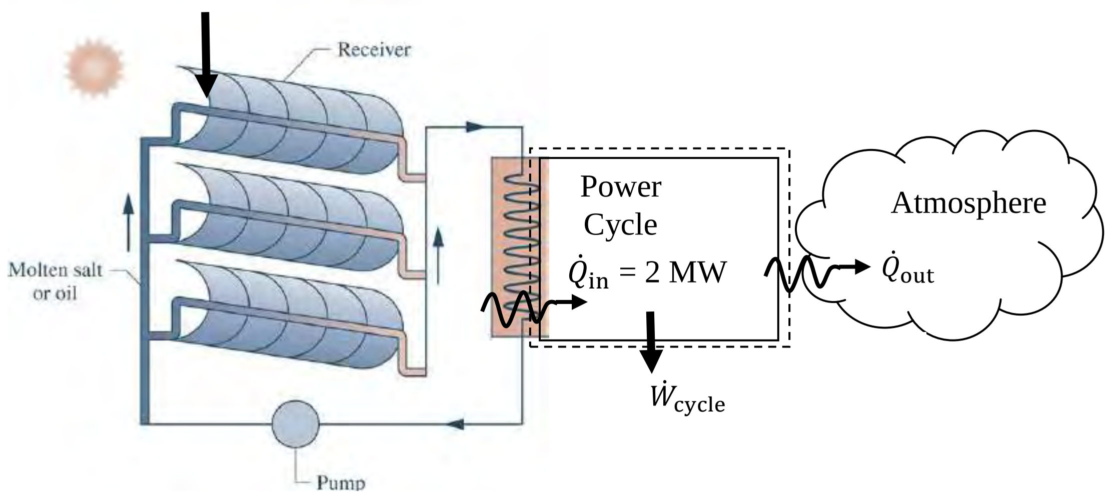
With given data,

(See unit conversions: 1 W = 3.413 Btu/h)

$$\eta = \left[\frac{1}{10^4 \frac{\text{Btu}}{\text{kW}\cdot\text{h}}}\right]\left|\frac{3413\ \text{Btu}}{1\ \text{kW}\cdot\text{h}}\right| = 0.3413 \quad (34.13\%)$$

Problem 2.85

A concentrating solar collector system, as shown in Fig. P2.85, provides energy by heat transfer to a power cycle at a rate of 2 MW. The cycle thermal efficiency is 36%. Determine the power developed by the cycle, in MW. What is the work output, in MW·h, for 4380 hours of steady-state operation? If the work is valued at \$0.08/kW·h, what is the total dollar value of the work output?



The power developed is

$$\dot{W}_{cycle} = \eta \dot{Q}_{in} = (0.36)\,(2\text{ MW}) = 0.72\text{ MW}$$

For 4380 hours of steady-state operation

$$W_{cycle} = \dot{W}_{cycle}\Delta t = (0.72\text{ MW})(4380\text{ h}) = 3153.6\text{ MW·h}$$

The total dollar value is

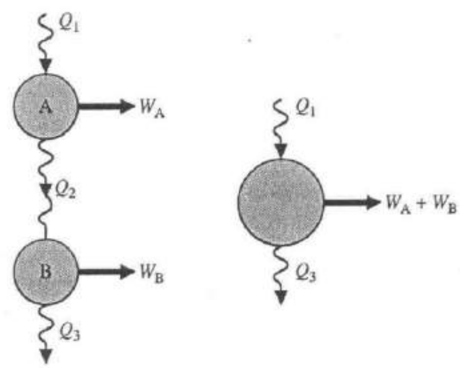
$$\$\text{ Value} = (3153.6\text{ MW·h})(\$0.08/\text{kW·h})\left|\frac{10^3\text{kW}}{1\text{ MW}}\right| = \$252{,}300$$

PROBLEM 2.86

KNOWN: Power cycles A and B operate in series.

FIND: Determine an expression for the thermal efficiency of an overall cycle consisting of A and B together in terms of η_A and η_B.

SCHEMATIC & GIVEN DATA:



(a) A and B in series (b) Overall cycle

ENGINEERING MODEL:

1. Cycles A, B and the overall cycle are power cycles.
2. Energy transfer is positive in the directions of the arrows on the schematic.

ANALYSIS:

$$\eta_A = \frac{W_A}{Q_1} = 1 - \frac{Q_2}{Q_1} \Rightarrow Q_2 = Q_1(1-\eta_A) \quad (1)$$

$$\eta_B = \frac{W_B}{Q_2} = 1 - \frac{Q_3}{Q_2} \Rightarrow Q_3 = Q_2(1-\eta_B) \quad (2)$$

$$= Q_1(1-\eta_A)(1-\eta_B) \quad (3)$$

$$\eta = \frac{(W_A + W_B)}{Q_1} = 1 - \frac{Q_3}{Q_1} \quad (4)$$

Introducing (3) into (4),

$$\eta = 1 - \frac{Q_1(1-\eta_A)(1-\eta_B)}{Q_1}$$

$$= 1 - (1-\eta_A)(1-\eta_B)$$

$$= 1 - (1 - \eta_A - \eta_B + \eta_A\eta_B)$$

① $\therefore \eta = \eta_A + \eta_B - \eta_A\eta_B$

1. Sample calculation: $\eta_A = 0.25$, $\eta_B = 0.32$

$$\eta = 0.25 + 0.32 - (0.25)(0.32)$$

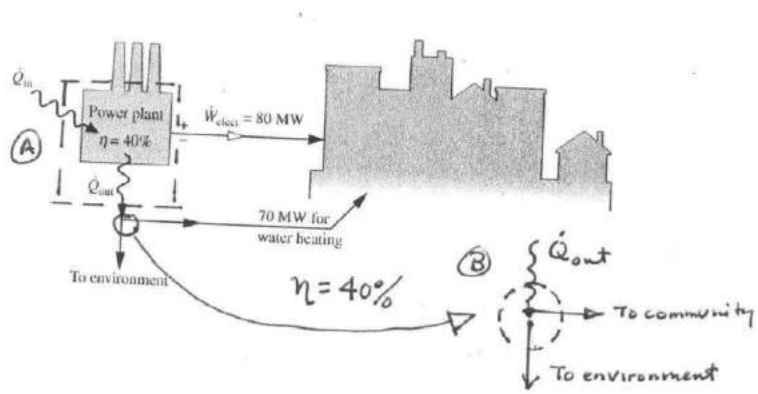
$$= 0.49 \ (49\%)$$

PROBLEM 2.87

KNOWN: Operating data are provided for a cogeneration power plant operating in a thermodynamic cycle at steady state.

FIND: Determine the rates at which energy is added by heat transfer and discarded to the environment. Also determine the value of the electricity generated.

SCHEMATIC & GIVEN DATA:



ENR. MODEL

1. As shown in the schematic, two systems — A and B — are considered.
2. Each system is at steady state. System A operates on a thermodynamic cycle.
3. Electricity is valued at \$0.08 per kW·h

ANALYSIS:

(a) $$\eta = \frac{\dot{W}_{elect}}{\dot{Q}_{in}} \Rightarrow \dot{Q}_{in} = \frac{\dot{W}_{elect}}{\eta} = \frac{80\text{ MW}}{0.40} = 200\text{ MW}$$ ← $\dot{Q}_{in}$

For the power plant, $\dot{W}_{cycle} = \dot{Q}_{cycle}$. That is

$$\dot{W}_{elect} = \dot{Q}_{in} - \dot{Q}_{out} \Rightarrow \dot{Q}_{out} = \dot{Q}_{in} - \dot{W}_{elect} = (200-80)\text{MW} = 120\text{MW}$$

(b) Considering system B,

$$\dot{Q}_{out} = \dot{Q}_{to\ environment} + \dot{Q}_{community}$$ (All terms are positive in the directions of the arrows.)

$$\Rightarrow \quad \dot{Q}_{to\ environment} = \dot{Q}_{out} - \dot{Q}_{community} = 120\text{MW} - 70\text{MW} = 50\text{MW}$$ ← To environment

(c) $$\begin{bmatrix}\text{Annual Value} \\ \text{of Electricity}\end{bmatrix} = (80\text{MW})\left|\frac{10^3\text{ kW}}{1\text{ MW}}\right|\left(365 \times 24\ \frac{\text{h}}{\text{year}}\right)\left(\frac{\$0.08}{\text{kW}\cdot\text{h}}\right)$$

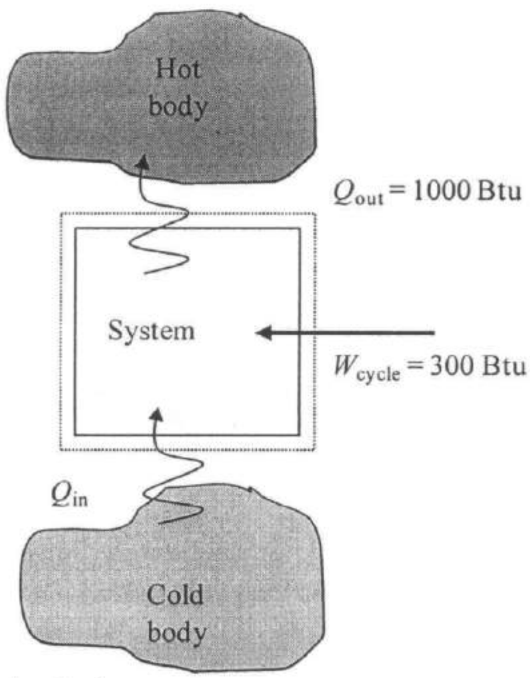
$$= \$\ 56.1\text{ M/year}$$ ← Annual value

PROBLEM 2.88

A refrigeration cycle operating as shown in Fig. 2.17*b* has $Q_{out} = 1000$ Btu and $W_{cycle} =$ 300 Btu. Determine the coefficient of performance for the cycle.

Solution:

Schematic and Given Data:



Analysis:

Using the following, determine β

$$\beta = \frac{Q_{in}}{W_{cycle}}$$

$$W_{cycle} = Q_{out} - Q_{in}$$

$$Q_{in} = Q_{out} - W_{cycle} = (1000 - 300)\,\text{Btu} = 700\,\text{Btu}$$

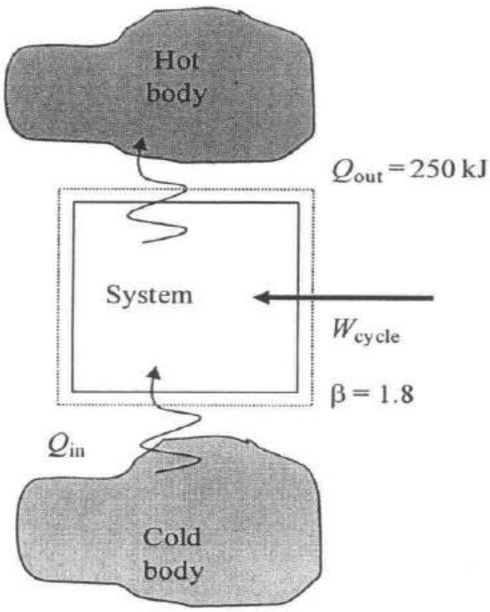
$$\beta = \frac{700\,\text{Btu}}{300\,\text{Btu}} = 2.33$$

Problem 2.89

A refrigeration cycle operating as shown in Fig. 2.17*b* has a coefficient of performance β = 1.8. For the cycle, $Q_{out} = 250$ kJ. Determine Q_{in} and W_{cycle}, each in kJ.

Solution:

Schematic and Given Data:



Analysis:

Using the following, determine Q_{out} and W_{cycle}, each in kJ

$$\beta = \frac{Q_{in}}{W_{cycle}} \text{ and } W_{cycle} = Q_{out} - Q_{in}$$

$$\beta = \frac{Q_{in}}{Q_{out} - Q_{in}}$$

$$Q_{in} = \beta(Q_{out} - Q_{in}) = Q_{out}\left(\frac{\beta}{1+\beta}\right) = 250\text{ kJ}\left(\frac{1.8}{1+1.8}\right) = 161\text{ kJ} \longleftarrow$$

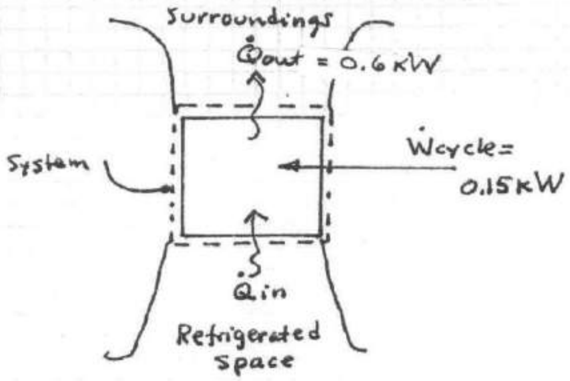
$$W_{cycle} = \frac{Q_{in}}{\beta} = \frac{161\text{ kJ}}{1.8} = 89\text{ kJ} \longleftarrow$$

PROBLEM 2.90

KNOWN: Steady-state operating data are provided for refrigerator.

FIND: Determine the rate energy is removed by heat transfer from the refrigerated space, in kW, and the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The system undergoes a refrigeration cycle.
2. Energy transfers are positive in the direction of the arrows on the schematic.
3. The cycle operates steadily.

ANALYSIS:

Applying Eq. 2.44 on a time rate basis:

$$\dot{W}_{cycle} = \dot{Q}_{out} - \dot{Q}_{in}$$

$$\Rightarrow \quad \dot{Q}_{in} = \dot{Q}_{out} - \dot{W}_{cycle}$$

$$= 0.6\text{ kW} - 0.15\text{ kW}$$

$$= 0.45\text{ kW} \qquad \longleftarrow \dot{Q}_{in}$$

Then, with Eq. 2.45 on a time rate basis,

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{cycle}}$$

$$= \frac{0.45\text{ kW}}{0.15\text{ kW}}$$

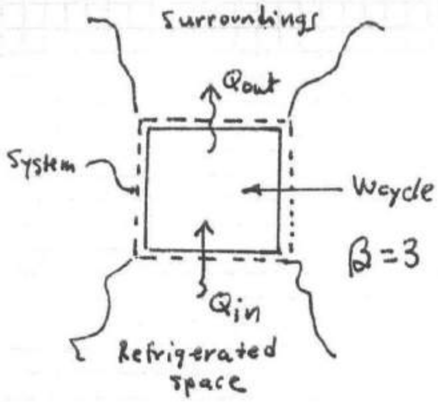
$$= 3 \qquad \longleftarrow \beta$$

PROBLEM 2.91

KNOWN: Operating and cost data are provided for a household refrigerator.

FIND: Determine the refrigerator's annual electricity requirement, in kW·h, and the amount of energy removed from its refrigerated space annually, in MJ.

SCHEMATIC & GIVEN DATA:



Annual cost of electricity = \$55.

Unit cost of electricity = 8 cents per kW·h

ENGINEERING MODEL:

1. The system undergoes a refrigeration cycle.
2. Energy transfers are positive in the direction of arrows on the schematic.
3. Electricity is valued at 8 cents per kW·h.

ANALYSIS:

(a)

$$\begin{bmatrix}\text{Annual}\\ \text{Cost of}\\ \text{Electricity}\end{bmatrix} = W_{cycle}\left[\frac{\$0.08}{\text{kW}\cdot\text{h}}\right]$$

in kW·h (annually)

$$\$55 = W_{cycle}\left[\frac{\$0.08}{\text{kW}\cdot\text{h}}\right]$$

$$\therefore \quad W_{cycle} = \left[\frac{\$55}{\$0.08/\text{kW}\cdot\text{h}}\right]$$

$$= 687.5 \text{ kW}\cdot\text{h (annually)}$$

(b) With Eq 2.45,

$$\beta = \frac{Q_{in}}{W_{cycle}}$$

$$\therefore Q_{in} = \beta W_{cycle}$$

$$= (3)(687.5 \text{ kW}\cdot\text{h})\left|\frac{1 \text{ kJ/s}}{1 \text{ kW}}\right|\left|\frac{3600 \text{ s}}{1 \text{ h}}\right|\left|\frac{1 \text{ MJ}}{10^3 \text{ kJ}}\right|$$

$$= 7425 \text{ MJ (annually)}$$

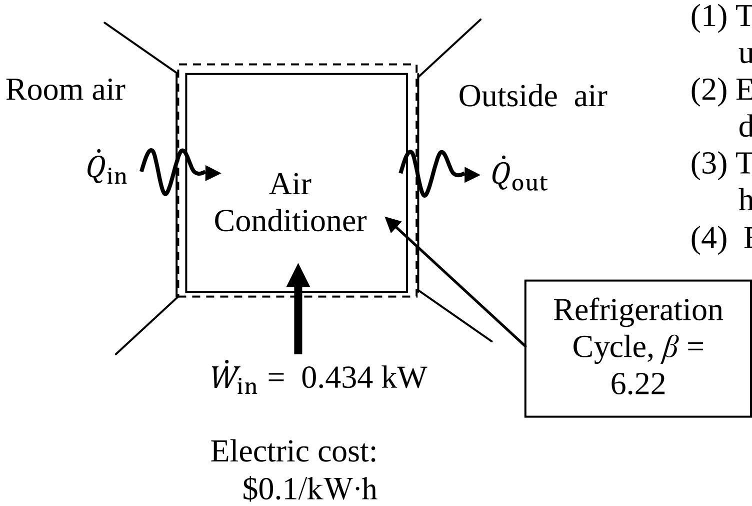
Problem 2.92

A window-mounted room air conditioner removes energy by heat transfer from a room and rejects energy by heat transfer to the outside air. For steady operation, the air conditioner cycle requires a power input of 0.434 kW and has a coefficient of performance of 6.22. Determine the rate that energy is removed from the room air, in kW. If electricity is valued at \$0.1/kW·h, determine the cost of operation for 24 hours of operation.

KNOWN: Steady-state operating data are provided for an air conditioner.

FIND: Determine the rate energy is removed from the room and air the cost of 24 hours of operation.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
(1) The system shown in the schematic undergoes a refrigeration cycle.
(2) Energy transfers are positive in the directions of the arrows.
(3) The cycle operates steadily for 24 hours.
(4) Electricity is valued at \$0.1/kW·h.

ANALYSIS: Using Eq. 2.45 on a time rate basis

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{cycle}} \quad \rightarrow \quad \dot{Q}_{in} = \beta \dot{W}_{cycle} = (6.22)(0.434 \text{ kW}) = 2.70 \text{ kW} \longleftarrow$$

The total amount of electric energy input by work for 24 h of operation is

$$W_{cycle} = \dot{W}_{cycle}\Delta t = (0.434 \text{ kW})(24 \text{ h}) = 10.42 \text{ kW·h}$$

Thus, the total cost is

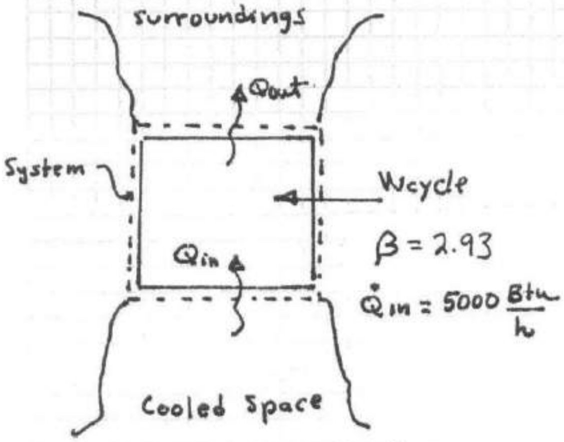
$$\text{Total cost} = (10.42 \text{ kW·h})(\$0.1/\text{kW·h}) = \$1.04 \text{ (for 24 hours)} \longleftarrow$$

PROBLEM 2.93

KNOWN: Operating and cost data are provided for an air-conditioner providing cooling.

FIND: Determine the cost, in \$, to operate the air-conditioner for a cooling season lasting 125 days.

SCHEMATIC & GIVEN DATA:



Unit cost of electricity = 10 cents per kW·h

ENGINEERING MODEL:

1. The system operates in a refrigeration cycle.
2. Energy transfers are positive in the direction of the arrows.
3. Electricity is valued at 10 cents per kW·h.

ANALYSIS:
$$\beta = \frac{Q_{in}}{W_{cycle}} \Rightarrow W_{cycle} = \frac{Q_{in}}{\beta}$$

$$\therefore \quad W_{cycle} = \frac{\left(5000\ \frac{Btu}{h}\right)\left(8\ \frac{h}{day}\right)\left(125\ \frac{days}{season}\right)}{2.93}\left|\frac{1\ kW}{3413\ Btu/h}\right|$$

Unit conversions, 1 W = 3.413 Btu/h

$$= 500\ \frac{kW \cdot h}{season}$$

Costing,

$$\dot{\$} = \left(500\ \frac{kW \cdot h}{season}\right)\left(\frac{\$0.10}{kW \cdot h}\right)$$

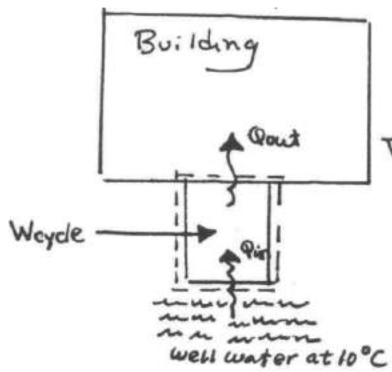
$$= \$50/\text{season}$$

PROBLEM 2.94

KNOWN: Operating data are provided for a heat pump cycle operating at steady state while receiving energy by heat transfer from well water.

FIND: Determine the amount of energy received from well water and the heat pump's coefficient of performance.

SCHEMATIC & GIVEN DATA:



$\dot{Q}_{out} = 120{,}000 \frac{kJ}{h}$

$W_{cycle} = 1490\ kW \cdot h$ (14 days)

ENGR. MODEL:

1. The heat pump is the system.
2. The building and well water play the roles of hot and cold bodies, respectively.
3. The heat pump operates at steady state.
4. All energy transfers are positive in the directions of the arrows.

ANALYSIS:

(a) Applying an energy balance using assumption 4 (Eq. 2.44),

$$W_{cycle} = Q_{out} - Q_{in} \Rightarrow Q_{in} = \underbrace{Q_{out}}_{(\dot{Q}_{out}\Delta t)} - W_{cycle}$$

$$Q_{out} = \left(120{,}000 \frac{kJ}{h}\right)\left|\frac{24\,h}{day}\right|(14\ days)$$

$$= 40.32 \times 10^6\ kJ$$

$$W_{cycle} = (1490\ kW \cdot h)\left|\frac{1\ kJ/s}{1\ kW}\right|\left|\frac{3600\,s}{1\,h}\right| = 5.364 \times 10^6\ kJ$$

Thus

$$Q_{in} = (40.36 - 5.364) \times 10^6\ kJ = 34.956 \times 10^6\ kJ \qquad \leftarrow Q_{in}$$

(b) Apply Eq. 2.47

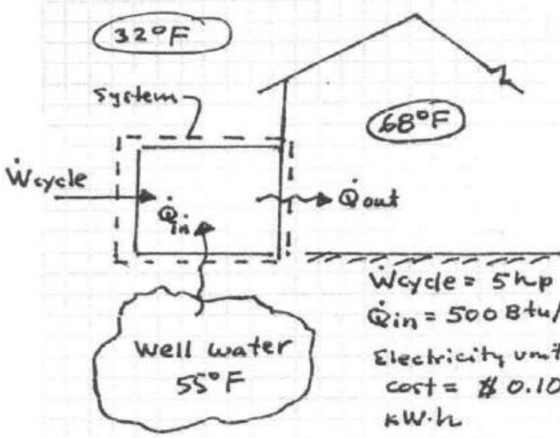
$$\gamma = \frac{Q_{out}}{W_{cycle}} = \frac{40.36 \times 10^6\ kJ}{5.364 \times 10^6\ kJ} = 7.52 \qquad \leftarrow \gamma$$

PROBLEM 2.95

KNOWN: Operating and cost data are provided for a heat pump.

FIND: Determine the coefficient of performance for the heat pump and the cost of electricity in a month when the heat pump operates for 300 hours.

SCHEMATIC & GIVEN DATA:



$\dot{W}_{cycle} = 5\text{ hp}$

$\dot{Q}_{in} = 500\text{ Btu/min}$

Electricity unit cost = \$0.10 per kW·h

ENGINEERING MODEL:

1. The system shown in the schematic operates in heat pump cycle.
2. Energy transfers are positive in the direction of the arrows.
3. The heat pump operates steadily.
4. Electricity is valued at \$0.10 per kW·h.

ANALYSIS:

(a) Using Eq. 2.47 on a time rate basis,

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_{cycle}} \qquad (1)$$

together with the cycle energy balance, Eq. 2.44, on a time rate basis: $\dot{W}_{cycle} = \dot{Q}_{out} - \dot{Q}_{in}$, we get:

$$\dot{Q}_{out} = \dot{W}_{cycle} + \dot{Q}_{in} = \underbrace{(5\text{ hp})\left|\frac{2545\text{ Btu/h}}{1\text{ hp}}\right|\left|\frac{1\text{ h}}{60\text{ min}}\right|}_{212.1\text{ Btu/min}} + 500\,\frac{\text{Btu}}{\text{min}}$$

$\therefore\ \dot{Q}_{out} = 712.1\text{ Btu/min}$

Eq. (1) gives,

$$\gamma = \frac{712.1\text{ Btu/min}}{212.1\text{ Btu/min}} = 3.36$$

Unit conversions: 1 W = 3.413 Btu/h

(b) $$\$ = \left(212.1\,\frac{\text{Btu}}{\text{min}}\right)\left[\frac{300\text{ h}}{\text{month}}\right]\left(\frac{\$0.10}{\text{kW·h}}\right)\left|\frac{60\text{ min}}{1\text{ h}}\right|\left|\frac{1\text{ kW}}{3413\text{ Btu/h}}\right|$$

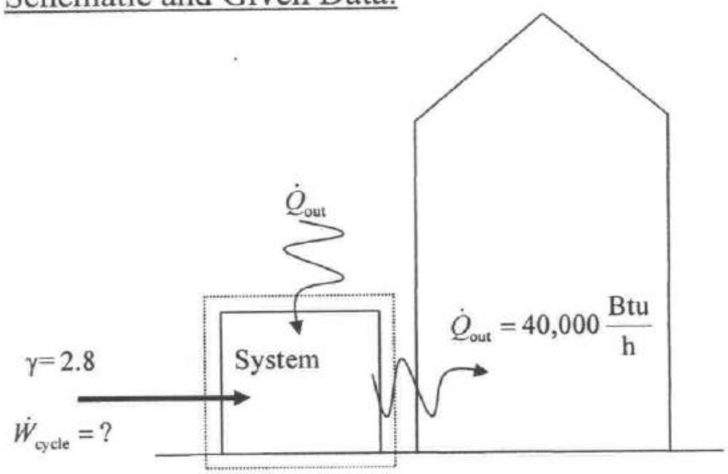
$= \$111.86/\text{month}$

PROBLEM 2.96

Known: Operating data are provided for a heat pump

Find: Determine the power input to the cycle, in hp, and heating season cost to operate the heat pump.

Schematic and Given Data:



Engineering Model:
(1) The system undergoes a heat pump cycle.
(2) The cycle operates steadily for 2000 h during the heating season.
(3) Electricity is valued at \$0.085/kW·h.

Analysis:

(a) Using the following, determine the power input to the cycle, in hp

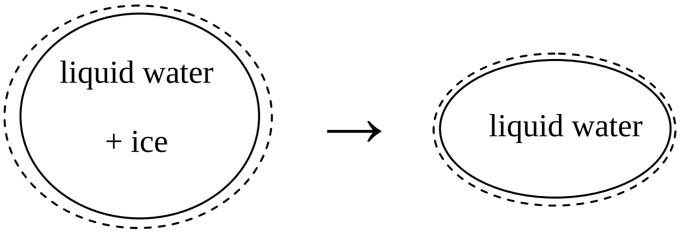
$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_{cycle}} \quad or \quad \dot{W}_{cycle} = \frac{\dot{Q}_{out}}{\gamma} = \frac{40{,}000 \frac{\text{Btu}}{\text{h}}}{2.8} \left| \frac{1\ \text{hp}}{2545 \frac{\text{Btu}}{\text{h}}} \right| = 5.61\ \text{hp} \longleftarrow$$

(b) Using assumptions 2 and 3, determine the heating season cost to operate the heat pump, in \$/heating season

$$\text{cost} = (5.61\ \text{hp}) \left| \frac{1\ \text{kW}}{1.341\ \text{hp}} \right| \left(\frac{2000\ \text{h}}{\text{heating season}} \right) \left(\frac{\$0.085}{\text{kW} \cdot \text{h}} \right) = \$711 / \text{heating season} \longleftarrow$$

PROBLEM 3.1

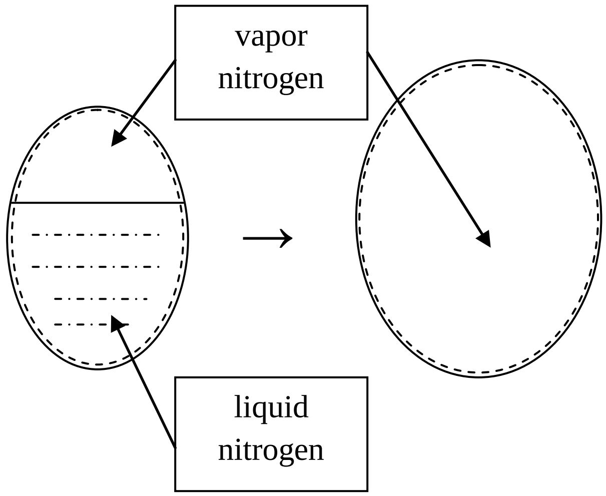
A system consisting of liquid water and ice undergoes a process. At the end of the process, the ice has melted and the system contains only liquid water. Can the system be considered a pure substance during the process? Explain.



The system is a pure substance. Although the phases change, the system remains of fixed chemical composition and is chemically homogenous.

PROBLEM 3.2

A system consists of liquid nitrogen in equilibrium with nitrogen vapor. How many phases are present? The system undergoes a process during which all of the liquid is vaporized. Can the system be viewed as a pure substance during the process? Explain.

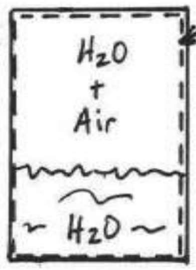


Initially, two phases are present: saturated liquid and saturated vapor.

The system is a pure substance. Although liquid is vaporized, the system remains fixed in chemical composition and is chemically homogenous.

PROBLEM 3.3

A system consists of liquid water in equilibrium with a gaseous mixture of air and water vapor. How many phases are present? Does the system consist of a pure substance? Explain. Repeat for a system consisting ice and liquid water in equilibrium with a gaseous mixture of air and water vapor.



system boundary

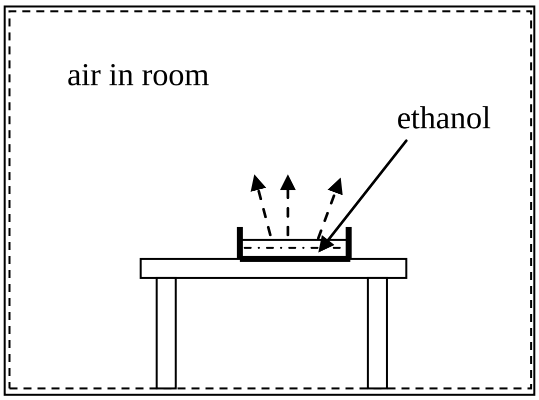
- two phases are present (liquid and gas).
- not a pure substance because composition is different in each phase.

system boundary

- three phases are present (solid, liquid, and gas).
- not a pure subtance because composition of gas phases is different than that of the solid and liquid phases.

PROBLEM 3.4

An open container of pure ethanol (ethyl alcohol) liquid is placed on a table in a room. Evaporation occurs until all of the ethanol is gone. Where did the alcohol go? If the ethanol liquid and the room air are taken to be a closed system, can the system be regarded as a pure substance during the process? How many phases are present initially and finally? Explain.



The ethanol vaporizes and diffuses into the room air.

The system is *not* a pure substance during the process since the composition of the gas phase changes as ethanol evaporates into the air. Also, the liquid and gas phases each have different chemical compositions, so the system is not chemically homogenous

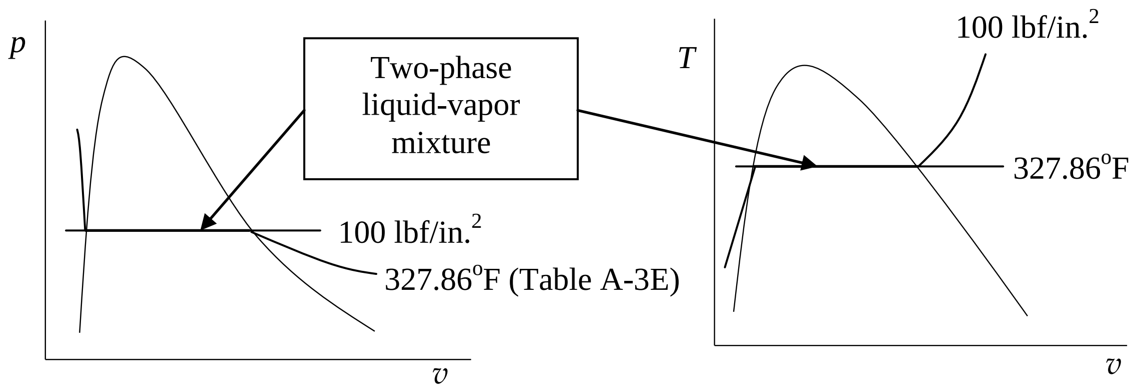
Once all of the ethanol evaporates, the gas phase comes to equilibrium and the composition becomes homogeneous. At this point, the gas phase can be treated as a pure substance.

PROBLEM 3.5

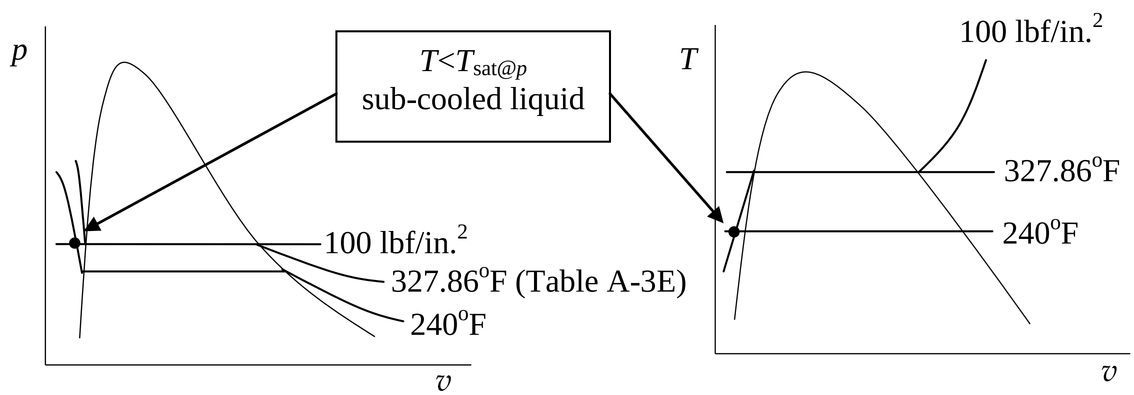
Determine the phase or phases in a system consisting of H_2O at the following conditions and sketch the p-v and T-v diagrams showing the location of each state.

(a) $p = 100\ \text{lbf/in.}^2$, $T = 327.86^\circ\text{F}$
(b) $p = 100\ \text{lbf/in.}^2$, $T = 240^\circ\text{F}$
(c) $T = 212^\circ\text{F}$, $p = 10\ \text{lbf/in.}^2$
(d) $T = 70^\circ\text{F}$, $p = 20\ \text{lbf/in.}^2$
(e) $p = 14.7\ \text{lbf/in.}^2$, $T = 20^\circ\text{F}$

(a) $p = 100\ \text{lbf/in.}^2$, $T = 327.86^\circ\text{F}$

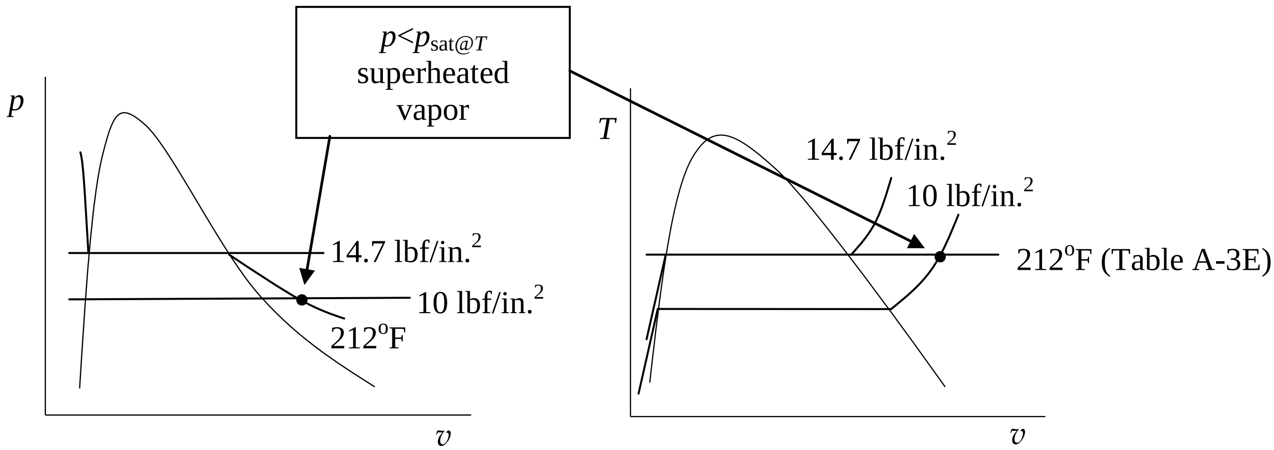


(b) $p = 100\ \text{lbf/in.}^2$, $T = 240^\circ\text{F}$

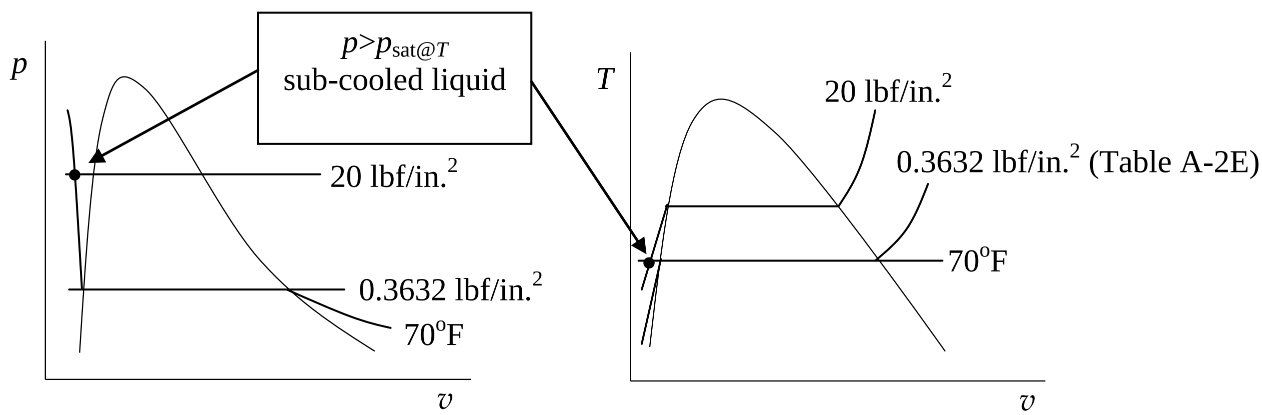


Problem 3.5 (Continued)

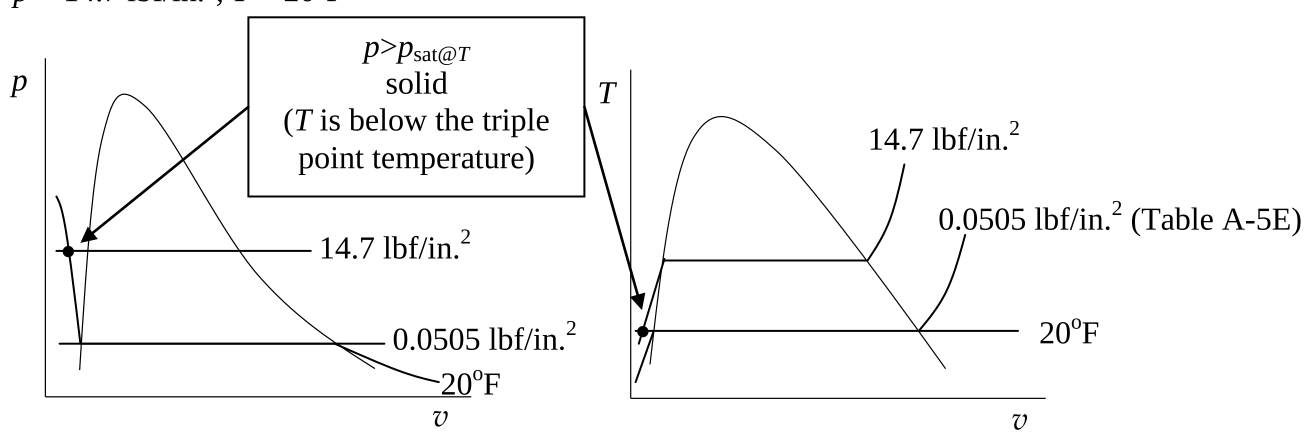
(c) $T = 212^{o}F$, $p = 10$ lbf/in.^{2}



(d) $T = 70^{o}F$, $p = 20$ lbf/in.^{2}



(e) $p = 14.7$ lbf/in.^{2}, $T = 20^{o}F$

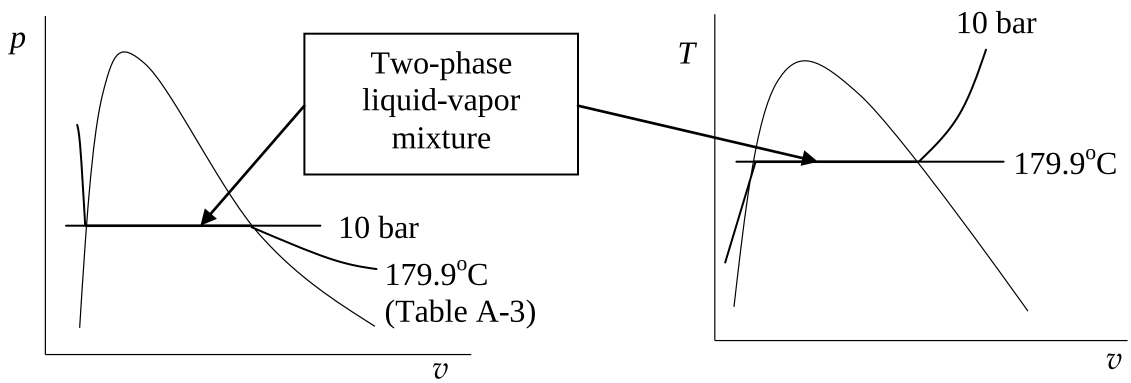


PROBLEM 3.6

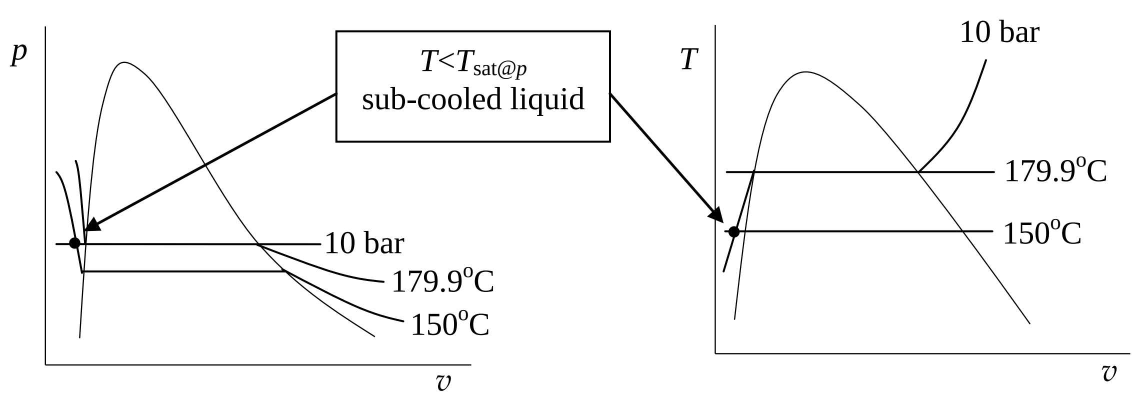
Determine the phase or phases in a system consisting of H_2O at the following conditions and sketch the p-v and T-v diagrams showing the location of each state.

(a) p = 10 bar, T = 179.9°C
(b) p = 10 bar, T = 150°C
(c) T = 100°C, p = 0.5 bar
(d) T = 20°C, p = 50 bar
(e) p = 1 bar, T = - 6°C

(a) p = 10 bar, T = 179.9°C

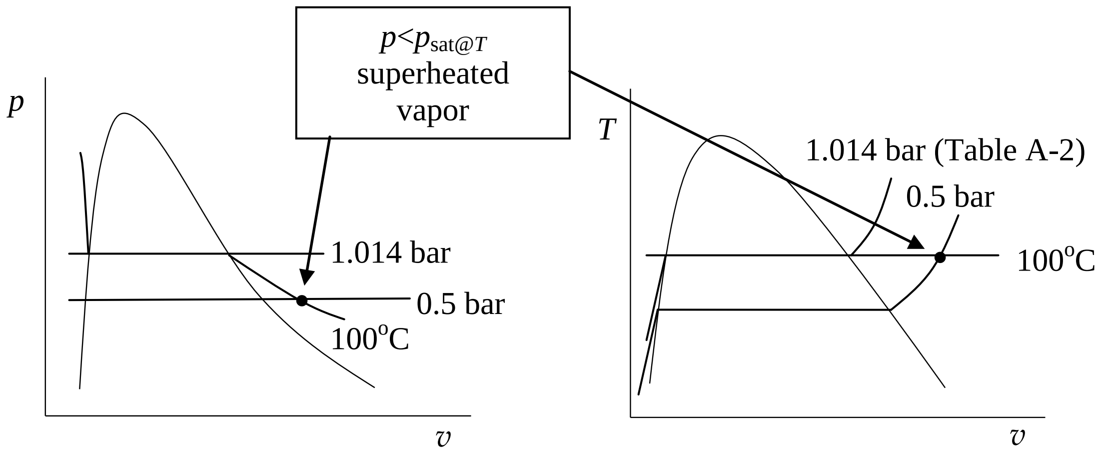


(b) p = 10 bar, T = 150°C

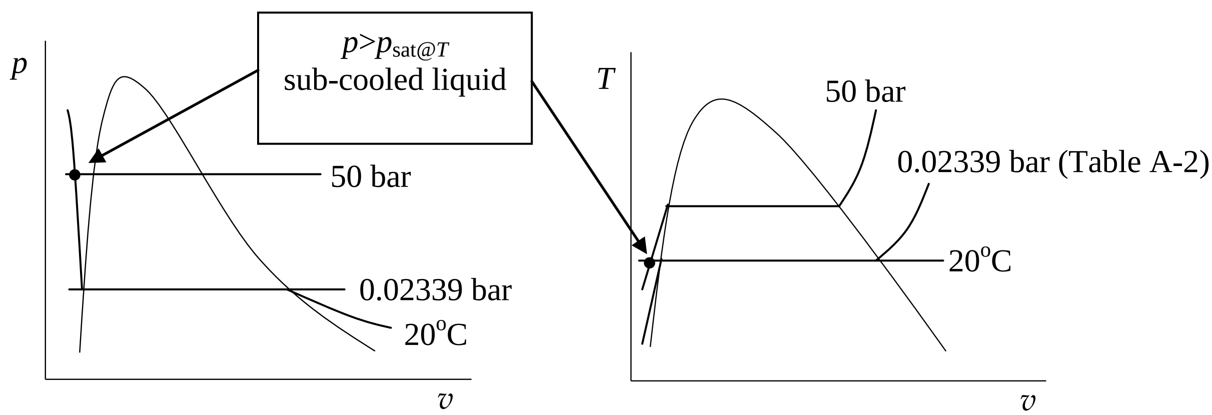


Problem 3.5 (Continued)

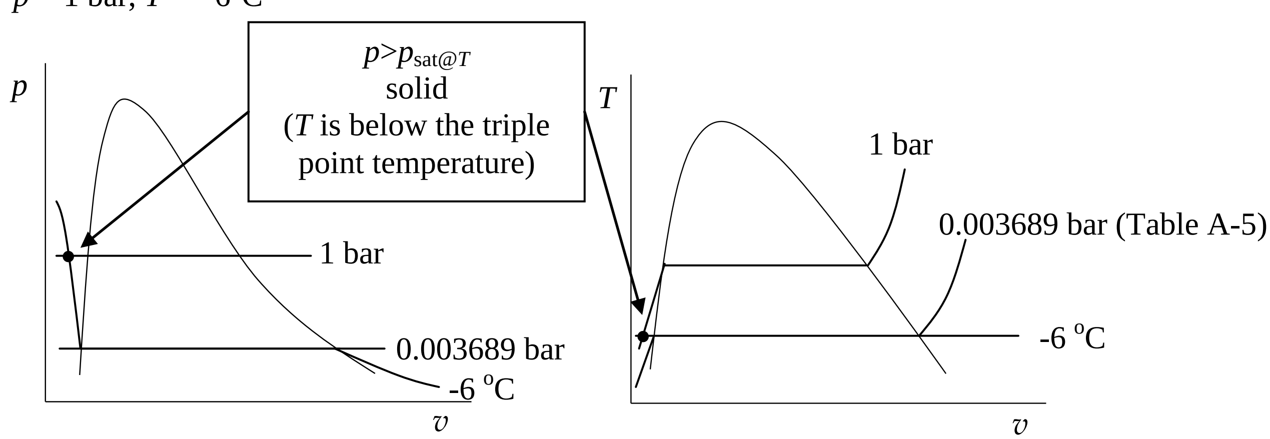
(c) $T = 100^{o}C$, $p = 0.5$ bar



(d) $T = 20^{o}C$, $p = 50$ bar

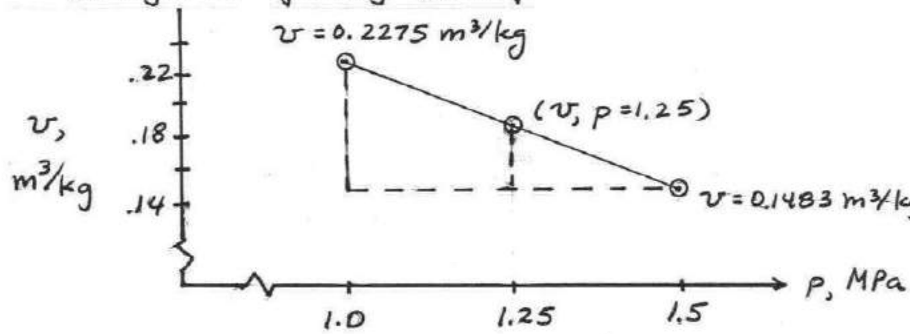


(e) $p = 1$ bar, $T = -6^{o}C$



PROBLEM 3.7

(a) At a temperature of 240°C, the specified pressure of 1.25 MPa falls between the tabe values of 1.0 and 1.5 MPa. To determine the specific volume corresponding to 1.25 MPa, we think of the slope of a straight line joining the adjacent table states, as follows:

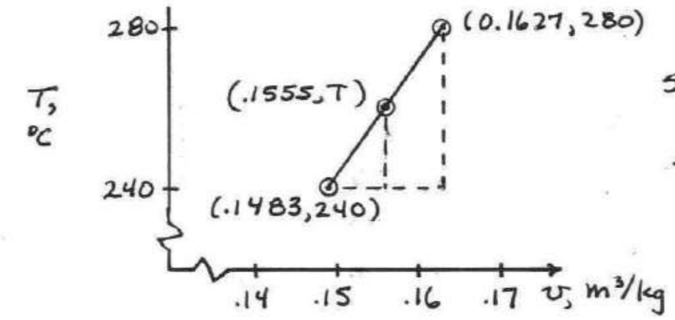


similar triangles:

$$|\text{slope}| = \frac{v - 0.1483}{1.5 - 1.25} = \frac{0.2275 - 0.1483}{1.5 - 1.0} \Rightarrow v = 0.1483 + \left(\frac{0.25}{0.50}\right)(0.2275 - 0.1483)$$

$$= 0.1879 \text{ m}^3/\text{kg} \quad \text{(a)}$$

(b) At a pressure of 1.5 MPa, the given specific volume of 0.1555 m³/kg falls between the table values of 240 and 280°C. To determine the temperature corresponding to the given specific volume, we think of the slope of a straight line joining the adjacent table states, as follows:



$$\text{Slope} = \frac{T - 240}{.1555 - .1483} = \frac{280 - 240}{.1627 - .1483}$$

$$\Rightarrow T = 240 + \left[\frac{.1555 - .1483}{.1627 - .1483}\right](40)$$

$$= 260°\text{C} \quad \text{(b)}$$

(c) In this case, the specified pressure falls between the table values of 1.0 and 1.5 MPa <u>and</u> the specified temperature falls between the table values of 200 and 240°C. Thus, <u>double</u> interpolation is required.

- At 220°C, the specific volume at each pressure is simply the average over the interval:

  at 1.0 MPa, 220°C; $v = \frac{.2060 + .2275}{2} = 0.21675 \text{ m}^3/\text{kg}$

  at 1.5 MPa, 220°C $\quad v = \frac{.1325 + .1483}{2} = 0.1404 \text{ m}^3/\text{kg}$

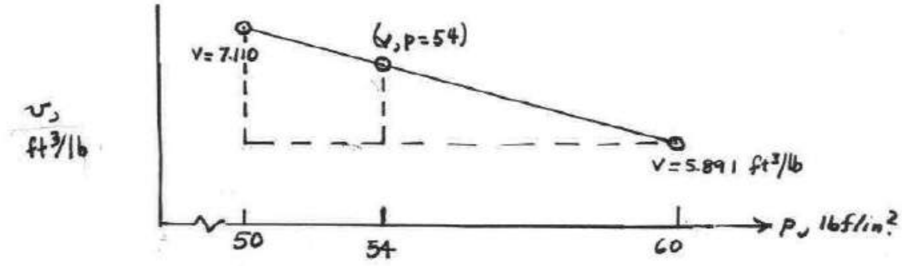
- Thus, with the same approach as in (a)

$$\frac{v - 0.1404}{1.5 - 1.4} = \frac{0.21675 - 0.1404}{1.5 - 1.0} \Rightarrow v = 0.1404 + \left(\frac{0.1}{0.5}\right)(0.21675 - 0.1404)$$

$$= 0.15567 \text{ m}^3/\text{kg} \quad \text{(c)}$$

PROBLEM 3.8

(a) At a temperature of 120°C, the specified pressure of 54 lbf/in.² falls between the table values of 50 and 60 lbf/in.² To determine the specific volume corresponding to 54 lbf/in.², we think of the slope of a straight line joining the adjacent table states, as follows:

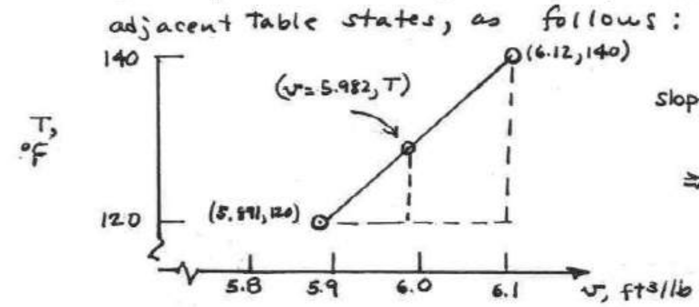


Similar triangles:

$$|\text{slope}| = \frac{v-5.891}{60-54} = \frac{7.110-5.891}{60-50} \Rightarrow v = 5.891 + \frac{6}{10}(7.110-5.891) = 6.622\,\frac{ft^3}{lb}$$

(a)

(b) At a pressure of 60 lbf/in.², the given specific volume of 5.982 ft³/lb falls between the table values of 120 and 140°F. To determine the temperature corresponding to the given specific volume, we think of the slope of a straight line joining the adjacent table states, as follows:



$$\text{slope} = \frac{T-120}{5.982-5.891} = \frac{140-120}{6.12-5.891}$$

$$\Rightarrow \quad T = 120 + \left[\frac{5.982-5.891}{6.12-5.891}\right](20)$$

$$= 127.9\,°F$$

(b)

(c) In this case, the specified pressure falls between the table values of 50 and 60 lbf/in² and the specified temperature falls between the table values of 100 and 120°F. Thus, double interpolation is required.

- At 110°F, the specific volume at each pressure is simply the average over the interval:

  at $50\,\frac{lbf}{in^2}$, 110°F; $v = \frac{7.110+6.836}{2} = 6.973\,ft^3/lb$

  at $60\,\frac{lbf}{in^2}$, 110°F; $v = \frac{5.891+5.659}{2} = 5.775\,ft^3/lb$

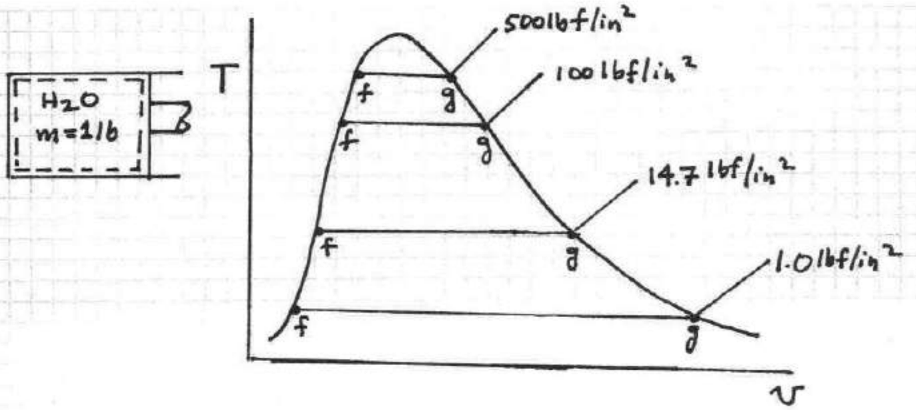
- Then, with the same approach as in (a)

$$\frac{v-5.775}{60-58} = \frac{6.973-5.775}{60-50} \Rightarrow v = 5.775 + \frac{2}{10}\left[6.973-5.775\right]$$

$$= 6.015\,ft^3/lb$$

(c)

PROBLEM 3.9



$\Delta V = m(v_g - v_f)$, where v_f and v_g are obtained from Table A-3E.

(a) $p = 1.0\ \text{lbf/in}^2$

$$\Delta V = (1\,\text{lb})(333.6 - 0.01614)\frac{\text{ft}^3}{\text{lb}} = 333.58\ \text{ft}^3 \longleftarrow$$

(b) $p = 14.7\ \text{lbf/in}^2$

$$\Delta V = (1\,\text{lb})(26.80 - 0.01672)\frac{\text{ft}^3}{\text{lb}} = 26.78\ \text{ft}^3 \longleftarrow$$

(c) $p = 100\ \text{lbf/in}^2$

$$\Delta V = (1\,\text{lb})(4.434 - 0.01774)\frac{\text{ft}^3}{\text{lb}} = 4.42\ \text{ft}^3 \longleftarrow$$

(d) $p = 500\ \text{lbf/in}^2$

$$\Delta V = (1\,\text{lb})(0.928 - 0.01975)\frac{\text{ft}^3}{\text{lb}} = 0.91\ \text{ft}^3 \longleftarrow$$

COMMENT: As the pressure increases, the difference in specific volume between saturated vapor and saturated liquid decreases. At the critical pressure, the two states coincide and the difference is zero.

PROBLEM 3.10

For H_2O, determine the specified property at the indicated state. Locate the state on a sketch of the T-v diagram.

(a) $T = 140^oC$, $v = 0.5$ m^3/kg. Find p, in bar.
(b) $p = 30$ MPa, $T = 100^oC$. Find v, in m^3/kg.
(c) $p = 10$ MPa, $T = 485^oC$. Find v, in m^3/kg.
(d) $T = 80^oC$, $x = 0.75$. Find p, in bar, and v, in m^3/kg.

(a) $T = 140^oC$, $v = 0.5$ m^3/kg. Find p, in bar.

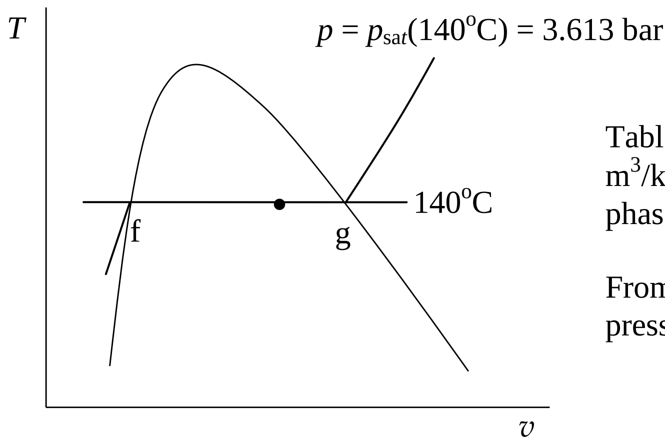
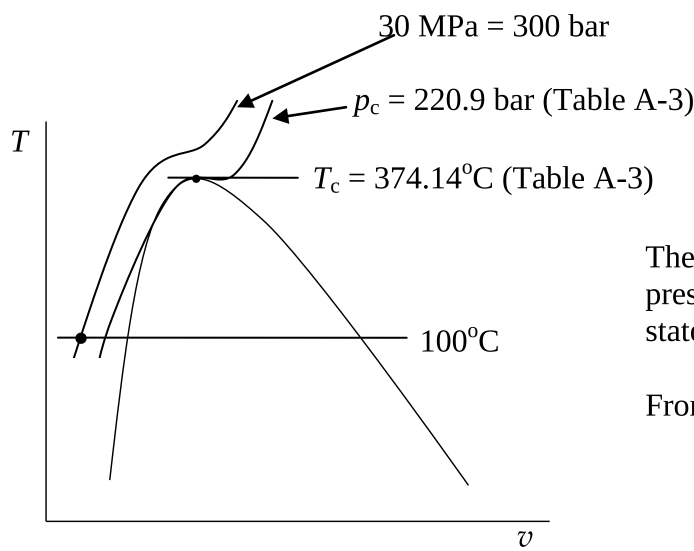


Table A-3: $v_f = 1.0435 \times 10^{-3}$ m^3/kg , $v_g = 1.673$ m^3/kg. Since $v_f < v < v_g$, the state is in the two-phase liquid-vapor region, as shown.

From Table A-3, the pressure is the saturation pressure at 140^oC: $p = 3.613$ bar.

(b) $p = 30$ MPa, $T = 100^oC$. Find v, in m^3/kg.

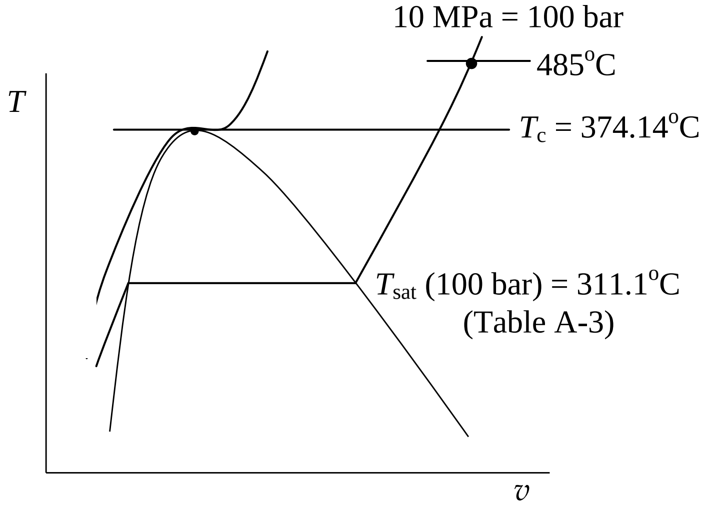


The pressure is higher than the critical pressure, as shown on the diagram. Hence, the state is in the compressed liquid region.

From Table A-5: $v = 1.0290$ m^3/kg.

Problem 3.10 (Continued)

(c) p = 10 MPa, T = 485°C. Find v, in m^3/kg.



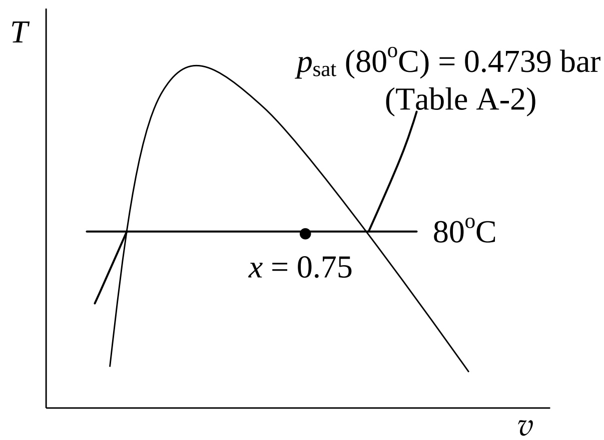
Since the temperature is higher than T_{sat} at 100 bar, the state is superheated vapor.

Interpolating in Table A-4, we get

v = 0.03160 +
(485 – 480)/(520 – 480)*(0.02343 – 0.3160)

= 0.03058 m^3/kg

(d) T = 80°C, x = 0.75. Find p, in bar, and v, in m^3/kg.



Eq. 3.2: $v_x = v_f + x(v_g - v_f)$

With data from Table A-2 at 80°C

v = 1.0291 x 10^{-3} + (0.75)(3.407 - 1.0291 x 10^{-3})

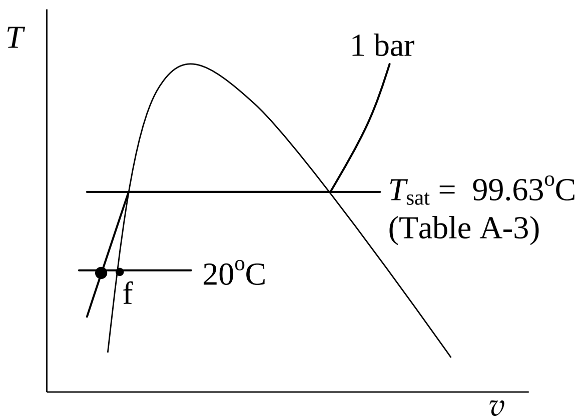
= 2.556 m^3/kg

PROBLEM 3.11

For each case, determine the specific volume at the indicated state. Locate the state on a sketch of the T-v diagram.

(a) Water at p = 1 bar, T = 20°C. Find v, in m^3/kg.
(b) Refrigerant-22 at p = 40 lbf/in.^{2}, x = 0.6. Find v, in ft^3/lb.
(c) Ammonia at p = 200 lbf/in.^{2}, T = 195°F. Find v, in ft^3/lb.

(a) Water at p = 1 bar, T = 20°C. Find v, in m^3/kg.



The state is in the compressed liquid region. Since p < 25 bar, Table A-5 cannot provide the needed data.

Using Eq. 3.11 as an approximation with data from Table A-2:

$v \approx v_f\,(20^\circ C) = 1.0018 \times 10^{-3}\ m^3/kg$ ←

(b) Refrigerant-22 at p = 40 lbf/in.^{2}, x = 0.6. Find v, in ft^3/lb.

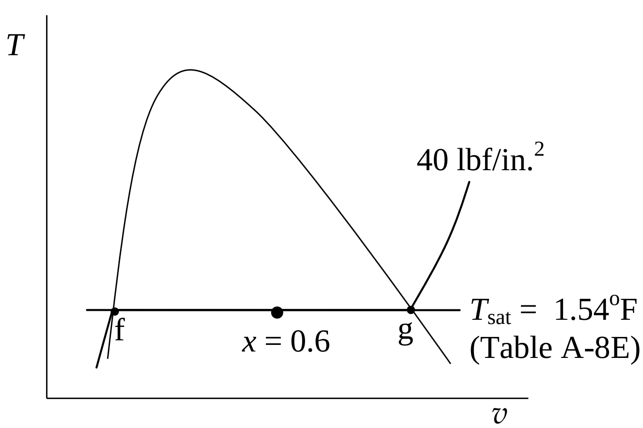


Table A-8E at 40 lbf/in.^{2} gives:
v_f = 0.01198 ft^3/lb, v_g = 1.3277 ft^3/lb

With Eq. 3.2

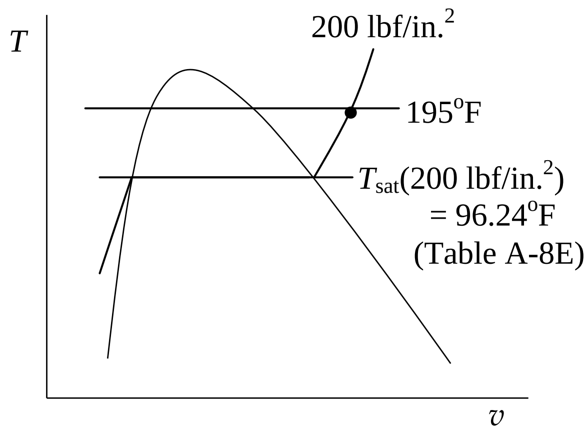
$$v_x = v_f + x(v_g - v_f)$$

$$= 0.01198 + (0.6)(1.3277 - 0.01198)$$

$$= 0.8014\ ft^3/lb$$ ←

PROBLEM 3.11 (CONTINUED)

(c) Ammonia at p = 200 lbf/in.^{2}, T = 195°F. Find v, in ft^3/lb.



Since $T > T_{sat}$, the state is in the superheated vapor region. Interpolating in Table A-9E:

$$v = 0.3497 + (195 - 180)/(200 - 180)*(0.3661 - 0.3497)$$

$$= 0.3620 \text{ ft}^3/\text{lb}$$

PROBLEM 3.12

(a) Water. $v = 0.5\ m^3/kg$, $p = 3\ bar$. Find T in °C.

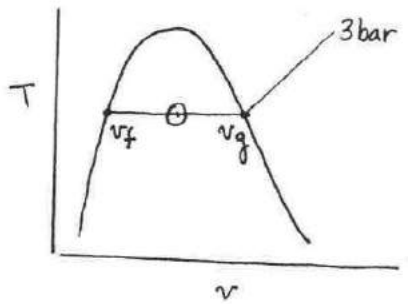


Table A-3, $v_f = 1.0732/10^3\ m^3/kg$, $v_g = 0.6058\ m^3/kg$.

Since $v_f < v < v_g$, the state is in the two-phase, liquid-vapor region — see T-v diagram.

Thus, $T = T_{sat}(3\ bar) = 133.6°C$ ⟵ T

(b) Ammonia. $p = 11\ lbf/in^2$, $T = -20°F$. Find v in ft^3/lb.

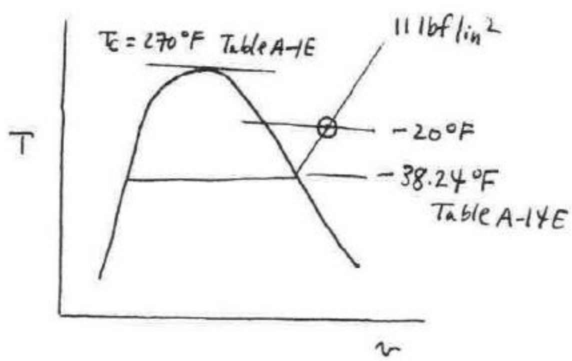


Table A-15E

	$10\ lbf/in^2$	$12\ lbf/in^2$
$-20°F$	$v = 27.241\ \frac{ft^3}{lb}$	$v = 22.621\ \frac{ft^3}{lb}$

$\Rightarrow$ at $11\ lbf/in^2$

$v = 24.931\ \frac{ft^3}{lb}$ ⟵ v

(c) Propane. $p = 1\ MPa$, $T = 85°C$. Find v in m^3/kg.

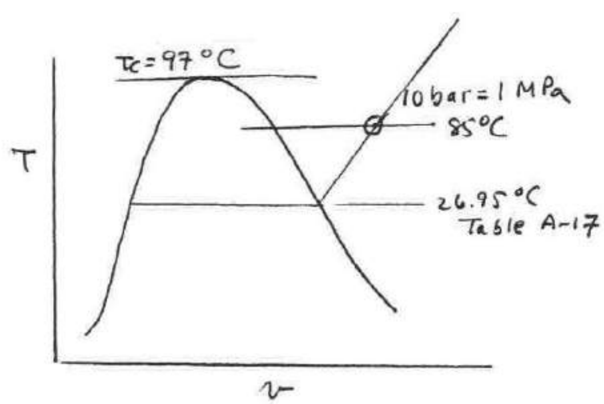
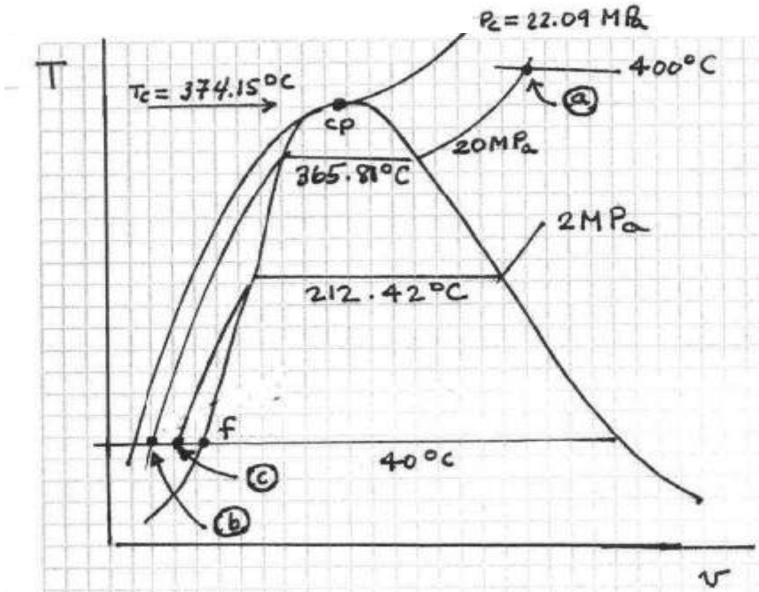


Table A-18 at 10 bar:

	10 bar
80°C	$v = 0.05992\ m^3/kg$
85°C	- - - -
90°C	$v = 0.06226\ m^3/kg$

$\Rightarrow v = 0.06109\ \frac{m^3}{kg}$ ⟵ v

PROBLEM 3.13



(a) Table A-4

$v = 9.94 \times 10^{-3}\ m^3/kg$

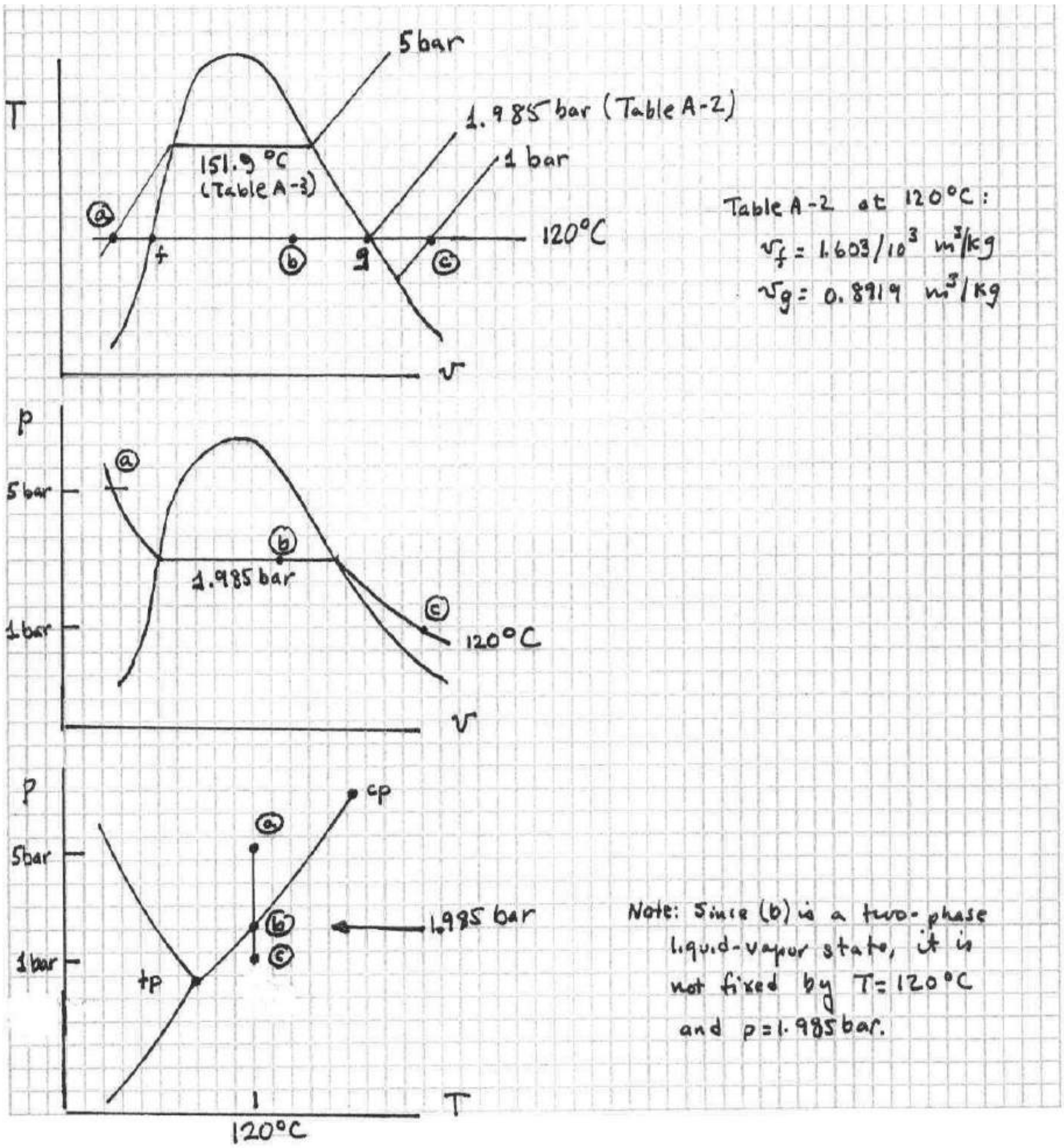
(b) Table A-5

$v = 0.9992 \times 10^{-3}\ m^3/kg$

(c) Table A-2 using Eq. 3.11

$v(T, p) \approx v_f(T) = 1.0078 \times 10^{-3}\ m^3/kg$

PROBLEM 3.14



T
5 bar
1.985 bar (Table A-2)
1 bar
151.9 °C
(Table A-3)
120°C
a
f
b
g
c
v
Table A-2 at 120°C:
v_f = 1.603/10^3 m^3/kg
v_g = 0.8919 m^3/kg
p
5 bar
1 bar
a
b
1.985 bar
c
120°C
v
p
cp
a
5bar
b
1.985 bar
c
1 bar
tp
T
120°C
Note: Since (b) is a two-phase liquid-vapor state, it is not fixed by T = 120°C and p = 1.985 bar.

PROBLEM 3.15

(a) Water at 100°C, m = 4 kg, V = 1 m³

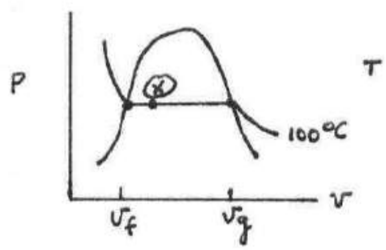
$$v = \frac{V}{m} = \frac{1\,m^3}{4\,kg} = 0.25\ m^3/kg$$

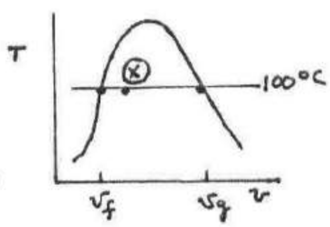
Table A-2 at 100°C: $v_f = 1.0435/10^3\ m^3/kg$, $v_g = 1.673\ m^3/kg$

Since $v_f < v < v_g$, the state is a two-phase, liquid-vapor state.

$$v = v_f + x(v_g - v_f) \Rightarrow x = \frac{v - v_f}{v_g - v_f} = \frac{0.25 - (1.0435/10^3)}{1.673 - (1.0435/10^3)} = 0.149$$

(14.9%)





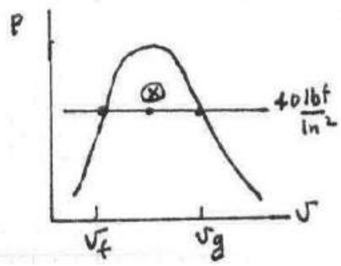
(b) Ammonia at 40 lbf/in², u = 308.75 Btu/lb

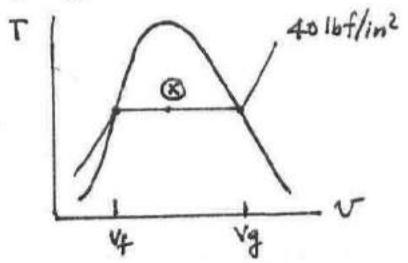
From Table A-14E at 40 lbf/in², $v_f = 0.0245\ ft^3/lb$, $v_g = 7.041\ ft^3/lb$
$u_f = 54.89$ Btu/lb, $u_g = 562.6$ Btu/lb

Since $u_f < u < u_g$, the state is a two-phase, liquid-vapor state.

$$u = u_f + x(u_g - u_f) \Rightarrow x = \frac{u - u_f}{u_g - u_f} = \frac{308.75 - 54.89}{562.6 - 54.89} = 0.5$$

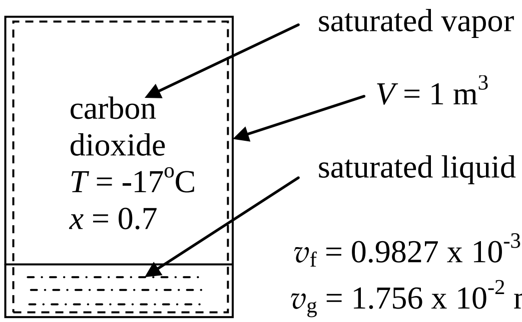
Then, $v = v_f + x(v_g - v_f) = 0.0245 + 0.5(7.041 - 0.0245) = 3.533\ \frac{ft^3}{lb}$





PROBLEM 3.16

A 1-m^3 tank holds a two-phase liquid-vapor mixture of carbon dioxide at – 17^{o}C. The quality of the mixture is 70%. For saturated carbon dioxide at – 17^{o}C, v_f = 0.9827 x 10^{-3} m^3/kg and v_g = 1.756 x 10^{-2} m^3/kg. Determine the masses of saturated liquid and saturated vapor, each in kg. What is the percent of the total volume occupied by saturated liquid?



v_f = 0.9827 x 10^{-3} m^3/kg
v_g = 1.756 x 10^{-2} m^3/kg

First, find the total mass as follows:

$$v_x = v_f + x\,(v_g - v_f) = 0.9827 \text{ x } 10^{-3} + (0.7)(1.756 \text{ x } 10^{-2} - 0.9827 \text{ x } 10^{-3}) = 0.01258 \text{ m}^3/\text{kg}$$

Thus

$$m = V/v_x = (1 \text{ m}^3)/(0.01258 \text{ m}^3/\text{kg}) = 79.46 \text{ kg}$$

Now, using the definition of quality

$$m_g = x\,m = (0.7)\,(79.46 \text{ kg}) = 55.62 \text{ kg} \longleftarrow$$

$$m_f = (1 - x)\,m = (1 – 0.7)\,(79.46 \text{ kg}) = 23.84 \text{ kg} \longleftarrow$$

The volume of saturated liquid is

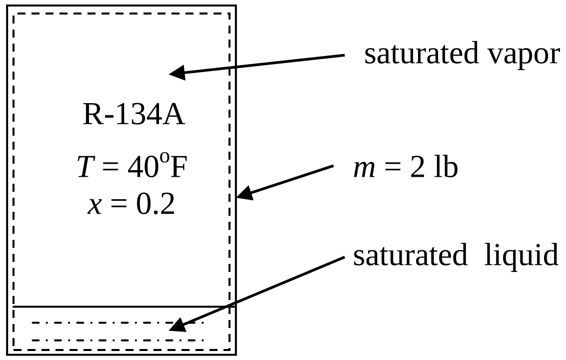
$$V_f = v_f \cdot m_f = (0.9827 \text{ x } 10^{-3} \text{ m}^3/\text{kg}) \cdot (23.84 \text{ kg}) = 0.0234 \text{ m}^3$$

The total volume is 1 m^3, so the percent of the total volume occupied saturated liquid is 2.34%. ⟵

Note: Although the liquid is 30% of the total mass, its specific volume is much less than that of the vapor. Consequently, the liquid occupies a very small fraction of the total volume.

PROBLEM 3.17

Determine the volume, in ft^3, of 2 lb of a two-phase liquid-vapor mixture of Refrigerant 134A at 40^oF with a quality of 20%. What is the pressure, in $lbf/in.^2$

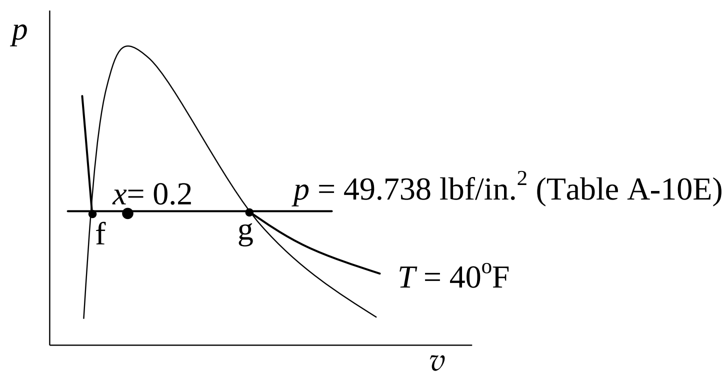


First, find the specific volume using Eq. 3.2 and data from Table A-10E at 40^oF.

$$v = v_f + x\,(v_f\ -\ v_g) = 0.01251 + (0.2)\,(0.9470 - 0.1251) = 0.28948\ ft^3/lb$$

Now

$$V = v\,m = (0.1994\ ft^3/lb)\,(2\ lb) = 0.579\ ft^3$$



PROBLEM 3.18

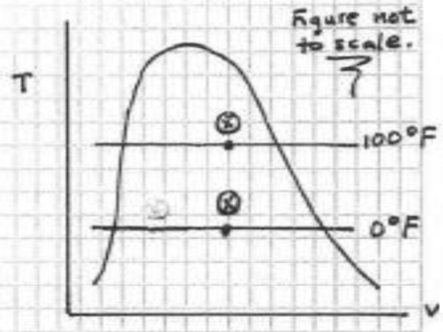
Ammonia: $v = 1\ ft^3/lb$ and (a) $100°F$, (b) $0°F$. Data from Table A-13E.

(a) $100°F$: $v_f = 0.02747\ ft^3/lb$, $v_g = 1.4168\ ft^3/lb$

$$x = \frac{v - v_f}{v_g - v_f} = \frac{1 - 0.02747}{1.4168 - 0.02747} = 0.7 \quad (70\%)$$

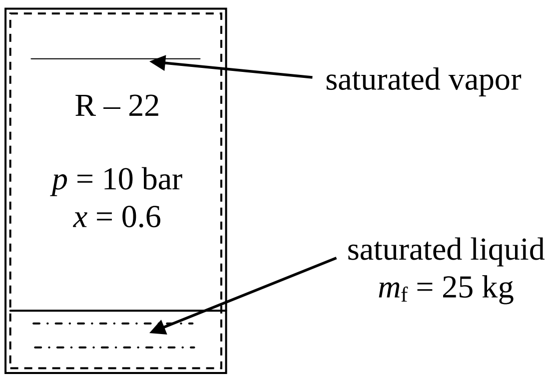
(b) $0°F$: $v_f = 0.02419\ ft^3/lb$, $v_g = 9.11\ ft^3/lb$

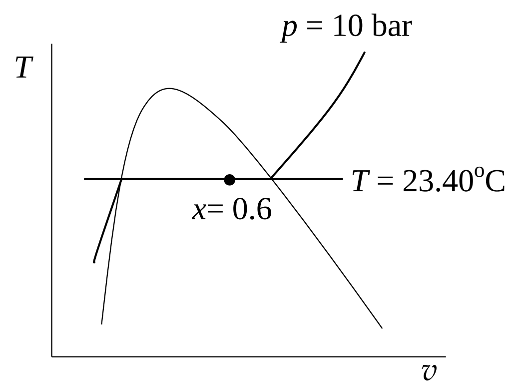
$$x = \frac{v - v_f}{v_g - v_f} = \frac{1 - 0.02419}{9.11 - 0.02419} = 0.11 \quad (11\%)$$



PROBLEM 3.19

A tank contains a two-phase liquid-vapor mixture of Refrigerant 22 at 10 bar. The mass of saturated liquid in the tank is 25 kg and the quality is 60%. Determine the volume of the tank, in m^3, and the fraction of the total volume occupied by saturated vapor.





First, determine the specific volume using Eq. 3.2 and data from Table A-8 at 10 bar.

$$v_x = v_x + x\,(v_g - v_f) = 0.8352 \times 10^{-3} + (0.6)\,(0.0236 - 0.8352 \times 10^{-3}) = 0.01449 \text{ m}^3/\text{kg}$$

The total mass is determined from the mass of saturated liquid and the definition of quality, as follows.

$$m = m_f / (1 - x) = (25 \text{ kg}) / (1 - 0.6) = 62.5 \text{ kg}$$

Now, the volume is

$$V = v_x\, m = (0.01449 \text{ m}^3/\text{kg})\,(62.5 \text{ kg}) = 0.9056 \text{ m}^3 \longleftarrow$$

The volume occupied by saturated vapor is

$$V_g = v_g\,(m - m_f) = (0.0236 \text{ m}^3/\text{kg})\,(62.5 - 25)\text{kg} = 0.885 \text{ m}^3$$

$$\text{fraction occupied by vapor} = (0.885)/(0.9056) = 0.977 \;(97.7\%) \longleftarrow$$

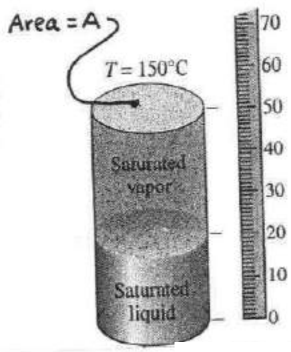
Note: Even though the vapor is only 60% of the mixture by mass, it occupies nearly the entire volume because the specific volume of saturated liquid is much smaller than the specific volume of saturated vapor at this pressure.

PROBLEM 3.20

KNOWN: A closed, rigid container holds different volumes of saturated liquid water and saturated vapor water.

FIND: Determine the quality of the mixture.

SCHEMATIC & GIVEN DATA:



ANALYSIS: $x = \dfrac{m_{vap}}{m_{vap} + m_{LIQ}}$, $m = V/v$. Thus, $m_{vap} = \dfrac{V_{vap}}{v_g}$, $m_{LIQ} = \dfrac{V_{LIQ}}{v_f}$

$$x = \frac{V_{vap}/v_g}{(V_{vap}/v_g) + (V_{liq}/v_f)}$$

$V_{vap} = 30\,A$ and $V_{LIQ} = 20\,A$, where area A is in the same units as the vertical measure shown. Then

$$x = \frac{(30A/v_g)}{(30A/v_g) + (20A/v_f)} = \frac{1}{1 + \frac{20}{30}\left(\frac{v_g}{v_f}\right)}$$

Since ratios appear in the last expression, the quantities can be in any consistent units.

(1) Using v_f and v_g from Table A-2 at 150°C,

$$v_f = 1.0905 \times 10^{-3}\ m^3/kg$$

$$v_g = 0.3928\ m^3/kg$$

$$x = \frac{1}{1 + \frac{20}{30}\left(\frac{0.3928}{1.0905 \times 10^{-3}}\right)} = 0.0041 \quad (0.41\%) \quad \longleftarrow x$$

1. Using v_f and v_g at 302°F (150°C) from Table A-2E: $v_f = 0.017468\ ft^3/lb$, $v_g = 6.292\ ft^3/lb$ gives the same value for x, as can be verified.

PROBLEM 3.21

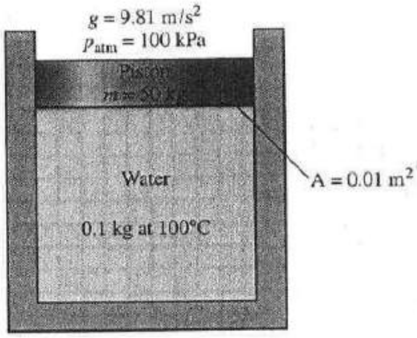
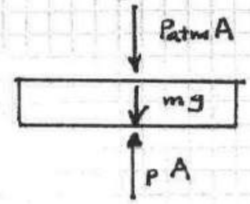


Fig. P3.22

Since the piston moves smoothly in the cylinder, the force of the pressure acting on the bottom of the piston balances the force of the atmosphere acting on the top of the piston and the piston weight:

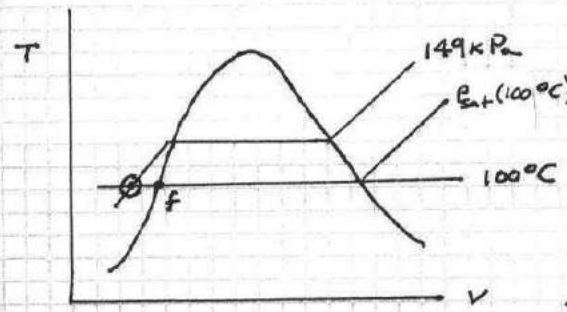


$$\Rightarrow \quad pA = p_{atm}A + mg \quad \Rightarrow \quad p = p_{atm} + mg/A$$

$$= 100\,\text{kPa} + (50\,\text{kg})(9.81\,\tfrac{\text{m}}{\text{s}^2})\left(\frac{1}{0.01\,\text{m}^2}\right)\left|\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1\,\text{kPa}}{10^3\,\text{N/m}^2}\right|$$

$$= 149\,\text{kPa} \longleftarrow$$

At $T = 100°C$ and $p = 149$ kPa, fix the state:



$p_{sat}(100°C) = 1.014$ bar (Table A-2)
$= 101.4$ kPa

The state falls in the liquid region, as shown on the T-v sketch.

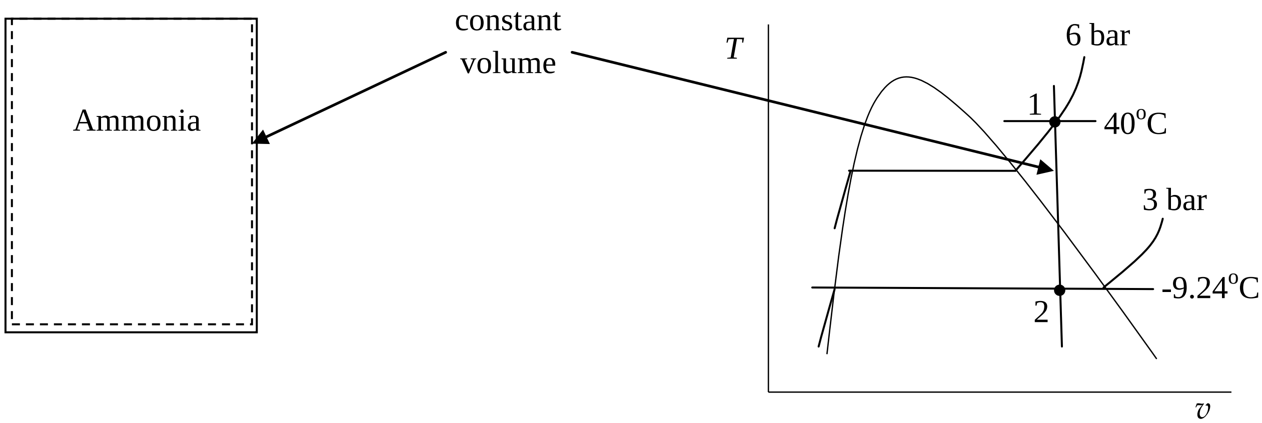
Using Eq. 3.11, $v \approx v_f(100°C) = 1.0435 \times 10^{-3}$ m^3/kg

Then, $V = mv = (0.1\,\text{kg})\left(\frac{1.0435}{10^3}\right)\frac{\text{m}^3}{\text{kg}}\left|\frac{10^2\,\text{cm}}{1\,\text{m}}\right|^3$

$= 104.35$ cm^3 $\longleftarrow$

PROBLEM 3.22

Ammonia, initially at 6 bar, 40°C, undergoes a constant volume process in a closed system to a final pressure of 3 bar. At the final state, determine the temperature, in °C, and the quality. Locate each state on a sketch of the *T*-*v* diagram.



The initial state is in the superheated vapor region. From Table A-25, $v_1 = 0.24118 \text{ m}^3/\text{kg}$. The system is a closed system (constant mass) and the volume is constant. Therefore, $v_2 = v_1$. From Table A-14 at $v_2 = 0.24118 \text{ m}^3/\text{kg}$, the state is in the two-phase liquid-vapor region, and

$$T_2 = T_{sat}(3 \text{ bar}) = -9.24^\circ\text{C}$$

The quality is

$$x_2 = \frac{v_2 - v_{f2}}{v_{g2} - v_{f2}} = \frac{0.24118 - 1.5361 \times 10^{-3}}{0.4061 - 1.5361 \times 10^{-3}} = 0.5924 \ (59.24\%)$$

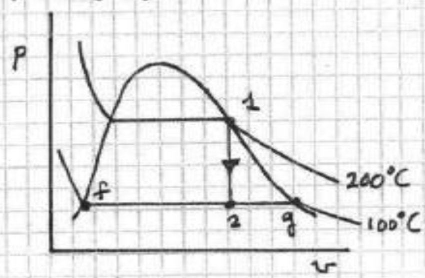
PROBLEM 3.23

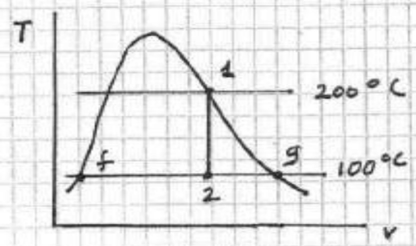
Water

Initially sat. vapor at 200°C

Finally, 100°C

Since total volume and total mass remain constant, the process of the water takes place at constant specific volume.





From Table A-2, $P_1 = P_{sat}(200°C) = 15.54$ bar

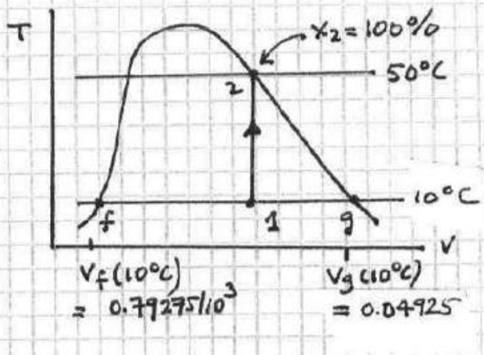
Also, $v_1 = v_g(200°C) = 0.1274$ m³/kg. Since $v_2 = v_1$, Table A-2 shows that $v_f < v_2 < v_g$, and thus state 2 falls in the two-phase liquid-vapor regim. The pressure is $P_2 = P_{sat}(100°C) = 1.014$ bar

PROBLEM 3.24

R134a
$T_1 = 10°C$, $x_1 = ?$
$T_2 = 50°C$,
$x_2 = 100\%$

$V = 1.5\ m^3$

Since the total volume and total mass remain constant, the process of the R134a takes place at constant specific volume. Using known data,



From Table A-10 at 50°C,

$v_2 = v_g(50°C) = 0.01505\ \frac{m^3}{kg}$

State 2: $v_2 = v_1$

$$\therefore x_1 = \frac{v_1 - v_f}{v_g - v_f} = \frac{v_2 - v_f}{v_g - v_f}$$

$$= \frac{0.01505 - (0.79275/10^3)}{0.04925 - (0.79275/10^3)}$$

$$= 0.294$$

At state 2, the tank contains only saturated vapor. Thus,

$$m_{vap,2} = \frac{V}{v_2} = \frac{1.5\ m^3}{0.01505\ m^3/kg} = 99.67\ kg$$

The quality at state 1 is the fraction of the total mass that is vapor. Thus,

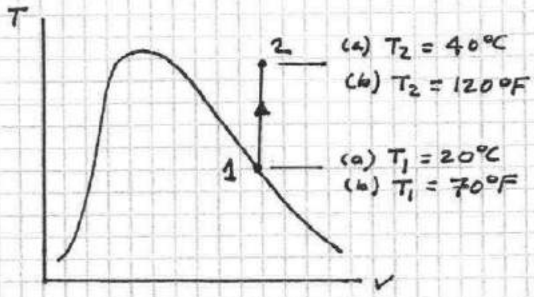
$$m_{vap,1} = x_1\, \underbrace{m_{total}}_{= m_{vap,2}}$$

$$= 0.294\,(99.67\ kg) = 29.3\ kg$$

PROBLEM 3.25

Ammonia

Initially, sat. vapor at T_1. Finally, $T_2 > T_1$.



Since the total volume and total mass remain constant, the process of the ammonia takes place at constant specific volume.

(a) $T_1 = 20°C$, $T_2 = 40°C$

IT SOLUTION:

```
T1=20
T2=40
x1=1
v2=v1
p1 = Psat_T("Ammonia", T1)
v1 = vsat_Px("Ammonia", p1, x1)
v2 = v_PT("Ammonia", p2, T2)
```

p2= 9.362 bar ←

Table Solution:

From Table A-13, $v_1 = 0.1492\ m^3/kg$.

Then, in Table A-15, at 40°C $v_2 = 0.1492\ m^3/kg$ falls between 9 bar and 10 bar, Interpolation gives

$p_2 = 9.368$ bar ←

(b) $T_1 = 70°F$, $T_2 = 120°F$

IT SOLUTION:

```
T1=70
T2=120
x1=1
v2=v1
p1 = Psat_T("Ammonia", T1)
v1 = vsat_Px("Ammonia", p1, x1)
v2 = v_PT("Ammonia", p2, T2)
```

p2= 145.2 $lbf/in.^2$ ←

Table Solution:

From Table A-13E, $v_1 = 2.3095\ ft^3/lb$.

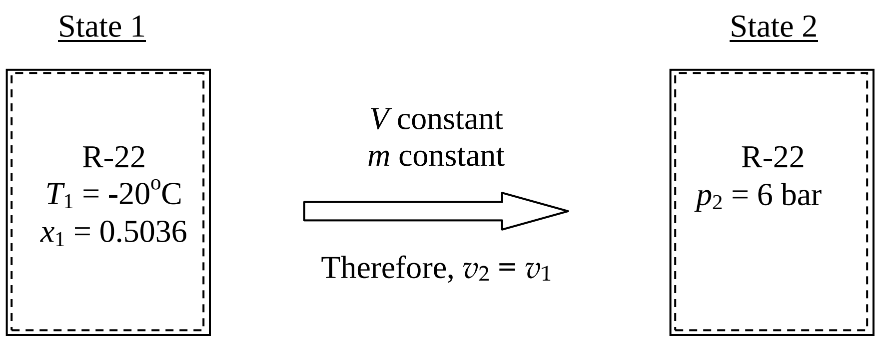
Then in Table A-15E, at 120°F $v_2 = 2.3095\ ft^3/lb$ falls between 140 and 150 lbf/in^2. Interpolation gives

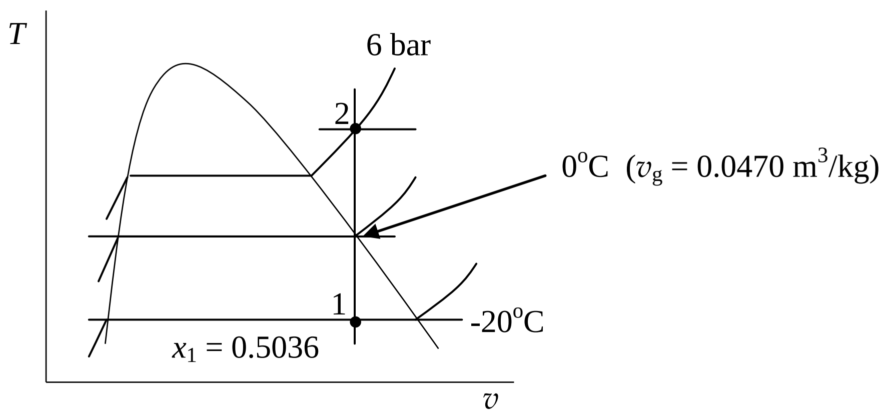
$p_2 = 145.2\ lbf/in.^2$ ←

COMMENT: The values obtained using *IT:Interactive Thermodynamics* compare favorably with those obtained using table data.

PROBLEM 3.26

A closed, rigid tank contains a two-phase liquid-vapor mixture of Refrigerant-22 initially at -20°C with a quality of 50.36%. Energy transfer by heat into the tank occurs until the refrigerant is at a final pressure of 6 bar. Determine the final temperature, in °C. If the final state is in the superheated vapor region, at what temperature, in °C, does the tank contain only saturated vapor?





First, using data from Table A-7 and Eq. 3.2, we can determine v_1 as follows

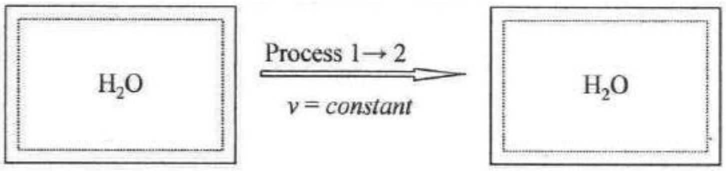
$$v_1 = v_{f1} + x_1(v_{g1} - v_{f1}) = 0.7427 \times 10^{-3} + (0.5036)(0.0926 - 0.7427 \times 10^{-3}) = 0.0470 \text{ m}^3/\text{kg}$$

Since $v_2 = v_1$, State 2 is in the superheated vapor region ($v_2 > v_{g@6bar}$). Thus, interpolating at 6 bar with v_2 = 0.0470 m^3/kg in Table A-9 we get

$T_2 \approx 43.75$°C ⟵

Since State 2 is superheated vapor, the tank contains only saturated vapor at the condition where v_g = 0.0470 m^3/kg. Referring to Table A-7, this occurs at T = 0°C. ⟵

PROBLEM 3.27



$T_1 = 520°C$
$p_1 = 100$ bar

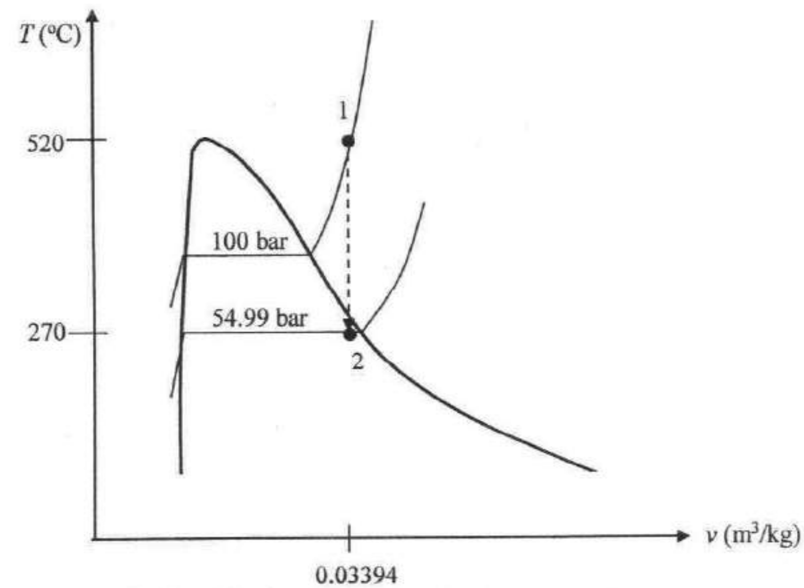
$v_2 = v_1$
$T_2 = 270°C$

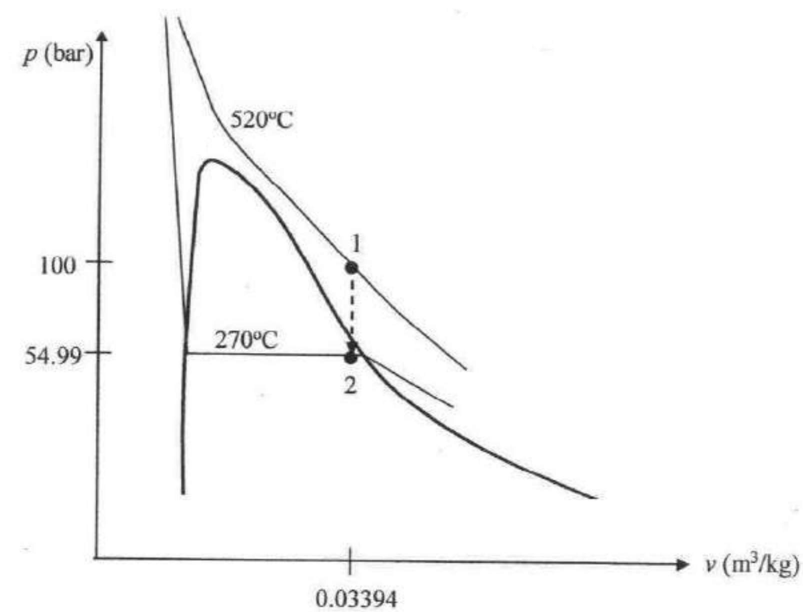
Since the process occurs at constant volume, $v_2 = v_1$. State 1 is fixed by the given property values, $T_1 = 520\,°C$ and $p_1 = 100$ bar. From Table A-4, $v_1 = 0.03394\ m^3/kg$.

State 2 now can be fixed by the properties, $v_2 = v_1 = 0.03394\ m^3/kg$; $T_2 = 270\,°C$. From Table A-2 at $T_2 = 270\,°C$, $v_{f2} < v_2 < v_{g2}$. Thus, State 2 is in the saturated mixture region where pressure and temperature are NOT independent of each other. From Table A-2

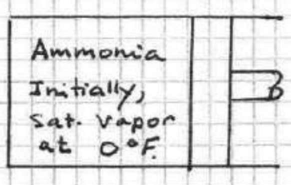
$$P_2 = P_{sat} = \underline{\mathbf{54.99\ bar}} \longleftarrow$$

Since volume remains constant during the process, the process begins in the superheated vapor region and follows a vertical path to 270°C on both the *T*-*v* and *p*-*v* diagrams.



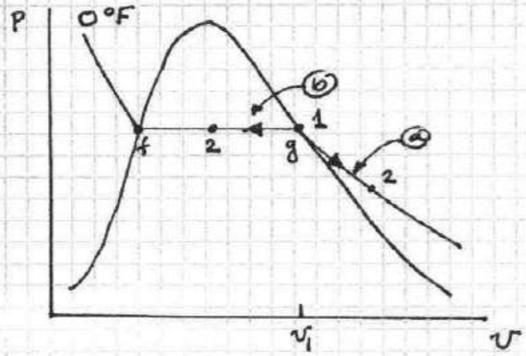


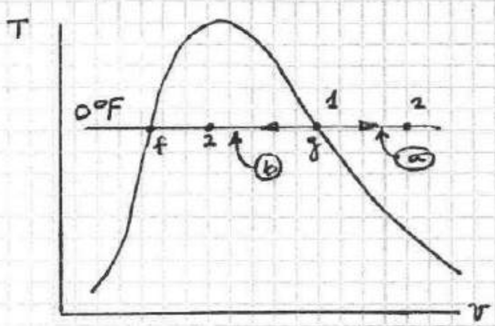
PROBLEM 3.28



(a) Isothermal process with $V_2 = 2V_1$

(b) Isothermal process with $V_2 = \frac{1}{2}V_1$





From Table A-13E, $v_1 = v_g(0°F) = 9.11 \text{ ft}^3/\text{lb}$

(a) $T = 0°C$, $v_2 = 2v_1 = 18.22 \text{ ft}^3/\text{lb}$

Interpolating in Table A-15E at 0°F: State 2 falls between 14 lbf/in² and 16 lbf/in². The pressure at state 2 is

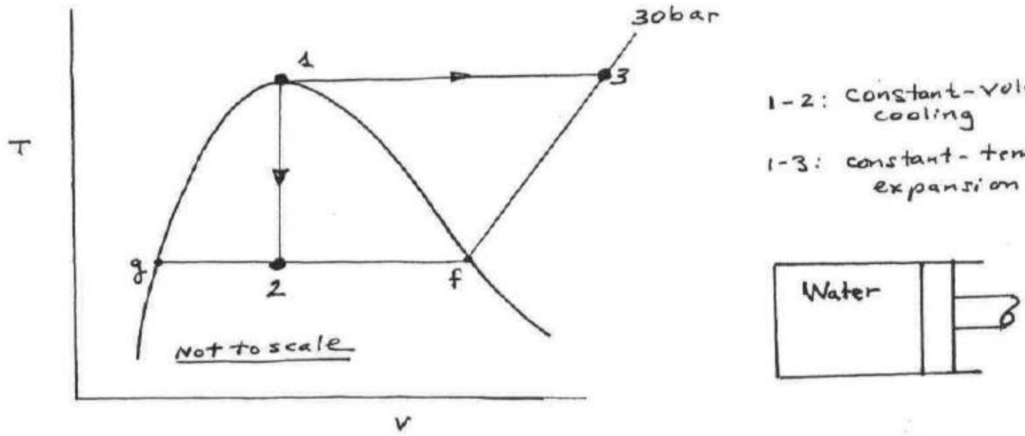
$$p_2 = 15.6 \text{ lbf/in}^2 \longleftarrow$$

(b) $T = 0°C$, $v_2 = \frac{1}{2}v_1 = 4.555 \text{ ft}^3/\text{lb}$. Since $v_f < v_2 < v_g$, state 2 is a two-phase, liquid-vapor mixture. With data from Table A-13E

$$x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{4.555 - 0.02419}{9.11 - 0.02419}$$

$$= 0.499 \; (49.9\%) \longleftarrow$$

PROBLEM 3.29



State 1. Critical point. From the last table entry of Table A-2, the critical point of water is described by

$T_c = 374.14°C$

$P_c = 220.9$ bar

$v_c = 3.155 \times 10^{-3}$ m³/kg

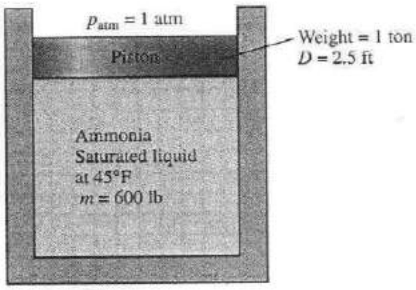
State 2. $P_2 = 30$ bar, $v_2 = v_c = 3.155 \times 10^{-3}$ m³/kg. With data from Table A-3 at 30 bar

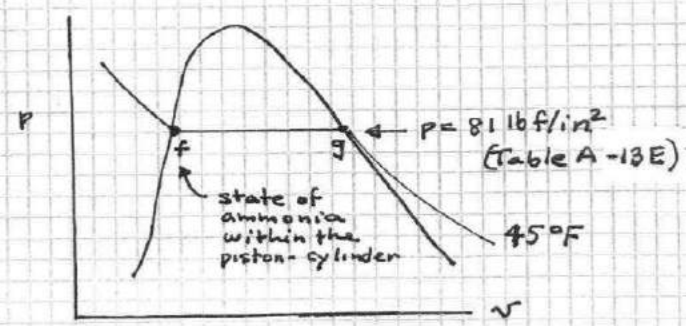
$$x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{(3.155 - 1.2165) \times 10^{-3}}{(66.68 - 1.2165) \times 10^{-3}} = 0.0296 \; (2.96\%) \leftarrow x_2$$

State 3 $P_3 = 30$ bar, $T_3 = 374.14°C$. Interpolation in Table A-4 gives

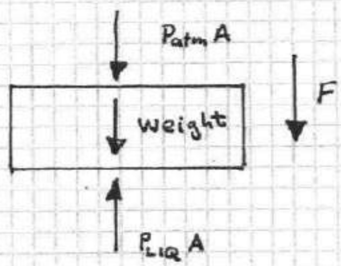
$v_3 = 0.0948$ m³/kg $\leftarrow v_3$

PROBLEM 3.30





In the absence of friction, the forces acting on the piston include the force of the pressure acting on the bottom of the piston, the piston weight, the force of the atmosphere acting on top of piston, and the force F, if any, required to hold the piston in place. Assuming the force F acts vertically downward, the piston force balance reads



$$P_{LIQ}A = P_{atm}A + \text{Weight} + F$$

$$\Rightarrow \quad F = (P_{LIQ} - P_{atm})A - \text{Weight}$$

$$A = \left(\frac{\pi D^2}{4}\right)$$

Calculating,

$$F = (81 - 14.7)\frac{\text{lbf}}{\text{in}^2}\left[\frac{\pi(2.5\,\text{ft})^2}{4}\right]\left|\frac{144\,\text{in}^2}{1\,\text{ft}^2}\right| - 1\,\text{ton}\left|\frac{2000\,\text{lbf}}{1\,\text{ton}}\right|$$

$$= 44{,}865\ \text{lbf}$$

Yes, mechanical attachments, such as stops, are required.

The volume occupied by the ammonia liquid is

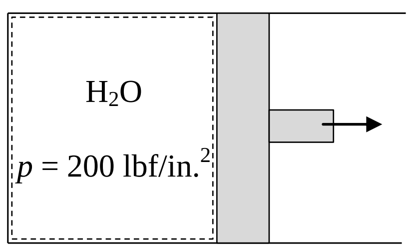
$$V = m\,v_f = (600\,\text{lb})\left(0.02548\,\frac{\text{ft}^3}{\text{lb}}\right) = 15.29\ \text{ft}^3$$

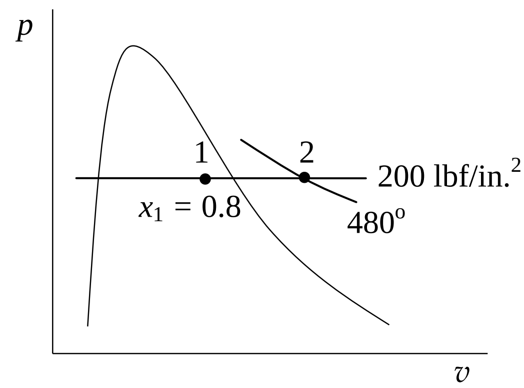
Table A-13E

PROBLEM 3.31

A piston-cylinder assembly contains a two-phase liquid-vapor mixture of H_2O at 200 lbf/in.^{2} with a quality of 80%. The mixture is heated and expands at constant pressure until a final temperature of 480^{o}F is reached. Determine the work for the process, in Btu per lb of H_2O present.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL: 1. The water is a closed system. 2. The pressure is constant.

ANALYSIS: Since the pressure is constant, Eq. 2.17 can be used to determine the work per unit mass of H_2O. First, fix each state and find the respective specific volumes.

For State 1, the specific volume can be determined using Eq. 3.2 with data from Table A-3E, as follows.

$$v_1 = v_{f1} + x_1(v_{g1} - v_{f1}) = 0.01839 + (0.8)(2.289 - 0.01839) = 1.835 \text{ ft}^3/\text{lb}$$

At State 2, $T_2 > T_{sat}$(200 lbf/in.^{2}), so the state is in the superheated vapor region. Interpolating in Table A-4e at 200 lbf/in.^{2} and 480^{o}F we get

$$v_2 \approx 2.548 + [(480 - 450)/(500 - 450)]*(2.724 - 2.548) = 2.654 \text{ ft}^3/\text{lb}$$

Now, with Eq. 2.17

$$W = \int_{V_1}^{V_2} p dV = p(V_2 - V_1)$$

Thus

$$W/m = \int_{v_1}^{v_2} p dv = p(v_2 - v_1)$$

$$= \left(200\ \frac{\text{lbf}}{\text{in.}^2}\right)(2.654 - 1.835)\frac{\text{ft}^3}{\text{lb}}\left|\frac{144 \text{ in.}^2}{1 \text{ ft}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right| = 30.32 \text{ Btu/lb (out)}$$

Note, the work is positive, indicating that the transfer of energy is out of the system as expected.

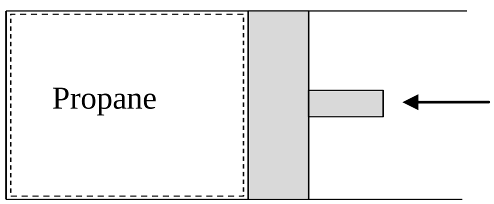
PROBLEM 3.32

Seven lb of propane in a piston-cylinder assembly, initially at p_1 = 200 lbf/in.^{2} and T_1 = 200°F, undergoes a constant-pressure process to a final state. The work for the process is – 88.84 Btu. At the final state, determine the temperature, in $^{\circ}$F, if superheated, and the quality if saturated.

KNOWN: Propane undergoes a process at constant pressure in a piston-cylinder assembly for which data are provided at the initial state and the work is specified.

FIND: Determine specified data at the final state.

SCHEMAITC AND GIVEN DATA:



ENGINEERING MODEL: (1) the given mass of propane is the closed system. (2) Volume change is the only work mode. (3) The process occurs at constant pressure.

m = 7 lb
p_1 = 200 lbf/in.^{2}
T_1 = 200°F
W = -88.84 Btu

ANALYSIS: Two properties are required to fix the State 2. One of these is the pressure, $p_2 = p_1$ = 200 lbf/in.^{2} The other is specific volume found from the given values for work as follows. Since volume change is the only work mode and the pressure is constant, we can use Eq. 2.17 to get

$$W = \int_{V_1}^{V_2} p\,dV = mp(v_2 - v_1)$$

Solving for v_2

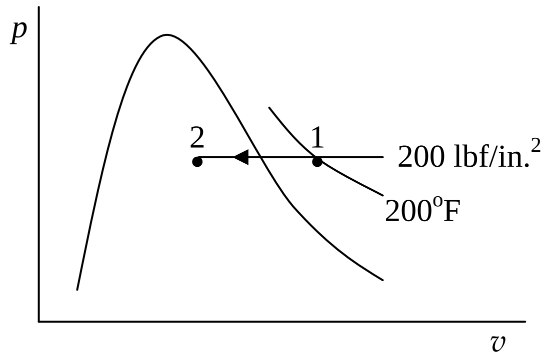
$$v_2 = W/mp + v_1$$

From Table A-18E, v_1 = 0.7026 ft^3/lb. Thus

$$v_2 = \frac{(-88.84\text{ Btu})}{(7\text{ lb})(200\frac{\text{lbf}}{\text{in}^2})}\left|\frac{778\text{ ft}\cdot\text{lbf}}{1\text{ Btu}}\right|\left|\frac{1\text{ ft}^2}{144\text{ in}^2}\right| + 0.7025\text{ ft}^3/\text{lb} = 0.3597\text{ ft}^3/\text{lb}$$

From Table A-17E we see that at 200 lbf/in.^{2}, $v_f < v_2 < v_g$. thus, State 2 is a two-phase liquid-vapor mixture, and the quality is

$$x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{0.3597 - 0.03432}{(0.5261 - 0.03432)} = 0.662\ (66.2\%)$$



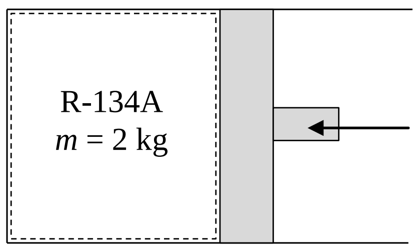
PROBLEM 3.33

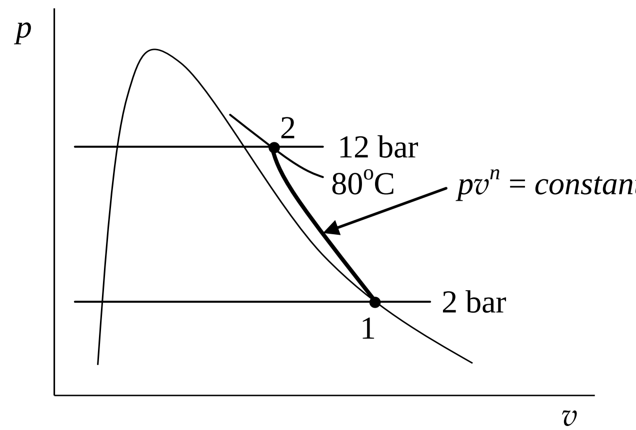
Two kg of Refrigerant 134A undergoes a polytropic process in a piston-cylinder assembly from an initial state of saturated vapor at 2 bar to a final state of 12 bar, 80°C. Determine the work for the process, in kJ.

KNOWN: Refrigerant 134A undergoes a polytropic process in a piston-cylinder assembly.

FIND: Determine the work.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL: 1. The refrigerant is a closed system. 2. The process is polytropic: pv^n = *constant*.

ANALYSIS: The work for the polytropic process is determined using Eq. 2.17, with pv^n = *constant*. Following the procedure of part (a) of Ex. 2.1

$$W = m\int_{v_1}^{v_2} p\,dv = \frac{m(p_2v_2 - p_1v_1)}{1-n} \qquad (*)$$

In order to evaluate this expression, we need to determine the specific volumes and the polytropic exponent, n.

State 1: From Table A-11; $v_1 = v_{g1} = 0.0993\ \text{m}^3/\text{kg}$
State 2: From Table A-12, at 12 bar, 80°C; $v_2 = 0.02051\ \text{m}^3/\text{kg}$

The polytropic exponent is found from pv^n = *constant* as follows.

$$p_1v_1^n = p_2v_2^n \quad \rightarrow \quad \left(\frac{p_1}{p_2}\right) = \left(\frac{v_2}{v_1}\right)^n \quad \rightarrow \quad n = \ln(p_1/p_2)\,/\,\ln(v_2/v_1)$$

$$n = \ln(2/12)\,/\,\ln(0.02051/0.0993) = 1.136$$

Inserting values in Eq. (*) and converting units, we get

PROBLEM 3.33 (CONTINUED)

$$W = \frac{(2\text{ kg})[(12\text{ bar})(0.02051\text{ m}^3/\text{kg}) - (2)(0.0993)]}{1 - 1.136}\left|\frac{10^5\text{N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{N}\cdot\text{m}}\right| = -69.88\text{ kJ (in)}$$

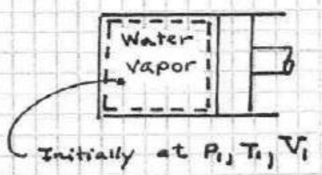
PROBLEM 3.34

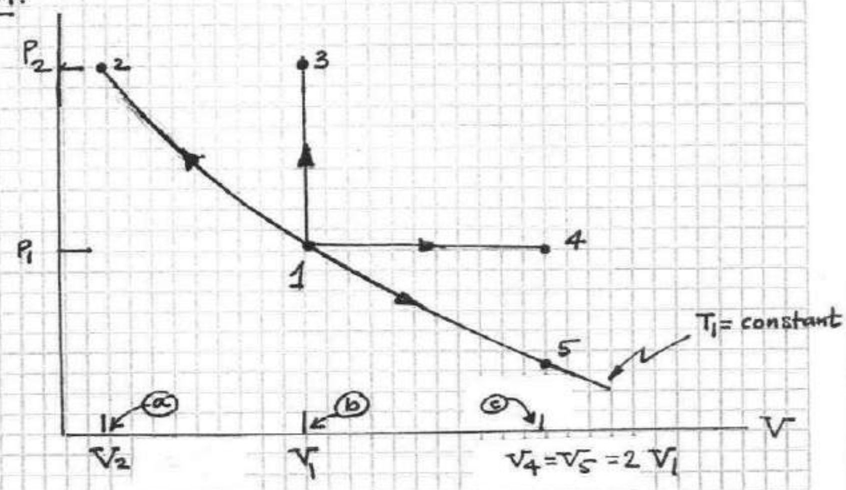
KNOWN: From an initial state, water vapor contained within a piston-cylinder assembly undergoes four different processes:

Process 1–2: Constant-temperature to $p_2 = 2p_1$.
Process 1–3: Constant-volume to $p_3 = 2p_1$.
Process 1–4: Constant-pressure to $V_4 = 2V_1$
Process 1–5: Constant-temperature to $V_5 = 2V_1$

FIND: Sketch each process on a p–V diagram, identify work by an area on the diagram, and comment.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL:
1. The water vapor is the closed system.
2. Volume change is the only work mode.

ANALYSIS: Since volume change is the only work mode, the work in this application is given by Eq. 2.17:

$$W = \int p\,dV$$

Process 1–2: Magnitude of work = Area(1–2–a–b–1). The water vapor is compressed, so work is done on the water vapor.

Process 1–3: There is no work in this constant-volume process.

Process 1–4: Magnitude of work = Area(1–4–c–b–1). The water vapor expands to a larger volume, so the water vapor does work.

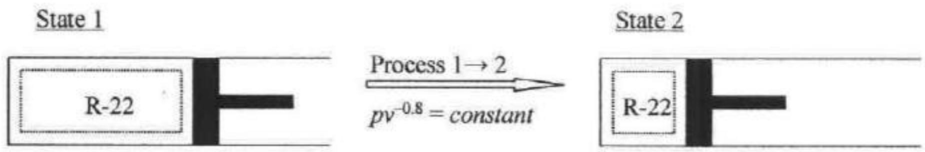
Process 1–5: Magnitude of work = Area(1–5–c–b–1). The water vapor expands to a larger volume, so the water vapor does work.

PROBLEM 3.35

KNOWN: Refrigerant 22 undergoes a polytropic process between an initial specified state and a specified final pressure.

FIND: Determine the work.

SCHEMATIC AND GIVEN DATA:



$m = 3$ kg
$p_1 = 12$ bar
$T_1 = 60°C$

$p_2 = 8$ bar

ENGINEERING MODEL:
1. The Refrigerant 22 is the system.
2. The system undergoes a polytropic process.

ANALYSIS:
Work during the process is due to expansion work

$$W = \int_{V_1}^{V_2} p dV = m\int_{v_1}^{v_2} p dv$$

From the pressure-volume relation, $pv^{-0.8} = constant$, pressure can be expressed in terms of specific volume

$$p = (constant)/v^{-0.8}$$

Substituting the expression for pressure in terms of volume into the work equation and integrating yield

$$W = m\int_{v_1}^{v_2} \frac{(constant)dv}{v^{-0.8}} = m\frac{(constant)v_2^{1-(-0.8)} - (constant)v_1^{1-(-0.8)}}{1-(-0.8)}$$

Substituting $constant = p_1 v_1^{-0.8} = p_2 v_2^{-0.8}$ and simplifying yield

$$W = m\frac{(p_2 v_2^{-0.8})v_2^{1-(-0.8)} - (p_1 v_1^{-0.8})v_1^{1-(-0.8)}}{1-(-0.8)} = m\frac{p_2 v_2 - p_1 v_1}{1-(-0.8)}$$

Specific volume at state 1 and state 2 are required to solve for work. From Table A-9, $v_1 = 0.02319$ m^3/kg. Rearranging the pressure-volume relation, $p_1 v_1^{-0.8} = p_2 v_2^{-0.8}$, to solve for v_2 yields

$$v_2 = v_1\left(\frac{p_1}{p_2}\right)^{\left(\frac{1}{-0.8}\right)} = \left(0.02319\frac{\text{m}^3}{\text{kg}}\right)\left(\frac{12\text{ bar}}{8\text{ bar}}\right)^{\left(\frac{1}{-0.8}\right)} = 0.01397 \text{ m}^3\text{/kg}$$

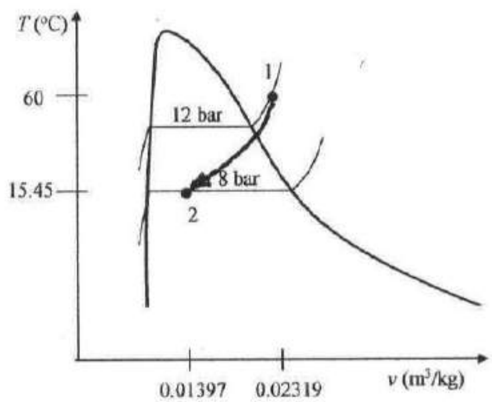
Solving for work yields

$$W = (3\text{ kg})\frac{(800\text{ kPa})\left(0.01397\frac{\text{m}^3}{\text{kg}}\right)-(1200\text{ kPa})\left(0.02319\frac{\text{m}^3}{\text{kg}}\right)}{1-(-0.8)}\left|\frac{10^3\frac{\text{N}}{\text{m}^2}}{1\text{ kPa}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| = \underline{\mathbf{-27.75\ kJ}} \longleftarrow$$

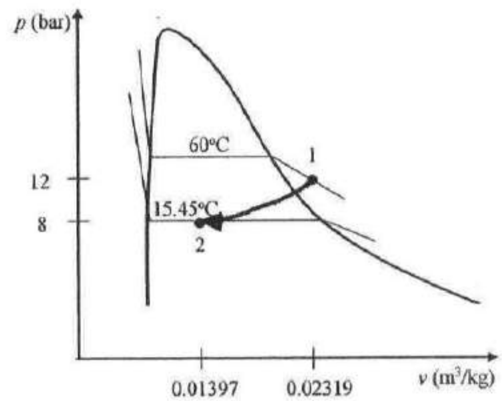
The negative sign for work denotes energy transfer into the system.

The process begins in the superheated vapor region and ends in the saturated mixture region as shown on the *T-v* and *p-v* diagrams.

<u>*T-v* Diagram</u>



<u>*p-v* Diagram</u>

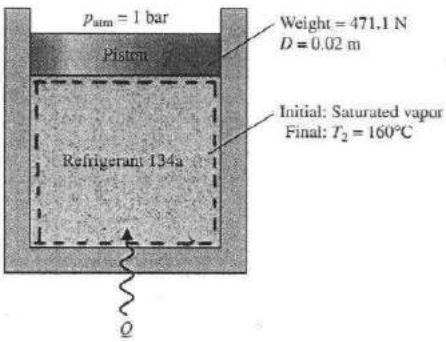


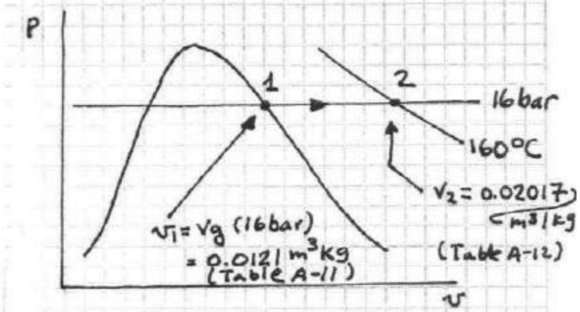
PROBLEM 3.36

KNOWN: Refrigerant 134 contained in a piston-cylinder assembly is slowly heated. Data is provided at the initial and final states.

FIND: Determine the work, in kJ/kg.

SCHEMATIC & GIVEN DATA:





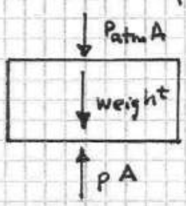
ENGINEERING MODEL:

1. The Refrigerant 134a is the closed system.
2. During the process the refrigerant is slowly heated and the piston moves smoothly in the cylinder.
3. Volume change is the only work mode.

ANALYSIS

With assumptions 2 and 3 the work can be evaluated from Eq. 2.17, expressed as $W/m = \int_1^2 p\,dv$. To apply this requires that states 1 and 2 be fixed and pressure-volume relation be determined.

Using a force balance for the piston, we conclude the pressure of the refrigerant remains constant during the process: Since the piston moves smoothly in the cylinder, the only forces are those shown:



$$\Sigma F = 0 \Rightarrow pA = p_{atm}A + \text{Weight}$$

$$\therefore\ p = p_{atm} + \frac{\text{Weight}}{A}$$

$$A = \frac{\pi D^2}{4} = \frac{\pi(0.02\,\text{m})^2}{4} = 3.14\times10^{-4}\,\text{m}^2$$

$$= 1\,\text{bar} + \left(\frac{471.1\,\text{N}}{3.14\times10^{-4}\,\text{m}^2}\right)\left|\frac{1\,\text{bar}}{10^5\,\text{N/m}^2}\right|$$

$$= 16\,\text{bar}.$$

The process is sketched on the accompanying p-v diagram. Since pressure is constant, we get

$$\frac{W}{m} = p(v_2 - v_1)$$

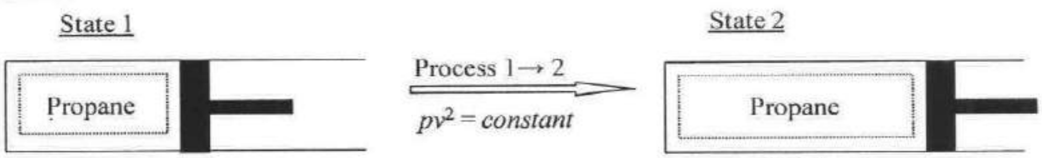
$$= (16\,\text{bar})\left|\frac{10^5\,\text{N/m}^2}{1\,\text{bar}}\right|(0.02017 - 0.0121)\frac{\text{m}^3}{\text{kg}}\left|\frac{1\,\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right| = 12.9\,\frac{\text{kJ}}{\text{kg}}$$

PROBLEM 3.37

Known: Propane is expanded in a piston-cylinder assembly from a known initial state to a known final pressure. The pressure-volume relationship for the process is given.

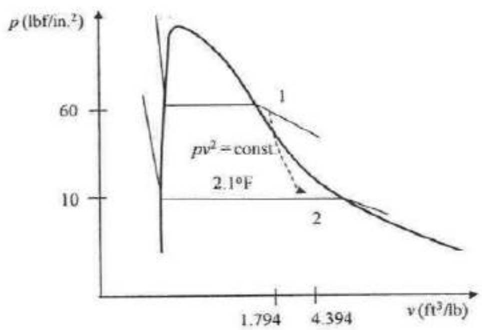
Find: Determine the energy transfer through work, in Btu.

Schematic and Given Data:



$m = 0.1$ lb
$p_1 = 60$ lbf/in.^{2}
$T_1 = 30°F$

$p_2 = 10$ lbf/in.^{2}



Engineering Model:
(1) The propane is a closed system.
(2) The process is polytropic with pv^2 = constant.

Analysis:

The work is obtained using Eq. 2.17 using pv^2 = constant. The result corresponds to Eq. (1) of Example 2.1, as can be verified:

$$W = \frac{m(p_2 v_2 - p_1 v_1)}{1-n} \tag{1}$$

Next find the specific volumes. State 1 is in superheated vapor region and from Table A-18E, v_1 = 1.794 ft^3/lb. The specific volume at state 2 is as follows:

$$p_1 v_1^2 = p_2 v_2^2 = \text{const.} \quad or \quad v_2 = v_1\sqrt{\frac{p_1}{p_2}} = 1.794\frac{\text{ft}^3}{\text{lb}}\sqrt{\frac{60}{10}} = 4.394\frac{\text{ft}^3}{\text{lb}}$$

The work is obtained using Eq. (1) with n=2, as follows:

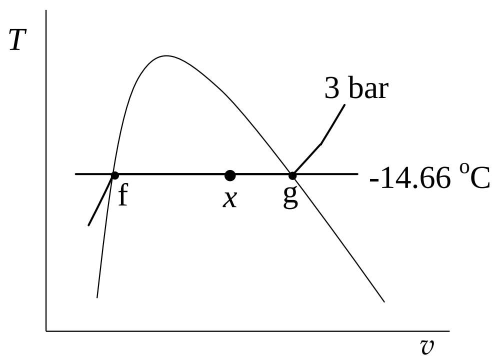
$$W = 0.1\text{lb}\left[\left(60\frac{\text{lbf}}{\text{in.}^2}\right)\left(1.794\frac{\text{ft}^3}{\text{lb}}\right) - \left(10\frac{\text{lbf}}{\text{in.}^2}\right)\left(4.394\frac{\text{ft}^3}{\text{lb}}\right)\right]\left|\frac{1\text{Btu}}{778\text{ft}\cdot\text{lbf}}\right|\left|\frac{144\text{in.}^2}{1\text{ft}^2}\right| = 1.18 \text{ Btu}$$

PROBLEM 3.38

For each of the following cases, determine the specified properties and show the states on a sketch of the T-v diagram.

(a) For Refrigerant 22 at p = 3 bar and v = 0.05 m^3/kg, determine T in ^{o}C and u in kJ/kg.
(b) For water at T = 200^{o}C and v = 0.2429 m^3/kg, determine p in bar and h in kJ/kg.
(c) For ammonia at p = 5 bar and u = 1400 kJ.kg, determine T in ^{o}C and v in m^3/kg.

(a) For Refrigerant 22 at p = 3 bar and v = 0.05 m^3/kg, determine T in ^{o}C and u in kJ/kg.

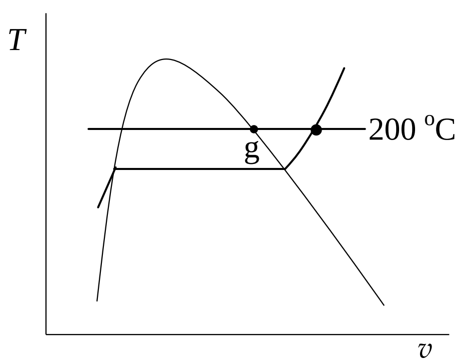


From Table A-8, $v_f < v < v_g$, so the state is in the two-phase liquid-vapor region and T = -14.66^{o}C.

The quality is
$x = (v - v_f) / (v_g - v_f)$
$= (0.05 – 0.7521 \text{ x } 10^{-3})/(0.0765 – 0.7521 \text{ x } 10^{-3}) = 0.65$

The specific internal energy is
$u = u_f + x(u_g – u_f)$
$= 27.99 + (0.65)(221.34 – 27.99) = 153.67$ kJ/kg

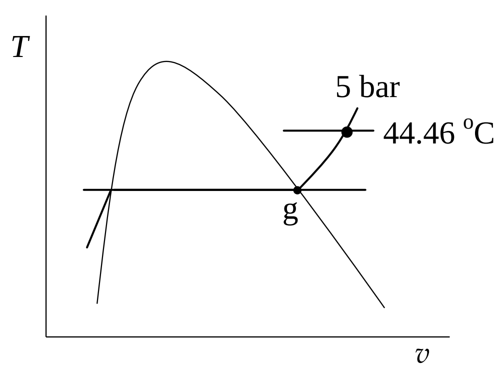
(b) For water at T = 200^{o}C and v = 0.4249 m^3/kg, determine p in bar and h in kJ/kg.



From Table A-2, $v > v_g$, so the state is in the superheated vapor region. Scanning across at a constant temperature of 200^{o}C in Table A-4, we find that v = 0.4249 m^3/kg at p = 5 bar.

The specific enthalpy is
h = 2855.4 kJ/kg

(c) For ammonia at p = 5 bar and u = 1400 kJ.kg, determine T in ^{o}C and v in m^3/kg.



From Table A-14, $u > u_g$ at 5 bar, so the state is in the superheated vapor region. Interpolating in Table A-15 with p = 5 bar and u = 1400 kJ/kg, we get

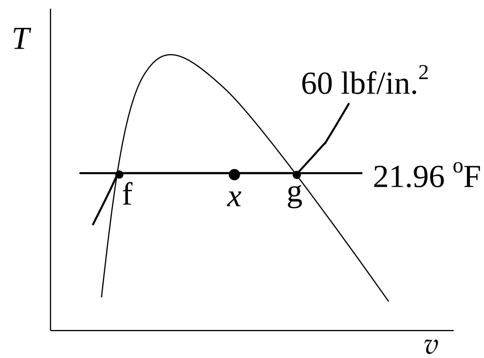
$T \approx$ 40 ^{o}C + [(1400 – 1391.74)/(1428.76 – 1391.74)] (60 – 40^{o}C)
= 44.46^{o}C

The specific volume is
$v \approx$ 0.29227 +
[(1400 – 1391.74)/(1428.76 – 1391.74)] (0.31410 – 0.29277)
= 0.2971 m^3/kg

PROBLEM 3.39

Determine the values of the specified properties at each of the following conditions and show the states on a sketch of the T-v diagram.
(a) For Refrigerant 22 at p = 60 lbf/in.^{2} and u = 50 Btu/lb, determine T in ^{o}F and v in ft^3/lb.
(b) For Refrigerant 134a at T = 120 ^{o}F and u = 114 Btu/lb, determine p in lbf/in.^{2} and v in ft^3/lb.
(c) For water vapor at p = 100 lbf/in.^{2} and h = 1240 Btu/lb, determine T in ^{o}F, v in ft^3/lb, and u in Btu/lb.

(a) For Refrigerant 22 at p = 60 lbf/in.^{2} and u = 50Btu/lb in kJ/kg, determine T in ^{o}F and v in ft^3/lb.

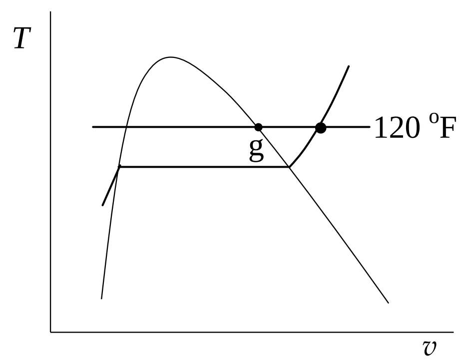


From Table A-8E, $u_f < u < u_g$, so the state is in the two-phase liquid-vapor region and T = 21.96 ^{o}F. ←

The quality is
$x = (u - u_f) / (u_g - u_f)$
$= (50 – 16.48)/(96.62 – 16.48) = 0.418$

The specific volume is
$v = v_f + x(v_g – v_f)$
$= 0.01232 + (0.418)(0.9014 – 0.01232) = 0.384$ ft^3/lb ←

(b) For Refrigerant 134a at T = 120^{o}F and u = 114 Btu/lb, determine p in lbf/in.^{2} and v in ft^3/lb.



From Table A-10E at 120^{o}F, $u > u_g$, so the state is in the superheated vapor region. Scanning across at a constant temperature of 120^{o}F in Table A-12E, we find that u = 114 m^3/kg between pressures of 60 and 70 lbf/in.^{2}

Interpolating at T = 120^{o}F and u = 114 Btu/lb, we get

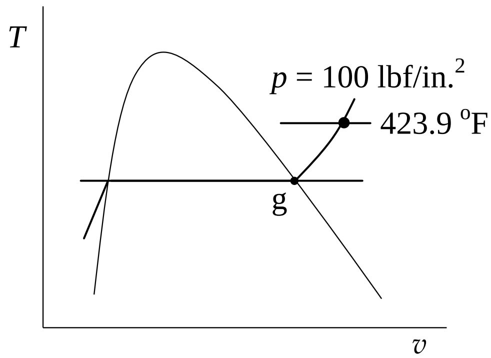
$p \approx 60 + [(114 – 113.96)/(114.35 - 113.96)] (70 – 60)$
$= 61.03$ lbf/in.^{2} ←

The specific volume is

$v \approx 0.9482 + [(114 – 113.96)/(114.35 - 113.96)](0.8023 - 0.9482)$
$= 0.9332$ ft^3/lb ←

PROBLEM 3.39 (CONTINUED) – PAGE 2

(c) For water vapor at p = 100 lbf/in.^{2} and h = 1240 Btu/lb, determine T in ^{o}F, v in ft^3/lb, and u in Btu/lb.



From Table A-3E, $h > h_g$ at 100 lbf/in.^{2}, so the state is in the superheated vapor region. Interpolating in Table A-4E with p = 100 lbf/in.^{2} and h = 1240 Btu/lb, we get

$$T \approx 400^{o}\text{F} + [(1240 - 1227.5)/(1253.6 - 1227.5)]\ (450 - 400)^{o}\text{F}$$
$$= 423.9^{o}\text{F}$$

The specific volume is

$$v \approx 4.934 + [(1240 - 1227.5)/(1253.6 - 1227.5)]\ (5.265 - 4.934)$$
$$= 5.093\ \text{ft}^3/\text{lb}$$

PROBLEM 3.40

Using *IT*, determine the specified property data at the indicated states. Compare with the results from the appropriate table.
(a) Cases (a), (b), and (c) of Problem 3.38.
(b) Cases (a), (b), and (c) of Problem 3.39.

Problem 3.38

Case (a)

```
p=3  // bar
v = 0.05  // m^3/kg
x = x_vP("R22", v, p)
u = usat_Px("R22", p, x)
```

IT Results	Table Results
T = -14.66°C	T = -14.66°C
x = 0.65	x = 0.65
u = 153.7 kJ/kg	u = 153.67 kJ/kg

Case (b)

```
T = 200  // C
v = 0.4229  // m^3/kg
v = v_PT("Water/Steam", p, T)
h = h_PT("Water/Steam", p, T)
```

IT Results	Table Results
p = 5.023 bar	p = 5 bar
h = 2855 kJ/kg	h = 2855.4 kJ/kg

Case (c)

```
p = 5  // bar
u = 1400  // kJ/kg
u = u_PT("Ammonia", p, T)
v = v_PT("Ammonia", p, T)
```

IT Results	Table Results
T = 44.42°C	T = 44.46°C
v = 0.2972 m^3/kg	v = 0.2971 m^3/kg

Problem 3.39

Case (a)

```
p = 60  // lbf/in^2
u = 50  // Btu/lb
T = Tsat_P("R22", p)
u = usat_Px("R22", p, x)
v = vsat_Px("R22", p, x)
```

IT Results	Table Results
T = 21.96°F	T = 21.96°F
x = 0.4187	x = 0.418
v = 0.3846 ft^3/lb	v = 0.384 ft^3/lb

Case (b)

```
T = 120  // oF
u = 114  // Btu/lb
u = u_PT("R134A", p, T)
v = v_PT("R134A", p, T)
```

IT Results	Table Results
p = 62 lbf/in.^{2}	p = 61.03 lbf/in.^{2}
v = 0.9153 ft^3/lb	v = 0.9332 ft^3/lb

Case (c)

```
p = 100  // lbf/in.^2
h = 1240  // Btu/lb
T = T_Ph("Water/Steam", p, h)
v = v_Ph("Water/Steam", p, h)
```

IT Results	Table Results
T = 424.1°F	T = 423.9°F
v = 5.095 ft^3/lb	v = 5.093 ft^3/lb

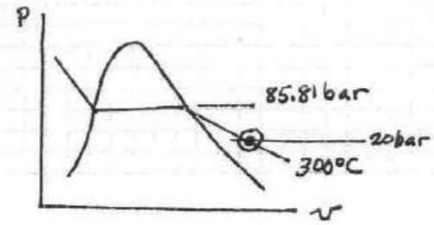
The results compare favorably. Note that finding the states using *IT* requires identifying the region (e.g. superheated vapor, two-phase liquid-vapor,…) in order to choose the correct functional forms for the data expressions.

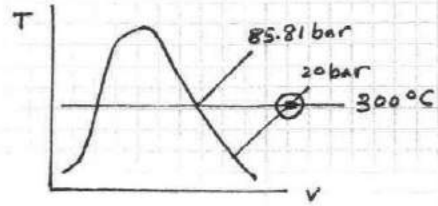
PROBLEM 3.41

(a) $p = 2$ MPa, $T = 300°C$, Find u, in kJ/kg.

Table A-4:

$\underline{u = 2772.15 \frac{kJ}{kg}}$

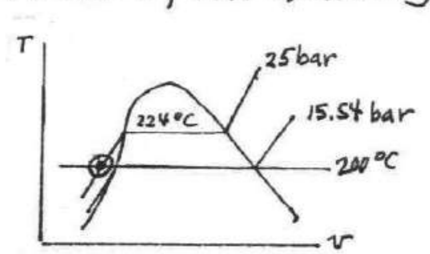


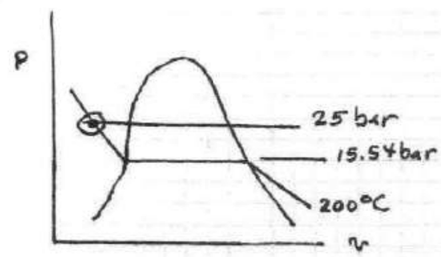


(b) $p = 2.5$ MPa, $T = 200°C$, Find u, in kJ/kg.

Table A-5:

$\underline{u = 849.9 \frac{kJ}{kg}}$





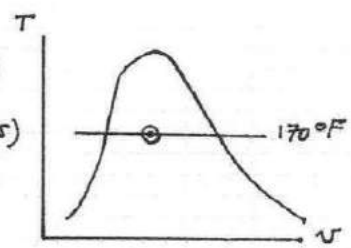
(c) $T = 170°F$, $x = 50\%$, find u, in Btu/lb.

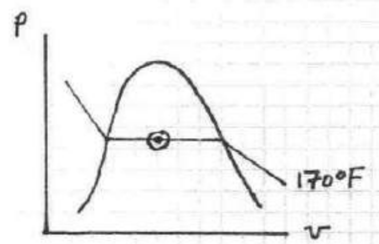
Table A-2E:

$u_x = u_f + x(u_g - u_f)$

$= 137.95 + 0.5(1065.4 - 137.95)$

$= \underline{601.68} \frac{Btu}{lb}$



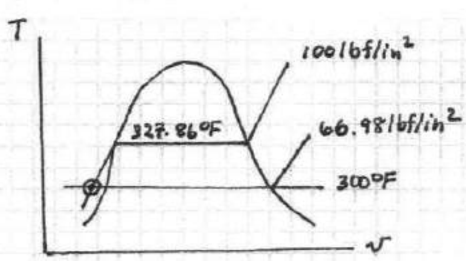


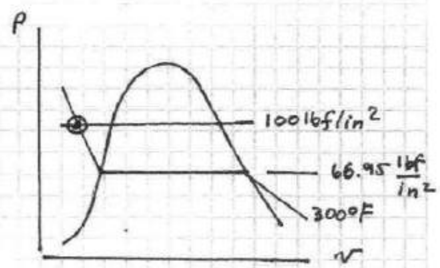
(d) $p = 100$ lbf/in^2, $T = 300°F$, find h, in Btu/lb

Table A-2E:
With Eq. 3.14,

$h \approx h_f(T)$

$= \underline{269.7}$ Btu/lb



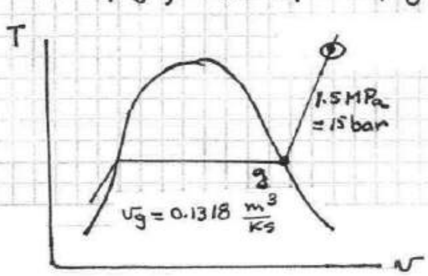


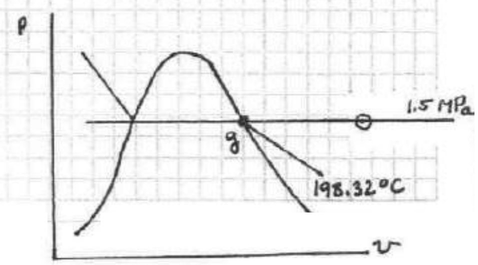
(e) $p = 1.5$ MPa, $v = 0.2095$ m^3/kg, find h, in kJ/kg.

Table A-4E:
$v_g = 0.1318$ m^3/kg.

$\Rightarrow v > v_g$

$\underline{h = 3299.15}$ kJ/kg

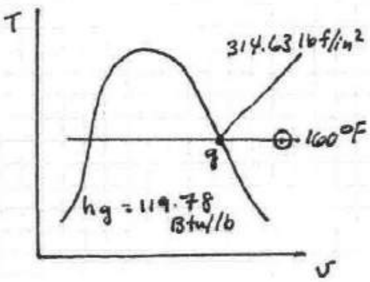


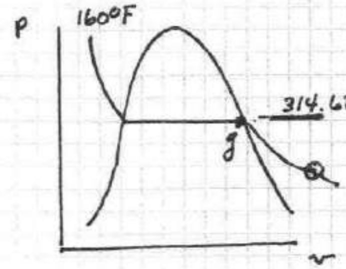


PROBLEM 3.42

(a) R134a at $T = 160°F$, $h = 127.7$ Btu/lb, find v, in ft^3/lb.

Table A-10E at 160°F,
$h_g = 119.78$ Btu/lb.
Then, from Table A-12E
at 160°F, $h = 127.7$ Btu/lb,
$v = 0.2636$ ft^3/lb
($p = 200$ lbf/in^2)



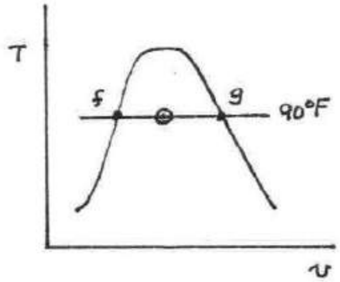


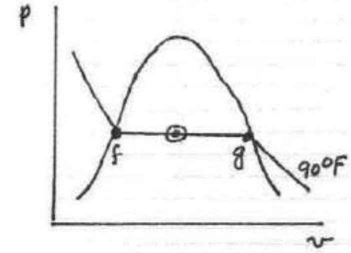
(b) R134a at $T = 90°F$, $u = 72.71$ Btu/lb, find h, in Btu/lb.

Table A-10E at 90°F
$u_f < u < u_g$

$$\therefore \; x = \frac{u-u_f}{u_g-u_f} = \frac{72.71-40.42}{105-40.42} = 0.5$$

$\Rightarrow$ $h = 77.24$ Btu/lb

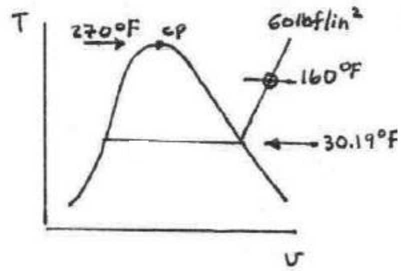


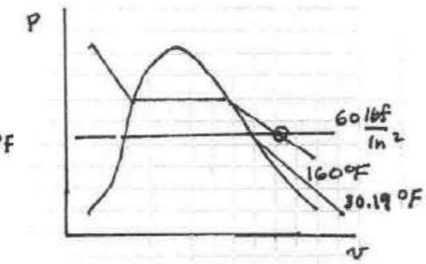


(c) Ammonia at $T = 160°F$, $p = 60$ lbf/in^2, find u, in Btu/lb

Table A-15E,

$u = 624.44$ Btu/lb

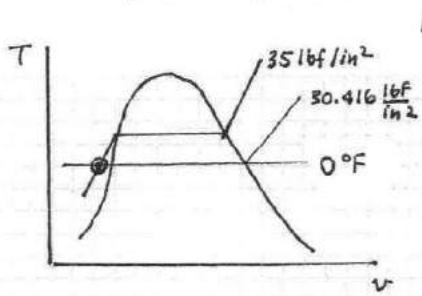


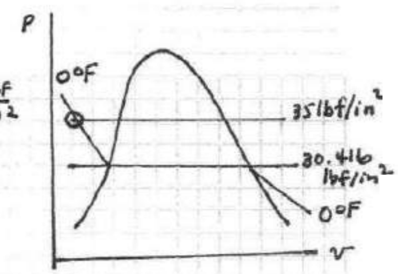


(d) Ammonia at $T = 0°F$, $p = 35$ lbf/in^2, find u, in Btu/lb.

This state is in the
liquid region. Using
Eq. 3.12, with Table A-13E

$u \approx u_f(T) = 42.32$ Btu/lb
0°F

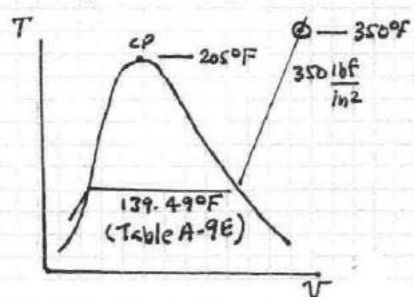


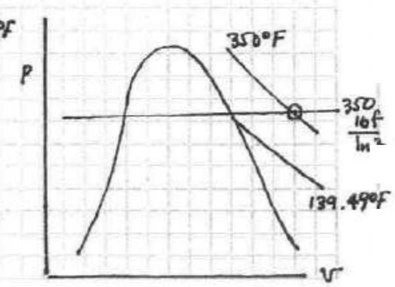


(e) Refrigerant 22 at $p = 350$ lbf/in^2, $T = 350°F$, find u, in Btu/lb

This is a vapor state.
Interpolation in Table
A-9E gives

$u = 143.14$ Btu/lb





PROBLEM 3.43

(a) $p = 3$ bar, $v = 0.5$ m³/kg

Table A-3; $v_g < v < v_f$

$\Rightarrow T = 133.6°C$

$$x = \frac{v - v_f}{v_g - v_f}$$

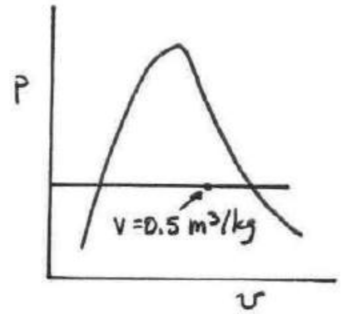
$$= \frac{0.5 - 1.0732 \times 10^{-3}}{0.6058 - 1.0732 \times 10^{-3}}$$

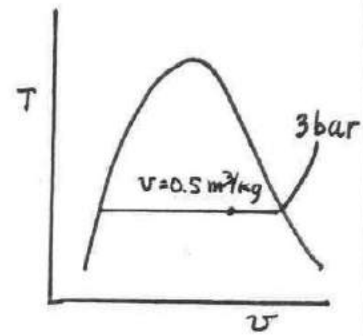
$$= 0.825$$

$$\therefore u = u_f + x(u_g - u_f)$$

$$= 561.15 + (.825)(2543.6 - 561.5)$$

$$= 2196.7 \text{ kJ/kg}$$





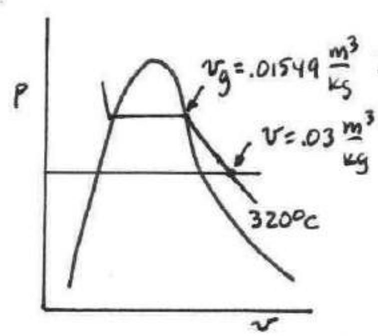
(b) $T = 320°C$, $v = 0.03$ m³/kg

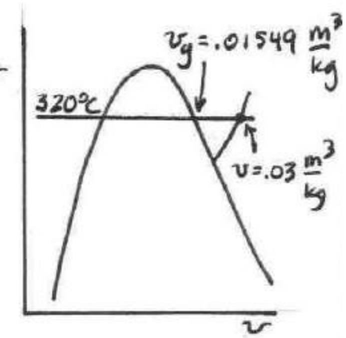
Table A-2; $v > v_g$ at 320°C

$\Rightarrow$ Table A-4; At 320°C the state falls between 60 and 80 bar. Interpolating,

$p = 74.67$ bar $= 7.467$ MPa

$u = 2678.0$ kJ/kg



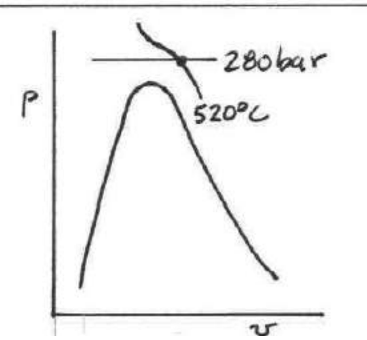


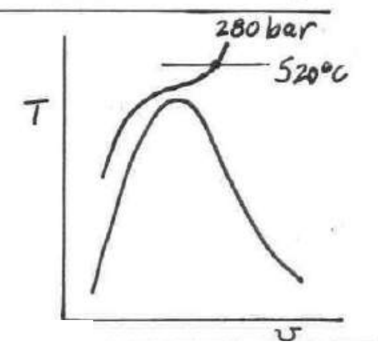
(c) $p = 28$ MPa $= 280$ bar, $T = 520°C$

Table A-2

$v = 0.01020$ m³/kg

$h = 3192.3$ kJ/kg





(d) $T = 10°C$, $v = 100$ m³/kg

Table A-2; $v_f < v < v_g$ at 10°C.

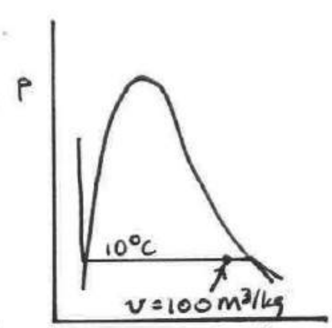
Thus, $p = 0.01228$ bar $= 1.228$ kPa

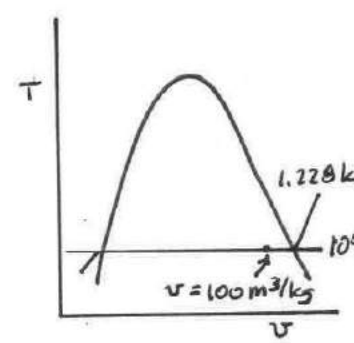
$$x = \frac{v - v_f}{v_g - v_f} = \frac{100 - 1.0004 \times 10^{-3}}{106.379 - 1.0004 \times 10^{-3}}$$

$$= 0.94$$

$$h = h_f + x\, h_{fg}$$

$$= 42.01 + (.94)(2477.7) = 2371 \text{ kJ/kg}$$





PROBLEM 3.44

(a) $p = 20\ \text{lbf/in}^2$, $v = 16\ \text{ft}^3/\text{lb}$

Table A-3E $v_f < v < v_g$

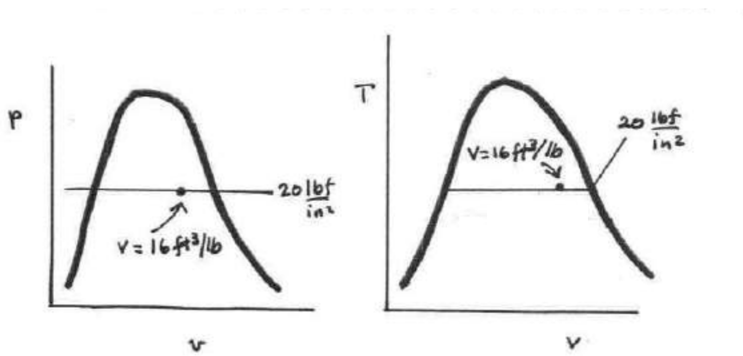
$\Rightarrow T = 227.96\ °F$

$$x = \frac{v - v_f}{v_g - v_f} = \frac{16 - 0.01683}{20.09 - 0.01683} = 0.796$$

$\therefore u = u_f + x(u_g - u_f)$

$= 196.19 + 0.796(1082 - 196.19)$

$= 901.29\ \text{Btu/lb}$

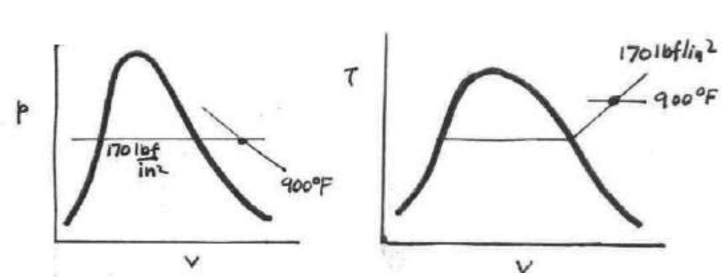


(b) $T = 900°F$, $p = 170\ \text{lbf/in}^2$

Table A-4E, interpolate at 900°F

$v = 4.734\ \text{ft}^3/\text{lb}$

$h = 1478.05\ \text{Btu/lb}$



(c) $= 600°F$, $v = 0.6\ \text{ft}^3/\text{lb}$

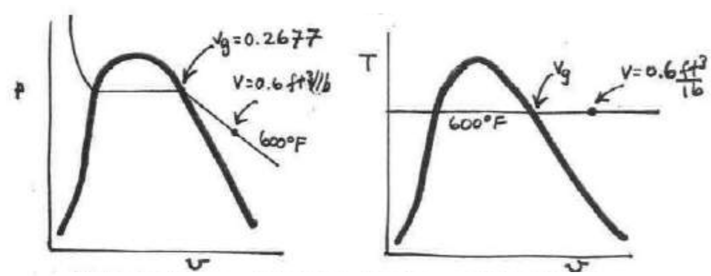
Table A-2E, $v > v_g$ at 600°F.

$\Rightarrow$ Table A-4E. At 600°F the state falls between 800 and 900 lbf/in².

Interpolating,

$p = 885.6\ \text{lbf/in}^2$

$u = 1163.34\ \text{Btu/lb}$



(d) $T = 40°F$, $v = 1950\ \frac{\text{ft}^3}{\text{lb}}$

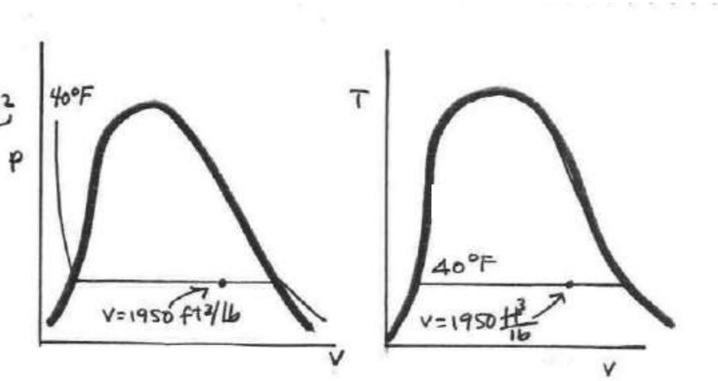
Table A-2E $v_f < v < v_g$ at 40°F. Thus, $p = 0.1217\ \text{lbf/in}^2$,

$$x = \frac{v - v_f}{v_g - v_f} = \frac{1950 - 0.016}{2445 - 0.016} = 0.798$$

$h = h_f + x\,h_{fg}$

$= 8.02 + 0.798(1070.9)$

$= 862.6\ \text{Btu/lb}$



PROBLEM 3.44 (CONTINUED)

(e) $p = 600\ lbf/in^2$; $T = 320°F$

Table A-3E at 600 lbf/in²

$T_{sat} = 486.33°F$

⇒ liquid state

Table A-5E – double interpolation

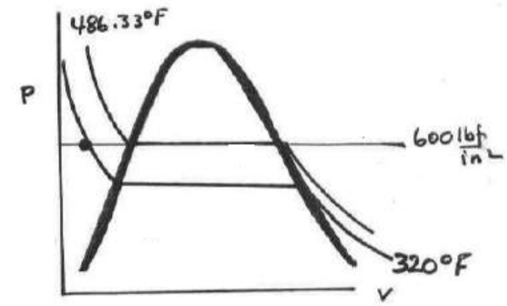
	p = 500 lbf/in²	p = 1000
T = 300°F	v = 0.017416 u = 268.92	0.017379 268.24
T = 400	v = 0.018608 u = 373.68	0.018550 372.55

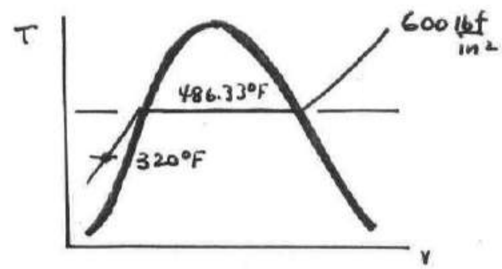
	at 600 lbf/in²
300°F	v = 0.017409 u = 268.78
400°F	v = 0.018596 u = 373.45

Then, at 600 lbf/in², 320°F

$v = 0.017646\ ft^3/lb$

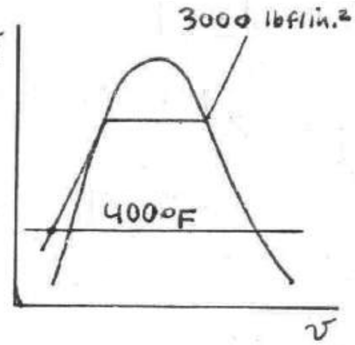
$u = 289.71\ Btu/lb$





PROBLEM 3.45

(a) Water



H_2O at $T = 400°F$, $p = 3000$ lbf/in²

from Table A-5E: $v = 0.018334$ ft³/lb

$h = 378.50$ Btu/lb

The specific volume and enthalpy values can also be estimated using saturation data from Table A-2E and Eqs. 3.11 and 3.14, respectively.

$$v = v_f(400\,°F) = \underline{0.01864 \text{ ft}^3/\text{lb}}$$

and

$$h \approx h_f(400\,°F) + v_f(400°F)\left[p - p_{sat@400°F}\right]$$

$$= 375.1\ \frac{\text{Btu}}{\text{lb}} + 0.01864\ \frac{\text{ft}^3}{\text{lb}}\left[3000\ \frac{\text{lbf}}{\text{in}^2} - 247.1\ \frac{\text{lbf}}{\text{in}^2}\right]\left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

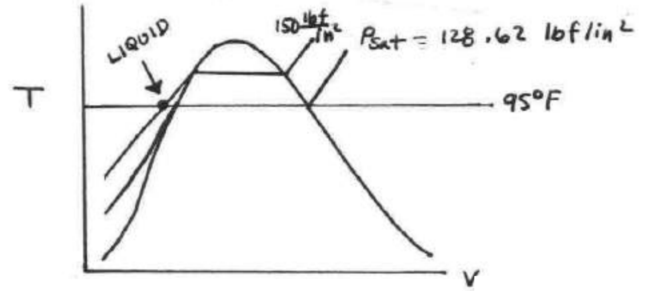
$$= \underline{384.6 \text{ Btu/lb}}$$

(b) Refrigerant 134a

Refrigerant 134a at 95°F, 150 lbf/in²

Table A-10E at 95°F; $p_{sat} = 128.62$ lbf/in²

⇒ liquid state when $p = 150$ lbf/in²



Using Eq. 3.11, $v \approx v_f(95°F) = \underline{\underline{0.01371 \text{ ft}^3/\text{lb}}}$ ← v

Using Eq. 3.14, $h \approx h_f(95°F) = \underline{\underline{42.47 \text{ Btu/lb}}}$ ← h

PROBLEM 3.45 (CONTINUED)

(c) Ammonia

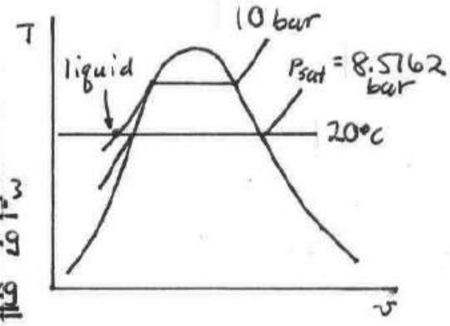
Ammonia at 20°C, 1 MPa = 10 bar

Table A-13 at 20°C; $p_{sat} = 8.5762$ bar

⇒ liquid state when p = 10 bar

Using Eq. 3.11; $v \approx v_f(20°C) = \underline{\underline{1.6386 \times 10^{-3}\ \frac{m^3}{kg}}}$

Using Eq. 3.14; $h \approx h_f(20°C) = \underline{\underline{274.26\ kJ/kg}}$



(d) Propane

Propane at 800 kPa, 0°C

Table A-17 at 8 bar, $T_{sat} = 18.33°C$

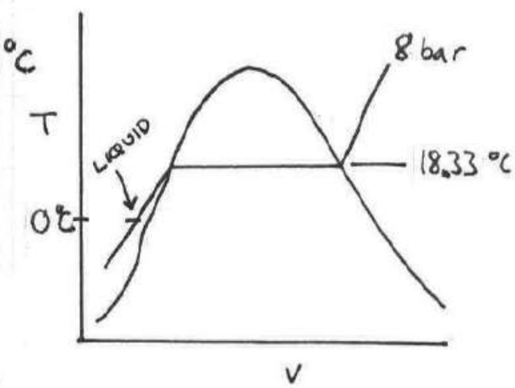
⇒ liquid state

Using Eq. 3.11, $v \approx v_f(0°C)$

$= \underline{\underline{1.890 \times 10^{-3}\ \frac{m^3}{kg}}}$

Using Eq. 3.14, $h \approx h_f(0°C)$

$= \underline{\underline{95.1\ kJ/kg}}$

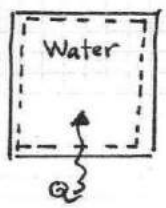


<u>PROBLEM 3.46</u>

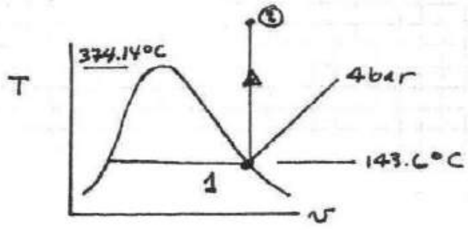
<u>KNOWN:</u> Water contained in a closed, rigid container is heated. State data is provided.

<u>FIND:</u> For the water, determine the heat transfer, in kJ/kg.

<u>SCHEMATIC & GIVEN DATA:</u>



Initial:
Sat. vapor at 4 bar
Final: T = 400°C



<u>ENGINEERING MODEL:</u>

1. The water in the container is the closed system.
2. The only energy transfer is by heat.
3. Kinetic and potential energy effects can be ignored.

<u>ANALYSIS:</u>

Since the total volume and total mass remain constant, the water undergoes a constant-specific volume process, as shown in the T-v diagram.

With 2 and 3, the energy balance reduces as follows:

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - \cancel{W}$$

$$\Rightarrow \quad Q = \Delta U$$

$$= m[u_2 - u_1]$$

or

$$\frac{Q}{m} = u_2 - u_1$$

From Table A-3 at 4 bar, $u_1 = u_g = 2553.6$ kJ/kg. Also, $v_1 = v_g = 0.4625$ m³/kg.
Interpolating in Table A-4 with T = 400°C and $v_2 = v_1 = 0.4625$ m³/kg,
$u_2 = 2961.2$ kJ/kg

$$\therefore Q = (2961.2 - 2553.6) \text{ kJ/kg}$$

$$= +407.6 \text{ kJ/kg}$$

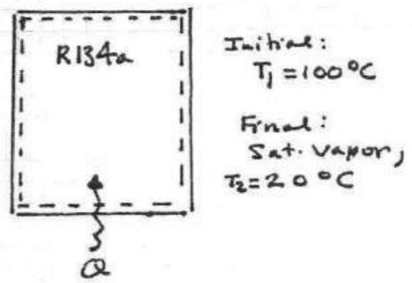
Energy transfer by heat <u>to</u> the water.

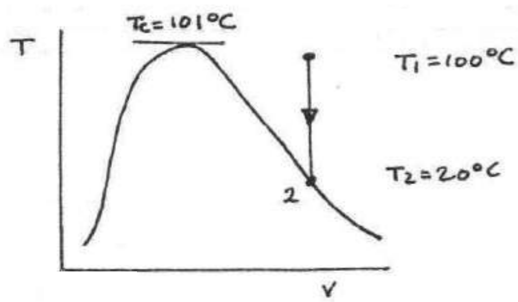
PROBLEM 3.47

KNOWN: R134a contained in a closed, rigid tank is cooled. State data is provided.

FIND: For the refrigerant, determine the initial and final pressures, each in bar, and the heat transfer, in kJ/kg.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL:

1. The R134a in the tank is the closed system.
2. The only energy transfer is by heat.
3. Kinetic and potential energy effects can be ignored.
4. The volume is constant.

ANALYSIS:

Since the total mass and total volume remain constant, the refrigerant undergoes a constant-specific volume process, as shown in the T-v diagram.

From Table A-10 at 20°C,

$$p_2 = p_{sat}(20°C) = 5.7160 \text{ bar} \longleftarrow$$
$$v_2 = v_g = 0.0358 \text{ m}^3/\text{kg}$$
$$u_2 = u_g = 237.91 \text{ kJ/kg}$$

State 1 is fixed by $v_2 = v_1$ and $T_1 = 100°C$. Interpolating in Table A-12,

$$p_1 = 7.89 \text{ bar} \longleftarrow$$
$$u_1 = 309.02 \text{ kJ/kg}$$

With 2 and 3 the energy balance reduces as follows:

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - \cancel{W} \Rightarrow Q = \Delta U, \text{ or}$$

$$Q = m(u_2 - u_1)$$

$$\Rightarrow \frac{Q}{m} = u_2 - u_1 = (237.91 - 309.02) \text{ kJ/kg}$$
$$= -71.11 \text{ kJ/kg} \longleftarrow$$

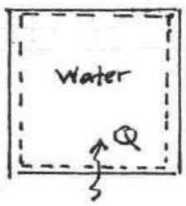
Energy transfer by heat from the refrigerant

PROBLEM 3.48

KNOWN: A closed, rigid tank is filled with water. The water is heated. State data is provided.

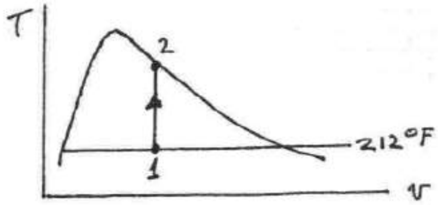
FIND: For the water, determine the quality at the initial state, temperature at the final state, in °F, and the heat transfer, in Btu.

SCHEMATIC & GIVEN DATA:



Initial: at 212°F
$9.9\ ft^3$ sat. vapor,
$0.1\ ft^3$ sat. liquid.

Final: The tank contains only sat. vapor



ENGINEERING MODEL:

1. The water in the tank is the closed system.
2. The only energy transfer is by heat.
3. Kinetic and potential energy effects can be ignored.

ANALYSIS:

Using v_f and v_g at 212°F from Table A-3E, the mass of the initial liquid and vapor can be determined:

$$m_{LIQ} = \frac{V_{LIQ}}{v_f} = \frac{0.1\ ft^3}{0.01672\ ft^3/lb} = 5.981\ lb$$

$$m_{VAP} = \frac{V_{VAP}}{v_g} = \frac{9.9\ ft^3}{26.8\ ft^3/lb} = 0.369\ lb$$

$$m = m_{LIQ} + m_{vap} = (5.981 + 0.369)\ lb = 6.35\ lb$$

The quality at the initial state is, $x = \frac{0.369}{6.35} = 0.058\ (5.8\%)$

Since the total volume and total mass remain constant, the water undergoes a constant-specific volume process, as shown in the T-v diagram: $v_2 = v_1$, where

$$v_2 = \frac{V_{tot}}{m} = \left(\frac{9.9\ ft^3 + 0.1\ ft^3}{6.35\ lb}\right) = 1.575\ \frac{ft^3}{lb}$$

Interpolating in Table A-2E with $v_2 = v_g$, $T_2 = 416\ °F$

Also, $u_2 = 1118\ Btu/lb$.

With 2 and 3, the energy balance reduces as follows:

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - \cancel{W} \Rightarrow Q = U_2 - U_1\text{, where}$$

$$U_1 = m_{vap} u_{g,1} + m_{LIQ} u_{f,1} = (0.369)(1077.6) + (5.981)(180.10) = 1474.8\ Btu$$

lb Btu/lb

$$U_2 = m u_2 = (6.35)(1118) = 7099.3\ Btu$$

So,

$$Q = (7099.3 - 1474.8)\ Btu = +5624.5\ Btu$$

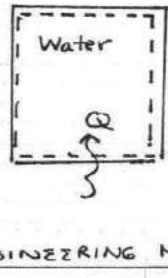
Energy transfer by heat to the water

PROBLEM 3.49

KNOWN: A closed, rigid tank is filled with water, initially at the critical point. The water is cooled to a given temperature

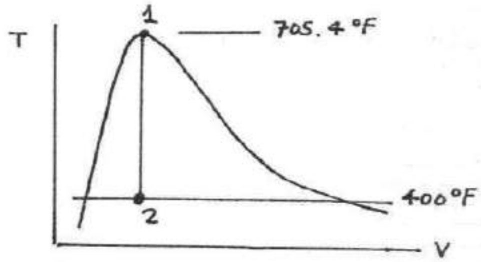
FIND: For the water, show the process on a T-v diagram and determine the heat transfer, in Btu/lb.

SCHEMATIC & GIVEN DATA:



Initially at the critical point.

Finally at 400°F



ENGINEERING MODEL

1. The water in the tank is the closed system.

2. The only energy transfer is by heat.

3. Kinetic and potential energy effects can be ignored.

ANALYSIS:

Since the total mass and total volume remain constant, the water undergoes a constant-volume process, as shown in the T-v diagram.

With 2 and 3, the energy balance reduces as follows:

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - \cancel{W} \implies Q = \Delta U = m(u_2 - u_1)\text{, or}$$

$$\frac{Q}{m} = u_2 - u_1 \qquad (1)$$

From Table A-2E, $u_1 = 872.6$ Btu/lb

To find u_2, first evaluate x_2 using $v_2 = v_1 = 0.05053$ ft³/lb.

(Table A-2 at 400°F)

$$x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{0.05053 - 0.01864}{1.866 - 0.01864} = 0.0173 \quad (1.73\%)$$

$$\Rightarrow \quad u_2 = u_f + x_2(u_g - u_f) = 374.3 + (0.0173)(1116.6 - 374.3) = 387.1 \text{ Btu/lb},$$

where u_f and u_g are from Table A-2 at 400°F.

Substituting values into Eq. (1),

$$\frac{Q}{m} = (387.1 - 872.6)\frac{\text{Btu}}{\text{lb}} = -485.5 \text{ Btu/lb}$$

Energy transfer by heat from the water

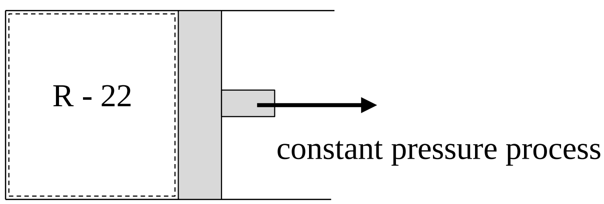
PROBLEM 3.50

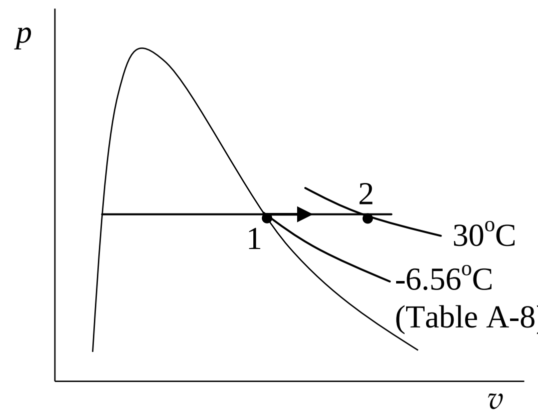
Refrigerant 22 undergoes a constant pressure process within a piston-cylinder assembly from saturated vapor at 4 bar to a final temperature of 30°C. Kinetic and potential energy effects are negligible. For the refrigerant, show the process on a *p*-*v* diagram. Evaluate the work and the heat transfer, each in kJ per kg of refrigerant.

KNOWN: Data are provided for a process of Refrigerant 22 in a piston-cylinder assembly.

FIND: Show the process on a *p*-*v* diagram and evaluate the work and heat transfer, each per unit mass of refrigerant.

SCEMATIC AND GIVEN DATA:





ENGINEERING MODEL: (1) The refrigerant is the closed system. (2) The refrigerant expands at constant pressure. (3) Volume change is the only work mode. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: The process is shown on the *p*-*v* diagram. Since volume change is the only work mode and pressure is constant, Eq. 2.17 is used to evaluate the work, as follows.

$$W_{12} = \int_1^2 p dV \quad \longrightarrow \quad (W_{12}/m) = \int_{v_1}^{v_2} p dv = p(v_2 - v_1)$$

From Table A-8, $v_1 = v_g$(4 bar) = 0.0581 m³/kg, and from Table A-9, v_2 = 0.06872 m³/kg. Inserting values and converting units, we get

$$W_{12}/m = (4 \text{ bar})(0.06872 - 0.0581) \text{ m}^3/\text{kg} \left|\frac{10^5 \text{N/m}^2}{1 \text{ bar}}\right| \left|\frac{1 \text{ kJ}}{10^3 \text{N·m}}\right| = 4.248 \text{ kJ/kg (out)} \longleftarrow$$

Now, the energy balance reduces to $\overset{0}{\cancel{\Delta KE}} + \overset{0}{\cancel{\Delta PE}} + \Delta U = Q - W$, or, with $\Delta U = m(u_2 - u_1)$

$$Q_{12}/m = (u_2 - u_1) + W_{12}/m$$

From Table A-8, $u_1 = u_g$(4 bar) = 224.24 kJ/kg, and from Table A-9, u_2 = 245.73 kJ/kg. Thus

$$Q_{12}/m = (245.73 - 224.24) \text{ kJ/kg} + (4.248 \text{ kJ/kg}) = 25.738 \text{ kJ/kg (in)} \longleftarrow$$

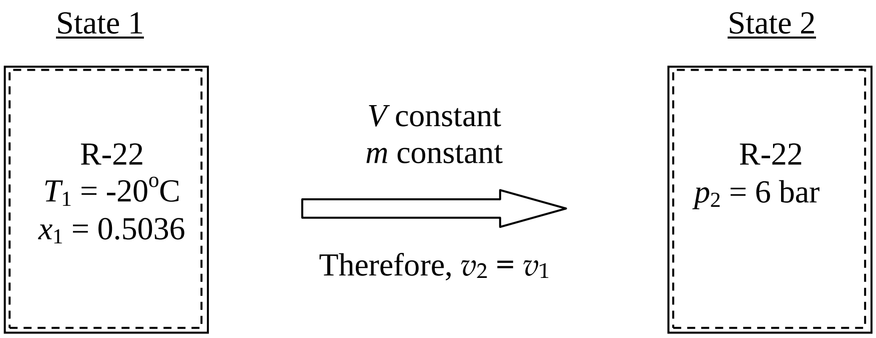
PROBLEM 3.51

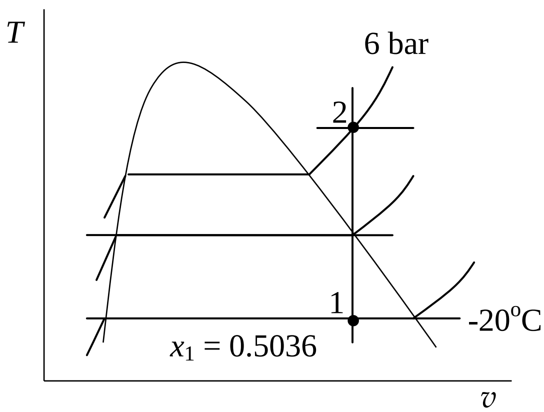
For the system of Problem 3.26, determine the amount of energy transfer by heat, in kJ per kg of refrigerant. Kinetic and potential energy effects can be neglected.

KNOWN: Refrigerant 22 undergoes a process involving energy transfer by heat in a closed, rigid tank.

FIND: Determine the amount of energy transfer by heat per kg of refrigerant.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL: (1) The refrigerant is the closed system. (2) The tank is rigid and there is no mechanism for work. (3) Kinetic and potential energy effect can be neglected.

ANALYSIS: To find the heat transfer, we start with the closed system energy balance:

$$\Delta U + \cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} = Q - \cancelto{0}{W} \qquad \rightarrow \qquad Q/m = (u_2 - u_1) \qquad (*)$$

Using data from Table A-7 with the given initial quality to determine the value of u_1

$$u_1 = u_{f1} + x_1(u_{g1} - u_{f1}) = 21.99 + 0.5036(219.37 - 21.99) = 121.39 \text{ kJ/kg}$$

From the solution to Problem 3.26; $v_2 = 0.0470 \text{ m}^3/\text{kg}$. Interpolating in Table A-9 at $p_2 = 6$ bar, we get $u_2 \approx 252.0$ kJ/kg.

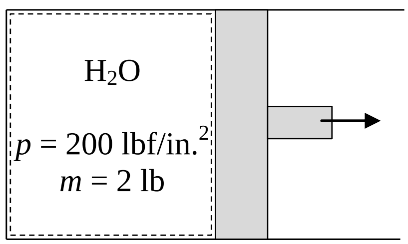
Inserting values into Eq. (*)

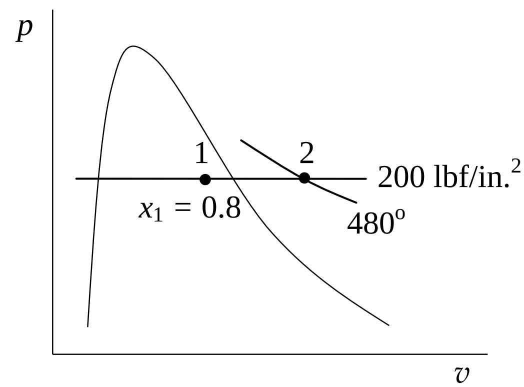
$$Q/m = (252.0 - 121.39) = 130.6 \text{ kJ/kg (in)}$$

PROBLEM 3.52

For the system of Problem 3.31, determine the amount of energy transfer by heat, in Btu, if the mass is 2 lb. Kinetic and potential energy effects can be neglected.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL: 1. The water is a closed system. 2. The pressure is constant. 3. Kinetic and potential energy effects can be neglected.

ANALYSIS: To find the heat transfer, we start with the closed system energy balance:

$$\Delta U + \cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} = Q - W \qquad \rightarrow \qquad Q = m(u_2 - u_1) + W \qquad (*)$$

The initial specific internal energy can be determined using data from Table A-3E, as follows.

$$u_1 = u_{f1} + x_1(u_{g1} - u_{f1}) = 354.9 + (0.8)(1114.6 - 354.9) = 962.7 \text{ Btu/lb}$$

Interpolating in Table A-4e at 200 lbf/in.^{2} and 480^{o}F, we get $u_2 \approx 1167.9$ Btu/lb

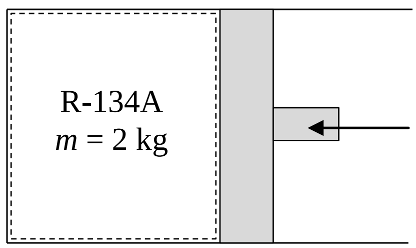
From the solution to Problem 3.31, W/m = 29.58 Btu/lb. Therefore, W = (2 lb) (29.58 Btu/lb) = 59.16 Btu. Inserting values into Eq. (*)

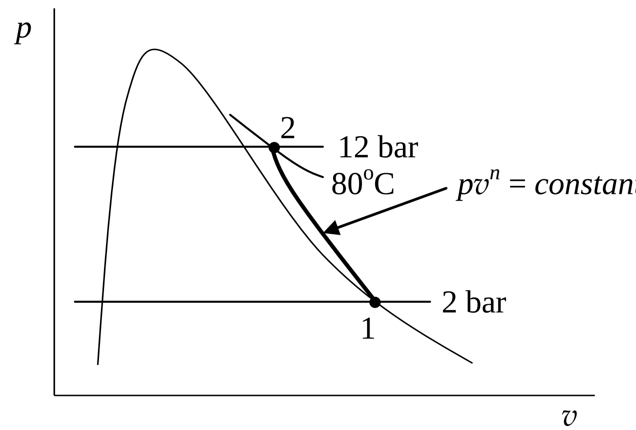
$$Q = (2 \text{ lb})(1167.9 - 962.7) \text{ Btu/lb} + (59.16 \text{ Btu}) = 469.6 \text{ Btu (in)} \longleftarrow$$

PROBLEM 3.53

For the system of Problem 3.33, determine the amount of energy transfer by heat, in kJ. Kinetic and potential energy effects can be neglected.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL: 1. The refrigerant is a closed system. 2. The process is polytropic: pv^n = *constant*. 3. Kinetic and potential energy effects can be neglected.

ANALYSIS: To find the heat transfer, we start with the closed system energy balance:

$$\Delta U + \cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} = Q - W \qquad \rightarrow \qquad Q = m(u_2 - u_1) + W \qquad (*)$$

Next, we find the specific internal energy values, as follows.

State 1: $u_1 = u_g$(2 bar) = 221.43 kJ/kg (Table A-11)

State 2: p_2 = 12 bar, T_2 = 80°C: u_2 = 285.62 kJ (Table A=12)

With W = -69.88 kJ (See the solution to Problem 3.33.), we get

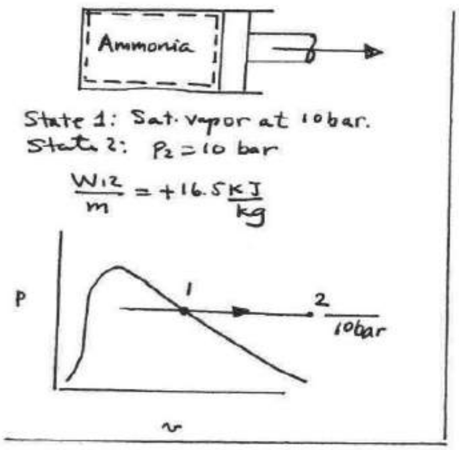
$$Q = (2 \text{ kg})(285.62 - 221.43) \text{ kJ} + (-69.88 \text{ kJ}) = 58.5 \text{ kJ (in)} \longleftarrow$$

PROBLEM 3.54

KNOWN: Data are provided for a process of ammonia contained in a piston-cylinder assembly.

FIND: Determine the final temperature and Q/m for the process.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The ammonia in the piston-cylinder assembly is the closed system.
2. The expansion occurs at constant pressure.
3. Volume change is the only work mode.
4. Changes in kinetic and potential energy are negligible.

ANALYSIS: Two property values are required to fix state 2. One is the pressure and the other is specific volume found from W_{12}/m.

$$\frac{W_{12}}{m} = \int_1^2 p\,dv = p(v_2 - v_1)$$

$$\Rightarrow \quad v_2 = \frac{W_{12}/m}{p} + v_1$$

$$= \left(\frac{16.5\ kJ/kg}{10 \times 10^5\ N/m^2}\right)\left|\frac{10^3\ N\cdot m}{1\ kJ}\right| + 0.1285\ m^3/kg \quad \text{(Table A-15)}$$

$$= 0.1450\ m^3/kg$$

So, from Table A-15 at 10 bar, $v_2 = 0.1450\ m^3/kg$,

$$T_2 = 50°C \quad \longleftarrow T_2$$

An energy balance reduces to $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{12} - W_{12}$, or

$$\frac{Q_{12}}{m} = u_2 - u_1 + \frac{W_{12}}{m}$$

$$= (1391.07 - 1334.66) + 16.5$$

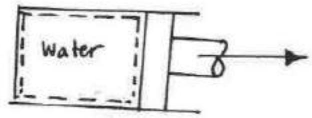
$$= 72.91\ kJ/kg \quad \longleftarrow Q_{12}/m$$

PROBLEM 3.55

KNOWN: Data are provided for a process of water contained in a piston-cylinder assembly.

FIND: Determine the mass of water and Q for the process.

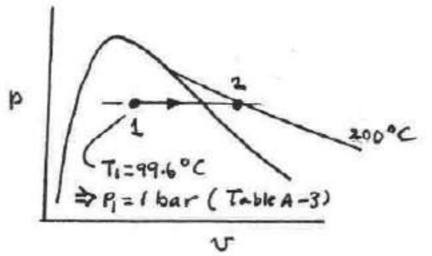
SCHEMATIC & GIVEN DATA:



$T_1 = 99.6°C$, $x_1 = 65\%$

$P_2 = P_1$, $T_2 = 200°C$

$W_{12} = +300$ kJ



ENGR. MODEL

1. The water in the piston-cylinder assembly is the closed system.
2. The process occurs at constant pressure.
3. Volume change is the only work mode.
4. Changes in kinetic and potential energy are negligible.

ANALYSIS: (a) Since volume change is the only work mode, $W_{12} = \int_1^2 p\,dV$ or

$$W_{12} = m\,p\,(v_2 - v_1).$$

Thus

$$m = \frac{W_{12}}{p(v_2 - v_1)} = \frac{300\text{ kJ}}{\left(10^5 \frac{N}{m^2}\right)(2.172 - 1.1015)\frac{m^3}{kg}}\left|\frac{10^3\,N\cdot m}{1\text{ kJ}}\right| = 2.80\text{ kg} \quad \longleftarrow m$$

where $v_1 = v_f + x_1(v_g - v_f) = \left(\frac{1.0432}{10^3}\right) + 0.65\left(1.694 - \frac{1.0432}{10^3}\right)$

$= 1.1015\ \frac{m^3}{kg}$ (Table A-3 data)

$v_2 = 2.172\ \frac{m^3}{kg}$ (Table A-4 data)

(b) An energy balance reduces to $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{12} - W_{12}$ or

$$Q_{12} = W_{12} + m(u_2 - u_1)$$

where $u_1 = u_f + x_1(u_g - u_f) = 417.36 + 0.65(2506.1 - 417.36)$

$= 1775.04\ \frac{kJ}{kg}$ (Table A-3 data)

$u_2 = 2658.1\ \frac{kJ}{kg}$ (Table A-4 data)

$\therefore\quad Q_{12} = 300\text{ kJ} + 2.8\text{ kg}\left[2658.1 - 1775.04\right]\frac{kJ}{kg}$

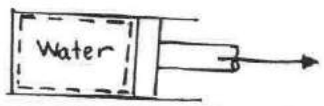
$= 2772.57$ kJ $\quad \longleftarrow Q_{12}$

PROBLEM 3.56

KNOWN: Data are provided for a process of water contained in a piston-cylinder assembly. The process inlvolves three parts.

FIND: Determine $\frac{Q}{m}$ and $\frac{W}{m}$ for each of the three parts of the overall process.

SCHEMATIC & GIVEN DATA:

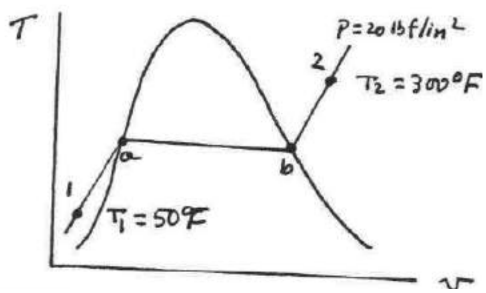


Constant pressure = 20 $\frac{lbf}{in^2}$

$1 \to a$: $T_1 = 50°F$ to sat. liquid

$b \to c$: Sat. Liquid to Sat. Vapor

$c \to d$: Sat. Vapor to $T_2 = 300°F$



ENGR. MODEL:

1. The water in the piston-cylinder assembly is the closed system.
2. The process occurs at constant pressure.
3. Volume change is the only work mode.
4. Kinetic and potential energy effects are negligible.

ANALYSIS: For each process we use

$$W = \int p\,dV = p\Delta V \Rightarrow \frac{W}{m} = p\Delta v \quad (1)$$

and the following expression obtained from an energy balance:

$$\frac{Q}{m} = \Delta u + \frac{W}{m} = \Delta u + p\Delta v \quad (2)$$

Data from Tables A-2, A-3, A-4. Note: At 1, $v_1 \sim v_f(T_1)$, $u_1 \sim u_f(T_1)$

	v (ft³/lb)	u (Btu/lb)
1	0.01602	18.06
a	0.01683	196.19
b	20.09	1082.0
2	22.36	1108.7

⊙ $1 \to a$:

$$\frac{W_{1a}}{m} = p(v_a - v_1)$$

$$= \left(20\frac{lbf}{in^2}\right)(0.01683 - 0.01602)\left(\frac{ft^3}{lb}\right)\left|\frac{144\,in^2}{1\,ft^2}\right|\left|\frac{1\,Btu}{778\,ft\cdot lbf}\right|$$

$$= 0.003\ \frac{Btu}{lb} \quad \text{(negligible)}$$

$$\frac{Q_{1a}}{m} = (196.19 - 18.06) + 0.003 = 178.13\ \frac{Btu}{lb}$$

$\leftarrow$ $1 \to a$

⊙ $a \to b$:

$$\frac{W_{ab}}{m} = 20(20.09 - 0.01683)\left|\frac{144}{778}\right| = 74.31\ \frac{Btu}{lb}$$

$$\frac{Q_{ab}}{m} = (1082.0 - 196.19) + 74.31 = 960.12\ \frac{Btu}{lb}$$

$\leftarrow$ a – b

⊙ b - 2:

$$\frac{W_{b2}}{m} = 20(22.36 - 20.09)\left|\frac{144}{778}\right| = 8.40\ \frac{Btu}{lb}$$

$$\frac{Q_{b2}}{m} = (1108.7 - 1082.0) + 8.40 = 35.10\ \frac{Btu}{lb}$$

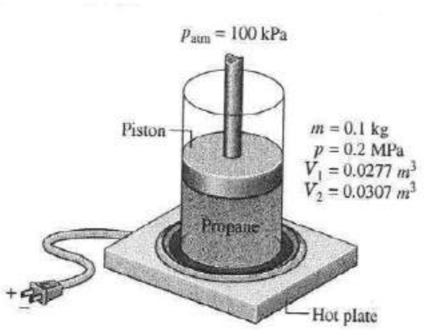
$\leftarrow$ b - 2

PROBLEM 3.57

KNOWN: Data are provided for a process of propane contained within a piston-cylinder assembly.

FIND: Determine the initial and final temperatures and W and Q for the process.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The propane within the piston-cylinder assembly is the closed system.
2. Friction between the piston and cylinder is negligible and the expansion occurs slowly at a constant pressure of 0.2 MPa.
3. Volume change is the only work mode.
4. For the system there are no significant kinetic and potential energy effects.

ANALYSIS: (a) To find T_1 and T_2 requires two property values to fix each state. Since pressure is constant, it is one of the properties. The specific volume provides the other: $v_1 = \frac{V_1}{m} = \frac{0.0277\,m^3}{0.1\,kg} = 0.277\,\frac{m^3}{kg}$, $v_2 = \frac{V_2}{m} = \frac{0.0307\,m^3}{0.1\,kg} = 0.307\,\frac{m^3}{kg}$.

Thus, from Table A-18 at 0.2 MPa = 2 bar, $T_1 = 30°C$, $T_2 = 60°C$. ← T_1, T_2

(b) Since volume change is the only work mode and pressure is constant,

$$W_{12} = \int_1^2 p\,dV = p(V_2 - V_1)$$

$$= 0.2\,MPa[0.0307 - 0.0277]\,m^3 \left|\frac{10^6\,N/m^2}{1\,MPa}\right|\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|$$

$$= 0.6\,kJ \quad \leftarrow W_{12}$$

(c) An energy balance reduces to give, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{12} - W_{12}$ or

$$Q_{12} = W_{12} + m(u_2 - u_1)$$

with data from Table A-18 at 0.2 MPa = 2 bar,

$$Q_{12} = 0.6\,kJ + 0.1\,kg\,(524.3 - 476.3)\frac{kJ}{kg}$$

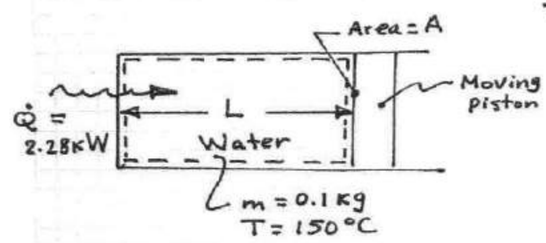
$$= 5.4\,kJ \quad \leftarrow Q_{12}$$

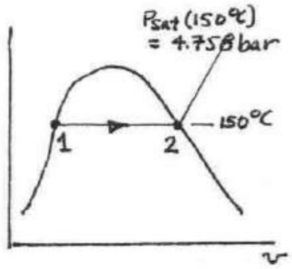
PROBLEM 3.58

KNOWN: A piston-cylinder assembly contains water, which is heated at constant temperature.

FIND: For a specified rate of heat transfer, determine the rate work is done by the water. For a specified total mass of water, determine the time required to execute the process of the water.

SCHEMATIC & GIVEN DATA:





Data from Table A-2, at 150°C:

$v_g - v_f = 0.3917 \text{ m}^3/\text{kg}$

$u_g - u_f = 1927.82 \text{ kJ/kg}$

$h_g - h_f = 2114.3 \text{ kJ/kg}$

ENGINEERING MODEL

1. The water in the piston-cylinder assembly is the closed system.
2. Volume change is the only work mode.
3. Kinetic and potential energy effects are ignored.

ANALYSIS:

(a) The energy rate balance is $\frac{dU}{dt} = \dot{Q} - \dot{W}$. With Eq. 2.13

$$\dot{W} = F \underbrace{V}_{\text{velocity of moving boundary}} = (pA)\frac{dL}{dt} = p\frac{d(AL)}{dt} = \underbrace{p}_{P_{sat}(150°C)}\frac{d\overbrace{V}^{\text{volume}}}{dt}$$

Collecting results,

$$\frac{dU}{dt} = \dot{Q} - p\frac{dV}{dt} \Rightarrow \dot{Q} = \frac{d}{dt}\left[U + pV\right] \Rightarrow \dot{Q} = \frac{d\overbrace{H}^{=mh}}{dt}$$

Finally, $\dot{Q} = m\frac{dh}{dt}$ (1)

Since, $V = mv$, we also have $\dot{W} = mp\frac{dv}{dt}$ (2)

With Eqs. (1) and (2)

$$\frac{\dot{W}}{\dot{Q}} = \frac{mp\, dv/dt}{m\, dh/dt} = \frac{p\, dv/dt}{dh/dt} \quad (3)$$

Since $v = v_f + x(v_g - v_f)$ and $h = h_f + x(h_g - h_f)$, we have

$$\frac{dv}{dt} = (v_g - v_f)\frac{dx}{dt} \quad \text{and} \quad \frac{dh}{dt} = (h_g - h_f)\frac{dx}{dt}$$

Inserting these expressions in Eq. (3), we get

$$\frac{\dot{W}}{\dot{Q}} = \frac{p(v_g - v_f)\, dx/dt}{(h_g - h_f)\, dx/dt} = \frac{p(v_g - v_f)}{(h_g - h_f)} = \frac{(4.758 \times 10^5 \text{ N/m}^2)(0.3917 \text{ m}^3/\text{kg})}{(2114.3 \frac{\text{kJ}}{\text{kg}})\left(\frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}}\right)} = 0.088$$

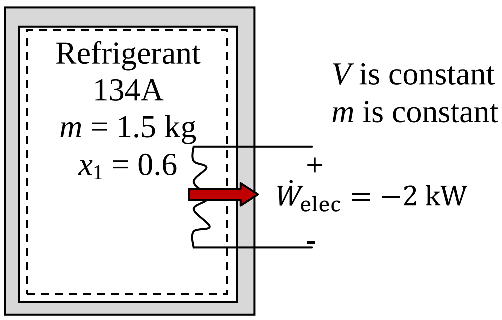
$$\Rightarrow \dot{W} = 0.088\,\dot{Q} = 0.088\,(2.28 \text{ kW}) = 0.2 \text{ kW}$$

PROBLEM 3.59

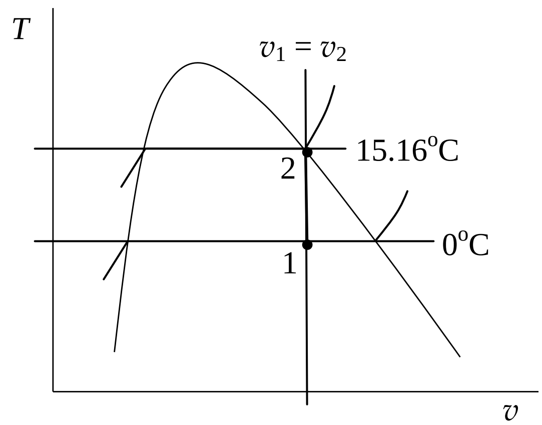
A well-insulated, rigid tank contains 1.5 kg of Refrigerant 134A initially a two-phase liquid-vapor mixture with a quality of 60% and a temperature of 0^{o}C. An electrical resistor transfers energy *to* the contents of the tank at a rate of 2 kW until the tank contains only saturated vapor. For the refrigerant, locate the initial and final states on a *T-v* diagram and determine the time it takes, in s, for the process.

KNOWN: An electrical resistor transfers energy to refrigerant in a well-insulated, rigid tank at a known rate. Data are given for the initial and final state of the refrigerant.

FIND: Determine the time it takes for the process.



V is constant
m is constant $\rightarrow$ $v_1 = v_2$



ENGINEERING MODEL: (1)The refrigerant is a closed system. (2) For the process, $Q = 0$. (3) The volume is constant. (4) Kinetic and potential energy effects can be neglected.

ANALYSIS: Since the volume is constant for the closed system, $v_1 = v_2$. At state 1, using data from Table A-10 at 0^{o}C:

$$v_1 = v_{f1} + x_1(v_{g1} - v_{f1}) = (0.7721 \text{ x } 10^{-3}) + (0.6)(0.0689 - 0.7721 \text{ x } 10^{-3}) = 0.04165 \text{ m}^3\text{/kg}$$

and

$$u_1 = u_{f1} + x_1(u_{g1} - u_{f1}) = 49.79 + (0.6)(227.06 - 49.79) = 156.15 \text{ kJ/kg}$$

To fix state 2, $v_2 = v_1 = v_g(T_2)$. Interpolating in Table A-10 with $v_g(T_2) = 0.04165$ m^3/kg; $T_2 \approx$ 15.16^{o}C, and $u_2 = u_g(15.16^oC) \approx 235.33$ kJ/kg.

Using the energy balance with assumptions 2 and 4, we get $\Delta \cancelto{0}{KE} + \Delta \cancelto{0}{PE} + \Delta U = \cancelto{0}{Q} - W_{elec}$
Thus

$$W_{elec} = m(u_1 - u_2) = (1.5 \text{ kg})(156.15 - 235.33)\text{kJ/kg} = -118.77 \text{ kJ}$$

The time is determined from $W_{elec} = \int \dot{W}_{elec} dt = \dot{W}_{elec} \Delta t$ as follows

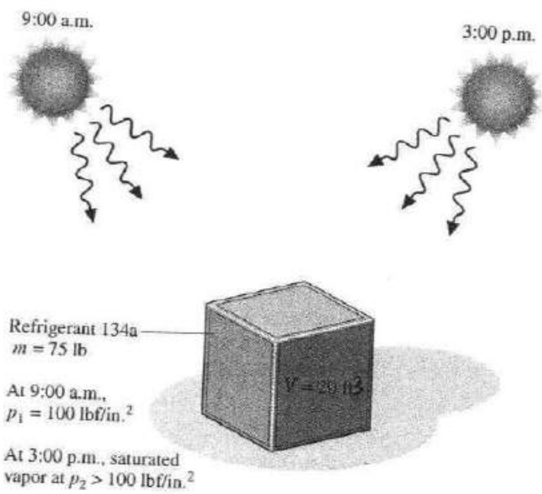
$$\Delta t = W_{elec}/\dot{W}_{elec} = (-118.77 \text{ kJ})/(-2 \text{ kW})\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right| = 59.4 \longleftarrow$$

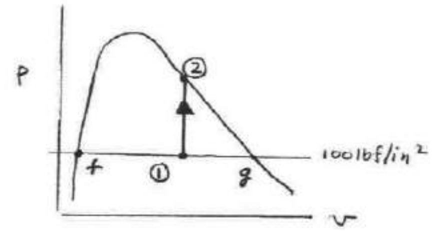
PROBLEM 3.60

KNOWN: Data are provided for Refrigeration 134a in a closed, rigid tank exposed to solar radiation.

FIND: For the process of the refrigerant, determine the initial temperature, final pressure, and Q.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL

1. The R134a in the tank is the closed system.
2. For the system, $W = 0$.
3. Kinetic and potential energy effects play no role.

ANALYSIS: At the initial state, $v_1 = \frac{V}{m} = \frac{20\,ft^3}{75\,lb} = 0.2667\,\frac{ft^3}{lb}$. Since $v_f < v_1 < v_g$, the initial state is in the two-phase, liquid-vapor region. Further, since mass and volume are each constant, there is no change in specific volume for the process: $v_2 = v_1$.

(a) Since the initial state is a two-phase, liquid-vapor mixture at 100 lbf/in², the initial temperature is the corresponding saturation temperature. From Table A-11E, $T_1 = 79.17°F$. $\longleftarrow T_1$

(b) Interpolating in Table A-11E with $v_2 = v_g = 0.2667\,ft^3/lb$, we get $P_2 = 174.4\,lbf/in^2$ and $u_2 = 107.93\,Btu/lb$. $\longleftarrow P_2$

(c) Reducing an energy balance $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{12} - \cancel{W_{12}}$, we get

$$Q_{12} = m(u_2 - u_1)$$

Finding u_1 requires the quality at state 1. That is, with v_f and v_g from Table A-11E at 100 lbf/in²,

$$x_1 = \frac{v_1 - v_f}{v_g - v_f} = \left(\frac{0.2667 - 0.01332}{0.4747 - 0.01332}\right) = 0.549$$

Then,

$$u_1 = u_f + x_1(u_g - u_f) = 36.75 + (0.549)(103.68 - 36.75) = 73.49\,Btu/lb$$

Finally,

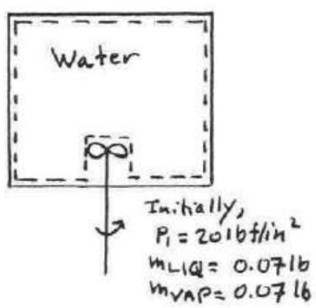
$$Q_{12} = 75\,lb(107.93 - 73.49)\,\frac{Btu}{lb} = 2583\,Btu$$ $\longleftarrow Q_{12}$

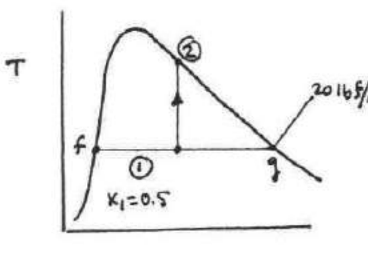
PROBLEM 3.61

KNOWN: Data are provided for water contained in a rigid, insulated tank fitted with a paddle wheel.

FIND: For the water determine the volume, initial temperature, final pressure, and work.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL:

1. The water in the tank is the closed system.
2. For the system, $Q = 0$
3. For the system there is no overall change in kinetic energy or potential energy.

ANALYSIS: Initially, the system consists of 0.07 lb of sat. liquid and 0.07 lb of saturated vapor. Thus,

$$x_1 = \frac{0.07\ lb}{0.14\ lb} = 0.5$$

Moreover, the initial temperature corresponds to the saturation temperature corresponding to the pressure of 20 lbf/in². That is, $T_1 = 227.96°F$ ← T_1

The total volume occupied is found using

$$V = m_{LIQ} v_f + m_{VAP} v_g$$
$$= (0.07)(0.01683) + (0.07)(20.09)$$
$$= 1.407\ ft^3 \quad \leftarrow V$$

All property data used thus far is from Table A-3E.

Since total volume and total mass are each constant, the specific volume at state 2 equals the specific volume at state 1: $v_2 = v_1$, where $v_1 = \frac{V}{m} = \frac{1.407\ ft^3}{0.14\ lb} = 10.05\ \frac{ft^3}{lb}$.
Then, interpolating with $v_2 = v_g$ in Table A-3E gives

$$P_2 = 42\ lbf/in^2 \quad \leftarrow P_2$$
$$u_2 = 1093\ Btu/lb$$

An energy balance reduces to read, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q_{12}} - W_{12}$, or

$$W_{12} = m(u_2 - u_1)$$

where $u_1 = u_f + x_1(u_g - u_f) = 196.19 + 0.5(1082 - 196.19) = 639.1\ Btu/lb$. Thus

$$W_{12} = 0.14\ lb\,(1093 - 639.1)\frac{Btu}{lb} = 63.5\ Btu \quad \leftarrow W_{12}$$

PROBLEM 3.62

KNOWN: A two-phase liquid-vapor mixture of water in a closed, rigid container is heated on a hot plate.

FIND: Referring to Fig. E 3.2, determine for a specified heat transfer rate (a) the time to bring the mixture from State 1 to State 2, (b) the time to bring the mixture from State 1 to State 3.

ENGINEERING MODEL: See Example 3.2. There is no change in kinetic or potential energy. W = 0.

ANALYSIS: An energy balance reduces to read $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - \cancel{W}$, or $Q = \Delta U$. Since the heat transfer rate is constant, $Q = \dot{Q}\Delta t$. Collecting results

$$\Delta t = \frac{m\Delta u}{\dot{Q}} \qquad (1)$$

where $m = 0.59$ kg, $x_1 = 0.5$ and $x_2 = 0.731$ (from the solution to Example 3.2). Thus,

at 1 bar: $u_1 = u_f + x_1(u_g - u_f) = 417.36 + 0.5(2506.1 - 417.36) = 1461.73 \frac{kJ}{kg}$

at 1.5 bar: $u_2 = u_f + x_2(u_g - u_f) = 466.94 + 0.731(2519.7 - 466.94) = 1967.51 \frac{kJ}{kg}$

State 3 is saturated vapor; so, interpolating with $v_g = 0.8475$ m³/kg, $u_3 = 2531.26 \frac{kJ}{kg}$

Then, with Eq.(1), the time required for Process 1-2 is

$$\Delta t = \frac{0.59\,\text{kg}\,(1967.51 - 1461.73)\,\text{kJ/kg}}{(0.1\,\text{kW})\left|\frac{1\,\text{kJ/s}}{1\,\text{kW}}\right|}\left|\frac{1\,\text{h}}{3600\,\text{s}}\right|$$

$$= 0.83\,\text{h} \qquad \longleftarrow (a)$$

With Eq.(1), the time required for Process 2-3 is

$$\Delta t = \frac{0.59\,\text{kg}\,(2531.26 - 1461.73)\,\text{kJ/kg}}{0.1\,\text{kW}\left|\frac{1\,\text{kJ/s}}{1\,\text{kW}}\right|}\left|\frac{1\,\text{h}}{3600\,\text{s}}\right|$$

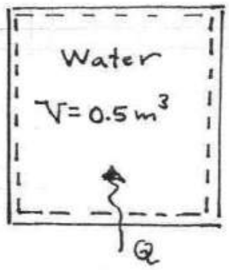
$$= 1.75\,\text{h} \qquad \longleftarrow (b)$$

PROBLEM 3.63

KNOWN: A closed rigid tank filled with water, initially a two-phase liquid-vapor mixture, is cooled. State data is provided.

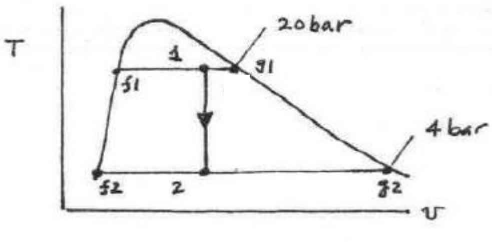
FIND: Show the process of the water on a T-v diagram and evaluate the accompanying heat transfer, in kJ.

SCHEMATIC & GIVEN DATA



Initial: $P_1 = 20$ bar, $x_1 = 0.8$

Final: $P_2 = 4$ bar



ENGINEERING MODEL:

1. The water in the tank is the closed system.
2. Energy transfer occurs only by heat.
3. Kinetic and potential energy effects can be ignored.

ANALYSIS:

Since the total mass and total volume each remain constant, the process occurs at constant specific volume, as shown in the T-v diagram.

An energy balance reduces as follows: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - \cancel{W}$

$\Rightarrow Q = \Delta U = m(u_2 - u_1)$. Accordingly, m, u_1 and u_2 are required.

Using data from Table A-3 at 20 bar,

$$v_1 = v_{f1} + x_1(v_{g1} - v_{f1}) = \frac{1.1767}{10^3} + 0.8\left(0.09963 - \frac{1.1767}{10^3}\right) = 0.07994\ m^3/kg$$

$$u_1 = u_{f1} + x_1(u_{g1} - u_{f1}) = 906.44 + 0.8(2600.3 - 906.44) = 2261.53\ kJ/kg$$

Using data from Table A-3 at 4 bar with $v_2 = v_1$,

$$x_2 = \frac{v_2 - v_{f2}}{v_{g2} - v_{f2}} = \frac{0.07994 - (1.0836/10^3)}{0.4625 - (1.0836/10^3)} = 0.1709$$

$$u_2 = u_{f2} + x_2(u_{g2} - u_{f2}) = 604.31 + 0.1709(2553.6 - 604.31) = 937.44\ kJ/kg$$

The mass is

$$m = \frac{V}{v_1} = \frac{0.5\,m^3}{0.07994\ m^3/kg} = 6.255\ kg$$

Finally

$$Q = m(u_2 - u_1) = (6.255\,kg)(937.44 - 2261.53)\ kJ/kg$$
$$= -8282\ kJ$$

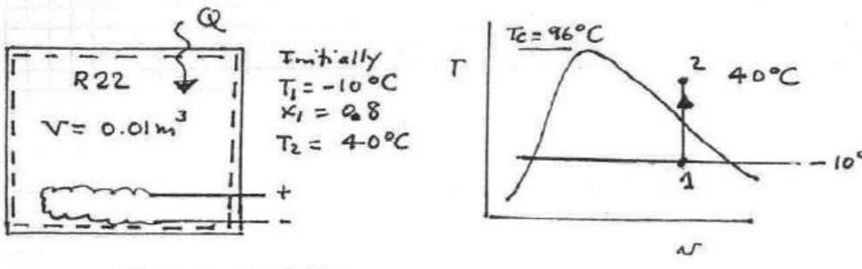
Energy transfer by heat is from the water.

PROBLEM 3.64

KNOWN: A closed, rigid tank fitted with a resistor is filled with Refrigerant 22, initially a two-phase liquid-vapor mixture. State data and operating data are provided.

FIND: Determine the heat transfer from the refrigerant, in kJ.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The closed system is the R22 plus resistor.
2. The mass and volume of the fine-wire resistor are negligible.
3. Kinetic and potential energy can be ignored.

ANALYSIS:

Since the total mass and volume of the R22 remain constant, the refrigerant undergoes a constant specific volume process, as shown in the T-v diagram.

An energy balance reduces as follows:

$$\left(\Delta U\right]_{R22} + \underbrace{\cancel{\Delta U}\big]_{Resistor}}_{\text{by assumption 2}}^{0}\Big) + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$$

$$\Rightarrow \quad Q = \Delta U\big]_{R22} + W = m(u_2 - u_1) + W \qquad (1)$$

With data from Table A-7,

$$\begin{cases} u_1 = u_f + x_1(u_g - u_f) = 33.27 + 0.8(223.02 - 33.27) = 185.07 \text{ kJ/kg} \\ v_1 = v_f + x_1(v_g - v_f) = (0.7606/10^3) + 0.8(0.0652 - (0.7606/10^3)) = 0.0523 \text{ m}^3/\text{kg} \end{cases}$$

Interpolating in Table A-9 with $T_2 = 40°C$ and $v_2 = v_1$, $u_2 = 250.33$ kJ/kg

The mass is $\quad m = \dfrac{V}{v_1} = \dfrac{0.01 \text{ m}^3}{0.0523 \text{ m}^3/\text{kg}} = 0.191 \text{ kg}$

With Eq. 2.21, $\quad W = -\mathcal{E} i \Delta t$

$$= -(12\text{ V})(5\text{ amp})(5\text{ min})\left|\frac{1\text{ Watt/amp}}{1\text{ volt}}\right|\left|\frac{60\text{ s}}{1\text{ min}}\right|\left|\frac{1\text{ J/s}}{1\text{ W}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ J}}\right|$$

$$= -18 \text{ kJ}$$

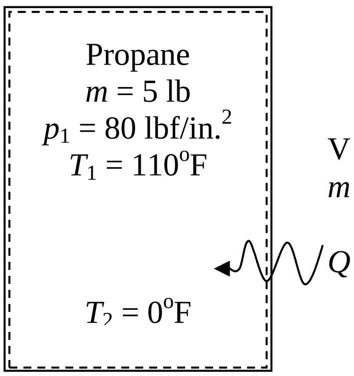
Collecting results, Eq. (1) gives

$$Q = (0.191 \text{ kg})(250.33 - 185.07)\frac{\text{kJ}}{\text{kg}} + (-18 \text{ kJ}) = -5.54 \text{ kJ}$$

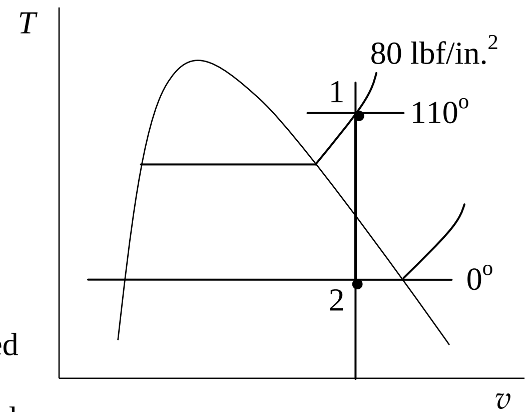
With assumption 2, we associate this heat transfer with the R22. Accordingly, the heat transfer from the refrigerant is 5.54 kJ.

PROBLEM 3.65

Five lb of propane is contained in a closed, rigid tank initially at 80 lbf/in.^{2}, 110°F. Heat transfer occurs until the final temperature in the tank is 0°F. Kinetic and potential energy effects are negligible. Show the initial and final states on a *T*- v diagram and determine the amount of energy transfer by heat, in Btu.



V is constant
m is constant $\rightarrow v_1 = v_2$



ENGINEERING MODEL: 1. The propane is a closed system. 2. Volume is constant. 3. Volume change is the only work mode, so $W = 0$. 4. Kinetic and potential energy effects can be neglected.

ANALYSIS: Since the volume and mass are constant, $v_1 = v_2$. At State 1, from Table A-18E;

$$v_1 = 1.599 \text{ ft}^3\text{/lb and } u_1 = 210.9 \text{ Btu/lb}$$

From Table A-16E at 0°F, v_2 = 1.599 ft^3/lb is between v_f and v_g. Thus, the state is a two-phase liquid-vapor mixture, and

$$x_2 = \frac{v_2 - v_{f2}}{v_{g2} - v_{f2}} = \frac{1.599 - 0.02901}{2.70 - 0.02901} = 0.5878$$

$$u_2 = u_{f2} + x_2(u_{g2} - u_{f2}) = 21.98 + (0.5878)(174.01 - 21.98) = 111.34 \text{ Btu/lb}$$

Applying the energy balance; $\Delta \cancelto{0}{KE} + \Delta \cancelto{0}{PE} + \Delta U = Q - \cancelto{0}{W}$. Thus

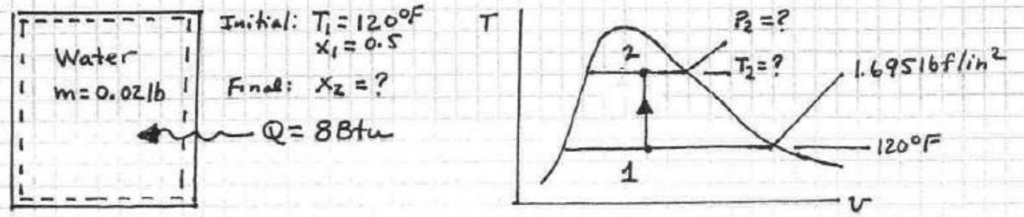
$$Q = m(u_2 - u_1) = (5 \text{ lb})(111.34 - 210.9) \text{ Btu/lb} = -\ 497.8 \text{ Btu (out)}$$

PROBLEM 3.66

KNOWN: A closed, rigid tank is filled with water, initially a two-phase liquid-volume mixture. State data and operating data are provided.

FIND: For the water, determine the final temperature, pressure, and quality.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The water in the tank is the closed system.
2. The only energy transfer is by heat.
3. Kinetic and potential energy effects can be ignored

ANALYSIS:

Since the total mass and total volume remain constant, the water undergoes a constant specific volume process, as shown in the T-v diagram. Reducing an energy balance: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - \cancel{W}$.

Thus, we get $Q = m(u_2 - u_1)$. (1)

Thinking initially of a solution using the "steam tables", solve Eq. (1) to obtain

$$u_2 = \frac{Q}{m} + u_1 \quad (2)$$

In Eq. (2), Q and m are known. Moreover, u_1 can be evaluated using data from Table A-2E and the known quality, x_1. Accordingly, the specific internal energy at state 2, u_2, can be evaluated. Moreover, since $V_2 = V_1$, we then have two independent intensive properties to fix state 2 — namely, u_2 and v_2. Using these two properties to determine T_2, p_2 and x_2 using "steam table" data requires an awkward trial solution. This can be avoided by using IT, as follows:

IT SOLUTION:

```
T1=120
x1 = 0.5
p1 = Psat_T("Water/Steam", T1)
v1 = vsat_Px("Water/Steam", p1, x1)
v2=v1
v2 = vsat_Px("Water/Steam", p2, x2)
m=0.02
Q=8
Q = m*(u2-u1)
u1 = usat_Px("Water/Steam", p1, x1)
u2 = usat_Px("Water/Steam", p2, x2)
T2 = Tsat_P("Water/Steam", p2)
```

Solution:

T2=143.9 °F

p2=3.193 lbf/in^2

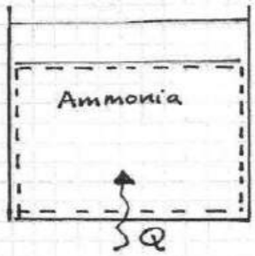
x2=0.9066

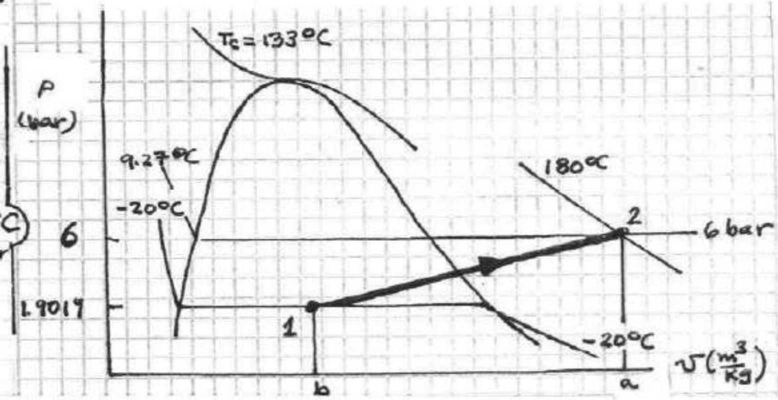
PROBLEM 3.67

KNOWN: A piston-cylinder assembly contains ammonia, which is slowly heated while the pressure varies linearly with specific volume. State data is provided.

FIND: Show the process of the ammonia on a p-v diagram. For the ammonia, evaluate the work and heat transfer, each in kJ/kg.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL:

1. The ammonia is the closed system.
2. For the ammonia pressure varies linearly with specific volume.
3. Volume change is the only work mode.
4. Kinetic and potential energy effects can be ignored.

ANALYSIS:

Using Eq. 2.17 with assumption 2,

$$\frac{W}{m} = \int_1^2 p\,dv = p_{ave}(v_2 - v_1) = \left(\frac{p_1+p_2}{2}\right)(v_2 - v_1) = \text{Area}\,(1-2-a-b-1)$$

$$v_1 = v_f + x_1(v_g - v_f) = \frac{1.5038}{10^3} + 0.5\left(0.6233 - \frac{1.5038}{10^3}\right) = 0.3124\ \frac{m^3}{kg} \quad \left(\begin{matrix}\text{Data from}\\ \text{Table A-13}\end{matrix}\right)$$

From Table A-15, $v_2 = 0.36390\ m^3/kg$

$$\therefore\ \frac{W}{m} = \frac{(6\text{ bar} + 1.9019\text{ bar})}{2}\left|\frac{10^5\ N/m^2}{1\text{ bar}}\right|(0.36390 - 0.3124)\frac{m^3}{kg}\left|\frac{1\text{ kJ}}{10^3\ N\cdot m}\right| = 20.35\ \frac{kJ}{kg}$$

An energy balance reduces as follows: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$

$$\Rightarrow\quad \frac{Q}{m} = \frac{W}{m} + (u_2 - u_1)$$

$$u_1 = u_f + x_1(u_g - u_f) = 88.40 + 0.5(1299.23 - 88.40) = 693.82\ \frac{kJ}{kg} \quad (\text{Table A-13})$$

$u_2 = 1649.22$ (Table A-15)

$$\therefore\ \frac{Q}{m} = 20.35\ \frac{kJ}{kg} + (1649.22 - 693.82)\frac{kJ}{kg}$$

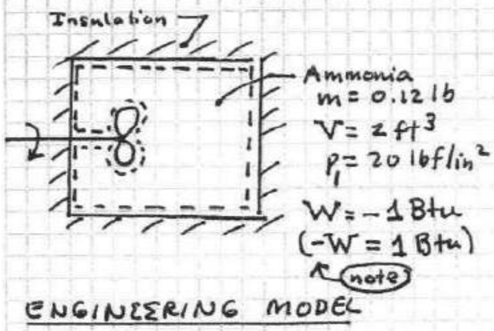
$$= 975.75\ \frac{kJ}{kg}$$

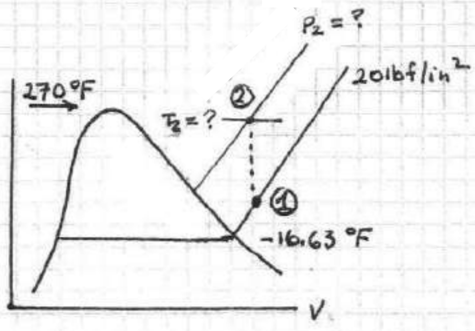
PROBLEM 3.68

KNOWN: Ammonia in a rigid, well-insulated container is stirred by a paddle wheel. State data and operating data are provided.

FIND: For the ammonia, determine the initial and final temperatures, each in °R, and the final pressure, in lbf/in².

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL

1. The ammonia is the closed system.
2. The ammonia undergoes an adiabatic process.
3. Kinetic and potential energy effects can be ignored.

ANALYSIS:

Since the total mass and total volume are constant, the initial and final states have the same specific volume: $v_2 = v_1$

Using given data, $v_1 = \dfrac{V}{m} = \dfrac{2\ \text{ft}^3}{0.12\ \text{lb}} = 16.667\ \dfrac{\text{ft}^3}{\text{lb}}$.

Interpolating in Table A-15E at 20 lbf/in² and $v_1 = 16.667\ \text{ft}^3/\text{lb}$, we get $T_1 = 77°\text{F} = 537°\text{R}$.

To fix state 2 requires 2 independent property values. One is $v_2 = v_1$, the other is u_2, obtained from an energy balance:

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - W \Rightarrow m(u_2 - u_1) = -W$$

$$\Rightarrow u_2 = u_1 + (-W/m) = 594.1 + (1/0.12) = 602.4\ \text{kJ/kg}$$

u_1 = 594.1 kJ/kg (Interpolation in Table A-15E).

Using the known values for v_2 and u_2 with the "steam tables" requires an awkward trial solution. So, we resort to IT:

```
m=0.12
V=2
v=V/m
p1=20
-W=1
- W=m*(u2-u1)

u1 = u_PT("Ammonia", p1, T1)
u2 = u_PT("Ammonia", p2, T2)
v = v_PT("Ammonia", p1, T1)
v = v_PT("Ammonia", p2, T2)
```

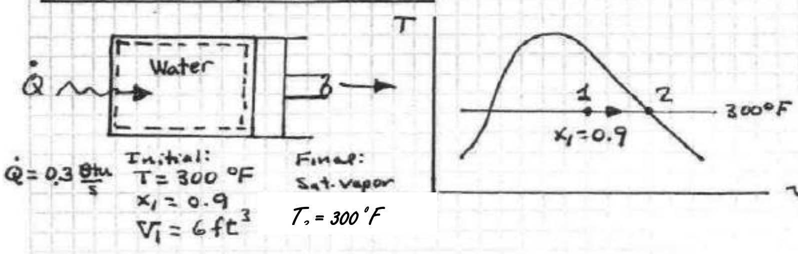
Solving:
T1=536.7 °R
T2=557.6 °R
p2=20.81 lbf/in²
u1=593.7 Btu/lb
u2=602 Btu/lb

PROBLEM 3.69

KNOWN: Water contained in a piston-cylinder assembly, initially a two-phase liquid-vapor mixture, is heated to saturated vapor. State data and operating data are provided.

FIND: Determine the time, in min, for the process of the water to occur.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The water is the closed system.
2. The temperature of the water (and thus the pressure) remains constant.
3. Kinetic and potential energy effects are negligible.

ANALYSIS: Reducing an energy balance, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$,

where $Q = \dot{Q}\,\Delta t$ and Δt is the time required for the process to occur.

Also, $W = \int_1^2 p\,dV = p[V_2 - V_1]$. Collecting results

$$\dot{Q}\,\Delta t = (U_2 - U_1) + p[V_2 - V_1]$$
$$= m[(u_2 - u_1) + p(v_2 - v_1)]$$
$$= m[(u_2 + pv_2) - (u_1 + pv_1)]$$

① $$= m[h_2 - h_1]$$

$$h_2 = h_g, \quad h_1 = h_f + x_1[h_g - h_f]$$

$$\therefore\ \dot{Q}\,\Delta t = m(1 - x_1)[h_g - h_f]$$

Finally

$$\Delta t = \frac{m(1-x_1)[h_g - h_f]}{\dot{Q}}, \text{ where } m = \frac{V}{v_1} = \frac{V}{v_f + x_1(v_g - v_f)}$$

With data from Table A-2E, $m = \left[\frac{6\ ft^3}{(0.01745 + 0.9(6.472 - 0.01745))\frac{ft^3}{lb}}\right] = 1.03\ lb$, and

$h_f = 269.7$ Btu/lb, $h_g = 1180.2$ Btu/lb

$$\Delta t = \frac{(1.03\ lb)(1 - 0.9)[1180.2 - 269.7]\ Btu/lb}{\left(0.3\ \frac{Btu}{s}\right)\left|\frac{60\ s}{1\ min}\right|} = 5.21\ min$$

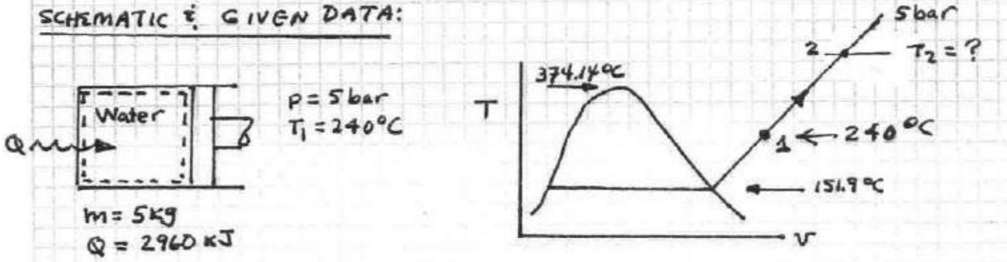
1. Introducing specific enthalpy is a convenient, but not necessary, solution step. An analysis using $[(u_2 - u_1) + p(v_2 - v_1)]$ is left as an exercise.

PROBLEM 3.70

KNOWN: Water contained in a piston-cylinder assembly is slowly heated at constant pressure. State data and operating data are provided.

FIND: For the water, determine the temperature at the final state, in °C, and the work, in kJ

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The water is the closed system.
2. The pressure of the water remains constant.
3. Kinetic and potential energy effects are negligible.

ANALYSIS:

The work is obtained using

$$W = \int_1^2 p\,dV = mp(\underbrace{v_2}_{\text{depends on } T_2} - v_1) \qquad (1)$$

An energy balance reduces as follows: $\Delta U + \cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0} = Q - W$

$\Rightarrow Q = \Delta U + W = m(u_2 - u_1) + mp(v_2 - v_1)$

(1) $\Rightarrow Q = m[(u_2 - u_1) + p(v_2 - v_1)] \Rightarrow \dfrac{2960\text{ kJ}}{5\text{ kg}} = (u_2 - u_1) + p(v_2 - v_1) \qquad (2)$

Depend on T_2

Solution Method #1

Rearrange Eq. (2) to read

$$\frac{2960\text{ kJ}}{5\text{ kg}} = \underbrace{(u_2 + p v_2)}_{h_2} - \underbrace{(u_1 + p v_1)}_{h_1} \Rightarrow h_2 = \left(\frac{2960}{5}\frac{\text{kJ}}{\text{kg}}\right) + h_1$$

From Table A-4, $h_1 = 2939.9$ kJ/kg. Thus, $h_2 = \left(\frac{2960}{5}\right)\frac{\text{kJ}}{\text{kg}} + 2939.9\frac{\text{kJ}}{\text{kg}} = 3531.9\frac{\text{kJ}}{\text{kg}}$

Interpolating in Table A-4 at p = 5 bar with h_2, we get $T_2 = 522°C$ ←

and $v_2 = 0.7314$ m³/kg.

With Eq. (1),

$$W = (5\text{ kg})\left(5\times10^5\frac{\text{N}}{\text{m}^2}\right)(0.7314 - 0.4646)\frac{\text{m}^3}{\text{kg}}\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

$$= 667\text{ kJ}$$

PROBLEM 3.70 (CONTINUED)

Solution Method #2

Based on Eqs. (1) and (2) develop an IT solution as follows:

```
T1=240
W=5*5*((v2-v1)*100) // 100 is a unit conversion factor        <—— See Eq. (1)

v1 = v_PT("Water/Steam", 5, T1)
v2 = v_PT("Water/Steam", 5, T2)

(2960/5) = (u2-u1) + ((5*(v2-v1)*100))                         <—— See Eq. (2)

u1 = u_PT("Water/Steam", 5, T1)
u2 = u_PT("Water/Steam", 5, T2)

SOLUTION:

W=667.8 kJ

T2=522.3  C

u1=2707   kJ/kg
u2=3166

v1=0.4646 m^3/kg
v2=0.7317
```

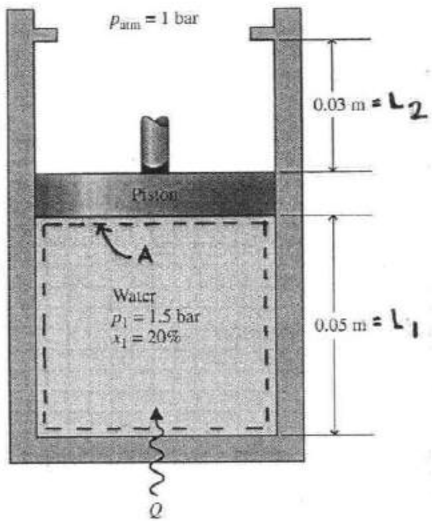
1. An iterative solution can be developed using Eq. (2) together with Steam Table data. However, this is less direct than Method #1, which centers only on interpolation with h_2.

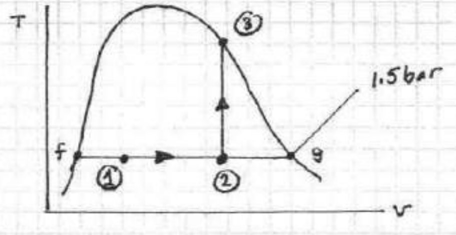
PROBLEM 3.71

KNOWN: Water contained in a piston-cylinder assembly, initially a two-phase liquid-vapor mixture undergoes two processes in series. State data is provided.

FIND: Show the two processes of the water in series on a T-v diagram. For the overall process of the water evaluate the work and heat transfer, each in kJ/kg.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL:

1. The water is the closed system.
2. Volume change is the only work mode.
3. Kinetic and potential energy effects are negligible

ANALYSIS: With assumption 2,

$$W = \int_1^3 p\,dV = \int_1^2 p\,dV + \cancel{\int_2^3 p\,dV}^{\,0}$$

$$\Rightarrow \frac{W}{m} = \int_1^2 p\,dv = p(v_2 - v_1) \qquad (1)$$

With data from Table A-3, $v_1 = v_f + x_1(v_g - v_f) = \left(\frac{1.0528}{10^3}\right) + 0.2\left[1.159 - \left(\frac{1.0528}{10^3}\right)\right] = 0.2326 \frac{m^3}{kg}$

With $v_1 = \frac{V_1}{m} = \frac{AL_1}{m}$, $v_2 = \frac{V_2}{m} = \frac{A(L_1+L_2)}{m}$ $\Big\rangle$ $\frac{v_2}{v_1} = \frac{L_1+L_2}{L_1} = \frac{0.08m}{0.05m} \Rightarrow v_2 = \left(\frac{0.08m}{0.05m}\right)v_1 = 0.3722 \frac{m^3}{kg}$

Also, $v_3 = v_2$.

Thus, Eq.(1) gives $\frac{W}{m} = 1.5\,\text{bar}\left|\frac{10^5 N/m^2}{1\,\text{bar}}\right|(0.3722 - 0.2326)\frac{m^3}{kg}\left|\frac{1\,kJ}{10^3 N\cdot m}\right| = 20.94 \frac{kJ}{kg}$ ←

Reducing an energy balance, $\Delta U + \Delta KE + \Delta PE = Q - W$, we get

$$Q = W + m(u_3 - u_1) \Rightarrow \frac{Q}{m} = \frac{W}{m} + (u_3 - u_1) \qquad (2)$$

With data from Table A-3, $u_1 = u_f + x_1(u_g - u_f) = 466.94 + 0.2[2519.7 - 466.94] = 877.49 \frac{kJ}{kg}$

Interpolation in Table A-3 with $v_3 (= v_2) = v_g$, $u_3 = 2561.48$ kJ/kg.

Then, Eq(2) gives

$$\frac{Q}{m} = 20.94 \frac{kJ}{kg} + (2561.48 - 877.49)\frac{kJ}{kg} = 1704.93 \frac{kJ}{kg}$$ ←

PROBLEM 3.72 REVISED 12-14

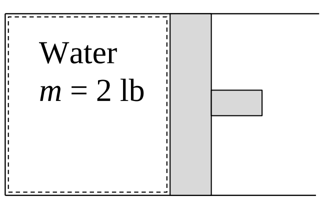
A piston-cylinder assembly contains 2 lb of water, initially at 100 lbf/in.^{2} and 400^{o}F. The water undergoes two processes in series: a constant pressure process followed by a constant volume process. At the of the constant volume process, the temperature is 300^{o}F and the water is a two-phase liquid-vapor mixture with a quality of 60%. Neglect kinetic and potential energy effects.

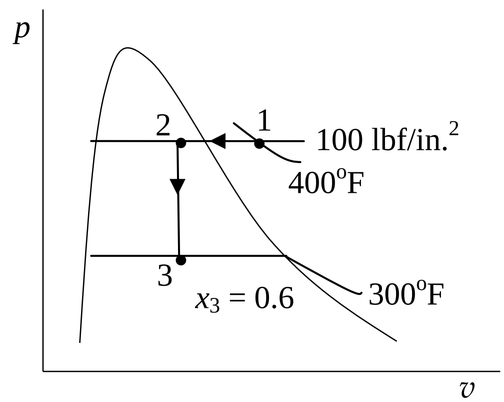
(a) Sketch T-v and p-v diagrams showing the key states and the processes.
(b) Determine the work and heat transfer for each process, all in Btu.

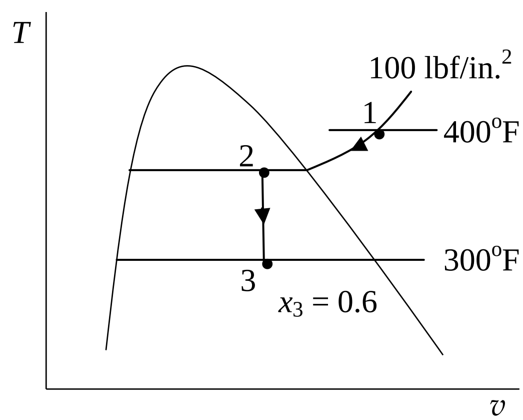
KNOWN: Water contained in a piston-cylinder assembly undergoes two processes in series.

FIND: Sketch the T-v and p-v diagrams and for each process determine Q and W.

SCHEMATIC AND GIVEN DATA:







ENGINEERING MODEL: 1. The `water is a closed system. 2. Volume change is the only work mode. 3. Process 1-2 occurs at constant pressure and Process 2-3 occurs at constant volume. 4. Kinetic and potential energy effects can be neglected.

ANALYSIS: First, we fix each state. State 1 is in the superheated vapor region. From Table A-4E; v_1 = 4.934 ft^3/lb and u_1 = 1136.2 Btu/lb.

With T_3 = 300^{o}F and x_3 = 0.6, we can evaluate v_3 and u_3 using data from Table A-2E at 300^{o}F as follows.

$$v_3 = v_{f3} + x_3(v_{g3} - v_{f3}) = 0.01745 + (0.6)(6.472 - 0.01745) = 3.89 \text{ ft}^3/\text{lb}$$

$$u_3 = u_{f3} + x_3(u_{g3} - u_{f3}) = 269.5 + (0.6)(1100.0 - 269.5) = 767.8 \text{ Btu/lb}$$

Note that $v_2 = v_3$ = 3.89 ft^3/lb, and from Table A-3E we see that $v_2 < v_g$(100 lbf/in.^{2}). Thus

$$x_2 = \frac{v_2 - v_{f2}}{v_{g2} - v_{f2}} = \frac{3.89 - 0.01774}{4.434 - 0.01774} = 0.8768$$

and

$u_2 = u_{f2} + x_2(u_{g2} - u_{f2}) = 298.3 + (0.8768)(1105.8 - 298.3) = 1006.3$ Btu/lb

Now, for Process 1-2 the pressure is constant. Thus

$$W_{12} = \int_1^2 p dV = m p_1 (v_2 - v_1) = (2 \text{ lb})\left(100 \ \frac{\text{lbf}}{\text{in.}^2}\right)(3.89 - 4.934)\frac{\text{ft}^3}{\text{lb}} \left|\frac{144 \text{ in.}^2}{1 \text{ ft}^2}\right| \left|\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right|$$

$= -38.65$ Btu (in)

An energy balance reduces to give

$Q_{12} = m(u_2 - u_1) + W_{12} = (2 \text{ lb})(1006.3 - 1136.2)$ Btu/lb + (– 38.65 Btu) = – 298.5 Btu (out)

Now, for Process 2-3, the volume is constant, so $W_{23} = 0$

And, the energy balance reduces to give

$Q_{23} = m(u_3 - u_2) = (2 \text{ lb})(767.8 - 1006.3)$But/lb = – 477 Btu (out)

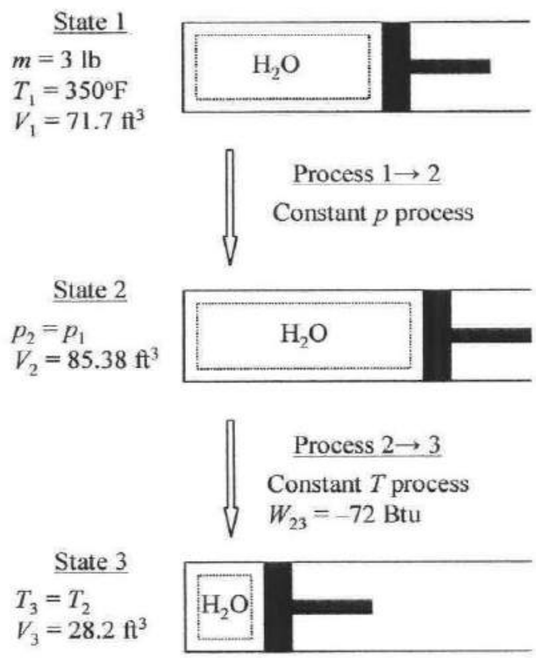
REVISED 12-14

PROBLEM 3.73

KNOWN: H_2O undergoes two processes. The initial state is known and the volume is specified for the second and third states. The first process is constant-pressure expansion. The second process is isothermal compression during which the work is known.

FIND: Determine the heat transfer for each process.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

1. The H_2O is the system.
2. Process 1-2 is constant pressure.
3. Process 2-3 is constant temperature.
4. Kinetic and potential energy effects are negligible

ANALYSIS:

For process 1-2, the heat transfer can be determined from the energy balance

$$(\Delta KE)_{12} + (\Delta PE)_{12} + (\Delta U)_{12} = Q_{12} - W_{12}$$

Neglecting changes in kinetic energy ($\Delta KE = 0$) and potential energy ($\Delta PE = 0$) and solving for heat transfer give

$$Q_{12} = (\Delta U)_{12} + W_{12} = m(u_2 - u_1) + W_{12}$$

State 1 is fixed by the two independent, intensive properties:
$T_1 = 350°F$
$v_1 = V_1/m = (71.7 \text{ ft}^3)/(3 \text{ lb}) = 23.9 \text{ ft}^3/\text{lb}$

Thus, State 1 is in the superheat vapor region. From Table A-4E:
$p_1 = 20 \text{ lbf/in.}^2$
$u_1 = 1126.9 \text{ Btu/lb}$

State 2 is fixed by the two independent, intensive properties:
$v_2 = V_2/m = (85.38 \text{ ft}^3)/(3 \text{ lb}) = 28.46 \text{ ft}^3/\text{lb}$
$p_2 = p_1 = 20 \text{ lbf/in.}^2$

Thus, State 2 is in the superheated vapor region. From Table A-4E:
$T_2 = 500°F$
$u_2 = 1181.5 \text{ Btu/lb}$

Work during process 1-2 is determined by

$$W_{12} = \int_{V_1}^{V_2} p\,dV$$

Since process 1-2 is constant pressure, integration and substitution of values give

$$W_{12} = p_1(V_2 - V_1) = \left(20\frac{\text{lbf}}{\text{in.}^2}\right)\left(85.38 \text{ ft}^3 - 71.7 \text{ ft}^3\right)\left|\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right| = 50.64 \text{ Btu}$$

Substituting values in the energy equation for process 1-2 yields

$$Q_{12} = (3 \text{ lb})[(1181.5 \text{ Btu/lb}) - (1126.9 \text{ Btu/lb})] + 50.64 \text{ Btu} = \underline{\mathbf{214.44 \text{ Btu}}} \longleftarrow$$

The positive sign indicates heat transfer is into the system during process 1-2.

For process 2-3, the heat transfer can be determined from the energy balance

$$(\Delta KE)_{23} + (\Delta PE)_{23} + (\Delta U)_{23} = Q_{23} - W_{23}$$

Neglecting changes in kinetic energy ($\Delta KE = 0$) and potential energy ($\Delta PE = 0$) and solving for heat transfer give

$$Q_{23} = (\Delta U)_{23} + W_{23} = m(u_3 - u_2) + W_{23}$$

PROBLEM 3.73 (CONTINUED) – PAGE 3

State 3 is fixed by the two independent, intensive properties:
$v_3 = V_3/m = (28.2\ \text{ft}^3)/(3\ \text{lb}) = 9.4\ \text{ft}^3/\text{lb}$
$T_3 = T_2 = 500°\text{F}$

Thus, State 3 is in the superheated vapor region. From Table A-4E:
$p_3 = 60\ \text{lbf/in.}^2$
$u_3 = 1178.6\ \text{Btu/lb}$

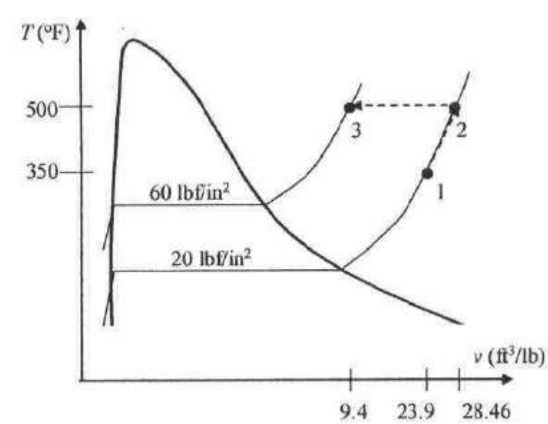
Work during process 2-3 is given as work into the system of 72 Btu. Since work is into the system, $W_{23} = -72$ Btu. Substituting values in the energy equation for process 2-3 yields

$$Q_{23} = (3\ \text{lb})[(1178.6\ \text{Btu/lb}) - (1181.5\ \text{Btu/lb})] + (-72\ \text{Btu}) = \underline{\mathbf{-80.7\ Btu}} \quad \leftarrow$$

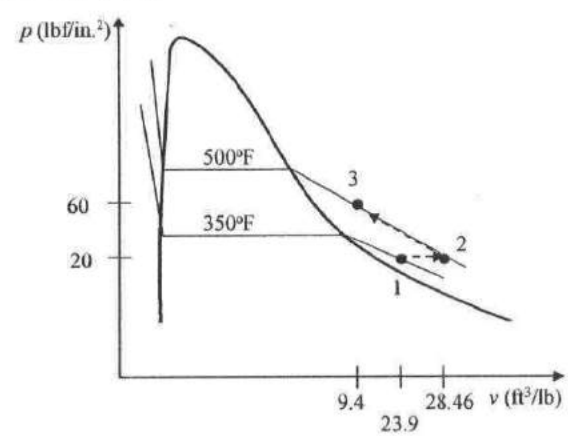
The negative sign indicates heat transfer is from the system during process 2-3.

The process begins and remains in the superheated vapor region during both processes. Process 1-2 is constant pressure, follows the constant pressure line of 20 lbf/in.^{2} on the T-v diagram and appears as a horizontal line on the p-v diagram. Process 2-3 is constant temperature, appears as a horizontal line on the T-v diagram, and follows the constant temperature line of 500°F on the p-v diagram.

<u>T-v Diagram</u>



<u>p-v Diagram</u>

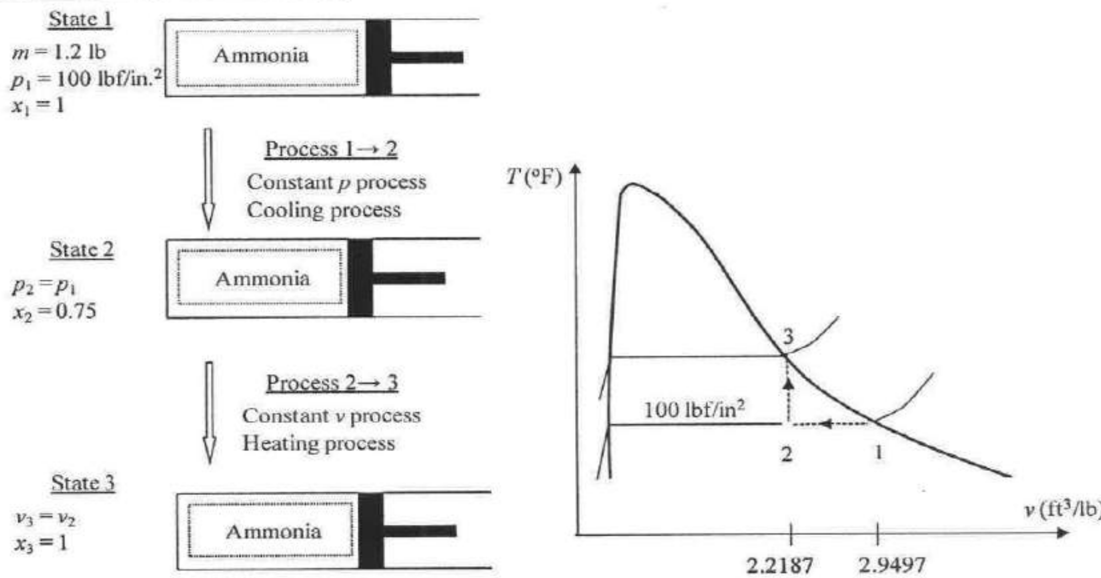


PROBLEM 3.74

Known: Ammonia in a piston-cylinder assembly undergoes two processes in series; the first at constant pressure and the second at constant volume. Data are known at each of the end states.

Find: Determine (a) the heat transfer and work for Process 1-2 and (b) the heat transfer for Process 2-3, all in Btu.

Schematic and Given Data:



Engineering Model:
(1) The ammonia is a closed system.
(2) The processes occur at constant pressure and constant volume, respectively.
(3) Volume change is the only work mode.
(4) Kinetic and potential energy effects are negligible.

Analysis:

The energy transfers are found using energy balances. First, fix each of the end states.
State 1: $p_1 = 100$ lbf/in.^{2} and $x_1 = 1$, using Table A-14E:

PROBLEM 3.74 (CONTINUED)

$u_1 = u_g = 571.21$ Btu/lb
$v_1 = v_g = 2.9497$ ft^3/lb

State 2: $p_2 = p_1 = 100$ lbf/in.^{2} and $x_2 = 0.75$, using Table A-14E:
$u_2 = u_f + x_2\,(u_g - u_f) = 103.87$ Btu/lb + 0.75(571.21 – 103.87) Btu/lb = 454.38 Btu/lb
$v_2 = v_f + x_2\,(v_g - v_f) = 0.02584$ ft^3/lb + 0.75(2.9497 – 0.02584) ft^3/lb = 2.2187 ft^3/lb

State 3: $v_3 = v_2$ and $x_3 = 1$, using Table A-14E, the value of $v_3 = v_g = 2.2187$ ft^3/lb exists between 130 and 140 lbf/in.^{2} Using linear interpolation, the corresponding $u_3 = 573.61$ Btu/lb.

(a) The energy balance for Process 1 to 2 reduces to:

$$\Delta\text{KE} + \Delta\text{PE} + m(u_2 - u_1) = Q_{12} - W_{12}$$
$$Q_{12} = m(u_2 - u_1) + W_{12} \qquad (1)$$

The work for Process 1 to 2 can be determined using Eq. 2.17:

$$W_{12} = m\int_{v_1}^{v_2} p\,dv = mp(v_2 - v_1) =$$

(#1) $$W_{12} = 1.2\ \text{lb}\left(100\frac{\text{lbf}}{\text{in.}^2}\right)(2.2187 - 2.9497)\frac{\text{ft}^3}{\text{lb}}\left|\frac{144\ \text{in.}^2}{1\text{ft}^2}\right|\left|\frac{1\ \text{Btu}}{778\text{ft}\cdot\text{lbf}}\right| = -16.24\ \text{Btu} \longleftarrow$$

The heat transfer for Process 1 to 2 can now be determined using Eq. (1).

$$Q_{12} = m(u_2 - u_1) + W = 1.2\ \text{lb}(454.38 - 571.21)\frac{\text{Btu}}{\text{lb}} + (-16.24\ \text{Btu}) =$$

(#2) $$Q_{12} = -140.2\ \text{Btu} + (-16.24)\text{Btu} = -156.44\ \text{Btu} \longleftarrow$$

(b) The energy balance for Process 2 to 3 reduces to:

(#3) $$Q_{23} = m(u_3 - u_2) = 1.2\ \text{lb}(573.61 - 454.38)\frac{\text{Btu}}{\text{lb}} = 143.08\ \text{Btu} \longleftarrow$$

---

1. The negative sign for W_{12} denotes energy transfer into the system, as expected during compression.
2. The negative sign for Q_{12} denotes energy rejected from the system by heat transfer, as expected during cooling.
3. The positive sign for Q_{23} denotes energy added to the system through heat transfer, as expected during heating.

PROBLEM 3.75

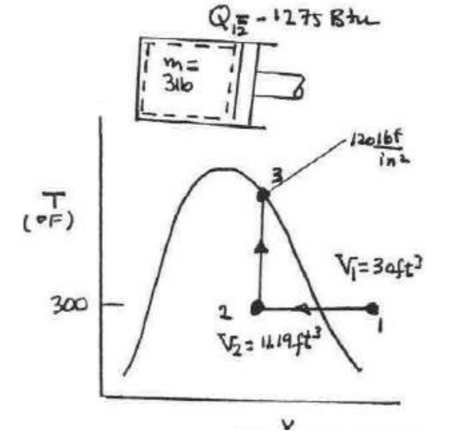
KNOWN: Water contained within a piston-cylinder assembly undergoes two processes in series:

Process 1-2: Constant-temperature compression to $V_2 = 11.19\ \text{ft}^3$, during which there is an energy transfer by heat *from* the water of 1275 Btu.

Process 2-3: Constant-volume heating to $p_3 = 120\ \text{lbf/in.}^2$.

FIND: Sketch the processes in series on a T-v diagram. Find W_{12} and Q_{23}.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The water within the piston-cylinder assembly is the closed system.
2. Volume change is the only work mode.
3. Kinetic and potential energy effects are negligible.

ANALYSIS: √State 1 is fixed by $T_1 = 300°F$ and $v_1 = V_1/m = \frac{30\ \text{ft}^3}{3\ \text{lb}} = 10\ \frac{\text{ft}^3}{\text{lb}}$. Interpolating in Table A-4E gives $u_1 = 1104.06\ \text{Btu/lb}$.

√State 2 is fixed by $T_2 = 300°F$ and $v_2 = \frac{V_2}{m}$ or $v_2 = \frac{11.19\ \text{ft}^3}{3\ \text{lb}} = 3.73\ \frac{\text{ft}^3}{\text{lb}}$. The quality at 2 is obtained with data at 300°F from Table A-2E

$$x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{3.73 - 0.01745}{6.472 - 0.01745} = 0.5752$$

Then, $u_2 = u_f + x_2(u_g - u_f) = 269.5 + 0.5752(1100.0 - 269.5)$
$= 747.2\ \text{Btu/lb}$

√State 3 is fixed by $v_3 = v_2$ and $P_3 = 120\ \text{lbf/in}^2$. From Table A-3E, we see state 3 is saturated vapor. So, $u_3 = u_g = 1108.3\ \text{Btu/lb}$.

An energy balance for Process 1-2 reads $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$. Thus

$$W_{12} = Q_{12} - m(u_2 - u_1) = -1275\ \text{Btu} - 3\ \text{lb}(747.2 - 1104.06)\frac{\text{Btu}}{\text{lb}}$$

$$= -204.42\ \text{Btu} \qquad \longleftarrow W_{12}$$

Since the piston does not move (volume is constant), $W_{23} = 0$. An energy balance for Process 2-3 then reads $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - \cancel{W}$ or

$$Q_{23} = m(u_3 - u_2) = 3\ \text{lb}(1108.3 - 747.2)\frac{\text{Btu}}{\text{lb}}$$

$$= 1083.3\ \text{Btu} \qquad \longleftarrow Q_{23}$$

PROBLEM 3.76

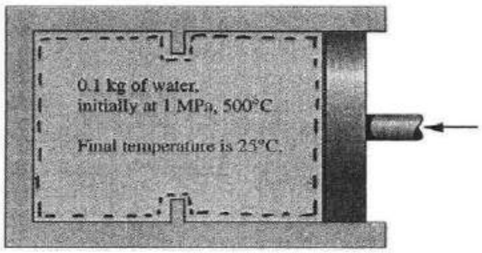
KNOWN: Water contained in a piston-cylinder assembly undergoes two processes in series.

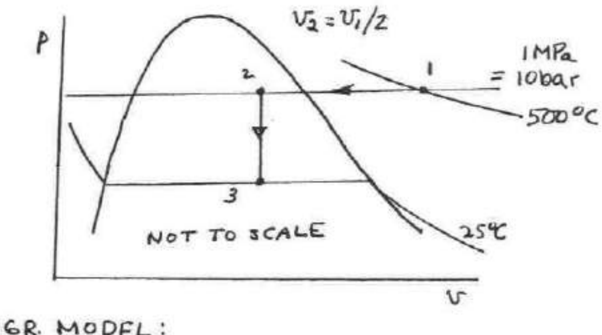
FIND: Evaluate Q and W for each process. Sketch the two processes in series on a p-v diagram.

SCHEMATIC & GIVEN DATA:

Process 1-2: Constant-pressure cooling until the piston face rests against the stops. The volume occupied by the water is then one-half its initial volume.

Process 2-3: With the piston face resting against the stops, the water cools to 25°C.





ENGR. MODEL:

1. The water in the piston cylinder assembly is the system.
2. Volume change is the only work mode.
3. Kinetic and potential energy changes are absent.

ANALYSIS: Since volume change is the only work mode, work can be evaluated from Eq. 2.17. Thus, $W_{23} = 0$ since the piston does not move (volume is constant) and

$$W_{12} = \int_1^2 p\,dV = mp(v_2 - v_1) \qquad \longleftarrow W_{23}$$

From Table A-4 at 10 bar, 500°C, $v_1 = 0.3541\ m^3/kg$, $u_1 = 3124.4\ kJ/kg$. At state 2,

$$v_2 = v_1/2 = \frac{0.3541}{2}\frac{m^3}{kg} = 0.17705\ m^3/kg \ . \text{ Then } x_2 = \frac{v_2 - v_f}{v_g - v_f} = \left(\frac{0.17705 - 1.1273/10^3}{0.1944 - 1.1273/10^3}\right) = 0.91$$

(Data from Table A-3)

Thus, $u_2 = u_f + x_2(u_g - u_f) = 761.68 + 0.91[2583.6 - 761.68] = 2419.63\ kJ/kg$.

Calculating,

$$W_{12} = mp(v_2 - v_1) = (0.1\ kg)(1\ MPa)\left|\frac{10^6 N/m^2}{1\ MPa}\right|\left(0.17705 - 0.3541\right)\frac{m^3}{kg}\left|\frac{1\ kJ}{10^3 N\cdot m}\right| = -17.71\ kJ \longleftarrow W_{12}$$

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{12} - W_{12} \Rightarrow Q_{12} = m(u_2 - u_1) + W_{12} = 0.1\ kg(2419.63 - 3124.4)\frac{kJ}{kg} - 17.71\ kJ$$

$$\therefore Q_{12} = -88.19\ kJ \qquad \longleftarrow Q_{12}$$

State 3 is fixed by $v_3 = v_2$ and $T_3 = 25°C$. Then $x_3 = \frac{v_3 - v_f}{v_g - v_f} = \frac{0.17705 - (1.0029/10^3)}{43.360 - (1.0029/10^3)} = 0.0041$

(Data from Table A-2)

Thus, $u_3 = u_f + x_3(u_g - u_f) = 104.88 + 0.0041(2409.8 - 104.88) = 114.33\ kJ/kg$.

For process 2-3,

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - \cancel{W}$$

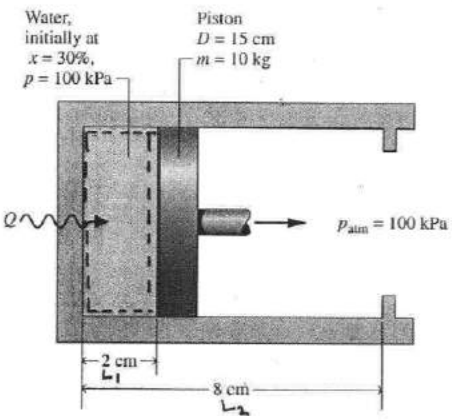
$$\Rightarrow Q_{23} = m(u_3 - u_2) = 0.1\ kg(114.33 - 2419.63)\frac{kJ}{kg} = -230.53\ kJ \qquad \longleftarrow Q_{23}$$

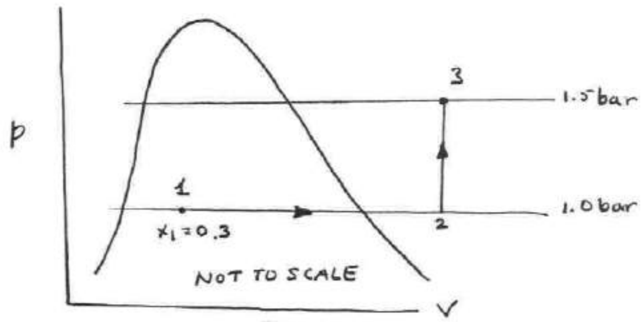
PROBLEM 3.77

KNOWN: Data are provided for water in a piston-cylinder assembly undergoing two processes in series: constant-pressure followed by constant-volume.

FIND: For the overall process find W and Q.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL:
1. The water in the piston-cylinder assembly is the system.
2. Friction and kinetic and potential energy effects are absent.
3. Volume change is the only work mode.

ANALYSIS:

Since volume change is the only work mode, work can be evaluated from Eq. 2.17. Thus

$$W_{12} = \int_1^2 p\,dV = p[V_2 - V_1] \qquad (1)$$

Note that $W_{23} = 0$ because there is no piston motion (Volume is constant).

An energy balance for the overall process reads.

$$(U_3 - U_1) + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{13} - W_{13} \Rightarrow Q_{13} = m(u_3 - u_1) + \underbrace{W_{13}}_{= W_{12}} \qquad (2)$$

Need V_1, V_2, m, u_1, u_3

$$\Rightarrow V_1 = \frac{\pi D^2}{4} L_1 = \frac{\pi (0.15\,m)^2 (0.02\,m)}{4} = 3.53 \times 10^{-4}\,m^3$$

$$\Rightarrow V_2 = \frac{\pi D^2}{4} L_2 = \frac{\pi (0.15\,m)^2 (0.08\,m)}{4} = 1.414 \times 10^{-3}\,m^3$$

with data from Table A-3,

$$v_1 = v_f + x(v_g - v_f) = \frac{1.0432}{10^3} + 0.3\left[1.694 - \frac{1.0432}{10^3}\right] = 0.50893\,m^3/kg$$

$$\Rightarrow m = \frac{V_1}{v_1} = 6.94 \times 10^{-4}\,kg$$

$$u_1 = u_f + x(u_g - u_f) = 417.36 + 0.3(2506.1 - 417.36) = 1043.98\,\frac{kJ}{kg}$$

$$v_3 = v_2 = \frac{V_2}{m} = \frac{1.414 \times 10^{-3}}{6.94 \times 10^{-4}} = 2.03746\,\frac{m^3}{kg}.$$

Interpolating in Table A-4 at 1.5 bar, $u_3 = 2952.1\,\frac{kJ}{kg}$

Using Eq (1),

$$W_{12} = p[V_2 - V_1] = (1\,bar)\left|\frac{10^5\,N/m^2}{1\,bar}\right|(14.14 - 3.53) \times 10^{-4}\,m^3 \left|\frac{1\,kJ}{10^3\,N \cdot m}\right| = 0.106\,kJ$$ ← W

Using Eq. (2)

$$Q_{13} = m(u_3 - u_1) + W_{13} = (6.94 \times 10^{-4}\,kg)(2952.1 - 1043.98)\frac{kJ}{kg} + 0.106\,kJ = 1.324 + 0.106 = 1.43\,kJ$$ ← Q

PROBLEM 3.78

A system consisting of 1 kg of H_2O undergoes a heat pump cycle composed of the following processes.

Process 1-2: Constant-volume heating from p_1 = 5 bar, T_1 = 160°C to p_2 = 10 bar.

Process 2-3: Constant-pressure cooling to saturated vapor.

Process 3-4: Constant-volume cooling to T_4 = 160°C.

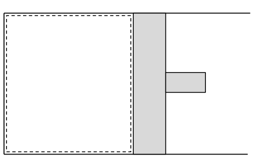
Process 4-1: Isothermal expansion with Q_{41} = 815.8 kJ.

Sketch the cycle on T-v and p-v diagrams. Neglecting kinetic and potential energy effects, determine the coefficient of performance.

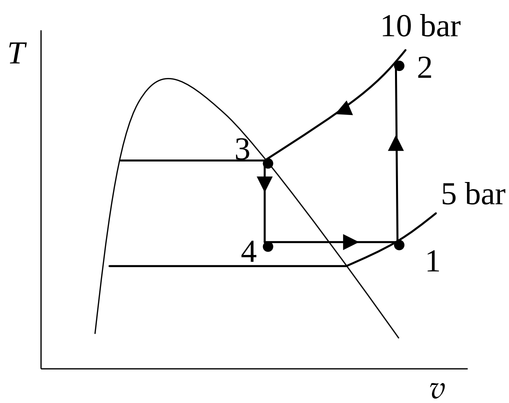
KNOWN: One kg of water undergoes a heat pump cycle consisting of four processes.

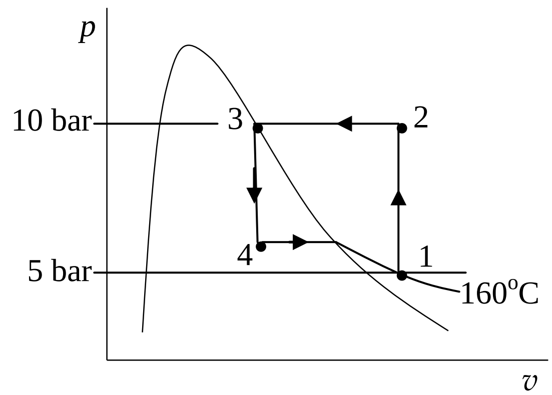
FIND: Sketch the cycle on T-v and p-v diagrams and determine the coefficient of performance.

SCHEMATIC AND GIVEN DATA:



Process 1-2: Constant-volume heating from p_1 = 5 bar, T_1 = 160°C to p_2 = 10 bar.
Process 2-3: Constant-pressure cooling to saturated vapor.
Process 3-4: Constant-volume cooling to T_4 = 160°C.
Process 4-1: Isothermal expansion with Q_{41} = 815.8 kJ.





ENGINEERING MODEL: 1. The water is the closed system. 2. Volume change is the only work mode. 3. Kinetic and potential energy effects can be ignored.

ANALYSIS: Referring to Sec. 2.6.3, the coefficient of performance of a heat pump cycle is $\gamma = Q_{out}/W_{cycle}$, where $W_{cycle} = W_{12} + W_{23} + W_{34} + W_{41}$.

Process 1-2: Referring to Table A-2 at 160°C, $p_1 < p_{sat}$ so State 1 is in the superheated vapor region. From Table A-4; $v_1 = 0.3835$ m³/kg and $u_1 = 2575.2$ kJ/kg.

State is fixed by $p_2 = 10$ bar and $v_2 = v_1 = 0.3835$ m³/kg. Interpolating in Table A-4, we get $u_2 = 3231.8$ kJ/kg.

Since the volume is constant, $W_{12} = 0$, and the energy balance reduces to give

$$Q_{12} = m(u_2 - u_1) = (1\text{ kg})(3231.8 - 2575.2)\text{ kJ/kg} = 656.6\text{ kJ}$$

Process 2-3: State 3 is saturated vapor at 10 bar. From Table A-3; $u_3 = u_{g3} = 2583.6$ kJ/kg and $v_3 = v_{g3} = 0.1944$ m³/kg.

For the constant-pressure cooling

$$W_{23} = \int_2^3 p dV = m\, p_2(v_3 - v_2) = (1\text{ kg})(10\text{ bar})(0.1944 - 0.3835)\frac{\text{m}^3}{\text{kg}}\left|\frac{10^5\text{N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{N}\cdot\text{m}}\right|$$
$$= -189.1\text{ kJ}$$

The energy balance reduces to

$$Q_{23} = m(u_3 - u_2) + W_{23} = (1\text{ kg})(2583.6 - 3231.8)\text{ kJ/kg} + (-189.1\text{kJ}) = -837.3\text{ kJ}$$

Process 3-4: State 4 is fixed by $T_4 = 160$°C and $v_4 = v_3 = 0.1944$ m³/kg. The state is in the two-phase liquid-vapor region, so

$$x_4 = \frac{v_4 - v_{f4}}{v_{g4} - v_{f4}} = \frac{0.1944 - 1.102 \times 10^{-3}}{0.3071 - 1.102 \times 10^{-3}} = 0.6317$$

and

$$u_4 = u_{f4} + x_4(u_{g4} - u_{f4}) = 674.86 + (0.6317)(2568.4 - 674.86) = 1871\text{ kJ/kg}$$

With $W_{34} = 0$, the energy balance reduces to give

$$Q_{34} = m(u_4 - u_3) = (1\text{ kg})(1871 - 2583.6)\text{kJ/kg} = -712.6\text{ kJ}$$

Process 4-1: $Q_{41} = 815.8$ kJ (given). From the energy balance, $m(u_1 - u_4) = Q_{41} - W_{41}$, so

① $$W_{41} = m(u_4 - u_1) + Q_{41} = (1\text{ kg})(1871 - 2575.2)\text{kJ/kg} + (815.8\text{ kJ}) = 111.6\text{ kJ}$$

Finally, the net work is $W_{cycle} = W_{12} + W_{23} + W_{34} + W_{41} = (0) + (-189.1) + (0) + (111.6) = -77.5$ kJ

PROBLEM 3.78 (CONTINUED) – PAGE 3

Now, $Q_{out} = Q_{23} + Q_{34}$ = (-837.3) + (-712.6) = -1549.9 kJ

So, the coefficient of performance is

$$\gamma = (-1549.9\text{ kJ})/(-77.5\text{ kJ}) = 20.0 \longleftarrow$$

---

① As a check, note that for every cycle, $Q_{cycle} = W_{cycle}$. So

$$Q_{cycle} = Q_{12} + Q_{23} + Q_{134} + Q_{41} = (656.6) + (-837.3) + (-712.6) + (815.8) = -77.5\text{ kJ}$$

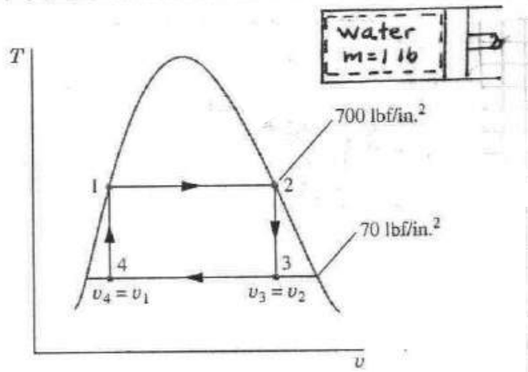
Which agrees with the value of W_{cycle} calculated using the work quantities, as expected.

PROBLEM 3.79

KNOWN: Water contained in a piston-cylinder undergoes a power cycle. State data are provided.

FIND: For each process of the cycle, evaluate work and heat transfer, in Btu; also evaluate the thermal efficiency.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL

1. The water is the closed system.
2. Kinetic and potential effects are ignored.

ANALYSIS:

The work for each process is evaluated using $W = \int p\,dV = m\int p\,dv$

Process 1-2:

$W_{12} = mp(v_2 - v_1)$. With data from Table A-3E

$$= (1\,lb)(700\times 144\,\tfrac{lbf}{in^2})\underbrace{(0.656 - 0.02051)}_{= 0.63549}\tfrac{ft^3}{lb}\left|\tfrac{1\,Btu}{778\,ft\cdot lbf}\right|$$

$= 82.34$ Btu

An energy balance reduces to give

$Q_{12} = \Delta U + W_{12} = m(u_2 - u_1) + W_{12}$

$= 1\,lb(1117 - 488.9)\frac{Btu}{lb} + 82.34$ Btu

$= 710.44$ Btu

Process 2-3: $W_{23} = 0$. An energy balance reduces to give $Q_{23} = m(u_3 - u_2)$

With $v_3 = v_2$, $x_3 = \frac{v_3 - v_f}{v_g - v_f} = \frac{0.656 - 0.01748}{6.209 - 0.01748} = 0.1031 \Rightarrow u_3 = u_f + x_3(u_g - u_f)$

$= 272.6 + 0.1031(1100.6 - 272.6)$

$= 357.97$ Btu/lb

$\Rightarrow Q_{23} = (1\,lb)(357.97 - 1117)\frac{Btu}{lb} = -759.03$ Btu

Process 3-4: $W_{34} = mp(v_4 - v_3)$. With $v_4 = v_1$, $x_4 = \frac{v_4 - v_f}{v_g - v_f} = \frac{0.02051 - 0.01748}{6.209 - 0.01748}$

$= 4.89\times 10^{-4}$

Noting that $(v_4 - v_3) = [v_f + x_4(v_g - v_f)] - [v_f + x_3(v_g - v_f)]$

$= (x_4 - x_3)(v_g - v_f)$

$W_{34} = mp(x_4 - x_3)(v_g - v_f)$

$= (1\,lb)(70\times 144\,\frac{lbf}{in^2})(4.89\times 10^{-4} - 0.1031)(6.1952\,\frac{ft^3}{lb})\left|\frac{1\,Btu}{778\,ft\,lbf}\right| = -8.24$ Btu

From an energy balance, $Q_{34} = m(u_4 - u_3) + W_{34}$, $u_4 = u_f + x_4(u_g - u_f) = 273$ Btu

$(= 272.6 + 4.89\times 10^{-4}(828\,Btu/lb))$

Thus, $Q_{34} = (1\,lb)(273 - 357.97)\frac{Btu}{lb} - 8.24$ Btu $= -93.21$ Btu

PROCESS 4-1: $W_{41} = 0$. $Q_{41} = m(u_1 - u_4) = 1\,lb\,(488.9 - 273)\frac{Btu}{lb} = 215.9$ Btu

(1) $$\eta = \frac{\text{Net Work Developed}}{\text{Total Heat Transfer to the system}} = \frac{W_{12} + W_{23} + W_{34} + W_{41}}{Q_{12} + Q_{41}} = \frac{82.34 + (0) + (-8.24) + (0)}{(710.44 + 215.9)} = 0.08\ (8\%)$$

1. For every thermodynamic cycle, Net Work = Net Heat Transfer.

Thus, Net Work $= W_{12} + W_{23} + W_{34} + W_{41} = 74.1$ Btu

Net Heat Transfer $= Q_{12} + Q_{23} + Q_{34} + Q_{41} = 710.44 + (-759.03) + (-93.21) + 215.9$

$= 74.1$ Btu (checks)

Also, note that the net work = Area (1-2-3-4). That is,

Net work $= m[P_{12} - P_{34}](v_2 - v_1)$

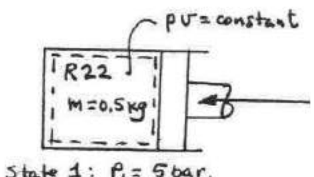
$= (1\,lb)[(700 - 70)(144)\frac{lbf}{ft^2}][0.63549\,\frac{ft^3}{lb}]\left|\frac{1\,Btu}{778\,ft\cdot lbf}\right| = 74.1$ Btu

PROBLEM 3.80

KNOWN: Data are provided for a process of Refrigerant 22 contained in a piston-cylinder assembly.

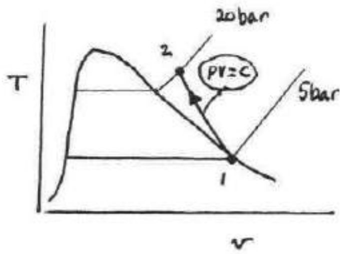
FIND: For the process, find W and Q.

SCHEMATIC & GIVEN DATA:



State 1: $p_1 = 5$ bar, sat. vapor.

State 2: $p_2 = 20$ bar



ENGR. MODEL:

1. The R22 contained in the piston-cylinder assembly is the closed system.
2. Volume change is the only work mode.
3. The process is described by pv = constant.
4. Kinetic and potential energy effects can be ignored.

ANALYSIS: The work is given by

$$W_{12} = \int_1^2 p\,dV = m\int_1^2 \underbrace{p}_{=\frac{c}{v}}\,dv = m\underbrace{c}_{=p_1 v_1}\ln\frac{v_2}{v_1}$$

or

$$W_{12} = m(p_1 v_1)\ln\frac{v_2}{v_1}$$

From Table A-8, at 5 bar, $v_1 = 0.0469\ m^3/kg$. Then, since $p_1 v_1 = p_2 v_2$, $v_2 = \left(\frac{p_1}{p_2}\right)v_1$. That is, $v_2 = \left(\frac{5\ bar}{20\ bar}\right)\left(0.0469\frac{m^3}{kg}\right) = 0.0117\ m^3/kg$. Accordingly

$$W_{12} = (0.5\ kg)\left(5\times10^5\frac{N}{m^2}\right)\left(0.0469\frac{m^3}{kg}\right)\ln(0.25)\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

$$= -16.25\ kJ \qquad \longleftarrow W_{12}$$

Reducing an energy balance, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$, or

$$Q_{12} = W_{12} + m(u_2 - u_1)$$

From Table A-8, $u_1 = 226.54\ kJ/kg$. Interpolating in Table A-9 at 20 bar and $v_2 = 0.0117\ m^3/kg$, $u_2 = 243.53\ kJ/kg$. Then

$$Q_{12} = -16.25\ kJ + 0.5\ kg\,(243.53 - 226.54)\frac{kJ}{kg}$$

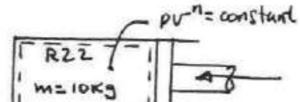
$$= -7.76\ kJ. \qquad \longleftarrow Q_{12}$$

PROBLEM 3.81

KNOWN: Data are provided for a process of Refrigerant 22 contained in a piston-cylinder assemby.

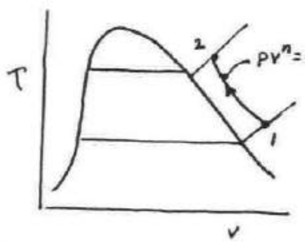
FIND: For the process, find W and Q.

SCHEMATIC & GIVEN DATA:



$P_1 = 400\,kPa$, $T_1 = -5°C$

$P_2 = 2000\,kPa$, $T_2 = 70°C$



ENGR. MODEL:

1. The R22 contained in the piston-cylinder assembly is the closed system.
2. Volume change is the only work mode.
3. The process is described by $pv^n = $ constant.
4. Kinetic and potential energy effects can be ignored.

ANALYSIS: Work is found using

$$W_{12} = \int_1^2 p\,dV = m\int_1^2 \underbrace{p}_{(= c/v^n)}\,dv$$

$$W_{12} = m\left[\frac{p_2 v_2 - p_1 v_1}{1-n}\right] \quad \text{(See Example 2.1)} \qquad (1)$$

The value for the exponent n can be obtained with p_1, p_2 and v_1, v_2 from Table A-9: At 4 bar, $-5°C$, $v_1 = 0.0586\ m^3/kg$. At 20 bar, $70°C$, $v_2 = 0.013\ m^3/kg$. Then, using $pv^n = $ constant, we get $p_1 v_1^n = $ constant and $p_2 v_2^n = $ constant, giving

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^n \Rightarrow \frac{20}{4} = \left(\frac{0.0586}{0.013}\right)^n. \quad \text{Solving, } n = 1.069.$$

Equation (1) gives

$$W_{12} = (10\,kg)\left[\frac{(20\,bar)(0.013\,\frac{m^3}{kg}) - (4\,bar)(0.0586\,\frac{m^3}{kg})}{1-1.069}\right]\left|\frac{10^5\,N/m^2}{1\,bar}\right|\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|$$

$$= -371\ kJ \qquad \longleftarrow W_{12}$$

An energy balance reduces to read, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{12} - W_{12}$, or

$$Q_{12} = W_{12} + m(u_2 - u_1)$$

$$= -371\,kJ + 10\,kg\,(255.35 - 225.16)\,\frac{kJ}{kg}$$

$$= -69.1\,kJ \qquad \longleftarrow Q_{12}$$

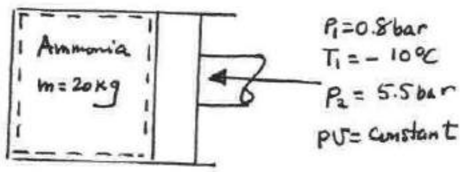
where u_1 and u_2 are from Table A-9.

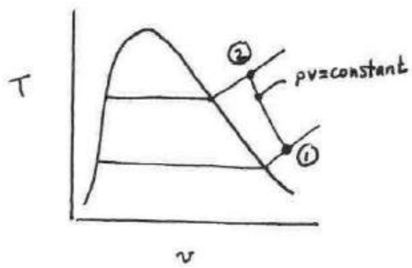
PROBLEM 3.82

KNOWN: Data are provided for a process of ammonia contained within a piston-cylinder assembly.

FIND: For the process find W and Q,

SCHEMATIC & GIVEN DATA:





ENGR. MODEL

1. The ammonia contained in the piston-cylinder assembly is the closed system.
2. Volume change is the only work mode.
3. The process is described by $pv =$ constant.
4. Kinetic and potential energy effects can be ignored.

ANALYSIS: The work is given by

$$W_{12} = \int_1^2 p\,dV = m\int_1^2 \underbrace{p}_{=\frac{C}{v}}\,dv = m\underbrace{C}_{=p_1v_1}\ln\frac{v_2}{v_1}$$

or

$$W_{12} = m(p_1v_1)\ln\frac{v_2}{v_1} \qquad (1)$$

From Table A-15 at $p_1 = 0.8$ bar, $T_1 = -10°C$, $v_1 = 1.5834\ m^3/kg$, $u_1 = 1325.85$ kJ/kg. Since $pv =$ constant, $p_2v_2 = p_1v_1$ or $v_2 = \left(\frac{p_1}{p_2}\right)v_1 = \left(\frac{0.8}{5.5}\right)(1.5834) = 0.2303\ m^3/kg$.

Interpolating in Table A-15 with v_2 gives $u_2 = 1326.21$ kJ/kg.

Using Eq.(1) we get

$$\frac{v_2}{v_1} = \frac{p_1}{p_2}$$

$$W_{12} = (20\ kg)(0.8\ bar)\left(1.5834\frac{m^3}{kg}\right)\ln\left(\frac{0.8}{5.5}\right)\left|\frac{10^5\ N/m^2}{1\ bar}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

$$= -4884\ kJ$$

$\longleftarrow W_{12}$

Reducing an energy balance, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{12} - W_{12}$,

$$Q_{12} = W_{12} + m(u_2 - u_1)$$

$$= -4884\ kJ + 20\ kg\,(1326.21 - 1325.85)\frac{kJ}{kg}$$

$$= -4877\ kJ$$

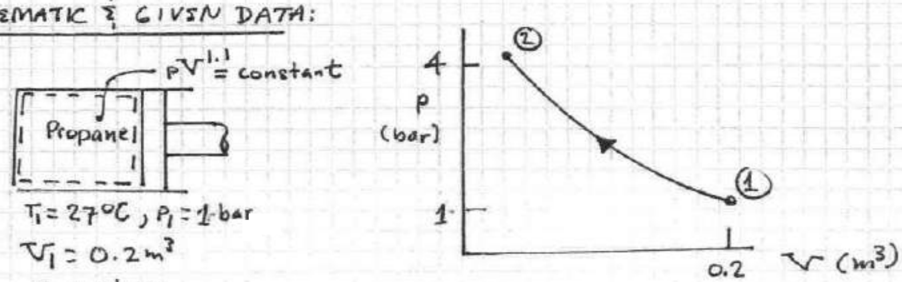
$\longleftarrow Q_{12}$

PROBLEM 3.83

KNOWN: Propane within a piston-cylinder assembly undergoes a process during which $pV^{1.1}$ = constant. State data are provided.

FIND: For the propane evaluate the work and heat transfer, in kJ

SCHEMATIC & GIVEN DATA:



$T_1 = 27°C$, $p_1 = 1$ bar
$V_1 = 0.2 m^3$
$p_2 = 4$ bar

ENGINEERING MODEL:

1. The propane is the closed system.
2. The process is described by $pV^{1.1}$ = constant
3. Kinetic and potential energy effects can be ignored.

ANALYSIS:

The work is evaluated from Eq. 2.17:

$$W = \int_1^2 p\,dV = \frac{p_2V_2 - p_1V_1}{(1-n)} \qquad (1)$$

See Example 2.1, Part(a), for the integration. In this case, n = 1.

To find V_2, write $p_1V_1^{1.1} = p_2V_2^{1.1} \Rightarrow V_2 = V_1\left[\frac{p_1}{p_2}\right]^{\frac{1}{1.1}} = (0.2 m^3)\left[\frac{1}{4}\right]^{\frac{1}{1.1}} = 0.0567 m^3$

Then, Eq. (1) gives

$$W = \frac{[(4\times10^5 N/m^2)(0.0567 m^3) - (10^5 N/m^2)(0.2 m^3)]}{(1-1.1)}\left|\frac{1 kJ}{10^3 N\cdot m}\right| = -26.8 kJ$$

An energy balance reduces to read: $Q = m(u_2 - u_1) + W$, where with data from Table A-18

$$m = \frac{V_1}{v_1} = \frac{0.2 m^3}{0.5571 m^3/kg} = 0.359 kg, \quad u_1 = 473.73 \frac{kJ}{kg}$$

Also, we have

① $m = \frac{V_1}{v_1} = \frac{V_2}{v_2} \Rightarrow v_2 = v_1\left[\frac{V_2}{V_1}\right] = 0.5571 \frac{m^3}{kg}\left(\frac{0.0567}{0.2}\right) = 0.1579 \frac{m^3}{kg}$

Interpolating in Table A-18 at 4 bar with v_2, $u_2 = 548.1$ kJ/kg

$$\therefore Q = (0.359 kg)(548.1 - 473.73)\frac{kJ}{kg} + (-26.8 kJ)$$

$$= -0.1 kJ$$

1. Alternatively,

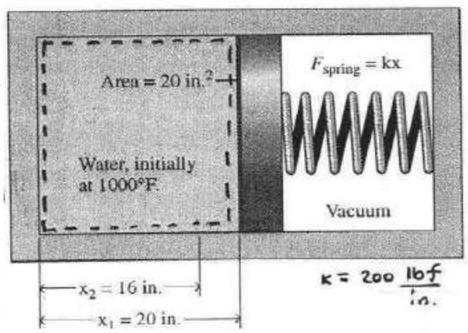
$$v_2 = v_1\left[\frac{p_1}{p_2}\right]^{\frac{1}{1.1}} = \left(0.5571 \frac{m^3}{kg}\right)\left(\frac{1}{4}\right)^{\frac{1}{1.1}} = 0.1579 m^3/kg$$

PROBLEM 3.84

KNOWN: Water contained within a piston-cylinder assembly fitted with a spring undergoes a process from state 1 to state 2.

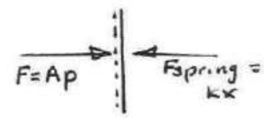
FIND: $P_1, P_2, m, W_{12}, Q_{12}$.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:
1. The water is the system.
2. Changes in kinetic and potential energy are absent. Friction between the piston and cylinder is negligible.
3. Cooling occurs slowly so no piston acceleration occurs.
4. Volume change is the only work mode.

ANALYSIS:
(a) To evaluate pressure consider a force balance at the piston face:



$\therefore \; Ap = kx \quad \text{or} \quad p = kx/A$

Then $P_1 = \frac{(200\,lbf/in.)(20\,in.)}{20\,in^2} = 200\,lbf/in^2$

$P_2 = \frac{(200\,lbf/in.)(16\,in.)}{20\,in^2} = 160\,lbf/in^2$ ← P_1, P_2

(b) To find the mass, m, write $m = V_1/v_1$, where v_1 is obtained from Table A-4E at $P_1 = 200\,lbf/in^2$, $T_1 = 1000°F$: $v_1 = 4.31\,ft^3/lb$. Thus,

$m = \frac{(20\,in^2)(20\,in)}{4.31\,ft^3/lb}\left|\left(\frac{1\,ft}{12\,in}\right)^3\right| = 0.054\,lb$ ← m

(c) $W_{12} = \int_{x_1}^{x_2} F\,dx = \int_{x_1}^{x_2} kx\,dx = \frac{k}{2}\left[x_2^2 - x_1^2\right] = \left(\frac{200\,lbf/in.}{2}\right)\left[(16\,in)^2 - (20\,in)^2\right]\left|\frac{1\,ft}{12\,in}\right|\left|\frac{1\,Btu}{778\,ft\cdot lbf}\right|$

$= -1.54\,Btu$ ← W_{12}

(d) $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{12} - W_{12} \Rightarrow Q_{12} = m(u_2 - u_1) + W_{12}$

From Table A-4E at $P_1 = 200\,lbf/in^2$, $T_1 = 1000°F$, $u_1 = 1369.8\,Btu/lb$.

State 2 is fixed by $P_2 = 160\,lbf/in^2$ and v_2, where $m = \frac{V_2}{v_2} = \frac{V_1}{v_1} \Rightarrow v_2 = \frac{V_2}{V_1} v_1$, or

$v_2 = \frac{(20\,in^2)(16\,in)}{(20\,in^2)(20\,in)}\left(4.31\,\frac{ft^3}{lb}\right) = 3.448\,\frac{ft^3}{lb}$. Interpolating in Table A-4E, $u_2 = 1171.98\,\frac{kJ}{kg}$.

Thus, $Q_{12} = 0.054\left[1171.98 - 1369.8\right]\frac{Btu}{lb} + (-1.54\,Btu)$

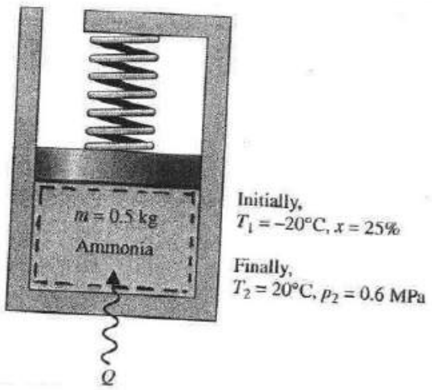
$= -12.22\,Btu$ ← Q_{12}

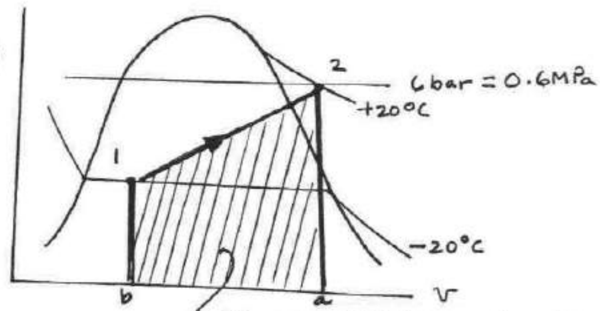
PROBLEM 3.85

KNOWN: Ammonia contained in a piston-cylinder assembly undergoes a process during which it is heated.

FIND: For the process, find Q and W.

SCHEMATIC & GIVEN DATA:





$W_{12} = m\left(\frac{P_1+P_2}{2}\right)(v_2 - v_1)$ = Area (1-2-a-b-1)

ENGINEERING MODEL:

1. The ammonia is the system.
2. Volume change is the only work mode. As shown by the p-v diagram, pressure varies linearly with volume.
3. There are no significant kinetic and potential energy effects.

ANALYSIS: Since volume change is the only work mode, $W_{12} = \int_1^2 p\,dV$, or $W_{12} = m\int_{v_1}^{v_2} p\,dv$. This can be evaluated geometrically as, $W_{12} = m\,P_{ave}(v_2 - v_1)$.

Thus,

$$W_{12} = m\left[\frac{P_1+P_2}{2}\right](v_2-v_1) = 0.5\,kg\left[\frac{1.9019+6}{2}\right]bar\left|\frac{10^5 N/m^2}{1\,bar}\right|(0.22155 - 0.157)\frac{m^3}{kg}\left|\frac{1\,kJ}{10^3 N\cdot m}\right| = 12.75\,kJ \qquad W_{12}$$

($P_1 = P_{sat}(T_1) = 1.9019$ bar)

Using x at the initial state, $v_1 = v_f + x(v_g - v_f) = \frac{1.5038}{10^3} + 0.25\left(0.6233 - \frac{1.5038}{10^3}\right) = 0.157\frac{m^3}{kg}$ (Data from Table A-13)

Then, from Table A-15, $v_2 = 0.22155\ m^3/kg$.

Writing an energy balance,

$$\Delta U + \cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0} = Q_{12} - W_{12} \Rightarrow Q_{12} = m(u_2 - u_1) + W_{12}$$

Using Table A-13 data, $u_1 = u_f + x(u_g - u_f) = 88.4 + 0.25(1299.23 - 88.40) = 391.11$ kJ/kg.

From Table A-15, $u_2 = 1347.62$ kJ/kg. Thus,

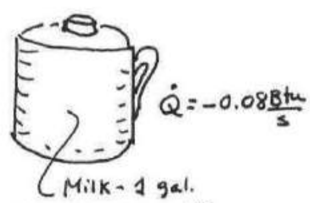
$$Q_{12} = 0.5\,kg\,(1347.62 - 391.11)\frac{kJ}{kg} + 12.75\,kJ$$
$$= 491.01\,kJ \qquad Q_{12}$$

PROBLEM 8.86

KNOWN: Data are provided for a gallon of milk placed in a refrigerator.

FIND: Determine the time, in min., for the milk to cool from 68°F to 40°F.

SCHEMATIC & GIVEN DATA:



$T_1 = 68°F$, $T_2 = 40°F$

$c = 0.94 \frac{Btu}{lb \cdot °R}$

$\rho = 64 \frac{lb}{ft^3}$

ENGR. MODEL:

1. The milk is the closed system.
2. For the system, $\dot{W} = 0$ and kinetic and potential energy play no role.
3. The milk is modeled as incompressible with constant specific heat, c.

ANALYSIS: An energy rate balance reduces as follows, $\frac{dU}{dt} + \cancel{\frac{dKE}{dt}} + \cancel{\frac{dPE}{dt}} = \dot{Q} - \cancel{\dot{W}}$,

or

$$\frac{dU}{dt} = \dot{Q} \Rightarrow \Delta U = \int \dot{Q}\, dt \Rightarrow m(u_2 - u_1) = \dot{Q}\Delta t.$$

In this expression, $m = \rho V = \left(64 \frac{lb}{ft^3}\right)(1\ gal.) \left| \frac{0.13368\ ft^3}{1\ gal.} \right| = 8.56\ lb$. Also, with Eq. 3.20a $(u_2 - u_1) = c(T_2 - T_1)$. Collecting results

$$\Delta t = \frac{mc(T_2 - T_1)}{\dot{Q}} = \frac{(8.56\ lb)\left(0.94 \frac{Btu}{lb \cdot °R}\right)(-28\ °R)}{(-0.08\ Btu/s)} \left| \frac{1\ min}{60\ s} \right|$$

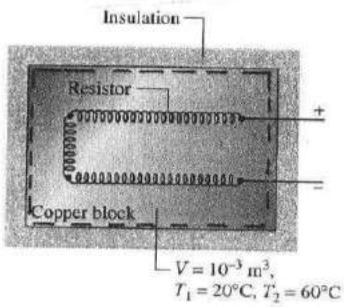
$$= 47\ min \quad \longleftarrow \Delta t$$

PROBLEM 3.87

KNOWN: Data are provided for an insulated copper block receiving energy from an embedded resistor.

FIND: Determine the time, in min., for the temperature of the block to increase to 60 °C from an initial temperature of 20 °C.

SCHEMATIC & GIVEN DATA:



$\dot{W} = -100\ W$

ENGR. MODEL:

1. The closed system is the copper block plus resistor. The mass and volume of the resistor are negligible.
2. For the system, $\dot{Q} = 0$ and kinetic and potential energy play no role.
3. The copper block is modeled as incompressible with constant specific heat, c.

ANALYSIS: An energy rate balance reduces as follows, $\frac{dU}{dt} + \cancel{\frac{dKE}{dt}} + \cancel{\frac{dPE}{dt}} = \cancel{\dot{Q}} - \dot{W}$

$\Rightarrow \frac{dU}{dt} = -\dot{W}$. Integrating, $\Delta U = -\int_0^t \dot{W}dt$. With Eq. 3.20a, $\Delta U = mc(T_2 - T_1)$.
Collecting results, $mc(T_2 - T_1) = -\int_0^t \dot{W}dt$. Since $\dot{W}$ is constant, $mc(T_2 - T_1) = -\dot{W}t$.

From Table A-19, for copper $\rho = 8930\ kg/m^3$, $c = 0.385\ kJ/kg \cdot K$. Finally

$$t = \frac{mc(T_2 - T_1)}{-\dot{W}} = \frac{(8930\ kg/m^3)(10^{-3}\ m^3)(0.385\ kJ/kg \cdot K)(40K)}{(100\ W)} \left|\frac{1\ W}{1\ J/s}\right| \left|\frac{10^3\ J}{1\ kJ}\right| \left|\frac{1\ min}{60\ s}\right|$$

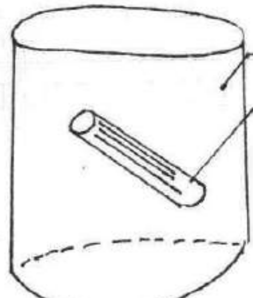
$$= 22.9\ min$$

PROBLEM 3.88

KNOWN: A metal part, initially at 1075K, is quenced in a closed tank filled with water. State data are provided.

FIND: Determine the final equilibrium temperature after quenching.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The closed system is the water and the metal part.
2. For the system, $Q = 0$, $W = 0$, and kinetic and potential energy play no role.
3. The water and metal part are each modeled as incompressible with constant specific heats: $c_m = 0.5$ kJ/kg·K, $c_w = 4.2$ kJ/kg·K.

ANALYSIS:

The energy balance reduces as follows:

$$\{\Delta U]_{water} + \Delta U]_{metal}\} + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - \cancel{W}$$

Using Eq. 3.20a, we get

$$m_w c_w [T_f - T_w] + m_m c_m [T_f - T_m] = 0$$

where T_f is the final equilibrium temperature.

Solving for T_f,

$$T_f = \frac{m_m c_m T_m + m_w c_w T_w}{m_m c_m + m_w c_w}$$

$$= \frac{(1\text{kg})(0.5\text{ kJ/kg}\cdot\text{K})(1075\text{K}) + (100\text{kg})(4.2\text{ kJ/kg}\cdot\text{K})(295)}{(1\text{kg})(0.5\text{ kJ/kg}\cdot\text{K}) + (100\text{kg})(4.2\text{ kJ/kg}\cdot\text{K})}$$

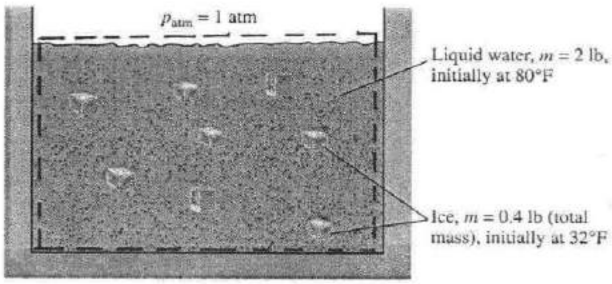
$$= 295.9\text{ K} \longleftarrow$$

PROBLEM 3.89

KNOWN: An open tank initially contains liquid water and ice. Eventually the ice melts. State data are provided.

FIND: Determine the final temperature, in °F, for the water in the tank.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The closed system is the liquid water and the ice.
2. For the system, heat transfer with the surroundings can be ignored. Kinetic and potential energy play no role.
3. The liquid water originally present and the ice melt are modeled as incompressible with constant specific heat.
4. Pressure remains constant throughout at 1 atm. The specific enthalpy change from solid to liquid at 32°F, 1 atm is 144 Btu/lb.

ANALYSIS: An energy balance reduces as follows:

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - W$$

$$W = \int p\,dV = p\Delta V$$

The tank is open to the atmosphere at p = 1 atm, and the volume of the tank's contents will change as the ice melts and as the resulting total amount of liquid interacts to a final equilibrium temperature.

$$\Rightarrow \quad \Delta U + p\Delta V = 0$$

Since pressure remains constant, this can be expressed as

$$\Delta H = 0 \qquad (1)$$

In evaluating Eq. (1), we recognize that the ice initially present melts, and the ice melt formed is raised in temperature as the temperature of the liquid initially present decreases. Eventually, all liquid achieves the final equilibrium temperature T_f. Accordingly, Eq. (1) can be expressed as

$$\left\{ [\Delta H]_{\substack{\text{ice to}\\ \text{liquid}\\ \text{at } 32°F}} + [\Delta H]_{\substack{\text{liquid:}\\ 32°F \text{ to}\\ T_f}} \right\} + \{\Delta H\}_{\substack{\text{original}\\ \text{liquid:}\\ 80°F \text{ to } T_f}} = 0 \qquad (2)$$

With assumption 4, c = 1 Btu/lb·°R for liquid water (from Table A-19E), and Eq. 3.20b, Eq. (2) becomes

$$\left[(0.4\text{ lb})\left(144\ \tfrac{\text{Btu}}{\text{lb}}\right) + (0.4\text{ lb})\left(1\ \tfrac{\text{Btu}}{\text{lb}\cdot°R}\right)(T_f - 492°R) \right] + \left[(2\text{ lb})\left(1\ \tfrac{\text{Btu}}{\text{lb}\cdot°R}\right)(T_f - 540°R) \right] = 0$$

Solving,

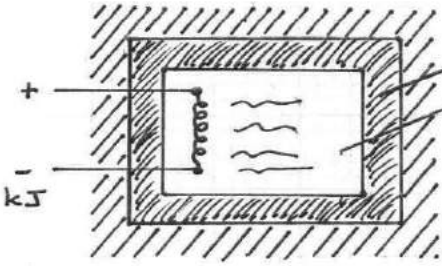
$$T_f = 508°R\ (48°F)$$

PROBLEM 3.90

KNOWN: An electric resistor transfers energy to water in a well-insulated copper tank.

FIND: Determine the final equilibrium temperature.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The copper tank and water are a closed system. (2) There is no heat transfer. (3) The copper and water behave as incompressible substances with constant specific heats. (4) Kinetic and potential energy effects are absent. (5) No energy is stored in the resistor: $\Delta U]_{resistor} = 0$.

ANALYSIS: The energy balance reduces as follows

$$\cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} + \Delta U = \cancelto{0}{Q} - W \quad \Rightarrow \quad \Delta U = -W$$

or $\cancelto{0}{\Delta U_{res}} + \Delta U_c + \Delta U_w = -W$. Then, with Eq. 3.20a, we get

$$m_c c_c (T_f - T_{ci}) + m_w c_w (T_f - T_{wi}) = -W$$

Solving for the final temperature, T_f

$$T_f = \frac{m_c c_c T_{ci} + m_w c_w T_{wi} - W}{m_c c_c + m_w c_w}$$

Using Table A-19, the specific heat of copper is $c_c = 0.385$ kJ/kg·K. And for water at 325 K, $c_w = 4.179$ kJ/kg·K. Thus

$$T_f = \frac{(13\text{ kg})(0.385\text{ kJ/kg·K})(27+273)\text{K} + (4)(4.179)(50+273) - (-100)}{(13\text{ kg})(0.385\text{ kJ/kg·K}) + (4)(4.179)}$$

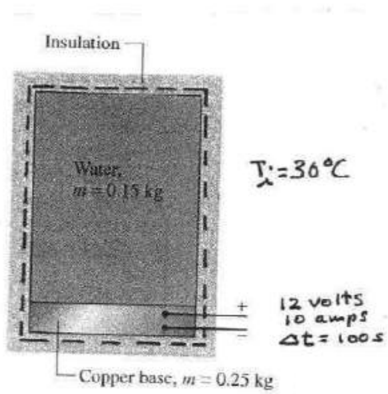
$$= 322.3\text{ K} = 49.3°C \longleftarrow T_f$$

PROBLEM 3.91

KNOWN: Data are provided for a closed, insulated tank containing liquid water. The tank has a copper base in which a heating element is embedded.

FIND: Determine the final temperature when equilibrium is reached.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The tank and its contents are the closed system.
2. For the system, $Q = 0$ and there are no effects of kinetic and potential energy.
3. The water and copper base are each modeled as incompressible with constant specific heat, c_w and c_{cu}, respectively.
4. The mass of the tank walls is negligible. The mass of the heating element also can be ignored.

ANALYSIS: An energy balance reduces to read $\cancel{\Delta KE + \Delta PE} + \Delta U = \cancel{Q} - W$, where $\Delta U = \Delta U]_{tank} + \Delta U]_{water} + \Delta U]_{copper}$. By assumption 4, $\Delta U]_{tank} = 0$. Also, with Eq. 3.20a, $\Delta U = mc\Delta T$. Collecting results

$$m_w c_w [T_f - T_i] + m_{cu} c_{cu} [T_f - T_i] = -W$$

Solving

$$T_f = T_i + \frac{(-W)}{(m_w c_w + m_{cu} c_{cu})} \qquad (1)$$

with Eq. 2.21,

$$W = -\mathcal{E} i \,\Delta t$$
$$= -(12\,V)(10\,amp)(100\,s)\left|\frac{1\,Watt/amp}{1\,Volt}\right|\left|\frac{1\,J/s}{1\,watt}\right|\left|\frac{1\,kJ}{10^3\,J}\right|$$
$$= -12\,kJ$$

From Table A-19, $c_{cu} = 0.385$ kJ/kg·K, $c_w = 4.18$ kJ/kg·K.

Inserting values, Eq. (1) gives

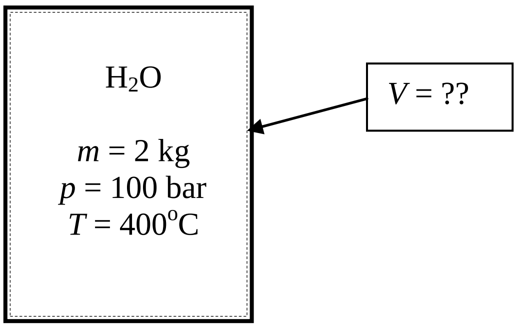
$$T_f = 303.15\,K + \frac{12\,kJ}{(0.15\,kg)\left(4.18\,\frac{kJ}{kg\cdot K}\right) + (0.25\,kg)\left(0.385\,\frac{kJ}{kg\cdot K}\right)}$$

$$= 319.74\,K \quad (46.6°C)$$

PROBLEM 3.92

Determine the volume, in m^3, occupied by 2 kg of H_2O at 100 bar, 400^{o}C, using (a) data from the compressibility chart, (b) data from the steam tables.

Compare the results of parts (a) and (b) and discuss.



(a) Using the compressibility chart, first we need to determine the reduced pressure and temperature. From Table A-1:

$$p_c = 220.9 \text{ bar} \quad \text{and} \quad T_c = 647.3 \text{ K}$$

$$\left.\begin{aligned} p_R &= p/p_c = (100)/(220.9) = 0.45 \\ T_R &= T/T_c = (400 + 273.15)/(647.3) = 1.04 \end{aligned}\right\} \rightarrow \text{(Figure A-1): } z \approx 0.86$$

Now, we can calculate the specific volume as follows.

$$v = Z\,\frac{\left(\frac{\bar{R}}{M}\right)T}{p} = (0.86)\left[\frac{\left(\frac{8.314}{18.02}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(673.15\text{ K})}{(100\text{ bar})}\right]\left|\frac{1\text{ bar}}{10^5\text{ N/m}^2}\right|\left|\frac{10^3\text{N}\cdot\text{m}}{1\text{ kJ}}\right| = 0.0267 \text{ m}^3\text{/kg}$$

So, the volume is: $V = m\,v = (2)(0.0267) = 0.0534$ m^3 ⟵

(b) From Table A-4 at 100 bar, 400^{o}C; $v = 0.02641$ m^3/kg

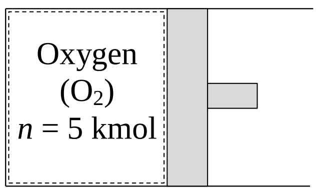
Thus, $V = m\,v = (2)(0.02641) = 0.05282$ m^3 ⟵

Comments: The compressibility chart gives a fairly accurate value considering the relative imprecision of reading values from the chart. The percent difference is approximately 1.1%.

Note also that the value of z is 0.86. Hence, the ideal gas model is not particularly applicable at this state. The ideal gas model would predict a volume of 0.03105 m^3, which is about 15% low.

PROBLEM 3.93

Five kmol of oxygen (O_2) gas undergoes a process in a closed system from p_1 = 50 bar, T_1 = 170 K to p_2 = 25 bar, T_2 = 200 K. Determine the change in volume, in m^3.



p_1 = 50 bar, T_1 = 170 K $\longrightarrow$ P_2 = 25 bar, T_2 = 200 K

ΔV = ?

ANALYSIS: Find the volumes at each state using $V = Zn\bar{R}T/p$, with data for Z from the compressibility chart.

From Table A-1 for oxygen (O_2): p_c = 50.5 bar, T_c = 154 K.

State 1: $p_{R1} = p_1/p_c = (50)/(50.5) \approx 1$ and $T_{R1} = T_1/T_c = (170)/(154) \approx 1.1$

From Figure A-1: $Z \approx 0.71$

Thus

$$V_1 = \frac{(0.71)(5\text{ kmol})\left(8.314\,\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(170\text{ K})}{(50\text{ bar})}\left|\frac{1\text{ bar}}{10^5\,\text{N/m}^2}\right|\left|\frac{10^3\,\text{N}\cdot\text{m}}{1\text{ kJ}}\right| = 1.00\text{ m}^3$$

State 2: $p_{R2} = p_2/p_c = (25)/(50.5) \approx 0.5$ and $T_{R1} = T_1/T_c = (200)/(154) \approx 1.13$

From Figure A-1: $Z \approx 0.93$

Thus

$$V_2 = \frac{(0.93)(5\text{ kmol})\left(8.314\,\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(200\text{ K})}{(25\text{ bar})}\left|\frac{1\text{ bar}}{10^5\,\text{N/m}^2}\right|\left|\frac{10^3\,\text{N}\cdot\text{m}}{1\text{ kJ}}\right| = 3.09\text{ m}^3$$

Finally

$$\Delta V = 3.09 - 1.00 = 2.09\text{ m}^3 \longleftarrow$$

Note that z ranges from 0.71 to 0.93. Hence, the ideal gas model would not be particularly suitable for this calculation.

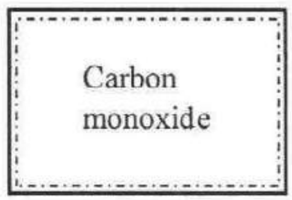
PROBLEM 3.94

Known: Carbon monoxide (CO) with known properties occupies a volume.

Find: Determine the volume, in ft^3.

Schematic and Given Data:

$V = ?\ ft^3$
$T = 500°R$
$p = 3500\ lbf/in.^2$
$m = 150\ lb$



Analysis:

Determine volume as follows

$$Z = \frac{pv}{RT} = \frac{pV}{mRT}$$

$$V = \frac{ZmRT}{p} \qquad (1)$$

Determine Z using Figure A-2 and data from Table A-1E for CO ($p_c = 34.5$ atm, $T_c = 239°R$, $M = 28.01$ lb/lbmol), as follows:

$$p_R = \frac{p}{p_c} = \frac{3500\frac{\text{lbf}}{\text{in.}^2}}{34.5\ \text{atm}}\left|\frac{1\ \text{atm}}{14.7\frac{\text{lbf}}{\text{in.}^2}}\right| = 6.9$$

$$T_R = \frac{T}{T_c} = \frac{500°\text{R}}{239°\text{R}} = 2.1$$

$$Z \approx 1.05$$

Substitute into Eq. (1):

$$V = \frac{ZmRT}{p} = \frac{(1.05)(150\ \text{lb})\left(\dfrac{1545\frac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot°\text{R}}}{28.01\frac{\text{lb}}{\text{lbmol}}}\right)(500°\text{R})}{3500\frac{\text{lbf}}{\text{in.}^2}\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right|} = 8.6\ \text{ft}^3$$

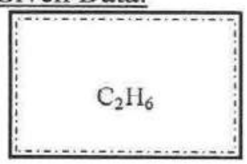
PROBLEM 3.95

Known: Ethane (C_2H_6) with known properties occupies a volume.

Find: Determine the temperature, in °F.

Schematic and Given Data:

$v = 0.4$ ft^3/lb
$T = ?$ °F
$p = 500$ lbf/in.^{2}



Analysis:

Determine temperature as follows:

$$Z = \frac{pv}{RT} \quad \text{or} \quad T = \frac{pv}{ZR} \qquad (1)$$

Determine Z using Figure A-1 and data from Table A-1E for C_2H_6 ($p_c = 48.2$ atm, $T_c =$ 549°R, $M = 30.07$ lb/lbmol), as follows:

$$p_R = \frac{p}{p_c} = \frac{500\,\frac{\text{lbf}}{\text{in.}^2}}{48.2\text{ atm}}\left|\frac{1\text{ atm}}{14.7\,\frac{\text{lbf}}{\text{in.}^2}}\right| = 0.71$$

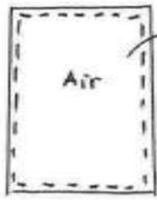
$$v'_R = \frac{vp_c}{RT_c} = \frac{\left(0.4\,\frac{\text{ft}^3}{\text{lb}}\right)48.2\text{ atm}\left|\frac{14.7\,\frac{\text{lbf}}{\text{in.}^2}}{1\text{ atm}}\right|\left|\frac{144\text{ in.}^2}{1\text{ ft}^2}\right|}{\left(\frac{1545\,\frac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot{}^\circ\text{R}}}{30.07\,\frac{\text{lb}}{\text{lbmol}}}\right)549^\circ\text{R}} = 1.45$$

$Z \approx 0.86$

Substitute into Eq. (1) to determine T, in °F.

$$T = \frac{pv}{ZR} = \frac{500\,\frac{\text{lbf}}{\text{in.}^2}\left|\frac{144\text{ in.}^2}{1\text{ ft}^2}\right|\left(0.4\,\frac{\text{ft}^3}{\text{lb}}\right)}{(0.86)\left(\frac{1545\,\frac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot{}^\circ\text{R}}}{30.07\,\frac{\text{lb}}{\text{lbmol}}}\right)} = 652^\circ\text{R} = 192^\circ\text{F}$$

PROBLEM 3.96



$V = 2 m^3$
$T = -93°C$
$P(gage) = 1.4 MPa$
$P_{atm} = 1 atm$

Determine Z:

$P = \underbrace{1.01325 bar}_{P_{atm}} + \underbrace{14 bar}_{P(gage)}$
$= 15.01325 bar$

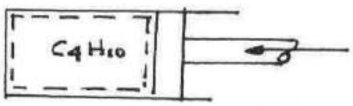
$P_R = \frac{P}{P_c} = \frac{15.01325}{37.7} = 0.40$

$T_R = \frac{180 K}{133 K} = 1.35$

Fig A-1 $\quad Z \sim 0.955$

$$m = \frac{p\bar{V}}{ZRT} = \frac{(15.01325 \times 10^5 N/m^2)(2 m^3)}{0.955\left(\frac{8314}{28.97} \frac{N \cdot m}{kg \cdot K}\right) 180 K} = 60.9 kg \longleftarrow$$

PROBLEM 3.97



$T_1 = 173°C = 446 K$

$P_1 = 1.9$ MPa

$P_2 = 2.5$ MPa

Compressibility chart data:

$$T_R = \frac{446\ K}{425\ K} = 1.05$$

$$P_{R1} = \frac{19\ bar}{38\ bar} = 0.5$$

$$P_{R2} = \frac{25\ bar}{38\ bar} = 0.66$$

From Fig. A-1,

$Z_1 \approx 0.835$

$Z_2 \approx 0.775$

The variation of Z with P_R for $T_R = 1.05$ is closely linear. Thus,

$Z_{ave} \approx 0.81$

$$W = \int_1^2 p\, dV$$

Using $p = m\left(\frac{ZRT}{V}\right)$,

$$\frac{W}{m} = RT\int_1^2 \frac{Z}{V}\, dV$$

$$= RT\int_1^2 Z\, d\ln V$$

$$\frac{W}{m} = RT Z_{ave} \ln\left[\frac{V_2}{V_1}\right]$$

Noting that $V = ZmRT/P$,

$V_2 = Z_2\, mRT/P_2$

$V_1 = Z_1\, mRT/P_1$

Collecting results

$$\frac{W}{m} = RT Z_{ave} \ln\left[\frac{Z_2}{Z_1}\cdot\frac{P_1}{P_2}\right]$$

Inserting values,

$$\frac{W}{m} = \left(\frac{8.314}{58.12}\frac{kJ}{kg\cdot K}\right)(446 K)(0.81)\ln\left[\frac{0.775}{0.835}\times\frac{1.9}{2.5}\right]$$

$$= -18\ \frac{kJ}{kg} \longleftarrow W/m$$

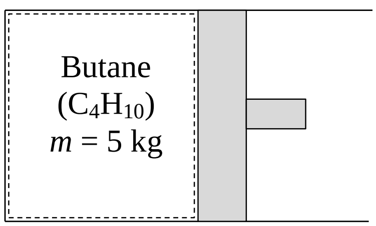
PROBLEM 3.98

Five kg of butane (C_4H_{10}) in a piston-cylinder assembly undergo a process from p_1 = 5 MPa, T_1 = 500 K to p_2 = 3 MPa during which the relationship between pressure and specific volume is pv = *constant*. Determine the work, in kJ.

KNOWN: Five kg of C_4H_{10} undergo a process for which pv = *constant* in a piston-cylinder assembly between two states.

FIND: Determine the work.

SCHEMATIC AND GIVEN DATA:



p_1 = 5 MPa = 50 bar
T_1 = 500 K

$\longrightarrow$ pv = *constant*

p_1 = 3 MPa = 30 bar

ENGINEERING MODEL: 1. The Butane is a closed system. 2. The process is polytropic with pv = *constant*. 3. Volume change is the only work mode.

ANALYSIS: The work is given by $W = m\int_{v_1}^{v_2} p\,dv$. With pv = *constant* = p_1v_1

$$W = m\int_{v_1}^{v_2} \frac{(p_1v_1)}{v}\,dv = (p_1v_1)\, m \ln\left(\frac{v_2}{v_1}\right) \qquad (*)$$

To evaluate W requires v_1 and v_2. The compressibility chart can be used to obtain v_1: From Table A-1; p_c = 38 bar, T_c = 425 K, M = 58.12 kg/kmol.

$$\left.\begin{array}{l} p_{R1} = p_1/p_c = (50)/(38) = 1.32 \\ \\ T_{R1} = T/T_c = (500)/(425) = 1.18 \end{array}\right\} \quad \text{Fig. A-2: } Z_1 \approx 0.67$$

Accordingly, with $v_1 = Z_1\,(RT_1/p_1)$, we get

$$v_1 = (0.67)\frac{\left(\frac{8.314\ \text{kJ}}{58.12\text{kg}\cdot\text{K}}\right)(500\ \text{K})}{(5\ \text{MPa})}\left|\frac{1\ \text{MPa}}{10^6\text{N/m}^2}\right|\left|\frac{10^3\text{N}\cdot\text{m}}{1\ \text{kJ}}\right| = 0.0096\ \ \text{m}^3/\text{kg}$$

Now, with pv = *constant*,

$$v_2 = (p_1v_1)/p_2 = [(5\ \text{MPa})(0.0096\ \text{m}^3)]/(3\ \text{MPa}) = 0.016\ \text{m}^3$$

Now, inserting values into (*), we get

PROBLEM 3.98 (CONTINUED) – PAGE 2

$$W = (5\ \text{MPa})(0.0096\ \text{m}^3/\text{kg})(5\ \text{kg})\ \ln[(0.016)/(0.0096)]\left|\frac{10^6\,\text{N/m}^2}{1\ \text{MPa}}\right|\left|\frac{1\ \text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right|$$

$$= 122.6\ \text{kJ (out)} \longleftarrow$$

Alternative Evaluation of v_1

$$\left.\begin{aligned} p_{R1} &= p_1/p_c = (50)/(38) = 1.32 \\ T_{R1} &= T/T_c = (500)/(425) = 1.18 \end{aligned}\right\} \quad \text{Fig. A-2: } v'_{R1} \approx 0.6$$

$$v_1 = v'_{R1}\left(\frac{RT_c}{p_c}\right) = (0.6)\left[\frac{\left(\frac{8.314\ \text{kJ}}{58.12\,\text{kg}\cdot\text{K}}\right)(425\ \text{K})}{(38\ \text{bar})}\right]\left|\frac{1\ \text{bar}}{10^5\,\text{N}/m^2}\right|\left|\frac{10^3\,\text{N}\cdot\text{m}}{1\ \text{kJ}}\right| = 0.0096\ \text{m}^3/\text{g}$$

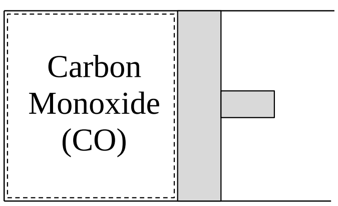
PROBLEM 3.99

In a cryogenic application, carbon monoxide (CO) gas undergoes a constant pressure process at 1000 lbf/in.^{2} in a piston-cylinder assembly from T_1 = -100^{o}F to T_2 = -30^{o}F. Determine the work for the process, in Btu per lb of carbon monoxide present.

KNOWN: Carbon monoxide undergoes a constant pressure process in a piston-cylinder assembly. Temperatures are known at the initial and final states.

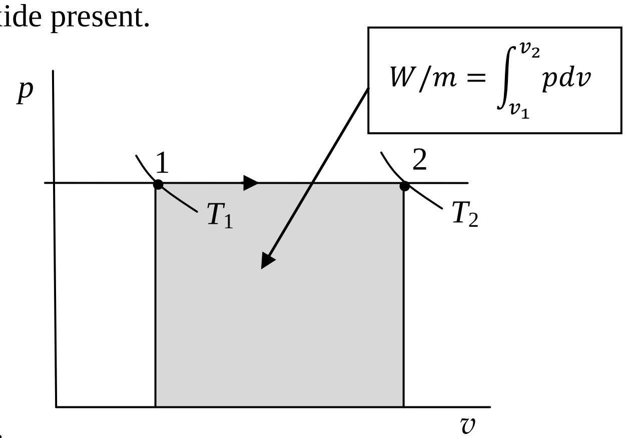
FIND: Determine the work per unit mass of carbon monoxide present.

SCHMEATIC AND GIVEN DATA:



T_1 = -100^{o}F = 360^{o}R
T_2 = -30^{o}F = 430^{o}R

p = 1000 lbf/in.^{2} = 68.0 atm



ENGINEERING MODEL: 1. The CO is a closed system.
2. The process is at constant pressure and volume change is the only work mode.

ANALYSIS: To evaluate the work, we note that $W/m = \int_{v_1}^{v_2} pdv = p(v_2 - v_1)$. We need to determine the specific volumes at the initial and final states.

From Table A-1E, for carbon monoxide: T_c = 239^{o}R, p_c = 34.5 atm and M = 28.01 lb/lbmol. Thus, using data from Figure A-1:

State 1 $\quad \left.\begin{array}{l} T_{R1} = (360)/(239) = 1.5 \\ p_{R1} = (68.0)/(34.5) = 1.97 \approx 2 \end{array}\right\} \quad Z_1 \approx 0.84$

State 2 $\quad \left.\begin{array}{l} T_{R2} = (430)/(239) = 1.8 \\ p_{R2} = (68.0)/(34.5) = 1.97 \approx 2 \end{array}\right\} \quad Z_1 \approx 0.94$

Now, the specific volumes are

$$v_1 = Z_1RT_1/p_1 = \frac{(0.84)\left(\frac{1545\ \text{ft}\cdot\text{lbf}}{28.01\text{lb}\cdot{}^{\text{o}}\text{R}}\right)(360^{\text{o}}\ \text{R})}{(1000\ \text{lbf/in}^2)}\left|\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right| = 11.58\ \text{ft}^3/\text{lb}$$

and

$$v_2 = Z_2RT_2/p_2 = \frac{(0.94)\left(\frac{1545\ \text{ft}\cdot\text{lbf}}{28.01\text{lb}\cdot{}^{\text{o}}\text{R}}\right)(430\ {}^{\text{o}}\text{R})}{(1000\ \text{lbf/in}^2)}\left|\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right| = 13.84\ \text{ft}^3/\text{lb}$$

PROBLEM 3.99 (CONTINUED) – PAGE 2

The work per unit mass is

$$W/m = p(v_2 - v_1) = (1000\ \text{lbf/in.}^2)(13.84 - 11.58)\frac{\text{ft}^3}{\text{lb}}\left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

1

$$= 418.3\ \text{Btu/lb (out)}$$

1. Note that if the ideal gas model (Z=1) is used, the specific volumes become v_1 = 13.79 ft^3/lb and v_2 = 14.72 ft^3/lb, respectively. The work is 172.1 Btu/lb, which is approximately a 59% difference. Thus, assuming the ideal gas model gives substantial error in this case.

PROBLEM 3.100

For what ranges of pressure and temperature can air be considered an ideal gas? Explain your reasoning. Repeat for H_2O.

The ideal gas equation is accurate for ranges of pressure and temperature for which $Z \approx 1$. We might arbitrarily consider the ideal gas model to be satisfactory if $0.96 \le Z \le 1.05$ (5% deviation). These are illustrated on the compressibility chart (Fig. A-2)below.

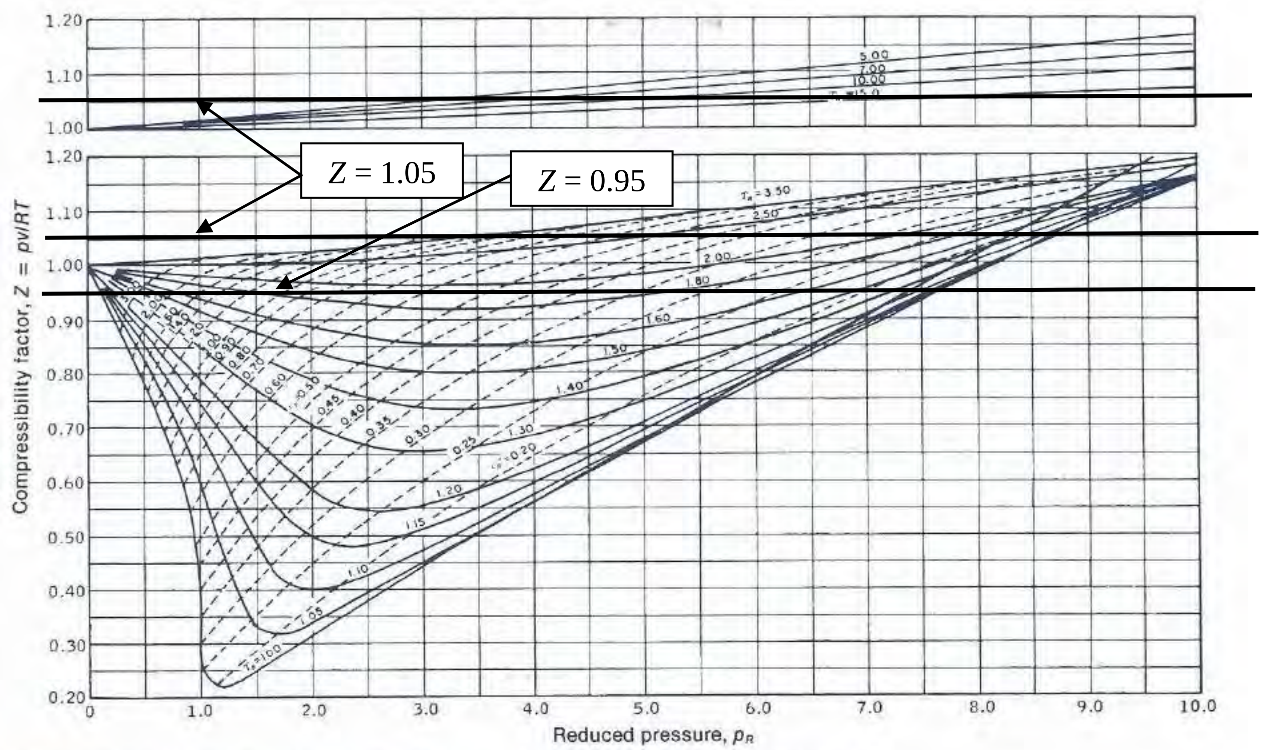


Figure A-2 Generalized compressibility chart, $p_R \le 10.0$. *Source:* **E. F. Obert, *Concepts of Thermodynamics*, McGraw-Hill, New York, 1960.**

We can use these envelopes to determine limits of p_R and T_R for which the ideal gas model is satisfactory.

- $p_R < 0.1$; Z is in the acceptable range for all temperatures
- for $T_R = 2.0$; $p_R < 7.2$
- for $T_R = 2.5$; $p_R < 5.0$
- for $T_R = 3.5$; $p_R < 3.5$
- for $T_R > 5.0$; $p_R < 3.2$
- for $T_R > 15$; $p_R < 7.0$

Since these conclusions are developed using the generalized compressibility chart, they apply for any substance. Now, we can look specifically at air and water.

<u>PROBLEM 3.100 (Continued)</u>

Air
Table A-1 gives: T_c = 133 K and p_c = 37.7 bar. Thus

- p < 3.77 bar; Z is in the acceptable range for all temperatures
- for T = 266 K; p < 271 bar
- for T = 333 K; p < 189 bar
- for T = 466 K; p < 132 bar
- for T > 665 K; p < 121 bar
- for T > 1995 K; p < 264 bar

Water
Table A-1 gives: T_c = 647.3 K and p_c = 220.9 bar. Thus

- p < 2.209 bar; Z is in the acceptable range for all temperatures
- for T = 1295 K; p < 1591 bar
- for T = 1618 K; p < 1145 bar
- for T = 2666 K; p < 773 bar
- for T > 3237 K; p < 1546 bar

PROBLEM 3.101

KNOWN: Nitrogen at specified volume, temperature, and pressure.

FIND: Mass of nitrogen using ideal gas model and compressibility chart and comment on applicability of ideal gas model for nitrogen at this state.

SCHEMATIC AND GIVEN DATA:

$V = 0.5\ \text{m}^3$
$T = -71.4^\circ\text{C}$
$P = 1356\ \text{kPa}$

ANALYSIS:
(a) Using the ideal gas model.

$$pV = mRT$$

Solving for mass yields

$$m = \frac{pV}{RT}$$

Values for pressure, volume, and temperature are given. Temperature must be converted to absolute scale: $T = -71^\circ\text{C} = 202\ \text{K}$. The gas constant is computed from the universal gas constant and molecular weight of nitrogen

$$R = \frac{\overline{R}}{M} = \frac{8.314 \dfrac{\text{kJ}}{\text{kmol}\cdot\text{K}}}{28.01 \dfrac{\text{kg}}{\text{kmol}}} = 0.2968 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

Substituting values into the ideal gas equation of state and applying appropriate conversion factors yield

$$m = \frac{(1356\ \text{kPa})(0.5\ \text{m}^3)}{\left(0.2968 \dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(202\ \text{K})} \left| \frac{10^3 \dfrac{\text{N}}{\text{m}^2}}{1\ \text{kPa}} \right| \left| \frac{1\text{kJ}}{10^3\ \text{N}\cdot\text{m}} \right| = \underline{\mathbf{11.31\ kg}} \longleftarrow$$

(b) Mass can be determined by dividing the volume of nitrogen by its specific volume

$$m = V/v$$

Volume, V, is given as $0.5\ \text{m}^3$. To determine specific volume, apply the definition of compressibility factor

$$Z = \frac{pv}{RT}$$

and solve for specific volume

$$v = \frac{ZRT}{p}$$

PROBLEM 3.102

Determine the percent error in using the ideal gas model to determine the specific volume of

(a) water vapor at 4000 lbf/in.^{2}, 1000°F.
(b) water vapor at 5 lbf/in.^{2}, 250°F.
(c) ammonia at 40 lbf/in.^{2}, 60°F.
(d) air at 1 atm, 560°R.
(e) Refrigerant 134a at 300 lbf/in.^{2}, 180°F.

Solution:

Analysis:

(a) Water vapor at 4000 lbf/in.^{2}, 1000°F
From Table A-4E: $v_{\text{table}} = 0.1752\ \text{ft}^3/\text{lb}$
Using ideal gas model

$$v_{\text{ideal gas}} = \frac{RT}{p} = \frac{\left(\dfrac{1545\dfrac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot{}^\circ\text{R}}}{18.02\dfrac{\text{lb}}{\text{lbmol}}}\right)1460^\circ\text{R}}{4000\dfrac{\text{lbf}}{\text{in.}^2}\left|\dfrac{144\ \text{in.}^2}{1\ \text{ft}^2}\right|} = 0.2173\frac{\text{ft}^3}{\text{lb}}$$

The percent error follows

$$\%\ \text{error} = \frac{v_{\text{ideal gas}} - v_{\text{table}}}{v_{\text{table}}}\times 100 = \frac{0.2173 - 0.1752}{0.1752}\times 100 = 24\% \quad \longleftarrow$$

(b) Water vapor at 5 lbf/in.^{2}, 250°F
From Table A-4E: $v_{\text{table}} = 84.21\ \text{ft}^3/\text{lb}$
Using ideal gas model

$$v_{\text{ideal gas}} = \frac{RT}{p} = \frac{\left(\dfrac{1545}{18.02}\dfrac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)710^\circ\text{R}}{5\dfrac{\text{lbf}}{\text{in.}^2}\left|\dfrac{144\ \text{in.}^2}{1\ \text{ft}^2}\right|} = 84.55\frac{\text{ft}^3}{\text{lb}}$$

The percent error follows

$$\%\ \text{error} = \frac{v_{\text{ideal gas}} - v_{\text{table}}}{v_{\text{table}}}\times 100 = \frac{84.55 - 84.21}{84.21}\times 100 = 0.40\% \quad \longleftarrow$$

PROBLEM 3.102

(c) Ammonia at 40 lbf/in.^{2}, 60°F
From Table A-15E: $v_{table} = 7.9134\ \mathrm{ft^3/lb}$
Using ideal gas model

$$v_{\text{ideal gas}} = \frac{RT}{p} = \frac{\left(\frac{1545}{17.03}\frac{\mathrm{ft\cdot lbf}}{\mathrm{lb\cdot{}^\circ R}}\right)520^\circ\mathrm{R}}{40\,\frac{\mathrm{lbf}}{\mathrm{in.^2}}\left|\frac{144\ \mathrm{in.^2}}{1\ \mathrm{ft^2}}\right|} = 8.19\frac{\mathrm{ft^3}}{\mathrm{lb}}$$

The percent error follows

$$\%\ \text{error} = \frac{v_{\text{ideal gas}} - v_{\text{table}}}{v_{\text{table}}}\times 100 = \frac{8.19-7.9134}{7.9134}\times 100 = 3.5\%$$

(d) Air at 1 atm, 560°R
Using Table A-1E and Figure A-2:

$$p_R = \frac{p}{p_c} = \frac{1\ \mathrm{atm}}{37.2\ \mathrm{atm}} = 0.027$$

$$T_R = \frac{T}{T_c} = \frac{560\ ^\circ\mathrm{R}}{239\ ^\circ\mathrm{R}} = 2.34$$

$$Z \approx 1.0$$

At this state air is closely modeled as an ideal gas.

(e) Refrigerant 134a at 300 lbf/in.^{2}, 180°F
From Table A-12E: $v_{table} = 0.1633\ \mathrm{ft^3/lb}$
Using ideal gas model

$$v_{\text{ideal gas}} = \frac{RT}{p} = \frac{\left(\frac{1545}{102.03}\frac{\mathrm{ft\cdot lbf}}{\mathrm{lb\cdot{}^\circ R}}\right)640^\circ\mathrm{R}}{300\,\frac{\mathrm{lbf}}{\mathrm{in.^2}}\left|\frac{144\ \mathrm{in.^2}}{1\ \mathrm{ft^2}}\right|} = 0.2243\frac{\mathrm{ft^3}}{\mathrm{lb}}$$

The percent error follows

$$\%\ \text{error} = \frac{v_{\text{ideal gas}} - v_{\text{table}}}{v_{\text{table}}}\times 100 = \frac{0.2243-0.1633}{0.1633}\times 100 = 37.4\%$$

PROBLEM 3.103

Check the applicability of the ideal gas model for
(a) for water at 600°F and pressures of 900 lbf/in.^{2} and 100 lbf/in.^{2}.
(b) for nitrogen at -20°C and pressures of 75 bar and 1 bar.

Check the applicability of the ideal gas model for

(a) Water at 600°F (1060°R), p_1 = 900 lbf/in.^{2}, p_2 = 100 lbf/in.^{2}
Method 1. Use Steam Table data.

For T = 600°F (1060°R), p_1 = 900 lbf/in.^{2}; Table A-4E gives v_1 = 0.587 ft^3/lb.
Using the ideal gas equation of state

$$v_1 = \frac{(\bar{R}/M)T}{p_1} = \frac{\left(\frac{1545\ \text{ft·lbf}}{18.02\ \text{lb·R}}\right)(1060^{\circ}\text{R})}{900\ \text{lbf/in}^2}\left|\frac{1\ \text{ft}^2}{144\ \text{in.}^2}\right| = 0.701\ \text{ft}^3/\text{lb}$$

In this case, the ideal gas model is *not* applicable.

For T = 600°F (1060°R), p_1 = 100 lbf/in.^{2}; Table A-4E gives v_2 = 6.216 ft^3/lb.
Using the ideal gas equation of state

$$v_2 = \frac{(\bar{R}/M)T}{p_2} = \frac{\left(\frac{1545\ \text{ft·lbf}}{18.02\ \text{lb·R}}\right)(1060^{\circ}\text{R})}{100\ \text{lbf/in}^2}\left|\frac{1\ \text{ft}^2}{144\ \text{in.}^2}\right| = 6.311\ \text{ft}^3/\text{lb}$$

The ideal gas value is about 1.5% higher than the steam table value. For many applications, this would be acceptable.

Method 2. Use the compressibility chart, Figure A-1. From Table A-1E: T_c = 1165°R, p_c = 218.0 atm.

$$T_R = T/T_c = (1060)/(1165) \approx 0.91$$

$$p_{R1} = p_1/p_c = \frac{(900\frac{\text{lbf}}{\text{in}^2})}{(218.0\ \text{atm})}\left|\frac{1\ \text{atm}}{14.7\ \text{lbf/in}^2}\right| = 0.28 \quad\longrightarrow\quad Z_1 \approx 0.87\ (\textit{not}\ \text{applicable})$$

$$p_{R2} = p_2/p_c = \frac{(100\frac{\text{lbf}}{\text{in}^2})}{(218.0\ \text{atm})}\left|\frac{1\ \text{atm}}{14.7\ \text{lbf/in}^2}\right| = 0.03 \quad\longrightarrow\quad Z_1 \approx 0.98\ (\text{acceptable})$$

PROBLEM 3.103 (CONTINUED)

(b) Nitrogen (N_2) at -20°C and p_1 = 75 bar, p_2 = 1 bar

Use the compressibility chart. From Table A-1: T_c = 227 K, p_c = 33.5 bar.

$$T_R = T/T_c = (273\text{-}20)/(227) \approx 1.1$$

$$p_{R1} = p_1/p_c = \frac{(75 \text{ bar})}{(33.5 \text{ bar})} = 2.24 \quad \longrightarrow \quad Z_1 \approx 0.42 \text{ (\textit{not} applicable)}$$

$$p_{R2} = p_2/p_c = \frac{(1 \text{ bar})}{(33.5 \text{ bar})} = 0.03 \quad \longrightarrow \quad Z_1 \approx 1 \text{ (acceptable)}$$

Problem 3.104

$T = 100\,°C$

$p = 16\,bar$

(a) Table A-12.

$v = 0.01601\ m^3/kg \longleftarrow$

(b) Figure A-1.

$T_R = \dfrac{T}{T_c} = \dfrac{373}{374} = 1.0$ ← Table A-1, $\quad P_R = \dfrac{P}{P_c} = \dfrac{16}{40.7} = 0.393$ ← Table A-1

$\Rightarrow\ Z \cong 0.86\ ,\quad Z = \dfrac{pv}{RT}\ \Rightarrow\ v = \dfrac{ZRT}{p} = 0.86\left[\dfrac{8314}{102.03}\dfrac{N\cdot m}{kg\cdot K}\right]\left[\dfrac{373 K}{16\times10^5 N/m^2}\right]$

$R = \bar{R}/M$

$= 0.01634\ m^3/kg \longleftarrow$

This is about 2% greater than the table value.

(c) Ideal gas model $\quad pv = RT \Rightarrow v = RT/p$.

$v = \left[\dfrac{8314}{102.03}\dfrac{N\cdot m}{kg\cdot K}\right]\left[\dfrac{373 K}{16\times10^5 N/m^2}\right] = 0.019\ m^3/kg \longleftarrow$

This is about 19% greater than the table value.

PROBLEM 3.105

$T = 50°C$
$p = 10$ bar

(a) Table A-15.

$v = 0.14499\ m^3/kg$ ←

(b) Figure A-1

$$T_R = \frac{T}{T_c} = \frac{323}{406} = 0.8, \quad p_R = \frac{p}{p_c} = \frac{10\ bar}{112.8\ bar} = 0.09$$

Table A-1   Table A-1

$$\Rightarrow Z \cong 0.93, \quad Z = \frac{pv}{RT} \Rightarrow \quad v = \frac{ZRT}{p} = 0.93\left[\frac{8314}{17.03}\ \frac{N\cdot m}{kg\cdot K}\right]\left[\frac{323\ K}{10\times 10^5\ N/m^2}\right]$$

$(= \bar{R}/M)$

$$= 0.14665\ m^3/kg \leftarrow$$

This is about 1% greater than the table value.

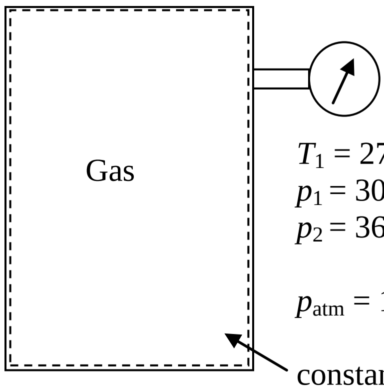
(c) Ideal gas model $\quad pv = RT \Rightarrow v = RT/p$

$$v = \left[\frac{8314}{17.03}\ \frac{N\cdot m}{kg\cdot K}\right]\left[\frac{323 K}{10\times 10^5\ N/m^2}\right] = 0.15769\ m^3/kg \leftarrow$$

This is about 9% greater than the table value.

PROBLEM 3.106

A closed, rigid tank is filled with a gas modeled as an ideal gas, initially at 27°C and a gage pressure of 300 kPa. The gas is heated, and the gage pressure at the final state is 367 kPa. Determine the final temperature, in °C. The local atmospheric pressure is 1 atm.



T_1 = 27°C
p_1 = 300 kPa (gage)
p_2 = 367 kPa (gage)

p_{atm} = 1 atm = 101.325 kPa

constant volume, fixed mass

Using the ideal gas equation of state

$$p_1V = mRT_1 \text{ and } p_2V = mRT_2 \longrightarrow p_2/p_1 = T_2/T_1 \text{ or } T_2 = (p_2/p_1)T_1 \qquad (*)$$

In this expression, the temperatures must be in K and the pressures must be on an absolute basis. Thus, T_1 = 27 + 273.15 = 300.15 K, and the absolute pressures are

$$p_1 = p_{atm} + p_1(\text{gage}) = 101.325 \text{ kPa} + 300 \text{ kPa} = 401.3 \text{ kPa}$$

and

$$p_2 = p_{atm} + p_2(\text{gage}) = 101.325 \text{ kPa} + 360 \text{ kPa} = 461.3 \text{ kPa}$$

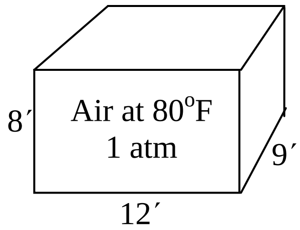
Finally

$$T_2 = (p_2/p_1)T_1 = (461.3/401.3)(300.15 \text{ K}) = 345.0 \text{ K}$$

Or, in °C

$$T_2 = 345.0 - 273.15 = 71.85^\circ\text{C} \longleftarrow$$

PROBLEM 3.107



$T_R = T/T_c = (540^{o}R)/(239^{o}R) = 2.26$; $p_R = p/p_c = (1\ atm)/(37.2\ atm) = 0.03$

Table A-1E

From Fig. A-1: $Z \approx 1$

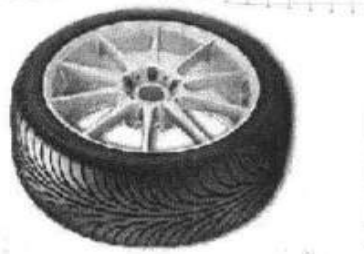
Thus, with $pV = mRT$

$$m = \frac{pV}{RT} = \frac{\left(14.696\frac{\text{lbf}}{\text{in.}^2}\right)(8\text{x}9\text{x}12\ \text{ft}^3)}{\left(\frac{1545\ \text{ft}\cdot\text{lbf}}{28.97\text{lb}\cdot{}^{o}\text{R}}\right)(540^{o}\text{R})}\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right| = 63.51\ \text{lb}$$

The weight is

$$F_{grav} = mg = (63.51\ \text{lb})(32.0\ \text{ft/s}^2)\left|\frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2}\right| = 63.12\ \text{lbf}$$

PROBLEM 3.108



Nitrogen, N_2: $T_C = 126\text{ K}$, $P_C = 33.9\text{ bar}$ (Table A-1)

Tire pressure = 180 kPa (gage)

$$p = p(\text{gage}) + p_{atm}$$

$$\therefore\ p = 1.80\text{ bar} + 1.01325\text{ bar} = 2.81\text{ bar}$$

$$\left.\begin{aligned} T_R &= \frac{T}{T_C} = \frac{298\text{ K}}{126\text{ K}} = 2.37 \\ P_R &= \frac{p}{p_C} = \frac{2.81\text{ bar}}{33.9\text{ bar}} = 0.08 \end{aligned}\right\} \quad \begin{array}{l}\text{Figure A-1} \\ Z \cong 1.0\end{array}$$

4 tires

$$\Rightarrow pV = mRT, \quad m = \frac{pV}{RT} = \frac{(2.81\text{ bar})(4 \times 0.6\text{ m}^3)}{\left(\frac{8314}{28.01}\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(298\text{ K})}\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right| = 7.62\text{ kg}$$

PROBLEM 3.109

Propane at $p = 2$ bar, $v = 0.307$ m³/kg, find T in K and °C.

⊙ Table A-18: $T = 60°C = 333.15 K$ ←

⊙ Figure A-1: $T_c = 370 K$, $p_c = 42.7$ bar (Table A-1)

$$\therefore \quad p_R = \frac{p}{p_c} = \frac{2 \text{ bar}}{42.7 \text{ bar}} = 0.047,$$

$$v_R' = \frac{v}{(RT_c/p_c)} = \frac{(0.307 \text{ m}^3/\text{kg})(42.7 \times 10^5 \text{ N/m}^2)}{\left(\frac{8314}{44.09} \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(370 \text{ K})} = 18.79$$

$\Rightarrow Z \cong 1 \Rightarrow pv = RT$ is a valid approximation.

$$\therefore \quad T = \frac{pv}{R} = \frac{(2 \times 10^5 \text{ N/m}^2)(0.307 \text{ m}^3/\text{kg})}{\left(\frac{8314}{44.09} \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)} = 325.6 \text{ K} \ (52.5°C)$$

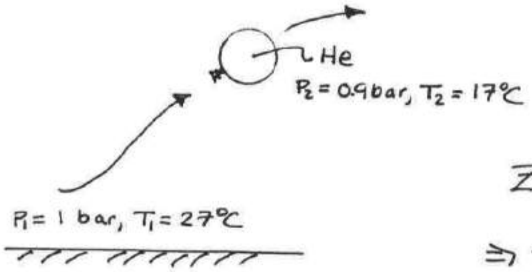
Comparison with values from Table A-18:

$$\text{In K: } \left(\frac{333.15 - 325.6}{333.15}\right)(100) = +2.3\%$$

$$\text{In °C: } \left(\frac{60 - 52.5}{60}\right)(100) = +12.5\%$$

According to the discussion of Fig. A-1 in Sec. 3.11.3, the departure between observed values and generalized compressibility figure values is typically only a few percent. The temperature, in K, determined using Fig. A-1 is in good agreement (about 2 % off) with that obtained from Table A-18. This should be expected since the Fig. A-1 correlation is developed in terms of absolute temperature. Such agreement should not be expected for temperatures in C (or F), and in this application is not achieved.

PROBLEM 3.110



$$Z = \frac{pV}{mRT}$$

$$\Rightarrow V = \frac{Z(mRT)}{p}$$

$$\therefore \quad \frac{V_2}{V_1} = \left[\frac{Z_2(mRT_2)/P_2}{Z_1(mRT_1)/P_1}\right]$$

$$= \left(\frac{Z_2}{Z_1}\right)\left(\frac{T_2}{T_1}\right)\left(\frac{P_1}{P_2}\right) \qquad (1)$$

For Helium, $T_c = 5.2\,K$, $P_c = 2.3$ bar. Thus, the ideal gas model applies because T_R is very large at both conditions:

$$\left.\begin{aligned} P_{R1} &= \frac{P_1}{P_c} = \frac{1\,\text{bar}}{2.3\,\text{bar}} = 0.43 \\ T_{R1} &= \frac{T_1}{T_c} = \frac{300}{5.2} = 57.7 \end{aligned}\right\} Z_1 \sim 1.0$$

$$\left.\begin{aligned} P_{R2} &= \frac{P_2}{P_c} = \frac{0.9}{2.3} = 0.39 \\ T_{R2} &= \frac{T_2}{T_c} = \frac{290}{5.2} = 55.8 \end{aligned}\right\} \rightarrow Z_2 \sim 1.0$$

Finally, inserting values, Eq. (1) gives

$$\frac{V_2}{V_1} = \left(\frac{T_2}{T_1}\right)\left(\frac{P_1}{P_2}\right) = \left(\frac{290\,K}{300\,K}\right)\left(\frac{1\,\text{bar}}{0.9\,\text{bar}}\right) = 1.074$$

$$\% = \frac{V_2 - V_1}{V_2} = +0.074 \qquad (+7.4\%)$$

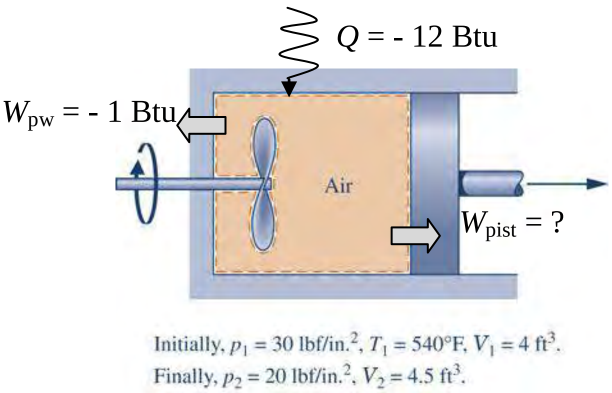
PROBLEM 3.111

As shown in Fig. 3.111, a piston-cylinder assembly fitted with a paddle wheel contains air, initially at p_1 = 30 lbf/in.^{2}, T_1 = 540^{o}F, and V_1 = 4 ft^3. The air undergoes a process to a final state where p_2 = 20 lbf/in.^{2}, V_2 = 4.5 ft^3. During the process, the paddle wheel transfers energy *to* the air by work in the amount 1 Btu, and there is energy transfer *to* the air by heat in the amount of 12 Btu. Assuming ideal gas behavior, and neglecting kinetic and potential energy effects, determine for the air (a) the temperature at state 2, in ^{o}R, and (b) the energy transfer by work from the air *to* the piston, in Btu.

KNOWN: Data are provided for air contained in a piston-cylinder assembly fitted with a paddle wheel.

FIND: For the process of the air, find the temperature at the final state and the energy transfer by work to the piston.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: 1. The air is the closed system. 2. Kinetic and potential energy effects are negligible. 3. The ideal gas model applies for the air.

ANALYSIS: (a) Using the ideal gas equation of state

$$m = \frac{p_1 V_1}{\left(\frac{\bar{R}}{M}\right)T_1} = \frac{(30\ \text{lbf/in.}^2)(4\ \text{ft}^3)}{\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^o\text{R}}\right)(1000^o\text{R})}\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right| = 0.324\ \text{lb}$$

and

$$T_2 = \frac{p_2 V_2}{m\left(\frac{\bar{R}}{M}\right)} = \frac{(20\ \text{lbf/in.}^2)(4.5\ \text{ft}^3)}{(0.324\ \text{lb})\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^o\text{R}}\right)}\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right| = 750^o\text{R}$$

(b) Noting that $W = W_{pw} + W_{pist}$ and $\Delta U = m(u_2 - u_1)$ the energy balance reduces as follows.

$$\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = Q - (W_{pw} + W_{pist})$$

$$W_{pist} = Q - W_{pw} - m(u_2 - u_1)$$

From Table A-22E: u_1 = 172.43 Btu/lb and u_2 = 128.25 Btu/lb. Thus

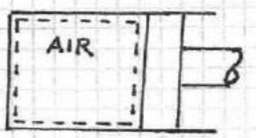
$$W_{pist} = (-12\ \text{Btu}) - (-1\ \text{Btu}) - (0.324\ \text{lb})(128.25 - 172.43)\text{Btu/lb} = 3.31\ \text{Btu (out)}$$

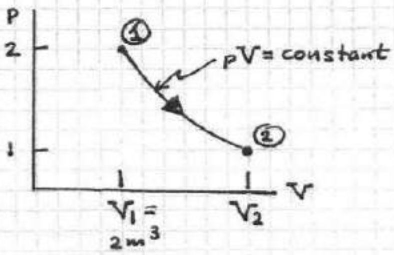
PROBLEM 3.112

KNOWN: Air in a piston-cylinder assembly undergoes a process described by pV = constant. State data are provided.

FIND: Determine the mass of the air, in kg, and the work and heat transfer, in kJ.

SCHEMATIC & GIVEN DATA:





$p_1 = 2\,\text{bar}$, $T_1 = 300\,\text{K}$

$V_1 = 2\,\text{m}^3$, $p_2 = 1\,\text{bar}$

ENGINEERING MODEL:

1. The air is the closed system.
2. The air undergoes a process described by pV = constant.
3. The air is modeled as an ideal gas.
4. Kinetic and potential energy effects are ignored.

ANALYSIS:

Using the ideal gas equation of state: $pV = mRT$,

$$m = \frac{p_1 V_1}{R T_1} = \frac{(2\times10^5\,\text{N/m}^2)(2\,\text{m}^3)}{\left(\frac{8314}{28.97}\,\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(300\,\text{K})} = 4.65\,\text{kg} \longleftarrow$$

Using Eq. 2.17,

$$W = \int_1^2 p\,dV = \int_1^2 \frac{C}{V}\,dV = \underbrace{C}_{= p_1V_1} \ln\frac{V_2}{V_1} = p_1V_1 \ln \underbrace{\frac{V_2}{V_1}}$$

$$p_1V_1 = p_2V_2 \Rightarrow \frac{V_2}{V_1} = \frac{p_1}{p_2} = 2$$

$$\therefore W = \left(2\times10^5\frac{\text{N}}{\text{m}^2}\right)(2\,\text{m}^3)\left|\frac{1\,\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right| \ln 2$$

$$= +277.3\,\text{kJ} \longleftarrow$$

An energy balance reduces as follows: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$

$$\Rightarrow \quad Q = \Delta U + W = m\left[u(T_2) - u(T_1)\right] + W \qquad (1)$$

Observing that pV = constant $\Rightarrow p_1V_1 = p_2V_2$. Thus, with the ideal gas equation of state: $pV = mRT$, it follows that $T_2 = T_1$. In Eq. (1), the term $[u(T_2) - u(T_1)] = 0$, leaving

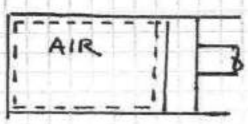
$$Q = W = +277.3\,\text{kJ} \longleftarrow$$

PROBLEM 3.113

KNOWN: Air in a piston-cylinder assembly undergoes a process during which the pressure-volume relationship is linear. State data are provided.

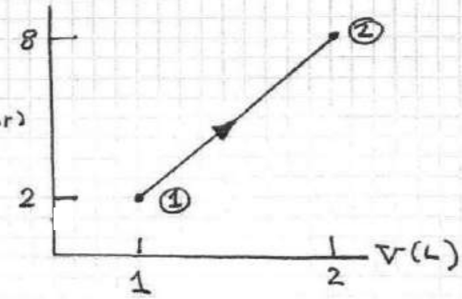
FIND: Determine the work and heat transfer, in kJ.

SCHEMATIC & GIVEN DATA:



$p_1 = 2\text{ bar}$, $T_1 = 200\text{K}$

$V_1 = 1\text{ L}$, $p_2 = 8\text{ bar}$, $V_2 = 2\text{ L}$



ENGINEERING MODEL:

1. The air is the closed system.
2. The air undergoes a process during which the pressure-volume relationship is linear.
3. The air is modeled as an ideal gas.
4. Kinetic and potential energy effects are ignored.

ANALYSIS:

Using Eq. 2.17

$$W = \int_1^2 p\,dV = p_{ave}[V_2 - V_1] = \left(\frac{p_1+p_2}{2}\right)[V_2 - V_1]$$

$$= \left[5\times10^5 \frac{N}{m^2}\right](2-1)\,L \left|\frac{10^{-3}\,m^3}{1\,L}\right|\left|\frac{1\,kJ}{10^3\,N\cdot m}\right| = 0.5\text{ kJ} \longleftarrow$$

An energy balance reduces as follows: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$

$$\Rightarrow Q = m[u(T_2) - u(T_1)] + W \qquad (1)$$

$$m = \frac{p_1 V_1}{R T_1} = \frac{(2\times10^5\,N/m^2)(1L)}{\left(\frac{8314}{28.97}\frac{N\cdot m}{kg\cdot K}\right)(200K)}\left|\frac{10^{-3}\,m^3}{1L}\right| = 0.0035\text{ kg}$$

$$\left.\begin{array}{l} p_1V_1 = mRT_1 \\ p_2V_2 = mRT_2 \end{array}\right\} \frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{V_2}{V_1} = \left(\frac{8}{2}\right)\left(\frac{2}{1}\right) = 8 \Rightarrow T_2 = 1600K.$$

Then, with specific internal values from Table A-22, Eq. (1) becomes

$$Q = 0.0035\,kg\,[1298.3 - 142.56]\,\frac{kJ}{kg} + 0.5\,kJ$$

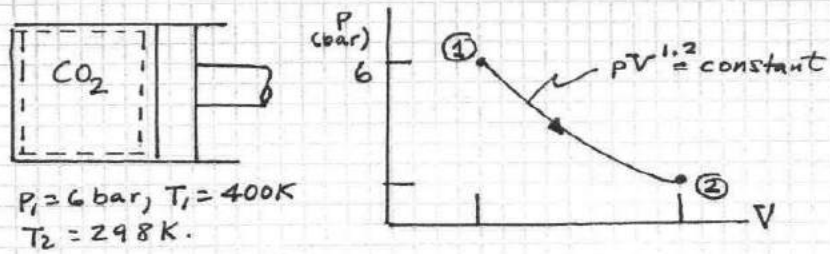
$$= 4.55\text{ kJ} \longleftarrow$$

PROBLEM 3.114

KNOWN: CO_2 in a piston-cylinder assembly undergoes a process during which $pV^{1.2}$ constant. State data are provided.

FIND: For the CO_2, determine the final pressure, in bar, and the work and heat transfer, in kJ/kg.

SCHEMATIC & GIVEN DATA:



$p_1 = 6$ bar, $T_1 = 400$K
$T_2 = 298$ K.

ENGINEERING MODEL:

1. The CO_2 is the closed system.
2. The CO_2 undergoes a process described by $pV^{1.2} =$ constant.
3. The CO_2 is modeled as an ideal gas.
4. Kinetic and potential energy effects are ignored.

ANALYSIS:

Using Eq. 3.56: $\frac{p_2}{p_1} = \left[\frac{T_2}{T_1}\right]^{\frac{n}{(n-1)}} \Rightarrow p_2 = (6\text{ bar})\left[\frac{298\text{K}}{400\text{K}}\right]^{\frac{1.2}{0.2}} = 1.03\text{ bar}$ ←

With Eq. 2.17,

$$W = \int_1^2 p\,dV = \frac{p_2V_2 - p_1V_1}{(1-n)} \quad \text{[See Example 2.1 for integration details.]}$$

With the ideal gas equation of state: $pV = mRT$, we get

$$W = \frac{mR[T_2 - T_1]}{(1-n)} \Rightarrow \frac{W}{m} = \frac{R[T_2-T_1]}{[1-n]} = \frac{\left[\frac{8.314}{44.01}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right][298-400]\text{ K}}{(-0.2)}$$

$$= +96.3\ \frac{\text{kJ}}{\text{kg}} \leftarrow$$

An energy balance reduces to read: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$

$$\Rightarrow \quad Q = \Delta U + W = m[u(T_2) - u(T_1)] + W$$

$$\therefore \quad \frac{Q}{m} = \left[u(T_2) - u(T_1)\right] + \frac{W}{m} \qquad (1)$$

Table A-23 gives specific internal energy on a molar basis. Thus, Eq. (1) becomes

$$\frac{Q}{m} = \left[\frac{\bar{u}(T_2) - \bar{u}(T_1)}{M}\right] + \frac{W}{m} \Rightarrow \frac{Q}{m} = \left[\frac{(6885 - 10{,}046)\frac{\text{kJ}}{\text{kmol}}}{44.01\frac{\text{kg}}{\text{kmol}}}\right] + 96.3\ \frac{\text{kJ}}{\text{kg}}$$

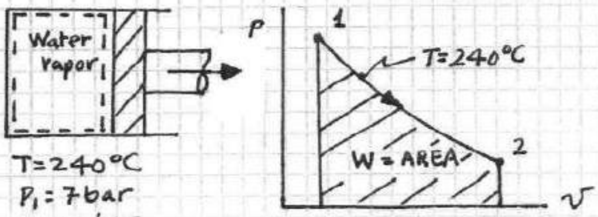
$$= 24.5\ \frac{\text{kJ}}{\text{kg}} \leftarrow$$

PROBLEM 3.115

KNOWN: Water vapor contained in a piston-cylinder assembly undergoes an isothermal expansion. State data are provided.

FIND: Evaluate the work, in kJ/kg, using two approaches:
(a) the ideal gas model, (b) IT with "water/steam" data.

SCHEMATIC & GIVEN DATA:



T = 240°C
P_1 = 7 bar
P_2 = 3 bar

ENGINEERING MODEL

1. The water vapor is the closed system.
2. Part(a) only: The water vapor is modeled as an ideal gas.
3. The water vapor undergoes an isothermal process

(a) Checking the ideal gas model assumption:

$$T_R = \frac{T}{T_c} = \frac{513K}{647K} = 0.79 \; , \; P_{R1} = \frac{P_1}{P_c} = \frac{7 bar}{221 bar} = 0.03, \; P_{R2} = \frac{3 bar}{221 bar} = 0.01 \Rightarrow \begin{matrix} Z_1 \cong 1 \\ Z_2 \cong 1 \end{matrix}$$

With Eq. 2.17 and $pv = RT$

$$\frac{W}{m} = \int_1^2 p\,dv = \int_1^2 \frac{RT}{v}dv = RT\ln\frac{v_2}{v_1} = RT\ln\frac{P_1}{P_2} = \left(\frac{8.314}{18.02}\frac{kJ}{kg\cdot K}\right)(513K)\ln\left(\frac{7}{3}\right) = 200.5\ \frac{kJ}{kg}$$

$$\left[\begin{matrix} P_1 v_1 = RT \\ P_2 v_2 = RT \end{matrix} > P_2 v_2 = P_1 v_1 \Rightarrow \frac{v_2}{v_1} = \frac{P_1}{P_2}\right]$$

(b) Approximating the area under the isothermal process curve on the p-v diagram by 8 trapezoids using "water/steam" data from IT,

```
T = 240  // C
        // pressures in bar
v1 = v_PT("Water/Steam", 7, T)
va = v_PT("Water/Steam", 6.5, T)
vb = v_PT("Water/Steam", 6, T)
vc = v_PT("Water/Steam", 5.5, T)
vd = v_PT("Water/Steam", 5, T)
ve = v_PT("Water/Steam", 4.5, T)
vf = v_PT("Water/Steam", 4, T)
vg = v_PT("Water/Steam", 3.5, T)
v2 = v_PT("Water/Steam", 3, T)

W=(6.75*(va-v1)+6.25*(vb-va)+5.75*(vc-vb)+5.25*(vd-vc)+4.75*(ve-vd)+4.25*(vf-ve)+3.75*(vg-vf)+3.25*(v2-vg)
*100
```

Answers:

W = 201.2 kJ/kg

v1=0.3292 m^3/kg
va=0.3552
vb=0.3856
vc=0.4215
vd=0.4646
ve=0.5173
vf= 0.5831
vg=0.6677
v2=0.7805

COMMENT: *Since the states visited in the process fall into the realm where the ideal gas model applies, the value of work determined using the ideal gas model agrees with Steam Table data, as expected.*

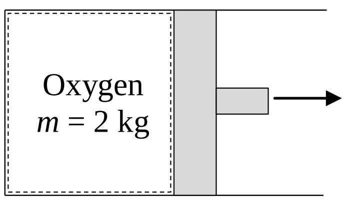
PROBLEM 3.116

Two kilograms of oxygen fills the cylinder of a piston-cylinder assembly. The initial volume and pressure are 2 m^3 and 1 bar, respectively. Heat transfer to the oxygen occurs at constant pressure until the volume is doubled. Determine the heat transfer for the process, in kJ, assuming the specific heat ratio is constant, $k = 1.35$. Kinetic and potential energy effects can be ignored.

KNOWN: Data are provided for oxygen contained within a piston-cylinder assembly undergoing a constant pressure process.

FIND: Determine the heat transfer.

SCHEMAITC AND GIVEN DATA:



$V_1 = 2\ \text{m}^3$

$p_1 = p_2 = 1$ bar

$V_2 = 2 \cdot V_1 = 4\ \text{m}^3$

ENGINEERING MODEL: 1. The oxygen is the closed system. 2. The pressure is constant. 3. The oxygen is modeled as an ideal gas and the specific heat ratio is constant: $k = 1.35$. 4. Kinetic and potential energy effects can be neglected.

ANALYSIS: Applying the energy balance: $\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = Q - W$.

The internal energy change can be expressed as $\Delta U = m(u_2 - u_1)$. Since k is constant, the specific heat c_v is also constant, so $\Delta U = m(u_2 - u_1) = m\, c_v(T_2 - T_1)$.

The work for the constant pressure expansion can be expressed as $W = \int_{V_1}^{V_2} p\,dV = p(V_2 - V_1)$.

Collecting results and solving for Q we get

$$Q = m\, c_v(T_2 - T_1) + p(V_2 - V_1) \qquad (*)$$

To evaluate T_1, we use the ideal gas equation of state

$$T_1 = \frac{pV_1}{m\left(\frac{\bar{R}}{M}\right)} = \frac{(1\text{ bar})(2\text{ m}^3)}{(2\text{ kg})\left(\frac{8.314}{32.0}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)}\left|\frac{10^5\text{N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{N}\cdot\text{m}}\right| = 384.9\text{ K}$$

Now, with $p_1 = p_2$

$$\frac{\cancel{m\left(\frac{\bar{R}}{M}\right)}T_1}{V_1} = \frac{\cancel{m\left(\frac{\bar{R}}{M}\right)}T_2}{V_2} \quad \longrightarrow \quad T_2 = (V_2/V_1)T_1 = (2)(384.9\text{ K}) = 769.8\text{ K}$$

Using Eq. 3.47b, $c_v = (\bar{R}/M)/(k - 1) = (8.314/32.00)/(1.35 - 1) = 0.742$ kJ/kg·K

PROBLEM 3.116 (CONTINUED)

Now, inserting values in (*)

$$Q = (2\text{ kg})(0.742\text{ kJ/kg}\cdot\text{K})(769.8 - 384.9)\text{K} + (1\text{ bar})(4 - 2)\text{m}^3\left|\frac{10^5\text{N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{N}\cdot\text{m}}\right|$$

$$= 771.2\text{ kJ (in)} \longleftarrow$$

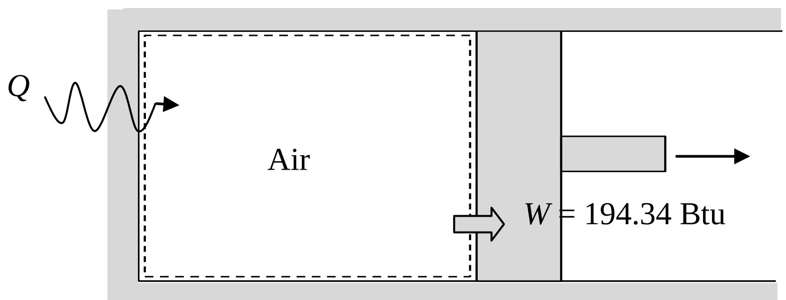
PROBLEM 3.117

As shown in Fig. P3.117, 20 ft^3 of air at T_1 = 600 ^{o}R, 100 lbf/in.^{2} undergoes a polytropic expansion to a final pressure of 51.4 lbf/in.^{2} The process follows $pV^{1.2}$ = constant. The work is W = 194.34 Btu. Assuming ideal gas behavior for the air, and neglecting kinetic and potential energy effects, determine
(a) the mass of air, in lb, and the final temperature, in ^{o}R.
(b) the heat transfer, in Btu.

KNOWN: Air undergoes a polytropic process in a piston-cylinder assembly. The work is known.

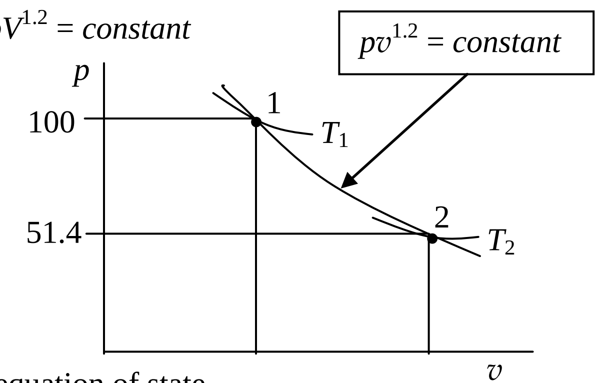
FIND: Determine the mass of air, the final temperature, and the heat transfer.

SCHEMATIC AND GIVEN DATA:



T_1 = 600^{o}R
p_1 = 100 lbf/in.^{2}
V_1 = 20 ft^3
P_2 = 51.4 lbf/in.^{2}

$pV^{1.2}$ = *constant*



ENGINEERING MODEL: 1. The air is a closed system. 2. Volume change is the only work mode. 3. The process is polytropic, with $pV^{1.2}$ = *constant* and W = 194.34 Btu. 4. Kinetic and potential energy effects can be neglected.

ANALYSIS: (a) The mass is determined using the ideal gas equation of state.

$$m = \frac{p_1 V_1}{RT_1} = \frac{\left(100\frac{\text{lbf}}{\text{in.}^2}\right)(20\text{ ft}^3)}{\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot\text{R}}\right)(600^o\text{R})}\left|\frac{144\text{ in.}^2}{1\text{ ft}^2}\right| = 9.00\text{ lb} \longleftarrow$$

To get the final temperature, we use the polytropic process, $pV^{1.2}$ = *constant*, to evaluate V_2 as follows.

$$V_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{1.2}} V_1 = \left(\frac{100}{51.4}\right)^{\frac{1}{1.2}}(20\text{ ft}^3) = 34.83\text{ ft}^3$$

Now

PROBLEM 3.117 (CONTINUED)

$$T_2 = \frac{p_2V_2}{mR} = \frac{(51.4\frac{\text{lbf}}{\text{in.}^2})(34.83\ \text{ft}^3)}{(9.00\ \text{lb})\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot\text{R}}\right)}\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right| = 537^{\circ}\text{R} \longleftarrow$$

Alternative solution for T_2

The work for the polytropic process can be evaluated using $W = \int_{V_2}^{V_1} p dV$. For the process $pV^{1.2}$ = *constant*, and incorporating the ideal gas equation of state, we get

$$W = \frac{p_2V_2 - p_1V_1}{1-1.2} = \frac{mR(T_2 - T_1)}{1-1.2}$$

Solving for T_2 and inserting values

$$T_2 = \frac{W(1-1.2)}{mR} + T_1 = \frac{(194.34\ \text{Btu})(1-1.2)}{(9.00\ \text{lb})(\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot\text{R}}\right)}\left|\frac{778\ \text{ft}\cdot\text{lbf}}{1\ \text{Btu}}\right| + (600^{\circ}\text{R}) = 537^{\circ}\text{R}$$

(b) Applying the energy balance; $\Delta \cancel{KE} + \Delta \cancel{PE} + \Delta U = Q - W$. With $\Delta U = m(u_2 - u_1)$, we get

$$Q = m(u_2 - u_1) + W$$

From Table A-22E: $u(600^{\circ}\text{R})$ = 102.34 Btu/lb and $u(537^{\circ}\text{R})$ = 91.53 Btu/lb. Thus,

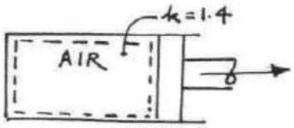
$$Q = (9.00\ \text{lb})(91.53 - 102.34)\text{Btu/lb} + (194.34\ \text{Btu}) = 97.05\ \text{Btu (in)} \longleftarrow$$

PROBLEM 3.118

KNOWN: Data are provided for air contained within a piston-cylinder assembly.

FIND: For the process of the air, find W and Q.

SCHEMATIC & GIVEN DATA:



$p = 30$ lbf/in²

$V_1 = 0.75$ ft³

$V_2 = 2V_1$

ENGR. MODEL

1. The air in the piston-cylinder assembly is the closed system.
2. For the air, the ideal gas model with $k = 1.4$ applies.
3. No change in kinetic or potential energy occurs during the process.
4. Volume change is the only work mode.

ANALYSIS:

(a) The work is evaluated from $W_{12} = \int_1^2 p\,dV$. Since pressure is constant

$$W_{12} = p[V_2 - V_1] = 30\frac{\text{lbf}}{\text{in}^2}\left|\frac{144\,\text{in}^2}{1\,\text{ft}^2}\right|\left[0.75\,\text{ft}^3\right]\left|\frac{1\,\text{Btu}}{778\,\text{ft}\cdot\text{lbf}}\right|$$

$$= 4.16\,\text{Btu}$$

$\longleftarrow W_{12}$

(b) Reducing an energy balance, $\Delta U + \Delta KE + \Delta PE = Q_{12} - W_{12}$, or

① $$Q_{12} = W_{12} + m(u_2 - u_1) \qquad (1)$$

With Eqs. 3.47b and 3.50, $c_v = R/(k-1)$ and $(u_2 - u_1) = c_v[T_2 - T_1]$, giving

$$Q_{12} = W_{12} + m\left[\frac{R}{k-1}\right](T_2 - T_1) \qquad (2)$$

The ideal gas equation of state also gives $pV_1 = mRT_1$ and $pV_2 = mRT_2$.
Thus, $p(V_2 - V_1) = mR(T_2 - T_1)$, and Eq. 2 becomes

$$Q_{12} = W_{12} + \frac{p(V_2 - V_1)}{k-1}$$

$$= 4.16\,\text{Btu} + \frac{4.16\,\text{Btu}}{1.4-1}$$

$$= 14.56\,\text{Btu}$$

$\longleftarrow Q_{12}$

---

1. Since $W_{12} = mp(v_2 - v_1)$, Eq. (1) can be expressed as

$$Q_{12} = m[(u_2 - u_1) + p(v_2 - v_1)]$$

$$= m[(u_2 + pv_2) - (u_1 + pv_1)]$$

$$= m[h_2 - h_1] = m\,c_p(T_2 - T_1)$$

$c_p = \frac{kR}{(k-1)}$ (Eq. 3.47a)

$$= \frac{k}{k-1}\underbrace{p(V_2 - V_1)}_{= 4.16\,\text{Btu}} = 14.56\,\text{Btu}$$

$\longleftarrow Q_{12}$

PROBLEM 3.119

KNOWN: Data are provided for air within an enclosure fitted with a fan.

FIND: For the air, determine the mass, final temperature, and final pressure.

SCHEMATIC & GIVEN DATA:

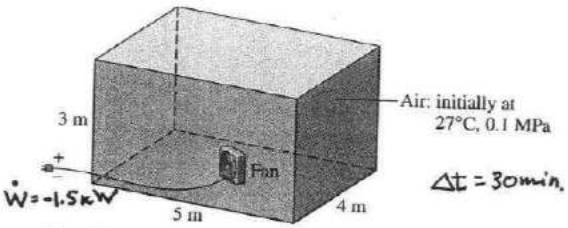


Fig. P3.117

ENG. MODEL

1. The contents of the enclosure are the closed system.
2. The volume occupied by the fan can be ignored and there is no overall change in internal energy of the fan.
3. The air can be modeled as an ideal gas.
4. For the system, $Q = 0$ and there are no overall changes in kinetic or potential energy.

ANALYSIS:

(a) Using the ideal gas equation of state with assumption 2, $p_1 V = m R T_1$, or

$$m = \frac{p_1 V}{R T_1} = \frac{(0.1\,MPa)(60\,m^3)}{\left(\frac{8314}{28.97}\frac{N\cdot m}{kg\cdot K}\right)(300\,K)}\left|\frac{10^6\,N/m^2}{1\,MPa}\right| = 69.69\,kg \quad \longleftarrow m$$

(b) Reducing an energy balance, $\Delta U + \cancel{\Delta U_{fan}}^{0} + \cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0} = \cancel{Q}^{0} - W$. Thus

$$-W = m[u_2 - u_1] \Rightarrow u_2 = u_1 - \frac{W}{m} \qquad (1)$$

where $W = \int \dot{W} dt = \dot{W} \Delta t$, or

$$W = \dot{W}\Delta t = (-1.5\,kW)(30\,min)\left|\frac{1\,kJ/s}{1\,kW}\right|\left|\frac{60\,s}{1\,min}\right| = -2700\,kJ$$

Also, from Table A-22, $u_1 = 214.07$ kJ/kg. Thus, Eq. (1) gives

$$u_2 = 214.07\,\frac{kJ}{kg} - \frac{(-2700\,kJ)}{69.69\,kg} = 252.81\,\frac{kJ}{kg}$$

Interpolating in Table A-22, $T_2 = 354\,K$ (81°C) $\longleftarrow T_2$

(c) Using the ideal gas equation of state again, $p_1 V = m R T_1$ and $p_2 V = m R T_2$,

$$\frac{p_2}{p_1} = \frac{T_2}{T_1} \Rightarrow p_2 = \frac{T_2}{T_1}\, p_1 = \left(\frac{354}{300}\right)(0.1\,MPa)$$

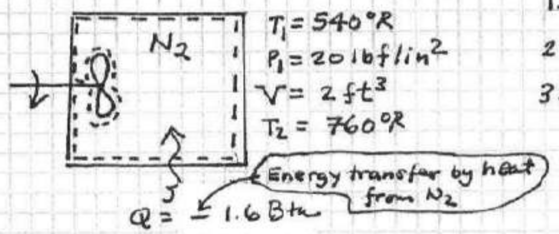
$$= 0.12\,MPa \quad \longleftarrow p_2$$

PROBLEM 3.120

KNOWN: N_2 fills a closed, rigid tank fitted with a paddle wheel. State data and the amount of energy transfer by heat from the N_2 are provided.

FIND: For the N_2, determine the mass, in lb, and the work, in Btu.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The N_2 is the closed system.
2. N_2 is modeled as an ideal gas.
3. Kinetic and potential energy effects are ignored.

ANALYSIS: Using the ideal gas equation of state, $m = \frac{P_1 V}{R T_1}$

$$\Rightarrow m = \frac{\left(20 \times 144 \frac{lbf}{ft^2}\right)(2 ft^3)}{\left(\frac{1545}{28.01} \frac{ft \cdot lbf}{lb \cdot °R}\right)(540°R)} = 0.193 \text{ lb}$$

Reducing an energy balance, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W \Rightarrow W = Q - \Delta U$

Since Table A-23E provides specific internal energy on a molar basis, this becomes

$$W = Q - m\left[\frac{\bar{u}(T_2) - \bar{u}(T_1)}{M}\right] \Rightarrow W = -1.6 \text{ Btu} - (0.193 \text{ lb})\left[\frac{(3774.9 - 2678) \text{ Btu/lbmol}}{28.01 \frac{lb}{lbmol}}\right]$$

$$= -9.16 \text{ Btu}$$

energy transfer by work to N_2

PROBLEM 3.121

KNOWN: Air is contained in a closed, rigid tank fitted with a paddle wheel. State data and operating data are provided.

FIND: Determine the work and heat transfer, in Btu.

SCHEMATIC & GIVEN DATA:

$\tau = 20$ ft·lbf

$\omega = 100$ RPM

$\Delta t = 60$ s

$T_1 = 540°R$,

$T_2 = 740°R$

ENGINEERING MODEL

1. The air is the closed system.
2. Air is modeled as an ideal gas.
3. There are no overall changes in kinetic or potential energy.

ANALYSIS:

With Eq. 2.20,

$$W = \overset{\text{Energy transfer by work to the air}}{-\tau\omega\Delta t}$$

$$= -[20\text{ ft·lbf}]\left(100\ \frac{\text{Rev}}{\text{min}}\right)\left|\frac{2\pi\text{ radians}}{1\text{ Rev}}\right|(60\text{ s})\left|\frac{1\text{ min}}{60\text{ s}}\right|\left|\frac{1\text{ Btu}}{778\text{ ft·lbf}}\right|$$

$$= -16.15\text{ Btu} \longleftarrow$$

Reducing an energy balance, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$

$$\Rightarrow Q = \Delta U + W$$

$$= m[u(T_2) - u(T_1)] + W$$

With data from Table A-22E,

$$Q = 0.4\text{ lb}\left[(126.51 - 92.04)\ \frac{\text{Btu}}{\text{lb}}\right] + (-16.15\text{ Btu})$$

$$= -2.36\text{ Btu} \longleftarrow$$

Heat transfer from the air

PROBLEM 3.122

KNOWN: Argon in a closed, rigid tank is heated. State data are provided.

FIND: For the argon, determine the final temperature, in °C, and the heat transfer, in kJ.

SCHEMATIC & GIVEN DATA:

$V = 2\,m^3$
$T_1 = 50°C$
$P_1 = 2\,bar$
$P_2 = 8\,bar$

ENGINEERING MODEL:

1. The argon is the closed system.
2. Argon is modeled as an ideal gas with $k = 1.67$.
3. For the system, $W = 0$ and kinetic and potential energy effects are ignored.

ANALYSIS: Using the ideal gas equation of state, $\left\{ \begin{array}{l} P_1 V = mRT_1 \\ P_2 V = mRT_2 \end{array} \right. \Rightarrow \dfrac{T_2}{T_1} = \dfrac{P_2}{P_1}$

$$\Rightarrow T_2 = T_1\left[\frac{P_2}{P_1}\right] = 323\,K\left[\frac{8\,bar}{2\,bar}\right] = 1292\,K\ (1019°C) \longleftarrow$$

Reducing an energy balance, $\Delta U + \cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0} = Q - \cancel{W}^{0} \Rightarrow Q = \Delta U$.

Noting that constant specific heat ratio corresponds to constant c_v,

$$Q = \underbrace{m}_{=\frac{P_1 V}{RT_1}}\Delta u = m\underbrace{c_v}_{=\frac{R}{(k-1)}\ (\text{Eq. } 3.47b)}[T_2 - T_1] \Rightarrow Q = \left[\frac{P_1 V}{RT_1}\right]\left[\frac{R}{k-1}\right](T_2 - T_1) = \frac{P_1 V}{(k-1)}\left[\underbrace{\frac{T_2}{T_1}}_{=P_2/P_1} - 1\right]$$

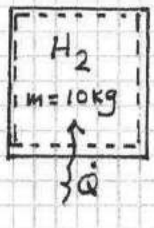
$$\therefore Q = \frac{(2 \times 10^5\,N/m^2)(2\,m^3)}{(1.67 - 1)}\left|\frac{1\,kJ}{10^3\,N \cdot m}\right|[4 - 1] = 1791\,kJ \longleftarrow$$

PROBLEM 3.123

KNOWN: H_2 is contained in a closed, rigid tank. State data and heat transfer data are provided.

FIND: Determine the final temperature of the H_2, in °C.

SCHEMATIC & GIVEN DATA



$T_1 = 20°C$

$k = 1.405$

$\dot{Q} = 400 W$

$\Delta t = 1 h$

ENGINEERING MODEL:

1. The H_2 is the closed system.
2. H_2 is modeled as an ideal gas with $k = 1.405$.
3. For the system, $W = 0$ and kinetic and potential energy effects are ignored.

ANALYSIS:

Using given data, $Q = \dot{Q}\Delta t = \left(400\frac{J}{s}\right)(1 h)\left|\frac{3600 s}{h}\right|\left|\frac{1 kJ}{10^3 J}\right| = 1440 kJ$

Reducing an energy balance, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - \cancel{W} \Rightarrow Q = \Delta U$

$$\rightarrow \quad Q = m\Delta u = m c_v [T_2 - T_1] = \frac{mR}{(k-1)}[T_2 - T_1]$$

$c_v = \frac{R}{(k-1)}$ (Eq. 3.47b)

$$\rightarrow \quad (T_2 - T_1) = \frac{(k-1)Q}{mR} = \frac{(1.405-1)(1440 kJ)}{10 kg\left(\frac{8.314}{2.016}\frac{kJ}{kg\cdot K}\right)} = 14.14 K$$

or, since a difference in temperature is evaluated, this is also 14.14°C.

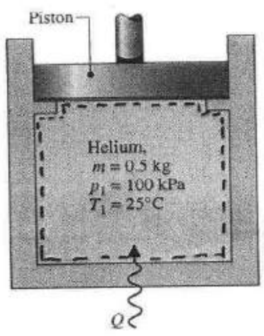
$$\Rightarrow T_2 = 34.14°C$$

PROBLEM 3.124

KNOWN: Data are provided for helium contained in a piston-cylinder assembly.

FIND: Determine the energy transfer by heat sufficient for the piston to start rising from rest on a set of stops.

SCHEMATIC & GIVEN DATA:



ENGR MODEL:

1. The helium is the closed system.
2. For the system, $W=0$ and $\Delta KE = \Delta PE = 0$.
3. The ideal gas model applies for the helium and $c_p = 5/2\, R$.

ANALYSIS: Heating occurs until the pressure of the helium becomes 500 kPa. The heating occurs at constant volume. Thus, $p_1 V = mRT_1$ and $p_2 V = mRT_2$, giving $T_2/T_1 = p_2/p_1$. That is, $T_2 = T_1\left(\frac{p_2}{p_1}\right) = 298\,K\left(\frac{500\,kPa}{100\,kPa}\right) = 1490\,K$.

An energy balance reads, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - \cancel{W}$ or $Q = m(u_2 - u_1)$. Since helium is a monatomic gas, its specific heat c_p is constant: $c_p = 5/2\, R$ (see Table A-21). Then, with Eq. 3.44, $c_v = 3/2\, R$.

Thus,

$$Q = m c_v (T_2 - T_1) = m\left(\frac{3}{2}R\right)(T_2 - T_1)$$

$$= (0.5\,kg)\left(\frac{3}{2}\cdot\frac{8.314}{4.003}\right)(1490 - 298)\,K$$

M from Table A-1

$$= 1857\,kJ$$

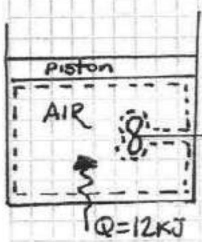
Q

PROBLEM 3.125

KNOWN: Air contained within a piston-cylinder assembly fitted with a paddle wheel undergoes a constant pressure process as it is slowly heated. State data and heat transfer data are provided.

FIND: Determine the work done by the paddle wheel on the air and the work done by the air to displace the piston, each in kJ.

SCHEMATIC & GIVEN DATA:



$m = 0.13\ \text{kg}$
$T_1 = 300\text{K}$
$T_2 = 400\text{K}$
$k = 1.4$
p = constant

ENGINEERING MODEL:

1. The air within the piston-cylinder assembly is the closed system.
2. The air is modeled as an ideal gas with $k = 1.4$.
3. Changes in kinetic and potential energy of the air are negligible.

ANALYSIS:

Since the air undergoes a constant-pressure process, the work done at the air-piston iterface can be obtained using

$$W_{piston} = \int_1^2 p\,dV = p[V_2 - V_1].$$ Using the ideal gas equation of state

$pV = mRT$, this becomes

$$W_{piston} = mR[T_2 - T_1] = (0.13\ \text{kg})\left[\frac{8.314}{28.97}\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right](400-300)\text{K} = 3.73\ \text{kJ}$$

An overall energy balance reduces to $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$,

where $W = W_{piston} + W_{\substack{paddle\\wheel}}$. Collecting results,

$$W_{\substack{paddle\\wheel}} = Q - \Delta U - W_{piston}$$

$$\Delta U = m\Delta u = mc_v[T_2 - T_1], \quad c_v = \frac{R}{(k-1)}\ (\text{Eq. } 3.47b)$$

(1) $$\therefore\ W_{\substack{paddle\\wheel}} = 12\ \text{kJ} - \underbrace{(0.13\ \text{kg})\left(\frac{(8.314/28.97)\ \text{kJ/kg}\cdot\text{K}}{(1.4-1)}\right)(100\text{K})}_{=9.33\ \text{kJ}} - 3.73\ \text{kJ}$$

$$= -1.06\ \text{kJ}$$

1. Energy balance sheet:

- Energy In:

✓ By heat transfer	12.00 kJ	(91.88%)
✓ By paddle wheel	1.06 kJ	(8.12%)
	13.06 kJ	

- Disposition of Energy In:

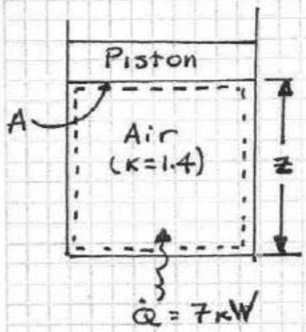
✓ Stored as internal energy	9.33	(71.44%)
✓ Transfer out to piston	3.73	(28.56%)
	13.06 kJ	

PROBLEM 3.126

KNOWN: A piston-cylinder assembly contains air, which is heated at constant pressure.

FIND: For a specified rate of heat transfer, determine the rate at which is done by the air.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The air within the piston-cylinder assembly is the closed system.
2. The air is modeled as an ideal gas with $\kappa = 1.4$.
3. For the air, kinetic and potential energy effects can be ignored.
4. For the air, pressure remains constant.

ANALYSIS: The energy rate balance is $\dfrac{dU}{dt} = \dot{Q} - \dot{W}$ (1).

Using Eq. 2.13, the rate work is done by the air on the piston is

$$\dot{W} = F\mathrm{V} = (pA)\frac{dz}{dt} = p\frac{d(Az)}{dt} = p\frac{dV}{dt} \qquad (2)$$

velocity of moving boundary

With $pV = mRT$, $\dfrac{dV}{dt} = \dfrac{mR}{p}\dfrac{dT}{dt}$ (pressure constant)

Collecting results,

$$\dot{W} = mR\frac{dT}{dt} \qquad (3)$$

Returning to Eq. (1), $\dfrac{dU}{dt} = m\dfrac{du}{dt} = m\underbrace{\dfrac{du}{dT}}_{c_v}\dfrac{dT}{dt} = mc_v\dfrac{dT}{dt}$. Using this, together with Eq. (3), Eq. (1) becomes

$$\dot{Q} = \frac{dU}{dt} + \dot{W} = mc_v\frac{dT}{dt}\ mR\frac{dT}{dt} = m\underbrace{(c_v + R)}_{= c_p \ (\text{Eq. } 3.44)}\frac{dT}{dt}.$$

$$= mc_p\frac{dT}{dt} \qquad (4)$$

Forming a ratio, Eqs. (3) and (4) give

$$\frac{\dot{W}}{\dot{Q}} = \frac{mR\, dT/dt}{mc_p\, dT/dt} = \frac{R}{c_p} \overset{\text{Eq. 3.47a}}{=} \frac{\kappa - 1}{\kappa}$$

$$\Rightarrow \quad \dot{W} = \left(\frac{\kappa-1}{\kappa}\right)\dot{Q} = \left(\frac{1.4-1}{1.4}\right)(0.7\ \text{kW})$$

$$= 0.2\ \text{kW}$$

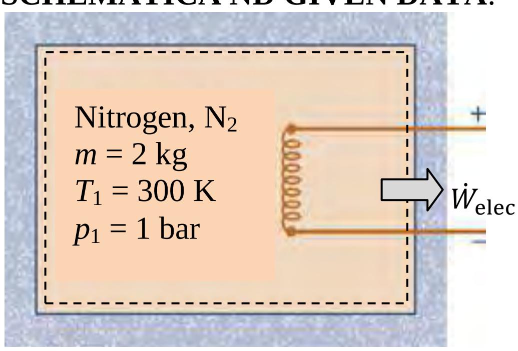
PROBLEM 3.127

As shown in Fig. P3.127, a well-insulated tank fitted with an electrical resistor of negligible mass holds 2 kg of nitrogen (N_2), initially at 300 K, 1 bar. Over a period of 10 minutes, electricity is provided to the resistor at a constant voltage of 120 volts and with a constant current of 1 ampere. Assuming ideal gas behavior, determine the nitrogen's final temperature, in K, and the final pressure, in bar.

KNOWN: Data are provided for nitrogen contained in a well-insulated tank fitted with and electrical resistor. Voltage is applied and a current flow for 10 minutes.

FIND: Determine the final temperature and pressure.

SCHEMATICA ND GIVEN DATA:



120 volt
1 ampere
Δt = 10 minutes

ENGINEERING MODEL: (1) The nitrogen in the tank is the closed system. (2) The tank is well-insulated, so we assume that $\dot{Q}$=0 (3) The resistor has negligible mass. (4) The nitrogen can be modeled as an ideal gas. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: Reducing the energy balance; $\Delta \cancel{KE} + \Delta \cancel{PE} + \Delta U = \cancel{Q} - W_{\text{elec}}$. Thus

$$\Delta U = -W_{\text{elec}} \quad \longrightarrow \quad n(\bar{u}_2 - \bar{u}_1) = -W_{\text{elec}}$$

where $n = m/M$ = (2 kg)/(28.01 lg/kmol) = 0.0714 kmol denotes the amount of N_2 on a molar basis. Solving for $\bar{u}_2$

$$\bar{u}_2 = \bar{u}_1 - W_{\text{elec}}/n \qquad (*)$$

The value $\bar{u}_1$ = 6229 kJ/kmol can be read from Table A-23. We will next evaluate the work and calculate the value of $\bar{u}_2$. Then, we can return to Table A-23 and interpolate to find T_2.

The *rate* of energy transfer by work (magnitude) in watts due to electric current flow through the resistance is

Rate of energy transfer in = (voltage)(amperage) = (120 volts)(1 ampere) = 120 watts

Thus

$\dot{W}_{\text{elec}}$ = -120 watts = - 0.12 kW

Since the voltage and current are constant, the power is constant and the total amount of energy transfer by work for the 10 minute period is

PROBLEM 3.127 (CONTINUED)

$$W_{\text{elec}} = \int \dot{W}_{\text{elec}} dt = \dot{W}_{\text{elec}} \Delta t = (-0.12 \text{ kW})(10 \text{ min}) \left| \frac{60 \text{ sec}}{1 \text{ min}} \right| \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| = -72 \text{ kJ}$$

Inserting values in (*)

$$\bar{u}_2 = (6229 \text{ kJ/kmol}) - (-72 \text{ kJ})/(0.0714 \text{ kmol}) = 7237.4 \text{ kmol}$$

From Table A-23; T_2 = 348.4 K ←

Since the volume is constant, the final pressure is

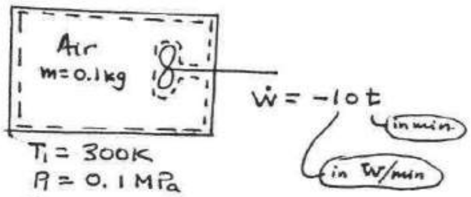
$$\frac{\cancel{n\bar{R}}T_1}{p_1} = V = \frac{\cancel{n\bar{R}}T_2}{p_2} \longrightarrow p_2 = (T_2/T_1)p_1 = (348.4/300)(1 \text{ bar}) = 1.16 \text{ bar}$$ ←

PROBLEM 3.128

KNOWN: Data are provided for air contained within a piston-cylinder assembly fitted with a paddlewheel.

FIND: For the air, find the final pressure, W and Q.

SCHEMATIC & GIVEN DATA:



$T_1 = 300K$
$P_1 = 0.1\,MPa$
$T_2 = 1060\,K$

ENGR. MODEL:

1. The air in the piston-cylinder assembly is the closed system.
2. The air is modeled as an ideal gas.
3. No changes in kinetic or potential energy occur.

ANALYSIS: (a) Using the ideal gas equation of state $P_1V = mRT_1$ and $P_2V = mRT$, we get

$$\frac{P_2}{P_1} = \frac{T_2}{T_1} \Rightarrow P_2 = P_1\left(\frac{T_2}{T_1}\right) = (0.1\,MPa)\left(\frac{1060K}{300K}\right) = 0.353\,MPa \longleftarrow P_2$$

(b)

$$W_{12} = \int_0^{t=20min.} \dot{W}\,dt = \int_0^{t=20min.} (-10t)\,dt = -\frac{10t^2}{2}\Big]_0^{t=20min.}$$

$$= -\frac{10\,W/min}{2}\left[20\,min\right]^2 = -2000\,W\cdot min\left|\frac{60s}{1\,min}\right|\left|\frac{1\,J/s}{1W}\right|\left|\frac{1\,kJ}{10^3 J}\right| = -120kJ \longleftarrow W_{12}$$

(c) Reducing an energy balance $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{12} - W_{12}$, or

$$Q_{12} = W_{12} + m(u_2 - u_1)$$

With data from Table A-22, $u_1 = 214.07\,kJ/kg$, $u_2 = 810.62\,kJ/kg$,

$$Q_{12} = -120\,kJ + 0.1\,kg\,(810.62 - 214.07)\,\frac{kJ}{kg}$$

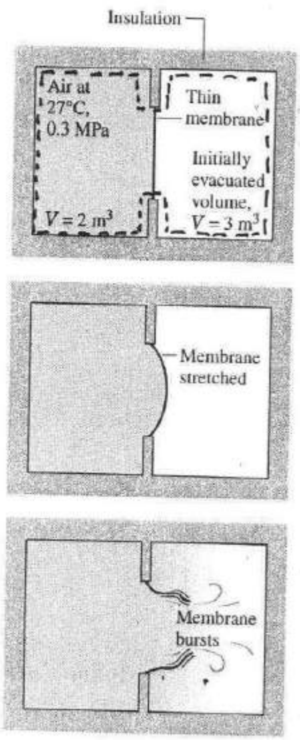
$$= -60.35\,kJ \longleftarrow Q_{12}$$

PROBLEM 3.129

KNOWN: Data is provided for air on one side of a rigid, insulated container. The other side of the container is initially evacuated.

FIND: For the air, determine the mass and the final temperature and pressure.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The closed system is the region within the container.
2. The air is modeled as an ideal gas.
3. The mass of the membrane can be ignored.
4. There are no significant overall kinetic or potential energy effects.

ANALYSIS:

(a) Using the ideal gas model equation of state,

$$m = \frac{P_1 V_1}{R T_1} = \frac{(0.3\,\text{MPa})(2\,\text{m}^3)}{\left(\frac{8314}{28.97}\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(300\,\text{K})}\left|\frac{10^6\,\frac{\text{N}}{\text{m}^2}}{1\,\text{MPa}}\right| = 6.97\,\text{kg} \longleftarrow m$$

(b) Reducing an energy balance,

$$\Delta U + \cancel{\Delta U}]_{\text{membrane}} + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - \cancel{W}$$

$$\Rightarrow \quad \Delta U = m(u_2 - u_1) = 0$$

$$\Rightarrow \quad u_2 = u_1$$

Since internal energy depends on temperature alone for an ideal gas,

$$T_2 = T_1 = 300\,\text{K} \qquad \longleftarrow T_2$$

(c) Using the ideal gas model equation of state again,

$$P_1 V_1 = m R T$$

$$P_2 V_2 = m R T$$

$$\Rightarrow \quad P_2 V_2 = P_1 V_1$$

or

$$P_2 = P_1\left[\frac{V_1}{V_2}\right]$$

$$= 0.3\,\text{MPa}\left[\frac{2\,\text{m}^3}{5\,\text{m}^3}\right]$$

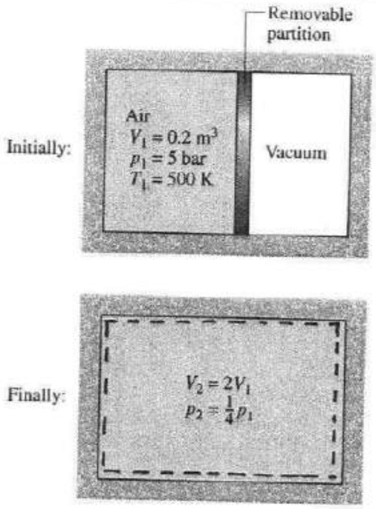
$$= 0.12\,\text{MPa} \qquad \longleftarrow P_2$$

PROBLEM 3.130

KNOWN: Data are provided for air on one side of a rigid container. The other side of the container is initially evacuated.

FIND: For the air, determine the final temperature and Q.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The closed system is the region within the container, ignoring the partition.
2. The air is modeled as an ideal gas.
3. There are no overall changes in kinetic or potential energy.
4. W = 0.

ANALYSIS:

(a) Using the ideal gas model equation of state, $P_1V_1 = mRT_1$, $P_2V_2 = mRT_2$. Thus,

$$T_2 = T_1\left[\frac{P_2\,V_2}{P_1\,V_1}\right] = 500\text{K}\left[\frac{1}{4}\right]\left[2\right] = 250\text{K} \qquad \longleftarrow T_2$$

(b) An energy balance reduces to $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - \cancel{W}$, or

$$Q = m\left(u(T_2) - u(T_1)\right)$$

where

$$m = \frac{P_1V_1}{RT_1} = \frac{(5\times10^5\,\text{N/m}^2)(0.2\,\text{m}^3)}{\left(\frac{8314}{28.97}\,\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(500\text{K})} = 0.7\,\text{kg}$$

So, with data from Table A-22

$$Q = 0.7\,\text{kg}\,(178.28 - 359.49)\,\frac{\text{kJ}}{\text{kg}}$$

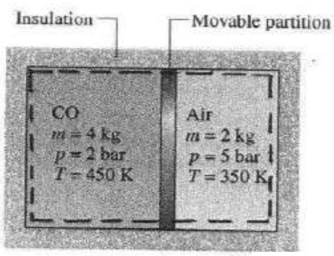
$$= -126.8\ \text{kJ} \qquad \longleftarrow Q$$

PROBLEM 3.131

KNOWN: Air and carbon monoxide are contained to opposite sides of a rigid, insulated container by a partition free to move.

FIND: Determine the final equilibrium temperature and pressure. Also, find the volume occupied by each gas finally.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The system is the container contents.
2. For the system, $W = Q = 0$ and there is no effect of kinetic and potential energy.
3. The partition is free to move, allows conduction from one gas to the other, and experiences no change in energy.
4. Each gas is modeled as an ideal gas with $k = 1.395$.

ANALYSIS: (a) An energy balance reads $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - \cancel{W}$, or $\Delta U = 0$, where $\Delta U = [\Delta U]_{co} + [\Delta U]_{Air} + [\cancel{\Delta U}]_{partition}$. That is,

$$[\Delta U]_{co} + [\Delta U]_{Air} = 0$$

With Eq. 3.50 this becomes

$$(mc_v)_{co}[T - 450K] + (mc_v)_{Air}[T - 350K] = 0 \qquad (1)$$

Using Eq 3.47b

$$c_{v,CO} = \frac{R}{k-1} = \frac{(8.314/28.01)\ kJ/kg\cdot K}{1.395-1} = 0.751\ \frac{kJ}{kg\cdot K}$$

$$c_{v,AIR} = \frac{R}{k-1} = \frac{(8.314/28.97)\ kJ/kg\cdot K}{1.395-1} = 0.727\ \frac{kJ}{kg\cdot K}$$

Solving Eq. (1),

$$T = \frac{(mc_v)_{co}(450K) + (mc_v)_{Air}(350K)}{(mc_v)_{co} + (mc_v)_{Air}}$$

$$= \frac{(4kg)(0.751\ kJ/kg\cdot K)(450K) + (2kg)(0.727\ kJ/kg\cdot K)(350K)}{(4kg)(0.751\ kJ/kg\cdot K) + (2kg)(0.727\ kJ/kg\cdot K)} = 417.4\ K \longleftarrow$$

(b) The total volume remains constant, where $V = V_{co} + V_{Air} = 3.07\ m^3$, and

$$V_{Air} = \left(\frac{mRT}{p}\right)_{Air} = \frac{(2\ kg)(8314/28.97)\left(\frac{N\cdot m}{kg\cdot K}\right)(350K)}{(5\times10^5\ N/m^2)} = 0.4\ m^3$$

$$V_{CO} = \left(\frac{mRT}{p}\right)_{CO} = \frac{(4)(8314/28.01)(450)}{(2\times10^5)} = 2.67\ m^3$$

At equilibrium, each gas is at $T = 417.4\ K$ and pressure p. With $V = mRT/p$,

$$3.07\ m^3 = \frac{(mR)_{Air}T}{p} + \frac{(mR)_{co}T}{p} \Rightarrow p = \frac{[(mR)_{Air} + (mR)_{co}]T}{3.07\ m^3}$$

Calculating

$$p = \frac{\left[2kg\left(8314/28.97\right)\left(\frac{N\cdot m}{kg\cdot K}\right) + 4kg\left(8314/28.01\right)\left(\frac{N\cdot m}{kg\cdot K}\right)\right]417.4K}{3.07\ m^3}\left|\frac{1\ bar}{10^5\ N/m^2}\right|$$

$$= 2.39\ bar \longleftarrow$$

(c) The final volumes occupied by the air and CO_2 are, respectively

$$V_{Air} = \frac{m_{Air} R T}{P} = \frac{(2\,kg)\left(\frac{8314}{28.97}\ \frac{N\cdot m}{kg\cdot K}\right)(417.4\,K)}{2.39\times10^5\ N/m^2} = 1\ m^3$$

$$V_{CO} = \frac{m_{CO} R T}{P} = \frac{(4\,kg)\left(\frac{8314}{28.01}\ \frac{N\cdot m}{kg\cdot K}\right)(417.4\,K)}{2.39\times10^5\ N/m^2} = 2.07\,m^3$$

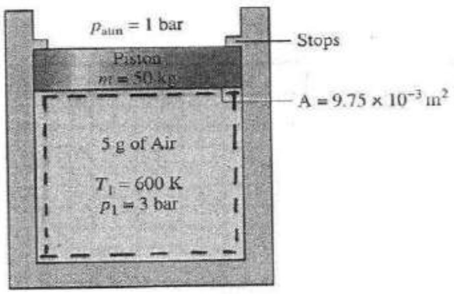
Note, $V_{AIR} + V_{CO} = 3.07\,m^3$, the total volume occupied initially.

PROBLEM 3.132

KNOWN: Air contained within a piston-cylinder is cooled until the piston, held in place by stops, just begins to move. State data are provided.

FIND: For the air, sketch the process on a p-V diagram and evaluate the heat transfer, in kJ.

SCHEMATIC & GIVEN DATA:

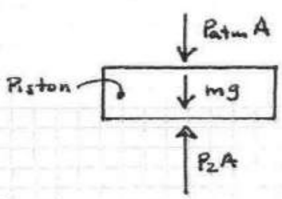


ENGINEERING MODEL:

1. The air within the piston-cylinder is the closed system.
2. The air is modeled as an ideal gas.
3. For the air kinetic and potential energy effects can be ignored.
4. The piston moves smoothly in the cylinder.
5. $g = 9.81\ m/s^2$

ANALYSIS: Since the volume of air remains constant until the piston just begins to move, $W = 0$. The energy balance reduces as follows: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - \cancel{W} \Rightarrow Q = \Delta U$, or $Q = m[u(T_2) - u(T_1)]$.

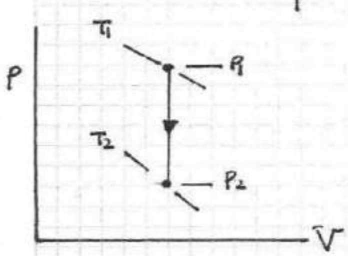
Applying the ideal gas equation of state: $P_1 V = mRT_1$ and $P_2 V = mRT_2$. Thus, $\frac{T_2}{T_1} = \frac{P_2}{P_1}$, where P_2 denotes the pressure of the air when the piston just begins to move. The pressure P_2 can be evaluated from a force balance on the smoothly moving piston:



$$\Rightarrow P_2 A = P_{atm} A + mg$$

$$\therefore \quad P_2 = P_{atm} + \frac{mg}{A}$$

$$P_2 = 1\,bar + \frac{(50\,kg)(9.81\,m/s^2)}{(9.75 \times 10^{-3}\,m^2)} \left| \frac{1\,N}{1\,kg \cdot m/s^2} \right| \left| \frac{1\,bar}{10^5\,N/m^2} \right| = 1.5\ bar$$



$$\Rightarrow T_2 = T_1 \left[\frac{P_2}{P_1} \right] = 600K \left[\frac{1.5\,bar}{3\,bar} \right] = 300K$$

$$\therefore \quad Q = m\,[u(T_2) - u(T_1)]$$

$$= \left(\frac{5}{1000}\,kg\right)\left[214.07 - 434.78\right] \frac{kJ}{kg} \qquad \text{(Data from Table A-22)}$$

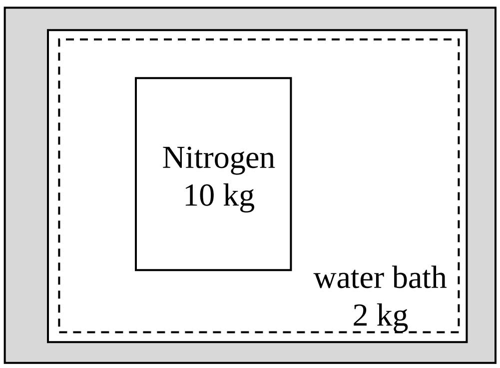
$$= -1.1\ kJ$$

Problem 3.133 Two kg of nitrogen (N_2) gas are contained in a closed, rigid tank surrounded by a 10-kg water bath, as shown in Fig. P1.133. Data for the initial states of the nitrogen and water are shown on the figure. The entire unit is well-insulated, and the nitrogen and water interact until thermal equilibrium is achieved. The measured final temperature is 34.1°C. The water can be modeled as an incompressible substance, with c = 4.179 kJ/kg·K, and the nitrogen is an ideal gas with constant c_v. From the measured data, determine the average value of the specific heat c_v, in kJ/kg·K.

KNOWN: A tank of nitrogen gas is surrounded by a well-insulated water bath. The gas and water are initially at different temperatures and they interact until equilibrium is achieved.

FIND: Determine the average values of the specific heat c_v for the nitrogen.

SCHEMATIC AND GIVEN DATA:



Nitrogen
T_{1,N_2} = 50°C

Water
$T_{1,w}$ = 20°C

ENGINEERING MODEL: (1) As shown on the accompanying sketch, the closed system consists of the nitrogen tank and the water bath. (2) W = 0 and Q = 0. (3) Kinetic and potential energy effects can be neglected. (4) The water is an incompressible substance with c_w = 4.179 kJ/kg·K. (5) The nitrogen is modeled as an ideal gas with constant c_v.

ANALYSIS: Reducing the energy balance

$$\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U_w + \Delta U_{N_2} = \cancel{Q} - \cancel{W} \longrightarrow \Delta U_w + \Delta U_{N_2} = 0$$

The final equilibrium temperature is T_2. With $\Delta U_w = m_w c_w (T_2 - T_{1,w})$ and $\Delta U_{N_2} = m_{N_2} c_v (T_2 - T_{1,N_2})$

$$m_w c_w (T_2 - T_{1,w}) + m_{N_2} c_v (T_2 - T_{1,N_2}) = 0$$

Solving for c_v

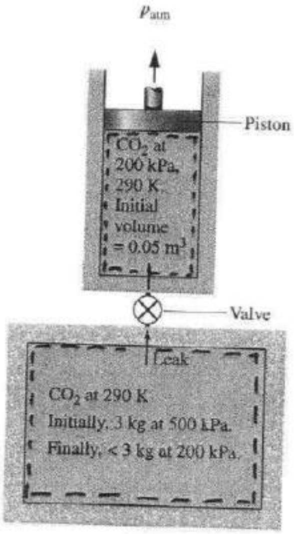
$$c_v = \frac{m_w c_w (T_{1,w} - T_2)}{m_{N_2}(T_2 - T_{1,N_2})} = \frac{(2\text{ kg})(4.179\text{ kJ/kg·K})(20 - 34.1)\text{K}}{(10\text{ kg})(34.1 - 50)\text{K}} = 0.741\text{ kJ/kg·K} \longleftarrow$$

PROBLEM 3.134

KNOWN: Data are provided for CO_2 contained in an assembly consisting of a piston-cylinder connected to a tank.

FIND: For the CO_2, evaluate Q and W.

SCHEMATIC & GIVEN DATA:



ENGR MODEL:

1. The total amount of CO_2 is the closed system.
2. The CO_2 is modeled as an ideal gas.
3. Volume change is the only work mode.
4. The CO_2 temperature remains constant.
5. The pressure of the CO_2 in the cylinder remains constant.
6. For the CO_2 there is no overall change in kinetic or potential energy.

ANALYSIS: Since volume change is the only work mode, $W = \int p\,dV$, where p is the pressure acting on the piston by the CO_2, which is 200 kPa. Thus

$$W = p\,\Delta V = \left(200\times10^3 \frac{N}{m^2}\right)\Delta V \qquad (1)$$

where ΔV is the change in volume of the CO_2.

Initial Volume: $V_{initial} = V_{cylinder} + V_{tank} = 0.05\,m^3 + \frac{m_T \bar{R} T}{P_T}$

$$= 0.05\,m^3 + \frac{(3\,kg)\left(\frac{8314}{44.01}\frac{N\cdot m}{kg\cdot K}\right)(290K)}{(500\,kPa)\left|\frac{10^3 N/m^2}{kPa}\right|}$$

$$= 0.379\,m^3$$

Final Volume: $V_{final} = \frac{m\bar{R}T}{p}$, where m is the total mass of CO_2. The initial mass in the tank is 3 kg. The initial mass in the cylinder is

$$m_{cyl} = \frac{p V_{cyl}}{RT} = \frac{(200\times10^3 N/m^2)(0.05\,m^3)}{\left(\frac{8314}{44.01}\frac{N\cdot m}{kg\cdot K}\right)(290K)} = 0.183\,kg.$$

So, $m = 3 + 0.183 = 3.183\,kg$. And

$$V_{final} = \frac{(3.183\,kg)\left(\frac{8314}{44.01}\frac{N\cdot m}{kg\cdot K}\right)(290K)}{(200\times10^3 N/m^2)} = 0.872\,m^3$$

Thus, Eq. (1) gives

$$W = \left(200\times10^3\frac{N}{m^2}\right)(0.872\,m^3 - 0.379\,m^3)\left|\frac{1\,kJ}{10^3 N\cdot m}\right| = 98.6\,kJ \longleftarrow W$$

An energy balance reads, $\Delta U + \Delta KE + \Delta PE = Q - W$, where $\Delta U = 0$ because the temperature of the CO_2 remains constant and the internal energy depends on temperature alone for an ideal gas. Thus

$$Q = W = 98.6\,kJ \longleftarrow Q$$

PROBLEM 3.135

KNOWN: Air contained within a closed rigid tank fitted with a paddle wheel experiences work and heat interactions. State data and operating data are provided.

FIND: Determine the final temperature of the air, in K.

SCHEMATIC & GIVEN DATA:

$\dot{W} = -1\ \text{kW}$

$\dot{Q} = +0.5\ \text{kW}$

$\Delta t = 5\ \text{min}$

$T_1 = 300\ \text{K}$

ENGINEERING MODEL:

1. The air within the tank is the closed system.
2. The air is modeled as an ideal gas.
3. For the process of the air there are no overall changes in kinetic or potential energy.

ANALYSIS:

Reducing an energy balance, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$

$$\Rightarrow \Delta U = Q - W$$

$$m(u(T_2) - u(T_1)) = Q - W$$

where

$$W = \int \dot{W}\,dt = \dot{W}\Delta t = (-1\ \text{kW})\left|\frac{1\ \text{kJ/s}}{1\ \text{kW}}\right|(5\ \text{min})\left|\frac{60\ \text{s}}{1\ \text{min}}\right|$$

$$= -300\ \text{kJ}$$

$$Q = \int \dot{Q}\,dt = \dot{Q}\Delta t = (+0.5\ \text{kW})\left|\frac{1\ \text{kJ/s}}{1\ \text{kW}}\right|(5\ \text{min})\left|\frac{60\ \text{s}}{1\ \text{min}}\right|$$

$$= +150\ \text{kJ}$$

Collecting results,

$$u(T_2) = u(T_1) + \frac{[Q - W]}{m}$$

$$= 214.07\ \frac{\text{kJ}}{\text{kg}} + \frac{[150 - (-300)]\ \text{kJ}}{2\ \text{kg}}$$

$$= 439.07\ \frac{\text{kJ}}{\text{kg}}$$

Interpolating in Table A-22 with $u(T_2) = 439.07\ \text{kJ/kg}$,

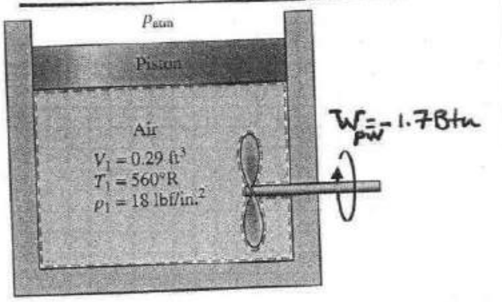
$$T_2 = 605.6\ \text{K} \longleftarrow$$

PROBLEM 3.136

KNOWN: Air within a piston-cylinder assembly fitted with a paddle wheel undergoes a process involving work but no heat transfer. State data and operating data are provided.

FIND: Determine the final temperature of the air, in °R.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The air within the piston-cylinder is the closed system.
2. The air is modeled as an ideal gas.
3. Since the piston moves smoothly in the cylinder the air undergoes a constant-pressure process. (confirmed with a force balance for the piston.)
4. For the air, $Q = 0$ and there are no significant effects of kinetic and potential energy.

ANALYSIS:

An energy balance reduces to give: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - W$

$\Rightarrow \Delta U = -W$ or $-\Delta U = W$.

where $W = W_{pw} + W_{piston}$. W_{piston} is the work done by the air on the piston. With assumption 3, $W_{piston} = \int p\,dV = p[V_2 - V_1]$

Collecting results,

$$-m[u(T_2) - u(T_1)] = [\underbrace{W_{pw}}_{= -1.7\,\text{Btu}} + pV_1[\tfrac{V_2}{V_1} - 1]] \qquad (1)$$

With the ideal gas model equation of state we get

$$m = \frac{pV_1}{RT_1} = \frac{(18 \times 144\ \text{lbf/ft}^2)(0.29\ \text{ft}^3)}{\left(\frac{1545}{28.97}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(560°\text{R})} = 0.025\ \text{lb}$$

and

$$\left.\begin{matrix} pV_1 = mRT_1 \\ pV_2 = mRT_2 \end{matrix}\right\rangle : \quad \frac{T_2}{T_1} = \frac{V_2}{V_1}$$

With these results, Eq. (1) becomes

$$-(0.025\ \text{lb})[u(T_2) - u(T_1)] = -1.7\ \text{Btu} + \left(2592\ \frac{\text{lbf}}{\text{ft}^2}\right)(0.29\ \text{ft}^3)\left[\frac{T_2}{T_1} - 1\right]\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

$$\boxed{\rightarrow \quad -(0.025\ \text{lb})[u(T_2) - u(T_1)] = -1.7\ \text{Btu} + (0.966\ \text{Btu})\left[\frac{T_2}{T_1} - 1\right] \qquad (2)}$$

Solving Eq. (2) using IT:

```
T1=560
u1 = u_T("Air",T1)
u2 = u_T("Air",T2)

- 0.025*(u2-u1)= 0.966*((T2/T1)-1)-1.7
```

(1)

Solving:

T2=840.8 R ←
u1=95.57 Btu/lb
u2=144.4 Btu/lb

---

1. Eq. (2) can be solved alternatively by iteration with Table A-22E data:

$$-(0.025\ \text{lb})[u(T_2) - 95.47] = (0.966)\left[\frac{T_2}{560} - 1\right] - 1.7$$

With this expression, trial values of T_2 are assumed. With each value, Table A-22 provides $u(T_2)$. The procedure continues until satisfactory closure is obtained. We get $T_2 \approx 841$°R.

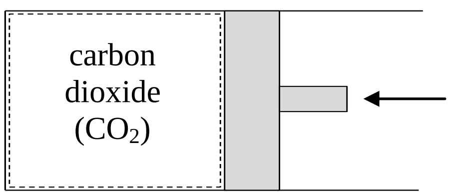
PROBLEM 3.137

Carbon dioxide (CO_2) is compressed in a piston-cylinder assembly from p_1 = 0.7 bar, T_1 = 280 K to p_2 = 11 bar. The initial volume is 0.262 m^3. The process is described by $pV^{1.25}$ = *constant*. Assuming ideal gas behavior and neglecting kinetic and potential energy effects, determine the work and heat transfer for the process, each in kJ, for (a) constant specific heats evaluated at 300 K, and (b) data from Table A-23. Compare the results and discuss.

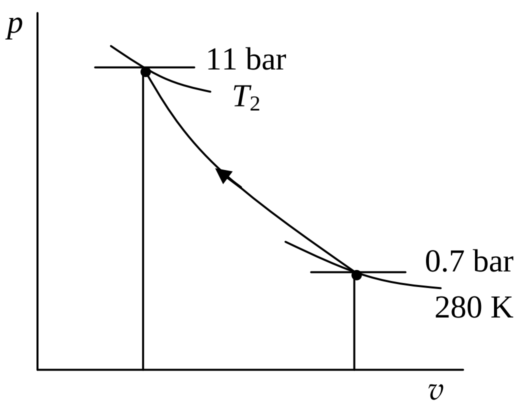
KNOWN: Data are provided for the polytropic compression of carbon dioxide in a piston-cylinder assembly.

FIND: Determine the work and heat transfer for the process using (a) constant specific heats, (b) data from Table A-23.

SCHEMATIC AND GIVEN DATA:



p_1 = 0.7 bar
T_1 = 280 K
V_1 = 0.262 m^3
$pV^{1.25}$ = *constant*
p_2 = 11 bar



ENGINEERING MODEL: (1) The carbon dioxide within the piston-cylinder assembly is a closed system. (2) The process is polytropic, with $pV^{1.25}$ = *constant*. (3) Volume change is the only work mode. (4) Kinetic and potential energy effects can be neglected. (5) The carbon dioxide is modeled as an ideal gas with (a) constant specific heats, and (b) with variable specific heats.

ANALYSIS: Work is evaluated from $W_{12} = \int_{V_1}^{V_2} p dV = \frac{(p_2V_2 - p_1V_1)}{1-1.25}$ (See Example 2.1 for details). For the polytropic process

$$V_2 = \left(\frac{p_1}{p_2}\right)^{1/1.25} V_1 = \left(\frac{0.7 \text{ bar}}{11 \text{ bar}}\right)^{1/1.25} (0.262 \text{ m}^3) = 0.02892 \text{ m}^3$$

The work is

$$W = \frac{(11 \text{ bar})(0.02892 \text{ m}^3) - (0.7)(0.262)}{1-1.25} \left|\frac{10^5 \frac{\text{N}}{\text{m}^2}}{1 \text{ bar}}\right| \left|\frac{1 \text{ kJ}}{10^3 \text{N}\cdot\text{m}}\right| = -\ 53.89 \text{ kJ (in)}$$

PROBLEM 3.137 (CONTINUED)

Reducing the energy balance gives: $Q_{12} = m(u_2 - u_1) + W_{12}$

The mass is

$$m = p_1V_1/RT_1 = \frac{(0.7\text{ bar})(0.262\text{ m}^3)}{\left(\frac{8.314}{44.01}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(280\text{ K})}\left|\frac{10^5\text{N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{N}\cdot\text{m}}\right| = 0.3467\text{ kg}$$

and the final temperature is

$$T_2 = p_2V_2/mR = \frac{(11\text{ bar})(0.02892\text{ m}^3)}{(0.3467\text{ kg})\left(\frac{8.314}{44.01}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)}\left|\frac{10^5\text{N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{N}\cdot\text{m}}\right| = 485.7\text{ K}$$

(a) From Table A-20 for CO_2 at 300 K; $c_v = 0.657$ kJ/kg·K. Using constant specific heats, the energy balance becomes

$$Q = mc_v(T_2 - T_1) + W_{12} = (0.3467\text{ kg})(0.657\text{ kJ/kg}\cdot\text{K})(485.7 - 280)\text{K} + (-53.89\text{ kJ})$$

$$= -7.035\text{ kJ (out)} \leftarrow$$

(b) Using data from Table A-23: $\bar{u}_1 = 6369$ kJ/kmol and interpolating; $\bar{u}_2 = 13004$ kJ/kmol

$$Q = m\left(\frac{\bar{u}(T_2)-\bar{u}(T_1)}{M}\right) + W_{12}$$

$$= (0.3467)\left(\frac{(13004-6369)\text{kJ/kmol}}{44.01\text{ kg/kmol}}\right) + (-53.89\text{ kJ}) = -1.62\text{ kJ (out)} \leftarrow$$

In this case, the assumption of constant specific heat results in a value for Q that differs from the variable specific heat value obtained using data from Table A-23 by approximately 334 %. The assumption of constant specific heat evaluated at 300 K is questionable in this case.

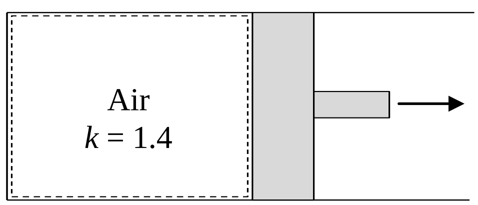
PROBLEM 3.138

Air is contained in a piston-cylinder assembly, initially at 40 lbf/in.^{2} and 660^{o}R. The air expands in a polytropic process with $n = k = 1.4$ until the volume is doubled. Modeling the air as an ideal gas with constant specific heats, determine (a) the final temperature, in ^{o}R, and pressure, in lbf/in.^{2}, and (b) the work and heat transfer, each in Btu per lb of air.

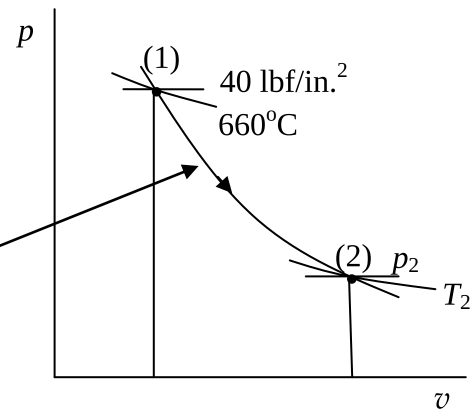
KNOWN: Data are provided for air undergoing a polytropic process while contained in a piston-cylinder assembly.

FIND: Determine the final temperature and pressure, and evaluate the work and heat transfer, each per unit mass of air present.

SCHEMATIC AND GIVEN DATA:



p_1 = 40 lbf/in.^{2}
T_1 = 660^{o}R
$V_2 = 2V_1$
$pv^k = constant$



ENGINEERING MODEL: (1) The air is the closed system.
(2) The process is polytropic, with $pv^k = constant$. (3) For the process, $\Delta KE = \Delta PE = 0$.
(4) The air is modeled as an ideal gas with constant specific heats and $k = 1.4$. (4) Volume change is the only work mode.

ANALYSIS: (a) For air as an ideal gas undergoing a polytropic process with $n = k$, we have from Sec. 3.15

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^k \quad \text{and} \quad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{(k-1)}$$

Thus

$$p_2 = p_1\left(\frac{V_1}{V_2}\right)^k = (40 \text{ lbf/in.}^2)(0.5)^{1.4} = 15.16 \text{ lbf/in.}^2 \longleftarrow$$

and

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{(k-1)} = (660^o\text{R})(0.5)^{(1.4-1)} = 500.2^o\text{R} \longleftarrow$$

(b) Since volume change is the only work mode, the work for the polytropic process can be expressed as $W = \int_{V_1}^{V_2} pdV = m\left[\frac{(p_2v_2 - p_1v_1)}{1-k}\right]$. With the ideal gas equation of state

$$W/m = \frac{R(T_2 - T_1)}{1-k} = \frac{\left(\frac{1.986}{28.97}\frac{\text{Btu}}{\text{lb}\cdot{}^o\text{R}}\right)(500.2-660)^o\text{R}}{1-1.4} = 27.39 \text{ Btu/lb} \longleftarrow$$

The energy balance reduces to $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$. Incorporating the above expression for work

$$Q/m = c_v\,(T_2 - T_1) + \frac{R(T_2 - T_1)}{1-k}$$

For an ideal gas; $c_v = R/(k\text{-}1)$ from Sec. 3.13.1. Thus

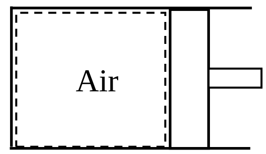
$$Q_{12}/m = \frac{R(T_2 - T_1)}{k-1} + \frac{R(T_2 - T_1)}{1-k} = 0$$

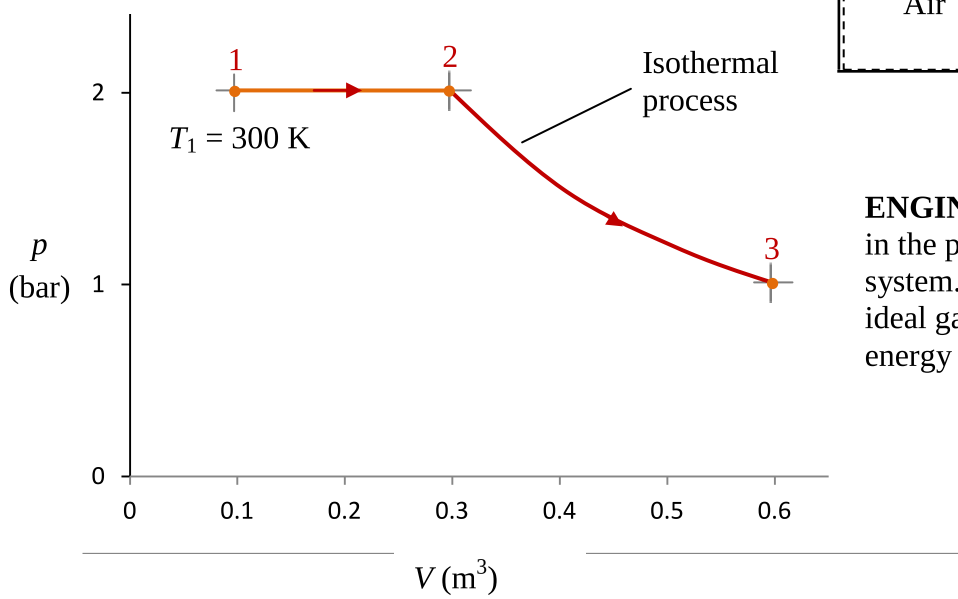
So, for an ideal gas with constant specific heats, the polytropic process with $n = k$ is adiabatic.

Problem 3.139 Air contained in a piston-cylinder assembly undergoes two processes in series , as shown in Fig. P3.139. Assuming ideal gas behavior for the air, the work and heat transfer for the overall process, each in kJ/kg.

KNOWN: Air contained in a piston-cylinder assembly undergoes two processes in series.

FIND: For the overall process of the air, find the work and the heat transfer, in kJ/kg.





ENGINEERING MODEL: (1) The air in the piston-cylinder is the closed system. (2) The air is modeled as an ideal gas. (3) Kinetic and potential energy effects are ignored.

ANALYSIS: For 1-2, the work determined using Eq. 2.17 is

pressure is constant

$$W_{12} = \int_{V_1}^{V_2} p dV = p(V_2 - V_1) = (2 \text{ bar})(0.3 - 0.1) \text{ m}^3\text{/kg} \left|\frac{10^5 \text{N/m}^2}{1 \text{ bar}}\right| \left|\frac{1 \text{ kJ}}{10^3 \text{N·m}}\right| = 40 \text{ kJ}$$

Using the ideal gas equation of state, the mass is

$$m = p_1V_1/RT_1 = \frac{(2 \text{ bar})(0.1 \text{ m}^3)}{\left(\frac{8.314 \text{ kJ}}{28.97 \text{kg·K}}\right)(300 \text{ K})} \left|\frac{10^5 \text{N/m}^2}{1 \text{ bar}}\right| \left|\frac{1 \text{ kJ}}{10^3 \text{N·m}}\right| = 0.2323 \text{ kg}$$

So, W_{12}/m = (40 kJ)/(0.2323 kg) = 172.2 kJ/kg

For 2-3, T is constant and $p = mRT/V$, so

$$W_{23} = \int_{V_2}^{V_3} p dV = mRT \int_{V_2}^{V_3} \frac{dV}{V} = mRT_2 \ln (V_3/V_2)$$

Problem 3.139 (Continued)

Process 1-2 is at constant pressure, so $\frac{RT_1}{V_1} = p_1 = p_2 = \frac{RT_2}{V_2} \longrightarrow T_2 = (V_2/V_1)T_1 = (0.3/0.1)(300\text{ K}) = 900\text{ K}$
Thus

$$W_{23}/m = RT_2 \ln (V_3/V_2) = (8.314/28.97\text{ kJ/kg·K})(900\text{ K}) \ln (0.6/0.3) = 179.0\text{ kJ/kg}$$

The total work is $W/m = W_{12}/m + W_{23}/m = 172.2 + 179.0 = 351.2$ kJ/kg ←

An energy balance for the overall process is obtained as follows: $\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = Q - W$

$$Q = \Delta U + W = m(u_3 - u_1) + W \longrightarrow Q/m = (u_3 - u_1) + W/m$$

With data from Table A-22

1 $Q/m = (674.58 - 214.07)\text{kJ/kg} + 351.2\text{ kJ/kg} = 811.7\text{ kJ/kg}$ ←

---

1 Note that the heat transfers Q_{12} and Q_{23} could be determined from the respective energy balances, as follows

$$Q_{12} = (u_2 - u_1) + W_{12}/m = (674.58 - 214.07) + (172.2) = 632.7\text{ kJ/kg}$$

and

$$Q_{23} = (u_3 - u_2) + W_{23}/m = (0) + 179.0 = 179.0\text{ kJ/kg}$$

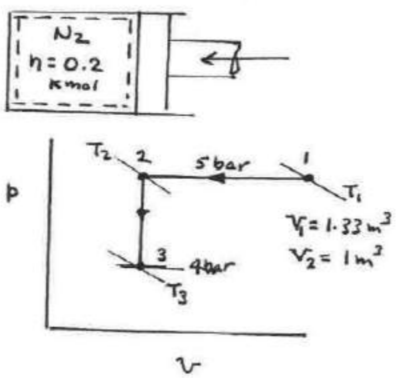
So, the total for the overall process is $Q = Q_{12} + Q_{23} = 632.7 + 179.0 = 811.7$ kJ/kg which is the same result as above, as expected.

PROBLEM 3.140

KNOWN: Data are provided for N_2 contained in a piston-cylinder assembly. The N_2 undergoes two processes in series.

FIND: For each process find W and Q, each in kJ.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The N_2 contained within the piston-cylinder assembly is the closed system.
2. Volume change is the only work mode.
3. The N_2 is modeled as an ideal gas.
4. Kinetic and potential energy effects play no role.

ANALYSIS: Process 1-2 occurs at constant pressure. Thus $W_{12} = \int_1^2 p\,dV = p[V_2 - V_1]$. That is

$$W_{12} = (5\text{ bar})\left[1\text{ m}^3 - 1.33\text{ m}^3\right]\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| = -165\text{ kJ} \longleftarrow W_{12}$$

Process 2-3 occurs at constant volume. Thus $W_{23} = 0$. $\longleftarrow W_{23}$

Reducing an energy balance, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$. Thus $Q = W + n\Delta\bar{u}$. This requires T_1, T_2, T_3. To find T_1 use the ideal gas equation of state:

$$T_1 = \frac{p_1 V_1}{n\bar{R}} = \frac{\left(5\times10^5 \frac{\text{N}}{\text{m}^2}\right)(1.33\text{ m}^3)}{(0.2\text{ kmol})\left(\frac{8314\text{ N}\cdot\text{m}}{\text{kmol}\cdot\text{K}}\right)} = 400\text{K}.$$

For process 1-2, p = constant; so the ideal gas equation of state gives

$pV_1 = n\bar{R}T_1$, $pV_2 = n\bar{R}T_2$, or

$$T_2 = T_1\left[\frac{V_2}{V_1}\right] = 400\left[\frac{1\text{ m}^3}{1.33\text{ m}^3}\right] = 301\text{K}$$

For process 2-3, V = constant, so $p_2 V = n\bar{R}T_2$, $p_3 V = n\bar{R}T_3$, or

$$T_3 = \left(\frac{p_3}{p_2}\right)T_2 = \left(\frac{4\text{ bar}}{5\text{ bar}}\right)(301\text{ K}) = 241\text{ K}$$

For process 1-2, with $\bar{u}_1$ and $\bar{u}_2$ from Table A-23

$$Q_{12} = W_{12} + n[\bar{u}_2 - \bar{u}_1] = -165\text{ kJ} + 0.2\text{ kmol}\,[6250 - 8314]\frac{\text{kJ}}{\text{kmol}}$$
$$= -578\text{ kJ} \longleftarrow Q_{12}$$

For process 2-3 $Q_{23} = \cancel{W_{23}} + n[\bar{u}_3 - \bar{u}_2] = 0.2\text{ kmol}\,[5000 - 6250]\frac{\text{kJ}}{\text{kmol}}$

$$= -250\text{ kJ} \longleftarrow Q_{23}$$

PROBLEM 3.141

KNOWN: Data are provided for air in a piston-cylinder assembly. The air undergoes two processes in series:

Process 1-2: Constant-temperature expansion until the volume is twice the initial volume.

Process 2-3: Constant-volume heating until the pressure is again 0.5 MPa.

FIND: Sketch the processes in series on a p-v diagram. Determine P_2, T_3, and W and Q for each process, in kJ.

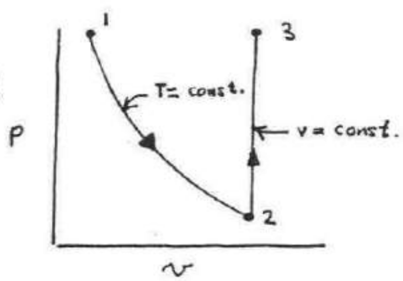
SCHEMATIC & GIVEN DATA:

$T_1 = 227°C$

$P_1 = P_3 = 0.5 MPa$

$V_2 = 2V_1 \Rightarrow$

$v_2 = 2v_1$.



ENGR. MODEL

1. The air within the piston-cylinder assembly is the closed system.
2. Volume change is the only work mode.
3. The air is modeled as an ideal gas.
4. Kinetic and potential energy play no significant role.

ANALYSIS: (a) For process 1-2, $P_1 v_1 = RT_1$ and $P_2 v_2 = RT_2$, where $v_2 = 2v_1$ and $T_2 = T_1$.

Thus, $P_2 v_2 = P_1 v_1$ and

$$P_2 = P_1\left(\frac{v_1}{v_2}\right) = 0.5\,MPa\left(\frac{1}{2}\right) = 0.25\,MPa \quad \longleftarrow P_2$$

(b) For process 2-3, $P_2 v_2 = RT_2$ and $P_3 v_3 = RT_3$, where $v_3 = v_2$. Thus,

$$\frac{T_3}{T_2} = \frac{P_3}{P_2} \Rightarrow T_3 = T_2\left(\frac{\overset{=P_1}{P_3}}{P_2}\right) = 500K\left(\frac{0.5\,MPa}{0.25\,MPa}\right) = 1000\,K \quad \longleftarrow T_3$$

(c) For Process 1-2, $W_{12} = \int_1^2 p\,dV = m\int_1^2 \underbrace{p}_{=\frac{RT}{v}\ (T \text{ is constant})}\,dv = mRT\int_{v_1}^{v_2}\frac{1}{v}\,dv$. Thus

$$W_{12} = m\,RT \ln\frac{v_2}{v_1} = (1\,kg)\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(500K)\ln 2 = 99.46\,kJ \quad \longleftarrow W_{12}$$

Reducing an energy balance, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{12} - W_{12}$, where $\Delta U = m\,(u(T_2) - u(T_1)) = 0$, since $T_1 = T_2$. Accordingly, $Q_{12} = W_{12} = 99.46\,kJ$ $\longleftarrow Q_{12}$

(d) For Process 2-3, the piston does not move (volume is constant). Thus, $W_{23} = 0$. $\longleftarrow W_{23}$

Reducing an energy balance, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q_{23} - \cancel{W_{23}}$. Or

$$Q_{23} = m[\,u(T_3) - u(T_2)\,]$$

With data from Table A-22,

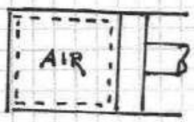
$$Q_{23} = (1\,kg)[\,758.94 - 359.49\,]\frac{kJ}{kg} = 399.45\,kJ \quad \longleftarrow Q_{23}$$

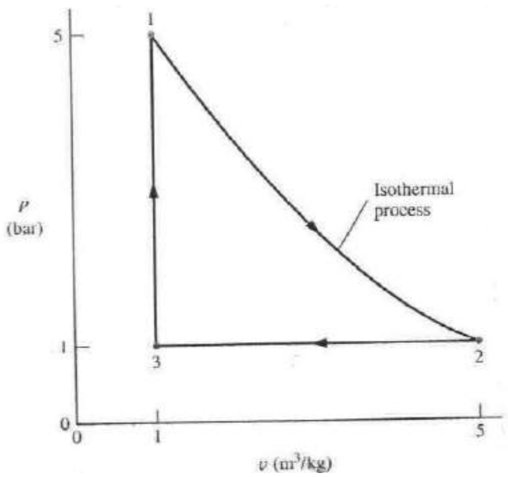
PROBLEM 3.142

KNOWN: Air within a piston-cylinder assembly undergoes a power cycle. State data are provided.

FIND: Evaluate the thermal efficiency of the cycle.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL:

1. The air within the piston-cylinder is the closed system.
2. The air is modeled as an ideal gas.
3. Kinetic and potential energy effects are ignored.

ANALYSIS:

The thermal efficiency of a power cycle is given by Eq. 2.42 and Eq. 2.43:

$$\eta = \frac{W_{cycle}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}} \qquad (1)$$

where the energy transfer terms are positive in the directions of the arrows on Fig. 2.17(a).

Next, we evaluate the work and heat transfer for each of the three processes of the cycle.

Process 1-2:

constant; Using $p_1V_1 = mRT$

$$W_{12} = \int_1^2 p\,dV = \int_1^2 \frac{mRT}{V}\,dV = mRT \ln\frac{V_2}{V_1} = p_1 V_1 \ln\frac{V_2}{V_1} \Rightarrow W_{12}/m = p_1 v_1 \ln\frac{v_2}{v_1}$$

$$\therefore \frac{W_{12}}{m} = \left(5\times10^5 \frac{N}{m^2}\right)\left(1\frac{m^3}{kg}\right)\left|\frac{1\ kJ}{10^3\ N\cdot m}\right| \ln\left(\frac{5}{1}\right) = 804.7\ \frac{kJ}{kg}$$

An energy balance reduces as follows: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$

$$\Rightarrow \Delta U = Q - W.$$

For process 1-2, temperature is constant. Thus with the ideal gas model

$$\Delta U = 0 \Rightarrow Q_{12}/m = W_{12}/m = 804.7\ kJ/kg$$

Process 2-3:

constant

$$W_{23} = \int_2^3 p\,dV = p\,[V_3 - V_2] \Rightarrow W_{23}/m = p(v_3 - v_2)$$

$$\therefore \frac{W_{23}}{m} = \left(1\times10^5 \frac{N}{m^2}\right)(1-5)\frac{m^3}{kg}\left|\frac{1\ kJ}{10^3\ N\cdot m}\right| = -400\ \frac{kJ}{kg}$$

The energy balance gives, $\frac{Q_{23}}{m} = [u(T_3) - u(T_2)] + \frac{W_{23}}{m}$

Using the ideal gas model equation of state, T_2 and T_3 can be determined:

$$T_2 = \frac{p_2 v_2}{R} = \frac{(10^5 N/m^2)(5\ m^3/kg)}{\left(\frac{8314}{28.97}\ \frac{N \cdot m}{kg \cdot K}\right)} = 1742.2\ K$$

Table A-22 gives $u_2 = 1432.5\ kJ/kg$

$$T_3 = \frac{p_3 v_3}{R} = \frac{(10^5 N/m^2)(1\ m^3/kg)}{\left(\frac{8314}{28.97}\ \frac{N \cdot m}{kg \cdot K}\right)} = 348.4\ K$$

Table A-22 gives $u_3 = 248.9\ kJ/kg$

Thus, $Q_{23}/m = (248.9 - 1432.5)\ kJ/kg + (-400\ kJ/kg) = -1583.6\ kJ/kg$

Process 3-1: Volume is constant.

$$W_{31} = \int p \, \cancelto{0}{dV} = 0.$$ The energy balance reduces to $Q_{31}/m = \underbrace{u_1}_{= u_2} - u_3$

$$\therefore \quad \frac{Q_{31}}{m} = (1432.5 - 248.9)\frac{kJ}{kg} = 1183.6\ \frac{kJ}{kg}$$

Collecting results,

Process	W/m	Q/m
1-2	804.7	804.7
2-3	− 400	−1583.6
3-1	0	1183.6
TOTAL	404.7 kJ/kg	404.7 kJ/kg

Agrees with the cycle energy balance, Eq. 2.40

Returning to the expression for the thermal efficiency, Eq. (1),

$$W_{cycle}/m = 404.7\ kJ/kg$$

$$\begin{aligned} Q_{in}/m &= Q_{12}/m + Q_{31}/m \\ &= (804.7 + 1183.6)\ kJ/kg \\ &= 1988.3\ kJ/kg \end{aligned}$$

Thus,

$$\eta = \frac{404.7}{1988.3} = 0.204 \quad (20.4\%)$$

PROBLEM 3.143

One pound mass of air undergoes a cycle consisting of the following processes:

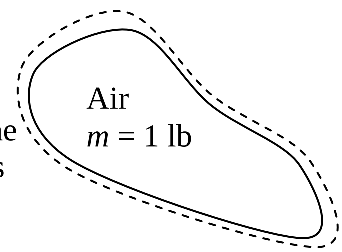
Process 1-2: constant-pressure expansion with p = 20 lbf/in.^{2} from T_1 = 500°R to $v_2 = 1.4\,v_1$
Process 2-3: adiabatic compression to $v_3 = v_1$ and T_3 = 820°R
Process 3-1: constant-volume process

Sketch the cycle on a carefully-labeled p- v diagram. Assuming ideal gas behavior, determine the energy transfers by heat and work for each process, in Btu.

KNOWN: Air undergoes a cycle consisting of three processes.

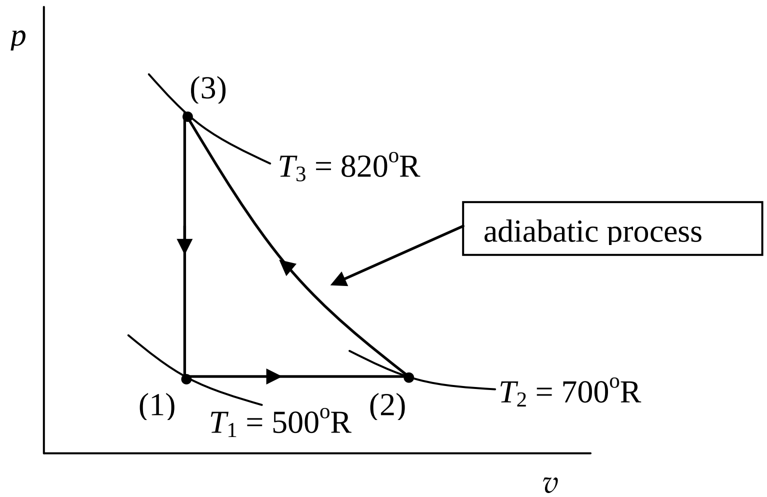
FIND: Sketch the cycle on a p- v diagram and calculate the energy transfers by heat and work for each process.

SCHEMATIC AND GIVEN DATA: The following data are given for each process:
Process 1-2: constant-pressure expansion with p = 20 lbf/in.^{2} from T_1 = 500°R to $v_2 = 1.4\,v_1$
Process 2-3: adiabatic compression to $v_3 = v_1$ and T_3 = 820°R
Process 3-1: constant-volume process



ENGINEERING MODEL: (1) the air is a closed system. (2) The air behaves as an ideal gas. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each principal state. For Process 1-2, the pressure is constant, so $T_2 = (v_2/\,v_1)T_1$ = (1.4)(500) = 700°R. The processes are shown on the accompanying p- v diagram:



Process 1-2 (constant pressure) $\longrightarrow$ $W_{12} = \int_{V_1}^{V_2} p\,dV = p(V_2 - V_1) = mR(T_2 - T_1)$

$$W_{12} = (1\text{ lb})\left(\frac{1545}{28.97}\ \frac{\text{ft·lbf}}{\text{lb·°R}}\right)(700 - 500)\left|\frac{1\text{ Btu}}{778\text{ ft·lbf}}\right| = 13.71\text{ Btu (out)}$$

PROBLEM 1.143 (CONTINUED)

Energy balance: $\Delta \cancel{KE} + \Delta \cancel{PE} + \Delta U = Q_{12} - W_{12}$. With data from Table A-22

$$Q_{12} = m(u_2 - u_1) + W_{12} = (1 \text{ lb})(119.58 - 85.20)\text{Btu/lb} + (13.71 \text{ Btu}) = 48.09 \text{ Btu}$$

Process 2-3: (adiabatic) $Q_{23} = 0$

$$W_{23} = -\, m(u_3 - u_2) = -\,(1 \text{ lb})(140.47 - 119.58)\text{Btu/lb} = -\,20.89 \text{ Btu (in)}$$

Process 3-1: (constant volume) $W_{31} = 0$

$$Q_{31} = m(u_1 - u_3) = (1 \text{ lb})(85.20 - 140.47)\text{Btu/lb} = -55.27 \text{ Btu (out)}$$

In Summary

$W_{cycle} = W_{12} + W_{23} + W_{31} = 13.71 + (-20.89) + 0 = -\,7.18$ Btu

$Q_{cycle} = Q_{12} + Q_{23} + Q_{31} = 48.09 + 0 + (-55.27) = -7.18$ Btu

$W_{cycle} = Q_{cycle}$ (as expected)

Note: The cycle is a refrigeration/heat pump cycle.

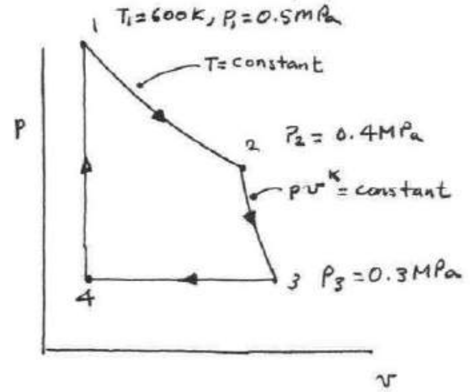
PROBLEM 3.144

KNOWN: Air contained within a piston-cylinder assembly undergoes a thermodynamic cycle consisting of four processes in series.

FIND: Sketch the cycle on p-v coordinates. Evaluate the work and heat transfer for each process and the thermal efficiency of the overall cycle.

SCHEMATIC & GIVEN DATA:

Process 1-2: Constant-temperature expansion at 600 K from $p_1 = 0.5$ MPa to $p_2 = 0.4$ MPa.
Process 2-3: Polytropic expansion with $n = k$ to $p_3 = 0.3$ MPa.
Process 3-4: Constant-pressure compression to $V_4 = V_3$.
Process 4-1: Constant-volume heating.



ENGR. MODEL:

1. The air is the closed system.
2. The air is modeled as an ideal gas with constant specific heat ratio, $k = 1.4$.
3. For each process $\Delta KE = \Delta PE = 0$.
4. Volume change is the only work mode.

ANALYSIS: (a) Each process is considered in turn:

Process 1-2: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$. Since the temperature remains constant and $u = u(T)$ for an ideal gas, $\Delta U = 0$, leaving $Q = W$. Using Eq. 2.17

$$W_{12} = \int_1^2 p\,dV = \int_1^2 \frac{mRT}{V}\,dV = mRT\ln\frac{V_2}{V_1}.$$ Thus, using $V = mRT/p$

$$\frac{Q_{12}}{m} = \frac{W_{12}}{m} = RT\ln\frac{V_2}{V_1} = RT\ln\left[\frac{mRT/P_2}{mRT/P_1}\right] = RT\ln\frac{P_1}{P_2}$$

$$= \left(\frac{8.314}{28.97}\,\frac{kJ}{kg\cdot K}\right)(600\,K)\ln\frac{0.5\,MPa}{0.4\,MPa} = 38.42\,\frac{kJ}{kg} \qquad \longleftarrow W_{12}, Q_{12}$$

Process 2-3: When $n = k$, integration of Eq. 2.17 gives $W = \frac{P_3V_3 - P_2V_2}{1-k}$ (see Example 2.1).

Then, with $pV = mRT$, this gives

$$\frac{W_{23}}{m} = \frac{R(T_3 - T_2)}{1-k}, \quad \text{where (Eq. 3.56)} \quad \frac{T_3}{T_2} = \left(\frac{P_3}{P_2}\right)^{\frac{k-1}{k}} \Rightarrow T_3 = 600\,K\left(\frac{0.3}{0.4}\right)^{\frac{1.4-1}{1.4}} = 553\,K$$

Then

$$\frac{W_{23}}{m} = \left(\frac{8.314}{28.97}\,\frac{kJ}{kg\cdot K}\right)\left(\frac{553-600}{1-1.4}\right)K = 33.72\,\frac{kJ}{kg} \qquad \longleftarrow W_{23}$$

Energy balance: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W \Rightarrow Q = \Delta U + W$.
Since k is constant, c_v is constant (Eq. 3.47a). Then, with Eq. 3.50 and the expression for W_{23}/m above,

$$\frac{Q_{23}}{m} = \underbrace{c_v}_{=\frac{R}{k-1}}(T_3 - T_2) + \frac{R(T_3 - T_2)}{1-k} \equiv 0 \qquad \longleftarrow Q_{23}$$

PROBLEM 3.144 (CONTINUED)

Process 3-4. Since pressure is constant, $W_{34} = p_3[V_4 - V_3]$. Using $pV = mRT$, this becomes

$$\frac{W_{34}}{m} = R[T_4 - T_3]$$

Also, $\left.\begin{matrix} p_4 V_4 = mRT_4 \\ p_1 V_1 = mRT_1 \end{matrix}\right\}$ $V_4 = V_1 \Rightarrow \frac{T_4}{T_1} = \frac{p_4}{p_1} \Rightarrow T_4 = \left(\frac{p_4}{p_1}\right)T_1 = 600\,K\left(\frac{0.3\,MPa}{0.5\,MPa}\right)$
$= 360\,K$

So, $$\frac{W_{34}}{m} = \left(\frac{8.314}{28.97}\frac{kJ}{kg \cdot K}\right)(360 - 553)K = -55.39\frac{kJ}{kg} \quad \leftarrow \frac{W_{34}}{}$$

Energy Balance: $\Delta U = Q - W \Rightarrow Q = \Delta U + W$. Since c_v is constant

$$\frac{Q_{34}}{m} = c_v(T_4 - T_3) + (W_{34}/m)$$
$$\boxed{c_v = \frac{R}{k-1} = \frac{(8.314/28.97)}{1.4-1}\frac{kJ}{kg \cdot K} = 0.717\frac{kJ}{kg \cdot K}}$$

$$= \left(0.717\frac{kJ}{kg \cdot K}\right)(360 - 553)K - 55.39\frac{kJ}{kg} = -193.77\frac{kJ}{kg} \quad \leftarrow \frac{Q_{34}}{}$$

Process 4-1. Since the piston does not move (volume is constant), $W_{41} = 0$. $\leftarrow \frac{W_{41}}{}$

Energy Balance: $\Delta U = Q - \cancel{W} \Rightarrow Q = \Delta U$. Since c_v is constant

$$\frac{Q_{41}}{m} = c_v(T_1 - T_4) = 0.717\frac{kJ}{kg \cdot K}(600 - 360) = 172.08\frac{kJ}{kg} \quad \leftarrow \frac{Q_{41}}{}$$

(b) For any cycle, the thermal efficiency is found from $\eta = W_{cycle}/Q_{in}$. Here,

$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{(W_{12} + W_{23} + W_{34} + W_{41})/m}{(Q_{12} + Q_{41})/m}$$

1 $$= \frac{38.42 + 33.72 - 55.39 + 0}{38.42 + 172.08} = \frac{16.75}{210.5} = 0.08 \; (8\%) \leftarrow \frac{\eta}{}$$

---

1. For any cycle, $W_{cycle} = Q_{cycle}$. In this case, $\frac{W_{cycle}}{m} = 16.75\,kJ/kg$.

And $$Q_{cycle} = Q_{12} + Q_{23} + Q_{34} + Q_{41} = 38.42 + 0 + (-193.77) + 172.08$$
$$= 16.73\frac{kJ}{kg}$$

which agrees within round off and provides a check on the calculations performed for the four processes.

PROBLEM 3.145

One lb of oxygen, O_2, undergoes a power cycle consisting of the following processes:

Process 1-2: Constant-volume from $p_1 = 20$ lbf/in.^{2}, T_1=500°R to T_2=820°R

Process 2-3: Adiabatic expansion to $v_3 = 1.432v_2$

Process 3-1: Constant-pressure compression to state 1.

Sketch the cycle on a *p-v* diagram. Assuming ideal gas behavior, determine

(a) the pressure at state 2, in lbf/in.^{2}

(b) the temperature at state 3, in °R.

(c) the heat transfer and work, each in Btu, for all processes.

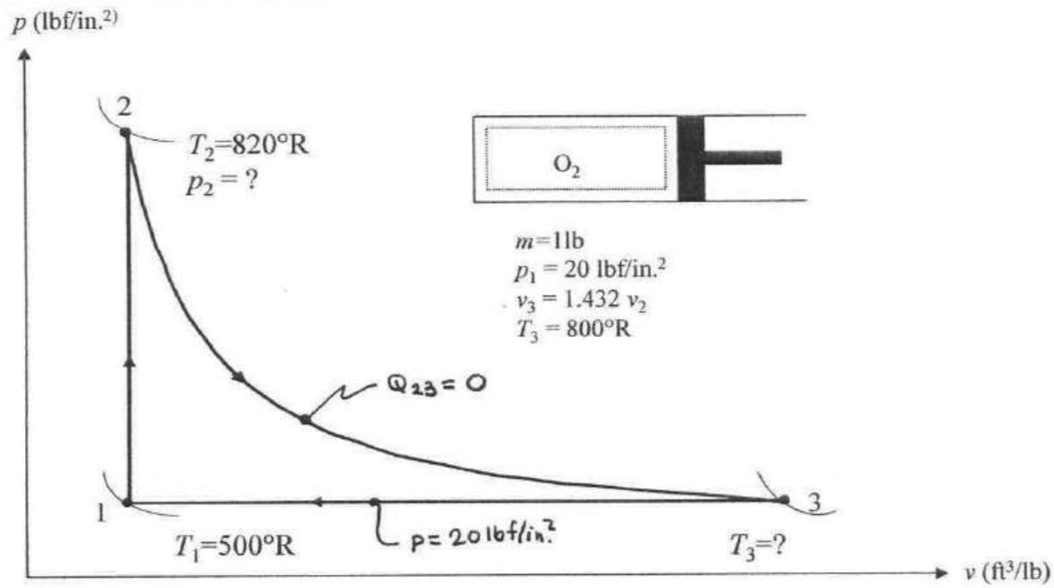
(d) the thermal efficiency of the cycle.

Solution:

Known: Oxygen undergoes a power cycle consisting of the three processes.

Find: Sketch the cycle on a *p-v* diagram and determine p_2 in lbf/in.^{2}; T_3 in °R; the heat transfer and work, each in Btu, for all processes; and the thermal efficiency of the cycle.

Schematic and Given Data:



Engineering Model:

(1) The oxygen is a closed system.

(2) The oxygen behaves as an ideal gas.

(3) Kinetic and potential energy effects are negligible.

(4) Volume change is the only work mode.

Analysis:

(a) For process 1 to 2, the volume is constant ($v_1 = v_2$) and for process 3 to 1, the pressure is constant ($p_1 = p_2$). Using the ideal gas equation of state:

$$p_1 v = RT_1 \quad \text{and} \quad p_2 v = RT_2$$

$$v = \frac{RT_1}{p_1} = \frac{RT_2}{p_2} \quad \text{or} \quad \frac{T_1}{p_1} = \frac{T_2}{p_2}$$

$$p_2 = \frac{p_1 T_2}{T_1} = \frac{\left(20\frac{\text{lbf}}{\text{in.}^2}\right)(820^\circ\text{R})}{(500^\circ\text{R})} = 32.8\frac{\text{lbf}}{\text{in.}^2}$$

(b) Again, using the ideal gas equation of state:

$p_1 v_1 = RT_1$ and $p_3 v_3 = RT_3$ since $p_1 = p_3$

$$\frac{T_3}{T_1} = \frac{v_3}{v_1} = 1.432$$

$$T_3 = (1.432)T_1 = 716^\circ \text{R}$$

(c) Determine the heat transfer and work, each in Btu, for each process.

Process 1 to 2: constant volume and therefore $W_{12} = 0$.
Using the energy balance and property data from Table A-1E and A-23E:

$$Q_{12} = W_{12} + \frac{m}{M}(\bar{u}_2 - \bar{u}_1) = (0) + \frac{1\text{ lb}}{32\frac{\text{lb}}{\text{lbmol}}}(4119.7 - 2473.2)\frac{\text{Btu}}{\text{lbmol}} = 51.45\text{ Btu}$$

Process 2 to 3: adiabatic expansion and therefore $Q_{23} = 0$.
Using the energy balance and property data from Table A-1E and A-23E:

$$W_{23} = -\frac{m}{M}(\bar{u}_3 - \bar{u}_2) = -\frac{1\text{ lb}}{32\frac{\text{lb}}{\text{lbmol}}}(3572.3 - 4119.7)\frac{\text{Btu}}{\text{lbmol}} = 17.11\text{ Btu}$$

Process 3 to 1: constant-pressure compression. Using Eq. 2.17 to determine W_{31}

$$W_{31} = m\int_{v_3}^{v_1} p\,dv = mp(v_1 - v_3) = mR(T_1 - T_3)$$

$$W_{31} = 1\text{ lb}\left(\frac{1545}{32}\frac{\text{ft}\cdot\text{lbf}}{\text{lb-}^\circ\text{R}}\right)\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right|(500 - 716)^\circ\text{R} = \text{-}13.40\text{ Btu}$$

Using the energy balance and property data from Table A-1E and A-23E:

$$Q_{31} = W_{31} + \frac{m}{M}(\bar{u}_1 - \bar{u}_3) = (\text{-}13.40\text{ Btu}) + \frac{(1\text{ lb})(2473.2 - 3572.3)\frac{\text{Btu}}{\text{lbmol}}}{32\frac{\text{lb}}{\text{lbmol}}} = -47.75\text{ Btu}$$

(d) Determine the thermal efficiency as follows:

#1 $$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{W_{12} + W_{23} + W_{31}}{Q_{12}} = \frac{0 + 17.11 + (-13.40)}{51.45} = 7.2\%$$

---

1. Check the above calculations using $Q_{cycle} = W_{cycle}$.

$$Q_{cycle} = Q_{12} + Q_{23} + Q_{31} = 3.7\text{ Btu}$$
$$W_{cycle} = W_{12} + W_{23} + W_{31} = 3.7\text{ Btu}$$

PROBLEM 3.146

A system consisting of 2 kg of carbon dioxide (CO_2) gas initially at p_1 = 1 bar, T_1 = 300 K, undergoes a power cycle with the following processes:

Process 1-2: constant volume to p_2 = 4 bar
Process 2-3: expansion with $pv^{1.28}$ = *constant*
Process 3-1: constant-pressure compression

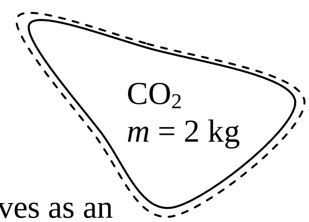
Assuming the ideal gas model and neglecting kinetic and potential energy effects,
(a) sketch the cycle on a *p-v* diagram and calculate the thermal efficiency.
(b) plot thermal efficiency versus p_2/p_1 ranging from 1.05 to 4.

KNOWN: A system consisting of carbon dioxide gas undergoes a power cycle made up of three processes.

FIND: (a) Sketch the cycle on a *p-v* diagram and calculate the thermal efficiency of the cycle. (b) Plot the thermal efficiency versus p_2/p_1 ranging from 1.05 to 4.

SCHEMATIC AND GIVEN DATA: The following data are known for each process

Process 1-2: constant volume from p_1 = 1 bar, T_1 = 300 K, to p_2 = 4 bar
Process 2-3: expansion with $pv^{1.28}$ = *constant*
Process 3-1: constant-pressure compression



ENGINEERING MODEL: (1) The CO_2 is a closed system. (2) The CO_2 behaves as an ideal gas. (3) Process 2-3 is polytropic with $pv^{1.28}$ = *constant*. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: (a) First, for Process 1-2: $V_2 = V_1 \longrightarrow T_2 = (p_2/p_1)T_1 = (4/1)(300\text{K}) = 1200$ K. Further

$$V_2 = V_1 = \frac{mRT_1}{p_1} = \frac{(2\text{ kg})\left(\frac{8.314}{44.01}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(300\text{K})}{(1\text{ bar})}\left|\frac{1\text{ bar}}{10^5\text{N/m}^2}\right|\left|\frac{10^3\text{N}\cdot\text{m}}{1\text{ kJ}}\right| = 1.134\text{ m}^3\text{/kg}$$

Using the given *p-v* relation for Process 2-3 with $v_2 = V_2/m$ = 0.567 m^3/kg, and noting that $p_3 = p_1$ =1 bar

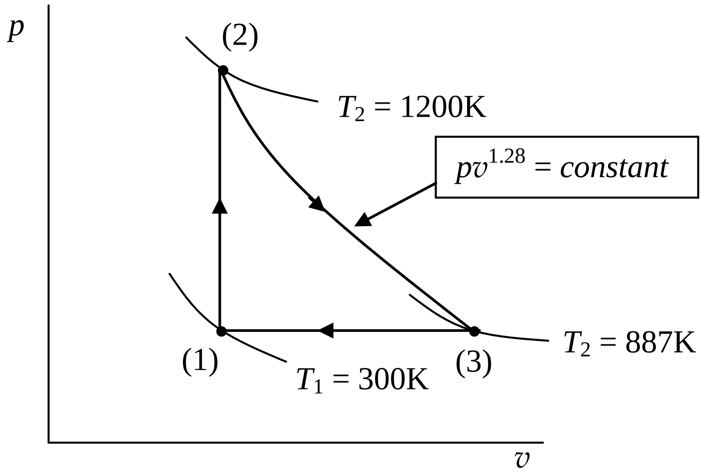
$$v_3 = \left(\frac{p_2}{p_3}\right)^{\frac{1}{1.28}} v_2 = \left(\frac{4\text{ bar}}{1\text{ bar}}\right)^{\frac{1}{1.28}}(0.567\text{ m}^3\text{/kg}) = 1.675\text{ m}^3\text{/kg}$$

and

$$T_3 = \frac{p_3 v_3}{R} = \frac{(1)(1.675)}{\left(\frac{8.314}{44.01}\right)}\left|\frac{10^5}{10^3}\right| = 887\text{K}$$

The *p-v* diagram is

PROBLEM 1.143 (CONTINUED) – PAGE 2



The thermal efficiency is $\eta = W_{cycle}/Q_{in}$, where W_{cycle} is the *net* work of the cycle and Q_{in} is the total heat transfer *into* the system during the cycle. Next, each process is analyzed.

Process 1-2: $W_{12} = 0$ (constant volume)

$$\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = Q_{12} - \cancel{W_{12}} \longrightarrow Q_{12} = m(u_2 - u_1)$$

With data from Table A-23

$$Q_{12} = (2\text{ kg})\left[\frac{(43871-6939)\text{kJ/kmol}}{(44.01\text{ kg/kmol})}\right] = 1678\text{ kJ (in)}$$

Process 2-3: The work is determined using Eq. 2.17 and the given p-v relation

$$W_{23} = \int_{V_2}^{V_3} p dV = m\int_{v_2}^{v_3}\frac{const}{v^{1.28}}dv = m\left(\frac{p_3v_3-p_2v_2}{1-1.28}\right) \overset{\text{Ideal gas}}{=} \frac{mR(T_3-T_2)}{1-1.28}$$

$$= \frac{(2\text{ kg})\left(\frac{8.314}{44.01}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(887\text{K}-1200\text{K})}{1-1.28} = 422.4\text{ kJ (out)}$$

Using the energy balance with $\Delta KE = \Delta PE = 0$: $m(u_3 - u_2) = Q_{23} - W_{23}$

$$Q_{23} = m\left(\frac{\bar{u}_3-\bar{u}_2}{M}\right) + W_{23} = (2\text{ kg})\frac{(29343-43871)\text{kJ/kmol}}{(44.01\frac{\text{kg}}{\text{kmol}})} + (422.4\text{ kJ}) = -237.8\text{ kJ (out)}$$

Process 3-1: Using Eq. 2.17

$$W_{31} = \int_{V_3}^{V_1} p dV = mp_3(v_1 - v_3) = mR(T_1 - T_3) = (2\text{ kg})\left(\frac{8.314}{44.01}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(300-887)\text{K}$$

$$= -221.8\text{ kJ (in)}$$

The energy balance reduces to

$$Q_{31} = m\left(\frac{\bar{u}_1-\bar{u}_3}{M}\right) + W_{31} = (2\text{ kg})\left[\frac{(6939-29343)\text{kJ/kmol}}{(44.01\frac{\text{kg}}{\text{kmol}})}\right] + (-221.8\text{ kJ}) = -1240\text{ kJ (out)}$$

Finally, $W_{cycle} = \cancel{W_{12}} + W_{23} + W_{31}$ and $Q_{in} = Q_{12}$, so

$$\eta = W_{cycle}/Q_{in} = (W_{23} + W_{31})/Q_{12} = [422.4 + (-221.8)]/(1678) = 0.1195\ (11.95\%) \longleftarrow$$

Note:

$$Q_{cycle} = Q_{12} + Q_{23} + Q_{31} = (1678) + (-237.8) + (-1240) = -200.2\text{ kJ}$$

$$W_{cycle} = 422.4 + (-221.8) = 200.4\text{ kJ}$$

so

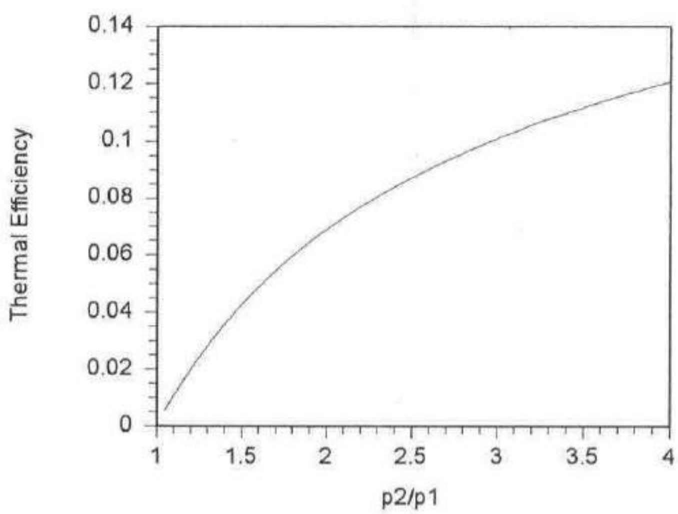
$W_{cycle} = Q_{cycle}$ as expected (The slight difference is due to round-off.)

(b) Using *IT:Interactive Thermodyanimcs*:

```
T1 = 300  // K
p1 = 1  // bar
m = 2  // kg
M = 44.01  // kg/kmol
R = 8.314 / M

W23 = m*R*(T3 - T2) / (1-1.28)
W31 = m*R*(T1 - T3)
m*(u2 - u1) = Q12
Wcycle = W23 + W31
eta = Wcycle / Q12
p2 / p1 = T2 / T1
p2 = r * p1
r = 4
p3 = p1
T3 / T2 = (p3 / p2)^((1.28-1) / 1.28)
u1 = u_T("CO2", T1)
u2 = u_T("CO2", T2)
u3 = u_T("CO2", T3)

/* Using the Explore button, sweep r from
   1.05 to 4 in steps of 0.01.   */
```

<u>*IT* Results</u> (r = 4)

$Q_{in} = 1678$ kJ
$T_2 = 1200$ K
$T_3 = 886.1$ K
$W_{23} = 423.6$ kJ
$W_{31} = -221.4$ kJ
$\eta = 0.1205$



PROBLEM 3.147

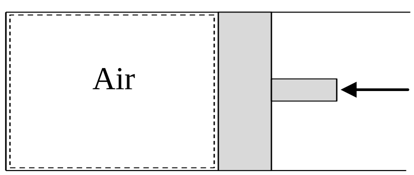
Air undergoes a polytropic process in a piston-cylinder assembly from p_1 = 1 bar, T_1 = 295K to p_2 = 7 bar. The air is modeled as an ideal gas and kinetic and potential energy effects are negligible. For a polytropic exponent of 1.6, determine the work and heat transfer, each in kJ per kg of air,
(a) assuming constant c_v evaluated at 300K.
(b) assuming variable specific heats.

Using *IT*, plot the work and heat transfer per unit mass of air for polytropic exponents ranging from 1.0 to 1.6. Investigate the error in the heat transfer introduced by assuming constant c_v.

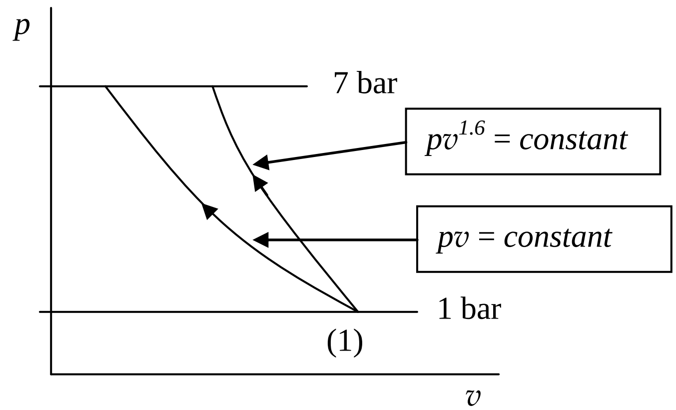
KNOWN: Air undergoes a polytropic process from a known initial state to a given final pressure.

FIND: Determine the heat transfer and work per unit mass of air for (a) constant specific heats and (b) variable specific heats. Plot the work and heat transfer per unit mass for a given range of polytropic exponents and investigate the error in heat transfer introduced by assuming constant c_v.

SCHEMATIC AND GIVEN DATA:



p_1 = 1 bar
T_1 = 300K
p_2 = 7 bar
pv^n = *constant*



ENGINEERING MODEL: (1) The air is a closed system. (2) The process is polytropic. (3) The air behaves as an ideal gas. (4) Volume change is the only work mode. (5) Kinetic and potential energy effects can be neglected.

ANALYSIS: Begin by fixing State 2. First, with $v_1 = RT_1/p_1$

$$v_1 = \frac{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg·K}}\right)(295\text{K})}{(1\text{ bar})}\left|\frac{1\text{ bar}}{10^5\text{N/m}^2}\right|\left|\frac{10^3\text{N·m}}{1\text{ kJ}}\right| = 0.8466\text{ m}^3/\text{kg}$$

From the polytropic process relation

$$p_1v_1^n = p_2v_2^n \longrightarrow v_2 = (p_1/p_2)^{1/n}\, v_1 = (1/7)^{1/1.6}(0.8466\text{ m}^3/\text{kg}) = 0.2509\text{ m}^3/\text{kg}$$

and

$$T_2 = p_2\, v_2/R = \frac{(7\text{ bar})(0.2509\text{ m}^3/\text{kg}}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg·K}}\right)}\left|\frac{10^5\text{N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{N·m}}\right| = 612\text{ K}$$

Now, the work is

$$W = \int_{V_1}^{V_2} pdV = m\int_{v_1}^{v_2} pdv = \frac{m(p_2v_2 - p_1v_1)}{1-n} = \frac{mR(T_2-T_1)}{1-n}$$

Thus

$$W/m = \frac{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(612-295)\text{K}}{1-1.6} = -\ 151.62\ \text{kJ/kg (in)} \longleftarrow$$

The heat transfer is found using the energy balance, as follows.

$$\Delta \cancel{KE} + \Delta \cancel{PE} + \Delta U = Q - W \longrightarrow Q/m = (u_2 - u_1) + W/m \qquad (*)$$

(a) From Table A-20, at 300K; c_v = 0.718 kJ/kg·K. Thus, with (*)

$$Q/m = c_v(T_2 - T_1) + W/m = (0.718\ \text{kJ/ kg}\cdot\text{K})(612 - 295)\text{K} + (-151.62\ \text{kJ/kg})$$

$$= 75.99\ \text{kJ/kg (in)} \longleftarrow$$

(b) Using data from Table A-22: u_1 = 210.49 kJ/kg and u_2 = 443.19 kJ/kg (interpolated)

and

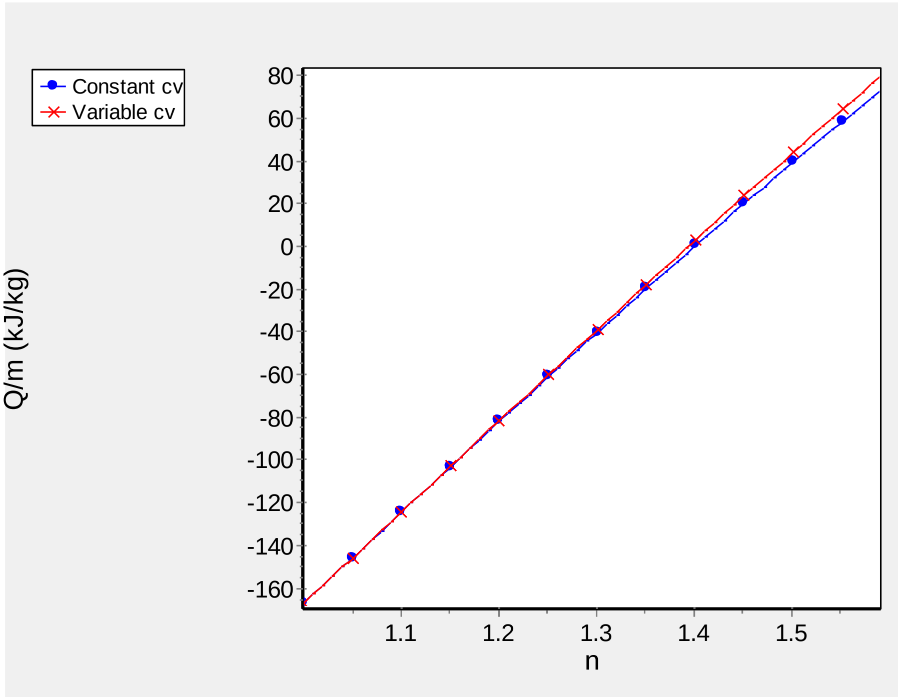
$$Q/m = (u_2 - u_1) + W/m = (443.19 - 210.49)\text{kJ/kg} + (-151.62\ \text{kJ/kg}) = 81.08\ \text{kJ/kg}$$

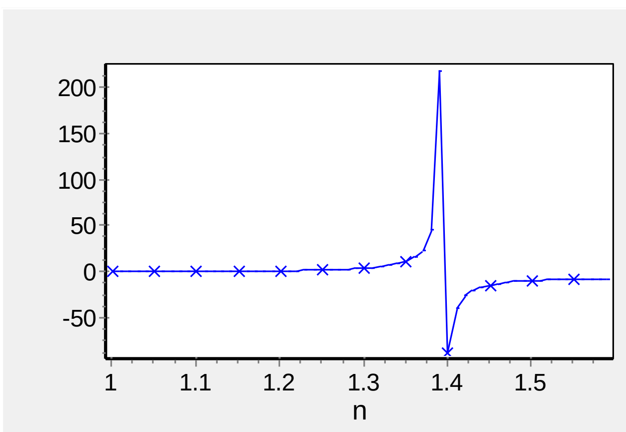
Note: The assumption of constant specific heat at 300 K leads to a value that is 6.28% lower than the values obtained using variable specific heats.

(c) The *IT* code is:

```
 p1 = 1 // bar
T1 = 295  // K
p2 = 7  // bar
n = 1.6
R = 8.314/28.97 // kJ/kg-K


v1 = v_Tp("Air",T1,p1)
v2 = (p1/p2)^(1/n)*v1
v2 = v_Tp("Air",T2,p2)

W = R*(T2 - T1)/(1-n)  // kJ/kg

cv = cv_T("Air",T1)
Qa = cv*(T2 - T1) + W  // kJ/kg

u1 = u_T("Air",T1)
u2 = u_T("Air",T2)
Qb = (u2 - u1) + W  // kJ/kg

Error = ((Qa – Qb)/Qb)*100

// Sweep n from 1.001 to 1.6 in steps of 0.01
```

<u>*IT* results for *n* = 1.6</u>
v_1 = 0.8466 m^3/kg
v_2 = 0.2509 m^3/kg
T_2 = 612 K
W/m = -151.6 kJ/kg
(a) Q_a/m = 75.66 kJ/kg
(b) Q_b/m = 81.83 kJ/kg
% Error = -7.541%





Discussion

The deviation of Q/m for constant specific heats and variable specific heats increases continually as n increases from 1 to 1.6.

Note that $Q/m = 0$ at $n \approx k = 1.4$.

The percent error is small for $n < 1.35$ and then increases greatly as n approaches 1.4. The error then becomes a large negative value at $n > 1.4$ and decreases rapidly, leveling out at about $n = 1.45$. However, when we look at the graph of Q/m versus n, we see that this is a numerical issue when Q/m approaches zero, since the absolute difference between the curves does not exhibit this phenomenon.

PROBLEM 3.148

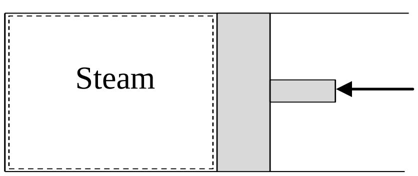
Steam, initially at 700 lbf/in.^{2}, 550^{o}F undergoes a polytropic process in a piston-cylinder assembly to a final pressure of 3000 lbf/in.^{2} Kinetic and potential energy effects are negligible. Determine the heat transfer, in Btu per lb of steam, for a polytropic exponent of 1.6,
(a) using data from the Steam Tables.
(b) assuming ideal gas behavior.

Using *IT*, plot the heat transfer per unit mass of steam for polytropic exponents ranging from 1.0 to 1.6. Investigate the error in the heat transfer introduced by assuming ideal gas behavior.

KNOWN: Steam undergoes a polytropic process from a known initial state to a given final pressure.

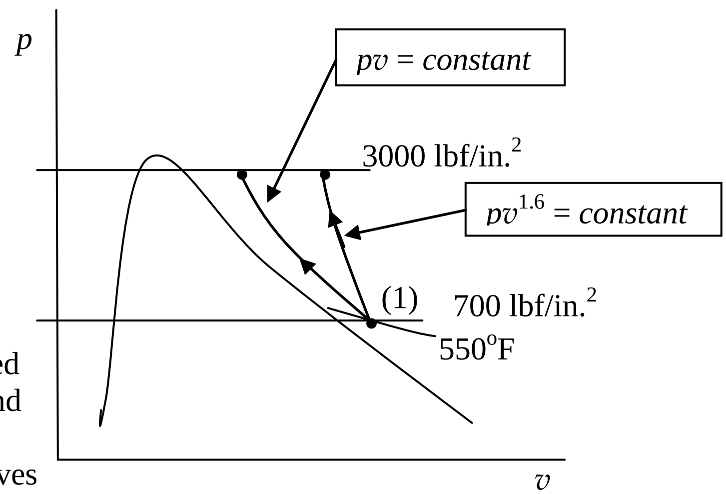
FIND: Determine the heat transfer per unit mass of steam (a) using Steam Table data and (b) assuming ideal gas behavior. Using *IT*, plot the heat transfer per unit mass of steam for polytropic exponents ranging from 1.0 to 1.6. Investigate the error in the heat transfer introduced by assuming ideal gas behavior.

SCHEMATIC AND GIVEN DATA:



p_1 = 700 lbf/in.^{2}
T_1 = 550^{o}F
p_2 = 3000 lbf/in.^{2}

pv^n = *constant*



ENGINEEIRNG MODEL: (1) The steam is a closed system. (2) The process is polytropic. (3) Kinetic and potential energy effects are negligible. (4) Volume is the only work mode. (5) For part (b), the steam behaves as an ideal gas.

SCHEMATIC AND GIVEN DATA: The work is determined from

$$W = \int_{V_1}^{V_2} p dV = m\int_{v_1}^{v_2} \frac{const}{v} dv \quad \longrightarrow \quad W/m \; = \frac{(p_2 v_2 - p_1 v_1)}{1-n} \qquad (*)$$

The energy balance reduces to $\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = Q - W$

$$Q/m = (u_2 - u_1) + W/m \qquad (**)$$

Further, from the polytropic process expression, $v_2 = (p_1/p_2)^{1/n}\, v_1$ (***)

(a) From Table A-4E: p_1 = 700 lbf/in.^{2}, T_1 = 550^{o}F; v_1 = 0.728 ft^3/lb and u_1 = 1149.0 Btu/lb

PROBLEM 3.148 (CONTINUED) – PAGE 2

$v_2 = (p_1/p_2)\, v_1 = (700/3000)^{1/1.6}(0.728\ \text{ft}^3/\text{lb}) = 0.2932\ \text{ft}^3/\text{lb}$

Now

$$W/m = \frac{(p_2v_2 - p_1v_1)}{1-n} = \frac{\left[\left(3000\frac{\text{lbf}}{\text{in.}^2}\right)(0.2932\ \text{ft}^3) - (700)(0.728)\right]}{1-1.6}\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

$$= -114.1\ \text{Btu/lb (out)}$$

Interpolating in Table A-4E with p_2 = 3000 lbf/in.^{2} and v_2 = 0.2932 ft^3/lb; u_2 = 1382.6 Btu/lb

$$Q/m = (u_2 - u_1) + W/m = (1382.6 - 1149.0)\text{Btu/lb} + (-114.1\ \text{Btu/lb}) = 119.5\ \text{Btu/lb (in)} \longleftarrow$$

(b) Using the ideal gas model for steam, Eq. (***) can be re-written as

$$T_2 = \left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)} T_1 = \left(\frac{3000}{700}\right)^{\frac{(1.6-1)}{1.6}}(1010^{\circ}\text{R}) = 1743.1^{\circ}\text{R}$$

And, Eq.(*) becomes

$$W/m = \frac{(p_2v_2 - p_1v_1)}{1-n} = \frac{R(T_2-T_1)}{1-n} = \frac{\left(\frac{1545}{18.02}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^{\text{o}}\text{R}}\right)(1743.1-1010)^{\text{o}}\text{R}}{1-1.6}\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right| = -134.6\ \text{Btu/lb}$$

From Table A-23E: $\bar{u}_1$ = 6158.9 Btu/lbmol and $\bar{u}_2$ = 11413 Btu/lbmol. Finally

$$Q/m = \frac{(\bar{u}_2 - \bar{u}_1)}{M} + W/m = \frac{(11413-6158.9)\text{Btu/lbmol}}{18.02\ \text{lb/lbmol}} + (-134.6\ \text{Btu/lb})$$

$$= 156.8\ \text{Btu/lb (in)} \longleftarrow$$

Note, the heat transfer value obtained using the ideal gas model is 23.8% higher than the value obtained using the Steam Tables. This is a significant error.

(c) *IT* solution code:

```
p1 = 700 // lbf/in.^2
T1 = 550 //F
p2 = 3000 // lbf/in.^2
n=1.6
// (a) Steam Table Solution
W = ((p2*v2 - p1*v1)/(1-n))*(144/778)  // Btu/lb
Q = (u2 - u1) + W  // Btu/lb
v1 = v_PT("Water/Steam", p1, T1)
v2 = ((p1/p2)^(1/n))*v1
v2 = v_PT("Water/Steam", p2, T2)
u2 = u_PT("Water/Steam", p2, T2)
u1 = u_PT("Water/Steam", p1, T1)
```

```
// (b)  Idea Gas Solution
R = 1545/(18.02*778)  // Btu/lb-R
W_IG = (R*(T2_IG - T1)/(1-n))  // Btu/lb
```

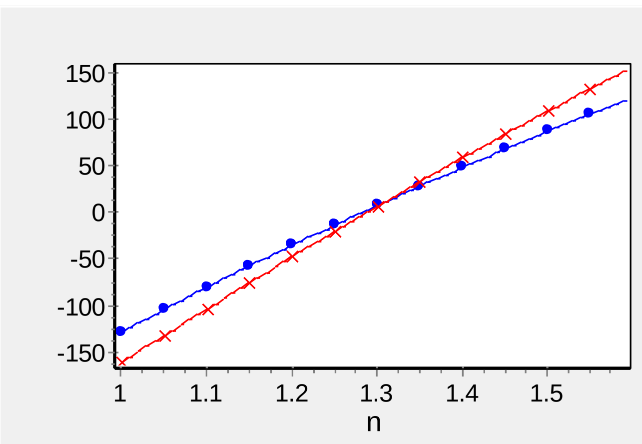
Problem 3.148 (Continued) – Page 3

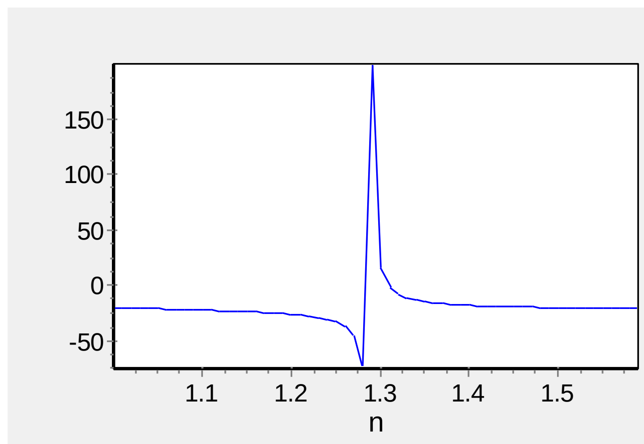
```
T2_IG = ((p2/p1)^((n-1)/n))*(T1+459.67) - 459.67
u2_IG = u_T("H2O",T2_IG)
u1_IG = u_T("H2O",T1)
Q_IG = (u2_IG - u1_IG) + W_IG  // Btu/lb
Error = ((Q - Q_IG)/Q_IG)*100
// Sweep n from 1.001 to 1.6 in steps of 0.01
```

IT Results for n = 1.6
(a) Steam Tables
v_1 = 0.7275 ft^3/lb
v_2 = 0.293 Btu/lb
u_1 = 1149 Btu/lb
u_2 = 1387 Btu/lb
Q/m = 123.8 Btu/lb

(b) Ideal Gas
T_2 = 1743^{o}R
Q/m = 156.2 Btu/lb

%Error = -20.7





Discussion

The curves for Q/m deviate significantly form each other for the Steam Table solution and the ideal gas solution as n approaches 1 and as n approaches 1.6. The deviation is minimal near n = 1.3 (Q/m = 0).

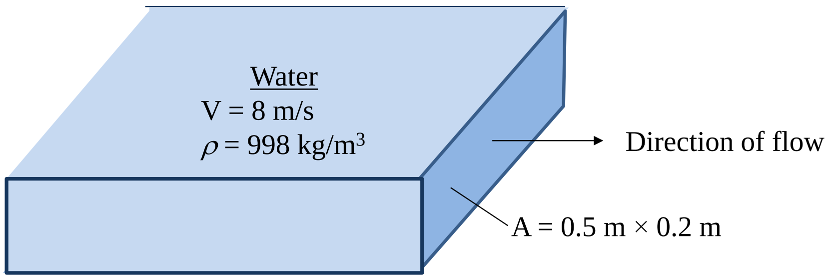
The percent error is increasingly negative for $n < 1.28$ and then fluctuates greatly as n approaches 11.3. The error then becomes a large positive value at $n > 1.3$ and decreases rapidly, leveling out at about n = 1.4. However, when we look at the graph of Q/m versus n, we see that this is a numerical issue when Q/m approaches zero, since the absolute difference between the curves does not exhibit this phenomenon.

4.1 A *laser Doppler velocimeter* measures a velocity of 8 m/s as water flows in an open channel. The channel has a rectangular cross-section of 0.5 m by 0.2 m in the flow direction. If the water density is a constant 998 kg/m^3, determine the mass flow rate, in kg/s.

KNOWN: Velocity, cross-sectional area, and density of water flowing in a channel.

FIND: Mass flow rate, in kg/s.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Flow is one-dimensional.
2. Water density is constant at 998 kg/m^3.

ANALYSIS:
The governing equation for one-dimensional flow in terms of density is

$$\dot{m} = \rho A V$$

Substituting values and solving for the mass flow rate yield

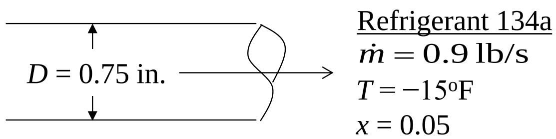
$$\dot{m} = (998\ \text{kg/m}^3)(0.5\ \text{m} \times 0.2\ \text{m})(8\ \text{m/s}) = \underline{\mathbf{798.4\ kg/s}}$$

4.2 Refrigerant 134a exits a heat exchanger through 0.75-in.-diameter tubing with a mass flow rate of 0.9 lb/s. The temperature and quality of the refrigerant are −15°F and 0.05, respectively. Determine the velocity of the refrigerant, in m/s.

KNOWN: Mass flow rate, temperature, and quality of refrigerant 134a exiting a heat exchanger through 0.75-in.-diameter tubing.

FIND: Velocity, in m/s.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Flow is one-dimensional.

ANALYSIS:
The governing equation for one-dimensional flow in terms of specific volume is

$$\dot{m} = \frac{\mathrm{AV}}{v}$$

Solving for the velocity gives

$$\mathrm{V} = \frac{\dot{m}v}{\mathrm{A}}$$

At $T = -15^{\circ}$F and $x = 0.05$, refrigerant 134a is liquid-vapor mixture. Determining specific volume using quality and values from Table A-10E: $v_{\mathrm{f}} = 0.01163\ \mathrm{ft^3/lb}$ and $v_{\mathrm{g}} = 3.0286\ \mathrm{ft^3/lb}$:

$$v = v_{\mathrm{f}} + x(v_{\mathrm{g}} - v_{\mathrm{f}}) = 0.01163\ \mathrm{ft^3/lb} + (0.05)(3.0286\ \mathrm{ft^3/lb} - 0.01163\ \mathrm{ft^3/lb}) = 0.16248\ \mathrm{ft^3/lb}$$

Cross-sectional area of the flow based on tubing diameter is

$$\mathrm{A} = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.75\ \mathrm{in.})^2 \left| \frac{1\ \mathrm{ft}^2}{144\ \mathrm{in.}^2} \right| = 0.00307\ \mathrm{ft}^2$$

Solving for the velocity yields

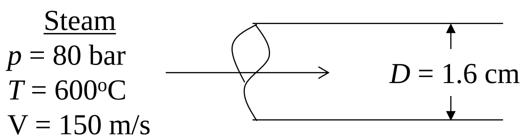
$$\mathrm{V} = \frac{(0.9\ \mathrm{lb/s})(0.16248\ \mathrm{ft^3/lb})}{0.00307\ \mathrm{ft}^2} = \underline{\mathbf{47.6\ ft/s}}$$

4.3 Steam enters a 1.6-cm-diameter pipe at 80 bar and 600°C with a velocity of 150 m/s. Determine the mass flow rate, in kg/s.

KNOWN: Pressure, temperature, and velocity of steam entering a 1.6-cm-diameter pipe.

FIND: Mass flow rate, in kg/s.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Flow is one-dimensional.

ANALYSIS:
The governing equation for one-dimensional flow in terms of specific volume is

$$\dot{m} = \frac{\mathrm{AV}}{v}$$

At p = 80 bar and T = 600°C, the steam is superheated vapor. Obtaining specific volume of the steam from Table A-4: v = 0.04845 m^3/kg.

Cross-sectional area of the flow based on pipe diameter is

$$\mathrm{A} = \frac{\pi}{4}D^2 = \frac{\pi}{4}(1.6\,\mathrm{cm})^2\left|\frac{1\,\mathrm{m}^2}{10000\,\mathrm{cm}^2}\right| = 0.00020\,\mathrm{m}^2$$

Solving for the mass flow rate yields

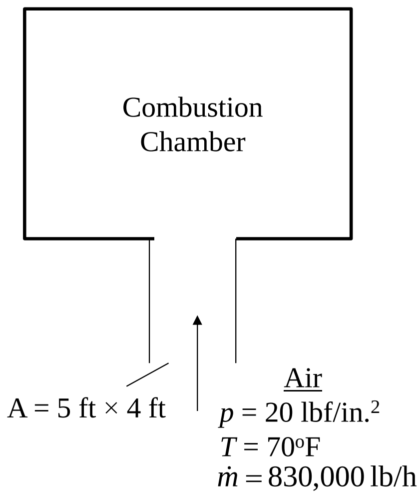
$$\dot{m} = \frac{(0.00020\,\mathrm{m}^2)(150\,\mathrm{m/s})}{0.04845\,\mathrm{m}^3/\mathrm{kg}} = \underline{\mathbf{0.62\ kg/s}}$$

4.4 Air modeled as an ideal gas enters a combustion chamber at 20 lbf/in.^{2} and 70^{o}F through a rectangular duct, 5 ft by 4 ft. If the mass flow rate of the air is 830,000 lb/h, determine the velocity, in ft/s.

KNOWN: Pressure, temperature, and mass flow rate of air entering a combustion chamber through a 5 ft by 4 ft rectangular duct.

FIND: Velocity, in ft/s.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Flow is one-dimensional.
2. Air is modeled as an ideal gas.

ANALYSIS:
The governing equation for one-dimensional flow in terms of specific volume is

$$\dot{m} = \frac{\mathrm{AV}}{v}$$

Solving for the velocity gives

$$\mathrm{V} = \frac{\dot{m}v}{\mathrm{A}}$$

Apply the ideal gas equation of state to solve for the specific volume of air. Converting temperature to Rankine scale: T = 70^{o}F + 460 = 530^{o}R. Substituting values to obtain specific volume yields

$$pv = RT \rightarrow v = \frac{RT}{p} = \frac{\left(\dfrac{1545\,\dfrac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot{}^\circ\text{R}}}{28.97\,\dfrac{\text{lb}}{\text{lbmol}}}\right)(530^\circ\text{R})}{20\,\dfrac{\text{lbf}}{\text{in.}^2}}\left|\frac{1\,\text{ft}^2}{144\,\text{in.}^2}\right| = 9.81\ \text{ft}^3/\text{lb}$$

Solving for the velocity yields

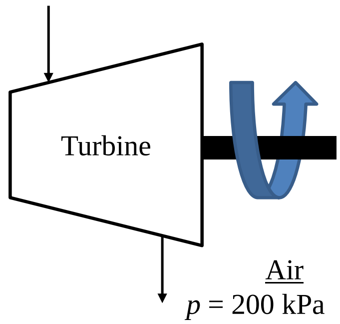
$$\text{V} = \frac{(830{,}000\,\text{lb/h})(9.81\,\text{ft}^3/\text{lb})}{(5\,\text{ft})(4\,\text{ft})}\left|\frac{1\,\text{h}}{3600\,\text{s}}\right| = \underline{\mathbf{113.1\ ft/s}}$$

4.5 Air exits a turbine at 200 kPa and 150°C with a volumetric flow rate of 7000 liters/s. Modeling air as an ideal gas, determine the mass flow rate, in kg/s.

KNOWN: Pressure, temperature, and volumetric flow rate of air exiting a turbine.

FIND: Mass flow rate, in kg/s.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Flow is one-dimensional.
2. Air is modeled as an ideal gas.

ANALYSIS:
The governing equation for one-dimensional flow in terms of specific volume is

$$\dot{m} = \frac{\mathrm{AV}}{v}$$

Apply the ideal gas equation of state to solve for the specific volume of air. Converting temperature to Kelvin scale: T = 150°C + 273 = 423 K. Substituting values to solve for specific volume gives

$$pv = RT \rightarrow v = \frac{RT}{p} = \frac{\left(\dfrac{8.314 \dfrac{\text{kJ}}{\text{kmol}\cdot\text{K}}}{28.97 \dfrac{\text{kg}}{\text{kmol}}}\right)(423\,\text{K})}{200\,\text{kPa}} \left| \frac{1000\ \text{N}\cdot\text{m}}{1\,\text{kJ}} \right| \left| \frac{1\,\text{kPa}}{1000 \dfrac{\text{N}}{\text{m}^2}} \right| = 0.6070\ \text{m}^3/\text{kg}$$

Solving for mass flow rate yields

$$\dot{m} = \frac{(7000\ \text{L/s})}{0.6070\ \text{m}^3/\text{kg}} \left| \frac{10^{-3}\ \text{m}^3}{1\ \text{L}} \right| = \underline{\mathbf{11.5\ kg/s}}$$

PROBLEM 4.6

KNOWN: A kitchen-sink water tap leaks one drop per second.

FIND: Determine on an annual basis the gallons of water wasted and the mass of the water, in lb.

ANALYSIS:

given

$$\begin{bmatrix}\text{Annual}\\ \text{Volumetric}\\ \text{Flow rate}\end{bmatrix} = \left(1\ \frac{\text{drop}}{\text{s}}\right)\left|\frac{1\ \text{gal}}{46{,}000\ \text{drops}}\right|\left|\frac{3600\,\text{s}}{1\ \text{h}}\right|\left|\frac{24\times 365\,\text{h}}{1\ \text{year}}\right| = 685.6\ \frac{\text{gallons}}{\text{year}}$$

$$\begin{bmatrix}\text{Annual}\\ \text{mass flow}\\ \text{rate}\end{bmatrix} = \left(685.6\ \frac{\text{gal}}{\text{year}}\right)\left(62.3\ \frac{\text{lb}}{\text{ft}^3}\right)\left|\frac{0.13368\ \text{ft}^3}{1\ \text{gal}}\right| = 5710\ \frac{\text{lb}}{\text{year}}$$

given

PROBLEM 4.7

KNOWN: Water enters and exits a tank. State data are provided.

FIND: Determine the mass flow rate at the inlet and exit, each in kg/s. Also determine the time rate of change of mass contained within the tank, in kg/s.

SCHEMATIC & GIVEN DATA:

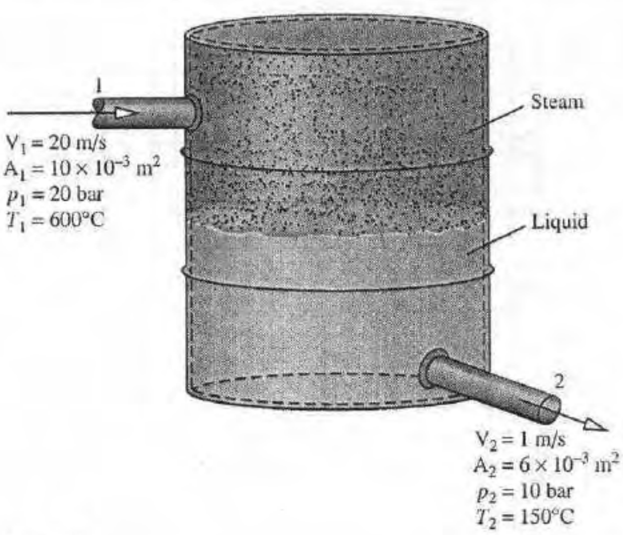
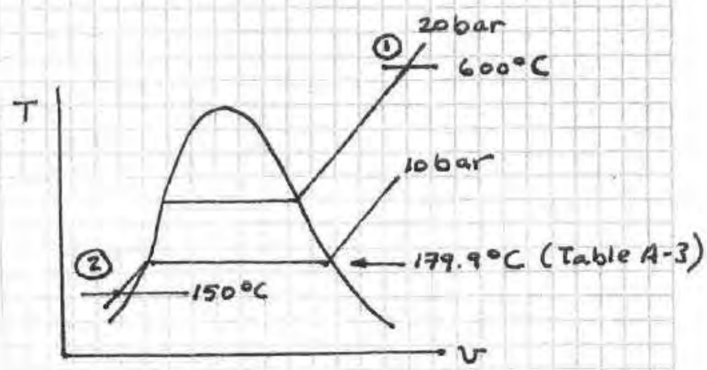


Fig. P4.7



ENGINEERING MODEL

1. The control volume is denoted by the dashed line on the figure.

ANALYSIS:

With data from Table A-4, $\dot{m}_1 = \frac{A_1 V_1}{v_1} = \frac{(10\times10^{-3}\,m^2)(20\,m/s)}{0.1996\,m^3/kg} = 1\,\frac{kg}{s}$

With $v_2 \approx v_f(T_2)$ and data from Table A-2,

$$\dot{m}_2 = \frac{A_2 V_2}{v_2} = \frac{(6\times10^{-3}\,m^2)(1\,m/s)}{(1.0905\times10^{-3}\,m^3/kg)} = 5.5\,\frac{kg}{s}$$

A mass rate balance reads,

$$\frac{dm_{cv}}{dt} = \dot{m}_1 - \dot{m}_2$$

$$= 1\,\frac{kg}{s} - 5.5\,\frac{kg}{s}$$

$$= -4.5\ kg/s$$

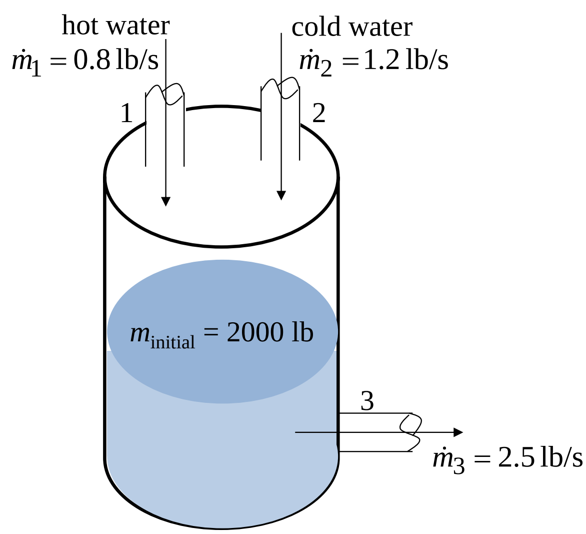
The mass contained within the tank decreases with time

4.8 Figure P4.8 shows a mixing tank initially containing 2000 lb of liquid water. The tank is fitted with two inlet pipes, one delivering hot water at a mass flow rate of 0.8 lb/s and the other delivering cold water at a mass flow rate of 1.2 lb/s. Water exits through a single exit pipe at a mass flow rate of 2.5 lb/s. Determine the mass of water, in lb, in the tank after 30 minutes.

KNOWN: Initial mass of water in tank, two inlet mass flow rates, and exit mass flow rate.

FIND: Mass of water, in lb, in the tank after 30 minutes.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume encloses the water in the tank and has two inlets and one exit.
2. The entering and exiting mass flow rates each remain constant.

ANALYSIS:
Apply a mass rate balance from the initial state when the tank contains 2000 lb of water until the final state after 30 minutes have elapsed and solve for the mass of water in the tank at the final state.

$$\frac{dm_{\text{cv}}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e$$

$$\frac{dm_{\text{cv}}}{dt} = \dot{m}_1 + \dot{m}_2 - \dot{m}_3$$

Integrating and solving for the final mass of water in the tank give

$$m_{\text{final}} - m_{\text{initial}} = (\dot{m}_1 + \dot{m}_2 - \dot{m}_3)(\Delta t)$$

$$m_{\text{final}} = m_{\text{initial}} + (\dot{m}_1 + \dot{m}_2 - \dot{m}_3)(\Delta t)$$

$$m_{\text{final}} = 2000\ \text{lb} + (0.8\frac{\text{lb}}{\text{s}} + 1.2\frac{\text{lb}}{\text{s}} - 2.5\frac{\text{lb}}{\text{s}})(30\ \text{min})\left|\frac{60\ \text{s}}{1\ \text{min}}\right| = \underline{\mathbf{1100\ lb}}$$

Problem 4.9

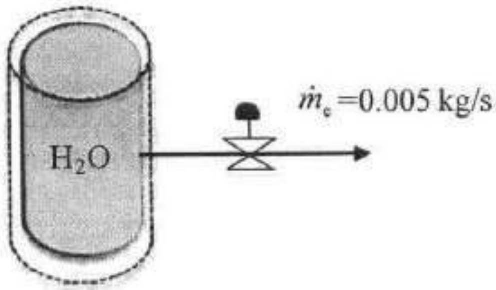
A 380 L tank contains steam, initially at 400°C, 3 bar. A valve is opened and steam flows out of the tank at a constant mass flow rate of 0.005 kg/s. During steam removal, a heater maintains the temperature within the tank constant. Determine the time, in s, at which 75% of the initial mass remains in the tank; and also determine the specific volume, in m^3/kg, and pressure, in bar, in the tank at that time.

KNOWN: Steam exits at a constant mass flow rate from a tank filled with steam at a constant temperature.

FIND: Determine the time, in s, when 75% of the initial mass remains in the tank, and the specific volume, in m^3/kg, and pressure, in bar, in the tank at that time.

SCHEMATIC AND GIVEN DATA:

$V = 380$ L
$p_1 = 3$ bar
$T_1 = 400°$ C
$m_2 = 0.75 m_1$



ENGINEERING MODEL:
(1) The control volume is shown on the accompanying diagram.
(2) The temperature in the tank remains constant.
(3) The tank volume is constant.

ANALYSIS:

Applying the mass rate balance:

$$\frac{dm_{cv}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e = -m_e = -0.005\frac{kg}{s}$$

Integrating over time and rearranging for t:

$$\int_{m_{cv}(0)}^{m_{cv}(t)} dm_{cv} = -\int_0^t \dot{m}_e dt = \left(-0.005\frac{kg}{s}\right)t$$

$$t = \frac{m_{cv}(t) - m_{cv}(0)}{\left(-0.005\frac{kg}{s}\right)} \qquad (1)$$

From Table A-4 at T_1 and p_1: $v_1 = 1.032\ m^3/kg$. Mass values are as follows:

Problem 4.9, continued

$$\mathrm{m_{cv}}(0) = \frac{V}{v_1} = \frac{380\ \mathrm{L}\left|\frac{10^{-3}\ \mathrm{m^3}}{1\ \mathrm{L}}\right|}{1.032\,\frac{\mathrm{m^3}}{\mathrm{kg}}} = 0.368\ \mathrm{kg}$$

$$\mathrm{m_{cv}}(t) = 0.75 \times \mathrm{m_{cv}}(0) = 0.75 \times 0.368\ \mathrm{kg} = 0.276\ \mathrm{kg}$$

Determine time, in s, using Eq. (1).

$$t = \frac{(0.276 - 0.368)\mathrm{kg}}{\left(-0.005\,\frac{\mathrm{kg}}{\mathrm{s}}\right)} = 18.4\mathrm{s} \quad \longleftarrow$$

To find p_2, first calculate v_2.

$$v_2 = \frac{V}{\mathrm{m_{cv}}(t)} = \frac{380\mathrm{L}\left|\frac{10^{-3}\ \mathrm{m^3}}{1\mathrm{L}}\right|}{0.276\mathrm{kg}} = 1.377\frac{\mathrm{m^3}}{\mathrm{kg}} \quad \longleftarrow$$

Interpolating in Table A-4 at $T_2 = 400°\mathrm{C}$, $v_2 = 0.1.377\ \mathrm{m^3/kg}$:

$p_2 \approx 2.5$ bar $\longleftarrow$

Problem 4.10

KNOWN: Data are provided for a crude oil storage tank.

FIND: After 24h, determine the mass and volume of oil in the tank.

SCHEMATIC & GIVEN DATA:

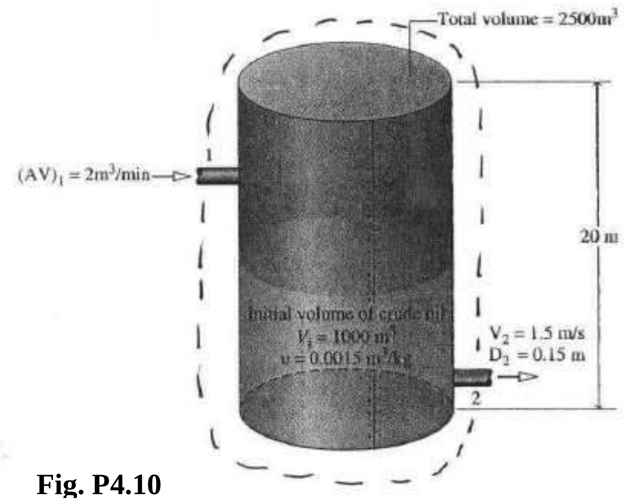


Fig. P4.10

ENGR. MODEL

1. As shown by the sketch, a control volume encloses the storage tank.
2. The specific volume of the oil is constant: $v = 0.0015\ \frac{m^3}{kg}$.

(a) Mass rate balance: $\frac{dm_{cv}}{dt} = \dot{m}_1 - \dot{m}_2$, where

$$\dot{m}_1 = \frac{(AV)_1}{v} = \left(\frac{2\ m^3/min}{0.0015\ m^3/kg}\right)\left|\frac{60\ min}{1\ h}\right| = 8\times10^4\ \frac{kg}{h}$$

$$\dot{m}_2 = \frac{A_2 V_2}{v} = \frac{(\pi D_2^2/4)(V_2)}{v} = \frac{\pi\,(0.15m)^2(1.5 m/s)}{4\,(0.0015 m^3/kg)}\left|\frac{3600s}{1h}\right| = 6.36\times10^4\ \frac{kg}{h}$$

$$\therefore\ \frac{dm_{cv}}{dt} = 1.64\times10^4\ \frac{kg}{h}$$

Integrating

$$\Rightarrow\quad m_{cv} - m_{cv}(0) = \left(1.64\times10^4\ \frac{kg}{h}\right)(24h) = 39.36\times10^4\ kg$$

$$\left(= \frac{V_i}{v} = \left[\frac{1000\ m^3}{0.0015\ m^3/kg}\right] = 66.67\times10^4\ kg\right.$$

So,

$$m_{cv}(24h) = (66.67 + 39.36)\times10^4\ kg = 1.06\times10^6\ kg \quad \longleftarrow m_{cv}$$

(b)

$$V(24h) = v\,m_{cv}(24h) = \left(0.0015\ \frac{m^3}{kg}\right)(1.06\times10^6\ kg)$$

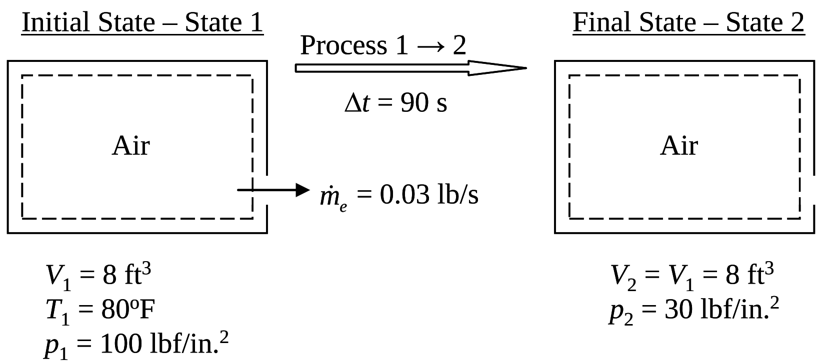
$$= 1590\ m^3 \quad \longleftarrow V$$

4.11 An 8-ft^3 tank contains air at an initial temperature of 80^{o}F and initial pressure of 100 lbf/in.^{2} The tank develops a small hole, and air leaks from the tank at a constant rate of 0.03 lb/s for 90 s until the pressure of the air remaining in the tank is 30 lbf/in.^{2} Employing the ideal gas model, determine the final temperature, in ^{o}F, of the air remaining in the tank.

KNOWN: Air at specified initial temperature and pressure leaks from rigid tank until a final specified pressure is attained by the air remaining in the tank.

FIND: Final temperature of air remaining in tank, in ^{o}F.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume is defined by the dashed line on the accompanying diagram.
2. Air can be modeled as an ideal gas.

ANALYSIS:
The ideal gas model can be applied to the final state, state 2, to determine the temperature of the air remaining in the tank.

$$p_2V_2 = m_2RT_2$$

Solving for temperature yields

$$T_2 = \frac{p_2V_2}{m_2R}$$

Pressure and volume are known at state 2. The mass in the tank at state 2, m_2, equals the initial mass in the tank, m_1, less the mass that leaks from the tank. Since the mass flow rate, $\dot{m}_e$, is constant, the amount of mass that leaks from the tank is

$$\dot{m}_e\,\Delta t = (0.03\text{ lb/s})(90\text{ s}) = 2.7\text{ lb}$$

The initial mass, m_1, is obtained using the ideal gas equation of state

$$m_1 = \frac{p_1 V_1}{R T_1}$$

The gas constant, R, is the universal gas constant divided by the molecular weight of air. Temperature must be expressed on an absolute scale, T_1 = 80 ^{o}F = 540^{o}R. Substituting values and applying the appropriate conversion factor yield

$$m_1 = \frac{\left(100\,\frac{\text{lbf}}{\text{in.}^2}\right)\left(8\,\text{ft}^3\right)}{\left(\frac{1545\,\frac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot{}^\circ\text{R}}}{28.97\,\frac{\text{lb}}{\text{lbmol}}}\right)\left(540^\circ\text{R}\right)}\left|\frac{144\,\text{in.}^2}{1\,\text{ft}^2}\right| = 4.0\text{ lb}$$

Collecting results

$$m_2 = 4.0\text{ lb} - 2.7\text{ lb} = 1.3\text{ lb}$$

Substituting m_2 to solve for T_2 yields

$$T_2 = \frac{\left(30\,\frac{\text{lbf}}{\text{in.}^2}\right)\left(8\,\text{ft}^3\right)}{\left(1.3\,\text{lb}\right)\left(\frac{1545\,\frac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot{}^\circ\text{R}}}{28.97\,\frac{\text{lb}}{\text{lbmol}}}\right)}\left|\frac{144\,\text{in.}^2}{1\,\text{ft}^2}\right| = 498.5^{o}\text{R} = \underline{\mathbf{38.5\ ^{o}F}}$$

Note the need to convert the final temperature from ^{o}R to ^{o}F to provide the answer in the requested units.

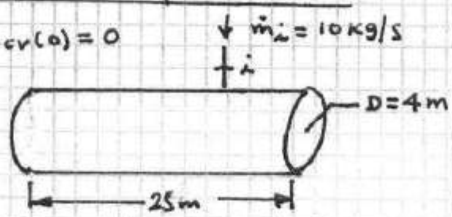
Problem 4.12

KNOWN: An initially-empty storage tank is filled with liquid propane at a constant mass flow rate. State data and tank dimensions are provided.

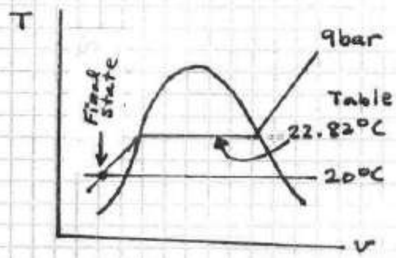
FIND: Determine the time in minutes to fill the tank.

SCHEMATIC & GIVEN DATA:

$m_{cv}(0) = 0$



Final condition in the tank: $T = 20°C$, $p = 9$ bar



ENGINEERING MODEL:

1. A control volume encloses the tank. There is a single inlet.
2. The mass flow rate is constant.

ANALYSIS:

For the single-inlet control volume, the mass rate balance reads

$$\frac{dm_{cv}}{dt} = \dot{m}_i$$

Then, since $\dot{m}_i$ is constant and the tank is empty initially,

$$m_{cv}(t_f) - \cancel{m_{cv}(0)}^{0} = \int_0^{t_f} \dot{m}_i \, dt = \dot{m}_i t_f$$

$$\Rightarrow m_{cv}(t_f) = \dot{m}_i t_f$$

Solving gives, $$t_f = \frac{m_{cv}(t_f)}{\dot{m}_i} \quad (1)$$

At t_f, the tank is filled with liquid propane at 9bar, 20°C. With $v \approx v_f(T_i) = (1.999/10^3)\ m^3/kg$ from Table A-16

$$m_{cv}(t_f) = \frac{V}{v} = \frac{\left[\frac{\pi}{4}(4\,m)^2(25\,m)\right]}{(1.999/10^3)\ m^3/kg} = 157,160\ kg$$

Equation (1) gives

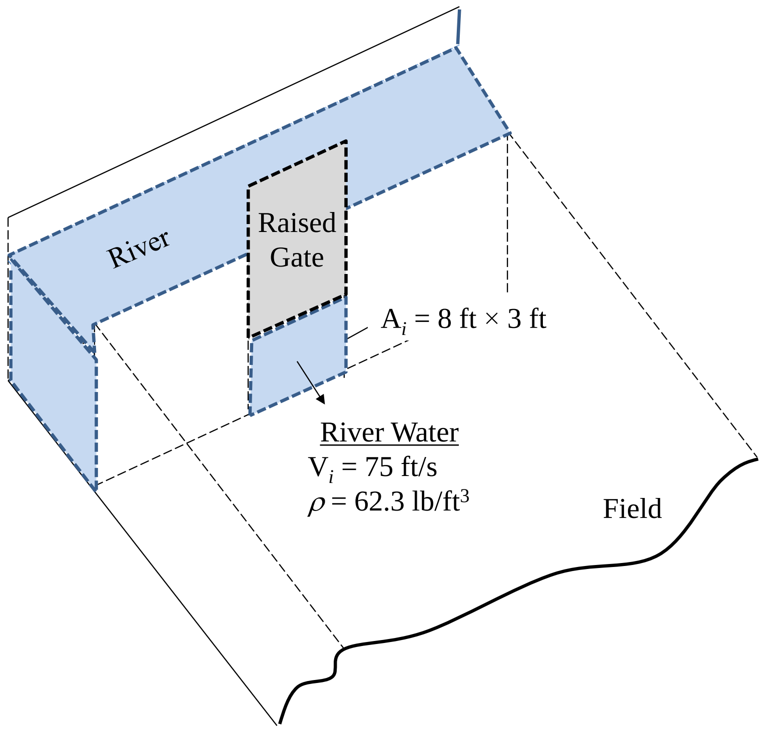
$$t_f = \frac{157,160\ kg}{10\ kg/s}\left|\frac{1\ min}{60\ s}\right| = 262\ min \quad \longleftarrow$$

4.13 As shown in Fig. P4.13, river water used to irrigate a field is controlled by a gate. When the gate is raised, water flows steadily with a velocity of 75 ft/s through an opening 8 ft by 3 ft. If the gate is raised for 24 hours, determine the volume of water, in gallons, provided for irrigation. Assume the density of river water is 62.3 lb/ft^3.

KNOWN: River water with velocity of 75 ft/s and density of 62.3 lb/ft^3 flows steadily through an 8 ft by 3 ft opening for 24 hours.

FIND: Volume, in gallons, of river water provided for irrigation.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume encloses the water in the field, which is initially empty.
2. The control volume has one inlet (through the gate) and no exits.
3. Water flows through the gate steadily and one-dimensionally.
4. Density of river water is constant at 62.3 lb/ft^3.

ANALYSIS:
Since mass of water provided for irrigation can be expressed in terms of water density and volume, determine the water volume based on the water mass and density.

$$m = \rho V \rightarrow V = \frac{m}{\rho}$$

The mass flow rate of water entering the field, $\dot{m}_i$, can be determined using the governing equation for one-dimensional flow in terms of density

$$\dot{m}_i = \rho \mathrm{A}_i \mathrm{V}_i = \left(62.3\frac{\mathrm{lb}}{\mathrm{ft}^3}\right)(8\,\mathrm{ft}\times 3\,\mathrm{ft})\left(75\frac{\mathrm{ft}}{\mathrm{s}}\right) = 112{,}140\ \mathrm{lb/s}$$

Apply a mass rate balance from the initial state when the gate is initially closed (mass of water in the field is zero) until the final state after 24 hours have elapsed with the gate open and solve for the mass of water at the final state.

$$\frac{dm_{\mathrm{cv}}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e$$

Since there is only one inlet and no exits, the mass rate balance reduces to

$$\frac{dm_{\mathrm{cv}}}{dt} = \dot{m}_i$$

Integrating, and solving yield

$$m_{\mathrm{final}} - m_{\mathrm{initial}} = \dot{m}_i(t_{\mathrm{final}} - t_{\mathrm{initial}})$$

$$m_{\mathrm{final}} = m_{\mathrm{initial}} + \dot{m}_i(t_{\mathrm{final}} - t_{\mathrm{initial}})$$

$$m_{\mathrm{final}} = 0\,\mathrm{lb} + \left(112{,}140\frac{\mathrm{lb}}{\mathrm{s}}\right)(24\,\mathrm{h})\left|\frac{3600\,\mathrm{s}}{\mathrm{h}}\right| = 9.69\times 10^9\ \mathrm{lb}$$

Solving for the volume gives

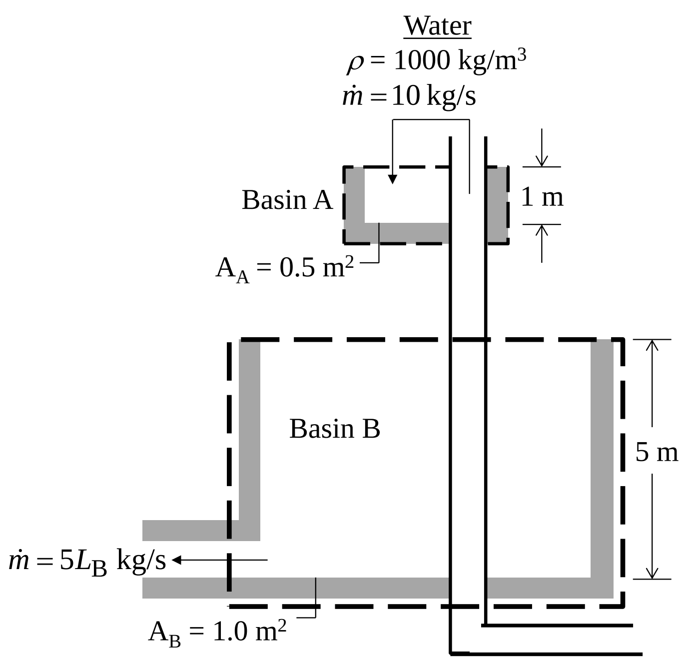
$$V = \frac{9.69\times 10^9\ \mathrm{lb}}{62.3\frac{\mathrm{lb}}{\mathrm{ft}^3}}\left|\frac{1\ \mathrm{gallon}}{0.13368\,\mathrm{ft}^3}\right| = \underline{\mathbf{1.16\times 10^9\ gallons}}$$

4.14 Figure P4.14 shows a two-tier fountain operating with basins A and B. Both basins are initially empty. When the fountain is turned on, water flows with a constant mass flow rate of 10 kg/s into basin A. Water overflows from basin A into basin B. Thereafter, water drains from basin B at a rate of $5L_B$ kg/s, where L_B is the height of the water in basin B, in m. Dimensions of the basins are indicated on the figure. Determine the variation of water height in each basin as a function of time. The density of water is constant at 1000 kg/m^3.

KNOWN: Water flows at a known rate into basin A. Eventually, water overflows into basin B, which drains at a rate proportional to the water level.

FIND: Determine the variation with time of the water level in each basin.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume boundaries are shown on the accompanying diagram.
2. The density of the water is constant at 1000 kg/m^3.

ANALYSIS:
Consider first the control volume for basin A. The mass rate balance is

$$\frac{dm_A}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e$$

Prior to basin A filling completely, there is no exiting mass flow. The mass rate balance reduces to

$$\frac{dm_A}{dt} = \dot{m}_i = 10 \text{ kg/s}$$

where m_A is the mass of water in basin A. Writing the mass in terms of water height (L_A), area (A_A), and density (ρ): $m_A = \rho A_A L_A$, and substituting

$$\rho A_A \frac{dL_A}{dt} = 10 \text{ kg/s}$$

Solving for the time rate of change of water height in basin A yields

$$\frac{dL_A}{dt} = \frac{10 \text{ kg/s}}{\rho A_A} = \frac{10 \text{ kg/s}}{(1000 \text{ kg/m}^3)(0.5 \text{ m}^2)} = 0.02 \text{ m/s}$$

Integrating

$$L_A(t) = 0.02[\text{m/s}]t + C$$

where C is a constant that depends on the initial condition. Thus, with $L_A = 0$ at $t = 0$ s, the constant vanishes yielding

$$L_A(t) = 0.02[\text{m/s}]t$$

When the tank is full, $L_A = 1$ m. Thus,

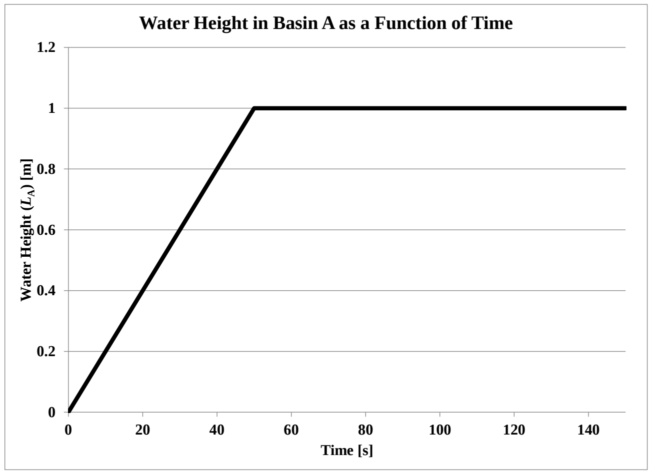
$$1 \text{ m} = 0.02[\text{m/s}]t_{\text{full}}$$

$$t_{\text{full}} = \frac{1 \text{ m}}{0.02 \text{ m/s}} = 50 \text{ s}$$

The variation of water height in basin A as a function of time is

$$\boldsymbol{L_A = \begin{cases} 0.02[\text{m/s}]t & 0 \le t \le 50 \text{ s} \\ 1 \text{ m} & t > 50 \text{ s} \end{cases}}$$

This result is shown graphically below:



For basin B after basin A is full (after 50 s have elapsed)

$$\frac{dm_B}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e$$

$$\frac{dm_B}{dt} = \dot{m}_i - \dot{m}_e = 10 \text{ kg/s} - 5L_B \text{ [kg/s]}$$

Writing the mass in terms of water height (L_B), area (A_B), and density (ρ): $m_B = \rho A_B L_B$, and substituting

$$\rho A_B \frac{dL_B}{dt} = 10 \text{ kg/s} - 5L_B \text{ [kg/s]}$$

Solving for the time rate of change of water height in basin B yields

$$\frac{dL_B}{dt} = \frac{10 \text{ kg/s}}{\rho A_B} - \frac{5L_B \text{ kg/s}}{\rho A_B} = \frac{10 \text{ kg/s}}{(1000 \text{ kg/m}^3)(1.0 \text{ m}^2)} - \frac{5L_B \text{ kg/s}}{(1000 \text{ kg/m}^3)(1.0 \text{ m}^2)}$$

$$\frac{dL_B}{dt} = 0.01 \text{ m/s} - 0.005L_B \text{ [m/s]}$$

The solution to this differential equation is of the form

$$L_B(t) = Ce^{-0.005t} + 2.0$$

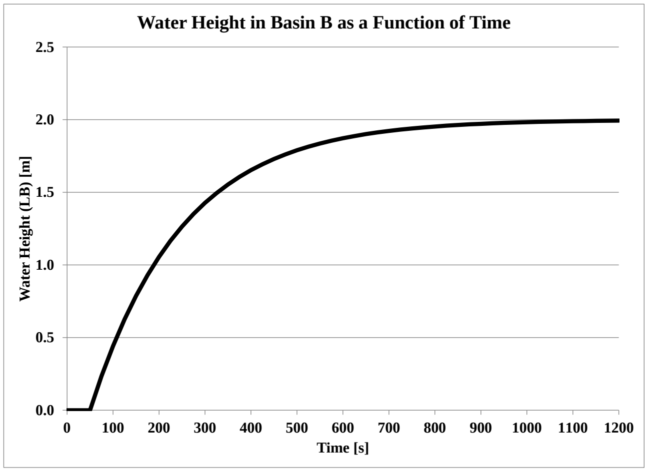
where C is a constant that depends on the initial condition. Thus, with $L_B = 0$ at $t = 50$ s

$$0 = Ce^{-(0.005)(50)} + 2.0 \rightarrow C = -2.568$$

The variation of water height in basin B as a function of time is

$$L_B = \begin{cases} 0 & 0 \le t \le 50 \text{ s} \\ -2.568e^{-0.005t} + 2.0 & t > 50 \text{ s} \end{cases}$$

This result is shown graphically below:



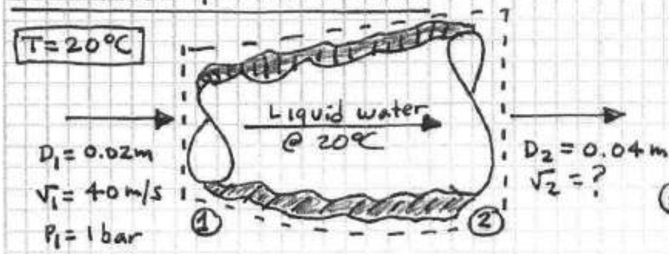
For time greater than about 1000 seconds, the level in basin B remains at 2 m.

Problem 4.15

KNOWN: Liquid water flows at 20°C through a duct at steady state. State data and duct dimensions are provided.

FIND: At the duct exit, determine the mass flow rate, in kg/s, and velocity, in m/s.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. A control volume encloses the duct shown in the figure.
2. The control volume is at steady state.
3. (1) $v \approx v_f(20°C) = \frac{1.0018}{10^3}\ \frac{m^3}{kg}$ (Table A-2).

ANALYSIS: At steady state the mass rate balance reduces to $\dot{m}_2 = \dot{m}_1$. Thus,

$$\dot{m}_2 = \frac{A_1 V_1}{v} = \frac{\left(\frac{\pi D_1^2}{4}\right) V_1}{v} = \frac{\left[\frac{\pi(0.02\text{ m})^2}{4}\right](40\text{ m/s})}{\left(\frac{1.0018}{10^3}\ \frac{m^3}{kg}\right)} = 12.54\ \frac{kg}{s}$$

Note assumption 3

With $\dot{m}_2 = \dot{m}_1$,

$$\frac{A_2 V_2}{v} = \frac{A_1 V_1}{v} \Rightarrow A_2 V_2 = A_1 V_1$$

$$\Rightarrow V_2 = \frac{A_1}{A_2} V_1 = \left[\frac{\pi(0.02)^2/4}{\pi(0.04)^2/4}\right]\left(40\ \frac{m}{s}\right) = 10\text{ m/s}$$

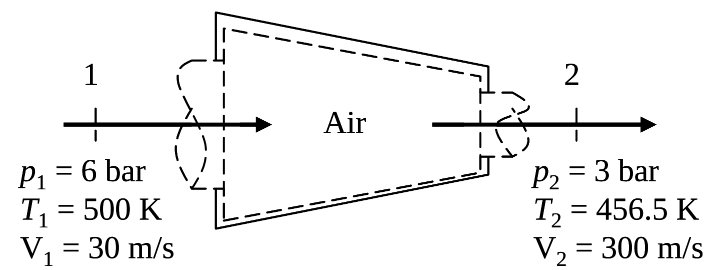
1. For a simple duct such as the one under consideration, we do not expect pressure P_2 to depart significantly from pressure P_1.

4.16 Air enters a one-inlet, one-exit control volume at 6 bar, 500 K, and 30 m/s through a flow area of 28 cm^2. At the exit, the pressure is 3 bar, the temperature is 456.5 K, and the velocity is 300 m/s. The air behaves as an ideal gas. For steady-state operation, determine
(a) the mass flow rate, in kg/s.
(b) the exit flow area, in cm^2.

KNOWN: Air flows through a one-inlet, one-exit control volume with known pressure, temperature, and velocity at the inlet and exit.

FIND: Determine the mass flow rate and exit flow area.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown on the accompanying figure is at steady state.
2. The ideal gas model applies for the air.

ANALYSIS:
(a) The mass rate balance for one-inlet, one-exit, steady flow is

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

For the inlet, state 1, the mass flow rate can be determined from given data and the ideal gas equation of state.

$$\dot{m}_1 = \frac{A_1 V_1}{v_1} = \frac{A_1 V_1 p_1}{\left(\overline{R}/M\right)T_1}$$

Substituting values yields

$$\dot{m}_1 = \frac{\left(28\ \text{cm}^2\right)\left(30\ \frac{\text{m}}{\text{s}}\right)\left(6\ \text{bar}\right)}{\left(\dfrac{8314\ \frac{\text{N}\cdot\text{m}}{\text{kmol}\cdot\text{K}}}{28.97\ \frac{\text{kg}}{\text{kmol}}}\right)\left(500\ \text{K}\right)}\left|\frac{10^5\ \frac{\text{N}}{\text{m}^2}}{\text{bar}}\right|\left|\frac{\text{m}^2}{10^4\ \text{cm}^2}\right| = \underline{\mathbf{0.351\ kg/s}}$$

(b) The exit flow area can be determined from given data and the ideal gas equation of state.

$$\dot{m}_2 = \frac{A_2 \mathrm{V}_2}{v_2} = \frac{A_2 \mathrm{V}_2\, p_2}{\left(\overline{R}/M\right)T_2}$$

Solving for area

$$A_2 = \frac{\dot{m}_2\left(\overline{R}/M\right)T_2}{\mathrm{V}_2\, p_2} = \frac{\left(0.351\,\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\dfrac{8314\,\frac{\mathrm{N\cdot m}}{\mathrm{kmol\cdot K}}}{28.97\,\frac{\mathrm{kg}}{\mathrm{kmol}}}\right)(456.5\,\mathrm{K})}{\left(300\,\frac{\mathrm{m}}{\mathrm{s}}\right)(3\,\mathrm{bar})}\left|\frac{\mathrm{bar}}{10^5\,\frac{\mathrm{N}}{\mathrm{m}^2}}\right|\left|\frac{10^4\ \mathrm{cm}^2}{\mathrm{m}^2}\right| = \underline{\mathbf{5.1\ cm^2}}$$

Problem 4.17

As shown in Fig. P4.17, air with a volumetric flow rate of 15,000 ft^3/min enters an air-handling unit at 35°F, 1 atm. The air-handling unit delivers air at 80°F, 1 atm to a duct system with three branches consisting of two 26-in.-diameter ducts and one 50-in. duct. The velocity in each 26-in. duct is 10 ft/s. Assuming ideal gas behavior for the air, determine at steady state
(a) the mass flow rate of air entering the air-handling unit, in lb/s.
(b) the volumetric flow rate in each 26-in. duct, in ft^3/min.
(c) the velocity in the 50-in.duct, in ft/s.

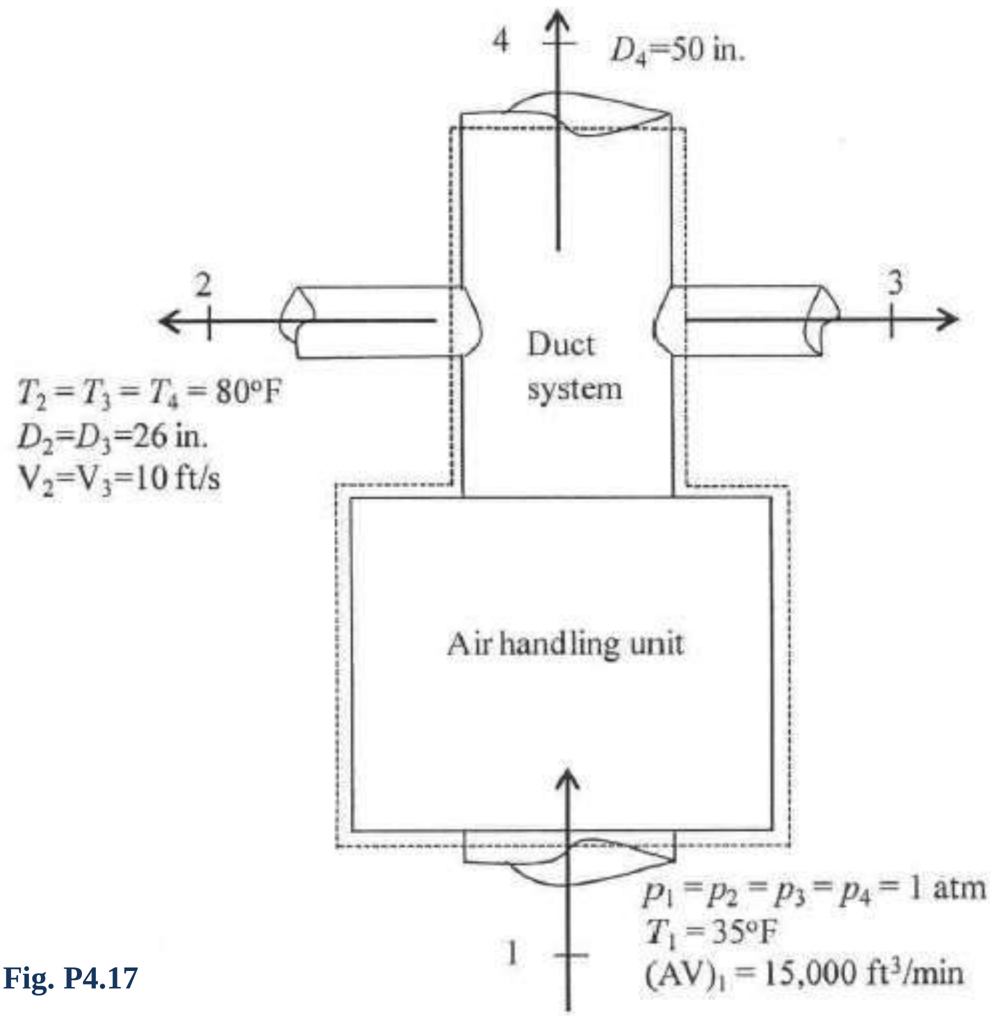


Fig. P4.17

KNOWN: At steady state, air enters an air-handling unit and exits from three ducts in an attached duct system.

FIND: Determine the mass flow rate of air entering the air handling unit, in lb/s; the volumetric flow rate in each 26-in. duct, in ft^3/min; and velocity in the 50-in.duct, in ft/s.

SCHEMATIC AND GIVEN DATA: See Fig. P4.17.

ENGINEERING MODEL:
(1) The control volume shown on the schematic is at steady state.
(2) The air behaves as an ideal gas.

Problem 4.17, continued

ANALYSIS:

(a) Determine the mass flow rate of air entering the air handling unit, in lb/s, as follows:

$$\dot{m}_1 = \frac{(AV)_1}{v_1} = \frac{(AV)_1 p_1}{RT_1} = \frac{\left(15000\frac{\text{ft}^3}{\text{min}}\right)(1\text{atm})}{\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot^\circ\text{R}}\right)(495^\circ\text{R})}\left|\frac{14.7\frac{\text{lbf}}{\text{in.}^2}}{1\text{atm}}\right|\left|\frac{144\text{in.}^2}{1\text{ft}^2}\right|\left|\frac{1\text{min}}{60\text{s}}\right| = 20.05\frac{\text{lb}}{\text{s}} \longleftarrow$$

(b) Determine the volumetric flow rate in each 26-in. duct, in ft^3/min, as follows:

$$(AV)_2 = (AV)_3 = \left(\frac{\pi D_2^{\,2}}{4}\right)V_2 = \left(\frac{\pi(26\text{ in.})^2}{4}\right)\left(10\frac{\text{ft}}{\text{s}}\right)\left|\frac{1\text{ ft}^2}{144\text{ in.}^2}\right|\left|\frac{60\text{ s}}{1\text{ min}}\right| = 2212.2\frac{\text{ft}^3}{\text{min}} \longleftarrow$$

(c) Determine the velocity in the 50-in.duct, in ft/s, as follows:

$$\frac{dm_{cv}}{dt} = 0 = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 - \dot{m}_4$$

$$\dot{m}_2 = \dot{m}_3 = \frac{(AV)_2}{v_2} = \frac{(AV)_2 p_2}{RT_2} = \frac{\left(2212.2\frac{\text{ft}^3}{\text{min}}\right)(1\text{atm})}{\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot^\circ\text{R}}\right)(540^\circ\text{R})}\left|\frac{14.7\frac{\text{lbf}}{\text{in.}^2}}{1\text{atm}}\right|\left|\frac{144\text{in.}^2}{1\text{ft}^2}\right|\left|\frac{1\text{min}}{60\text{s}}\right| = 2.71\frac{\text{lb}}{\text{s}}$$

$$\dot{m}_4 = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 = 20.05\frac{\text{lb}}{\text{s}} - 2\left(2.71\frac{\text{lb}}{\text{s}}\right) = 14.63\frac{\text{lb}}{\text{s}}$$

$$\dot{m}_4 = \frac{(AV)_4}{v_4}$$

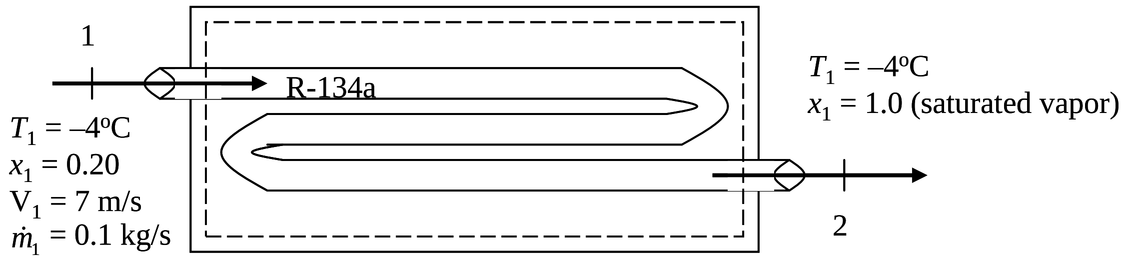
$$V_4 = \frac{\dot{m}_4 v_4}{A_4} = \frac{\dot{m}_4 RT_4}{p_4\left(\frac{\pi D_4^{\,2}}{4}\right)} = \frac{\left(14.63\frac{\text{lb}}{\text{s}}\right)\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot^\circ\text{R}}\right)(540^\circ\text{R})}{(1\text{ atm})\left(\frac{\pi(50\text{ in.})^2}{4}\right)}\left|\frac{1\text{ atm}}{14.7\frac{\text{lbf}}{\text{in.}^2}}\right| = 14.6\frac{\text{ft}}{\text{s}} \longleftarrow$$

4.18 Refrigerant 134a enters the evaporator of a refrigeration system operating at steady state at –4°C and quality of 20% at a velocity of 7 m/s. At the exit, the refrigerant is a saturated vapor at a temperature of –4°C. The evaporator flow channel has constant diameter. If the mass flow rate of the entering refrigerant is 0.1 kg/s, determine
(a) the diameter of the evaporator flow channel, in cm.
(b) the velocity at the exit, in m/s.

KNOWN: Refrigerant 134a flows through a constant-diameter evaporator entering as a saturated mixture at given temperature, quality, and velocity and exiting as a saturated vapor at a given temperature.

FIND: Determine the diameter of the flow channel and the velocity at the exit.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown on the accompanying figure is at steady state.

ANALYSIS:
(a) The diameter of the flow channel can be determined from the mass flow rate at the inlet, state 1

$$\dot{m}_1 = \frac{A_1 V_1}{v_1} = \frac{(\pi/4) D_1^2 V_1}{v_1} \Rightarrow D_1 = \left(\frac{4\dot{m}_1 v_1}{\pi V_1}\right)^{1/2}$$

Apply the quality relation to determine the specific volume at state 1. From Table A-10, v_{f1} = 0.0007644 m³/kg, v_{g1} = 0.0794 m³/kg. Substituting to determine specific volume

$$v_1 = v_{f1} + x_1(v_{g1} - v_{f1})$$

$$v_1 = 0.0007644\ \text{m}^3/\text{kg} + (0.20)(0.0794\ \text{m}^3/\text{kg} - 0.0007644\ \text{m}^3/\text{kg}) = 0.01649\ \text{m}^3/\text{kg}$$

Substituting, applying the appropriate conversion factor, and solving for the diameter

$$D_1 = \left(\frac{4\left(0.1\frac{\text{kg}}{\text{s}}\right)\left(0.01649\frac{\text{m}^3}{\text{kg}}\right)}{\pi\left(7\frac{\text{m}}{\text{s}}\right)}\left|\frac{10^4\ \text{cm}^2}{\text{m}^2}\right|\right)^{1/2} = \underline{\mathbf{1.732\ cm}}$$

(b) The exit flow velocity can be determined from the mass flow rate being equal at inlet and exit:

$$\dot{m} = \frac{A_1 \text{V}_1}{v_1} = \frac{A_2 \text{V}_2}{v_2}$$

Since the diameter is constant throughout the channel, the inlet and exit areas are the same. Since the refrigerant is a saturated vapor at the exit, from Table A-10, $v_2 = v_{g2} = 0.0794\ \text{m}^3/\text{kg}$. Solving for the exit velocity

$$\text{V}_2 = \text{V}_1\left(\frac{v_2}{v_1}\right) = \left(7\frac{\text{m}}{\text{s}}\right)\left(\frac{0.0794\frac{\text{m}^3}{\text{kg}}}{0.01649\frac{\text{m}^3}{\text{kg}}}\right) = \underline{\mathbf{33.7\ m/s}}$$

As an alternative solution, the exit flow velocity can be determined from the mass flow rate at the exit, state 2

$$\dot{m}_2 = \frac{A_2 \text{V}_2}{v_2} = \frac{(\pi/4)D_2^2 \text{V}_2}{v_2} \Rightarrow \text{V}_2 = \frac{4\dot{m}_2 v_2}{\pi D_2^2}$$

The mass flow rate is the same at the inlet and the exit based on the mass rate balance for one-inlet, one-exit, steady flow. The diameter is the same at the inlet and exit since the diameter is constant through the evaporator. Since the refrigerant is a saturated vapor at the exit, from Table A-10, $v_2 = v_{g2} = 0.0794\ \text{m}^3/\text{kg}$. Substituting values and applying the appropriate conversion factor

$$\text{V}_2 = \frac{4\left(0.1\frac{\text{kg}}{\text{s}}\right)\left(0.0794\frac{\text{m}^3}{\text{kg}}\right)}{\pi(1.732\ \text{cm})^2}\left|\frac{10^4\ \text{cm}^2}{\text{m}^2}\right| = \underline{\mathbf{33.7\ m/s}}$$

Problem 4.19

As shown in Fig. P4.19, steam at 80 bar, 440°C, enters a turbine operating at steady state with a volumetric flow rate of 236 m^3/min. Twenty percent of the entering mass flow exits through a diameter of 0.25 m at 60 bar, 400°C. The rest exits through a diameter of 1.5 m with a pressure of 0.7 bar and a quality of 90%. Determine the velocity at each exit duct, in m/s.

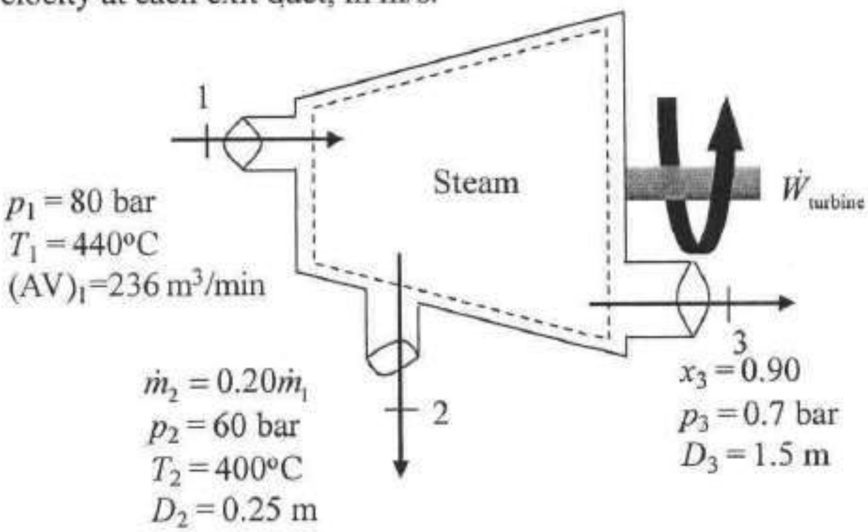


Fig. P4.19

KNOWN: Data are given for steam flowing through a turbine with one inlet and two exits.

FIND: Determine the velocity, in m/s, at each exit duct.

SCHEMATIC AND GIVEN DATA:

Refer to Fig. P4.13 for schematic and given information.

ENGINEERING MODEL:

(1) The control volume shown in the accompanying schematic operates at steady state.
(2) The flow at the inlet and each exit are one-dimensional.

ANALYSIS:

The mass flow rate at the inlet and each exit is given by an expression of the form

$$\dot{m} = \frac{(AV)}{v} \qquad (1)$$

From Table A-4, $v_1 = 0.03742$ m^3/kg. Thus,

Problem 4.19, continued

$$\dot{m}_1 = \frac{\left(236\,\frac{\text{m}^3}{\text{min}}\right)}{0.03742\,\frac{\text{m}^3}{\text{kg}}}\left|\frac{1\text{ min}}{60\text{ s}}\right| = 105.1\,\frac{\text{kg}}{\text{s}}$$

$$\dot{m}_2 = 0.2\dot{m}_1 = 21.0\,\frac{\text{kg}}{\text{s}}$$

$$\frac{dm_{cv}}{dt} = 0 = \dot{m}_1 - \dot{m}_2 - \dot{m}_3$$

$$\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = (105.1 - 21.0)\,\frac{\text{kg}}{\text{s}} = 84.1\,\frac{\text{kg}}{\text{s}}$$

Rearrange Eq. (1) to determine velocity at exit 2 and 3. From Table A-4, $v_2 = 0.04739$ m^3/kg.

$$\text{V}_2 = \frac{\dot{m}_2 v_2}{\text{A}_2} = \frac{\left(21.0\,\frac{\text{kg}}{\text{s}}\right)\left(0.04739\,\frac{\text{m}^3}{\text{kg}}\right)}{\left(\frac{\pi(0.25\text{ m})^2}{4}\right)} = 20.3\,\frac{\text{m}}{\text{s}} \longleftarrow$$

From Table A-3, at $p_3 = 0.7$ bar and $x_3 = 0.90$:

$$v_3 = v_{f_3} + x_3\left(v_{g_3} - v_{f_3}\right) = 0.001036 + 0.90(2.365 - 0.001036) = 2.1286\,\frac{\text{m}^3}{\text{kg}}$$

$$\text{V}_3 = \frac{\dot{m}_3 v_3}{\text{A}_3} = \frac{\left(84.1\,\frac{\text{kg}}{\text{s}}\right)\left(2.1286\,\frac{\text{m}^3}{\text{kg}}\right)}{\left(\frac{\pi(1.5\text{ m})^2}{4}\right)} = 101.3\,\frac{\text{m}}{\text{s}} \longleftarrow$$

Problem 4.20

KNOWN: Water vapor flows through a piping configuration with one inlet and two exits. Steady state operating and state data are provided.

FIND: Determine the mass flow rate at the inlet and each of the exits, all in kg/s.

SCHEMATIC & GIVEN DATA:

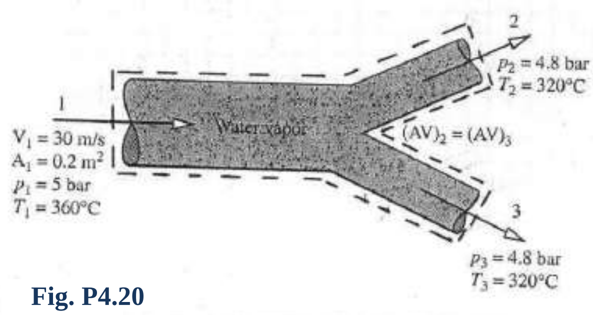
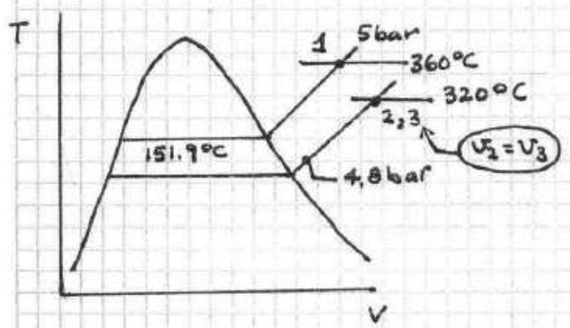


Fig. P4.20



ENGINEERING MODEL:

1. The control volume is shown in the schematic by a dashed line.
2. The control volume is at steady state.

ANALYSIS:

With v_1 from Table A-4, $v_1 = 0.5796$ m³/kg

$$\dot{m}_1 = \frac{A_1 V_1}{v_1} = \frac{(0.2\ \text{m}^2)(30\ \text{m/s})}{(0.5796\ \text{m}^3/\text{kg})} = 10.352\ \frac{\text{kg}}{\text{s}}$$

At steady state, $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$, where

$$\dot{m}_2 = \frac{(AV)_2}{v_2}, \quad \dot{m}_3 = \frac{(AV)_3}{v_3}$$

Since $v_2 = v_3$ (see T-v diagram) and the volumetric flow rates are equal (see control volume sketch), we have $\dot{m}_2 = \dot{m}_3$. Thus

$$\dot{m}_1 = \dot{m}_2 + \underbrace{\dot{m}_3}_{= \dot{m}_2} = 2\dot{m}_2$$

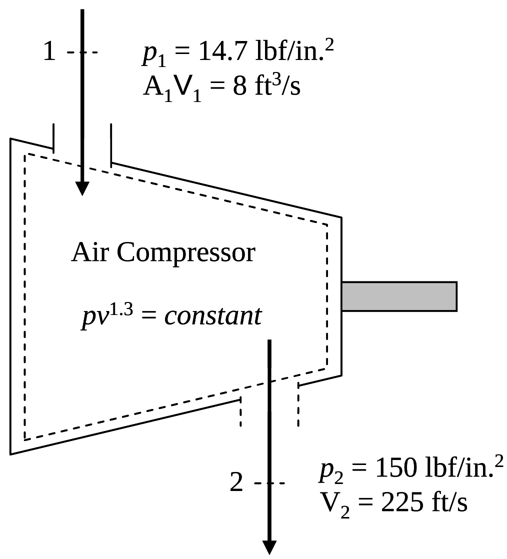
$$\therefore \dot{m}_2 = \dot{m}_3 = \frac{\dot{m}_1}{2} = 5.176\ \text{kg/s}$$

4.21 Air enters a compressor operating at steady state with a pressure of 14.7 lbf/in.^{2} and a volumetric flow rate of 8 ft^3/s. The air velocity in the exit pipe is 225 ft/s and the exit pressure is 150 lbf/in.^{2} If each unit mass of air passing from inlet to exit undergoes a process described by $pv^{1.3}$ = *constant*, determine the diameter of the exit pipe, in inches.

KNOWN: An air compressor operates at steady state with specified inlet pressure and volumetric flow rate and exit velocity and pressure.

FIND: The exit pipe diameter.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown on the accompanying figure is at steady state.
2. The air undergoes a polytropic process, $pv^{1.3}$ = *constant*, through the compressor.

ANALYSIS:
The mass flow rate is the same at the inlet and the exit based on the mass rate balance for one-inlet, one-exit, steady flow.

$$\dot{m}_1 = \dot{m}_2$$

Then, with $\dot{m} = (AV)/v$ and $A_2 = (\pi/4)D^2$, this becomes

$$\frac{A_1V_1}{v_1} = \frac{(\pi/4)D_2^2V_2}{v_2}$$

Solving for the exit diameter, D_2,

$$D_2 = \left(\left(\frac{4}{\pi}\right)\left(\frac{A_1 \mathrm{V}_1}{\mathrm{V}_2}\right)\left(\frac{v_2}{v_1}\right)\right)^{1/2}$$

The ratio of the specific volumes can be determined from the polytropic relationship, $pv^{1.3} = constant$.

$$p_1 v_1^{\ 1.3} = p_2 v_2^{\ 1.3} \Rightarrow \frac{v_2}{v_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{1.3}}$$

Substituting for the specific volume ratio

$$D_2 = \left(\left(\frac{4}{\pi}\right)\left(\frac{A_1 \mathrm{V}_1}{\mathrm{V}_2}\right)\left(\frac{p_1}{p_2}\right)^{\frac{1}{1.3}}\right)^{1/2}$$

Substituting values and applying the appropriate conversion factor give

$$D_2 = \left(\left(\frac{4}{\pi}\right)\left(\frac{8\,\frac{\mathrm{ft}^3}{\mathrm{s}}}{225\,\frac{\mathrm{ft}}{\mathrm{s}}}\right)\left(\frac{14.7\,\frac{\mathrm{lbf}}{\mathrm{in.}^2}}{150\,\frac{\mathrm{lbf}}{\mathrm{in.}^2}}\right)^{\frac{1}{1.3}}\left|\frac{144\,\mathrm{in.}^2}{\mathrm{ft}^2}\right|\right)^{1/2} = \underline{\mathbf{1.04\ in.}}$$

Problem 4.22

Ammonia enters a control volume operating at steady state at $p_1 = 16$ bar, $T_1 = 32°C$, with a mass flow rate of 1.5 kg/s. Saturated vapor at 6 bar leaves through one exit and saturated liquid at 6 bar leaves through a second exit with a volumetric flow rate of 0.10 m^3/min. Determine

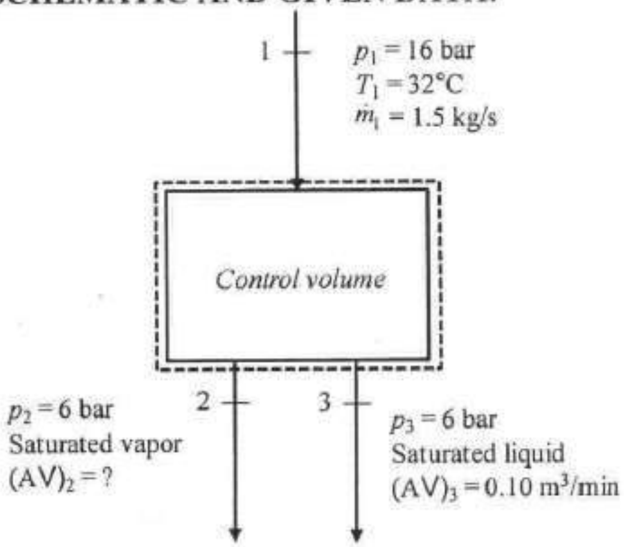
(a) the minimum diameter of the inlet pipe, in cm, so the ammonia velocity at the inlet does not exceed 18 m/s.

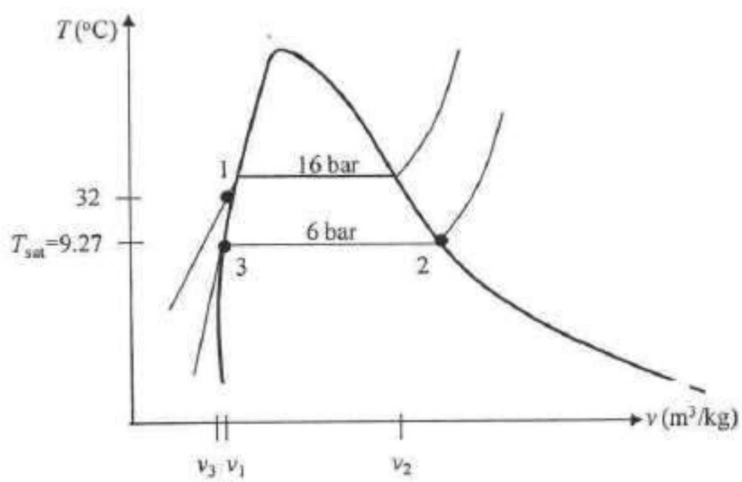
(b) the volumetric flow rate of the exiting saturated vapor, in m^3/min.

KNOWN: Ammonia enters a control volume operating at steady state with known conditions. Saturated vapor leaves through one exit and saturated liquid leaves through a second exit.

FIND: Determine the minimum diameter, in cm, of the inlet pipe so the inlet velocity does not exceed 18 m/s; and the volumetric flow rate of the exiting saturated vapor, in m^3/min.

SCHEMATIC AND GIVEN DATA:





Problem 4.22, continued

ENGINEERING MODEL:

(1) The control volume shown in the accompanying schematic operates at steady state.
(2) For compressed liquid ammonia data, use saturated liquid data (Table A-13).

ANALYSIS:

(a) Relate mass flow rate, V_1, and D_1 as follows:

$$\mathrm{V}_1 = \frac{\dot{m}_1 v_1}{\mathrm{A}_1} = \frac{\dot{m}_1 v_1}{\left(\frac{\pi D_1^2}{4}\right)}$$

Solving for D_1:

$$D_1 = \sqrt{\frac{4\dot{m}_1 v_1}{\pi \mathrm{V}_1}}$$

Since V_1 cannot exceed 18 m/s, it follows that

$$D_1 \geq \sqrt{\frac{4\dot{m}_1 v_1}{\pi \mathrm{V}_1}}$$

where V_1= 18m/s.

From Table A-14, $T_1 < T_{sat}$ at p_1, therefore state 1 is in the compressed liquid region (see assumption 2). Using Table A-13 at T_1: $v_1 \approx v_f = 1.6887 \times 10^{-3}\ \mathrm{m^3/kg}$.

$$D_1 \geq \sqrt{\frac{4\dot{m}_1 v_1}{\pi \mathrm{V}_1}} = \sqrt{\frac{4\left(1.5\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(1.6887\times10^{-3}\frac{\mathrm{m}^3}{\mathrm{kg}}\right)}{\pi\left(18\frac{\mathrm{m}}{\mathrm{s}}\right)}} = 0.0134\ \mathrm{m} = 1.34\ \mathrm{cm} \longleftarrow$$

(b) To determine $(\mathrm{AV})_2$, begin with the mass balance.

$$\frac{dm_{cv}}{dt} = 0 = \dot{m}_1 - \dot{m}_2 - \dot{m}_3$$

$$\dot{m}_2 = \dot{m}_1 - \dot{m}_3$$

$$\dot{m}_2 = \frac{(\mathrm{AV})_2}{v_2} \quad or$$

$$(\mathrm{AV})_2 = v_2\dot{m}_2 = v_2(\dot{m}_1 - \dot{m}_3) = v_2\left(\dot{m}_1 - \frac{(\mathrm{AV})_3}{v_3}\right)$$

Problem 4.22, continued

From Table A-14 at 6 bar: $v_2 = 0.2104\ \mathrm{m^3/kg}$ and $v_3 = 1.5982 \times 10^{-3}\ \mathrm{m^3/kg}$.

$$(\mathrm{AV})_2 = 0.2104\frac{\mathrm{m^3}}{\mathrm{kg}}\left(1.5\frac{\mathrm{kg}}{\mathrm{s}}\left|\frac{60\ \mathrm{s}}{1\ \mathrm{min}}\right| - \frac{0.10\dfrac{\mathrm{m^3}}{\mathrm{min}}}{1.5982\times10^{-3}\dfrac{\mathrm{m^3}}{\mathrm{kg}}}\right) = 5.771\frac{\mathrm{m^3}}{\mathrm{min}} \longleftarrow$$

Problem 4.23

KNOWN: Air flows through a rectangular duct at steady state. State data and duct dimensions are provided.

FIND: At the inlet, determine the volumetric flow rate, in ft^3/s, and mass flow rate, in kg/s. Determine the same quantities at the exit, if possible; otherwise explain.

SCHEMATIC & GIVEN DATA:

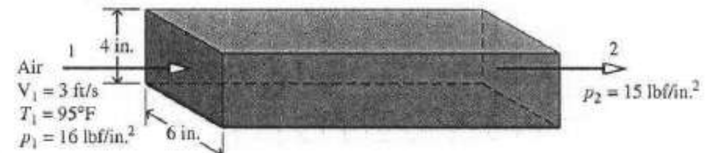


Fig. P4.18

ENGINEERING MODEL:

1. A control volume encloses the duct shown.
2. The control volume is at steady state.
3. The ideal gas model applies to the air flowing through the duct.

ANALYSIS:

At the inlet,

$$(AV_1) = \left[\left(\frac{4}{12}\text{ ft}\right)\left(\frac{6}{12}\text{ ft}\right)\right]\left(3\,\frac{\text{ft}}{\text{s}}\right) = 0.5\,\frac{\text{ft}^3}{\text{s}}$$

$$\dot{m}_1 = \frac{(AV_1)}{v_1} = \frac{(AV_1)}{(RT_1/p_1)} = \frac{(AV)_1\,p_1}{RT_1}$$

$$= \frac{(0.5\text{ ft}^3/\text{s})(16\times144\text{ lbf/ft}^2)}{\left(\frac{1545}{28.97}\,\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(555°\text{R})} = 0.039\,\frac{\text{lb}}{\text{s}}$$

At steady state, the mass rate balance reduces to $\dot{m}_2 = \dot{m}_1$. Thus,

$$\dot{m}_2 = 0.039\,\frac{\text{lb}}{\text{s}}$$

With $\dot{m}_2$ known,

$$\dot{m}_2 = \frac{AV_2}{v_2} \Rightarrow (AV_2) = \dot{m}_2 v_2$$

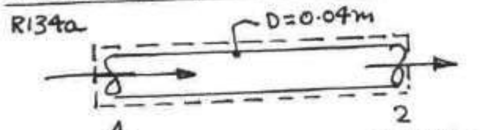
$$(AV_2) = \dot{m}_2\left[\frac{RT_2}{P_2}\right]$$

To find the volumetric flow rate at 2, need V_2 or T_2. $\Rightarrow$ (AV_2) cannot be determined without additional information.

Problem 4.24

Refrigerant 134a enters a horizontal pipe operating at steady state at 40°C, 300 kPa and a velocity of 40 m/s. At the exit, the temperature is 50°C and the pressure is 240 kPa. The pipe diameter is 0.04 m. Determine (a) the mass flow rate of the refrigerant, in kg/s, (b) the velocity at the exit, in m/s, and (c) the rate of heat transfer between the pipe and its surroundings, in kW.

SCHEMATIC & GIVEN DATA:



$T_1 = 40°C$ $\quad$ $T_2 = 50°C$

$p_1 = 300 kPa$ $\quad$ $p_2 = 240 kPa$

$V_1 = 40 m/s$

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.

2. For the control volume, $\dot{W}_{cv} \equiv 0$ and $\Delta pe = 0$ (horizontal).

ANALYSIS: (a) Using Eq. 4.4a,

$$\dot{m}_1 = \frac{A V_1}{v_1} = \frac{\left(\frac{\pi (0.04 m)^2}{4}\right)\left(40 \frac{m}{s}\right)}{0.08089 \frac{m^3}{kg}}$$

Table A-12

$$\dot{m}_1 = 0.621 \text{ kg/s} \longleftarrow \text{(a)}$$

(b) $\dot{m}_1 = \dot{m}_2$ (steady state)

$$\Rightarrow \frac{A V_1}{v_1} = \frac{A V_2}{v_2} \Rightarrow V_2 = \frac{v_2}{v_1} V_1$$

$$\therefore V_2 = \left(\frac{0.10562}{0.08089}\right)(40 m/s) = 52.23 m/s \longleftarrow \text{(b)}$$

(c) Reducing Eq. 4.20a,

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g\cancel{(z_1 - z_2)}\right]$$

$$\Rightarrow \dot{Q}_{cv} = \dot{m}\left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}\right]$$

unit conversions on k.e. term

$$= 0.621 \frac{kg}{s}\left[(294.47 - 284.05)\frac{kJ}{kg} + \left[\frac{(52.23)^2 - (40)^2}{2}\left(\frac{m^2}{s^2}\right)\left|\frac{1 N}{1 kg \cdot m/s^2}\right|\left|\frac{1 kJ}{10^3 N \cdot m}\right|\right]\right]$$

(Table A-12)

$$= 0.621 \frac{kg}{s}\left[10.42 + 0.56\right]\frac{kJ}{kg}\left|\frac{1 kW}{1 kJ/s}\right| = +6.82 kW \longleftarrow \text{(c)}$$

PROBLEM 4.25

As shown in Fig. P4.25, air enters a pipe at 25°C, 100 kPa with a volumetric flow rate of 23 m³/h. On the outer pipe surface is an electrical resistor covered with insulation. With a voltage of 120 V, the resistor draws a current of 4 amps. Assuming the ideal gas model with $c_p = 1.005$ kJ/kg · K for air and ignoring kinetic and potential energy effects, determine (a) the mass flow rate of the air, in kg/h, and (b) the temperature of the air at exit, in °C.

SCHEMATIC & GIVEN DATA:

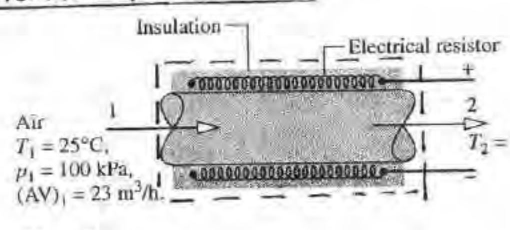


Fig. P4.25

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer and kinetic and potential energy effects can be ignored.
3. The air can be modeled as an ideal gas with constant specific heat cp.

ANALYSIS:

(a) $$\dot{m} = \frac{(AV)_1}{v_1} = \frac{p_1 (AV)_1}{RT_1} = \frac{(10^5 \text{N/m}^2)(23 \text{ m}^3/\text{h})}{\left(\frac{8314}{28.97}\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(298\text{K})} = 26.89 \frac{\text{kg}}{\text{h}} \quad \text{(a)}$$

(b) Reducing Eq. 4.20a using listed assumptions,

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

where $(h_1 - h_2) = c_{pa}(T_1 - T_2)$ and

$$\dot{W}_{cv} = -(\text{voltage})(\text{current})$$
$$= -(120 \text{volts})(4 \text{amps}) \left| \frac{1 \text{ Watt/amp}}{1 \text{ Volt}} \right| \left| \frac{1 \text{ kW}}{10^3 \text{ Watt}} \right|$$
$$= -0.48 \text{ kW}$$

Collecting results, and solving for T_2,

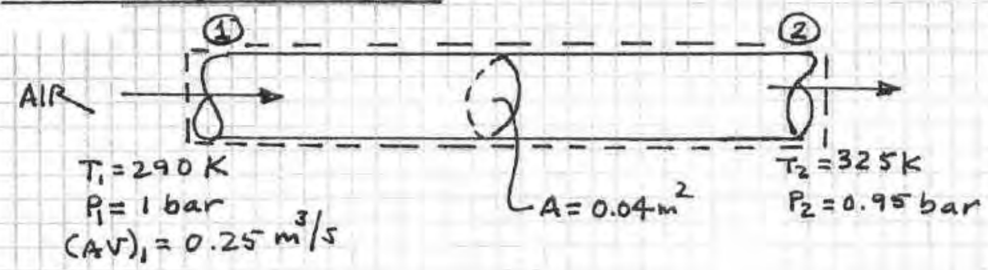
$$T_2 = T_1 - \frac{\dot{W}_{cv}}{\dot{m} c_{pa}}$$
$$= 298 - \frac{(-0.48 \text{kW})}{\left(26.89 \frac{\text{kg}}{\text{h}}\right)\left(1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)} \left| \frac{3600 \text{s}}{1 \text{h}} \right| \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right|$$
$$= 362 \text{K} \; (89°\text{C}) \quad \text{(b)}$$

PROBLEM 4.26

KNOWN: Air flows through a horizontal, constant-diameter duct operating at steady state. State data and duct flow area are provided.

FIND: Determine the mass flow rate of the air, in kg/s, the inlet and exit velocities, in m/s, and the rate of heat transfer, in kW.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The control volume shown in the schematic operates at steady state.
2. The air is modeled as an ideal gas with k = 1.4

ANALYSIS:

(a) $$\dot{m}_1 = \frac{(AV)_1}{v_1} = \frac{(AV)_1}{(RT_1/P_1)} = \frac{(AV)_1 P_1}{RT_1} = \frac{(0.25\ m^3/s)(10^5\ N/m^2)}{\left(\frac{8314}{28.97}\ \frac{N\cdot m}{kg\cdot K}\right)(290K)} = 0.3\ \ kg/s$$

(b) $(AV)_1 = 0.25\ m^3/s$

$$\Rightarrow V_1 = \frac{0.25\ m^3/s}{0.04\ m^2} = 6.25\ m/s$$

At steady state, the mass rate balance reduces to $\dot{m}_1 = \dot{m}_2$. Thus

$$\frac{AV_1}{v_1} = \frac{AV_2}{v_2} \Rightarrow \text{With } v = RT/P,\ \frac{AV_1 P_1}{RT_1} = \frac{AV_2 P_2}{RT_2} \Rightarrow V_2 = \left(\frac{T_2}{T_1}\right)\left(\frac{P_1}{P_2}\right) V_1$$

$$= \left(\frac{325}{290}\right)\left(\frac{1}{0.95}\right)(6.25\ \tfrac{m}{s}) = 7.37\ \frac{m}{s}$$

(c) Reducing Eq. 4.20a,

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}\left[h_1 - h_2 + \left(\frac{V_1^2 - V_2^2}{2}\right) + \cancel{g(z_1 - z_2)}\right]$$

$$\dot{Q} = \dot{m}\left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}\right], \text{ or with } (h_2 - h_1) = c_p(T_2 - T_1)$$

$$\dot{Q} = \dot{m}\left[c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}\right], \text{ where, with Eq. 3.47a: } c_p = \frac{kR}{(k-1)} = \frac{1.4\left(\frac{8.314}{28.97}\right)}{(1.4-1)} = 1.004\ \frac{kJ}{kg\cdot K}$$

○ $c_p(T_2 - T_1) = \left(1.004\ \frac{kJ}{kg\cdot K}\right)(325 - 290)\ K = 35.14\ kJ/kg$

① ○ $$\left[\frac{V_2^2 - V_1^2}{2}\right] = \frac{(7.37)^2 - (6.25)^2}{2}\left[\frac{m}{s}\right]^2 \left|\frac{1\ N}{1\ kg\cdot m/s^2}\right| \left|\frac{1\ kJ}{10^3\ N\cdot m}\right| = 0.0076\ \frac{kJ}{kg}$$

Finally,

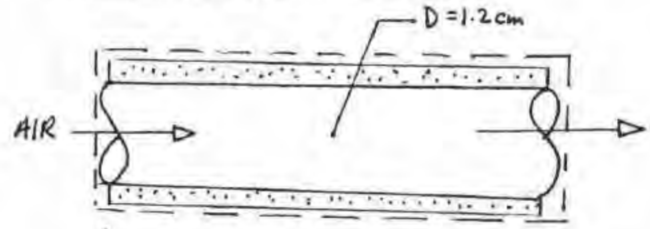
$$\dot{Q} = \left(0.3\ \frac{kg}{s}\right)\left[35.14 + 0.0076\right]\frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right| = 10.54\ kW$$

1. In this case, by direct calculation the effects of kinetic energy are shown to be negligible.

PROBLEM 4.27

Air at 600 kPa, 330 K enters a well-insulated, horizontal pipe having a diameter of 1.2 cm and exits at 120 kPa, 300 K. Applying the ideal gas model for air, determine at steady state (a) the inlet and exit velocities, each in m/s, and (b) the mass flow rate, in kg/s.

SCHEMATIC & GIVEN DATA:



$T_1 = 330\text{K}$ $\quad$ $T_2 = 300\text{K}$

$P_1 = 600\text{kPa}$ $\quad$ $P_2 = 120\text{kPa}$

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer can be ignored. Also, $\Delta pe = 0$ (horizontal); $\dot{W}_{cv} = 0$.
3. The air can be modeled as an ideal gas

ANALYSIS: (a) Mass rate balance, $\dot{m}_1 = \dot{m}_2$. That is

$$\frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2} \Rightarrow \quad V_1 = \frac{v_1}{v_2} V_2 = \frac{(RT_1/P_1)}{(RT_2/P_2)} V_2$$

$$V_1 = \left(\frac{P_2}{P_1}\right)\left(\frac{T_1}{T_2}\right) V_2 = \left(\frac{120\text{kPa}}{600\text{kPa}}\right)\left(\frac{330\text{K}}{300\text{K}}\right) V_2$$

$$V_1 = 0.22\, V_2 \qquad (1)$$

Energy rate balance,

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g\cancel{(z_1 - z_2)}\right]$$

$$\Rightarrow \quad 0 = (h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} \Rightarrow \quad 0 = (h_1 - h_2) + \frac{(0.22 V_2)^2 - V_2^2}{2}$$

Or, with data from Table A-22 for h_1 and h_2,

$$\frac{0.9516 V_2^2}{2} = (330.34 - 300.19)\frac{\text{kJ}}{\text{kg}}$$

$$\Rightarrow \quad V_2 = \sqrt{\frac{2(330.34 - 300.19)\text{ kJ/kg}}{0.9516}\left|\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right|\left|\frac{10^3\text{ N}\cdot\text{m}}{1\text{ kJ}}\right|}$$

$$= 251.73 \text{ m/s} \qquad \text{(a)}$$

And with Eq. (1) $\quad V_1 = 0.22\, V_2 = 55.38 \text{ m/s}$

(b) Then, with

$$\dot{m} = \frac{A_2 V_2}{v_2} = \frac{P_2 A_2 V_2}{RT_2}$$

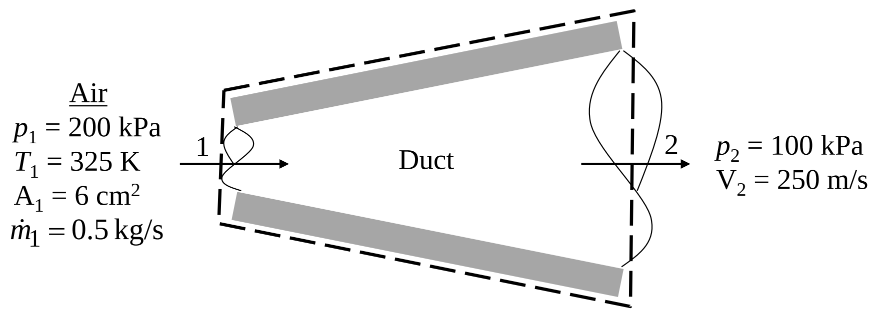
$$= \frac{\left(120\times10^3 \frac{\text{N}}{\text{m}^2}\right)\left(\frac{\pi}{4}\left(\frac{1.2}{100}\text{m}\right)^2\right)(251.73 \text{ m/s})}{\left(\frac{8314}{28.97}\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(300\text{K})} = 0.04 \frac{\text{kg}}{\text{s}} \qquad \text{(b)}$$

4.28 At steady state, air at 200 kPa, 325 K, and mass flow rate of 0.5 kg/s enters an insulated duct having differing inlet and exit cross-sectional areas. The inlet cross-sectional area is 6 cm^2. At the duct exit, the pressure of the air is 100 kPa and the velocity is 250 m/s. Neglecting potential energy effects and modeling air as an ideal gas with constant c_p = 1.008 kJ/kg·K, determine
(a) the velocity of the air at the inlet, in m/s.
(b) the temperature of the air at the exit, in K.
(c) the exit cross-sectional area, in cm^2.

KNOWN: Air flow through a duct with varying cross-sectional area.

FIND: (a) the velocity of the air at the inlet, in m/s, (b) the temperature of the air at the exit, in K, and (c) the exit cross-sectional area, in cm^2.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown with the schematic is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$ and stray heat transfer can be ignored.
3. Δpe = 0.
4. The air is modeled as an ideal gas with constant c_p = 1.008 kJ/kg·K.

ANALYSIS:

(a) Mass flow rate at the inlet is $\dot{m}_1 = \frac{A_1 V_1}{v_1}$. Substituting $v_1 = \frac{RT_1}{p_1}$ from the ideal gas equation of state and solving for the velocity give

$$V_1 = \frac{\dot{m}_1 R T_1}{A_1 p_1}$$

$$V_1 = \frac{(0.5\ \text{kg/s})\left(\dfrac{8.314\ \dfrac{\text{kJ}}{\text{kmol}\cdot\text{K}}}{28.97\ \dfrac{\text{kg}}{\text{kmol}}}\right)(325\ \text{K})}{(6\ \text{cm}^2)(200\ \text{kPa})}\left|\frac{10000\ \text{cm}^2}{1\ \text{m}^2}\right|\left|\frac{1000\ \text{N}\cdot\text{m}}{1\ \text{kJ}}\right|\left|\frac{1\ \text{kPa}}{1000\ \dfrac{\text{N}}{\text{m}^2}}\right|$$

V_1 = **388.6 m/s**

(b) Since $\Delta h = c_p\,\Delta T$ for air with constant specific heat, the energy rate balance

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\,[(h_1 - h_2) + \tfrac{1}{2}(\mathrm{V}_1^2 - \mathrm{V}_2^2) + g(\cancel{z_1 - z_2})]$$

simplifies to

$$0 = [c_p(T_1 - T_2) + \tfrac{1}{2}(\mathrm{V}_1^2 - \mathrm{V}_2^2)]$$

Solving for exit temperature gives

$$T_2 = T_1 - \frac{\left(\mathrm{V}_2^2 - \mathrm{V}_1^2\right)}{2c_p} = 325\,\mathrm{K} - \frac{\left(250\,\dfrac{\mathrm{m}}{\mathrm{s}}\right)^2 - \left(388.6\,\dfrac{\mathrm{m}}{\mathrm{s}}\right)^2}{2\left(1.008\,\dfrac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\right)}\left|\frac{1\,\mathrm{kJ}}{1000\,\mathrm{N\cdot m}}\right|\left|\frac{1\,\mathrm{N}}{1\,\dfrac{\mathrm{kg\cdot m}}{\mathrm{s}^2}}\right|$$

$T_2 =$ **368.9 K**

(c) From the mass rate balance and ideal gas equation of state

$$\dot{m}_2 = \dot{m}_1 = \frac{\mathrm{A}_2\mathrm{V}_2}{v_2} = \frac{\mathrm{A}_2\mathrm{V}_2 p_2}{RT_2}$$

Solving for the exit area yields

$$\mathrm{A}_2 = \frac{\dot{m}_1 R T_2}{\mathrm{V}_2 p_2}$$

$$\mathrm{A}_2 = \frac{(0.5\,\mathrm{kg/s})\left(\dfrac{8.314\,\dfrac{\mathrm{kJ}}{\mathrm{kmol\cdot K}}}{28.97\,\dfrac{\mathrm{kg}}{\mathrm{kmol}}}\right)(368.9\,\mathrm{K})}{(250\,\mathrm{m/s})(100\,\mathrm{kPa})}\left|\frac{10000\,\mathrm{cm}^2}{1\,\mathrm{m}^2}\right|\left|\frac{1000\,\mathrm{N\cdot m}}{1\,\mathrm{kJ}}\right|\left|\frac{1\,\mathrm{kPa}}{1000\,\dfrac{\mathrm{N}}{\mathrm{m}^2}}\right|$$

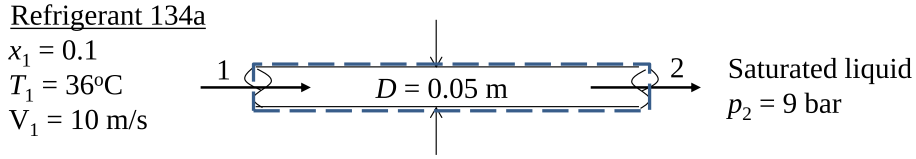
$\mathrm{A}_2 =$ **21.2 cm²**

4.29 Refrigerant 134a flows at steady state through a horizontal tube having an inside diameter of 0.05 m. The refrigerant enters the tube with a quality of 0.1, temperature of 36°C, and velocity of 10 m/s. The refrigerant exits the tube at 9 bar as a saturated liquid. Determine
(a) the mass flow rate of the refrigerant, in kg/s.
(b) the velocity of the refrigerant at the exit, in m/s.
(b) the rate of heat transfer, in kW, and its associated direction with respect to the refrigerant.

KNOWN: Refrigerant 134a flows through a tube.

FIND: (a) the mass flow rate of the refrigerant, in kg/s, (b) the velocity of the refrigerant at the exit, in m/s, and (c) the rate of heat transfer, in kW, and its associated direction with respect to the refrigerant.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown with the schematic is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$, and Δpe = 0.

ANALYSIS:

(a) Mass flow rate at the inlet is $\dot{m}_1 = \dfrac{A_1 V_1}{v_1} = \dfrac{\left(\frac{\pi}{4} D^2\right) V_1}{v_1}$.

Specific volume of the refrigerant at the inlet is determined using quality

$$v_1 = v_{f1} + x_1(v_{g1} - v_{f1})$$

From Table A-10 at T_1 = 36°C, v_{f1} = 0.0008590 m^3/kg and v_{g1} = 0.0223 m^3/kg. Solving

$$v_1 = 0.0008590 \text{ m}^3/\text{kg} + (0.1)(0.0223 \text{ m}^3/\text{kg} - 0.0008590 \text{ m}^3/\text{kg}) = 0.003003 \text{ m}^3/\text{kg}$$

Substituting values and solving for mass flow rate give

$$\dot{m}_1 = \frac{\left(\frac{\pi}{4}\right)(0.05\text{ m})^2\left(10\frac{\text{m}}{\text{s}}\right)}{0.003003\frac{\text{m}^3}{\text{kg}}}$$

$$\dot{m}_1 = \mathbf{6.54\ kg/s}$$

(b) From the mass rate balance

$$\dot{m}_2 = \dot{m}_1 = \frac{\cancel{A_2}V_2}{v_2} = \frac{\cancel{A_1}V_1}{v_1}$$

Since the tube diameter is constant, the areas cancel. Solving for exit velocity yields

$$V_2 = V_1\left(\frac{v_2}{v_1}\right)$$

Specific volume for the saturated liquid refrigerant at the exit is obtained from Table A-11:

$$v_2 = v_{f2} = 0.0008576 \text{ m}^3\text{/kg}$$

Substituting and solving for the exit velocity give

$$V_2 = \left(10\frac{\text{m}}{\text{s}}\right)\left(\frac{0.0008576\frac{\text{m}^3}{\text{kg}}}{0.003003\frac{\text{m}^3}{\text{kg}}}\right)$$

V_2 = **2.86 m/s**

(c) The energy rate balance

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}[(h_1 - h_2) + ½ (V_1^2 - V_2^2) + g\cancel{(z_1 - z_2)}]$$

simplifies to

$$\dot{Q}_{cv} = \dot{m}[(h_2 - h_1) + ½ (V_2^2 - V_1^2)]$$

Specific enthalpy of the refrigerant at the inlet is determined using quality

$$h_1 = h_{f1} + x_1 h_{fg1}$$

From Table A-10 at T_1 = 36°C, h_{f1} = 100.25 kJ/kg and h_{fg1} = 166.15 kJ /kg. Solving

$$h_1 = 100.25 \text{ kJ /kg} + (0.1)(166.15 \text{ kJ /kg}) = 116.87 \text{ kJ /kg}$$

Specific enthalpy for the saturated liquid refrigerant at the exit is obtained from Table A-11:

$$h_2 = h_{f2} = 99.56 \text{ kJ/kg}$$

Substituting values and solving for rate of heat transfer give

$$\dot{Q}_{cv} = \left(6.54\frac{kg}{s}\right)\left[(99.56 - 116.87)\frac{kJ}{kg} + \frac{\left(2.86\frac{m}{s}\right)^2 - \left(10\frac{m}{s}\right)^2}{2}\left|\frac{1\,kJ}{1000\,N\cdot m}\right|\left|\frac{1\,N}{1\frac{kg\cdot m}{s^2}}\right|\right]\left|\frac{kW}{\left(\frac{kJ}{s}\right)}\right|$$

$$\dot{Q}_{cv} = \mathbf{-113.5\ kW}$$

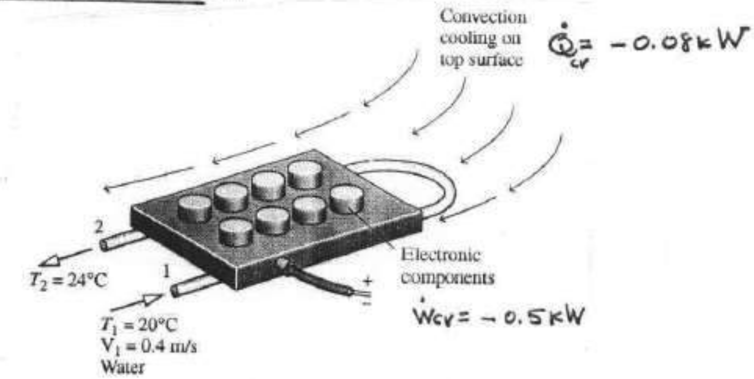
Since the heat transfer rate is negative, heat transfer is from the refrigerant.

Problem 4.30

KNOWN: Data are provided for electronic components mounted on a plate that are cooled by convection to the surroundings and water circulating through a tube bonded to the plate. Operation is at steady state.

FIND: Determine the tube diameter.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. A control volume encloses the plate-mounted electronic components with an inlet at 1 and an exit at 2. 2. The control volume ① is at steady state. 3. For the water entering and exiting, $h \approx h_f(T)$, $v \approx v_f(T)$. 4. Kinetic and potential energy effects can be ignored.

ANALYSIS: At steady state, $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Also,

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{(\pi D^2/4)V_1}{v_f(T_1)}$$

An energy rate balance reads

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)} + \cancel{g(z_1 - z_2)}\right]$$

or

$$\dot{m} = \frac{[\dot{Q}_{cv} - \dot{W}_{cv}]}{h_2 - h_1} = \frac{[\dot{Q}_{cv} - \dot{W}_{cv}]}{h_f(T_2) - h_f(T_1)}$$

Collecting results

$$D = \sqrt{\frac{4\,v_f(T_1)}{\pi V_1}\left[\frac{(\dot{Q}_{cv} - \dot{W}_{cv})}{h_f(T_2) - h_f(T_1)}\right]}$$

with data from Table A-2: $v_f(20°C) = (1.0018/10^3)\,m^3/kg$, $h_f(T_1) = 83.96$ kJ/kg, $h_f(T_2) = 100.7$ kJ/kg

$$D = \sqrt{\frac{(4)(1.0018/10^3)\,m^3/kg}{\pi\,(0.4\,m/s)}\left[\frac{(-0.08 - (-0.5))\,kJ/s}{(100.7 - 83.96)\,kJ/kg}\right]}$$

$$= 0.0089\,m \left|\frac{10^2\,cm}{m}\right|$$

$$= 0.89\,cm$$

1. Alternatively, the incompressible model can be used, with Eq. 3.20b and c from Table A-19.

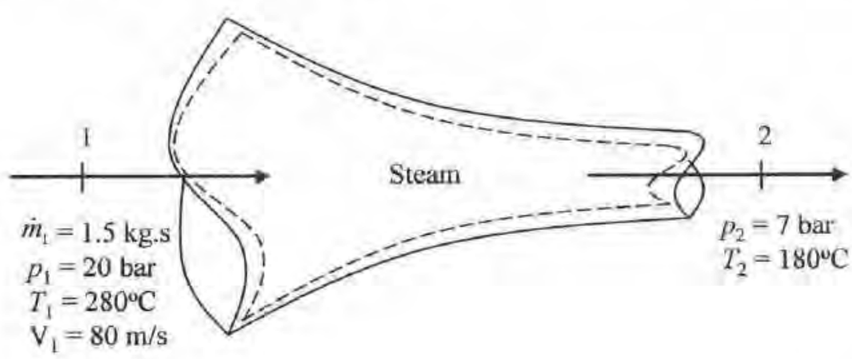
PROBLEM 4.31

4.31 Steam enters a nozzle operating at steady state at 20 bar, 280°C, with a velocity of 80 m/s. The exit pressure and temperature are 7 bar and 180°C, respectively. The mass flow rate is 1.5 kg/s. Neglecting heat transfer and potential energy, determine
(a) the exit velocity, in m/s.
(b) the inlet and exit flow areas, in cm^2.

KNOWN: Steam at a specified mass flow rate flows through a nozzle with known pressure, temperature, and velocity at the inlet and known pressure and temperature at the exit.

FIND: Determine the exit velocity and the inlet and exit flow areas.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown on the accompanying figure is at steady state.
2. Heat transfer and the change in potential energy from inlet to exit can be neglected.
3. $\dot{W}_{cv} = 0$.

ANALYSIS:
State 1 is fixed by T_1 and p_1 and is in the superheated vapor region. From Table A-4, $h_1 = 2976.4$ kJ/kg and $v_1 = 0.1200$ m³/kg.

State 2 is fixed by T_2 and p_2 and also is in the superheated vapor region. From Table A-4, $h_2 = 2799.1$ kJ/kg and $v_2 = 0.2847$ m³/kg.

(a) The steady-state, one-inlet, one-exit energy balance gives

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\,[(h_1 - h_2) + \tfrac{1}{2}(V_1^2 - V_2^2) + g(z_1 - z_2)]$$

Neglecting heat transfer and potential energy change, recognizing no work is associated with a nozzle, and dividing by the mass flow rate, the energy balance simplifies to

$$0 = (h_1 - h_2) + \tfrac{1}{2}(V_1^2 - V_2^2)$$

Solving for the exit velocity

PROBLEM 4.31 (Continued)

$$V_2 = [V_1^2 + 2(h_1 - h_2)]^{1/2}$$

Substituting values and applying the appropriate conversion factors

$$V_2 = \left[\left(80\frac{\text{m}}{\text{s}}\right)^2 + 2\left(2976.4\frac{\text{kJ}}{\text{kg}} - 2799.1\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{10^3\ \text{N}\cdot\text{m}}{\text{kJ}}\right|\left|\frac{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}{\text{N}}\right|\right]^{1/2} = \underline{\mathbf{600.8\ m/s}}$$

(b) The inlet and exit flow areas can be determined from the mass flow rate, $\dot{m}$

$$\dot{m} = \frac{AV}{v}$$

For the inlet

$$A_1 = (\dot{m}\, v_1)/V_1 = \frac{\left(1.5\frac{\text{kg}}{\text{s}}\right)\left(0.1200\frac{\text{m}^3}{\text{kg}}\right)}{80\frac{\text{m}}{\text{s}}}\left|\frac{10^4\ \text{cm}^2}{\text{m}^2}\right| = \underline{\mathbf{22.5\ cm^2}}$$

For the exit

$$A_2 = (\dot{m}\, v_2)/V_2 = \frac{\left(1.5\frac{\text{kg}}{\text{s}}\right)\left(0.2847\frac{\text{m}^3}{\text{kg}}\right)}{600.8\frac{\text{m}}{\text{s}}}\left|\frac{10^4\ \text{cm}^2}{\text{m}^2}\right| = \underline{\mathbf{7.11\ cm^2}}$$

As an alternative to solve for A_2, since $(A_1V_1)/\,v_1 = (A_2V_2)/\,v_2$,

$$A_2 = A_1(v_2/v_1)(V_1/V_2) = \left(22.5\,\text{cm}^2\right)\left(\frac{0.2847\frac{\text{m}^3}{\text{kg}}}{0.1200\frac{\text{m}^3}{\text{kg}}}\right)\left(\frac{80\frac{\text{m}}{\text{s}}}{600.8\frac{\text{m}}{\text{s}}}\right) = \underline{\mathbf{7.11\ cm^2}}$$

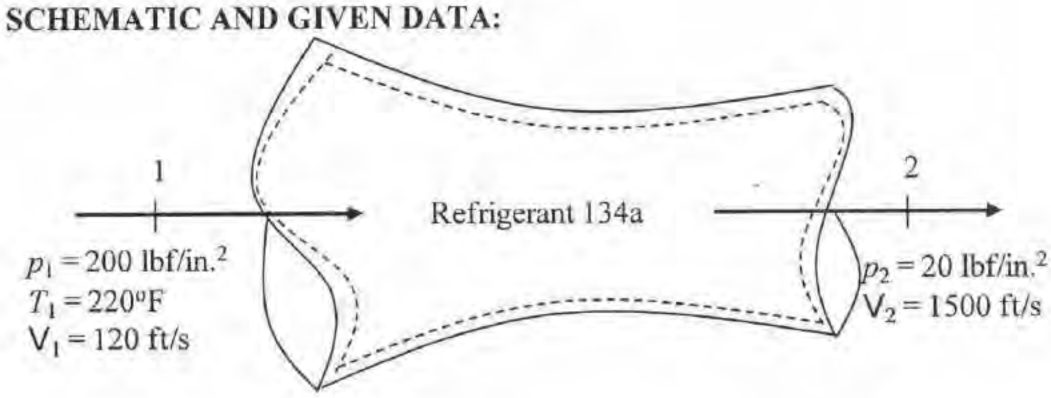
PROBLEM 4.32

4.32 Refrigerant 134a enters a well-insulated nozzle at 200 lbf/in.^{2}, 220°F, with a velocity of 120 ft/s and exits at 20 lbf/in.^{2} with a velocity of 1500 ft/s. For steady-state operation, and neglecting potential energy effects, determine the exit temperature, in °F.

KNOWN: Refrigerant 134a enters a well-insulated nozzle at known inlet and exit conditions.

FIND: Determine the exit temperature, in °F.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

(1) The control volume shown in the accompanying schematic operates at steady state.
(2) There are no energy transfers via heat or work, $\dot{W}_{cv}$.
(3) Potential energy change from inlet to exit can be neglected.

ANALYSIS:

To fix state 1, use the steady-state energy balance to determine the enthalpy at the exit.

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)\right]$$

$$\dot{m} = \dot{m}_1 = \dot{m}_2$$

Simplify based on assumptions and solve for h_2.

$$h_2 = h_1 + \left(\frac{V_1^2 - V_2^2}{2}\right)$$

From Table A-12E, h_1 = 144.15 Btu/lb. Thus,

$$h_2 = 144.15\frac{\text{Btu}}{\text{lb}} + \left(\frac{120^2 - 1500^2}{2}\right)\frac{\text{ft}^2}{\text{s}^2}\left|\frac{1\text{ lbf}}{32.2\,\frac{\text{lb}\cdot\text{ft}}{\text{s}^2}}\right|\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right| = 99.53\frac{\text{Btu}}{\text{lb}}$$

From Table A-11E at 20 lbf/in.^{2} $h_f < h_2 < h_g$ and therefore state 2 falls in the two-phase liquid-vapor region. At the exit, $T_2 = T_{sat}$ = -2.48°F with x_2 = 98%.

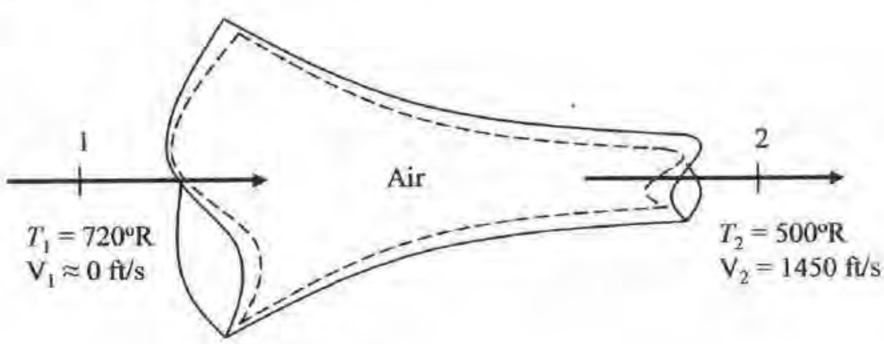
PROBLEM 4.33

4.33 Air enters a nozzle operating at steady state at 720°R with negligible velocity and exits the nozzle at 500°R with a velocity of 1450 ft/s. Assuming ideal gas behavior and neglecting potential energy effects, determine the heat transfer, in Btu per lb of air flowing.

KNOWN: Air flows through a nozzle with known temperature and velocity at the inlet and exit.

FIND: Determine the heat transfer per unit mass of air flowing.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown on the accompanying figure is at steady state.
2. Change in potential energy from inlet to exit can be neglected.
3. $\dot{W}_{cv} = 0$.
4. Air can be modeled as an ideal gas.
5. The inlet velocity is negligible.

ANALYSIS:
Since enthalpy of an ideal gas is a function of only temperature, the enthalpy values at State 1 and State 2 are determined from Table A-22E: $h_1 = 172.39$ Btu/lb and $h_2 = 119.48$ Btu/lb.

The steady-state, one-inlet, one-exit energy balance gives

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\,[(h_1 - h_2) + \tfrac{1}{2}(V_1^2 - V_2^2) + g(z_1 - z_2)]$$

Neglecting potential energy change, setting $V_1 = 0$, recognizing no work is associated with a nozzle, solving for rate of heat transfer, and dividing by the mass flow rate, the energy balance simplifies to

$$\frac{\dot{Q}_{cv}}{\dot{m}} = (h_2 - h_1) + \tfrac{1}{2} V_2^2$$

Substituting values and applying the appropriate conversion factors give

$$\frac{\dot{Q}_{cv}}{\dot{m}} = \left(119.48\frac{\text{Btu}}{\text{lb}} - 172.39\frac{\text{Btu}}{\text{lb}}\right) + \frac{\left(1450\frac{\text{ft}}{\text{s}}\right)^2}{2}\left|\frac{\text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|\left|\frac{\text{lbf}}{32.2\frac{\text{lb}\cdot\text{ft}}{\text{s}^2}}\right| = \underline{\mathbf{-10.9\ Btu/lb}}$$

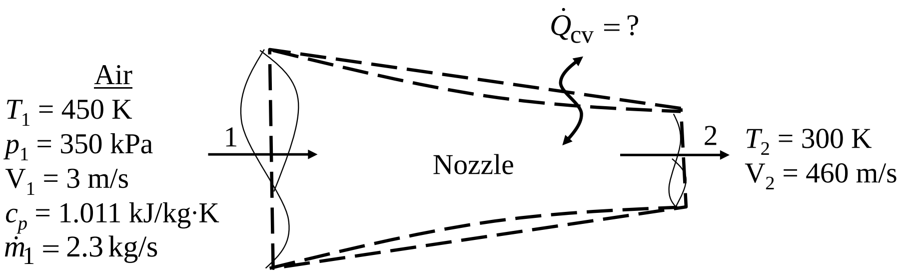
The negative sign indicates energy transfer by heat from the nozzle.

4.34 Air with a mass flow rate of 2.3 kg/s enters a horizontal nozzle operating at steady state at 450 K, 350 kPa, and velocity of 3 m/s. At the exit, the temperature is 300 K and the velocity is 460 m/s. Using the ideal gas model for air with constant c_p = 1.011 kJ/kg·K, determine
(a) the area at the inlet, in m^2.
(b) the heat transfer between the nozzle and its surroundings, in kW. Specify whether the heat transfer is to or from the air.

KNOWN: Air flows through a nozzle.

FIND: (a) the area at the inlet, in m^2, and (b) the heat transfer between the nozzle and its surroundings, in kW. Specify whether the heat transfer is to or from the air.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown with the schematic is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$, and Δpe = 0.
3. Model air as an ideal gas with constant c_p = 1.011 kJ/kg·K.

ANALYSIS:

(a) Mass flow rate at the inlet is $\dot{m}_1 = \dfrac{A_1V_1}{v_1}$. Substituting $v_1 = \dfrac{RT_1}{p_1}$ from the ideal gas equation of state and solving for the area give

$$A_1 = \frac{\dot{m}_1 R T_1}{V_1 p_1}$$

$$A_1 = \frac{(2.3\,\text{kg/s})\left(\dfrac{8.314\,\dfrac{\text{kJ}}{\text{kmol}\cdot\text{K}}}{28.97\,\dfrac{\text{kg}}{\text{kmol}}}\right)(450\,\text{K})}{(3\,\dfrac{\text{m}}{\text{s}})(350\,\text{kPa})}\left|\frac{1000\,\text{N}\cdot\text{m}}{1\,\text{kJ}}\right|\left|\frac{1\,\text{kPa}}{1000\,\dfrac{\text{N}}{\text{m}^2}}\right|$$

A_1 = **0.2829 m^2**

(b) Since $\Delta h = c_p \, \Delta T$ for air with constant specific heat, the energy rate balance

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}[(h_1 - h_2) + ½\,(V_1^2 - V_2^2) + g(\cancel{z_1 - z_2})]$$

simplifies to

$$\dot{Q}_{cv} = \dot{m}\,[c_p(T_2 - T_1) + ½\,(V_2^2 - V_1^2)]$$

Substituting values and solving for rate of heat transfer give

$$\dot{Q}_{cv} = \left(2.3\frac{\text{kg}}{\text{s}}\right)\left[\left(1.011\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(300 - 450)\,\text{K} + \frac{\left(460\frac{\text{m}}{\text{s}}\right)^2 - \left(3\frac{\text{m}}{\text{s}}\right)^2}{2}\left|\frac{1\,\text{kJ}}{1000\,\text{N}\cdot\text{m}}\right|\left|\frac{1\,\text{N}}{1\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right|\right]\left|\frac{\text{kW}}{\left(\frac{\text{kJ}}{\text{s}}\right)}\right|$$

$$\dot{Q}_{cv} = \underline{\mathbf{-105.5\ kW}}$$

<u>Since the heat transfer rate is negative, heat transfer is from the air.</u>

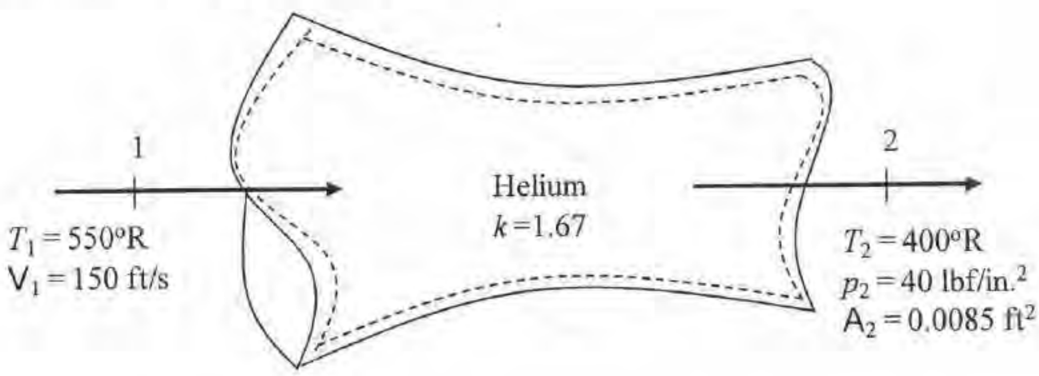
PROBLEM 4.35

4.35 Helium gas flows through a well-insulated nozzle at steady state. The temperature and velocity at the inlet are 550°R and 150 ft/s, respectively. At the exit, the temperature is 400°R and the pressure is 40 lbf/in.^{2} The area of the exit is 0.0085 ft^2. Using the ideal gas model with k=1.67, and neglecting potential energy effects, determine the mass flow rate, in lb/s, through the nozzle.

KNOWN: Helium enters and exits a well-insulated nozzle at known conditions.

FIND: Determine the mass flow rate, in lb/s.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
(1) The control volume shown in the accompanying schematic operates at steady state.
(2) There are no energy transfers via heat or work, $\dot{W}_{cv}$.
(3) The helium behaves as an ideal gas with $k = 1.67$.
(4) Potential energy change from inlet to exit can be neglected.

ANALYSIS:

From the mass balance and using the ideal gas equation of state:

$$\dot{m} = \dot{m}_1 = \dot{m}_2$$

$$\dot{m} = \frac{(AV)_2}{v_2} = \frac{(AV)_2 p_2}{RT_2}$$

The exit velocity is found using the steady state energy balance.

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)\right]$$

Simplify based on assumptions and solve for V_2.

$$V_2 = \sqrt{2(h_1 - h_2) + V_1^2}$$

PROBLEM 4.35 (Continued)

Determine the specific heat, c_p, for helium using Eq. 3.47a, as follows:

$$c_p(T) = \frac{kR}{k-1} = \frac{(1.67)\left(\dfrac{1.986}{4.003}\dfrac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)}{(1.67-1)} = 1.24\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$

Since c_p is constant,

$$V_2 = \sqrt{2c_p(T_1 - T_2) + V_1^2}$$

$$= \sqrt{2\left(1.24\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)(550-400)^\circ\text{R}\left|\frac{32.2\frac{\text{lb}\cdot\text{ft}}{\text{s}^2}}{1\,\text{lbf}}\right|\left|\frac{778\,\text{ft}\cdot\text{lbf}}{1\,\text{Btu}}\right| + (150^2)\frac{\text{ft}^2}{\text{s}^2}} = 3056\frac{\text{ft}}{\text{s}}$$

Solving for mass flow rate:

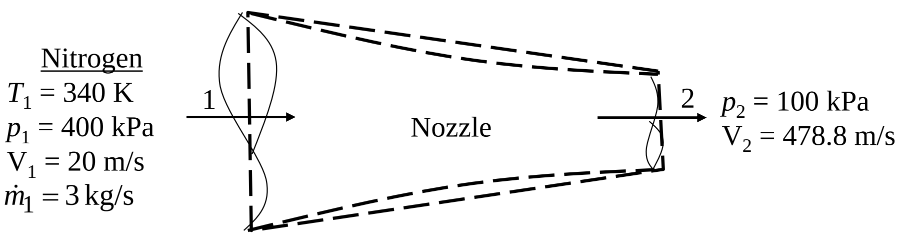
$$\dot{m} = \frac{(\text{AV})_2\, p_2}{RT_2} = \frac{(0.0085\text{ft}^2)\left(3056\frac{\text{ft}}{\text{s}}\right)\left(40\frac{\text{lbf}}{\text{in.}^2}\right)}{\left(\dfrac{1545}{4.003}\dfrac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(400^\circ\text{R})}\left|\frac{144\,\text{in.}^2}{1\text{ft}^2}\right| = 0.97\frac{\text{lb}}{\text{s}} \longleftarrow$$

4.36 Nitrogen, modeled as an ideal gas, flows at a rate of 3 kg/s through a well-insulated horizontal nozzle operating at steady state. The nitrogen enters the nozzle with a velocity of 20 m/s at 340 K, 400 kPa and exits the nozzle at 100 kPa. To achieve an exit velocity of 478.8 m/s, determine
(a) the exit temperature, in K.
(b) the exit area, in m^2.

KNOWN: Nitrogen flows through a nozzle.

FIND: (a) the exit temperature, in K, and (b) the exit area, in m^2.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown with the schematic is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$, $\dot{Q}_{cv} = 0$, and Δpe = 0.
3. Model nitrogen as an ideal gas.

ANALYSIS:
(a) The energy rate balance

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}[(h_1 - h_2) + ½ (V_1^2 - V_2^2) + g\cancel{(z_1 - z_2)}]$$

simplifies to

$$0 = [(h_2 - h_1) + ½ (V_2^2 - V_1^2)]$$

Since enthalpy values for nitrogen are provided on a molar basis in Table A-23, the energy rate balance is expressed in terms of molar enthalpy

$$0 = [\frac{(\bar{h}_2 - \bar{h}_1)}{M} + ½ (V_2^2 - V_1^2)]$$

Solving for exit molar enthalpy, which is a function of only temperature, gives

$$\bar{h}_2 = \bar{h}_1 - \frac{M}{2}(V_2^2 - V_1^2)$$

From Table A-23, $\bar{h}_1 = 9888$ kJ/kmol
From Table A-1 (nitrogen): M = 28.01 kJ/kmol

Substituting values and solving for exit molar enthalpy give

$$\bar{h}_2 = \left[\left(9888\frac{\text{kJ}}{\text{kmol}}\right) - \left(\frac{28.01\frac{\text{kg}}{\text{kmol}}}{2}\right)\left[\left(478.8\frac{\text{m}}{\text{s}}\right)^2 - \left(20\frac{\text{m}}{\text{s}}\right)^2\right]\left|\frac{1\,\text{kJ}}{1000\,\text{N}\cdot\text{m}}\right|\left|\frac{1\,\text{N}}{1\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right|\right]$$

$$\bar{h}_2 = 6683 \text{ kJ/kmol}$$

From Table A-23, the temperature that corresponds to the exit molar enthalpy is $\underline{\boldsymbol{T_2 = 230\ \text{K}}}$.

(b) From the mass rate balance

$$\dot{m}_2 = \dot{m}_1 = \frac{\text{A}_2\text{V}_2}{v_2}$$

Substituting $v_2 = \dfrac{RT_2}{p_2}$ from the ideal gas equation of state and solving for the area give

$$\text{A}_2 = \frac{\dot{m}_1 R T_2}{\text{V}_2 p_2}$$

$$\text{A}_2 = \frac{(3\,\text{kg/s})\left(\frac{8.314\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}}{28.01\frac{\text{kg}}{\text{kmol}}}\right)(230\,\text{K})}{(478.8\frac{\text{m}}{\text{s}})(100\,\text{kPa})}\left|\frac{1000\,\text{N}\cdot\text{m}}{1\,\text{kJ}}\right|\left|\frac{1\,\text{kPa}}{1000\,\frac{\text{N}}{\text{m}^2}}\right|$$

$\text{A}_2 = \underline{\mathbf{0.004278\ m^2}}$

PROBLEM 4.37

KNOWN: Steady-state operating data are provided for a jet engine.

FIND: Determine the velocity at the diffuser exit.

SCHEMATIC & GIVEN DATA:

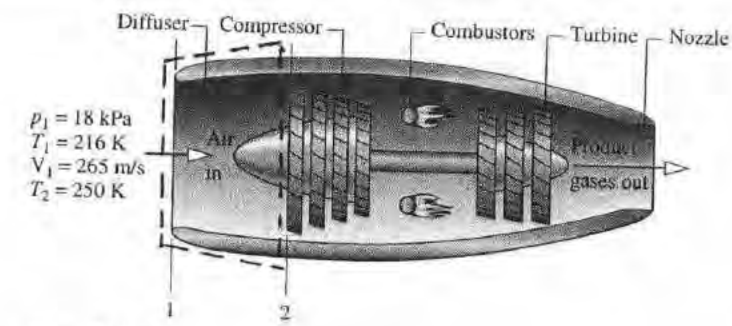


Fig. P4.37

ENGR. MODEL

1. As shown in the sketch, a control volume encloses the diffuser.
2. The control volume is at steady state.
3. For the control volume, $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$ and potential energy effects can be ignored.
4. The air is modeled as an ideal gas.

ANALYSIS: For the control volume, $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_a$. An energy rate balance reads

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_a\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\cancel{(z_1 - z_2)}\right]$$

Accordingly,

$$V_2 = \sqrt{V_1^2 + 2(h_1 - h_2)}$$

$$= \sqrt{\left(265 \frac{m}{s}\right)^2 + 2\,\underbrace{(215.97 - 250.05)}_{\text{Table A-22}} \frac{kJ}{kg} \left| \frac{10^3 N\cdot m}{1 kJ} \right| \left| \frac{1 kg\cdot m/s^2}{1 N} \right|}$$

$$= 45 \text{ m/s}$$

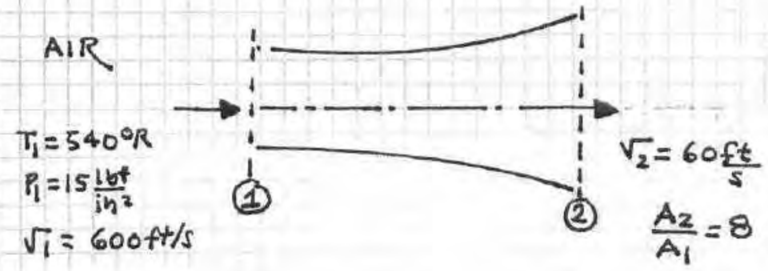
V_2

PROBLEM 4.38

KNOWN: Air flows through a diffuser operating at steady state. State data and ratio of exit area to inlet area are provided.

FIND: At the exit, determine temperature, in °R, and pressure, in lbf/in.²

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. A control volume encloses the diffuser.
2. The control volume is at steady state.
3. For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. Potential energy effects are ignored
4. The air is modeled as an ideal gas.

ANALYSIS: At steady state the mass rate balance reduces to give

$$\dot{m}_1 = \dot{m}_2 \Rightarrow \frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2}. \text{ With } pv = RT, \quad \frac{A_1 V_1 P_1}{RT_1} = \frac{A_2 V_2 P_2}{RT_2}$$

$$\Rightarrow P_2 = P_1\left[\frac{A_1}{A_2}\right]\left[\frac{V_1}{V_2}\right]\left[\frac{T_2}{T_1}\right] \qquad (1)$$

Reducing Eq. 4.20a, $0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(\cancel{z_1 - z_2})\right]$,

$$h_2 = h_1 + \left[\frac{V_1^2 - V_2^2}{2}\right] \qquad (2)$$

With h_1 from Table A-22E

$$h_2 = 129.06\ \frac{Btu}{lb} + \left[\frac{(600 ft/s)^2 - (60 ft/s)^2}{2}\right]\left(\frac{ft}{s}\right)^2 \left|\frac{1\ lbf}{32.2\ lb\cdot ft/s^2}\right|\left|\frac{1\ Btu}{778\ ft\cdot lbf}\right|$$

$$= 136.17\ \frac{Btu}{lb}$$

Interpolating in Table A-22E, $T_2 = 569.6\ °R$ ◄

Then, Eq. (1) gives

$$P_2 = \left[15\ \frac{lbf}{in^2}\right]\left[\frac{1}{8}\right]\left[\frac{600 ft/s}{60 ft/s}\right]\left[\frac{569.6\ °R}{540\ °R}\right]$$

$$= 19.78\ \frac{lbf}{in^2}$$ ◄

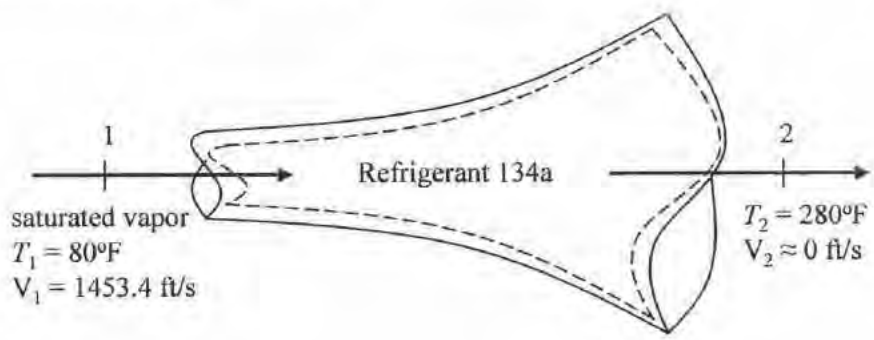
PROBLEM 4.39

4.39 Refrigerant 134a enters an insulated diffuser as a saturated vapor at 80°F with a velocity of 1453.4 ft/s. At the exit, the temperature is 280°F and the velocity is negligible. The diffuser operates at steady state and potential energy effects can be neglected. Determine the exit pressure, in lbf/in.^{2}

KNOWN: Refrigerant 134a flows through an insulated diffuser with known temperature and velocity at the inlet and exit.

FIND: Determine the exit pressure.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

1. The control volume shown on the accompanying figure is at steady state.
2. Change in potential energy from inlet to exit can be neglected.
3. $\dot{W}_{cv} = 0$.
4. The exit velocity is negligible.

ANALYSIS:

State 1 is saturated vapor at 80°F. Thus, from Table A-10E, $h_1 = h_{g1} = 112.56$ Btu/lb.

The steady-state, one-inlet, one-exit energy balance gives

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\,[(h_1 - h_2) + ½\,(V_1^2 - V_2^2) + g(z_1 - z_2)]$$

Neglecting heat transfer and potential energy change, setting $V_2 = 0$, recognizing no work is associated with a diffuser, and dividing by the mass flow rate, the energy balance simplifies to

$$0 = (h_1 - h_2) + ½\,V_1^2$$

Solving for the exit enthalpy

$$h_2 = h_1 + ½\,V_1^2$$

Substituting values and applying appropriate conversion factors give

$$h_2 = 112.56\,\frac{\text{Btu}}{\text{lb}} + \frac{\left(1453.4\,\frac{\text{ft}}{\text{s}}\right)^2}{2}\left|\frac{\text{Btu}}{778\text{ ft}\cdot\text{lbf}}\right|\left|\frac{\text{lbf}}{32.2\,\frac{\text{lb}\cdot\text{ft}}{\text{s}^2}}\right| = 154.72\text{ Btu/lb}$$

State 2, now fixed by T_2 and h_2, is in the superheated vapor region. From Table A-12E, <u>$p_2 = 400$ lbf/in.^{2}</u> ←

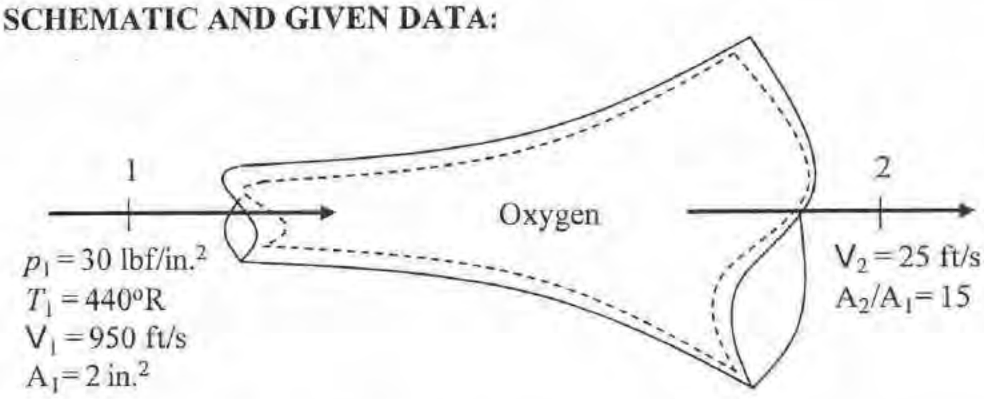
PROBLEM 4.40

4.40 Oxygen gas enters a well-insulated diffuser at 30 lbf/in.^{2}, 440°R, with a velocity of 950 ft/s through a flow area of 2.0 in.^{2} At the exit, the flow area is 15 times the inlet area, and the velocity is 25 ft/s. The potential energy change from inlet to exit is negligible. Assuming ideal gas behavior for the oxygen and steady-state operation of the nozzle, determine the exit temperature, in °R, the exit pressure, in lbf/in.^{2}, and the mass flow rate, in lb/s.

KNOWN: Oxygen enters and exits a well-insulated diffuser at known conditions.

FIND: Determine the exit temperature, in °R, the exit pressure, in lbf/in.^{2}, and the mass flow rate, in lb/s.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

(1) The control volume shown in the accompanying schematic operates at steady state.
(2) There are no energy transfers via heat or work.
#1 (3) The oxygen behaves as an ideal gas.
(4) Potential energy change from inlet to exit can be neglected.

ANALYSIS:

For an ideal gas, $h=h(T)$. Start with the steady state form of the energy balance.

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)\right]$$

Simplify based on assumptions and solve for h_2 noting that $\bar{h}_1 = h_1 M$. M is the molecular weight for oxygen and $\bar{h}_1$ from Table A-23E is $\bar{h}_1$=3047.5 Btu/lbmol.

$$h_2 = h_1 + \left(\frac{V_1^2 - V_2^2}{2}\right) \rightarrow \bar{h}_2 = \bar{h}_1 + \left(\frac{V_1^2 - V_2^2}{2}\right)M$$

$$\bar{h}_2 = 3047.5\frac{\text{Btu}}{\text{lbmol}} + \left(\frac{950^2 - 25^2}{2}\frac{\text{ft}^2}{\text{s}^2}\right)\left(32\frac{\text{lb}}{\text{lbmol}}\right)\left|\frac{1\,\text{lbf}}{32.2\frac{\text{lb}\cdot\text{ft}}{\text{s}^2}}\right|\left|\frac{1\,\text{Btu}}{778\text{ft}\cdot\text{lbf}}\right| = 3623.5\frac{\text{Btu}}{\text{lbmol}}$$

Interpolating from Table A-23E: $T_2 \approx 522.5°R$.

Obtain p_2 from the mass balance and using the ideal gas equation of state.

$$\dot{m} = \dot{m}_1 = \dot{m}_2$$

$$\dot{m} = \frac{(AV)_2}{v_2} = \frac{(AV)_2 p_2}{RT_2} = \frac{(AV)_1 p_1}{RT_1}$$

$$p_2 = \frac{T_2(A_1V_1)p_1}{T_1(A_2V_2)} = \left(\frac{522.5}{440}\right)\left(\frac{1}{15}\right)\left(\frac{950}{25}\right)\left(30\frac{\text{lbf}}{\text{in.}^2}\right) = 90.25\frac{\text{lbf}}{\text{in.}^2}$$

Solve for mass flow rate.

$$\dot{m} = \frac{(AV)_1 p_1}{RT_1} = \frac{(2\text{ in.}^2)\left(950\frac{\text{ft}}{\text{s}}\right)\left(30\frac{\text{lbf}}{\text{in.}^2}\right)}{\left(\frac{1545}{32}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(440°\text{R})} = 2.683\frac{\text{lb}}{\text{s}}$$

---

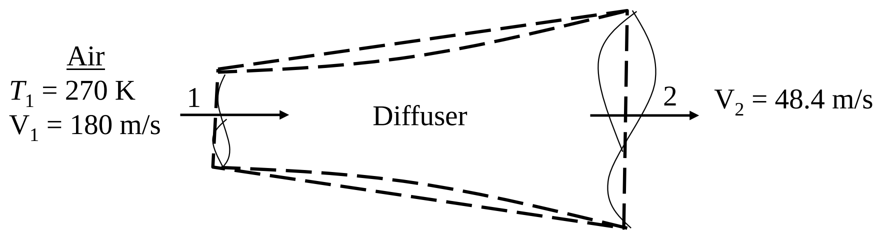
1. The applicability of the ideal gas model can be checked by reference to the compressibility chart.

4.41 Air modeled as an ideal gas enters a well-insulated diffuser operating at steady state at 270 K with a velocity of 180 m/s and exits with a velocity of 48.4 m/s. For negligible potential energy effects, determine the exit temperature, in K.

KNOWN: Air flows through a diffuser.

FIND: The exit temperature, in K.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown with the schematic is at steady state.
2. No stray heat transfer occurs between the air and its surroundings.
3. Model the air as an ideal gas.
4. For the control volume, Δpe = 0.

ANALYSIS:
The energy rate balance

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}[(h_1 - h_2) + ½\,(V_1^2 - V_2^2) + g\cancel{(z_1 - z_2)}]$$

simplifies to

$$0 = (h_1 - h_2) + ½\,(V_1^2 - V_2^2)$$

For an ideal gas, specific enthalpy is a function of only temperature. Solving for the exit specific enthalpy

$$h_2 = h_1 + ½\,(V_1^2 - V_2^2)$$

From Table A-22 at T_1 = 270 K, h_2 = 270.11 kJ/kg. Substituting values and solving for exit specific enthalpy

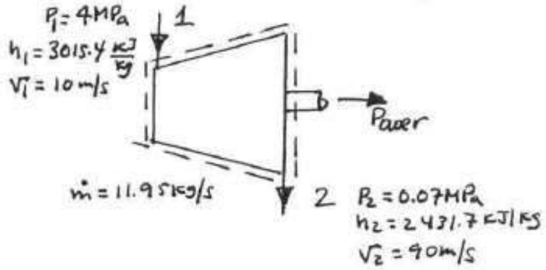
$$h_2 = \left(270.11\frac{\text{kJ}}{\text{kg}}\right) + \left[\frac{\left(180\frac{\text{m}}{\text{s}}\right)^2 - \left(48.4\frac{\text{m}}{\text{s}}\right)^2}{2}\right]\left|\frac{1\,\text{kJ}}{1000\,\text{N}\cdot\text{m}}\right|\left|\frac{1\,\text{N}}{1\,\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right| = 285.14 \text{ kJ/kg}$$

From Table A-22 at h_2 = 285.14 kJ/kg, the corresponding temperature is T_2 = **285 K**.

Problem 4.42

Steam enters a well-insulated turbine operating at steady state at 4 MPa with a specific enthalpy of 3015.4 kJ/kg and a velocity of 10 m/s. The steam expands to the turbine exit where the pressure is 0.07 MPa, specific enthalpy is 2431.7 kJ/kg, and the velocity is 90 m/s. The mass flow rate is 11.95 kg/s. Neglecting potential energy effects, determine the power developed by the turbine, in kW.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$ and $\Delta pe \approx 0$.

ANALYSIS: Reducing Eq. 4.20a

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + \cancel{g(z_1 - z_2)}\right)$$

$$\Rightarrow \dot{W}_{cv} = \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}\right]$$

$$= \left(11.95\ \frac{kg}{s}\right)\left\{\left(3015.4\ \frac{kJ}{kg} - 2431.7\ \frac{kJ}{kg}\right) + \left[\frac{\left(10\frac{m}{s}\right)^2 - \left(90\frac{m}{s}\right)^2}{2}\right]\left|\frac{1N}{1kg \cdot m/s^2}\right|\left|\frac{1 kJ}{10^3 N \cdot m}\right|\right\}$$

$$= 11.95\ \frac{kg}{s}\left[583.7\ \frac{kJ}{kg} - 4\ \frac{kJ}{kg}\right]\left|\frac{1 kW}{1 kJ/s}\right|$$

$$= 6927\ kW$$

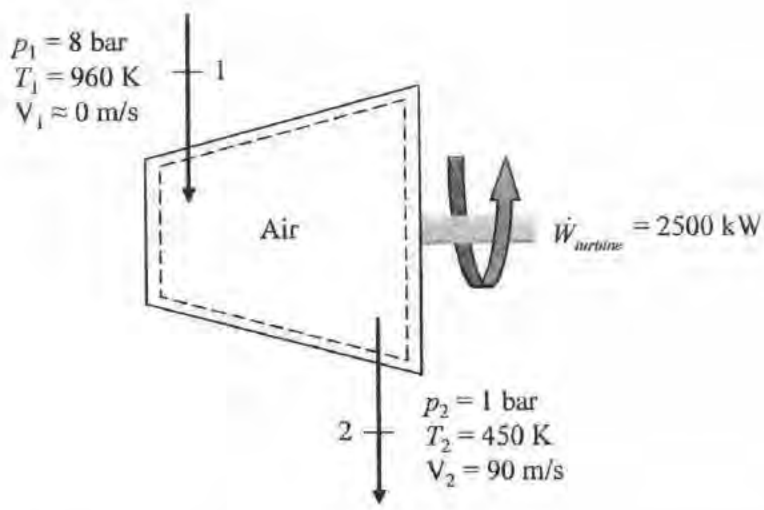
PROBLEM 4.43

4.43 Air expands through a turbine from 8 bar, 960 K to 1 bar, 450 K. The inlet velocity is small compared to the exit velocity of 90 m/s. The turbine operates at steady state and develops a power output of 2500 kW. Heat transfer between the turbine and its surroundings and potential energy effects are negligible. Modeling air as an ideal gas, calculate the mass flow rate of air, in kg/s, and the exit area, in m^2.

KNOWN: Air expands through a turbine with known temperature, pressure, and velocity at the inlet and exit to produce a specified amount of power.

FIND: Determine the mass flow rate of air and the exit area.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown on the accompanying figure is at steady state.
2. Heat transfer and the change in potential energy from inlet to exit can be neglected.
3. The inlet velocity is negligible.

ANALYSIS:
The steady-state, one-inlet, one-exit energy balance gives

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\,[(h_1 - h_2) + \tfrac{1}{2}(V_1^2 - V_2^2) + g(z_1 - z_2)]$$

Neglecting heat transfer and potential energy change, and setting $V_1 = 0$, the energy balance simplifies to

$$0 = -\dot{W}_{cv} + \dot{m}\,[(h_1 - h_2) - \tfrac{1}{2} V_2^2]$$

Solving for the mass flow rate gives

$$\dot{m} = \frac{\dot{W}_{cv}}{h_1 - h_2 - \dfrac{V_2^2}{2}}$$

Since air can be modeled as an ideal gas, Table A-22 applies: $h_1 = 1000.55$ kJ/kg and $h_2 = 451.80$ kJ/kg. Substituting values and applying appropriate conversion factors yield

$$\dot{m} = \frac{2500\,\text{kW}}{1000.55\dfrac{\text{kJ}}{\text{kg}} - 451.80\dfrac{\text{kJ}}{\text{kg}} - \left(\dfrac{\left(90\dfrac{\text{m}}{\text{s}}\right)^2}{2}\left|\dfrac{\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right|\left|\dfrac{\text{N}}{\dfrac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right|\right)}\left|\dfrac{\dfrac{\text{kJ}}{\text{s}}}{\text{kW}}\right| = \underline{\mathbf{4.59\ kg/s}}$$

The mass flow rate can be written as

$$\dot{m} = \frac{A_2 V_2}{v_2}$$

From the ideal gas equation of state, $pv = RT$, specific volume can be written as $v = RT/p$ where R is the gas constant, $\overline{R}/M$. Substituting for specific volume gives

$$\dot{m} = \frac{A_2 V_2 p_2}{\left(\overline{R}/M\right)T_2}$$

Solving for exit area, A_2 yields

$$A_2 = \frac{\dot{m}\left(\overline{R}/M\right)T_2}{V_2 p_2}$$

Substituting values and applying the appropriate conversion factor yield

$$A_2 = \frac{\left(4.59\dfrac{\text{kg}}{\text{s}}\right)\left(\dfrac{8314\dfrac{\text{N}\cdot\text{m}}{\text{kmol}\cdot\text{K}}}{28.97\dfrac{\text{kg}}{\text{kmol}}}\right)(450\,\text{K})}{\left(90\dfrac{\text{m}}{\text{s}}\right)(1\,\text{bar})}\left|\dfrac{\text{bar}}{10^5\dfrac{\text{N}}{\text{m}^2}}\right| = \underline{\mathbf{0.066\ m^2}}$$

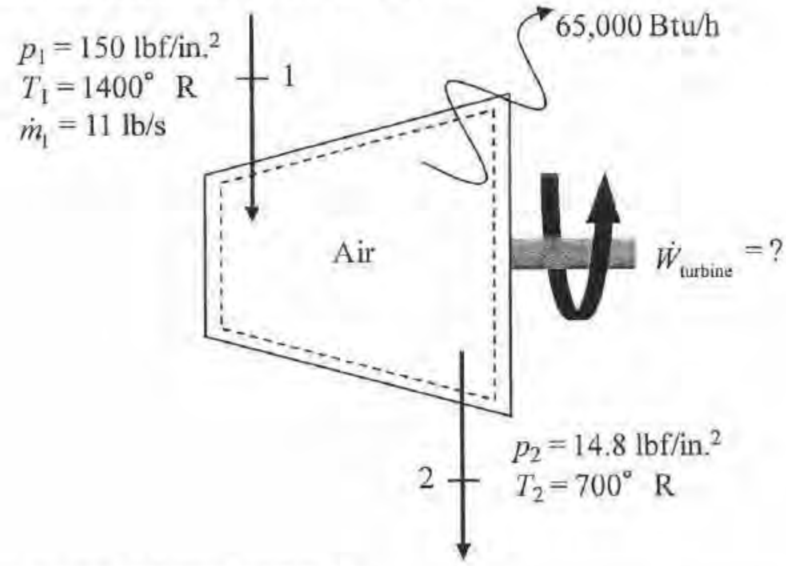
PROBLEM 4.44

4.44 Air expands through a turbine operating at steady state. At the inlet, $p_1 = 150$ lbf/in.^{2}, $T_1 = 1400°R$, and at the exit, p_2=14.8 lbf/in.^{2}, $T_2 = 700°R$. The mass flow rate of air entering the turbine is 11 lb/s, and 65,000 Btu/h of energy is rejected by heat transfer. Neglecting kinetic and potential energy effects, determine the power developed, in hp.

KNOWN: Air expands through a turbine with known conditions at the inlet and exit. The inlet mass flow rate and heat transfer rate are given.

FIND: Determine the power developed, in hp.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

(1) The control volume shown in the accompanying schematic operates at steady state.

#1 (2) The air behaves as an ideal gas.

(3) Kinetic and Potential energy change from inlet to exit can be neglected.

ANALYSIS:

For an ideal gas, $h=h(T)$. Start with the steady state form of the energy balance.

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)\right]$$

Simplify based on assumptions and solve for power developed in turbine. From Table A-22E is h_1=342.9 Btu/lb and h_2=167.56 Btu/lb.

$$\dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}(h_1 - h_2)$$

$$= \left(-65{,}000\frac{\text{Btu}}{\text{h}}\right) + 11\frac{\text{lb}}{\text{s}}(342.9 - 167.56)\frac{\text{Btu}}{\text{lb}}\left|\frac{3600\text{ s}}{1\text{h}}\right| = 6{,}878{,}464\frac{\text{Btu}}{\text{h}}\left|\frac{1\text{ hp}}{2545\frac{\text{Btu}}{\text{h}}}\right| = 2703\text{ hp}$$

1. Table A-1E gives $p_c = 37.2$ atm = 546.69 lbf/in.^{2}, $T_c = 239°$ R for air. Therefore, $p_{R1} = 0.274$, $T_{R1} = 5.86$. Referring to Fig. A-1, the value of the compressibility factor at this state is $Z \approx 1$. The same conclusion results when state 2 is checked. Accordingly, $pv = RT$ adequately describes the p-v-T relation for the air at those states.

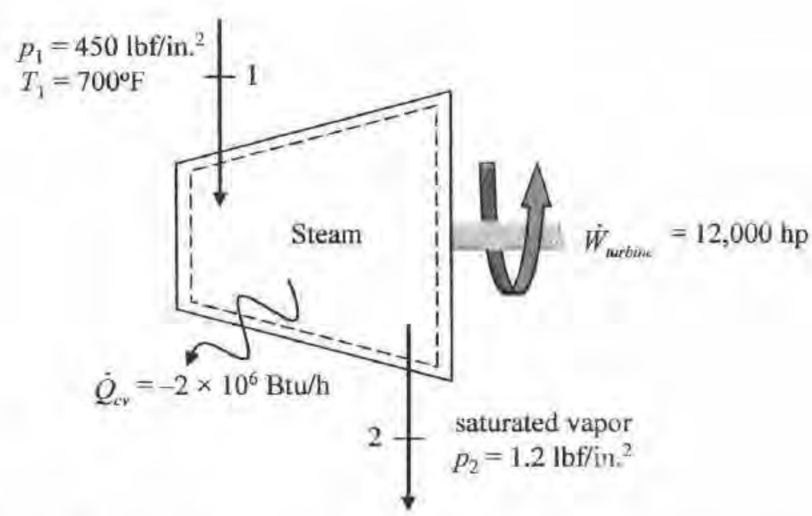
PROBLEM 4.45

4.45 Steam enters a turbine operating at steady state at 700°F and 450 lbf/in.^{2} and leaves as a saturated vapor at 1.2 lbf/in.^{2} The turbine develops 12,000 hp, and heat transfer from the turbine to the surroundings occurs at a rate of 2×10^6 Btu/h. Neglecting kinetic and potential energy changes from inlet to exit, determine the volumetric flow rate of the steam at the inlet, in ft^3/s.

KNOWN: Steam expands through a turbine with known inlet temperature and pressure and known exit pressure. Power produced and heat transfer rate are known.

FIND: Determine the volumetric flow rate of steam at the inlet.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown on the accompanying figure is at steady state.
2. The change in kinetic and potential energy from inlet to exit can be neglected.

ANALYSIS:
The steady-state, one-inlet, one-exit energy balance gives

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\,[(h_1 - h_2) + \tfrac{1}{2}(V_1^2 - V_2^2) + g(z_1 - z_2)]$$

Neglecting kinetic and potential energy changes, the energy balance simplifies to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\,(h_1 - h_2)$$

Solving for the mass flow rate gives

$$\dot{m} = \frac{\dot{Q}_{cv} - \dot{W}_{cv}}{h_2 - h_1}$$

PROBLEM 4.45 (Continued)

Steam at the inlet is superheated. From Table A-4E, $h_1 = 1359.6$ Btu/lb and $v_1 = 1.458$ ft^3/lb. The exit stream is saturated vapor. Thus, from Table A-3E, $h_2 = h_{g2} = 1108.4$ Btu/lb. Substituting values and applying the appropriate conversion factor yield

$$\dot{m} = \frac{-2\times10^6 \frac{\text{Btu}}{\text{h}} - (12{,}000\ \text{hp})\left|\frac{2545\frac{\text{Btu}}{\text{h}}}{\text{hp}}\right|}{1108.4\frac{\text{Btu}}{\text{lb}} - 1359.6\frac{\text{Btu}}{\text{lb}}}\left|\frac{\text{h}}{3600\ \text{s}}\right| = 35.98\ \text{lb/s}$$

The inlet volumetric flow rate, AV, can be determined from the mass flow rate: $\dot{m} = \dfrac{A\text{V}}{v}$

$$(A\text{V})_1 = \dot{m}\, v_1 = (35.98\ \text{lb/s})(1.458\ \text{ft}^3/\text{lb}) = \underline{\mathbf{52.46\ ft^3/s}}$$

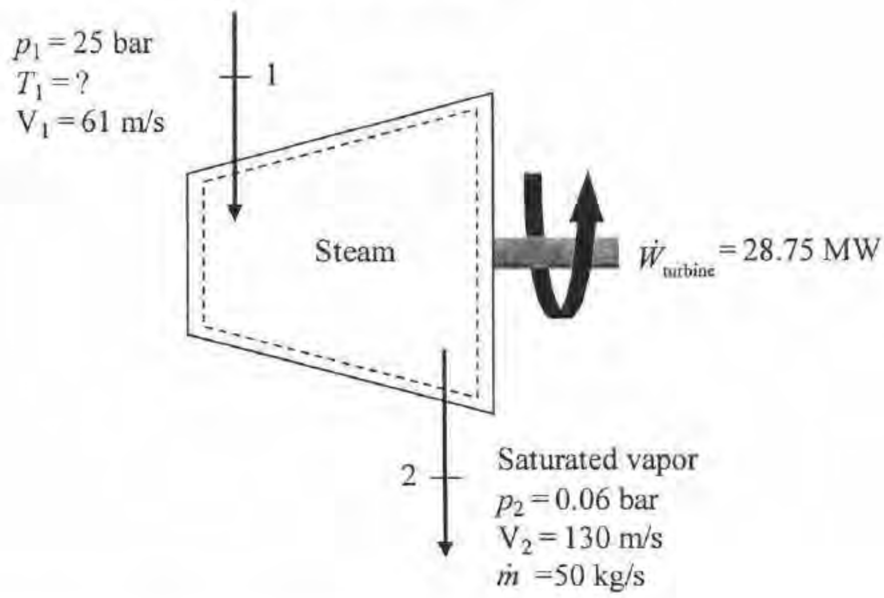
PROBLEM 4.46

4.46 A well-insulated turbine operating at steady state develops 28.75 MW of power for a steam flow rate of 50 kg/s. The steam enters at 25 bar with a velocity of 61 m/s and exits as saturated vapor at 0.06 bar with a velocity of 130 m/s. Neglecting potential energy effects, determine the inlet temperature, in °C.

KNOWN: A well-insulated turbine operates at steady state with known inlet and exit conditions. The power output and mass flow rate are specified.

FIND: Determine the inlet temperature, in °C.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

(1) The control volume shown in the accompanying schematic operates at steady state.
(2) Heat transfer can be neglected.
(3) Potential energy change from inlet to exit can be neglected.

ANALYSIS:

The pressure is known at the inlet. To fix the inlet state, h_1 is determined using steady state forms of the mass and energy balances.

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)\right]$$

Simplify based on assumptions and solve for h_1.

$$h_1 = \frac{\dot{W}_{cv}}{\dot{m}} + h_2 + \left(\frac{V_2^2 - V_1^2}{2}\right)$$

From Table A-3 for saturated vapor at p_2=0.06 bar; h_2=2567.4 kJ/kg.

$$h_1 = \frac{28.75\text{MW}}{50\frac{\text{kg}}{\text{s}}}\left|\frac{1000\frac{\text{kJ}}{\text{s}}}{1\text{MW}}\right| + 2567.4\frac{\text{kJ}}{\text{kg}} + \left(\frac{130^2 - 61^2}{2}\right)\frac{\text{m}^2}{\text{s}^2}\left|\frac{1\text{N}}{1\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right|\left|\frac{1\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right| = 3149\frac{\text{kJ}}{\text{kg}}$$

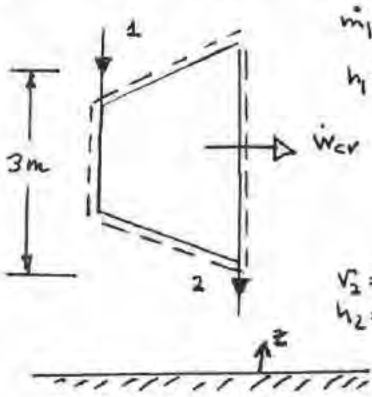
Superheated steam enters a steam turbine, therefore interpolate from Table A-4 with p_1 = 25 bar; $T_1 \approx 360$°C

PROBLEM 4.47

KNOWN: Steady-state data are provided for a steam turbine.

FIND: Determine the power developed by the turbine.

SCHEMATIC & GIVEN DATA



$\dot{m}_1 = 10 \frac{kg}{min}$, $V_1 = 30 m/s$

$h_1 = 3100 kJ/kg$

$\frac{\dot{Q}_{cv}}{\dot{m}} = -1.1 \frac{kJ}{kg}$

$V_2 = 45 m/s$

$h_2 = 2300 kJ/kg$

ENGR. MODEL:

1. As shown in the sketch, a control volume encloses the turbine.
2. The control volume is at steady state.
3. $g = 9.81 m/s^2$.

ANALYSIS: Mass rate balance at steady state, $\dot{m}_2 = \dot{m}_1$. Energy rate balance at steady state,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)\right]$$

$$\Rightarrow \quad \dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)\right]$$

$$\Rightarrow \quad \frac{\dot{W}_{cv}}{\dot{m}} = \frac{\dot{Q}_{cv}}{\dot{m}} + \left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)\right]$$

where

$$\frac{\dot{Q}_{cv}}{\dot{m}} = -1.1 \frac{kJ}{kg}$$

$$(h_1 - h_2) = (3100 - 2300)\frac{kJ}{kg} = 800 \frac{kJ}{kg}$$

$$\frac{V_1^2 - V_2^2}{2} = \frac{[(30 m/s)^2 - (45 m/s)^2]}{2}\left|\frac{1 N}{1 kg\cdot m/s^2}\right|\left|\frac{1 kJ}{10^3 N\cdot m}\right| = -0.56 \frac{kJ}{kg}$$

$$g(z_1 - z_2) = \left(9.81 \frac{m}{s^2}\right)(3 m)\left|\frac{1 N}{1 kg\cdot m/s^2}\right|\left|\frac{1 kJ}{10^3 N\cdot m}\right| = 0.03 \frac{kJ}{kg}$$

Collecting results,

(1)
$$\frac{\dot{W}_{cv}}{\dot{m}} = [-1.1 + 800 - 0.56 + 0.03]\frac{kJ}{kg} = 798.37 \frac{kJ}{kg}$$

$$\Rightarrow \quad \dot{W}_{cv} = \left(10 \frac{kg}{min}\right)\left(798.3 \frac{kJ}{kg}\right)\left|\frac{1 min}{60 s}\right|\left|\frac{1 kW}{1 kJ/s}\right| = 133.1 kW$$

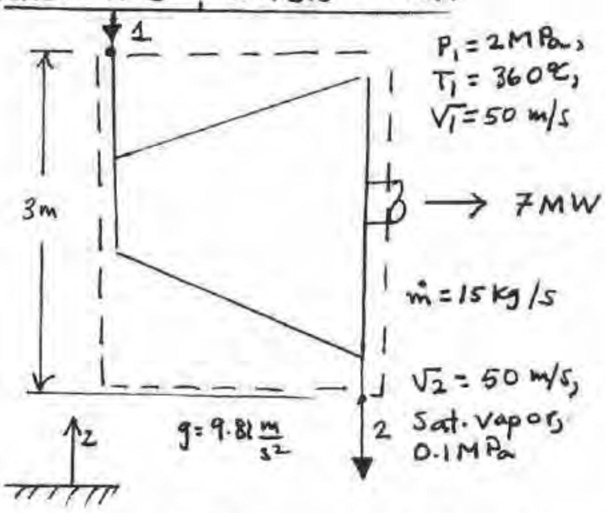
$\dot{W}_{cv}$

1. Enthalpy change is clearly the dominant effect.

PROBLEM 4.48

Steam enters a turbine operating at steady state at 2 MPa, 360°C with a velocity of 100 m/s. Saturated vapor exits at 0.1 MPa and a velocity of 50 m/s. The elevation of the inlet is 3 m higher than at the exit. The mass flow rate of the steam is 15 kg/s, and the power developed is 7 MW. Let $g = 9.81$ m/s^2. Determine (a) the area at the inlet, in m^2, and (b) the rate of heat transfer between the turbine and its surroundings, in kW.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. $g = 9.81$ m/s^2

ANALYSIS:

(a) $\dot{m} = \frac{A_1 V_1}{v_1} \Rightarrow A_1 = \frac{\dot{m} v_1}{V_1} = \frac{(15 kg/s)(0.1411 m^3/kg)}{(100 m/s)} = 0.021 m^2$ (a)

Table A-4

(b) Considering Eq. 4.20a,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)\right]$$

$$\Rightarrow \quad \dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}\left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right] \quad (1)$$

where, with data from Tables A-3 and A-4,

$$h_2 - h_1 = (2675.5 - 3159.3)\frac{kJ}{kg} = -483.8 \frac{kJ}{kg}$$

$$\frac{V_2^2 - V_1^2}{2} = \left[\frac{(50 m/s)^2 - (100 m/s)^2}{2}\right]\left|\frac{1 N}{1 kg\cdot m/s^2}\right|\left|\frac{1 kJ}{10^3 N\cdot m}\right| = -3.75 \frac{kJ}{kg}$$

① $$g(z_2 - z_1) = \left(9.8 \frac{m}{s^2}\right)(-3 m)\left|\frac{1 N}{1 kg\cdot m/s^2}\right|\left|\frac{1 kJ}{10^3 N\cdot m}\right| = -0.03 \frac{kJ}{kg}$$

Inserting values in Eq. (1)

$$\dot{Q}_{cv} = 7000 kW + 15\frac{kg}{s}\left[-483.8 - 3.75 - 0.03\right]\frac{kJ}{kg}\left|\frac{1 kW}{1 kJ/s}\right|$$

$$= -313.7 kW$$ (b)

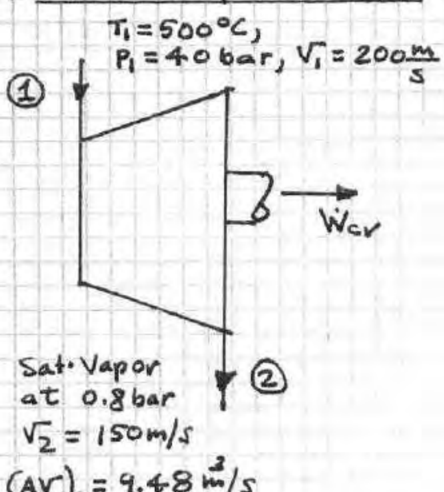
1. Potential energy is a minor effect, as expected.

PROBLEM 4.49

KNOWN: State data and operating data are provided for a steam turbine operating at steady state.

FIND: Determine the power developed, in kW.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. A control volume encloses the turbine.
2. The control volume is at steady state.
3. The expansion through the turbine occurs adiabatically.
4. Potential energy effects are ignored.

ANALYSIS: Reducing Eq. 4.20a

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g\cancel{(z_1 - z_2)}\right]$$

$$\Rightarrow \dot{W}_{cv} = \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2}\right] \qquad (1)$$

where $\dot{m} = \frac{(AV)_2}{v} = \frac{9.48\ m^3/s}{2.087\ m^3/kg} = 4.54\ kg/s$

$v = v_g(0.8\text{ bar})$ from Table A-3

Also, from Tables A-4 and A-3, $h_1 = 3445.3$ kJ/kg, $h_2 = 2665.8$ kJ/kg, respectively. Substituting values into Eq. (1),

$$\dot{W}_{cv} = \left(4.54\frac{kg}{s}\right)\left[(3445.3 - 2665.8)\frac{kJ}{kg} + \left[\left(\frac{(200)^2 - (150)^2}{2}\right)\left(\frac{m}{s}\right)^2\right]\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|\right]$$

$$= \left(4.54\frac{kg}{s}\right)\left[779.5\frac{kJ}{kg} + 8.75\frac{kJ}{kg}\right]\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 3578.7\ kW$$

Answer is (c).

PROBLEM 4.50

KNOWN: Steady-state operating data are provided for a two-stage turbine with a reheater.

FIND: Determine the steam mass flow rate, the total power developed, and rate of heat transfer for the steam flowing through the reheater.

SCHEMATIC & GIVEN DATA:

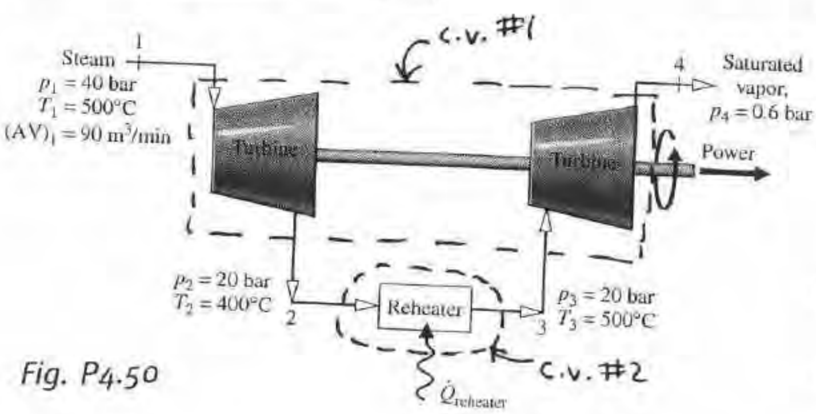


Fig. P4.50

ENGR. MODEL:

1. As shown in the sketch, two control volumes are under consideration.
2. Each control volume is at steady state.
3. Kinetic and potential energy effects can be ignored.
4. For control volume #1, stray heat transfer can be ignored.

ANALYSIS: (a) The mass flow rate at inlet 1 is found from $\dot{m}_1 = \frac{(AV)_1}{v_1}$. From Table A-4 at 40 bar, 500 °C, $v_1 = 0.08643\ m^3/kg$. Then,

$$\dot{m}_1 = \frac{90\ m^3/min}{0.08643\ m^3/kg}\left|\frac{60\ min}{1\ h}\right| = 6.248 \times 10^4\ \frac{kg}{h}$$ ← $\dot{m}$

(b) An energy rate balance for control volume #1 reads

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}[h_1 - h_2] + \dot{m}[h_3 - h_4]$$

$$\Rightarrow \quad \dot{W}_{cv} = \dot{m}[(h_1 - h_2) + (h_3 - h_4)]$$

With data from Tables A-3 and A-4, $h_1 = 3445.3$ kJ/kg, $h_2 = 3247.6$ kJ/kg, $h_3 = 3467.6$ kJ/kg, $h_4 = 2653.5$ kJ/kg.

$$\Rightarrow \quad \dot{W}_{cv} = \dot{m}\left[(3445.3 - 3247.6) + (3467.6 - 2653.5)\right]\frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 6.248 \times 10^4 \frac{kg}{h}\left|\frac{1\ h}{3600\ s}\right| = 17.36\ \frac{kg}{s}$$

$$= 17{,}565\ kW$$ ← $\dot{W}_{cv}$

(c) An energy rate balance for control volume #2 reads

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}(h_2 - h_3) \Rightarrow \dot{Q}_{cv} = \dot{m}(h_3 - h_2)$$

$$\Rightarrow \quad \dot{Q}_{cv} = 17.36\ \frac{kg}{s}(3467.6 - 3247.6)\frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 3{,}819\ kW$$ ← $\dot{Q}_{cv}$

PROBLEM 4.51

4.51 Steam at 1800 lbf/in.^{2} and 1100°F enters a turbine operating at steady state. As shown in Fig. P4.51, 20% of the entering mass flow is extracted at 600 lbf/in.^{2} and 500°F. The rest of the steam exits as a saturated vapor at 1 lbf/in.^{2} The turbine develops a power output of 6.8×10^6 Btu/h. Heat transfer from the turbine to the surroundings occurs at a rate of 5×10^4 Btu/h. Neglecting kinetic and potential energy effects, determine the mass flow rate of the steam entering the turbine, in lb/s.

KNOWN: Steam enters a turbine with known inlet temperature and pressure. A per cent of the steam is extracted at known temperature and pressure with the remaining steam exiting at known pressure as a saturated vapor. Power produced and heat transfer rate are known.

FIND: Determine the mass flow rate of steam at the inlet.

SCHEMATIC AND GIVEN DATA:

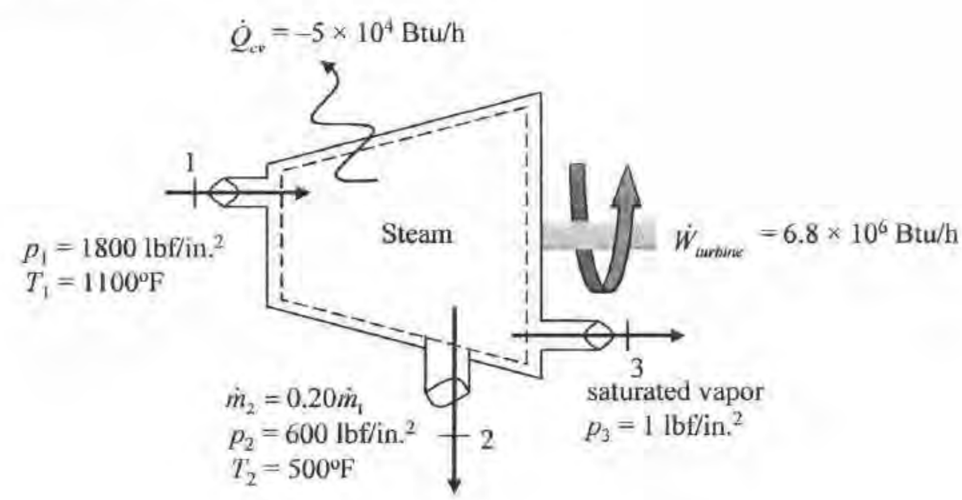


Fig. P4.51

ENGINEERING MODEL:
1. The control volume shown on the accompanying figure is at steady state.
2. Kinetic and potential energy effects are neglected.

ANALYSIS:
The steady-state mass rate balance gives

$$\sum_i \dot{m}_i = \sum_e \dot{m}_e$$

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$$

Solving for $\dot{m}_3$

PROBLEM 4.51 (Continued)

$$\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = \dot{m}_1 - 0.20\dot{m}_1 = 0.80\dot{m}_1$$

The steady-state energy balance gives

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \,(h_i + ½\,V_i^2 + gz_i) - \sum_e \dot{m}_e \,(h_e + ½\,V_e^2 + gz_e)$$

Neglecting kinetic and potential energy changes, the energy balance simplifies to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \, h_i - \sum_e \dot{m}_e \, h_e$$

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3$$

Substituting for $\dot{m}_2$ and $\dot{m}_3$

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 h_1 - (0.20\,\dot{m}_1)h_2 - (0.80\,\dot{m}_1)h_3$$

Solving for $\dot{m}_1$

$$\dot{m}_1 = \frac{\dot{Q}_{cv} - \dot{W}_{cv}}{0.20h_2 + 0.80h_3 - h_1}$$

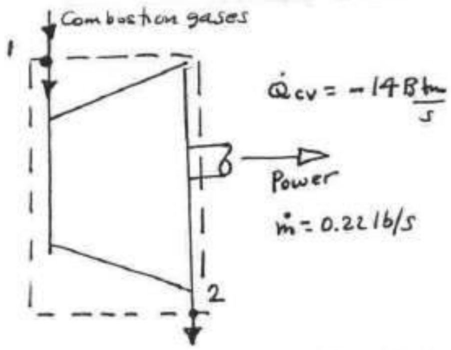
Steam at the inlet, State 1, and extracted, State 2, is superheated. From Table A-4E, $h_1 = 1542.5$ Btu/lb and $h_2 = 1216.2$ Btu/lb. Saturated vapor exits at State 3. From Table A-3E, $h_3 = h_{g3} = 1105.8$ Btu/lb. Substituting values

$$\dot{m}_1 = \frac{\left(-5\times10^4\,\frac{\text{Btu}}{\text{h}}\right) - \left(6.8\times10^6\,\frac{\text{Btu}}{\text{h}}\right)}{0.20\left(1216.2\,\frac{\text{Btu}}{\text{lb}}\right) + 0.80\left(1105.8\,\frac{\text{Btu}}{\text{lb}}\right) - 1542.5\,\frac{\text{Btu}}{\text{lb}}}\left|\frac{\text{h}}{3600\text{ s}}\right| = \underline{\mathbf{4.59\ lb/s}} \longleftarrow$$

Problem 4.52

Hot combustion gases, modeled as air behaving as an ideal gas, enter a turbine at 145 lbf/in.^{2}, 2700°R with a mass flow rate of 0.22 lb/s and exit at 29 lbf/in.^{2} and 1620°R. If heat transfer from the turbine to its surroundings occurs at a rate of 14 Btu/s, determine the power output of the turbine, in hp.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The control volume shown in the figure is at steady state.
2. Kinetic and potential energy effects are ignored.
3. The combustion gases are modeled as air as an ideal gas.

ANALYSIS: Reducing Eq. 4.20a,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + g(\cancel{z_1 - z_2}) \right]$$

$$\Rightarrow \quad \dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}(h_1 - h_2)$$

With enthalpy data from Table A-22E,

$$\dot{W}_{cv} = -14\ \frac{Btu}{s} + 0.22\ \frac{lb}{s}\left[703.35 - 401.10\right]\frac{Btu}{lb}$$

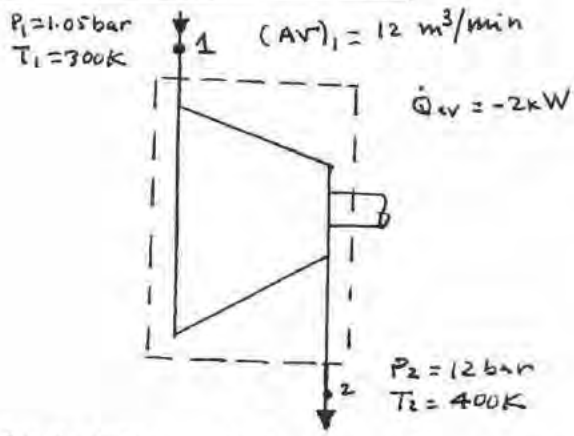
$$= 52.495\ \frac{Btu}{s} \left| \frac{3600\ s}{1\ h} \right| \left| \frac{1\ hp}{2545\ Btu/h} \right|$$

$$= 74.26\ hp$$

PROBLEM 4.53

Air enters a compressor operating at steady state at 1.05 bar, 300 K, with a volumetric flow rate of 12 m^3/min and exits at 12 bar, 400 K. Heat transfer occurs at a rate of 2 kW from the compressor to its surroundings. Assuming the ideal gas model for air and neglecting kinetic and potential energy effects, determine the power input, in kW.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. The air is modeled as an ideal gas.
3. Kinetic and potential energy effects are neglected.

ANALYSIS: Reducing Eq. 4.20a,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

$$\Rightarrow \quad \dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}[h_1 - h_2] \qquad (1)$$

where

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{(AV)_1 p_1}{RT_1} = \frac{(12\,m^3/min)(1.05\times10^5\,N/m^2)}{\left(\frac{8314}{28.97}\,\frac{N\cdot m}{kg\cdot K}\right)(300K)}\left|\frac{1\,min}{60s}\right|$$

$$= 0.2439\,\frac{kg}{s}$$

Then, with enthalpy data from Table A-22, Eq. (1) gives

$$\dot{W}_{cv} = -2kW + \left(0.2439\,\frac{kg}{s}\right)[300.19 - 400.98]\,\frac{kJ}{kg}\left|\frac{1\,kW}{1\,kJ/s}\right|$$

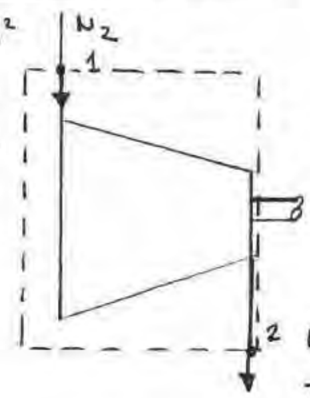
$$= -26.6\,kW$$

PROBLEM 4.54

Nitrogen is compressed in an axial-flow compressor operating at steady state from a pressure of 15 lbf/in.^{2} and a temperature of 50°F to a pressure 60 lbf/in.^{2}. The gas enters the compressor through a 6-in.-diameter duct with a velocity of 30 ft/s and exits at 198°F with a velocity of 80 ft/s. Using the ideal gas model, and neglecting stray heat transfer and potential energy effects, determine the compressor power input, in hp.

SCHEMATIC & GIVEN DATA:

$P_1 = 15\,\text{lbf/in}^2$
$T_1 = 50°F$
$D_1 = 6\,\text{in.}$
$V_1 = 30\,\frac{ft}{s}$



$P_2 = 60\,\frac{lbf}{in^2}$
$T_2 = 198°F$
$V_2 = 80\,ft/s$

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer and potential energy effects can be ignored.
3. The nitrogen is modeled as an ideal gas.

ANALYSIS: Reducing Eq. 4.20a,

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + \cancel{g(z_1 - z_2)}\right]$$

$$\Rightarrow \quad \dot{W}_{cv} = \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}\right]$$

where

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{(\pi D_1^2/4)(V_1)}{RT_1/P_1} = \frac{P_1(\pi D_1^2/4)V_1}{RT_1} = \frac{(15 \times 144\,\frac{lbf}{ft^2})\left(\frac{\pi}{4}(0.5\,ft)^2\right)\left(30\,\frac{ft}{s}\right)}{\left(\frac{1545}{28.01}\,\frac{ft\cdot lbf}{lb\cdot °R}\right)(510\,°R)}$$

$$= 0.45\,lb/s$$

Then with specific enthalpies on a molar basis from Table A-23E,

$$\dot{W}_{cv} = \left(0.45\,\frac{lb}{s}\right)\left[\frac{(3541.8 - 4571.9)}{28.01}\,\frac{Btu}{lb} + \left(\frac{(30)^2 - (80)^2}{2}\left(\frac{ft^2}{s^2}\right)\left|\frac{1\,lbf}{32.2\,lb\cdot ft/s^2}\right|\left|\frac{1\,Btu}{778\,ft\,lbf}\right|\right)\right]$$

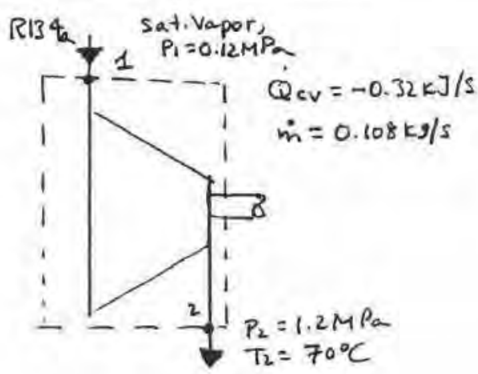
$$= \left(0.45\,\frac{lb}{s}\right)\left[-36.78 - 0.11\right]\frac{Btu}{lb}\left|\frac{3600\,s}{h}\right|\left|\frac{1\,hp}{2545\,Btu/h}\right|$$

$$= -23.48\,hp$$

PROBLEM 4.55

Refrigerant 134a enters a compressor operating at steady state as saturated vapor at 0.12 MPa and exits at 1.2 MPa and 70°C at a mass flow rate of 0.108 kg/s. As the refrigerant passes through the compressor, heat transfer to the surroundings occurs at a rate of 0.32 kJ/s. Determine at steady state the power input to the compressor, in kW.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, kinetic and potential energy effects can be ignored.

ANALYSIS: Reducing Eq. 4.20a,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + g\cancel{(z_1 - z_2)}\right]$$

$$\Rightarrow \dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}\left[h_1 - h_2\right]$$

With data from Tables A-11 and A-12,

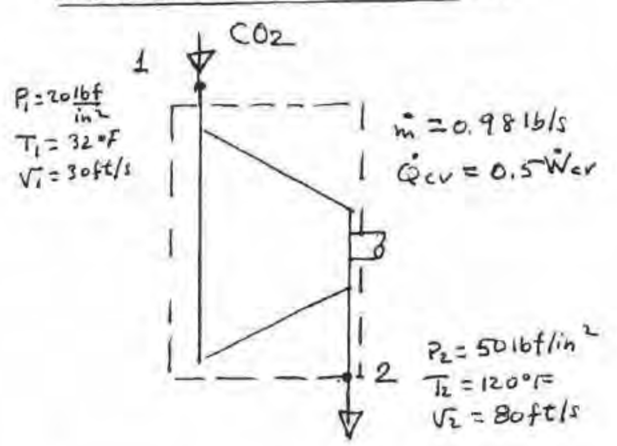
$$\dot{W}_{cv} = -0.32\,\frac{kJ}{s} + 0.108\,\frac{kg}{s}\left[233.86 - 298.96\right]\frac{kJ}{kg}$$

$$= -7.35\,\frac{kJ}{s}\left|\frac{1\ kW}{1\ kJ/s}\right| = -7.35\ kW$$

PROBLEM 4.56

Carbon dioxide gas is compressed at steady state from a pressure of 20 lbf/in.^{2} and a temperature of 32°F to a pressure of 50 lbf/in.^{2} and a temperature of 120°F. The gas enters the compressor with a velocity of 30 ft/s and exits with a velocity of 80 ft/s. The mass flow rate is 0.98 lb/s. The magnitude of the heat transfer rate from the compressor to its surroundings is 5% of the compressor power input. Using the ideal gas model with $c_p = 0.21$ Btu/lb · °R and neglecting potential energy effects, determine the compressor power input, in horsepower.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, potential energy effects can be neglected.
3. The CO_2 is modeled as an ideal gas with constant $c_p = 0.21$ Btu/lb·°R.

ANALYSIS: Reducing Eq. 4.20a,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g\,\cancel{(z_1 - z_2)}\right]$$

$\llcorner\ \dot{Q}_{cv} = 0.5\dot{W}_{cv}$

$$\Rightarrow \quad \dot{W}_{cv} = \frac{\dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}\right]}{0.95} = \frac{\dot{m}\left[c_p(T_1 - T_2) + \frac{V_1^2 - V_2^2}{2}\right]}{0.95}$$

Inserting values,

$$\dot{W}_{cv} = \frac{\left(0.98 \frac{lb}{s}\right)\left[0.21 \frac{Btu}{lb \cdot °R}(-88°R) + \left[\frac{(30)^2 - (80)^2}{2}\left(\frac{ft}{s}\right)^2 \left|\frac{1\,lbf}{32.2\,lb \cdot ft/s^2}\right| \left|\frac{1\,Btu}{778\,ft \cdot lbf}\right|\right]\right]}{0.95}$$

① $$= \frac{\left(0.98 \frac{lb}{s}\right)\left[-18.48 \frac{Btu}{lb} - 0.11 \frac{Btu}{lb}\right]}{0.95} \left|\frac{3600\,s}{1\,h}\right| \left|\frac{1\,hp}{2545\,Btu/h}\right|$$

$$= -27.1 \text{ hp}$$

The power input is 27.1 hp. ◄

---

1. Kinetic energy change is not a significant contributor in this application as may be expected.

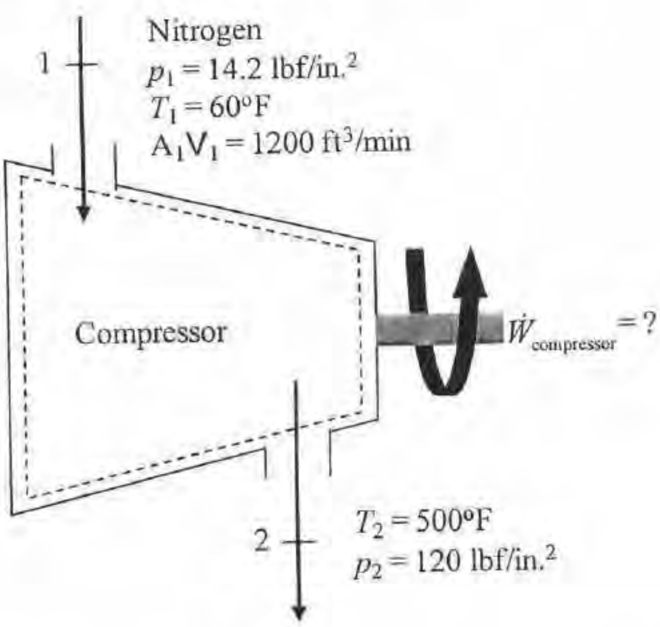
PROBLEM 4.57

4.57 At steady state, a well-insulated compressor takes in nitrogen at 60°F, 14.2 lbf/in.^{2}, with a volumetric flow rate of 1200 ft^3/min. Compressed nitrogen exits at 500°F, 120 lbf/in.^{2} Kinetic and potential energy changes from inlet to exit can be neglected. Determine the compressor power, in hp, and the volumetric flow rate at the exit, in ft^3/min.

KNOWN: Nitrogen with known inlet and exit conditions flows through a well-insulated compressor operating at steady state.

FIND: Determine the compressor power, in hp, and the volumetric flow rate at the exit, in ft^3/min.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
(1) The control volume shown in the accompanying schematic operates at steady state.
(2) Heat transfer can be neglected.
#1 (3) Nitrogen behaves as an ideal gas.
(4) Potential and kinetic energy changes from inlet to exit can be neglected.

ANALYSIS:
To determine the power, begin with the steady state mass and energy balances.

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)\right]$$

With listed assumptions, we solve for $\dot{W}_{cv}$.

PROBLEM 4.57 (Continued)

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2)$$

Note that $\bar{h}$ equals (h/M), where M is the molecular weight for nitrogen and $\bar{h}$ is from Table A-23E: $\bar{h}_1$=3611.3Btu/lbmol and $\bar{h}_2$=6693.1 Btu/lbmol.

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2) \rightarrow \dot{W}_{cv} = \frac{\dot{m}}{M}\left(\bar{h}_1 - \bar{h}_2\right) \qquad (1)$$

Obtain $\dot{m}$, in lb/h, using the ideal gas equation of state.

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{(AV)_1 p_1}{RT_1} = \frac{\left(1200\frac{ft^3}{min}\right)\left(14.2\frac{lbf}{in.^2}\right)}{\left(\frac{1545}{28.01}\frac{ft \cdot lbf}{lb \cdot {}^\circ R}\right)(520^\circ R)}\left|\frac{60min}{1h}\right|\left|\frac{144in.^2}{1ft^2}\right| = 5133\frac{lb}{h}$$

Using Eq. (1), solve for $\dot{W}_{cv}$, in hp.

(#2) $$\dot{W}_{cv} = \frac{5133\frac{lb}{h}}{28.01\frac{lb}{lbmol}}(3611.3 - 6693.1)\frac{Btu}{lbmol}\left|\frac{1\,hp}{2545\frac{Btu}{h}}\right| = -221.9\,hp$$

The exit volumetric flow rate, in ft^3/min, is as follows:

$$(AV)_2 = \dot{m}v_2 = \dot{m}\left(\frac{RT_2}{p_2}\right) = 5133\frac{lb}{h}\frac{\left(\frac{1545}{28.01}\frac{ft \cdot lbf}{lb \cdot {}^\circ R}\right)(960^\circ R)}{\left(120\frac{lbf}{in.^2}\right)}\left|\frac{1h}{60min}\right|\left|\frac{1ft^2}{144in.^2}\right| = 262.2\frac{ft^3}{min}$$

---

1. Table A-1E gives p_c = 33.5 atm = 492.32 lbf/in.^{2}, T_c = 227° R for nitrogen. Therefore, p_{R1} = 0.029, T_{R1} = 2.29. Referring to Fig. A-1, the value of the compressibility factor at this state is $Z \approx 1$. The same conclusion results when state 2 is checked. Accordingly, $pv = RT$ adequately describes the p-v-T relation for the air at those states.
2. $\dot{W}_{cv}$ is negative, as expected for a compressor. Here, energy is added to the system through work.

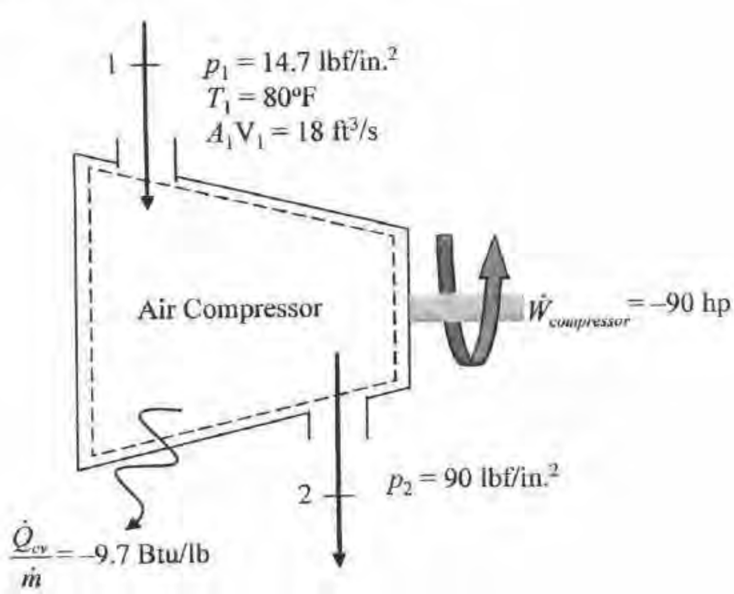
PROBLEM 4.58

4.58 Air enters a compressor operating at steady state with a pressure of 14.7 lbf/in.^{2}, a temperature of 80°F, and a volumetric flow rate of 18 ft^3/s. The air exits the compressor at a pressure of 90 lbf/in.^{2} Heat transfer from the compressor to its surroundings occurs at a rate of 9.7 Btu per lb of air flowing. The compressor power input is 90 hp. Neglecting kinetic and potential energy effects and modeling air as an ideal gas, determine the exit temperature, in °F.

KNOWN: An air compressor operates at steady state with specified inlet pressure, temperature, and volumetric flow rate and exit pressure. Heat transfer rate per lb of air flowing and compressor input power are known.

FIND: The exit temperature.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown on the accompanying figure is at steady state.
2. Kinetic and potential energy effects are neglected.
3. The air is modeled as an ideal gas.

ANALYSIS:
The steady-state, one-inlet, one-exit energy balance gives

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\,[(h_1 - h_2) + \tfrac{1}{2}(V_1^2 - V_2^2) + g(z_1 - z_2)]$$

Neglecting kinetic and potential energy changes, the energy balance simplifies to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\,(h_1 - h_2)$$

Solving for the exit enthalpy gives

PROBLEM 4.58 (Continued)

$$h_2 = \frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}} + h_1$$

For the inlet, state 1, the mass flow rate can be determined from given data and the ideal gas equation of state:

$$\dot{m}_1 = \frac{A_1 V_1}{v_1} = \frac{A_1 V_1 p_1}{\left(\overline{R}/M\right)T_1}$$

Substituting values yields

$$\dot{m}_1 = \frac{\left(18\frac{\text{ft}^3}{\text{s}}\right)\left(14.7\frac{\text{lbf}}{\text{in.}^2}\right)}{\left(\dfrac{1545\frac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot{}^\circ\text{R}}}{28.97\frac{\text{lb}}{\text{lbmol}}}\right)(540^\circ\text{R})}\left|\frac{144\,\text{in.}^2}{\text{ft}^2}\right| = 1.32\text{ lb/s}$$

Since air can be treated as an ideal gas, from Table A-22E, $h_1 = 129.06$ Btu/lb. Substituting values and applying appropriate conversion factors yield

$$h_2 = \left(-9.7\frac{\text{Btu}}{\text{lb}}\right) - \left(\frac{-90\text{ hp}}{1.32\frac{\text{lb}}{\text{s}}}\right)\left|\frac{2545\frac{\text{Btu}}{\text{h}}}{\text{hp}}\right|\left|\frac{\text{h}}{3600s}\right| + 129.06\frac{\text{Btu}}{\text{lb}} = 167.56\text{ Btu/lb}$$

From Table A-22E, $T_2 = 700^\circ\text{R}$ = **240°F**

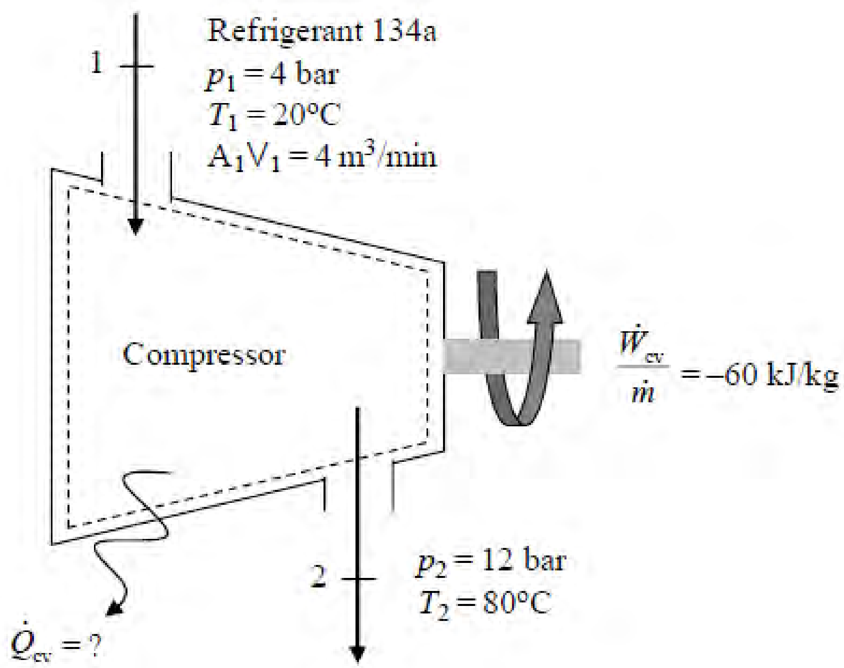
Problem 4.59

4.59 Refrigerant 134a enters an air conditioner compressor at 4 bar, 20°C, and is compressed at steady state to 12 bar, 80°C. The volumetric flow rate of the refrigerant entering is 4 m^3/min. The power input to the compressor is 60 kJ per kg of refrigerant flowing. Neglecting kinetic and potential energy effects, determine the heat transfer rate, in kW.

KNOWN: Refrigerant 134a with known inlet and exit conditions flows through a compressor operating at steady state.

FIND: Determine the heat transfer rate, in kW.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
(1) The control volume shown in the accompanying schematic operates at steady state.
(2) Potential and kinetic energy changes from inlet to exit can be neglected.

ANALYSIS:
To determine the heat transfer rate, begin with the steady state mass and energy balances.

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)\right]$$

Problem 4.59, continued

Simplify based on assumptions and solve for $\dot{Q}_{cv}$.

$$\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}(h_2 - h_1) = \dot{m}\left[\frac{\dot{W}_{cv}}{\dot{m}} + (h_2 - h_1)\right] \qquad (1)$$

To obtain $\dot{m}$, in lb/s, fix state 1 by referencing Table A-11 at 4 bar. $T_1 > T_{sat}$ and therefore a superheated condition exists at state 1. From Table A-12 at T_1 and p_1: v_1 = 0.05397 m^3/kg.

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{4\frac{m^3}{min}}{0.05397\frac{m^3}{kg}}\left|\frac{1\,min}{60s}\right| = 1.235\frac{kg}{s}$$

From Table A-12: h_1 = 262.96 kJ/kg and h_2 = 310.24 kJ/kg. To obtain $\dot{Q}_{cv}$, in kW, substitute into Eq. (1).

#1

$$\dot{Q}_{cv} = 1.235\frac{kg}{s}\left[-60\frac{kJ}{kg} + (310.24 - 262.96)\frac{kJ}{kg}\right]\left|\frac{1\,kW}{1\frac{kJ}{s}}\right| = -15.71\,kW$$

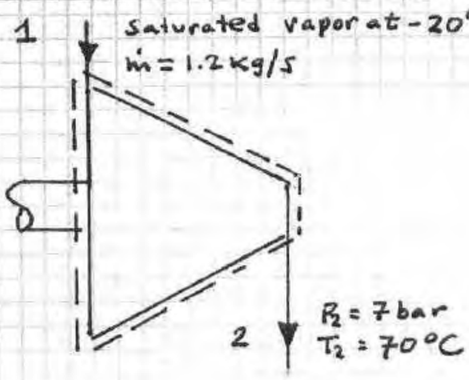
1. The negative sign indicates that there is energy rejected from the system by heat transfer.

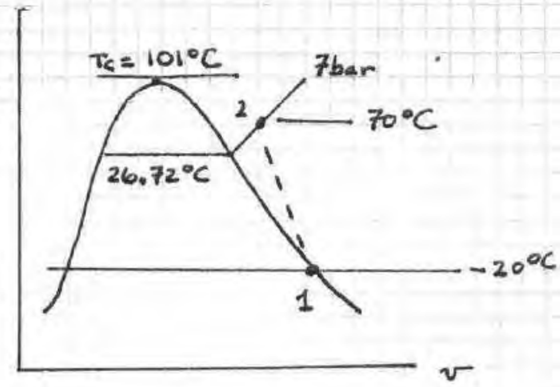
PROBLEM 4.60

KNOWN: Steady-state data are provided for an insulated compressor operating with R-134a.

FIND: For the compressor, determine the inlet and exit volumetric flow rates, in m^3/s, and the power input, in kW.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL:

1. The control volume enclosing the compressor is at steady state.
2. For the control volume $\dot{Q}_{cv} = 0$.
3. Kinetic and potential energy changes from inlet to exit are ignored.

ANALYSIS:

At steady state the mass rate balance reads $\dot{m}_2 = \dot{m}_1 \; (= \dot{m})$. Also,

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{(AV)_2}{v_2}$$

Table A-10 at -20°C

$$\Rightarrow (AV)_1 = \dot{m} v_1 = \left(1.2 \frac{kg}{s}\right)(0.1464 \; m^3/kg) = 0.1757 \frac{m^3}{kg}$$

$$(AV)_2 = \dot{m} v_2 = \left(1.2 \frac{kg}{s}\right)(0.03634 \; m^3/kg) = 0.0436 \frac{m^3}{kg}$$

Table A-12

Reducing Eq. 4.20 a,

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

$$\dot{W}_{cv} = \dot{m}\left[h_1 - h_2\right]$$

$$= \left(1.2 \frac{kg}{s}\right)\left[235.31 - 307.01\right] \frac{kJ}{kg} \left| \frac{1 \; kW}{1 \; kJ/s} \right|$$

$$= -86.04 \; kW$$

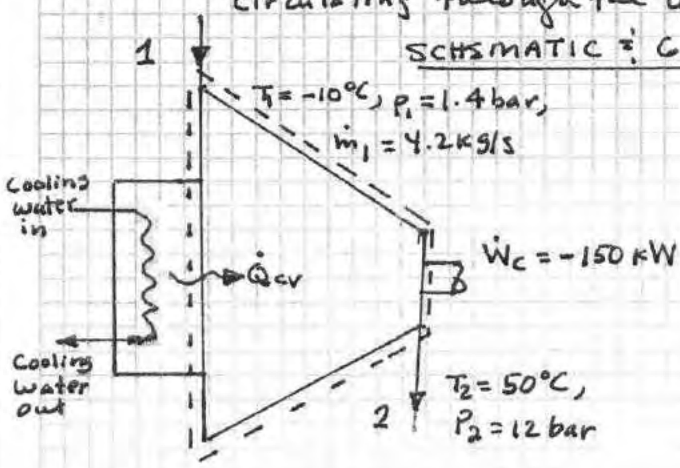
$$\therefore \; (-\dot{W}_{cv}) = +86.04 \; kW \quad \text{(Power input)}$$

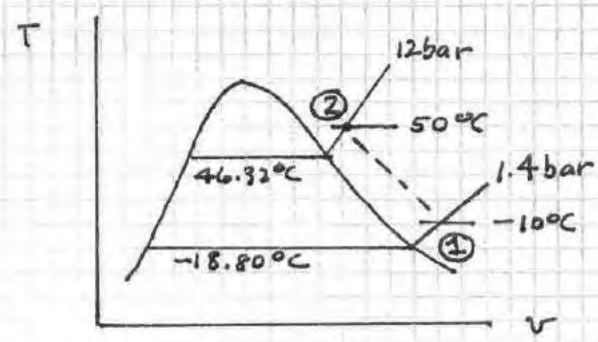
PROBLEM 4.61

KNOWN: Steady-state data are provided for a water-jacketed compressor operating with R-134a.

FIND: Determine the rate of heat transfer to the cooling water circulating through the water jacket, in kW.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL:

1. As shown in the schematic a control volume encloses the compressor only.
2. The control volume is at steady state.
3. Kinetic and potential energy effects are ignored.

ANALYSIS: At steady state a mass rate balance reduces to $\dot{m}_1 = \dot{m}_2 = (\dot{m})$. Eq. 4.20a reduces as follows

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

$$\Rightarrow \quad \dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}[h_2 - h_1]$$

With data from Table A-12

$$\dot{Q}_{cv} = -150\,\text{kW} + \left(4.2\,\frac{\text{kg}}{\text{s}}\right)\left[275.52 - 243.4\right]\frac{\text{kJ}}{\text{kg}}\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right|$$

$$= -15.1\,\text{kW}$$

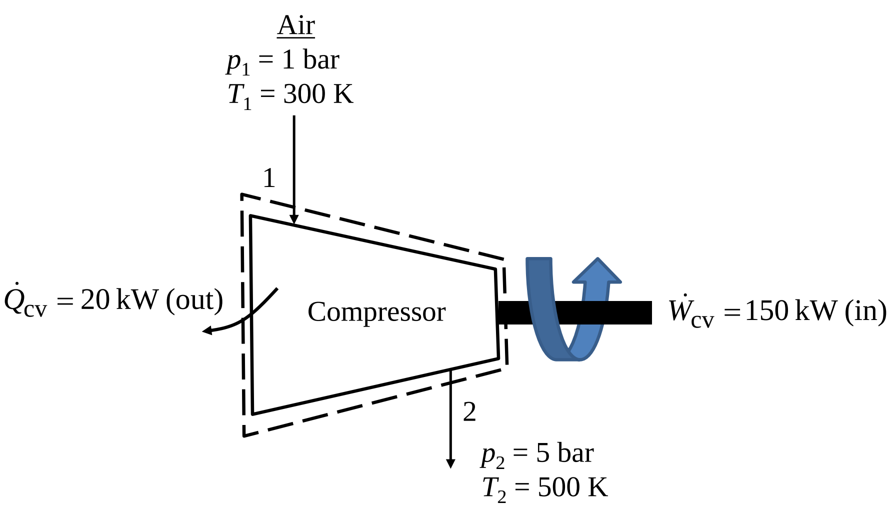
Heat transfer to the cooling water

4.62 Air, modeled as an ideal gas, is compressed at steady state from 1 bar, 300 K, to 5 bar, 500 K, with 150 kW of power input. Heat transfer occurs at a rate of 20 kW from the air to cooling water circulating in a water jacket enclosing the compressor. Neglecting kinetic and potential energy effects, determine the mass flow rate of the air, in kg/s.

KNOWN: Air flows through a compressor.

FIND: The mass flow rate of the air, in kg/s.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown with the schematic is at steady state.
2. For the control volume, Δke = 0 and Δpe = 0.
3. Model air as an ideal gas.

ANALYSIS:
The energy rate balance

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}[(h_1 - h_2) + \tfrac{1}{2}\cancel{(V_1^2 - V_2^2)} + g\cancel{(z_1 - z_2)}]$$

simplifies to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2)$$

Solving for mass flow rate gives

$$\dot{m} = \frac{\dot{Q}_{cv} - \dot{W}_{cv}}{h_2 - h_1}$$

Since heat transfer is from the control volume, $\dot{Q}_{cv} = -20\,\text{kW}$. Since power is to the control volume, $\dot{W}_{cv} = -150\,\text{kW}$. Using inlet and exit temperatures, specific enthalpy values are obtained from Table A-22: h_1 = 300.10 kJ/kg and h_2 = 503.02 kJ/kg. Substituting values and solving for mass flow rate give

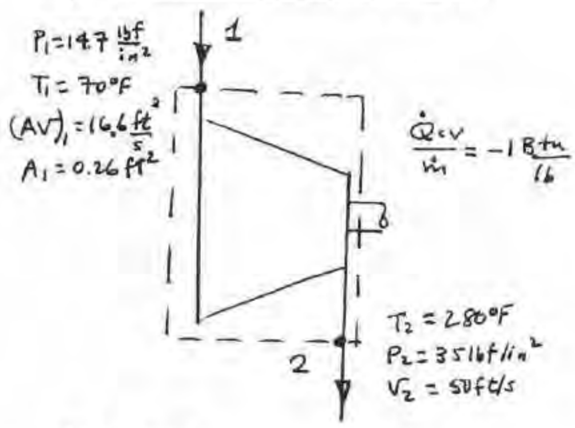
$$\dot{m} = \left[\frac{(-20\,\text{kW}) - (-150\,\text{kW})}{(503.02 - 300.19)\frac{\text{kJ}}{\text{kg}}}\right]\left|\frac{\left(\frac{\text{kJ}}{\text{s}}\right)}{\text{kW}}\right|$$

$\dot{m}$ = **0.64 kg/s**

PROBLEM 4.63

Air enters a compressor operating at steady state with a pressure of 14.7 lbf/in.^{2} and a temperature of 70°F. The volumetric flow rate at the inlet is 16.6 ft^3/s, and the flow area is 0.26 ft^2. At the exit, the pressure is 35 lbf/in.^{2}, the temperature is 280°F, and the velocity is 50 ft/s. Heat transfer from the compressor to its surroundings occurs at a rate of 1.0 Btu per lb of air flowing. Potential energy effects are negligible, and the ideal gas model can be assumed for the air. Determine (a) the velocity of the air at the inlet, in ft/s, (b) the mass flow rate, in lb/s, and (c) the compressor power, in Btu/s and hp.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, potential energy effects are negligible.
3. The air is modeled as an ideal gas.

ANALYSIS:

(a) Using given data at the inlet,

$$V_1 = \frac{(AV)_1}{A_1} = \frac{16.6\ ft^3/s}{0.26\ ft^2} = 63.85\ \frac{ft}{s} \qquad (a)$$

(b)

$$\dot{m}_1 = \frac{(AV)_1}{v_1} = \frac{p_1 (AV)_1}{RT_1} = \frac{\left(14.7 \times 144\ \frac{lbf}{ft^2}\right)\left(16.6\ \frac{ft^3}{s}\right)}{\left(\frac{1545}{28.97}\ \frac{ft\cdot lbf}{lb\cdot °R}\right)(530°R)} = 1.24\ \frac{lb}{s} \qquad (b)$$

(c) Reducing Eq. 4.20a

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + \cancel{g(z_1 - z_2)}\right]$$

$$\Rightarrow \quad \dot{W}_{cv} = \dot{m}\left[\frac{\dot{Q}_{cv}}{\dot{m}} + (h_1 - h_2) + \frac{V_1^2 - V_2^2}{2}\right]$$

Inserting values from Table A-22E and other data,

$$\dot{W}_{cv} = 1.24\frac{lb}{s}\left(-1\ \frac{Btu}{lb} + (126.67 - 177.23)\frac{Btu}{lb} + \left[\frac{(63.85)^2 - (50)^2}{2}\right]\left(\frac{ft^2}{s^2}\right)\left|\frac{1\ lbf}{32.2\ lb\cdot ft/s^2}\right|\left|\frac{1\ Btu}{778\ ft\cdot lbf}\right|\right)$$

① $$= 1.24\frac{lb}{s}\left[-1 - 50.56 + 0.03\right]\frac{Btu}{lb}$$

$$= -63.9\ \frac{Btu}{s}$$

or

$$\dot{W}_{cv} = -63.9\ \frac{Btu}{s}\left|\frac{3600\ s}{1\ h}\right|\left|\frac{1\ hp}{2545\ Btu/h}\right|$$

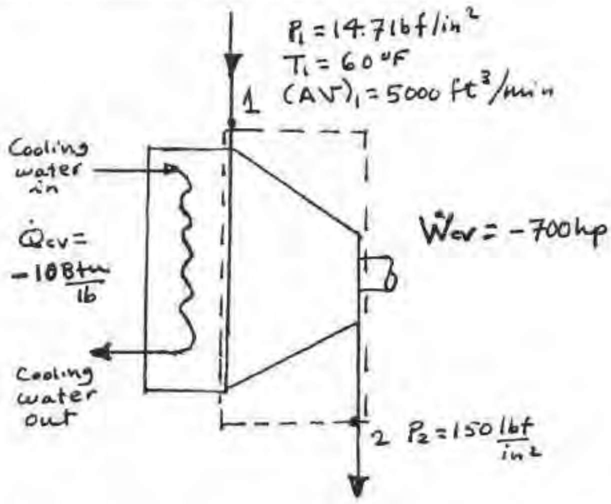
$$= -90.4\ hp$$

1. The kinetic energy change is a minor contributor, as may be expected.

PROBLEM 4.64

Air enters a compressor operating at steady state at 14.7 lbf/in.^{2} and 60°F and is compressed to a pressure of 150 lbf/in.^{2}. As the air passes through the compressor, it is cooled at a rate of 10 Btu per lb of air flowing by water circulated through the compressor casing. The volumetric flow rate of the air at the inlet is 5000 ft^3/min, and the power input to the compressor is 700 hp. The air behaves as an ideal gas, there is no stray heat transfer, and kinetic and potential effects are negligible. Determine (a) the mass flow rate of the air, lb/s, and (b) the temperature of the air at the compressor exit, in °F.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, kinetic and potential energy effects are negligible.
3. The air is modeled as an ideal gas.

ANALYSIS: (a) The mass flow rate is

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{(AV)_1 p_1}{RT_1} = \frac{(14.7\times144\ \mathrm{lbf/ft^2})(5000\ \mathrm{ft^3/min})}{\left(\frac{1545}{28.97}\ \frac{\mathrm{ft\cdot lbf}}{\mathrm{lb\cdot °R}}\right)(520\,°\mathrm{R})}\left|\frac{60\,\mathrm{s}}{1\,\mathrm{min}}\right|$$

$$= 6.36\ \frac{\mathrm{lb}}{\mathrm{s}} \qquad \text{(a)}$$

(b) Reducing an energy rate balance,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

$$\Rightarrow \qquad h_2 = h_1 + \frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}}$$

With h_1 from Table A-22E and other known data,

$$h_2 = 124.27\ \frac{\mathrm{Btu}}{\mathrm{lb}} - 10\ \frac{\mathrm{Btu}}{\mathrm{lb}} - \frac{[-700\,\mathrm{hp}]}{6.36\,\mathrm{lb/s}}\left|\frac{2545\,\mathrm{Btu/h}}{1\,\mathrm{hp}}\right|\left|\frac{1\,\mathrm{h}}{3600\,\mathrm{s}}\right|$$

$$= 192.08\ \mathrm{Btu/lb}$$

Interpolation in Table A-22E gives

$$T_2 = 801\,°\mathrm{R}\ (341\,°\mathrm{F}) \qquad \text{(b)}$$

PROBLEM 4.65

KNOWN: Operating and state data are provided for a pump and accompanying piping at steady state.

FIND: Determine the rate of heat transfer between the pump and its surroundings, in hp and Btu/min.

SCHEMATIC & GIVEN DATA:

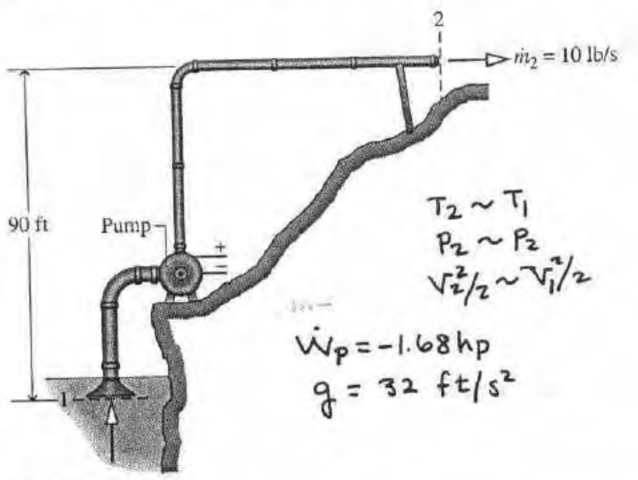


Fig. P4.65

$T_2 \sim T_1$
$P_2 \sim P_2$
$V_2^2/2 \sim V_1^2/2$
$\dot{W}_p = -1.68$ hp
$g = 32$ ft/s^2

ENGINEERING MODEL

1. A control volume includes the pump and the piping shown in the schematic.
2. The control volume is at steady state.
3. $T_2 \sim T_1$, $P_2 \sim P_1$, $V_2^2/2 \sim V_1^2/2$
4. $g = 32$ ft/s^2

ANALYSIS: At steady state $\dot{m}_2 = \dot{m}_1 = (\dot{m})$. Too, Eq. 4.20a reduces as follows,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \overset{0}{\cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)}} + g(z_1 - z_2)\right]$$

Using Eq. 3.13, the enthalpy term is expressed as

$$(h_1 - h_2) = [h_f(T_1) + v_f(T_1)[P_1 - P_{sat}(T_1)]] - [h_f(T_2) + v_f(T_1)[P_2 - P_{sat}(T_2)]]$$

$$\approx 0 \quad \text{because } T_2 \sim T_1 \text{ and } P_2 \sim P_1.$$

$\Rightarrow$

$$\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m} g (z_2 - z_1)$$

$$= -1.68 \text{ hp} + \left(10 \frac{\text{lb}}{\text{s}}\right)\left(32 \frac{\text{ft}}{\text{s}^2}\right)(90 \text{ ft})\left|\frac{1 \text{ lbf}}{32.2 \text{ lb}\cdot\text{ft/s}^2}\right|\left|\frac{1 \text{ hp}}{550 \text{ ft}\cdot\text{lbf/s}}\right|$$

$$= -1.68 \text{ hp} + 1.63 \text{ hp}$$

1 $$= -0.05 \text{ hp}$$ $\blacktriangleleft$

or

$$\dot{Q}_{cv} = -0.05 \text{ hp}\left|\frac{2545 \text{ Btu/h}}{1 \text{ hp}}\right|\left|\frac{1 \text{ h}}{60 \text{ min}}\right|$$

$$= -2.12 \frac{\text{Btu}}{\text{min}}$$ $\blacktriangleleft$

1 The heat transfer stems from friction and like effects, primarily within the pump.

PROBLEM 4.66

KNOWN: Operating and state data are provided for a pump and accompanying piping at steady state.

FIND: Determine the power required, in kW, to draw water from a reservoir and deliver it as required.

SCHEMATIC & GIVEN DATA:

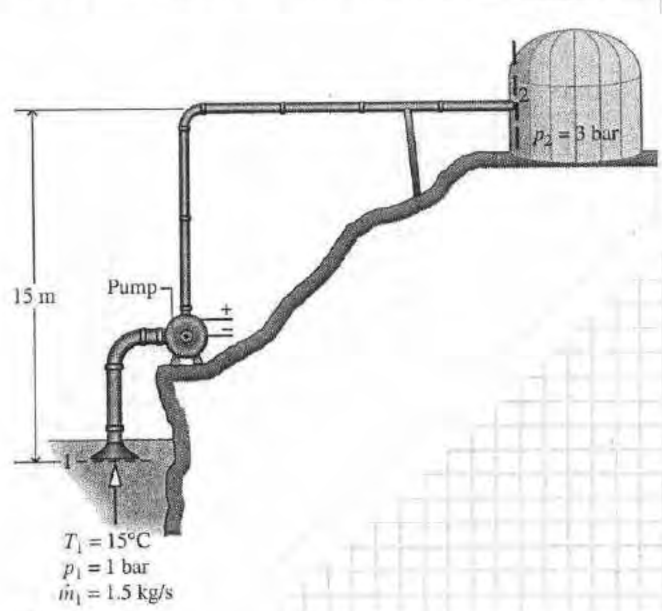


Fig. P4.66

ENGINEERING MODEL:

1. A control volume includes the pump and the piping shown in the schematic.
2. The control volume is at steady state.
3. For the control volume, $\dot{Q}_{cv}$ can be ignored.
4. The water temperature remains nearly constant from inlet to exit.
5. There is no significant change in kinetic energy from inlet to exit.
6. $g = 9.81\ m/s^2$

ANALYSIS: At steady state, $\dot{m}_1 = \dot{m}_2$ ($= \dot{m}$). Too, Eq. 4.20a reduces as follows:

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)} + g(z_1 - z_2)\right] \qquad (1)$$

Using Eq. 3.13, the enthalpy term is expressed as

$$(h_1 - h_2) = \left[h_f(T_1) + v_f(T_1)[p_1 - p_{sat}(T_1)]\right] - \left[h_f(T_2) + v_f(T_2)[p_2 - p_{sat}(T_2)]\right]$$

Since $T_2 \sim T_1$, this reduces to

$$(h_1 - h_2) = v_f(\overset{15°C}{T})[p_1 - p_2] \qquad (2)$$

Collecting results

① $$\dot{W}_{cv} = \dot{m}\left[\underbrace{v_f(T)}_{v_f \text{ at } 15°C \text{ from Table A-2}}[p_1 - p_2] + g(z_1 - z_2)\right]$$

$$= 1.5\frac{kg}{s}\left[\left(\frac{1.0009}{10^3}\right)\frac{m^3}{kg}(1-3)\,bar\left|\frac{10^5\ N/m^2}{1\ bar}\right| + \left(9.81\frac{m}{s^2}\right)(-15\,m)\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|\right]\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

$$= 1.5\frac{kg}{s}\left[-200.18\frac{N\cdot m}{kg} - 147.15\frac{N\cdot m}{kg}\right]\left|\frac{1\ kJ}{10^3\ N\cdot m}\right| = -0.52\frac{kJ}{s}$$

➡ $$\dot{W}_{cv} = -0.52\frac{kJ}{s}\left|\frac{1\ kW}{1\ kJ/s}\right| = -0.52\ kW \Rightarrow (-\dot{W}_{cv}) = 0.52\ kW$$

Power required by the control volume.

1. Recall from Sec. 4.4.2 that the product pv at a control volume inlet or exit represents work: flow work.

PROBLEM 4.67

KNOWN: Operating and state data are provided for a pump and accompanying piping at steady state.

FIND: Determine the power required, in kW.

SCHEMATIC & GIVEN DATA:

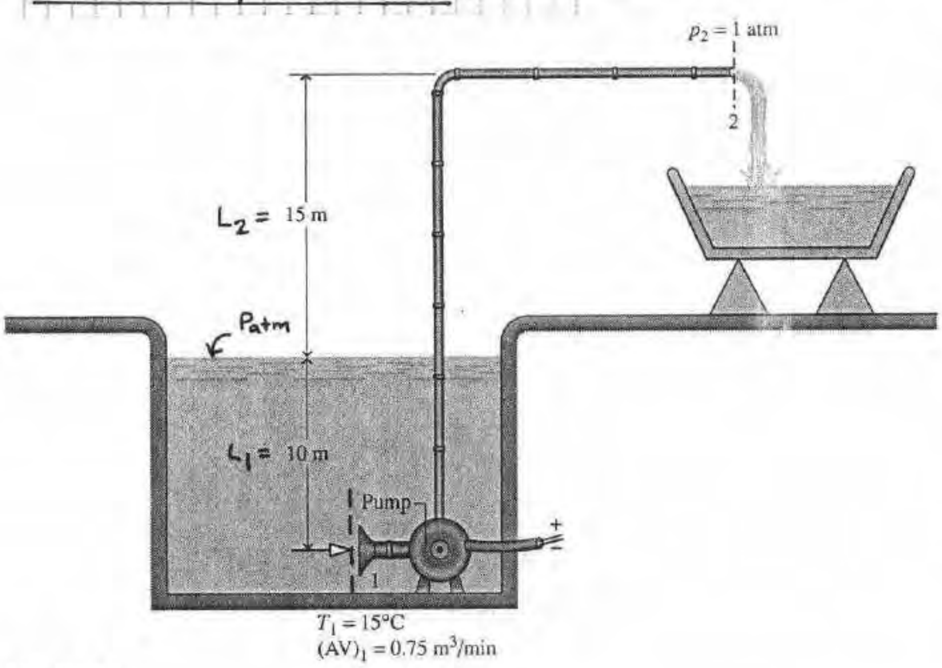


Fig. P4.67

ENGINEERING MODEL

1. A control volume includes the pump and piping shown in the schematic.
2. The control volume is at steady state.
3. For the control volume, $\dot{Q}_{cv}$ is negligible.
4. There is no significant change in water temperature or kinetic energy from inlet to exit.
5. $g = 9.81$ m/s²

ANALYSIS: At steady state, $\dot{m}_2 = \dot{m}_1 \; (= \dot{m})$. Thus,

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{(AV)_1}{v_f(15°C)} = \frac{0.75\,\text{m}^3/\text{min}}{\left(\frac{1.0009}{10^3}\frac{\text{m}^3}{\text{kg}}\right)}\left|\frac{1\,\text{min}}{60\,\text{s}}\right| = 12.49\,\text{kg/s}$$

Equation 4.20a reduces as follows: $0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)} + g(z_1 - z_2)\right]$

Using Eq. 3.13, the enthalpy term is expressed as

$$(h_1 - h_2) = \left[h_f(T_1) + v_f(T_1)[P_1 - P_{sat}(T_1)]\right] - \left[h_f(T_2) + v_f(T_2)[P_2 - P_{sat}(T_2)]\right]$$

Since $T_2 \sim T_1$, this gives

① $$(h_1 - h_2) = v_f(\underset{15°C}{T})\,[P_1 - P_2] \qquad (*)$$

With pressure concepts from Sec. 1.6.1, $P_1 = P_{atm} + \rho g L_1$. Then, since $P_2 = 1$ atm and $\rho = 1/v$,

$P_1 = P_2 + \rho g L_1 \Rightarrow v(P_1 - P_2) = gL_1$. Eq. (*) then reads, $(h_1 - h_2) = gL_1$.

Collecting results,

$$\dot{W}_{cv} = \dot{m}\left[(h_1 - h_2) + g\underbrace{(z_1 - z_2)}_{=-(L_1+L_2)}\right] = \dot{m}\left[gL_1 + g(-(L_1 + L_2))\right]$$

$$\therefore \dot{W}_{cv} = -\dot{m}\,g\,L_2$$

$$= -\left(12.49\frac{\text{kg}}{\text{s}}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)(15\,\text{m})\left|\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1\,\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right|\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right| =$$

$$= -1.84\,\text{kW} \Rightarrow (-\dot{W}_{cv}) = 1.84\,\text{kW}$$

The power required by the control volume.

1. Recall from Sec. 4.4.2 that the product pv at a control volume inlet or exit represents work: flow work.

PROBLEM 4.68

As shown in Fig. P4.68, a power washer used to clean the siding of a house has water entering through a hose at 20°C, 1 atm and a velocity of 0.2 m/s. A jet of water exits with a velocity of 20 m/s at an average elevation of 5 m with no significant change in temperature or pressure. At steady state, the magnitude of the heat transfer rate *from* the power washer to the surroundings is 10% of the electrical power *input*. Evaluating electricity at 8 cents per kW · h, determine the cost of the power required, in cents per liter of water delivered. Compare with the cost of water, assuming 0.05 cents per liter, and comment.

SCHEMATIC & GIVEN DATA:

Fig. 4.68

ENGR. MODEL:

1. A control volume encloses the power washer, including the inlet and delivery hoses.
2. The control volume is at steady state.
3. For liquid water, $v \approx v_f(T)$ and $h \approx h_f(T)$.
4. There is no significant change in temperature or pressure from inlet to exit.
5. The cost of electricity is 8 cents per kW·h. The cost of water is 0.05 cents per liter.
6. $g = 9.8\ \text{m/s}^2$.

ANALYSIS: Reducing an energy rate balance,

$$0 = \underbrace{\dot{Q}_{cv}}_{=0.1\dot{W}_{cv}} - \dot{W}_{cv} + \underbrace{\dot{m}}_{=\frac{(AV)_1}{v_1}}\left[\underbrace{(h_1-h_2)}_{\approx 0 \text{ by assumptions 3 and 4}} + \left(\frac{V_1^2-V_2^2}{2}\right) + g(z_1-z_2)\right]$$

$$\Rightarrow \quad \frac{\dot{W}_{cv}}{(AV)_1} = \frac{1}{0.9\,v_1}\left[\left(\frac{V_1^2-V_2^2}{2}\right) + g(z_1-z_2)\right]$$

Inserting data, including $v_1 \approx v_f(20°C) = 1.0018\times 10^{-3}\ \text{m}^3/\text{kg}$ from Table A-2,

$$\frac{\dot{W}_{cv}}{(AV)_1} = \frac{10^3}{0.9\,(1.0018)\ \text{m}^3/\text{kg}}\left[\frac{(0.2)^2-(20)^2}{2}\left(\frac{\text{m}}{\text{s}}\right)^2 + 9.8\frac{\text{m}}{\text{s}^2}(-5\,\text{m})\right]\left|\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1\,\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right|$$

$$= -276\ \frac{\text{kJ}}{\text{m}^3}\left|\frac{1\,\text{h}}{3600\,\text{s}}\right|\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right|\left|\frac{10^{-3}\,\text{m}^3}{1\,\text{L}}\right| = -7.7\times 10^{-5}\ \frac{\text{kW}\cdot\text{h}}{\text{L}}$$

Costing:

$$\begin{bmatrix}\text{Electricity}\\ \text{per Liter}\end{bmatrix} = 7.7\times 10^{-5}\ \frac{\text{kW}\cdot\text{h}}{\text{L}}\left(\frac{8\ \text{cents}}{\text{kW}\cdot\text{h}}\right) = 6.2\times 10^{-4}\ \frac{\text{cents}}{\text{L}}$$

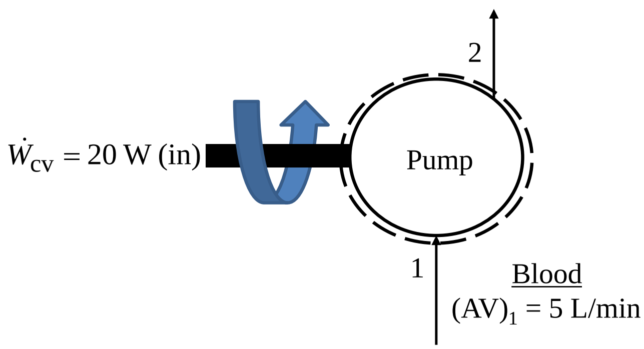
Comment: The cost of the water is significantly greater than the cost of electricity to deliver it: $(0.05\ \text{cents/L})/(6.2\times 10^{-4}\ \text{cents/L}) = 81$.

4.69 During cardiac surgery, a heart-lung machine achieves *extracorporeal circulation* of the patient's blood using a pump operating at steady state. Blood enters the well-insulated pump at a rate of 5 liters/min. The temperature change of the blood is negligible as it flows through the pump. The pump requires 20 W of power input. Modeling the blood as an incompressible substance with negligible kinetic and potential energy effects, determine the pressure change, in kPa, of the blood as it flows through the pump.

KNOWN: Blood flows through a pump.

FIND: The change in pressure, in kPa, of the blood as it flows through the pump.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown with the schematic is at steady state.
2. No stray heat transfer occurs between the blood and its surroundings.
3. The temperature of blood does not change as it flows through the pump.
4. Model the blood as an incompressible substance.
5. For the control volume, Δke = 0 and Δpe = 0.

ANALYSIS:
The energy rate balance

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}[(h_1 - h_2) + ½ (V_1^2 \cancel{-} V_2^2) + g(z_1 \cancel{-} z_2)]$$

simplifies to

$$0 = -\dot{W}_{cv} + \dot{m}(h_1 - h_2)$$

The change in specific enthalpy of an incompressible substance is determined using Eq. 3.19

$$h_2 - h_1 = \cancel{\int_{T_1}^{T_2} c(T)\, dT} + v(p_2 - p_1)$$

which simplifies to

$$h_2 - h_1 = v(p_2 - p_1)$$

since temperature does not change. Substituting into the energy rate balance

$$0 = -\dot{W}_{cv} + \dot{m}v(p_1 - p_2)$$

From the mass rate balance, $\dot{m}_2 = \dot{m}_1$ and $\dot{m} = \frac{\mathrm{AV}}{v}$. Since blood is modeled as incompressible, $v_2 = v_1$. Thus, volumetric flow rate, (AV), is constant. Substituting $\dot{m}v = \mathrm{AV}$ gives

$$0 = -\dot{W}_{cv} + (\mathrm{AV})(p_1 - p_2)$$

Since power is to the control volume, $\dot{W}_{cv} = -20\ \mathrm{W}$. Solving for the pressure change

$$(p_2 - p_1) = \frac{-\dot{W}_{cv}}{(\mathrm{AV})} = \frac{-(-20\ \mathrm{W})}{\left(5\ \frac{\mathrm{L}}{\mathrm{min}}\right)}\left|\frac{1\ \mathrm{L}}{10^{-3}\ \mathrm{m}^3}\right|\left|\frac{1\ \frac{\mathrm{J}}{\mathrm{s}}}{1\ \mathrm{W}}\right|\left|\frac{1\ \mathrm{N}\cdot\mathrm{m}}{1\ \mathrm{J}}\right|\left|\frac{1\ \mathrm{kPa}}{1000\ \frac{\mathrm{N}}{\mathrm{m}^2}}\right|\left|\frac{60\ \mathrm{s}}{1\ \mathrm{min}}\right|$$

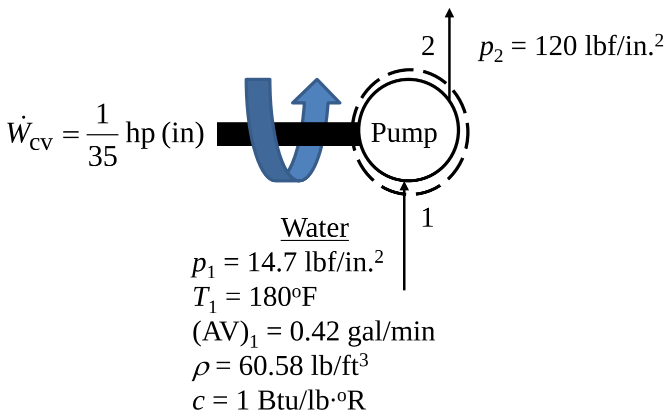
$(p_2 - p_1)$ = **240 kPa**

4.70 A pump is used to circulate hot water in a home heating system. Water enters the well-insulated pump operating at steady state at a rate of 0.42 gal/min. The inlet pressure and temperature are 14.7 lbf/in.^{2}, and 180°F, respectively; at the exit the pressure is 120 lbf/in.^{2} The pump requires 1/35 hp of power input. Water can be modeled as an incompressible substance with constant density of 60.58 lb/ft^3 and constant specific heat of 1 Btu/lb·°R. Neglecting kinetic and potential energy effects, determine the temperature change, in °R, as the water flows through the pump. Comment on this change.

KNOWN: Water flows through a circulation pump.

FIND: The change in temperature, in °R, as the water flows through the pump.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown with the schematic is at steady state.
2. No stray heat transfer occurs between the water and its surroundings.
3. Model the water as an incompressible substance with constant specific heat of 1 Btu/lb·°R and constant density of 60.58 lb/ft^3.
4. For the control volume, Δke = 0 and Δpe = 0.

ANALYSIS:
The energy rate balance

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \tfrac{1}{2}\,(\cancel{V_1^2 - V_2^2}) + g(\cancel{z_1 - z_2})\right]$$

simplifies to

$$0 = -\dot{W}_{cv} + \dot{m}\,(h_1 - h_2)$$

The change in specific enthalpy of an incompressible substance with constant specific heat is given by Eq. 3.20b

$$h_2 - h_1 = c(T_2 - T_1) + v(p_2 - p_1)$$

Substituting into the energy rate balance and solving for the temperature difference yield

$$0 = -\dot{W}_{cv} + \dot{m}[c(T_1 - T_2) + v(p_1 - p_2)]$$

From the mass rate balance, $\dot{m}_2 = \dot{m}_1$ and $\dot{m} = \rho AV$. For water modeled as an incompressible substance, $\rho_2 = \rho_1$. Solving for mass flow rate gives

$$\dot{m} = \left(60.58 \frac{\text{lb}}{\text{ft}^3}\right)\left(0.42 \frac{\text{gal}}{\text{min}}\right)\left|\frac{1\,\text{min}}{60\,\text{s}}\right|\left|\frac{0.13368\,\text{ft}^3}{1\,\text{gal}}\right| = 0.0567\ \text{lb/s}$$

Substituting for specific volume, $v = 1/\rho$, and solving for temperature change give

$$(T_2 - T_1) = \frac{\left(\frac{-\dot{W}_{cv}}{\dot{m}} + \frac{(p_1 - p_2)}{\rho}\right)}{c}$$

Since power is to the control volume, $\dot{W}_{cv} = -1/35\ \text{hp}$. Solving for temperature change

$$(T_2 - T_1) = \frac{\frac{-(-1/35\ \text{hp})}{\left(0.0567 \frac{\text{lb}}{\text{s}}\right)}\left|\frac{2545 \frac{\text{Btu}}{\text{h}}}{1\,\text{hp}}\right|\left|\frac{1\,\text{h}}{3600\,\text{s}}\right| + \left(\frac{(14.7 - 120)\frac{\text{lbf}}{\text{in.}^2}}{60.58 \frac{\text{lb}}{\text{ft}^3}}\right)\left|\frac{1\,\text{Btu}}{778.17\,\text{ft}\cdot\text{lbf}}\right|\left|\frac{144\,\text{in.}^2}{1\,\text{ft}^2}\right|}{1 \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}}$$

$(T_2 - T_1) =$ **0.0346°R**

The temperature change of the water as it flows through the pump is a negligible increase.

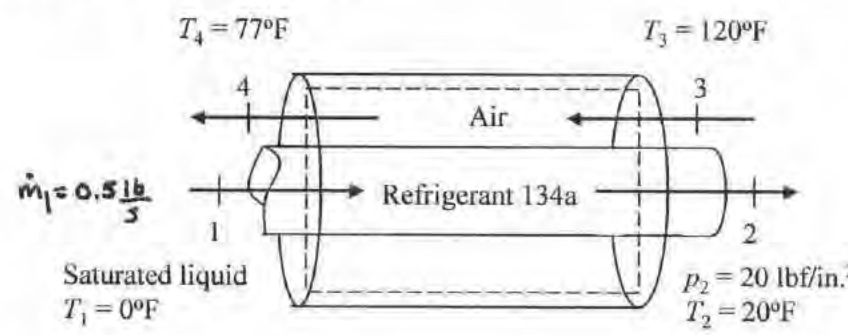
PROBLEM 4.71

4.71 Refrigerant 134a at a flow rate of 0.5 lb/s enters a heat exchanger in a refrigeration system operating at steady state as saturated liquid at 0°F and exits at 20°F at a pressure of 20 lbf/in.^{2} A separate air stream passes in counterflow to the Refrigerant 134a stream, entering at 120°F and exiting at 77°F. The outside of the heat exchanger is well-insulated. Neglecting kinetic and potential energy effects and modeling the air as an ideal gas, determine the mass flow rate of air, in lb/s.

KNOWN: Refrigerant 134a flows through a heat exchanger entering as saturated liquid at a given temperature and exiting at a given temperature and pressure. Air with given inlet and exit temperatures flows counter to the Refrigerant 134a flow.

FIND: Determine the mass flow rate of air.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown on the accompanying figure is at steady state.
2. Air can be modeled as an ideal gas.
3. Heat transfer and kinetic and potential energy effects can be neglected.
4. $\dot{W}_{cv} = 0$.

ANALYSIS:
Since the Refrigerant 134a stream and the air stream do not mix, each flow has one inlet and one exit. The steady-state mass rate balance gives

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_{134a} = 0.5\frac{\text{lb}}{\text{s}}$$

and

$$\dot{m}_3 = \dot{m}_4 = \dot{m}_{Air}$$

The steady-state energy balance gives

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i\,(h_i + \tfrac{1}{2}\,V_i^2 + gz_i) - \sum_e \dot{m}_e\,(h_e + \tfrac{1}{2}\,V_e^2 + gz_e)$$

PROBLEM 4.71 (Continued)

Neglecting heat transfer and kinetic and potential energy effects and recognizing no work $\dot{W}_{cv}$ is associated with a heat exchanger, the energy balance simplifies to

$$0 = \sum_i \dot{m}_i h_i - \sum_e \dot{m}_e h_e$$

$$0 = \dot{m}_{134a} h_1 + \dot{m}_{Air} h_3 - \dot{m}_{134a} h_2 - \dot{m}_{Air} h_4$$

Solving for the mass flow rate of air gives

$$\dot{m}_{Air} = \frac{\dot{m}_{134a}(h_2 - h_1)}{h_3 - h_4}$$

The Refrigerant 134a at State 1 is a saturated liquid. From Table A-10E, $h_1 = h_{1f} = 11.63$ Btu/lb. The Refrigerant 134a at State 2 is superheated vapor. From Table A-12E, $h_2 = 105.88$ Btu/lb. Since air can be modeled as an ideal gas, enthalpy values are obtained from Table A-22E: at $T_3 = 580^{\circ}\text{R}$, $h_3 = 138.66$ Btu/lb and at $T_4 = 537^{\circ}\text{R}$, $h_4 = 128.34$ Btu/lb.

Substituting values yields

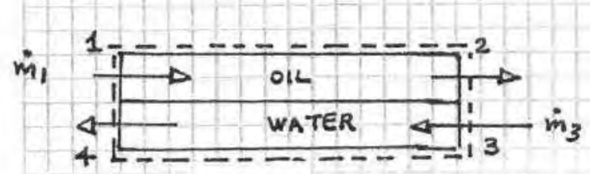
$$\dot{m}_{Air} = \frac{\left(0.5 \frac{\text{lb}}{\text{s}}\right)\left(105.88 \frac{\text{Btu}}{\text{lb}} - 11.63 \frac{\text{Btu}}{\text{lb}}\right)}{138.66 \frac{\text{Btu}}{\text{lb}} - 128.34 \frac{\text{Btu}}{\text{lb}}} = \underline{\mathbf{4.6\ lb/s}}$$

PROBLEM 4.72

KNOWN: State data are provided for a counterflow heat exchanger having oil and water passing in the two streams.

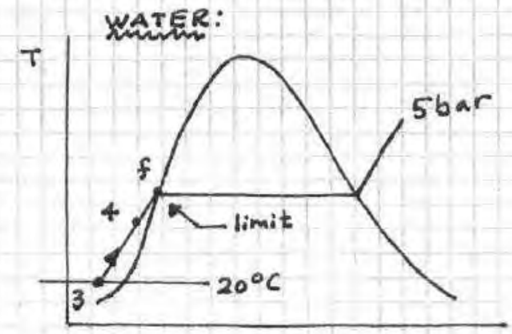
FIND: Determine the range of water mass flow rate, in kg/s, that ensures no water vapor is present in the exiting water stream.

SCHEMATIC & GIVEN DATA:



$T_1 = 450$ K
$\dot{m}_1 = 10$ kg/s
$T_2 = 350$ K

$T_3 = 20°C$
$P_3 = 5$ bar



ENGINEERING MODEL:

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv}$ is negligible and kinetic and potential energy effects can be ignored.
3. Each stream experiences no significant change in pressure.
4. The oil is modeled as incompressible with $c = 2$ kJ/kg·K.

ANALYSIS: Mass rate balances for the streams give $\dot{m}_4 = \dot{m}_3$, $\dot{m}_2 = \dot{m}_1$. The energy rate balance Eq. 4.18 reduces to give

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1\left[(h_1 - h_2) + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right] + \dot{m}_3\left[(h_3 - h_4) + \cancel{\frac{V_3^2 - V_4^2}{2}} + \cancel{g(z_3 - z_4)}\right]$$

$$\therefore\ 0 = \dot{m}_1[h_1 - h_2] + \dot{m}_3[h_3 - h_4]$$

$$\dot{m}_3 = \dot{m}_1\left[\frac{h_1 - h_2}{h_4 - h_3}\right] \qquad (1)$$

Using Eq. 3.20b, $(h_2 - h_1) = c[T_2 - T_1] + \overset{0}{\cancel{v(P_2 - P_1)}} = c[T_2 - T_1]$. Also, $h_3 \approx h_f(T_3)$. Eq. (1) becomes

$$\dot{m}_3 = \frac{\dot{m}_1 c[T_1 - T_2]}{[h_4 - h_f(T_3)]} \qquad (2)$$

For stream 4 to exit with no vapor, $h_4 \le h_f(5\text{bar})$. Thus,

$$\dot{m}_3 \ge \frac{\dot{m}_1 c[T_1 - T_2]}{[h_f(5\text{bar}) - h_f(T_3)]} = \frac{(10\text{ kg/s})(2\text{ kJ/kg}\cdot\text{K})(100\text{ K})}{[640.23 - 83.96]\text{ kJ/kg}}$$

$$\ge 3.6\text{ kg/s} \longleftarrow$$

PROBLEM 4.73

4.73 As shown in Fig. P4.73, Refrigerant 134a enters a condenser operating at steady state at 70 lbf/in.^{2}, and 160°F and is condensed to saturated liquid at 60 lbf/in.^{2} on the outside of tubes through which cooling water flows. In passing through the tubes, the cooling water increases in temperature by 20°F and experiences no significant pressure drop. Cooling water can be modeled as incompressible with $v = 0.0161$ ft^3/lb and $c = 1$ Btu/lb·°R. The mass flow rate of the refrigerant is 3100 lb/h. Neglecting kinetic and potential energy effects and ignoring heat transfer from the outside of the condenser, determine

(a) the volumetric flow rate of the entering cooling water, in gal/min.

(b) the rate of heat transfer, in Btu/h, to the cooling water from the condensing refrigerant.

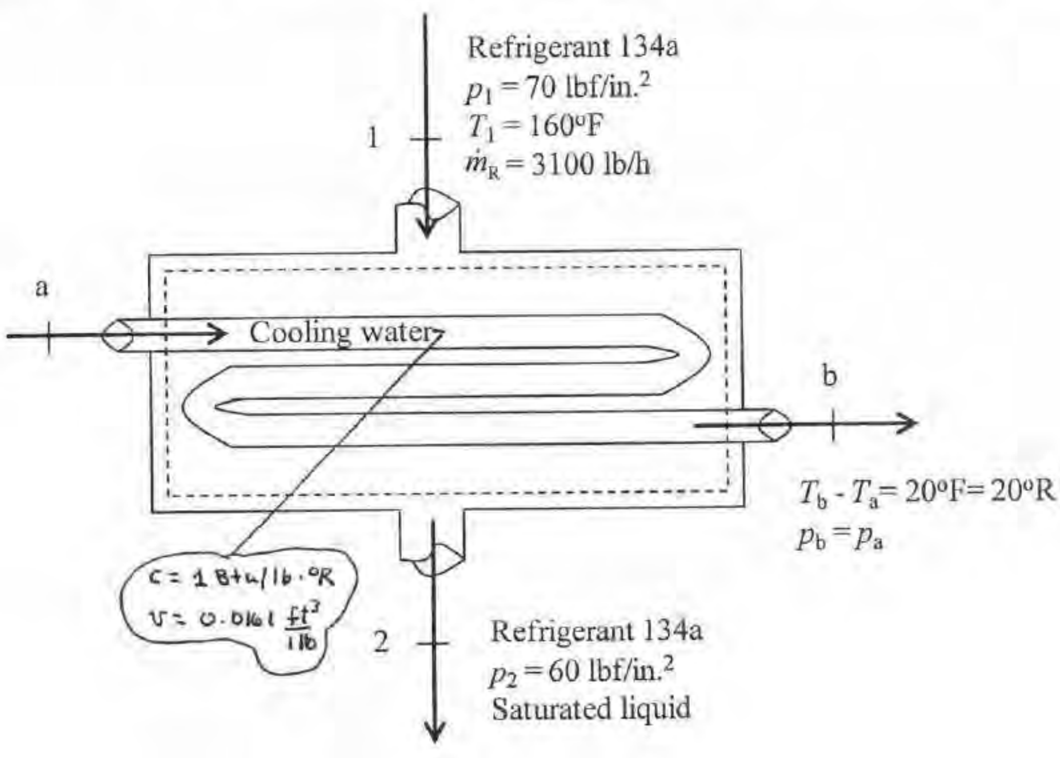


Fig. P4.73

KNOWN: Refrigerant 134a and cooling water pass in separate streams through a condenser (heat exchanger). The mass flow rate of the refrigerant and other data are given at the inlets and exits.

FIND: Determine the volumetric flow rate of the entering cooling water, in gal/min; and the rate of heat transfer, in Btu/h, to the cooling water from the condensing refrigerant.

SCHEMATIC AND GIVEN DATA:
Refer to Fig. P4.73.

ENGINEERING MODEL:
(1) The control volume shown in the accompanying schematic operates at steady state.
(2) Heat transfer from the condenser is negligible and $\dot{W}_{cv} = 0$.

PROBLEM 4.73 (Continued – 1)

(3) Potential and kinetic energy changes from inlet to exit can be neglected.
(4) The cooling water is modeled as an incompressible liquid with constant specific heat, and experiences no significant change in pressure.

ANALYSIS:

(a) Because the cooling water and refrigerant are separate streams, the steady state mass balances reduce to:

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_R$$

$$\dot{m}_a = \dot{m}_b = \dot{m}_{CW}$$

The mass flow rate of the cooling water is found using the steady state energy balance.

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_R\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)\right] + \dot{m}_{CW}\left[\begin{array}{l}(h_a - h_b) + \left(\dfrac{V_a^2 - V_b^2}{2}\right) \\ + g(z_a - z_b)\end{array}\right]$$

Simplify based on assumptions and solve for $\dot{m}_{CW}$.

$$\dot{m}_{CW} = \frac{\dot{m}_R(h_1 - h_2)}{(h_b - h_a)} \qquad (1)$$

Obtain $(h_b - h_a)$ for the cooling water using Eq. 3.20b with the given specific volume and specific heat values as follows:

$$(h_b - h_a) = c(T_b - T_a) + v\overset{0}{\cancel{(p_b - p_a)}} = c(T_b - T_a) = 1\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}(20^\circ\text{R}) = 20\frac{\text{Btu}}{\text{lb}}$$

To calculate the mass flow rate of the cooling water, fix state 1 by referencing Table A-11E at 70 lbf/in.^{2} where $T_1 > T_{sat}$, therefore state 1 is superheated. From Table A-12E at T_1 and p_1: $h_1 = 133.82$ Btu/lb. At state 2, saturated liquid exits and from Table A-11E: $h_2 = 27.24$ Btu/lb. Substituting into Eq. (1):

$$\dot{m}_{CW} = \frac{3100\dfrac{\text{lb}}{\text{h}}(133.82 - 27.24)\dfrac{\text{Btu}}{\text{lb}}}{\left(20\dfrac{\text{Btu}}{\text{lb}}\right)} = 16.520\times 10^3\,\frac{\text{lb}}{\text{h}}$$

Since the specific volume of the cooling water is constant, the volumetric flow rate is also constant. Determine the volumetric flow rate of the cooling water, in gal/min, as follows:

$$(\text{AV})_{CW} = \dot{m}_{CW}v_{CW} = \left(16.520\times 10^3\,\frac{\text{lb}}{\text{h}}\right)\left(0.0161\frac{\text{ft}^3}{\text{lb}}\right)\left|\frac{1\text{h}}{60\text{ min}}\right|\left|\frac{1\text{ gal}}{0.13368\text{ ft}^3}\right| = 33.2\,\frac{\text{gal}}{\text{min}} \longleftarrow$$

(b) To obtain the rate of energy transfer, in Btu/h, to the cooling water from the condensing refrigerant, use the steady state energy balance on a control volume enclosing only the cooling water, as follows:

$$0 = \dot{Q}_{CW} - \dot{W}_{CW} + \dot{m}_{CW}\left[(h_a - h_b) + \left(\frac{V_a^2 - V_b^2}{2}\right) + g(z_a - z_b)\right] \qquad (2)$$

PROBLEM 4.73 (Continued–2)

Reduce Eq. (2) based on assumptions noting that $\dot{Q}_{CW}$ denotes the heat transfer rate for the cooling water only.

#1
#2
$$\dot{Q}_{CW} = \dot{m}_{CW}(h_b - h_a) = 16520\frac{\text{lb}}{\text{h}}\left(20\frac{\text{Btu}}{\text{lb}}\right) = 3.304\times10^5\ \frac{\text{Btu}}{\text{h}}$$

---

1. The positive sign indicates that there is energy added to the cooling water by heat transfer.
2. For a control volume enclosing only the refrigerant stream

$$\dot{Q}_R = \dot{m}_R(h_2 - h_1) = 3100\frac{\text{lb}}{\text{hr}}(27.24 - 133.82)\frac{\text{Btu}}{\text{lb}} = -3.304\times10^5\ \frac{\text{Btu}}{\text{hr}}$$

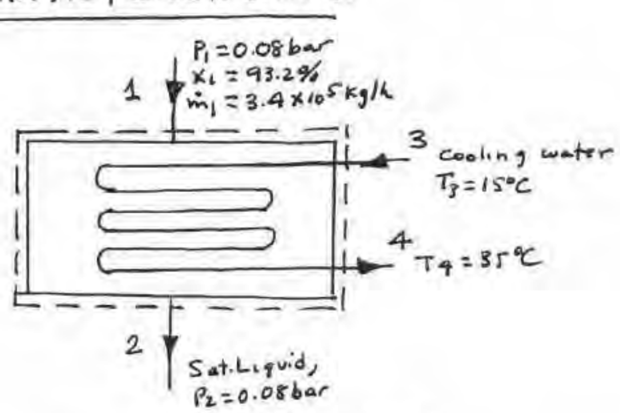
$$\dot{Q}_R = -\dot{Q}_{CW}$$

This result is expected

PROBLEM 4.74

Steam at a pressure of 0.08 bar and a quality of 93.2% enters a shell-and-tube heat exchanger where it condenses on the outside of tubes through which cooling water flows, exiting as saturated liquid at 0.08 bar. The mass flow rate of the condensing steam is 3.4×10^5 kg/h. Cooling water enters the tubes at 15°C and exits at 35°C with negligible change in pressure. Neglecting stray heat transfer and ignoring kinetic and potential energy effects, determine the mass flow rate of the cooling water, in kg/h, for steady-state operation.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$, $\dot{Q}_{cv} \approx 0$, and kinetic and potential energy effects can be ignored.
3. For the cooling water, $h \approx h_f(T)$.

ANALYSIS: Reducing Eq. 4.18,

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right] + \dot{m}_3\left[h_3 - h_4 + \cancel{\frac{V_3^2 - V_4^2}{2}} + \cancel{g(z_3 - z_4)}\right]$$

$$\Rightarrow \quad \dot{m}_3 = \dot{m}_1 \frac{[h_1 - h_2]}{[h_4 - h_3]} \qquad (1)$$

where with data from Table A-3,

$$h_1 = h_f + x_1(h_g - h_f) = 173.88 + 0.932(2403.1) = 2413.6 \frac{kJ}{kg}.$$

$$h_f = h_2$$

And with data from Table A-2, $h_3 \approx h_f(15°C) = 62.99 \frac{kJ}{kg}$, $h_4 \approx h_f(35°C) = 146.68 \frac{kJ}{kg}$.

Inserting values into Eq. (1), we get

$$\dot{m}_3 = 3.4 \times 10^5 \frac{kg}{h}\left[\frac{2413.6 - 173.88}{146.68 - 62.99}\right]$$

$$= 9.06 \times 10^6 \frac{kg}{h}$$

PROBLEM 4.75

4.75 An air conditioning system is shown in Fig. P4.75 in which air flows over tubes carrying Refrigerant 134a. Air enters with a volumetric flow rate of 50 m^3/min at 32°C, 1 bar, and exits at 22°C, 0.95 bar. Refrigerant enters the tubes at 5 bar with a quality of 20% and exits at 5 bar, 20°C. Ignoring heat transfer at the outer surface of the air conditioner, and neglecting kinetic and potential energy effects, determine at steady state
(a) the mass flow rate of the refrigerant, in kg/min.
(b) the rate of heat transfer, in kJ/min, between the air and refrigerant.

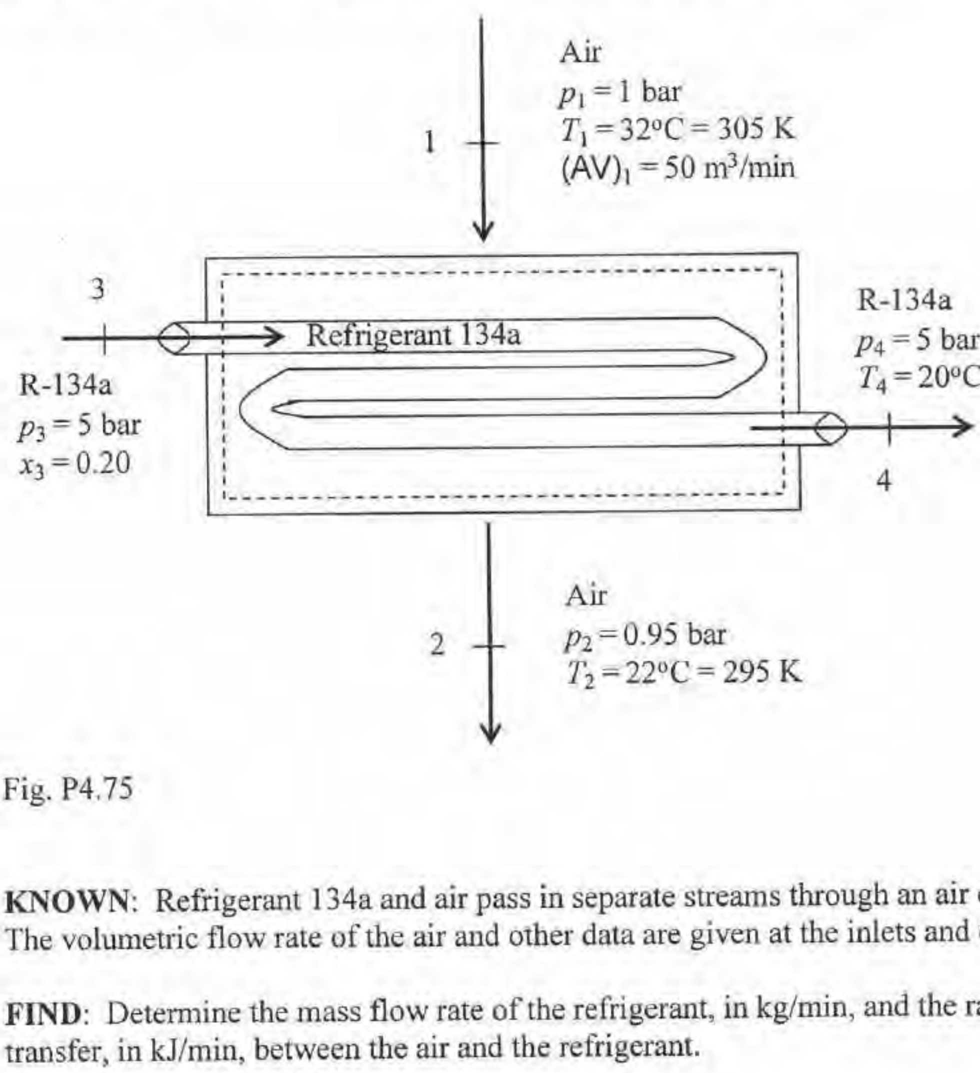


Fig. P4.75

KNOWN: Refrigerant 134a and air pass in separate streams through an air conditioner. The volumetric flow rate of the air and other data are given at the inlets and exits.

FIND: Determine the mass flow rate of the refrigerant, in kg/min, and the rate of heat transfer, in kJ/min, between the air and the refrigerant.

SCHEMATIC AND GIVEN DATA:
Refer to Fig. P4.75.

ENGINEERING MODEL:
(1) The control volume shown in the accompanying schematic operates at steady state.
(2) Heat transfer from the outer surface of the air conditioner is negligible, and $\dot{W}_{cv} = 0$.
(3) Potential and kinetic energy changes from inlet to exit can be neglected.
#1 (4) The air behaves as an ideal gas.

ANALYSIS:

(a) Because the air and refrigerant are separate streams, the steady state mass balances reduce to:

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_{air}$$
$$\dot{m}_3 = \dot{m}_4 = \dot{m}_R$$

The mass flow rate of the refrigerant is found using the steady state energy balance.

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_{air}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)\right] + \dot{m}_R\left[(h_3 - h_4) + \left(\frac{V_3^2 - V_4^2}{2}\right) + g(z_3 - z_4)\right]$$

Simplify based on assumptions and solve for $\dot{m}_R$.

$$\dot{m}_R = \dot{m}_{air}\frac{(h_1 - h_2)}{(h_4 - h_3)} \qquad (1)$$

Obtain $\dot{m}_{air}$, using the ideal gas equation of state.

$$\dot{m}_{air} = \frac{(AV)_1}{v_1} = \frac{(AV)_1 p_1}{RT_1} = \frac{\left(50\frac{m^3}{min}\right)(1\text{ bar})}{\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(305\text{ K})}\left|\frac{10^5\frac{N}{m^2}}{1\text{ bar}}\right|\left|\frac{1\text{kJ}}{10^3\text{N}\cdot\text{m}}\right| = 57.12\frac{kg}{min}$$

From Table A-22: h_1 = 305.22 kJ/kg and h_2=295.17 kJ/kg. Further, using data from Table A-11:

$$h_3 = h_{f3} + x_3 h_{fg3} = 71.33 + 0.20(184.74) = 108.28\frac{kJ}{kg}$$

Because $T_4 > T_{sat}$ at p_4 (Table A-11), from Table A-12: h_4 = 260.34 kJ/kg. Substitute into Eq. (1):

$$\dot{m}_R = 57.12\frac{kg}{min}\frac{(305.22 - 295.17)}{(260.34 - 108.28)} = 3.775\frac{kg}{min} \longleftarrow$$

(b) To obtain the rate of heat transfer, in kJ/min, between the air and the refrigerant, use the steady state energy balance on a control volume enclosing only the air, as follows:

$$0 = \dot{Q}_{air} - \dot{W}_{air} + \dot{m}_{air}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)\right] \qquad (2)$$

Reduce Eq. (2) based on assumptions noting that $\dot{Q}_{air}$ denotes the energy transfer due to heat transfer for the air only.

#2 #3
$$\dot{Q}_{air} = \dot{m}_{air}(h_2 - h_1) = 57.12\frac{kg}{min}(295.17 - 305.22)\frac{kJ}{kg} = -574\frac{kJ}{min} \longleftarrow$$

---

1. Table A-1 gives p_c = 37.7 bar, T_c = 133 K for air. Therefore, p_{R1} = 0.027, T_{R1} = 2.29. Referring to Fig. A-1, the value of the compressibility factor at this state is $Z \approx 1$. The same conclusion results when state 2 is checked. Accordingly, $pv = RT$ adequately describes the p-v-T relation for the air at those states.

2. The negative sign indicates that there is energy rejected from the air to the refrigerant by heat transfer.
3. For a control volume enclosing only the refrigerant stream

$$\dot{Q}_R = \dot{m}_R(h_4 - h_3) = 3.775\frac{kg}{min}(260.34 - 108.28)\frac{kJ}{kg} = 574\frac{kJ}{min}$$

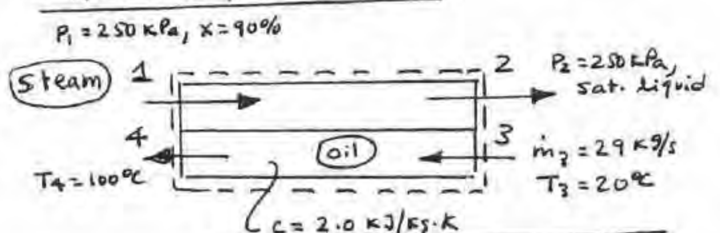
$$\dot{Q}_R = -\dot{Q}_{CW}$$

This result is expected.

PROBLEM 4.76

Steam enters a heat exchanger operating at steady state at 250 kPa and a quality of 90% and exits as saturated liquid at the same pressure. A separate stream of oil with a mass flow rate of 29 kg/s enters at 20°C and exits at 100°C with no significant change in pressure. The specific heat of the oil is $c = 2.0$ kJ/kg · K. Kinetic and potential energy effects are negligible. If heat transfer from the heat exchanger to its surroundings is 10% of the energy required to increase the temperature of the oil, determine the steam mass flow rate, in kg/s.

SCHEMATIC & GIVEN DATA:



$$\dot{Q}_{cv} = -0.10[\underbrace{\dot{m}_3(h_4-h_3)}_{\text{energy required to increase the oil temperature.}}] \quad (1)$$

(1)

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$ and kinetic and potential energy effects are negligible.
3. The oil is modeled as incompressible with $c = 2.0$ kJ/kg·K and no significant change in pressure.

ANALYSIS: Reducing Eq. 4.18,

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}_1\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)}\right] + \dot{m}_3\left[h_3 - h_4 + \cancel{\frac{V_3^2 - V_4^2}{2} + g(z_3 - z_4)}\right]$$

$$\Rightarrow \qquad \dot{m}_1 = \frac{-\dot{Q}_{cv} + \dot{m}_3[h_4 - h_3]}{h_1 - h_2}$$

Introducing Eq. (1),

$$\dot{m}_1 = \frac{+0.10\,\dot{m}_3[h_4 - h_3] + \dot{m}_3[h_4 - h_3]}{h_1 - h_2}$$

$$= \frac{(1.10)\,\dot{m}_3[h_4 - h_3]}{h_1 - h_2} \qquad (2)$$

With data from Table A-3, $h_1 = h_f + x_1 h_{fg}$, $h_2 = h_f \Rightarrow h_1 - h_2 = x_1 h_{fg}$. That is, $h_1 - h_2 = 0.90(2181.5\text{ kJ/kg}) = 1963.4$ kJ/kg.

Inserting values into Eq. (2), using Eq. 3.20b: $(h_4 - h_3) = c(T_4 - T_3) + v\cancelto{0}{(p_4 - p_3)}$, we get

(2) $$\dot{m}_1 = \frac{1.10(29\text{ kg/s})(2.0\text{ kJ/kg·K})(80\text{ K})}{1963.4\text{ kJ/kg}} = 2.6\text{ kg/s}$$

---

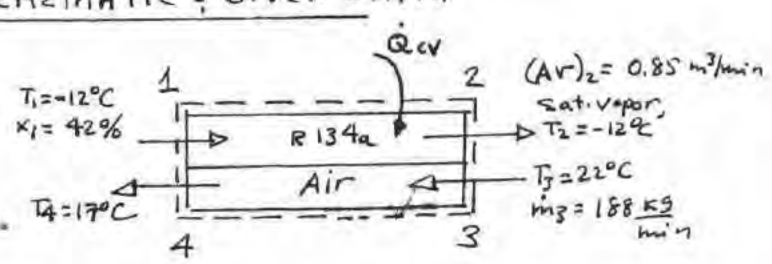
1. Consider a control volume consisting of the oil side of the heat exchanger. Assume steady state with $\dot{W}_{cv} = 0$ and ignore kinetic and potential energy effects. An energy rate balance gives $\dot{Q}_{oil} = \dot{m}_3(h_4 - h_3)$, which is the energy that would be required just to increase the oil temperature.

2. If $\dot{Q}_{cv} = 0$, the required steam mass flow rate is 2.36 kg/s. The details are left as an exercise.

PROBLEM 4.77.

Refrigerant 134a enters a heat exchanger at −12°C and a quality of 42% and exits as saturated vapor at the same temperature with a volumetric flow rate of 0.85 m³/min. A separate stream of air enters at 22°C with a mass flow rate of 188 kg/min and exits at 17°C. Assuming the ideal gas model for air and ignoring kinetic and potential energy effects, determine (a) the mass flow rate of the Refrigerant 134a, in kg/min, and (b) the heat transfer between the heat exchanger and its surroundings, in kJ/min.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$ and kinetic and potential effects are negligible.
3. The air is modeled as an ideal gas.

ANALYSIS:

(a) $\dot{m}_2 = \frac{(AV)_2}{v_2} = \frac{0.85\ m^3/min}{0.1068\ m^3/kg} = 7.96\ \frac{kg}{min}\ (= \dot{m}_1)$ ◄ (a)

(Table A-10)

(b) Reducing Eq. 4.18,

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}_1\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right] + \dot{m}_3\left[h_3 - h_4 + \cancel{\frac{V_3^2 - V_4^2}{2}} + \cancel{g(z_3 - z_4)}\right]$$

$$\Rightarrow \dot{Q}_{cv} = \dot{m}_1[h_2 - h_1] + \dot{m}_3[h_4 - h_3] \qquad (1)$$

With data from Table A-10, $h_2 = 240.15$ kJ/kg and $h_1 = h_f + x_1 h_{fg} = 34.39 + 0.42(205.77) = 120.81$ kJ/kg. Also, from Table A-22, $h_3 = 295.17$ kJ/kg and $h_4 = 290.16$ kJ/kg.

Substituting values into Eq. (1), we get

$$\dot{Q}_{cv} = 7.96\ \frac{kg}{min}(240.15 - 120.81)\frac{kJ}{kg} + 188\frac{kg}{min}(290.16 - 295.17)\frac{kJ}{kg}$$

$$= 949.95\ \frac{kJ}{min} - 941.88\ \frac{kJ}{kg}$$

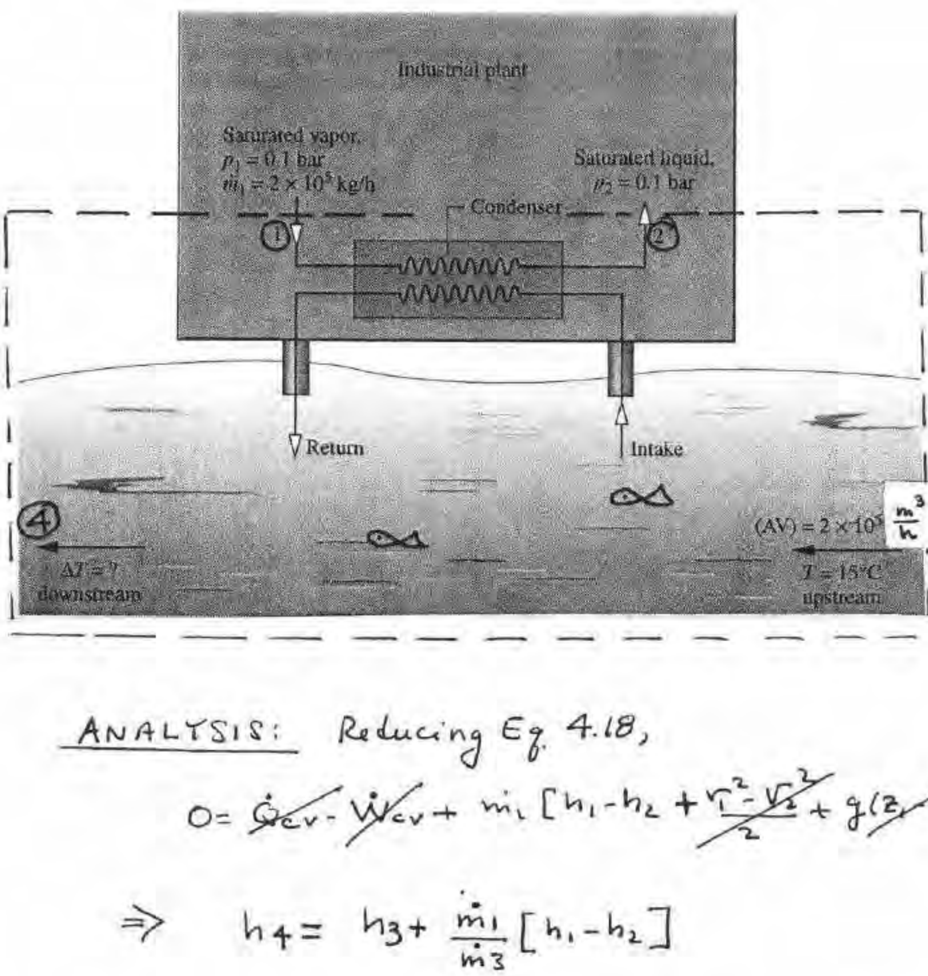
1 $$= +8.1\ \frac{kJ}{kg}$$ ◄ (b)

1. A positive sign here is reasonable owing to the low interior temperature of the heat exchanger relative to the temperature of the entering air.

PROBLEM 4.78

As sketched in Fig. P4.78, a condenser using river water to condense steam with a mass flow rate of 2×10^5 kg/h from saturated vapor to saturated liquid at a pressure of 0.1 bar is proposed for an industrial plant. Measurements indicate that several hundred meters upstream of the plant, the river has a volumetric flow rate of 2×10^5 m³/h and a temperature of 15°C. For operation at steady state and ignoring changes in kinetic and potential energy, determine the river-water temperature rise, in °C, downstream of the plant traceable to use of such a condenser, and comment.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$, there is no net heat transfer, and kinetic and potential energy effects can be ignored.
3. At 3 and 4, $v \approx v_f(T)$, $h \approx h_f(T)$.

ANALYSIS: Reducing Eq. 4.18,

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right] + \dot{m}_3\left[h_3 - h_4 + \cancel{\frac{V_3^2 - V_4^2}{2}} + \cancel{g(z_3 - z_4)}\right]$$

$$\Rightarrow \quad h_4 = h_3 + \frac{\dot{m}_1}{\dot{m}_3}[h_1 - h_2]$$

where

$$\dot{m}_3 = \frac{(AV)_3}{v_3} = \frac{2 \times 10^5 \text{ m}^3/\text{h}}{\left(\frac{1.0009}{10^3} \frac{\text{m}^3}{\text{kg}}\right)} = 2 \times 10^8 \frac{\text{kg}}{\text{h}}$$
(Table A-2)

Then, with $h_1 - h_2 = h_{fg}$ @ 0.1 bar, $h_1 - h_2 = 2392.8 \frac{kJ}{kg}$ (Table A-3). Also, from Table A-2, $h_3 = 62.99$ kJ/kg.

$$h_4 = 62.99 + \left(\frac{2 \times 10^5}{2 \times 10^8}\right)(2392.8) = 65.38 \frac{kJ}{kg}$$

① Interpolation in Table A-3 with $h_4 \approx h_f(T_4)$ gives $T_4 = 15.6°C$

$\Rightarrow$ Temperature rise $= 0.6°C$ ←

---

1. Comment: As discussed in Sec. 2.6.2, adverse environmental consequences can result when large quantities of warm water are returned to a river or lake.

PROBLEM 4.79

Figure P4.79 shows a solar collector panel embedded in a roof. The panel, which has a surface area of 24 ft^2, receives energy from the sun at a rate of 200 Btu/h per ft^2 of collector surface. Twenty-five percent of the incoming energy is lost to the surroundings. The remaining energy is used to heat domestic hot water from 90 to 120°F. The water passes through the solar collector with a negligible pressure drop. Neglecting kinetic and potential effects, determine at steady state how many gallons of water at 120°F the collector generates per hour.

SCHEMATIC & GIVEN DATA:

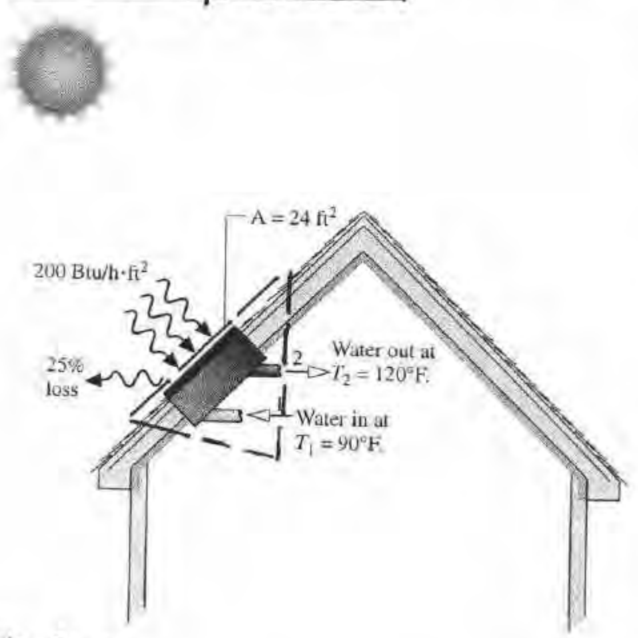


Fig. P4.79

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$ and kinetic and potential energy can be neglected.
3. For liquid water, $v \approx v_f(T)$ and $h \approx h_f(T)$.

ANALYSIS: Reducing Eq. 4.20a,

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

$$\Rightarrow \quad \dot{m} = \frac{\dot{Q}_{cv}}{h_2 - h_1} \qquad (1)$$

where

$$\dot{Q}_{cv} = (0.75)\left(200 \frac{\text{Btu}}{\text{h}\cdot\text{ft}^2}\right)(24\,\text{ft}^2) = 3600\ \text{Btu/h}$$

Then with $h_1 \approx h_f(90°F) = 58.07$ Btu/lb, $h_2 \approx h_f(120°F) = 88.0$ Btu/lb from Table A-2E.

Inserting values in Eq. (1),

$$\dot{m} = \frac{3600\ \text{Btu/h}}{(88.0 - 58.07)\ \text{Btu/lb}} = 120.3\ \frac{\text{lb}}{\text{h}}$$

The volumetric flow rate of the water delivered at 120°F is then

$$(AV)_2 = v_2\dot{m} = v_f(120°F)\,\dot{m}$$
$$= \left(0.01621 \frac{\text{ft}^3}{\text{lb}}\right)\left(120.3 \frac{\text{lb}}{\text{h}}\right)$$
$$= 1.95\ \text{ft}^3/\text{h}$$

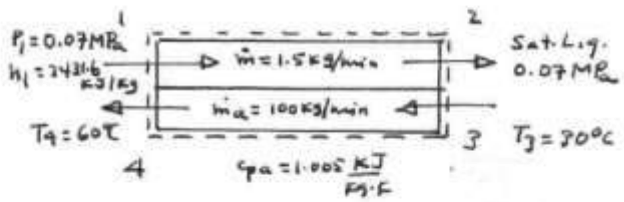
Converting to gallons/h,

$$(AV)_2 = 1.95 \frac{\text{ft}^3}{\text{h}} \left|\frac{1\ \text{gal}}{0.13368\ \text{ft}^3}\right| = 14.59\ \frac{\text{gal}}{\text{h}}$$

Problem 4.80

Steam enters a heat exchanger operating at steady state at 0.07 MPa with a specific enthalpy of 2431.6 kJ/kg and exits at the same pressure as saturated liquid. The steam mass flow rate is 1.5 kg/min. A separate stream of air with a mass flow rate of 100 kg/min enters at 30°C and exits at 60°C. The ideal gas model with $c_p = 1.005$ kJ/kg · K can be assumed for air. Kinetic and potential energy effects are negligible. Determine (a) the quality of the entering steam and (b) the rate of heat transfer between the heat exchanger and its surroundings, in kW.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$ and all kinetic and potential energy effects are negligible.
3. The air is modeled as an ideal gas with constant c_{pa}.

ANALYSIS:

(a) From Table A-3 at 0.07 MPa, $h_f = 376.70$ kJ/kg, $h_g = 2660.0$ kJ/kg.
Thus, the steam is a two-phase liquid-vapor mixture at 1. Then,

$$x_1 = \frac{h_1 - h_f}{h_g - h_f} = \frac{2431.6 - 376.70}{2660.0 - 376.70} = 0.9$$

(b) Reducing Eq. 4.18,

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + g\cancel{(z_1 - z_2)}\right] + \dot{m}_a\left[h_3 - h_4 + \cancel{\frac{V_3^2 - V_4^2}{2}} + g\cancel{(z_3 - z_4)}\right]$$

$$\Rightarrow \dot{Q}_{cv} = \dot{m}[h_2 - h_1] + \dot{m}_a\underbrace{[h_4 - h_3]}_{c_{pa}(T_4 - T_3)}$$

$$= \dot{m}[h_2 - h_1] + \dot{m}_a c_{pa}(T_4 - T_3)$$

Inserting values,

$$\dot{Q}_{cv} = \left(1.5 \frac{kg}{min}\right)\left[376.7 - 2431.6\right]\frac{kJ}{kg} + \left(100 \frac{kg}{min}\right)\left(1.005 \frac{kJ}{kg \cdot K}\right)(+30 K)$$

$$= -3082.4 \frac{kJ}{min} + 3015 \frac{kJ}{min}$$

$$= -67.35 \frac{kJ}{min}\left|\frac{1 min}{60 s}\right|\left|\frac{1 kW}{1 kJ/s}\right|$$

$$= -1.12 \text{ kW}$$

Problem 4.81

KNOWN: State data are provided for a parallel flow heat exchanger operating at steady state with separate streams of air and water. Each stream exits at the same temperature.

FIND: Find the temperature of the exiting streams, in K.

SCHEMATIC & GIVEN DATA:

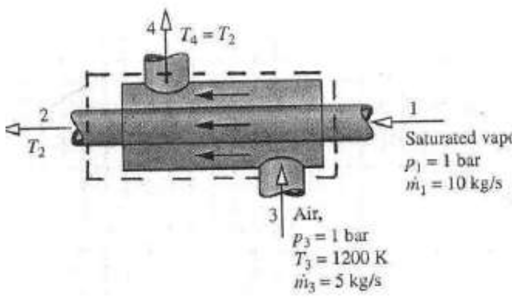


Fig. P4.85

ENGINEERING MODEL:

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv}$ is negligible and kinetic and potential energy effects are ignored.
3. The ideal gas model applies to the air.
4. For each stream there is no significant change in pressure.

ANALYSIS: Mass rate balances for the streams give $\dot{m}_2 = \dot{m}_1$ and $\dot{m}_4 = \dot{m}_3$.

An energy rate balance reduces as follows, $0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_4)$

Thus, $0 = \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_4)$, giving

$$0 = \left(10\frac{kg}{s}\right)[h_1 - h_2] + \left(5\frac{kg}{s}\right)(h_3 - h_4) \Rightarrow 0 = 2[h_1 - h_2] + [h_3 - h_4] \quad (1)$$

In Eq. (1), $h_1 = h_g(1\text{bar})$, $h_2 = h(T_2, 1\text{ bar})$ for water. Also, $h_3 = h(T_3)$, $h_4 = h(\underbrace{T_4}_{=T_2})$ for air modeled as an ideal gas. In principle, solution of Eq. (1) can be obtained iteratively with data from the "steam tables" and "air tables". A more direct approach is to use IT, as follows:

```
p1=1
p2=1
x=1
T3=1200

2*(h1-h2)+(h3-h4) = 0
h3 = h_T("Air",T3)
h4 = h_T("Air",T)
```

①
```
h1 = hsat_Px("Water/Steam", p1, x)
h2 = h_PT("Water/Steam", p2, T)
```

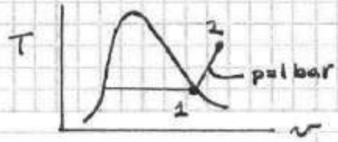
where $T = T_2 = T_4$.

② SOLUTION:

T=553.5 K
h1=2675 kJ/kg
h2=3034
h3=1277
h4=558.2

Problem 4.81, continued

1 The water receives energy by heat transfer from the air. Thus the temperature of the water increases as the temperature of the air decreases. The process of the water is shown on T-v coordinates as:



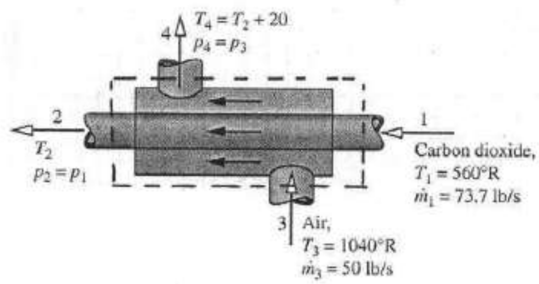
2. An iterative solution using the "steam tables" and the "air tables" is left as an exercise.

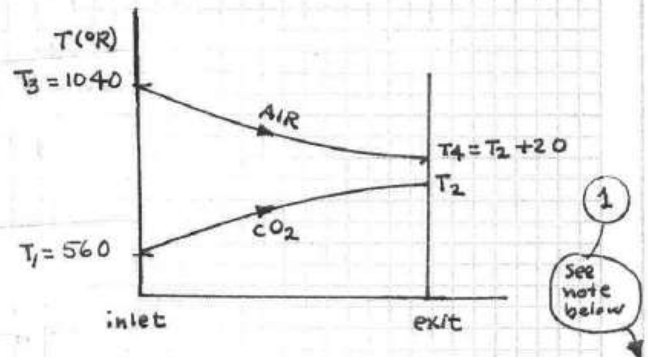
Problem 4.82

KNOWN: State data are provided for a parallel flow heat exchanger operating at steady state with separate streams of air and carbon dioxide, each a gas.

FIND: Subject to the constraint that the temperature of the exiting air must be 20 degrees greater than the temperature of the exiting CO_2, determine the exit temperature of each stream, in °R.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv}$ is negligible and kinetic and potential energy effects are ignored.
3. The ideal gas model applies to each gas.

ANALYSIS: Mass rate balances for the streams give $\dot{m}_2 = \dot{m}_1$ and $\dot{m}_4 = \dot{m}_3$. An energy rate balance reduces as follows: $0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_4)$.

That is, $0 = \underbrace{\dot{m}_1(h_1 - h_2)}_{CO_2} + \underbrace{\dot{m}_3(h_3 - h_4)}_{Air}$, where $h_1 = h_{CO_2}(T_1)$ and $h_3 = h_{air}(T_3)$.

Also, $h_2 = h_{CO_2}(T_2)$ and $h_4 = h_{AIR}(T_2 + 20)$. T_2 can be determined iteratively

(2) using data from Tables A-22E and A-23E. A more direct approach is to use IT, as follows:

```
T1=560      // °R
m1=73.7     // lb/s
T3=1040
m3=50

m1*(h1-h2) + m3*(h3-h4)=0

T4=T2+20

h1 = h_T("CO2",T1)
h2= h_T("CO2",T2)

h3 = h_T("Air",T3)
h4= h_T("Air",T4)
```

SOLUTION:

T2=759.9 R
T4=779.9 R

1. To meet a size constraint for a parallel flow heat exchanger can be challenging, for as the temperatures of the two streams approach each other the area required for heat transfer can become very large.

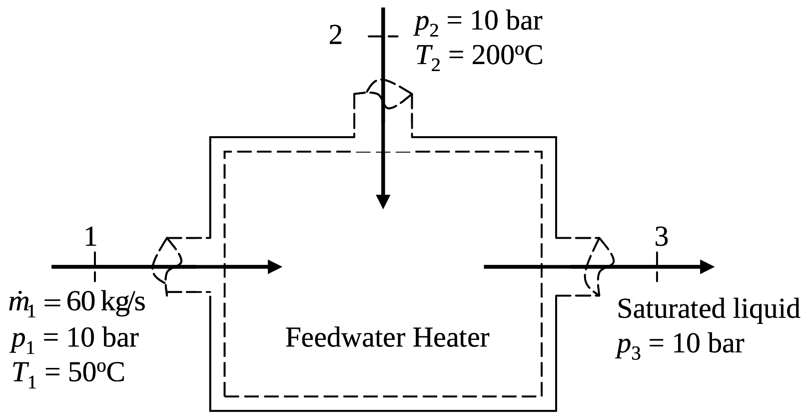
2. An iterative solution using Tables A-22E and A-23E is left as an exercise.

4.83 An open feedwater heater operates at steady state with liquid water entering inlet 1 at 10 bar, 50°C, and a mass flow rate of 60 kg/s. A separate stream of steam enters inlet 2 at 10 bar and 200°C. Saturated liquid at 10 bar exits the feedwater heater at exit 3. Ignoring heat transfer with the surroundings and neglecting kinetic and potential energy effects, determine the mass flow rate, in kg/s, of the steam at inlet 2.

KNOWN: Liquid water at given pressure and temperature and steam at given pressure and temperature enter a feedwater heater. Saturated liquid exits the feedwater heater at given pressure.

FIND: Determine the mass flow rate of steam entering at inlet 2.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown on the accompanying figure is at steady state.
2. Heat transfer and kinetic and potential energy effects can be neglected.
3. $\dot{W}_{cv}$ = 0 since a feedwater heater has no work associated with it.

ANALYSIS:
The steady-state mass rate balance gives

$$\sum_i \dot{m}_i = \sum_e \dot{m}_e$$

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

The steady-state energy balance gives

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left(h_i + ½\, V_i^2 + gz_i\right) - \sum_e \dot{m}_e \left(h_e + ½\, V_e^2 + gz_e\right)$$

Neglecting heat transfer and kinetic and potential energy effects and recognizing no work is associated with a feedwater heater, the energy balance simplifies to

$$0 = \sum_i \dot{m}_i h_i - \sum_e \dot{m}_e h_e$$

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$$

Substituting for $\dot{m}_3$ from the mass rate balance

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3$$

Solving for $\dot{m}_2$

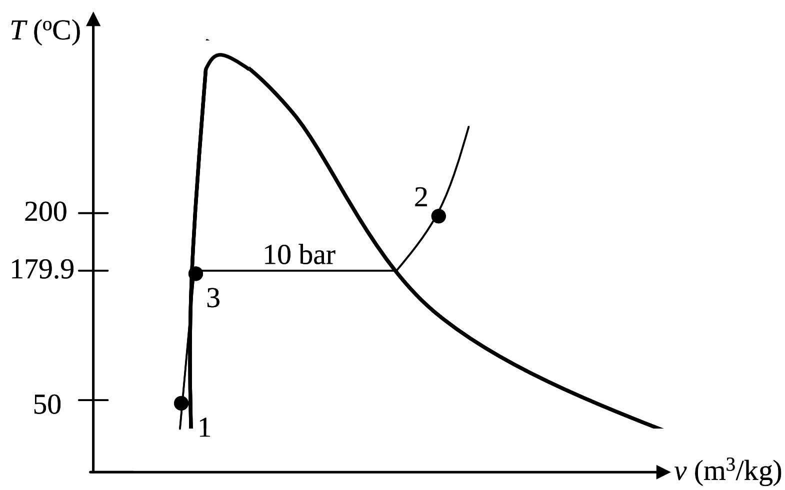
$$\dot{m}_2 = \frac{\dot{m}_1 (h_3 - h_1)}{h_2 - h_3}$$

At inlet 1, the water is compressed liquid. From Table A-2, $h_1 \approx h_{f1}$ = 209.33 kJ/kg.
At inlet 2, the steam is superheated. From Table A-4, h_2 = 2827.9 kJ/kg.
At exit 3, the water is saturated liquid. From Table A-3, $h_3 = h_{f3}$ = 762.81 kJ/kg.

Substituting values yields

$$\dot{m}_2 = \frac{\left(60\,\frac{\text{kg}}{\text{s}}\right)\left(762.81\,\frac{\text{kJ}}{\text{kg}} - 209.33\,\frac{\text{kJ}}{\text{kg}}\right)}{2827.9\,\frac{\text{kJ}}{\text{kg}} - 762.81\,\frac{\text{kJ}}{\text{kg}}} = \underline{\mathbf{16.08\ kg/s}}$$

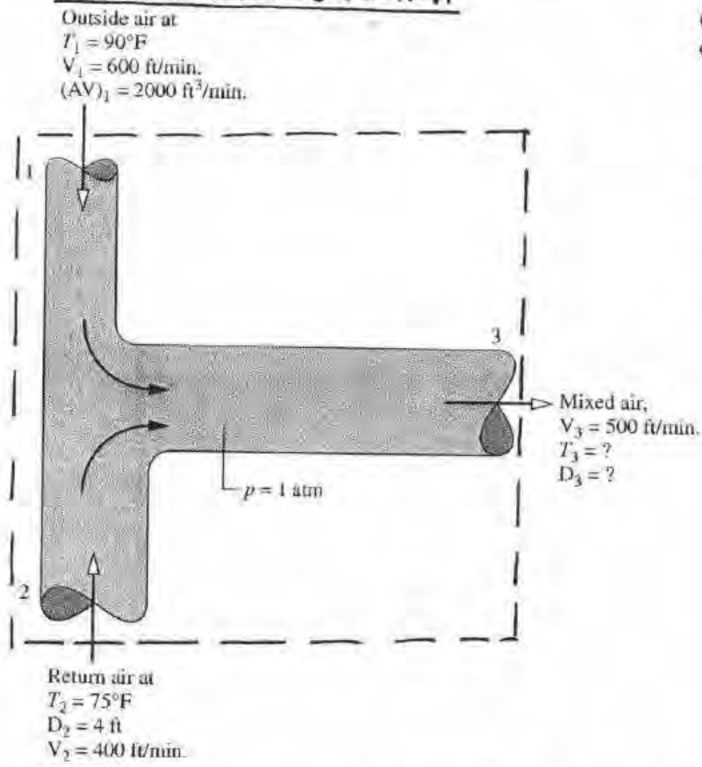
The *T-v* diagram for the three state points is shown below.



PROBLEM 4.84

Figure P4.84 provides steady-state data for the ducting ahead of the chiller coils in an air conditioning system. Outside air at 90°F is mixed with return air at 75°F. Stray heat transfer is negligible, kinetic and potential energy effects can be ignored, and the pressure throughout is 1 atm. Modeling the air as an ideal gas with $c_p = 0.24$ Btu/lb · R, determine (a) the mixed-air temperature, in °F, and (b) the diameter of the mixed-air duct, in ft.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. Stray heat transfer and kinetic and potential energy effects are negligible. $\dot{W}_{cv} = 0$.
3. The air is modeled as an ideal gas with $c_p = 0.24$ Btu/lb.

ANALYSIS: Mass rate balance: $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$, where

$$\dot{m}_1 = \frac{(AV)_1}{v_1} = \frac{p_1 (AV)_1}{RT_1} = \frac{\left(14.7 \times 144 \frac{lbf}{ft^2}\right)\left(2000 \frac{ft^3}{min}\right)}{\left(\frac{1545}{28.97} \frac{ft \cdot lbf}{lb \cdot °R}\right)(550°R)} = 144.33 \frac{lb}{min}$$

$$\dot{m}_2 = \frac{A_2 V_2}{v_2} = \frac{\left(\frac{\pi D_2^2}{4}\right)(V_2)}{\bar{R}T_2/p_2} = \frac{p_2 \left(\frac{\pi D_2^2}{4}\right)(V_2)}{\bar{R}\,T_2} = \frac{\left(14.7 \times 144 \frac{lbf}{ft^2}\right)\left(\frac{\pi}{4}(4\,ft)^2\right)\left(400 \frac{ft}{min}\right)}{\left(\frac{1545}{28.97} \frac{ft \cdot lbf}{lb \cdot °R}\right)(535°R)} = 372.92 \frac{lb}{min}$$

(a) Energy rate balance:

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1\left(h_1 + \cancel{\frac{V_1^2}{2}} + \cancel{gz_1}\right) + \dot{m}_2\left(h_2 + \cancel{\frac{V_2^2}{2}} + \cancel{gz_2}\right) - (\dot{m}_1 + \dot{m}_2)\left(h_3 + \cancel{\frac{V_3^2}{2}} + \cancel{gz_3}\right)$$

$$\Rightarrow \quad 0 = \dot{m}_1[h_1 - h_3] + \dot{m}_2[h_2 - h_3] \Rightarrow 0 = \dot{m}_1 c_p [T_1 - T_3] + \dot{m}_2 c_p [T_2 - T_3]$$

$$\Rightarrow \quad T_3 = \frac{\dot{m}_1 T_1 + \dot{m}_2 T_2}{\dot{m}_1 + \dot{m}_2} = \frac{(144.33)(550°R) + (372.92)(535°R)}{(144.33 + 372.92)} = 539°R\,(79°F) \qquad \text{(a)}$$

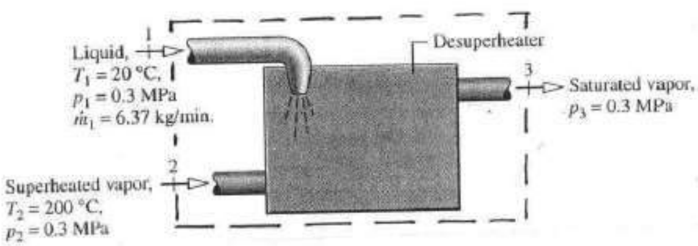
(b) $\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 517.25 \frac{lb}{min}$. Also, $\dot{m}_3 = \frac{A_3 V_3}{v_3} = \frac{A_3 V_3 \, p_3}{R\,T_3}$ and $A_3 = \frac{\pi D_3^2}{4}$. Collecting results

$$D_3 = \sqrt{\frac{4}{\pi}\frac{\dot{m}_3 R T_3}{p_3 V_3}} = \sqrt{\frac{4}{\pi}\frac{\left(517.25 \frac{lb}{min}\right)\left(\frac{1545}{28.97} \frac{ft \cdot lbf}{lb \cdot °R}\right)(539°R)}{(14.7 \times 144\ lbf/ft^2)(500\ ft/min)}} = 4.23\ ft \qquad \text{(b)}$$

Problem 4.85

For the *desuperheater* shown in Fig. P4.82, liquid water at state 1 is injected into a stream of superheated vapor entering at state 2. As a result, saturated vapor exits at state 3. Data for steady state operation are shown on the figure. Ignoring stray heat transfer and kinetic and potential energy effects, determine the mass flow rate of the incoming superheated vapor, in kg/min.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer and kinetic and potential energy effects can be ignored. $\dot{W}_{cv} \equiv 0$.
3. At state 1, $h_1 \approx h_f(T_1)$.

ANALYSIS:

Mass rate balance: $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$ (1)

Energy rate balance,

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1\left[h_1 + \cancel{\frac{V_1^2}{2}} + \cancel{gz_1}\right] + \dot{m}_2\left[h_2 + \cancel{\frac{V_2^2}{2}} + \cancel{gz_2}\right] - \dot{m}_3\left[h_3 + \cancel{\frac{V_3^2}{2}} + \cancel{gz_3}\right]$$

$$\Rightarrow 0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3 \qquad (2)$$

Combining Eqs. (1), (2)

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3$$

$$\Rightarrow \dot{m}_2 = \dot{m}_1 \left[\frac{h_3 - h_1}{h_2 - h_3}\right] \qquad (3)$$

From Table A-2, $h_1 \approx 83.96$ kJ/kg. From Table A-3, $h_3 = 2725.3$ kJ/kg. From Table A-4, $h_2 = 2865.5$ kJ/kg. Inserting values in Eq. (3),

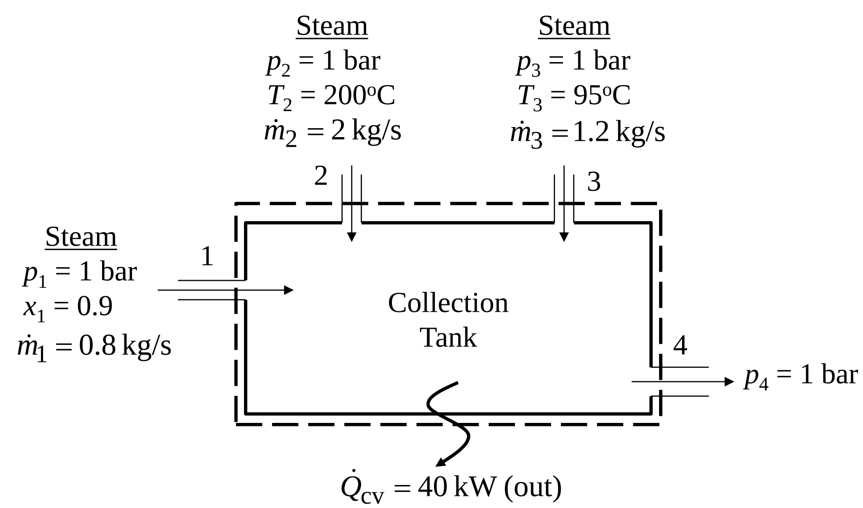
$$\dot{m}_2 = 6.37 \frac{kg}{min}\left[\frac{2725.3 - 83.96}{2865.5 - 2725.3}\right] = 120.01 \frac{kg}{min}$$

4.86 Three return steam lines in a chemical processing plant enter a collection tank operating at steady state at 1 bar. Steam enters inlet 1 with flow rate of 0.8 kg/s and quality of 0.9. Steam enters inlet 2 with flow rate of 2 kg/s at 200°C. Steam enters inlet 3 with flow rate of 1.2 kg/s at 95°C. Steam exits the tank at 1 bar. The rate of heat transfer from the collection tank is 40 kW. Neglecting kinetic and potential energy effects, determine for the steam exiting the tank
(a) the mass flow rate, in kg/s.
(b) the temperature, in °C.

KNOWN: Three return steam lines enter a collection tank with one exit.

FIND: For the exiting steam (a) the mass flow rate, in kg/s, and (b) the temperature, in °C.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown with the schematic is at steady state.
2. For the control volume, kinetic and potential energy effects can be ignored and $\dot{W}_{cv} = 0$.

ANALYSIS:
(a) The steady state form of the mass rate balance

$$\sum_i \dot{m}_i = \sum_e \dot{m}_e$$

reduces to

$$\dot{m}_1 + \dot{m}_2 + \dot{m}_3 = \dot{m}_4$$

Substituting values gives

$$\dot{m}_4 = 0.8 \text{ kg/s} + 2 \text{ kg/s} + 1.2 \text{ kg/s} = \mathbf{4\ kg/s}$$

(b) The steady state form of the energy rate balance

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \sum_i \dot{m}_i\left(h_i + \cancel{\frac{V_i^2}{2}} + \cancel{gz_i}\right) - \sum_e \dot{m}_e\left(h_e + \cancel{\frac{V_e^2}{2}} + \cancel{gz_e}\right)$$

simplifies to

$$0 = \dot{Q}_{cv} + \dot{m}_1 h_1 + \dot{m}_2 h_2 + \dot{m}_3 h_3 - \dot{m}_4 h_4$$

Solving for exit enthalpy, h_4, gives

$$h_4 = \frac{\dot{Q}_{cv} + \dot{m}_1 h_1 + \dot{m}_2 h_2 + \dot{m}_3 h_3}{\dot{m}_4}$$

Specific enthalpy of the steam at inlet 1 is determined using quality

$$h_1 = h_{f1} + x_1 h_{fg1}$$

From Table A-3 at p_1 = 1 bar, h_{f1} = 417.46 kJ/kg and h_{fg1} = 2258.0 kJ/kg. Solving

$$h_1 = 417.46 \text{ kJ/kg} + (0.9)(2258.0\text{kJ/kg}) = 2449.66 \text{ kJ/kg}$$

Since steam at inlet 2 is superheated vapor, specific enthalpy, h_2, is determined from Table A-4:

$$h_2 = 2875.3 \text{ kJ/kg}$$

Since steam at inlet 3 is compressed liquid, specific enthalpy, h_3, is determined using Table A-2 by

$$h_3 \approx h_{f3} = 397.96 \text{ kJ/kg}$$

Since heat transfer is from the control volume, $\dot{Q}_{cv} = -40\,\text{kW}$. Substituting values into the energy rate balance and solving for the exit specific enthalpy yield

$$h_4 = \frac{(-40\,\text{kW})\left|\frac{1\frac{\text{kJ}}{\text{s}}}{1\,\text{kW}}\right| + \left(0.8\frac{\text{kg}}{\text{s}}\right)\left(2449.66\frac{\text{kJ}}{\text{kg}}\right) + \left(2\frac{\text{kg}}{\text{s}}\right)\left(2875.3\frac{\text{kJ}}{\text{kg}}\right) + \left(1.2\frac{\text{kg}}{\text{s}}\right)\left(397.96\frac{\text{kJ}}{\text{kg}}\right)}{4\frac{\text{kg}}{\text{s}}}$$

$$h_4 = 2036.97 \text{ kJ/kg}$$

From Table A-3 at p_4 = 1 bar, h_{f4} = 417.46 kJ/kg and h_{g4} = 2675.5 kJ/kg. Since, $h_{f4} \le h_4 \le h_{g4}$, state 4 is a two-phase, liquid-vapor mixture and $T_4 = T_{sat4}$ at p_4 = 1 bar. From Table A-3,

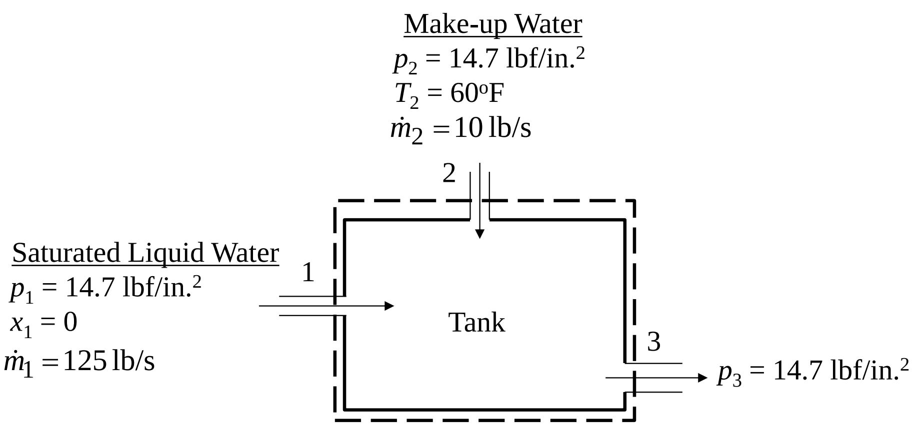
$T_4 = 99.63^{\circ}$C

4.87 A well-insulated tank in a vapor power plant operates at steady state. Water enters at inlet 1 at a rate of 125 lb/s at 14.7 lbf/in.^{2} Make-up water to replenish steam losses from the plant enters at inlet 2 at a rate of 10 lb/s at 14.7 lbf/in.^{2} and 60^{o}F. Water exits the tank at 14.7 lbf/in.^{2} Neglecting kinetic and potential energy effects, determine for the water exiting the tank
(a) the mass flow rate, in lb/s.
(b) the specific enthalpy, in Btu/lb.
(c) The temperature, in ^{o}F.

KNOWN: Two water lines enter a well-insulated tank with one exit.

FIND: For the exiting water (a) the mass flow rate, in lb/s, and (b) the specific enthalpy, in Btu/lb.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown with the schematic is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$, $\dot{Q}_{cv} = 0$, and kinetic and potential energy effects can be ignored.

ANALYSIS:
(a) The steady state form of the mass rate balance

$$\sum_i \dot{m}_i = \sum_e \dot{m}_e$$

reduces to

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

Substituting values gives

$$\dot{m}_3 = 125 \text{ lb/s} + 10 \text{ lb/s} = \mathbf{135 \text{ lb/s}}$$

(b) The steady state form of the energy rate balance

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \sum_i \dot{m}_i \left(h_i + \cancel{\frac{V_i^2}{2}} + \cancel{gz_i} \right) - \sum_e \dot{m}_e \left(h_e + \cancel{\frac{V_e^2}{2}} + \cancel{gz_e} \right)$$

simplifies to

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$$

Solving for exit enthalpy, h_3, gives

$$h_3 = \frac{\dot{m}_1 h_1 + \dot{m}_2 h_2}{\dot{m}_3}$$

Specific enthalpy of saturated liquid water at inlet 1 is obtained from Table A-3E at p_1 = 14.7 lbf/in.^{2}

$$h_1 = h_{f1} = 180.15 \text{ Btu/lb}$$

Make-up water at inlet 2 is a liquid. Thus, specific enthalpy, h_2, is

$$h_2 \approx h_f\,(60^{o}\text{F}) = 28.08 \text{ Btu/lb}$$

Substituting values into the energy rate balance and solving for the exit specific enthalpy yield

$$h_3 = \frac{\left(125 \frac{\text{lb}}{\text{s}}\right)\left(180.15 \frac{\text{Btu}}{\text{lb}}\right) + \left(10 \frac{\text{lb}}{\text{s}}\right)\left(28.08 \frac{\text{Btu}}{\text{lb}}\right)}{135 \frac{\text{lb}}{\text{s}}}$$

h_3 = **168.89 Btu/lb**

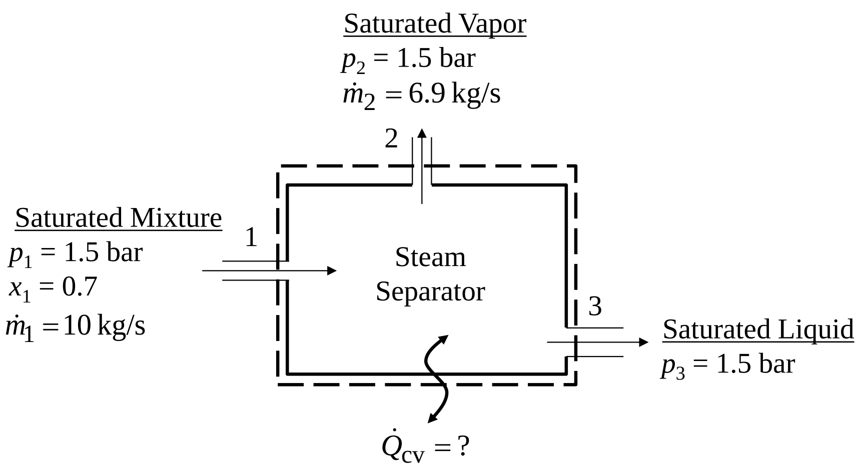
(c) Since, $h_3 < h_{f4}$, state 3 is a compressed liquid. The corresponding temperature can be determined from the approximation $h_3 \approx h_{f3}$ at T_3. Interpolating in Table A-2E, ***T_3* = 200.8^{o}C**.

4.88 Steam with a quality of 0.7, pressure of 1.5 bar, and flow rate of 10 kg/s enters a steam separator operating at steady state. Saturated vapor at 1.5 bar exits the separator at state 2 at a rate of 6.9 kg/s while saturated liquid at 1.5 bar exits the separator at state 3. Neglecting kinetic and potential energy effects, determine the rate of heat transfer, in kW, and its associated direction.

KNOWN: A steam separator operates with one inlet and two exits.

FIND: The rate of heat transfer, in kW, and its associated direction.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown with the schematic is at steady state.
2. For the control volume, kinetic and potential energy effects can be ignored, and $\dot{W}_{cv} = 0$.

ANALYSIS:
The steady state form of the mass rate balance

$$\sum_i \dot{m}_i = \sum_e \dot{m}_e$$

reduces to

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$$

Solving for $\dot{m}_3$ and substituting values give

$$\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 10 \text{ kg/s} - 6.9 \text{ kg/s} = 3.1 \text{ kg/s}$$

The steady state form of the energy rate balance

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \sum_i \dot{m}_i \left(h_i + \cancel{\frac{V_i^2}{2}} + \cancel{gz_i} \right) - \sum_e \dot{m}_e \left(h_e + \cancel{\frac{V_e^2}{2}} + \cancel{gz_e} \right)$$

simplifies to

$$0 = \dot{Q}_{cv} + \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3$$

Solving for rate of heat transfer gives

$$\dot{Q}_{cv} = \dot{m}_2 h_2 + \dot{m}_3 h_3 - \dot{m}_1 h_1$$

Specific enthalpy of the steam at inlet 1 is determined using quality

$$h_1 = h_{f1} + x_1 h_{fg1}$$

From Table A-3 at p_1 = 1.5 bar, h_{f1} = 467.11 kJ/kg and h_{fg1} = 2226.5 kJ/kg. Solving

$$h_1 = 467.11 \text{ kJ/kg} + (0.7)(2226.5 \text{ kJ/kg}) = 2025.66 \text{ kJ/kg}$$

At exit 2 steam is saturated vapor. Specific enthalpy, h_2, is determined from Table A-3 at p_2 = 1.5 bar:

$$h_2 = h_{g2} = 2693.6 \text{ kJ/kg}$$

At exit 3 steam is saturated liquid. Specific enthalpy, h_3, is determined from Table A-3 at p_3 = 1.5 bar:

$$h_3 = h_{f3} = 467.11 \text{ kJ/kg}$$

Substituting values into the energy rate balance and solving for rate of heat transfer yield

$$\dot{Q}_{cv} = \left[\left(6.9 \frac{\text{kg}}{\text{s}} \right) \left(2693.6 \frac{\text{kJ}}{\text{kg}} \right) + \left(3.1 \frac{\text{kg}}{\text{s}} \right) \left(467.11 \frac{\text{kJ}}{\text{kg}} \right) - \left(10 \frac{\text{kg}}{\text{s}} \right) \left(2025.66 \frac{\text{kJ}}{\text{kg}} \right) \right] \left| \frac{1 \text{ kW}}{1 \frac{\text{kJ}}{\text{s}}} \right|$$

$$\dot{Q}_{cv} = \mathbf{-222.7 \text{ kW}}$$

Since the rate of heat transfer has a negative sign, heat transfer is from the steam separator to the surroundings.

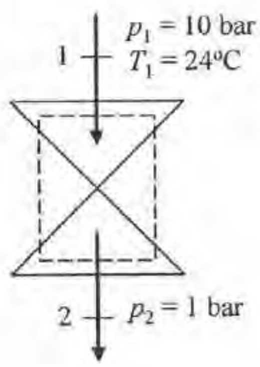
PROBLEM 4.89

4.89 Ammonia enters the expansion valve of a refrigeration system at a pressure of 10 bar and a temperature of 24°C and exits at 1 bar. If the refrigerant undergoes a throttling process, what is the quality of the refrigerant exiting the expansion valve?

KNOWN: Ammonia enters an expansion valve at given pressure and temperature and exits at given pressure.

FIND: Determine the quality of the refrigerant at the exit.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown on the accompanying figure is at steady state.
2. Heat transfer and kinetic and potential energy effects can be neglected.
3. $\dot{W}_{cv} = 0$.

ANALYSIS:
The steady-state, one-inlet, one-exit energy balance gives

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\,[(h_1 - h_2) + \tfrac{1}{2}(V_1^2 - V_2^2) + g(z_1 - z_2)]$$

Neglecting heat transfer and kinetic and potential energy effects and recognizing no work $\dot{W}_{cv}$ is associated with an expansion valve, the energy balance simplifies to

$$0 = h_1 - h_2$$

Solving for the exit enthalpy gives

$$h_2 = h_1$$

at 24°C

The ammonia at State 1 is compressed liquid. From Table A-13, $h_1 \approx h_{f1} = 293.45$ kJ/kg.

Thus, $$h_2 = 293.45 \text{ kJ/kg}.$$

The quality at state 2 can be determined from the relation

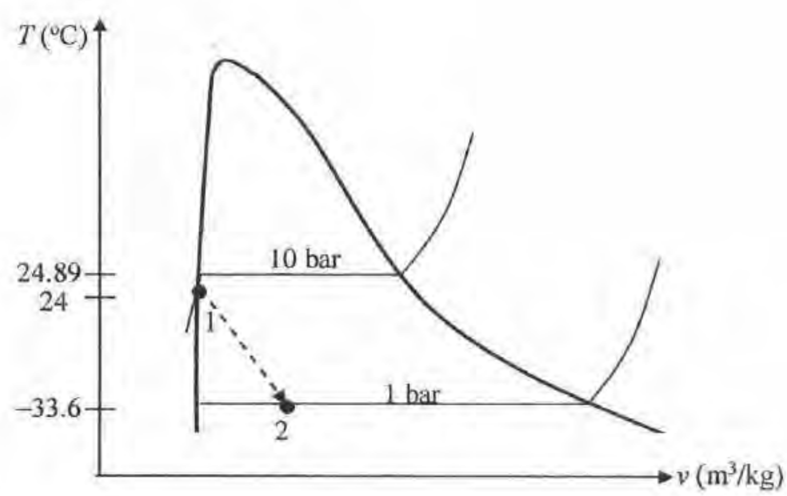
PROBLEM 4.89 (Continued)

$$x_2 = \frac{h_2 - h_{f2}}{h_{fg2}}$$

From Table A-14, $h_{f2} = 28.18$ kJ/kg and $h_{fg2} = 1370.23$ kJ/kg. Substituting in the quality relation gives

$$x_2 = \frac{293.45\dfrac{\text{kJ}}{\text{kg}} - 28.18\dfrac{\text{kJ}}{\text{kg}}}{1370.23\dfrac{\text{kJ}}{\text{kg}}} = \underline{\mathbf{0.1936}}$$

The *T*-*v* diagram for the process is shown below.



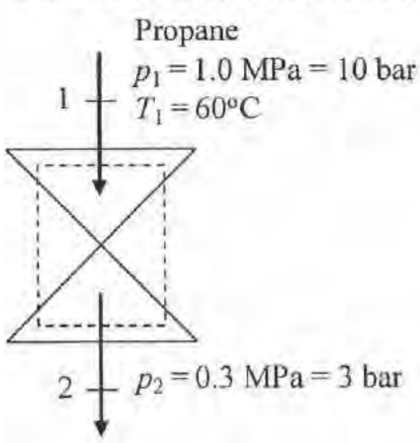
PROBLEM 4.90

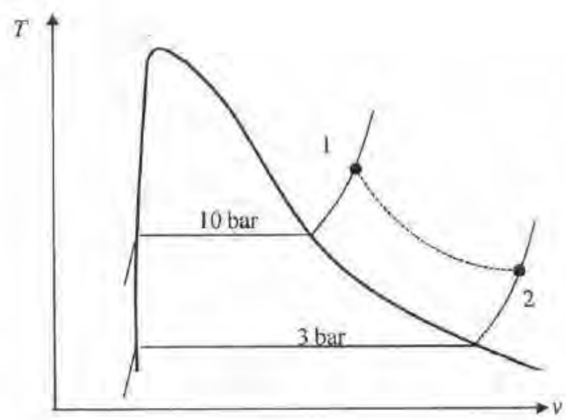
4.90 Propane vapor enters a valve at 1.0 MPa, 60°C, and leaves at 0.3 MPa. If the propane undergoes a throttling process, what is the temperature of the propane leaving the valve, in °C?

KNOWN: Propane expands through a valve from a known inlet state to a known exit pressure.

FIND: Determine the temperature of the propane leaving the valve, in °C.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL:

(1) The control volume shown in the accompanying schematic operates at steady state.
(2) Heat transfer from the valve is negligible, and $\dot{W}_{cv} = 0$.
(3) Potential and kinetic energy changes from inlet to exit can be neglected.
(4) Throttling process occurs through the expansion valve ($h_1 = h_2$).

ANALYSIS:

In accordance with assumptions for a throttling process, $h_1 = h_2$.
Using data from Table A-18: $h_1 = 565.2$ kJ/kg $= h_2$. Interpolating in Table A-18 at $p_2 = 3$ bar, $h_2 = 568.2$ kJ/kg:

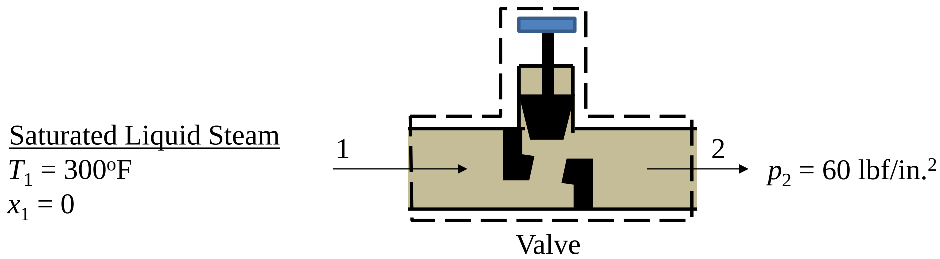
$T_2 \approx 50.2$°C

4.91 Steam enters a partially-open valve operating at steady state as saturated liquid at 300°F and exits at 60 lbf/in.^{2} Neglecting kinetic and potential energy effects and any stray heat transfer with the surroundings, determine the temperature, in °F, and the quality of the steam exiting the valve.

KNOWN: Steam flows through a partially-open valve.

FIND: The temperature, in °F, and the quality of the steam exiting the valve.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown with the schematic is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$, $\dot{Q}_{cv} = 0$, Δke = 0, and Δpe = 0.

ANALYSIS:
The energy rate balance

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}[(h_1 - h_2) + \tfrac{1}{2}(V_1^2 \cancel{-} V_2^2) + g(z_1 \cancel{-} z_2)]$$

simplifies to

$$h_2 = h_1$$

Specific enthalpy of saturated liquid steam at inlet 1 is obtained from Table A-2E at T_1 = 300°F

$$h_1 = h_{f1} = 269.7 \text{ Btu/lb}$$

Thus, h_2 = 269.7 Btu/lb.

From Table A-3E at p_2 = 60 lbf/in.^{2}, h_{f2} = 262.2 Btu/lb and h_{g2} = 1178.0 kJ/kg.
Since, $h_{f2} \le h_2 \le h_{g2}$, state 2 is a two-phase, liquid-vapor mixture and $T_2 = T_{sat2}$ at p_2 = 60 lbf/in.^{2}
From Table A-3,

$$\boxed{T_2 = 292.73^{\circ}\text{C}}$$

Quality of the steam at the exit is determined using

$$x_2 = \frac{h_2 - h_{f2}}{h_{fg2}}$$

From Table A-3E at p_2 = 60 lbf/in.^{2}, h_{fg2} = 915.8 Btu/lb. Substituting values and solving for quality give

$$x_2 = \frac{269.7\,\frac{\text{Btu}}{\text{lb}} - 262.2\,\frac{\text{Btu}}{\text{lb}}}{915.8\,\frac{\text{Btu}}{\text{lb}}}$$

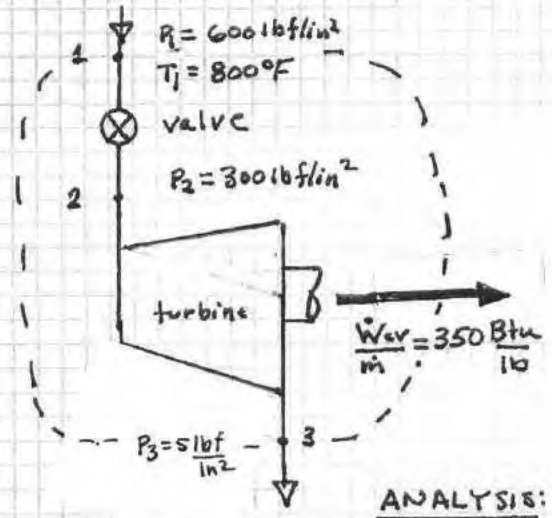
x_2 = **0.0082 (0.82%)**

PROBLEM 4.92

KNOWN: State data are provided for a throttling valve in series with a steam turbine, each at steady state. The power developed by the turbine per lb of steam flowing is specified.

FIND: Fix the state at the turbine exit.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. A control volume enclosing the valve and turbine is considered.
2. The control volume is at steady state.
3. For the control volume, $\dot{Q}_{cv}$ is negligible and the effects of kinetic and potential energy are ignored where mass crosses the boundary of the control volume.

ANALYSIS: At 3, P_3 is known. To fix the state requires an additional property value at 3. In this case, it is h_3.

To begin, mass rate balances give $\dot{m}_1 = \dot{m}_2 = \dot{m}_3\ (= \dot{m})$. An energy rate balance for the control volume reduces as follows:

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_3) + \cancel{\frac{(V_1^2 - V_3^2)}{2}} + \cancel{g(z_1 - z_3)}\right]$$

$$\Rightarrow \quad h_3 = h_1 - (\dot{W}_{cv}/\dot{m})$$

with h_1 from Table A-4E,

$$h_3 = 1407.6\,\frac{Btu}{lb} - 350\,\frac{Btu}{lb} = 1057.6\ Btu/lb$$

With data from Table A-3E at $5\,lbf/in^2$, $h_f = 130.17\ Btu/lb$, $h_g = 1131\ Btu/lb$; thus we see state 3 falls in the two-phase, liquid-vapor region. Accordingly,

$$x_3 = \frac{h_3 - h_f}{h_g - h_f} = \frac{1057.6 - 130.17}{1131 - 130.17} = 0.927\ (92.7\%)$$

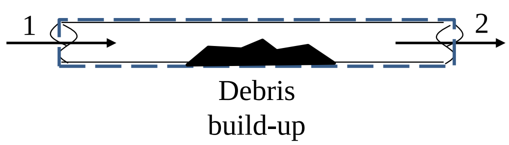
4.93 A horizontal constant-diameter pipe with a build-up of debris is shown in Fig. P4.93. Air modeled as an ideal gas enters at 320 K, 900 kPa, with a velocity of 30 m/s and exits at 305 K. Assuming steady state and neglecting stray heat transfer, determine for the air exiting the pipe section
(a) the velocity, in m/s.
(b) the pressure, in kPa.

KNOWN: Air flows through a pipe section with a build-up of debris.

FIND: For the air exiting the pipe section (a) the velocity, in m/s, and (b) the pressure, in kPa.

SCHEMATIC AND GIVEN DATA:

Air
T_1 = 320 K
p_1 = 900 kPa
V_1 = 30 m/s



T_2 = 305 K
$D_2 = D_1$

ENGINEERING MODEL:
1. The control volume shown with the schematic is at steady state.
2. Air is modeled as an ideal gas.
3. For the control volume, $\dot{W}_{cv} = 0$, $\dot{Q}_{cv} = 0$, and Δpe = 0.

ANALYSIS:
(a) The energy rate balance

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}[(h_1 - h_2) + ½\,(V_1^2 - V_2^2) + g(\cancel{z_1 - z_2})]$$

simplifies to

$$0 = (h_1 - h_2) + ½\,(V_1^2 - V_2^2)$$

Solving for exit velocity gives

$$V_2 = \sqrt{V_1^2 + 2(h_1 - h_2)}$$

From Table A-22, h_1 = 320.29 kJ/kg and h_2 = 305.22 kJ/kg. Solving for exit velocity

$$V_2 = \sqrt{\left(30\frac{\text{m}}{\text{s}}\right)^2 + 2\left(320.29\frac{\text{kJ}}{\text{kg}} - 305.22\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{1000\ \text{N}\cdot\text{m}}{1\ \text{kJ}}\right|\left|\frac{1\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}{1\ \text{N}}\right|}$$

$$V_2 = \underline{\mathbf{176.2\ m/s}}$$

(b) From the mass rate balance and ideal gas equation of state

$$\dot{m}_2 = \dot{m}_1 = \frac{A_1V_1}{v_1} = \frac{A_2V_2}{v_2} = \frac{\cancel{A_1}V_1p_1}{\cancel{R}T_1} = \frac{\cancel{A_2}V_2p_2}{\cancel{R}T_2}$$

Since the inlet and exit diameters are the same, the areas cancel as well as the gas constant. Solving for exit pressure yields

$$p_2 = p_1\left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_1}\right)$$

$$p_2 = (900\ \text{kPa})\left(\frac{30\ \text{m/s}}{176.2\ \text{m/s}}\right)\left(\frac{305\ \text{K}}{320\ \text{K}}\right)$$

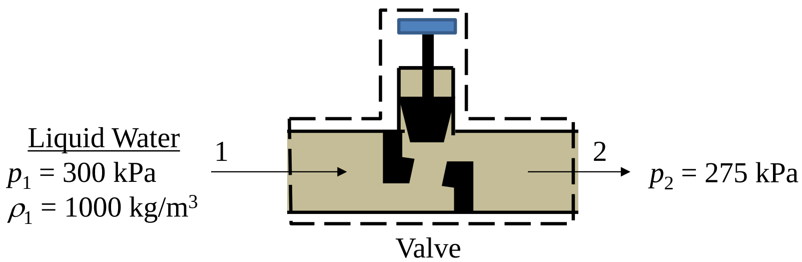
$$p_2 = \underline{\mathbf{146.1\ kPa}}$$

4.94 Liquid water enters a valve at 300 kPa and exits at 275 kPa. As water flows through the valve, the change in its temperature, stray heat transfer with the surroundings, and potential energy effects are each negligible. Operation is at steady state. Modeling the water as incompressible with constant $\rho = 1000$ kg/m^3, determine the change in kinetic energy per unit mass of water flowing through the valve, in kJ/kg.

KNOWN: Water flows through a valve.

FIND: The change in kinetic energy per unit mass of water flowing through the valve, in kJ/kg.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown with the schematic is at steady state.
2. No stray heat transfer occurs between the water and its surroundings.
3. The temperature of water does not change as it flows through the valve.
4. Model the water as an incompressible substance with constant $\rho = 1000$ kg/m^3.
5. For the control volume, Δpe = 0.

ANALYSIS:
The energy rate balance

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\left[(h_1 - h_2) + \tfrac{1}{2}(V_1^2 - V_2^2) + g\cancel{(z_1 - z_2)}\right]$$

simplifies to

$$0 = (h_1 - h_2) + \tfrac{1}{2}(V_1^2 - V_2^2)$$

The change in specific enthalpy of an incompressible substance, Eq. 3.19,

$$h_2 - h_1 = \cancel{\int_{T_1}^{T_2} c(T)\,dT} + v(p_2 - p_1)$$

simplifies to

$$h_2 - h_1 = v(p_2 - p_1)$$

because temperature does not change. Since water is modeled as incompressible, $v_2 = v_1$. Substituting $v = 1/\rho$ for specific volume, substituting for the change in specific enthalpy into the energy rate balance, and solving for the change in kinetic energy give

$$0 = v(p_1 - p_2) + \tfrac{1}{2}(V_1^2 - V_2^2)$$

$$\Delta ke = \tfrac{1}{2}(V_2^2 - V_1^2) = \frac{(p_1 - p_2)}{\rho}$$

$$\Delta ke = \frac{(300\ \text{kPa} - 275\ \text{kPa})}{\left(1000\ \frac{\text{kg}}{\text{m}^3}\right)} \left| \frac{1000\ \frac{\text{N}}{\text{m}^2}}{1\ \text{kPa}} \right| \left| \frac{1\ \text{kJ}}{1000\ \text{N} \cdot \text{m}} \right|$$

$\Delta ke =$ **0.025 kJ/kg**

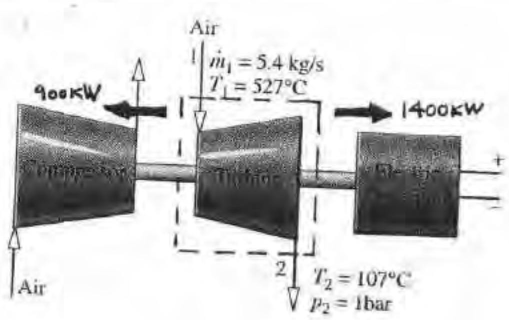
The kinetic energy of the water increases slightly as it flows from the inlet to the exit of the valve.

PROBLEM 4.95

KNOWN: Steady-state operating data are provided for a turbine powering a compressor and electric generator.

FIND: For the turbine determine (a) the volumetric flow rate of the air at the exit and (b) the heat transfer rate.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. As shown in the sketch, a control volume encloses the turbine.
2. The control volume is at steady state.
3. The air can be modeled as an ideal gas, and kinetic and potential energy changes are negligible.

ANALYSIS: (a) At steady state, $\dot{m}_1 = \dot{m}_2$. Then, with $\dot{m}_2 = (AV)_2/v_2$,

$$(AV)_2 = \dot{m}_2 v_2 = \dot{m}_2\left[\frac{RT_2}{P_2}\right] = \left(5.4\frac{kg}{s}\right)\left[\frac{\left(\frac{8314}{28.97}\frac{N \cdot m}{kg \cdot K}\right)(380\,K)}{10^5\,N/m^2}\right] = 5.89\frac{m^3}{s}$$

(b) An energy rate balance reads

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

where $\dot{W}_{cv} = 900KW + 1400KW = 2300KW$. With h_1 and h_2 from Table A-22,

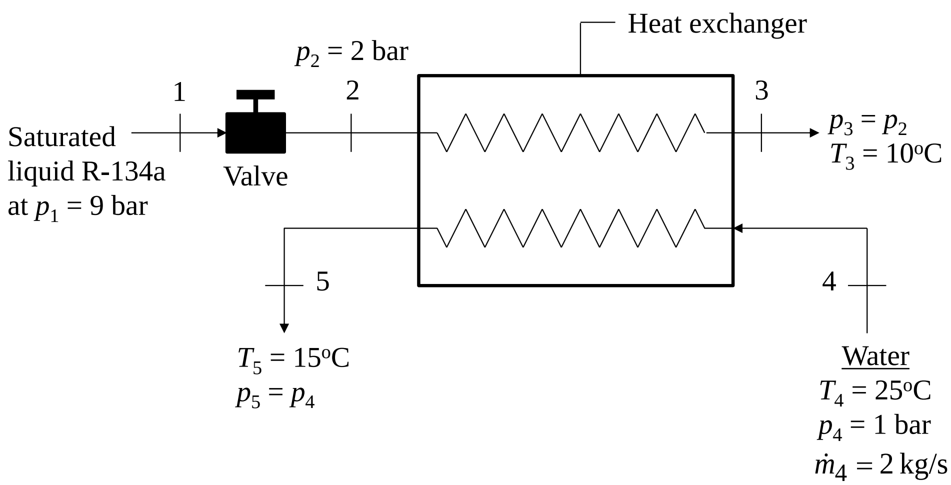
$$\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}(h_2 - h_1)$$
$$= 2300\,kW + 5.4\frac{kg}{s}\left(380.77 - 821.95\right)\frac{kJ}{kg}\left|\frac{1\,kW}{1\,kJ/s}\right|$$
$$= -82\,kW$$

4.96 Figure P4.96 provides steady-state data for a throttling valve in series with a heat exchanger. Saturated liquid Refrigerant 134a enters the valve at a pressure of 9 bar and is throttled to a pressure of 2 bar. The refrigerant then enters the heat exchanger, exiting at a temperature of 10°C with no significant decrease in pressure. In a separate stream, liquid water at 1 bar enters the heat exchanger at a temperature of 25°C with a mass flow rate of 2 kg/s and exits at 1 bar as liquid at a temperature of 15°C. Stray heat transfer and kinetic and potential energy effects can be ignored. Determine
(a) the temperature, in °C, of the refrigerant at the exit of the valve.
(b) the mass flow rate of the refrigerant, in kg/s.

KNOWN: Refrigerant 134a flows through a throttling valve in series with a heat exchanger while liquid water flows through the same heat exchanger in a separate line.

FIND: (a) the temperature, in °C, of the refrigerant at the exit of the valve and (b) the mass flow rate of the refrigerant, in kg/s.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Each component operates at steady state.
2. No stray heat transfer occurs between the components and their surroundings.
3. For both components, kinetic and potential energy effects can be ignored.
4. For both components, $\dot{W}_{cv} = 0$ and $\dot{Q}_{cv} = 0$.

ANALYSIS:
(a) The energy rate balance for the valve

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}[(h_1 - h_2) + \tfrac{1}{2}(V_1^2 \cancel{-} V_2^2) + g(z_1 \cancel{-} z_2)]$$

simplifies to

$$h_2 = h_1$$

Specific enthalpy of saturated liquid refrigerant 134a at inlet 1 is obtained from Table A-11 at

$p_1 = 9$ bar

$$h_1 = h_{f1} = 99.56 \text{ kJ/kg}$$

Thus, $h_2 = 99.56$ kJ/kg.

From Table A-11 at $p_2 = 2$ bar, $h_{f2} = 36.84$ kJ/kg and $h_{g2} = 241.30$ kJ/kg. Since, $h_{f2} \leq h_2 \leq h_{g2}$, state 2 is liquid-vapor mixture and $T_2 = T_{sat2}$ at $p_2 = 2$ bar. From Table A-11,

$$\underline{\mathbf{T_2 = -10.09^{o}C}}$$

(b) Since the refrigerant and water do not mix in the heat exchanger, the steady state mass balance reduces to

$$\dot{m}_2 = \dot{m}_3$$

$$\dot{m}_4 = \dot{m}_5 = 2 \text{ kg/s}$$

The steady state form of the energy rate balance for the heat exchanger

$$0 = \cancel{\dot{Q}_{\text{cv}}} - \cancel{\dot{W}_{\text{cv}}} + \sum_i \dot{m}_i\left(h_i + \cancel{\frac{V_i^2}{2}} + \cancel{gz_i}\right) - \sum_e \dot{m}_e\left(h_e + \cancel{\frac{V_e^2}{2}} + \cancel{gz_e}\right)$$

simplifies to

$$0 = \dot{m}_2 h_2 + \dot{m}_4 h_4 - \dot{m}_3 h_3 - \dot{m}_5 h_5$$

or substituting results from the mass balance

$$0 = \dot{m}_2(h_2 - h_3) + \dot{m}_4(h_4 - h_5)$$

Solving for the mass flow rate of refrigerant gives

$$\dot{m}_2 = \frac{\dot{m}_4(h_4 - h_5)}{(h_3 - h_2)}$$

Since the refrigerant is superheated vapor at state 3, the specific enthalpy at state 3 is determined from Table A-12: $h_3 = 258.89$ kJ/kg.

The specific enthalpies of liquid water for state 4 and state 5 are obtained from Table A-2: $h_4 \approx h_{f4} = 104.89$ kJ/kg and $h_5 \approx h_{f5} = 62.99$ kJ/kg. Substituting values gives

$$\dot{m}_2 = \frac{\left(2\frac{\text{kg}}{\text{s}}\right)\left(104.89\frac{\text{kJ}}{\text{kg}} - 62.99\frac{\text{kJ}}{\text{kg}}\right)}{(258.89\frac{\text{kJ}}{\text{kg}} - 99.56\frac{\text{kJ}}{\text{kg}})}$$

$\dot{m}_2 =$ **<u>0.526 kg/s</u>**

PROBLEM 4.97

KNOWN: Steady-state operating data are provided for a compressor in series with a condenser.

FIND: Determine the mass flow rate of the Refrigerant 22 and the compressor power.

SCHEMATIC & GIVEN DATA:

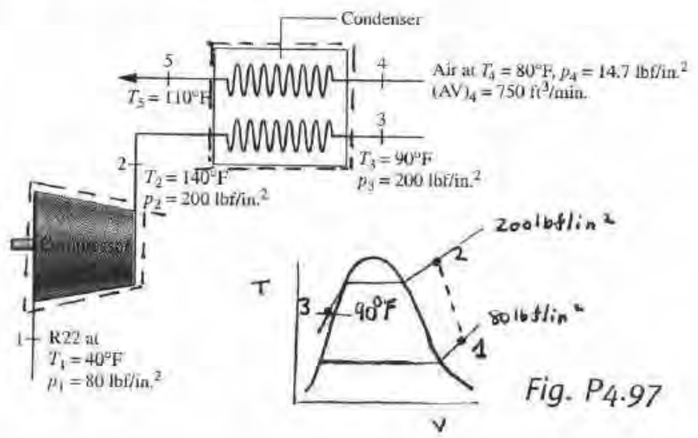


Fig. P4.97

ENGR. MODEL

1. As shown in the sketch, two control volumes are under consideration.
2. Each control volume operates at steady state.
3. For each control volume stray heat transfer and kinetic and potential energy effects can be ignored. For the condenser, $\dot{W}_{cv} = 0$.
4. The air is modeled as an ideal gas.

ANALYSIS:

(a) Mass and energy balances for the control volume enclosing the condenser read

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_R[h_2 - h_3] + \dot{m}_A[h_4 - h_5]$$

$$\Rightarrow \quad \dot{m}_R = \dot{m}_A\left[\frac{h_5 - h_4}{h_2 - h_3}\right]$$

where

$$\dot{m}_A = \frac{[AV]_4}{v_4} = \frac{p_4[AV]_4}{RT_4} = \frac{(750\ \text{ft}^3/\text{min})(14.7 \times 144\ \text{lbf/ft}^2)}{\left(\frac{1545}{28.97}\ \frac{\text{ft·lbf}}{\text{lb·°R}}\right)(540°\text{R})} = 55.13\ \frac{\text{lb}}{\text{min}}$$

$$\left(= \frac{RT_4}{p_4}\right)$$

Then, with $h_2 = 121.25$ Btu/lb from Table A-9E and $h_3 \approx h_f(90°F) = 36.32$ Btu/lb from Table A-7E, together with $h_4 = 129.06$ Btu/lb and $h_5 = 136.26$ Btu/lb from Table A-22E, we get

$$\dot{m}_R = 55.13\ \frac{\text{lb}}{\text{min}}\left[\frac{136.26 - 129.06}{121.25 - 36.32}\right] = 4.67\ \frac{\text{lb}}{\text{min}} \quad \longleftarrow \dot{m}_R$$

(b) Mass and energy rate balances for the control volume enclosing the compressor reduce to

$$\dot{W}_{cv} = \dot{m}_R[\underbrace{h_1}_{= 108.42\ \text{Btu/lb (Table A-9E)}} - h_2]$$

$$= 4.67\ \frac{\text{lb}}{\text{min}}\ (108.42 - 121.25)\ \frac{\text{Btu}}{\text{lb}}\left|\frac{60\ \text{min}}{1\ \text{h}}\right|\left|\frac{1\ \text{hp}}{2545\ \text{Btu/h}}\right| \quad \longleftarrow \dot{W}_{cv}$$

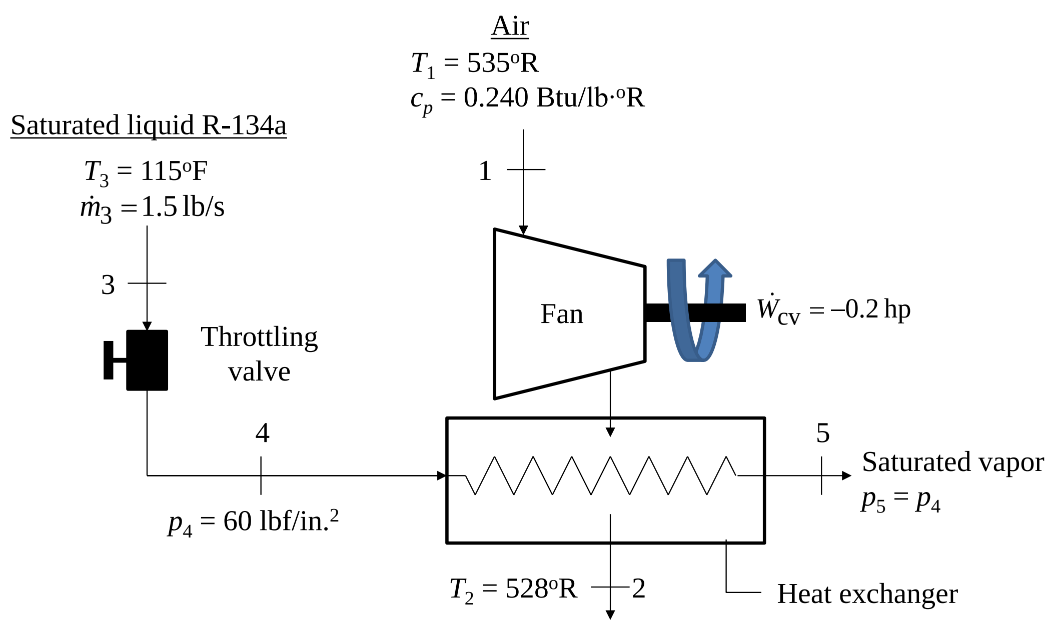
$$= -1.41\ \text{hp}$$

4.98 Figure P4.98 shows three components of an air-conditioning system. Refrigerant 134a flows through a throttling valve and a heat exchanger while air flows through a fan and the same heat exchanger. Data for steady-state operation are given on the figure. There is no significant heat transfer between any of the components and the surroundings. Kinetic and potential energy effects are negligible. Modeling air as an ideal gas with constant c_p = 0.240 Btu/lb·°R, determine the mass flow rate of the air, in lb/s.

KNOWN: An air-conditioning system section operates with refrigerant 134a flowing through a throttling value and heat exchanger and air flowing through a fan and the same heat exchanger.

FIND: The mass flow rate of the air, in lb/s.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. All components operate at steady state.
2. No stray heat transfer occurs between any of the components and their surroundings.
3. For all components, kinetic and potential energy effects can be ignored.
4. Air is modeled as an ideal gas with constant c_p = 0.240 Btu/lb·°R.
5. Define control volume A to encompass the fan and the heat exchanger.

ANALYSIS:
A mass balance for the valve gives

$$\dot{m}_4 = \dot{m}_3 = 1.5 \text{ lb/s}$$

Since the air and refrigerant do not mix in control volume A, the steady state mass balance reduces to

$$\dot{m}_4 = \dot{m}_5 = 1.5 \text{ lb/s}$$

$$\dot{m}_1 = \dot{m}_2$$

The steady state form of the energy rate balance for control volume A

$$0 = \cancel{\dot{Q}_{\text{cv}}} - \dot{W}_{\text{cv}} + \sum_i \dot{m}_i\left(h_i + \cancel{\frac{V_i^2}{2}} + \cancel{gz_i}\right) - \sum_e \dot{m}_e\left(h_e + \cancel{\frac{V_e^2}{2}} + \cancel{gz_e}\right)$$

simplifies to

$$0 = -\dot{W}_{\text{cv}} + \dot{m}_1 h_1 + \dot{m}_4 h_4 - \dot{m}_2 h_2 - \dot{m}_5 h_5$$

or substituting results from the mass balance

$$0 = -\dot{W}_{\text{cv}} + \dot{m}_1(h_1 - h_2) + \dot{m}_4(h_4 - h_5)$$

Since air is modeled as an ideal gas with constant c_p = 0.240 Btu/lb·°R,

$$h_1 - h_2 = c_p(T_1 - T_2)$$

Substituting and solving for the mass flow rate of air give

$$0 = -\dot{W}_{\text{cv}} + \dot{m}_1 c_p(T_1 - T_2) + \dot{m}_4(h_4 - h_5)$$

$$\dot{m}_1 = \frac{\dot{W}_{\text{cv}} + \dot{m}_4(h_5 - h_4)}{c_p(T_1 - T_2)}$$

The enthalpy at state 4 can be determined by analyzing the throttling valve with the energy rate balance

$$0 = \cancel{\dot{Q}_{\text{cv}}} - \cancel{\dot{W}_{\text{cv}}} + \dot{m}[(h_3 - h_4) + ½\,(V_3^2 \cancel{-} V_4^2) + g(z_3 \cancel{-} z_4)]$$

which simplifies to

$$h_4 = h_3$$

The specific enthalpy at state 3 is obtained from Table A-10E: $h_3 = h_{f3}$ = 49.63 Btu/lb. Thus,

$$h_4 = 49.63 \text{ Btu/lb}$$

Since the refrigerant is saturated vapor at state 5, the specific enthalpy at state 5 is obtained from Table A-11E: $h_5 = h_{g5}$ = 108.72 Btu/lb. Substituting values gives

$$\dot{m}_1 = \frac{(-2\,\text{hp})\left|\dfrac{2545\dfrac{\text{Btu}}{\text{h}}}{1\,\text{hp}}\right|\left|\dfrac{1\,\text{h}}{3600\,\text{s}}\right| + \left(1.5\,\dfrac{\text{lb}}{\text{s}}\right)\left(108.72\,\dfrac{\text{Btu}}{\text{lb}} - 49.63\,\dfrac{\text{Btu}}{\text{lb}}\right)}{\left(0.240\,\dfrac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)(535^\circ\text{R} - 528^\circ\text{R})}$$

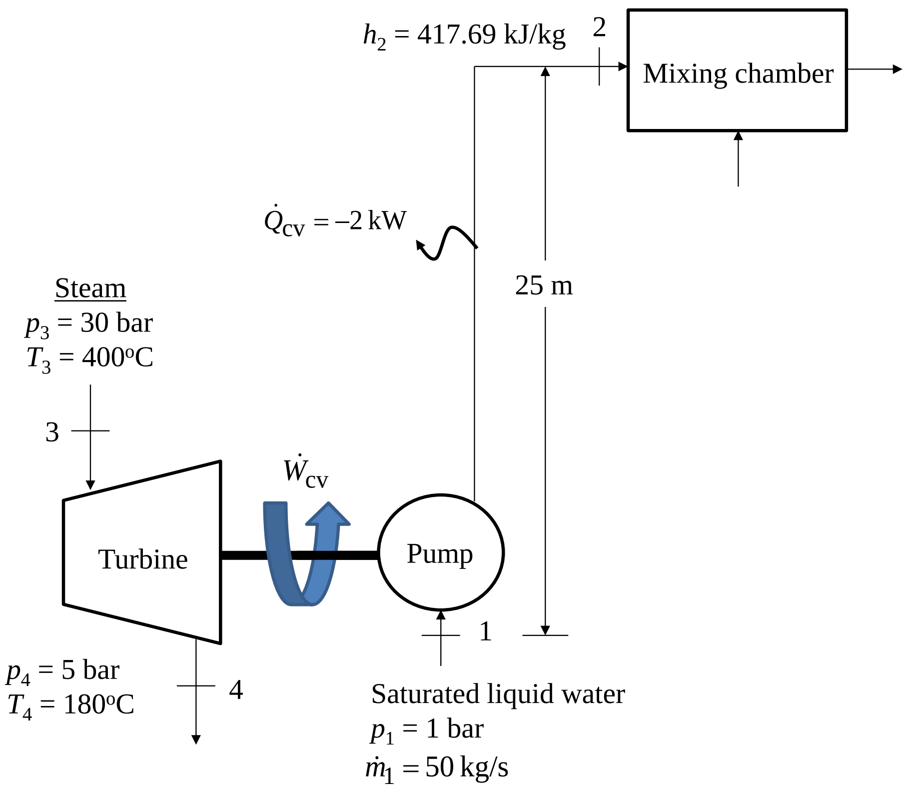
$\dot{m}_1 =$ **51.9 lb/s**

4.99 Figure P4.99 shows a turbine-driven pump that provides water to a mixing chamber located 25 m higher than the pump. Steady-state operating data for the turbine and pump are labeled on the figure. Heat transfer from the water to its surroundings occurs at a rate of 2 kW. For the turbine, heat transfer with the surroundings and potential energy effects are negligible. Kinetic energy effects at all numbered states can be ignored. Determine
(a) The power required by the pump, in kW, to supply water to the inlet of the mixing chamber.
(b) The mass flow rate of steam, in kg/s, that flows through the turbine.

KNOWN: A steam turbine drives a pump through which water flows to a mixing chamber located 25 m higher than the pump.

FIND: (a) The power required by the pump, in kW, to supply water to the inlet of the mixing chamber and (b) the mass flow rate of steam, in kg/s, that flows through the turbine.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. All components operate at steady state.
2. Define control volume A to encompass the pump and the line to the mixing chamber.
3. Define control volume B to encompass the turbine.
4. For both control volumes A and B, kinetic energy effects can be ignored.
5. No stray heat transfer occurs between control volume B and its surroundings.
6. For control volume B, potential energy effects can be ignored.

ANALYSIS:
(a) The energy rate balance for control volume A

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}[(h_1 - h_2) + ½\,(\cancel{V_1^2 - V_2^2}) + g(z_1 - z_2)]$$

simplifies to

$$\dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}_1[(h_1 - h_2) + g(z_1 - z_2)]$$

since $\dot{m}_1 = \dot{m}_2$. The specific enthalpy at state 1 is obtained from Table A-3: $h_1 = h_{f1} = 417.46$ kJ/kg. Since the elevation at state 2 is higher than the elevation at state 1, $(z_1 - z_2) = -25$ m. Substituting values and solving yield

$$\dot{W}_{cv} = -2\text{ kW} + \left(50\frac{\text{kg}}{\text{s}}\right)\left[\left(417.46\frac{\text{kJ}}{\text{kg}} - 417.69\frac{\text{kJ}}{\text{kg}}\right) + \left(9.81\frac{\text{m}}{\text{s}^2}\right)(-25\text{ m})\left|\frac{1\text{ N}}{1\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right|\left|\frac{1\text{ kJ}}{1000\text{ N}\cdot\text{m}}\right|\right]\left|\frac{1\text{ kW}}{1\frac{\text{kJ}}{\text{s}}}\right|$$

$$\dot{W}_{cv} = \mathbf{-25.8\ kW}$$

Since the value for power is negative, work is to the pump.

(b) The energy rate balance for control volume B

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}[(h_3 - h_4) + ½\,(\cancel{V_3^2 - V_4^2}) + g(\cancel{z_3 - z_4})]$$

simplifies to

$$0 = -\dot{W}_{cv} + \dot{m}_3(h_3 - h_4)$$

since $\dot{m}_3 = \dot{m}_4$. Solving for mass flow rate of steam yields

$$\dot{m}_3 = \frac{\dot{W}_{cv}}{h_3 - h_4}$$

Since both states 3 and 4 are superheated vapor, their specific enthalpies are obtained from Table A-4: $h_3 = 3230.9$ kJ/kg and $h_4 = 2812.0$ kJ/kg. Since the turbine produces the power required by the pump, $\dot{W}_{cv}\,(\text{turbine}) = -\dot{W}_{cv}\,(\text{pump}) = -(-25.8\text{ kW}) = 25.8\text{ kW}$. Substituting values and solving yield

$$\dot{m}_3 = \frac{25.8\text{ kW}}{3230.9\frac{\text{kJ}}{\text{kg}} - 2812.0\frac{\text{kJ}}{\text{kg}}}\left|\frac{1\frac{\text{kJ}}{\text{s}}}{1\text{ kW}}\right|$$

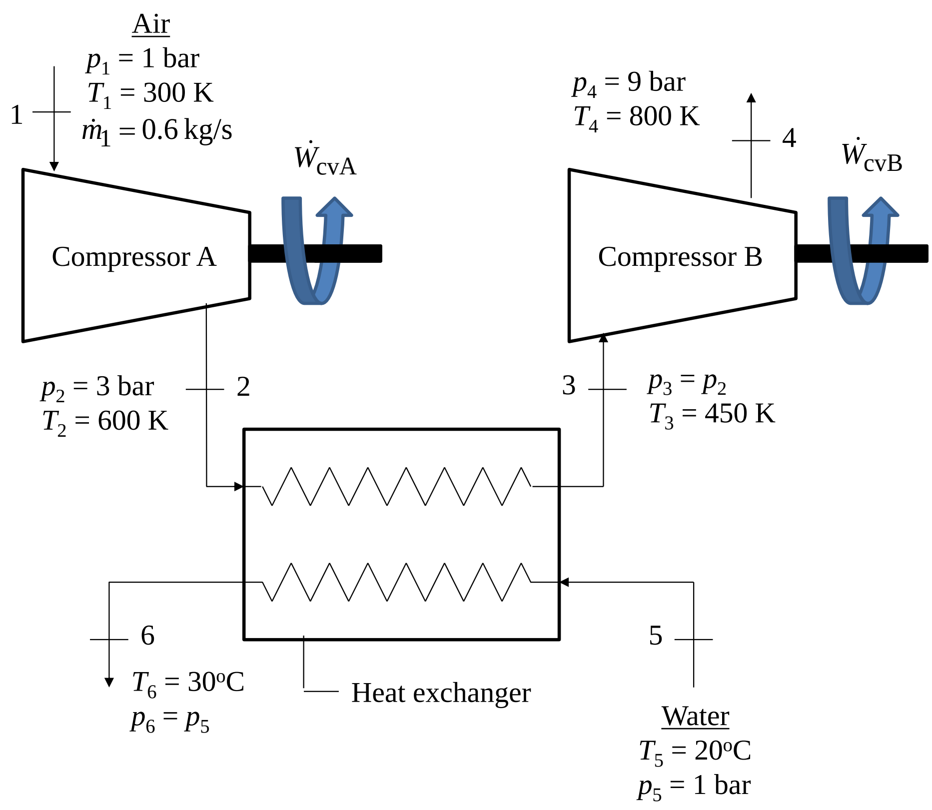
$$\dot{m}_3 = \mathbf{0.0616\ kg/s}$$

4.100 Separate streams of air and water flow through the compressor and heat exchanger arrangement shown in Fig. P4.100. Steady-state operating data are provided on the figure. Heat transfer with the surroundings can be neglected, as can all kinetic and potential energy effects. The air is modeled as an ideal gas. Determine
(a) the total power required by both compressors, in kW.
(b) the mass flow rate of the water, in kg/s.

KNOWN: Separate streams of air and water flow through a compressor and heat exchanger arrangement.

FIND: (a) The total power required by both compressors, in kW, and (b) the mass flow rate of the water, in kg/s.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Control volumes at steady state enclose the compressors and heat exchanger.
2. For each control volume, heat transfer with the surroundings is negligible and kinetic and potential effects can be ignored.
3. The air is modeled as an ideal gas.

ANALYSIS:

(a) A mass balance for the air flowing through compressor A, the heat exchanger, and compressor B gives

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}_4 = 0.6 \text{ kg/s}$$

The energy rate balance for compressor A

$$0 = \cancel{\dot{Q}_{cvA}} - \dot{W}_{cvA} + \dot{m}\,[(h_1 - h_2) + \tfrac{1}{2}\,(\cancel{V_1^2 - V_2^2}) + g(\cancel{z_1 - z_2})]$$

simplifies to

$$\dot{W}_{cvA} = \dot{m}(h_1 - h_2)$$

The specific enthalpies for air at state 1 and state 2 are obtained from Table A-22: h_1 = 300.19 kJ/kg and h_2 = 607.02 kJ/kg. Substituting values and solving yield

$$\dot{W}_{cvA} = \left(0.6\frac{\text{kg}}{\text{s}}\right)\left(300.19\frac{\text{kJ}}{\text{kg}} - 607.02\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{1\,\text{kW}}{1\,\frac{\text{kJ}}{\text{s}}}\right| = -184.1 \text{ kW}$$

Similarly for compressor B,

$$\dot{W}_{cvB} = \dot{m}(h_3 - h_4)$$

The specific enthalpies for air at state 3 and state 4 are obtained from Table A-22: h_3 = 451.80 kJ/kg and h_4 = 821.95 kJ/kg. Substituting values and solving yield

$$\dot{W}_{cvB} = \left(0.6\frac{\text{kg}}{\text{s}}\right)\left(451.80\frac{\text{kJ}}{\text{kg}} - 821.95\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{1\,\text{kW}}{1\,\frac{\text{kJ}}{\text{s}}}\right| = -222.1 \text{ kW}$$

The total power required by both compressors is

$$\dot{W}_{total} = \dot{W}_{cvA} + \dot{W}_{cvB} = (-184.1 \text{ kW}) + (-222.1 \text{ kW}) = \underline{\mathbf{-406.2\ kW}}$$

The negative sign indicates power is to the compressors.

(b) Since the air and water do not mix in the heat exchanger, the steady state mass balance reduces to

$$\dot{m}_2 = \dot{m}_3 = 0.6 \text{ kg/s}$$

$$\dot{m}_5 = \dot{m}_6$$

The steady state form of the energy rate balance

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \sum_i \dot{m}_i \left(h_i + \cancel{\frac{V_i^2}{2}} + \cancel{gz_i} \right) - \sum_e \dot{m}_e \left(h_e + \cancel{\frac{V_e^2}{2}} + \cancel{gz_e} \right)$$

simplifies to

$$0 = \dot{m}_2 h_2 + \dot{m}_5 h_5 - \dot{m}_3 h_3 - \dot{m}_6 h_6$$

or substituting results from the mass balance

$$0 = \dot{m}_2 (h_2 - h_3) + \dot{m}_5 (h_5 - h_6)$$

Solving for the mass flow rate of water gives

$$\dot{m}_5 = \frac{\dot{m}_2 (h_2 - h_3)}{h_6 - h_5}$$

The specific enthalpies of water at state 5 and state 6 are obtained from Table A-2: $h_5 \approx h_{f5}$ = 83.96 kJ/kg and $h_6 \approx h_{f6}$ = 125.79 kJ/kg. Substituting values and solving yield

$$\dot{m}_5 = \frac{\left(0.6 \frac{\text{kg}}{\text{s}}\right)\left(607.02 \frac{\text{kJ}}{\text{kg}} - 451.80 \frac{\text{kJ}}{\text{kg}}\right)}{125.79 \frac{\text{kJ}}{\text{kg}} - 83.96 \frac{\text{kJ}}{\text{kg}}}$$

$\dot{m}_5$ = **2.23 kg/s**

PROBLEM 4.101

KNOWN: Steady-state data are provided for a pumped-hydro system delivery water from a lower reservoir to a higher reservoir using off-peak electricity.

FIND: Determine the pump power required, in MW. Discuss.

SCHEMATIC & GIVEN DATA:

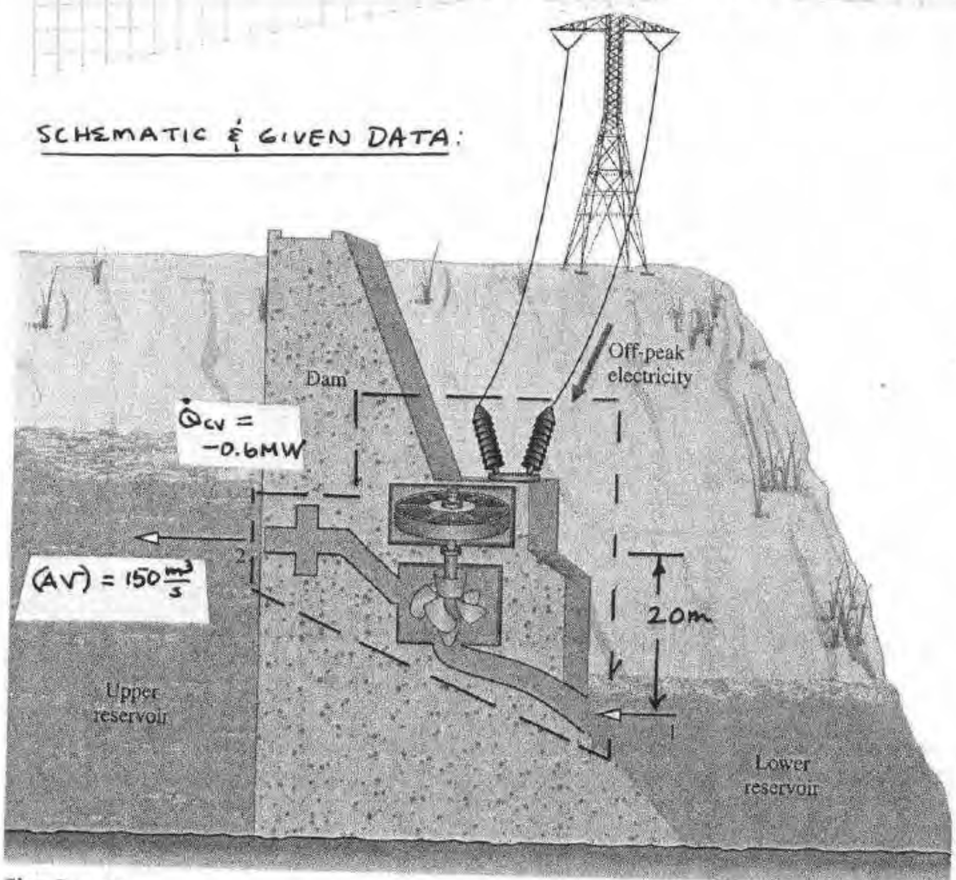


Fig. P4.101

ENGINEERING MODEL:

1. The control volume shown in the schematic is at steady state.
2. For the control volume there is no significant change in temperature, pressure, or kinetic energy from inlet to exit.
3. $g = 9.81\,m/s^2$
4. Water is modeled as incompressible with $\rho = 1000\,kg/m^3$ (Table A-19).

ANALYSIS: The mass rate balance reduces to give, $\dot{m}_1 = \dot{m}_2 \; (= \dot{m})$

The energy rate balance reduces as follows:

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[\underbrace{(h_1 - h_2)}_{=0\ (\text{Eq. 3.20b with } T_2 \sim T_1 \text{ and } P_2 \sim P_1)} + \overbrace{\frac{(V_1^2 - V_2^2)}{2}}^{0} + g(z_1 - z_2)\right]$$

$$\dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}\, g(z_1 - z_2) \quad , \text{ where } \dot{m} = \rho(AV).$$

$$= -0.6\,MW + \left[\left[\left(1000\,\frac{kg}{m^3}\right)\left(150\,\frac{m^3}{s}\right)\right]\left(9.81\,\frac{m}{s^2}\right)(-20\,m)\right]\left|\frac{1\,N}{1\,kg\cdot m/s^2}\right|\left|\frac{1\,MW}{10^6\,N\cdot m/s}\right|$$

(1) $= -30\,MW$. Thus, the power required = 30 MW

The heat transfer in this instance can be traced to frictional effects, including electrical resistance. Such effects also will be present as the control volume generates on-peak electricity using stored water. Accordingly, for the same volumetric flow rate, the rate power is developed will be less than 30 MW.

1. In the absence of heat transfer from the system the power required is 29.4 MW.

PROBLEM 4.102

KNOWN: Steady-state data are provided for a simple steam power plant.

FIND: Determine the thermal efficiency and the mass flow rate of the cooling water per kg of steam flowing.

SCHEMATIC & GIVEN DATA:

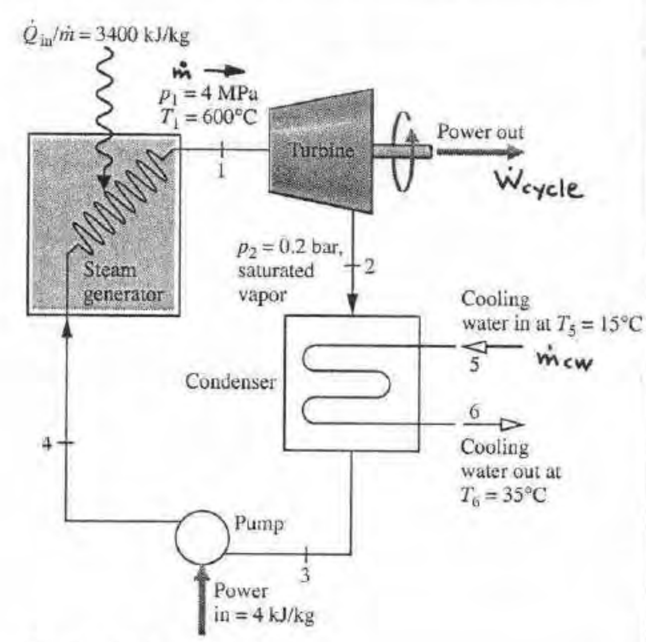


Fig. P4.102

ENGINEERING MODEL:

1. Control volumes enclosing the turbine and the overall power plant are considered.
2. The control volumes are at steady state.
3. For the control volumes, stray heat transfer and kinetic energy and potential energy effects can be ignored.
4. For the cooling water, $h \approx h_f(T)$.

ANALYSIS: (a) On a time-rate basis the thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \frac{\dot{W}_t - \dot{W}_p}{\dot{Q}_{in}}$$

Applying mass and energy balances to the turbine with assumptions listed in the model, we get $(\dot{W}_t/\dot{m}) = h_1 - h_2$. Then with data from Tables A-3, 4, $(\dot{W}_t/\dot{m}) = (3674.4 - 2609.7)$ kJ/kg $= 1064.7$ kJ/kg. With $\dot{W}_p/\dot{m} = -4$ kJ/kg and the given value for $\dot{Q}_{in}/\dot{m} = 3400$ kJ/kg, we get

$$\eta = \frac{(1064.7 - 4)\ \text{kJ/kg}}{3400\ \text{kJ/kg}} = 0.312 \quad (31.2\%)$$

(b) Applying mass and energy balances to a control volume enclosing the overall power plant with assumptions listed in the model, we get

$$0 = \dot{Q}_{in} - \dot{W}_{cycle} + \dot{m}_{cw}(h_5 - h_6) \quad \text{or} \quad 0 = \frac{\dot{Q}_{in}}{\dot{m}} - \frac{\dot{W}_{cycle}}{\dot{m}} + \frac{\dot{m}_{cw}}{\dot{m}}(h_5 - h_6)$$

Solving

$$\frac{\dot{m}_{cw}}{\dot{m}} = \frac{(\dot{Q}_{in}/\dot{m}) - (\dot{W}_{cycle}/\dot{m})}{(h_6 - h_5)} = \frac{(\dot{Q}_{in}/\dot{m}) - (\dot{W}_{cycle}/\dot{m})}{(h_f(T_6) - h_f(T_5))}$$

$$= \frac{(3400\ \text{kJ/kg}) - (1064.7 - 4)\ \text{kJ/kg}}{(146.68 - 62.99)\ \text{kJ/kg}} = \frac{2339.3\ \text{kJ/kg}}{83.69\ \text{kJ/kg}}$$

$$= 27.95\ \frac{\text{kg (cooling water)}}{\text{kg (steam)}}$$

PROBLEM 4.103

KNOWN: Steady-state data are provided for a compressor and heat exchanger in series. Two gases are involved: N_2 and helium.

FIND: Determine the mass flow rate of the helium, in kg/s.

SCHEMATIC & GIVEN DATA:

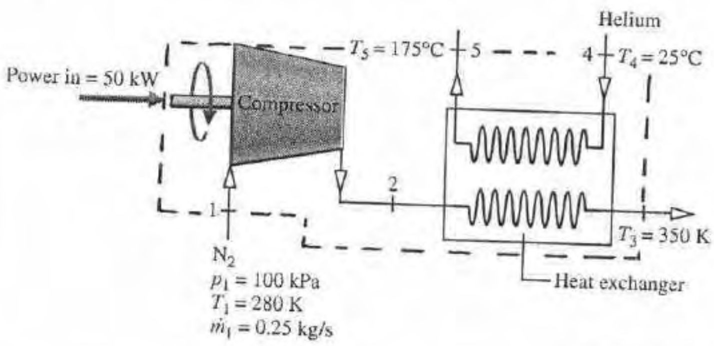


Fig. P4.103

ENGINEERING MODEL:

1. A control volume enclosing both components is considered.
2. The control volume is at steady state.
3. For the control volume, $\dot{Q}_{cv}$ is negligible and kinetic and potential energy effects are ignored.
4. The ideal gas model applies to each gas. For helium, $k = 1.67$.

ANALYSIS: Applying mass and energy rate balances to a control volume enclosing both components we get,

$$0 = \cancel{\dot{Q}_{cv}}^{\,0} - \dot{W}_{cv} + \dot{m}_1[h_1 - h_3] + \dot{m}_4[h_4 - h_5]$$

$$\Rightarrow \quad \dot{m}_4 = \frac{(-\dot{W}_{cv}) + \dot{m}_1[h_1 - h_3]}{[h_5 - h_4]}, \quad \text{where } (-\dot{W}_{cv}) = 50\,\text{kW}.$$

Evaluating properties, data from Table A-23 is used for N_2:

$$(h_1 - h_3) = \left[\frac{\bar{h}_1 - \bar{h}_3}{M}\right] = \left[\frac{(8141 - 10{,}180)\ \text{kJ/kmol}}{28.01\ \text{kg/kmol}}\right] = -72.8\ \text{kJ/kg}$$

For helium,

$$(h_5 - h_4) = c_p[T_5 - T_4] = \left(\frac{1.67}{0.67}\right)\left(\frac{8.314}{4.003}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(150\,\text{K}) = 776.5\ \text{kJ/kg}$$

$\hookrightarrow \frac{kR}{(k-1)}$ (Eq. 3.47a)

Collecting results,

$$\dot{m}_4 = \frac{50\,\text{kW}\left|\frac{1\,\text{kJ/s}}{1\,\text{kW}}\right| + (0.25\frac{\text{kg}}{\text{s}})(-72.8\,\text{kJ/kg})}{776.5\ \text{kJ/kg}}$$

$$= 0.041\ \text{kg/s}$$

PROBLEM 4.104

KNOWN: Steady-state data are provided for a cogeneration system operating with water as the working fluid.

FIND: Determine the rate of heat transfer between the system and its surroundings, in MW

SCHEMATIC & GIVEN DATA:

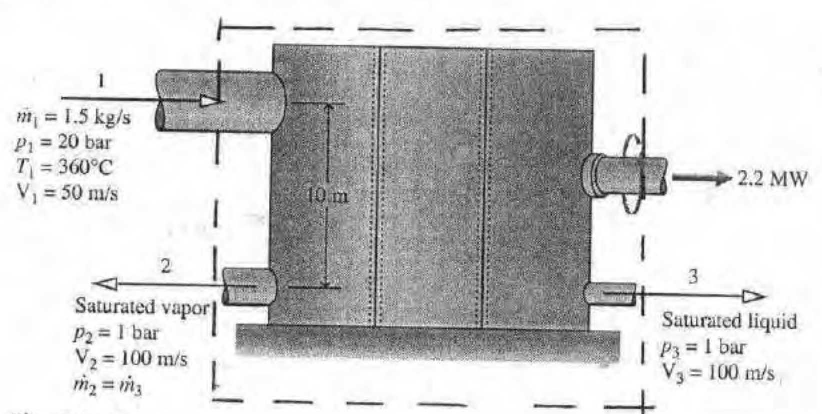


Fig. P4.104

ENGINEERING MODEL:

1. The control volume shown in the schematic is at steady state.
2. $g = 9.81 \text{ m/s}^2$

ANALYSIS: A mass rate balance reads, $\dot m_1 = \dot m_2 + \dot m_3$. Since $\dot m_2 = \dot m_3$, we get
$\dot m_2 = \dot m_3 = \dot m_1/2$

An energy rate balance reduces to give this expression:

$$0 = \dot Q_{cv} - \dot W_{cv} + \dot m_1\left[h_1 + \frac{V_1^2}{2} + gz_1\right] - \overset{(=\dot m_1/2)}{\dot m_2}\left[h_2 + \frac{V_2^2}{2} + gz_2\right] - \overset{(=\dot m_1/2)}{\dot m_3}\left[h_3 + \frac{V_3^2}{2} + gz_3\right]$$

$$0 = \dot Q_{cv} - \dot W_{cv} + \dot m_1\underbrace{\left[h_1 - \frac{h_2}{2} - \frac{h_3}{2}\right]}_{(i)} + \dot m_1\underbrace{\left[\frac{V_1^2}{2} - \frac{V_2^2}{4} - \frac{V_3^2}{4}\right]}_{(ii)} + \dot m_1 g\underbrace{\left[z_1 - \frac{z_2}{2} - \frac{z_3}{2}\right]}_{(iii)}$$

where,

(i) $\left[h_1 - \frac{h_2}{2} - \frac{h_3}{2}\right] = \left[3159.3 - \frac{2675.5}{2} - \frac{417.46}{2}\right] = 1612.82 \text{ kJ/kg}$ (Data from Tables A-3,4)

(ii) $\left[\frac{V_1^2}{2} - \frac{V_2^2}{4} - \frac{V_3^2}{4}\right] = \left[\frac{(50)^2}{2} - \frac{(100)^2}{4} - \frac{(100)^2}{4}\right]\left(\frac{m}{s}\right)^2 \left|\frac{1 N}{1 kg\cdot m/s^2}\right| \left|\frac{1 kJ}{10^3 N\cdot m}\right| = -3.75 \frac{kJ}{kg}$

(iii) $g\left[z_1 - \frac{z_2}{2} - \frac{z_3}{2}\right] = \left(9.81\frac{m}{s^2}\right)[10 m] \left|\frac{1 N}{1 kg\, m/s^2}\right| \left|\frac{1 kJ}{10^3 N\cdot m}\right| = 0.1 \frac{kJ}{kg}$

Collecting results.

$$0 = \dot Q_{cv} - 2.2 \text{ MW} + \left(1.5\frac{kg}{s}\right)\left[1612.82 - 3.75 + 0.1\right]\frac{kJ}{kg}\left|\frac{1 MW}{10^3 kJ/s}\right|$$

① $\Rightarrow \dot Q_{cv} = -0.21$ MW ←

1. The effects of kinetic and potential energy are minor in this application.

PROBLEM 4.105

KNOWN: Steady-state data are provided for three components in series: valve, flash chamber, and turbine.

FIND: Determine the power developed by the turbine, in hp.

SCHEMATIC & GIVEN DATA:

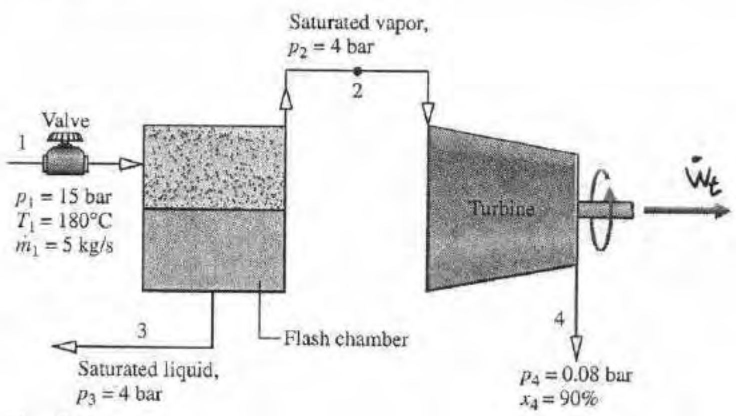


Fig. P4.105

ENGINEERING MODEL:

1. Control volumes enclosing the turbine and the valve plus flash chamber are considered.
2. The control volumes are at steady state.
3. For each control volume, $\dot{Q}_{cv}$ is negligible and kinetic and potential energy effects are ignored.

ANLYSIS: For a control volume enclosing the turbine, mass and energy rate balance combine to give, $\dot{W}_t = \dot{m}_2(h_2 - h_4)$. (1)

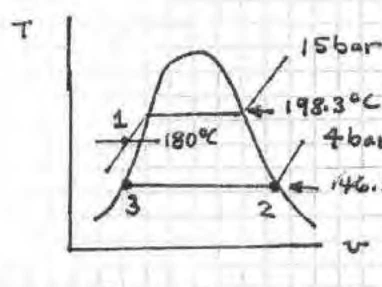
Since $\dot{m}_2$ is unknown, a control volume enclosing the valve and flash chamber is considered: The mass rate balance reduces to give $\dot{m}_1 = \dot{m}_2 + \dot{m}_3 \Rightarrow \dot{m}_3 = \dot{m}_1 - \dot{m}_2$. An energy rate balance reduces with assumptions listed as follows: $0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3$.

Then, with $\dot{m}_3 = \dot{m}_1 - \dot{m}_2$, we get

$$0 = \dot{m}_1 h_1 - \dot{m}_2 h_2 - (\dot{m}_1 - \dot{m}_2) h_3$$

Solving,

$$\dot{m}_2 = \frac{\dot{m}_1[h_1 - h_3]}{[h_2 - h_3]}$$



With $h_1 \approx h_f(180°C)$ from Table A-2, and $h_2 = h_g$ and $h_3 = h_f$ at 4 bar from Table A-3,

$$\dot{m}_2 = \frac{5\,kg/s\,[763.22 - 604.74]\,kJ/kg}{[2738.6 - 604.74]\,kJ/kg} = 0.371\,kg/s$$

In Eq. (1), h_4 is also needed: $h_4 = h_f + x(h_g - h_f)$ where h_f and h_g are at 0.08 bar.

$$\therefore\ h_4 = 173.88 + 0.9[2577 - 173.88] = 2336.69\,kJ/kg$$

Substituting values into Eq. (1),

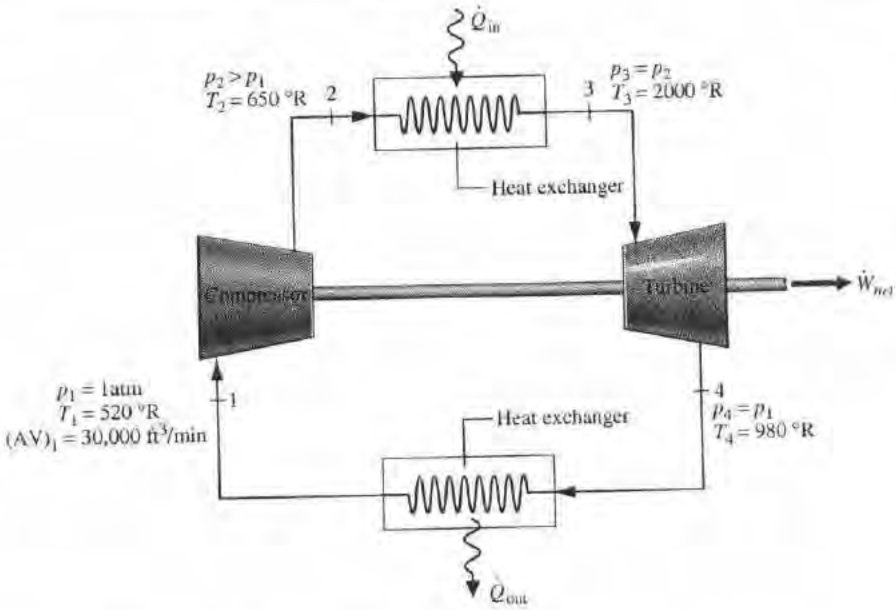
$$\dot{W}_t = \left(0.371\,\frac{kg}{s}\right)[2738.6 - 2336.69]\,\frac{kJ}{kg} = 149.1\,\frac{kJ}{s}\left|\frac{1\,kW}{1\,kJ/s}\right| = 149.1\,kW$$

$$= 149.1\,kW\left|\frac{1\,hp}{0.7457\,kW}\right| = 199.9\,hp$$

PROBLEM 4.106

A simple gas turbine power cycle operating at steady state with air as the working substance is shown in Fig. P4.106. The cycle components include an air compressor mounted on the same shaft as the turbine. The air is heated in the high-pressure heat exchanger before entering the turbine. The air exiting the turbine is cooled in the low-pressure heat exchanger before returning to the compressor. Kinetic and potential effects are negligible. The compressor and turbine are adiabatic. Using the ideal gas model for air, determine the (a) power required for the compressor, in hp, (b) power output of turbine, in hp, and (c) thermal efficiency of the cycle.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. Control volumes at steady state enclose the compressor, turbine and high-pressure heat exchanger.
2. For the compressor and turbine, $\dot{Q}_{cv} = 0$. For the heat exchanger, $\dot{W}_{cv} = 0$.
3. Kinetic and potential energy effects are negligible.
4. The air is modeled as an ideal gas.

ANALYSIS: Find the mass flow rate as follows:

$$\dot{m} = \frac{(AV)_1}{\underbrace{v_1}_{(=RT_1/p_1)}} = \frac{p_1(AV)_1}{RT_1} = \frac{(14.7 \times 144\ \frac{lbf}{in^2})(30{,}000\ ft^3/min)}{\left(\frac{1545}{28.97}\ \frac{ft\cdot lbf}{lb\cdot °R}\right)(520\ °R)}\left|\frac{60\ min}{1\ h}\right| = 137{,}394\ \frac{lb}{h}$$

(a) An energy rate balance for the compressor reduces to read

$$\dot{W}_c = \dot{m}(h_1 - h_2) = \left(137{,}394\ \frac{lb}{h}\right)(124.27 - 155.51)\frac{Btu}{lb}\left|\frac{1\ hp}{2545\ Btu/h}\right| = -1687\ hp \quad \text{(a)}$$

where specific enthalpies are from Table A-22E.

(b) An energy rate balance for the turbine reduces to read

$$\dot{W}_t = \dot{m}(h_3 - h_4) = \left(137{,}394\ \frac{lb}{h}\right)(504.71 - 236.02)\frac{Btu}{lb}\left|\frac{1\ hp}{2545\ Btu/h}\right| = 14{,}505\ hp \quad \text{(b)}$$

(c) $$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}_t - |\dot{W}_c|}{\dot{Q}_{in}}, \quad \text{where } \dot{Q}_{in} = \dot{m}(h_3 - h_2)$$

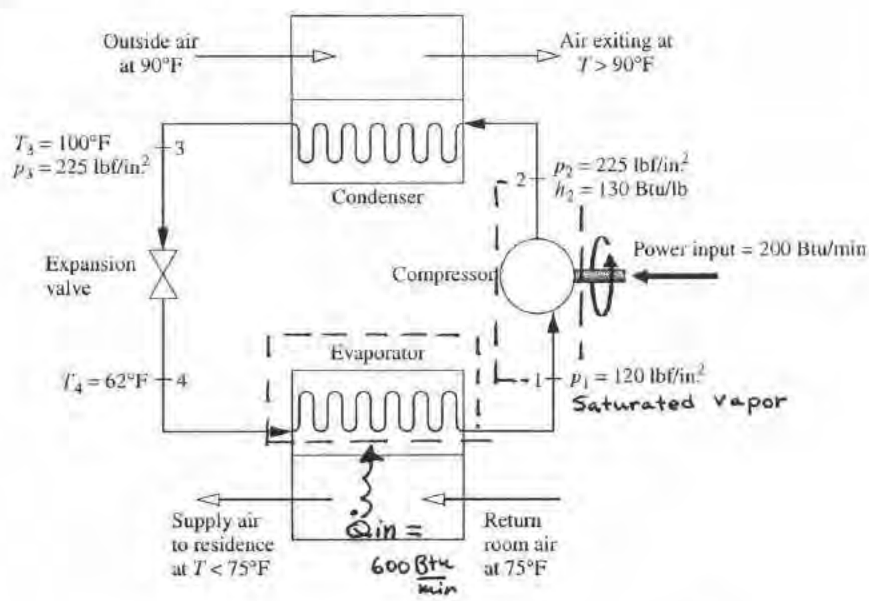
$$\therefore \quad \eta = \frac{\dot{m}(h_3 - h_4) - \dot{m}(h_2 - h_1)}{\dot{m}(h_3 - h_2)}$$

$$= \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_2)} = \frac{268.69 - 31.24}{349.20} = 0.68\ (68\%) \quad \text{(c)}$$

PROBLEM 4.107

A residential air conditioning system operates at steady state, as shown in Fig. P4.107. Refrigerant 22 circulates through the components of the system. Property data at key locations are given on the figure. If the evaporator removes energy by heat transfer from the room air at a rate of 600 Btu/min, determine (a) the rate of heat transfer between the compressor and the surroundings, in Btu/min, and (b) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. Control volumes at steady state enclose the compressor and the refrigerant side of the evaporator.
2. Kinetic and potential energy effects can be ignored. For the evaporator, $\dot{W}_{cv} = 0$.
3. The expansion through the valve is a throttling process: $h_4 = h_3$.
4. At state 3, $h_3 \approx h_f(T_3)$

ANALYSIS: (a) To find the mass flow rate of the refrigerant, write an energy rate balance for the control volume enclosing the refrigerant side of the evaporator:

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) \Rightarrow \dot{m} = \frac{\dot{Q}_{in}}{h_1 - h_4} \qquad (1)$$

From Table A-9E, h_1 = 109.88 Btu/lb (saturated vapor value). Using $h_4 = h_3$, Table A-7E gives $h_3 \approx h_f(T_3)$ = 39.41 Btu/lb. Then, Eq. (1) yields

$$\dot{m} = \frac{600 \text{ Btu/min}}{(109.88 - 39.41)} = 8.5 \frac{\text{lb}}{\text{min}}$$

An energy rate balance for the compressor reduces to give

$$\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}(h_2 - h_1) = -200 \frac{\text{Btu}}{\text{min}} + 8.5 \frac{\text{lb}}{\text{min}}(130 - 109.88)\frac{\text{Btu}}{\text{lb}}$$

$$= -29 \text{ Btu/min} \qquad \text{(a)}$$

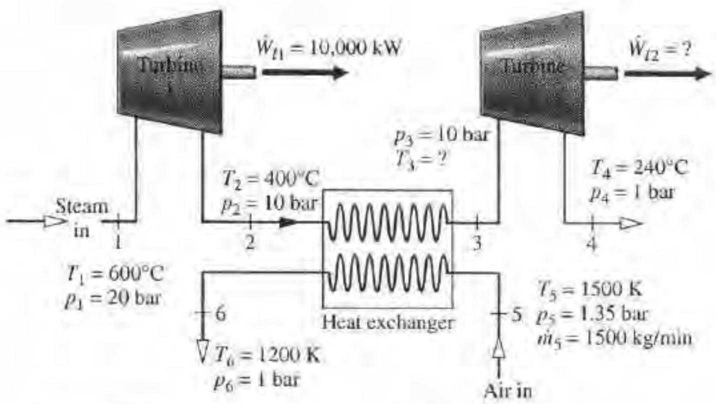
(b) The coefficient of performance is given by (see Sec. 2.6.3)

$$\beta = \frac{\dot{Q}_{in}}{|\dot{W}_{comp}|} = \frac{600 \text{ Btu/min}}{200 \text{ Btu/min}} = 3.0 \qquad \text{(b)}$$

PROBLEM 4.108

Separate streams of steam and air flow through the turbine and heat exchanger arrangement shown in Fig. P4.108. Steady-state operating data are provided on the figure. Heat transfer with the surroundings can be neglected, as can all kinetic and potential energy effects. Determine (a) T_3, in K, and (b) the power output of the second turbine, in kW.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. Consider a control volume at steady state enclosing each of the three components.
2. For each control volume, ignore stray heat transfer and kinetic and potential energy effects.
3. The ideal gas model applies for the air. (This can be verified from the compressibility chart.)

ANALYSIS:

(a) To determine the steam mass flow rate write an energy rate balance for turbine 1 and use data from Table A-4:

$$\dot{W}_{t1} = \dot{m}_1(h_1 - h_2) \Rightarrow \quad \dot{m}_1 = \frac{\dot{W}_{t1}}{h_1 - h_2} = \frac{10{,}000 \text{ kW}\left|\frac{1 \text{ kJ/s}}{1 \text{ kW}}\right|}{(3690.1 - 3263.9)\text{ kJ/kg}}$$

$$= 23.46 \text{ kg/s}$$

Next, an energy rate balance for the heat exchanger reduces to

$$0 = \underbrace{\dot{m}_2}_{=\dot{m}_1}[h_2 - h_3] + \dot{m}_5[h_5 - h_6]$$

with data from Table A-22,

$$\Rightarrow h_3 = h_2 + \frac{\dot{m}_5}{\dot{m}_2}[h_5 - h_6] = 3263.9\frac{\text{kJ}}{\text{kg}} + \frac{(1500/60)\text{ kg/s}}{23.46 \text{ kg/s}}(1635.97 - 1277.79)\frac{\text{kJ}}{\text{kg}}$$

$$= 3645.6 \frac{\text{kJ}}{\text{kg}}$$

Interpolating in Table A-4 at 10 bar gives, $T_3 = 576°\text{C}\ (849 \text{ K})$ ← (a)

(b) An energy rate balance for turbine 2 is

$$\dot{W}_{t1} = \underbrace{\dot{m}_3}_{=\dot{m}_1}(h_3 - h_4) = 23.46\frac{\text{kg}}{\text{s}}(3645.6 - \overset{\text{Table A-4}}{2954.5})\frac{\text{kJ}}{\text{kg}}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$

$$= 16{,}213 \text{ kW}$$

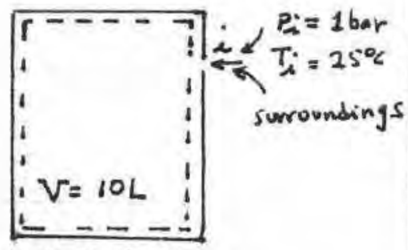
← (b)

PROBLEM 4.109

KNOWN: A pinhole develops in the wall of an initially evacuated tank, and air from the surroundings at 1 bar, 25°C enters until the pressure in the tank is 1 bar.

FIND: Determine the final temperature in the tank, in °C, and the amount of air that leaks in, in g.

SCHEMATIC & GIVEN DATA:



Tank is initially evacuated. The pressure of the air within the tank finally is 1bar

ENGINEERING MODEL

1. The control volume is defined by the dashed line on the schematic.
2. For the control volume, $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$, and kinetic and potential energy effects are negligible.
3. Air is modeled as an ideal gas with k=1.4

ANALYSIS: (a) For the one-inlet control volume the mass rate balance reads

$$\frac{dm_{cv}}{dt} = \dot{m}_i$$

The energy rate balance reduces with specified assumptions to

$$\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_i h_i$$

Combining the mass and energy rate balances, $\frac{dU_{cv}}{dt} = h_i \frac{dm_{cv}}{dt}$

Since h_i remains constant as air from the surroundings leaks into the tank, integrating with respect to time gives

$$\Delta U_{cv} = h_i \int_1^2 \frac{dm_{cv}}{dt} dt \Rightarrow \underbrace{\Delta U_{cv}}_{=(m_2u_2 - m_1u_1)} = h_i(m_2 - m_1) \rightarrow (m_2u_2 - m_1u_1) = h_i(m_2 - m_1)$$

where m_1 and m_2 denote the initial and final amounts of mass in the tank.

Thus, we get with $m_1 = 0$

$$(m_2u_2 - \cancel{m_1u_1}) = h_i(m_2 - \cancel{m_1}) \Rightarrow \boxed{u_2 = h_i} \qquad (1)$$

That is, the final specific internal energy of the air in the tank equals the specific enthalpy of the air leaking into the tank.

(1) Since $h_i = u_i + (pv)_i$, Eq. (1) becomes $\boxed{u_2 = u_i + \underline{(pv)_i}}$. (2)

Then, with the ideal gas equation of state, $(pv)_i = RT_i$ we get $(u_2 - u_i) = RT_i$. Finally, since the specific heat ratio k is constant, the specific heat c_v is also constant and is given by Eq. 3.47b: $c_v = R/(k-1)$. Accordingly,

$$(u_2 - u_i) = RT_i \Rightarrow c_v[T_2 - T_i] = RT_i \Rightarrow \frac{R}{(k-1)}[T_2 - T_i] = RT_i \Rightarrow \boxed{T_2 = kT_i} \qquad (3)$$

Inserting values, $T_2 = 1.4[298.15K] = 417.41K \; (= 144.26°C)$ ← (a)

(b) Since the tank is initially evacuated, the amount of air that leaks into the tank must equal the mass in the tank finally. That is,

$$m_2 = \frac{p_2 V}{RT_2} = \frac{(10^5 N/m^2)(10L)}{\left(\frac{8314}{28.97}\frac{N\cdot m}{kg\cdot K}\right)(417.41K)} \left|\frac{10^{-3}m^3}{1L}\right| \left|\frac{10^3 g}{1kg}\right| = 8.35g \leftarrow (b)$$

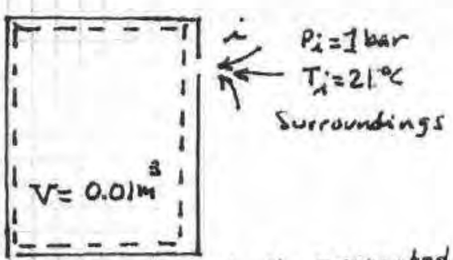
1. To interpret the temperature rise of the air within the tank during the filling process, refer to Eq. (2). The underlined term is the flow work discussed in Sec. 4.4.2. The temperature rise of the air is due to energy entering the tank at the inlet by flow work. This energy originates in the surroundings and is stored within the tank as internal energy.

PROBLEM 4.110

KNOWN: A pinhole develops in the wall of an initially evacuated tank, and air from the surroundings at 1 bar, 21°C enters until the final pressure in the tank is 1 bar. The final temperature of the air in the tank is 21°C.

FIND: Determine the final mass in the tank, in g, and the heat transfer between the tank contents and surroundings, in kJ.

SCHEMATIC & GIVEN DATA:



The tank is initially evacuated. The final condition in the tank is 1 bar, 21°C.

ENGINEERING MODEL:

1. The control volume is defined by the dashed line on the schematic.
2. For the control volume $\dot{W}_{cv} = 0$, and kinetic and potential energy effects are negligible.
3. Air is modeled as an ideal gas.

ANALYSIS: (a) Since the tank is initially evacuated, the amount of air that leaks into the tank and thus the mass in the tank finally is

$$m_2 = \frac{P_2 V}{R T_2} = \frac{(10^5 N/m^2)(0.01 m^3)}{\left(\frac{8314}{28.97} \frac{N \cdot m}{kg \cdot K}\right)(294.15 K)} \left| \frac{10^3 g}{1 kg} \right| = 11.85 g$$

(b) For the one-inlet control volume, mass and energy rate balances reduce with listed assumptions to read, respectively, $\frac{dm_{cv}}{dt} = \dot{m}_i$; $\frac{dU_{cv}}{dt} = \dot{Q}_{cv} + \dot{m}_i h_i$. Combining these expressions and integrating with respect to time,

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} + \underbrace{h_i}_{\text{remains constant as air enters}} \frac{dm_{cv}}{dt} \Rightarrow \underbrace{\Delta U_{cv}}_{(m_2 u_2 - m_1 u_1)} = Q_{cv} + h_i (m_2 - m_1)$$

, which is a special case of Eq. 4.28 in Sec. 4.12.2.

Since $m_1 = 0$, the previous expression reduces to $m_2 u_2 = Q_{cv} + m_2 h_i$, giving

$$Q_{cv} = m_2 \left[u_2 - h_i \right] \quad (1)$$

In Eq. (1), $h_i = u_i + (pv)_i$. Thus, Eq. (1) reads, $Q_{cv} = m_2[\underbrace{(u_2 - u_i)}_{= 0 \text{ (since } T_2 = T_i)} - (pv)_i]$. Finally,

① ②
$$Q_{cv} = - m_2 \left[\underline{\underline{pv}} \right] \quad (2)$$

where pv is evaluated at 21°C, 1 bar. Using the ideal gas equation of state, $pv = RT$, where $T = 21°C$ (294.15 K).

$$Q_{cv} = -m_2 [R \underbrace{T}_{= T_i = T_2}]$$

$$= -11.85 g \left| \frac{1 kg}{10^3 g} \right| \left(\frac{8.314}{28.97} \frac{kJ}{kg \cdot K} \right) (294.15 K) = -1 kJ$$

Heat transfer from the tank contents

1. The underlined term in Eq. (2) is the flow work discussed in Sec. 4.4.2. The energy entering the control volume at the inlet by flow work is eventually returned to the surroundings via heat transfer.

2. Alternatively, Eq. (2) can be expressed as $Q_{cv} = -pV$, where $p = P_i = P_2$ and V is the tank volume:

$$Q_{cv} = -pV = -\left(10^5 \frac{N}{m^2}\right)(0.01 m^3) \left| \frac{1 kJ}{10^3 N \cdot m} \right| = -1 kJ$$

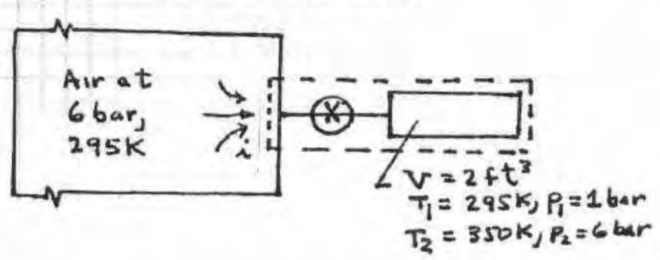
which agrees the previous result, as expected.

PROBLEM 4.111

KNOWN: A rigid tank initially containing air is connected by a valve to a large vessel holding air at a higher pressure. The valve is opened and additional air flows into the rigid tank. State data are provided.

FIND: Find the heat transfer between the tank contents and its surroundings, in kJ.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The control volume is defined by the dashed line on the schematic. The condition at i remains constant at 6bar, 295 K. The valve serves only to adjust the flow.
2. For the control volume, $\dot{W}_{cv} = 0$ and kinetic and potential energy effects are negligible.
3. Air is modeled as an ideal gas.

ANALYSIS: For the one-inlet control volume, mass and energy rate balances reduce with listed assumptions to read, respectively, $\frac{dm_{cv}}{dt} = \dot{m}_i$, $\frac{dU_{cv}}{dt} = \dot{Q}_{cv} + \dot{m}_i h_i$.

Combining these expressions and integrating with respect to time,

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} + \underset{\text{Remains constant}}{h_i}\frac{dm_{cv}}{dt} \Rightarrow \underbrace{\Delta U_{cv}}_{(m_2u_2 - m_1u_1)} = Q_{cv} + h_i[m_2 - m_1]$$

, which is a special case of Eq. 4.28 in Sec. 4.12.2.

Solving for Q_{cv},

$$Q_{cv} = (m_2u_2 - m_1u_1) - h_i(m_2 - m_1) \tag{1}$$

where

$$m_1 = \frac{P_1 V}{R T_1} = \frac{(10^5\,N/m^2)(2ft^3)}{\left(\frac{8314}{28.97}\,\frac{N\cdot m}{kg\cdot K}\right)(295K)} = 2.36\,kg, \quad m_2 = \frac{P_2 V}{R T_2} = \frac{(6\times10^5\,N/m^2)(2ft^3)}{\left(\frac{8314}{28.97}\,\frac{N\cdot m}{kg\cdot K}\right)(350K)} = 11.95\,kg$$

From Table A-22, $u_1 = 210.49\,\frac{kJ}{kg}$, $u_2 = 250.02\,\frac{kJ}{kg}$, $h_i = 295.17\,\frac{kJ}{kg}$.

Inserting values into Eq. (1)

$$Q_{cv} = \left[(11.95\,kg)(250.02\,\tfrac{kJ}{kg}) - (2.36\,kg)(210.49\,\tfrac{kJ}{kg}\right) - \left(295.17\,\tfrac{kJ}{kg}\right)(11.95 - 2.36)\,kJ$$

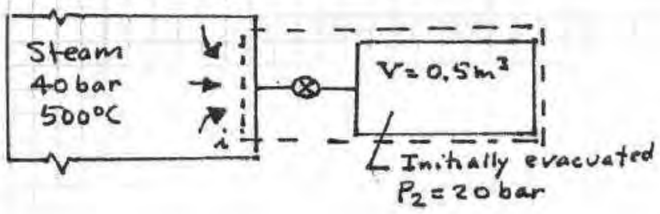
$$= -339.7\ kJ \longleftarrow$$

PROBLEM 4.112

KNOWN: An insulated, rigid tank, initially evacuated, is connected by a valve to a large vessel holding steam. The valve is opened and steam flows into the tank. State data are provided.

FIND: Determine the final temperature of the steam in the tank, in °C, and the final mass of the steam in the tank, in kg.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The control volume is defined by the dashed line on the schematic. The condition at i remains constant at 40 bar, 500°C. The valve serves only to adjust the flow.
2. For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and kinetic and potential energy effects are negligible.

ANALYSIS: For the one-inlet control volume, mass and energy rate balances reduce with listed assumptions to read, respectively, $\frac{dm_{cv}}{dt} = \dot{m}_i$, $\frac{dU_{cv}}{dt} = \dot{m}_i h_i$. Combining these expressions and integrating with respect to time,

$$\frac{dU_{cv}}{dt} = \underbrace{h_i}_{\text{Remains constant}} \frac{dm_{cv}}{dt} \Rightarrow \underbrace{\Delta U_{cv}}_{(m_2u_2 - m_1u_1)} = h_i[m_2 - m_1]$$

, which is a special case of Eq. 4.28 in Sec. 4.12.2.

(1) Since $m_1 = 0$, $m_2u_2 - \cancel{m_1}^{0} u_1 = h_i[m_2 - \cancel{m_1}^{0}] \Rightarrow \boxed{u_2 = h_i}$. That is, the final specific internal energy of the steam in the tank equals the specific enthalpy of the entering steam.

From Table A-4, $h_i = 3445.3 \frac{kJ}{kg}$.

Then, $u_2 = 3445.3$ kJ/kg, $p_2 = 20$ bar fix the final state of the steam in the tank. Interpolating with u_2 at 20 bar gives $\begin{cases} T_2 = 685.9\ °C \\ v_2 = 0.2199\ m^3/kg \end{cases}$ ←

The final mass in the tank is

$$m_2 = \frac{V}{v_2} = \frac{0.5\ m^3}{0.2199\ m^3/kg} = 2.27\ kg$$ ←

---

1. Since $h_i = u_i + (pv)_i$, the result $u_2 = h_i$ can be expressed as

$$u_2 = u_i + \underline{(pv)_i}$$

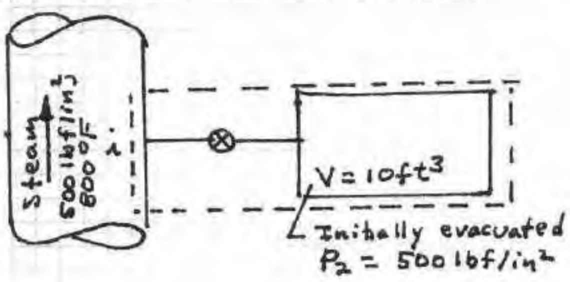
The underlined term is the flow work discussed in Sec. 4.4.2. The increase in temperature of the tank contents over that of the steam in the vessel is due to energy entering the tank at the inlet by flow work.

PROBLEM 4.113

KNOWN: An insulated, rigid tank, initially evacuated, is connected by a valve to a large steam line through which steam flows at 500 lbf/in². The valve is opened and steam flows into the tank until the pressure is 500 lbf/in². State data are provided.

FIND: Determine the final temperature of the steam in the tank, in °F, and the final mass of the steam in the tank, in lb.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The control volume is defined by the dashed line on the schematic. The condition at i remains constant at 500 lbf/in², 800°F. The valve serves only to adjust the flow.
2. For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and kinetic and potential energy effects are negligible.

ANALYSIS:

For the one-inlet control volume, mass and energy balances reduce with listed assumptions to read, respectively, $\frac{dm_{cv}}{dt} = \dot{m}_i$, $\frac{dU_{cv}}{dt} = \dot{m}_i h_i$. Combining these expressions and integrating with respect to time,

$$\frac{dU_{cv}}{dt} = \underbrace{h_i}_{\text{Remains constant}} \frac{dm_{cv}}{dt} \Rightarrow \underbrace{\Delta U_{cv}}_{(m_2u_2 - m_1u_1)} = h_i[m_2 - m_1]$$, which is a special case of Eq. 4.28 in Sec. 4.12.2.

(1) Since $m_1 = 0$, $(m_2u_2 - \cancel{m_1}u_1) = h_i(m_2 - \cancel{m_1}) \Rightarrow \boxed{u_2 = h_i}$. That is, the final specific internal energy of the steam in the tank equals the specific enthalpy of the entering steam.

From Table A-4E, $h_i = 1412.1$ Btu/lb.

Then, $u_2 = 1412.1$ Btu/lb, $P_2 = 500$ lbf/in² fix the final state of the steam in the tank. Interpolating with u_2 at 500 lbf/in² gives,

$$\begin{cases} T_2 = 1114.1\,°F \\ v_2 = 1.8447\ \frac{ft^3}{lb} \end{cases}$$

The final mass in the tank is

$$m_2 = \frac{V}{v_2} = \frac{10\ ft^3}{1.8447\ ft^3/lb} = 5.42\ lb$$

1. Since $h_i = u_i + (pv)_i$, the result $u_2 = h_i$ can be expressed as

$$u_2 = u_i + (\underline{pv})_i$$

The underlined term is the flow work discussed in Sec. 4.4.2. The increase in temperature of the tank contents over that of the steam flowing in the line is due to energy entering the tank at the inlet by flow work.

PROBLEM 4.114

KNOWN: The compressor of a compressed-air energy storage system delivers air to a cavern. State data and cavern size are provided.

FIND: Determine the initial and final mass of air in the cavern, each in kg, and the work required by the compressor, in GJ.

SCHEMATIC & GIVEN DATA:

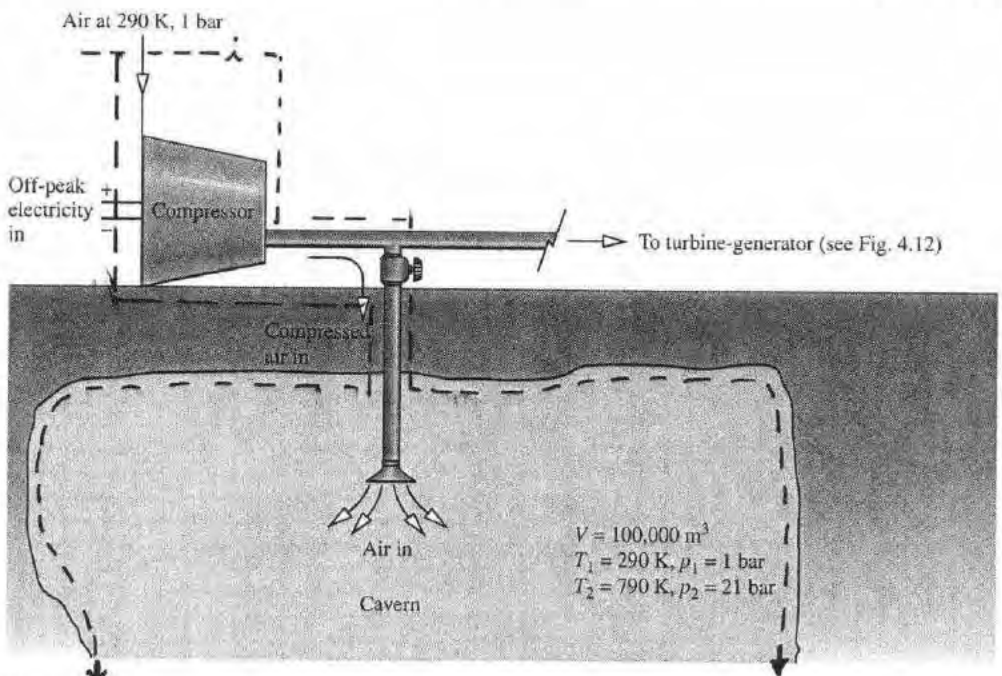


Fig. P4.114

ENGINEERING MODEL:

1. The control volume is defined by the dashed line on the schematic. The condition at i remains constant at 290 K, 1 bar.
2. For the control volume, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are ignored.
3. The air is modeled as an ideal gas.

ANALYSIS: For the one-inlet control volume, mass and energy rate balances reduce with listed assumptions to read, respectively, $\frac{dm_{cv}}{dt} = \dot{m}_i$,

$\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}_i h_i$. Combining these expressions and integrating,

$$\frac{dU_{cv}}{dt} = -\dot{W}_{cv} + h_i \frac{dm_{cv}}{dt} \Rightarrow \underbrace{\Delta U_{cv}}_{(m_2u_2 - m_1u_1)} = -W_{cv} + h_i\,[m_2 - m_1]$$

, which is a special case of Eq. 4.28 in Sec. 4.12.2.

Remains constant

$$\Rightarrow (-W_{cv}) = (m_2u_2 - m_1u_1) - h_i\,[m_2 - m_1] \quad (1)$$

where

$$m_1 = \frac{P_1 V}{RT_1} = \frac{(10^5\,N/m^2)(10^5\,m^3)}{\left(\frac{8314}{28.97}\,\frac{N\cdot m}{kg\cdot K}\right)(290\,K)} = 1.2 \times 10^5\ kg$$

$$m_2 = \frac{P_2 V}{RT_2} = \frac{(21\times10^5\,N/m^2)(10^5\,m^3)}{\left(\frac{8314}{28.97}\,\frac{N\cdot m}{kg\cdot K}\right)(790\,K)} = 9.26 \times 10^5\ kg$$

PROBLEM 4.114 (Continued)

With data from Table A-22,

$$u_1 = 206.91 \text{ kJ/kg}$$
$$u_2 = 584.21 \text{ kJ/kg}$$
$$h_i = 290.16 \text{ kJ/kg}$$

Eq.(1) gives

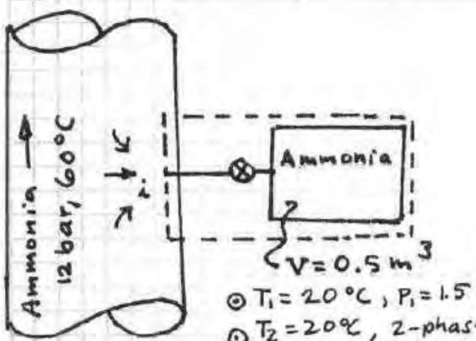
$$(-W_{cv}) = [(9.26 \times 10^5 \text{ kg})(584.21 \text{ kJ/kg}) - (1.2 \times 10^5 \text{ kg})(206.91 \tfrac{\text{kJ}}{\text{kg}})]$$
$$- (290.16 \tfrac{\text{kJ}}{\text{kg}})[9.26 \times 10^5 - 1.2 \times 10^5] \text{ kg}$$
$$= [5.41 \times 10^8 \text{ kJ} - 0.25 \times 10^8 \text{ kJ}] - 2.34 \times 10^8 \text{ kJ}$$
$$= 2.82 \times 10^8 \text{ kJ} \left| \frac{1 \text{ GJ}}{10^6 \text{ kJ}} \right|$$
$$= 282 \text{ GJ}$$

PROBLEM 4.115

KNOWN: A rigid tank containing ammonia is connected by a valve to a large supply line carrying ammonia. The valve is opened and additional ammonia enters the tank, bringing the total mass of ammonia in the tank to 143.36 kg.

FIND: Determine the heat transfer between the tank contents and its surroundings, in kJ.

SCHEMATIC & GIVEN DATA:



① $T_1 = 20°C$, $P_1 = 1.5$ bar

② $T_2 = 20°C$, 2-phase liquid-vapor mixture

$m_2 = 143.36$ kg

ENGINEERING MODEL

1. The control volume is defined by the dashed line on the schematic. The condition at i remains constant at 12 bar, 60°C. The valve serves only to adjust the flow.
2. For the control volume, $\dot{W}_{cv} = 0$ and kinetic and potential energy effects are negligible.

ANALYSIS: For the one-inlet control volume, mass and energy rate balances reduce with listed assumptions to read, respectively, $\frac{dm_{cv}}{dt} = \dot{m}_i$; $\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}_i h_i$. Combining these expressions and integrating with respect to time, $\frac{dU_{cv}}{dt} = \dot{Q}_{cv} + \underbrace{h_i}_{\text{Remains constant}} \frac{dm_{cv}}{dt} \Rightarrow \underbrace{\Delta U_{cv}}_{(m_2u_2 - m_1u_1)} = Q_{cv} + h_i(m_2 - m_1)$, which is a special case of Eq. 4.28.

Solving, $$Q_{cv} = (m_2u_2 - m_1u_1) - h_i(m_2 - m_1) \qquad (1)$$

From Table A-15, $v_1 = 0.9382$ m³/kg, $u_1 = 1371.79$ kJ/kg, $h_i = 1553.07$ kJ/kg.

$$m_1 = \frac{V}{v_1} = \frac{0.5\ \text{m}^3}{0.9382\ \text{m}^3/\text{kg}} = 0.533\ \text{kg}$$

$$m_2 = 143.36\ \text{kg (given)}$$

$$\therefore\ v_2 = \frac{0.5\ \text{m}^3}{143.36\ \text{kg}} = 0.0035\ \text{m}^3/\text{kg}$$

At state 2, there is a two-phase liquid-vapor mixture; using data from Table A-13

$$x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{0.0035 - (1.6386/10^3)}{0.1492 - (1.6386/10^3)} = 0.0126\ (1.26\%).$$

$$u_2 = u_f + x_2(u_g - u_f) = 272.86 + (0.0126)[1331.94 - 272.86]$$
$$= 286.2\ \text{kJ/kg}$$

Substituting values into Eq (1),

$$Q_{cv} = [(143.36\ \text{kg})(286.2\ \text{kJ/kg}) - (0.533\ \text{kg})(1371.79\ \text{kJ/kg}) - (1553.07\ \text{kJ/kg})[143.36 - 0.533]\ \text{kg}$$

$$= -181{,}522\ \text{kJ}$$

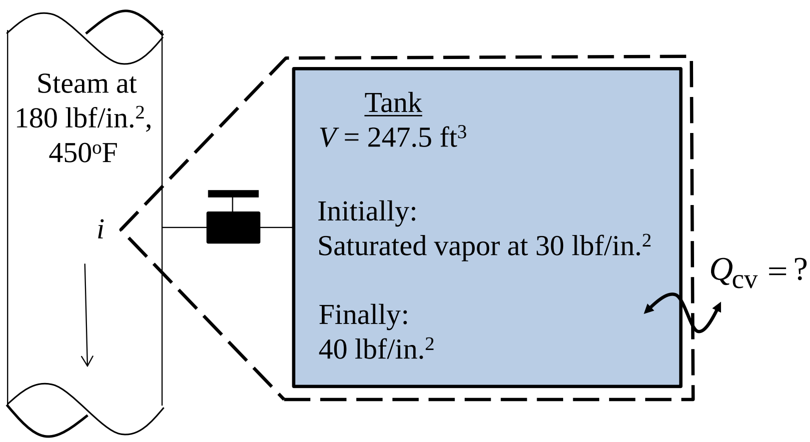
Energy transfer by heat to the surroundings

4.116 As shown in Fig. P4.116, a 247.5-ft^3 tank contains saturated vapor water initially at 30 lbf/in.^{2} The tank is connected to a large line carrying steam at 180 lbf/in.^{2}, 450^{o}F. Steam flows into the tank through a valve until 2.9 lb of steam have been added to the tank. The valve is then closed and the pressure in the tank is 40 lbf/in.^{2} Determine the specific volume, in ft^3/lb, at the final state of the control volume and the magnitude and direction of the heat transfer between the tank and its surroundings, in Btu.

KNOWN: Steam flows into a tank.

FIND: Specific volume, in ft^3/lb, at the final state of the control volume and the magnitude and direction of the heat transfer between the tank and its surroundings, in Btu.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume is defined by the dashed line on the accompanying diagram.
2. Conditions within the steam line remain constant.
3. For the control volume, kinetic and potential energy effects can be ignored and $\dot{W}_{cv} = 0$.

ANALYSIS: Applying the mass rate balance

$$\frac{dm_{cv}}{dt} = \dot{m}_i \rightarrow m_2 - m_1 = \int_{t_1}^{t_2} \dot{m}_i dt = 2.9 \text{ lb}$$

Solving for the final mass in the tank gives

$$m_2 = m_1 + 2.9 \text{ lb}$$

In this expression $m = V/v$. At the initial state with data from Table A-3E, $v_1 = v_{g1}$ = 13.75 ft^3/lb.

$$m_1 = \frac{247.5\ \text{ft}^3}{13.75\frac{\text{ft}^3}{\text{lb}}} = 18.0\ \text{lb}$$

Solving for the final mass

$$m_2 = 18.0\ \text{lb} + 2.9\ \text{lb} = 20.9\ \text{lb}$$

The specific volume at the final state is

$$v_2 = \frac{V}{m_2} = \frac{247.5\ \text{ft}^3}{20.9\ \text{lb}} = \underline{\mathbf{11.84\ ft^3/lb}}$$

The energy rate balance reduces to

$$\frac{dU_{\text{cv}}}{dt} = \dot{Q}_{\text{cv}} - \cancel{\dot{W}_{\text{cv}}} + \dot{m}_i h_i - \cancel{\dot{m}_e} h_e$$

There is no power, and no mass exits the control volume. Noting h_i remains constant, combining mass and energy rate balances results in

$$m_2 u_2 - m_1 u_1 = \dot{Q}_{\text{cv}} \Delta t + h_i \int_{t_1}^{t_2} \dot{m}_i dt = Q_{\text{cv}} + h_i (m_2 - m_1)$$

$$Q_{\text{cv}} = m_2 (u_2 - h_i) - m_1 (u_1 - h_i)$$

For the initial state, Table A-3E applies: $u_1 = u_{g1} = 1088.0$ Btu/lb.

For the final state, specific internal energy is determined from Table A-4E at $p_2 = 40$ lbf/in.^{2} and $v_2 = 11.84$ ft^3/lb: $u_2 = 1124.2$ Btu/lb.

For the steam entering the control volume, specific enthalpy is determined from Table A-4E: $h_i = 1243.4$ Btu/lb.

Substituting values gives

$$Q_{\text{cv}} = (20.9\ \text{lb})\left(1124.2\frac{\text{Btu}}{\text{lb}} - 1243.4\frac{\text{Btu}}{\text{lb}}\right) - (18.0\ \text{lb})\left(1088.0\frac{\text{Btu}}{\text{lb}} - 1243.4\frac{\text{Btu}}{\text{lb}}\right) = \underline{\mathbf{305.92\ Btu}}$$

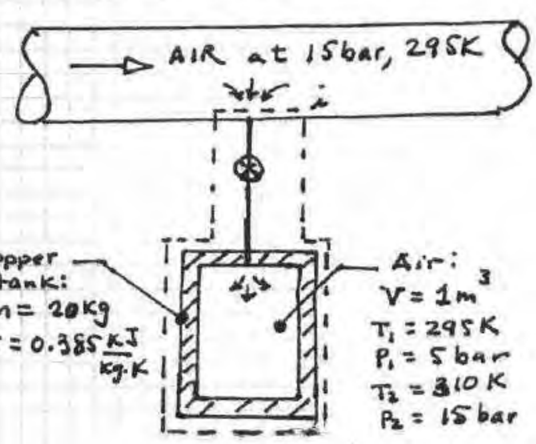
Since the sign associated with heat transfer is positive, heat transfer is into the control volume.

PROBLEM 4.117

KNOWN: A rigid copper tank containing air at 5 bar is connected to a large supply line carrying air at 15 bar. An interconnecting valve is opened, allowing air to enter the tank until the pressure of the air in the tank is 15 bar. State data are provided.

FIND: Determine (a) the initial and final mass within the tank, in kg, and (b) the heat transfer to the surroundings from the tank and its contents, in kJ.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The control volume is defined by the dashed line in the schematic. The condition at i remains constant at 15 bar, 295K.
2. For the control volume, $\dot{W}_{cv} = 0$ and kinetic and potential energy effects can be ignored.
3. The air is modeled as an ideal gas.
4. The copper tank is at the same temperature as the air in the tank, initially and finally.

ANALYSIS: For the one-inlet control volume, mass and energy rate balances reduce with listed assumptions to read, respectively, $\frac{dm_{cv}}{dt} = \dot{m}_i$ and $\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i h_i$. Combining these expressions and integrating with respect to time, $\frac{dU_{cv}}{dt} = \dot{Q}_{cv} + h_i \frac{dm_{cv}}{dt} \Rightarrow \Delta U_{cv} = Q_{cv} + h_i(m_2 - m_1)$, which is a special case of Eq. 4.28. Here ΔU_{cv} is the sum of the internal energy changes of the tank and the air contained within the tank: $\Delta U_{cv} = \Delta U_{tank} + \Delta U_{Air}$. Solving

Remains constant

$$Q_{cv} = \Delta U_{tank} + \Delta U_{air} - h_i[m_2 - m_1] \tag{1}$$

$$= \underbrace{mc[T_2 - T_1]}_{\text{(tank)}} + \underbrace{[m_2u_2 - m_1u_1]}_{\text{(air in tank)}} - h_i[m_2 - m_1] \tag{2}$$

From Table A-22, $u_1 = 210.49$ kJ/kg, $u_2 = 221.25$ kJ/kg, $h_i = 295.17$ kJ/kg. Also,

$$m_1 = \frac{P_1 V}{RT_1} = \frac{(5\times10^5 N/m^2)(1m^3)}{\left(\frac{8314}{28.97}\frac{N\cdot m}{kg\cdot K}\right)(295K)} = 5.91 kg, \quad m_2 = \frac{P_2 V}{RT_2} = \frac{(15\times10^5 N/m^2)(1m^3)}{\left(\frac{8314}{28.97}\frac{N\cdot m}{kg\cdot K}\right)(310K)} = 16.86 kg$$

Inserting values in Eq. (2),

$$Q_{cv} = (20kg)(0.385 \tfrac{kJ}{kg\cdot K})[15K] + \left[(16.86kg)(221.25 \tfrac{kJ}{kg}) - (5.91kg)(210.49 \tfrac{kJ}{kg})\right] - (295.17 \tfrac{kJ}{kg})(10.95kg)$$

$$= -630.3 \text{ kJ}$$

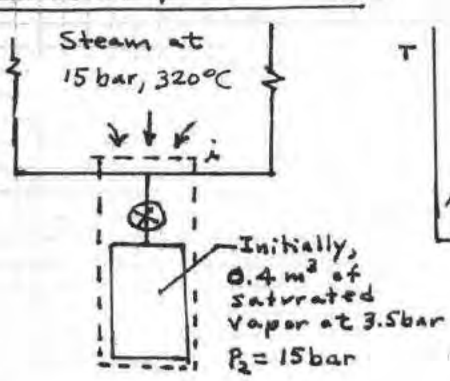
Energy transfer by heat from the control volume

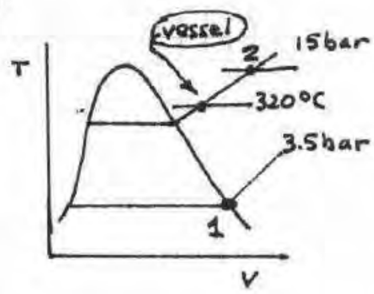
PROBLEM 4.118

KNOWN: A rigid, insulated tank, initially holding saturated water vapor at 3.5 bar, is connected by a valve to a large vessel holding steam at 15 bar. The valve is opened and additional matter enters until the pressure in the tank is 15 bar.

FIND: Determine the final mass in the tank, in kg, and its temperature, in °C.

SCHEMATIC & GIVEN DATA





ENGINEERING MODEL:

1. The control volume is defined by the dashed line on the schematic. The condition at i remains constant at 15 bar, 320°C. The valve serves only to adjust the flow.
2. For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and kinetic and potential energy are ignored.

ANALYSIS: For the one-inlet control volume, mass and energy rate balances reduce with listed assumptions to read, respectively, $\frac{dm_{cv}}{dt} = \dot{m}_i$, $\frac{dU_{cv}}{dt} = \dot{m}_i h_i$. Combining these expressions and integrating with respect to time,

$$\frac{dU_{cv}}{dt} = h_i \frac{dm_{cv}}{dt} \Rightarrow \underbrace{\Delta U_{cv}}_{(m_2u_2 - m_1u_1)} = h_i[m_2 - m_1] \rightarrow (m_2u_2 - m_1u_1) = h_i(m_2 - m_1) \quad (1)$$

Remains constant

Eq. (1) is a special case of Eq. 4.28.

With data from Tables A-3 and A-4, $u_1 = 2546.9 \frac{kJ}{kg}$, $v_1 = 0.5243 \frac{m^3}{kg}$, $h_i = 3081.9 \frac{kJ}{kg}$,

$$m_1 = \frac{V}{v_1} = \frac{0.4\ m^3}{0.5243\ m^3/kg} = 0.763\ kg \quad , \quad m_2 = \frac{V}{v_2} = \frac{0.4\ m^3}{v_2}$$

Take note that u_2 and v_2 remain unknown. Rearranging Eq. (1) and inserting values shows that an iterative solution is required:

$$m_2[h_i - u_2] = m_1[h_i - u_1] \Rightarrow \left(\frac{0.4\ m^3}{v_2}\right)[3081.9 - u_2]\frac{kJ}{kg} = \underbrace{(0.763\ kg)[3081.9 - 2546.9]\frac{kJ}{kg}}_{408.2\ kJ}$$

or

$$0.4[3081.9 - u_2] = 408.2\, v_2 \quad (2)$$

(m^3) $\left(\frac{kJ}{kg}\right)$ (kJ) (m^3/kg)

An iterative solution of Eq. (2) can be obtained using "Steam table" data at 15 bar. For each iteration, u_2 and the corresponding value of v_2 is obtained from the tables. The procedure continues until Eq. (2) is satisfied. This is left as an exercise. Alternatively, IT can be used as follows:

```
p2 = 15
0.4*(3081.9 - u2) = 408.2*v2
m2 = 0.4/v2
v2 = v_PT("Water/Steam", p2, T2)
u2 = u_PT("Water/Steam", p2, T2)
```

SOLUTION:

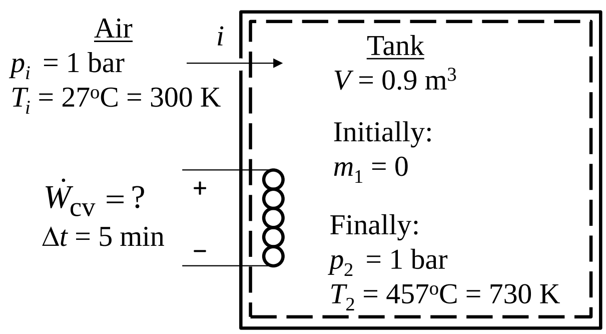
```
m2 = 2.099 kg   ←
T2 = 362.1 C    ←
u2 = 2887
v2 = 0.1906
```

4.119 A rigid, well-insulated tank of volume 0.9 m^3 is initially evacuated. At time t = 0, air from the surroundings at 1 bar, 27^{o}C begins to flow into the tank. An electric resistor transfers energy to the air in the tank at a constant rate for 5 minutes, after which time the pressure in the tank is 1 bar and the temperature is 457^{o}C. Modeling air as an ideal gas, determine the power input to the tank, in kW.

KNOWN: Air flows into a tank.

FIND: Determine the power input to the tank, in kW.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume is defined by the dashed line on the accompanying diagram.
2. Air is modeled as an ideal gas.
3. The condition of the air entering the tank remains constant.
4. For the control volume, kinetic and potential energy effects can be ignored and $\dot{Q}_{cv} = 0$.
5. For the resistor, $\Delta U_{resistor}$ = 0.

ANALYSIS: The mass rate balance reduces to

$$\frac{dm_{cv}}{dt} = \dot{m}_i$$

The energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}_i h_i - \cancel{\dot{m}_e} h_e$$

There is no heat transfer, and no mass exits the control volume. Noting h_i remains constant, combining mass and energy rate balances results in

$$m_2 u_2 - \cancel{m_1} u_1 = -\dot{W}_{cv}\Delta t + h_i \int_{t_1}^{t_2} \dot{m}_i dt = -\dot{W}_{cv}\Delta t + h_i (m_2 - \cancel{m_1})$$

$$\dot{W}_{cv} = \frac{m_2(h_i - u_2)}{\Delta t}$$

Applying the ideal gas equation of state to the final state to solve for m_2 gives

$$m_2 = \frac{p_2V_2}{RT_2} = \frac{(1\,\text{bar})(0.9\,\text{m}^3)}{\left(\dfrac{8.314\dfrac{\text{kJ}}{\text{kmol}\cdot\text{K}}}{28.97\dfrac{\text{kg}}{\text{kmol}}}\right)(730\,\text{K})}\left|\frac{10^5\,\dfrac{\text{N}}{\text{m}^2}}{1\,\text{bar}}\right|\left|\frac{1\,\text{kJ}}{1000\,\text{N}\cdot\text{m}}\right| = 0.43\ \text{kg}$$

Specific internal energy at the final state and specific enthalpy for the inlet state are determined from Table A-22: u_2 = 536.07 kJ/kg and h_i = 300.19 kJ/kg.

Substituting values and solving for power give

$$\dot{W}_{cv} = \frac{(0.43\,\text{kg})\left(300.19\dfrac{\text{kJ}}{\text{kg}} - 536.07\dfrac{\text{kJ}}{\text{kg}}\right)}{5\,\text{min}}\left|\frac{1\,\text{min}}{60\,\text{s}}\right|\left|\frac{1\,\text{kW}}{1\dfrac{\text{kJ}}{\text{s}}}\right| = -\underline{\mathbf{0.338\ kW}}$$

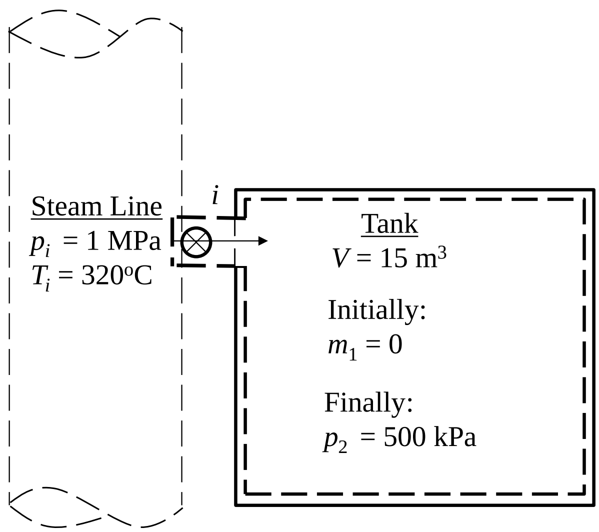
Since the sign associated with power is negative, power is into the control volume as expected.

4.120 A well-insulated, rigid tank of volume 15 m^3 is connected to a large steam line through which steam flows at 1 MPa, 320°C. The tank is initially evacuated. Steam is allowed to flow into the tank until the pressure inside is *p*.
(a) Determine the amount of mass in the tank, in kg, and the temperature in the tank, in °C, when p = 500 kPa.
(b) Plot the quantities of part (a) versus *p* ranging from 0 kPa to 500 kPa.

KNOWN: Steam flows into a tank.

FIND: (a) The amount of mass in the tank, in kg, and the temperature in the tank, in °C, when *p* = 500 kPa and (b) plot the quantities of part (a) versus *p* ranging from 0 kPa to 500 kPa.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume is defined by the dashed line on the accompanying diagram.
2. The condition of the steam entering the tank remains constant.
3. For the control volume, kinetic and potential energy effects can be ignored, $\dot{Q}_{cv} = 0$, and $\dot{W}_{cv} = 0$.

ANALYSIS: (a) The mass rate balance reduces to

$$\frac{dm_{cv}}{dt} = \dot{m}_i$$

The energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_i h_i - \cancel{\dot{m}_e} h_e$$

There is no heat transfer and no work, and no mass exits the control volume. Noting h_i remains constant, combining mass and energy rate balances results in

$$m_2 u_2 - \cancel{m_1} u_1 = h_i \int_{t_1}^{t_2} \dot{m}_i dt = h_i (m_2 - \cancel{m_1})$$

$$u_2 = h_\mathrm{i}$$

Specific enthalpy at the inlet is obtained from Table A-4: h_i = 3093.9 kJ/kg.

For p_2 = 500 kPa and u_2 = 3093.9 kJ/kg, Table A-4 (interpolated) gives ***T_2 = 479.3°C*** and v_2 = 0.6915 m^3/kg.

Solving for mass gives

$$m_2 = \frac{V}{v_2} = \frac{15\,\mathrm{m}^3}{0.6915 \frac{\mathrm{m}^3}{\mathrm{kg}}} = \mathbf{21.7\ kg}$$

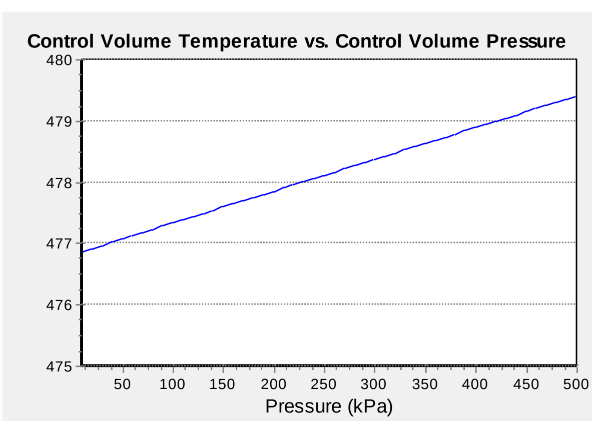
(b) IT is used to generate the plots:

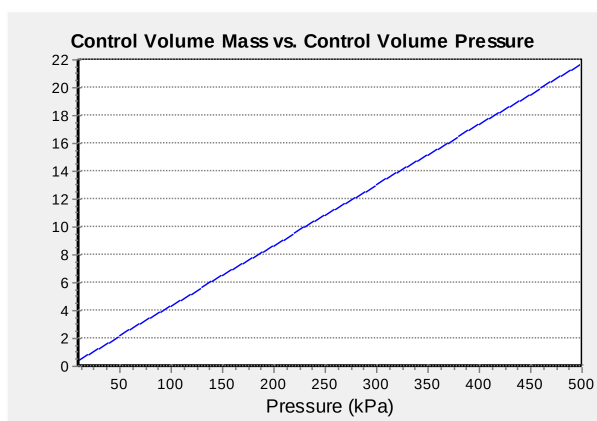
IT Code

```
//Given Values:
V = 15//m3
p_i = 1000//kPa
T_i = 320//oC
p_2 = 500//kPa

//Properties
h_i = h_PT("Water/Steam", p_i, T_i)
u_2 = h_i
u_2 = u_PT("Water/Steam", p_2, T_2)
v_2 = v_PT("Water/Steam", p_2, T_2)
m_2 = V/v_2
```

IT Results: p_2 = 500 kPa

m_2	21.69
T_2	479.4
u_2	3093
v_2	0.6917
p_2	500
p_i	1000
T_i	320
V	15



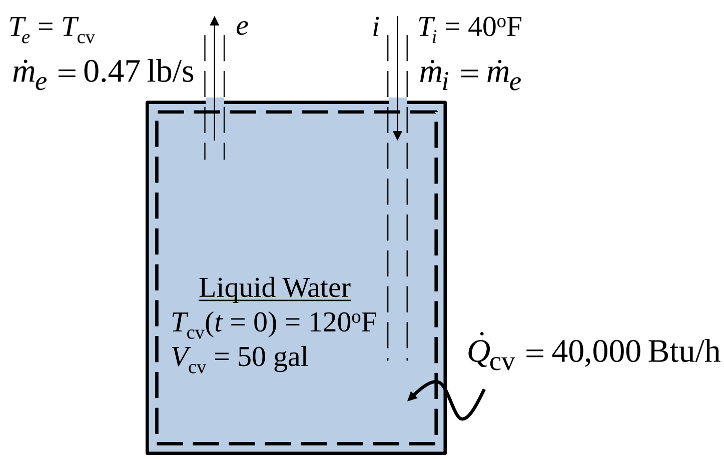


4.121 A 50-gallon-capacity hot water heater is shown in Fig. P4.121. Water in the tank of the heater initially has a temperature of 120°F. When someone turns on the shower faucet, water flows from the tank at a rate of 0.47 lb/s, and replenishment water at 40°F flows into the tank from the municipal water distribution system. Water in the tank receives an energy input at a rate of 40,000 Btu/h from electrical resistors. If the water within the tank is well mixed, the temperature at any time can be taken as uniform throughout. The tank is well insulated so stray heat transfer with the surroundings is negligible. Neglecting kinetic and potential energy effects, assuming negligible change in pressure from inlet to exit of the tank, and modeling water as an incompressible substance with density of 62.28 lb/ft^3 and specific heat of 1.0 Btu/lb·°R, plot the temperature, in °F, of the water in the tank versus time from t = 0 to 20 min (1200 s).

KNOWN: Water flows through a hot water heater tank.

FIND: Plot the temperature, in °F, of the water in the tank versus time from t = 0 to 20 min.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume is defined by the dashed line on the accompanying diagram.
2. Pressure change from inlet to exit is negligible.
3. Water is modeled as an incompressible substance with density of 62.28 lb/ft^3 and specific heat of 1 Btu/ lb·°R.
4. For the control volume, kinetic and potential energy effects can be ignored and $\dot{W}_{cv} = 0$.
5. The state of the water entering the tank remains constant.
6. Water in the tank is well-mixed.
7. Stray heat transfer with the surroundings is negligible.

ANALYSIS: Since the control volume has a single inlet and single exit, $\dot{m}_i = \dot{m}_e = \dot{m}$, and so

$$\frac{dm_{cv}}{dt} = 0$$

The energy rate balance reduces to

$$\frac{dU_{\text{cv}}}{dt} = \dot{Q}_{\text{cv}} - \cancel{\dot{W}_{\text{cv}}} + \dot{m}_i h_i - \dot{m}_e h_e$$

Combining mass and energy rate balances result in

$$\frac{d(m_{\text{cv}} u_{\text{cv}})}{dt} = \dot{Q}_{\text{cv}} + \dot{m}[h_i - h_e(t)]$$

$$m_{\text{cv}} \frac{du_{\text{cv}}}{dt} + u_{\text{cv}} \cancel{\frac{dm_{\text{cv}}}{dt}} = \dot{Q}_{\text{cv}} + \dot{m}[h_i - h_e(t)]$$

$$m_{\text{cv}} \frac{du_{\text{cv}}}{dt} = \dot{Q}_{\text{cv}} + \dot{m}[h_i - h_e(t)]$$

For an incompressible substance with negligible pressure change, Eqs. 3.20a and 3.20b apply. Substituting $T_{\text{cv}} = T_e = T(t) = T$, $du_{\text{cv}} = c\text{d}T$, and $h_i - h_e(t) = c(T_i - T)$ gives

$$m_{\text{cv}} c \frac{dT}{dt} = \dot{Q}_{\text{cv}} + \dot{m}c(T_i - T)$$

$$\frac{dT}{dt} + \left(\frac{\dot{m}}{m_{\text{cv}}}\right) T = \left(\frac{\dot{m}}{m_{\text{cv}}}\right) T_i + \frac{\dot{Q}_{\text{cv}}}{m_{\text{cv}} c}$$

The solution of this differential equation takes the form

$$T(t) = \left(T_i + \frac{\dot{Q}_{\text{cv}}}{\dot{m}c}\right) + C \exp\left[\left(-\frac{\dot{m}}{m_{\text{cv}}}\right) t\right]$$

To determine the constant C, apply the initial condition $T(t = 0) = 120^{\circ}\text{F} = 580^{\circ}\text{R}$.

$$580^{\circ}\text{R} = \left(T_i + \frac{\dot{Q}_{\text{cv}}}{\dot{m}c}\right) + C$$

$$C = (580^{\circ}\text{R} - T_i) - \frac{\dot{Q}_{\text{cv}}}{\dot{m}c}$$

Thus, the solution is

$$T(t) = \left(T_i + \frac{\dot{Q}_{\text{cv}}}{\dot{m}c}\right) + \left[(580^{\circ}\text{R} - T_i) - \left(\frac{\dot{Q}_{\text{cv}}}{\dot{m}c}\right)\right] \exp\left[\left(-\frac{\dot{m}}{m_{\text{cv}}}\right) t\right]$$

Since the mass of the control volume is constant, its mass can be determined at t=0 using the relationship

$$m_{cv} = \rho V_{cv}$$

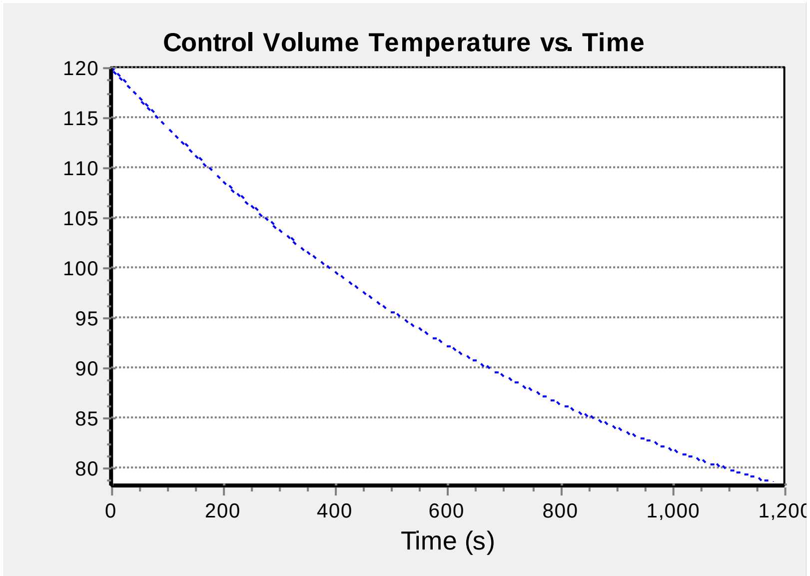
Using the given water density (which can be obtained from Table A-19E), the mass of water in the control volume is

$$m_{cv} = \left(62.28 \frac{\text{lb}}{\text{ft}^3}\right)(50\text{ gal})\left|\frac{0.13368\text{ ft}^3}{1\text{ gal}}\right| = 416.3\text{ lb}$$

The inlet temperature remains a constant 40°F = 500°R.

Entering in IT the given values and the equation to determine control volume temperature (T) as a function of time and varying the range for time from 0 s to 1200 s (20 minutes) yield the plot below.

```
//Given Data
T_i = 500//oR
T_i_F = 40//oF
m_dot = 0.47//lb/s
c = 1.0//Btu/lb·oR
m_cv = 416.3//lb
Q_dot = 40000//Btu/hr
t = 1200//s

//Solve for temperature
T = (T_i + (Q_dot/(m_dot*c*3600))) + ((580 - T_i) - (Q_dot/(m_dot*c*3600)))*exp(-(m_dot/m_cv)*t)
T_F = T - 460
```



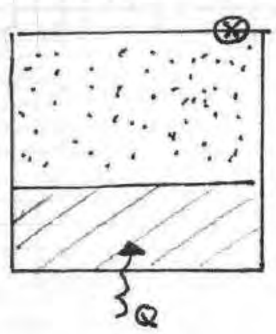
Note that continuous use of hot water results in lower water temperature and eventually cold showers!

PROBLEM 4.122

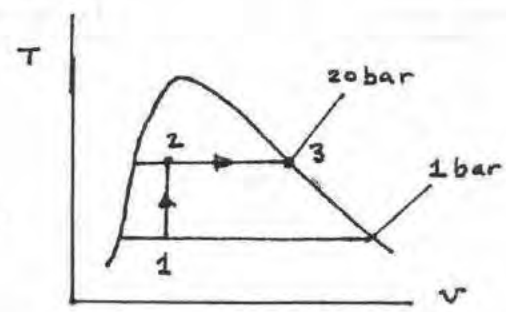
KNOWN: A rigid tank, initially containing a two-phase liquid-vapor mixture, is heated in two stages: constant-volume heating followed by heating while saturated water vapor is withdrawn at a constant pressure. State data is provided.

FIND: For each stage of heating, evaluate the heat transfer, in kJ.

SCHEMATIC & GIVEN DATA:



$V = 0.1\ m^3$
$P_1 = 1$ bar
$x_1 = 1\%$
$P_2 = P_3 = 20$ bar
$x_3 = 100\%$



ENGINEERING MODEL:

1. For process 1-2, a closed system is considered that consists of the initial tank contents.
2. For process 2-3, a one-exit control volume is considered for which the condition at the exit is saturated vapor at 20 bar.
3. For each system, $\dot{W}_{cv} = 0$ and kinetic and potential energy effects are ignored.

ANALYSIS: We begin by finding key property values at states 1, 2, 3:

$$v_1 = v_f + x_1(v_g - v_f) = \frac{1.0432}{10^3} + 0.01\left[1.694 - \frac{1.0432}{10^3}\right] = 0.018\ m^3/kg$$

$$v_2 = v_1 \Rightarrow \quad x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{0.018 - (1.1767/10^3)}{0.09963 - (1.1767/10^3)} = 0.17$$

$$m_1 = \frac{V}{v_1} = \frac{0.1\ m^3}{0.018\ m^3/kg} = 5.56\ kg, \quad m_2 = m_1$$

$$m_3 = \frac{V}{v_{g3}} = \frac{0.1\ m^3}{0.09963\ m^3/kg} = 1.0\ kg$$

$$u_1 = u_f + x_1[u_g - u_f] = 417.36 + 0.01[2506.1 - 417.36] = 438.25\ kJ/kg$$

$$u_2 = u_f + x_2[u_g - u_f] = 906.44 + 0.17[2600.3 - 906.44] = 1194.4\ kJ/kg$$

$$h_e = h_{g3} = 2799.5\ kJ/kg, \quad u_3 = u_{g3} = 2600.3\ kJ/kg$$

PROCESS 1-2:

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - \cancel{W} \Rightarrow Q = \underbrace{m_1}_{=m_2}[u_2 - u_1]$$

$$= 5.56\ kg\,[1194.4 - 438.25]\ \frac{kJ}{kg}$$

$$= 4204.2\ kJ$$

PROCESS 2-3

Mass and energy rate balances for the one-exit control volume reduce to, $\frac{dm_{cv}}{dt} = -\dot{m}_e$ and $\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{m}_e h_e$

Combining them gives, $\frac{dU_{cv}}{dt} = \dot{Q}_{cv} + \underbrace{h_e}_{\text{remains constant}} \frac{dm_{cv}}{dt}$

Integrating with respect to time,

$$\underbrace{\Delta U_{cv}}_{m_3u_3 - m_2u_2} = Q_{cv} + h_e[m_3 - m_2] \Rightarrow (m_3u_3 - m_2u_2) = Q_{cv} + h_e[m_3 - m_2]$$

Solving, $Q_{cv} = (m_3u_3 - m_2u_2] - h_e[m_3 - m_2]$, where $m_2 = m_1$.

Inserting values,

$$Q_{cv} = \left[(1\ kg)(2600.3\ \tfrac{kJ}{kg}) - (5.56\ kg)(1194.4\ \tfrac{kJ}{kg})\right] - (2799.5\ \tfrac{kJ}{kg})\left[1 - 5.56\right] kg$$

1 $= 8725.2\ kJ$

---

1. The total heat transfer is $(4204.2\ kJ + 8725.2\ kJ) = 12{,}929.4\ kJ$

An overall energy balance reads,

$$Q_{cv} = (m_3u_3 - m_1u_1) - h_e(m_3 - m_1)$$

$$= \left[(1\ kg)(2600.3\ \tfrac{kJ}{kg}) - (5.56\ kg)(438.25\ \tfrac{kJ}{kg})\right] - (2799.5\ \tfrac{kJ}{kg})(1 - 5.56)\ kg$$

$$= 12{,}929.4\ kJ$$

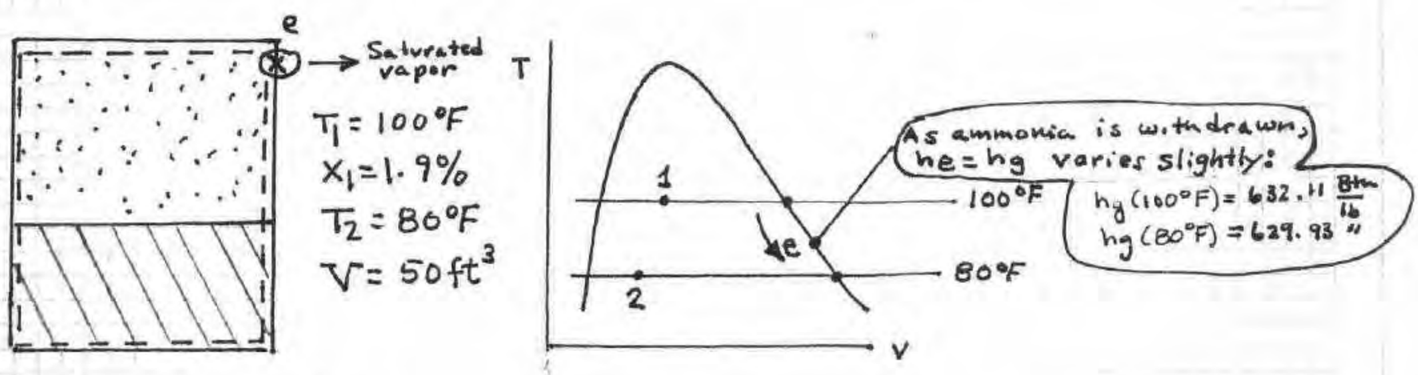
which checks the total given previously.

PROBLEM 4.123

KNOWN: A rigid, insulated tank initially contains a two-phase liquid-vapor mixture of ammonia. Saturated vapor is slowly withdrawn until a two-phase liquid-vapor mixture at a lower temperature remains. State data are provided.

FIND: Determine the initial and final amounts of mass in the tank, in lb.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. A one-exit control volume is considered, as shown in the schematic.
2. For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and kinetic and potential energy effects are ignored.
3. At the exit $h_e = [h_g(100°F) + h_g(80°F)]/2$. That is, the average h_g value is used.

ANALYSIS: We begin by evaluating key property data, using Table A-13E.

$$h_e = \frac{h_g(100°F) + h_g(80°F)}{2} = \frac{(632.11 + 629.93)}{2}\ \frac{Btu}{lb} = 631.02\ \frac{Btu}{lb}$$

$$v_1 = v_{f1} + x_1[v_{g1} - v_{f1}] = 0.02747 + 0.019[1.4168 - 0.02747] = 0.0539\ \frac{ft^3}{lb}$$

$$u_1 = u_{f1} + x_1[u_{g1} - u_{f1}] = 153.98 + 0.019[576.51 - 153.98] = 162.01\ \frac{Btu}{lb}$$

$$m_1 = \frac{V}{v_1} = \frac{50\ ft^3}{0.0539\ ft^3/lb} = 927.6\ lb \longleftarrow$$

$$m_2 = \frac{V}{v_2} = \frac{50\ ft^3}{v_2}$$

Mass and energy rate balances for the one-exit control volume reduce to $\frac{dm_{cv}}{dt} = -\dot{m}_e$ and $\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} - \dot{m}_e h_e$. Combining them gives $\frac{dU_{cv}}{dt} = h_e \frac{dm_{cv}}{dt}$. Integrating with respect to time, we get

Remains constant

$$m_2 u_2 - m_1 u_1 = h_e[m_2 - m_1] \qquad (1)$$

or, on rearrangement

$$m_2[h_e - u_2] = m_1[h_e - u_1] \qquad (2)$$

PROBLEM 4.123 (Continued)

Inserting know quantities, Eq.(2) reads

$$\left(\frac{50\,ft^3}{v_2}\right)\left[631.02 - u_2\right]\frac{Btu}{lb} = (927.6\,lb)\left[631.02 - 162.01\right]\frac{Btu}{lb}$$

$$(ft^3/lb) \qquad = 435{,}053.7\ Btu$$

Alternatively,

$$50\left[631.02 - u_2\right] = (435{,}053.7)\, v_2 \qquad (3)$$

$$ft^3 \quad Btu/lb \qquad Btu \quad ft^3/lb$$

① Steam Table Solution: Since

$$v_2 = v_{f2} + x_2(v_{g2} - v_{f2})$$
$$u_2 = u_{f2} + x_2(u_{g2} - u_{f2})$$

Eq.(3) is one equation in one unknown: x_2. Accordingly, with data from Table A-13 at 80°F, x_2 can be evaluated. This is left as an exercise.

① IT Solution:

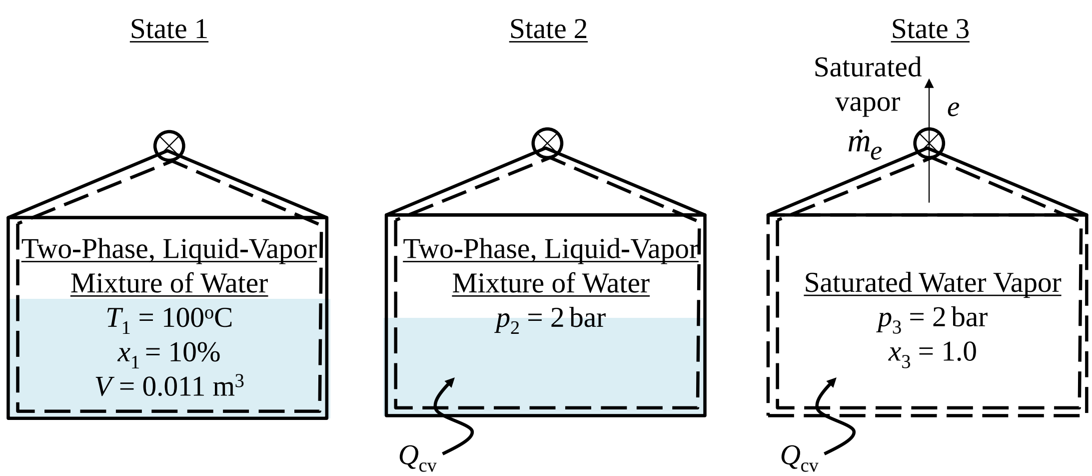
```
50* (631.02-u2)= 435053.7*v2
p2=153.13
v2 = vsat_Px("Ammonia", p2, x2)
u2 = usat_Px("Ammonia", p2, x2)
m2=50/v2

SOLUTION:

m2 = 882.1 lb
u2 = 137.8
v2 = 0.05668
x2 = 0.01558
```

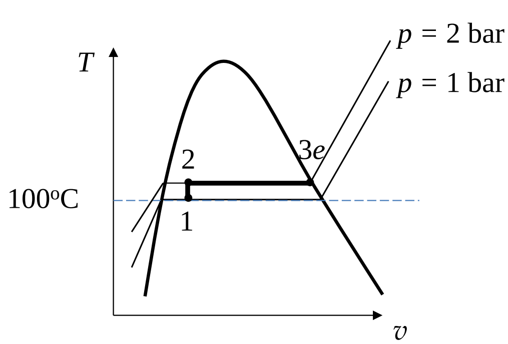
4.124 A pressure cooker has a volume of 0.011 m^3 and initially contains a two-phase liquid-vapor mixture of H_2O at a temperature of 100°C and a quality of 10%. As the water is heated at constant volume the pressure rises to 2 bar and the quality becomes 18.9%. With further heating a pressure-regulating valve keeps the pressure constant in the cooker at 2 bar by allowing saturated vapor at 2 bar to escape. Neglecting kinetic and potential energy effects
(a) determine the quality of the H_2O at the initial onset of vapor escape (state 2) and the amount of heat transfer, in kJ, to reach this state.
(b) determine the final mass in the cooker, in kg, and the additional amount of heat transfer, in kJ, if heating continues from state 2 until the final quality is 1.0.
(c) plot the quantities of part (b) versus quality increasing from the value at state 2 to 100%.

KNOWN: Water is heated in a pressure cooker that allows steam to escape.

FIND: (a) Determine the quality of the H_2O when at the initial onset of vapor escape (state 2) and the amount of heat transfer, in kJ, to reach this state, (b) determine the final mass in the cooker, in kg, and the additional amount of heat transfer, in kJ, if heating continues from state 2 until the final quality is 1.0, and (c) plot the quantities of part (b) versus quality increasing from the value at state 2 to 100%.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL:

1. For part (a) the closed system during process 1-2 is defined by the dashed line on the accompanying figure. For part (b) the control volume during process 2-3 is defined by the dashed line on the accompanying figure.
2. For the system and the control volume, kinetic and potential energy effects can be ignored and $\dot{W}_{\text{cv}} = 0$.
3. For part (b), saturated vapor exits the control volume at 2 bar.

ANALYSIS: (a) Process 1-2 from the initial state until the onset of vapor escape is analyzed as a closed system. Since the mass is constant during process 1-2 and with assumption 2 in the engineering model, the energy rate balance for process 1-2 reduces to

$$\Delta U = Q$$

$$m(u_2 - u_1) = Q$$

The mass of water can be determined from the volume of the cooker and the specific volume of the water at state 1. At x_1 = 10% and T_1 = 100°C, water is two-phase liquid-vapor mixture. From Table A-2, v_{f1} = 0.0010435 m^3/kg and v_{g1} = 1.673 m^3/kg. Solving for specific volume at state 1 gives

$$v_1 = v_{f1} + x_1(v_{g1} - v_{f1})$$

$$v_1 = 0.0010435\ \text{m}^3/\text{kg} + (0.1)(1.673\ \text{m}^3/\text{kg} - 0.0010435\ \text{m}^3/\text{kg}) = 0.16823915\ \text{m}^3/\text{kg}$$

Solving for the mass gives

$$m_1 = \frac{V}{v_1} = \frac{(0.011\,\text{m}^3)}{0.16823915\,\frac{\text{m}^3}{\text{kg}}} = 0.0654\ \text{kg}$$

The specific internal energy at state 1 is determined from

$$u_1 = u_{f1} + x_1(u_{g1} - u_{f1})$$

From Table A-2, u_{f1} = 418.94 kJ/kg and u_{g1} = 2506.5 kJ/kg. Solving for specific internal energy gives

$$u_1 = 418.94\ \text{kJ/kg} + (0.1)(2506.5\ \text{kJ/kg} - 418.94\ \text{kJ/kg}) = 627.70\ \text{kJ/kg}$$

Since volume and mass are constant during process 1-2, $v_2 = v_1$ = 0.16823915 m^3/kg.

State 2 is fixed using p_2 = 2 bar and v_2 = 0.16823915 m^3/kg. Since state 2 is a two-phase liquid-vapor mixture, quality is determined by

$$x_2 = \frac{v_2 - v_{f\,2}}{v_{g2} - v_{f\,2}}$$

From Table A-3, v_{f2} = 0.0010605 m^3/kg and v_{g2} = 0.8857 m^3/kg. Solving for quality

$$x_2 = \frac{(0.16823915\,\frac{\text{m}^3}{\text{kg}} - 0.0010605\,\frac{\text{m}^3}{\text{kg}})}{(0.8857\,\frac{\text{m}^3}{\text{kg}} - 0.0010605\,\frac{\text{m}^3}{\text{kg}})} = \underline{\mathbf{0.1890}}$$

The specific internal energy at state 2 is determined from

$$u_2 = u_{f2} + x_2(u_{g2} - u_{f2})$$

From Table A-3, u_{f2} = 504.49 kJ/kg and u_{g2} = 2529.5 kJ/kg. Solving for specific internal energy gives

$$u_2 = 504.49 \text{ kJ/kg} + (0.1890)(2529.5 \text{ kJ/kg} - 504.49 \text{ kJ/kg}) = 887.2 \text{ kJ/kg}$$

Solving for heat transfer during process 1-2

$$Q = (0.0654 \text{ kg})(887.2 \text{ kJ/kg} - 627.70 \text{ kJ/kg}) = \underline{\mathbf{16.97 \text{ kJ}}}$$

(b) For process 2-3 the mass rate balance reduces to

$$\frac{dm_{\text{cv}}}{dt} = -\dot{m}_e$$

The energy rate balance reduces to

$$\frac{dU_{\text{cv}}}{dt} = \dot{Q}_{\text{cv}} - \cancel{\dot{W}_{\text{cv}}} + \cancel{\dot{m}_i} h_i - \dot{m}_e h_e$$

There is no work, and no mass enters the control volume. Noting h_e remains constant, combining mass and energy rate balances results in

$$\frac{dU_{\text{cv}}}{dt} = \dot{Q}_{\text{cv}} + \frac{dm_{\text{cv}}}{dt} h_e$$

$$(m_3 u_3 - m_2 u_2) = \dot{Q}_{\text{cv}} \Delta t + h_e(m_3 - m_2) = Q_{\text{cv}} + h_e(m_3 - m_2)$$

$$Q_{\text{cv}} = m_3(u_3 - h_e) - m_2(u_2 - h_e)$$

The final mass of water can be determined from the volume of the cooker and the specific volume of the water at state 3. From Table A-3, $v_3 = v_{g3} = 0.8857\ \text{m}^3/\text{kg}$. Solving for the mass gives

$$m_3 = \frac{V}{v_3} = \frac{(0.011\,\text{m}^3)}{0.8857\,\frac{\text{m}^3}{\text{kg}}} = 0.0124\ \text{kg}$$

The specific internal energy for saturated vapor at state 3 is obtained from Table A-3 at $p_3 = 2$ bar: $u_3 = u_{g3} = 2529.5$ kJ/kg.

The specific enthalpy for saturated vapor at the exit is obtained from Table A-3 at $p_e = 2$ bar: $h_e = h_{ge} = 2706.7$ kJ/kg.

Solving for heat transfer during process 2-3

$$Q_{\text{cv}} = (0.0124\,\text{kg})(2529.5\frac{\text{kJ}}{\text{kg}} - 2706.7\frac{\text{kJ}}{\text{kg}}) - (0.0654\,\text{kg})(887.1\frac{\text{kJ}}{\text{kg}} - 2706.7\frac{\text{kJ}}{\text{kg}}) = \underline{\mathbf{116.80\ kJ}}$$

(c) IT is used to generate the plots:

<u>IT Code</u>

```
//Given
V = 0.011//m^3

//State 1
T_1 = 100//oC
x_1 = 0.1
p_1 = Psat_T("Water/Steam", T_1)
v_1 = vsat_Px("Water/Steam", p_1, x_1)
u_1 = usat_Px("Water/Steam", p_1, x_1)
m_1 = V/v_1

//State 2
p_2 = 2//bar
v_2 = v_1
m_2 = m_1
x_2 = x_vP("Water/Steam", v_2, p_2)
u_2 = usat_Px("Water/Steam", p_2, x_2)

//State 3
p_3 = p_2
x_3 = 1.0
v_3 = vsat_Px("Water/Steam", p_3, x_3)
u_3 = usat_Px("Water/Steam", p_3, x_3)
m_3 = V/v_3

//State e (exit)
p_e = p_3
x_e = 1
h_e = hsat_Px("Water/Steam", p_e, x_e)
```

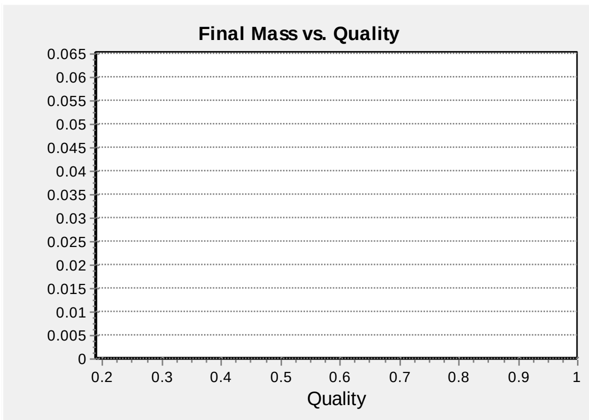
<u>IT Results: $x_3 = 1.0$</u>

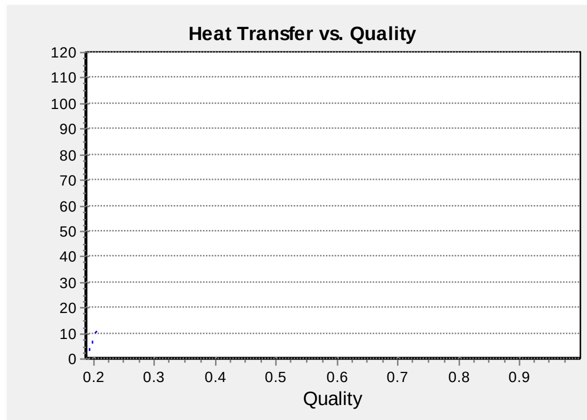
h_e	2706
m_1	0.06539
m_2	0.06539
m_3	0.01242
p_1	1.013
p_3	2
p_e	2
Q_12	16.93
Q_23	116.7
u_1	628.1
u_2	887
u_3	2529
v_1	0.1682
v_2	0.1682
v_3	0.8857
x_2	0.189
p_2	2
T_1	100
V	0.011
x_1	0.1
x_3	1

//Heat Transfers
Q_12 = m_1*(u_2 - u_1)
Q_23 = m_3*(u_3 - h_e) - m_2*(u_2 - h_e)

The results for x_3 = 1.0 agree with the result of part (b).

Now using the Explore button, sweep x_3 from 0.189 to 1.0 in steps of 0.001. The following plots are constructed from the data.





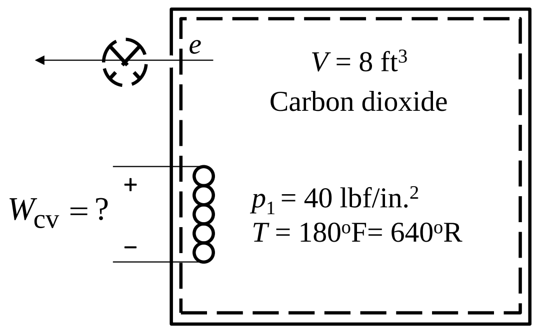
We see from the plots that as the quality of the mixture within the tank increases the mass in the tank drops rapidly while heat transfer to the tank rapidly increases.

4.125 A well-insulated rigid tank of volume 8 ft^3 initially contains carbon dioxide at 180^oF and 40 lb/in.^{2} A valve connected to the tank is opened, and carbon dioxide is withdrawn slowly until the pressure within the tank drops to p. An electrical resistor inside the tank maintains the temperature at 180^oF. Modeling carbon dioxide as an ideal gas and neglecting kinetic and potential energy effects
(a) determine the mass of carbon dioxide withdrawn, in lb, and the energy input to the resistor, in Btu, when p = 22 lb/in.^{2}
(b) plot the quantities of part (a) versus p ranging from 15 to 40 lbf/in.^{2}

KNOWN: Carbon dioxide is withdrawn slowly from a tank.

FIND: (a) Determine the mass of carbon dioxide withdrawn, in lb, and the energy input to the resistor, in Btu, when p = 22 lb/in.^{2} and (b) plot the quantities of part (a) versus p ranging from 15 to 40 lbf/in.^{2}

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume is defined by the dashed line on the accompanying figure.
2. For the control volume, kinetic and potential energy effects can be ignored and $\dot{Q}_{cv} = 0$.
3. Carbon dioxide is modeled as an ideal gas.
4. The mass of the resistor is small enough to be ignored.

ANALYSIS: (a) The mass of the carbon dioxide withdrawn over a time interval equals the difference between the initial amount of mass in the tank and the mass in the tank at a later time:

$$\text{(Amount Withdrawn)} = m_1 - m_2$$

Since $T_1 = T_2$, the ideal gas equation of state can be used to rewrite the expression for (Amount Withdrawn) as follows where $p_2 = p$:

$$\text{(Amount Withdrawn)} = \frac{p_1 V}{RT} - \frac{p_2 V}{RT} = (p_1 - p_2)\left(\frac{V}{RT}\right)$$

$$(\text{Amount Withdrawn}) = \left(40\,\frac{\text{lbf}}{\text{in.}^2} - p\right)\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right|\left(\frac{8\ \text{ft}^3}{\left(\dfrac{1545\,\dfrac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot{}^\circ\text{R}}}{44.01\,\dfrac{\text{lb}}{\text{lbmol}}}\right)(640^\circ\text{R})}\right)$$

$$(\text{Amount Withdrawn}) = 0.0513(40 - p)\ \text{lb} \qquad (1)$$

The mass rate balance reduces to

$$\frac{dm_{\text{cv}}}{dt} = -\dot{m}_e$$

The energy rate balance reduces to

$$\frac{dU_{\text{cv}}}{dt} = \cancel{\dot{Q}_{\text{cv}}} - \dot{W}_{\text{cv}} + \cancel{\dot{m}_i h_i} - \dot{m}_e h_e$$

Introducing the mass rate balance and noting that h_e remains constant since temperature remains constant result in

$$\frac{dU_{\text{cv}}}{dt} = -\dot{W}_{\text{cv}} + \frac{dm_{\text{cv}}}{dt} h_e$$

$$(m_2 u_2 - m_1 u_1) = -W_{\text{cv}} + h_e(m_2 - m_1)$$

$$W_{\text{cv}} = -(m_2 u_2 - m_1 u_1) + h_e(m_2 - m_1)$$

Since temperature is constant, $u_2 = u(T)$, $u_1 = u(T)$, and $h_e = \text{u}(T) + (pv)_e$. Thus,

$$W_{\text{cv}} = -m_2 u(T) + m_1 u(T) + m_2\big((u(T) + (pv)_e\big) - m_1\big((u(T) + (pv)_e\big)$$

$$W_{\text{cv}} = m_2\big(\cancel{u(T) - u(T)}\big) + m_1\big(\cancel{u(T) - u(T)}\big) + (m_2 - m_1)(pv)_e$$

1 $$W_{\text{cv}} = (m_2 - m_1)(pv)_e$$

Substituting $m_2 - m_1 = -(\text{Amount Withdrawn}) = -0.0513(40 - p)$ lb and $(pv)_e = RT$ gives

$$W_{\text{cv}} = \big(-0.0513(40 - p)\ \text{lb}\big)RT$$

$$W_{cv} = (-0.0513(40-p)\text{ lb})\left(\frac{1545\dfrac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot{}^\circ\text{R}}}{44.01\dfrac{\text{lb}}{\text{lbmol}}}\right)(640^\circ\text{R})\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right|$$

$$W_{cv} = -1.481(40 - p)\text{ Btu} \qquad (2)$$

When p = 22 lbf/in.^{2}, Eq. (1) gives

(Amount Withdrawn) = **0.9234 lb**

Eq. (2) gives

W_{cv} = **–26.66 Btu**

The negative sign associated with work indicates work is to the control volume as expected.

(b) The final working expressions for both the amount of mass withdrawn and the energy input to the resistor are linear in p. One can either hand-sketch a plot based on sample data or use IT to generate the plots. Results using IT are:

IT Code

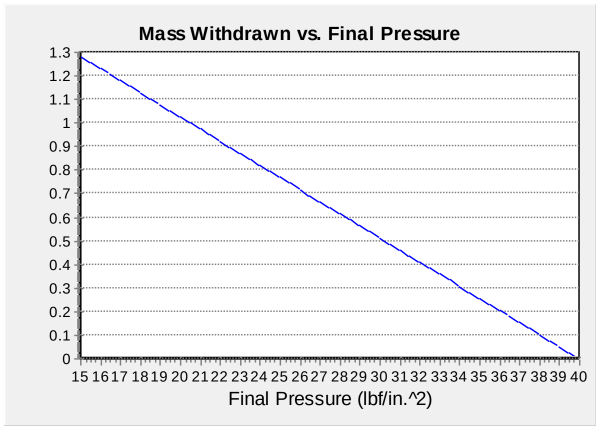
```
//Given
V = 8//ft^3

//State 1
T = 640//oR
p_1 = 40//lbf/in.^2

//State 2
p = 22//lbf/in.^2

//Calclulations
m_Withdrawn = 0.0513*(40 - p)//lb
W_cv = -1.481*(40-p)//Btu
```

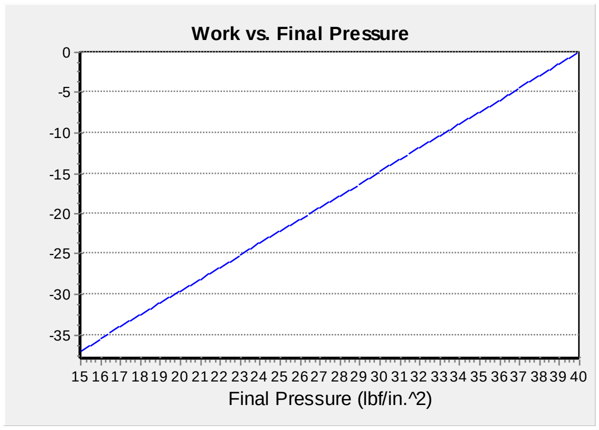
IT Results: p = 22 lbf/in.^{2}

m_Withdrawn	0.9234
W_cv	-26.66
p	22
p_1	40
T	640
V	8

The results for p = 22 lbf/in.^{2} compare favorably with the result of part (b).

Now using the Explore button, sweep p from 15 lbf/in.^{2} to 40 lbf/in.^{2} in steps of 0.05. The following plots are constructed from the data.





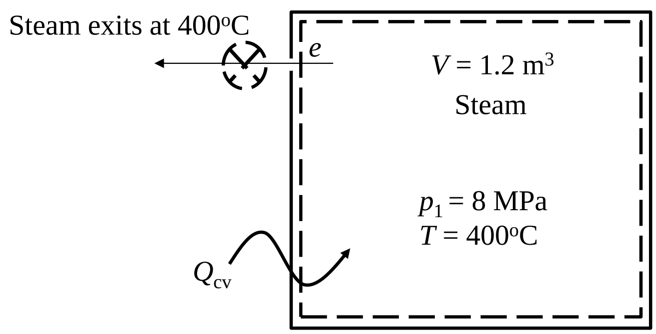
1 ***To maintain the temperature of the carbon dioxide within the tank constant, the resistor must provide energy to the carbon dioxide equal to the energy carried out at e by the specific flow work, which is denoted $(pv)_e$ in this expression.***

4.126 A tank of volume 1.2 m^3 initially contains steam at 8 MPa and 400^{o}C. Steam is withdrawn slowly from the tank until the pressure drops to p. Heat transfer to the tank contents maintains the temperature at 400^{o}C. Neglecting all kinetic and potential energy effects and assuming specific enthalpy h_e is nearly linear with the mass in the tank
(a) determine the heat transfer, in kJ, if p = 2 MPa.
(b) plot the heat transfer, in kJ, versus p ranging from 0.5 to 8 MPa.

KNOWN: Steam is withdrawn slowly from a tank.

FIND: (a) Determine the heat transfer, in kJ, if p = 2 MPa and (b) plot the heat transfer, in kJ, versus p ranging from 0.5 to 8 MPa.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume is defined by the dashed line on the accompanying figure.
2. For the control volume, kinetic and potential energy effects can be ignored and $\dot{W}_{cv} = 0$.
3. At each instant, pressure is uniform throughout the steam.
4. Specific enthalpy h_e is nearly linear with the mass in the tank.

ANALYSIS: (a) The mass rate balance reduces to

$$\frac{dm_{cv}}{dt} = -\dot{m}_e$$

While the energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \cancel{\dot{m}_i h_i} - \dot{m}_e h_e$$

Introducing the mass rate balance and integrating

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} + \frac{dm_{cv}}{dt} h_e$$

$$(m_2u_2 - m_1u_1) = Q_{cv} + \int_1^2 h_e dm$$

$$Q_{cv} = m_2u_2 - m_1u_1 - \int_1^2 h_e dm \tag{1}$$

where m denotes the mass contained within the tank.

Since the specific enthalpy h_e is nearly linear with the mass in the tank, the average value of h_e can be used to evaluate $-\int_1^2 h_e dm = \left(\frac{(h_e)_1 + (h_e)_2}{2}\right)(m_1 - m_2)$.

At any instant $m = V/v$ where v is specific volume determined at that instant by 400°C and the tank pressure. Initially, p_1 = 8 MPa, so Table A-4 gives v_1 = 0.03432 m³/kg, u_1 = 2863.8 kJ/kg, and $(h_e)_1$ = 3138.3 kJ/kg. Thus,

$$m_1 = \frac{V}{v_1} = \frac{1.2\ \text{m}^3}{0.03432\ \frac{\text{m}^3}{\text{kg}}} = 34.965\ \text{kg}$$

Finally, p_2 = 2 MPa, so at 400°C Table A-4 gives v_2 = 0.1512 m³/kg, u_2 = 2945.2 kJ/kg, and $(h_e)_2$ = 3247.6 kJ/kg. Thus,

$$m_2 = \frac{V}{v_2} = \frac{1.2\ \text{m}^3}{0.1512\ \frac{\text{m}^3}{\text{kg}}} = 7.9365\ \text{kg}$$

Substituting values gives

$$-\int_1^2 h_e dm = \left(\frac{(h_e)_1 + (h_e)_2}{2}\right)(m_1 - m_2) = \left(\frac{3138.3\ \frac{\text{kJ}}{\text{kg}} + 3247.6\ \frac{\text{kJ}}{\text{kg}}}{2}\right)(34.965\ \text{kg} - 7.9364\ \text{kg})$$

$$-\int_1^2 h_e dm = 86{,}301\ \text{kJ}$$

Inserting values into Eq. (1)

$$Q_{cv} = (7.9364\ \text{kg})\left(2945.2\ \frac{\text{kJ}}{\text{kg}}\right) - (34.965\ \text{kg})\left(2863.8\ \frac{\text{kJ}}{\text{kg}}\right) + 86{,}301\ \text{kJ} = \underline{\mathbf{9542.5\ kJ}}$$

Note: The nearly linear relationship between specific enthalpy h_e and mass in the tank can be shown using IT. For each mass in the interval 7.9365 kg < m < 34.965 kg, IT can determine the corresponding exit enthalpy h_e by the two independent, intensive properties, T = 400°C and v = V/m. These values allow the plot of h_e vs. m to be constructed as done below using IT.

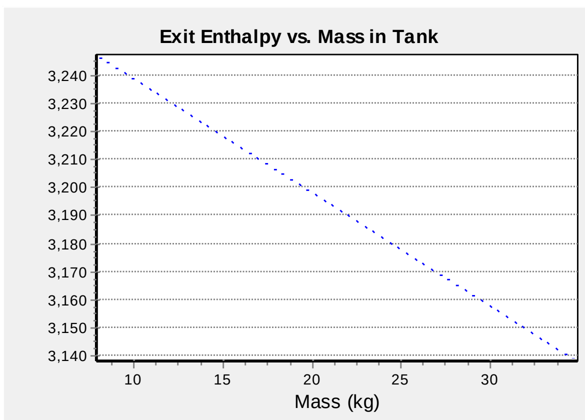
IT Code

```
//Given
V = 1.2//m^3

//Range of mass spans 7.9365 kg to 34.965 kg

T = 400//oC
m = 7.9365//kg
v = V/m
v = v_PT("Water/Steam", p, T)
h_e = h_PT("Water/Steam", p, T)
```

IT Results: m = 7.9365 kg

h_e	3247
m	7.937
v	0.1512
p	2000
T	400
V	1.2

Now using the Explore button, sweep m from 7.9364 to 34.965 kg in steps of 0.1. The following plot is constructed from the data.



The area under the line from 1 to 2 equals the term $-\int_1^2 h_e dm$ of Eq. (1). As this variation is very nearly linear, the average value of h_e is appropriate to evaluate this term.

$$-\int_1^2 h_e dm = \left(\frac{(h_e)_1 + (h_e)_2}{2}\right)(m_1 - m_2)$$

(b) IT is used to generate the plot:

IT Code

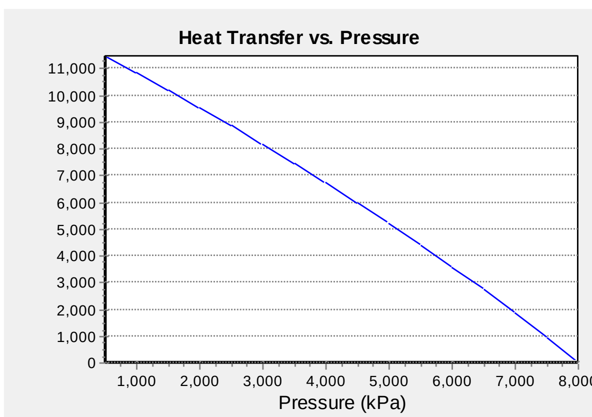
```
//Given
V = 1.2//m^3
T = 400//°C

//Initial State 1
p_1 = 8000//kPa
v_1 = v_PT("Water/Steam", p_1, T)
u_1 = u_PT("Water/Steam", p_1, T)
h_e_1 = h_PT("Water/Steam", p_1, T)
m_1 = V/v_1

//Final State
p = 2000//kPa
v = v_PT("Water/Steam", p, T)
u = u_PT("Water/Steam", p, T)
h_e = h_PT("Water/Steam", p, T)
m = V/v

//Calculation
Q_cv = (m*u) - (m_1*u_1) + ((h_e_1 + h_e)/2)*(m_1 - m)
```

IT Results: p = 2000 kPa

h_e	3247
h_e_1	3138
m	7.937
m_1	34.97
Q_cv	9545
u	2945
u_1	2863
v	0.1512
v_1	0.03432
p	2000
p_1	8000
T	400
V	1.2

The results for p = 2000 kPa compare favorably with the result of part (a).

Now using the Explore button, sweep p from 500 kPa to 8000 kPa in steps of 500. The following plot is constructed from the data.



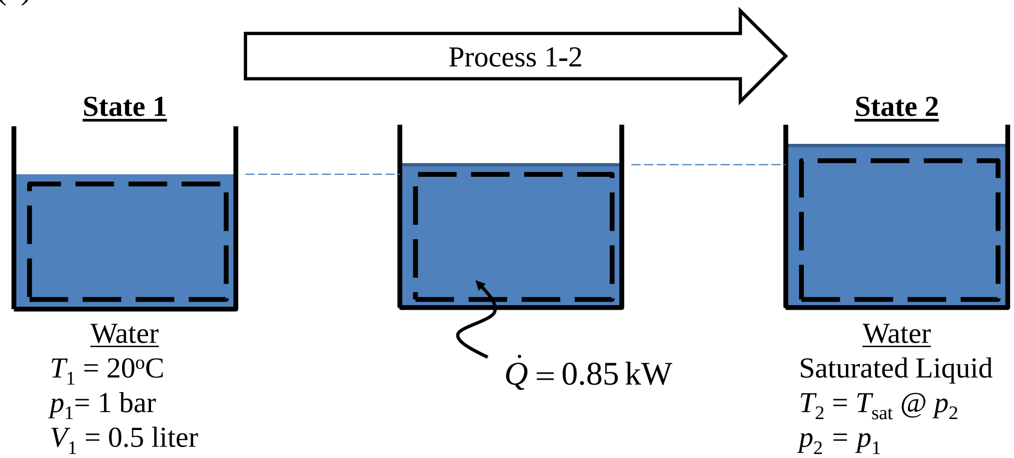
The pressure within the tank decreases as mass is withdrawn while the temperature remains constant.

4.127 An open cooking pot containing 0.5 liter of water at 20°C, 1 bar sits on a stove burner. Once the burner is turned on the water is gradually heated at a rate of 0.85 kW while pressure remains constant. After a period of time, the water starts boiling and continues to do so until all of the water has evaporated. Determine
(a) the time required for the onset of evaporation, in s.
(b) the time required for all of the water to evaporate, in s, once evaporation starts.

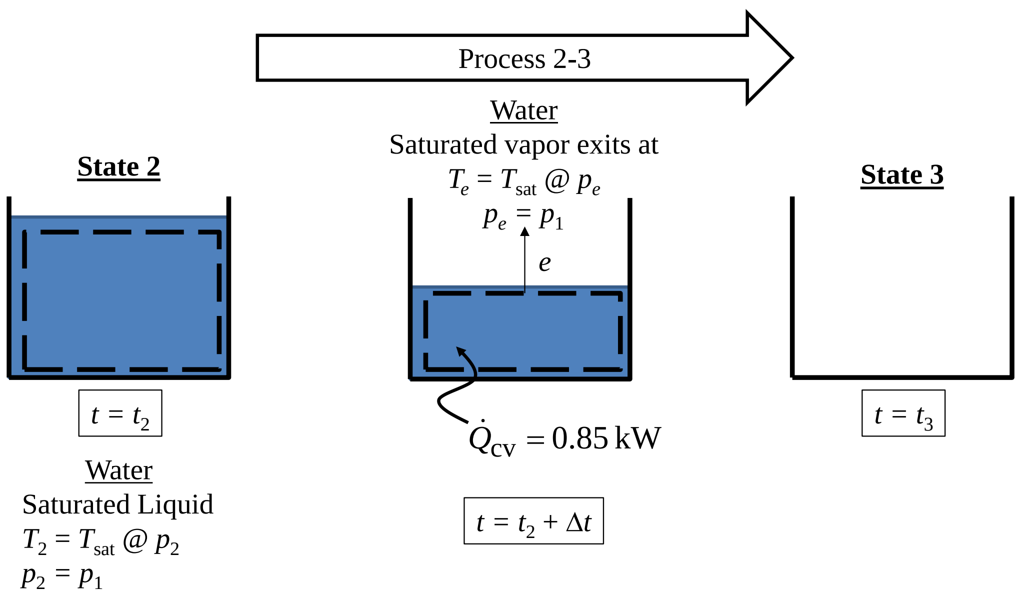
KNOWN: Water is heated in an open pot on a burner.

FIND: (a) The time required for the onset of evaporation, in s, and (b) the time required for all of the water to evaporate, in s, once evaporation starts.

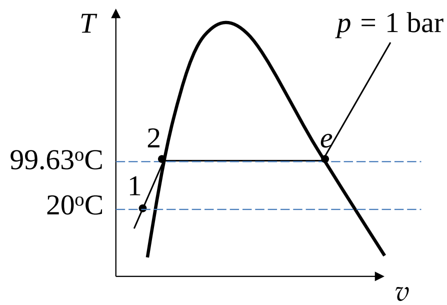
SCHEMATIC AND GIVEN DATA:
Part (a)



Part (b)



The temperature-specific volume diagram below indicates the states associated with the overall process:



ENGINEERING MODEL:

1. For part (a) the liquid water inside the pot defines the system during process 1-2 as shown by the dashed line on the accompanying figure. For part (b) the control volume is defined by the boundary of the liquid water inside the pot during process 2-3 as shown by the dashed line on the accompanying figure.
2. For the system and control volume, kinetic and potential effects can be ignored.
3. As shown in the figure of part (b), saturated vapor exits the control volume at 1 bar.
4. Water is assumed to remain at 1 bar throughout the entire process.
5. Work associated with system boundary movement (part a) and work associated with control volume boundary movement (part b) are not ignored.

ANALYSIS:

(a) During process 1-2, heat transfer results in a temperature increase from 20°C to 99.63°C (the saturation temperature that corresponds to 1 bar). Since evaporation has not yet started, there is no mass flow of water into or from the system, and the water can be analyzed as a closed system. Since the mass is constant during process 1-2 and with assumption 2 in the engineering model, the energy rate balance for process 1-2 reduces to

1

$$\Delta U = Q - W$$

or

$$m(u_2 - u_1) = \dot{Q}\Delta t - W$$

Solving for Δt gives

$$\Delta t = \frac{m(u_2 - u_1) + W}{\dot{Q}}$$

The mass of water can be determined from the volume of the liquid water and its corresponding specific volume at state 1. At p_1 = 1 bar and T_1 = 20°C, water is compressed liquid. From Table A-2, $v_1 \approx v_{f1}$ = 0.0010018 m^3/kg

$$m = \frac{V_1}{v_1} = \frac{(0.5\,\text{L})}{0.0010018\,\frac{\text{m}^3}{\text{kg}}}\left|\frac{10^{-3}\,\text{m}^3}{1\,\text{L}}\right| = 0.50\text{ kg}$$

The specific internal energy at state 1 is determined from Table A-2, $u_1 \approx u_{f1}$ = 83.95 kJ/kg. The water is saturated liquid at state 2. From Table A-3 at p_2 = 1 bar, $u_2 = u_{f2}$ = 417.36 kJ/kg.

As the liquid water is heated, its volume increases slightly. Thus, *work is done by the system at the top surface* during process 1-2

$$W = \int_1^2 p dV = mp(v_2 - v_1)$$

From Table A-3, $v_2 = v_{f2}$ = 0.0010432 m^3/kg. Solving for work

$$W = (0.50\text{ kg})(1\text{ bar})(0.0010432\frac{\text{m}^3}{\text{kg}} - 0.0010018\frac{\text{m}^3}{\text{kg}})\left|\frac{100\text{ kPa}}{1\text{ bar}}\right|\left|\frac{1000\ \frac{\text{N}}{\text{m}^2}}{1\text{ kPa}}\right|\left|\frac{1\text{ kJ}}{1000\text{ N}\cdot\text{m}}\right|$$

$$W = 0.00207\text{ kJ}$$

Substituting values and solving for Δt give

$$\Delta t = \frac{(0.50\text{ kg})(417.36\frac{\text{kJ}}{\text{kg}} - 83.95\frac{\text{kJ}}{\text{kg}}) + 0.00207\text{ kJ}}{0.85\text{ kW}}\left|\frac{1\text{ kW}}{1\frac{\text{kJ}}{\text{s}}}\right| = \underline{\mathbf{196.1\ s}}\ \mathbf{(3.27\ min)}$$

(b) During process 2-3, heat transfer results in phase change from liquid to vapor at 99.63$^{\text{o}}$C (the saturation temperature that corresponds to 1 bar). Mass flow from the control volume is the saturated vapor resulting from evaporation of the liquid. The mass rate balance takes the form

$$\frac{dm_{\text{cv}}}{dt} = -\dot{m}_e$$

The energy rate balance for process 2-3 reduces to

$$\frac{dU_{\text{cv}}}{dt} = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} - m_e h_e$$

Combining mass and energy rate balances results in

$$\frac{dU_{\text{cv}}}{dt} = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \frac{dm_{\text{cv}}}{dt} h_e$$

By assumption 3 of the engineering model, the specific enthalpy at the exit is constant. Accordingly, integration gives

$$\Delta U_{\text{cv}} = \dot{Q}_{\text{cv}}\Delta t - \dot{W}_{\text{cv}}\Delta t + \Delta m_{\text{cv}} h_e$$

$$m_3u_3 - m_2u_2 = \dot{Q}_{cv}\Delta t - W_{cv} + (m_3 - m_2)h_e$$

Mass at state 2 is the system mass during process 1-2: $m_2 = 0.50$ kg. Since no mass remains in the control volume at state 3, $m_3 = 0$. The equation reduces to

2

$$-m_2u_2 = \dot{Q}_{cv}\Delta t - W_{cv} - m_2h_e$$

Solving for time associated with evaporation during process 2-3 yields

$$\Delta t = \frac{m_2(h_e - u_2) + W_{cv}}{\dot{Q}_{cv}}$$

The specific enthalpy for saturated vapor at the exit is obtained from Table A-3 at $p_e = 1$ bar: $h_e = h_{ge} = 2675.5$ kJ/kg.

The liquid volume decreases as vapor exits the control volume. Thus, *work is done on the control volume at the moving portion of the boundary* during process 2-3

$$W_{cv} = \int_2^3 pdV = p(V_3 - V_2) = -pV_2 = -m_2p_2v_2$$

Substituting values and solving for work give

$$W_{cv} = -(0.50\,\text{kg})(1\,\text{bar})(0.0010432\frac{\text{m}^3}{\text{kg}})\left|\frac{100\,\text{kPa}}{1\,\text{bar}}\right|\left|\frac{1000\,\frac{\text{N}}{\text{m}^2}}{1\,\text{kPa}}\right|\left|\frac{1\,\text{kJ}}{1000\,\text{N}\cdot\text{m}}\right|$$

$$W_{cv} = -0.0522\text{ kJ}$$

Solving for time associated with evaporation during process 2-3 yields

$$\Delta t = \frac{(0.50\,\text{kg})(2675.5\frac{\text{kJ}}{\text{kg}} - 417.36\frac{\text{kJ}}{\text{kg}}) + (-0.0522\,\text{kJ})}{0.85\,\text{kW}}\left|\frac{1\,\text{kW}}{1\frac{\text{kJ}}{\text{s}}}\right| = \underline{\mathbf{1328.3\ s}}\ \mathbf{(22.1\ min)}$$

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1 For process 1-2 a combination of the energy balance and the work expression gives an expression for energy represented in terms of specific enthalpy only.

$$m(u_2 - u_1) = \dot{Q}\Delta t - W = \dot{Q}\Delta t - mp(v_2 - v_1)$$

$$m[(u_2 + pv_2) - (u_1 + pv_1)] = \dot{Q}\Delta t$$

$$m(h_2 - h_1) = \dot{Q}\Delta t$$

2 For process 2-3 a combination of the energy balance and the work expression gives an expression for energy represented in terms of specific enthalpy only.

$$-m_2u_2 = \dot{Q}_{\text{cv}}\Delta t - W_{\text{cv}} - m_2h_e = \dot{Q}_{\text{cv}}\Delta t - (-m_2p_2v_2) - m_2h_e$$

$$-m_2(u_2 + p_2v_2) = \dot{Q}_{\text{cv}}\Delta t - m_2h_e$$

$$-m_2h_2 = \dot{Q}_{\text{cv}}\Delta t - m_2h_e$$

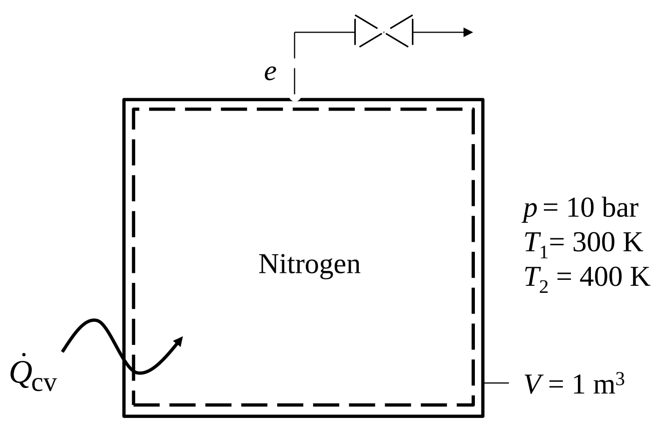
$$-m_2(h_2 - h_e) = \dot{Q}_{\text{cv}}\Delta t$$

4.128 Nitrogen gas is contained in a rigid 1-m^3 tank, initially at 10 bar, 300 K. Heat transfer to the contents of the tank occurs until the temperature has increased to 400 K. During the process, a pressure-relief valve allows nitrogen to escape, maintaining constant pressure in the tank. Neglecting kinetic and potential energy effects, and using the ideal gas model with constant specific heats evaluated at 350 K, determine the mass of nitrogen that escapes, in kg, and the amount of energy transfer by heat, in kJ.

KNOWN: Heat transfer occurs to nitrogen gas contained in a rigid tank. Gas escapes through a pressure relief valve, maintaining constant pressure in the tank. The initial and final temperatures are specified.

FIND: Determine the mass of nitrogen that escapes, in kg, and the amount of energy transfer by heat, in kJ.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

1. For the control volume shown, $\dot{W}_{cv} = 0$.
2. The state in the control volume can be assumed to be uniform at any time during the process
3. Kinetic and potential energy effects can be neglected.
4. The nitrogen behaves as an ideal gas with constant specific heats evaluated at 350 K.

ANALYSIS:

The mass rate balance takes the form, $\dfrac{dm_{cv}}{dt} = -\dot{m}_e$. Thus, the mass that escapes is

$$\int_1^2 \dot{m}_e dt = -\int_{m_1}^{m_2} dm_{cv} = m_1 - m_2$$

With the ideal gas equation of state

$$\int_1^2 \dot{m}_e dt = \frac{pV}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\int_1^2 \dot{m}_e dt = \frac{(10\text{ bar})(1\text{ m}^3)}{\left(\frac{8.314}{28.01}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)}\left[\frac{1}{300\text{ K}} - \frac{1}{400\text{ K}}\right]\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| = \underline{\mathbf{2.808\ kg}}$$

With the assumptions listed, the energy rate balance is

$$\frac{dU_{\text{cv}}}{dt} = \dot{Q}_{\text{cv}} - \dot{m}_e h_e$$

Noting that $U_{\text{cv}} = mu$ and $h_e = u + RT$, and with the mass rate balance

$$m\frac{du}{dt} + \cancel{u\frac{dm}{dt}} = \dot{Q}_{\text{cv}} + (\cancel{u} + RT)\frac{dm}{dt}$$

$$\dot{Q}_{\text{cv}} = m\frac{du}{dt} - RT\frac{dm}{dt}$$

For an ideal gas, $du = c_v dT$. Also, with $m = pV/RT$

$$dm = \left(\frac{pV}{R}\right)\frac{dT}{(-T^2)}$$

Thus

$$\dot{Q}_{\text{cv}} = \left(\frac{pV}{RT}\right)c_v\frac{dT}{dt} + \left(\frac{pV}{RT}\right)R\frac{dT}{dt} = \left(\frac{pV}{R}\right)(c_v + R)\frac{1}{T}\frac{dT}{dt}$$

Noting that $c_v + R = c_p$

$$\dot{Q}_{\text{cv}}dt = \left(\frac{pVc_p}{R}\right)\frac{dT}{T}$$

From Table A-20, $c_p = 1.041$ kJ/kg·K. Integrating and inserting values

$$\int_1^2 \dot{Q}_{\text{cv}}dt = \left(\frac{pVc_p}{R}\right)\int_{T_1}^{T_2}\frac{dT}{T}$$

$$Q_{\text{cv}} = \left(\frac{pVc_p}{R}\right)\ln\left(\frac{T_2}{T_1}\right)$$

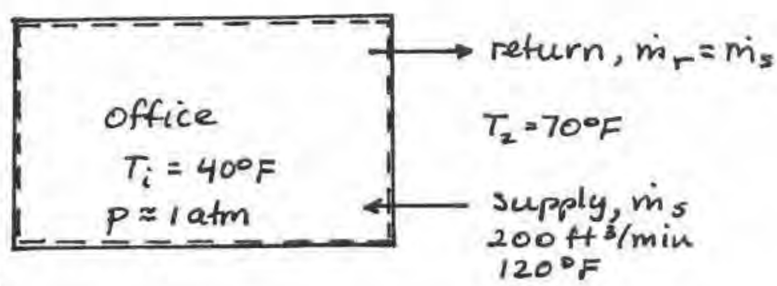
$$Q_{\text{cv}} = \left(\frac{(10\text{ bar})(1\text{ m}^3)(1.041\text{ kJ/kg}\cdot\text{K})}{\frac{8.314}{28.01}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}}\right)\ln\left(\frac{400\text{ K}}{300\text{ K}}\right)\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| = \underline{\mathbf{1009\ kJ}}$$

PROBLEM 4.129

KNOWN: The air supply to an office is shut off overnight and the room temperature drops. In the morning, the thermostat is reset and a known volumetric flow rate of heated air is supplied.

FIND: Estimate the time it takes for the room to reach 70°F, and plot room temperature as a function of time.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) For the control volume shown, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) The air behaves as an ideal gas with constant specific heats. (3) The pressure is taken as 1 atm everywhere. (4) Kinetic and potential energy effects can be neglected. (5) The room air is well-mixed.

ANALYSIS: The mass rate balance takes the form $dm/dt = \dot{m}_s - \dot{m}_r$. With $\dot{m}_s = \dot{m}_r$, $dm/dt = 0$. Thus, the mass in the room is constant with time. The energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}_s h_s - \dot{m}_r h_r$$

or, with $U_{cv} = mu$ and $h_r = h(T)$

$$m\frac{du}{dt} + u\cancelto{0}{\frac{dm}{dt}} = \dot{m}_s(h_s - h(T))$$

Further, $du = c_v\,dT$ and $h_s - h(T) = c_p(T_s - T)$. Thus

$$mc_v\frac{dT}{dt} = \dot{m}_s c_p(T_s - T)$$

With $dT = -d(T_s - T)$ and $c_p/c_v = k$

$$-\frac{m\,d(T_s - T)}{k\,(T_s - T)} = \dot{m}_s\,dt \qquad (1)$$

Evaluating $\dot{m}_s$

$$\dot{m}_s = \frac{(AV)_s}{v_s} = \frac{p\,(AV)_s}{RT_s}$$

$$= \frac{(14.7\ lbf/in^2)(200\ ft^3/min)}{\left(\frac{1545\ ft\cdot lbf}{28.97\ lb\cdot °R}\right)(580°R)}\left|\frac{144\ in^2}{1\ ft^2}\right| = 13.69\ lb/min$$

and

$$m = \frac{pV}{RT} = \frac{(14.7)(20{,}000)\,|144|}{\left(\frac{1545}{28.97}\right)(515)} = 154\ lb$$

where the average of the initial and final temperatures is used to estimate the mass. Also, from A-20E, $k = 1.4$.

Integrating Eq. (1) from $t=0$ ($T=T_i=40°F$) to any time t

$$-\left(\frac{m}{k\cdot \dot{m}_s}\right)\ln\left(\frac{T_s-T}{T_s-T_i}\right)=t$$

or, solving for T

$$T=T_s-(T_s-T_i)\exp\left[\left(-\frac{\dot{m}_s k}{m}\right)t\right]$$

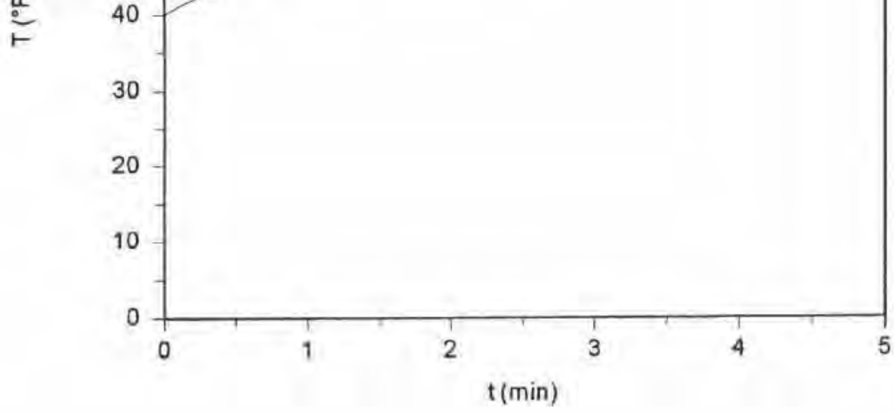
Inserting values

$$T=120-80\exp\left[(-0.1245)t\right] \qquad (2)$$

Solving for t_2 when $T_2=70°F$ gives

$$t_2=3.79\text{ min} \longleftarrow t_2$$

Eq.(2) can readily be plotted using software. The following plot was constructed using IT:

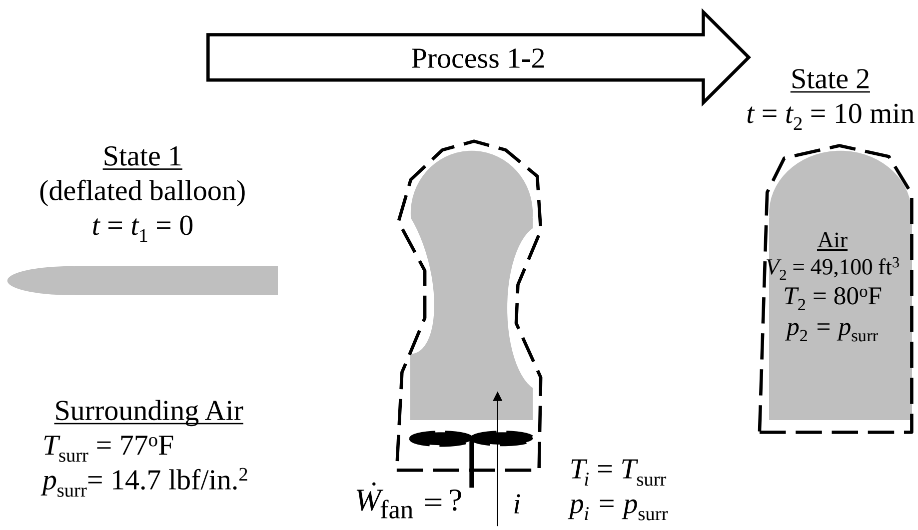


4.130 The procedure to inflate a hot air balloon requires a fan to move an initial amount of air into the balloon envelope followed by heat transfer from a propane burner to complete the inflation process. After a fan operates for 10 minutes with negligible heat transfer with the surroundings, the air in an initially deflated balloon achieves a temperature of 80°F and a volume of 49,100 ft^3. Next the propane burner provides heat transfer as air continues to flow into the balloon without use of the fan until the air in the balloon reaches a volume of 65,425 ft^3 and a temperature of 210°F. Air at 77°F and 14.7 lbf/in.^{2} surrounds the balloon. The net rate of heat transfer is 7×10^6 Btu/h. Ignoring effects due to kinetic and potential energy, modeling the air as an ideal gas, and assuming the pressure of the air inside the balloon remains the same as that of the surrounding air, determine
(a) the power required by the fan, in hp.
(b) the time required for full inflation of the balloon, in min.

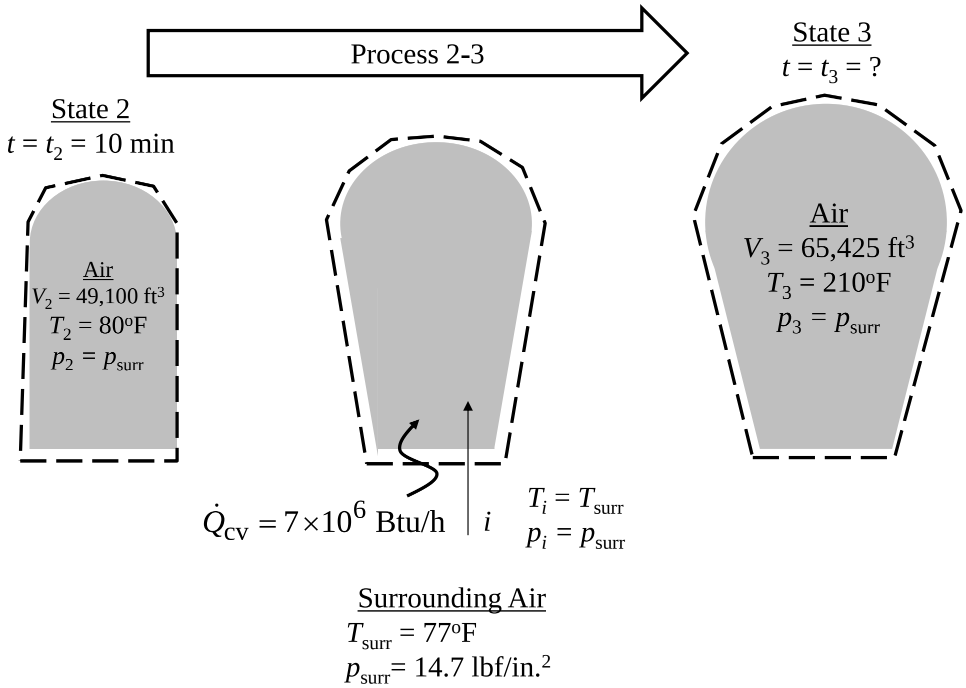
KNOWN: A hot air balloon undergoes inflation.

FIND: (a) The power required by the fan, in hp, and (b) the time required for full inflation of the balloon, in min.

SCHEMATIC AND GIVEN DATA:
Part (a)



Part (b)



ENGINEERING MODEL:

1. The control volume is defined by the dashed line on the accompanying diagrams.
2. Air is modeled as an ideal gas.
3. For the control volume, kinetic and potential energy effects can be ignored.
4. The state of the air entering the balloon remains constant and equivalent to the state of the surrounding air.
5. Air inside the balloon has the same pressure as the surrounding air during the entire process.
6. Heat transfer with the surroundings is negligible during process 1-2.

ANALYSIS:

(a) Since the control volume has a single inlet and no exit, the mass rate balance for process 1-2 reduces to

$$\frac{dm_{cv}}{dt} = \dot{m}_i$$

The energy rate balance for process 1-2 reduces to

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i h_i$$

Neglecting heat transfer with the surroundings and combining mass and energy rate balances result in

$$\frac{dU_{cv}}{dt} = -\dot{W}_{cv} + \frac{dm_{cv}}{dt} h_i$$

By assumption 4 of the engineering model, the specific enthalpy at the inlet is constant. Accordingly, integration gives

$$\Delta U_{cv} = -\dot{W}_{cv}\Delta t + \Delta m_{cv} h_i$$

$$m_2 u_2 - m_1 u_1 = -\dot{W}_{cv}\Delta t + (m_2 - m_1) h_i$$

Since the balloon is deflated at state 1, $m_1 = 0$. The equation reduces to

$$m_2 u_2 = -\dot{W}_{cv}\Delta t + m_2 h_i$$

Work associated with the control volume is due to expansion of the control volume and the work required by the fan. Thus, $W_{cv} = W_{fan} + W_{expansion}$. The work associated with the control volume due to expansion of the air at constant pressure is

$$W_{expansion} = \int_1^2 p\,dV = p(V_2 - \cancel{V_1}) = pV_2 = \left(14.7 \frac{\text{lbf}}{\text{in.}^2}\right)(49{,}100\ \text{ft}^3)\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

$$W_{expansion} = 133{,}592\ \text{Btu}$$

The sign is positive as expected for work due to expansion.

The work associated with the fan is $\dot{W}_{fan}\,\Delta t$. Substituting values and solving for the fan power give

$$m_2 u_2 = -(\dot{W}_{fan}\,\Delta t + W_{expansion}) + m_2 h_i$$

$$\dot{W}_{fan} = \frac{m_2 (h_i - u_2) - W_{expansion}}{\Delta t}$$

The mass in the control volume at state 2 can be determined using the ideal gas equation of state. Converting air temperature to absolute scale, $T_2 = 80^{\circ}\text{F} = 540^{\circ}\text{R}$ and substituting

$$p_2 V_2 = m_2 R T_2 \Rightarrow m_2 = \frac{p_2 V_2}{R T_2}$$

$$m_2 = \frac{(14.7 \frac{\text{lbf}}{\text{in.}^2})(49{,}100\ \text{ft}^3)}{\left(\dfrac{1545 \frac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot{}^{\circ}\text{R}}}{28.97 \frac{\text{lb}}{\text{lbmol}}}\right)(540^{\circ}\text{R})}\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right| = 3609\ \text{lb}$$

The specific enthalpy of the inlet air ($T_i = 77^{\circ}\text{F} = 537^{\circ}\text{R}$) and the specific internal energy of the air at state 2 ($T_2 = 540^{\circ}\text{R}$) are determined from Table A-22E: $h_i = 128.34$ Btu/lb and

u_2 = 92.04 Btu/lb. Solving for fan power

$$\dot{W}_{\text{fan}} = \frac{(3609\text{ lb})(128.34\frac{\text{Btu}}{\text{lb}} - 92.04\frac{\text{Btu}}{\text{lb}}) - (133{,}592\text{ Btu})}{10\text{ min}}\left|\frac{60\text{ min}}{1\text{ h}}\right|\left|\frac{1\text{ hp}}{2545\frac{\text{Btu}}{\text{h}}}\right| = \underline{\mathbf{-6.10\ hp}}$$

The sign is negative as expected since the fan requires power to operate.

(b) During process 2-3, the mass rate balance takes the form

$$\frac{dm_{\text{cv}}}{dt} = \dot{m}_i$$

The energy rate balance for process 2-3 reduces to

$$\frac{dU_{\text{cv}}}{dt} = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \dot{m}_i h_i$$

Combining mass and energy rate balances results in

$$\frac{dU_{\text{cv}}}{dt} = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \frac{dm_{\text{cv}}}{dt}h_i$$

By assumption 4 of the engineering model, the specific enthalpy at the inlet is constant. Accordingly, integration gives

$$\Delta U_{\text{cv}} = \dot{Q}_{\text{cv}}\Delta t - \dot{W}_{\text{cv}}\Delta t + \Delta m_{\text{cv}} h_i$$

$$m_3 u_3 - m_2 u_2 = \dot{Q}_{\text{cv}}\Delta t - W_{\text{cv}} + (m_3 - m_2)h_i$$

Rearranging and solving for time associated with process 2-3 yields

$$\Delta t = \frac{m_3(u_3 - h_i) - m_2(u_2 - h_i) + W_{\text{cv}}}{\dot{Q}_{\text{cv}}}$$

The mass in the control volume at state 3 can be determined using the ideal gas equation of state. Converting air temperature to absolute scale, T_3 = 210°F = 670°R and substituting

$$p_3 V_3 = m_3 R T_3 \Rightarrow m_3 = \frac{p_3 V_3}{R T_3}$$

$$m_3 = \frac{\left(14.7\ \frac{\text{lbf}}{\text{in.}^2}\right)(65{,}425\ \text{ft}^3)}{\left(\frac{1545\ \frac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot{}^\circ\text{R}}}{28.97\ \frac{\text{lb}}{\text{lbmol}}}\right)(670^\circ\text{R})}\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right| = 3876\ \text{lb}$$

The specific internal energy of the air at state 3 (T_3 = 670°R) is determined from Table A-22E using interpolation: u_3 = 114.40 kJ/kg.

The work associated with the control volume is due to expansion of the balloon at constant pressure

$$W_{cv} = \int_2^3 p dV = p(V_3 - V_2) = \left(14.7\frac{\text{lbf}}{\text{in.}^2}\right)(65{,}425\ \text{ft}^3 - 49{,}100\ \text{ft}^3)\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

$$W_{cv} = 44{,}417\ \text{Btu}$$

Substituting values and solving for time associated with process 2-3 give

$$\Delta t = \frac{(3876\ \text{lb})(114.4\frac{\text{Btu}}{\text{lb}} - 128.34\frac{\text{Btu}}{\text{lb}}) - (3609\ \text{lb})(92.04\frac{\text{Btu}}{\text{lb}} - 128.34\frac{\text{Btu}}{\text{lb}}) + (44{,}417\ \text{Btu})}{7\times10^6\ \frac{\text{Btu}}{\text{h}}}\left|\frac{60\ \text{min}}{1\ \text{h}}\right|$$

$$\Delta t = 1.04\ \text{min}$$

Solving for time associated with the entire inflation process yields

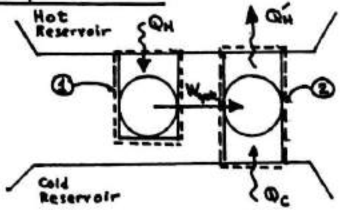
$t_3 = t_2 + (\Delta t)_{23}$ = 10 min + 1.04 min = **11.04 min**

PROBLEM 5.1

KNOWN: A system undergoes a cycle in violation of the Kelvin-Planck statement of the second law.

FIND: Show that a violation of the Clausius statement of the second law is a consequence.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) System 1 undergoes a cycle while receiving energy Q_H from the hot reservoir and developing work W_{cycle}. (This is in violation of the Kelvin-Planck statement of the second law.) (2) System 2 undergoes a cycle while removing energy Q_C from the cold reservoir and discharging energy Q_H' to the hot reservor. The work developed by System 1 is used to drive System 2.

ANALYSIS: An energy balance for system 1 reduces to

$$W_{cycle} = Q_H$$

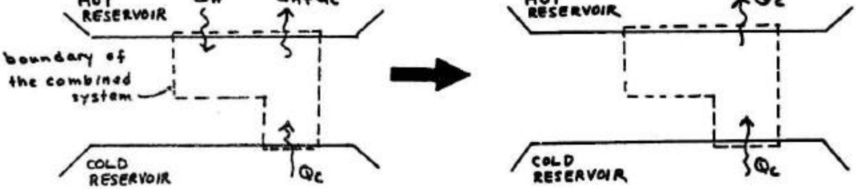
For System 2 an energy balance reads

$$W_{cycle} = Q_H' - Q_C$$

Combining these expressions

$$Q_H' = Q_H + Q_C$$

Next, consider a combined system consisting of Systems 1 and 2:



The sole result of the combined system is that a net amount of energy Q_C is transferred from the cold reservoir to the hot reservoir. This is a violation of the Clausius statement of the second law.

PROBLEM 5.2

5.2 Shown in Fig. P5.2 is a proposed system that undergoes a cycle while operating between cold and hot reservoirs. The system receives 500 kJ from the cold reservoir and discharges 400 kJ to the hot reservoir while delivering net work to its surroundings in the amount of 100 kJ. There are no other energy transfers between the system and its surroundings. Evaluate the performance of the system using

(a) the Clausius statement of the second law.
(b) the Kelvin-Planck statement of the second law.

SCHEMATIC & GIVEN DATA:

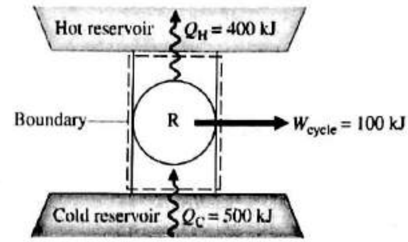
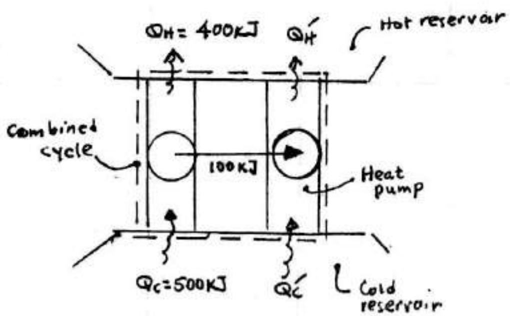


Fig. P5.2

KNOWN: Data are provided for a proposed system that undergoes a cycle while operating between cold and hot reservoirs.

FIND: Evaluate performance using (a) the Clausius statement, (b) the Kelvin-Planck statement.

(a) Clausius statement: Consider a combined cycle where the given cycle drives a heat pump cycle operating between the same two reservoirs:



An energy balance for the combined cycle reads

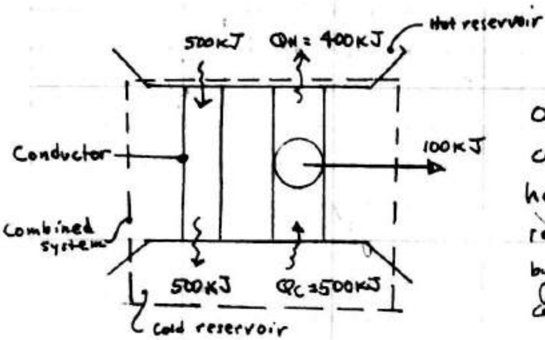
$$400\text{ kJ} + Q_H' = 500\text{ kJ} + Q_C'$$

$$\Rightarrow \quad Q_H' = 100\text{ kJ} + Q_C'$$

Thus, the combined cycle receives energy by heat transfer from the cold reservoir in the amount, $500\text{ kJ} + Q_C'$, and discharges energy by heat transfer to the hot reservoir in an equal amount: $Q_H + Q_H' = 500\text{ kJ} + Q_C'$ with no other effect. This is in violation of the Clausius statement, and therefore the system shown in Fig. P5.2 cannot operate as indicated.

PROBLEM 5.2 (Continued)

(b) Kelvin-Planck statement: Consider a combined system consisting of the given cycle, a thermal conductor at steady state, and the cold reservoir:



Operating at steady state, the conductor receives 500 KJ by heat transfer from the hot reservoir and delivers 500 KJ by heat transfer to the cold reservoir.

The combined system is composed of a cycle plus two parts that experience no net change in state: the conductor and the cold reservoir. Accordingly the combined system undergoes a thermodynamic cycle. This cycle receives a net amount of energy from the hot reservoir: 100 KJ and produces net work in the same amount. This is in violation of the Kelvin-Planck statement. Therefore the system shown in Fig. P5.2 cannot operate as indicated.

PROBLEM 5.3

5.3 Classify the following processes of a closed system as *possible*, *impossible*, or *indeterminate*.

	Entropy Change	Entropy Transfer	Entropy Production
(a)	>0	0	
(b)	<0		>0
(c)	0	>0	
(d)	>0	>0	
(e)	0	<0	
(f)	>0		<0
(g)	<0	<0	

Eq. 5.2 for a closed system:

$$\begin{bmatrix}\text{Entropy}\\ \text{Change}\end{bmatrix} = \begin{bmatrix}\text{Entropy}\\ \text{Transfer}\end{bmatrix} + \begin{bmatrix}\text{Entropy}\\ \text{Production}\end{bmatrix}$$

(a) $[>0] = [0] + \begin{bmatrix}\text{Entropy}\\ \text{Production}\end{bmatrix} \Rightarrow \begin{bmatrix}\text{Entropy}\\ \text{Production}\end{bmatrix} > 0$. Possible.

(b) $\begin{bmatrix}\text{Entropy}\\ \text{Production}\end{bmatrix} > 0$. Possible.

(c) $[0] = [>0] + \begin{bmatrix}\text{Entropy}\\ \text{Production}\end{bmatrix} \Rightarrow \begin{bmatrix}\text{Entropy}\\ \text{Production}\end{bmatrix} < 0$. Impossible.

(d) $[>0] = [>0] + \begin{bmatrix}\text{Entropy}\\ \text{Production}\end{bmatrix}$. No conclusion about $\begin{bmatrix}\text{Entropy}\\ \text{Production}\end{bmatrix}$ can be reached without actual values for the other terms. Indeterminate.

(e) $[0] = [<0] + \begin{bmatrix}\text{Entropy}\\ \text{Production}\end{bmatrix} \Rightarrow \begin{bmatrix}\text{Entropy}\\ \text{Production}\end{bmatrix} > 0$. Possible.

(f) $\begin{bmatrix}\text{Entropy}\\ \text{Production}\end{bmatrix} < 0$. Impossible.

(g) $[<0] = [<0] + \begin{bmatrix}\text{Entropy}\\ \text{Production}\end{bmatrix}$. No conclusion about $\begin{bmatrix}\text{Entropy}\\ \text{Production}\end{bmatrix}$ can be reached without actual values for the other terms. Indeterminate.

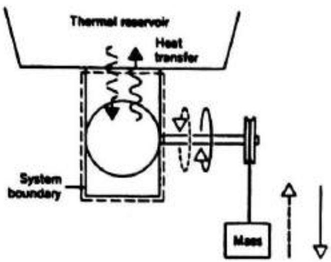
PROBLEM 5.4

.. Complete the discussion of the Kelvin–Planck statement of the second law in the box of Sec. 5.4 by showing that if a system undergoes a thermodynamic cycle reversibly while communicating thermally with a single reservoir, the equality in Eq. 5.3 applies.

KNOWN: A system undergoes a cycle reversibly while communicating thermally with a single reservoir.

FIND: Show that $W_{cycle} = 0$.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system shown in the accompanying figure undergoes a reversible cycle. (2) During the cycle the system communicates thermally only with a single reservoir.

ANALYSIS: Let the system undergo one cycle. According to the Kelvin-Planck statement of the second law, the work for the cycle is zero or negative in value: $W_{cycle} \leq 0$.

As the cycle is reversible, it is possible to return both the system and its surroundings to their initial states. Accordingly, there would be no net change in the condition of the reservoir or the elevation of the mass used to store energy in the surroundings.

① The above statements are consistent only if the sign of equality is used: $W_{cycle} = 0$.

---

1. In Sec. 5.4, the converse is demonstrated: If the sign of equality applies: $W_{cycle} = 0$, then the cycle is reversible. Taken together, these two demonstrations establish the following proposition: If, and only if, the sign of equality applies in Eq. 5.3, the cycle is reversible.

PROBLEM 5.5

5.5 As shown in Fig. P5.5, a reversible power cycle R and an irreversible power cycle I operate between the same hot and cold thermal reservoirs. Cycle I has a thermal efficiency equal to one-third of the thermal efficiency of cycle R.

(a) If each cycle receives the same amount of energy by heat transfer from the hot reservoir, determine which cycle (i) develops greater net work, (ii) discharges greater energy by heat transfer to the cold reservoir.
(b) If each cycle develops the same net work, determine which cycle (i) receives greater energy by heat transfer from the hot reservoir, (ii) discharges greater energy by heat transfer to the cold reservoir.

SCHEMATIC & GIVEN DATA:

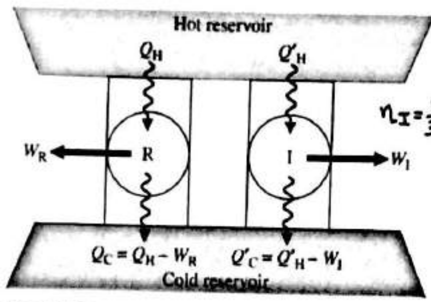


Fig. P5.5

$\eta_I = \frac{1}{3}\eta_R$

$\eta_R = \frac{W_R}{Q_H}$, $\eta_I = \frac{W_I}{Q'_H}$

ANALYSIS:

(a) $Q_H = Q'_H$

(i) $\eta_I = \frac{1}{3}\eta_R$

$\frac{W_I}{Q_H} = \frac{1}{3}\frac{W_R}{Q_H} \Rightarrow W_I = \frac{1}{3}W_R \Rightarrow$ cycle R develops greater net work.

(ii) Energy balances give

$$\left.\begin{array}{l} Q_C = Q_H - W_R \\ Q_C' = Q_H - W_I \end{array}\right\} \Rightarrow Q_C' - Q_C = W_R - W_I$$
$$= W_R - \frac{1}{3}W_R$$
$$= \frac{2}{3}W_R$$

$\Rightarrow$ Cycle I discharges greater energy by heat transfer to the cold reservoir

Summary: Cycle I develops less net work than cycle R but discharges greater heat transfer to the cold reservoir.

(b) $W_I = W_R$

(i) $\eta_I = \frac{1}{3}\eta_R$

$\frac{W_R}{Q_H'} = \frac{1}{3}\frac{W_R}{Q_H} \Rightarrow Q_H' = 3Q_H$ Cycle I receives greater energy by heat transfer from the hot reservoir.

(iii) Energy balances give

$$\left.\begin{aligned} Q_C &= Q_H - W_R \\ Q_C' &= Q_H' - W_R \end{aligned}\right\} \quad \begin{aligned} Q_C' - Q_C &= Q_H' - Q_H \\ &= 3Q_H - Q_H \\ &= 2Q_H \end{aligned}$$

⇒ Cycle I transfers greater energy by heat transfer to the cold reservoir.

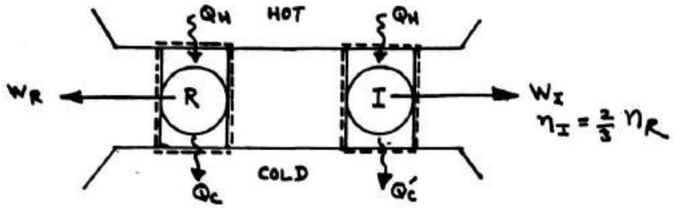
Summary: For equal net work developed, cycle I receives greater energy by heat transfer from the hot reservoir and discharges greater energy by heat transfer to the cold reservoir.

PROBLEM 5.6

KNOWN: A power cycle I and a reversible power cycle R operate between the same two reservoirs. For these cycles, $\eta_I = \frac{2}{3}\eta_R$.

FIND: Show that cycle I must be irreversible.

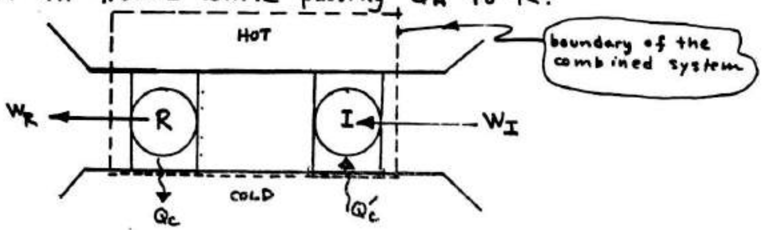
SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system denoted by R in the accompanying figure undergoes a reversible power cycle while system I undergoes a power cycle such that $\eta_I = \frac{2}{3}\eta_R$. (2) Both cycles receive the same energy Q_H from the hot reservoir.

ANALYSIS: If $\eta_I = 2/3\,\eta_R$ and both cycles receive Q_H from the hot reservoir, $W_I = 2/3\,W_R < W_R$.

In this proof by contradiction, assume I is reversible. Then, if I operates in the opposite direction as a refrigeration (or heat pump) cycle, the magnitudes of the energy transfers Q_H, Q_C', and W_I would remain the same but would be oppositely directed as shown in the figure below. With I operating in the opposite direction, the hot reservoir would experience no net change since it would receive Q_H from I while passing Q_H to R.



As the combined system shown in the figure above is made up of parts that execute cycles or experience no net change, the combined system operates in a cycle. Moreover, it exchanges energy by heat transfer with a single reservoir: the cold reservoir. Accordingly, the combined system must satisfy Eq. 5.3 expressed as $W_{cycle} = 0$, where the sign of equality is used because R is reversible and I has been assumed reversible. Evaluating W_{cycle} for the combined system in terms of the work amounts W_R and W_I, $W_{cycle} = W_R - W_I$, it follows that

$$W_I = W_R$$

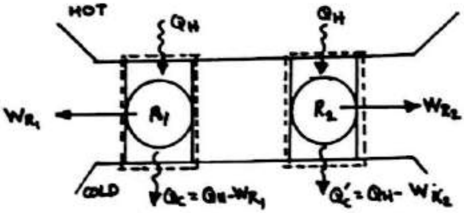
As this conclusion is not in agreement with the requirement that $W_I < W_R$, it can be concluded that the hypothesis is false and I must be irreversible.

PROBLEM 5.7

KNOWN: The Kelvin-Planck statement of the second law: $W_{cycle} \leq 0$ (single reservoir).

FIND: Show that all reversible power cycles operating between the same two reservoirs have the same thermal efficiency.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: The systems shown in the accompanying figure each undergo reversible power cycles while receiving the same amount of energy Q_H from the hot reservoir.

ANALYSIS: Let cycle R_1 operate as a reversible refrigeration (or heat pump cycle). The magnitudes of W_{R1}, Q_H, and Q_C remain the same but are now oppositely directed. Further, with R_1 working in the opposite direction, the hot reservoir would experience no net change in its condition since it would receive Q_H from R_1 while passing Q_H to R_2.

The demonstration is completed by considering the combined system consisting of the two cycles and the hot reservoir. Since its parts execute cycles or experience no net change, the combined system operates in a cycle. Moreover, it exchanges energy by heat transfer with a single reservoir: the cold reservoir. Accordingly, the combined system must satisfy the Kelvin-Planck statement expressed as

$$W_{cycle} = 0$$

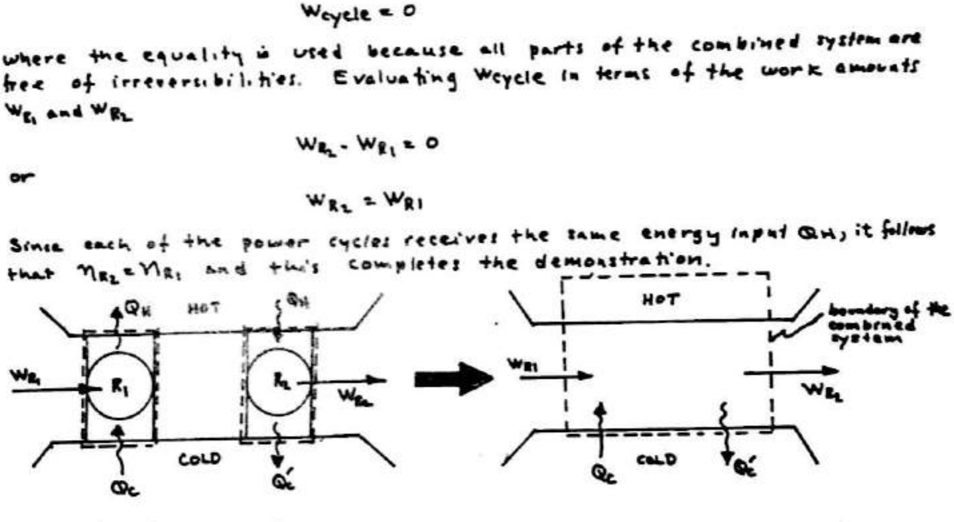
where the equality is used because all parts of the combined system are free of irreversibilities. Evaluating W_{cycle} in terms of the work amounts W_{R_1} and W_{R_2}

$$W_{R_2} - W_{R_1} = 0$$

or

$$W_{R_2} = W_{R1}$$

Since each of the power cycles receives the same energy input Q_H, it follows that $\eta_{R_2} = \eta_{R_1}$ and this completes the demonstration.

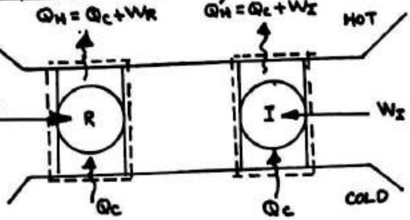


PROBLEM 5.8

KNOWN: The Kelvin-Planck statement of the second law: $W_{cycle} \le 0$ (single reservoir).

FIND: Show that (a) the coefficient of performance of an irreversible refrigeration cycle is less than the coefficient of performance of a reversible refrigeration cycle when both exchange energy by heat transfer with the same two reservoirs, (b) all reversible refrigeration cycles operating between the same two reservoirs have the same coefficient of performance. For parts (c), (d), see next page.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: The systems shown in the accompanying figure undergo refrigeration cycles. Each cycle removes energy Q_C from the cold reservoir. R accomplishes this reversibly while I is irreversible.

ANALYSIS: (a) Let R operate as a reversible power cycle. The magnitudes of W_R, Q_C and Q_H remain the same but are now oppositely directed. Further, with R working in the opposite direction, the cold reservoir would experience no net change in its condition since it would receive Q_C from R while passing Q_C to I.

The demonstration is completed by considering the combined system consisting of the two cycles and the cold reservoir. Since its parts execute cycles or experience no net change, the combined system operates in a cycle. Moreover, it exchanges energy by heat transfer with a single reservoir: the hot reservoir. Accordingly, the combined system must satisfy the Kelvin-Planck statement expressed as

$$W_{cycle} < 0$$

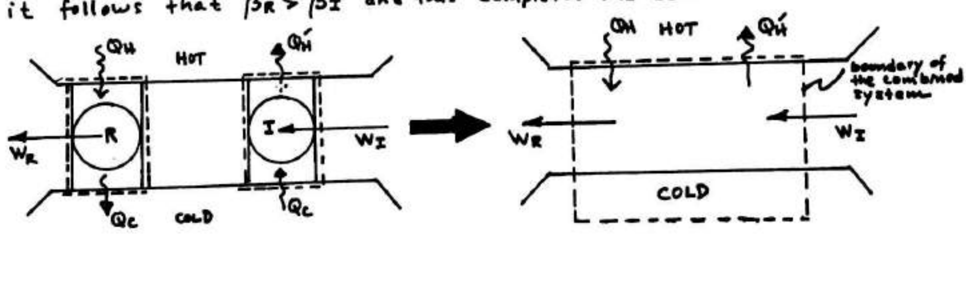
where the inequality is used because irreversible cycle I is included. Evaluating W_{cycle} in terms of the work amounts W_R and W_I

$$W_R - W_I < 0$$

or

$$W_R < W_I$$

Since each of the refrigeration cycles receives the same energy input Q_C, it follows that $\beta_R > \beta_I$ and this completes the demonstration.



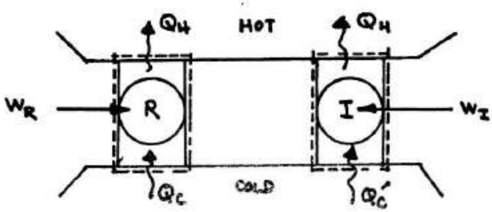
PROBLEM 5.8 (Contd.)

ANALYSIS (b) This proposition can be demonstrated in a parallel way by considering two reversible refrigeration cycles R_1 and R_2 operating between the same two reservoirs. Then, letting R_1 play the role of R and R_2 the role of I in the previous development, a combined system consisting of the two cycles and the hot reservoir may be formed which must satisfy $W_{cycle} = 0$, where the equality is used because all parts of the combined system are free of irreversibilities. Thus, it can be concluded that $W_{R_1} = W_{R_2}$, and therefore $\beta_{R_1} = \beta_{R_2}$.

Parts (c), (d):

FIND: Show that (c) the coefficient of performance of an irreversible heat pump cycle is less than the coefficient of performance a reversible heat pump cycle when both exchange energy by heat transfer with the same two reservoirs, (d) all reversible heat pump cycles operating between the same two reservoirs have the same coefficient of performance.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The systems shown in the accompanying figure undergo heat pump cycles. (2) Each cycle provides energy Q_H to the hot reservoir. R accomplishes this reversibly while I is irreversible. (3) Both cycles operate between the same two reservoirs.

ANALYSIS: (c) The demonstration parallels the approach used in part(a). However, in the present case the combined system would consist of the two cycles and the hot reservoir. (d). The demonstration parallels the approach used in part (b).

PROBLEM 5.9

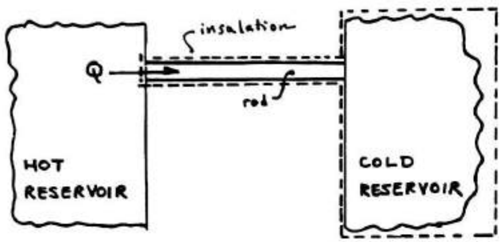
5.9. Use the Kelvin–Planck statement of the second law to show that the specified process is irreversible.

(a) As shown in Fig. P5.9*a*, a hot thermal reservoir is separated from a cold thermal reservoir by a cylindrical rod insulate on its lateral surface. Energy transfer by conduction between the two reservoirs takes place through the rod, which remains at steady state.

KNOWN: Energy transfer by conduction from a hot reservoir to a cold reservoir takes place spontaneously.

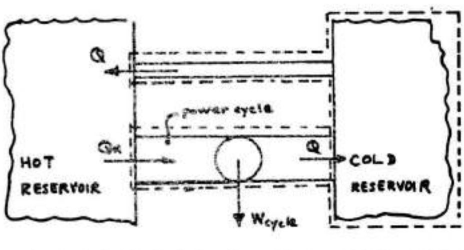
FIND: Using the Kelvin - Planck statement of the second law, show that such a process is irreversible.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system shown in the accompanying figure receives energy Q from the hot reservoir which passes by conduction through a rod at steady state to the cold reservoir. (2) A power cycle is available for use in demonstrating that the process is irreversible.

ANALYSIS: The objective in this proof by contradiction is to devise a system that undergoes a cycle which develops work while the system communicates thermally with a single reservoir, thereby violating the Kelvin-Planck statement.



Note: Q represents the energy transferred from the cold reservoir to the hot reservoir without any other effect.

Note: Q represents the energy discharged to the cold reservoir from the power cycle.

The system shown above consists of the original system plus a system capable of undergoing a power cycle (assumption 2). This combined system undergoes a cycle as follows. (1) An amount of energy Q_H is transferred from the hot reservoir to the power cycle, producing work W_{cycle} and discharging energy Q to the cold reservoir. (2) An amount of energy Q is transferred from the cold reservoir to the hot reservoir without any other effects. (This would be possible only if the process described in assumption 1 were reversible.)

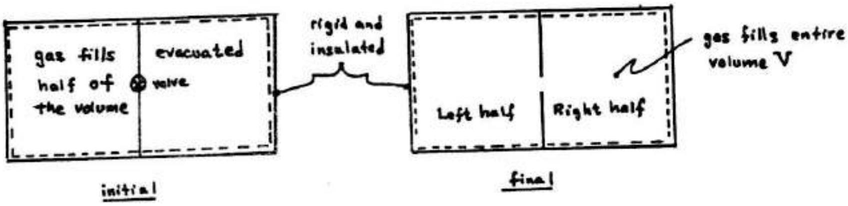
Since Q is added to the cold reservoir in the first process of the cycle and the same amount of energy is removed from the cold reservoir in the second process of the cycle, this reservoir experiences no net change in its condition. The power cycle enclosed within the combined system also undergoes a cycle. Accordingly, the combined system undergoes a cycle in which work Wcycle is produced while exchanging energy by heat transfer with a single reservoir (the hot reservoir). Such a cycle violates the Kelvin-Planck statement of the second law, and thus is impossible. It follows that one, or both, of the processes making up the cycle executed by the combined system must be impossible. However, as the first process involving the power cycle can occur, it is the second process that must be impossible: energy Q cannot be transferred from the cold to the hot reservoir without other effects. By definition, then, the transfer of energy Q by conduction from the hot to the cold reservoir is irreversible.

(b) As shown in Fig. P5.9*b*, a rigid insulated tank is divided into halves by a partition. On one side of the partition is a gas. The other side is initially evacuated. A valve in the partition is opened and the gas expands to fill the entire volume.

KNOWN: When an interconnecting valve is opened, a gas expands from one half of a tank to the other half, which is initially evacuated.

FIND: Using the Kelvin-Planck statement of the second law, show that such a process is irreversible.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system shown in the accompanying figure undergoes a process in which the gas expands spontaneously to fill the entire volume V. (2) During the process Q = W = 0. (3) The initial and final states are equilibrium states. There is no change in kinetic or potential energy between these states. (4) A turbine and a thermal reservoir are available for use in demonstrating that the process is irreversible.

ANALYSIS: The object in this proof by contradiction is to devise a system that undergoes a power cycle while the system communicates thermally with a single reservoir, thereby violating the Kelvin-Planck statement of the second law.

Before considering such a system, note that an energy balance for the spontaneous process is

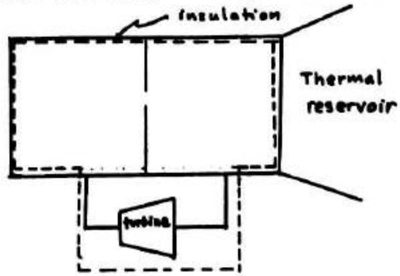
$$U_{final} - U_{initial} = \cancel{Q}^{0} - \cancel{W}^{0}$$

where assumptions 2 and 3 have been used. Accordingly,

$$U_{final} = U_{initial}$$

That is, the internal energy of the system does not change in the free expansion.

As shown in the figure below, modify the system to include a turbine and introduce a thermal reservoir in the surroundings (assumption 4).



Starting with the gas filling the entire volume V, let the modified system undergo a cycle consisting of three processes.

Process 1. Let the reverse of the free expansion occur without any other effects. That is, the gas passes spontaneously from the right half of the tank until it fills only the left half. (This would be possible only if the process described in assumption 1 were reversible.)

Process 2. Let part of the gas expand through the turbine into the right half of the tank until the pressure in both halves is the same. In expanding through the turbine the gas does work so that its internal energy is decreased: $U < U_{initial}$.

Process 3. Remove part of the tank insulation and add energy by heat transfer from the thermal reservoir until the internal energy of the gas is restored to its initial value. Thus, a cycle is completed.

The net result of this cycle is to draw energy from a single reservoir by heat transfer and produce an equivalent amount of work. Such a cycle violates the Kelvin-Planck statement of the second law, and thus is impossible. Since both the heating of the gas by the reservoir (process 3) and the development of work as gas passes through the turbine (process 2) are possible, it can be concluded that it is process 1 that is impossible. Since process 1 is the reverse of the original free expansion, it follows that the original process is irreversible.

PROBLEM 5.10

5.10 Figure P5.10 shows two power cycles, denoted 1 and 2, operating in series, together with three thermal reservoirs. The energy transfer by heat into cycle 2 is equal in magnitude to the energy transfer by heat from cycle 1. All energy transfers are positive in the directions of the arrows.

(a) Determine an expression for the thermal efficiency of an overall cycle consisting of cycles 1 and 2 expressed in terms of their individual thermal efficiencies.
(b) If cycles 1 and 2 are each reversible, use the result of part (a) to obtain an expression for the thermal efficiency of the overall cycle in terms of the temperatures of the three reservoirs, T_H, T, and T_C, as required. Comment.
(c) If cycles 1 and 2 are each reversible and have the same thermal efficiency, obtain an expression for the intermediate temperature T in terms of T_H and T_C.

SCHEMATIC & GIVEN DATA

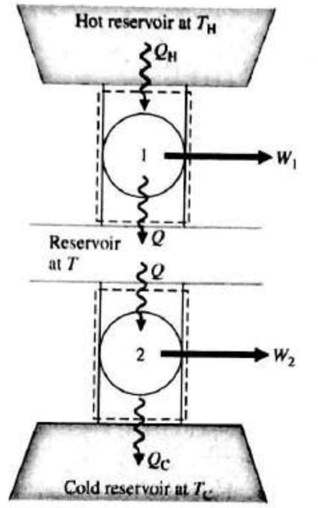


Fig.P5.10

KNOWN: Two power cycles operate in series.

FIND: For three scenarios obtain expressions for the thermal efficiency of the overall cycle formed from the two.

ENGINEERING MODEL:

1. The systems denoted by 1 and 2 in the figure each undergo power cycles.
2. There are three thermal reservoirs at temperatures T_H, T, and T_C, respectively.
3. All energy transfers are positive in the direction of the accompanying arrow.

ANALYSIS:

(a) For the overall cycle $(W_1 + W_2) = Q_H - Q_C$. Moreover,

$$\eta = \frac{W_1 + W_2}{Q_H} = 1 - \frac{Q_C}{Q_H}$$

For cycle 1,

$$\eta_1 = \frac{W_1}{Q_H} = 1 - \frac{Q}{Q_H} \Rightarrow Q_H = \frac{Q}{(1-\eta_1)}$$

For cycle 2

$$\eta_2 = \frac{W_2}{Q} = 1 - \frac{Q_C}{Q} \Rightarrow Q_C = Q(1-\eta_2)$$

$$\Rightarrow \frac{Q_C}{Q_H} = (1-\eta_1)(1-\eta_2)$$

Collecting results

$$\eta = 1 - (1-\eta_1)(1-\eta_2) = 1 - (1 - \eta_1 - \eta_2 + \eta_1 \eta_2)$$

$$= \eta_1 + \eta_2 - \eta_1 \eta_2 \longleftarrow (a)$$

① (b) If cycles 1 and 2 are each reversible, then Eq. 5.9 gives

$$\eta_1 = 1 - \frac{T}{T_H}, \quad \eta_2 = 1 - \frac{T_C}{T}$$

Substituting into the result of part (a)

$$\eta = \left(1 - \frac{T}{T_H}\right) + \left(1 - \frac{T_C}{T}\right) - \left(1 - \frac{T}{T_H}\right)\left(1 - \frac{T_C}{T}\right)$$

PROBLEM 5.10 (Continued)

On reduction

$$\eta = 1-\frac{T}{T_H} + 1 - \frac{T_C}{T} - \left[1 - \frac{T}{T_H} - \frac{T_C}{T} + \frac{T}{T_H}\frac{T_C}{T}\right]$$

(2) $$= 1 - \frac{T_C}{T_H} \qquad \longleftarrow \text{(b)}$$

(c) If cycles 1 and 2 are each reversible

$$\eta_1 = 1-\frac{T}{T_H}, \quad \eta_2 = 1-\frac{T_C}{T}$$

If $\eta_1 = \eta_2$,

$$1-\frac{T}{T_H} = 1-\frac{T_C}{T}$$

$$\Rightarrow \quad T^2 = T_C T_H$$

(3) $$\Rightarrow \quad T = \sqrt{T_C T_H} \qquad \longleftarrow \text{(c)}$$

---

1 Sample calculation: $\eta_1 = 0.25$, $\eta_2 = 0.40$

$$\eta = 0.25 + 0.40 - (0.25)(0.40) = 0.55$$

The combined cycle thermal efficiency is greater than that for each of individual cycles.

2. Referring again to Eq. 5.9, this result indicates that the combined cycle formed from two reversible cycles, as shown in the figure, is also reversible, as expected.

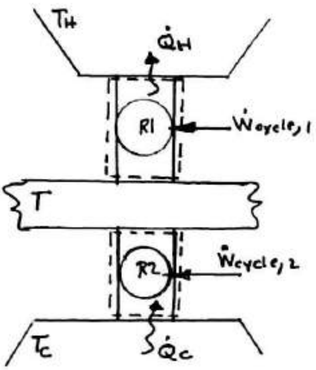
3 Sample Calculation: $T_C = 360\text{K}$, $T_H = 1000\text{K}$

$$T = \sqrt{(360\text{K})(1000\text{K})} = 600\text{K}$$

PROBLEM 5.11

Two reversible refrigeration cycles are arranged in series. The first cycle receives energy by heat transfer from a cold reservoir at temperature T_C and rejects energy by heat transfer to a reservoir at an intermediate temperature T greater than T_C. The second cycle receives energy by heat transfer from the reservoir at temperature T and rejects energy by heat transfer to a higher-temperature reservoir at T_H. Obtain an expression for the coefficient of performance of a *single* reversible refrigeration cycle operating directly between cold and hot reservoirs at T_C and T_H, respectively, in terms of the coefficients of performance of the two cycles.

SCHEMATIC & GIVEN DATA:



ANALYSIS:

Using Eq. 5.10

$$\beta_1 = \frac{T}{T_H - T} \Rightarrow \frac{T_H}{T} = 1 + \frac{1}{\beta_1}$$

$$\beta_2 = \frac{T_C}{T - T_C} \Rightarrow \frac{T}{T_C} = 1 + \frac{1}{\beta_2}$$

$$\beta = \frac{T_C}{T_H - T_C} \Rightarrow \frac{T_H}{T_C} = 1 + \frac{1}{\beta}$$

Then with

$$\frac{T_H}{T_C} = \left(\frac{T_H}{T}\right)\left(\frac{T}{T_C}\right)$$

$$1 + \frac{1}{\beta} = \left(1 + \frac{1}{\beta_1}\right)\left(1 + \frac{1}{\beta_2}\right)$$

$$\Rightarrow \frac{1}{\beta} = \frac{(\beta_1 + 1)(\beta_2 + 1)}{\beta_1 \beta_2} - 1$$

$$\Rightarrow \frac{1}{\beta} = \frac{(\beta_1\beta_2 + \beta_1 + \beta_2 + 1) - \beta_1\beta_2}{\beta_1 \beta_2}$$

$$\Rightarrow \beta = \frac{\beta_1 \beta_2}{\beta_1 + \beta_2 + 1}$$

PROBLEM 5.12

5.12 Repeat Problem 5.11 for the case of two reversible heat pump cycles.

SCHEMATIC & GIVEN DATA:

See the figure in the solution to Problem 5.11.

ANALYSIS:

Using Eq 5.11

$$\gamma_1 = \frac{T_H}{T_H - T} \Rightarrow \frac{T}{T_H} = 1 - \frac{1}{\gamma_1}$$

$$\gamma_2 = \frac{T}{T - T_C} \Rightarrow \frac{T_C}{T} = 1 - \frac{1}{\gamma_2}$$

$$\gamma = \frac{T_H}{T_H - T_C} \Rightarrow \frac{T_C}{T_H} = 1 - \frac{1}{\gamma}$$

Then with

$$\frac{T_C}{T_H} = \left(\frac{T}{T_H}\right)\left(\frac{T_C}{T}\right)$$

$$1 - \frac{1}{\gamma} = \left(1 - \frac{1}{\gamma_1}\right)\left(1 - \frac{1}{\gamma_2}\right)$$

$$\Rightarrow \frac{1}{\gamma} = 1 - \left(1 - \frac{1}{\gamma_1}\right)\left(1 - \frac{1}{\gamma_2}\right)$$

$$= \frac{\gamma_1\gamma_2 - (\gamma_1 - 1)(\gamma_2 - 1)}{\gamma_1\gamma_2}$$

$$= \frac{\gamma_1\gamma_2 - \gamma_1\gamma_2 + \gamma_1 + \gamma_2 - 1}{\gamma_1\gamma_2}$$

$$\Rightarrow \gamma = \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2 - 1} \longleftarrow$$

PROBLEM 5.13

KNOWN: Two reversible cycles operate between hot and cold reservoirs at T_H and T_C, respectively.

(a) If one is a power cycle and the other is a heat pump cycle, what is the relation between the coefficient of performance of the heat pump cycle and the thermal efficiency of the power cycle?

Power Cycle: $\eta = \dfrac{T_H - T_C}{T_H}$ (Eq. 5.9)

Heat Pump: $\gamma = \dfrac{T_H}{T_H - T_C}$ (Eq. 5.11)

By inspection,

$$\Rightarrow \gamma = \frac{1}{\eta} \quad \longleftarrow \text{(a)}$$

(b) If one is a refrigeration cycle and the other is a heat pump cycle, what is the relation between their coefficients of performance?

Refrigeration Cycle: $\beta = \dfrac{T_C}{T_H - T_C}$ (Eq. 5.10)

Heat pump cycle: $\gamma = \dfrac{T_H}{T_H - T_C}$ (Eq. 5.11)

Multiply each expression by $(T_H - T_C)$ and subtract,

$$\beta(T_H - T_C) = T_C$$

$$\gamma(T_H - T_C) = T_H$$

$$\Rightarrow (T_H - T_C)(\gamma - \beta) = (T_H - T_C)$$

$$\Rightarrow \gamma - \beta = 1 \Rightarrow \gamma = \beta + 1 \quad \longleftarrow \text{(b)}$$

PROBLEM 5.14

5.14 Figure P5.14 shows a system consisting of a reversible power cycle driving a reversible heat pump. The power cycle receives Q_s by heat transfer at T_s from a high-temperature source and delivers Q_1 to a dwelling at T_d. The heat pump receives Q_0 from the outdoors at T_0 and delivers Q_2 to the dwelling. Obtain an expression for the ratio of the total heating provided to the dwelling to the heat transfer supplied from the high-temperature source: $(Q_1 + Q_2)/ Q_s$ in terms of the temperatures T_s, T_d, and T_0.

SCHEMATIC & GIVEN DATA

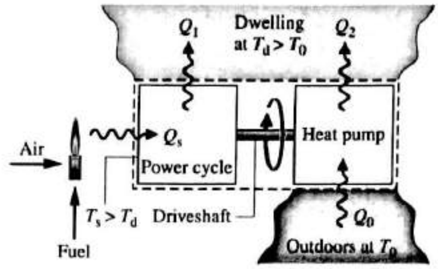


Fig. P5.14

Known: A power cycle combines with a heat pump cycle to heat a dwelling. Each cycle is reversible.

Find: Evaluate the ratio $(Q_1+Q_2)/Q_s$ in terms of the temperatures T_s, T_d, T_0.

Engineering Model:

1. The power cycle and heat pump cycles operate reversibly.
2. Three thermal reservoirs participate: a hot reservoir at T_s, the dwelling at T_d, and the outdoors at T_0.
3. As shown by the dashed line, the system consists of the power cycle and the heat pump.
4. Energy transfers by heat are in the directions of the arrows.

Analysis: Applying an energy balance to the system,

$$Q_1 + Q_2 = Q_s + Q_0 \tag{a}$$

Alternatively,

$$\frac{Q_1 + Q_2}{Q_s} = 1 + \frac{Q_0}{Q_s} \tag{b}$$

Applying the second law in the form of Eq. 5.7 to the reversible cycles,

$$\frac{Q_1}{Q_s} = \frac{T_d}{T_s} \quad , \quad \frac{Q_2}{Q_0} = \frac{T_d}{T_0} \tag{c}$$

Introducing Eqs. (c) on the left of Eq. (a) and reducing

$$\frac{T_d}{T_s} Q_s + \frac{T_d}{T_0} Q_0 = Q_s + Q_0$$

$$\left[\frac{T_d}{T_s} - 1\right] Q_s = \left[1 - \frac{T_d}{T_0}\right] Q_0$$

$$\Rightarrow \quad \frac{Q_0}{Q_s} = \frac{T_0\left[T_d - T_s\right]}{T_s\left[T_0 - T_d\right]} \tag{d}$$

Combining Eqs.(b) and (d)

$$\frac{Q_1 + Q_2}{Q_s} = 1 + \frac{T_0}{T_s}\left[\frac{T_d - T_s}{T_0 - T_d}\right]$$

$$= \frac{T_s[T_0 - T_d] + T_0[T_d - T_s]}{T_s[T_0 - T_d]}$$

① ② $\Rightarrow$ $$\frac{Q_1 + Q_2}{Q_s} = \frac{T_d}{T_s}\left[\frac{T_s - T_0}{T_d - T_0}\right] \qquad (e)$$

Eq.(e) is the desired result.

---

1. Since the power cycle and heat pump cycle are reversible, we can infer that Eq.(e) gives the maximum theoretical value for the ratio.

2. As a sample calculation, if $T_s = 1000\text{K}$, $T_d = 295\text{K}$, and $T_0 = 275\text{K}$, Eq. (e) gives

$$\frac{Q_1 + Q_2}{Q_s} = \frac{295}{1000}\left[\frac{725}{20}\right]$$

$$= 10.7$$

If $T_s = 375\text{K}$ and T_d, T_0 remain the same as above

$$\frac{Q_1 + Q_2}{Q_s} = \frac{295}{375}\left[\frac{100}{20}\right]$$

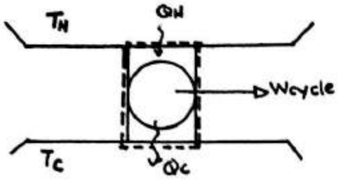
$$= 3.9$$

PROBLEM 5.15

KNOWN: A system undergoes a reversible power cycle while receiving and discharging energy by heat transfer with two thermal reservoirs.

FIND: To increase the thermal efficiency, determine whether it would be better to increase T_H or decrease T_C.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: The system shown in the accompanying figure undergoes a reversible power cycle while receiving and discharging energy by heat transfer with two thermal reservoirs.

ANALYSIS: Trends can be determined by differentiation of Eq. 5.9, which is applicable in this case

$$\left(\frac{\partial\eta}{\partial T_H}\right)_{T_C} = \frac{T_C}{(T_H)^2} \Rightarrow \text{As } T_H \text{ increases, } \eta \text{ increases.}$$

$$\left(\frac{\partial\eta}{\partial T_C}\right)_{T_H} = -\frac{1}{T_H} \Rightarrow \text{As } T_C \text{ decreases, } \eta \text{ increases.}$$

Quantitative evaluations also can be had using Eq. 5.9

$$\eta = 1 - \frac{T_C}{T_H}$$

If T_C is decreased by ϵ degrees, the thermal efficiency is increased

$$\eta_{(-)} = 1 - \frac{(T_C - \epsilon)}{T_H} = \eta + \frac{\epsilon}{T_H} \quad (1)$$

If T_H is increased by ϵ degrees, the thermal efficiency is increased

$$\eta_{(+)} = 1 - \frac{T_C}{T_H + \epsilon}$$

$$= \frac{T_H - T_C + \epsilon}{T_H + \epsilon} = \frac{\eta T_H + \epsilon}{T_H + \epsilon} = \frac{\eta + \epsilon/T_H}{1 + \epsilon/T_H} \quad (2)$$

Combining Eqs. (1), (2)

$$\eta_{(+)} = \frac{\eta_{(-)}}{1 + \epsilon/T_H} \Rightarrow \eta_{(+)} < \eta_{(-)}$$

① ② **If the objective is to increase the thermal efficiency, this indicates that it is better to decrease TC than increase TH.**

---

1. Observe that increasing the thermal efficiency of a power cycle by reducing TC below the ambient temperature is not viable. For instance, reducing TC below the ambient using an actual refrigeration cycle requires a work input to the refrigeration cycle that will exceed the increase in work of the power cycle, giving a lower net work output.

2. An increase in thermal efficiency by increasing TH is limited by the properties of the materials used to fabricate the system undergoing the cycle.

PROBLEM 5.16

KNOWN: The symbol Θ denotes temperature on Kelvin's logarithmic scale.

FIND: (a) Show that $\Theta = \ln T + C$, where T is temperature on the Kelvin scale. (b) Determine the range of temperature values on the logarithmic scale. (c) Obtain an expression for the thermal efficiency of a reversible power cycle operating between reservoirs at Θ_H and Θ_C on the logarithmic scale.

ANALYSIS: The two scales arise from different specifications of the function Ψ in Eq. (a) in Sec. 5.8.1:

$$\left(\frac{Q_C}{Q_H}\right)_{\substack{rev\\cycle}} = \Psi$$

That is

$$\left(\frac{Q_C}{Q_H}\right)_{\substack{rev\\cycle}} = \frac{T_C}{T_H} \qquad \text{Kelvin scale}$$

$$\left(\frac{Q_C}{Q_H}\right)_{\substack{rev\\cycle}} = \frac{\exp \Theta_C}{\exp \Theta_H} \qquad \text{Logarithmic scale}$$

(a) By comparison of the last two equations

$$\frac{T_C}{T_H} = \frac{\exp \Theta_C}{\exp \Theta_H} = \exp[\Theta_C - \Theta_H]$$

Thus

$$\ln T_C - \ln T_H = \Theta_C - \Theta_H$$

and therefore

$$\Theta = \ln T + C \qquad \text{(a)}$$

where C is a constant determining the level of temperature corresponding to zero on the logarithmic scale.

(b) Temperatures on the Kelvin scale vary from 0 to $+\infty$. With the relationship of part (a), temperatures on the logarithmic scale vary from $-\infty$ to $+\infty$. (b)

(c) Use of

$$\left(\frac{Q_C}{Q_H}\right)_{\substack{rev\\cycle}} = \frac{\exp \Theta_C}{\exp \Theta_H}$$

in Eq 5.4 gives an expression for the thermal efficiency of a reversible power cycle while operating between reservoirs at temperatures Θ_H and Θ_C on the logarithmic scale

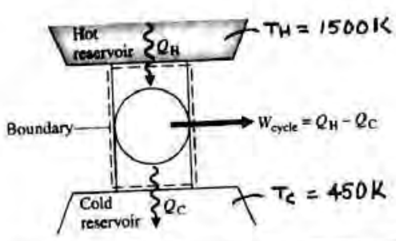
$$\eta = 1 - \frac{\exp \Theta_C}{\exp \Theta_H} \qquad \text{(c)}$$

PROBLEM 5.17

KNOWN: Data are provided for a power cycle operating between hot and cold reservoirs.

FIND: In each case, determine whether the cycle operates reversibly, operates irreversibly, or is impossible.

SCHEMATIC & GIVEN DATA:



(a) $Q_H = 600kJ$, $W_{cycle} = 300kJ$, $Q_C = 300kJ$.

Check energy balance:

$$W_{cycle} = Q_H - Q_C = 600 - 300 = 300kJ.$$

With the given data,

$$\eta = \frac{W_{cycle}}{Q_H} = \frac{300kJ}{600kJ} = 0.5$$

The maximum thermal efficiency is

$$\eta_{MAX} = 1 - \frac{T_C}{T_H} = 1 - \frac{450}{1500} = 0.7$$

Since $\eta < \eta_{MAX}$, the cycle operates irreversibly.

(b) $Q_H = 400kJ$, $W_{cycle} = 280kJ$, $Q_C = 120kJ$.

Check energy balance:

$$W_{cycle} = Q_H - Q_C = 400 - 120 = 280kJ \quad \text{OK}$$

With the given data

$$\eta = \frac{W_{cycle}}{Q_H} = \frac{280}{400} = 0.7$$

From part (a), $\eta_{MAX} = 0.7$.
Since $\eta = \eta_{MAX}$, the cycle operates reversibly.

(c) $Q_H = 700kJ$, $W_{cycle} = 300kJ$, $Q_C = 500kJ$.

Check energy balance:

$$W_{cycle} = Q_H - Q_C = 700 - 500 = 200kJ$$

This does not agree with the claimed value. The cycle cannot operate as claimed. Impossible.

(d) $Q_H = 800kJ$, $W_{cycle} = 600kJ$, $Q_C = 200kJ$.

Check energy balance:

$$W_{cycle} = Q_H - Q_C = 800kJ - 200kJ = 600kJ \quad \text{OK}$$

With the given data

$$\eta = \frac{W_{cycle}}{Q_H} = \frac{600}{800} = 0.75$$

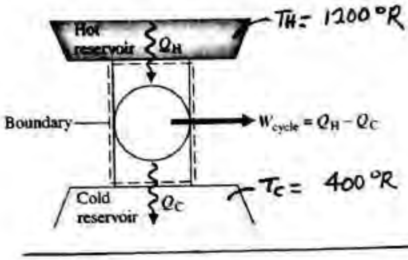
From part (a), $\eta_{MAX} = 0.70$. Since $\eta > \eta_{MAX}$, the cycle cannot operate as claimed. Impossible.

PROBLEM 5.18

KNOWN: Data are provided for a power cycle operating between hot and cold reservoirs.

FIND: In each case, determine whether the cycle operates reversibly, operates irreversibly, or is impossible.

SCHEMATIC & GIVEN DATA:



(a) $Q_H = 900$ Btu, $W_{cycle} = 450$ Btu

With the given data,

$$\eta = \frac{W_{cycle}}{Q_H} = \frac{450\text{ Btu}}{900\text{ Btu}} = 0.5$$

The maximum thermal efficiency is

$$\eta_{MAX} = 1 - \frac{T_C}{T_H} = 1 - \frac{400}{1200} = 0.667$$

Since $\eta < \eta_{MAX}$, the cycle operates irreversibly.

(b) $Q_H = 900$ Btu, $Q_C = 300$ Btu

From the energy balance

$$W_{cycle} = Q_H - Q_C = 900 - 300 = 600 \text{ Btu}$$

With these data, the thermal efficiency is

$$\eta = \frac{W_{cycle}}{Q_H} = \frac{600}{900} = 0.667$$

Since $\underset{(\text{Part(a)})}{\eta = \eta_{MAX}}$, the cycle operates reversibly.

(c) $W_{cycle} = 600$ Btu, $Q_C = 400$ Btu

From the energy balance

$$W_{cycle} = Q_H - Q_C$$

$$\Rightarrow Q_H = W_{cycle} + Q_C = 600 + 400 = 1000 \text{ Btu}$$

With these data, the thermal efficiency is

$$\eta = \frac{600}{1000} = 0.60$$

Since $\underset{(\text{Part(a)})}{\eta < \eta_{MAX}}$, the cycle operates irreversibly.

(d) $\eta = 70\%$

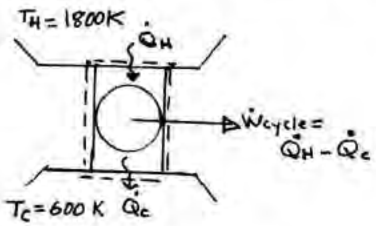
Since $\eta > \eta_{MAX} = 0.667$ (Part(a)), this cycle cannot operate as claimed. Impossible.

PROBLEM 5.19

KNOWN: Data are provided for a power cycle operating between hot and cold reservoirs. The data are for steady-state operation.

FIND: In each case, determine whether the cycle operates reversibly, operates irreversibly, or is impossible.

SCHEMATIC & GIVEN DATA:



ANALYSIS:

Using Eq. 5.9, the maximum theoretical thermal efficiency any such cycle can achieve is

$$\eta_{MAX} = 1 - \frac{T_c}{T_H}$$
$$= 1 - \frac{600}{1800} = 0.667 \quad (66.7\%)$$

(a) $\dot{Q}_H = 500\,kW, \dot{Q}_c = 100\,kW$

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H}$$
$$= 1 - \frac{\dot{Q}_c}{\dot{Q}_H} = 1 - \frac{100\,kW}{500\,kW} = 0.8\ (80\%)$$

Since $\eta > \eta_{MAX}$, this case is impossible.

(b) $\dot{Q}_H = 500\,kW,\ \dot{W}_{cycle} = 250\,kW,\ \dot{Q}_C = 200\,kW$

Check energy balance:

$$\dot{W}_{cycle} = \dot{Q}_H - \dot{Q}_c$$
$$= 500\,kW - 200\,kW = 300\,kW$$

Since $\dot{W}_{cycle}$ is given as 250 kW, the energy balance is not satisfied. This case is impossible.

(c) $\dot{W}_{cycle} = 350\,kW,\ \dot{Q}_c = 150\,kW$

Using the energy balance,

$$\dot{Q}_H = \dot{W}_{cycle} + \dot{Q}_c$$
$$= 350\,kW + 150\,kW = 500\,kW$$

Then

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H} = \frac{350\,kW}{500\,kW} = 0.7\ (70\%)$$

Since $\eta > \eta_{max}$, this case is impossible.

(d) $\dot{Q}_H = 500\,kW,\ \dot{Q}_c = 200\,kW$

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H} = 1 - \frac{\dot{Q}_c}{\dot{Q}_H} = 1 - \frac{200\,kW}{500\,kW} = 0.6\ (60\%)$$

Since $\eta < \eta_{MAX}$, this case corresponds to irreversible operation.

PROBLEM 5.20

KNOWN: Data are provided for four cases involving reversible power cycles operating between hot and cold reservoirs.

SCHEMATIC & GIVEN DATA:

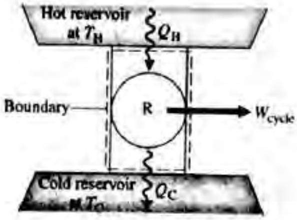


Fig. P5.20

(a) $T_H = 1600K$, $T_C = 400K$, find η

With Eq. 5.9, $\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{400K}{1600K} = 0.75\ (75\%)$ ← (a)

(b) $T_H = 500°C$, $T_C = 20°C$, $W_{cycle} = 1000 kJ$. Find Q_C and Q_H.

With Eq. 5.9 and $\eta = W_{cycle}/Q_H$, which applies for any power cycle,

$$\frac{W_{cycle}}{Q_H} = 1 - \frac{T_C}{T_H} \Rightarrow Q_H = \frac{W_{cycle}}{1 - \frac{T_C}{T_H}} = \frac{1000 kJ}{1 - \frac{293K}{773K}} = 1610 kJ$$

(b)

For any power cycle, $W_{cycle} = Q_H - Q_C$; thus,

$$Q_C = Q_H - W_{cycle} = 1610 kJ - 1000 kJ = 610 kJ$$

(c) $\eta = 60\%$, $T_C = 40°F$. Find T_H.

With Eq. 5.9,

$$\eta = 1 - \frac{T_C}{T_H} \Rightarrow 0.60 = 1 - \frac{500°R}{T_H}$$

$\Rightarrow T_H = 1250°R\ (790°F)$ ← (c)

(d) $\eta = 40\%$, $T_H = 727°C$. Find T_C.

With Eq 5.9,

$$\eta = 1 - \frac{T_C}{T_H} \Rightarrow 0.40 = 1 - \frac{T_C}{1000K} \Rightarrow T_C = 600K\ (327°C)$$ ← (d)

PROBLEM 5.21

KNOWN: Data are provided for a reversible powercycle operating between hot and cold reservoirs at temperatures T_H and T_C.

FIND: Determine the energy rejected, in kJ, and T_C, in K.

ANALYSIS: The thermal efficiency is given. Thus, since Q_H is also given

(1) $$\underbrace{\eta}_{0.4} = \frac{W_{cycle}}{Q_H} = 1 - \frac{Q_C}{Q_H} \Rightarrow \frac{Q_C}{Q_H} = 0.6 \Rightarrow Q_C = 0.6(50\text{ kJ}) = 30\text{ kJ} \leftarrow$$

Since the cycle is reversible, Eq 5.9 applies:

$$\eta = 1 - \frac{T_C}{T_H} \Rightarrow \frac{T_C}{T_H} = 1 - \underbrace{\eta}_{=0.4} = 0.6 \Rightarrow T_C = 0.6(600\text{K}) = 360\text{ K} \leftarrow$$

---

1. For every reversible cycle operating between reservoirs at T_H, T_C,

(Eq. 5.7): $$\frac{Q_C}{Q_H} = \frac{T_C}{T_H} \Rightarrow T_C = \frac{Q_C}{Q_H} T_H = 0.6(600\text{K}) = 360\text{ K} \leftarrow$$

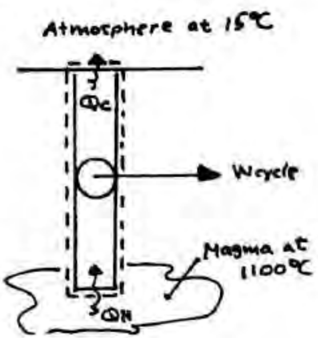
CHECK: $\eta_{rev} = 1 - \frac{T_C}{T_H} = 1 - \frac{360}{600} = 0.4$

PROBLEM 5.22

KNOWN: At a particular locale, the temperatures at a depth below ground and at the Earth's surface are 1100°C and 15°C, respectively.

FIND: Determine the maximum thermal efficiency for any power cycle operating between hot and cold reservoirs at these temperatures.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The system shown in the figure undergoes a power cycle.
2. The magma plays the role of the hot reservoir.
3. The atmosphere plays the role of the cold reservoir.

ANALYSIS: The maximum thermal efficiency corresponds to a reversible power cycle for which Eq. 5.9 applies:

$$\eta_{MAX} = 1 - \frac{T_C}{T_H}$$

① $$= 1 - \frac{288\text{K}}{1373\text{K}}$$

② $$= 0.79 \ (79\%)$$

---

1. The temperatures T_C and T_H must be in K or °R.

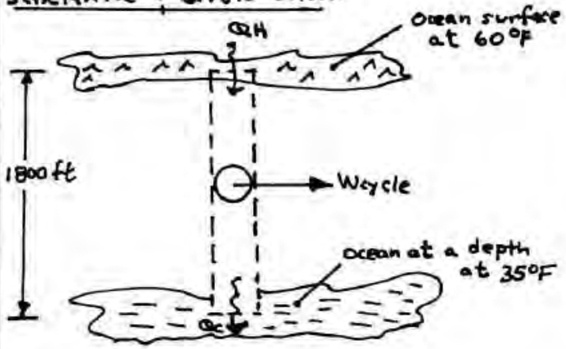
2. Based on this value, tapping into the magma to generate power looks promising. However, other issues must be considered, including but not limited to costs and engineering challenges associated with drilling to the magma site.

PROBLEM 5.23

KNOWN: At a particular locale, the temperatures at the surface of the ocean and at a depth of 1800 ft are 60 and 35 °F, respectively.

FIND: Determine the maximum thermal efficiency for any power cycle operating between hot and cold reservoirs at these temperatures.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The system shown in the figure undergoes a power cycle.
2. The ocean surface water plays the role of the hot reservoir.
3. The ocean water at a depth plays the role of the cold reservoir.

ANALYSIS: The maximum thermal efficiency corresponds to a reversible power cycle for which Eq. 5.9 applies:

① $$\eta_{MAX} = 1 - \frac{T_C}{T_H}$$

$$= 1 - \frac{495\,°R}{520\,°R}$$

② $$= 0.048 \quad (4.8\%)$$ ←

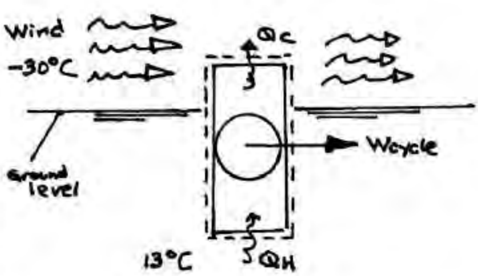
1. The temperatures T_C and T_H must be in °R or K.
2. Since the thermal efficiency of an actual power cycle will be less than the calculated value, the practicality of any such power cycle may be in doubt.

PROBLEM 5.24

KNOWN: At a particular location winds at −30°C exist while underground the temperature is 13°C.

FIND: Investigate whether a power cycle working between these temperatures might have a thermal efficiency of 5%.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The system shown in the figure undergoes a power cycle.
2. The ground at 13°C plays the role of the hot reservoir.
3. The atmosphere at −30°C plays the role of the cold reservoir.

ANALYSIS: For any power cycle operating between hot and cold reservoirs at the specified temperatures, the thermal efficiency cannot be greater than that of a reversible power cycle operating between the same reservoirs. That is,

$$\eta \le \eta_{max} = 1 - \frac{T_C}{T_H}$$

① $$\Rightarrow \eta \le 1 - \frac{243\,K}{286\,K}$$

② $$\le 0.15 \; (15\%)$$

Since the claimed thermal efficiency, 5%, is less than the calculated maximum theoretical value, the inventor's power cycle is not denied by the second law of thermodynamics. Still, at 5% the practicality of such a power cycle in these harsh conditions may be in doubt.

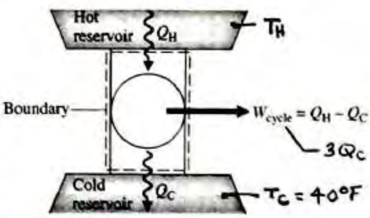
1. The temperatures T_C and T_H must be in K or °R.
2. Alternatively, power can be produced at this site using a wind turbine. Wind turbines do not operate as thermodynamic cycles and thus are not limited by the notion of a maximum theoretical thermal efficiency.

PROBLEM 5.25

KNOWN: Data are provided for a reversible cycle operating as in Figure 5.5 : $T_C = 40°F$, $W_{cycle} = 3Q_c$.

Find : Determine η and T_H, in °F.

Schematic and Given Data:



Engineering model:

1. The system shown in the figure undergoes a reversible power cycle.

Analysis:

(a) $$\eta = \frac{W_{cycle}}{Q_H}$$

For the cycle $$\left.\begin{array}{l} W_{cycle} = Q_H - Q_c \\ W_{cycle} = 3Q_c \end{array}\right\} \Rightarrow Q_H = 4Q_c$$

$$\therefore \eta = \frac{3Q_c}{4Q_c} = = 0.75 \ (75\%)$$ ←

(b) Since the cycle operates reversibly, Eq. 5.9 applies:

① $$\eta = 1 - \frac{T_c}{T_H}$$

$$\Rightarrow 0.75 = 1 - \frac{500°R}{T_H} \Rightarrow T_H = 2000°R \ (1540°F)$$ ←

---

1. T must be in °R or K.

PROBLEM 5.26

KNOWN: Data are provided for two reversible power cycle in series that produce the same net work.

FIND: Determine (a) the intermediate temperature T and the thermal efficiency for each of the two power cycles. (b) For a single cycle, determine the thermal efficiency and net work.

SCHEMATIC & GIVEN DATA:

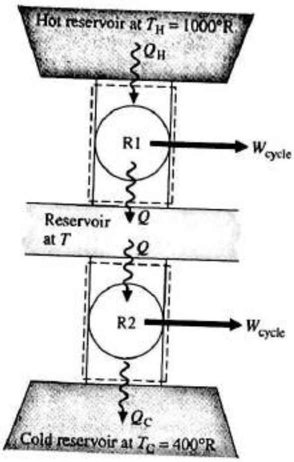


Fig. P5.26

ANALYSIS:

(a) Energy balances: Since W_{cycle} is the net work developed by each cycle

$$W_{cycle} = Q_H - Q = Q - Q_C$$

$$\Rightarrow \quad Q = \frac{Q_H + Q_C}{2} \qquad (1)$$

Since the cycles are reversible, Eq. 5.7 applies to each:

$$\frac{Q}{Q_H} = \frac{T}{T_H}, \quad \frac{Q_C}{Q} = \frac{T_C}{T}$$

$$\Rightarrow \quad Q_H = \frac{T_H}{T} Q$$

$$Q_C = \frac{T_C}{T} Q$$

Substituting these results into Eq. (1)

$$Q = \frac{\frac{T_H}{T} Q + \frac{T_C}{T} Q}{2}$$

$$\Rightarrow \quad T = \frac{T_H + T_C}{2} = \frac{1000°R + 400°R}{2} = 700°R$$

Then, for R1:

$$\eta_1 = 1 - \frac{T}{T_H} = 1 - \frac{700°R}{1000°R} = 0.3 \ (30\%)$$

For R2:

$$\eta_2 = 1 - \frac{T_C}{T} = 1 - \frac{400°R}{700°R} = 0.43 \ (43\%)$$

(b) For the single reversible cycle operating between $T_H = 1000°R$ and $T_C = 400°R$,

① $$\eta_{MAX} = 1 - \frac{T_C}{T_H} = 1 - \frac{400°R}{1000°R} = 0.6 \ (60\%)$$

Also, the net work of this cycle is

$$W'_{cycle} = Q_H - Q_C$$

$$= (Q_H - Q) + (Q - Q_C)$$

$$= 2 W_{cycle}$$

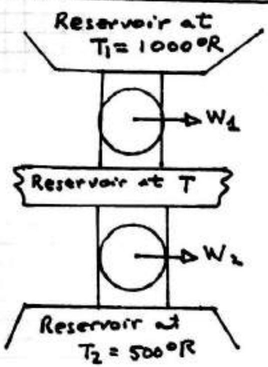
1. The combined cycle thermal efficiency is greater than either of the individual cycles denoted R1 and R2.

PROBLEM 5.27

KNOWN: Data are provided for two reversible cycles in series, connected by a thermal reservoir at temperature T.

FIND: Determine, (a) T, in °R, and the thermal efficiency of of each power cycle, (b) the thermal efficiency of a single overall power cycle.

SCHEMATIC & GIVEN DATA:



$\eta_1 = 1.5\eta_2$

ANALYSIS: (a) Since each cycle is reversible,

$$\eta_1 = 1 - \frac{T}{1000°R}, \quad \eta_2 = 1 - \frac{500°R}{T}$$

Also, $\eta_1 = 1.5\,\eta_2$. Collecting results

$$\left(1 - \frac{T}{1000}\right) = 1.5\left(1 - \frac{500}{T}\right)$$

Solving for T, a quadratic equation results

$$T^2 + 500T - 750{,}000 = 0$$

① Solving the equation, $T = 651.39$ °R ←

Using this value,

$$\eta_1 = 1 - \frac{651.39}{1000} = 0.349 \quad (34.9\%)$$ ←

$$\eta_2 = 1 - \frac{500}{651.39} = 0.232 \quad (23.2\%)$$

(b) For a single overall cycle operating between reservoirs at T_1 and T_2

② $$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{500}{1000} = 0.5 \quad (50\%)$$ ←

---

1. The quadratic equation also has a negative root. This can be ignored because the absolute temperature, in °R, cannot be negative.

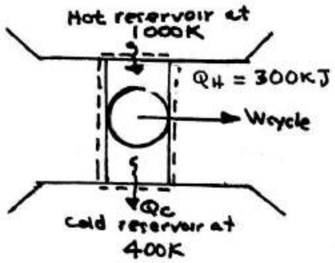
2. The combined cycle thermal efficiency is greater than either of the individual cycles denoted 1 and 2.

PROBLEM 5.28

KNOWN: Data are provided for power cycles operating between hot and cold reservoirs at 1000K and 400K, respectively.

FIND: For each of four cases, determine whether the cycle is in keeping with the first and second laws of thermodynamics.

SCHEMATIC & GIVEN DATA:



W_{cycle} (KJ): (a) 160, (b) 180, (c) 170, (d) 200
Q_C (KJ): (a) 140, (b) 120, (c) 140, (d) 100

ENGINEERING MODEL:

1. The system shown in the figure undergoes a power cycle.
2. First law constraint:
$$W_{cycle} = Q_H - Q_C$$
3. Second law constraint:
$$\eta \leq \eta_{max} = 1 - \frac{T_C}{T_H}$$
$$\eta \leq 1 - \frac{400K}{1000K} = 0.6$$

ANALYSIS: For each case the power cycle is in keeping with the first and second laws only if the respective constraints are satisfied. Otherwise, the power cycle cannot occur.

(a) First law constraint: $W_{cycle} = 160\text{ KJ} = \underbrace{(300-140)\text{KJ}}_{160\text{ KJ}}$. Satisfied.

Second law constraint:
$$\eta = \frac{W_{cycle}}{Q_H} = \frac{160\text{KJ}}{300\text{KJ}} = 0.53 \Rightarrow \eta < \eta_{max}.\quad \text{Satisfied}$$

This cycle satisfies both constraints and thus is allowed.

(b) First law constraint: $W_{cycle} = 180\text{KJ} = \underbrace{(300-120)}_{180\text{ KJ}}\text{KJ}$. Satisfied.

Second law constraint:
$$\eta = \frac{W_{cycle}}{Q_H} = \frac{180\text{KJ}}{300\text{KJ}} = 0.6 \Rightarrow \eta = \eta_{max}\quad \text{Satisfied.}$$

This cycle satisfies both constraints and thus is allowed. Note that since $\eta = \eta_{max}$, the cycle is reversible.

(c) First law constraint: $W_{cycle} = 170\text{KJ} \neq \underbrace{(300-140)\text{KJ}}_{160\text{ KJ}}$. Not satisfied

Second law constraint:
$$\eta = \frac{W_{cycle}}{Q_H} = \frac{170\text{KJ}}{300\text{KJ}} = 0.57 \Rightarrow \eta < \eta_{max}.\quad \text{Satisfied.}$$

Since the cycle does not satisfy both constraints, it cannot occur.

(d) First law constraint: $W_{cycle} = 200\text{KJ} = \underbrace{(300-100)}_{200\text{ KJ}}\text{KJ}$. Satisfied.

Second law constraint:
$$\eta = \frac{W_{cycle}}{Q_H} = \frac{200\text{KJ}}{300\text{KJ}} = 0.67 \Rightarrow \eta > \eta_{max}.\quad \text{Not satisfied.}$$

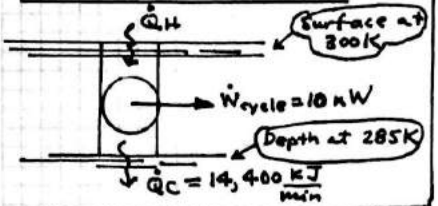
Since the cycle does not satisfy both constraints, it cannot occur.

PROBLEM 5.29

KNOWN: Data are provided for a power cycle operating between a lake's surface water at 300K and water at a depth where the temperature is 285K.

FIND: Determine the thermal efficiency of the cycle and the thermal efficiency of a reversible cycle operating between hot and cold reservoirs at these temperatures.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL

1. The power cycle operates at steady state.
2. Surface water and water at a depth play the roles of hot and cold reservoirs.

ANALYSIS: (a) An energy rate balance is

$$\dot{W}_{cycle} = \dot{Q}_H - \dot{Q}_C \rightarrow \dot{Q}_H = \dot{W}_{cycle} + \dot{Q}_C$$

$$\therefore \quad \dot{Q}_H = 10\ kW + \left(14{,}400 \frac{kJ}{min}\right)\left|\frac{1\ min}{60\ s}\right|\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 250\ kW$$

Then, $\eta = \dfrac{\dot{W}_{cycle}}{\dot{Q}_H} = \dfrac{10\ kW}{250\ kW} = 0.04\ (4\%)$ ← Actual thermal efficiency

(b) The thermal efficiency of a reversible power cycle operating between hot and cold reservoirs at 300K and 285 K, respectively, is

$$\eta_{MAX} = 1 - \frac{T_C}{T_H} = 1 - \frac{285}{300} = 0.05\ (5\%)$$ ← Maximum

PROBLEM 5.30

KNOWN: An inventor claims to have a power cycle operating between hot and cold reservoirs at T_H and $T_C = 300K$, respectively, for which $\eta = 40\%$

FIND: Evaluate the claim if T_H is (a) 900K, (b) 500K, (c) 375K.

ANALYSIS: For the claim to be possible, $\eta \leq \eta_{MAX}$. That is, $\eta \leq 1 - \frac{T_C}{T_H}$ or $0.4 \leq \eta_{max}$.

(a) $T_H = 900K$:

$$\eta_{MAX} = 1 - \frac{300K}{900K} = 0.667 (66.7\%)$$

Possible. The cycle would be operating reversibly.

(b) $T_H = 500K$:

$$\eta_{MAX} = 1 - \frac{300K}{500K} = 0.4 (40\%).$$

Possible but the cycle would be operating reversibly. Thus, the claim is unlikely.

(c) $T_H = 375K$: $\eta_{MAX} = 1 - \frac{300K}{375K} = 0.2 (20\%)$

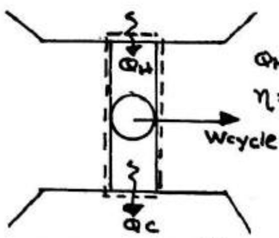
Impossible. The claimed performance is greater than the maximum theoretical value.

PROBLEM 5.31

KNOWN: Data are provided for a power cycle working between hot and cold reservoirs.

FIND: For the specified cycle and a reversible power cycle working between the same two reservoirs, determine the thermal efficiency and energy discharged to the cold reservoir.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The system shown in the figure undergoes a power cycle.

2. The power cycle exchanges energy by heat transfer with hot and cold reservoirs at 1000 and 300 F, respectively.

ANALYSIS: For each cycle, actual and reversible, an energy balance reads

$Q_C = Q_H - W_{cycle}$, where $W_{cycle} = \eta Q_H$. Collecting results

$$Q_C = Q_H(1-\eta) \qquad (a)$$

(a) For the actual cycle, $\eta = 0.75\,\eta_{max}$, where

$$\eta_{max} = \left[1 - \frac{T_C}{T_H}\right] = \left[1 - \frac{760°R}{1460°R}\right] = 0.48 \; (48\%)$$

Thus, $\eta = 0.75(0.48) = 0.36$ (36 %)

With Eq. (a), $Q_C = 1000\text{ Btu}(1-0.36) = 640$ Btu

(b) For the reversible cycle, Eq. (a) gives,

① $Q_C = 1000\text{ Btu}(1-0.48) = 520$ Btu

---

1. Compared to the actual cycle, the reversible cycle develops greater net work while rejecting less energy by heat transfer to the cold reservoir.

PROBLEM 5.32

KNOWN: Data are provided for the cycle of Fig. 5.13.

FIND: Determine T_C, in K, the net work of the cycle, in kJ, and η.

SCHEMATIC & GIVEN DATA:

$P_1 = 2\text{ bar}$
$v_1 = 0.31\text{ m}^3/\text{kg}$
$T_H = 475\text{ K}$
$Q_H = 150\text{ kJ}$

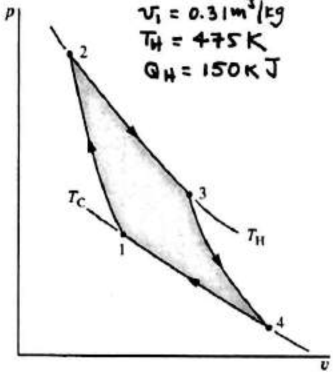


Fig. 5.13 p–v diagram for a Carnot gas power cycle.

ENGINEERING MODEL:

1. Air executes a reversible power cycle while operating between hot and cold reservoirs at T_H and T_C, respectively.
2. Air is modeled as an ideal gas.

ANALYSIS:

Since $T_C = T_1$, the ideal gas equation of state gives

$$T_1 = \frac{P_1 v_1}{R} = \frac{(2\times10^5\text{ N/m}^2)(0.31\text{ m}^3/\text{kg})}{(8314/28.97)\frac{\text{N}\cdot\text{m}}{\text{kg}}} = 216\text{ K}$$

Using, (Power cycle is reversible)

$$\eta = \frac{W}{Q_H} \stackrel{*}{=} 1 - \frac{T_C}{T_H}$$

$$= 1 - \frac{216}{475} = 0.545\ (54.5\%)$$

Then, with

$$W = \eta\, Q_H = (0.545)(150\text{ kJ})$$

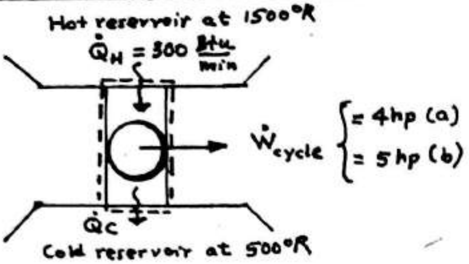
$$= 81.75\text{ kJ}$$

PROBLEM 5.33

KNOWN: Operating data are claimed by an inventor for a power cycle working between hot and cold reservoirs.

FIND: For each of two claims, evaluate performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The system shown in the figure operates as a power cycle.
2. The data provided are for steady-state operation.
3. The power cycle exchanges energy by heat transfer with hot and old reservoirs at 1500 and 500 °R, respectively.

ANALYSIS: The thermal efficiency of the power cycle must obey $\eta \le \eta_{MAX}$, where

$$\eta_{MAX} = 1 - \frac{T_C}{T_H} = 1 - \frac{500°R}{1500°R} = 0.667 \; (66.7\%)$$

(a) If $\dot{W}_{cycle} = 4 hp$,

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H} = \frac{4 hp \left| \frac{2545 \; Btu/h}{1 \; hp} \right| \left| \frac{1 \; h}{60 \; min} \right|}{300 \; Btu/min} = 0.566 \; (56.6\%)$$

Since $\eta < \eta_{max}$, the claimed performance is in accord with the second law.

(b) If $\dot{W}_{cycle} = 5 hp$,

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H} = \frac{5 hp \left| \frac{2545 \; Btu/h}{1 \; hp} \right| \left| \frac{1 \; h}{60 \; min} \right|}{300 \; Btu/min} = 0.707 \; (70.7\%)$$

Since $\eta > \eta_{max}$, the claimed performance is denied by the second law.

PROBLEM 5.34

KNOWN: A power cycle operates between hot and cold reservoirs at 500 K and 310 K, respectively. The power developed is provided: 0.1 MW.

FIND: Determine the minimum theoretical rate at which energy is rejected by heat transfer to the cold reservoir, in MW.

ANALYSIS: At steady state the energy rate balance is $\dot{W}_{cycle} = \dot{Q}_H - \dot{Q}_C$.

$$\Rightarrow \dot{Q}_H = \dot{W}_{cycle} + \dot{Q}_C \quad (1)$$

We know that

$$\eta \leq \eta_{MAX} = \left(1 - \frac{T_C}{T_H}\right) \quad \text{where} \quad \eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H}$$

$$\therefore \quad \frac{\dot{W}_{cycle}}{\dot{Q}_H} \leq \left(1 - \frac{T_C}{T_H}\right). \text{ With Eq. (1), } \frac{\dot{W}_{cycle}}{[\dot{W}_{cycle} + \dot{Q}_C]} \leq \left(1 - \frac{T_C}{T_H}\right).$$

Finally,

$$\frac{\dot{W}_{cycle}[T_C/T_H]}{\left[1 - \frac{T_C}{T_H}\right]} \leq \dot{Q}_C \quad (2)$$

With known values, we get

$$\frac{0.1\,\text{MW}[310/500]}{\left[1 - \frac{310}{500}\right]} \leq \dot{Q}_C$$

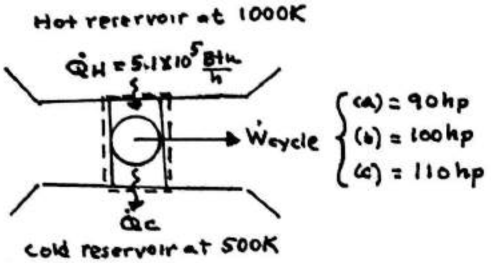
$$0.163\,\text{MW} \leq \dot{Q}_C \quad \Rightarrow \quad (\dot{Q}_C)_{MIN} = 0.163\,\text{MW} \longleftarrow$$

PROBLEM 5.35

KNOWN: Operating data are claimed by an inventor for a power cycle working between hot and cold reservoirs.

FIND: For each of three claims, evaluate performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The system shown in the figure operates as a power cycle.
2. The data provided are for steady-state operation.
3. The power cycle exchanges energy by heat transfer with hot and cold reservoirs at 1000 and 500K, respectively.

ANALYSIS: The thermal efficiency of the power cycle must obey

① $$\eta \le \eta_{MAX} = 1 - \frac{T_C}{T_H} = 1 - \frac{500K}{1000K} = 0.5\ (50\%)$$

(a) If $\dot{W}_{cycle} = 90$ hp,

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H} = \frac{90\text{ hp}}{5.1 \times 10^5\text{ Btu/h}}\left|\frac{2545\text{ Btu/h}}{1\text{ hp}}\right| = 0.45\ (45\%)$$

Since $\eta < \eta_{max}$, the claimed performance is in accord with the second law.

(b) If $\dot{W}_{cycle} = 100$hp,

$$\eta = \frac{100\text{ hp}}{5.1 \times 10^5\text{ Btu/h}}\left|\frac{2545\text{ Btu/h}}{1\text{ hp}}\right| = 0.50\ (50\%)$$

Since $\eta = \eta_{max}$, the inventor's claim corresponds to reversible operation. While such operation is not denied by the second law, it is extremely unlikely in a real-world system.

(c) If $\dot{W}_{cycle} = 110$ hp,

$$\eta = \frac{110\text{hp}}{5.1 \times 10^5\text{ Btu/h}}\left|\frac{2545\text{ Btu/h}}{1\text{ hp}}\right| = 0.55\ (55\%)$$

Since $\eta > \eta_{max}$, the claimed performance is denied by the second law.

---

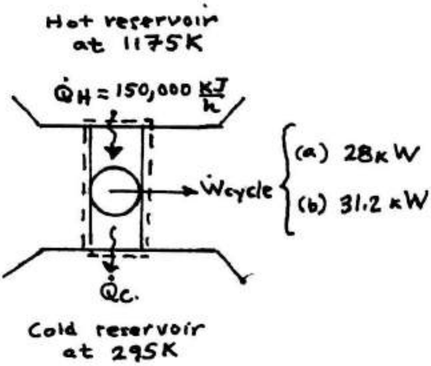
1. Here temperatures can be in K or °R. In the present case the temperatures are given in K.

PROBLEM 5.36

KNOWN: Operating data are claimed by an inventor for a power cycle working between hot and cold reservoirs.

FIND: For each of two claims, evaluate performance.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL

1. The system shown in the figure operates as a power cycle.
2. The data provided are for steady-state operation.
3. The power cycle exchanges energy by heat transfer with hot and cold reservoirs at 1175 and 295K, respectively.

ANALYSIS: The thermal efficiency of the power cycle must obey

$$\eta \leq \eta_{max} = 1 - \frac{T_C}{T_H} = 1 - \frac{295K}{1175K} = 0.749\ (74.9\%)$$

(a) If $\dot{W}_{cycle} = 28\,kW$.

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H} = \frac{28\,kW}{\left(150{,}000\ \frac{kJ}{h}\right)}\left|\frac{1\ kJ/s}{1\,kW}\right|\left|\frac{3600s}{1\,h}\right| = 0.672\ (67.2\%)$$

Since $\eta < \eta_{max}$, the claimed performance is in accord with the second law.

(b) If $\dot{W}_{cycle} = 31.2\,kW$

$$\eta = \frac{31.2\,kW}{\left(150{,}000\ \frac{kJ}{h}\right)}\left|\frac{1\,kJ/s}{1\,kW}\right|\left|\frac{3600\,s}{1\,h}\right| = 0.749\ (74.9\%)$$

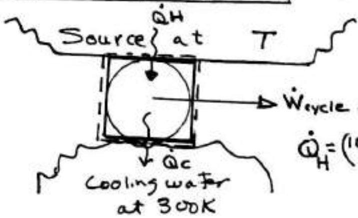
Since $\eta = \eta_{max}$, the inventor's claim corresponds to reversible operation. While such operation is not denied by the second law, it is extremely unlikely in a real-world system.

PROBLEM 5.37

KNOWN: Steady-state operating data are provided for a power cycle receiving energy by heat transfer from a source at temperature T and rejecting energy by heat transfer to cooling water at 300 K.

FIND: Determine the minimum theoretical value for T, in K.

SCHEMATIC & GIVEN DATA:



$$\dot{Q}_H = \left(10\,\frac{kJ}{cycle}\right)\left(\frac{100\,cycles}{min}\right)$$

ENGR. MODEL

1. The system shown in the sketch undergoes a power cycle.
2. The data provided are for steady-state operation.
3. The source and cooling water play the roles of hot and cold reservoirs, respectively.

ANALYSIS:

The power developed must be less than, or equal to, the power developed by a reversible power cycle operating between thermal reservoirs at the specified temperatures. That is, with Eq. 5.9

$$\eta \le \eta_{MAX} = 1 - \frac{T_C}{T_H} = 1 - \frac{300}{T}$$

where

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H} = \frac{10\,kW}{\left(10\,\frac{kJ}{cycle}\right)\left(100\,\frac{cycles}{min}\right)\left(\frac{1\,min}{60\,s}\right)}\left|\frac{1\,kJ/s}{1\,kW}\right|$$

$$= 0.6$$

$$\therefore \quad 0.6 \le 1 - \frac{300}{T}$$

$$\Rightarrow T \ge 750\,K$$

The minimum theoretical value of T is 750 K. ←

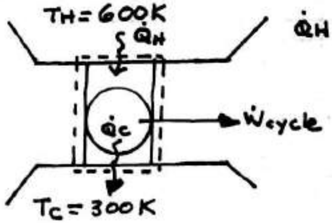
PROBLEM 5.38

A power cycle operates between hot and cold reservoirs at 600 K and 300 K, respectively. At steady state the cycle develops a power output of 0.45 MW while receiving energy by heat transfer from the hot reservoir at the rate of 1 MW.

(a) Determine the thermal efficiency and the rate at which energy is rejected by heat transfer to the cold reservoir, in MW.

(b) Compare the results of part (a) with those of a reversible power cycle operating between these reservoirs and receiving the same rate of heat transfer from the hot reservoir.

SCHEMATIC & GIVEN DATA:



$\dot{Q}_H = 1\,MW$

ANALYSIS: (a) $\dot{Q}_H = 1\,MW$, $\dot{W}_{cycle} = 0.45\,MW$. The actual thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H} = 0.45 \longleftarrow$$

Energy rate balance: $\dot{W}_{cycle} = \dot{Q}_H - \dot{Q}_C$

$$\Rightarrow \dot{Q}_C = \dot{Q}_H - \dot{W}_{cycle} = 0.55\,MW \longleftarrow$$

(b) $$\eta_{MAX} = 1 - \frac{T_C}{T_H} = 1 - \frac{300K}{600K} = 0.5 \longleftarrow$$

$$\therefore \quad \dot{W}_{cycle} = \eta_{MAX}\,\dot{Q}_H = 0.5\,(1\,MW) = 0.5\,MW$$

An energy rate balance gives (as in part (a))

$$\dot{Q}_C = \dot{Q}_H - \dot{W}_{cycle}$$

1 $$= 1\,MW - 0.5\,MW = 0.5\,MW \longleftarrow$$

---

1. From the results of parts (a) and (b) we see that the actual cycle produces less power and discharges more energy to the cold reservoir by heat transfer than the reversible cycle.

PROBLEM 5.39

KNOWN: A system undergoes a power cycle while receiving energy by heat transfer from condensing steam and discharging energy by heat transfer to a lake.

FIND: Determine the minimum theoretical steam mass flow rate for a net power output of 1 MW.

SCHEMATIC & GIVEN DATA:

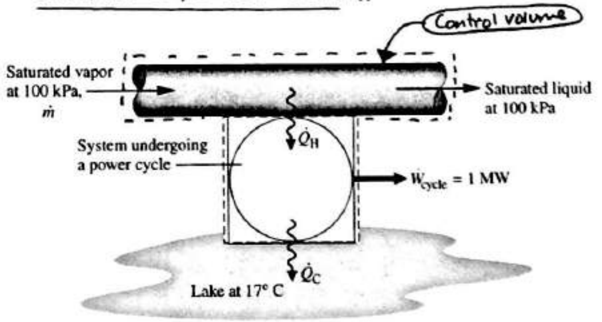


Fig. P5.39

ENGR. MODEL:

1. As shown by the sketch, two systems are under consideration, each operating at steady state.
2. The condensing steam and the lake play the roles of hot and cold reservoirs, respectively.
3. For the control volume, kinetic and potential energy effects are ignored. Steam condenses at 100 kPa.

ANALYSIS: For the system undergoing the power cycle, we have

$$\frac{\dot{W}_{cycle}}{\dot{Q}_H} \leq 1 - \frac{T_C}{T_H} \quad , \text{ where } T_C = 290\,K \text{ and } T_H = T_{sat}(100\,kPa) = 99.63°C = 373\,K$$

(Table A-3 at 100 kPa)

An energy rate balance for the control volume gives $\dot{Q}_H = \dot{m}[h_g - h_f]$, where $h_g - h_f = 2258$ kJ/kg. Collecting results

$$\frac{1\,MW \left| \frac{10^3\,kJ/s}{1\,MW} \right|}{\dot{m}\,(2258\,kJ/kg)} \leq 1 - \frac{290\,K}{373\,K} = 0.223$$

Thus,

$$\frac{10^3\,kJ/s}{(2258\,kJ/kg)(0.223)} \leq \dot{m}$$

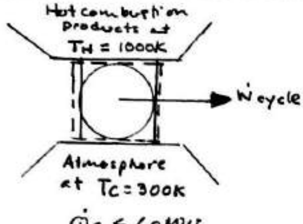
$$\Rightarrow \dot{m} \geq 1.99\,kg/s \quad \longleftarrow \dot{m}_{MIN}$$

PROBLEM 5.40

KNOWN: Steady-state operating and cost data are provided for a power cycle.

FIND: Determine, in \$/year, the maximum value of the power developed and the corresponding fuel cost.

SCHEMATIC & GIVEN DATA:



$\dot{Q}_C \leq 60 MW$

ENGR. MODEL

1. The system shown by the dashed line undergoes a power cycle.
2. The system operates at steady state.
3. The hot combustion gases and atmosphere play the roles of hot and cold reservoirs, respectively.
3. The cycle operates 8000 h/year. The value of the power developed is \$0.10 per kW·h. The cost to provide the heat transfer is \$4.50 per GJ.

ANALYSIS: (a) For any such cycle the maximum power developed occurs when the cycle operates reversibly. Accordingly, the thermal efficiency is given by Eq. 5.9. Then, with $\dot{Q}_H = \dot{W}_{cycle} + \dot{Q}_C$ from an energy balance, we have

$$\frac{\dot{W}_{cycle}}{\dot{W}_{cycle} + \dot{Q}_C} = 1 - \frac{T_C}{T_H} = 1 - \frac{300}{1000} = 0.7 \Rightarrow \dot{W}_{cycle} = 0.7(\dot{W}_{cycle} + \dot{Q}_C) \text{ or}$$

$$\dot{W}_{cycle} = \frac{0.7}{0.3}\dot{Q}_C \leq \frac{0.7}{0.3}(60 MW) = 140 MW.$$ It follows that the value of the maximum power that can be developed is

$$\$]_{\dot{W}_{cycle}} = (140 MW)\left|\frac{10^3 kW}{1 MW}\right|\left(8000\frac{h}{year}\right)\left|\frac{\$0.10}{kW \cdot h}\right| = \$112M/year.$$ ←

(b) The corresponding value of $\dot{Q}_H$ is $\dot{Q}_H = 140 MW + 60 MW$. Accordingly, the corresponding cost is

$$\$]_{\dot{Q}_H} = (200 MW)\left|\frac{1 GJ/s}{10^3 MW}\right|\left|\frac{3600 s}{1 h}\right|\left(8000\frac{h}{year}\right)\left(\frac{\$4.50}{GJ}\right)$$

① $$= \$25.92 M/year$$

---

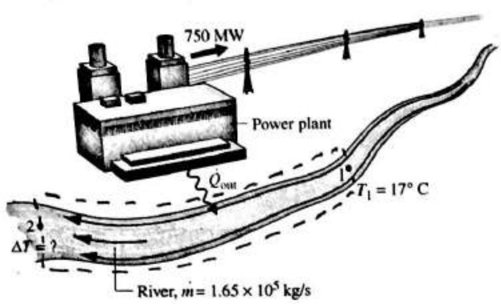
1 The fuel cost is not the only cost. For example, the annual cost of owning the plant is significant.

PROBLEM 5.41

KNOWN: Steady-state operating data are provided for a power plant discharging energy by heat transfer to a river.

FIND: Plot the increase in the temperature of the river traceable to such heat transfer versus the thermal efficiency of the plant.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The power plant is modeled as a power cycle operating between hot and cold thermal reservoirs and at steady state. See ANALYSIS for more.
2. The combustion products provide energy by heat transfer to the power cycle. Energy is discharged by heat transfer. These are the only heat transfers.
3. The river water is modeled as incompressible with specific heat $c = 4.2$ kJ/kg·K (Table A-19).

ANALYSIS: For a control volume enclosing a section of the river, as shown in the sketch, an energy rate balance reduces to give

$$\dot{Q}_{out} = \dot{m}[h_2 - h_1]$$

where $\dot{Q}_{out}$ is a positive number accounting for the energy discharged by heat transfer from the power cycle to the river. With Eq. 3.20b

1 this becomes

$$\dot{Q}_{out} = \dot{m}c[T_2 - T_1] \tag{1}$$

For the power cycle, $\dot{Q}_{out} = \dot{Q}_{in} - \dot{W}_{cycle}$ and $\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}}$. Collecting results, we get

$$\dot{Q}_{out} = \dot{W}_{cycle}\left[\frac{1}{\eta} - 1\right] \tag{2}$$

Combining Eqs. (1), (2)

$$(T_2 - T_1) = \frac{\dot{W}_{cycle}\left[\frac{1}{\eta} - 1\right]}{\dot{m}c} \tag{3}$$

Inserting known values

$$(T_2 - T_1) = \frac{750\,\text{MW}\left[\frac{1}{\eta} - 1\right]}{\left(1.65 \times 10^5 \frac{\text{kg}}{\text{s}}\right)\left(4.2 \frac{\text{kJ}}{\text{kg·K}}\right)}\left|\frac{10^3\,\text{kJ/s}}{1\,\text{MW}}\right|$$

$$= (1.08\ \text{K})\left[\frac{1}{\eta} - 1\right] \tag{4}$$

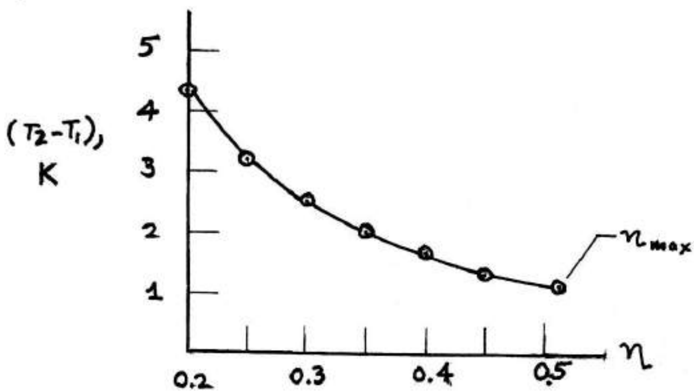
PROBLEM 5.41 (Continued)

In Eq. (4), η is the thermal efficiency of the power plant. Its value is limited by a Carnot efficiency, provided hot and cold reservoirs can be identified. From the problem statement, a plausible choice for T_H is the average temperature of heat addition: $T_H = 590\,K$ (317°C). Thinking of the undisturbed river as the cold reservoir, $T_C \approx 290\,K$ (17°C). Then,

$$\eta_{MAX} = 1 - \frac{T_C}{T_H} \approx 1 - \frac{290K}{590K} = 0.51 \ (51\%)$$

These considerations represent an additional contribution to the Engineering Model for this problem.

Letting η range from 0.2 to 0.51, we get

② 

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1. Assuming additionally that $P_2 \approx P_1$.

2. Even if the cycle were to approach reversibility the river water temperature would increase by about 1 K.

PROBLEM 5.42

KNOWN: Data are provided for a solar-activated power plant.

FIND: Plot the value of the electricity generated annually versus the power plant thermal efficiency. Comment.

SCHEMATIC & GIVEN DATA:

See Fig. P5.42

- The solar collector receives an average annual daily solar input of 4 kW-h per m^2 of collector area.
- The collector measures 15 m by 25 m.
- $T_H = 400\ K$, $T_C = 285\ K$ (see Model, #3)

ENGINEERING MODEL:

1. The solar collector receives a solar input at a known average annual daily rate.
2. The collected energy is delivered without loss to the storage unit.
3. The storage unit and and surroundings play the roles of hot and cold reservoirs.
4. The electricity generated can be sold for 8 cents per kW·h.

ANALYSIS: With assumptions 1, 2, and 3 we can focus on a system undergoing a power cycle that receives energy by heat transfer from the storage unit and discharges energy by heat transfer to the surroundings.

For the power cycle on a daily basis, $W_{cycle} = \eta\, Q_H$, where Q_H is the energy received from the storage unit each day. With assumption 2

① $$Q_H = \left[\frac{4\ kW\cdot h/m^2}{day}\right]\left[\underbrace{15\,m \times 25\,m}_{375\,m^2}\right]$$

$$= 1500\ \frac{kW\cdot h}{day}$$

Accordingly,

$$W_{cycle} = \eta\left(1500\ \frac{kW\cdot h}{day}\right)$$

The annual value is

$$\#/year = \eta\left(1500\ \frac{kW\cdot h}{day}\right)\left(\frac{365\ days}{year}\right)\left(\#0.08\ per\ kW\cdot h\right)$$

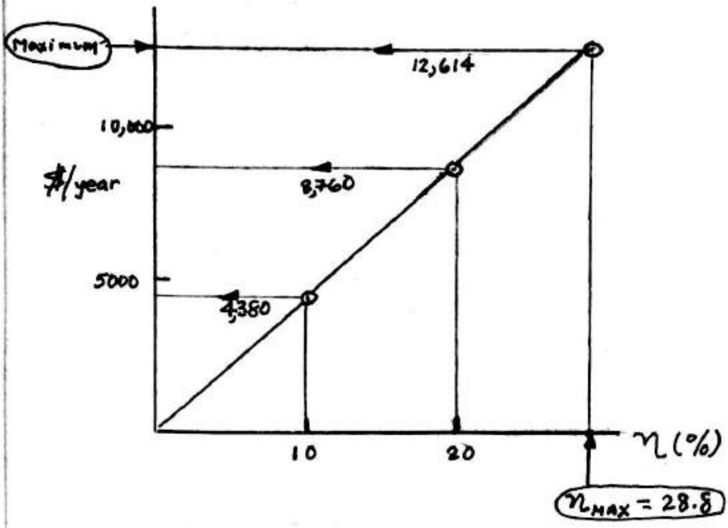
$$= \eta\,(\#43{,}800/year) \qquad (1)$$

The thermal efficiency η must obey

$$\eta \le \eta_{MAX} = \left(1 - \frac{T_C}{T_H}\right) = \left(1 - \frac{285}{400}\right) = 0.288 \quad (28.8\%) \qquad (2)$$

② With these results the following plot is constructed.

PROBLEM 5.42 Continued



1. The collector area is 375 m^2. For comparison, the average ~~total~~ floor area of new single family houses in the U.S in 2010 is 222 m^2.

2. These values do not represent profit, for the annual cost of owning and operating such a system must be considered. At lower values for η profitability could be in doubt.

PROBLEM 5.43

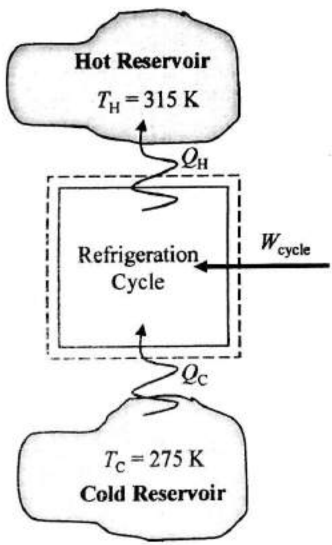
5.43 A refrigeration cycle operating between two reservoirs receives energy Q_C from a cold reservoir at $T_C = 275$ K and rejects energy Q_H to a hot reservoir at $T_H = 315$ K. For each of the following cases determine whether the cycle operates *reversibly*, operates *irreversibly*, or is *impossible*:

(a) $Q_C = 1000$ kJ, $W_{cycle} = 80$ kJ.
(b) $Q_C = 1200$ kJ, $Q_H = 2000$ kJ.
(c) $Q_H = 1575$ kJ, $W_{cycle} = 200$ kJ.
(d) $\beta = 6$.

KNOWN: A refrigeration cycle operates between two reservoirs with specified temperatures.

FIND: Whether each of four cycles operates *reversibly*, operates *irreversibly*, or is *impossible*.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume defined by the dashed line on the accompanying diagram undergoes a refrigeration cycle.

ANALYSIS:
The maximum coefficient of performance for a refrigeration cycle operating between two reservoirs is

$$\beta_{max} = \frac{T_C}{T_H - T_C} = \frac{275\text{ K}}{315\text{ K} - 275\text{ K}} = 6.875$$

Coefficient of performance for any refrigeration cycle is

$$\beta = \frac{Q_C}{W_{cycle}}$$

(a) Given $Q_C = 1000$ kJ, $W_{cycle} = 80$ kJ, the coefficient of performance determined using these energy data is

$$\beta = \frac{1000\text{ kJ}}{80\text{ kJ}} = 12.5$$

Since $\beta = 12.5 > \beta_{max} = 6.875$, the cycle is impossible. ←

(b) Given $Q_C = 1200$ kJ, $Q_H = 2000$ kJ, cycle work can be determined from

$$W_{cycle} = Q_H - Q_C = 2000\text{ kJ} - 1200\text{ kJ} = 800\text{ kJ}$$

The coefficient of performance determined using these energy data is

$$\beta = \frac{1200\text{ kJ}}{800\text{ kJ}} = 1.5$$

Since $\beta = 1.5 < \beta_{max} = 6.875$, the cycle operates irreversibly. ←

(c) Given $Q_H = 1575$ kJ, $W_{cycle} = 200$ kJ, heat transfer associated with the cold reservoir can be determined from

$$W_{cycle} = Q_H - Q_C \rightarrow Q_C = Q_H - W_{cycle} = 1575\text{ kJ} - 200\text{ kJ} = 1375\text{ kJ}$$

The coefficient of performance determined using these energy data is

$$\beta = \frac{1375\text{ kJ}}{200\text{ kJ}} = 6.875$$

Since $\beta = \beta_{max} = 6.875$, the cycle operates reversibly. ←

(d) Given $\beta = 6$, the cycle is irreversible.

Since $\beta = 6 < \beta_{max} = 6.875$, the cycle operates irreversibly. ←

PROBLEM 5.44

KNOWN: A reversible refrigeration cycle operates between cold and hot reservoirs at T_C and T_H.

FIND: For each of 5 sets of data, obtain a specified result.

ANALYSIS:

(a) $\beta = 3.5$, $T_C = -40°F$, find T_H. Since the cycle is reversible, Eq. 5.10 applies:

$$\beta_{MAX} = \frac{T_C}{T_H - T_C} = 3.5 \Rightarrow \frac{420°R}{T_H - 420°R} = 3.5 \Rightarrow T_H = 540°R\ (80°F) \quad \text{(a)}$$

(b) $T_C = -30°C$, $T_H = 30°C$, find β.

$$\beta = \beta_{MAX} = \frac{T_C}{T_H - T_C} = \frac{243\,K}{(303 - 243)\,K} = 4.05 \quad \text{(b)}$$

(c) $Q_C = 500\,Btu$, $Q_H = 800\,Btu$, $T_C = 20°F$, find T_H.

with Eqs. 5.5 and 5.10,

$$\beta = \beta_{MAX} \Rightarrow \frac{Q_C}{Q_H - Q_C} = \frac{T_C}{T_H - T_C}$$

$$\frac{500\,Btu}{800\,Btu - 500\,Btu} = \frac{480°R}{T_H - 480°R} \Rightarrow T_H = 768°R\ (308°F) \quad \text{(c)}$$

(d) $T_C = 30°F$, $T_H = 100°F$, find β

$$\beta = \beta_{MAX} = \frac{T_C}{T_H - T_C} = \frac{490°R}{560°R - 490°R} = 7 \quad \text{(d)}$$

(e) $\beta = 8.9$, $T_C = -5°C$, find T_H.

$$\beta_{MAX} = \frac{T_C}{T_H - T_C} = 8.9$$

$$\Rightarrow \frac{268K}{T_H - 268K} = 8.9 \Rightarrow T_H = 298K\ (25°C) \quad \text{(e)}$$

PROBLEM 5.45

KNOWN: Data are provided for a reversible heat pump operating at steady state.

FIND: For each of 3 sets of data, obtain a specified result.

ANALYSIS: In these cases, Eq. 5.10 is applicable.

(a) $T_H = 13°C$, $T_C = 2°C$, find β.

$$\beta = \beta_{MAX} = \frac{T_C}{T_H - T_C} = \frac{275\ K}{11\ K} = 25 \longleftarrow \text{(a)}$$

(b) $\dot{Q}_H = 10.5\ kW$, $\dot{Q}_C = 8.75\ kW$, $T_C = 0°C$, find T_H.

With Eqs. 5.5 and 5.10,

$$\beta = \beta_{MAX} \Rightarrow \frac{\dot{Q}_C}{\dot{Q}_H - \dot{Q}_C} = \frac{T_C}{T_H - T_C}$$

$$\frac{8.75\ kW}{10.5\ kW - 8.75\ kW} = \frac{273\ K}{T_H - 273\ K} \Rightarrow T_H = 328\ K \ (55°C) \longleftarrow \text{(b)}$$

(c) $\beta = 10$, $T_H = 27°C$, find T_C.

$$\beta = \beta_{MAX} \Rightarrow 10 = \frac{T_C}{T_H - T_C} \Rightarrow 10 = \frac{T_C}{300\ K - T_C}$$

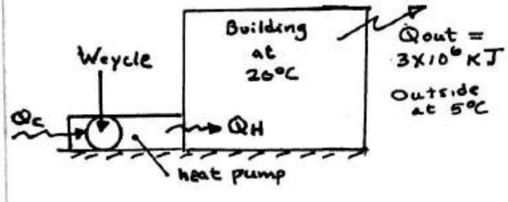
$$T_C = 273\ K \ (0°C) \longleftarrow \text{(c)}$$

PROBLEM 5.46

KNOWN: Operating data are provided for three heat pumps for heating a building.

FIND: Evaluate the minimum theoretical work input required by each heat pump.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. For the heat pump shown in the figure, the dwelling plays the role of the hot reservoir.
2. The ground, a pond, and the outside air play the roles of the cold reservoir.

ANALYSIS: To maintain the building at 20°C, it is necessary for $Q_H = Q_{out} = 3 \times 10^6$ kJ. For any such heat pump, Eqs. 5.6 and 5.11 require

$$\gamma \leq \gamma_{MAX} = \frac{T_H}{T_H - T_C} \Rightarrow \frac{Q_H}{W_{cycle}} \leq \frac{T_H}{T_H - T_C} \Rightarrow Q_H \left[\frac{T_H - T_C}{T_H} \right] \leq W_{cycle}$$

where $T_H = 293$ K (20°C). Finally,

$$\Rightarrow \left[\frac{3 \times 10^6 \text{ kJ}}{293 \text{K}} \right] (293\text{K} - T_c) \leq W_{cycle}$$

(a) $T_c = 288$K

$$\left[\frac{3 \times 10^6 \text{ kJ}}{293 \text{ K}} \right] (5\text{K}) = 5.12 \times 10^4 \text{ kJ}$$

$$\Rightarrow (W_{cycle})_{min} = 5.12 \times 10^4 \text{ kJ} \longleftarrow$$

(b) $T_c = 283$K

$$\left[\frac{3 \times 10^6 \text{ kJ}}{293 \text{K}} \right] (10\text{K}) = 10.24 \times 10^4 \text{ kJ}$$

$$\Rightarrow (W_{cycle})_{min} = 10.24 \times 10^4 \text{ kJ} \longleftarrow$$

(c) $T_c = 278$K

$$\left[\frac{3 \times 10^6 \text{ kJ}}{293 \text{ K}} \right] (15\text{K}) = 15.36 \times 10^4 \text{ kJ}$$

$$\Rightarrow (W_{cycle})_{min} = 15.36 \times^4 \text{ kJ} \longleftarrow$$

PROBLEM 5.47

A refrigeration cycle rejects $Q_H = 500$ Btu per cycle to a hot reservoir at $T_H = 540°R$, while receiving $Q_C = 375$ Btu per cycle from a cold reservoir at temperature T_C. For 10 cycles of operation, determine (a) the net work input, in Btu, and (b) the minimum theoretical temperature T_C, in °R.

ANALYSIS:

(a) Applying Eq. 2.44

$$W_{cycle} = Q_H - Q_C = [500 - 375] \frac{\text{Btu}}{\text{cycle}} (10 \text{ cycles})$$

$$= 1250 \text{ Btu} \qquad \text{(a)}$$

(b) For any refrigeration cycle,

$$\beta \leq \beta_{MAX} = \frac{T_C}{T_H - T_C}$$

With Eq. 5.5

$$\frac{Q_C}{Q_H - Q_C} \leq \frac{T_C}{T_H - T_C}$$

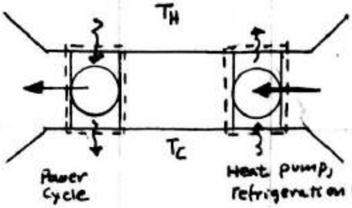
$$\frac{3750 \text{ Btu}}{1250 \text{ Btu}} \leq \frac{T_C}{540°R - T_C} \Rightarrow 405°R \leq T_C \qquad \text{(b)}$$

PROBLEM 5.48

KNOWN: Systems undergo reversible power, refrigeration, and heat pump cycles while operating between the same two thermal reservoirs. The thermal efficiency of the power cycle is 20%.

FIND: Determine the coefficients of performance for the refrigeration and heat pump cycles.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The systems shown in the accompanying figure undergo reversible power, heat pump, and refrigeration cycles while working between hot and cold reservoirs at T_H and T_C, respectively.

ANALYSIS: For the power cycle, $\eta_{max} = 1 - \frac{T_C}{T_H} \Rightarrow \frac{T_C}{T_H} = 1 - \eta_{max}$
$= 1 - 0.2 = 0.8$

(a) For the refrigeration cycle, Eq. 5.10 reads

$$\beta_{max} = \frac{T_C}{T_H - T_C} = \frac{T_C/T_H}{1 - T_C/T_H}$$

$$= \frac{0.8}{1 - 0.8} = 4.0 \longleftarrow$$

(b) For the heat pump cycle, Eq. 5.11 reads

$$\gamma_{max} = \frac{T_H}{T_H - T_C} = \frac{1}{1 - (T_C/T_H)}$$

① ②

$$= \frac{1}{1 - 0.8} = 5.0 \longleftarrow$$

1. $\gamma_{MAX} - \beta_{MAX} = \left(\frac{T_H}{T_H - T_C}\right) - \left(\frac{T_C}{T_H - T_C}\right) = 1 \Rightarrow \gamma_{MAX} = \beta_{MAX} + 1$

2. $\gamma_{max} = 1/\eta_{max}$

PROBLEM 5.49

KNOWN: Two sets of steady-state operating data are provided for a system consisting of a power cycle and a heat pump cycle, each working between hot and cold reservoirs at 500 K and 300 K, respectively.

FIND: For each set of data determine whether the first and second laws are satisfied.

SCHEMATIC & GIVEN DATA:

	Power cycle			Heat pump cycle		
	$\dot{Q}_H$	$\dot{Q}_C$	$\dot{W}_{cycle}$	$\dot{Q}'_H$	$\dot{Q}'_C$	$\dot{W}'_{cycle}$
(a)	60	40	20	80	60	20
(b)	120	80	40	100	80	20

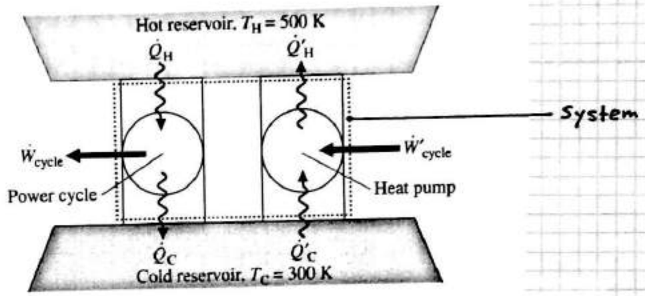


Fig.P5.49

ENGINEERING MODEL: 1. The system shown by the dashed line operates at steady state. 2. Each energy transfer is positive in the direction of the accompanying arrow.

ANALYSIS: (a) To check the first law, apply Eqs. 2.41 and 2.44, each on a time-rate basis, using data set (a). By inspection, the energy rate balance is satisfied for each of the cycles and thus for the system shown by the dashed line in Fig. P5.49. An energy rate balance applied to the system leads to the same conclusion.

Considering the second law next, observe that the system receives energy by heat transfer from the cold reservoir at a net rate of 20 kJ/s, while it delivers energy by heat transfer to the hot reservoir at the same net rate. Moreover, the net work rate for the system is 0 kJ/s. In this mode the system operates as a heat pump with no work input required. This mode of operation is denied by the Clausius statement of the second law, and thus the Kelvin-Planck statement, which is equivalent.

Alternatively, since the system operates as a heat pump with no work input required, its coefficient of performance, Eq. 5.6, is infinite. The maximum theoretical coefficient of performance for any heat pump operating between the two reservoirs is limited by Eq. 5.11: $\gamma_{max} = 2.5$. Accordingly, the second law is not satisfied from this perspective as well.

PROBLEM 5.49 (Continued)

(b) To check the first law, apply Eqs. 2.41 and 2.44, each on a time-rate basis, using data set (b). By inspection, the energy rate balance is satisfied for each of the cycles and thus for the system. An energy rate balance applied to the system leads to the same conclusion.

Considering the second law next, observe that the system generates power at a net rate of 20 kJ/s, while it receives energy by heat transfer from the hot reservoir at the same net rate. There is no net heat transfer with the cold reservoir. This mode of operation is denied by the Kelvin-Planck statement of the second law, and thus the Clausius statement, which is equivalent.

Alternatively, since the system operates as a power cycle with no heat transfer with the cold reservoir, its thermal efficiency, Eq. 5.4, is 100%. The maximum thermal efficiency for any power cycle operating between the two reservoirs is limited by Eq. 5.9: $\eta_{max} = 40\%$. Accordingly, the second law is not satisfied from this perspective as well.

PROBLEM 5.50

An inventor has developed a refrigerator capable of maintaining its freezer compartment at 20°F while operating in a kitchen at 70°F, and claims the device has a coefficient of performance of (a) 10, (b) 9.6, (c) 4. Evaluate the claim in each of the three cases.

ANALYSIS:

$$\beta \leq \beta_{MAX} = \frac{T_C}{T_H - T_C} \qquad \text{(Eq. 5.10)}$$

$$\Rightarrow \beta \leq \frac{480°R}{50°R} = 9.6$$

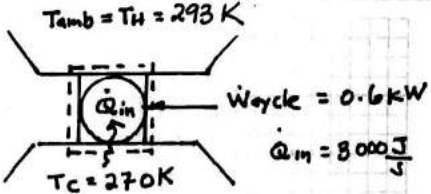
(a) $\beta = 10$. Not a valid claim. β must be less than 9.6

(b) $\beta = 9.6$. Claim allowed by the second law of thermodynamics only if the cycle can operate reversibly. Unlikely.

(c) $\beta = 4$. Claim allowed by second law.

PROBLEM 5.51

An inventor claims to have developed a food freezer that at steady state requires a power input of 0.6 kW to extract energy by heat transfer at a rate of 3000 J/s from freezer contents at 270 K. Evaluate this claim for an ambient temperature of 293 K.



$$\beta \leq \beta_{MAX}$$

$$\frac{\dot{Q}_{in}}{\dot{W}_{cycle}} \leq \frac{T_C}{T_H - T_C} \quad \rightarrow \quad \frac{(3000\ \text{J/s})\left|\frac{1\ \text{kJ}}{10^3\ \text{J}}\right|}{0.6\ \text{kW}} \leq \frac{270}{293-270}$$

$$5 \leq 11.74$$

The claim is in accord with the second law. ①

1. While operation is in accord with the second law, a more indepth analysis might reveal practical and/or economic shortcomings.

PROBLEM 5.52

An inventor claims to have developed a refrigerator that at steady state requires a net power input of 0.7 horsepower to remove 12,000 Btu/h of energy by heat transfer from the freezer compartment at 0°F and discharge energy by heat transfer to a kitchen at 70°F. Evaluate this claim.

ANALYSIS:

$$\beta \le \beta_{MAX} = \frac{T_C}{T_H - T_C} \Rightarrow \underbrace{\frac{\dot{Q}_C}{\dot{W}_{cycle}}}_{\beta} \le \underbrace{\frac{T_C}{T_H - T_C}}_{\beta_{max}}$$

$$\underbrace{\frac{12{,}000\ \frac{\text{Btu}}{\text{h}}}{0.7\ \text{hp}}\left|\frac{1\ \text{hp}}{2545\ \frac{\text{Btu}}{\text{h}}}\right|}_{=6.74} \le \underbrace{\frac{460^\circ\text{R}}{70^\circ\text{R}}}_{=6.57}$$

No. The claim cannot be allowed. The claimed β exceeds the maximum theoretical value.

PROBLEM 5.53

An inventor claims to have devised a refrigeration cycle operating between hot and cold reservoirs at 300 K and 250 K, respectively, that removes an amount of energy Q_C by heat transfer from the cold reservoir that is a multiple of the net work input—that is, $Q_C = NW_{cycle,}$ where all quantities are positive. Determine the maximum theoretical value of the number N for any such cycle.

ANALYSIS:

$$\underbrace{\beta}_{= \frac{Q_c}{W_{cycle}}} \leq \beta_{MAX} = \frac{T_c}{T_H - T_c} \Rightarrow \frac{\overbrace{Q_c}^{= NW_{cycle}}}{W_{cycle}} \leq \frac{T_c}{T_H - T_c}$$

$$\therefore \; N \leq \frac{250 K}{50 K} = 5 \qquad \therefore \; N \leq 5$$

$$\Rightarrow N_{max} = 5$$

PROBLEM 5.54

Data are provided for two reversible refrigeration cycles. One cycle operates between hot and cold reservoirs at 27°C and −8°C, respectively. The other cycle operates between the same hot reservoir at 27°C and a cold reservoir at −28°C. If each refrigerator removes the same amount of energy by heat transfer from its cold reservoir, determine the ratio of the net work input values of the two cycles.

ANALYSIS:

Refrigeration cycle #1 : $\frac{Q}{W_{cycle,1}} = \frac{T_C}{T_H - T_C} = \frac{265\,K}{35\,K} = 7.57$

Refrigeration cycle #2 : $\frac{Q}{W_{cycle,2}} = \frac{T_C'}{T_H - T_C} = \frac{245\,K}{55\,K} = 4.45$

$$\Rightarrow \frac{Q/W_{cycle,1}}{Q/W_{cycle,2}} = \frac{7.57}{4.45} \Rightarrow \frac{W_{cycle,2}}{W_{cycle,1}} = 1.7$$

PROBLEM 5.55

By removing energy by heat transfer from its freezer compartment at a rate of 1.25 kW, a refrigerator maintains the freezer at −26°C on a day when the temperature of the surroundings is 22°C. Determine the minimum theoretical power, in kW, required by the refrigerator at steady state.

ANALYSIS:

$$\beta \le \beta_{MAX} = \frac{T_C}{T_H - T_C} \Rightarrow \frac{\dot{Q}_C}{\dot{W}_{cycle}} \le \frac{T_C}{T_H - T_C}$$

$\frac{\dot{Q}_C}{\dot{W}_{cycle}}$

$$\Rightarrow \dot{Q}_C \left[\frac{T_H - T_C}{T_C} \right] \le \dot{W}_{cycle}$$

$$\therefore \quad 1.25\,kW \left[\frac{48K}{247K} \right] \le \dot{W}_{cycle}$$

$$\Rightarrow \dot{W}_{cycle} \ge 0.243\,kW$$

$$(\dot{W}_{cycle})_{min} = 0.243\,kW$$

PROBLEM 5.56

5.56 At steady state, a refrigeration cycle maintains a *clean room* at 55°F by removing energy entering the room by heat transfer from adjacent spaces at the rate of 0.12 Btu/s. The cycle rejects energy by heat transfer to the outdoors where the temperature is 80°F.

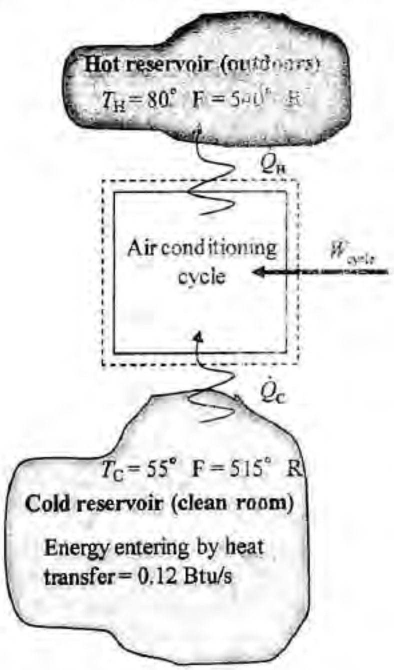
(a) If the rate at which the cycle rejects energy by heat transfer to the outdoors is 0.16 Btu/s, determine the power required, in Btu/s.

(b) Determine the power required to maintain the clean room's temperature by a reversible refrigeration cycle operating between cold and hot reservoirs at 55°F and 80°F, respectively, and the corresponding rate at which energy is rejected by heat transfer to the outdoors, each in Btu/s.

KNOWN: A refrigeration cycle maintains a clean room at known conditions.

FIND: Determine the power required by a refrigeration cycle rejecting energy by heat transfer to the outdoors at the rate of 0.16 Btu/s. Also determine the power required by a reversible refrigeration cycle and the corresponding rate of heat transfer to the outdoors.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

(1) The system shown on the accompanying diagram undergoes a refrigeration cycle in steady state.

(2) The clean room is at steady state.

(3) The clean room and outdoors act as the cold and hot reservoirs, respectively.

PROBLEM 5.56 (Continued)

ANALYSIS:

To maintain the clean room temperature at steady state, we have $\dot{Q}_C = 0.12$ Btu/s.

(a) In this case, $\dot{Q}_H = 0.16$ Btu/s. Applying the energy rate balance for refrigeration cycles, the power required is as follows:

$$\dot{W}_{cycle} = \dot{Q}_H - \dot{Q}_E = 0.16\frac{\text{Btu}}{\text{s}} - 0.12\frac{\text{Btu}}{\text{s}} = 0.04\frac{\text{Btu}}{\text{s}} \longleftarrow$$

(b) To determine power required by a reversible refrigeration cycle, begin by noting that its coefficient of performance is given by Eq. 5.10. Combining this with Eq. 5.5 on a rate basis we get

$$\frac{T_C}{T_H - T_C} = \frac{\dot{Q}_C}{\dot{W}_{cycle}}$$

Rearranging

$$\dot{W}_{cycle} = \frac{\dot{Q}_C(T_H - T_C)}{T_C} = \frac{0.12\frac{\text{Btu}}{\text{s}}(540°\text{R} - 515°\text{R})}{515°\text{R}} = 0.0058\frac{\text{Btu}}{\text{s}}$$

Thus the power required by a reversible cycle is 0.0058 Btu/s. $\longleftarrow$

From an energy rate balance, the energy rejected by heat transfer to the outdoors for the reversible cycle case = (0.0058 +0.12) Btu/s = 0.1258 Btu/s. $\longleftarrow$

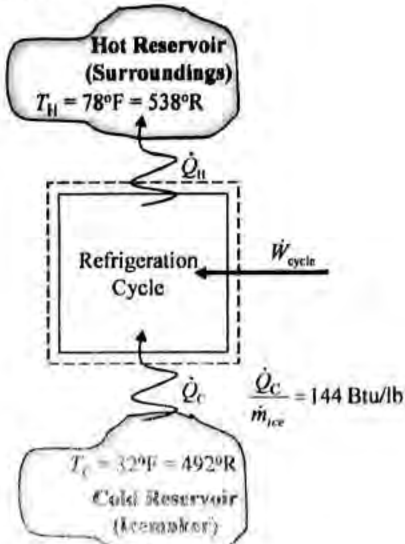
PROBLEM 5.57

5.57 For each kW of power input to an ice maker at steady state, determine the maximum rate that ice can be produced, in lb/h, from liquid water at 32°F. Assume that 144 Btu/lb of energy must be removed by heat transfer to freeze water at 32°C, and that the surroundings are at 78°F.

KNOWN: An ice maker (refrigeration cycle) operates between two reservoirs with specified temperatures with specified energy rate per mass rate removal to make ice.

FIND: Determine the maximum rate ice can be produced

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

1. The control volume defined by the dashed line on the accompanying diagram undergoes a refrigeration cycle.
2. The operation is at steady-state.
3. Water at 32°F is the cold reservoir.
4. The surroundings at 78°F are the hot reservoir.

ANALYSIS:

The maximum rate ice can be produced would correspond to maximum coefficient of performance for a refrigeration cycle operating between the two specified reservoirs. Thus

$$\beta \le \beta_{max}$$

Coefficient of performance for any refrigeration cycle is

PROBLEM 5.57 (Continued)

$$\beta = \frac{\dot{Q}_C}{\dot{W}_{cycle}} = \frac{\dot{m}_{ice}\left(\frac{\dot{Q}_C}{\dot{m}_{ice}}\right)}{\dot{W}_{cycle}}$$

where $\dot{m}_{ice}$ is the rate of ice formation. The maximum coefficient of performance for a refrigeration cycle operating between two reservoirs is

$$\beta_{max} = \frac{T_C}{T_H - T_C} = \frac{492°R}{538°R - 492°R} = 10.70$$

Thus

$$\frac{\dot{m}_{ice}\left(\frac{\dot{Q}_C}{\dot{m}_{ice}}\right)}{\dot{W}_{cycle}} \le 10.70$$

$$\frac{\dot{m}_{ice}}{\dot{W}_{cycle}} \le \frac{10.70}{\left(\frac{\dot{Q}_C}{\dot{m}_{ice}}\right)} = \frac{10.70}{144\,\frac{\text{Btu}}{\text{lb}}}\left|\frac{\text{Btu}}{1.0551\text{ kJ}}\right|\left|\frac{\frac{\text{kJ}}{\text{s}}}{\text{kW}}\right|\left|\frac{3600\text{ s}}{\text{h}}\right| = \underline{\mathbf{253.5\ (lb/h)/kW}} \longleftarrow$$

The maximum rate that ice can be made is 253.5 (lb/h)/kW of power input.

PROBLEM 5.58

At steady state, a refrigeration cycle operating between hot and cold reservoirs at 300 K and 275 K, respectively, removes energy by heat transfer from the cold reservoir at a rate of 600 kW.

(a) If the cycle's coefficient of performance is 4, determine the power input required, in kW.
(b) Determine the minimum theoretical power required, in kW, for *any* such cycle.

ANALYSIS:

(a) $$\beta = \frac{\dot{Q}_C}{\dot{W}_{cycle}} \Rightarrow \dot{W}_{cycle} = \frac{\dot{Q}_C}{\beta} = \frac{600\,kW}{4} = 150\,kW \quad \longleftarrow (a)$$

(b) $$\beta \leq \beta_{MAX} \Rightarrow \frac{\dot{Q}_C}{\dot{W}_{cycle}} \leq \frac{T_C}{T_H - T_C} \Rightarrow \dot{Q}_C\left[\frac{T_H - T_C}{T_C}\right] \leq \dot{W}_{cycle}$$

$$600\,kW\left[\frac{25\,K}{275\,K}\right] \leq \dot{W}_{cycle}$$

$$\Rightarrow \dot{W}_{cycle} \geq 54.5\,kW \quad \longleftarrow (b)$$

PROBLEM 5.59

An air conditioner operating at steady state maintains a dwelling at 20°C on a day when the outside temperature is 35°C. Energy is removed by heat transfer from the dwelling at a rate of 2800 J/s while the air conditioner's power input is 0.8 kW. Determine (a) the coefficient of performance of the air conditioner and (b) the power input required by a reversible refrigeration cycle providing the same cooling effect while operating between hot and cold reservoirs at 35°C and 20°C, respectively.

(a)

$$\beta = \frac{\dot{Q}_C}{\dot{W}_{cycle}} = \frac{(2800\ \text{J/s})\left|\frac{1\ \text{kJ}}{10^3\ \text{J}}\right|\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right|}{0.8\ \text{kW}} = 3.5 \longleftarrow$$

(b)

$$\beta_{MAX} = \frac{T_C}{T_H - T_C} = \frac{293\text{K}}{15\text{K}} = 19.5$$

For the same $\dot{Q}_C$: 2.8 kW, the reversible refrigeration cycle requires a power input of

$$(\dot{W}_{cycle})_{min} = \frac{\dot{Q}_C}{\beta_{MAX}} = \frac{2.8\ \text{kW}}{19.5} = 0.14\ \text{kW} \longleftarrow$$

PROBLEM 5.60

A heat pump is under consideration for heating a research station located on an Antarctica ice shelf. The interior of the station is to be kept at 15°C. Determine the maximum theoretical rate of heating provided by a heat pump, in kW per kW of power input, in each of two cases: The role of the cold reservoir is played by (a) the atmosphere at −20°C, (b) ocean water at 5°C.

For any heat pump on a time-rate basis, $\gamma = \frac{\dot{Q}_H}{\dot{W}_{in}}$.

Also, $\gamma_{MAX} = \frac{T_H}{T_H - T_C}$, where $T_H = 288\text{K}\ (15°\text{C})$.

Collecting results

$$\left(\frac{\dot{Q}_H}{\dot{W}_{in}}\right)_{MAX} = \left[\frac{288\text{ K}}{288\text{K} - T_C}\right] \qquad (1)$$

(a) Atmosphere: $T_C = 253\text{ K}\ (-20°\text{C})$. Eq. (1) gives

$$\left(\frac{\dot{Q}_H}{\dot{W}_{in}}\right)_{MAX} = \frac{288\text{ K}}{(288-253)\text{K}} = 8.23 \longleftarrow$$

(b) Ocean water: $T_C = 278\text{K}\ (5°\text{C})$. Eq. (1) gives

1 $$\left(\frac{\dot{Q}_H}{\dot{W}_{in}}\right)_{MAX} = \frac{288\text{K}}{(288-278)\text{K}} = 28.8 \longleftarrow$$

---

1. Use of ocean water as the cold reservoir appears to be advantageous thermodynamically. However, actual performance will depart significantly from these maximum values, and cost will be a factor in deciding on the preferred course of action.

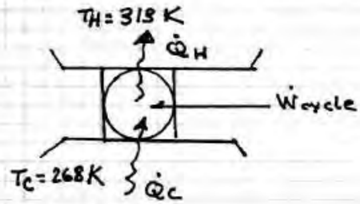
PROBLEM 5.61

A refrigeration cycle has a coefficient of performance equal to 75% of the value for a reversible refrigeration cycle operating between cold and hot reservoirs at −5°C and 40°C, respectively. For operation at steady state, determine the net power input, in kW per kW of cooling, required by (a) the actual refrigeration cycle and (b) the reversible refrigeration cycle. Compare values.

KNOWN: Data are provided for a refrigeration cycle and a reversible refrigeration cycle operating between cold and hot reservoirs at specified temperatures.

FIND: For each cycle evaluate the net power input required, in kW per kW of cooling. Compare the values.

SCHEMATIC & GIVEN DATA:



Engineering Model:

1. The heat pump shown in the figure operates at steady state.
2. Compared to a reversible heat pump operating between the same two reservoirs the coefficient of performance is 75% of that for the reversible heat pump.

ANALYSIS:

(a) $$\beta = 0.75\,\beta_{max} = 0.75\Big[\underbrace{\frac{T_C}{T_H - T_C}}_{\beta_{max}}\Big] = 0.75\Big[\underbrace{\frac{268\text{ K}}{45\text{ K}}}_{5.96}\Big] = 4.47$$

Since $$\beta = \frac{\dot{Q}_C}{\dot{W}_{cycle}} \Rightarrow \frac{\dot{W}_{cycle}}{\dot{Q}_C} = \frac{1}{4.47} = 0.224 \longleftarrow \text{(a)}$$

(b) For the reversible heat pump

$$\frac{\dot{W}_{cycle}}{\dot{Q}_C} = \frac{1}{5.96} = 0.168 \longleftarrow \text{(b)}$$

from part(a)

Comparing values

$$\frac{(\dot{W}_{cycle}/\dot{Q}_C)_{act}}{(\dot{W}_{cycle}/\dot{Q}_C)_{rev}} = \frac{0.224}{0.168} = 1.33$$

33% more work is required in the actual heat pump per unit of cooling than in a reversible heat pump

PROBLEM 5.62

By removing energy by heat transfer from a room, a window air conditioner maintains the room at 22°C on a day when the outside temperature is 32°C.

(a) Determine, in kW per kW of cooling, the *minimum* theoretical power required by the air conditioner.

(b) To achieve required rates of heat transfer with practical-sized units, air conditioners typically receive energy by heat transfer at a temperature *below* that of the room being cooled and discharge energy by heat transfer at a temperature *above* that of the surroundings. Consider the effect of this by determining the *minimum* theoretical power, in kW per kW of cooling, required when $T_C = 18°C$ and $T_H = 36°C$, and compare with the value found in part (a).

Assuming the room interior and the outside are playing the roles of the cold and hot reservoirs, respectively,

(a)
$$\beta \le \beta_{MAX} = \frac{T_C}{T_H - T_C} = \frac{(22+273)\,K}{10\,K} = 29.5$$

$\frac{\dot{Q}_C}{\dot{W}_{cycle}}$

$$\Rightarrow \left(\frac{\dot{W}_{cycle}}{\dot{Q}_C}\right)_{min} = \frac{1}{29.5} = 3.39 \times 10^{-2}$$

(b)
$$\beta \le \beta_{MAX} = \frac{T_C'}{T_H' - T_C'} = \frac{(18+273)\,K}{18\,K} = 16.17$$

$$\Rightarrow \left(\frac{\dot{W}_{cycle}}{\dot{Q}_C}\right)_{min} = \frac{1}{16.17} = 6.18 \times 10^{-2}$$

Comparing values,

$$\frac{(\dot{W}_{cycle}/\dot{Q}_C)\,(b)}{(\dot{W}_{cycle}/\dot{Q}_C)\,(a)} = \frac{6.18 \times 10^{-2}}{3.39 \times 10^{-2}} = 1.82$$

$\Rightarrow$ 82% more work required owing to the effect of "external" irreversibilities related to heat transfer to and from the refrigerator.

PROBLEM 5.63

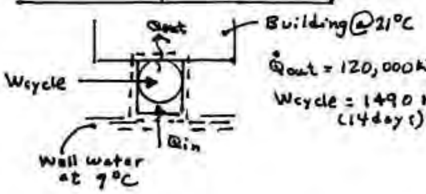
A heat pump cycle is used to maintain the interior of a building at 21°C. At steady state, the heat pump receives energy by heat transfer from well water at 9°C and discharges energy by heat transfer to the building at a rate of 120,000 kJ/h. Over a period of 14 days, an electric meter records that 1490 kW · h of electricity is provided to the heat pump. Determine

(a) the amount of energy that the heat pump receives over the 14-day period from the well water by heat transfer, in kJ.
(b) the heat pump's coefficient of performance.
(c) the coefficient of performance of a reversible heat pump cycle operating between hot and cold reservoirs at 21°C and 9°C.

KNOWN: Operating data are provided for a heat pump at steady state that heats a building while receiving energy by heat transfer from well water.

FIND: Determine the amount of energy received from the well water, the heat pump's coefficient of performance, and the coefficient of performance of a reversible heat pump working between the same temperature levels

SCHEMATIC & GIVEN DATA:



$\dot{Q}_{out} = 120{,}000\ kJ/h$

$W_{cycle} = 1490\ kW$ (14 days)

ENGINEERING MODEL:

1. The heat pump is the system.
2. The building and well water play the roles of hot and cold reservoirs, respectively.
3. The system is at steady state.
4. All energy transfers are positive in the directions of the arrows.

ANALYSIS: (a) Applying an energy balance using assumption 4,

$$W_{cycle} = Q_{out} - Q_{in} \Rightarrow Q_{in} = Q_{out} - W_{cycle}$$

where

$$Q_{out} = \left(120{,}000\ \frac{kJ}{h}\right)\left(\frac{24 h}{day}\right)(14\ days) = 40.32 \times 10^6\ kJ$$

$$W_{cycle} = (1490\ kW\cdot h)\left|\frac{1\ kJ/s}{1\ kW}\right|\left|\frac{3600 s}{1 h}\right| = 5.36 \times 10^6\ kJ$$

$$\Rightarrow \quad Q_{in} = (40.32 - 5.36)\times 10^6\ kJ = 34.96 \times 10^6\ kJ \quad \longleftarrow Q_{in}$$

(b) The coefficient of performance is

$$\gamma = \frac{Q_{out}}{W_{cycle}} = \frac{40.32 \times 10^6\ kJ}{5.36 \times 10^6\ kJ} = 7.52 \quad \longleftarrow \gamma$$

(c) For a reversible heat pump operating between reservoirs at $T_H = 273°C + 21°C = 294 K$ and $T_C = 273°C + 9°C = 282 K$,

$$\gamma_{MAX} = \frac{T_H}{T_H - T_C} = \frac{294 K}{12 K} = 24.5 \quad \longleftarrow \gamma_{max}$$

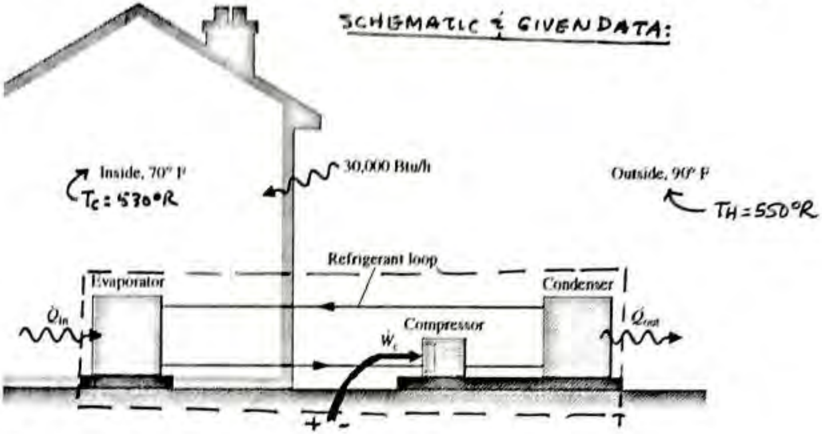
PROBLEM 5.64

As shown in Fig P5.64, an air conditioner operating at steady state maintains a dwelling at 70°F on a day when the outside temperature is 90°F. If the rate of heat transfer into the dwelling through the walls and roof is 30,000 Btu/h, might a net power input to the air conditioner compressor of 3 hp be sufficient? If yes, determine the coefficient of performance. If no, determine the minimum theoretical power input, in hp.

ENGR. MODEL:

1. The dashed line defines the air conditioning (refrigeration) system.
2. The system is at steady state.
3. The inside and outside air play the roles of the cold and hot reservoirs, respectively.

SCHEMATIC & GIVEN DATA:



ANALYSIS: To maintain the dwelling at 70°F, the air conditioning system must remove energy by heat transfer at a rate of 30,000 Btu/h. Thus, $\dot{Q}_{in} = 30,000$ Btu/h.

Also,

$$\beta \leq \beta_{MAX} = \frac{T_C}{T_H - T_C} = \frac{530°R}{20°R} = 26.5$$

$$\Rightarrow \quad \frac{\dot{Q}_{in}}{\dot{W}_c} \leq 26.5 \Rightarrow \dot{W}_c \geq \frac{30{,}000 \text{ Btu/h}}{26.5} \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| = 0.445 \text{ hp}$$

MIN. POWER Required

Yes, a power input of 3 hp might be sufficient. The corresponding coefficient of performance is then

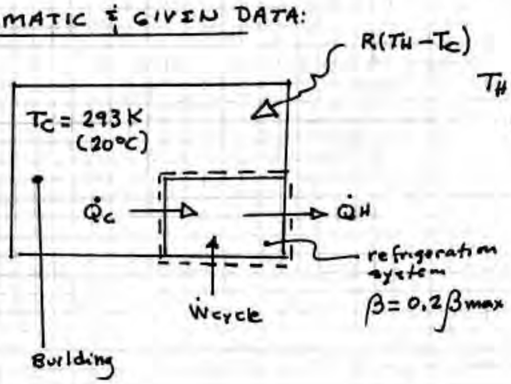
$$\beta = \frac{\dot{Q}_c}{\dot{W}_{cycle}} = \left(\frac{30{,}000 \text{ Btu/h}}{3 \text{ hp}}\right) \left| \frac{1 \text{ hp}}{2545 \frac{\text{Btu}}{\text{h}}} \right| = 3.93.$$

PROBLEM 5.65

KNOWN: Steady-state data are provided for a refrigeration cycle that maintains a building at 20°C when the outside temperature is 35°C. The rate of heat transfer into the building is $R(T_H - T_C)$, where R is a constant.

FIND: (a) If the power input to the motor driving the cycle is 3 kW, evaluate R. (b) If R is reduced by 5%, determine the required power input.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The dashed line defines the refrigeration system, which is at steady state.
2. The inside and outside of the building play the roles of the cold and hot reservoirs.
3. The rate of heat transfer into the building is given by $R[T_H - T_C]$.
4. For the refrigeration cycle, $\beta = 0.2\,\beta_{max}$.

ANALYSIS: (a) To maintain the temperature of the interior, energy must be removed by the refrigeration cycle at the rate $R(T_H - T_C)$. Additionally, $\beta = 0.2\,\beta_{max}$. With Eqs. 5.5 and 5.10,

$$\beta = \frac{R(T_H - T_C)}{\dot{W}_C} = 0.2\left[\frac{T_C}{T_H - T_C}\right] \Rightarrow \quad R = 0.2\,\dot{W}_C\left[\frac{T_C}{(T_H - T_C)^2}\right] \qquad \text{(a)}$$

Inserting values,

$$R = 0.2\,(3\,\text{kW})\left[\frac{293\,\text{K}}{(15\,\text{K})^2}\right] = 0.781\ \frac{\text{kW}}{\text{K}} \longleftarrow$$

(b) If R is reduced by 5%, R = 0.742 kW/K. Then, on rearranging Eq. (a)

① ②
$$\dot{W}_{cycle} = \left(\frac{R}{0.2}\right)\left(\frac{(T_H - T_C)^2}{T_C}\right) = \left(\frac{0.742\,\text{kW/K}}{0.2}\right)\left(\frac{(15\,\text{K})^2}{293\,\text{K}}\right)$$
$$= 2.85\ \text{kW} \longleftarrow$$

---

1. A reduction in R means that the rate of heat transfer into the building is reduced, and thus the power required is also reduced. The value of R can be reduced by adding insulation, plugging leaks, and other measures. The power required also can be reduced by employing a refrigeration cycle with a greater coefficient of performance.

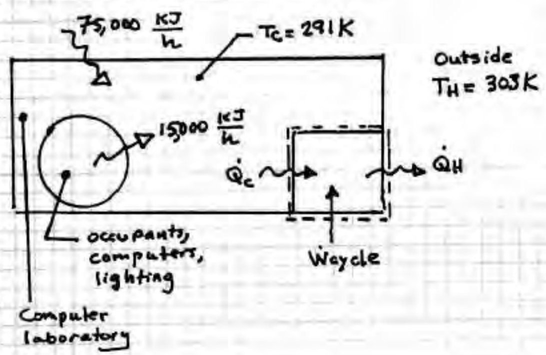
2. Reducing R by 5% while keeping all other data the same results in a 5% reduction of the power required, as expected.

PROBLEM 5.66

KNOWN: A refrigeration cycle maintains a computer laboratory at a specified temperature, 291K (18°C), for an outside temperature of 303K (30°C). Thermal load data and electricity cost data are provided.

FIND: Determine (a) the minimum theoretical power required, (b) the coefficient of performance if the actual power input is 8.3 kW, (c) For each of (a), (b) the cost for 100 hours of operation.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The system shown by the dashed line undergoes a refrigeration cycle.
2. All data are for operation at steady state.
3. The laboratory interior and the outside play the roles of cold and hot reservoirs, respectively.
4. Electricity is valued at 13¢ per kW·h
5. 100 hours of operation.

ANALYSIS: To maintain the laboratory at 291 K (18°C) the refrigeration cycle must remove energy from it by heat transfer at the total rate energy enters from all sources. Thus $\dot{Q}_C = (75000 + 15000)\frac{kJ}{h} = 90{,}000\ kJ/h$.

(a) The maximum theoretical coefficient of performance is given by Eq. 5.10:

$$\beta_{MAX} = \frac{T_C}{T_H - T_C} = \frac{291K}{(12K)} = 24.25.$$

The minimum theoretical power required is then

$$(\dot{W}_{cycle})_{min} = \frac{\dot{Q}_C}{\beta_{MAX}} = \left(\frac{90{,}000\ kJ/h}{24.25}\right)\left|\frac{1h}{3600s}\right|\left|\frac{1kW}{1kJ/s}\right| = 1.03\ kW \longleftarrow$$

(b) If the actual power required is 8.3 kW, then

$$\beta = \frac{\dot{Q}_C}{\dot{W}_{cycle}} = \frac{(90{,}000\ kJ/h)}{(8.3\ kW)}\left|\frac{1h}{3600s}\right|\left|\frac{1kW}{1kJ/s}\right| = 3.01 \longleftarrow$$

(c) For case (a): $\text{Cost} = (1.03\ kW)(100\ h)\left(0.13\ \frac{\$}{kW\cdot h}\right) = \$13.4 \longleftarrow$

For case (b): $\text{Cost} = (8.3\ kW)(100\ h)\left(0.13\ \frac{\$}{kW\cdot h}\right) = \$107.9 \longleftarrow$

unit cost

100 hours of operation

power

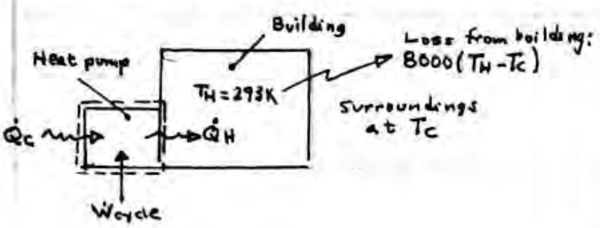
PROBLEM 5.67

At steady state, a heat pump driven by an electric motor maintains the interior of a building at $T_H = 293$ K. The rate of heat transfer, in kJ/h, from the building through its walls and roof is given by $8000(T_H - T_C)$, where T_C is the outdoor temperature. Plot the minimum theoretical electric power, in kW, required to drive the heat pump versus T_C ranging from 273 K to 293 K.

KNOWN: Steady-state data are provided for a heat pump that maintains the interior of a building at 293 K when the outside temperature is $273 \le T_C \le 293$ K.

FIND: Plot the minimum theoretical power required by the heat pump versus T_C.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The system shown in the accompanying figure undergoes a heat pump cycle.
2. The system is at steady state.
3. The building interior and the surroundings play the roles of hot and cold reservoirs.
4. The rate of heat transfer from the building is $8000(T_H - T_C)$, where the factor 8000 has units of kJ/h·K.

ANALYSIS: The heat pump must provide energy by heat transfer to the building at the same rate energy is lost to the surroundings. That is, $\dot{Q}_H = 8000(T_H - T_C)$. Moreover, with Eqs. 5.6 and 5.11

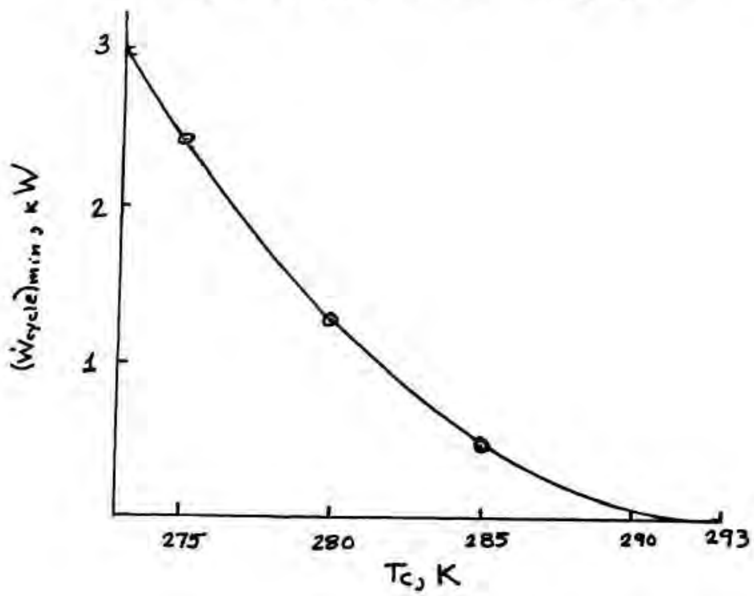
$$\gamma = \frac{\dot{Q}_H}{\dot{W}_{cycle}} \le \frac{T_H}{T_H - T_C} \Rightarrow \dot{W}_{cycle} \ge \left(\frac{T_H - T_C}{T_H}\right)\dot{Q}_H = (8000)\frac{(T_H - T_C)^2}{T_H}$$

Thus

$$(\dot{W}_{cycle})_{min} = \left(8000\frac{\text{kJ/h}}{\text{K}}\right)\left(\frac{(293\text{ K} - T_C)^2}{293\text{ K}}\right)\left|\frac{1\text{ h}}{3600\text{ s}}\right|\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$

$$= \left(7.58 \times 10^{-3}\ \frac{\text{kW}}{\text{K}^2}\right)(293 - T_C)^2\ \text{K}^2$$

Plotting $(\dot{W}_{cycle})_{min}$ vs. T_C ranging from 273K to 293K

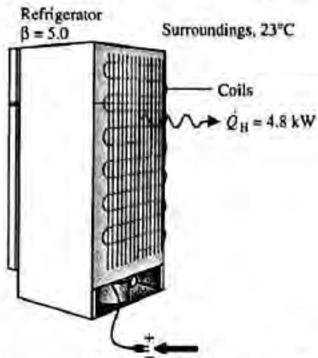


PROBLEM 5.68

The refrigerator shown in Fig. P5.68 operates at steady state with a coefficient of performance of 5.0 within a kitchen at 23°C. The refrigerator rejects 4.8 kW by heat transfer to its surroundings from metal coils located on its exterior. Determine

(a) the power input, in kW.

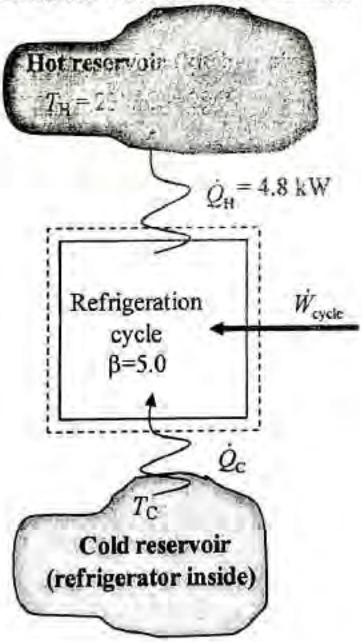
(b) the lowest theoretical temperature *inside* the refrigerator, in K.



KNOWN: Refrigeration system operates at known conditions with a known heat transfer rate to surroundings.

FIND: Determine the power input, in kW and the lowest theoretical temperature *inside* the refrigerator, in K.

SCHEMATIC AND GIVEN DATA:



PROBLEM 5.68 (Continued)

ENGINEERING MODEL:

(1) The system shown on the accompanying diagram undergoes a refrigeration cycle in steady state.

(2) The kitchen and refrigerator interior act as the hot and cold reservoirs, respectively.

ANALYSIS:

(a) Determine the power input, in kW, where $\dot{Q}_C$ represents the energy into the circulating refrigerant from the food and other contents within the refrigerator. With Eq. 5.5 on a time rate basis:

$$\beta = \frac{\dot{Q}_C}{\dot{W}_{cycle}} = \frac{\dot{Q}_C}{\dot{Q}_H - \dot{Q}_C} \qquad (1)$$

Rearrange to solve for $\dot{Q}_C$.

$$\dot{Q}_C = \left(\frac{\beta}{1+\beta}\right)\dot{Q}_H = \left(\frac{5}{6}\right)4.8\,\text{kW} = 4\,\text{kW}$$

Substitute into Eq. (1) and rearrange to solve for power required for the cycle, in kW.

#1 $$\dot{W}_{cycle} = \frac{\dot{Q}_C}{\beta} = \frac{4\,\text{kW}}{5} = 0.8\,\text{kW} \longleftarrow$$

(b) To determine the minimum theoretical temperature inside the refrigerator, begin by noting that the actual coefficient of performance is less than or equal to the maximum theoretical coefficient of performance:

$\beta_{max} \geq \beta = 5.0$

Then with Eq.5.10

$$\frac{T_C}{T_H - T_C} \geq 5.0$$

$$T_C \geq \left(\frac{5}{6}\right)T_H = \left(\frac{5}{6}\right)296\text{K}$$

$$T_C \geq 246.7\,\text{K} \longleftarrow$$

The minimum theoretical temperature is 246.7 K.

---

1. Alternatively, the net power for the cycle can also be found from an energy rate balance: $\dot{Q}_H = \dot{Q}_C + \dot{W}_{cycle} \Rightarrow \dot{W}_{cycle} = 4.8\,\text{kW} - 4\,\text{kW} = 0.8\,\text{kW}$.

PROBLEM 5.69

At steady state, a heat pump provides energy by heat transfer at the rate of 25,000 Btu/h to maintain a dwelling at 70°F on a day when the outside temperature is 30°F. The power input to the heat pump is 4.5 hp. Determine

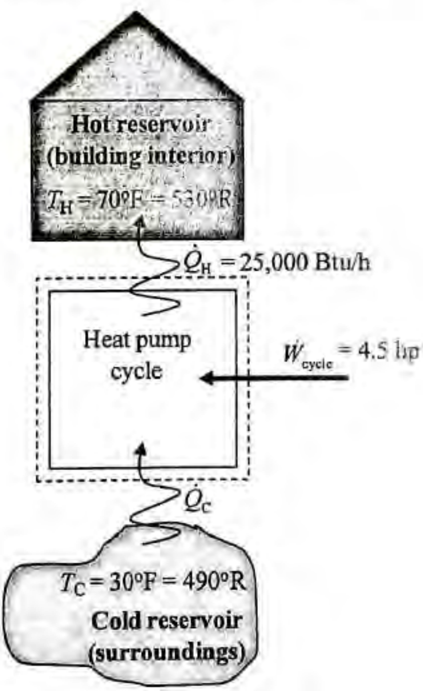
(a) the coefficient of performance of the heat pump.

(b) the coefficient of performance of a reversible heat pump operating between hot and cold reservoirs at 70°F and 30°F, respectively, and the corresponding rate at which energy would be provided by heat transfer to the dwelling for a power input of 4.5 hp.

KNOWN: A heat pump maintains the temperature within a dwelling under known conditions.

FIND: Determine the coefficient of performance of the heat pump. Also determine the coefficient of performance of a reversible heat pump operating between reservoirs at the given temperatures and the corresponding heating that would be provided to the dwelling for a power input of 4.5 hp.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

(1) The system shown on the accompanying diagram undergoes a heat pump cycle in steady state.

(2) The dwelling's interior and outdoors act as the cold and hot reservoirs, respectively.

PROBLEM 5.69 (Continued)

ANALYSIS:

(a) Using Eq. 5.6 on a time rate basis:

$$\gamma = \frac{\dot{Q}_H}{\dot{W}_{cycle}} = \frac{25{,}000 \frac{\text{Btu}}{\text{h}}}{4.5\,\text{hp}\left|\frac{2545 \frac{\text{Btu}}{\text{h}}}{1\,\text{hp}}\right|} = 2.19$$

(b) For a reversible heat pump, the coefficient of performance is given by Eq. 5.11.

$$\gamma_{max} = \frac{T_H}{T_H - T_c} = \frac{530°\text{R}}{530°\text{R} - 490°\text{R}} = 13.25$$

Then with Eq. 5.6 on a rate basis:

$$\overset{\gamma_{max}}{13.25} = \frac{\dot{Q}_H}{\dot{W}_{cycle}}$$

$$\dot{Q}_H = 13.25(\dot{W}_{cycle}) = 13.25(4.5\,\text{hp})\left|\frac{2545 \frac{\text{Btu}}{\text{h}}}{1\,\text{hp}}\right| = 151{,}746 \frac{\text{Btu}}{\text{h}}$$

① Thus for a power input of 4.5 hp, the maximum theoretical rate of heating is 151,746 Btu/h.

---

1. The minimum theoretical power required to provide heating at the rate of 25,000 Btu/h is

$$(\dot{W}_{cycle})_{MIN} = \frac{\dot{Q}_H}{\gamma_{MAX}} = \frac{25{,}000\,\text{Btu/h}}{13.25}\left|\frac{1\,\text{hp}}{2545\,\text{Btu/h}}\right|$$

$$= 0.74\,\text{hp}$$

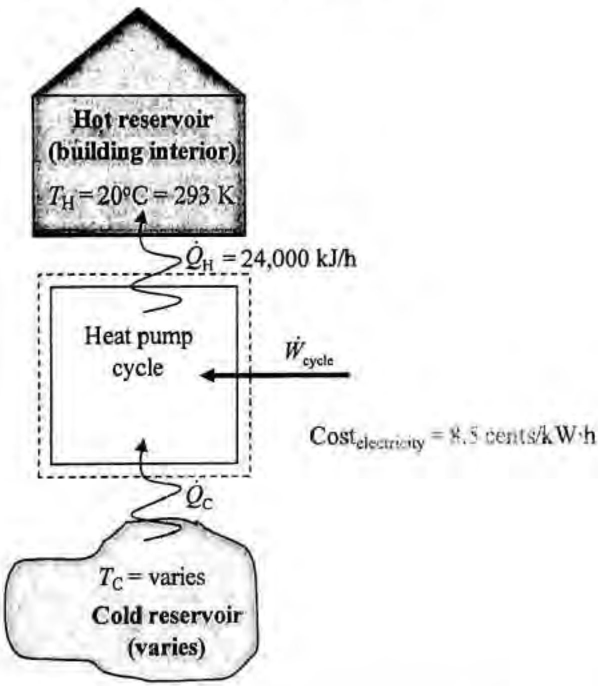
PROBLEM 5.70

By supplying energy at an average rate of 24,000 kJ/h, a heat pump maintains the temperature of a dwelling at 20°C. If electricity costs 8.5 cents per kW·h, determine the minimum theoretical operating cost for each day of operation if the heat pump receives energy by heat transfer from
(a) the outdoor air, at -7°C.
(b) the ground, at 5°C.

KNOWN: A heat pump maintains the temperature within a dwelling under known conditions.

FIND: Determine the minimum theoretical operating cost for each day of operation if the heat pump receives energy by heat transfer from the outdoor air, at -7°C; and the ground at 5°C.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
(1) The system shown on the accompanying diagram undergoes a heat pump cycle in steady state.
(2) The dwelling's interior acts as the hot reservoir.
(3) The outdoor air and ground act as the cold reservoirs in parts a and b, respectively.

PROBLEM 5.70 (Continued)

ANALYSIS:
The coefficient of performance for the heat pump is less than, or equal to, the coefficient of performance for a reversible heat pump operating between reservoirs at T_C and T_H. Then with Eq. 5.6 on a time rate basis and Eq. 5.11 we get.

$$\gamma_{max} \geq \gamma$$

$$\frac{T_H}{T_H - T_c} \geq \frac{\dot{Q}_H}{\dot{W}_{cycle}}$$

$$\dot{W}_{cycle} \geq \frac{\dot{Q}_H}{\dfrac{T_H}{T_H - T_c}} = \frac{\dot{Q}_H(T_H - T_c)}{T_H} = \frac{24{,}000\dfrac{kJ}{h}(293K - T_c)}{293K}\left|\frac{1\,h}{3600\,s}\right|\left|\frac{1\,kW}{1\dfrac{kJ}{s}}\right| \qquad (1)$$

The corresponding minimum theoretical operating cost for each day of operation is as follows.

#1 $$\$ \geq (\dot{W}_{cycle})\left|\frac{24\,h}{1\,day}\right|\left|\frac{\$0.085}{kW \cdot h}\right| \qquad (2)$$

Use Eqs. (1) and (2) to determine the minimum theoretical operating cost for each day of operation if the heat pump receives energy by heat transfer from different cold reservoirs.

(a) For a cold reservoir at T_C=-7°C=266 K,

$$\dot{W}_{cycle} \geq \frac{24{,}000\dfrac{kJ}{h}(293\,K - 266\,K)}{293\,K}\left|\frac{1\,h}{3600\,s}\right|\left|\frac{1\,kW}{1\dfrac{kJ}{s}}\right| = 0.614\,kW$$

$$\$ \geq (0.614\,kW)\left|\frac{24\,h}{1\,day}\right|\left|\frac{\$0.085}{kW \cdot h}\right| = \$1.25/day \longleftarrow$$

(b) For a cold reservoir at 5°C=278 K,

$$\dot{W}_{cycle} \geq \frac{24{,}000\dfrac{kJ}{h}(293\,K - 278\,K)}{293K}\left|\frac{1\,h}{3600\,s}\right|\left|\frac{1\,kW}{1\dfrac{kJ}{s}}\right| = 0.341\,kW$$

$$\$ \geq (0.341\,kW)\left|\frac{24\,h}{1\,day}\right|\left|\frac{\$0.085}{kW \cdot h}\right| = \$0.70/day \longleftarrow$$

---

1. Owing to effects of irreversibilities, the daily operating cost of an actual heat pump providing the specified heating will be significantly greater than the calculated minimum values.

PROBLEM 5.71

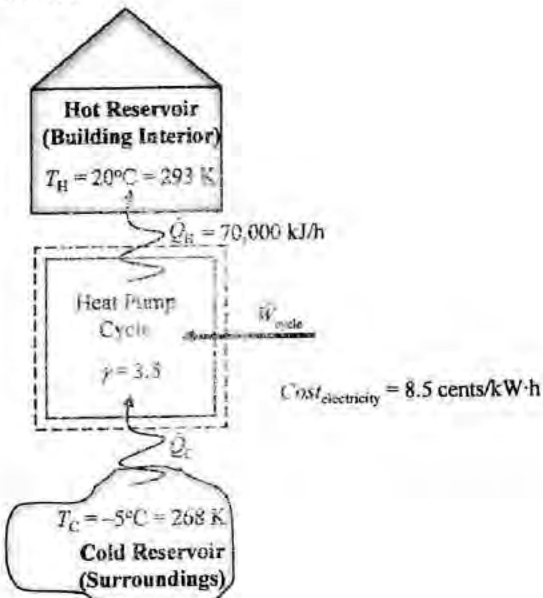
A heat pump with a coefficient of performance of 3.5 provides energy at an average rate of 70,000 kJ/h to maintain a building at 20°C on a day when the outside temperature is –5°C. If electricity costs 8.5 cents per kW·h,

(a) determine the actual operating cost and the minimum theoretical operating cost, each in $/day.
(b) compare the results of part (a) with the cost of electrical-resistance heating.

KNOWN: A heat pump cycle with specified coefficient of performance provides known heat transfer to a building while operating between two reservoirs with specified temperatures.

FIND: Determine the actual operating cost, the minimum theoretical operating cost, and electrical-resistance heating cost to provide the required heat transfer.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The system defined by the dashed line on the accompanying diagram undergoes a heat pump cycle.
2. The surroundings at –5°C are the cold reservoir.
3. The building interior at 20°C is the hot reservoir.

ANALYSIS:
(a) Actual operating cost depends on the amount of electricity required to operate the actual heat pump cycle. Coefficient of performance for an actual heat pump cycle is

PROBLEM 5.71 (Continued)

$$\gamma = \frac{\dot{Q}_H}{\dot{W}_{cycle}}$$

Solving for actual cycle power yields

$$\dot{W}_{cycle} = \frac{\dot{Q}_H}{\gamma} = \frac{70{,}000\,\frac{kJ}{h}}{3.5} = 20{,}000\ \text{kJ/h}$$

Cost to provide actual cycle power is

$$Cost_{Actual} = \left(20{,}000\,\frac{kJ}{h}\right)\left(8.5\,\frac{cents}{kW\cdot h}\right)\left|\frac{24\,h}{day}\right|\left|\frac{\$}{100\ cents}\right|\left|\frac{kW}{\frac{kJ}{s}}\right|\left|\frac{h}{3600\,s}\right| = \mathbf{\$11.33\ per\ day}$$

Minimum theoretical operating cost depends on the amount of electricity required to operate the theoretical heat pump cycle operating at maximum coefficient of performance. Maximum coefficient of performance for a heat pump cycle operating between two reservoirs is

$$\gamma_{max} = \frac{T_H}{T_H - T_C} = \frac{293\ K}{293\ K - 268\ K} = 11.72$$

The minimum theoretical power requirement for the heat pump cycle is determined from

$$\gamma_{max} = \frac{\dot{Q}_H}{\dot{W}_{cycle(min)}}$$

Solving for minimum theoretical cycle power yields

$$\dot{W}_{cycle(min)} = \frac{\dot{Q}_H}{\gamma_{max}} = \frac{70{,}000\,\frac{kJ}{h}}{11.72} = 5973\ \text{kJ/h}$$

Cost to provide minimum theoretical cycle power is

$$Cost_{Theoretical} = \left(5973\,\frac{kJ}{h}\right)\left(8.5\,\frac{cents}{kW\cdot h}\right)\left|\frac{24\,h}{day}\right|\left|\frac{\$}{100\ cents}\right|\left|\frac{kW}{\frac{kJ}{s}}\right|\left|\frac{h}{3600\,s}\right| = \mathbf{\$3.38\ per\ day}$$

(b) For electrical resistance heating, electricity provides all energy required to achieve required heat transfer to the building. The cost of electrical resistance heating is

$$Cost_{Electrical\ Resistance} = \left(70{,}000\,\frac{kJ}{h}\right)\left(8.5\,\frac{cents}{kW\cdot h}\right)\left|\frac{24\,h}{day}\right|\left|\frac{\$}{100\ cents}\right|\left|\frac{kW}{\frac{kJ}{s}}\right|\left|\frac{h}{3600\,s}\right| = \mathbf{\$39.67\ per\ day}$$

Use of a heat pump saves over two-thirds the cost of electrical-resistance heating for this situation. If the heat pump could operate at maximum coefficient of performance, the savings would be over 90% of the cost of electrical-resistance heating.

PROBLEM 5.72

As shown in Fig. P5.72, a heat pump provides energy by heat transfer to water vaporizing from saturated liquid to saturated vapor at a pressure of 2 bar and a mass flow rate of 0.05 kg/s. The heat pump receives energy by heat transfer from a pond at 16°C. These are the only significant heat transfers. Kinetic and potential energy effects can be ignored. A faded, hard-to-read data sheet indicates the power required by the heat pump at steady state is 35 kW. Can this value be correct? Explain.

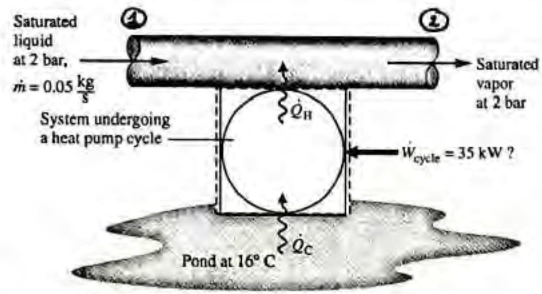


Fig. P5.70

ANALYSIS:

We recognize that

$$\gamma \le \gamma_{MAX} = \frac{T_H}{T_H - T_C} \qquad (1)$$

where $T_H = T_{sat}(2\text{bar}) = 120.2\,°C$ from Table A-3. And $T_H = 16°C$. Expressing T_H and T_C in K, gives $T_H = 393.35\,K$, $T_C = 289.15\,K$.

Eq. (1) takes the form,

$$\frac{\dot{Q}_H}{\dot{W}_{cycle}} \le \frac{393.35\,K}{(393.35 - 289.15)\,K}$$

$$\le 3.775$$

Applying mass and energy rate balances to a control volume enclosing the tube carrying the saturated water, we get with data from Table A-3,

$$\dot{Q}_H = \dot{m}\,[\underbrace{h_2}_{=h_g(2\,bar)} - \underbrace{h_1}_{=h_f(2\,bar)}] = \left(0.05\,\frac{kg}{s}\right)\left[2202\,\frac{kJ}{kg}\right]\left|\frac{1\,kW}{1\,kJ/s}\right| = 110.1\,kW$$

Collecting results

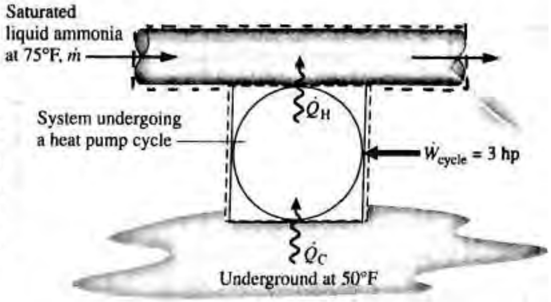
$$\frac{110.1\,kW}{\dot{W}_{cycle}} \le 3.775 \Rightarrow \frac{110.1\,kW}{3.775} \le \dot{W}_{cycle}$$

$$29.2\,kW \le \dot{W}_{cycle}$$

Accordingly, the the power required by the heat pump could be 35 kW.

PROBLEM 5.73

As shown in Fig. P 5.73, a heat pump receives energy by heat transfer from below Earth's surface where the temperature is 50°F and delivers energy by heat transfer to ammonia vaporizing from saturated liquid to saturated vapor at 75°F. These are the only significant heat transfer. At steady state, the power input to the heat pump is 3 hp. Determine the maximum theoretical ammonia mass flow rate, in lb/min, for any such heat pump. For the ammonia ignore kinetic and potential energy effects.



KNOWN: Steady-state data are provided for a heat pump that delivers energy by heat transfer to ammonia vaporizing from saturated liquid to saturated vapor.

FIND: Determine the maximum theoretical ammonia mass flow rate.

ENGINEERING MODEL: 1. A control volume encloses the tube carrying the ammonia. The heat pump is another system. 2. The systems operate at steady state. 3. For the ammonia, kinetic and potential energy effects can be ignored and there are no stray heat transfers.

ANALYSIS: The coefficient of performance for the heat pump must satisfy

$$\gamma = \frac{\dot{Q}_H}{\dot{W}_{cycle}} \leq \frac{T_H}{T_H - T_C} \Rightarrow \dot{Q}_H \leq \dot{W}_{cycle}\left[\frac{T_H}{T_H - T_C}\right]$$

$$\leq (3\text{ hp})\left[\frac{535\text{°R}}{25\text{°R}}\right]\left|\frac{2545\text{ Btu/h}}{1\text{ hp}}\right|\left|\frac{1\text{ h}}{60\text{ min}}\right|$$

$$\leq 2723\text{ Btu/min}$$

An energy rate balance for the tube enclosing the tube carrying the ammonia reduces to $\dot{Q}_H = \dot{m}(h_2 - h_1)$. Collecting results

$$\dot{m}(h_2 - h_1) \leq 2723\text{ Btu/min}$$

With data from Table A-13-E, $(h_2 - h_1) = 503.18$ Btu/lb. Thus,

$$\dot{m} \leq \frac{2723\text{ Btu/min}}{503.18\text{ Btu/lb}} = 5.41\ \frac{\text{lb}}{\text{min}}.$$

$$\Rightarrow (\dot{m})_{MAX} = 5.41\ \frac{\text{lb}}{\text{min}}$$

PROBLEM 5.74

To maintain a dwelling steadily at 68°F on a day when the outside temperature is 32°F, heating must be provided at an average rate of 700 Btu/min. Compare the electrical power required, in kW, to deliver the heating using (a) electrical-resistance heating, (b) a heat pump whose coefficient of performance is 3.5, (c) a reversible heat pump.

(a) Electrical resistance heating:

In this case, the heating is fully provided by electricity. That is

$$\dot{W}_{elec} = \dot{Q}_H = 700 \frac{Btu}{min} \left| \frac{60\ min}{1\ h} \right| \left| \frac{1\ kW}{3413\ Btu/h} \right|$$

$$= 12.31\ kW \longleftarrow$$

(b) Heat pump with $\gamma = 3.5$:

$$\gamma = \frac{\dot{Q}_H}{\dot{W}_{elec}} \Rightarrow \dot{W}_{elec} = \frac{\dot{Q}_H}{\gamma} = \frac{12.31\ kW}{3.5}$$

$$= 3.52\ kW \longleftarrow$$

(c) Reversible heat pump:

In this case, $\gamma_{MAX} = \frac{T_H}{T_H - T_C} = \frac{528\ °R}{36\ °R} = 14.67$

Also,

$$\dot{W}_{elec}\Big]_{MIN} = \frac{\dot{Q}_H}{\gamma_{MAX}} = \frac{12.31\ kW}{14.67}$$

$$= 0.84\ kW \longleftarrow$$

PROBLEM 5.75

A heating system must maintain the interior of a building at $T_H = 20°C$ when the outside temperature is $T_C = 2°C$. If the rate of heat transfer from the building through its walls and roof is 16.4 kW, determine the electrical power required, in kW, to heat the building using (a) electrical-resistance heating, (b) a heat pump whose coefficient of performance is 3.0, (c) a reversible heat pump operating between hot and cold reservoirs at 20°C and 2°C, respectively.

ANALYSIS: To maintain the building at 20°C, it is necessary to provide energy by heat transfer at the rate energy exits through the roof and walls. That is, at a rate of 16.4 kW.

(a) Electrical-resistance heating: In this case, the heating is fully provided by electricity. Thus, $\dot{W}_{elec} = 16.4 \text{ kW}$ ←

(b) Heat pump with $\gamma = 3.0$:

$$\gamma = \frac{\dot{Q}_H}{\dot{W}_{cycle}} \Rightarrow \dot{W}_{cycle} = \frac{\dot{Q}_H}{\gamma} = \frac{16.4 \text{ kW}}{3} = 5.47 \text{ kW}$$ ←

(c) Reversible heat pump:

$$\gamma_{MAX} = \frac{T_H}{T_H - T_C} = \frac{293 \text{ K}}{18 \text{ K}} = 16.28$$

Then

$$(\dot{W}_{cycle})_{MIN} = \frac{\dot{Q}_H}{\gamma_{MAX}} = \frac{16.4 \text{ kW}}{16.28} = 1.01 \text{ kW}$$ ←

PROBLEM 5.76

KNOWN: A gas within a piston-cylinder assembly undergoes a Carnot power cycle.

FIND: For $T_H = 600K$ and $T_C = 300K$, determine thermal efficiency. Also determine thermal efficiency for cases involving % changes in the T_H and T_C values from those given above.

ANALYSIS:

The thermal efficiency is given by Eq. 5.9: $\eta_{max} = 1 - \frac{T_C}{T_H}$.

(a) $T_H = 600K$, $T_C = 300K$

$$\eta_{MAX} = 1 - \frac{300K}{600K} = 0.5 \quad (50\%)$$

(b) $T_H = 1.15(600K) = 690K$, $T_C = 300K$

$$\eta_{MAX} = 1 - \frac{300K}{690K} = 0.565 \quad (56.5\%)$$ 13% Increase

(c) $T_H = 600K$, $T_C = 0.85(300K) = 255K$

$$\eta_{MAX} = 1 - \frac{255K}{600K} = 0.575 \quad (57.5\%)$$ 15% Increase

(d) $T_H = 1.15(600K) = 690K$, $T_C = 0.85(300K) = 255K$

$$\eta_{MAX} = 1 - \frac{255K}{690K} = 0.63 \quad (63\%)$$ 26% Increase

PROBLEM 5.77

Referring to the heat pump cycle of Fig. 5.16, if $p_1 = 14.7$ and $p_4 = 18.7$, each in lbf/in.^{2}, $v_1 = 12.6$ and $v_4 = 10.6$, each in ft^3/lb, and the gas is air obeying the ideal gas model, determine T_H and T_C, each in °R, and the coefficient of performance.

KNOWN: Data are provided for the cycle of Fig. 5.16.

FIND: Determine T_H and T_C, each in °R, and γ.

SCHEMATIC & GIVEN DATA

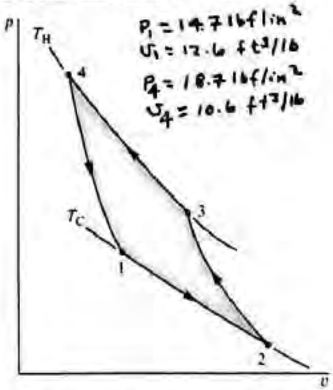


Fig. 5.16 p–v diagram for a Carnot gas refrigeration or heat pump cycle.

ENGINEERING MODEL:

1. Air executes a reversible heat pump cycle while operating between hot and cold reservoirs at T_H and T_C, respectively.
2. Air is modeled as an ideal gas.

ANALYSIS: Since $T_C = T_1$ and $T_H = T_4$, the ideal gas equation of state yields

$$T_1 = \frac{p_1 v_1}{R} = \frac{(14.7 \times 144 \text{ lbf/ft}^2)(12.6 \text{ ft}^3/\text{lb})}{\left(\frac{1545}{28.97} \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)} = 500.1\,°\text{R} \quad T_C$$

T_C T_H

$$T_4 = \frac{p_4 v_4}{R} = \frac{(18.7 \times 144 \text{ lbf/ft}^2)(10.6 \text{ ft}^3/\text{lb})}{\left(\frac{1545}{28.97} \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)} = 535.2\,°\text{R} \quad T_H$$

Also, the coefficient of performance is

$$\gamma = \frac{T_H}{T_H - T_C} = \frac{535.2\,°\text{R}}{(535.2 - 500.1)\,°\text{R}} = 15.25 \quad \longleftarrow$$

PROBLEM 5.78

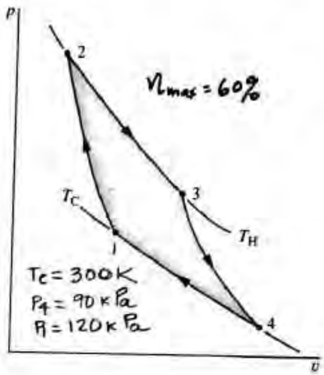
KNOWN: Data are provided for the power cycle of Fig. 5.13.

FIND: Determine (a) T_H, in K, (b) W_{cycle}/n, in kJ/kmol.

ENGINEERING MODEL:

1. An ideal gas in a piston-cylinder assembly executes a Carnot power cycle.
2. Kinetic and potential effects are ignored.

SCHEMATIC & GIVEN DATA:



ANALYSIS

(a) Using Eq. 5.9 with $\eta_{max} = 0.60$

$$0.6 = 1 - \frac{T_C}{T_H} \Rightarrow 0.4 = \frac{300K}{T_H}$$

$$\Rightarrow T_H = 750K \longleftarrow$$

(b) For every power cycle $W_{cycle} = Q_{cycle}$. Since processes 1-2, 3-4 occur adiabatically, $W_{cycle} = Q_{23} - |Q_{41}|$. From Eq. 5.7,

magnitude

$$\frac{Q_{23}}{T_H} = \frac{|Q_{41}|}{T_C}. \text{ Collecting results, } W_{cycle} = |Q_{41}|\left[\frac{T_H - T_C}{T_C}\right] \qquad (a)$$

For process 4-1, an energy balance reads, $\underbrace{(U_1 - U_4)}_{=0 \text{ since } T \text{ is constant}} = Q_{41} - W_{41}$

Accordingly, $Q_{41} = W_{41}$.

Moreover,

$$W_{41} = \int_4^1 p\,dV = \int_4^1 \overbrace{\frac{n\bar{R}T_C}{V}}^{\text{ideal gas}}\,dV = n\bar{R}T_C \ln\left(\frac{V_1}{V_4}\right) \quad : \left\{\begin{array}{l} P_1V_1 = nRT_C \\ P_4V_4 = nRT_C \end{array} \Rightarrow \frac{V_1}{V_4} = \frac{P_4}{P_1}\right\}$$

$$= n\bar{R}T_C \ln\left(\frac{P_4}{P_1}\right)$$

Collecting results, $|Q_{41}| = |W_{41}| = n\bar{R}T_C \ln\left(\frac{P_1}{P_4}\right) \quad (P_1 > P_4) \qquad (b)$

Combining Eqs. (a), (b)

$$\frac{W_{cycle}}{n} = \bar{R}(T_H - T_C)\ln\frac{P_1}{P_4} \qquad (c)$$

$$= \left(8.314\,\frac{kJ}{kmol\cdot K}\right)(750-300)K \ln\left(\frac{120kPa}{90kPa}\right)$$

$$= 1076.3\,\frac{kJ}{kmol} \longleftarrow$$

PROBLEM 5.79

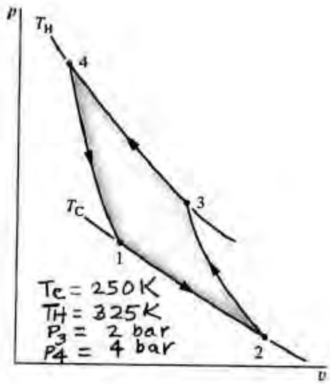
KNOWN: Data are provided for the refrigeration cycle of Fig. 5.16

FIND: Determine (a) the coefficient of performance, (b) Q_{12}/n, in kJ/kmol, (c) $|W_{cycle}/n|$, in kJ/kmol.

ENGINEERING MODEL:

1. An ideal gas in a piston-cylinder assembly executes a Carnot refrigeration cycle.
2. Kinetic and potential energy effects are ignored.

SCHEMATIC & GIVEN DATA:



ANALYSIS:

(a) Using Eq. 5.10,

$$\beta_{max} = \frac{T_C}{T_H - T_C} = \frac{250\,K}{(325-250)\,K} = 3.33 \longleftarrow$$

(b) With Eq. 5.7, $\underbrace{\frac{Q_{12}}{|Q_{34}|}}_{\text{magnitude}} = \frac{T_C}{T_H} \Rightarrow Q_{12} = \frac{T_C}{T_H}|Q_{34}|$ (a)

For process 3-4 an energy balance reads,

$$\underbrace{U_4 - U_3}_{=0 \text{ since } T \text{ is constant}} = Q_{34} - W_{34} \Rightarrow Q_{34} = W_{34}.$$

Moreover, $W_{34} = \int_3^4 p\,dV = \int_3^4 \frac{n\bar{R}T_H}{V}dV = n\bar{R}T_H \ln\left(\frac{V_4}{V_3}\right)$: $\left\{ \begin{array}{l} P_3V_3 = n\bar{R}T_H \\ P_4V_4 = n\bar{R}T_H \end{array} \Rightarrow \frac{P_3}{P_4} = \frac{V_4}{V_3} \right\}$

$$\therefore\ W_{34} = n\bar{R}T_H \ln\left(\frac{P_3}{P_4}\right)$$

Collecting results, $Q_{34} = n\bar{R}T_H \ln\left(\frac{P_3}{P_4}\right) \Rightarrow |Q_{34}| = n\bar{R}T_H(P_4/P_3)$ $(P_4 > P_3)$ (b)

Combining Eqs. (a), (b)

$$\frac{Q_{12}}{n} = \left(\frac{T_C}{T_H}\right)\bar{R}T_H \ln\frac{P_4}{P_3} = \bar{R}T_C \ln\left(\frac{P_4}{P_3}\right)$$

$$= \left(8.314\frac{kJ}{kmol\cdot K}\right)(250\,K)\ln\left(\frac{4\,bar}{2\,bar}\right)$$

$$= 1440.71\ \frac{kJ}{kmol} \longleftarrow$$

(c) For the refrigeration cycle, $\beta = \frac{Q_{12}/n}{|W_{cycle}/n|}$. Using the results of parts (a), (b), we get

$$|W_{cycle}/n| = \frac{1440.71\ kJ/kmol}{3.33}$$

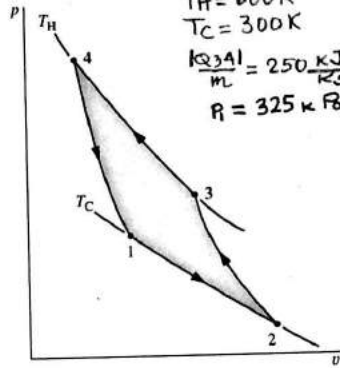
$$= 432.65\ \frac{kJ}{kmol} \longleftarrow$$

PROBLEM 5.80

KNOWN: Data are provided for the heat pump cycle of Fig. 5.16.

FIND: Determine (a) the magnitude of the net work input, in kJ/kg,
(b) the pressure, in kPa, at the end of the isothermal expansion.

Schematic & Given Data:



$T_H = 600\,K$
$T_C = 300\,K$
$\frac{|Q_{34}|}{m} = 250\,\frac{kJ}{kg}$
$P_1 = 325\,kPa$

ENGINEERING MODEL:

1. Air modeled as an ideal gas executes a Carnot refrigeration cycle within a piston-cylinder assembly.
2. Kinetic and potential energy effects are ignored.

ANALYSIS: (a) Using Eq. 5.11, we have for the Carnot heat pump cycle

magnitudes

$$\gamma_{MAX} = \frac{|Q_{34}/m|}{|W_{cycle}/m|} = \frac{T_H}{T_H - T_C}$$

$$\Rightarrow \quad |W_{cycle}/m| = \left(\frac{T_H - T_C}{T_H}\right)|Q_{34}/m|$$

$$= \frac{(600-300)K}{600K}\left(250\frac{kJ}{kg}\right) = 125\frac{kJ}{kg}$$

(b) Since processes 2-3 and 4-1 occur adiabatically, an energy balance for the overall cycle gives $Q_{12}/m = 125\ kJ/kg$.
Moreover, an energy balance for process 1-2 reads, $\underbrace{(U_2 - U_1)}_{\substack{=0 \text{ since} \\ \text{temperature} \\ \text{is constant}}} = Q_{12} - W_{12}$

$$\Rightarrow \quad Q_{12} = W_{12} = \int_1^2 p\,dV = \int_1^2 \frac{mRT_C}{V}\,dV = mRT_C \ln\left(\frac{V_2}{V_1}\right)$$

Since $P_1V_1 = mRT_C$ and $P_2V_2 = mRT_C$, $(V_2/V_1) = (P_1/P_2)$. Collecting results

$$Q_{12} = mRT_C \ln\left(\frac{P_1}{P_2}\right) \Rightarrow \ln\left(\frac{325\,kPa}{P_2}\right) = \frac{Q_{12}/m}{RT_C}$$

$$\Rightarrow \ln\left(\frac{325\,kPa}{P_2}\right) = \frac{125\,kJ/kg}{\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(300K)} \Rightarrow P_2 = 76.1\,kPa$$

PROBLEM 5.81

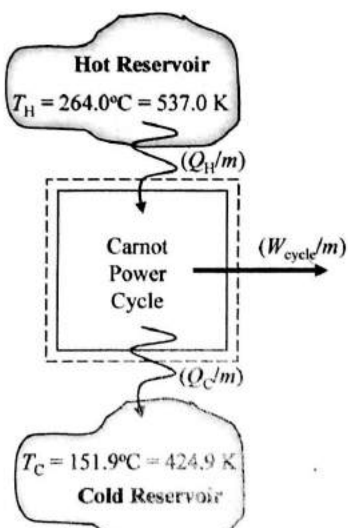
5.77 A quantity of water within a piston-cylinder assembly executes a Carnot power cycle. During isothermal expansion, the water is heated from saturated liquid at 50 bar until it is a saturated vapor. The vapor then expands adiabatically to a pressure of 5 bar while doing 364.31 kJ/kg of work.

(a) Sketch the cycle on p-v coordinates.
(b) Evaluate the heat transfer per unit mass and work per unit mass for each process, in kJ/kg.
(c) Evaluate the thermal efficiency.

KNOWN: Water executes a closed system Carnot power cycle between two known pressures and with specified work produced during one process.

FIND: Sketch the cycle on p-v coordinates, determine heat and work during each process, and evaluate thermal efficiency.

SCHEMATIC AND GIVEN DATA:



Known Data		
Process	***Q/m* [kJ/kg]**	***W/m* [kJ/kg]**
1-2		
2-3	0	364.31
3-4		
4-1	0	

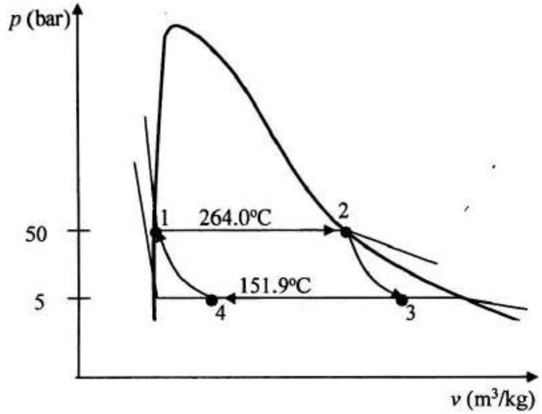
State	***p* [bar]**	***T* [°C]**	***x* [%]**
1	50	264.0	0
2	50	264.0	1
3	5	151.9	
4	5	151.9	

ENGINEERING MODEL:
1. The closed system defined by the dashed line on the accompanying diagram undergoes a Carnot power cycle.
2. Kinetic and potential energy effects can be neglected.

ANALYSIS:
(a) The p-v diagram is shown below. Process 1-2 is isothermal expansion from State 1, a saturated liquid, to State 2, a saturated vapor, both at 50 bar. In the saturated mixture region temperature and pressure are dependent and thus constant during process 1-2. Temperature is

the saturation temperature at 50 bar. Specific volume increases during process 1-2 expansion. Process 2-3 is adiabatic expansion in which specific volume continues to increase while pressure drops to 5 bar. Process 3-4 is isothermal compression during which specific volume decreases. Since the water is still in the saturated mixture region, pressure and temperature are both constant. The temperature is the saturation temperature at 5 bar. Process 4-1 is adiabatic compression, during which specific volume continues to decrease while pressure increases back to 50 bar.

<u>p-v Diagram</u>



To determine the energy transfers during each process, the energy balance for a closed system and the expansion/compression work equation apply.

Process 1-2 (Isothermal Expansion)

State 1 is saturated liquid at $p_1 = 50$ bar. From Table A-3:
$v_1 = v_{f1} = 0.0012859\ \text{m}^3/\text{kg}$,
$u_1 = u_{f1} = 1147.8$ kJ/kg

State 2 is saturated vapor at $p_2 = 50$ bar. From Table A-3:
$v_2 = v_{g2} = 0.03944\ \text{m}^3/\text{kg}$,
$u_2 = u_{g2} = 2597.1$ kJ/kg

Applying Eq. 2.17 for the constant-pressure expansion

$$\frac{W_{12}}{m} = p_1\left(v_2 - v_1\right)$$

Substituting values and applying appropriate conversion factors give

$$\frac{W_{12}}{m} = (50\text{ bar})\left(0.03944\frac{\text{m}^3}{\text{kg}} - 0.0012859\frac{\text{m}^3}{\text{kg}}\right)\left|\frac{10^5\ \frac{\text{N}}{\text{m}^2}}{\text{bar}}\right|\left|\frac{\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right| = \underline{\mathbf{190.77\ kJ/kg}}$$

The positive sign associated with work indicates work is from the system.

Applying the closed system energy balance (neglecting kinetic and potential energy effects):

$$u_2 - u_1 = (Q_{12}/m) - (W_{12}/m)$$

Solving for heat transfer per unit mass gives

$$(Q_{12}/m) = (W_{12}/m) + u_2 - u_1$$

Substituting values gives

$$(Q_{12}/m) = 190.77\text{ kJ/kg} + 2597.1\text{ kJ/kg} - 1147.8\text{ kJ/kg} = \underline{\mathbf{1640.07\ kJ/kg}}$$

The positive sign associated with energy transfer by heat indicates heat is into the system.

Process 2-3 (Adiabatic Expansion)
(Q_{23}/m) = 0 kJ/kg (adiabatic process)
(W_{23}/m) = 364.31 kJ/kg (given)

To fix State 3, a property in addition to pressure is required. Applying the closed system energy balance (neglecting kinetic and potential energy effects):

$$u_3 - u_2 = (Q_{23}/m) - (W_{23}/m)$$

Solving for internal energy at State 3 gives:

$$u_3 = (Q_{23}/m) - (W_{23}/m) + u_2$$

$$u_3 = 0\text{ kJ/kg} - 364.31\text{ kJ/kg} + 2597.1\text{ kJ/kg} = 2232.8\text{ kJ kg}$$

State 3 is a two-phase mixture at p_3 = 5 bar. From Table A-3:
u_{f3} = 639.68 kJ/kg
u_{g3} = 2561.2 kJ/kg
u_3 = 2232.8 kJ/kg (computed from energy balance for process 2-3)
v_{f3} = 0.0010926 m^3/kg
v_{g3} = 0.3749 m^3/kg
h_{f3} = 640.23 kJ/kg
h_{fg3} = 2108.5 kJ/kg

Quality is determined from the relationship

$$x = \frac{u - u_f}{u_g - u_f}$$

Solving for quality at State 3 gives

$$x_3 = \frac{2232.8\,\frac{\text{kJ}}{\text{kg}} - 639.68\,\frac{\text{kJ}}{\text{kg}}}{2561.2\,\frac{\text{kJ}}{\text{kg}} - 639.68\,\frac{\text{kJ}}{\text{kg}}} = 0.8291$$

Specific volume at State 3 can be determined from the quality relation

$$v_3 = v_{f3} + x_3(v_{g3} - v_{f3})$$

$$v_3 = 0.0010926\ \text{m}^3/\text{kg} + (0.8291)(0.3749\ \text{m}^3/\text{kg} - 0.0010926\ \text{m}^3/\text{kg}) = 0.3110\ \text{m}^3/\text{kg}$$

Specific enthalpy at State 3 is convenient later in the analysis. It can be determined from the quality relation

$$h_3 = h_{f3} + x_3 h_{fg3}$$

$$h_3 = 640.23\ \text{kJ/kg} + (0.8291)(2108.5\ \text{kJ/kg}) = 2388.39\ \text{kJ/kg}$$

Process 3-4 (Isothermal Compression)

The heat transfer during Process 3-4 is the heat transfer of the cycle associated with the cold reservoir while heat transfer during Process 1-2 is the heat transfer associated with the hot reservoir. Since the cycle is a Carnot cycle, and thus reversible, the following relationship introduced in Sec. 5.8.1 is applicable:

$$\left(\frac{Q_C}{Q_H}\right)_{\substack{rev\\cycle}} = \frac{T_C}{T_H}$$

Expressing heat transfer on a per unit mass basis and solving for (Q_C/m) yield

$$\frac{Q_C}{m} = \left(\frac{Q_H}{m}\right)\left(\frac{T_C}{T_H}\right) = \left(1640.07\,\frac{\text{kJ}}{\text{kg}}\right)\left(\frac{424.9\ \text{K}}{537.0\ \text{K}}\right) = 1297.70\ \text{kJ/kg}$$

Here Q_C is a *magnitude*. Heat transfer during process 3-4 is from the system to the cold reservoir and thus is out of the system. Consequently, **$(Q_{34}/m) = -1297.70$ kJ/kg.**

To determine the work per unit mass during process 3-4, apply the closed system energy balance (neglecting kinetic and potential energy effects):

$$u_4 - u_3 = (Q_{34}/m) - (W_{34}/m)$$

Applying Eq. 2.17 for the constant-pressure compression

$$\frac{W_{34}}{m} = p_3(v_4 - v_3) \tag{1}$$

Substituting for work per unit mass in the energy equation gives

$$u_4 - u_3 = (Q_{34}/m) - p_3(v_4 - v_3)$$

Rearranging terms gives

$$0 = (Q_{34}/m) + (u_3 + p_3 v_3) - (u_4 + p_4 v_4)$$

1 Substituting enthalpy, $h = u + pv$, yields

$$0 = (Q_{34}/m) + h_3 - h_4$$

Solving for the exit enthalpy, h_4, gives

$$h_4 = (Q_{34}/m) + h_3$$

Substituting values yields

$$h_4 = (-1297.70 \text{ kJ/kg}) + 2388.39 \text{ kJ/kg} = 1090.69 \text{ kJ/kg}$$

State 4 is a two-phase mixture at $p_4 = 5$ bar. From Table A-3:
$h_{f4} = 640.23$ kJ/kg
$h_{fg4} = 2108.5$ kJ/kg
$h_4 = 1090.69$ kJ/kg (computed from energy balance for process 3-4)
$v_{f4} = 0.0010926$ m^3/kg
$v_{g4} = 0.3749$ m^3/kg
$u_{f4} = 639.68$ kJ/kg
$u_{g4} = 2561.2$ kJ/kg

State 4 now can be fixed by h_4 and p_4. Quality at State 4 is determined from the relationship

$$x = \frac{h - h_f}{h_{fg}}$$

Solving for quality at State 4 gives

$$x_4 = \frac{1090.69\,\dfrac{\text{kJ}}{\text{kg}} - 640.23\,\dfrac{\text{kJ}}{\text{kg}}}{2108.5\,\dfrac{\text{kJ}}{\text{kg}}} = 0.2136$$

Specific volume at State 4 can be determined from the quality relation

$$v_4 = v_{f4} + x_4(v_{g4} - v_{f4})$$

$$v_4 = 0.0010926\ \text{m}^3/\text{kg} + (0.2136)(0.3749\ \text{m}^3/\text{kg} - 0.0010926\ \text{m}^3/\text{kg}) = 0.08094\ \text{m}^3/\text{kg}$$

Substituting values into Eq. (1) and applying appropriate conversion factors to solve for work per unit mass give

$$\frac{W_{34}}{m} = (5\ \text{bar})\left(0.08094\,\frac{\text{m}^3}{\text{kg}} - 0.3110\,\frac{\text{m}^3}{\text{kg}}\right)\left|\frac{10^5\,\dfrac{\text{N}}{\text{m}^2}}{\text{bar}}\right|\left|\frac{\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right| = \underline{\mathbf{-115.03\ kJ/kg}}$$

The negative sign associated with work indicates work is to the system.

<u>Process 4-1 (Adiabatic Compression)</u>
<u>$(Q_{41}/m) = 0$ kJ/kg (adiabatic process)</u>

To determine the work per unit mass during process 4-1, apply the closed system energy balance (neglecting kinetic and potential energy effects):

$$u_1 - u_4 = (Q_{41}/m) - (W_{41}/m)$$

Substituting $(Q_{41}/m) = 0$ and solving for work give

$$(W_{41}/m) = u_4 - u_1$$

Applying the quality relation to determine u_4 gives

$$u_4 = u_{f4} + x_4(u_{g4} - u_{f4})$$

$$u_4 = 639.68\ \text{kJ/kg} + (0.2136)(2561.2\ \text{kJ/kg} - 639.68\ \text{kJ/kg}) = 1050.12\ \text{kJ/kg}$$

Substituting values and solving for work per unit mass give

$$(W_{41}/m) = 1050.12\ \text{kJ/kg} - 1147.8\ \text{kJ/kg} = \underline{\mathbf{-97.68\ kJ/kg}}$$

The negative sign associated with work indicates work is to the system.

Summarizing energy transfers

PROBLEM 5.81 - Continued (7)

Process	Q/m [kJ/kg]	W/m [kJ/kg]
1-2	1640.07	190.77
2-3	0	364.31
3-4	–1297.70	–115.03
4-1	0	–97.68
Sum	342.37	342.37

As expected for a cycle, the net heat transfer is equal to the net work.

(c) By definition, power cycle thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{H}}} = \frac{\left(\dfrac{W_{\text{cycle}}}{m}\right)}{\left(\dfrac{Q_{\text{H}}}{m}\right)} = \frac{342.37\,\dfrac{\text{kJ}}{\text{kg}}}{1640.07\,\dfrac{\text{kJ}}{\text{kg}}} = \underline{\mathbf{0.2088}}$$

Alternatively, since the cycle is a Carnot cycle, the thermal efficiency can be determined from the maximum thermal efficiency equation

$$\eta_{\max} = 1 - \frac{T_{\text{C}}}{T_{\text{H}}} = 1 - \frac{424.9\text{ K}}{537.0\text{ K}} = \underline{\mathbf{0.2088}}$$

---

❶ The introduction of enthalpy here is only for computational convenience using table data.

PROBLEM 5.82

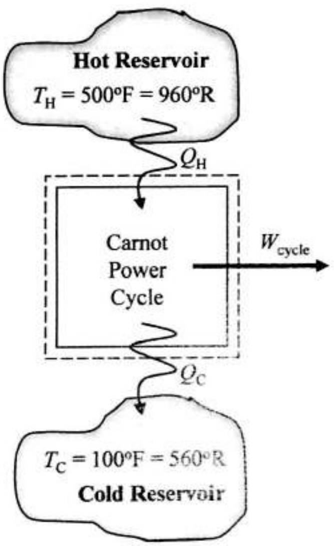
5.78 One and one-half pounds of water within a piston-cylinder assembly execute a Carnot power cycle. During isothermal expansion, the water is heated at 500°F from saturated liquid to saturated vapor. The vapor then expands adiabatically to a temperature of 100°F and quality of 70.38%.

(a) Sketch the cycle on *p-v* coordinates.
(b) Evaluate the heat transfer and work for each process, in Btu.
(c) Evaluate the thermal efficiency.

KNOWN: Water executes a closed system Carnot power cycle between two known temperatures. Quality at State 3 is specified.

FIND: Sketch the cycle on *p-v* coordinates, determine heat and work during each process, and evaluate thermal efficiency.

SCHEMATIC AND GIVEN DATA:



Known Data		
Process	***Q* [Btu]**	***W* [Btu]**
1-2		
2-3	0	
3-4		
4-1	0	

State	***p*** **[lbf/in.^{2}]**	***T*** **[°F]**	***x*** **[%]**
1	680.0	500	0
2	680.0	500	1
3	0.9503	100	70.38
4	0.9503	100	

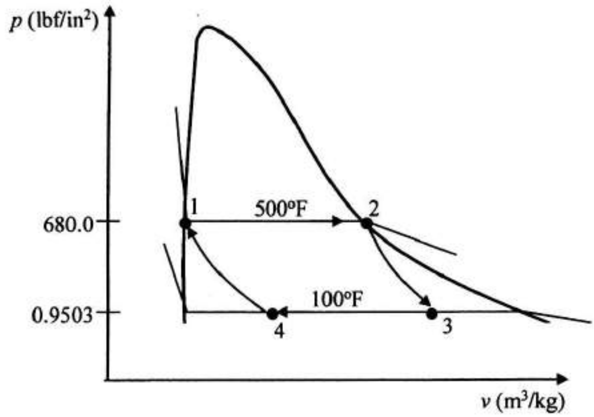
ENGINEERING MODEL:
1. The closed system defined by the dashed line on the accompanying diagram undergoes a Carnot power cycle.
2. Kinetic and potential energy effects can be neglected.

ANALYSIS:
(a) The *p-v* diagram is shown below. Process 1-2 is isothermal expansion from State 1, a saturated liquid, to State 2, a saturated vapor, both at 500°F. In the saturated mixture region temperature and pressure are dependent and thus constant during process 1-2. Pressure is the

PROBLEM 5.82 (2)

saturation pressure at 500°F. Specific volume increases during process 1-2 expansion. Process 2-3 is adiabatic expansion in which specific volume continues to increase while temperature drops to 100°F. Process 3-4 is isothermal compression during which specific volume decreases. Since the water is still in the saturated mixture region, pressure and temperature are both constant. Pressure is saturation pressure at 100°F. Process 4-1 is adiabatic compression, during which specific volume continues to decrease while temperature increases back to 500°F.

<u>*p-v* Diagram</u>



To determine the energy transfers during each process, the energy balance for a closed system and the expansion/compression work equation apply.

Process 1-2 (Isothermal Expansion)

State 1 is saturated liquid at $T_1 = 500°F$. From Table A-2E:
$v_1 = v_{f1} = 0.02043\ ft^3/lb$
$u_1 = u_{f1} = 485.1$ Btu/lb

State 2 is saturated vapor at $T_2 = 500°F$. From Table A-2E:
$v_2 = v_{g2} = 0.6761\ ft^3/lb$
$u_2 = u_{g2} = 1117.4$ Btu/lb

Applying Eq. 2.17 for the constant-pressure expansion

$$W_{12} = mp_1(v_2 - v_1)$$

Substituting values and applying appropriate conversion factors give

$$W_{12} = (1.5\ \text{lb})\left(680\frac{\text{lbf}}{\text{in.}^2}\right)\left(0.6761\frac{\text{ft}^3}{\text{lb}} - 0.02043\frac{\text{ft}^3}{\text{lb}}\right)\left|\frac{144\text{in.}^2}{\text{ft}^2}\right|\left|\frac{\text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right| = \underline{\mathbf{123.79\ Btu}}$$

The positive sign associated with work indicates work is from the system.

PROBLEM 5.82 (3)

Applying the closed system energy balance (neglecting kinetic and potential energy effects):

$$m(u_2 - u_1) = Q_{12} - W_{12}$$

Solving for heat transfer gives

$$Q_{12} = W_{12} + m(u_2 - u_1)$$

Substituting values gives

$$Q_{12} = 123.79 \text{ Btu} + (1.5 \text{ lb})(1117.4 \text{ Btu/lb} - 485.1 \text{ Btu/lb}) = \underline{\mathbf{1072.24 \text{ Btu}}}$$

The positive sign associated with energy transfer by heat indicates heat is into the system.

<u>Process 2-3 (Adiabatic Expansion)</u>
<u>Q_{23} = 0 Btu (adiabatic process)</u>
State 3 is a two-phase mixture at $T_3 = 100°F$. From Table A-2E:
$u_{f3} = 68.04$ Btu/lb
$u_{g3} = 1043.5$ Btu/lb
$v_{f3} = 0.01613$ ft^3/lb
$v_{g3} = 350.0$ ft^3/lb
$h_{f3} = 68.05$ Btu/lb
$h_{fg3} = 1037.0$ Btu/lb
$x_3 = 0.7038$ (given)

Work during process 2-3 can be determined by applying the closed system energy balance (neglecting kinetic and potential energy effects):

$$m(u_3 - u_2) = Q_{23} - W_{23}$$

Substituting $Q_{23} = 0$ and solving for work give:

$$W_{23} = m(u_2 - u_3)$$

The internal energy at State 3 can be determined from the quality relation

$$u_3 = u_{f3} + x_3(u_{g3} - u_{f3})$$

$$u_3 = 68.04 \text{ Btu/lb} + (0.7038)(1043.5 \text{ Btu/lb} - 68.04 \text{ Btu/lb}) = 754.57 \text{ Btu/lb}$$

Substituting values gives

$$W_{23} = (1.5 \text{ lb})(1117.4 \text{ Btu/lb} - 754.57 \text{ Btu/lb}) = \underline{\mathbf{544.25 \text{ Btu}}}$$

The positive sign associated with work indicates work is from the system.

PROBLEM 5.82 (4)

Specific volume at State 3 can be determined from the quality relation

$$v_3 = v_{f3} + x_3(v_{g3} - v_{f3})$$

$$v_3 = 0.01613 \text{ ft}^3/\text{lb} + (0.7038)(350.0 \text{ ft}^3/\text{lb} - 0.01613 \text{ ft}^3/\text{lb}) = 246.33 \text{ ft}^3/\text{lb}$$

Specific enthalpy at State 3 is convenient later in the analysis. It can be determined from the quality relation

$$h_3 = h_{f3} + x_3 h_{fg3}$$

$$h_3 = 68.05 \text{ Btu/lb} + (0.7038)(1037.0 \text{ Btu/lb}) = 797.89 \text{ Btu/lb}$$

Process 3-4 (Isothermal Compression)

The heat transfer during Process 3-4 is the heat transfer of the cycle associated with the cold reservoir while heat transfer during Process 1-2 is the heat transfer associated with the hot reservoir. Since the cycle is a Carnot cycle, and thus reversible, the following relationship introduced in Sec. 5.8.1 is applicable:

$$\left(\frac{Q_C}{Q_H}\right)_{\substack{rev\\cycle}} = \frac{T_C}{T_H}$$

Solving for Q_C yields

$$Q_C = Q_H\left(\frac{T_C}{T_H}\right) = (1072.24 \text{ Btu})\left(\frac{560°\text{R}}{960°\text{R}}\right) = 625.47 \text{ Btu}$$

Here Q_C is a *magnitude*. Heat transfer during process 3-4 is from the system to the cold reservoir and thus is out of the system. Consequently, **Q_{34} = –625.47 Btu.**

To determine the work during process 3-4, apply the closed system energy balance (neglecting kinetic and potential energy effects):

$$m(u_4 - u_3) = Q_{34} - W_{34}$$

Applying Eq. 2.17 for the constant-pressure compression

$$W_{34} = mp_3(v_4 - v_3) \qquad (1)$$

Substituting for work in the energy equation gives

$$m(u_4 - u_3) = Q_{34} - mp_3(v_4 - v_3)$$

Rearranging terms gives

$$0 = Q_{34} + m(u_3 + p_3\, v_3) - m(u_4 + p_4\, v_4)$$

PROBLEM 5.82 (5)

❶ Substituting enthalpy, $h = u + pv$, yields

$$0 = Q_{34} + m(h_3 - h_4)$$

Solving for the exit enthalpy, h_4, gives

$$h_4 = (Q_{34}/m) + h_3$$

Substituting values yields

$$h_4 = (-625.47 \text{ Btu})/(1.5 \text{ lb}) + 797.89 \text{ Btu/lb} = 380.91 \text{ Btu/lb}$$

State 4 is a two-phase mixture at $T_4 = 100^{\circ}$F. From Table A-2E:
$h_{f4} = 68.05$ Btu/lb
$h_{fg4} = 1037.0$ Btu/lb
$h_4 = 380.91$ Btu/lb (computed from energy balance for process 3-4)
$v_{f4} = 0.01613$ ft^3/lb
$v_{g4} = 350.0$ ft^3/lb
$u_{f4} = 68.04$ Btu/lb
$u_{g4} = 1043.5$ Btu/lb

State 4 now can be fixed by h_4 and T_4. Quality at State 4 is determined from the relationship

$$x = \frac{h - h_f}{h_{fg}}$$

Solving for quality at State 4 gives

$$x_4 = \frac{380.91\dfrac{\text{Btu}}{\text{lb}} - 68.05\dfrac{\text{Btu}}{\text{lb}}}{1037.0\dfrac{\text{Btu}}{\text{lb}}} = 0.3017$$

Specific volume at State 4 can be determined from the quality relation

$$v_4 = v_{f4} + x_4(v_{g4} - v_{f4})$$

$$v_4 = 0.01613 \text{ ft}^3/\text{lb} + (0.3017)(350.0 \text{ ft}^3/\text{lb} - 0.01613 \text{ ft}^3/\text{lb}) = 105.61 \text{ ft}^3/\text{lb}$$

Substituting values into Eq. (1) and applying appropriate conversion factors to solve for work give

$$W_{34} = (1.5 \text{ lb})\left(0.9503\frac{\text{lbf}}{\text{in.}^2}\right)\left(105.61\frac{\text{ft}^3}{\text{lb}} - 246.33\frac{\text{ft}^3}{\text{lb}}\right)\left|\frac{144\text{in.}^2}{\text{ft}^2}\right|\left|\frac{\text{Btu}}{778 \text{ ft}\cdot\text{lbf}}\right| = \underline{\mathbf{-37.13 \text{ Btu}}}$$

The negative sign associated with work indicates work is to the system.

PROBLEM 5.82 (6)

Process 4-1 (Adiabatic Compression)
Q_{41} = 0 kJ/kg (adiabatic process)

To determine the work during process 4-1, apply the closed system energy balance (neglecting kinetic and potential energy effects):

$$m(u_1 - u_4) = Q_{41} - W_{41}$$

Substituting $Q_{41} = 0$ and solving for work give

$$W_{41} = m(u_4 - u_1)$$

Applying the quality relation to determine u_4 gives

$$u_4 = u_{f4} + x_4(u_{g4} - u_{f4})$$

$$u_4 = 68.04 \text{ Btu/lb} + (0.3017)(1043.5 \text{ Btu/lb} - 68.04 \text{ Btu/lb}) = 362.34 \text{ Btu/lb}$$

Substituting values and solving for work give

$$W_{41} = (1.5 \text{ lb})(362.34 \text{ Btu/lb} - 485.1 \text{ Btu/lb}) = \underline{\mathbf{-184.14 \text{ Btu}}}$$

The negative sign associated with work indicates work is to the system.

Summarizing energy transfers

Process	***Q* [Btu]**	***W* [Btu]**
1-2	1072.24	123.79
2-3	0	544.25
3-4	–625.47	–37.13
4-1	0	–184.14
Sum	446.77	446.77

As expected for a cycle, the net heat transfer is equal to the net work.

(c) By definition, power cycle thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{H}}} = \frac{446.77 \text{ Btu}}{1072.24 \text{ Btu}} = \underline{\mathbf{0.4167}}$$

Alternatively, since the cycle is a Carnot cycle, the thermal efficiency can be determined from the maximum thermal efficiency equation

$$\eta_{\max} = 1 - \frac{T_{\text{C}}}{T_{\text{H}}} = 1 - \frac{560°\text{R}}{960°\text{R}} = \underline{\mathbf{0.4167}}$$

---

❶ The introduction of enthalpy here is only for computational convenience using table data.

PROBLEM 5.83

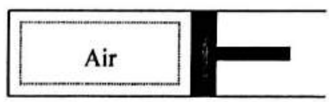
5.83 Two kilograms of air within a piston-cylinder assembly executes a Carnot power cycle with maximum and minimum temperatures of 750 K and 300 K, respectively. The heat transfer to the air during the isothermal expansion is 60 kJ. At the end of the isothermal expansion, the volume is 0.4 m^3. Assuming the ideal gas model for the air, determine

(a) the thermal efficiency.
(b) the pressure and volume at the beginning of the isothermal expansion, in kPa and m^3, respectively.
(c) the work and heat transfer for each of the four processes , in kJ.
(d) Sketch the cycle on p - V coordinates.

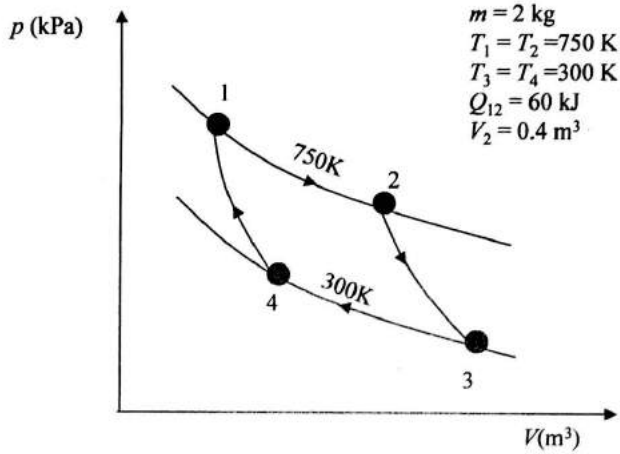
KNOWN: Two kg of air undergoes a Carnot cycle with known operating conditions.

FIND: Determine the thermal efficiency; the pressure and volume at the beginning of the isothermal expansion in kPa and m^3, respectively; and the work and heat transfer for each of the four processes, in kJ. Sketch the cycle on p-V coordinates.

SCHEMATIC AND GIVEN DATA:



$m = 2$ kg
$T_1 = T_2 = 750$ K
$T_3 = T_4 = 300$ K
$Q_{12} = 60$ kJ
$V_2 = 0.4$ m^3



ENGINEERING MODEL:
(1) The system under consideration includes air modeled as an ideal gas within a piston-cylinder assembly as shown in accompanying diagram.
(2) Volume change is the only work mode.
(3) Kinetic and potential energy are ignored.

PROBLEM 5.83 (2)

ANALYSIS:

(a) Using Eq. 5.9, the thermal efficiency is

$$\eta = \frac{T_H - T_C}{T_H} = \frac{750-300}{750} = 0.60 = 60\%$$

(b) Determine p_1, in bar and V_1, in m^3, where $Q_{12} = 60$ kJ. Use an energy balance for a closed system assuming negligible effects due to kinetic and potential energy.

$$Q_{12} - W_{12} = m(u_2 - u_1)$$

For an ideal gas, specific internal energy is a function of temperature only and therefore $u_2 = u_1$ and $Q_{12} = W_{12} = 60\text{kJ}$.

To fix state 1, use Eq. 2.17 and solve for an isothermal process involving an ideal gas, as follows:

$$W_{12} = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{mRT_H}{V}\,dV = mRT_H \ln\left(\frac{V_2}{V_1}\right) = 60\text{ kJ}$$

$$\ln\left(\frac{V_2}{V_1}\right) = \frac{60\text{ kJ}}{(2\text{ kg})\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kgK}}\right)(750\text{ K})} = 0.1394$$

$$V_1 = 0.348\,\text{m}^3$$

Since $T_1 = T_2$, the ideal gas equation of state gives:

$$p_2V_2 = p_1V_1 \quad \text{or} \quad p_1 = \frac{p_2V_2}{V_1}$$

Determine p_2 using ideal gas model and then solve for p_1

$$p_2 = \frac{mRT_2}{V_2} = \frac{(2\text{ kg})\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kgK}}\right)(750\text{ K})}{0.4\text{ m}^3}\left|\frac{1000\text{ N}\cdot\text{m}}{1\text{ kJ}}\right|\left|\frac{\text{Pa}}{1\text{ N/m}^2}\right|\left|\frac{1\text{ kPa}}{1000\text{ Pa}}\right| = 1076.2\text{ kPa}$$

$$p_1 = \frac{p_2V_2}{V_1} = \frac{(1076.2\text{ kPa})(0.4\text{ m}^3)}{(0.348\text{ m}^3)} = 1237.0\text{ kPa}$$

(c) To determine the work and heat transfer for each of the four processes, in kJ, analyze each process within the cycle.

<u>Process 1 to 2 (isothermal expansion)</u>
Use an energy balance for a closed system assuming negligible effects due to kinetic and potential energy.

PROBLEM 5.83 (3)

$$Q_{12} - W_{12} = m(u_2 - u_1)$$

For an ideal gas, specific internal energy is a function of temperature only and because $T_1 = T_2$, then $u_1 = u_2$ and $Q_{12} = W_{12} = 60\text{kJ}$.

Process 2 to 3 (adiabatic expansion)
Recognize that the process is adiabatic and use a closed system energy balance and property data from Table A-22, as follows:

$$Q_{23} = 0$$

$$W_{23} = m(u_2 - u_3) = 2\,\text{kg}\,(551.99 - 214.07)\frac{\text{kJ}}{\text{kg}} = 676\,\text{kJ}$$

Process 3 to 4 (isothermal compression)
For reversible cycles operating between two thermal reservoirs, Eq. 5.7 applies as follows:

$$\left(\frac{Q_C}{Q_H}\right)_{\substack{\text{rev}\\\text{cycle}}} = \frac{T_C}{T_H} \rightarrow \left(\frac{|Q_{34}|}{Q_{12}}\right) = \frac{300\,\text{K}}{750\,\text{K}}$$

$$|Q_{34}| = 60\,\text{kJ}\left(\frac{300\,\text{K}}{750\,\text{K}}\right) = 24\,\text{kJ}$$

For an ideal gas, specific internal energy is a function of temperature only and since $T_3 = T_4$, then $u_3 = u_4$ and,

$$Q_{34} = W_{34} = -24\,\text{kJ}$$

Process 4 to 1 (adiabatic compression)
Recognize that the process is adiabatic and use a closed system energy balance and property data from Table A-22 for air. Observe that $u_4 = u_3$ since $T_4 = T_3$ and $u_1 = u_2$ since $T_1 = T_2$.

$$Q_{41} = 0$$

#1 $$W_{41} = m(u_4 - u_1) = -W_{23} = -676\,\text{kJ}$$

1. Check the above calculations using $W_{\text{cycle}} = Q_{\text{cycle}}$ and $\eta = W_{\text{cycle}}/Q_{\text{in}}$ as follows:

$$Q_{\text{cycle}} = Q_{12} + Q_{23} + Q_{34} + Q_{41} = 36\,\text{kJ}$$

$$W_{\text{cycle}} = W_{12} + W_{23} + W_{34} + W_{41} = 36\,\text{kJ}$$

Therefore, as required by every thermodynamic cycle, $Q_{\text{cycle}} = W_{\text{cycle}}$.

Finally, with $W_{\text{cycle}} = 36$ kJ and $Q_{\text{in}} = Q_{12}$,

$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{in}}} = \frac{36\,\text{kJ}}{60\,\text{kJ}} = 60\%$$

This is in agreement with part (a), as expected.

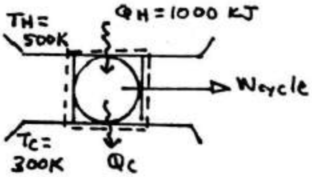
PROBLEM 5.84

A system executes a power cycle while receiving 1000 kJ by heat transfer at a temperature of 500 K and discharging energy by heat transfer at a temperature of 300 K. There are no other heat transfers. Applying Eq. 5.13, determine σ_{cycle} if the thermal efficiency is (a) 100%, (b) 40%, (c) 25%. Identify cases (if any) that are internally reversible or impossible.

KNOWN: Data are provided for a system executing a power cycle.

FIND: Applying Eq. 5.13 determine σ_{cycle} for each of three cases.

SCHEMATIC & GIVEN DATA:

① 

ENGINEERING MODEL:

1. The system shown in the schematic undergoes a power cycle while receiving Q_H at T_H and discharging Q_C at T_C
2. Q_H and Q_C are positive in the direction of the arrows. These are the only heat transfers.

ANALYSIS:

For the cycle, $\eta = 1 - \frac{Q_C}{Q_H} \Rightarrow Q_C = (1-\eta)Q_H$. Then, Eq. 5.13 reads

$$\sigma_{cycle} = -\oint\left(\frac{\delta Q}{T}\right)_b$$

$$= -\left[\frac{Q_H}{T_H} - \frac{Q_C}{T_C}\right] = -\left[\frac{Q_H}{T_H} - \frac{(1-\eta)Q_H}{T_C}\right]$$

$$= -Q_H\left[\frac{1}{T_H} - \frac{(1-\eta)}{T_C}\right] = -1000\,\text{kJ}\left[\frac{1}{500\,\text{K}} - \frac{(1-\eta)}{300\,\text{K}}\right] \qquad (1)$$

(a) $\eta = 1$ (100%)

$$\sigma_{cycle} = -1000\,\text{kJ}\left[\frac{1}{500\,\text{K}} - 0\right] = -2\,\frac{\text{kJ}}{\text{K}} \longleftarrow$$

Since σ_{cycle} cannot be negative, this case is impossible.

(b) $\eta = 0.4$ (40%)

$$\sigma_{cycle} = -1000\,\text{kJ}\left[\frac{1}{500\,\text{K}} - \frac{(1-0.4)}{300\,\text{K}}\right] = 0 \longleftarrow$$

Since $\sigma_{cycle} = 0$, this case corresponds to internally reversible operation.

(c) $\eta = 0.25$ (25%)

$$\sigma_{cycle} = -1000\,\text{kJ}\left[\frac{1}{500\,\text{K}} - \frac{(1-0.25)}{300\,\text{K}}\right] = 0.5\,\frac{\text{kJ}}{\text{K}} \longleftarrow$$

Since $\sigma_{cycle} > 0$, this case corresponds to the presence of internal irreversibilities.

---

1. The maximum thermal efficiency any such cycle can have is

$$\eta_{MAX} = \left[1 - \frac{T_C}{T_H}\right] = \left[1 - \frac{300\,\text{K}}{500\,\text{K}}\right] = 0.4\ (40\%)$$

PROBLEM 5.85

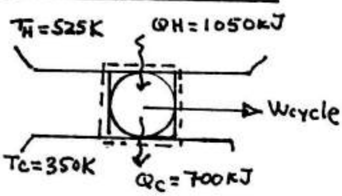
A system executes a power cycle while receiving 1050 kJ by heat transfer at a temperature of 525 K and discharging 700 kJ by heat transfer at 350 K. There are no other heat transfers.

(a) Using Eq. 5.13, determine whether the cycle is *internally reversible*, *irreversible*, or *impossible*.
(b) Determine the thermal efficiency using Eq. 5.4 and the given heat transfer data. Compare this value with the *Carnot efficiency* calculated using Eq. 5.9 and comment.

KNOWN: Data are provided for a system executing a power cycle.

FIND: Determine whether the cycle is internally reversible, irreversible, or impossible. Also evaluate the thermal efficiency and compare with the Carnot efficiency.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL

1. The system shown in the schematic undergoes a power cycle while receiving Q_H at T_H and discharging Q_C at T_C.
2. Q_H and Q_C are positive in the direction of the arrows. These are the only heat transfers.

ANALYSIS:

(a) Applying Eq. 5.13

$$\sigma_{cycle} = -\oint \frac{\delta Q}{T}$$

$$= -\left[\frac{Q_H}{T_H} - \frac{Q_C}{T_C}\right] = -\left[\frac{1050 kJ}{525K} - \frac{700 kJ}{350K}\right]$$

$$= 0$$

This cycle is internally reversible. ←

(b) For any such cycle,

$$\eta = 1 - \frac{Q_C}{Q_H}$$

$$= 1 - \frac{700kJ}{1050kJ} = 0.333 \ (33.3\%)$$ ←

The Carnot efficiency is

$$\eta_{MAX} = 1 - \frac{T_C}{T_H}$$

$$= 1 - \frac{350K}{525K} = 0.333 \ (33.3\%)$$ ←

The cycle is internally reversible

COMMENT: In this application the nature of the specified cycle can be investigated with one (or both) of the two approaches used in (a) and (b).

PROBLEM 5.86

For the refrigerator of Example 5.2, apply Eq. 5.13 on a time-rate basis to determine whether the cycle operates reversibly, operates irreversibly, or is impossible. Repeat for the case where there is no power input.

ANALYSIS: For the refrigerator of Example 5.2 an energy rate balance reads, $\dot{Q}_H = \dot{Q}_C + \dot{W}_{cycle}$. Thus, with given data, $\dot{Q}_H = 11{,}200$ kJ/h. Then, on a time-rate basis Eq. 5.13 reads

$$\left[\underbrace{\frac{\dot{Q}_C}{T_C}}_{\text{in}} - \underbrace{\frac{\dot{Q}_H}{T_H}}_{\text{out}}\right] = -\dot{\sigma}_{cycle}$$

$$\Rightarrow \quad \dot{\sigma}_{cycle} = \left[\frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_C}{T_C}\right] = \left[\frac{11{,}200\ \text{kJ/h}}{295\ \text{K}} - \frac{8{,}000\ \text{kJ/h}}{268\ \text{K}}\right]$$

① $$= 8.12\ \frac{\text{kJ/h}}{\text{K}}$$

Since $\dot{\sigma}_{cycle} > 0$, irreversibilities are present within the system undergoing the cycle. This finding agrees with the solution to Example 5.2.

If $\dot{W}_{cycle} = 0$, then $\dot{Q}_H = \dot{Q}_C$, and Eq. 5.13 gives

$$\dot{\sigma}_{cycle} = \left[\frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_C}{T_C}\right] = \dot{Q}_C\left[\frac{1}{T_H} - \frac{1}{T_C}\right]$$

$$= 8000\ \frac{\text{kJ}}{\text{h}}\left[\frac{1}{295\ \text{K}} - \frac{1}{268\ \text{K}}\right] = -2.33\ \frac{\text{kJ/h}}{\text{K}}$$

② Since $\dot{\sigma}_{cycle} < 0$, this mode of operation is impossible.

---

1. This is the result obtained in the 'For Example' of Sec. 5.11.

2. In this case, energy is removed by heat transfer from the freezer compartment and delivered by heat transfer to the higher-temperature surroundings with there being no other effect. This mode of operation is denied by the Clausius statement of the second law.

PROBLEM 5.87

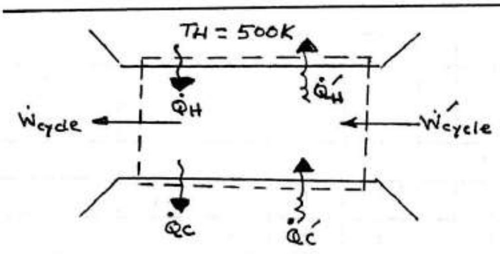
For each data set of Prob. 5.49, apply Eq. 5.13 on a time-rate basis to determine whether the system operates reversibly, operates irreversibly, or is impossible.

KNOWN: Two sets of steady-state operating data are provided for a system consisting of a power cycle and a heat pump cycle, each working between hot and cold reservoirs at 500 K and 300 K, respectively.

FIND: Apply Eq. 5.13 to determine whether the system operates reversibly, irreversibly, or is impossible.

SCHEMATIC & GIVEN DATA:

	Power cycle			Heat pump cycle		
	$\dot{Q}_H$	$\dot{Q}_C$	$\dot{W}_{cycle}$	$\dot{Q}'_H$	$\dot{Q}'_C$	$\dot{W}'_{cycle}$
(a)	60	40	20	80	60	20
(b)	120	80	40	100	80	20



ENGINEERING MODEL:

1. The system shown in the figure is at steady state.
2. Energy transfers are positive in the directions of the arrows.
3. All energy transfers are in kW

ANALYSIS: For each set of data, Eq. 5.13 on a time-rate basis reads

$$\dot{\sigma}_{cycle} = \underbrace{\left(\frac{\dot{Q}'_H}{T_H} + \frac{\dot{Q}_C}{T_C}\right)}_{\text{out}} - \underbrace{\left(\frac{\dot{Q}_H}{T_H} + \frac{\dot{Q}'_C}{T_C}\right)}_{\text{in}}$$

(a) $$\dot{\sigma}_{cycle} = \left(\frac{80\text{ kW}}{500\text{K}} + \frac{40\text{ kW}}{300\text{K}}\right) - \left(\frac{60\text{ kW}}{500\text{K}} + \frac{60\text{ kW}}{300\text{ K}}\right) = -0.03\ \frac{\text{kW}}{\text{K}}$$

negative $\Rightarrow$ the cycle is impossible. This is in accord with the conclusion of Prob. 5.49.

(b) $$\dot{\sigma}_{cycle} = \left(\frac{100\text{ kW}}{500\text{K}} + \frac{80\text{ kW}}{300\text{K}}\right) - \left(\frac{120\text{ kW}}{500\text{K}} + \frac{80\text{ kW}}{300\text{K}}\right) = -0.04\ \frac{\text{kW}}{\text{K}}$$

PROBLEM 5.88

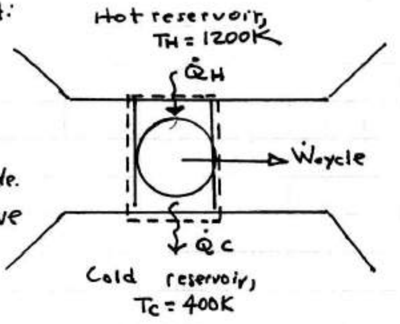
The steady-state data listed below are claimed for a power cycle operating between hot and cold reservoirs at 1200 K and 400 K, respectively. For each case, evaluate the net power developed by the cycle, in kW, and the thermal efficiency. Also in each case apply Eq. 5.13 on a time-rate basis to determine whether the cycle operates reversibly, operates irreversibly, or is impossible.

(a) $\dot{Q}_H = 600$ kW, $\dot{Q}_C = 400$ kW
(b) $\dot{Q}_H = 600$ kW, $\dot{Q}_C = 0$ kW
(c) $\dot{Q}_H = 600$ kW, $\dot{Q}_C = 200$ kW

KNOWN: Steady-state data are claimed for a power cycle operating between hot and cold reservoirs at 1200K and 400K, respectively.

FIND: For each case, determine $\dot{W}_{cycle}$, in kW, η, and use Eq. 5.13 on a time-rate basis to determine if the cycle is reversible, irreversible, or impossible.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:
1. The system shown in the figure executes a power cycle.
2. Each energy transfer is positive in the direction of the accompanying arrow.

ANALYSIS:

---

1 (a) $\dot{W}_{cycle} = \dot{Q}_H - \dot{Q}_C = 200\text{kW}$, $\eta = \dfrac{\dot{W}_{cycle}}{\dot{Q}_H} = \dfrac{200\text{kW}}{600\text{kW}} = 0.333\ (33.3\%)$

Using Eq. 5.13 on a time-rate basis,

$$\dot{\sigma}_{cycle} = \underset{\text{out}}{\frac{\dot{Q}_C}{T_C}} - \underset{\text{in}}{\frac{\dot{Q}_H}{T_H}} = \frac{400\text{ kW}}{400\text{ K}} - \frac{600\text{ kW}}{1200\text{K}} = +0.5\ \frac{\text{kW}}{\text{K}}$$

plus: Cycle operates irreversibly.

(b) $\dot{W}_{cycle} = \dot{Q}_H - \dot{Q}_C = (600 - 0)\text{kW} = 600\text{ kW}$, $\eta = \dfrac{\dot{W}_c}{\dot{Q}_H} = 1\ (100\%)$

2 Using Eq. 5.13,

$$\dot{\sigma}_{cycle} = \frac{\dot{Q}_C}{T_C} - \frac{\dot{Q}_H}{T_H} = \frac{0\text{ kW}}{400\text{K}} - \frac{600\text{kW}}{1200\text{K}} = -0.5\ \frac{\text{kW}}{\text{K}}$$

minus: impossible

---

3 (c) $\dot{W}_{cycle} = \dot{Q}_H - \dot{Q}_C = (600 - 200) = 400\text{ kW}$, $\eta = \dfrac{\dot{W}_c}{\dot{Q}_H} = \dfrac{400\text{kW}}{600\text{kW}} = 0.667\ (66.7\%)$

Using Eq. 5.13,

$$\dot{\sigma}_{cycle} = \frac{\dot{Q}_C}{T_C} - \frac{\dot{Q}_H}{T_H} = \frac{200\text{ kW}}{400\text{K}} - \frac{600\text{kW}}{1200\text{ K}} = 0\ \frac{\text{kW}}{\text{K}}$$

no irreversibilities present
$\Rightarrow$ cycle operates reversibly.

PROBLEM 5.88 (Continued)

Comments:

1. Alternatively, we can apply Eq. 5.9 to determine the maximum thermal efficiency for any power cycle operating between the hot and cold reservoirs:

$$\eta_{max} = 1 - \frac{T_C}{T_H} = 1 - \frac{400\,K}{1200\,K}$$

$$= 0.667 \; (66.7\%)$$

Since $\eta < \eta_{max}$, we conclude that the power cycle of part (a) operates irreversibly.

2. Alternatively, this conclusion follows directly from the Kelvin-Planck statement of the second law.

3. Using $\eta_{max} = 0.667$ from note 1 above, we see that in part (c) $\eta = \eta_{max}$. Accordingly the cycle operates reversibly.

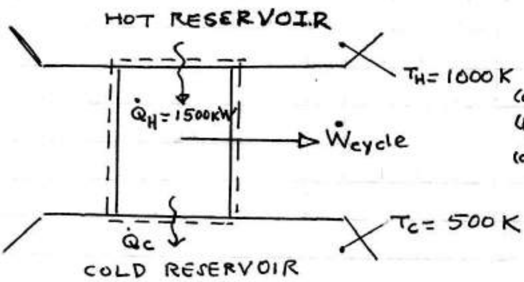
PROBLEM 5.89

At steady state, a thermodynamic cycle operating between hot and cold reservoirs at 1000 K and 500 K, respectively, receives energy by heat transfer from the hot reservoir at a rate of 1500 kW, discharges energy by heat transfer to the cold reservoir, and develops power at a rate of (a) 1000 kW, (b) 750 kW, (c) 0 kW. For each case, apply Eq. 5.13 on a time-rate basis to determine whether the cycle operates reversibly, operates irreversibly, or is impossible.

KNOWN: Steady-state data are provided for a power cycle operating between hot and cold reservoirs.

FIND: For each case apply Eq. 5.13 on a time-rate basis to investigate the claimed operation.

Schematic & Given Data



(a) $\dot{W}_{cycle} = 1000 kW$
(b) $\dot{W}_{cycle} = 750 kW$
(c) $\dot{W}_{cycle} = 0 kW$

Engineering Model: 1. The system shown by the dashed line is at steady state. 2. All energy transfers are positive in the direction of the arrows.

ANALYSIS: (a) An energy rate balance reads, $\dot{W}_{cycle} = \dot{Q}_H - \dot{Q}_C$; thus $\dot{Q}_C = 500 kW$. Applying Eq. 5.13,

① $$\dot{\sigma}_{cycle} = \underset{\text{out}}{\frac{\dot{Q}_C}{T_C}} - \underset{\text{in}}{\frac{\dot{Q}_H}{T_H}} = \frac{500 kW}{500 K} - \frac{1500 kW}{1000 K} = -0.5 \frac{kW}{K}$$

Impossible

(b) An energy rate balance gives, $\dot{Q}_C = 750 kW$. Applying Eq. 5.13

② $$\dot{\sigma}_{cycle} = \frac{750 kW}{500 K} - \frac{1500 kW}{1000 kW} = 0$$

No irreversibilities within the system.

③ (c) An energy rate balance gives $\dot{Q}_H = \dot{Q}_C = 1500 kW$. Applying Eq. 5.13

$$\dot{\sigma}_{cycle} = \frac{\dot{Q}_C}{T_C} - \frac{\dot{Q}_H}{T_H} = \dot{Q}_H \left[\frac{1}{T_C} - \frac{1}{T_H}\right]$$

$$= 1500 kW \left[\frac{1}{500 K} - \frac{1}{1000 K}\right] = +1.5 \frac{kW}{K}$$

Irreversibilities within the system.

Comments:

1. Alternatively, we can apply Eq. 5.9 to determine the maximum thermal efficiency for any power cycle operating between the hot and cold reservoirs:

$$\eta_{max} = 1 - \frac{T_C}{T_H} = 1 - \frac{500\text{K}}{1000\text{K}} = 0.5\ (50\%)$$

In part (a),

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H} = \frac{1000\text{kW}}{1500\text{kW}} = 0.667\ (66.7\%)$$

Since $\eta > \eta_{max}$, the cycle is impossible.

2. In part (b)

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H} = \frac{750\text{kW}}{1500\text{kW}} = 0.5\ (50\%)$$

Since $\eta = \eta_{max}$, the cycle is reversible.

3. The system operates at steady state and thus can be regarded as a special case of the cycle concept introduced in Sec. 2.6. Moreover, as $\dot{W}_{cycle} = 0$, the thermal efficiency is zero as well. Finally, since $\eta < \eta_{max}$, the cycle operates irreversibly.

Alternatively, since $\dot{W}_{cycle} = 0$, $\dot{Q}_H = \dot{Q}_C$. Accordingly the system allows a spontaneous heat transfer from the hot reservoir to the cold reservoir. By the discussion of Sec. 5.3.1, such heat transfer is irreversible.

PROBLEMS 5.90 - 5.92

5.90 Figure P5.90 gives the schematic of a vapor power plant in which water steadily circulates through the four components shown. The water flows through the boiler and condenser at constant pressure and through the turbine and pump adiabatically. Kinetic and potential energy effects can be ignored. Process data follow:

Process 4–1: constant-pressure at 1 MPa from saturated liquid to saturated vapor

Process 2–3: constant-pressure at 20 kPa from x_2 = 88% to x_3 = 18%

(a) Using Eq. 5.13 expressed on a time-rate basis, determine if the cycle is *internally reversible, irreversible,* or *impossible.*

(b) Determine the thermal efficiency using Eq. 5.4 expressed on a time-rate basis and steam table data.
(c) Compare the result of part (b) with the *Carnot efficiency* calculated using Eq. 5.9 with the boiler and condenser temperatures and comment.

SCHEMATIC & GIVEN DATA:

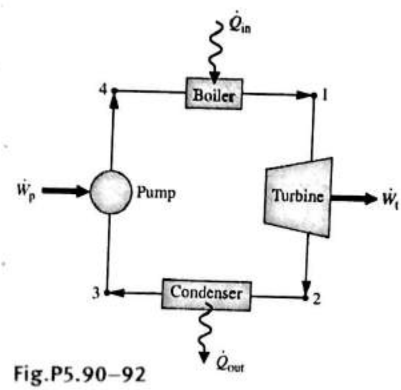
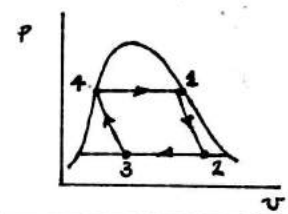


Fig.P5.90–92

ENGINEERING MODEL:

1. A control volume at steady state encloses each of the four components
2. Power and heat transfer rates are positive in the direction of the arrows.
3. Stray heat transfer and kinetic and potential energy effects can be ignored



ANALYSIS:

(a) Expressed on a time-rate basis, Eq. 5.13 takes the form:

$$\dot{\sigma}_{cycle} = \frac{\dot{Q}_{23}}{T_{23}} - \frac{\dot{Q}_{41}}{T_{41}}$$

where

$$\dot{Q}_{41} = \dot{m}(h_1 - h_4) = \dot{m}[h_{fg}]_{41}$$

$$\dot{Q}_{23} = \dot{m}[h_2 - h_3] = \dot{m}[(h_f + x_2 h_{fg}) - (h_f + x_3 h_{fg})]$$

$$= \dot{m}\{(x_2 - x_3)][h_{fg}]_{23}\} \quad = (h_g - h_f) @ T_{23}$$

Collecting results,

$$\boxed{\frac{\dot{\sigma}_{cycle}}{\dot{m}} = \frac{(x_2 - x_3)(h_{fg})_{23}}{T_{23}} - \frac{(h_{fg})_{41}}{T_{41}}} \qquad (1)$$

With data from Table A-3, $T_{41} = 179.9\,°C$, $(h_{fg})_{41} = 2015.3$ kJ/kg

$T_{23} = 60.06\,°C$, $(h_{fg})_{23} = 2358.3$ kJ/kg

$$\frac{\dot{\sigma}_{cycle}}{\dot{m}} = \frac{\overbrace{(0.88 - 0.18)(2358.3)(\text{kJ/kg})}^{1650.81}}{333.21\,\text{K}} - \frac{2015.3\ \text{kJ/kg}}{453.05\ \text{K}} = 0.51\ \frac{\text{kJ/kg}}{\text{K}}$$ irreversible

(b) $$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}} = \boxed{1 - \frac{(x_2 - x_3)[h_{fg}]_{23}}{[h_{fg}]_{41}}} \qquad (2)$$

$$\Rightarrow \quad \eta = 1 - \frac{(0.88-0.18)(2358.3)}{2015.3} = 0.18 \quad (18\%)$$

(c) $$\eta_{MAX} = 1 - \frac{T_C}{T_H} = \boxed{1 - \frac{T_{23}}{T_{41}}} \qquad (3)$$

$$= 1 - \frac{333.21}{453.05} = 0.265 \quad (26.5\%)$$

Since $\eta < \eta_{MAX}$, the cycle is irreversible.

5.91 Repeat Problem 5.90 for the following case:

Process 4–1: constant pressure at 8 MPa from saturated liquid to saturated vapor

Process 2–3: constant pressure at 8 kPa from x_2 = 67.5% to x_3 = 34.2%

Data from Table A-3:

$T_{41} = 295.1°C$, $(h_{fg})_{41} = 1441.3 \frac{kJ}{kg}$

$T_{23} = 41.51°C$, $(h_{fg})_{23} = 2403.1 \frac{kJ}{kg}$

(a) Using Eq.(1), $\frac{\dot{\sigma}_{cycle}}{\dot{m}} = \frac{800.23}{314.66} - \frac{1441.3}{568.25} \doteq 0$ (rounded value) internally reversible

(b) Using Eq.(2), $\eta = 1 - \frac{800.23}{1441.3} = 0.445$ (44.5%)

(c) Using Eq.(3): $\eta_{MAX} = 1 - \frac{314.66}{568.25} = 0.446$ (44.6%)

The results of (b) and (c) agree to within round-off. The cycle is internally reversible.

5.92 Repeat Problem 5.90 for the following case:

Process 4–1: constant pressure at 0.15 MPa from saturated liquid to saturated vapor

Process 2–3: constant pressure at 20 kPa from x_2 = 90% to x_3 = 10%

Data from Table A-3:

$T_{41} = 111.4°C$, $(h_{fg})_{41} = 2226.5$ kJ/kg

$T_{23} = 60.06°C$, $(h_{fg})_{23} = 2358.3$ kJ/kg

(a) Using Eq.(1), $\dot{\sigma}_{cycle} = \frac{1886.64}{333.21} - \frac{2226.5}{384.55} = -0.13 \frac{kJ/kg}{K}$ (impossible

(b) Using Eq.(2), $\eta = 1 - \frac{1886.64}{2226.5} = 0.153$ (15.3%)

(c) Using Eq.(3), $\eta_{MAX} = 1 - \frac{333.21}{384.55} = 0.134$ (13.4%)

Since $\eta > \eta_{MAX}$, the cycle is impossible.

PROBLEM 5.93

As shown in Fig. P5.93, a system executes a power cycle while receiving 750 kJ by heat transfer at a temperature of 1500 K and discharging 100 kJ by heat transfer at a temperature of 500 K. Another heat transfer from the system occurs at a temperature of 1000 K. Using Eq. 5.13, plot the thermal efficiency of the cycle versus σ_{cycle}, in kJ/K.

Schematic and Given Data:

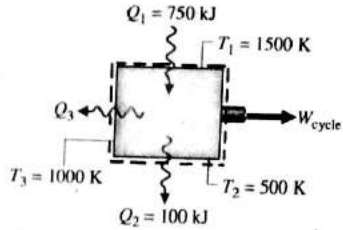


Fig.P5.93

KNOWN: Heat transfer and temperature data are provided for a power cycle.

FIND: Using Eq. 5.13, plot η vs σ_{cycle}.

ENGINEERING MODEL:

1. The system shown in the schematic executes a power cycle.
2. All heat transfers occur at the indicated temperatures and are in the direction of the accompanying arrow

ANALYSIS: For any power cycle, $\eta = W_{cycle}/Q_{in}$. In the present case $Q_{in} = Q_1$ and $W_{cycle} = Q_{cycle} = Q_1 - Q_2 - Q_3$. Thus

$$\eta = \frac{Q_1 - Q_2 - Q_3}{Q_1} = \frac{750 - 100 - Q_3}{750}$$

① $$= \frac{(650 - Q_3)\,\mathrm{kJ}}{750\,\mathrm{kJ}} \qquad (1)$$

Using Eq. 5.13

$$\frac{Q_1}{T_1} - \frac{Q_2}{T_2} - \frac{Q_3}{T_3} = -\sigma_{cycle}$$

$$\Rightarrow \quad Q_3 = T_3\left[\frac{Q_1}{T_1} - \frac{Q_2}{T_2} + \sigma_{cycle}\right]$$

② $$= 1000\,\mathrm{K}\left[\frac{750\,\mathrm{kJ}}{1500\,\mathrm{K}} - \frac{100\,\mathrm{kJ}}{500\,\mathrm{K}} + \sigma_{cycle}\right]$$

$$= 1000\,\mathrm{K}\left[0.3 + \sigma_{cycle}\right]\frac{\mathrm{kJ}}{\mathrm{K}} \qquad (2)$$

Collecting results, inserting Eq.(2) into Eq.(1), we get

$$\eta = \frac{650\,\mathrm{kJ} - 1000\,\mathrm{K}\left[0.3 + \sigma_{cycle}\right]\frac{\mathrm{kJ}}{\mathrm{K}}}{750\,\mathrm{kJ}}$$

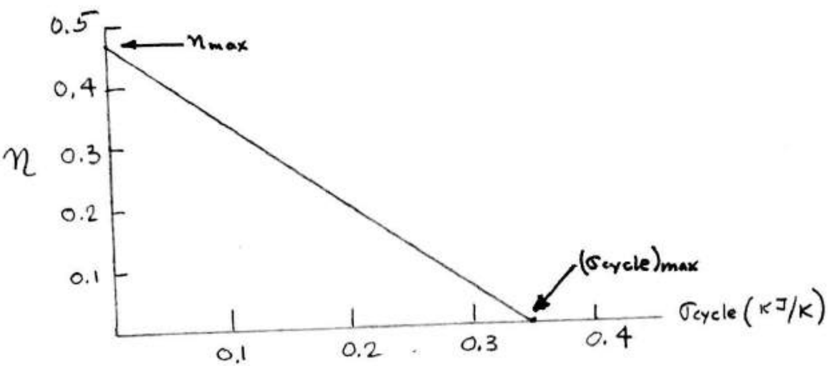
$$= 0.467 - (1.333\,\mathrm{K/kJ})\,\sigma_{cycle} \qquad (3)$$

where $\sigma_{cycle} \geq 0$ and $0 \leq \eta \leq \eta_{max}$.

PROBLEM 5.93 (Continued)

By inspection of Eq. (3), the case $\sigma_{cycle} = 0$ gives $\eta_{max} = 0.467$
Moreover, the case $\eta = 0$ gives $(\sigma_{cycle})_{max} = 0.35$ kJ/K.

Observing Eq. (3) is a linear relation, we get



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1. This is a first law constraint.

2. This is a second law constraint

PROBLEM 5.94

Shown in Fig. P5.94 is a system that executes a power cycle while receiving 600 Btu by heat transfer at a temperature of 1000°R and discharging 400 Btu by heat transfer at a temperature of 800°R. A third heat transfer occurs at a temperature of 600°R. These are the only heat transfers experienced by the system.

(a) Applying an energy balance together with Eq. 5.13, determine the direction and allowed range of values, in Btu, for the heat transfer at 600°R.

(b) For the power cycle, evaluate the maximum theoretical thermal efficiency.

SCHEMATIC & GIVEN DATA:

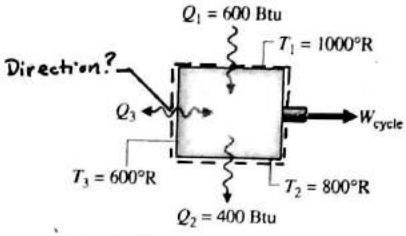


Fig.P5.94

KNOWN: Data are provided for a power cycle experiencing heat tranfers at three different temperatures.

FIND: (a) For the heat transfer at 600°R, determine the direction and allowed range of values. (b) For the power cycle, evaluate the maximum theoretical thermal efficiency.

ENGINEERING MODEL: 1. The heat transfers Q_1 and Q_2 are each in the direction of the accompanying arrow. 2. The heat transfer Q_3 will be assumed into the system at T_3, pending determination of its actual direction. 3. Q_1, Q_2, and Q_3 are the only heat transfers. 4. The system shown by the dashed line executes a power cycle: $W_{cycle} \geq 0$.

ANALYSIS: (a) An energy balance reads, $W_{cycle} = Q_1 - Q_2 + Q_3$, giving

$$Q_3 = W_{cycle} + Q_2 - Q_1$$
$$= W_{cycle} + 400\,\text{Btu} - 600\,\text{Btu}$$
$$= W_{cycle} - 200\,\text{Btu}$$

(a)

(1) Since, $W_{cycle} \geq 0$, $Q_3 \geq -200$ Btu.

Applying Eq. 5.13

$$\sigma_{cycle} = \left[\frac{Q_2}{T_2} - \left(\frac{Q_1}{T_1} + \frac{Q_3}{T_3}\right)\right]$$

out, in

$$= \left[\frac{400\,\text{Btu}}{800°R} - \frac{600\,\text{Btu}}{1000°R} - \frac{Q_3}{600°R}\right]$$

(b)

$$\Rightarrow \quad Q_3 = -60\,\text{Btu} - (600°R)\,\sigma_{cycle}$$

(c)

(2) Since $\sigma_{cycle} \geq 0$, we conclude that $Q_3 \leq -60$ Btu.

Collecting results,

$$-200\,\text{Btu} \leq Q_3 \leq -60\,\text{Btu}$$

The direction of Q_3 is out of the system.

PROBLEM 5.94 (Continued)

(b) The thermal efficiency is

$$\eta = \frac{Q_{cycle}}{Q_1} = \frac{Q_1 + Q_2 + Q_3}{Q_1}$$

$$= \frac{600\text{ Btu} - 400\text{ Btu} + [-60\text{ Btu} - (600°R)\sigma_{cycle}]}{600\text{ Btu}}$$

$$= \frac{140\text{ Btu} - (600°R)\sigma_{cycle}}{600\text{ Btu}}$$

$$\eta = \frac{140\text{ Btu}}{600\text{ Btu}} - \left[\frac{(600°R)\sigma_{cycle}}{600\text{ Btu}}\right]$$

Since $\sigma_{cycle} \geq 0$

$$\eta_{max} = \frac{140\text{ Btu}}{600\text{ Btu}} = 0.233 \quad (23.3\%)$$

---

1. This is a constraint imposed by the first law of thermodynamics.

2. This is a constraint imposed by the second law of thermodynamics.

PROBLEM 6.1

Construct a plot, to scale, showing constant-pressure lines of 5.0 and 10 MPa ranging from 100 to 400°C on a *T*-*s* diagram for water.

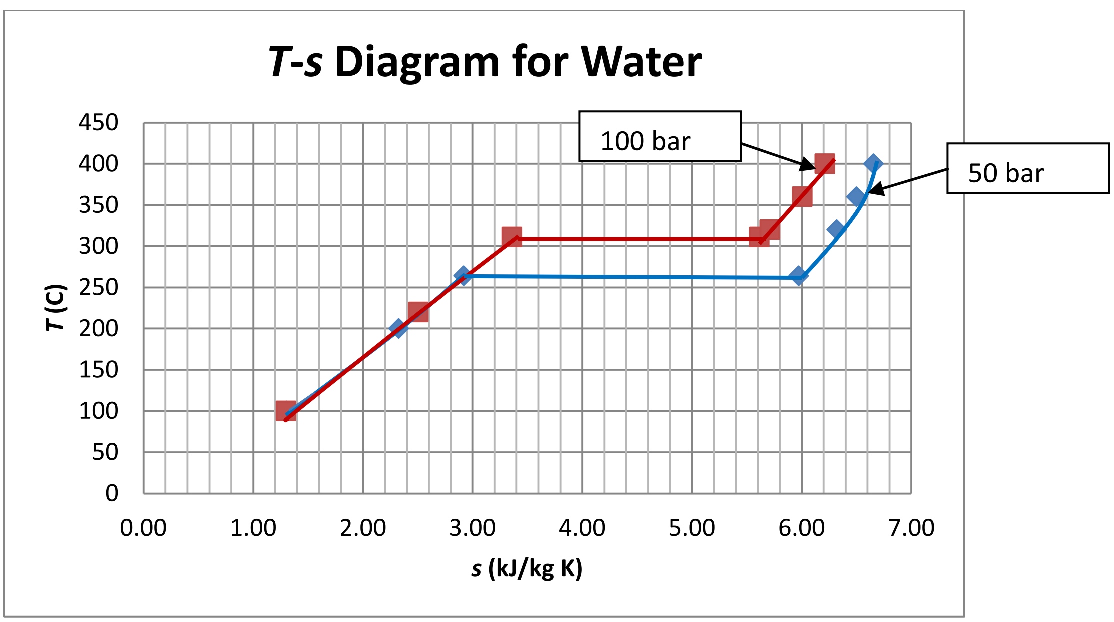
Data Table

***p* = 5.0 MPa = 50 bar**

T (°C)	*s* (kJ/kg·K)	Table/Comments
100	1.3030	Table A-5/compressed liquid
200	2.3255	Table A-5/compressed liquid
264.0	2.9202	Table A-3/saturated liquid @ 50 bar
264.0	5.9734	Table A-3/saturated vapor @ 50 bar
320	6.320	Table 4-4/superheated vapor - interpolating
360	6.500	Table 4-4/superheated vapor - interpolating
400	6.655	Table 4-4/superheated vapor - interpolating

***p* = 10.0 MPa = 100 bar**

T (°C)	*s* (kJ/kg·K)	Table/Comments
100	1.2992	Table A-5/compressed liquid
220	2.5039	Table A-5/compressed liquid
311.1	3.3596	Table A-3/saturated liquid @ 50 bar
311.1	5.6141	Table A-3/saturated vapor @ 50 bar
320	5.7103	Table 4-4/superheated vapor
360	6.0060	Table 4-4/superheated vapor
400	6.2120	Table 4-4/superheated vapor



PROBLEM 6.2

Construct a plot, to scale, showing constant-pressure lines of 1000 and 1500 lbf/in.^{2} ranging from 300 to 1000^{o}F on a *T*-*s* diagram for water.

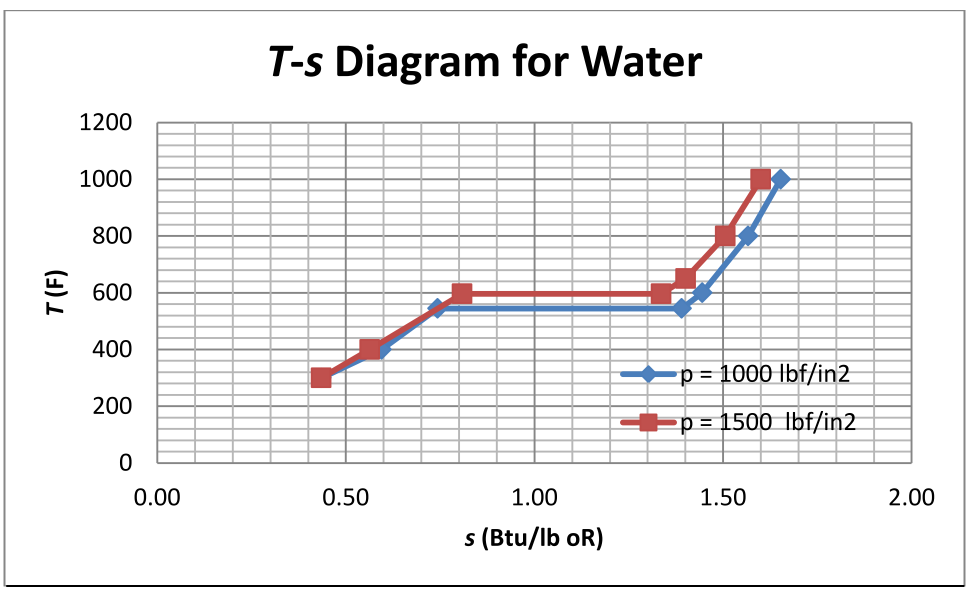
Data Table

***p* = 1000 lbf/in.^{2}**

T (^{o}F)	*s* (Btu/lb·^{o}R)	Table/Comments
300	0.43552	Table A-5E/compressed liquid
400	0.59472	Table A-5E/compressed liquid
544.75	0.7432	Table A-3E/saturated liquid @ 1000 lbf/in.^{2}
544.75	1.3903	Table A-3E/saturated vapor @ 1000 lbf/in.^{2}
600	1.4450	Table 4-4E/superheated vapor
800	1.5665	Table 4-4E/superheated vapor
1000	1.6530	Table 4-E/superheated vapor

***p* = 1500 lbf/in.^{2}**

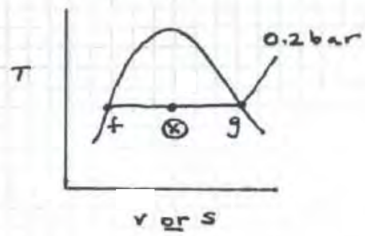
T (^{o}F)	*s* (Btu/lb·^{o}R)	Table/Comments
300	0.43463	Table A-5E/compressed liquid
400	0.56343	Table A-5E/compressed liquid
596.39	0.8082	Table A-3E/saturated liquid @ 50 bar
596.39	1.3359	Table A-3e/saturated vapor @ 50 bar
650	1.401	Table 4-4E/superheated vapor - interpolating
800	1.506	Table 4-4E/superheated vapor - interpolating
1000	1.600	Table 4-4E/superheated vapor - interpolating



PROBLEM 6.3

FIND: Determine the indicated property in the indicated units. Locate each state on T-v and T-s diagrams.

(a) Water at $p = 0.2$ bar, $s = 4.3703$ kJ/kg·K. Find h, in kJ/kg.



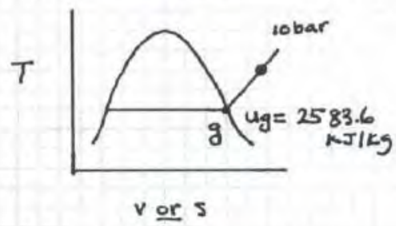
With data from Table A-3,

$$x = \frac{s - s_f}{s_g - s_f} = \frac{4.3703 - 0.8320}{7.9085 - 0.8320} = 0.5$$

$$h = h_f + x(h_g - h_f)$$

$$= 251.4 + 0.5(2358.3) = 1430.55 \ \frac{kJ}{kg}$$

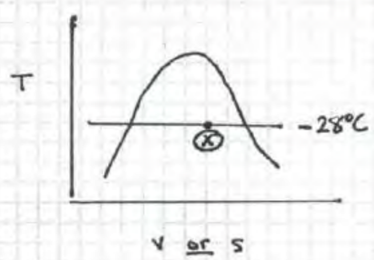
(b) Water at $p = 10$ bar, $u = 3124.4$ kJ/kg. Find s, in kJ/kg·K.



With data from Table A-4,

$$s = 7.7622 \text{ kJ/kg·K}$$

(c) R134a at $T = -28°C$, $x = 0.8$. Find s, in kJ/kg·K.



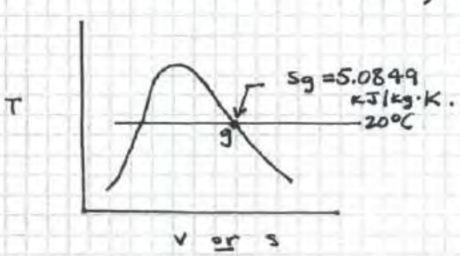
With data from Table A-10,

$$s = s_f + x(s_g - s_f)$$

$$= (0.0600) + 0.8(0.9411 - 0.0600)$$

$$= 0.7649 \text{ kJ/kg·K}$$

(d) Ammonia at $T = 20°C$, $s = 5.0849$ kJ/kg·K. Find u, in kJ/kg.



With data from Table A-13,

$$u = u_g(20°C) = 1331.94 \ \frac{kJ}{kg}$$

PROBLEM 6.4

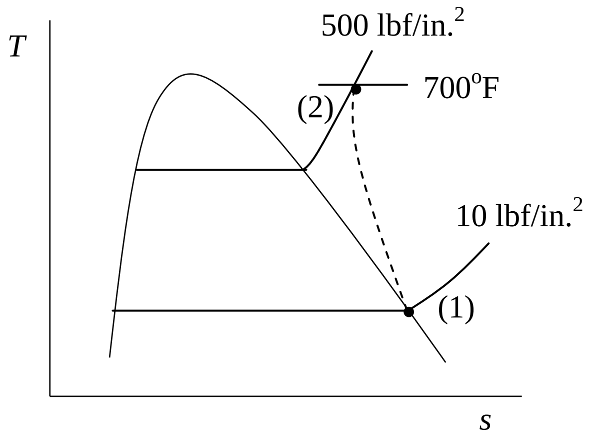
Using the appropriate tables, determine the change in specific entropy between the specified states, in Btu/lb·°R. Show the states on a sketch of the *T*-*s* diagram.

(a) water, p_1 = 10 lbf/in.^{2}, saturated vapor; p_2 = 500 lbf/in.^{2}, T_2 = 700°F.
(b) ammonia, p_1 = 140 lbf/in.^{2}, T_1 = 160°F; T_2 = - 10°F, h_2 = 590 Btu/lb.
(c) air as an ideal gas, T_1 = 80°F, p_1 = 1 atm; T_2 = 340°F, p = 5 atm.
(d) oxygen as an ideal gas, T_1 = T_2 = 520°R, p_1 = 10 atm, p_2 = 5 atm.

(a) water, p_1 = 10 lbf/in.^{2}, saturated vapor; p_2 = 500 lbf/in.^{2}, T_2 = 700°F.

State 1: Table A-3E: s_1 = 1.7877 Btu/ lb·°R

State 2: Table A-4E: s_2 = 1.6112 Btu/ lb·°R ⟶ Δs = - 0.1765 Btu/ lb·°R ◀



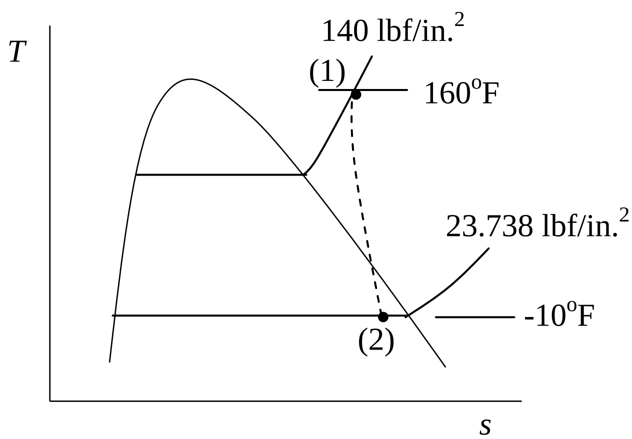
(b) ammonia, p_1 = 140 lbf/in.^{2}, T_1 = 160°F; T_2 = - 10°F, h_2 = 590 Btu/lb.

State 1: Table A-15E: s_1 = 1.3025 Btu/ lb·°R

State 2: Table A-113E: $h_2 < h_{g@\,-10}$°F. The quality is $x_2 = \dfrac{590 - 31.73}{607.99 - 31.73} = 0.969$

$s_2 = s_{f2} + x_2(s_{g2} - s_{f2})$ = 0.0729 + (0.969)(1.3544 – 0.0729) = 1.3147 Btu/ lb·°R

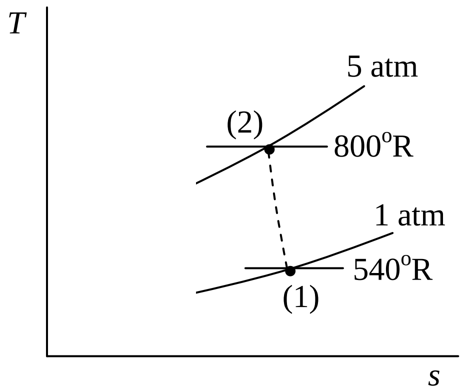
Δs = 0.0122 Btu/ lb·°R ◀



PROBLEM 6.4 (CONTINUED)

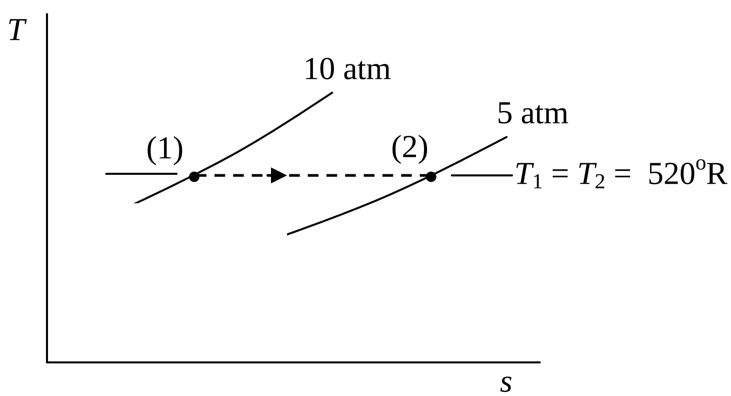
(c) air as an ideal gas, T_1 = 80°F = 540 °R, p_1 = 1 atm; T_2 = 340°F = 800 °R, p = 5 atm.
For air as an ideal gas; $\Delta s = s^o(T_2) - s^o(T_1) - R \ln (p_2/p_1)$. With data from Table A-22E

$$\Delta s = (0.69558) - (0.60078) - (1.986/28.97) \ln(5/1) = -0.01553 \text{ Btu/lb}\cdot{}^o\text{R}$$



(d) oxygen as an ideal gas, $T_1 = T_2$ = 520°R, p_1 = 10 atm, p_2 = 5 atm.
For oxygen as an ideal gas; $\Delta s = \left(\frac{\bar{s}^o(T_2) - \bar{s}^o(T_1)}{M}\right) - R \ln (p_2/p_1)$. Since $T_1 = T_2$

$$\Delta s = -(1.986/32.00) \ln(5/10) = 0.04302 \text{ Btu/lb}\cdot{}^o\text{R}$$



PROBLEM 6.5

```
//(a)
pa = 0.20  //bar
sa = 4.3703  // kJ/kg K
xa = x_sP("Water/Steam", sa, pa)
ha = hsat_Px("Water/Steam", pa, xa)
```

Results
IT: h = 1431 kJ/kg
Problem 6.3: h = 1430.55 kJ/kg

```
//(b)
pb = 10 //bar
ub = 3124.4  // kJ/kg
ub = u_PT("Water/Steam", pb, Tb)
sb = s_PT("Water/Steam", pb, Tb)
```

Results
IT: s = 7.762 kJ/kg·K
Problem 6.3: s = 7.7622 kJ/kg·K

```
//(c)
Tc = -28  // oC
xc = 0.8
pc = Psat_T("R134A", Tc)
sc = ssat_Px("R134A", pc, xc)
```

Results
IT: s = 0.7649 kJ/kg·K
Problem 6.3: s = 0.7649 kJ/kg·K

```
//(d)
Td = 20  // oC
sd = 5.0849  // kJ/kg K
pd = Psat_T("Ammonia", Td)
sd = ssat_Px("Ammonia", pd, xd)
ud = usat_Px("Ammonia", pd, xd)
```

Results
IT: u = 1332 kJ/kg
Problem 6.3: u = 1331.94 kJ/kg

PROBLEM 6.6

```
// (a)
pa1 = 10  // lbf/in^2
xa1 = 1
pa2 = 500  // lbf/in^2
Ta2 = 700  // oF
sa1 = ssat_Px("Water/Steam", pa1, xa1)
sa2 = s_PT("Water/Steam", pa2, Ta2)
dels_a = sa2 - sa1
```

Results
IT: Δs = - 0.1765 Btu/lb·°R
Problem 6.4: Δs = - 0.1765 Btu/lb·°R

```
// (b)
p1b = 140  // lbf/in^2
T1b = 169  // oF
T2b = -10  // oF
h2b = 590  // Btu/lb
s1b = s_PT("Ammonia", p1b, T1b)
p2b = Psat_T("Ammonia", T2b)
h2b = hsat_Px("Ammonia", p2b, x2b)
s2b = ssat_Px("Ammonia", p2b, x2b)
dels_b = s2b - s1b
```

Results
IT: Δs = 0.003552 Btu/lb·°R
Problem 6.4: Δs = 0.0122 Btu/lb·°R
Note: the change is entropy is very small, so round-off errors result in a significant difference in the two values in this case.

```
// (c)
Tc1 = 80  // oF
pc1 = 14.696  // lbf/in^2
Tc2 = 340  // oF
pc2 = 5*14.696  // lbf/in^2
sc1 = s_Tp("Air",Tc1,pc1)
sc2 = s_Tp("Air",Tc2,pc2)
dels_c = sc2 - sc1
```

Results
IT: Δs = -0.01545 Btu/lb·°R
Problem 6.4: Δs = -0.01553 Btu/lb·°R

```
// (d)
Td = 520 - 460  // oF
pd1 = 10*14.696  // lbf/in^2
pd2 = 5*14.696  // lbf/in^2
sd1 = s_Tp("O2",Td,pd1)
sd2 = s_Tp("O2",Td,pd2)
dels_d = sd2 - sd1
```

Results
IT: Δs = 0.04304 Btu/lb·°R
Problem 6.4: Δs = 0.04302 Btu/lb·°R

PROBLEM 6.7

Using *steam table* data, determine the indicated property data for a process in which there is no change in specific entropy between state 1 and state 2. In each case, locate the states on a sketch of the T–s diagram.

(a) $T_1 = 40°C$, $x_1 = 100\%$, $p_2 = 150$ kPa. Find T_2, in °C, and Δh, in kJ/kg.

(b) $T_1 = 10°C$, $x_1 = 75\%$, $p_2 = 1$ MPa. Find T_2, in °C, and Δu, in kJ/kg.

(a)

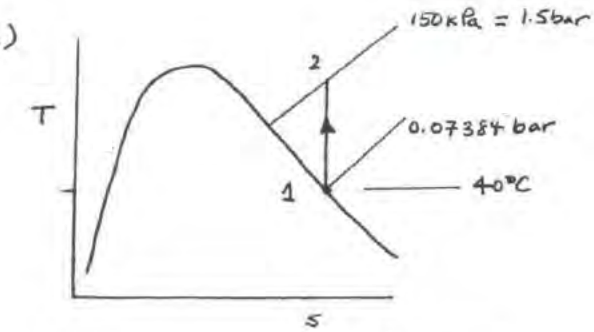


Table A-2,

$s_1 = 8.2570$ kJ/kg·K

$h_1 = 2574.3$ kJ/kg

Interpolating in Table A-4,

$T_2 = 368.8°C$

$h_2 = 3213.1 \ \frac{kJ}{kg}$

$\Delta h = 638.8 \ \frac{kJ}{kg}$

(b)

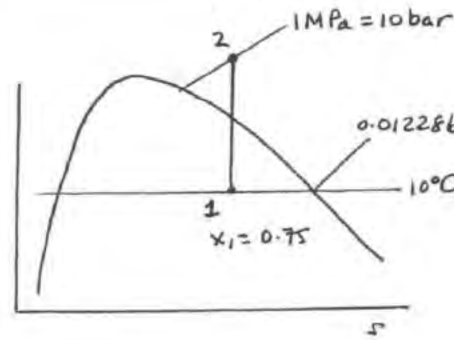


Table A-2,

$s_1 = s_f + x_1 (s_g - s_f)$

$= 0.1510 + 0.75(8.9008 - 0.1510)$

$= 6.71335$ kJ/kg·K

$u_1 = u_f + x_1 (u_g - u_f)$

$= 42.0 + 0.75(2389.2 - 42.0)$

$= 1802.4$ kJ/kg

Interpolating in Table A-4,

$T_2 = 204.1°C$

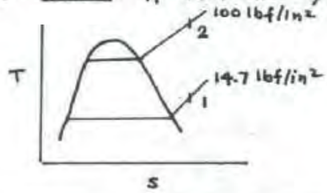
$u_2 = 2629.2$ kJ/kg

$\Delta u = 826.8$ kJ/kg

PROBLEM 6.8

FIND: Determine the indicated property for a process where $s_2 = s_1$.

(a) Water. $P_1 = 14.7\ lbf/in^2$, $T_1 = 500°F$, $P_2 = 100\ lbf/in^2$. Find T_2.

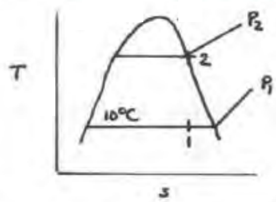


From Table A-4E, $s_1 = 1.9263\ Btu/lb \cdot °R$.

Interpolating with $s_2 = s_1$ in Table A-4E at $100\ lbf/in^2$,

$T_2 = 1017\ °F$. (a)

(b) Water. $T_1 = 10°C$, $x_1 = 0.75$. Saturated vapor at state 2. Find P_2.



Using data from Table A-2

$$s_1 = s_f + x_1(s_g - s_f) = 0.1510 + 0.75(8.9008 - 0.1510)$$

$$= 6.71335\ kJ/kg \cdot K$$

Then, interpolating with $s_2 = s_1$ in Table A-3, $P_2 = 6.897\ bar$ (b)

(c) Air as an ideal gas. $T_1 = 300K\ (27°C)$, $P_1 = 1.5\ bar$, $T_2 = 400K\ (127°C)$. Find P_2.

Since $s_2 = s_1$, Eq. 6.20a gives

$$\ln \frac{P_2}{P_1} = \frac{s°(T_2) - s°(T_1)}{R}$$

With $s°$ data from Table A-22

$$\ln \frac{P_2}{P_1} = \frac{1.99194 - 1.70203}{(8.314/28.97)}$$

$$\Rightarrow \quad \frac{P_2}{P_1} = 2.746 \quad \Rightarrow \quad P_2 = 4.119\ bar$$ (c)

(d) Air as an ideal gas. $T_1 = 560°R\ (100°F)$, $P_1 = 3\ atm$, $P_2 = 2\ atm$. Find T_2.

Since $s_2 = s_1$, Eq. 6.20a gives

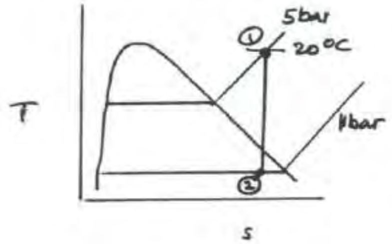
$$s°(T_2) = s°(T_1) + R \ln \frac{P_2}{P_1}$$

With $s°$ at T_1 from Table A-22E

$$s°(T_2) = 0.60950 + \frac{1.986}{28.97} \ln \frac{2}{3} = 0.5817\ Btu/lb \cdot °R$$

Interpolating with $s°(T_2)$ in Table A-22E, $T_2 = 498.7°R\ (39°F)$. (d)

(e) R134a. $T_1 = 20°C$, $P_1 = 5\ bar$, $P_2 = 1\ bar$. Find v_2



From Table A-12, $s_1 = 0.9264\ kJ/kg \cdot K$

Then, with data from Table A-11 and $s_2 = s_1$

$$x_2 = \frac{0.9264 - 0.0678}{0.9395 - 0.0678} = 0.985$$

And

$$v_2 = v_f + x_2(v_g - v_f)$$

$$= \frac{0.7258}{10^3} + 0.985\left[0.1917 - \frac{0.7258}{10^3}\right]$$

$$= 0.188\ m^3/kg$$ (e)

PROBLEM 6.9

Using IT, obtain the property data requested in

(a) Problem 6.7.

(b) Problem 6.8

Problem 6.7 Using IT :

Problem 6.7(a)

```
T1=40
x1=1
p2=150
s2=s1
deltah=h2-h1
p1 = Psat_T("Water/Steam", T1)
s1 = ssat_Px("Water/Steam", p1, x1)
s2 = s_PT("Water/Steam", p2, T2)
h1 = hsat_Px("Water/Steam", p1, x1)
h2 = h_PT("Water/Steam", p2, T2)

Solving...
T2=368.6
deltah=638.4
```

Problem 6.7 (b)

```
T1=10
x1=0.75
p2=10 // bar
s2=s1
deltau=u2-u1
p1 = Psat_T("Water/Steam", T1)
s1 = ssat_Px("Water/Steam", p1, x1)
u1 = usat_Px("Water/Steam", p1, x1)
s2 = s_PT("Water/Steam", p2, T2)
u2 = u_PT("Water/Steam", p2, T2)

Solving...
T2=203.7
deltau=826.5
```

PROBLEM 6.8 Using IT:

IT Code

```
/* (a) Water
p1 = 14.7 // lbf/in.²
T1 = 500 // °F
p2 = 100
s1 = s_PT("Water/Steam", p1, T1)
s2 = s_PT("Water/Steam", p2, T2)
s2 = s1

(b) Water
T1 = 10 // °C
x1 = 0.75
x2 = 1
p1 = Psat_T("Water/Steam", T1)
s1 = ssat_Px("Water/Steam", p1, x1)
s2 = ssat_Px("Water/Steam", p2, x2)
s2 = s1

(c) Air
T1 = 27 // °C
p1 = 1.5 // bar
T2 = 127
s1 = s_TP("Air", T1, p1)
s2 = s_TP("Air", T2, p2)
s2 = s1

(d) Air
T1 = 100 // °F
p1 = 3 // atm
p2 = 2
s1 = s_TP("Air", T1, p1)
s2 = s_TP("Air", T2, p2)
s2 = s1

(e) R-134a
*/
T1 = 20 // °C
p1 = 5 // bar
p2 = 1
s1 = s_PT("R134A", p1, T1)
x2 = x_sP("R134A", s2, p2)
v2 = vsat_Px("R134A", p2, x2)
s2 = s1
```

IT Results

(a) s = 1.926Btu/lb·°R; T_2 = 1016°F
(b) s = 6.712 kJ/kg·K; p_2 = 6.903 bar
(c) p_2 = 4.117 bar
(d) T_2 = 38.75°F
(e) s = 0.9264 kJ/kg·K; x_2 = 0.985, v_2 = 0.1888 m³/kg

Comment: As expected, good agreement exists between data obtained from the Appendix tables and data from IT.

PROBLEM 6.10

Propane undergoes a process from state 1, where p_1 = 1.4 MPa, T_1 = 60°C, to state 2, where p_2 = 1.0 MPa, during which the change in specific entropy is $s_2 - s_1$ = - 0.035 kJ/kg·K. At state 2, determine the temperature, in °C, and the specific enthalpy, in kJ/kg.

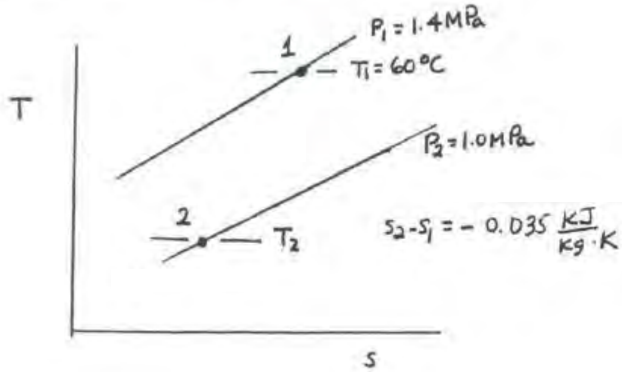


Table A-18:

$s_1 = 1.845$ kJ/kg·K

Given: $s_2 - s_1 = -0.035$ kJ/kg·K

$\Rightarrow s_2 = (1.845 - 0.035)$ kJ/kg·K

= 1.81 kJ/kg·K

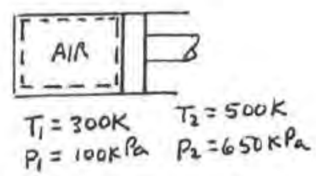
Then, at $P_2 = 1.0$ MPa,

$T_2 = 40°C$

$h_2 = 524.6$ kJ/kg.

PROBLEM 6.11

Air in a piston–cylinder assembly undergoes a process from state 1, where $T_1 = 300$ K, $p_1 = 100$ kPa, to state 2, where $T_2 = 500$ K, $p_2 = 650$ kPa. Using the ideal gas model for air, determine the change in specific entropy between these states, in kJ/kg · K, if the process occurs (a) without internal irreversibilities, (b) with internal irreversibilities.



Entropy is a property. Thus, the change in specific entropy has the same value for fixed end states whether or not irreversibilities are present during a process between the end states.

With Eq. 6.20a and data from Table A-22,

$$\Delta s = s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{P_2}{P_1}$$

$$= \left[2.21952 - 1.70203 - \frac{8.314}{28.97} \ln\left(\frac{650}{100}\right)\right] \frac{kJ}{kg \cdot K}$$

$$= -0.0197 \frac{kJ}{kg \cdot K}$$

PROBLEM 6.12

Water contained in a closed, rigid tank, initially at 100 lbf/in.^{2}, 800°F, is cooled to a final state where the pressure is 20 lbf/in.^{2} Determine the change in specific entropy, in Btu/lb · °R, and show the process on sketches of the T–v and T–s diagrams.

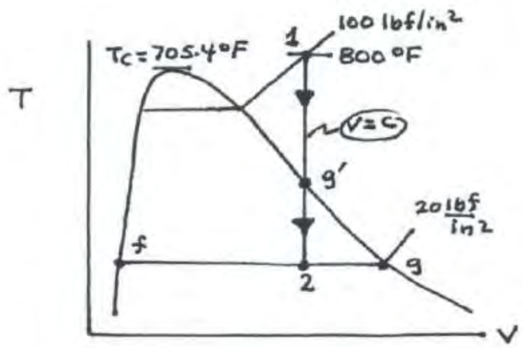
KNOWN: Water in a closed, rigid tank is cooled from an initial state where $p = 100$ lbf/in^2, $T = 800°F$ to a final pressure of 20 lbf/in^2.

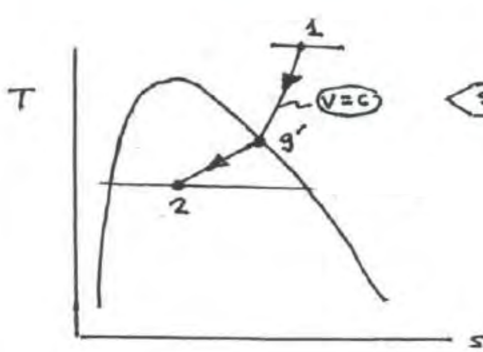
FIND: Determine Δs, in Btu/lb·°R, and show the process on T–V and T–s diagrams.

SCHEMATIC & GIVEN DATA:

$P_1 = 100$ lbf/in^2
$T_1 = 800°F$

$P_2 = 20$ lbf/in^2





see Fig. 6.2

ANALYSIS:

Table A-4E: $v_1 = 7.445 \frac{ft^3}{lb}$, $s_1 = 1.8449$ Btu/lb·°R

With $v_2 = v_1$,

$$x_2 = \frac{v_2 - v_f}{v_g - v_f}$$

Using v_f, v_g from Table A-3E at 20 lbf/in^2,

$$x_2 = \frac{7.445 - 0.01683}{20.09 - 0.01683}$$

$$= 0.37$$

Then, $s_2 = s_f + x_2(s_g - s_f)$

$$s_2 = 0.3358 + 0.37(1.732 - 0.3358)$$

$$= 0.8524 \text{ Btu/lb·°R}$$

$$\therefore \quad s_2 - s_1 = (0.8524 - 1.8449) \frac{Btu}{lb·°R}$$

$$= -0.9925 \frac{Btu}{lb·°R}$$

PROBLEM 6.13

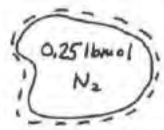
One-quarter lbmol of nitrogen gas (N_2) undergoes a process from $p_1 = 20$ lbf/in.^{2}, $T_1 = 500°R$ to $p_2 = 150$ lbf/in.^{2}. For the process $W = -500$ Btu and $Q = -125.9$ Btu. Employing the ideal gas model, determine

(a) T_2, in °R.

(b) the change in entropy, in Btu/°R.

Show the initial and final states on a T–s diagram.

SCHEMATIC & GIVEN DATA:



$W = -500$ Btu

$Q = -125.9$ Btu

ENGR. MODEL: (1) The ideal gas model applies to the N_2. (2) Ignore changes in kinetic & potential energy.

ANALYSIS:

(a) $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$

$\Rightarrow n[\bar{u}(T_2) - \bar{u}(T_1)] = Q - W$

$\Rightarrow \bar{u}(T_2) = \bar{u}(T_1) + \dfrac{Q - W}{n}$

With $\bar{u}(500°R) = 2479.3$ Btu/lbmol from Table A-23E,

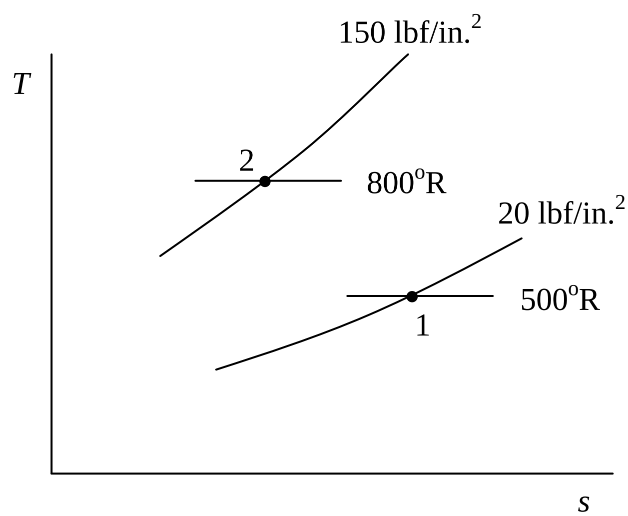
$$\bar{u}(T_2) = 2479.3 + \frac{[-125.9 - (-500)]\ \text{Btu}}{0.25\ \text{lbmol}} = 3975.7\ \frac{\text{Btu}}{\text{lbmol}}$$

From Table A-23E, $T_2 = 800°R$. ← T_2

(b) $\Delta S = n[\bar{s}°(T_2) - \bar{s}°(T_1) - \bar{R}\ln p_2/p_1]$

$$= 0.25\ \text{lbmol}\left[48.522 - 45.246 - 8.314\ln\left(\frac{150}{20}\right)\right]\frac{\text{Btu}}{\text{lbmol}\cdot°R}$$

$$= -3.369\ \frac{\text{Btu}}{°R}$$ ← ΔS



PROBLEM 6.14

Five kg of nitrogen (N_2) undergoes a process from p_1 = 5 bar, T_1 = 400 K to p_2 = 2 bar, T_2 = 500 K. Assuming ideal gas behavior, determine the change in entropy, in kJ/K, with (a) constant specific heats evaluated at 450 K, and (b) variable specific heats.
Compare results and discuss.

(a) For an ideal gas with constant specific heats; $\Delta s = c_p \ln(T_2/T_1) - R \ln(p_2/p1)$. From Table A-20 for nitrogen at 450 K: c_p = 1.049 kJ/kg·K. Thus

$$\Delta S = m\,[c_p \ln(T_2/T_1) - R \ln(p_2/p1)]$$

$$= (5\text{ kg})\,[(1.049\text{ kJ/kg·K}) \ln(500/400) - (8.314/28.01\text{ kJ/kg·K}) \ln(2/5)] = 2.530\text{ kJ/K} \longleftarrow$$

(b) Data from Table A-23 can be used to account for specific heat variation. Thus

$$\Delta S = m\left\{\left[\frac{\bar{s}^{\circ}(T_2) - \bar{s}^{\circ}(T_1)}{M}\right] - R \ln(p_2/p1)\right\}$$

$$= (5\text{ kg})\left\{\left[\frac{(206.630 - 200.071)}{28.01}\right]\frac{\text{kJ}}{\text{kg·K}} - (8.314/28.01\text{ kJ/kg·K}) \ln(2/5)\right\} = 2.531\text{ kJ/K} \longleftarrow$$

Discussion

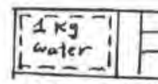
The two values are almost exactly the same, indicating that using the c_p value at the midpoint temperature and the constant specific heat relationship is extremely accurate in this case. The result could be significantly different if the temperature range were greater.

PROBLEM 6.15

One kilogram of water contained in a piston–cylinder assembly, initially at 160°C, 150 kPa, undergoes an isothermal compression process to saturated liquid. For the process, $W = -471.5$ kJ. Determine for the process,

(a) the heat transfer, in kJ.
(b) the change in entropy, in kJ/K.
Show the process on a sketch of the T–s diagram.

SCHEMATIC & GIVEN DATA:



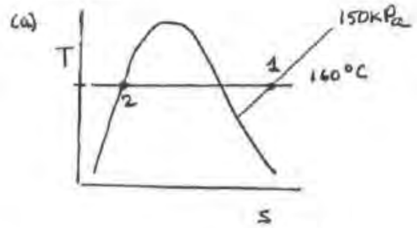
$W = -471.5$ kJ

$T = 160°C$
$P_1 = 150$ kPa
State 2: Saturated liquid at 160°C

ENGR. MODEL: Ignore changes in kinetic and potential energy.

ANALYSIS:

(a)



(b) $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$

$\Rightarrow Q = W + \Delta U$

$\Rightarrow Q = W + m(u_2 - u_1)$

Table A-4, $u_1 = 2595.2$ kJ/kg
Table A-2, $u_2 = 674.86$ kJ/kg

$\therefore Q = -471.5 + (1\text{ kg})(674.86 - 2595.2)\frac{\text{kJ}}{\text{kg}}$

$= -2391.84$ kJ $\leftarrow Q$

(c) Table A-4, $s_1 = 7.4665 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$
Table A-2, $s_2 = 1.9427 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

$\Delta S = 1\text{ kg}(1.9427 - 7.4665)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

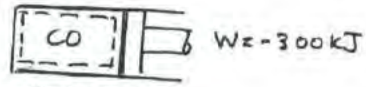
$= -5.5238 \frac{\text{kJ}}{\text{K}} \leftarrow \Delta S$

PROBLEM 6.16

One-tenth kmol of carbon monoxide (CO) in a piston-cylinder assembly undergoes a process from $p_1 = 150$ kPa, $T_1 = 300$ K to $p_2 = 500$ kPa, $T_2 = 370$ K. For the process, $W = -300$ kJ. Employing the ideal gas model, determine

(a) the heat transfer, in kJ.
(b) the change in entropy, in kJ/K.
Show the process on a sketch of the T–s diagram.

SCHEMATIC & GIVEN DATA:

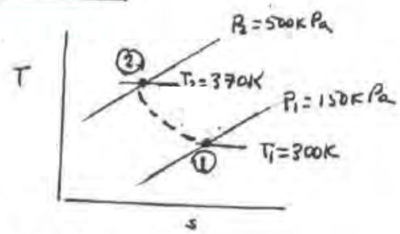


$n = 0.1$ kmol

$P_1 = 150$ kPa $P_2 = 500$ kPa
$T_1 = 300$ K $T_2 = 370$ K

ENGR. MODEL 1. The CO is modeled as an ideal gas. 2. Kinetic and potential energy effects can be ignored.

ANALYSIS:



(a) Energy balance:

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$$

$$\Rightarrow Q = \Delta U + W$$
$$= n[\bar{u}(T_2) - \bar{u}(T_1)] + W$$

With data from Table A-23,

$$Q = 0.1 \text{ kmol}\left[7689 - 6229\right]\frac{\text{kJ}}{\text{kmol}} + (-300 \text{ kJ})$$
$$= -154 \text{ kJ}$$

Q

(b) With data from Table A-23,

$$\Delta S = n\left[\bar{s}^{\circ}(T_2) - \bar{s}^{\circ}(T_1) - \bar{R}\ln\frac{P_2}{P_1}\right]$$

$$\Delta S = (0.1 \text{ kmol})\left[203.842 - 197.723 - 8.314 \ln\left(\frac{500}{150}\right)\right]\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}$$

$$= -0.3891 \frac{\text{kJ}}{\text{K}}$$

ΔS

PROBLEM 6.17

Argon in a piston–cylinder assembly is compressed from state 1, where $T_1 = 300$ K, $V_1 = 1$ m^3, to state 2, where $T_2 = 200$ K. If the change in specific entropy is $s_2 - s_1 = -0.27$ kJ/kg · K, determine the final volume, in m^3. Assume the ideal gas model with $k = 1.67$.

SCHEMATIC & GIVEN DATA:

$T_1 = 300$ K $\quad T_2 = 200$K

$V_1 = 1$m^3 $\quad V_2 = ?$

$s_2 - s_1 = -0.27$ kJ/kg·K

ENGR. MODEL: The argon is modeled as an ideal gas with $k = 1.67$.

ANALYSIS: If k is constant, then $c_v = \frac{R}{k-1}$ is also constant (Eq. 3.47b). With Eq. 6.21,

$$s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$$

where

$$c_v = \frac{R}{k-1} = \frac{(8.314/39.94)\text{ kJ/kg}\cdot\text{K}}{1.67-1} = 0.3107 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

That is

$$-0.27 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 0.3107 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \ln\left(\frac{200\text{K}}{300\text{K}}\right) + \frac{8.314}{39.94}\frac{\text{kJ}}{\text{kg}\cdot\text{K}} \ln\left(\frac{V_2}{1\text{m}^3}\right)$$

Solving,

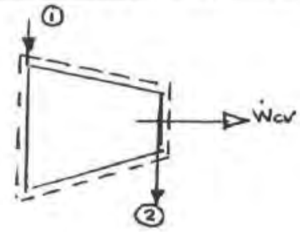
$$V_2 = 0.5\text{m}^3$$

V_2

PROBLEM 6.18

Steam enters a turbine operating at steady state at 1 MPa, 200°C and exits at 40°C with a quality of 83%. Stray heat transfer and kinetic and potential energy effects are negligible. Determine (a) the power developed by the turbine, in kJ per kg of steam flowing, (b) the change in specific entropy from inlet to exit, in kJ/K per kg of steam flowing.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. Stray heat transfer and kinetic and potential energy effects are negligible.

ANALYSIS:

(a) An energy rate balance reduces to read, $\dot{W}_{cv}/\dot{m} = h_1 - h_2$, where

$h_1 = 2827.9$ kJ/kg (Table A-4) and $h_2 = h_f + x_2 h_{fg} = 167.57 + 0.83(2406.7) = 2165.1 \frac{kJ}{kg}$ (Table A-2). Thus,

$$\frac{\dot{W}_{cv}}{\dot{m}} = (2827.9 - 2165.1)\frac{kJ}{kg} = 662.8 \frac{kJ}{kg} \quad \longleftarrow \text{(a)}$$

(b) $s_1 = 6.6940$ kJ/kg·K, $s_2 = s_f + x_2 s_{fg} = 0.5720 + 0.83(8.2570 - 0.5720) = 6.9506$ kJ/kg·K

$$\therefore \quad s_2 - s_1 = (6.9506 - 6.6940) = 0.2566 \text{ kJ/kg·K} \quad \longleftarrow \text{(b)}$$

PROBLEM 6.19

Ethylene gas (C_2H_4) enters a compressor operating at steady state at 310K, 1 bar and is compressed to 600K, 5 bar. Assuming the ideal gas model, determine the change in specific entropy of the gas from inlet to exit, in kJ/kg·K.

Assuming the ideal gas model for ethylene gas at the stated conditions, the change in specific entropy is

$$\Delta s = \int_{T_1}^{T_2} \frac{c_{p(T)}}{T} dT - R \ln (p_2/p_1)$$

Table A-21 lists the coefficients of the fourth-order polynomial c_p function for ethylene gas. Evaluating the integral

$$\int_{T_1}^{T_2} \frac{c_{p(T)}}{T} dT = R\left[\alpha \ln\left(\frac{T_2}{T_1}\right) + \beta(T_2 - T_1) + \gamma\left(\frac{T_2^2 - T_1^2}{2}\right) + \delta\left(\frac{T_2^3 - T_1^3}{3}\right) + \varepsilon\left(\frac{T_2^4 - T_1^4}{4}\right)\right] =$$

$$(8.314/28.05)\left[((1.426)\ln\left(\frac{600}{310}\right) + (11.383 \text{ x } 10^{-3})(600 - 310) + (7.989 \text{ x } 10^{-6})\left(\frac{600^2 - 310^2}{2}\right) + (-16.254 \text{ x } 10^{-9})\left(\frac{600^3 - 310^3}{3}\right) + (6.749 \text{ x } 10^{-12})\left(\frac{600^4 - 310^4}{4}\right)\right]$$

$$= 1.3311 \text{ kJ/kg·K}$$

Now

$$\Delta s = \int_{T_1}^{T_2} \frac{c_{p(T)}}{T} dT - R \ln (p_2/p_1) = 1.3311 - (8.314/28.05) \ln(5/1) = 0.8541 \text{ kJ/kg·K} \longleftarrow$$

Checking the Applicability of the Ideal Gas Model

From Table A-1: T_c = 283 K and p_c = 51.2 bar

State 1: $p_{R1} = 1/51.2 \approx 0.02$ and T_{R1} $310/283 \approx 1.1$ $\longrightarrow$ (Fig. A-1) $Z_1 \approx 1$

State 1: $p_{R2} = 5/51.2 \approx 0.1$ and $T_{R2} = 310/283 \approx 2.12$ $\longrightarrow$ (Fig. A-1) $Z_2 \approx 1$

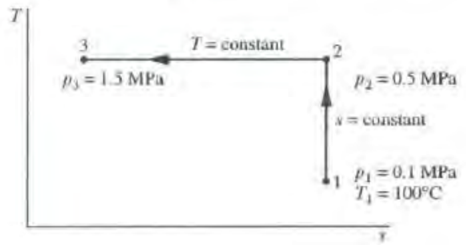
Alternative *IT* Solution

```
T1 = 310
p1 = 1
T2 = 600
p2 = 5

s1 = s_Tp("C2H4",T1,p1)
s2 = s_Tp("C2H4",T2,p2)
dels = s2 – s1
```

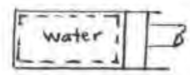
IT Result
Δs = 0.8528 kJ/kg·K

PROBLEM 6.20

One kilogram of water in a piston-cylinder assembly undergoes the two internally reversible processes in series shown in Fig. P6.20. For each process, determine, in kJ, the heat transfer and the work.



SCHEMATIC & GIVEN DATA



$m = 1\ kg$

ENGR. MODEL:

1. The water is the closed system.
2. For the system, kinetic and potential energy changes can be ignored.
3. The processes are internally reversible.

ANALYSIS:

Process 1-2. With Eq. 6.23, $Q = 0$. An energy balance reduces to give $\Delta U = \cancel{Q}^{0} - W$

$\Rightarrow W = -\Delta U = -m(u_2 - u_1)$

Table A-4, $u_1 = 2506.7\ \frac{kJ}{kg}$, $s_1 = 7.3614\ \frac{kJ}{kg \cdot K}$. Interpolating with $s_2 = s_1$ at 0.5MPa, $u_2 = 2760.95\ kJ/kg$, $T_2 = 273.56°C$.

$$W = -1kg(2760.95 - 2506.7) = -254.25\ \frac{kJ}{kg} \leftarrow$$

Process 2-3. With Eq. 6.23, $Q = m\,T(s_3 - s_2)$. From Table A-4 at 1.5MPa, $T = 273.56°C$, $s_3 = 6.8099\ kJ/kg \cdot K$, $u_3 = 2737.05\ kJ/kg$. Thus,

$$Q = (1kg)(546.71\ K)(6.8099 - 7.3614)\frac{kJ}{kg \cdot K} =$$

$$= -301.51\ kJ \leftarrow$$

An energy balance reduces to read, $\Delta U = Q - W$; or

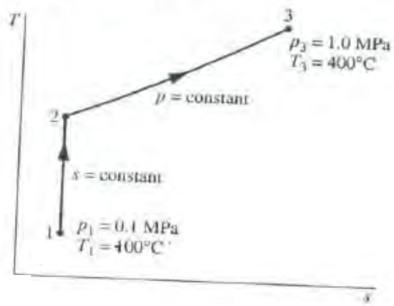
$$W = Q - m(u_3 - u_2)$$

$$= -301.51\ kJ - (1\ kg)(2737.05 - 2760.95)\frac{kJ}{kg}$$

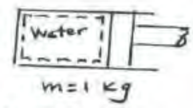
$$= -277.61\ kJ \leftarrow$$

PROBLEM 6.21

One kilogram of water in a piston–cylinder assembly undergoes the two internally reversible processes in series shown in Fig. P6.21. For each process, determine, in kJ, the heat transfer and the work.



SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The water is the closed system.
2. For the system, kinetic and potential energy changes can be ignored.
3. The processes are internally reversible.

ANALYSIS:

Process 1-2. With Eq. 6.23, Q = 0. An energy balance reduces to give $\Delta U = \cancel{Q}^{0} - W$

$$\Rightarrow \quad W = -\Delta U = -m(u_2 - u_1)$$

From Table A-4, $s_1 = 7.3614$ kJ/kg·K, $u_1 = 2506.7$ kJ/kg. Interpolating at 1.0 MPa with $s_2 = s_1$, $u_2 = 2904.97$ kJ/kg, $v_2 = 0.2912$ m³/kg

$$\therefore \quad W = -(1\,\text{kg})(2904.97 - 2506.7)\frac{\text{kJ}}{\text{kg}} = -398.27\ \text{kJ}$$

Process 2-3. $W = \int_2^3 p\,dV = mp(v_3 - v_2)$. From Table A-4, $v_3 = 0.3066$ m³/kg.

$$\therefore \quad W = (1\,\text{kg})\left(10^6 \frac{\text{N}}{\text{m}^2}\right)(0.3066 - 0.2912)\frac{\text{m}^3}{\text{kg}}\left|\frac{1\,\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right| = 15.4\ \text{kJ}$$

An energy balance reduces to read, $\Delta U = Q - W \Rightarrow Q = \Delta U + W$

Thus, with $u_3 = 2957.3$ kJ/kg

$$Q = m(u_3 - u_2) + W$$
$$= (1\,\text{kg})(2957.3 - 2904.97) + 15.4\ \text{kJ}$$
$$= 67.73\ \text{kJ}$$

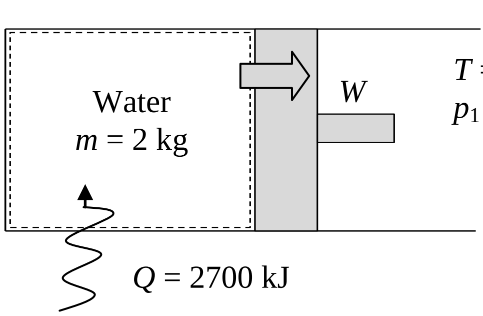
PROBLEM 6.22

A system consisting of 2 kg of water initially at 160°C, 10 bar, undergoes an internally reversible, isothermal expansion during which there is energy transfer by heat *into* the system of 2700 kJ. Determine the final pressure, in bar, and the work, in kJ.

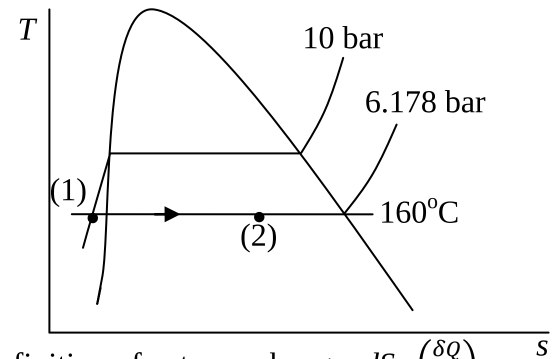
KNOWN: Two kg of water undergoes an isothermal process from a specified initial state. The energy transfer by heat is given.

FIND: Determine the final pressure and the work.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The water is the closed system. (2) The expansion takes place isothermally. (3) The process is internally reversible. (4) Kinetic and potential energy effects can be neglected.



ANALYSIS: Using assumptions (2) and (3), and the definition of entropy change: $dS = \left(\frac{\delta Q}{T}\right)_{\substack{\text{int}\\ \text{rev}}}$

$$Q_{\substack{\text{int}\\ \text{rev}}} = T(S_2 - S_1) = mT(s_2 - s_1)$$

or, on rearrangement and with data from Table A-2: $s_1 \approx s_{f@160\,C} = 1.9427$ kJ/kg·K

$$s_2 = s_1 + Q_{\substack{\text{int}\\ \text{rev}}}/mT = 1.9427 \text{ kJ/kg·K} + (2700 \text{ kJ})/[(2 \text{ kg})(433 \text{ K})] = 5.0605 \text{ kJ/kg·K}$$

Since $s_f < s_2 < s_g$ at 160°C, state 2 is in the two-phase liquid-vapor region, and the quality is

$$x_2 = \frac{s_2 - s_{f2}}{s_{g2} - s_{f2}} = \frac{5.0605 - 1.9427}{6.7502 - 1.9427} = 0.649$$

and the pressure is the saturation pressure at 160°C: $p_2 = 6.178$ bar ←

With modeling assumption (4), the energy balance reduces to give

$$W = Q + m(u_1 - u_2)$$

With data from Table A-2; $u_1 = u_f(160^\circ\text{C}) = 674.86$ kJ/kg and

$$u_2 = u_{f2} + x_2(u_{g2} - u_{f2}) = 674.86 + (0.649)(2568.4 - 674.86) = 1903.77 \text{ kJ/kg}$$

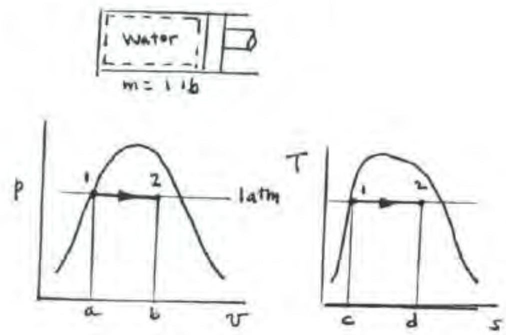
Finally

$$W = (2700 \text{ kJ}) + (2 \text{ kg})(674.86 - 1903.77)\text{kJ/kg} = 242.2 \text{ kJ (out)}$$ ←

<u>PROBLEM 6.23</u>

One pound mass of water initially a saturated liquid at 1 atm undergoes a constant-pressure, internally reversible expansion to $x = 90\%$. Determine the work and heat transfer, each in Btu. Sketch the process on p–v and T–s coordinates. Associate the work and heat transfer with areas on these diagrams.

SCHEMATIC & GIVEN DATA:



Area 1-2-b-a-1 = W/m

Area 1-2-d-c-1 = Q/m

ENGR. MODEL:

1. The water is the closed system.
2. The process is internally reversible.

ANALYSIS:

$$W = \int_1^2 p\,dV = mp(v_2 - v_1) = mp\,x_2(v_g - v_f)$$

$v_2 = v_f + x_2(v_g - v_f)$, $v_1 = v_f$

$$= (1\,\text{lb})\left(14.7 \times 144\,\frac{\text{lbf}}{\text{ft}^2}\right)(0.90)(26.80 - 0.01672)\frac{\text{ft}^3}{\text{lb}}\left|\frac{1\,\text{Btu}}{778\,\text{ft}\cdot\text{lbf}}\right| = 65.6\ \text{Btu}$$

Using Eq. 6.23,

$$Q = \int_1^2 T\,dS = mT(s_2 - s_1) = mT\,x_2(s_g - s_f)$$

$s_2 = s_f + x_2(s_g - s_f)$, $s_1 = s_f$

(1) $$= (1\,\text{lb})(671.66\,°\text{R})(0.9)\left(1.4446\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right) = 873.3\ \text{Btu}$$

$671.66 = 211.99 + 459.67$

---

1. Alternatively, an energy balance reduces to read $\Delta U = Q - W$ or

$$m[u_2 - u_1] = Q - W \Rightarrow m x_2(u_g - u_f) = Q - W$$

$u_2 = u_f + x_2(u_g - u_f)$, $u_1 = u_f$

$$\Rightarrow Q - W = (1\,\text{lb})(0.9)(1077.6 - 180.10)$$

$$= 807.8\ \text{Btu}$$

Thus, knowing Q or W allows the other quantity to be evaluated from an energy balance.

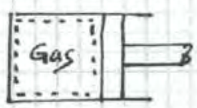
PROBLEM 6.24

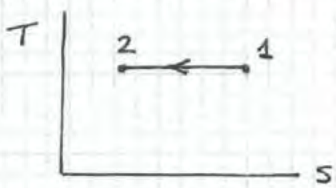
A gas within a piston–cylinder assembly undergoes an isothermal process at 400 K during which the change in entropy is −0.3 kJ/K. Assuming the ideal gas model for the gas and negligible kinetic and potential energy effects, evaluate the work, in kJ.

KNOWN: The change in entropy is known for an isothermal process of a gas within a piston-cylinder assembly.

FIND: Evaluate the work for the process of the gas, in kJ.

SCHEMATIC & GIVEN DATA:





$$(S_2 - S_1) = -0.3 \ \frac{kJ}{K}$$

ENGINEERING MODEL:

1. The gas is the closed system, and the ideal gas model applies.
2. For the system, kinetic and potential energy effects can be ignored.
3. The process of the gas is isothermal and thus internally reversible.

ANALYSIS: An energy balance reduces as follows:

$$\underbrace{\Delta U}_{=0} + \underbrace{\cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0}}_{\text{See model \#2}} = Q - W$$

= 0 since T = constant and the ideal gas model applies.

$$\Rightarrow \quad W = Q$$

With Eq. 6.23 we get $Q = \int_1^2 T\,dS$ (Model #3)

$$\therefore W = \int_1^2 T\,dS$$

$$= T\,\Delta S$$

$$= 400\,K\left(-0.3\ \frac{kJ}{kg}\right)$$

$$= -120\,kJ$$

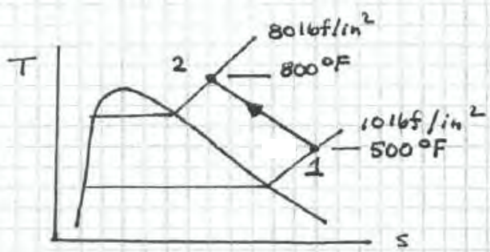
PROBLEM 6.25

Water within a piston–cylinder assembly, initially at 10 lbf/in.^{2}, 500°F, undergoes an internally reversible process to 80 lbf/in.^{2}, 800°F, during which the temperature varies linearly with specific entropy. For the water, determine the work and heat transfer, each in Btu/lb. Neglect kinetic and potential energy effects.

KNOWN: Water in a piston-cylinder assembly undergoes an internally reversible process during which temperature varies linearly with specific entropy. State data are provided.

FIND: For the water, determine Q and W, each in Btu/lb

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The water is the closed system.
2. The water undergoes an internally reversible process during which T varies linearly with s, as shown in the T-s diagram.
3. For the water, kinetic and potential energy effects can be neglected.

ANALYSIS: With Eq. 6.23

T varies linearly with s

$$Q_{12} = \int_1^2 T\,dS \Rightarrow \frac{Q_{12}}{m} = \int_1^2 T\,ds = T_{ave}(s_2 - s_1) \qquad (1)$$

With data from Table A-4E,

$$s_1 = 1.9690 \text{ Btu/lb}\cdot°R, \quad u_1 = 1182.2 \text{ Btu/lb}$$

$$s_2 = 1.8700 \text{ Btu/lb}\cdot°R, \quad u_2 = 1292.4 \text{ Btu/lb}$$

Eq. (1) gives

$$\frac{Q_{12}}{m} = \left[\frac{(959.67 + 1259.67)°R}{2}\right](1.8700 - 1.9690)\frac{\text{Btu}}{\text{lb}\cdot°R}$$

$$= -109.86 \text{ Btu/lb} \longleftarrow$$

An energy balance reduces as follows: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$

$$\Rightarrow \frac{W_{12}}{m} = \frac{Q_{12}}{m} - (u_2 - u_1)$$

$$= -109.86 - (1292.4 - 1182.2)$$

$$= -220.06 \text{ Btu/lb} \longleftarrow$$

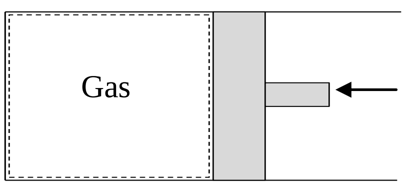
PROBLEM 6.26

A gas initially at 2.8 bar and 60°C is compressed to a final pressure of 14 bar in an isothermal internally reversible process. Determine the work and heat transfer, each in kJ per kg of gas, if the gas is (a) Refrigerant 134a, (b) air as an ideal gas. Sketch the process on *p*-*v* and *T*-*s* coordinates.

KNOWN: A gas is compressed isothermally with no internal irreversibilities from a specified initial state to a specified final pressure.

FIND: Determine the work and the heat transfer, per unit mass of gas, for (a) R-134a), (b) air as an ideal gas. Sketch the process on *p*-*v* and *T*-*s* coordinates.

SCHEMATIC AND GIVEN DATA:



$T = 60^{\circ}\mathrm{C}$
$p_1 = 2.8$ bar

$p_2 = 14$ bar

ENGINEERING MODEL: (1) The gas is a closed system. (2) The compression takes place isothermally and with no internal irreversibilities. (3) Kinetic and potential energy effects are negligible. (4) For part (b), the air behaves as an ideal gas.

ANALYSIS: Using modeling assumption (2), the definition of entropy change: $dS = \left(\frac{\delta Q}{T}\right)_{\substack{\text{int}\\\text{rev}}}$ can be used to obtain

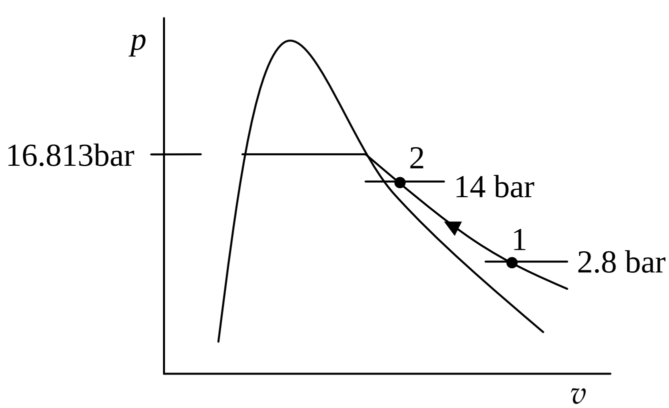
$$Q = \int_1^2 T dS = mT(s_2 - s_1) \longrightarrow Q/m = T(s_2 - s_1) \qquad (*)$$

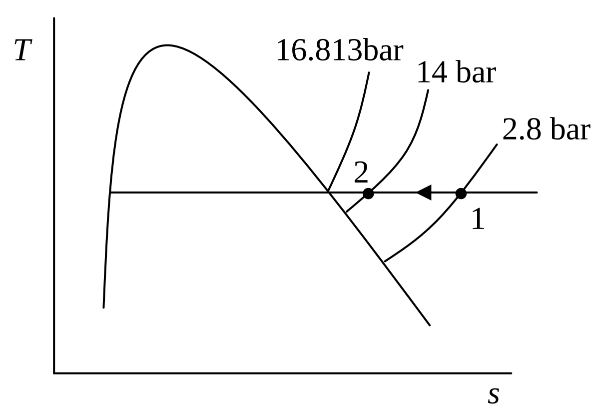
The energy balance reduces to give: $\Delta U = Q - W \longrightarrow W/m = Q/m + (u_1 - u_2) \qquad (**)$

(a) R-134a From Table A-12: $u_1 = 277.23$ kJ/kg, $s_1 = 1.1079$ kJ/kg·K; $u_2 = 262.17$ kJ/kg, $s_1 = 0.9297$kJ/kg·K
Thus

$$Q/m = (60 + 273)\mathrm{K}(0.9297 - 1.1079)\ \mathrm{kJ/kg\cdot K} = -59.34\ \mathrm{kJ/kg\ (out)}$$

$$W/m = (-59.34\ \mathrm{kJ/kg}) + (277.23 - 262.17)\mathrm{kJ/kg} = -\ 44.28\ \mathrm{kJ/kg\ (in)}$$





PROBLEM 6.26 (CONTINUED)

(b) Air Since $T_1 = T_2$, the specific entropy change of the air reduces to

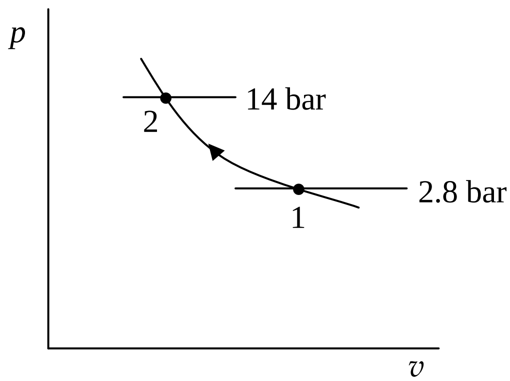
$$s_2 - s_1 = -R \ln (p_2/p_1) = -\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\ln\left(\frac{14\ \text{bar}}{2.8\ \text{bar}}\right) = -0.46139\ \text{kJ/kg}\cdot\text{K}$$

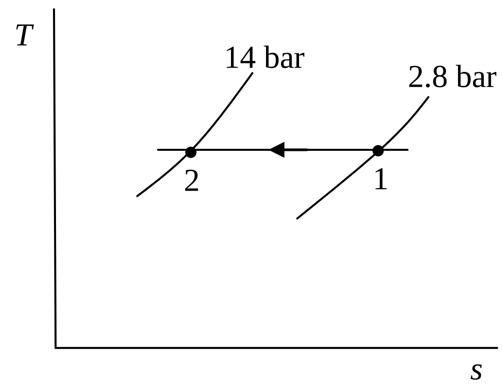
and

$$Q/m = T(s_2 - s_1) = (333\ \text{K})(-0.46139\ \text{kJ/kg}\cdot\text{K}) = -153.81\ \text{kJ/kg (out)}$$

Since $T_1 = T_2$, the change in specific internal energy is zero, and

$$W/m = Q/m + \cancel{(u_1 - u_2)} = -153.81\ \text{kJ/kg (in)}$$





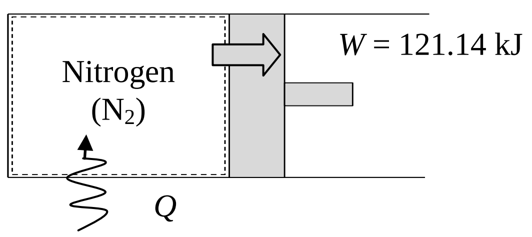
PROBLEM 6.27

Nitrogen (N_2) undergoes an internally reversible process from 6 bar, 247°C during which $pv^{1.20}$ = *constant*. The initial volume is 0.1 m³ and the work for the process is 121.14 kJ. Assuming ideal gas behavior, and neglecting kinetic and potential energy effects, determine heat transfer, in kJ, and the entropy change, in kJ/K. Show the process on a *T-s* diagram.

KNOWN: Nitrogen at specified initial volume, pressure, and temperature undergoes a polytropic process with known work.

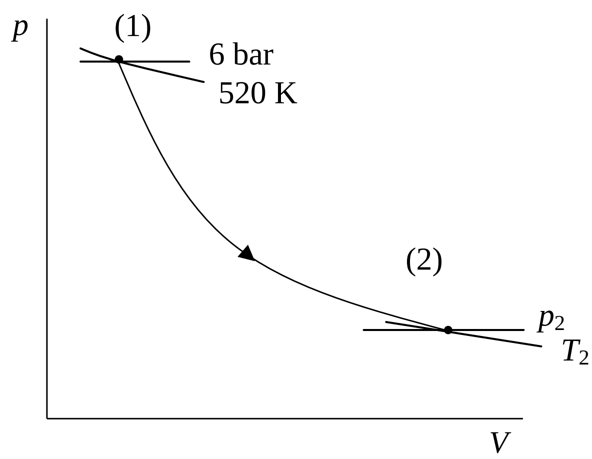
FIND: Determine heat transfer and the entropy change. Show the process on a *T-s* diagram.

SCHEMATIC AND GIVEN DATA:



p_1 = 6 bar, T_1 = 247°C = 520 K, V_1 = 0.1 m³

$pv^{1.20}$ = *constant*



ENGINEERING MODEL: (1)The nitrogen is a closed system. (2) Nitrogen can be modeled as an ideal gas. (2)The nitrogen undergoes an internally reversible polytropic process in which $pv^{1.20}$ =*constant*. (3) The system is at an equilibrium state initially and finally. (4) There is no change in kinetic or potential energy between the initial and final states.

ANALYSIS: For the polytropic process, the work can be expressed using the ideal gas equation of state as

$$W = \int_{V_1}^{V_2} p\,dV = \frac{(p_2V_2 - p_1V_1)}{(1-1.20)} = \frac{mR(T_2-T_1)}{1-1.20} \quad \longrightarrow \quad T_2 = \frac{W(1-1.20)}{mR} + T_1$$

The mass is

$$m = \frac{p_1V_1}{RT_1} = \frac{(6\text{ bar})(0.1\text{ m}^3)}{\left(\frac{8.314}{28.01}\frac{\text{kJ}}{\text{kg·K}}\right)(520\text{ K})} = 0.3887\text{ kg}$$

and

$$T_2 = \frac{(121.14\text{ kJ})(1-1.20)}{(0.3887\text{ kg})\left(\frac{8.314}{28.01}\frac{\text{kJ}}{\text{kg·K}}\right)} + (520\text{ K}) = 310\text{ K}$$

Now, since the final temperature is known, the final pressure can be determined from Eq. 3.56, as follows

$$p_2 = p_1\left(\frac{T_2}{T_1}\right)^{\frac{1.20}{1.20-1}} = (6\text{ bar})\left(\frac{310}{520}\right)^{\frac{1.20}{1.20-1}} = 0.2693\text{ bar}$$

The energy balance reduces to: $\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = Q - W$. With $\Delta U = m(u_2 - u_1)$

$$Q = m(u_2 - u_1) + W$$

Values from Table A-23 are expressed on a molar basis: $\bar{u}_1$ = 10,848 kJ/kmol and $\bar{u}_2$ = 6,437 kJ/kmol. Thus

$$Q = (0.3887\text{ kg})\left(\frac{6{,}437\frac{\text{kJ}}{\text{kmol}} - 10{,}848\text{kJ/kmol}}{28.01\text{ kg/kmol}}\right) + (121.14\text{ kJ}) = 59.93\text{ kJ} \longleftarrow$$

PROBLEM 6.27 (CONTINUED)

The change in entropy is

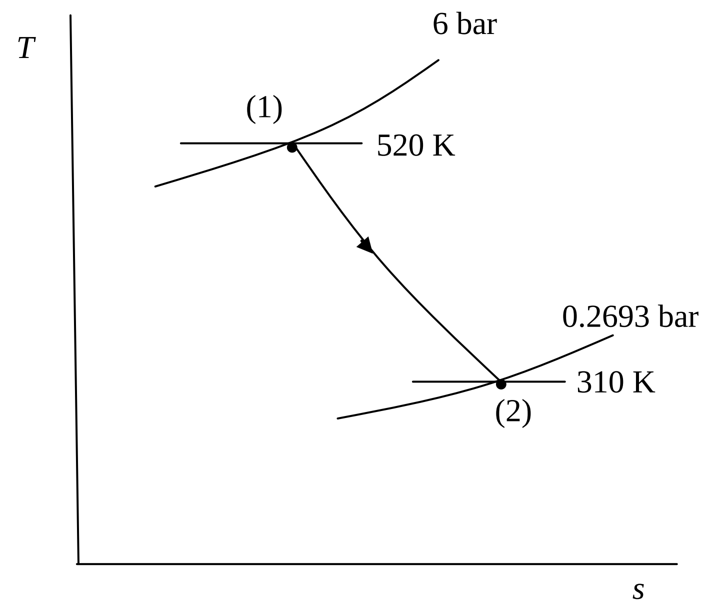
$$S(T_2,p_2) - S(T_1,p_1) = m\left[\frac{\bar{s}^o(T_2)-\bar{s}^o(T_1)}{M} - R\ln\left(\frac{p_2}{p_1}\right)\right]$$

From Table A-23: $\bar{s}^o(T_1) = 207.792$ kJ/kmol·K and $\bar{s}^o(T_2) = 192.638$ kJ/kmol·K. Thus

$$S(T_2,p_2) - S(T_1,p_1) = (0.3887\text{ kg})\left[\left(\frac{192.638-207.792}{28.01}\right)\frac{\text{kJ}}{\text{kg·K}} - \left(\frac{8.314}{28.10}\frac{\text{kJ}}{\text{kg·K}}\right)\ln\left(\frac{0.2693\text{ bar}}{6\text{ bar}}\right)\right]$$

$$= 0.1478\text{ kJ/K}$$

The T-s diagram is



The positive entropy change indicates the entropy of the system increased during the process.

PROBLEM 6.28

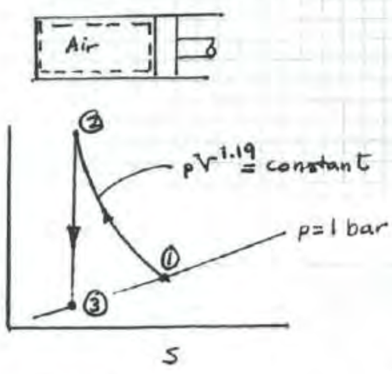
Air in a piston–cylinder assembly and modeled as an ideal gas undergoes two internally reversible processes in series from state 1, where $T_1 = 290$ K, $p_1 = 1$ bar.

Process 1-2: Compression to $p_2 = 5$ bar during which $pV^{1.19} = constant$

Process 2-3: Isentropic expansion to $p_3 = 1$ bar.

(a) Sketch the two processes in series on T–s coordinates.
(b) Determine the temperature at state 2, in K.
(c) Determine the net work, in kJ/kg.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The air is the closed system.
2. The processes are internally reversible.
3. Kinetic and potential energy play no role.
4. The air is modeled as an ideal gas.

ANALYSIS:

(b) With Eq. 3.56, $T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{(n-1)/n} = (290\text{K})(5)^{(1.19-1)/1.19} = 375\text{K}$ ← T_2

$u_2 = 268.08 \frac{\text{kJ}}{\text{kg}}$ (Table A-22).

(c) Following Example 2.1,

$$\frac{W_{12}}{m} = \frac{p_2 v_2 - p_1 v_1}{1-n} \overset{\text{Ideal gas model: } pv = RT}{=} \frac{R(T_2 - T_1)}{1-n}$$

$$= \frac{(8.314/28.97)}{1-1.19}\left(\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(375-290)\text{K} = -128.39 \frac{\text{kJ}}{\text{kg}}$$

With Eq. 6.23, $Q_{23} = 0$. Thus, an energy balance reduces to read

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - W \Rightarrow W = -\Delta U = -m(u_3 - u_2).$$

To find u_3, apply Eq. 6.20a: $\underbrace{s_3 - s_2}_{=0} = s^\circ(T_3) - s^\circ(T_2) - R\ln(p_3/p_2)$

$$\Rightarrow s^\circ(T_3) = s^\circ(T_2) + R\ln(p_3/p_2) = \underset{(\text{Table A-22})}{1.92657} + \frac{8.314}{28.97}\ln\left(\frac{1}{5}\right) = 1.46468 \text{ kJ/kg}\cdot\text{K}$$

$$\Rightarrow u_3 = 168.86 \text{ kJ/kg}$$

$$\therefore \frac{W_{23}}{m} = -(168.86 - 268.08) = 99.22 \frac{\text{kJ}}{\text{kg}}$$

$$\frac{W_{net}}{m} = \frac{W_{12}}{m} + \frac{W_{23}}{m} = -128.39 \frac{\text{kJ}}{\text{kg}} + 99.22 \frac{\text{kJ}}{\text{kg}} = -29.17 \frac{\text{kJ}}{\text{kg}}$$ ← W_{net}/m

<u>PROBLEM 6.29</u>

One lb of oxygen, O_2, in a piston-cylinder assembly undergoes a cycle consisting of the following processes:

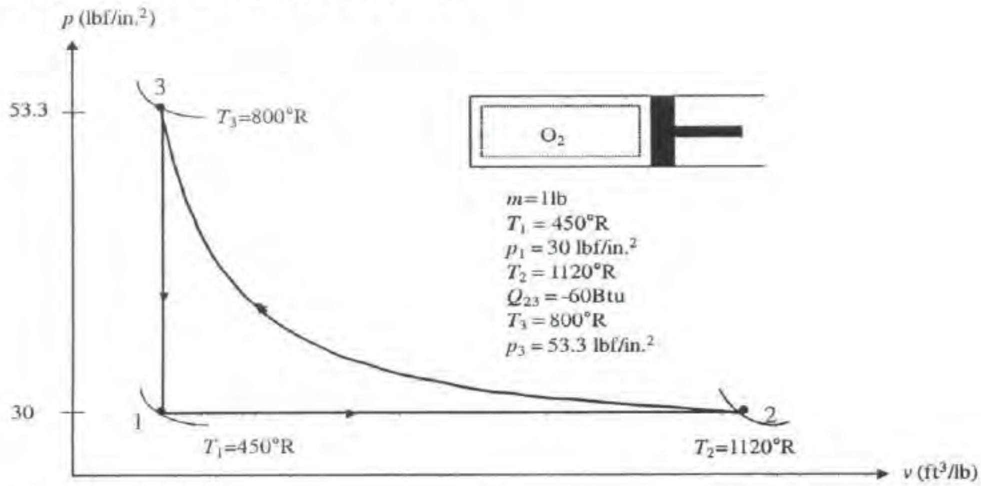
Process 1-2: Constant-pressure expansion from $T_1 = 450°R$, $p_1 = 30$ lbf/in.^{2} to T_2=1120°R.
Process 2-3: Compression to $T_3 = 800°R$ and $p_3 = 53.3$ lbf/in.^{2} with $Q_{23} = -60$ Btu.
Process 3-1: Constant-volume cooling to state 1.

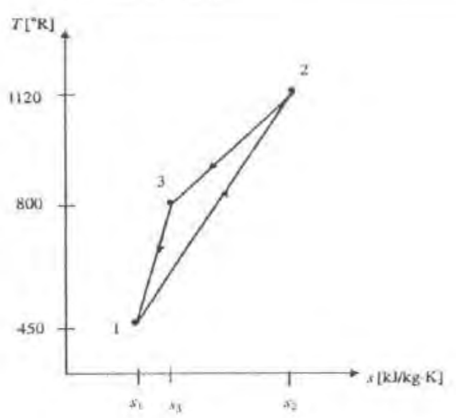
Employing the ideal gas model with c_p evaluated at T_1, determine the change in specific entropy, in Btu/lb·°R, for each process. Sketch the cycle on a p-v and T-s coordinates.

KNOWN: Oxygen in a piston-cylinder assembly undergoes a cycle consisting of three processes.

FIND: Sketch the cycle on a p-v and T-s coordinates and determine change in specific entropy, in kJ/kg·K, for each process.

SCHEMATIC AND GIVEN DATA:





PROBLEM 6.29 (CONTINUED)

ENGINEERING MODEL:

(1) The O_2 is a closed system as shown in the accompanying figure.
(2) The oxygen behaves as an ideal gas with c_p=0.233 Btu/lb·°R.

ANALYSIS:

Process 1 to 2: constant pressure expansion from T_1 = 450°R, p_1 = 30 lbf/in.^{2} to T_2=1120°R. Use Eq. 6.22 to determine change is specific entropy, simplify using $p_1 = p_2$.

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right) = c_p \ln\left(\frac{T_2}{T_1}\right) = 0.233\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\ln\left(\frac{1120}{450}\right) = 0.2125\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$

Process 2 to 3: compression from T_2=1120°R, p_2 = 30 lbf/in.^{2} to T_3 = 800°R, p_3 = 53.3 lbf/in^2. Use Eq. 6.22 to determine change is specific entropy.

$$s_3 - s_2 = c_p \ln\left(\frac{T_3}{T_2}\right) - R\ln\left(\frac{p_3}{p_2}\right) = 0.233\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\ln\left(\frac{800}{1120}\right) - \left(\frac{1.986}{32}\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln\left(\frac{53.3}{30}\right)$$

$$s_3 - s_2 = -0.1141\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

Process 3 to 1: constant volume cooling to state 1. Use Eq. 6.21 to determine change is specific entropy, simplify using $v_3 = v_1$.

$$s_1 - s_3 = c_v \ln\left(\frac{T_1}{T_3}\right) + R\ln\left(\frac{v_1}{v_3}\right) = c_v \ln\left(\frac{T_1}{T_3}\right)$$

Using Eq. 3.44,

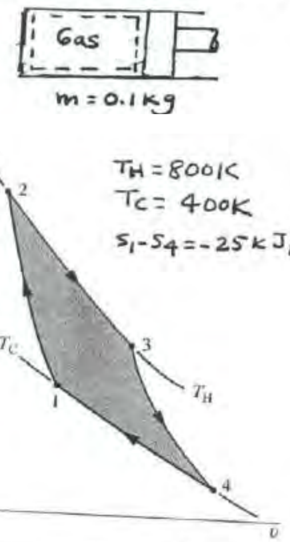
$$c_v = c_p - R = 0.233\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} - \left(\frac{1.986}{32}\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right) = 0.171\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$

$$s_1 - s_3 = \left(0.171\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln\left(\frac{450}{800}\right) = -0.0984\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$

PROBLEM 6.30

One-tenth kilogram of a gas in a piston–cylinder assembly undergoes a Carnot power cycle for which the isothermal expansion occurs at 800 K. The change in specific entropy of the gas during the isothermal compression, which occurs at 400 K, is −25 kJ/kg · K. Determine (a) the net work developed per cycle, in kJ, and (b) the thermal efficiency.

SCHEMATIC & GIVEN DATA:



$T_H = 800K$

$T_C = 400K$

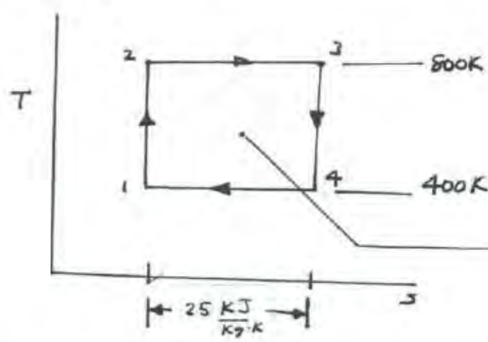
$s_1 - s_4 = -25\ kJ/kg \cdot K$

ENGR. MODEL:

1. The gas is the closed system.
2. The gas undergoes a Carnot power cycle.

ANALYSIS:

On T-s coordinates, the Carnot cycle is



For any cycle, $W_{cycle} = Q_{cycle}$. For a power cycle, the net work done equals the net heat transfer of energy supplied.

The enclosed area is the net work developed per unit of mass. That is

$$W_{net}/m = \text{Area } 1\text{-}2\text{-}3\text{-}4\text{-}1$$
$$= (800K - 400K)\left(25\ \frac{kJ}{kg \cdot K}\right) = 10{,}000\ \frac{kJ}{kg}/\text{cycle}$$

With $m = 0.1\ kg$,

$$W_{net} = 1000\ kJ/\text{cycle} \qquad \longleftarrow W_{net}$$

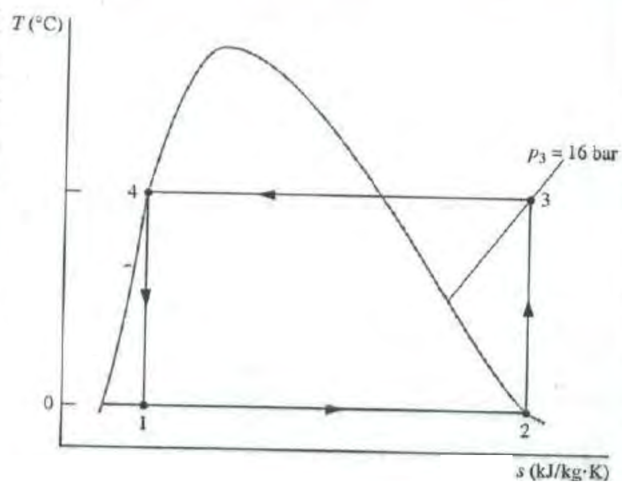
With Eq. 5.9,

① $$\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{400}{800} = 0.5 \quad (50\%) \qquad \longleftarrow \eta$$

1. Alternatively, η can be found in terms of areas on the T-s diagram. See Sec. 6.6.2 for discussion.

PROBLEM 6.31

Figure P6.30 provides the T–s diagram of a Carnot refrigeration cycle for which the substance is Refrigerant 134a. Determine the coefficient of performance.



ANALYSIS: See Sec. 6.6.2 for discussion of Carnot refrigeration cycles. From Eq. 5.10,

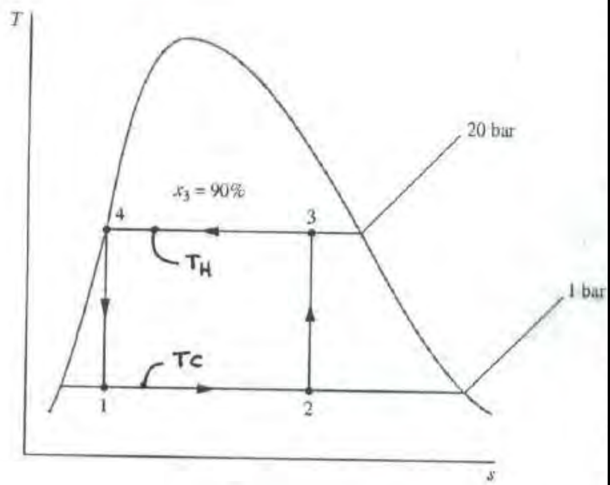
$$\beta = \frac{T_C}{T_H - T_C}.$$

$T_C = 273.15\,K$ (0°C). Also, $T_H = T_3$. To determine T_3, apply $s_3 = s_2$ to fix the state at 3: $s_2 = 0.9190$ kJ/kg·K (Table A-10). Then, with $P_3 = 16$ bar, Table A-12 gives $T_3 = 63.12\,°C$, or $T_3 = 336.27\,K$. Finally

$$\beta = \frac{273.15}{(336.27 - 273.15)} = 4.33$$

PROBLEM 6.32

Figure P6.31 provides the T–s diagram of a Carnot heat pump cycle for which the substance is ammonia. Determine the net work input required, in kJ, for 50 cycles of operation and 0.1 kg of substance.



From Table A-14,

$T_C = T_{sat}(1\text{ bar}) = -33.60°C \; (239.55K)$

$T_H = T_{sat}(20\text{bar}) = 49.37°C \; (322.52K)$

$\Rightarrow T_H - T_C = 82.97\text{ K}$

KNOWN: The T-s diagram of a Carnot heat pump cycle is given.

FIND: Determine the net work input for 50 cycles.

ANALYSIS: The discussion of Fig. 6.5 interprets heat transfer and net work as areas on the T-s diagram. For a Carnot heat pump the enclosed area 1-2-3-4-1 is the net work per cycle. That is,

$$W_{cycle} = m[s_3 - s_4][T_H - T_C] \quad (1)$$

$$s_3 = s_f + x_3(s_g - s_f)$$
$$s_4 = s_f$$
$$\therefore \; s_3 - s_4 = x_3(s_g - s_f)$$
$$= 0.90(4.7670 - 1.5012)\ \frac{kJ}{kg \cdot K}$$
$$= 2.9392\ kJ/kg \cdot K$$

Eq. (1) gives

$$W_{cycle} = (0.1kg)(2.9392\ \frac{kJ}{kg.K})(82.97K)$$
$$= 24.387\text{ kJ per cycle}$$

For 50 cycles,

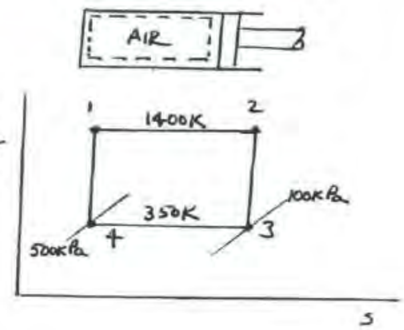
$$W_{cycle} = 1219.3\text{ kJ}$$

PROBLEM 6.33

Air in a piston–cylinder assembly undergoes a Carnot power cycle. The isothermal expansion and compression processes occur at 1400 K and 350 K, respectively. The pressures at the beginning and end of the isothermal compression are 100 kPa and 500 kPa, respectively. Assuming the ideal gas model with $c_p = 1.005$ kJ/kg · K, determine

(a) the pressures at the beginning and end of the isothermal expansion, each in kPa.
(b) the heat transfer and work, in kJ/kg, for each process.
(c) the thermal efficiency.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The air is the closed system.
2. The air undergoes a Carnot power cycle.
3. Kinetic and potential energy play no role.
4. The air is modeled as an ideal gas with $c_p = 1.005$ kJ/kg·K

ANALYSIS: (a) Use the relations $s_2 - s_3 = 0$ and $s_1 - s_4 = 0$ together with Eq. 6.22. Thus

$$s_2 - s_3 = 0 = c_p \ln\frac{T_2}{T_3} - R\ln\frac{P_2}{P_3} \Rightarrow \ln\frac{P_2}{P_3} = \frac{c_p}{R}\ln\frac{T_2}{T_3}$$

$$= \left(\frac{1.005}{8.314/28.97}\right)\ln 4 \Rightarrow P_2 = 12830 \text{ kPa} \longleftarrow P_2$$

$$s_1 - s_4 = 0 = c_p \ln\frac{T_1}{T_4} - R\ln\frac{P_1}{P_4} \Rightarrow \ln\frac{P_1}{P_4} = \frac{c_p}{R}\ln\frac{T_1}{T_4} \Rightarrow P_1 = 64{,}150 \text{ kPa} \longleftarrow P_1$$

(b) Process 1-2:

$$\frac{Q_{12}}{m} = \int_1^2 T\,ds = T(s_2 - s_1) \Rightarrow \frac{Q_{12}}{m} = T\left[\cancel{c_p \ln\frac{T_2}{T_1}}^{\,0} - R\ln\frac{P_2}{P_1}\right] = -1400\text{K}\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\ln(0.2)$$

$$\Rightarrow \frac{Q_{12}}{m} = 646.64 \frac{\text{kJ}}{\text{kg}} \longleftarrow \text{process 1-2}$$

An energy balance reads, $\underset{(T=C)}{\cancel{\Delta U}^{\,0}} = Q_{12} - W_{12} \Rightarrow$ $\quad \frac{W_{12}}{m} = 646.64 \frac{\text{kJ}}{\text{kg}}$

Process 2-3: $Q_{23} = \int_2^3 T\,ds = 0$. An energy balance reduces to $\Delta U = \cancel{Q_{23}}^{\,0} - W_{23}$

$$\Rightarrow W_{23} = -m[u_3 - u_2] = -m c_v (T_3 - T_2) \Rightarrow \frac{W_{23}}{m} = -0.718\frac{\text{kJ}}{\text{kg}\cdot\text{K}}(350 - 1400)\text{K} \longleftarrow \text{Process 2-3}$$

Eq. 3.44: $c_v = c_p - R = 0.718 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

$$= 753.9 \frac{\text{kJ}}{\text{kg}}$$

Process 3-4: $\frac{Q_{34}}{m} = T(s_4 - s_3) = T\left[\cancel{c_p \ln\frac{T_4}{T_3}}^{\,0} - R\ln\frac{P_4}{P_3}\right]$

$$= -(350\text{K})\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\ln\left(\frac{500}{100}\right) = -161.66 \frac{\text{kJ}}{\text{kg}} \longleftarrow \text{Process 3-4}$$

Energy balance: $\underset{(T=C)}{\cancel{\Delta U}^{\,0}} = Q_{34} - W_{34} \Rightarrow$ $\quad \frac{W_{34}}{m} = -161.66 \frac{\text{kJ}}{\text{kg}}$

PROBLEM 6.33 (CONTINUED)

Process 4-1: $Q_{41} = \int_4^1 T\,dS = 0$. Energy balance: $\Delta U = \cancel{Q_{41}}^{0} - W_{41}$

(1) $\Rightarrow W_{41} = -m(u_1 - u_4) \Rightarrow \frac{W_{41}}{m} = -c_v(T_1 - T_4)$ ← Process 4-1

$$= -0.718 \frac{kJ}{kg \cdot K}(1400 - 350) = -753.9 \frac{kJ}{kg}$$

(c) $\eta = \frac{W_{net}/m}{Q_{in}/m} = \frac{646.64 + 753.9 - 161.66 - 753.9}{646.64} = 0.75 \quad (75\%)$ ← η

Alternatively, apply Eq. 5.9

$$\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{350}{1400} = 0.75 \quad (75\%)$$

---

1. For any cycle $W_{net} = Q_{net}$. Thus, to check the current calculations,

$$\frac{W_{net}}{m} = 646.64 + 753.9 - 161.66 - 753.9 = 484.98 \frac{kJ}{kg}$$

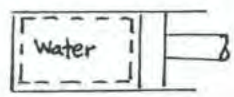
$$\frac{Q_{net}}{m} = 646.64 + 0 - 161.66 + 0 = 484.98 \frac{kJ}{kg}$$

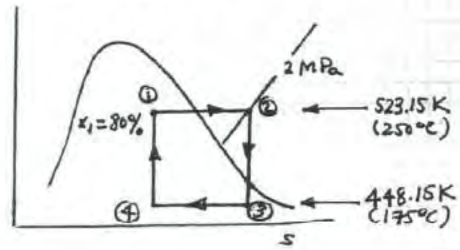
PROBLEM 6.34

Water in a piston–cylinder assembly undergoes a Carnot power cycle. At the beginning of the isothermal expansion, the temperature is 250°C and the quality is 80%. The isothermal expansion continues until the pressure is 2 MPa. The adiabatic expansion then occurs to a final temperature of 175°C.

(a) Sketch the cycle on T–s coordinates.
(b) Determine the heat transfer and work, in kJ/kg, for each process.
(c) Evaluate the thermal efficiency.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL

1. The water is the closed system.
2. The water undergoes a Carnot cycle.
3. Kinetic and potential energy play no role.

ANALYSIS:

(b) Process 1-2

$$\frac{Q_{12}}{m} = \int_1^2 T\,ds = T(s_2 - s_1) = 523.15\,K(6.5421 - 5.417)\frac{kJ}{kg\cdot K} = 588.6\,kJ/kg \quad \leftarrow Q_{12}/m$$

Table A-2 at 250°C:

$$s_1 = s_f + x_1(s_g - s_f) = 2.7927 + 0.8(6.0730 - 2.7927) = 5.417\,kJ/kg\cdot K$$

$$u_1 = u_f + x_1(u_g - u_f) = 1080.4 + 0.8(2602.4 - 1080.4) = 2298\,kJ/kg$$

Table A-4 at 2 MPa, 250°C: $s_2 = 6.5421\,kJ/kg\cdot K$, $u_2 = 2678.8\,kJ/kg$

An energy balance reduces to read, $\Delta U = Q_{12} - W_{12} \Rightarrow$

$$\frac{W_{12}}{m} = \frac{Q_{12}}{m} - (u_2 - u_1) = 588.6 - (2678.8 - 2298) = 207.8\frac{kJ}{kg} \quad \leftarrow W_{12}/m$$

Process 2-3. $Q_{23}/m = \int_2^3 T\,ds = 0$. An energy balance reads, $\leftarrow Q_{23}/m$

$$\Delta U = Q_{23} - W_{23} \Rightarrow \frac{W_{23}}{m} = -(u_3 - u_2) = -(2546.1 - 2678.8) = 132.7\,kJ/kg \quad \leftarrow W_{23}/m$$

To find u_3, use $s_3 = s_2$. Thus $x_3 = \frac{s_g - s_f}{s_g - s_f} = \frac{6.5421 - 2.09075}{6.626 - 2.09075} = 0.9815$

So, $u_3 = u_f + x_3 u_{fg} = 740.21 + 0.9815(2580.1 - 740.21) = 2546.1\,kJ/kg$.

Process 3-4.

$$\frac{Q_{34}}{m} = \int_3^4 T\,ds = T(\underbrace{s_4 - s_3}_{= -(s_2 - s_1)}) = -448.15\,K(6.5421 - 5.417)\frac{kJ}{kg\cdot K} = -504.21\frac{kJ}{kg} \quad \leftarrow Q_{34}/m$$

An energy balance gives $\Delta U = Q_{34} - W_{34} \Rightarrow \frac{W_{34}}{m} = \frac{Q_{34}}{m} - (u_4 - u_3)$

To get u_4, use $s_4 = s_1 = 5.417\,kJ/kg\cdot K$. Then

$$x_4 = \frac{5.417 - 2.09075}{6.626 - 2.09075} = 0.7334 \Rightarrow u_4 = 740.21 + (0.7334)(2580.1 - 740.21) = 2089.6\frac{kJ}{kg}$$

Finally

$$\frac{W_{34}}{m} = (-504.21) - (2089.6 - 2546.1) = -47.7\frac{kJ}{kg}$$

PROBLEM 6.34 (CONTINUED)

Process 4-1. $Q_{41}/m = \int_4^1 T\,dS = 0$. $\leftarrow Q_{41}/m$

An energy balance reads, $\Delta U = \cancel{Q_{41}} - W_{41}$. So

① $$\frac{W_{41}}{m} = -(u_1 - u_4) = -(2298 - 2089.6) = -208.4 \frac{kJ}{kg}$$ $\leftarrow W_{41}/m$

(c) $$\eta = \frac{W_{net}/m}{Q_{in}/m} = \frac{207.8 + 132.7 - 47.7 - 208.4}{588.6} = 0.143 \quad (14.3\%)$$

Alternatively, apply Eq. 5.9

$$\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{448.15}{523.15} = 0.143 \quad (14.3\%)$$

---

1. For any cycle $W_{net} = Q_{net}$. Thus, to check the calculations,

$$\frac{W_{net}}{m} = 207.8 + 132.7 - 47.7 - 208.4 = 84.4 \text{ kJ/kg}$$

$$\frac{Q_{net}}{m} = 588.6 + 0 - 504.21 + 0 = 84.4 \text{ kJ/kg}$$

PROBLEM 6.35

One lb of water contained in a piston–cylinder assembly, initially saturated vapor at 1 atm, is condensed at constant pressure to saturated liquid. Evaluate the heat transfer, in Btu, and the entropy production, in Btu/°R, for

(a) the water as the system.
(b) an enlarged system consisting of the water and enough of the nearby surroundings that heat transfer occurs only at the ambient temperature, 80°F.

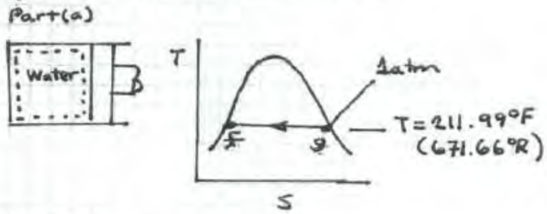
Assume the state of the nearby surroundings does not change during the process of the water, and ignore kinetic and potential energy.

KNOWN: One lb of water in a piston-cylinder assembly is condensed at constant pressure. State data are provided.

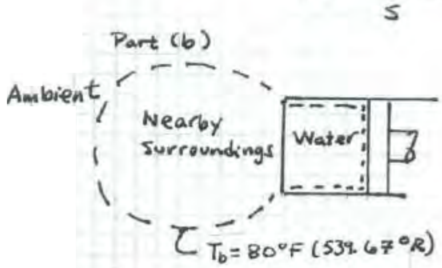
FIND: Evaluate Q and σ for (a) the water as the system, (b) an enlarged system.

SCHEMATIC & GIVEN DATA:

Part(a)



Part (b)



ENGINEERING MODEL:

1. In part(a) the water alone is the system. In part (b), the system is enlarged, as shown in the schematic.
2. The pressure of the water (and thus its temperature) remains constant.
3. The effects of kinetic and potential energy are ignored.
4. The state of the nearby surroundings does not change during the process of the water.

ANALYSIS:

(a) For the water as the system, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - \overset{\int p\,dV = p\Delta V}{W}$

$$\Rightarrow \quad Q = \Delta U + p\Delta V = m[(u_f - u_g) + p(v_f - v_g)] = m[(u_f + pv_f) - (u_g + pv_g)]$$

$$= m[h_f - h_g] = (1\text{ lb})(180.15 - 1150.5)\text{ Btu/lb} = -970.35\text{ Btu}$$

(Table A-3E)

An entropy balance reads

$$\Delta S = \int \frac{\delta Q}{T} + \sigma \Rightarrow \Delta S = \frac{Q}{T} + \sigma \Rightarrow \sigma = \Delta S - \frac{Q}{T}$$

With data from Tables A-3E

$$\sigma = \underbrace{(1\text{ lb})[0.3121 - 1.7567]\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}}_{(= -1.4446\text{ Btu/°R})} - \frac{(-970.35\text{ Btu})}{671.66°\text{R}} = 0$$

The process of the water is internally reversible.

(b) For the enlarged system, $Q = -970.35$ Btu (the state of the nearby surroundings does not change)

The entropy balance reads,

$$(\Delta S)_{water} + \overset{0}{\cancel{(\Delta S)_{surr}}} = \frac{Q}{T_b} + \sigma \Rightarrow \sigma = (\Delta S)_{water} - \frac{Q}{T_b}$$

① $\therefore \quad \sigma = \underset{(\text{from (a)})}{-1.4446\text{ Btu/°R}} - \frac{(-970.35\text{ Btu})}{539.67°\text{R}} = 0.353\ \frac{\text{Btu}}{°\text{R}}$

---

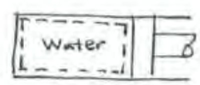
① In (b), the nonzero value for σ accounts for the spontaneous (and thus irreversible) heat transfer between the condensing water and the ambient at 80°F. For more, see Sec. 6.7.2.

PROBLEM 6.36

Five kg of water contained in a piston-cylinder assembly expand from an initial state where $T_1 = 400°C$, $p_1 = 700$ kPa to a final state where $T_2 = 200°C$, $p_2 = 300$ kPa, with no significant effects of kinetic and potential energy. The accompanying table provides additional data at the two states. It is claimed that the water undergoes an adiabatic process between these states, while developing work. Evaluate this claim.

State	T (°C)	p (kPa)	v (m³/kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg · K)
1	400	700	0.4397	2960.9	3268.7	7.6350
2	200	300	0.7160	2650.7	2865.5	7.3115

SCHEMATIC & GIVEN DATA:



m = 5 kg

ENGR. MODEL:

1. The water is the closed system.
2. For the system, Q = 0 and there are no significant effects of kinetic and potential energy.

ANALYSIS:

Every process between the given end states must satisfy the second law. That is,

$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{\,0} + \sigma \Rightarrow \sigma = \Delta S = m(s_2 - s_1)$$

$$\sigma = 5\text{ kg}(7.3115 - 7.6350)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

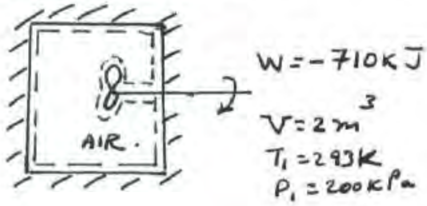
$$= -1.6175 \frac{\text{kJ}}{\text{K}}$$

Since $\sigma < 0$, this process cannot occur as claimed

PROBLEM 6.37

Two m^3 of air in a rigid, insulated container fitted with a paddle wheel is initially at 293 K, 200 kPa. The air receives 710 kJ by work from the paddle wheel. Assuming the ideal gas model with $c_v = 0.72$ kJ/kg · K, determine for the air (a) the mass, in kg, (b) final temperature, in K, and (c) the amount of entropy produced, in kJ/K.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The air is the closed system.
2. For the air, $Q = 0$ and there are no overall changes in KE or PE.
3. The air is modeled as an ideal gas with $c_v = 0.72$ kJ/kg·K.

ANALYSIS: Using the ideal gas equation of state, the mass, m, of the air is

$$m = \frac{P_1 V}{R T_1} = \frac{(200 \times 10^3 \text{ N/m}^2)(2 \text{ m}^3)}{\left(\frac{8314}{28.97} \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(293 \text{ K})} = 4.76 \text{ kg} \longleftarrow$$

An energy balance reduces as, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - W$. Thus

$$W = -\Delta U = -m c_v (T_2 - T_1) \Rightarrow T_2 = T_1 - \frac{W}{m c_v} = 293 - \frac{(-710 \text{ kJ})}{(4.76 \text{ kg})(0.72 \frac{\text{kJ}}{\text{kg}\cdot\text{K}})}$$

$$= 500 \text{ K} \longleftarrow T_2$$

An entropy balance reduces as, $\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{\,0} + \sigma \Rightarrow \sigma = \Delta S = m(s_2 - s_1)$ with Eq. 6.21 and $V_2 = V_1$,

$$\sigma = m c_v \ln \frac{T_2}{T_1} = (4.76 \text{ kg})(0.72 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}) \ln \frac{500}{293}$$

$$= 1.832 \frac{\text{kJ}}{\text{K}} \longleftarrow \sigma$$

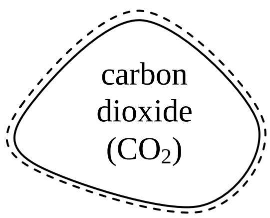
PROBLEM 6.38

Carbon dioxide (CO_2) gas undergoes a process in a closed system from T_1 = 100°F, p_1 = 20 lbf/in.^{2}, to T_2 = 400°R, p_2 = 50 lbf/in.^{2} The entropy produced due to internal irreversiblities during the process is determined to be 0.15 Btu/°R per lb of gas. The carbon dioxide can be modeled as an ideal gas. Determine if the energy transfer by heat, Q, is positive (into the system), negative (out of the system), or zero.

KNOWN: Carbon dioxide gas undergoes a process in a closed system between two specified states. The entropy produced per unit mass is known.

FIND: Determine the direction of the energy transfer Q.

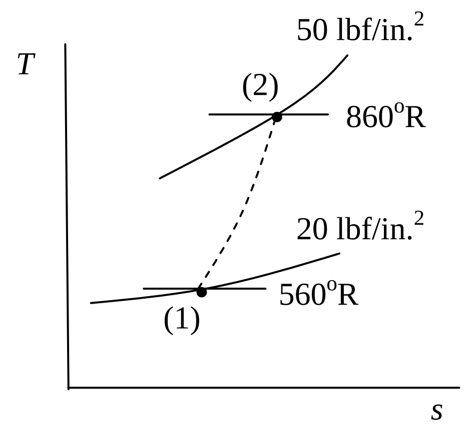
SCHEMATIC AND GIVEN DATA:



T_1 = 100°F = 560°R
p_1 = 20 lbf/in.^{2}

T_2 = 400°F = 860 °R
p_2 = 50 lbf/in.^{2}

σ/m = 0.15 Btu/lb·°R



ENGINEERING MODEL: (1) The carbon dioxide is a closed system. (2) The carbon dioxide can be modeled as an ideal gas.

ANALYSIS: To determine the direction of Q for the process, we can use the entropy balance to determine the direction of the entropy transfer associated with heat, which is in the same direction as Q.

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma \quad \longrightarrow \quad \frac{1}{m}\int_1^2 \left(\frac{\delta Q}{T}\right)_b = \Delta s - \sigma/m$$

Next, we determine Δs using data from Table A-23E as follows

$$\Delta s = \frac{\bar{s}^o(T_2) - \bar{s}^o(T_1)}{M} - R \ln\left(\frac{p_2}{p_1}\right)$$

$$= \left(\frac{55.589 - 49.698}{44.01}\right)\frac{\text{Btu}}{\text{lb}\cdot{}^o\text{R}} - \left(\frac{1545}{44.01}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^o\text{R}}\right)\left|\frac{1\ \text{ft}\cdot\text{lbf}}{778\ \text{Btu}}\right| \ln\left(\frac{50\frac{\text{lbf}}{\text{in}^2}}{20\frac{\text{lbf}}{\text{in}^2}}\right)$$

$$= 0.09251\ \text{Btu/lb}\cdot{}^o\text{R}$$

Thus

$$\frac{1}{m}\int_1^2 \left(\frac{\delta Q}{T}\right)_b = \Delta s - \sigma/m = 0.09251 - 0.15 = -\ 0.05749\ \text{Btu/lb}\cdot{}^o\text{R}$$

Since the entropy transfer by heat is negative, $Q < 0$ (out of the system). ←

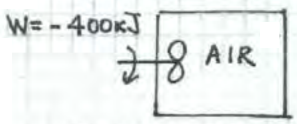
PROBLEM 6.39

Air contained in a rigid, insulated tank fitted with a paddle wheel, initially at 1 bar, 330 K and a volume of 1.93 m^3, receives an energy transfer by work from the paddle wheel in an amount of 400 kJ. Assuming the ideal gas model for the air, determine (a) the final temperature, in K, (b) the final pressure, in bar, and (c) the amount of entropy produced, in kJ/K. Ignore kinetic and potential energy.

KNOWN: Air in a rigid, insulated tank is stirred by a paddle wheel. State data and W are given.

FIND: Determine the final temperature and pressure and σ.

SCHEMATIC & GIVEN DATA:



$P_1 = 1$ bar, $T_1 = 330$K

$V = 1.93$ m^3

ENGINEERING MODEL:

1. The air in the tank is the closed system.
2. For the system, $Q = 0$ and there are no effects of kinetic and potential energy.
3. The air is modeled as an ideal gas.

ANALYSIS:

(a) Applying an energy balance, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - W$

$$\Rightarrow \Delta U = -W \text{. or, } m(u_2 - u_1) = -W \Rightarrow u_2 = u_1 - W/m \qquad (1)$$

We get u_1 from Table A-22 : $u_1 = 235.61$ kJ/kg. To obtain m, apply the ideal gas model equation of state:

$$m = \frac{P_1 V}{R T_1} = \frac{(10^5 N/m^2)(1.93 m^3)}{\left(\frac{8314}{28.97}\frac{N \cdot m}{kg \cdot K}\right)(330K)} = 2.04 \text{ kg}$$

Then Eq. (1) gives

$$u_2 = \left[235.61 - \frac{(-400)}{2.04}\right]\frac{kJ}{kg} = 431.68 \frac{kJ}{kg}$$

Interpolating in Table A-22, $T_2 = 596$K

(b) Applying the ideal gas model equation of state,

$$\left.\begin{array}{l} P_1 V = m R T_1 \\ P_2 V = m R T_2 \end{array}\right\rangle \frac{P_2}{P_1} = \frac{T_2}{T_1} \Rightarrow P_2 = P_1\left[\frac{T_2}{T_1}\right] = 1 \text{ bar}\left[\frac{596K}{330K}\right] = 1.81 \text{ bar}$$

(c) Applying an entropy balance, $\Delta S = \cancel{\int \frac{\delta Q}{T}}^{0} + \sigma$

$$\Rightarrow \sigma = m[s_2 - s_1] = m\left[s_2^o - s_1^o - R \ln \frac{P_2}{P_1}\right]$$

With s^o data from Table A-22,

$$\sigma = 2.04 \text{kg}\left[2.40188 - 1.79783 - \frac{8.314}{28.97}\ln(1.81)\right]\frac{kJ}{kg \cdot K}$$

$$= 0.885 \frac{kJ}{K}$$

PROBLEM 6.41

Air contained in a rigid, insulated tank fitted with a paddle wheel, initially at 300 K, 2 bar, and a volume of 2 m^3, is stirred until its temperature is 500 K. Assuming the ideal gas model for the air, and ignoring kinetic and potential energy, determine (a) the final pressure, in bar, (b) the work, in kJ, and (c) the amount of entropy produced, in kJ/K. Solve using

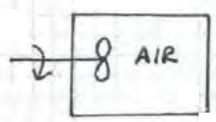
(a) data from Table A-22.
(b) constant c_v read from Table A-20 at 400 K.

Compare the results of parts (a) and (b).

KNOWN: Air in a rigid, insulated tank is stirred by a paddle wheel. State data are given.

FIND: Determine final pressure, W and σ.

SCHEMATIC & GIVEN DATA:



$V = 2 m^3$
$T_1 = 300 K$
$P_1 = 2 bar$
$T_2 = 500 K$

ENGINEERING MODEL:

1. The air in the tank is the closed system.
2. For the system, $Q = 0$ and kinetic and potential energy are ignored.
3. The air is modeled as an ideal gas, using (a) data from Table A-22, (b) constant c_v at 400 K from Table A-20.

ANALYSIS:

(a) Using the ideal gas equation of state, $\left\{\begin{matrix} P_1 V = mRT_1 \\ P_2 V = mRT_2 \end{matrix}\right\} \frac{P_2}{P_1} = \frac{T_2}{T_1}$

$$\therefore \quad P_2 = P_1\left[\frac{T_2}{T_1}\right] = (2 bar)\left[\frac{500K}{300K}\right] = 3.33 \text{ bar}$$

Also, $m = P_1 V / RT_1$. Thus

$$m = \frac{(2\times10^5 N/m^2)(2 m^3)}{\left(\frac{8314}{28.97}\frac{N\cdot m}{kg\cdot K}\right)(300 K)} = 4.65 kg$$

Preliminaries for (b), (c): Energy balance: $\Delta U + \Delta KE + \Delta PE = \cancel{Q} - W$

$$\Rightarrow W = -m[u_2 - u_1] \qquad (1)$$

Entropy balance: $\Delta S = \int \frac{\delta Q}{T}^{\,0} + \sigma \Rightarrow \sigma = m[s_2 - s_1] \qquad (2)$

(b) Table A-22 Data:

Eq. (1): $W = -m[u_2 - u_1] = -4.65 kg[359.49 - 214.07]\frac{kJ}{kg} = -676.2 kJ$

Eq. (2) with Eq. 6.20(a): $\sigma = m[s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1}]$ $\left(= \frac{T_2}{T_1}\right)$

$$= (4.65 kg)\left[2.21952 - 1.70203 - \frac{8.314}{28.97}\ln\left(\frac{500}{300}\right)\right]\frac{kJ}{kg\cdot K}$$

$$= 1.725 \text{ kJ/K}$$

(b) c_v at 400K from Table A-20: $c_v = 0.726$ kJ/kg·K.

Eq. (1): $W = -m c_v [T_2 - T_1] = -(4.65 kg)(0.726 \frac{kJ}{kg\cdot K})(200 K) = -675.2 kJ$

Eq. (2) with Eq. 6.21: $\sigma = m[c_v \ln T_2/T_1 + R \ln V_2/V_1^{\,0}] = m c_v \ln \frac{T_2}{T_1}$

$$= (4.65 kg)\left(0.726 \frac{kJ}{kg\cdot K}\right)\ln\left(\frac{500}{300}\right)$$

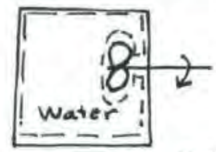
$$= 1.724 \text{ kJ/K}$$

COMMENT: In this application, the results for W and σ obtained using Table A-22 are in very good agreement with those obtained using c_v at the average temperature, 400K. This is to be expected when the temperature range from T_1 to T_2 is relatively small, as it is in the present application.

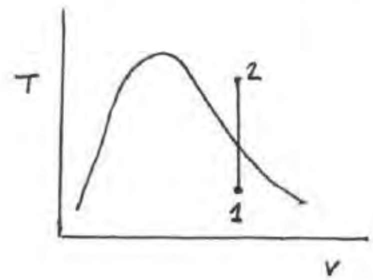
PROBLEM 6.42

A rigid, insulated container fitted with a paddle wheel contains 5 lb of water, initially at 260°F and a quality of 60%. The water is stirred until the temperature is 350°F. For the water, determine (a) the work, in Btu, and (b) the amount of entropy produced, in Btu/°R.

SCHEMATIC & GIVEN DATA:



$m = 5\ lb \quad T_1 = 260°F,\ x_1 = 0.6$
$T_2 = 350°F$



ENGR. MODEL:

1. The water is the closed system.
2. For the system, $Q = 0$ and there are no overall changes in KE or PE.

ANALYSIS:

With data from Table A-2E,

$$v_1 = (1-x_1)v_f + x_1 v_g$$
$$= (0.4)(0.0178) + 0.6(11.77) = 7.07\ \frac{ft^3}{lb}$$
$$u_1 = (0.4)(228.6) + 0.6(1090.5) = 745.74\ \frac{Btu}{lb}$$
$$s_1 = (0.4)(0.3819) + 0.6(1.6864) = 1.1646\ Btu/lb\cdot°R$$

Since $v_2 = v_1$ and $T_2 = 350°F$, Table A-4E gives

$$u_2 = 1120.32\ Btu/lb,\quad s_2 = 1.6699\ Btu/lb\cdot°R.$$

(a) An energy balance reads,

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - W$$
$$\Rightarrow W = -m(u_2 - u_1) = -5\ lb\,(1120.32 - 745.74)\frac{Btu}{lb}$$
$$= -1872.9\ Btu \qquad \longleftarrow W$$

(b) An entropy balance reads,

$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{\,0} + \sigma$$
$$\Rightarrow \sigma = \Delta S = m(s_2 - s_1)$$
$$= 5\ lb\,(1.6699 - 1.1646)\frac{Btu}{lb\cdot°R}$$
$$= 2.5265\ \frac{Btu}{°R} \qquad \longleftarrow \sigma$$

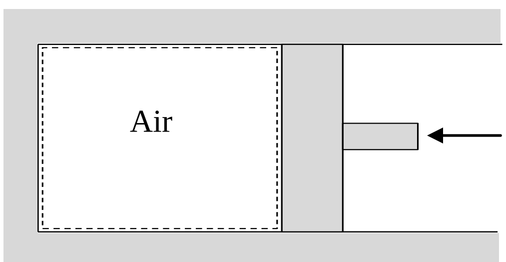
PROBLEM 6.43

Air is compressed adiabatically in a piston-cylinder assembly from 1 bar, 300 K to 10 bar, 600 K. The air can be modeled as an ideal gas and kinetic and potential energy effects are negligible. Determine the amount of entropy produced, in kJ/K per kg of air, for the compression. What is the minimum theoretical work input, in kJ per kg of air, for an adiabatic compression from the given initial state to a final pressure of 10 bar?

KNOWN: Air is compressed adiabatically in a piston-cylinder. The initial and final states are specified.

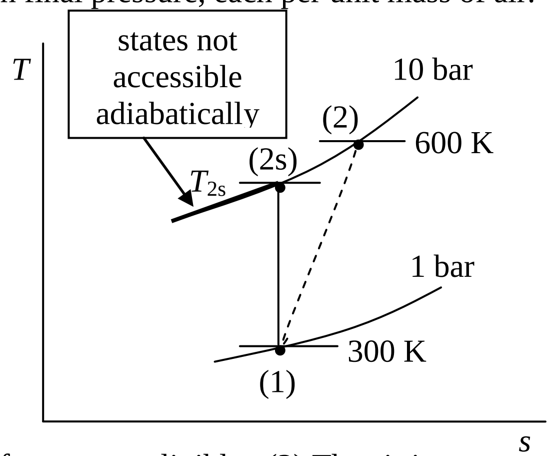
FIND: Determine the amount of entropy produced and the minimum theoretical work input for adiabatic compression from the initial state to the given final pressure, each per unit mass of air.

SCHEMATIC AND GIVEN DATA:



p_1 = 1 bar
T_1 = 300 K

p_2 = 10 bar
T_2 = 600 K



ENGINEERING MODEL: (1) The air is a closed system. (2) $Q = 0$ and kinetic and potential energy effects are negligible. (3) The air is modeled as an ideal gas.

ANALYSIS: To find the entropy produced, begin with the entropy balance: $\Delta S = \int_1^1 \cancel{\left(\frac{\delta Q}{T}\right)_b} + \sigma$

Thus

$$\sigma/m = s_2 - s_1 = s^o(T_2) - s^o(T_1) - R\ln(p_2/p_1)$$

With data from Table A-22

$$\sigma/m = (2.40902 - 1.70203)\text{kJ/kg·K} - \left(\frac{8.314}{28.97}\ \frac{\text{kJ}}{\text{kg·K}}\right)\ln\left(\frac{10\text{ bar}}{1\text{ bar}}\right) = 0.04618\text{ kJ/kg·K} \longleftarrow$$

The work is determined using the energy balance, as follows: $\Delta \cancel{KE} + \Delta \cancel{PE} + \Delta U = \cancel{Q} - W$

$$W/m = (u_1 - u_2)$$

As u varies with T for an ideal gas, the work input *decreases* as T_2 decreases. From the entropy balance

$$s_2 - s_1 = \sigma/m \geq 0 \longrightarrow \boxed{s_2 \geq s_1}$$

PROBLEM 6.43 (CONTINUED)

As shown on the accompanying *T*-*s* diagram, only states at 10 bar to the right of state 2s (isentropic compression – with $\sigma/m = 0$) are accessible in an adiabatic compression, and the corresponding temperature T_{2s} is the lowest possible temperature. Hence, compression to state 2s gives the *minimum theoretical work input*.

For $s_2 - s_1$: $s^o(T_{2s}) = s^o(T_1) + R \ln(p_2/p_1) = 1.70203 + (8.314/28.97) \ln(10/1) = 2.36284$ kJ/kg·K

Interpolating in Table A-22: $T_{2s} \approx 564.1$ K and $u_2 \approx 407.55$ kJ/kg. With $u_1 = 214.07$ kJ/kg

#1 $$(W/m)_{\text{min input}} = (u_{2s} - u_1) = 407.55 - 214.07 = 193.48 \text{ kJ/kg}$$

1. Note: The work for actual process from state 1 to state 2 is

$$(W/m)_{\text{input}} = (u_2 - u_1) = 434.78 - 214.07 = 220.71 \text{ kJ/kg}$$

which is *greater* than the theoretical minimum, as expected.

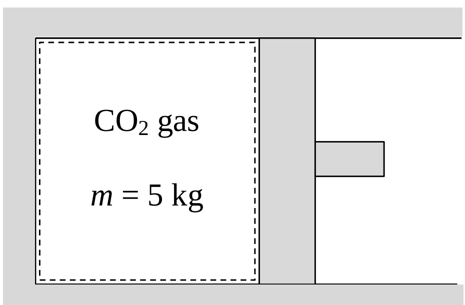
PROBLEM 6.44

Five kg of carbon dioxide (CO_2) gas undergoes a process in a well-insulated piston-cylinder assembly from 2 bar, 280K to 20 bar, 520 K. If the carbon dioxide behaves as an ideal gas, determine the amount of entropy produced, in kJ/K, assuming
(a) constant specific heats with c_p = 0.939 kJ/kg·K.
(b) variable specific heats.
Compare the results of parts (a) and (b).

KNOWN: Carbon dioxide gas undergoes an adiabatic process in a piston-cylinder assembly. The initial and final states are specified.

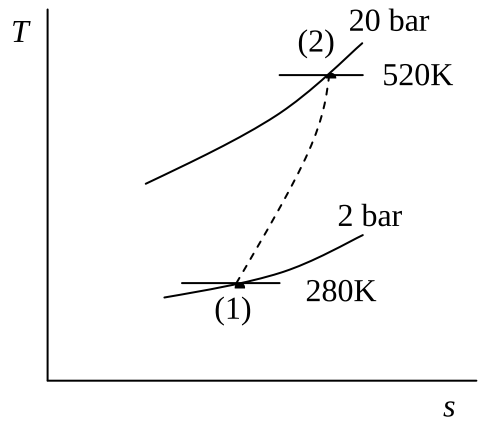
FIND: Determine the amount of entropy produced assuming the ideal gas model with (a) constant specific heats, and (b) variable specific heats. Comment.

SCHEMATIC AND GIVEN DATA:



P_1 = 2 bar
T_1 = 280K

p_2 = 20 bar
T_2 = 520K



ENGINEERING MODEL: (1) The CO_2 is a closed system. (2) $Q = 0$. (3) $\Delta KE = \Delta PE = 0$. (4) The CO_2 behaves as an ideal gas with (a) constant specific heats, and (b) variable specific heats.

ANALYSIS: To determine the amount of entropy produced, we reduce the entropy balance, as follows

$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b} + \sigma \longrightarrow \sigma = \Delta S = m\,\Delta s$$

(a) For constant c_p, the change in specific entropy is

$$\Delta s = c_p \ln(T_2/T_1) - R\ln(p_2/p_1) = (0.939 \text{ kJ/kg·K}) \ln(520/280) - \left(\frac{8.314}{44.01}\ \frac{\text{kJ}}{\text{kg·K}}\right) \ln(20/2)$$

$$= 0.14629 \text{ kJ/kg·K} \longrightarrow \sigma = \Delta S = m\,\Delta s = (5 \text{ kg})(0.14629 \text{ kJ/kg·K}) = 0.73145 \text{ kJ/K} \longleftarrow$$

(b) For variable specific heats use data from Table A-23 (Note: the data are on a molar basis):

$$\Delta s = \frac{\bar{s}^o(T_2) - \bar{s}^o(T_1)}{M} - R\ln(p_2/p_1)$$

PROBLEM 6.44 (CONTINUED)

$$= \frac{(236.575 - 211.376)\text{kJ/kmol}\cdot\text{K}}{44.01\ \text{kg/kmol}} - \left(\frac{8.314}{44.01}\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\ln\left(\frac{20}{2}\right) = 0.13795\ \text{kJ/kg}\cdot\text{K}$$

Thus

$$\sigma = \Delta S = m\,\Delta s = (5\ \text{kg})(0.13795\ \text{kJ/kg}\cdot\text{K}) = 0.68975\ \text{kJ/K}$$

Comment

The value for entropy production obtained using constant specific heats is approximately 6% higher than the value obtained when accounting explicitly for the variation in specific heats. This error is unnecessary, since the tabulated values are available.

For reference, the value of c_p provided was for c_p at 400K, which is the average of the initial and final temperatures.

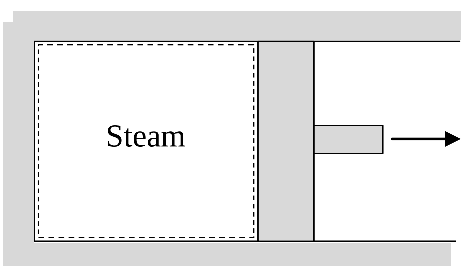
PROBLEM 6.45

Steam undergoes an adiabatic expansion in a piston-cylinder assembly from 100 bar, 360^{o}C to 1 bar, 160^{o}C. What is work in kJ per kg of steam for the process? Calculate the amount of entropy produced, in kJ/K per kg of steam. What is the maximum theoretical work that could be obtained from the given initial state to the same final pressure. Show both processes on a properly-labeled sketch of the *T*-*s* diagram.

KNOWN: Steam undergoes an adiabatic expansion in a piston-cylinder assembly. Data are given at the initial and final states.

FIND: Determine the work and entropy produced, each per unit mass of steam. Find the maximum theoretical work that could be obtained from the given initial state to the same final pressure. Show both processes on a *T*-*s* diagram.

SCHEMATIC AND GIVEN DATA:



p_1 = 100 bar
T_1 = 360^{o}C

p_2 1 bar
T_2 = 160^{o}C

ENGINEERING MODEL: (1) The steam is a closed system. (2) Q = 0. (3) $\Delta KE + \Delta PE = 0$.

ANALYSIS: The work is determined by reducing the energy balance, as follows.

$$\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = \cancel{Q} - W \longrightarrow \quad W/m = (u_1 - u_2)$$

From Table A-3: p_1 = 100 bar and T_1 = 360^{o}C; u_1 = 2729.1 kJ/kg and s_1 = 6.0060 kJ/kg·K

From Table A-3: p_2 = 1 bar and T_2 = 160^{o}C; u_2 = 2597.8 kJ/kg and s_2 = 7.6597 kJ/kg·K

$$W/m = (2729.1 - 2597.8) = 131.3 \text{ kJ/kg (out, as expected)}$$

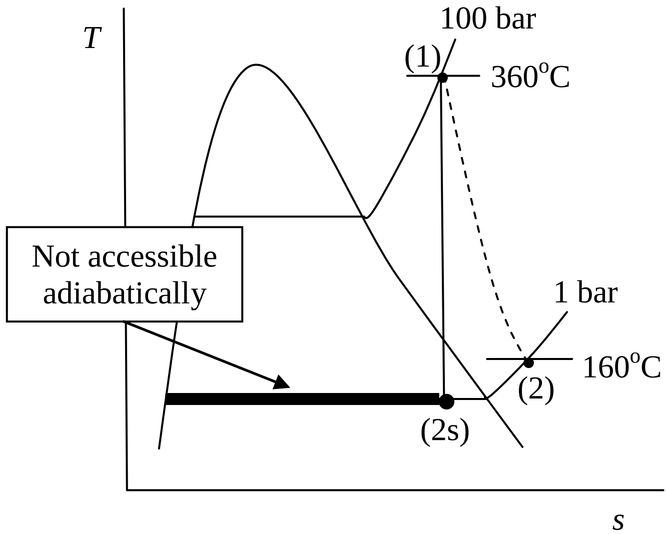
Now, the entropy production can be evaluated using the entropy balance, as follows.

$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b} + \sigma \longrightarrow \quad \sigma/m = (s_2 - s_1) = (7.6597 - 6.0060) \text{ kJ/kg·K} = 1.6537 \text{ kJ/kg·K}$$

To find the maximum theoretical work, we note that since $(s_2 - s_1) = \sigma/m \geq 0$; $\boxed{s_2 \geq s_1}$

PROBLEM 6.45 (CONTINUED)

Graphically



Note that since $s_2 \geq s_1$, state 2s is the limiting case when $s_{2s} = s_1$ (*reversible expansion with no entropy production*). It is not possible to access states to the left of state 2s adiabatically from state 1.

Expansion to state 2s give the biggest difference in *u*, and hence the *maximum theoretical work*.

From Table A-3: p_2 = 1 bar, s_{2s} = 6.0060 kJ/kg·K gives

$$x_{2s} = \frac{s_{2s} - s_{f@1\text{ bar}}}{s_{g@1\text{ bar}} - s_{f@1\text{ bar}}} = \frac{6.0060 - 1.3026}{7.3594 - 1.3026} = 0.7765$$

and

$$u_{2s} = u_{f@1\text{ bar}} + x_{2s}(u_{g@1\text{ bar}} - u_{f@1\text{ bar}}) = 417.36 + (0.7765)(2506.1 - 417.36) = 2039.3 \text{ kJ/kg}$$

Finally, the maximum theoretical work is

$$(W/m)_{max} = (u_1 - u_{2s}) = (2729.1 - 2039.3) = 689.8 \text{ kJ/kg}$$

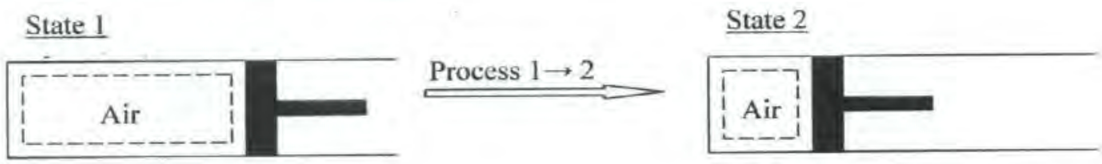
PROBLEM 6.46

Two kilograms of air contained in a piston-cylinder assembly are initially at 1.5 bar and 400 K. Can a final state at 6 bar and 500 K be attained in an adiabatic process?

KNOWN: Air at specified initial pressure and temperature undergoes a process to a final specified pressure and temperature.

FIND: Whether the process can occur adiabatically.

SCHEMATIC AND GIVEN DATA:



$m = 2$ kg
$p_1 = 1.5$ bar
$T_1 = 400$ K

$p_2 = 6$ bar
$T_2 = 500$ K

ENGINEERING MODEL:
1. The air is a closed system defined by the dashed line on the accompanying diagram.
2. Air can be modeled as an ideal gas.

ANALYSIS:
The closed system entropy balance can be used to determine whether process 1-2 can occur adiabatically. The closed system entropy balance is

$$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma$$

If the process is adiabatic, the term, $\int_1^2 \left(\frac{\delta Q}{T}\right)_b$, is zero. Solving for entropy production gives

$$\sigma = S_2 - S_1 = m(s_2 - s_1)$$

Since air can be modeled as an ideal gas, the entropy change can be determined from Eq. 6.20a

$$S(T_2, p_2) - S(T_1, p_1) = m[s^{\circ}(T_2) - s^{\circ}(T_1) - R \ln \frac{p_2}{p_1}]$$

Thus

$$\sigma = m[s^{\circ}(T_2) - s^{\circ}(T_1) - R \ln \frac{p_2}{p_1}]$$

Values for s° for air are a function of only temperature and are obtained from ideal gas table for air, Table A-22.
State 1 ($T_1 = 400$ K): $s^{\circ}(T_1) = 1.99194$ kJ/(kg·K)
State 2 ($T_2 = 500$ K): $s^{\circ}(T_2) = 2.21952$ kJ/(kg·K)

Substituting values and solving give

$$\sigma = (2\text{ kg})\left[2.21952\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 1.99194\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - \left(\frac{8.314\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}}{28.97\frac{\text{kg}}{\text{kmol}}}\right)\ln\left(\frac{6\text{ bar}}{1.5\text{ bar}}\right)\right]$$

$$\sigma = -0.3405 \text{ kJ/K}$$

Since entropy production is negative, the final state cannot be attained in an adiabatic process. ◄

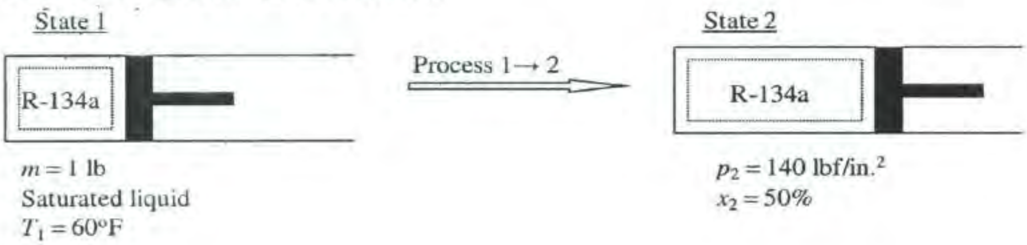
PROBLEM 6.47

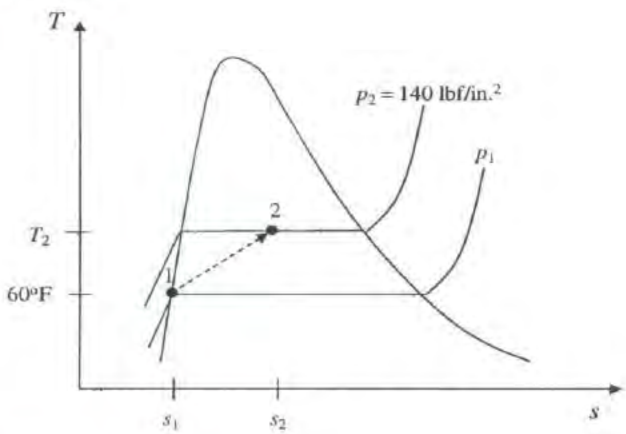
One pound mass of Refrigerant 134a contained within a piston-cylinder assembly undergoes a process from a state where the temperature is 60°F and the refrigerant is saturated liquid to a state where the pressure is 140 lbf/in.^{2} and quality 50%. Determine the change in specific entropy of the refrigerant, in Btu/lb·°R. Can this process be accomplished adiabatically?

KNOWN: Refrigerant 134a undergoes a process with known conditions.

FIND: Determine the change in specific entropy of the refrigerant, in Btu/lb·°R. Can this process be accomplished adiabatically?

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL:
(1) The R-134a is a closed system as shown in the accompanying figure.

ANALYSIS:

To determine change is specific entropy for R-134a, use data from Table A-10E and Table A-11E as follows.

$$s_1 = s_f = 0.0648 \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$

$$s_2 = s_f + x_2(s_g - s_f) = 0.0902 \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} + 0.5(0.2161 - 0.0902)\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} = 0.15315 \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$

$$s_2 - s_1 = (0.15315 - 0.0648)\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} = 0.08835 \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$

To determine if the process is possible if adiabatic, begin with the entropy balance:

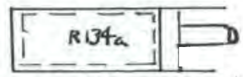
$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma$$

If adiabatic, the first term on the right side drops out, leaving $\Delta S = \sigma$. In this case, $\Delta S = m(\Delta s)$ is positive and therefore, σ is positive which indicates that an adiabatic process is possible.

PROBLEM 6.48

Refrigerant 134a contained in a piston–cylinder assembly rapidly expands from an initial state where $T_1 = 140°F$, $p_1 = 200$ lbf/in.^{2} to a final state where $p_2 = 5$ lbf/in.^{2} and the quality, x_2, is (a) 99%, (b) 95%. In each case, determine if the process can occur adiabatically. If yes, determine the work, in Btu/lb, for an adiabatic expansion between these states. If no, determine the direction of the heat transfer.

SCHEMATIC & GIVEN DATA



$T_1 = 140°F$, $P_1 = 200$ lb/in^2
$P_2 = 5$ lb/in^2
x_2: (a) 99% (b) 95%

ENGR. MODEL:

1. The R134a is the closed system.
2. There are no overall changes in KE or PE.

ANALYSIS: An entropy balance reads

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma \Rightarrow m(s_2 - s_1) = \underbrace{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}_{+,0,-} + \overbrace{\sigma}^{+ \text{ or } 0, \text{ never } -} \qquad (1)$$

From Table A-12E, $u_1 = 112.87$ Btu/lb, $s_1 = 0.2226$ Btu/lb·°R.

(a) $x_2 = 0.99$. Data from Table A-11E

$$s_2 = s_f + x_2(s_g - s_f) = -0.0090 + 0.99(0.2311 - (-0.0090)) = 0.2287 \text{ Btu/lb·°R}$$

So, Eq. (1) reads

$$m(\underbrace{0.2287 - 0.2226}_{+0.0061})\frac{\text{Btu}}{\text{lb·°R}} = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma$$

If the process occurs adiabatically, we get $\sigma = m\left(0.0061 \frac{\text{Btu}}{\text{lb·°R}}\right)$, which is positive. Thus, such a process is allowed.

For an adiabatic process between these states, an energy balance reduces to, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - W$, or

$$W = -m(u_2 - u_1) \Rightarrow \frac{W}{m} = -(u_2 - u_1) = -(85.17 - 112.87)\frac{\text{Btu}}{\text{lb}}$$
$$= +27.7 \text{ Btu/lb}$$

where $u_2 = u_f + x_2(u_g - u_f) = -3.74 + 0.99(86.07 - (-3.74)) = 85.17$ Btu/lb.

(b) $x = 0.95$. Data from Table A-11E

$$s_2 = s_f + x_2(s_g - s_f) = -0.0090 + 0.95(0.2311 - (-0.0090)) = 0.2191 \text{ Btu/lb·°R}.$$

So, Eq. (1) reads

$$m(\underbrace{0.2191 - 0.2226}_{-0.0035})\frac{\text{Btu}}{\text{lb·°R}} = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma$$

In this case, if the process occurred adiabatically, $\sigma < 0$. Such a process is not allowed. Entropy transfer accompanying heat transfer from the system must occur if $\Delta S < 0$.

PROBLEM 6.49

One kg of air contained in a piston–cylinder assembly undergoes a process from an initial state where $T_1 = 300$ K, $v_1 = 0.8$ m^3/kg to a final state where $T_2 = 420$ K, $v_2 = 0.2$ m^3/kg. Can this process occur adiabatically? If yes, determine the work, in kJ, for an adiabatic process between these states. If no, determine the direction of the heat transfer. Assume the ideal gas model for air.

SCHEMATIC & GIVEN DATA:

$m = 1$ kg

ENGR. MODEL:

1. The air is the closed system.
2. The air is modeled as an ideal gas.

ANALYSIS:

An entropy balance reads,

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma \qquad (1)$$

where

$$\Delta S = (s_2 - s_1) = s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{P_2}{P_1}$$

For an ideal gas, $P = RT/v$, so $\frac{P_2}{P_1} = \frac{T_2}{T_1} \frac{v_1}{v_2}$

$$\Rightarrow \Delta s = s^\circ(T_2) - s^\circ(T_1) - R \ln\left(\frac{T_2}{T_1} \frac{v_1}{v_2}\right)$$

With data from Table A-22

$$\Delta s = \left(2.04142 - 1.70203 - \frac{8.314}{28.97} \ln\left(\frac{420}{300} \times \frac{0.8}{0.2}\right)\right) \frac{kJ}{kg \cdot K}$$

$$= -0.155 \text{ kJ/kg} \cdot \text{K}$$

Thus, in Eq. (1), $\Delta S < 0$. Accordingly, since $\sigma \geq 0$, this process cannot occur adiabatically. Entropy transfer accompanying heat transfer from the system must occur if $\Delta S < 0$.

PROBLEM 6.50

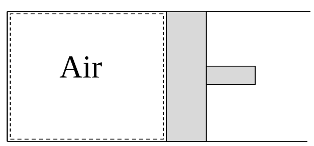
Air as an ideal gas contained within a piston-cylinder assembly is ocmpressed between two specified states. In each of the following cases, cand the process occur adiabatically? If yes, determine the work in appropriate units for an adiabatic process between these states. If no, determine the direction of heat transfer.

(a) State 1: p_1 = 0.1 MPa, T_1 = 27°C. State 2: p_1 = 0.5 MPa, T_1 = 207°C. Use Table A-22 data.
(b) State 1: p_1 = 3 atm, T_1 = 80°F. State 2: p_1 = 10 atm, T_1 = 240°F. Assume c_p = 0.241 Btu/lb·°R.

KNOWN: Air is compressed in a pistopn-cylinder assembly between two specified states.

FIND: Determine if the process can occur adiabatically. If yes, determine the work. If no, determine the direction of heat transfer.

SCHEMATIC AND GIVEN DATA:



(a) State 1: p_1 = 0.1 MPa, T_1 = 27°C. State 2: p_1 = 0.5 MPa, T_1 = 207°C.
(b) State 1: p_1 = 3 atm, T_1 = 80°F. State 2: p_1 = 10 atm, T_1 = 240°F.

ENGINEERING MODEL: (1) The air is a closed system. (2) Ignore kinetic and potential energy effects. (3) The sir is modeled as an ideal gas. For part (a), ause variable specific heats and Table A-22 data. For part (b) assume c_p = 0.241 Btu/lb·°R.

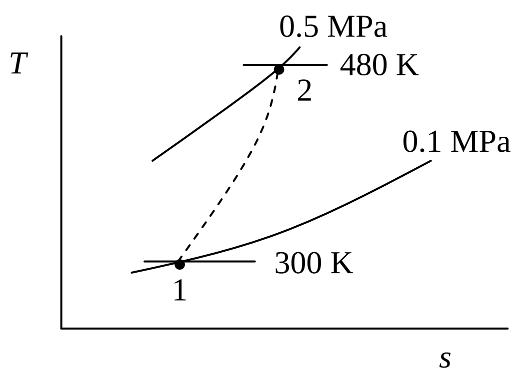
ANALYSIS: To determine if the process can occur adiabatically, begin with the closed system entropy balance: $\Delta S = \underline{\int \left(\frac{\delta Q}{T}\right)_b} + \sigma$. In an adiabatic process, the underlined term vanishes, giving $\Delta S = \sigma$. Since $\sigma \geq 0$, ΔS cannot be negative.

(a) With Eq. 6.20a and s^o data from Table A-22

$$\Delta S/m = \Delta s = s^o(T_2) - s^o(T_1) - R\ln(p_2/p_1) = 2.17776 - 1.70203 - (8.314/28.97)\ln(0.5/0.1)$$
$$= +0.0137 \text{ kJ/kg·K}$$

Accordingly, an adiabatic process between these state is allowed. The energy balance gives $\cancel{\Delta KE} + \cancel{\Delta PE} + m(u_2 - u_1) = \cancel{Q} - W$. Thus, with data from Table A-22

$$W/m = (u_1 - u_2) = 214.07 - 344.7 = -130.6 \text{ kJ/kg}$$



PROBLEM 6.50 (CONTINUED)

(b) With Eq. 6.22

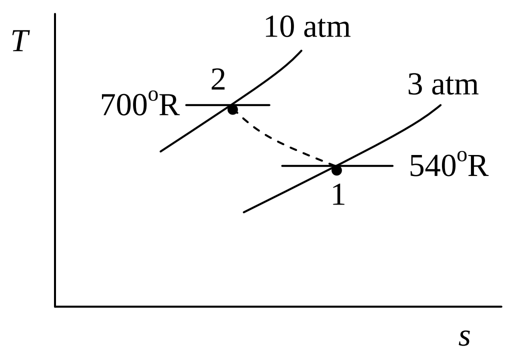
$$\Delta S/m = \Delta s = c_p \ln(T_2/T_1) - R \ln(p_2/p_1) = (0.241) \ln(700/540) - (1.986/28.97) \text{ lm}(10/3)$$

$$= -0.02 \text{ Btu/lb}\cdot{}^{\circ}\text{R}$$

From the entropy balance

$$(-0.02 \text{ Btu/lb}\cdot{}^{\circ}\text{R}) = \int \left(\frac{\delta Q}{T}\right)_{\text{b}} + \underbrace{\sigma}_{\boxed{\sigma \geq 0}}$$

The entropy transfer term *must* be negative, since the change in entropy is negative. Therefore, entropy transfer by heat occurs *from* the system *to* its surroundings, and the direction of energy transfer by heat must be *out* of the system.



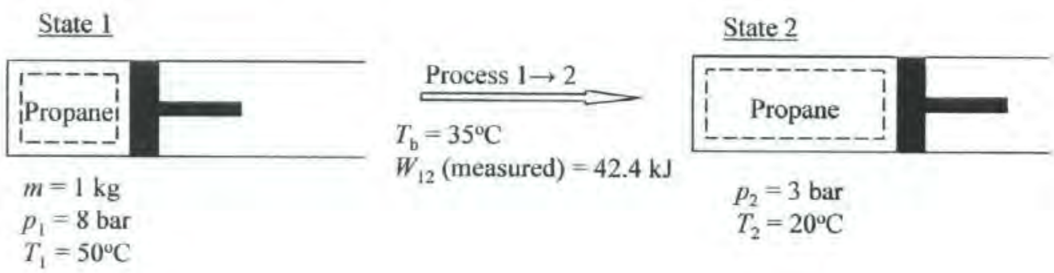
PROBLEM 6.51

One kilogram of propane initially at 8 bar and 50°C undergoes a process to 3 bar, 20°C while being rapidly expanded in a piston-cylinder assembly. Heat transfer between the propane and its surroundings occurs at an average temperature of 35°C. The work done by the propane is measured as 42.4 kJ. Kinetic and potential energy effects can be ignored. Determine whether it is possible for the work measurement to be correct.

KNOWN: Propane at specified initial pressure and temperature undergoes rapid expansion with heat transfer and work to a final specified pressure and temperature.

FIND: Whether it is possible for the work measurement to be correct.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The propane is a closed system defined by the dashed line on the accompanying diagram.
2. Kinetic or potential energy effects can be ignored.
3. Heat transfer between the propane and its surroundings occurs at 308 K (35°C).

ANALYSIS:
The closed system entropy balance can be used to determine whether process 1-2 can occur as indicated. The closed system entropy balance is

$$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma$$

The amount of entropy production indicates whether a process is possible. Solving for entropy production

$$\sigma = S_2 - S_1 - \int_1^2 \left(\frac{\delta Q}{T}\right)_b = m(s_2 - s_1) - \int_1^2 \left(\frac{\delta Q}{T}\right)_b$$

The heat transfer associated with process 1-2 corresponding to the claimed measured work, 42.4 kJ, can be determined from a closed system energy balance. The energy balance for a closed system is

$$\Delta KE + \Delta PE + \Delta U = Q - W$$

PROBLEM 6.51 (CONTINUED)

Ignoring kinetic and potential energy changes, expressing the energy balance for process 1-2, and rewriting $\Delta U = m(u_2 - u_1)$ give

$$m(u_2 - u_1) = Q_{12} - W_{12}$$

Solving for heat transfer

$$Q_{12} = m(u_2 - u_1) + W_{12}$$

States 1 and 2, both superheated vapor, are defined by pressure and temperature. Values for internal energy and entropy for propane are obtained from Table A-18.
State 1: $u_1 = 496.7$ kJ/kg; $s_1 = 1.930$ kJ/(kg·K)
State 2: $u_2 = 458.8$ kJ/kg; $s_2 = 1.971$ kJ/(kg·K)

Substituting values and solving for heat transfer give

$$Q_{12} = (1\ \text{kg})\left(458.8\frac{\text{kJ}}{\text{kg}} - 496.7\frac{\text{kJ}}{\text{kg}}\right) + 42.4\ \text{kJ} = \underline{\mathbf{4.5\ kJ}}$$

The positive sign associated with heat transfer indicates heat transfer is into the system.

Since heat transfer occurs at only one boundary temperature, integration of $\int_1^2\left(\frac{\delta Q}{T}\right)_b$ gives $\frac{Q_{12}}{T_b}$.
Substituting, the relationship for entropy production becomes

$$\sigma = m(s_2 - s_1) - \frac{Q_{12}}{T_b}$$

where $T_b = 308$ K (35°C). Substituting values gives

$$\sigma = (1\ \text{kg})\left(1.971\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 1.930\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right) - \left(\frac{4.5\ \text{kJ}}{308\ \text{K}}\right) = 0.0264\ \text{kJ/K}$$

Since entropy production is positive, the work measurement is possible.

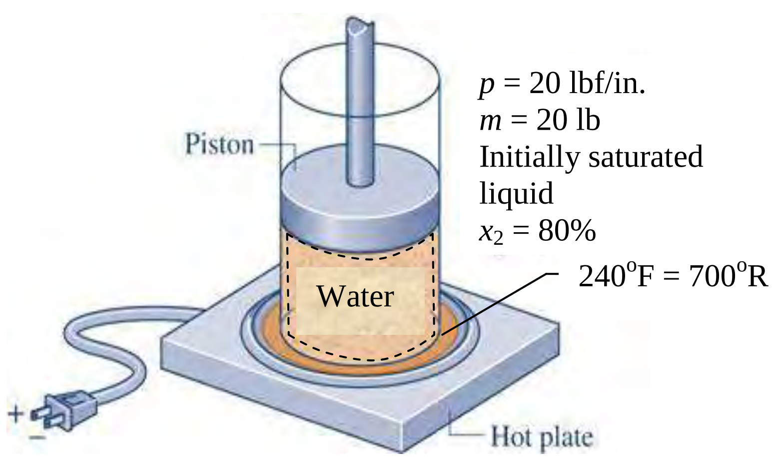
PROBLEM 6.52

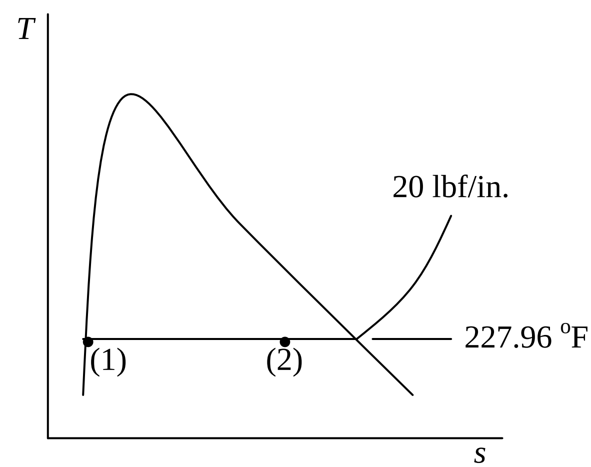
Figure P6.52 shows a piston-cylinder assembly containing 20 lb of water, initially a saturated liquid at 20 lbf/in.^{2} in contact with a hot plate. Heat transfer occurs slowly from the hot plate to the contents of the cylinder, and the pressure of the water remains nearly constant as phase change occurs. The process continues until the quality is 80%. There is no significant heat transfer across the vertical surface of the cylinder or to the piston, and kinetic and potential energy effects are negligible.
(a) For the water as the system, determine the work and heat transfer, each in Btu.
(b) Consider an enlarged system that includes the bottom of the piston-cylinder assembly wall in contact with the hot plate such that the boundary temperature is 240°F. Neglecting any change of state of the cylinder wall material, calculate the entropy production for the enlarged system, in Btu/°R.

KNOWN: Water in a piston-cylinder assembly is heated at constant pressure on a hot plate. Initially, the water is saturated liquid and the final quality is known.

FIND: For the water as the system, determine the work, heat transfer and the amount of entropy produced. Consider an enlarged system that includes the bottom of the cylinder in contact with the hot plate and calculate the entropy production for the enlarged system.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL: (1) For part (a), the water is the closed system. For part (b), the system includes the bottom of the piston-cylinder assembly and the boundary temperature is 240°F. (2) There is no significant heat transfer across the vertical surface of the piston-cylinder assembly or to the piston. (3) Kinetic and potential energy effects are negligible. (4) Neglect any change of state of the cylinder wall material in contact with the hot plate.

ANALYSIS: (a) For the water as the system, the work is evaluated as follows. Since the pressure is constant

PROBLEM 6.52 (CONTINUED)

$$W = \int_{V_1}^{V_2} p\,dV = mp(v_2 - v_1)$$

From Table A-3E: p = 20 lbf/in.^{2}; $v_1 = v_f$ = 0.01683 ft^3/lb and

$$v_2 = v_f + x_2(v_g - v_f) = 0.01683 + (0.8)(20.09 - 0.01683) = 16.075 \text{ ft}^3/\text{lb}$$

Thus

$$W = (20 \text{ lb})(20 \text{ lbf/in.}^2)(16.075 - 0.01683) \text{ ft}^3/\text{lb}\left|\frac{1 \text{ Btu}}{778 \text{ ft·lbf}}\right|\left|\frac{144 \text{ in.}^2}{1 \text{ ft}^2}\right| = 1188.9 \text{ Btu (out)} \longleftarrow$$

With model assumption (3), the heat transfer is

$$Q = m(u_2 - u_1) + W$$

Again, referring to Table A-3E at p = 20 lbf/in.^{2}: $u_1 = u_f$ = 196.19 Btu/lb and

$$u_2 = u_f + x_2(u_g - u_f) = = 196.19 + (0.8)(1082.0 - 196.19) = 904.84 \text{ Btu/lb}$$

Thus

$$Q = (20 \text{ lb})(904.84 - 196.19)\text{Btu/lb} + (1188.9 \text{ Btu}) = 15{,}362 \text{ Btu (in)} \longleftarrow$$

(b) The entropy produced is determined using the entropy balance:

$$m(s_2 - s_1) = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma$$

Since $T_b = T_{\text{hot plate}}$, the entropy production is

$$\sigma = m(s_2 - s_1) - \frac{Q}{T_{\text{hot plate}}}$$

The pressure is constant and the entropies are $s_1 = s_f$ = 0.3358 Btu/lb·^{o}R and s_2 = 0.3358 + (0.8)(1.7320 – 0.3358) = 1.4528 Btu/lb·^{o}R. Thus

$$\sigma = m(s_2 - s_1) - \frac{Q}{T_{\text{hot plate}}} = (20)(1.4528 - 0.3358) - (15{,}362)/(700) = 0.3943 \text{ Btu/}^o\text{R} \longleftarrow$$

In this case, the enlarged system has positive entropy production because it includes the irreversibility of the heat transfer across the bottom of the cylinder due to the fact that the temperature of the hot plate is greater than that of the water.

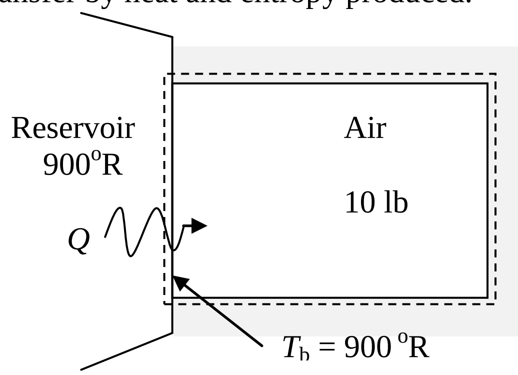
PROBLEM 6.53

A system consisting of 10 lb of air contained within a closed, rigid tank is initially at 1 atm and 600°R. Energy is transferred to the system by heat transfer from a reservoir at 900°R until the temperature of the air is 800°R. During the process, the temperature of the system boundary where the heat transfer occurs is 900°R. Using the ideal gas model, determine the amount of energy transfer by heat, in Btu, and the amount of entropy produced, in Btu/°R.

KNOWN: A system consisting of a fixed volume of air is heated from a specified initial state to a specified final temperature. Heat transfer occurs from a reservoir at a given temperature and the boundary temperature where the heat transfer occurs is known.

FIND: Determine the amounts of energy transfer by heat and entropy produced.

SCHEMATIC AND GIVEN DATA:



$T_1 = 600°R$
$p_1 = 1$ atm

$T_2 = 800°R$

ENGINEERING MODEL: (1) The air is a closed system. (2) The air is modeled as an ideal gas. (3) The volume is constant and $W = 0$. (4) The boundary temperature where heat transfer occurs is $T_b = 900°R$. (5) Kinetic and potential energy effects are negligible.

ANALYSIS: The energy balance reduces to give: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - \cancel{W}$. With data from Table A-22

$$Q = m\,(u_2 - u_1) = (10 \text{ lb})(136.97 - 102.34)\text{Btu/lb} = 346.3 \text{ Btu (in)} \longleftarrow$$

With modeling assumption 4, the entropy balance gives

$$\Delta S = Q/T_b + \sigma \quad \longrightarrow \quad \sigma = m\,(s_2 - s_1) - Q/T_b$$

For the air as an ideal gas; $(s_2 - s_1) = s^o(T_2) - s^o(T_1) - R \ln (p_2/p_1)$. Since the volume is constant, $p_2/p_1 = T_2/T_1$. Thus

$$\sigma = m\,[s^o(T_2) - s^o(T_1) - R \ln (T_2/T_1)] - Q/T_b$$

$$= (10 \text{ lb})[(0.69558 - 0.62607 - (1.986/28.97) \ln(800/600)]\text{Btu/lb}\cdot°\text{R} \; - (346.3 \text{ Btu}/900°\text{R})$$

$$= \; 0.1131 \text{ Btu}/°\text{R} \longleftarrow$$

PROBLEM 6.54

An inventor claims that the device shown generates electricity while receiving a heat transfer at the rate of 250 Btu/s at a temperature of 500°R, a second heat transfer at the rate of 350 Btu/s at 700°R, and a third at the rate of 500 Btu/s at 1000°R. For operation at steady state, evaluate this claim.

SCHEMATIC & GIVEN DATA:

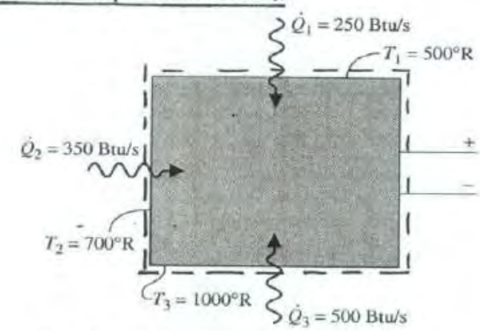


Fig. P6.53

ENGR. MODEL:

1. The system is shown by the dashed line on the sketch.
2. The system is at steady state.
3. Heat transfer rates are each positive in the direction of the accompanying arrow.

ANALYSIS: Applying Eq. 6.28,

$$\cancel{\frac{dS}{dt}}^{\,0} = \frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} + \frac{\dot{Q}_3}{T_3} + \dot{\sigma}$$

$$\Rightarrow \quad \dot{\sigma} = -\left[\frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} + \frac{\dot{Q}_3}{T_3}\right]$$

$$= -\left[\frac{250\text{ Btu/s}}{500°R} + \frac{350\text{ Btu/s}}{700°R} + \frac{500\text{ Btu/s}}{1000°R}\right]$$

$$= -\left[0.5 + 0.5 + 0.5\right]\frac{\text{Btu/s}}{°R}$$

$$= -1.5\ \frac{\text{Btu/s}}{°R}$$

Since $\dot{\sigma} < 0$, the claim cannot be valid.

PROBLEM 6.55

KNOWN: Data are provided for a silicon chip at steady state.

FIND: Determine the rate of entropy production and identify the principal source of irreversibility.

SCHEMATIC & GIVEN DATA: See Fig. E 2.5.

ENGR. MODEL: 1. The chip is a closed system at steady state. 2. There is no heat transfer between the chip and the substrate. All heat transfer is by convection from the top surface at T_b.

ANALYSIS: An entropy rate balance at steady state reads

$$\cancelto{0}{\frac{dS}{dt}} = \frac{\dot{Q}}{T_b} + \dot{\sigma} \Rightarrow \dot{\sigma} = -\frac{\dot{Q}}{T_b} \quad \text{where} \quad \dot{Q} = -0.225\ \text{W (from Example 2.5)}.$$

Thus

$$\dot{\sigma} = \frac{-(-0.225)}{353\ \text{K}} = 6.37 \times 10^{-4}\ \frac{\text{watt}}{\text{K}} \left| \frac{1\ \text{kW}}{10^3\ \text{W}} \right| = 6.37 \times 10^{-7}\ \frac{\text{kW}}{\text{K}} \longleftarrow$$

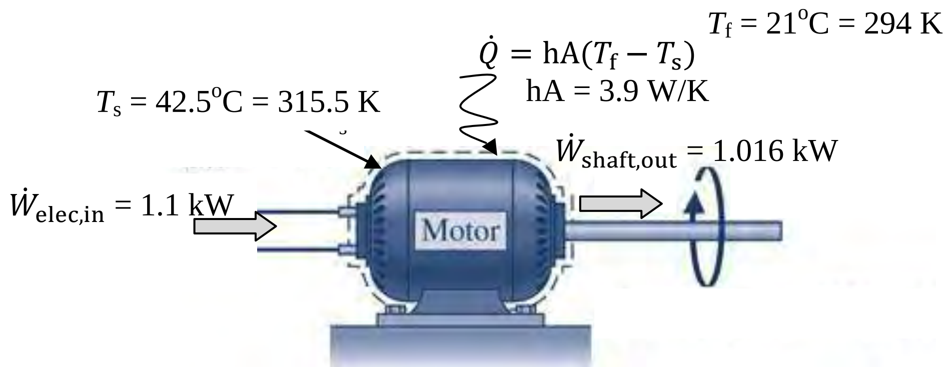
The principal source of internal irreversibility is electric current flow through a resistance.

PROBLEM 6.56

Data are provided for steady-state operation of an electric motor in Fig. P6.56. Determine for the motor the rate of entropy production, in kW/K. Repeat for an enlarged system boundary such that the heat transfer occurs in the nearby surroundings at T_f = 21°F.

KNOWN: Operating data are provided for an electric motor at steady state.

FIND: Determine (a) the electric power required, (b) the power developed by the output shaft, and (c) average the surface temperature.



ENGINEERING MODEL: (1) The motor is the closed system. (2) The system is at steady state.

ANALYSIS: For the motor as the system, the closed system entropy rate balance reduces as follows:

$$\cancel{\frac{dS}{dt}} = \frac{\dot{Q}}{T_s} + \dot{\sigma} \quad \longrightarrow \quad \dot{\sigma} = -\frac{\text{hA}(T_f - T_s)}{T_s} = \frac{\left(3.9\frac{\text{kW}}{\text{K}}\right)(315.5 - 294)\text{K}}{315.5 \text{ K}} = 0.2658 \text{ kW/K} \longleftarrow$$

For an enlarged control volume, the heat transfer rate is unchanged, but the boundary temperature is T_f. The entropy production rate becomes

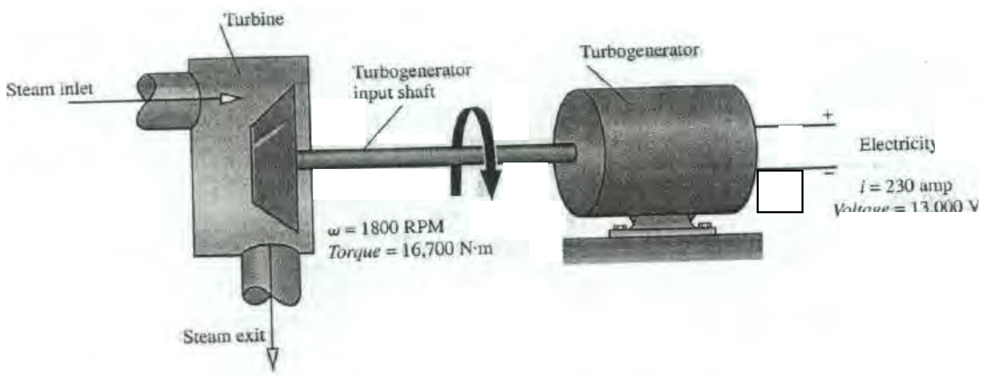
$$\dot{\sigma}_{\substack{\text{enlarged}\\ \text{system}}} = -\frac{\text{hA}(T_f - T_s)}{T_f} = \frac{\left(3.9\frac{\text{kW}}{\text{K}}\right)(315.5 - 294)\text{K}}{294 \text{ K}} = 0.2852 \text{ kW/K} \longleftarrow$$

The entropy production rate increases because the enlarged system boundary includes the irreversibility associated with the heat transfer from the surface of the motor to the surroundings, which is not included in original system.

PROBLEM 6.57

A power plant has a turbogenerator operating at steady state with an input shaft rotating at 1800 RPM with a torque of 16,700 N·m. The turbogenerator produces current at 230 amp with a voltage of 13,000 V. The rate of heat transfer between the turbogenerator and its surroundings is related to the surface temperature T_b and the lower ambient temperature T_0, and is given by $\dot{Q} = -hA(T_b - T_0)$, where h = 110 W/m^2·K, A = 32 m^2, and T_0 = 298 K.

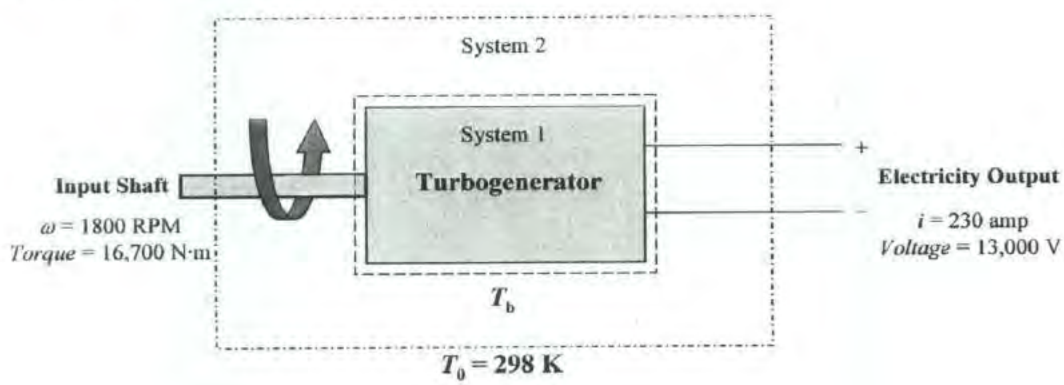
(a) Determine the temperature T_b, in K.
(b) For the turbogenerator as the system, determine the rate of entropy production, in kW/K.
(c) If the system boundary is located to take in enough of the nearby surroundings for heat transfer to take place at temperature T_0, determine the rate of entropy production, in kW/K, for the enlarged system.



KNOWN: A turbogenerator operates at steady state with specified operating data.

FIND: Boundary temperature and rate of entropy generation for a system consisting of the turbogenerator and for an enlarged system also including the nearby surroundings.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

1. The turbogenerator operates at steady state and is considered as System 1.
2. System boundary temperature T_b is uniform.
3. The turbogenerator with its nearby surroundings is considered as System 2.

ANALYSIS:

(a) The energy rate balance for a closed system can be used to determine the rate of heat transfer, which is needed to determine the boundary temperature, T_b. The energy rate balance for a closed system is

$$\frac{dE}{dt} = \dot{Q} - \dot{W}$$

Since the process occurs at steady state, the left-hand term is zero. Power associated with the system consists of shaft power into the turbogenerator and electric power from the turbogenerator. With these substitutions, the energy rate balance for a closed system at steady state becomes

$$0 = \dot{Q} - \left(\dot{W}_{shaft} + \dot{W}_{electric}\right)$$

Solving for rate of heat transfer

$$\dot{Q} = \dot{W}_{shaft} + \dot{W}_{electric}$$

Shaft power can be determined using Eq. 2.20, written with a minus sign since the shaft power is *into* the turbogenerator.

$$\dot{W}_{shaft} = -\mathscr{T}\omega$$

Substituting values

$$\dot{W}_{shaft} = -(16{,}700\ \text{N}\cdot\text{m})\left(1800\,\frac{\text{rev}}{\text{min}}\right)\left(2\pi\,\frac{\text{rad}}{\text{rev}}\right)\left|\frac{\text{min}}{60\ \text{s}}\right|\left|\frac{\text{kJ}}{1000\ \text{N}\cdot\text{m}}\right|\left|\frac{\text{kW}}{\frac{\text{kJ}}{\text{s}}}\right| = -3{,}148\ \text{kW}$$

Electric power can be determined using Eq. 2.21, written with a positive sign since the electric power is *from* the system.

$$\dot{W}_{electric} = \mathscr{E}\,i$$

Substituting values

$$\dot{W}_{electric} = (13{,}000\ \text{volts})(230\ \text{amperes})\left|\frac{1\,\frac{\text{watt}}{\text{ampere}}}{1\ \text{volt}}\right|\left|\frac{\text{kW}}{1000\ \text{W}}\right| = 2{,}990\ \text{kW}$$

Substituting values for power and solving for rate of heat transfer yield

$$\dot{Q} = -3{,}148\ \text{kW} + 2{,}990\ \text{kW} = -158\ \text{kW}$$

The negative sign indicates heat transfer from the system. Since the rate of heat transfer between the turbogenerator and its surroundings is $\dot{Q} = -\text{hA}(T_b - T_0)$, we get

$$-158\ \text{kW} = -\text{h}A(T_b - T_0)$$

Solving for T_b and substituting values

$$T_b = T_0 + \frac{158\ \text{kW}}{\text{hA}}$$

$$T_b = 298\ \text{K} + \frac{158\ \text{kW}}{\left(110\ \frac{\text{W}}{\text{m}^2 \cdot \text{K}}\right)(32\ \text{m}^2)} \left|\frac{1000\ \text{W}}{\text{kW}}\right| = \underline{\mathbf{342.9\ K}}$$

(b) The rate of entropy production for the turbogenerator as the system (System 1) can be determined from the closed system entropy rate balance

$$\frac{dS}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{\sigma}_{cv}$$

Since the process occurs at steady state, the left-hand term is zero. Since heat transfer with the surroundings occurs only at $T_b = 342.9$ K, the first term, $\sum_j \frac{\dot{Q}_j}{T_j}$, becomes $\frac{\dot{Q}}{T_b}$. Solving for the rate of entropy generation yields

$$\dot{\sigma}_{cv} = -\frac{\dot{Q}}{T_b}$$

Substituting values gives

$$\dot{\sigma}_{cv} = -\left(\frac{-158\ \text{kW}}{342.9\ \text{K}}\right) = \underline{\mathbf{0.4608\ kW/K}}$$

(c) The rate of entropy production for the turbogenerator with its nearby surroundings as the system (System 2) can be determined from the closed system entropy rate balance

$$\frac{dS}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{\sigma}_{cv}$$

Since the process occurs at steady state, the left-hand term is zero. Since heat transfer with the surroundings now occurs only at $T_0 = 298$ K, the first term, $\sum_j \frac{\dot{Q}_j}{T_j}$, becomes $\frac{\dot{Q}}{T_0}$. Solving for the rate of entropy generation for the enlarged system yields

$$\dot{\sigma}_{cv} = -\frac{\dot{Q}}{T_0}$$

Substituting values gives

$$\dot{\sigma}_{cv} = -\left(\frac{-158\ \text{kW}}{298\ \text{K}}\right) = \underline{\mathbf{0.5302\ kW/K}}$$

The rate of entropy generation is greater for the enlarged system due to additional sources of irreversibility associated with heat transfer from the turbogenerator at T_b to the surroundings at T_0.

PROBLEM 6.58

A 33.8-lb aluminum bar, initially at 200°F, is placed in a tank together with 249 lb of liquid water, initially at 70°F, and allowed to achieve thermal equilibrium. The aluminum bar and water can be modeled as incompressible with specific heats 0.216 Btu/lb · °R and 0.998 Btu/lb · °R, respectively. For the aluminum bar and water as the system, determine (a) the final temperature, in °F, and (b) the amount of entropy produced within the tank, in Btu/°R. Ignore heat transfer between the system and its surroundings.

KNOWN: An aluminum bar is quenched in a tank of water.

FIND: Determine the final temperature and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:

Aluminum bar:
$m_a = 33.8$ lb
$T_a = 660°R$ (200°F)
water:
$m_w = 249$ lb
$T_w = 530°R$ (70°F)

ENGINEERING MODEL:

1. The closed system is the water plus bar. The total volume remains constant.
2. For the system, $Q = 0$, $W = 0$. Kinetic and potential energy play no role.
3. The water and bar are each modeled as incompressible with specific heats $c_a = 0.216$ Btu/lb·°R, $c_w = 0.998$ Btu/lb·°R.

ANALYSIS: (a) The energy balance reduces as follows: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - \cancel{W}$. Thus, $\Delta U]_{aluminum} + \Delta U]_{water} = 0$. Using Eq. 3.20a, we get

$$m_a c_a [T_f - T_a] + m_w c_w [T_f - T_w] = 0$$

Solving for T_f, the final temperature,

$$T_f = \frac{m_a c_a T_a + m_w c_w T_w}{m_a c_a + m_w c_w}$$

$$= \frac{(33.8\,lb)(0.216\,Btu/lb\cdot°R)(660°R) + (249\,lb)(0.998\,Btu/lb\cdot°R)(530°R)}{((33.8)(0.216) + (249)(0.998))\,Btu/°R}$$

$$= 533.7\ °R \quad (74°F)$$

(b) Applying an entropy balance, $\Delta S = \int_1^2 \delta Q/T + \sigma$. Or $\sigma = (\Delta S)_a + (\Delta S)_w$. With Eq. 6.13

$$\sigma = m_a c_a \ln\frac{T_f}{T_a} + m_w c_w \ln\frac{T_f}{T_w}$$

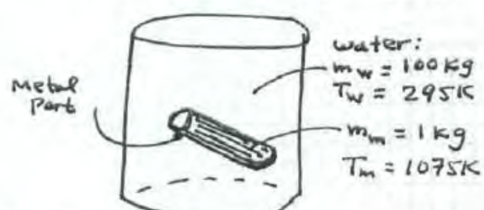
$$= (33.8\,lb)\left(0.216\,\frac{Btu}{lb\cdot °R}\right)\ln\left[\frac{533.7°R}{660°R}\right] + (249)(0.998)\ln\left[\frac{533.7}{530}\right]$$

$$= 0.1789\ \frac{Btu}{°R}$$

PROBLEM 6.59

In a heat-treating process, a 1-kg metal part, initially at 1075 K, is quenched in a tank containing 100 kg of water, initially at 295 K. There is negligible heat transfer between the contents of the tank and their surroundings. Taking the specific heat of the metal part and water as constant at 0.5 kJ/kg · K and 4.2 kJ/kg · K, respectively, determine (a) the final equilibrium temperature after quenching, in K, and (b) the amount of entropy produced within the tank, in kJ/K.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The closed system is the water plus metal part. Total volume is constant.
2. For the system, $Q = 0$, $W = 0$. Kinetic and potential energy play no role.
3. The water and metal part are each modeled as incompressible with specific heats $c_m = 0.5$ kJ/kg·K, $c_w = 4.2$ kJ/kg·K, respectively.

ANALYSIS: (a) The energy balance reduces as follows: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - \cancel{W}$, or $\Delta U]_{water} + \Delta U]_{metal} = 0$. Using Eq. 3.20a we get

$$m_m c_m [T_f - T_m] + m_w c_w [T_f - T_w] = 0$$

Solving for T_f, the final temperature,

$$T_f = \frac{m_m c_m T_m + m_w c_w T_w}{m_m c_m + m_w c_w}$$

$$= \frac{(1\,kg)(0.5\,kJ/kg\cdot K)(1075\,K) + (100\,kg)(4.2\,kJ/kg\cdot K)(295\,K)}{(1\,kg)(0.5\,kJ/kg\cdot K) + (100\,kg)(4.2\,kJ/kg\cdot K)}$$

$$= 295.9\ K \qquad \longleftarrow T_f$$

(b) An entropy balance reduces as follows: $\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{\,0} + \sigma$. Or $\sigma = (\Delta S)_m + (\Delta S)_w$. With Eq. 6.13,

$$\sigma = m_w c_w \ln\frac{T_f}{T_w} + m_m c_m \ln\frac{T_f}{T_m}$$

$$= (100\,kg)\left(4.2\,\frac{kJ}{kg\cdot K}\right)\ln\left(\frac{295.9}{295}\right) + (1\,kg)\left(0.5\,\frac{kJ}{kg\cdot K}\right)\ln\left(\frac{295.9}{1075}\right)$$

$$= 0.615\ \frac{kJ}{K} \qquad \longleftarrow \sigma$$

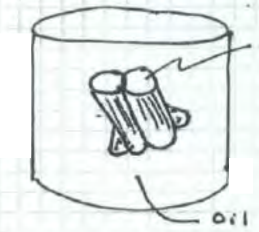
PROBLEM 6.60

A 50-lb iron casting, initially at 700°F, is quenched in a tank filled with 2121 lb of oil, initially at 80°F. The iron casting and oil can be modeled as incompressible with specific heats 0.10 Btu/lb · °R, and 0.45 Btu/lb · °R, respectively. For the iron casting and oil as the system, determine (a) the final equilibrium temperature, in °F, and (b) the amount of entropy produced within the tank, in Btu/°R. Ignore heat transfer between the system and its surroundings.

KNOWN: An iron casting is quenched in a tank filled with oil.

FIND: Determine the final temperature and the amount of entropy produced.

SCHEMATIC & GIVEN DATA



$m_i = 50$ lb
$T_i = 1160$°R (700°F)
$c_i = 0.10$ Btu/lb·°R

$m_o = 2112$ lb
$T_o = 540$°R (80°F)
$c_o = 0.45$ Btu/lb·°R

ENGINEERING MODEL:

1. The closed system is the oil plus casting. The total volume remains constant.
2. For the system, $Q = 0$, $W = 0$. Kinetic and potential energy play no role.
3. The oil and casting are each modeled as incompressible with specific heats 0.45 Btu/lb·°R; 0.10 Btu/lb·°R.

ANALYSIS: (a) The energy balance reduces as follows: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - \cancel{W}$.

Thus, $\Delta U]_{iron} + \Delta U]_{oil} = 0$. Using Eq. 3.20a, we get

$$m_i c_i [T_f - T_i] + m_o c_o [T_f - T_o] = 0$$

Solving for T_f, the final temperature,

$$T_f = \frac{m_i c_i T_i + m_o c_o T_o}{m_i c_i + m_o c_o}$$

$$= \frac{(50\,\text{lb})(0.1\,\text{Btu/lb·°R})(1160\,\text{°R}) + (2112\,\text{lb})(0.45\,\text{Btu/lb·°R})(540\,\text{°R})}{[(50\,\text{lb})(0.1\,\text{Btu/lb·°R}) + (2121\,\text{lb})(0.45\,\text{Btu/lb·°R})]}$$

$$= 543.2\,\text{°R}\;(83\,\text{°F})$$

(b) Applying the entropy balance, $\Delta S = \int_1^2 \cancel{\delta Q}^{\,0}/T + \sigma$. or

$\sigma = \Delta S]_i + \Delta S]_o$. With Eq. 6.13,

$$\sigma = m_i c_i \ln \frac{T_f}{T_i} + m_o c_o \ln \frac{T_f}{T_o}$$

$$= (50\,\text{lb})\left(0.10\,\frac{\text{Btu}}{\text{lb·°R}}\right) \ln\left(\frac{543.2}{1160}\right) + (2121)(0.45) \ln\left(\frac{543.2}{540}\right)$$

$$= 1.8378\,\text{Btu/°R}$$

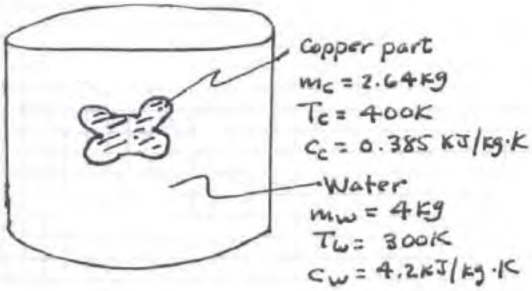
PROBLEM 6.61

A 2.64-kg copper part, initially at 400 K, is plunged into a tank containing 4 kg of liquid water, initially at 300 K. The copper part and water can be modeled as incompressible with specific heats 0.385 kJ/kg · K and 4.2 kJ/kg · K, respectively. For the copper part and water as the system, determine (a) the final equilibrium temperature, in K, and (b) the amount of entropy produced within the tank, in kJ/K. Ignore heat transfer between the system and its surroundings.

KNOWN: A copper part is quenched in a tank filled with water.

FIND: Determine the final temperature and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The closed system is the water plus copper part. Total volume remains constant.
2. For the system $Q = 0$, $W = 0$. Kinetic and potential energy play no role.
3. The water and copper part are each modeled as incompressible with specific heats 4.2 kJ/kg·K, 0.385 kJ/kg·K

ANALYSIS: (a) The energy balance reduces as follows: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - \cancel{W}$. Thus, $\Delta U]_{copper} + \Delta U]_{water} = 0$. Using Eq. 3.20a, we get

$$m_c c_c [T_f - T_c] + m_w c_w [T_f - T_w] = 0$$

Solving for T_f, the final temperature,

$$T_f = \frac{m_c c_c T_c + m_w c_w T_w}{m_c c_c + m_w c_w}$$

$$= \frac{(2.64\text{ kg})(0.385\text{ kJ/kg·K})(400\text{K}) + (4\text{ kg})(4.2\text{ kJ/kg·K})(300\text{K})}{(2.64\text{ kg})(0.385\text{ kJ/kg·K}) + (4\text{ kg})(4.2\text{ kJ/kg·K})}$$

$$= 305.6\text{ K}$$

(b) Applying the entropy balance, $\Delta S = \int_1^2 \cancel{\frac{\delta Q}{T}}^{\,0} + \sigma$. or

$\sigma = \Delta S]_c + \Delta S]_w$. With Eq 6.13

$$\sigma = m_c c_c \ln \frac{T_f}{T_c} + m_w c_w \ln \frac{T_f}{T_w}$$

$$= (2.64\text{ kg})(0.385 \frac{\text{kJ}}{\text{kg·K}}) \ln \frac{305.6}{400} + (4\text{ kg})(4.2 \frac{\text{kJ}}{\text{kg·K}}) \ln \frac{305.6}{300}$$

$$= 0.037 \frac{\text{kJ}}{\text{kg}}$$

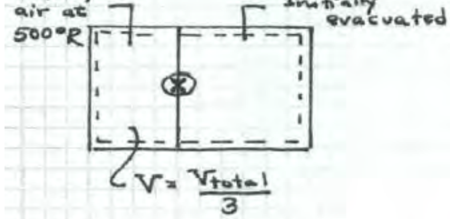
PROBLEM 6.62

A rigid, insulated vessel is divided into two compartments connected by a valve. Initially, one compartment, occupying one-third of the total volume, contains air at 500°R, and the other is evacuated. The valve is opened and the air is allowed to fill the entire volume. Assuming the ideal gas model, determine the final temperature of the air, in °R, and the amount of entropy produced, in Btu/°R per lb of air.

KNOWN: A rigid, insulated vessel is divided into two compartments, one initially contains air and the other is evacuated. An interconnecting valve is opened and air eventually fills the total volume.

FIND: Determine the final temperature of the air and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The system is the region within the tank, including its contents.
2. For the system, $Q = 0$, $W = 0$ (negligible work required to open the valve).
3. No overall changes in kinetic or potential energy.
4. The air is modeled as an ideal gas.

ANALYSIS:

Applying an energy balance: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - \cancel{W}$

$\Rightarrow \Delta U = m(u_2 - u_1) = 0 \quad \Rightarrow \quad T_2 = T_1$ (assumption 4).

Applying an entropy balance $\quad \Delta S = \int \frac{\delta Q}{T_b}^{\,0} + \sigma \quad$ with Eq. 6.20a:

$$\Rightarrow \sigma = \Delta S$$

$$= m\left[\underbrace{s^\circ(T_2) - s^\circ(T_1)}_{\substack{=0 \\ (T_1 = T_2)}} - R \ln \frac{P_2}{P_1}\right]$$

Applying the ideal gas equation of state:

$P_1 V = m R T_1$

$P_2 V_{tot} = m R T_2$

$V = V_{tot}/3$ and $T_1 = T_2$

$\Rightarrow P_2/P_1 = 1/3$

$$\therefore \sigma = -R \ln \frac{P_2}{P_1}$$

$$= -\frac{1.986}{28.97} \frac{\text{Btu}}{\text{lb} \cdot {}^\circ\text{R}} \ln\left(\frac{1}{3}\right)$$

① $$= 0.075 \frac{\text{Btu}}{\text{lb} \cdot {}^\circ\text{R}}$$

1. Entropy is produced in an unrestrained expansion of a gas to a lower pressure.

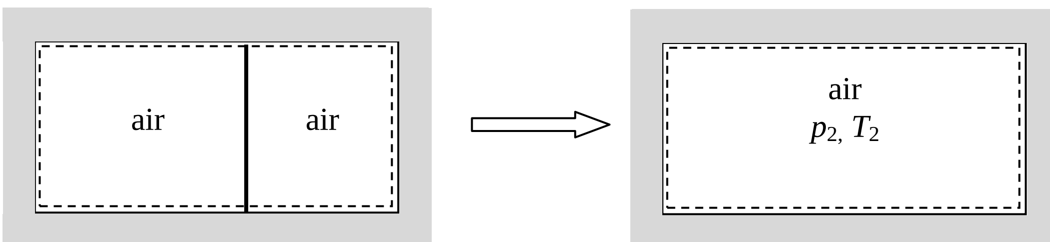
PROBLEM 6.63

A rigid, well-insulated tank contains air. A partition in the tank separates 12 ft^3 of air at 14.7 lbf/in.^{2}, 40^{o}F from 10 ft^3 of air at 50 lbf/in.^{2}, 200^{o}F, as illustrated in fig. P6.63. The partition is removed and air from the two sides mix until a final equilibrium state is attained. The air can be modeled as an ideal gas, and kinetic and potential energy effects can be neglected. Determine the final temperature, in ^{o}F and pressure, in lbf/in.^{2} Calculate the amount of entropy produced, in Btu/^{o}R.

KNOWN: Air is contained in a rigid, well-insulated tank on two sides of a partition. The initial states of the air on each side are specified. The partition is removed and equilibrium is attained.

FIND: Determine the final temperature and pressure and the amount of entropy produced.

SCHEMATIC AND GIVEN DATA:



V_L = 12 ft^3 $\quad$ V_R = 10 ft^3
p_L = 14.7 lbf/in.^{2} $\quad$ P_R = 50 lbf/in.^{2}
T_L = 40^{o}F = 500^{o}R $\quad$ T_R = 200^{o}F = 660^{o}R

ENGINEERING MODEL: (1) The air on both sides of the partition are a closed system. (2) The air is modeled as an ideal gas. (3) Kinetic and potential energy effects can be neglected. (4) There is no heat transfer from the contents of the tank to the surroundings and $W = 0$.

ANALYSIS: First, we use the ideal gas equation of state to calculate the masses on the left and right sides of the partition, respectively.

$$m_L = \frac{p_L V_L}{R T_L} = \frac{\left(14.7\frac{\text{lbf}}{\text{in.}^2}\right)(12\text{ ft}^3)}{\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^o\text{R}}\right)(500^o\text{R})}\left|\frac{144\text{in.}^2}{1\text{ ft}^2}\right| = 0.9526 \text{ lb}$$

and

$$m_R = \frac{p_R V_R}{R T_R} = \frac{(50)(10)}{\left(\frac{1545}{28.97}\right)(660)}\left|\frac{144}{1}\right| = 2.0455 \text{ lb}$$

$$m_{tot} = 2.9981 \text{ lb}$$

Now, using the energy balance: $\Delta \cancel{KE} + \Delta \cancel{PE} + \Delta U = \cancel{Q} - \cancel{W} \longrightarrow \boxed{\Delta U = 0}$

Thus

$$U_1 = m_L u_L + m_R u_R \quad \text{and } U_2 = m_{tot} u_2$$

So

$$u_2 = (m_L u_L + m_R u_R)/m_{tot}$$

PROBLEM 6.61 (CONTINUED)

With data from Table A-22: u_L = 85.20 Btu/lb and u_R = 112.67 Btu/lb

$$u_2 = [(0.9526 \text{ lb})(85.20 \text{ Btu/lb}) + (2.0455)(112.67)]/(2.9981 \text{ lb}) = 103.94 \text{ Btu/lb}$$

Interpolating in Table A-22: $T_2 \approx 609.3^{o}R = 149.3^{o}F$ ←

The final pressure is

$$p_2 = \frac{m_{tot}RT_2}{V_{tot}} = \frac{(2.9981 \text{ lb})\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^{o}\text{R}}\right)(609.3^{o}\text{R})}{22 \text{ ft}^3}\left|\frac{1\text{ft}^2}{144 \text{ in.}^2}\right| = 30.75 \text{ lbf/in.}^2 \leftarrow$$

Now, the entropy balance reduces to: $\Delta S = \cancel{\int_1^2\left(\frac{\delta Q}{T}\right)_b} + \sigma \quad \longrightarrow \quad \sigma = \Delta S = m_L(s_2 - s_L) + m_R(s_2 - s_R)$

$$\sigma = \Delta S = m_L[s^o(T_2) - s^o(T_L) - R \ln (p_2/p_L)] + m_R[s^o(T_2) - s^o(T_R) - R \ln (p_2/p_R)]$$

From Table A-22: $s^o(T_L)$ = 0.58233 Btu/lb·°R, $s^o(T_R)$ = 0.64902 Btu/lb·°R, and $s^o(T_2) \approx$ 0.62973 Btu/lb·°R
Thus

$$\sigma = (0.9526 \text{ lb})[(0.62973 - 0.58233) \text{ Btu/lb}\cdot{}^{o}\text{R} - (1545/28.97) \text{ Btu/lb}\cdot{}^{o}\text{R}\left|\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right|\ln(30.75/14.7)]$$

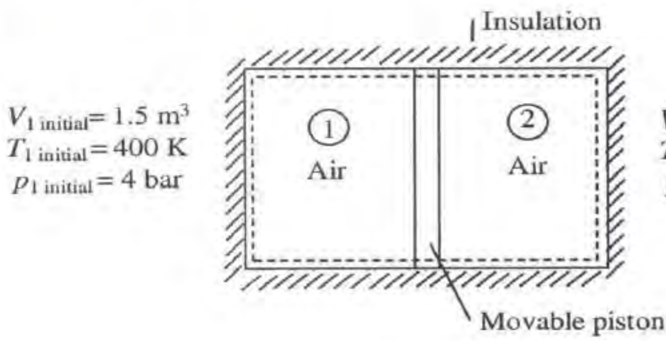
$$+ (2.0455)[(0.62973 - 0.64902) - (1545/28.97)\left|\frac{1}{778}\right|\ln(30.75/50)]$$

$$= 0.1056 \text{ Btu/}^{o}\text{R} \leftarrow$$

The entropy production indicates that the mixing process is irreversible.

PROBLEM 6.64

An insulated box is initially divided into halves by a frictionless, thermally conducting piston. On one side of the piston is 1.5 m^3 of air at 400 K, 4 bar. On the other side is 1.5 m^3 of air at 400 K, 2 bar. The piston is released and equilibrium is attained, with the piston experiencing no overall change in its state. Employing the ideal gas model for the air, determine the
(a) final temperature of the air, in K.
(b) final pressure of the air, in bar.
(c) the amount of entropy produced, in kJ/kg.



KNOWN: An insulated box is initially divided into halves by a frictionless, thermally conducting piston. Air with equal volumes and temperatures are on either side of the piston. The piston is released and equilibrium is attained, with the piston experiencing no change in its overall state.

FIND: Using the ideal gas model for the air, determine the final temperature, in K; the final pressure, in bar; and the amount of entropy produced, in kJ/kg.

SCHEMATIC AND GIVEN DATA:
See Fig. P6.66

ENGINEERING MODEL:
(1) The system consists of the piston and the total amount of air in the box.
(2) After the piston is released, equilibrium is attained, with the piston experiencing no change in state.
(3) The air is modeled as an ideal gas.
(4) The piston moves freely and is thermally conducting.
(5) For the system, $W=Q=0$ and there are no overall changes in kinetic or potential energy.

ANALYSIS:

(a) Applying the closed system energy balance with assumptions listed above, we get

PROBLEM 6.64 (CONTINUED)

$$\Delta U = Q - W = 0$$
$$\Delta U = \Delta U_1 + \Delta U_2 + \Delta U_{\text{piston}} \text{ where } \Delta U_{\text{piston}} = 0$$
$$\Delta U = m_1(u_{1\text{final}} - u_{1\text{initial}}) + m_2(u_{2\text{final}} - u_{2\text{initial}}) = 0$$

Since the initial temperature of each quantity of air is 400 K, we have $u_{1\text{initial}} = u_{2\text{initial}} = u(400\text{ K})$. At equilibrium we expect the final temperature of each quantity of air to be the same, and so $u_{1\text{final}} = u_{2\text{final}} = u(T_{\text{final}})$. Collecting results,

$$(m_1 + m_2)u(T_{\text{initial}}) = (m_1 + m_2)u(T_{\text{final}})$$

Thus $T_{\text{final}} = 400$ K, as expected.

(b) At equilibrium we also expect the final pressure of each quantity of air to be the same: $p_{\text{final}} = p_{1\text{final}} = p_{2\text{final}}$. Moreover, the total mass of air is $m = m_1 + m_2$ and the total volume of air is $V = V_1 + V_2 = 3\text{ m}^3$. Thus with the ideal gas equation of state

$$p_{\text{final}} = \frac{mRT_{\text{final}}}{V} = \frac{(m_1 + m_2)RT_{\text{final}}}{V} \tag{1}$$

where

$$m_1 = \frac{p_{1\text{initial}}V_{1\text{initial}}}{RT_{1\text{initial}}} = \frac{p_{1\text{initial}}\left(V/2\right)}{RT_{\text{final}}} \quad \text{and} \quad m_2 = \frac{p_{2\text{initial}}\left(V/2\right)}{RT_{\text{final}}} \tag{2}$$

Substituting Eq. (2) into Eq. (1):

$$p_{\text{final}} = \frac{p_{1\text{initial}}\left(V/2\right) + p_{2\text{initial}}\left(V/2\right)}{V} = \frac{p_{1\text{initial}} + p_{2\text{initial}}}{2} = \frac{4\text{ bar} + 2\text{ bar}}{2}$$
$$p_{\text{final}} = p_{1\text{final}} = p_{2\text{final}} = 3\text{ bar}$$

(c) To determine the amount of entropy produced, in kJ/kg, during the adiabatic process, an entropy balance reduces as follows:

$$\Delta S = \sigma$$
$$\sigma = \Delta S_1 + \Delta S_2 + \Delta S_{\text{piston}} \text{ where} \Delta S_{\text{piston}} = 0$$
$$\sigma = m_1(s_{1\text{final}} - s_{1\text{initial}}) + m_2(s_{2\text{final}} - s_{2\text{initial}})$$
$$\sigma = m_1\left(s^\circ_{1\text{final}} - s^\circ_{1\text{initial}} - R\ln\frac{p_{1\text{final}}}{p_{1\text{initial}}}\right) + m_2\left(s^\circ_{2\text{final}} - s^\circ_{2\text{initial}} - R\ln\frac{p_{2\text{final}}}{p_{2\text{initial}}}\right)$$

Since the initial and final temperatures are equal (400 K), the s° terms cancel and we get

$$\sigma = m_1\left(-R\ln\frac{p_{1\text{final}}}{p_{1\text{initial}}}\right) + m_2\left(-R\ln\frac{p_{2\text{final}}}{p_{2\text{initial}}}\right) \tag{3}$$

Substituting Eq. (2) into Eq. (3) and rearranging:

$$\sigma = -\frac{V}{2T_{\text{final}}}\left[p_{1\text{initial}}\left(\ln\frac{p_{1\text{final}}}{p_{1\text{initial}}}\right) + p_{2\text{initial}}\left(\ln\frac{p_{2\text{final}}}{p_{2\text{initial}}}\right)\right]$$

$$\sigma = -\frac{3\text{ m}^3}{2(400\text{ K})}\left[4\text{ bar}\left(\ln\frac{3}{4}\right) + 2\text{ bar}\left(\ln\frac{3}{2}\right)\right]\left|\frac{100\frac{\text{kN}}{\text{m}^2}}{1\text{bar}}\right|\left|\frac{1\text{ kJ}}{1\text{kN}\cdot\text{m}}\right| = 0.1274\frac{\text{kJ}}{\text{K}}$$

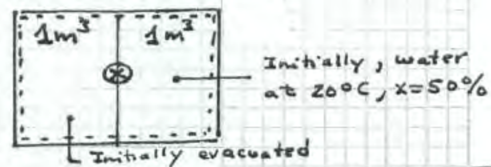
PROBLEM 6.65

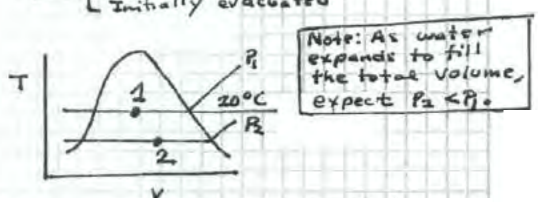
A rigid, insulated vessel is divided into two equal-volume compartments connected by a valve. Initially, one compartment contains 1 m^3 of water at 20°C, x = 50%, and the other is evacuated. The valve is opened and the water is allowed to fill the entire volume. For the water, determine the final temperature, in °C, and the amount of entropy produced, in kJ/K.

KNOWN: A rigid, insulated vessel is divided into two compartments, one initially contains water and the other is evacuated. An interconnecting valve is opened and water eventually fills the total volume.

FIND: Determine the final temperature of the water and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL:

1. The system is the region within the vessel, including its contents.
2. For the system $Q = 0$, $W = 0$ (negligible work required to open the valve).
3. There is no overall change in kinetic or potential energy.

ANALYSIS: Reducing the energy balance, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - \cancel{W}$

$\Rightarrow u_2 = u_1$ (1)

Also, since the volume of the water doubles, $V_2 = 2V_1$ (2)

With data from Table A-2,

$$v_1 = v_f + \underbrace{x_1}_{0.5}(v_g - v_f) = 0.5(v_f + v_g)$$

$$= 0.5\left(\frac{1.0018}{10^3} + 57.791\right) = 28.896 \text{ m}^3/\text{kg}$$

$u_1 = 0.5(u_f + u_g) = 0.5(83.95 + 2402.9) = 1243.4$ kJ/kg, $v_2 = 57.792$ m^3/kg.

Since $P_1 = 0.02339$ bar and $P_2 < P_1$, inspection of v_g values in Table A-2 using $v_2 = 57.792$ m^3/kg suggests that state 2 is also in the two-phase liquid vapor region. Collecting results,

$$\begin{cases} u_2 = 1243.4 \frac{\text{kJ}}{\text{kg}} = [u_f + x_2(u_g - u_f)] & (3) \\ v_2 = 57.792 \frac{\text{m}^3}{\text{kg}} = [v_f + x_2(v_g - v_f)] & (4) \end{cases}$$

where u_f, u_g, v_f, and v_g are evaluated at T_2 (or P_2).

*Steam Table Solution: Specify T_2, solve for x_2 using Eq(3) or Eq.(4). Check for closure with the other equation. This is left as an exercise.

IT Solution:

```
u2 = usat_Px("Water/Steam", p2, x2)
u2 = 1243.4
v2 = vsat_Px("Water/Steam", p2, x2)
v2 = 57.792
T2 = Tsat_P("Water/Steam", p2)
s2 = ssat_Px("Water/Steam", p2, x2)
```

SOLUTION:

T2 = 9.107 C ←
p2 = 0.01156 bar
x2 = 0.5132
s2 = 4.644 kJ/kg-K

$$\Delta S = \int_1^2 \cancelto{0}{\frac{\delta Q}{T}} + \sigma$$

$\Rightarrow \sigma = m(s_2 - s_1)$, where

$$m = \left(\frac{V}{v}\right)_2 = \left(\frac{2 \text{ m}^3}{57.792 \text{ m}^3/\text{kg}}\right) = 0.0346 \text{ kg}$$

$$s_1 = s_f + x_1(s_g - s_f) = 0.5(s_f + s_g)$$
$$= 0.5(0.2966 + 8.6672)$$
$$= 4.4819 \text{ kJ/kg·K}$$

Finally,

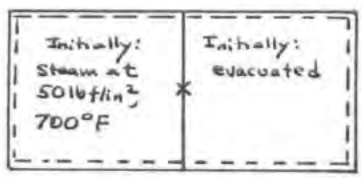
$$\sigma = [0.0346 \text{ kg}](4.644 - 4.4819)\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 0.0056 \text{ kJ/K} \leftarrow$$

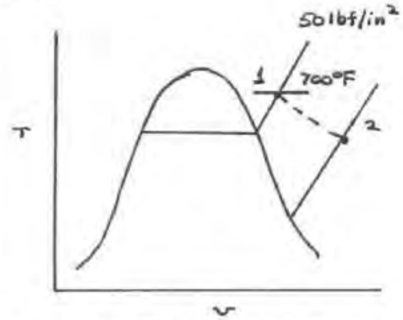
PROBLEM 6.66

KNOWN: An insulated vessel is divided into equal-sized compartments connected by a valve. Initially one compartment contains steam at a known state and the other is evacuated. The valve is opened and the steam fills the entire volume.

FIND: Determine (a) the final temperature and (b) the amount of entropy produced per unit mass of steam.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL: (1) The system is shown by the dashed line in the figure above. (2) For the system, $Q = 0$, $W = 0$, and kinetic/potential energy effects can be ignored.

ANALYSIS: To fix the final state requires the values of two independent intensive properties: u_2 and v_2. Thus, since the steam expands to fill twice the initial volume occupied by the steam, $V_2 = 2V_1$. And from an energy balance

$$\Delta U = \cancel{Q}^{\,0} - \cancel{W}^{\,0} \Rightarrow u_2 = u_1$$

An entropy balance reduces to

$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{\,0} + \sigma \Rightarrow \frac{\sigma}{m} = (s_2 - s_1) \qquad (1)$$

As the use of the pair of independent properties u_2, v_2 to fix the state with tabular steam table data is cumbersome, *IT* is employed to evaluate σ/m using Eq. (1):

IT Code

```
p1 = 50  // lbf/in.²
T1 = 700  // °F

u2 = u1
v2 = 2 * v1
u1 = u_PT("Water/Steam", p1, T1)
v1 = v_PT("Water/Steam", p1, T1)
u2 = u_PT("Water/Steam", p2, T2)
v2 = v_PT("Water/Steam", p2, T2)

sigma / m = s2 - s1
m = 1  // lb
s1 = s_PT("Water/Steam", p1, T1)
s2 = s_PT("Water/Steam", p2, T2)
```

IT Results

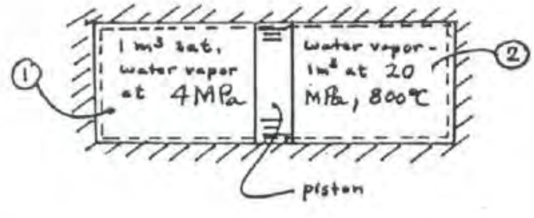
$T_2 = 697.4$ °F ← T_2
$p_2 = 25.01$ lbf/in.²
$s_1 = 1.881$ Btu/lb·°R
$s_2 = 1.957$ Btu/lb·°R
$u_2 = 1255$ Btu/lb
$v_2 = 27.48$ ft³/lb
$\sigma/m = 0.07611$ Btu/lb·°R ← σ/m

PROBLEM 6.67

KNOWN: An insulated tank is divided into two compartments by a piston. On either side of the piston is water vapor at known states. The piston is released and equilibrium is attained.

FIND: Determine (a) the final pressure, (b) the final temperature, and (c) the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The closed system consists of both quantities of water vapor and the piston, but the piston experiences no change of state. (2) The piston moves freely and is thermally conducting. (3) For the system, $Q=0$, $W=0$, $\Delta KE = \Delta PE = 0$.

ANALYSIS: (a),(b) To fix the final state of the water vapor, we begin with an energy balance. That is, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - \cancel{W} \Rightarrow \Delta U = 0$. Thus

$$\Delta U\big]_1 + \Delta U\big]_2 + \cancel{\Delta U\big]_{piston}}^{\,0} = 0 \Rightarrow m_1(\Delta u)_1 + m_2(\Delta u)_2 = 0 \qquad (1)$$

For each side, $m = V/v$. With data from Table A-3 at the initial state

$$m_1 = \frac{V_1}{v_1} = \frac{1\ m^3}{0.04978\ m^3/kg} = 20.09\ kg$$

Similarly, for side 2 initially Table A-4 gives $v_2 = 0.02385\ m^3/kg$. Thus

$$m_2 = \frac{V_2}{v_2} = \frac{1\ m^3}{0.02385\ m^3/kg} = 41.93\ kg$$

It follows from assumption 2 that at equilibrium, the states of water vapor on either side of the piston are the same. Thus, from (1)

$$m_1(u - u_1) + m_2(u - u_2) = 0$$

From Table A-3, $u_1 = 2602.3$ kJ/kg and from Table A-4, $u_2 = 3592.7$ kJ/kg. Inserting values and solving for u we get

$$20.09(u - 2602.3) + 41.93(u - 3592.7) = 0 \Rightarrow u = 3271.9\ kJ/kg$$

The overall system volume is constant: $2\ m^3$, and the total mass of water vapor is fixed. Thus, at equilibrium $v = (2\ m^3)/(20.09 + 41.93)\ kg = 0.03225\ m^3/kg$. The equilibrium state on both sides of the piston or fixed by $u = 3271.9$ kJ/kg and $v = 0.03225\ m^3/kg$. Interpolation in Table A-4 with u and v is very inconvenient. Instead, we use IT to get

$$p = 121.5\ bar$$
$$T = 622.6\ °C \qquad \longleftarrow \qquad (a)\ (b)$$
$$s = 6.861\ kJ/kg \cdot K$$

(c) The entropy produced can be determined from an entropy balance which reduces with assumptions (1) and (3) to give

$$\sigma = \Delta S \Rightarrow \sigma = \Delta S]_1 + \Delta S]_2 + \cancel{\Delta S}]_{piston}^{0} \Rightarrow \sigma = m_1(\Delta s)_1 + m_2(\Delta s)_2$$

From Table A-3, $s_1 = 6.0701$ kJ/kg·K, and from Table A-4, $s_2 = 7.0544$ kJ/kg·K. Thus

$$\sigma = 20.09\,[6.861 - 6.0701] + 41.93\,[6.861 - 7.0544] = 7.78 \text{ kJ/kg·K}$$

σ

ALTERNATIVE IT SOLUTION

*IT*Code

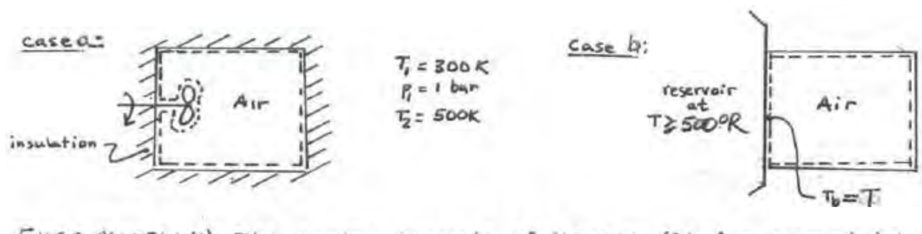
```
V1 = 1  // m³
x1 = 1
p1 = 40  // bar
V2 = 1  // m³
p2 = 200  // bar
T2 = 800  // °C

m1 = V1 / v1
v1 = vsat_Px("Water/Steam", p1, x1)
m2 = V2 / v2
v2 = v_PT("Water/Steam", p2, T2)


m1 * (u - u1) + m2 * (u - u2) = 0
u1 = usat_Px("Water/Steam", p1, x1)
u2 = u_PT("Water/Steam", p2, T2)
v = (V1 + V2) / (m1 + m2)

u = u_PT("Water/Steam", p, T)
v = v_PT("Water/Steam", p, T)


sigma = m1 * (s - s1) + m2 * (s - s2)
s1 = ssat_Px("Water/Steam",    p1, x1)
s2 = s_PT("Water/Steam", p2, T2)
s = s_PT("Water/Steam",p, T)
```

PROBLEM 6.68

KNOWN: A system consisting of air at a specified state undergoes constant volume processes in which the temperature increases in each of two ways.

FIND: (a) When the air is stirred adiabatically, determine the entropy produced. (b) When the air is heated by a reservoir at temperature T, plot the entropy produced versus T. Compare and discuss.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system consists of the air. (2) Air is modeled as an ideal gas. (3) In case (a), $Q = 0$, $W \neq 0$. In case (b), $Q \neq 0$, $W = 0$.

ANALYSIS: The change in entropy of the air is required in the evaluation of σ in each case. Thus, with data from Table A-22 and the ideal gas equation which gives $p_2/p_1 = T_2/T_1$

$$s_2 - s_1 = s°(T_2) - s°(T_1) - R \ln \frac{p_2}{p_1} = 2.21952 - 1.70203 - \frac{8.314}{28.97} \ln \frac{500}{300} = 0.3709 \text{ kJ/kg·K}$$

CASE a: An entropy balance reduces to give

$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{\,0} + \sigma_a \Rightarrow \frac{\sigma_a}{m} = s_2 - s_1 = 0.3709 \text{ kJ/kg·K} \quad \text{(a)}$$

CASE b: An entropy balance reduces to give

$$\Delta S = \frac{Q}{T_b} + \sigma_b \Rightarrow \frac{\sigma_b}{m} = (s_2 - s_1) - \frac{Q/m}{T_b}$$

To find Q, write an energy balance: $\Delta U = Q - \cancel{W}^{\,0}$. Thus, with data from Table A-22

$$\frac{Q}{m} = u(T_2) - u(T_1) = 359.49 - 214.07 = 145.42 \text{ kJ/kg}$$

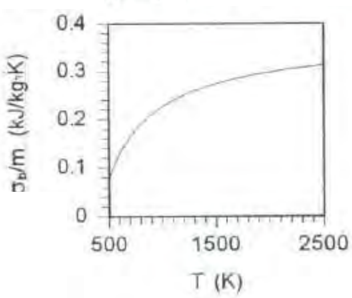
Thus

$$\frac{\sigma_b}{m} = 0.3709 - \frac{145.42}{T} \quad \text{(b)}$$

Sample calculation: When $T = 500K$, $\sigma_b/m = 0.08$ kJ/kg·K. Equation (b) is plotted below. Comparing the two cases, we get

$$(\sigma_b/m) = (\sigma_a/m) - (145.42/T).$$

Thus, the entropy produced by heating is always less than the entropy produced by stirring. The entropy produced by heating approaches the entropy produced stirring at higher reservoir temperatures (mathematically, as $T \to \infty$). See the accompanying plot.



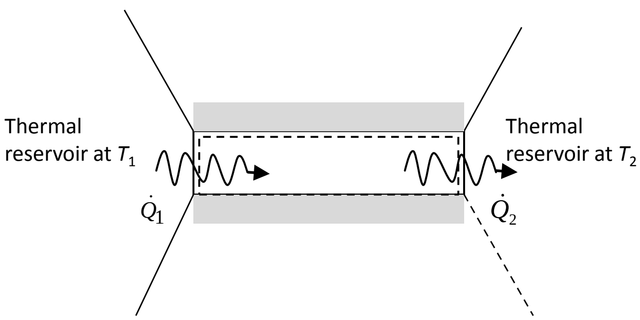
PROBLEM 6.69

Consider the solid rod at steady state shown in Fig. P6.69. The rod is insulated on its lateral surfaces, but energy transfer occurs at the rate $\dot{Q}_1$ *into* the rod at location 1, and energy transfer occurs at the rate $\dot{Q}_2$ *out* of the rod at location 2. Applying the energy and entropy rate balances to the rod as a system, determine which temperature, T_1 or T_2, is greater.

KNOWN: A well-insulated rod is exposed to thermal reservoir at each end with different temperatures. Heat transfer occurs and the rod is at steady state.

FIND: Determine which temperature is greater, T_1 or T_2.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The rod is a closed system. (2) The system is at steady state. (3) There is no work and kinetic and potential energy effects are negligible.

ANALYSIS: The energy rage balance reduces to $\cancel{\frac{dKE}{dt}} + \cancel{\frac{dPE}{dt}} + \cancel{\frac{dU}{dt}} = \dot{Q}_1 - \dot{Q}_2 - \cancel{\dot{W}}$

where the heat transfer rates are taken as the *magnitudes* of the respective rates. Thus

$$\dot{Q}_1 = \dot{Q}_2$$

Now, the entropy rate balance reduces to $\cancel{\frac{dS}{dt}} = \frac{\dot{Q}_1}{T_1} - \frac{\dot{Q}_2}{T_2} + \dot{\sigma}$

Thus, with $\dot{Q}_1 = \dot{Q}_2$

$$0 = \frac{\dot{Q}_1}{T_1} - \frac{\dot{Q}_2}{T_2} + \dot{\sigma} \longrightarrow 0 = \dot{Q}_1\left[\frac{1}{T_1} - \frac{1}{T_2}\right] + \dot{\sigma}$$

Solving for $T_2 - T_1$

Both positive in value

$$T_2 - T_1 = -\frac{(T_1 T_2)}{\dot{Q}_1}\,\dot{\sigma}$$

Negative sign

Heat transfer at 1 is *in*, so it is *positive* in value

By the Second Law, the entropy production rate must be ≥0

PROBLEM 6.69 (CONTINUED)

So, we conclude that $T_2 - T_1 \leq 0$ and

$$\boxed{T_1 \geq T_2} \longleftarrow$$

For the heat transfer in the direction shown, T_1 must be greater than T_2 or the Second Law would be violated. The Clausius Statement of the Second Law is: ***It is impossible for any system to operate in such a way that the sole result would be an energy transfer by heat from a cooler to a hotter body.***

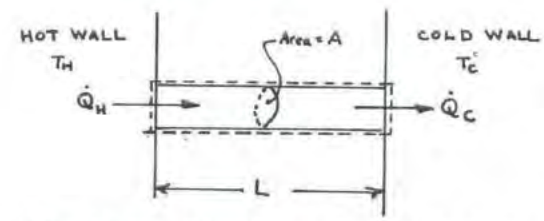
Hence, this analysis illustrates the equivalence of the Clausius Statement of the Second Law (directionality of heat transfer) to the notion of entropy production associated with heat transfer.

PROBLEM 6.70

KNOWN: Energy is conducted steadily through a copper rod from a hot wall to a cold wall. The rate of heat transfer, temperatures, and geometrical parameters are specified

FIND: (a) Determine an expression for the rate of entropy production within the rod in terms of specified quantities, and (b) plot $\dot{Q}_H$ and $\dot{\sigma}$ versus L for a given set of numerical values for these quantities.

SCHEMATIC & GIVEN DATA:



$$\dot{Q}_H = \kappa A (T_H - T_C)/L \qquad (1)$$

$T_H = 600$ K (327°C)
$T_C = 350$ K (77°C)
$\kappa = 0.4$ kW/m·K
$A = 0.1\ m^2$

ENGR. MODEL: (1) As shown in the accompanying figure, the system is the copper rod. (2) The system is at steady state. (3) The rod is insulated on its lateral surface. (4) An expression for evaluating $\dot{Q}_H$ is provided.

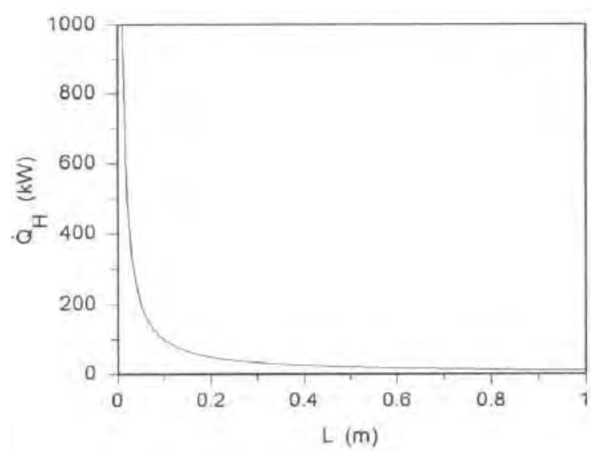
ANALYSIS: (a) At steady state an entropy rate balance reduces to give

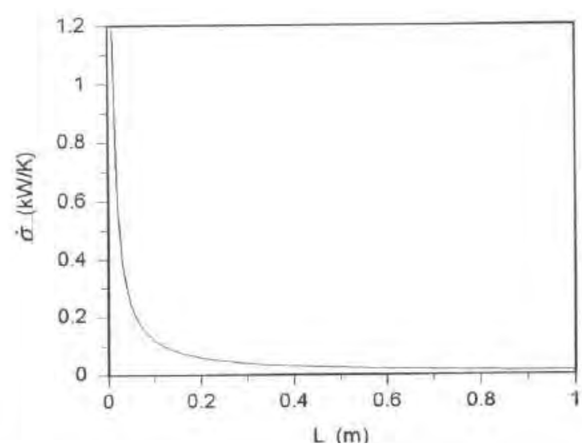
$$\cancel{\frac{dS}{dt}}^{0} = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_C}{T_C} + \dot{\sigma} \quad \Rightarrow \quad \dot{\sigma} = \frac{\dot{Q}_C}{T_C} - \frac{\dot{Q}_H}{T_H}$$

Noting that the energy transfers are positive in the directions of the arrows, an energy rate balance gives $\dot{Q}_C = \dot{Q}_H$. Collecting results

$$\dot{\sigma} = \dot{Q}_H \left[\frac{1}{T_C} - \frac{1}{T_H}\right] = \frac{\kappa A (T_H - T_C)^2}{L\ T_H T_C} \qquad (2)$$

(b) If $T_H = 600$K (327°C), $T_C = 350$K (77°C), $\kappa = 0.4$ kW/m·K, $A = 0.1\ m^2$, the variations of $\dot{Q}$ and $\dot{\sigma}$ with L are





The plots show that both heat transfer and entropy production rate decrease rapidly with increasing L. $\dot{Q}_H$ decreases due to greater resistance to heat transfer as the length of the rod increases. The variation of $\dot{\sigma}$ is associated with the changing temperature gradient, dT/dx, in the rod. Higher temperature gradient ($L \to 0$; $dT/dx \to \infty$) corresponds to high rates of entropy production, and conversely. As a final point, Eq. (2) above indicates that $\dot{\sigma}$ is positive, illustrating the irreversible nature of heat tranfer.

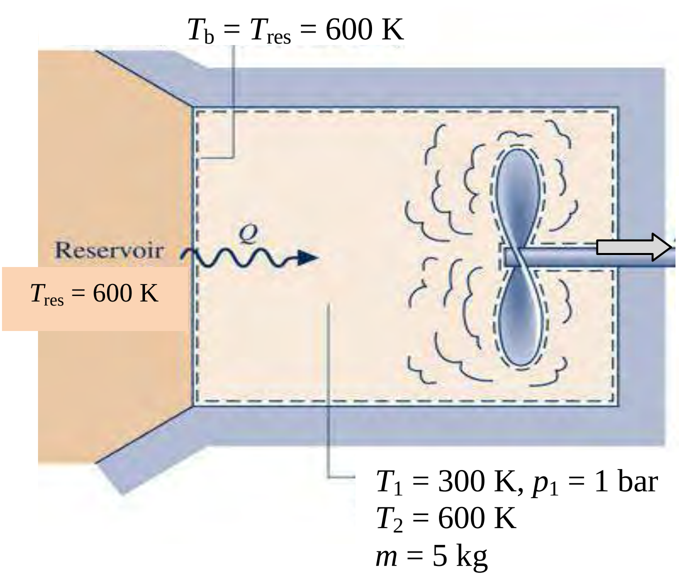
PROBLEM 6.71

A closed, rigid tank contains 5 kg of air initially at 300 K, 1 bar. As illustrated in Fig. 6.71, the tank is in contact with a thermal reservoir at 600 K and heat transfer occurs at the boundary where the temperature is 600K. A stirring rod transfers 600 kJ of energy to the air. The final temperature is 600 K. The air can be modeled as an ideal gas with $c_v = 0.733$ kJ/kg·K and kinetic and potential energy effects are negligible. Determine the amount of entropy transferred into the air and the amount of entropy produced, each in kJ/K.

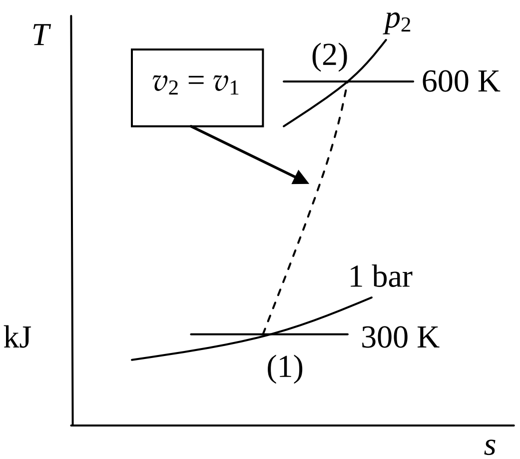
KNOWN: Air undergoes a process in a closed rigid tank. The work is specified and the air receives energy input from a thermal reservoir at a given temperature.

FIND: Determine the amount of entropy transferred into the air and the amount of entropy produced.

SCHEMATIC AND GIVEN DATA:



$W = -600$ kJ



EINGINEERING MODEL: (1) The air is a closed system. (2) The air is modeled as an ideal gas with $c_v = 0.733$ kJ/kg·K. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: Since the volume is constant for the closed system, $v_2 = v_1$. Therefore, by the ideal gas equation of state; $p_2 = (T_2/T_1)p_1 = (600/300)(1 \text{ bar}) = 2$ bar.

To determine the amount of entropy transfer by heat into the air, we begin with the energy balance: $\Delta \cancel{KE} + \Delta \cancel{PE} + \Delta U = Q - W \longrightarrow Q = m(u_2 - u_1) + W$. For constant specific heats

$$Q = mc_v(T_2 - T_1) + W = (5 \text{ kg})(0.733 \text{ kJ/kg·K})(600 - 300)\text{K} + (-600 \text{ kJ}) = 499.5 \text{ kJ}$$

The boundary temperature where the heat transfer occurs is constant, so the amount of entropy transfer associated with the heat transfer is

$$\int_1^2 \left(\frac{\delta Q}{T}\right)_b = Q/T_b = (499.5 \text{ kJ})/(600 \text{ K}) = 0.8325 \text{ kJ/K} \longleftarrow$$

PROBLEM 6.69 (CONTINUED)

The entropy produced is found using the entropy balance: $(S_2 - S_1) = Q/T_b + \sigma$

Or

$$\sigma = m(s_s - s_1) - Q/T_b$$

The change in specific entropy can be calculated for constant specific heats using

$$(s_s - s_1) = c_v \ln (T_2/T_1) + R \cancel{\ln(v_2 /v_1)} = (0.733 \text{ kJ/kg·K}) \ln (600/300) = 0.5081 \text{ kJ/kg·K}$$

Thus

$$\sigma = m(s_s - s_1) - Q/T_b = (5 \text{ kg})(0.50808 \text{ kJ/kg·K}) - (0.8325 \text{ kJ/K}) = 1.7079 \text{ kJ/K} \longleftarrow$$

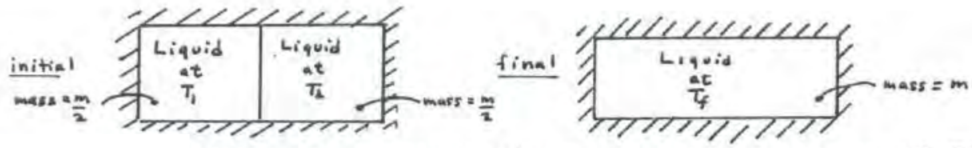
The *net* entropy increase is $S_2 - S_1 = 1.7079 + 0.8325 = 2.5404$ kJ/K

PROBLEM 6.72

KNOWN: An isolated system of total mass m is formed by mixing equal masses of the same liquid initially at temperatures T_1 and T_2.

FIND: (a) Show that the amount of entropy produced is $\sigma = mc \ln\left[(T_1+T_2)/2\,(T_1 T_2)^{1/2}\right]$ and (b) that σ must be positive.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system consists of the total mass of liquid. (2) The system is isolated. (3) The liquid is incompressible with constant specific heat c

ANALYSIS: (a) The final temperature T_f can be evaluated from an energy balance: $\Delta U = \cancel{Q}^0 - \cancel{W}^0$. Thus $\Delta U = 0$, or

$$m\,u(T_f) - \left[\frac{m}{2}u(T_1) + \frac{m}{2}u(T_2)\right] = 0$$

$$\Rightarrow \quad \frac{m}{2}\left[u(T_f) - u(T_1)\right] + \frac{m}{2}\left[u(T_f) - u(T_2)\right] = 0$$

Since the liquid is incompressible with constant specific heat c

$$\frac{m}{2}c\left[T_f - T_1\right] + \frac{m}{2}c\left[T_f - T_2\right] = 0$$

$$\Rightarrow \quad T_f = \frac{T_1 + T_2}{2}$$

An entropy balance gives

$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{\,0} + \sigma$$

or

$$\sigma = m s_f - \left[\frac{m}{2}s_1 + \frac{m}{2}s_2\right]$$

$$= \frac{m}{2}\left[(s_f - s_1) + (s_f - s_2)\right]$$

Using Eq. 6.13

$$\sigma = \frac{m}{2}c\left[\ln\frac{T_f}{T_1} + \ln\frac{T_f}{T_2}\right] = \frac{m}{2}c\ln\left[\frac{T_f^2}{T_1T_2}\right]$$

$$= mc\ln\left[\frac{T_f}{(T_1T_2)^{1/2}}\right]$$

$$= mc\ln\left[\frac{T_1+T_2}{2\,(T_1T_2)^{1/2}}\right] \longleftarrow \text{(a)}$$

(b) $\sigma \geq 0$ when $\ln\left[\frac{T_1+T_2}{2(T_1T_2)^{1/2}}\right] \geq 0$

$$\Rightarrow \quad \frac{T_1+T_2}{2(T_1T_2)^{1/2}} \geq 1 \quad \text{or} \quad T_1 + T_2 \geq 2(T_1T_2)^{1/2}$$

Squaring both sides

$$(T_1+T_2)^2 \geq 4(T_1T_2) \quad \text{or} \quad T_1^2 + 2T_1T_2 + T_2^2 \geq 4T_1T_2$$

$$\Rightarrow \quad T_1^2 - 2T_1T_1 + T_2^2 \geq 0$$

$$\Rightarrow \quad (T_1 - T_2)^2 \geq 0$$

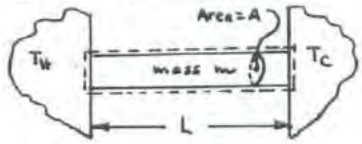
The inequality is satisfied for either $T_1 > T_2$ or $T_2 > T_1$. The equality applies only when $T_1 = T_2$ ← (b)

PROBLEM 6.73

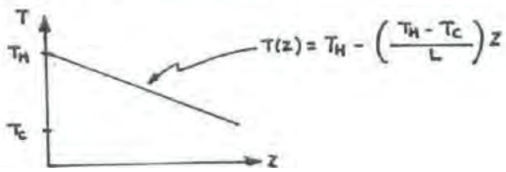
KNOWN: The temperature within a rod initially in contact with hot and cold walls at its ends is linear with position. The rod is insulated overall and eventually comes to a final equilibrium state where the temperature is T_f.

FIND: Evaluate the final temperature and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



$T_C < T_H$



ENGR. MODEL: (1) The system is the rod which is insulated on its lateral surface. (2) The rod is modeled as incompressible with constant specific heat c. (3) Initially, the temperature within the rod varies linearly from T_H to T_C.

ANALYSIS: The final temperature can be determined using an energy balance which reduces to give $\Delta U = \cancel{Q}^0 - \cancel{W}^0$ or $\Delta U = 0$. Each element of rod dz changes temperature from T(z) to the final temperature T_f, and thus contributes to the change in internal energy

$$dU = dm\, c\,(T_f - T(z))$$
$$= (\rho A\, dz)\, c\,(T_f - T(z))$$

Accordingly

$$\Delta U = \int_0^L (\rho A\, dz)\, c\,(T_f - T(z))$$
$$= \rho A c \int_0^L \left[T_f - T_H + \left(\frac{T_H - T_C}{L}\right)z\right]dz$$
$$= \rho A c \left[(T_f - T_H)z + \left(\frac{T_H - T_C}{L}\right)\frac{z^2}{2}\right]_0^L$$
$$= \rho A c L\left[(T_f - T_H) + \left(\frac{T_H - T_C}{2}\right)\right]$$

Since $\Delta U = 0$, $T_f = (T_H + T_C)/2$. ← T_f

To find the entropy production, an entropy balance reduces to give $\Delta S = \cancel{\int_1^2 \frac{\delta Q}{T}}^0 + \sigma$ or $\sigma = \Delta S$. With Eq. 6.13 the entropy change of an element of rod dz is

$$dS = dm\, c \ln\frac{T_f}{T(z)}$$
$$= (\rho A\, dz)\, c \ln\frac{T_f}{T(z)}$$

Accordingly

$$\sigma = \rho A c \int_0^L (\ln T_f - \ln T(z))\,dz = \rho A c\left[(\ln T_f)L - \int_0^L (\ln T(z))\,dz\right]$$

Using the given temperature distribution, the variable of integration can be changed from z to T:

$$dT = -\left(\frac{T_H - T_C}{L}\right)dz \quad \Rightarrow \quad dz = -\left(\frac{L}{T_H - T_C}\right)dT$$

With this, the integral can be expressed as

$$\int_0^L (\ln T(z))\,dz = \int_{T_H}^{T_C} (\ln T)\left(\frac{-L}{T_H - T_C}\right) dT$$

$$= \frac{L}{(T_H - T_C)} \int_{T_C}^{T_H} \ln T \, dT$$

$$= \frac{L}{(T_H - T_C)} \Big[T \ln T - T \Big]_{T_C}^{T_H}$$

$$= \frac{L}{T_H - T_C} \Big[(T_H \ln T_H - T_H) - (T_C \ln T_C - T_C) \Big]$$

$$= L \left[\frac{T_H \ln T_H}{T_H - T_C} - \frac{T_C \ln T_C}{T_H - T_C} - 1 \right]$$

Collecting results

$$\sigma = \rho A c \left[(\ln T_f) L - L \left[\frac{T_H \ln T_H}{T_H - T_C} - \frac{T_C \ln T_C}{T_H - T_C} - 1 \right] \right]$$

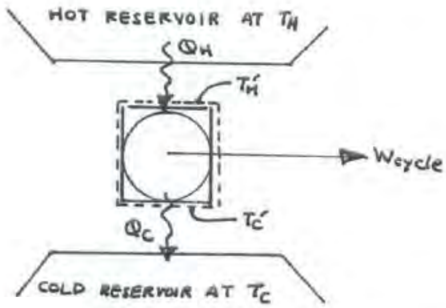
$$= mc \left[1 + \ln T_f + \frac{T_C \ln T_C}{T_H - T_C} - \frac{T_H \ln T_H}{T_H - T_C} \right] \longleftarrow \sigma$$

PROBLEM 6.74

KNOWN: A system undergoes a cycle while receiving Q_H at T_H' and discharging Q_C at T_C'. Q_H and Q_C are with hot and cold reservoirs at T_H and T_C, respectively.

FIND: (a) Determine an expression for W_{cycle} in terms of Q_H, T_C', T_H', and σ. (b) State the relationship of T_H' to T_H and T_C' to T_C. (c) Obtain an expression for W_{cycle} when there are (i) no internal irreversibilities, (ii) no irreversibilities.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: The system shown in the accompanying figure undergoes a power cycle while receiving Q_H at T_H' and discharging Q_C at T_C'. There are no other heat transfers.

ANALYSIS: (a) An energy balance gives

$$W_{cycle} = Q_H - Q_C \qquad (1)$$

An entropy balance gives

$$\cancel{\Delta S_{cycle}}^{\,0} = \frac{Q_H}{T_H'} - \frac{Q_C}{T_C'} + \sigma_{cycle} \qquad (2)$$

where σ is the amount of entropy produced within the system per cycle and $\Delta S = 0$ because the system undergoes a cycle. Solving Eq. (2) for Q_C and substituting the result into Eq. (1)

$$W_{cycle} = Q_H\left[1 - \frac{T_C'}{T_H'}\right] - T_C'\sigma_{cycle} \qquad (3) \quad \longleftarrow (a)$$

(b) For heat transfer to occur from the hot reservoir to the system $T_H \geq T_H'$. For heat transfer to occur from the system to the cold reservoir $T_C' \geq T_C$. $\longleftarrow$ (b)

(c) If there were no irreversibilities within the system during the cycle, the term σ_{cycle} in Eq. (3) would vanish leaving

$$W_{cycle} = Q_H\left[1 - \frac{T_C'}{T_H'}\right] \qquad (4) \quad \longleftarrow (c\text{-}i)$$

External irreversibilities are associated with heat transfer between the reservoirs and the system. If these are also absent, $T_H' = T_H$ and $T_C' = T_C$, and Eq. (4) becomes

$$W_{cycle} = Q_H\left[1 - \frac{T_C}{T_H}\right] \qquad (5) \quad \longleftarrow (c\text{-}ii)$$

which is the maximum theoretical work that can be obtained.

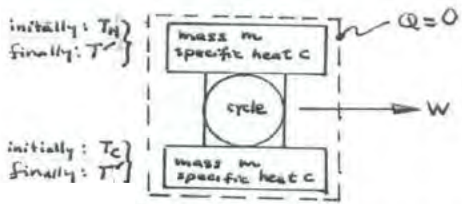
PROBLEM 6.75

A thermodynamic power cycle receives energy by heat transfer from an incompressible body of mass *m* and specific heat *c* initially at temperature T_H. The cycle discharges energy by heat transfer to another incompressible body of mass *m* and specific heat *c* initially at a lower temperature T_C. There are no other heat transfers. Works is developed by the cycle until the temperature of each of the two bodies is the same. Develop an expression for the maximum theoretical amount of work that can be developed, W_{max}, in terms of *m*, *c*, T_H. and T_c.

KNOWN: A thermodynamic power cycle operates between two incompressible bodies, each initially at different temperatures.

FIND: Develop an expression for the maximum theoretical work that could be developed as energy is exchanged and the two bodies reach the same final temperature.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system shown in the accompany figure is composed of three subsystems: An incompressible body of mass m and specific heat c initially at T_H. An incompressible body of mass m and specific heat c initially at T_C. A system that undergoes a power cycle. (2) For the overall system, $Q = 0$.

ANALYSIS: An energy balance gives $\Delta U = \cancel{Q}^{0} - W \Rightarrow$

$$W = -[\Delta U]_{HOT} + \cancel{\Delta U]_{cycle}}^{0} + \Delta U]_{COLD}] = -[mc(T' - T_H) + mc(T' - T_C)]$$

or

$$W = mc[T_H + T_C - 2T'] \qquad (1)$$

An entropy balance gives $\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{0} + \sigma$. Then, with Eq. 6.13

$$\sigma = [\Delta S]_{HOT} + \cancel{\Delta S]_{cycle}}^{0} + \Delta S]_{COLD}] = \left[mc \ln \frac{T'}{T_H} + mc \ln \frac{T'}{T_C}\right]$$

or

$$\sigma = mc \ln\left[\frac{(T')^2}{T_H T_C}\right]$$

Solving for T'

$$T' = [T_H T_C \exp(\sigma/mc)]^{1/2} \qquad (2)$$

Substituting Eq. (2) into Eq. (1)

$$W = mc[T_H + T_C - 2[T_H T_C \exp(\sigma/mc)]^{1/2}]$$

Since $\sigma \geq 0$, it follows that W is maximum when $\sigma = 0$, giving

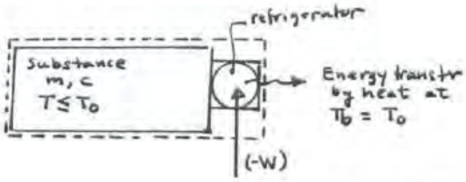
$$W_{MAX} = mc[T_H + T_C - 2(T_H T_C)^{1/2}] \longleftarrow$$

PROBLEM 6.76

KNOWN: The temperature of an incompressible substance of mass m and specific heat c is reduced from T_0 to T ($<T_0$) by a refrigeration cycle.

FIND: Plot (W_{min}/mcT_0) versus T/T_0, where W_{min} is the minimum theoretical work input required.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The system shown in the accompanying figure is composed of an incompressible substance and a system that undergoes a refrigeration cycle. 2. The cycle discharges energy to the surroundings at T_0.

ANALYSIS: An energy balance gives $(-W) = \Delta U - Q$, where ΔU is the change in internal energy of the substance because $(\Delta U)_{cycle} = 0$. With Eq. 3.20a

$$(-W) = mc[T - T_0] - Q \qquad (1)$$

An entropy balance reads $\Delta S = (Q/T_0) + \sigma$, where ΔS is the change in entropy of the substance because $(\Delta S)_{cycle} = 0$. With Eq. 6.13

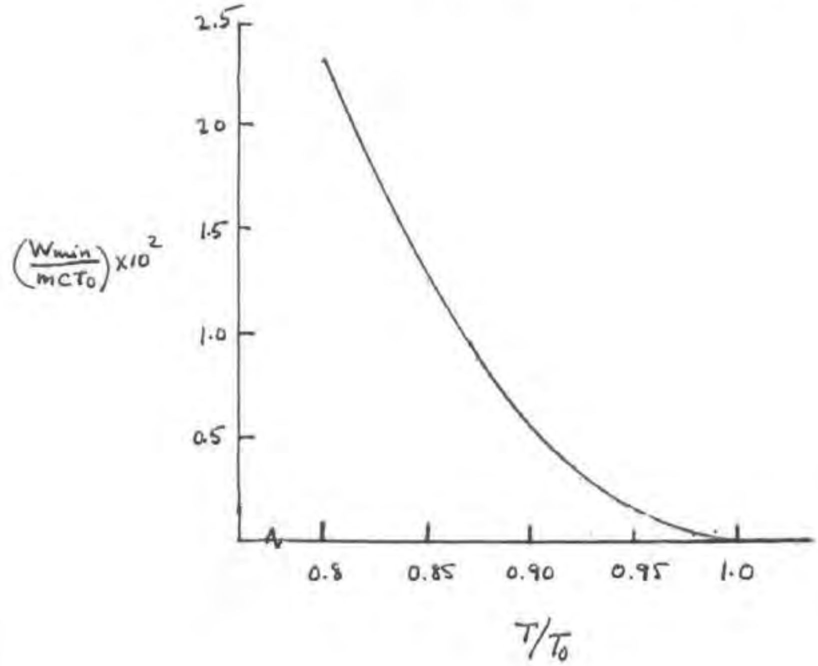
$$mc \ln\left(\frac{T}{T_0}\right) = \frac{Q}{T_0} + \sigma \Rightarrow Q = T_0 mc \ln\left(\frac{T}{T_0}\right) - T_0 \sigma \qquad (2)$$

Collecting Eqs. (1), (2)

$$(-W) = mc[T - T_0] - T_0 mc \ln\left(\frac{T}{T_0}\right) + T_0 \sigma$$

Since $\sigma \geq 0$, the minimum work input corresponds to $\sigma = 0$, giving

$$W_{min} = mc\left[T - T_0 - T_0 \ln\frac{T}{T_0}\right] \Rightarrow \left(\frac{W_{min}}{mcT_0}\right) = \frac{T}{T_0} - 1 - \ln\frac{T}{T_0} \qquad (3)$$



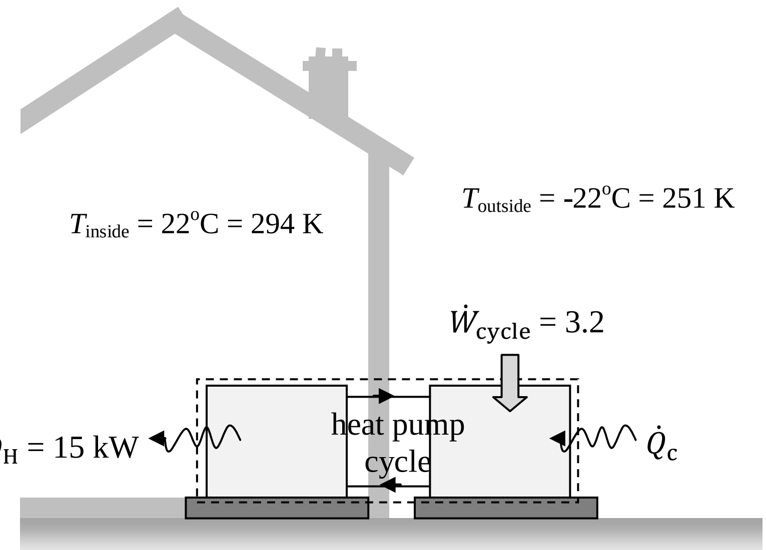
PROBLEM 6.77

The heat pump cycle shown in Fig. P6.77 operates at steady state and provides energy by heat transfer at a rate of 15 kW to maintain a dwelling at 22°C when the outside temperature is -22°C. The manufacturer claims that the power input required for this operating condition is 3.2 kW. Applying energy and entropy rate balances evaluate this claim.

KNOWN: A heat pump cycle provides energy to a dwelling at a constant rate for given inside and outside temperatures. The power requirement is specified by the manufacturer.

FIND: Applying an entropy rate balance, evaluate the claim.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) the heat pump cycle is a closed system, operating steadily. (2) The heat transfers occur at $T_{\text{inside}} = T_{\text{H}}$ and $T_{\text{outside}} = T_{\text{C}}$, respectively. (3) The energy transfers are positive in the direction of the arrows on the accompanying diagram.

ANALYSIS: First, we use the energy rate balance to find $\dot{Q}_{\text{c}}$. For the cycle

$$\dot{Q}_{\text{cycle}} = \dot{Q}_{\text{H}} - \dot{Q}_{\text{C}} \quad \text{and } \dot{Q}_{\text{cycle}} = \dot{W}_{\text{cycle}} \quad \longrightarrow \quad \dot{Q}_{\text{C}} = \dot{Q}_{\text{H}} - \dot{W}_{\text{cycle}} = 15 \text{ kW} - 3.2 \text{ kW} = 11.8 \text{ kW}$$

This rate of heat transfer would satisfy the First Law of Thermodynamics.

The entropy rate balance reduces to: $0 = \dfrac{\dot{Q}_{\text{H}}}{T_{\text{H}}} - \dfrac{\dot{Q}_{\text{C}}}{T_{\text{C}}} + \sigma_{\text{cycle}}$

$$\sigma_{\text{cycle}} = \frac{\dot{Q}_{\text{C}}}{T_{\text{C}}} - \frac{\dot{Q}_{\text{H}}}{T_{\text{H}}} = \frac{15 \text{ kW}}{294 \text{ K}} - \frac{11.8 \text{ kW}}{251 \text{ K}} = 0.004 \text{ kW}$$

Since $\sigma_{\text{cycle}} > 0$ the claimed performance satisfies the Second Law of Thermodynamics. ⟵

PROBLEM 6.75 (CONTINUED)

Alternative Solution Using γ_{max}

The maximum possible coefficient of performance for a heat pump cycle operating between T_C and T_H is

$$\gamma_{max} = T_H/(T_H - T_C) = (294)/ (294 - 251) = 6.682$$

For the claimed cycle performance

$$\gamma_{actual} = \dot{Q}_H/\dot{W}_{cycle} = (15 \text{ kW})/(3.2 \text{ kW}) = 4.6875$$

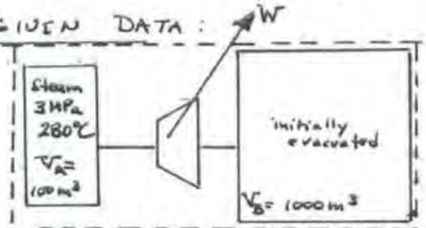
Since $\gamma_{actual} < \gamma_{max}$, the claimed performance satisfies the Second Law of Thermodynamics.

PROBLEM 6.78

KNOWN: A turbine is located between two tanks, initially one tank is filled with steam and the other is evacuated. Steam is allowed to flow through the turbine until equilibrium is established.

FIND: Determine the maximum theoretical work that can be developed.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system is shown by the dashed line above. (2) Heat transfer can be ignored. (3) The initial and final states are equilibrium states. At the final state the mass of steam contained within the turbine and interconnecting piping can be ignored.

ANALYSIS: An energy balance reduces to $\Delta U = \cancel{Q} - W$, or $W = m(u_1 - u_2)$, where $m = \frac{V_A}{v_1}$. Thus, with steam table data, $u_1 = 2709.9$ kJ/kg, $v_1 = 0.0771$ m³/kg, $s_1 = 6.4462$ kJ/kg·K,

$$W = \left(\frac{100\ m^3}{0.0771\ m^3/kg}\right)(2709.9 - u_2)\frac{kJ}{kg} = 1297(2709.9 - u_2)\ kJ \qquad (1)$$

The final specific volume is

$$v_2 = \frac{V_A + V_B}{m} = v_1\left[\frac{V_A + V_B}{V_A}\right] = (0.0771)\left(\frac{1100}{100}\right) = 0.8481\ \frac{m^3}{kg} \qquad (2)$$

An entropy balance reduces to $\Delta S = \int_1^2 \left(\frac{\cancel{\delta Q}}{T}\right)_b^{0} + \sigma \rightarrow m(s_2 - s_1) = \sigma$. (3)

By inspection of Eq.(1), the maximum value of W is attained when u_2 assumes the smallest allowed value at the value of v_2 given by Eq.(2). Since
1 u and s change in the same direction at fixed v, the smallest allowed value for u_2 corresponds to the smallest allowed value for s_2. Since $\sigma \geq 0$, Eq.(3) indicates that the smallest value of s_2 corresponds to $\sigma = 0$: $s_2 = s_1$. The final state of the steam is then fixed by: $v_2 = .8481$ m³/kg, $s_2 = 6.4462$ kJ/kg·K.

As this pair of independent properties is cumbersome to use with tabular steam table data, IT is employed to obtain $u_2 = 2269$ kJ/kg. Returning to Eq.(1)

$$W_{MAX} = 1297(2709.9 - 2269) = 5.72 \times 10^5\ kJ$$

The final state is a two-phase liquid-vapor mixture with $T_2 = 117°C$, $P_2 = 1.815$ bar, $x_2 = 0.8741$.

When the entire problem is solved using IT, we get

IT Solution:

```
p1 = 30 bar
T1 = 280 °C
VA = 100 m³
VB = 1000 m³

v1 = v_PT("Water/Steam", p1, T1)
u1 = u_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
m = VA / v1

W = m * (u1 - u2)
v2 = (VA + VB) / m
s2 = s1

v2 = vsat_Px("Water/Steam", p2, x2)
s2 = ssat_Px("Water/Steam", p2, x2)
u2 = usat_Px("Water/Steam", p2, x2)
T2 = Tsat_P("Water/Steam", p2)
```

Note: These are the IT results when the entire problem is solved using IT

T2	117.2
W	5.707E5
s2	6.445
u2	2269
v2	0.8483

---

1. When v is fixed, the first Tds equation — Eq. 6.10a — reads $Tds = du$. Thus, changes in u and s occur in the same direction.

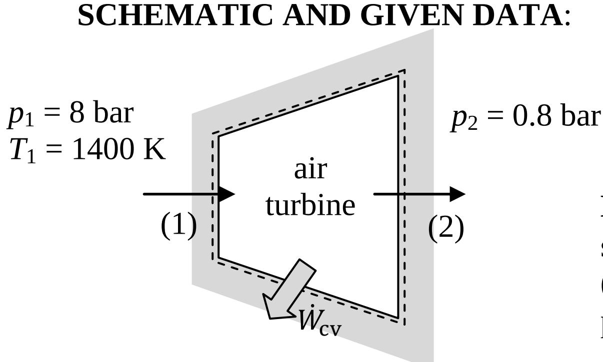
PROBLEM 6.79

Air enters a turbine operating at steady state at 8 bar, 1400 K and expands to 0.8 bar. The turbine is well-insulated, and kinetic and potential energy effects can be neglected. Assuming ideal gas behavior for the air, what is the maximum theoretical work that could be developed by the turbine in kJ per kg of air flowing?

KNOWN: Air expands through a turbine operating at steady state from a given initial state to a given final pressure.

FIND: Determine the maximum theoretical work, per unit mass of air flowing.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The control volume is at steady state. (2) The turbine is well-insulated, so $\dot{Q}_{cv} = 0$. (3) The air behaves as an ideal gas. (4) Kinetic and potential energy effects can be neglected.

ANALYSIS: To find the work, start with mass and energy balances: $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$ and

$$\cancel{\frac{dE_{cv}}{dt}} = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)} + g\cancel{(z_1 - z_2)}\right]$$

or

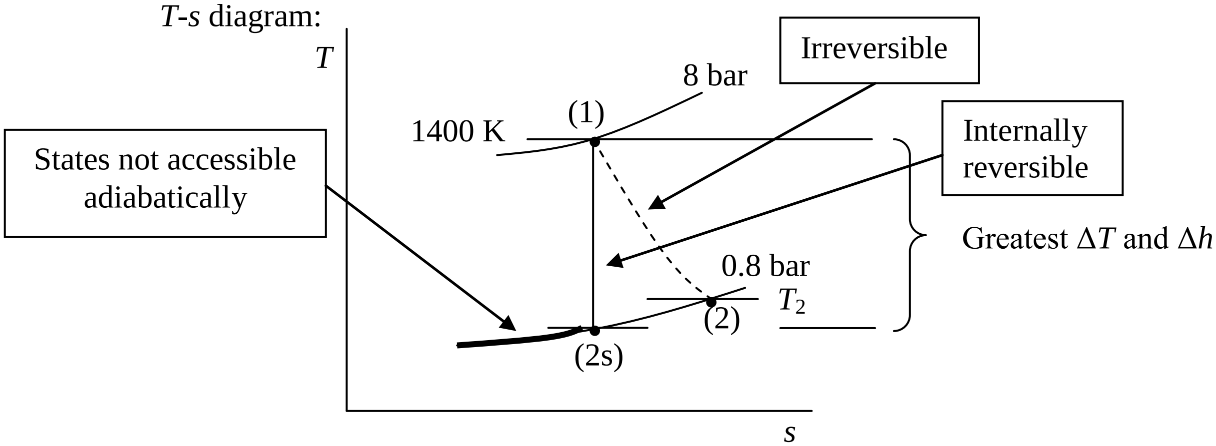
$$\dot{W}_{cv}/\dot{m} = (h_1 - h_2) \qquad (*)$$

We see that the work increases as h_2 decreases, since state 1, and hence h_1, is fixed.

To find the minimum value for h_2, we use the entropy rate balance:

$$\cancel{\frac{dS_{cv}}{dt}} = \sum_j \cancel{\left(\frac{\dot{Q}}{T}\right)_j} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \longrightarrow \dot{\sigma}_{cv}/\dot{m} = (s_2 - s_1) \geq 0 \longrightarrow \boxed{s_2 \geq s_1}$$

Thus, the Second Law constraint that $\dot{\sigma}_{cv} \geq 0$ restricts possible exits states as illustrated on the T-s diagram:



PROBLEM 6.77 (CONTINUED)

The greatest change in specific enthalpy, which corresponds to the maximum work, is obtained when the expansion occurs in an internally reversible process. The corresponding exit state is determined by $s_{2s} = s_1$ (isentropic process). Thus

$$\cancel{s(T_{2s},p_2) - s(T_1,p_1)} = s^o(T_{2s}) - s^o(T_1) - R\ln(p_2/p_1) \quad \longrightarrow \quad s^o(T_{2s}) = s^o(T_1) + R\ln(p_2/p_1)$$

With data from Table A-22

$$s^o(T_{2s}) = 3.36200 \text{ kJ/kg·K} - (8.314/28.97) \text{ kJ/kg·K} \ln(8/0.8) = 2.70119 \text{ kJ/kg·K}$$

Interpolating in Table A-22: $T_{2s} \approx 788.0$ K and $h_{2s} \approx 808.8$ kJ/kg

Finally, from (*)

$$(\dot{W}_{cv}/\dot{m})_{max} = (h_1 - h_{2s}) = 1515.42 - 808.8 = 706.6 \text{ kJ/kg}$$

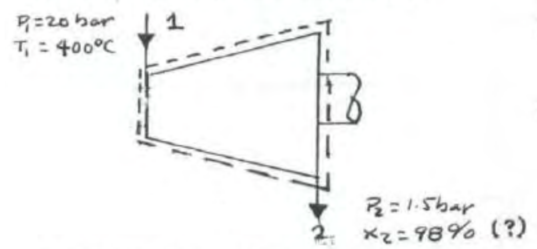
PROBLEM 6.80

Water at 20 bar, 400°C enters a turbine operating at steady state and exits at 1.5 bar. Stray heat transfer and kinetic and potential energy effects are negligible. A hard-to-read data sheet indicates that the quality at the turbine exit is 98%. Can this quality value be correct? If no, explain. If yes, determine the power developed by the turbine, in kJ per kg of water flowing.

KNOWN: Steady-state data are provided for turbine. The quality of the water at the turbine exit is claimed to be 98%

FIND: Determine if the quality value can be correct. If no, explain. If yes, determine the power developed

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$.

ANALYSIS: Applying an entropy rate balance, we get at steady state

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{\,0} + \dot{m}(s_1 - s_2) + \dot{\sigma}$$

$$\Rightarrow \quad \frac{\dot{\sigma}}{\dot{m}} = s_2 - s_1 \qquad (1)$$

From Table A-4,

$$s_1 = 7.1271 \text{ kJ/kg}\cdot\text{K}$$

With data from Table A-3,

$$s_2 = s_f + x_2(s_g - s_f)$$

$$= 1.4336 + 0.98(7.2233 - 1.4336)$$

$$= 7.1075 \text{ kJ/kg}\cdot\text{K}$$

Inserting values in Eq. (1)

$$\frac{\dot{\sigma}}{\dot{m}} = (7.1075 - 7.1271)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$= -0.02 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

Since $\dot{\sigma}/\dot{m}$ cannot be negative, the claimed value cannot be correct.

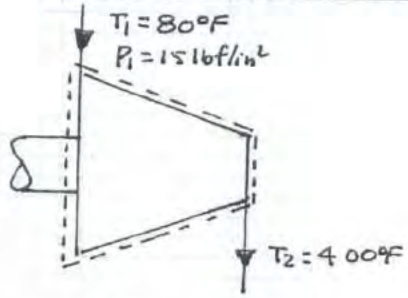
PROBLEM 6.81

Air enters a compressor operating at steady state at 15 lbf/in.^{2}, 80^{o}F and exits at 400^{o}F. Stray heat transfer and kinetic and potential energy effects are negligible. Assuming the ideal gas model applies for the air, determine the maximum theoretical pressure at the exit, in lbf/in.^{2}

KNOWN: Steady-state data are provided for an air compressor.

FIND: Determine the maximum theoretical pressure of the air exiting the compressor.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$
3. The air is modeled as an ideal gas.

ANALYSIS: An entropy balance for the control volume at steady state reduces as follows:

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{\,0} + \dot{m}_1 (s_1 - s_2) + \dot{\sigma}$$

$$\Rightarrow \frac{\dot{\sigma}}{\dot{m}} = s_2 - s_1$$

With Eq. 6.20a,

$$\frac{\dot{\sigma}}{\dot{m}} = s°(T_2) - s°(T_1) - R \ln \frac{P_2}{P_1}$$

$$\Rightarrow \ln \frac{P_2}{P_1} = \frac{s°(T_2) - s°(T_1)}{R} - \left(\frac{\dot{\sigma}/\dot{m}}{R}\right)$$

With data from Table A-22E,

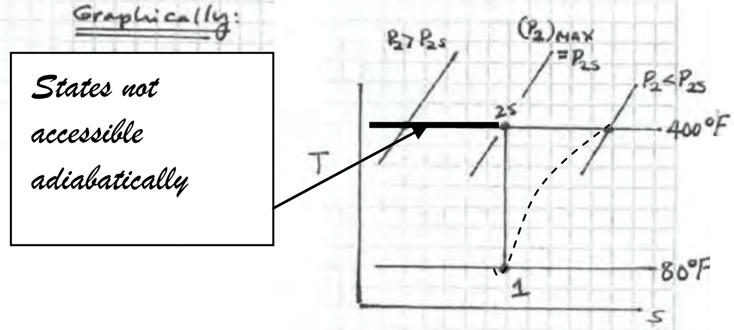
$$\ln \frac{P_2}{P_1} = \frac{(0.71323 - 0.60078)\ Btu/lb \cdot °R}{\left(\frac{1.986}{28.97}\right) \frac{Btu}{lb \cdot °R}} - \left(\frac{\dot{\sigma}/\dot{m}}{R}\right)$$

$$\ln \frac{P_2}{P_1} = 1.64032 - \underbrace{\left(\frac{\dot{\sigma}/\dot{m}}{R}\right)}_{\geq 0}$$

Since the second term on the right cannot be negative, the maximum value for P_2 corresponds a value of zero for the term. This also corresponds to a process in which $s_2 = s_1$. Thus

$$\ln \frac{(P_2)_{MAX}}{\underset{15\ lbf/in^2}{P_1}} = 1.64032 \Rightarrow (P_2)_{MAX} = 77.35\ lbf/in^2$$

Graphically:

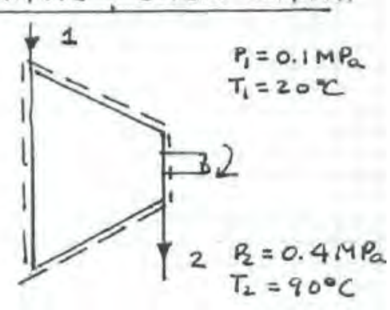


PROBLEM 6.82

Propane at 0.1 MPa, 20°C enters an insulated compressor operating at steady state and exits at 0.4 MPa, 90°C. Neglecting kinetic and potential energy effects, determine

(a) the power required by the compressor, in kJ per kg of propane flowing,
(b) the rate of entropy production within the compressor, in kJ/K per kg of propane flowing.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown with the sketch is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects can be ignored.

ANALYSIS: Table A-18, $h_1 = 517.6$ kJ/kg, $s_1 = 2.194$ kJ/kg·K, $h_2 = 639.2$ kJ/kg, $s_2 = 2.311$ kJ/kg·K.

An energy rate balance reduces as follows: $0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$

$$\Rightarrow \frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 = (517.6 - 639.2)\frac{kJ}{kg} = -121.6\ \frac{kJ}{kg} \quad \longleftarrow (a)$$

An entropy rate balance reduces as follows:

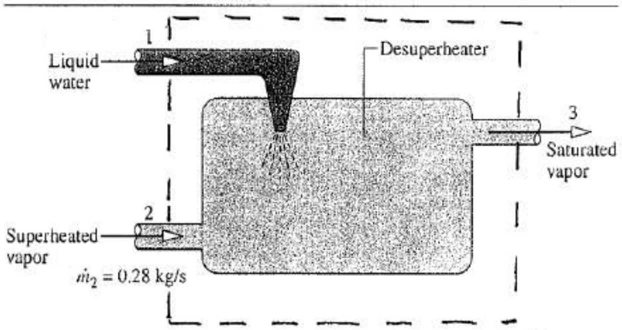
$$0 = \cancel{\sum_j \frac{\dot{Q}_j}{T_j}}^{0} + \dot{m}(s_1 - s_2) + \dot{\sigma}$$

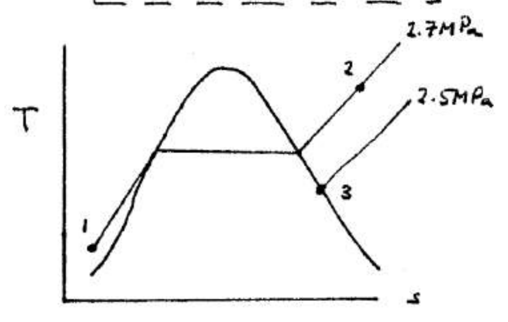
$$\Rightarrow \frac{\dot{\sigma}}{\dot{m}} = s_2 - s_1 = (2.311 - 2.194)\frac{kJ}{kg\cdot K} = 0.117\ \frac{kJ}{kg\cdot K} \quad \longleftarrow (b)$$

PROBLEM 6.83

By injecting liquid water into superheated steam, the *desuperheater* has a saturated vapor stream at its exit. Steady-state operating data are provided in the accompanying table. Stray heat transfer and all kinetic and potential energy effects are negligible. (a) Locate states 1, 2, and 3 on a sketch of the T–s diagram. (b) Determine the rate of entropy production within the desuperheater, in kW/K.

State	p(MPa)	T(°C)	$v \times 10^3$(m^3/kg)	u(kJ/kg)	h(kJ/kg)	s(kJ/kg·K)
1	2.7	40	1.0066	167.2	169.9	0.5714
2	2.7	300	91.01	2757.0	3002.8	6.6001
3	2.5	sat. vap.	79.98	2603.1	2803.1	6.2575





ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. $\dot{W}_{cv} = 0$. $\dot{Q}_{cv}$ and the effects of kinetic and potential energy can be ignored.

ANALYSIS: An entropy rate balance reduces as follows:

$$0 = \sum_j \cancel{\dot{Q}_j / T_j} + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$$

$$\Rightarrow \dot{\sigma}_{cv} = \underbrace{\dot{m}_3}_{= (\dot{m}_1 + \dot{m}_2)} s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2$$

$$\Rightarrow \dot{\sigma}_{cv} = (\dot{m}_1 + \dot{m}_2) s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2 \quad (1)$$

To find $\dot{m}_1$, apply an energy rate balance:

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1 \left[h_1 + \cancel{\frac{V_1^2}{2}} + \cancel{g z_1} \right] + \dot{m}_2 \left[h_2 + \cancel{\frac{V_2^2}{2}} + \cancel{g z_2} \right] - \dot{m}_3 \left[h_3 + \cancel{\frac{V_3^2}{2}} + \cancel{g z_3} \right]$$

$$\Rightarrow 0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \underbrace{\dot{m}_3}_{= (\dot{m}_1 + \dot{m}_2)} h_3$$

$$\Rightarrow \dot{m}_1 = \dot{m}_2 \left[\frac{h_2 - h_3}{h_3 - h_1} \right]$$

$$= 0.28 \frac{kg}{s} \left[\frac{3002.8 - 2803.1}{2803.1 - 169.9} \right]$$

$$= 0.021 \text{ kg/s}$$

Inserting values, Eq. (1) gives

$$\dot{\sigma}_{cv} = \left(0.301 \frac{kg}{s}\right)\left(6.2575 \frac{kJ}{kg \cdot K}\right) - (0.021)(0.5714) - (0.28)(6.6001)$$

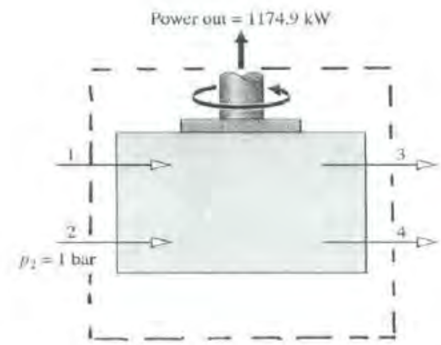
$$= 0.0235 \frac{kJ/s}{K} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 0.0235 \frac{kW}{K}$$

PROBLEM 6.84

An inventor claims that at steady state the device shown develops power from entering and exiting streams of water at a rate of 1174.9 kW. The accompanying table provides data for inlet 1 and exits 3 and 4. The pressure at inlet 2 is 1 bar. Stray heat transfer and kinetic and potential energy effects are negligible. Evaluate the inventor's claim.

State	$\dot{m}$(kg/s)	p (bar)	T(°C)	v (m³/kg)	u (kJ/kg)	h (kJ/kg)	s(kJ/kg·K)
1	4	1	450	3.334	3049.0	3382.4	8.6926
3	5	2	200	1.080	2654.4	2870.5	7.5066
4	3	4	400	0.773	2964.4	3273.4	7.8985



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer and kinetic and potential energy effects are negligible.

ANALYSIS: Begin by fixing the state at 2. This requires another property value to complement $p_2 = 1$ bar. To do this, adopt the claimed power output an apply an energy rate balance.

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3 - \dot{m}_4 h_4$$

mass rate balance: $\dot{m}_2 = \dot{m}_3 + \dot{m}_4 - \dot{m}_1$
$= 4$ kg/s

Solving the energy balance,

$$h_2 = \frac{\dot{W}_{cv} - \dot{m}_1 h_1 + \dot{m}_3 h_3 + \dot{m}_4 h_4}{\dot{m}_2}$$

$$= \frac{(1174.9 - 4(3382.4) + 5(2870.5) + 3(3273.4)}{4}$$

$$= 2954.5 \text{ kJ/kg}$$

With P_2 and h_2, Table A-4 gives $s_2 = 7.9949$ kJ/kg·K.

Now, checking the 2nd law via an entropy rate balance:

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{\,0} + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 - \dot{m}_4 s_4 + \dot{\sigma}_{cv}$$

$$\Rightarrow \dot{\sigma}_{cv} = \dot{m}_3 s_3 + \dot{m}_4 s_4 - \dot{m}_1 s_1 - \dot{m}_2 s_2$$

$$= (5)(7.5066) + (3)(7.8985) - (4)(8.6926) - 4(7.9949)$$

$$= -5.52 \ \frac{\text{kJ/s}}{\text{K}}$$

Since $\dot{\sigma}_{cv}$ must be positive for an actual device, the claimed performance is not allowed by the second law. The claimed power output and/or the given data must be in error.

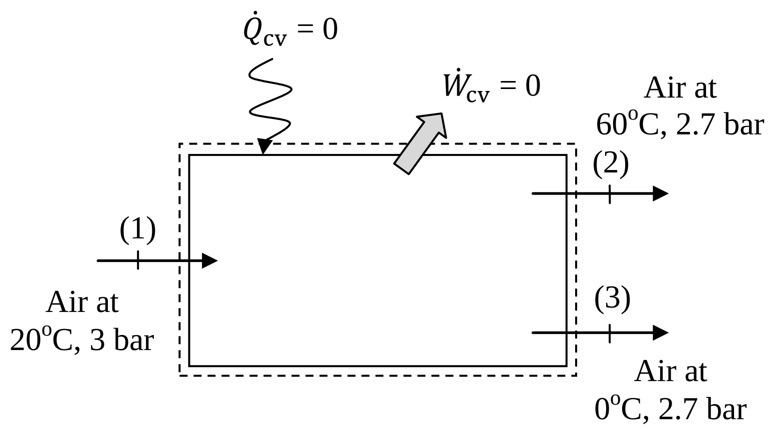
PROBLEM 6.85

An inventor claims to have developed a device requiring no work or heat transfer input, yet able to produce hot and cold air streams at steady state. Data claimed by the inventor are shown on the control volume in Fig. P6.85. The ideal gas model can be used for the air, and kinetic and potential energy effects can be neglected. Evaluate this claim.

KNOWN: An inventor claims to have developed a device that at steady state produces hot and cold air streams with no work or heat transfer. Data are provided.

FIND: Evaluate this claim.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The control volume operates at steady state. (2) $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (3) The air can be modeled as an ideal gas. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: The device must satisfy conservation and mass and energy. Applying mass and energy rate balances for the one-inlet, two-exit control volume at steady state:

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$$

and, neglecting kinetic and potential energy effects

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$$

Introducing $y = \dot{m}_2/\dot{m}_1$ we can combine the mass and energy rate balances to give

$$y = \frac{h_1 - h_3}{h_2 - h_3}$$

With data from Table A-22: $y = \frac{h_1 - h_3}{h_2 - h_3} = \frac{293.2 - 273.1}{333.3 - 273.1} = 0.334$

Accordingly, to satisfy conservation of mass and the First Law of Thermodynamics, the mass flow rates must be in definite proportions: $\dot{m}_2/\dot{m}_1 = 0.334$ and $\dot{m}_3/\dot{m}_1 = 0.666$.

PROBLEM 6.83 (CONTINUED)

The operation must also satisfy the Second law of Thermodynamics. The entropy rate balance at steady state reduces to give

$$0 = \sum_j \cancel{\left(\frac{\dot{Q}}{T}\right)_j} + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$$

Thus

$$\dot{\sigma}_{cv}/\dot{m}_1 = (\dot{m}_3/\dot{m}_1)s_3 - (\dot{m}_2/\dot{m}_1)s_2 - s_1$$

or

$$\dot{\sigma}_{cv}/\dot{m}_1 = (1 - y)\, s_3 - y s_2 - s_1 = (s_3 - s_1) + y\,(s_2 - s_3)$$

With data from Table A-22:

$$s_3 - s_1 = s^o(T_3) - s_o(T_1) - R \ln (p_3/p_1) = 1.6073 - 1.6783 - (8.314/28.97) \ln(2.7/3) = -0.040763 \text{ kJ/kg·K}$$

and

$$s_2 - s_3 = s^o(T_2) - s_o(T_3) - R \ln (p_2/p_3) = 1.8069 - 1.6073 - (8.314/28.97) \cancel{\ln(2.7/2.7)} = 0.1996 \text{ kJ/kg·K}$$

Finally

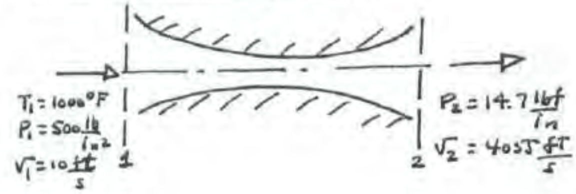
$$\dot{\sigma}_{cv}/\dot{m}_1 = (s_3 - s_1) + y\,(s_2 - s_3) = -0.040763 + (0.334)(0.1996) = 0.0295 \text{ kJ/kg·K}$$

Accordingly, the Second Law of thermodynamics would be satisfied, since $\dot{\sigma}_{cv}/\dot{m}_1 \geq 0$. Therefore, the claimed performance is consistent with conservation of mass as well as the First and Second Laws of Thermodynamics.

PROBLEM 6.86

Steam enters a well-insulated nozzle operating at steady state at 1000°F, 500 lbf/in.^{2} and a velocity of 10 ft/s. At the nozzle exit, the pressure is 14.7 lbf/in.^{2} and the velocity is 4055 ft/s. Determine the rate of entropy production, in Btu/°R per lb of steam flowing.

SCHEMATIC & GIVEN DATA:



ENGR MODEL:

1. A control volume at steady state encloses the nozzle.
2. For the control volume, $\dot{Q}_{cv}=0$, $\dot{W}_{cv}=0$, and potential energy effects play no role.

ANALYSIS: Reducing an entropy rate balance, $0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$

$$\Rightarrow \dot{\sigma}_{cv}/\dot{m} = s_2 - s_1 \qquad (1)$$

From Table A-4E, $s_1 = 1.7371$ Btu/lb·°R, $h = 1520.7$ Btu/lb.

To fix state 2, another property is needed to complement the pressure. This is specific enthalpy, h_2 found from an energy rate balance:

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + g\cancel{(z_1 - z_2)}\right]$$

$$\Rightarrow \quad h_2 = h_1 + \frac{V_1^2 - V_2^2}{2} = 1520.7 \frac{Btu}{lb} + \left[\frac{(10\,ft/s)^2 - (4055 \frac{ft}{s})^2}{2}\right]\left|\frac{1\,lbf}{32.2\,\frac{lb \cdot ft}{s^2}}\right|\left|\frac{1\,Btu}{778\,ft\cdot lbf}\right|$$

$$= 1192.5 \frac{Btu}{lb}$$

From Table A-4E at $P_2 = 14.7 \frac{lbf}{in^2}$, $s_2 = 1.8156$ Btu/lb·°R

Eq. (1) then gives

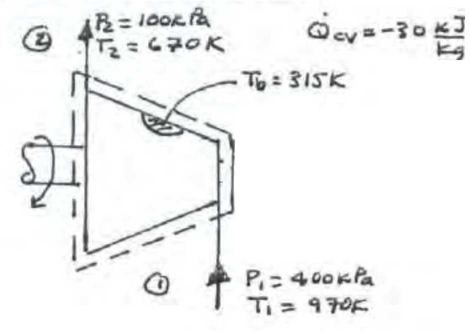
$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = (1.8156 - 1.7371) \frac{Btu}{lb \cdot °R}$$

$$= 0.079 \frac{Btu}{lb \cdot °R}$$

PROBLEM 6.87

Air at 400 kPa, 970 K enters a turbine operating at steady state and exits at 100 kPa, 670 K. Heat transfer from the turbine occurs at an average outer surface temperature of 315 K at the rate of 30 kJ per kg of air flowing. Kinetic and potential energy effects are negligible. For air as an ideal gas with $c_p = 1.1$ kJ/kg · K, determine (a) the rate power is developed, in kJ per kg of air flowing, and (b) the rate of entropy production within the turbine, in kJ/K per kg of air flowing.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. Heat transfer occurs at $T_b = 315$ K.
3. Kinetic and potential energy effects are negligible.
4. The air is modeled as an ideal gas with $c_p = 1.1$ kJ/kg·K.

ANALYSIS: (a) An energy rate balance reads, $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2)$

or

$$\frac{\dot{W}_{cv}}{\dot{m}} = \frac{\dot{Q}_{cv}}{\dot{m}} + (h_1 - h_2)$$

$$= \frac{\dot{Q}_{cv}}{\dot{m}} + c_p(T_1 - T_2) = -30\frac{kJ}{kg} + 1.1\frac{kJ}{kg \cdot K}(970 - 670)K$$

$$= 300 \text{ kJ/kg} \qquad (a)$$

(b) An entropy rate balance reads,

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

or

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = -\frac{\dot{Q}_{cv}/\dot{m}}{T_b} + (s_2 - s_1)$$

With Eq. 6.22

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = -\frac{\dot{Q}_{cv}/\dot{m}}{T_b} + \left[c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}\right]$$

$$= -\frac{(-30 \text{ kJ/kg})}{315 K} + \left[1.1\frac{kJ}{kg \cdot K}\ln\frac{670}{970} - \frac{8.314}{28.97}\frac{kJ}{kg \cdot K}\ln\left(\frac{100}{400}\right)\right]$$

$$= 0.086 \frac{kJ}{kg \cdot K} \qquad (b)$$

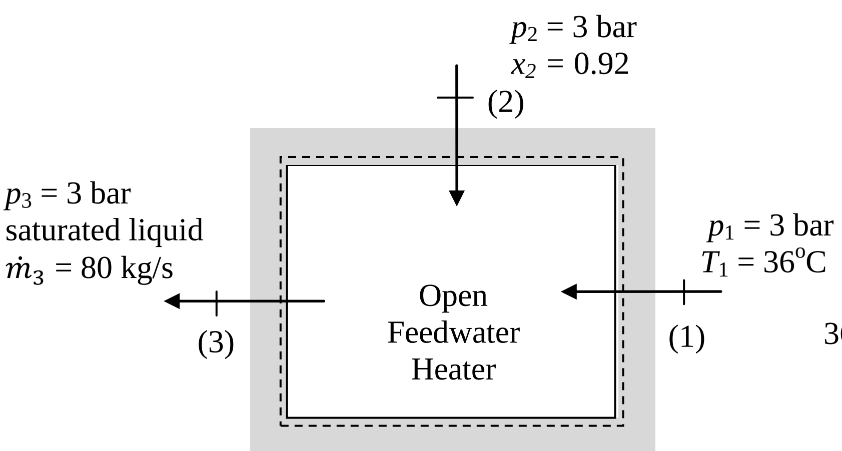
PROBLEM 6.88

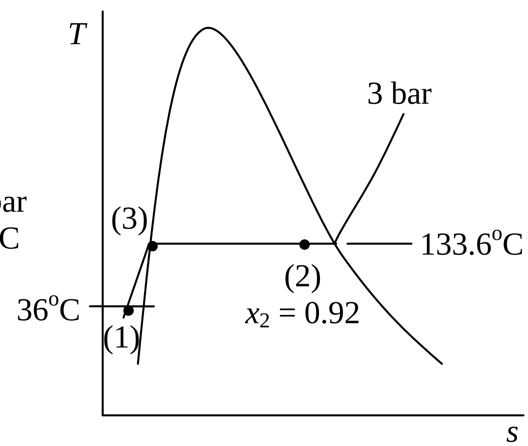
An *open feedwater heater* is a direct-contact heat exchanger used in vapor power plants. Shown in Fig. P6.88 are operating data for an open heater with H_2O as the working fluid operating at steady state. Ignoring stray heat transfer from the outside of the heat exchanger to its surroundings and kinetic and potential energy effects, determine the rate of entropy production, in kW/K.

KNOWN: Data are provided for the steady-state operation of an open feedwater heater.

FIND: Determine the rate of entropy production.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL: (1) The control volume is at steady state. (2) For the open heater, $\dot{Q}_{\text{cv}} = \dot{W}_{\text{cv}} = 0$. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: First, let's fix the states using appropriate table data.
State 1: The state is compressed liquid. Treating the liquid as virtually incompressible, the following approximations can be used: $h_1 \approx h_f(36^\circ\text{C}) = 150.86$ kJ/kg and $s_1 \approx s_f(36^\circ\text{C}) = 0.5188$ kJ/kg·K.

State 2: Using data from Table A-3

$$h_2 = h_{f2} + x_2(h_{g2} - h_{f2}) = 561.47 + (0.92)(2725.3 - 561.47) = 2552.2 \text{ kJ/kg}$$

and

$$s_2 = s_{f2} + x_2(s_{g2} - s_{f2}) = 1.6718 + (0.92)(6.9919 - 1.6718) = 6.5663 \text{ kJ/kg·K}$$

State 3: p_3 = 3 bar, saturated liquid: $h_3 = 561.47$ kJ/kg and $s_3 = 1.6718$ kJ/kg·K

Applying mass and energy balances: $0 = \dot{m}_1 + \dot{m}_2 - \dot{m}_3 \longrightarrow \dot{m}_1 = \dot{m}_3 - \dot{m}_2$
Neglecting kinetic and potential energy effects

$$0 = \cancel{\dot{Q}_{\text{cv}}} - \cancel{\dot{W}_{\text{cv}}} + \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3 \longrightarrow 0 = (\dot{m}_3 - \dot{m}_2)h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$$

PROBLEM 6.88 (CONTINUED)

Or

$$\dot{m}_2 = (\dot{m}_3)\left(\frac{h_3 - h_1}{h_2 - h_1}\right) = (80 \text{ kg/s})\left(\frac{561.47 - 150.86}{2552.2 - 150.86}\right) = 13.68 \text{ kg/s}$$

$$\dot{m}_1 = \dot{m}_3 - \dot{m}_2 = 80 - 13.68 = 66.32 \text{ kg/s}$$

Now, applying the entropy rate balance to the control volume at steady state

$$0 = \cancel{\sum_j \left(\frac{\dot{Q}}{T}\right)_j} + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv} \quad \longrightarrow \quad \dot{\sigma}_{cv} = \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2$$

Inserting values

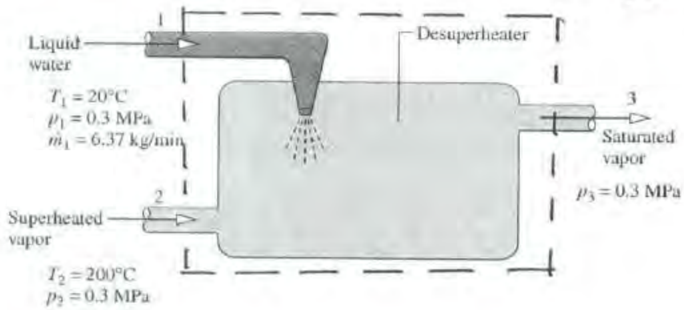
$$\dot{\sigma}_{cv} = [\,(80\text{kg/s})(\,1.6718 \text{ kJ/kg·K}) - (66.32)(\,0.5188) - (13.68)(\,6.5663)]\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$
$$= 9.51 \text{ kW/K} \longleftarrow$$

The entropy production rate is positive, indicating that the mixing of streams at different states is irreversible, as expected.

PROBLEM 6.89

By injecting liquid water into superheated vapor, the *desuperheater* shown has a saturated vapor stream at its exit. Steady-state operating data are shown on the figure. Ignoring stray heat transfer and kinetic and potential energy effects, determine (a) the mass flow rate of the superheated vapor stream, in kg/min, and (b) the rate of entropy production within the desuperheater, in kW/K.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the figure is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$. Also, stray heat transfer and kinetic and potential energy effects are ignored.
3. At state 1, $h \approx h_f(T_1)$, $s \approx s_f(T_1)$

ANALYSIS: Property data:

Table A-2: $h_1 \approx h_f(T_1) = 83.96$ kJ/kg $s_1 \approx s_f(T_1) = 0.2966$ kJ/kg·K	Table A-4: $h_2 = 2865.5$ kJ/kg $s_2 = 7.3115$ kJ/kg·K	Table A-3: $h_3 = 2725.3$ kJ/kg $s_3 = 6.9919$ kJ/kg·K

(a) To obtain the value of $\dot{m}_2$, write a mass rate balance: $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$ and an energy rate balance: $0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$, obtaining

$$\dot{m}_2 = \dot{m}_1 \left[\frac{h_3 - h_1}{h_2 - h_3}\right] = 6.37\frac{kg}{min}\left[\frac{2725.3 - 83.96}{2865.5 - 2725.3}\right] = 120.01 \frac{kg}{min}$$ (a)

(b) To obtain the rate of entropy production, write an entropy rate balance:

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{0} + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$$

$$\Rightarrow \quad \dot{\sigma}_{cv} = \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2$$

$$= (120.01 + 6.37)\frac{kg}{min}\left(6.9919 \frac{kJ}{kg \cdot K}\right) - (6.37)(0.2966) - (120.01)(7.3115)$$

$$= 4.3 \frac{kJ/min}{K}\left|\frac{1\ min}{60\ s}\right|\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 0.072 \frac{kW}{K}$$ (b)

PROBLEM 6.90

Air at 600 kPa, 330 K enters a well-insulated, horizontal pipe having a diameter of 1.2 cm and exits at 120 kPa, 300 K. Applying the ideal gas model for air, determine at steady state (a) the inlet and exit velocities, each in m/s, (b) the mass flow rate, in kg/s, and (c) the rate of entropy production, in kW/K.

See the solution to Problem 4.27:

(a) $V_1 = 55.38\ m/s$, $V_2 = 251.73\ m/s$, (b) $\dot{m} = 0.04\ kg/s$.

(c) Reducing an entropy rate balance, $0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{\,0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$

$$\Rightarrow \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1) = \dot{m}\left(s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{P_2}{P_1}\right)$$

$$= 0.04 \frac{kg}{s}\left[1.70203 - 1.79783 - \frac{8.314}{28.97} \ln\left(\frac{120}{600}\right)\right] \frac{kJ}{kg \cdot K}$$

$$= 0.0146 \frac{kJ/s}{K} \left| \frac{1\ kW}{1\ kJ/s} \right|$$

$$= 0.0146 \frac{kW}{K} \qquad \text{(c)}$$

PROBLEM 6.91

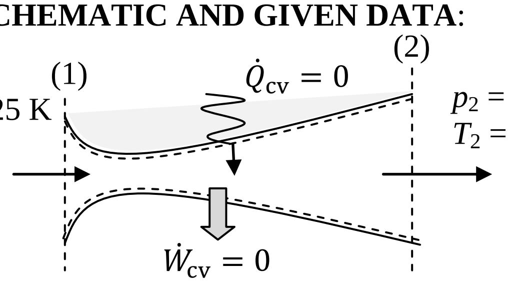
Air at 200 kPa, 52°C, and a velocity of 355 m/s enters an insulated duct of varying cross-sectional area. The air exits at 100 kPa, 82°C. At the inlet, the cross-sectional area is 6.57 cm^2. Assuming the ideal gas model for the air, determine
(a) the exit velocity, in m/s.
(b) the rate of entropy production within the duct, in kW/K.

KNOWN: Data are provided for air flowing through a variable-area, insulated duct at steady state.

FIND: Determine the exit velocity and the rate of entropy production within the duct.

SCHEMATIC AND GIVEN DATA:

p_1 = 200 kPa
T_1 = 52°C = 325 K
V_1 = 355 m/s
A_1 = 6.57 cm^2



p_2 = 100 kPa
T_2 = 82°C = 355 K

ENGINEERING MODEL: (1) The control volume is at steady state. (2) $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (3) Potential energy effects are negligible. (4) The air is modeled as an ideal gas.

ANALYSIS: (a) To get the exit velocity, we apply mass and energy rate balances at steady state as follows. $0 = \dot{m}_1 - \dot{m}_2 \equiv \dot{m}$ and

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}[(h_1 - h_2) + (V_1^2 - V_2^2)/2 + g\cancel{(z_1 - z_2)}]$$

or

$$V_2 = \sqrt{V_1^2 + 2(h_1 - h_2)}$$

With specific enthalpy values from Table A-22

$$V_2 = \sqrt{(355)^2(\text{m}^2/\text{s}^2) + 2(325.31 - 355.94)\frac{\text{kJ}}{\text{kg}}\left|\frac{10^3\,\text{N}\cdot\text{m}}{1\,\text{kJ}}\right|\left|\frac{1\,\text{kg}\cdot\text{m/s}^2}{1\,\text{N}}\right|} = 254.5 \text{ m/s} \longleftarrow$$

(b) Reducing the entropy rate balance; $0 = \cancel{\sum_j \left(\frac{\dot{Q}}{T}\right)_j} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$

or

$$\dot{\sigma}_{cv} = \dot{m}(s_2 - s_1) = \dot{m}[(s^\circ(T_2) - s^\circ(T_1) - R \ln(p_2/p_1)]$$

$$= (0.5 \text{ kg/s})[(1.87269 - 1.78249) - (8.314/28.97) \ln(100/200)]\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$

$$= 0.144 \text{ kW/K} \longleftarrow$$

PROBLEM 9.92

KNOWN: Operating data are provided for an electronics enclosure at steady state.

FIND: For the control volume of Example 4.8 determine the rate of entropy production when air exits at 32°C.

SCHEMATIC & GIVEN DATA: See Fig. E4.8

ENGINEERING MODEL: See Example 4.8. Ignore the change in pressure from inlet to exit.

ANALYSIS: At steady state, mass and entropy rate balances reduce to give

$$0 = \cancel{\sum_j \frac{\dot{Q}_j}{T_j}}^{\,0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

$$\Rightarrow \quad \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1)$$

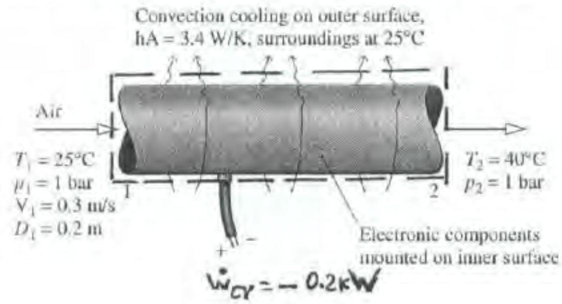
Introducing $\dot{m}$ from Example 4.8 and evaluating $(s_2 - s_1)$ with Eq. 6.22

$$\dot{\sigma}_{cv} = \left[\frac{(-\dot{W}_{cv})}{c_p(T_2 - T_1)}\right]\left(c_p \ln\frac{T_2}{T_1} - \cancel{R \ln\frac{p_2}{p_1}}^{\,0}\right) \Rightarrow \dot{\sigma}_{cv} = \frac{(-\dot{W}_{cv})}{(T_2 - T_1)} \ln\frac{T_2}{T_1}$$

$$\Rightarrow \quad \dot{\sigma}_{cv} = \frac{(98\,W)}{(305 - 293)K} \ln\frac{305}{293} = 0.328\,\frac{W}{K} \longleftarrow \dot{\sigma}_{cv}$$

PROBLEM 6.93

Electronic components are mounted on the inner surface of a horizontal cylindrical duct whose inner diameter is 0.2 m, To prevent overheating of the electronics, the cylinder is cooled by a stream of air flowing through it and by convection from its outer surface. Air enters the duct at 25°C, 1 bar and a velocity of 0.3 m/s and exits at 40°C with negligible changes in kinetic energy and pressure. Convective cooling occurs on the outer surface to the surroundings, which are at 25°C, in accord with hA = 3.4 W/K, where h is the heat transfer coefficient and A is the surface area. The electronic components require 0.20 kW of electric power. For a control volume enclosing the cylinder, determine at steady state (a) the mass flow rate of the air, in kg/s, (b) the temperature on the outer surface of the duct, in °C, and (c) the rate of entropy production, in W/K. Assume the ideal gas model for air.



ENGR. MODEL:

1. The control volume shown in the figure is at steady state.
2. For the control volume, kinetic and potential energy effects can be ignored.
3. The air is modeled as an ideal gas.

ANALYSIS: (a) At steady state, $\dot{m}_1 = \dot{m}_2 = \dot{m}$, where

$$\dot{m}_1 = \frac{A_1 V_1}{v_1} = \frac{(\pi D_1^2/4)V_1}{RT_1/P_1}$$

$$= \frac{\pi(0.2\,\text{m})^2(0.3\,\text{m/s})(10^5\,\text{N/m}^2)}{4\left(\frac{8314}{28.97}\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(298\,\text{K})} = 0.011\ \frac{\text{kg}}{\text{s}} \longleftarrow \text{(a)}$$

(b) An energy rate balance is

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

or

$$\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}(h_2 - h_1)$$
$$= \dot{W}_{cv} + \dot{m}(h(T_2) - h(T_1))$$

Continued →

PROBLEM 6.93 (CONTINUED)

$$\dot{Q}_{cv} = (-0.2\text{ kW}) + \left(0.011\ \frac{\text{kg}}{\text{s}}\right)\left[h(313\text{ K}) - h(298\text{ K})\right]\left(\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$

$$= (-0.2\text{ kW}) + \left(0.011\ \frac{\text{kg}}{\text{s}}\right)\left[(313.3 - 298.2)\ \frac{\text{kJ}}{\text{kg}}\right]\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$

$$= -0.034\text{ kW} = -34\text{ W}$$

Invoking Eq. 2.34, $|\dot{Q}_{cv}| = hA(T_b - T_f)$ where $hA = 3.4\ \frac{W}{K}$ and $T_f = 298K$.

$$\Rightarrow T_b = \frac{|\dot{Q}_{cv}|}{hA} + T_f = \frac{34\text{ W}}{3.4\text{ W/K}} + 298\text{K} = \begin{matrix}308K \\ (35^\circ C)\end{matrix} \quad \text{(b)}$$

(c) Mass and entropy rate balances reduce to read,

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

$$\Rightarrow \quad \dot{\sigma}_{cv} = -\frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_2 - s_1)$$

$$= -\frac{\dot{Q}_{cv}}{T_b} + \dot{m}\left(s^\circ(T_2) - s^\circ(T_1) - \cancelto{0\ (p_2 = p_1)}{R\ln\frac{p_2}{p_1}}\right)$$

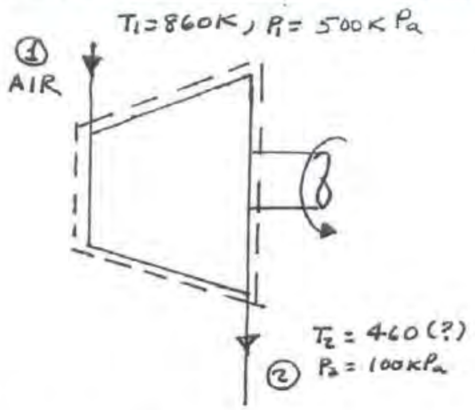
$$= -\frac{(-34\text{ W})}{308\text{ K}} + 0.011\ \frac{\text{kg}}{\text{s}}(1.7446 - 1.6953)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{10^3\text{ J}}{1\text{ kJ}}\right|\left|\frac{1\text{ W}}{1\text{ J/s}}\right|$$

$$= 0.653\ \frac{W}{K}$$

PROBLEM 6.94

Air enters a turbine operating at steady state at 500 kPa, 860 K and exits at 100 kPa. A temperature sensor indicates that the exit air temperature is 460 K. Stray heat transfer and kinetic and potential energy effects are negligible, and the air can be modeled as an ideal gas. Determine if the exit temperature reading can be correct. If yes, determine the power developed by the turbine for an expansion between these states, in kJ per kg of air flowing. If no, provide an explanation with supporting calculations.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer and kinetic and potential effects are negligible.
3. The air can be modeled as an ideal gas.

ANALYSIS: Apply an entropy rate balance:

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

$$\Rightarrow \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1)$$

$$= \dot{m}\left(s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1}\right)$$

or

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1}$$

$$= \left(2.13407 - 2.79783 - \frac{8.314}{28.97} \ln \frac{100}{500}\right) \frac{kJ}{kg \cdot K}$$

$$= -0.202 \frac{kJ}{kg \cdot K}$$

Since $\dot{\sigma}_{cv}/\dot{m}$ cannot be negative, the exit temperature reading cannot be correct.

PROBLEM 6.95

The figure below provides steady-state test data for a control volume in which two entering streams of air mix to form a single exiting stream. Stray heat transfer and kinetic and potential energy effects are negligible. A hard-to-read photocopy of the data sheet indicates that the pressure of the exiting stream is either 1.0 MPa or 1.8 MPa. Assuming the ideal gas model for air with $c_p = 1.02$ kJ/kg · K, determine if either or both of these pressure values can be correct.

ENGR. MODEL:

1. The control volume shown in the figure is at steady state.
2. Stray heat transfer and kinetic and potential energy effects are negligible.
3. The air is modeled as an ideal gas with $c_p = 1.02$ kJ/kg·K.

ANALYSIS: Mass rate balance: $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$. To find T_3, apply an energy rate balance:

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3 \Rightarrow 0 = \dot{m}_1 \underbrace{(h_1 - h_3)}_{c_p(T_1 - T_3)} + \dot{m}_2 \underbrace{(h_2 - h_3)}_{c_p(T_2 - T_3)}$$

$$\Rightarrow \quad T_3 = \frac{\dot{m}_1 T_1 + \dot{m}_2 T_2}{(\dot{m}_1 + \dot{m}_2)} = \frac{(1\,kg/s)(800K) + (2\,kg/s)(650K)}{3\,kg/s} = 700K$$

An entropy rate balance reduces to read, $0 = \sum \cancel{\dot{Q}_j/T_j} + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$

$$\Rightarrow \dot{\sigma}_{cv} = (\dot{m}_1 + \dot{m}_2) s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2 = \dot{m}_1 (s_3 - s_1) + \dot{m}_2 (s_3 - s_2)$$

With Eq. 6.22

$$\dot{\sigma}_{cv} = \dot{m}_1 \left[c_p \ln \frac{T_3}{T_1} - R \ln \frac{P_3}{P_1} \right] + \dot{m}_2 \left[c_p \ln \frac{T_3}{T_2} - R \ln \frac{P_3}{P_2} \right]$$

$$\dot{\sigma}_{cv} = c_p \left[\dot{m}_1 \ln \frac{T_3}{T_1} + \dot{m}_2 \ln \frac{T_3}{T_2} \right] - R \left[\dot{m}_1 \ln \frac{P_3}{P_1} + \dot{m}_2 \ln \frac{P_3}{P_2} \right]$$

Inserting values

$$\dot{\sigma}_{cv} = 1.02 \frac{kJ}{kg \cdot K} \left[1\frac{kg}{s} \ln \frac{700}{800} + 2\frac{kg}{s} \ln \frac{700}{650} \right] - \frac{8.314}{28.97} \frac{kJ}{kg \cdot K} \left[1\frac{kg}{s} \ln \frac{P_3}{1.8} + 2\frac{kg}{s} \ln \frac{P_3}{1.0} \right]$$

$$= 0.01499 \frac{kJ/s}{K} - 0.287 \frac{kJ}{kg \cdot K} \left[1\frac{kg}{s} \ln \frac{P_3}{1.8} + 2\frac{kg}{s} \ln \frac{P_3}{1.0} \right] \qquad (1)$$

Check $P_3 = 1.0$ MPa. Eq.(1) gives

$$\dot{\sigma}_{cv} = 0.01499 \frac{kJ/s}{K} - 0.287 \frac{kJ}{kg \cdot K} \left[1\frac{kg}{s} \ln \frac{1.0}{1.8} \right] = 0.1837 \frac{kJ/s}{K}$$

Yes. This value for P_3 can be correct.

Check $P_3 = 1.8$ MPa. Eq.(1) gives

$$\dot{\sigma}_{cv} = 0.01499 \frac{kJ/s}{K} - 0.287 \frac{kJ}{kg \cdot K} \left[2\frac{kg}{s} \ln \frac{1.8}{1.0} \right] = -0.3224 \frac{kJ/s}{K}$$

No. This value for P_3 cannot be correct.

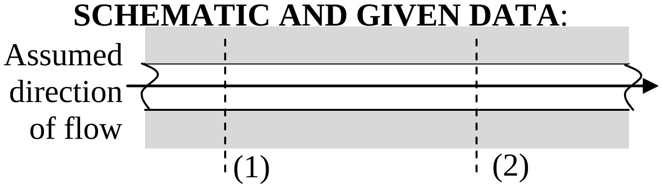
PROBLEM 6.96

Students in a laboratory are studying air flowing at steady state through a horizontal, insulated duct. One student group reports the measured pressure, temperature, and velocity at one location in the duct as 0.95 bar, 67°C, and 75 m/s, respectively. The group reports the following values at another location in the duct: 0.8 bar, 22°C, and 310 m/s. The group neglected to note the direction of flow on the data sheet, however. Using the data provided, determine the direction of flow.

KNOWN: Data are known for air flow at two locations in a horizontal, well-insulated duct.

FIND: Determine the direction of flow.

SCHEMATIC AND GIVEN DATA:



p_1 = 0.95 bar
T_1 = 67°C = 340 K
V_1 = 75 m/s

p_2 = 0.8 bar
T_2 = 22°C = 295 K
V_2 = 310 m/s

ENGINEERING MODEL: (1) The control volume is at steady state. (2) $\dot{Q}_{cv}$ = 0. (3) The air is modeled as an ideal gas.

ANALYSIS: To determine the direction of flow, let us apply the entropy rate balance at steady state, assuming the direction of flow as shown above and calculate $\dot{\sigma}_{cv}/\dot{m}$. If the value is greater than zero, the assumption is correct. If the value is less than zero, the flow is in the opposite direction to the direction assumed.

At steady state: $0 = \cancel{\sum_j \frac{\dot{Q}_j}{T_j}} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \longrightarrow \dot{\sigma}_{cv}/\dot{m} = (s_2 - s_1) \geq 0$

For air as an ideal gas, with data from Table A-22

$$s_2 - s_1 = s^o(T_2) - s^o(T_1) - R \ln(p_2/p_1)$$

$$= (1.68515 - 1.82790)\text{k/kg·K} - (8.314/28.97)(\text{ k/kg·K}) \ln(0.8/0.95)$$

$$= -0.0934 \text{ k/kg·K}$$

Accordingly, the assumption that the flow is from left to right is incorrect. The direction of flow is *from right to left*.

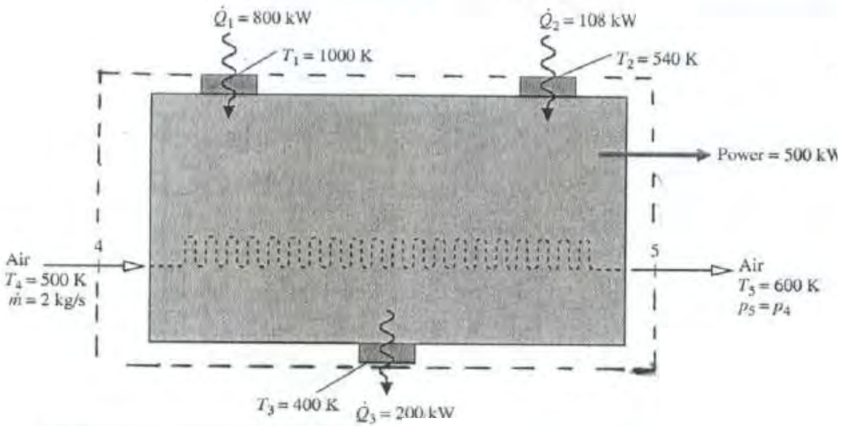
PROBLEM 9.97

An inventor has provided the steady-state operating data for a *cogeneration* system producing power and increasing the temperature of a stream of air. The system receives and discharges energy by heat transfer at the rates and temperatures indicated on the figure. All heat transfers are in the directions of the accompanying arrows. The ideal gas model applies to the air. Kinetic and potential energy effects are negligible. Using energy and entropy rate balances, evaluate the thermodynamic performance of the system.

KNOWN: Steady-state operating data are provided for a cogeneration system.

FIND: Evaluate the thermodynamic performance of the system using energy and entropy balances.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The control volume shown in the schematic is at steady state.
2. The only heat transfers are those shown on the schematic.
3. Kinetic and potential energy effects are negligible.
4. The air is modeled as an ideal gas.

ANALYSIS: For the control volume under consideration, $\dot{m}_4 = \dot{m}_5 = \dot{m}$. An energy rate balance reads, $0 = [\dot{Q}_1 + \dot{Q}_2 - \dot{Q}_3] - \dot{W} + \dot{m}[h_4 - h_5]$. Thus

$$\dot{W} = [\dot{Q}_1 + \dot{Q}_2 - \dot{Q}_3] + \dot{m}[h_4 - h_5]$$, where h_4 and h_5 are obtained from Table A-22.

$$= [800 + 108 - 200]\,\text{kW} + 2\frac{\text{kg}}{\text{s}}[503.02 - 607.02]\frac{\text{kJ}}{\text{kg}}\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right| = 500\,\text{kW}$$

Accordingly, the given data agree with the conservation of energy principle.

An entropy rate balance reads

$$0 = \left[\frac{\dot{Q}_1}{T_1} + \frac{\dot{Q}_2}{T_2} - \frac{\dot{Q}_3}{T_3}\right] + \dot{m}[s_4 - s_5] + \dot{\sigma}$$

$$\Rightarrow \quad \dot{\sigma} = \left[\frac{\dot{Q}_3}{T_3} - \frac{\dot{Q}_1}{T_1} - \frac{\dot{Q}_2}{T_2}\right] + \dot{m}\Big[s_5^\circ - s_4^\circ - \underbrace{\cancel{R\ln\frac{p_5}{p_4}}}_{=0\ (p_5 = p_4)}\Big]$$

$$= \left[\frac{200}{400} - \frac{800}{1000} - \frac{108}{540}\right]\frac{\text{kW}}{\text{K}} + 2\frac{\text{kg}}{\text{s}}[2.40902 - 2.21952]\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right|$$

$$= [0.50 - 0.8 - 0.2]\frac{\text{kW}}{\text{K}} + 0.38\frac{\text{kW}}{\text{K}}$$

$$= -0.12\frac{\text{kW}}{\text{K}}$$

*

Since the entropy production rate is negative, the given data do not agree with the second law. In sum, the system cannot perform in accordance with the operating data provided.

PROBLEM 6.98

Steam at 550 lbf/in.^{2}, 700^{o}F enters a turbine operating at steady state and exits at 1 lbf/in.^{2} The turbine produces 500 hp. For the turbine, heat transfer is negligible as are kinetic and potential energy effects.

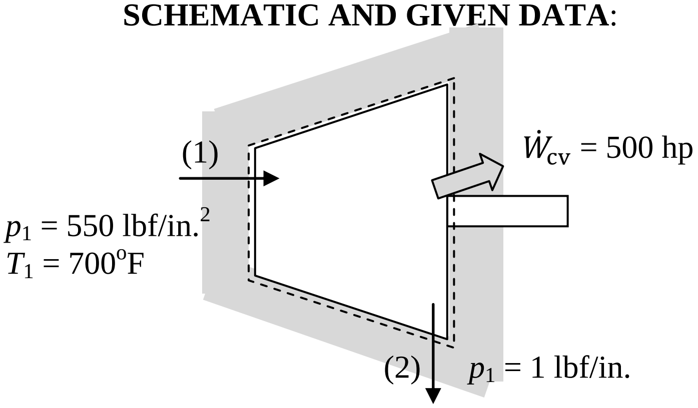
(a) Determine the quality of the steam at the turbine exit, the mass flow rate, in lb/s, and the entropy production rate, in Btu/s·^{o}R, if the turbine operates without internal irreversibilities.

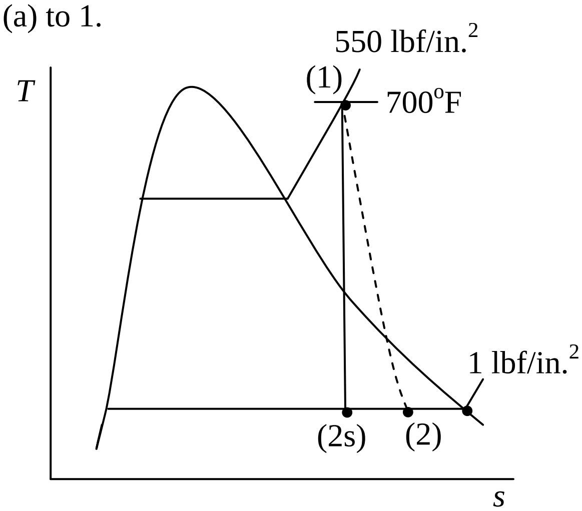
(b) Plot the mass flow rate, in lb/s, and the entropy production rate, in Btu/s·^{o}R, for exit qualities ranging from the value calculated in part (a) to 1.

KNOWN: Data are given for a steam turbine operating at steady state. The power is pecified.

FIND: (a) For internally reversible operation, determine the exit quality, the mass flow rate, and the rate of entropy production. (b) Plot the mass flow rate and entropy production rate versus exit quality ranging from the value calculated in part (a) to 1.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL: (1) The control volume is at steady state. (2) $\dot{Q}_{cv} = 0$. (3) Kinetic and potential energy effects can be neglected.

ANALYSIS: For the control volume at steady state: $\dot{m}_1 = m_2 \equiv \dot{m}$ and the entropy rate balance reduces to give

$$0 = \cancel{\sum_j \left(\frac{\dot{Q}_j}{T_i}\right)} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \longrightarrow \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1) \qquad (*)$$

(a) For an *internally reversible* expansion, $\boxed{\dot{\sigma}_{cv} = 0}$ and $s_2 = s_1$. Let's call this state 2s as shown on the *T-s* diagram. Interpolating in Table A-4; h_1 = 1353.6 Btu/lb and s_1 = 1.5992 Btu/lb·^{o}R. For state 2s, we use data from Table A-3:

$$x_{2s} = \frac{s_{2s} - s_{f2}}{s_{g2} - s_{f2}} = \frac{1.5992 - 0.1327}{1.9779 - 0.1327} = 0.7948$$

and

$$h_{2s} = h_{f2} + x_{2s}(h_{g2} - h_{f2}) = 69.74 + (0.7948)(1105.8 - 69.74) = 893.2 \text{ Btu/lb}$$

PROBLEM 6.98 (CONTINUED)

The energy rate balance reduces to: $0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\frac{(V_1^2 - V_2^2)}{2}} + g(\cancel{z_1 - z_2})\right]$

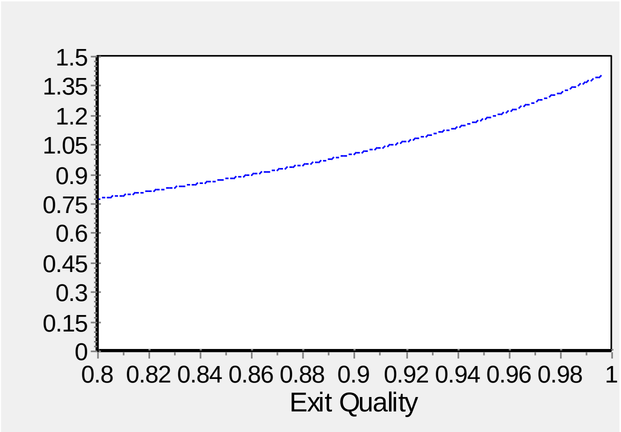
Thus

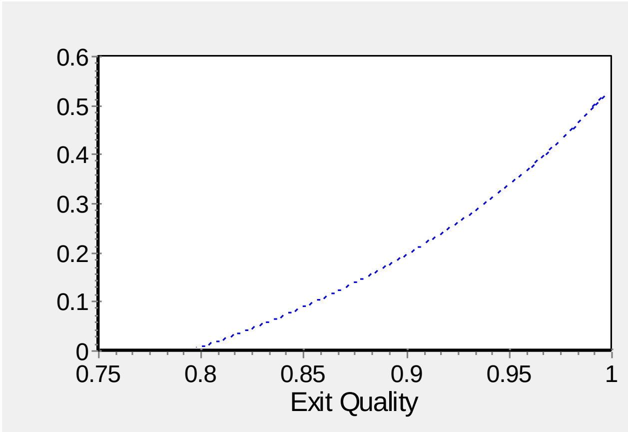
$$\dot{m} = \frac{\dot{W}_{cv}}{(h_1 - h_2)} = \frac{(500\ \text{hp})}{(1353.6 - 893.2)\text{Btu/lb}}\left|\frac{2545\ \text{Btu/h}}{1\ \text{hp}}\right|\left|\frac{1\ \text{h}}{3600\ \text{s}}\right| = 0.7678\ \text{lb/s}$$

(b) The following is the *IT* code used to generate the data for the plots.

```
// Given Data
p1 = 550  //lbf/in^2
T1 = 700  //F
p2 = 1  //lb/in^2
Wdot = 500 //hp

// Determine Properties
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
s2s = s1
s2s = ssat_Px("Water/Steam", p2, x2s)
h2s = hsat_Px("Water/Steam", p2, x2s)

x2 = 1
h2 = hsat_Px("Water/Steam", p2, x2)
s2 = ssat_Px("Water/Steam", p2, x2)

// Energy and Entropy Balances
Wdot = mdot*(h1 - h2)*(3600/2545)
sigmadot = mdot*(s2 - s1)

// Sweep x2 from 0.795 to 1.0 in steps of 0.01
```

	$x_2 = 0.7945$	$x_2 = 0.8$	$x_2 = 0.9$	$x_2 = 1$
h_2 (Btu/lb)	892.7	898.4	1002	1106
s_2 (Btu/lb·°R)	1.599	1.609	1.793	1.978
$\dot{\sigma}_{cv}$ (Btu/s·°R)	0	0.007834	0.1957	0.5406
$\dot{m}$ (lb/s)	0.7678	0.7767	1.006	1.426





The values in the table above compare favorably with the values calculated in Part (a) for the base case of internally reversible operation. The graphs indicate that as the exit quality increases, the process is increasingly irreversible, as indicated by the increased entropy production rate. The required mass flow rate increases in order to maintain constant power output as the change in specific enthalpy decreases from inlet to exit with increasing irreversibility.

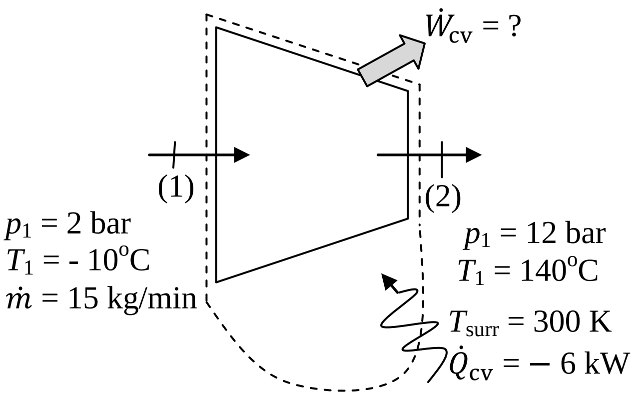
PROBLEM 6.99

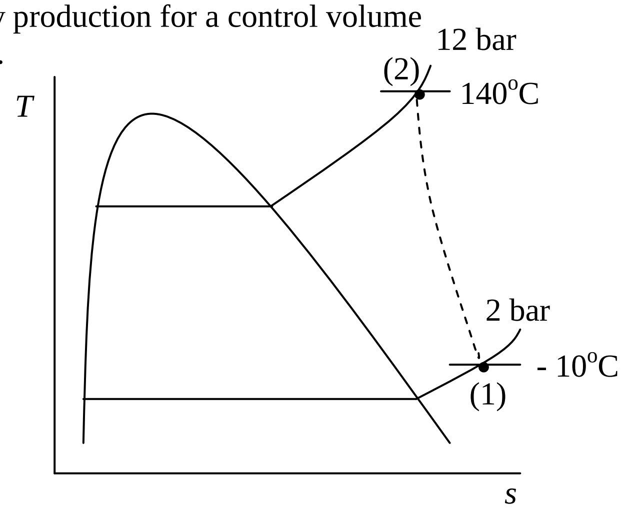
Ammonia enters the compressor of an industrial refrigeration plant at 2 bar, -10°C with a mass flow rate of 15 kg/min and is compressed to 12 bar, 140°C. Heat transfer occurs from the compressor to its surroundings at a rate of 6 kW. For steady state operation with negligible kinetic and potential energy effects, determine (a) the power input to the compressor, an KW, and (b) the rate of entropy production, in kW/K, for a control volume enclosing the compressor and its immediate surroundings such that the heat transfer occurs at 300 K.

KNOWN: Ammonia is compressed in a compressor. Data are provided for steady state operation.

FIND: Determine the power input and the rate of entropy production for a control volume enclosing the compressor and its immediate surroundings.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL: (1) The control volume shown is at steady state. (2) Heat transfer occurs to the surroundings at 300 K. (3) Kinetic and potential energy effects can be ignored.

ANALYSIS: First, we fix the states. Both states are in the superheated vapor region, as can be verified. From Table A-15: h_1 = 1440.31 kJ/kg, s_1 = 5.6781 kJ/kg·K and h_2 = 1757.26 kJ/kg, s_2 = 5.7850 kJ/kg·K.

(a) To determine the power input, we use the steady state control volume energy rate balance, which reduces to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\frac{(V_1^2 - V_2^2)}{2}} + g\cancel{(z_1 - z_2)}\right]$$

or

$$\dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}(h_1 - h_2) = (-6 \text{ kW}) + (15 \text{ kg/min})\left|\frac{1 \text{ min}}{60 \text{ s}}\right|(1440.31 - 1757.26)\text{kJ/kg}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$

$$= -85.24 \text{ kW (in)}$$

PROBLEM 6.99 (CONTINUED)

(b) The steady state control volume entropy rate balance reduces to

$$0 = \frac{\dot{Q}_{\text{cv}}}{T_{\text{surr}}} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{\text{cv}}$$

or

$$\dot{\sigma}_{\text{cv}} = -\frac{\dot{Q}_{\text{cv}}}{T_{\text{surr}}} + \dot{m}(s_2 - s_1) = -\left(\frac{-6\text{ kW}}{300\text{K}}\right) + (15/60)\text{kg/s}(5.7850 - 5.6781)\text{ kJ/kg}\cdot\text{K}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$

$$= 0.04673\text{ kW/K}$$

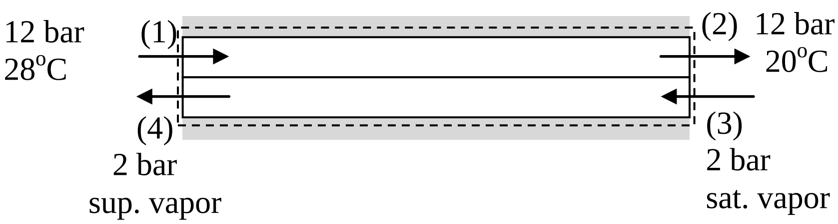
PROBLEM 6.100

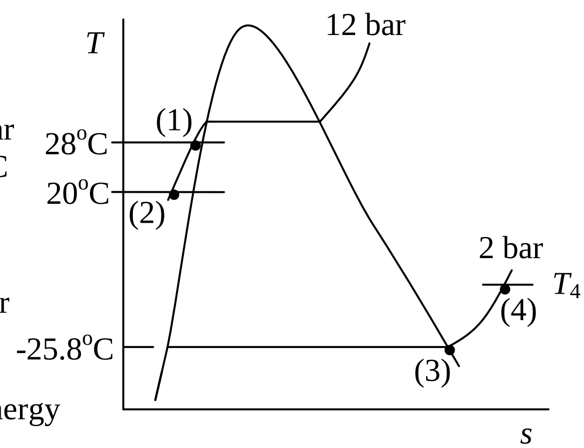
Refrigerant 22 in a refrigeration system enters one side of a counter-flow heat exchanger at 12 bar, 28°C. The refrigerant exits at 22 bar, 20°C. A separate stream of R-22 enters the other side of the heat exchanger as saturated vapor at 2 bar and exits as superheated vapor at 2 bar. The mass flow rates of the two streams are equal. Stray heat transfer from the heat exchanger to its surroundings and kinetic and potential energy effects are negligible. Determine the entropy production in the heat exchanger, in kJ/K per kg of refrigerant flowing. What gives rise to the entropy production in this application?

KNOWN: Two streams of R-22 pass though opposite sides of a counter-flow heat exchanger operating at steady state with equal mass flow rates. Data are known for each stream.

FIND: Determine the entropy production for the heat exchanger per unit mass of refrigerant flowing.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL: (1) The control volume is at steady state. (2) $\dot{W}_{cv} = \dot{Q}_{cv} = 0$. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: To fix state 4, we write mass and energy rate balances. The mass balances reduce at steady state to $\dot{m}_1 = \dot{m}_2$ and $\dot{m}_3 = \dot{m}_4$. Further, $\dot{m}_1 = \dot{m}_3 \equiv \dot{m}$

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}[(h_1 - h_2) + (h_3 - h_4)] \longrightarrow h_4 = h_1 - h_2 + h_3$$

From Table A-7: $h_1 \approx h_f(28^\circ C) = 79.05$ kJ/kg and $s_1 \approx s_f(28^\circ C) = 0.2936$ kJ/kg·K
$h_2 \approx h_f(20^\circ C) = 69.09$ kJ/kg and $s_2 \approx s_f(20^\circ C) = 0.2607$ kJ/kg·K

From Table A-8: $h_3 = h_g(2\text{ bar}) = 239.88$ kJ/kg and $s_3 = s_g(2\text{ bar}) = 0.9691$ kJ/kg·K

$$h_4 = h_1 - h_2 + h_3 = 79.05 - 69.09 + 239.88 = 249.84 \text{ kJ/kg}$$

Interpolating in Table A-9: $T_4 \approx -9.815^\circ C$ and $s_4 \approx 1.0081$ kJ/kg·K

The entropy rate balance reduces as follows: $0 = \cancel{\sum_j \left(\frac{\dot{Q}_j}{T_j}\right)} + \dot{m}[(s_1 - s_2) + (s_3 - s_4)] + \dot{\sigma}_{cv}$

Thus

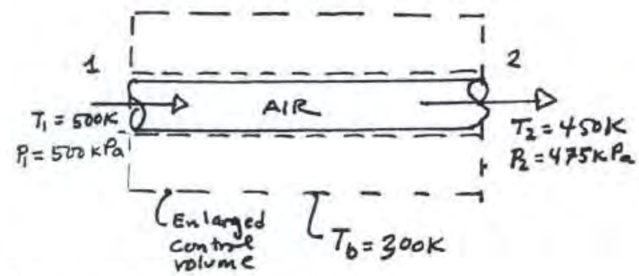
$$\dot{\sigma}_{cv}/\dot{m} = (0.2607 - 0.2936) + (1.0081 - 0.9691) = 0.0061 \text{ kJ/ kg·K} \longleftarrow$$

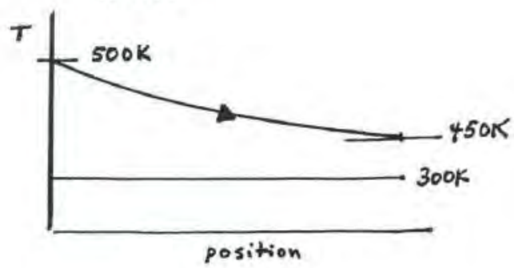
The entropy production is due to irreversible heat transfer between the two streams. There would be a small effect of frictional pressure drop, but pressure drops have been ignored.

PROBLEM 6.101

Air at 500 kPa, 500 K and a mass flow of 600 kg/h enters a pipe passing overhead in a factory space. At the pipe exit, the pressure and temperature of the air are 475 kPa and 450 K, respectively. Air can be modeled as an ideal gas with $k = 1.39$. Kinetic and potential energy effects can be ignored. Determine at steady state, (a) the rate of heat transfer, in kW, for a control volume comprising the pipe and its contents, and (b) the rate of entropy production, in kW/K, for an enlarged control volume that includes the pipe and enough of its surroundings that heat transfer occurs at the ambient temperature, 300 K.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL:

1. The control volumes shown in the sketch are at steady state.
2. For the control volumes, $\dot{W}_{cv} = 0$ and kinetic and potential energy effects can be ignored.
3. Heat transfer from the enlarged control volume occurs at $T_b = 300K$.
4. The air is modeled as an ideal gas with $k = 1.39$

ANALYSIS: (a) For the control volume enclosing the pipe an energy rate balance reads, $0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}(h_1 - h_2)$. Thus,

$$\dot{Q}_{cv} = \dot{m}(h_2 - h_1) = \dot{m} c_p (T_2 - T_1)$$

$$c_p = \frac{kR}{k-1} = \frac{1.39(8.314/28.97)}{1.39-1} \frac{kJ}{kg \cdot K} = 1.023 \frac{kJ}{kg \cdot K}$$

(Eq. 3.47a)

$$\therefore \quad \dot{Q}_{cv} = 600 \frac{kg}{h} \left| \frac{1h}{3600s} \right| \left(1.023 \frac{kJ}{kg \cdot K}\right)(450 - 500)K \left| \frac{1 kW}{1 kJ/s} \right|$$

$$= -8.53 \text{ kW} \quad \text{(a)}$$

(b) An entropy rate balance for the enlarged control volume reads

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \dot{\sigma}_{cv} = -\frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_2 - s_1)$$

Or, with Eq. 6.22,

$$\dot{\sigma}_{cv} = -\frac{\dot{Q}_{cv}}{T_b} + \dot{m}\left[c_p \ln\frac{T_2}{T_1} - R \ln\frac{P_2}{P_1}\right]$$

$$= \frac{8.53 kW}{300 K} - \left(\frac{600}{3600} \frac{kg}{s}\right)\left[1.023 \ln\frac{450}{500} - \frac{8.314}{28.97} \ln\frac{475}{500}\right] \frac{kJ}{kg \cdot K} \left| \frac{1 kW}{1 kJ/s} \right|$$

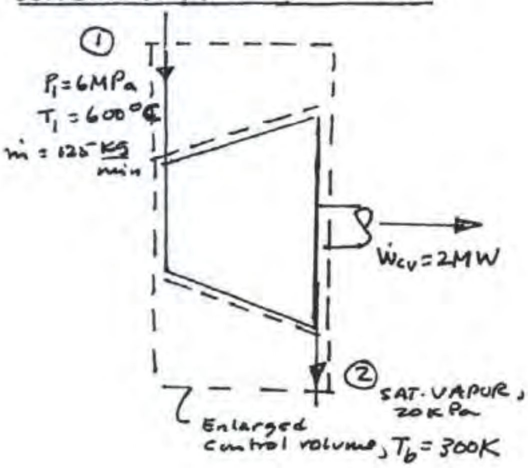
$$= (0.0284 - 0.0155) \frac{kW}{K}$$

$$= 0.0129 \frac{kW}{K} \quad \text{(b)}$$

PROBLEM 6.102

Steam enters a turbine operating at steady state at 6 MPa, 600°C with a mass flow rate of 125 kg/min and exits as saturated vapor at 20 kPa, producing power at a rate of 2 MW. Kinetic and potential energy effects can be ignored. Determine (a) the rate of heat transfer, in kW, for a control volume including the turbine and its contents, and (b) the rate of entropy production, in kW/K, for an enlarged control volume that includes the turbine and enough of its surroundings that heat transfer occurs at the ambient temperature, 27°C.

SCHEMATIC & GIVEN DATA



ENGR. MODEL:

1. The control volumes shown in the sketch are at steady state.
2. Kinetic and potential energy effects can be ignored.
3. For the enlarged control volume heat transfer occurs at $T_b = 300K$.

ANALYSIS: At 6MPa, 600°C, Table A-4 gives $h_1 = 3658.4$ kJ/kg, $s_1 = 7.1677$ kJ/kg·K.
At 20 kPa, saturated vapor Table A-3 gives $h_2 = 2609.7$ kJ/kg, $s_2 = 7.9085$ kJ/kg·K.

(a) An energy balance for the control volume enclosing just the turbine reads, $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2)$. This gives

$$\dot{Q}_{cv} = \dot{W}_{cv} - \dot{m}(h_1 - h_2)$$

$$= 2MW \left| \frac{10^3 kW}{1 MW} \right| - 125\frac{kg}{min}\left|\frac{1 min}{60 s}\right|(3658.4 - 2609.7)\frac{kJ}{kg}\left|\frac{1 kW}{1 kJ/s}\right|$$

$$= -184.8 kW \qquad \text{(a)}$$

(b) An entropy rate balance for the enlarged control volume reads,

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

$$\Rightarrow \quad \dot{\sigma}_{cv} = -\frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_2 - s_1)$$

$$= -\frac{(-184.8 kW)}{300 K} + \frac{125}{60}\frac{kg}{s}(7.9085 - 7.1677)\frac{kJ}{kg \cdot K}$$

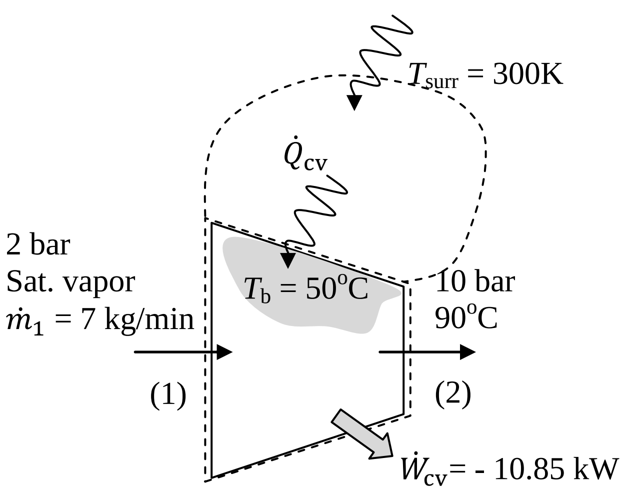
$$= 2.16 \frac{kW}{K} \qquad \text{(b)}$$

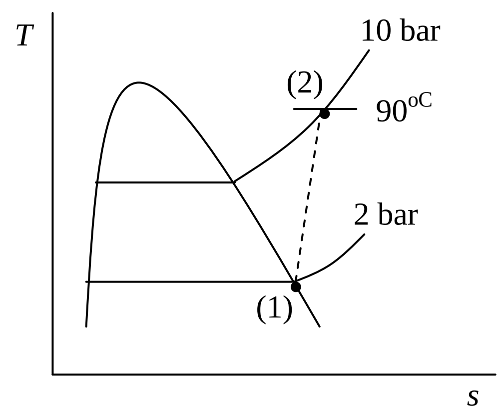
PROBLEM 6.103

Refrigerant 134a is compressed from 2 bar, saturated vapor, to 10 bar, 90°C, in a compressor operating at steady state. The mass flow rate of refrigerant entering the compressor is 7 kg/min, and the power *input* is 10.85 kW. Kinetic and potential energy effects can be neglected.
(a) Determine the rate of heat transfer, in kW.
(b) If the heat transfer occurs at an average surface temperature of 50°C, determine the rate of entropy production, in kW/K.
(c) Determine the rate of entropy production for an enlarged control volume that includes the compressor and its immediate surroundings such that the heat transfer occurs at 300K. Compare the results of parts (b) and (c) and discuss.

KNOWN: Refrigerant 134a is compressed at steady state. Data are known at the inlet and exit, the power is given, and the mass flow rate is known.

FIND: Determine the heat transfer rate, and the rate of entropy production for a control volume enclosing just the compressor and for an enlarged control volume that includes the compressor and its immediate surroundings. Discuss.





ENGINEERING MODEL: (1) The control volume is at steady state. (2) Kinetic and potential energy effects are negligible.

ANALYSIS: (a) To find the rate of heat transfer, start with the mass and energy rate balances. For the one-inlet, one-exit control volume at steady state: $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$ and

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)} + \cancel{g(z_1 - z_2)}\right]$$

Solving

$$\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}(h_2 - h_1) \qquad (*)$$

State 1: 2 bar, sat. vapor; h_1 = 241.30 kJ/kg and s_1 = 0.9253 kJ/kg·K

State 2: 10 bar, 90°C; superheated vapor; h_2 = 324.01 kJ/kg and s_2 = 1.0707 kJ/kg·K

Thus, inserting values into (*) and converting units

PROBLEM 6.103 (CONTINUED)

$$\dot{Q}_{cv} = (-10.85\text{ kW}) + (7\text{ kg/min})(324.01 - 241.30)\text{kJ/kg}\left|\frac{1\text{ min}}{60\text{ s}}\right|\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = -1.2\text{ kW (out)} \longleftarrow$$

(b) Now, applying the steady-state entropy balance to the one-inlet, one-exit control volume enclosing the compressor

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

or

$$\dot{\sigma}_{cv} = -\frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_2 - s_1)$$

$$= -\frac{(-1.2\text{ kW})}{(323\text{ K})} + (7\text{ kg/min})(1.0707 - 0.9253)\text{kJ/kg·K}\left|\frac{1\text{ min}}{60\text{ s}}\right|\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = 0.02068\text{ kW/K} \longleftarrow$$

(c) For the enlarged control volume, the heat transfer rate is unchanged (steady state conditions dictate that the heat transfer rate out of the compressor equals the heat transfer rate into the surroundings). However, the entropy transfer occurs at T_{surr} instead of T_b. Thus

$$\dot{\sigma}_{cv} = -\frac{\dot{Q}_{cv}}{T_{surr}} + \dot{m}(s_2 - s_1)$$

$$= -\frac{(-1.2\text{ kW})}{(300\text{ K})} + (7\text{ kg/min})(1.0707 - 0.9253)\text{kJ/kg·K}\left|\frac{1\text{ min}}{60\text{ s}}\right|\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = 0.02096\text{ kW/K} \longleftarrow$$

Discussion

The rate of entropy production for the compressor accounts for irreversibilities within the compressor, such as fluid friction and internal heat transfer. Associated with the heat transfer is entropy transfer to the surroundings that is not included in the entropy production.

For the enlarged control volume, the irreversibility associated with heat transfer from the surface of the compressor to the surroundings is included in the control volume, resulting in increased entropy production.

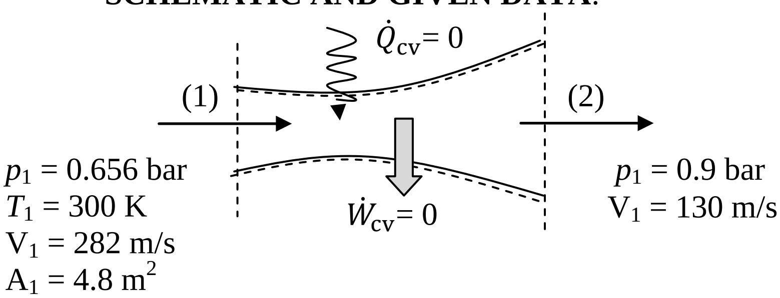
PROBLEM 6.104

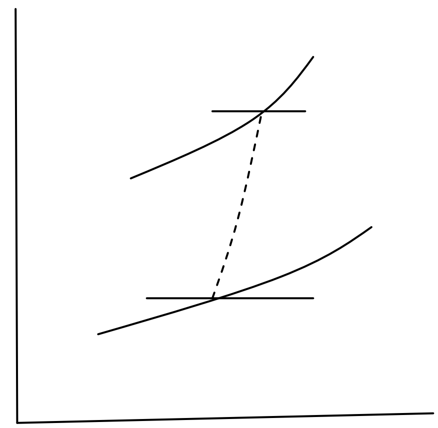
Nitrogen (N_2) enters a well-insulated diffuser operating at steady state at 0.656 bar, 300 K with a velocity of 282 m/s. The inlet area is 4.8 x 10^{-3} m^2. At the diffuser exit, the pressure is 0.9 bar and the velocity is 130 m/s. The nitrogen behaves as an ideal gas with k = 1.4. Determine the exit temperature, in K, and the exit area, in m^2. For a control volume enclosing the diffuser, determine the rate of entropy production, in kJ/K per kg of nitrogen flowing.

KNOWN: Nitrogen gas flows through a well-insulated diffuser operating at steady state. Conditions are known at the inlet and exit.

FIND: Determine the exit temperature and area and the rate of entropy production for a control volume enclosing the diffuser.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL: (1) The control volume is at steady state. (3) $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (3) The nitrogen is modeled as an ideal gas with k = 1.4. (4) Potential energy effects can be neglected.

ANALYSIS: To find the exit temperature, applying mass and energy rate balances, as follows.

$$\dot{m}_1 = \dot{m}_2 \equiv \dot{m} \text{ and } 0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + \cancel{g(z_1 - z_2)}\right]$$

With $h_2 - h_1 = c_p(T_2 - T_1)$ we get

$$T_2 = \left(\frac{1}{c_p}\right)\left(\frac{V_1^2 - V_2^2}{2}\right) + T_1$$

For k = 1.4: $c_p = kR/(k-1) = (1.4)(8.314/28.01)/(1 - 1.4)$ kJ/kg·K = 1.0389 kJ/kg·K

$$T_2 = \left(\frac{1}{1.0389}\frac{\text{kg·K}}{\text{kJ}}\right)\left(\frac{282^2 - 130^2}{2}\frac{\text{m}^2}{\text{s}^2}\right)\left|\frac{1\text{ kJ}}{10^3\text{N·m}}\right|\left|\frac{1\text{ N}}{1\text{ kg·m/s}^2}\right| + (300\text{ K}) = 330.33\text{ K} \longleftarrow$$

The exit area is found from

$$(A_2V_2)/v_2 = (A_1V_1)/v_1 \quad \longrightarrow \quad A_2 = A_1(V_1/V_2)(v_2/v_1)$$

PROBLEM 6.104 (CONTINUED)

With the ideal gas equation of state: $v_2/v_1 = (T_2/T_1)(p_1/p_2)$

$$A_2 = A_1(V_1/V_2)\,(T_2/T_1)(p_1/p_2)$$

$$= (4.8 \text{ x } 10^{-3}\text{ m}^2)(282/130)(330.33/300)(0.656/0.9) = 8.357 \text{ x } 10^{-3}\text{ m}^2 \longleftarrow$$

Applying the entropy rate balance to a control volume enclosing the diffuser

$$0 = \cancel{\sum_j \left(\frac{\dot{Q}_j}{T_j}\right)} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{\text{cv}} \longrightarrow \dot{\sigma}_{\text{cv}} = \dot{m}(s_2 - s_1)$$

For an ideal gas with constant specific heats

$$\dot{\sigma}_{\text{cv}} = \dot{m}(s_2 - s_1) = \dot{m}\left[c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)\right]$$

The mass flow rate is

$$\dot{m} = (A_1V_1)/(v_1) = (A_1V_1)(RT_1/p_1)$$

$$= (4.8 \text{ x } 10^{-3}\text{m}^2)\left(282\,\frac{\text{m}}{\text{s}}\right)\frac{\left(\frac{8.314}{28.01}\frac{\text{kJ}}{\text{kg·K}}\right)(300\text{ K})}{(0.656\text{ bar})}\left|\frac{1\text{ bar}}{10^5\text{N/m}^2}\right|\left|\frac{10^3\text{N·m}}{1\text{ kJ}}\right| = 1.837\text{ kg/s}$$

and

$$\dot{\sigma}_{\text{cv}} = (1.837\text{kg/s})[(1.0389\text{ kJ/kg·K})\ln(330.33/300) - (8.314/28.01\text{ kJ/kg·K})\ln(0.9/0.656)]\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$

$$= 0.01132\text{ kW/K} \longleftarrow$$

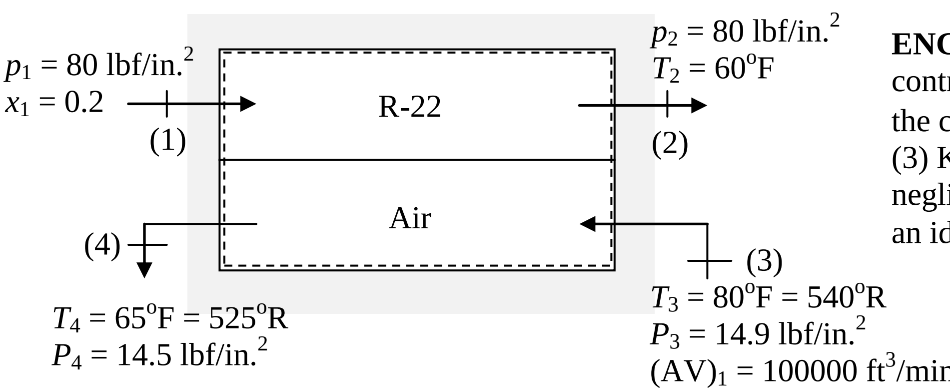
PROBLEM 6.105

Refrigerant 22 enters the evaporator heat exchanger of an air conditioning system at 80 lbf/in.^{2}, with a quality of 0.2. The refrigerant stream exits at 80 lbf/in.^{2}, 60^{o}F. Air flows in counterflow through the heat exchanger, entering at 14.9 lbf/in.^{2}, 80^{o}F, with a volumetric flow rate of 100,000 ft^3/min and exiting at 14.5 lbf/in.^{2}, 65^{o}F. Operation is at steady state, stray heat transfer from the outside of the heat exchanger to the surroundings can be neglected, and kinetic and potential energy effects are negligible. Assuming ideal gas behavior for the air, determine the rate of entropy production in the evaporator, in Btu/min·^{o}R.

KNOWN: Data are provided for the steady state operation of an evaporator heat exchanger with R-22 and air as the two fluid streams.

FIND: Determine the rate of entropy production in the evaporator.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The control volume is at steady state. (2) For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (3) Kinetic and potential energy effects are negligible. (4) The air can be modeled as an ideal gas.

ANALYSIS: Mass rate balances for each stream reduce, respectively to $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_{ref}$ and $\dot{m}_3 = \dot{m}_4 \equiv \dot{m}_{air}$. The air mass flow rate is

$$\dot{m}_{air} = \frac{(AV)_1}{v_1} = \frac{(AV)_1 p_1}{RT_1} = \frac{\left(100000\frac{m^3}{min}\right)\left(14.9\frac{lbf}{in.^2}\right)}{\left(\frac{1545}{28.97}\frac{ft\cdot lbf}{lb\cdot {}^{o}R}\right)(540^{o}R)}\left|\frac{144\ in.^2}{1\ ft^2}\right| = 7450\ lb/min$$

To determine the mass flow rate of refrigerant, we reduce the energy rate balance as follows.

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_{ref}\,(h_1 - h_2) + \dot{m}_{air}\,(h_3 - h_4)$$

or

$$\dot{m}_{ref} = \dot{m}_{air}\,[(h_3 - h_4)/(h_2 - h_1)] \qquad (*)$$

From Table A-8E; $h_1 = h_{f1} + x_1(h_{g1} - h_{f1})$ = 21.01 + (0.2)(108.00 – 21.01) = 38.408 Btu/lb. From Table A-9E; h_2 = 111.97 Btu/lb. For the air, we use Table A-22E; h_3 = 129.06 Btu/lb and $h_4 \approx$ 125.47 Btu/lb. The mass flow rate of refrigerant is

$$\dot{m}_{ref} = (7450\ lb/min)[(129.06 - 125.47)/(111.97 - 38.408) = 363.6\ lb/min$$

PROBLEM 6.105 (CONTINUED)

Now, applying the entropy rate balance

$$0 = \cancel{\sum_j \left(\frac{\dot{Q}_j}{T_i}\right)} + \dot{m}_{\text{ref}}\,(s_1 - s_2) + \dot{m}_{\text{air}}\,(s_3 - s_4) + \dot{\sigma}_{\text{cv}} \;\longrightarrow\; \boxed{\dot{\sigma}_{\text{cv}} = \dot{m}_{\text{ref}}\,(s_2 - s_1) + \dot{m}_{\text{air}}\,(s_4 - s_3)}$$

For the refrigerant, $s_1 = s_{f1} + x_1(s_{g1} - s_{f1}) = 0.0456 + (0.2)(0.2205 - 0.0456) = 0.08058$ Btu/lb·°R and $s_2 = 0.2283$ Btu/lb·°R. For the air

$$s_4 - s_3 = s^{o}(T_4) - s^{o}(T_3) - R\ln(p_4/p_3)$$

Thus, with data from Table A-22E

$$\dot{\sigma}_{\text{cv}} = (363.6 \text{ lb/min})(0.2283 - 0.08058) \text{ Btu/lb·°R}$$

$$+ (7450 \text{ lb/min})[(0.59399 - 0.60078) - (1545/28.97)\text{ft·lbf/lb·°R}\left|\frac{1 \text{ Btu}}{778 \text{ ft·lbf}}\right| \ln\left(\frac{14.5}{14.9}\right)$$

$$= 17.023 \text{ Btu/min·°R}$$

The entropy production rate is positive, indicating irreversibilities within the heat exchanger. The primary irreversibility is the heat transfer between the two streams. The pressure drop on the air side due to friction contributes a small amount as well.

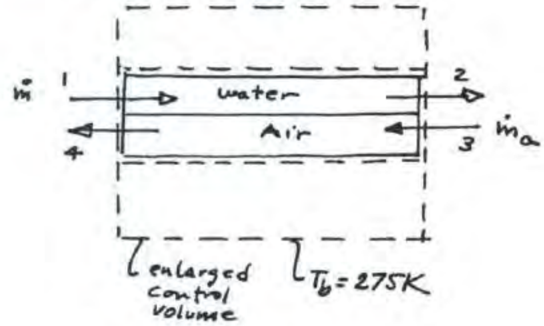
PROBLEM 6.106

Saturated water vapor at 100 kPa enters a counterflow heat exchanger operating at steady state and exits at 20°C with a negligible change in pressure. Ambient air at 275 K, 1 atm enters in a separate stream and exits at 290 K, 1 atm. The air mass flow rate is 170 times that of the water. The air can be modeled as an ideal gas with $c_p = 1.005$ kJ/kg · K. Kinetic and potential energy effects can be ignored.

(a) For a control volume enclosing the heat exchanger, evaluate the rate of heat transfer, in kJ per kg of water flowing.

(b) For an enlarged control volume that includes the heat exchanger and enough of its immediate surroundings that heat transfer from the control volume occurs at the ambient temperature, 275 K, determine the rate of entropy production, in kJ/K per kg of water flowing.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volumes shown in the sketch are at steady state.
2. Kinetic and potential energy effects are ignored. $\dot{W}_{cv} = 0$.
3. For the enlarged control volume heat transfer occurs at $T_b = 275K$.
4. The air is modeled as an ideal gas with $c_p = 1.005$ kJ/kg·K.
5. For liquid water $h \sim h_f(T)$, $s \sim s_f(T)$.

ANALYSIS:

From Table A-3, $h_1 = h_g$ (at 100 kPa) = 2778.1 kJ/kg, and $s_1 = 6.5863$ kJ/kg·K. From Table A-2, $h_2 \sim h_f(20°C) = 83.96 \frac{kJ}{kg}$, $s_2 \sim s_f(20°C) = 0.2966$ kJ/kg·K.

(a) An energy rate balance for a control volume enclosing just the heat-exchanger reads,

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}(h_1 - h_2) + \dot{m}_a(h_3 - h_4) \Rightarrow \dot{Q}_{cv} = \dot{m}(h_2 - h_1) + \dot{m}_a(h_4 - h_3).$$

Since $\dot{m}_a/\dot{m} = 170$, and using assumption 4,

$$\frac{\dot{Q}_{cv}}{\dot{m}} = (h_2 - h_1) + 170\left(c_{pa}(T_4 - T_3)\right) = [(83.96 - 2778.1) + 170(1.005)(290 - 275)] \frac{kJ}{kg(w)}$$

(water)

$$= -131.4 \text{ kJ/kg(w)} \quad \text{(a)}$$

(b) An entropy rate balance for the enlarged control volume reads,

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_1 - s_2) + \dot{m}_a(s_3 - s_4) + \dot{\sigma}_{cv}$$

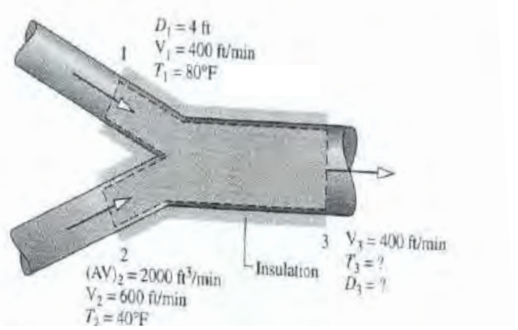
$$\Rightarrow \dot{\sigma}_{cv}/\dot{m} = \frac{(-\dot{Q}_{cv}/\dot{m})}{T_b} + (s_2 - s_1) + 170\left[c_p \ln\frac{T_4}{T_3} - \cancel{R \ln\frac{P_4}{P_3}}^{\,0\,(P_4 = P_3)}\right]$$

$$= \left[\frac{131.4}{275} + (0.2966 - 6.5863) + 170\,(1.005) \ln\frac{290}{275}\right] \frac{kJ/kg(w)}{K}$$

$$= 3.26 \frac{kJ/kg(w)}{K} \quad \text{(b)}$$

PROBLEM 6.107

The figure shows data for a portion of the ducting in a ventilation system operating at steady state. The ducts are well insulated and the pressure is very nearly 1 atm throughout. Assuming the ideal gas model for air with $c_p = 0.24$ Btu/lb · °R, and ignoring kinetic and potential energy effects, determine (a) the temperature of the air at the exit, in °F, (b) the exit diameter, in ft, and (c) the rate of entropy production within the duct, in Btu/min · °R.



ENGR. MODEL

1. The control volume shown in the figure is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$, and kinetic and potential energy effects can be ignored.
3. The air is modeled as an ideal gas with $c_p = 0.24$ Btu/lb·°R.

ANALYSIS: To find T_3, begin with steady-state mass and energy balances

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}_1\left(h_1 + \frac{V_1^2}{2} + gz_1\right) + \dot{m}_2\left(h_2 + \frac{V_2^2}{2} + gz_2\right) - \dot{m}_3\left(h_3 + \frac{V_3^2}{2} + gz_3\right)$$

and
$$\dot{m}_1 + \dot{m}_3 - \dot{m}_3 = 0 \Rightarrow \dot{m}_3 = \dot{m}_1 + \dot{m}_2$$

Combining and incorporating assumption (2)

$$0 = \dot{m}_1\left[(h_1 - h_3) + \cancel{\left(\frac{V_1^2 - V_3^2}{2}\right)}^{0}\right] + \dot{m}_2\left[(h_2 - h_3) + \cancel{\left(\frac{V_2^2 - V_3^2}{2}\right)}^{0}\right]$$

Using $\Delta h = c_p \Delta T$, this becomes

$$0 = \dot{m}_1\left[c_p(T_1 - T_3)\right] + \dot{m}_2\left[c_p(T_2 - T_3)\right] \quad (*)$$

The mass flow rates are evaluated using Eq. 4-4b and the ideal gas equation of state

$$\dot{m}_1 = \frac{A_1 V_1}{v_1} = \frac{\left(\frac{\pi D_1^2}{4}\right) V_1 P_1}{R T_1} = \frac{\left(\frac{\pi (4^2)\,ft^2}{4}\right)(400\ ft/min)(14.7\ lbf/in^2)}{\left(\frac{1545}{28.97}\frac{ft\cdot lbf}{lb\cdot °R}\right)(540°R)}\left|\frac{144\ in^2}{1\ ft^2}\right|$$

$$= 369.5\ lb/min$$

$$\dot{m}_2 = \frac{(AV)_2 P_2}{R T_2} = \frac{(2000)(14.7)|144|}{\left(\frac{1545}{28.97}\right)(500)} = 158.8\ lb/min$$

Returning to (*) and solving for T_3

$$T_3 = \frac{\dot{m}_1 \cancel{c_p} T_1 + \dot{m}_2 \cancel{c_p} T_2}{(\dot{m}_1 + \dot{m}_2)\cancel{c_p}} = \frac{\dot{m}_1 T_1 + \dot{m}_2 T_2}{(\dot{m}_1 + \dot{m}_2)}$$

PROBLEM 6.107 (CONTINUED)

Inserting values

$$T_3 = \frac{(369.5\ \text{lb/min})(540°R) + (158.8)(500)}{(369.5 + 158.8)\ \text{lb/min}}$$

$= 528\ °R = 68\ °F \longleftarrow$ T_3

To get D_3, note that

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 528.3\ \text{lb/min}$$

Thus

$$A_3 = \frac{v_3 \dot{m}_3}{V_3} = \frac{R T_3 \dot{m}_3}{p_3 V_3} = \frac{\left(\frac{1545}{28.97}\right)(528)(528.3)}{(14.7)(144)(400)} = 17.57\ \text{ft}^2$$

and $D_3 = \sqrt{\dfrac{4 A_3}{\pi}} = 4.73\ \text{ft} \longleftarrow$ D_3

An entropy rate balance at steady state reduces to read

$$0 = \cancel{\sum_j \frac{\dot{Q}_j}{T_j}}^{\,0} + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$$

Since $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$ at steady state

$$\dot{\sigma}_{cv} = (\dot{m}_1 + \dot{m}_2) s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2$$

$$= \dot{m}_1 (s_3 - s_1) + \dot{m}_2 (s_3 - s_2)$$

Introducing Eq. 6.22 and noting that pressure is nearly 1 atm throughout,

$$\dot{\sigma}_{cv} = \dot{m}_1 \left[c_p \ln \frac{T_3}{T_1} - \cancel{R \ln \frac{p_3}{p_1}}^{\,0} \right] + \dot{m}_2 \left[c_p \ln \frac{T_3}{T_2} - \cancel{R \ln \frac{p_3}{p_2}}^{\,0} \right]$$

$$= c_p \left[\dot{m}_1 \ln \frac{T_3}{T_1} + \dot{m}_2 \ln \frac{T_3}{T_2} \right]$$

$$= 0.24 \frac{\text{Btu}}{\text{lb} \cdot °R} \left[\left(369.5 \frac{\text{lb}}{\text{min}}\right) \ln \frac{528}{540} + \left(158.8 \frac{\text{lb}}{\text{min}}\right) \ln \frac{528}{500} \right]$$

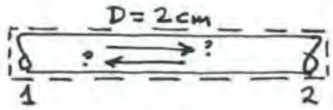
$$= 0.084 \frac{\text{Btu}}{\text{min}\ °R}$$

$\longleftarrow$ $\dot{\sigma}_{cv}$

PROBLEM 6.108

Air flows through an insulated circular duct having a diameter of 2 cm. Steady-state pressure and temperature data obtained by measurements at two locations, denoted as 1 and 2, are given in the accompanying table. Modeling air as an ideal gas with $c_p = 1.005$ kJ/kg · K, determine (a) the direction of the flow, (b) the velocity of the air, in m/s, at each of the two locations, and (c) the mass flow rate of the air, in kg/s.

Measurement location	1	2
Pressure (kPa)	100	500
Temperature (°C)	20	50



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$ and potential energy effects are negligible.
3. The air is modeled as an ideal gas with $c_p = 1.005$ kJ/kg · K.

ANALYSIS: As discussed in Secs. 5.1 and 6.8, directionality normally can be established using the 2nd law. Here, a direction is assumed and the associated entropy production is evaluated. Taking the inlet at 1 and the exit at 2, an entropy rate balance reads, $0 = \sum_j \frac{\dot{Q}_j}{T_j}^{\,0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$

$$\Rightarrow \frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} = 1.005 \ln\frac{323}{293} - \frac{8.314}{28.97}\ln\frac{500}{100} = -0.364\ \frac{kJ}{kg \cdot K}$$

Since $\dot{\sigma}_{cv}/\dot{m}$ cannot be negative, the flow direction can only be from 2 to 1. ← (a)

A mass rate balance reads, $\dot{m}_1 = \dot{m}_2 \Rightarrow \frac{AV_1}{v_1} = \frac{AV_2}{v_2} \Rightarrow \frac{V_2}{V_1} = \frac{P_1}{P_2}\frac{T_2}{T_1}$, where $v_1 = RT_1/P_1$ and $v_2 = RT_2/P_2$.

Inserting values, $V_2 = \left(\frac{100}{500}\right)\left(\frac{323}{293}\right)V_1 = 0.2205 V_1$.

An energy rate balance reads, $0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}\right]$. Or

$$0 = c_p(T_2 - T_1) + \frac{(0.2205V_1)^2 - V_1^2}{2} \Rightarrow 0 = c_p(T_2 - T_1) - \frac{0.9514\,V_1^2}{2}.$$ Solving, we get

$$V_1 = \left[\frac{2\left(1.005\frac{kJ}{kg \cdot K}\right)(30\,K)}{0.9514}\left|\frac{10^3\,N \cdot m}{1\,kJ}\right|\left|\frac{1\,kg \cdot m/s^2}{1\,N}\right|\right]^{\frac{1}{2}} = 251.75\ m/s$$

And

$$V_2 = 0.2205\,V_1 = 55.51\ m/s$$ ← (b)

(c) $$\dot{m} = \frac{AV_1}{v_1} = \frac{P_1(\pi D^2/4)V_1}{RT_1} = \frac{(10^5\,N/m^2)\left(\frac{\pi}{4}(.02\,m)^2\right)251.75\,m/s}{\left(\frac{8314}{28.97}\frac{N \cdot m}{kg \cdot K}\right)(293\,K)} = 0.094\ \frac{kg}{s}$$ ← (c)

where $v_1 = RT_1/P_1$.

PROBLEM 6.109

Determine the rates of entropy production, in Btu/min · °R, for the steam generator and turbine of Example 4.10. Identify the component that contributes more to inefficient operation of the overall system.

KNOWN: Steady-state operating data are provided by the solution of Example 4.10.

FIND: Determine the rates of entropy production for the steam generator and turbine.

SCHEMATIC & GIVEN DATA: See Fig. E4.10

ENGINEERING MODEL:

1. Control volumes enclosing the steam generator and turbine are at steady state.
2. For each control volume $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible.
3. The combustion products are modeled as air as an ideal gas.

ANALYSIS: At steady state the entropy rate balance reduces to give

$$\dot{\sigma}_t = \dot{m}_3(s_5 - s_4) \qquad \text{(turbine)}$$

$$\dot{\sigma}_{HX} = \dot{m}_1(s_2 - s_1) + \dot{m}_3(s_4 - s_3) \qquad \text{(steam generator)}$$

From the solution to Example 4.10, $h_4 = 1213.6$ Btu/lb, $p_4 = 40$ lbf/in^2; thus Table A-4E gives $s_4 = 1.7334$ Btu/lb·°R. With data from Table A-3E

$s_5 = s_f + x_5(s_g - s_f) = 0.1327 + 0.93(1.8453) = 1.8488$ Btu/lb·°R. Thus,

$$\dot{\sigma}_t = \left(275 \frac{lb}{min}\right)(1.8488 - 1.7334)\frac{Btu}{lb\cdot°R}\left|\frac{1\,min}{60\,s}\right| = 0.529 \frac{Btu}{s\cdot°R}$$

With $s°$ data from Table A-22E and $s_3 = s_f(T_3)$ from Table A-2E

$$\dot{\sigma}_{HX} = \left[\left(9230.6 \frac{lb}{min}\right)\left[0.67002 - 0.71323 - R\cancel{\ln p_2/p_1}^{\,0}\right]\frac{Btu}{lb\cdot°R} + 275(1.7334 - 0.1331)\right]\left|\frac{1\,min}{60\,s}\right|$$

$$= 0.687 \frac{Btu}{s\cdot°R}$$

The steam generator contributes more to inefficient operation of the overall system than the turbine. Still, each is a significant contributor.

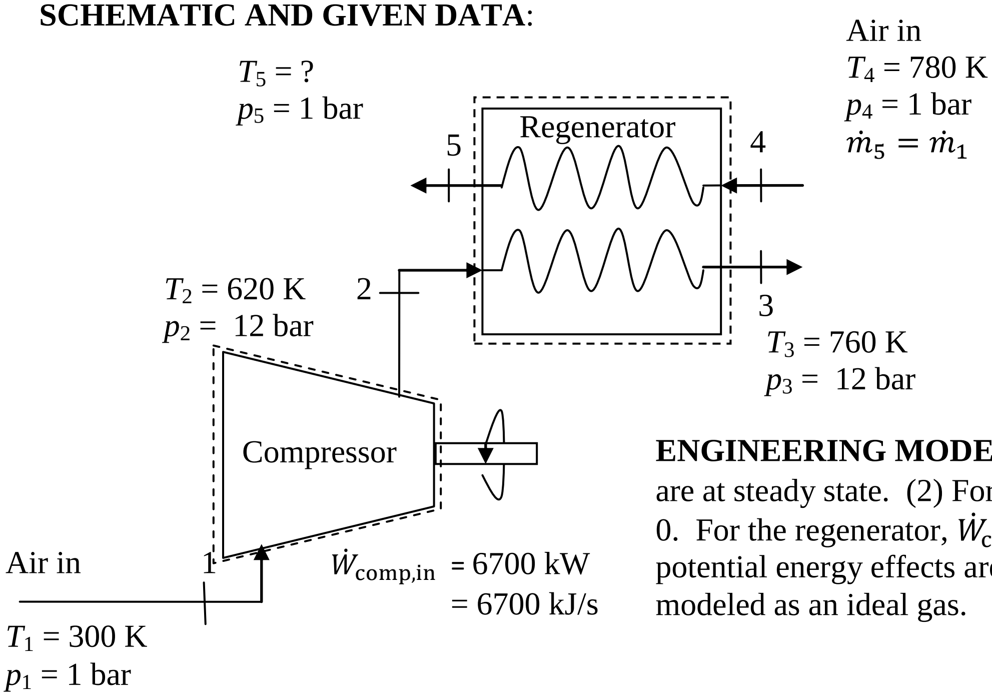
PROBLEM 6.110

Figure P6.110 shows an air compressor and regenerative heat exchanger in a gas turbine system operating at steady state. Air flows from the compressor through the regenerator, and a separate stream of air passes though the regenerator in counterflow. Operating data are provided on the figure. Stray heat transfer to the surroundings and kinetic and potential energy effects can be neglected. The compressor power *input* is 6700 kW. Determine the mass flow rate of air entering the compressor, in kg/s, the temperature of the air exiting the regenerator at state 5, in K, and the rates of entropy production in the compressor and regenerator, in kW/K.

KNOWN: Data are provided for steady-state operation of an air compressor and regenerative heat exchanger.

FIND: Determine the mass flow rate of air entering the compressor, the temperature of the air exiting the regenerator, and the rates of entropy production in the compressor and regenerator.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The control volumes are at steady state. (2) For each control volume, $\dot{Q}_{\text{cv}}$ = 0. For the regenerator, $\dot{W}_{\text{cv}}$ = 0. (3) Kinetic and potential energy effects are neglected. (4) The air is modeled as an ideal gas.

ANALYSIS: Energy and mass rate balances for the control volume enclosing the compressor reduce to

$$0 = \cancel{\dot{Q}_{\text{cv}}} - \dot{W}_{\text{cv}} + \dot{m}_1(h_1 - h_2) \;\longrightarrow\; \dot{m}_1 = \dot{W}_{\text{cv}}/(h_1 - h_2)$$

From Table A-22: h_1 = 300.19 kJ/kg and h_2 = 628.07 kJ/kg. Thus

$$\dot{m}_1 = \text{(-6700 kJ/s)(300.19 – 628.07)kJ/kg} = 20.43 \text{ kg/s} \longleftarrow$$

Now, applying the energy rate balance to the control volume enclosing the regenerator, with $\dot{m}_5 = \dot{m}_1$, we get

PROBLEM 6.110 (CONTINUED)

$$0 = \dot{m}_1[(h_2 - h_3) + (h_4 - h_5)] \longrightarrow h_5 = h_2 - h_3 + h_4$$

From Table A-22: h_3 = 778.18 kJ/kg and h_4 = 800.03 kJ/kg. Thus

$$h_5 = 628.07 - 778.18 + 800.03 = 649.92 \text{ kJ/kg}$$

Interpolating in Table A-22: $T_5 \approx 640$ K ⟵

Now, the entropy rate balance for the compressor reduces to

$$0 = \cancel{\sum_j \left(\frac{\dot{Q}_j}{T_j}\right)} + \dot{m}_1(s_1 - s_2) + (\dot{\sigma}_{cv})_{comp} \longrightarrow (\dot{\sigma}_{cv})_{comp} = \dot{m}_1 [(s^o(T_2) - s^o(T_1) - R \ln (p_2/p_1)]$$

From Table A-22; $s^o(T_1)$ = 1.70203 kJ/kg·K and $s^o(T_2)$ = 2.44356 kJ/kg·K . Thus

$$(\dot{\sigma}_{cv})_{comp} = (20.43 \text{ kg/s})[(2.44356 - 1.70203) - (8.314/28.97) \ln(12/1)]\text{kJ/kg·K}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$
$$= 0.5801 \text{ kW/K} \longleftarrow$$

For the regenerative heat exchanger

$$0 = \cancel{\sum_j \left(\frac{\dot{Q}_j}{T_j}\right)} + \dot{m}_1[(s_2 - s_3) + (s_4 - s_5)] + (\dot{\sigma}_{cv})_{HX}$$

$$(\dot{\sigma}_{cv})_{HX} = \dot{m}_1\{[(s^o(T_3) - s^o(T_2) - R \ln\cancel{(p_3/p_2)}] + [(s^o(T_5) - s^o(T_4) - R \ln\cancel{(p_5/p_4)}]]\}$$

From Table A-22: $s^o(T_3)$ = 2.66176 kJ/kg·K, $s^o(T_4)$ = 2.69013 kJ/kg·K, $s^o(T_5)$ = 2.47716 kJ/kg·K

Finally

$$(\dot{\sigma}_{cv})_{HX} = (20.43)[(2.66176 - 2.44356) + (2.47716 - 2.69013)] = 0.1068 \text{ kW/K} \longleftarrow$$

PROBLEM 6.111

The figure shows several components in series, all operating at steady state. Liquid water enters the boiler at 60 bar. Steam exits the boiler at 60 bar, 540°C and undergoes a throttling process to 40 bar before entering the turbine. Steam expands adiabatically through the turbine to 5 bar, 240°C, and then undergoes a throttling process to 1 bar before entering the condenser. Kinetic and potential energy effects can be ignored.

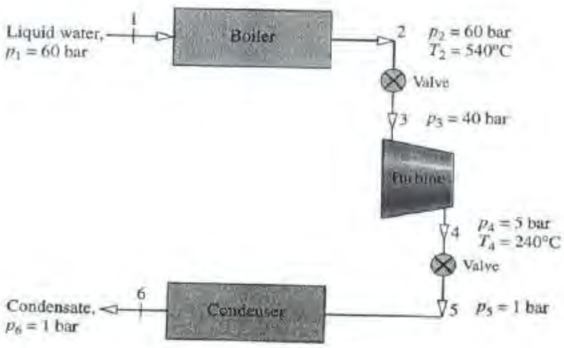
(a) Locate each of the states 2–5 on a sketch of the T–s diagram.

(b) Determine the power developed by the turbine, in kJ per kg of steam flowing.

(c) For the valves and the turbine, evaluate the rate of entropy production, each in kJ/K per kg of steam flowing.

(d) Using the result of part (c), place the components in rank order, beginning with the component contributing the most to inefficient operation of the overall system. Comment.

(e) If the goal is to increase the power developed per kg of steam flowing, which of the components (if any) might be eliminated? Explain.

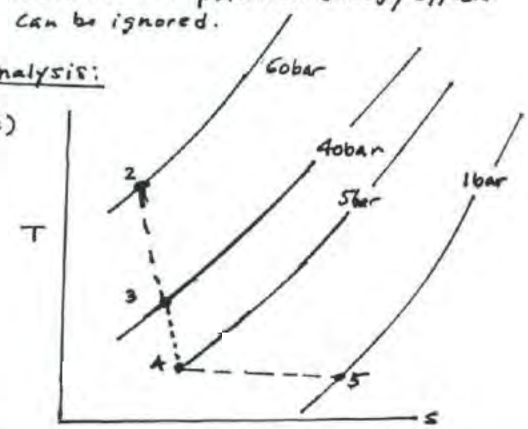


ENGR. MODEL:

1. A control volume at steady state encloses each valve and the turbine.
2. For the valves, the expansion is a throttling process: $h_3 = h_2$, $h_5 = h_4$
3. For the turbine, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects can be ignored.

Analysis:

(a)



(b) Mass and energy rate balances reduce for the turbine to give $\dot{W}_t/\dot{m} = h_3 - h_4$. But since $h_3 = h_2$, we have $\dot{W}_t/\dot{m} = h_2 - h_4 = (3517 - 2939.9)\ \frac{kJ}{kg}$ or $\dot{W}_t/\dot{m} = 577.1$ kJ/kg, where h_2 and h_4 are from Table A-4.

(c) Since each of the three control volumes is at steady state, experiences no heat transfer, and has a single inlet and single exit, the entropy rate balance reduces to give for each one, $\dot{\sigma}_{cv}/\dot{m} = s_e - s_i$.

Valve #1: $\dot{\sigma}_{cv}/\dot{m} = (7.1805 - 6.9999)$ kJ/kg·K $= 0.1806$ kJ/kg·K

Turbine: $\dot{\sigma}_{cv}/\dot{m} = (7.2307 - 7.1805)$ kJ/kg·K $= 0.0502$ kJ/kg·K

Valve #2: $\dot{\sigma}_{cv}/\dot{m} = (7.9653 - 7.2307)$ kJ/kg·K $= 0.7346$ kJ/kg·K

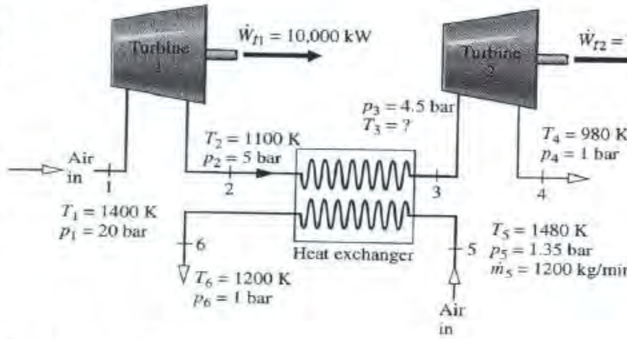
(d)
- Valve #2
- Valve #1
- Turbine

(e) The two valves are clearly significant sources of irreversibility. Still, a valve placed before a turbine is often used for flow control purposes, whereas a valve following a turbine is rarely (if ever) encountered. Accordingly, valve #2 should be eliminated.

PROBLEM 6.112

Air as an ideal gas flows through the turbine and heat exchanger arrangement shown . Steady-state data are given on the figure. Stray heat transfer and kinetic and potential energy effects can be ignored. Determine

(a) temperature T_3, in K.
(b) the power output of the second turbine, in kW.
(c) the rates of entropy production, each in kW/K, for the turbines and heat exchanger.
(d) Using the result of part (c), place the components in rank order, beginning with the component contributing most to inefficient operation of the overall system.



ENGR. MODEL:

1. A control volume at steady state encloses each turbine and the heat exchanger.
2. Stray heat transfer and kinetic and potential energy can be ignored.
3. The air is modeled as an ideal gas.

ANALYSIS: (a) First, find the air flow rate at 1. Begin with steady-state energy and mass balances for turbine 1

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \dot{W}_{t1} + \dot{m}_1\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)}^{0} + g\cancel{(z_1 - z_2)}^{0}\right]$$

and $\quad 0 = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2$

Solving for $\dot{m}_1$

$$\dot{m}_1 = \frac{\dot{W}_{t1}}{(h_1 - h_2)}$$

From Table A-22; $h_1 = 1515.42$ kJ/kg and $h_2 = 1161.07$ kJ/kg. Thus

$$\dot{m}_1 = \frac{(10{,}000 \text{ kW})}{(1515.42 - 1161.07) \text{ kJ/kg}}\left|\frac{1 \text{ kJ/s}}{1 \text{ kW}}\right| = 28.22 \text{ kg/s}$$

Turning next to the heat exchanger

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}_4$$
$$\dot{m}_5 = \dot{m}_6$$

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}_2\left[(h_2 - h_3) + \cancel{\left(\frac{V_2^2 - V_3^2}{2}\right)}^{0} + g\cancel{(z_2 - z_3)}^{0}\right] + \dot{m}_5\left[(h_5 - h_6) + \cancel{\left(\frac{V_5^2 - V_6^2}{2}\right)}^{0} + g\cancel{(z_5 - z_6)}^{0}\right]$$

$$0 = \dot{m}_2(h_2 - h_3) + \dot{m}_5(h_5 - h_6)$$

or $\quad h_3 = h_2 + \dfrac{\dot{m}_5}{\dot{m}_2}(h_5 - h_6)$

Again, from Table A-22; $h_5 = 1611.79$ kJ/kg and $h_6 = 1277.79$ kJ/kg. Thus

$$h_3 = 1161.07 + \left[\frac{(1200 \text{ kg/min})}{(28.22 \text{ kg/s})|60 \text{ s/min}|}\right](1611.79 - 1277.79)$$
$$= 1397.8 \text{ kJ/kg}$$

Interpolating in Table A-22

$$T_3 = 1301.5 \text{ K} \longleftarrow \quad T_3$$

(b) Now, writing the steady-state energy balance for turbine 2

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \dot{W}_{t_2} + \dot{m}_3\left[(h_3 - h_4) + \cancel{\left(\frac{V_3^2 - V_4^2}{2}\right)}^{0} + g\cancel{(z_3 - z_4)}^{0}\right]$$

or

$$\dot{W}_{t_2} = \dot{m}_3 (h_3 - h_4)$$

From Table A-22; $h_4 = 1023.25$ kJ/kg, and

$$\dot{W}_{t_2} = (28.22\ \text{kg/s})(1397.8 - 1023.25)\ \text{kJ/kg}\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right|$$

$$= 10{,}570\ \text{kW} \longleftarrow \dot{W}_{t_2}$$

(c) Mass and entropy rate balances for the first turbine reduce to give

$$\dot{\sigma}_{t_1} = \dot{m}_1 (s_2 - s_1) = \dot{m}_1 \left(s^\circ(T_2) - s^\circ(T_1) - R \ln p_2/p_1 \right)$$

$$= \left(28.22\ \frac{\text{kg}}{\text{s}}\right)\left[3.07732 - 3.3620 - \frac{8.314}{28.97}\ln\frac{5}{20}\right]\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right|$$

$$= 3.1936\ \text{kW/K} \longleftarrow \dot{\sigma}_{t_1}$$

where Eq. 6.20a and data from Table A-22 have been used. Likewise for turbine 2

$$\dot{\sigma}_{t_2} = \dot{m}_1 (s_4 - s_3) = \dot{m}_1 \left[s^\circ(T_4) - s^\circ(T_3) - R \ln p_4/p_3 \right]$$

$$= (28.22)\left[2.94468 - 3.27481 - \frac{8.314}{28.97}\ln\frac{1}{4.5}\right] = 2.8649\ \text{kW/K} \longleftarrow \dot{\sigma}_{t_2}$$

Mass and entropy rate balances for the interconnecting heat exchanger give

$$\dot{\sigma}_{HX} = \dot{m}_1 [s_3 - s_2] + \dot{m}_5 [s_6 - s_5]$$

$$= \dot{m}_1 \left[s^\circ(T_3) - s^\circ(T_2) - R \ln\frac{p_3}{p_2} \right] + \dot{m}_5 \left[s^\circ(T_6) - s^\circ(T_5) - R \ln\frac{p_6}{p_5} \right]$$

$$= 28.22\left[3.27481 - 3.07732 - \frac{8.314}{28.97}\ln\frac{4.5}{5}\right] +$$

$$\left(\frac{1200}{60}\right)\left[3.17868 - 3.42892 - \frac{8.314}{28.97}\ln\frac{1}{1.35}\right]$$

$$= 6.4265 - 3.2783 = 3.1482\ \text{kW/K} \longleftarrow \dot{\sigma}_{HX}$$

(d)

In rank order: Turbine 1, heat exchanger, turbine 2.

PROBLEM 6.113

A rigid, insulated tank whose volume is 10 L is initially evacuated. A pinhole leak develops and air from the surroundings at 1 bar, 25°C enters the tank until the pressure in the tank becomes 1 bar. Assuming the ideal gas model with $k = 1.4$ for the air, determine (a) the final temperature in the tank, in °C, (b) the amount of air that leaks into the tank, in g, and (c) the amount entropy produced, in J/K.

See solution to Problem 4.109 for the KNOWN, SCHEMATIC & GIVEN DATA, and ENGINEERING MODEL. The solution also gives (a) $T_2 = 417.41$ K, (b) $m_2 = 8.35$g.

ANALYSIS for part (c):

An entropy rate balance reads,

$$\frac{dS_{cv}}{dt} = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{0} + \dot{m}_i s_i + \dot{\sigma}_{cv}$$

Mass rate balance: $\frac{dm_{cv}}{dt} = \dot{m}_i$

$$\Rightarrow \quad \frac{dS_{cv}}{dt} = s_i \frac{dm_{cv}}{dt} + \dot{\sigma}_{cv} \quad \Rightarrow \quad \Delta S_{cv} = s_i(m_2 - m_1) + \sigma_{cv}$$

specific entropy of the entering air remains constant

The last expression reduces as follows:

$$m_2 s_2 - \cancel{m_1}^{0} s_1 = s_i(m_2 - \cancel{m_1}^{0}) + \sigma_{cv}$$

Thus,

$$\sigma_{cv} = m_2(s_2 - s_i)$$

Then, with Eq. 6.22

$$\sigma_{cv} = m_2\left[c_p \ln \frac{T_2}{T_i} - R \ln \cancel{\frac{P_2}{P_i}}^{0\ (P_2 = P_i)}\right]$$

(Eq. 3.47a): $\frac{kR}{(k-1)}$ $= k$ (see solution to Problem 4.109)

$$\therefore \quad \sigma_{cv} = m_2\left(\frac{kR}{k-1}\right)\ln k$$

$$= (8.35\text{g})\left[\left(\frac{1.4}{0.4}\right)\left(\frac{8.314}{28.97}\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left|\frac{10^3\text{J}}{1\text{kJ}}\right|\left|\frac{1\text{kg}}{10^3\text{g}}\right|\ln(1.4)\right]$$

① $$= 2.82\ \frac{\text{J}}{\text{K}}$$

---

1. This accounts for entropy produced within the tank owing to the free expansion of air within the tank from the surroundings.

PROBLEM 6.114

An insulated, rigid tank whose volume is 0.5 m^3 is connected by a valve to a large vessel holding steam at 40 bar, 500°C. The tank is initially evacuated. The valve is opened only as long as required to fill the tank with steam to a pressure of 20 bar. Determine (a) the final temperature of the steam in the tank, in °C, (b) the final mass of the steam in the tank, in kg, and (c) the amount of entropy produced, in kJ/K.

See the solution of Problem 4.112 for the KNOWN, SCHEMATIC & GIVEN DATA, and ENGINEERING MODEL. The solution also gives (a) $T_2 = 685.9\,°C$, (b) $m_2 = 2.27\,kg$.

ANALYSIS: (c) An entropy rate balance reads

$$\frac{dS_{cv}}{dt} = \cancel{\sum_j \frac{\dot{Q}_j}{T_j}}^{\,0} + \dot{m}_i s_i + \dot{\sigma}_{cv}$$

Mass rate balance: $\frac{dm_{cv}}{dt} = \dot{m}_i$

$$\Rightarrow \quad \frac{dS_{cv}}{dt} = s_i \frac{dm_{cv}}{dt} + \dot{\sigma}_{cv} \qquad \Rightarrow \quad \Delta S_{cv} = s_i (m_2 - m_1) + \sigma_{cv}$$

specific entropy of the entering steam remains constant

The last expression reduces as follows:

$$m_2 s_2 - \cancel{m_1}^{\,0} s_1 = s_i (m_2 - \cancel{m_1}^{\,0}) + \sigma_{cv}$$

Thus,

$$\sigma_{cv} = m_2 (s_2 - s_i)$$

The value of s_i is obtained from Table A-4 at 40 bar, 500°C: 7.0901 kJ/kg·K. The value of s_2 is obtained from Table A-4 at 20 bar and $u_2 = h_i = 3445.3$ kJ/kg (see solution to Problem 4.112): 7.9145 kJ/kg·K.

Thus

$$\sigma_{cv} = (2.27\,kg)\left[7.9145 - 7.0901\right] \frac{kJ}{kg \cdot K}$$

① $$= 1.87\ \frac{kJ}{K}$$

1 This accounts for the entropy produced within the tank owing to the free expansion of steam within the tank from the large vessel.

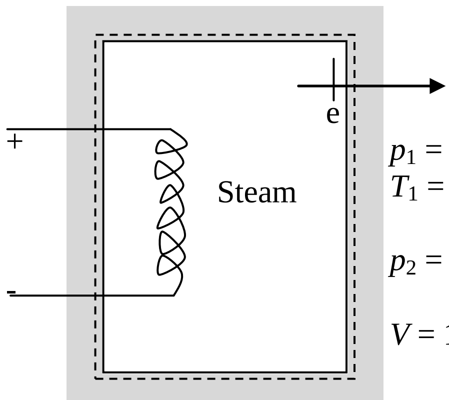
PROBLEM 6.115

A tank of volume 1 m^3 initially contains steam at 60 bar, 320°C. Steam is withdrawn slowly from the tank until the pressure drops to 15 bar. An electric resistor in the tank transfers energy to the steam maintaining the temperature constant at 320°C during the process. Neglecting kinetic and potential energy effects, determine the amount of entropy produced, in kJ/K.

KNOWN: Steam is withdrawn slowly from a rigid tank. The initial state is specified and the final pressure is known. A resistor within the tank maintains a constant temperature during the process.

FIND: Determine the amount of entropy produced.

SCHEMATIC AND GIVEN DATA:



p_1 = 60 bar
T_1 = 320°C

p_2 = 15 bar

V = 1 m^3

ENGINEERING MODEL: (1) For the control volume shown, $\dot{Q}_{cv} = 0$. (2) Kinetic and potential energy effects can be ignored. (3) The process happens slowly, so at each instant the pressure can be assumed uniform throughout the control volume.

ANALYSIS: The mass rate balance takes the form $dm_{cv}/dt = -\dot{m}_e \longrightarrow dm_{cv} = -\dot{m}_e dt$
Using the modeling assumptions listed, the entropy rate balance reduces to

$$\frac{dS_{cv}}{dt} = \cancel{\sum_j \left(\frac{\dot{Q}_j}{T_j}\right)} - \dot{m}_e s_e + \dot{\sigma}_{cv} \quad \longrightarrow \quad dS_{cv} = -s_e \dot{m}_e dt + \dot{\sigma}_{cv} dt$$

Combining these and solving for the entropy production term

$$\dot{\sigma}_{cv} dt = dS_{cv} + s_e dm_{cv}$$

Integrating

$$\sigma_{cv} = (m_2 s_2 - m_1 s_1) + \int_{m_1}^{m_2} s_e dm_{cv} \qquad (*)$$

The mass in the tank at any time is $m = V/v$, where v is the specific volume at that instant determined by 320°C and the tank pressure. Initially p_1 = 60 bar, so v_1 = 0.03876 m^3/kg and s_1 = 6.1846 kJ/kg·K. Finally p_1 = 60 bar, so v_2 = 0.1765 m^3/kg and s_2 = 6.9938 kJ/kg·K. Thus

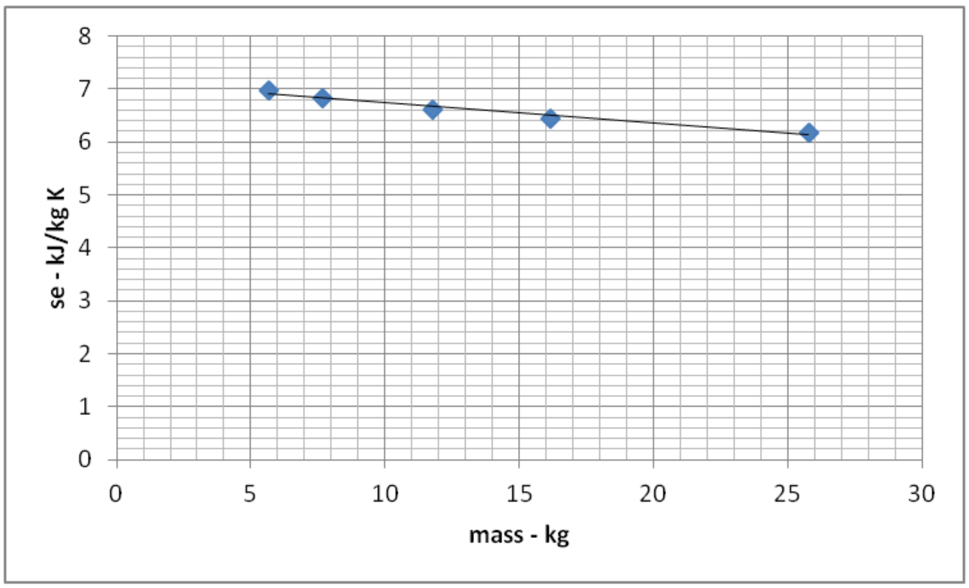
$$m_1 = 1/(0.03876) = 25.8 \text{ kg} \quad \text{and} \quad m_2 = 1/(0.1765) = 5.67 \text{ kg}$$

PROBLEM 6.115 (CONTINUED)

Note that the integral in (*) cannot be evaluated directly since s_e varies as the state changes within the tank. The following table provides data for pressures between 60 bar and 15 bar at 320°C:

p (bar)	v (m^3/kg)	s_e (kJ/kg·K)	m (kg)
60	.03876	6.1846	25.8
40	0.06199	6.4553	16.13
30	0.0850	6.6245	11.76
20	0.1308	6.8452	7.65
15	0.1765	6.9938	5.67

These data are shown in the following plot:



The value of the integral $\int_{m_1}^{m_2} s_e dm_{cv}$ is the area under the curve. Since the variation is nearly linear, the integral can be approximated as

$$\int_{m_1}^{m_2} s_e dm_{cv} \approx \left[\frac{((s_e)_1 + (s_e)_2)}{2}\right](m_2 - m_1) = \left[\frac{6.1846 + 6.9938}{2}\right](5.67 - 25.8) = -132.64 \text{ kJ/K}$$

Thus, the entropy production is

$$\sigma_{cv} = (m_2 s_2 - m_1 s_1) - \int_{m_1}^{m_2} s_e dm_{cv} = [(5.67)(6.9938) - (25.8)(6.1846)] - (-132.64)$$

$$= 12.73 \text{ kJ/K}$$

The process is irreversible.

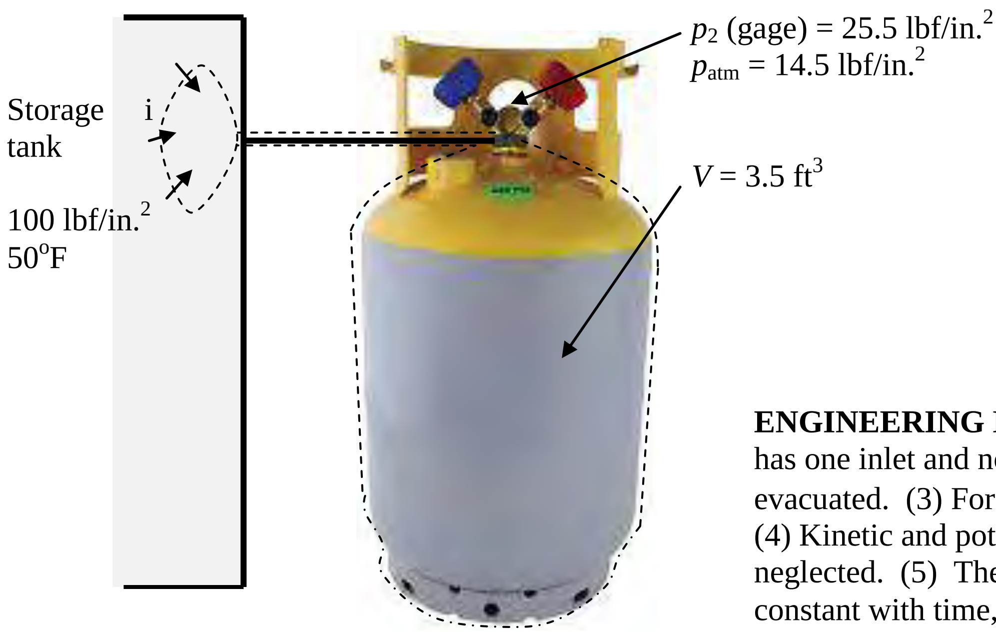
PROBLEM 6.116

A two-phase liquid-vapor mixture of Refrigerant 134a is held in a large storage tank at 100 lbf/in.^{2}, 50^{o}F, as illustrated in Fig. P6.116. A technician fills a 3-ft^3 cylinder that is initially evacuated to take on a service call. The technician opens a valve and lets refrigerant from the storage tank flow into the cylinder until the pressure gage on the tank reads 25.5 lbf/in.^{2} (gage). The surrounding atmospheric pressure is 14.5 lbf/in.^{2} Assuming no heat transfer and neglecting kinetic and potential energy effects, determine the final mass of refrigerant in the cylinder, in lb, and the amount of entropy produced, in Btu/^{o}R.

KNOWN: Refrigerant 13a from a large storage tank fills an initially evacuated cylinder.

FIND: Determine the final mass in the cylinder and the amount of entropy produced.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The control volume has one inlet and no exit. (2) The cylinder is initially evacuated. (3) For the control volume, $\dot{Q}_{\text{cv}} = \dot{W}_{\text{cv}} = 0$. (4) Kinetic and potential energy effects can be neglected. (5) The conditions in the storage tank are constant with time, so h_i is constant.

ANALYSIS: The mass rate balance reduces to $dm_{cv}/dt = \dot{m}_i \longrightarrow m_2 - \cancel{m_1} = \int \dot{m}_i dt$
And, the energy rate balance is

$$dU_{cv}/dt = \dot{m}_i h_i \longrightarrow m_2 u_2 = \int h_i \dot{m}_i dt \longrightarrow m_2 u_2 = m_2 h_i \longrightarrow \boxed{u_2 = h_i}$$

From Table A-11E; T_i = 50^{o}F is less than T_{sat} = 79.17^{o}F at 100 lbf/in.^{2} Therefore, the refrigerant is at a compressed liquid state. From Table A-10E, the specific enthalpy is $h_i \approx h_f(50^oF)$ = 27.28 Btu/lb.

Now, the final pressure (absolute) is $p_2 = p_2(\text{gage}) + p_{atm}$ = 25.5 + 14.5 = 40 lbf/in.^{2} From the energy balance, $u_2 = h_i$ = 27.28 Btu/lb. Therefore, with data from table A-12E;

$$x_2 = (u_2 - u_{f2})/(u_{g2} - u_{f2}) = (27.28 - 20.48)/(97.23 - 20.48) = 0.0886$$

Thus

$$v_2 = v_{f2} + x_2(v_{g2} - v_{f2}) = 0.01232 + (0.0886)(1.1692 - 0.01232) = 0.11482 \text{ ft}^3/\text{lb}$$

and the final mass is

$$m_2 = V/v_2 = (3.5\text{ lb})/(\ 0.11482\text{ ft}^3/\text{lb}) = 30.48\text{ lb} \longleftarrow$$

The entropy rate balance reduces to $dS_{cv}/dt \ = \ \dot{m}_i s_i + \dot{\sigma}_{cv} \ \longrightarrow \ m_2 s_2 = \int s\dot{m}_i dt + \sigma_{cv}$

Now

$$m_2 s_2 = m_2 s_i + \sigma_{cv} \ \longrightarrow \ \sigma_{cv} = m_2(s_2 - s_i)$$

From Table A-10E: $s_i \approx s_f(50^oF) = 0.0585$ Btu/lb·^{o}R. At state 2;

$$s_2 = s_{f2} + x_2(s_{g2} - s_{f2}) = 0.0452 + (0.0886)(0.2197 - 0.0452) = 0.06066\text{ Btu/lb}\cdot{}^o\text{R}$$

and

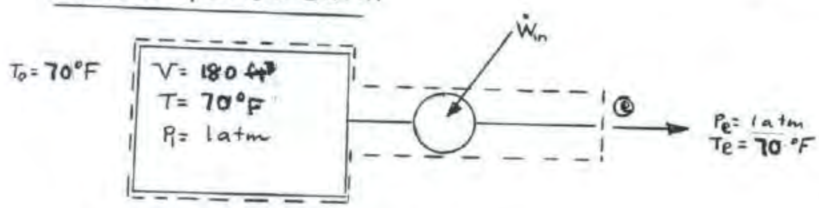
$$\sigma_{cv} = m_2(s_2 - s_i) = (30.48\text{ lb})(0.06066 - 0.0585)\text{ Btu/lb}\cdot{}^o\text{R} = 0.06584\text{ Btu/}^o\text{R} \longleftarrow$$

PROBLEM 6.117

KNOWN: A tank initially filled with air is evacuated by pumping out the air.

FIND: Determine the minimum theoretical work required.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is shown in the figure. (2) Heat transfer with the surroundings maintains the temperature of the air in the tank and at the discharge of the pump at 70°F. (2) Kinetic and potential energy effects can be ignored. (3) The air is modeled as an ideal gas.

ANALYSIS: A mass balance reduces

$dU/dt = \dot{Q} + \dot{W}_{in} - \dot{m}_e h_e$. Combining these expressions

$$\frac{dU}{dt} = \dot{Q} + \dot{W}_{in} + h_e \frac{dm}{dt} \Rightarrow \Delta U = Q + W_{in} + h_e \Delta m \qquad (1)$$

where h_e is determined by T_e and thus is constant.

An entropy balance reads $dS/dt = \dot{Q}/T_0 - \dot{m}_e s_e + \dot{\sigma}$. With $-\dot{m}_e = dm/dt$

$$\frac{dS}{dt} = \frac{\dot{Q}}{T_0} + s_e \frac{dm}{dt} + \dot{\sigma} \Rightarrow \Delta S = \frac{Q}{T_0} + s_e \Delta m + \sigma \qquad (2)$$

where s_e is determined by T_e, p_e and thus is constant

Eliminating Q between Eqs. (1), (2)

$$W_{in} = \Delta U - T_0 \Delta S + T_0 s_e \Delta m - h_e \Delta m + T_0 \sigma$$

$$= [\cancel{m_2}^{0} u_2 - m_1 u_1] - T_0(\cancel{m_2}^{0} s_2 - m_1 s_1) + T_0 s_e [\cancel{m_2}^{0} - m_1] - h_e(\cancel{m_2}^{0} - m_1) + T_0 \sigma$$

$$= m_1 T_0 (s_1 - s_e) + m_1 (h_e - u_1) + T_0 \sigma \qquad (3)$$

Since the states are the same initially and at the exit, $s_1 = s_e$. Also, with $h_e = u_e + p_e v_e$ $(h_e - u_1) = (u_e - u_1) + p_e v_e = p_e v_e$, since $T_1 = T_e$. With these, Eq. (3) reduces to

$$W_{in} = m_1 (p_e v_e) + T_0 \sigma = p_e V + T_0 \sigma \qquad (4)$$

The specific volume at the exit is the same as the initial specific volume in the tank, and so $V = m_1 v_e$.

Finally, since $\sigma \geq 0$, the minimum theoretical value corresponds to $\sigma = 0$:

$$(W_{in})_{MIN} = p_e V$$

$$= \left(14.7 \times 144 \frac{lbf}{ft^2}\right)(180\ ft^3)\left(\frac{1\ Btu}{778\ ft \cdot lbf}\right)$$

$$= 489.7\ Btu \qquad \longleftarrow (W_{in})_{MIN}$$

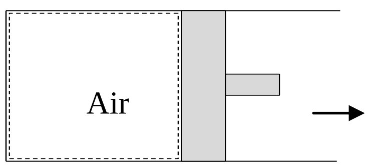
*<u>PROBLEM 6.118</u>

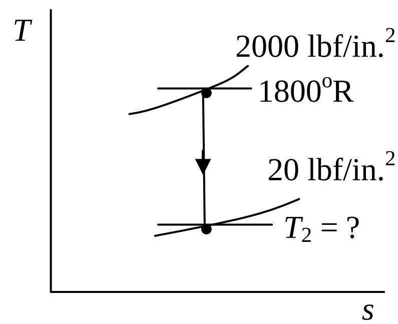
Air in a piston-cylinder assembly is expands isentropically from T_1 = 1800°R, p_1 = 2000 lbf/in.^{2}, to p_2 = 20 lbf/in.^{2} Assuming the ideal gas model, determine the temperature at state 2, in °R, using (a) data from Table A-22E, and (b) a constant specific heat ratio, k = 1.4. Compare the values in pats (a) and (b) and comment.

KNOWN: Air expand isentropically in a piston-cylinder assembly from a known initial state to a specified final pressure.

FIND: Determine the final temperature using (a) data from Table A-22E, and (b) constant specific heat ratio k = 1.4.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL: The air is a closed system. (2) The air undergoes an isentropic process. (3) The air is modeled as an ideal gas.

ANALYSIS: (a) For the isentropic process: $p_2/p_1 = p_r(T_2)/p_r(T_1)$. Thus, with data from Table A-22E

$$P_r(T_2) = p_r(T_1)(p_2/p_1) = (114.0)(20/2000) = 1.14$$

Interpolating in Table A-22E gives; T_2 = 510.4 °R ←

(b) From Eq. 6.43

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{(1-k)}{k}} = (1800\ ^{o}\mathrm{R})\left(\frac{20}{2000}\right)^{\frac{(1-1,4)}{1.4}} = 482.9\ ^{o}\mathrm{R}$$ ←

The value obtained in part (b) is about 5.4% smaller than the values obtained in part (a). The approach of part (a) accounts for the variation in specific heat with temperature, whereas the approach in part (b) does not.

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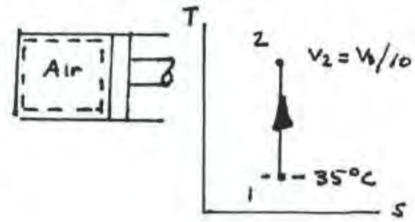
* In the first printing of the 8th Edition, p_1 was incorrectly given as 20 and p_2 was incorrectly given as 2000. The values in this solution are the correct values.

PROBLEM 6.119

Air in a piston–cylinder assembly is compressed isentropically from state 1, where $T_1 = 35°C$, to state 2, where the specific volume is one-tenth of the specific volume at state 1. Applying the ideal gas model with $k = 1.4$, determine (a) T_2, in °C and (b) the work, in kJ/kg.

SCHEMATIC & GIVEN DATA



ENGR. MODEL:

1. The air is the closed system.
2. The air undergoes an isentropic process.
3. The air is modeled as an ideal gas with $k = 1.4$.
4. Kinetic and potential energy play no role.

ANALYSIS: (a) With Eq. 6.44,

$$T_2 = T_1 \left(\frac{v_1}{v_2}\right)^{k-1} = (308.15\,K)(10)^{1.4-1} = 774.07\,K \;(500.9\,°C)$$

(b) Reducing an energy balance for the adiabatic process,

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - W \Rightarrow W = -\Delta U = -mc_v(T_2 - T_1).$$

From Eq. 3.47b, $c_v = R/(k-1)$. So

$$\frac{W}{m} = -\frac{R}{(k-1)}(T_2 - T_1) = -\frac{8.314}{28.97}\left(\frac{308.15 - 774.07}{1.4-1}\right)\frac{kJ}{kg} = -334.3\,\frac{kJ}{kg}$$

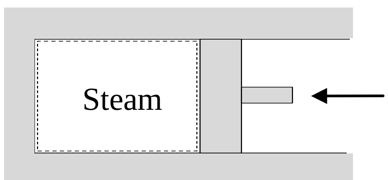
PROBLEM 6.120

Steam undergoes an isentropic compression in an insulated piton-cylinder assembly from an initial state where $T_1 = 120^{o}C$, $p_1 = 1$ bar to a final state where the pressure is $p_2 = 100$ bar. Determine the final temperature, in ^{o}C, and the work, in kJ per kg of steam.

KNOWN: Steam undergoes an isentropic compression in an insulated piston-cylinder assembly. The initial state is fixed and the final pressure is specified.

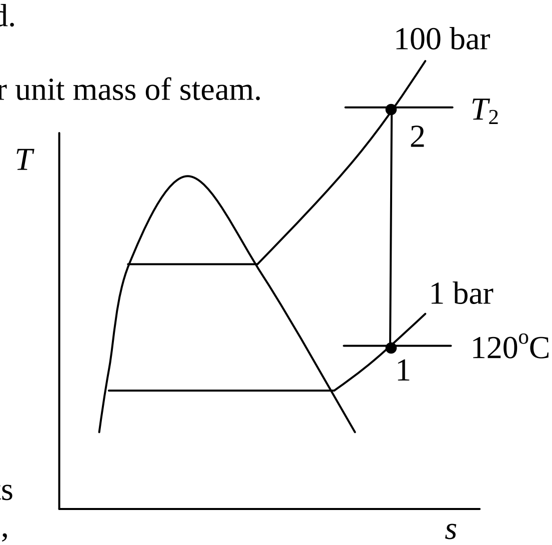
FIND: Determine the final temperature and the work per unit mass of steam.

SCHEMATIC AND GIVEN DATA:



$T_1 = 120^{o}C$
$p_1 = 1$ bar

$p_2 = 100$ bar
$s_2 = s_1$



ENGINEERING MODEL: (1) The steam is a closed system. (2) $Q = 0$ and kinetic and potential energy effects can be neglected. (3) The process is internally reversible, and $s_2 = s_1$.

ANALYSIS: To fix state 2, we use the pressure, 100 bar, and the specific entropy: $s_2 = s_1$. From Table A-4, at $p_1 = 1$ bar, $T_1 = 120^{o}F$; $s_1 = 7.4668$ kJ/kg·K. Also, $u_1 = 2537.3$ kJ/kg.

The highest specific entropy value in Table A-4 at 100 bar is 7.2670 kJ/kg·K (at $740^{o}C$). Extrapolating ; $T_2 \approx 821.3^{o}C$. Further, $u_2 \approx 3669.4$ kJ/kg.

Using *IT* the values are $T_2 = 826.1^{o}C$ and $u_2 = 3680$ kJ/kg. These values are more accurate, and will be used for further calculations.

The work is obtained using the closed system energy balance, which reduces as follows:

$$\Delta \cancel{KE} + \Delta \cancel{PE} + \Delta U = \cancel{Q} - W \quad \longrightarrow \quad W/m = (u_1 - u_2)$$

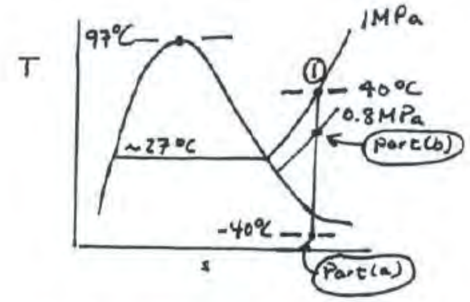
or

$$W/m = 2537.3 - 3680 = -1142.7 \text{ kJ/kg}$$

PROBLEM 6.121

Propane undergoes an isentropic expansion from an initial state where $T_1 = 40°C$, $p_1 = 1$ MPa to a final state where the temperature and pressure are T_2, p_2, respectively. Determine

(a) p_2, in kPa, when $T_2 = -40°C$
(b) T_2, in °C, when $p_2 = 0.8$ MPa



(a) From Table A-18, $s_1 = 1.810$ kJ/kg·K.

From Table A-16 at $-40°C$,

$s_f = 0.000$ kJ/kg·K

$s_g = 1.815$ kJ/kg·K

So, state 2 is in the two-phase region

and $P_2 = P_{sat}(-40°C) = 1.11$ bar

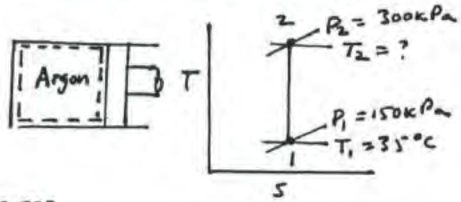
$= 111$ kPa ← (a)

(b) From Table A-18 at 0.8 MPa, interpolation gives $T_2 = 30.6°C$ ← (b)

PROBLEM 6.122

Argon in a piston–cylinder assembly is compressed isentropically from state 1, where $p_1 = 150$ kPa, $T_1 = 35°C$, to state 2, where $p_2 = 300$ kPa. Assuming the ideal gas model with $k = 1.67$, determine (a) T_2, in °C, and (b) the work, in kJ per kg of argon.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The argon is the closed system.
2. The argon undergoes an isentropic process.
3. The argon is modeled as an ideal gas with $k = 1.67$.
4. Kinetic and potential energy play no role.

ANALYSIS:

(a) With Eq. 6.43, $T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(k-1)/k} = (308\text{ K})\left(\frac{300}{150}\right)^{\frac{1.67-1}{1.67}} = 407\text{K}\ (134°C)$ ← (a)

(b) Reducing an energy balance for the adiabatic process,

$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - W \Rightarrow W = -m\Delta u = -mc_v(T_2 - T_1)$.

From Eq. 3.47b, $c_v = R/(k-1)$. So

$$\frac{W}{m} = -\frac{R}{(k-1)}(T_2 - T_1) = -\left(\frac{8.314}{39.94}\ \frac{kJ}{kg\cdot K}\right)\left(\frac{407-308}{1.67-1}\right)K$$

$$= -30.8\ \frac{kJ}{kg}$$ ← (b)

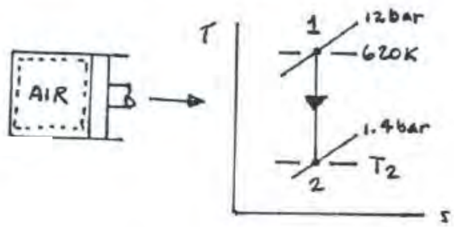
PROBLEM 6.123

Air within a piston–cylinder assembly, initially at 12 bar, 620 K, undergoes an isentropic expansion to 1.4 bar. Assuming the ideal gas model for the air, determine the final temperature, in K, and the work, in kJ/kg. Solve two ways: using (a) data from Table A-22 and (b) $k = 1.4$.

KNOWN: Air in a piston-cylinder assembly undergoes an isentropic expansion.

FIND: Determine the final temperature and the work.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The closed system is the air within the piston-cylinder.
2. The air undergoes an isentropic (and thus adiabatic) expansion.
3. Kinetic and potential energy effects are ignored.
4. The air is modeled as an ideal gas using (a) Table A-22 data, (b) $k=1.4$

ANALYSIS: An energy balance reduces to give $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - W$.

Thus $W = -\Delta U = -m(u_2 - u_1) \Rightarrow W/m = -(u_2 - u_1)$.

(a) Table A-22

$$p_r(T_2) = p_r(T_1)\left(\frac{p_2}{p_1}\right) = 18.36\left(\frac{1.4}{12}\right) = 2.142$$

Interpolation gives $T_2 = 339.7\ \text{K}$ ←

$u_2 = 242.58\ \text{kJ/kg}$

Then, $$W/m = -[242.58 - 450.09]\ \frac{\text{kJ}}{\text{kg}} = 207.5\ \frac{\text{kJ}}{\text{kg}}$$ ←

(b) $k=1.4$

Using Eq. 6.43, $$T_2 = T_1\left[\frac{p_2}{p_1}\right]^{\left(\frac{k-1}{k}\right)}$$

$$= 620\text{K}\left[\frac{1.4}{12}\right]^{(0.4/1.4)} = 335.6\ \text{K}$$ ←

Then,

$$\frac{W}{m} = -c_v[T_2 - T_1] = -\frac{R}{(k-1)}[T_2 - T_1]$$

$\left(c_v = \frac{R}{(k-1)}\ \ (\text{Eq. } 3.47b)\right)$

$$= -\frac{(8.314/28.97)\ \text{kJ/kg}\cdot\text{K}}{(1.4-1)}[335.6 - 620]\ \text{K}$$

$$= 204.0\ \frac{\text{kJ}}{\text{kg}}$$ ←

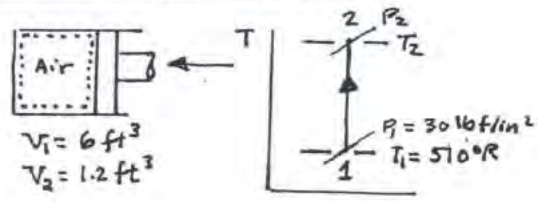
PROBLEM 6.124

Air within a piston–cylinder assembly, initially at 30 lbf/in.^{2}, 510°R, and a volume of 6 ft^3, is compressed isentropically to a final volume of 1.2 ft^3. Assuming the ideal gas model with $k = 1.4$ for the air, determine the (a) mass, in lb, (b) final pressure, in lbf/in.^{2}, (c) final temperature, in °R, and (d) work, in Btu.

KNOWN: Air in a piston-cylinder assembly undergoes an isentropic compression.

FIND: The mass of the air and its final pressure, temperature, and the work.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The closed system is the air within the piston-cylinder.
2. The air undergoes an isentropic (and thus adiabatic) process.
3. Kinetic and potential energy effects are ignored.
4. The air is modeled as an ideal gas with $k = 1.4$.

ANALYSIS:

(a) Using the ideal gas model equation of state,

$$m = \frac{P_1 V_1}{R T_1} = \frac{(30 \times 144\ lbf/ft^2)(6\ ft^3)}{\left(\frac{1545}{28.97}\ \frac{ft\cdot lbf}{lb\cdot °R}\right)(510°R)} = 0.953\ lb \longleftarrow$$

(b) Applying Eq. 6.45,

$$P_2 = P_1 \left[\frac{V_1}{V_2}\right]^k = \left(30\ \frac{lbf}{in^2}\right)\left(\frac{6\ ft^3}{1.2\ ft^3}\right)^{1.4} = 285.55\ \frac{lbf}{in^2} \longleftarrow$$

(c) With the ideal gas model equation of state,

① $$\left.\begin{matrix} P_1 V_1 = m R T_1 \\ P_2 V_2 = m R T_2 \end{matrix}\right\rangle \quad T_2 = T_1 \left[\frac{P_2}{P_1}\right]\left[\frac{V_2}{V_1}\right] = 510°R \left[\frac{285.55\ lbf/in^2}{30\ lbf/in^2}\right]\left[\frac{1.2\ ft^3}{6\ ft^3}\right]$$

$$= 970.9°R \longleftarrow$$

(d) Reducing an energy balance, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - W$

$$\Rightarrow \quad W = -\Delta U = -m\, c_v [T_2 - T_1]$$

$$c_v = \frac{R}{(k-1)} \quad \text{(Eq. 3.47(b))}$$

$$\therefore W = -m\left(\frac{R}{k-1}\right)(T_2 - T_1)$$

$$= -(0.953\ lb)\left(\frac{1.986/28.97}{1.4-1}\ \frac{Btu}{lb\cdot °R}\right)(970.9 - 510)°R$$

$$= -75.3\ Btu \longleftarrow$$

1. Alternatively, applying Eq. 6.44,

$$T_2 = T_1 \left[\frac{V_1}{V_2}\right]^{(k-1)} = (510°R)\left[\frac{6}{1.2}\right]^{0.4} = 970.9°R$$

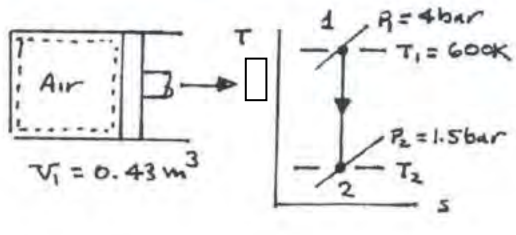
PROBLEM 6.125

Air contained in a piston–cylinder assembly, initially at 4 bar, 600 K and a volume of 0.43 m^3, expands isentropically to a pressure of 1.5 bar. Assuming the ideal gas model for the air, determine the (a) mass, in kg, (b) final temperature, in K, and (c) work, in kJ.

KNOWN: Air in a piston-cylinder assembly expands isentropically.

FIND: Determine the mass of the air, its final temperature, and the work.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The closed system is the air within the piston-cylinder.
2. The air undergoes an isentropic (and thus adiabatic) expansion.
3. Kinetic and potential energy effects are ignored.
4. The air is modeled as an ideal gas

ANALYSIS:

(a) Using the ideal gas model equation of state

$$m = \frac{P_1 V_1}{R T_1} = \frac{(4 \times 10^5\ N/m^2)(0.43\ m^3)}{\left(\frac{8314}{28.97}\ \frac{N \cdot m}{kg \cdot K}\right)(600K)} = 1\ kg$$

(b) With data from Table A-22

$$P_r(T_2) = P_r(T_1)\frac{P_2}{P_1} = 16.28\left[\frac{1.5\ bar}{4\ bar}\right] = 6.105$$

Interpolation in Table A-22 gives

$$T_2 = 457\ K$$

$$u_2 = 327.78\ kJ/kg$$

(c) Reducing an energy balance, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - W$

$$\Rightarrow W = -\Delta U = -m[u_2 - u_1]$$

$$= -(1kg)[327.78 - 434.78]\ \frac{kJ}{kg}$$

$$= 107\ kJ$$

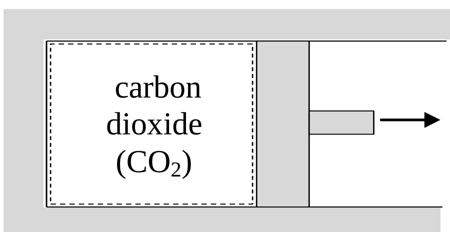
PROBLEM 6.126

Carbon dioxide (CO_2) expands isentropically in a well-insulated piston-cylinder assembly from p_1 = 200 lbf/in.^{2}, T_1 = 800°R to a final specific volume of v_2 = 1.8 ft^3/lb. Determine the work, in Btu per lb of carbon dioxide, assuming the ideal gas model with
(a) constant specific heats evaluated at 600°R.
(b) variable specific heats using *IT:Interactive Thermodynamics*.

KNOWN: Carbon dioxide expands isentropically in a piston-cylinder assembly from a known initial state to a specified final specific volume.

FIND: Determine the work per unit mass of carbon dioxide using the ideal gas model with (a) constant specific heats and (b) variable specific heats and *IT:Interactice Thermodynamics*.

SCHEMATIC AND GIVEN DATA:



p_1 = 200 lbf/in.^{2}
T_1 = 800°R

v_2 = 1.8 ft^3/lb
$s_2 = s_1$

ENGINEERING MODEL: (1) The carbon dioxide is a closed system. (2) The process is isentropic, and Q = 0. (3) The gas is modeled as an ideal gas with (a) constant specific heats and (b) variable specific heats. (4) Kinetic and potential energy effects can be neglected.

ANALYSIS: The specific volume at state 1 is

$$v_1 = \frac{RT_1}{p_1} = \frac{\left(\frac{1545}{44.01}\frac{\text{ft}\cdot\text{lbf}}{\text{lb.}^\text{o}\text{R}}\right)(800^\text{o}\text{R})}{\left(200\frac{\text{lbf}}{\text{in.}^2}\right)}\left|\frac{1\ \text{ft}^2}{144\ \text{in.}^2}\right| = 0.9752\ \text{ft}^3/\text{lb}$$

(a) For constant specific heats, Eq. 6.44 relates temperature and specific volume at states of equal entropy. Thus, with data from Table A-20E at 600°R

$$T_2 = (v_1/\,v_2)^{k\text{-}1} = (0.9752/1.8)^{(1.202\,-\,1)}(800^\text{o}\text{R}) = 706.8^\text{o}\text{R}$$

The energy balance reduces to

$$W/m = (u_1 - u_2) = c_v(T_1 - T_2) = (0.224\ \text{Btu/lb}\cdot^\text{o}\text{R})(800 - 706.8)^\text{o}\text{R} = 20.88\ \text{Btu/lb} \longleftarrow$$

(b) Using variable specific heats, for $s_2 = s_1$

$$0 = s^\text{o}(T_2) - s^\text{o}(T_1) - R\ln(p_2/p_1)$$

This, combines with $v_2 = RT_2/p_2$ gives two equations in two unknowns: T_2 and p_2. Table A-23E can be used to solve for these unknowns, but the process would be highly iterative. Instead, we can use *IT: Interactive Thermodynamics*.

PROBLEM 6.126 (CONTINUED)

The code is

```
p1 = 200  //lbf/in^2
T1 = 800  //R
v2 = 1.8  //ft^3/lb

v1 = v_Tp("CO2",T1,p1)
s1 = s_Tp("CO2",T1,p1)
s2 = s_Tp("CO2",T2,p2)
s2 = s1
v2 = v_Tp("CO2",T2,p2)

W/m = u1 – u2
u1 = u_T("CO2",T1)
u2 = u_T("CO2",T2)
m= 1
```

The results are:

p_2 = 93.04 lbf/in.^{2}
T_2 = 686.8^{o}R
$u_1 - u_2$ = 20.54 Btu/lb
v_1 = 0.9753 ft^3/lb

The work is

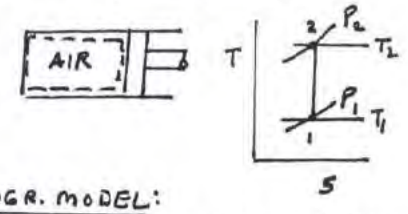
$$W/m = (u_1 - u_2) = 20.54 \text{ Btu/lb}$$

The results compare favorably. For the temperature range of this problem, choosing the specific heat as constant evaluated at 600^o R gives a reasonable approximation.

PROBLEM 6.127

Air in a piston–cylinder assembly is compressed isentropically from an initial state where $T_1 = 340$ K to a final state where the pressure is 90% greater than at state 1. Assuming the ideal gas model, determine (a) T_2, in K, and (b) the work, in kJ/kg.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The air is the closed system
2. The air undergoes an isentropic process.
3. The air is modeled as an ideal gas.
4. Kinetic and potential energy play no role.

ANALYSIS: (a) With $p_2/p_1 = 1.9$, Eq. 6.41 gives

$$p_{r2} = p_{r1}\left(p_2/p_1\right) = 2.149(1.9) = 4.083$$

Interpolating in Table A-22, $T_2 = 408\text{ K}$, ←

$u_2 = 291.96\text{ kJ/kg}$

(b) An energy balance reduces to read,

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - W \Rightarrow W = -m(u_2 - u_1).$$

$$\Rightarrow \frac{W}{m} = u_1 - u_2 = 242.82 - 291.96 = -49.14\text{ kJ/kg}$$ ←

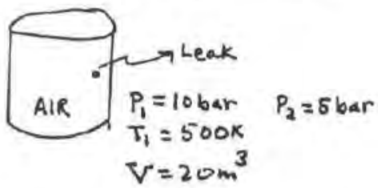
PROBLEM 6.128

A rigid, insulated tank with a volume of 20 m^3 is filled initially with air at 10 bar, 500 K. A leak develops, and air slowly escapes until the pressure of the air remaining in the tank is 5 bar. Employing the ideal gas model with $k = 1.4$ for the air, determine the amount of mass remaining in the tank, in kg, and its temperature, in K.

KNOWN: Air slowly leaks from a rigid, insulated tank until the remaining air is at 5 bar.

FIND: Determine the amount of air remaining in the tank and its temperature.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. As in Example 6.10, the closed system is the mass initially in the tank that remains in the tank; call it m_2.
2. For the system, $Q = 0$.
3. Irreversibilities within the system can be ignored as air slowly escapes.
4. The air is modeled as an ideal gas with $k = 1.4$

ANALYSIS: (a) For the closed system under consideration, the entropy balance reduces with assumptions 2 and 3 to read,

$$\Delta S = \int \cancel{\frac{\delta Q}{T}}^{0} + \cancel{\sigma}^{0} \Rightarrow \Delta S = \underbrace{m_2}_{\text{denotes mass remaining in the tank}} \Delta s = 0 \Rightarrow \Delta s = 0 \Rightarrow$$

The specific entropy of the air remaining in the tank is constant; it undergoes an isentropic process.

Then, applying Eq. 6.43

$$T_2 = T_1 \left[\frac{P_2}{P_1}\right]^{(k-1)/k} = (500\text{K})\left[\frac{5\text{ bar}}{10\text{ bar}}\right]^{0.4/1.4} = 410.2\text{ K} \longleftarrow$$

(b) Applying the ideal gas equation of state, the final amount of mass in the tank is

① $$m_2 = \frac{P_2 V}{R T_2} = \frac{(5\times10^5\text{N/m}^2)(20\text{ m}^3)}{\left(\frac{8314}{28.97}\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(410.2\text{K})} = 84.95\text{ kg} \longleftarrow$$

---

1. The initial mass in the tank is

$$m_1 = \frac{P_1 V}{R T_1} = \frac{(10\times10^5\text{N/m}^2)(20\text{m}^3)}{\left(\frac{8314}{28.97}\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(500\text{K})} = 139.38\text{kg}.$$

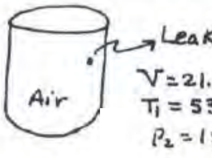
PROBLEM 6.129

A rigid, insulated tank with a volume of 21.61 ft^3 is filled initially with air at 110 lbf/in.^{2}, 535°R. A leak develops, and air slowly escapes until the pressure of the air remaining in the tank is 15 lbf/in.^{2} Employing the ideal gas model with $k = 1.4$ for the air, determine the amount of mass remaining in the tank, in lb, and its temperature, in °R.

KNOWN: Air slowly leaks from a rigid, insulated tank until the remaining air is at 15 lbf/in²

FIND: Determine the amount of air remaining in the tank and its temperature.

SCHEMATIC & GIVEN DATA:



$V = 21.61 ft^3$

$T_1 = 535°R$, $P_1 = 110 lbf/in^2$

$P_2 = 15 lbf/in^2$

ENGINEERING MODEL:

1. As in Example 6.10, the closed system is the air initially in the tank that remains in the tank, call it m_2.
2. For the system, $Q = 0$.
3. Irreversibilities within the system can be ignored as air slowly escapes.
4. The air is modeled as an ideal gas with $k = 1.4$.

ANALYSIS: (a) For the closed system under consideration, the entropy balance reduces with assumptions 2 and 3 to read,

$$\Delta S = \cancel{\int \frac{\delta Q}{T}}^{0} + \cancel{\sigma}^{0} \Rightarrow \Delta S = \underbrace{m_2}_{\text{denotes mass remaining in the tank}} \Delta s \Rightarrow \Delta s = 0 \Rightarrow$$

The specific entropy of the air remaining in the tank is constant: it undergoes an isentropic process.

Then, applying Eq. 6.43, $T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = 635°R \left(\frac{15}{110}\right)^{(0.4/1.4)} = 302.8°R$ ←

(b) Applying the ideal gas equation of state, the final amount of mass in the tank is

$$m_2 = \frac{P_2 V}{R T_2} = \frac{(15)(144\ lbf/ft^2)(21.61\ ft^3)}{\left(\frac{1545}{28.97}\ \frac{ft \cdot lbf}{lb \cdot °R}\right)(302.8°R)} = 2.89\ lb$$ ←

PROBLEM 6.130

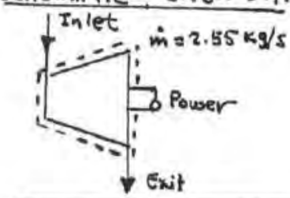
The accompanying table provides steady-state data for an isentropic expansion of steam through a turbine. For a mass flow rate of 2.55 kg/s, determine the power developed by the turbine, in MW. Ignore the effects of potential energy.

	p(bar)	T(°C)	V(m/s)	h(kJ/kg)	s(kJ/kg · K)
Inlet	10	300	25	3051.1	7.1214
Exit	1.5	—	100		7.1214

KNOWN: Steam expands isentropically through a turbine.

FIND: Determine the power developed.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The control volume shown in the schematic is at steady state.
2. The steam expands isentropically through the turbine. Thus, $\dot{Q} = 0$.
3. Potential energy effects can be ignored

ANALYSIS: The control volume energy rate balance at steady state reduces to

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_i - h_e) + \frac{V_i^2 - V_e^2}{2} + \cancel{g(z_i - z_e)}\right]$$

$$\Rightarrow \quad \dot{W}_{cv} = \dot{m}\left[(h_i - h_e) + \frac{V_i^2 - V_e^2}{2}\right] \qquad (1)$$

All data required by Eq.(1) is known except h_e. This can be evaluated using x_e obtained from the specific entropy at the exit together with s_f and s_g values from Table A-3 at 1.5 bar:

$$x_e = \frac{s_e - s_f}{s_g - s_f} = \frac{7.1214 - 1.4336}{7.2233 - 1.4336} = 0.9824 \Rightarrow h_e = 467.11 + 0.9824(2226.5) = 2654.4\ \text{kJ/kg}$$

Inserting values, Eq.(1) reads,

$$\dot{W}_{cv} = 2.55\frac{\text{kg}}{\text{s}}\left[(3051.1 - 2654.4)\frac{\text{kJ}}{\text{kg}} + \left\{\left[\frac{(25)^2 - (100)^2}{2}\right]\left(\frac{\text{m}}{\text{s}}\right)^2\right\}\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|\right]$$

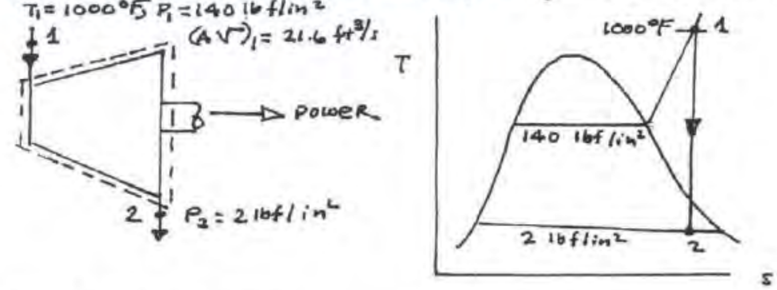
$$= 2.55\frac{\text{kg}}{\text{s}}\left[396.7\frac{\text{kJ}}{\text{kg}} - 4.7\frac{\text{kJ}}{\text{kg}}\right]\left|\frac{1\ \text{MW}}{10^3\ \text{kJ/s}}\right| = 1\ \text{MW}$$

PROBLEM 6.131

Water vapor enters a turbine operating at steady state at 1000°F, 140 lbf/in.^{2}, with a volumetric flow rate of 21.6 ft^3/s, and expands isentropically to 2 lbf/in.^{2} Determine the power developed by the turbine, in hp. Ignore kinetic and potential energy effects.

KNOWN: Steam expands isentropically through a turbine operating at steady state.

FIND: Determine the power developed by the turbine, in hp



ENGINEERING MODEL:

1. The control volume shown in the schematic operates at steady state.
2. The expansion is isentropic, and thus $\dot{Q}_{cv} = 0$.
3. Kinetic and potential energy effects are ignored.

ANALYSIS: The one-inlet, one-exit energy rate balance reduces at steady state to give

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\frac{V_1^2 - V_2^2}{2}} + g\cancel{(z_1 - z_2)}\right]$$

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2) \qquad (1)$$

From Table A-4E, $h_1 = 1531 \frac{\text{Btu}}{\text{lb}}$, $v_1 = 6.173 \frac{\text{ft}^3}{\text{lb}}$, $s_1 = 1.8827 \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$

Then,

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{21.6 \text{ ft}^3/\text{s}}{6.173 \text{ ft}^3/\text{lb}} = 3.5 \text{ lb/s}$$

Since $s_2 = s_1$,

$$x_2 = \frac{s_2 - s_f}{s_g - s_f} = \frac{1.8827 - 0.1750}{1.7448} = 0.9787$$

where data are from Table A-3E. Continuing,

$$h_2 = h_f + x_2(h_g - h_f) = 94.02 + 0.9787(1022.1) = 1094.3 \text{ Btu/lb}$$

Inserting values, Eq. (1) gives

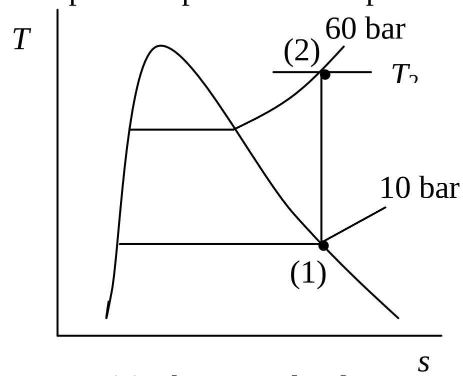
$$\dot{W}_{cv} = \left(3.5 \frac{\text{lb}}{\text{s}}\right)(1531 - 1094.3)\frac{\text{Btu}}{\text{lb}}\left|\frac{3600 \text{ s}}{\text{h}}\right|\left|\frac{1 \text{ hp}}{2545 \text{ Btu/h}}\right|$$

$$= 2162 \text{ hp}$$

PROBLEM 6.132

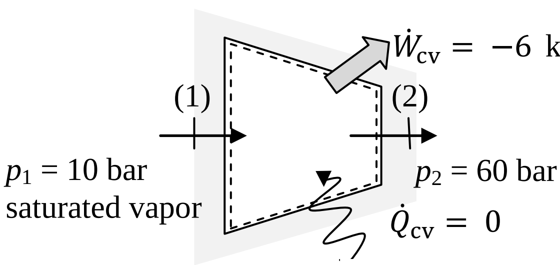
Refrigerant 22 is enters a compressor operating at steady state as saturated vapor at 10 bar and compressed adiabatically in an internally reversible process to 16 bar. Ignoring kinetic and potential energy effects, determine the required mass flow rate of refrigerant, in kg/s, if the compressor power *input* is 6 kW.

KNOWN: Refrigerant 22 is compressed at steady state from a known initial state to a given final pressure in an internally reversible, adiabatic process. The power input to the compressor is specified.



FIND: Determine the mass flow rate of refrigerant.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The control volume is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and there are no internal irreversibilities. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: Since the process is adiabatic and internally reversible, the entropy rate balance reduces to give $0 = \cancel{\sum_j \left(\frac{\dot{Q}_j}{T_j}\right)} + \dot{m}(s_1 - s_2) + \cancel{\dot{\sigma}_{cv}} \longrightarrow \boxed{s_2 = s_1}$

From Table A-8, at p_1 = 10 bar, saturated vapor; h_1 = 257.28 kJ/kg and s_1 = 0.8952 kJ/kg·K. With $s_2 = s_1$ and p_2 = 16 bar, Table A-9 gives h_2 = 268.58 kJ/kg.

The mass and energy rate balances reduce to give

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\frac{(V_1^2 - V_2^2)}{2}} + \cancel{g(z_1 - z_2)}\right]$$

Thus

$$\dot{m} = \dot{W}_{cv}/(h_1 - h_2) = (-6 \text{ kW})/(257.28 - 268.58)\text{kJ/kg} \left|\frac{1 \text{ kJ/s}}{1 \text{ kW}}\right| = 0.531 \text{ kg/s}$$

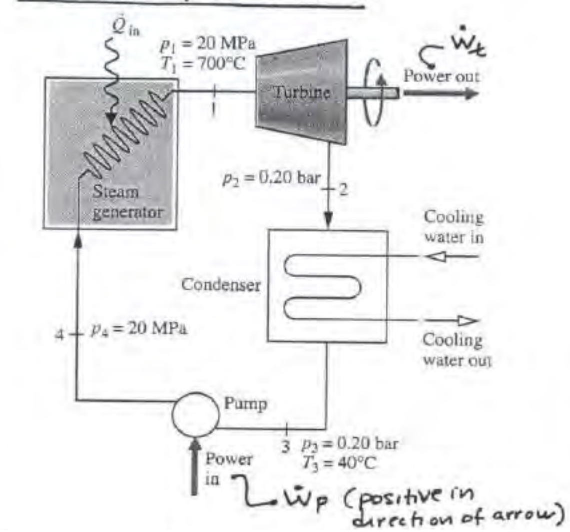
PROBLEM 6.133

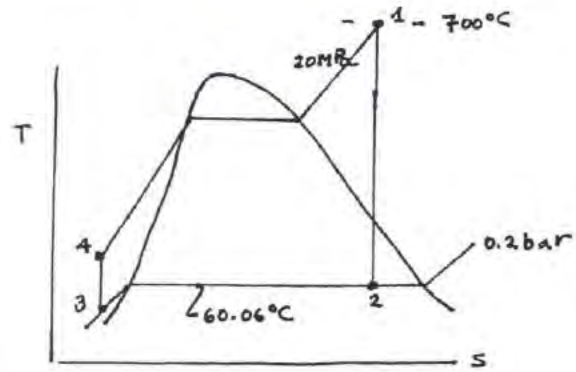
Shown below is a simple vapor power cycle operating at steady state with water as the working fluid. Data at key locations are given on the figure. Flow through the turbine and pump occurs isentropically. Flow through the steam generator and condenser occurs at constant pressure. Stray heat transfer and kinetic and potential energy effects are negligible. Sketch the four processes of this cycle in series on a T–s diagram. Determine the thermal efficiency.

KNOWN: Steady-state operating data are provided for a simple vapor power cycle.

FIND: Determine the thermal efficiency.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL:

1. Control volumes at steady state enclose the steam generator, turbine, and pump.
2. Flow through the turbine and pump is isentropic. Flow through the steam generator and condenser is at constant pressure.
3. Stray heat transfer and kinetic and potential energy effects are negligible.
4. At state 3, $h \approx h_f(T_3)$, $s \approx s_f(T_3)$.

ANALYSIS: The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{(\dot{W}_t/\dot{m}) - (\dot{W}_p/\dot{m})}{\dot{Q}_{in}/\dot{m}} \qquad (1)$$

Applying one-inlet, one-exit energy rate balances, Eq. (1) becomes

$$\eta = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)} \qquad (2)$$

State 1. Table A-4 gives $h_1 = 3809 \frac{kJ}{kg}$, $s_1 = 6.7993 \frac{kJ}{kg \cdot K}$

State 2. $s_2 = s_1$. With data from Table A-3

$$x_2 = \frac{6.7993 - 0.8320}{7.9085 - 0.8320} = 0.843$$

$$h_2 = h_f + x_2(h_g - h_f) = 251.4 + 0.843(2358.3)$$
$$= 2239.4 \text{ kJ/kg}$$

State 3. $h_3 \approx h_f(T_3) = 167.57$ kJ/kg.

State 4. Interpolating in Table A-5, using $s_4 = s_3 = s_f(40°C) = 0.5725 \frac{kJ}{kg \cdot K}$

$h_4 = 187.8$ kJ/kg

Inserting values, Eq. (2) becomes

$$\eta = \frac{(3809 - 2239.4) - (187.8 - 167.57)}{(3809 - 187.8)}$$

$$= \frac{1569.6 - 20.2}{3621.2}$$

$$= 0.428 \ (42.8\%) \qquad ①$$

1. The value obtained for η assumes no internal irreversibilities in any of the four processes. Owing to the effects of irreversibilities, the actual thermal efficiency would be less than that calculated here.

PROBLEM 6.134

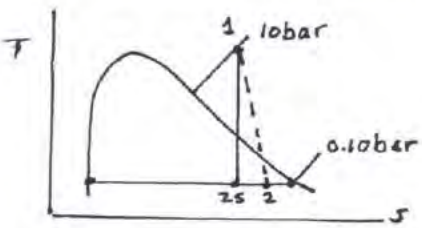
The accompanying table provides steady-state data for steam expanding adiabatically though a turbine. The states are numbered as in Fig. 6.11. Kinetic and potential energy effects can be ignored. Determine for the turbine (a) the work developed per unit mass of steam flowing, in kJ/kg, (b) the amount of entropy produced per unit mass of steam flowing, in kJ/kg · K, and (c) the isentropic turbine efficiency.

State	p(bar)	T(°C)	x(%)	h(kJ/kg)	s(kJ/kg · K)
1	10	300	—	3051	7.121
2s	0.10	45.81	86.3	—	7.121
2	0.10	45.81	90.0	—	7.400

KNOWN: Steady-state data are provided for steam expanding adiabatically through a turbine.

FIND: Determine $\dot{W}_{cv}/\dot{m}$, $\dot{\sigma}/\dot{m}$, and η_t.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. A control volume encloses the turbine.
2. The control volume is at steady state with $\dot{Q}_{cv}=0$ and negligible effects of kinetic and potential energy.

ANALYSIS: Reducing one-inlet, one-exit energy and entropy rate balances at steady state we get, respectively

$$\dot{W}_t/\dot{m} = h_1 - h_2 \quad , \quad \dot{\sigma}/\dot{m} = s_2 - s_1$$

Also, the isentropic turbine efficiency is given by Eq. 6.46: $\eta_t = \dfrac{\overbrace{h_1 - h_2}^{=\dot{W}_t/\dot{m}}}{h_1 - h_{2s}}$.

Evaluating needed properties with data from Table A-3,

$$h_2 = h_f + x_2(h_g - h_f) = 191.83 + 0.9\,(2584.7 - 191.83) = 2345.4 \text{ kJ/kg}.$$

$$h_{2s} = h_f + x_{2s}(h_g - h_f) = 191.83 + 0.863\,(2584.7 - 191.83) = 2256.9 \text{ kJ/kg}$$

Then,

$$\dot{W}_t/\dot{m} = (3051 - 2345.4) \text{ kJ/kg} = 705.6 \text{ kJ/kg} \quad \longleftarrow$$

$$\dot{\sigma}/\dot{m} = (7.4 - 7.121) \text{ kJ/kg}\cdot\text{K} = 0.279 \text{ kJ/kg}\cdot\text{K} \quad \longleftarrow$$

$$\eta_t = \frac{705.6 \text{ kJ/kg}}{(3051 - 2256.9)} = 0.889 \quad (88.9\%) \quad \longleftarrow$$

PROBLEM 6.135

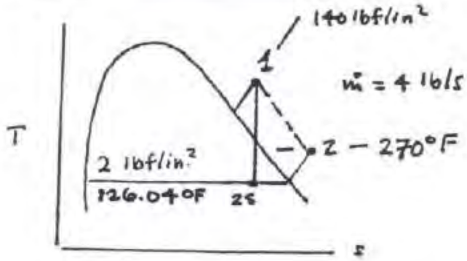
The accompanying table provides steady-state data for steam expanding adiabatically with a mass flow rate of 4 lb/s through a turbine. Kinetic and potential energy effects can be ignored. Determine for the turbine (a) the power developed, in hp, (b) the rate of entropy production, in hp/°R, and (c) the isentropic turbine efficiency.

	p(lbf/in.^{2})	T(°F)	u(Btu/lb)	h(Btu/lb)	s(Btu/lb · °R)
Inlet	140	1000	1371.0	1531.0	1.8827
Exit	2	270	1101.4	1181.7	2.0199

KNOWN: Steady-state data are provided for steam expanding adiabatically through a turbine.

FIND: Determine $\dot{W}_t$, $\dot{\sigma}_t$, and η_t.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. A control volume encloses the turbine.
2. The control volume is at steady state with $\dot{Q}_{cv}$ and negligible effects of kinetic and potential energy.

ANALYSIS: Reducing one-inlet, one-exit energy and rate balances at steady state, we get

$$\dot{W}_t = \dot{m}(h_1 - h_2) \quad , \quad \dot{\sigma}_t = \dot{m}(s_2 - s_1)$$

Also, the isentropic turbine efficiency is given by Eq. 6.46: $\eta_t = \dfrac{h_1 - h_2}{h_1 - h_{2s}}$.

Then,

$$\dot{W}_t = \left(4\,\frac{lb}{s}\right)(1531 - 1181.7)\,\frac{Btu}{lb}\left|\frac{3600\,s}{1\,h}\right|\left|\frac{1\,hp}{2545\,Btu/h}\right| = 1976\,hp$$

$$\dot{\sigma}_t = \left(4\,\frac{lb}{s}\right)(2.0199 - 1.8827)\,\frac{Btu}{lb \cdot °R}\left|\frac{3600\,s}{1\,h}\right|\left|\frac{1\,hp}{2545\,Btu/h}\right| = 0.776\,\frac{hp}{°R}$$

To find h_{2s}, use $s_{2s} = s_1$ and data from Table A-3E,

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{(1.8827 - 0.1750)}{1.7448} = 0.9787$$

$$h_{2s} = h_f + x_{2s}(h_g - h_f) = 94.02 + 0.9787(1022.1) = 1094.3\ Btu/lb.$$

$$\therefore \quad \eta = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{1531 - 1181.7}{1531 - 1094.3} = 0.8\ (80\%)$$

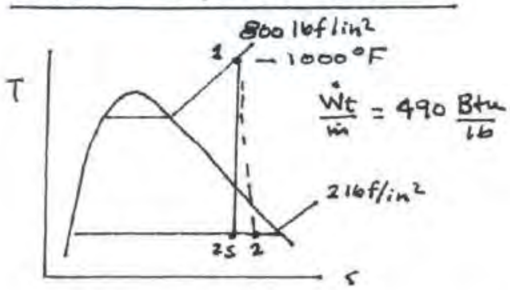
PROBLEM 6.136

Water vapor at 800 lbf/in.², 1000°F enters a turbine operating at steady state and expands adiabatically to 2 lbf/in.², developing work at a rate of 490 Btu per lb of vapor flowing. Determine the condition at the turbine exit: two-phase liquid–vapor or superheated vapor? Also, evaluate the isentropic turbine efficiency. Kinetic and potential energy effects are negligible.

KNOWN: Steady-state data are provided for a turbine.

FIND: Determine the state at the turbine exit and the isentropic turbine efficiency.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. A control volume encloses the turbine.
2. The turbine is at steady-state.
3. $\dot{Q}_{cv} = 0$ and there are no significant effects of kinetic and potential energy.

ANALYSIS: Reducing a one-inlet, one-exit energy rate balance for the control volume, $\dot{W}_t/\dot{m} = h_1 - h_2$. Thus, at the exit

$$h_2 = h_1 - \left[\frac{\dot{W}_t}{\dot{m}}\right] = \underset{(\text{Table A-4})}{1511.9 \frac{\text{Btu}}{\text{lb}}} - 490 \frac{\text{Btu}}{\text{lb}} = 1021.9 \frac{\text{Btu}}{\text{lb}}$$

Since $h_f < h < h_g$ (h_f, h_g from Table A-3E at 2 lbf/in²), state 2 falls in the two-phase liquid-vapor region, as shown in the schematic.

The isentropic turbine efficiency, Eq. 6.46, takes the form, $\eta = \frac{\dot{W}_t/\dot{m}}{(h_1 - h_{2s})}$

Since $s_{2s} = s_1 = 1.6807$ Btu/lb·°R (Table A-4E), then with s_f, s_g at 2 lbf/in² from Table A-3E

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{(1.6807 - 0.1750)}{1.7448} = 0.863$$

$$\Rightarrow h_{2s} = h_f + x_{2s}(h_g - h_f) = 94.02 + 0.863(1022.1) = 976.1 \text{ Btu/lb}$$

Finally,

$$\eta_t = \frac{490 \text{ Btu/lb}}{(1511.9 - 976.1)\text{Btu/lb}} = 0.915 \quad (91.5\%)$$

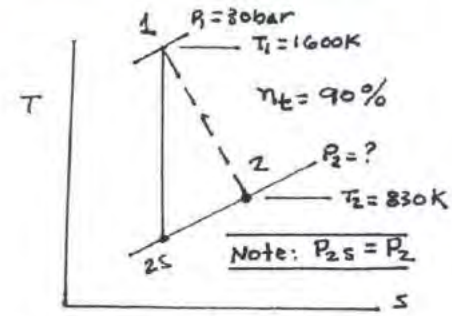
PROBLEM 6.137

Air at 1600 K, 30 bar enters a turbine operating at steady state and expands adiabatically to the exit, where the temperature is 830 K. If the isentropic turbine efficiency is 90%, determine (a) the pressure at the exit, in bar, and (b) the work developed, in kJ per kg of air flowing. Assume ideal gas behavior for the air and ignore kinetic and potential energy effects.

KNOWN: Steady-state and isentropic turbine efficiency data are provided for a turbine.

FIND: Determine the pressure at the turbine exit and $\dot{W}_t/\dot{m}$.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. A control volume at steady state encloses the turbine.
2. Operation is at steady state.
3. $\dot{Q}_{cv} = 0$ and kinetic and potential energy can be ignored.
3. The air is modeled as an ideal gas.

ANALYSIS: A one-inlet, one-exit control volume energy rate balance reduces at steady state to give

$$\frac{\dot{W}_t}{\dot{m}} = h_1 - h_2 = (1757.57 - 855.03)\frac{\text{kJ}}{\text{kg}} = 902.54\frac{\text{kJ}}{\text{kg}} \qquad \text{(b)}$$

where h_1 and h_2 are from Table A-22.

The isentropic turbine efficiency can be used to obtain P_2, as follows:

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_{2s} = h_1 - \frac{(h_1 - h_2)}{\eta_t} = 1757.57 - \frac{(902.54)}{0.9} = 754.75\frac{\text{kJ}}{\text{kg}}$$

Also, with Eq. (6.41)

$$\frac{\overbrace{P_2}^{=P_{2s}}}{P_1} = \frac{P_r(2s)}{P_r(1)} \Rightarrow P_2 = P_1\left[\frac{P_r(2s)}{P_r(1)}\right]$$

Interpolation in Table A-22 with h_{2s}

$$= (30\text{ bar})\left[\frac{35.22}{791.2}\right]$$

$$= 1.34 \text{ bar} \qquad \text{(a)}$$

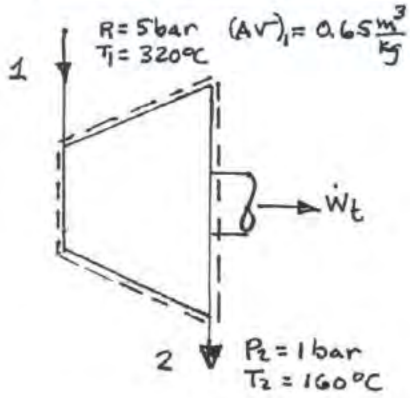
PROBLEM 6.138

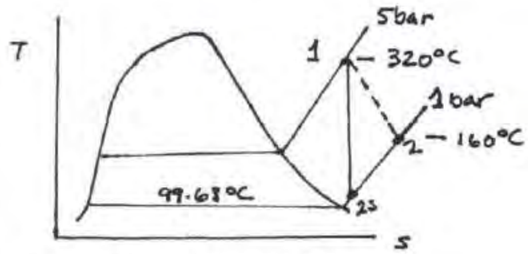
Water vapor at 5 bar, 320°C enters a turbine operating at steady state with a volumetric flow rate of 0.65 m^3/s and expands adiabatically to an exit state of 1 bar, 160°C. Kinetic and potential energy effects are negligible. Determine for the turbine (a) the power developed, in kW, (b) the rate of entropy production, in kW/K, and (c) the isentropic turbine efficiency.

KNOWN: Steady-state operating data are provided for a turbine.

FIND: $\dot{W}_t$, $\dot{\sigma}_t$, and η_t.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL:

1. A control volume encloses the turbine.
2. Operation is at steady state.
3. $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible.

ANALYSIS: (a) A one-inlet, one-exit control volume energy rate equation reduces at steady state to give $\dot{W}_t = \dot{m}(h_1 - h_2)$.

With data from Table A-4

$$\dot{W}_t = \dot{m}(h_1 - h_2) = 1.2\frac{kg}{s}(3105.6 - 2796.2)\frac{kJ}{kg}\left|\frac{1\,kW}{1\,kJ/s}\right| = 371.3\text{ kW}$$

$$\boxed{\dot{m} = \frac{(AV)_1}{v_1} = \frac{0.65\,m^3/kg}{0.5416\,m^3/kg} = 1.2\text{ kg/s}}$$

(b) A one-inlet, one-exit control volume entropy rate equation reduces at steady-state to give $\dot{\sigma}_t = \dot{m}(s_2 - s_1)$

With data from Table A-4

$$\dot{\sigma}_t = (1.2\text{ kg/s})(7.6597 - 7.5308)\frac{kJ}{kg \cdot K}\left|\frac{1\,kW}{1\,kJ/s}\right| = 0.155\ \frac{kW}{K}$$

(c) The isentropic turbine efficiency is $\eta_t = \dfrac{h_1 - h_2}{h_1 - h_{2s}}$.

To obtain h_{2s}, interpolate at 1 bar with $s_{2s} = s_1 = 7.5308$ kJ/kg·K. We get $h_{2s} = 2743$ kJ/kg. Then

$$\eta_t = \frac{3105.6 - 2796.2}{3105.6 - 2743} = 0.853\ (85.3\%)$$

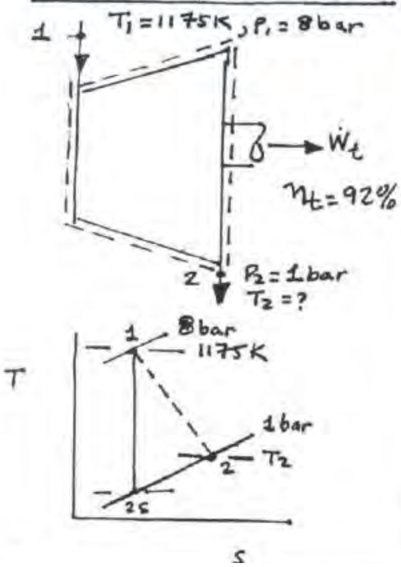
PROBLEM 6.139

Air at 1175 K, 8 bar enters a turbine operating at steady state and expands adiabatically to 1 bar. The isentropic turbine efficiency is 92%. Employing the ideal gas model with $k = 1.4$ for the air, determine (a) the work developed by the turbine, in kJ per kg of air flowing, and (b) the temperature at the exit, in K. Ignore kinetic and potential energy effects.

KNOWN: Steady-state data plus η_t are provided for a turbine.

FIND: Determine $\dot{W}_t/\dot{m}$ and the temperature at the turbine exit.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. A control volume encloses the turbine.
2. operation is at steady state.
3. $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects can be ignored
4. The air is modeled as an ideal gas with $k = 1.4$.

ANALYSIS: (a) A one-inlet, one-exit control volume energy rate balance reduces at steady state to give with Eq. 3.47a

$$\frac{\dot{W}}{\dot{m}} = h_1 - h_2 = c_p[T_1 - T_2] \qquad (1)$$

$$\boxed{c_p = \frac{kR}{(k-1)} = \frac{1.4}{0.4}\left[\frac{8.314}{28.97}\frac{kJ}{kg \cdot K}\right] = 1.004\frac{kJ}{kg \cdot K}}$$

Also,

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{c_p[T_1 - T_2]}{c_p[T_1 - T_{2s}]} = \frac{T_1 - T_2}{T_1 - T_{2s}} \qquad (2)$$

Combining Eqs (1) and (2),

$$\frac{\dot{W}}{\dot{m}} = c_p \eta_t [T_1 - T_{2s}] \qquad (3)$$

Then, with Eq. 6.43,

$$T_{2s} = T_1\left[\frac{P_2}{P_1}\right]^{\frac{k-1}{k}} = (1175K)\left(\frac{1}{8}\right)^{0.4/1.4} = 648.6\text{ K}$$

Inserting values into Eq. (3),

$$\frac{\dot{W}}{\dot{m}} = \left(1.004\frac{kJ}{kg \cdot K}\right)(0.92)[1175 - 648.6]\text{ K} = 486.2\frac{kJ}{kg} \longleftarrow$$

(b) Using Eq (1) and solving for T_2,

$$T_2 = T_1 - \frac{\dot{W}/\dot{m}}{c_p} = 1175K - \frac{486.2\text{ kJ/kg}}{1.004\text{ kJ/kg}\cdot\text{K}} = 690.7K \longleftarrow$$

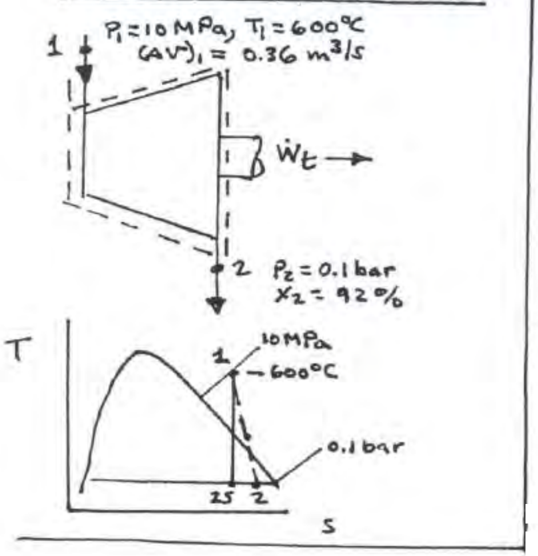
PROBLEM 6.140

Water vapor at 10 MPa, 600°C enters a turbine operating at steady state with a volumetric flow rate of 0.36 m^3/s and exits at 0.1 bar and a quality of 92%. Stray heat transfer and kinetic and potential energy effects are negligible. Determine for the turbine (a) the mass flow rate, in kg/s, (b) the power developed by the turbine, in MW, (c) the rate at which entropy is produced, in kW/K, and (d) the isentropic turbine efficiency.

KNOWN: Steady-state operating data are provided for a turbine.

FIND: Determine $\dot{m}$, $\dot{W}_t$, $\dot{\sigma}$, and η_t.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. A control volume encloses the turbine.
2. Operation is at steady state.
3. $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible

ANALYSIS: (a) with v_1 from Table A-4

$$\dot{m} = \frac{0.36\,m^3/s}{0.03837\,m^3/kg} = 9.38\,kg/s$$

(b) A one-inlet, one-exit control volume energy rate equation reduces at steady state to give

$$\dot{W}_t = \dot{m}(h_1 - h_2)$$

From Table A-4, $h_1 = 3625.3$ kJ/kg. Then with h_f, h_g at 0.1 bar from Table A-3

$$h_2 = 191.83 + 0.92(2392.87) = 2393.3\ kJ/kg$$

Finally, $\dot{W}_t = 9.38\,kg/s\,(3625.3 - 2393.3)\frac{kJ}{kg}\left|\frac{1\,MW}{10^3\,kJ/s}\right|$

$$= 11.56\,MW$$

(c) A one-inlet, one-exit control volume entropy rate equation reduces at steady state to give

$$\dot{\sigma}_t = \dot{m}(s_2 - s_1)$$

Table A-4 gives $s_1 = 6.9029$ kJ/kg·K. Then, with s_f and s_g at 0.1 bar from Table A-3, $s_2 = 0.6493 + 0.92(8.1502 - 0.6493) = 7.5501$ kJ/kg·K. Finally,

$$\dot{\sigma}_t = \left(9.38\,\frac{kg}{s}\right)(7.5501 - 6.9029)\frac{kJ}{kg \cdot K}\left|\frac{1\,kW}{1\,kJ/s}\right| = 6.07\,\frac{kW}{K}$$

(d) The isentropic turbine efficiency is $\eta = \dfrac{h_1 - h_2}{h_1 - h_{2s}}$

To obtain h_{2s}, use $s_{2s} = s_1$ and s_f, s_g data from Table A-3:

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{6.9029 - 0.6493}{8.1502 - 0.6493} = 0.834$$

$$\therefore \quad h_{2s} = h_f + x_{2s}(h_g - h_f) = 191.83 + 0.834(2392.87) = 2187.5\ kJ/kg$$

Finally,

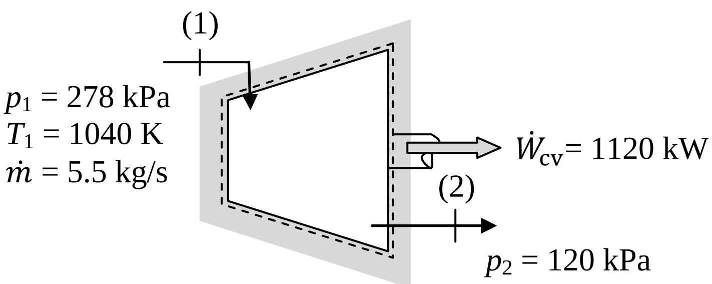
$$\eta = \frac{(3625.3 - 2393.3)}{(3625.3 - 2187.5)} = 0.857\ (85.7\%)$$

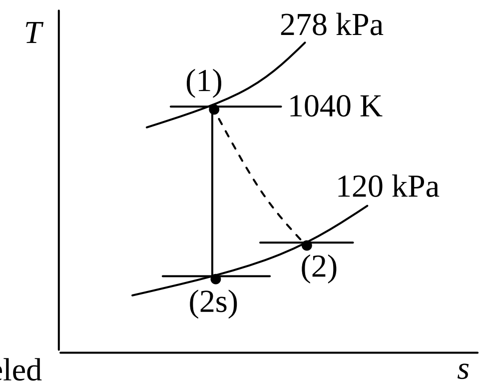
PROBLEM 6.141

Air modeled as an ideal gas enters a turbine operating at steady state at 1040 K, 278 kPa and exits at 120 kPa. The mass flow rate is 5.5 kg/s, and the power developed is 1120 kW. Stray heat transfer and kinetic and potential energy effects are negligible. Assuming $k = 1.4$, determine (a) the temperature of the air at the turbine exit, in K, and (b) the isentropic turbine efficiency.

KNOWN: Air expands adiabatically through a turbine operating at steady state. Operating data are known.

FIND: Determine the exit temperature and the isentropic turbine efficiency.





ENGINEERING MODEL: (1) The control volume is at state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects can be neglected. (3) The air is modeled as an ideal gas with constant specific heats: $k = 1.4$.

ANALYSIS: (a) Mass and energy rate balances reduce to give: $0 = -\dot{W}_{cv} + \dot{m}(h_1 - h_2)$. With $h_1 - h_2 = c_p(T_1 - T_2)$

$$T_2 = T_1 - \dot{W}_{cv}/\dot{m}c_p$$

From Sec. 3.13.1; $c_p = kR/(k-1) = (1.4)(8.314/28.97)/(1.4 - 1) = 1.004$ kJ/kg·K and

$$T_2 = T_1 - \dot{W}_{cv}/(\dot{m}c_p) = 1040\text{ K} - (1120\text{ kW})/[(5.5\text{ kg/s})(1.004\text{ kJ/kg·K})\left|\frac{1\text{ kJ/s}}{1\text{ kW}}\right|$$
$$= 837.2\text{ K}$$

(b) The isentropic efficiency is $\eta_t = (h_1 - h_2)/(h_1 - h_{2s}) = \cancel{c_p}\,(T_1 - T_2)/\,\cancel{c_p}\,(T_1 - T_{2s})$. To get T_{2s} we note that for an isentropic process of an ideal gas with constant specific heats

$$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k} \longrightarrow T_{2s} = \left(\frac{120}{278}\right)^{\frac{1.4-1}{1.4}}(1040\text{ K}) = 818.1\text{ K}$$

Thus, the isentropic efficiency is

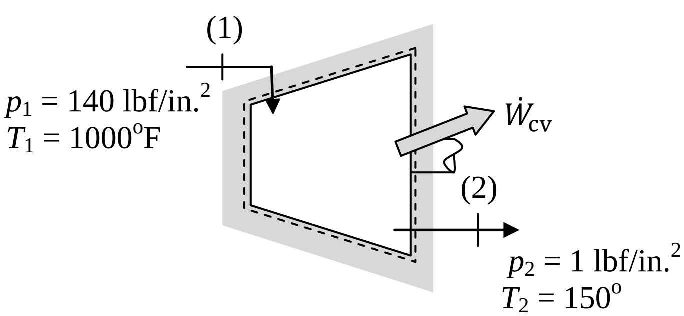
$$\eta_t = (1040 - 837.2)/(1040 - 818.1) = 0.914\ (91.4\%)$$

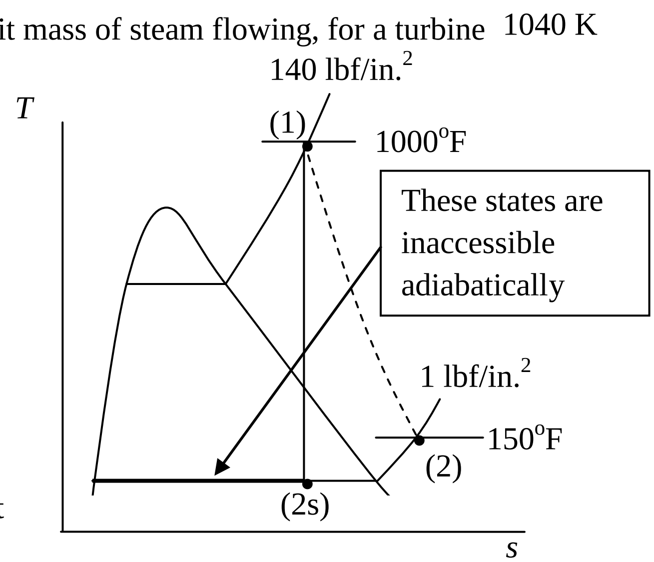
PROBLEM 6.142

Water vapor at 1000°F, 140 lbf/in.^{2} enters a turbine operating at steady state and expands to 2 lbf/in.^{2}, 150°F. Stray heat transfer and kinetic and potential energy effects are negligible. Determine the actual work and the maximum theoretical work that could be developed for a turbine with the same inlet state and exit pressure, each in Btu per lb of water vapor flowing.

KNOWN: Water vapor expands adiabatically through a turbine operating at steady state. Operating data are known.

FIND: Determine the maximum theoretical work, per unit mass of steam flowing, for a turbine with the same inlet state and exit pressure.





ENGINEERING MODEL: (1) The control volume is at state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects can be neglected. (3) For the maximum work, the expansion occurs without internal irreversibilities.

ANALYSIS: The mass and energy rate balances reduce to give

$$\dot{W}_{cv}/\dot{m} = h_1 - h_2$$

From Table A-4E: h_1 = 1531.0 Btu/lb and h_2 = 1127.5 Btu/lb. Thus

$$\dot{W}_{cv}/\dot{m} = (1531.0 - 1127.5) = 403.5 \text{ Btu/lb}$$

To determine the maximum theoretical work, we use the entropy balance, which reduces as follows for the one-inlet, one-exit control volume at steady state: $0 = \cancel{\sum_j \left(\frac{\dot{Q}_j}{T_j}\right)} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$

Thus

$$(s_2 - s_1) = \dot{\sigma}_{cv}/\dot{m} \geq 0 \text{ and } \boxed{s_2 \geq s_1}$$

The states to the left of point 2s are therefore inaccessible adiabatically. For the maximum work; $\left(\dot{W}_{cv}/\dot{m}\right)_s = h_1 - h_{2s}$, where h_{2s} is the entropy for isentropic expansion to state 2s (constant entropy). Using data from Table A-3e, with $s_{2s} = s_1$ = 1.8827 Btu/lb·°R

PROBLEM 6.142 (CONTINUED)

$$x_{2s} = (s_{2s} - s_{f2})/(s_{g2} - s_{f2}) = (1.8827 - 0.1327)/(1.9779 - 0.1327) = 0.9484$$

and

$$h_{2s} = h_{f2} + x_{2s}(h_{g2} - h_{f2}) = (69.74 + (0.9484)(1105.8 - 69.74) = 1052.3 \text{ Btu/lb}$$

Finally

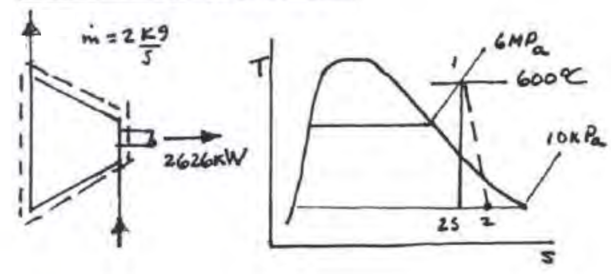
$$\left(\dot{W}_{cv}/\dot{m}\right)_s = (1531.0 - 1052.3) = 478.7 \text{ Btu/lb.}$$

Note: The isentropic efficiency is $\eta_t = \left(\dot{W}_{cv}/\dot{m}\right)/\left(\dot{W}_{cv}/\dot{m}\right)_s$ = 403.5/478.7 = 0.843 (84.3%)

PROBLEM 6.143

Water vapor at 6 MPa, 600°C enters a turbine operating at steady state and expands to 10 kPa. The mass flow rate is 2 kg/s, and the power developed is 2626 kW. Stray heat transfer and kinetic and potential energy effects are negligible. Determine (a) the isentropic turbine efficiency and (b) the rate of entropy production within the turbine, in kW/K.

SCHEMATIC & GIVEN DATA



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, stray heat transfer and kinetic and potential energy effects are negligible.

Analysis: (a) To fix the exit state requires two independent intensive properties. One is the pressure. The other is h_2 found from applying the mass and energy rate balances:

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)} + \cancel{g(z_1 - z_2)}\right] \Rightarrow h_2 = h_1 - (\dot{W}_{cv}/\dot{m})$$

Table A-4, $h_1 = 3658.4$ kJ/kg, $s_1 = 7.1677$ kJ/kg·K

$$\therefore \quad h_2 = 3658.4 - \left[\frac{2626 \text{ kJ/s}}{2 \text{ kg/s}}\right] = 2345.4 \text{ kJ/kg}.$$ Since $h_f < h_2 < h_g$, state 2 is a 2-phase liquid-vapor mixture:

$$\therefore \quad x_2 = \frac{h_2 - h_f}{h_g - h_f} = \frac{2345.4 - 191.83}{2392.8} = \underline{0.9}$$ (data from Table A-3) @ 0.1 bar

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}}.$$ To find h_{2s}, use $s_{2s} = s_1 \Rightarrow$ $$x_{2s} = \frac{\overset{(=s_1)}{s_{2s}} - s_f}{s_g - s_f} = \frac{7.1677 - 0.6493}{8.1502 - 0.6493} = \underline{0.869}$$

$$\therefore \quad h_{2s} = h_f + x_{2s}(h_g - h_f) = 191.83 + 0.869\,(2392.8) = 2271.2 \frac{\text{kJ}}{\text{kg}}$$

$$\therefore \quad \eta_t = \frac{3658.4 - 2345.4}{3658.4 - 2271.2} = \frac{1313}{1387.2} = \underline{0.947} \quad (94.7\%)$$ ← (a)

(b) An entropy rate balance reduces to, $$0 = \sum \cancel{\dot{Q}_j}^{\,0}/T_j + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

$$\Rightarrow \dot{\sigma}_{cv} = \dot{m}(s_1 - s_2),$$ where $$s_2 = s_f + x_2(s_g - s_f) = 0.6493 + 0.9(7.5009) = 7.4 \frac{\text{kJ}}{\text{kg·K}}.$$

So,

$$\dot{\sigma}_{cv} = \left(2\,\frac{\text{kg}}{\text{s}}\right)(7.4 - 7.1677)\frac{\text{kJ}}{\text{kg·K}}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right| = 0.465 \frac{\text{kW}}{\text{K}}$$ ← (b)

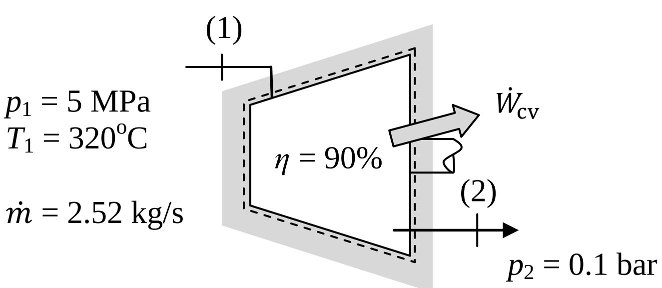
PROBLEM 6.144

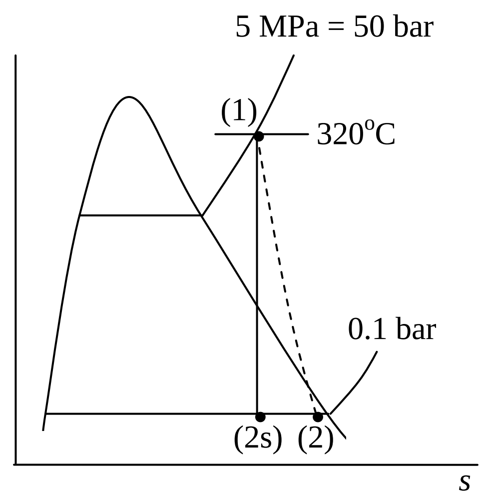
Water vapor at 5 MPa, 320°C enters a turbine operating at steady state and expands to 0.1bar. The mass flow rate is 2.52 kg/s, and the isentropic turbine efficiency is 92%. Stray heat transfer and kinetic and potential energy effects are negligible. Determine the power developed by the turbine, in kW.

KNOWN: Data are given for a turbine operating adiabatically at steady state. The mass flow rate and the isentropic turbine efficiency are specified.

FIND: Determine the power developed by the turbine.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL: (1) The control volume is at state. (2) For the control volume, $\dot{Q}_{cv}$ = 0 and kinetic and potential energy effects can be neglected.

ANALYSIS: Reducing the mass and energy rate balances; $\dot{W}_{cv} = \dot{m}(h_1 - h_2)$. Also, the isentropic turbine efficiency is $\eta_t = (h_1 - h_2)/(h_1 - h_{2s})$. Combining these expressions

$$\dot{W}_{cv} = \dot{m}\eta_t(h_1 - h_2) \qquad (*)$$

Interpolating in Table A-4 at 50 bar, 320°C; $h_1 \approx 2984$ kJ/kg and $s_1 \approx 6.320$ kJ/kg·K

With data from Table A-3

$$x_{2s} = (s_{2s} - s_{f2})/(s_{g2} - s_{f2}) = (6.320 - 0.6493)/(8.1502 - 0.6493) = 0.756$$

Thus

$$h_{2s} = h_{f2} + x_{2s}(h_{g2} - h_{f2}) = 191.83 + (0.756)(2584.7 - 191.83) = 2000.8 \text{ kJ/kg}$$

Substituting values into Eq. (*)

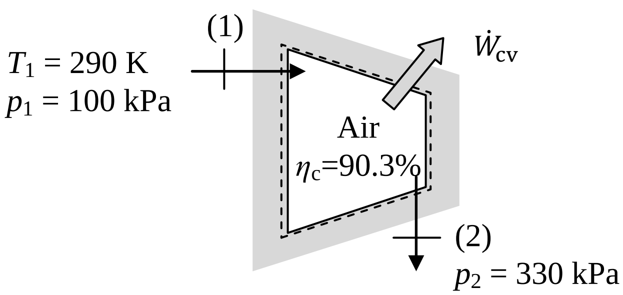
$$\dot{W}_{cv} = (2.52 \text{ kg/s})(0.92)(2984 - 2000.8)\text{kJ/kg}\left|\frac{1 \text{ kW}}{1 \text{ kj/s}}\right| = 2279 \text{ kW} \longleftarrow$$

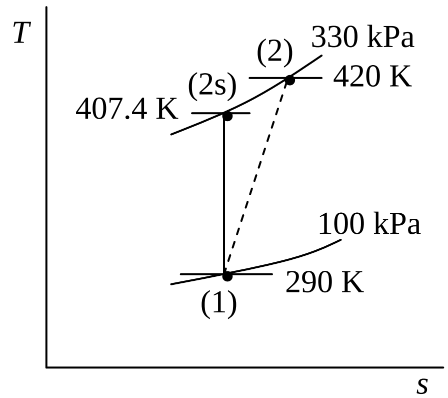
PROBLEM 6.145

Air enters the compressor of a gas turbine power plant operating at steady state at 290 K, 100 kPa and exits at 330 kPa. Stray heat transfer and kinetic and potential energy effects are negligible. The isentropic compressor efficiency is 90.3%. Using the ideal gas model for air, determine the work input, in kJ per kg of air flowing.

KNOWN: Air is compressed adiabatically. The state is known at the inlet and the exit pressure is specified. The isentropic compressor efficiency is known.

FIND: Determine the work per unit mass of air flowing.





ENGINEERING MODEL: (1) The control volume is at steady state. (2) $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible. (3) The air is modeled as an ideal gas.

ANAYSIS: The mass and energy rate balances reduce to give $\dot{W}_{cv}/\dot{m} = (h_1 - h_2)$. With the isentropic compressor efficiency: $\eta_c = (h_1 - h_{2s})/(h_1 - h_2)$

$$\dot{W}_{cv}/\dot{m} = (h_1 - h_{2s})/\eta_c$$

To find h_{2s}, use Eq. 6.41 and data from Table A-22: $p_{r2} = p_{r1}(p2/p_1) = 1.2311\ (330/100) = 4.0626$
Interpolating in Table A-22; $h_{2s} \approx 408.5$ kJ/kg. Also, at 290 K, $h_1 = 290.16$ kJ/kg. Thus

$$\dot{W}_{cv}/\dot{m} = (290.16 - 408.5)/(0.903) = -131.05 \text{ kJ/kg (in)} \longleftarrow$$

Note: As indicated on the *T*-*s* diagram…

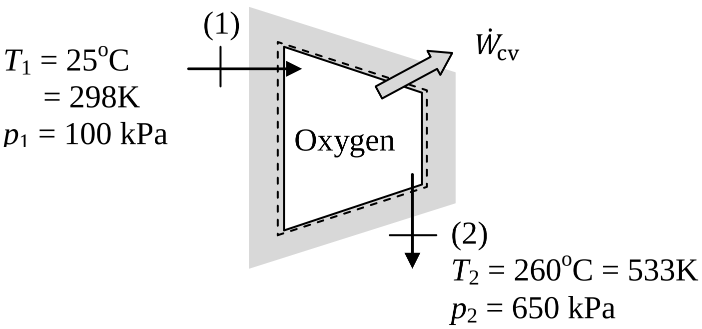
$$T_{2s} \approx 407.4 \text{ K and } h_2 = 290.16 + 131.05 = 421.2 \text{ kJ/kg} \quad \longrightarrow \quad T_2 \approx 420 \text{ K}$$

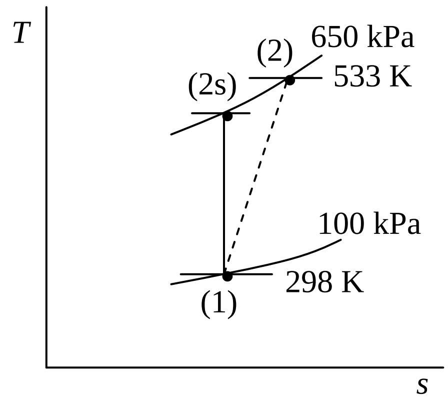
PROBLEM 6.146

Oxygen (O_2) at 25°C, 100 kPa enters a compressor operating at steady state and exits at 260°C, 650 kPa. Stray heat transfer and kinetic and potential energy effects are negligible. Modeling the oxygen as an ideal gas with k = 1.379, determine the isentropic compressor efficiency and the work in kJ per kg of oxygen flowing.

KNOWN: Oxygen is compressed adiabatically in a compressor operating at steady state. The inlet and exit states are specified.

FIND: Determine the isentropic compressor efficiency and the work per unit mass of oxygen flowing.





ENGINEERING MODEL: (1) The control volume is at steady state. (2) $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible. (3) The oxygen is modeled as an ideal gas, with k = 1.379.

ANALYSIS: By Eq. 3.47a, $c_p = kR/(k-1)$. So, if k is constant, c_p is also constant. For the given value of k

$$c_p = [(1.379)(8.314/32.00)]/(1.379 - 1) = 0.9453 \text{ kJ/kg·K}$$

Accordingly, the isentropic compressor efficiency is

$$\eta_c = (h_{2s} - h_1)/(h_2 - h1) = [\cancel{c_p}(T_{2s} - T_1)]/[\cancel{c_p}(T_2 - T_1)] = (T_{2s} - T_1)/(T_2 - T_1) \qquad (*)$$

With Eq. 6.47

$$T_{2s} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = (298 \text{ K})\left(\frac{650}{100}\right)^{\frac{1.379-1}{1.379}} = 498.5 \text{ K}$$

Thus, from (*)

$$\eta_c = (498.5 - 298)/(533 - 298) = 0.8532 \ (85.32\ \%) \longleftarrow$$

The work per unit mass of oxygen flowing is

$$\dot{W}_{cv}/\dot{m} = (h_1 - h_2) = c_p(T_1 - T_2) = (0.9453 \text{ kJ/kg·K})(298 - 533)\text{K} = -222.1 \text{ kJ/kg (in)} \longleftarrow$$

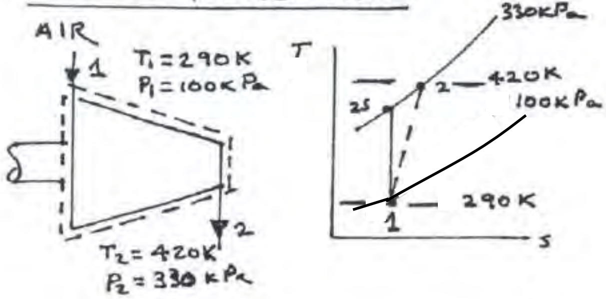
PROBLEM 6.147

Air at 290 K, 100 kPa enters a compressor operating at steady state and is compressed adiabatically to an exit state of 420 K, 330 kPa. The air is modeled as an ideal gas, and kinetic and potential energy effects are negligible. For the compressor, (a) determine the rate of entropy production, in kJ/K per kg of air flowing, and (b) the isentropic compressor efficiency.

KNOWN: Steady-state data are provided for an air compressor.

FIND: Determine $\dot{\sigma}/\dot{m}$ and η_c.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. A control volume encloses the air compressor.
2. Operation is at steady state.
3. $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible.
4. The air is modeled as an ideal gas.

ANALYSIS: (a) A one-inlet one-exit control volume entropy rate balance reduces to give $\dot{\sigma}_{cv} = \dot{m}(s_2 - s_1)$. Then with Eq. 6.20a

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = s^\circ(T_2) - s^\circ(T_1) - R \ln P_2/P_1$$

with data from Table A-22

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = (2.04142 - 1.66802) \frac{kJ}{kg \cdot K} - \left(\frac{8.314}{28.97} \frac{kJ}{kg \cdot K}\right) \ln\left(\frac{330 \, kPa}{100 \, kPa}\right)$$

$$= 0.0308 \frac{kJ}{kg \cdot K}$$

(b) The isentropic compressor efficiency is given by Eq. 6.48,

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1}$$

Using data from Table A-22, $h_1 = 290.16$ kJ/kg, $h_2 = 421.26$ kJ/kg

$$p_r(2s) = p_r(1)\left(\frac{\overbrace{P_{2s}}^{=P_2}}{P_1}\right) = 1.2311\left(\frac{330 \, kPa}{100 \, kPa}\right) = 4.0626$$

Interpolating in Table A-22 with $p_r(2s)$, $h_{2s} = 408.48$ kJ/kg.

$$\therefore \eta_c = \frac{408.48 - 290.16}{421.26 - 290.16} = 0.903 \quad (90.3\%)$$

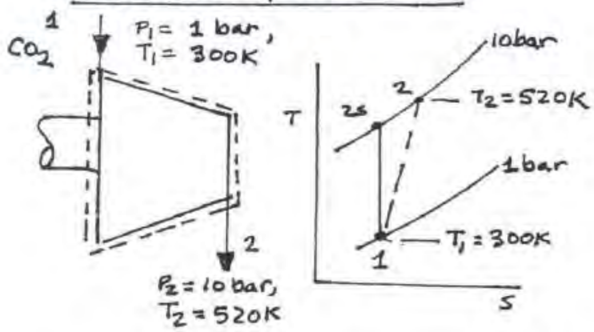
PROBLEM 6.148

Carbon dioxide (CO_2) at 1 bar, 300 K enters a compressor operating at steady state and is compressed adiabatically to an exit state of 10 bar, 520 K. The CO_2 is modeled as an ideal gas, and kinetic and potential energy effects are negligible. For the compressor, determine (a) the work input, in kJ per kg of CO_2 flowing, (b) the rate of entropy production, in kJ/K per kg of CO_2 flowing, and (c) the isentropic compressor efficiency.

KNOWN: Steady-state data are provided for a compressor for which CO_2 is the gas.

FIND: Determine $\dot{W}_c/\dot{m}$, $\dot{\sigma}_c/\dot{m}$, and η_c.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. A control volume encloses the compressor.
2. Operation is at steady state.
3. $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible.
4. The CO_2 is modeled as an ideal gas

ANALYSIS: (a) An energy rate balance at steady state reduces to give

$$\underset{\text{(work input)}}{\left(-\frac{\dot{W}_{cv}}{\dot{m}}\right)} = h_2 - h_1 = \frac{\bar{h}_2 - \bar{h}_1}{M} \overset{\text{Data from Table A-23}}{=} \frac{(18{,}576 - 9431)\text{ kJ/kmol}}{44.01\text{ kg/kmol}} = 207.8\ \frac{\text{kJ}}{\text{kg}}$$

(b) An entropy rate balance at steady state reduces to give

$$\left(\dot{\sigma}_{cv}/\dot{m}\right) = s_2 - s_1 \underset{\text{Eq. 6.20 b}}{=} \frac{\bar{s}^\circ(T_2) - \bar{s}^\circ(T_1) - \bar{R}\ln(P_2/P_1)}{M}$$

With data from Table A-23

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = \frac{(236.575 - 213.915 - 8.314\ln(10))\text{ kJ/kmol·K}}{44.01\text{ kg/kmol}} = 0.08\ \frac{\text{kJ}}{\text{kg·K}}$$

(c) The isentropic compressor efficiency, Eq. 6.48, can be expressed as

$$\eta_c = \frac{\bar{h}_{2s} - \bar{h}_1}{\bar{h}_2 - \bar{h}_1}$$

To find $\bar{h}_{2s}$, apply Eq. 6.20b for the process 1–2s:

$$0 = \bar{s}^\circ_{2s} - \bar{s}^\circ_1 - \bar{R}\ln\frac{P_2}{P_1} \Rightarrow \bar{s}^\circ_{2s} = \bar{s}^\circ_1 + \bar{R}\ln(P_2/P_1)$$

$$= 213.915 + 8.314\ln(10) = 233.059$$

Interpolating in Table A-23, $\bar{h}_{2s} = 16{,}818$ kJ/kmol.

Finally, with $\bar{h}_1$ and $\bar{h}_2$ from Table A-23

$$\eta_c = \frac{16{,}818 - 9431}{18{,}576 - 9431} = 0.808 \quad (80.8\%)$$

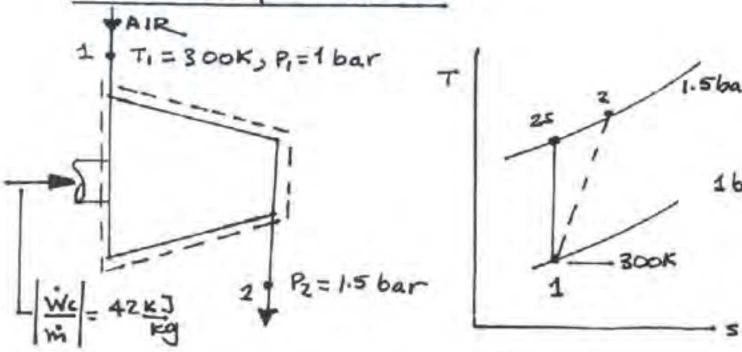
PROBLEM 6.149

Air at 300 K, 1 bar enters a compressor operating at steady state and is compressed adiabatically to 1.5 bar. The power input is 42 kJ per kg of air flowing. Employing the ideal gas model with $k = 1.4$ for the air, determine for the compressor (a) the rate of entropy production, in kJ/K per kg of air flowing, and (b) the isentropic compressor efficiency. Ignore kinetic and potential energy effects.

KNOWN: Data are provided for an air compressor at steady state.

FIND: $\dot{\sigma}_c/\dot{m}$, η_c

SCHEMATIC & GIVEN DATA



ENGINEERING MODEL:

1. A control volume encloses the compressor.
2. Operation is at steady state.
3. $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible.
4. The air is modeled as an ideal gas with $k = 1.4$

ANALYSIS: (a) Reducing energy and entropy rate balances for the one-inlet, one-exit control volume at steady state, we get with assumption 4,

(work input): $\left|\frac{\dot{W}_c}{\dot{m}}\right| = h_2 - h_1 = c_p[T_2 - T_1]$ (1) and $\frac{\dot{\sigma}_c}{\dot{m}} = s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$ (2)

With Eq. 3.47a, $c_p = \frac{kR}{k-1} = \frac{(1.4)(8.314/28.97 \text{ kJ/kg}\cdot\text{K})}{(0.4)} = 1.004 \frac{kJ}{kg\cdot K}$

Then, solving Eq. (1) for T_2,

$$T_2 = T_1 + \frac{|\dot{W}_c/\dot{m}|}{c_p} = 300K + \frac{42 \text{ kJ/kg}}{1.004 \text{ kJ/kg}\cdot\text{K}} = 341.8K$$

Then, Eq. (2) gives

$$\frac{\dot{\sigma}_c}{\dot{m}} = \left(1.004 \frac{kJ}{kg\cdot K}\right)\ln\left(\frac{341.8}{300}\right) - \left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)\ln\left(\frac{1.5}{1}\right) = 0.0146 \frac{kJ}{kg\cdot K}$$

(b) The compressor efficiency is

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{c_p(T_{2s} - T_1)}{c_p(T_2 - T_1)} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

With Eq. 6.43,

$$T_{2s} = T_1\left[\frac{\overbrace{P_{2s}}^{=P_2}}{P_1}\right]^{(k-1/k)} = 300K\,[1.5]^{(0.4/1.4)} = 336.8K$$

$$\therefore \eta_c = \frac{336.8 - 300}{341.8 - 300} = 0.88\ (88\%)$$

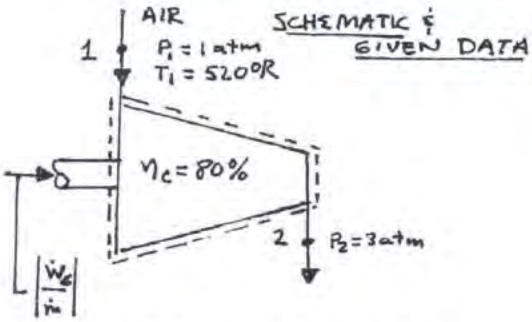
PROBLEM 6.150

Air at 1 atm, 520°R enters a compressor operating at steady state and is compressed adiabatically to 3 atm. The isentropic compressor efficiency is 80%. Employing the ideal gas model with $k = 1.4$ for the air, determine for the compressor (a) the power input, in Btu per lb of air flowing, and (b) the amount of entropy produced, in Btu/°R per lb of air flowing. Ignore kinetic and potential energy effects.

KNOWN: Steady-state data are provided for an air compressor.

FIND: $|\dot{W}_c/\dot{m}|$, $\dot{\sigma}/\dot{m}$

SCHEMATIC & GIVEN DATA



ENGINEERING MODEL:

1. A control volume encloses the air compressor.
2. Operation is at steady state.
3. $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are ignored.
4. The air is modeled as an ideal gas with $k = 1.4$.

ANALYSIS: (a) Reducing an energy rate balance for the one-inlet one-exit control volume at steady state

$$\left|\frac{\dot{W}_c}{\dot{m}}\right| = h_2 - h_1 = c_p\,[T_2 - T_1] \quad \text{(Assumption 4)}$$

We also have,

$$\eta_c = \frac{h_{2s}-h_1}{h_2-h_1} = \frac{c_p\,[T_{2s}-T_1]}{c_p\,[T_2-T_1]} = \frac{T_{2s}-T_1}{T_2-T_1}$$

$$\left|\frac{\dot{W}_c}{\dot{m}}\right| = c_p\frac{(T_{2s}-1)}{\eta_c} \qquad (1)$$

With Eq. 3.47a, $c_p = \frac{kR}{k-1} = \frac{(1.4)}{(0.4)}\left(\frac{1.986}{28.97}\ \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right) = 0.24\ \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$

with Eq. 6.43

$$T_{2s} = T_1\left[\frac{\overset{(=P_2)}{P_{2s}}}{P_1}\right]^{\frac{k-1}{k}} = 520°\text{R}\left(\frac{3\ \text{atm}}{1\ \text{atm}}\right)^{0.4/1.4} = 711.7\ °\text{R}$$

With these values, Eq. (1) gives

$$\left|\frac{\dot{W}_c}{\dot{m}}\right| = \frac{(0.24\ \text{Btu/lb}°\text{R})(711.7-520)°\text{R}}{0.80} = 57.5\ \frac{\text{Btu}}{\text{lb}}$$

(b) Reducing an entropy rate balance for the control volume and using Eq. 6.22

$$\frac{\dot{\sigma}}{\dot{m}} = s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} \qquad (2)$$

To find T_2, begin with $\left|\frac{\dot{W}_c}{\dot{m}}\right| = c_p(T_2 - T_1)$. Solving $T_2 = \frac{|\dot{W}_c/\dot{m}|}{c_p} + T_1$

Then,

$$T_2 = \frac{57.5\ \text{Btu/lb}}{0.24\ \text{Btu/lb}\cdot°\text{R}} + 520°\text{R} = 759.6\ °\text{R}$$

With known values, Eq. (2) gives

$$\frac{\dot{\sigma}}{\dot{m}} = \left(0.24\ \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)\ln\left(\frac{759.6}{520}\right) - \left(\frac{1.986}{28.97}\ \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)\ln(3)$$

$$= 0.0156\ \text{Btu/lb}\cdot°\text{R}$$

PROBLEM 6.151

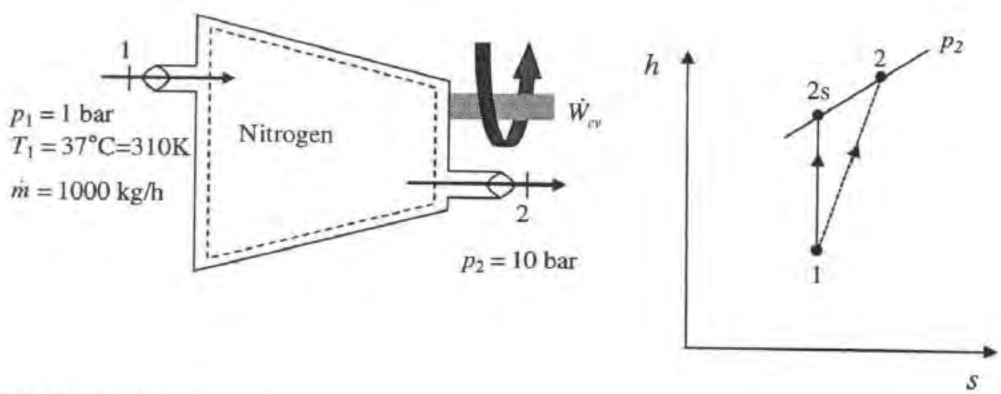
Nitrogen enters an insulated compressor operating at steady state at 1 bar, 37°C with a mass flow rate of 1000 kg/h and exits at 10 bar. Kinetic and potential energy effects are negligible. The nitrogen can be modeled as an ideal gas with k=1.391.

(a) Determine the minimum theoretical power input required, in kW, and the corresponding exit temperature, in °C.
(b) If the exit temperature is 397°C, determine the power input, in kW, and the isentropic compressor efficiency.

KNOWN: Nitrogen enters and exits an insulated compressor operating at steady state with known inlet and exit conditions. Kinetic and potential energy effects are negligible and nitrogen is modeled as an ideal gas with constant k.

FIND: Determine the minimum theoretical power input required, in kW, and the corresponding exit temperature, in °C. If the exit temperature is 397°C, determine the power input, in kW, and the isentropic compressor efficiency.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

(1) The compressor operates at steady state.
(2) No heat transfer occurs at the outer surface of the compressor.
(3) There are negligible kinetic and potential energy effects.
(4) Nitrogen is modeled as an ideal gas with k=1.391.

ANALYSIS:

(a) Evaluate the power required using a steady-state energy rate balance and conservation of mass simplified based on assumptions.

$\dot{m}_1 = \dot{m}_2 = \dot{m}$

$\dot{W}_{cv} = \dot{m}(h_1 - h_2)$

Assuming constant specific heat, use Eq. 3.51 for $(h_1 - h_2)$, as follows:

$$\dot{W}_{cv} = \dot{m}c_p(T_1 - T_2) \qquad (1)$$

PROBLEM 6.151 (CONTINUED)

The minimum theoretical power input is associated with an isentropic compressor where Eq. (1) becomes:

$$\left(\dot{W}_{cv}\right)_s = \dot{m}c_p\left(T_1 - T_{2s}\right) \qquad (2)$$

Use an isentropic relationship for an ideal gas with constant k, Eq. 6.43, to determine T_{2s}.

$$T_{2s} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = 310\,\mathrm{K}\left(\frac{10}{1}\right)^{\frac{1.391-1}{1.391}} = 592.2\,\mathrm{K} = 319.2^\circ\mathrm{C}$$

The exit temperature corresponding with a minimum power input is 319.2°C. ←

Substitute calculated and known values into Eq. (2) to determine the minimum theoretical power input required, in kW. Using Eq. 3.47a and k=1.391, c_p =1.056 kJ/kg·K.

$$\left(\dot{W}_{cv}\right)_s = 1000\frac{\mathrm{kg}}{\mathrm{h}}\left(1.056\frac{\mathrm{kJ}}{\mathrm{kg}\cdot\mathrm{K}}\right)(310-592.2)\,\mathrm{K}\left|\frac{1\,\mathrm{h}}{3600\,\mathrm{s}}\right|\left|\frac{1\,\mathrm{kW}}{1\frac{\mathrm{kJ}}{\mathrm{s}}}\right| = -82.78\,\mathrm{kW} \qquad \longleftarrow$$

(b) Assuming the actual exit temperature is 397° C = 670 K, determine the power input, in kW, using Eq. (1).

$$\dot{W}_{cv} = 1000\frac{\mathrm{kg}}{\mathrm{h}}\left(1.056\frac{\mathrm{kJ}}{\mathrm{kg}\cdot\mathrm{K}}\right)(310-670)\,\mathrm{K}\left|\frac{1\,\mathrm{h}}{3600\,\mathrm{s}}\right|\left|\frac{1\,\mathrm{kW}}{1\frac{\mathrm{kJ}}{\mathrm{s}}}\right| = -105.6\,\mathrm{kW} \qquad \longleftarrow$$

Using Eq. 6.48, the isentropic efficiency for the compressor is as follows:

$$\eta_c = \frac{\left(\dot{W}_{cv}\right)_s}{\dot{W}_{cv}} = \frac{-82.78\,\mathrm{kW}}{-105.6\,\mathrm{kW}} = 78.4\% \qquad \longleftarrow$$

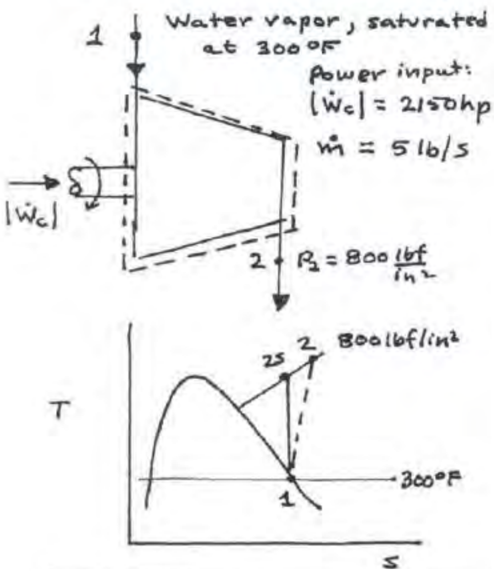
PROBLEM 6.152

Saturated water vapor at 300°F enters a compressor operating at steady state with a mass flow rate of 5 lb/s and is compressed adiabatically to 800 lbf/in.[2] If the power input is 2150 hp, determine for the compressor (a) the isentropic compressor efficiency and (b) the rate of entropy production, in hp/°R. Ignore kinetic and potential energy effects.

KNOWN: Steady-state data are provided for a compressor

FIND: η_c, $\dot{\sigma}$.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. A control volume encloses the compressor.
2. Operation is at steady state.
3. $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects can be ignored.

ANALYSIS: Reducing energy and entropy rate balances for the control volume, we get

$$|\dot{W}_c| = \dot{m}(h_2 - h_1) \quad (1), \qquad \dot{\sigma} = \dot{m}(s_2 - s_1) \quad (2)$$

From Table A-2E at 300°F, $h_1 = 1180.2$ Btu/lb, $s_1 = 1.6356$ Btu/lb·°R. Then, since $s_{2s} = s_1$, interpolation in Table A-4E gives, $h_{2s} = 1448.8 \frac{Btu}{lb}$

(a) The isentropic compressor efficiency, Eq. 6.28, takes the form

$$\eta_c = \frac{\dot{m}(h_{2s} - h_1)}{|\dot{W}_c|} = \frac{(5\,lb/s)(1448.8 - 1180.2)\frac{Btu}{lb}\left|\frac{3600\,s}{h}\right|}{(2150\,hp)\left|\frac{2545\,Btu/h}{hp}\right|} = 0.884\ (88.4\%)$$

(b) Using Eq.(1),

$$h_2 = h_1 + \frac{|\dot{W}_c|}{\dot{m}} = 1180.2\frac{Btu}{lb} + \frac{(2150\,hp)}{(5\,lb/s)}\left|\frac{2545\,Btu/h}{hp}\right|\left|\frac{1\,h}{3600\,s}\right|$$

$$= 1484.2\ \frac{Btu}{lb}$$

Interpolating in Table A-4E at 800 lbf/in² with h_2, we get $s_2 = 1.6611 \frac{Btu}{lb\cdot °R}$

Then, Eq.(2) gives

$$\dot{\sigma} = \left(5\frac{lb}{s}\right)(1.6611 - 1.6356)\ \frac{Btu}{lb\cdot °R}\left|\frac{3600\,s}{1\,h}\right|\left|\frac{1\,hp}{2545\,Btu/h}\right|$$

$$= 0.1804\ \frac{hp}{°R}$$

PROBLEM 6.153

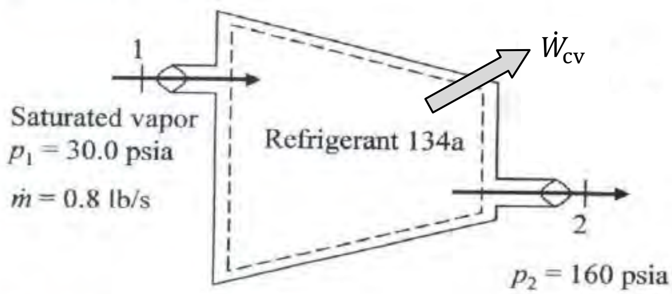
Refrigerant 134a at a rate of 0.8 lb/s enters a compressor operating at steady state as saturated vapor at 30 psia and exits at a pressure of 160 psia. There is no significant heat transfer with the surroundings, and kinetic and potential energy effects can be ignored.

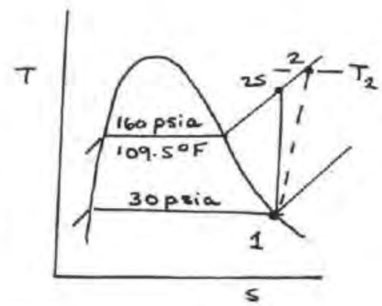
(a) Determine the minimum theoretical power input required, in Btu/s, and the corresponding exit temperature, in °F.
(b) If the refrigerant exits at a temperature of 130°F, determine the actual power, in Btu/s, and the isentropic compressor efficiency.

KNOWN: Refrigerant 134a as saturated vapor at specified mass flow rate and inlet pressure flows through a compressor exiting at a specified pressure.

FIND: Minimum theoretical power input required with corresponding exit temperature; the actual power and the isentropic compressor efficiency at specified exit temperature.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL:
1. The control volume encloses the compressor as shown in the schematic.
2. The control volume operates adiabatically at steady state.
3. Kinetic and potential energy effects can be ignored.

ANALYSIS:
(a) For a specified mass flow rate, the minimum theoretical power input corresponds to an isentropic compression from the given inlet state to the specified exit pressure (Sec. 6.12.3). Thus, $s_{2s} = s_1$ together with p_2 fixes the final state of the isentropic compression. The minimum theoretical power input can be determined from the steady state energy balance

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left(h_i + \tfrac{1}{2} V_i^2 + gz_i\right) - \sum_e \dot{m}_e \left(h_e + \tfrac{1}{2} V_e^2 + gz_e\right)$$

Setting rate of heat transfer to zero, ignoring kinetic and potential energy effects, writing the energy balance for the isentropic compressor between inlet state 1 and exit state 2*s*, and solving for power give

$$\dot{W}_{cv\,(\text{minimum})} = \dot{m}\,(h_1 - h_{2s})$$

PROBLEM 6.153 (CONTINUED)

At state 1, Refrigerant 134a is saturated vapor. From Table A-11E, $h_1 = h_g = 103.96$ Btu/lb; $s_1 = s_g = 0.2209$ Btu/(lb·°R).

State 2s is fixed by $p_{2s} = p_2 = 160$ psia and $s_{2s} = s_1 = 0.2209$ Btu/(lb·°R). From Table A-12E, $h_{2s} = 118.89$ Btu/lb and $T_{2s} = 120$°F.

Substituting values to solve for minimum theoretical power gives

$$\dot{W}_{cv\,(\text{minimum})} = \left(0.8\frac{\text{lb}}{\text{s}}\right)\left(103.96\frac{\text{Btu}}{\text{lb}} - 118.89\frac{\text{Btu}}{\text{lb}}\right) = \underline{\mathbf{-11.94\ Btu/s}}$$

The negative sign associated with power indicates power is into the control volume.

The temperature corresponding to the minimum theoretical power is $\underline{\boldsymbol{T_{2s} = 120^{\circ}\mathrm{F}}}$.

(b) For an exit temperature of 130°F, the actual power is determined from the energy balance for the actual compressor between inlet state 1 and exit state 2.

$$\dot{W}_{cv} = \dot{m}\,(h_1 - h_2)$$

State 2 is fixed by $p_2 = 160$ psia and $T_2 = 130$°F. From Table A-12E (Interpolated), $h_2 = 121.65$ Btu/lb.

Substituting values gives

$$\dot{W}_{cv} = \left(0.8\frac{\text{lb}}{\text{s}}\right)\left(103.96\frac{\text{Btu}}{\text{lb}} - 121.65\frac{\text{Btu}}{\text{lb}}\right) = \underline{\mathbf{-14.15\ Btu/s}}$$

The negative sign associated with power indicates power is into the control volume.

The isentropic compressor efficiency is determined from the ratio of the isentropic power to the actual power

$$\eta_c = \frac{\left(-\dot{W}_{cv}\right)_s}{\left(-\dot{W}_{cv}\right)}$$

Substituting values gives

$$\eta_c = \frac{-\left(-11.94\dfrac{\text{Btu}}{\text{s}}\right)}{-\left(-14.15\dfrac{\text{Btu}}{\text{s}}\right)} = 0.8438 = \underline{\mathbf{84.38\%}}$$

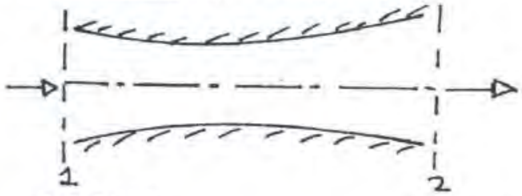
PROBLEM 6.154

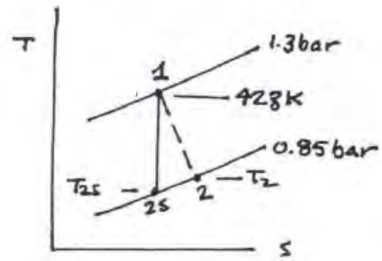
Air at 1.3 bar, 423 K and a velocity of 40 m/s enters a nozzle operating at steady state and expands adiabatically to the exit, where the pressure is 0.85 bar and velocity is 307 m/s. For air modeled as an ideal gas with $k = 1.4$, determine for the nozzle (a) the temperature at the exit, in K, and (b) the isentropic nozzle efficiency.

KNOWN: Steady-state data are provided for a nozzle.

Find: Determine the temperature at the nozzle exit and the isentropic nozzle efficiency.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL:

1. A control volume encloses the nozzle.
2. Operation is at steady state.
3. $\dot{Q}_{cv} = 0$ and potential energy effects are ignored.
4. The air is modeled as an ideal gas with $k = 1.4$.

ANALYSIS: (a) Reducing an energy rate balance at steady state:

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + \cancel{g(z_1 - z_2)}\right] \Rightarrow 0 = (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right)$$

With assumption # 4,

$$0 = c_p(T_1 - T_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) \Rightarrow T_2 = T_1 + \left(\frac{V_1^2 - V_2^2}{2c_p}\right)$$

$$\boxed{\text{Eq. 3.47a}: \; c_p = \frac{kR}{(k-1)} = \left(\frac{1.4}{0.4}\right)\left(\frac{8.314}{28.97}\right)\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 1.004 \text{ kJ/kg}\cdot\text{K}}$$

$$\therefore \; T_2 = 423\text{ K} + \left[\frac{(40)^2 - (307)^2}{(2)(1.004 \text{ kJ/kg}\cdot\text{K})}\right]\left(\frac{\text{m}}{\text{s}}\right)^2 \left|\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}}\right|\left|\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right| = 376.9 \text{ K} \longleftarrow$$

(b) The isentropic nozzle efficiency is given by Eq. 6.47: $\eta_{nozzle} = \dfrac{V_2^2/2}{V_{2s}^2/2}$

To find V_{2s} apply the energy rate balance to an isentropic expansion to obtain

$$0 = c_p(T_1 - T_{2s}) + \left(\frac{V_1^2 - V_{2s}^2}{2}\right) \Rightarrow V_{2s} = \sqrt{V_1^2 + 2c_p(T_1 - T_{2s})}$$

, where T_{2s} is evaluated from Eq. 6.43:

$$T_{2s} = T_1\left[\frac{\underbrace{P_{2s}}_{=P_2}}{P_1}\right]^{\left(\frac{k-1}{k}\right)} = 423\text{K}\left[\frac{0.85 \text{ bar}}{1.3 \text{ bar}}\right]^{\left(\frac{0.4}{1.4}\right)} = 374.6\text{K}$$

Then

$$V_{2s} = \sqrt{(40\tfrac{\text{m}}{\text{s}})^2 + 2\left(1.004\tfrac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(423 - 374.6)\text{K}\left|\frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}}\right|\left|\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right|} = 314.3 \text{ m/s}$$

Finally,

$$\eta = \frac{V_2^2/2}{V_{2s}^2/2} = \left(\frac{V_2}{V_{2s}}\right)^2 = \left(\frac{307}{314.3}\right)^2 = 0.954 \quad (95.4\%) \longleftarrow$$

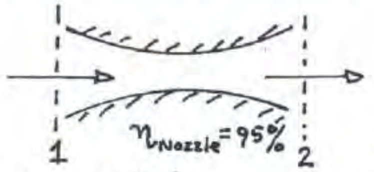
PROBLEM 6.155

Water vapor at 100 lbf/in.², 500°F and a velocity of 100 ft/s enters a nozzle operating at steady state and expands adiabatically to the exit, where the pressure is 40 lbf/in.² If the isentropic nozzle efficiency is 95%, determine for the nozzle (a) the velocity of the steam at the exit, in ft/s, and (b) the amount of entropy produced, in Btu/°R per lb of steam flowing.

KNOWN: Steady-state data are provided for a nozzle.

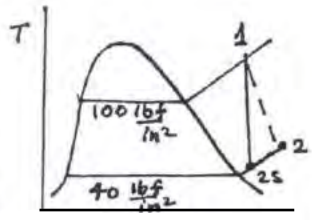
FIND: Determine the velocity of the steam at the exit and $\dot{\sigma}/\dot{m}$.

SCHEMATIC & GIVEN DATA:



$P_1 = 100$ lbf/in², $T_1 = 500°F$, $V_1 = 100$ f/s

$P_2 = 40$ lbf/in²



ENGINEERING MODEL

1. A control volume encloses the nozzle.
2. Operation is at steady state.
3. $\dot{Q}_{cv} = 0$ and potential energy can be ignored.

ANALYSIS: (a) Eq. 6.47 gives, $V_2 = V_{2s}\sqrt{\eta_{nozzle}}$. To find V_{2s} reduce a one-inlet, one-exit control volume energy rate balance at steady state:

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\left[h_1 - h_{2s} + \left(\frac{V_1^2 - V_{2s}^2}{2}\right) + g\cancel{(z_1 - z_2)}\right] \Rightarrow V_{2s} = \sqrt{V_1^2 + 2(h_1 - h_{2s})}$$

From Table A-4E, $h_1 = 1279.1$ Btu/lb, $s_1 = 1.7085$ Btu/lb·°R. To find h_{2s}, interpolate at 40 lbf/in² with $s_{2s} = s_1$: $h_{2s} = 1194$ Btu/lb. Thus,

$$V_{2s} = \sqrt{\left(100\frac{ft}{s}\right)^2 + 2(1279.1 - 1194)\frac{Btu}{lb}\left|\frac{778\ ft\cdot lbf}{1\ Btu}\right|\left|\frac{32.2\ lb\cdot ft/s^2}{1\ lbf}\right|} = 2067.3\ \frac{ft}{s}$$

Finally, $V_2 = \sqrt{0.95}\,(2067.3\ ft/s) = 2015\ ft/s$

(b) Reducing an entropy rate balance for the control volume, we get $(\dot{\sigma}_{cv}/\dot{m}) = s_2 - s_1$. To find s_2 requires state 2 to be fixed. In this case, state 2 is fixed by p_2 and h_2, where h_2 is found from an energy rate balance for the actual expansion:

$$0 = \left[h_1 - h_2 + \left(\frac{V_1^2 - V_2^2}{2}\right)\right] \Rightarrow h_2 = h_1 + \left[\frac{V_1^2 - V_2^2}{2}\right].$$

That is,

$$h_2 = 1279.1\ \frac{Btu}{lb} + \left[\frac{(100)^2 - (2015)^2}{2}\right]\left[\frac{ft}{s}\right]^2\left|\frac{1\ Btu}{778\ ft\cdot lbf}\right|\left|\frac{1\ lbf}{32.2\ lb\cdot ft/s^2}\right|$$

$$= 1198.3\ Btu/lb$$

Interpolating in Table A-4E at 40 lbf/in², $s_2 = 1.7140$ Btu/lb·°R.

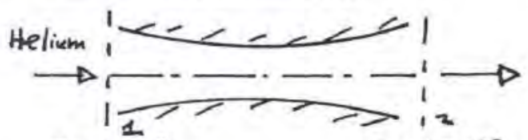
Finally,

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = (1.740 - 1.7085)\ \frac{Btu}{lb\cdot°R} = 0.0055\ \frac{Btu}{lb\cdot°R}$$

PROBLEM 6.156

Helium gas at 810°R, 45 lbf/in.^{2} and a velocity of 10 ft/s enters an insulated nozzle operating at steady state and exits at 670°R, 25 lbf/in.^{2}. Modeling helium as an ideal gas with $k = 1.67$, determine (a) the velocity at the nozzle exit, in ft/s, (b) the isentropic nozzle efficiency, and (c) the rate of entropy production within the nozzle, in Btu/°R per lb of helium flowing.

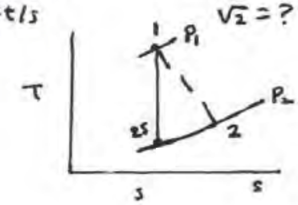
SCHEMATIC & GIVEN DATA:



$T_1 = 810\,°R$ $\quad$ $T_2 = 670\,°R$

$P_1 = 45\ lbf/in^2$ $\quad$ $P_2 = 25\ lbf/in^2$

$V_1 = 10\ ft/s$ $\quad$ $V_2 = ?$



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. For the control volume, $\dot{Q}_{cv}$ and potential energy effects can be ignored.
3. The helium is modeled as an ideal gas with $k = 1.67$.

ANALYSIS: (a) Mass an energy rate balances reduce to give,

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + \cancel{g(z_1 - z_2)}\right)$$

$$\Rightarrow V_2 = \sqrt{V_1^2 + 2(h_1 - h_2)}$$

$$= \sqrt{V_1^2 + 2c_p(T_1 - T_2)} \qquad (1)$$

With Eq. 3.47a, $c_p = \frac{kR}{k-1} = \left(\frac{1.67}{1.67-1}\right)\left(\frac{1.986}{4.003}\frac{Btu}{lb\cdot°R}\right) = 1.24\ \frac{Btu}{lb\cdot°R}$. So, with $(T_1 - T_2) = 140°R$,

$$V_2 = \sqrt{\left(10\frac{ft}{s}\right)^2 + 2\left(1.24\frac{Btu}{lb\cdot°R}\right)(140°R)\left|\frac{778\,ft\cdot lbf}{1\,Btu}\right|\left|\frac{32.2\ lb\cdot ft/s^2}{1\,lbf}\right|}$$

$$= 2949\ ft/s \qquad \text{(a)}$$

(b) The isentropic nozzle efficiency is given by Eq. 6.47: $\eta_{nozzle} = \frac{V_2^2/2}{V_{2s}^2/2}$.

To find V_{2s}, T_{2s} is required. Using Eq. 6.43,

$$T_{2s} = T_1\left(P_2/P_1\right)^{(k-1)/k} = 810°R\,(25/45)^{0.67/1.67} = 639.8\,°R$$

Then, Eq. (1) gives

$$V_{2s} = \sqrt{V_1^2 + 2c_p(T_1 - T_{2s})} = \sqrt{(10\,ft/s)^2 + 2(1.24)(810 - 639.8)|778||32.2|} = 3252\ ft/s$$

$$\Rightarrow \eta_{nozzle} = \frac{(2949)^2/2}{(3252)^2/2} = 0.822 \quad (82.2\%) \qquad \text{(b)}$$

(c) Mass and entropy rate balances reduce to give $\dot{\sigma}_{cv}/\dot{m} = s_2 - s_1$. Or

$$\dot{\sigma}_{cv}/\dot{m} = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} = 1.24 \ln\frac{670}{810} - \frac{1.986}{4.003}\ln\frac{25}{45}$$

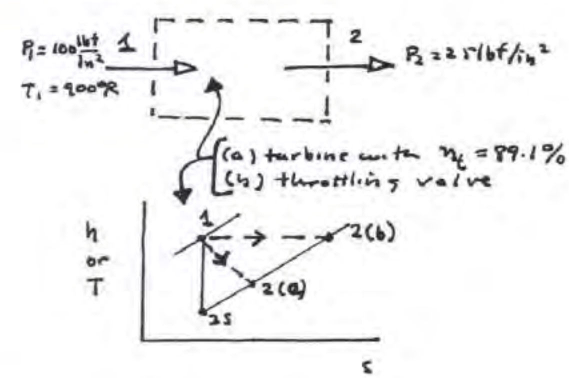
$$= 0.056\ \frac{Btu}{lb\cdot°R} \qquad \text{(c)}$$

PROBLEM 6.157

Air modeled as an ideal gas enters a one-inlet, one-exit control volume operating at steady state at 100 lbf/in.^{2}, 900°R and expands adiabatically to 25 lbf/in.^{2}. Kinetic and potential energy effects are negligible. Determine the rate of entropy production, in Btu/°R per lb of air flowing,

(a) if the control volume encloses a turbine having an isentropic turbine efficiency of 89.1%.
(b) if the control volume encloses a throttling valve.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The control volume shown in the sketch is at steady state.
2. In each case, the expansion occurs adiabatically, and kinetic and potential energy effects are negligible.
3. The air is modeled as an ideal gas.

ANALYSIS: Mass and entropy rate balances reduce to give, with Eq. 6.20a,

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1 = s°(T_2) - s°(T_1) - R \ln P_2/P_1 \qquad (1)$$

(a) Turbine with $\eta_t = 0.891$. To find $s°(T_2)$, use $\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow$

$h_2 = h_1 - \eta_t (h_1 - h_{2s})$. To find h_{2s}, use $P_r(2s) = P_r(1)\frac{P_2}{P_1} = 8.411\left(\frac{25}{100}\right) = 2.103$ ← Table A-22E

Interpolation in Table A-22E gives, $h_{2s} = 145.4$ Btu/lb. Then,

$h_2 = 216.26 - 0.891(216.26 - 145.4) = 153.12$ Btu/lb $\Rightarrow s°(T_2) \cong 0.64159$

Then, Eq. (1) gives

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = 0.64159 - 0.72438 - \underbrace{\left[\frac{1.987}{28.97} \ln\left(\frac{25}{100}\right)\right]}_{(-0.095\ Btu/lb\cdot°R)}$$

$$= 0.0122\ Btu/lb\cdot°R \qquad \longleftarrow (a)$$

(b) Throttling valve. Since $h_2 = h_1$, it follows that for an ideal gas $T_2 = T_1$. So, Eq. (1) reads

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = \underbrace{s°(T_2) - s°(T_1)}_{=0} - R \ln \frac{P_2}{P_1}$$

1 $$= -R \ln \frac{P_2}{P_1} = 0.095\ Btu/lb\cdot°R \qquad \longleftarrow (b)$$

See (a) for calculation

---

1. Entropy production associated with expansion through the throttling valve is significantly greater than in the more controlled expansion through the turbine.

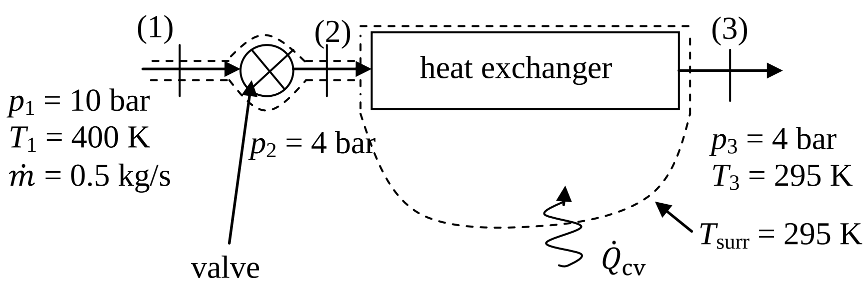
PROBLEM 6.158

As part of an industrial process, air as an ideal gas at 10 bar, 400K expands at steady state through a valve to a pressure of 4 bar. The mass flow rate of air is 0.5 kg/s. The air then passes through a heat exchanger where it is cooled to a temperature of 295K with negligible change in pressure. The valve can be modeled as a throttling process, and kinetic and potential energy effects can be neglected.
(a) For a control volume enclosing the valve and heat exchanger and enough of the local surroundings that the heat transfer occurs at the ambient temperature of 295 K, determine the rate of entropy production, in kW/K.
(b) If the expansion valve were replaced by an adiabatic turbine operating isentropically, what would be the entropy production?
Compare the results of parts (a) and (b) and discuss.

KNOWN: As part of an industrial process, air is first expanded though a valve and then cooled to the ambient temperature at steady state. Data are known at various steps in the process.

FIND: For a control volume enclosing the valve and heat exchanger and enough of the local surroundings that the heat transfer occurs at the ambient temperature, volume enclosing the valve and heat exchanger and enough of the local surroundings that the heat transfer occurs at the ambient temperature. Repeat if the valve were replaced by an isentropic turbine. Comment.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The control volumes shown are at steady state. (2) The air passing through the valve undergoes a throttling process. (3) For the heat exchanger, $\dot{W}_{cv} = 0$. (4) The air can be modeled as an ideal gas. (5) Kinetic and potential energy effects can be neglected.

ANALYSIS: (a) To begin, we consider separate control volumes for the valve and the heat exchanger. For the valve: the process is a *throttling process*, so $h_2 = h_1$ (See Section 4.10.1). Since the air behaves as an ideal gas, $T_2 = T_1$. The entropy rate balance reduces to:

$$0 = \cancel{\frac{\dot{Q}_{\text{valve}}}{T_b}} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{\text{valve}}.$$

Thus

$$\dot{\sigma}_{\text{valve}} = \dot{m}(s_2 - s_1) = \dot{m}[\cancel{(s^o(T_2) - s^o(T_1)} - R\ln(p_2/p_1)]$$

$$= \dot{m}[-R\ln(p_2/p_1)] = (0.5\text{ kg/s})[-(8.314/28.97)\ln(4/10)]\text{kJ/kg·K}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = 0.13148\text{ kW/K}$$

Now, for the heat exchanger: $0 = \dot{Q}_{cv} + \dot{m}(h_2 - h_3)$. With data from Table A-22

$$\dot{Q}_{cv} = \dot{m}(h_3 - h_2) = (0.5\text{ kg/s})(295.17 - 400.98)\text{kJ/kg}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = -52.91\text{ kW (out)}$$

PROBLEM 6.158 (CONTINUED) – PAGE 2

The entropy rate balance for the heat exchanger becomes: $0 = \frac{\dot{Q}_{cv}}{T_{surr}} + \dot{m}(s_1 - s_3) + \dot{\sigma}_{HX}$. Thus

$$\dot{\sigma}_{HX} = -\frac{\dot{Q}_{cv}}{T_{surr}} + \dot{m}(s_3 - s_2) = -\frac{\dot{Q}_{cv}}{T_{surr}} + \dot{m}[(s^o(T_3) - s^o(T_2) - \cancel{R \ln(p_3/p_2)})$$

With data from Table A-22

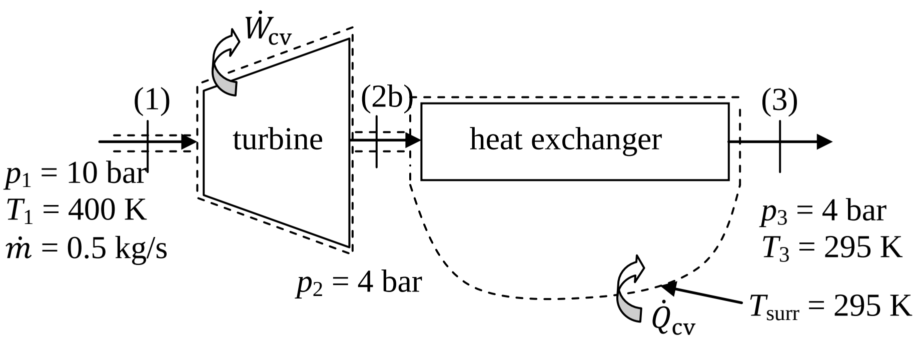
$$\dot{\sigma}_{HX} = -\frac{\dot{Q}_{cv}}{T_{surr}} + \dot{m}[s^o(T_3) - s^o(T_2)]$$

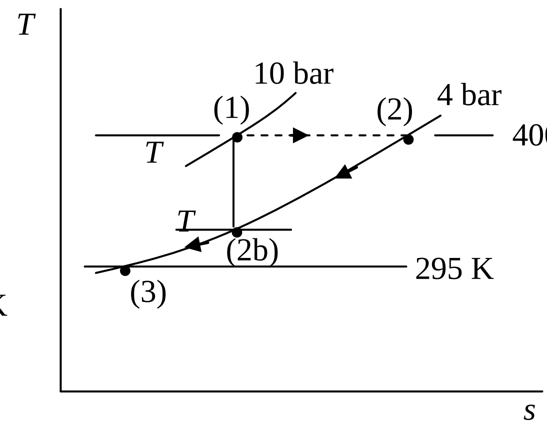
$$= -(-52.91 \text{ kW})/(295 \text{ K}) + (0.5 \text{ kg/s})(1.68515 - 1.99194)\text{kJ/kg}\cdot\text{K}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right| = 0.02596 \text{ kW/K}$$

Thus, the total rate of entropy production is $\dot{\sigma}_{cv} = 0.13148 + 0.02596 = 0.15744$ kW/K ⟵

Note that 83.5% of the entropy production occurs in the valve.

(b) Turning now to the case where the valve is replaced by a turbine operating *isentropically*, the schematic becomes





Using Eq. 6.41 with data from Table AA-22: $p_{r2} = p_{r1}(p_2/p_1) = (3.806)(4/10) = 1.5224$
Thus: $T_{2b} \approx 308.1$ K and $h_{2b} \approx 308.36$ kJ/kg. The heat transfer rate for the heat exchanger becomes

$$\dot{Q}_{cv,b} = \dot{m}(h_3 - h_{2b}) = (0.5 \text{ kg/s})(295.17 - 308.36)\text{kJ/kg}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right| = -6.595 \text{ kW (out)}$$

and the entropy production rate is

$$\dot{\sigma}_{HX.b} = -\frac{\dot{Q}_{cv,b}}{T_{surr}} + \dot{m}[s^o(T_3) - s^o(T_{2b})]$$

$$= -(-6.595)/(295) + (0.5)[(1.68515) - (1.72887)] = 4.959 \times 10^{-4} \text{ kW/K}$$

Since the turbine is isentropic, the total rate of entropy production for this case is 4.959×10^{-4} kW/K ⟵

PROBLEM 6.158 (CONTINUED) – PAGE 3

Comments

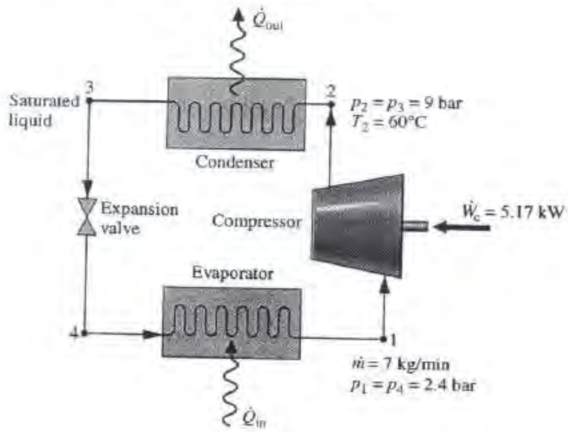
The entropy production rate has decreased substantially with the elimination of the highly irreversible process in the valve. Further, the turbine produces power at a rate of

$$\dot{W}_{\text{cv}} = \dot{m}(h_1 - h_2) = (0.5)(400.98 - 308.36) = 46.31\ \text{kW}$$

This power could have significant economic value in the industrial setting that might justify the additional capital and maintenance costs. A full economic evaluation would need to be done to determine if this option should be adopted.

PROBLEM 6.159

The figure below provides the schematic of a heat pump using Refrigerant 134a as the working fluid, together with steady-state data at key points. The mass flow rate of the refrigerant is 7 kg/min, and the power input to the compressor is 5.17 kW. (a) Determine the coefficient of performance for the heat pump. (b) If the valve were replaced by a turbine, power could be produced, reducing thereby the power requirement of the heat pump system. Would you recommend this *power-saving* measure? Explain.



ENGR. MODEL:

1. The heat pump system shown in the figure operates at steady state.
2. Stray heat transfer and kinetic and potential energy effects can be ignored.
3. $\dot{W}_c$ and $\dot{Q}_{out}$ are positive in the directions of the arrows.

ANALYSIS: (a) Using Eq. 2.47 expressed on a rate basis,

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_c}$$

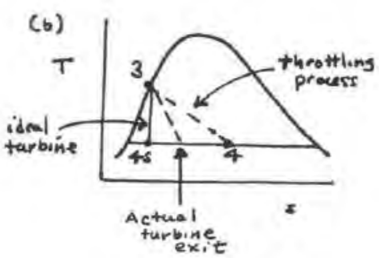
An energy rate balance for the condenser gives, $\dot{Q}_{out} = \dot{m}(h_2 - h_3)$.

So,

$$\gamma = \frac{\dot{m}(h_2-h_3)}{\dot{W}_c} = \frac{7\frac{kg}{min}\left|\frac{1\,min}{60\,s}\right|(293.21 - 99.56)\frac{kJ}{kg}}{5.17\,kJ/s}$$

$$= 4.37 \quad \leftarrow (a)$$

(b)



To assess the merit of a power-recovery turbine, consider an ideal turbine: one whose isentropic turbine efficiency is 100%. Then,

$\dot{W}_t = \dot{m}(h_3 - h_{4s})$, where $h_3 = 99.56\frac{kJ}{kg}$, $s_3 = 0.3656\ \frac{kJ}{kg\cdot K}$.

Then, since $s_{4s} = s_3$,

$$x_{4s} = \frac{s_{4s} - s_f}{s_g - s_f} = \frac{0.3656 - 0.1710}{0.9222 - 0.1710} = 0.259$$

$$\Rightarrow h_{4s} = h_f + x_{4s}h_{fg} = 42.95 + 0.259(201.14) = 95.05\ \frac{kJ}{kg}$$

Then

$$\dot{W}_t = \left(7\frac{kg}{min}\right)\left|\frac{1\,min}{60\,s}\right|\left(99.56\frac{kJ}{kg} - 95.05\frac{kJ}{kg}\right)\left|\frac{1\,kW}{1\,kJ/s}\right|$$

$$= 0.53\,kW$$

DISCUSSION: This is the maximum theoretical power that any power-recovery turbine would be able to develop. At best, the turbine could offset about 10% of the compressor power requirement. In most heat pump applications, such a turbine is not implemented owing to the turbine cost and operating difficulties related to the low-quality refrigerant that would be expanding through such a turbine.

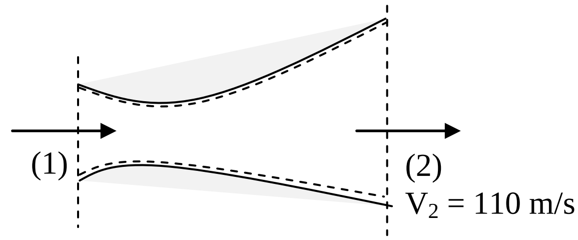
PROBLEM 6.160

Air as an ideal gas enters a diffuser operating at steady state at 4 bar, 290K with a velocity of 512 m/s. The exit velocity is 110 m/s. For adiabatic operation with no internal irreversibilities, determine the exit temperature, in K, and the exit pressure, in bar, for
(a) constant specific heats with k = 1.4.
(b) variable specific heats using data from Table A-22.

KNOWN: Air flows adiabatically through a diffuser operating at steady state with no internal irreversibilities. Data are specified at the inlet and exit.

FIND: Determine the exit temperature and pressure for (a) constant specific heats, and (b) variable specific heats.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The control volume is at steady state. (2) For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$, and potential energy effects can be neglected. (3) The air undergoes an internally reversible process in passing through the diffuser. (4) The air is modeled as an ideal gas (a) with constant specific heats and (b) with variable specific heats.

ANALYSIS: For adiabatic, internally reversible operation at steady state

$$0 = \cancel{\sum_j \left(\frac{\dot{Q}_j}{T_j}\right)} + \dot{m}(s_1 - s_2) + \cancel{\dot{\sigma}_{cv}} \quad \longrightarrow \quad \boxed{s_2 = s_1}$$

Further

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + \cancel{g(z_1 - z_2)}\right] \quad \longrightarrow \quad 0 = (h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2}$$

(a) For k = 1.4; $c_p = kR/(k-1)$ = [(1.4)(8.314/28.97)]/(1.4 – 1) = 1.004 kJ/kg·K. Further, $\Delta h = \Delta T$. Thus

$$0 = c_p(T_1 - T_2) + \frac{(V_1^2 - V_2^2)}{2} \quad \longrightarrow \quad T_2 = T_1 + \frac{(V_1^2 - V_2^2)}{2c_p}$$

Inserting values and converting units

$$T_2 = (290\text{ K}) + \left[\frac{(512^2 - 110^2)(\text{m}^2/\text{s}^2)}{2\left(1.004 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)}\right]\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right| = 414.5\text{ K} \longleftarrow$$

Now, using Eq. 6.43

$$p_2 = p_1\left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = (4\text{ bar})\left(\frac{414.5}{290}\right)^{\frac{1.4}{1-1.4}} = 13.96\text{ bar} \longleftarrow$$

PROBLEM 6.160 (CONTINUED)

(b) For variable specific heats, we use data from Table A-22 for the specific enthalpies, and

$$h_2 = h_1 + \frac{(V_1^2 - V_2^2)}{2} = 290.16 \text{ kJ/kg} + \left[\frac{(512^2 - 110^2)(\text{m}^2/\text{s}^2)}{2}\right] \left|\frac{1 \text{ kJ}}{10^3 \text{N·m}}\right| \left|\frac{1 \text{ N}}{1 \text{ kg·m/s}^2}\right| = 415.18 \text{ kJ/kg}$$

From Table A-22: $T_2 \approx 414$ K

The pressure is found from the isentropic relation: $0 = s^o(T_2) - s^o(T_1) - R \ln(p_2/p_1)$. Solving for p_2

$$p_2 = \exp\left[\frac{s^o(T_2) - s^o(T_1)}{R}\right] p_1 = \exp\left[\frac{2.02676 - 1.66802}{8.314/28.97}\right](4 \text{ bar}) = 13.96 \text{ bar}$$

Note: In this case, the assumption of constant specific heats was quite accurate.

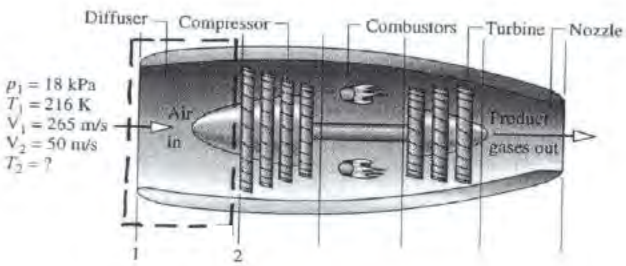
PROBLEM 6.161

As shown in   air enters the diffuser of a jet engine at 18 kPa, 216 K with a velocity of 265 m/s, all data corresponding to high-altitude flight. The air flows adiabatically through the diffuser, decelerating to a velocity of 50 m/s at the diffuser exit. Assume steady-state operation, the ideal gas model for air, and negligible potential energy effects.

(a) Determine the temperature of the air at the exit of the diffuser, in K.

(b) If the air would undergo an isentropic process as it flows through the diffuser, determine the pressure of the air at the diffuser exit, in kPa.

(c) If friction were present, would the pressure of the air at the diffuser exit be greater than, less than, or equal to the value found in part (b)? Explain.



ENGR. MODEL:

1. The control volume shown in the figure is at steady state.
2. For the control volume, $\dot{Q}_{cv}$ and potential energy effects can be ignored. $\dot{W}_{cv} = 0$.
3. The air is modeled as an ideal gas.

ANALYSIS: (a) Mass and energy rate balances reduce to give

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\left[(h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} + \cancel{g(z_1 - z_2)}\right] \Rightarrow h_2 = h_1 + \frac{V_1^2 - V_2^2}{2}$$

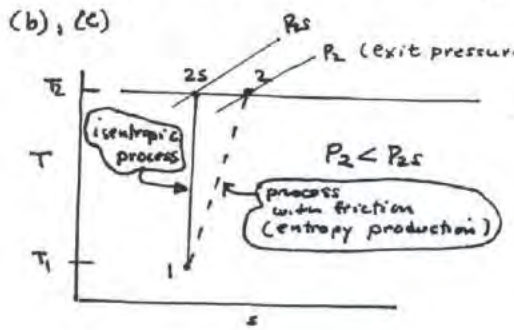
With data from Table A-22,

$$h_2 = 215.97\,\frac{kJ}{kg} + \frac{(265\,m/s)^2 - (50\,m/s)^2}{2}\left|\frac{1\,N}{1\,kg\cdot m/s^2}\right|\left|\frac{1\,kJ}{10^3\,N\cdot m}\right| = 249.8\,\frac{kJ}{kg}$$

Interpolation in Table A-22 gives $T_2 = 249.8\,K$ ← (a)

(b), (c)

← (c)



To find P_{2s}, use Eq. 6.41:

$$\frac{P_{2s}}{P_1} = \frac{P_r(2s)}{P_r(1)}$$

$$P_{2s} = P_1\,\frac{P_r(2s)}{P_r(1)}$$

$$= (18\,kPa)\left(\frac{0.7510}{0.4469}\right)$$

$$= 30\,kPa$$ ← (b)

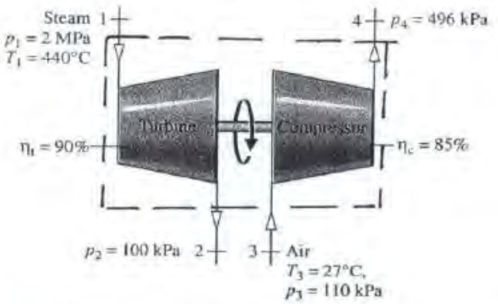
(exit pressure for an isentropic process)

PROBLEM 6.162

As shown a steam turbine having an isentropic turbine efficiency of 90% drives an air compressor having an isentropic compressor efficiency of 85%. Steady-state operating data are provided on the figure. Assume the ideal gas model for air, and ignore stray heat transfer and kinetic and potential energy effects.

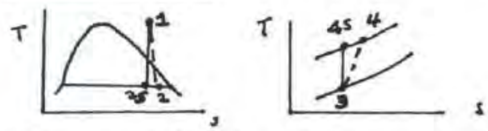
(a) Determine the mass flow rate of the steam entering the turbine, in kg of steam per kg of air exiting the compressor.

(b) Repeat part (a) if $\eta_t = \eta_c = 100\%$



ENGR. MODEL:

1. The control volume shown in the figure is at steady state.
2. For the control volume, $\dot{W}_{cv} = 0$. Also, stray heat transfer and kinetic and potential energy effects can be ignored.
3. The air is modeled as an ideal gas.



ANALYSIS: Mass balances give, $\dot{m}_1 = \dot{m}_2$ and $\dot{m}_3 = \dot{m}_4$. An energy rate balance gives

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1[h_1 - h_2] + \dot{m}_4[h_3 - h_4] \Rightarrow \dot{m}_1 = \dot{m}_4\left[\frac{h_4 - h_3}{h_1 - h_2}\right] \quad (1)$$

The respective isentropic efficiencies are, $\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}}$, $\eta_c = \frac{h_{4s} - h_3}{h_4 - h_3}$. With these Eq. (1) becomes

1 $$\frac{\dot{m}_1}{\dot{m}_4} = \frac{1}{\eta_t \eta_c}\left[\frac{h_{4s} - h_3}{h_1 - h_{2s}}\right] \quad (2)$$

Property data: Table A-4 $h_1 = 3335.5$ kJ/kg, $s_1 = 7.2540$ kJ/kg·K

$s_{2s} = s_1 \Rightarrow x_{2s} = \frac{7.2540 - 1.3026}{7.3594 - 1.3026} = .9826 \Rightarrow h_{2s} = 417.46 + 0.9826(2258) = 2636.2$ kJ/kg

Table A-22 $h_3 = 300.19$ kJ/kg, $p_{r3} = 1.3860$, $p_{r4} = p_{r3}\left(p_4/p_3\right) = 1.3860(4.96/1.1) = 6.2496$

$\Rightarrow h_{4s} = 462.1$ kJ/kg

(a) $\eta_t = 90\%, \eta_c = 85\%$

$$\frac{\dot{m}_1}{\dot{m}_4} = \frac{1}{(0.9)(0.85)}\underbrace{\left[\frac{462.1 - 300.19}{3335.5 - 2636.2}\right]}_{0.232} = 0.303 \qquad \text{(a)}$$

(b) $\eta_t = 100\%$, $\eta_c = 100\%$

$$\frac{\dot{m}_1}{\dot{m}_4} = 0.232 \qquad \text{(b)}$$

1. Note that

$$\frac{\dot{m}_1}{\dot{m}_4} = \frac{1}{\eta_t \eta_c}\left(\frac{\dot{m}_1}{\dot{m}_4}\right)_{\substack{int\\rev}}$$

PROBLEM 6.163

The figure below shows a simple vapor power plant operating at steady state with water as the working fluid. Data at key locations are given on the figure. The mass flow rate of the water circulating through the components is 109 kg/s. Stray heat transfer and kinetic and potential energy effects can be ignored. Determine

(a) the net power developed, in MW.
(b) the thermal efficiency.
(c) the isentropic turbine efficiency.
(d) the isentropic pump efficiency.
(e) the mass flow rate of the cooling water, in kg/s.
(f) the rates of entropy production, each in kW/K, for the turbine, condenser, and pump.

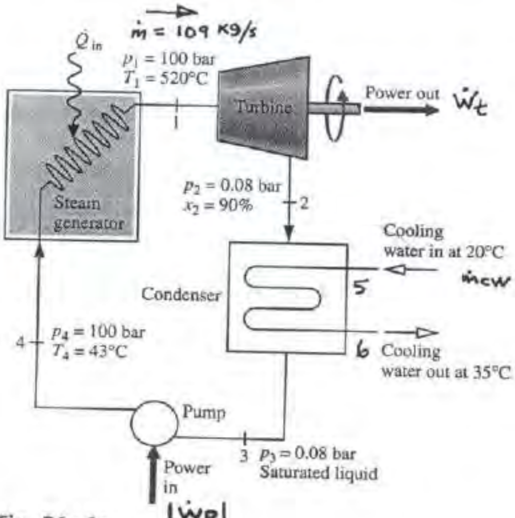


Fig. P6.165

KNOWN: Steady-state data are provided for a simple vapor power plant.

FIND: $\dot{W}_{net}$, η, η_t, η_p, $\dot{m}_{cw}$, $\dot{\sigma}_t$, $\dot{\sigma}_{cond}$, $\dot{\sigma}_{pump}$. Rank order $\dot{\sigma}_t$, $\dot{\sigma}_{cond}$, $\dot{\sigma}_{pump}$.

ENGINEERING MODEL:

1. Control volumes at steady state enclose all four components.
2. Stray heat transfer and kinetic and potential energy effects are ignored.
3. For the cooling water, $h \approx h_f(T)$, $s \approx s_f(T)$
4. All energy transfers are positive in the directions of the arrows.

ANALYSIS: Begin by tabulating h and s at the principal states. These values are obtained from the tables indicated.

State	h (kJ/kg)	s (kJ/kg·K)	Comment
1	3425.1	6.6622	Table A-4
2	2336.7	7.4651	Table A-3
3	173.9	0.5926	Table A-3
4	188.9	0.6061	Table A-5
5	83.96	0.2966	Table A-2
6	146.68	0.5053	Table A-2

(a) $\dot{W}_{net} = \dot{W}_t - |\dot{W}_p|$

$$\dot{W}_t = \dot{m}(h_1 - h_2) = \left(109\ \frac{kg}{s}\right)(3425.1 - 2336.7)\frac{kJ}{kg} = 118{,}636\ \frac{kJ}{s}$$

$$|\dot{W}_p| = \dot{m}(h_4 - h_3) = \left(109\ \frac{kg}{s}\right)(188.9 - 173.9)\ \frac{kJ}{kg} = 1635\ \frac{kJ}{s}$$

$$\therefore\ \dot{W}_{net} = (118{,}636 - 1{,}635)\frac{kJ}{s}\left|\frac{1\ MW}{10^3\ kJ/s}\right| = 117\ MW$$

(b) $\eta = \dfrac{\dot{W}_{net}}{\dot{Q}_{in}}$, where $\dot{Q}_{in} = \dot{m}(h_1 - h_4) = \left(109\ \frac{kg}{s}\right)(3425.1 - 188.9)\frac{kJ}{kg}\left|\frac{1\ MW}{10^3\ kJ/s}\right|$

$$= 352.75\ MW$$

$$\therefore\ \eta = \frac{117\ MW}{352.75\ MW} = 0.332 \quad (33.2\%)$$

(c) $\eta_t = \dfrac{h_1 - h_2}{h_1 - h_{2s}}$. To find h_{2s}, note $s_{2s} = s_1 = 6.6622$ kJ/kg·K.

Then $x_{2s} = \dfrac{s_{2s} - s_f}{s_g - s_f} = \dfrac{6.6622 - 0.5926}{8.2287 - 0.5926} = 0.795$

$\therefore h_{2s} = h_f + x_{2s}(h_g - h_f) = 173.88 + 0.795(2403.1) = 2084.3 \frac{kJ}{kg}$

where h_f, h_g, s_f and s_g are from Table A-3 at 0.08 bar.

$\therefore \quad \eta_t = \left(\dfrac{3425.1 - 2336.7}{3425.1 - 2084.3}\right) = 0.812 \qquad (81.2\%)$ ←

(d) $\eta_p = \dfrac{h_{4s} - h_3}{h_4 - h_3}$. To find h_{4s}, note $s_{4s} = s_3 = 0.5926$ kJ/kg·K.

Then, interpolating in Table A-5 at 100 bar, $h_{4s} = 184.4$ kJ/kg.

$\therefore \eta_p = \left(\dfrac{184.4 - 173.9}{188.9 - 173.9}\right) = 0.7 \qquad (70\%)$ ←

(e) Mass and energy rate balances for the condenser reduce to give

$\dot{m}_{cw} = \dot{m}\left[\dfrac{h_2 - h_3}{h_6 - h_5}\right] = 109 \frac{kg}{s}\left[\dfrac{2336.7 - 173.9}{146.68 - 83.96}\right] = 3758.7$ kg/s ←

(f) Applying mass and entropy rate balances, we get

turbine: $\dot{\sigma}_t = \dot{m}(s_2 - s_1) = (109 \text{ kg/s})(7.4651 - 6.6622)\frac{kJ}{kg \cdot K}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right| = 87.52 \frac{kW}{K}$

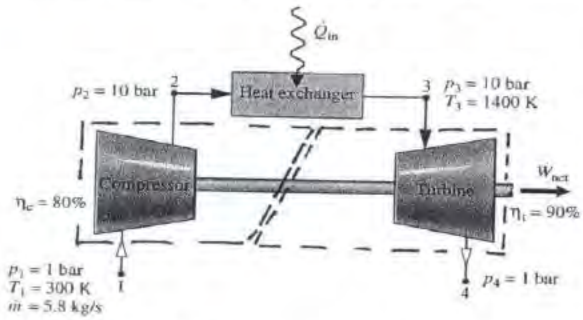
pump: $\dot{\sigma}_p = \dot{m}(s_4 - s_3) = (109)(0.6061 - 0.5926)|1| = 1.47 \frac{kW}{K}$ ←

condenser: $\dot{\sigma}_c = \dot{m}(s_3 - s_2) + \dot{m}_{cw}(s_6 - s_5)$

$= (109)(0.5926 - 7.4651) + 3758.7(0.5053 - 0.2966) = 35.34 \frac{kW}{K}$

PROBLEM 6.164

Shown in the figure below is a power system operating at steady state consisting of three components in series: an air compressor having an isentropic compressor efficiency of 80%, a heat exchanger, and a turbine having an isentropic turbine efficiency of 90%. Air enters the compressor at 1 bar, 300K with a mass flow rate of 5.8 kg/s and exits at a pressure of 10 bar. Air enters the turbine at 10 bar, 1400 K and exits at a pressure of 1 bar. Air can be modeled as an ideal gas. Stray heat transfer and kinetic and potential energy effects are negligible. Determine, in kW, (a) the power required by the compressor, (b) the power developed by the turbine, and (c) the *net* power output of the overall power system.



ENGR. MODEL:

1. Control volumes at steady state enclose the turbine and compressor.
2. For the control volumes, stray heat transfer and kinetic and potential energy effects are negligible.
3. The air is modeled as an ideal gas.

ANALYSIS: Mass and energy rate balances reduce to give,

turbine: $\dot{W}_t = \dot{m}(h_3 - h_4)$ compressor: $\dot{W}_c = \dot{m}(h_1 - h_2)$

Isentropic efficiencies,

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} \qquad \eta_c = \frac{h_{2s} - h_1}{h_2 - h_1}$$

Collecting results

$$\dot{W}_t = \dot{m}\,\eta_t (h_3 - h_{4s}) \quad (1) \qquad \dot{W}_c = -\frac{\dot{m}(h_{2s} - h_1)}{\eta_c} \quad (2)$$

From Table A-22, $h_1 = 300.19$ kJ/kg, $h_3 = 1515.4$ kJ/kg; to find h_{2s} and h_{4s}, use

$$p_r(4s) = p_r(3)\left[\frac{p_4}{p_3}\right] = 450.5\left[\frac{1}{10}\right] = 45.05 \Rightarrow h_{4s} = 808.5 \frac{kJ}{kg}$$

$$p_r(2s) = p_r(1)\left[\frac{p_2}{p_1}\right] = 1.386\left[\frac{10}{1}\right] = 13.86 \Rightarrow h_{2s} = 579.9 \frac{kJ}{kg}$$

Inserting values into Eqs. (1), (2)

$$\dot{W}_c = -\left(5.8\frac{kg}{s}\right)\frac{(579.9 - 300.19)}{0.80}\frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right| = -2028 \quad \text{(a)}$$

$$\dot{W}_t = (5.8)(0.9)(1515.4 - 808.5)|1| = 3690\ kW \quad \text{(b)}$$

$$\dot{W}_{net} = 3690\ kW - 2028\ kW = 1662\ kW \quad \text{(c)}$$

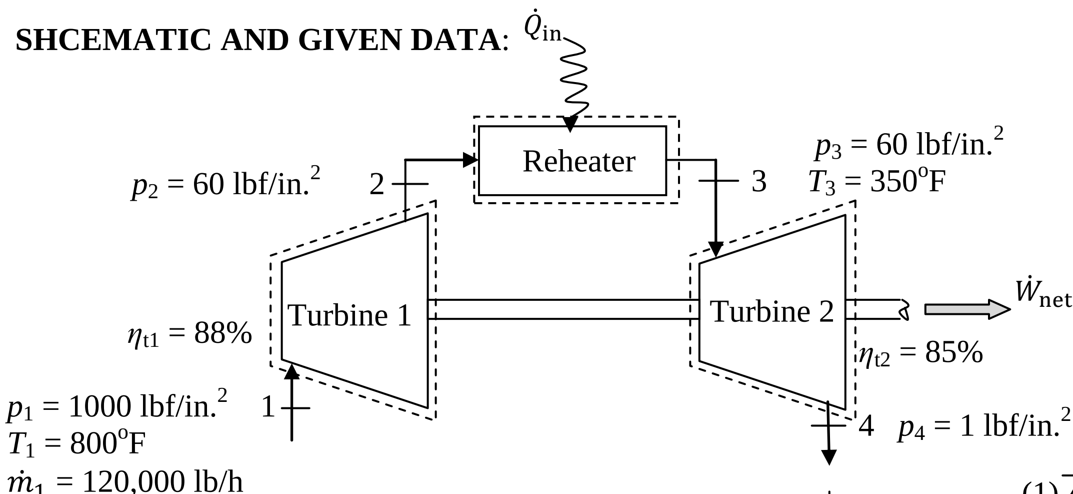
PROBLEM 6.165

Steam enters a two-stage turbine with reheat operating at steady state as shown in Fig. P6.165. The steam enters turbine 1 with a mass flow rate of 120,000 lb/h at 1000 lbf/in.^{2}, 800 ^{o}F and expands to a pressure of 60 lbf/in.^{2}. From there, the steam enters the reheater where it is heated at constant pressure to 350^{o}C before entering turbine 2 and expanding to a final pressure of 1 lbf/in.^{2} . The turbines operate adiabatically, with isentropic efficiencies of 88% and 85%, respectively. Kinetic and potential energy effects can be neglected. Determine the net power developed by the two turbines and the rate of heat transfer in the reheater, each in Btu/h.

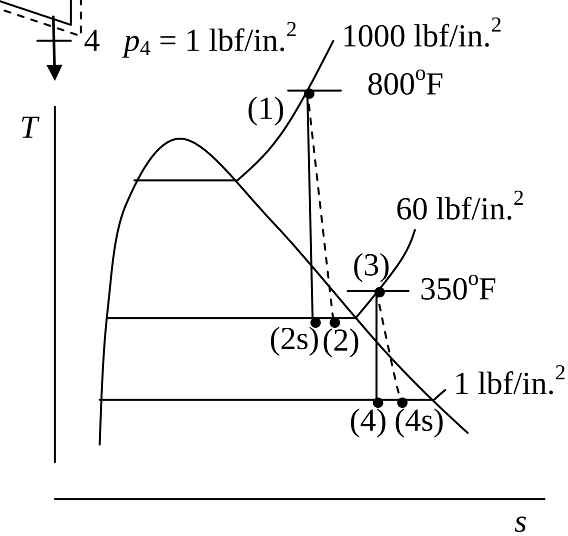
KNOWN: Data are provided for the steady state operation of a two-stage turbine with reheat between the stages. Steam conditions are given at various locations and the isentropic turbine efficiencies are specified.

FIND: Determine the net power developed and the rate of heat transfer in the reaheater.

SHCEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The control volumes are at steady state. (2) The turbines operate adiabatically. (3) Kinetic and potential energy effects are negligible.



ANALYSIS: First, fix each of the numbered states.

State 1 p_1 = 1000 lbf/in.^{2}, T_1 = 800^{o}F

h_1 = 1388.5 Btu/lb, s_1 = 1.5665 Btu/lb·^{o}R

State 2 p_2 = 60 lbf/in.^{2}, $s_{2s} = s_1$ = 1.5665 Btu/lb·^{o}R

$$x_{2s} = (s_{2s} - s_{f2})/(s_{g2} - s_{f2}) = (1.5665 - 1.2170)/(1.6443 - 1.2170) = 0.8062$$

$$h_{2s} = h_{f2} + x_{2s}(h_{g2} - h_{f2}) = 262.2 + (0.8062)(1178.0 - 262.2) = 1000.5 \text{ Btu/lb}$$

Now, from the definition of isentropic turbine efficiency; $\eta_{t1} = (h_1 - h_{2s})/(h_1 - h_2)$. Thus

PROBLEM 6.165 (CONTINUED)

$h_2 = h_1 - \eta_{t1}(h_1 - h_{2s})$ = 1388.5 – (0.88)(1388.5 – 1000.5) = 1047.06 Btu/lb

State 3 p_3 = 60 lbf/in.^{2}, T_3 = 350°F; h_3 = 1208.2 Btu/lb and s_2 = 1.6830 Btu/lb·°R

State 4 p_4 = 1 lbf/in.^{2}, $s_{4s} = s_3$ = 1.6830 Btu/lb·°R

$x_{4s} = (s_{4s} - s_{f4})/(s_{g4} - s_{f4})$ = (1.6830 – 0.1327)/(1.9779 – 0.1327) = 0.840

$h_{4s} = h_{f4} + x_{4s}(h_{g4} - h_{f4})$ = 69.74 + (0.840)(1105.8 – 69.74) = 940.0 Btu/lb

Now, from the definition of isentropic turbine efficiency; $\eta_{t2} = (h_3 - h_{4s})/(h_3 - h_4)$. Thus

$h_4 = h_3 - \eta_{t2}(h_3 - h_{4s})$ = 1208.2 – (0.85)(1208.2 – 940.0) = 980.2 Btu/lb

At steady state, the mass flow rates for each component are equal: $\dot{m} = \dot{m}_1$ = 120,000 lb/h. The energy rate balances for each turbine reduce, respectively, to

$\dot{W}_{\mathrm{t1}} = \dot{m}(h_1 - h_2)$ = (120,000 lb/h)(1388.5 – 1047.06)Btu/lb = 4.097 x 10^7 Btu/h

$\dot{W}_{t2} = \dot{m}(h_3 - h_4)$ = (120,000 lb/h)(1208.2 – 980.2)Btu/lb = 2.736 x 10^7 Btu/h

The net power is: $\dot{W}_{\mathrm{net}} = \dot{W}_{\mathrm{t1}} + \dot{W}_{\mathrm{t2}}$ = 6.833 x 10^7 Btu/h ←

Finally, the heat transfer rate in the reheater is

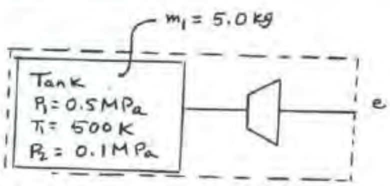
$\dot{Q}_{\mathrm{in}} = \dot{m}(h_3 - h_2)$ = (120,000 lb/h)(1208.2 – 1047.06)Btu/lb = 1.934 x 10^7 Btu/h ←

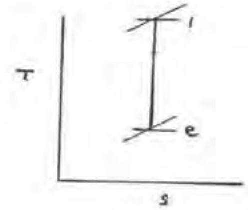
PROBLEM 6.166

KNOWN: A tank initially filled with air is allowed to discharge through a turbine until the pressure in the tank becomes atmospheric

FIND: Determine the maximum theoretical work that could be developed

SCHEMATIC & GIVEN DATA:





ENGR. MODEL: (1) The control volume is shown above. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) The air is modeled as an ideal gas. (4) The control volume is free of irreversibilities.

ANALYSIS: A mass rate balance reduces to $dm/dt = -\dot{m}_e$. Using this, an energy rate balance reads

$$\frac{dU}{dt} = \cancel{\dot{Q}_{cv}}^{\,0} - \dot{W}_{cv} - \dot{m}_e h_e \quad , \quad \frac{dU}{dt} = -\dot{W}_{cv} + h_e \frac{dm}{dt}$$

or

$$\dot{W}_{cv} = -\frac{dU}{dt} + h_e\frac{dm}{dt} \Rightarrow W_{cv} = -\Delta U + \int h_e\, dm \qquad (1)$$

We expect that the maximum theoretical work would be developed in the absence of irreversibilities within the tank and the turbine (assumption (4)). The data given are recognized as corresponding to those for Example 6.10. The solution to Example 6.10 shows that a typical unit of mass remaining in the tank would undergo an isentropic expansion from T_1, P_1 to T_2, P_2. Moreover, each unit of mass passing through the turbine expands isentropically. Accordingly, if P, T denote the pressure and temperature within the tank at a particular instant

$$\frac{p_r(T)}{p_r(T_1)} = \frac{P}{P_1} \qquad (2)$$

But P, T would also correspond to the condition of the mass entering the turbine. So

$$\frac{p_r(T_e)}{p_r(T)} = \frac{P_e}{P} \qquad (3)$$

Combining Eqs. (2), (3)

$$\frac{p_r(T_e)}{p_r(T_1)} = \frac{P_e}{P_1} \quad \rightarrow \quad \frac{p_r(T_e)}{p_r(T_1)} = \frac{P_2}{P_1} \quad \text{since } P_e = P_2$$

Then, with the result of Example 6.10, $T_e = T_2 = 317\text{ K}$.

Returning to Eq. (1), h_e is fixed by T_e, and thus a constant:

$$(W_{cv})_{MAX} = -\Delta U + h_e \Delta m$$

$$= (m_1 u_1 - m_2 u_2) + h_e(m_2 - m_1)$$

From Example 6.10, $m_2 = 1.58\text{ kg}$. With data from Table A-22

$$(W_{cv})_{MAX} = (5\text{ kg})\left(359.49\ \frac{kJ}{kg}\right) - (1.58\text{ kg})\left(226.28\ \frac{kJ}{kg}\right) + \left(317.28\ \frac{kJ}{kg}\right)(1.58 - 5)\text{ kg}$$

$$= 1797.45\text{ kJ} - 357.52\text{ kJ} - 1085.1\text{ kJ}$$

$$= 354.83\text{ kJ} \longleftarrow \text{MAX}$$

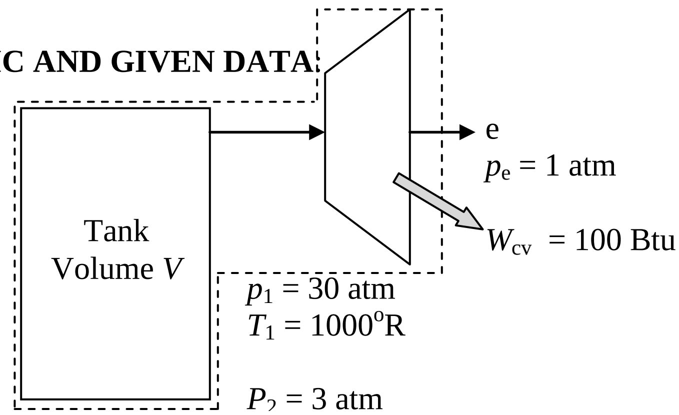
PROBLEM 6.167

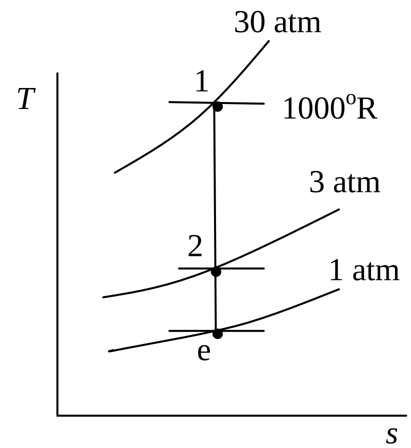
A tank initially containing air at 30 atm and 1000°R is connected to a small turbine. Air discharges from the tank through the turbine, which produces work in the amount of 100 Btu. The pressure in the tank falls to 3 atm during the process, and the turbine exhausts to the atmosphere at 1 atm. Employing the ideal gas model for the air with $k = 1.4$, and ignoring irreversibilities within the tank and the turbine, determine the volume of the tank, in ft^3. Heat transfer with the atmosphere and changes in kinetic and potential energy are negligible.

KNOWN: A tank initially filled with air is allowed to discharge through a turbine, developing work, until the pressure in the tank becomes 3 atm.

FIND: Determine the volume of the tank in the absence of irreversibilities.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL: (1) The control volume is shown on the accompanying figure. (2) For the control volume, $\dot{Q}_{\text{cv}} = 0$ and kinetic and potential energy effects are negligible. (3) The air is modeled as an ideal gas with constant specific heats ($k = 1.4$). (4) Irreversibilities are absent within the control volume.

ANALYSIS: A mass rate balance reduces to $dm/dt = -\dot{m}_{\text{e}}$. With this result, the energy rate balance can be expressed as

$$\frac{dU}{dt} = \cancel{\dot{Q}_{\text{cv}}} - \dot{W}_{\text{cv}} - \dot{m}_{\text{e}}h_{\text{e}}\ ;\ \frac{dU}{dt} = -\dot{W}_{\text{cv}} + h_{\text{e}}dm/dt \quad \longrightarrow \quad W_{\text{cv}} = -\Delta U + \int h_{\text{e}}dm \qquad (*)$$

We will evaluate the volume by using modeling assumption (4). Following the reasoning of Example 6.10, a typical unit of mass remaining in the tank would undergo an isentropic expansion from p_1, T_1 to p_2. Moreover, each unit of mass passing through the turbine expands isentropically. Hence, the relationship for the states in the tank, at the inlet to the turbine, and at the exit e is isentropic as illustrated on the T-s diagram.

Accordingly, for an ideal gas with constant specific heats, the pressure p and temperature T within the tank at any time and within the turbine are related by Eq. 6.43. Thus

$$\frac{T}{T_1} = \left(\frac{p}{p_1}\right)^{\frac{k-1}{k}}$$

PROBLEM 6.167 (CONTINUED)

At the exit

$$T_e = T_1\left(\frac{p_e}{p_1}\right)^{\frac{k-1}{k}} = (1000^{o}\text{R})\left(\frac{1\ \text{atm}}{30\ \text{atm}}\right)^{\frac{1.4-1}{1.4}} = 378.4^{o}\text{R}$$

and at the final state

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = (1000^{o}\text{R})\left(\frac{3\ \text{atm}}{30\ \text{atm}}\right)^{\frac{1.4-1}{1.4}} = 517.9^{o}\text{R}$$

Since the exit state is constant with time, and with $\Delta U = m_2u_2 - m_1u_1$, Eq. (*) integrates further to give

$$W_{cv} = m_1u_1 - m_2u_2 + h_e(m_2 - m_1)$$

For an ideal gas with constant specific heats, $u = c_v(T - T_{ref})$ and $h = c_p(T - T_{ref})$, where T_{ref} is an arbitrary reference temperature. Setting $T_{ref} = 0$, we get $u = c_vT$ and $h = c_pT$. Also, with the ideal gas equation of state $m_1 = p_1V/RT_1$ and $m_2 = p_2V/RT_2$. Incorporating these expression in the energy balance

$$W_{cv} = m_1c_vT_1 - m_2c_vT_2 + c_pT_e(m_2 - m_1)$$

$$= \left(\frac{p_1V}{RT_1}\right)c_vT_1 - \left(\frac{p_2V}{RT_2}\right)c_vT_2 + c_pT_e\left[\left(\frac{p_2V}{RT_2}\right) - \left(\frac{p_1V}{RT_1}\right)\right]$$

$$= \frac{V}{R}\left[c_v(p_1 - p_2) + c_pT_e\left(\frac{p_2}{T_2} - \frac{p_1}{T_1}\right)\right]$$

Now, solving for V

$$V = \frac{RW_{cv}}{\left[c_v(p_1 - p_2) + c_pT_e\left(\frac{p_2}{T_2} - \frac{p_1}{T_1}\right)\right]}$$

For air with $k = 1.4$: $c_v = R/(k - 1) = (1.986/29.97)/(1.4 - 1) = 0.1714$ Btu/lb·°R and $c_p = c_v + R = (0.1714) + (1.986/28.97) = 0.240$ Btu/lb·°R. With $p_1 = 30*14.7 = 441$ lbf/in.^{2}, $p_2 = 44.1$ lbf/in.^{2}, and $p_e = 14.7$ lbf/in.^{2}

$$V = \frac{\left(\frac{1545}{28.97}\right)\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^{o}\text{R}}(100\ \text{Btu})}{\left[\left(0.1714\frac{\text{Btu}}{\text{lb}\cdot{}^{o}\text{R}}\right)(441 - 44.1)\frac{\text{lbf}}{\text{in.}^2} + (0.240)(378.4)\left(\frac{44.1}{517.9} - \frac{441}{1000}\right)\right]\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right|} = 1.037\ \text{ft}^3$$

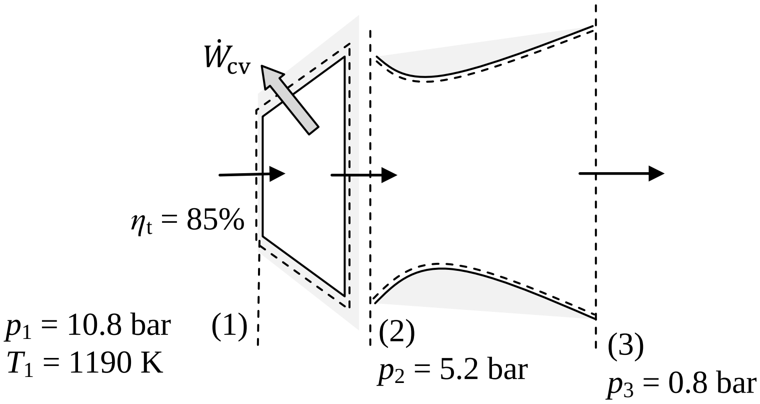
PROBLEM 6.168

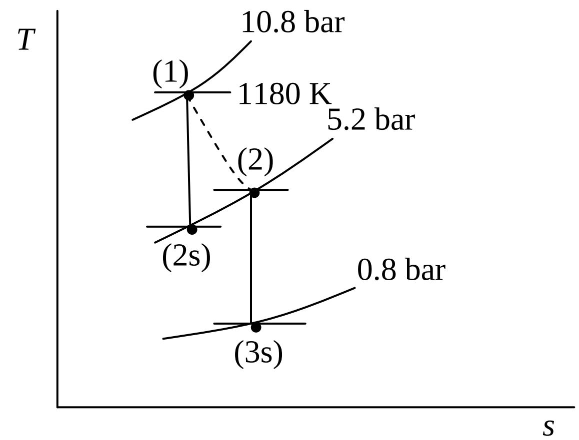
Air in a jet engine enters the turbine at 1190 K, 10.8 bar and expands to 5.2 bar. The air then flows through a nozzle and exits at 0.8 bar. Operation is at steady state, and the flow through the turbine and nozzle is adiabatic. The nozzle operates with no internal irreversibilities, and the isentropic turbine efficiency is 85%. The air velocities at the turbine inlet and exit are negligible. Assuming the ideal gas model for the air, determine the velocity of the air exiting the nozzle, in m/s.

KNOWN: Air flows in series through a turbine and a nozzle. Operating conditions at steady state are known.

FIND: Determine the velocity of the air at the exit of the nozzle.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL: (1) The control volumes are at steady state. (2) For both control volumes, $\dot{Q}_{cv}$ = 0, and for the nozzle $\dot{W}_{cv}$= 0. (3) Potential energy effects are negligible, and kinetic energy is neglected everywhere except at the nozzle exit. (4) The air is modeled as an ideal gas. (5) The nozzle operates with no internal irreversibilities.

ANALYSIS: First, fix each state. To find state 2, we use the isentropic turbine efficiency as follows

$$\eta_t = (h_1 - h_2)/(h_1 - h_{2s}) \quad \longrightarrow \quad h_2 = h_1 - \eta_t(h_1 - h_{2s})$$

To find h_{2s} we use the isentropic relation $p_r(T_{2s}) = p_r(T_1)(p_2/p_1)$. With data from Table A-22

$$p_r(T_{2s}) = (230.1)(5.2/10.8) = 110.79 \quad \longrightarrow \quad h_{2s} \approx 1037.7 \text{ kJ/kg } (T_{2s} \approx 992.7 \text{ K})$$

Now

$$h_2 = 1266.1 - (0.85)(1266.1 - 1037.7) = 1072.0 \text{ kJ/kg}$$

Interpolating: $T_2 \approx 1022.7$ K and $p_r(T_2) \approx 124.74$. Thus

$$p_r(T_{3s}) = p_r(T_2)(p_3/p_2) = (124.74)(0.8/5.2) = 19.191 \quad \longrightarrow \quad h_{3s} \approx 634.0 \text{ kJ/kg } (T_{3s} \approx 625.6 \text{ K})$$

PROBLEM 6.168 (CONTINUED)

The energy rate balance for the nozzle reduces to give

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\left[(h_2 - h_{3s}) + \frac{(\cancel{V_2^2} - V_3^2)}{2} + g\cancel{(z_2 - z_3)}\right]$$

Solving for the exit velocity

$$V_2 = \sqrt{2(h_2 - h_{3s})}$$

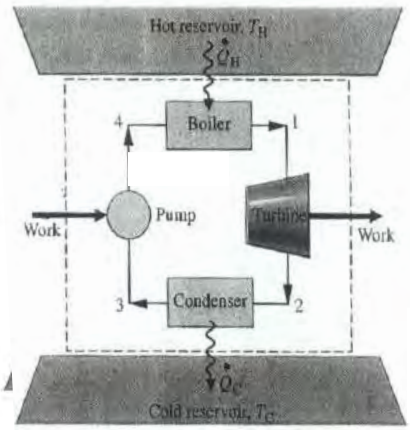
$$= \sqrt{2(1072.0 - 634.0)\text{kJ/kg}\left|\frac{10^3\ \text{N}\cdot\text{m}}{1\ \text{kJ}}\right|\left|\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right|} = 935.9\ \text{m/s} \longleftarrow$$

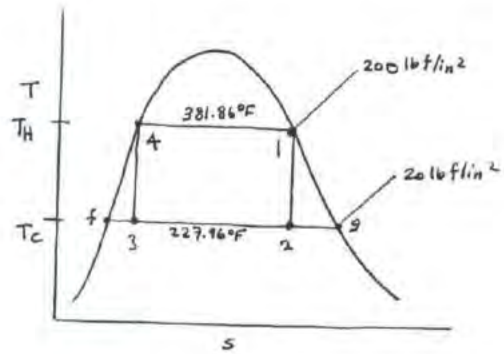
PROBLEM 6.169

KNOWN: Steady-state operating data are provided for a Carnot power cycle.

FIND: Sketch the cycle on T-s coordinates. Also, evaluate $\dot{Q}/\dot{m}$ and $\dot{W}/\dot{m}$ for each process and the thermal efficiency of the overall cycle

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL

1. A control volume at steady state encloses each of the four components.
2. For the turbine and pump, $\dot{Q}_{cv} = 0$.
3. Kinetic and potential energy effects are ignored.

ANALYSIS: From Table A-4E,

$h_1 = 1199.3$ Btu/lb, $s_1 = 1.5465$ Btu/lb·°R.

$h_4 = 355.6$ Btu/lb, $s_4 = 0.544$ Btu/lb·°R

$s_2 = s_1 \Rightarrow x_2 = \dfrac{s_2 - s_f}{s_g - s_f} = \dfrac{1.5465 - 0.3358}{1.3962} = 0.867$

$s_4 = s_3 \Rightarrow x_3 = \dfrac{s_3 - s_f}{s_g - s_f} = \dfrac{0.544 - 0.3358}{1.3962} = 0.149$

Process 1-2: $\dot{Q}_{12}/\dot{m} = 0$.

$\dfrac{\dot{W}_t}{\dot{m}} = h_1 - h_2$, $h_2 = h_f + x_2(h_g - h_f)$
$= 196.26 + 0.867(960.1)$
$= 1028.67$ Btu/lb

$\therefore \dfrac{\dot{W}_t}{\dot{m}} = (1199.3 - 1028.67) = 170.63 \dfrac{\text{Btu}}{\text{lb}}$

Process 3-4: $\dot{Q}_{34}/\dot{m} = 0$

$\dfrac{\dot{W}_p}{\dot{m}} = h_3 - h_4$, $h_3 = h_f + x_3(h_g - h_f)$
$= 196.26 + 0.149(960.1)$
$= 339.31$ Btu/lb

$\therefore \dfrac{\dot{W}_p}{\dot{m}} = (339.31 - 355.6) = -16.29 \dfrac{\text{Btu}}{\text{lb}}$

Process 2-3: $\dot{W}_{23}/\dot{m} = 0$.

$\dfrac{\dot{Q}_{23}}{\dot{m}} = h_3 - h_2 = 339.31 - 1028.67$
$= -689.36$ Btu/lb

or $\dfrac{\dot{Q}_{23}}{\dot{m}} = T_C(s_3 - s_2) = 687.63°R\,(0.544 - 1.5465)\dfrac{\text{Btu}}{\text{lb·°R}}$
$= -689.35$ Btu/lb

Process 4-1: $\dot{W}_{41}/\dot{m} = 0$

$\dfrac{\dot{Q}_{41}}{\dot{m}} = h_1 - h_4 = (1199.3 - 355.6)$
$= 843.7$ Btu/lb

or $\dfrac{\dot{Q}_{41}}{\dot{m}} = T_H(s_1 - s_4) = 841.53°R\,(1.5465 - 0.544)\dfrac{\text{Btu}}{\text{lb·°R}}$
$= 843.63$ Btu/lb

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \frac{170.63 - 16.29}{843.7} = 0.183$$

or

$$\eta = \frac{T_H - T_C}{T_H} = \frac{841.53 - 687.63}{841.53} = 0.183$$

PROBLEM 6.170

The figure shows a Carnot heat pump cycle operating at steady state with ammonia as the working fluid. The condenser temperature is 120°F, with saturated vapor entering and saturated liquid exiting. The evaporator temperature is 10°F.

(a) Determine the heat transfer and work for each process, in Btu per lb of ammonia flowing.
(b) Evaluate the coefficient of performance for the heat pump.
(c) Evaluate the coefficient of performance for a Carnot refrigeration cycle operating as shown in the figure.

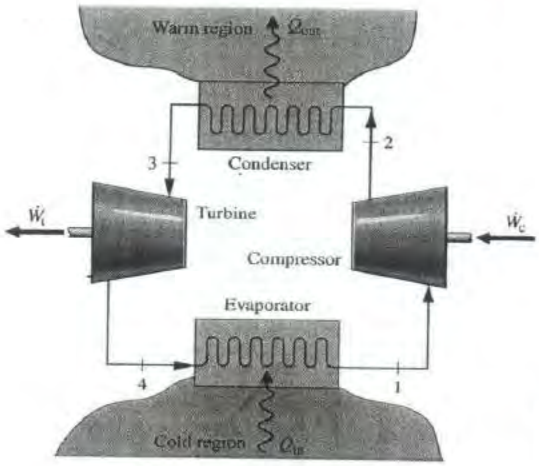
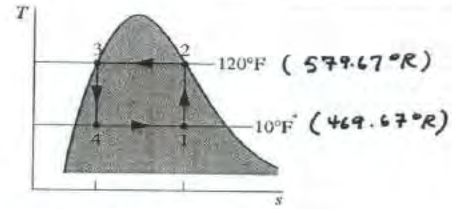


Fig P 6.35



ENGR. MODEL:

1. Each of the four components is enclosed by a control volume at steady state.
2. Ammonia circulating through the components executes a Carnot heat pump cycle.
3. Energy transfers are positive in the directions of the arrows.

ANALYSIS:

(a) Process 1-2,

$$\frac{\dot{Q}_{12}}{\dot{m}} = \int_1^2 T ds = 0$$

An energy rate balance reduces to give $0 = \cancel{\dot{Q}_{12}} - \dot{W}_{12} + \dot{m}(h_1 - h_2) \Rightarrow \frac{\dot{W}_{12}}{\dot{m}} = h_1 - h_2$.

$h_2 = 632.95$ Btu/lb, $s_2 = 1.1405$ Btu/lb·°R (Table A-13E). To find h_1 use $s_1 = s_2$.

Then $x_1 = \frac{1.1405 - 0.1196}{1.3141 - 0.1196} = 0.8547 \Rightarrow h_1 = h_f + x_1 h_{fg} = 53.27 + 0.8547(561) = 532.76 \frac{\text{Btu}}{\text{lb}}$

So, $$\frac{\dot{W}_{12}}{\dot{m}} = h_1 - h_2 = 532.76 - 632.95 = -100.19 \text{ Btu/lb}$$

Process 2-3. $\dot{W}_{23}/\dot{m} = 0$

① $$\frac{\dot{Q}_{23}}{\dot{m}} = \int_2^3 T ds = T(s_3 - s_2) = 579.67°R(0.3570 - 1.1405) \frac{\text{Btu}}{\text{lb}\cdot°R} = -454.17 \frac{\text{Btu}}{\text{lb}}$$

Process 3-4.

$$\frac{\dot{Q}_{34}}{\dot{m}} = \int_3^4 T ds = 0$$

$$\frac{\dot{W}_{34}}{\dot{m}} = h_3 - h_4$$

To find h_4 use $s_4 = s_3 = 0.3570$. Then, $x_4 = \frac{0.3570 - 0.1196}{1.3141 - 0.1196} = 0.1987$ and $h_4 = h_f + x_4 h_{fg} = 53.27 + 0.1987(561) = 164.74$. Finally

$$\frac{\dot{W}_{34}}{\dot{m}} = 178.79 - 164.74 = 14.05 \text{ Btu/lb}$$

Process 4-1. $\dot{W}_{41}/\dot{m} = 0$

② $\frac{\dot{Q}_{41}}{\dot{m}} = \int_4^1 T\,ds = T\underbrace{(s_1 - s_4)}_{=(s_2-s_3)} = 469.67°R(1.1405 - 0.3570)\frac{Btu}{lb \cdot °R} = 367.99\ \frac{Btu}{lb}$

(b) The coefficient of performance for the heat pump shown in the figure is

$$\gamma = \frac{\dot{Q}_{out}}{\underbrace{\dot{W}_c - \dot{W}_t}_{\text{All quantities are positive}}} = \frac{|\dot{Q}_{23}/\dot{m}|}{\left|\frac{\dot{W}_{12}}{\dot{m}}\right| - \frac{\dot{W}_{34}}{\dot{m}}} = \frac{454.17}{100.19 - 14.05} = 5.27$$

Alternatively, Eq. 5.11 can be used

$$\gamma = \frac{T_H}{T_H - T_C} = \frac{579.67°R}{110°R} = 5.27$$

(c) The coefficient of performance for a Carnot refrigeration cycle obtained using Eq. 5.10 is

③ $$\beta = \frac{T_C}{T_H - T_C} = \frac{469.67°R}{110°R} = 4.27$$

---

1. Applying an energy rate balance, $\frac{\dot{Q}_{23}}{\dot{m}} = h_3 - h_2 = h_f - h_g = -454.16$ Btu/lb.

2. For any cycle $\dot{Q}_{net}/\dot{m} = \dot{W}_{net}/\dot{m}$. Thus, to check the calculations

$$\dot{Q}_{net}/\dot{m} = 0 + (-454.17) + 0 + (367.99) = -86.18\ Btu/lb$$

$$\dot{W}_{net}/\dot{m} = (-100.19) + 0 + (14.05) + 0 = -86.14\ Btu/lb$$

3. Alternatively

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_c - \dot{W}_t} = \frac{\dot{Q}_{41}/\dot{m}}{\left|\frac{\dot{W}_{12}}{\dot{m}}\right| - \frac{\dot{W}_{34}}{\dot{m}}} = \frac{367.99}{100.19 - 14.05} = 4.27$$

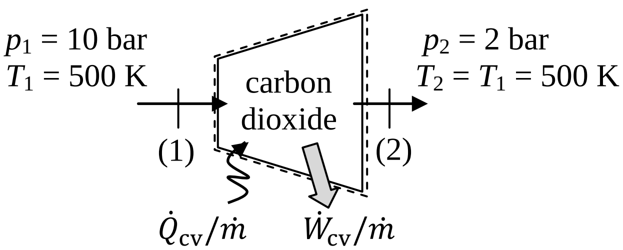
PROBLEM 6.171

Carbon dioxide (CO_2) expands isothermally at steady state with no irreversibilities through a turbine from 10 bar, 500 K to 2 bar. Assuming the ideal gas model and neglecting kinetic and potential energy effects, determine the heat transfer and work, each in kJ per kg of carbon dioxide flowing.

KNOWN: Carbon dioxide expands isothermally at steady state though a turbine. The inlet state and exit pressure are specified.

FIND: Determine the work and heat transfer, each per unit mass of carbon dioxide flowing.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The control volume is at steady state. (2) The process is isothermal and occurs with no internal irreversibilities. (3) The carbon dioxide is modeled as an ideal gas. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: Applying Eq. 6.49 for the internally reversible isothermal process from inlet to exit

$$\left(\frac{\dot{Q}_{cv}}{\dot{m}}\right)_{\substack{\text{int}\\\text{rev}}} = \int_1^2 T\,ds = T\,(s_2 - s_1) = T\,[\cancel{s^o(T_2) - s^o(T_1)} - R\ln(p_2/p1)]$$

$$= (500\text{ K})[-(8.314/44.01)\text{ kJ/kg·K}\ln(2/10)] = 152.02\text{ kJ/kg (in)} \longleftarrow$$

Now, the mass and energy rate balances reduce for the isothermal process of the ideal gas to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[\cancel{(h_1 - h_2)} + \cancel{\frac{(V_1^2 - V_2^2)}{2}} + \cancel{g(z_1 - z_2)}\right]$$

Thus

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{\text{int}\\\text{rev}}} = \left(\frac{\dot{Q}_{cv}}{\dot{m}}\right)_{\substack{\text{int}\\\text{rev}}} = 152.02\text{ kJ/kg (out)} \longleftarrow$$

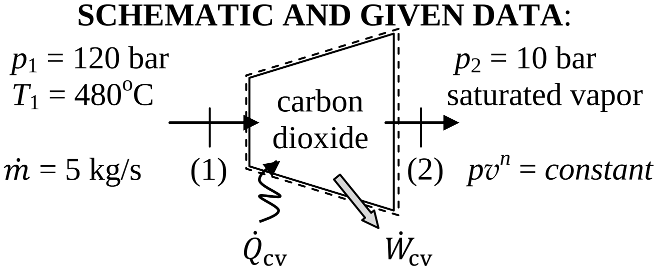
PROBLEM 6.172

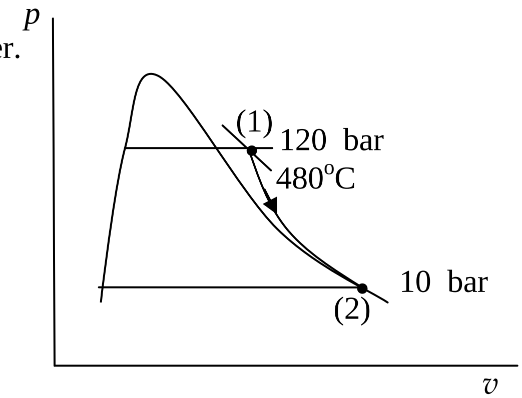
Steam at 12.0 MPa, 480°C expands through a turbine operating at steady state to 10 bar, saturated vapor. The process follows $pv^n = constant$ and occurs with negligible effects of kinetic or potential energy. The mass flow rate of steam is 5 kg/s. Determine the power developed and the rate of heat transfer, each in kW.

KNOWN: Steam enters a turbine operating at steady state and expands in an internally reversible process according to $pv^n = constant$.

FIND: Determine the power developed and the rate of heat transfer.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL: (1) The control volume is at steady state. (2) The process is internally reversible with $pv^n = constant$. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: The power developed is determined by using Eq. 6.53, as follows

$$\left(\dot{W}_{cv}\right)_{\substack{int\\rev}} = -\dot{m}\left(\frac{n}{n-1}\right)(p_2v_2 - p_1v_1)$$

From Table A-4: v_1 = 0.02576 m³/kg and h_1 = 3293.5 kJ/kg. From Table A-3: v_2 = 0.1944 m³/kg and h_2 = 2778.1 kJ/kg. For the polytropic process

$$p_2/p1 = (v_1/v_2)^n \longrightarrow n = \ln(p_2/p_1)/\ln(v_1/v_2) = \ln(10/120)/\ln(0.02576/0.1944) = 1.229$$

Thus

$$\dot{W}_{cv} = -(5\text{ kg/s})(1.229/0.229)[(10\text{ bar})(0.1944\text{ m}^3\text{/kg}) - (120)(0.02576)]\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$

$$= 3078\text{ kW} \longleftarrow$$

Now, applying the mass and energy rate balances with modeling assumption (3)

$$\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}(h_2 - h_1)$$

$$= (3078\text{ kW}) + (5\text{ kg/s})(2778.1 - 3293.5)\text{kJ/kg}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = 501\text{ kW} \longleftarrow$$

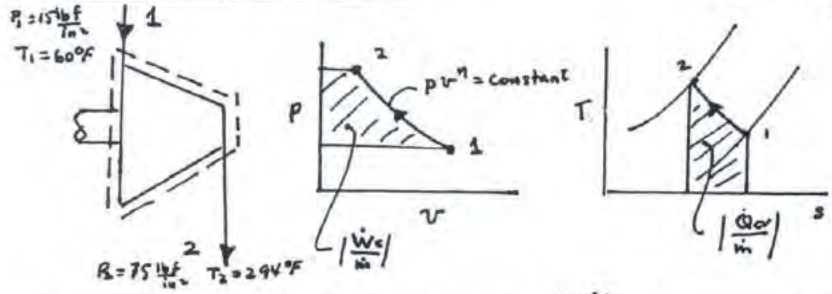
PROBLEM 6.173

An air compressor operates at steady state with air entering at $p_1 = 15$ lbf/in.^{2}, $T_1 = 60°F$. The air undergoes a polytropic process, and exits at $p_2 = 75$ lbf/in.^{2}, $T_2 = 294°F$. (a) Evaluate the work and heat transfer, each in Btu per lb of air flowing. (b) Sketch the process on p–v and T–s diagrams and associate areas on the diagrams with work and heat transfer, respectively. Assume the ideal gas model for air and neglect changes in kinetic and potential energy.

ENGR MODEL:

1. The control volume shown in the sketch is at steady state.
2. The air undergoes a process described by $pv^n = \text{constant}$.
3. Kinetic and potential energy effects can be neglected.
4. The air is modeled as an ideal gas.

SCHEMATIC & GIVEN DATA:



(b)

ANALYSIS: With Eq. 3.56, $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(n-1)/n} \Rightarrow \frac{754°R}{520°R} = \left(\frac{75\ lbf/in^2}{15\ lbf/in^2}\right)^{(n-1)/n} \Rightarrow n = 1.3$

(a) With Eq. 6.55a

$$\frac{\dot W_{cv}}{\dot m} = \frac{-nR}{(n-1)}(T_2 - T_1)$$

$$= \frac{-1.3}{(1.3-1)}\left(\frac{1.986}{28.97}\ \frac{Btu}{lb\cdot °R}\right)(754-520)°R = -69.5\ \frac{Btu}{lb}$$

Reducing an energy balance, $0 = \dot Q_{cv} - \dot W_{cv} + \dot m(h_1 - h_2)$

$$\Rightarrow \quad \dot Q_{cv} = \dot W_{cv} + \dot m(h_2 - h_1)$$

$$\frac{\dot Q_{cv}}{\dot m} = \frac{\dot W_{cv}}{\dot m} + (h_2 - h_1)$$

$$= -69.5\ \frac{Btu}{lb} + (180.63 - 124.27)\ \frac{Btu}{lb}$$

$$= -13.14\ Btu/lb$$

(a)

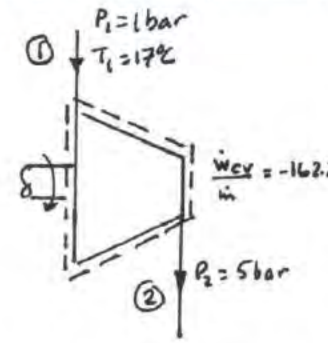
where h_1 and h_2 are from Table A-22E.

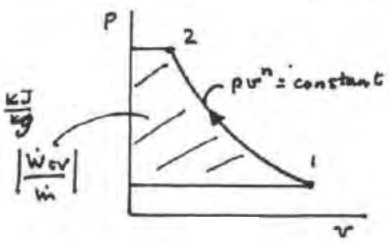
PROBLEM 6.174

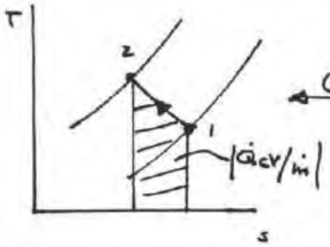
An air compressor operates at steady state with air entering at $p_1 = 1$ bar, $T_1 = 17°C$ and exiting at $p_2 = 5$ bar. The air undergoes a polytropic process for which the compressor work input is 162.2 kJ per kg of air flowing. Determine (a) the temperature of the air at the compressor exit, in °C, and (b) the heat transfer, in kJ per kg of air flowing. (c) Sketch the process on p–v and T–s diagrams and associate areas on the diagrams with work and heat transfer, respectively. Assume the ideal gas model for air and neglect changes in kinetic and potential energy.

ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. The air undergoes a process described by $pv^n = \text{constant}$.
3. Kinetic and potential energy effects can be neglected.
4. The air is modeled as an ideal gas.







← (c)

ANALYSIS: Using Eq. 6.55b

$$\frac{\dot{W}_{cv}}{\dot{m}} = -\frac{nRT_1}{(n-1)}\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] \Rightarrow -162.2\ \frac{kJ}{kg} = \frac{-n}{(n-1)}\left(\frac{8.314}{28.97}\ \frac{kJ}{kg \cdot K}\right)(290\,K)\left[\left(\frac{5}{1}\right)^{\frac{(n-1)}{n}} - 1\right]$$

$$\Rightarrow n = 1.3$$

(a) With Eq. 3.56,

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} = 290\,K\left(\frac{5}{1}\right)^{0.3/1.3} = 420.4\,K \quad \longleftarrow (a)$$

(b) Reducing an energy rate balance, $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2)$

$$\Rightarrow \frac{\dot{Q}_{cv}}{\dot{m}} = \frac{\dot{W}_{cv}}{\dot{m}} + (h_2 - h_1)$$

$$= -162.2\ \frac{kJ}{kg} + (421.67 - 290.16)$$

$$= -30.69\ \frac{kJ}{kg} \quad \longleftarrow (b)$$

where h_1 and h_2 are from Table A-22.

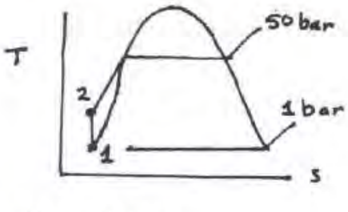
PROBLEM 6.175

Water as saturated liquid at 1 bar enters a pump operating at steady state and is pumped isentropically to a pressure of 50 bar. Kinetic and potential energy effects are negligible. Determine the pump work input, in kJ per kg of water flowing, using (a) Eq. 6.51c, (b) an energy balance. Obtain data from Table A-3 and A-5, as appropriate. Compare the results of parts (a) and (b), and comment.

KNOWN: Steady-state data are provided for a pump operating isentropically.

FIND: Determine the pump work input, in kJ per kg of mass flowing, two specified ways.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. A control volume encloses the pump.
2. Operation is at steady state.
3. Water flows through the pump isentropically, and thus $\dot{Q}_{cv} = 0$.
4. Kinetic and potential energy effects are ignored.

ANALYSIS:

(a) Eq. 6.51c applies for internally reversible processes, with or without heat transfer. In this application, the special case of an isentropic (internally reversible and adiabatic) process is considered. Taking the specific volume as $v_f(1\,bar)$, we get

$$\left(-\frac{\dot{W}}{\dot{m}}\right) = \int_1^2 v\,dp = v_1(P_2 - P_1) = \left(\frac{1.0432}{10^3}\,\frac{m^3}{kg}\right)(50-1)\,bar\left|\frac{10^5 N/m^2}{1\,bar}\right|\left|\frac{1\,kJ}{10^3 N\cdot m}\right|$$

1
$$= 5.11\,\frac{kJ}{kg}$$

(b) Reducing mass and energy rate balances,

$$\left(-\frac{\dot{W}}{\dot{m}}\right) = h_{2s} - h_1$$

From Table A-3 at 1 bar, $h_1 = h_f = 417.46\,\frac{kJ}{kg}$, $s_1 = s_f = 1.3026\,\frac{kJ}{kg\cdot K}$

Interpolating in Table A-5 at 50 bar, $h_2 = 422.57\,kJ/kg$, $v_2 = \left(\frac{1.041}{10^3}\right)\frac{m^3}{kg}$.

Then

$$\left(-\frac{\dot{W}}{\dot{m}}\right) = (422.57 - 417.46)\,\frac{kJ}{kg} = 5.11\,\frac{kJ}{kg}$$

Discussion: The value of pump work obtained by these methods are in agreement, as expected. Observe that the Tds equation, Eq. 6.10b, reduces when entropy is constant as follows:

$$\cancel{T}ds = dh - v\,dp$$

1
$$\Rightarrow \quad h_2 - h_1 = \int v\,dp = v_{ave}\Delta p$$

---

1. In this application, the average specific volume: $(v_1 + v_2)/2 = \left(\frac{1.0421}{10^3}\right)\frac{m^3}{kg}$ gives $v_{ave}\Delta p = 5.10\,kJ/kg$.

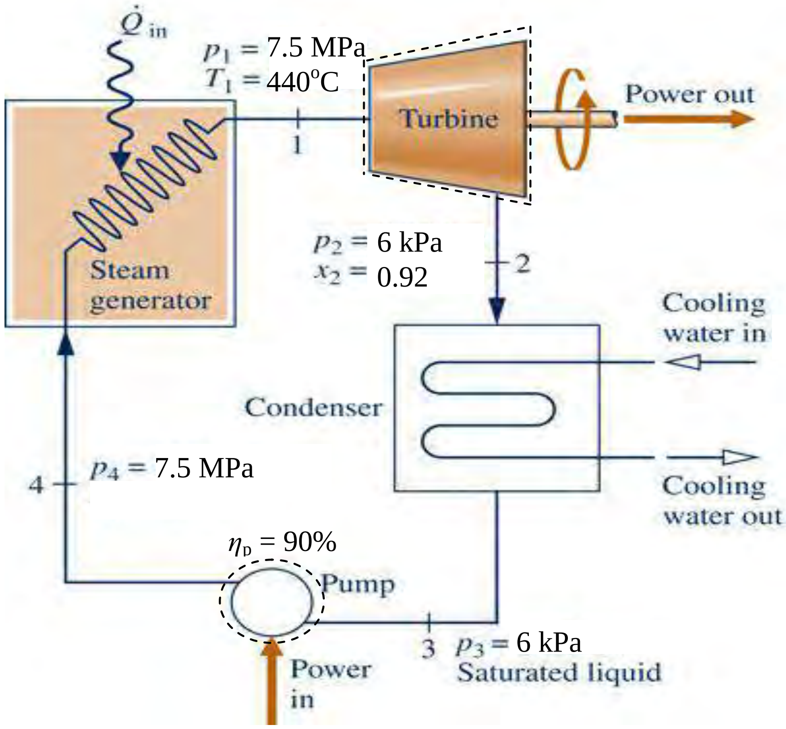
PROBLEM 6.176

Fig. P6.176 shows a vapor power plant operating at steady state. Data at key locations are given on the figure. The turbine and pump operate adiabatically, and kinetic and potential energy effects can be neglected. The isentropic pump efficiency is 90%. For such a vapor power cycle, the *back-work ratio* is the ratio of the pump work input to the turbine work output. Determine the back-work ratio
(a) using data interpolated from Table A-5 to obtain the specific enthalpy state 4.
(b) using the approximation of Eq. 6.51c to obtain the specific enthalpy at state 4.
Compare the results of parts (a) and (b) and discuss.

KNOWN: Data are known for the steady-state operation of a vapor power plant.

FIND: Determine the back-work ratio using data from Table A-5 and the approximation of Eq. 6.51c to obtain the specific enthalpy at the exit of the pump.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The control volumes are at steady state. (2) For the turbine and the pump, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible.

ANALYSIS: First, determine the turbine work. For the turbine, mass and energy rate balances reduce to give: $\dot{W}_t/\dot{m} = (h_1 - h_2)$.

From Table A-4: h_1 = 3253.9 kJ/kg. With data from Table A-2:

$$h_2 = h_{f2} + x_2(h_{g2} - h_{f2}) = 151.53 + (0.92)(2567.4 - 151.53) = 2374.1 \text{ kJ/kg}$$

The turbine work is

$$\dot{W}_t/\dot{m} = 3253.9 - 2374.1 = 879.8 \text{ kJ/kg}$$

PROBLEM 6.176 (CONTINUED)

The pump work *input* is $\dot{W}_p/\dot{m}$ = (h_4 – h_3). From Table A-2: h_3 = h_{f3} = 151.53 kJ/kg. To determine h_4 we use the isentropic pump efficiency: $\eta_p = (\dot{W}_p/\dot{m})_s/(\dot{W}_p/\dot{m}) = (h_{4s} - h_3)/(h_4 - h_3)$

(a) Interpolating in Table A-5 with $s_{4s} = s_3$ = 0.5210 kJ/kg·K and p_4 = 7.5 MPa we get $h_{4s} \approx$ 159.53 kJ/kg. Thus

$$h_4 = h_3 + (h_{4s} - h_3)/\eta_p = 151.53 + (159.53 - 151.53)/(0.9) = 160.42 \text{ kJ/kg}$$

and

$$\dot{W}_p/\dot{m} = 160.42 - 151.53 = 8.89 \text{ kJ/kg}$$

The back-work ratio is

$$bwr = (\dot{W}_p/\dot{m})/(\dot{W}_t/\dot{m}) = 8.89/879.8 = 0.010\ (1.0\%) \longleftarrow$$

(b) Approximating the ideal pump work using Eq. 6.51c we get

$$(\dot{W}_p/\dot{m})_s \approx v_3(p_4 - p_3)$$

$$= (1.0064 \times 10^{-3}\ \text{m}^3/\text{kg})(75 - 0.06)\text{bar}\left|\frac{10^5\,\text{N/m}^2}{1\ \text{bar}}\right|\left|\frac{1\ \text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right| = 7.542 \text{ kJ/kg}$$

Now, $h_4 = h_3 + (\dot{W}_p/\dot{m})_s/\eta_p$ = 151.53 + 7.542/0.9 = 159.91 kJ/kg. Thus

$$\dot{W}_p/\dot{m} = 159.91 - 151.53 = 8.38 \text{ kJ/kg}$$

The back-work ratio is

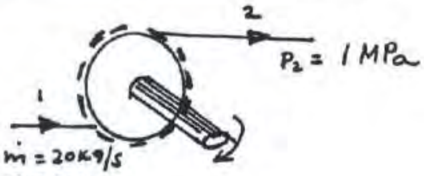
$$bwr = (\dot{W}_p/\dot{m})/(\dot{W}_t/\dot{m}) = 8.38/879.8 = 0.0095\ (0.95\%) \longleftarrow$$

Note: The values obtained using the two methods are very close to each other. The method of approximation in part (b) is commonly used in vapor power plant analyses (See Chapter 8).

PROBLEM 6.177

A pump operating at steady state receives saturated liquid water at 50°C with a mass flow rate of 20 kg/s. The pressure of the water at the pump exit is 1 MPa. If the pump operates with negligible internal irreversibilities and negligible changes in kinetic and potential energy, determine the power required in kW.

SCHEMATIC & GIVEN DATA:



$\dot{m} = 20\ kg/s$
$T_1 = 50°C$
Sat. liquid

ENGR. MODEL:

1. The control volume shown in the sketch operates at steady state.
2. Internal irreversibilities are negligible.
3. Kinetic and potential energy effects can be ignored.
4. $v \approx v_f(T_1)$.

ANALYSIS: Invoking Eq. 6.51c

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = -\underbrace{v}_{\approx v_f(50°C)}(P_2 - \overset{P_{sat}\ at\ 50°C}{P_1}) = -\left(\frac{1.0121}{10^3}\frac{m^3}{kg}\right)\left(10^6\frac{N}{m^2} - 0.1235\times 10^5\frac{N}{m^2}\right)\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

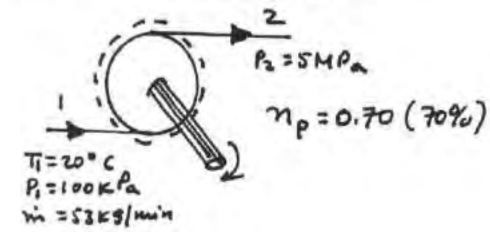
$$= -1\ \frac{kJ}{kg}$$

$$\therefore\ (\dot{W}_{cv})_{\substack{int\\rev}} = 20\ \frac{kg}{s}\left(-1\ \frac{kJ}{kg}\right)\left|\frac{1\ kW}{1\ kJ/s}\right| = -20\ kW \longleftarrow$$

PROBLEM 6.178

A pump operating at steady state receives liquid water at 20 °C, 100 kPa with a mass flow rate of 53 kg/min. The pressure of the water at the pump exit is 5 MPa. The isentropic pump efficiency is 70%. Stray heat transfer and changes in kinetic and potential energy are negligible. Determine the power required by the pump, in kW.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The control volume shown in the sketch operates at steady state.
2. Kinetic and potential energy effects can be ignored.
3. $v \approx v_f(T_1)$

ANALYSIS: Invoking Eq. 6.51c,

$$(\dot{W}_{cv})_{\substack{int\\rev}} = -\dot{m}\,v(P_2 - P_1) = -53\frac{kg}{min}\left(\frac{1.0018}{10^3}\frac{m^3}{kg}\right)\left(5\times10^6\frac{N}{m^2} - 10^5\frac{N}{m^2}\right)\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|\left|\frac{1\,kW}{1\,kJ/s}\right|\left|\frac{1\,min}{60\,s}\right|$$

$$= -4.34\,kW$$

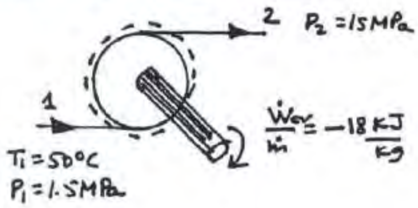
Using the pump efficiency,

$$-\dot{W} = \frac{(-\dot{W}_{cv})_{\substack{int\\rev}}}{\eta_p} = \frac{4.34\,kW}{0.70} = 6.19\,kW$$

PROBLEM 6.179

A pump operating at steady state receives liquid water at 50°C, 1.5 MPa. The pressure of the water at the pump exit is 15 MPa. The magnitude of the work required by the pump is 18 kJ per kg of water flowing. Stray heat transfer and changes in kinetic and potential energy are negligible. Determine the isentropic pump efficiency.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL

1. The control volume shown in the sketch operates at steady state.
2. Kinetic and potential energy effects can be ignored.
3. $v \approx v_f(T_1)$.

ANALYSIS: Invoking Eq. 6.51c,

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = -v(P_2 - P_1)$$

$$= -\left(\frac{1.0121}{10^3}\frac{m^3}{kg}\right)\left(15\times10^6\frac{N}{m^2} - 1.5\times10^6\frac{N}{m^2}\right)\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|$$

$$= -13.66\,\frac{kJ}{kg}$$

Then,

$$\eta_{pump} = \frac{(\dot{W}_{cv}/\dot{m})_{\substack{int\\rev}}}{(\dot{W}_{cv}/\dot{m})} = \frac{13.66}{18} = 0.759 \quad (75.9\%) \longleftarrow$$

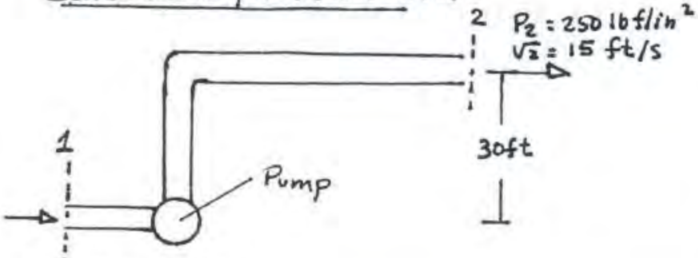
PROBLEM 6.180

Liquid water at 70°F, 14.7 lbf/in.^{2} and a velocity of 30 ft/s enters a system at steady state consisting of a pump and attached piping and exits at a point 30 ft above the inlet at 250 lbf/in.^{2}, a velocity of 15 ft/s, and no significant change in temperature. (a) In the absence of internal irreversibilities, determine the power input required by the system, in Btu per lb of liquid water flowing. (b) For the same inlet and exit states, in the presence of friction would the power input be greater, or less, than determined in part (a)? Explain. Let g = 32.2 ft/s^2.

KNOWN: Steady-state data are provided for pumping system that delivers water at a higher elevation.

FIND: Determine the power input required per lb of liquid flowing in the absence of internal irreversibilities and comment.

SCHEMATIC & GIVEN DATA:



Liquid water
$T_1 = 70°F$
$P_1 = 14.7$ lbf/in^2
$V_1 = 30$ ft/s

ENGINEERING MODEL:
1. A control volume encloses the pumping system.
2. Operation is at steady state.
3. Liquid water having a specific volume of $v_f(70°F)$ flows throughout.
4. In part (a), there are no internal irreversibilities.

ANALYSIS: (a) In the absence of internal irreversibilities Eq. 6.51a applies. Then, with assumption 3 we get

$$\left(-\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = v_f\,[P_2 - P_1] + \frac{V_2^2 - V_1^2}{2} + g\,(z_2 - z_1)$$

Table A-2E at 70°F

$$= \left(0.01605\,\frac{ft^3}{lb}\right)(250 - 14.7)\,\frac{lbf}{in^2}\left|\frac{12\,in}{1\,ft}\right|^2 + \left[\frac{(15)^2 - (30)^2}{2}\right]\left[\frac{ft}{s}\right]^2\left|\frac{1\,lbf}{32.2\,lb\cdot ft/s^2}\right|$$

$$+ \left(32.2\,\frac{ft}{s^2}\right)(30\,ft)\left|\frac{1\,lbf}{32.2\,lb\cdot ft/s^2}\right|$$

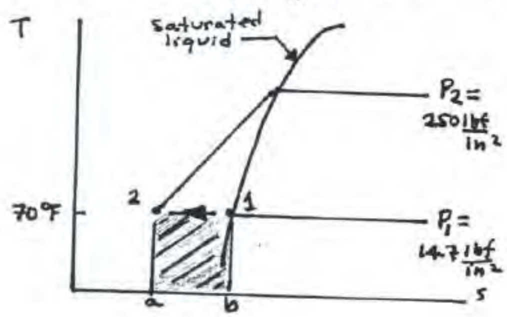
① $$= [543.8 - 10.5 + 30]\,\frac{ft\cdot lbf}{lb}\left|\frac{1\,Btu}{778\,ft\cdot lbf}\right|$$

$$= 0.72\ \text{Btu/lb}$$

(b) Since internal irreversibilities are absent, we expect the value obtained to be the minimum theoretical work input. The effect of friction and like irreversibilities is expected to require a greater work input and thus a greater accompanying heat transfer from the system.

---

1. By showing the constant-temperature process on a T-s diagram, we can represent the accompanying heat transfer per lb flowing as an area:



$$\left(\frac{\dot{Q}_{\substack{int\\rev}}}{\dot{m}}\right) = \int_1^2 T\,ds = T(s_2 - s_1)$$

⇒ Heat transfer is from the system. It's magnitude is represented by area 1-2-a-b-1.

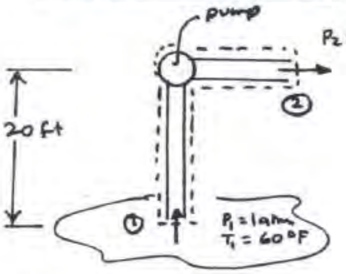
PROBLEM 6.181

A 3-hp pump operating at steady state draws in liquid water at 1 atm, 60°F and delivers it at 5 atm at an elevation 20 ft above the inlet. There is no significant change in velocity between the inlet and exit, and the local acceleration of gravity is 32.2 ft/s². Would it be possible to pump 1000 gal in 10 min or less? Explain.

KNOWN: Steady-state operating data are provided for a pump.

FIND: Determine if it would be possible to pump 1000 gallons of water in 10 min or less.

SCHEMATIC & GIVEN DATA:



$\dot{W}_{cv} = -3\,hp$

$g = 32.2\,ft/s^2$

ASSUMPTIONS: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, kinetic energy effects can be ignored. 3. Liquid water is modeled as incompressible with $v = v_f(T_1) = 0.01604\ ft^3/lb$ (Table A-2E).

ANALYSIS: To pump 1000 gal in 10 minutes, or less, would require a volumetric flow rate (AV)

$$(AV) \geq \left(\frac{1000\ gal}{10\ min}\right)\left|\frac{0.13368\ ft^3}{gal}\right|\left|\frac{1\ min}{60\ s}\right| = 0.2228\ \frac{ft^3}{s}$$

which corresponds to a mass flow rate

$$\dot{m} \geq \frac{0.2228\ ft^3/s}{0.01604\ ft^3/lb} = 13.89\ \frac{lb}{s} \qquad (1)$$

In the absence of internal irreversibilities, the power input required would be given by the following expression obtained from Eq. 6.51a using assumptions 2, 3

$$(-\dot{W}_{cv})_{\substack{int\\rev}} = \dot{m}\left[v(p_2 - p_1) + g(z_2 - z_1) + \cancelto{0}{\frac{(V_2^2 - V_1^2)}{2}}\right]$$

Then, with (1)

$$(-\dot{W}_{cv})_{\substack{int\\rev}} \geq \left(13.89\ \frac{lb}{s}\right)\left[\left(0.01604\ \frac{ft^3}{lb}\right)\left(4 \times 14.7 \times 144\ \frac{lbf}{ft^2}\right) + \left(32.2\ \frac{ft}{s^2}\right)[20\ ft]\left|\frac{1\ lbf}{32.2\ lb\cdot ft/s^2}\right|\right]$$

$$\geq \left(13.89\ \frac{lb}{s}\right)\left[135.81\ \frac{ft\cdot lbf}{lb} + 20\ \frac{ft\cdot lbf}{lb}\right]\left|\frac{1\ hp}{550\ ft\cdot lbf/s}\right|$$

$$\geq 3.9\ hp$$

To overcome the effect of internal irreversibilities, such as friction between the flowing liquid and pipe wall, we expect the actual power input to the pump to be greater than calculated here. Since, only a 3-hp pump is available, the desired flow rate cannot be accomodated. ◄

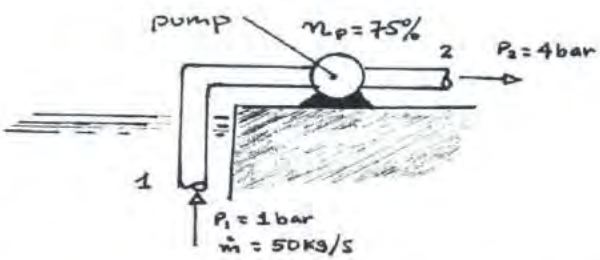
PROBLEM 6.182

An electrically driven pump operating at steady state draws water from a pond at a pressure of 1 bar and a rate of 50 kg/s and delivers the water at a pressure of 4 bar. There is no significant heat transfer with the surroundings, and changes in kinetic and potential energy can be neglected. The isentropic pump efficiency is 75%. Evaluating electricity at 8.5 cents per kW · h, estimate the hourly cost of running the pump.

KNOWN: Steady-state operating data are provided for a pump that draws water from a pond.

FIND: Determine the hourly cost of operating the pump.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. A control volume encloses the pumping system.
2. Operation is at steady state.
3. Changes in kinetic and potential energy from pipe inlet to pipe exit can be ignored.
4. Electricity is valued at 8.5 cents per kW·h.
5. Liquid water is modeled as incompressible with $\rho = 10^3$ kg/m^3 (Table A-19).

ANALYSIS: Using the pump efficiency, the magnitude of the power required by the pump is

$$|\dot{W}_{pump}| = \frac{\left(-\dot{W}_{\substack{int\\rev}}\right)}{\eta_p}$$

where the numerator is obtained by reducing Eq. 6.51a using assumption 3:

$$\left(-\frac{\dot{W}}{\dot{m}}\right)_{\substack{int\\rev}} = v\,(P_2 - P_1) = \frac{(P_2 - P_1)}{\rho}$$

$$\Rightarrow \quad |\dot{W}_{pump}| = \dot{m}\,\frac{(P_2 - P_1)}{\rho\,\eta_p}$$

$$= \frac{(50\text{ kg/s})(3\times10^5\text{ N/m}^2)}{(10^3\text{ kg/m}^3)(0.75)}\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$

$$= 20\text{ kW}$$

The hourly cost of operating the pump is

$$(\text{cost}) = (20\text{ kW})(1\text{ h})\left(0.085\,\frac{\$}{\text{kW}\cdot\text{h}}\right)$$

$$= \$1.70$$

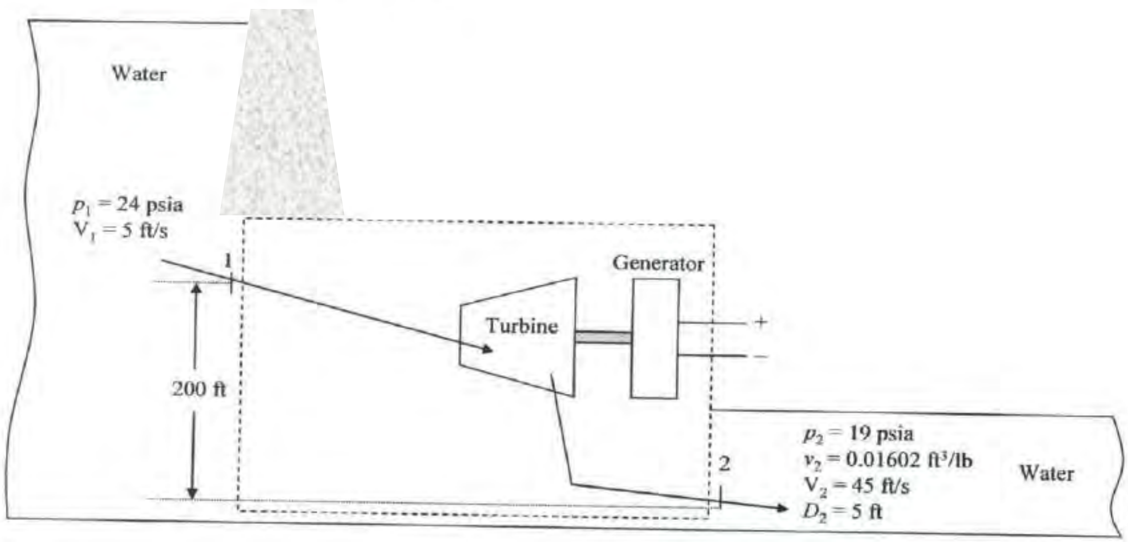
PROBLEM 6.183

As shown in Fig. P6.183, water behind a dam enters an intake pipe at a pressure of 24 psia and velocity of 5 ft/s, flows through a hydraulic turbine-generator, and exits at a point 200 ft below the intake at 19 psia, 45 ft/s, and a specific volume of 0.01602 ft^3/lb. The diameter of the exit pipe is 5 ft and the local acceleration of gravity is 32.2 ft/s^2. Evaluating the electricity generated at 8.5 cents per kW·h, determine the value of the power produced, in \$/day, for operation at steady state and in the absence of internal irreversibilities.

KNOWN: Water at specified pressure and velocity enters a turbine exiting at known elevation difference, pressure, velocity, and specific volume.

FIND: The economic value of the power produced.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

1. The control volume encloses the piping and turbine-generator as shown in the schematic.
2. The control volume operates with no internal irreversibilities at steady state.
3. Water can be treated as an incompressible substance with constant specific volume (density).

ANALYSIS:

For a steady-state control volume with no internal irreversibilities and a fluid with constant specific volume, Eq. 6.51a applies and can be used to determine the power

$$\left(\dot{W}_{cv}\right)_{\substack{int\\rev}} = \dot{m}\left[v(p_1 - p_2) + \tfrac{1}{2}\left(V_1^2 - V_2^2\right) + g(z_1 - z_2)\right]$$

The constant mass flow rate can be determined by

$$\dot{m} = \frac{AV}{v}$$

The cross-sectional area, A, is $\frac{1}{4}\pi d^2$. Applying the mass flow rate expression to the exit

$$\dot{m} = \frac{\left(\frac{\pi}{4}\right)(5\text{ ft})^2\left(45\frac{\text{ft}}{\text{s}}\right)}{0.01602\frac{\text{ft}^3}{\text{lb}}} = 55{,}154.4\text{ lb/s}$$

Substituting values, applying appropriate conversion factors, and solving for power give

$$\left(\dot{W}_{cv}\right)_{\substack{\text{int}\\\text{rev}}} =$$

$$\left(55{,}154.4\frac{\text{lb}}{\text{s}}\right)\left[\left(0.01602\frac{\text{ft}^3}{\text{lb}}\right)\left(24\frac{\text{lbf}}{\text{in}^2} - 19\frac{\text{lbf}}{\text{in}^2}\right)\left|\frac{144\text{ in}^2}{\text{ft}^2}\right| + \left(\frac{\left(5\frac{\text{ft}}{\text{s}}\right)^2 - \left(45\frac{\text{ft}}{\text{s}}\right)^2}{2} + \left(32.2\frac{\text{ft}}{\text{s}^2}\right)(200\text{ ft})\right)\left|\frac{\text{lbf}}{32.2\frac{\text{lb}\cdot\text{ft}}{\text{s}^2}}\right|\right]\left|\frac{\text{hp}}{550\frac{\text{ft}\cdot\text{lbf}}{\text{s}}}\right|\left|\frac{\text{kW}}{1.341\text{ hp}}\right|$$

$$= 13{,}496\text{ kW}$$

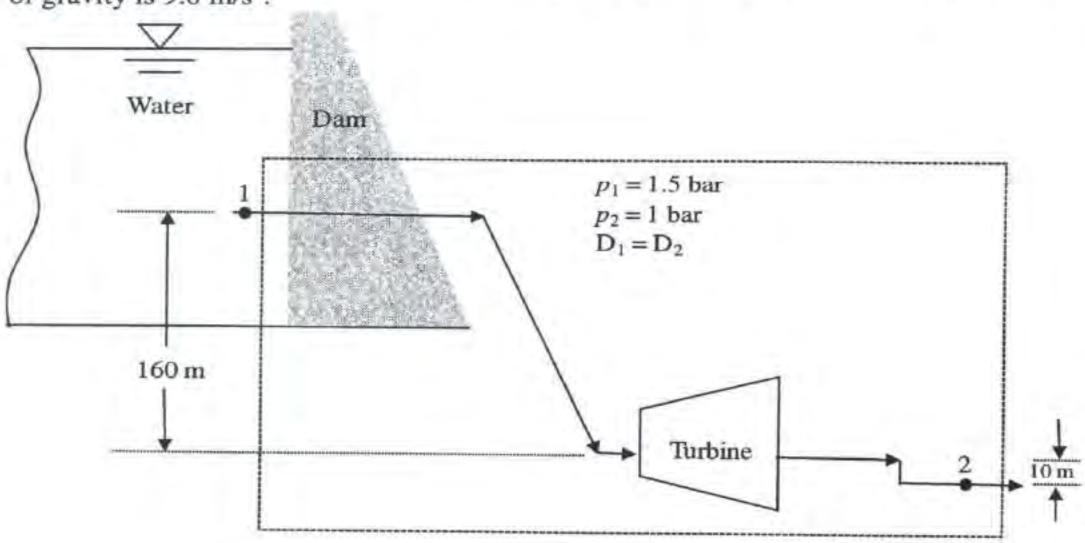
The positive sign associated with power indicates power is produced by the control volume. The generator converts turbine shaft power to electricity. The value of electricity per day is

$$Value = (13{,}496\text{ kW})\left(8.5\frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)\left(\frac{\$}{100\text{ cents}}\right)\left(\frac{24\text{ h}}{\text{day}}\right) = \underline{\textbf{\$27,532 per day}}$$

The effect of irreversibilities is expected to reduce the power produced and thus the daily economic value.

PROBLEM 6.194

As shown in Figure P6.184, water flows from an elevated reservoir through a hydraulic turbine operating at steady state. Determine the maximum power output, in MW, associated with a mass flow rate of 950 kg/s. The inlet and exit diameters are equal. The water can be modeled as incompressible with $v = 10^{-3}$ m^3/kg. The local acceleration of gravity is 9.8 m/s^2.



KNOWN: Water flows from an elevated reservoir through a hydraulic turbine at steady state. Mass flow rate is known.

FIND: Determine the maximum power output, in MW.

SCHEMATIC AND GIVEN DATA:
Refer to Fig. P6.184

ENGINEERING MODEL:
(1) A one-inlet, one-exit control volume with inlet at 1 and exit at 2, and enclosing the turbine, is at steady state.
(2) Liquid water is modeled as incompressible, with $v \approx v_f \approx 10^{-3}$ m^3/kg.

ANALYSIS:
The maximum power output of the actual hydraulic turbine is limited by the power produced by an internally reversible turbine, as follows:

$$\left(\dot{W}_t\right)_{\text{actual}} \leq \left(\dot{W}_t\right)_{\text{int rev}}$$

The relationship for internally reversible work is given in Eq. 6.51a and simplified based on assumption 2, as follows:

$$\left(\dot{W}_t\right)_{\text{actual}} \leq \dot{m}\left[-v(p_2 - p_1) + g(z_1 - z_2) + \frac{\left(V_1^{\,2} - V_2^{\,2}\right)}{2}\right] \qquad (1)$$

In Eq. (1), the kinetic energy term is eliminated because the diameter, specific volume, and mass flow rate are equal at states 1 and 2. Substituting known values:

$$\left(\dot{W}_t\right)_{\text{actual}} \leq 950\frac{\text{kg}}{\text{s}}\left[-\left(10^{-3}\frac{\text{m}^3}{\text{kg}}\right)(-0.5\text{ bar})\left|\frac{10^5\ \frac{\text{N}}{\text{m}^2}}{1\text{bar}}\right| + 9.8\frac{\text{m}}{\text{s}^2}(160\text{ m})\left|\frac{1\text{N}}{1\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right|\right]\left|\frac{1\text{ MW}}{10^6\ \frac{\text{N}\cdot\text{m}}{\text{s}}}\right| = 1.54\text{ MW}$$

The maximum power output of the hydraulic turbine is 1.54 MW.

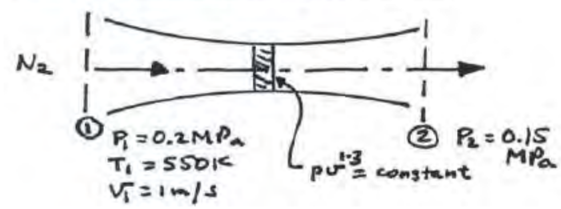
PROBLEM 6.185

Nitrogen (N_2) enters a nozzle operating at steady state at 0.2 MPa, 550K with a velocity of 1 m/s and undergoes a polytropic expansion with $n = 1.3$ to 0.15 MPa. Using the ideal gas model with $k = 1.4$, and ignoring potential energy effects, determine (a) the exit velocity, in m/s, and (b) the rate of heat transfer, in kJ per kg of gas flowing.

KNOWN: *Nitrogen undergoes a polytropic expansion in flowing through a nozzle at steady state. Data are known at the inlet and the exit.*

FIND: *Determine the exit velocity and the rate of heat transfer per unit mass of nitrogen flowing.*

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. The N_2 undergoes an expansion with $pv^{1.3}$ = constant.
3. Potential energy effects can be ignored. $\dot{W}_{cv} = 0$.
4. The N_2 is modeled as an ideal gas with $k = 1.4$

ANALYSIS: (a) Invoking Eq. 6.51a with $(\dot{W}_{cv}/\dot{m})_{\substack{int\\rev}} = 0$ and dropping the potential energy term, we get

$$\int_1^2 v\,dp + \frac{V_2^2 - V_1^2}{2} = 0$$

Then, with pv^n = constant, $\int_1^2 v\,dp = \frac{n}{n-1}(p_2v_2 - p_1v_1)$, Eq. 6.53. Using the ideal gas equation of state, and collecting results,

$$\frac{nR}{n-1}[T_2 - T_1] + \frac{V_2^2 - V_1^2}{2} = 0$$

Solving for V_2,

$$V_2 = \sqrt{V_1^2 + \frac{2nR}{(n-1)}(T_1 - T_2)} \qquad (1)$$

With Eq. 3.56

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(n-1)/n} = 550\text{ K}\left(\frac{0.15\text{ MPa}}{0.2\text{ MPa}}\right)^{\frac{(1.3-1)}{1.3}} = 514.7\text{ K}$$

Inserting values in Eq. (1),

$$V_2 = \sqrt{\left(1\frac{m}{s}\right)^2 + 2\left(\frac{1.3}{0.3}\right)\left(\frac{8.314}{28.01}\frac{kJ}{kg\cdot K}\right)(550 - 514.7)K\left|\frac{10^3 N\cdot m}{1\,kJ}\right|\left|\frac{1\,kg\cdot m/s^2}{1\,N}\right|}$$
$$= 301.3\text{ m/s} \qquad (a)$$

(b) Reducing an energy rate balance,

$$\dot{Q}_{cv} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} = \underbrace{c_p}_{=\frac{kR}{k-1}\ (\text{Eq } 3.47a)}(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}$$

$$\Rightarrow \quad \dot{Q}_{cv} = \frac{kR}{k-1}(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}$$
$$= \left(\frac{1.4}{0.4}\right)\left(\frac{8.314}{28.01}\right)(514.7 - 550)\frac{kJ}{kg} + \left[\frac{(301.3)^2 - (1)^2}{2}\right]\left(\frac{m}{s}\right)^2\left|\frac{1\,N}{1\,kg\,m/s^2}\right|\left|\frac{1\,kJ}{10^3 N\cdot m}\right|$$
$$= +8.72\ \frac{kJ}{kg} \qquad (b)$$

PROBLEM 6.186

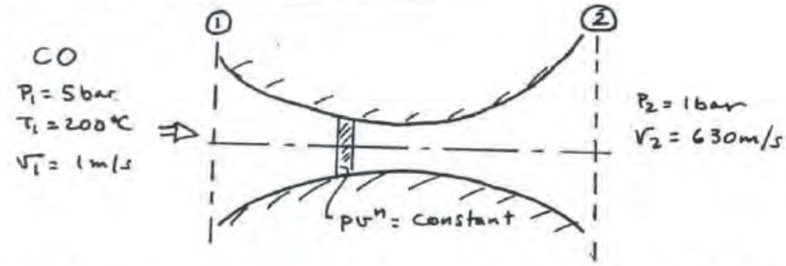
Carbon monoxide enters a nozzle operating at steady state at 5 bar, 200°C with a velocity of 1 m/s and undergoes a polytropic expansion to 1 bar and an exit velocity of 630 m/s. Using the ideal gas model and ignoring potential energy effects, determine

(a) the exit temperature, in °C.
(b) the rate of heat transfer, in kJ per kg of gas flowing.

KNOWN: *Carbon monoxide enters a nozzle operating at steady state and undergoes a polytropic expansion. Data are given at the inlet and exit.*

FIND: *Determine the exit temperature and the rate of heat transfer per unit mass of carbon monoxide flowing.*

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The nozzle is at steady state. (2) The expansion is described by pv^n = constant. (3) CO is modeled as an ideal gas. (4) Potential energy effects are ignored.

ANALYSIS: (a) As a polytropic process is internally reversible, Eq. 6.51a is applicable. Then, with $\dot{W}_{cv} = 0$, the Bernoulli equation, Eq. 6.52 results:

$$\int_1^2 v\,dp + \frac{V_2^2 - V_1^2}{2} + \cancel{g(z_2 - z_1)}^{\,0} = 0 \qquad (1)$$

Using the relationship pv^n = constant, the integral can be performed (Sec. 6.13), giving Eq. 6.53. Then,

$$\int_1^2 v\,dp = \frac{n}{n-1}(P_2 v_2 - P_1 v_1)$$

$$= \frac{nR}{n-1}(T_2 - T_1) \qquad (2)$$

where the ideal gas equation of state has been used to obtain the last expression. For a polytropic process of an ideal gas (Eq. 3.56)

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(n-1)/n} \qquad (3)$$

With Eqs. (1) – (3),

$$\frac{nR}{(n-1)}(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2} = 0 \Rightarrow \frac{nRT_1}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right] + \frac{V_2^2 - V_1^2}{2} = 0.$$

Inserting values,

$$\left(\frac{n}{n-1}\right)\left(\frac{8.314}{28.01}\frac{kJ}{kg\cdot K}\right)(473K)\left[\left(\frac{1}{5}\right)^{(n-1)/n} - 1\right] + \left[\frac{(630)^2 - (1)^2}{2}\right]\left(\frac{m}{s}\right)^2\left|\frac{1\,kJ}{10^3 N\cdot m}\right|\left|\frac{1\,N}{1\,kg\cdot m/s^2}\right| = 0$$

Solving, $n = 1.2$. Then, with Eq. (3), $T_2 = 473\,K\left(\frac{1}{5}\right)^{\frac{1.2-1}{1.2}} = 362\,K\ (89°C)$ ← (a)

(b) An energy balance reduces to give $\dot{Q}_{cv}/\dot{m} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}$

With data from Table A-23

$$\frac{\dot{Q}_{cv}}{\dot{m}} = \left(\frac{10531 - 13797}{28.01}\right)\frac{kJ}{kg} + \left[\frac{(630)^2 - (1)^2}{2}\right]\left(\frac{m}{s}\right)^2\left|\frac{1\,kJ}{10^3 N\cdot m}\right|\left|\frac{1\,N}{1\,kg\cdot m/s^2}\right| = +81.8\frac{kJ}{kg}$$ ←

Problem 7.1

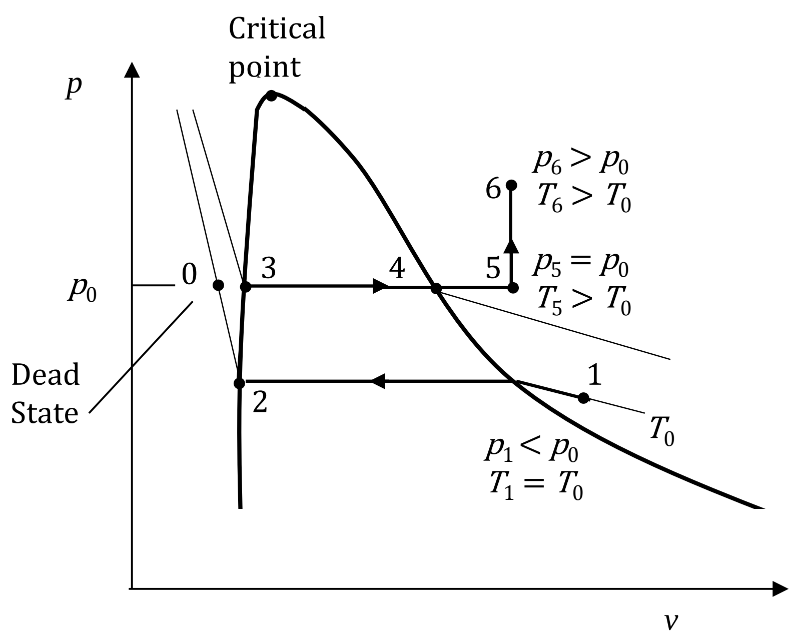
By inspection of Fig. P7.1 giving a *p*-*v* diagram for water, indicate whether exergy would increase, decrease, or remain the same in (a) Process 1-2, (b) Process 3-4, (c) Process 5-6. Explain.

Solution:

Known:
p-*v* diagram with multiple states provided.

Find:
Determine if exergy increases, decreases, or remains the same in (a) Process 1-2, (b) Process 3-4, (c) Process 5-6. Explain.

Schematic and Known Data:



Analysis:
Process 1-2:
Exergy decreases
p moves towards p_0 while $T = T_0$
Process 3-4:
Exergy increases
T moves further from T_0 while $p = p_0$
Process 5-6:
Exergy increases
p increases from p_0 and T moves further from T_0

Problem 7.2

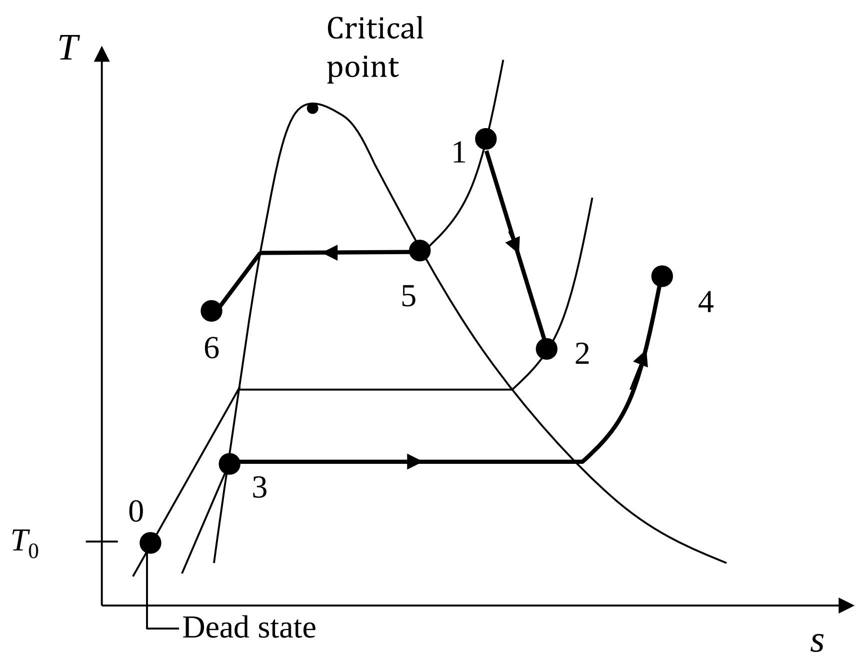
By inspection of Fig. P7.2 giving a *T-s* diagram for R-134a, indicate whether exergy would increase, decrease, or remain the same in (a) Process 1-2, (b) Process 3-4, (c) Process 5-6. Explain.

Solution:

Known:
T-s diagram with multiple states provided.

Find:
Determine if exergy increases, decreases, or remains the same in (a) Process 1-2, (b) Process 3-4, (c) Process 5-6. Explain.

Schematic and Known Data:



Analysis:
Process 1-2:
Exergy decreases
p moves towards p_0 while T moves towards T_0
Process 3-4:
Exergy increases
T moves further from T_0 while p remains constant
Process 5-6:
Exergy decreases
T moves towards T_0 while p remains constant

PROBLEM 7.3

KNOWN: An ideal gas is stored in a closed vessel at p, T.

FIND: (a) If $T = T_0$, derive $e = f(p, p_0, T_0, R)$

(b) If $p = p_0$, derive $e = f(T, T_0, c_p)$

ENGR. MODEL: (1) The gas obeys the ideal gas model. In part (b), c_p is constant as well. (2) The effects of motion and gravity can be ignored

ANALYSIS:

(a) With assumptions (1), (2), Eq. 7.2 becomes

$$e = \cancel{(u - u_0)}^{\,0} + p_0(v - v_0) - T_0(s - s_0)$$

The first term vanishes because u of an ideal gas depends only on temperature, and $T = T_0$. Also,

$$s - s_0 = \cancel{(s^\circ(T) - s^\circ(T_0))}^{\,0} - R \ln \frac{p}{p_0}$$

collecting results

$$e = p_0(v - v_0) + RT_0 \ln \frac{p}{p_0}$$

With $v = RT_0/p$ and $v_0 = RT_0/p_0$

$$e = p_0\left[\frac{RT_0}{p} - \frac{RT_0}{p_0}\right] + RT_0 \ln \frac{p}{p_0}$$

$$= RT_0\left[\frac{p_0}{p} - 1 + \ln \frac{p}{p_0}\right]$$

(b) With assumption (2), Eq. 7.2 becomes

$$e = u - u_0 + p_0(v - v_0) - T_0(s - s_0)$$

Introducing ideal gas relations, with $p = p_0$

$$e = u(T) - u(T_0) + p_0\left[\frac{RT}{p_0} - \frac{RT_0}{p_0}\right] - T_0\left[\int_{T_0}^{T} \frac{c_p}{T}\,dT - \cancel{R \ln \frac{p_0}{p_0}}^{\,0}\right]$$

If c_p is constant, so is c_v and the two differ by the gas constant R. Thus

$$e = c_v[T - T_0] + R[T - T_0] - T_0 c_p \ln \frac{T}{T_0}$$

$$= \underbrace{(c_v + R)}_{c_p}[T - T_0] - T_0 c_p \ln \frac{T}{T_0}$$

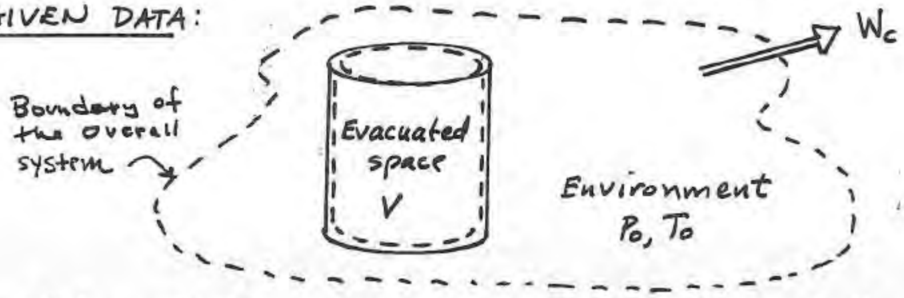
$$= c_p T_0\left[\frac{T}{T_0} - 1 - \ln \frac{T}{T_0}\right]$$

PROBLEM 7.4

KNOWN: An evacuated container of known volume is under consideration.

FIND: For the space inside the tank, determine the exergy.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) An overall system is considered consisting of the evacuated space (a closed system) and the environment. (2) The volume of the overall system is constant. (3) For the overall system, $Q = 0$. (4) For the environment, p_0 and T_0 are constant. (5) The effects of gravity and motion are ignored.

ANALYSIS: Consider a process where the evacuated space collapses and work is developed by the overall system. An energy balance for the overall system reduces with assumption 3 to give

$$W_c = -\Delta E_c$$

Since $\Delta U = 0$ for the evacuated space, $\Delta E_c = \Delta U_e$. With the first TdS equation, we get

$$W_c = -[T_0 \Delta S_e - p_0 \Delta V_e]$$

By assumption 2, $\Delta V_e = -\Delta V = -(\cancelto{0}{V_0} - V) = V$. Further, an entropy balance for the overall system gives

$$\Delta S_c = \sigma_c \Rightarrow \cancelto{0}{\Delta S} + \Delta S_e = \sigma_c$$

Collecting results

$$W_c = p_0 V - T_0 \sigma_c$$

Since $\sigma_c \geq 0$, the maximum theoretical work, or exergy, is obtained when $\sigma_c = 0$. Thus

$$\mathsf{E} = p_0 V \qquad (1)$$

Alternatively, Eq. (1) can be regarded as the minimum theoretical work required to produce the evacuated space.

Problem 7.5

Equal molar amounts of carbon monoxide and neon are maintained at the same temperature and pressure. Which has the greater value for exergy relative to the same reference environment? Assume the ideal gas model with constant c_v for each gas. There are no significant effects of motion and gravity.

Known:
Equal molar amounts of CO and Ne are at the same T, p.

Find:
Determine which gas has the greater exergy value, ē.

Engineering Model:
(1) Each gas obeys the ideal gas model with constant $\bar{c}_v$.
(2) There are no significant effects of motion or gravity.

Analysis:
With assumption (2), Eq. 7.2 reduces to give on a molar basis:

#1 $$\bar{e} = \bar{u} - \bar{u}_0 + p_0(\bar{v} - \bar{v}_0) - T_0(\bar{s} - \bar{s}_0)$$

Then, with assumption (1):

$$\begin{aligned}\bar{e} &= \bar{c}_v(T - T_0) + \bar{R}T_0\left[\frac{T}{T_0}\cdot\frac{p_0}{p} - 1\right] - T_0\left[\bar{c}_p \ln\frac{T}{T_0} - \bar{R}\ln\frac{p}{p_0}\right] \\ &= \bar{c}_v(T - T_0) - T_0\bar{c}_p \ln\frac{T}{T_0} + \bar{R}T_0\left[\frac{T}{T_0}\cdot\frac{p_0}{p} - 1 + \ln\frac{p}{p_0}\right]\end{aligned}$$

Applying this to each of CO and Ne, and subtracting the resulting equation gives:

$$\bar{e}_{CO} - \bar{e}_{Ne} = \left[\bar{c}_{v_{CO}} - \bar{c}_{v_{Ne}}\right](T - T_0) - \left[\bar{c}_{p_{CO}} - \bar{c}_{p_{Ne}}\right]T_0 \ln\frac{T}{T_0}$$

However, $\bar{c}_{v_{CO}} - \bar{c}_{v_{Ne}} = \bar{c}_{p_{CO}} - \bar{c}_{p_{Ne}}$ using Eq. 3.45 ($\bar{c}_p = \bar{c}_v + \bar{R}$) and simplifying

$$\bar{e}_{CO} - \bar{e}_{Ne} = \left[\bar{c}_{p_{CO}} - \bar{c}_{p_{Ne}}\right](T - T_0) - \left[\bar{c}_{p_{CO}} - \bar{c}_{p_{Ne}}\right]T_0 \ln\frac{T}{T_0}$$

$$\bar{e}_{CO} - \bar{e}_{Ne} = \left[\bar{c}_{p_{CO}} - \bar{c}_{p_{Ne}}\right]\left[T - T_0 - T_0 \ln\frac{T}{T_0}\right]$$

#2 By inspection of Figure 3.13, $\bar{c}_{p_{CO}} > \bar{c}_{p_{Ne}}$ giving $\bar{e}_{CO} > \bar{e}_{Ne}$ ←

Comments:

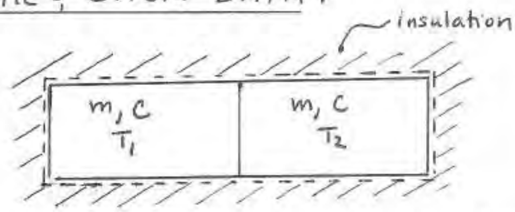
1. Here only the thermo-mechanical component is considered. The chemical contribution to exergy developed in Chapter 13 is not included in the present discussion.
2. But note that the molecular weights are different: $M_{CO} = 28.01$, $M_{Ne} = 20.18$.

PROBLEM 7.6

KNOWN: Two solid blocks of mass m and specific heat c are brought into contact and attain thermal equilibrium.

FIND: Derive an expression for the exergy destruction; demonstrate that the exergy destruction cannot be negative, and discuss.

SCHEMATIC & GIVEN DATA:



initially

ENGR. MODEL: (1) As shown in the figure, the two solid blocks form a closed system. (2) The blocks are modeled as incompressible with constant specific heat c. (3) For the system, $Q = W = 0$ and there are no kinetic and potential energy effects.

ANALYSIS: (a) Using assumptions (3), the energy balance reduces to

$$\Delta U = \cancel{Q}^{\,0} - \cancel{W}^{\,0} \Rightarrow \Delta U = 0 \Rightarrow \left[(2m)u(T_f) - \left[m u(T_1) + m u(T_2)\right]\right] = 0$$

where T_f is the final temperature at equilibrium. Accordingly

$$\left[2u(T_f) - u(T_1) - u(T_2)\right] = 0 \quad \Rightarrow \left[u(T_f) - u(T_1)\right] + \left[u(T_f) - u(T_2)\right] = 0$$

$$\Rightarrow c\left[T_f - T_1\right] + c\left[T_f - T_2\right] = 0 \Rightarrow T_f = \frac{T_1 + T_2}{2}.$$

The amount of exergy destroyed is $E_d = T_0 \sigma$, where σ is the amount of entropy produced, obtained from an entropy balance:

$$\Delta S = \cancel{\int \left(\frac{\delta Q}{T}\right)_b}^{\,0} + \sigma \Rightarrow \left[(2m)s(T_f) - \left[m s(T_1) + m s(T_2)\right]\right] = \sigma$$

or

$$\sigma = m\left[\left[s(T_f) - s(T_1)\right] + \left[s(T_f) - s(T_2)\right]\right]$$

$$= mc \ln \frac{T_f}{T_1} + mc \ln \frac{T_f}{T_2}$$

$$= mc \ln\left[\frac{T_f^2}{T_1 T_2}\right] = mc \ln\left[\frac{(T_1 + T_2)^2}{4 T_1 T_2}\right]$$

Finally

$$E_d = mc\, T_0 \ln\left[\frac{(T_1 + T_2)^2}{4 T_1 T_2}\right] \longleftarrow E_d$$

(b) The value of E_d would be negative only if $\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right]$ were less than unity. Considering this possibility...

$$\frac{(T_1 + T_2)^2}{4T_1T_2} < 1 \Rightarrow (T_1 + T_2)^2 < 4T_1T_2 \Rightarrow T_1^2 + 2T_1T_2 + T_2^2 < 4T_1T_2$$

or

$$T_1^2 - 2T_1T_2 + T_2^2 < 0 \Rightarrow (T_1 - T_2)^2 < 0$$

Since this inequality cannot be satisfied, we can conclude that

$E_d \geq 0$.

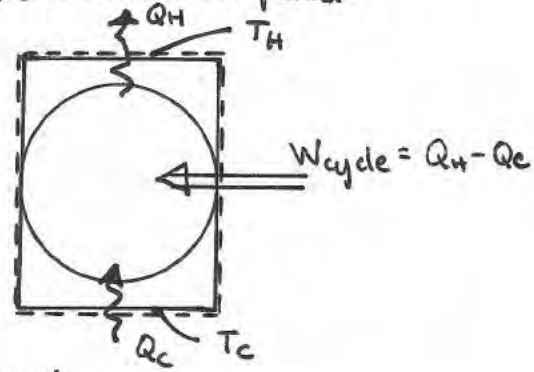
(c) The exergy destruction in this case can be traced to the spontaneous heat transfer that takes place <u>within</u> the two blocks as they come to thermal equilibrium.

PROBLEM 7.7

KNOWN: A system undergoes a refrigeration cycle while receiving Q_c at temperature T_C and discharging Q_H at T_H, where $T_H > T_C$. Q_c and Q_H are the only heat transfers.

FIND: (a) Show that W_{cycle} cannot be zero. (b) Obtain a specified expression for coefficient of performance β. (c) Determine β_{max}.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The system shown undergoes a refrigeration cycle. (2) Q_c and Q_H are the only heat transfers and are in the directions of the arrows. (3) T_c and T_H are constant and $T_H > T_C$. (4) The environment temperature is T_0.

ANALYSIS: (a) An exergy balance for the cycle reads

$$\cancel{\Delta E}\Big]_{cycle}^{0} = \left[1 - \frac{T_0}{T_c}\right]Q_c - \left[1 - \frac{T_0}{T_H}\right]Q_H - \left[W_{cycle} - p_0 \cancel{\Delta V}^{0}\Big]_{cycle}\right] - E_d$$

where $\Delta E = \Delta V = 0$ for a cycle. Introducing the energy balance, $Q_H = W_{cycle} + Q_c$ we get

$$0 = \left[1 - \frac{T_0}{T_c}\right]Q_c - \left[1 - \frac{T_0}{T_H}\right](W_{cycle} + Q_c) - W_{cycle} - E_d$$

$$= \left[\left(\cancel{1} - \frac{T_0}{T_c}\right) - \left(\cancel{1} - \frac{T_0}{T_H}\right)\right]Q_c + \left[\cancel{1} - \left(\cancel{1} - \frac{T_0}{T_H}\right)\right]W_{cycle} - E_d$$

$$= T_0\left[\frac{1}{T_H} - \frac{1}{T_c}\right]Q_c + \frac{T_0}{T_H}W_{cycle} - E_d \qquad (1)$$

Solving for E_d, and setting W_{cycle} to zero

$$E_d = \underbrace{T_0}_{pos.}\underbrace{\left[\frac{1}{T_H} - \frac{1}{T_c}\right]}_{neg.}\underbrace{Q_c}_{pos} + \frac{T_0}{T_H}\cancel{W_{cycle}}^{0} \Rightarrow E_d < 0 \quad \text{impossible!}$$

Thus W_{cycle} cannot be zero. ——— (a)

(b) Solving (1) for $\beta = Q_c / W_{cycle}$

$$\left[\frac{T_H - T_c}{T_H T_c}\right]Q_c = \frac{W_{cycle}}{T_H} - E_d/T_0$$

$$\beta = \frac{Q_c}{W_{cycle}} = \left[\frac{T_c}{T_H - T_c}\right]\left[1 - \frac{T_H E_d}{T_0 W_{cycle}}\right] = \left[\frac{T_c}{T_H - T_c}\right]\left[1 - \frac{T_H E_d}{T_0 (Q_H - Q_c)}\right] \qquad (b)$$

(c) From the result of part (b), β increases as $E_d \to 0$. Thus, when $E_d = 0$

$$\beta_{max} = \frac{T_c}{T_H - T_c}$$

as expected.

PROBLEM 7.8

When matter flows across the boundary of a control volume, an energy transfer by work, called *flow work,* occurs. The rate is $\dot{m}(pv)$ where $\dot{m}$, p, and v denote the mass flow rate, pressure, and specific volume, respectively, of the matter crossing the boundary (see Sec. 4.4.2). Show that the *exergy transfer accompanying flow work* is given by $\dot{m}(pv - p_0 v)$, where p_0 is the pressure at the dead state.

ANALYSIS: The objective is to demonstrate the following expression:

$$\begin{bmatrix} \text{time rate of exergy transfer} \\ \text{accompanying flow work} \end{bmatrix} = \dot{m}(pv - p_0 v) \tag{1}$$

Let us develop Eq. (1) for the case shown in the figure. The figure shows a closed system that occupies different regions at time t and a later time $t + \Delta t$. The fixed quantity of matter under consideration is shown shaded. During the time interval Δt, some of the mass initially within the region labeled *control volume* exits to fill the small region e adjacent to the control volume, as shown in Fig. (b). We assume that the increase in the volume of the closed system in the time interval Δt is equal to the volume of region e and, for further simplicity, that the only work is associated with this volume change. With Eq. 7.6, the exergy transfer accompanying work is

$$\begin{bmatrix} \text{exergy transfer} \\ \text{accompanying work} \end{bmatrix} = W - p_0 \,\Delta V \tag{2}$$

where ΔV is the volume change of the system. The volume change of the system equals the volume of region e. Thus, we may write $\Delta V = m_e v_e$, where m_e is the mass within region e and v_e is the specific volume, which is regarded as uniform throughout region e. With this expression for ΔV, Eq. (2) becomes

$$\begin{bmatrix} \text{exergy transfer} \\ \text{accompanying work} \end{bmatrix} = W - m_e(p_0 v_e) \tag{3}$$

Equation (3) can be placed on a time rate basis by dividing each term by the time interval Δt and taking the limit as Δt approaches zero. That is

$$\begin{bmatrix} \text{time rate of exergy} \\ \text{transfer accompanying work} \end{bmatrix} = \lim_{\Delta t \to 0}\left(\frac{W}{\Delta t}\right) - \lim_{\Delta t \to 0}\left[\frac{m_e}{\Delta t}(p_0 v_e)\right] \tag{4}$$

In the limit as Δt approaches zero, the boundaries of the closed system and control volume coincide. Accordingly, in this limit the rate of energy transfer by work from the closed system is also the rate of energy transfer by work from the control volume. For the present case, this is just the flow work. Thus, the first term on the right side of Eq. (4) becomes

$$\lim_{\Delta t \to 0}\left(\frac{W}{\Delta t}\right) = \dot{m}_e(p_e v_e) \tag{5}$$

where $\dot{m}_e$ is the mass flow rate at the exit of the control volume. In the limit as Δt approaches zero, the second term on the right side of Eq. (4) becomes

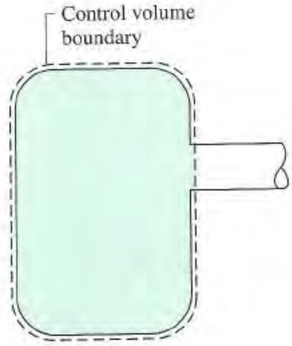
$$\lim_{\Delta t \to 0}\left[\frac{m_e}{\Delta t}(p_0 v_e)\right] = \dot{m}_e(p_0 v_e) \tag{6}$$

In this limit, the assumption of uniform specific volume throughout region e corresponds to the assumption of uniform specific volume across the exit (one-dimensional flow).

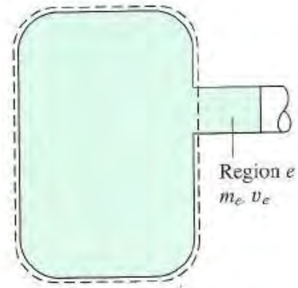
Substituting Eqs. (5) and (6) into Eq. (4) gives

$$\begin{aligned} \begin{bmatrix} \text{time rate of exergy transfer} \\ \text{accompanying flow work} \end{bmatrix} &= \dot{m}_e(p_e v_e) - \dot{m}_e(p_0 v_e) \\ &= \dot{m}_e(p_e v_e - p_0 v_e) \end{aligned} \tag{7}$$

Extending the reasoning given here, it can be shown that an expression having the same form as Eq. (7) accounts for the transfer of exergy accompanying flow work at inlets to control volumes as well. The general result applying at both inlets and exits is given by Eq. (1).



(a) Time t.



(b) Time $t + \Delta t$.

PROBLEM 7.9

When matter flows across the boundary of a control volume, an exergy transfer accompanying mass flow occurs, which is given by $\dot{m}$e, where e is the specific exergy (Eq. 7.2) and $\dot{m}$ is the mass flow rate. An exergy transfer accompanying flow work, which is given by the result of Problem 7.8, also occurs at the boundary. Show that the sum of these exergy transfers is given by e_f, where e_f is the specific flow exergy (Eq. 7.14).

ANALYSIS:

When mass enters or exits a control volume, there is an accompanying exergy transfer given by

$$\begin{bmatrix}\text{time rate of exergy transfer}\\ \text{accompanying mass flow}\end{bmatrix} = \dot{m}\underbrace{\mathsf{e}}_{\text{specific exergy}}$$
$$= \dot{m}[(\underbrace{e}_{\text{specific energy}} - u_0) + p_0(v - v_0) - T_0(s - s_0)] \quad (1)$$

where Eq. 7.2 for specific exergy is written compactly in terms of specific energy: $e = u + V^2/2 + gz$. In addition to an exergy transfer accompanying mass flow, an exergy transfer accompanying flow work takes place at locations where mass enters or exits a control volume. Transfers of exergy accompanying flow work are accounted for by $\dot{m}(pv - p_0 v)$, which is developed in Problem 7.8.

Since transfers of exergy accompanying mass flow and flow work occur at locations where mass enters or exits a control volume, a single expression giving the sum of these effects is convenient. Thus,

$$\begin{bmatrix}\text{time rate of exergy transfer}\\ \text{accompanying mass flow } \textit{and} \text{ flow work}\end{bmatrix} = \dot{m}[\mathsf{e} + (pv - p_0 v)]$$
$$= \dot{m}[\underline{(e - u_0) + p_0(v - v_0) - T_0(s - s_0)}$$
$$\underline{+ (pv - p_0 v)}] \quad (2)$$

The underlined terms in Eq. (2) represent, per unit of mass, the exergy transfer accompanying mass flow and flow work, respectively. The sum identified by underlining is the specific flow exergy e_f. That is

① $$\mathsf{e}_f = (e - u_0) + p_0(v - v_0) - T_0(s - s_0) + (pv - p_0 v) \quad (3)$$

The specific flow exergy can be placed in a more convenient form for calculation by introducing $e = u + V^2/2 + gz$ in Eq. (3) and simplifying to obtain

$$\mathsf{e}_f = \left(u + \frac{V^2}{2} + gz - u_0\right) + (pv - p_0 v_0) - T_0(s - s_0)$$
$$= \underbrace{(u + pv)}_{h} - \underbrace{(u_0 + p_0 v_0)}_{h_0} - T_0(s - s_0) + \frac{V^2}{2} + gz \quad (4)$$

Finally, with $h = u + pv$ and $h_0 = u_0 + p_0 v_0$, Eq. (4) gives Eq. 7.14, which is the objective.

---

1. The specific flow exergy, e_f, and specific exergy, e, are related by

$$\underbrace{\mathsf{e}_f}_{\text{specific flow exergy}} = \underbrace{\mathsf{e}}_{\text{specific exergy}} + \underbrace{v(p - p_0)}_{\text{flow work contribution — negative when } p < p_0.} \quad (5)$$

Although the specific exergy cannot be negative, the specific flow exergy can take on negative values when the local pressure is less than the pressure at the dead state: $p < p_0$. In such cases, the net transfer of exergy where matter crosses the boundary is opposite to the direction of mass flow.

PROBLEM 7.10

KNOWN: An ideal gas with constant specific heat ratio k.

FIND: Show that in the absence of motion and gravity effects

$$\frac{e_f}{c_p T_0} = \frac{T}{T_0} - 1 - \ln\frac{T}{T_0} + \ln\left(\frac{p}{p_0}\right)^{(k-1)/k}$$

and develop plots for $k = 1.2, 1.3, 1.4$ of $e_f / c_p T_0$ vs. T/T_0 for $p/p_0 = 0.25, 0.5, 1, 2, 4$. Discuss the significance of $e_f < 0$.

ENGR. MODEL: (1) The system consists of an ideal gas with constant specific heat ratio. (2) The effects of motion and gravity are not significant.

ANALYSIS: With assumption (2) and ideal gas relationships, Eq. 7.14 gives

$$e_f = h - h_0 - T_0 (s - s_0)$$

$$= c_p [T - T_0] - T_0 \left[c_p \ln\frac{T}{T_0} - R \ln\frac{p}{p_0} \right]$$

$$R = c_p \left[\frac{k-1}{k}\right] \quad \text{(Eq. 3.47a)}$$

Thus

$$\frac{e_f}{c_p T_0} = \frac{T}{T_0} - 1 - \ln\frac{T}{T_0} + \ln\left[\frac{p}{p_0}\right]^{(k-1)/k}$$

(a) See plots on the next page.

(b) Discussion.

The specific exergy is

$$e = (u - u_0) + p_0 (v - v_0) - T_0 (s - s_0)$$

and the specific flow exergy is

$$e_f = h - h_0 - T_0 (s - s_0)$$

$$= (u + pv) - (u_0 + p_0 v_0) - T_0 (s - s_0) = (u - u_0) + (pv - p_0 v_0) - T_0 (s - s_0)$$

Subtracting e from e_f gives

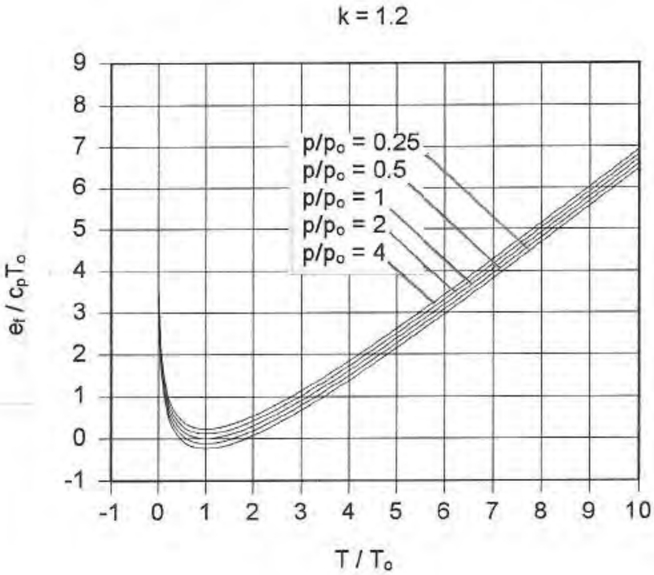
$$e_f = e + v(p - p_0)$$

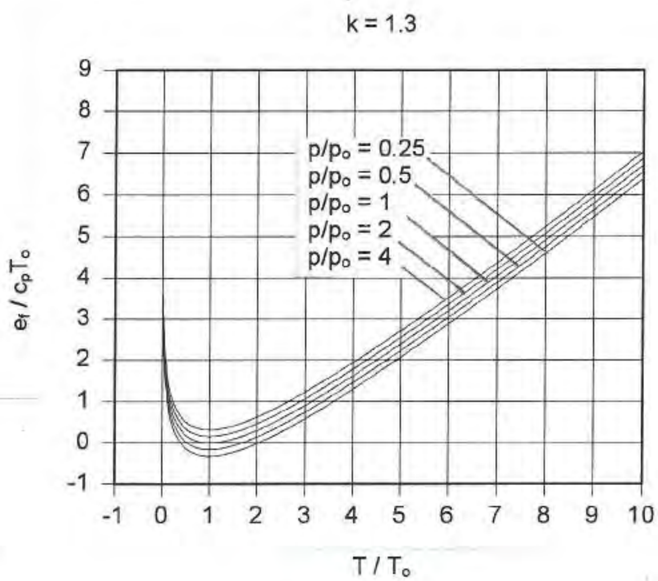
Thus, although the specific exergy e cannot be negative, e_f can take on negative values when $p < p_0$, as shown in the plots below.

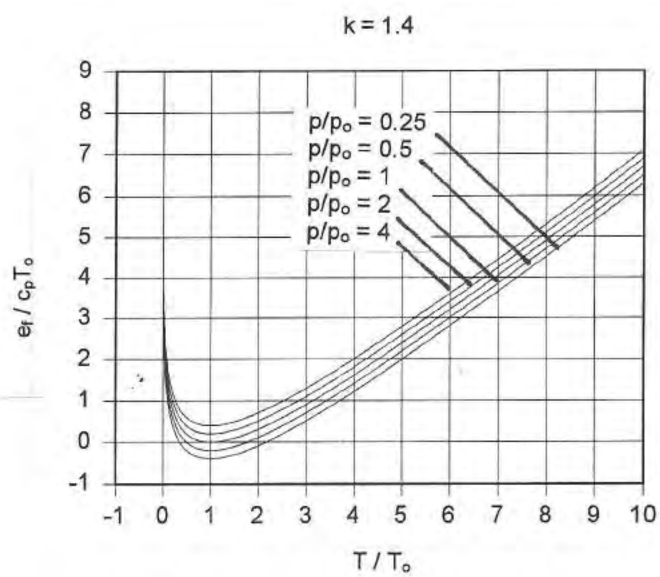
From the developments of Problems 7.8 and 7.9, the specific flow exergy can be viewed as the sum of exergy transfers accompanying mass flow and flow work. The value of e_f can be negative, then, when these contributions have opposite signs: the flow work contribution is opposite to the direction of flow.

PROBLEM 7.10 (Contd.)

PLOTS:







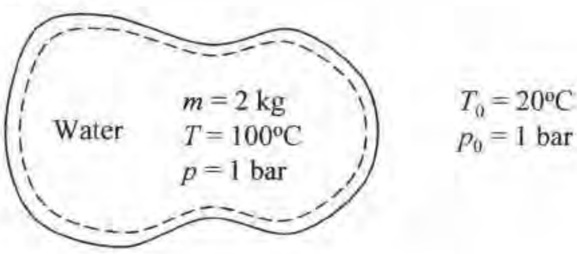
PROBLEM 7.11

7.11 A system consists of 2 kg of water at 100°C and 1 bar. Determine the exergy, in kJ, if the system is at rest and zero elevation relative to an exergy reference environment for which T_0 = 20°C, p_0 = 1 bar.

KNOWN: System of water at specified temperature and pressure exists in a reference environment with specified temperature and pressure.

FIND: Exergy of the system.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The water is a closed system defined by the dashed line on the accompanying diagram.
2. The effects of motion and gravity can be ignored.
3. T_0 = 20°C = 293 K and p_0 = 1 bar.

ANALYSIS:
The exergy of the system can be determined from Eq. 7.1

$$\mathsf{E} = (U - U_0) + p_0(V - V_0) - T_0(S - S_0) + \mathrm{KE} + \mathrm{PE}$$

Ignoring motion and gravity effects and rewriting extensive properties in terms of mass times specific properties gives

$$\mathsf{E} = m[(u - u_0) + p_0(v - v_0) - T_0(s - s_0)]$$

The water in the system is superheated vapor. From Table A-4, u = 2506.7 kJ/kg, v = 1.696 m^3/kg, s = 7.3614 kJ/(kg·K).

Water at the reference state is compressed liquid. From Table A-2 at T_0 = 20°C, $u_0 \approx u_{f0}$ = 83.95 kJ/kg, $v_0 \approx v_{f0}$ = 0.0010018 m^3/kg, $s_0 \approx s_{f0}$ = 0.2966 kJ/(kg·K). Substituting values and applying appropriate conversion factors give

$$\mathsf{E} =$$

$$(2\,\mathrm{kg})\left[(2506.7 - 83.95)\frac{\mathrm{kJ}}{\mathrm{kg}} + (1\,\mathrm{bar})(1.696 - 0.0010018)\frac{\mathrm{m^3}}{\mathrm{kg}}\left|\frac{10^5\ \frac{\mathrm{N}}{\mathrm{m^2}}}{\mathrm{bar}}\right|\left|\frac{\mathrm{kJ}}{10^3\,\mathrm{N\cdot m}}\right| - (293\,\mathrm{K})(7.3614 - 0.2966)\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\right]$$

$$\mathsf{E} = \underline{\mathbf{1044.5\ kJ}}$$

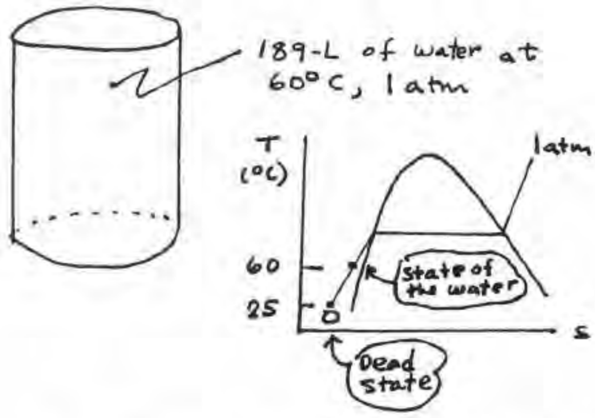
PROBLEM 7.12

7.12 A domestic water heater holds 189 L of water at 60°C, 1 atm. Determine the exergy of the hot water, in kJ. To what elevation, in *m*, would a 1000-kg mass have to be raised from zero elevation relative to the reference environment for its exergy to equal that of the hot water? Let $T_0 = 298$ K, $p_0 = 1$ atm, $g = 9.81$ m/s^2.

KNOWN: Data are provided for a domestic water heater.

FIND: Determine the exergy of the hot water. Also, determine the elevation to which a 1000-kg mass would have to be raised for its exergy to equal that of the hot water.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The system is the water within the water heater.
2. For the water, $u \approx u_f(T)$, $s \approx s_f(T)$, $v \approx v_f(T)$.
3. For the system, the effects of motion and gravity are ignored.
4. $T_0 = 298\,\text{K}$, $p_0 = 1\,\text{atm}$, $g = 9.81\,\text{m/s}^2$

ANALYSIS: With data from Table A-2,

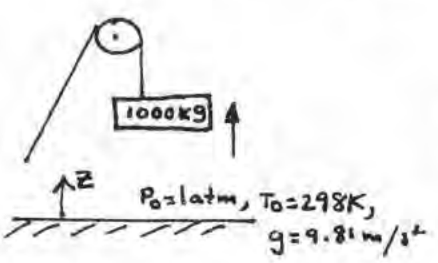
$$u_0 = u_f(25°C) = 104.88\ \text{kJ/kg} \qquad u = u_f(60°C) = 251.11\ \text{kJ/kg}$$
$$v_0 = v_f(25°C) = (1.0029/10^3)\ \frac{\text{m}^3}{\text{kg}} \qquad v = v_f(60°C) = (1.0172/10^3)\ \frac{\text{m}^3}{\text{kg}}$$
$$s_0 = s_f(25°C) = 0.3674\ \text{kJ/kg·K} \qquad s = s_f(60°C) = 0.8312\ \text{kJ/kg·K}$$

Then,

$$m = \frac{V}{v} = \frac{(189\,\text{L})\left|\frac{10^{-3}\,\text{m}^3}{1\,\text{L}}\right|}{(1.0172/10^3\ \text{m}^3/\text{kg})} = 185.8\ \text{kg}$$

Also, with Eq. 7.2 reduced using assumption # 3, we get

$$E = m\left[(u-u_0) + p_0(v-v_0) - T_0(s-s_0)\right]$$
$$= 185.8\,\text{kg}\left[(251.11 - 104.88)\frac{\text{kJ}}{\text{kg}} + \left(1.01325\times10^5\frac{\text{N}}{\text{m}^2}\right)\left(\frac{1.0172 - 1.0029}{10^3}\right)\frac{\text{m}^3}{\text{kg}}\left|\frac{1\,\text{kJ}}{10^3\,\text{N·m}}\right| - 298\,\text{K}\,(0.8312 - 0.3674)\frac{\text{kJ}}{\text{kg·K}}\right]$$
$$= 185.8\ \text{kg}\left[146.23 + 0.001 - 138.21\right] = 1490\ \text{kJ}$$



The elevation, Z, to which a 1000kg mass would have to be raised for its exergy to equal 1490 kJ is found as follows:

$$E = mgZ$$

$$\Rightarrow \qquad Z = \frac{E}{mg} = \frac{1490\ \text{kJ}}{(1000\,\text{kg})(9.81\,\text{m/s}^2)}\left|\frac{10^3\,\text{N·m}}{1\,\text{kJ}}\right|\left|\frac{1\,\text{kg·m/s}^2}{1\,\text{N}}\right|$$
$$= 151.9\ \text{m}$$

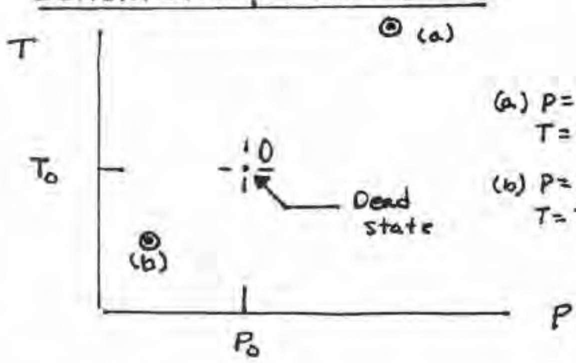
PROBLEM 7.13

7.13 Determine the specific exergy of argon at (a) $p = 2\,p_0$, $T = 2\,T_0$, (b) $p = p_0/2$, $T = T_0/2$. Locate each state relative to the dead state on temperature–pressure coordinates. Assume ideal gas behavior with $k = 1.67$. Let $T_0 = 537°R$, $p_0 = 1$ atm.

KNOWN: State data are provided for argon.

FIND: Determine the specific exergy at these states and locate the states on T-P coordinates also locating the dead state.

SCHEMATIC & GIVEN DATA:



(a) $P = 2P_0$, $T = 2T_0$

(b) $P = P_0/2$, $T = T_0/2$

ENGINEERING MODEL:

1. Argon is modeled as an ideal gas with $k = 1.67$.
2. The effects of motion and gravity are ignored.
3. $T_0 = 537°R$, $P_0 = 1$ atm

ANALYSIS: Reducing Eq. 7.2 with assumption #2 and applying assumption #1,

$$e = (u - u_0) + P_0(v - v_0) - T_0(s - s_0)$$

$$= \underbrace{c_v}_{=\frac{R}{k-1}}[T - T_0] + P_0\left[\frac{RT}{P} - \frac{RT_0}{P_0}\right] - T_0\left[\underbrace{c_p}_{=\frac{kR}{k-1}} \ln\frac{T}{T_0} - R\ln\frac{P}{P_0}\right]$$

$$= \frac{RT_0}{(k-1)}\left[\frac{T}{T_0} - 1\right] + RT_0\left[\frac{P_0}{P}\frac{T}{T_0} - 1\right] - RT_0\left[\frac{k}{(k-1)}\ln\frac{T}{T_0} - \ln\frac{P}{P_0}\right]$$

$$= RT_0\left\{\frac{1}{(k-1)}\left[\frac{T}{T_0} - 1\right] + \left[\frac{P_0}{P}\frac{T}{T_0} - 1\right] - \frac{k}{(k-1)}\ln\frac{T}{T_0} + \ln\frac{P}{P_0}\right\} \qquad (1)$$

(a) When $\frac{T}{T_0} = 2$ and $\frac{P}{P_0} = 2$, Eq. (1) gives

$$e = \overbrace{\left(\frac{1.986}{39.94}\,\frac{Btu}{lb\cdot°R}\right)(537°R)}^{26.702\ Btu/lb}\left\{\frac{1}{0.67}(2-1) + \underbrace{\left[\left(\tfrac{1}{2}\right)(2) - 1\right]}_{=0} - \frac{1.67}{0.67}\ln 2 + \ln 2\right\}$$

$$= 12.23\ Btu/lb$$

(b) When $\frac{T}{T_0} = 0.5$ and $\frac{P}{P_0} = 0.5$, Eq. (1) gives

$$e = \left(26.702\ \frac{Btu}{lb}\right)\left\{\frac{1}{0.67}(0.5-1) + \underbrace{\left[(2)(0.5) - 1\right]}_{=0} - \frac{1.67}{0.67}\ln(0.5) + \ln(0.5)\right\}$$

① $= 7.72\ Btu/lb$

---

1. Exergy is positive at all states other than the dead state. In this case, $P < P_0$ and $T < T_0$. Still, exergy is positive.

PROBLEM 7.14

7.14 Determine the specific exergy, in Btu, of one pound mass of
(a) saturated liquid Refrigerant-134a at -5°F.
(b) saturated vapor Refrigerant-134a at 140°F.
(c) Refrigerant-134a at 60°F, 20 lbf/in.^{2}
(d) Refrigerant-134a at 60°F, 10 lbf/in.^{2}
In each case, consider a fixed mass at rest and zero elevation relative to an exergy reference environment for which $T_0 = 60°F$, $p_0 = 15$ lbf/in.^{2}

KNOWN: Systems at specified states are defined.

FIND: Determine the specific exergy, in Btu, for 1 lb of each substance.

ENGINEERING MODEL:
(1) The closed systems are at rest and zero elevation relative to the environment at $T_0 = 60°F = 520°R$, $p_0 = 15$ lbf/in.^{2}

ANALYSIS:
Determine exergy using a form of Eq. 7.2.

$$\mathrm{E} = m\left[(u-u_0)+p_0(v-v_0)-T_0(s-s_0)+\frac{\mathrm{V}^2}{2}+gz\right]$$

Simplify based on assumption:

$$\mathrm{E} = m[(u-u_0)+p_0(v-v_0)-T_0(s-s_0)] \qquad (1)$$

(a) Saturated liquid Refrigerant-134a at -5°F

Fix properties of R-134a at the dead state. Using Table A-11E at p_0, $T_0 > T_{sat}$ and therefore superheated vapor exists. From Table A-12E referenced at p_0, T_0:

$$v_0 = 3.5533\frac{\text{ft}^3}{\text{lb}} \quad u_0 = 104.54\frac{\text{Btu}}{\text{lb}} \quad s_0 = 0.2548\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$$

Fix properties of R-134a using Table A-10E referenced at -5°F:

$$v = v_f = 0.01178\frac{\text{ft}^3}{\text{lb}} \quad u = u_f = 10.09\frac{\text{Btu}}{\text{lb}} \quad s = s_f = 0.0231\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$$

Substitute values into Eq. (1).

$$\mathbf{E} = 1\,\text{lb}\left[\begin{array}{l}(10.09-104.54)\frac{\text{Btu}}{\text{lb}}+15\frac{\text{lbf}}{\text{in.}^2}(0.01178-3.5533)\frac{\text{ft}^3}{\text{lb}}\left|\frac{144\,\text{in.}^2}{1\,\text{ft}^2}\right|\left|\frac{1\,\text{Btu}}{778\,\text{ft}\cdot\text{lbf}}\right| \\ -520°\text{R}(0.0231-0.2548)\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\end{array}\right]$$

#1 $$\mathbf{E} = 1\,\text{lb}[(-94.45)+(-9.83)-(-120.48)]\frac{\text{Btu}}{\text{lb}} = 16.2\,\text{Btu}$$

(b) Saturated vapor Refrigerant-134a at 140°F

Dead state properties are the same as part (a). Fix properties of R-134a using Table A-10E referenced at 140°F:

$$v = v_g = 0.1827\frac{\text{ft}^3}{\text{lb}} \quad u = u_g = 110.41\frac{\text{Btu}}{\text{lb}} \quad s = s_g = 0.2143\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$$

PROBLEM 7.14 (Continued)

Substitute values into Eq. (1).

$$\mathbf{E} = 1\,\text{lb}\left[\begin{array}{l}(110.41 - 104.54)\dfrac{\text{Btu}}{\text{lb}} + 15\dfrac{\text{lbf}}{\text{in.}^2}(0.1827 - 3.5533)\dfrac{\text{ft}^3}{\text{lb}}\left|\dfrac{144\,\text{in.}^2}{1\,\text{ft}^2}\right|\left|\dfrac{1\,\text{Btu}}{778\,\text{ft}\cdot\text{lbf}}\right| \\ -520^\circ\text{R}(0.2143 - 0.2548)\dfrac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\end{array}\right]$$

$$\mathbf{E} = 1\,\text{lb}[(5.87) + (-9.36) - (-21.06)]\frac{\text{Btu}}{\text{lb}} = 17.57\,\text{Btu}$$ ←

(c) Refrigerant-134a at 60°F, 20 lbf/in.^{2}

Dead state properties are the same as part (a). Fix properties of R-134a using Table A-12E referenced at 60°F, 20 lbf/in.^{2}:

$$v = 2.6416\frac{\text{ft}^3}{\text{lb}} \quad u = 104.28\frac{\text{Btu}}{\text{lb}} \quad s = 0.2487\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$

Substitute values into Eq. (1).

$$\mathbf{E} = 1\,\text{lb}\left[\begin{array}{l}(104.28 - 104.54)\dfrac{\text{Btu}}{\text{lb}} + 15\dfrac{\text{lbf}}{\text{in.}^2}(2.6416 - 3.5533)\dfrac{\text{ft}^3}{\text{lb}}\left|\dfrac{144\,\text{in.}^2}{1\,\text{ft}^2}\right|\left|\dfrac{1\,\text{Btu}}{778\,\text{ft}\cdot\text{lbf}}\right| \\ -520^\circ\text{R}(0.2487 - 0.2548)\dfrac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\end{array}\right]$$

#2 $$\mathbf{E} = 1\,\text{lb}[(-0.26) + (-2.53) - (-3.17)]\frac{\text{Btu}}{\text{lb}} = 0.38\,\text{Btu}$$ ←

(d) Refrigerant-134a at 60°F, 10 lbf/in.^{2}

Dead state properties are the same as part (a). Fix properties of R-134a using Table A-12E referenced at 60°F, 10 lbf/in.^{2}

$$v = 5.3758\frac{\text{ft}^3}{\text{lb}} \quad u = 104.80\frac{\text{Btu}}{\text{lb}} \quad s = 0.2632\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$

Substitute values into Eq. (1).

$$\mathbf{E} = 1\,\text{lb}\left[\begin{array}{l}(104.80 - 104.54)\dfrac{\text{Btu}}{\text{lb}} + 15\dfrac{\text{lbf}}{\text{in.}^2}(5.3758 - 3.5533)\dfrac{\text{ft}^3}{\text{lb}}\left|\dfrac{144\,\text{in.}^2}{1\,\text{ft}^2}\right|\left|\dfrac{1\,\text{Btu}}{778\,\text{ft}\cdot\text{lbf}}\right| \\ -520^\circ\text{R}(0.2632 - 0.2548)\dfrac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\end{array}\right]$$

#3 $$\mathbf{E} = 1\,\text{lb}[(0.26) + (5.06) - (4.37)]\frac{\text{Btu}}{\text{lb}} = 0.95\,\text{Btu}$$ ←

---

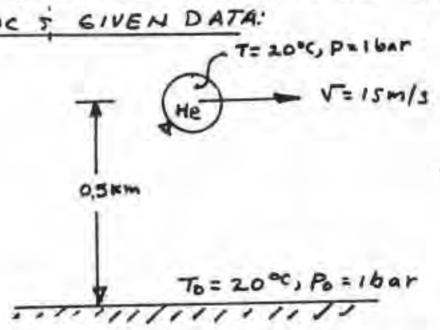
1. Even though $T < T_0$, the value of exergy is positive, as expected. See discussion of Figure 7.4.
2. Although $T = T_0$, the value of exergy is positive, as expected, since $p \neq p_0$. See discussion of Figure 7.4.
3. Although $T = T_0$ and $p < p_0$ the value of exergy is positive, as expected, since $p \neq p_0$. See discussion of Figure 7.4.

PROBLEM 7.15

KNOWN: A balloon filled with helium is at a specified state.

FIND: Determine the specific exergy, in kJ/kg.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The closed system is the helium within the baloon. (2) Helium is modeled as an ideal gas with $k=1.67$. (3) $T_0 = 20°C$, $P_0 = 1$ bar, $g = 9.807\ m/s^2$.

ANALYSIS: The specific exergy is given by Eq. 7.2:

$$e = u - u_0 + P_0(v - v_0) - T_0(s - s_0) + \frac{V^2}{2} + gz$$

Since k is constant, c_v and c_p are also constant (see Eqs. 3.47). Thus, we get

$$e = \underline{c_v[T - T_0] + P_0\left[\frac{RT}{P} - \frac{RT_0}{P_0}\right] - T_0\left[c_p \ln\frac{T}{T_0} - R\ln\frac{P}{P_0}\right]} + \frac{V^2}{2} + gz$$

Since $T = T_0$ and $p = P_0$, the underlined term vanishes, leaving

$$e = \frac{V^2}{2} + gz$$

$$= \left[\frac{(15\ m/s)^2}{2} + (9.807\ m/s^2)(500\ m)\right]\left|\frac{1\ N}{1\ kg \cdot \frac{m}{s^2}}\right|\left|\frac{kJ}{10^3\ N\cdot m}\right|$$

$$= 5.016\ \frac{kJ}{kg} \longleftarrow e$$

Problem 7.16

A vessel contains carbon dioxide. Using the ideal gas model:

(a) determine the specific exergy of the gas, in Btu/lb, at p = 80 lbf/in.^{2}, for T = 180°F.

(b) plot the specific exergy of the gas, in Btu/lb, versus pressure ranging from 15 to 80 lbf/in.^{2}, for T = 80°F.

(c) plot the specific exergy of the gas, in Btu/lb, versus temperature ranging from 80 to 180°F, for p = 15 lbf/in.^{2}

The gas is at rest and zero elevation relative to an exergy reference environment for which T_0 = 80°F, p_0 = 15 lbf/in.^{2}

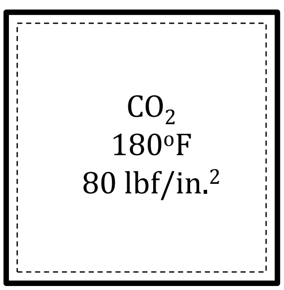
Known:

A vessel contains carbon dioxide (CO_2).

Find:

(a) Determine the specific exergy of the gas, in Btu/lb, at p = 80 lbf/in.^{2}, for T = 180°F. (b) Plot the specific exergy versus pressure for T = 80°F. (c) Plot the specific exergy versus temperature for p = 15 lbf/in.^{2}.

Schematic and Known Data:



$T_0 = 80$°F
$p_0 = 15$ lbf/in.^{2}

Engineering Model:

(1) The CO_2 is a closed system at rest and zero elevation relative to the reference environment at T_0 = 80°F, p_0 = 15 lbf/in.^{2}

(2) The CO_2 is modeled as an ideal gas.

Analysis:

Eq. 7.2 reduces to give:

$$\mathrm{e} = (u - u_0) + p_0(v - v_0) - T_0(s - s_0) \qquad (1)$$

(a) When using gas table data,

$$u - u_0 = \frac{\bar{u}(T) - \bar{u}(T_0)}{M}, \quad s - s_0 = \frac{\bar{s}^o(T) - \bar{s}^o(T_0) - \bar{R}\ln\frac{p}{p_0}}{M}$$

Thus with ideal gas relations, (1) becomes:

$$e = \frac{1}{M}\left\{[\bar{u}(T) - \bar{u}(T_0)] + p_0\left[\frac{\bar{R}T}{p} - \frac{\bar{R}T_0}{p_0}\right] - T_0\left[\bar{s}^o(T) - \bar{s}^o(T_0) - \bar{R}\ln\frac{p}{p_0}\right]\right\}$$

$$= \frac{1}{M}\left\{[\bar{u}(T) - \bar{u}(T_0)] + \bar{R}T\left[\frac{p_0}{p} - \frac{T_0}{T}\right] - T_0\left[\bar{s}^o(T) - \bar{s}^o(T_0) - \bar{R}\ln\frac{p}{p_0}\right]\right\}$$

Using data from Table A-23E:

$$\bar{u}(T) - \bar{u}(T_0) = 3704.0 - 2984.4 = 719.6\,\frac{\text{Btu}}{\text{lbmol}}$$

$$\bar{R}T\left[\frac{p_0}{p} - \frac{T_0}{T}\right] = \left(1.986\,\frac{\text{Btu}}{\text{lbmol}\cdot{}^\circ\text{R}}\right)(640^\circ\text{R})\left[\frac{15}{80} - \frac{540}{640}\right] = -834.12\,\frac{\text{Btu}}{\text{lbmol}}$$

$$T_0\left[\bar{s}^o(T) - \bar{s}^o(T_0) - \bar{R}\ln\frac{p}{p_0}\right] = (540^\circ\text{R})\left[(52.641 - 51.082) - (1.986)\ln\frac{80}{15}\right]\left|\frac{\text{Btu}}{\text{lbmol}\cdot{}^\circ\text{R}}\right|$$

$$= -953.38\,\frac{\text{Btu}}{\text{lbmol}}$$

Finally:

$$e = \left(\frac{1}{44.01\,\frac{\text{lb}}{\text{lbmol}}}\right)(719.6 - 834.12 + 953.38)\,\frac{\text{Btu}}{\text{lbmol}} = 19.06\,\frac{\text{Btu}}{\text{lb}}$$

(b) and (c) The following IT code is used to generate data for the required plots. The evaluations are based on Eq. (1) and internal functions in IT for *u*, *v*, and *s* of CO_2 as an ideal gas.

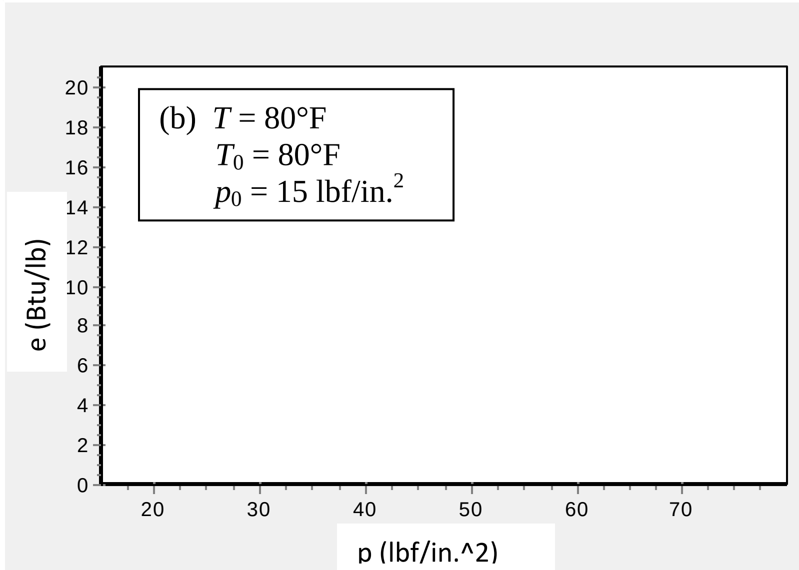
<u>IT Code</u>

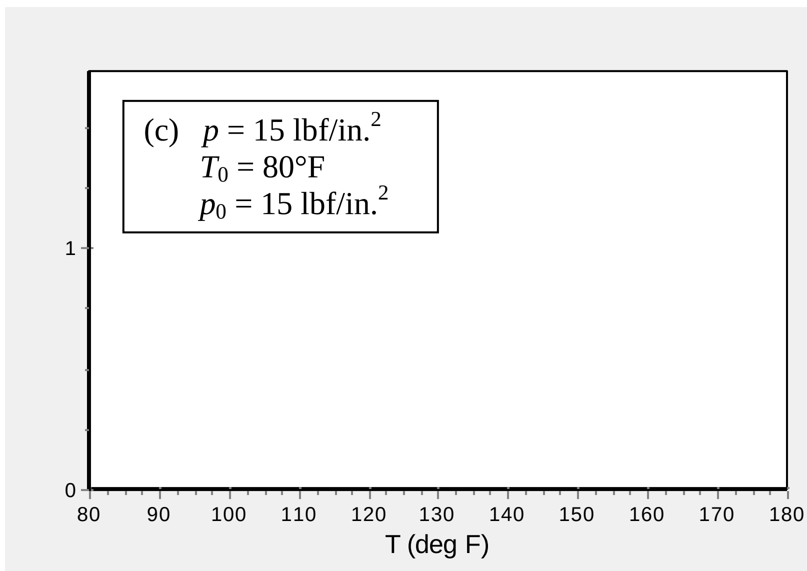
```
T = 180 // °F
p = 80 // lbf/in.^2
T0 = 80 // °F
p0 = 15 // lbf/in.^2

e = (u - u0) + (p0 * (144 / 778)) * (v - v0) - (T0 + 460) * (s - s0)
u = u_T("CO2",T)
u0 = u_T("CO2",T0)
v = v_Tp("CO2",T,p)
v0 = v_Tp("CO2",T0,p0)
s = s_Tp("CO2",T,p)
s0 = s_Tp("CO2",T0,p0)
```

<u>IT Solution for p = 80 lbf/in.^{2}, T = 180°F</u>

#1 e = 19.08 Btu/lb

Plots:





Comment:

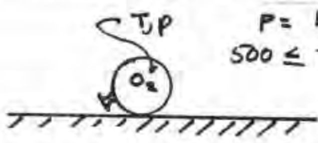
1. The IT result compares very favorably with the ideal gas table result in part (a), as expected.

PROBLEM 7.17

KNOWN: Oxygen (O_2) at T, 1atm fills a balloon at rest on the surface of the earth where the ambient temperature and pressure are known.

FIND: Plot specific exergy versus T.

SCHEMATIC & GIVEN DATA:



$p = 1\text{atm}$, $p_0 = 1\text{atm}$, $T_0 = 500°R$

$500 \le T \le 600°R$

ENGR. MODEL: (1) The O_2 is at rest and zero elevation relative to the environment for which $T_0 = 500°R$, $p_0 = 1\text{atm}$. (2) O_2 is modeled as an ideal gas with constant c_p.

ANALYSIS: Eq. 7.2 reduces to

$$e = u - u_0 + p_0(v - v_0) - T_0(s - s_0)$$

With ideal gas relations, and noting that $p = p_0$

$$e = u(T) - u(T_0) + p_0\left(\frac{RT}{p} - \frac{RT_0}{p_0}\right) - T_0\left(s°(T) - s°(T_0) - \cancel{R \ln\frac{p}{p_0}}^{\,0}\right)$$

$$= u(T) - u(T_0) + R(T - T_0) - T_0(s°(T) - s°(T_0)) \qquad (1)$$

(1) Since c_p is constant: $c_p = 0.22\,\text{Btu/lb}\cdot°R$, Eq. (1) can be expressed as

$$e = c_v[T - T_0] + R[T - T_0] - T_0 c_p \ln\frac{T}{T_0}$$

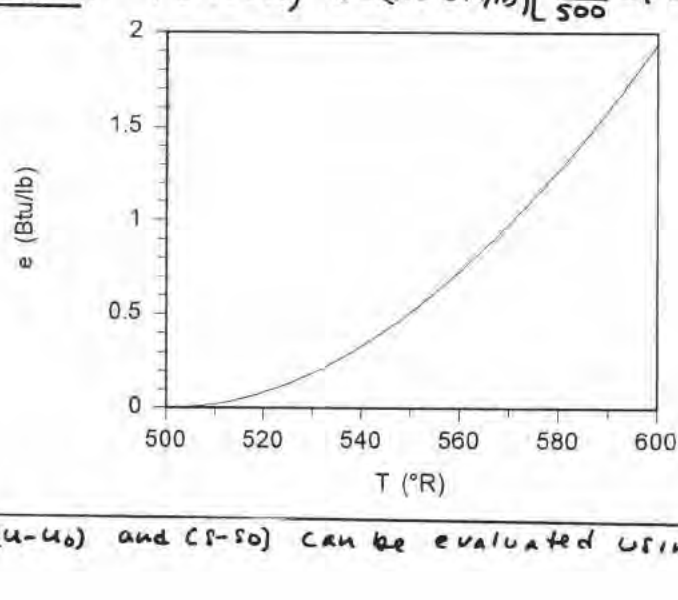
$$= \underbrace{(c_v + R)}_{c_p \text{ (Eq. 3.44)}}[T - T_0] - T_0 c_p \ln\frac{T}{T_0}$$

or

$$e = T_0 c_p\left[\frac{T}{T_0} - 1 - \ln\frac{T}{T_0}\right]$$

$$= \underbrace{\left(0.22\,\frac{\text{Btu}}{\text{lb}\cdot°R}\right)(500°R)}_{110\,\text{Btu/lb}}\left[\frac{T}{T_0} - 1 - \ln\frac{T}{T_0}\right] = \left(110\,\frac{\text{Btu}}{\text{lb}}\right)\left[\frac{T}{T_0} - 1 - \ln\frac{T}{T_0}\right] \qquad (2)$$

SAMPLE CALCULATION: $T = 600°R$, $e = (110\,\text{Btu/lb})\left[\frac{600}{500} - 1 - \ln\frac{600}{500}\right] = 1.945\,\text{Btu/lb}$



1. Alternatively, $(u - u_0)$ and $(s - s_0)$ can be evaluated using data from Table A-23E.

PROBLEM 7.18

KNOWN: A vessel contains a known amount of air at pressure p and 200°F.

FIND: Plot the specific exergy versus p ranging from 0.5 to 2 atm.

SCHEMATIC & GIVEN DATA:

$T = 200°F$
$0.5 \le p \le 2$ atm
$T_0 = 60°F$
$p_0 = 1$ atm

ENGR. MODEL: (1) The air is a closed system, at rest and zero elevation relative to the reference environment which is at $T_0 = 60°F$, $p_0 = 1$ atm. (2) The air is modeled as an ideal gas.

ANALYSIS: Equation 7.2 reduces to $e = (u - u_0) + p_0(v - v_0) - T_0(s - s_0)$ (1)

SAMPLE CALCULATION: This calculation will be done using gas table data. That is, $u - u_0 = u(T) - u(T_0)$, $s - s_0 = s^\circ(T) - s^\circ(T_0) - R \ln p/p_0$. With the ideal gas equation of state

$$e = u(T) - u(T_0) + p_0\left(\frac{RT}{p} - \frac{RT_0}{p_0}\right) - T_0\left[s^\circ(T) - s^\circ(T_0) - R \ln p/p_0\right]$$

Using data from Table A-22E, for the case $T = 200°F$, $p = 0.5$ atm

$$u(T) - u(T_0) = 112.67 - 88.62 = 24.05 \text{ Btu/lb}$$

$$p_0 R\left(\frac{T}{p} - \frac{T_0}{p_0}\right) = (1 \text{ atm})\left(\frac{1.986}{28.97}\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)\left(\frac{660°\text{R}}{0.5 \text{ atm}} - \frac{520°\text{R}}{1 \text{ atm}}\right) = 54.843 \text{ Btu/lb}$$

$$T_0\left[s^\circ(T) - s^\circ(T_0) - R \ln p/p_0\right] = (520°\text{R})\left[(0.64902 - 0.59172) - \left(\frac{1.986}{28.97}\right)\ln\left(\frac{0.5}{1}\right)\right]\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$$

$$= 54.505 \text{ Btu/lb}$$

Finally

$$e = 24.05 + 54.843 - 54.505 = 24.388 \text{ Btu/lb}$$

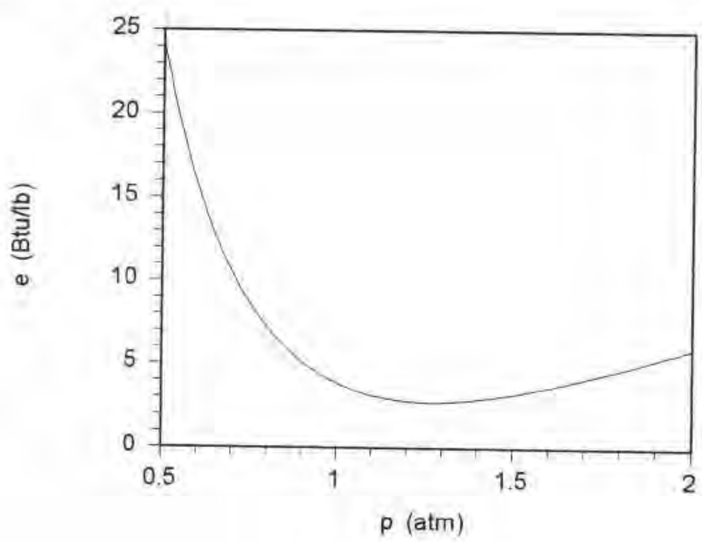
Using IT to generate the data required, the evaluations are based on Eq. (1) above and internal IT functions for u, v, and s, as follows:

IT Code

```
T = 200 // °F
p = 0.5 // atm
To = 60 // °F
po = 1 // atm

e = (u - uo) + po * (v - vo) * (14.696*144/778) - (To + 460) * (s - so)
u = u_T("Air", T)
uo = u_T("Air", To)
v = v_TP("Air",T,p)
vo = v_TP("Air",To,po)
s = s_TP("Air", T, p)
so = s_TP("Air", To, po)
```

IT Result for p = 0.5 atm, T = 200°F

(1) e = 24.37 Btu/lb



1. The IT result is in close agreement with the sample calculation based on Table A-22E, as expected.

PROBLEM 7.19

7.19 Determine the exergy, in Btu, of a sample of water as saturated solid at 10°F, measuring 2.25 in. × 0.75 in. × 0.75 in. Let $T_0 = 537°R$ and $p_0 = 1$ atm.

KNOWN: Data are provided for a sample of water as saturated solid.

FIND: Determine the exergy, in Btu

ENGINEERING MODEL:

1. The closed system is the sample of water.
2. $T_0 = 537°R$, $P_0 = 1$ atm. The effects of motion and gravity are not significant.
3. $u \approx u_f(T)$, $v \approx v_f(T)$, $s \approx s_f(T)$. All for liquid water.

ANALYSIS: At $T_0 = 537°R$, $P_0 = 1$ atm, water is a liquid. Thus, with data from Table A-2E and assumption #3,

$$u_0 \approx u_f\,(77°F) = 45.09\ \text{Btu/lb}$$
$$v_0 \approx v_f\,(77°F) = 0.01607\ \text{ft}^3/\text{lb}$$
$$s_0 \approx u_f\,(77°F) = 0.08775\ \text{Btu/lb}\cdot°R$$

For saturated solid at 10°F, Table A-6E gives $u = -154.17$ Btu/lb, $v = 0.01744$ ft³/lb, $s = -0.314 \frac{\text{Btu}}{\text{lb}\cdot°R}$

Then, with assumption #2, Eq. 7.2 reduces to read

$$e = (u - u_0) + p_0(v - v_0) - T_0(s - s_0)$$

$$= \left[-154.17 - 45.09\right]\frac{\text{Btu}}{\text{lb}} + \left(14.7 \times 144\ \frac{\text{lbf}}{\text{ft}^2}\right)\left(0.01744 - 0.01607\right)\frac{\text{ft}^3}{\text{lb}}\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

$$- 537°R\left(-0.314 - 0.08775\right)\frac{\text{Btu}}{\text{lb}\cdot°R}$$

$$= \left(-199.26 + 0.004 + 215.74\right)\frac{\text{Btu}}{\text{lb}}$$

$$= 16.48\ \frac{\text{Btu}}{\text{lb}}$$

The system mass is,

$$m = \frac{V}{v} = \frac{(2.25\ \text{in.})(0.75\ \text{in.})(0.75\ \text{in.})}{(0.01744\ \text{ft}^3/\text{lb})}\left|\frac{1\ \text{ft}}{12\ \text{in.}}\right|^3 = 0.042\ \text{lb}$$

Collecting results,

$$E = me = (0.042\ \text{lb})\left(16.48\ \frac{\text{Btu}}{\text{lb}}\right)$$

① $$= 0.692\ \text{Btu}$$

1. This result can be interpreted as the minimum theoretical work input, in Btu, required to bring the system from the dead state where it is a liquid to the specified state where it is a solid (ice).

PROBLEM 7.20

7.20 Determine the exergy, in kJ, of the contents of a 1.5-m^3 storage tank, if the tank is filled with
(a) air as an ideal gas at 440°C and 0.70 bar.
(b) water vapor at 440°C and 0.70 bar.
Ignore the effects of motion and gravity and let T_0=22°C, p_0=1 bar.

KNOWN: Systems at specified states are defined and contained within a rigid tank with known volume.

FIND: Determine the exergy, in kJ, for each substance.

SCHEMATIC AND GIVEN DATA:

$T_0 = 22°C$
$p_0 = 1$ bar

$T_1 = 440°C = 713$ K
$p_1 = 0.7$ bar
$V = 1.5$ m^3

ENGINEERING MODEL:
(1) In part (a), air is modeled as an ideal gas.
(2) The closed systems are at rest and zero elevation relative to the environment at T_0= 22°C = 295K, p_0=1 bar.

ANALYSIS:
Determine exergy using a form of Eq. 7.2.

$$\mathbf{E} = m\left[(u_1 - u_0) + p_0(v_1 - v_0) - T_0(s_1 - s_0) + \frac{\mathrm{V}_1^2}{2} + gz_1\right]$$

Simplify based on assumption:

$$\mathbf{E} = m[(u_1 - u_0) + p_0(v_1 - v_0) - T_0(s_1 - s_0)] \qquad (1)$$

(a) Air as an ideal gas at 440°C and 0.70 bar
Fix properties of air at the dead state and given state using Table A-22 referenced at 295 K and 713 K, respectively.

$$u_0 = 210.49\frac{\text{kJ}}{\text{kg}} \quad s_0^{\,o} = 1.68515\frac{\text{kJ}}{\text{kg}\cdot\text{K}} \quad u_1 = 522.60\frac{\text{kJ}}{\text{kg}} \quad s_1^{\,o} = 2.59263\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

Determine the change in specific entropy using Eq. 6.20a.

$$s_1 - s_0 = s_1^{\,o} - s_0^{\,o} - \frac{\overline{R}}{M}\ln\frac{p_1}{p_0} = (2.59263 - 1.68515)\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - \frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\ln\frac{0.70}{1} = 1.00984\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

Fix values for m, v_1, and v_0 using the ideal gas model, as follows:

$$m = \frac{p_1 V M}{\overline{R}T_1} = \frac{(0.70\,\text{bar})(1.5\,\text{m}^3)\left(28.97\frac{\text{kg}}{\text{kmol}}\right)}{\left(8.314\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(713\text{K})}\left|\frac{10^5\frac{\text{N}}{\text{m}^2}}{1\,\text{bar}}\right|\left|\frac{1\,\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right| = 0.513\,\text{kg}$$

$$v_1 = \frac{V}{m} = \frac{1.5\,\text{m}^3}{0.513\,\text{kg}} = 2.924\frac{\text{m}^3}{\text{kg}}$$

$$v_0 = \frac{\overline{R}T_0}{Mp_0} = \frac{\left(8.314\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(295\,\text{K})}{\left(28.97\frac{\text{kg}}{\text{kmol}}\right)(1\,\text{bar})}\left|\frac{1\,\text{bar}}{10^5\frac{\text{N}}{\text{m}^2}}\right|\left|\frac{10^3\,\text{N}\cdot\text{m}}{1\,\text{kJ}}\right| = 0.8466\frac{\text{m}^3}{\text{kg}}$$

PROBLEM 7.20 (Continued)

Substitute values into Eq. (1).

$$\mathbf{E} = 0.513\,\text{kg}\left[(522.60-210.49)\frac{\text{kJ}}{\text{kg}} + 1\,\text{bar}(2.924-0.8466)\frac{\text{m}^3}{\text{kg}}\left|\frac{10^5\,\frac{\text{N}}{\text{m}^2}}{1\,\text{bar}}\right|\left|\frac{1\,\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right| - 295\,\text{K}\left(1.00984\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\right]$$

#1 $$\mathbf{E} = 0.513\,\text{kg}[(312.11)+(207.74)-(297.90)]\frac{\text{kJ}}{\text{kg}} = 113.86\,\text{kJ}$$

(b) Water vapor at 440°C and 0.70 bar

Fix properties of water vapor at the dead state. Using Table A-3 at p_0, $T_0 < T_{sat}$, therefore compressed liquid exists. Use Table A-2 as follows:

$$u_0 = u_f = 92.32\frac{\text{kJ}}{\text{kg}} \quad s_0 = s_f = 0.3251\frac{\text{kJ}}{\text{kg}\cdot\text{K}} \quad v_0 = v_f = 1.0022\times10^{-3}\frac{\text{m}^3}{\text{kg}}$$

Fix properties of water vapor at the given state. Using Table A-3 at p_1, $T_1 > T_{sat}$ therefore superheated vapor exists. Using Table A-4:

$$u_1 = 3032.9\frac{\text{kJ}}{\text{kg}} \quad s_1 = 8.8286\frac{\text{kJ}}{\text{kg}\cdot\text{K}} \quad v_1 = 4.698\frac{\text{m}^3}{\text{kg}}$$

Determine mass as follows:

$$m = \frac{V}{v_1} = \frac{1.5\,\text{m}^3}{4.698\frac{\text{m}^3}{\text{kg}}} = 0.3193\,\text{kg}$$

Substitute values into Eq. (1).

$$\mathbf{E} = 0.3193\,\text{kg}\left[(3032.9-92.32)\frac{\text{kJ}}{\text{kg}} + 1\,\text{bar}(4.698-0.0010022)\frac{\text{m}^3}{\text{kg}}\left|\frac{10^5\,\frac{\text{N}}{\text{m}^2}}{1\,\text{bar}}\right|\left|\frac{1\,\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right| - 295\text{K}(8.8286-0.3251)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]$$

#1 $$\mathbf{E} = 0.3193\,\text{kg}[(2940.58)+(469.7)-(2508.53)]\frac{\text{kJ}}{\text{kg}} = 287.93\,\text{kJ}$$

---

1. Exergy is positive, as expected. In comparing the exergy values from parts (a) and (b), the exergy of the water vapor is over two times greater than that of air at the same pressure and temperature.

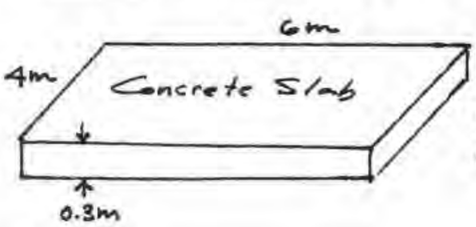
PROBLEM 7.21

7.21 A concrete slab measuring 0.3 m × 4 m × 6 m, initially at 298 K, is exposed to the sun for several hours, after which its temperature is 301 K. The density of the concrete is 2300 kg/m^3 and its specific heat is c = 0.88 kJ/kg · K. (a) Determine the increase in exergy of the slab, in kJ. (b) To what elevation, in m, would a 1000-kg mass have to be raised from zero elevation relative to the reference environment for its exergy to equal the exergy increase of the slab? Let T_0 = 298 K, p_0 = 1 atm, g = 9.81 m/s^2.

KNOWN: Data are provided for a concrete slab warmed by the sun.

FIND: Determine the exergy of the slab. Also, determine the elevation to which a 1000-kg mass would have to be raised for its exergy to equal that of the slab.

SCHEMATIC & GIVEN DATA:



$T_1 = 298K$
$T_2 = 301K$
$\rho = 2300\ kg/m^3$
$c = 0.88 \frac{kJ}{kg \cdot K}$

ENGINEERING MODEL:

1. The slab is the closed system.
2. For the system, the effects of motion and gravity are not significant. $T_0 = 298K$, $P_0 = 1\ atm$.
3. The slab is modeled as incompressible with known values of ρ and specific heat.

ANALYSIS: With assumption #2, Eq. 7.3 reduces to give

$$E_2 - E_1 = m\left[(u_2-u_1) + p_0(v_2-v_1) - T_0(s_2-s_1)\right]$$

Since the slab is modeled as incompressible, $(u_2-u_1) = c(T_2-T_1)$, $(v_2-v_1) = 0$, and $(s_2-s_1) = c \ln\frac{T_2}{T_1}$.

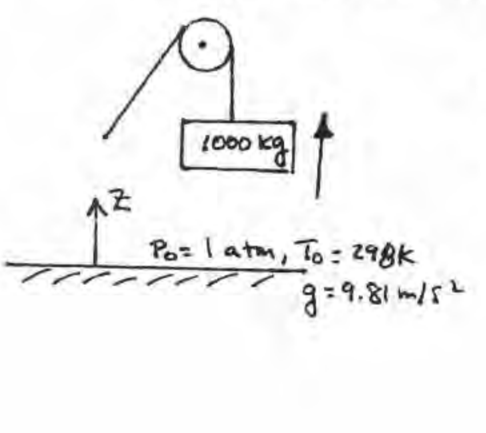
Collecting results,

$$E_2 - E_1 = m c\left[(T_2-T_1) - T_0 \ln\frac{T_2}{T_1}\right] \qquad (1)$$

$$m = \rho V = \left(2300 \frac{kg}{m^3}\right)\left((6m)(4m)(0.3m)\right)$$

$$= 16{,}560\ kg$$

$$\Rightarrow \quad E_2 - E_1 = (16{,}560\ kg)\left(0.88\frac{kJ}{kg \cdot K}\right)\left[3\ K - 298K \ln\left(\frac{301}{298}\right)\right] = 218.6\ kJ$$



The elevation to which a 1000-kg mass would have to be raised for its exergy to equal 218.6 kJ is found as follows:

$$\Delta E = mgz$$

$$\Rightarrow \quad z = \frac{\Delta E}{mg}$$

$$= \frac{218.6\ kJ}{(1000\ kg)(9.81\ m/s^2)} \left|\frac{10^3 N \cdot m}{1 kJ}\right|\left|\frac{1\ kg \cdot m/s^2}{1\ N}\right|$$

$$= 22.3\ m$$

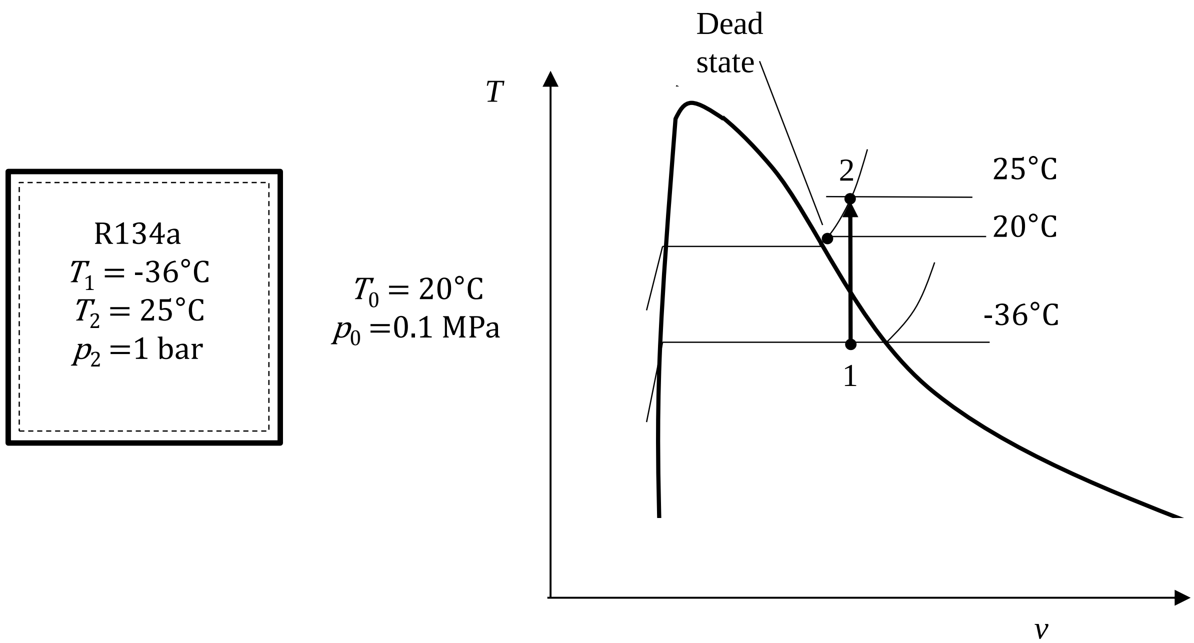
Problem 7.22

Refrigerant 134a initially at -36°C fills a rigid vessel. The refrigerant is heated until the temperature becomes 25°C and the pressure is 1 bar. There is no work during the process. For the refrigerant, determine the heat transfer per unit mass and the change in specific exergy, each in kJ/kg. Comment. Let T_0 = 20°C, p_0 = 0.1 MPa and ignore the effects of motion and gravity.

Known:
Refrigerant 134a in a rigid vessel is heated from its initial conditions.

Find:
Determine the heat transfer per unit mass and the change in specific exergy.

Schematic and Known Data:



Engineering Model:
(1) The R134a is the closed system.
(2) The volume remains constant in the process.
(3) For the system, W = 0 and the effects of motion and gravity can be ignored.
(4) The environment is at T_0 = 20°C, p_0 = 1 bar.

Analysis:

State 2 is in superheated region. Obtaining property data from Table A-12, using 1 bar, 25°C and interpolating:

$$v_2 = 0.23783\,\frac{\mathrm{m}^3}{\mathrm{kg}},\quad u_2 = 250.61\,\frac{\mathrm{kJ}}{\mathrm{kg}},\quad s_2 = 1.0976\,\frac{\mathrm{kJ}}{\mathrm{kg}\cdot\mathrm{K}}$$

According to assumption (2), $v_2 = v_1$ for the process. Using Table A-10 at -36°C:

$$x_1 = \frac{v_1 - v_f}{v_g - v_f} = \frac{0.23783 - 0.7113 \cdot 10^{-3}}{0.2947 - 0.7113 \cdot 10^{-3}} = 0.81$$

$$u_1 = u_f + x_1(u_g - u_f) = 4.68 + 0.81(206.73 - 4.68) = 168.34\,\frac{\text{kJ}}{\text{kg}}$$

$$s_1 = s_f + x_1(s_g - s_f) = 0.0201 + 0.81(0.9506 - 0.0201) = 0.7738\,\frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

Reducing an energy balance:

$$\frac{Q}{m} = u_2 - u_1 = 250.61 - 168.34 = 82.27\,\frac{\text{kJ}}{\text{kg}}$$

With Eq. 7.2:

$$\Delta\text{e} = (u_2 - u_1) + \underbrace{p_0(v_2 - v_1)}_{=0} - T_0(s_2 - s_1) = 82.27\,\frac{\text{kJ}}{\text{kg}} - (293\text{ K})(1.0976 - 0.7738)\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$\Delta\text{e} = -12.60\,\frac{\text{kJ}}{\text{kg}}$$

In keeping with the discussion of Fig. 7.4, the exergy decreases because the system is brought closer to the dead state with $p_2 = p_0$ and T_2 approaching T_0.

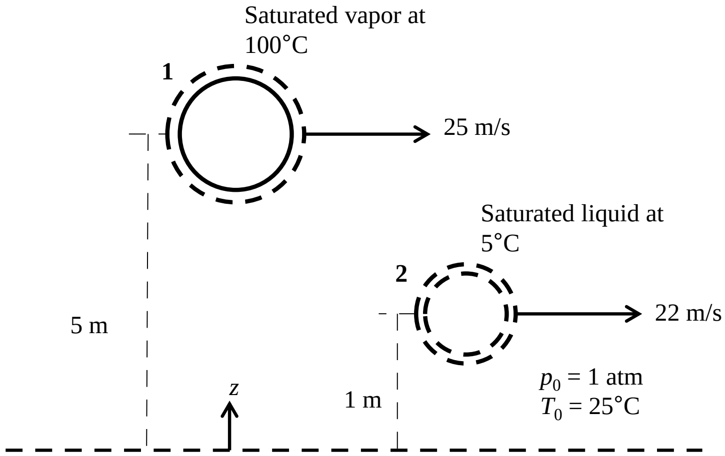
Problem 7.23

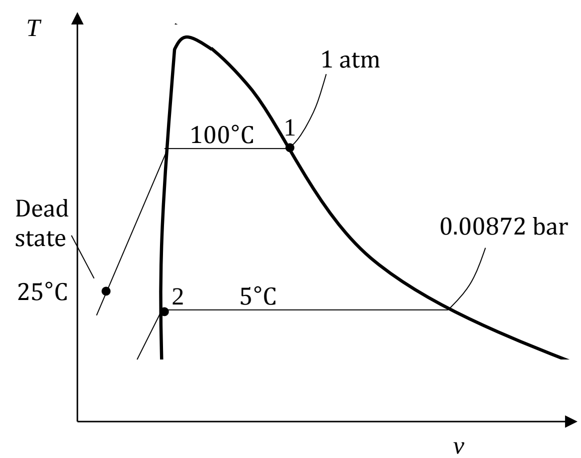
As shown in Fig. P7.23, one kilogram of water undergoes a process from an initial state where the water is saturated vapor at 100°C, the velocity is 25 m/s, and the elevation is 5 m to a final state where the water is saturated liquid at 5°C, the velocity is 22 m/s, and the elevation is 1 m. Determine in kJ, (a) the exergy at the initial state, (b) the exergy at the final state, and (c) the change in exergy. Let T_0 = 25°C, p_0 = 1 atm, and $g = 9.8$ m/s^2.

Known:
One kg of water undergoes a process between specified states.

Find:
Determine the exergy at the initial and final states and the change in exergy.

Schematic and Known Data:





Engineering Model:
(1) The water is a closed system at equilibrium states initially and finally.
(2) The velocities and elevations are measured relative to the environment.
(3) T_0 = 25°C, p_0 = 1 atm, $v_0 \approx v_f(T_0)$, $u_0 \approx u_f(T_0)$, $s_0 \approx s_f(T_0)$.

Analysis:
The exergy at the initial and final states can be calculated using Eq. 7.2:

$$E_1 = m\left[(u_1 - u_0) + p_0(v_1 - v_0) - T_0(s_1 - s_0) + \frac{V_1^{\,2}}{2} + gz_1\right]$$

#1
$$E_2 = m\left[(u_2 - u_0) + p_0(v_2 - v_0) - T_0(s_2 - s_0) + \frac{V_2^{\,2}}{2} + gz_2\right]$$

(a) For saturated vapor at 100°C, Table A-2 gives:

$$v_1 = 1.673\,\frac{\text{m}^3}{\text{kg}},\quad u_1 = 2506.5\,\frac{\text{kJ}}{\text{kg}},\quad s_1 = 7.3549\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

At the dead state using Table A-2 and assumption (3):

$$v_0 = 1.0029 \cdot 10^{-3}\,\frac{\mathrm{m}^3}{\mathrm{kg}},\quad u_0 = 104.88\,\frac{\mathrm{kJ}}{\mathrm{kg}},\quad s_0 = 0.3674\,\frac{\mathrm{kJ}}{\mathrm{kg}\cdot\mathrm{K}}$$

Using these values:

$$\mathrm{E}_1 = (1\ \mathrm{kg})\left[(2506.5 - 104.88)\frac{\mathrm{kJ}}{\mathrm{kg}} + \left(1.013 \cdot 10^5\,\frac{\mathrm{N}}{\mathrm{m}^2}\right)(1.673 - 1.0029 \cdot 10^{-3})\frac{\mathrm{m}^3}{\mathrm{kg}} \cdot \left|\frac{1\ \mathrm{kJ}}{10^3\mathrm{N}\cdot\mathrm{m}}\right| - (298\ \mathrm{K})(7.3549 - 0.3674)\frac{\mathrm{kJ}}{\mathrm{kg}\cdot\mathrm{K}} + \left[\frac{\left(25\,\frac{\mathrm{m}}{\mathrm{s}}\right)^2}{2} + \left(9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)(5\ \mathrm{m})\right]\left|\frac{1\ \mathrm{N}}{1\,\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2}}\right|\left|\frac{\mathrm{kJ}}{10^3\mathrm{N}\cdot\mathrm{m}}\right|\right]$$

$$= (1\ \mathrm{kg})[2401.62 + 169.37 - 2082.28 + 0.36]\,\frac{\mathrm{kJ}}{\mathrm{kg}} = 489.07\ \mathrm{kJ} \longleftarrow$$

#2 (b) For saturated liquid at 5°C, Table A-2 gives:

$$v_2 = 1.0001 \cdot 10^{-3}\,\frac{\mathrm{m}^3}{\mathrm{kg}},\quad u_2 = 20.97\,\frac{\mathrm{kJ}}{\mathrm{kg}},\quad s_2 = 0.0761\,\frac{\mathrm{kJ}}{\mathrm{kg}\cdot\mathrm{K}}$$

Using these values in Eq. 7.2:

$$\mathrm{E}_2 = (1\mathrm{kg})\left[(20.97 - 104.88)\frac{\mathrm{kJ}}{\mathrm{kg}} + \left(1.013 \cdot 10^5\,\frac{\mathrm{N}}{\mathrm{m}^2}\right)(1.0001 \cdot 10^{-3} - 1.0029 \cdot 10^{-3})\frac{\mathrm{m}^3}{\mathrm{kg}} \cdot \left|\frac{\mathrm{kJ}}{10^3\mathrm{N}\cdot\mathrm{m}}\right| - (298\ \mathrm{K})(0.0761 - 0.3674)\frac{\mathrm{kJ}}{\mathrm{kg}\cdot\mathrm{K}} + \left[\frac{\left(22\frac{\mathrm{m}}{\mathrm{s}}\right)^2}{2} + \left(9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)(1\ \mathrm{m})\right]\left|\frac{1\ \mathrm{N}}{1\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2}}\right|\left|\frac{\mathrm{kJ}}{10^3\mathrm{N}\cdot\mathrm{m}}\right|\right] \longleftarrow$$

$$= (1\ \mathrm{kg})[-83.91 + (-0.00028) + 86.81 + 0.25]\,\frac{\mathrm{kJ}}{\mathrm{kg}} = 3.15\ \mathrm{kJ}$$

#3 (c) Finally, the change in exergy is:

$$\Delta\mathrm{E} = \mathrm{E}_2 - \mathrm{E}_1 = 3.15 - 489.07 = -485.92\ \mathrm{kJ} \longleftarrow$$

<u>Comments:</u>

1. The kinetic and potential energies measured relative to the environment contribute their full magnitudes to the value of exergy, for in principle each could be completely converted to work were the system brought to rest at zero elevation relative to the environment.

2. Exergy is a measure of the departure of the state of the system from that of the environment. At all states, E $\geq$ 0. This applies when $T > T_0$, $p = p_0$, as in part (a), and when $T < T_0$, $p < p_0$ as in part (b).
3. Alternatively, Eq. 7.3 can be used. This requires dead state property values <u>only</u> for T_0 and p_0. In parts (a) and (b) above, u_0, v_0, and s_0 are also required; so more computation is needed with the presented approach.

Problem 7.24

Three pounds of carbon monoxide initially at 180°F and 40 lbf/in.^{2} undergo two processes in series:

Process 1-2: Constant pressure to T_2 = -10°F

Process 2-3: Isothermal to p_3 = 10 lbf/in.^{2}

Employing the ideal gas model,

(a) represent each process on a *p*-*v* diagram and indicate the dead state.

(b) determine the change in exergy for each process, in Btu.

Let T_0 = 77°F, p_0 = 14.7 lbf/in.^{2} and ignore the effects of motion and gravity.

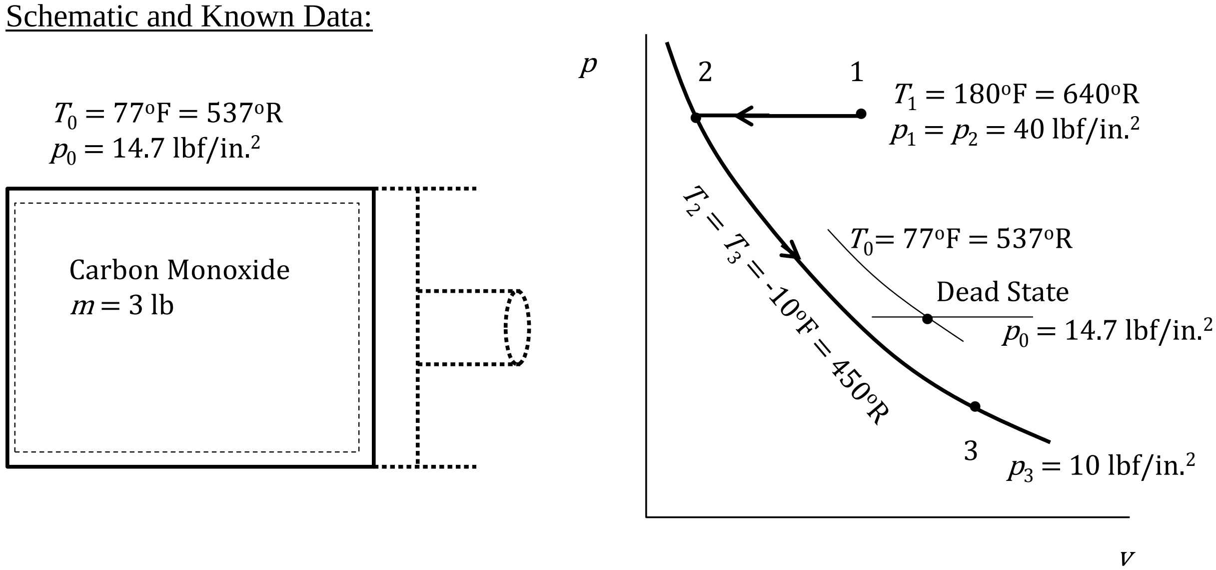
Known:

Carbon monoxide undergoes two specified processes in series.

Find:

(a) Represent each process on a *p*-*v* diagram and indicate the dead state. (b) Determine the change in exergy for each process.

Schematic and Known Data:



Engineering Model:

(1) The carbon monoxide is the closed system.

(2) The effects of motion and gravity can be ignored.

(3) The carbon monoxide is modeled as an ideal gas.

(4) For the environment, T_0 = 77°F, p_0 = 14.7 lbf/in.^{2}

Analysis:

(a) From the given information, p_1 = 40 lbf/in.^{2}, T_1 = 180°F = 640°R. Thus:

$$v_1 = \frac{RT_1}{p_1} = \frac{\left(\frac{1545}{28.01}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(640°\text{R})}{40\,\frac{\text{lbf}}{\text{in.}^2}}\left|\frac{\text{ft}^2}{144\text{ in.}^2}\right| = 6.13\,\frac{\text{ft}^3}{\text{lb}}$$

Similarly:

$$v_2 = \frac{RT_2}{p_2} = \frac{\left(\frac{1545}{28.01}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(450^\circ\text{R})}{40\,\frac{\text{lbf}}{\text{in.}^2}}\left|\frac{\text{ft}^2}{144\ \text{in.}^2}\right| = 4.31\,\frac{\text{ft}^3}{\text{lb}}$$

$$v_3 = \frac{RT_3}{p_3} = \frac{\left(\frac{1545}{28.01}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(450^\circ\text{R})}{10\,\frac{\text{lbf}}{\text{in.}^2}}\left|\frac{\text{ft}^2}{144\ \text{in.}^2}\right| = 17.24\,\frac{\text{ft}^3}{\text{lb}}$$

(b) Using Eq. 7.3 and ideal gas relations:

$$\text{E}_2 - \text{E}_1 = m\left[\frac{\bar{u}(T_2) - \bar{u}(T_1)}{M} + p_0(v_2 - v_1) - T_0\left(\frac{\bar{s}^o(T_2) - \bar{s}^o(T_1)}{M} - \underbrace{R\ln\frac{p_2}{p_1}}_{=0}\right)\right]$$

$$\text{E}_3 - \text{E}_2 = m\left[\underbrace{\frac{\bar{u}(T_3) - \bar{u}(T_2)}{M}}_{=0} + p_0(v_3 - v_2) - T_0\left(\underbrace{\frac{\bar{s}^o(T_3) - \bar{s}^o(T_2)}{M}}_{=0} - R\ln\frac{p_3}{p_2}\right)\right]$$

With data from Table A-23E and interpolation as needed:

$$\begin{aligned}\text{E}_2 - \text{E}_1 = (3\ \text{lb})\ &\left[\frac{(2230.85 - 3175.9)\,\frac{\text{Btu}}{\text{lbmol}}}{28.01\,\frac{\text{lb}}{\text{lbmol}}}\right.\\ &+ \left(14.7\,\frac{\text{lbf}}{\text{in.}^2}\cdot\left|\frac{144\ \text{in.}^2}{\text{ft}^2}\right|\cdot\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|\right)(4.31 - 6.13)\,\frac{\text{ft}^3}{\text{lb}}\\ &\left.- (537^\circ\text{R})\,\frac{(46.04 - 48.494)\,\frac{\text{Btu}}{\text{lbmol}\cdot{}^\circ\text{R}}}{28.01\,\frac{\text{lb}}{\text{lbmol}}}\right]\end{aligned}$$

$$= (3)[-33.74 - 4.95 + 47.05] = 25.1\ \text{Btu} \longleftarrow$$

$$\begin{aligned}\text{E}_3 - \text{E}_2 = (3\ \text{lb})\ &\left[\left(14.7\,\frac{\text{lbf}}{\text{in.}^2}\cdot\left|\frac{144\ \text{in.}^2}{\text{ft}^2}\right|\cdot\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|\right)(17.24 - 4.31)\,\frac{\text{ft}^3}{\text{lb}}\right.\\ &\left.- (537^\circ\text{R})\left(-\frac{1.986}{28.01}\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\ln\frac{10}{40}\right)\right] = (3)[35.18 - 52.78] = -52.8\ \text{Btu}\end{aligned}$$

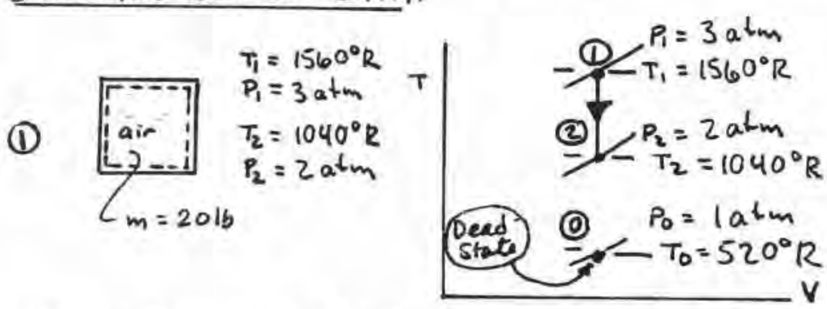
$$\longleftarrow$$

PROBLEM 7.25

KNOWN: 20 lb of air fills a rigid tank. The air is cooled from a known initial state to a specified final state.

FIND: Locate the initial, final, and dead states on a T-v diagram. Evaluate the heat transfer and the exergy change, and interpret the sign of the exergy change using the p-v diagram.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 1. The air is the closed system. 2. For the system, W = 0 and the effects of motion and gravity can be ignored. 3. Air is modeled as an ideal gas. 4. $T_0 = 520°R$, $P_0 = 1$ atm.

ANALYSIS: (a) Since volume and mass are fixed, the air undergoes a constant specific volume cooling process, as shown in the T-v diagram.

(b) Reducing an energy balance and using data from Table A-22E

$$m(u_2 - u_1) = Q - \cancel{W}^{0} \Rightarrow Q = m(u_2 - u_1) = (20\text{ lb})(179.66 - 278.13)\frac{\text{Btu}}{\text{lb}} = -1969.4\text{ Btu}$$

(c) Using Eq. 7.3, the change in exergy is

$$E_2 - E_1 = m\left[(u_2 - u_1) + \underbrace{p_0(v_2 - v_1)}_{=0 \text{ since } v_2 = v_1} - T_0(s_2 - s_1)\right] = m\left[(u_2 - u_1) - T_0(s_2 - s_1)\right]$$

where the kinetic/potential energy terms have been dropped by assumption 2. With Eq. 6.20a, this becomes

$$E_2 - E_1 = m\left[(u_2 - u_1) - T_0\left(s°(T_2) - s°(T_1) - R\ln\frac{p_2}{p_1}\right)\right]$$

$$= (20\text{ lb})\left[(179.66 - 278.13)\frac{\text{Btu}}{\text{lb}} - 520°R\left(0.76019 - 0.86456 - \frac{1.986}{28.97}\ln\frac{2}{3}\right)\frac{\text{Btu}}{\text{lb}°R}\right]$$

$$= -1173\text{ Btu}$$

The T-v diagram shows that the state of the system is brought "closer" to the dead state as the air is cooled. Since the value of exergy is a measure of the departure of the state of the system from the state of the environment, exergy decreases in the process from 1 to 2.

---

1. Since $pv = RT$, $\frac{v}{R} = \frac{T}{p}$. Then, using given data we get

$$\frac{v_1}{R} = \frac{T_1}{P_1} = \frac{1560°R}{3\text{ atm}} = 520, \quad \frac{v_2}{R} = \frac{T_2}{P_2} = \frac{1040°R}{2\text{ atm}} = 520, \quad \frac{v_0}{R} = \frac{T_0}{P_0} = \frac{520°R}{1\text{ atm}} = 520$$

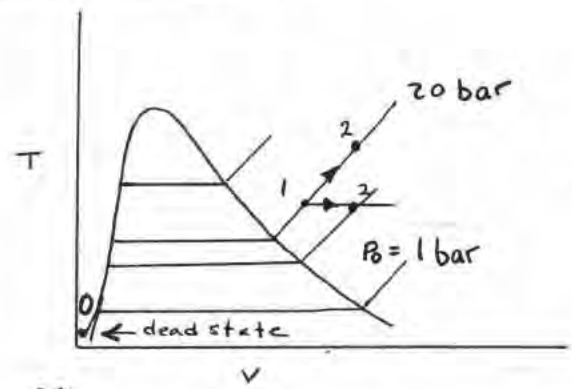
Thus, states 1, 2, and 0 fall as shown on the T-v diagram.

PROBLEM 7.26

KNOWN: 100 kg of steam initially at 20 bar and 240°C undergoes two different processes.

FIND: For each process determine the change in exergy.

SCHEMATIC & GIVEN DATA:



$T_1 = 240°C$

1-2: constant pressure, $v_2 = 2v_1$

1-2': isothermal, $v_2 = 2v_1$

ENGR. MODEL: (1) The system is the mass of steam. (2) For the environment, $T_0 = 20°C$, $p_0 = 1$ bar. (3) Ignore the effects of motion and gravity.

ANALYSIS: Equation 7.3 reduces to give $E_2 - E_1 = m[u_2 - u_1 + p_0(V_2 - V_1) - T_0(s_2 - s_1)]$. From Table A-4, $u_1 = 2659.6$ kJ/kg, $v_1 = 0.1085$ m³/kg, $s_1 = 6.4952$ kJ/kg K. For each process $v_2 = 2v_1 = 0.217$ m³/kg.

(a) Constant pressure process: Interpolating in Table A-4 at 20 bar with $v_2 = 0.217$ m³/kg, $u_2 = 3423.1$ kJ/kg, $s_2 = 7.8849$ kJ/kg·K

$$E_2 - E_1 = (100\,\text{kg})\left[(3423.1 - 2659.6)\frac{\text{kJ}}{\text{kg}} + \left(100\frac{\text{kN}}{\text{m}^2}\right)(0.217 - 0.1085)\frac{\text{m}^3}{\text{kg}}\left|\frac{1\,\text{kJ}}{1\,\text{kN·m}}\right| - 293\,\text{K}\,(7.8849 - 6.4952)\frac{\text{kJ}}{\text{kg·K}}\right]$$

① $$= (100\,\text{kg})[763.5 + 10.9 - 407.2]\frac{\text{kJ}}{\text{kg}} = 36{,}720\,\text{kJ} \longleftarrow \text{(a)}$$

(b) Isothermal Process: Interpolating in Table A-4 at 240°C with $v_2 = 0.217$ m³/kg, $u_2 = 2690.8$ kJ/kg, $s_2 = 6.8527$ kJ/kg·K

$$E_2 - E_1 = (100)(2690.8 - 2659.6) + (100)(0.1085) - 293(6.8527 - 6.4952)]$$

② $$= (100\,\text{kg})[31.2 + 10.9 - 104.7]\frac{\text{kJ}}{\text{kg}} = -6{,}260\,\text{kJ} \longleftarrow \text{(b)}$$

---

1. Here, pressure is kept constant and temperature increases relative to T_0. Exergy increases in the process.

2. Here, temperature is kept constant and pressure decreases relative to P_0. Exergy decreases in the process.

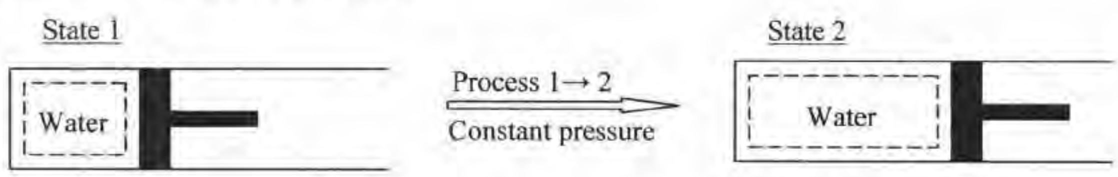
PROBLEM 7.27

7.27 Two kilograms of water in a piston-cylinder assembly, initially at 2 bar and 120°C, are heated at constant pressure with no internal irreversibilities to a final state where the water is a saturated vapor. For the water as the system, determine the work, the heat transfer, and the amounts of exergy transfer accompanying work and heat transfer, each in kJ. Let $T_0 = 20°C$, $p_0 =$ 1 bar and ignore the effects of motion and gravity.

KNOWN: System of water at specified pressure and temperature undergoes constant pressure process until reaching saturated vapor state.

FIND: The work, the heat transfer, and the amounts of exergy transfer accompanying work and heat transfer.

SCHEMATIC AND GIVEN DATA:



$m = 2$ kg
$p_1 = 2$ bar
$T_1 = 120°C$

$T_0 = 20°C = 293$ K
$p_0 = 1$ bar

Saturated vapor
$p_2 = p_1 = 2$ bar

ENGINEERING MODEL:
1. The water in the piston-cylinder assembly is a closed system defined by the dashed line on the accompanying diagram.
2. The process is internally reversible.
3. Pressure is constant during the process.
4. The effects of motion and gravity can be ignored.
5. $T_0 = 20°C$ and $p_0 = 1$ bar.

ANALYSIS:
The water at State 1 is compressed liquid. From Table A-2 at $T_1 = 120°C$,
$v_1 \approx v_{f1} = 0.0010603$ m^3/kg, $u_1 \approx u_{f1} = 503.50$ kJ/kg, $s_1 \approx s_{f1} = 1.5276$ kJ/(kg·K).

The water at State 2 is saturated vapor. From Table A-3, $v_2 = v_{g2} = 0.8857$ m^3/kg,
$u_2 = u_{g2} = 2529.5$ kJ/kg, $s_2 = s_{g2} = 7.1271$ kJ/(kg·K).

Water at the reference state is compressed liquid. From Table A-2 at $T_0 = 20°C$,
$u_0 \approx u_{f0} = 83.95$ kJ/kg, $v_0 \approx v_{f0} = 0.0010018$ m^3/kg, $s_0 \approx s_{f0} = 0.2966$ kJ/(kg·K).

Process 1-2 is constant pressure; thus $W_{12} = \int_{V_1}^{V_2} p dV = p_1(V_2 - V_1) = mp_1(v_2 - v_1)$. Substituting values, applying appropriate conversion factors, and solving for work yield

PROBLEM 7.27 (Continued, p.2)

$$W_{12} = (2\text{ kg})(2\text{ bar})\left(0.8857\frac{\text{m}^3}{\text{kg}} - 0.0010603\frac{\text{m}^3}{\text{kg}}\right)\left|\frac{10^5\frac{\text{N}}{\text{m}^2}}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| = \underline{\mathbf{353.9\text{ kJ}}}$$

The positive sign associated with work indicates work is done by the system.

Heat transfer can be determined from the energy balance for a closed system.

$$\Delta\text{KE} + \Delta\text{PE} + \Delta U = Q - W$$

Neglecting changes in kinetic and potential energy and expressing internal energy in terms of mass and specific internal energy give

$$m(u_2 - u_1) = Q_{12} - W_{12}$$

Solving for heat transfer and substituting values give

$$Q_{12} = m(u_2 - u_1) + W_{12} = (2\text{ kg})(2529.5\text{ kJ/kg} - 503.50\text{ kJ/kg}) + (353.9\text{ kJ}) = \underline{\mathbf{4{,}405.9\text{ kJ}}}$$

The positive sign indicates heat transfer to the system.

Exergy transfer accompanying work is determined from Eq. 7.6.

$$\mathsf{E}_w = [W - p_0(V_2 - V_1)] = [W - mp_0(v_2 - v_1)]$$

Substituting values and applying appropriate conversion factors give

$$\mathsf{E}_w = 353.9\text{ kJ} - (2\text{ kg})(1\text{ bar})\left(0.8857\frac{\text{m}^3}{\text{kg}} - 0.0010603\frac{\text{m}^3}{\text{kg}}\right)\left|\frac{10^5\frac{\text{N}}{\text{m}^2}}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| = \underline{\mathbf{177.0\text{ kJ}}}$$

The positive sign indicates exergy transfer accompanying work is from the system.

Exergy transfer accompanying heat is determined from Eq. 7.5.

$$\mathsf{E}_q = \int_1^2\left(1 - \frac{T_0}{T_b}\right)\delta Q = Q_{12} - T_0\int_1^2\frac{\delta Q}{T_b}$$

The integral, $\int_1^2\frac{\delta Q}{T_b}$, can be determined from the entropy balance for a closed system

1 $$m(s_2 - s_1) = \int_1^2\frac{\delta Q}{T_b} + \sigma$$

PROBLEM 7.27 (Continued- p.3)

Since Process 1-2 has no internal irreversibilities, $\sigma = 0$. Substituting for $\int_1^2 \frac{\delta Q}{T_b}$ in the expression for exergy transfer accompanying heat gives

$$E_q = Q_{12} - mT_0(s_2 - s_1)$$

Substituting values and solving for exergy transfer accompanying heat give

$$E_q = 4{,}405.9 \text{ kJ} - (2 \text{ kg})(293 \text{ K})(7.1271 \text{ kJ/(kg·K)} - 1.5276 \text{ kJ/(kg·K)}) = \underline{\mathbf{1124.6 \text{ kJ}}}$$

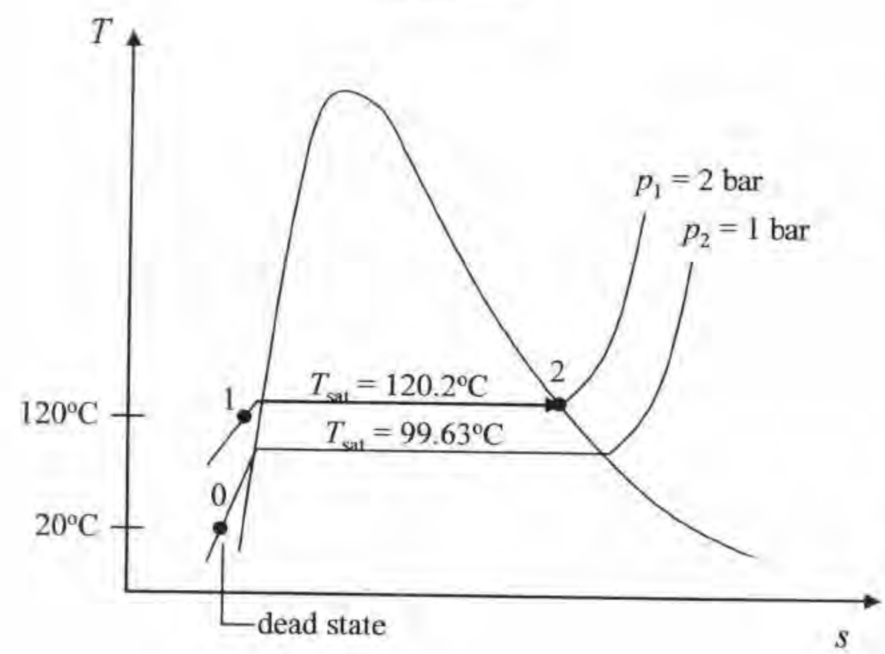
The positive sign indicates exergy transfer accompanying heat is into the system.

Substituting E_q = 1124.6 kJ, E_w = 177.0 kJ, and E_d = 0 into the closed system exergy balance, Eq. 7.4b, gives ($E_2 - E_1$) = 947.6 kJ, which agrees with the value calculated from Eq. 7.3, as can be verified.

---

1 Alternatively, since the process is internally reversible, the integral can be obtained directly from Eq. 6.2a.

<u>*T-s* Diagram</u>



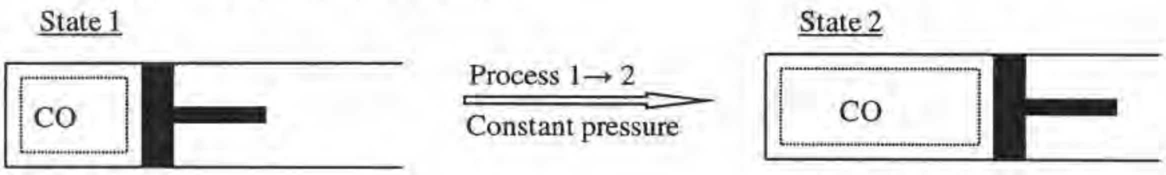
PROBLEM 7.28

7.28 Two kilograms of carbon monoxide in a piston-cylinder assembly, initially at 1.0 bar and 27°C, is heated at constant pressure with no internal irreversibilities to a final temperature of 227°C. Employing the ideal gas model, determine the work, the heat transfer, and the amounts of exergy transfer accompanying work and heat transfer, each in kJ. Let T_0= 300 K, p_0=1 bar and ignore the effects of motion and gravity.

KNOWN: 2 kg of CO is heated at constant pressure with no internal irreversibilities.

FIND: Determine the work, the heat transfer, and the amounts of exergy transfer accompanying work and heat transfer, each in kJ.

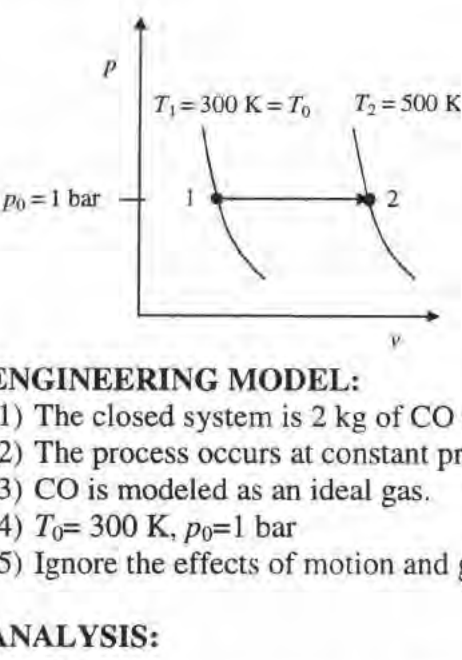
SCHEMATIC AND GIVEN DATA:

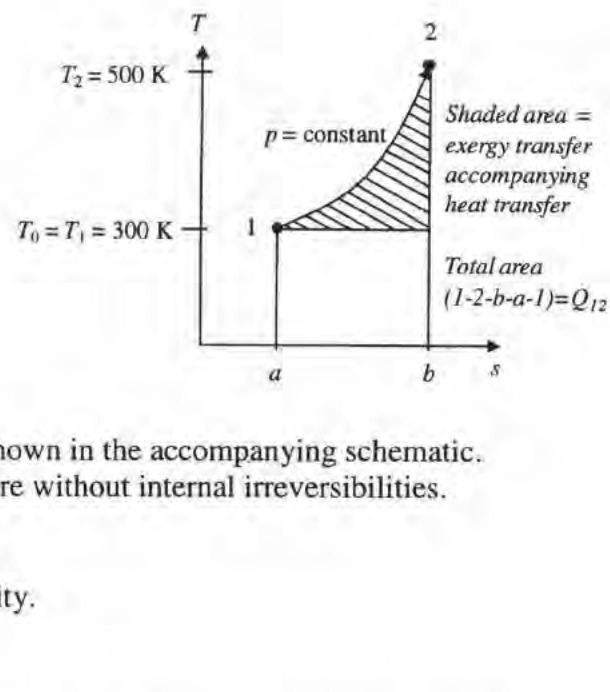


$m = 2$ kg
$p_1 = 1$ bar
$T_1 = 27°C=300$ K

$T_0 = 300$ K
$p_0 = 1$ bar

$T_2 = 227°C=500$ K
$p_2 = p_1 = 1$ bar





ENGINEERING MODEL:
(1) The closed system is 2 kg of CO as shown in the accompanying schematic.
(2) The process occurs at constant pressure without internal irreversibilities.
(3) CO is modeled as an ideal gas.
(4) T_0= 300 K, p_0=1 bar
(5) Ignore the effects of motion and gravity.

ANALYSIS:

Determine the work, in kJ, using Eq. 2.17 with an ideal gas undergoing an isobaric process.

$$W_{12} = \int_{V_1}^{V_2} pdV = mp(v_2 - v_1) = m\frac{\overline{R}}{M}(T_2 - T_1) = 2\,\text{kg}\left(\frac{8.314}{28.01}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(500-300)\text{K} = 118.7\,\text{kJ}$$

Using Eq. 7.6, the accompanying exergy transfer is

PROBLEM 7.28 (Continued)

$$\mathbf{E}_w = \left[\begin{array}{l}\text{Exergy transfer}\\ \text{accompanying work}\end{array}\right] = W_{12} - p_0\Delta V = mp(v_2 - v_1) - mp_0(v_2 - v_1)$$

$$= m(p - p_0)(v_2 - v_1) = 0 \quad \text{since } p = p_0$$

For the carbon monoxide as the system, evaluate the heat transfer using the energy balance simplified based on assumptions.

$$Q_{12} = m(u_2 - u_1) + W_{12} = \frac{m}{M}(\bar{u}_2 - \bar{u}_1) + W_{12}$$

Using data from Tables A-1 and A-23:

(#1) $$Q_{12} = \frac{2\,\text{kg}}{28.01\frac{\text{kg}}{\text{kmol}}}(10443 - 6229)\frac{\text{kJ}}{\text{kmol}} + 118.7\,\text{kJ} = 419.6\,\text{kJ}$$

Using Eq. 7.5, the accompanying exergy transfer is

$$\mathbf{E}_q = \left[\begin{array}{l}\text{Exergytransfer}\\ \text{accompanyingheat}\end{array}\right] = \int_1^2 \left[1 - \frac{T_0}{T_b}\right]\delta Q = Q_{12} - T_0\int_1^2\left(\frac{\delta Q}{T_b}\right) \tag{1}$$

However, since the process is internally reversible, Eq. 6.2a gives,

$$m(s_2 - s_1) = \int_1^2\left(\frac{\delta Q}{T_b}\right)$$

Eq. (1) simplifies to:

$$\mathbf{E}_q = Q_{12} - mT_0(s_2 - s_1) = Q_{12} - \frac{mT_0}{M}\left[\left(\bar{s}_2^{\,o} - \bar{s}_1^{\,o}\right) - \bar{R}\ln\frac{p_2}{p_1}\right]$$

Using data from Tables A-1 and A-23:

$$\mathbf{E}_q = 419.6\,\text{kJ} - \frac{2\,\text{kg}(300\,\text{K})}{28.01\frac{\text{kg}}{\text{kmol}}}(212.719 - 197.723)\frac{\text{kJ}}{\text{kmol}\cdot\text{K}} = 98.4\,\text{kJ}$$

---

1. Alternatively for a constant pressure process:

$$Q_{12} = m(u_2 - u_1) + W_{12} = m(u_2 - u_1) + mp(v_2 - v_1) = m(h_2 - h_1) = \frac{m}{M}(\bar{h}_2 - \bar{h}_1)$$

$$= \frac{2\,\text{kg}}{28.01\frac{\text{kg}}{\text{kmol}}}(14600 - 8723)\frac{\text{kJ}}{\text{kmol}} = 419.6\,\text{kJ}$$

Problem 7.29

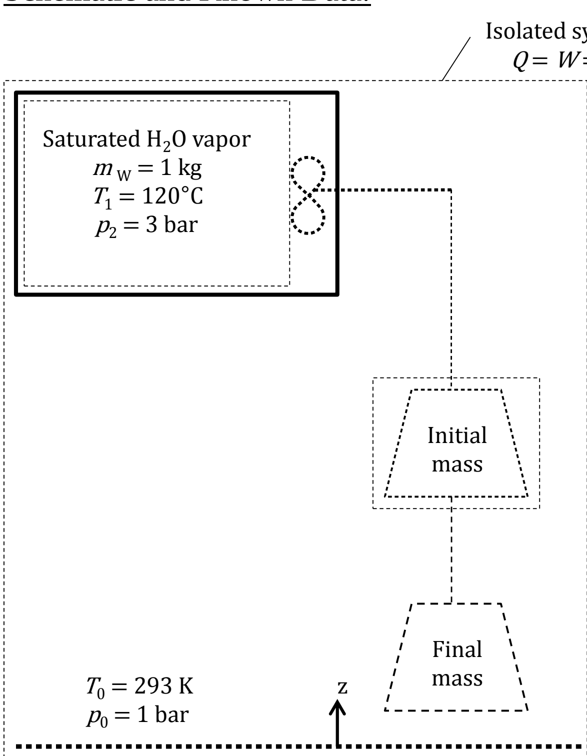
As shown in Fig. P7.29, 1 kg of H_2O is contained in a rigid, insulated cylindrical vessel. The H_2O is initially saturated vapor at 120°C. The vessel is fitted with a paddle wheel from which a mass is suspended. As the mass descends a certain distance, the H_2O is stirred until it attains a final equilibrium state at a pressure of 3 bar. The only significant changes in state are experienced by the H_2O and the suspended mass. Determine, in kJ,
(a) the change in exergy of the H_2O.
(b) the change in exergy of the suspended mass.
(c) the change in exergy of an isolated system of the vessel and pulley-mass assembly.
(d) the destruction of exergy within the isolated system.

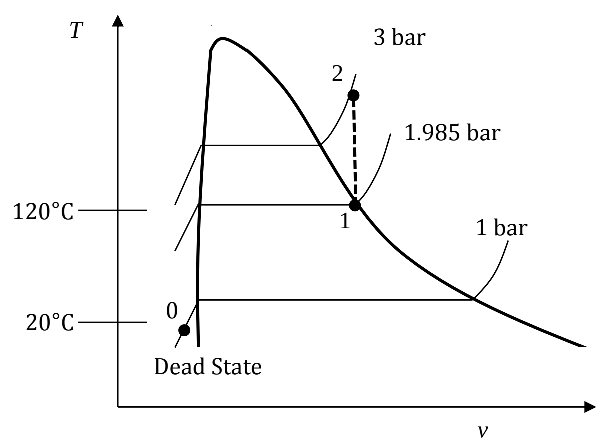
Let T_0 = 293 K, p_0 = 1 bar.

<u>Known:</u>
H_2O is in a rigid vessel with a paddle wheel inside that is attached to a dropping mass.

<u>Find:</u>
(a) the change in exergy of the H_2O, (b) the change in exergy of the suspended mass, (c) the change in exergy of an isolated system of the vessel and pulley-mass assembly, and (d) the destruction of exergy within the isolated system.

<u>Schematic and Known Data:</u>





<u>Engineering Model:</u>

(1) As shown in the schematic, three systems are under consideration: the H_2O, the suspended mass, and an isolated system consisting of the vessel and pulley-mass assembly. For the isolated system, $Q = W = 0$.
(2) The only significant changes of state are experienced by the H_2O and the suspended mass. For the H_2O, there is no change in kinetic or potential energy. For the suspended mass, there is no change in kinetic or internal energy. Elevation is the only intensive property of the suspended mass that changes.
(3) For the environment, T_0 = 293 K, p_0 = 1 bar.

Analysis:

(a) The initial and final exergies of the H_2O can be evaluated using Eq. 7.2. From assumption (2), it follows that for the H_2O there are no significant effects of motion or gravity, thus the exergy at the initial state is:

$$\mathrm{E}_1 = m_W[(u_1 - u_0) + p_0(v_1 - v_0) - T_0(s_1 - s_0)]$$

The initial and final states of the H_2O are shown on the accompanying T-v diagram. From Tables A-2 and A-4:

$$u_1 = 2529.3\frac{\mathrm{kJ}}{\mathrm{kg}},\quad v_1 = 0.8919\frac{\mathrm{m}^3}{\mathrm{kg}},\quad s_1 = 7.1296\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}$$

$$u_0 = 83.95\frac{\mathrm{kJ}}{\mathrm{kg}},\quad v_0 = 1.0018\cdot 10^{-3}\frac{\mathrm{m}^3}{\mathrm{kg}},\quad s_0 = 0.2966\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}$$

Then:

$$\mathrm{E}_1 = (1\ \mathrm{kg})\left[(2529.3 - 83.95)\frac{\mathrm{kJ}}{\mathrm{kg}} + \left(10^5\frac{\mathrm{N}}{\mathrm{m}^2}\right)(0.8919 - 1.0018\cdot 10^{-3})\left(\frac{\mathrm{m}^3}{\mathrm{kg}}\right)\left|\frac{1\ \mathrm{kJ}}{10^3\mathrm{N\cdot m}}\right| - (293\ \mathrm{K})(7.1296 - 0.2966)\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\right] = (1)[2445.35 + 89.09 - 2002.07] = 532.37\ \mathrm{kJ}$$

#1

The final state of the H_2O is fixed by p_2 = 3 bar and $v_2 = v_1$. Using interpolation and values from Table A-4:

$$u_2 = 2823.07\frac{\mathrm{kJ}}{\mathrm{kg}},\quad s_2 = 7.7381\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}$$

Then:

$$\mathrm{E}_2 = (1\ \mathrm{kg})\left[(2823.07 - 83.95)\frac{\mathrm{kJ}}{\mathrm{kg}} + \left(10^5\frac{\mathrm{N}}{\mathrm{m}^2}\right)(0.8919 - 1.0018\cdot 10^{-3})\left(\frac{\mathrm{m}^3}{\mathrm{kg}}\right)\left|\frac{\mathrm{kJ}}{10^3\mathrm{N\cdot m}}\right| - (293\ \mathrm{K})(7.7381 - 0.2966)\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\right]$$
$$= (1)[2739.12 + 89.09 - 2180.36] = 647.85\ \mathrm{kJ}$$

For the H_2O, the change in exergy is:

$$\Delta\mathrm{E}_{H_2O} = \mathrm{E}_2 - \mathrm{E}_1 = 647.85 - 532.37 = 115.48\ \mathrm{kJ}$$

#2 (b) With assumption 2, Eq. 7.3 reduces to give the exergy change for the suspended mass:

$$\Delta \mathrm{E_m} = \underbrace{\Delta U + p_0 \Delta V - T_0 \Delta S + \Delta \mathrm{KE}}_{=0} + \Delta \mathrm{PE_m} = \Delta \mathrm{PE_m}$$

Thus, the exergy change for the suspended mass equals the change in potential energy. The change in potential energy of the suspended mass is obtained from an energy balance for the isolated system as follows: the change in energy of the isolated system is the sum
#3 of the energy changes of the H_2O and suspended mass. There is no heat transfer or work, and with assumption 2 we have:

$$\left(\underbrace{\Delta \mathrm{PE} + \Delta \mathrm{KE}}_{=0} + \Delta U\right)_{\mathrm{H_2O}} + \left(\underbrace{\Delta U + \Delta \mathrm{KE}}_{=0} + \Delta \mathrm{PE}\right)_{\mathrm{m}} = \underbrace{Q - W}_{=0}$$

Solve for $\Delta \mathrm{PE_m}$ and using previously determined values for the specific internal energy of the H_2O:

$$\Delta \mathrm{PE_m} = -\Delta U_{\mathrm{H_2O}} = -(1\ \mathrm{kg})(2823.07 - 2529.3)\frac{\mathrm{kJ}}{\mathrm{kg}} = -293.77\ \mathrm{kJ}$$

The exergy of the mass decreases because its elevation decreases.

(c) The change in exergy of the isolated system is the sum of the exergy changes of the water and suspended mass. With the results from parts (a) and (b):

$$\Delta \mathrm{E_{sys}} = \Delta \mathrm{E_{H_2O}} + \Delta \mathrm{E_m} = 115.48 - 293.77 = -178.29\ \mathrm{kJ}$$

(d) The exergy of the isolated system decreases. With Eq. 7.9, $(\mathrm{E_d})_{\mathrm{sys}}$ = 178.29 kJ

To summarize:

	Energy Change (kJ)	**Exergy Change (kJ)**
H_2O	293.77	115.48
Suspended Mass	-293.77	-293.77
Isolated System	0	-178.29

For the isolated system there is no net change in energy. The increase in the internal energy of the H_2O equals the decrease in potential energy of the suspended mass. However, the *increase* in exergy of the H_2O is much less than the *decrease* in exergy of the mass. For the isolated system, exergy decreases because stirring destroys exergy.

Comments:

1. Exergy is a measure of the departure of the state of the system from that of the environment. At all states, $\mathrm{E} \geq 0$. This applies when $T > T_0$, $p > p_0$, as at states 1 and 2, and when $T < T_0$, $p < p_0$.
2. Alternatively, Eq. 7.3 can be used. This requires dead state property values only for T_0 and p_0. In part (a), u_0, v_0, and s_0 are also required; so more computation is needed with the approach shown in this problem's solution.
3. The change in potential energy of the suspended mass cannot be determined from Eq. 2.10 (Sec. 2.1) since the mass and change in elevation are unknown. Moreover, for the suspended mass as the system, change in potential energy of the suspended mass cannot be obtained from an energy balance without first evaluating the work. Thus, we resort

here to an energy balance for the isolated system, which does not require such information.

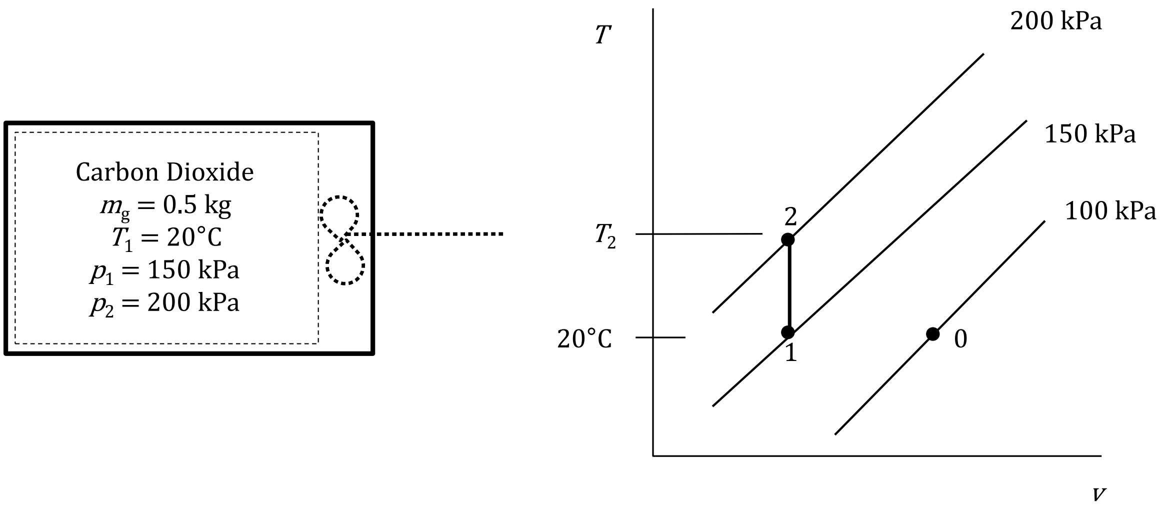
Problem 7.30

A rigid insulated tank contains 0.5 kg of carbon dioxide, initially at 150 kPa, 20°C. The carbon dioxide is stirred by a paddle wheel until its pressure is 200 kPa. Using the ideal gas model with c_v = 0.65 kJ/kg·K, determine, in kJ, (a) the work, (b) the change in exergy of the carbon dioxide, and (c) the amount of exergy destroyed. Ignore the effects of motion and gravity, and let T_0 = 20°C, p_0 = 100 kPa.

Known:
Carbon dioxide is in a rigid tank with a paddle wheel.

Find:
Determine (a) the work, (b) the change in exergy of the carbon dioxide, and (c) the amount of exergy destroyed.

Schematic and Known Data:



Engineering Model:
(1) The carbon dioxide is the closed system and the volume remains constant throughout the process (isometric).
(2) For the system, Q = 0 and the effects of motion and gravity can be ignored.
(3) For the environment, T_0 = 20°C = 293 K, p_0 = 100 kPa.

Analysis:
Using assumption (1) and the ideal gas model equation of state to obtain T_2:

$$\frac{p_1 V}{T_1} = \frac{p_2 V}{T_2} \Rightarrow T_2 = T_1 \frac{p_2}{p_1} = 293 \frac{200}{150} = 390.7 \text{ K}$$

(a) An energy balance reduces to give, $\Delta U + \underbrace{\Delta KE}_{=0} + \underbrace{\Delta PE}_{=0} = \underbrace{Q}_{=0} - W$, therefore:

$$W = -m(u_2 - u_1) = -mc_v(T_2 - T_1) = -(0.5 \text{ kg})\left(0.65 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right)(390.7 - 293)\text{K}$$
$$= -31.75 \text{ kJ}$$

(b) With Eq. 7.3:

$$\Delta\mathrm{E} = m\left[u_2 - u_1 + \underbrace{p_0(v_2 - v_1)}_{=0} - T_0(s_2 - s_2)\right]$$

Then, with Eq. 6.21:

$$\Delta\mathrm{E} = m\left[c_v(T_2 - T_1) - T_0\left(c_v \ln\frac{T_2}{T_1} + \underbrace{R\ln\frac{v_2}{v_1}}_{=0}\right)\right] = (0.5\ \mathrm{kg})\left[\left(0.65\,\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\right)(390.7 - 293)\mathrm{K} - (293\ \mathrm{K})\left[\left(0.65\,\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\right)\ln\frac{390.7}{293}\right]\right] = 4.35\ \mathrm{kJ}$$

(c) $\mathrm{E_d} = T_0\sigma$, where σ is obtained from an entropy balance:

$$\sigma = m(s_2 - s_1) = m\left(c_v \ln\frac{T_2}{T_1} + \underbrace{R\ln\frac{v_2}{v_1}}_{=0}\right) = (0.5\ \mathrm{kg})\left(\left(0.65\,\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\right)\ln\frac{390.7}{293}\right)$$

$$= 0.0935\,\frac{\mathrm{kJ}}{\mathrm{k}}$$

Finally,

$$\mathrm{E_d} = (293\ \mathrm{K})\left(0.0935\,\frac{\mathrm{kJ}}{\mathrm{K}}\right) = 27.4\ \mathrm{kJ}$$

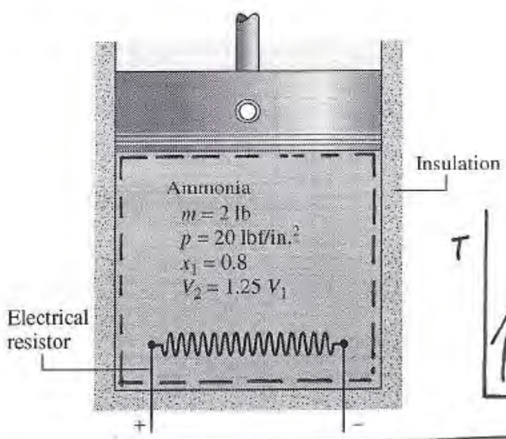
Alternatively, an exergy balance reduces to $\Delta\mathrm{E} = -\mathrm{E_W} - \mathrm{E_d} \Longrightarrow \mathrm{E_d} = -W - \Delta\mathrm{E}$. This equation could also be used to solve part (c).

PROBLEM 7.31

As shown in Fig. P7.31, two lb of ammonia is contained in a well-insulated piston-cylinder assembly fitted with an electrical resistor of negligible mass. The ammonia is initially at 20 lbf/in.^{2} and a quality of 80%. The resistor is activated until the volume of the ammonia increases by 25%, while its pressure varies negligibly. Determine, in Btu,

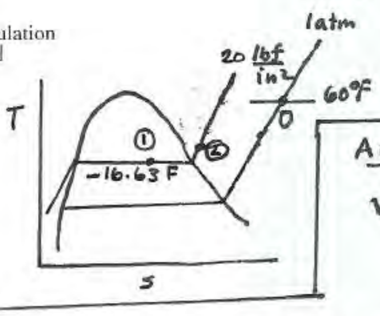
(a) the amount of energy transfer by electrical work and the accompanying exergy transfer.
(b) the amount of energy transfer by work to the piston and the accompanying exergy transfer.
(c) the change in exergy of the ammonia.
(d) the amount of exergy destruction.

Ignore the effects of motion and gravity and let $T_0 = 60°F$, $p_0 = 1$ atm.



ENGR. MODEL

1. The ammonia is the closed system.
2. For the system, $Q = 0$ and the effects of motion and gravity can be ignored.
3. The electrical resistor has negligible mass
4. For the environment, $T_0 = 520°R$ $(60°F)$, $p_0 = 14.7\ lbf/in^2$.



ANALYSIS: Property data:

$v_1 = v_f + x_1(v_g - v_f)$

$= 0.02377 + 0.8(13.497 - 0.02377)$

$= 10.8\ ft^3/lb$

$u_1 = u_f + x_1(u_g - u_f)$

$= 24.58 + 0.8(555.78 - 24.58) = 449.54\ \frac{Btu}{lb}$

$s_1 = s_f + x_1(s_g - s_f)$

$= 0.0571 + 0.8(1.3687 - 0.0571)$

$= 1.1064\ Btu/lb \cdot °R$

$v_2 = 1.25\, v_1 = 1.25(10.8\ ft^3/lb)$

$= 13.5\ ft^3/lb$

Note: State 2 is closely saturated vapor at 20 lbf/in². So,

$u_2 \cong u_g = 555.78\ \frac{Btu}{lb}$, $s_2 \cong s_g = 1.3687\ \frac{Btu}{lb \cdot °R}$

(a) To get W_{elec}, apply an energy balance:

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - [W_{piston} + W_{elec}]$$

where

$$W_{piston} = m\int_1^2 p\,dv = mp(v_2 - v_1)$$

$$= (2\,lb)\left(20\frac{lbf}{in^2}\right)(13.5 - 10.8)\frac{ft^3}{lb}\left|\frac{144\,in^2}{1\,ft^2}\right|\left|\frac{1\,Btu}{778\,ft\cdot lbf}\right|$$

$$= 19.99\ Btu$$

Thus,

$$W_{elec} = -W_{pist} - \Delta U$$

$$= -19.99\ Btu - 2\,lb(555.78 - 449.54)\frac{Btu}{lb}$$

$$= -232.47\ Btu$$

$$\boxed{E_w}_{elect} = -232.47\ Btu \quad \text{(no accompanying volume change)}$$

$\cdot(W_{piston} + W_{elec})$

(b) W_{piston} is evaluated in part (a). Then

$$\boxed{E_w}_{piston} = W_{piston} - p_0 \Delta V$$

$$= W_{piston} - p_0 m(v_2 - v_1)$$

$$= 19.99\ Btu - (14.7)(2)(13.5 - 10.8)\left|\frac{144}{778}\right| = 5.3\ Btu$$

PROBLEM 7.31 (Contd)

(c) $\Delta E = m\left((u_2-u_1) + p_0(v_2-v_1) - T_0(s_2-s_1)\right)$

$$= 2\left[(555.78-449.54) + 14.7\left|\frac{144}{778}\right|(13.5-10.8) - 520(1.3687-1.1064)\right] \text{Btu}$$

① $$= 2\left[106.24 + 7.35 - 136.40\right] = -45.62 \text{ Btu}$$

(d) An exergy balance reads

$$\Delta E = \cancel{E_q}^{0} - E_W - E_d$$

$$\Rightarrow E_d = -E_W - \Delta E$$

$$= -\left[5.3 - 232.47\right]\text{Btu} - \left[-45.62 \text{ Btu}\right]$$

$$= 272.8 \text{ Btu}$$

Alternatively, $E_d = T_0\sigma$, where σ is obtained from an entropy balance as $\sigma = m(s_2-s_1)$. So

$$E_d = T_0 m(s_2-s_1) = (520°R)(2\,lb)(1.3687-1.1064)\frac{\text{Btu}}{lb\cdot°R}$$

$$= 272.8 \text{ Btu}$$

---

1. As suggested by the location of state 2 on the T-s diagram, and gauged by the values of v, u, and s at states 1 and 2 relative to the respective values at the dead state, state 2 is closer to the dead state than state 1. Accordingly, the change in exergy from state 1 to state 2 is negative, as calculated here.

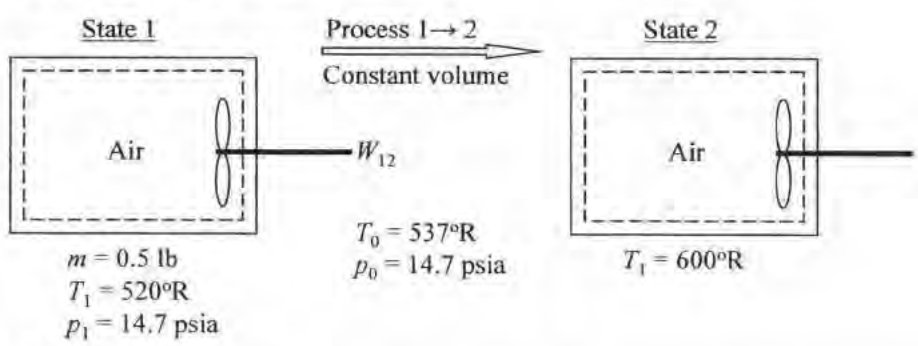
PROBLEM 7.32

7.32 One-half pound of air is contained in a closed, rigid, insulated tank. Initially the temperature is 520°R and the pressure is 14.7 psia. The air is stirred by a paddle wheel until its temperature is 600°R. Using the ideal gas model, determine for the air the change in exergy, the transfer of exergy accompanying work, and the exergy destruction, all in Btu. Ignore the effects of motion and gravity and let T_0 = 537°R, p_0 = 14.7 psia.

KNOWN: System of air at specified temperature and pressure undergoes constant volume process until reaching a specified temperature.

FIND: The change in exergy, the transfer of exergy accompanying work, and the exergy destruction.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The air in the rigid tank is a closed system defined by the dashed line on the accompanying diagram.
2. Air can be modeled as an ideal gas.
3. There is no heat transfer.
4. Volume is constant during the process.
5. The effects of motion and gravity can be ignored.
6. T_0 = 537°R and p_0 = 14.7 psia.

ANALYSIS:
The change in exergy can be determined from Eq. 7.3

$$\mathsf{E}_2 - \mathsf{E}_1 = (U_2 - U_1) + p_0(V_2 - V_1) - T_0(S_2 - S_1) + (\mathrm{KE}_2 - \mathrm{KE}_1) + (\mathrm{PE}_2 - \mathrm{PE}_1)$$

Ignoring changes in kinetic and potential energy, recognizing $V_2 - V_1 = 0$ since the process is constant volume, and rewriting extensive properties in terms of mass and specific properties gives

$$\mathsf{E}_2 - \mathsf{E}_1 = m[(u_2 - u_1) - T_0(s_2 - s_1)]$$

Since air can be modeled as an ideal gas, the change in entropy can be written by applying Eq. 6.20a

$$s_2 - s_1 = s(T_2, p_2) - s(T_1, p_1) = s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{p_2}{p_1}$$

Substituting for entropy change, the change in exergy becomes

$$\mathsf{E}_2 - \mathsf{E}_1 = m[(u_2 - u_1) - T_0(s^o(T_2) - s^o(T_1) - R\ln\frac{p_2}{p_1})]$$

From Table A-22E, $u_1 = 88.62$ Btu/lb, $s^o(T_1) = 0.59172$ Btu/(lb·°R), $u_2 = 102.34$ Btu/lb, $s^o(T_2) = 0.62607$ Btu/(lb·°R). The pressure at State 2 is determined from the ideal gas equation of state.

$$pv = RT$$

Since specific volume and the gas constant are constant, the ratio of temperature to pressure is the same for State 1 and State 2. Solving for pressure at State 2 gives

$$\frac{v}{R} = \frac{T_1}{p_1} = \frac{T_2}{p_2} \Rightarrow p_2 = p_1\left(\frac{T_2}{T_1}\right)$$

Substituting values yields

$$p_2 = (14.7\text{ psia})\left(\frac{600°\text{R}}{520°\text{R}}\right) = 16.96\text{ psia}$$

Substituting values and solving for change in exergy give

$$\mathsf{E}_2 - \mathsf{E}_1 = (0.5\text{ lb})\left[(102.34 - 88.62)\frac{\text{Btu}}{\text{lb}} - (537°\text{R})\left((0.62607 - 0.59172)\frac{\text{Btu}}{\text{lb}\cdot°\text{R}} - \left(\frac{1.986\frac{\text{Btu}}{\text{lbmol}\cdot°\text{R}}}{28.97\frac{\text{lb}}{\text{lbmol}}}\right)\ln\left(\frac{16.96\text{ psia}}{14.7\text{ psia}}\right)\right)\right]$$

$$\mathsf{E}_2 - \mathsf{E}_1 = \underline{\mathbf{0.27\ Btu}}$$

Exergy transfer accompanying work is determined from Eq. 7.6.

$$\mathsf{E}_w = [W - p_0(V_2 - V_1)]$$

Since the volume is constant during Process 1-2, the exergy transfer accompany work is the work

$$\mathsf{E}_w = W$$

The work can be determined from the energy balance for a closed system.

$$\Delta KE + \Delta PE + \Delta U = Q - W$$

Neglecting changes in kinetic and potential energy and heat transfer and expressing internal energy in terms of mass and specific internal energy give

$$m(u_2 - u_1) = -W_{12}$$

PROBLEM 7.32 (Continued, p.3)

Solving for work and substituting values give

$$W_{12} = m(u_1 - u_2) = (0.5\text{ lb})(88.62\text{ Btu/lb} - 102.34\text{ Btu/lb}) = -6.86\text{ Btu}$$

The negative sign indicates work is done on the system. The exergy transfer accompanying work is

$$\mathsf{E}_w = W = \underline{\mathbf{-6.86\ Btu}}$$

The negative sign indicates exergy transfer is into the system accompanying work.

Exergy destruction can be determined from Eq. 7.4b

$$\mathsf{E}_2 - \mathsf{E}_1 = \mathsf{E}_q - \mathsf{E}_w - \mathsf{E}_d$$

Since there is no heat transfer, E_q is zero. Solving for exergy destruction, E_d, gives

$$\mathsf{E}_d = -\mathsf{E}_w - (\mathsf{E}_2 - \mathsf{E}_1) = -(-6.86\text{ Btu}) - (0.27\text{ Btu}) = \underline{\mathbf{6.59\ Btu}}$$

Alternatively, exergy destruction can be determined from Eq. 7.7

$$\mathsf{E}_d = T_0\sigma$$

Entropy production, σ, can be determined from the closed system entropy balance

$$S_2 - S_1 = \int_1^2\left(\frac{\delta Q}{T}\right)_b + \sigma$$

Since the process is adiabatic, the term, $\int_1^2\left(\frac{\delta Q}{T}\right)_b$, is zero. Solving for entropy production and substituting entropy change for an ideal gas give

$$\sigma = S_2 - S_1 = m(s_2 - s_1) = m(s^\circ(T_2) - s^\circ(T_1) - R\ln\frac{p_2}{p_1})$$

Substituting values gives

$$\sigma = (0.5\text{ lb})\left((0.62607 - 0.59172)\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} - \left(\frac{1.986\frac{\text{Btu}}{\text{lbmol}\cdot{}^\circ\text{R}}}{28.97\frac{\text{lb}}{\text{lbmol}}}\right)\ln\left(\frac{16.96\text{ psia}}{14.7\text{ psia}}\right)\right) = 0.01227\text{ Btu/}^\circ\text{R}$$

Substituting values and solving for exergy destruction give

$$\mathsf{E}_d = (537^\circ\text{R})(0.01227\text{ Btu/}^\circ\text{R}) = \underline{\mathbf{6.59\ Btu}}$$

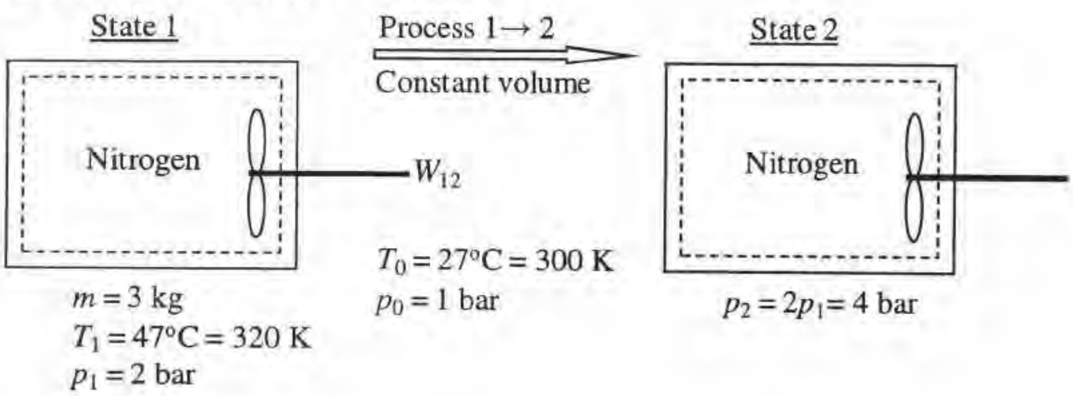
PROBLEM 7.33

7.33 Three kilograms of nitrogen initially at 47°C and 2 bar is contained within a rigid, insulated tank. The nitrogen is stirred by a paddle wheel until its pressure doubles. Employing the ideal gas model with constant specific heat evaluated at 300 K, determine the work and exergy destruction for the nitrogen, each in kJ. Ignore the effects of motion and gravity and let T_0= 300 K, p_0=1 bar.

KNOWN: Nitrogen in a closed, rigid, insulated tank is stirred by a paddle wheel.

FIND: Determine the work and exergy destruction for the nitrogen, each in kJ.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
(1) The closed system shown in the accompanying figure is 3 kg of N_2 with $Q_{12} = 0$.
(2) The process occurs at constant volume.
(3) N_2 is modeled as an ideal gas with constant specific heat evaluated at 300 K.
(4) T_0= 300 K, p_0=1 bar.
(5) Ignore the effects of motion and gravity.

ANALYSIS:

For the nitrogen as the system, evaluate the work using the energy balance simplified based on assumptions.

$$W_{12} = m(u_1 - u_2) = mc_v(T_1 - T_2) \qquad (1)$$

Using Table A-20 for N_2 at 300 K, $c_v = 0.743$ kJ/kg·K. Substituting into Eq. (1):

$$W_{12} = 3\,\text{kg}\left(0.743\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(320-640)\text{K} = -713.3\,\text{kJ} \longleftarrow$$

Exergy destroyed can be determined from an exergy balance or from $\mathbf{E}_d = T_0\sigma$, where σ is the entropy produced as obtained from the entropy balance. Using the second of these approaches:

$$\Delta S = \overset{0}{\cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}} + \sigma$$

Rearranging and simplifying for an insulated system yields:

$$\sigma = \Delta S = m(s_2 - s_1)$$

Using Eq. 6.21 and noting $v_1 = v_2$:

$$\sigma = m(s_2 - s_1) = m\left(c_v \ln\left(\frac{T_2}{T_1}\right) + \frac{\overline{R}}{M}\ln\left(\frac{v_2}{v_1}\right)\right) = mc_v \ln\left(\frac{T_2}{T_1}\right) = 3\,\text{kg}\left(0.743\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\ln\left(\frac{640}{320}\right) = 1.545\frac{\text{kJ}}{\text{K}}$$

Therefore,

$$\mathbf{E}_d = T_0\sigma = (300\,\text{K})1.545\frac{\text{kJ}}{\text{K}} = 463.5\,\text{kJ} \longleftarrow$$

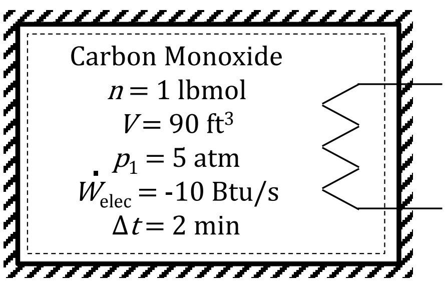
Problem 7.34

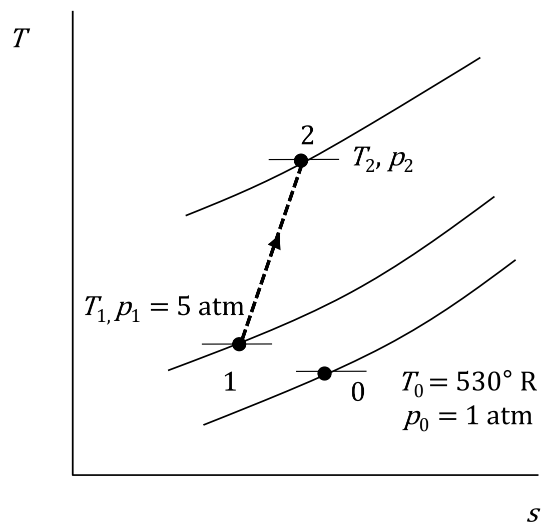
One lbmol of carbon monoxide gas is contained in a 90-ft^3 rigid, insulated vessel initially at 5 atm. An electric resistor of negligible mass transfers energy to the gas at a constant rate of 10 Btu/s for 2 min. Employing the ideal gas model and ignoring the effects of motion and gravity, determine (a) the change in exergy of the gas, (b) the electrical work, and (c) the exergy destruction, each in Btu. Let T_0 = 70°F, p_0 = 1 atm.

Known:
A known amount of carbon monoxide gas is contained in a rigid, insulated vessel of known volume and at a specified initial pressure. An electric resistor in the vessel transfers energy to the gas at a known rate for a specified period of time.

Find:
Determine (a) the change in exergy of the gas, (b) the electrical work, and (c) the exergy destruction.

Schematic and Known Data:





Engineering Model:
(1) The system consists of the gas and the resistor.
(2) The volume is constant and kinetic and potential energy effects are neglected.
(3) The resistor is of negligible mass and thus undergoes no change of state.
(4) The carbon monoxide is modeled as an ideal gas.
(5) For the system, Q = 0.

Analysis:
(a) First, fix both states as follows:

$$T_1 = \frac{p_1 V}{n\bar{R}} = \frac{\left(5\text{ atm} \cdot \left|\frac{14.696\,\frac{\text{lbf}}{\text{in.}^2}}{1\text{ atm}}\right|\right)\left(90\text{ ft}^3 \cdot \left|\frac{144\text{ in.}^2}{1\text{ ft}^2}\right|\right)}{(1\text{ lbmol})\left(1545\,\frac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot°\text{R}}\right)} = 616°\text{R}$$

To determine T_2, begin with an energy balance: $\Delta U = Q - W$. With $U_{res} = 0$:
$\underbrace{Q}_{=0} - W = n(\bar{u}_2 - \bar{u}_1)$

To obtain W:

$$W = \int_1^2 \dot{W}dt = \dot{W}_{elec}\Delta t = \left(-10\frac{\text{Btu}}{\text{s}}\right)\left(2\text{ min}\cdot\left|\frac{60\text{ s}}{\text{min}}\right|\right) = -1200\text{ Btu}$$

Thus, solving for $\bar{u}_2$ and inserting data from Table A-23E using interpolation:

$$\bar{u}_2 = -\frac{W}{n} + \bar{u}_1 = -\frac{-1200\text{ Btu}}{1\text{ lbmol}} + 3056.3\frac{\text{Btu}}{\text{lbmol}} = 4256.3\frac{\text{Btu}}{\text{lbmol}}$$

Interpolating in Table A-23E, $T_2 = 854.5°\text{R}$. Thus:

$$p_2 = \frac{n\bar{R}T_2}{V} = \frac{(1\text{ lbmol})\left(1545\frac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot°\text{R}}\right)(854.5°\text{R})}{90\text{ ft}^3\cdot\left|\frac{144\text{ in.}^2}{1\text{ ft}^2}\right|}\left|\frac{1\text{ atm}}{14.696\frac{\text{lbf}}{\text{in.}^2}}\right| = 6.93\text{ atm}$$

Now, the change in exergy of the gas is obtained using Eq. 7.3 and ideal gas relations:

$$\text{E}_2 - \text{E}_1 = n\left[\bar{u}(T_2) - \bar{u}(T_1) + p_0\underbrace{(V_2 - V_1)}_{=0} - T_0\left(\bar{s}^o(T_2) - \bar{s}^o(T_1) - \bar{R}\ln\frac{p_2}{p_1}\right)\right]$$

Inserting data interpolated from Table A-23E:

$$\begin{aligned}\text{E}_2 - \text{E}_1 \\ &= (1\text{ lbmol})\left[(4256.3 - 3056.3)\frac{\text{Btu}}{\text{lbmol}}\right. \\ &\left. - (530°\text{R})\left[\left(50.523 - 48.266 - 1.986\ln\frac{6.93}{5}\right)\frac{\text{Btu}}{\text{lbmol}\cdot°\text{R}}\right]\right] \\ &= 347.38\text{ Btu}\end{aligned}$$

As state 2 is further from the dead state than state 1, an increase in exergy occurs during this process.

(b) The exergy transfer accompanying work, E_W, is:
$\text{E}_\text{W} = W - p_0\underbrace{\Delta V}_{=0} = -1200\text{ Btu}$

(c) The exergy destruction, E_d, is determined by solving the exergy balance:

$$\text{E}_2 - \text{E}_1 = \int_1^2\left(1 - \frac{T_0}{T_b}\right)\underbrace{\delta Q}_{=0} - \left(W - p_0\underbrace{\Delta V}_{=0}\right) - \text{E}_\text{d}$$

Rearranging:

#1 #2 $\text{E}_\text{d} = -(\text{E}_2 - \text{E}_1) - W = -347.38\text{ Btu} - (-1200\text{ Btu}) = 852.62\text{ Btu}$

Comments:

1. Note that the exergy input by work (1200 Btu) results in only 347.38 Btu being stored as increased exergy of the gas. The rest, 852.62 Btu, is destroyed by irreversibilities.
2. Alternatively, $\mathrm{E_d} = T_0\sigma$, where $\sigma = n(\bar{s}_2 - \bar{s}_1)$, giving $\mathrm{E_d}$ = 852.62 Btu.

PROBLEM 7.35

KNOWN: A rigid insulated tank has two equal-volume compartments separated by a valve. Initially, one is evacuated and the other holds a gas at a known condition. The valve is opened and the gas fills the entire volume.

FIND: Determine the final temperature and pressure, and evaluate the exergy destruction. Discuss.

SCHEMATIC & GIVEN DATA:

ENGR. MODEL: 1. The closed system consists of both volumes, as shown in the schematic. 2. For the system, $Q = W = 0$ and effects of motion and gravity can be ignored. 3. The gas is modeled as an ideal gas. 4. For the environment, $T_0 = 530°R$, $p_0 = 1\,atm$.

ANALYSIS:

(a) With assumptions 1, 2 the energy balance reduces to $\Delta U = \cancel{Q}^{0} - \cancel{W}^{0} \Rightarrow \Delta U$. Since the specific internal energy of an ideal gas depends on temperature alone: $T_2 = T_1 = 560\,°R$

$T_2 = 100°F$ ←

Using the ideal gas equation of state

initially: $p_1 V = m R T_1$

finally: $p_2 (2V)\ m R T_2$

but $T_2 = T_1$, giving

$$p_2(2V) = p_1 V \Rightarrow p_2 = \tfrac{1}{2} p_1 = 25\ lbf/in^2$$ ←

(b) The exergy destruction can be found from an exergy balance or using $E_d = T_0 \sigma$, where σ is obtained from an entropy balance. That is,

$$\Delta S = \int_1^2 \cancel{\left(\frac{\delta Q}{T}\right)_b}^{0} + \sigma$$

$$\Rightarrow \quad \sigma = \Delta S = n\,[\underbrace{\bar{s}°(T_2) - \bar{s}°(T_1)}_{=0 \text{ since } T_2 = T_1} - \bar{R} \ln p_2/p_1]$$

$$\sigma = -n \bar{R} \ln \frac{p_2}{p_1} = -(0.25\,lbmol)\left(1.986 \frac{Btu}{lbmol \cdot °R}\right) \ln\left(\tfrac{1}{2}\right)$$

$$= 0.344\ Btu/°R$$

Finally

$$E_d = T_0 \sigma$$

$$= (530°R)\left(0.344 \frac{Btu}{°R}\right)$$

$$= 182.3\ Btu$$ ←

(c) Exergy is destroyed in this case because the gas undergoes an unrestrained, and thus irreversible, expansion to a lower pressure.

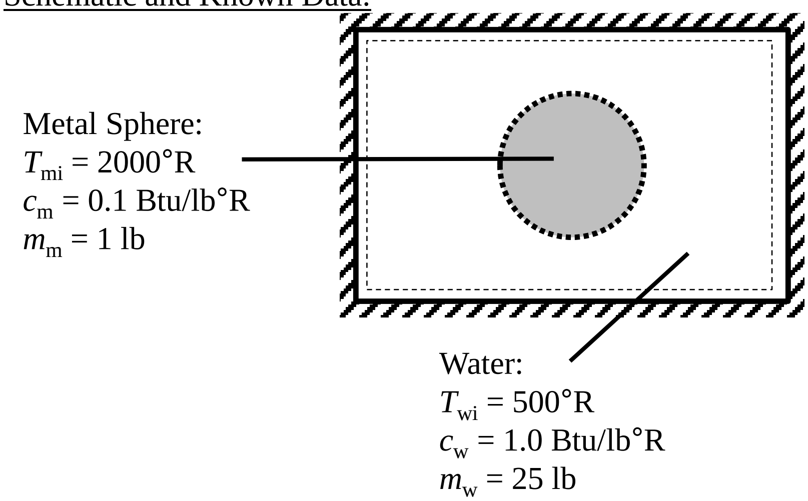
Problem 7.36

As shown in ig. P7.36, a 1 lb metal sphere initially at 2000°R is removed from an oven and quenched by immersing it in a closed tank containing 25 lb of water initially at 500°R. Each substance can be modeled as incompressible. An appropriate constant specific heat for the water is $c_w = 1.0 \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$, and an appropriate value for the metal is $c_m = 0.1 \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$. Heat transfer from the tank contents can be neglected. Determine the exergy destruction, in Btu. Let T_0 = 77°F.

Known:
A hot metal sphere is quenched by immersing it in a tank of water.

Find:
Determine the exergy destruction.

Schematic and Known Data:



Engineering Model:

#1 (1) As shown in the schematic, the metal sphere and water form a closed system.
(2) For the system, $Q = W = 0$ and there are no effects of motion or gravity.
(3) The metal sphere and water are each modeled as incompressible.
(4) T_0 = 537°R (77°F).

Analysis:
An energy balance for the system reduces to give:

$$\Delta U_m + \Delta U_w = 0$$

$$\text{or } \Delta U_m = -\Delta U_w \qquad (1)$$

An exergy balance reduces to give:

$$\Delta \text{E} = \int_1^2 \left(1 - \frac{T_0}{T_b}\right) \underbrace{\delta Q}_{=0} - \underbrace{[W - p_0 \Delta V]}_{=0} - \text{E}_d \Rightarrow \text{E}_d = -\Delta \text{E}$$

Since exergy is an extensive property, $\Delta \text{E} = \Delta \text{E}_m + \Delta \text{E}_w$. Then, with Eq. 7.3:

$$\mathrm{E_d} = -\left[\left(\Delta U + p_0 \underbrace{\Delta V}_{=0} - T_0 \Delta S\right)_m + \left(\Delta U + p_0 \underbrace{\Delta V}_{=0} - T_0 \Delta S\right)_w\right]$$

Using Eq. (1) this becomes:

#2 $\mathrm{E_d} = T_0[\Delta S_m + \Delta S_w]$ (2)

The term in square brackets is the amount of entropy produced, σ, which is evaluated using Eq. 6.13 for incompressible substances.

$$\sigma = m_\mathrm{w} c_\mathrm{w} \ln \frac{T_\mathrm{f}}{T_{\mathrm{w_i}}} + m_\mathrm{m} c_\mathrm{m} \ln \frac{T_\mathrm{f}}{T_{\mathrm{m_i}}} \qquad (3)$$

Where T_f is the final equilibrium temperature which is determined from Eq. (1) and using Eq. 3.20a to evaluate the internal energy changes of the water and metal in terms of the constant specific heats:

$$m_\mathrm{w} c_\mathrm{w}\left(T_\mathrm{f} - T_{\mathrm{w_i}}\right) + m_\mathrm{m} c_\mathrm{m}\left(T_\mathrm{f} - T_{\mathrm{m_i}}\right) = 0$$

Where $T_{\mathrm{w_i}}$ and $T_{\mathrm{m_i}}$ are the initial temperatures of the water and metal, respectively. Solving for T_f and inserting values:

$$T_\mathrm{f} = \frac{m_\mathrm{w} c_\mathrm{w} T_{\mathrm{w_i}} + m_\mathrm{m} c_\mathrm{m} T_{\mathrm{m_i}}}{m_\mathrm{w} c_\mathrm{w} + m_\mathrm{m} c_\mathrm{m}} = \frac{(25\ \mathrm{lb})\left(1.0\ \frac{\mathrm{Btu}}{\mathrm{lb \cdot °R}}\right) 500\ \mathrm{°R} + (1\ \mathrm{lb})\left(0.1\ \frac{\mathrm{Btu}}{\mathrm{lb \cdot °R}}\right) 2000\ \mathrm{°R}}{(25\ \mathrm{lb})\left(1.0\ \frac{\mathrm{Btu}}{\mathrm{lb \cdot °R}}\right) + (1\ \mathrm{lb})\left(0.1\ \frac{\mathrm{Btu}}{\mathrm{lb \cdot °R}}\right)}$$

$$= 506\mathrm{°R}$$

Solving of entropy produced by inserting values into Eq. (3):

$$\sigma = m_\mathrm{w} c_\mathrm{w} \ln \frac{T_\mathrm{f}}{T_{\mathrm{w_i}}} + m_\mathrm{m} c_\mathrm{m} \ln \frac{T_\mathrm{f}}{T_{\mathrm{m_i}}}$$

$$= (25\ \mathrm{lb})\left(1 \frac{\mathrm{Btu}}{\mathrm{lb \cdot °R}}\right) \ln \frac{506}{500} + (1\ \mathrm{lb})\left(0.1 \frac{\mathrm{Btu}}{\mathrm{lb \cdot °R}}\right) \ln \frac{506}{2000} = 0.1608 \frac{\mathrm{Btu}}{\mathrm{°R}}$$

Finally, inserting values into Eq. (2):

$$\mathrm{E_d} = (537\mathrm{°R})\left(0.1608 \frac{\mathrm{Btu}}{\mathrm{°R}}\right) = 86.3\ \mathrm{Btu}$$

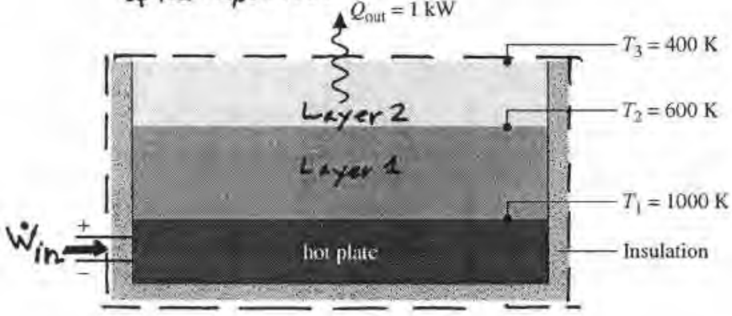
Comment:

1. With the indicated assumptions, the system is isolated. Eq. (1) indicates that the total energy of the system remains constant. Eq. (2) indicates that the exergy of the system does not remain constant because exergy is destroyed.
2. Alternatively, Eq. (2) can be expressed as $\mathrm{E_d} = T_0[\sigma]$, where σ is the amount of entropy produced within the system.

PROBLEM 7.37

, Figure P7.37 provides steady-state data for a composite of a hot plate and two solid layers. Perform a full exergy accounting, in kW, of the electrical power provided to the composite, including the exergy transfer accompanying heat transfer from the composite and the destruction of exergy in the hot plate and each of the two layers. Let $T_0 = 300$ K.

$\dot{W}_{in}$ and $\dot{Q}_{out}$ are taken as positive in the directions of the respective arrows...



ENGR. MODEL:

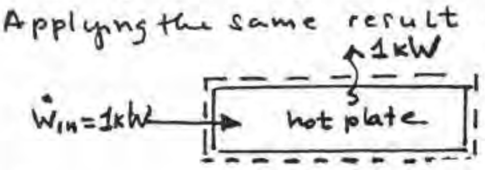
1. Closed systems consist of the hot plate, layer 1, layer 2, and the overall composite.
2. Each system is at steady state.
3. For the environment, $T_0 = 300K$

ANALYSIS:

For any closed system at steady state, $\cancel{dE/dt}^{\,0} = \dot{Q} - \dot{W} \Rightarrow \quad \dot{W} = \dot{Q}$. (1)

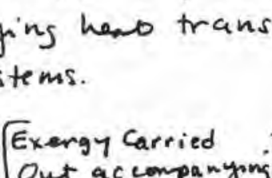
When Eq.(1) is applied to the overall composite, we get $\dot{W}_{in} = \dot{Q}_{out} = 1$ kW.

Applying the same result to the hot plate,



(work and heat transfer positive with the directions of the respective arrows)

For each layer, $\dot{W} = 0$; so $\dot{Q} = 0$: the rate of heat transfer into the layer equals the rate of heat transfer out of the layer:



(heat transfers positive in the directions of the respective arrows)

With these preliminaries, we can account for the exergy carried in electrically, $\dot{W}_{in}$, in terms of the exergy carried out accompanying heat transfer and the exergy destroyed in each of the three subsystems.

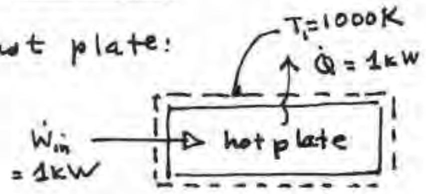
$$\begin{bmatrix} \text{Exergy Carried} \\ \text{Out accompanying} \\ \text{heat transfer} \end{bmatrix} = \left[1 - \frac{T_0}{T_3}\right]\dot{Q}_{out}$$

$$= \left[1 - \frac{300}{400}\right](1\text{kW}) = 0.25\text{ kW}$$

The exergy destroyed in each of the three subsystems is conveniently evaluated as $\dot{E}_d = T_0\dot{\sigma}$, where $\dot{\sigma}$ is the rate of entropy production.

PROBLEM 7.37 (Contd.)

For the hot plate:

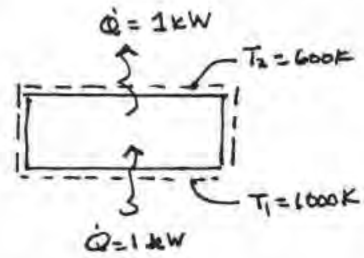


where $\dot{Q}$ is positive in the direction of the arrow. An entropy rate balance reads,

$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{\sigma} \Rightarrow 0 = \frac{-\dot{Q}}{T_1} + \dot{\sigma} \Rightarrow \dot{\sigma} = \frac{\dot{Q}}{T_1}$$

$$\therefore \dot{E}_d = T_0 \dot{\sigma} = \frac{T_0}{T_1}\dot{Q}_1 = \left(\frac{300}{1000}\right)(1\,kW) = 0.3\,kW$$

For Layer 1,



$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{\sigma} \Rightarrow 0 = \frac{\dot{Q}}{T_1} - \frac{\dot{Q}}{T_2} + \dot{\sigma} \Rightarrow \dot{\sigma} = \left[\frac{1}{T_2} - \frac{1}{T_1}\right]\dot{Q}$$

$$\therefore \dot{E}_d = T_0 \dot{\sigma} = T_0\left[\frac{1}{T_2} - \frac{1}{T_1}\right]\dot{Q} = 300\left[\frac{1}{600} - \frac{1}{1000}\right](1\,kW) = 0.2\,kW$$

With the same approach for Layer 2,

$$\dot{E}_d = T_0 \dot{\sigma} = T_0\left[\frac{1}{T_3} - \frac{1}{T_2}\right]\dot{Q} = 300\left[\frac{1}{400} - \frac{1}{600}\right](1\,kW) = 0.25\,kW$$

(1) Thus, an exergy accounting for the overall composite reads:

Exergy supplied:

$$\begin{bmatrix}\text{Rate exergy} \\ \text{is carried in} \\ \text{electrically}\end{bmatrix} = 1.0\,kW$$

Disposition of the exergy carried in:

- $$\begin{bmatrix}\text{Exergy carried} \\ \text{out accompanying} \\ \text{heat transfer}\end{bmatrix} = 0.25\,kW$$

- Exergy destroyed:

$$\left.\begin{array}{ll} \checkmark \text{ Hot plate} & = 0.30\,kW \\ \checkmark \text{ Layer 1} & = 0.20\,kW \\ \checkmark \text{ Layer 2} & = 0.25\,kW \end{array}\right\} 0.75\,kW$$

$$1.0\,kW$$

---

1. On an exergy basis, the efficiency of the composite can be evaluated as

$$\frac{\begin{bmatrix}\text{Exergy carried out} \\ \text{accompanying heat transfer}\end{bmatrix}}{\begin{bmatrix}\text{Exergy carried in} \\ \text{electrically}\end{bmatrix}} = 0.25 \quad (25\%)$$

PROBLEM 7.38

7.38 As shown in Fig. P7.38, heat transfer at a rate of 1000 Btu/h takes place through the inner surface of a wall. Measurements made during steady-state operation reveal temperatures of $T_1 = 2500°R$ and $T_2 = 500°R$ at the inner and outer surfaces, respectively. Determine, in Btu/h

(a) the rates of exergy transfer accompanying heat at the inner and outer surfaces of the wall.
(b) the rate of exergy destruction.
(c) What is the cause of exergy destruction in this case?

Let $T_0 = 500°R$.

SCHEMATIC & GIVEN DATA:

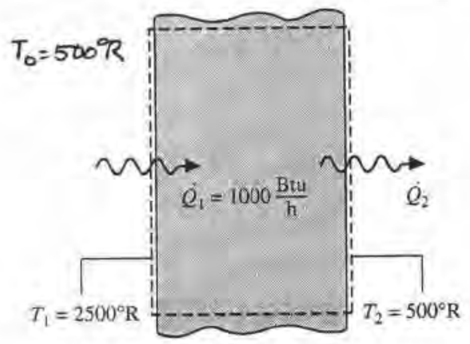


Fig. P7.38

KNOWN: Heat transfer rate and temperature data are provided for a wall at steady state.

FIND: Determine the rates of exergy transfer at the inner and outer wall surfaces and the rate of exergy destruction within the wall. Discuss

ENGINEERING MODEL:

1. The closed system is shown by the dashed line on the on the schematic.
2. The system is at steady state.
3. For the system, $\dot{W} = 0$ and there are no effects of motion or gravity.
4. The directions of the heat transfers are indicated by the arrows.
5. $T_0 = 500°R$

ANALYSIS:

Reducing an energy rate balance at steady state, $\cancel{\frac{dE}{dt}}^{0} = \dot{Q} - \cancel{\dot{W}}^{0} \Rightarrow \dot{Q} = 0$
$\Rightarrow \dot{Q}_1 = \dot{Q}_2 = 1000$ Btu/h.

(a) Applying Eq. 7.12 at the inner surface, we get

① $$\dot{E}_{q,1} = \left[1 - \frac{T_0}{T_1}\right]\dot{Q}_1 = \left[1 - \frac{500}{2500}\right]\left(1000 \frac{\text{Btu}}{\text{h}}\right) = 800 \frac{\text{Btu}}{\text{h}}$$

Then, at the outer surface we get

② $$\dot{E}_{q,2} = \left[1 - \frac{\overset{T_0}{500}}{\underset{T_2}{500}}\right]\left(1000 \frac{\text{Btu}}{\text{h}}\right) = 0 \ \frac{\text{Btu}}{\text{h}}$$

(b) Applying Eq. 7.11a,

③ $$\dot{E}_d = \left[1 - \frac{T_0}{T_1}\right]\dot{Q}_1 - \underbrace{\left[1 - \frac{T_0}{T_2}\right]\dot{Q}_2}_{=0} - \cancel{\dot{W}}^{0} \Rightarrow \dot{E}_d = 800 \frac{\text{Btu}}{\text{h}}$$

(c) Exergy is destroyed because heat transfer through a finite temperature difference occurs spontaneously.

---

1. $\dot{E}_{q,1}$ is the power that could be obtained in principle by providing $\dot{Q}_1$ to a reversible power cycle operating between T_1 and T_0.
2. At the outer surface, $\dot{Q}_2$ is nonzero but $\dot{E}_{q,2} = 0$. No power can be developed by providing $\dot{Q}_2$ to a reversible power cycle operating with $T_2 (= T_0)$ and T_0.
3. Exergy enters the wall at the inner surface and none is carried out at the outer surface nor is there any useful exergy product. All of the entering exergy is destroyed.

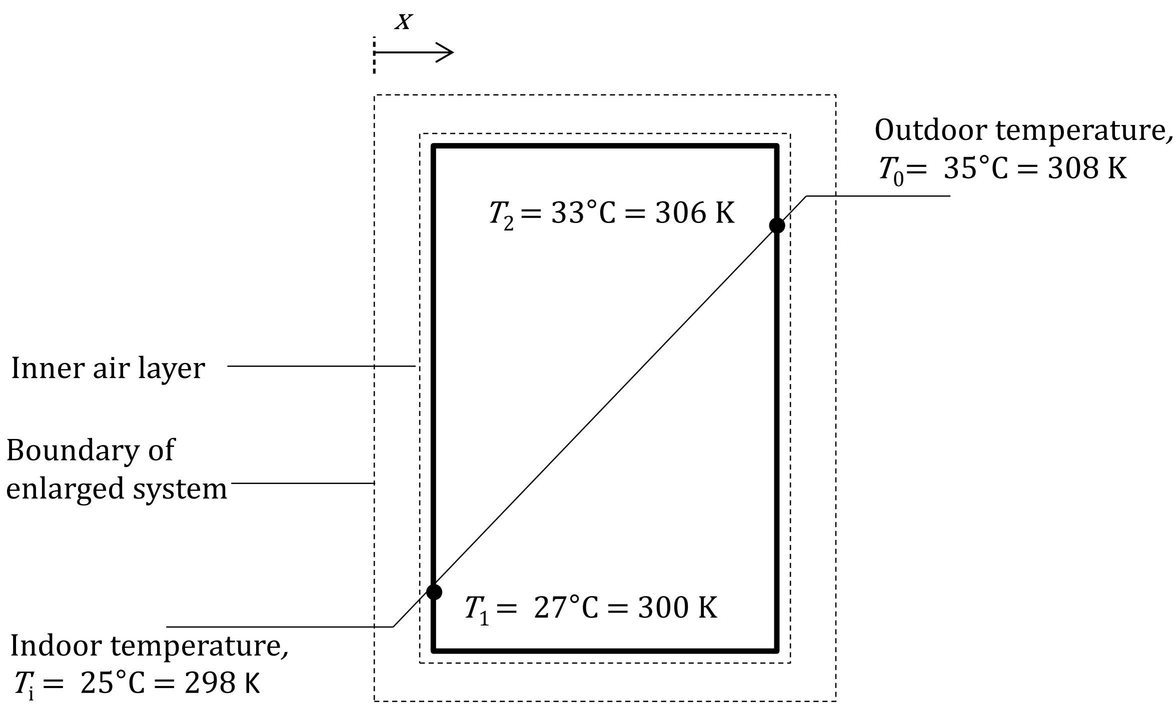
Problem 7.39

Figure P7.39 provides steady state data for the outer wall of a dwelling on a day when the indoor temperature is maintained at 25°C and the outdoor temperature is 35°C. The heat transfer rate through the wall is 1000 W. Determine, in W, the rate of exergy destruction (a) within the wall and (b) within the enlarged system shown on the figure by the dashed line. Comment. Let T_0 = 35°C.

Known:
A dwelling is to be kept cool on a day when the temperature outside is higher than the temperature inside.

Find:
Determine the exergy destruction (a) within the wall, and (b) within the enlarged system shown on the figure.

Schematic and Known Data:



Engineering Model:
(1) As shown in the figure, two closed systems are considered: one is a schematic of the wall; the other is an enlarged system.
(2) The systems are at steady state.
(3) Heat transfer at a rate of 1000 W takes place in the negative x direction.
(4) For the environment, T_0 = 308 K.

Analysis:
(a) An energy balance for the wall reads:

$$\underbrace{\frac{dE}{dt}}_{=0} = \dot{Q} - \underbrace{\dot{W}}_{=0} \Longrightarrow \dot{Q} = 0$$

That is, $\dot{Q}_1 = \dot{Q}_2 = 1000\ W$, where the heat transfer rates at the inner, $\dot{Q}_1$, and outer, $\dot{Q}_2$, surfaces of the wall are negative in the negative x direction.

An exergy rate balance at steady state reads:

$$0 = \left(1 - \frac{T_0}{T_1}\right)\dot{Q}_1 - \left(1 - \frac{T_0}{T_2}\right)\dot{Q}_2 - \underbrace{\dot{W}}_{=0} - \dot{\mathrm{E}}_\mathrm{d}$$

$$\dot{\mathrm{E}}_\mathrm{d} = \left(1 - \frac{T_0}{T_1}\right)\dot{Q}_1 - \left(1 - \frac{T_0}{T_2}\right)\dot{Q}_2$$

$$= \left(1 - \frac{308\ \mathrm{K}}{300\ \mathrm{K}}\right)(-1000\ \mathrm{W}) - \left(1 - \frac{308\ \mathrm{K}}{306\ \mathrm{K}}\right)(-1000\ \mathrm{W}) = 20.13\ \mathrm{W}$$

(b) An energy rate balance for the enlarged system reads:

$$\underbrace{\frac{dE}{dt}}_{=0} = \dot{Q} - \underbrace{\dot{W}}_{=0} \Longrightarrow \dot{Q} = 0$$

That is, $\dot{Q}_i = \dot{Q}_o = 1000$ W, where the heat transfer rates at the inner, $\dot{Q}_i$, and outer, $\dot{Q}_o$, surfaces of the enlarged system are negative in the negative *x* direction. An exergy rate balance at steady state reads:

$$0 = \left(1 - \frac{T_0}{T_i}\right)\dot{Q}_i - \left(1 - \underbrace{\frac{T_0}{T_o}}_{=0}\right)\dot{Q}_o - \underbrace{\dot{W}}_{=0} - \dot{\mathrm{E}}_\mathrm{d}$$

$$\dot{\mathrm{E}}_\mathrm{d} = \left(1 - \frac{T_0}{T_i}\right)\dot{Q}_i = \left(1 - \frac{308\ \mathrm{K}}{298\ \mathrm{K}}\right)(-1000\ \mathrm{W}) = 33.56\ \mathrm{W}$$

The exergy destruction in the enlarged system is greater than for the wall alone because of exergy destruction associated with heat transfer though the air layers at each of the wall surfaces.

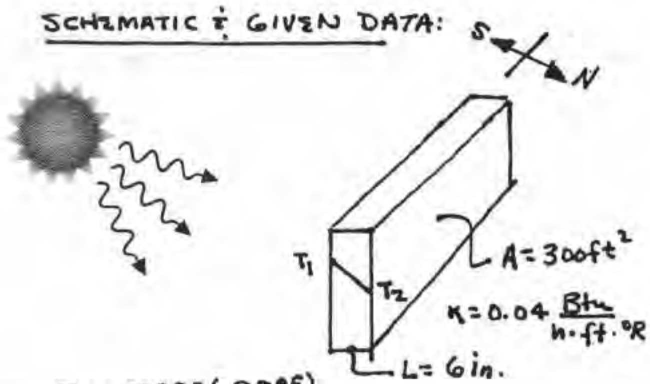
PROBLEM 7.40

7.40 The sun shines on a 300-ft^2 south-facing wall, maintaining that surface at 98°F. Temperature varies linearly through the wall and is 77°F at its other surface. The wall thickness is 6 inches and its thermal conductivity is 0.04 Btu/h · ft · R. Assuming steady state, determine the rate of exergy destruction within the wall, in Btu/h. Let T_0 = 77°F.

KNOWN: Data are provided for a wall at steady state warmed by the sun.

FIND: Determine the rate of exergy destruction within the wall.

SCHEMATIC & GIVEN DATA:



$T_1 = 558°R$ (98°F)
$T_2 = 537°R$ (77°F)
$T_0 = 537°R$ (77°F)

ENGINEERING MODEL:

1. The closed system is the wall.
2. The system is at steady state.
3. $\dot{W} = 0$ and there are no effects of motion and gravity.
4. Temperature varies linearly through the wall.
5. $T_0 = 537°R$ (77°F)

ANALYSIS: Reducing an energy rate balance at steady state: $\cancel{\frac{dE}{dt}}^{0} = \dot{Q} - \cancel{\dot{W}}^{0}$
$\Rightarrow \dot{Q} = 0 \Rightarrow \dot{Q}_1 = \dot{Q}_2$. Then, with Eq. 2.31 and assumption # 4,

$$\dot{Q}_1 = \dot{Q}_2 = -\kappa A \left[\frac{T_2 - T_1}{L}\right] = \left(0.04 \frac{Btu}{h \cdot ft \cdot °R}\right)(300 ft^2)\left(\frac{(558-537)°R}{0.5 ft}\right) = 504 \frac{Btu}{h}$$

Applying Eq. 7.12 at surface 1, we get

① $$\dot{E}_{q,1} = \left[1 - \frac{T_0}{T_1}\right]\dot{Q}_1 = \left[1 - \frac{537}{558}\right]\left(504 \frac{Btu}{h}\right) = 18.97 \frac{Btu}{h}$$

Then, at surface 2,

② $$\dot{E}_{q,2} = \left[1 - \frac{T_0}{T_2}\right]\dot{Q}_2 = 0 \ \frac{Btu}{h}$$

$(T_0 = T_2)$

Applying Eq. 7.11a

$$\dot{E}_d = \dot{E}_{q,1} - \cancel{\dot{E}_{q,2}}^{0} - \cancel{\dot{W}}^{0}$$

③ $$\Rightarrow \dot{E}_d = \dot{E}_{q,1} = 18.97 \frac{Btu}{h}$$

---

1. $\dot{E}_{q,1}$ is the power that could be obtained in principle by providing $\dot{Q}_1$ to a reversible power cycle operating between T_1 and T_0.
2. At surface 2, $\dot{Q}_2$ is nonzero but $\dot{E}_{q,2} = 0$. No power can be developed by providing $\dot{Q}_2$ to a reversible power cycle operating with T_2 (= T_0) and T_0.
3. Exergy enters the wall at surface 1 and none is carried out at surface 2 nor is there any useful exergy product. All of the entering exergy is destroyed.

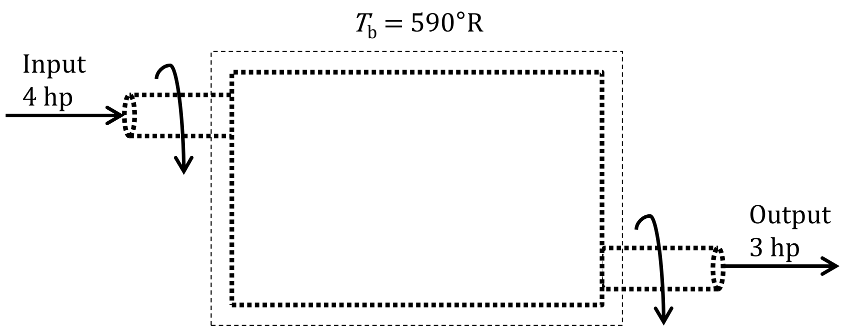
Problem 7.41

A gearbox operating at steady state receives 4 hp along the input shaft and delivers 3 hp along the output shaft. The outer surface of the gearbox is at 130°F. For the gearbox, (a) determine, in Btu/s, the rate of heat transfer and (b) perform a full exergy accounting, in Btu/s, of the input power. Let T_0 = 70°F.

Known:
A gearbox operating at steady state has one input and one output.

Find:
Determine (a) the rate of heat transfer, and (b) perform a full exergy accounting of the input power.

Schematic and Known Data:



Engineering Model:
(1) The system shown in the schematic is at steady state.
(2) Heat transfer occurs only at T_b.
(3) For the environment, T_0 = 530°R (70°F).

Analysis:

(a) At steady state, an energy balance reduces to read:

$$\underbrace{\frac{dE}{dt}}_{=0} = \dot{Q} - \dot{W} \Longrightarrow \dot{Q} = \dot{W}$$

$$\dot{Q} = (3-4)\text{hp}\left|\frac{2545\frac{\text{Btu}}{\text{h}}}{1\text{ hp}}\right| \cdot \left|\frac{1\text{ h}}{3600\text{ s}}\right| = -0.707\text{ Btu/s}$$

(b) The rate exergy enters the system equals the input power (4 hp). Expressing this in units of Btu/s:

$$\dot{W}_{\text{in}} = (4\text{ hp})\left|\frac{2545\frac{\text{Btu}}{\text{h}}}{1\text{ hp}}\right| \cdot \left|\frac{1\text{ h}}{3600\text{ s}}\right| = 2.83\frac{\text{Btu}}{\text{s}}$$

Exergy exits the systems via output power of 3 hp:

$$\dot{W}_{\text{out}} = (3\text{ hp})\left|\frac{2545\,\frac{\text{Btu}}{\text{h}}}{1\text{ hp}}\right| \cdot \left|\frac{1\text{ h}}{3600\text{ s}}\right| = 2.12\,\frac{\text{Btu}}{\text{s}}$$

Exergy also exits accompanying the heat transfer, that is:

$$\left[1-\frac{T_0}{T_\text{b}}\right]\left|\dot{Q}\right| = \left[1-\frac{530}{590}\right](0.707) = 0.072\,\frac{\text{Btu}}{\text{s}}$$

Finally, exergy is destroyed within the gearbox. This is conveniently evaluated in terms of the rate of entropy production, $\dot{\text{E}}_\text{d} = T_0\dot{\sigma}$,

where $\dot{\sigma} = -\frac{\dot{Q}}{T_\text{b}}$, giving:

$$\dot{\text{E}}_\text{d} = \frac{T_0}{T_\text{b}}\dot{Q} = \frac{530}{590}\left(0.707\,\frac{\text{Btu}}{\text{s}}\right) = 0.635\,\frac{\text{Btu}}{\text{s}}$$

Exergy Accounting Summary:

Rate exergy is input: $2.83\,\frac{\text{Btu}}{\text{s}}$

Disposition of exergy:

Output Power:	$2.12\,\frac{\text{Btu}}{\text{s}}$	75.0 %
Heat Transfer:	$0.072\,\frac{\text{Btu}}{\text{s}}$	2.54 %
Destroyed:	$0.635\,\frac{\text{Btu}}{\text{s}}$	22.44 %
Total:	$= 2.83\,\frac{\text{Btu}}{\text{s}}$	= 100 %

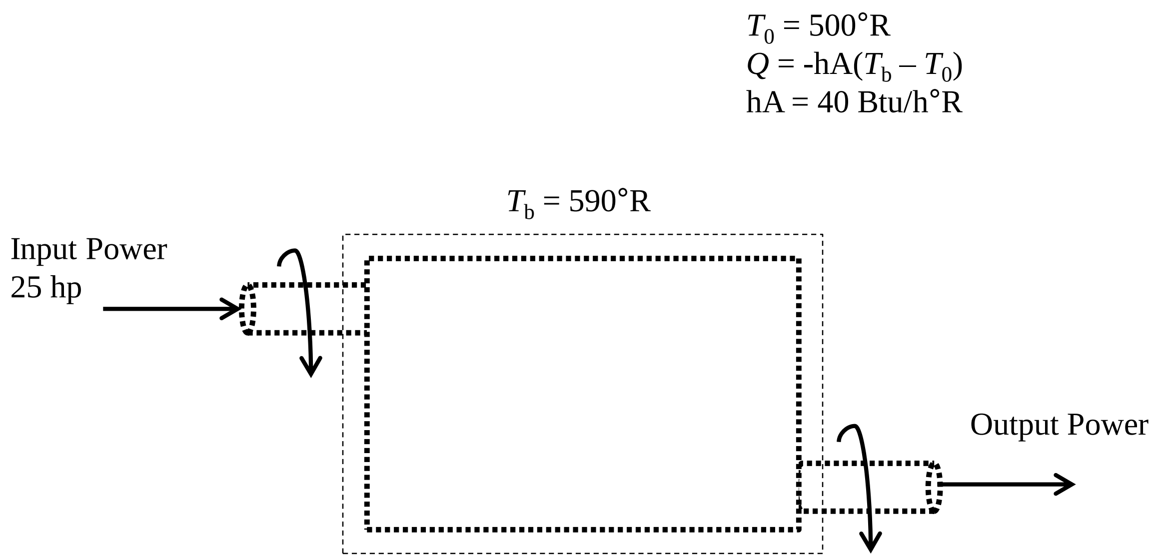
Problem 7.42

A gearbox operating at steady state receives 25 horsepower along its input shaft, delivers power along its output shaft, and is cooled on its outer surface according to hA(T_b – T_0), where T_b = 130°F is the temperature of the outer surface and T_0 = 40°F is the temperature of the surroundings far from the gearbox. The product of the heat transfer coefficient h and outer surface area A is 40 Btu/h°R. For the gearbox, determine, in hp, a full exergy accounting of the input power. Let T_0 = 40°F.

Known:
A gearbox operates at steady state. Performance data are provided.

Find:
Determine a full exergy accounting of the input power.

Schematic and Known Data:



Engineering Model:
(1) The system shown in the schematic is at steady state.
(2) The temperature of the outer surface of the gearbox and the temperature of the surroundings are each uniform.
(3) For the environment, T_0 = 500°R (40°F).

Analysis:
At steady state, an energy balance reduces to read:

$$\underbrace{\frac{dE}{dt}}_{=0} = \dot{Q} - \dot{W} \Longrightarrow \dot{Q} = \dot{W} = \dot{W}_{\text{out}} - \dot{W}_{\text{in}}$$

Then, inserting given information:

$$\dot{W}_{\text{out}} = \dot{Q} + \dot{W}_{\text{in}} = -\text{hA}[T_{\text{b}} - T_0] + \dot{W}_{\text{in}} = -\left(40\,\frac{\text{Btu}}{\text{h}\cdot{}^{\circ}\text{R}}\right)(590-500)^{\circ}\text{R} + (25\ \text{hp})\left|\frac{2545\,\frac{\text{Btu}}{\text{h}}}{1\ \text{hp}}\right|$$

$$= \underbrace{(-3600)\,\frac{\text{Btu}}{\text{h}}}_{=\dot{Q}} + \underbrace{63625\,\frac{\text{Btu}}{\text{h}}}_{=\dot{W}_{\text{in}}} = 60025\,\frac{\text{Btu}}{\text{h}}$$

The magnitude of the rate of exergy transfer accompanying heat is:

$$\left[1-\frac{T_0}{T_{\text{b}}}\right]\left|\dot{Q}\right| = \left[1-\frac{500}{590}\right](3600) = 549.15\,\frac{\text{Btu}}{\text{h}}$$

The rate of exergy destruction can be found from $\dot{\text{E}}_{\text{d}} = T_0\dot{\sigma}$, where $\dot{\sigma}$ is the rate of entropy production, or by reducing the exergy rate balance at steady state to obtain:

$$\dot{\text{E}}_{\text{d}} = \left[1-\frac{T_0}{T_{\text{b}}}\right]\dot{Q} - \dot{W} = \left[1-\frac{T_0}{T_{\text{b}}}\right]\dot{Q} - (\dot{W}_{\text{out}} - \dot{W}_{\text{in}}) = -549.15 - (60025 - 63625)$$

$$= 3050.85\,\frac{\text{Btu}}{\text{h}}$$

<u>Exergy Accounting Summary:</u> Expressing values as percentages of the input power, the following list accounts for the disposition of the input power.

Rate exergy is input:	$63625\,\frac{\text{Btu}}{\text{h}}$	
Disposition of exergy:		
Output Power:	$60025\,\frac{\text{Btu}}{\text{h}}$	94.34 %
#1 Heat Transfer:	$549.15\,\frac{\text{Btu}}{\text{h}}$	0.86 %
Destroyed:	$3050.85\,\frac{\text{Btu}}{\text{h}}$	4.80 %
Total:	$=\ 63625\,\frac{\text{Btu}}{\text{h}}$	100 %

<u>Comment:</u>

1. This represents the true thermodynamic value of the heat loss. An energy analysis overstates its significance:

$$\frac{\left|\dot{Q}\right|}{\dot{W}_{\text{in}}} = \frac{3600}{63625}\cdot 100 = 5.7\%$$

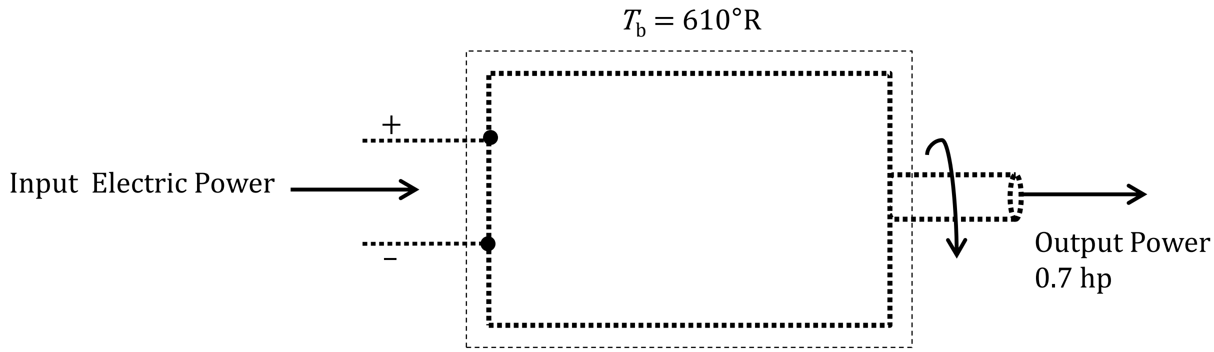
Problem 7.43

At steady state, an electric pump motor develops power along its output shaft of 0.7 hp while drawing 6 amps at 100 V. The outer surface of the motor is at 150°F. For the motor:
(a) determine, in Btu/h, the rate of heat transfer and (b) perform a full exergy accounting, in Btu/h, of the electrical power input. Let T_0 = 40°F.

Known:
An electric pump motor operates at steady state. Performance data are provided.

Find:
Determine (a) the rate of heat transfer, and (b) perform a full exergy accounting of the electrical power input.

Schematic and Known Data:



Engineering Model:
(1) The system shown in the schematic is at steady state.
(2) Heat transfer occurs at T_b = 610°R and the power quantities are positive with the directions of the arrows.
(3) For the environment, T_0 = 500°R (40°F).

Analysis:
(a) At steady state, an energy balance reduces to read:

$$\underbrace{\frac{dE}{dt}}_{=0} = \dot{Q} - \dot{W} \Longrightarrow \dot{Q} = \dot{W} \text{ or}$$

$$\dot{Q} = \dot{W}_{\text{shaft}} - \dot{W}_{\text{elec}} = (0.7\text{ hp})\left|\frac{2545\frac{\text{Btu}}{\text{h}}}{1\text{ hp}}\right| - \left[(6\text{ A})(100\text{ V})\left|\frac{\text{W}}{\text{A}\cdot\text{V}}\right|\left|\frac{3.412\frac{\text{Btu}}{\text{h}}}{1\text{ W}}\right|\right] = \underbrace{1781.5\frac{\text{Btu}}{\text{h}}}_{=\dot{W}_{\text{shaft}}} - \underbrace{2047.2\frac{\text{Btu}}{\text{h}}}_{=\dot{W}_{\text{elec}}} = -265.7\frac{\text{Btu}}{\text{h}}$$

(b) Exergy is carried into the pump motor by electric power ($2047.2\frac{\text{Btu}}{\text{h}}$). Exergy is carried out of the motor by shaft power ($1781.5\frac{\text{Btu}}{\text{h}}$). Exergy is also carried out accompanying heat transfer; the rate is:

$$\left[1-\frac{T_0}{T_\mathrm{b}}\right]\left|\dot{Q}\right| = \left[1-\frac{500}{610}\right](265.7) = 47.91\,\frac{\mathrm{Btu}}{\mathrm{h}}$$

Finally, exergy is destroyed within the motor. This is conveniently evaluated in terms of entropy production: $\dot{\mathrm{E}}_\mathrm{d} = T_0\dot{\sigma}$, where $\dot{\sigma} = -\frac{\dot{Q}}{T_\mathrm{b}}$, giving:

$$\dot{\mathrm{E}}_\mathrm{d} = \frac{T_0}{T_\mathrm{b}}\left(-\dot{Q}\right) = \left(\frac{500}{610}\right)\left(265.7\,\frac{\mathrm{Btu}}{\mathrm{h}}\right) = 217.79\,\frac{\mathrm{Btu}}{\mathrm{h}}$$

Exergy Accounting Summary: Expressing values as percentages of the input power, the following list accounts for the disposition of the input power.

Rate exergy input: $2047.2\ \frac{\mathrm{Btu}}{\mathrm{h}}$

Disposition of exergy:

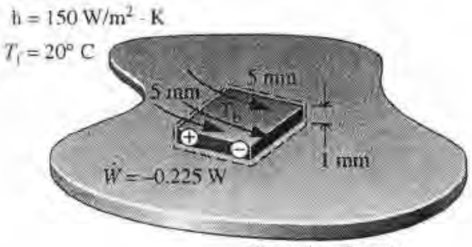
Output Power:	$1781.5\ \frac{\mathrm{Btu}}{\mathrm{h}}$	87.02 %
#1		
Heat Transfer:	$47.19\,\frac{\mathrm{Btu}}{\mathrm{h}}$	2.31 %
Destroyed:	$217.79\ \frac{\mathrm{Btu}}{\mathrm{h}}$	10.64 %
#2 **Total**:	$= 2046.5\ \frac{\mathrm{Btu}}{\mathrm{h}}$	= 99.97 %

Comments:

1. Electric motors typically develop a greater fraction of output power from the input power than determined in this case.
2. The answer to part (b) is slightly off due to round-off errors when using conversion factors.

PROBLEM 7.44

As shown in Fig. P7.44, a silicon chip measuring 5 mm on a side and 1 mm in thickness is embedded in a ceramic substrate. At steady state, the chip has an electrical power input of 0.225 W. The top surface of the chip is exposed to a coolant whose temperature is 20°C. The heat transfer coefficient for convection between the chip and the coolant is 150 W/m^2 · K. Heat transfer by conduction between the chip and the substrate is negligible. Determine (a) the surface temperature of the chip, in °C, and (b) the rate of exergy destruction within the chip, in W. What causes the exergy destruction in this case? Let T_0 = 293 K.



Ceramic substrate

Note: This application is the subject of Example 2.5 and Problem 6.54.

(a) From Example 2.5, $T_b = 353\,K\,(80°C)$ ← (a)

(b) From Problem 6.54, $\dot{\sigma} = 6.37 \times 10^{-7}\,kW/K$

The principal source of entropy production — and exergy destruction — is electric current flow through a resistance. The rate of exergy destruction is

$$\dot{E}_d = T_0 \dot{\sigma} = 293\,K \left(6.37 \times 10^{-7} \frac{kW}{K}\right) \left| \frac{10^3 W}{1 kW} \right|$$

$$= 0.187\,W$$ ← (b)

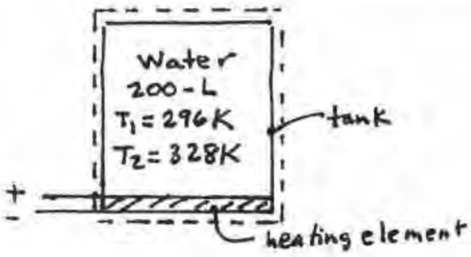
PROBLEM 7.45

7.45 An electric water heater having a 200-L capacity heats water from 23 to 55°C. Heat transfer from the outside of the water heater is negligible, and the states of the electrical heating element and the tank holding the water do not change significantly. Perform a full exergy accounting, in kJ, of the electricity supplied to the water heater. Model the water as incompressible with a specific heat c = 4.18 kJ/kg · K. Let T_0 = 23°C.

KNOWN: Operating data are provided for an electric water heater.

FIND: Perform a full exergy accounting of the electricity supplied to the water heater.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The system is shown by the dashed line on the schematic.
2. For the system, stray heat transfer and the effects of motion and gravity are ignored.
3. The water is modeled as incompressible with $c = 4.18$ kJ/kg·K, $v = v_f(T_{ave})$.
4. The states of the heating element and tank do not change significantly.
5. $T_0 = 296$ K

ANALYSIS: The mass of water is determined as follows:

$$m = \frac{V}{v} = \frac{(200\,\text{L})\left|\frac{10^{-3}\,\text{m}^3}{1\,\text{L}}\right|}{(1.008\times10^{-3}\,\text{m}^3/\text{kg})} = 198.4\ \text{kg} \quad \text{, where } v = v_f(39°C)\ (T_{ave})$$

With listed assumptions, an energy balance reduces to: $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - W$

$$\therefore\ W = -\Delta U = -m\Delta u = -mc[T_2 - T_1] = -(198.4\,\text{kg})(4.18\,\tfrac{\text{kJ}}{\text{kg}\cdot\text{K}})(328-296)\,\text{K}$$

$$= -26{,}538\ \text{kJ}$$

The rate of exergy destruction is $E_d = T_0\sigma$, where σ is obtained from an entropy balance:

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)^0 + \sigma \Rightarrow \sigma = \Delta S = m\Delta s = mc\ln\frac{T_2}{T_1}\ \ (\text{Eq. 6.13})$$

$$\therefore\ \sigma = (198.4\,\text{kg})(4.18\,\tfrac{\text{kJ}}{\text{kg}\cdot\text{K}})\ln\frac{328}{296} = 85.13\ \frac{\text{kJ}}{\text{K}}$$

Then,

$$E_d = T_0\sigma = (296\,\text{K})(85.13\,\tfrac{\text{kJ}}{\text{K}}) = 25{,}198\ \text{kJ}$$

The exergy stored in the water is determined using Eq. 7.3 and previously calculated values:

$$E_2 - E_1 = (U_2 - U_1) + p_0(\cancel{V_2 - V_1})^0 - T_0(S_2 - S_1)$$

$$= 26{,}538 + 0 - 25{,}198 = 1340\ \text{kJ}$$

Exergy Accounting:

- Exergy carried into the system electrically: 26,538 kJ
- Disposition of the exergy:

①

✓ Exergy Stored	1,340 kJ	(5%)
✓ Exergy Destroyed	25,198 kJ	(95%)
	26,538 kJ	

1 The wasteful nature of water heaters is illustrated by the relatively small fraction of the supplied exergy that is stored in this application.

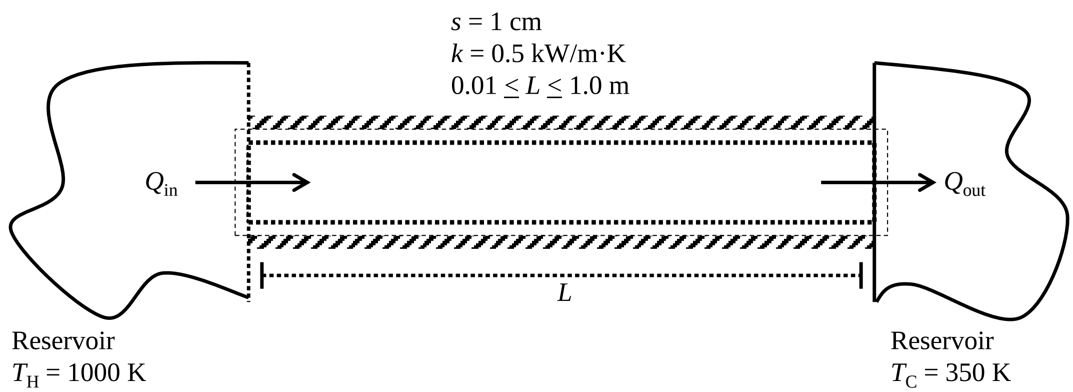
Problem 7.46

A thermal reservoir at 1000 K is separated from another thermal reservoir at 350 K by a 1cm by 1 cm square-cross section rod insulated on its lateral surfaces. At steady state, energy transfer by conduction takes place through the rod. The rod length is L, and the thermal conductivity is 0.5 kW/m·K. Plot the following quantities, each in kW, versus L ranging from 0.01 to 1 m: the rate of conduction through the rod, the rates of exergy transfer accompanying heat transfer into and out of the rod, and the rate of exergy destruction. Let T_0 = 300 K.

Known:
Energy is conducted from a thermal reservoir through a square-cross section rod at steady state to another thermal reservoir at a lower temperature.

Find:
Plot the rate of conduction through the rod, the rates of exergy transfer accompanying heat transfer into and out of the rod, and the rate of exergy destruction, each versus the rod length, L.

Schematic and Known Data:



Engineering Model:
(1) The system shown in the schematic is at steady state.
(2) Energy transfer is in the directions of the arrows only.
(3) For the environment, T_0 = 300 K.

Analysis:
An energy rate balance reduces to give:

$$\underbrace{\frac{dE}{dt}}_{=0} = \dot{Q} - \underbrace{\dot{W}}_{=0} \Rightarrow \dot{Q} = 0 \Rightarrow \dot{Q}_{\text{in}} = \dot{Q}_{\text{out}}$$

Using Eq. 2.31, the heat transfer rate by conduction is:

$$\dot{Q}_{\text{in}} = \dot{Q}_{\text{out}} = k\text{A}\left[\frac{T_{\text{H}} - T_{\text{C}}}{L}\right] = \left(0.5\,\frac{\text{kW}}{\text{m}\cdot\text{K}}\right)(0.01\text{ m})^2\left(\frac{1000 - 350}{L}\right)\frac{\text{K}}{\text{m}} = \frac{0.0325}{L}\,\text{kW}$$

The rates of exergy transfer accompanying heat transfer are, respectively:

$$[\text{Rate of exergy transfer in}] = \left[1 - \frac{T_0}{T_H}\right]\dot{Q}_{in} = \left[1 - \frac{300}{1000}\right]\dot{Q}_{in} = 0.7\dot{Q}_{in}$$

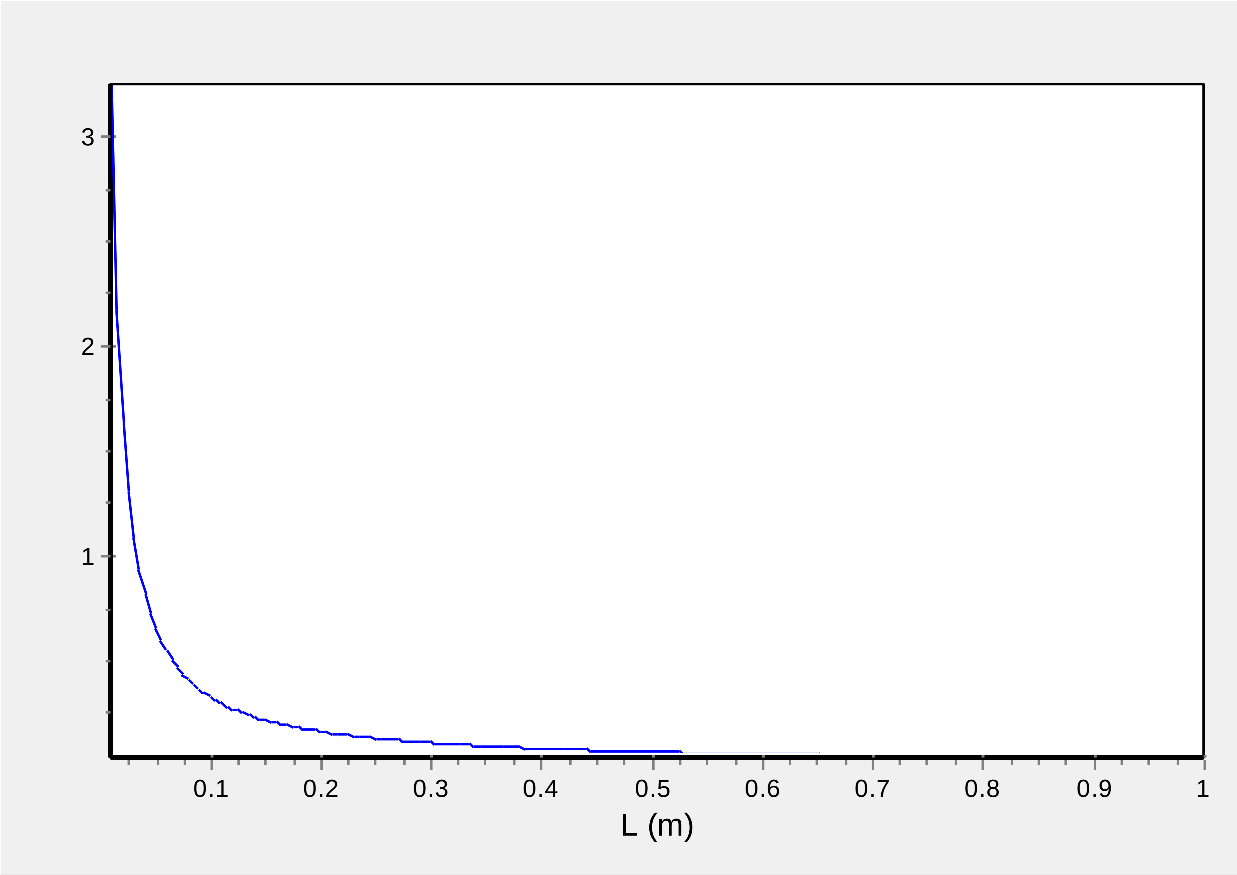
$$[\text{Rate of exergy transfer out}] = \left[1 - \frac{T_0}{T_C}\right]\dot{Q}_{in} = \left[1 - \frac{300}{350}\right]\dot{Q}_{in} = 0.143\dot{Q}_{in}$$

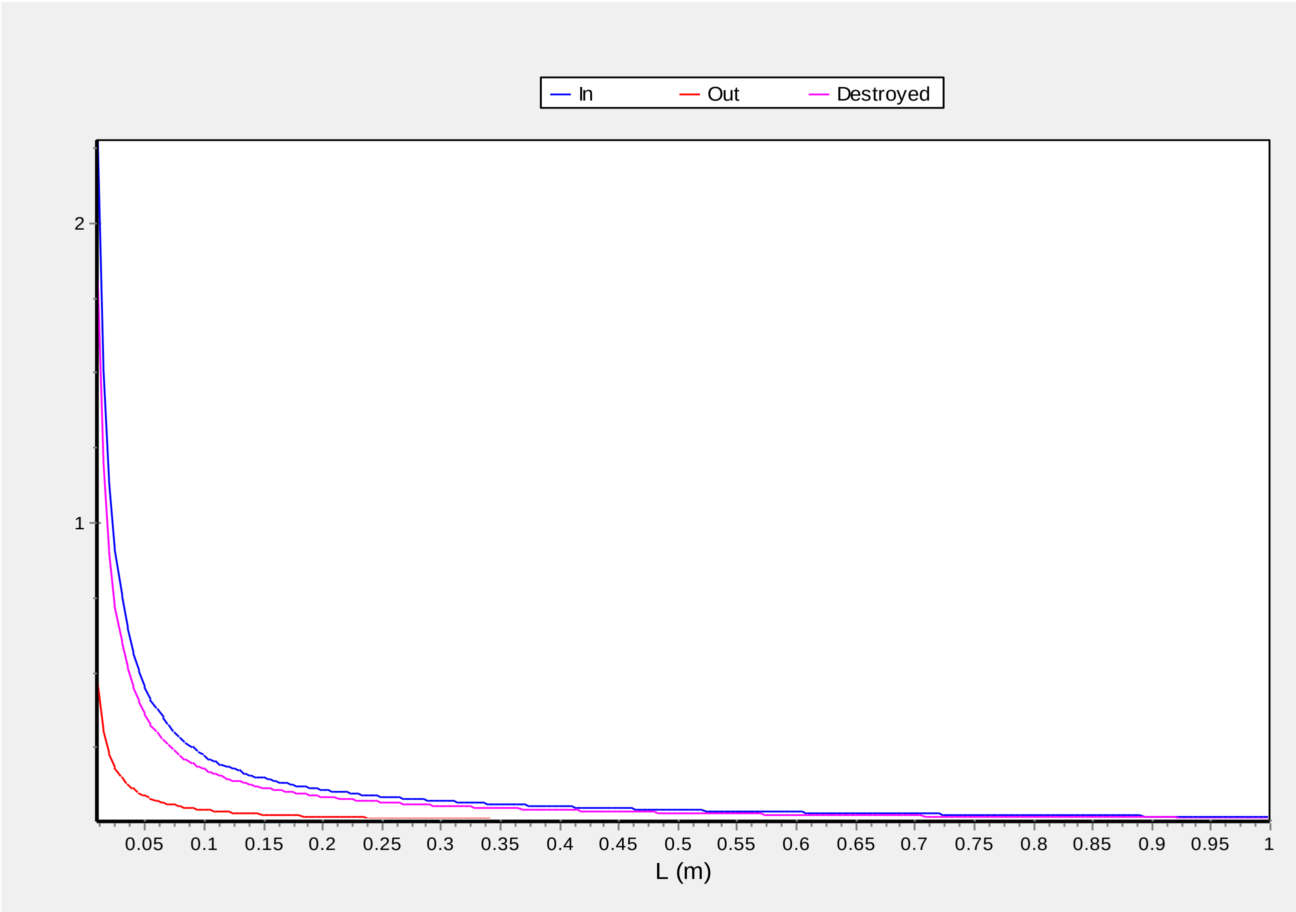
The rate of exergy destruction can be obtained by reducing an exergy rate balance, which in this case corresponds to finding the difference between the exergy entering and exiting the rod. Thus:

$$\dot{E}_d = [\text{Rate of exergy transfer in}] - [\text{Rate of exergy transfer out}] = (0.7 - 0.143)\dot{Q}_{in} = 0.557\dot{Q}_{in}$$

<u>Sample Calculation</u>: L = 1m, $\dot{Q}_{in} = 0.0325$ kW, $\dot{E}_d = 0.0181$ kW

<u>Plots (done in IT):</u>





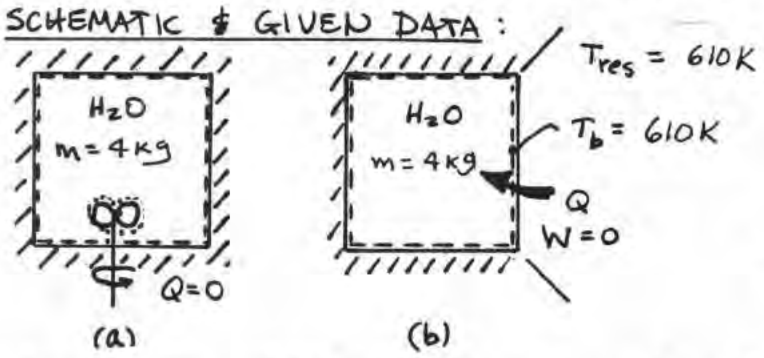
Discussion:
As the length of the rod increases, the heat transfer rate and the exergy destruction rate decrease rapidly. Since exergy can be viewed as having economic value (see Sec. 7.7), a reduction in exergy destruction can be viewed as a cost saving. However, the cost of the rod increases with length, so an economic tradeoff is inherent in selecting a value for *L*.

PROBLEM 7.47

KNOWN: Four kilograms of a two-phase liquid-vapor mixture of water undergo two different processes at constant volume between the same end states: (a) adiabatic process, stirring with a paddle wheel, (b) heat transfer from a thermal reservoir at 610K.

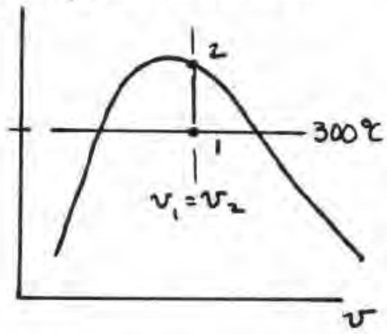
FIND: For each case, determine the change in exergy, the net amounts of exergy transfer by work and heat, and the amount of exergy destruction.

SCHEMATIC & GIVEN DATA:



$T_1 = 300°C$
$x_1 = 0.5$
$x_2 = 1.0$

$T_0 = 300K$
$p_0 = 1$ bar



ENGR MODEL: (1) The water is a closed system. (2) The volume is constant. (3) In case (a); Q = 0, in case (b); W = 0. (4) Kinetic and potential energy effects are negligible. (5) For the environment, $T_0 = 300K$, $p_0 = 1bar$.

ANALYSIS: For both cases, the initial and final states are the same. Let us begin by obtaining data at each state. From Table A-2 at 300°C

$$v_1 = v_{f1} + x_1(v_{g1} - v_{f1}) = 1.4036\times10^{-3} + (.5)(0.02167 - 1.4036\times10^{-3}) = 0.01154\ m^3/kg$$

$$u_1 = u_{f1} + x_1(u_{g1} - u_{f1}) = 1332.0 + (.5)(2563.0 - 1332.0) = 1947.5\ kJ/kg$$

$$s_1 = s_{f1} + x_1(s_{g1} - s_{f1}) = 3.2534 + (.5)(5.7045 - 3.2534) = 4.479\ kJ/kg\cdot K$$

By assumptions (1) and (2), $v_2 = v_1 = v_g(T_2)$. From Table A-2, $T_2 = 336.8$ °C and

$$u_2 = 2474.2\ kJ/kg,\quad s_2 = 5.3673\ kJ/kg\cdot K$$

(a) In this case, the volume is constant. Thus

$$\Delta E = m[(u_2 - u_1) + p_0\cancelto{0}{(v_2 - v_1)} - T_0(s_2 - s_1)]$$

$$= (4kg)[(2474.2 - 1947.5) - (300)(5.3673 - 4.479)]\frac{kJ}{kg} = 1040.8\ kJ \longleftarrow \Delta E$$

$E_W = (W - p_0\cancelto{0}{\Delta V}) = W$. From an energy balance, $m(u_2 - u_1) = \cancelto{0}{Q} - W$. Thus

$$W = -m(u_2 - u_1) = -(4kg)(2474.2 - 1947.5)\frac{kJ}{kg} = -2106.8\ kJ \longleftarrow E_W$$

The exergy transfer accompanying heat transfer, denoted E_Q, is

$$E_Q = \int_1^2 \left(1 - \frac{T_0}{T_b}\right)\cancelto{0}{\delta Q} = 0 \longleftarrow E_Q$$

Now, from an exergy balance

$$\Delta E = \cancelto{0}{E_Q} - E_W - E_d$$

$$E_d = -E_W - \Delta E = -(-2106.8) - (1040.8)$$

$$= 1066\ kJ \longleftarrow E_d$$

(b) Since the change of state is the same, the change in exergy is the same as in part (a). That is

$$\Delta \mathsf{E} = 1040.8 \text{ kJ} \longleftarrow \Delta \mathsf{E}$$

Since the work is zero, $\mathsf{E}_w = 0 \longleftarrow \mathsf{E}_w$

The exergy transfer accompanying heat transfer is evaluated at the boundary temperature, $T_b = 610$K. Thus

$$\mathsf{E}_Q = \left(1 - \frac{T_0}{T_b}\right) Q$$

In this case, $m(u_2 - u_1) = Q - \cancel{W}^{0} \Rightarrow Q = 2106.8$ kJ and

$$\mathsf{E}_Q = \left(1 - \frac{300}{610}\right)(2106.8 \text{ kJ}) = 1070.7 \text{ kJ} \longleftarrow \mathsf{E}_Q$$

Now, from an exergy balance

$$\Delta \mathsf{E} = \mathsf{E}_Q - \cancel{\mathsf{E}_w}^{0} - \mathsf{E}_d$$

$$\mathsf{E}_d = \mathsf{E}_Q - \Delta \mathsf{E} = (1070.7 - 1040.8) = 29.9 \text{ kJ} \longleftarrow \mathsf{E}_d$$

Discussion

The exergy destruction for case (b) is significantly less than for case (a). Since the exergy change is the same in both cases, case (b) allows the exergy change to be brought about by less exergy transfer from the surroundings.

PROBLEM 7.48

7.48 As shown in Fig. P7.48, one-half pound of nitrogen (N_2), in a piston-cylinder assembly, initially at 80°F, 20 lbf/in.^{2}, is compressed isothermally to a final pressure of 100 lbf/in.^{2} During compression, the nitrogen rejects energy by heat transfer through the cylinder's end wall, which has inner and outer temperatures of 80 °F and 70°F, respectively.

(a) For the nitrogen as the system, evaluate the work, heat transfer, exergy transfers accompanying work and heat transfer, and amount of exergy destruction, each in Btu.
(b) Evaluate the amount of exergy destruction, in Btu, for an enlarged system that includes the nitrogen and the wall, assuming the state of the wall remains unchanged. Comment.

Use the ideal gas model for the nitrogen and let T_0 = 70°F, p_0 = 14.7 lbf/in.^{2}

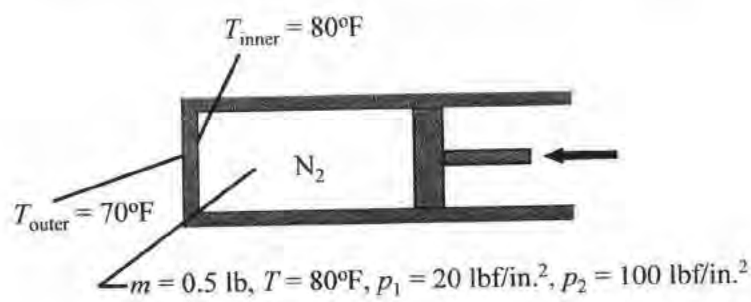


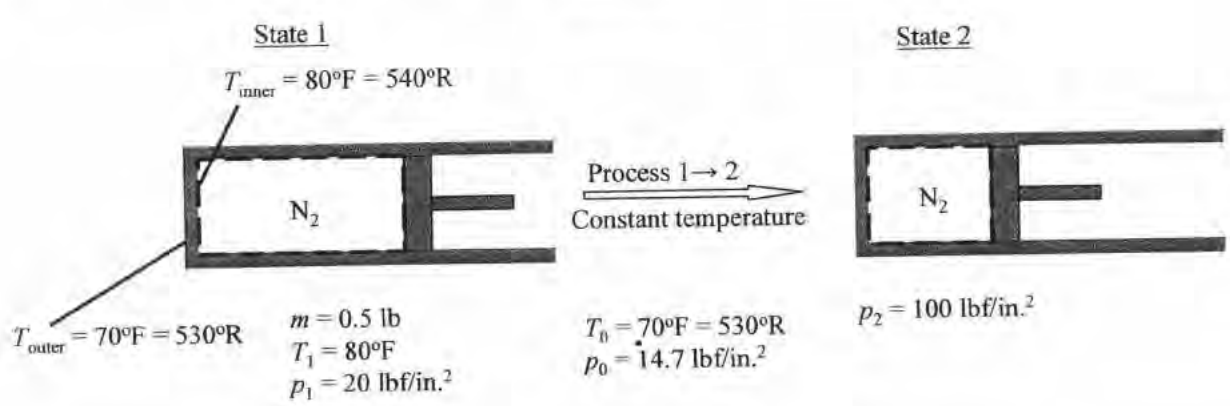
Fig. P7.48

KNOWN: System of nitrogen at specified temperature and pressure undergoes constant temperature process until reaching a specified pressure.

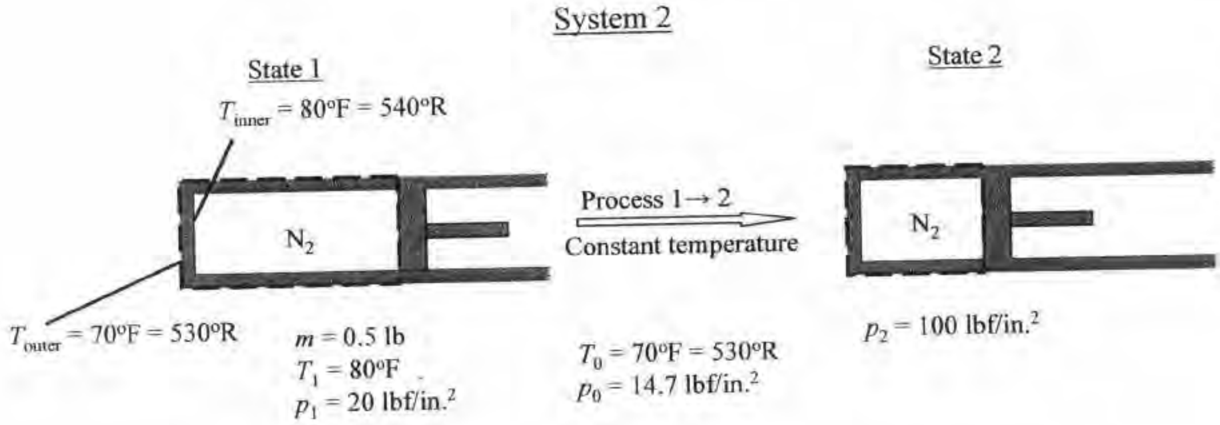
FIND: The work, the heat transfer, the transfer of exergy accompanying work and heat transfer, and the exergy destruction for the nitrogen as the system and the exergy destruction for an enlarged system consisting of the nitrogen and wall.

SCHEMATIC AND GIVEN DATA:

System 1



System 2



PROBLEM 7.48 (Continued, p.2)

ENGINEERING MODEL:
1. System 1 is the nitrogen, a closed system defined by the dashed line on the accompanying diagram.
2. System 2 is the nitrogen and cylinder wall, a closed system defined by the dashed line on the accompanying diagram.
3. Nitrogen can be modeled as an ideal gas.
4. Temperature of the nitrogen is constant.
5. The state of the wall does not change.
6. The effects of motion and gravity can be ignored.
7. $T_0 = 530^\circ$R and $p_0 = 14.7$ psia.

ANALYSIS:
(a) Work can be determined from

$$W_{12} = \int_{V_1}^{V_2} p\,dV$$

From the ideal gas model,

$$pV = mRT \quad \rightarrow \quad p = \frac{mRT}{V}$$

Substituting for pressure and integrating to solve for work gives

$$W_{12} = \int_{V_1}^{V_2} \frac{mRT}{V} dV = mRT \int_{V_1}^{V_2} \frac{dV}{V} = mRT \ln\left(\frac{V_2}{V_1}\right)$$

For an ideal gas undergoing an isothermal process,

$$mRT = p_1 V_1 = p_2 V_2 \rightarrow \frac{V_2}{V_1} = \frac{p_1}{p_2}$$

Thus, work can be determined by

$$W_{12} = mRT \ln\left(\frac{p_1}{p_2}\right) \tag{1}$$

Substituting values and solving for work yield

$$W_{12} = (0.5\text{ lb})\left(\frac{1.986 \dfrac{\text{Btu}}{\text{lbmol}\cdot{}^\circ\text{R}}}{28.01 \dfrac{\text{lb}}{\text{lbmol}}}\right)(540^\circ\text{R})\ln\left(\frac{20 \dfrac{\text{lbf}}{\text{in.}^2}}{100 \dfrac{\text{lbf}}{\text{in.}^2}}\right) = \underline{\mathbf{-30.81\ Btu}}$$

The negative sign indicates work is done on the system.
Heat transfer can be determined from the closed system energy balance.

$$\Delta\text{KE} + \Delta\text{PE} + \Delta U = Q - W$$

Neglecting changes in kinetic and potential energy and expressing internal energy in terms of mass and specific internal energy give

$$m(u_2 - u_1) = Q_{12} - W_{12}$$

For an ideal gas, internal energy is a function of only temperature. Since the process is isothermal, $u_2 - u_1 = 0$. Thus the heat transfer equals the work

$$Q_{12} = W_{12} = \underline{\mathbf{-30.81\ Btu}}$$

The negative sign indicates heat transfer from the system.

PROBLEM 7.48 (Continued, p. 3)

Exergy transfer accompanying work is determined from Eq. 7.6.

$$\mathsf{E}_w = [W - p_0(V_2 - V_1)]$$

Volume can be determined from the ideal gas equation of state

$$V_1 = \frac{mRT}{p_1} = \frac{(0.5\text{ lb})\left(\dfrac{1545\dfrac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot{}^\circ\text{R}}}{28.01\dfrac{\text{lb}}{\text{lbmol}}}\right)(540^\circ\text{R})}{20\dfrac{\text{lbf}}{\text{in.}^2}}\left|\frac{\text{ft}^2}{144\text{ in.}^2}\right| = 5.171\text{ ft}^3$$

$$V_2 = \frac{mRT}{p_2} = \frac{(0.5\text{ lb})\left(\dfrac{1545\dfrac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot{}^\circ\text{R}}}{28.01\dfrac{\text{lb}}{\text{lbmol}}}\right)(540^\circ\text{R})}{100\dfrac{\text{lbf}}{\text{in.}^2}}\left|\frac{\text{ft}^2}{144\text{ in.}^2}\right| = 1.034\text{ ft}^3$$

Substituting values and applying appropriate conversion factors yield

$$\mathsf{E}_w = (-30.81\text{ Btu}) - \left(14.7\frac{\text{lbf}}{\text{in.}^2}\right)(1.034\text{ ft}^3 - 5.171\text{ ft}^3)\left|\frac{144\text{ in.}^2}{\text{ft}^2}\right|\left|\frac{\text{Btu}}{778\text{ ft}\cdot\text{lbf}}\right| = \underline{\mathbf{-19.55\text{ Btu}}}$$

The negative sign indicates exergy transfer accompanying work is into the system.
Exergy transfer accompanying heat is determined from Eq. 7.5.

$$\mathsf{E}_q = \int_1^2\left(1 - \frac{T_0}{T_b}\right)\delta Q = \left(1 - \frac{T_0}{T_b}\right)Q_{12} = \left(1 - \frac{530^\circ\text{R}}{540^\circ\text{R}}\right)(-30.81\text{ Btu}) = \underline{\mathbf{-0.57\text{ Btu}}}$$

The negative sign indicates exergy transfer accompanying heat is from the system.
Exergy destruction is determined from Eq. 7.7

$$\mathsf{E}_d = T_0\sigma$$

Entropy production, σ, can be determined from the closed system entropy balance

$$m(s_2 - s_1) = \int_1^2\left(\frac{\delta Q}{T}\right)_b + \sigma$$

Solving for entropy production gives

$$\sigma = m(s_2 - s_1) - \int_1^2\left(\frac{\delta Q}{T}\right)_b = m(s_2 - s_1) - \frac{Q_{12}}{T_b}$$

Since nitrogen is modeled as an ideal gas, entropy change for an ideal gas is substituted for $s_2 - s_1$. Noting that $Q_{12} = W_{12}$, use Eq. (1) to express entropy production in terms of T_1, T_2, and pressure ratios

$$\sigma = m(s^o(T_2) - s^o(T_1) - R\ln\frac{p_2}{p_1}) - \frac{1}{T_b}\left(mRT\ln\left(\frac{p_1}{p_2}\right)\right)$$

Since the process is isothermal $s^o(T_2) = s^o(T_1)$ so these terms cancel each other. Since $T_b = T$, it's evident that σ vanishes.

PROBLEM 7.48 (Continued, p.4)

$$\sigma = m(-R\ln\frac{p_2}{p_1}) - mR\ln\left(\frac{p_1}{p_2}\right) = -mR\ln\frac{p_2}{p_1} - \left(-mR\ln\left(\frac{p_2}{p_1}\right)\right) = 0$$

The process has no entropy production and thus is internally reversible. Consequently,

$$\mathsf{E}_d = T_0\sigma = \underline{\mathbf{0\ Btu}}$$

The process has no exergy destruction.

(b) For an enlarged system that includes the nitrogen and the wall, the exergy destruction is determined from Eq. 7.7

$$\mathsf{E}_d = T_0\sigma$$

Entropy production, σ, can be determined from the closed system entropy balance applied to the nitrogen and the wall

$$\Delta S_{N2} + \Delta S_{wall} = \int_1^2\left(\frac{\delta Q}{T}\right)_b + \sigma$$

Since the state of the wall does not change, $\Delta S_{wall} = 0$. The entropy change of the nitrogen is $m(s_2 - s_1)$. Substituting and solving for entropy production give

$$\sigma = m(s_2 - s_1) - \int_1^2\left(\frac{\delta Q}{T}\right)_b = m(s_2 - s_1) - \frac{Q_{12}}{T_b}$$

Since nitrogen is modeled as an ideal gas, entropy change for an ideal gas is substituted for $s_2 - s_1$.

$$\sigma = m(s^o(T_2) - s^o(T_1) - R\ln\frac{p_2}{p_1}) - \frac{Q_{12}}{T_b}$$

Since the process is isothermal $s^o(T_2) = s^o(T_1)$ so these terms cancel each other.

$$\sigma = -mR\ln\frac{p_2}{p_1} - \frac{Q_{12}}{T_b}$$

The boundary temperature for the enlarged system is the reservoir temperature, 530°R. Substituting values and solving for entropy production give

$$\sigma = -(0.5\text{ lb})\left(\frac{1.986\dfrac{\text{Btu}}{\text{lbmol}\cdot{}^\circ\text{R}}}{28.01\dfrac{\text{lb}}{\text{lbmol}}}\right)\ln\left(\frac{100\dfrac{\text{lbf}}{\text{in.}^2}}{20\dfrac{\text{lbf}}{\text{in.}^2}}\right) - \frac{-30.81\text{ Btu}}{530^\circ\text{R}} = 1.07\times10^{-3}\text{ Btu/}^\circ\text{R}$$

Substituting values for exergy destruction gives

$$\mathsf{E}_d = (530^\circ\text{R})(1.07\times10^{-3}\text{ Btu/}^\circ\text{R}) = \underline{\mathbf{0.57\ Btu}}$$

Exergy is destroyed within the enlarged system due to heat transfer through a finite temperature difference through the wall. For the enlarged system, this is the only source of exergy destruction.

PROBLEM 7.49

7.49 Air initially at 1 atm and 500°R with a mass of 2.5 lb is contained within a closed, rigid tank. The air is slowly warmed, receiving 100 Btu by heat transfer through a wall separating the gas from a thermal reservoir at 800°R. This is the only energy transfer. Assuming the air undergoes an internally reversible process and using the ideal gas model,

(a) determine the change in exergy and the exergy transfer accompanying heat, each in Btu, for the air as the system.

(b) determine the exergy transfer accompanying heat and the exergy destruction, each in Btu, for an enlarged system that includes the air and the wall, assuming that the state of the wall remains unchanged. Compare with part (a) and comment.

Let T_0= 90°F, p_0=1 atm.

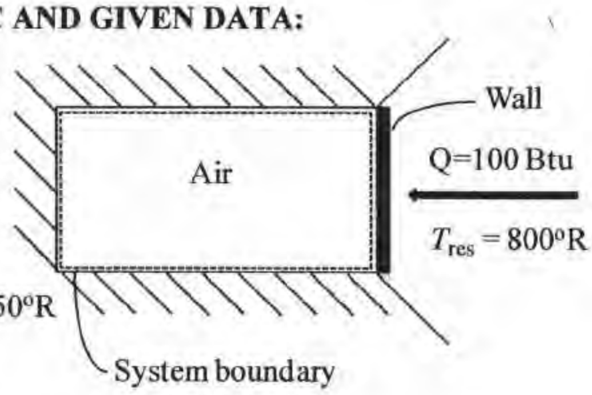
KNOWN: Air at a known initial state is contained in a closed, rigid tank. The air receives a specified energy transfer by heat transfer through a wall separating the air from a thermal reservoir at 800°R.

FIND: (a) For the air as the system, determine the exergy change and the exergy transfer accompanying heat, each in Btu. (b) Determine the exergy transfer accompanying heat and the exergy destruction for an enlarged system including the wall. Compare and discuss.

SCHEMATIC AND GIVEN DATA:

$T_1 = 500$°R
$p_1 = 1$ atm
$m = 2.5$ lb

$T_0 = 90$°F$=550$°R
$p_0 = 1$ atm



ENGINEERING MODEL:

(1) Closed systems are under consideration where the effects of motion and gravity can be ignored.
(2) The volume is constant and there is no work.
(3) The air behaves as an ideal gas.
(4) The state of the wall remains unchanged.
(5) The air undergoes an internally reversible process.
(6) Heat transfer occurs across one of the tank's sides as shown in the accompanying schematic.

ANALYSIS:

(a) For the air as the system, fix state 2 using the energy balance simplified based on assumptions.

$$Q_{12} = m(u_2 - u_1) \quad \text{or} \quad u_2 = \frac{Q_{12}}{m} + u_1$$

Inserting values from Table A-22E:

$$u_2 = \frac{Q_{12}}{m} + u_1 = \frac{100\text{ Btu}}{2.5\text{lb}} + 85.20\frac{\text{Btu}}{\text{lb}} = 125.20\frac{\text{Btu}}{\text{lb}}$$

Interpolating from Table A-22E; $T_2 = 732.45$ °R, $s_2^\circ = 0.67415$ Btu/lb·°R.

PROBLEM 7.49 (Continued)

The exergy change of the air is determined using a form of Eq. 7.2 simplified based on assumptions.

$$\Delta \mathbf{E}_{\text{air}} = m[u_2 - u_1 + p_0(v_2 - v_1) - T_0(s_2 - s_1)]$$

Recognizing $v_2 = v_1$ and using Eq. 6.20a for entropy change of an ideal gas:

$$\Delta \mathbf{E}_{\text{air}} = m\left[u_2 - u_1 - T_0\left(s_2^{\,o} - s_1^{\,o} - \frac{\overline{R}}{M}\ln\frac{p_2}{p_1}\right)\right]$$

Using data from Table A-22E and the relationship $p_2/p_1 = T_2/T_1$, obtained using the ideal gas equation of state with $v_2 = v_1$:

$$\Delta \mathbf{E}_{\text{air}} = 2.5\,\text{lb}\left[(125.20 - 85.20)\frac{\text{Btu}}{\text{lb}} - 550^{\circ}\text{R}\left(0.67415 - 0.58233 - \frac{1.986}{28.97}\ln\frac{732.45}{500}\right)\frac{\text{Btu}}{\text{lb}\cdot{}^{\circ}\text{R}}\right]$$

$$= 9.74\,\text{Btu}$$

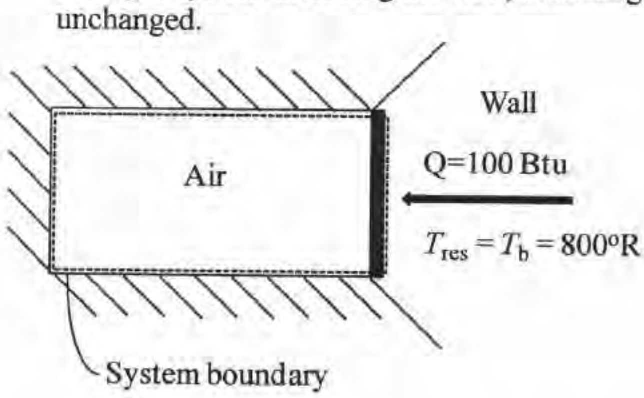
Denoting the exergy transfer accompanying heat transfer as $\mathbf{E}_{\text{Q,air}}$, the exergy balance using a form of Eq. 7.4a gives

$$\Delta \mathbf{E}_{\text{air}} = \mathbf{E}_{\text{Q,air}} - \cancel{(W - p_0\Delta V)}^{\,0} - \cancel{\mathbf{E}_{\text{d,air}}}^{\,0} \quad \Rightarrow \Delta \mathbf{E}_{\text{air}} = \mathbf{E}_{\text{Q,air}}$$

Then,

$$\mathbf{E}_{\text{Q,air}} = \Delta \mathbf{E}_{\text{air}} = 9.74\,\text{Btu}$$

(b) Apply the exergy balance to the enlarged system including the air and the wall to determine the exergy transfer accompanying heat and the exergy destruction for an enlarged system including the wall, assuming that the state of the wall remains unchanged.



Applying Eq. 7.4a again

$$\Delta \mathbf{E}_{\text{air}} + \Delta \mathbf{E}_{\text{wall}} = \left(1 - \frac{T_0}{T_{\text{b}}}\right)Q - (W - p_0\Delta V) - \mathbf{E}_{\text{d,air\&wall}}$$

The exergy transfer accompanying heat transfer, $\mathbf{E}_{\text{Q,air\&wall}}$, follows from Eq. 7.5.

$$\mathbf{E}_{\text{Q,air\&wall}} = \left(1 - \frac{T_0}{T_{\text{b}}}\right)Q = \left(1 - \frac{550}{800}\right)(100\,\text{Btu}) = 31.25\,\text{Btu}$$

Substitute into the exergy balance and simplify based on assumptions.

$$\mathbf{E}_{\text{d,air\&wall}} = \mathbf{E}_{\text{Q,air\&wall}} - \Delta \mathbf{E}_{\text{air}} = 31.25 - 9.74 = 21.51\,\text{Btu}$$

From the analysis of the enlarged system, the exergy transfer accompanying heat transfer to the wall is 31.25 Btu. This occurs at the reservoir temperature of 800°R. The exergy transfer from the wall to the air, $\mathbf{E}_{\text{Q,air}}$, occurs at much lower temperatures, and amounts to only 9.74 Btu. The difference is accounted for by the exergy destruction associated with the temperature gradient across the wall.

PROBLEM 7.50

7.50 Determine the specific flow exergy, in Btu/lbmol and Btu/lb, at 440°F, 73.5 lbf/in.^{2} for (a) nitrogen (N_2) and (b) carbon dioxide (CO_2), each modeled as an ideal gas, and relative to an exergy reference environment for which T_0 = 77°F, p_0 = 14.7 lbf/in.^{2} Ignore the effects of motion and gravity.

ANALYSIS: We apply Eq. 7.14 on a molar basis, dropping the $V^2/2$ and gz terms:

$$\bar{e}_f = \bar{h} - \bar{h}_0 - T_0(\bar{s} - \bar{s}_0)$$

$$= \bar{h} - \bar{h}_0 - T_0\left(\bar{s}^\circ - \bar{s}_0^\circ - \bar{R}\ln\frac{p}{p_0}\right) \quad (1)$$

where $T = 900°R$, $T_0 = 537°R$, $p_0 = 14.7 \frac{lbf}{in^2}$

(a) N_2. Using Eq.(1) and data from Table A-23E,

$$\bar{e}_f = (6268.1 - 3729.5)\frac{Btu}{lbmol} - 537°R\left(49.352 - 45.743 - 1.986 \ln \overset{p/p_0}{5}\right)\frac{Btu}{lbmol \cdot °R}$$

$$= 2317 \frac{Btu}{lbmol} \longleftarrow$$

Converting to a mass basis

$$e_f = \frac{\bar{e}_f}{M} = \frac{2317\ Btu/lbmol}{28.01\ lb/lbmol} = 82.7 \frac{Btu}{lb} \longleftarrow$$

(b) CO_2. Using Eq.(1) and data from Table A-23E,

$$\bar{e}_f = (7597.6 - 4027.5)\frac{Btu}{lbmol} - 537°R\left(56.070 - 51.032 - 1.986 \ln(5)\right)\frac{Btu}{lbmol \cdot °R}$$

$$= 2581 \frac{Btu}{lbmol} \longleftarrow$$

Converting to a mass basis

$$e_f = \frac{\bar{e}_f}{M} = \frac{2581\ Btu/lbmol}{44.01\ lb/lbmol} = 58.6 \frac{Btu}{lb} \longleftarrow$$

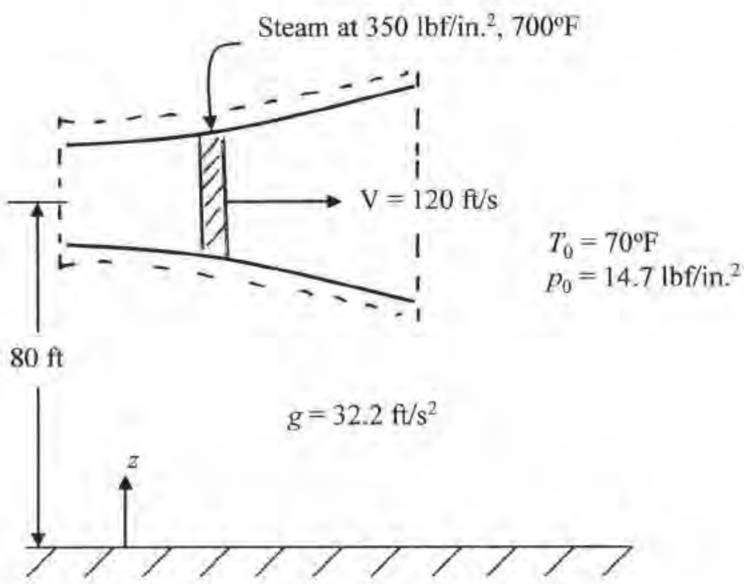
PROBLEM 7.51

7.51 Determine the specific exergy and the specific flow exergy, each in Btu/lb, for steam at 350 lbf/in.^{2}, 700°F, with V = 120 ft/s and z = 80 ft. The velocity and elevation are relative to an exergy reference environment for which T_0 = 70°F, p_0 = 14.7 lbf/in.^{2}, and g = 32.2 ft/s^2.

KNOWN: Steam at specified pressure, temperature, velocity, and elevation exists in a reference environment with specified temperature and pressure.

FIND: Specific exergy and the specific flow exergy of the system.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume defined by the dashed line on the accompanying diagram is used to evaluate the specific flow exergy of the steam.
2. T_0 = 530°R, p_0 = 14.7 lbf/in.^{2}, $V_0 = 0$, $z_0 = 0$, and g = 32.2 ft/s^2.

ANALYSIS:
The steam is superheated vapor. From Table A-4E, u = 1242.5 Btu/lb, v = 1.898 ft^3/lb, s = 1.6562 Btu/(lb·°R), h = 1365.4 Btu/lb.

Water at the reference state is compressed liquid. From Table A-2E at T_0 = 70°F, $u_0 \approx u_{f0}$ = 38.09 Btu/lb, $v_0 \approx v_{f0}$ = 0.01605 ft^3/lb, $s_0 \approx s_{f0}$ = 0.07463 Btu/(lb·°R), $h_0 \approx h_{f0}$ = 38.09 Btu/lb.

The specific exergy of the steam can be determined from Eq. 7.2

$$\mathsf{e} = (u - u_0) + p_0(v - v_0) - T_0(s - s_0) + V^2/2 + gz \qquad (1)$$

Substituting values and applying appropriate conversion factors give

$$\text{e} = (1242.5 - 38.09)\frac{\text{Btu}}{\text{lb}} + \left(14.7\frac{\text{lbf}}{\text{in.}^2}\right)(1.898 - 0.01605)\frac{\text{ft}^3}{\text{lb}}\left|\frac{144\text{ in.}^2}{\text{ft}^2}\right|\left|\frac{\text{Btu}}{778\text{ ft}\cdot\text{lbf}}\right|$$

$$- (530°\text{R})(1.6562 - 0.07463)\frac{\text{Btu}}{\text{lb}\cdot°\text{R}} + \frac{\left(120\frac{\text{ft}}{\text{s}}\right)^2}{2}\left|\frac{\text{lbf}}{32.2\frac{\text{lb}\cdot\text{ft}}{\text{s}^2}}\right|\left|\frac{\text{Btu}}{778\text{ ft}\cdot\text{lbf}}\right|$$

$$+ \left(32.2\frac{\text{ft}}{\text{s}^2}\right)(80\text{ ft})\left|\frac{\text{lbf}}{32.2\frac{\text{lb}\cdot\text{ft}}{\text{s}^2}}\right|\left|\frac{\text{Btu}}{778\text{ ft}\cdot\text{lbf}}\right|$$

e = **371.7 Btu/lb**

The specific flow exergy of the system can be determined from Eq. 7.14

$$\text{e}_f = (h - h_0) - T_0(s - s_0) + V^2/2 + gz \qquad (2)$$

Substituting values and applying appropriate conversion factors give

$$\text{e}_f = (1365.4 - 38.09)\frac{\text{Btu}}{\text{lb}} - (530°\text{R})(1.6562 - 0.07463)\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$$

$$+ \frac{\left(120\frac{\text{ft}}{\text{s}}\right)^2}{2}\left|\frac{\text{lbf}}{32.2\frac{\text{lb}\cdot\text{ft}}{\text{s}^2}}\right|\left|\frac{\text{Btu}}{778\text{ ft}\cdot\text{lbf}}\right| + \left(32.2\frac{\text{ft}}{\text{s}^2}\right)(80\text{ ft})\left|\frac{\text{lbf}}{32.2\frac{\text{lb}\cdot\text{ft}}{\text{s}^2}}\right|\left|\frac{\text{Btu}}{778\text{ ft}\cdot\text{lbf}}\right|$$

e_f = **489.5 Btu/lb**

Subtracting Eq. (1) from Eq. (2) gives $\text{e}_f = \text{e} + v(p\text{-}p_0)$. Using this, the e_f value can be determined from the e value, and conversely – as can be verified.

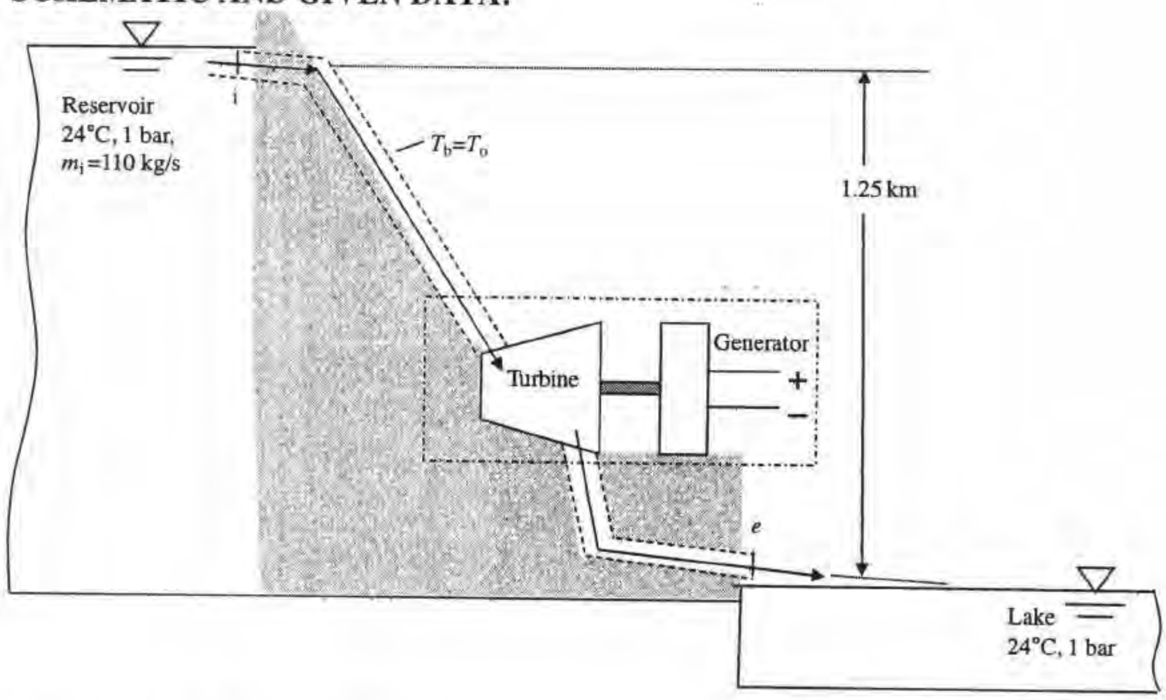
PROBLEM 7.52

7.52 Water at 24°C, 1 bar is drawn from a reservoir 1.25 km above a valley and allowed to flow through a hydraulic-turbine-generator into a lake on the valley floor. For operation at steady state, determine the maximum theoretical rate electricity is generated, in MW, for a flow rate of 110 kg/s. Let T_0= 24°C, p_0=1 bar and ignore the effects of motion.

KNOWN: Water drawn from a reservoir flows through a hydraulic-turbine-generator to a lake located 1.25 km below the reservoir.

FIND: Determine the maximum theoretical rate electricity is generated, in MW, for steady-state operation and a flow rate of 110 kg/s.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

(1) The control volume shown in the accompanying schematic is at steady state. Heat transfer (if any) occurs only at T_b=T_o.

(2) At the exit, e, the state corresponds to the dead state: $T_e = T_0$, $p_e = p_0$, $s_e = s_0$, $z_e = z_0$, h_e=h_0, and V_e= 0.

(3) The kinetic energy at the inlet can be ignored.

(4) For the environment, T_0 = 24°C, p_0 = 1 bar, and g = 9.8 m/s^2.

ANALYSIS:

Applying the steady-state control volume exergy rate balance, Eq. 7.17:

$$0 = \sum_j \left[1 - \frac{T_o}{T_j}\right]\dot{Q}_j - \dot{W}_{cv} + \dot{m}(e_{fi} - e_{fe}) - \dot{E}_d$$

where the first term vanishes using assumption (1), and with assumption (2), $e_{fe} = 0$. Simplifying and rearranging:

$$\dot{W}_{cv} = \dot{m}(e_{fi}) - \dot{E}_d$$

By inspection for fixed inlet state and mass flow rate, the maximum theoretical rate of electricity generated corresponds to $\dot{E}_d = 0$. Simplifying and incorporating Eq. 7.14 for specific flow exergy:

$$\dot{W}_{cv,max} = \dot{m}(e_{fi}) = \dot{m}\left[(h_i - h_0) - T_0(s_i - s_0) + \frac{V_i^2}{2} + gz_i\right] \qquad (1)$$

Since $p_i = p_0$, $T_i = T_0$, and $V_i^2/2$ can be ignored, Eq. (1) reduces to

$$\dot{W}_{cv,max} = \dot{m}[gz_i] = \left(110\frac{kg}{s}\right)\left(9.8\frac{m}{s^2}\right)(1250\ m)\left|\frac{1\ N}{1\frac{kg \cdot m}{s^2}}\right|\left|\frac{1\ J}{1\ N \cdot m}\right|\left|\frac{1\ MW}{10^6\frac{J}{s}}\right| = 1.35\ MW \longleftarrow$$

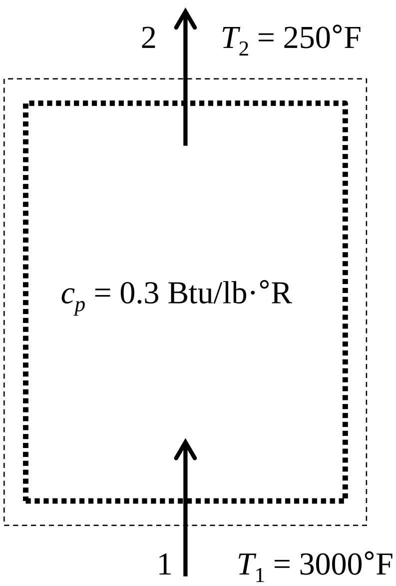
Problem 7.53

At steady state, hot gaseous products of combustion from a gas turbine cool from 3000°F to 250°F as they flow through a stack. Owing to negligible fluid friction, the flow occurs at nearly constant pressure. Applying the ideal gas model with c_p = 0.3 Btu/lb·°R, determine the exergy transfer from the gas, in Btu per lb of gas flowing. Let T_0 = 80°F and ignore the effects of motion and gravity.

Known:
A stack is cooling gas from a gas turbine, with provided conditions.

Find:
Determine the exergy transfer from the gas.

Schematic and Known Data:



Engineering Model:
(1) The control volume shown in the schematic is at steady state.
(2) The flow is internally reversible. Fluid friction is negligible and the gas flows at constant pressure.
(3) The gas is modeled as an ideal gas with c_p = 0.3 Btu/lb·°R.
(4) Ignore the effects of motion and gravity.
(5) For the environment, T_0 = 540°R (80°F).

Analysis:
Applying the steady state control volume exergy rate balance, Eq. 7.13:

$$0 = \dot{\mathrm{E}}_\mathrm{q} - \underbrace{\dot{W}_\mathrm{cv}}_{=0} + \dot{m}\left[\mathrm{e}_{\mathrm{f}_1} - \mathrm{e}_{\mathrm{f}_2}\right] - \underbrace{\dot{\mathrm{E}}_\mathrm{d}}_{=0}$$

#1 $$\frac{\dot{\mathrm{E}}_{\mathrm{q}}}{\dot{m}} = \mathrm{e}_{\mathrm{f}_2} - \mathrm{e}_{\mathrm{f}_1} = (h_2 - h_1) - T_0(s_2 - s_1) = c_p(T_2 - T_1) - T_0\left[c_p \ln\frac{T_2}{T_1} - \underbrace{R\ln\frac{p_2}{p_1}}_{=0}\right]$$

$$\frac{\dot{\mathrm{E}}_{\mathrm{q}}}{\dot{m}} = c_p\left[T_2 - T_1 - T_0\ln\frac{T_2}{T_1}\right] = \left(0.3\frac{\mathrm{Btu}}{\mathrm{lb}\cdot{}^\circ\mathrm{R}}\right)\left[710 - 3460 - (540)\ln\frac{710}{3460}\right]{}^\circ\mathrm{R}$$

$$\frac{\dot{\mathrm{E}}_{\mathrm{q}}}{\dot{m}} = -568.43\,\frac{\mathrm{Btu}}{\mathrm{lb}}$$

$\longleftarrow$

Comment:

1. Alternatively:

$$\frac{\dot{\mathrm{E}}_{\mathrm{q}}}{\dot{m}} = \int_{T_1}^{T_2}\left[1 - \frac{T_0}{T}\right]\delta q_{\text{internal rev}}$$

From Eq. 6.49:

$$\delta q_{\text{internal rev}} = T\,ds$$

From Eq. 6.10b and using assumption 2:

$$T\,ds = dh - v\underbrace{dp}_{=0,}$$

with ideal gas assumption $\quad \delta q_{\text{internal rev}} = dh = c_p\,dT$

Therefore:

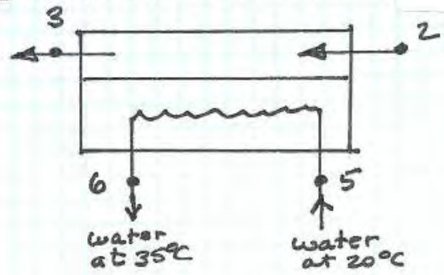
$$\frac{\dot{\mathrm{E}}_{\mathrm{q}}}{\dot{m}} = \int_{T_1}^{T_2}\left[1 - \frac{T_0}{T}\right]c_p\,dT = c_p\left[(T_2 - T_1) - T_0\ln\frac{T_2}{T_1}\right]$$

PROBLEM 7.54

7.54 For the simple vapor power plant of Problem 6.163 evaluate, in MW, (a) the *net* rate energy exits the plant with the cooling water and (b) the *net* rate exergy exits the plant with the cooling water. Comment. Let $T_0 = 20°C, p_0 = 1$ atm and ignore the effects of motion and gravity.

SCHEMATIC & GIVEN DATA:

States numbered as in Problem 6.163.



$\dot{m}_{cw} = 3758.7$ kg/s (from Problem 6.165)

For the power plant, the net power output $= 117$ MW (Problem 6.163)

ENGR. MODEL:

1. See model of Problem 6.163.
2. For the environment, $T_0 = 20°C$, $P_0 = 1$ atm.

ANALYSIS:

(a) The net rate energy exits with the cooling water

$$= \dot{m}(h_6 - h_5)$$

$$= 3758.7\frac{kg}{s}(146.68 - 83.96)\frac{kJ}{kg}\left|\frac{1\,MW}{10^3\,kJ/s}\right| = 235.7\ MW$$

$h \approx h(T)$

(b) The net rate exergy exits with the cooling water

$$= \dot{m}\left[(h_6 - h_5) - T_0(s_6 - s_5)\right]$$

$$= 3758.7\frac{kg}{s}\left[(146.68 - 83.96) - 293(0.5053 - 0.2966)\right]\frac{kJ}{kg}\left|\frac{1\,MW}{10^3\,kJ/s}\right|$$

$s \approx s_f(T)$

$$= 5.9\ MW$$

COMMENT: Part (a) shows that considerable energy is carried out with the cooling water. However, since the exiting water temperature is only 35°C, it has little thermodynamic utility. Part (b) confirms this conclusion by quantifying the net exergy carried out. In keeping with expectations, the net exergy carried out with the cooling water is much less than the net power developed by the plant.

PROBLEM 7.55

7.55 Water vapor enters a valve with a mass flow rate of 2 kg/s at a temperature of 320°C and a pressure of 60 bar and undergoes a throttling process to 40 bar.

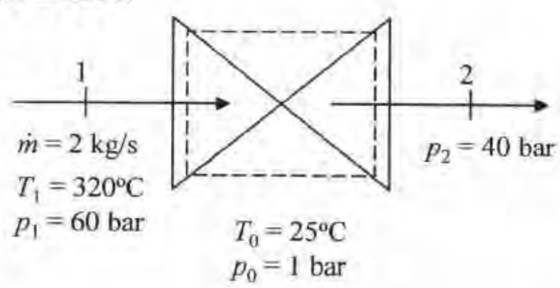
(a) Determine the flow exergy rates at the valve inlet and exit and the rate of exergy destruction, each in kW.
(b) Evaluating exergy at 8.5 cents per kW·h, determine the annual cost, in $/year, associated with the exergy destruction, assuming 8400 hours of operation annually.

Let $T_0 = 25°C$, $p_0 = 1$ bar.

KNOWN: Water vapor at specified temperature and pressure undergoes a throttling process to a specified pressure.

FIND: Inlet and exit flow exergy rates, the rate of exergy destruction, and the annual cost associated with exergy destruction.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume defined by the dashed line on the accompanying diagram is at steady state.
2. For the throttling process, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$, and the effects of motion and gravity can be ignored.
3. Exergy is evaluated at 8.5 cents per kW·h.
4. $T_0 = 25°C$, $p_0 = 1$ bar.

ANALYSIS:
(a) At the inlet the water is superheated vapor. From Table A-4, $h_1 = 2952.6$ kJ/kg and $s_1 = 6.1846$ kJ/(kg·K).

Water at the reference state is compressed liquid. From Table A-2 at $T_0 = 25°C$, $h_0 \approx h_{f0} = 104.89$ kJ/kg and $s_0 \approx s_{f0} = 0.3674$ kJ/(kg·K).

Pressure is known at the exit so one additional property is required to fix State 2. The exit enthalpy can be determined from the steady-state, one-inlet, one-exit energy balance

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\,[(h_1 - h_2) + \tfrac{1}{2}(V_1^2 - V_2^2) + g(z_1 - z_2)]$$

Neglecting heat transfer rate, power, and kinetic and potential energy effects, the energy balance simplifies to

PROBLEM 7.55 (Continued)

$$0 = h_1 - h_2$$

Solving for the exit enthalpy gives

$$h_2 = h_1 = 2952.6 \text{ kJ/kg}$$

State 2 is superheated vapor. From Table A-4 (interpolated), $s_2 = 6.3456$ kJ/(kg·K).

The flow exergy rate at the valve inlet (neglecting kinetic and potential energy effects) is determined from

$$\dot{\mathsf{E}}_{fi} = \dot{m}\,[h_1 - h_0 - T_0(s_1 - s_0)]$$

Substituting values gives

$$\dot{\mathsf{E}}_{fi} = \left(2\frac{\text{kg}}{\text{s}}\right)\left[2952.6\frac{\text{kJ}}{\text{kg}} - 104.89\frac{\text{kJ}}{\text{kg}} - (298\text{ K})\left(6.1846\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 0.3674\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\right]\left|\frac{\text{kW}}{\frac{\text{kJ}}{\text{s}}}\right|$$

$$\dot{\mathsf{E}}_{fi} = \underline{\mathbf{2228.4\ kW}}$$

The flow exergy rate at the valve exit (neglecting kinetic and potential energy effects) is determined from

$$\dot{\mathsf{E}}_{fe} = \dot{m}\,[h_2 - h_0 - T_0(s_2 - s_0)]$$

Substituting values gives

$$\dot{\mathsf{E}}_{fe} = \left(2\frac{\text{kg}}{\text{s}}\right)\left[2952.6\frac{\text{kJ}}{\text{kg}} - 104.89\frac{\text{kJ}}{\text{kg}} - (298\text{ K})\left(6.3456\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 0.3674\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\right]\left|\frac{\text{kW}}{\frac{\text{kJ}}{\text{s}}}\right|$$

$$\dot{\mathsf{E}}_{fe} = \underline{\mathbf{2132.4\ kW}}$$

The rate of exergy destruction is determined from the steady-state control volume exergy rate balance for one inlet and one exit. Since there is no heat transfer and no work, the exergy rate balance Eq. 7.13b reduces to

$$0 = \dot{\mathsf{E}}_{fi} - \dot{\mathsf{E}}_{fe} - \dot{\mathsf{E}}_{d}$$

Solving for rate of exergy destruction gives

1 $$\dot{\mathsf{E}}_{d} = \dot{\mathsf{E}}_{fi} - \dot{\mathsf{E}}_{fe} = 2228.4\text{ kW} - 2132.4\text{ kW} = \underline{\mathbf{96.0\ kW}}$$

The annual economic cost of exergy destruction is

$$Cost = (96\text{ kW})\left(8.5\frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)\left(8400\frac{\text{h}}{\text{year}}\right)\left(\frac{\$}{100\text{ cents}}\right) = \underline{\mathbf{\$68{,}544/year}}$$

---

1 Alternatively, the rate of exergy destruction can be evaluated from $\dot{\mathsf{E}}_d = T_0\dot{\sigma}_{cv}$, which reduces in the present case to $\dot{\mathsf{E}}_d = T_0\dot{m}\,(s_2 - s_1)$. This calculation is left as an exercise.

Problem 7.56

R-134A at 100 lbf/in.^{2}, 200°F enters a valve operating at steady state and undergoes a throttling process. (a) Determine the exit temperature, in °F, and the exergy destruction rate, in Btu per lb of R-134A flowing, for an exit pressure of 50 lbf/in.^{2} (b) Plot the exit temperature, in °F, and the exergy destruction rate, in Btu per lb of R-134A flowing, each versus exit pressure ranging from 50 to 100 lbf/in.^{2} Let T_0 = 70°F, p_0 = 14.7 lbf/in.^{2}

Known:

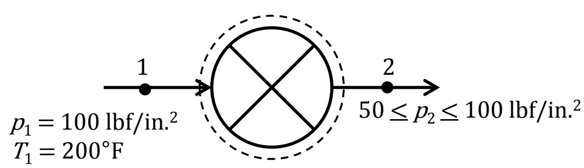
R-134A at a known state enters a valve operating at steady state and undergoes a throttling process to pressure p_2.

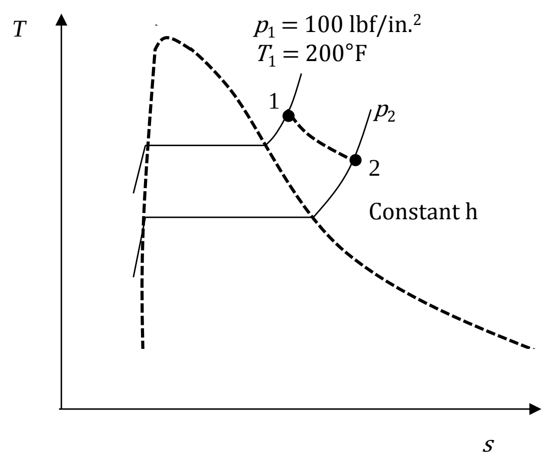
Find:

(a) For p_2 = 50 lbf/in.^{2}, determine the exit temperature and the exergy destruction per unit of steam flowing.

(b) Plot these quantities versus p_2 ranging from 50 to 100 lbf/in.^{2}

Schematic and Known Data:





Engineering Model:

(1) The control volume shown in the schematic is at steady state.

(2) The R-134A undergoes a throttling process in passing through the valve.

(3) For the environment, T_0 = 70°F, p_0 = 14.7 lbf/in.^{2}

Analysis:

State 1 is fixed by T_1 = 200°F, p_1 = 100 lbf/in.^{2} To fix state 2, we use assumption (2) and $h_1 = h_2$. Thus, with p_2 and h_2, state 2 is fixed as well and T_2 can be determined.

From Table A-12E, h_1 = 142.45 Btu/lb and s_1 = 0.2671 Btu/lb·°R.
At p_2 = 50 lbf/in.^{2} and $h_2 = h_1$ with interpolation:

$$T_2 = 193.01°\text{F}$$

$$s_2 = 0.27995 \frac{\text{Btu}}{\text{lb} \cdot °\text{R}}$$

To obtain $\frac{\dot{\mathrm{E}}_\mathrm{d}}{\dot{m}}$, we begin with an exergy balance at steady state:

$$0 = \sum_j \left(1 - \frac{T_0}{T_j}\right)\underbrace{\dot{Q}_j}_{=0} - \underbrace{\dot{W}}_{=0} + \dot{m}\left[\mathrm{e}_{\mathrm{f}_1} - \mathrm{e}_{\mathrm{f}_2}\right] - \dot{\mathrm{E}}_\mathrm{d} \quad \text{or}$$

$$\frac{\dot{\mathrm{E}}_\mathrm{d}}{\dot{m}} = \mathrm{e}_{\mathrm{f}_1} - \mathrm{e}_{\mathrm{f}_2} = \underbrace{(h_1 - h_2)}_{=0} - T_0(s_1 - s_2) = T_0(s_2 - s_1) \qquad (1)$$

Now, from Eq. (1) above:

$$\frac{\dot{\mathrm{E}}_\mathrm{d}}{\dot{m}} = T_0(s_2 - s_1) = (530°\mathrm{R})(0.27995 - 0.2671)\frac{\mathrm{Btu}}{\mathrm{lb}\cdot°\mathrm{R}} = 6.81\,\frac{\mathrm{Btu}}{\mathrm{lb}}$$

(a) The data to construct the required plots are obtained using IT, as follows:

IT Code

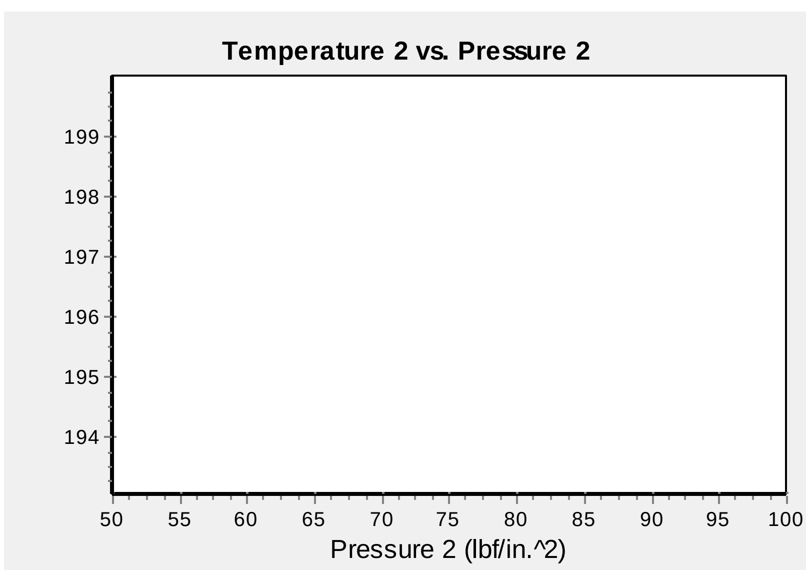
```
p1 = 100 // lbf/in.^2
T1 = 200 // F
p2 = 50 // lbf/in.^2
T0 = 530 // F

h1 = h_PT("R134A", p1, T1)
h2 = h_PT("R134A", p2, T2)
h1 = h2
s1 = s_PT("R134A", p1, T1)
s2 = s_PT("R134A", p2, T2)

Ed = T0 * (s2 - s1)
```

IT Solution for p_2 = 50 lbf/in.^{2}

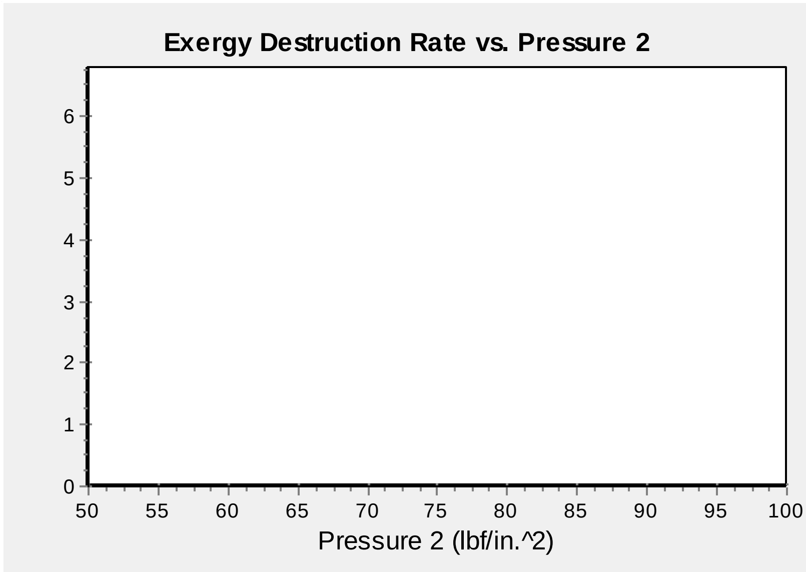
h1	s1	s2	T2	Ed
142.4	0.2671	0.28	193	6.796

Plots:

#1



#2



Comments:

1. As seen on the accompanying *T*-*s* diagram, the exit temperature decreases as p_2 decreases for fixed *h*. Thus, the plot of exit temperature exhibits the expected behavior.
2. The rate of exergy destruction per unit mass of R-134A flowing increases with decreasing p_2. This is also expected based on greater entropy production and hence increasing s_2 (see *T*-*s* diagram).

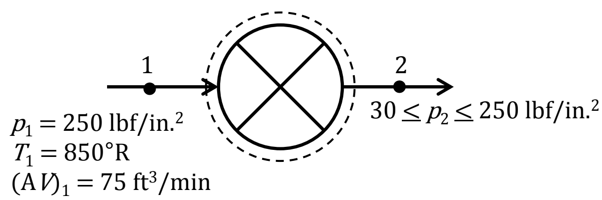
Problem 7.57

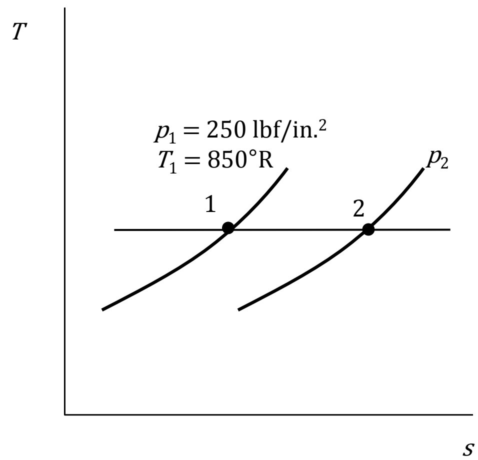
Carbon monoxide at 250 lbf/in.^{2}, 850°R, and a volumetric flow rate of 75 ft^3/min enters a valve operating at steady state and undergoes a throttling process. Assuming ideal gas behavior: (a) determine the rate of exergy destruction, in Btu/min, for an exit pressure of 30 lbf/in.^{2} and (b) plot the exergy destruction rate, in Btu/min, versus exit pressure ranging from 30 to 250 lbf/in.^{2} Let T_0 = 530°R, p_0 = 15 lbf/in.^{2}

Known:
Carbon monoxide at a known state enters a valve operating at steady state with a known volumetric flow rate. The carbon monoxide undergoes a throttling process to exit pressure p_2.

Find:
(a) For p_2 = 30 lbf/in.^{2}, determine the exergy destruction rate and (b) plot the exergy destruction rate versus p_2 ranging from 30 to 250 lbf/in.^{2}

Schematic and Known Data:





Engineering Model:
(1) The control volume shown in the schematic is at steady state.
(2) The carbon monoxide undergoes a throttling process in passing through the valve.
(3) The carbon monoxide is modeled as an ideal gas.
(4) For the environment, T_0 = 530°F, p_0 = 15 lbf/in.^{2}

Analysis:

(a) State 1 is fixed by T_1 = 850°R, p_1 = 250 lbf/in.^{2} To fix state 2, we use assumption (2) and $h_1 = h_2$. By assumptions (2) and (3), $T_2 = T_1$. To determine $\dot{\mathrm{E}}_\mathrm{d}$, we begin with an exergy balance at steady state. With $\dot{m}_1 = \dot{m}_2 = \dot{m}$:

$$0 = \sum_j \left(1 - \frac{T_0}{T_j}\right)\underbrace{\dot{Q}_j}_{=0} - \underbrace{\dot{W}}_{=0} + \dot{m}\left[\mathrm{e}_{\mathrm{f}_1} - \mathrm{e}_{\mathrm{f}_2}\right] - \dot{\mathrm{E}}_\mathrm{d}$$

Or:

$$\dot{\mathrm{E}}_{\mathrm{d}} = \dot{m}\left(\mathrm{e}_{\mathrm{f}_1} - \mathrm{e}_{\mathrm{f}_2}\right) = \dot{m}\left[\underbrace{(h_1 - h_2)}_{=0} - T_0(s_1 - s_2)\right] = \dot{m}T_0(s_2 - s_1) \qquad (1)$$

To determine $\dot{m}$ we use we $\dot{m} = \frac{(AV)_1}{v_1}$ and the ideal gas equation for v_1:

$$\dot{m} = \frac{p_1(\mathrm{AV})_1}{RT_1} = \frac{\left(250\,\frac{\mathrm{lbf}}{\mathrm{in.}^2}\cdot\left|\frac{144\ \mathrm{in.}^2}{1\ \mathrm{ft}^2}\right|\right)\left(75\,\frac{\mathrm{ft}^3}{\mathrm{min}}\right)}{\left(\frac{1545}{28.01}\,\frac{(\mathrm{ft}\cdot\mathrm{lbf})}{\mathrm{lb}\cdot{}^\circ\mathrm{R}}\right)(850^\circ\mathrm{R})} = 57.59\,\frac{\mathrm{lb}}{\mathrm{min}}$$

Using ideal gas relations with Eq. (1) and inserting values:

$$\dot{\mathrm{E}}_d = \dot{m}T_0\left[\underbrace{s^o(T_2) - s^o(T_1)}_{=0} - \mathrm{R}\ln\frac{p_2}{p_1}\right] = \left(57.59\,\frac{\mathrm{lb}}{\mathrm{min}}\right)(530^\circ\mathrm{R})\left[-\frac{1.986}{28.01}\,\frac{\mathrm{Btu}}{\mathrm{lb}\cdot{}^\circ\mathrm{R}}\ln\frac{30}{250}\right]$$
$$= 4588.6\,\frac{\mathrm{Btu}}{\mathrm{min}}$$

(b) The data to construct the required plots are obtained using IT, as follows. For the IT solutions, $v_1(T_1, p_1)$ and the entropy values $s_1(T_1, p_1)$ and $s_2(T_2, p_2)$ are evaluated directly using internal IT property functions.

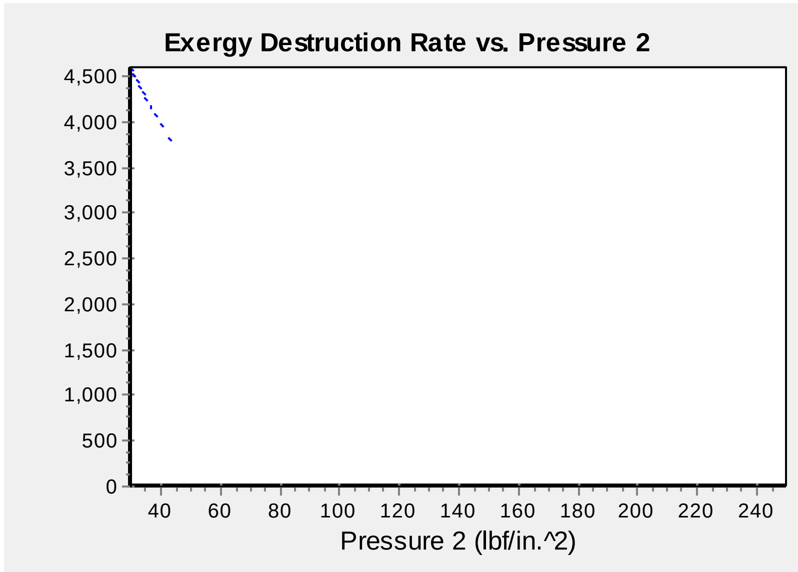
<u>IT Code</u>

```
p1 = 250 // lbf/in.^2
T1 = 850 // R
p2 = 30 // lbf/in.^2
AV1 = 75 // ft^3 / min
T0 = 530 // R

T2 = T1
v1 = v_Tp("CO",T1,p1)
s1 = s_Tp("CO",T1,p1)
s2 = s_Tp("CO",T2,p2)

mdot = AV1/v1
Ed = mdot * T0 * (s2 - s1)
```

<u>IT Solution for P_2 = 50 lbf/in.^{2}</u>

Ed	mdot	s1	s2	v1
4590	57.58	1.598	1.749	1.303

Plot:

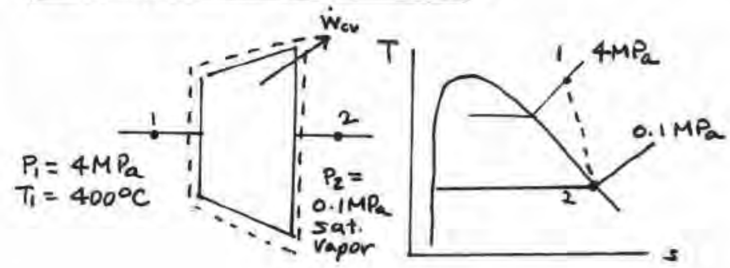


Discussion:
From the plot, we see that the rate of exergy destruction increases with decreasing p_2. This is expected based on greater entropy production and hence increasing s_2.

PROBLEM 7.58

Water vapor at 4.0 MPa and 400°C enters an insulated turbine operating at steady state and expands to saturated vapor at 0.1 MPa. The effects of motion and gravity can be neglected. Determine the work developed and the exergy destruction, each in kJ per kg of water vapor passing through the turbine. Let $T_0 = 27°C$, $p_0 = 0.1$ MPa.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv}$ and the effects of motion and gravity can be ignored.
3. For the environment, $T_0 = 300K$ (27°C), $P_0 = 0.1 MPa$

ANALYSIS: (a) Reducing mass and energy rate balances, we get

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 = (3213.6 - 2675.5)\frac{kJ}{kg} = 538.1\ kJ/kg$$

Reducing mass and entropy rate balances, the rate of entropy production is

$$\frac{\dot{\sigma}}{\dot{m}} = s_2 - s_1 = (7.3594 - 6.7690)\frac{kJ}{kg \cdot K} = 0.5904\frac{kJ}{kg \cdot K}$$

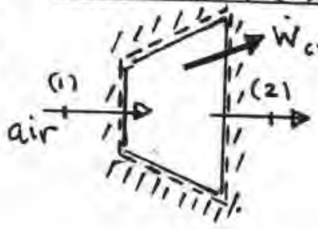
The rate of exergy destruction is then

$$\frac{\dot{E}_d}{\dot{m}} = T_0\left(\frac{\dot{\sigma}}{\dot{m}}\right) = 300K(0.5904)\frac{kJ}{kg \cdot K} = 177.12\frac{kJ}{kg}$$

PROBLEM 7.59

Air enters an insulated turbine operating at steady state at 8 bar, 500 K, and 150 m/s. At the exit the conditions are 1 bar, 320 K, and 10 m/s. There is no significant change in elevation. Determine the work developed and the exergy destruction, each in kJ per kg of air flowing. Let $T_0 = 300$ K, $p_0 = 1$ bar.

SCHEMATIC & GIVEN DATA:



$p_1 = 8$ bar $p_2 = 1$ bar
$T_1 = 500$K $T_2 = 320$K
$V_1 = 150$ m/s $V_2 = 10$ m/s
$T_0 = 300$K, $p_0 = 1$ bar

ENGR. MODEL:

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv}$ and the effect of gravity can be ignored.
3. For the environment, $T_0 = 300$K, $p_0 = 1$ bar.

ANALYSIS: The mass and energy rate balances reduce with assumptions (1)-(3) to give

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - V_2^2)}{2} + g\cancelto{0}{(z_1 - z_2)}\right] \Rightarrow \frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 + \frac{(V_1^2 - V_2^2)}{2} \qquad (1)$$

With data from Table A-22

$$\frac{\dot{W}_{cv}}{\dot{m}} = (503.02 - 320.29)\frac{kJ}{kg} + \left(\frac{150^2 - 10^2}{2}\right)\frac{m^2}{s^2}\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right| = 193.93\ \frac{kJ}{kg} \longleftarrow (\dot{W}_{cv}/\dot{m})$$

The exergy destruction rate is found using $\dot{E}_d = T_0\dot{\sigma}_{cv}$ and an entropy balance.

$$0 = \cancelto{0}{\sum_j \left(\frac{\dot{Q}}{T}\right)_j} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \dot{E}_d/\dot{m} = T_0(s_2 - s_1) \qquad (2)$$

With ideal gas relations and data from Table A-22

$$\frac{\dot{E}_d}{\dot{m}} = T_0\left[s^\circ(T_2) - s^\circ(T_1) - R\ln p_2/p_1\right]$$

$$= (300\text{K})\left[(1.76690 - 2.21952) - \left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)\ln\left(\frac{1}{8}\right)\right] = 43.25\ kJ/kg \longleftarrow \dot{E}_d/\dot{m}$$

PROBLEM 7.60

Air enters a turbine operating at steady state with a pressure of 75 lbf/in.^{2}, a temperature of 800°R, and a velocity of 400 ft/s. At the turbine exit, the conditions are 15 lbf/in.^{2}, 600°R, and 100 ft/s. Heat transfer from the turbine to its surroundings takes place at an average surface temperature of 620°R. The rate of heat transfer is 2 Btu per lb of air passing through the turbine. For the turbine, determine the work developed and the exergy destruction, each in Btu per lb of air flowing. Let $T_0 = 40°F$, $p_0 = 15$ lbf/in.^{2}.

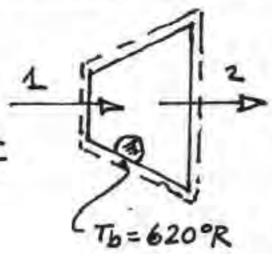
SCHEMATIC & GIVEN DATA:

AIR

$P_1 = 75\ \frac{lbf}{in^2}$

$T_1 = 800°R$

$V_1 = 400\ \frac{ft}{s}$



$P_2 = 15\ lbf/in^2$

$T_2 = 600°R$

$V_2 = 100\ ft/s$

$\frac{\dot{Q}_{cv}}{\dot{m}} = -2\ \frac{Btu}{lb}$

ENGR. MODEL:

1. The control volume shown in the schematic is at steady state.
2. Heat transfer takes place at temperature T_b.
3. The effect of gravity can be ignored.
4. Air is modeled as an ideal gas. (This can be checked using the generalized compressibility chart.)

ANALYSIS: At steady state an energy rate balance reduces to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + \cancel{g(z_1 - z_2)}^{0}\right]$$

or

$$\frac{\dot{W}_{cv}}{\dot{m}} = \frac{\dot{Q}_{cv}}{\dot{m}} + (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) \qquad (1)$$

With data from Table A-22E

$$\frac{\dot{W}_{cv}}{\dot{m}} = -2\ \frac{Btu}{lb} + (191.81 - 143.47)\ \frac{Btu}{lb} + \frac{[(400)^2 - (100)^2](ft^2/s^2)}{(2)\ 32.2\ \frac{lb \cdot ft}{lbf \cdot s^2}\ \ 778\ \frac{ft \cdot lbf}{Btu}}$$

$$= -2\ \frac{Btu}{lb} + 48.34\ \frac{Btu}{lb} + 2.99\ \frac{Btu}{lb} = 49.33\ \frac{Btu}{lb} \longleftarrow \dot{W}_{cv}/\dot{m}$$

The exergy destruction can be evaluated by reducing an exergy rate balance. Thus

$$\frac{\dot{E}_d}{\dot{m}} = \left[1 - \frac{T_0}{T_b}\right]\left(\frac{\dot{Q}_{cv}}{\dot{m}}\right) - \frac{\dot{W}_{cv}}{\dot{m}} + \left[(h_1 - h_2) - T_0(s_1 - s_2) + \frac{V_1^2 - V_2^2}{2}\right]$$

Introducing Eq. (1) and simplifying

(1) $$\frac{\dot{E}_d}{\dot{m}} = T_0\left[(s_2 - s_1) - \frac{\dot{Q}_{cv}/\dot{m}}{T_b}\right] \qquad (2)$$

or, for an ideal gas (assumptions)

$$\frac{\dot{E}_d}{\dot{m}} = T_0\left[\left(s^\circ(T_2) - s^\circ(T_1) - \frac{\bar{R}}{M}\ln\frac{P_2}{P_1}\right) - \frac{\dot{Q}_{cv}/\dot{m}}{T_b}\right] \qquad (3)$$

with data from Table A-22E

$$\frac{\dot{E}_d}{\dot{m}} = 500°R\left[\left(0.62607 - 0.69558 - \frac{1.986}{28.97}\ln\frac{15}{75}\right)\frac{Btu}{lb \cdot °R} - \frac{(-2\ Btu/lb)}{620}\right] \qquad \dot{E}_d/\dot{m}$$

$$= 500°R\left[0.04082 + 0.0032\right]\frac{Btu}{lb \cdot °R} = 22.01\ Btu/lb \longleftarrow$$

1. This corresponds to $\dot{E}_d/\dot{m} = T_0\,\dot{\sigma}/\dot{m}$

PROBLEM 7.61

7.61 Steam enters a turbine operating at steady state at 4 MPa, 500°C with a mass flow rate of 50 kg/s. Saturated vapor exits at 10 kPa and the corresponding power developed is 42 MW. The effects of motion and gravity are negligible.

(a) For a control volume enclosing the turbine, determine the rate of heat transfer, in MW, from the turbine to its surroundings. Assuming an average turbine outer surface temperature of 50°C, determine the rate of exergy destruction, in MW.

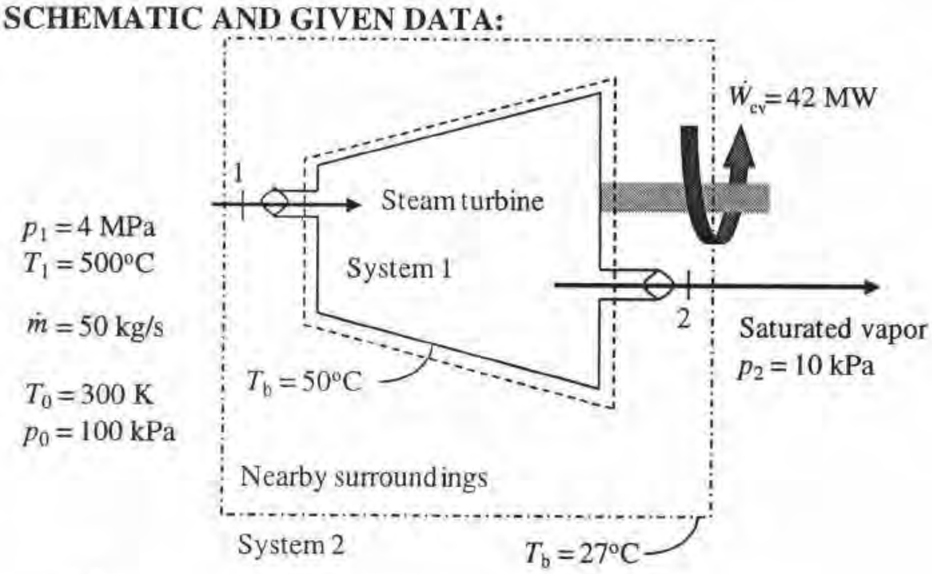
(b) If the turbine is located in a facility where the ambient temperature is 27°C, determine the rate of exergy destruction for an enlarged control volume including the turbine and its immediate surroundings so heat transfer takes place at ambient temperature. Explain why the exergy destruction values in parts (a) and (b) differ.

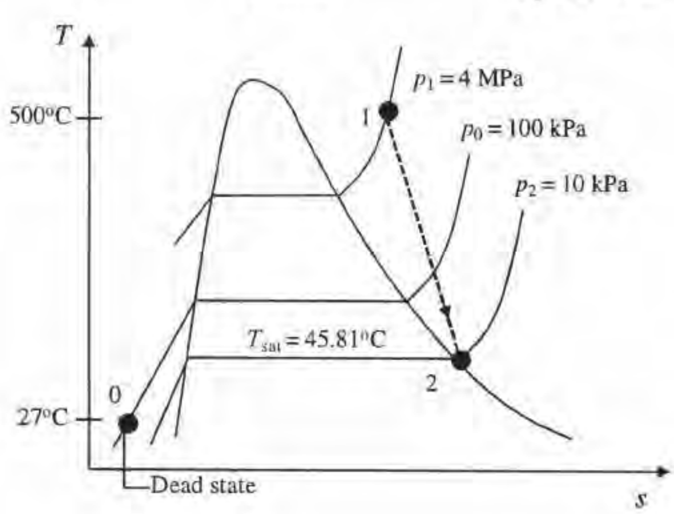
Let T_0= 300 K, p_0=100 kPa.

KNOWN: Steady-state operating data are provided for a steam turbine.

FIND: Determine the heat transfer, in MW. For each of the specified control volumes, evaluate rate of exergy destruction, in MW, and compare.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL:
(1) The control volumes shown in the accompanying schematic are at steady state.
(2) In each case, heat transfer occurs at the specified value of T_b.
(3) Effects of motion and gravity can be ignored.
(4) For the environment, $T_0 = 300$ K and $p_0 = 100$ kPa.

ANALYSIS:
(a) Mass and energy balances at steady state reduce to:

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2)$$

Rearrange and solve using data from Tables A-3 and A-4.

$$\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}(h_2 - h_1) = 42\text{ MW} + 50\frac{\text{kg}}{\text{s}}(2584.7 - 3445.3)\frac{\text{kJ}}{\text{kg}}\left|\frac{1\text{ MW}}{10^3\frac{\text{kJ}}{\text{s}}}\right| = -1.03\text{ MW}$$

The rate of exergy destruction can be obtained by using an exergy rate balance or using $\dot{\text{E}}_d = \dot{\sigma}T_o$. Choosing the second approach, an entropy rate balance is used to determine the rate of entropy produced.

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_1 - s_2) + \dot{\sigma}$$

$$\dot{\sigma} = -\frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_2 - s_1) \qquad (1)$$

$$\dot{\sigma} = -\left(\frac{-1.03\text{ MW}}{323\text{ K}}\right) + 50\frac{\text{kg}}{\text{s}}(8.1502 - 7.0901)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\text{MW}}{10^3\frac{\text{kJ}}{\text{s}}}\right| = 0.05619\frac{\text{MW}}{\text{K}}$$

The exergy destruction rate can be determined, as follows:

$$\dot{\text{E}}_d = \left(0.05619\frac{\text{MW}}{\text{K}}\right)300\text{K} = 16.86\text{ MW}$$

(b) Eq. (1) also applies for the enlarged control volume. Thus with known values we get

$$\dot{\sigma} = -\left(\frac{-1.03\text{MW}}{300\text{K}}\right) + 50\frac{\text{kg}}{\text{s}}(8.1502 - 7.0901)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\text{MW}}{10^3\frac{\text{kJ}}{\text{s}}}\right| = 0.05644\frac{\text{MW}}{\text{K}}$$

$$\dot{\text{E}}_d = \left(0.05644\frac{\text{MW}}{\text{K}}\right)300\text{K} = 16.93\text{ MW}$$

The exergy destruction rate in part (b) is greater because there is an additional source of irreversibility associated with the heat transfer from the outer surface of the turbine to the ambient. For the control volume described in part (a), this same heat transfer effect is an <u>external</u> irreversibility.

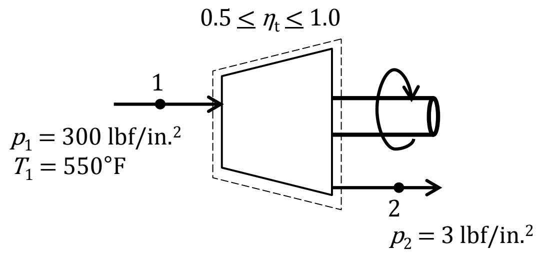
Problem 7.62

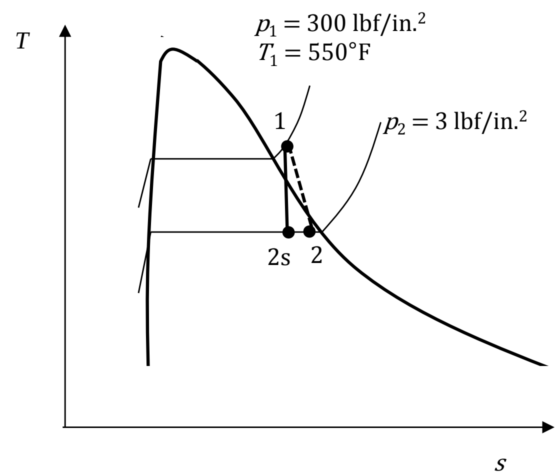
An insulated turbine operating at steady state receives steam at 300 lbf/in.^{2}, 550°F and exhausts at 3 lbf/in.^{2} Plot the exergy destruction rate, in Btu per lb of steam flowing, versus turbine isentropic efficiency ranging from 50 to 100%. The effects of motion and gravity can be ignored and T_0 = 60°F, p_0 = 1 atm.

Known:
An insulated steam turbine operating at steady state receives steam at a known state and exhausts at a given pressure.

Find:
Plot the exergy destruction rate per unit of steam flowing versus isentropic turbine efficiency ranging from 50 to 100%.

Schematic and Known Data:





Engineering Model:
(1) The control volume shown in the schematic is at steady state.
(2) For the turbine, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects can be neglected.
(3) For the exergy reference environment, T_0 = 520°R, p_0 = 1 atm.

Analysis:
To determine the exergy destruction rate, we begin with mass and entropy rate balances which reduce to give:

$$0 = \sum_j \underbrace{\frac{\dot{Q}_j}{T_j}}_{=0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1$$

With $\dot{\mathrm{E}}_{\mathrm{d}} = T_0 \dot{\sigma}_{cv}$

$$\frac{\dot{\mathrm{E}}_{\mathrm{d}}}{\dot{m}} = T_0(s_2 - s_1) \qquad (1)$$

Since p_1 = 300 lbf/in.^{2}, T_1 = 550°F, state 1 is in the superheated vapor region and the state is fixed. To fix state 2, we use the isentropic turbine efficiency:

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \Longrightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s}) \qquad (2)$$

Where h_{2s} is determined using p_2 = 3 lbf/in.^{2} and $s_{2s} = s_1$. Then, with p_2 and h_2 known from Eq. (2), s_2 can be determined.

<u>Sample calculation for η_t = 0.5:</u>
From Table A-4E, $h_1 = 1286.7\,\frac{\text{Btu}}{\text{lb}}, s_1 = 1.5997\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$. With $s_{2s} = s_1 = 1.5997\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$ and using Table A-3E at p_2:

$$x_{2\text{s}} = \frac{s_{2\text{s}} - s_{\text{f}2}}{s_{\text{fg}_2}} = \frac{1.5997 - 0.2009}{1.6852} = 0.830$$

$$h_{2\text{s}} = h_{\text{f}2} + x_{2\text{s}} h_{\text{fg}_2} = 109.39 + (0.830)(1013.1) = 950.26\,\frac{\text{Btu}}{\text{lb}}$$

From Eq. (2):

$$h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 1286.7 - 0.5(1286.7 - 950.26) = 1118.48\,\frac{\text{Btu}}{\text{lb}}$$

With p_2 = 3 lbf/in.^{2}:

$$x_2 = \frac{h_2 - h_{\text{f}2}}{h_{\text{fg}_2}} = \frac{1118.48 - 109.39}{1013.1} = 0.996$$

$$s_2 = s_{\text{f}2} + x_2 s_{\text{fg}_2} = 0.2009 + (0.996)(1.6852) = 1.8794\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$$

Thus:

$$\frac{\dot{\text{E}}_\text{d}}{\dot{m}} = T_0(s_2 - s_1) = (520°\text{R})(1.8794 - 1.5997)\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}} = 145.44\,\frac{\text{Btu}}{\text{lb}}$$

The data to construct the required plot are obtained using IT, as follows:

<u>IT Code</u>

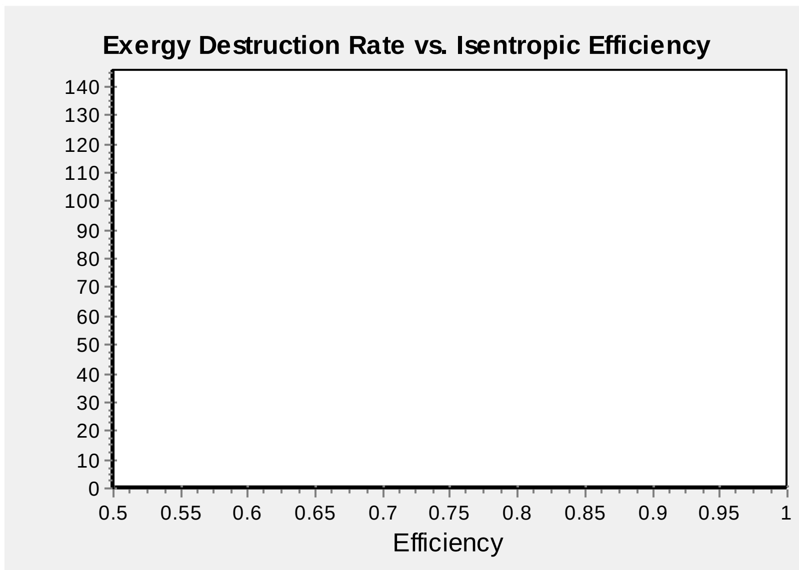
```
p1 = 300 // lbf/in.^2
T1 = 550 // F
p2 = 3 // lbf/in.^2
eff = 0.5
T0 = 520 // R

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
s2s = s1
h2s = h_Ps("Water/Steam", p2, s2s)
```

h2 = h1 - eff * (h1 - h2s)
Ed = T0 * (s2 - s1)
h2 = h_Ps("Water/Steam", p2, s2)

<u>IT Solution for η_t = 0.5:</u>

h1	h2	h2s	s1	s2	s2s	Ed
1286	1118	950.1	1.599	1.879	1.599	145.5

Plot:



<u>Discussion:</u>
From the plot, we see that lower values of isentropic turbine efficiency correspond to increased exergy destruction, as expected.

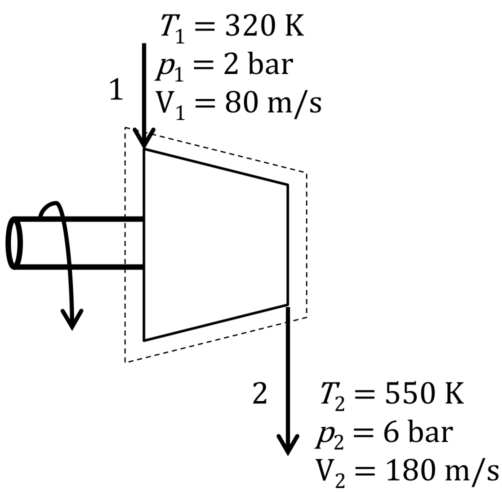
Problem 7.63

Air enters a compressor operating at steady state at T_1 = 320 K, p_1 = 2 bar with a velocity of 80 m/s. At the exit, T_2 = 550 K, p_2 = 6 bar and the velocity is 180 m/s. The air can be modeled as an ideal gas with c_p = 1.01 kJ/kg·K. Stray heat transfer can be ignored. Determine, in kJ per kg of air flowing, (a) the power required by the compressor and (b) the rate of exergy destruction within the compressor. Let T_0 = 300 K, p_0 = 1 bar. Ignore the effects of motion and gravity.

Known:
An air compressor operates at steady state with known initial conditions.

Find:
Determine (a) the power required by the compressor, and (b) the rate of exergy destruction within the compressor.

Schematic and Known Data:



Engineering Model:
(1) The control volume shown in the schematic is at steady state.
(2) For the compressor, $\dot{Q}_{cv} = 0$ and the effects of gravity can be neglected.
(3) The air can be modeled as an ideal gas with c_p = 1.01 kJ/kg·K
(4) For the environment, T_0 = 300 K, p_0 = 1 bar.

Analysis:
(a) Reducing an energy rate balance:

$$0 = \underbrace{\dot{Q}_{cv}}_{=0} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \left(\frac{V_1^2}{2} - \frac{V_2^2}{2}\right) + g\underbrace{(z_1 - z_2)}_{=0}\right] \Longrightarrow$$

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} = c_p(T_1 - T_2) + \frac{V_1^2 - V_2^2}{2} \Longrightarrow$$

$$\frac{\dot{W}_{\text{cv}}}{\dot{m}} = \left(1.01 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right)(320 - 550)\text{K} + \left(\frac{80^2 - 180^2}{2}\right)\frac{\text{m}^2}{\text{s}^2}\left|\frac{1\ \text{N}}{\text{kg} \cdot \frac{\text{m}}{\text{s}^2}}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N} \cdot \text{m}}\right| = -245.3 \frac{\text{kJ}}{\text{kg}} \longleftarrow$$

(b) The rate of exergy destruction is $\dot{\text{E}}_{\text{d}} = T_0\dot{\sigma}$, where $\dot{\sigma}$ is the rate of entropy production. From an entropy rate balance, $\dot{\sigma} = \dot{m}(s_2 - s_1)$. Thus:

$$\frac{\dot{\text{E}}_{\text{d}}}{\dot{m}} = T_0(s_2 - s_1) = T_0\left(c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}\right)$$

$$= (300\ \text{K})\left[\left(1.01 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right)\ln\frac{550}{320} - \left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right)\ln\frac{6}{2}\right] = 69.52 \frac{\text{kJ}}{\text{kg}} \longleftarrow$$

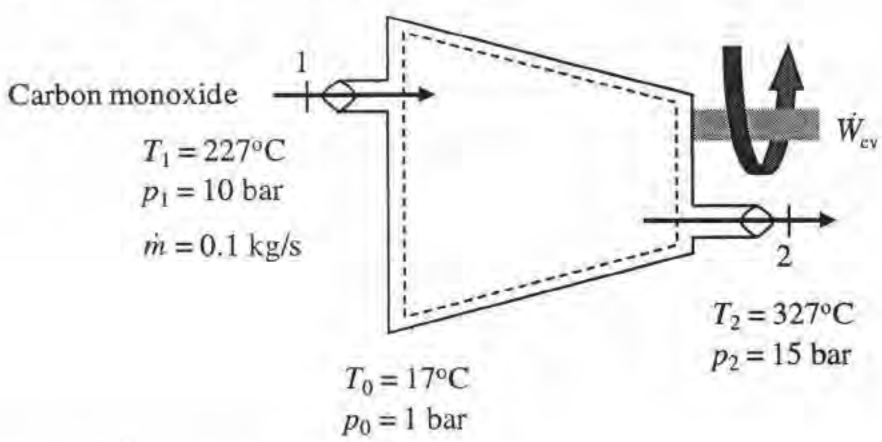
PROBLEM 7.64

7.64 Carbon monoxide (CO) enters an insulated compressor operating at steady state at 10 bar, 227°C, and a mass flow rate of 0.1 kg/s and exits at 15 bar, 327°C. Determine the power required by the compressor and the rate of exergy destruction, each in kW. Ignore the effects of motion and gravity. Let T_0= 17°C, p_0=1 bar.

KNOWN: Carbon monoxide (CO) enters and exits a compressor operating at steady sate with inlet and exit conditions provided.

FIND: Determine the power required by the compressor and the rate of exergy destruction.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

(1) The control volume shown in the accompanying schematic is at steady state and adiabatic.

#1 (2) CO is modeled as an ideal gas.

(3) Effects of motion and gravity can be ignored.

(4) For the environment, T_0 =290 K and p_0 = 1 bar.

ANALYSIS:

Mass and energy rate balances reduce to

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2)$$

with data from Tables A-1 and A-23

$$\dot{W}_{cv} = \frac{\dot{m}}{M}(\bar{h}_1 - \bar{h}_2) = \frac{\left(0.1\frac{\text{kg}}{\text{s}}\right)}{\left(28.01\frac{\text{kg}}{\text{kmol}}\right)}(14600 - 17611)\frac{\text{kJ}}{\text{kmol}}\left|\frac{1\,\text{kW}}{1\frac{\text{kJ}}{\text{s}}}\right| = -10.75\,\text{kW}$$

PROBLEM 7.64 (Continued).

The rate of exergy destruction can be obtained by using an exergy rate balance or using $\dot{E}_d = \dot{\sigma} T_o$ where $\dot{\sigma}$ is the rate of entropy production using an entropy balance. Using the entropy approach simplified based on assumptions

$$0 = \dot{m}(s_1 - s_2) + \dot{\sigma}$$

$$\dot{\sigma} = \dot{m}(s_2 - s_1) = \frac{\dot{m}}{M}\left(\bar{s}_2^{\,o} - \bar{s}_1^{\,o} - \bar{R}\,ln\frac{p_2}{p_1}\right)$$

$$\dot{\sigma} = \frac{\left(0.1\dfrac{\text{kg}}{\text{s}}\right)}{\left(28.01\dfrac{\text{kg}}{\text{kmol}}\right)}\left(218.204 - 212.719 - (8.314)\ln\left(\frac{15}{10}\right)\right)\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\left|\frac{1\,\text{kW}}{1\dfrac{\text{kJ}}{\text{s}}}\right| = 7.547\times10^{-3}\,\frac{\text{kW}}{\text{K}}$$

The exergy destruction rate can be determined, as follows:

$$\dot{\mathbf{E}}_d = \left(7.547\times10^{-3}\,\frac{\text{kW}}{\text{K}}\right)290\text{K} = 2.19\,\text{kW} \qquad \longleftarrow$$

---

1. Can be verified using the generalized compressibility chart.

PROBLEM 7.65

7.65 Refrigerant 134a at 10°C, 1.8 bar, and a mass flow rate of 5 kg/min enters an insulated compressor operating at steady state and exits at 5 bar. The isentropic compressor efficiency is 76.04%. Determine

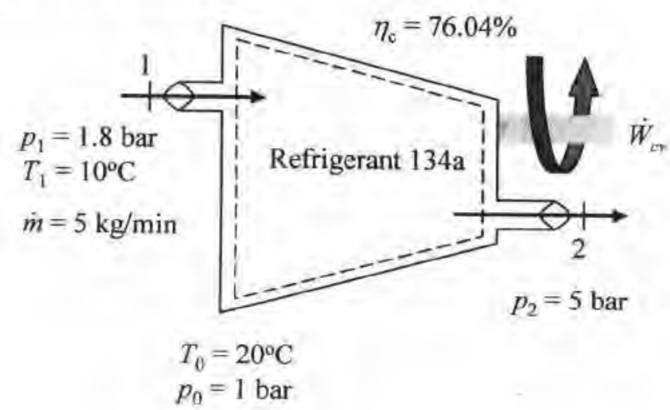
(a) the temperature of the refrigerant exiting the compressor, in °C.
(b) the power input to the compressor, in kW.
(c) the rate of exergy destruction, in kW.

Ignore the effects of motion and gravity and let $T_0 = 20°C$, $p_0 = 1$ bar.

KNOWN: Refrigerant 134a at specified temperature, pressure, and mass flow rate enters a compressor with known isentropic efficiency and exits at specified pressure.

FIND: The temperature of the refrigerant exiting the compressor, the power input to the compressor, and the rate of exergy destruction.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume defined by the dashed line on the accompanying diagram is at steady state.
2. Heat transfer and the effects of motion and gravity can be ignored.
3. $T_0 = 20°C$, $p_0 = 1$ bar.

ANALYSIS:
At the inlet the refrigerant is superheated vapor. From Table A-12, $h_1 = 259.41$ kJ/kg and $s_1 = 0.9998$ kJ/(kg·K).

At state 2s (see T-s diagram), the specific entropy is the same as at the inlet. Thus, $s_{2s} = s_1 = 0.9998$ kJ/(kg·K). For $p_{2s} = p_2 = 5$ bar, State 2s is superheated vapor. From Table A-12 (interpolated), $h_{2s} = 282.70$ kJ/kg

Refrigerant 134a at the reference state is superheated vapor. From Table A-12, $h_0 = 270.02$ kJ/kg and $s_0 = 1.0829$ kJ/(kg·K).

PROBLEM 7.65 (Continued, p. 2)

(a) The exit temperature can be determined from Eq. 6.48, isentropic compressor efficiency, by solving for the exit enthalpy, h_2, to fix State 2

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1}$$

Solving for the exit enthalpy and substituting values gives

$$h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c} = 259.41\frac{\text{kJ}}{\text{kg}} + \frac{282.70\frac{\text{kJ}}{\text{kg}} - 259.41\frac{\text{kJ}}{\text{kg}}}{0.7604} = 290.04\ \text{kJ/kg}$$

For $p_2 = 5$ bar, $h_2 = 290.04$ kJ/kg, State 2 is superheated vapor. From Table A-12, $\underline{\mathbf{T_2 = 50^\circ C}}$ and $s_2 = 1.0229$ kJ/(kg·K).

(b) The power input to the compressor can be determined from the steady state energy balance

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i\,(h_i + ½\,V_i^2 + gz_i) - \sum_e \dot{m}_e\,(h_e + ½\,V_e^2 + gz_e)$$

Setting rate of heat transfer to zero, ignoring kinetic and potential energy effects, and writing the energy balance for one inlet and one exit give

$$0 = -\dot{W}_{cv} + \dot{m}\,(h_1 - h_2)$$

Solving for power, substituting values, and applying appropriate conversion factors give

$$\dot{W}_{cv} = \dot{m}\,(h_1 - h_2) = \left(5\frac{\text{kg}}{\text{min}}\right)\left(259.41\frac{\text{kJ}}{\text{kg}} - 290.04\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{\text{min}}{60\ \text{s}}\right|\left|\frac{\text{kW}}{\frac{\text{kJ}}{\text{s}}}\right| = \underline{\mathbf{-2.55\ kW}}$$

The negative sign indicates power is into the compressor.

The rate of exergy destruction can be determined from the entropy production rate

$$\dot{\mathsf{E}}_d = T_0\,\dot{\sigma}_{cv}$$

Entropy production rate can be determined from the steady-state control volume entropy rate balance

$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{cv}$$

Neglecting heat transfer rate, entropy production rate for one inlet and one exit becomes

$$\dot{\sigma}_{cv} = \dot{m}\,(s_2 - s_1)$$

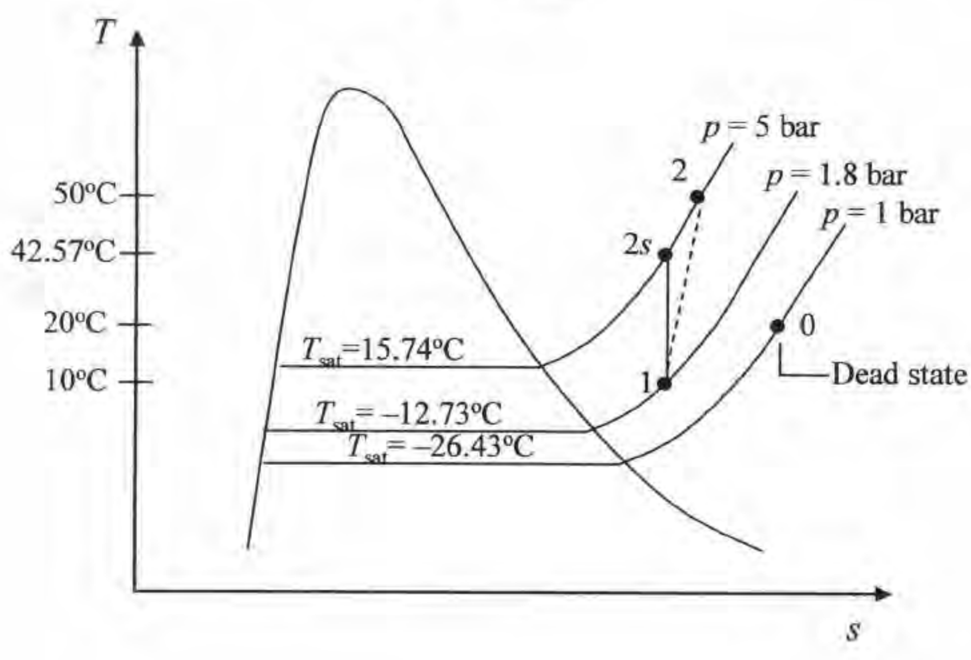
Substituting values and applying appropriate conversion factors give

$$\dot{\sigma}_{cv} = \left(5\,\frac{\text{kg}}{\text{min}}\right)\left(1.0229\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 0.9998\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left|\frac{\text{min}}{60\text{ s}}\right|\left|\frac{\text{kW}}{\frac{\text{kJ}}{\text{s}}}\right| = 0.001925\text{ kW/K}$$

Substituting values and solving for exergy destruction give

$$\dot{E}_d = (293\text{ K})(0.001925\text{ kW/K}) = \underline{\mathbf{0.5640\ kW}}$$

***T-s* Diagram**



Problem 7.66

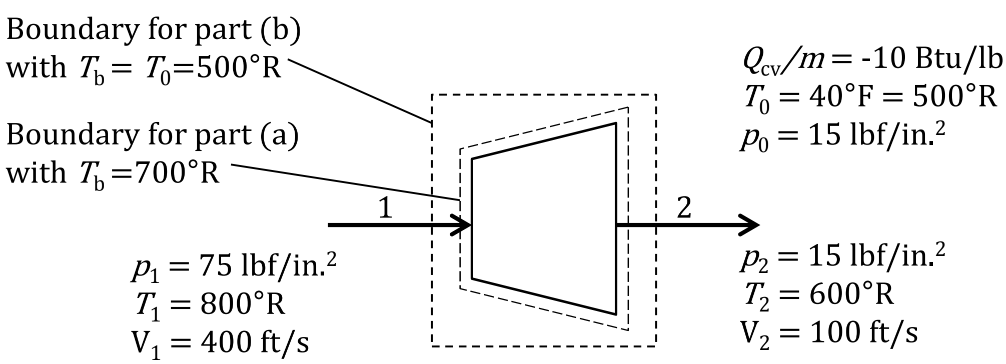
Air enters a turbine operating at steady state at a pressure of 75 lbf/in.^{2}, a temperature of 800°R, and a velocity of 400 ft/s. At the exit, the conditions are 15 lbf/in.^{2}, 600°R, and 100 ft/s. There is no significant change in elevation. Heat transfer from the turbine to its surroundings at a rate of 10 Btu per lb of air flowing takes place at an average surface temperature of 700°R.
(a) Determine, in Btu per lb of air passing through the turbine, the work developed and the exergy destruction rate.
(b) Expand the boundary of the control volume to include both the turbine and a portion of its immediate surroundings so that heat transfer occurs at a temperature T_0. Determine, in Btu per lb of air passing through the turbine, the work developed and the exergy destruction rate.
(c) Explain why the exergy destruction rates in parts (a) and (b) are different.

Let T_0 = 40°F, p_0 = 15 lbf/in.^{2}

Known:
Data are provided for a turbine operating at steady state.

Find:
Determine per unit mass of air flowing, the work developed and the exergy destruction rate at various boundary temperatures. Explain why these values are different.

Schematic and Known Data:



Engineering Model:
(1) The control volume shown in the schematic is at steady state.
(2) Heat transfer takes place at temperature T_b.
(3) There are no effects from gravity.
(4) The air can be modeled as an ideal gas.
(5) For the environment, T_0 = 600°R (40°F), p_0 = 15 lbf/in.^{2}

Analysis:
(a) At steady state, an energy rate balance reduces to give:

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\,\underbrace{(z_1 - z_2)}_{=0}\right]$$, rearranging:

$$\frac{\dot{W}_{cv}}{\dot{m}} = \frac{\dot{Q}_{cv}}{\dot{m}} + h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} \qquad (1)$$

With given information and data from Table A-22E:

$$\frac{\dot{W}_{cv}}{\dot{m}} = -10\frac{\text{Btu}}{\text{lb}} + (191.81 - 143.47) + \left(\frac{400^2 - 100^2}{2}\frac{\text{ft}^2}{\text{s}^2} \cdot \left|\frac{1\ \text{lbf} \cdot \text{s}^2}{32.2\ \text{lb} \cdot \text{ft}}\right| \cdot \left|\frac{1\ \text{Btu}}{778\ \text{ft} \cdot \text{lbf}}\right|\right)$$
$$= 41.33\frac{\text{Btu}}{\text{lb}}$$

The exergy destruction can be evaluated by reducing an exergy rate balance. Rearranging and expanding the flow exergy terms, Eq. 7.13a becomes:

$$\frac{\dot{\text{E}}_{\text{d}}}{\dot{m}} = \left[1 - \frac{T_0}{T_b}\right]\frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}} + \left[h_1 - h_2 - T_0(s_1 - s_2) + \frac{V_1^2 - V_2^2}{2}\right]$$

Introducing Eq. (1) and simplifying:

$$\frac{\dot{\text{E}}_{\text{d}}}{\dot{m}} = T_0\left[(s_2 - s_1) - \frac{\dot{Q}_{cv}}{\dot{m}T_{\text{b}}}\right]$$

Or with assumption (4):

$$\frac{\dot{\text{E}}_{\text{d}}}{\dot{m}} = T_0\left[s^o(T_2) - s^o(T_1) - \frac{\bar{R}}{M}\ln\frac{p_2}{p_1} - \frac{\dot{Q}_{cv}}{\dot{m}T_{\text{b}}}\right]$$

Using T_{b} = 700°R and data from Table A-22E:

$$\frac{\dot{\text{E}}_{\text{d}}}{\dot{m}} = (500°\text{R})\left[\left(0.62607 - 0.69558 - \frac{1.986}{28.97}\ln\frac{15}{75}\right)\frac{\text{Btu}}{\text{lb} \cdot °\text{R}} - \frac{-10\frac{\text{Btu}}{\text{lb}}}{700°\text{R}}\right] = 27.55\frac{\text{Btu}}{\text{lb}}$$

(b) Using T_{b} = 500°R and data from Table A-22E:

$$\frac{\dot{\text{E}}_{\text{d}}}{\dot{m}} = (500°\text{R})\left[\left(0.62607 - 0.69558 - \frac{1.986}{28.97}\ln\frac{15}{75}\right)\frac{\text{Btu}}{\text{lb} \cdot °\text{R}} - \frac{-10\frac{\text{Btu}}{\text{lb}}}{500°\text{R}}\right] = 30.41\frac{\text{Btu}}{\text{lb}}$$

(c) The exergy destruction rate is greater in part (b) because the enlarged control volume in part (b) has an additional source of irreversibility associated with the heat transfer from the outer surface of the turbine to the ambient. For the control volume described in part (a), this same heat transfer effect is an external irreversibility.

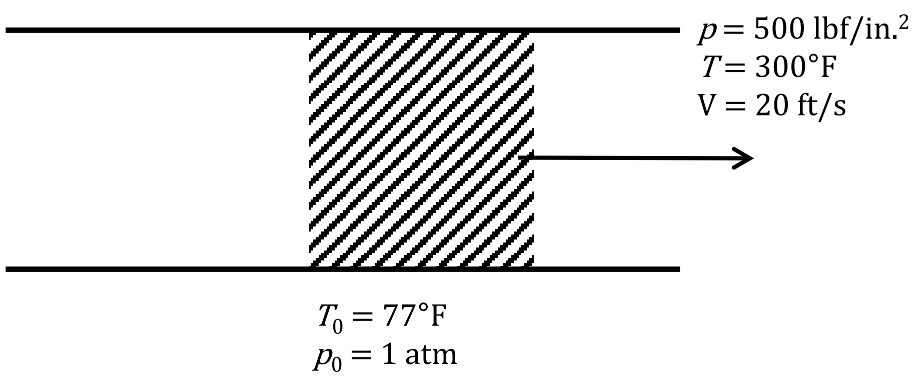
Problem 7.67

A stream of hot water at 300°F, 500 lbf/in.^{2}, and a velocity of 20 ft/s is obtained from a geothermal supply. Determine the specific flow exergy, in Btu/lb. The velocity is relative to the exergy reference environment for which T_0 = 77°F, p_0 = 1 atm. Neglect the effect of gravity.

Known:
A stream of hot water at a specified state is obtained from a geothermal source.

Find:
Determine the specific flow exergy.

Schematic and Known Data:



Engineering Model:
(1) The effects of gravity can be neglected.
(2) The velocity is relative to the exergy reference environment for which T_0 = 77°F, p_0 = 1 atm.

Analysis:
Obtain specific flow exergy using:

$$\mathrm{e_f} = h - h_0 - T_0(s - s_0) + \frac{\mathrm{V}^2}{2} \qquad (1)$$

Interpolating from Table A-2E for conditions at the dead state where 77°F:

$$h_0 \approx h_\mathrm{f}(T_0) = 45.09\,\frac{\mathrm{Btu}}{\mathrm{lb}},\, s_0 \approx s_\mathrm{f}(T_0) = 0.08775\,\frac{\mathrm{Btu}}{\mathrm{lb\cdot °R}}$$

The given state is compressed liquid and from Table A-5E:

$$h = 270.53\,\frac{\mathrm{Btu}}{\mathrm{lb}},\, s = 0.43641\,\frac{\mathrm{Btu}}{\mathrm{lb{\cdot}°R}}$$

Inserting values and with assumption (2), Eq. (1) becomes:

$$\mathrm{e_f} = (270.53 - 45.09)\,\frac{\mathrm{Btu}}{\mathrm{lb}} - (537\mathrm{°R})(0.43641 - 0.08775)\,\frac{\mathrm{Btu}}{\mathrm{lb\cdot °R}} + \left(\frac{20^2}{2}\,\frac{\mathrm{ft}^2}{\mathrm{s}^2}\cdot\frac{1\ \mathrm{lbf\cdot s^2}}{32.2\ \mathrm{lb\cdot ft}}\cdot\frac{1\ \mathrm{Btu}}{778\ \mathrm{ft\cdot lbf}}\right) = 38.22\,\frac{\mathrm{Btu}}{\mathrm{lb}}$$

PROBLEM 7.68

Determine the rate of exergy destruction, in Btu/min, for the duct system of Problem 6.107. Let $T_0 = 500°R$, $p_0 = 1$ atm.

ANALYSIS: From the solution to Problem 6.107 $\dot{\sigma}_{cv} = 0.084 \frac{Btu}{min \cdot °R}$.

Then, with $\dot{E}_d = T_0 \dot{\sigma}_{cv}$ we get

$$\dot{E}_d = 500°R \left(0.084 \frac{Btu}{min \cdot °R}\right) = 42 \frac{Btu}{min}$$

PROBLEM 7.69

For the *vortex tube* of Example 6.7, determine the rate of exergy destruction, in Btu per lb of air entering. Referring to this value for exergy destruction, comment on the inventor's claim. Let $T_0 = 530°R$, $p_0 = 1$ atm.

KNOWN: Steady-state operating data are provided for a vortex tube.

FIND: Evaluate the rate of exergy destruction per unit mass of air entering.

SCHEMATIC & GIVEN DATA: See Figure E 6.7

ENGR. MODEL: 1. The assumptions of Example 6.7 apply. 2. $T_0 = 530°R$, $p_0 = 1$ atm.

ANALYSIS: Using $\dot{E}_d = T_0 \dot{\sigma}_{cv}$ and $\dot{\sigma}_{cv}/\dot{m}_1 = 0.1086$ Btu/lb·°R from the solution to Example 6.7, we get

$$\frac{\dot{E}_d}{\dot{m}_1} = T_0 \frac{\dot{\sigma}_{cv}}{\dot{m}} = (530\,°R)(0.1086)\frac{Btu}{lb \cdot °R}$$

$$= 57.6\ \frac{Btu}{lb}$$

The inventor claims the device operates without work or heat transfer. However, the exergy destroyed within the vortex tube can be traced to a device, or devices, located upstream of the vortex tube that would require some combination of work, heat transfer, and fuel input. So, in a larger sense, the inventor's claim is not valid.

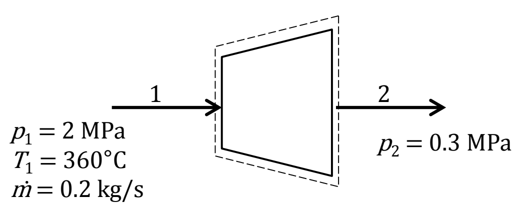
Problem 7.70

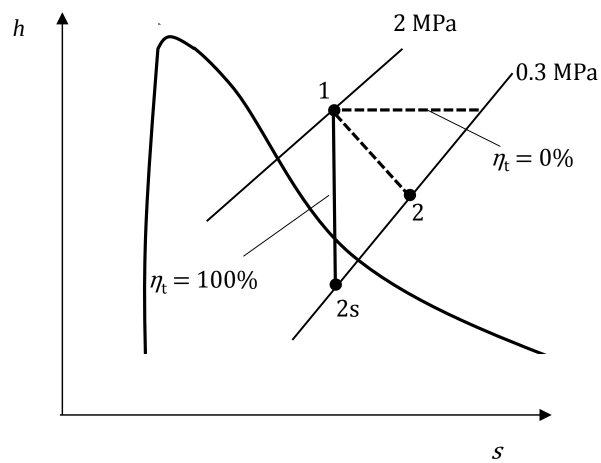
Steam at 2 MPa and 360°C with a mass flow rate of 0.2 kg/s enters an insulated turbine operating at steady state and exhausts at 300 kPa. Plot the temperature of the exhaust steam, in °C, the power developed by the turbine, in kW, and the rate of exergy destruction within the turbine, in kW, each versus the isentropic turbine efficiency ranging from 0 to 100%. Ignore the effects of motion and gravity. Let T_0 = 30°C, p_0 = 0.1 MPa.

Known:
Steady state operating data are provided for a steam turbine.

Find:
Plot the isentropic turbine efficiency ranging from 0 to 100%, the turbine exit temperature, power developed, and the rate of exergy destruction within the turbine.

Schematic and Known Given Data:





Engineering Model:
(1) The control volume shown in the schematic is at steady state.
(2) For the turbine, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects can be neglected.
(3) For the exergy reference environment, T_0 = 303 K, p_0 = 0.1 MPa.

Analysis:
At steady state, mass and energy rate balances reduce to give $\dot{W}_{cv} = \dot{m}(h_1 - h_2)$. Introducing the isentropic turbine efficiency:

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \Longrightarrow \qquad (1)$$

$$\dot{W}_{cv} = \dot{m}\eta_t(h_1 - h_{2s}) \qquad (2)$$

Note that Eq. (1) gives h_2, which together with p_2 fixes T_2. Mass and entropy rate balances reduce to give:

$$\dot{E}_d = \dot{m}T_0(s_2 - s_1) \qquad (3)$$

Sample Calculation at η_t = 80%

From Table A-4:

$$h_1 = 3159.3\,\frac{kJ}{kg}, s_1 = 6.9917\,\frac{kJ}{kg \cdot K}$$

Then, with h_{2s} determined with $s_{2s} = s_1$ and table data from Table A-3 at $p_2 = 0.3$ MPa (3 bar):

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{6.9917 - 1.6718}{6.9919 - 1.6718} = 0.99996,$$

$$h_{2s} = 561.47 + 0.99996(2163.8) = 2725.18\,\frac{\text{kJ}}{\text{kg}}$$

$$h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 3159.3 - 0.8(3159.3 - 2725.18) = 2812.00\,\frac{\text{kJ}}{\text{kg}}$$

State 2 is in superheated vapor region and using Table A-4 and interpolation:

$$\Longrightarrow\ T_2 = 174.28\text{°C}, \qquad s_2 = 7.1933\,\frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

So, with Eqs. (2) and (3):

$$\dot{W}_{\text{cv}} = \left(0.2\,\frac{\text{kg}}{\text{s}}\right)(0.8)(3159.3 - 2725.18)\,\frac{\text{kJ}}{\text{kg}} = 69.46\text{ kW}$$

$$\dot{\text{E}}_{\text{d}} = \left(0.2\,\frac{\text{kg}}{\text{s}}\right)(303\text{ K})(7.1933 - 6.9917)\,\frac{\text{kJ}}{\text{kg}} = 12.22\text{ kW}$$

<u>IT Code</u>:

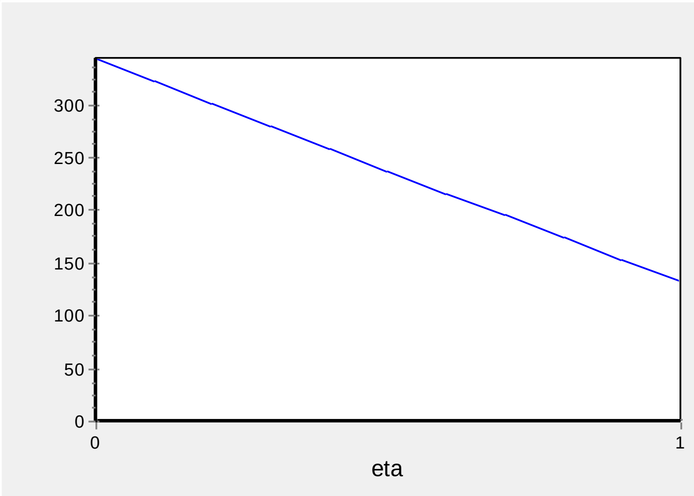
η_t = 80%

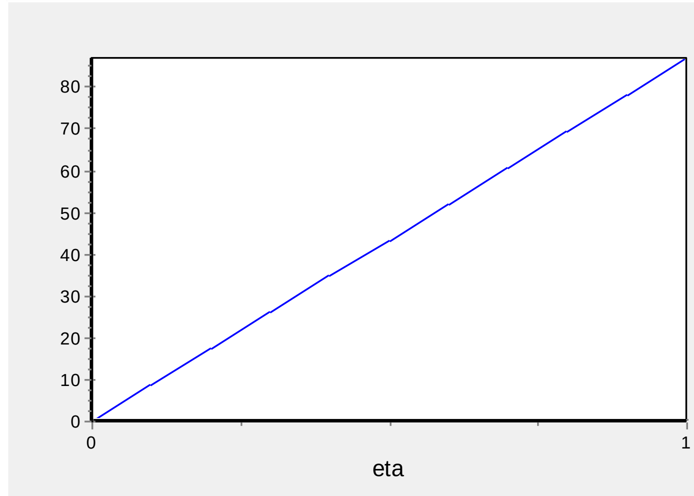
```
p1 = 2000 // kPa
T1 = 360 // C
p2 = 300 // kPa
eta = 0.8
mdot = 0.2 // kg/s
T0 = 303 // K

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
s2s = s1
h2s = h_Ps("Water/Steam", p2, s2s)
h2 = h1 - eta * (h1 - h2s)
s2 = s_Ph("Water/Steam", p2, h2)

T2 = T_Ph("Water/Steam", p2, h2)
Wdot = mdot * (h1 - h2)
Edot = mdot * T0 * (s2 - s1)
```

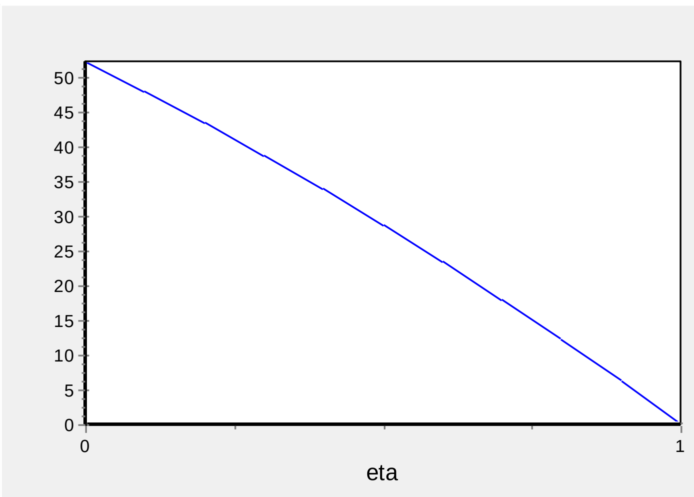
<u>IT Solution for η_t = 0.8:</u>

h2	h2s	s1	T2	Wdot	Edot
2812	2725	6.991	174.2	69.44	12.33

Plots:







Discussion:

1. The case of η_t = 100% corresponds to maximum power and zero exergy destruction.
2. In the case of η_t = 0%, there is no work, and the turbine acts as a throttling process with $h_2 = h_1$.

Problem 7.71

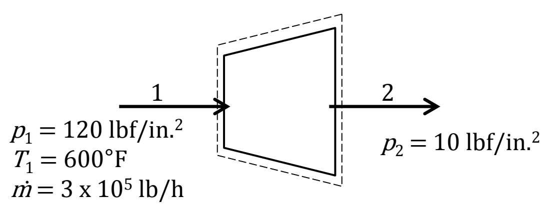
Steam enters an insulated turbine operating at steady state at 120 lbf/in.^{2}, 600°F, with a mass flow rate of 3x10^5 lb/h and expands to a pressure of 10 lbf/in.^{2} The isentropic turbine efficiency is 80%. If exergy is valued at 8 cents per kW·h, determine:
(a) the value of the power produced, in \$/h.
(b) the cost of the exergy destroyed, in \$/h.
(c) plot the values of the power produced and the exergy destroyed, each in \$/h, versus isentropic efficiency ranging from 80 to 100%.

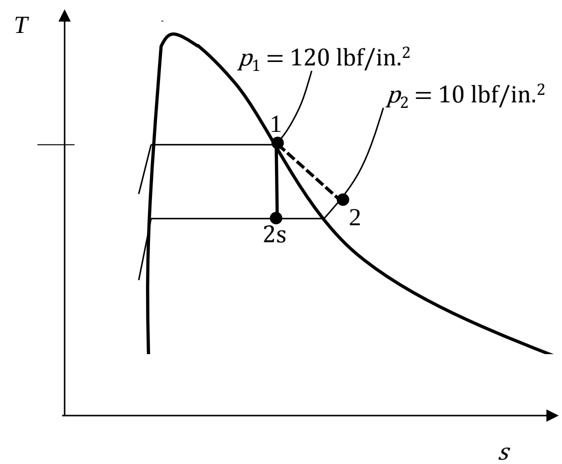
Ignore the effects of motion and gravity. Let T_0 = 70°F, p_0 = 1 atm.

Known:
Steady state operating data are provided for a steam turbine.

Find:
If the isentropic efficiency is 80% and exergy is valued at 8 cents per kW·h, determine the value of the power produced and the cost of the exergy destroyed. Plot the quantities versus isentropic efficiency ranging from 80 to 100%.

Schematic and Known Data:





Engineering Model:
(1) The control volume shown in the schematic is at steady state.
(2) For the turbine, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects can be neglected.
(3) For the exergy reference environment, T_0 = 530°R, p_0 = 1 atm.
(4) Exergy is valued at 8 cents per kW·h.

Analysis:
Reducing mass and energy rate balances and using the definition of isentropic turbine efficiency:
$$\dot{W}_t = \dot{m}(h_1 - h_2) = \eta_t \dot{m}(h_1 - h_{2s}) \qquad (1)$$

Using $\dot{\mathrm{E}}_\mathrm{d} = T_0 \dot{\sigma}_{cv}$ and $\dot{\sigma}_{cv}$ from the mass and entropy balances:
$$\dot{\mathrm{E}}_\mathrm{d} = T_0 \dot{m}(s_2 - s_1) \qquad (2)$$

(a) From Table A-4E, $h_1 = 1327.8\,\frac{\text{Btu}}{\text{lb}}$, $s_1 = 1.7371\,\frac{\text{Btu}}{\text{lb°R}}$.

Then, with h_{2s} determined with $s_{2s} = s_1$ and table data from Table A-3E at $p_2 = 10$ lbf/in.^{2}:

$$x_{2s} = \frac{s_{2s} - s_f}{s_{fg}} = \frac{1.7371 - 0.2836}{1.5041} = 0.9664$$

Then:

$$h_{2s} = h_f + x_{2s}(h_{fg}) = 161.23 + (0.9664)(982.1) = 1110.33\,\frac{\text{Btu}}{\text{lb}}$$

Substituting into Eq. (1):

$$\dot{W}_t = (0.8)\left(3 \cdot 10^5\,\frac{\text{lb}}{\text{h}}\right)(1327.8 - 1110.33)\,\frac{\text{Btu}}{\text{lb}} = 521.93 \cdot 10^5\,\frac{\text{Btu}}{\text{h}}$$

The value is:

$$\dot{\$} = \left(521.93 \cdot 10^5\,\frac{\text{Btu}}{\text{h}} \cdot \left|\frac{1\text{ kW}\cdot\text{h}}{3413\text{ Btu}}\right|\right)\left(0.08\,\frac{\$}{\text{kW}\cdot\text{h}}\right) = 1223.39\,\frac{\$}{\text{h}} \longleftarrow$$

(b) To fix state 2 use Eq. (1) to write

$$h_2 = h_1 - \frac{\dot{W}_t}{\dot{m}} = 1327.8 - \left(\frac{521.93 \cdot 10^5}{3 \cdot 10^5}\right) = 1153.82\,\frac{\text{Btu}}{\text{lb}}$$

Then at $p_2 = 10$ lbf/in.^{2} and using Table A-3E, state 2 is superheated vapor. Using Table A-4E at p_2 and h_2 and interpolating:

$$s_2 = 1.8033\,\frac{\text{Btu}}{\text{lb}\cdot\text{°R}}$$

Substituting into Eq. (2):

$$\dot{\text{E}}_d = (530\text{°R})\left(3 \cdot 10^5\,\frac{\text{lb}}{\text{h}}\right)(1.8033 - 1.7371)\,\frac{\text{Btu}}{\text{lb}\cdot\text{°R}} = 105.26 \cdot 10^5\,\frac{\text{Btu}}{\text{h}} \longleftarrow$$

The cost rate is:

$$\dot{\$} = \left(105.26 \cdot 10^5\,\frac{\text{Btu}}{\text{h}} \cdot \left|\frac{1\text{ kW}\cdot\text{h}}{3413\text{ Btu}}\right|\right)\left(0.08\,\frac{\$}{\text{kW}\cdot\text{h}}\right) = 246.72\,\frac{\$}{\text{h}}$$

(c) <u>IT Code</u>

```
p1 = 120 // lbf/in.^2
T1 = 600 // F
p2 = 10 // lbf/in.^2
eta = 80 // %
mdot = 300000 // lb/h
T0 = 530 // R

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
```

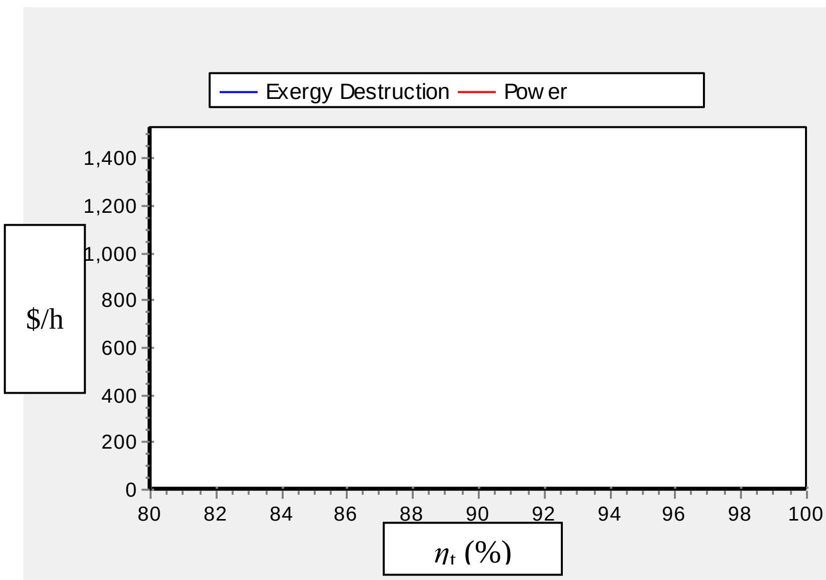
s2s = s1
h2s = h_Ps("Water/Steam", p2, s2s)
h2 = h1 - (eta / 100) * (h1 - h2s)
s2 = s_Ph("Water/Steam", p2, h2)

Wdot = mdot * (h1 - h2)
CostW = Wdot * (0.08 / 3413)
Edot = mdot * T0 * (s2 - s1)
CostE = Edot * (0.08 / 3413)

<u>IT Solution for $\eta_t = 0.8$:</u>

h2	s2	Wdot	CostE	CostW	Edot
1154	1.803	5.222E7	247.4	1224	1.056E7

Plots:



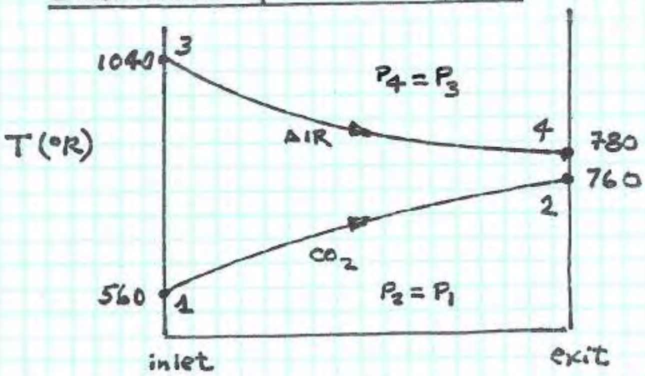
<u>Discussion:</u>
At higher values of η_t, there is less exergy destruction and greater power output, as expected.

PROBLEM 7.72

7.72 Consider the parallel flow heat exchanger of Prob. 4.82. Verify that the stream of air exits at 780°R and the stream of carbon dioxide exits at 760°R. Pressure is constant for each stream. For the heat exchanger, determine the rate of exergy destruction, in Btu/s. Let T_0 = 537°R, p_0 = 1 atm.

See solution to Prob. 4.82 for details, including the evaluation of the two exiting temperatures given here as 780°R, 760°R.

SCHEMATIC & GIVEN DATA:



$\dot{m}_1 = 73.7$ lb/s , $\dot{m}_3 = 50$ lb/s

ANALYSIS: The rate of exergy destruction can be found from an exergy rate balance or using $\dot{E}_d = T_0 \dot{\sigma}$, where $\dot{\sigma}$ is from an entropy rate balance. Selecting the second of these approaches, an entropy rate balance reduces at steady state to give

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{\,0} + \dot{m}(s_1 - s_2) + \dot{m}_3(s_3 - s_4) + \dot{\sigma}$$

Solving for $\dot{\sigma}$ and introducing expressions for entropy change of ideal gases,

$$\dot{\sigma} = \dot{m}_1(s_2 - s_1) + \dot{m}_3(s_4 - s_3)$$

$$= \dot{m}_1\left[\frac{\bar{s}_2^\circ - \bar{s}_1^\circ - \bar{R}\cancel{\ln p_2/p_1}^{\,0}}{M_{CO_2}}\right] + \dot{m}_3\left[s_4^\circ - s_3^\circ - \frac{\bar{R}}{M_{air}}\cancel{\ln p_4/p_3}^{\,0}\right]$$

$$= \dot{m}_1\left[\frac{\bar{s}_2^\circ - \bar{s}_1^\circ}{M_{CO_2}}\right] + \dot{m}_3\left[s_4^\circ - s_3^\circ\right]$$

with data from Tables A-22E and A-23E,

$$\dot{\sigma} = \left(\frac{73.7\ \text{lb/s}}{44.01\ \text{lb/lbmol}}\right)\left(54.319 - 51.408\right)\frac{\text{Btu}}{\text{lbmol}\cdot{}^\circ\text{R}} + \left(50\frac{\text{lb}}{\text{s}}\right)\left(0.68942 - 0.76019\right)\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$

$$= [4.8748 - 3.5385]\ \frac{\text{Btu}}{\text{s}\cdot{}^\circ\text{R}} = 1.3363\ \frac{\text{Btu}}{\text{s}\cdot{}^\circ\text{R}}$$

Finally,

$$\dot{E}_d = T_0\,\dot{\sigma} = (537^\circ\text{R})\left(1.3363\ \frac{\text{Btu}}{\text{s}\cdot{}^\circ\text{R}}\right)$$

$$= 717.6\ \frac{\text{Btu}}{\text{s}}$$

Problem 7.73

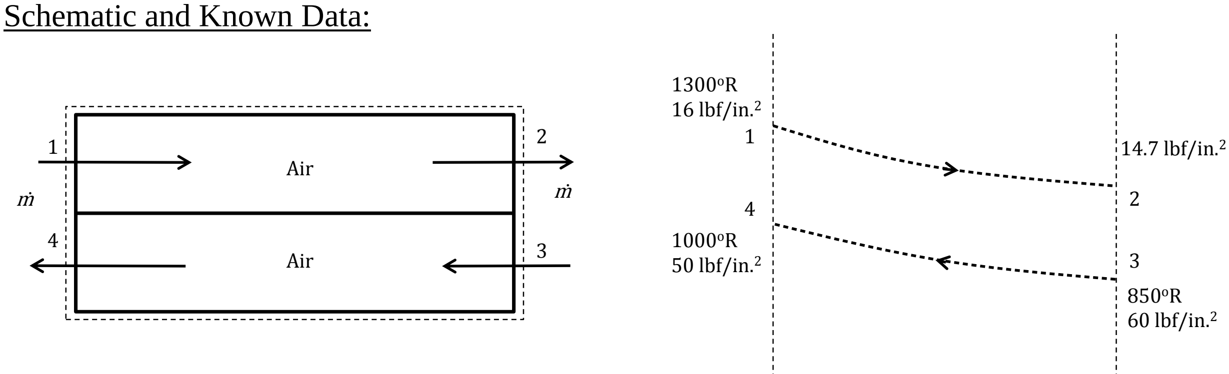
Air at T_1 = 1300°R, p_1 = 16 lbf/in.^{2} enters a counterflow heat exchanger operating at steady state and exits p_2 = 14.7 lbf/in.^{2} A separate stream of air enters at T_3 = 850°R, p_3 = 60 lbf/in.^{2} and exits at T_4 = 1000°R, p_4 = 50 lbf/in.^{2} The mass flow rate of the streams are equal. Stray heat transfer and the effects of motion and gravity can be ignored. Assuming the ideal gas model with c_p = 0.24 Btu/lb·°R, determine (a) T_2, in °R, (b) the rate of exergy destruction within the heat exchanger, in Btu per pound of air flowing, and (c) plot the exergy destruction rate (in Btu per lb of air flowing) versus p_2 ranging from 1 to 50 lbf/in.^{2}

Let T_0 = 520°R, p_0 = 1 atm.

Known:
Steady state operating data are provided for a heat exchanger with two separate streams of air flowing through it.

Find:
Determine (a) the temperature at the outlet of the first stream, and (b) the rate of exergy destruction within the heat exchanger. (c) Plot the rate of exergy destruction versus p_2 ranging from 1 to 50 lbf/in.^{2}

Schematic and Known Data:



Engineering Model:
(1) The control volume shown in the schematic is at steady state.
(2) For the control volume, stray heat transfer and the effects of motion and gravity can be neglected.
(3) The air is modeled as an ideal gas with c_p = 0.24 Btu/lb·°R.
(4) For the exergy reference environment, T_0 = 520°R, p_0 = 1 atm.

Analysis:
(a) An exergy rate balance reduces to read:

$$0 = \underbrace{\dot{Q}_{cv}}_{=0} - \underbrace{\dot{W}_{cv}}_{=0} + \dot{m}(h_1 - h_2) + \dot{m}(h_3 - h_4) \Longrightarrow 0 = c_p(T_1 - T_2) + c_p(T_3 - T_4)$$

$$T_2 = T_1 + T_3 - T_4 = 1300 + 850 - 1000 = 1150°R$$

(b) To determine $\dot{E}_d$ it is convenient to use $\dot{E}_d = T_0\dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is obtained from an entropy rate balance:

$$0 = \sum \underbrace{\frac{\dot{Q}_{\mathrm{j}}}{T_{\mathrm{j}}}}_{=0} + \dot{m}(s_1 - s_2) + \dot{m}(s_3 - s_4) + \sigma_{\mathrm{cv}} \Longrightarrow$$

$$\frac{\sigma_{\mathrm{cv}}}{\dot{m}} = s_2 - s_1 + s_4 - s_3$$

Combining:

$$\frac{\dot{\mathrm{E}}_{\mathrm{d}}}{\dot{m}} = T_0(s_2 - s_1 + s_4 - s_3) = T_0\left[c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} + c_p \ln\frac{T_4}{T_3} - R\ln\frac{p_4}{p_3}\right]$$
$$= (520°\mathrm{R})\left[\left(0.24\frac{\mathrm{Btu}}{\mathrm{lb}\cdot°\mathrm{R}}\right)\ln\frac{1150}{1300} - \left(\frac{1.986}{28.97}\frac{\mathrm{Btu}}{\mathrm{lb}\cdot°\mathrm{R}}\right)\ln\frac{14.7}{16}\right.$$
$$\left. + \left(0.24\frac{\mathrm{Btu}}{\mathrm{lb}\cdot°\mathrm{R}}\right)\ln\frac{1000}{850} - \left(\frac{1.986}{28.97}\frac{\mathrm{Btu}}{\mathrm{lb}\cdot°\mathrm{R}}\right)\ln\frac{50}{60}\right] = 14.5\frac{\mathrm{Btu}}{\mathrm{lb}}$$

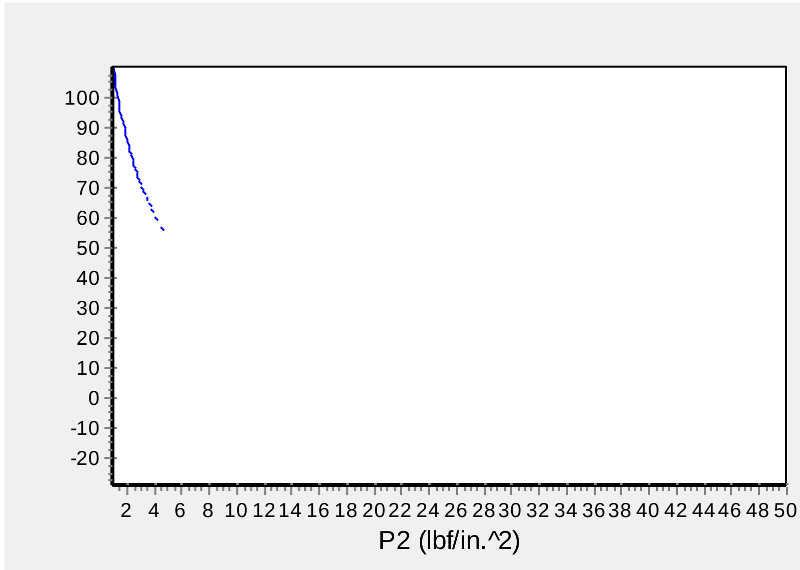
(c) IT Code

```
p1 = 16 // lbf/in.^2
T1 = 1300 // R
p2 = 14.7 // lbf/in.^2
T3 = 850 // R
p3 = 60 // lbf/in.^2
T4 = 1000 // R
p4 = 50 // lbf/in.^2
T0 = 520 // R

T2 = T1 + T3 - T4
s1 = s_Tp("Air",T1,p1)
s2 = s_Tp("Air",T2,p2)
s3 = s_Tp("Air",T3,p3)
s4 = s_Tp("Air",T4,p4)

Ed = T0 * (s2 - s1 + s4 - s3)
```

IT Solution for p_2 = 14.7:

Ed	s1	s2	s3	s4	T2
14.06	0.8108	0.7853	0.6139	0.6665	1150

Plot:



Comment:

1. The answer to part (c) is slightly different than the answer attained in part (b) due to rounding errors associated with assumption (3).
2. The plot obtained in part (c) eventually reaches a negative exergy destruction rate at $p_2 \approx 21.9\,\frac{lbf}{in.^2}$. This indicates that the second law is violated once p_2 exceeds this value, meaning that the problem setup is impossible at that point. Otherwise, the plot conforms to the expected shape, as the exergy destruction rate decreases as the pressure at point two increases.
3. In an actual application, achieving a p_2 value which exceeds the p_1 value of $16\,\frac{lbf}{in.^2}$ would require a device within the control volume to increase the stream's air pressure.

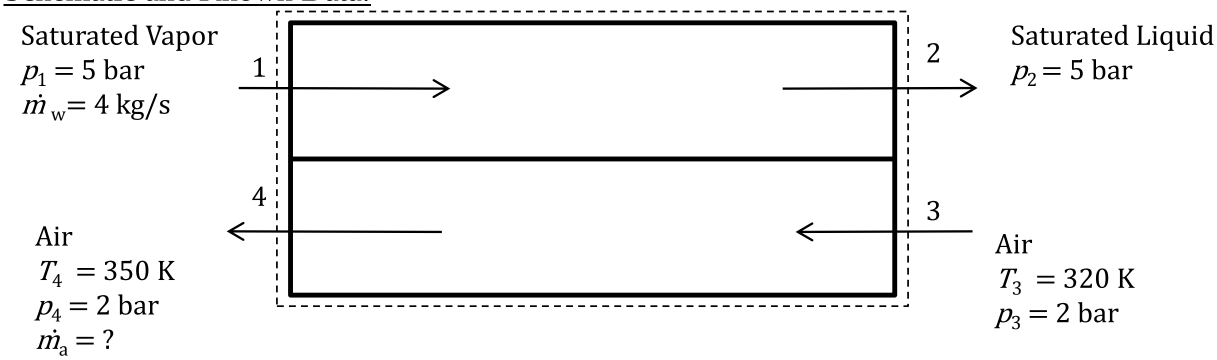
Problem 7.74

A counterflow heat exchanger operating at steady state has water entering as saturated vapor at 5 bar with a mass flow rate of 4 kg/s and exiting as saturated liquid at 5 bar. Air enters in a separate stream at 320 K, 2 bar and exits at 350 K with negligible change in pressure. Heat transfer between the heat exchanger and its surroundings is negligible. Determine (a) the change in the flow exergy rate of each stream, in kW and (b) the rate of exergy destruction in the heat exchanger, in kW. Ignore the effects of motion and gravity. Let T_0 = 300 K, p_0 = 1 atm.

Known:
Water and air flow on opposite sides of a counterflow heat exchanger operating at steady state. Operating data are provided.

Find:
Determine (a) the change in flow exergy rate of each stream, and (b) the rate of exergy destruction.

Schematic and Known Data:



Engineering Model:
(1) The control volume shown in the schematic is at steady state.
(2) For the control volume, $\dot{W}_{cv} = \dot{Q}_{cv} = 0$ and the effects of kinetic and potential energy can be neglected.
(3) The air is modeled as an ideal gas.
(4) For the environment, T_0 = 300 K, p_0 = 1 atm.

Analysis:
First we use mass and energy rate balance to determine $\dot{m}_\mathrm{a}$. That is $\dot{m}_1 = \dot{m}_2 = \dot{m}_\mathrm{w}$ and $\dot{m}_3 = \dot{m}_4 = \dot{m}_\mathrm{a}$. Thus, with assumptions (1) and (2):

$$0 = \underbrace{\dot{Q}_{cv}}_{=0} - \underbrace{\dot{W}_{cv}}_{=0} + \dot{m}(h_1 - h_2) + \dot{m}_\mathrm{a}(h_3 - h_4) \qquad (1)$$

Or, solving for $\dot{m}_a$ and inserting enthalpy data from Tables A-3 and A-22:

$$\dot{m}_\mathrm{a} = \dot{m}_\mathrm{w}\left(\frac{h_1 - h_2}{h_4 - h_3}\right) = \left(4\,\frac{\mathrm{kg}}{\mathrm{s}}\right)\left(\frac{2108.5}{350.49 - 320.29}\right) = 279.27\,\frac{\mathrm{kg}}{\mathrm{s}}$$

(a) The change in flow exergy rate for the water stream is:

$$\begin{aligned}\dot{\mathrm{E}}_{\mathrm{f}_2} - \dot{\mathrm{E}}_{\mathrm{f}_1} &= \dot{m}_{\mathrm{w}}(\mathrm{e}_2 - \mathrm{e}_1) = \dot{m}_{\mathrm{w}}[h_2 - h_1 - T_0(s_2 - s_1)] \\ &= \left(4\,\frac{\mathrm{kg}}{\mathrm{s}}\right)\left[-2108.5\,\frac{\mathrm{kJ}}{\mathrm{kg}} - (300\ \mathrm{K})(1.8607 - 6.8212)\,\frac{\mathrm{kJ}}{\mathrm{kg}\cdot\mathrm{K}}\right] \\ &= -2481.4\ \mathrm{kW}\end{aligned}$$

For the air stream, using data from Table A-22:

$$\begin{aligned}\dot{\mathrm{E}}_{\mathrm{f}_4} - \dot{\mathrm{E}}_{\mathrm{f}_3} &= \dot{m}_{\mathrm{a}}(\mathrm{e}_4 - \mathrm{e}_3) = \dot{m}_{\mathrm{a}}\left[h_4 - h_3 - T_0\left(s_4^o - s_3^o - \underbrace{R\ln\frac{p_4}{p_3}}_{=0}\right)\right] \\ &= \left(279.27\,\frac{\mathrm{kg}}{\mathrm{s}}\right)\left[(350.49 - 320.29)\,\frac{\mathrm{kJ}}{\mathrm{kg}}\right. \\ &\quad \left. - (300\ \mathrm{K})(1.85708 - 1.76690)\,\frac{\mathrm{kJ}}{\mathrm{kg}\cdot\mathrm{K}}\right] = 878.58\ \mathrm{kW}\end{aligned}$$

(b) At steady state, the exergy rate balances reduces to:

$$0 = \sum_j \left[1 - \frac{T_0}{T_j}\right]\underbrace{\dot{Q}_j}_{=0} - \left[\underbrace{\dot{W}_{cv}}_{=0} - p_0\underbrace{\frac{dV}{dT}}_{=0}\right] + \dot{m}\left(\mathrm{e}_{\mathrm{f}_1} - \mathrm{e}_{\mathrm{f}_2}\right) + \dot{m}_a\left(\mathrm{e}_{\mathrm{f}_3} - \mathrm{e}_{\mathrm{f}_4}\right) - \dot{\mathrm{E}}_{\mathrm{d}}$$

Thus:

#1 $$\dot{\mathrm{E}}_{\mathrm{d}} = -\left[\dot{\mathrm{E}}_{\mathrm{f}_2} - \dot{\mathrm{E}}_{\mathrm{f}_1}\right] - \left[\dot{\mathrm{E}}_{\mathrm{f}_4} - \dot{\mathrm{E}}_{\mathrm{f}_3}\right] = -(-2481.4) - 878.58 = 1602.82\ \mathrm{kW}$$

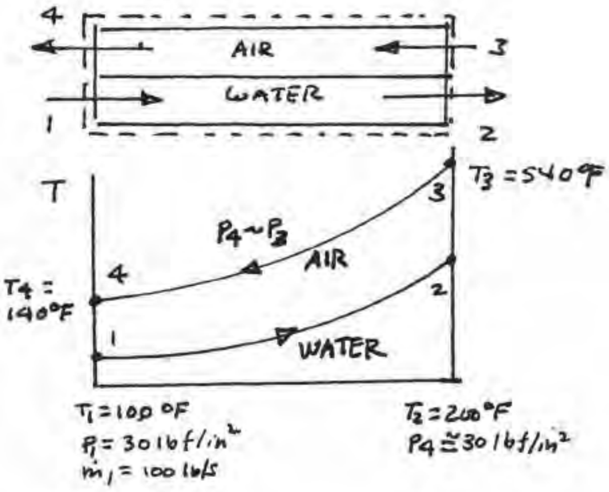
Comment**:**

1. The decrease in energy in the water stream exactly equals the increase in energy in the air stream, for energy is conserved—see Eq. (1). However, because exergy is destroyed, the decrease in exergy of the water stream differs significantly from the increase of exergy of the air stream. Indeed, the decrease in exergy of the water stream is accounted for as the increase in exergy of the air stream plus the exergy destroyed.

PROBLEM 7.75

Water at $T_1 = 100°F$, $p_1 = 30$ lbf/in.^{2} enters a counterflow heat exchanger operating at steady state with a mass flow rate of 100 lb/s and exits at $T_2 = 200°F$ with closely the same pressure. Air enters in a separate stream at $T_3 = 540°F$ and exits at $T_4 = 140°F$ with no significant change in pressure. Air can be modeled as an ideal gas and stray heat transfer can be ignored. Determine (a) the mass flow rate of the air, in lb/s, and (b) the rate of exergy destruction within the heat exchanger, in Btu/s. Ignore the effects of motion and gravity and let $T_0 = 60°F$, $p_0 = 1$ atm.

SCHEMATIC + GIVEN DATA:



ENGR MODEL:

1. The control volume shown in the sketch is at steady state.
2. Stray heat transfer and the effects of motion and gravity can be ignored.
3. The air is modeled as an ideal gas.
4. For the liquid water at 1 and 2, $h \approx h_f(T)$, $s \approx s_f(T)$.
5. For the environment, $T_0 = 520°R$, $p_0 = 1$ atm

ANALYSIS: At 200 lbf/in^2, $P_{sat} = 381.86°F$. Thus, the water is a liquid at 1 and 2.

(a) An energy rate balance reduces to, $0 = \cancel{\dot Q_{cv}} - \cancel{\dot W_{cv}} + \dot m_1 (h_1 - h_2) + \dot m_3 (h_3 - h_4)$

$$\Rightarrow \quad \dot m_3 = \dot m_1 \left[\frac{h_2 - h_1}{h_3 - h_4}\right] = 100\frac{lb}{s}\left[\frac{168.07 - 68.05}{240.98 - 143.47}\right] = 102.6\,\frac{lb}{s} \qquad (a)$$

where data are from Tables A-2E and A-22E.

(b) To find $\dot E_d$, it is convenient to use $\dot E_d = T_0 \dot\sigma_{cv}$, where $\dot\sigma_{cv}$ is from an entropy rate balance:

$$\dot E_d = T_0 \dot\sigma = T_0\left[\dot m_1 (s_2 - s_1) + \dot m_3 (s_4 - s_3)\right]$$

$$= T_0\left[\dot m_1 (s_f(T_2) - s_f(T_1)) + \dot m_3 \left(s_4^o - s_3^o - \overbrace{\cancel{R \ln P_4/P_3}}^{0}\right)\right]$$

$$= 520°R\left[100\frac{lb}{s}(0.2940 - 0.1296)\frac{Btu}{lb\cdot°R} + 102.6(0.62607 - 0.75042)\right]$$

$$= 1914.5\ \frac{Btu}{s} \qquad (b)$$

Problem 7.76

Air enters a counterflow heat exchanger operating at steady state at 27°C, 0.3 MPa and exits at 12°C. Refrigerant 134a enters at 0.4 MPa, a quality of 0.3, and a mass flow rate of 35 kg/h. Refrigerant exits at 10°C. Stray heat transfer is negligible and there is no significant change in pressure for either stream.
(a) For the Refrigerant 134a stream, determine the rate of heat transfer, in kJ/h.
(b) For each of the streams, evaluate the change in flow exergy rate, in kJ/h, and interpret its value and sign.

Let T_0 = 22°C, p_0 = 0.1 MPa, and ignore the effects of motion and gravity.

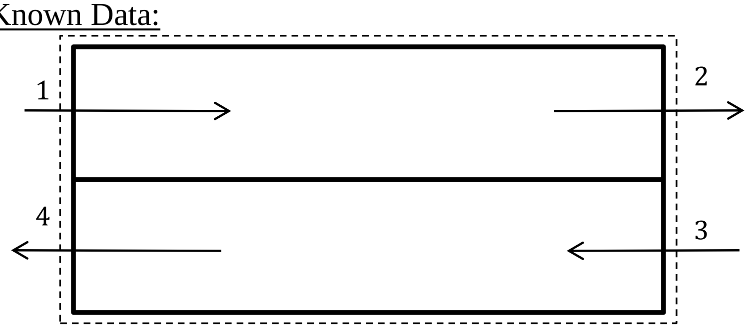
<u>Known:</u>
Operating data are provided for a counterflow heat exchanger at steady state. One stream is R-134a and the other is air.

<u>Find:</u>
(a) For the R-134a, determine the rate of heat transfer, and (b) for each of the two streams, evaluate the change in flow exergy rate and discuss.

<u>Schematic and Known Data:</u>

Air
$T_1 = 27°\text{C}$
$p_1 = 0.3$ MPa
$\dot{m}_a = ?$

R-134a
$T_4 = 10°\text{C}$
$p_4 = p_3$



Air
$T_2 = 12°\text{C}$
$p_2 = p_1$

R-134a
$p_3 = 0.4$ MPa = 4 bar
$x_3 = 0.3$
$\dot{m}_{\text{ref}} = 35$ kg/h

<u>Engineering Model:</u>
(1) The control volume shown in the schematic is at steady state. With $\dot{W}_{\text{cv}} = \dot{Q}_{\text{cv}} = 0$ and negligible effects of motion and gravity.
(2) For a control volume enclosing only the refrigerant stream, the foregoing applies, except $\dot{Q}_{\text{cv}} \neq 0$.
(3) For each stream, pressure change is negligible.
(4) The air is modeled as an ideal gas.
(5) For the environment, T_0 = 22°C =295 K, p_0 = 0.1 MPa.

<u>Analysis:</u>
(a) For a control volume enclosing only the refrigerant stream, the mass and energy rate balances reduce to give:

$$0 = \dot{Q}_{\text{cv}} - \underbrace{\dot{W}_{\text{cv}}}_{=0} + \dot{m}_{\text{ref}}(h_3 - h_4) \Longrightarrow \dot{Q}_{\text{cv}} = \dot{m}_{\text{ref}}(h_4 - h_3)$$

From Tables A-11 and A-12 at the provided conditions, $h_3 = 119.096\,\frac{\text{kJ}}{\text{kg}}$, $h_4 = 253.35\,\frac{\text{kJ}}{\text{kg}}$. Thus:

$$\dot{Q}_{\text{cv}} = \left(35\,\frac{\text{kg}}{\text{h}}\right)(253.35 - 119.096)\,\frac{\text{kJ}}{\text{kg}} = 4698.9\,\frac{\text{kJ}}{\text{h}}$$

#1 (b) The change in flow exergy rate for the refrigerant stream is, with data from Tables A-11 and A-12:

$$\left(\Delta\dot{\text{E}}_{\text{f}}\right)_{\text{ref}} = \dot{m}_{\text{ref}}[(h_4 - h_3) - T_0(s_4 - s_3)]$$

$$= \left(35\,\frac{\text{kg}}{\text{h}}\right)\left[(253.35 - 119.096)\,\frac{\text{kJ}}{\text{kg}} - (295\text{ K})(0.9182 - 0.44228)\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right] = -215\,\frac{\text{kJ}}{\text{h}}$$

To determine the changes in flow exergy for the air, we first evaluate $\dot{m}_{\text{a}}$ using mass and energy rate balances for the overall control volume.

$$0 = \underbrace{\dot{Q}_{\text{cv}}}_{=0} - \underbrace{\dot{W}_{\text{cv}}}_{=0} + \dot{m}_{\text{a}}(h_1 - h_2) + \dot{m}_{\text{ref}}(h_3 - h_4)$$

Rearranging and inserting values from Tables A-11, 12, and 22:

$$\dot{m}_{\text{a}} = \dot{m}_{\text{ref}}\left[\frac{h_4 - h_3}{h_1 - h_2}\right] = \left(35\,\frac{\text{kg}}{\text{h}}\right)\left[\frac{253.35 - 119.096}{300.19 - 285.14}\right] = 312.22\,\frac{\text{kg}}{\text{h}}$$

Now, the change in exergy rate of the air is:

$$\left(\Delta\dot{\text{E}}_{\text{f}}\right)_{\text{air}} = \dot{m}_{\text{a}}\left(\text{e}_{\text{f}_2} - \text{e}_{\text{f}_1}\right) = \dot{m}_{\text{a}}[h_2 - h_1 - T_0(s_2 - s_1)]$$

$$= \dot{m}_{\text{a}}\left[h_2 - h_1 - T_0\left(s^o(T_2) - s^o(T_1) - R\,\underbrace{\ln\frac{p_2}{p_1}}_{=0}\right)\right]$$

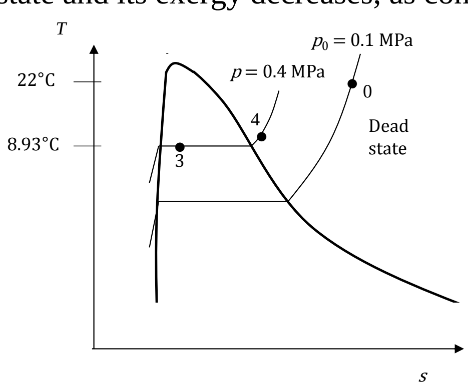
With data from Table A-22:

#2 $$\left(\Delta\dot{\text{E}}_{\text{f}}\right)_{\text{air}} = \left(312.22\,\frac{\text{kg}}{\text{h}}\right)\left[(285.14 - 300.19)\,\frac{\text{kJ}}{\text{kg}} - (295\text{ K})(1.65055 - 1.70203)\,\frac{\text{kJ}}{\text{kg·K}}\right]$$

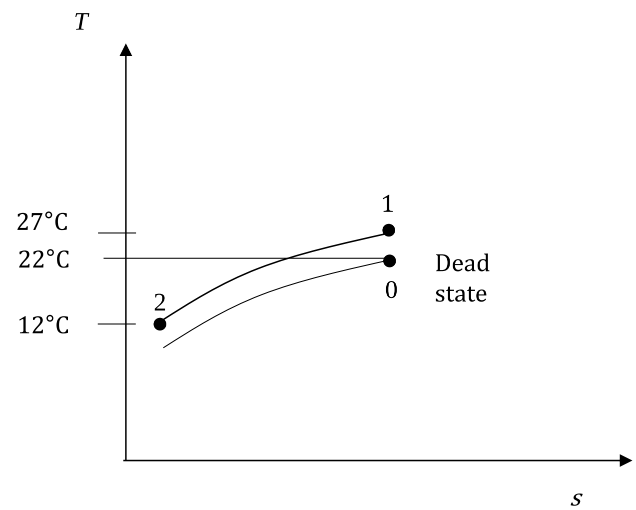
#3 $$\left(\Delta\dot{\text{E}}_{\text{f}}\right)_{\text{air}} = 42.65\,\frac{\text{kJ}}{\text{h}}$$

Comments:

1. As the refrigerant flows through the heat exchanger, its state is brought closer to the dead state and its exergy decreases, as confirmed by this calculation.



2. As the air is brought at constant pressure from above the dead state to a lower temperature, its exergy increases as confirmed by this calculation.

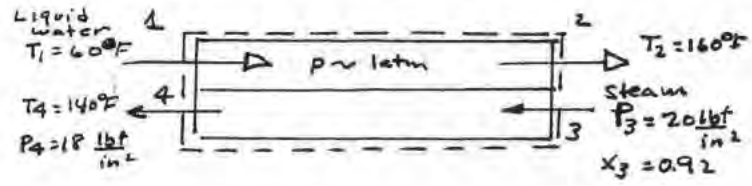


3. When heat transfer occurs at temperatures below T_0, as in the present case (generally), the accompanying exergy transfer occurs from the warmer air to the cooler refrigerant. Still, exergy transfer is from the refrigerant. Further, the exergy transferred from the refrigerant is accounted for by the exergy increase of the air and by the exergy destroyed within the heat exchanger owing to spontaneous heat transfer.

PROBLEM 7.77

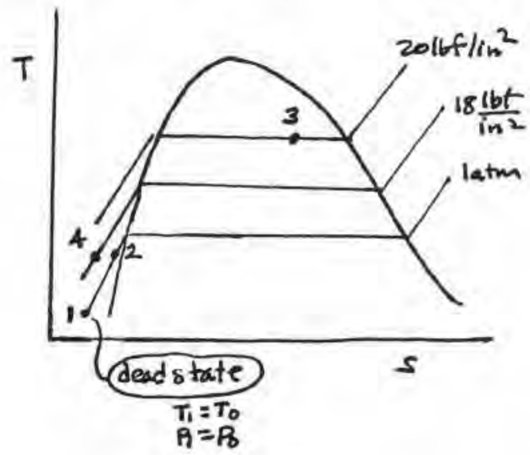
Liquid water enters a heat exchanger operating at steady state at $T_1 = 60°F$, $p_1 = 1$ atm and exits at $T_2 = 160°F$ with a negligible change in pressure. In a separate stream, steam enters at $T_3 = 20$ lbf/in², $x_3 = 92\%$ and exits at $T_4 = 140°F$, $p_4 = 18$ lbf/in². Stray heat transfer and the effects of motion and gravity are negligible. Let $T_0 = 60°F$, $p_0 = 1$ atm. Determine (a) the ratio of the mass flow rates of the two streams and (b) the rate of exergy destruction, in Btu per lb of steam entering the heat exchanger.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the schematic is at steady state.
2. Stray heat transfer and the effects of motion and gravity are negligible.
3. At states 1, 2, and 4, $h \sim h_f(T)$, $s \sim s_f(T)$.
4. Let $T_0 = 520°R\,(60°F)$, $p_0 = 1$ atm.



ANALYSIS:

(a) An energy rate balance reads: $0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_4)$

$$\Rightarrow \quad \frac{\dot{m}_1}{\dot{m}_3} = \frac{h_3 - h_4}{h_2 - h_1} = \frac{1079.55 - 107.96}{127.96 - 28.08} = 9.73 \ \frac{\text{lb(liquid)}}{\text{lb(steam)}} \qquad \text{(a)}$$

where $h_3 = h_f + x_3(h_g - h_f) = 196.26 + 0.92(960.1) = 1079.55$ Btu/lb

(b) To find $\dot{E}_d$ it is convenient to use $\dot{E}_d = T_0\dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is obtained from an entropy balance. That is,

$$\dot{E}_d = T_0\dot{\sigma}_{cv} = T_0[\dot{m}_1(s_2 - s_1) + \dot{m}_3(s_4 - s_3)]$$

$$\Rightarrow \quad \frac{\dot{E}_d}{\dot{m}_3} = T_0\left[\frac{\dot{m}_1}{\dot{m}_3}(s_2 - s_1) + (s_4 - s_3)\right]$$

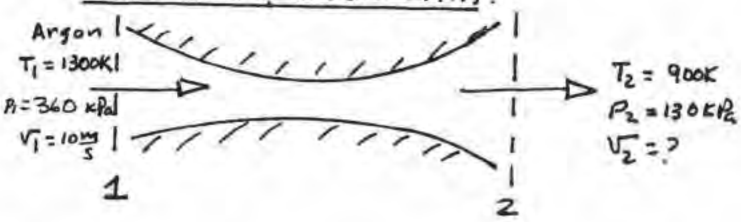
$$= 520°R\left[9.73(0.2313 - 0.05555) + (0.1985 - 1.6203)\right]\frac{\text{Btu/°R}}{\text{lb(steam)}} = 149.9 \ \frac{\text{Btu}}{\text{lb(steam)}} \qquad \text{(b)}$$

where $s_3 = s_f + x_3(s_g - s_f) = 0.3358 + 0.92(1.3962) = 1.6203$ Btu/lb·°R.

PROBLEM 7.78

Argon enters a nozzle operating at steady state at 1300 K, 360 kPa with a velocity of 10 m/s and exits the nozzle at 900 K, 130 kPa. Stray heat transfer can be ignored. Modeling argon as an ideal gas with $k = 1.67$, determine (a) the velocity at the exit, in m/s, and (b) the rate of exergy destruction, in kJ per kg of argon flowing. Let $T_0 = 293$ K, $p_0 = 1$ bar.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the figure is at steady state.
2. Stray heat transfer and the effects of gravity can be ignored.
3. The argon is modeled as an ideal gas with $k = 1.67$.
4. For the environment, $T_0 = 293$ K, $P_0 = 1$ bar.

ANALYSIS: With Eq. 3.47a, $c_p = \frac{kR}{k-1} = \left(\frac{1.67}{0.67}\right)\frac{8.314}{39.94}\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 0.52\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

(a) Reducing an energy rate balance,

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + \cancel{g(z_1 - z_2)}\right]$$

$$\Rightarrow \quad V_2 = \sqrt{V_1^2 + 2\underbrace{(h_1 - h_2)}_{c_p(T_1 - T_2)}} = \sqrt{V_1^2 + 2c_p(T_1 - T_2)}$$

$$\therefore \quad V_2 = \sqrt{\left(10\frac{\text{m}}{\text{s}}\right)^2 + 2\left(0.52\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(1300 - 900)\text{K}\left|\frac{10^3\,\text{N}\cdot\text{m}}{1\,\text{kJ}}\right|\left|\frac{1\,\text{kg}\cdot\text{m/s}^2}{1\,\text{N}}\right|} = 645\frac{\text{m}}{\text{s}} \quad \longleftarrow \text{(a)}$$

(b) To find $\dot{E}_d$ it is convenient to use $\dot{E}_d = T_0\dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is obtained from an entropy rate balance. Thus, with Eq. 6.22,

$$\frac{\dot{E}_d}{\dot{m}} = T_0[s_2 - s_1] = T_0\left[c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}\right]$$

$$= 293\text{K}\left[0.52\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\ln\frac{900}{1300} - \frac{8.314}{39.94}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\ln\left(\frac{130}{360}\right)\right]$$

$$= 6.10\frac{\text{kJ}}{\text{kg}} \quad \longleftarrow \text{(b)}$$

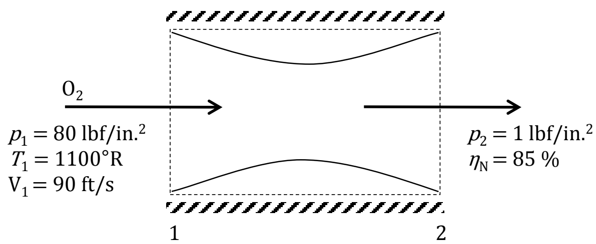
Problem 7.79

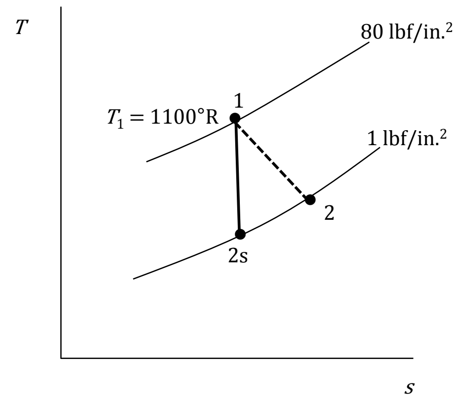
Oxygen (O_2) enters a well insulated nozzle operating at steady state at 80 lbf/in.^{2}, 1100°R, 90 ft/s. At the nozzle exit, the pressure is 1 lbf/in.^{2} The isentropic nozzle efficiency is 85%. For the nozzle, determine the exit velocity, in ft/s, and the exergy destruction rate, in Btu per lb of oxygen flowing. Let T_0 = 70°F, p_0 = 14.7 lbf/in.^{2}

Known:
Oxygen enters a well insulated nozzle operating at steady state at a given state and expands to a specified pressure. The isentropic nozzle efficiency is known.

Find:
Determine the exit velocity and the exergy destruction rate.

Schematic and Known Data:





Engineering Model:
(1) The control volume shown in the schematic is at steady state.
(2) For the control volume, $\dot{W}_{cv} = \dot{Q}_{cv} = 0$ and potential energy effects are negligible.
(3) The oxygen is modeled as an ideal gas.
(4) For the environment, T_0 = 530°R, p_0 = 14.7 lbf/in.^{2}

Analysis:
To determine the exergy destruction rate, we use mass and entropy rate balances and $\dot{\mathrm{E}}_{\mathrm{d}} = T_0\dot{\sigma}_{cv}$ to get:

$$0 = \sum_j \underbrace{\left(\frac{\dot{Q}}{T}\right)_j}_{=0} + \dot{m}(s_1 - s_2) + \dot{\sigma} \Longrightarrow \frac{\dot{E}_{\mathrm{d}}}{\dot{m}} = T_0(s_2 - s_1) \qquad (1)$$

To fix state 2, consider mass and energy balances which reduce as follows:

$$0 = \underbrace{\dot{Q}_{\mathrm{cv}}}_{=0} - \underbrace{\dot{W}_{\mathrm{cv}}}_{=0} + \dot{m}\left[\frac{\bar{h}_1 - \bar{h}_2}{M} + \frac{\mathrm{V}_1^2 - \mathrm{V}_2^2}{2} + g\,\underbrace{(z_1 - z_2)}_{=0}\right]$$

Or:

$$\bar{h}_2 = \bar{h}_1 + M\left(\frac{\mathrm{V}_1^2 - \mathrm{V}_2^2}{2}\right) \qquad (2)$$

To obtain V_2 we use:

$$\eta_N = \frac{\left(\frac{V_2^2}{2}\right)}{\left(\frac{V_2^2}{2}\right)_s}$$

For the isentropic process from 1 to 2s:

$0 = \bar{s}^0\left(T_{2_s}\right) - \bar{s}^o(T_1) - \bar{R}\ln\frac{p_2}{p_1}$, rearranging and inserting $\bar{s}^o$ values from Table A-23E

$$\bar{s}^0\left(T_{2_s}\right) = \bar{s}^o(T_1) + \bar{R}\ln\frac{p_2}{p_1} = 54.204 + 1.986\ln\frac{1}{80} = 45.501\,\frac{\text{Btu}}{\text{lbmol}\cdot\text{°R}}$$

Interpolating in Table A-23E gives $T_{2s} = 325.97\text{°R}$, $h_{2s} = 2254.13\,\frac{\text{Btu}}{\text{lbmol}}$. Now:

$$V_{2s} = \sqrt{\frac{2\left(\bar{h}_1 - \bar{h}_{2s}\right)}{M} + V_1^2}$$

$$= \sqrt{\frac{2(7850.4 - 2254.13)}{32.00}\frac{\text{Btu}}{\text{lb}}\cdot\left|\frac{778\text{ ft}\cdot\text{lbf}}{1\text{ Btu}}\right|\cdot\left|\frac{32.2\text{ lb}\cdot\text{ft}}{1\text{ lbf}\cdot\text{s}^2}\right| + \left(90\,\frac{\text{ft}}{\text{s}}\right)^2}$$

$$= 2961.5\,\frac{\text{ft}}{\text{s}}$$

Thus:

$$V_2 = V_{2s}\sqrt{\eta_N} = 2961.5\sqrt{0.85} = 2730.4\,\frac{\text{ft}}{\text{s}}$$

From Eq. (2):

$$\bar{h}_2 = 7850.4\,\frac{\text{Btu}}{\text{lb}} + \left(\frac{90^2 - 2730.4^2}{2}\right)\frac{\text{ft}^2}{\text{s}^2}\cdot\left|\frac{1\text{ lbf}\cdot\text{s}^2}{32.2\text{ lb}\cdot\text{ft}}\right|\cdot\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right|\cdot\frac{32.00\text{ lb}}{\text{lbmol}}$$

$$= 3094.15\,\frac{\text{Btu}}{\text{lbmol}}$$

Using the value for $\bar{h}_2$ and interpolating from Table A-23E, $\bar{s}^o(T_2) = 47.694\,\frac{\text{Btu}}{\text{lbmol}\cdot\text{°R}}$.

Finally, from Eq. (1):

$$\frac{\dot{E}_d}{\dot{m}} = \frac{T_0}{M}\left[\bar{s}^o(T_2) - \bar{s}^o(T_1) - \bar{R}\ln\frac{p_2}{p_1}\right]$$

$$= \left(\frac{530\text{°R}}{32.00\,\frac{\text{lb}}{\text{lbmol}}}\right)\left[47.694 - 54.204 - 1.986\ln\frac{1}{80}\right]\frac{\text{Btu}}{\text{lbmol}\cdot\text{°R}} = 36.32\,\frac{\text{Btu}}{\text{lb}}$$

PROBLEM 7.80

7.80 Steady-state operating data are shown in Fig. P7.80 for an open feedwater heater. Heat transfer from the feedwater heater to its surroundings occurs at an average outer surface temperature of 50°C at a rate of 100 kW. Ignore the effects of motion and gravity and let T_0= 25°C, p_0=1 bar. Determine
(a) the ratio of the incoming mass flow rates ($\dot{m}_1/\dot{m}_2$).
(b) the rate of exergy destruction, in kW.

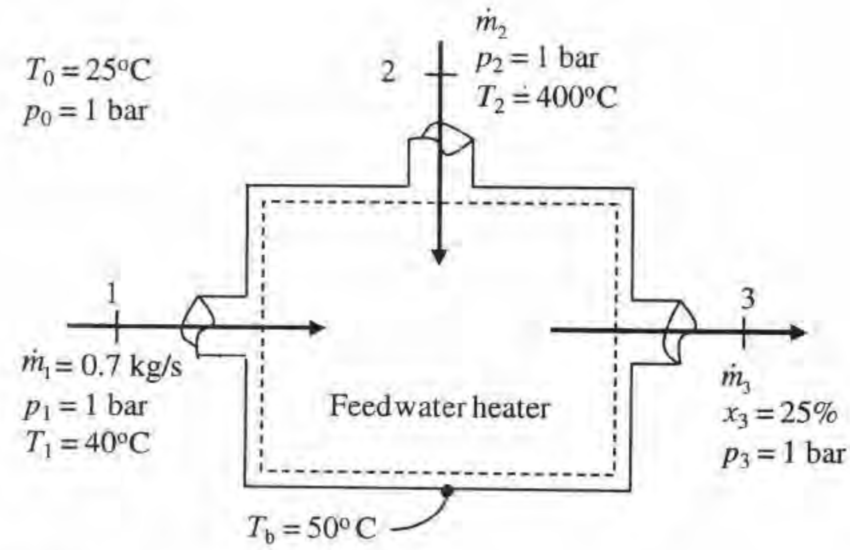
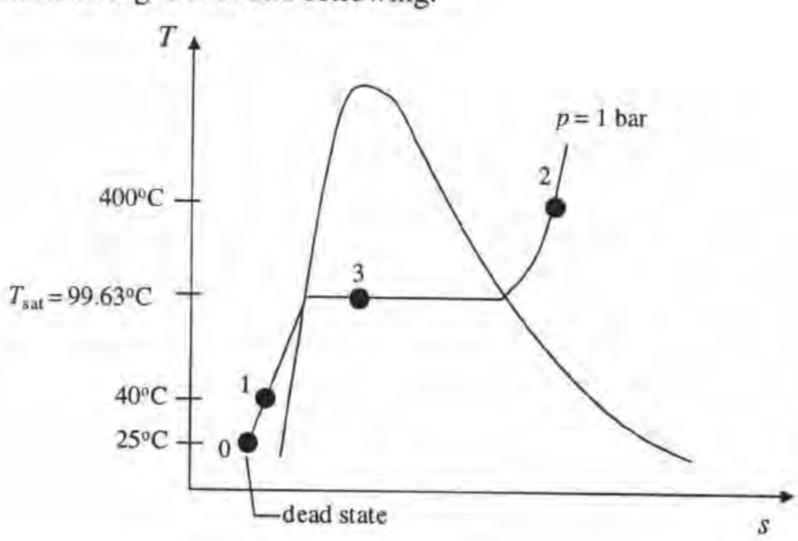


Fig. P7.80

KNOWN: Steady state operating data are provided for an open feedwater heater.

FIND: Determine the ratio of the incoming mass flow rates and the rate of exergy destruction.

SCHEMATIC AND GIVEN DATA:
Refer to Fig. P7.80 and following:



ENGINEERING MODEL:
(1) The control volume shown in the schematic is at steady state.
(2) Heat transfer from the control volume occurs at T_b = 323 K (50°C).
(3) For the control volume, $\dot{W}_{cv} = 0$.
(4) The effects of motion and gravity are ignored.
(5) For the environment, T_0 =298 K and p_0 = 1 bar.

PROBLEM 7.80 (Continued)

ANALYSIS:

(a) The simplified mass balance reduces to

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2 \quad (1)$$

An energy balance reduces to

$$0 = \dot{Q}_{cv} + \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3 \quad (2)$$

Substituting Eq. (1) into Eq. (2) and rearranging yields

$$\dot{m}_2 = \frac{\dot{m}_1(h_1 - h_3) + \dot{Q}_{cv}}{(h_3 - h_2)} \quad (3)$$

where

$$h_1 = h_f(T_1) = 167.57\frac{\text{kJ}}{\text{kg}} \text{ from Table A - 2 and } h_2 = 3278.2\frac{\text{kJ}}{\text{kg}} \text{ from Table A - 4}$$

Using Table A - 3, $h_3 = h_f + x_3(h_{fg}) = 417.46 + 0.25(2258.0) = 981.96\frac{\text{kJ}}{\text{kg}}$

Substituting into Eq. (3)

$$\dot{m}_2 = \frac{0.7\frac{\text{kg}}{\text{s}}(167.57 - 981.96)\frac{\text{kJ}}{\text{kg}} + (-100\,\text{kW})\left|\frac{1\frac{\text{kJ}}{\text{s}}}{1\,\text{kW}}\right|}{(981.96 - 3278.2)\frac{\text{kJ}}{\text{kg}}} = 0.292\frac{\text{kg}}{\text{s}}$$

Therefore, $\dot{m}_1/\dot{m}_2 = 0.7/0.292 = 2.4$ ⟵

(b) The rate of exergy destruction can be obtained by using an exergy rate balance or using $\dot{E}_d = \dot{\sigma}T_o$ where $\dot{\sigma}$ is the rate of entropy production obtained from an entropy balance. Using an entropy balance, simplified based on assumptions and rearranged yields

$$\dot{\sigma} = -\frac{\dot{Q}_{cv}}{T_b} - \dot{m}_1 s_1 - \dot{m}_2 s_2 + \dot{m}_3 s_3 \quad (4)$$

where

$$s_1 = s_f(T_1) = 0.5725\frac{\text{kJ}}{\text{kg}\cdot\text{K}} \text{ from Table A - 2 and } s_2 = 8.5435\frac{\text{kJ}}{\text{kg}\cdot\text{K}} \text{ from Table A - 4}$$

Using Table A - 3, $s_3 = s_f + x_3(s_g - s_f) = 1.3026 + 0.25(7.3594 - 1.3026) = 2.8168\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

From Eq. (1) and result from part (a)

$$\dot{m}_3 = 0.992\frac{\text{kg}}{\text{s}}$$

Substituting into Eq. (4)

$$\dot{\sigma} = \left(-\frac{(-100)}{323}\frac{\text{kW}}{\text{K}} - \left[\begin{matrix}\left(0.7\frac{\text{kg}}{\text{s}}\right)0.5725\frac{\text{kJ}}{\text{kg}\cdot\text{K}} + \left(0.292\frac{\text{kg}}{\text{s}}\right)8.5435\frac{\text{kJ}}{\text{kg}\cdot\text{K}} \\ -(0.992\frac{\text{kg}}{\text{s}})\,2.8168\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\end{matrix}\right]\left|\frac{1\,\text{kW}}{1\frac{\text{kJ}}{\text{s}}}\right|\right)$$

$$\dot{\sigma} = 0.20841\frac{\text{kW}}{\text{K}}$$

The exergy destruction can be determined, as follows:

$$\dot{E}_d = T_0(\dot{\sigma}) = 298\text{K}\left(0.20841\frac{\text{kW}}{\text{K}}\right) = 62.11\,\text{kW}$$ ⟵

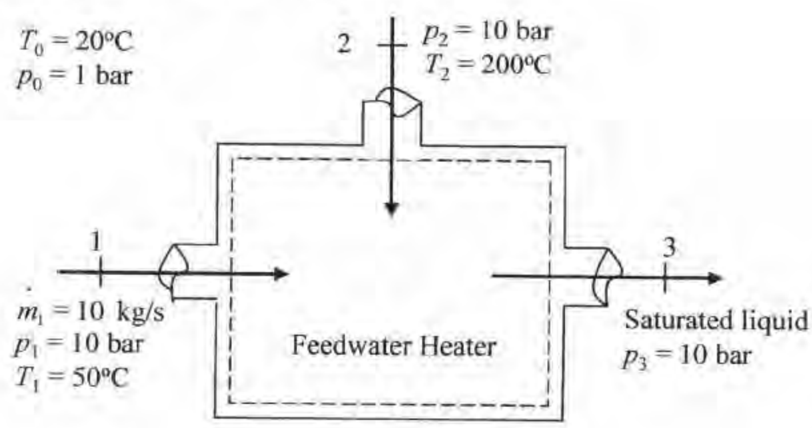
PROBLEM 7.81

7.81 An open feedwater heater operates at steady state with liquid water entering inlet 1 at 10 bar, 50°C, and a mass flow rate of 10 kg/s. A separate stream of steam enters inlet 2 at 10 bar and 200°C. Saturated liquid at 10 bar exits the feedwater heater. Stray heat transfer and the effects of motion and gravity can be ignored. Let $T_0 = 20°C$, $p_0 = 1$ bar. Determine (a) the mass flow rate of the steams at inlet 2 and the exit, each in kg/s, (b) the rate of exergy destruction, in kW, and (c) the cost of the exergy destroyed, in \$/year, for 8400 hours of operation annually. Evaluate exergy at 8.5 cents per kW·h.

KNOWN: Liquid water at given pressure and temperature and steam at given pressure and temperature enter a feedwater heater. Saturated liquid exits the feedwater heater at given pressure.

FIND: Determine the mass flow rate of the streams at inlet 2 and the exit, the rate of exergy destruction, and the cost of the exergy destroyed.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume shown on the accompanying figure is at steady state.
2. Heat transfer and effects of motion and gravity can be ignored.
3. $\dot{W}_{cv} = 0$ since a feedwater heater has no work associated with it.
4. Exergy is valued at 8.5 cents per kW·h.
5. $T_0 = 20°C$, $p_0 = 1$ bar.

ANALYSIS:
At inlet 1, the water is compressed liquid. From Table A-2, $h_1 \approx h_{f1} = 209.33$ kJ/kg, $s_1 \approx s_{f1} = 0.7038$ kJ/(kg·K).
At inlet 2, the steam is superheated. From Table A-4, $h_2 = 2827.9$ kJ/kg, $s_2 = 6.6940$ kJ/(kg·K).
At exit 3, the water is saturated liquid. From Table A-3, $h_3 = h_{f3} = 762.81$ kJ/kg, $s_3 = s_{f3} = 2.1387$ kJ/(kg·K).

(a) The steady-state mass rate balance gives

$$\sum_i \dot{m}_i = \sum_e \dot{m}_e$$

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

The steady-state energy balance gives

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \,(h_i + \tfrac{1}{2}\,\mathrm{V}_i^2 + gz_i) - \sum_e \dot{m}_e \,(h_e + \tfrac{1}{2}\,\mathrm{V}_e^2 + gz_e)$$

Neglecting heat transfer and effects of motion and gravity and recognizing no work is associated with a feedwater heater, the energy balance simplifies to

$$0 = \sum_i \dot{m}_i \, h_i - \sum_e \dot{m}_e \, h_e$$

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$$

Substituting for $\dot{m}_3$ from the mass rate balance

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3$$

Solving for $\dot{m}_2$

$$\dot{m}_2 = \frac{\dot{m}_1 (h_3 - h_1)}{h_2 - h_3}$$

Substituting values yields

$$\dot{m}_2 = \frac{\left(10\frac{\text{kg}}{\text{s}}\right)\left(762.81\frac{\text{kJ}}{\text{kg}} - 209.33\frac{\text{kJ}}{\text{kg}}\right)}{2827.9\frac{\text{kJ}}{\text{kg}} - 762.81\frac{\text{kJ}}{\text{kg}}} = \underline{\mathbf{2.68\ kg/s}}$$

Substituting values in the steady-state mass rate balance and solving for $\dot{m}_3$ give

$$\dot{m}_3 = 10 \text{ kg/s} + 2.68 \text{ kg/s} = \underline{\mathbf{12.68\ kg/s}}$$

(b) The rate of exergy destruction can be determined from the entropy production rate

$$\dot{\mathsf{E}}_d = T_0 \dot{\sigma}_{cv}$$

Entropy production rate can be determined from the steady-state control volume entropy rate balance

PROBLEM 7.81 (Continued – p.3)

$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{cv}$$

Neglecting heat transfer rate, entropy production rate becomes

$$\dot{\sigma}_{cv} = \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2$$

Substituting values and applying the appropriate conversion factor give.

$$\dot{\sigma}_{cv} = \left[\left(12.68\frac{\text{kg}}{\text{s}}\right)\left(2.1387\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right) - \left(10\frac{\text{kg}}{\text{s}}\right)\left(0.7038\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right) - \left(2.68\frac{\text{kg}}{\text{s}}\right)\left(6.6940\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\right]\left|\frac{\text{kW}}{\frac{\text{kJ}}{\text{s}}}\right|$$

$$\dot{\sigma}_{cv} = 2.14 \text{ kW/K}$$

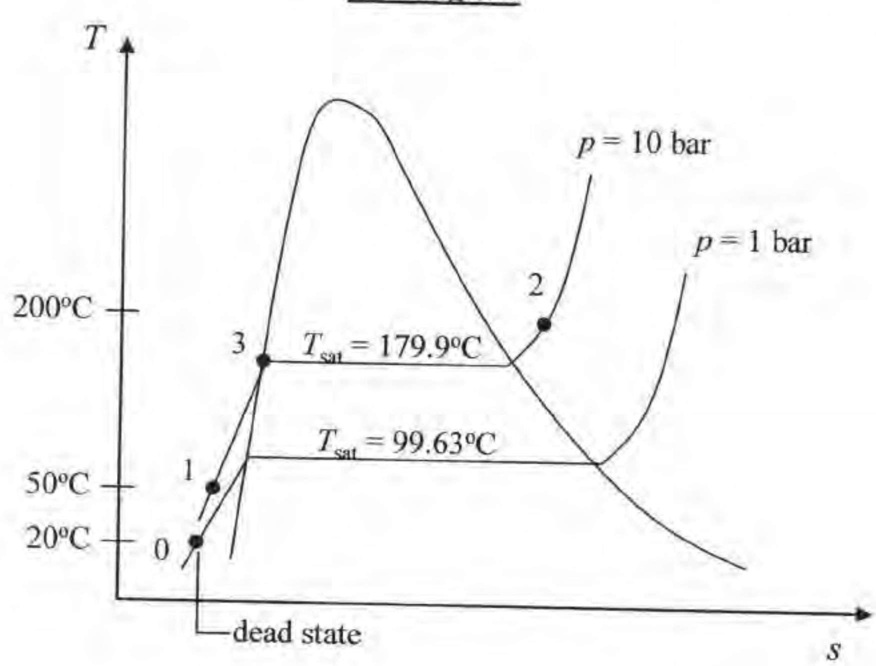
Substituting values and solving for rate of exergy destruction give

$$\dot{E}_d = (293 \text{ K})(2.14 \text{ kW/K}) = \underline{\mathbf{627.02\ kW}}$$

(c) The economic cost of exergy destroyed is

$$Cost = (627.02 \text{ kW})\left(8.5\frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)\left(8400\frac{\text{h}}{\text{year}}\right)\left(\frac{\$}{100 \text{ cents}}\right) = \underline{\mathbf{\$447{,}692/year}}$$

***T-s* Diagram**



PROBLEM 7.82

7.82 Figure P7.82 and the accompanying table provide a schematic and steady-state operating data for a mixer that combines two streams of air. The stream entering at 1500 K has a mass flow rate of 2 kg/s. Stray heat transfer and the effects of motion and gravity are negligible. Assuming the ideal gas model for the air, determine the rate of exergy destruction, in kW. Let T_0 = 300 K, p_0 = 1 bar.

State	T(K)	p(bar)	h(kJ/kg)	s°(kJ/kg · K)[a]
1	1500	2	1635.97	3.4452
2	300	2	300.19	1.7020
3	—	1.9	968.08	2.8869

[a] s° is the variable appearing in Eq. 6.20a and Table A-22.

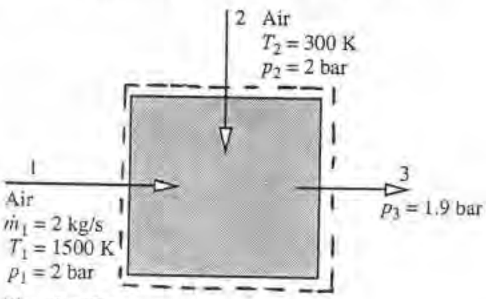


Fig. P7.82

KNOWN: Steady-state data are provided for a mixer combining two air streams.

FIND: For the mixer determine the rate of exergy destruction.

ENGINEERING MODEL

1. A control volume at steady state encloses the mixer.
2. For the control volume, stray heat transfer and the effects of motion and gravity are negligible.
3. The air is modeled as an ideal gas.
4. $T_0 = 300K$, $P_0 = 1\,bar$

ANALYSIS: In this application, it is convenient to evaluate the rate of exergy destruction via $\dot{E}_d = T_0 \dot{\sigma}$, where $\dot{\sigma}$ is the rate of entropy production obtained from an entropy rate balance. To implement this, the mass flow rate at 2 is required. Reducing an energy rate balance, we get

$$\dot{m}_2 = \frac{\dot{m}_1[h_1 - h_3]}{[h_3 - h_2]} = \frac{(2\,kg/s)[1635.97 - 968.08]\,kJ/kg}{[968.08 - 300.19]\,kJ/kg} = 2\,kg/s, \quad \dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 4\,kg/s$$

Reducing an entropy rate balance, $0 = \sum_j \overset{0}{\dot{Q}_j}/T_j + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \underbrace{\dot{m}_3}_{=\dot{m}_1+\dot{m}_2} s_3 + \dot{\sigma}$

$$\Rightarrow \quad \dot{\sigma} = (\dot{m}_1 + \dot{m}_2)s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2 = \dot{m}_1(s_3 - s_1) + \dot{m}_2(s_3 - s_2)$$

With Eq. 6.20a,

$$\dot{\sigma} = \dot{m}_1\left[s_3^\circ - s_1^\circ - R\ln\frac{P_3}{P_1}\right] + \dot{m}_2\left[s_3^\circ - s_2^\circ - R\ln\frac{P_3}{P_2}\right]$$

$$= \left(2\,\frac{kg}{s}\right)\left(2.8869 - 3.4452 - \frac{8.314}{28.97}\ln\frac{1.9}{2}\right)\frac{kJ}{kg\cdot K}$$

$$\left(2\,\frac{kg}{s}\right)\left(2.8869 - 1.7020 - \frac{8.314}{28.97}\ln\frac{1.9}{2}\right)\frac{kJ}{kg\cdot K}$$

$$= 1.312\,\frac{kJ/s}{K}\left|\frac{1\,kW}{1\,kJ/s}\right| = 1.312\,\frac{kW}{K}$$

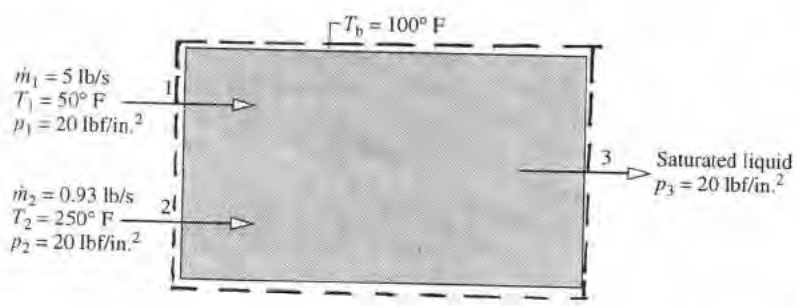
Finally

$$\dot{E}_d = T_0\dot{\sigma} = (300K)\left(1.312\,\frac{kW}{K}\right)$$

$$= 393.6\,kW$$

PROBLEM 7.83

Figure P7.83 provides steady-state operating data for a mixing chamber in which entering liquid and vapor streams of water mix to form an exiting saturated liquid stream. Heat transfer from the mixing chamber to its surroundings occurs at an average surface temperature of 100°F. The effects of motion and gravity are negligible. Let $T_0 = 70°F$, $p_0 = 1$ atm. For the mixing chamber, determine, each in Btu/s, (a) the rate of heat transfer and the accompanying rate of exergy transfer and (b) the rate of exergy destruction.



ENGR. MODEL:

1. The control volume shown in the figure is at steady state.
2. $\dot{W}_{cv} = 0$ and the effects of gravity and motion can be ignored.
3. All heat transfer from the control volume occurs at T_b.
4. For the environment, $T_0 = 530°R$ (70°F), $P_0 = 1$ atm.

ANALYSIS: From a mass rate balance, $\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 5.93$ lb/s. Also, using Tables A-2E, 3E, 4E:

	h (Btu/lb)	s (Btu/lb·°R)
1	18.06 (~h_f)	0.03607 (~s_f)
2	1167.2	1.7475
3	196.26	0.3358

(a) An energy rate balance reduces to read, $0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$

$$\Rightarrow \dot{Q}_{cv} = \dot{m}_3 h_3 - \dot{m}_1 h_1 - \dot{m}_2 h_2$$

$$= (5.93 \tfrac{lb}{s})(196.26 \tfrac{Btu}{lb}) - 5(18.06) - 0.93(1167.2)$$

$$= -12 \text{ Btu/s} \quad \text{(a)}$$

$$\dot{E}_q = \left[1 - \frac{T_0}{T_b}\right]\dot{Q}_{cv}$$

① $$= \left[1 - \frac{530}{560}\right]\left(-12 \tfrac{Btu}{s}\right) = -0.64 \tfrac{Btu}{s}$$

(b) To get $\dot{E}_d$ it is convenient to use $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production from an entropy rate balance:

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$$

$$\Rightarrow \quad \dot{E}_d = T_0\left[-\frac{\dot{Q}_{cv}}{T_b} + \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2\right]$$

$$= 530°R\left[-\frac{(-12 \text{ Btu/s})}{560°R} + (5.93 \tfrac{lb}{s})(0.3358 \tfrac{Btu}{lb \cdot °R}) - 5(0.03607) - 0.93(1.7475)\right]$$

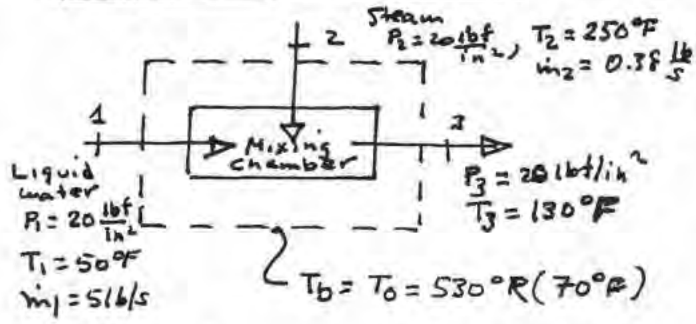
$$= 109.8 \text{ Btu/s} \quad \text{(b)}$$

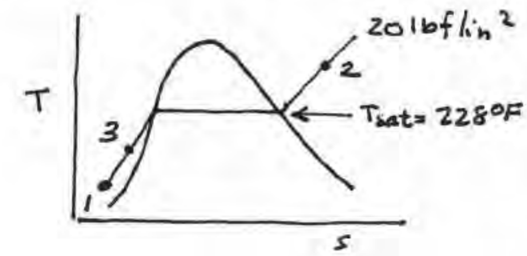
1. In comparison to the value of $\dot{E}_d$ found in part (b), the exergy transfer accompanying heat transfer is a minor effect.

PROBLEM 7.84

Liquid water at 20 lbf/in.², 50°F enters a mixing chamber operating at steady state with a mass flow rate of 5 lb/s and mixes with a separate stream of steam entering at 20 lbf/in.², 250°F with a mass flow rate of 0.38 lb/s. A single mixed stream exits at 20 lbf/in.², 130°F. Heat transfer from the mixing chamber occurs to its surroundings. Neglect the effects of motion and gravity and let $T_0 = 70°F$, $p_0 = 1$ atm. Determine the rate of exergy destruction, in Btu/s, for a control volume including the mixing chamber and enough of its immediate surroundings that heat transfer occurs at 70°F.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. Heat transfer occurs at $T_b = T_0 = 530°R$ only. $\dot{W}_{cv} = 0$.
3. The effects of motion and gravity can be ignored.
4. For the environment, $T_0 = 530°R$, $p_0 = 1$ atm.

ANALYSIS: An energy rate balance reduces to, $0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$

or

$$\dot{Q}_{cv} = \underbrace{\dot{m}_3}_{(=\dot{m}_1+\dot{m}_2)} \overbrace{h_3}^{\approx h_f(T_3)} - \dot{m}_1 \overbrace{h_1}^{\approx h_f(T_1)} - \dot{m}_2 h_2 = 5.38\frac{lb}{s}\left(97.98\frac{Btu}{lb}\right) - (5)(18.06) - 0.38(1167.2)$$

$$= -6.71 \text{ Btu/s}$$

To find $\dot{E}_d$, it is convenient to use $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production from an entropy rate balance. Thus, with $T_b = 530°R$,

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$$

$$\Rightarrow \quad \dot{E}_d = T_0\left[-\frac{\dot{Q}_{cv}}{T_b} + \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2\right]$$

$$= 530°R\left[-\frac{(-6.71 Btu/s)}{530°R} + \left(5.38\frac{lb}{s}\right)\left(0.1817\frac{Btu}{lb\cdot°R}\right) - (5)(0.03607) - 0.38(1.7475)\right]$$

① $$= 77.3 \frac{Btu}{s}$$

---

1. This value accounts for exergy destruction within the mixing chamber and exergy destruction in the immediate surroundings of the mixing chamber owing to spontaneous heat transfer there. To evaluate the rate of exergy destruction for the mixing chamber alone requires information about the temperature(s) at which heat transfer occurs from it.

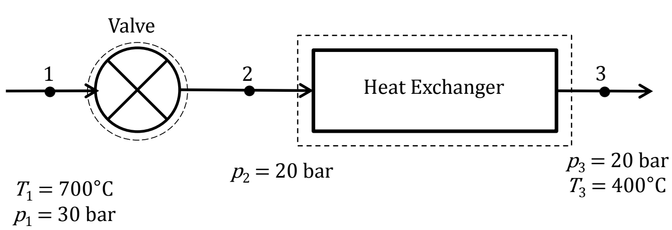
Problem 7.85

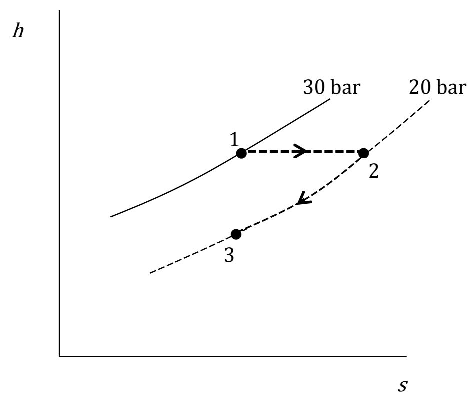
Steam at 30 bar and 700°C is available at one location in an industrial plant. At another location, steam at 20 bar and 400°C is required for use in a certain process. An engineer suggests that steam at this condition can be provided by allowing the higher pressure steam to expand through a valve to 20 bar and then cool to 400°C through a heat exchanger with heat transfer to the surroundings, which are at 20°C.
(a) Evaluate this suggestion by determining the associated exergy destruction rate per mass flow rate of steam (kJ/kg) for the valve and heat exchanger. Discuss.
(b) Evaluating exergy at 8 cents per kW·h and assuming continuous operation, estimate the total annual cost of the exergy destruction for a mass flow rate of 1 kg/s.
(c) Suggest an alternative method for obtaining steam at the desired condition that would be preferable thermodynamically, and determine the total amount cost, in \$, of the exergy destruction for a mass flow rate of 1 kg/s. Let T_0 = 20°C, p_0 = 1 atm.

Known:
A method is suggested for providing steam at 20 bar, 400°C from an available source at 30 bar, 700°C.

Find:
(a) Using exergy principles, evaluate this suggestion.
(b) Estimate the annual cost of this suggestion using data provided.
(c) Suggest alternative method that would be preferable thermodynamically and discuss.

Schematic and Known Data:





Engineering Model:
(1) The expansion across the valve is a throttling process.
(2) The heat exchanger operates at steady state with negligible effects of motion and gravity.
(3) For the environment, T_0 = 20°C, p_0 = 1 atm.

Analysis:
(a) Considering the valve first, the rate of exergy destruction is:

$$\frac{\dot{E}_d}{\dot{m}} = T_0 \frac{\dot{\sigma}_{cv}}{\dot{m}} \qquad (1)$$

where $\frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1 \qquad (2)$

State 2 is fixed by p_2 and $h_2 = h_1$ = 3911.7 kJ/kg (from assumption 1). Thus, with data from Table A-4, s_1 = 7.7571 kJ/kg-K and the value of s_2 = 7.9427is obtained using p_2, h_2 and interpolation. Inserting values into Eq. (1):

$$\left(\frac{\dot{\text{E}}_\text{d}}{\dot{m}}\right)_\text{valve} = (293\text{ K})(7.9427 - 7.7571)\frac{\text{kJ}}{\text{kg·K}} = 54.38\frac{\text{kJ}}{\text{kg}}$$

Then, since the heat transfer as the stream goes from state 2 to state 3 is discarded without use, it is reasonable to locate the control volume enclosing the heat exchanger so that heat transfer occurs at the temperature of the surroundings, as shown in the accompanying figure. For this control volume, mass and exergy rate balances at steady state give:

$$0 = \left[\underbrace{1 - \frac{T_0}{T_b}}_{=0}\right]\dot{Q}_{cv} - \underbrace{\dot{W}_{cv}}_{=0} + \dot{m}\left(\text{e}_{\text{f}_2} - \text{e}_{\text{f}_3}\right) - \dot{\text{E}}_\text{d}/\dot{m} \qquad (3)$$

Thus inserting values from above and Table A-4 into Eq. (3):

$$\left(\frac{\dot{\text{E}}_\text{d}}{\dot{m}}\right)_\text{heat exchanger} = (h_2 - h_3) - T_0(s_2 - s_3)$$

$$= (3911.7 - 3247.6)\frac{\text{kJ}}{\text{kg}} - (293\text{ K})(7.9427 - 7.1271)\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 425.13\frac{\text{kJ}}{\text{kg}}$$

These exergy destruction values confirm what may be apparent: there are two significant sources of exergy destruction in the suggested method. One is related to the expansion across the valve and the other is related to heat transfer to the surroundings. It may come as a surprise; however, that the second of these is the far more significant source of exergy destruction.

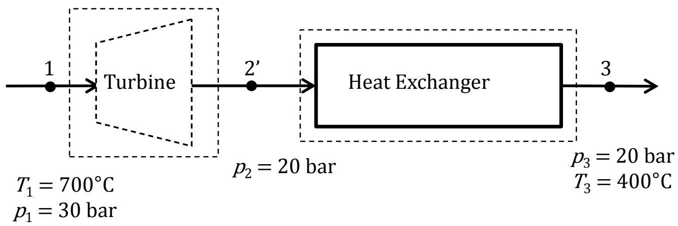
(b) The total exergy destruction rate is the sum of the values determined in part (a). For a mass flow rate of 1 kg/s:

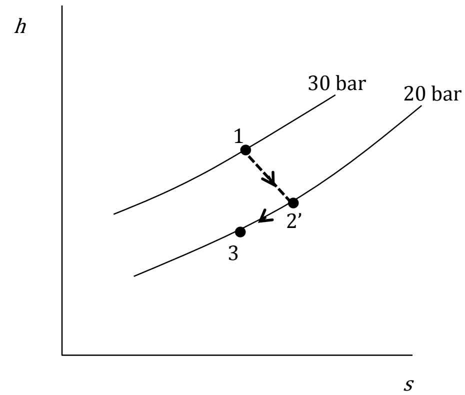
$$\left(\dot{\text{E}}_\text{d}\right)_\text{cv} = \left(\frac{1\text{kg}}{\text{s}}\right)(54.38 + 425.13)\frac{\text{kJ}}{\text{kg}} = 479.51\text{ kW}$$

Evaluating exergy destruction at \$0.08 per kW·h, the annual cost is:

$$\text{Anual cost in \$} = (479.51\text{ kW})\left(\frac{24\text{ h}}{1\text{ day}}\cdot\frac{365\text{ days}}{1\text{ year}}\right)\left(\frac{\$0.08}{\text{kW}\cdot\text{h}}\right) = \$336{,}041$$

(c) The annual cost calculated in part (b) shows there is an economic incentive for considering other alternatives. For example, the valve could be replaced by a power recovery turbine, as shown in the figure below:





For a turbine with an isentropic turbine efficiency of 80%, the specific enthalpy and entropy at state 2' would be $h_{2'} = 3773.5\,\frac{\text{kJ}}{\text{kg}}, s_{2'} = 7.7958\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$. The turbine exergy destruction rate is:

$$\left(\frac{\dot{\text{E}}_{\text{d}}}{\dot{m}}\right)_{\text{turbine}} = T_0(s_{2'} - s_1) = (293\ \text{K})(7.7958 - 7.7571)\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 11.34\,\frac{\text{kJ}}{\text{kg}}$$

For the subsequent heat exchanger:

$$\left(\frac{\dot{\text{E}}_{\text{d}}}{\dot{m}}\right)_{\text{heat exchanger}} = (h_{2'} - h_3) - T_0(s_{2'} - s_3)$$

$$= (3773.5 - 3247.6)\,\frac{\text{kJ}}{\text{kg}} - (293\ \text{K})(7.7958 - 7.1271)\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 329.97\,\frac{\text{kJ}}{\text{kg}}$$

The annual cost of the exergy destroyed for a flow rate of 1 kg/s is:

$$\left(\dot{\text{E}}_{\text{d}}\right)_{\text{cv}} = \left(\frac{1\text{kg}}{\text{s}}\right)(11.34 + 329.97)\,\frac{\text{kJ}}{\text{kg}} = 341.31\ \text{kW}$$

$$\text{Anual cost in \$} = (351.31\ \text{kW})\left(\frac{24\ \text{h}}{1\ \text{day}}\cdot\frac{365\ \text{days}}{1\ \text{year}}\right)\left(\frac{\$0.08}{\text{kW}\cdot\text{h}}\right) = \$239{,}190$$

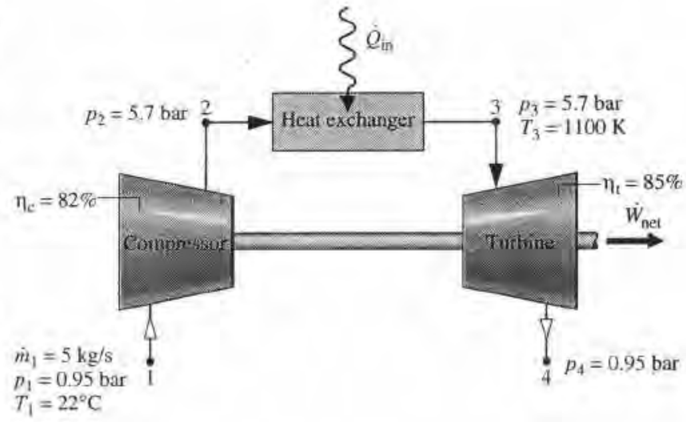
It is evident that the annual cost associated with the exergy destroyed has been reduced with this alternative. However, it must be recognized that there is an annual cost associate with owning and operating the power recovery turbine. In practice, proposals for improving energy resource utilization may not be approved unless they pay for themselves in a relatively short time possibly as few as three years. It may be noted that with the arrangement in the figure above there is still a significant exergy destruction rate associated with heat transfer. This source of exergy destruction can be reduced by means of a waste heat recovery device in place of direct heat transfer to the surroundings. However, it must be recognized that there is an annual cost associated with owning and operating such a device.

PROBLEM 7.86

A gas turbine operating at steady state is shown in Fig. P7.86. Air enters the compressor with a mass flow rate of 5 kg/s at 0.95 bar and 22°C and exits at 5.7 bar. The air then passes through a heat exchanger before entering the turbine at 1100 K, 5.7 bar. Air exits the turbine at 0.95 bar. The compressor and turbine operate adiabatically and the effects of motion and and gravity can be ignored. The compressor and turbine isentropic efficiencies are 82 and 85%, respectively. Using the ideal gas model for air, determine, each in kW,

(a) the *net* power developed.
(b) the rates of exergy destruction for the compressor and turbine.
(c) the *net* rate exergy is carried out of the plant at the turbine exit, $(\dot{E}_{f4} - \dot{E}_{f1})$.

Let $T_0 = 22°C$, $p_0 = 0.95$ bar.



ENGR. MODEL:

1. The gas turbine operates at steady state.
2. The compressor and turbine operate adiabatically.
3. The effects of motion and gravity can be ignored.
4. The air is modeled as an ideal gas.
5. For the environment, $T_0 = 295K$, $P_0 = 0.95 bar$

ANALYSIS: Property evaluation: From Table A-22, $h_1 = 295.17$ kJ/kg, $s_1^o = 1.68515$ kJ/kg·K, $P_{r1} = 1.3068$. $h_3 = 1161.07$ kJ/kg, $s_3^o = 3.07732$ kJ/kg·K, $P_{r3} = 167.1$.

$$P_{r2} = P_{r1}\left[\frac{P_2}{P_1}\right] = 1.3068\left[\frac{5.7}{0.95}\right] = 7.8408 \Rightarrow h_{2s} = 493.03 \text{ kJ/kg}$$

$$P_{r4} = P_{r3}\left[\frac{P_4}{P_3}\right] = 167.1\left[\frac{0.95}{5.7}\right] = 27.85 \Rightarrow h_{4s} = 706.51 \text{ kJ/kg}$$

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} \Rightarrow h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 774.69 \frac{kJ}{kg}, \; s_4^o = 2.6571 \frac{kJ}{kg \cdot K}, \; T_4 = 757 K.$$

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c} = 536.46 \frac{kJ}{kg}, \; s_2^o = 2.2843 \frac{kJ}{kg \cdot K}$$

(a) The net power developed $= \dot{W}_t - \dot{W}_c = \dot{m}[(h_3 - h_4) - (h_2 - h_1)]$

taken as positive since an input

$$\dot{W}_{net} = 5\frac{kg}{s}\left[(1161.07 - 774.69) - (536.46 - 295.17)\right]\frac{kJ}{kg}\left|\frac{1 kW}{1 kJ/s}\right| = 725.5 \text{ kW} \quad \text{(a)}$$

(b) $\dot{E}_d$ is conveniently determined as $\dot{E}_d = T_0 \dot{\sigma}_{cv}$. For the compressor, $\dot{E}_d = T_0 \dot{m}(s_2 - s_1)$. For the turbine $\dot{E}_d = T_0 \dot{m}(s_4 - s_3)$. That is,

Compressor: $\dot{E}_d = T_0 \dot{m}(s_2^o - s_1^o - R \ln P_2/P_1) = 295K(5\frac{kg}{s})\left(2.2843 - 1.68515 - \frac{8.314}{28.97} \ln \frac{5.7}{0.95}\right)\frac{kJ}{kg \cdot K}$

$= 125.3$ kW

Turbine: $\dot{E}_d = T_0 \dot{m}[s_4^o - s_3^o - R \ln P_4/P_3] = (295)(5)\left[2.6571 - 3.07732 - \frac{8.314}{28.97} \ln\left(\frac{0.95}{5.7}\right)\right]$

$= 138.6$ kW

(c) $\dot{E}_{f4} - \dot{E}_{f1} = \dot{m}[(h_4 - h_1) - T_0(s_4 - s_1)] = \dot{m}[(h_4 - h_1) - T_0(s_4^o - s_1^o - \underbrace{R \ln P_4/P_1}_{0 \; (P_4 = P_1)}))$

(1) $= \dot{m}[(h_4 - h_1) - T_0(s_4^o - s_1^o)] = 5\frac{kg}{s}[(774.69 - 295.17) - 295(2.6571 - 1.68515)]\frac{kJ}{kg}$

$= 964$ kW

---

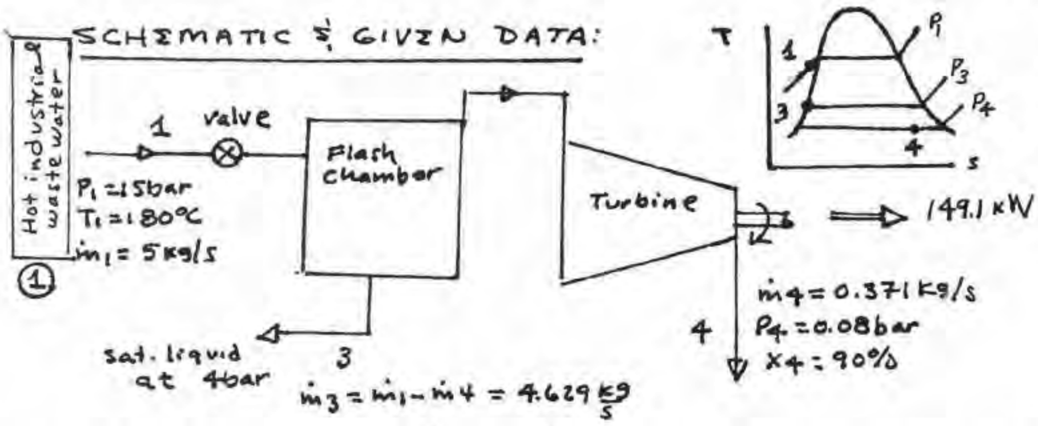
1. Consideration should be given to means for utilizing the considerable exergy of the high-temperature gas exiting the turbine at 4. See Secs. 9.7, 9.9 for options.

PROBLEM 7.87

7.87 Consider the *flash chamber* and turbine of Problem 4.105. Verify that the mass flow rate of saturated vapor entering the turbine is 0.371 kg/s and the power developed by the turbine is 149 kW. Determine the total rate of exergy destruction within the flash chamber and turbine, in kW. Comment. Let $T_0 = 298$ K.

See the solution to Prob. 4.105 for details, including the evaluation of the mass flow rates and power developed given here.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. A control volume at steady state encloses the valve, flash chamber and turbine.
2. $\dot Q_{cv} = 0$ and the effects of motion and gravity are ignored.
3. $T_0 = 298$ K

ANALYSIS: We use $\dot E_d = T_0 \dot\sigma$, where $\dot\sigma$ is the rate of entropy production for the overall configuration. Reducing an entropy rate balance,

$$0 = \cancel{\sum \frac{\dot Q_j}{T_j}} + \dot m_1 s_1 - \dot m_3 s_3 - \dot m_4 s_4 + \dot\sigma \Rightarrow \dot\sigma = \dot m_4 s_4 + \dot m_3 s_3 - \dot m_1 s_1 \quad (1)$$

At state 1, $s_1 \approx s_f(T_1) = 2.1396$ kJ/kg·K (Table A-2). From Table A-3 at 4 bar, $s_3 = 1.7766$ kJ/kg·K. Then, with data from Table A-3 at 0.08 bar

$$s_4 = s_f + x_4(s_g - s_f) = 0.5926 + 0.9(8.2287 - 0.5926) = 7.4651\ \frac{kJ}{kg \cdot K}$$

Substituting values into Eq. (1), we get

$$\dot\sigma = \left(0.371\ \frac{kg}{s}\right)\left(7.4651\ \frac{kJ}{kg\cdot K}\right) + \left(4.629\ \frac{kg}{s}\right)\left(1.7766\ \frac{kJ}{kg\cdot K}\right) - \left(5\ \frac{kg}{s}\right)\left(2.1396\ \frac{kJ}{kg\cdot K}\right)$$

$$= 0.2954\ \frac{kJ/s}{K} \left| \frac{1\ kW}{1\ kJ/s} \right| = 0.2954\ \frac{kW}{K}$$

Finally,

$$\dot E_d = T_0 \dot\sigma = 298K\left(0.2954\ \frac{kW}{K}\right)$$

$$= 88\ kW$$

1 Although the overall system produces power using hot industrial waste water, costs should be carefully considered, including the cost of the components and particularly the turbine cost.

PROBLEM 7.88

7.88 Figure P7.88 and the accompanying table provide the schematic and steady-state operating data for a *flash chamber* fitted with an inlet valve that produces saturated vapor and saturated liquid streams from a single entering stream of liquid water. Stray heat transfer and the effects of motion and gravity are negligible. Determine (a) the mass flow rate, in lb/s, for each of the streams exiting the flash chamber and (b) the total rate of exergy destruction, in Btu/s. Let $T_0 = 77°F$, $p_0 = 1$ atm.

State	Condition	T(°F)	p(lbf/in.²)	h(Btu/lb)	s(Btu/lb · R)
1	liquid	300	80	269.7	0.4372
2	sat. vapor	—	30	1164.3	1.6996
3	sat. liquid	—	30	218.9	0.3682

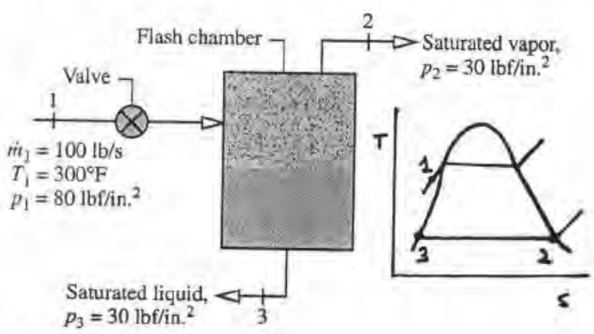


Fig. P7.88

KNOWN: Steady-state data are provided for an overall configuration consisting of a valve and flash chamber.

FIND: Determine the mass flow rates $\dot{m}_2$ and $\dot{m}_3$; also evaluate the total rate of exergy destruction.

ENGINEERING MODEL:

1. A control volume at steady state encloses the overall configuration.
2. For the control volume $\dot{Q}_{cv} = 0$ and the effects of motion and gravity are ignored.
3. $T_0 = 77°F$, $p_0 = 1$ atm.

ANALYSIS: (a) A mass rate balance gives $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$. An energy rate balance reduces to

$$\dot{m}_3 = \dot{m}_1 \left[\frac{h_2 - h_1}{h_2 - h_3}\right] = 100\frac{lb}{s}\left[\frac{1164.3 - 269.7}{1164.3 - 218.9}\right]$$

$$= 94.627 \frac{lb}{s} \Rightarrow \dot{m}_2 = 5.373 \frac{lb}{s}$$

(a)

(b) To find $\dot{E}_d$ it is convenient to use $\dot{E}_d = T_0 \dot{\sigma}$, where $\dot{\sigma}$ is the rate of entropy production for the control volume. Reducing an entropy rate balance for the control volume, we get,

$$0 = \sum \frac{\dot{Q}_j}{T_j}^{0} + \dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}$$

$$\Rightarrow \quad \dot{\sigma} = \dot{m}_2 s_2 + \dot{m}_3 s_3 - \dot{m}_1 s_1$$

$$= \left(5.373 \frac{lb}{s}\right)\left(1.6996 \frac{Btu}{lb \cdot °R}\right) + \left(94.627 \frac{lb}{s}\right)\left(0.3682 \frac{Btu}{lb \cdot °R}\right) - \left(100 \frac{lb}{s}\right)\left(0.4372 \frac{Btu}{lb \cdot °R}\right)$$

$$= 0.2537 \frac{Btu/s}{°R}$$

Finally

$$\dot{E}_d = T_0 \dot{\sigma}$$

$$= (537°R)\left(0.2537 \frac{Btu/s}{°R}\right)$$

$$= 136.2 \frac{Btu}{s}$$

(b)

PROBLEM 7.89

7.89 Figure P7.89 shows a gas turbine power plant operating at steady state consisting of a compressor, a heat exchanger, and a turbine. Air enters the compressor with a mass flow rate of 3.9 kg/s at 0.95 bar, 22°C and exits the turbine at 0.95 bar, 421°C. Heat transfer to the air as it flows through the heat exchanger occurs at an average temperature of 488°C. The compressor and turbine operate adiabatically. Using the ideal gas model for the air, and neglecting the effects of motion and gravity, determine, in MW,

(a) the rate of exergy transfer accompanying heat transfer to the air flowing through the heat exchanger.
(b) the *net* rate exergy is carried out of the plant at the turbine exit, $(\dot{E}_{f2} - \dot{E}_{f1})$.
(c) the rate of exergy destruction within the power plant.
(d) Using the results of parts (a)–(c), perform a full exergy accounting of the exergy supplied to the power plant accompanying heat transfer. Comment.

Let T_0 = 295 K(22°C), p_0 = 0.95 bar.

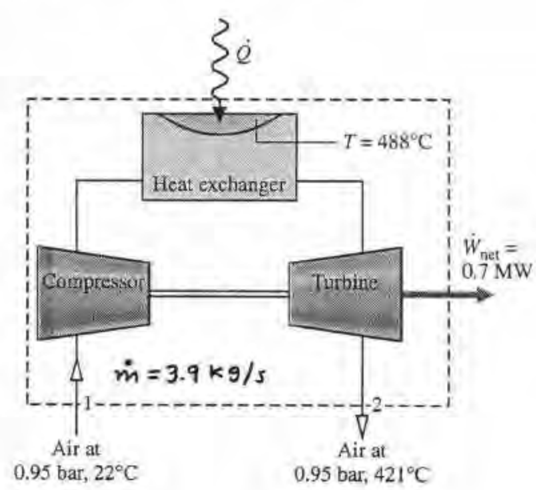


Fig. P7.89

ENGR. MODEL:

1. As shown in the sketch, consider a control volume enclosing the gas turbine power plant.
2. The control volume is at steady state.
3. Heat transfer occurs only at $T_b = 761$ K
4. The effects of motion and gravity can be ignored.
5. The air is modeled as an ideal gas.
6. For the environment, $T_0 = 295$ K, $p_0 = 0.95$ bar.

ANALYSIS: (a) Mass and energy rate balances reduce to give
$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2) \Rightarrow \dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}(h_2 - h_1)$. Then with data from Table A.22

$$\dot{Q}_{cv} = 0.7\,\text{MW} + \left(3.9\,\frac{\text{kg}}{\text{s}}\right)\left[706.8 - 295.17\right]\frac{\text{kJ}}{\text{kg}}\left|\frac{1\,\text{MW}}{10^3\,\text{kJ/s}}\right| = 2.305\,\text{MW}$$

Using Eq. 7.15

$$\dot{E}_q = \left[1 - \frac{T_0}{T_b}\right]\dot{Q}_{cv} = \left[1 - \frac{295}{761}\right](2.305\,\text{MW}) = 1.411\,\text{MW} \qquad \leftarrow (a)$$

(b) With Eq. 7.18,

$$\dot{E}_{f2} - \dot{E}_{f1} = \dot{m}\left[(h_2 - h_1) - T_0(s_2 - s_1)\right] = \dot{m}\left[(h_2 - h_1) - T_0\left(s^\circ(T_2) - s^\circ(T_1) - R\,\overbrace{\cancel{\ln\frac{p_2}{p_1}}}^{0}\right)\right]$$

$$= \left(3.9\,\frac{\text{kg}}{\text{s}}\right)\left[(706.8 - 295.17) - 295\,(2.5635 - 1.68515)\right]\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\,\text{MW}}{10^3\,\text{kJ/s}}\right|$$

$$= 0.595\,\text{MW} \qquad \leftarrow (b)$$

PROBLEM 7.89 (Continued)

(c) The total rate of exergy destruction can be obtained by use of the exergy rate balance, 7.13b

$$\dot{E}_d = \dot{E}_q - \dot{W}_{cv} + (\dot{E}_{f1} - \dot{E}_{f2})$$

$$= (1.411 - 0.7 - 0.595)\ \text{MW}$$

$$= 0.116\ \text{MW} \qquad \longleftarrow (c)$$

(d) The foregoing results provide the basis for an exergy accounting of the gas turbine power plant:

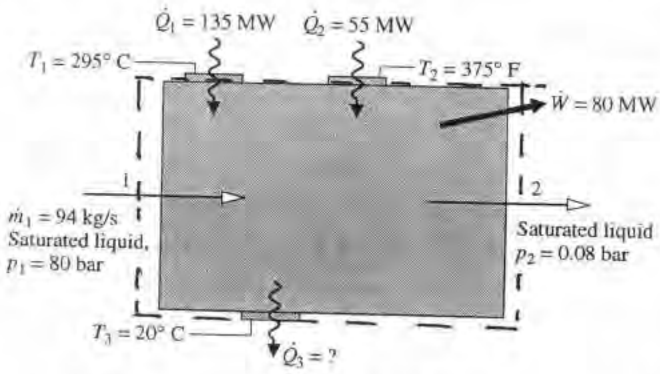
- Rate exergy is supplied accompanying heat transfer. — 1.411 MW
- Disposition of the exergy supplied.

✓ Power developed	0.700 MW (49.6%)
✓ Exergy destroyed	0.116 MW (8.2%)
① ✓ Net exergy carried out the stream exiting the turbine	0.595 MW (42.2%)
	1.411 MW

1. The net exergy carried out by the exiting stream is significant. Consideration should be given to means for cost-effectively using it. For options, see Secs. 9.7 and 9.9.

PROBLEM 7.90

Figure P7.90 shows a power-generating system at steady state. Saturated liquid water enters at 80 bar with a mass flow rate of 94 kg/s. Saturated liquid exits at 0.08 bar with the same mass flow rate. As indicated by arrows, three heat transfers occur, each at a specified temperature in the direction of the arrow: The first adds 135 MW at 295°C, the second adds 55 MW at 375°C, and the third removes energy at 20°C. The system generates power at the rate of 80 MW. The effects of motion and gravity can be ignored. Let $T_0 = 20°C$, $p_0 = 1$ atm. Determine, in MW, (a) the rate of heat transfer $\dot{Q}_3$ and the accompanying rate of exergy transfer and (b) a full exergy accounting of the *total* exergy supplied to the system with the two heat additions and with the *net* exergy, $(\dot{E}_{f1} - \dot{E}_{f2})$, carried in by the water stream as it passes from inlet to exit.



ENGR. MODEL:

1. The control volume shown in the figure is at steady state
2. Heat transfer occurs only at the specified temperatures and are positive in the directions of the arrows
3. The effects of motion and gravity can be ignored.
4. For the environment, $T_0 = 293K$, P_0: 1atm

ANALYSIS: At state 1, $h_1 = 1316.6$ kJ/kg, $s_1 = 3.2068$ kJ/kg·K. At state 2, $h_2 = 173.9$ kJ/kg, $s_2 = 0.5926$ kJ/kg·K.

(a) Energy rate balance: $0 = (\dot{Q}_1 + \dot{Q}_2 - \dot{Q}_3) - \dot{W} + \dot{m}(h_1 - h_2)$

$$\Rightarrow \quad \dot{Q}_3 = \dot{Q}_1 + \dot{Q}_2 - \dot{W} + \dot{m}(h_1 - h_2) = 135\text{MW} + 55\text{MW} - 80\text{MW} + 94\tfrac{\text{kg}}{\text{s}}(1316.6 - 173.9)\tfrac{\text{kJ}}{\text{kg}}\left|\tfrac{1\text{MW}}{10^3\text{kJ/s}}\right|$$

$$= 217\text{MW}$$

(a)

$$\dot{E}_q = \left[1 - \frac{T_0}{T_3}\right]\dot{Q}_3 = 0 \quad \text{since } T_3 = T_0$$

Comment: There is considerable energy exiting the control volume by heat transfer at 3, but the exergetic value is zero because heat transfer occurs at T_0 $(= T_3)$.

(b) Exergy supplied = 65.4MW + 30.1MW + 35.4MW = 130.9 MW

✓ $$\left[1 - \frac{T_0}{T_1}\right]\dot{Q}_1 = \left[1 - \frac{293}{568}\right](135\text{MW}) = 65.4\text{ MW}$$

✓ $$\left[1 - \frac{T_0}{T_2}\right]\dot{Q}_2 = \left[1 - \frac{293}{648}\right](55\text{MW}) = 30.1\text{ MW}$$

✓ $$\dot{E}_{f1} - \dot{E}_{f2} = \dot{m}\left[h_1 - h_2 - T_0(s_1 - s_2)\right] = \left(94\tfrac{\text{kg}}{\text{s}}\right)\left[(1316.6 - 173.9) - 293(3.2068 - 0.5926)\right]\tfrac{\text{kJ}}{\text{kg}}\left|\tfrac{1\text{MW}}{10^3\text{kJ/s}}\right|$$

$$= 35.4\text{MW}$$

The exergy destruction is obtained from an exergy rate balance, Eq. 7.13a,

$$\dot{E}_d = \left[\{\text{Rate exergy is supplied}\} - \dot{W}\right] = 130.9\text{MW} - 80\text{MW} = 50.9\text{MW}.$$

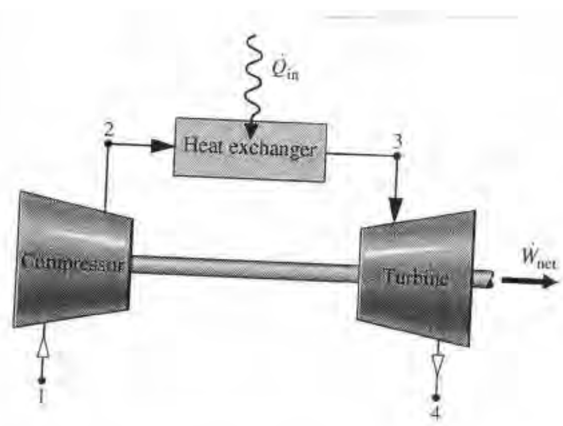
Summary:

- Exergy Supplied: 130.9 MW
- Disposition of Exergy:
  - ✓ Power developed: 80 MW (61.1%)
  - ✓ Exergy destroyed: 50.9 MW (38.9%)

130.9 MW

PROBLEM 7.91

Figure P7.91 shows a gas turbine power plant using air as the working fluid. The accompanying table gives steady-state operating data. Air can be modeled as an ideal gas. Stray heat transfer and the effects of motion and gravity can be ignored. Let $T_0 = 290$ K, $p_0 = 100$ kPa. Determine, each in kJ per kg of air flowing, (a) the *net* power developed, (b) the *net* exergy increase of the air passing through the heat exchanger, $(e_{f3} - e_{f2})$, and (c) a full exergy accounting based on the exergy supplied to the plant found in part (b). Comment.



State	p (kPa)	T (K)	h (kJ/kg)	s^0 (kJ/kg K)[a]
1	100	290	290.16	1.6680
2	500	505	508.17	2.2297
3	500	875	904.99	2.8170
4	100	635	643.93	2.4688

[a] s^0 is the variable appearing in Eq. 6.20a and Table A-22.

ENGR. MODEL:

1. The gas turbine power plant operates at steady state. Control volumes at steady state enclose each of the three components and the overall power plant.
2. Effects of motion and gravity can be ignored.
3. Air is modeled as an ideal gas.
4. $T_0 = 290$ K, $P_0 = 100$ kPa

ANALYSIS:

(a) The net power developed equals the power developed by the turbine less the power required by the compressor:

$$\frac{\dot{W}_{net}}{\dot{m}} = (h_3 - h_4) - (h_2 - h_1) = (904.99 - 643.93) - (508.17 - 290.16)$$
$$= 43.05 \text{ kJ/kg} \quad \longleftarrow \text{(a)}$$

1 (b)
$$\frac{\dot{E}_{f3} - \dot{E}_{f2}}{\dot{m}} = (h_3 - h_2) - T_0(s_3 - s_2) = (h_3 - h_2) - T_0(s_3^\circ - s_2^\circ - R \ln \cancelto{0}{P_3/P_2}) \quad (P_3 = P_2)$$
$$= (904.99 - 508.17) - 290(2.8170 - 2.2297) = 226.5 \text{ kJ/kg} \quad \longleftarrow \text{(b)}$$

(c) The net exergy carried out with the gas exiting the turbine:

$$\frac{\dot{E}_{f4} - \dot{E}_{f1}}{\dot{m}} = (h_4 - h_1) - T_0(s_4 - s_1) = (h_4 - h_1) - T_0(s_4^\circ - s_1^\circ - R \ln \cancelto{0}{P_4/P_1}) \quad (P_4 = P_1)$$
$$= (643.93 - 290.16) - 290(2.4688 - 1.6680) = 121.54 \text{ kJ/kg}$$

Rate exergy is destroyed within the turbine and compressor:

$$\left(\frac{\dot{E}_d}{\dot{m}}\right)_{comp} = T_0 \frac{\dot{\sigma}_{cv}}{\dot{m}} = T_0(s_2 - s_1) = T_0[s_2^\circ - s_1^\circ - R \ln P_2/P_1]$$
$$= 290\text{K}\left[2.2297 - 1.6680 - \frac{8.314}{28.97} \ln[5/1]\right] = 28.95 \frac{\text{kJ}}{\text{kg}}$$

$$\left(\frac{\dot{E}_d}{\dot{m}}\right)_{turb} = T_0[s_4^\circ - s_3^\circ - R \ln P_4/P_3] = 290\left[2.4688 - 2.8170 - \frac{8.314}{28.97} \ln(1/5)\right]$$
$$= 32.97 \text{ kJ/kg}$$

Summary: $\longleftarrow$ (c)

- Exergy supplied: 226.5 kJ/kg
- Disposition of exergy:
  - ✓ power developed: 43.05 kJ/kg (19%)
  - ✓ exergy destroyed:
    - compressor: 28.95 kJ/kg (12.8%)
    - turbine: 32.97 kJ/kg (14.6%)
  - ✓ Net exergy carried out at turbine exit: 121.54 kJ/kg (53.7%)
  - Total: 226.5 kJ/kg

For a stand-alone unit the performance of the gas turbine is poor: relatively little power is developed while considerable exergy is carried out at the turbine exit. But see note 2.

2

1. The net increase in exergy of the air passing through the heat exchanger results from an exergy transfer accompanying the heat transfer, $\dot{Q}_{in}$, which may be from an external source or combustion of a fuel. The associated exergy destruction is not considered in this analysis.

2. Considerable exergy is carried from the power plant with the air exiting the turbine. Steps should be taken to utilize this exergy cost-effectively. For options, see Secs. 9.7 and 9.9.

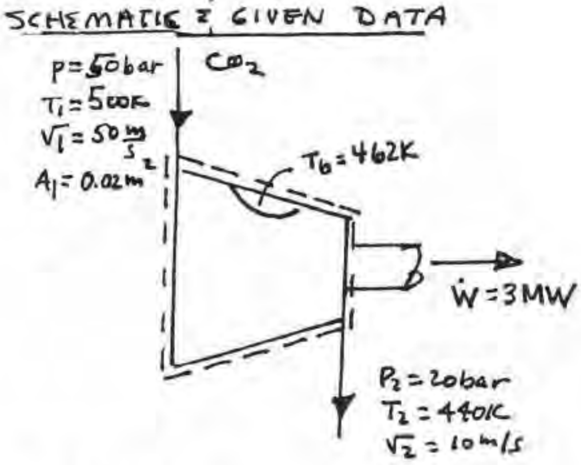
PROBLEM 7.92

Carbon dioxide (CO_2) gas enters a turbine operating at steady state at 50 bar, 500 K with a velocity of 50 m/s. The inlet area is 0.02 m^2. At the exit, the pressure is 20 bar, the temperature is 440 K, and the velocity is 10 m/s. The power developed by the turbine is 3 MW, and heat transfer occurs across a portion of the surface where the average temperature is 462 K. Assume ideal gas behavior for the carbon dioxide and neglect the effect of gravity. Let $T_0 = 298$ K, $p_0 = 1$ bar.

(a) Determine the rate of heat transfer, in kW.

(b) Perform a full exergy accounting, in kW, based on the *net* rate exergy is carried into the turbine by the carbon dioxide.

SCHEMATIC & GIVEN DATA



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. Heat transfer occurs only at temperature $T_b = 462$ K.
3. The effect of gravity can be ignored.
4. CO_2 is modeled as an ideal gas.
5. For the environment, $T_0 = 298$K, $P_0 = 1$bar.

ANALYSIS: (a) An energy rate balance reduces to give

$$\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}\left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2}\right] \quad (1)$$

where

$$\dot{m} = \frac{A V_1}{v_1} = \frac{P_1 A_1 V_1}{R T_1} = \frac{(5\times10^6\,N/m^2)(0.02\,m^2)(50\,\frac{m}{s})}{\left(\frac{8314}{44.01}\,\frac{N\cdot m}{kg\cdot K}\right)(500K)} = 52.93\,\frac{kg}{s}$$

$$v_1 = RT_1/P_1$$

From Table A-23, $\bar{h}_1 = 17,678$ kJ/kmol, $\bar{h}_2 = 15,054$ kJ/kmol. Thus, Eq. (1) gives

$$\dot{Q}_{cv} = 3\times10^3\,kW + 52.93\,\frac{kg}{s}\left[\frac{15,054 - 17,678}{44.01}\,\frac{kJ}{kg} + \left[\frac{(10)^2 - (50)^2}{2}\right]\left(\frac{m}{s}\right)^2\left|\frac{1\,kg\cdot m/s^2}{1\,N}\right|\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|\right]$$

$$= -219\,kW \quad (a)$$

(b) The net rate exergy is carried into the turbine $= \dot{E}_{f1} - \dot{E}_{f2}$, or

$$(\dot{E}_{f1} - \dot{E}_{f2}) = \dot{m}\left[(h_1 - h_2) - T_0(s_1 - s_2) + \frac{V_1^2 - V_2^2}{2}\right]$$

$$= \dot{m}\left[(h_1 - h_2) - T_0\left[s_1^\circ - s_2^\circ - R\ln P_1/P_2\right] + \frac{V_1^2 - V_2^2}{2}\right]$$

From Table A-23, $\bar{s}_1^\circ = 234.814$ kJ/kmol·K, $\bar{s}_2^\circ = 229.230$ kJ/kmol·K. Thus

$$\dot{E}_{f1} - \dot{E}_{f2} = 52.93\,\frac{kg}{s}\left[\frac{(17,678 - 15,054)}{44.01}\,\frac{kJ}{kg} - 298K\left(\frac{234.814 - 229.230}{44.01}\,\frac{kJ}{kg\cdot K} - \frac{8.314}{44.01}\ln\frac{50}{20}\right) + \left[\left[\frac{(50)^2 - (10)^2}{2}\right]\left(\frac{m}{s}\right)^2\left|\frac{1\,kg\cdot m/s^2}{1\,N}\right|\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|\right]\right]$$

$$= 3948\,kW$$

The rate exergy is carried from the control volume accompanying heat transfer is

PROBLEM 7.92 (Continued)

$$\dot{E}_q = \left[1 - \frac{T_0}{T_b}\right]\dot{Q}_{cv} = \left[1 - \frac{298}{462}\right](-219\text{ kW}) = -78\text{ kW}$$

The rate of exergy destruction can be found from an exergy rate balance or using $\dot{E}_d = T_0 \dot{\sigma}_{cv}$. Selecting the former,

$$\dot{E}_d = \dot{E}_q - \dot{W} + (\dot{E}_{f1} - \dot{E}_{f2})$$

$$= (-78) + (-3000) + 3948 = 870\text{ kW}$$

Summary: (b)

- Net exergy supplied to the turbine: 3948 kW

- Disposition of exergy supplied:

✓ Power developed:	3000 kW	(76%)
✓ Exergy carried out via heat transfer	78 kW	(2%)
✓ Exergy destroyed	870 kW	(22%)
	3948 kW	

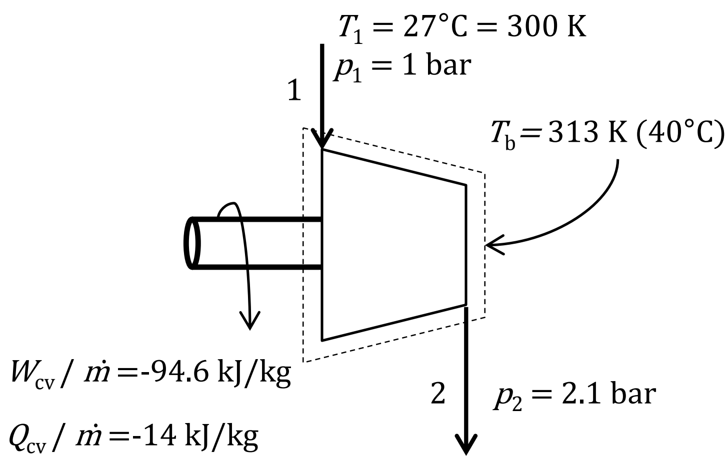
Problem 7.93

Air is compressed in an axial-flow compressor operating at steady state from 27°C, 1 bar to a pressure of 2.1 bar. The work required is 94.6 kJ per kg of air flowing. Heat transfer from the compressor occurs at an average surface temperature of 40°C at the rate of 14 kJ per kg of air flowing. The effects of motion and gravity can be ignored. Let T_0 = 20°C, p_0 = 1 bar. Assuming ideal gas behavior, (a) determine the temperature of the air at the exit, in °C, (b) determine the rate of exergy destruction within the compressor, in kJ per kg of air flowing, and (c) perform a full exergy accounting, in kJ per kg of air flowing, based on work input.

Known:
Operating data are provided for an air compressor at steady state.

Find:
Determine (a) the temperature of the air leaving the compressor, and (b) the rate of exergy destruction per kg of air flowing. (c) Perform a full exergy accounting in, kJ per kg of air flowing, based on work input.

Schematic & Known Data:



Engineering model:
(1) The control volume shown in the accompanying figure is at steady state.
(2) Heat transfer takes place at T_b only.
(3) Kinetic and potential energy changes from inlet to exit can be ignored.
(4) Air is modeled as an ideal gas.
(5) For the environment, T_0=293 K (20°C), p_0=1 bar

Analysis:

(a) At steady state, with assumptions given and $\dot{m}_1 = \dot{m}_2 = \dot{m}$, and with data from Table A-22, $h_1 = 300.19\ \frac{\text{kJ}}{\text{kg}}$, an energy balance reduced to:

$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2)$, rearranging

$$h_2 = h_1 + \frac{\dot{Q}_{\mathrm{cv}}}{\dot{m}} - \frac{\dot{W}_{\mathrm{cv}}}{\dot{m}} = (300.19 + (-14) - (-94.6))\frac{\mathrm{kJ}}{\mathrm{kg}} = 380.79\ \frac{\mathrm{kJ}}{\mathrm{kg}}$$

Interpolating from Table A-22 at $h_2 = 380.79\frac{\mathrm{kJ}}{\mathrm{kg}}$, $s°_2 = 1.94006$, and $T_2 = 380.02\ \mathrm{K}$

(b) Determine the rate of exergy destruction using Eq. 7.11a:

$$0 = \frac{\dot{Q}_{\mathrm{cv}}}{T_{\mathrm{b}}} - \dot{m}(s_1 - s_2) + \dot{\sigma}_{\mathrm{cv}}$$

Rearranging and using values from Table A-22, $s°_1 = 1.70203$

$$\frac{\dot{\sigma}_{\mathrm{cv}}}{\dot{m}} = \frac{\left(-\frac{\dot{Q}_{\mathrm{cv}}}{\dot{m}}\right)}{T_{\mathrm{b}}} + s_2 - s_1 = \frac{\left(-\frac{\dot{Q}_{\mathrm{cv}}}{\dot{m}}\right)}{T_{\mathrm{b}}} + \left(s°_2 - s°_1 - R\ln\frac{p_2}{p_1}\right)$$

$$= \left(\frac{14\,\frac{\mathrm{kJ}}{\mathrm{kg}}}{313\ \mathrm{K}}\right) + \left(1.94006 - 1.70203 - \frac{8.314}{28.97}\ln\frac{2.1}{1}\right)\ \frac{\mathrm{kJ}}{\mathrm{kg}\cdot\mathrm{K}}$$

$$= (0.0447 + 0.0251)\ \frac{\mathrm{kJ}}{\mathrm{kg}\cdot\mathrm{K}} = 0.0698\ \frac{\mathrm{kJ}}{\mathrm{kg}\cdot\mathrm{K}}$$

$$\frac{\dot{E}_{\mathrm{d}}}{\dot{m}} = T_0\frac{\dot{\sigma}_{\mathrm{cv}}}{\dot{m}} = 293\ \mathrm{k}\left(0.0698\frac{\mathrm{kJ}}{\mathrm{kg}\cdot\mathrm{K}}\right) = 20.45\ \frac{\mathrm{kJ}}{\mathrm{kg}}$$

(c) In order to perform a full exergy accounting, the following must first be determined:

The net change in exergy of the air stream is

$$\frac{\left(\dot{\mathrm{E}}_{\mathrm{f2}} - \dot{\mathrm{E}}_{\mathrm{f1}}\right)}{\dot{m}} = (h_2 - h_1) - T_0(s_2 - s_1) = (380.79 - 300.19) - 293\ \mathrm{K}(0.0251) = 73.25\ \frac{\mathrm{kJ}}{\mathrm{kg}}$$

The exergy carried out via heat transfer is

$$= \frac{\dot{Q}_{\mathrm{cv}}}{\dot{m}}\left(1 - \frac{T_0}{T_{\mathrm{b}}}\right) = 14\frac{\mathrm{kJ}}{\mathrm{kg}}\left(1 - \frac{293}{313}\right) = 0.895\ \frac{\mathrm{kJ}}{\mathrm{kg}}$$

<u>Full Exergy Accounting:</u>

Exergy Supplied to compressor = work: $94.6\,\frac{\mathrm{kJ}}{\mathrm{kg}}$

Disposition of exergy:

Exergy increase of air stream:	$73.25\,\frac{\mathrm{kJ}}{\mathrm{kg}}$	(77%)
Exergy carried out via heat transfer:	$0.895\,\frac{\mathrm{kJ}}{\mathrm{kg}}$	(1%)
Exergy destroyed:	$20.45\,\frac{\mathrm{kJ}}{\mathrm{kg}}$	(22%)

Problem 7.94

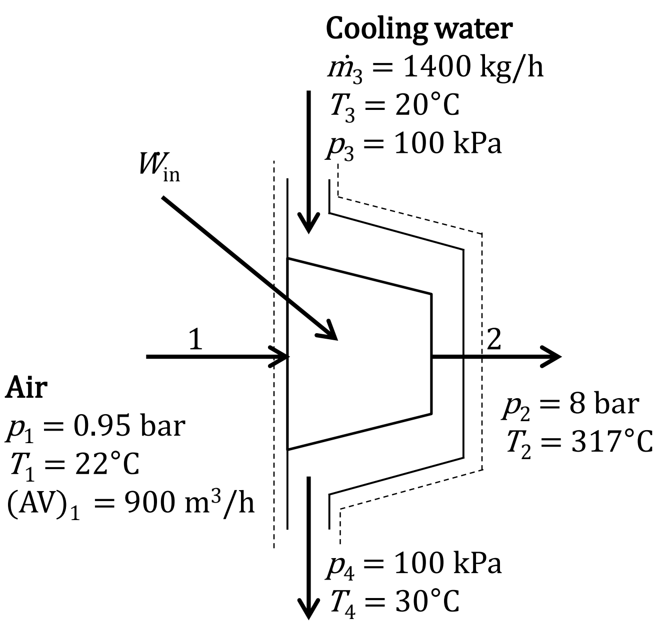
Figure P7.94 shows a compressor fitted with a water jacket. The compressor operates at steady state and takes in air with a volumetric flow rate of 900 m^3/h at 22°C, 0.95 bar and discharges air at 317°C, 8 bar. Cooling water enters the water jacket at 20°C, 100 kPa with a mass flow rate of 1400 kg/h and exits at 30°C and essentially the same pressure. The accompanying table gives steady-state operating data. There is no significant heat transfer from the outer surface of the water jacket to its surroundings, and the effects of motion and gravity can be ignored. For the water-jacketed compressor, perform a full exergy accounting of the power input. Let T_0 = 20°C, p_0 = 1 atm.

Known:
Steady-state operating data are provided for a water-jacketed air compressor.

Find:
Perform an exergy accounting of the power input.

Schematic and Known Data:

State	**Fluid type**	***T***	***p***	***h* (kJ/kg)**	**s^o (kJ/kg·K)**
1	Air	22° C	0.95 bar	295.17	1.68515
2	Air	317° C	8 bar	596.52	2.39140



Engineering Model:
(1) The control volume shown in the schematic operates at steady state with negligible kinetic and potential energy effects and $\dot{Q}_{\text{cv}} = 0$.

(2) Air is modeled as an ideal gas.
#1 (3) Water is modeled as incompressible with c = 4.19 kJ/kg (Table A-19).
(4) For the environment, T_0 = 293 K, p_0 = 1 atm.

<u>Analysis:</u>
The exergy entering the control volume with the power input has the following disposition: the exergy of the air stream is increased, the exergy of the water stream is increased, and the exergy is destroyed by irreversibilities within the control volume. These quantities are now evaluated:

At steady state, $\dot{m}_1 = \dot{m}_2$, $\dot{m}_3 = \dot{m}_4$. The energy rate balance reduces to give:

$$\dot{W}_{cv} = \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_4) \qquad (1)$$

With the ideal gas model equation of state:

$$\dot{m}_1 = \frac{(\mathrm{AV})_1}{v_1} = \frac{(\mathrm{AV})_1}{\frac{RT_1}{p_1}} = \frac{p_1(\mathrm{AV})_1}{RT_1} = \frac{\left((0.95\ \mathrm{bar})\left|\frac{10^5\ \frac{\mathrm{N}}{\mathrm{m}^2}}{1\ \mathrm{bar}}\right|\right)\left(900\ \frac{\mathrm{m}^3}{\mathrm{h}}\right)}{\left(\frac{8.314}{28.97}\frac{(\mathrm{N\cdot m})}{\mathrm{kg\cdot K}}\right)(295\ \mathrm{K})} = 1009.9\,\frac{\mathrm{kg}}{\mathrm{h}}$$

Then, insert values into Eq. (1) with provided h_1 and h_2 values from accompanying table, and Eq. 3.20b:

$$\dot{W}_{\mathrm{cv}} = \dot{m}_1(h_1 - h_2) + \dot{m}_3 c\left[(T_3 - T_4) + v\underbrace{(p_3 - p_4)}_{=0}\right]$$

$$\dot{W}_{\mathrm{cv}} = \left(1009.9\,\frac{\mathrm{kg}}{\mathrm{h}}\right)(295.17 - 596.52)\frac{\mathrm{kJ}}{\mathrm{kg}} + \left(1400\,\frac{\mathrm{kg}}{\mathrm{h}}\right)\left(4.19\,\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\right)(-10\ \mathrm{K})$$
$$= (-304333 - 58660)\frac{\mathrm{kJ}}{\mathrm{h}}\cdot\left|\frac{1\ \mathrm{kW}}{3600\,\frac{\mathrm{kJ}}{\mathrm{h}}}\right| = -100.83\ \mathrm{kW}$$

The rate of exergy destruction can be determined using an exergy rate balance or $\dot{\mathrm{E}}_{\mathrm{d}} = T_0\dot{\sigma}$ and an entropy rate balance. At steady state, the entropy rate balance reduces to give:

$$0 = \sum_j \underbrace{\frac{\dot{Q}_j}{T_j}}_{=0} + \dot{m}_1(s_1 - s_2) + \dot{m}_3(s_3 - s_4) + \dot{\sigma}_{cv}$$

Or with s^{o} data from the provided table and Eq. 6.13:

$$\dot{\sigma}_{\mathrm{cv}} = \dot{m}_1\left[s^o(T_2) - s^o(T_1) - R\ln\frac{p_2}{p_1}\right] + \dot{m}_3 c\ln\frac{T_4}{T_3}$$
$$= \left(1009.9\,\frac{\mathrm{kg}}{\mathrm{h}}\right)\left(2.39140 - 1.68515 - \left(\frac{8.314}{28.97}\right)\ln\frac{8}{0.95}\right)\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}$$
$$+ \left(1400\,\frac{\mathrm{kg}}{\mathrm{h}}\right)\left(4.19\,\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\right)\ln\frac{303}{293}$$

$$= (95.70\ + 196.86)\frac{\text{kJ}}{\text{h}\cdot\text{K}}\cdot\left|\frac{1\ \text{kW}}{3600\,\frac{\text{kJ}}{\text{h}}}\right| = 0.08127\,\frac{\text{kW}}{\text{K}}$$

Therefore:

$$\dot{\text{E}}_{\text{d}} = T_0\dot{\sigma}_{\text{cv}} = (293\ \text{K})\left(0.08127\,\frac{\text{kW}}{\text{K}}\right) = 23.81\ \text{kW}$$

For the air, the increase in flow exergy rate from inlet to exit is:

$$\dot{\text{E}}_{\text{f}_2} - \dot{\text{E}}_{\text{f}_1} = \dot{m}_1\left(\text{e}_{\text{f}_2} - \text{e}_{\text{f}_1}\right) = \dot{m}_1[h_2 - h_1 - T_0(s_2 - s_1)] = \underbrace{\dot{m}_1(h_2 - h_1)}_{=304333\,\frac{\text{kJ}}{\text{h}}} - T_0\,\underbrace{\dot{m}_1(s_2 - s_1)}_{=95.70\,\frac{\text{kJ}}{\text{h}\cdot\text{K}}}$$

Using values calculated in intermediate steps of previous calculations listed above:

$$\dot{\text{E}}_{\text{f}_2} - \dot{\text{E}}_{\text{f}_1} = \left[304333\,\frac{\text{kJ}}{\text{h}} - (293\ \text{K})\left(95.70\,\frac{\text{kJ}}{\text{h}\cdot\text{K}}\right)\right]\cdot\left|\frac{1\ \text{kW}}{3600\,\frac{\text{kJ}}{\text{h}}}\right| = 76.75\ \text{kW}$$

For the water, the increase in flow exergy rate from inlet to exit is:

$$\dot{\text{E}}_{\text{f}_4} - \dot{\text{E}}_{\text{f}_3} = \dot{m}_3\left(\text{e}_{\text{f}_4} - \text{e}_{\text{f}_3}\right) = \dot{m}_3[h_4 - h_3 - T_0(s_4 - s_3)] = \underbrace{\dot{m}_3(h_4 - h_3)}_{=58660\,\frac{\text{kJ}}{\text{h}}} - T_0\,\underbrace{\dot{m}_3(s_4 - s_3)}_{=196.86\,\frac{\text{kJ}}{\text{h}\cdot\text{K}}}$$

Using values calculated in intermediate steps of previous calculations listed above:

$$\dot{\text{E}}_{\text{f}_4} - \dot{\text{E}}_{\text{f}_3} = \left[58660\,\frac{\text{kJ}}{\text{h}} - (293\ \text{K})\left(196.86\,\frac{\text{kJ}}{\text{h}\cdot\text{K}}\right)\right]\cdot\left|\frac{1\ \text{kW}}{3600\,\frac{\text{kJ}}{\text{h}}}\right| = 0.27\ \text{kW}$$

Exergy Accounting:

• Power Input		100.83 kW	
• Disposition			
○ Air Stream		76.75 kW	76.1%
○ Water Stream		0.27 kW	0.3%
○ Exergy Destruction	+	23.81 kW	23.6%
	=	100.83 kW	100%

Comment**:**

1. Alternatively, for the liquid water, $h \sim h_f(T)$, $s \sim s_f(T)$.

Problem 7.95

Argon enters an insulated turbine operating at steady state at 1000°C and 2 MPa and exhausts at 350 kPa. The mass flow rate is 0.5 kg/s and the turbine develops power at the rate of 120 kW. Determine:

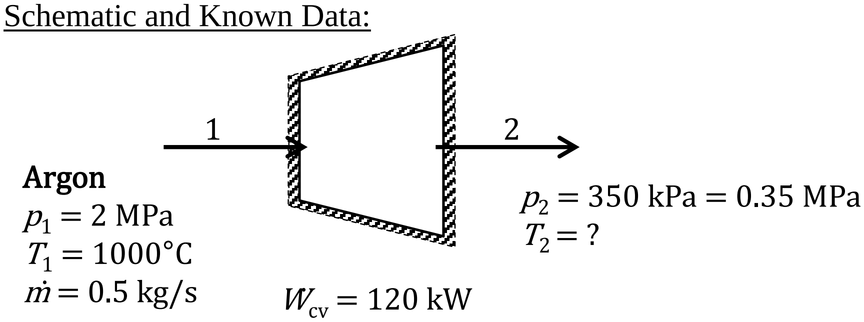
(a) the temperature of the argon at the turbine exit, in °C.
(b) the exergy destruction rate of the turbine, in kW.
(c) the turbine exergetic efficiency.

Neglect kinetic and potential energy effects. Let T_0 = 20°C, p_0 = 1 bar.

Known:
Operating data are provided for an insulated turbine at steady state through which argon is expanding.

Find:
Determine (a) the temperature at the turbine exit, (b) the exergy destruction rate, and (c) the turbine exergetic efficiency.

Schematic and Known Data:



Engineering Model:
(1) The turbine is insulated and at steady state.
(2) Kinetic and potential energy effects can be neglected.
(3) Argon is modeled as an ideal gas.
(4) For the environment, T_0 = 20°C, p_0 = 1 bar.

Analysis:

(a) With assumptions (1) and (2), mass and energy rate balances reduce to give

$\dot{W}_{cv} = \dot{m}(h_1 - h_2)$

Accordingly, with $c_p = \frac{5}{2}R = \left(\frac{5}{2}\right)\frac{\bar{R}}{M}$ from Table A-21:

$$h_2 - h_1 = -\frac{\dot{W}_{cv}}{\dot{m}} \Rightarrow \frac{5}{2}R\,(T_2 - T_1) = -\frac{\dot{W}_{cv}}{\dot{m}} \Rightarrow$$

$$T_2 = T_1 - \frac{\dot{W}_{cv}}{\dot{m}\frac{5}{2}R} = 1273\text{ K} - \frac{120\,\frac{\text{kJ}}{\text{s}}}{\left(0.5\,\frac{\text{kg}}{\text{s}}\right)\left(\frac{5}{2}\right)\left(\frac{8.314}{39.94}\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)} = 811.8\text{ K}$$

(b) The exergy rate is $\dot{\mathrm{E}}_\mathrm{d} = T_0\dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production. With stated assumptions, mass and entropy balances give $\dot{\sigma}_{cv} = \dot{m}(s_2 - s_1)$. Thus:

$$\dot{\mathrm{E}}_\mathrm{d} = \dot{m}T_0(s_2 - s_1) = \dot{m}T_0\left(c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}\right) = \dot{m}T_0\frac{\bar{R}}{M}\left(2.5\ln\frac{T_2}{T_1} - \ln\frac{p_2}{p_1}\right)$$

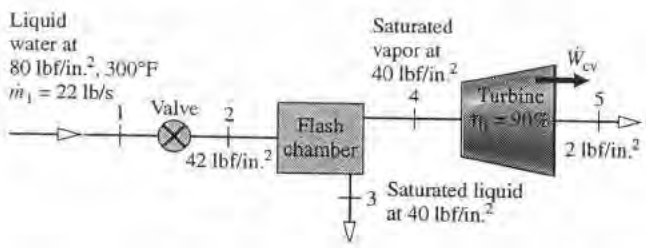
$$= \left(0.5\,\frac{\mathrm{kg}}{\mathrm{s}}\right)(293\ \mathrm{K})\left(\frac{8.314}{39.94}\frac{\mathrm{kJ}}{\mathrm{kg}\cdot\mathrm{K}}\right)\left(2.5\ln\frac{811.8}{1273} - \ln\frac{0.35}{2}\right) = 18.86\ \mathrm{kW}$$

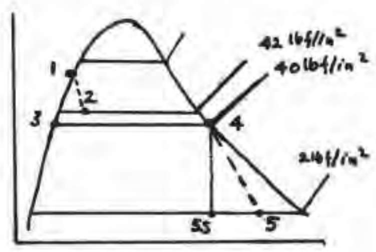
(c) The exergetic efficiency is given by Eq. 7.24:

$$\epsilon = \frac{\dot{W}_{cv}}{\dot{m}\left(\mathrm{e_{f_1}} - \mathrm{e_{f_2}}\right)} = \frac{\dot{W}_{cv}}{\dot{m}\left(h_1 - h_2 - T_0(s_1 - s_2)\right)} = \frac{120\ \mathrm{kW}}{(120 + 18.86)\mathrm{kW}} = 0.864 = 86.4\%$$

PROBLEM 7.96

Figure P7.96 shows liquid water at 80 lbf/in.², 300°F entering a flash chamber though a valve at the rate of 22 lb/s. At the valve exit, the pressure is 42 lbf/in.². Saturated liquid at 40 lbf/in.² exits from the bottom of the flash chamber and saturated vapor at 40 lbf/in.² exits from near the top. The vapor stream is fed to a steam turbine having an isentropic efficiency of 90% and an exit pressure of 2 lbf/in.². For steady-state operation, negligible heat transfer with the surroundings, and no significant effects of motion and gravity, perform a full exergy accounting, in Btu/s, of the *net* rate at which exergy is supplied: $(\dot{E}_{f1} - \dot{E}_{f3} - \dot{E}_{f5})$. Let $T_0 = 500°R$, $p_0 = 1$ atm.





ENGR. MODEL: (1) Each component operates at steady state with negligible heat transfer between each component and its surroundings. (2) Kinetic and potential energy effects can be ignored. (3) The expansion across the valve is a throttling process. (4) $T_0 = 500°R$, $P_0 = 1$ atm.

ANALYSIS At steady state, mass rate balances reduce to give $\dot{m}_2 = \dot{m}_1$, $\dot{m}_3 + \dot{m}_4 = \dot{m}_2$, $\dot{m}_4 = \dot{m}_5$. An entropy rate balance for a control volume enclosing the flash chamber takes the form

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{\,0} + \dot{m}_2 s_2 - \dot{m}_3 s_3 - \dot{m}_4 s_4 + \dot{\sigma}_{cv}$$

or

$$\dot{\sigma}_{cv} = \dot{m}_4 s_4 + \dot{m}_3 s_3 - \dot{m}_2 s_2 \qquad \text{[flash chamber]} \qquad (1)$$

Similarly, for the valve and turbine, the rates of entropy production are

$$\dot{\sigma}_{cv} = \dot{m}_1 (s_2 - s_1) \qquad \text{[valve]} \qquad (2)$$

$$\dot{\sigma}_{cv} = \dot{m}_4 (s_5 - s_4) \qquad \text{[turbine]} \qquad (3)$$

To evaluate Eqs. (1)-(3) requires $\dot{m}_3$, $\dot{m}_4$, s_1, s_2, s_3, s_4, and s_5. These will now be evaluated in turn.

Using the mass rate balance together with listed assumptions, an energy rate balance for a control volume enclosing the flash chamber reduces to

$$0 = \dot{m}_2 h_2 - \dot{m}_3 h_3 - \dot{m}_4 h_4$$

$$0 = \dot{m}_1 h_2 - \dot{m}_3 h_3 - (\dot{m}_1 - \dot{m}_3) h_4$$

Across the valve $h_2 = h_1$, so on solving the above equation

$$\dot{m}_3 = \dot{m}_1 \frac{[h_1 - h_4]}{[h_3 - h_4]} = \left(22 \frac{lb}{s}\right)\left[\frac{269.7 - 1170}{236.16 - 1170}\right] = 21.21 \text{ lb/s}$$

where $h_1 \approx h_f(T_1)$ from Table A-2E and h_3 and h_4 are from Table A-3E. Then, $\dot{m}_4 = \dot{m}_2 - \dot{m}_3 = 22 - 21.21 = 0.79$ lb/s.

From Table A-2E, $s_1 \approx s_f(T_1) = 0.4372$ Btu/lb·°R. State 2 is fixed by 42 lbf/in² and $h_2 = h_1 = 269.7$ Btu/lb. Thus, with data from Table A-3E

$$x_2 = \frac{h_2 - h_f}{h_{fg}} = \frac{269.7 - 239.1}{931.8} = 0.0328$$

Thus

$$s_2 = s_f + x_2 s_{fg} = 0.3961 + 0.0328(1.2767) = 0.438 \text{ Btu/lb·°R}$$

From Table A-3E at 40 lbf/in², $s_3 = 0.3921$ Btu/lb·°R, $s_4 = 1.6767$ Btu/lb·°R. State 5 is fixed by 2 lbf/in² and h_5. Using the turbine isentropic efficiency,

$$\eta_t = \frac{h_4 - h_5}{h_4 - h_{5s}} \qquad (4)$$

where h_{5s} is the specific entropy at the turbine exit for an isentropic expansion from state 4 to 2 lbf/in². The value of h_{5s} is determined by 2 lbf/in² and $s_{5s} = s_4$.

PROBLEM 7.96 (Continued)

Thus, with data from Table A-3E at 2 lbf/in²

$$x_{5s} = \frac{s_{5s} - s_f}{s_{fg}} = \frac{1.6767 - 0.175}{1.7448} = 0.861 \Rightarrow h_{5s} = h_f + x_{5s} h_{fg}$$
$$= 94.02 + (0.861)(1022.1) = 974.05 \ \frac{Btu}{lb}$$

Then, with data from Table A-3E at 2 lbf/in²

$x_{5s} = \frac{s_{5s} - s_f}{s_{fg}} = \frac{1.6767 - 0.175}{1.7448} = 0.861$, $h_{5s} = h_f + x_{5s} h_{fg} = 94.02 + 0.861(1022.1) = 974.05$ Btu/lb

Solving Eq. (4) for h_5 and inserting values

$$h_5 = h_4 - \eta_t (h_4 - h_{5s}) = 1170 - 0.9(1170 - 974.05) = 993.65 \ \text{Btu/lb}$$

Using this together with data from Table A-3E at 2 lbf/in²

$$x_5 = \frac{h_5 - h_f}{h_{fg}} = \frac{993.65 - 94.02}{1022.1} = 0.88$$

$s_5 = s_f + x_5 s_{fg} = 0.175 + 0.88(1.7448) = 1.7104$ Btu/lb·°R.

Finally, substituting values into Eqs. (1) – (3)

- flash chamber: $\dot{\sigma}_{cv} = (0.79)[1.6767] + (21.21)[0.3921] - (22)[0.438] = 0.0054 \ \frac{Btu}{s \cdot °R}$
- valve: $\dot{\sigma}_{cv} = (22)[0.438 - 0.4372] = 0.0176 \ \frac{Btu}{s \cdot °R}$
- turbine: $\dot{\sigma}_{cv} = (0.79)[1.7104 - 1.6767] = 0.0266 \ \frac{Btu}{s \cdot °R}$

Mass and energy rate balances reduce for the turbine to give

$$\dot{W}_t = \dot{m}_4 [h_4 - h_5] = \left(0.79 \ \frac{lb}{s}\right)[1170 - 993.65] = 139.3 \ \frac{Btu}{s}$$

Using the entropy production rates, exergy destruction rates are found from $\dot{E}_d = T_0 \dot{\sigma}_{cv}$:

- flash chamber: $\dot{E}_d = 500°R\left(0.0054 \ \frac{Btu}{s \cdot °R}\right) = 2.7$ Btu/s
- valve: $\dot{E}_d = 500°R\left(0.0176 \ \frac{Btu}{s \cdot °R}\right) = 8.8$ Btu/s
- turbine: $\dot{E}_d = 500°R\left(0.0266 \ \frac{Btu}{s \cdot °R}\right) = 13.3$ Btu/s

The net rate at which exergy is supplied is $(\dot{E}_{f1} - \dot{E}_{f3} - \dot{E}_{f5}) = \underbrace{\dot{m}_1}_{= \dot{m}_3 + \dot{m}_5} e_{f1} - \dot{m}_3 e_{f3} - \dot{m}_5 e_{f5}$

$$\Rightarrow (\dot{E}_{f1} - \dot{E}_{f3} - \dot{E}_{f5}) = \dot{m}_3(e_{f1} - e_{f3}) + \dot{m}_5(e_{f1} - e_{f5})$$
$$= \dot{m}_3[(h_1 - h_3) - T_0(s_1 - s_3)] + \dot{m}_5[(h_1 - h_5) - T_0(s_1 - s_5)]$$
$$= 21.21 \frac{lb}{s}\left[(269.7 - 236.16) - 500(0.4372 - 0.3921)\right]\frac{Btu}{lb} + 0.79\left[(269.7 - 993.65) - 500(0.4372 - 1.7104)\right]$$
$$= 164.1 \ \text{Btu/s}$$

Summary:

- Net rate exergy is supplied: 164.1 Btu/s
- Disposition:
  - ✓ Power developed: 139.3 Btu/s (84.9%)
  - ✓ Exergy destruction
    - turbine: 13.3 Btu/s (8.1%)
    - valve: 8.8 Btu/s (5.4%)
    - flash chamber: 2.7 Btu/s (1.6%)

Total: 164.1 Btu/s

Problem 7.97

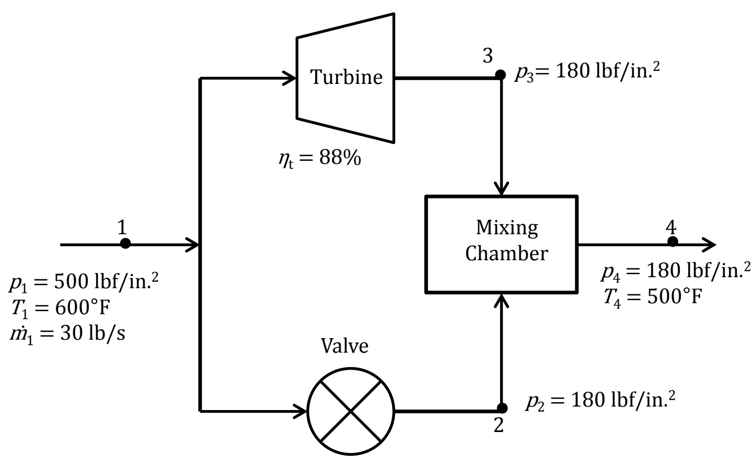
Figure P7.97 provides steady-state operating data for a throttling valve in parallel with a steam turbine having an isentropic turbine efficiency of 88%. The streams exiting the valve and the turbine mix in a mixing chamber. Heat transfer with the surroundings and the effects of motion and gravity can be neglected. Determine:
(a) the power developed by the turbine, in Btu/s.
(b) the mass flow rates through the turbine and valve, each in lb/s.
(c) a full exergy accounting, in Btu/s, of the net rate at which exergy is supplied: $(\dot{E}_{f1} - \dot{E}_{f4})$.

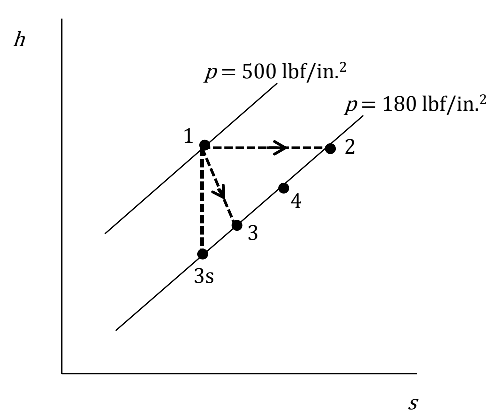
Let T_0 = 500°R, p_0 = 1 atm.

Known:
Operating data are provided for two streams of steam flowing through a throttling valve in parallel with a turbine that combines in a mixing chamber.

Find:
Determine (a) the power developed by the turbine, (b) the mass flow rates through the turbine and valve, (c) a full exergy accounting.

Schematic and Known Data:





State	*p* (lbf/in.^{2})	*T* (°F)	*h* (Btu/lb)	*s* (Btu/lb·°R)
1	500	600	1298.3	1.5585
2	180	551.5	1298.3	1.6650
3	180	-	1212.2	1.5723
3s	180	-	1200.5	1.5585
4	180	400	1214.4	1.5749

Engineering Model:

(1) Control volumes enclosing the turbine, valve, mixing chamber, and overall configuration are at steady state.
(2) Heat transfer with the surroundings and the effects of motion and gravity can be neglected.
(3) The expansion across the valve is a throttling process.
(4) For the environment, T_0 = 500°R, p_0 = 1 atm.

Analysis:

(a) Considering an overall control volume at steady state including the turbine, valve, and mixing chamber, mass and energy rate balances reduce to give $\dot{W}_t = \dot{m}(h_1 - h_4)$. With data from the provided table:

$$\dot{W}_t = \left(30\,\frac{\text{lb}}{\text{s}}\right)(1298.3 - 1214.4)\,\frac{\text{Btu}}{\text{lb}} = 2517\,\frac{\text{Btu}}{\text{s}} \cdot \left|\frac{3600\text{ s}}{1\text{ h}}\right| \cdot \left|\frac{1\text{ hp}}{2545\,\frac{\text{Btu}}{\text{h}}}\right| = 3560\text{ hp}$$

(b) Then mass and energy rate balances for a control volume enclosing only the turbine give $\dot{W}_t = \dot{m}_t(h_1 - h_3)$. Using the isentropic turbine efficiency:

$$\eta_t = \frac{h_1 - h_3}{h_1 - h_{3s}} \Longrightarrow (h_1 - h_3) = \eta_t(h_1 - h_{3s})$$

$$\dot{W}_t = \dot{m}_t \eta_t (h_1 - h_{3s}) \Longrightarrow \dot{m}_t = \frac{\dot{W}_t}{\eta_t(h_1 - h_{3s})}$$

Inserting values from the provided table:

$$\dot{m}_t = \frac{\left(30\,\frac{\text{lb}}{\text{s}}\right)(1298.3 - 1214.4)\,\frac{\text{Btu}}{\text{lb}}}{(0.88)(1298.3 - 1200.5)\,\frac{\text{Btu}}{\text{lb}}} = 29.25\,\frac{\text{lb}}{\text{s}}$$

The mass flow rate through the valve is then:

$$\dot{m} = \dot{m}_t + \dot{m}_v \Longrightarrow \dot{m}_v = \dot{m} - \dot{m}_t = 30 - 29.25 = 0.75\,\frac{\text{lb}}{\text{s}}$$

(c) Mass and entropy rate balances for the valve give:

$$\dot{\sigma}_{\text{valve}} = \dot{m}_v[s_2 - s_1]$$

Using data from the provided table:

$$\dot{\sigma}_{\text{valve}} = \left(0.75\,\frac{\text{lb}}{\text{s}}\right)(1.6650 - 1.5585)\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} = 0.0799\,\frac{\text{Btu}}{\text{s}\cdot{}^\circ\text{R}}$$

Mass and entropy rate balances for the turbine give:
$\dot{\sigma}_{\text{turbine}} = \dot{m}_t[s_3 - s_1]$

Using data from the provided table:

$$\dot{\sigma}_{\text{turbine}} = \left(29.25\,\frac{\text{lb}}{\text{s}}\right)(1.5723 - 1.5585)\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} = 0.4037\,\frac{\text{Btu}}{\text{s}\cdot{}^\circ\text{R}}$$

Mass and entropy rate balances for the mixing chamber give:

$$\dot{\sigma}_{\text{mix}} = \dot{m}_4 s_4 - \dot{m}_t s_3 - \dot{m}_v s_2 = (30)(1.5749) - (29.25)(1.5723) - (0.75)(1.6650)$$
$$= 0.0085\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$

The rates of exergy destruction in the three components can be evaluated using $\dot{\text{E}}_{\text{d}} = T_0\dot{\sigma}_{cv}$. Thus:

$$\dot{\text{E}}_{\text{d}_\text{v}} = (500{}^\circ\text{R})(0.0799)\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} = 39.95\,\frac{\text{Btu}}{\text{s}}$$
$$\dot{\text{E}}_{\text{d}_\text{t}} = (500{}^\circ\text{R})(0.4037)\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} = 201.85\,\frac{\text{Btu}}{\text{s}}$$
$$\dot{\text{E}}_{\text{d}_\text{m}} = (500{}^\circ\text{R})(0.0085)\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} = 4.25\,\frac{\text{Btu}}{\text{s}}$$

The net exergy carried in the water stream is:

$$\dot{\text{E}}_{\text{f}_1} - \dot{\text{E}}_{\text{f}_4} = \dot{m}[(h_1 - h_4) - T_0(s_1 - s_4)]$$
$$= \left(30\,\frac{\text{lb}}{\text{s}}\right)\left[(1298.3 - 1214.4)\,\frac{\text{Btu}}{\text{lb}} - (500{}^\circ\text{R})(1.5585 - 1.5749)\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right]$$
$$= 2763\,\frac{\text{Btu}}{\text{s}}$$

Exergy Accounting:

- Power Input 2763 Btu/s
- Disposition

Power Developed		2517 Btu/s	91.1%
Valve Destruction		39.95 Btu/s	1.5%
Turbine Destruction		201.85 Btu/s	7.3%
Mixer Destruction	+	4.25 Btu/s	0.2%
	=	2763 Btu/s	100.1%

PROBLEM 7.98

For the water heater of Problem 7.45, devise and evaluate an exergetic efficiency.

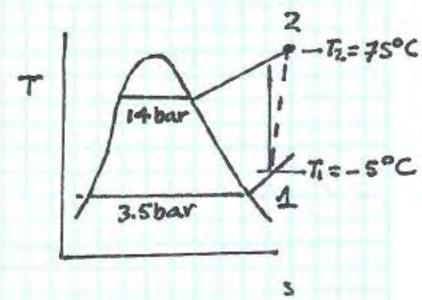
See solution to Problem 7.45 for data. Then,

(1) $$\epsilon = \frac{(\text{Exergy Stored})}{\left(\begin{array}{c}\text{Exergy Carried}\\ \text{In Electrically}\end{array}\right)} = \frac{1340\text{ kJ}}{26538\text{ kJ}} = 0.05\ (5\%)$$

1. This value is typical for the exergetic efficiencies exhibited by domestic water heaters operating from an input of electricity or natural gas.

PROBLEM 7.99

7.99 For the compressor of Example 6.14, evaluate the exergetic efficiency given by Eq. 7.25. Let $T_0 = 20°C$, $p_0 = 1$ atm.



Eq. 7.25:
$$\epsilon = \frac{e_{f2} - e_{f1}}{(-\dot{W}_{cv}/\dot{m})} = \frac{(h_2 - h_1) - T_0(s_2 - s_1)}{(h_2 - h_1)}$$

$$\epsilon = 1 - T_0\left(\frac{s_2 - s_1}{h_2 - h_1}\right)$$

With data from Table A-9,
$$\epsilon = 1 - 293K\left(\frac{0.98225 - 0.9572}{294.18 - 249.75}\right)\frac{kJ/kg\cdot K}{kJ/kg}$$

$$= 0.835 \; (83.5\%)$$

PROBLEM 7.100

7.100 For the heat exchanger of Example 7.6, evaluate the exergetic efficiency given by Eq. 7.27 with states numbered for the case at hand.

Eq. 7.27 regards the hot stream as supplying the exergy increase of the cold stream as well as the exergy destroyed. When expressed in the numbering of Example 7.6 it is (see Fig. E7.6)

$$\epsilon = \frac{\dot{m}\,(e_{f2} - e_{f1})}{\dot{m}\,(e_{f3} - e_{f4})}$$

With data from the solution to Example 7.6,

$$\epsilon = \frac{14.1\ \text{MW}}{16.93\ \text{MW}} = 0.833 \qquad (83.3\%)$$

PROBLE 7.101

7.101 Referring to the discussion of Sec. 7.6.2 as required, evaluate the exergetic efficiency for each of the following cases, assuming steady-state operation with negligible effects of heat transfer with the surroundings:

(a) Turbine: $\dot{W}_{cv}$ = 1200 hp, e_{f1} = 250 Btu//lb, e_{f2} = 15 Btu/lb, $\dot{m}$ = 240 lb/min.
(b) Compressor: $\dot{W}_{cv}/\dot{m}$ = −105 kJ/kg, e_{f1} = 5 kJ/kg, e_{f2} = 90 kJ/kg, $\dot{m}$ = 2 kg/s.
(c) Counterflow heat exchanger: $\dot{m}_h$ = 3 kg/s, $\dot{m}_c$ = 10 kg/s, e_{f1} = 2100 kJ/kg, e_{f2} = 300 kJ/kg, $\dot{E}_d$ = 3.4 MW.
(d) Direct contact heat exchanger: $\dot{m}_1$ = 10 lb/s, $\dot{m}_3$ = 15 lb/s, e_{f1} = 1000 Btu/lb, e_{f2} = 50 Btu/lb, e_{f3} = 400 Btu/lb.

(a) Apply Eq. 7.24.

$$\epsilon = \frac{\dot{W}_{cv}}{\dot{m}(e_{f1}-e_{f2})} = \frac{1200\,hp \left|\frac{2545\,Btu/h}{1\,hp}\right|\left|\frac{1\,h}{3600\,s}\right|}{\left(240\,\frac{lb}{min}\right)\left|\frac{1\,min}{60\,s}\right|(250-15)\,\frac{Btu}{lb}} = 0.902 \quad (90.2\%)$$

(b) Apply Eq. 7.25.

$$\epsilon = \frac{e_{f2}-e_{f1}}{(-\dot{W}_{cv}/\dot{m})} = \frac{(90-5)\,kJ/kg}{105\,kJ/kg} = 0.809 \quad (80.9\%)$$

(c) Apply Eq. 7.27, numbering as in Fig. 7.10.

$$\epsilon = \frac{\dot{m}_c(e_{f4}-e_{f3})}{\dot{m}_h(e_{f1}-e_{f2})}$$

An exergy rate balance reduces to give,

$$\dot{m}_c(e_{f4}-e_{f3}) = \dot{m}_h(e_{f1}-e_{f2}) - \dot{E}_d$$

Thus, Eq. 7.27 can be expressed as

$$\epsilon = \frac{\dot{m}_h(e_{f1}-e_{f2})-\dot{E}_d}{\dot{m}_h(e_{f1}-e_{f2})} = \frac{(3\,kg/s)(2100-300)\,\frac{kJ}{kg} - 3.4\,MW\left|\frac{10^3\,kJ/s}{1\,MW}\right|}{(3\,kg/s)(2100-300)\,\frac{kJ}{kg}} = 0.37 \quad (37\%)$$

(d) Apply Eq. 7.29, numbering as in Fig. 7.11.

$$\epsilon = \frac{\dot{m}_2(e_{f3}-e_{f2})}{\dot{m}_1(e_{f1}-e_{f3})}.$$

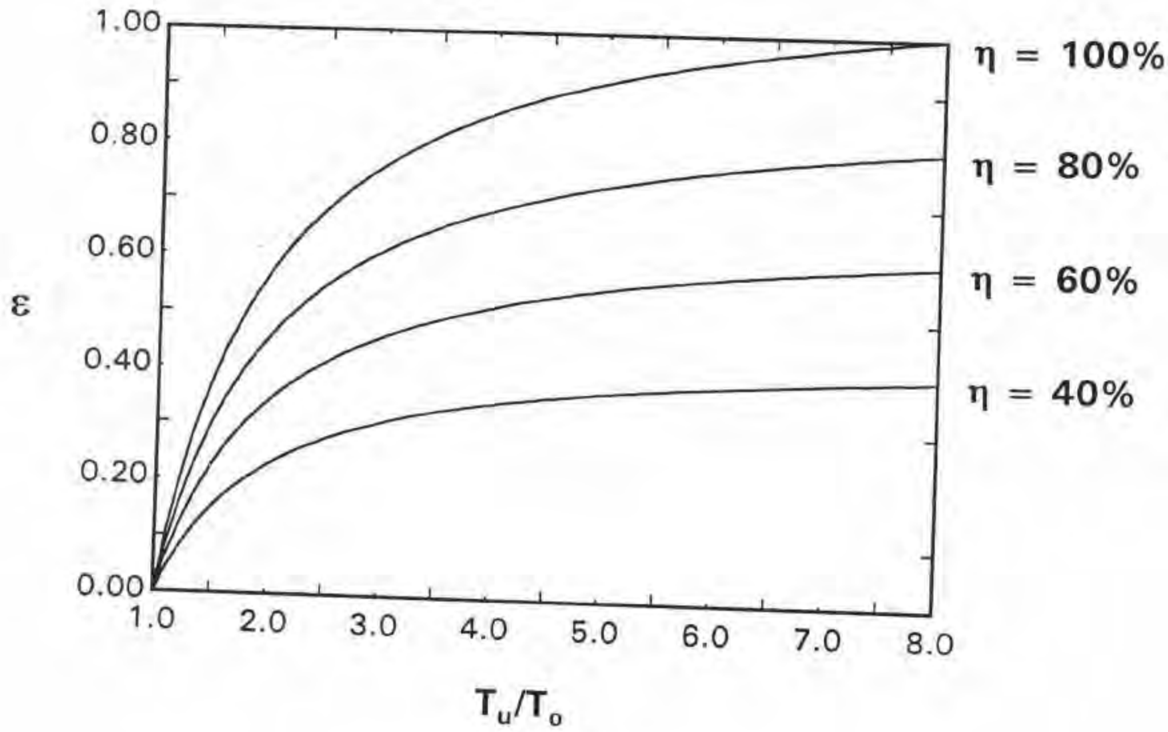
Note: $\dot{m}_3 = \dot{m}_1 + \dot{m}_2 \Rightarrow \dot{m}_2 = \dot{m}_3 - \dot{m}_1 = 15\,lb/s - 10\,lb/s = 5\,lb/s.$

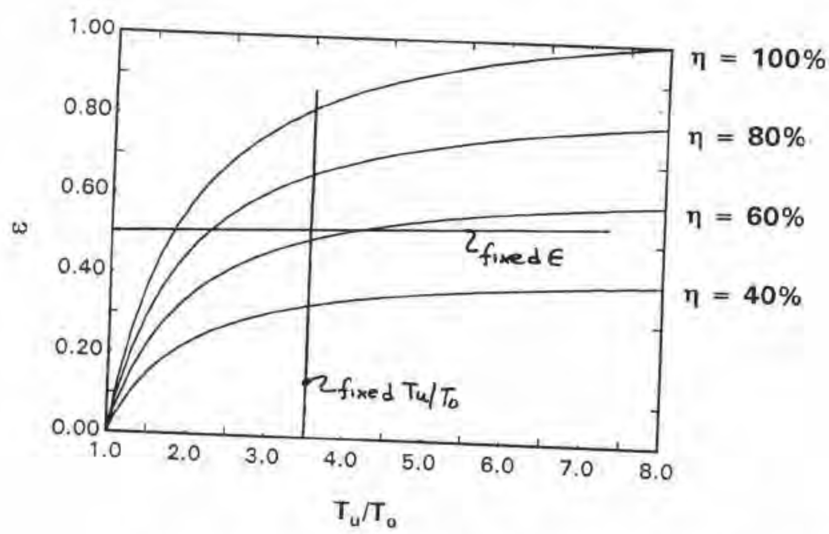
$$\epsilon = \frac{(5\,lb/s)(400-50)\,Btu/lb}{(10\,lb/s)(1000-400)\,Btu/lb} = 0.292 \quad (29.2\%)$$

PROBLEM 7.102

Plot the exergetic efficiency given by Eq. 7.21b versus T_u/T_0 for $T_s/T_0 = 8.0$ and $\eta = 0.4, 0.6, 0.8, 1.0$. What can be learned from the plot when T_u/T_0 is fixed? When ε is fixed? Discuss.

Eq. 7.21b $\quad \epsilon = \eta \left[\frac{1 - T_0/T_u}{\underbrace{1 - T_0/T_s}_{=1/8}} \right] = \eta \left[\frac{1 - T_0/T_u}{1 - \frac{1}{8}} \right] = \frac{8}{7}\eta \left[1 - \frac{T_0}{T_u} \right]$





- when ϵ is fixed, as η increases, lower use temperatures are allowed.
- when T_u/T_0 is fixed, as η increases, ϵ increases.

In sum, a high first-law efficiency, η, has practical benefit. So, both first-law and second-law considerations are important in proper energy resource utilization.

Problem 7.103

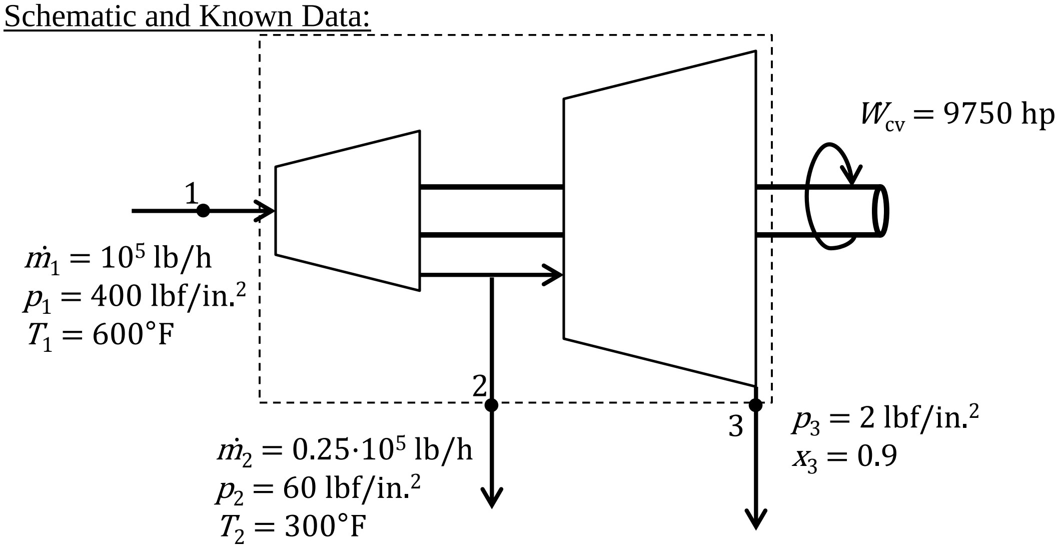
A steam turbine operating at steady state develops 9750 hp. The turbine receives 100,000 pounds of steam per hour at 400 lbf/in.^{2} and 600°F. At a point in the turbine where the pressure is 60 lbf/in.^{2} and the temperature is 300°F, steam is bled off at the rate of 25,000 lb/h. The remaining steam continues to expand through the turbine, exiting at 2 lbf/in.^{2} and 90% quality.
(a) Determine the rate of heat transfer between the turbine and its surroundings, in Btu/h.
(b) Devise and evaluate an exergetic efficiency for the turbine.

Kinetic and potential energy effects can be ignored. Let T_0 = 77°F, p_0 = 1 atm.

Known:
Operating data are provided for a steam turbine at steady state from which steam is bled off at a specified state and flow rate.

Find:
(a) Determine the rate of heat transfer between the turbine and its surroundings. (b) Devise and evaluate an exergetic efficiency.

Schematic and Known Data:



Engineering Model:
(1) The control volume shown in the accompanying figure operates at steady state.
(2) Kinetic and potential energy effects can be neglected.
(3) For the environment, T_0 = 77°F, p_0 = 1 atm.

Analysis:
(a) At steady state a mass rate balance reads $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$, or:

$$\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 10^5 - 0.25 \cdot 10^5 = 0.75 \cdot 10^5 \, \frac{\text{lb}}{\text{h}}$$

An energy rate balance with assumptions (1) and (2) gives:

$0 = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 \Longrightarrow$

$$\dot{Q}_{\text{cv}} = \dot{W}_{\text{cv}} - \dot{m}_1 h_1 + \dot{m}_2 h_2 + \dot{m}_3 h_3 \qquad (1)$$

From Table A-4E, $h_1 = 1306.6\,\frac{\text{Btu}}{\text{lb}}, h_2 = 1181.9\,\frac{\text{Btu}}{\text{lb}}, s_1 = 1.5892\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}, s_2 = 1.6496\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$. Then, with data from Table A-3E:

$$h_3 = h_f + x_3 h_{fg} = 94.02 + 0.9(1022.1) = 1013.91\,\frac{\text{Btu}}{\text{lb}}$$

$$s_3 = s_f + x_3 s_{fg} = 0.1750 + 0.9(1.7448) = 1.7453\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$$

Inserting values into Eq. (1):

$$\begin{aligned}\dot{Q}_{\text{cv}} = &\left(9750\text{ hp}\cdot\left|\frac{2545\,\frac{\text{Btu}}{\text{h}}}{1\text{ hp}}\right|\right) \\ &-\left[\left(10^5\,\frac{\text{lb}}{\text{h}}\right)\left(1306.6\,\frac{\text{Btu}}{\text{lb}}\right) - \left(0.25\cdot 10^5\,\frac{\text{lb}}{\text{h}}\right)\left(1181.9\,\frac{\text{Btu}}{\text{lb}}\right)\right. \\ &\left.-\left(0.75\cdot 10^5\,\frac{\text{lb}}{\text{h}}\right)\left(1013.91\,\frac{\text{Btu}}{\text{lb}}\right)\right] \\ &= 245.5825\cdot 10^5\,\frac{\text{Btu}}{\text{h}} - 248.138\cdot 10^5\,\frac{\text{Btu}}{\text{h}} = -2.555\cdot 10^5\,\frac{\text{Btu}}{\text{h}}\end{aligned}$$

(b) An exergy rate balance at steady state reads:

$$0 = \sum\left[1 - \frac{T_0}{T_j}\right]\dot{Q}_j + \dot{m}_1 \mathrm{e}_{\mathrm{f}_1} - \dot{m}_2 \mathrm{e}_{\mathrm{f}_2} - \dot{m}_3 \mathrm{e}_{\mathrm{f}_3} - \dot{W}_{cv} - \dot{\mathrm{E}}_{\mathrm{d}}$$

Or:

$$\dot{m}_1 \mathrm{e}_{\mathrm{f}_1} - \dot{m}_2 \mathrm{e}_{\mathrm{f}_2} - \dot{m}_3 \mathrm{e}_{\mathrm{f}_3} = \dot{W}_{cv} + \dot{\mathrm{E}}_{\mathrm{d}} - \sum\left[1 - \frac{T_0}{T_j}\right]\dot{Q}_j \qquad (2)$$

The left side of Eq. (2) is the net exergy supplied to the turbine. The right side of Eq. (2) shows that the net supply can be accounted for as power developed, exergy destroyed, and exergy lost in heat transfer to the surroundings. Accordingly, an exergetic efficiency that gauges how effectively the net supply is converted to power is:

$$\begin{aligned}\epsilon &= \frac{\dot{W}_{\text{cv}}}{\dot{m}_1 \mathrm{e}_{\mathrm{f}_1} - \dot{m}_2 \mathrm{e}_{\mathrm{f}_2} - \dot{m}_3 \mathrm{e}_{\mathrm{f}_3}} \\ &= \frac{\dot{W}_{\text{cv}}}{\dot{m}_1[h_1 - h_0 - T_0(s_1 - s_0)] - \dot{m}_2[h_2 - h_0 - T_0(s_2 - s_0)] - \dot{m}_3[h_3 - h_0 - T_0(s_3 - s_0)]} \\ &= \frac{\dot{W}_{\text{cv}}}{(\dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3) - T_0(\dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{m}_3 s_3)}\end{aligned}$$

Solving for the denominator:

$(\dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3) - T_0(\dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{m}_3 s_3) = \left(248.138 \cdot 10^5 \frac{\text{Btu}}{\text{h}}\right) - (537°\text{R}) \left[\left(10^5 \frac{\text{lb}}{\text{h}}\right)\left(1.5892 \frac{\text{Btu}}{\text{lb·°R}}\right) - \left(0.25 \cdot 10^5 \frac{\text{lb}}{\text{h}}\right)\left(1.6496 \frac{\text{Btu}}{\text{lb·°R}}\right) - \left(0.75 \cdot 10^5 \frac{\text{lb}}{\text{h}}\right)\left(1.7453 \frac{\text{Btu}}{\text{lb·°R}}\right)\right] = 319.1155 \cdot 10^5 \frac{\text{Btu}}{\text{h}}$

Inserting value for numerator from above and solving:

$$\epsilon = \frac{245.5825 \cdot 10^5 \frac{\text{Btu}}{\text{h}}}{319.1155 \cdot 10^5 \frac{\text{Btu}}{\text{h}}} = 0.77 = 77\%$$

PROBLEM 7.104

7.104 Figure P7.104 provides two options for generating hot water at steady state. In (a), water heating is achieved by utilizing *industrial waste heat* supplied at a temperature of 500 K. In (b), water heating is achieved by an electrical resistor. For each case, devise and evaluate an exergetic efficiency. Compare the calculated efficiency values and comment. Stray heat transfer and the effects of motion and gravity are negligible. Let $T_0 = 20°C$, $p_0 = 1$ bar.

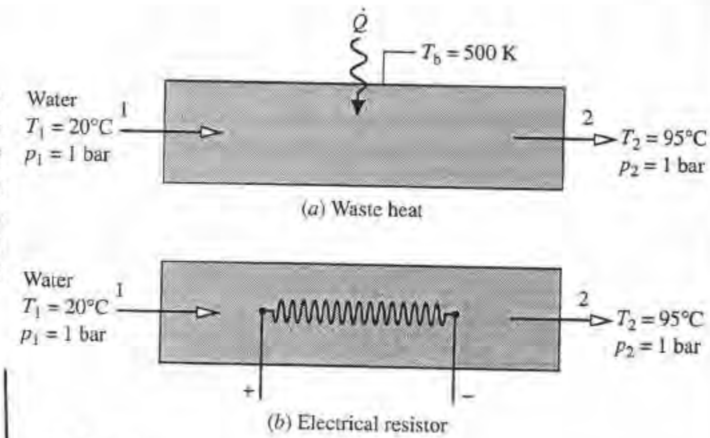


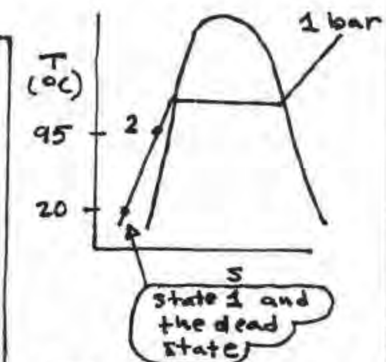
Fig. P7.104

KNOWN: Data are provided for two ways for generating hot water.

FIND: For each option, devise and evaluate an exergetic efficiency. Compare calculated values and comment.

ENGINEERING MODEL:

1. In each case, a control volume at steady state encloses the option: (a) and (b).
2. Stray heat transfer and the effects of motion and gravity are ignored. In(a), $\dot{W}=0$. In (b), $\dot{Q}_{cv}=0$.
3. $T_0 = 20°C$, $p_0 = 1$ bar



$h_1 \approx h_f(20°C) = 83.96$ kJ/kg

$s_1 \approx s_f(20°C) = 0.2966 \frac{kJ}{kg\cdot K}$

$h_2 \approx h_f(95°C) = 397.96$ kJ/kg

$s_2 \approx s_f(95°C) = 1.25 \frac{kJ}{kg\cdot K}$

ANALYSIS:

CASE (a) An exergy rate balance reduces as follows:

$$0 = \left[1 - \frac{T_0}{T_b}\right]\dot{Q} - \cancel{\dot{W}}^{\,0} + \dot{m}(e_{f1} - e_{f2}) - \dot{E}_d$$

① $$\therefore \left[1 - \frac{T_0}{T_b}\right]\dot{Q} = \dot{m}(e_{f2} - e_{f1}) + \dot{E}_d$$

$$\Rightarrow \epsilon = \frac{\dot{m}(e_{f2} - e_{f1})}{\left[1 - \frac{T_0}{T_b}\right]\dot{Q}} = \frac{(h_2 - h_1) - T_0(s_2 - s_1)}{\left[1 - \frac{T_0}{T_b}\right](\dot{Q}/\dot{m})}$$

An energy rate balance reduces to give $(\dot{Q}/\dot{m}) = h_2 - h_1$. Collecting results

$$\epsilon = \frac{[(h_2 - h_1) - T_0(s_2 - s_1)]}{\left[1 - \frac{T_0}{T_b}\right](h_2 - h_1)} = \frac{\overbrace{(397.96 - 83.96)}^{314}\frac{kJ}{kg} - (293K)(1.25 - 0.2966)\frac{kJ}{kg\cdot K}}{\left[1 - \frac{293}{500}\right](397.96 - 83.96)\frac{kJ}{kg}}$$

$$= 0.267 \quad (26.7\%)$$

CASE (b) An exergy rate balance reduces as follows:

$$0 = \sum\left[1 - \frac{T_0}{T_j}\right]\dot{Q}_j^{\,0} - \dot{W} + \dot{m}(e_{f1} - e_{f2}) - \dot{E}_d$$

② $$\therefore (-\dot{W}) = \dot{m}(e_{f2} - e_{f1}) + \dot{E}_d$$

$$\Rightarrow \epsilon = \frac{e_{f2} - e_{f1}}{(-\dot{W}/\dot{m})}$$

An energy rate balance reduces to give $(-\dot{W}) = \dot{m}(h_2 - h_1)$ or $(-\dot{W}/\dot{m}) = h_2 - h_1$.

PROBLEM 7.104 (Continued)

Collecting results,

$$\epsilon = \frac{(h_2 - h_1) - T_0(s_2 - s_1)}{h_2 - h_1}$$

$$= \frac{(397.96 - 83.96)\,\frac{kJ}{kg} - 293\,K\,(1.25 - 0.2966)\,\frac{kJ}{kg \cdot K}}{(397.96 - 83.96)\,\frac{kJ}{kg}}$$

$$= 0.11 \quad (11\%)$$

Discussion: Based on the calculated exergetic efficiency values, the waste heat method (option(a)) is better matched to the water heating task than the resistor method (option (b)). Still, other factors should be taken into account: costs, maintenance, and safety, among others.

---

1. In this option, $[1 - \frac{T_0}{T_b}]\dot{Q}$ is the exergy supplied. Using values from the subsequent solution,

$$\left[1 - \frac{T_0}{T_b}\right]\dot{Q}/\dot{m} = \left(1 - \frac{293}{500}\right)\left(314\,\frac{kJ}{kg}\right) = 130\,\frac{kJ}{kg}$$

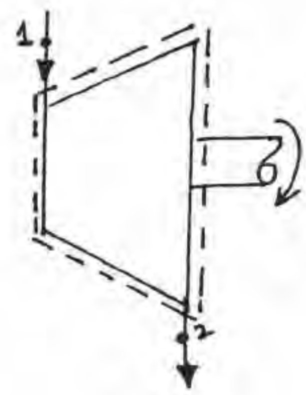
2. In this option $(-\dot{W})$ is the exergy supplied. Using values from the subsequent solution, $(-\dot{W}/\dot{m}) = 214$ kJ/kg.

PROBLEM 7.105

Steam enters a turbine operating at steady state at $p_1 = 12$ MPa, $T_1 = 700°C$ and exits at $p_2 = 0.6$ MPa. The isentropic turbine efficiency is 88%. Property data are provided in the accompanying table. Stray heat transfer and the effects of motion and gravity are negligible. Let $T_0 = 300$ K, $p_0 = 100$ kPa. Determine (a) the power developed and the rate of exergy destruction, each in kJ per kg of steam flowing, and (b) the exergetic turbine efficiency.

	State	p (MPa)	T(°C)	h (kJ/kg)	s (kJ/kg · K)
1	Turbine inlet	12	700	3858.4	7.0749
2	Turbine exit	0.6	(η_t = 88%)	3017.5	7.2938

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. Stray heat transfer and the effects of motion and gravity are negligible.
3. $T_0 = 300K$, $p_0 = 100kPa$

ANALYSIS: (a) An energy rate balance reduces to

$$\dot{W}_{cv}/\dot{m} = h_1 - h_2 = (3858.4 - 3017.5)\frac{kJ}{kg}$$
$$= 840.9 \text{ kJ/kg} \quad \text{(a)}$$

(b) The rate of exergy destruction can be found from $\dot{E}_d = T_0\dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production found from an entropy rate balance. Thus,

$$\frac{\dot{E}_d}{\dot{m}} = T_0(s_2 - s_1) = 300K(7.2938 - 7.0749)\frac{kJ}{kg\cdot K}$$
$$= 65.67 \text{ kJ/kg}$$

(c) The exergetic turbine efficiency is

$$\epsilon = \frac{\dot{W}_{cv}}{\dot{m}[e_{f1} - e_{f2}]} = \frac{\overbrace{\dot{W}_{cv}/\dot{m}}^{=(h_1-h_2) \text{ from (a)}}}{(h_1 - h_2) - \underbrace{T_0(s_1 - s_2)}_{=-\dot{E}_d/\dot{m} \text{ from (b)}}}$$

$$= \frac{(h_1 - h_2)}{(h_1 - h_2) + \dot{E}_d/\dot{m}} = \frac{840.9}{840.9 + 65.67}$$

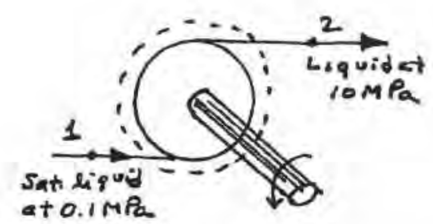
$$= 0.928 \;(92.8\%)$$

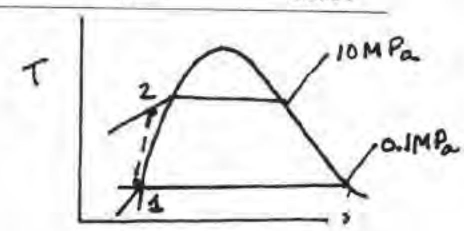
PROBLEM 7.106

Saturated liquid water at 0.01 MPa enters a power plant pump operating at a steady state. Liquid water exits the pump at 10 MPa. The isentropic pump efficiency is 90%. Property data are provided in the accompanying table. Stray heat transfer and the effects of motion and gravity are negligible. Let $T_0 = 300$ K, $p_0 = 100$ kPa. Determine (a) the power required by the pump and the rate of exergy destruction, each in kJ per kg of water flowing, and (b) the exergetic pump efficiency.

State	p (MPa)	h (kJ/kg)	s (kJ/kg·K)
1 Pump inlet	0.01	191.8	0.6493
2 Pump exit	10	204.5	0.6531

SCHEMATIC & GIVEN DATA:





ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. Stray heat transfer and the effects of motion and gravity are negligible.
3. $T_0 = 300$K, $P_0 = 100$ KPa

ANALYSIS:

(a) An energy rate balance reduces to

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 = (191.8 - 204.5)\ \frac{kJ}{kg}$$

$$= -12.7\ kJ/kg \qquad \longleftarrow \text{(a)}$$

(b) The rate of exergy destruction can be found from $\dot{E}_d = T_0\,\dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production found from an entropy rate balance. Thus

$$\frac{\dot{E}_d}{\dot{m}} = T_0(s_2 - s_1) = 300K\,(0.6531 - 0.6493)\ \frac{kJ}{kg\cdot K}$$

$$= 1.14\ kJ/kg \qquad \longleftarrow \text{(a)}$$

(c) The exergetic pump efficiency is

$$\epsilon = \frac{e_{f2} - e_{f1}}{(-\dot{W}_{cv}/\dot{m})} = \frac{(h_2 - h_1) - \overbrace{T_0(s_2 - s_1)}^{= \dot{E}_d/\dot{m} \text{ from (b)}}}{\underbrace{(-\dot{W}_{cv}/\dot{m})}_{= (h_2 - h_1) \text{ from (a)}}}$$

$$= \frac{(h_2 - h_1) - T_0(s_2 - s_1)}{(h_2 - h_1)}$$

$$= \frac{12.7 - 1.14}{12.7} = 0.91 \ (91\%) \qquad \longleftarrow \text{(b)}$$

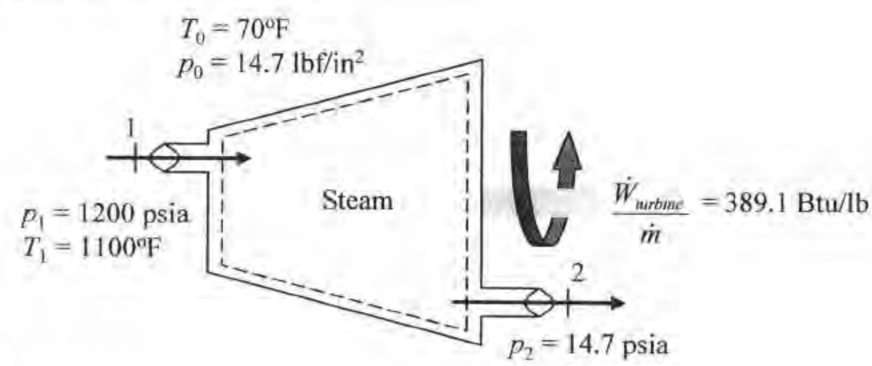
PROBLEM 7.107

7.107 At steady state, an insulated steam turbine develops work at a rate of 389.1 Btu per lb of steam flowing through the turbine. Steam enters at 1200 psia and 1100°F and exits at 14.7 psia. Evaluate the isentropic turbine efficiency and the exergetic turbine efficiency. Ignore the effects of motion and gravity. Let T_0 = 70°F, p_0 = 14.7 psia.

KNOWN: Steam at specified temperature and pressure enters a turbine with known work production and exits at specified pressure.

FIND: The isentropic turbine efficiency and the exergetic turbine efficiency.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume defined by the dashed line on the accompanying diagram is at steady state.
2. Heat transfer and the effects of motion and gravity can be ignored.
3. T_0 = 530°R, p_0 = 14.7 psia.

ANALYSIS:
At the inlet the steam is superheated vapor. From Table A-4E, h_1 = 1557.9 Btu/lb and s_1 = 1.6682 Btu/(lb·°R).

The isentropic turbine efficiency can be determined from Eq. 6.46

$$\eta_t = \frac{\dot{W}_{cv}/\dot{m}}{\left(\dot{W}_{cv}/\dot{m}\right)_s} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

At state 2s (see T-s diagram), the specific entropy is the same as at the inlet. Thus, $s_{2s} = s_1$ = 1.6682 Btu/(lb·°R). For $p_{2s} = p_2$ = 14.7 psia, State 2s is a liquid-vapor mixture. From Table A-3E, h_{f2s} = 180.15 Btu/lb, h_{fg2s} = 970.4 Btu/lb, s_{f2s} = 0.3121 Btu/(lb·°R), s_{fg2s} = 1.4446 Btu/(lb·°R).

Since State 2s is a liquid-vapor mixture, the enthalpy at the final state of an isentropic expansion to 14.7 psia can be determined from the quality relations. The quality is determined using the known pressure and entropy, s_{2s}

PROBLEM 7.107 (Continued, p.2)

$$x_{2s} = \frac{s_{2s} - s_{f2s}}{s_{fg2s}}$$

Substituting values yields

$$x_{2s} = \frac{1.6682\frac{\text{Btu}}{\text{lb}\cdot°\text{R}} - 0.3121\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}}{1.4446\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}} = 0.9387$$

The enthalpy is determined from the relationship

$$h_{2s} = h_{f2s} + x_{2s}h_{fg2s}$$

Substituting values yields

$$h_{2s} = 180.15 \text{ Btu/lb} + (0.9387)(970.4 \text{ Btu/lb}) = 1091.1 \text{ Btu/lb}$$

Since the rate of work per lb of steam flowing through the turbine is known, the isentropic turbine efficiency can be determined by

$$\eta_t = \frac{\dot{W}_{cv}/\dot{m}}{h_1 - h_{2s}} = \frac{389.1\frac{\text{Btu}}{\text{lb}}}{1557.9\frac{\text{Btu}}{\text{lb}} - 1091.1\frac{\text{Btu}}{\text{lb}}} = \underline{\mathbf{0.8335}}$$

The exergetic turbine efficiency is determined from Eq. 7.24

$$\varepsilon = \frac{\dot{W}_{cv}/\dot{m}}{\mathsf{e}_{f1} - \mathsf{e}_{f2}}$$

where the change in specific flow exergy is defined by Eq. 7.18 (ignoring effects of motion and gravity)

$$\mathsf{e}_{f1} - \mathsf{e}_{f2} = (h_1 - h_2) - T_0(s_1 - s_2)$$

The exit enthalpy can be determined from the steady-state, one-inlet, one-exit energy balance

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\,[(h_1 - h_2) + \tfrac{1}{2}(V_1^2 - V_2^2) + g(z_1 - z_2)]$$

Neglecting heat transfer rate and kinetic and potential energy effects, the energy balance simplifies to

$$0 = -\dot{W}_{cv} + \dot{m}\,(h_1 - h_2)$$

Solving for exit enthalpy and substituting values give

PROBLEM 7.107 (Continued, p.3)

$$h_2 = h_1 - \frac{\dot{W}_{cv}}{\dot{m}} = 1557.9 \text{ Btu/lb} - 389.1 \text{ Btu/lb} = 1168.8 \text{ Btu/lb}$$

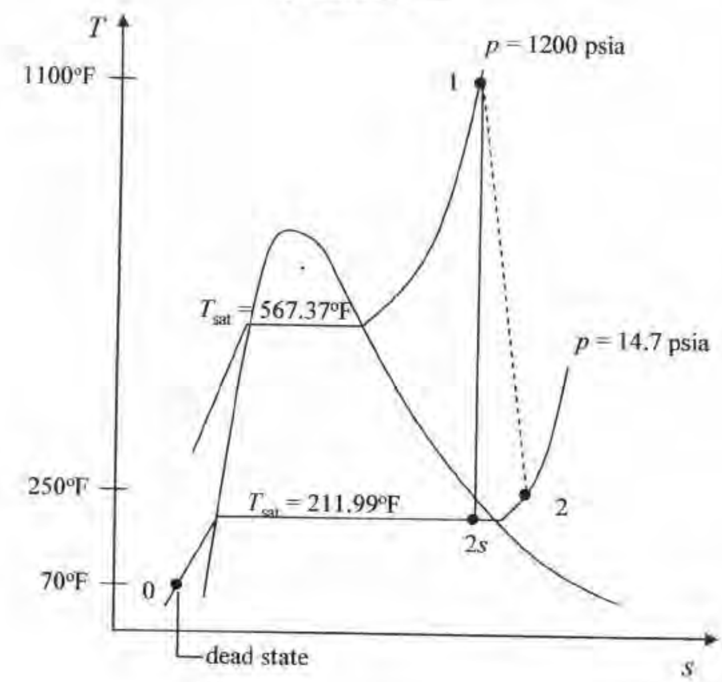
State 2 is superheated vapor. From Table A-4E, $s_2 = 1.7832$ Btu/(lb·°R). Substituting values gives

$$e_{f1} - e_{f2} = 1557.9\frac{\text{Btu}}{\text{lb}} - 1168.8\frac{\text{Btu}}{\text{lb}} - (530°\text{R})\left(1.6682\frac{\text{Btu}}{\text{lb}\cdot°\text{R}} - 1.7832\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right) = 450.05 \text{ Btu/lb}$$

Substituting values and solving for exergetic turbine efficiency give

$$\varepsilon = \frac{389.1\frac{\text{Btu}}{\text{lb}}}{450.05\frac{\text{Btu}}{\text{lb}}} = \underline{\mathbf{0.8646}}$$

***T-s* Diagram**



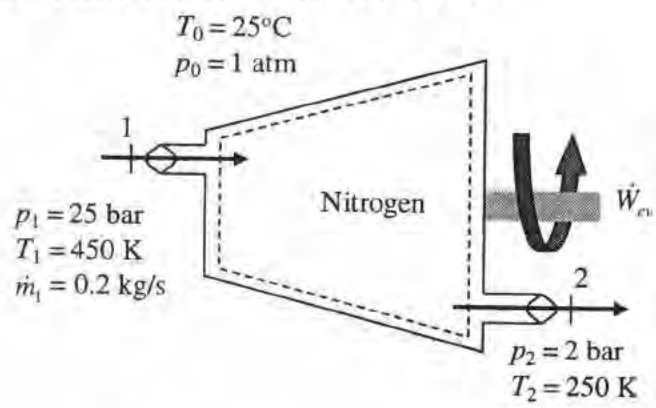
PROBLEM 7.108

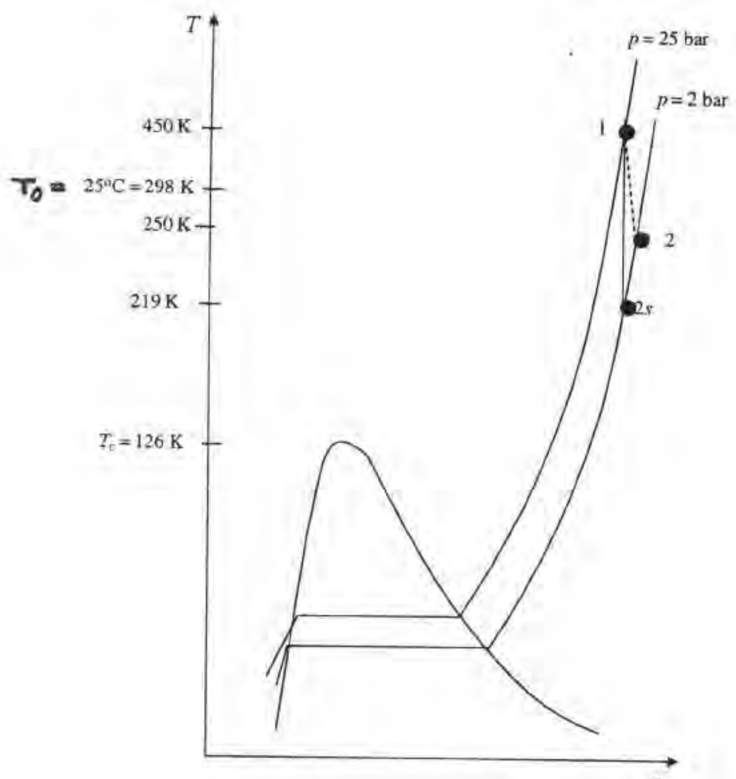
7.108 Nitrogen (N_2) at 25 bar, 450 K enters a turbine and expands to 2 bar, 250 K with a mass flow rate of 0.2 kg/s. The turbine operates at steady state with negligible heat transfer with its surroundings. Assuming the ideal gas model with k =1.399 and ignoring effects of motion and gravity, determine
(a) the isentropic turbine efficiency.
(b) the exergetic turbine efficiency.
Let T_0= 25°C, p_0=1 atm.

KNOWN: Steady state operating data are provided for a nitrogen turbine.

FIND: Determine (a) the isentropic turbine efficiency and (b) the turbine exergetic.

SCHEMATIC AND GIVEN DATA:





PROBLEM 7.108 (Continued)

ENGINEERING MODEL:

(1) The control volume enclosing the turbine shown on the schematic is at steady state.
(2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are ignored.
(3) Nitrogen is modeled as an ideal gas with k = 1.399.
(4) The exergy reference environment is T_0 =298 K and p_0 = 1 atm.

ANALYSIS:

(a) The turbine isentropic efficiency is given by Eq. 6.46. Then, since c_p is constant when k is constant, we get

$$\eta_t = \frac{(h_1 - h_2)}{(h_1 - h_{2s})} = \frac{c_p(T_1 - T_2)}{c_p(T_1 - T_{2s})} = \frac{(T_1 - T_2)}{(T_1 - T_{2s})} \qquad (1)$$

Using Eq. 6.43

$$T_{2s} = T_1\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = 450\text{ K}\left(\frac{2}{25}\right)^{\frac{0.399}{1.399}} = 219\text{ K}$$

Substituting into Eq. (1)

$$\eta_t = \frac{(450 - 250)}{(450 - 219)} = 0.87$$

(b) The exergetic efficiency is given by Eq. 7.24, which for an ideal gas with constant k becomes

$$\varepsilon = \frac{(h_1 - h_2)}{(h_1 - h_2) - T_0(s_1 - s_2)} = \frac{c_p(T_1 - T_2)}{c_p(T_1 - T_2) - T_0\left(c_p \ln\frac{T_1}{T_2} - \frac{\overline{R}}{M}\ln\frac{P_1}{P_2}\right)} \qquad (2)$$

Using the following form of Eq. 3.47a

$$\frac{\overline{R}}{M} = c_p\left(\frac{k-1}{k}\right)$$

Eq. (2) becomes

$$\varepsilon = \frac{(T_1 - T_2)}{(T_1 - T_2) - T_0\left(\ln\frac{T_1}{T_2} - \left(\frac{k-1}{k}\right)\ln\frac{P_1}{P_2}\right)}$$

$$= \frac{(450 - 250)}{(450 - 250) - 298\left(\ln\frac{450}{250} - \left(\frac{0.399}{1.399}\right)\ln\frac{25}{2}\right)} = 0.835$$

Problem 7.109

Air enters the insulated duct shown in Fig. P7.109 at a temperature of 60°F and a pressure of 1 atm and exits at a temperature of 140°F and a pressure only slightly less than 1 atm. Electricity is provided to the resistor at a rate of 0.1 kW. Kinetic and potential energy effects can be ignored. For steady state operation,

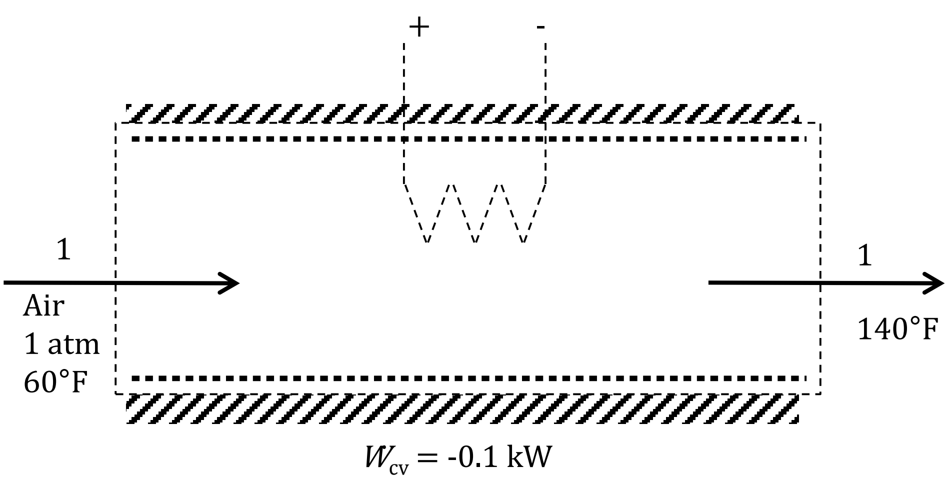
(a) determine the exergetic destruction rate, in kW.
(b) devise and evaluate an exergetic efficiency for the heater.

Let T_0 = 60°F, p_0 = 1 atm.

Known:
Air flowing through an insulated duct is heated by an electrical resistor.

Find:
(a) Determine the exergy destruction rate and (b) devise and evaluate an exergetic efficiency for the heater.

Schematic and Known Data:



Engineering Model:

(1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{\text{cv}} = 0$ and negligible effects of kinetic and potential energy.
(2) Air is modeled as an ideal gas.
(3) For the environment, T_0 = 60°F, p_0 = 1 atm.
(4) There is negligible change in pressure

Analysis:

(a) The exergy destruction rate is $\dot{\mathrm{E}}_{\mathrm{d}} = T_0 \dot{\sigma}_{\text{cv}}$, where $\dot{\sigma}_{\text{cv}}$ is the rate of entropy production from an entropy balance. Thus, $\dot{\mathrm{E}}_{\mathrm{d}} = T_0 \dot{m}(s_2 - s_1)$. The mass flow rate $\dot{m}$ can be determined using mass and energy rate balances:

$$0 = \underbrace{\dot{Q}_{\text{cv}}}_{=0} - \dot{W}_{\text{cv}} + \dot{m}(h_1 - h_2) \Rightarrow \dot{m} = -\frac{\dot{W}_{\text{cv}}}{h_2 - h_1}$$

Thus, with data from Table A-13E:

$$\dot{m} = \frac{-(-0.1\ \text{kW})\left|\frac{3413\ \frac{\text{Btu}}{\text{h}}}{1\ \text{kW}}\right|}{(143.47 - 124.27)\ \frac{\text{Btu}}{\text{lb}}} = 17.78\ \frac{\text{lb}}{\text{h}}$$

Accordingly:

$$\dot{\text{E}}_{\text{d}} = \dot{m}T_0(s_2 - s_1) = \dot{m}T_0\left(s_2^o - s_1^o - \underbrace{R\ln\frac{p_2}{p_1}}_{=0}\right)$$

$$= \left(17.78\ \frac{\text{lb}}{\text{h}}\right)(520°\text{R})(0.62607 - 0.59172)\ \frac{\text{Btu}}{\text{lb}\cdot°\text{R}} \cdot \left|\frac{1\ \text{kW}}{3413\ \frac{\text{Btu}}{\text{h}}}\right|$$

$$= 0.093\ \text{kW}$$

(b) At steady state an exergy rate balance reduces to give:

$$0 = \sum\left[1 - \frac{T_0}{T_j}\right]\underbrace{\dot{Q}_j}_{=0} - \dot{W}_{\text{cv}} + \dot{m}\left(\text{e}_{\text{f}_1} - \text{e}_{\text{f}_2}\right) - \dot{\text{E}}_{\text{d}}$$

Thus:

$$-\dot{W}_{\text{cv}} = \dot{m}\left(\text{e}_{\text{f}_2} - \text{e}_{\text{f}_1}\right) + \dot{\text{E}}_{\text{d}}$$

That is, the exergy entering by electrical work either goes into increasing the flow exergy of the air or is destroyed by irreversibilities. An exergetic efficiency can be defined as the ratio of the flow exergy increase to the exergy supplied:

$$\epsilon = \frac{\dot{m}\left(\text{e}_{\text{f}_2} - \text{e}_{\text{f}_1}\right)}{-\dot{W}_{\text{cv}}} = \frac{-\dot{W}_{\text{cv}} - \dot{\text{E}}_{\text{d}}}{-\dot{W}_{\text{cv}}} = 1 - \frac{\dot{\text{E}}_{\text{d}}}{-\dot{W}_{\text{cv}}} = 1 - \frac{0.093}{0.1} = 0.07 = 7\%$$

PROBLEM 7.110

7.110 Air enters an insulated turbine operating at steady state with a pressure of 5 bar, a temperature of 500 K, and a volumetric flow rate of 3 m^3/s. At the exit, the pressure is 1 bar. The isentropic turbine efficiency is 76.7%. Assuming the ideal gas model and ignoring the effects of motion and gravity, determine

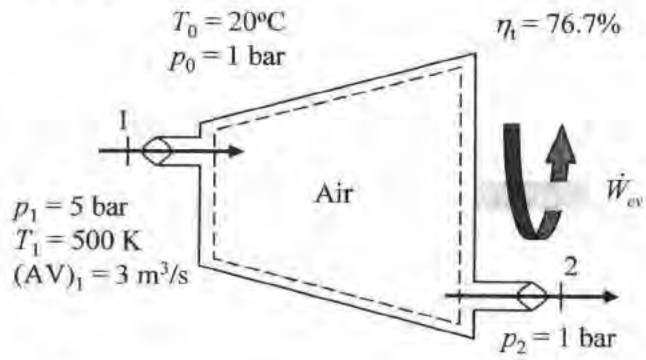
(a) the power developed and the exergy destruction rate, each in kW.
(b) the exergetic turbine efficiency.

Let $T_0 = 20^\circ$C, $p_0 = 1$ bar.

KNOWN: Air at specified pressure, temperature, and volumetric flow rate enters a turbine with known isentropic efficiency and exits at specified pressure.

FIND: The power developed, the exergy destruction rate, and the exergetic turbine efficiency.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume defined by the dashed line on the accompanying diagram is at steady state.
2. Air can be modeled as an ideal gas.
3. Heat transfer and the effects of motion and gravity can be ignored.
4. $T_0 = 293$ K, $p_0 = 1$ bar.

ANALYSIS:
(a) The power developed can be determined from the isentropic turbine efficiency

$$\eta_t = \frac{\dot{W}_{cv}}{(\dot{W}_{cv})_s} = \frac{\dot{m}(h_1 - h_2)}{\dot{m}(h_1 - h_{2s})}$$

Solving for power gives

$$\dot{W}_{cv} = \eta_t \dot{m}\,(h_1 - h_{2s}) \qquad (1)$$

The mass flow rate can be determined from the volumetric flow rate at the inlet

$$\dot{m} = \frac{(AV)_1}{v_1}$$

The inlet specific volume can be determined from the ideal gas equation of state.

$$pv = RT$$

Solving for the inlet specific volume, substituting values, and applying the appropriate conversion factors yield

$$v_1 = \frac{RT_1}{p_1} = \frac{\left(\dfrac{8.314\dfrac{\text{kJ}}{\text{kmol}\cdot\text{K}}}{28.97\dfrac{\text{kg}}{\text{kmol}}}\right)(500\text{ K})}{5\text{ bar}}\left|\frac{\text{bar}}{10^5\dfrac{\text{N}}{\text{m}^2}}\right|\left|\frac{10^3\text{ N}\cdot\text{m}}{\text{kJ}}\right| = 0.2870\text{ m}^3\text{/kg}$$

PROBLEM 7.110 (Continued, p.2)

Solving for mass flow rate gives

$$\dot{m} = \frac{3\dfrac{\text{m}^3}{\text{s}}}{0.2870\dfrac{\text{m}^3}{\text{kg}}} = 10.45 \text{ kg/s}$$

The isentropic relations can be used to determine relative pressure, p_{r2s}, which then can be used to determine h_{2s}.

$$\frac{p_2}{p_1} = \frac{p_{r2s}}{p_{r1}}$$

Solving for relative pressure at State 2s gives

$$p_{r2s} = p_{r1}\frac{p_2}{p_1}$$

From Table A-22, p_{r1} = 8.411 and h_1 = 503.02 kJ/kg. Substituting values and solving give

$$p_{r2s} = (8.411)\left(\frac{1 \text{ bar}}{5 \text{ bar}}\right) = 1.6822$$

From Table A-22 (interpolating), h_{2s} = 317.31 kJ/kg. Substituting values in Eq. (1) and solving for power developed give

$$\dot{W}_{cv} = (0.767)\left(10.45\frac{\text{kg}}{\text{s}}\right)\left(503.02\frac{\text{kJ}}{\text{kg}} - 317.31\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{\text{kW}}{\dfrac{\text{kJ}}{\text{s}}}\right| = \underline{\mathbf{1488.5\ kW}}$$

The exergy destruction rate can be determined from the entropy production rate

$$\dot{E}_d = T_0\dot{\sigma}_{cv}$$

Entropy production rate can be determined from the steady-state control volume entropy rate balance

$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{cv}$$

Neglecting heat transfer rate, entropy production rate for one inlet and one exit becomes

$$\dot{\sigma}_{cv} = \dot{m}\,(s_2 - s_1)$$

For an ideal gas, $s_2 - s_1 = s(T_2, p_2) - s(T_1, p_1) = [s^{\text{o}}(T_2) - s^{\text{o}}(T_1) - R \ln\frac{p_2}{p_1}]$. Thus,

$$\dot{\sigma}_{cv} = \dot{m}\,[s^{\text{o}}(T_2) - s^{\text{o}}(T_1) - R \ln\frac{p_2}{p_1}] \tag{2}$$

The exit temperature can be determined by solving the steady-state energy balance for h_2

$$h_2 = h_1 - \frac{\dot{W}_{cv}}{\dot{m}}$$

Substituting values and solving give

$$h_2 = 503.02\frac{\text{kJ}}{\text{kg}} - \frac{1488.5\text{ kW}}{10.45\frac{\text{kg}}{\text{s}}}\left|\frac{\frac{\text{kJ}}{\text{s}}}{\text{kW}}\right| = 360.58\text{ kJ/kg}$$

From Table A-22, $T_2 = 360$ K and $s^o(T_2) = 1.88543$ kJ(kg·K); $s^o(T_1) = 2.21952$ kJ(kg·K).

Substituting values in Eq. (2) and solving for entropy production rate give

$$\dot{\sigma}_{cv} = \left(10.45\frac{\text{kg}}{\text{s}}\right)\left(1.88543\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 2.21952\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - \left(\frac{8.314\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}}{28.97\frac{\text{kg}}{\text{kmol}}}\right)\ln\left(\frac{1\text{ bar}}{5\text{ bar}}\right)\right)\left|\frac{\text{kW}}{\frac{\text{kJ}}{\text{s}}}\right|$$

$$\dot{\sigma}_{cv} = 1.335\text{ kW/K}$$

Substituting values into Eq. 2 and solving for the exergy destruction rate give

$$\dot{E}_d = (293\text{ K})(1.335\text{ kW/K}) = \underline{\mathbf{391.2\text{ kW}}}$$

The exergetic turbine efficiency is determined from Eq. 7.24

$$\varepsilon = \frac{\dot{W}_{cv}/\dot{m}}{e_{f1} - e_{f2}}$$

where the change in specific flow exergy is defined by Eq. 7.18 (ignoring effects of motion and gravity)

$$e_{f1} - e_{f2} = (h_1 - h_2) - T_0(s_1 - s_2)$$

Substituting $s_2 - s_1 = s(T_2, p_2) - s(T_1, p_1) = [s^o(T_2) - s^o(T_1) - R\ln\frac{p_2}{p_1}]$ gives

$$e_{f1} - e_{f2} = (h_1 - h_2) - T_0[s^o(T_1) - s^o(T_2) - R\ln\frac{p_1}{p_2}]$$

Substituting values and solving give

$$e_{f1} - e_{f2} =$$

$$503.02\frac{\text{kJ}}{\text{kg}} - 360.58\frac{\text{kJ}}{\text{kg}} - (293\text{ K})\left(2.21952\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 1.88543\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - \left(\frac{8.314\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}}{28.97\frac{\text{kg}}{\text{kmol}}}\right)\ln\left(\frac{5\text{ bar}}{1\text{ bar}}\right)\right)$$

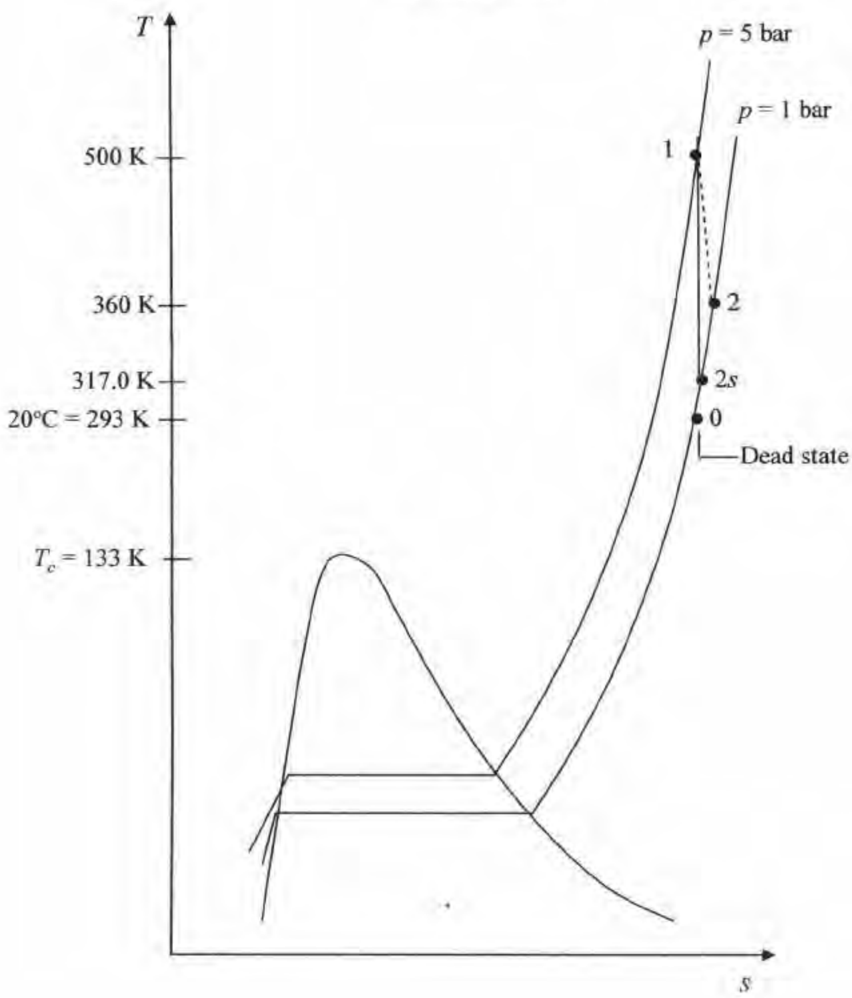
$$e_{f1} - e_{f2} = 179.88\text{ kJ/kg}$$

The exergetic turbine efficiency is

$$\varepsilon = \frac{\left(\frac{1488.5\text{ kW}}{10.45\frac{\text{kg}}{\text{s}}}\right)}{179.88\frac{\text{kJ}}{\text{kg}}}\left|\frac{\frac{\text{kJ}}{\text{s}}}{\text{kW}}\right| = \underline{\mathbf{0.7919}}$$

PROBLEM 7.110 (Continued, p. 4)

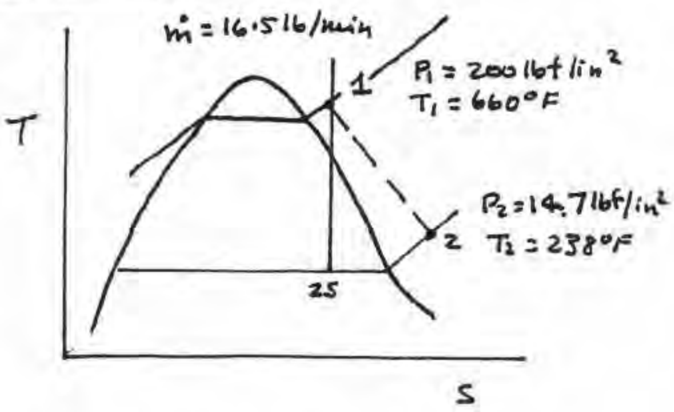
_T-s_ Diagram



PROBLEM 7.111

Steam at 200 lbf/in.^{2}, 660°F enters a turbine operating at steady state with a mass flow rate of 16.5 lb/min and exits at 14.7 lbf/in.^{2}, 238°F. Stray heat transfer and the effects of motion and gravity can be ignored. Let $T_0 = 537°R$, $p_0 = 14.7$ lbf/in.^{2}. Determine for the turbine (a) the power developed and the rate of exergy destruction, each in Btu/min, and (b) the isentropic and exergetic turbine efficiencies.

SCHEMATIC & GIVEN DATA



ENGR. MODEL:

1. A control volume at steady state encloses the turbine.
2. Stray heat transfer and the effects of motion and gravity can be ignored.

$T_0 = 537°R$, $P_0 = 14.7$ lbf/in^2

ANALYSIS: From Table A-4E, $h_1 = 1353.12$ Btu/lb, $s_1 = 1.7047$ Btu/lb·°R.

$h_2 = 1163.02 \frac{Btu}{lb}$, $s_2 = 1.7748$ Btu/lb·°R.

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{1.7047 - 0.3121}{1.4446} = 0.964 \Rightarrow h_{2s} = 180.15 + 0.964(970.4) = 1115.62 \frac{Btu}{lb}$$

(a) $$\dot{W}_t = \dot{m}(h_1 - h_2) = 16.5 \frac{lb}{min}\left(1353.12 - 1163.02\right)\frac{Btu}{lb} = 3136.65 \frac{Btu}{min} \longleftarrow \dot{W}_t$$

$$\dot{E}_d = T_0\, \dot{\sigma}_{cv} = \dot{m} T_0 (s_2 - s_1) = \left(16.5 \frac{lb}{min}\right)(537°R)(1.7748 - 1.7047)\frac{Btu}{lb \cdot °R}$$

$$= 621.12 \frac{Btu}{min} \longleftarrow \dot{E}_d$$

(b) $$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{1353.12 - 1163.02}{1353.12 - 1115.62} = \frac{190.1}{237.5} = 0.8 \; (80\%) \longleftarrow \eta_t$$

$$\epsilon = \frac{h_1 - h_2}{h_1 - h_2 - T_0(s_1 - s_2)} = \frac{190.1}{190.1 - 537(1.7047 - 1.7748)} = 0.835 \; (83.5\%) \longleftarrow \epsilon$$

PROBLEM 7.112

7.112 Water vapor at 6 MPa, 600°C enters a turbine operating at steady state and expands adiabatically to 10 kPa. The mass flow rate is 2 kg/s and the isentropic turbine efficiency is 94.7%. Kinetic and potential energy effects are negligible. Determine

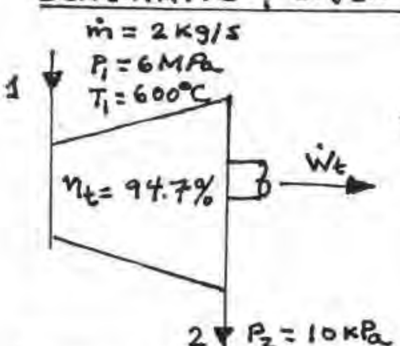
(a) the power developed by the turbine, in kW.
(b) the rate at which exergy is destroyed within the turbine, in kW.
(c) the exergetic turbine efficiency.

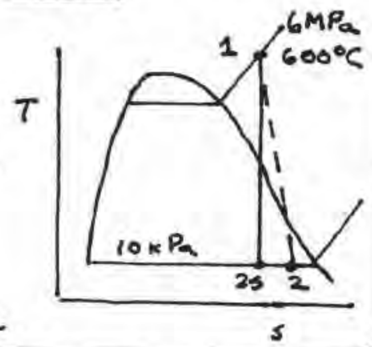
Let $T_0 = 298$ K, $p_0 = 1$ atm.

KNOWN: Data are provided for a steam turbine at steady state.

FIND: For the turbine, determine $\dot{W}_t$, $\dot{E}_d$, and ϵ.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL:

1. A control volume at steady state encloses the turbine.
2. For the control volume, $\dot{Q}_{cv} = 0$ and the effects of motion and gravity are negligible.
3. $T_0 = 298$ K, $P_0 = 1$ atm

ANALYSIS: (a) Reducing mass and energy rate balances, we get $\dot{W}_t = \dot{m}(h_1 - h_2)$. Also, $\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}}$. Collecting results, $\dot{W}_t = \dot{m}\eta_t(h_1 - h_{2s})$ (1)

Table A-4 gives $h_1 = 3658.4$ kJ/kg, $s_1 = 7.1677$ kJ/kg·K. Using data from Table A-3 with $s_{2s} = s_1$, $x_{2s} = \frac{7.1677 - 0.6493}{8.1502 - 0.6493} = 0.869$. Then $h_{2s} = 191.83 + 0.869(2392.8) = 2271.2$ kJ/kg

Inserting values, Eq. (1) gives

$$\dot{W}_t = \left(2\,\frac{\text{kg}}{\text{s}}\right)(0.947)(3658.4 - 2271.2)\frac{\text{kJ}}{\text{kg}}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = 2627.4\text{ kW}$$

(b) $\dot{E}_d = T_0\dot{\sigma}$. With $\dot{\sigma} = \dot{m}(s_2 - s_1)$ from an entropy rate balance, $\dot{E}_d = T_0\dot{m}(s_2 - s_1)$ (2)

To find s_2, fix the exit state using the expression for power from Part (a):

$$\frac{\dot{W}_t}{\dot{m}} = h_1 - h_2 \Rightarrow h_2 = h_1 - \dot{W}/\dot{m} = \left[3658.4\,\frac{\text{kJ}}{\text{kg}} - \frac{2627.4\text{ kJ/s}}{2\text{ kg/s}}\right] = 2344.7\text{ kJ/kg}$$

Then,

$$x_2 = \left(\frac{2344.7 - 191.83}{2392.8}\right) = 0.9 \Rightarrow s_2 = 0.6493 + 0.9(8.1502 - 0.6493) = 7.4001\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

Inserting values, Eq. (2) gives

$$\dot{E}_d = (298\text{ K})(2\text{ kg/s})(7.4001 - 7.1677)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = 138.5\text{ kW}$$

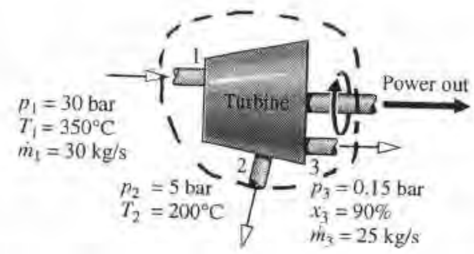
(c) An exergy rate balance gives, $0 = \sum_j\left(1 - \frac{T_0}{T_j}\right)\cancel{\dot{Q}_j}^{\,0} - \dot{W}_t + \dot{m}(e_{f1} - e_{f2}) - \dot{E}_d$

Thus, $\dot{m}(e_{f1} - e_{f2}) = \dot{W}_t + \dot{E}_d$ and Eq. 7.24 becomes,

$$\epsilon = \frac{\dot{W}_t}{\dot{m}(e_{f1} - e_{f2})} = \left(\frac{\dot{W}_t}{\dot{W}_t + \dot{E}_d}\right) = \left(\frac{2627.4}{2627.4 + 138.5}\right) = 0.95\ (95\%)$$

PROBLEM 7.113

Figure P7.113 shows a turbine operating at steady state with steam entering at $p_1 = 30$ bar, $T_1 = 350°C$ and a mass flow rate of 30 kg/s. Process steam is extracted at $p_2 = 5$ bar, $T_2 = 200°C$. The remaining steam exits at $p_3 = 0.15$ bar, $x_3 = 90\%$, and a mass flow rate of 25 kg/s. Stray heat transfer and the effects of motion and gravity are negligible. Let $T_0 = 25°C$, $p_0 = 1$ bar. The accompanying table provides property data at key states. For the turbine, determine the power developed and rate of exergy destruction, each in MW. Also devise and evaluate an exergetic efficiency for the turbine.



State	p (bar)	T(°C)	h (kJ/kg)	s (kJ/kg · K)
1	30	350	3115.3	6.7428
2	5	200	2855.4	7.0592
3	0.15	(x = 90%)	2361.7	7.2831

ENGR MODEL:

1. The control volume shown in the sketch is at steady state.
2. Stray heat transfer and the effects of motion and gravity are negligible.
3. $T_0 = 298K$, $P_0 = 1$ bar

ANALYSIS: $\dot{m}_2 = \dot{m}_1 - \dot{m}_3 = 5$ kg/s.

An energy rate balance reduces to give

$$\dot{W}_{cv} = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 = \left[\left(30\frac{kg}{s}\right)\left(3115.3\frac{kJ}{kg}\right) - (5)(2855.4) - (25)(2361.7)\right]\left|\frac{1\,MW}{10^3\,kJ/s}\right|$$

$$= 20.14\ MW \quad \longleftarrow \dot{W}_{cv}$$

$\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is obtained from an entropy rate balance. Thus

$$\dot{E}_d = T_0\left[\dot{m}_2 s_2 + \dot{m}_3 s_3 - \dot{m}_1 s_1\right] = 298K\left[\left(5\frac{kg}{s}\right)\left(7.0592\frac{kJ}{kg\cdot K}\right) + (25)(7.2831) - (30)(6.7428)\right]\left|\frac{1MW}{10^3\frac{kJ}{s}}\right|$$

$$= 4.5MW \quad \longleftarrow \dot{E}_d$$

Writing an exergy rate balance,

$$0 = \cancel{\dot{E}_q}^{\,0} - \dot{W}_{cv} + \dot{m}_1 e_{f1} - \dot{m}_2 e_{f2} - \dot{m}_3 e_{f3} - \dot{E}_d$$

$$\Rightarrow \underbrace{\dot{m}_1 e_{f1} - \dot{m}_2 e_{f2} - \dot{m}_3 e_{f3}}_{\text{net rate exergy is supplied}} = \dot{W}_{cv} + \dot{E}_d$$

① $$\Rightarrow \epsilon = \frac{\dot{W}_{cv}}{(\dot{m}_1 e_{f1} - \dot{m}_2 e_{f2} - \dot{m}_3 e_{f3})} = \frac{\dot{W}_{cv}}{\dot{W}_{cv} + \dot{E}_d} = \frac{20.14}{20.14 + 4.5} = 0.817\ (81.7\%) \quad \longleftarrow \epsilon$$

---

1. Alternatively, note that

$$\underbrace{\dot{m}_1}_{(\dot{m}_2+\dot{m}_3)} e_{f1} - \dot{m}_2 e_{f2} - \dot{m}_3 e_{f3} = \dot{m}_2 (e_{f1} - e_{f2}) + \dot{m}_3 (e_{f1} - e_{f3})$$

$$= \dot{m}_2\left[(h_1 - h_2) - T_0(s_1 - s_2)\right] + \dot{m}_3\left[(h_1 - h_3) - T_0(s_1 - s_3)\right]$$

PROBLEM 7.114

For the turbine and heat exchanger arrangement of Problem 6.112 evaluate an exergetic efficiency for (a) each turbine, (b) the heat exchanger, and (c) an overall control volume enclosing the turbines and heat exchanger. Comment. Let $T_0 = 300$ K, $p_0 = 1$ bar.

FROM Problem 6.112:

State	h (kJ/kg)	s° (kJ/kg·K)
1	1515.42	3.3620
2	1161.07	3.07732
3	1397.78	3.27481
4	1023.25	2.94468
5	1611.79	3.42892
6	1277.79	3.17888

$\dot{W}_{t1} = 10{,}000$ kW

$\dot{m}_1 = 28.22$ kg/s , $\dot{W}_{t2} = 10{,}570$ kW

$\dot{\sigma}]_{\text{turbine } 1} = 3.1936$ kW/K

$\dot{\sigma}]_{\text{turbine } 2} = 2.8649$ kW/K

$\dot{\sigma}]_{HX} = 3.1482$ kW/K

ANALYSIS: Using the known entropy production values, the rates of exergy destruction can be found using $\dot{E}_d = T_0\dot{\sigma}$:

$$(\dot{E}_d)_{\text{turbine } 1} = (300\text{ K})(3.1936\text{ kW/K}) = 958\text{ kW}$$

$$(\dot{E}_d)_{\text{turbine } 2} = (300\text{ K})(2.8649\text{ kW/K}) = 859\text{ kW}$$

$$(\dot{E}_d)_{HX} = (300\text{ K})(3.1482\text{ kW/K}) = 944\text{ kW}$$

(a) With Eqs. 7.23 and 7.24, $\epsilon = \dfrac{\dot{W}_{cv}}{\dot{W}_{cv} + \dot{E}_d}$

Turbine 1: $\epsilon_1 = \dfrac{10{,}000\text{ kW}}{(10{,}000 + 958)\text{ kW}} = 0.913 \quad (91.3\%)$ ← (a)

Turbine 2: $\epsilon_2 = \dfrac{10{,}570\text{ kW}}{(10{,}570 + 859)\text{ kW}} = 0.925 \quad (92.5\%)$

(b) With Eqs. 7.26 and 7.27

$$\epsilon_{HX} = \frac{\dot{m}_1(e_{f3} - e_{f2})}{\dot{m}_5(e_{f5} - e_{f6})} = \frac{\dot{m}_1(e_{f3} - e_{f2})}{\dot{m}_1(e_{f3} - e_{f2}) + \dot{E}_d} \quad (1)$$

where

$$\dot{m}_1(e_{f3} - e_{f2}) = \dot{m}_1[h_3 - h_2 - T_0(s_3 - s_2)]$$

$$= \dot{m}_1[h_3 - h_2 - T_0(s_3^\circ - s_2^\circ - R \ln p_3/p_2]$$

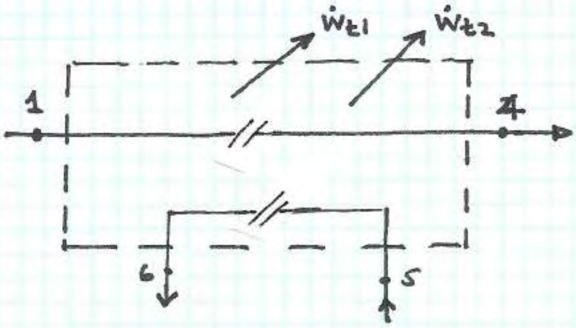
$$= \left(28.22\frac{\text{kg}}{\text{s}}\right)\left[1397.78 - 1161.07 - 300\left(3.27481 - 3.07732 - \frac{8.314}{28.97}\ln\frac{4.5}{5.0}\right)\right]\frac{\text{kJ}}{\text{kg}}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$

$$= 4752\text{ kW}$$

Then, Eq. 1 gives

$$\epsilon = \frac{4752 \text{ kW}}{(4752+944)} = 0.834 \ (83.4\%) \longleftarrow \text{(b)}$$

(c) Taking an overall control volume,



An exergy rate balance reads

$$0 = \cancel{\dot{E}_q}^{\,0} - (\dot{W}_{t1} + \dot{W}_{t2}) + \dot{m}_1(e_{f1} - e_{f4}) + \dot{m}_5(e_{f5} - e_{f6}) - (\Sigma \dot{E}_d)$$

$$\Rightarrow \underbrace{\dot{m}_1(e_{f1} - e_{f4}) + \dot{m}_5(e_{f5} - e_{f6})}_{\substack{\text{net exergy supplied} \\ \text{to the control volume}}} = (\dot{W}_{t1} + \dot{W}_{t2}) + (\Sigma \dot{E}_d)$$

$$\Rightarrow \quad \epsilon = \frac{(\dot{W}_{t1} + \dot{W}_{t2})}{\dot{m}_1(e_{f1} - e_{f4}) + \dot{m}_5(e_{f5} - e_{f6})} = \frac{(\dot{W}_{t1} + \dot{W}_{t2})}{(\dot{W}_{t1} + \dot{W}_{t2}) + (\Sigma \dot{E}_d)}$$

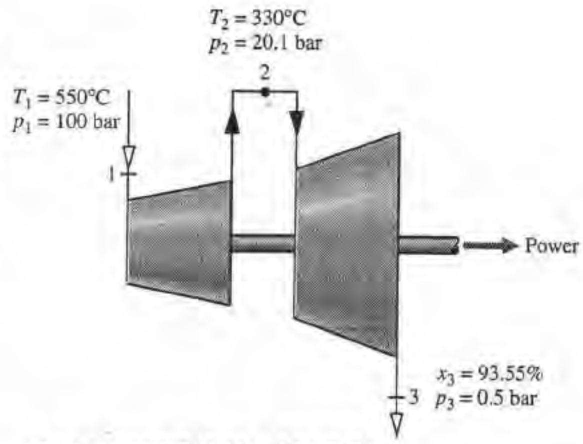
$$= \frac{(10{,}000 + 10{,}570) \text{ kW}}{(20{,}570 + \underline{(958 + 859 + 944)}) \text{ kW}} = 0.882 \ (88.2\%)$$

When an overall control volume is under consideration, its exergetic efficiency is impacted by all sources of exergy destruction present within the control volume.

PROBLEM 7.115

7.115 Figure P7.115 and the accompanying table provide steady-state operating data for a two-stage steam turbine. Stray heat transfer and the effects of motion and gravity are negligible. For each turbine stage, determine the work developed, in kJ per kg of steam flowing, and the exergetic turbine efficiency. For the overall two-stage turbine, devise and evaluate an exergetic efficiency. Let $T_0 = 298$ K, $p_0 = 1$ atm.

State	T(°C)	p(bar)	h(kJ/kg)	s(kJ/kg · K)
1	550	100	3500	6.755
2	330	20.1	3090	6.878
3	(x = 93.55%)	0.5	2497	7.174



KNOWN: Steady-state data are provided for a two-stage steam turbine.

FIND: For each turbine, evaluate $\dot{W}_t/\dot{m}$ and ϵ. Also, devise and evaluate an exergetic efficiency for the overall two-stage turbine.

ENGINEERING MODEL:

1. Control volumes at steady state enclose each turbine and the overall configuration.
2. Stray heat transfer and the effects of motion and gravity are ignored.
3. $T_0 = 298$K, $p_0 = 1$ atm.

ANALYSIS:

TURBINE 1

$$\frac{\dot{W}_t}{\dot{m}} = h_1 - h_2 = (3500-3090)\frac{kJ}{kg} = 410\ \frac{kJ}{kg}$$

With Eq. Eq. 7.24,

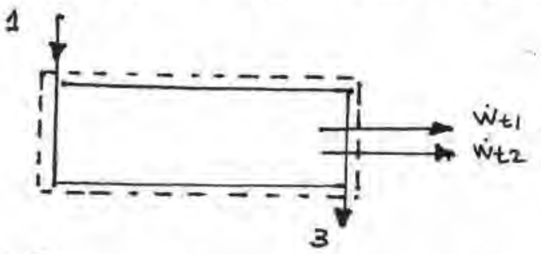
$$\epsilon = \frac{\dot{W}_t/\dot{m}}{e_{f1}-e_{f2}} = \frac{\dot{W}_t/\dot{m}}{(h_1-h_2)-T_0(s_1-s_2)} = \frac{410}{410-298(6.755-6.878)} = 0.918\ (91.8\%)$$

TURBINE 2

$$\frac{\dot{W}_t}{\dot{m}} = h_2 - h_3 = (3090-2497)\frac{kJ}{kg} = 593\ \frac{kJ}{kg}$$

$$\epsilon = \frac{\dot{W}_t/\dot{m}}{(h_2-h_3)-T_0(s_2-s_3)} = \frac{593}{593-298(6.878-7.174)} = 0.871\ (87.1\%)$$

OVERALL



Exergy rate balance:

$$0 = \sum_j \left[1-\frac{T_0}{T_j}\right]\cancel{\dot{Q}_j}^{\,0} - (\dot{W}_{t1}+\dot{W}_{t2}) + \dot{m}[e_{f1}-e_{f3}] - \dot{E}_d$$

$$\therefore\ \dot{m}[e_{f1}-e_{f3}] = (\dot{W}_{t1}+\dot{W}_{t2}) + \dot{E}_d \Rightarrow \epsilon = \frac{\dot{W}_{t1}+\dot{W}_{t2}}{\dot{m}(e_{f1}-e_{f3})} \quad (1)$$

Or,

$$\epsilon = \frac{(\dot{W}_{t1}/\dot{m})+(\dot{W}_{t2}/\dot{m})}{(h_1-h_3)-T_0(s_1-s_3)} = \frac{(410+593)}{(3500-2497)-298(6.755-7.174)} = 0.889\ (88.9\%)$$

---

1. Using the exergy rate balance, the exergetic efficiency can be written as

$$\epsilon = \frac{(\dot{W}_{t1}+\dot{W}_{t2})}{(\dot{W}_{t1}+\dot{W}_{t2})+\dot{E}_d}$$, where $\dot{E}_d$ is the total rate of exergy destruction.

Problem 7.116

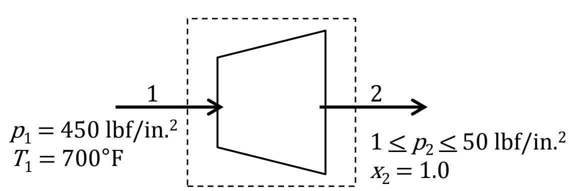
Steam at 450 lbf/in.^{2}, 700°F enters a well-insulated turbine operating at steady state and exits as saturated vapor at a pressure, p.
(a) For p = 50 lbf/in.^{2}, determine the exergy destruction rate, in Btu per lb of steam expanding through the turbine, and the turbine exergetic and isentropic efficiencies.
(b) Plot the exergy destruction rate, in Btu per lb of steam flowing, and the exergetic efficiency and isentropic efficiency, each versus pressure p ranging from 1 to 50 lbf/in.^{2}

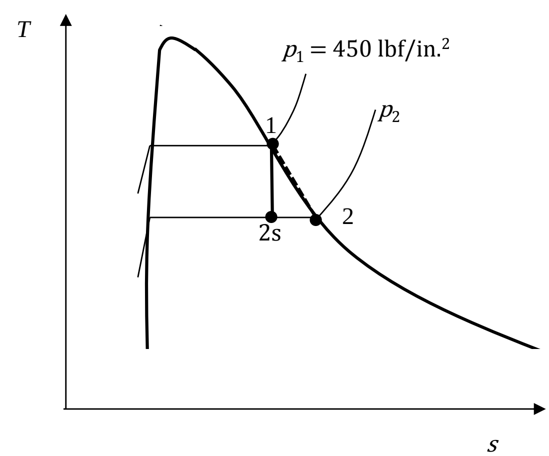
Ignore the effects of motion and gravity and let T_0 = 70°F, p_0 = 1 atm.

Known:
Steam enters a well insulated turbine operating at steady state at a specified state and exits at pressure p.

Find:
(a) For p = 50 lbf/in.^{2}, determine the exergy destruction rate per unit mass of steam flowing and the isentropic and exergetic turbine efficiencies. (b) Plot each of these quantities versus p ranging from 1 to 50 lbf/in.^{2}

Schematic and Known Data:





Engineering model:
(1) The control volume shown in the schematic is at steady state.
(2) For the control volume, $\dot{Q}_{\text{cv}} = 0$ and kinetic and potential energy effects are negligible.
(3) For the environment, T_0 = 70°F, p_0 = 1 atm.

Analysis:
With assumptions (1) and (2), the mass and energy balances reduce to give:

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 \qquad (1)$$

Further, mass and entropy rate balances reduce to give $\frac{\dot{\sigma}}{\dot{m}} = s_2 - s_1$. The exergy destruction rate is:

$$\frac{\dot{\mathrm{E}}_\mathrm{d}}{\dot{m}} = T_0(s_2 - s_1) \qquad (2)$$

Furthermore, the isentropic and exergetic turbine efficiencies are, respectively:

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \qquad (3)$$

$$\epsilon = \frac{\frac{\dot{W}_{cv}}{\dot{m}}}{\mathrm{e}_{\mathrm{f}_1} - \mathrm{e}_{\mathrm{f}_2}} = \frac{h_1 - h_2}{h_1 - h_2 - T_0(s_1 - s_2)} \qquad (4)$$

(a) From Table A-4E, $h_1 = 1359.6\,\frac{\text{Btu}}{\text{lb}}, s_1 = 1.6248\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$. Further, from Table A-3E at 50 lbf/in.^{2}, $h_2 = h_g$ = 1174.4 Btu/lb, $s_2 = s_g$ = 1.6589 Btu/lb·°R. Accordingly:

$$\frac{\dot{\mathrm{E}}_\mathrm{d}}{\dot{m}} = (530°\text{R})(1.6589 - 1.6248)\frac{\text{Btu}}{\text{lb}\cdot°\text{R}} = 18.073\,\frac{\text{Btu}}{\text{lb}}$$

State 2s is fixed by p_2 and $s_{2s} = s_1$, therefore:

$$x_{2s} = \frac{s_{2s} - s_f}{s_{fg}} = \frac{1.6248 - 0.4113}{1.2476} = 0.97267$$

Therefore:

$$h_{2s} = h_f + x_{2s}h_{fg} = 250.24 + (0.97267)(924.2) = 1149.18\,\frac{\text{Btu}}{\text{lb}}$$

Thus, inserting values into Eq. (3):

$$\eta_t = \frac{1359.6 - 1174.4}{1359.6 - 1149.18} = 0.8801 = 88.01\%$$

Now, from Eq. (4):

$$\epsilon = \frac{1359.6 - 1174.4}{1359.6 - 1174.4 - (520)(1.6248 - 1.6589)} = 0.9126 = 91.11\%$$

(b) The data for the required plots are obtained using IT, as follows:

<u>IT Code</u>:

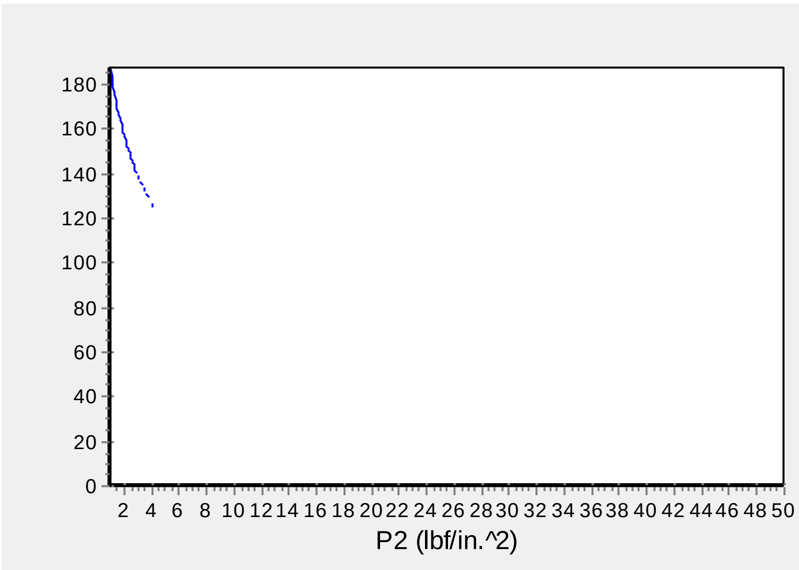
```
p1 = 450 // lbf/in.^2
T1 = 700 // F
x2 = 1.0
p2 = 50 //lbf/in.^2
T0 = 70 + 460 // R

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2 = hsat_Px("Water/Steam", p2, x2)
s2 = ssat_Px("Water/Steam", p2, x2)
s2s = s_Ph("Water/Steam", p2, h2s)
s2s = s1

Ed = T0 * (s2 - s1)
```

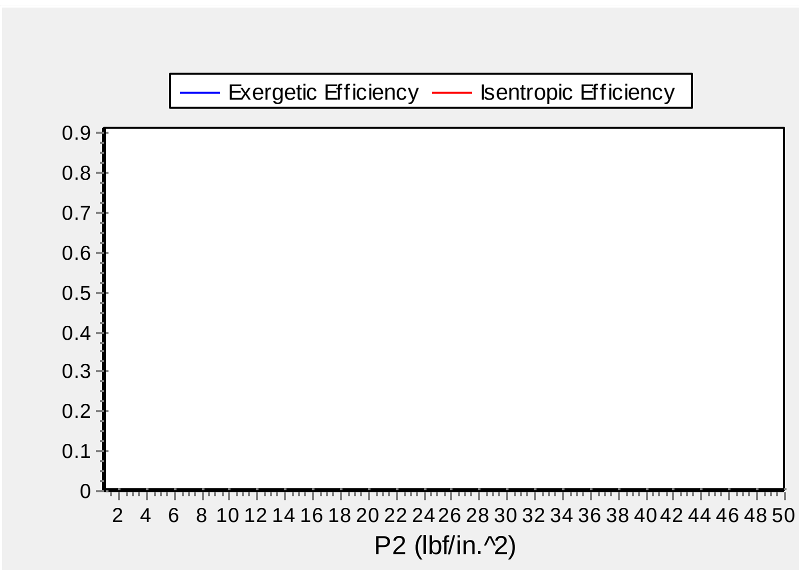
eta = (h1 - h2) / (h1 - h2s)
eff = (h1 - h2) / ((h1 - h2) - T0 * (s1 - s2))

IT Results for p_2 = 50 lbf/in.^{2}:

Ed	eff	eta	h2	h2s	s2	s2s
18.1	0.911	0.8799	1174	1149	1.659	1.625

Plots:





Discussion:
As expected, increased efficiency corresponds with decreased exergy destruction.

Problem 7.117

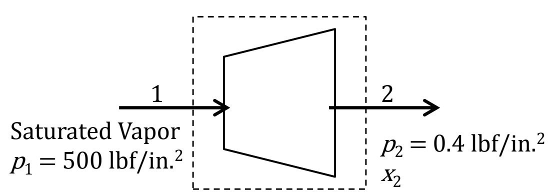
Saturated water vapor at 500 lbf/in.^{2} enters an insulated turbine operating at steady state. A two phase liquid-vapor mixture exits at 0.4 lbf/in.^{2} Plot each of the following versus the steam quality at the turbine exit ranging from 75 to 100%:
(a) the power developed and the rate of exergy destruction, each in Btu per lb of steam flowing.
(b) the isentropic turbine efficiency.
(c) the exergetic turbine efficiency.

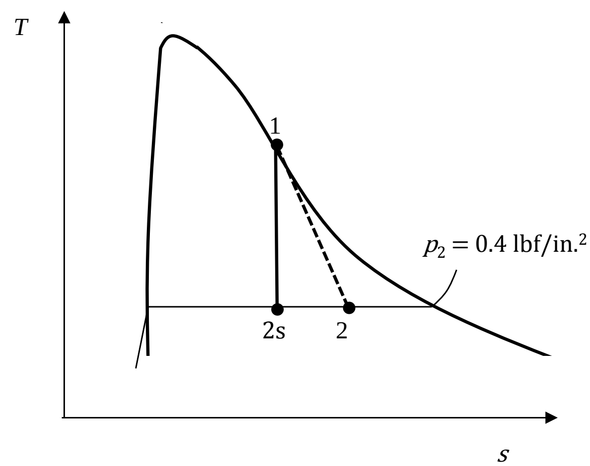
Let T_0 = 70°F, p_0 = 1 atm. Ignore the effects of motion and gravity.

Known:
Steady-state operating data are provided for a steam turbine.

Find:
Plot versus the steam quality at the turbine exit: (a) the power developed and rate of exergy destruction, each per unit mass of steam flowing, (b) the isentropic turbine efficiency, and (c) the exergetic turbine efficiency.

Schematic and Known Data:





Engineering Model:
(1) The control volume shown in the schematic is at steady state.
(2) For the control volume, $\dot{Q}_{\mathrm{cv}} = 0$ and kinetic and potential energy effects are negligible.
(3) For the environment, T_0 = 530°R, p_0 = 1 atm.

Analysis:
Reducing mass, energy, and entropy balances gives:

$$\frac{\dot{W}_{\mathrm{cv}}}{\dot{m}} = h_1 - h_2 \qquad (1)$$

$$\frac{\dot{\mathrm{E}}_{\mathrm{d}}}{\dot{m}} = T_0 \frac{\dot{\sigma}_{\mathrm{cv}}}{\dot{m}} = T_0(s_2 - s_1) \qquad (2)$$

h_1 and s_1 are fixed by p_1, saturated vapor. h_2 and s_2 are fixed by p_2, x_2. Using Eq. 6.46,

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \qquad (3)$$

Where h_{2s} is fixed by p_2, $s_{2s} = s_1$. Then, with Eq. 7.24:

$$\epsilon = \frac{\frac{\dot{W}_{\text{cv}}}{\dot{m}}}{\mathrm{e}_{\mathrm{f}_1} - \mathrm{e}_{\mathrm{f}_2}} = \frac{h_1 - h_2}{h_1 - h_2 - T_0(s_1 - s_2)} \qquad (4)$$

<u>Sample calculation:</u>

x_2 = 0.81. From Table A-3E, $h_1 = 1205.3\,\frac{\text{Btu}}{\text{lb}}, s_1 = 1.4644\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$ and:

$$h_2 = h_f + x_2 h_{fg} = 40.94 + (0.81)(1052.3) = 893.30\,\frac{\text{Btu}}{\text{lb}}$$

$$s_2 = s_f + x_2 s_{fg} = 0.0800 + (0.81)(1.9760) = 1.6806\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$$

With $s_{2s} = s_1$:

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{1.4644 - 0.0800}{1.9760} = 0.7006 \Longrightarrow$$

$$h_{2s} = 40.94 + (0.7006)(1052.3) = 778.18\,\frac{\text{Btu}}{\text{lb}}$$

Then, we get from Eqs. (1), (2), (3), and (4), respectively:

$$\frac{\dot{W}_{\text{cv}}}{\dot{m}} = 1205.3 - 893.30 = 312\,\frac{\text{Btu}}{\text{lb}}$$

$$\frac{\dot{\mathrm{E}}_{\mathrm{d}}}{\dot{m}} = (530°\text{R})(1.6806 - 1.4644)\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}} = 114.59\,\frac{\text{Btu}}{\text{lb}}$$

$$\eta_t = \frac{1205.3 - 893.30}{1205.3 - 778.18} = 0.73$$

#1

$$\epsilon = \frac{1205.3 - 893.30}{1205.3 - 893.3 - (530)(1.4644 - 1.6806)} = 0.73$$

Data required for the plots are obtained with the following IT code:

<u>IT Code</u>:

```
p1 = 500 // lbf/in.^2
x1 = 1
x2 = 0.81
p2 = 0.4 //lbf/in.^2
T0 = 70 + 460 // R

h1 = hsat_Px("Water/Steam", p1, x1)
s1 = ssat_Px("Water/Steam", p1, x1)
h2 = hsat_Px("Water/Steam", p2, x2)
s2 = ssat_Px("Water/Steam", p2, x2)
```

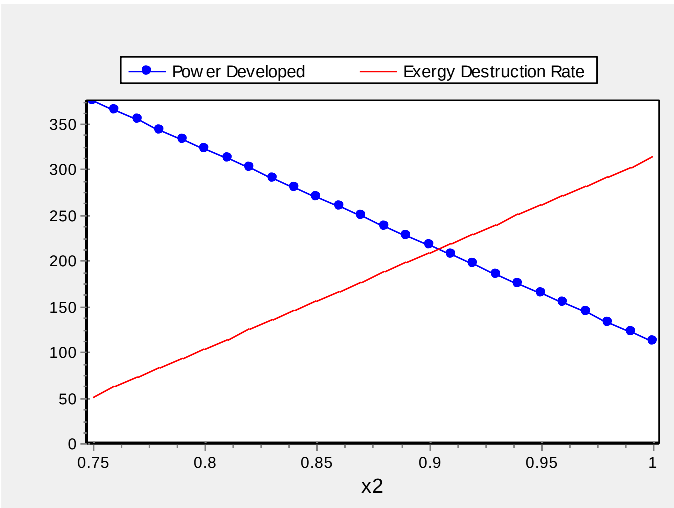
s2s = s_Ph("Water/Steam", p2, h2s)
s2s = s1

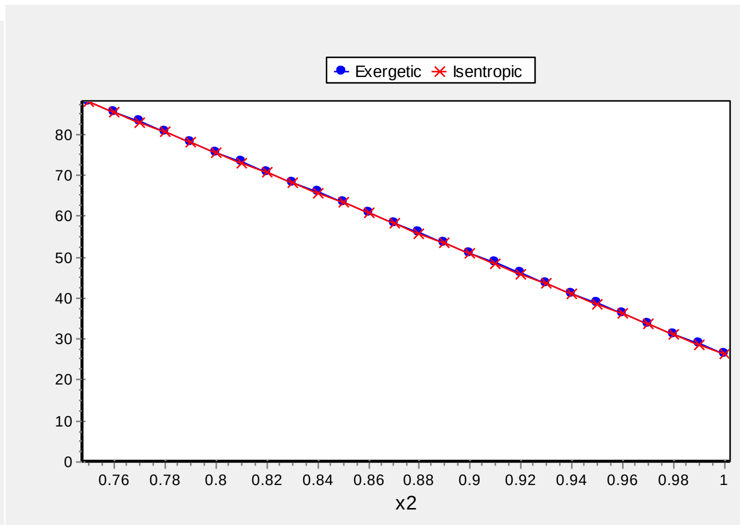
Wcv = h1-h2
Ed = T0 * (s2 - s1)
eta = (h1 - h2) / (h1 - h2s)
eff = (h1 - h2) / ((h1 - h2) - T0 * (s1 - s2))

IT Results for p_2 = 50 lbf/in.^{2}:

Ed	eff	eta	h2	h2s	s2	s2s
114.4	0.7317	0.7307	893.1	778.1	1.68	1.464

Plots:





Discussion:
As expected, increased exergy destruction corresponds with decreased power output and efficiency.

Comment:
1. In this case the numerical value for ε and η_t for each x_2 are nearly the same, but this is not observed generally.

Problem 7.118

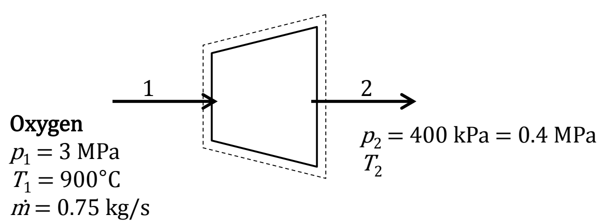
Oxygen enters an insulated turbine operating at steady state at 900°C and 3 MPa and exhausts at 400 kPa. The mass flow rate is 0.75 kg/s. Plot each of the following versus the turbine exit temperature, in °C:
(a) the power developed, in kW.
(b) the rate of exergy destruction in the turbine, in kW.
(c) the exergetic turbine efficiency.

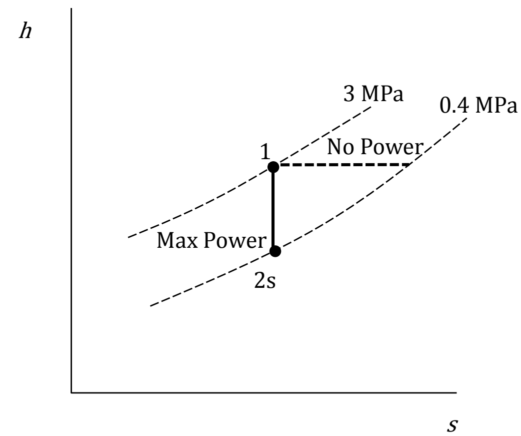
For oxygen, use the ideal gas model with k = 1.395. Ignore the effects of motion and gravity. Let T_0 = 30°C, p_0 = 1 bar.

Known:
Steady-state operating data are provided for a turbine through which oxygen is expanding.

Find:
Plot versus the turbine exit temperature, (a) the power developed, (b) the rate of exergy destruction, and (c) the exergetic turbine efficiency.

Schematic and Known Data:





Engineering Model:
(1) The control volume shown in the schematic is at steady state.
(2) For the control volume, $\dot{Q}_{\text{cv}} = 0$ and kinetic and potential energy effects are negligible.
(3) Oxygen is modeled as an ideal gas with k = 1.395 which corresponds to c_p = 0.918 (Table A-20).
(4) For the environment, T_0 = 30° C = 303 K, p_0 = 1 bar.

Analysis:
Reducing mass, energy, and entropy balances together with ideal gas model relations for c_p constant:

$$\dot{W}_{\text{cv}} = \dot{m}(h_1 - h_2) = \dot{m}c_p(T_2 - T_1)$$

$$\dot{\text{E}}_{\text{d}} = T_0\dot{\sigma}_{\text{cv}} = \dot{m}T_0(s_2 - s_1) = \dot{m}T_0\left[c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}\right]$$

$$\epsilon = \frac{\dot{W}_{cv}}{\dot{m}[e_{f_1} - e_{f_2}]} = \frac{\dot{W}_{cv}}{\dot{m}[h_1 - h_2 - T_0(s_1 - s_2)]} = \frac{\dot{W}_{cv}}{\dot{W}_{cv} + \dot{E}_d}$$

Observe that T_2 can range from T_{2s}, corresponding to an isentropic expansion, to $T_2 = T_1$, corresponding to no power development. With Eq. 6.43:

$$T_{2s} = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = (1173\text{ K}) \left(\frac{0.4}{3}\right)^{\frac{1.395-1}{1.395}} = 663\text{ K}$$

So, 663 K $\leq T_2 \leq$ 1173 K

The data for the plots are obtained using IT as follows:

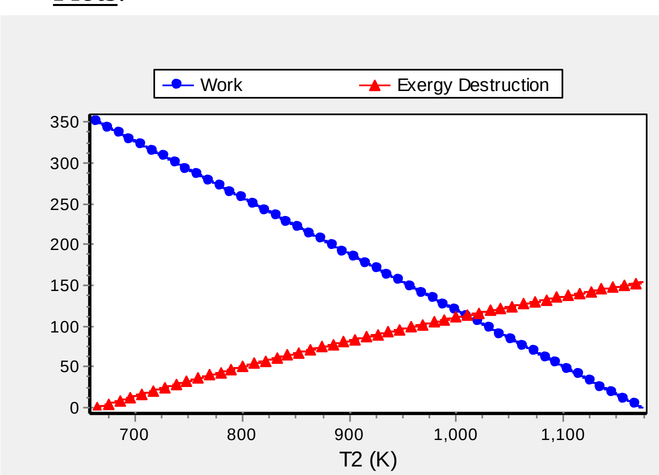
<u>IT Code</u>:

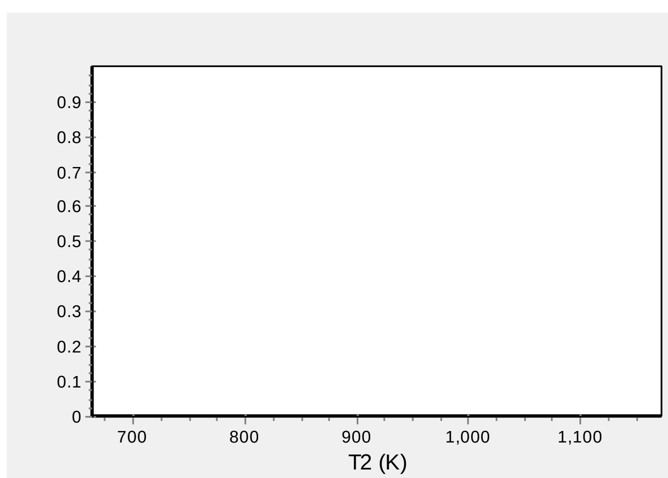
```
p1 = 3 // MPa
T1 = 900 + 273 // K
p2 = 0.4 // MPa
T2 = 664 // K
T0 = 20 + 273 // K
mdot = 0.75 // kg/s
cp = 0.9177
R = 8.314 / 32 // kJ/kg K

Wdot = mdot * cp * (T1 - T2)
delta_s = cp * ln(T2 / T1) - R * ln(p2 / p1)
Edot = T0 * delta_s
eff = Wdot / (Wdot + Edot)
```

<u>IT Results for p_2 = 50 lbf/in.^{2}</u>:

delta_s	Edot	eff	Wdot
0.001291	0.3783	0.9989	350.3

<u>Plots</u>:



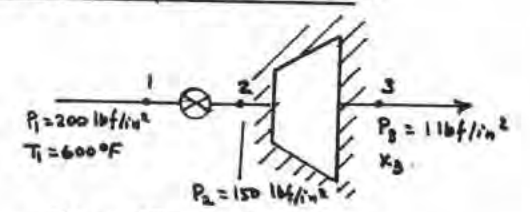


PROBLEM 7.119

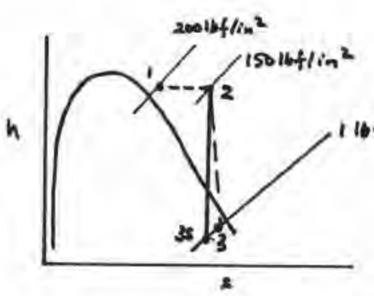
KNOWN: Steady-state operating data are provided for a throttling valve and turbine in series.

FIND: For the turbine plot the rate of exergy destruction and exergetic efficiency versus quality ranging from 90 to 100%.

SCHEMATIC & GIVEN DATA:



$T_0 = 60$°F, $P_0 = 1$ atm



ENGR. MODEL: (1) The turbine is insulated and at steady state. $\dot{Q}_{cv} = 0$. 2. The expansion across the valve is a throttling process. (3) For the environment, $T_0 = 60$°F, $P_0 = 1$ atm.

ANALYSIS: For a 1-inlet, 1-exit control volume at steady state for which there is no heat transfer, an entropy rate balance reduces to give

$$0 = \sum \frac{\cancel{\dot{Q}_j}^{\,0}}{T_j} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1)$$

Then, with $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, we get

$$\text{Valve: } \frac{\dot{E}_d}{\dot{m}} = T_0(s_2 - s_1) \quad (1) \qquad \text{Turbine: } \frac{\dot{E}_d}{\dot{m}} = T_0(s_3 - s_2) \quad (2)$$

Sample calculation: $x_3 = 0.98$. From Table A-4E, $h_1 = 1322.1$ Btu/lb, $s_1 = 1.6767$ Btu/lb·°R. With assumption 2, $h_2 \sim h_1$. Interpolating in Table A-4E at 150 lbf/in² with $h_2 = 1322.1$ Btu/lb, $s_2 = 1.7078$ Btu/lb·°R. With data from Table A-3E, $s_3 = s_f + x_3 s_{fg}$ or $s_3 = 0.1327 + 0.98(1.8453) = 1.9411$ Btu/lb·°R. Inserting values into Eqs. (1), (2)

$$(\dot{E}_d/\dot{m})_{valve} = 520°R\,(1.7078 - 1.6767)\frac{\text{Btu}}{\text{lb}\cdot°R} = 16.17\ \frac{\text{Btu}}{\text{lb}} \quad \leftarrow \text{valve}$$

$$(\dot{E}_d/\dot{m})_{turbine} = 520°R\,(1.9411 - 1.7078)\frac{\text{Btu}}{\text{lb}\cdot°R} = 121.32\ \frac{\text{Btu}}{\text{lb}} \quad \leftarrow \text{turbine}$$

Note that the valve is considered only for comparison.

IT Code

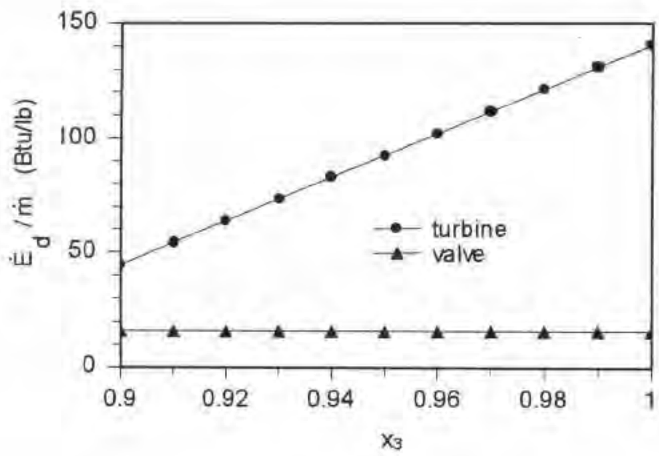
```
p1 = 200  // lbf/in.²
T1 = 600  // °F
p2 = 150  // lbf/in.²
p3 = 1  //  lbf/in.²
To = 60 + 460  // °R
x3 = 0.98

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2 = h_Ps("Water/Steam", p2, s2)
h2 = h1
s3 = ssat_Px("Water/Steam", p3, x3)
Edvalve= To * (s2 - s1)
Edturb= To * (s3 - s2)
```

IT Results for $x_3 = 0.98$

h1= 1322 Btu/lb
s1 = 1.676 Btu/lb·°R
s2 = 1.707 Btu/lb·°R
s3 = 1.941 Btu/lb·°R
$(\dot{E}_d/\dot{m})_{valve}$ = 121.4 Btu/lb
$(\dot{E}_d/\dot{m})_{turbine}$ = 16.04 Btu/lb

PROBLEM 7.119 (Continued)

PLOT:



- Note that the exergy destruction in the valve is independent of x_3.
- The exergy destruction in the turbine increases as x_3 increases, corresponding to a decrease in isentropic turbine efficiency, as expected.

The turbine exergetic efficiency is given by Eq. 7.24. With indicated assumptions, this can be expressed as

$$\epsilon = \frac{h_2 - h_3}{(h_2 - h_3) - T_0(s_2 - s_3)}$$. Since $h_2 = h_1$ for the throttling process, we get

$$\epsilon = \frac{h_1 - h_3}{(h_1 - h_3) - T_0(s_2 - s_3)} \quad (1).$$ In Eq. (1) h_1 is fixed by T_1, P_1, s_2 is fixed by $h_2 = h_1, P_2$, and h_3 and s_3 are fixed by P_3, X_3.

Sample Calculation: $x_3 = 0.98$. From Table A-4E, $h_1 = 1322.1$ Btu/lb. Interpolating in Table A-4E with $h_2 = h_1$ at 150 lbf/in.^{2}, $s_2 = 1.7078$ Btu/lb·°R. With data from Table A-3E at 1 lbf/in.^{2}, $h_3 = 69.74 + 0.98(1036) = 1085$ Btu/lb, $s_3 = 0.1327 + 0.98(1.8453) = 1.9411$ Btu/lb·°R. Inserting values, Eq. (1) gives

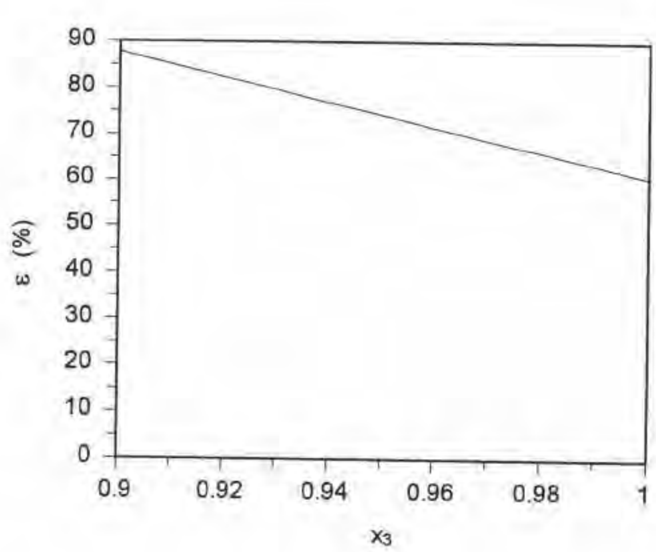
$$\epsilon = \frac{(1322.1 - 1085)}{(1322.1 - 1085) - 520(1.7078 - 1.9411)} = \frac{237.1}{358.4} = 0.662 \quad (66.2\%)$$

IT Code

```
p1 = 200  // lbf/in.²
T1 = 600  // °F
p2 = 150  // lbf/in.²
p3 = 1  // lbf/in.²
x3 = 0.98
To = 60 + 460  // °R

h1 = h_PT("Water/Steam", p1, T1)
s2 = s_Ph("Water/Steam", p2, h1)
s3 = ssat_Px("Water/Steam", p3, x3)
h3 = hsat_Px("Water/Steam", p3, x3)
eff = (h1 - h3) / ((h1 - h3) - To * (s2 - s3))
```

IT Results for $x_3 = 0.98$

eff	0.6613
h1	1322
h3	1085
s2	1.707
s3	1.941



PROBLEM 7.120

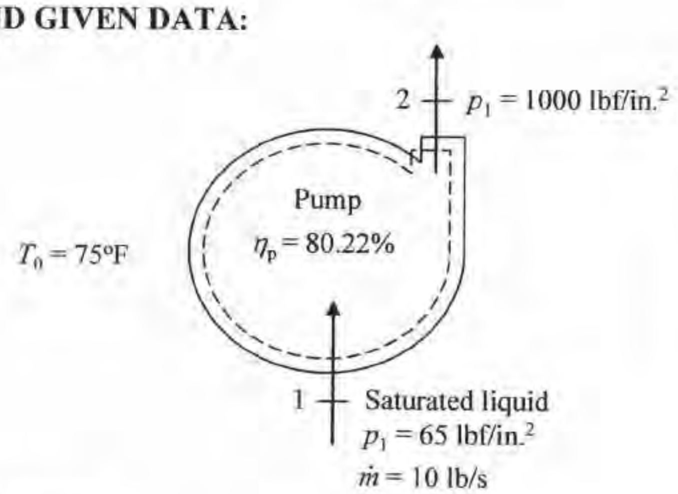
7.120 A pump operating at steady state takes in saturated liquid water at 65 lbf/in.^{2} at a rate of 10 lb/s and discharges water at 1000 lbf/in.^{2} The isentropic pump efficiency is 80.22%. Heat transfer with the surroundings and the effects of motion and gravity can be neglected. If T_0 = 75°F, determine for the pump

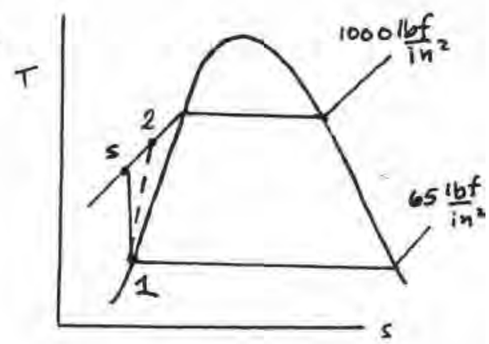
(a) the exergy destruction rate, in Btu/s.
(b) the exergetic efficiency.

KNOWN: Water at specified pressure and mass flow rate enters a pump with known isentropic efficiency and exits at specified pressure.

FIND: The exergy destruction rate and the exergetic efficiency.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL:
1. The control volume defined by the dashed line on the accompanying diagram is at steady state.
2. Heat transfer and the effects of motion and gravity can be ignored.
3. The specific volume of the water is assumed constant.
4. T_0 = 75°F = 535°R.

ANALYSIS:
(a) The exergy destruction rate can be determined from the entropy production rate

$$\dot{E}_d = T_0 \dot{\sigma}_{cv}$$

Entropy production rate can be determined from the steady-state control volume entropy rate balance

$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{cv}$$

Neglecting heat transfer rate, entropy production rate for one inlet and one exit becomes

$$\dot{\sigma}_{cv} = \dot{m}\,(s_2 - s_1)$$

State 1 is saturated liquid. From Table A-3E, $v_1 = v_{f1}$ = 0.01743 ft^3/lb, $h_1 = h_{f1}$ = 267.7 Btu/lb, $s_1 = s_{f1}$ = 0.4345 Btu/(lb·°R).

To fix State 2 a second independent, intensive property is required. The specific enthalpy at the pump exit can be determined from the isentropic pump efficiency

Eq. 6.51c

$$\eta_p = \frac{\left(-\dot{W}_{cv}/\dot{m}\right)_s}{\left(-\dot{W}_{cv}/\dot{m}\right)} = \frac{v(p_2 - p_1)}{h_2 - h_1}$$

❶ Using the assumption of constant specific volume, the numerator of the above expression is evaluated with Eq. 6.51c. Solving for the exit enthalpy gives

$$h_2 = h_1 + \frac{v(p_2 - p_1)}{\eta_p}$$

Substituting values and applying appropriate conversion factors give

$$h_2 = 267.7\frac{\text{Btu}}{\text{lb}} + \frac{\left(0.01743\frac{\text{ft}^3}{\text{lb}}\right)\left(1000\frac{\text{lbf}}{\text{in.}^2} - 65\frac{\text{lbf}}{\text{in.}^2}\right)}{0.8022}\left|\frac{144\text{ in.}^2}{\text{ft}^2}\right|\left|\frac{\text{Btu}}{778\text{ ft}\cdot\text{lbf}}\right| = 271.46\text{ Btu/lb}$$

State 2 is compressed liquid. From Table A-5E, $s_2 = 0.43552$ Btu/(lb·°R). Substituting values and solving for rate of entropy produced give

$$\dot{\sigma}_{cv} = (10\text{ lb/s})[0.43552\text{ Btu/(lb·°R)} - 0.4345\text{ Btu/(lb·°R)}] = 0.0102\text{ Btu/(s·°R)}$$

The rate of exergy destruction is

$$\dot{E}_d = (535°\text{R})[0.0102\text{ Btu/(s·°R)}] = \underline{\mathbf{5.457\text{ Btu/s}}}$$

(b) The exergetic pump efficiency is determined from Eq. 7.25

$$\varepsilon = \frac{e_{f2} - e_{f1}}{\left(-\dot{W}_{cv}/\dot{m}\right)}$$

where the change in specific flow exergy is defined by Eq. 7.18 (ignoring effects of motion and gravity)

$$e_{f2} - e_{f1} = 271.46\frac{\text{Btu}}{\text{lb}} - 267.7\frac{\text{Btu}}{\text{lb}} - (535°\text{R})\left(0.43552\frac{\text{Btu}}{\text{lb}\cdot°\text{R}} - 0.4345\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)$$

$$e_{f2} - e_{f1} = 3.21\text{ Btu/lb}$$

The power per unit mass of water flowing can be determined from the steady-state energy balance

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i\,(h_i + ½\,V_i^2 + gz_i) - \sum_e \dot{m}_e\,(h_e + ½\,V_e^2 + gz_e)$$

Setting rate of heat transfer to zero, ignoring kinetic and potential energy effects, and writing the energy balance for one inlet and one exit give

$$0 = -\dot{W}_{cv} + \dot{m}\,(h_1 - h_2)$$

Solving for power per unit mass of water flowing gives

$$(-\dot{W}_{cv}/\dot{m}) = h_2 - h_1$$

Substituting values gives

$$(-\dot{W}_{cv}/\dot{m}) = 271.46\text{ Btu/lb} - 267.7\text{ Btu/lb} = 3.76\text{ Btu/lb}$$

The exergetic pump efficiency is

$$\varepsilon = \frac{3.21\frac{\text{Btu}}{\text{lb}}}{3.76\frac{\text{Btu}}{\text{lb}}} = \underline{\mathbf{0.8537}}$$

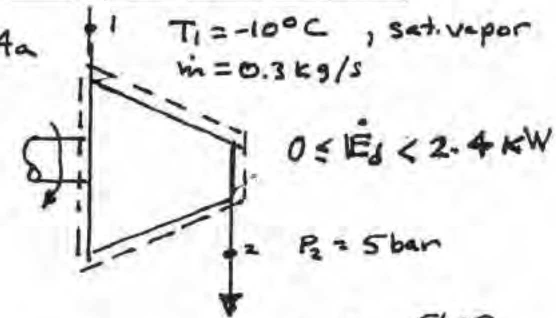
---

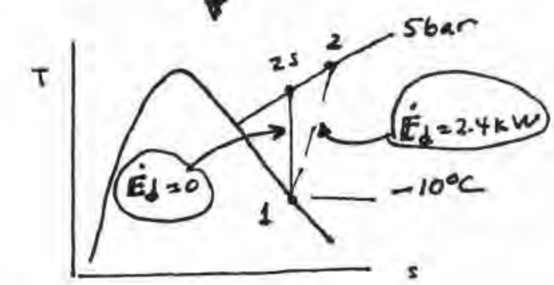
❶ Equation 6.51c requires *only* an internally reversible flow; so it applies whether there is heat transfer or not. When there is no heat transfer, Eq. 6.51c gives the correct expression for the isentropic case.

PROBLEM 7.121

7.121 Refrigerant 134a as saturated vapor at −10°C enters a compressor operating at steady state with a mass flow rate of 0.3 kg/s. At the compressor exit the pressure of the refrigerant is 5 bar. Stray heat transfer and the effects of motion and gravity can be ignored. If the rate of exergy destruction within the compressor must be kept less than 2.4 kW, determine the allowed ranges for (a) the power required by the compressor, in kW, and (b) the exergetic compressor efficiency. Let T_0 = 298 K, p_0 = 1 bar.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL:

1. The control volume shown in the sketch is at steady state.
2. Stray heat transfer and the effects of motion and gravity can be ignored.
3. $T_0 = 298$ K, $P_0 = 1$ bar

ANALYSIS: $\dot{E}_d = T_0 \dot{\sigma}_{cv} = T_0 \dot{m}(s_2 - s_1)$. So, $T_0 \dot{m}(s_2 - s_1) < 2.4$ kW, or

$$s_2 < s_1 + \frac{2.4\text{ kW}}{\dot{m}\, T_0} \Rightarrow s_2 < 0.9253 + \frac{2.4\text{ kJ/s}}{(0.3\frac{kg}{s})(298K)} = 0.9521 \frac{kJ}{kg \cdot K}$$

Thus, from Table A-12 at 5 bar, $h_2 < 268.01$ kJ/kg

(a) Reducing mass and energy rate balances, $(-\dot{W}_c) = h_2 - h_1$. Thus,

$$\dot{m}[\underbrace{h_{2s} - h_1}_{\dot{E}_d = 0}] \le (-\dot{W}_{cv}) < \dot{m}(\underbrace{h_2 - h_1}_{h_2 < 268.01\text{ kJ/kg}}) \qquad (1)$$

From Table A-10, $h_1 = 241.35$ kJ/kg, $s_1 = 0.9253$ kJ/kg·K. Then, with $s_{2s} = s_1$, $h_{2s} = 260.02$ kJ/kg. Eq. (1) gives

$$\left(0.3\frac{kg}{s}\right)(260.02 - 241.35)\frac{kJ}{kg}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| \le (-\dot{W}_c) < 0.3(268.01 - 241.35)|1|$$

or

$$5.6\text{ kW} \le (-\dot{W}_c) < 8\text{ kW} \qquad \leftarrow (a)$$

(b) Invoking Eq. 7.25,

$$\epsilon = \frac{e_{f2} - e_{f1}}{-\dot{W}_c} = \frac{(h_2 - h_1) - T_0(s_2 - s_1)}{(h_2 - h_1)}$$

When $\dot{E}_d = 0$, $\epsilon = 1$ (100%). Otherwise,

$$\epsilon > \frac{(268.01 - 241.35) - 298(0.9521 - 0.9253)}{(268.01 - 241.35)} = 0.7 \quad (70\%)$$

$$\Rightarrow \quad 70\% < \epsilon \le 100\% \qquad \leftarrow (b)$$

Problem 7.122

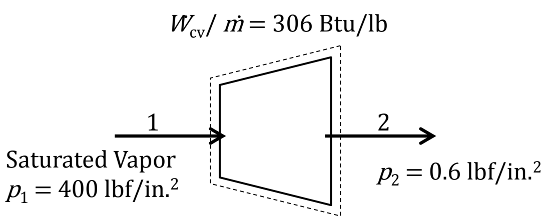
Saturated water vapor at 400 lbf/in.^{2} enters an insulated turbine operating at steady state. At the turbine exit the pressure is 0.6 lbf/in.^{2} The work developed equals 306 Btu per pound of steam passing through the turbine. Kinetic and potential energy effects can be neglected. Let T_0 = 60°F, p_0 = 1 atm. Determine:

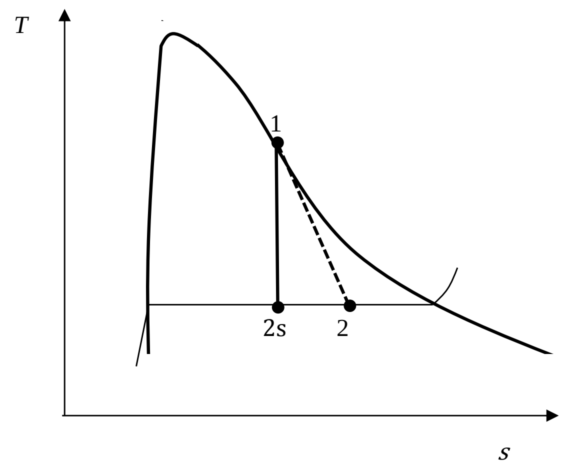
(a) the exergy destruction rate, in Btu per pound of steam expanding through the turbine.
(b) the isentropic turbine efficiency.
(c) the exergetic turbine efficiency.

Known:
Operating data are provided for a steam turbine at steady state.

Find:
(a) Determine the exergy destruction rate per pound of steam flowing, (b) the isentropic turbine efficiency, and (c) the exergetic turbine efficiency.

Schematic and Known Data:





Engineering Model:
(1) The turbine operates at steady state and is insulated.
(2) Kinetic and potential energy effects can be neglected.
(3) For the environment, T_0 = 60°F, p_0 = 1 atm.

Analysis:

(a) The exergy destruction rate is $\dot{\mathrm{E}}_{\mathrm{d}} = T_0\dot{\sigma}_{\mathrm{cv}}$ where $\dot{\sigma}_{\mathrm{cv}}$ is the rate of entropy production. With stated assumptions, the entropy rate balance reduces to:

$$\dot{\sigma}_{\mathrm{cv}} = \dot{m}(s_2 - s_1) \Longrightarrow \frac{\dot{\mathrm{E}}_{\mathrm{d}}}{\dot{m}} = T_0(s_2 - s_1)$$

From Table A-4E: $h_1 = 1205.5\,\frac{\mathrm{Btu}}{\mathrm{lb}}, s_1 = 1.4856\,\frac{\mathrm{Btu}}{\mathrm{lb{\cdot}°R}}$. To fix state 2, use mass and energy rate balances to write $\frac{\dot{W}_{\mathrm{cv}}}{\dot{m}} = h_1 - h_2$. Thus:

$$h_2 = h_1 - \frac{\dot{W}_{\text{cv}}}{\dot{m}} = 1205.5 - 306 = 899.5\,\frac{\text{Btu}}{\text{lb}}$$

$$x_2 = \frac{h_2 - h_f}{h_{fg}} = \frac{899.5 - 53.27}{1045.4} = 0.8095$$

$$s_2 = s_f + x_2 s_{fg} = 0.1029 + (0.8095)(1.9184) = 1.656\,\frac{\text{Btu}}{\text{lb} \cdot {}^\circ\text{R}}$$

Finally:

$$\frac{\dot{\text{E}}_{\text{d}}}{\dot{m}} = (520^\circ\text{R})(1.656 - 1.4856)\,\frac{\text{Btu}}{\text{lb} \cdot {}^\circ\text{R}} = 88.6\,\frac{\text{Btu}}{\text{lb}}$$

(b) The isentropic turbine efficiency is:

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

To find h_{2s}, use $s_{2s} = s_1$ to write:

$$x_{2s} = \frac{s_{2s} - s_f}{s_{fg}} = \frac{1.4856 - 0.1029}{1.9184} = 0.7208$$

So:

$$h_{2s} = h_f + x_{2s} h_{fg} = 53.27 + (0.7208)(1045.4) = 806.8\,\frac{\text{Btu}}{\text{lb}}$$

Accordingly:

$$\eta_t = \frac{306}{1205.5 - 806.8} = \frac{306}{398.7} = 0.767 = 76.7\%$$

(c) The exergetic turbine efficiency is given by Eq. 7.24

$$\epsilon = \frac{\frac{\dot{W}_{\text{cv}}}{\dot{m}}}{\text{e}_{\text{f}_1} - \text{e}_{\text{f}_2}} = \frac{\frac{\dot{W}_{\text{cv}}}{\dot{m}}}{h_1 - h_2 - T_0(s_1 - s_2)} = \frac{306}{306 + 88.6} = 0.775 = 77.5\%$$

PROBLEM 7.123

7.123 Figure P7.123 shows an insulated counterflow heat exchanger with carbon dioxide (CO_2) and air flowing through the inner and outer channels, respectively. The figure provides data for operation at steady state. The heat exchanger is a component of an overall system operating in an arctic region where the average annual ambient temperature is 20°F. Heat transfer between the heat exchanger and its surroundings can be ignored, as can effects of motion and gravity. Evaluate for the heat exchanger
(a) the rate of exergy destruction, in Btu/s.
(b) the exergetic efficiency given by Eq. 7.27.
Let T_0= 20°F, p_0=1 atm.

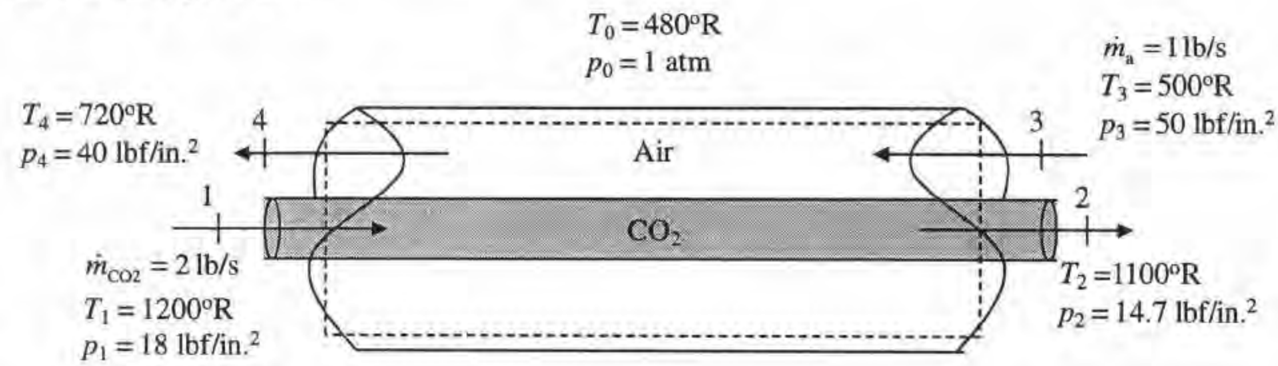


Fig. P7.123

KNOWN: Steady state operating data is provided for an annular counterflow heat exchanger.

FIND: Determine (a) the rate of exergy destruction, and (b) the exergetic efficiency given by Eq. 7.27.

SCHEMATIC AND GIVEN DATA:
Refer to Fig. P7.123

State	T (°R)	p (lbf/in.²)	$\bar{h}$ (Btu/lbmol)	h (Btu/lb)	s^o (Btu/lb-°R)	$\bar{s}^o$ (Btu/lbmol-°R)
	given	*given*	Tab A-23E	Tab A-22E	Tab A-23E	Tab A-22E
1	1200	18	10955.3	-	59.283	-
2	1100	14.7	9802.6	-	58.281	-
3	500	50	-	119.48	-	0.58233
4	720	40	-	172.39	-	0.67002

ENGINEERING MODEL:
(1) The control volume shown in the schematic is at steady state.
(2) For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$, and the effects of motion and gravity can be ignored.
(3) Air and CO_2 are modeled as ideal gases.
(4) The exergy reference environment is T_0 = 480°R, p_0=1 atm

ANALYSIS:

To check the consistency of the given data, we use the mass and energy rate balances as follows:

$$0 = \dot{m}_{CO2}\left(\frac{\bar{h}_2 - \bar{h}_1}{M}\right) + \dot{m}_a(h_4 - h_3) \qquad (1)$$

With enthalpy data from Tables A-22E and A-23E listed in the accompanying table, the values when substituted into Eq. (1) are consistent with the conservation of mass and energy principles.

PROBLEM 7.123 (Continued)

(a) Applying the steady-state control volume exergy rate balance, Eq. 7.13a, simplifying based on assumptions, and rearranging:

(#1) $$\dot{\mathbf{E}}_d = \dot{m}_{CO2}(e_{f1} - e_{f2}) + \dot{m}_a(e_{f3} - e_{f4}) \qquad (2)$$

Solving for the CO_2 change in flow exergy term using Eq. 7.18 and data from Table A-23E:

$$(e_{f1} - e_{f2}) = \frac{1}{M}\left[\bar{h}_1 - \bar{h}_2 - T_0\left(\bar{s}_1^o - \bar{s}_2^o - \bar{R}\ln\frac{P_1}{P_2}\right)\right]$$

$$= \frac{1}{44.01\frac{\text{lb}}{\text{lbmol}}}\left[(10955.3 - 9802.6)\frac{\text{Btu}}{\text{lbmol}} - 480^\circ\text{R}\left(59.283 - 58.281 - 1.986\ln\frac{18}{14.7}\right)\frac{\text{Btu}}{\text{lbmol}\cdot^\circ\text{R}}\right]$$

$$= 19.65\frac{\text{Btu}}{\text{lb}}$$

Repeating for air using data from Table A-22E:

$$(e_{f3} - e_{f4}) = \left[h_3 - h_4 - T_0\left(s_3^o - s_4^o - \frac{\bar{R}}{M}\ln\frac{P_3}{P_4}\right)\right]$$

$$= \left[(119.48 - 172.39)\frac{\text{Btu}}{\text{lb}} - 480^\circ\text{R}\left(0.58233 - 0.67002 - \frac{1.986}{28.97}\ln\frac{50}{40}\right)\frac{\text{Btu}}{\text{lb}\cdot^\circ\text{R}}\right] = -3.476\frac{\text{Btu}}{\text{lb}}$$

Substituting values into Eq. (2)

$$\dot{\mathbf{E}}_d = 2\frac{\text{lb}}{\text{s}}\left(19.65\frac{\text{Btu}}{\text{lb}}\right) + 1\frac{\text{lb}}{\text{s}}\left(-3.476\frac{\text{Btu}}{\text{lb}}\right) = 35.82\frac{\text{Btu}}{\text{s}} \longleftarrow$$

(b) The exergetic efficiency given by Eq. 7.27

(#2 #3) $$\varepsilon = \frac{\dot{m}_a(e_{f4} - e_{f3})}{\dot{m}_{CO2}(e_{f1} - e_{f2})} = \frac{1\frac{\text{lb}}{\text{s}}\left(3.476\frac{\text{Btu}}{\text{lb}}\right)}{2\frac{\text{lb}}{\text{s}}\left(19.65\frac{\text{Btu}}{\text{lb}}\right)} = 0.0884 = 8.84\% \longleftarrow$$

---

1. This corresponds to $\dot{E}_d = \dot{\sigma}T_o$.
2. In accord with the discussion of Eq. 7.27, both streams are at temperatures above T_0.
3. Nearly all of the decrease in flow exergy associated with the hot stream resulted in exergy destruction and therefore the exergetic efficiency is very low. The engineers operating this system should investigate the significant heat exchanger exergy destruction and, if warranted, take corrective steps.

PROBLEM 7.124

7.124 A counterflow flow heat exchanger operating at steady state has oil and liquid water flowing in separate streams. The oil is cooled from 700 to 580°R while the water temperature increases from 530 to 560°R. Neither stream experiences a significant pressure change. The mass flow rate of the water is 3 lb/s. The oil and water can be regarded as incompressible with constant specific heats of 0.51 and 1.00 Btu/(lb·°R), respectively. Heat transfer between the heat exchanger and its surroundings can be ignored, as can the effects of motion and gravity. Determine

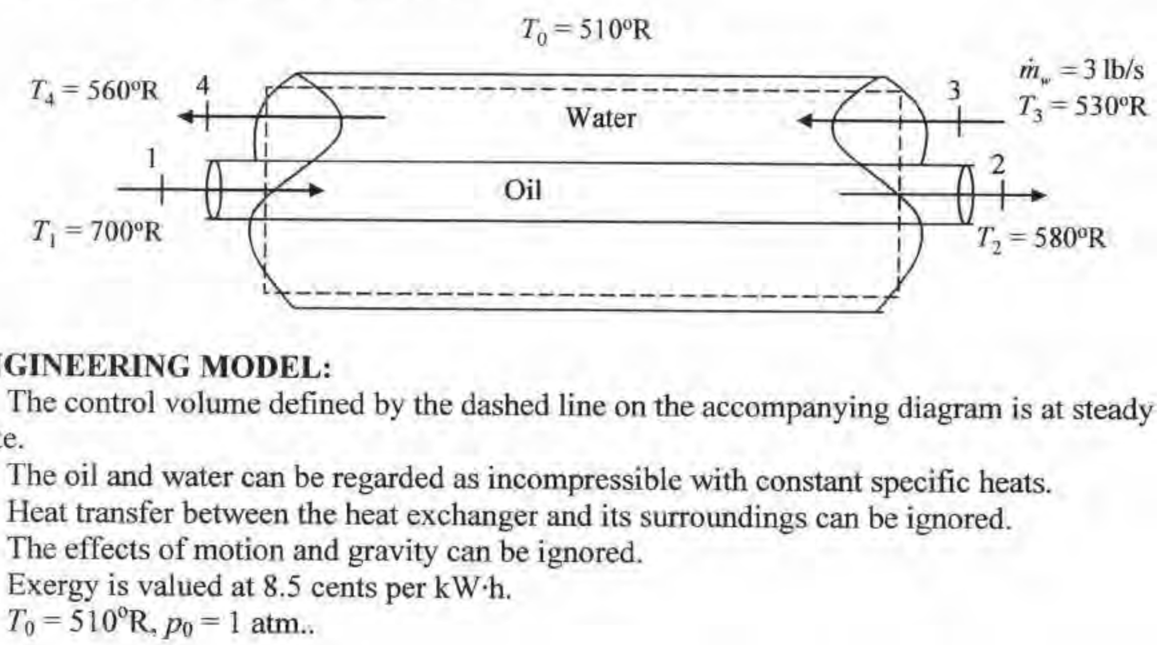
(a) the mass flow rate of the oil, in lb/s.
(b) the exergetic efficiency given by Eq. 7.27.
(c) the hourly cost of exergy destruction if exergy is valued at 8.5 cents per kW·h.

Let $T_0 = 50°F$, $p_0 = 1$ atm.

KNOWN: Water with known mass flow rate and oil flow through a counterflow heat exchanger with known inlet and exit temperatures.

FIND: The mass flow rate of the oil, the exergetic efficiency, and the hourly cost of exergy destruction.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The control volume defined by the dashed line on the accompanying diagram is at steady state.
2. The oil and water can be regarded as incompressible with constant specific heats.
3. Heat transfer between the heat exchanger and its surroundings can be ignored.
4. The effects of motion and gravity can be ignored.
5. Exergy is valued at 8.5 cents per kW·h.
6. $T_0 = 510°R$, $p_0 = 1$ atm..

ANALYSIS:
(a) The mass flow rate of oil can be determined from the steady-state energy balance

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i\,(h_i + ½\,V_i^2 + gz_i) - \sum_e \dot{m}_e\,(h_e + ½\,V_e^2 + gz_e)$$

Setting rate of heat transfer and power to zero, ignoring kinetic and potential energy effects, and writing the energy balance for the oil stream and the water stream give

$$0 = \dot{m}_o(h_1 - h_2) + \dot{m}_w(h_3 - h_4)$$

Since specific heats and pressures are constant, the changes in enthalpy are found by reducing Eq. 3.20b, giving

$$0 = \dot{m}_o c_o(T_1 - T_2) + \dot{m}_w c_w(T_3 - T_4)$$

Solving for the mass flow rate of oil gives

$$\dot{m}_o = \frac{\dot{m}_w c_w (T_4 - T_3)}{c_o(T_1 - T_2)} = \frac{\left(3\frac{\text{lb}}{\text{s}}\right)\left(1.00\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)(560^\circ\text{R} - 530^\circ\text{R})}{\left(0.51\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)(700^\circ\text{R} - 580^\circ\text{R})} = \underline{\mathbf{1.47\ lb/s}} \longleftarrow$$

(b) The exergetic efficiency can be determined by Eq. 7.27 with the water as the cold stream and the oil as the hot stream.

$$\varepsilon = \frac{\dot{m}_w(\mathsf{e}_{f4} - \mathsf{e}_{f3})}{\dot{m}_o(\mathsf{e}_{f1} - \mathsf{e}_{f2})}$$

The change in specific flow exergy is defined by Eq. 7.18 (ignoring effects of motion and gravity). Applying Eq. 7.18 to the water stream

$$\mathsf{e}_{f4} - \mathsf{e}_{f3} = (h_4 - h_3) - T_0(s_4 - s_3)$$

For an incompressible substance with constant specific heat c, the change in entropy can be determined by Eq. 6.13. Substituting for constant specific heats gives

$$\mathsf{e}_{f4} - \mathsf{e}_{f3} = c(T_4 - T_3) - T_0(c \ln \frac{T_4}{T_3})$$

Applying similar analysis to the oil stream, the exergetic efficiency can be rewritten as

$$\varepsilon = \frac{\dot{m}_w c_w \left(T_4 - T_3 - T_0 \ln \frac{T_4}{T_3}\right)}{\dot{m}_o c_o \left(T_1 - T_2 - T_0 \ln \frac{T_1}{T_2}\right)}$$

Substituting values and solving for exergetic efficiency give

$$\varepsilon = \frac{\left(3\frac{\text{lb}}{\text{s}}\right)\left(1.00\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)\left(560°\text{R} - 530°\text{R} - (510°\text{R})\ln\frac{560°\text{R}}{530°\text{R}}\right)}{\left(1.47\frac{\text{lb}}{\text{s}}\right)\left(0.51\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)\left(700°\text{R} - 580°\text{R} - (510°\text{R})\ln\frac{700°\text{R}}{580°\text{R}}\right)} = \frac{\left(5.759\frac{\text{Btu}}{\text{s}}\right)}{\left(18.063\frac{\text{Btu}}{\text{s}}\right)} = \underline{\mathbf{0.3188}}$$

(c) The exergy destruction rate is required to determine the hourly cost of exergy destruction. Exergy destruction rate is determined by reducing the exergy rate balance, Eq. 7.13a, to get.

$$\dot{E}_d = \dot{m}_o(e_{f1} - e_{f2}) - \dot{m}_w(e_{f4} - e_{f3})$$

Substituting values gives

$$\dot{E}_d = \left(18.063\frac{\text{Btu}}{\text{s}}\right) - \left(5.759\frac{\text{Btu}}{\text{s}}\right) = 12.304\ \text{Btu/s}$$

The economic cost of exergy destruction per hour is

$$Cost = \left(12.304\frac{\text{Btu}}{\text{s}}\right)\left(8.5\frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)\left(\frac{1.0551\ \text{kJ}}{\text{Btu}}\right)\left(\frac{\text{kW}}{\frac{\text{kJ}}{\text{s}}}\right)\left(\frac{\$}{100\ \text{cents}}\right) = \underline{\mathbf{\$1.10/hour}}$$

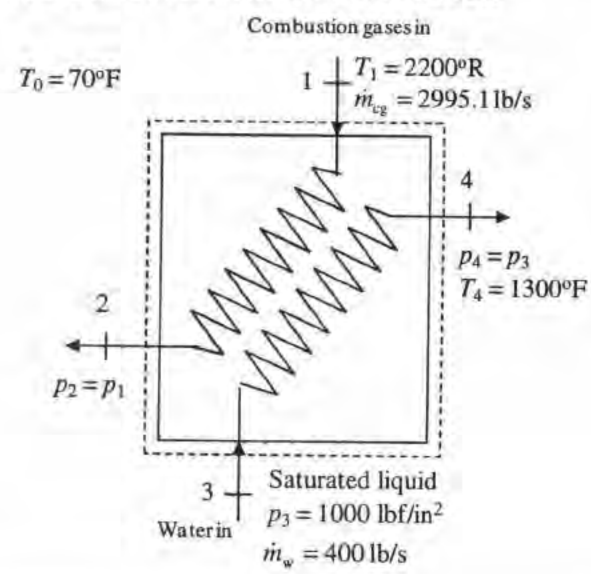
PROBLEM 7.125

7.125 In the boiler of a power plant are tubes through which water flows as it is brought from 0.6 MPa, 130°C to 200°C at essentially constant pressure. Combustion gases with a mass flow rate of 400 kg/s pass over the tubes and cool from 827°C to 327°C at essentially constant pressure. The combustion gases can be modeled as air as an ideal gas. There is no significant heat transfer from the boiler to its surroundings. Assuming steady state and neglecting the effects of motion and gravity, determine
(a) the mass flow rate of the water, in kg/s.
(b) the rate of exergy destruction, in kJ/s.
(c) the exergetic efficiency given by Eq. 7.27.
Let T_0= 25°C, p_0=1 atm.

KNOWN: Steady-state operating data are provided for boiler tubes carrying water over which combustion gases flow.

FIND: Determine (a) the mass flow rate of the water, (b) the rate of exergy destruction, and (c) the exergetic efficiency given by Eq. 7.27.

SCHEMATIC AND GIVEN DATA:



State	T	p (MPa)	h (kJ/kg)	s^o (kJ/kg-K)	s (kJ/kg-K)
1	1100K	p_1=p_2	1161.07 (Tab A-22)	3.07732	-
2	600K	p_1=p_2	607.02 (Tab A-22)	2.40902	-
3	130°C	0.6	546.31 (h_3=$h_f(T_3)$ Tab A-2)	-	1.6344 (s_3=$s_f(T_3)$)
4	200°C	0.6	2850.1 (Tab A-4)	-	6.9729

ENGINEERING MODEL:
(1) The control volume shown in the schematic is at steady state.
(2) For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$, and the effects of motion and gravity can be ignored.

PROBLEM 7.125 (Continued)

(3) The combustion gases are modeled as air as an ideal gas.
(4) There is no significant change in pressure for either stream.
(5) The exergy reference environment is T_0 = 298K, p_0=1 atm.

ANALYSIS:

(a) Mass and energy balances reduce to

$$0 = \dot{m}_{cg}(h_1 - h_2) + \dot{m}_w(h_3 - h_4)$$

Rearrange and solve using enthalpy data listed in the accompanying table:

$$\dot{m}_w = \frac{\dot{m}_{cg}(h_2 - h_1)}{(h_3 - h_4)} = \frac{400\frac{\text{kg}}{\text{s}}(607.02 - 1161.07)\frac{\text{kJ}}{\text{kg}}}{(546.31 - 2850.1)\frac{\text{kJ}}{\text{kg}}} = 96.2\frac{\text{kg}}{\text{s}}$$ ←

(b) Applying the steady-state control volume exergy rate balance, Eq. 7.13a, simplifying based on assumptions, and rearranging:

(#1) $$\dot{\text{E}}_d = \dot{m}_{cg}(\text{e}_{f1} - \text{e}_{f2}) + \dot{m}_w(\text{e}_{f3} - \text{e}_{f4}) \quad (1)$$

Solving for the combustion gas change in flow exergy term using Eq. 7.18 and data from Table A-22:

$$(\text{e}_{f1} - \text{e}_{f2}) = h_1 - h_2 - T_0\left(s_1^o - s_2^o - \frac{\overline{R}}{M}\ln\frac{P_1}{P_2}\right)$$

$$= (1161.07 - 607.02)\frac{\text{kJ}}{\text{kg}} - 298\text{ K}(3.07732 - 2.40902 - 0)\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 354.9\frac{\text{kJ}}{\text{kg}}$$

Repeating for water using data from Table A-2 and A-4:

$$(\text{e}_{f3} - \text{e}_{f4}) = h_3 - h_4 - T_0(s_3 - s_4)$$

$$= \left[(546.31 - 2850.1)\frac{\text{kJ}}{\text{kg}} - 298\text{K}(1.6344 - 6.9729)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right] = -712.9\frac{\text{kJ}}{\text{kg}}$$

Substituting values into Eq. (1)

$$\dot{\text{E}}_d = \left[400\frac{\text{kg}}{\text{s}}\left(354.9\frac{\text{kJ}}{\text{kg}}\right) + 96.2\frac{\text{kg}}{\text{s}}\left(-712.9\frac{\text{kJ}}{\text{kg}}\right)\right]\left|\frac{1\text{ kW}}{1\frac{\text{kJ}}{\text{s}}}\right| = 73379\text{ kW}$$ ←

(c) The exergetic efficiency given by Eq. 7.27

(#2) $$\varepsilon = \frac{\dot{m}_w(\text{e}_{f4} - \text{e}_{f3})}{\dot{m}_{cg}(\text{e}_{f1} - \text{e}_{f2})} = \frac{96.2\frac{\text{kg}}{\text{s}}\left(712.9\frac{\text{kJ}}{\text{kg}}\right)}{400\frac{\text{kg}}{\text{s}}\left(354.9\frac{\text{kJ}}{\text{kg}}\right)} = 0.483 = 48.3\%$$ ←

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1. This corresponds to $\dot{\text{E}}_d = \dot{\sigma}T_o$.
2. In accord with the discussion of Eq. 7.27, both streams are at temperatures above T_0.

PROBLEM 7.126

7.126 In the boiler of a power plant are tubes through which water flows as it is brought from a saturated liquid condition at 1000 lbf/in.^{2} to 1300°F at essentially constant pressure. Combustion gases passing over the tubes cool from 1740°F to temperature T at essentially constant pressure. The mass flow rates of the water and combustion gases are 400 lb/s and 2995.1 lb/s, respectively. The combustion gases can be modeled as air behaving as an ideal gas. There is no significant heat transfer from the boiler to its surroundings. Assuming steady state and neglecting the effects of motion and gravity, determine

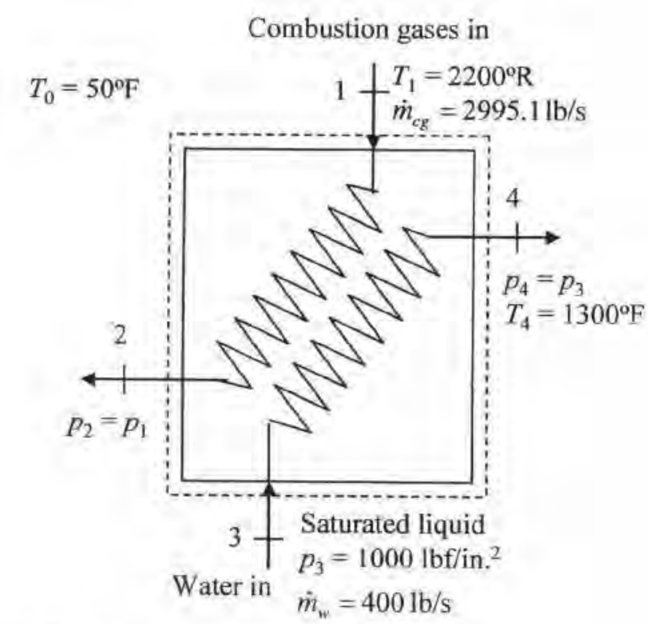
(a) the exit temperature of combustion gases, in °F.
(b) the exergy destruction rate, in Btu/s.
(c) the exergetic efficiency given by Eq. 7.27.

Let T_0 = 50°F, p_0 = 1 atm.

KNOWN: Water with known mass flow rate and inlet and exit states and combustion gases with known mass flow rate and inlet temperature flow through a boiler.

FIND: The exit temperature of combustion gases, the exergy destruction rate, and the exergetic efficiency.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

1. The control volume defined by the dashed line on the accompanying diagram is at steady state.
2. The combustion gases can be modeled as air as an ideal gas.
3. The combustion gas stream and the water stream each flow at constant pressure.
4. Heat transfer between the control volume and its surroundings can be ignored.
5. The effects of motion and gravity can be ignored.
6. T_0 = 510°R.

ANALYSIS:

(a) The exit temperature of combustion gases can be determined from the steady-state energy balance

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i\,(h_i + \tfrac{1}{2}\mathrm{V}_i^2 + gz_i) - \sum_e \dot{m}_e\,(h_e + \tfrac{1}{2}\mathrm{V}_e^2 + gz_e)$$

Setting rate of heat transfer and power to zero, ignoring kinetic and potential energy effects, and writing the energy balance for the combustion gases stream and the water stream give

$$0 = \dot{m}_{cg}\,(h_1 - h_2) + \dot{m}_w\,(h_3 - h_4)$$

Solving for the combustion gas exit enthalpy, a function of temperature only, gives

$$h_2 = h_1 - \frac{\dot{m}_w\left(h_4 - h_3\right)}{\dot{m}_{cg}}$$

From Table A-22E at $T_1 = 2200^{\circ}\mathrm{R}$, $h_1 = 560.59$ Btu/lb and $s^{\circ}(T_1) = 0.95868$ Btu(lb·°R).
State 3 is saturated liquid. From Table A-3E, $h_3 = h_{f3} = 542.4$ Btu/lb,
$s_3 = s_{f3} = 0.7432$ Btu/(lb·°R).
State 4 is superheated vapor. From Table A-4E, $h_4 = 1676.5$ Btu/lb, $s_4 = 1.7593$ Btu/(lb·°R).
Substituting values gives

$$h_2 = 560.59\frac{\mathrm{Btu}}{\mathrm{lb}} - \frac{\left(400\dfrac{\mathrm{lb}}{\mathrm{s}}\right)\left(1676.5\dfrac{\mathrm{Btu}}{\mathrm{lb}} - 542.4\dfrac{\mathrm{Btu}}{\mathrm{lb}}\right)}{\left(2995.1\dfrac{\mathrm{lb}}{\mathrm{s}}\right)} = 409.13\ \mathrm{Btu/lb}$$

From Table A-22E, the exit temperature of the combustion gases is $T_2 = 1650^{\circ}\mathrm{R}$ = **<u>1190 °F</u>** and $s^{\circ}(T_2) = 0.87954$ Btu(lb·°R).

(b) Exergy destruction rate is determined by reducing the exergy rate balance, Eq. 7.13a, to get.

$$\dot{\mathsf{E}}_{\mathrm{d}} = \dot{m}_{cg}\,(\mathsf{e}_{f1} - \mathsf{e}_{f2}) - \dot{m}_w\,(\mathsf{e}_{f4} - \mathsf{e}_{f3})$$

The change in specific flow exergy is defined by Eq. 7.18. Ignoring effects of motion and gravity, the change in specific flow exergy for the water stream is

$$\mathsf{e}_{f4} - \mathsf{e}_{f3} = (h_4 - h_3) - T_0(s_4 - s_3)$$

Substituting values gives

$$\mathsf{e}_{f3} - \mathsf{e}_{f4} = (1676.5\ \mathrm{Btu/lb} - 542.4\ \mathrm{Btu/lb}) - (510^{\circ}\mathrm{R})[1.7593\ \mathrm{Btu/(lb\cdot^{\circ}R)} - 0.7432\ \mathrm{Btu/(lb\cdot^{\circ}R)}]$$

$$\mathsf{e}_{f4} - \mathsf{e}_{f3} = 615.889\ \mathrm{Btu/lb}$$

PROBLEM 7.126 (Continued, p.3)

For the combustion gas stream

$$e_{f1} - e_{f2} = (h_1 - h_2) - T_0(s_1 - s_2)$$

Since the combustion gases are modeled as air as an ideal gas, the change in entropy is expressed as

$$s_1 - s_2 = s(T_1, p_1) - s(T_2, p_2) = [s^o(T_1) - s^o(T_2) - R \ln \frac{p_1}{p_2}]$$

Since pressure is constant, the last term on the right-hand side is zero. The change in specific flow exergy for the combustion gases reduces to

$$e_{f1} - e_{f2} = (h_1 - h_2) - T_0[s^o(T_1) - s^o(T_2)]$$

Substituting values gives

$$e_{f1} - e_{f2} = (560.59 \text{ Btu/lb} - 409.13 \text{ Btu/lb}) - (510^\circ\text{R})[0.95868 \text{ Btu/(lb}\cdot{}^\circ\text{R}) - 0.87954 \text{ Btu/(lb}\cdot{}^\circ\text{R})]$$

$$e_{f1} - e_{f2} = 111.099 \text{ Btu/lb}$$

Substituting values for the exergy destruction rate gives

$$\dot{E}_d = (2995.1 \text{ lb/s})(111.099 \text{ Btu/lb}) - (400 \text{ lb/s})(615.889 \text{ Btu/lb}) = \underline{\mathbf{86{,}397 \text{ Btu/s}}}$$ ←

(c) The exergetic efficiency can be determined by Eq. 7.27 with the water as the cold stream and the combustion gases as the hot stream.

1 $$\varepsilon = \frac{\dot{m}_w(e_{f4} - e_{f3})}{\dot{m}_{cg}(e_{f1} - e_{f2})}$$

Substituting values gives

$$\varepsilon = \frac{\left(400 \frac{\text{lb}}{\text{s}}\right)\left(615.889 \frac{\text{Btu}}{\text{lb}}\right)}{\left(2995.1 \frac{\text{lb}}{\text{s}}\right)\left(111.099 \frac{\text{Btu}}{\text{lb}}\right)} = \underline{\mathbf{0.7404}}$$ ←

1. In accord with the discussion of Eq. 7.27, both streams are at temperatures above T_0.

Problem 7.127

Refrigerant 134a enters a counterflow heat exchanger operating at steady state at -32°C and a quality of 40% and exits as saturated vapor at -32°C. Air enters as a separate stream with a mass flow rate of 5 kg/s and is cooled at a constant pressure of 1 bar from 300 to 250 K. Heat transfer between the heat exchanger and its surroundings can be ignored, as can the effects of motion and gravity.

(a) As in Fig. E7.6, sketch the variation with position of the temperature of each stream. Locate T_0 on the sketch.

(b) Determine the rate of exergy destruction within the heat exchanger, in kW.

(c) Devise and evaluate an exergetic efficiency for the heat exchanger.

Let T_0 = 300 K, p_0 = 1 bar.

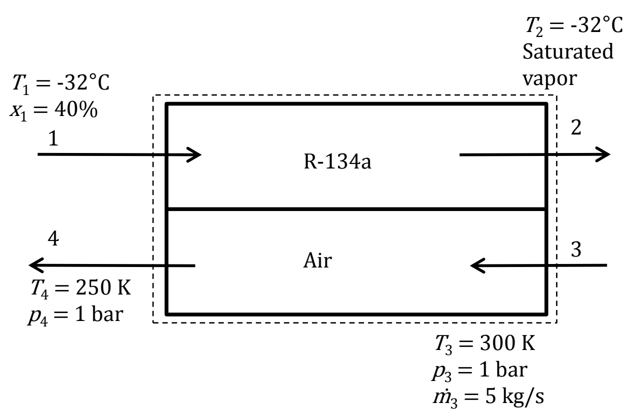
Known:

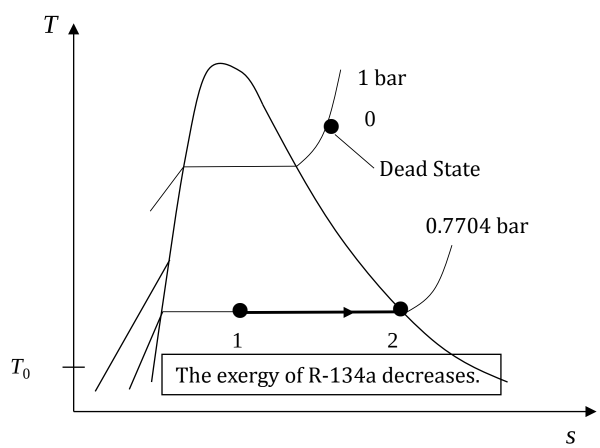
Steady-state operating data are provided for a counterflow heat exchanger.

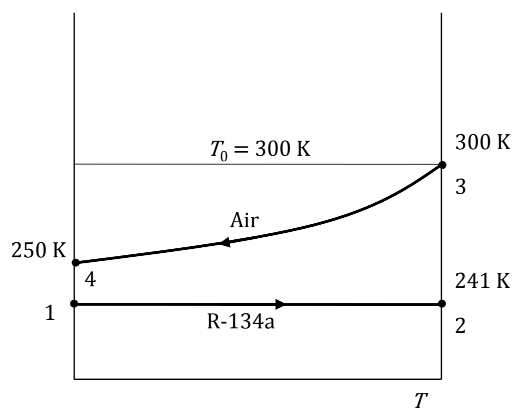
Find:

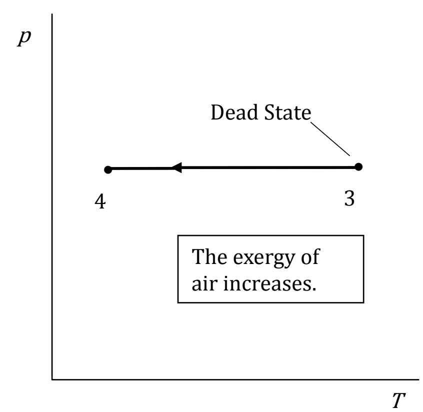
(a) Sketch the variation of the temperature of each stream with position. Locate T_0. (b) Determine the rate of exergy destruction. (c) Devise and evaluate exergetic efficiency.

Schematic and Known Data:









Engineering Model:

(1) The control volume shown in the schematic is at steady state.
(2) For the control volume, $\dot{Q}_{\text{cv}} = 0$ and kinetic and potential energy effects are negligible.
(3) Air is modeled as an ideal gas.
(4) For the environment, T_0 = 300 K, p_0 = 1 atm.

Analysis:

The refrigerant mass flow rate can be found by reducing mass and energy rate balances:

$$0 = \underbrace{\dot{Q}_{\text{cv}}}_{=0} - \underbrace{\dot{W}_{\text{cv}}}_{=0} + \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_4)$$

With data from Tables A-10 and A-22:

$$\dot{m}_1 = \dot{m}_3 \frac{h_3 - h_4}{h_2 - h_1} = \left(5\,\frac{\text{kg}}{\text{s}}\right)\frac{300.19 - 250.05}{227.90 - \underbrace{96.868}_{=9.52+0.4(218.37)}} = 1.913\,\frac{\text{kg}}{\text{s}}$$

(a) The solution to part (a) can be seen in the above provided schematics.

(b) The rate of exergy destruction can be determined by reducing an exergy rate balance:

$$0 = \sum\left[1 - \frac{T_0}{T_j}\right]\underbrace{\dot{Q}_j}_{=0} - \underbrace{\dot{W}_{cv}}_{=0} + \dot{m}_1(\mathrm{e}_{\mathrm{f}_1} - \mathrm{e}_{\mathrm{f}_2}) + \dot{m}_3(\mathrm{e}_{\mathrm{f}_3} - \mathrm{e}_{\mathrm{f}_4}) - \dot{\mathrm{E}}_{\mathrm{d}}, \text{ rearranging}$$

$$\dot{\mathrm{E}}_{\mathrm{d}} = \dot{m}_1[h_1 - h_2 - T_0(s_1 - s_2)] + \dot{m}_3[h_3 - h_4 - T_0(s_3 - s_4)]$$

$$= \dot{m}_1[h_1 - h_2 - T_0(s_1 - s_2)] + \dot{m}_3\left[h_3 - h_4 - T_0\left(s_3^o - s_4^o - R\ln\underbrace{\frac{p_3}{p_4}}_{=0}\right)\right]$$

$$= \left(1.913\,\frac{\text{kg}}{\text{s}}\right)\left[(96.868 - 227.90)\,\frac{\text{kJ}}{\text{kg}} - (300\text{ K})(0.4023 - 0.9456)\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]$$

$$+ \left(5\,\frac{\text{kg}}{\text{s}}\right)\left[(300.19 - 250.05)\,\frac{\text{kJ}}{\text{kg}} - (300\text{ K})(1.70203 - 1.51917)\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]$$

$$= \left(1.913\,\frac{\text{kg}}{\text{s}}\right)\left[31.96\,\frac{\text{kJ}}{\text{kg}}\right] + \left(5\,\frac{\text{kg}}{\text{s}}\right)\left[-4.72\,\frac{\text{kJ}}{\text{kg}}\right] = 37.5\text{ kW}$$

(c) Since the air is brought at constant pressure from the dead state to a lower temperature, its exergy increases in the process. The state of the R-134a is brought closer to the dead state, so its exergy decreases. These points are confirmed by the calculation of part (b) that show;

$$\dot{m}_3(\mathrm{e}_{\mathrm{f}_4} - \mathrm{e}_{\mathrm{f}_3}) = 23.6\text{ kW}, \quad \dot{m}_1(\mathrm{e}_{\mathrm{f}_1} - \mathrm{e}_{\mathrm{f}_2}) = 61.14\text{ kW}$$

Thus, the colder refrigerant provides the exergy that is either transferred to the warmer air
#1 or is destroyed by irreversibilities within the control volume.

$$\epsilon = \frac{\dot{m}_3(\mathrm{e}_{\mathrm{f}_4} - \mathrm{e}_{\mathrm{f}_3})}{\dot{m}_1(\mathrm{e}_{\mathrm{f}_1} - \mathrm{e}_{\mathrm{f}_2})} = \frac{23.6}{61.14} = 0.386 = 38.6\%$$

Comment:

1. Since the streams are each below T_0, this expression is formulated differently than Eq. 7.27, which regards the cold stream as receiving exergy from the hot stream. When heat transfer occurs below T_0, as in the present case, the accompanying exergy transfer is oppositely directed. This can be seen from study of Eq. 7.15. In the present case, heat transfer occurs from the warmer air to the cooler refrigerant. Still, exergy transfer is from the refrigerant to the air. Further, the exergy transferred from the refrigerant is accounted for by the exergy increase of the air and by the exergy destroyed within the heat exchanger owing to spontaneous heat transfer.

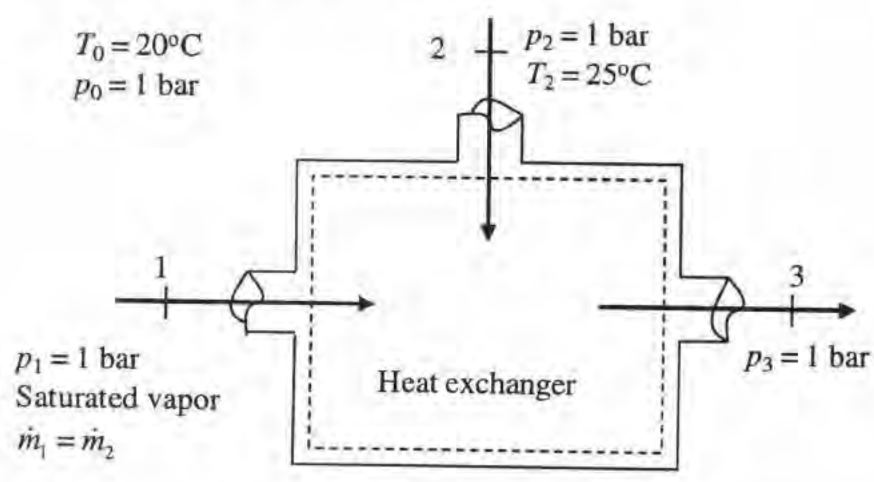
PROBLEM 7.128

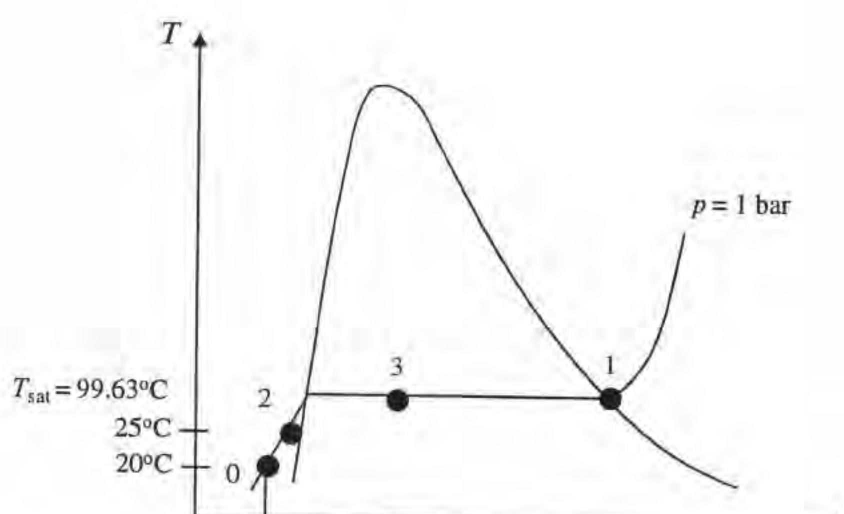
7.128 Saturated water vapor at 1 bar enters a direct-contact heat exchanger operating at steady state and mixes with a stream of liquid water entering at 25°C, 1 bar. A two-phase liquid-vapor mixture exits at 1 bar. The entering streams have equal mass flow rates. Neglecting heat transfer with the surroundings and effects of motion and gravity, determine for the heat exchanger
(a) the rate of exergy destruction, in kJ per kg of mixture exiting.
(b) the exergetic efficiency given by Eq. 7.29.
Let T_0= 20°C, p_0=1 bar.

KNOWN: Steady-state operating data are provided for a direct-contact heat exchanger.

FIND: Determine (a)the rate of exergy destruction per unit mass exiting, and (b) the exergetic efficiency given by Eq. 7.29.

SCHEMATIC AND GIVEN DATA:





State	T (°C)	p (bar)	h (kJ/kg)	s (kJ/kg-K)
1	-	1	2675.5 (h_1=$h_g(p_1)$ Tab A-3)	7.3594 (s_1=$s_g(p_1)$ Tab A-3)
2	25	1	104.89 (h_2=$h_f(T_2)$ Tab A-2)	0.3674 (s_2=$s_f(T_2)$ Tab A-2)
3		1	See solution	See solution

ENGINEERING MODEL:

(1) The control volume shown in the schematic is at steady state.

(2) For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$, and the effects of motion and gravity can be ignored.

(3) The exergy reference environment is $T_0 = 20$°C, p_0=1 bar.

ANALYSIS:

(a) Reducing the energy rate balance based on assumptions:

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$$

where the mass balance reduces to

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

Then with $\dot{m} = \dot{m}_1 = \dot{m}_2$

$$\dot{m} = \frac{\dot{m}_3}{2}$$

Substituting the above into Eq. (1) and rearranging yields:

$$0 = \dot{m}(h_1 + h_2) - \dot{m}_3(h_3) = \dot{m}_3\left[\frac{(h_1 + h_2)}{2} - h_3\right]$$

Rearrange and substitute data values listed in the accompanying table:

$$h_3 = \frac{(h_1 + h_2)}{2} = \frac{(2675.5 + 104.89)}{2} = 1390.195\frac{\text{kJ}}{\text{kg}}$$

Since $h_f < h_3 < h_g$ at p_3, state 3 is in the saturated mixture region and x_3 follows:

$$x_3 = \frac{(h_3 - h_f)}{h_{fg}} = \frac{(1390.195 - 417.46)}{2258.0} = 0.431 = 43.1\%$$

The corresponding entropy, s_3 is

$$s_3 = s_f + x_3(s_g - s_f) = 1.3026 + 0.431(7.3594 - 1.3026) = 3.9131\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

The rate of exergy destruction can be obtained by using an exergy rate balance or using $\dot{\text{E}}_d = \dot{\sigma}T_o$ where $\dot{\sigma}$ is the rate of entropy production using an entropy balance. Using the entropy approach simplified based on assumptions

$$\dot{\sigma} = \dot{m}_3 s_3 - \dot{m}(s_1 + s_2) = \dot{m}_3 s_3 - \frac{\dot{m}_3}{2}(s_1 + s_2) = \dot{m}_3\left(s_3 - \frac{(s_1 + s_2)}{2}\right)$$

$$\frac{\dot{\sigma}}{\dot{m}_3} = s_3 - \frac{(s_1 + s_2)}{2} = 3.9131 - \frac{(7.3594 + 0.3674)}{2} = 0.0497 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

The exergy destruction rate per unit mass exiting can be determined, as follows:

$$\frac{\dot{\mathbf{E}}_d}{\dot{m}_3} = \frac{\dot{\sigma}}{\dot{m}_3} T_0 = \left(0.0497 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right) 293\text{K} = 14.562 \frac{\text{kJ}}{\text{kg}}$$ ←

(a) The exergetic efficiency given by Eq. 7.29

$$\varepsilon = \frac{\dot{m}(e_{f3} - e_{f2})}{\dot{m}(e_{f1} - e_{f3})} = \frac{h_3 - h_2 - T_0(s_3 - s_2)}{h_1 - h_3 - T_0(s_1 - s_3)}$$

$$= \frac{(1390.195 - 104.89)\frac{\text{kJ}}{\text{kg}} - 293\,\text{K}(3.9131 - 0.3674)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}}{(2675.5 - 1390.195)\frac{\text{kJ}}{\text{kg}} - 293\,\text{K}(7.3594 - 3.9131)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}} = 0.894 = 89.4\%$$ ←

PROBLEM 7.129

7.129 Figure P7.129 and the accompanying table provide steady-state operating data for a direct-contact heat exchanger fitted with a valve. Water is the substance. The mass flow rate of the exiting stream is 20 lb/s. Stray heat transfer and the effects of motion and gravity are negligible. For an overall control volume, (a) evaluate the rate of exergy destruction, in Btu/s, and (b) devise and evaluate an exergetic efficiency. Let $T_0 = 60°F$, $p_0 = 14.7$ lbf/in.^{2}

State	T(°F)	p(lbf/in.^{2})	h(Btu/lb)	s(Btu/lb · R)
1	60	14.7	28.1	0.0556
2	500	20.0	1286.8	1.8919
3	320	14.7	1202.1	1.8274

SCHEMATIC & GIVEN DATA:

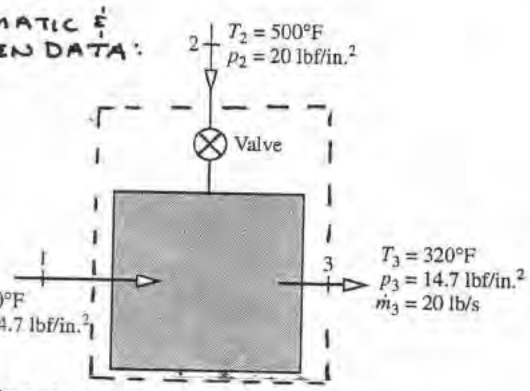


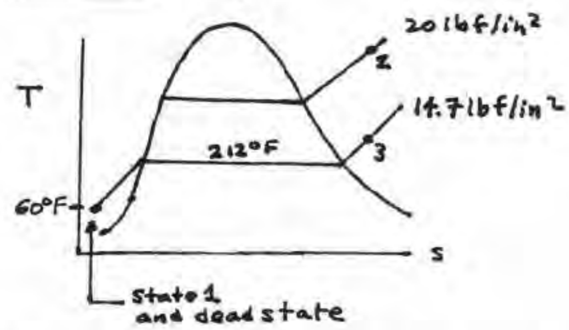
Fig. P7.129

KNOWN: Steady-state data are provided for a direct-contact heat exchanger fitted with a valve.

FIND: For an overall control volume, evaluate $\dot{E}_d$ and ϵ.

ENGINEERING MODEL:

1. The control volume shown in the figure is at steady state.
2. For the control volume, stray heat transfer and the effects of motion and gravity are ignored.
3. At 1, $h_1 \approx h_f(T_1)$, $s_1 \approx s_f(T_1)$
4. $T_0 = 60°F$, $p_0 = 14.7$ lbf/in^2.



ANALYSIS: To find $\dot{m}_1$ and $\dot{m}_2$, apply mass and energy rate balances to obtain, $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$, and

$$\dot{m}_2 = \dot{m}_3\left[\frac{h_3 - h_1}{h_2 - h_1}\right] = 20\frac{lb}{s}\left[\frac{1202.1 - 28.1}{1286.8 - 28.1}\right] = 18.654\ \frac{lb}{s}$$

Then, $\dot{m}_1 = \dot{m}_3 - \dot{m}_2 = 1.346$ lb/s.

(a) $\dot{E}_d = T_0\dot{\sigma}$, where $\dot{\sigma}$ is obtained from an entropy rate balance:

$$\dot{\sigma} = \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2 = \left(20\frac{lb}{s}\right)\left(1.8274\ \frac{Btu}{lb \cdot °R}\right) - (1.346)(0.0556) - (18.654)(1.8919)$$

$$\therefore\ \dot{\sigma} = 1.181\ \frac{Btu/s}{°R}$$

$$\Rightarrow\ \dot{E}_d = T_0\dot{\sigma} = (520°R)\left(1.181\ \frac{Btu/s}{°R}\right) = 614.1\ Btu/s$$

(b) An exergy rate balance reduces as follows: $0 = \sum\left[1 - \frac{T_0}{T_j}\right]\cancel{\dot{Q}_j} - \cancel{\dot{W}} + \dot{m}_1 e_{f1} + \dot{m}_2 e_{f2} - \dot{m}_3 e_{f3} - \dot{E}_d$

$$\therefore\ 0 = \dot{m}_1 e_{f1} + \dot{m}_2 e_{f2} - \underbrace{\dot{m}_3}_{=(\dot{m}_1 + \dot{m}_2)} e_{f3} - \dot{E}_d$$

$$\therefore\ 0 = \dot{m}_1 e_{f1} + \dot{m}_2 e_{f2} - (\dot{m}_1 + \dot{m}_2) e_{f3} - \dot{E}_d \Rightarrow 0 = \dot{m}_1(e_{f1} - e_{f3}) + \dot{m}_2(e_{f2} - e_{f3}) - \dot{E}_d$$

$$\Rightarrow\ \dot{m}_2(e_{f2} - e_{f3}) = \dot{m}_1(e_{f3} - e_{f1}) + \dot{E}_d$$

The exergy decrease of the hot stream provides for the exergy increase of the cold stream and the exergy destroyed

$$\Rightarrow\ \epsilon = \frac{\dot{m}_1(e_{f3} - e_{f1})}{\dot{m}_2(e_{f2} - e_{f3})}$$

$$\dot{m}_1(e_{f3} - e_{f1}) = \dot{m}_1\left[(h_3 - h_1) - T_0(s_3 - s_1)\right] = 1.346\frac{lb}{s}\left[(1202.1 - 28.1) - 520(1.8274 - 0.0556)\right]\frac{Btu}{lb}$$
$$= 340.1\ Btu/s$$

$$\dot{m}_2(e_{f2} - e_{f3}) = \left(18.654\frac{lb}{s}\right)\left((1286.8 - 1202.1) - 520(1.8919 - 1.8274)\right)\frac{Btu}{lb} = 954.3\ Btu/s$$

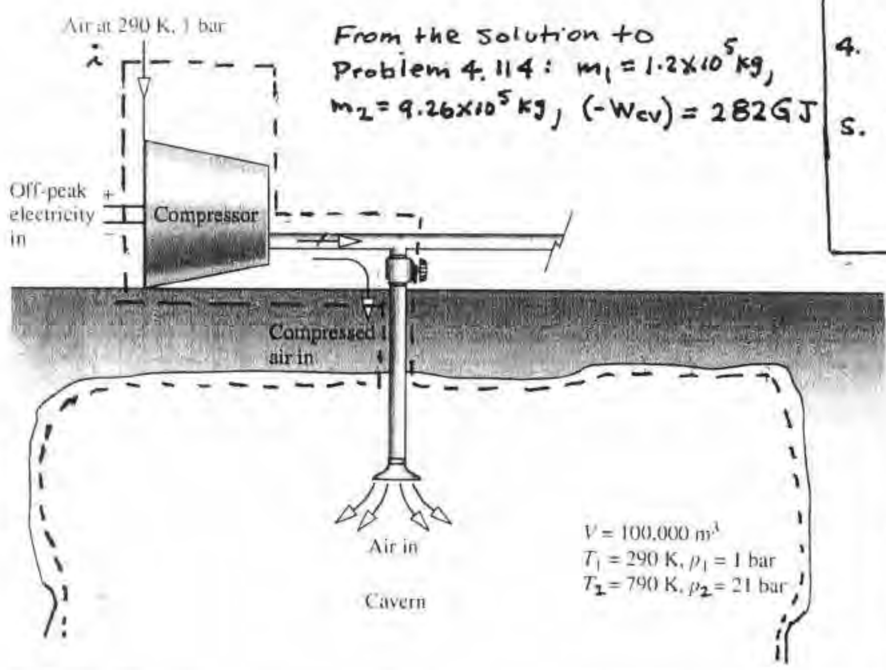
① Then, $$\epsilon = \frac{340.1\ Btu/s}{954.3\ Btu/s} = 0.356\ (35.6\%)$$

1. Observe that $\dot{E}_d = \dot{m}_2(e_{f2} - e_{f3}) - \dot{m}_1(e_{f3} - e_{f1}) = (954.3 - 340.1)$ Btu/s $= 614.2$ Btu/s, which agrees with the value obtained in part (a) to within round-off.

PROBLEM 7.130

7.130 For the compressed-air energy storage system of Problem 4.114, determine the amount of exergy destruction associated with filling the cavern, in GJ. Devise and evaluate the accompanying exergetic efficiency. Comment. Let $T_0 = 290$ K, $p_0 = 1$ bar.

SCHEMATIC & GIVEN DATA:



From the solution to Problem 4.114: $m_1 = 1.2\times10^5$ kg, $m_2 = 9.26\times10^5$ kg, $(-W_{cv}) = 282$ GJ

ENGINEERING MODEL:

1. The control volume is defined by the dashed line.
2. At inlet i, the state remains constant at 290K, 1bar.
3. For the control volume, $\dot{Q}_{cv}=0$ and the effects of motion and gravity are negligible.
4. The air is modeled as an ideal gas.
5. $T_0 = 290$K, $P_0 = 1$ bar.

ANALYSIS:

Mass rate balance,

$$\frac{dm_{cv}}{dt} = \dot{m}_i \qquad (1)$$

Exergy rate balance,

$$\frac{d\mathsf{E}_{cv}}{dt} = \sum_j\left(1-\frac{T_0}{T_j}\right)\cancel{\dot{Q}_j}^{0} - \dot{W}_{cv} + \dot{m}_i e_{fi} - \dot{\mathsf{E}}_d \qquad (2)$$

Inserting Eq. (1) into Eq. (2) and integrating with respect to time

$$\frac{d\mathsf{E}_{cv}}{dt} = (-\dot{W}_{cv}) + e_{fi}\frac{dm_{cv}}{dt} - \dot{\mathsf{E}}_d \quad\Rightarrow\quad \underbrace{\Delta \mathsf{E}_{cv}}_{(m_2e_2 - m_1e_1)} = (-W_{cv}) + e_{fi}(m_2-m_1) - \mathsf{E}_d \qquad (3)$$

Remains constant since the state at i does not change

$(m_2e_2 - m_1e_1)$, where e is specific exergy.

Since states i and 1 correspond to the dead state, where $T_0 = 290$K, $P_0 = 1$ bar, we have $e_{fi} = 0$ and $e_1 = 0$. Accordingly, Eq. (3) reduces to

$$\boxed{\underset{(a)}{m_2e_2} = \underset{(b)}{(-W_{cv})} - \underset{(c)}{\mathsf{E}_d}} \qquad (4)$$

where (a) accounts for the exergy stored in the cavern, (b) accounts for the exergy transfer to the control volume accompanying work, and (c) accounts for exergy destruction within the control volume as the cavern fills.

Rearranging Eq. (4), $\mathsf{E}_d = (-W_{cv}) - m_2e_2$, where e_2 is determined using Eq. 7.2:

$$e_2 = (u_2-u_0) + P_0(v_2-v_0) - T_0(s_2-s_0)$$

With ideal gas model relations, this becomes

$$e_2 = (u_2-u_0) + P_0\left[\frac{RT_2}{P_2} - \frac{RT_0}{P_0}\right] - T_0\left[s^\circ(T_2) - s^\circ(T_0) - R\ln\frac{P_2}{P_0}\right]$$

$$= (u_2-u_0) + R\left[\left(\frac{P_0}{P_2}\right)T_2 - T_0\right] - T_0\left[s^\circ(T_2) - s^\circ(T_0) - R\ln\frac{P_2}{P_0}\right]$$

PROBLEM 7.130 (Continued)

With data from Table A-22,

$$e_2 = (584.21 - 206.91)\frac{kJ}{kg} + \frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\left[\left(\frac{1\,bar}{21\,bar}\right)(790K) - (290K)\right]$$

$$- 290K\left[2.704 - 1.66802 - \frac{8.314}{28.97}\ln\left(\frac{21}{1}\right)\right]\frac{kJ}{kg\cdot K}$$

$$= [(377.3) + (-72.43) - (47.05)]\frac{kJ}{kg}$$

$$= 257.82\ \frac{kJ}{kg}$$

Then,

$$m_2 e_2 = (9.26 \times 10^5\ kg)\left(257.82\frac{kJ}{kg}\right)\left|\frac{1\ GJ}{10^6\ kJ}\right| = 239\ GJ$$

from solution to Problem 4.114

Finally,

$$E_d = (-W_{cv}) - m_2 e_2$$

1 $$= 282\ GJ - 239\ GJ = 43\ GJ$$

from solution to Problem 4.114

Rearranging Eq. (4),

$$\underset{\boxed{\text{Exergy supplied}}}{(-W_{cv})} = \underset{\boxed{\text{Exergy stored}}}{m_2 e_2} + \underset{\boxed{\text{Exergy destroyed}}}{E_d}$$

$$\Rightarrow \quad \epsilon = \frac{m_2 e_2}{(-W_{cv})} = \frac{239\ GJ}{282\ GJ} = 0.848\ \ (84.8\%)$$

---

1. When the cavern is discharged to generate power, additional exergy destruction occurs. Also, note that the exergy destruction can be evaluated from $E_d = T_0 \sigma$, where σ is obtained from an entropy balance as $\sigma = m_2(s_2 - s_i)$. The details are left as an exercise.

PROBLEM 7.131

7.131 Figure P7.131 and the accompanying table provide steady-state operating data for a *cogeneration system* that produces power and 50,000 lb/h of process steam. Stray heat transfer and the effects of motion and gravity are negligible. The isentropic pump efficiency is 100%. Determine

(a) the net power developed, in Btu/h.
(b) the net exergy increase of the water passing through the steam generator, $\dot{m}_1(e_{f1} - e_{f4})$, in Btu/h.
(c) a full exergy accounting based on the net exergy supplied to the system found in part (b).
(d) Using the result of part (c), devise and evaluate an exergetic efficiency for the overall cogeneration system. Comment.

Let $T_0 = 70°F$, $p_0 = 1$ atm.

State	T(°F)	p(lbf/in.²)	h(Btu/lb)	s(Btu/lb · R)
1	700	800	1338	1.5471
2	—	180	1221	1.5818
3	(x_3 = 0%)	180	346	0.5329
4	—	800	348	0.5329
5	250	140	219	0.3677
6	(x_6 = 100%)	140	1194	1.5761

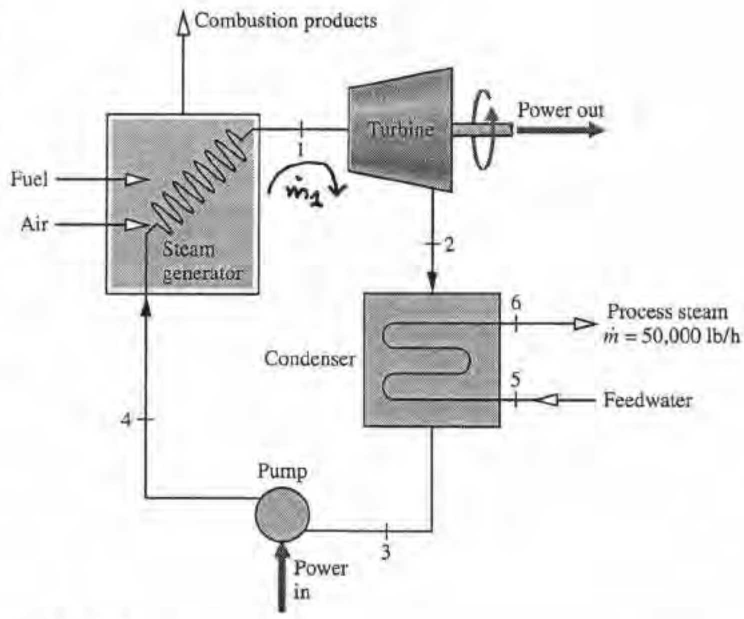


Fig. P7.131

ENGINEERING MODEL:

1. Control volumes at steady state enclose the turbine, pump, and condenser.
2. Stray heat transfer and the effects of motion and gravity are negligible.
3. $T_0 = 530°R\ (70°F)$, $P_0 = 1$ atm

ANALYSIS:

To determine the mass flow rate $\dot{m}_1$ of the water flowing through the components of the power cycle, apply mass and energy rate balances to the condenser:

$$\dot{m}_1 = \frac{\dot{m}[h_6 - h_5]}{(h_2 - h_3)} = \left(5\times10^4 \frac{lb}{h}\right)\left[\frac{1194-219}{1221-346}\right]$$
$$= 5.57\times10^4 \frac{lb}{h}$$

(a) To evaluate the net power developed, consider the turbine and pump:

$$\dot{W}_{net} = \dot{W}_t - |\dot{W}_p| = \dot{m}_1[(h_1-h_2)-(h_4-h_3)] = \left(5.57\times10^4\frac{lb}{h}\right)\left[(1338-1221)-(348-346)\right]\frac{Btu}{lb}$$
$$= 6.406\times10^6\ Btu/h \qquad \text{(a)}$$

(Net exergy carried out by the process steam) $= \dot{m}[e_{f6} - e_{f5}]$

$$= \dot{m}\left((h_6-h_5) - T_0(s_6-s_5)\right) = \left(5\times10^4\frac{lb}{h}\right)\left[(1194-219) - 530(1.5761-0.3677)\right]\frac{Btu}{lb}$$
$$= 16.727\times10^6\ Btu/h$$

(b) (Net exergy increase of the water passing through the steam generator)

$$= \dot{m}_1(e_{f1} - e_{f4}) = \dot{m}_1\left((h_1-h_4) - T_0(s_1-s_4)\right)$$
$$= \left(5.57\times10^4\frac{lb}{h}\right)\left((1338-348) - 530(1.5471-0.5329)\right)\frac{Btu}{lb}$$
$$= 25.203\times10^6\ \frac{Btu}{h} \qquad \text{(b)}$$

(c) Begin by determining the exergy destruction in the turbine and condenser. In each case, $\dot{E}_d = T_0\dot{\sigma}$, where $\dot{\sigma}$ is obtained from an entropy rate balance.

TURBINE:
$$\dot{E}_d = T_0\dot{\sigma} = T_0\dot{m}_1(s_2 - s_1)$$
$$= (530°R)\left(5.57\times10^4\frac{Btu}{h}\right)(1.5818-1.5471)\frac{Btu}{lb\cdot°R}$$
$$= 1.024\times10^6\ \frac{Btu}{h}$$

PROBLEM 7.131 (Continued)

CONDENSER: $\dot{E}_d = T_0\dot{\sigma} = T_0[\dot{m}_1(s_3-s_2) + \dot{m}(s_6-s_5)]$

$= 530°R[5.57\times10^4(0.5329-1.5818) + 5\times10^4(1.5761-0.3677)]\ \frac{Btu/h}{°R}$

$= 1.058\times10^6\ \frac{Btu}{h}$ ⟵

EXERGY ACCOUNTING ⟵ (c)

Net exergy carried into the cogeneration plant at the steam generator	$25.203\times10^6\ \frac{Btu}{h}$
Disposition of the exergy carried into the plant.	Based on the above value for exergy in.
✓ Net power developed	$6.407\times10^6\ \frac{Btu}{h}$ (25.42%)
✓ Exergy carried out by process steam	$16.727\times10^6\ \frac{Btu}{h}$ (66.37%)
✓ Exergy Destruction	
⊙ Turbine	$1.024\times10^6\ \frac{Btu}{h}$ (4.06%)
⊙ Condenser	$1.058\times10^6\ \frac{Btu}{h}$ (4.2%)
① ⊙ Pump	0.0
②	$25.216\times10^6\ \frac{Btu}{h}$

(d) In this application,

$$\epsilon = \frac{\begin{bmatrix}\text{Net power}\\ \text{developed}\end{bmatrix} + \begin{bmatrix}\text{Exergy carried out}\\ \text{by process steam}\end{bmatrix}}{[\text{Net exergy carried in}]}$$

$$= \frac{[(6.407)+(16.727)]\times10^6\ Btu/h}{25.203\times10^6\ Btu/h}$$

③④ $= 0.918\ (91.8\%)$

1. The pump is assumed to have an isentropic efficiency of 100% and thus $\dot{E}_d = 0$.
2. There is agreement to within round-off.
3. This value for exergetic efficiency does not account for significant exergy destruction taking place in the steam generator. See Sec. 8.6 for an introduction.
4. For more on co-generation, see Secs. 7.7.3 and 8.5.2.

PROBLEM 7.132

Figure P7.132 shows a *cogeneration* system producing two useful products: net power and process steam. The accompanying table provides steady-state mass flow rate, temperature, pressure, and flow exergy data at the 10 numbered states on the figure. Stray heat transfer and the effects of motion and gravity can be ignored. Let $T_0 = 298.15$ K, $p_0 = 1.013$ bar. Determine, in MW,

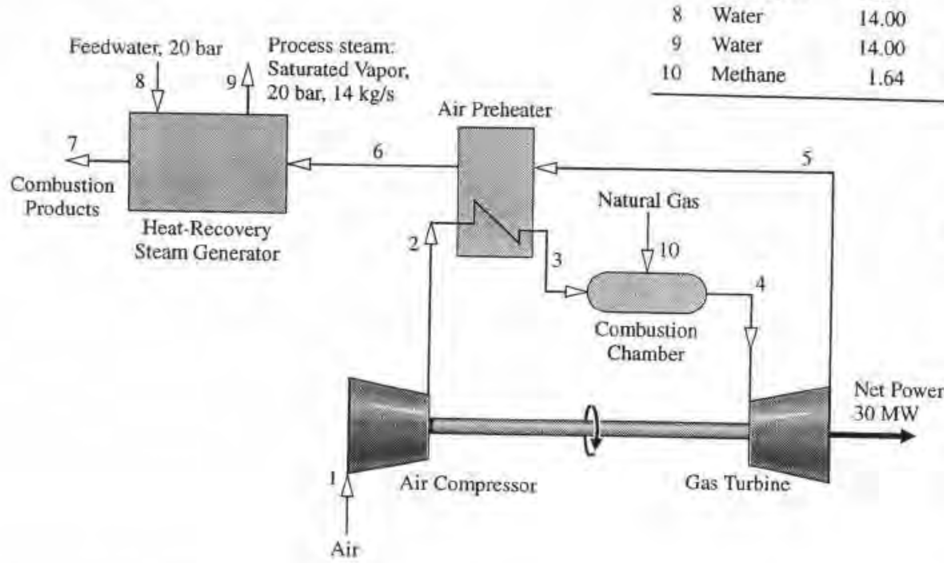
(a) the *net* rate exergy is carried out with the process steam, $(\dot{E}_{f9} - \dot{E}_{f8})$.

(b) the *net* rate exergy is carried out with the combustion products, $(\dot{E}_{f7} - \dot{E}_{f1})$.

(c) the rates of exergy destruction in the air preheater, heat-recovery steam generator, and the combustion chamber.

Devise and evaluate an exergetic efficiency for the overall cogeneration system.

State	Substance	Mass Flow Rate (kg/s)	Temperature (K)	Pressure (bar)	Flow Exergy Rate, $\dot{E}_f$ (MW)
1	Air	91.28	298.15	1.013	0.00
2	Air	91.28	603.74	10.130	27.54
3	Air	91.28	850.00	9.623	41.94
4	Combustion products	92.92	1520.00	9.142	101.45
5	Combustion products	92.92	1006.16	1.099	38.78
6	Combustion products	92.92	779.78	1.066	21.75
7	Combustion products	92.92	426.90	1.013	2.77
8	Water	14.00	298.15	20.000	0.06
9	Water	14.00	485.57	20.000	12.81
10	Methane	1.64	298.15	12.000	84.99



ENGR. MODEL:

1. Consider control volumes enclosing the overall system and, individually, around the preheater, HRSG, and combustion chamber.
2. All control volumes are at steady state.
3. Ignore stray heat transfer and the effects of motion and gravity.
4. $T_0 = 298.15$ K, $p_0 = 1.013$ bar.
5. The net exergy carried out with the combustion products is a loss.

ANALYSIS:

(a) $(\dot{E}_{f9} - \dot{E}_{f8}) = 12.81 - 0.06 = 12.75$ MW.

(b) $(\dot{E}_{f7} - \dot{E}_{f1}) = 2.77 - 0.00 = 2.77$ MW

(c) Exergy rate balances with assumption 3:

Preheater:

$$0 = \cancelto{0}{\sum\left(1-\frac{T_0}{T_j}\right)\dot{Q}_j} - \cancelto{0}{\dot{W}_{cv}} + \dot{E}_{f2} - \dot{E}_{f3} + \dot{E}_{f5} - \dot{E}_{f6} - \dot{E}_d$$

$$\Rightarrow \dot{E}_d = \dot{E}_{f2} - \dot{E}_{f3} + \dot{E}_{f5} - \dot{E}_{f6}$$

$$= 27.54 - 41.94 + 38.78 - 21.75 = \underline{\underline{2.63 \text{ MW}}}$$

(1) HRSG:

$$\Rightarrow \dot{E}_d = \dot{E}_{f6} - \dot{E}_{f7} + \dot{E}_{f8} - \dot{E}_{f9}$$

$$= 21.75 - 2.77 + 0.06 - 12.81 = \underline{\underline{6.23 \text{ MW}}}$$

COMB. CHAMBER:

$$\Rightarrow \dot{E}_d = \dot{E}_{f3} + \dot{E}_{f10} - \dot{E}_{f4}$$

$$= 41.94 + 84.99 - 101.45 = \underline{\underline{25.48 \text{ MW}}}$$

PROBLEM 7.132 (Continued)

Exergetic Efficiency:

This is a cogeneration system having two products: the net power developed and the net exergy carried out with the process steam.

In accordance with assumption 5, the net exergy carried out with the combustion products — calculated in part(b) — is regarded here as a loss.

The exergy supplied to the overall system is the exergy provided by the methane.

Thus, in summary

② $$\epsilon = \frac{\text{exergy products}}{\text{exergy supplied}} = \frac{\dot{W}_{net} + (\dot{E}_{f9} - \dot{E}_{f8})}{\dot{E}_{f,10}}$$

$$= \frac{30\,MW + 12.75\,MW}{84.99\,MW} = 0.503 \quad (50.3\%)$$

---

1. An exergy rate balance for the compressor/turbine gives the rate of exergy destruction in these components:

$$\dot{E}_d = (\dot{E}_{f1} - \dot{E}_{f2}) + (\dot{E}_{f4} - \dot{E}_{f5}) - \dot{W}_{net}$$
$$= (0 - 27.54) + (101.45 - 38.78) - 30 = 5.13\,MW$$

2. An exergy accounting for the cogeneration system reads,

⊙ Exergy supplied by the fuel: 84.99 MW

⊙ Disposition of the exergy supplied:

✓ Products		
Power Developed	30.00 MW	35.3%
Steam	12.75 MW	15.0%
	$\}$	$\}$ 50.3%
✓ Loss associated with combustion products	2.77 MW	3.3%
✓ Exergy Destruction		
– Preheater	2.63 MW	3.1%
– HRSG	6.23 MW	7.3%
– Combustion Chamber	25.48 MW	30.0%
– Compressor/turbine (from note 1 above)	5.13 MW	6.0%
	84.99 MW	

Note that the most significant source of exergy destruction is combustion

PROBLEM 7.133

Figure P7.133 shows a *combined* gas turbine-vapor power plant operating at steady state. The gas turbine is numbered 1-5. The vapor power plant is numbered 6-9. The accompanying table gives data at these numbered states. The total *net* power output is 45 MW and the mass flow rate of the water flowing through the vapor power plant is 15.6 kg/s. The ideal gas model applies to the air. Stray heat transfer and the effects of motion and gravity can be ignored. Let $T_0 = 300$ K, $p_0 = 100$ kPa. Determine

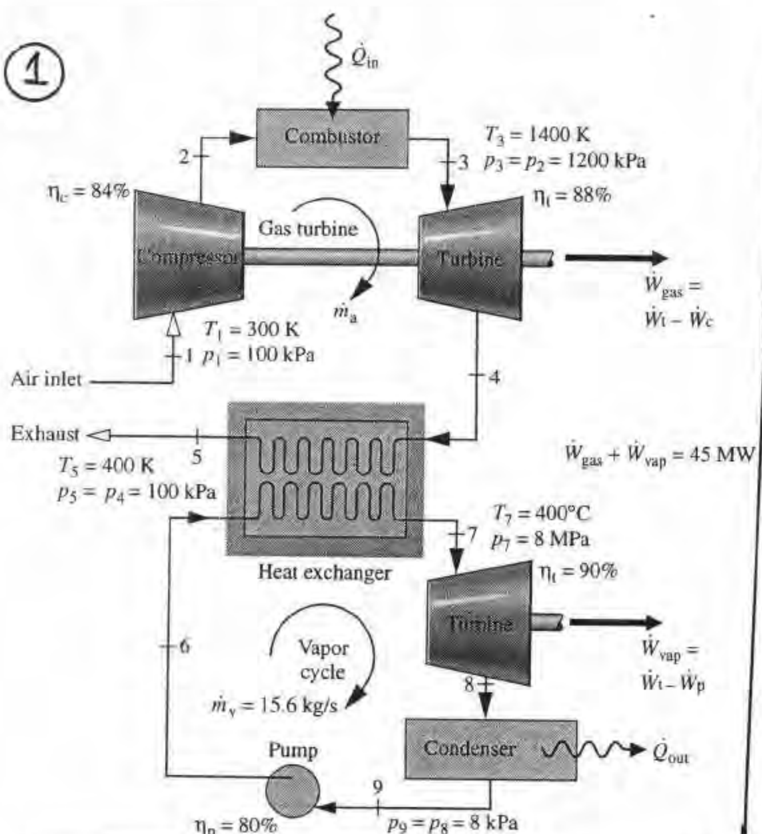
(a) the mass flow rate of the air flowing through the gas turbine, in kg/s.
(b) the *net* rate exergy is carried out with the exhaust air stream, $(\dot{E}_{f5} - \dot{E}_{f1})$, in MW.
(c) the rate of exergy destruction in the compressor and pump, each in MW.
(d) the *net* rate of exergy increase of the air flowing through the combustor, $(\dot{E}_{f3} - \dot{E}_{f2})$, in MW.

Devise and evaluate an exergetic efficiency for the overall combined power plant.

Gas Turbine[a]			Vapor Cycle		
State	h (kJ/kg)	s^0 (kJ/kg · K)	State	h (kJ/kg)	s (kJ/kg · K)
1	300.19	1.7020	6	183.96	0.5975
2	669.79	2.5088	7	3138.30	6.3634
3	1515.42	3.3620	8	2104.74	6.7282
4	858.02	2.7620	9	173.88	0.5926
5	400.98	1.9919			

[a] s^0 is the variable appearing in Eq. 6.20a and Table A-22.

1



ENGR. MODEL:

1. Control volumes at steady state enclose principal components and the overall system.
2. Stray heat transfer and the effects of motion and gravity can be ignored.
3. $T_0 = 300$ K, $p_0 = 100$ kPa.
4. The air is modeled as an ideal gas.

ANALYSIS:

(a) An energy rate balance for the heat exchanger reduces to

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_a(h_4 - h_5) + \dot{m}_v(h_6 - h_7)$$

$$\Rightarrow \dot{m}_a = \dot{m}_v\left[\frac{h_7 - h_6}{h_4 - h_5}\right]$$

$$= \left(15.6\frac{kg}{s}\right)\left[\frac{3138.3 - 183.96}{858.02 - 400.98}\right]$$

$$= 100.84\frac{kg}{s} \quad \text{(a)}$$

(b) $$(\dot{E}_5 - \dot{E}_{f1}) = \dot{m}_a\left[h_5 - h_1 - T_0(s_5 - s_1)\right]$$

$$= \dot{m}_a\left[h_5 - h_1 - T_0\left(s_5^\circ - s_1^\circ - \cancel{R\ln\frac{p_5}{p_1}}^{\,0}\right)\right]$$

$$= \left(100.84\frac{kg}{s}\right)\left[400.98 - 300.19 - 300(1.9919 - 1.7020)\right]\frac{kJ}{kg}\left|\frac{1\,MW}{10^3\,kJ/s}\right|$$

$$= 1.39\,MW$$

PROBLEM 7.133 (Continued)

(c) Rate of exergy destruction.

Compressor:

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{\,0} + \dot{m}\,(s_1 - s_2) + \dot{\sigma}_{cv} \quad , \quad \dot{E}_d = T_0 \dot{\sigma}_{cv}$$

$$\dot{E}_d = \dot{m}_a T_0 (s_2 - s_1) = \dot{m}_a T_0 \left(s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{P_2}{P_1} \right)$$

$$= \left(100.84 \frac{kg}{s}\right)(300 K) \left[2.5088 - 1.7020 - \frac{8.314}{28.97} \ln \frac{1200}{100} \right] \frac{kJ}{kg \cdot K} \left| \frac{1 MW}{10^3 kJ/s} \right|$$

$$= \underline{2.83 \text{ MW}}$$

Pump:

$$\dot{E}_d = T_0 \dot{\sigma}_{cv} = T_0 \dot{m}_v (s_6 - s_9)$$

$$= (300 K)(15.6 \text{ kg/s}) (.5975 - .5926) \frac{kJ}{kg \cdot K} \left| \frac{1 MW}{10^3 kJ/s} \right|$$

$$= \underline{\underline{0.02 \text{ MW}}}$$

(d)

$$\dot{E}_{f3} - \dot{E}_{f2} = \dot{m}_a \left[(h_3 - h_2) - T_0 (s_3 - s_2) \right]$$

$$= \left(100.84 \frac{kg}{s}\right) \left[(1515.42 - 669.79) - 300 (3.3620 - 2.5088) \right] \frac{kJ}{kg \cdot K} \left| \frac{1 MW}{10^3 kJ/s} \right|$$

$$= \underline{\underline{59.46 \text{ MW}}}$$

(e) Although exergy is carried from the overall combined system via the exhaust stream (part (b)) and via the cooling water, they are assumed to be losses here. The product of the overall system is then the net power developed.
The exergy provided to the overall system is obtained in part (d). Thus, the exergetic efficiency is

(2) $$\epsilon = \frac{\dot{W}_{net}}{\dot{E}_{f3} - \dot{E}_{f2}} = \frac{45 \text{ MW}}{59.46 \text{ MW}} = 0.757 \; (75.7\%)$$

---

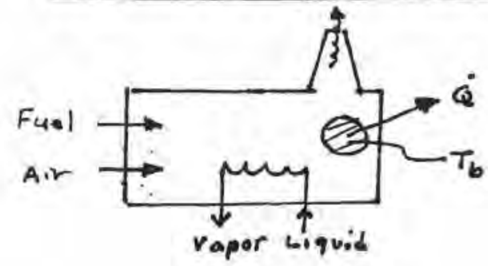
1. See Sec. 9.9 for further discussion of the combined gas turbine - vapor cycle.

2. This value does not account for irreversibilities with the combustor. See Example 9.12 for discussion.

PROBLEM 7.134

7.134 A high-pressure (HP) boiler and a low-pressure (LP) boiler will be added to a plant's steam-generating system. Both boilers use the same fuel and at steady state have approximately the same rate of energy loss by heat transfer. The average temperature of the combustion gases is less in the LP boiler than in the HP boiler. In comparison to the LP boiler, might you spend more, the same, or less to insulate the HP boiler? Explain.

SCHEMATIC & GIVEN DATA



ENGINEERING MODEL:

1. Each boiler uses the same fuel.
2. The same rate of energy loss by heat transfer is experienced by both types of boiler.
3. The average temperature of the combustion gases is less for the LP boiler than the HP boiler.
4. The exergy loss accompanying heat transfer is covered by the supply of extra fuel to the boiler, and the unit cost of providing fuel is constant.

ANALYSIS: Following the discussion of costing heat loss in Sec. 7.6.1, we apply Eq. 7.22 where T_b denotes the temperature on the boundary,

$$[\text{Cost Rate - HP}] = c_F\left[1 - \frac{T_0}{T_{b,HP}}\right]\dot{Q}$$

$$[\text{Cost Rate - LP}] = c_F\left[1 - \frac{T_0}{T_{b,LP}}\right]\dot{Q}$$

Forming a ratio,

$$\frac{[\text{Cost Rate - HP}]}{[\text{Cost Rate - LP}]} = \frac{c_F\left[1 - \frac{T_0}{T_{b,HP}}\right]\dot{Q}}{c_F\left[1 - \frac{T_0}{T_{b,LP}}\right]\dot{Q}} = \frac{\left[1 - \frac{T_0}{T_{b,HP}}\right]}{\left[1 - \frac{T_0}{T_{b,LP}}\right]}$$

Since $T_{b,LP} < T_{b,HP}$, it follows that

$$[\text{Cost Rate - HP}] > [\text{Cost Rate - LP}]$$

Accordingly, it may be justified to spend more for insulation of the HP boiler.

PROBLEM 7.135

7.135 Reconsider Example 7.10 for a turbine exit state fixed by p_2 = 2 bar, h_2 = 2723.7 kJ/kg, s_2 = 7.1699 kJ/kg · K. The cost of owning and operating the turbine is $\dot{Z}_t = 7.2\,\dot{W}_e$, in \$/h, where $\dot{W}_e$ is in MW. All other data remain unchanged. Determine

(a) the power developed by the turbine, in MW.
(b) the exergy destroyed within the turbine, in MW.
(c) the exergetic turbine efficiency.
(d) the unit cost of the turbine power, in cents per kW · h of exergy.

See Example 7.10 for Schematic and Engineering Model.

ANALYSIS:

(a) Mass and energy rate balances reduce to give

$$\dot{W}_e = \dot{m}(h_1 - h_2) = \left(26.15\,\frac{kg}{s}\right)(3353.54 - 2723.7)\frac{kJ}{kg}\left|\frac{1\,MW}{10^3\,kJ/s}\right|$$
$$= 16.47\ MW$$

(b) Mass and entropy rate balances reduce to give

$$\dot{E}_d = \dot{m}T_0(s_2 - s_1) = \left(26.15\,\frac{kg}{s}\right)(298K)(7.1699 - 6.8773)\frac{kJ}{kg\cdot K}\left|\frac{1\,MW}{10^3\,kJ/s}\right|$$
$$= 2.28\,MW$$

(c) Applying Eq. 7.24

$$\epsilon = \frac{\dot{W}_e}{\dot{m}(e_{f1} - e_{f2})} = \frac{16.47\,MW}{18.75\,MW} = 0.878 \quad (87.8\%)$$

where

① $$\dot{m}(e_{f1} - e_{f2}) = \dot{m}\left[(h_1 - h_2) - T_0(s_1 - s_2)\right]$$
$$= \left(26.15\,\frac{kg}{s}\right)\left[(3353.34 - 2723.7) - 298(6.8773 - 7.1699)\right]\frac{kJ}{kg}\left|\frac{1\,MW}{10^3\,kJ/s}\right|$$
$$= 18.75\,MW$$

(d) The cost rate balance for the turbine is given by Eq. 7.34b. Then, with the exergetic efficiency we get Eq. 7.34c:

$$c_e = \frac{\overbrace{c_1}^{\text{From Example 7.10}}}{\epsilon} + \frac{\dot{Z}_t}{\dot{W}_e}$$
$$= \left(\frac{1}{0.878}\right)\left(\frac{7.2\ cents}{kW\cdot h}\right) + \frac{(7.2\,\dot{W}_e)\ \$/h}{\dot{W}_e}\left|\frac{10^2\,cents}{1\,\$}\right|\left|\frac{1\,MW}{10^3\,kW}\right|$$

(in MW) (in MW)

$$= (8.2 + 0.72)\ \frac{cents}{kW\cdot h}$$
$$= 8.92\ \frac{cents}{kW\cdot h}$$

---

1. Alternatively, with an exergy rate balance

$$0 = \sum_j\left[1 - \frac{T_0}{T_j}\right]\cancel{\dot{Q}_j}^{\,0} - \dot{W}_e + \dot{m}(e_{f1} - e_{f2}) - \dot{E}_d$$

$$\Rightarrow \quad \dot{m}(e_{f1} - e_{f2}) = \dot{W}_e + \dot{E}_d = (16.47\,MW) + (2.28\,MW) = 18.75\,MW$$

PROBLEM 7.136

7.136 At steady state, a turbine with an exergetic efficiency of 90% develops 7×10^7 kW · h of work annually (8000 operating hours). The annual cost of owning and operating the turbine is $\$2.5 \times 10^5$. The steam entering the turbine has a specific flow exergy of 559 Btu/lb, a mass flow rate of 12.55×10^4 lb/h, and is valued at \$0.0165 per kW · h of exergy.

(a) Using Eq. 7.34c, evaluate the unit cost of the power developed, in \$ per kW · h.

(b) Evaluate the unit cost based on exergy of the steam entering and exiting the turbine, each in cents per lb of steam flowing through the turbine.

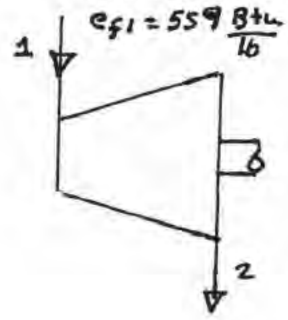
KNOWN: Data are provided for a turbine operating at steady state.

FIND: Evaluate unit costs based on exergy of the power developed and the steam entering and exiting the turbine.

SCHEMATIC & GIVEN DATA:

$\dot{m} = 12.55 \times 10^4 \frac{lb}{h}$, $c_s = \$0.0165/kW\cdot h$

$e_{f1} = 559 \frac{Btu}{lb}$,



$\epsilon = 90\%$

$\dot{Z}_t = \$2.5 \times 10^5/year$

$\dot{W}_e = 7 \times 10^7 \frac{kW\cdot h}{year}$

ENGINEERING MODEL:

1. A control volume at steady state encloses the turbine.
2. Stray heat transfer and the effects of motion and gravity are ignored.
3. 1 year = 8000 operating hours.

ANALYSIS: (a) Applying Eq. 7.34c,

$$c_e = \frac{c_s}{\epsilon} + \frac{\dot{Z}_t}{\dot{W}_e} = \frac{(0.0165)\,\$/kW\cdot h}{0.9} + \frac{\$2.5 \times 10^5/year}{7 \times 10^7\ kW\cdot h/year}$$

$$= [0.0183 + 0.0036]\frac{\$}{kW\cdot h} = 0.022\frac{\$}{kW\cdot h}$$

(b) By assumption leading to Eq. 7.34c, the same unit cost based on exergy applies to the steam at the inlet and at the exit — namely, 0.0165 \$/kW·h, or 1.65 cents/kW·h. Thus, the unit cost of steam at the inlet, in cents per lb of steam flowing is,

$$\begin{bmatrix}\text{unit cost of} \\ \text{steam per lb}\end{bmatrix}_{inlet} = \left(1.65\frac{cents}{kW\cdot h}\right)\left|\frac{1\ kW\cdot h}{3413\ Btu}\right|\left(559\frac{Btu}{lb}\right)$$

$$= 0.27\ \frac{cents}{lb}$$

The same approach applies for the exiting steam, but first its specific exergy value is required: Using Eq. 7.24,

$$\epsilon = \frac{\dot{W}_e}{\dot{m}(e_{f1} - e_{f2})} \Rightarrow e_{f2} = e_{f1} - \frac{\dot{W}_e/\dot{m}}{\epsilon}$$

$$\therefore\ e_{f2} = 559\frac{Btu}{lb} - \frac{(7 \times 10^7 kW\cdot h/yr)\left|\frac{3413\ Btu}{kW\cdot h}\right|\frac{1\ yr}{8000\ h}\right|}{(12.55 \times 10^4 \frac{lb}{h})(0.9)}$$

$$= 295\ \frac{Btu}{lb}$$

Then

$$\begin{bmatrix}\text{unit cost of} \\ \text{steam per lb}\end{bmatrix}_{exit} = \left(1.65\frac{cents}{kW\cdot h}\right)\left|\frac{1\ kW\cdot h}{3413\ Btu}\right|\left(295\frac{Btu}{lb}\right)$$

$$= 0.14\ \frac{cents}{lb}$$

PROBLEM 7.137

7.137 Figure P7.137 shows a boiler at steady state. Steam having a specific flow exergy of 1300 kJ/kg exits the boiler at a mass flow rate of 5.69×10^4 kg/h. The cost of owning and operating the boiler is \$91/h. The ratio of the exiting steam exergy to the entering fuel exergy is 0.45. The unit cost of the fuel based on exergy is \$1.50 per 10^6 kJ. If the cost rates of the combustion air, feedwater, heat transfer with the surroundings, and exiting combustion products are ignored, develop

(a) an expression in terms of exergetic efficiency and other pertinent quantities for the unit cost based on exergy of the steam exiting the boiler.

(b) Using the result of part (a), determine the unit cost of the steam, in cents per kg of steam flowing.

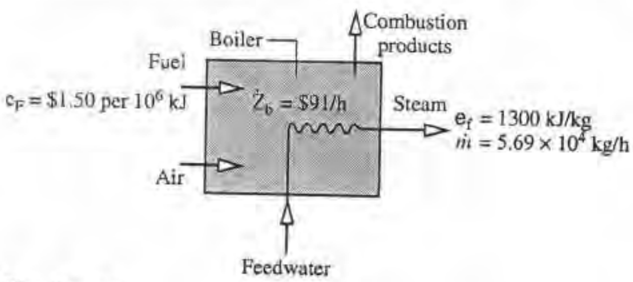


Fig. P7.137

KNOWN: Data are provided for a boiler operating at steady state.

FIND: Determine an expression for the unit cost based on exergy of the steam exiting the boiler. Using this expression, calculate the unit cost of the exiting steam, in cents per kg of steam flowing.

ENGINEERING MODEL:

1. A control volume at steady state encloses the boiler.
2. Cost rates of the combustion air, feedwater, and stray heat transfer are ignored.

ANALYSIS: With the engineering model specified, the boiler analysis of Sec. 7.7.3 reduces to give Eq. 7.32 b. ⟵ (a)

(b) Thus, the unit cost of the exiting steam is calculated as follows:

$$c_s = c_F \left[\frac{\dot{E}_{fF}}{\dot{E}_{fs}} \right] + \frac{\dot{Z}}{\dot{E}_{fs}}$$, where subscripts denotes the steam.

We have $(\dot{E}_{fs}/\dot{E}_{fF}) = 0.45$, $\dot{Z} = \$91/h$, and

$$\dot{E}_{fs} = \dot{m}\, e_f = \left(5.69 \times 10^4 \frac{kg}{h}\right)\left(1300 \frac{kJ}{kg}\right) = 73.97 \times 10^6 \frac{kJ}{h}$$

Calculating,

$$c_s = \left[\frac{\$1.50}{10^6\,kJ}\right]\left[\frac{1}{0.45}\right] + \left[\frac{\$91/h}{73.97 \times 10^6\,kJ/h}\right] = \left[\frac{3.33 + 1.23}{10^6}\right] \frac{\$}{kJ} = \left(\frac{4.56}{10^6}\right) \frac{\$}{kJ}$$

Using $e_f = 1300$ kJ/kg and converting to cents

① $$\left(\begin{array}{l}\text{Unit cost of the}\\ \text{exiting steam}\\ \text{per kg}\end{array}\right) = \left(1300 \frac{kJ}{kg}\right)\left(\frac{4.56}{10^6}\right)\frac{\$}{kJ} \left| \frac{100\,\text{cents}}{1\,\$} \right| = 0.59 \frac{\text{cents}}{kg}$$ ⟵ (b)

---

1. Converting to a per lb basis, we get

$$\left(\begin{array}{l}\text{Unit cost of the}\\ \text{exiting steam}\\ \text{per lb}\end{array}\right) = 0.59 \frac{\text{cents}}{kg} \left| \frac{1\,kg}{2.2\,lb} \right| = 0.27 \frac{\text{cents}}{lb}$$

This value corresponds to the unit cost of the steam entering the turbine of Problem 7.136 (Part (b)).

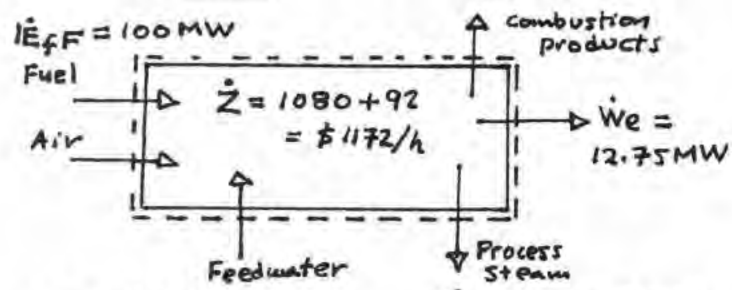
PROBLEM 7.138

7.138 Consider an *overall* control volume comprising the boiler and steam turbine of the cogeneration system of Example 7.10. Assuming the power and process steam each have the same unit cost based on exergy: $c_e = c_2$, evaluate the unit cost, in cents per kW · h. Compare with the respective values obtained in Example 7.10 and comment.

KNOWN: Data are provided in Example 7.10 for a cogeneration system.

FIND: Consider an overall control volume and take the unit cost based on exergy to be the same for power and steam produced. Evaluate that unit cost and compare with the values of Example 7.10.

SCHEMATIC & GIVEN DATA:



$c_F = 1.44 \frac{\text{cents}}{\text{kW·h}}$ $\qquad \dot{E}_{f2} = 20.67$ MW

Note: Values given on the schematic are from Example 7.10.

ENGINEERING MODEL:

1. The control volume at steady state shown in the schematic encloses the boiler and turbine of Example 7.10.
2. The process steam and the power developed have the same unit cost based on exergy.
3. Other relevant assumptions listed in Example 7.10 apply.

ANALYSIS: With the engineering model given, there is one valuable input: fuel and two valuable outputs: power and process steam. A cost rate balance reads,

$$c\dot{W}_e + c\dot{E}_{f2} = c_F\dot{E}_{fF} + \dot{Z}$$

where c denotes the common unit cost based on exergy of the power and process steam (assumption 2). Solving for c, we get

$$c = \frac{c_F\dot{E}_{fF} + \dot{Z}}{\dot{W}_e + \dot{E}_{f2}} \qquad (1)$$

Substituting known values, Eq. (1) gives

$$c = \frac{(1.44 \frac{\text{cents}}{\text{kW·h}})(100\text{MW})}{(12.75 + 20.67)\text{MW}} + \frac{(1172 \frac{\$}{h})}{(12.75 + 20.67)\text{MW}}\left|\frac{1\text{MW}}{10^3\text{kW}}\right|\left|\frac{100\text{ cents}}{1\ \$}\right|$$

$$= (4.309 + 3.507)\frac{\text{cents}}{\text{kW·h}} = 7.82 \frac{\text{cents}}{\text{kW·h}}$$

From the solution to Example 7.10, the unit cost for power and process steam are 8.81 cents per kW-h and 7.2 cents per kw-h, respectively. When an overall control volume is considered, the unit cost for power is less than before because it no longer bears the full burdens of these costs (see note 1 of Example 7.10). On the other hand, the unit cost of the process steam is higher than before because it now bears a part of these costs.

PROBLEM 7.139

7.139 A cogeneration system operating at steady state is shown schematically in Fig. P7.139. The exergy transfer rates of the entering and exiting streams are shown on the figure, in MW. The fuel, produced by reacting coal with steam, has a unit cost of 5.85 cents per kW · h of exergy. The cost of owning and operating the system is \$1800/h. The feedwater and combustion air enter with negligible exergy and cost. Expenses related to proper disposal of the combustion products are included with the cost of owning and operating the system.

(a) Determine the rate of exergy destruction within the cogeneration system, in MW.

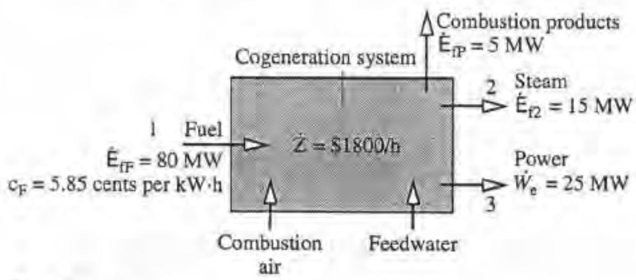


Fig. P7.139

(b) Devise and evaluate an exergetic efficiency for the system.
(c) Assuming the power and steam each have the same unit cost based on exergy, evaluate the unit cost, in cents per kW · h. Also evaluate the cost rates of the power and steam, each in \$/h.

KNOWN: Data are provided for a cogeneration system at steady state.

FIND: (a) Determine the rate of exergy destruction, (b) Devise and evaluate an exergetic efficiency. (c) Evaluate the unit cost, and the cost rates of the power and steam generated, each on an exergy basis.

ENGINEERING MODEL:
1. A control volume at steady state encloses the cogeneration system.
2. The exiting steam and power each have the same unit cost based on exergy.
3. Feedwater and combustion air enter with negligible exergy and cost. Expenses related to the combustion products are included with $\dot{Z}$. Stray heat transfer is ignored.

ANALYSIS:

(a) With assumptions listed, the exergy rate balance reduces to give

$$\dot{E}_d = \dot{E}_{fF} - \dot{E}_{fP} - \dot{E}_{f2} - \dot{W}_e \qquad (1)$$

$$= 80 - 5 - 15 - 25 = 35\,\text{MW} \qquad \text{(a)}$$

(b) Rearranging Eq. (1)

$$\underbrace{\dot{E}_{fF}}_{\text{(exergy supplied)}} = \underbrace{\dot{E}_{f2} + \dot{W}_e}_{\text{(exergy products)}} - \underbrace{\dot{E}_{fP}}_{\text{(exergy loss)}} - \underbrace{\dot{E}_d}_{\text{(exergy destruction)}} \implies \epsilon = \frac{\dot{E}_{f2} + \dot{W}_e}{\dot{E}_{fF}} = \frac{(15+25)}{80} = 0.5 \ (50\%) \qquad \text{(b)}$$

(c) A cost rate balance reduces with given assumptions to read

$$c\,\dot{E}_{f2} + c\,\dot{W}_e = c_F\,\dot{E}_{fF} + \dot{Z}$$, where c is the unit cost based on exergy of the power and steam.

Solving

$$c = c_F\left[\frac{\dot{E}_{fF}}{\dot{E}_{f2} + \dot{W}_e}\right] + \frac{\dot{Z}}{[\dot{E}_{f2} + \dot{W}_e]} = \frac{c_F}{\epsilon} + \frac{\dot{Z}}{[\dot{E}_{f2} + \dot{W}_e]}$$

$$= \left[\frac{5.85\ \frac{\text{cents}}{\text{kW-h}}}{0.5}\right] + \left[\frac{1800\ \frac{\$}{h}}{40\,\text{MW}}\right]\left|\frac{1\,\text{MW}}{10^3\,\text{kW}}\right|\left|\frac{100\,\text{cents}}{1\,\$}\right| = (11.7 + 4.5)\frac{\text{cents}}{\text{kW}\cdot\text{h}} = 16.2\ \frac{\text{cents}}{\text{kW}\cdot\text{h}} \qquad \text{(c)}$$

The cost rates are,

- Steam: $\dot{C}_2 = c\,\dot{E}_{f2} = (16.2\ \frac{\text{cents}}{\text{kW}\cdot\text{h}})(15\,\text{MW})\left|\frac{10^3\,\text{kW}}{1\,\text{MW}}\right|\left|\frac{1\,\$}{100\,\text{cents}}\right| = \$2430/h$
- Power: $\dot{C}_3 = c\,\dot{W}_e = (16.2\ \frac{\text{cents}}{\text{kW}\cdot\text{h}})(25\,\text{MW})\left|\frac{10^3\,\text{kW}}{1\,\text{MW}}\right|\left|\frac{1\,\$}{100\,\text{cents}}\right| = \$4050/h$

PROBLEM 7.140

7.140 Figure P7.140 provides steady-state operating data for a coal gasification system fueling a cogeneration system that produces power and process steam. The numbers given for each of the seven streams, in MW, represent rates of exergy flow. The unit cost based on exergy of stream 1 is $c_1 = 1.08$ cents per kW · h. On the advice of a *cost engineer*, the unit costs based on exergy of the process steam (stream 4) and power (stream 5) are assumed to be equal, and no cost is associated with combustion air and water (stream 7). The costs of owning and operating the gasification and cogeneration systems are \$3600/h and \$1800/h, respectively. These figures include expenses related to discharge of ash (stream 3) and flue gas (stream 6) to the surroundings. Determine the

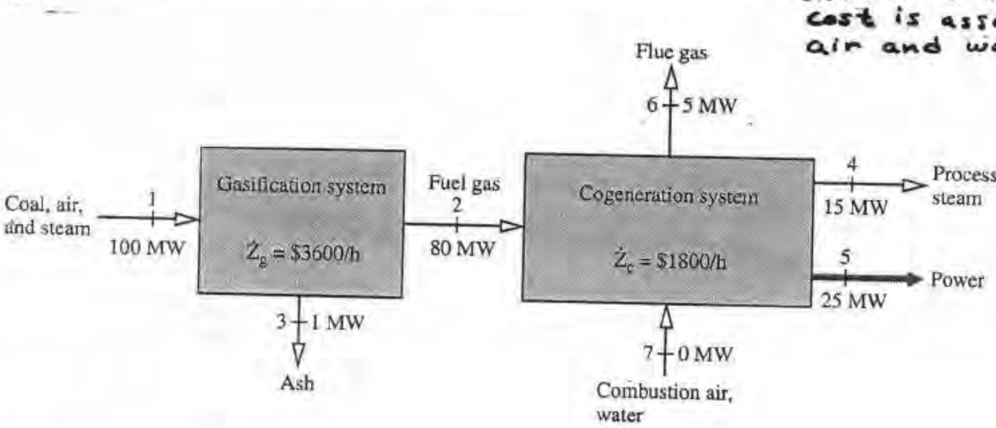
(a) rate of exergy destruction, in MW, for each system.
(b) exergetic efficiency for each system and for an overall system formed by the two systems.
(c) unit cost based on exergy, in cents/kW · h, for each of streams 2, 4, and 5.
(d) cost rate, in \$/h, associated with each of streams 1, 2, 4 and 5.

KNOWN: Steady-state operating data are provided for a coal gasification system fueling a cogeneration system.

FIND: Determine $\dot{E}_d$ for each system. Also determine ϵ for each system and for an overall system. Also determine the unit cost based on exergy for streams 2, 4, 5, and the cost rate for streams 1, 2, 4, 5.

ENGINEERING MODEL:

1. A control volume at steady state encloses each of the two units and an overall control volume encloses the two units.
2. The unit cost based on exergy of streams 4 and 5 are equal and no cost is associated with combustion air and water (stream 7).



ANALYSIS:

(a) Exergy rate balances reduce at steady state to give,

gasification system $\}$ $\quad \dot{E}_d = \dot{E}_1 - \dot{E}_2 - \dot{E}_3 = (100 - 80 - 1)\,\text{MW} = 19\,\text{MW}$

cogeneration system $\}$ $\quad \dot{E}_d = \dot{E}_2 + \dot{E}_7 - \dot{E}_6 - \dot{E}_4 - \dot{E}_5 = (80 + 0 - 5 - 15 - 25) = 35\,\text{MW}$

(b) Gasification: $\quad \epsilon_g = \dfrac{\dot{E}_2}{\dot{E}_1} = \dfrac{80}{100} = 0.8\ (80\%)$

Cogeneration: $\quad \epsilon_c = \dfrac{\dot{E}_4 + \dot{E}_5}{\dot{E}_2} = \dfrac{(15+25)}{80} = 0.5\ (50\%)$

Overall:

$$\epsilon_o = \frac{\dot{E}_4 + \dot{E}_5}{\dot{E}_1} = \frac{(15+25)}{100} = 0.4 \quad (40\%)$$

Alternatively,

$$\epsilon_o = \left[\frac{\dot{E}_2}{\dot{E}_1}\right]\left[\frac{\dot{E}_4 + \dot{E}_5}{\dot{E}_2}\right] = (0.8)(0.5) = 0.4$$

$(\epsilon_o = \epsilon_g \epsilon_c)$

(c) Cost rate bances,

Gasification system:

$$c_2 \dot{E}_2 = c_1 \dot{E}_1 + Z_g \Rightarrow c_2 = \frac{c_1 \dot{E}_1 + \dot{Z}_g}{\dot{E}_2}$$

$$\therefore \quad c_2 = \frac{\left(1.08 \frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)(100\text{MW})}{(80\text{ MW})} + \frac{\$\,3600/h}{(80\text{MW})}\left|\frac{1\text{MW}}{10^3\text{kW}}\right|\left|\frac{100\text{ cents}}{1\ \$}\right|$$

$$= (1.35 + 4.5)\ \frac{\text{cents}}{\text{kW}\cdot\text{h}} = 5.85\ \frac{\text{cents}}{\text{kW}\cdot\text{h}}$$

Cogeneration system:

$$\underbrace{c_4 \dot{E}_4 + c_5}_{=c\ \text{(assumption \#2)}} \dot{E}_5 = c_2 \dot{E}_2 + \dot{Z}_c \Rightarrow c\left[\dot{E}_4 + \dot{E}_5\right] = c_2 \dot{E}_2 + \dot{Z}_c$$

$$\therefore \quad c = \frac{c_2 \dot{E}_2 + \dot{Z}_c}{(\dot{E}_4 + \dot{E}_5)} = \left(5.85 \frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)\left(\frac{80\text{MW}}{40\text{MW}}\right) + \frac{\$1800h}{(40\text{MW})}\left|\frac{1\text{ MW}}{10^3\text{kW}}\right|\left|\frac{100\text{cents}}{1\ \$}\right|$$

$$= (11.7 + 4.5)\ \frac{\text{cents}}{\text{kW}\cdot\text{h}} = 16.2\ \frac{\text{cents}}{\text{kW}\cdot\text{h}}$$

1 (d) Cost rates,

Stream 1: $c_1 \dot{E}_1 = \left(1.08 \frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)(100\text{ MW})\left|\frac{10^3\text{kW}}{1\text{ MW}}\right|\left|\frac{1\ \$}{100\text{ cents}}\right|$

$= \$\,1080/h$

Stream 2: $c_2 \dot{E}_2 = \left(5.85 \frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)(80\text{ MW})\left|\frac{10^3\text{kW}}{1\text{MW}}\right|\left|\frac{1\ \$}{100\text{cents}}\right|$

$= \$\,4680/h$

Stream 4: $c_4 \dot{E}_4 = \left(16.2 \frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)(15\text{MW})\left|\frac{10^3\text{ kW}}{1\text{MW}}\right|\left|\frac{1\ \$}{100\text{ cents}}\right|$

$= \$\,2430/h$

Stream 5: $c_5 \dot{E}_5 = \left(16.2 \frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)(25\text{MW})\left|\frac{10^3\text{kW}}{1\text{MW}}\right|\left|\frac{1\ \$}{100\text{ cents}}\right|$

$= \$\,4050/h$

---

1 On an overall basis:

EXPENSES:

$c_1 \dot{E}_1 = 1080/h$ (16.7%)

$\dot{Z}_g = \$3600/h$ (55.6%)

$\dot{Z}_c = \$1800/h$ (27.8%)

$\$\,6480/h$

RETURN:

$c_4 \dot{E}_4 = \$2430/h$ (37.5%)

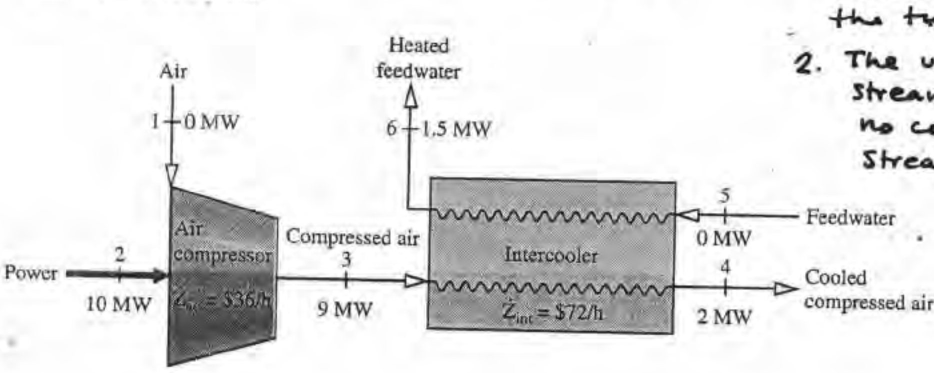
$c_5 \dot{E}_5 = \$4050/h$ (62.5%)

$\$6480/h$

PROBLEM 7.141

7.141 Figure P7.141 provides steady-state operating data for an air compressor–intercooler system. The numbers given for each of the six streams, in MW, represent rates of exergy flow. The unit cost of the power input is c_2 = 3.6 cents per kW · h. On the advice of a *cost engineer*, the unit costs based on exergy of the compressed air (stream 3) and cooled compressed air (stream 4) are assumed to be equal, and no costs are associated with the incoming air (stream 1) and feedwater (stream 5). The costs of owning and operating the air compressor and intercooler are \$36/h and \$72/h, respectively. Determine the

(a) rate of exergy destruction for the air compressor and intercooler, each in MW.
(b) exergetic efficiency for the air compressor, the intercooler, and an overall system formed from the two components.
(c) unit cost based on exergy, in cents/kW · h, for each of streams 3, 4, and 6.
(d) cost rate, in \$/h, associated with each of streams 2, 3, 4, and 6, and comment.



KNOWN: Steady-state operating data are provided for an air compressor–intercooler system.

FIND: Determine $\dot{E}_d$ for each unit. Also, determine ϵ for each unit and for an overall system. Also, determine the unit cost based on exergy for streams 3, 4, 6. Finally determine the cost rate for streams 2, 3, 4, 6.

ENGINEERING MODEL:
1. A control volume at steady state encloses each of the two units and an overall control volume encloses the two units.
2. The unit cost based on exergy of streams 3 and 4 are equal and no costs are associated with streams 1 and 5.

ANALYSIS:

(a) Exergy rate balances reduce at steady state to give,

air compressor } $\dot{E}_d = \dot{E}_1 + \dot{W}_{in} - \dot{E}_3 = (0 + 10 - 9)\,MW = 1\,MW$ ←

intercooler } $\dot{E}_d = \dot{E}_3 + \dot{E}_5 - \dot{E}_4 - \dot{E}_6 = (9 + 0 - 2 - 1.5)MW = 5.5MW$ ←

(b) air compressor: $\epsilon_{ac} = \dfrac{\dot{E}_3 - \dot{E}_1}{\dot{W}_{in}} = \dfrac{(9-0)MW}{10\,MW} = 0.9\ (90\%)$ ←

intercooler: $\epsilon_{int} = \dfrac{\dot{E}_6 - \dot{E}_5}{\dot{E}_3 - \dot{E}_4} = \dfrac{(1.5-0)\,MW}{(9-2)\,MW} = 0.214\ (21.4\%)$ ←

overall: An exergy rate balance for the overall control volume reduces to read,

$$\dot{W}_{in} = \underbrace{(\dot{E}_4 - \dot{E}_1)}_{\boxed{\text{exergy increase of the air stream}}} + \underbrace{(\dot{E}_6 - \dot{E}_5)}_{\boxed{\text{exergy increase of the water stream}}} + \underbrace{\dot{E}_d}_{\boxed{\text{exergy destruction}}}$$

$$\Rightarrow \epsilon_o = \frac{(\dot{E}_4 - \dot{E}_1) + (\dot{E}_6 - \dot{E}_5)}{\dot{W}_{in}} = \frac{(2-0)MW + (1.5-0)MW}{10MW} = 0.35\,(35\%) \leftarrow$$

PROBLEM 7.141 (Continued)

(c) Cost rate balances:

air compressor: $c_3\dot{E}_3 = c_2\dot{W}_{in} + \dot{Z}_{ac} \Rightarrow c_3 = c_2\left[\frac{\dot{W}_{in}}{\dot{E}_3}\right] + \frac{\dot{Z}_{ac}}{\dot{E}_3}$

$$\therefore \quad c_3 = \left(3.6\ \frac{\text{cents}}{\text{kW·h}}\right)\left(\frac{10\text{MW}}{9\text{MW}}\right) + \left(\frac{\$36/h}{9\text{ MW}}\right)\left|\frac{1\text{MW}}{10^3\text{kW}}\right|\left|\frac{100\text{ cents}}{1\ \$}\right|$$

$$= (4 + 0.4)\ \frac{\text{cents}}{\text{kW·h}} = 4.4\ \frac{\text{cents}}{\text{kW·h}} \longleftarrow$$

intercooler: $\underbrace{c_4\dot{E}_4 + c_6\dot{E}_6}_{=c_3\text{ (assumption \#2)}} = c_3\dot{E}_3 + \dot{Z}_{int}$

$$\Rightarrow \quad c_6 = \frac{c_3[\dot{E}_3 - \dot{E}_4]}{\dot{E}_6} + \frac{\dot{Z}_{int}}{\dot{E}_6}. \quad \text{Thus,} \quad c_6 = \left(4.4\ \frac{\text{cents}}{\text{kW·h}}\right)\left(\frac{(9-2)\text{MW}}{1.5\text{MW}}\right) + \left(\frac{\$72/h}{1.5\text{MW}}\right)\left|\frac{1\text{MW}}{10^3\text{kW}}\right|\left|\frac{100\text{cents}}{1\ \$}\right|$$

$$\therefore \quad c_6 = (20.53 + 4.8)\ \frac{\text{cents}}{\text{kW·h}} = 25.33\ \frac{\text{cents}}{\text{kW·h}} \longleftarrow$$

(d) Cost rates:

Stream 2: $c_2\dot{E}_2 = \left(3.6\ \frac{\text{cents}}{\text{kW·h}}\right)(10\text{MW})\left|\frac{10^3\text{kW}}{1\text{MW}}\right|\left|\frac{\$1}{100\text{ cents}}\right| = \$360/h \longleftarrow$

Stream 3: $c_3\dot{E}_3 = \left(4.4\ \frac{\text{cents}}{\text{kW·h}}\right)(9\text{MW})\left|\frac{10^3\text{kW}}{1\text{MW}}\right|\left|\frac{\$1}{100\text{cents}}\right| = \$396/h \longleftarrow$

Stream 4: $c_4\dot{E}_4: \left(4.4\ \frac{\text{cents}}{\text{kW·h}}\right)(2\text{MW})\left|\frac{10^3\text{kW}}{1\text{MW}}\right|\left|\frac{\$1}{100\text{ cents}}\right| = \$88/h \longleftarrow$

① ② Stream 6: $c_6\dot{E}_L = \left(25.33\ \frac{\text{cents}}{\text{kW·h}}\right)(1.5\text{MW})\left|\frac{10^3\text{kW}}{1\text{MW}}\right|\left|\frac{\$1}{100\text{cents}}\right| = \$380/h \longleftarrow$

---

1. With the assumption $c_4 = c_3$, the cost rate of the cooled compressed air is relatively low because this stream does not bear costs related to the intercooler, whereas the cost rate of the heated water is relatively high because this stream does bear such costs.

2. On an overall basis:

EXPENSES:			RETURN:		
$c_2\dot{E}_2 =$	\$360/h	(76.9%)	$c_4\dot{E}_4 =$	\$88/h	(18.8%)
$\dot{Z}_{ac} =$	\$36/h	(7.7%)	$c_6\dot{E}_L =$	\$380/h	(81.2%)
$\dot{Z}_{int} =$	\$72/h	(15.4%)		\$468/h	
	\$468/h				

PROBLEM 7.142

7.142 Repeat parts (c) and (d) of Problem 7.141 as follows: On the advice of a *cost engineer*, assume $c_4 = c_6$. That is, the unit cost based on exergy of the cooled-compressed air is the same as the unit cost based on exergy of the heated feedwater.

Revise Problem 7.141 ENGINEERING MODEL:

2. The unit cost based on exergy of streams 4 and 6 are equal.

ANALYSIS:

(c) Determine the unit cost based on exergy for each of streams 3, 4, 6. From the solution to Problem 7.141, we have $c_3 = 4.4\ \frac{\text{cents}}{\text{kW}\cdot\text{h}}$.

The cost rate balance for the intercooler reads

$$\underbrace{c_4 \dot{E}_4 + c_6 \dot{E}_6}_{= c \text{ (assumption \#2)}} = c_3 \dot{E}_3 + \dot{Z}_{int}$$

$$\Rightarrow \quad c = c_3\left[\frac{\dot{E}_3}{\dot{E}_4+\dot{E}_6}\right] + \frac{\dot{Z}_{int}}{[\dot{E}_4+\dot{E}_6]}$$

$$\therefore \quad c = \left(4.4\ \frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)\left[\frac{9\text{ MW}}{(2+1.5)\text{ MW}}\right] + \left[\frac{\$72/h}{(2+1.5)\text{ MW}}\right]\left|\frac{1\text{ MW}}{10^3\text{ kW}}\right|\left|\frac{100\text{ cents}}{\$1}\right|$$

$$= (11.31 + 2.06)\ \frac{\text{cents}}{\text{kW}\cdot\text{h}} = 13.37\ \frac{\text{cents}}{\text{kW}\cdot\text{h}}$$

(d) Cost rates:

Stream 2: $c_2 \dot{E}_2 = \$360/h$ } same as in Problem 7.141

Stream 3: $c_3 \dot{E}_3 = \$396/h$

Stream 4: $c_4 \dot{E}_4 = \left(13.37\ \frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)(2\text{ MW})\left[\frac{10^3\text{ kW}}{1\text{ MW}}\right]\left|\frac{\$1}{100\text{ cents}}\right| = \$267.4/h$

① ② Stream 6: $c_6 \dot{E}_6 = \left(13.37\ \frac{\text{cents}}{\text{kW}\cdot\text{h}}\right)(1.5\text{ MW})\left|\frac{10^3\text{ kW}}{1\text{ MW}}\right|\left|\frac{\$1}{100\text{ cents}}\right| = \$200.6/h$

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1. In this application, where $c_4 = c_6$, the costs related to the intercooler are shared by streams 4 and 6. Their cost rates are proportional to their rates of exergy flow.

2. On an overall basis:

EXPENSES:

$c_2\dot{E}_2 = \$360/h$ (76.9%)

$\dot{Z}_{ac} = \$36/h$ (7.7%)

$\dot{Z}_{int} = \$72/h$ (15.4%)

$\$468/h$

RETURN:

$c_4\dot{E}_4 = \$267.4/h$ (57.1%)

$c_6\dot{E}_4 = \$200.6/h$ (42.9%)

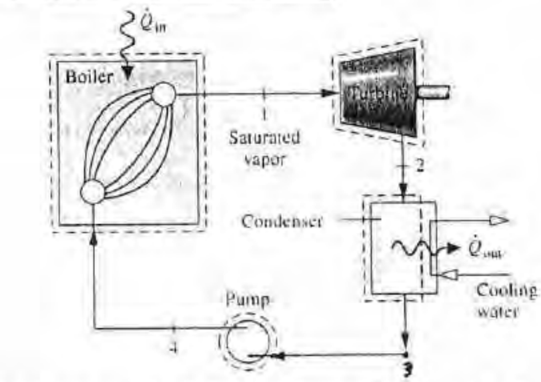
$\$468/h$

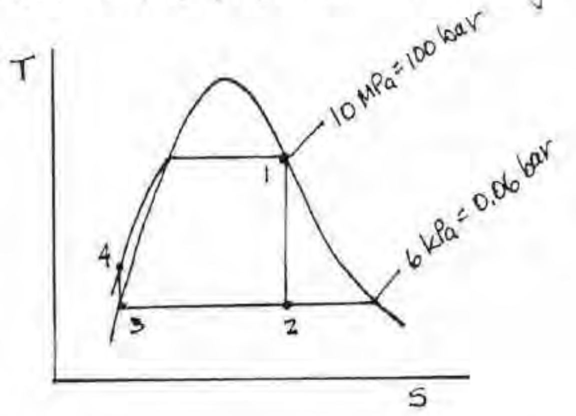
PROBLEM 8.1

KNOWN: Water is the working fluid in an ideal Rankine cycle. The states are specified at the condenser exit and the turbine inlet.

FIND: Determine the heat transfer rates, per unit of mass flowing, for the boiler and condenser as well as the thermal efficiency.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 8.1

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 100$ bar, sat. vapor $\Rightarrow h_1 = 2724.7$ kJ/kg, $s_1 = 5.6141 \frac{kJ}{kg \cdot K}$

State 2: $p_2 = 0.06$ bar, $s_2 = s_1 \Rightarrow x_2 = \frac{s_2 - s_{f2}}{s_{g2} - s_{f2}} = 0.6522$, $h_2 = 1727.2 \frac{kJ}{kg}$

State 3: $p_3 = p_2 = 0.06$ bar, sat. liquid $\Rightarrow h_3 = 151.53$ kJ/kg

State 4: $h_4 \approx h_3 + v_3(p_4 - p_3)$; $h_3 = 151.53$ kJ/kg, $v_3 = 1.0064 \times 10^{-3}$ m³/kg

$$h_4 = 151.53 \frac{kJ}{kg} + (1.0064 \times 10^{-3} \frac{m^3}{kg})(100 - 0.06)\,bar \left|\frac{10^5 N/m^2}{1\,bar}\right| \left|\frac{1\,kJ}{10^3 N \cdot m}\right|$$
$$= 151.53 + 10.06 = 161.59 \text{ kJ/kg}$$

For the control volume enclosing the boiler

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) \Rightarrow \dot{Q}_{in}/\dot{m} = h_1 - h_4 = 2563.1 \text{ kJ/kg} \leftarrow \dot{Q}_{in}/\dot{m}$$

For the condenser

$$\dot{Q}_{out} = \dot{m}(h_2 - h_3) \Rightarrow \dot{Q}_{out}/\dot{m} = h_2 - h_3 = 1575.7 \text{ kJ/kg} \leftarrow \dot{Q}_{out}/\dot{m}$$

The thermal efficiency is

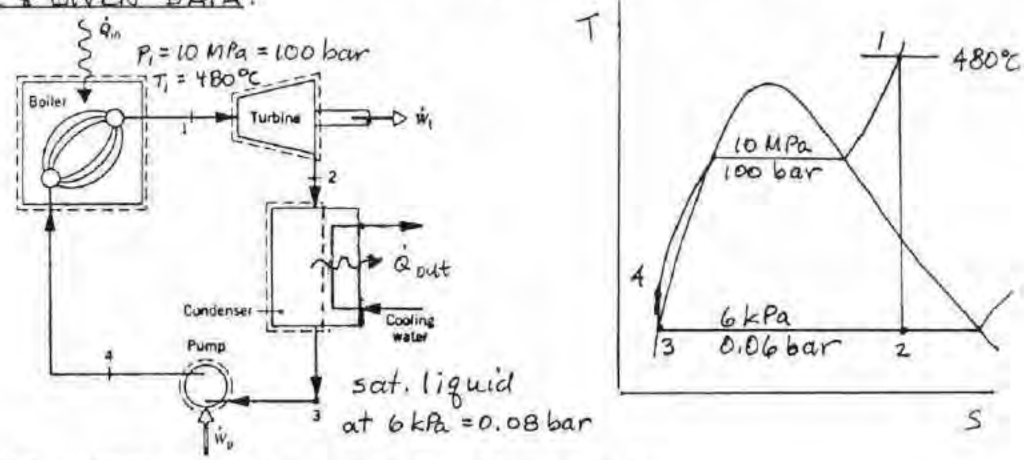
$$\eta = 1 - \frac{\dot{Q}_{out}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = 1 - \frac{1575.7}{2562.5} = 0.385\ (38.5\%) \leftarrow \eta$$

PROBLEM 8.2

KNOWN: Water is the working fluid in an ideal Rankine cycle with superheated vapor entering the turbine. The states at the turbine inlet and condenser exit are specified.

FIND: Determine (a) the heat tranfer rate for the steam generator, per kg of steam flowing, (b) the thermal efficiency, and (c) the rate of heat transfer for the condenser, per kg of steam condensing.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 8.1, items 1-4

ANALYSIS: First, fix each principal state.

STATE 1: $p_1 = 100$ bar, $T_1 = 480°C \Rightarrow h_1 = 3321.4$ kJ/kg, $s_1 = 6.5282$ kJ/kg·K

STATE 2: $p_2 = 0.06$ bar, $s_2 = s_1 \Rightarrow x_2 = \dfrac{s_2 - s_{f2}}{s_{g2} - s_{f2}} = 0.7692$, $h_2 = 2009.8$ kJ/kg

STATE 3: $p_3 = 0.06$ bar, sat. liquid $\Rightarrow h_3 = 151.53$ kJ/kg, $v_3 = 1.0064 \times 10^{-3} \frac{m^3}{kg}$

STATE 4: $h_4 \approx h_3 + v_3(p_4 - p_3)$

$$= 151.53 \frac{kJ}{kg} + (1.0064 \times 10^{-3} \frac{m^3}{kg})(100 - 0.06)\text{bar} \left|\frac{10^5 \text{ N/m}^2}{1 \text{ bar}}\right| \left|\frac{1 \text{ kJ}}{10^3 \text{ N·m}}\right|$$

$$= 151.53 + 10.06 = 161.59 \text{ kJ/kg}$$

(a) For the control enclosing the steam generator

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) \Rightarrow \frac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_4 = 3321.4 - 161.59 = 3159.8 \text{ kJ/kg} \leftarrow \dot{Q}_{in}/\dot{m}$$

(b) The thermal efficiency is $\eta = \dfrac{\dot{W}_{net}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \dfrac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$

① $$\dot{W}_{net}/\dot{m} = (h_1 - h_2) - (h_4 - h_3) = 1311.6 - 10.06 = 1301.5 \text{ kJ/kg}$$

$$\eta = \frac{1301.5}{3159.8} = 0.412 \,(41.2\%) \leftarrow \eta$$

(c) For the condenser

$$\dot{Q}_{out} = \dot{m}(h_2 - h_3) \Rightarrow \frac{\dot{Q}_{out}}{\dot{m}} = h_2 - h_3 = 2009.8 - 151.53 = 1858.3 \frac{kJ}{kg} \leftarrow \dot{Q}_{out}/\dot{m}$$

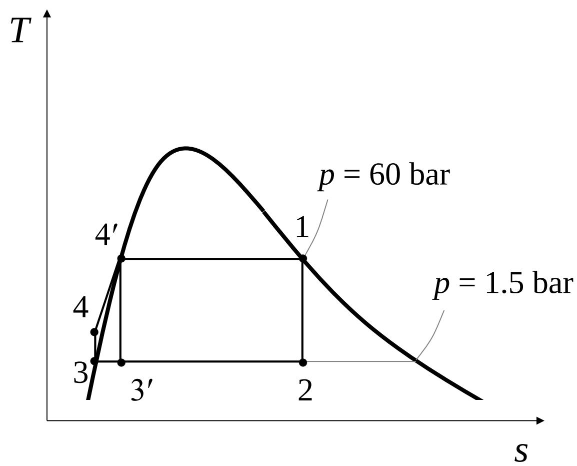
1. Comparing these results with those of Problem 8.1 illustrates some of the effects of superheating the vapor at the turbine inlet for an ideal Rankine cycle.

8.3 Steam is the working fluid in the ideal Rankine cycle 1-2-3-4-1 and in the Carnot cycle 1-2-3′-4′-1 that both operate between pressures of 1.5 bar and 60 bar as shown in the *T*-*s* diagram in Fig. P8.3. Both cycles incorporate the steady flow devices shown in Fig. 8.2. For each cycle determine (a) the net power developed per unit mass of steam flowing, in kJ/kg, and (b) the thermal efficiency. Compare results and comment.

KNOWN: An ideal Rankine cycle and a Carnot cycle operate between specified pressures of 1.5 bar and 60 bar.

FIND: Determine for both cycles (a) the net power developed per unit mass of steam flowing, in kJ/kg, and (b) the thermal efficiency. Compare results and comment.

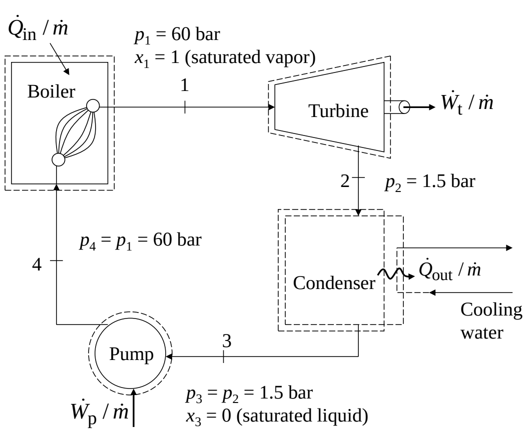
SCHEMATIC AND GIVEN DATA:



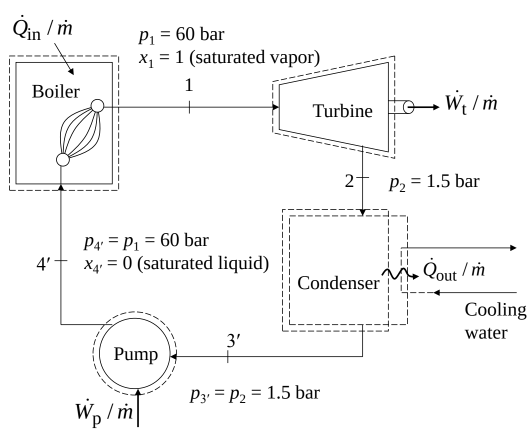
State	*p* (bar)	*h* (kJ/kg)	*x*	*v* (m^3/kg)
1	60	2784.3	1	
2	1.5	2180.6	0.7696	
3′	1.5	1079.8	0.2752	
3	1.5	467.11	0	0.0010528
4	60	473.27	--	
4′	60	1213.4	0	

Fig. P8.3

<u>Ideal Rankine Cycle</u>



<u>Carnot Cycle</u>



ENGINEERING MODEL:

1. Each component of both cycles is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. For both cycles, all processes of the working fluid are internally reversible.
3. The turbine and pump operate adiabatically for both cycles.
4. Kinetic and potential energy effects are negligible for both cycles.
5. Saturated vapor enters the turbine for both cycles.
6. Condensate exits the condenser as saturated liquid for the ideal Rankine cycle.
7. Water exits the pump as saturated liquid for the Carnot cycle.

ANALYSIS:

(a) The net power developed by the cycle per unit mass of steam flowing is given for both cycles by

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = \frac{\dot{W}_{\text{t}}}{\dot{m}} - \frac{\dot{W}_{\text{p}}}{\dot{m}}$$

Mass and energy rate balances for control volumes around the turbine for both cycles give

$$\frac{\dot{W}_{\text{t}}}{\dot{m}} = h_1 - h_2$$

where $\dot{m}$ is the mass flow rate of the steam.

Mass and energy rate balances for control volumes around the pump for the Rankine cycle and the Carnot cycle, respectively, give

Rankine cycle: $$\frac{\dot{W}_{\text{p}}}{\dot{m}} = h_4 - h_3$$

Carnot cycle: $$\frac{\dot{W}_{\text{p}}}{\dot{m}} = h_{4'} - h_{3'}$$

Solving for the net power developed per unit mass of steam flowing for each cycle yields

Rankine cycle:

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = (h_1 - h_2) - (h_4 - h_3)$$

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = (2784.3 - 2180.6)\frac{\text{kJ}}{\text{kg}} - (473.27 - 467.11)\frac{\text{kJ}}{\text{kg}} = \textbf{597.5 kJ/kg}$$

Carnot cycle:

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = (h_1 - h_2) - (h_{4'} - h_{3'})$$

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = (2784.3 - 2180.6)\frac{\text{kJ}}{\text{kg}} - (1213.4 - 1079.8)\frac{\text{kJ}}{\text{kg}} = \underline{\mathbf{470.1\ kJ/kg}} \qquad \textbf{(1)}$$

(b) The thermal efficiency is given by

$$\eta = \frac{\dot{W}_{\text{cycle}}/\dot{m}}{\dot{Q}_{\text{in}}/\dot{m}}$$

Mass and energy rate balances for control volumes around the boiler for the Rankine cycle and the Carnot cycle, respectively, give

Rankine cycle:

$$\frac{\dot{Q}_{\text{in}}}{\dot{m}} = h_1 - h_4$$

$$\frac{\dot{Q}_{\text{in}}}{\dot{m}} = (2784.3 - 473.27)\frac{\text{kJ}}{\text{kg}} = \underline{\mathbf{2311.0\ kJ/kg}}$$

Carnot cycle:

$$\frac{\dot{Q}_{\text{in}}}{\dot{m}} = h_1 - h_{4'}$$

$$\frac{\dot{Q}_{\text{in}}}{\dot{m}} = (2784.3 - 1213.4)\frac{\text{kJ}}{\text{kg}} = \underline{\mathbf{1570.9\ kJ/kg}}$$

The thermal efficiency for each cycle is then

Rankine cycle:

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{597.5\ \text{kW}}{2311.0\ \text{kW}} = \underline{\mathbf{0.2585\ (25.85\%)}}$$

Carnot cycle:

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{470.1\ \text{kW}}{1570.9\ \text{kW}} = \underline{\mathbf{0.2993\ (29.93\%)}} \qquad \textbf{(2)}$$

Note (1): For an ideal cycle, the enclosed area on the *T-s* diagram represents the net power developed per unit of mass flowing. By inspection of Fig. P8.3, the Rankine cycle enclosed area 1-2-3-4-1 is greater than the Carnot cycle enclosed area 1-2-3'-4'-1 and thus the Rankine cycle has the greater net power per unit mass flowing. See Sec. 8.2.2 for discussion.

Although the turbine produces the same power per unit mass of steam flowing in both cycles, the Carnot cycle pump requires more power input per unit mass of steam flowing (for it is a liquid-vapor mixture) than the pump in the Rankine cycle (for it is a liquid only) to achieve the same pressure increase.

Note (2): For an ideal cycle, the thermal efficiency tends to increase as the average temperature at which energy is added by heat transfer increases (see discussion in the box of Sec. 8.2.3). By inspection of Fig. P8.3, the Carnot cycle has the higher average temperature at which energy is added by heat addition and thus the greater thermal efficiency.

Although it produces more power per unit mass of steam flowing, the Rankine cycle requires a greater rate of heat transfer into the cycle since energy input is required to raise the temperature of the liquid exiting the pump to the saturation temperature as well as to achieve phase change from saturated liquid to saturated vapor. The Carnot cycle requires a lower rate of heat transfer into the cycle, since energy input is required only to achieve phase change from saturated liquid to saturated vapor.

PROBLEM 8.4

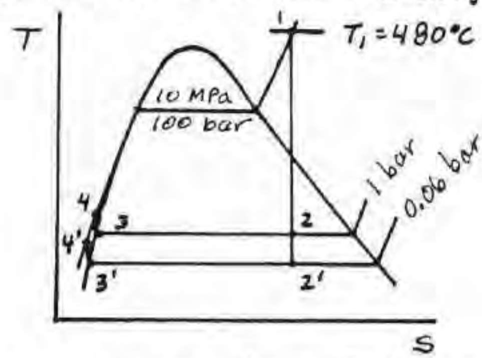
KNOWN: Water is the working fluid in an ideal Rankine cycle with superheated vapor entering the turbine. Data are given in Problem 8.2.

FIND: Plot each of the quantities calculated in Problem 8.2 versus condenser pressure ranging from 6 kPa to 0.1 MPa.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.2.

ENGINEERING MODEL: See solution to Problem 8.2.

ANLYSIS: The procedure for fixing each of the principal states is the same as in Problem 8.2. State 1 is fixed, and states 2, 3, and 4 all vary with condenser pressure, as illustrated on the T-s diagram below



For a sample calculation, see the solution to Problem 8.2.

IT Code:

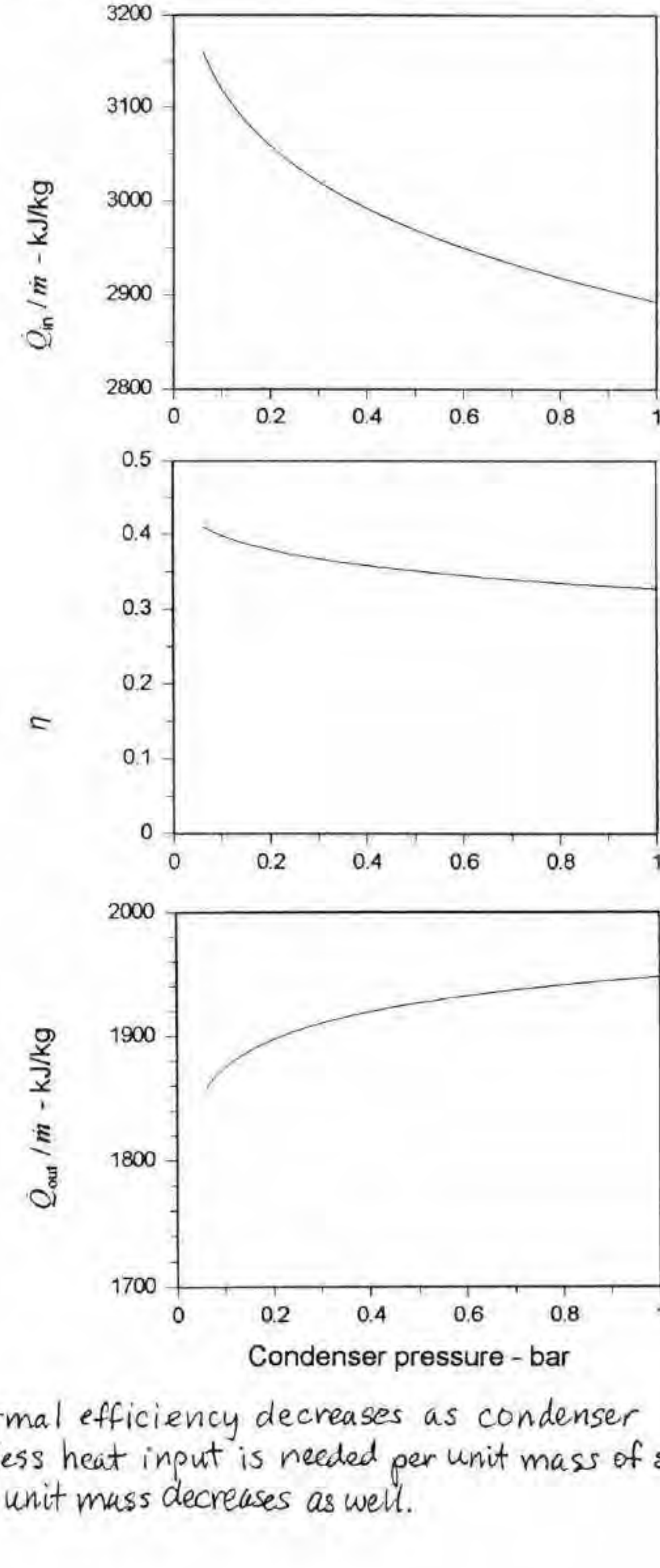
```
p1 = 100  // bar
T1 = 480  // °C
p2 = 0.06  // bar
mdot = 1
// Turbine
h1 = h_PT("Steam", p1, T1)
s1 = s_PT("Steam", p1, T1)
s2 = s1
h2 = h_Ps("Water", p2, s2)
Wdott = mdot * (h1 - h2)
// Condenser
p3 = p2
x3 = 0
h3 = hsat_Px("Water", p3, x3)
Qdotout = mdot * (h2 - h3)
// Pump
v3 = vsat_Px("Water", p3, x3)
p4 = p1
h4 = h3 + v3*(p4 - p3)*(10^5/10^3)
Wdotp = mdot * (h4 - h3)
// Cycle parameters
Wdotnet = Wdott - Wdotp
eta = Wdotnet / Qdotin
Qdotin = mdot * (h1 - h4)
```

IT Results:

p_2 (bar)	$\dot{Q}_{in}/\dot{m}$	$\dot{Q}_{out}/\dot{m}$	$\dot{W}_t/\dot{m}$	$\dot{W}_p/\dot{m}$	$\dot{W}_{net}/\dot{m}$	η	h_1	h_2	h_3	h_4
0.06	3160	1858	1311	10.06	1301	0.4119	3321	2009	151	161.1
1	2893	1947	955.8	10.33	945.5	0.3269	3321	2365	417.9	428.3

PROBLEM 8.4 (Cont'd)

Plotting for the entire range of condenser pressures



The thermal efficiency decreases as condenser pressure increases. Although less heat input is needed per unit mass of steam flowing, the net work per unit mass decreases as well.

PROBLEM 8.5

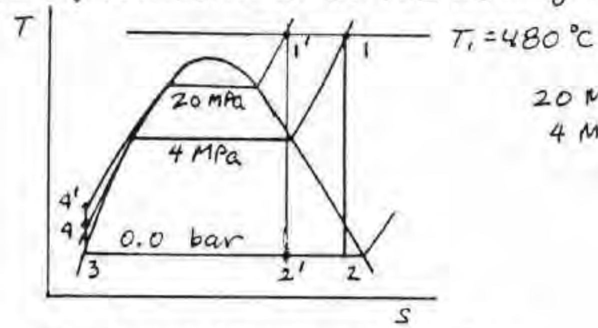
KNOWN: Water is the working fluid in an ideal Rankine cycle with superheated vapor at the turbine inlet. Data are given in Problem 8.2.

FIND: Plot each of the quantities calculated in Problem 8.2 versus steam generator pressure ranging from 4 to 20 MPa while maintaining constant turbine inlet temperature.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.2.

ENGINEERING MODEL: See solution to Problem 8.2.

ANALYSIS: The procedure for fixing each of the principal states is the same as in Problem 8.2. State 3 is fixed, and states 1, 2, and 4 all vary with steam generator pressure, as illustrated on the T-s diagram below



20 MPa = 200 bar
4 MPa = 40 bar

For a sample calculation, see the solution to problem 8.2.

IT Code

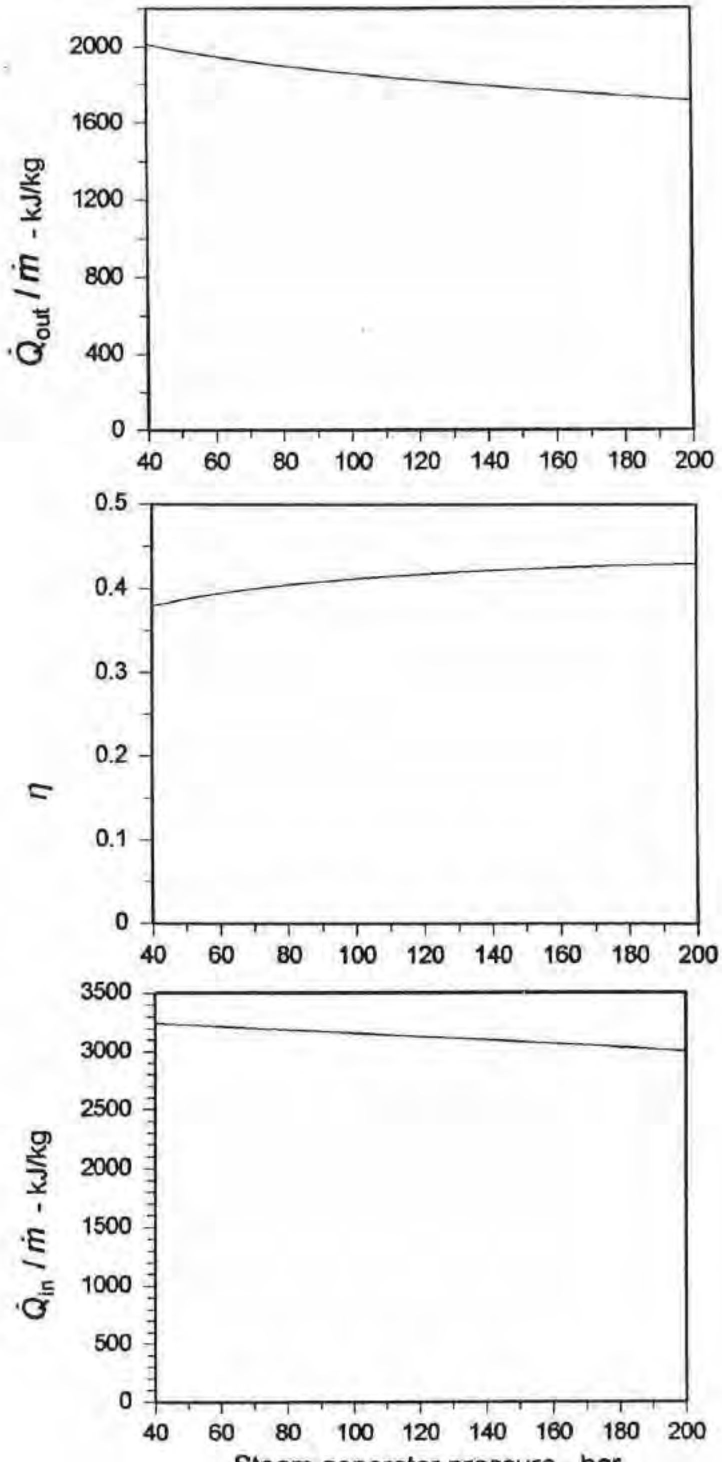
```
p1 = 100  // bar
T1 = 480  // deg C
p2 = 0.06  // bar
mdot = 1
// Turbine
h1 = h_PT("Steam", p1, T1)
s1 = s_PT("Steam", p1, T1)
s2 = s1
h2 = h_Ps("Water", p2, s2)
Wdott = mdot * (h1 - h2)
// Condenser
p3 = p2
x3 = 0
h3 = hsat_Px("Water", p3, x3)
Qdotout = mdot * (h2 - h3)
// Pump
v3 = vsat_Px("Water", p3, x3)
p4 = p1
h4 = h3+v3*(p4 - p3)*(10^5/10^3)
Qdotin = mdot * (h1 - h4)
Wdotp = mdot * (h4 - h3)
Wdotnet = Wdott - Wdotp
eta = Wdotnet / Qdotin
```

IT Results:

p_1 (bar)	$\dot{Q}_{in}/\dot{m}$	$\dot{Q}_{out}/\dot{m}$	$\dot{W}_t/\dot{m}$	$\dot{W}_p/\dot{m}$	$\dot{W}_{net}/\dot{m}$	η	h_1	h_2	h_3	h_4
40	3244	2014	1234	4.02	1230	0.3792	3399	2165	151	155.1
200	3160	1858	1311	10.06	1301	0.4119	3321	2009	151	161.1

PROBLEM 8.5 (Cont'd)

Plotting for the entire range of steam generator pressures



The thermal efficiency increases with increasing steam generator pressure. Although less heat input is need per unit mass of steam flowing, the net work per unit mass decreases as well.

8.6 A Carnot vapor power cycle operates with water as the working fluid. Saturated liquid enters the boiler at 1800 lbf/in.^{2}, and saturated vapor enters the turbine (State 1). The condenser pressure is 1.2 lbf/in.^{2} The mass flow rate of steam is 1 x 10^6 lb/h. Data at key points in the cycle are provided in Fig. P8.6. Determine

(a) the thermal efficiency.
(b) the back work ratio.
(c) the net power developed, in Btu/h.
(d) the rate of heat transfer to the working fluid passing through the boiler, in Btu/h.

State	p (lbf/in.^{2})	h (Btu/lb)
1	1800	1150.4
2	1.2	734.4
3	1.2	840.9
4	1800	648.3

Fig. P8.6

KNOWN: Water is the working fluid in a Carnot vapor power cycle. Data are given at key state points in the cycle, and the mass flow rate of steam is known.

FIND: Determine (a) the thermal efficiency, (b) the back work ratio, (c) the net power developed, and (d) the rate of heat transfer to the working fluid passing through the boiler.

SCHEMATIC AND GIVEN DATA:

State	p (lbf/in.^{2})	h (Btu/lb)
1	1800	1150.4
2	1.2	734.4
3	1.2	840.9
4	1800	648.3

$\dot{m}$ = 1 x 10^6 lb/h
$p_1 = p_4 = p_{boiler}$ = 1800 lbf/in.^{2}
$p_2 = p_3 = p_{condenser}$ = 1.2 lbf/in.^{2}

See Cycle 1-2-3'-4' of Fig. 8.5 for a diagram of the cycle.

ENGINEERING MODEL:
(1) Each component is analyzed as a control volume at steady state.
(2) All processes of the working fluid are internally reversible
(3) The turbine and pump operate adiabatically
(4) Kinetic and potential energy effects are negligible.

ANALYSIS:

The specific enthalpy at each of the principle states is given. The temperatures of the boiler and condenser are the *saturation temperatures* corresponding to the respective pressures. From Table A-3E

$T_1 = T_H = 621.21\ ^oF = 1080.88\ ^oR$
$T_2 = T_C = 107.88\ ^oF = 567.55\ ^oR$

(a) To find the thermal efficiency, use the boiler and condenser temperatures with the expression for the Carnot cycle thermal efficiency, as follows:

$$\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{567.55}{1080.88} = 0.475\ (47.5\%)$$

(b) The back work ratio is

$$bwr = \frac{\dot{m}\ (h_4 - h_3)}{\dot{m}\ (h_1 - h_2)} = \frac{(648.3 - 472.0)}{(1150.4 - 735.7)} = 0.425$$

(c) The net power developed is

$$\dot{W}_{cycle} = \dot{m}[(h_1 - h_2) - (h_4 - h_3)]$$

$$\dot{W}_{cycle} = \left(1\times10^6 \frac{\text{lb}}{\text{h}}\right)[(1150.4 - 735.7) - (648.3 - 472.0)]\frac{\text{Btu}}{\text{lb}} = 238.4\times10^8 \text{ Btu/h}$$

(d) The rate of heat transfer to the working fluid passing through the boiler is

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = \left(1\times10^6 \frac{\text{lb}}{\text{h}}\right)(1150.4 - 648.3)\frac{\text{Btu}}{\text{lb}} = 502.1\times10^8 \text{ Btu/h}$$

The thermal efficiency can be calculated alternatively as

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \frac{238.4\ \text{x}10^8}{502.1\ \text{x}10^8} = 0.475(47.5\%)$$

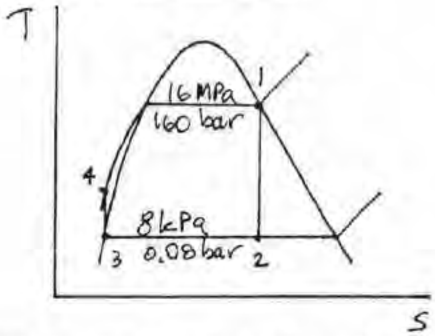
which is identical to the result of part (a), as expected.

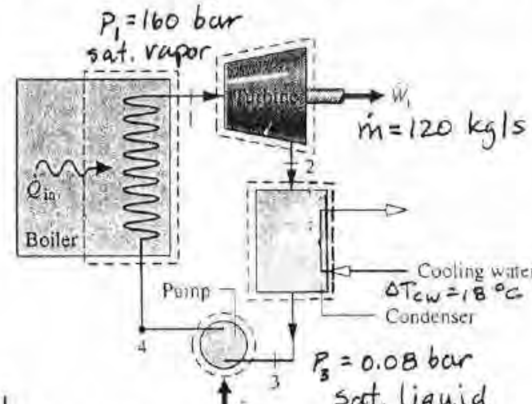
PROBLEM 8.7

KNOWN: Water is the working fluid in an ideal Rankine cycle. The condenser pressure and the turbine inlet state are specified. The mass flow of steam is given.

FIND: Determine (a) the net power, (b) the rate of heat transfer to the steam passing through the boiler, (c) the thermal efficiency, and (d) the mass flow rate of cooling water passing through the condenser.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 8.1.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 160$ bar, sat. vapor $\Rightarrow h_1 = 2580.6$ kJ/kg, $s_1 = 5.2455 \frac{kJ}{kg \cdot K}$

State 2: $p_2 = 0.08$ bar, $s_2 = s_1 \Rightarrow x_2 = \frac{s_2 - s_{f2}}{s_{g2} - s_{f2}} = 0.6091$, $h_2 = 1637.6$ kJ/kg

State 3: $p_3 = 0.08$ bar, sat. liquid $\Rightarrow h_3 = 173.88$ kJ/kg

State 4: $p_4 = 160$ bar, $h_4 \approx h_3 + v_3(p_4 - p_3)$

$$= 173.88 \frac{kJ}{kg} + (1.0084 \times 10^{-3} \frac{m^3}{kg})(160 - 0.08)\,\text{bar} \left| \frac{10^5\,N/m^2}{1\,\text{bar}} \right| \left| \frac{1\,kJ}{10^3\,N \cdot m} \right|$$

$$= 173.88 + 16.13 = 190.01\ \text{kJ/kg}$$

(a) The net power developed is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p = \dot{m}[(h_1 - h_2) - (h_4 - h_3)]$$

$$= 120 \frac{kJ}{kg}[(2580.6 - 1637.6) - (190.01 - 173.88)] \frac{kJ}{kg} \left| \frac{1\,kW}{1\,kJ/s} \right|$$

$$= 1.112 \times 10^5\ \text{kW} \longleftarrow \dot{W}_{cycle}$$

(b) For the steam passing through the boiler

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = (120)(2580.6 - 190.01) = 2.869 \times 10^5\ \text{kW} \longleftarrow \dot{Q}_{in}$$

(c) $$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = 0.388\ (38.8\%) \longleftarrow \eta$$

(d) For the control volume enclosing the condenser (assuming $\Delta h_{cw} = c_w \Delta T_{cw}$)

$$\dot{m}_{cw} = \frac{\dot{m}(h_2 - h_3)}{c_w \Delta T_{cw}}$$

With $c_w = 4.179$ kJ/kg·K from Table A-19

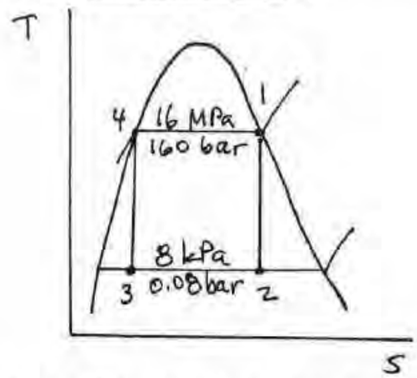
$$\dot{m}_{cw} = \frac{(120)(1637.6 - 173.88)}{(4.179)(18)} = 2335\ \frac{kg}{s} \longleftarrow \dot{m}_{cw}$$

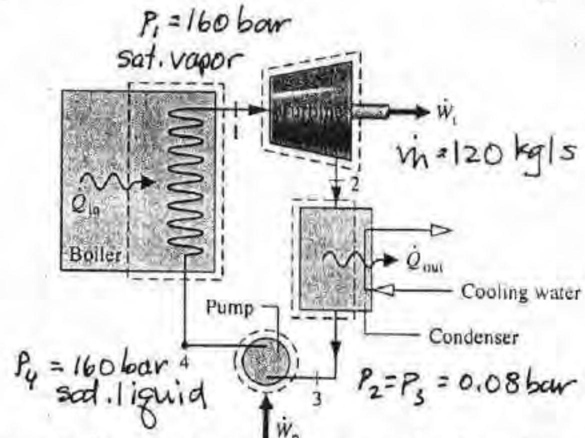
PROBLEM 8.8

KNOWN: Water is the working fluid in a Carnot vapor power cyle. Saturated liquid enters the boiler and saturated vapor enters the turbine, and the boiler pressure is specified. The condenser pressure is specified, and the mass flow rate of steam is given.

FIND: Determine (a) the thermal efficiency, (b) the back work ratio, (c) the net power, and (d) the rate of heat transfer from the working fluid passing through the condenser.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) Each component is analyzed as a control volume at steady state. (2) All processes are internally reversible. (3) the turbine and the pump operate adiabatically. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 160$ bar, sat. vapor $\Rightarrow h_1 = 2580.6$ kJ/kg, $s_1 = 5.2455$ kJ/kg·K

State 2: $p_2 = 0.08$ bar, $s_2 = s_1 \Rightarrow x_2 = \dfrac{s_2 - s_{f2}}{s_{g2} - s_{f2}} = 0.6091$, $h_2 = 1637.6$ kJ/kg

State 3: $p_3 = 0.08$ bar, $s_3 = s_4 = s_{f\,@\,160\,bar} = 3.7461$ kJ/kg·K

$$\Rightarrow x_3 = \frac{s_3 - s_{f3}}{s_{g3} - s_{f3}} = 0.4130, \quad h_3 = 1166.4 \text{ kJ/kg}$$

State 4: $p_4 = 160$ bar, sat. liquid $\Rightarrow h_4 = 1650.1$ kJ/kg

(a) The thermal efficiency of the Carnot cycle is given by Eq. 5.9. With data from Table A-3; $T_H = T_{sat\,@\,160\,bar} = 620.55$ K and $T_C = T_{sat\,@\,0.08\,bar} = 314.66$ K. Thus

① $$\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{314.66}{620.55} = 0.493 \ (49.3\%) \longleftarrow \quad \eta$$

(b) The back work ratio is

$$bwr = \frac{\dot{W}_P}{\dot{W}_t} = \frac{\cancel{\dot{m}}(h_4 - h_3)}{\cancel{\dot{m}}(h_1 - h_2)} = \frac{1650.1 - 1166.4}{2580.6 - 1637.6} = 0.513 \longleftarrow \quad bwr$$

(c) The net power developed is

② $$\dot{W}_{cycle} = \dot{m}[(h_1 - h_2) - (h_4 - h_3)] = \left(120 \frac{kg}{s}\right)[(2580.6 - 1637.6) - (1650.1 - 1166.4)] \frac{kJ}{kg} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 55{,}120 \text{ kW} \longleftarrow \quad \dot{W}_{cycle}$$

(d) For the condenser: $\dot{Q}_{out} = \dot{m}(h_2 - h_3) = (120)(1637.6 - 1166.4) = 56{,}540$ kW $\longleftarrow \quad \dot{Q}_{out}$

1. Alternatively, $\eta = \dot{W}_{cycle}/\dot{Q}_{in} = \dot{W}_{cycle}/(\dot{W}_{cycle} + \dot{Q}_{out}) = 0.4936$.

2. These results can be compared with the Rankine cycle of Probem 8.7.

PROBLEM 8.9

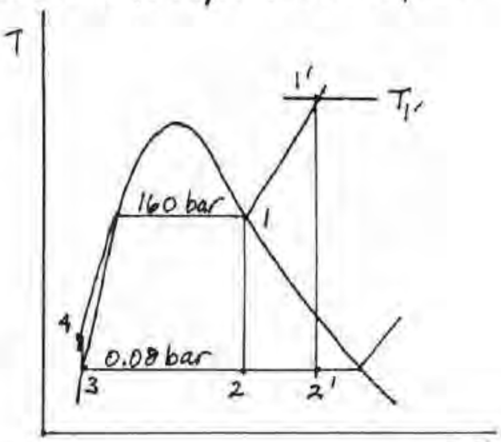
KNOWN: Water is the working fluid in an ideal Rankine cycle. The condenser and steam generator pressures are specified as in Problem 8.7.

FIND: Plot each of the quantities calculated in Problem 8.7 versus turbine inlet temperature ranging from saturation to superheated vapor.

SCHEMATIC & GIVEN DATA: See Problem 8.7.

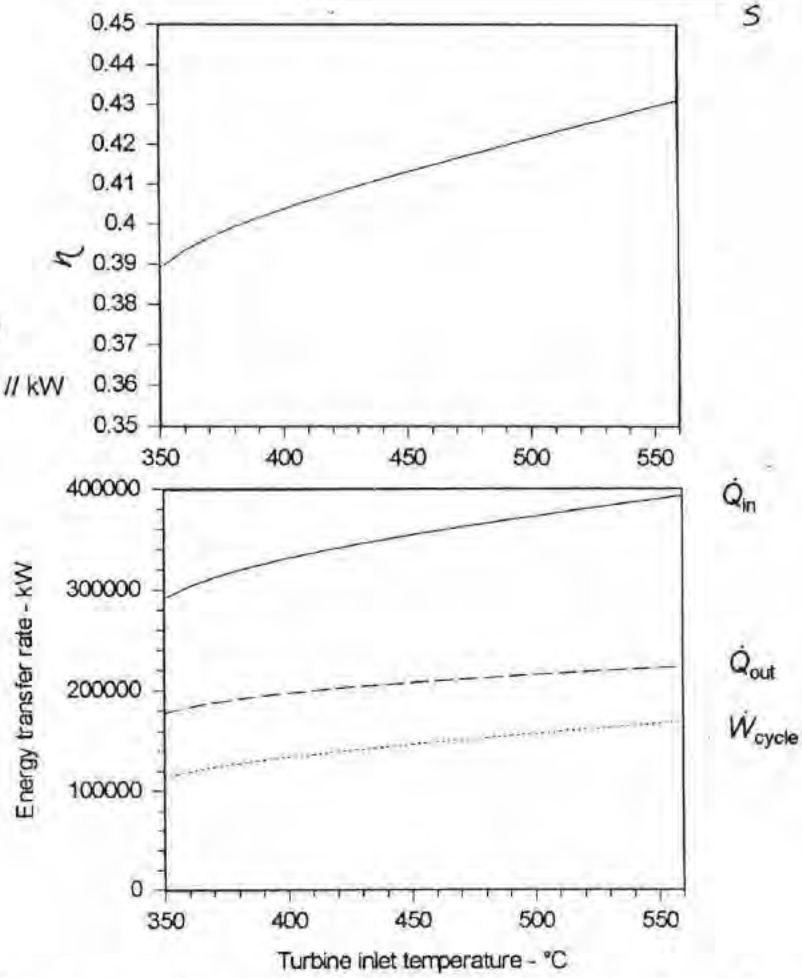
ENGINEERING MODEL: See Problem 8.7.

ANALYSIS: The turbine inlet and exit states change as in the T-s diagram:



IT Code

```
p1 =160  // bar
p2 = 0.08  // bar
mdot = 120  // kg/s
// Turbine
T1sat = Tsat_P("Steam", p1)
T1 = 350
h1 = h_PT("Steam", p1, T1)
s1 = s_PT("Steam", p1, T1)
s2 = s1
h2 = h_Ps("Water", p2, s2)
// Condenser
p3 = p2
x3 = 0
h3 = hsat_Px("Water", p3, x3)
// Pump
v3 = vsat_Px("Water", p3, x3)
p4 = p1
h4 = h3 + v3 * (p4 - p3) * (10^5 / 10^3)
// Cycle parameters
Wdotnet = mdot * ((h1 - h2) - (h4-h3))  // kW
Qdotin = mdot * (h1 - h4)  // kW
eta = Wdotnet/Qdotin
Qdotout = mdot * (h2 - h3)  // kW
```



IT Results ($T_1 = T_{sat\,@\,160\,bar}$)

$\dot{Q}_{in}$ = 2.868E5 kW

$\dot{Q}_{out}$ = 1.757E5 kW

$\dot{W}_{cycle}$ = 1.111E5 kW

η = 0.3875

h_1 = 2580 kJ/kJ

h_2 = 1638 kJ/kg

h_3 = 173.6 kJ/kg

h_4 = 189.8 kJ/kg

The cycle with superheating (1'-2'-3-4-1') has a higher average temperature of heat addition than the cycle without superheating (1-2-3-4-1), so the thermal efficiency is higher. The energy transfer rates vary accordingly.

8.10 Water is the working fluid in an ideal Rankine cycle. Steam enters the turbine at 1400 lbf/in.^{2} and 1000°F. The condenser pressure is 2 lbf/in.^{2} The net power output of the cycle is 1×10^9 Btu/h. Cooling water experiences a temperature increase from 60°F to 76°F, with negligible pressure drop, as it passes through the condenser. Determine for the cycle

(a) the mass flow rate of steam, in lb/h.

(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the steam generator.

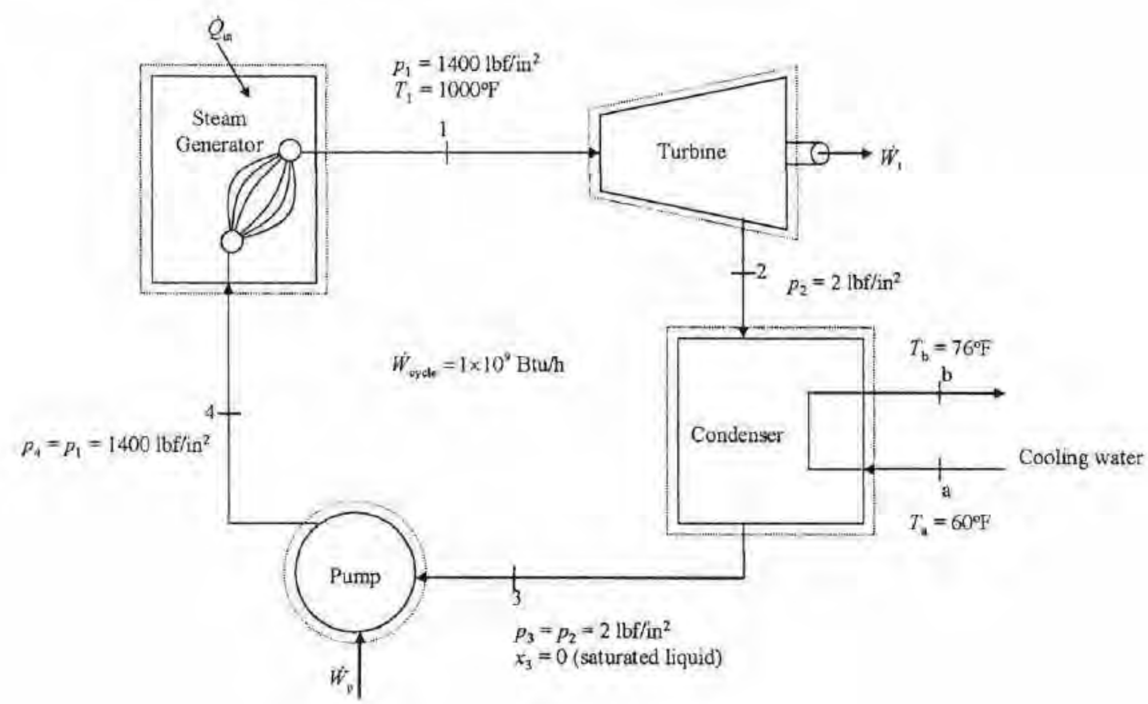
(c) the thermal efficiency.

(d) the mass flow rate of cooling water, in lb/h.

KNOWN: An ideal Rankine cycle with superheat operates with water as the working fluid. The net power output of the cycle is given.

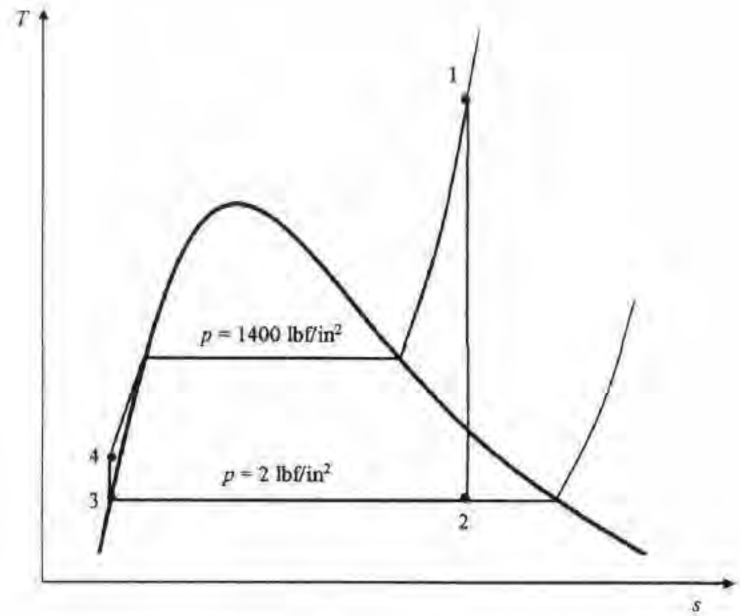
FIND: Determine the mass flow rate of steam, the rate of heat transfer to the working fluid passing through the steam generator, the thermal efficiency, and the mass flow rate of cooling water.

SCHEMATIC AND GIVEN DATA:



Problem 8.10 (Continued) – Page 2

T-s diagram



ENGINEERING MODEL:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. All processes of the working fluid are internally reversible.
3. The turbine and pump operate adiabatically.
4. Kinetic and potential energy effects are negligible.
5. Condensate exits the condenser as saturated liquid.
6. There is no heat transfer between the outside of the condenser and the surroundings.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 1400$ lbf/in.^{2}, $T_1 = 1000^\circ$F $\rightarrow$ From Table A-4E: $h_1 = 1493.5$ Btu/lb and $s_1 = 1.6094$ Btu/(lb·°R)

State 2: $p_2 = 2$ lbf/in.^{2}, $s_2 = s_1 \rightarrow$ From Table A-3E: $x_2 = (1.6094 - 0.1750)/1.7448 = 0.8221$ and $h_2 = h_{f2} + x_2 h_{fg2} = 94.02 + (0.8221)(1022.1) = 934.29$ Btu/lb

State 3: $p_3 = p_2 = 2$ lbf/in.^{2}, sat liq. $\rightarrow$ From Table A-3E: $h_3 = h_{f3} = 94.02$ Btu/lb and $v_3 = v_{f3} = 0.01623$ ft^3/lb

State 4: $h_4 \approx h_3 + v_3(p_4 - p_3)$

$$h_4 = 94.02\ \text{Btu/lb} + 0.01623\left(\frac{\text{ft}^3}{\text{lb}}\right)(1400-2)\left(\frac{\text{lbf}}{\text{in}^2}\right)\left|\frac{144\ \text{in}^2}{\text{ft}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right| = 98.22\ \text{Btu/lb}$$

Problem 8.10 (Continued) – Page 3

(a) The mass flow rate of steam is found as follows. Mass and energy rate balances for control volumes enclosing the turbine and pump give

$$\dot{W}_t = \dot{m}(h_1 - h_2) \quad \text{and} \quad \dot{W}_p = \dot{m}(h_4 - h_3)$$

The net power of the cycle is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p = \dot{m}[(h_1 - h_2) - (h_4 - h_3)]$$

Solving for $\dot{m}$

$$\dot{m} = \frac{\dot{W}_{cycle}}{[(h_1 - h_2) - (h_4 - h_3)]}$$

Inserting values

$$\dot{m} = \frac{1\times10^9 \dfrac{\text{Btu}}{\text{h}}}{\left(1493.5\dfrac{\text{Btu}}{\text{lb}} - 934.29\dfrac{\text{Btu}}{\text{lb}}\right) - \left(98.22\dfrac{\text{Btu}}{\text{lb}} - 94.02\dfrac{\text{Btu}}{\text{lb}}\right)} = \underline{\mathbf{1.80 \times 10^6\ lb/h}}$$

(b) The rate of heat transfer to the working fluid passing through the steam generator can be determined by applying mass and energy balances to a control volume around the steam generator to give

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = (1.80\times10^6\ \text{lb/h})(1493.5\ \text{Btu/lb} - 98.22\ \text{Btu/lb}) = \underline{\mathbf{2.51\times10^9\ Btu/h}}$$

(c) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = (1\times10^9\ \text{Btu/h})/(2.51\times10^9\ \text{Btu/h}) = \underline{\mathbf{0.3984\ (39.84\%)}}$$

(d) The mass flow rate of cooling water through the condenser is determined by applying steady state mass and energy rate balances to a control volume enclosing the condenser.

$$\dot{m}_2 = \dot{m}_3 \equiv \dot{m} \text{ and } \dot{m}_a = \dot{m}_b \equiv \dot{m}_{cw}$$

and

$$0 = \dot{m}(h_2 - h_3) + \dot{m}_{cw}(h_a - h_b)$$

Solving for $\dot{m}_{cw}$

$$\dot{m}_{cw} = \frac{\dot{m}(h_2 - h_3)}{h_b - h_a}$$

Problem 8.10 (Continued) – Page 4

For the cooling water, $h \approx h_f(T)$. Consequently, from Table A-2E

$h_a = 28.08$ Btu/lb
$h_b = 44.09$ Btu/lb

Substituting values

$$\dot{m}_{cw} = \frac{\left(1.80 \times 10^6 \dfrac{\text{lb}}{\text{h}}\right)\left(934.29 \dfrac{\text{Btu}}{\text{lb}} - 94.02 \dfrac{\text{Btu}}{\text{lb}}\right)}{44.09 \dfrac{\text{Btu}}{\text{lb}} - 28.08 \dfrac{\text{Btu}}{\text{lb}}} = \underline{\mathbf{9.45 \times 10^7\ lb/h}}$$

8.11 Plot each of the quantities calculated in Problem 8.10 versus condenser pressure ranging from 0.3 lbf/in.^{2} to 14.7 lbf/in.^{2} Maintain constant net power output. Discuss.

IT Code

```
/*
Problem 8.10

KNOWN:  Water is working fluid in an ideal Rankine cycle.  Data are given in Problem 8.10.

FIND:  Plot each of the quantities calculated in Problem 8.10 versus condenser pressure ranging from 0.3
to 14.7 lbf/in^2.  Maintain constant net power output.

SCHEMATIC AND GIVEN DATA:  See solution to Problem 8.10.

ENGINEERING MODEL:
1.  Each component is analyzed as a control volume at steady state.
2.  All processes of the working fluid are internally reversible.
3.  The turbine and pump operate adiabatically.
4.  Kinetic and potential energy effects are negligible.
5.  Condensate exits the condenser as saturated liquid.
6.  There is no heat transfer between the outside of the condenser and the surroundings.

*/

p1 = 1400 // lbf/in^2
T1 = 1000 // deg F
p2 = 2 // lbf/in^2
p3 = p2
x3 = 0
p4 = p1
Wdotcycle = 1.0E09 // Btu/h
Twin = 60 // deg F
Twout = 76 // deg F

h1 = h_PT("Water/Steam", p1, T1) // Btu/lb
s1 = s_PT("Water/Steam", p1, T1) // Btu/(lb*deg R)
s2 = s1 // Btu/(lb * deg R)
h2 = h_Ps("Water/Steam", p2, s2) // Btu/lb
h3 = hsat_Px("Water/Steam", p3, x3) // Btu/lb
v3 = vsat_Px("Water/Steam", p3, x3) // ft^3/lb
h4 = h3 + v3 * (p4 - p3) * (144/778.17) // BTU/lb
pwin = Psat_T("Water/Steam", Twin) // lbf/in^2
hwin = hsat_Px("Water/Steam", pwin, 0) // Btu/lb
pwout = Psat_T("Water/Steam", Twout) // lbf/in^2
hwout = hsat_Px("Water/Steam", pwout, 0) // Btu/lb

mdot = Wdotcycle / ((h1 - h2) - (h4 - h3)) // lb/h
Wdott = mdot * (h1 - h2) // Btu/h
Wdotp = mdot * (h4 - h3) // Btu/h

Qdotin = mdot * (h1 - h4) // Btu/h
eta = Wdotcycle/Qdotin
mdotw = (mdot * (h2 - h3))/(hwout - hwin) // lb/h
```

Problem 8.11 (Continued) – Page 2

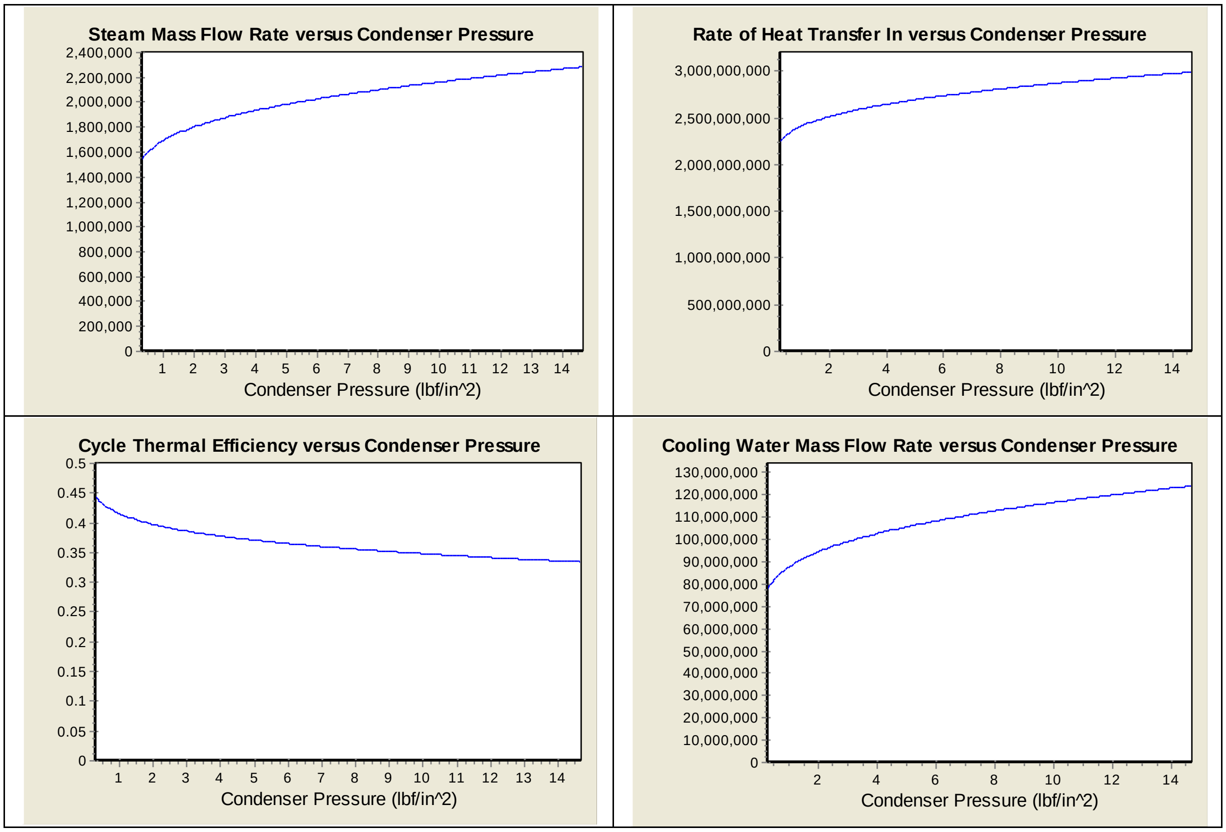
IT Results for p_2 = 2 lbf/in.^{2}

eta	0.3979
h1	1493
h2	934.1
h3	94.12
h4	98.32
hwin	27.63
hwout	43.67
mdot	1.80E+06
mdotw	9.43E+07
p3	2
p4	1400
pwin	0.2563
pwout	0.4446
Qdotin	2.51E+09
s1	1.609
s2	1.609
v3	0.01623
Wdotp	7.57E+06
Wdott	1.01E+09
p1	1400
p2	2
T1	1000
Twin	60
Twout	76
Wdotcycle	1.00E+09
x3	0

The results compare favorably with those of Problem 8.10.

Problem 8.11 (Continued) – Page 3

<u>Plots</u>



<u>Discussion</u>

The plots indicate that as condenser pressure is increased while maintaining constant net power output: steam mass flow rate increases, heat transfer rate into the cycle increases, and cycle thermal efficiency decreases. As thermal efficiency decreases with increasing condenser pressure, the rate energy is rejected by the condenser must increase, as evidenced by the increasing cooling water flow rate.

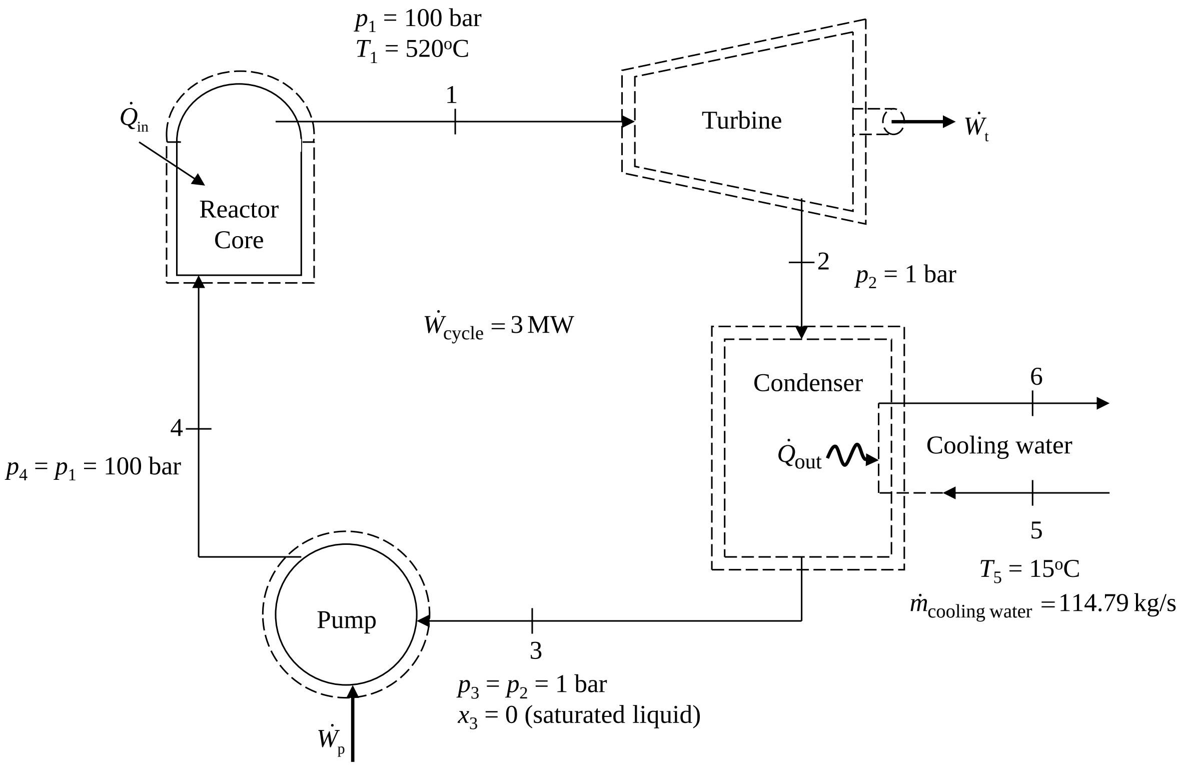
From a practical standpoint, reducing condenser pressure while maintaining constant net power output is desirable since steam mass flow rate decreases, heat transfer rate into the cycle decreases, and cycle thermal efficiency increases. Since less energy is rejected in the condenser at lower condenser pressure, required cooling water mass flow rate is reduced.

8.12 A nuclear power plant based on the Rankine cycle operates with a boiling-water reactor to develop net cycle power of 3 MW. Steam exits the reactor core at 100 bar, 520°C and expands through the turbine to the condenser pressure of 1 bar. Saturated liquid exits the condenser and is pumped to the reactor pressure of 100 bar. Isentropic efficiencies of the turbine and pump are 81% and 78%, respectively. Cooling water enters the condenser at 15°C with a mass flow rate of 114.79 kg/s. Determine
(a) the thermal efficiency.
(b) the temperature of the cooling water exiting the condenser, in °C.

KNOWN: A nuclear power plant based on the Rankine cycle operates with a boiling-water reactor between specified pressures of 100 bar and 1 bar to produce 3 MW of net power.

FIND: Determine (a) the thermal efficiency and (b) the temperature of the cooling water exiting the condenser, in °C.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. Flow through the reactor core and condenser occurs at constant pressure.
3. Stray heat transfer in the turbine, condenser, and pump is ignored.
4. Kinetic and potential energy effects are negligible.
5. Condensate exits the condenser as saturated liquid.
6. For the cooling water, $h \approx h_f(T)$.

ANALYSIS: First fix each principal state.

State 1: p_1 = 100 bar, T_1 = 520°C → h_1 = 3425.1 kJ/kg, s_1 = 6.6622 kJ/kg·K

State 2s: $p_{2s} = p_2$ = 1 bar, $s_{2s} = s_1$ = 6.6622 kJ/kg·K → x_{2s} = 0.8849, h_{2s} = 2415.6 kJ/kg

State 2: p_2 = 1 bar, h_2 = 2607.4 kJ/kg (see below)

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \rightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 3425.1\frac{\text{kJ}}{\text{kg}} - (0.81)(3425.1 - 2415.6)\frac{\text{kJ}}{\text{kg}} = 2607.4 \text{ kJ/kg}$$

State 3: p_3 = 1 bar, saturated liquid → $h_3 = h_{f3}$ = 417.46 kJ/kg, $v_3 = v_{f3}$ = 0.0010432 m³/kg

State 4: p_4 = 100 bar, h_4 = 430.70 kJ/kg (see below)

$$\eta_p = \frac{v_3(p_4 - p_3)}{h_4 - h_3} \rightarrow h_4 = h_3 + \frac{v_3(p_4 - p_3)}{\eta_p}$$

$$h_4 = 417.46\frac{\text{kJ}}{\text{kg}} + \frac{(0.0010432\frac{\text{m}^3}{\text{kg}})(100-1)\text{ bar}}{0.78}\left|\frac{100\text{ kPa}}{1\text{ bar}}\right|\left|\frac{1000\frac{\text{N}}{\text{m}^2}}{1\text{ kPa}}\right|\left|\frac{1\text{ kJ}}{1000\text{ N}\cdot\text{m}}\right| = 430.70 \text{ kJ/kg}$$

State 5: T_5 = 15°C, liquid → $h_5 \approx h_{f5}$ at T_5 = 62.99 kJ/kg

(a) The thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m} - \dot{W}_p/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$$

Substituting enthalpy values and solving yield

$$\eta = \frac{(3425.1 - 2607.4)\text{ kJ/kg} - (430.70 - 417.46)\text{ kJ/kg}}{(3425.1 - 430.70)\text{ kJ/kg}} = \mathbf{0.2687\ (26.87\%)}$$

(b) The temperature of the cooling water exiting the condenser is determined by writing an energy balance for the condenser, solving for the specific enthalpy of the cooling water exiting the condenser, and finding the corresponding temperature for the specific enthalpy of the liquid in Table 2. With no stray heat transfer with the surroundings and no work, the energy balance for the condenser reduces to

$$0 = \dot{m}(h_2 - h_3) + \dot{m}_{cw}(h_5 - h_6)$$

where $\dot{m}$ is the mass flow rate of the steam and $\dot{m}_{\text{cw}}$ is the mass flow rate of the cooling water. Rearranging to solve for the specific enthalpy of the cooling water exiting the condenser, h_6, gives

$$h_6 = h_5 + \frac{\dot{m}(h_2 - h_3)}{\dot{m}_{\text{cw}}}$$

The mass flow rate of steam, $\dot{m}$, is determined from the net cycle power

$$\dot{m} = \frac{\dot{W}_{\text{cycle}}}{\dot{W}_{\text{t}}/\dot{m} - \dot{W}_{\text{p}}/\dot{m}} = \frac{\dot{W}_{\text{cycle}}}{(h_1 - h_2) - (h_4 - h_3)}$$

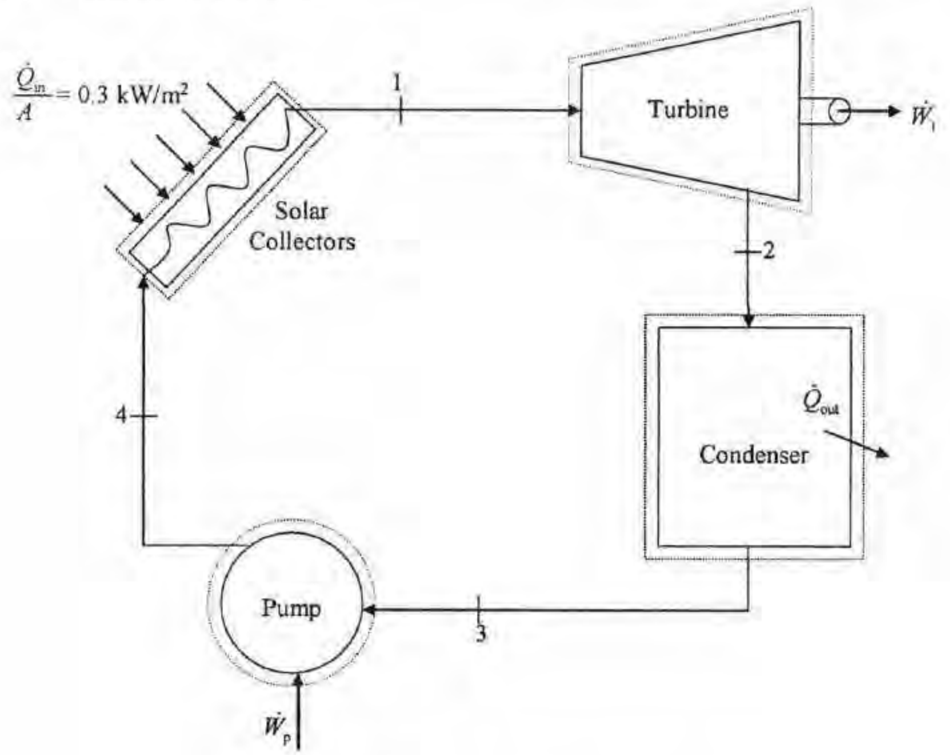
$$\dot{m} = \frac{3000\ \text{kW}}{(3425.1 - 2607.4)\ \text{kJ/kg} - (430.70 - 417.46)\ \text{kJ/kg}}\left|\frac{1\ \text{kJ/s}}{1\ \text{kW}}\right| = 3.73\ \text{kg/s}$$

Solving for the specific enthalpy at the condenser exit gives

$$h_6 = 62.99\ \text{kJ/kg} + \frac{(3.73\ \text{kg/s})(2607.4 - 417.46)\ \text{kJ/kg}}{114.79\ \text{kg/s}} = 134.15\ \text{kJ/kg}$$

Since the cooling water is liquid, $h_6 \approx h_{f6}$ at T_6. From Table A-2, **$T_6 \approx 32^{\text{o}}\text{C}$**.

8.13 Figure P8.13 provides steady-state operating data for a solar power plant that operates on a Rankine cycle with Refrigerant 134a as its working fluid. The turbine and pump operate adiabatically. The rate of energy input to the collectors from solar radiation is 0.3 kW per m^2 of collector surface area, with 60% of the solar input to the collectors absorbed by the refrigerant as it passes through the collectors. Determine the solar collector surface area, in m^2 per kW of power developed by the plant. Discuss possible operational improvements that could reduce the required collector surface area.



State	P (bar)	h (kJ/kg)	x
1	18	276.83	1
2	7	261.01	0.9952
3	7	86.78	0
4	18	87.93	--

Fig P8.13

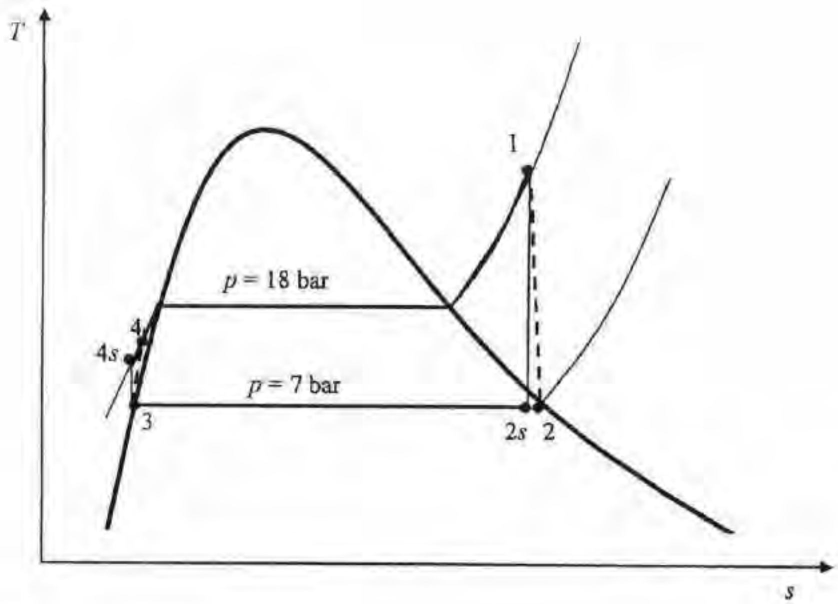
KNOWN: A Rankine cycle operates with Refrigerant 134a as the working fluid. The rate of energy input to the collectors per m^2 of collector surface area from solar radiation is given. The percentage of energy input to the solar collectors absorbed by Refrigerant 134a is given.

FIND: Determine the solar collector surface area per kW of power developed by the plant.

SCHEMATIC AND GIVEN DATA: See problem statement above and T-s diagram below.

Problem 8.13 (Continued) – Page 2

T-s diagram



ENGINEERING MODEL:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The turbine and pumps operate adiabatically.
3. Kinetic and potential energy effects are negligible.
4. Refrigerant enters the turbine as saturated vapor.
5. Refrigerant exits the condenser as saturated liquid.
6. There is no heat transfer between the outside of the condenser and the surroundings.

ANALYSIS: The mass flow rate of refrigerant per kW of power produced can be determined from the cycle power

$$\dot{W}_{cycle} = \dot{m}\left[\frac{\dot{W}_t}{\dot{m}} - \frac{\dot{W}_p}{\dot{m}}\right] = \dot{m}\left[(h_1 - h_2) - (h_4 - h_3)\right]$$

$$\dot{m} = \frac{\dot{W}_{cycle}}{(h_1 - h_2) - (h_4 - h_3)} = \frac{1\,\frac{\text{kJ}}{\text{s}}}{\left[(276.83 - 261.01) - (87.93 - 86.78)\right]\frac{\text{kJ}}{\text{kg}}} = 0.06817 \text{ kg/s}$$

Thus, rate of heat transfer into the refrigerant, $\dot{Q}_{in}$, per kW of power produced as refrigerant passes through the solar collectors is

Problem 8.13 (Continued) – Page 3

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = \left(0.06817\,\frac{\text{kg}}{\text{s}}\right)\left[(276.83 - 87.93)\frac{\text{kJ}}{\text{kg}}\right]\left|\frac{\text{kW}}{\frac{\text{kJ}}{\text{s}}}\right| = 12.88\ \text{kW}$$

Since 60% of the solar energy received by the collectors is absorbed by the refrigerant,

$$\dot{Q}_{in} = 0.6\left(\dot{Q}_{in}\right)_{collectors}$$

Solving for rate of energy input to the solar collectors per kW of power produced

$$\left(\dot{Q}_{in}\right)_{collectors} = \frac{\dot{Q}_{in}}{0.6} = \frac{12.88\ \text{kW}}{0.6} = 21.47\ \text{kW}$$

The collector area is determined from the rate of energy input to the collectors from solar radiation per m^2 of collector surface area.

$$\frac{\left(\dot{Q}_{in}\right)_{collectors}}{A} = 0.3\,\frac{\text{kW}}{\text{m}^2}$$

Solving for collector area and substituting the value for heat transfer rate in give collector area per kW of power produced

$$A = \frac{\left(\dot{Q}_{in}\right)_{collectors}}{0.3\,\frac{\text{kW}}{\text{m}^2}} = \frac{21.47\ \text{kW}}{0.3\,\frac{\text{kW}}{\text{m}^2}} = \underline{\mathbf{71.57\ m^2}}$$

Discussion

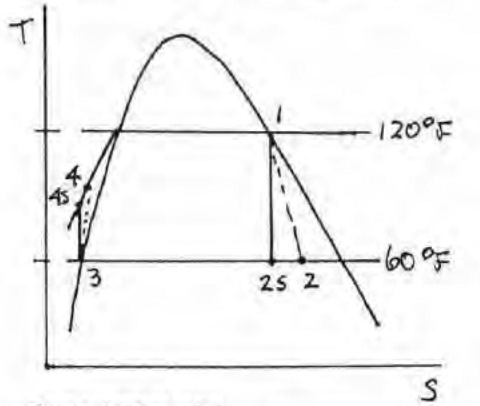
To reduce required collector surface area, one could improve the cycle, the solar collector, or both. Improved cycle thermal efficiency can be achieved by reducing irreversibilities associated with components in the cycle. Improved solar collector performance can be achieved by reducing irreversibilities and stray heat transfer from the collector to allow the refrigerant to absorb more of the incident energy reaching the collector.

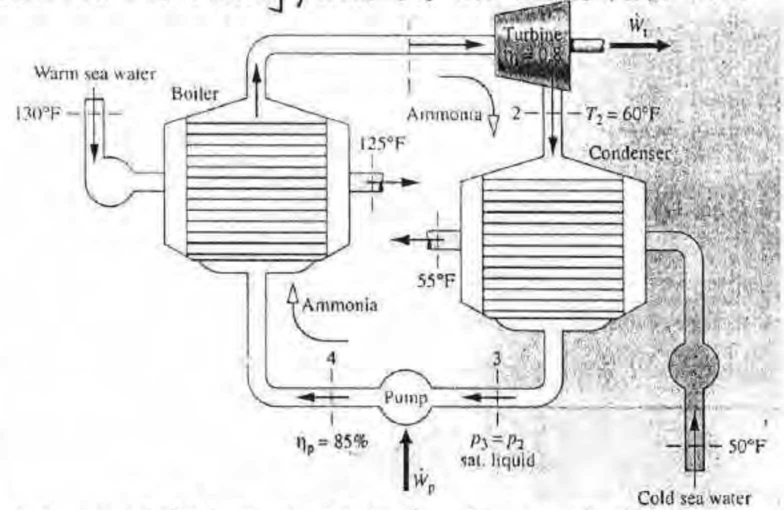
PROBLEM 8.14

KNOWN: A Rankine cycle power plant with ammonia is the working fluid is used to generate power based on a naturally-occuring temperature gradient in the ocean. Data are given at principal states in the cycle. The net power output is specified.

FIND: Determine (a) the thermal efficiency, and (b) the mass flow rate of ammonia.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) Each component is modeled as a control volume at steady state, (2) The turbine and pump operate adiabatically, (3) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states. Use data from Table A-13E.

State 1: $T_1 = 120°F$, sat. vapor $\Rightarrow h_1 = 632.95\ \frac{Btu}{lb}$, $s_1 = 1.1405\ \frac{Btu}{lb\cdot°R}$, $P_1 = 286.47\ lbf/in^2$

State 2: $T_2 = 60°F$, $s_{2s} = s_1 \Rightarrow x_{2s} = \frac{s_{2s} - s_{f2}}{s_{g2} - s_{f2}} = 0.9125$; $h_{2s} = 581.31$ Btu/lb

$$h_2 = h_1 - \eta_t (h_1 - h_{2s}) = 632.95 - (0.8)(632.95 - 581.31)$$
$$= 591.64 \text{ Btu/lb}$$

State 3: $T_3 = 60°F$, sat. liquid $\Rightarrow h_3 = 108.87$ Btu/lb, $P_3 = 107.66\ lbf/in.^2$

State 4: $h_{4s} \approx h_3 + v_3 (p_4 - P_3)$

$$= 108.87\ \frac{Btu}{lb} + (0.02597\ \frac{ft^3}{lb})(286.47 - 107.66)\frac{lbf}{in^2}\left|\frac{144\ in^2}{1\ ft^2}\right|\left|\frac{1\ Btu}{778\ ft\cdot lb_f}\right|$$
$$= 108.87 + 0.8595 = 109.73 \text{ Btu/lb}$$

$$h_4 = h_3 + (h_{4s} - h_3)/\eta_p = 108.87 + \frac{(109.73 - 108.87)}{0.85}$$
$$= 108.87 + 1.012 = 109.88 \text{ Btu/lb}$$

(a) The thermal efficiency is $\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}}$

$$\dot{W}_{cycle}/\dot{m} = (h_1 - h_2) - (h_4 - h_3) = 41.31 - 1.012 = 40.298 \text{ Btu/lb}$$
$$\dot{Q}_{in}/\dot{m} = h_1 - h_4 = 523.1 \text{ Btu/lb}$$

① Thus $\eta = \frac{40.298}{523.1} = 0.077\ (7.7\%) \longleftarrow \eta$

PROBLEM 8.14 (Cont'd)

(b) For $\dot{W}_{cycle} = 300$ hp, the mass flow rate of ammonia can be determined from

$$\dot{m} = \frac{\dot{W}_{cycle}}{(h_1 - h_2) - (h_4 - h_3)}$$

$$= \frac{300 \text{ hp}}{(41.31 - 1.012)\frac{\text{Btu}}{\text{lb}}} \left| \frac{2545 \text{ Btu/h}}{1 \text{ hp}} \right| \left| \frac{1 \text{ h}}{60 \text{ min}} \right|$$

$$= 315.8 \text{ lb/min} \longleftarrow \dot{m}$$

---

1. If the seawater temperatures were assumed to be thermal reservoir temperatures, $T_C = 50°F = 510°R$ and $T_H = 130°F = 590°R$. For a Carnot cycle operating between T_C and T_H, the thermal efficiency would be

$$\eta_{max} = 1 - \frac{T_C}{T_H} = 1 - \frac{510}{590} = 0.136 \; (13.6\%)$$

Thus, even the most efficient cycle would still have a relatively low thermal efficiency. The usefulness of the naturally-occurring temperature gradients in the oceans is limited and costly to exploit.

Further, for the heat exchangers, the mass flow rate of seawater would be

$$(\dot{m}_{sw})_{boiler} = \frac{\dot{m}(h_1 - h_4)}{c_{sw}\,\Delta T_{sw}} = \frac{(315.8 \frac{\text{lb}}{\text{min}})(632.95 - 109.88)\frac{\text{Btu}}{\text{lb}}}{(1 \text{ Btu/lb}\cdot°)(5°)}$$

$$= 33{,}040 \text{ lb/min}$$

and $$(\dot{m}_{sw})_{condenser} = \frac{(315.8)(591.64 - 108.87)}{(1)(5)} = 30{,}490 \text{ lb/min}$$

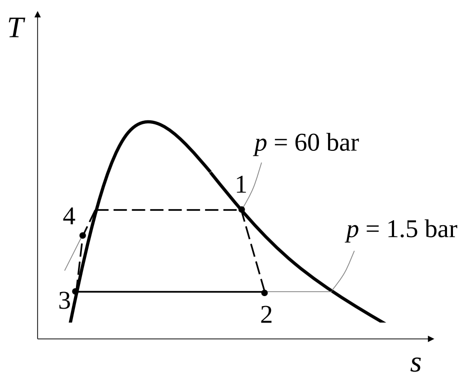
The mass flow rates of seawater are quite high, and a great deal of pumping power would be required to circulate seawater through the heat exchangers.

8.15 The ideal Rankine cycle 1-2-3-4-1 of problem 8.3 is modified to include the effects of irreversibilities in the adiabatic expansion and compression processes as shown in the *T-s* diagram in Fig. P8.15. Let T_0 = 300 K, p_0 = 1 bar. Determine
(a) the isentropic turbine efficiency.
(b) the rate of exergy destruction per unit mass of steam flowing in the turbine, in kJ/kg.
(c) the isentropic pump efficiency.
(d) the thermal efficiency.

KNOWN: A modified Rankine cycle that includes the effects of irreversibilities in the adiabatic expansion and compression processes operates between specified pressures of 1.5 bar and 60 bar.

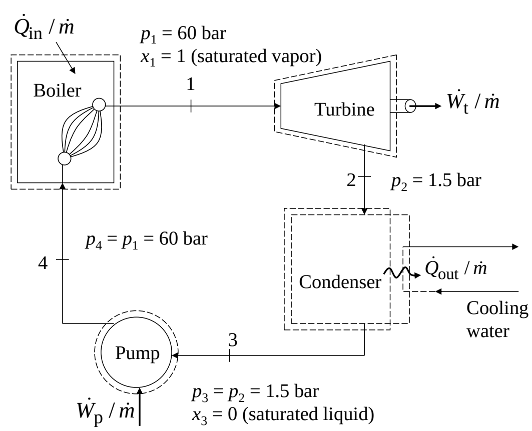
FIND: Determine (a) the isentropic turbine efficiency, (b) the rate of exergy destruction per unit mass of steam flowing in the turbine, in kJ/kg, (c) the isentropic pump efficiency, and (d) the thermal efficiency.

SCHEMATIC AND GIVEN DATA:



State	*p* (bar)	*h* (kJ/kg)	*x*	*v* (m³/kg)	*s* (kJ/kg·K)
1	60	2784.3	1		5.8892
2	1.5	2262.8	0.8065		6.1030
3	1.5	467.11	0	0.0010528	
4	60	474.14	--		

Fig. P8.15



ENGINEERING MODEL:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. Processes of the working fluid are internally reversible through the condenser and the boiler.
3. The turbine and pump operate adiabatically.
4. Kinetic and potential energy effects are negligible.
5. Saturated vapor enters the turbine. Condensate exits the condenser as saturated liquid.
6. T_0 = 300 K, p_0 = 1 bar.

ANALYSIS:

(a) The turbine isentropic efficiency is given by

$$\eta_t = \frac{(\dot{W}_t/\dot{m})}{(\dot{W}_t/\dot{m})_s} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

State 2s corresponds to the exit state for isentropic expansion to the turbine exhaust pressure, which is the same as State 2 in Problem 8.3: h_{2s}=2180.6 kJ/kg. Substituting values and solving give

$$\eta_t = \frac{(2784.3 - 2262.8)\text{kJ/kg}}{(2784.3 - 2180.6)\text{kJ/kg}} = \underline{\mathbf{0.8638\ (86.38\%)}}$$

(b) The rate of exergy destruction per unit mass of steam flowing in the turbine is given by

$$\dot{\mathsf{E}}_d = T_0\dot{\sigma} = T_0\left[\dot{m}(s_2 - s_1) - \cancel{\frac{\dot{Q}_{cv}}{T_b}}\right]$$

Since the turbine is adiabatic, the rate of heat transfer term cancels and the rate of exergy destruction per unit mass of steam flowing in the turbine is given by

$$\frac{\dot{\mathsf{E}}_d}{\dot{m}} = T_0(s_2 - s_1)$$

Substituting values and solving give

$$\frac{\dot{\mathsf{E}}_d}{\dot{m}} = (300\text{ K})(6.1030 - 5.8892)\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = \underline{\mathbf{64.14\ kJ/kg}}$$

(c) The isentropic pump efficiency is given by Eq. 8.10b

$$\eta_p = \frac{(\dot{W}_p/\dot{m})_s}{(\dot{W}_p/\dot{m})} = \frac{v_3(p_4 - p_3)}{h_4 - h_3}$$

Substituting values and solving give

$$\eta_p = \frac{0.0010528\frac{\text{m}^3}{\text{kg}}(60-1.5)\text{bar}}{(474.14-467.11)\text{kJ/kg}}\left|\frac{10^5\ \frac{\text{N}}{\text{m}^2}}{1\ \text{bar}}\right|\left|\frac{1\ \text{kJ}}{1000\ \text{N}\cdot\text{m}}\right| = \mathbf{\underline{0.8761\ (87.61\%)}}$$

(d) The thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m} - \dot{W}_p/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$$

Substituting enthalpy values and solving yield

$$\eta = \frac{(2784.3-2262.8)\ \text{kJ/kg} - (474.14-467.11)\ \text{kJ/kg}}{(2784.3-474.14)\ \text{kJ/kg}} = \mathbf{\underline{0.2227\ (22.27\%)}} \qquad \mathbf{(1)}$$

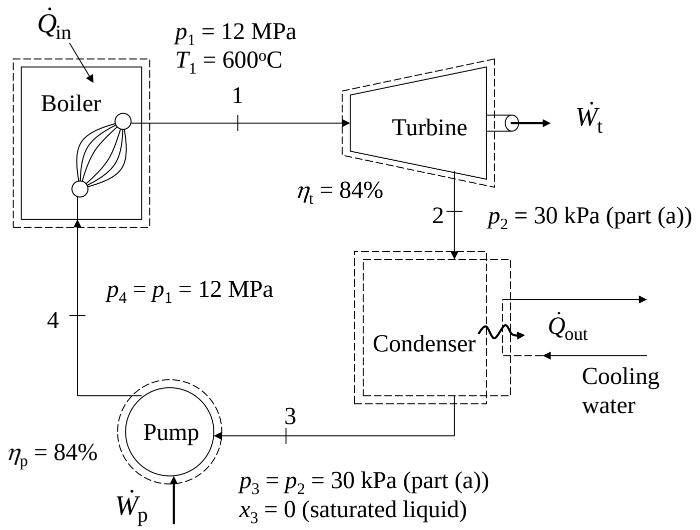
Note (1): The effects of irreversibilities in the adiabatic expansion and compression processes result in lower cycle thermal efficiency (22.27%) compared to the thermal efficiency of the ideal Rankine cycle (25.85%) in Problem 8.3.

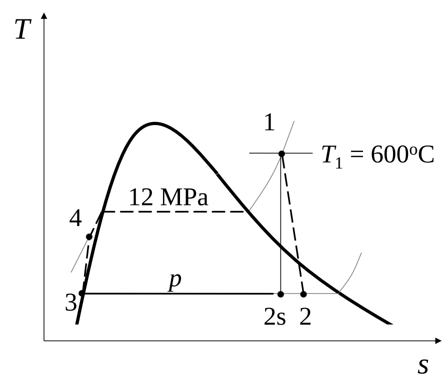
8.16 Steam enters the turbine of a simple vapor power plant with a pressure of 12 MPa and a temperature of 600°C and expands adiabatically to condenser pressure, p. Saturated liquid exits the condenser at pressure p. The isentropic efficiency of both the turbine and the pump is 84%.
(a) For p = 30 kPa, determine the turbine exit quality and the cycle thermal efficiency.
(b) Plot the quantities of part (a) versus p ranging from 6 kPa to 100 kPa.

KNOWN: Water is the working fluid in a simple vapor power plant. Data are given at various states in the cycle. The condenser pressure is p.

FIND: (a) For p = 30 kPa, determine the turbine exit quality and the cycle thermal efficiency, (b) plot the quantities of part (a) versus p ranging from 6 kPa to 100 kPa.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL:
1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. Flow through the boiler and condenser occurs at constant pressure.
3. Stray heat transfer in the turbine, condenser, and pump is ignored.
4. Kinetic and potential energy effects are negligible.
5. Condensate exits the condenser as saturated liquid.

ANALYSIS: First fix each principal state with p_2 = 30 kPa.

__State 1__: p_1 = 12 MPa (120 bar), T_1 = 600°C → h_1 = 3608.3 kJ/kg, s_1 = 6.8037 kJ/kg·K

__State 2s__: $p_{2s} = p_2$ = 30 kPa (0.3 bar), $s_{2s} = s_1$ = 6. 8037 kJ/kg·K → x_{2s} = 0.8586,
h_{2s} = 2295.0 kJ/kg

__State 2__: p_2 = 30 kPa (0.3 bar), h_2 = 2505.1 kJ/kg (see below)

$$\eta_{\mathrm{t}} = \frac{h_1 - h_2}{h_1 - h_{2s}} \rightarrow h_2 = h_1 - \eta_{\mathrm{t}}(h_1 - h_{2s}) = 3608.3\frac{\mathrm{kJ}}{\mathrm{kg}} - (0.84)(3608.3 - 2295.0)\frac{\mathrm{kJ}}{\mathrm{kg}} = 2505.1 \text{ kJ/kg}$$

State 3: p_3 = 30 kPa (0.3 bar), saturated liquid → $h_3 = h_{f3}$ = 289.23 kJ/kg,
$v_3 = v_{f3}$ = 0.0010223 m^3/kg,

State 4: $p_4 = p_1$ = 12 MPa (120 bar), h_4 = 303.80 kJ/kg (see below)

$$\eta_{\mathrm{p}} = \frac{v_3(p_4 - p_3)}{h_4 - h_3} \rightarrow h_4 = h_3 + \frac{v_3(p_4 - p_3)}{\eta_{\mathrm{p}}}$$

$$h_4 = 289.23\frac{\mathrm{kJ}}{\mathrm{kg}} + \frac{(0.0010223\frac{\mathrm{m^3}}{\mathrm{kg}})(12000 - 30)\ \mathrm{kPa}}{0.84}\left|\frac{1000\frac{\mathrm{N}}{\mathrm{m^2}}}{1\ \mathrm{kPa}}\right|\left|\frac{1\ \mathrm{kJ}}{1000\ \mathrm{N \cdot m}}\right| = 303.80 \text{ kJ/kg}$$

(a) The turbine exit quality, x_2, is

$$x_2 = \frac{h_2 - h_{f2}}{h_{fg2}}$$

Substituting values from Table 3, h_{f2} = 289.23 kJ/kg and h_{fg2} = 2336.1 kJ/kg, gives

$$x_2 = \frac{(2505.1 - 289.23)\ \mathrm{kJ/kg}}{2336.1\ \mathrm{kJ/kg}} = \mathbf{\underline{0.9485\ (94.85\%)}}$$

The thermal efficiency is

$$\eta = \frac{\dot{W}_{\mathrm{t}}/\dot{m} - \dot{W}_{\mathrm{p}}/\dot{m}}{\dot{Q}_{\mathrm{in}}/\dot{m}} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$$

Substituting enthalpy values and solving yield

$$\eta = \frac{(3608.3 - 2505.1)\ \mathrm{kJ/kg} - (303.80 - 289.23)\ \mathrm{kJ/kg}}{(3608.3 - 303.80)\ \mathrm{kJ/kg}} = \mathbf{\underline{0.3294\ (32.94\%)}}$$

The data for the required plots are obtained using IT as follows:

IT Code

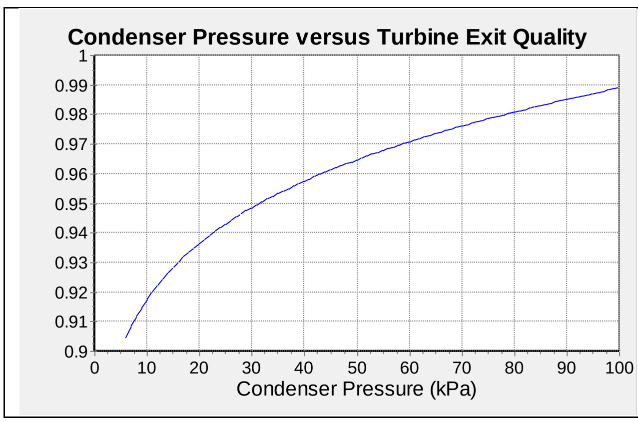
```
p1 = 12000 // kPa
T1 = 600 // oC
p2 = 30 // kPa
eff_t = 0.84
eff_p = 0.84
p3 = p2
x3 = 0
p4 = p1

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)

s2s = s1
p2s = p2
h2s = h_Ps("Water/Steam", p2s, s2s)

h2 = h1 - eff_t*(h1 - h2s)
x2 = x_hP("Water/Steam", h2, p2)

h3 = hsat_Px("Water/Steam", p3, x3)
v3 = vsat_Px("Water/Steam", p3, x3)

h4 = h3 + (v3*(p4 - p3))/eff_p

eta = ((h1 - h2) - (h4 - h3))/(h1 - h4)
```

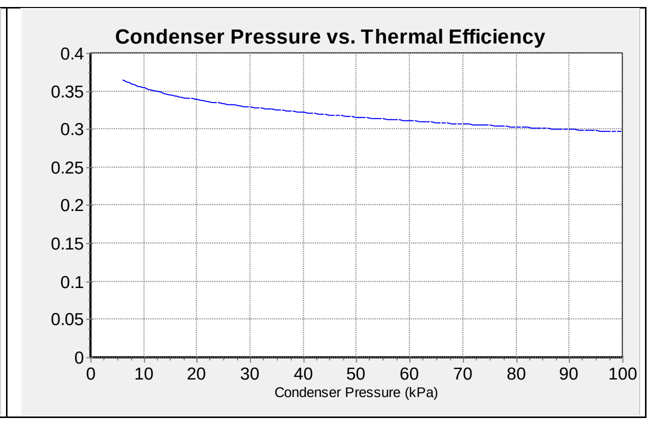
IT Results for p_2 = 30 kPa

eta	0.3295
h1	3608
h2	2505
h2s	2295
h3	289.9
h4	304.5
p2s	30
p3	30
p4	1.2E4
s1	6.803
s2s	6.803
v3	0.001022
x2	0.9486
eff_p	0.84
eff_t	0.84
p1	1.2E4
p2	30
T1	600
x3	0

IT results are consistent with the calculations in part (a).

Plots:





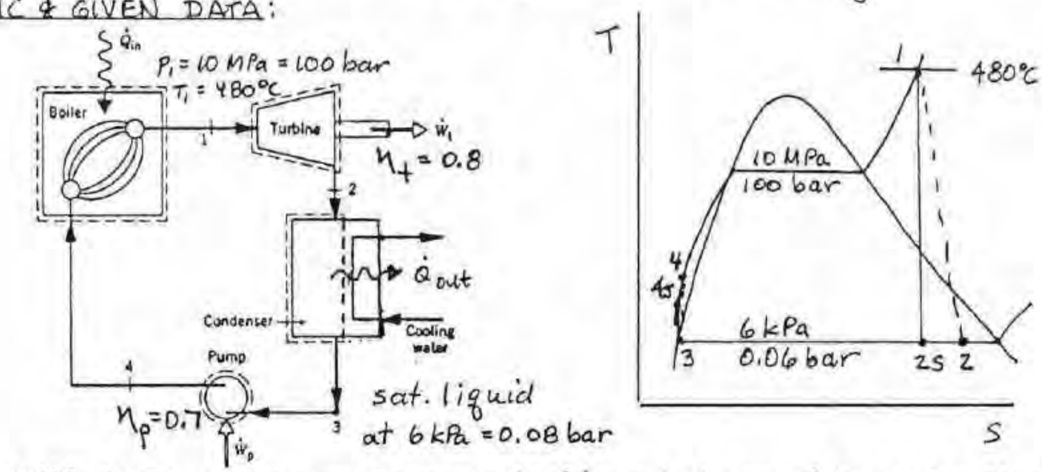
From the T-s diagram, we see that within the range of pressures considered the working fluid is a liquid-vapor mixture ($0 \le x \le 1$) and as p_2 increases, points 2s and 2 move to the right. Thus, x_2 increases as indicated in the plot above. Further as p_2 increases, the average temperature of heat rejection increases, lowering thermal efficiency. Thus, η decreases as indicated in the plot above.

PROBLEM 8.17

KNOWN: Water is the working fluid in a Rankine cycle with superheated vapor entering the turbine. The states at the turbine inlet and condenser exit are specified and turbine and pump isentropic efficiencies are given.

FIND: Determine (a) the heat tranfer rate for the steam generator, per kg of steam flowing, (b) the thermal efficiency, and (c) the rate of heat transfer for the condenser, per kg of steam condensing.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: Same as Example 8.1, items 1–4, except $\eta_t = 0.8$ and $\eta_p = 0.7$.

ANALYSIS: From Problem 8.2, $h_1 = 3321.4$ kJ/kg and $h_3 = 151.53$ kJ/kg. The specific enthalpy at state 2 is found using the turbine efficiency

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s})$$

From Probem 8.2, $h_{2s} = 2009.8$ kJ/kg. Thus, $h_2 = 2272.1$ kJ/kg. Similarly,

$$\eta_p = \frac{h_{4s} - h_3}{h_4 - h_3} \Rightarrow h_4 = h_3 + (h_{4s} - h_3)/\eta_p$$

From Problem 8.2, $h_{4s} = 161.59$. Thus, $h_4 = 165.9$ kJ/kg.

(a) For the control enclosing the steam generator

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) \Rightarrow \frac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_4 = 3321.4 - 165.9 = 3155.5 \text{ kJ/kg} \leftarrow \dot{Q}_{in}/\dot{m}$$

(b) The thermal efficiency is $\eta = \frac{\dot{W}_{net}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$

① $$\dot{W}_{net}/\dot{m} = (h_1 - h_2) - (h_4 - h_3) = 1049.3 - 14.37 = 1034.9 \text{ kJ/kg}$$

$$\eta = \frac{1034.9}{3155.5} = 0.328\ (32.8\%) \leftarrow \eta$$

(c) For the condenser

$$\dot{Q}_{out} = \dot{m}(h_2 - h_3) \Rightarrow \frac{\dot{Q}_{out}}{\dot{m}} = h_2 - h_3 = 2272.1 - 151.53 = 2120.6 \ \frac{\text{kJ}}{\text{kg}} \leftarrow \dot{Q}_{out}/\dot{m}$$

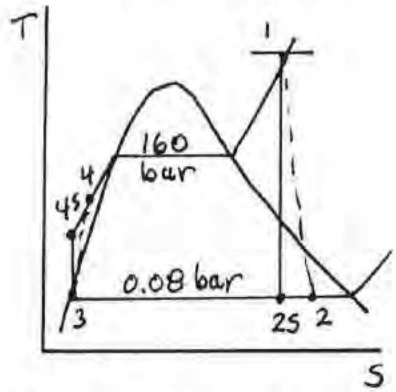
1. These results can be compared with those of Problem 8.2 to see some of the effects of irreversibilities in the turbine and pump on the ideal Rankine cycle.

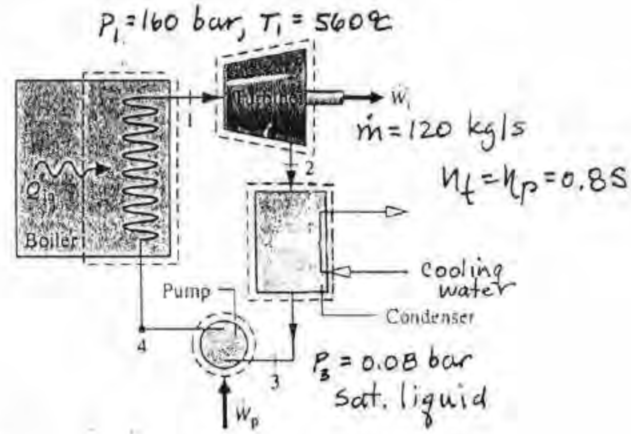
PROBLEM 8.18

KNOWN: Water is the working fluid in a Rankine cycle. The condenser pressure and the turbine inlet state are specified. The mass flow of steam is given, and the turbine and pump efficiencies are specified

FIND: Determine (a) the net power, (b) the rate of heat transfer to the steam passing through the boiler, and (c) the thermal efficiency.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 8.2.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 160$ bar, $T_1 = 560$°C $\Rightarrow$ $h_1 = 3465.4$ kJ/kg, $s_1 = 6.5132$ kJ/kg·K

State 2: $p_2 = 0.08$ bar, $s_{2s} = s_1 \Rightarrow x_{2s} = \dfrac{s_{2s} - s_{f2}}{s_{g2} - s_{f2}} = 0.7753$, $h_{2s} = 2037.0 \frac{kJ}{kg}$

The specific enthalpy h_2 is found using the isentropic turbine efficiency.

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 2251.3 \text{ kJ/kg}$$

State 3: $p_3 = 0.08$ bar, sat. liquid $\Rightarrow h_3 = 173.88$ kJ/kg

State 4: $p_4 = 160$ bar, $h_{4s} \approx h_3 + v_3(p_4 - p_3)$

$$= 173.88 \frac{kJ}{kg} + (1.0084 \times 10^{-3}) \frac{m^3}{kg}(160 - 0.08)\text{bar} \left| \frac{10^5 N/m^2}{1 \text{ bar}} \right| \left| \frac{1 kJ}{10^3 N \cdot m} \right|$$

$$= 190.01 \text{ kJ/kg}$$

With the pump efficiency: $\eta_p = (h_{4s} - h_3)/(h_4 - h_3)$

$$h_4 = h_3 + (h_{4s} - h_3)/\eta_p = 192.86 \text{ kJ/kg}$$

(a) The net power is $\dot{W}_{cycle} = \dot{m}[(h_1 - h_2) - (h_4 - h_3)]$

$$= (120 \tfrac{kg}{s})[(3465.4 - 2251.3) - (192.86 - 173.88)] \frac{kJ}{kg} \left| \frac{1 kW}{1 kJ/s} \right|$$

$$= 1.434 \times 10^5 \text{ kW} \longleftarrow \dot{W}_{cycle}$$

(b) For the steam passing through the steam generator

$$\dot{Q}_{in} = \dot{m}[h_1 - h_4] = (120)[3465.4 - 192.86] \left| \frac{1}{1} \right| = 3.927 \times 10^5 \text{ kW} \longleftarrow \dot{Q}_{in}$$

(c) The thermal efficiency is $\eta = \dfrac{\dot{W}_{cycle}}{\dot{Q}_{in}} = 0.365 \, (36.5\%) \longleftarrow \eta$

PROBLEM 8.18 (Cont'd)

The plots are developed using IT, as follows:

IT Code

```
p1 =160   // bar
p2 = 0.08   // bar
mdot = 120 // kg/s
T1 = 560  // °C
etat = 0.85
etap = etat
// Turbine
T1sat = Tsat_P("Steam", p1)
h1 = h_PT("Steam", p1, T1)
s1 = s_PT("Steam", p1, T1)
s2s = s1
h2s = h_Ps("Water", p2, s2s)
h2 = h1 - etat * (h1 - h2s)
// Condenser
p3 = p2
x3 = 0
h3 = hsat_Px("Water", p3, x3)
// Pump
v3 = vsat_Px("Water", p3, x3)
p4 = p1
h4s = h3 + v3 * (p4 - p3) * (10^5 / 10^3)
h4 = h3 + (h4s - h3) / etap
// Cycle parameters
Wdotnet = mdot * ((h1 - h2) - (h4-h3))    // kW
Qdotin = mdot * (h1 - h4)    // kW
eta = Wdotnet/Qdotin
```

IT Results

$\dot{Q}_{in} = 3.927 \times 10^5$ kW

$\dot{W}_{cycle} = 1.434 \times 10^5$ kW

eta = 0.3652

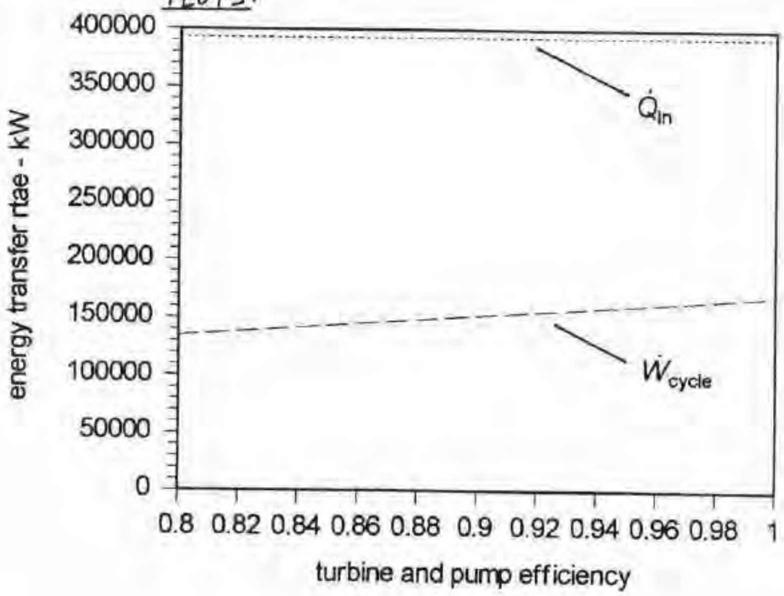
h_1 = 3465 kJ/kg

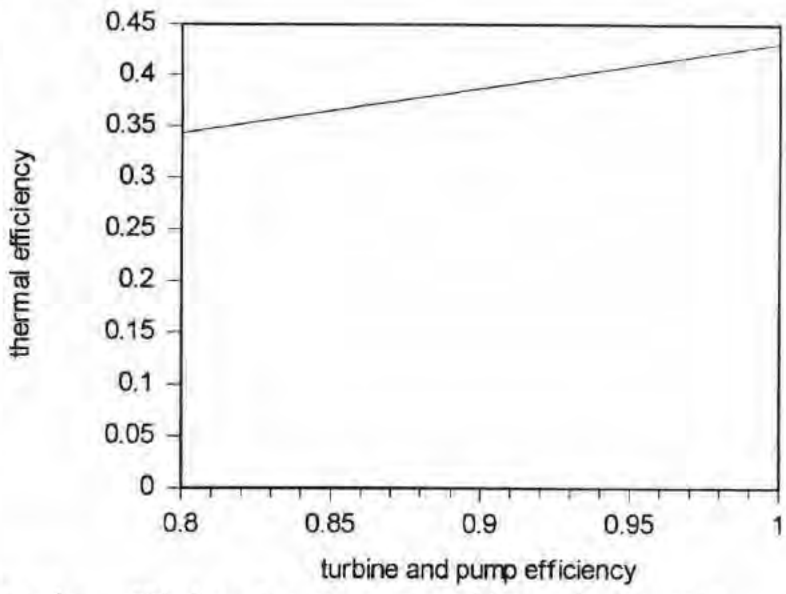
h_2 = 2251 kJ/kg

h_3 = 173.6 kJ/kg

h_4 = 192.6 kJ/kg

PLOTS:





As turbine and pump isentropic efficiencies decrease, the net power developed decreases significantly, resulting in reduced thermal efficiency. The heat input changes only slightly.

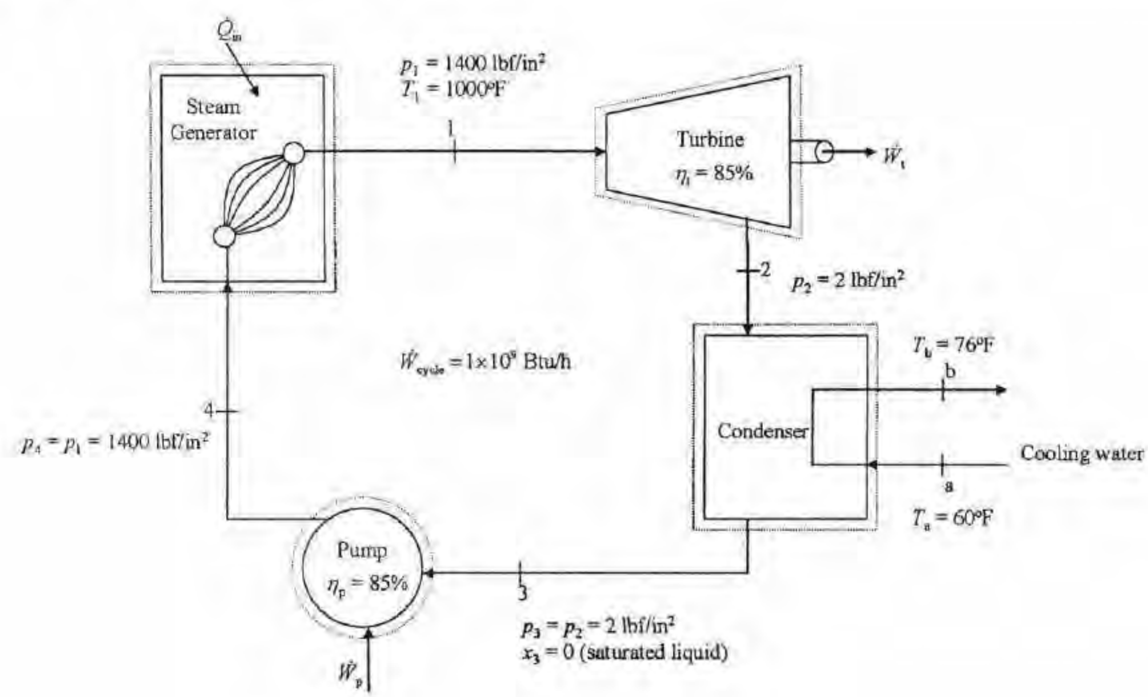
8.19 Water is the working fluid in a Rankine cycle. Steam enters the turbine at 1400 lbf/in.^{2} and 1000°F. The condenser pressure is 2 lbf/in.^{2} Both the turbine and pump have isentropic efficiencies of 85%. The working fluid has negligible pressure drop in passing through the steam generator. The net power output of the cycle is 1×10^9 Btu/h. Cooling water experiences a temperature increase from 60°F to 76°F, with negligible pressure drop, as it passes through the condenser. Determine for the cycle

(a) the mass flow rate of steam, in lb/h.
(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the generator.
(c) the thermal efficiency.
(d) the mass flow rate of cooling water, in lb/h.

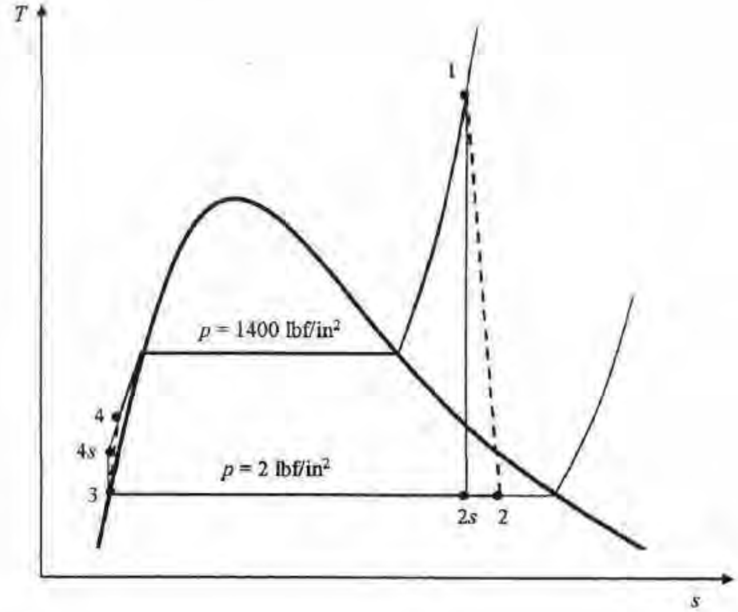
KNOWN: A Rankine cycle with superheat operates with water as the working fluid. The net power output of the cycle is given.

FIND: Determine the mass flow rate of steam, the rate of heat transfer to the working fluid passing through the steam generator, the thermal efficiency, and the mass flow rate of cooling water.

SCHEMATIC AND GIVEN DATA:



T-s diagram



ENGINEERING MODEL:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The steam generator and condenser operate with constant pressure.
3. The turbine and pump each operate adiabatically with an isentropic efficiency of 85%.
4. Kinetic and potential energy effects are negligible.
5. Condensate exits the condenser as saturated liquid.
6. There is no heat transfer between the outside of the condenser and the surroundings.

ANALYSIS: First, fix each of the principal states.

State 1: p_1 = 1400 lbf/in.^{2}, T_1 = 1000°F → From Table A-4E: h_1 = 1493.5 Btu/lb and s_1 = 1.6094 Btu/(lb·°R)

State 2s: p_{2s} = 2 lbf/in.^{2}, $s_{2s} = s_1$ → From Table A-3E: x_{2s} = (1.6094 – 0.1750)/1.7448 = 0.8221 and $h_{2s} = h_{f2s} + x_{2s}h_{fg2s}$ = 94.02 + (0.8221)(1022.1) = 934.29 Btu/lb

State 2: p_2 = 2 lbf/in.^{2},
$h_2 = h_1 - \eta_t(h_1 - h_{2s})$ = 1493.5 Btu/lb – (0.85)(1493.5 Btu/lb – 934.29 Btu/lb) = 1018.17 Btu/lb

State 3: $p_3 = p_2$ = 2 lbf/in.^{2}, sat liq. → From Table A-3E: $h_3 = h_{f3}$ = 94.02 Btu/lb and $v_3 = v_{f3}$ = 0.01623 ft^3/lb

State 4: $p_4 = p_1$ = 1400 lbf/in.^{2}, $h_4 = h_3 + \dfrac{v_3(p_4 - p_3)}{\eta_p}$

Problem 8.19 (Continued) – Page 3

$$h_4 = 94.02\frac{\text{Btu}}{\text{lb}} + \frac{\left(0.01623\frac{\text{ft}^3}{\text{lb}}\right)(1400-2)\left(\frac{\text{lbf}}{\text{in}^2}\right)\left|\frac{144\ \text{in}^2}{\text{ft}^2}\right|\left|\frac{\text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|}{0.85} = 98.96\ \text{Btu/lb}$$

(a) The mass flow rate of steam is found as follows. Mass and energy rate balances for control volumes enclosing the turbine and pump give

$$\dot{W}_t = \dot{m}(h_1 - h_2) \quad \text{and} \quad \dot{W}_p = \dot{m}(h_4 - h_3)$$

The net power of the cycle is

$$\dot{W}_{\text{cycle}} = \dot{W}_t - \dot{W}_p = \dot{m}[(h_1 - h_2) - (h_4 - h_3)]$$

Solving for $\dot{m}$

$$\dot{m} = \frac{\dot{W}_{\text{cycle}}}{[(h_1 - h_2) - (h_4 - h_3)]}$$

Inserting values

$$\dot{m} = \frac{1\times10^9\ \frac{\text{Btu}}{\text{h}}}{\left(1493.5\frac{\text{Btu}}{\text{lb}} - 1018.17\frac{\text{Btu}}{\text{lb}}\right) - \left(98.96\frac{\text{Btu}}{\text{lb}} - 94.02\frac{\text{Btu}}{\text{lb}}\right)} = \underline{\mathbf{2.13\times10^6\ lb/h}}$$

(b) The rate of heat transfer to the working fluid passing through the boiler can be determined by applying mass and energy balances to a control volume around the steam generator to give

$$\dot{Q}_{\text{in}} = \dot{m}(h_1 - h_4) = (2.13\times10^6\ \text{lb/h})(1493.5\ \text{Btu/lb} - 98.96\ \text{Btu/lb}) = \underline{\mathbf{2.97\times10^9\ Btu/h}}$$

(c) The thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = (1\times10^9\ \text{Btu/h})/(2.97\times10^9\ \text{Btu/h}) = \underline{\mathbf{0.3367\ (33.67\%)}}$$

(d) The mass flow rate of cooling water through the condenser is determined by applying steady state mass and energy rate balances to a control volume enclosing the condenser.

$$\dot{m}_2 = \dot{m}_3 \equiv \dot{m} \text{ and } \dot{m}_a = \dot{m}_b \equiv \dot{m}_{cw}$$

and

$$0 = \dot{m}(h_2 - h_3) + \dot{m}_{cw}(h_a - h_b)$$

Solving for $\dot{m}_{cw}$

Problem 8.19 (Continued) – Page 4

$$\dot{m}_{cw} = \frac{\dot{m}(h_2 - h_3)}{h_b - h_a}$$

For the cooling water, $h \approx h_f(T)$. Consequently, from Table A-2E

$h_a = 28.08$ Btu/lb
$h_b = 44.09$ Btu/lb

Substituting values

$$\dot{m}_{cw} = \frac{\left(2.13\times10^6 \frac{\text{lb}}{\text{h}}\right)\left(1018.17\frac{\text{Btu}}{\text{lb}} - 94.02\frac{\text{Btu}}{\text{lb}}\right)}{44.09\frac{\text{Btu}}{\text{lb}} - 28.08\frac{\text{Btu}}{\text{lb}}} = \underline{\mathbf{1.23 \times 10^8\ lb/h}}$$

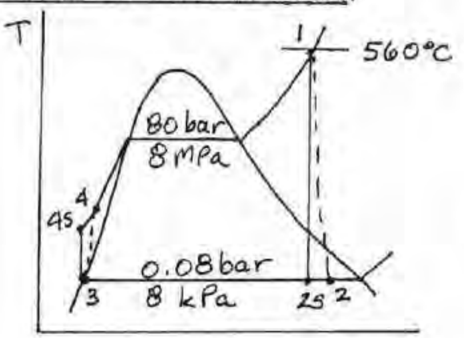
The results of Problem 8.19 can be compared to the results of Problem 8.10 to see some of the effects of irreversibilities on the performance of a Rankine cycle for cycles with the same net power output. In this case, the irreversibilities in the turbine and pump result in lower thermal efficiency, greater steam flow rate, increased heat addition, and increased mass flow rate of cooling water.

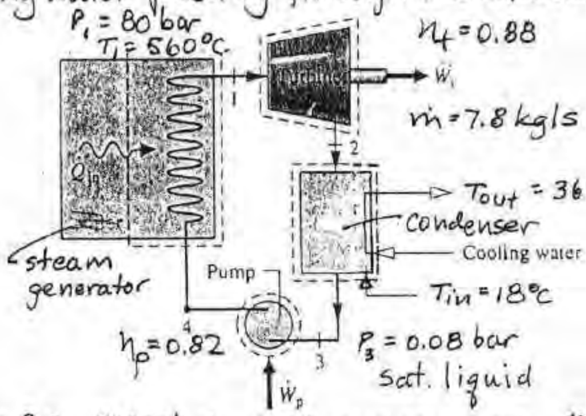
PROBLEM 8.20

KNOWN: Water is the working fluid in a Rankine cycle. The condenser pressure and the turbine inlet state are specified. The mass flow rate of steam is given and the turbine and pump isentropic efficiencies are known. Temperature are given for cooling water entering and exiting the condenser.

FIND: Determine (a) the net power, (b) the thermal efficiency, and (c) the mass flow rate of cooling water passing through the condencer.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: Same as Problem 8.2, except $\eta_t = 0.88$ and $\eta_p = 0.82$. Also, we assume that the pressure drop is negligable for the cooling water passing through the condenser, and that $h \approx h_f(T)$.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 80$ bar, $T_1 = 560°C \Rightarrow h_1 = 3545.3$ kJ/kg, $s_1 = 6.9072$ kJ/kg·K

State 2: $p_2 = 0.08$ bar, $s_{2s} = s_1 \Rightarrow x_{2s} = \dfrac{s_{2s} - s_{f2}}{s_{g2} - s_{f2}} = 0.8269$, $h_{2s} = 2161.0$ kJ/kg

Using the isentropic turbine efficiency to get h_2

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 2327.1 \text{ kJ/kg}$$

State 3: $p_3 = 0.08$ bar, sat. liquid $\Rightarrow h_3 = 173.88$ kJ/kg

State 4: $p_4 = 80$ bar, $h_4 \approx h_3 + v_3(p_4 - p_3)$

$$= 173.88 \frac{\text{kJ}}{\text{kg}} + (1.0084\times10^{-3})\frac{\text{m}^3}{\text{kg}}(80-0.08)\text{bar}\left|\frac{10^5 \text{ N/m}^2}{1 \text{ bar}}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N·m}}\right|$$

$$= 181.94 \text{ kJ/kg}$$

Using the isentropic pump efficiency to get h_4

$$\eta_p = \frac{h_{4s} - h_3}{h_4 - h_3} \Rightarrow h_4 = h_3 + (h_{4s} - h_3)/\eta_p = 183.58 \text{ kJ/kg}$$

(a) The net power developed is

$$\dot{W}_{cycle} = \dot{m}[(h_1 - h_2) - (h_4 - h_3)]$$

$$= (7.8 \text{ kg/s})[(3545.3 - 2327.1) - (183.58 - 173.88)]\frac{\text{kJ}}{\text{kg}}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$

$$= 9426 \text{ kW}$$ ← $\dot{W}_{cycle}$

(b) To find the thermal efficiency, first get $\dot{Q}_{in}$.

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = (7.8 \tfrac{\text{kJ}}{\text{kg}})(3545.3 - 183.58)\frac{\text{kJ}}{\text{kg}}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right| = 26{,}221 \text{ kW}$$

Thus $\eta = \dot{W}_{cycle}/\dot{Q}_{in} = 9426/26{,}221 = 0.359$ (35.9%) ← η

PROBLEM 8.20 (Cont'd)

(c) The mass flow rate of cooling water passing through the condenser is

$$\dot{m}_{cw} = \frac{\dot{m}(h_2 - h_3)}{h_{cw,in} - h_{cw,out}}$$

with $h_{cw} \approx h_f(T)$

$$\dot{m}_{cw} = \frac{(7.8 \text{ kg/s})(2327.1 - 173.88) \text{ kJ/kg}}{(150.86 - 75.58) \text{ kJ/kg}}$$

$$= 223.1 \text{ kg/s} \longleftarrow \dot{m}_{cw}$$

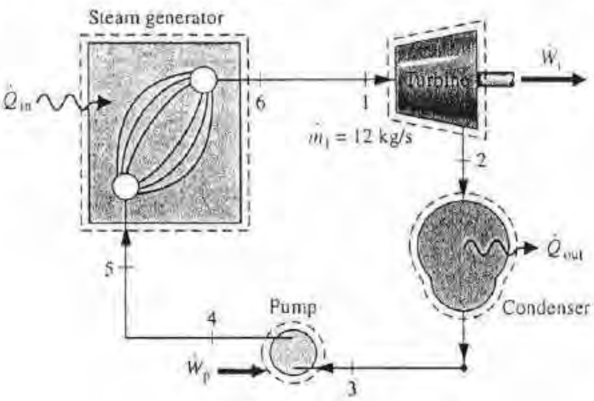
PROBLEM 8.21

KNOWN: A vapor power plant operates at steady state with water as the working fluid. Data are known at principal states in the cycle.

FIND: Determine the (a) thermal efficiency, and (b) the rates of heat transfer $\dot{Q}_{in}$ and $\dot{Q}_{out}$.

SCHEMATIC & GIVEN DATA:

State	p	T (°C)	h (kJ/kg)
1	6 MPa	500	3422.2
2	10 kPa	---	1633.3
3	10 kPa	Sat.	191.83
4	7.5 MPa	---	199.4
5	7 MPa	40	167.57
6	6 MPa	550	3545.3



ENGINEERING MODEL: (1) Each control volume operates at steady state, (2) The turbine and pump operate adiabatically, (3) Kinetic and potential energy effects can be neglected.

ANALYSIS: (a) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_6 - h_5)}$$

$$= \frac{(3422.2 - 1633.3) - (199.4 - 191.83)}{(3545.3 - 167.57)} = 0.527\ (52.7\%) \leftarrow \eta$$

(b) For the steam generator

① $$\dot{Q}_{in} = \dot{m}(h_6 - h_5) = \left(12\ \frac{kg}{s}\right)(3545.3 - 167.57)\frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 4.054 \times 10^4\ kW \leftarrow \dot{Q}_{in}$$

For the condenser

$$\dot{Q}_{out} = \dot{m}(h_2 - h_3) = (12)(1633.3 - 191.83) = 1.73 \times 10^4\ kW \leftarrow \dot{Q}_{out}$$

---

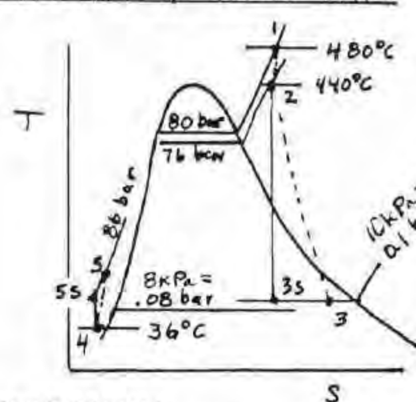
1. Note that stray heat loss from the piping between the pump and the steam generator and the steam generator and turbine result in more energy input ($\dot{Q}_{in}$) without additional power being developed.

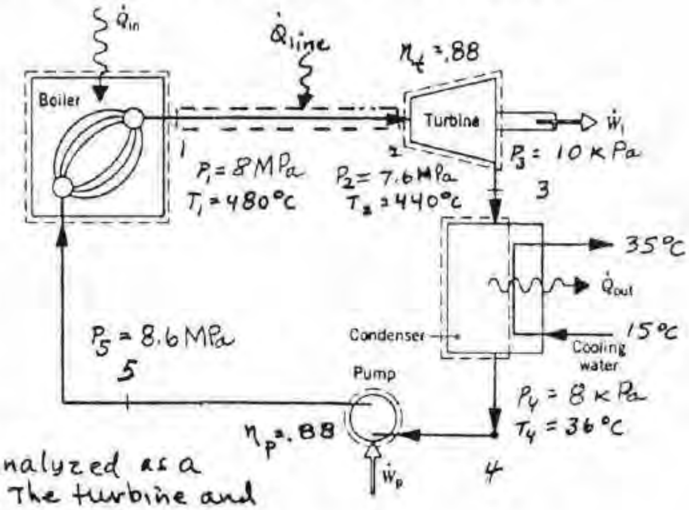
PROBLEM 8.22

KNOWN: Water is the working fluid in a vapor power plant. Data are known at various locations, and the mass flow rate is given.

FIND: Determine (a) the net power output, (b) thermal efficiency, (c) the rate of heat transfer from the line connecting the steam generator and the turbine, and (d) the mass flow rate of condenser cooling water.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The turbine and pump operate adiabatically (3) Kinetic & potential energy effects are negligible. (4) For the cooling water, assume $h_{cw} \approx h_f(T)$.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 8 MPa$, $T_1 = 480°C \Rightarrow h_1 = 3348.4$ kJ/kg, $s_1 = 6.6586$ kJ/kg·K

State 2: $p_2 = 7.6 MPa$, $T_2 = 440°C \Rightarrow$ Interpolating in Table A-4;

$h_2 = 3252.3$ kJ/kg, $s_2 \approx 6.5526$ kJ/kg·K

State 3: State 3 is fixed using the turbine efficiency. First, at $p_3 = 10 kPa$, $s_{3s} = s_2 = 6.5526 \Rightarrow x_{3s} = 0.787$; $h_{3s} = 2075.0$ kJ/kg. Thus

$$\eta_t = \frac{(\dot{W}_t/\dot{m})}{(\dot{W}_t/\dot{m})_s} = \frac{h_2 - h_3}{h_2 - h_{3s}} \Rightarrow h_3 = h_2 - \eta_t(h_2 - h_{3s}) = 2216.3 \text{ kJ/kg}$$

Further, with $h_3 = 2216.3$ kJ/kg; $x_3 = .8461 \Rightarrow s_3 = 6.9958$ kJ/kg·K

State 4: $p_4 = 8 kPa$, $T_4 = 36°C \Rightarrow h_4 \approx h_f(T_4) = 150.86$ kJ/kg

State 5: $h_{5s} \approx h_4 + v_4(p_5 - p_4)$

$$= 150.86 + \left(1.0063 \times 10^{-3} \frac{m^3}{kg}\right)(86 - .08) \text{ bars} \left(\frac{10^5 N/m^2}{1 \text{ bar}}\right)\left(\frac{1 kJ}{10^3 N\cdot m}\right)$$

$$= 150.86 + 8.646 = 159.51 \text{ kJ/kg}$$

Thus, using the pump efficiency

$$\eta_p = \frac{(\dot{W}_p/\dot{m})_s}{(\dot{W}_p/\dot{m})} = \frac{h_{5s} - h_4}{h_5 - h_4} \Rightarrow h_5 = h_4 + \left(\frac{h_{5s} - h_4}{\eta_p}\right)$$

$$= 150.86 + \left(\frac{159.51 - 150.86}{.88}\right)$$

$$= 160.69 \text{ kJ/kg}$$

PROBLEM 8.22 (Cont'd.)

(a) To determine the net power output, we use energy and mass balances for the control volumes surrounding the turbine and pump to get

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p = \dot{m}\left[(h_2 - h_3) - (h_5 - h_4)\right]$$

Inserting values

$$\dot{W}_{cycle} = \left(79.53\ \frac{kg}{s}\right)\left[(3252.3 - 2216.3) - (160.69 - 150.86)\right]\frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 8.161 \times 10^4\ kW \longleftarrow \dot{W}_{cycle}$$

(b) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \frac{\dot{W}_{cycle}}{\dot{m}(h_1 - h_5)} = \frac{8.161 \times 10^4}{(79.53)(3348.4 - 160.69)\left|\frac{1}{1}\right|}$$

$$= 0.322\ (32.2\%) \longleftarrow \eta$$

(c) For a control volume enclosing the line connecting the steam generator and the turbine

$$0 = \dot{Q}_{line} + \dot{m}(h_1 - h_2)$$

or

$$\dot{Q}_{line} = \dot{m}(h_2 - h_1) = \left(79.53\ \frac{kJ}{kg}\right)(3252.3 - 3348.4)\frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= -7643\ kW \longleftarrow \dot{Q}_{line}$$

(d) The mass flow rate of cooling water is found from

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}(h_3 - h_4) + \dot{m}_{cw}(h_{cw,in} - h_{cw,out})$$

or

$$\dot{m}_{cw} = \dot{m}\frac{(h_3 - h_4)}{(h_{cw,out} - h_{cw,in})}$$

From Table A-2, $h_{cw,out} \approx h_f(35°C) = 146.68$ kJ/kg and $h_{cw,in} \approx h_f(15°C) = 62.99$ kJ/kg. Inserting values

$$\dot{m}_{cw} = \left(79.53\ \frac{kg}{s}\right)\frac{(2216.3 - 150.86)}{(146.68 - 62.99)} = 1963\ kg/s \longleftarrow \dot{m}_{cw}$$

8.23 Water is the working fluid in a Rankine cycle. Steam exits the steam generator at 1500 lbf/in.^{2} and 1100°F. Due to heat transfer and frictional effects in the line connecting the steam generator and turbine, the pressure and temperature at the turbine inlet are reduced to 1400 lbf/in.^{2} and 1000°F, respectively. Both the turbine and pump have isentropic efficiencies of 85%. Pressure at the condenser inlet is 2 lbf/in.^{2}, but due to frictional effects the condensate exits the condenser at a pressure of 1.5 lbf/in.^{2} and a temperature of 110°F. The condensate is pumped to 1600 lbf/in.^{2} before entering the steam generator. The net power output of the cycle is 1×10^9 Btu/h. Cooling water experiences a temperature increase from 60°F to 76°F, with negligible pressure drop, as it passes through the condenser. Determine for the cycle

(a) the mass flow rate of steam, in lb/h.

(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the steam generator.

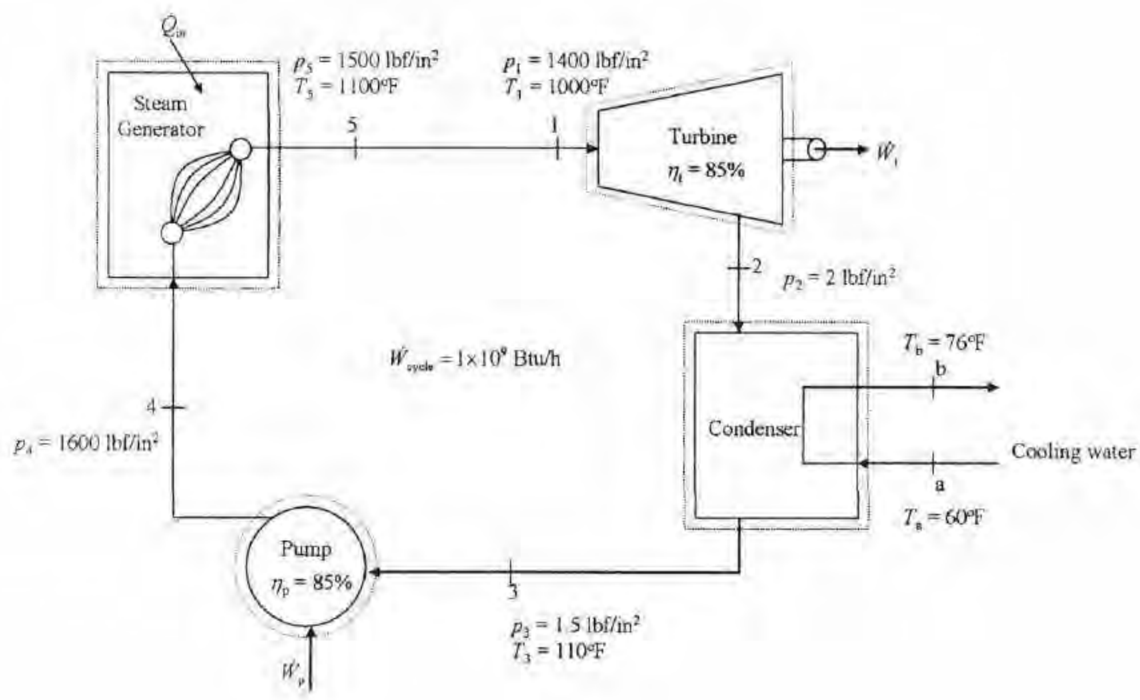
(c) the thermal efficiency.

(d) the mass flow rate of cooling water, in lb/h.

KNOWN: A Rankine cycle operates with water as the working fluid. The net power output of the cycle is given. Frictional effects cause pressure drops in the line connecting the steam generator to the turbine and in the flows through the condenser and steam generator.

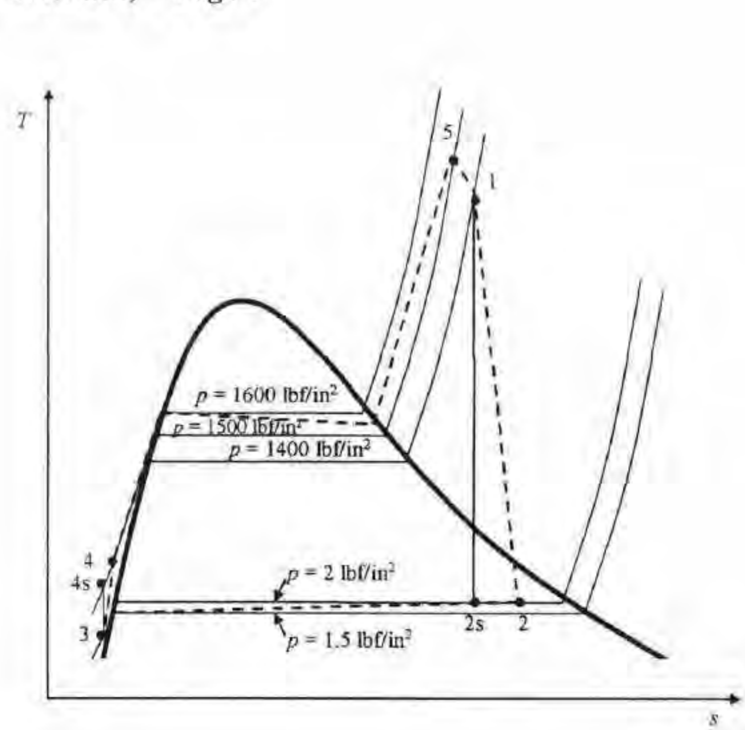
FIND: Determine the mass flow rate of steam, the rate of heat transfer to the working fluid passing through the steam generator, the thermal efficiency, and the mass flow rate of cooling water.

SCHEMATIC AND GIVEN DATA:



Problem 8.23 (Continued) – Page 2

T-s diagram



ENGINEER MODEL:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The turbine and pump each operate adiabatically with an isentropic efficiency of 85%.
3. Kinetic and potential energy effects are negligible.
4. There is no heat transfer between the outside of the condenser and the surroundings.

ANALYSIS: First, fix each of the principal states.

State 1: p_1 = 1400 lbf/in.^{2}, T_1 = 1000°F → From Table A-4E: h_1 = 1493.5 Btu/lb and s_1 = 1.6094 Btu/(lb·°R)

State 2s: p_{2s} = 2 lbf/in.^{2}, $s_{2s} = s_1$ → From Table A-3E: x_{2s} = (1.6094 – 0.1750)/1.7448 = 0.8221 and $h_{2s} = h_{f2s} + x_{2s}h_{fg2s}$ = 94.02 + (0.8221)(1022.1) = 934.29 Btu/lb

State 2: p_2 = 2 lbf/in.^{2},
$h_2 = h_1 - \eta_t(h_1 - h_{2s})$ = 1493.5 Btu/lb – (0.85)(1493.5 Btu/lb – 934.29 Btu/lb) = 1018.17 Btu/lb

State 3: p_3 = 1.5 lbf/in.^{2}, T_3 = 110°F (Compressed Liq.) → From Table A-2E: $h_3 \approx h_{f3}$ = 78.02 Btu/lb and $v_3 \approx v_{f3}$ = 0.01617 ft^3/lb

State 4: p_4 = 1600 lbf/in.^{2}, $h_4 = h_3 + \dfrac{v_3(p_4 - p_3)}{\eta_p}$

Problem 8.23 (Continued) – Page 3

$$h_4 = 78.02\,\frac{\text{Btu}}{\text{lb}} + \frac{\left(0.01617\,\frac{\text{ft}^3}{\text{lb}}\right)(1600 - 1.5)\left(\frac{\text{lbf}}{\text{in}^2}\right)\left|\frac{144\ \text{in}^2}{\text{ft}^2}\right|\left|\frac{\text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|}{0.85} = 83.65\ \text{Btu/lb}$$

State 5: p_5 = 1500 lbf/in.^{2}, T_5 = 1100°F → From Table A-4E (interpolated): h_5 = 1550.25 Btu/lb

(a) The mass flow rate of steam is found as follows. Mass and energy rate balances for control volumes enclosing the turbine and pump give

$$\dot{W}_t = \dot{m}(h_1 - h_2) \quad \text{and} \quad \dot{W}_p = \dot{m}(h_4 - h_3)$$

The net power of the cycle is

$$\dot{W}_{\text{cycle}} = \dot{W}_t - \dot{W}_p = \dot{m}[(h_1 - h_2) - (h_4 - h_3)]$$

Solving for $\dot{m}$

$$\dot{m} = \frac{\dot{W}_{\text{cycle}}}{[(h_1 - h_2) - (h_4 - h_3)]}$$

Inserting values

$$\dot{m} = \frac{1\times 10^9\,\frac{\text{Btu}}{\text{h}}}{\left(1493.5\,\frac{\text{Btu}}{\text{lb}} - 1018.17\,\frac{\text{Btu}}{\text{lb}}\right) - \left(83.65\,\frac{\text{Btu}}{\text{lb}} - 78.02\,\frac{\text{Btu}}{\text{lb}}\right)} = \underline{\mathbf{2.13\times 10^6\ lb/h}}$$

(b) The rate of heat transfer to the working fluid passing through the steam generator can be determined by applying mass and energy balances to a control volume around the steam generator to give

$$\dot{Q}_{\text{in}} = \dot{m}(h_5 - h_4) = (2.13\times 10^6\ \text{lb/h})(1550.25\ \text{Btu/lb} - 83.65\ \text{Btu/lb} = \underline{\mathbf{3.12\times 10^9\ Btu/h}}$$

(c) The thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = (1\times 10^9\ \text{Btu/h})/(3.12\times 10^9\ \text{Btu/h}) = \underline{\mathbf{0.3205\ (32.05\%)}}$$

(d) The mass flow rate of cooling water through the condenser is determined by applying steady state mass and energy rate balances to a control volume enclosing the condenser.

$$\dot{m}_2 = \dot{m}_3 \equiv \dot{m} \text{ and } \dot{m}_a = \dot{m}_b \equiv \dot{m}_{cw}$$

and

Problem 8.23 (Continued) – Page 4

$$0 = \dot{m}(h_2 - h_3) + \dot{m}_{cw}(h_a - h_b)$$

Solving for $\dot{m}_{cw}$

$$\dot{m}_{cw} = \frac{\dot{m}(h_2 - h_3)}{h_b - h_a}$$

For the cooling water, $h \approx h_f(T)$. Consequently, from Table A-2E

h_a = 28.08 Btu/lb
h_b = 44.09 Btu/lb

Substituting values

$$\dot{m}_{cw} = \frac{\left(2.13 \times 10^6 \dfrac{\text{lb}}{\text{h}}\right)\left(1018.17 \dfrac{\text{Btu}}{\text{lb}} - 78.02 \dfrac{\text{Btu}}{\text{lb}}\right)}{44.09 \dfrac{\text{Btu}}{\text{lb}} - 28.08 \dfrac{\text{Btu}}{\text{lb}}} = \underline{\mathbf{1.25 \times 10^8\ lb/h}}$$

The results of Problem 8.23 can be compared to the results of Problem 8.10 to see some of the effects of irreversibilities on the performance of a Rankine cycle for cycles with the same net power output. In this case, frictional pressure drop and stray heat loss in the line connecting the steam generator and the turbine, frictional pressure drop of the working fluid in passing through the condenser, and irreversibilities in the turbine and pump result in lower thermal efficiency, greater steam flow rate, increased heat addition, and increased mass flow rate of cooling water.

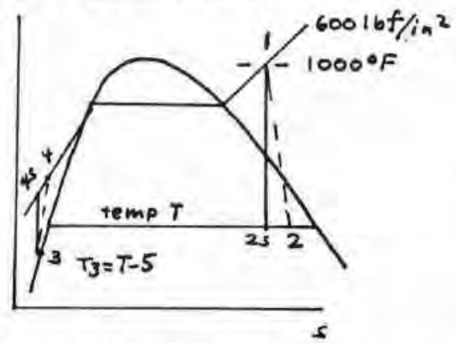
PROBLEM 8.24

KNOWN: Steady-state operating data are provided for a vapor power plant in which steam exits the turbine as a two-phase liquid-vapor mixture at temperature T, and condensate exits the condenser at 5°F lower than T.

FIND: (a) For T = 80°F, determine the steam quality at the turbine exit, the mass flow rate, and the cycle thermal efficiency. (b) Plot the quantities of (a) versus T ranging from 80 to 105°F.

SCHEMATIC & GIVEN DATA:

$\eta_t = 90\%$
$\eta_p = 80\%$
$\dot{W}_{net} = 1\,MW$



ENGINEERING MODEL:

1. Control volumes enclosing each of the principal components is at steady state.
2. The pump and turbine operate adiabatically.
3. Kinetic/potential energy can be ignored.
4. Condensate exits the condenser and enters the pump as sat. liquid.

ANALYSIS: First fix each of the principal states for T = 80°F: From Table A-4E, $h_1 = 1517.8$ Btu/lb, $s_1 = 1.7155$ Btu/lb·°R. Then, with $s_{2s} = s_1$ and data from Table A-2E

$$x_{2s} = \frac{s_1 - s_f}{s_g - s_f} = \frac{1.7155 - 0.09332}{2.0356 - 0.09332} = 0.835 \Rightarrow h_{2s} = 48.09 + (0.835)(1048.3) = 923.4 \text{ Btu/lb}$$

Then, with

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 1517.8 - 0.9(1517.8 - 923.4) = 982.8 \text{ Btu/lb}$$

$$\Rightarrow x_2 = \frac{h_2 - h_f}{h_{fg}} = \frac{982.8 - 48.09}{1048.3} = 0.892 \longleftarrow x_2$$

At state 3, $h_3 = h_f(75°F) = 43.1$ Btu/lb. Too, $h_{4s} \approx h_3 + v_3(P_4 - P_3)$, or

$$h_{4s} = 43.9 \frac{\text{Btu}}{\text{lb}} + \left(0.0161 \frac{\text{ft}^3}{\text{lb}}\right)(600 - 0.43)\frac{\text{lbf}}{\text{in}^2}\left|\frac{144\,\text{in}^2}{1\,\text{ft}^2}\right|\left|\frac{1\,\text{Btu}}{778\,\text{ft}\cdot\text{lbf}}\right| = 44.9 \text{ Btu/lb}.$$

Then, with

$$\eta_p = \frac{h_{4s} - h_3}{h_4 - h_3} \Rightarrow h_4 = h_3 + \frac{(h_{4s} - h_3)}{\eta_p} = 43.1 + \frac{(44.9 - 43.1)}{0.8} = 45.4 \text{ Btu/lb}.$$

The thermal efficiency is obtained as

$$\eta = \frac{\dot{W}_t/\dot{m} - \dot{W}_p/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4} = \frac{(1517.8 - 982.8) - (45.4 - 43.1)}{1517.8 - 45.4} = 0.362 \longleftarrow \eta$$

The mass flow rate is

$$\dot{m} = \frac{\dot{W}_{net}}{(\dot{W}_t/\dot{m}) - (\dot{W}_p/\dot{m})} = \frac{(10^3\,\text{kW})\left|\frac{3413\,\text{Btu/h}}{1\,\text{kW}}\right|}{(532.7\,\text{Btu/lb})} = 6.41 \times 10^3 \frac{\text{lb}}{\text{h}} \longleftarrow \dot{m}$$

The data for the required plots are obtained using IT, as follows:

PROBLEM 8.24 (Cont'd.)

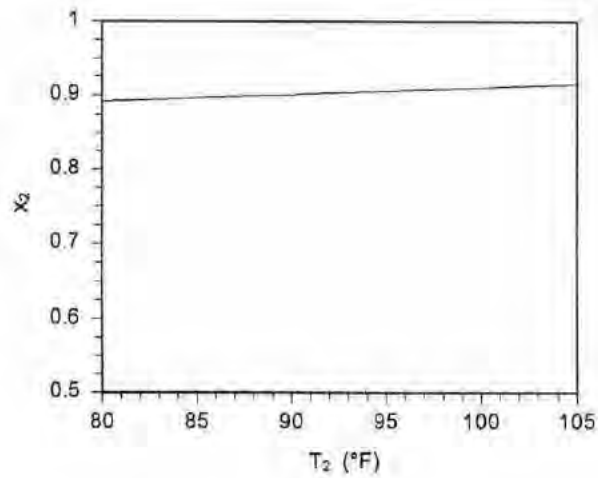
IT Code

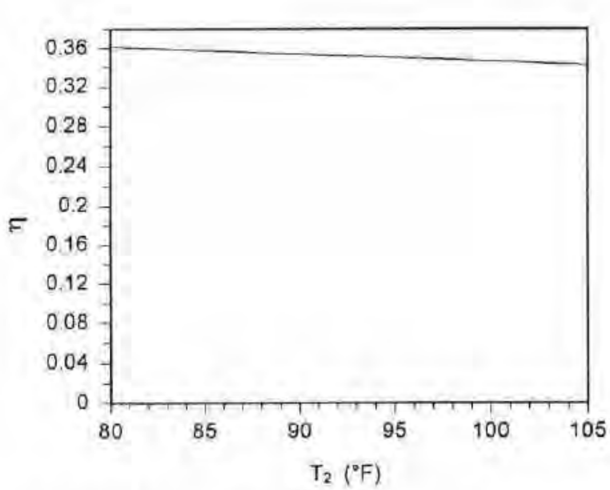
```
p1 = 600  // lbf/in.²
T1 = 1000  // °F
T2 = 80
T3 = T2 - 5
p3 = p2
p4 = p1
efft = 0.9
effp = 0.8
Wdotcycle = 1 * 1000 * 3413  // Btu/h

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
p2 = Psat_T("Water/Steam", T2)
h2s = h_Ps("Water/Steam", p2, s1)
h2 = h1 - (h1 - h2s) * efft
x2 = x_hP("Water/Steam", h2, p2)
psat = Psat_T("Water/Steam", T3)
h3 = hsat_Px("Water/Steam", psat, 0)
v3 = vsat_Px("Water/Steam", psat, 0)
h4 = h3 + v3 * (p4 - p3) * (144 / 778) /
effp

Wcycle = (h1 - h2) - (h4 - h3)
eta = Wcycle / (h1 - h4)
Wdotcycle = mdot * (Wcycle)
```

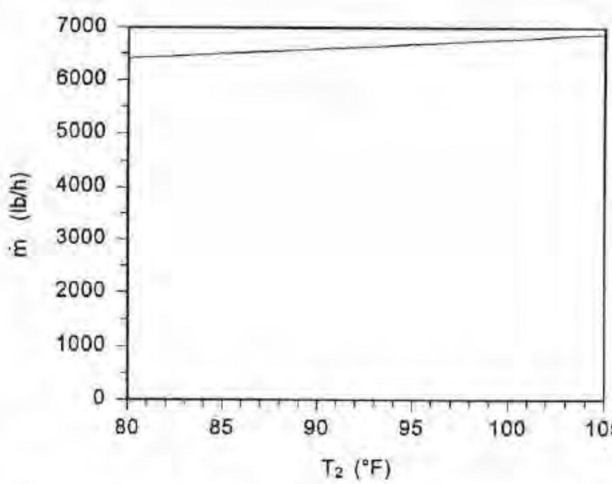
IT Results for T_1 = 80°F

h1 = 1518 Btu/lb
h2 = 982.9 Btu/lb
h3 = 42.67 Btu/lb
h4 = 44.9 Btu/lb
x2 = 0.8919
η = 0.3616
$\dot{m}$ = 6409 lb/h

PLOTS:







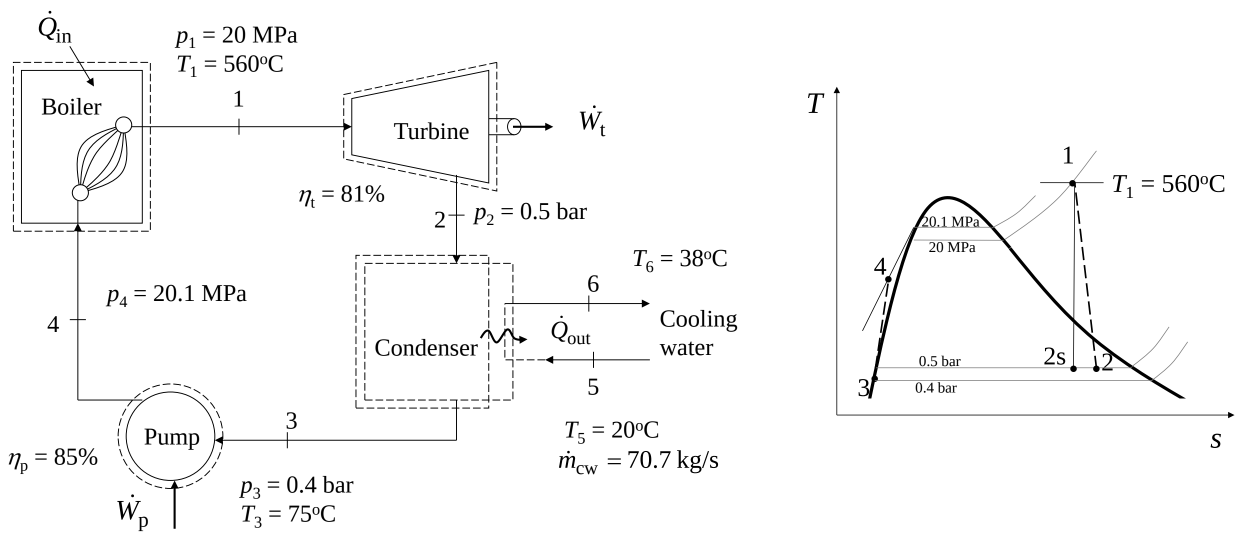
As the condenser temperature increases, the cycle thermal efficiency decreases, as expected. As a result, the turbine exit quality increases and the required mass flow rate increases.

8.25 Superheated steam at 20 MPa, 560°C enters the turbine of a vapor power plant. The pressure at the exit of the turbine is 0.5 bar, and liquid leaves the condenser at 0.4 bar at 75°C. The pressure is increased to 20.1 MPa across the pump. The turbine and pump have isentropic efficiencies of 81 and 85%, respectively. Cooling water enters the condenser at 20°C with a mass flow rate of 70.7 kg/s and exits the condenser at 38°C. For the cycle, determine
(a) the mass flow rate of steam, in kg/s.
(b) the thermal efficiency.

KNOWN: Water is the working fluid in a vapor power plant. Data are given at various states in the cycle.

FIND: (a) the mass flow rate of steam, in kg/s and (b) the thermal efficiency.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. Stray heat transfer in the turbine, condenser, and pump is ignored.
3. Kinetic and potential energy effects are negligible.

ANALYSIS: First fix each principal state.

State 1: p_1 = 20 MPa (200 bar), T_1 = 560°C → h_1 = 3423.0 kJ/kg, s_1 = 6.3705 kJ/kg·K

State 2s: $p_{2s} = p_2$ = 0.5 bar, $s_{2s} = s_1$ = 6.3705 kJ/kg·K → x_{2s} = 0.8119, h_{2s} = 2212.2 kJ/kg

State 2: p_2 = 0.5 bar, h_2 = 2442.3 kJ/kg (see below)

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \rightarrow h_2 = h_1 - \eta_t (h_1 - h_{2s}) = 3423.0\frac{\text{kJ}}{\text{kg}} - (0.81)(3423.0 - 2212.2)\frac{\text{kJ}}{\text{kg}} = 2442.3 \text{ kJ/kg}$$

State 3: p_3 = 0.4 bar, T_3 = 75°C → From Table A-2 $p_3 > p_{sat}$ @ 75°C. Thus, state 3 is a sub-cooled liquid state. Since the pressure is low, $h_3 \approx h_{f3}$ at 75°C = 313.93 kJ/kg, $v_3 \approx v_{f3}$ at 75°C = 0.0010259 m^3/kg

State 4: p_4 = 20.1 MPa (201 bar), h_4 = 338.14 kJ/kg (see below)

$$\eta_p = \frac{v_3(p_4 - p_3)}{h_4 - h_3} \rightarrow h_4 = h_3 + \frac{v_3(p_4 - p_3)}{\eta_p}$$

$$h_4 = 313.93\frac{\text{kJ}}{\text{kg}} + \frac{(0.0010259\frac{\text{m}^3}{\text{kg}})(201 - 0.4)\text{ bar}}{0.85}\left|\frac{100\text{ kPa}}{1\text{ bar}}\right|\left|\frac{1000\frac{\text{N}}{\text{m}^2}}{1\text{ kPa}}\right|\left|\frac{1\text{ kJ}}{1000\text{ N}\cdot\text{m}}\right| = 338.14 \text{ kJ/kg}$$

State 5: T_5 = 20°C, liquid → $h_5 \approx h_{f5}$ at 20°C = 83.96 kJ/kg

State 6: T_6 = 38°C, liquid → $h_6 \approx h_{f6}$ at 38°C = 159.21 kJ/kg

(a) The mass flow rate of the steam can be determined by writing an energy balance for the condenser. With no stray heat transfer with the surroundings and no work, the energy balance for the condenser reduces to

$$0 = \dot{m}(h_2 - h_3) + \dot{m}_{cw}(h_5 - h_6)$$

where $\dot{m}$ is the mass flow rate of the steam and $\dot{m}_{cw}$ is the mass flow rate of the cooling water. Rearranging to solve for the mass flow rate of steam gives

$$\dot{m} = \frac{\dot{m}_{cw}(h_6 - h_5)}{(h_2 - h_3)}$$

Substituting values and solving give

$$\dot{m} = \frac{(70.7\text{ kg/s})(159.21 - 83.96)\text{ kJ/kg}}{(2442.3 - 313.93)\text{ kJ/kg}} = \mathbf{\underline{2.50\ kg/s}}$$

(b) The thermal efficiency is

$$\eta = \frac{\dot{W}_{\mathrm{t}} / \dot{m} - \dot{W}_{\mathrm{p}} / \dot{m}}{\dot{Q}_{\mathrm{in}} / \dot{m}} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$$

Substituting enthalpy values and solving yield

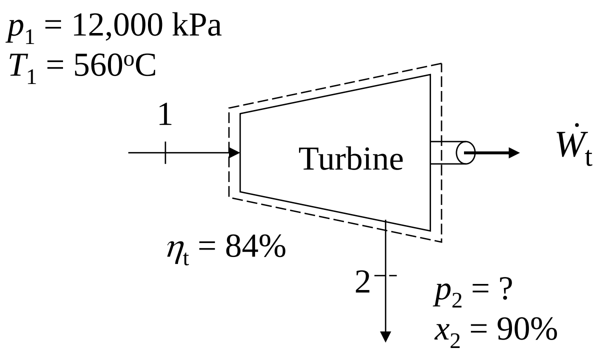
$$\eta = \frac{(3423.0 - 2442.3)\ \mathrm{kJ/kg} - (338.14 - 313.93)\ \mathrm{kJ/kg}}{(3423.0 - 338.14)\ \mathrm{kJ/kg}} = \underline{\mathbf{0.3101\ (31.01\%)}}$$

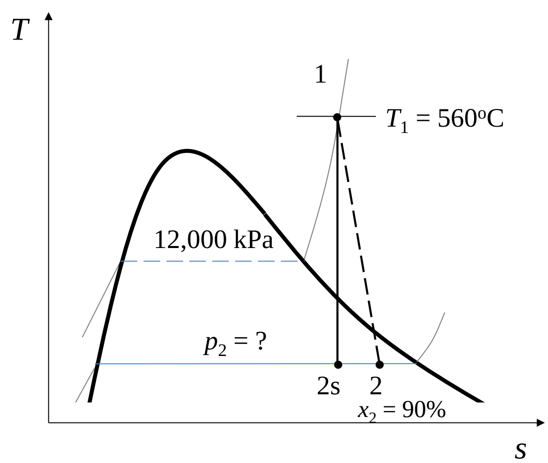
8.26 In the preliminary design of a power plant, water is chosen as the working fluid. It is determined that the turbine inlet temperature and pressure should be 560°C and 12,000 kPa, respectively. The quality of steam exiting the turbine should be at least 90%. If the isentropic turbine efficiency is 84%, determine the minimum condenser pressure allowable, in kPa.

KNOWN: Turbine inlet temperature and pressure and exit quality are specified in the preliminary design of a vapor power plant with water as the working fluid. The isentropic turbine efficiency is also known.

FIND: the minimum allowable condenser pressure, in kPa.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL:
1. The control volume is at steady state.
2. The turbine operates adiabatically.
3. Kinetic and potential energy effects are negligible.
4. At the turbine exit the water is a two-phase liquid-vapor mixture whose quality is at least 90%.

ANALYSIS: For fixed turbine inlet temperature and pressure, the values of specific enthalpy and specific entropy can be determined.

<u>State 1</u>: p_1 = 12,000 kPa, T_1 = 560°C → h_1 = 3506.2 kJ/kg, s_1 = 6.6840 kJ/kg·K

A trial and error procedure can be used to determine p_2 corresponding to x_2 = 90% using table data. Assume a value for p_2, calculate x_2 using the isentropic turbine efficiency as follows:

<u>State 2s</u>: $p_{2s} = p_2$ = ***assumed value***, $s_{2s} = s_1$ = 6.6840 kJ/kg·K → Solve for x_{2s} and then h_{2s}

<u>State 2</u>: Solve for h_2 as using isentropic turbine efficiency shown below

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \rightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s})$$

Next calculate x_2 using p_2 and h_2. Compare results. If the calculated value of x_2 is less than 90%, select a higher pressure for p_2. If the calculated value for x_2 is greater than 90% select a lower pressure for p_2.

For instance, assume p_2 = 10 kPa.

Now calculate h_2 using the isentropic turbine efficiency.

State 2s: $p_{2s} = p_2$ = 10 kPa, $s_{2s} = s_1$ = 6.6840 kJ/kg·K $\rightarrow x_{2s}$ = 0.8045, h_{2s} = 2116.8 kJ/kg

$$h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 3506.2\frac{\text{kJ}}{\text{kg}} - (0.84)(3506.2 - 2116.8)\frac{\text{kJ}}{\text{kg}} = 2339.1 \text{ kJ/kg.}$$

State 2: p_2 = 10 kPa, h_2 = 2339.1 kJ/kg $\rightarrow x_2$ = 0.8974

Since 0.8974 < 0.9, the next assumed pressure value should be higher. Continue this process until the calculated x_2 = 0.9.

Alternatively, the following IT code can be used to automatically converge values for h_2 to determine p_2 = **11.08 kPa**.

IT Code

```
T1 = 560 // oC
p1 = 12000 // kPa
eff_t = 0.84
x2 = 0.9

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
s2s = s1
p2s = p2
x2s = x_sP("Water/Steam", s2s, p2s)
h2s = h_Ps("Water/Steam", p2s, s2s)
h2 = h1 - eff_t*(h1 - h2s)
x2_calc = x_hP("Water/Steam", h2, p2)
x2_calc = x2
```

IT Results

h1	3506
h2	2349
h2s	2129
p2	11.08
p2s	11.08
s1	6.683
s2s	6.683
x2_calc	0.9
x2s	0.8077
eff_t	0.84
p1	1.2E4
T1	560
x2	0.9

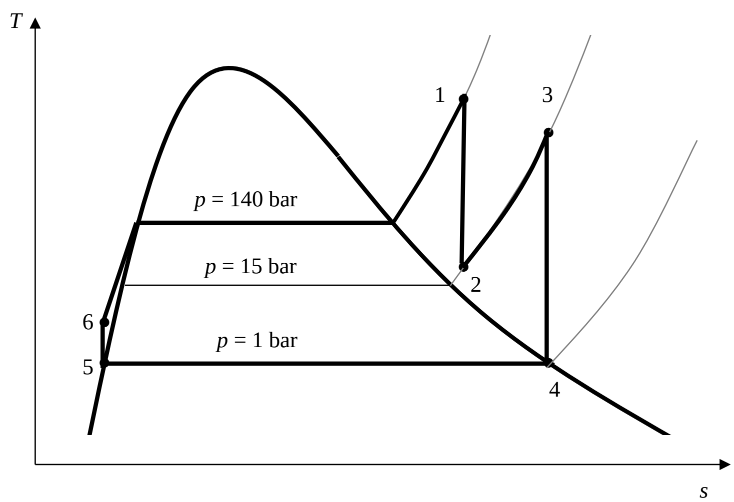
It is common practice to maintain at least 90% quality at the exit of a steam turbine to avoid undesirable effect.

8.27 Steam is the working fluid in the ideal reheat cycle shown in Fig. P8.27 together with operational data. If the mass flow rate is 1.3 kg/s, determine the power developed by the cycle, in kW, and the cycle thermal efficiency.

KNOWN: An ideal reheat cycle operates with steam as the working fluid. Operational data are provided.

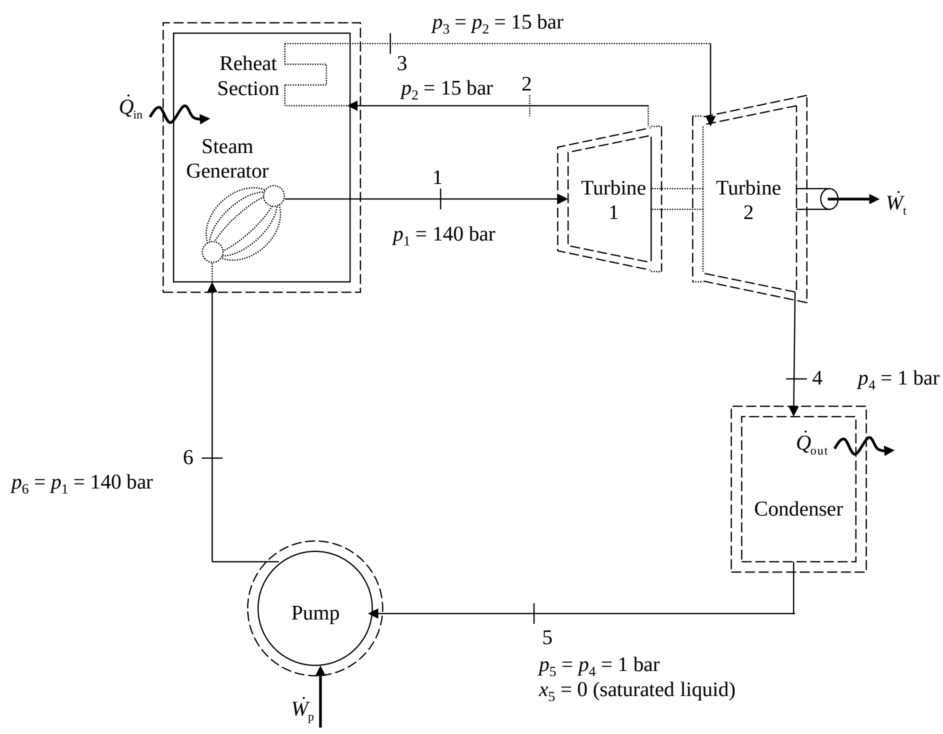
FIND: Determine the power developed by the cycle, in kW, and the cycle thermal efficiency.

SCHEMATIC AND GIVEN DATA:



State	p (bar)	T (°C)	h (kJ/kg)
1	140	520.0	3377.8
2	15	201.2	2800.0
3	15	428.9	3318.5
4	1	99.63	2675.5
5	1	99.63	417.46
6	140		431.96

Fig. P8.27



ENGINEERING MODEL:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. All processes of the working fluid are internally reversible.
3. The turbine and pump operate adiabatically.
4. Kinetic and potential energy effects are negligible.
5. Condensate exits the condenser as saturated liquid.

ANALYSIS:

The net power developed by the cycle is

$$\dot{W}_{\text{cycle}} = \dot{W}_{\text{t1}} + \dot{W}_{\text{t2}} - \dot{W}_{\text{p}}$$

Mass and energy rate balances for control volumes around the two turbine stages and the pump give, respectively,

$$\text{Turbine 1:}\quad \dot{W}_{\text{t1}} = \dot{m}(h_1 - h_2)$$

$$\text{Turbine 2:}\quad \dot{W}_{\text{t2}} = \dot{m}(h_3 - h_4)$$

$$\text{Pump:}\quad \dot{W}_{\text{p}} = \dot{m}(h_6 - h_5)$$

where $\dot{m}$ is the mass flow rate of the steam. Solving for the net power developed by the cycle yields

$$\dot{W}_{\text{cycle}} = \dot{m}[(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)]$$

$$\dot{W}_{\text{cycle}} = \left(1.3\frac{\text{kg}}{\text{s}}\right)\left[(3377.8 - 2800.0)\frac{\text{kJ}}{\text{kg}} + (3318.5 - 2675.5)\frac{\text{kJ}}{\text{kg}} - (431.96 - 417.46)\frac{\text{kJ}}{\text{kg}}\right]\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right|$$

$$\dot{W}_{\text{cycle}} = \underline{\mathbf{1568.2\ kW}}$$

The thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}}$$

The total rate of heat transfer to the working fluid as it passes through the steam generator and reheater is determined using mass and energy rate balances as

$$\dot{Q}_{\text{in}} = \dot{m}\left[(h_1 - h_6) + (h_3 - h_2)\right]$$

Solving for rate of heat transfer gives

$$\dot{Q}_{\text{in}} = \left(1.3\frac{\text{kg}}{\text{s}}\right)\left[(3377.8 - 431.96) + (3318.5 - 2800.0)\right]\frac{\text{kJ}}{\text{kg}}\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right| = 4503.6\ \text{kW}$$

The thermal efficiency is then

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{1568.2\,\text{kW}}{4503.6\,\text{kW}} = \underline{\mathbf{0.3482\ (34.82\%)}}$$

8.28 Water is the working fluid in an ideal Rankine cycle with superheat and reheat. Steam enters the first-stage turbine at 1400 lbf/in.^{2} and 1000°F, expands to a pressure of 350 lbf/in.^{2}, and is reheated to 900°F before entering the second-stage turbine. The condenser pressure is 2 lbf/in.^{2} The net power output of the cycle is 1×10^9 Btu/h. Determine for the cycle

(a) the mass flow rate of steam, in lb/h.

(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the steam generator.

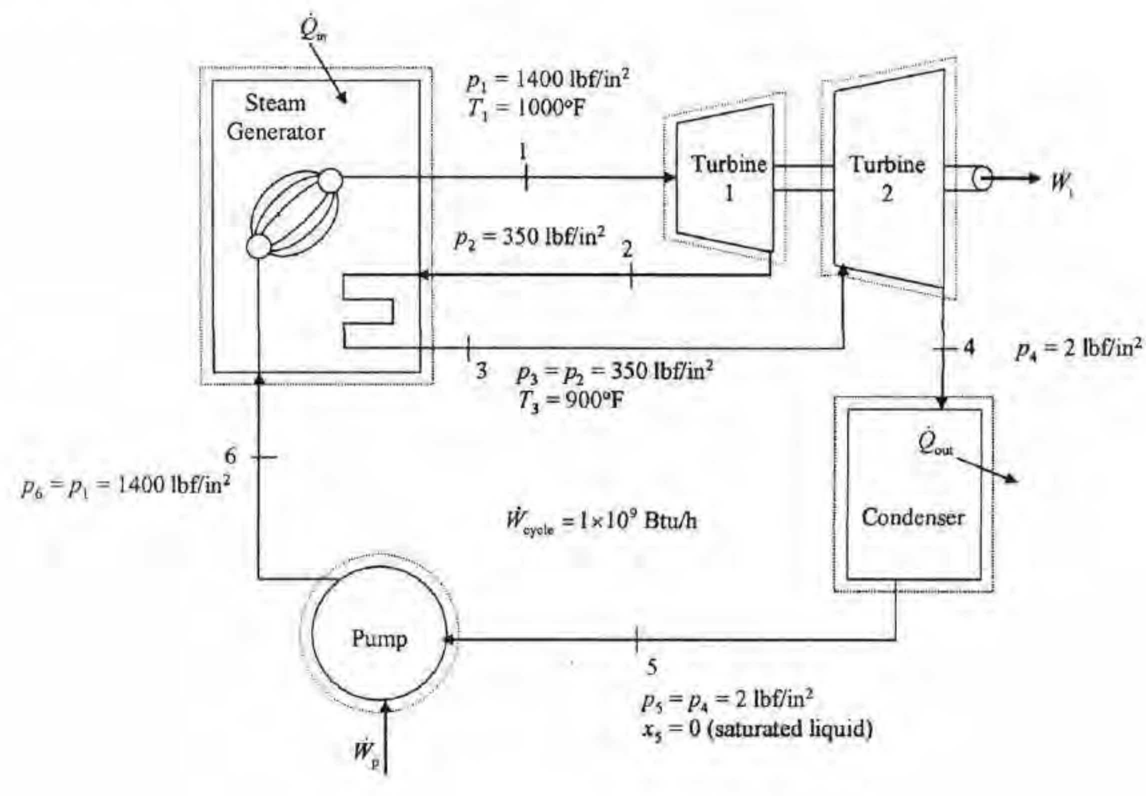
(c) the rate of heat transfer, in Btu/h, to the working fluid passing through the reheater.

(d) the thermal efficiency.

KNOWN: An ideal Rankine cycle with superheat and reheat operates with water as the working fluid. The net power output of the cycle is given.

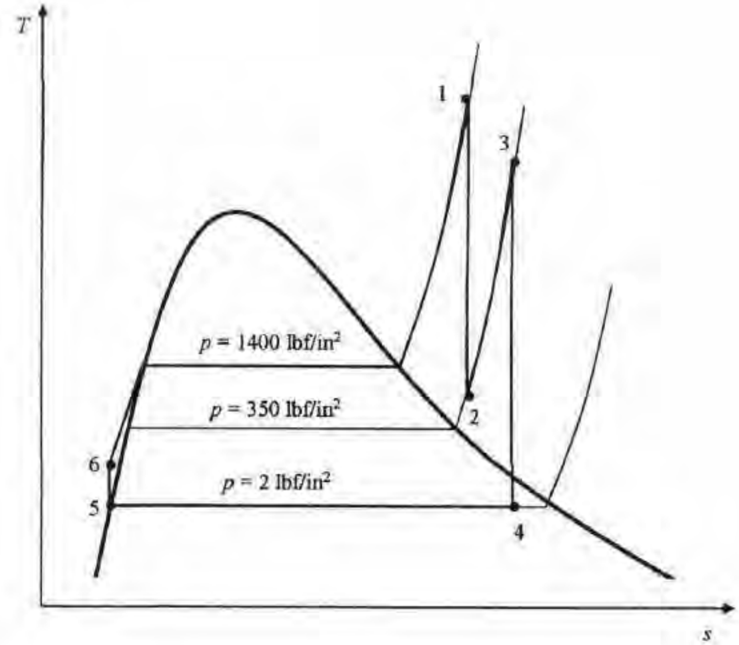
FIND: Determine the mass flow rate of steam, the rate of heat transfer to the working fluid passing through the steam generator and the reheater, and the thermal efficiency.

SCHEMATIC AND GIVEN DATA:



Problem 8.28 (Continued) – Page 2

T-s diagram



ENGINEERING MODEL:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. All processes of the working fluid are internally reversible.
3. The turbine and pump operate adiabatically.
4. Kinetic and potential energy effects are negligible.
5. Condensate exits the condenser as saturated liquid.
6. The steam generator and the compressor operate at constant pressure.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 1400$ lbf/in.^{2}, $T_1 = 1000$°F → From Table A-4E: $h_1 = 1493.5$ Btu/lb and $s_1 = 1.6094$ Btu/(lb·°R)

State 2: $p_2 = 350$ lbf/in.^{2}, $s_2 = s_1 = 1.6094$ Btu/(lb·R) → From Table A-4E (interpolated): $h_2 = 1313.48$ Btu/lb

State 3: $p_3 = p_2 = 350$ lbf/in.^{2}, $T_3 = 900$°F → From Table A-4E: $h_3 = 1471.8$ Btu/lb and $s_1 = 1.7409$ Btu/(lb·°R)

State 4: $p_4 = 2$ lbf/in.^{2}, $s_4 = s_3 = 1.7409$ Btu/(lb·°R) → From Table A-3E: $x_4 = (1.7409 - 0.1750)/1.7448 = 0.8975$ and $h_4 = h_{f4} + x_4 h_{fg4} = 94.02 + (0.8975)(1022.1) =$ 1011.35 Btu/lb

Problem 8.28 (Continued) – Page 3

State 5: $p_5 = p_4 = 2$ lbf/in.^{2}, sat liq. → From Table A-3E: $h_5 = h_{f5} = 94.02$ Btu/lb and $v_5 = v_{f5} = 0.01623$ ft^3/lb

State 6: $h_6 \approx h_5 + v_5(p_6 - p_5)$

$$h_6 = 94.02 \text{ Btu/lb} + 0.01623\left(\frac{\text{ft}^3}{\text{lb}}\right)(1400-2)\left(\frac{\text{lbf}}{\text{in}^2}\right)\left|\frac{144 \text{ in}^2}{\text{ft}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right| = 98.22 \text{ Btu/lb}$$

(a) The mass flow rate of steam is found as follows. Mass and energy rate balances for control volumes enclosing the turbine and pump give

$$\dot{W}_{t1} = \dot{m}(h_1 - h_2), \quad \dot{W}_{t2} = \dot{m}(h_3 - h_4), \quad \text{and} \quad \dot{W}_p = \dot{m}(h_6 - h_5)$$

The net power of the cycle is

$$\dot{W}_{cycle} = \dot{W}_{t1} + \dot{W}_{t2} - \dot{W}_p = \dot{m}[(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)]$$

Solving for $\dot{m}$

$$\dot{m} = \frac{\dot{W}_{cycle}}{[(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)]}$$

Inserting values

$$\dot{m} = \frac{1\times10^9 \frac{\text{Btu}}{\text{h}}}{\left(1493.5\frac{\text{Btu}}{\text{lb}} - 1313.48\frac{\text{Btu}}{\text{lb}}\right) + \left(1471.8\frac{\text{Btu}}{\text{lb}} - 1011.35\frac{\text{Btu}}{\text{lb}}\right) - \left(98.22\frac{\text{Btu}}{\text{lb}} - 94.02\frac{\text{Btu}}{\text{lb}}\right)}$$

$$\dot{m} = \underline{\mathbf{1.57\times10^6\ lb/hr}}$$

(b) The rate of heat transfer to the working fluid passing through the steam generator can be determined by applying mass and energy balances to a control volume around the steam generator to give

$$\dot{Q}_{inSG} = \dot{m}(h_1 - h_6) = (1.57\times10^6 \text{ lb/h})(1493.5 \text{ Btu/lb} - 98.22 \text{ Btu/lb}) = \underline{\mathbf{2.19\times10^9\ Btu/h}}$$

(c) The rate of heat transfer to the working fluid passing through the reheater can be determined by applying mass and energy balances to a control volume around the reheater to give

$$\dot{Q}_{inRH} = \dot{m}(h_3 - h_2) = (1.57\times10^6 \text{ lb/h})(1471.8 \text{ Btu/lb} - 1313.48 \text{ Btu/lb}) = \underline{\mathbf{0.249\times10^9\ Btu/h}}$$

(c) The thermal efficiency is

Problem 8.28 (Continued) – Page 4

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = (1\times10^9 \text{ Btu/h})/(2.19\times10^9 \text{ Btu/h} + 0.249\times10^9 \text{ Btu/h}) = \underline{\mathbf{0.4100\ (41.00\%)}}$$

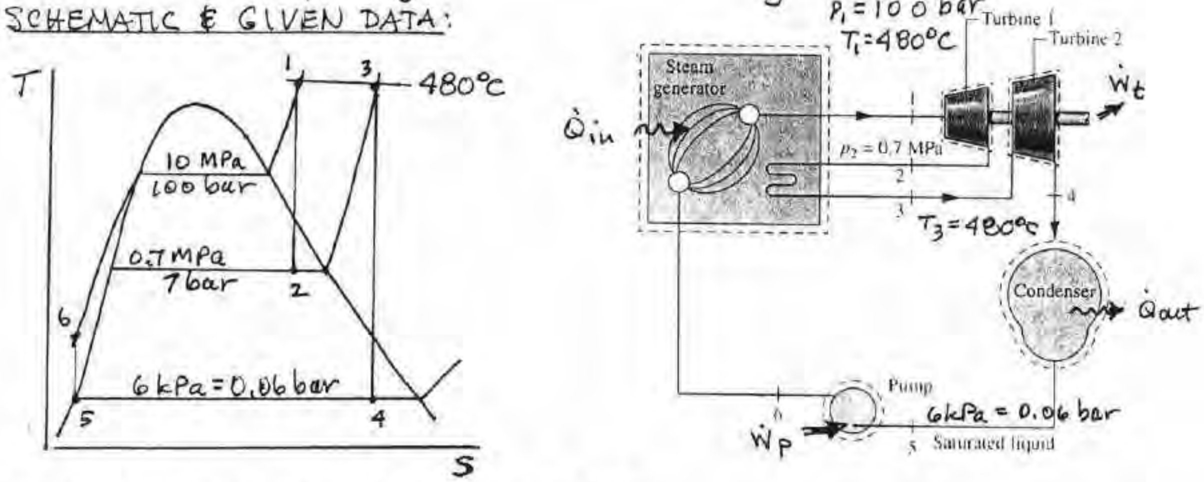
The results of Problem 8.28 can be compared to the results of Problem 8.10 to see some of the effects of reheat on the performance of a Rankine cycle for cycles with the same net power output. In this case, reheat results in higher thermal efficiency, lower steam flow rate, and lower heat addition.

PROBLEM 8.29

KNOWN: Water is the working fluid in an ideal Rankine cycle with reheat. The states at the inlets to both turbine stages and the condenser exit are specified.

FIND: Determine (a) the rate of heat addition per kg of steam flowing, (b) the thermal efficiency, (c) the rate of heat transfer for the condenser per kg of steam condensing.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 8.3.

ANALYSIS: First, fix all of the principal states.

State 1: $p_1 = 100$ bar, $T_1 = 480°C \Rightarrow h_1 = 3321.4$ kJ/kg, $s_1 = 6.5282$ kJ/kg·K

State 2: $p_2 = 7$ bar, $s_2 = s_1 \Rightarrow x_2 = \dfrac{s_2 - s_{f2}}{s_{g2} - s_{f2}} = 0.9619$, $h_2 = 2684.8$ kJ/kg

State 3: $p_3 = 7$ bar, $T_3 = 480°C \Rightarrow h_3 = 3438.9$ kJ/kg, $s_3 = 7.8723$ kJ/kg·K

State 4: $p_4 = 0.06$ bar, $s_4 = s_3 \Rightarrow x_4 = \dfrac{s_4 - s_{f4}}{s_{g4} - s_{f4}} = 0.9413$, $h_4 = 2425.6\ \frac{kJ}{kg}$

State 5: $p_5 = 0.06$ bar, sat. liquid $\Rightarrow h_5 = 151.53$ kJ/kg

State 6:
$$h_6 \approx h_5 + v_5(p_6 - p_5)$$
$$= 151.53\ \frac{kJ}{kg} + (1.006\times10^{-3})\ \frac{m^3}{kg}(100 - 0.06)\,\text{bar}\left|\frac{10^5\ N/m^2}{1\ \text{bar}}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$
$$= 151.53 + 10.06 = 161.59\ \text{kJ/kg}$$

(a) For the control volume enclosing the steam generator

$$\dot{Q}_{in} = \dot{m}\left[(h_1 - h_6) + (h_3 - h_2)\right] \Rightarrow \dot{Q}_{in}/\dot{m} = (h_1 - h_6) + (h_3 - h_2)$$

$$\dot{Q}_{in}/\dot{m} = (3321.4 - 161.59) + (3438.9 - 2684.8)$$
$$= 3913.9\ \text{kJ/kg} \leftarrow \quad \dot{Q}_{in}/\dot{m}$$

(b) The thermal efficiency is $\eta = \dfrac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}}$

$$\dot{W}_{cycle}/\dot{m} = (h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)$$
$$= 636.6 + 1013.3 - 10.06 = 1639.8\ \text{kJ/kg}$$

Thus
$$\eta = \frac{1639.8}{3913.9} = 0.419\ (41.9\%) \leftarrow \quad \eta$$

PROBLEM 8.29 (Cont'd)

(c) For the condenser

$$\dot{Q}_{out} = \dot{m}(h_4 - h_5) \Rightarrow \frac{\dot{Q}_{out}}{\dot{m}} = h_4 - h_5$$

$$= 2425.6 - 151.53 = 2274.1 \ \frac{kJ}{kg} \leftarrow \dot{Q}_{out}/\dot{m}$$

Alternatively

$$\dot{Q}_{out}/\dot{m} = \dot{Q}_{in}/\dot{m} - \dot{W}_{cycle}/\dot{m}$$

① $$= 3913.9 - 1639.8 = 2274.1 \text{ kJ/kg}$$

---

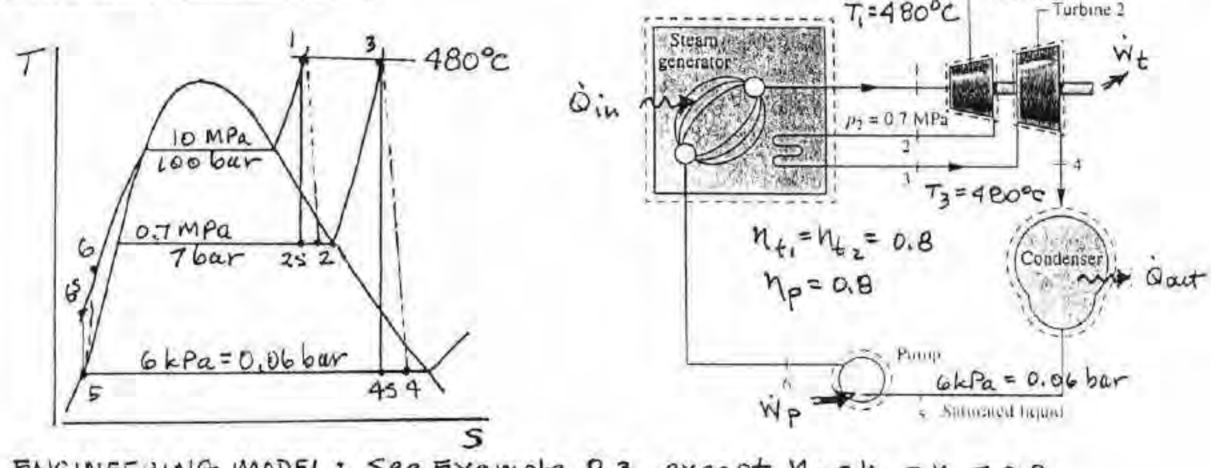
1. The results of this analysis can be compared to the results of Problem 8.2 to see some of the effects of incorporating reheat into the ideal Rankine cycle.

PROBLEM 8.30

KNOWN: The ideal Rankine cycle with reheat of problem 8.29 is modified to have turbine stage and pump isentropic efficiencies of 80%.

FIND: Answer the same questions as in Problem 8.29 for the modified cycle.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 8.3, except $\eta_{t_1} = \eta_{t_2} = \eta_p = 0.8$.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 100$ bar, $T_1 = 480°C \Rightarrow h_1 = 3321.4$ kJ/kg, $s_1 = 6.5282$ kJ/kg·K

State 2: Using the isentropic efficiency of the first turbine stage

$$\eta_{t_1} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_{t_1}(h_1 - h_{2s})$$

With $h_{2s} = 2684.8$ kJ/kg from Problem 8.29, $h_2 = 2812.1$ kJ/kg

State 3: $p_3 = 7$ bar, $T_3 = 480°C \Rightarrow h_3 = 3438.9$ kJ/kg, $s_3 = 7.8723$ kJ/kg·K

State 4: For the second turbine stage, $h_4 = h_3 - \eta_{t_2}(h_3 - h_{4s})$

With $h_{4s} = 2425.6$ kJ/kg from Problem 8.29, $h_4 = 2628.3$ kJ/kg

State 5: $p_5 = 0.06$ bar, sat. liquid $\Rightarrow h_5 = 151.53$ kJ/kg

State 6: Using the isentropic pump efficiency

$$\eta_p = \frac{h_{6s} - h_5}{h_6 - h_5} \Rightarrow h_6 = h_5 + (h_{6s} - h_5)/\eta_p$$

With $h_{6s} = 161.59$ kJ/kg from Problem 8.29, $h_6 = 164.11$ kJ/kg

(a) The heat addition is

$$\dot{Q}_{in}/\dot{m} = (h_1 - h_6) + (h_3 - h_2) = 3786.6 \text{ kJ/kg} \leftarrow \qquad \dot{Q}_{in}/\dot{m}$$

(b) the net power developed, per kg of steam flowing, is

$$\dot{W}_{cycle}/\dot{m} = (h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5) = 1307.3 \text{ kJ/kg}$$

The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = 0.345 (34.5\%) \leftarrow \qquad \eta$$

(c) For the condenser

① $$\dot{Q}_{out}/\dot{m} = h_4 - h_5) = 2479.3 \text{ kJ/kg} \leftarrow \qquad \dot{Q}_{out}/\dot{m}$$

1. These results can be compared with those of Problem 8.29 to see some of the effects of turbine and pump irreversibilities on the performance of the Rankine cycle with reheat.

PROBLEM 8.31

(a) Refer to Problem 8.29.

IT Code

```
p1 = 100  // bar
T1 = 480  // °C
p2 = 7  // bar
p3 = p2
T3 = 480  // °C
p4 = 0.06  // bar
p5 = p4
p6 = p1
mdot = 1

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2 = h_Ps("Water/Steam", p2, s2)
s2 = s1
h3 = h_PT("Water/Steam", p3, T3)
s3 = s_PT("Water/Steam", p3, T3)
h4 = h_Ps("Water/Steam", p4, s4)
s4 = s3
h5 = hsat_Px("Water/Steam", p5, 0)
v5 = vsat_Px("Water/Steam", p5, 0)
h6s = h5 + v5 * (p6 - p5) * 100

Wdott = mdot * ((h1 - h2) + (h3 - h4))
Wdotp = mdot * (h6 - h5)
Wdotcycle = Wdott - Wdotp
Qdotin = mdot * ((h1 - h6) + (h3 - h2))
eta = Wdotcycle / Qdotin
Qdotout = mdot * (h4 - h5)
```

IT Results (p_2 = 7 bar, T_3 = 480 °C)

$\dot{Q}_{in} / \dot{m}$ = 3914 kJ/kg

$\dot{Q}_{out} / \dot{m}$ = 2275 kJ/kg

$\dot{W}_{cycle} / \dot{m}$ = 1639 kJ/kg

η = 0.4188

h_1 = 3321 kJ/kg

h_2 = 2684 kJ/kg

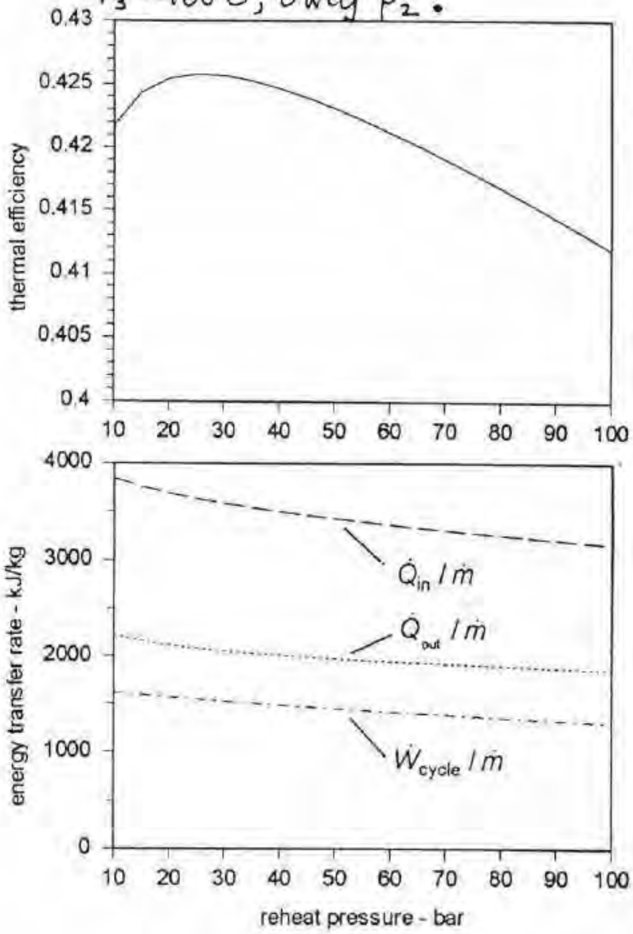
h_3 = 3438 kJ/kg

h_4 = 2426 kJ/kg

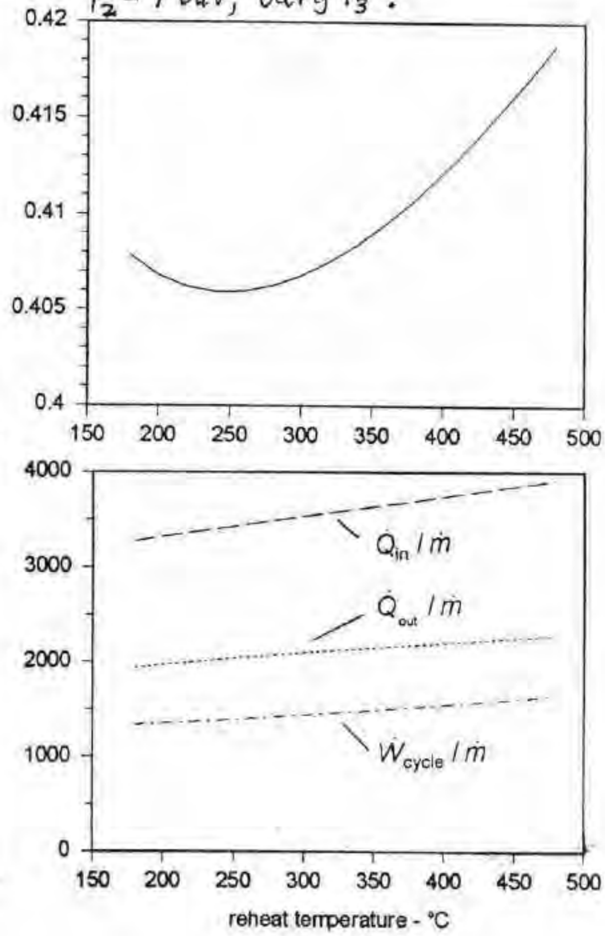
h_5 = 151 kJ/kg

h_6 = 161.1 kJ/kg

T_3 = 480°C, vary p_2:



p_2 = 7 bar, vary T_3:



PROBLEM 8.31 (Cont'd)

(b) Refer to Problem 8.30.

IT Code

```
p1 = 100  // bar
T1 = 480  // °C
p2 = 7  // bar
p3 = p2
T3 = 480  // °C
p4 = 0.06  // bar
p5 = p4
p6 = p1
etat1 = 0.8
etat2 = etat1
etap = etat1
mdot = 1

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2s = h_Ps("Water/Steam", p2, s2s)
s2s = s1
h2 = h1 - etat1 * (h1 - h2s)

h3 = h_PT("Water/Steam", p3, T3)
s3 = s_PT("Water/Steam", p3, T3)
h4s = h_Ps("Water/Steam", p4, s4s)
s4s = s3
h4 = h3 - etat2 * (h3 - h4s)

h5 = hsat_Px("Water/Steam", p5, 0)
v5 = vsat_Px("Water/Steam", p5, 0)
h6s = h5 + v5 * (p6 - p5) * 100
h6 = h5 + (h6s - h5) / etap

Wdott = mdot * ((h1 - h2) + (h3 - h4))
Wdotp = mdot * (h6 - h5)
Wdotcycle = Wdott - Wdotp
Qdotin = mdot * ((h1 - h6) + (h3 - h2))
eta = Wdotcycle / Qdotin
Qdotout = mdot * (h4 - h5)
```

IT Results (p_2 = 7 bar, T_3 = 480 °C)

$\dot{Q}_{in} / \dot{m}$ = 3784 kJ/kg

$\dot{Q}_{out} / \dot{m}$ = 2477 kJ/kg

$\dot{W}_{cycle} / \dot{m}$ = 1307 kJ/kg

η = 0.3454

h_1 = 3321 kJ/kg

h_2 = 2812 kJ/kg

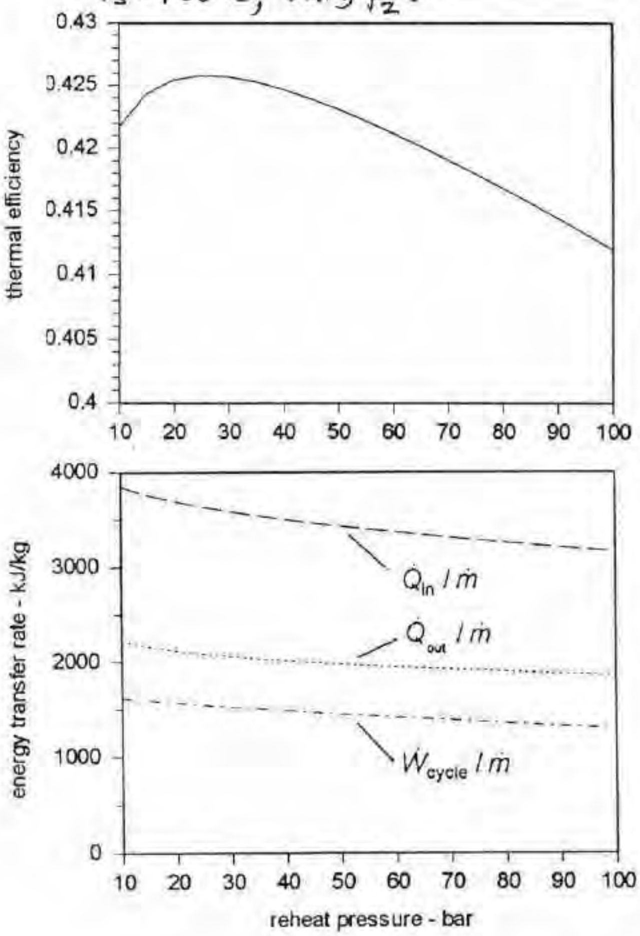
h_3 = 3438 kJ/kg

h_4 = 2628 kJ/kg

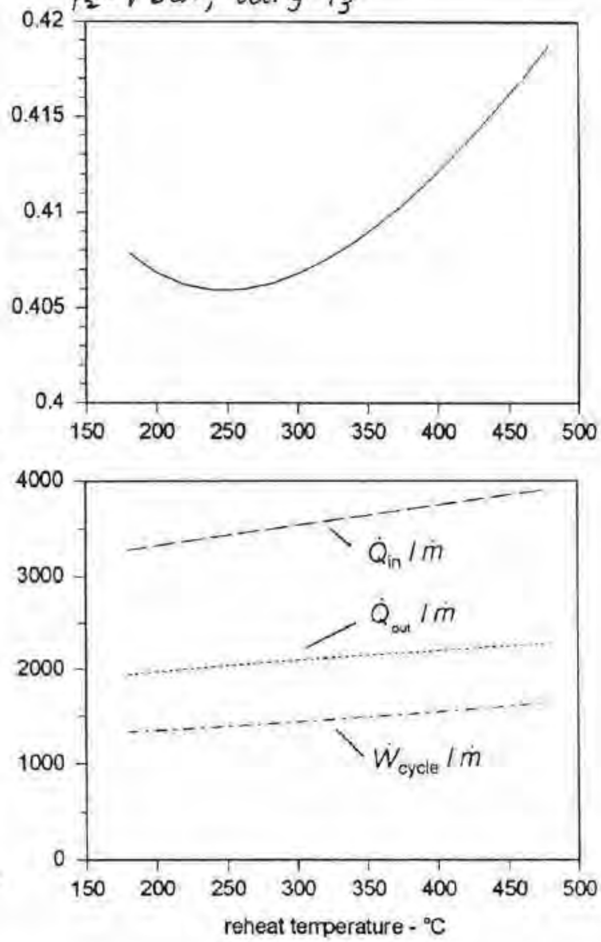
h_5 = 151 kJ/kg

h_6 = 163.6 kJ/kg

T_3 = 480°C, vary p_2:



p_2 = 7 bar, vary T_3

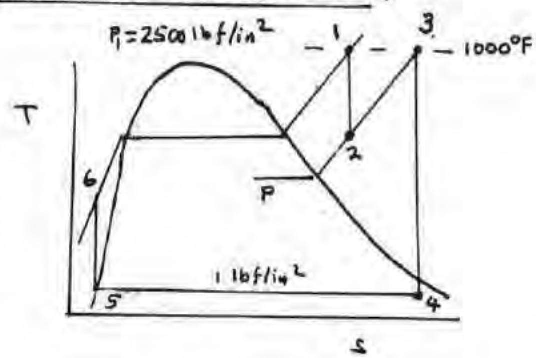


PROBLEM 8.32

KNOWN: Operating data are provided for an ideal Rankine cycle that has reheat at pressure p between the turbine stages.

FIND: (a) If $p/p_1 = 0.2$, where p_1 is the turbine inlet pressure, determine the cycle thermal efficiency and steam quality at the exit of the second turbine stage. (b) Plot the quantities of (a) versus p/p_1 ranging from 0.05 to 1.0.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. Control volumes enclosing each of the principal components are at steady state.
2. Each process is internally reversible, and the turbine stages and pump operate adiabatically.
3. Kinetic/potential energy can be ignored.

ANALYSIS: (a) For the case $p/p_1 = 0.2$, or $p = 500$ lbf/in.², the principal states are fixed as follows: From Table A-4E, $h_1 = 1457.2$ Btu/lb, $s_1 = 1.5262$ Btu/lb·°R. With $s_2 = s_1$, interpolation in Table A-4E at 500 lbf/in.² gives $h_2 = 1264.9$ Btu/lb.

① At state 3, $h_3 = 1520.7$ Btu/lb, $s_3 = 1.7371$ Btu/lb·°R. Then, with data from Table A-3E at 1 lbf/in.², and $s_4 = s_3$

$$x_4 = \frac{s_3 - s_f}{s_g - s_f} = \frac{1.7371 - 0.1327}{1.8453} = 0.8695 \Rightarrow h_4 = 69.74 + .8695\,(1036.0) = 970.5 \text{ Btu/lb}$$

x_4

From Table A-3E, $h_5 = 69.7$ Btu/lb and $h_6 \approx h_5 + v_5(p_6 - p_5)$, or

$$h_6 = 69.7\,\frac{\text{Btu}}{\text{lb}} + 0.01614\,\frac{\text{ft}^3}{\text{lb}}\,(2500-1)\,\frac{\text{lbf}}{\text{in}^2}\left|\frac{144\,\text{in}^2}{1\,\text{ft}^2}\right|\left|\frac{1\,\text{Btu}}{778\,\text{ft}\cdot\text{lbf}}\right| = 77.2\,\frac{\text{Btu}}{\text{lb}}$$

The thermal efficiency is given by

$$\eta = \frac{(h_1-h_2)+(h_3-h_4)-(h_6-h_5)}{(h_1-h_6)+(h_3-h_2)} = \frac{(1457.2-1264.9)+(1520.7-970.5)-(77.2-69.7)}{(1457.2-77.2)+(1520.7-1264.9)} = 0.449\ (44.9\%)$$

η

(b) The data for the required plots are obtained using IT, as follows:

IT Code

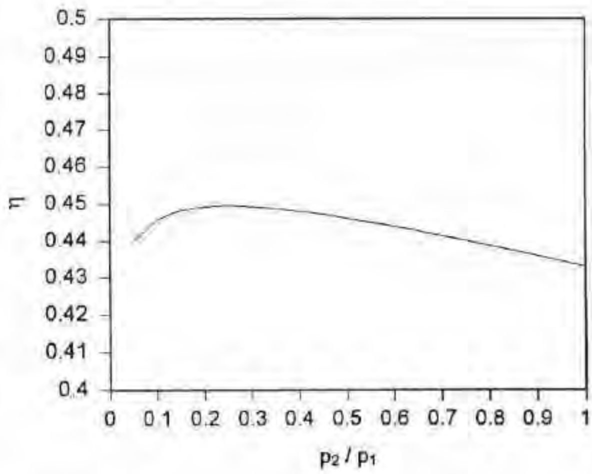
```
p1 = 2500  // lbf/in.²
T1 = 1000  // °F
pratio = p2 / p1
pratio = 0.2
p3 = p2
T3 = 1000  // °F
p4 = 1  // lbf/in.²
p5 = p4
p6 = p1

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2 = h_Ps("Water/Steam", p2, s2)
s2 = s1

h3 = h_PT("Water/Steam", p3, T3)
s3 = s_PT("Water/Steam", p3, T3)
h4 = h_Ps("Water/Steam", p4, s4)
s4 = s3
h5 = hsat_Px("Water/Steam", p5, 0)
v5 = vsat_Px("Water/Steam", p5, 0)
h6 = h5 + v5 * (p6 - p5) * (144 / 778)

x4 = x_hP("Water/Steam", h4, p4)
eta = ((h1 - h2) + (h3 - h4) - (h6 - h5)) / ((h1 - h6) + (h3 - h2))
```

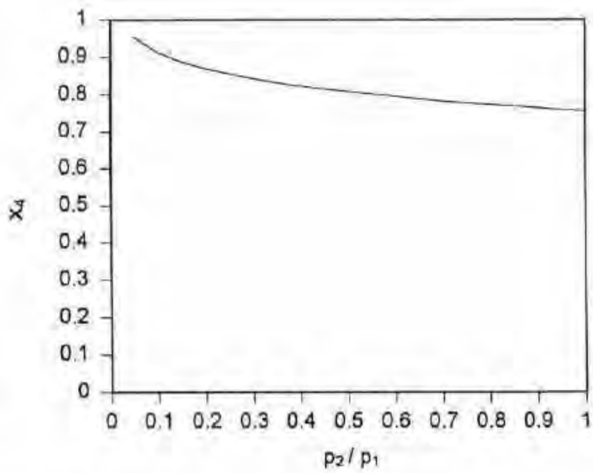
PROBLEM 8.32 (Cont'd.)

IT Results for $p_2/p_1 = 0.2$ ($p_2 = 500$ lbf/in.^{2})

$h_1 = 1457$ Btu/lb
$s_1 = 1.526$ Btu/lb·°R
$h_2 = 1265$ Btu/lb
$h_3 = 1521$ Btu/lb
$s_3 = 1.737$ Btu/lb·°R
$h_4 = 970.4$ Btu/lb
$h_5 = 69.58$ Btu/lb
$h_6 = 77.05$ Btu/lb
$v_5 = 0.01614$ ft^3/lb
$\eta = 0.4493$
$x_4 = 0.8695$

PLOTS:





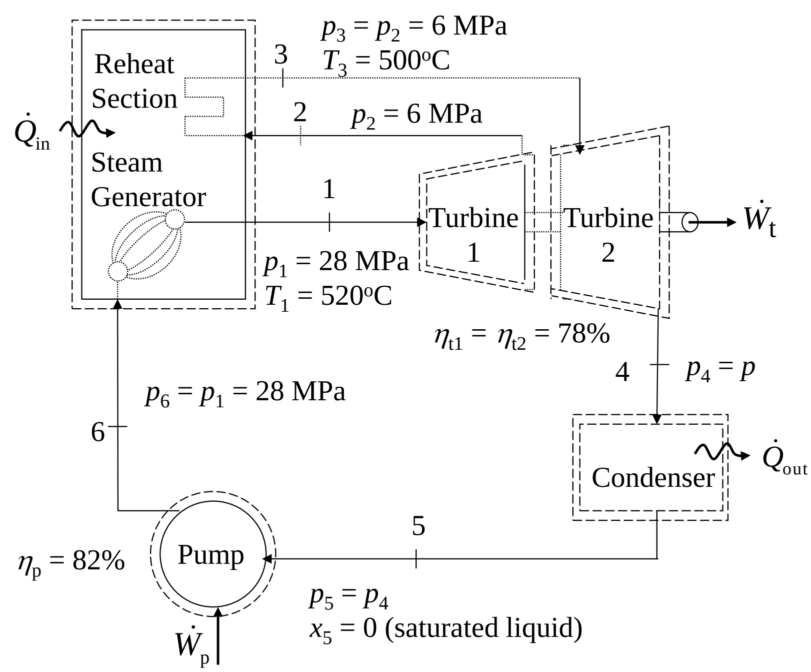
The plots show the thermal efficiency is affected by reheat pressure, with a maximum at $p_2/p_1 \approx 0.2$ to 0.25. Also, as the reheat pressure increases the quality at the turbine exit decreases, which is consistent with point 4 on the T-s diagram moving to the left.

8.33 Steam heated at constant pressure in a steam generator enters the first stage of a supercritical reheat cycle at 28 MPa, 520°C. Steam exiting the first-stage turbine at 6 MPa is reheated at constant pressure to 500°C. Each turbine stage has an isentropic efficiency of 78% while the pump has an isentropic efficiency of 82%. Saturated liquid exits the condenser that operates at constant pressure, *p*.
(a) For p = 6 kPa, determine the quality of the steam exiting the second stage of the turbine and the thermal efficiency.
(b) Plot the quantities of part (a) versus *p* ranging from 4 kPa to 70 kPa.

KNOWN: A supercritical reheat cycle operates with steam as the working fluid.

FIND: (a) For p = 6 kPa, determine the quality of the steam exiting the second stage of the turbine and the thermal efficiency, and (b) plot the quantities of part (a) versus *p* ranging from 4 kPa to 70 kPa.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. For all components stray heat transfer is ignored.
3. Flow through the steam generator, reheater, and condenser is at constant pressure.
4. Kinetic and potential energy effects are negligible.

ANALYSIS: First fix each principal state.

<u>State 1:</u> p_1 = 28 MPa, T_1 = 520°C → h_1 = 3192.3 kJ/kg, s_1 = 5.9566 kJ/kg·K

State 2s: $p_{2s} = p_2 = 6$ MPa, $s_{2s} = s_1 = 5.9566$ kJ/kg·K $\rightarrow h_{2s} = 2822.2$ kJ/kg

State 2: $p_2 = 6$ MPa, $h_2 = 2903.6$ kJ/kg (see below)

$$\eta_{t1} = \frac{h_1 - h_2}{h_1 - h_{2s}} \rightarrow h_2 = h_1 - \eta_{t1}(h_1 - h_{2s}) = 3192.3\,\frac{\text{kJ}}{\text{kg}} - (0.78)(3192.3 - 2822.2)\,\frac{\text{kJ}}{\text{kg}} = 2903.6 \text{ kJ/kg}$$

State 3: $p_3 = 6$ MPa, $T_3 = 500^{o}$C $\rightarrow h_3 = 3422.2$ kJ/kg, $s_3 = 6.8803$ kJ/kg·K

State 4s: $p_{4s} = p_4 = 6$ kPa, $s_{4s} = s_3 = 6.8803$ kJ/kg·K $\rightarrow x_{4s} = 0.8143$, $h_{4s} = 2118.8$ kJ/kg

State 4: $p_4 = 6$ MPa, $h_4 = 2405.5$ kJ/kg (see below)

$$\eta_{t2} = \frac{h_3 - h_4}{h_3 - h_{4s}} \rightarrow h_4 = h_3 - \eta_{t2}(h_3 - h_{4s}) = 3422.2\,\frac{\text{kJ}}{\text{kg}} - (0.78)(3422.2 - 2118.8)\,\frac{\text{kJ}}{\text{kg}} = 2405.5 \text{ kJ/kg}$$

State 5: $p_5 = 6$ kPa, saturated liquid $\rightarrow h_5 = h_{f5} = 151.53$ kJ/kg, $v_5 = v_{f5} = 0.0010064$ m^3/kg

State 6: $p_6 = p_1 = 28$ MPa, $h_6 = 185.89$ kJ/kg (see below)

$$\eta_p = \frac{v_5(p_6 - p_5)}{h_6 - h_5} \rightarrow h_6 = h_5 + \frac{v_5(p_6 - p_5)}{\eta_p}$$

$$h_6 = 151.53\text{ kJ/kg} + \frac{0.0010064\,\frac{\text{m}^3}{\text{kg}}(28{,}000 - 6)\text{kPa}}{0.82}\left|\frac{1000\,\frac{\text{N}}{\text{m}^2}}{1\,\text{kPa}}\right|\left|\frac{1\,\text{kJ}}{1000\,\text{N}\cdot\text{m}}\right| = 185.89 \text{ kJ/kg}$$

(a) The quality of the steam at the exit of the second stage of the turbine (state 4) is determined using values from Table A-4, $h_{f4} = 151.53$ kJ/kg and $h_{fg4} = 2415.9$ kJ/kg, as follows:

$$x_4 = \frac{h_4 - h_{f4}}{h_{fg4}} = \frac{(2405.5 - 151.53)\text{ kJ/kg}}{2415.9\text{ kJ/kg}} = \mathbf{\underline{0.9330}}$$

The cycle thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{\dot{W}_{t1}/\dot{m} + \dot{W}_{t2}/\dot{m} - \dot{W}_p/\dot{m}}{\dot{Q}_{61}/\dot{m} + \dot{Q}_{23}/\dot{m}}$$

$$\eta = \frac{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)}{(h_1 - h_6) + (h_3 - h_2)}$$

Substituting enthalpy values and solving yield a thermal efficiency of

$$\eta = \frac{(3192.3 - 2903.6)\text{ kJ/kg} + (3422.2 - 2405.5)\text{ kJ/kg} - (185.89 - 151.53)\text{ kJ/kg}}{(3192.3 - 185.89)\text{ kJ/kg} + (3422.2 - 2903.6)\text{ kJ/kg}}$$

$$\eta = \underline{\mathbf{0.3606\ (36.06\%)}}$$

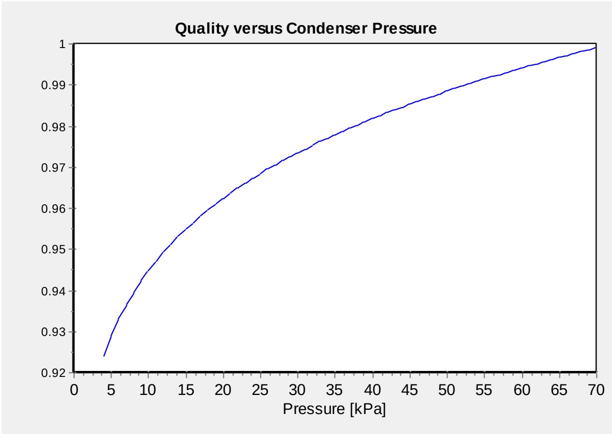
(b) For p ranging from 4 kPa to 70 kPa, IT gives the following results:

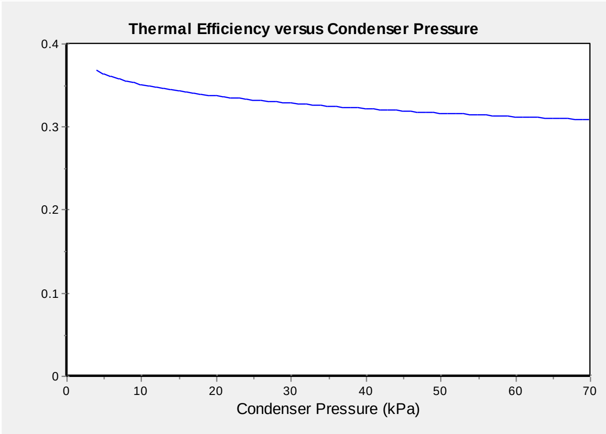
IT Code	**IT Output for p_4 = 6 kPa**

<pre>// Known Properties
 	
p1 = 28000 // kPa
T1 = 520 // oC
p6 = p1
p3 = 6000 // kPa
T3 = 500 // oC
p2 = p3
p2s = p2
p4 = 6 // kPa
p4s = p4
p5 = p4
x5 = 0
 	
// Known Operating Parameters
eff_t1 = 0.78
eff_t2 = 0.78
eff_p = 0.82
 	
// Calculations for Quality at State 4
 	
h3 = h_PT("Water/Steam", p3, T3)
s3 = s_PT("Water/Steam", p3, T3)
s4s = s3
h4s = h_Ps("Water/Steam", p4s, s4s)
h4 = h3 - eff_t2*(h3 - h4s)
x4 = x_hP("Water/Steam", h4, p4)
 	
// Calculations for Thermal Efficiency
 	
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
s2s = s1
h2s = h_Ps("Water/Steam", p2s, s2s)
h2 = h1 - eff_t1*(h1 - h2s)
 	
h5 = hsat_Px("Water/Steam", p5, x5)
v5 = vsat_Px("Water/Steam", p5, x5)
h6 = h5 + (v5*(p6 - p5)/eff_p)
 	
eff_thermal = ((h1 - h2) + (h3 - h4) - (h6 - h5))/((h1 - h6) + (h3 - h2))</pre>
	<pre>eff_thermal 0.3606
	h1 3192
	h2 2903
	h2s 2821
	h3 3422
	h4 2405
	h4s 2118
	h5 151
	h6 185.4
	p2 6000
	p2s 6000
	p4s 6
	p5 6
	p6 2.8E4
	s1 5.956
	s2s 5.956
	s3 6.879
	s4s 6.879
	v5 0.001007
	x4 0.933
	eff_p 0.82
	eff_t1 0.78
	eff_t2 0.78
	p1 2.8E4
	p3 6000
	p4 6
	T1 520
	T3 500
	x5 0</pre>

Results from IT for p_4 = 6 kPa correspond closely to the results obtained using steam tables and hand calculations.

Graphical Results from IT are shown below:





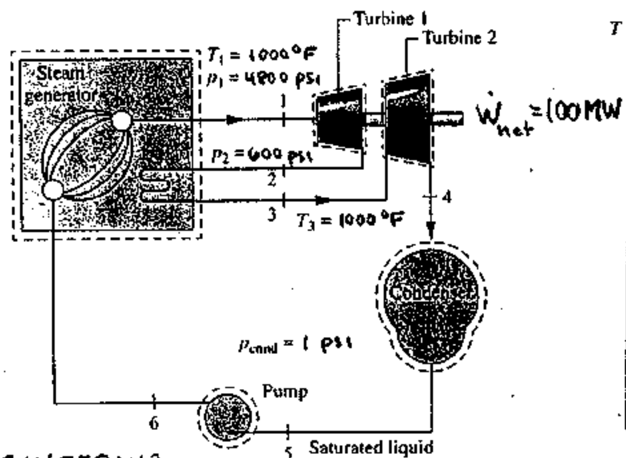
In general, as the condenser pressure increases the quality of the steam increases and the thermal efficiency decreases since the average temperature of heat rejection is higher. As shown by the Quality versus Condenser Pressure Graph for the conditions of this problem, when the condenser pressure reaches approximately 70 kPa, the liquid-vapor mixture quality becomes 1. Above this pressure steam is superheated vapor and the quality is not defined.

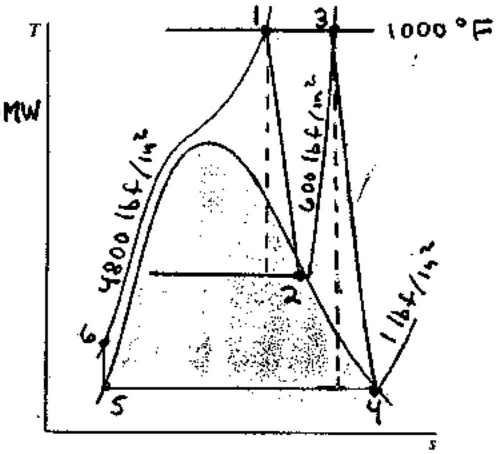
PROBLEM 8.34

KNOWN: Water is the working fluid in an ideal Rankine cycle modified to include two turbine stages with reheat between the stages. The turbine and pump efficiencies are 85%.

FIND: (a) the rate of heat transfer to the working fluid passing through the steam generator, (b) the rate of heat transfer from the working fluid passing through the condenser, and (c) the cycle efficiency.

SCHEMATIC + GIVEN DATA:





ENGINEERING MODEL: Same as in Example 8.3

ANALYSIS: First, fix each of the principal states.

state 1: $p_1 = 4800\ lbf/in^2$, $T_1 = 1000°F \Rightarrow h_1 = 1317.4\ Btu/lb$, $s_1 = 1.4078\ Btu/lb°R$

state 2: $p_2 = 600\ lbf/in^2$, $s_{2s} = s_1 \Rightarrow x_{2s} = 0.9500$, $h_{2s} = 1167.49\ Btu/lb$

$$\eta_t = 0.85 = \frac{(h_1 - h_2)}{(h_1 - h_{2s})} \Rightarrow h_2 = 1189.98\ Btu/lb$$

state 3: $p_3 = 600\ lbf/in^2$, $T_3 = 1000°F \Rightarrow h_3 = 1517.8\ Btu/lb$, $s_3 = 1.7155\ Btu/lb°R$

state 4: $p_4 = 1\ lbf/in^2$, $s_{4s} = s_3 \Rightarrow x_{4s} = 0.8577$, $h_{4s} = 958.32\ Btu/lb$

$$\eta_t = 0.85 = \frac{(h_3 - h_4)}{(h_3 - h_{4s})} \Rightarrow h_4 = 1042.24\ Btu/lb$$

state 5: $p_5 = 1\ lbf/in^2$, sat. liq. $\Rightarrow h_5 = 69.74\ Btu/lb$, $v_5 = 0.01614\ ft^3/lb$

state 6: $p_6 = 4800\ lbf/in^2$, $h_6 \approx h_5 + v_5(p_6 - p_5)/\eta_p$

$$h_6 = 69.74\ \frac{Btu}{lb} + \left(\frac{0.01614}{0.85}\right)\frac{ft^3}{lb}(4800-1)(144)\frac{lbf}{in^2}\left|\frac{1\ Btu}{778\ ft{\cdot}lbf}\right| = 69.74 + 16.87 = 86.61\ \frac{Btu}{lb}$$

Next, determine the flow rate of the working fluid.

$$\dot{W}_{net} = \dot{W}_t - \dot{W}_p = \dot{m}[(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)]$$

$$\dot{m} = \frac{\dot{W}_{net}}{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)} = \frac{(100\ MW)\left|\frac{1000\ kJ/s}{1\ MW}\right|\left|\frac{1\ Btu}{1.0551\ kJ}\right|}{[(1317.4 - 1189.98) + (1517.8 - 1042.24) - (86.61 - 69.74)]\ \frac{Btu}{lb}}$$

$$= 161.7\ lb/s$$

(a) The rate of heat transfer to the working fluid in the steam generator is

$$\dot{Q}_{in} = \dot{m}(h_1 - h_6 + h_3 - h_2) = (161.7)\frac{lb}{s}(1317.4 - 86.61 + 1517.8 - 1189.98)\frac{Btu}{lb}$$

$$= 2.52 \times 10^5\ \frac{Btu}{s}\left|\frac{1.0551\ kJ}{1\ Btu}\right|\left|\frac{1\ MW}{1000\ kJ/s}\right| = 265.9\ MW \longleftarrow \dot{Q}_{in}$$

PROBLEM 8.34 (cont'd)

(b) The rate of heat transfer from the working fluid in the condenser is

$$\dot{Q}_{out} = \dot{m}(h_4 - h_5) = (161.7)(1042.24 - 69.74)\left|\frac{1.0551}{1000}\right|$$

① $$\dot{Q}_{out} = 165.9 \text{ MW} \longleftarrow \dot{Q}_{out}$$

(c) The cycle thermal efficiency is

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{sg}} = \frac{100 \text{ MW}}{265.9 \text{ MW}} = 0.376\ (37.6\%) \longleftarrow \eta$$

---

1. The overall energy balance holds for the cycle:

$$\dot{Q}_{in} = \dot{W}_{net} + \dot{Q}_{out}$$

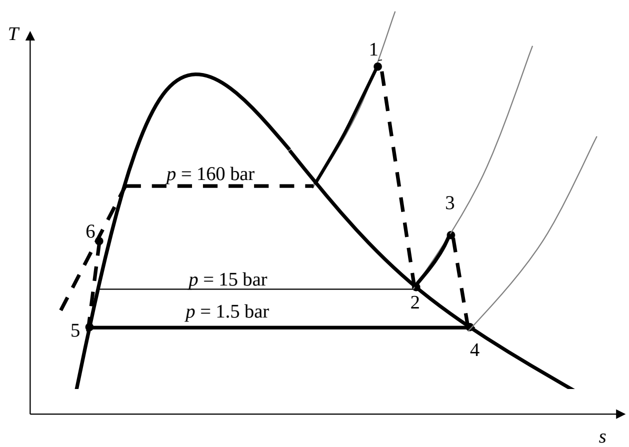
$$265.9 \text{ MW} = 100 \text{ MW} + 165.9 \text{ MW}$$

8.35 Steam is the working fluid in the vapor power cycle with reheat shown in Fig. P8.35 with operational data. The mass flow rate is 2.3 kg/s, and the turbines and pump operate adiabatically. Steam exits both turbine 1 and turbine 2 as saturated vapor. If the reheat pressure is 15 bar, determine the power developed by the cycle, in kW, and the cycle thermal efficiency.

KNOWN: A vapor power cycle with reheat operates with steam as the working fluid. Operational data are provided.

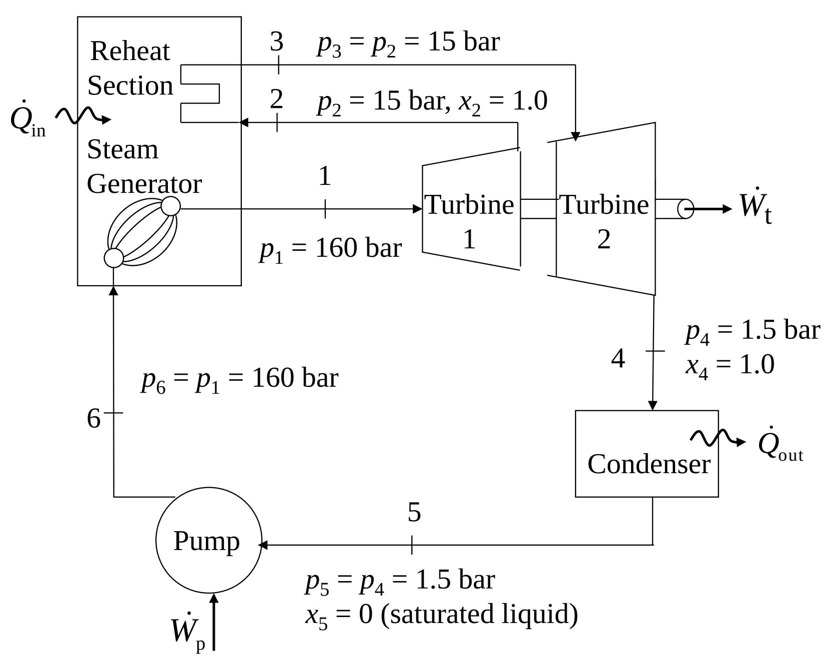
FIND: Determine the power developed by the cycle, in kW, and the cycle thermal efficiency.

SCHEMATIC AND GIVEN DATA:



State	*p* (bar)	*h* (kJ/kg)	*x*
1	160	3353.3	--
2	15	2792.2	1.0
3	15	3169.2	--
4	1.5	2693.6	1.0
5	1.5	467.11	0
6	160	486.74	--

Fig. P8.35



ENGINEERING MODEL:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The turbines and pump operate adiabatically.
3. Kinetic and potential energy effects are negligible.

ANALYSIS:

The net power developed by the cycle is

$$\dot{W}_{\text{cycle}} = \dot{W}_{\text{t1}} + \dot{W}_{\text{t2}} - \dot{W}_{\text{p}}$$

For a steam mass flow rate of $\dot{m} = 2.3\ \text{kg/s}$, mass and energy rate balances for control volumes around Turbine 1, Turbine 2, and the pump give, respectively,

$$\text{Turbine 1:}\quad \dot{W}_{\text{t1}} = \dot{m}(h_1 - h_2) = \left(2.3\frac{\text{kg}}{\text{s}}\right)(3353.3 - 2792.2)\frac{\text{kJ}}{\text{kg}}\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right| = 1290.53\ \text{kW}$$

$$\text{Turbine 2:}\quad \dot{W}_{\text{t2}} = \dot{m}(h_3 - h_4) = \left(2.3\frac{\text{kg}}{\text{s}}\right)(3169.2 - 2693.6)\frac{\text{kJ}}{\text{kg}}\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right| = 1093.88\ \text{kW}$$

$$\text{Pump:}\quad \dot{W}_{\text{p}} = \dot{m}(h_6 - h_5) = \left(2.3\frac{\text{kg}}{\text{s}}\right)(486.74 - 467.11)\frac{\text{kJ}}{\text{kg}}\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right| = 45.15\ \text{kW}$$

Substituting and solving for the net power developed by the cycle give

$$\dot{W}_{\text{cycle}} = 1290.53\ \text{kW} + 1093.88\ \text{kW} - 45.15\ \text{kW} = \mathbf{2339.3\ kW}$$

The thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}}$$

The total rate of heat transfer to the working fluid as it passes through the steam generator and reheater section is determined using mass and energy rate balances as

$$\dot{Q}_{\text{in}} = \dot{m}\left[(h_1 - h_6) + (h_3 - h_2)\right]$$

Solving for rate of heat transfer gives

$$\dot{Q}_{\text{in}} = \left(2.3\frac{\text{kg}}{\text{s}}\right)\left[(3353.3 - 486.74) + (3169.2 - 2792.2)\right]\frac{\text{kJ}}{\text{kg}}\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right| = 7460.2\ \text{kW}$$

The thermal efficiency is then

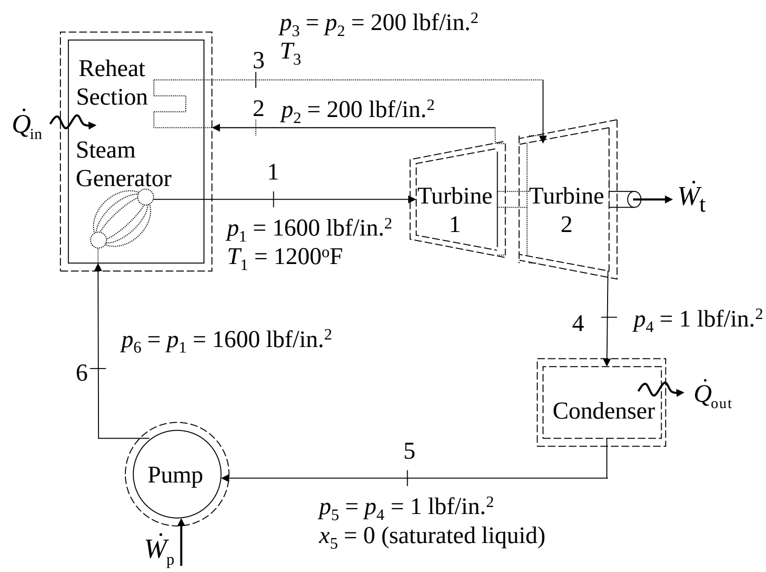
$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{2339.3\,\text{kW}}{7460.2\,\text{kW}} = \underline{\mathbf{0.314\ (31.4\%)}}$$

8.36 An ideal Rankine cycle with reheat uses water as the working fluid. As shown in Fig. P8.36, the conditions at the inlet to the first turbine stage are 1600 lbf/in.^{2}, 1200^{o}F and the steam is reheated to temperature T_3 between the turbine stages at a pressure of 200 lbf/in.^{2} For a condenser pressure of 1 lbf/in.^{2}, plot the cycle thermal efficiency versus reheat temperature and plot the cycle thermal efficiency versus quality of the steam at the second stage turbine exit for the reheat temperature ranging from 600^{o}F to 1200^{o}F.

KNOWN: An ideal reheat cycle uses steam as the working fluid. Operating pressures and temperatures are given.

FIND: Plot thermal efficiency versus reheat temperature and plot the cycle thermal efficiency versus quality of the steam at the second stage turbine exit for temperatures ranging from 600^{o}F to 1200^{o}F.

SCHEMATIC AND GIVEN DATA:



P8.36

ENGINEERING MODEL:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. All processes of the working fluid are internally reversible.
3. The turbines and pump operate adiabatically.
4. Condensate exits the condenser as saturated liquid.
5. Kinetic and potential energy effects are negligible.

ANALYSIS: First fix each principal state. Assume T_3 = 800^{o}F as sample calculation.

State 1: p_1 = 1600 lbf/in.^{2}, T_1 = 1200^{o}F $\rightarrow$ h_1 = 1607.1 Btu/lb, s_1 = 1.6684 Btu/lb $\cdot^o$R

State 2: p_2 = 200 lbf/in.^{2}, $s_2 = s_1$ = 1.6684 Btu/lb $\cdot^o$R $\rightarrow$ h_2 = 1313.5 Btu/lb

State 3: p_3 =200 lbf/in.^{2}, T_3 = 800^{o}F $\longrightarrow$ h_3 = 1425.3 Btu/lb, s_3 = 1.7660 Btu/lb ·^{o}R

State 4: p_4 = 1 lbf/in.^{2}, s_4 = s_3 = 1.7660 Btu/lb ·^{o}R → **x_4 = 0.8851**, h_4 = 986.80 Btu/lb

State 5: p_5 = 1 lbf/in.^{2}, saturated liquid ⟶ h_5 = h_{f5} = 69.74 Btu/lb, v_5 = v_{f5} = 0. 01614 ft^3/lb

State 6: p_6 = p_1 = 1600 lbf/in.^{2}, h_6 ≈ 185.89 Btu/lb (see below)

$$h_6 \approx h_5 + v_5(p_6 - p_5)$$

$$h_6 \approx 69.74\text{ Btu/lb} + \left(0.01614\frac{\text{ft}^3}{\text{lb}}\right)(1600-1)\frac{\text{lbf}}{\text{in.}^2}\left|\frac{144\text{ in.}^2}{1\text{ ft}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right| = 74.52\text{ Btu/lb}$$

The work per unit mass of steam flow is

$$\frac{\dot{W}_t}{\dot{m}} = (h_1 - h_2) + (h_3 - h_4)$$

$$\frac{\dot{W}_t}{\dot{m}} = (1607.1 - 1313.5)\frac{\text{Btu}}{\text{lb}} + (1425.3 - 986.80)\frac{\text{Btu}}{\text{lb}} = 732.1\text{ Btu/lb}$$

For the pump

$$\frac{\dot{W}_p}{\dot{m}} = h_6 - h_5$$

$$\frac{\dot{W}_p}{\dot{m}} = (74.52 - 69.74)\frac{\text{Btu}}{\text{lb}} = 4.78\text{ Btu/lb}$$

The heat input per unit of mass flowing to the steam generator and reheat process is

$$\frac{\dot{Q}_{in}}{\dot{m}} = (h_1 - h_6) + (h_3 - h_2)$$

$$\frac{\dot{Q}_{in}}{\dot{m}} = (1607.1 - 74.52)\frac{\text{Btu}}{\text{lb}} + (1425.3 - 1313.5)\frac{\text{Btu}}{\text{lb}} = 1644.4\text{ Btu/lb}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m} - \dot{W}_p/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{(732.1 - 4.78)\text{ Btu/lb}}{1644.4\text{ Btu/lb}} = \mathbf{0.4423\ (44.23\%)}$$

The following IT code is used to obtain data for the required plot.

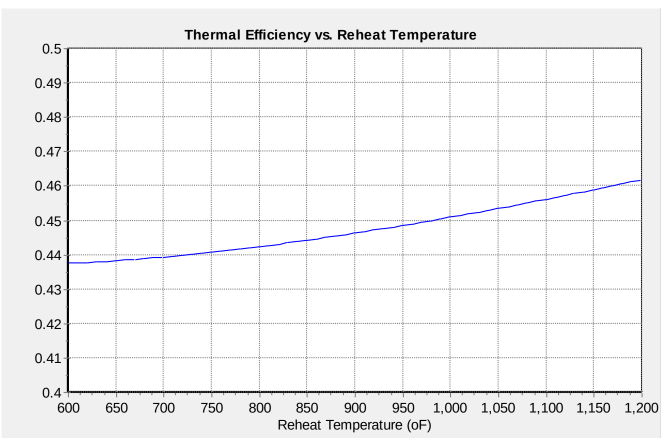
IT Code

```
p1 = 1600 // lbf/in.^2
T1 = 1200 // oF
p2 = 200 // lbf/in.^2
p3 = p2
T3 = 800 // oF
p4 = 1 // lbf/in.^2
p5 = p4
p6 = p1

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)

s2 = s1
h2 = h_Ps("Water/Steam", p2, s2)
x2 = x_sP("Water/Steam", s2, p2)

h3 = h_PT("Water/Steam", p3, T3)
s3 = s_PT("Water/Steam", p3, T3)

s4 = s3
h4 = h_Ps("Water/Steam", p4, s4)
x4 = x_sP("Water/Steam", s4, p4)

x5 = 0
h5 = hsat_Px("Water/Steam", p5, x5)
v5 = vsat_Px("Water/Steam", p5, x5)

h6 = h5 + v5*(p6 - p5)*(144/778)

Wt_m = (h1 - h2) + (h3 - h4)
Wp_m = h6 - h5
Qin_m = (h1 - h6) + (h3 - h2)
eta = (Wt_m - Wp_m)/Qin_m
```

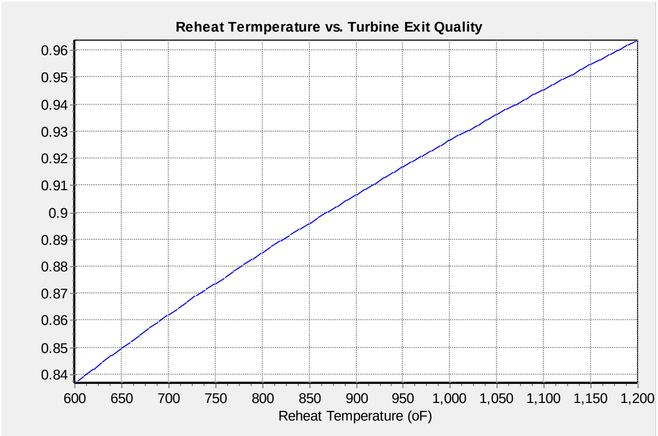
IT Results

eta	0.4424
h1	1607
h2	1313
h3	1425
h4	986.6
h5	69.58
h6	74.36
p3	200
p5	1
p6	1600
Qin_m	1645
s1	1.668
s2	1.668
s3	1.766
s4	1.766
v5	0.01614
Wp_m	4.776
Wt_m	732.3
x2	1
x4	0.8851
p1	1600
p2	200
p4	1
T1	1200
T3	800
x5	0

The plot showing the results of sweeping T_3 from 600^{o}F to 1200^{o}F and the effects on thermal efficiency and quality are shown below.



For the specified reheat pressure, as reheat temperature increases, thermal efficiency increases since the average temperature of heat addition is higher. The material properties of the turbine blades will dictate an upper limit of the reheat temperature.



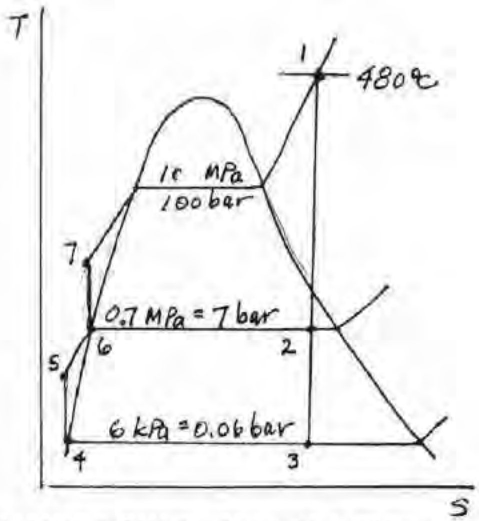
For the specified reheat pressure, as reheat temperature increases, the quality of the steam exiting the second turbine stage increases. Since it is common practice to maintain at least 90% quality at the exit of a steam turbine to avoid undesirable effects, the reheat temperature should be higher than approximately 875°F.

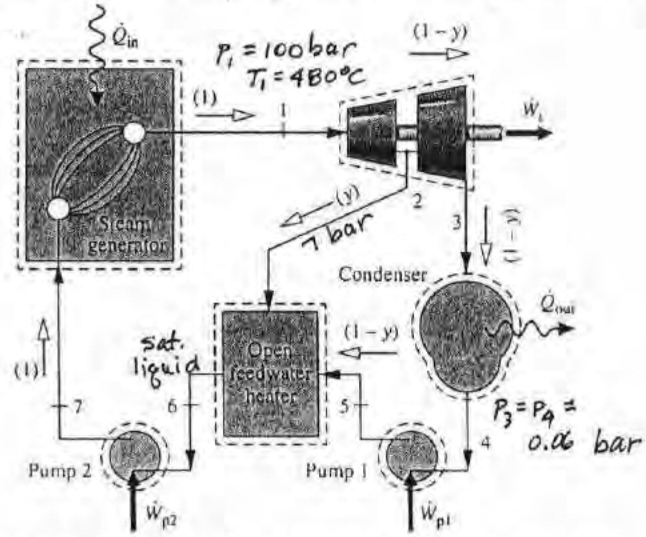
PROBLEM 8.37

KNOWN: Water is the working fluid in an ideal regenerative Rankine cycle with one open feedwater heater. Data at various locations are known.

FIND: Determine (a) the rate of heat addition per kg of steam entering the first-stage turbine, (b) the thermal efficiency, and (c) the rate of heat transfer for the condenser per kg of steam entering the first-stage turbine.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: Same as Example 8.5, except turbine stages and pumps operate in an internally reversible manner.

ANALYSIS: First, fix each principal state.

State 1: $p_1 = 100$ bar, $T_1 = 480°C \Rightarrow h_1 = 3321.4$ kJ/kg, $s_1 = 6.5282$ kJ/kg·K

State 2: $p_2 = 7$ bar, $s_2 = s_1 \Rightarrow x_2 = \dfrac{s_2 - s_{f2}}{s_{g2} - s_{f2}} = 0.9619$, $h_2 = 2684.8$ kJ/kg

State 3: $p_3 = 0.06$ bar, $s_3 = s_2 \Rightarrow x_3 = \dfrac{s_2 - s_{f3}}{s_{g3} - s_{f3}} = 0.7692$, $h_3 = 2009.8$ kJ/kg

State 4: $p_4 = 0.06$ bar, sat. liquid $\Rightarrow h_4 = 151.53$ kJ/kg

State 5: $h_5 \approx h_4 + v_4(p_5 - p_4)$

$$= 151.53 + (1.0064 \times 10^{-3})\frac{m^3}{kg}(7 - 0.06)\,\text{bar}\left|\frac{10^5\,N/m^2}{1\,\text{bar}}\right|\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|$$

$$= 151.53 + 0.698 = 152.23 \text{ kJ/kg}$$

State 6: $p_6 = 7$ bar, sat. liquid $\Rightarrow h_6 = 697.22$ kJ/kg

State 7: $h_7 \approx h_6 + v_6(p_7 - p_6) = 697.22 + (1.1080 \times 10^{-3})(100 - 7)\left|\dfrac{10^5}{10^3}\right| = 707.52 \dfrac{kJ}{kg}$

(a) For the steam generator

$$\dot{Q}_{in}/\dot{m}_1 = h_1 - h_7 = (3321.4 - 707.52) = 2613.9 \text{ kJ/kg} \leftarrow \dot{Q}_{in}/\dot{m}$$

(b) Applying mass and energy balances to the control volume enclosing the open feedwater heater

$$y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{697.22 - 152.23}{2684.8 - 152.23} = 0.2152$$

For the control volume enclosing the turbine stages

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1 - y)(h_2 - h_3)$$

$$= (3321.4 - 2684.8) + (1 - 0.2152)(2684.8 - 2009.8) = 1166.3 \text{ kJ/kg}$$

PROBLEM 8.37 (Cont'd)

For the pumps

$$\dot{W}_p = \dot{W}_{p1} + \dot{W}_{p2} = \dot{m}_1[(1-y)(h_5-h_4) + (h_7-h_6)]$$

or

$$\dot{W}_p/\dot{m}_1 = (1-y)(h_5-h_4) + (h_7-h_6)$$
$$= (1-0.2152)(152.23-151.53)+(707.52-697.22)$$
$$= 10.85 \text{ kJ/kg}$$

Thus, the net power developed, per unit mass entering the first-stage turbine is

$$\dot{W}_{cycle}/\dot{m}_1 = \dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1 = 1166.3 - 10.85 = 1155.3 \text{ kJ/kg}$$

And the thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{1155.3}{2613.9} = 0.442 \ (44.2\%) \longleftarrow \eta$$

(c) For the condenser

$$\dot{Q}_{out} = \dot{m}_1(1-y)(h_3-h_4)$$

or

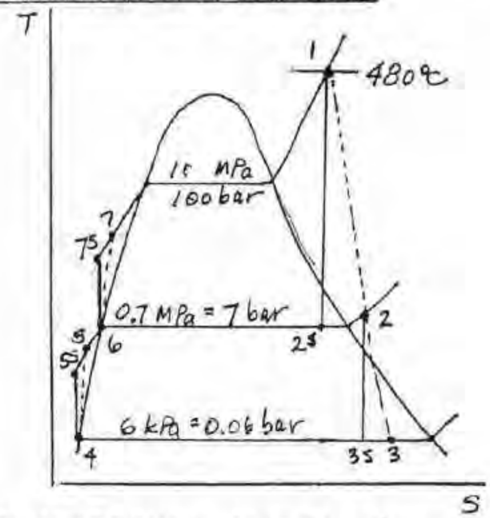
$$\dot{Q}_{out}/\dot{m}_1 = (1-y)(h_3-h_4) = (1-0.2152)(2009.8-151.53)$$
$$= 1458.4 \text{ kJ/kg} \longleftarrow \dot{Q}_{out}/\dot{m}_1$$

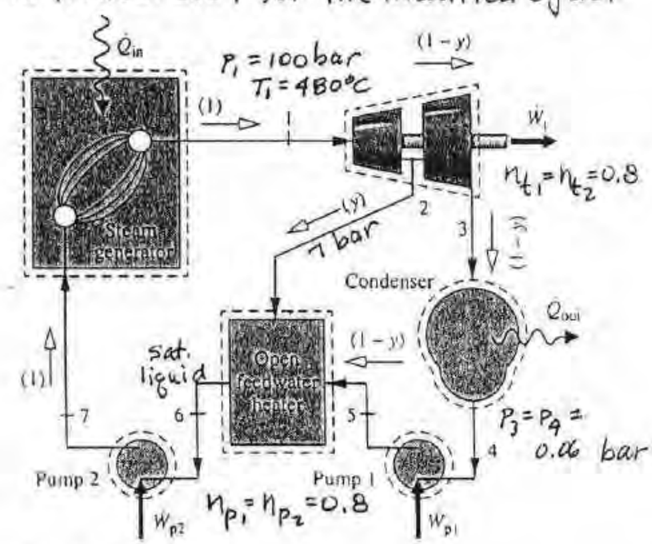
PROBLEM 8.38

KNOWN: The ideal regenerative Rankine cycle of Problem 8.37 is modified to include turbine stage and pump isentropic efficiencies of 0.8.

FIND: Answer the same questions as in Problem 8.37 for the modified cycle.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: Same as Example 8.5

ANALYSIS: First, fix each principal state.

State 1: $p_1 = 100$ bar, $T_1 = 480°C \Rightarrow h_1 = 3321.4$ kJ/kg, $s_1 = 6.5282$ kJ/kg·K

State 2: Using the isentropic efficiency of the first turbine stage

$$\eta_{t_1} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_{t_1}(h_1 - h_{2s})$$

With $h_{2s} = 2684.8$ kJ/kg from Problem 8.37, $h_2 = 2812.1$ kJ/kg

State 3: $p_3 = 0.06$ bar, $s_{3s} = s_2$. To get s_2, interpolate in Table A-3: $s_2 = 6.810 \frac{kJ}{kg \cdot K}$

Thus $x_{3s} = \frac{s_{3s} - s_{f3}}{s_{g3} - s_{f3}} = 0.8061$; $h_{3s} = 2099.0$ kJ/kg

Using the second-stage turbine isentropic efficiency

$$h_3 = h_2 - \eta_{t_2}(h_2 - h_{3s}) = 2241.6 \text{ kJ/kg}$$

State 4: $p_4 = 0.06$ bar, sat. liquid $\Rightarrow h_4 = 151.53$ kJ/kg

State 5: Using the isentropic pump efficiency; $\eta_{p_1} = (h_{5s} - h_4)/(h_5 - h_4)$

Thus $h_5 = h_4 + (h_{5s} - h_4)/\eta_{p_1} = 152.41$ kJ/kg

State 6: $p_6 = 7$ bar, sat. liquid $\Rightarrow h_6 = 697.22$ kJ/kg

State 7: With $\eta_{p_2} = (h_{7s} - h_6)/(h_7 - h_6)$; $h_7 = h_6 + (h_{7s} - h_6)/\eta_{p_2} = 710.1$ kJ/kg

(a) $\dot{Q}_{in}/\dot{m}_1 = h_1 - h_7 = 2611.3$ kJ/kg ← $\dot{Q}_{in}/\dot{m}_1$

(b) $y = \frac{h_6 - h_5}{h_2 - h_5} = 0.2048$ and $\dot{W}_t/\dot{m}_1 = (h_1 - h_2) + (1-y)(h_2 - h_3) = 962.96$ kJ/kg

For the pumps, $\dot{W}_p/\dot{m}_1 = (1-y)(h_5 - h_4) + (h_7 - h_6) = 13.58$ kJ/kg

And $\dot{W}_{cycle}/\dot{m}_1 = \dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1 = 949.38$ kJ/kg

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{949.3}{2611.3} = 0.364 \ (36.4\%)$$ ← η

(1)

(c) $\dot{Q}_{out}/\dot{m}_1 = (1-y)(h_3 - h_4) = 1662$ kJ/kg ← $\dot{Q}_{out}/\dot{m}_1$

1. These results can be compared with those of Problem 8.37 to see some of the effects of turbine and pump irreversibilities on cycle performance.

PROBLEM 8.39

(a) Refer to Problem 8.37.

IT Code

```
p1 = 100 // bar
T1 = 480 // °C
p2 = 7 // bar
p3 = 0.06 // bar
p4 = p3
p5 = p2
p6 = p5
p7 = p1
mdot1 = 1

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2s = h_Ps("Water/Steam", p2, s2s)
s2s = s1
s3s = s2s
h3s = h_Ps("Water/Steam", p3, s3s)
h4 = hsat_Px("Water/Steam", p4, 0)
v4 = vsat_Px("Water/Steam", p5, 0)
h5s = h4 + v4 * (p5 - p4) * 100
h6 = hsat_Px("Water/Steam", p6, 0)
v6 = vsat_Px("Water/Steam", p6, 0)
h7s = h6 + v6 * (p7 - p6) * 100

Qdotin = mdot1 * (h1 - h7s)
y = (h6 - h5s) / (h2s - h5s)
Wdott = mdot1 * ((h1 - h2s) + (1 - y) * (h2s - h3s))
Wdotp = mdot1 * ((1 - y) * (h5s - h4) + (h7s - h6))
Wdotcycle = Wdott - Wdotp
eta = Wdotcycle / Qdotin
Qdotout = mdot1 * (1 - y) * (h3s - h4)
```

IT Results (p_2 = 7 bar)

$\dot{Q}_{in} / \dot{m}_1$ = 2614 kJ/kg

$\dot{Q}_{out} / \dot{m}_1$ = 1459 kJ/kg

$\dot{W}_{cycle} / \dot{m}_1$ = 1155 kJ/kg

η = 0.442

h_1 = 3321 kJ/kg

h_{2s} = 2684 kJ/kg

h_{3s} = 2009 kJ/kg

h_4 = 151 kJ/kg

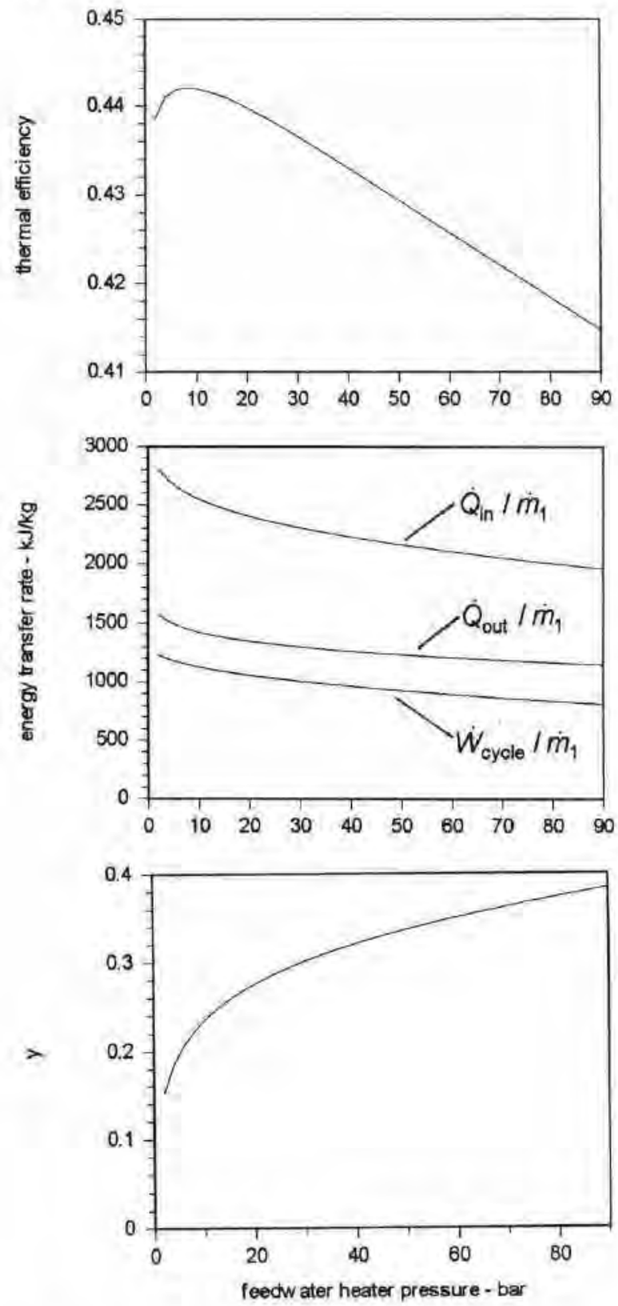
h_{5s} = 151.8 kJ/kg

h_6 = 696.8 kJ/kg

h_{7s} = 707.1 kJ/kg

y = 0.2152

Plots:



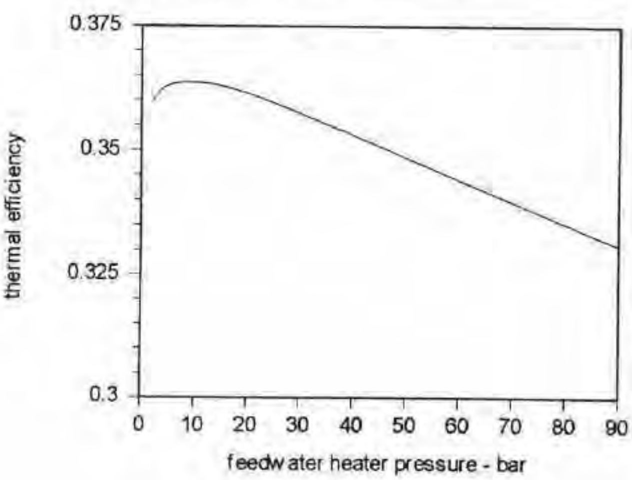
From the plots, we see that the thermal efficiency exhibits a maximum in the range of feedwater heater pressures studied. Also, the fraction of steam extracted increases with increasing feedwater heater pressure.

PROBLEM 8.39 (Cont'd)

(b) Refer to Prolem 8.38.

IT Code

```
p1 = 100  // bar
T1 = 480  // °C
p2 = 7  // bar
p3 = 0.06 // bar
p4 = p3
p5 = p2
p6 = p5
p7 = p1
etat = 0.8
etap = etat
mdot1 = 1

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
s2s = s1
h2s = h_Ps("Water/Steam", p2, s2s)
h2 = h1 - etat * (h1 - h2s)
s2 = s_Ph("Water/Steam", p2, h2)
s3s = s2
h3s = h_Ps("Water/Steam", p3, s3s)
h3 = h2 - etat * (h2 - h3s)
h4 = hsat_Px("Water/Steam", p4, 0)
v4 = vsat_Px("Water/Steam", p5, 0)
h5s = h4 + v4 * (p5 - p4) * 100
h5 = h4 + (h5s - h4) / etap
h6 = hsat_Px("Water/Steam", p6, 0)
v6 = vsat_Px("Water/Steam", p6, 0)
h7s = h6 + v6 * (p7 - p6) * 100
h7 = h6 + (h7s - h6) / etap

Qdotin = mdot1 * (h1 - h7)
y = (h6 - h5s) / (h2 - h5)
Wdott = mdot1 * ((h1 - h2) + (1 - y) * (h2 - h3))
Wdotp = mdot1 * ((1 - y) * (h5 - h4) + (h7 - h6))
Wdotcycle = Wdott - Wdotp
eta = Wdotcycle / Qdotin
Qdotout = mdot1 * (1 - y) * (h3 - h4)
```



IT Results (p_2 = 7 bar)

$\dot{Q}_{in} / \dot{m}_1$ = 2611 kJ/kg

$\dot{Q}_{out} / \dot{m}_1$ = 1662 kJ/kg

$\dot{W}_{cycle} / \dot{m}_1$ = 949.2 kJ/kg

η = 0.3635

h_1 = 3321 kJ/kg

h_2 = 2812 kJ/kg

h_3 = 2241 kJ/kg

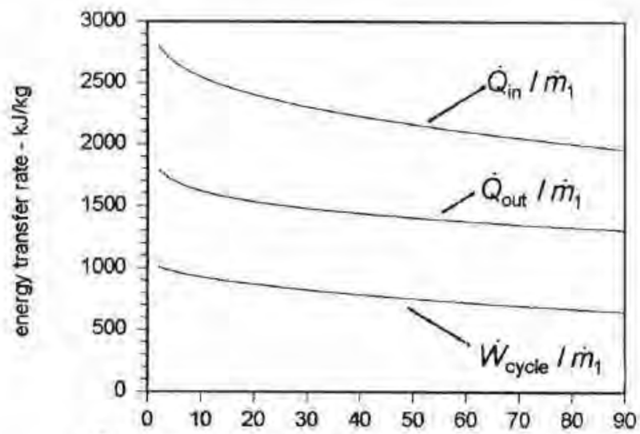
h_4 = 151 kJ/kg

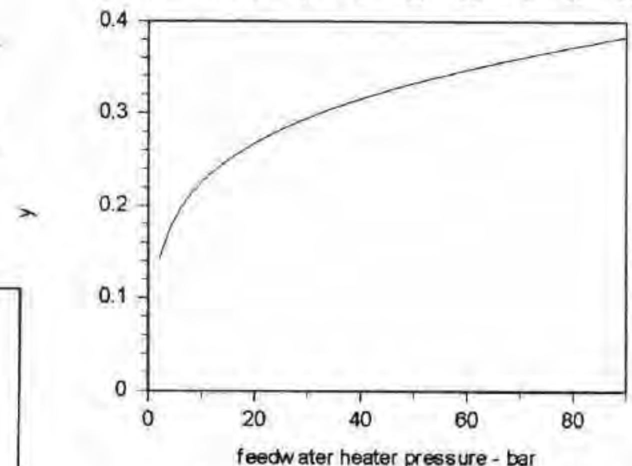
h_5 = 152 kJ/kg

h_6 = 696.8 kJ/kg

h_7 = 709.6 kJ/kg

y = 0.2049





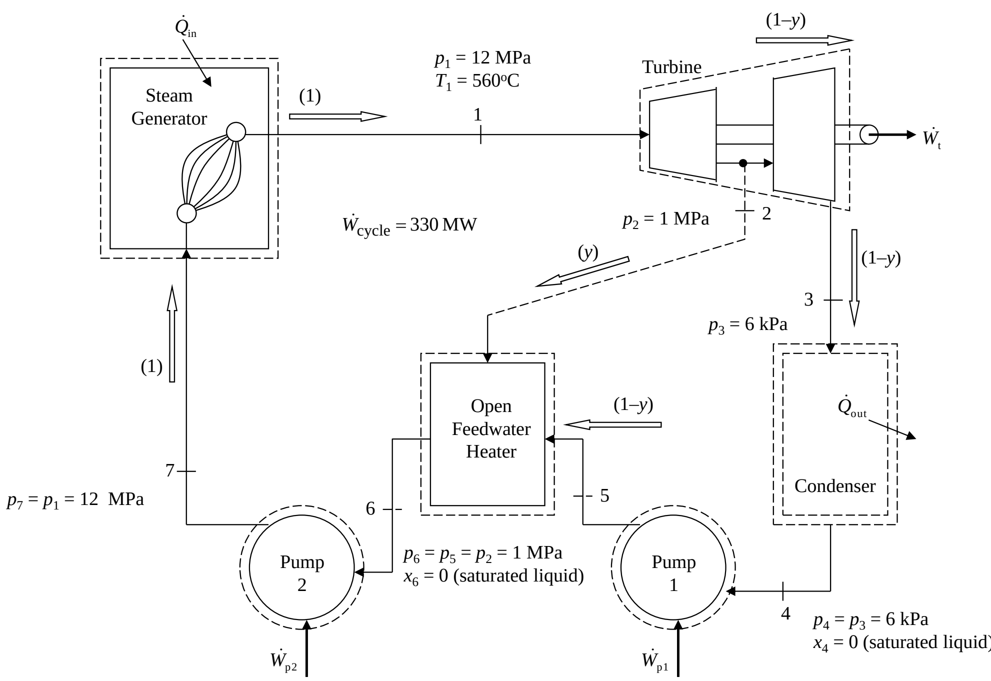
From the plots, we see that the thermal efficiency exhibits a maximum and the fraction of steam extracted increases with p_2 for the range of pressures studied. Further, these results can be compared to those of part (a) to see some of the effects of turbine stage and pump irreversibilities on cycle performance.

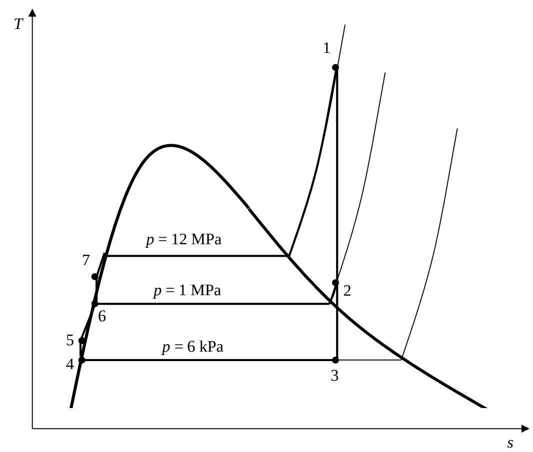
8.40 A power plant operates on a regenerative vapor power cycle with one open feedwater heater. Steam enters the first turbine stage at 12 MPa, 560°C and expands to 1 MPa, where some of the steam is extracted and diverted to the open feedwater heater operating at 1 MPa. The remaining steam expands through the second turbine stage to the condenser pressure of 6 kPa. Saturated liquid exits the open feedwater heater at 1 MPa. The net power output for the cycle is 330 MW. For isentropic processes in the turbines and pumps, determine
(a) the cycle thermal efficiency.
(b) the mass flow rate into the first turbine stage, in kg/s.
(c) the rate of entropy production in the open feedwater heater, in kW/K.

KNOWN: A regenerative vapor power cycle with one open feedwater heater operates with steam as the working fluid. Operational data are provided.

FIND: Determine (a) the cycle thermal efficiency, (b) the mass flow rate into the first turbine stage, in kg/s, and (c) the rate of entropy production in the open feedwater heater, in kW/K.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. All processes of the working fluid are internally reversible except for mixing in the open feedwater heater.
3. The turbines, pumps, and open feedwater heater operate adiabatically.
4. Kinetic and potential energy effects are negligible.
5. Saturated liquid exits the open feedwater heater, and saturated liquid exits the condenser.

ANALYSIS:

First fix each principal state.

State 1: p_1 = 12 MPa (120 bar), T_1 = 560°C → h_1 = 3506.2 kJ/kg, s_1 = 6.6840 kJ/kg·K

State 2: p_2 = 1 MPa (10 bar), $s_2 = s_1$ = 6.6840 kJ/kg·K → h_2 = 2823.3 kJ/kg

State 3: p_3 = 6 kPa (0.06 bar), $s_3 = s_1$ = 6.6840 kJ/kg·K → x_3 = 0.7892, h_3 = 2058.2 kJ/kg

State 4: p_4 = 6 kPa (0.06 bar), saturated liquid → h_4 = 151.53 kJ/kg, s_4 = 0.5210 kJ/kg·K,
v_4 = 0.0010064 m³/kg

State 5: $p_5 = p_2$ = 1 MPa (10 bar), $s_5 = s_4$ = 0.5210 kJ/kg·K → $h_5 \approx h_4 + v_4(p_5 - p_4)$

$$h_5 = 151.53\frac{\text{kJ}}{\text{kg}} + \left(0.0010064\frac{\text{m}^3}{\text{kg}}\right)(1000-6)\text{kPa}\left|\frac{1000\ \text{N/m}^2}{1\,\text{kPa}}\right|\left|\frac{1\,\text{kJ}}{1000\ \text{N}\cdot\text{m}}\right| = 152.53\ \text{kJ/kg}$$

State 6: p_6 = 1 MPa (10 bar), saturated liquid → h_6 = 762.81 kJ/kg, s_6 = 2.1387 kJ/kg·K,
v_6 = 0.0011273 m³/kg

State 7: $p_7 = p_1$ = 12 MPa (120 bar), $s_7 = s_6$ = 2.1387 kJ/kg·K → $h_7 \approx h_6 + v_6(p_7 - p_6)$

$$h_7 = 762.81\frac{\text{kJ}}{\text{kg}} + \left(0.0011273\frac{\text{m}^3}{\text{kg}}\right)(12000 - 1000)\text{kPa}\left|\frac{1000\ \text{N/m}^2}{1\,\text{kPa}}\right|\left|\frac{1\,\text{kJ}}{1000\ \text{N}\cdot\text{m}}\right| = 775.21\ \text{kJ/kg}$$

(a) Applying energy and mass balances to the control volume enclosing the open feedwater heater, the fraction of flow, y, extracted at location 2 is

$$y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{(762.81 - 152.53)\ \text{kJ/kg}}{(2823.3 - 152.53)\ \text{kJ/kg}} = 0.2285$$

For the control volume surrounding the turbine stages

$$\frac{\dot{W}_\text{t}}{\dot{m}_1} = (h_1 - h_2) + (1 - y)(h_2 - h_3)$$

$$\frac{\dot{W}_\text{t}}{\dot{m}_1} = (3506.2 - 2823.3)\frac{\text{kJ}}{\text{kg}} + (1 - 0.2285)(2823.3 - 2058.2)\frac{\text{kJ}}{\text{kg}} = 1273.2\ \text{kJ/kg}$$

For the pumps

$$\frac{\dot{W}_\text{p}}{\dot{m}_1} = (h_7 - h_6) + (1 - y)(h_5 - h_4)$$

$$\frac{\dot{W}_\text{p}}{\dot{m}_1} = (775.21 - 762.81)\frac{\text{kJ}}{\text{kg}} + (1 - 0.2285)(152.53 - 151.53)\frac{\text{kJ}}{\text{kg}} = 13.17\ \text{kJ/kg}$$

For the working fluid passing through the steam generator

$$\frac{\dot{Q}_\text{in}}{\dot{m}_1} = h_1 - h_7 = (3506.2 - 775.21)\frac{\text{kJ}}{\text{kg}} = 2731.0\ \text{kJ/kg}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_\text{t}/\dot{m}_1 - \dot{W}_\text{p}/\dot{m}_1}{\dot{Q}_\text{in}/\dot{m}_1} = \frac{(1273.2 - 13.17)\ \text{kJ/kg}}{2731.0\ \text{kJ/kg}} = \mathbf{\underline{0.461\ (46.1\%)}}$$

(b) The ***net*** power developed is

$$\dot{W}_{\text{cycle}} = \dot{m}_1(\dot{W}_{\text{t}}/\dot{m}_1 - \dot{W}_{\text{p}}/\dot{m}_1)$$

Thus,

$$\dot{m}_1 = \frac{\dot{W}_{\text{cycle}}}{(\dot{W}_{\text{t}}/\dot{m}_1 - \dot{W}_{\text{p}}/\dot{m}_1)}$$

$$\dot{m}_1 = \frac{330\ \text{MW}}{(1273.2 - 13.17)\frac{\text{kJ}}{\text{kg}}}\left|\frac{1000\frac{\text{kJ}}{\text{s}}}{1\ \text{MW}}\right| = \underline{\mathbf{261.9\ kg/s}}$$

(c) The rate of entropy production in the open feedwater heater is determined using the steady-state form of the entropy rate balance:

$$0 = \cancel{\sum_j \frac{\dot{Q}_j}{T_j}} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{\text{cv}}$$

Since the feedwater heater is adiabatic, the heat transfer term drops. Thus,

$$\dot{\sigma}_{\text{cv}} = \sum_e \dot{m}_e s_e - \sum_i \dot{m}_i s_i = \dot{m}_6 s_6 - \dot{m}_2 s_2 - \dot{m}_5 s_5$$

$$\dot{\sigma}_{\text{cv}} = \dot{m}_1[s_6 - y s_2 - (1-y)s_5]$$

$$\dot{\sigma}_{\text{cv}} = 261.9\frac{\text{kg}}{\text{s}}[2.1387 - (0.2285)(6.6840) - (1 - 0.2285)(0.5210)]\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right| = \underline{\mathbf{54.86\ kW/K}}$$

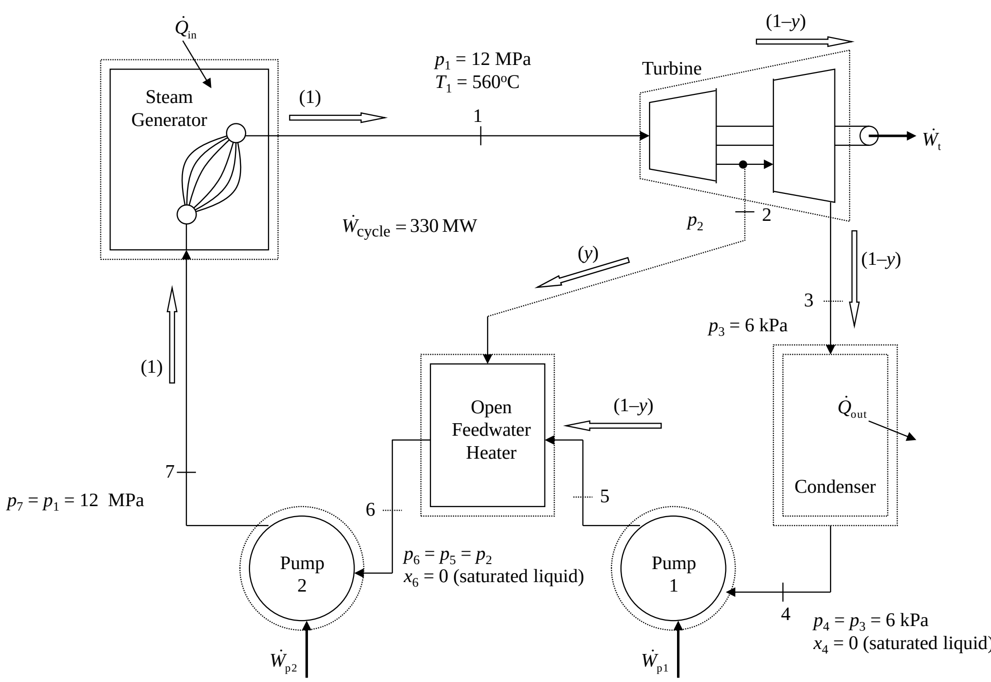
Mixing of streams within the open feedwater heater is a source of irreversibility that produces entropy.

8.41 Reconsider the cycle of Problem 8.40 as the feedwater heater pressure takes on other values. Plot the cycle thermal efficiency, cycle work per unit mass entering the turbine, in kJ/kg, the heat transfer into the cycle per unit mass entering the turbine, in kJ/kg, the fraction of steam extracted and sent to the feedwater heater, the mass flow rate into the first turbine stage, in kg/s, and the rate of entropy production in the open feedwater heater, in kW/K, versus feedwater heater pressure ranging from 0.3 to 10 MPa.

KNOWN: Steady-state operating data are provided for the regenerative cycle of Problem 8.40.

FIND: Plot the cycle thermal efficiency, cycle work per unit mass entering the turbine, in kJ/kg, the heat transfer into the cycle per unit mass entering the turbine, in kJ/kg, the fraction of steam extracted and sent to the feedwater heater, the mass flow rate into the first turbine stage, in kg/s, and the rate of entropy production in the open feedwater heater, in kW/K, versus feedwater heater pressure ranging from 0.3 to 10 MPa (3 to 100 bar).

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. All processes of the working fluid are internally reversible except for mixing in the open feedwater heater.
3. The turbines, pumps, and open feedwater heater operate adiabatically.
4. Kinetic and potential energy effects are negligible.
5. Saturated liquid exits the open feedwater heater, and saturated liquid exits the condenser.

ANALYSIS: For a sample calculation, let p_2 = 1 MPa (10 bar) (Problem 8.40).

First fix each principal state.

State 1: p_1 = 12 MPa (120 bar), T_1 = 560°C → h_1 = 3506.2 kJ/kg, s_1 = 6.6840 kJ/kg·K

State 2: p_2 = 1 MPa (10 bar), $s_2 = s_1$ = 6.6840 kJ/kg·K → h_2 = 2823.3 kJ/kg

State 3: p_3 = 6 kPa (0.06 bar), $s_3 = s_1$ = 6.6840 kJ/kg·K → x_3 = 0.7892, h_3 = 2058.2 kJ/kg

State 4: p_4 = 6 kPa (0.06 bar), saturated liquid → h_4 = 151.53 kJ/kg, s_4 = 0.5210 kJ/kg·K,
v_4 = 0.0010064 m³/kg

State 5: $p_5 = p_2$ = 1 MPa (10 bar), $s_5 = s_4$ = 0.5210 kJ/kg·K → $h_5 \approx h_4 + v_4(p_5 - p_4)$

$$h_5 = 151.53\frac{\text{kJ}}{\text{kg}} + \left(0.0010064\frac{\text{m}^3}{\text{kg}}\right)(1000-6)\text{kPa}\left|\frac{1000\ \text{N/m}^2}{1\,\text{kPa}}\right|\left|\frac{1\,\text{kJ}}{1000\ \text{N}\cdot\text{m}}\right| = 152.53\ \text{kJ/kg}$$

State 6: p_6 = 1 MPa (10 bar), saturated liquid → h_6 = 762.81 kJ/kg, s_6 = 2.1387 kJ/kg·K,
v_6 = 0.0011273 m³/kg

State 7: $p_7 = p_1$ = 12 MPa (120 bar), $s_7 = s_6$ = 2.1387 kJ/kg·K → $h_7 \approx h_6 + v_6(p_7 - p_6)$

$$h_7 = 762.81\frac{\text{kJ}}{\text{kg}} + \left(0.0011273\frac{\text{m}^3}{\text{kg}}\right)(12000-1000)\text{kPa}\left|\frac{1000\ \text{N/m}^2}{1\,\text{kPa}}\right|\left|\frac{1\,\text{kJ}}{1000\ \text{N}\cdot\text{m}}\right| = 775.21\ \text{kJ/kg}$$

(a) Applying energy and mass balances to the control volume enclosing the open feedwater heater, the fraction of flow, y, extracted at location 2 is

$$y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{(762.81-152.53)\ \text{kJ/kg}}{(2823.3-152.53)\ \text{kJ/kg}} = \mathbf{\underline{0.2285}}$$

For the control volume surrounding the turbine stages

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1-y)(h_2 - h_3)$$

$$\frac{\dot{W}_t}{\dot{m}_1} = (3506.2-2823.3)\frac{\text{kJ}}{\text{kg}} + (1-0.2285)(2823.3-2058.2)\frac{\text{kJ}}{\text{kg}} = 1273.2\ \text{kJ/kg}$$

For the pumps

$$\frac{\dot{W}_p}{\dot{m}_1} = (h_7 - h_6) + (1-y)(h_5 - h_4)$$

$$\frac{\dot{W}_p}{\dot{m}_1} = (775.21 - 762.81)\frac{\text{kJ}}{\text{kg}} + (1 - 0.2285)(152.53 - 151.53)\frac{\text{kJ}}{\text{kg}} = 13.17 \text{ kJ/kg}$$

The cycle work per unit mass entering the turbine is

$$\dot{W}_{\text{cycle}} / \dot{m}_1 = \dot{W}_t / \dot{m}_1 - \dot{W}_p / \dot{m}_1 = (1273.2 - 13.17)\ \text{kJ/kg} = \underline{\mathbf{1260.03\ kJ/kg}}$$

For the working fluid passing through the steam generator

$$\frac{\dot{Q}_{\text{in}}}{\dot{m}_1} = h_1 - h_7 = (3506.2 - 775.21)\frac{\text{kJ}}{\text{kg}} = \underline{\mathbf{2731.0\ kJ/kg}}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t / \dot{m}_1 - \dot{W}_p / \dot{m}_1}{\dot{Q}_{\text{in}} / \dot{m}_1} = \frac{(1273.2 - 13.17)\ \text{kJ/kg}}{2731.0\ \text{kJ/kg}} = \underline{\mathbf{0.461\ (46.1\%)}}$$

(b) The ***net*** power developed is

$$\dot{W}_{\text{cycle}} = \dot{m}_1(\dot{W}_t / \dot{m}_1 - \dot{W}_p / \dot{m}_1)$$

Thus,

$$\dot{m}_1 = \frac{\dot{W}_{\text{cycle}}}{(\dot{W}_t / \dot{m}_1 - \dot{W}_p / \dot{m}_1)}$$

$$\dot{m}_1 = \frac{330\ \text{MW}}{(1273.2 - 13.17)\frac{\text{kJ}}{\text{kg}}}\left|\frac{1000\frac{\text{kJ}}{\text{s}}}{1\ \text{MW}}\right| = \underline{\mathbf{261.9\ kg/s}}$$

(c) The rate of entropy production in the open feedwater heater is determined using the steady-state form of the entropy rate balance:

$$0 = \cancel{\sum_j \frac{\dot{Q}_j}{T_j}} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{\text{cv}}$$

Since the feedwater heater is adiabatic, the heat transfer term drops. Thus,

$$\dot{\sigma}_{\text{cv}} = \sum_e \dot{m}_e s_e - \sum_i \dot{m}_i s_i = \dot{m}_6 s_6 - \dot{m}_2 s_2 - \dot{m}_5 s_5$$

$$\dot{\sigma}_{\text{cv}} = \dot{m}_1[s_6 - y s_2 - (1 - y)s_5]$$

$$\dot{\sigma}_{cv} = 261.9\frac{\text{kg}}{\text{s}}[2.1387-(0.2285)(6.6840)-(1-0.2285)(0.5210)]\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right| = \underline{\mathbf{54.86\ kW/K}}$$

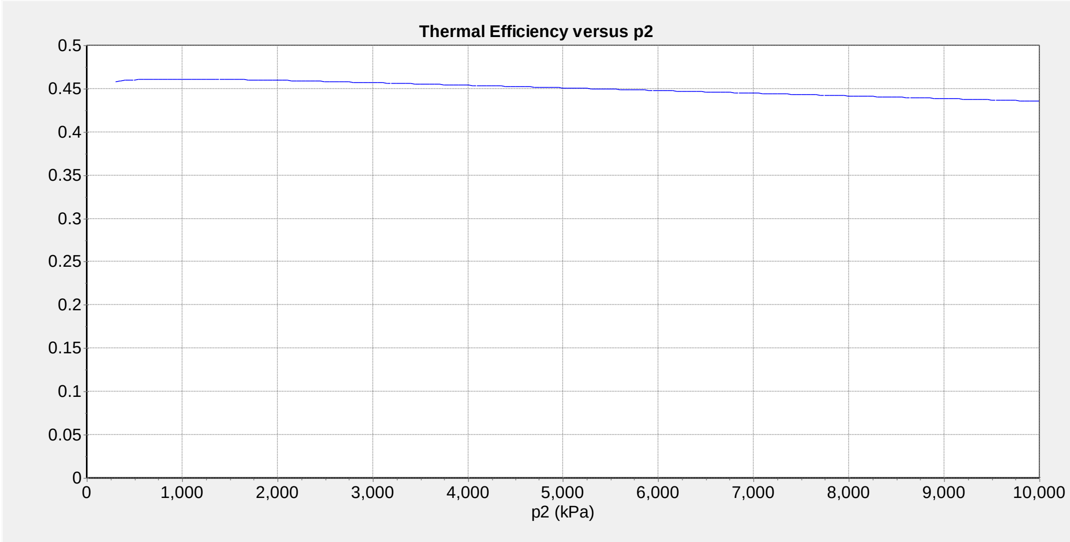
To obtain the data required for the plots, use IT as follows:

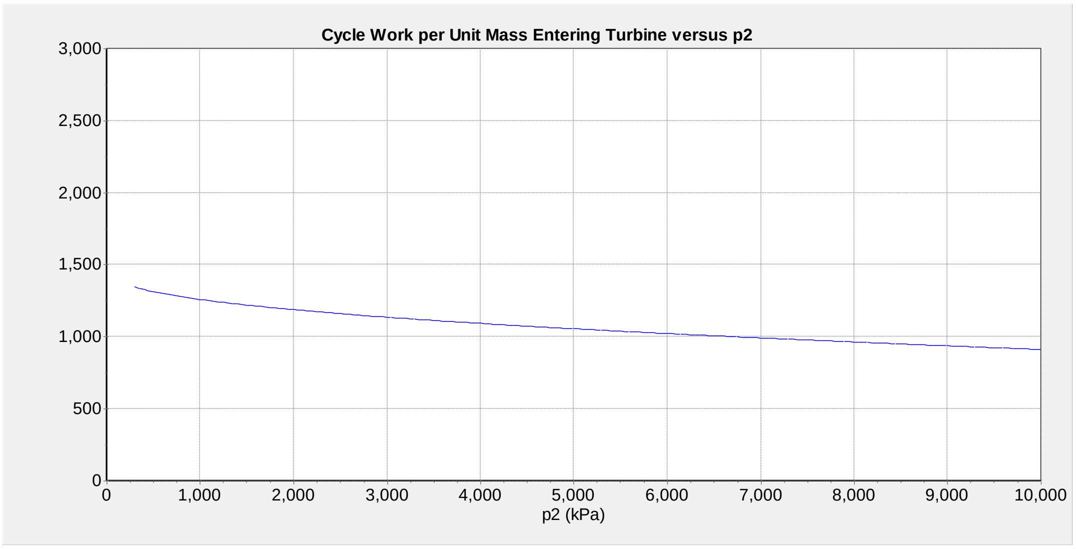
IT Code	IT Output

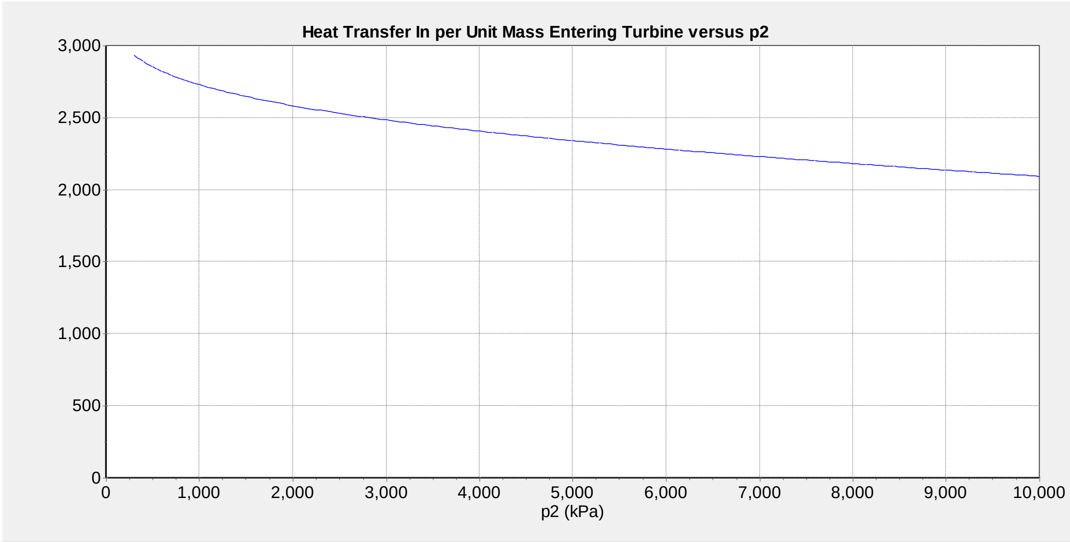
//Given Data
p1 = 12000 // kPa
T1 = 560 // oC
p2 = 1000 // kPa
p3 = 6 // kPa
p4 = p3
p5 = p2
p6 = p2
p7 = p1
x4 = 0
x6 = 0
W_dot_cycle = 330000 // kW
 	
//Determine required properties
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
s2 = s1
h2 = h_Ps("Water/Steam", p2, s2)
s3 = s1
h3 = h_Ps("Water/Steam", p3, s3)
h4 = hsat_Px("Water/Steam", p4, x4)
v4 = vsat_Px("Water/Steam", p4, x4)
s4 = ssat_Px("Water/Steam", p4, x4)
h5 = h4 + v4 * (p5 - p4)
s5 = s4
v6 = vsat_Px("Water/Steam", p6, x6)
h6 = hsat_Px("Water/Steam", p6, x6)
s6 = ssat_Px("Water/Steam", p6, x6)
h7 = h6 + v6 * (p7 - p6)
 	
//Determine energy transfers and performance parameters
y = (h6 - h5) / (h2 - h5)
Wdotturb_per_mdot1 = (h1 - h2) + (1 - y)*(h2 - h3)
Wdotpump_per_mdot1 = (h7 - h6) + (1 - y)*(h5 - h4)
Wdotcycle_per_mdot1 = Wdotturb_per_mdot1 - Wdotpump_per_mdot1
Qdotin_per_mdot1 = h1 - h7
eta = (Wdotturb_per_mdot1 - Wdotpump_per_mdot1)/Qdotin_per_mdot1
mdot1 = W_dot_cycle/(Wdotturb_per_mdot1 - Wdotpump_per_mdot1)
Sdotprod_FWH = mdot1 * (s6 - y*s2 - (1-y)*s5)
	eta 0.4614
	h1 3506
	h2 2823
	h3 2058
	h4 151
	h5 152
	h6 762.5
	h7 774.9
	mdot1 261.9
	p4 6
	p5 1000
	p6 1000
	p7 1.2E4
	Qdotin_per_mdot1 2731
	s1 6.683
	s2 6.683
	s3 6.683
	s4 0.519
	s5 0.519
	s6 2.138
	Sdotprod_FWH 54.93
	v4 0.001007
	v6 0.001127
	Wdotcycle_per_mdot1 1260
	Wdotpump_per_mdot1 13.17
	Wdotturb_per_mdot1 1273
	y 0.2286
	p1 1.2E4
	p2 1000
	p3 6
	T1 560
	W_dot_cycle 3.3E5
	x4 0
	x6 0

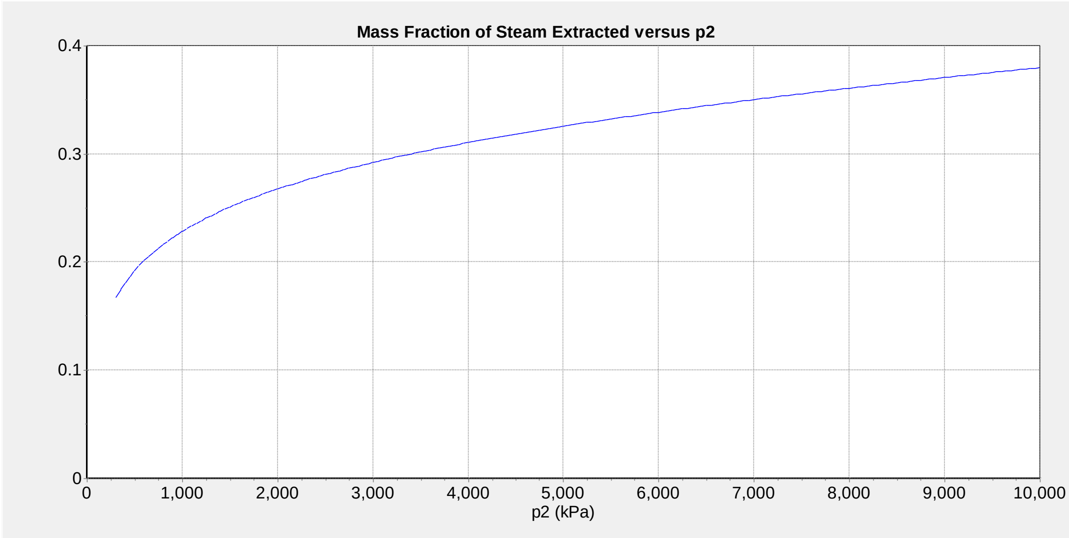
Note the results for thermal efficiency, cycle work per unit mass entering the turbine, heat transfer per unit mass entering the turbine, mass fraction of steam extracted from the turbine and sent to the feedwater heater, mass flow rate entering the turbine, and rate of entropy production are in good agreement with the sample calculations.

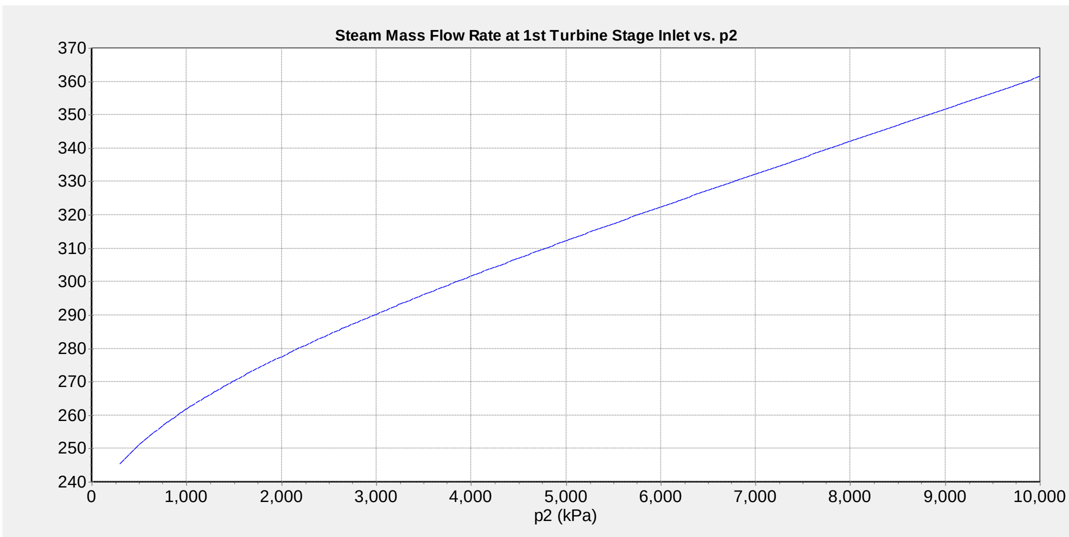
Plots:

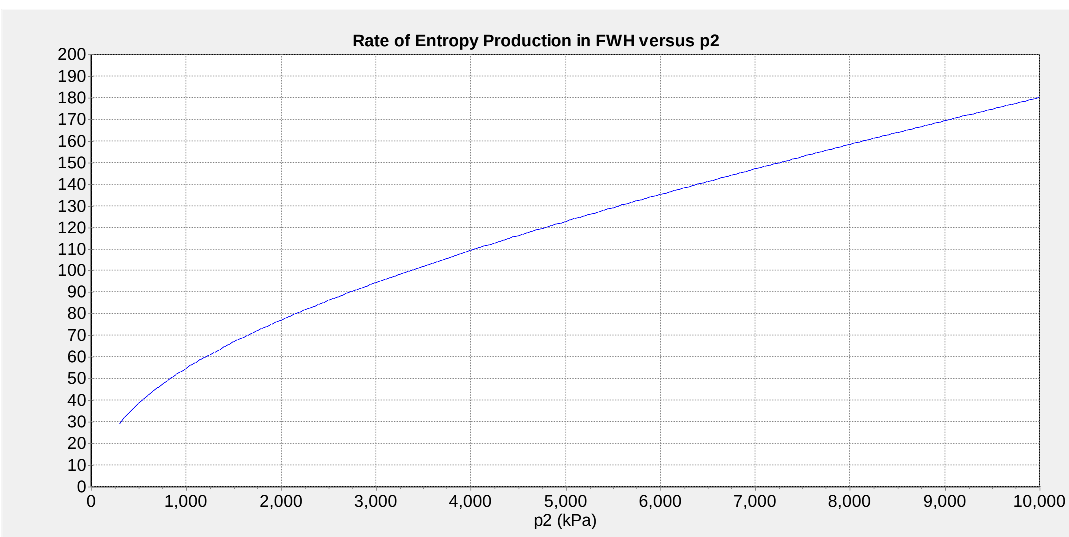

Thermal Efficiency versus p2
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1,000
2,000
3,000
4,000
5,000
6,000
7,000
8,000
9,000
10,000
p2 (kPa)


Cycle Work per Unit Mass Entering Turbine versus p2
3,000
2,500
2,000
1,500
1,000
500
0
0
1,000
2,000
3,000
4,000
5,000
6,000
7,000
8,000
9,000
10,000
p2 (kPa)


Heat Transfer In per Unit Mass Entering Turbine versus p2
3,000
2,500
2,000
1,500
1,000
500
0
0
1,000
2,000
3,000
4,000
5,000
6,000
7,000
8,000
9,000
10,000
p2 (kPa)


Mass Fraction of Steam Extracted versus p2
0.4
0.3
0.2
0.1
0
0
1,000
2,000
3,000
4,000
5,000
6,000
7,000
8,000
9,000
10,000
p2 (kPa)


Steam Mass Flow Rate at 1st Turbine Stage Inlet vs. p2
370
360
350
340
330
320
310
300
290
280
270
260
250
240
0
1,000
2,000
3,000
4,000
5,000
6,000
7,000
8,000
9,000
10,000
p2 (kPa)


Rate of Entropy Production in FWH versus p2
200
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
0
1,000
2,000
3,000
4,000
5,000
6,000
7,000
8,000
9,000
10,000
p2 (kPa)

From the plots we see that for a fixed cycle power output of 330 MW, the thermal efficiency gradually decreases over the range of feedwater heaters studied.

The required mass flow rate of steam entering the first stage turbine increases with increasing p_2. To maintain a fixed cycle power output of 330 MW, this increase in flow rate is required since the cycle work per unit of mass decreases as more steam is diverted to the feedwater heater with increasing p2.

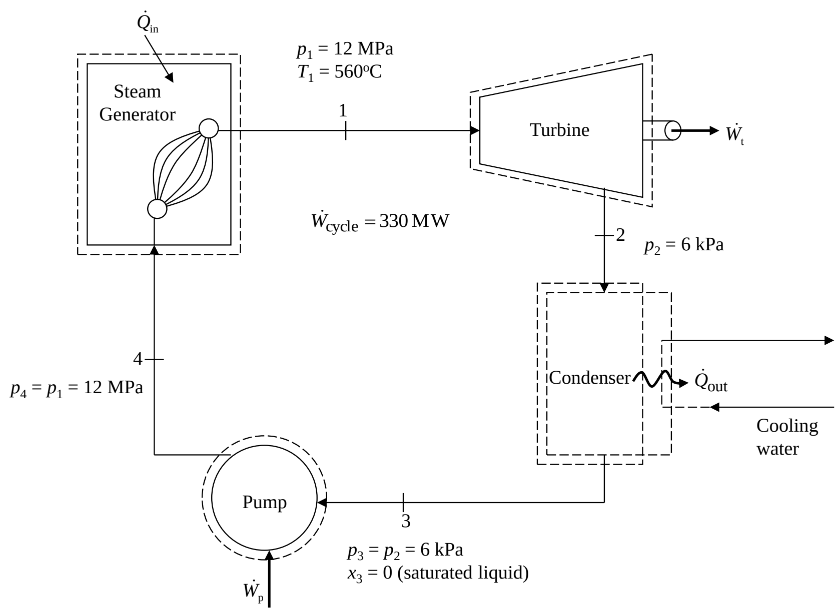
The rate of entropy production in the feedwater heater increases with increasing p_2 as more mass is diverted from the turbine to mix in the feedwater heater.

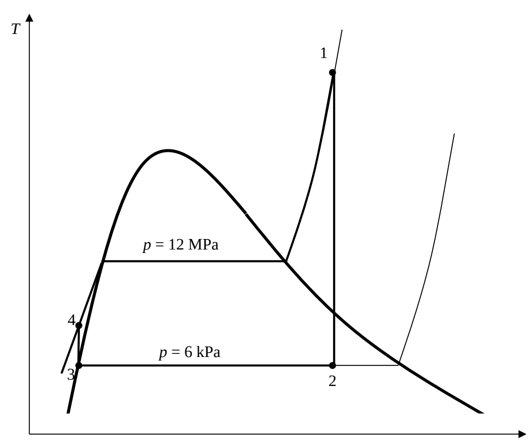
8.42 Compare the results of problem 8.40 (a) and (b) with those for an ideal Rankine cycle having the same turbine inlet conditions and condenser pressure but no regenerator.

KNOWN: An ideal Rankine cycle operates with steam as the working fluid. Operational data are provided.

FIND: Determine (a) the cycle thermal efficiency and (b) the mass flow rate into the first turbine stage, in kg/s. Compare results with those from problem 8.40.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. All processes of the working fluid are internally reversible.

3. The turbine and pump operate adiabatically.
4. Kinetic and potential energy effects are negligible.
5. Saturated liquid exits the condenser.

ANALYSIS:

First fix each principal state.

State 1: p_1 = 12 MPa (120 bar), T_1 = 560°C → h_1 = 3506.2 kJ/kg, s_1 = 6.6840 kJ/kg·K

State 2: p_2 = 6 kPa (0.06 bar), $s_2 = s_1$ = 6.6840 kJ/kg·K → x_2 = 0.7892, h_2 = 2058.2 kJ/kg

State 3: p_3 = 6 kPa (0.06 bar), saturated liquid → h_3 = 151.53 kJ/kg, s_3 = 0.5210 kJ/kg·K, v_3 = 0.0010064 m³/kg

State 4: $p_4 = p_1$ = 12 MPa (120 bar), $s_4 = s_3$ = 0.5210 kJ/kg·K → $h_4 \approx h_3 + v_3(p_4 - p_3)$

$$h_4 = 151.53\frac{\text{kJ}}{\text{kg}} + \left(0.0010064\frac{\text{m}^3}{\text{kg}}\right)(12000-6)\text{kPa}\left|\frac{1000\ \text{N/m}^2}{1\,\text{kPa}}\right|\left|\frac{1\,\text{kJ}}{1000\ \text{N}\cdot\text{m}}\right| = 163.60\ \text{kJ/kg}$$

(a) For the control volume surrounding the turbine

$$\frac{\dot{W}_\text{t}}{\dot{m}} = h_1 - h_2$$

$$\frac{\dot{W}_\text{t}}{\dot{m}} = (3506.2 - 2058.2)\frac{\text{kJ}}{\text{kg}} = 1448\ \text{kJ/kg}$$

For the pump

$$\frac{\dot{W}_\text{p}}{\dot{m}} = h_4 - h_3$$

$$\frac{\dot{W}_\text{p}}{\dot{m}} = (163.60 - 151.53)\frac{\text{kJ}}{\text{kg}} = 12.07\ \text{kJ/kg}$$

For the working fluid passing through the steam generator

$$\frac{\dot{Q}_\text{in}}{\dot{m}} = h_1 - h_4 = (3506.2 - 163.60)\frac{\text{kJ}}{\text{kg}} = 3342.6\ \text{kJ/kg}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t / \dot{m} - \dot{W}_p / \dot{m}}{\dot{Q}_{in} / \dot{m}} = \frac{(1448 - 12.07)\ \text{kJ/kg}}{3342.6\ \text{kJ/kg}} = \underline{\mathbf{0.430\ (43.0\%)}}$$

(b) The ***net*** power developed is

$$\dot{W}_{cycle} = \dot{m}(\dot{W}_t / \dot{m} - \dot{W}_p / \dot{m})$$

Thus,

$$\dot{m} = \frac{\dot{W}_{cycle}}{(\dot{W}_t / \dot{m} - \dot{W}_p / \dot{m})}$$

$$\dot{m} = \frac{330\ \text{MW}}{(1448 - 12.07)\dfrac{\text{kJ}}{\text{kg}}} \left| \frac{1000\ \dfrac{\text{kJ}}{\text{s}}}{1\ \text{MW}} \right| = \underline{\mathbf{229.8\ kg/s}}$$

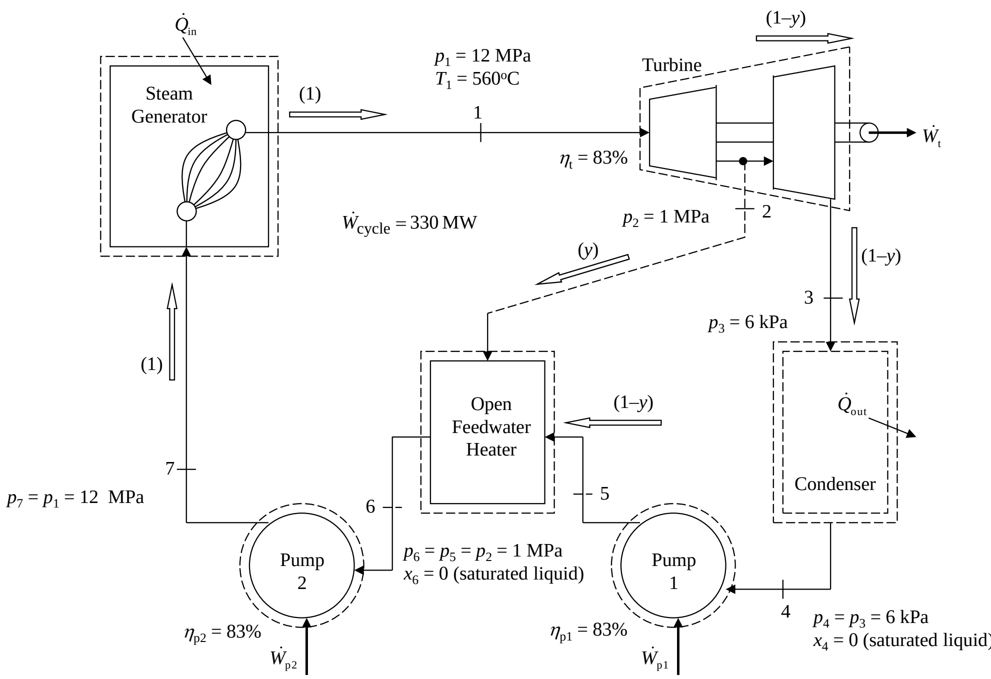
For fixed net power output by the cycle, introduction of the open feedwater heater into the basic cycle results in increased thermal efficiency and higher mass flow rate into the first turbine stage.

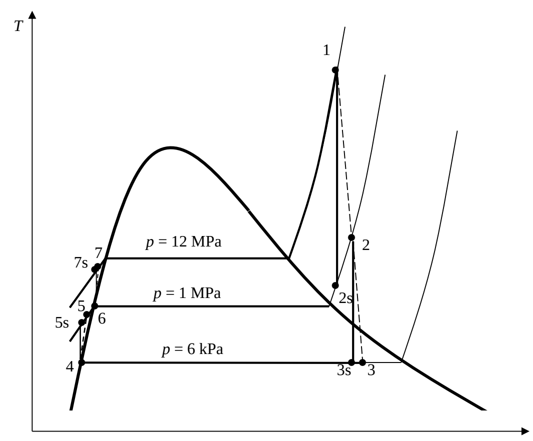
8.43 Compare the results of Problem 8.40 with those for the same cycle whose processes of the working fluid are not internally reversible in the turbines and pumps. Assume that both turbine stages and both pumps have an isentropic efficiency of 83%.

KNOWN: A regenerative vapor power cycle with one open feedwater heater operates with steam as the working fluid. Operational data are provided.

FIND: Determine (a) the cycle thermal efficiency, (b) the mass flow rate into the first turbine stage, in kg/s, and (c) the rate of entropy production in the open feedwater heater, in kW/K. Compare results with those of Problem 8.40.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. All processes of the working fluid are internally reversible except for processes in the turbines and pumps and mixing in the open feedwater heater.
3. The turbines, pumps, and open feedwater heater operate adiabatically.
4. Kinetic and potential energy effects are negligible.
5. Saturated liquid exits the open feedwater heater, and saturated liquid exits the condenser.

ANALYSIS: First fix each principal state.

<u>State 1</u>: p_1 = 12 MPa (120 bar), T_1 = 560°C → h_1 = 3506.2 kJ/kg, s_1 = 6.6840 kJ/kg·K

<u>State 2s</u>: $p_{2s} = p_2$ = 1 MPa (10 bar), $s_{2s} = s_1$ = 6.6840 kJ/kg·K → h_{2s} = 2823.3 kJ/kg

<u>State 2</u>: p_2 = 1 MPa (10 bar), h_2 = 2939.4 kJ/kg (see below) → s_2 = 6.9174 kJ/kg·K

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \rightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 3506.2\frac{\text{kJ}}{\text{kg}} - (0.83)(3506.2 - 2823.3)\frac{\text{kJ}}{\text{kg}} = 2939.4 \text{ kJ/kg}$$

<u>State 3s</u>: $p_{3s} = p_3$ = 6 kPa (0.06 bar), $s_{3s} = s_2$ = 6.9174 kJ/kg·K → x_{3s} = 0.8191, h_{3s} = 2130.4 kJ/kg

<u>State 3</u>: p_3 = 6 kPa (0.06 bar), h_3 = 2267.9 kJ/kg (see below) → x_3 = 0.8760, s_3 = 7.3620 kJ/kg·K

$$\eta_t = \frac{h_2 - h_3}{h_2 - h_{3s}} \rightarrow h_3 = h_2 - \eta_t(h_2 - h_{3s}) = 2939.4\frac{\text{kJ}}{\text{kg}} - (0.83)(2939.4 - 2130.4)\frac{\text{kJ}}{\text{kg}} = 2267.9 \text{ kJ/kg}$$

<u>State 4</u>: p_4 = 6 kPa (0.06 bar), saturated liquid → h_4 = 151.53 kJ/kg, v_4 = 0.0010064 m³/kg, s_4 = 0.5210 kJ/kg·K

<u>State 5</u>: $p_5 = p_2$ = 1 MPa (10 bar), h_5 = 152.74 kJ/kg (see below) → $s_5 \approx$ 0.5249 kJ/kg·K (***assuming the saturated liquid state corresponding to $h_5 = h_f$ in Table A-2 and interpolating for $s_5 = s_f$***)

$$\eta_{p1} = \frac{v_4(p_5 - p_4)}{h_5 - h_4} \rightarrow h_5 = h_4 + \frac{v_4(p_5 - p_4)}{\eta_{p1}}$$

$$h_5 = 151.53\frac{\text{kJ}}{\text{kg}} + \frac{(0.0010064\frac{\text{m}^3}{\text{kg}})(1000 - 6)\text{ kPa}}{0.83}\left|\frac{1000\frac{\text{N}}{\text{m}^2}}{1\text{ kPa}}\right|\left|\frac{1\text{ kJ}}{1000\text{ N}\cdot\text{m}}\right| = 152.74 \text{ kJ/kg}$$

<u>State 6</u>: p_6 = 1 MPa (10 bar), saturated liquid → h_6 = 762.81 kJ/kg, s_6 = 2.1387 kJ/kg·K,

v_6 = 0.0011273 m^3/kg

State 7: $p_7 = p_1$ = 12 MPa (120 bar), h_7 = 777.75 kJ/kg (see below)

$$\eta_{\text{p2}} = \frac{v_6(p_7 - p_6)}{h_7 - h_6} \rightarrow h_7 = h_6 + \frac{v_6(p_7 - p_6)}{\eta_{\text{p2}}}$$

$$h_7 = 762.81\frac{\text{kJ}}{\text{kg}} + \frac{(0.0011273\frac{\text{m}^3}{\text{kg}})(12000 - 1000)\text{ kPa}}{0.83}\left|\frac{1000\frac{\text{N}}{\text{m}^2}}{1\text{ kPa}}\right|\left|\frac{1\text{ kJ}}{1000\text{ N}\cdot\text{m}}\right| = 777.75 \text{ kJ/kg}$$

(a) Applying energy and mass balances to the control volume enclosing the open feedwater heater, the fraction of flow, y, extracted at location 2 is

$$y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{(762.81 - 152.74)\text{ kJ/kg}}{(2939.4 - 152.74)\text{ kJ/kg}} = 0.2189$$

For the control volume surrounding the turbine stages

$$\frac{\dot{W}_{\text{t}}}{\dot{m}_1} = (h_1 - h_2) + (1 - y)(h_2 - h_3)$$

$$\frac{\dot{W}_{\text{t}}}{\dot{m}_1} = (3506.2 - 2939.4)\frac{\text{kJ}}{\text{kg}} + (1 - 0.2189)(2939.4 - 2267.9)\frac{\text{kJ}}{\text{kg}} = 1091.3 \text{ kJ/kg}$$

For the pumps

$$\frac{\dot{W}_{\text{p}}}{\dot{m}_1} = (h_7 - h_6) + (1 - y)(h_5 - h_4)$$

$$\frac{\dot{W}_{\text{p}}}{\dot{m}_1} = (777.75 - 762.81)\frac{\text{kJ}}{\text{kg}} + (1 - 0.2189)(152.74 - 151.53)\frac{\text{kJ}}{\text{kg}} = 15.89 \text{ kJ/kg}$$

For the working fluid passing through the steam generator

$$\frac{\dot{Q}_{\text{in}}}{\dot{m}_1} = h_1 - h_7 = (3506.2 - 777.75)\frac{\text{kJ}}{\text{kg}} = 2728.5 \text{ kJ/kg}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{(1091.3 - 15.89)\text{ kJ/kg}}{2728.5\text{ kJ/kg}} = \boxed{\underline{\mathbf{0.394\ (39.4\%)}}}$$

(b) The ***net*** power developed is

$$\dot{W}_{cycle} = \dot{m}_1(\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1)$$

Thus,

$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{(\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1)}$$

$$\dot{m}_1 = \frac{330\text{ MW}}{(1091.3 - 15.89)\frac{\text{kJ}}{\text{kg}}}\left|\frac{1000\frac{\text{kJ}}{\text{s}}}{1\text{ MW}}\right| = \boxed{\underline{\mathbf{306.9\ kg/s}}}$$

(c) The rate of entropy production in the open feedwater heater is determined using the steady-state form of the entropy rate balance:

$$0 = \cancel{\sum_j \frac{\dot{Q}_j}{T_j}} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{cv}$$

Since the feedwater heater is adiabatic, the heat transfer term drops. Thus,

$$\dot{\sigma}_{cv} = \sum_e \dot{m}_e s_e - \sum_i \dot{m}_i s_i = \dot{m}_6 s_6 - \dot{m}_2 s_2 - \dot{m}_5 s_5$$

$$\dot{\sigma}_{cv} = \dot{m}_1[s_6 - y s_2 - (1-y)s_5]$$

$$\dot{\sigma}_{cv} = 306.9\frac{\text{kg}}{\text{s}}[2.1387 - (0.2189)(6.9174) - (1 - 0.2189)(0.5249)]\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = \boxed{\underline{\mathbf{65.82\ kW/K}}}$$

Compared to the ideal cycle in problem 8.40, the presence of internal irreversibilities in the turbine stages and the pumps results in lower cycle thermal efficiency, higher required mass flow rate of steam entering the first-stage turbine, and greater rate of entropy production in the open feedwater heater.

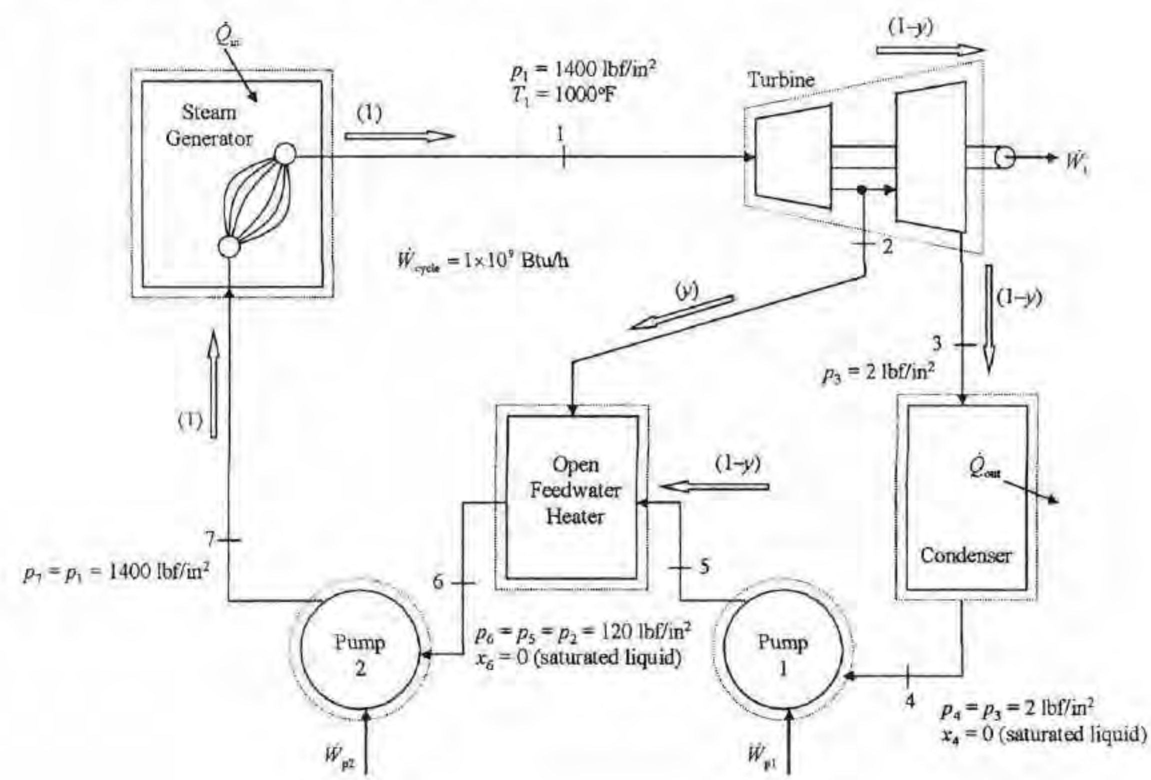
8.44 Water is the working fluid in an ideal regenerative Rankine cycle with one open feedwater heater. Steam enters the turbine at 1400 lbf/in.^{2} and 1000°F and expands it to 120 lbf/in.^{2}, where some of the steam is extracted and diverted to the open feedwater heater operating at 120 lbf/in.^{2} The remaining steam expands through the second-stage turbine to the condenser pressure of 2 lbf/in.^{2} Saturated liquid exits the open feedwater heater at 120 lbf/in.^{2} The net power output of the cycle is 1×10^9 Btu/h. Determine for the cycle

(a) the mass flow rate of steam entering the first stage of the turbine, in lb/h.
(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the steam generator.
(c) the thermal efficiency.

KNOWN: An ideal regenerative Rankine cycle with one open feedwater heater and superheat operates with water as the working fluid. The net power output of the cycle is given.

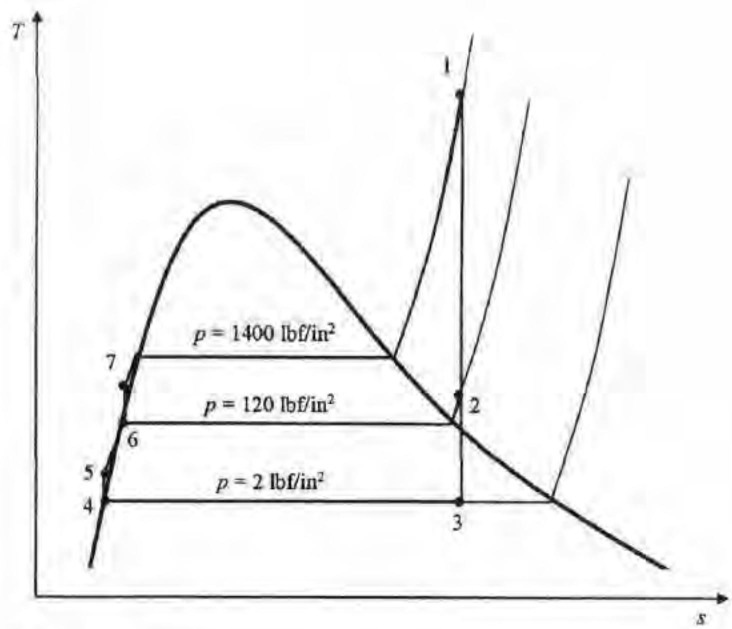
FIND: Determine the mass flow rate of steam entering the first stage of the turbine, the rate of heat transfer to the working fluid passing through the steam generator, and the thermal efficiency.

SCHEMATIC AND GIVEN DATA:



Problem 8.44 (Continued) – Page 2

T-s diagram



ENGINEERING MODEL:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. All processes of the working fluid are internally reversible.
3. The turbine and pump operate adiabatically.
4. Kinetic and potential energy effects are negligible.
5. Condensate exits the condenser as saturated liquid.
6. There is no heat transfer between the outside of the condenser and the surroundings.
7. There is no heat transfer between the outside of the open feedwater heater and the surroundings.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 1400$ lbf/in.^{2}, $T_1 = 1000°F \rightarrow$ From Table A-4E: $h_1 = 1493.5$ Btu/lb and $s_1 = 1.6094$ Btu/(lb·°R)

State 2: $p_2 = 120$ lbf/in.^{2}, $s_2 = s_1 \rightarrow$ From Table A-4E (interpolated): $h_2 = 1208.21$ Btu/lb

State 3: $p_3 = 2$ lbf/in.^{2}, $s_3 = s_1 \rightarrow$ From Table A-3E: $x_3 = (1.6094 - 0.1750)/1.7448 = 0.8221$ and $h_3 = h_{f3} + x_3 h_{fg3} = 94.02 + (0.8221)(1022.1) = 934.29$ Btu/lb

State 4: $p_4 = p_3 = 2$ lbf/in.^{2}, sat liq. $\rightarrow$ From Table A-3E: $h_4 = h_{f4} = 94.02$ Btu/lb and $v_4 = v_{f4} = 0.01623$ ft^3/lb

State 5: $p_5 = p_2 = 120$ lbf/in.^{2}, $h_5 \approx h_4 + v_4(p_5 - p_4)$

$$h_5 = 94.02 \text{ Btu/lb} + 0.01623\left(\frac{\text{ft}^3}{\text{lb}}\right)(120-2)\left(\frac{\text{lbf}}{\text{in}^2}\right)\left|\frac{144 \text{ in}^2}{\text{ft}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right| = 94.37 \text{ Btu/lb}$$

State 6: $p_6 = p_2 = 120$ lbf/in.^{2}, sat liq. → From Table A-3E: $h_6 = h_{f6} = 312.7$ Btu/lb and $v_6 = v_{f6} = 0.01789$ ft^3/lb

State 7: $p_7 = p_1 = 1400$ lbf/in.^{2}, $h_7 \approx h_6 + v_6(p_7 - p_6)$

$$h_7 = 312.7 \text{ Btu/lb} + 0.01789\left(\frac{\text{ft}^3}{\text{lb}}\right)(1400-120)\left(\frac{\text{lbf}}{\text{in}^2}\right)\left|\frac{144 \text{ in}^2}{\text{ft}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right| = 316.94 \text{ Btu/lb}$$

(a) The mass flow rate of steam entering the first stage of the turbine can be determined from the net power output of the cycle

$$\dot{W}_{cycle} = \dot{W}_{t1} + \dot{W}_{t2} - \dot{W}_{p1} - \dot{W}_{p2}$$

Let $\dot{m}_1$ be the mass flow rate of the steam entering the first stage of the turbine. Mass and energy balances for control volumes around the two turbine stages and the two pumps give, respectively

$$\dot{W}_{t1} = \dot{m}_1 (h_1 - h_2)$$
$$\dot{W}_{t2} = \dot{m}_1 (1-y)(h_2 - h_3)$$
$$\dot{W}_{p1} = \dot{m}_1 (1-y)(h_5 - h_4)$$
$$\dot{W}_{p2} = \dot{m}_1 (h_7 - h_6)$$

where y is the fraction of flow entering the first-stage turbine extracted at 2.

Substituting for net power output of the cycle

$$\dot{W}_{cycle} = \dot{m}_1[(h_1 - h_2) + (1-y)(h_2 - h_3) - (1-y)(h_5 - h_4) - (h_7 - h_6)]$$

Solving for $\dot{m}_1$ yields

$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{(h_1 - h_2) + (1-y)(h_2 - h_3) - (1-y)(h_5 - h_4) - (h_7 - h_6)} \quad (1)$$

The mass fraction of steam (y) extracted after the first stage of the turbine is unknown. Analyze the open feedwater heater to determine y. Mass and energy balances for a control volume around the open feedwater heater give

Problem 8.44 (Continued) – Page 4

$$0 = \dot{Q} - \dot{W} + \dot{m}_1\left[(y)h_2 + (1-y)h_5 - h_6\right]$$

Since there is no transfer of energy by heat or work, we can solve for y and substitute values for specific enthalpy to yield

$$y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{312.7\frac{\text{Btu}}{\text{lb}} - 94.37\frac{\text{Btu}}{\text{lb}}}{1208.21\frac{\text{Btu}}{\text{lb}} - 94.37\frac{\text{Btu}}{\text{lb}}} = 0.196$$

Substituting values into (1) gives

$$\dot{m}_1 = \frac{1\times10^9\ \frac{\text{Btu}}{\text{h}}}{(1493.5-1208.21)\frac{\text{Btu}}{\text{lb}} + (1-0.196)(1208.21-934.29)\frac{\text{Btu}}{\text{lb}} - (1-0.196)(94.37-94.02)\frac{\text{Btu}}{\text{lb}} - (316.94-312.7)\frac{\text{Btu}}{\text{lb}}}$$

$$\dot{m}_1 = \underline{\mathbf{2.00\times10^6\ lb/h}}$$

(b) The rate of heat transfer to the working fluid passing through the steam generator can be determined by applying mass and energy balances to a control volume around the steam generator to give

$$\dot{Q}_{\text{in}} = \dot{m}_1(h_1 - h_7) = (2.00\times10^6\ \text{lb/h})(1493.5\ \text{Btu/lb} - 316.94\ \text{Btu/lb}) = \underline{\mathbf{2.35\times10^9\ Btu/h}}$$

(c) The thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = (1\times10^9\ \text{Btu/h})/(2.35\times10^9\ \text{Btu/h}) = \underline{\mathbf{0.4255\ (42.55\%)}}$$

The results of Problem 8.44 can be compared to the results of Problem 8.10 to see some of the effects of incorporating an open feedwater heater on the performance of a Rankine cycle for cycles with the same net power output. In this case, use of an open feedwater heater results in higher thermal efficiency and less heat addition, but the steam flow rate entering the first-stage turbine increases.

8.45 Water is the working fluid in a regenerative Rankine cycle with one open feedwater heater. Steam enters the turbine at 1400 lbf/in.^{2} and 1000°F and expands to 120 lbf/in.^{2}, where some of the steam is extracted and diverted to the open feedwater heater operating at 120 lbf/in.^{2} The remaining steam expands through the second-stage turbine to the condenser pressure of 2 lbf/in.^{2} Each turbine stage and both pumps have isentropic efficiencies of 85%. Flow through the condenser, open feedwater heater, and steam generator is at constant pressure. Saturated liquid exits the open feedwater heater at 120 lbf/in.^{2} The net power output of the cycle is 1×10^9 Btu/h. Determine for the cycle

(a) the mass flow rate of steam entering the first stage of the turbine, in lb/h.

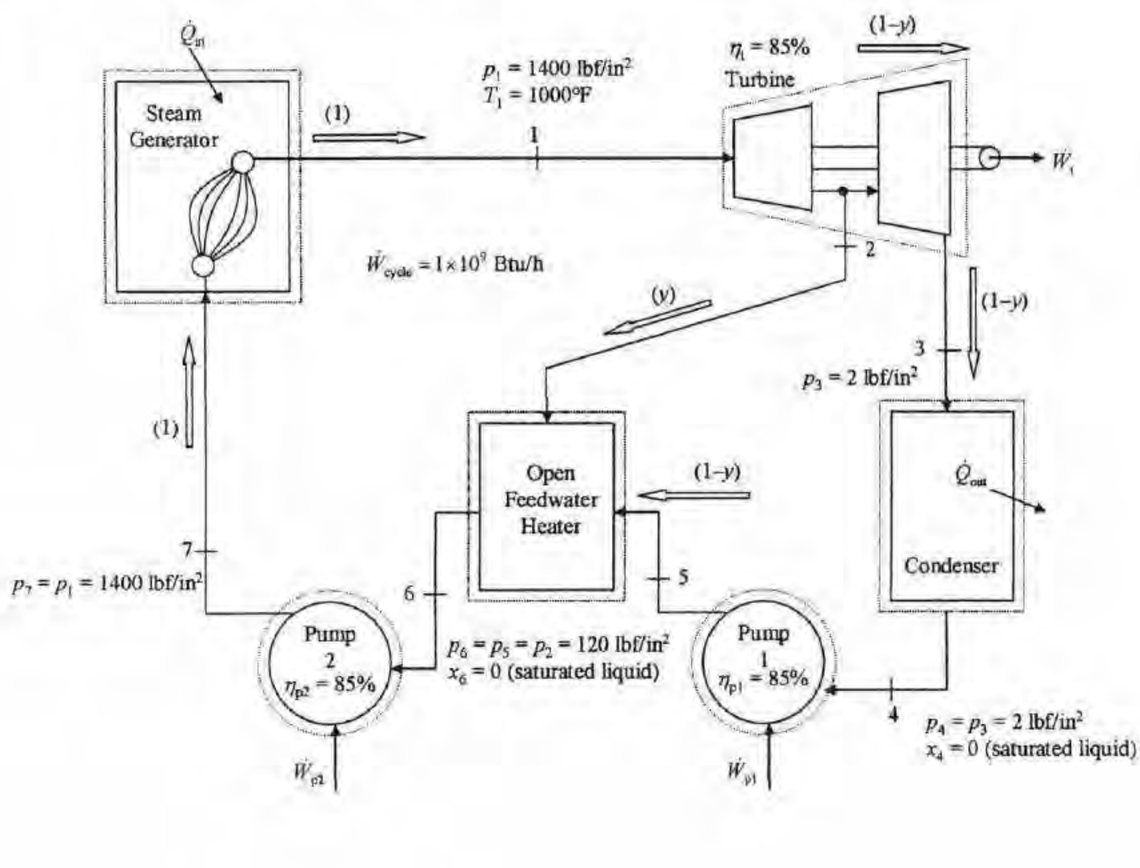
(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the steam generator.

(c) the thermal efficiency.

KNOWN: A regenerative Rankine cycle with one open feedwater heater and superheat operates with water as the working fluid. The net power output of the cycle is given.

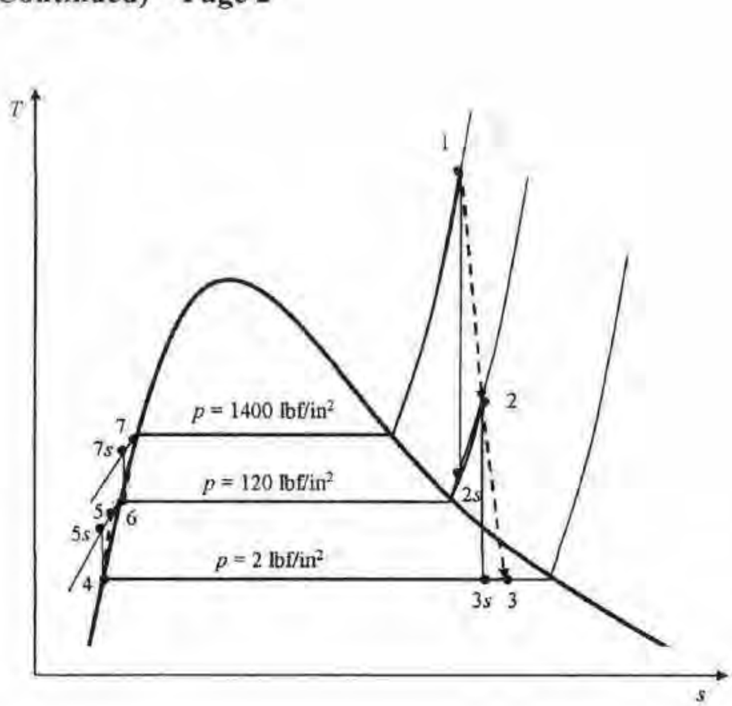
FIND: Determine the mass flow rate of steam entering the first stage of the turbine, the rate of heat transfer to the working fluid passing through the steam generator, and the thermal efficiency.

SCHEMATIC AND GIVEN DATA:



Problem 8.45 (Continued) – Page 2

T-s diagram



ENGINEERING MODEL:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The turbine stages and pumps each operate adiabatically with an isentropic efficiency of 85%.
3. Kinetic and potential energy effects are negligible.
4. Condensate exits the condenser as saturated liquid.
5. There is no heat transfer between the outside of the condenser and the surroundings.
6. There is no heat transfer between the outside of the open feedwater heater and the surroundings.

ANALYSIS: First, fix each of the principal states.

State 1: p_1 = 1400 lbf/in.^{2}, T_1 = 1000°F → From Table A-4E: h_1 = 1493.5 Btu/lb and s_1 = 1.6094 Btu/(lb·°R)

State 2s: $p_{2s} = p_2$ = 120 lbf/in.^{2}, $s_{2s} = s_1$ → From Table A-4E (interpolated): h_{2s} = 1208.21 Btu/lb

State 2: p_2 = 120 lbf/in.^{2}, $h_2 = h_1 - \eta_t(h_1 - h_{2s})$ = 1493.5 – 0.85(1493.5 – 1208.21) = 1251.0 Btu/lb and → From Table A-4E (interpolated): s_2 = 1.6587 Btu/(lb·°R)

State 3s: $p_{3s} = p_3$ = 2 lbf/in.^{2}, $s_{3s} = s_2$ → From Table A-3E: x_{3s} = (1.6587 – 0.1750)/1.7448 = 0.8504 and $h_{3s} = h_{f3s} + x_3h_{fg3s}$ = 94.02 + (0.8504)(1022.1) = 963.21 Btu/lb

State 3: p_3 = 2 lbf/in.^{2}, $h_3 = h_2 - \eta_t(h_2 - h_{3s})$ = 1251.0 – 0.85(1251.0 – 963.21) = 1006.4 Btu/lb

State 4: $p_4 = p_3 = 2$ lbf/in.^{2}, sat liq. → From Table A-3E: $h_4 = h_{f4} = 94.02$ Btu/lb and $v_4 = v_{f4} = 0.01623$ ft^3/lb

State 5: $p_5 = p_6 = 120$ lbf/in.^{2}, $h_5 = h_4 + \dfrac{v_4(p_5 - p_4)}{\eta_{p1}}$

$$h_5 = 94.02 \text{ Btu/lb} + \frac{0.01623\left(\frac{\text{ft}^3}{\text{lb}}\right)(120-2)\left(\frac{\text{lbf}}{\text{in}^2}\right)}{0.85}\left|\frac{144\,\text{in}^2}{\text{ft}^2}\right|\left|\frac{1\,\text{Btu}}{778\,\text{ft}\cdot\text{lbf}}\right| = 96.27 \text{ Btu/lb}$$

State 6: $p_6 = p_2 = 120$ lbf/in.^{2}, sat liq. → From Table A-3E: $h_6 = h_{f6} = 312.7$ Btu/lb and $v_6 = v_{f6} = 0.01789$ ft^3/lb

State 7: $p_7 = p_1 = 1400$ lbf/in.^{2}, $h_7 = h_6 + \dfrac{v_6(p_7 - p_6)}{\eta_{p2}}$

$$h_7 = 312.7 \text{ Btu/lb} + \frac{0.01789\left(\frac{\text{ft}^3}{\text{lb}}\right)(1400-120)\left(\frac{\text{lbf}}{\text{in}^2}\right)}{0.85}\left|\frac{144\,\text{in}^2}{\text{ft}^2}\right|\left|\frac{1\,\text{Btu}}{778\,\text{ft}\cdot\text{lbf}}\right| = 339.64 \text{ Btu/lb}$$

(a) The mass flow rate of steam entering the first stage of the turbine can be determined from the net power output of the cycle

$$\dot{W}_{cycle} = \dot{W}_{t1} + \dot{W}_{t2} - \dot{W}_{p1} - \dot{W}_{p2}$$

Let $\dot{m}_1$ be the mass flow rate of the steam entering the first stage of the turbine. Mass and energy balances for control volumes around the two turbine stages and the two pumps give, respectively

$$\dot{W}_{t1} = \dot{m}_1 (h_1 - h_2)$$
$$\dot{W}_{t2} = \dot{m}_1 (1-y)(h_2 - h_3)$$
$$\dot{W}_{p1} = \dot{m}_1 (1-y)(h_5 - h_4)$$
$$\dot{W}_{p2} = \dot{m}_1 (h_7 - h_6)$$

where y is the fraction of the flow entering the first-stage turbine that is extracted at 2.

Substituting for net power output of the cycle

$$\dot{W}_{cycle} = \dot{m}_1\left[(h_1 - h_2) + (1-y)(h_2 - h_3) - (1-y)(h_5 - h_4) - (h_7 - h_6)\right]$$

Solving for $\dot{m}_1$

$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{(h_1 - h_2) + (1-y)(h_2 - h_3) - (1-y)(h_5 - h_4) - (h_7 - h_6)} \quad (1)$$

The mass fraction of steam (y) extracted after the first stage of the turbine is unknown. Analyze the open feedwater heater to determine y. Mass and energy balances for a control volume around the open feedwater heater give

$$0 = \dot{Q} - \dot{W} + \dot{m}_1\left[(y)h_2 + (1-y)h_5 - h_6\right]$$

Since there is no transfer of energy by heat or work, we can solve for y and substitute values for specific enthalpy to yield

$$y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{312.7\frac{\text{Btu}}{\text{lb}} - 96.27\frac{\text{Btu}}{\text{lb}}}{1251.0\frac{\text{Btu}}{\text{lb}} - 96.27\frac{\text{Btu}}{\text{lb}}} = 0.187$$

Substituting values into (1)

$\dot{m}_1 =$

$$\frac{1\times10^9\ \frac{\text{Btu}}{\text{h}}}{(1493.5-1251.0)\frac{\text{Btu}}{\text{lb}} + (1-0.187)(1251.0-1006.4)\frac{\text{Btu}}{\text{lb}} - (1-0.187)(96.27-94.02)\frac{\text{Btu}}{\text{lb}} - (339.64-312.7)\frac{\text{Btu}}{\text{lb}}}$$

$$\dot{m}_1 = \underline{\mathbf{2.42\times10^6\ lb/h}}$$

(b) The rate of heat transfer to the working fluid passing through the steam generator can be determined by applying mass and energy balances to a control volume around the steam generator to give

$$\dot{Q}_{in} = \dot{m}_1(h_1 - h_7) = (2.42\times10^6\ \text{lb/h})(1493.5\ \text{Btu/lb} - 339.64\ \text{Btu/lb}) = \underline{\mathbf{2.79\times10^9\ Btu/h}}$$

(c) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = (1\times10^9\ \text{Btu/h})/(2.79\times10^9\ \text{Btu/h}) = \underline{\mathbf{0.3584\ (35.84\%)}}$$

The results of Problem 8.45 can be compared to the results of Problem 8.19 to see some of the effects of incorporating an open feedwater heater on the performance of a Rankine cycle for cycles with the same net power output. In this case, use of an open feedwater heater results in higher thermal efficiency and less heat addition, but the steam flow rate entering the first-stage

Problem 8.45 (Continued) – Page 5

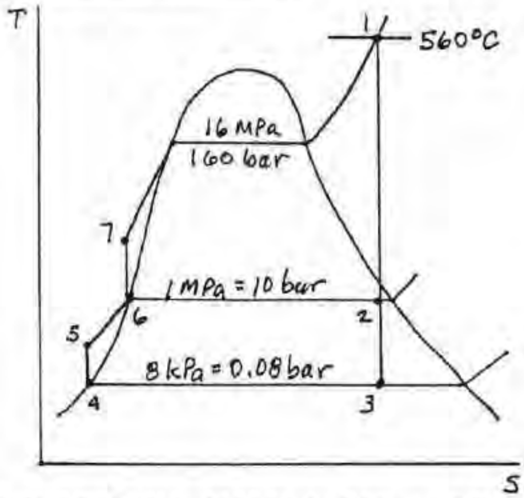
turbine is increased. By comparing the results with those of Prob. 8.44, we see the impact of turbine stage and pump irreversibilities on cycle performance. In this case, irreversibilities result in lower thermal efficiency, increased heat addition, and increased steam flow rate.

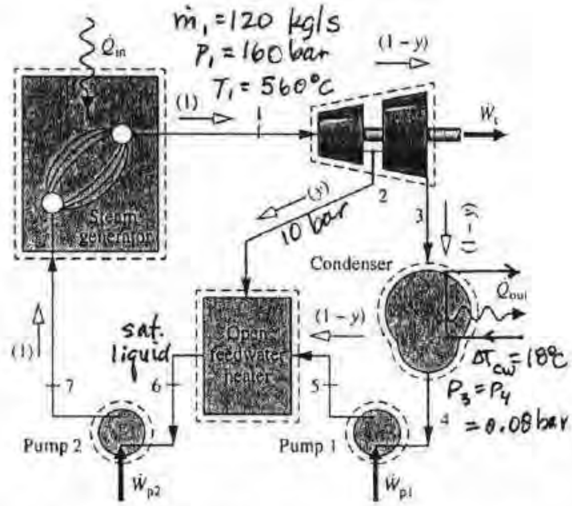
PROBLEM 8.46

KNOWN: Water is the working fluid in an ideal regenerative Rankine cycle with one open feedwater heater. Data at various locations are known. The mass flow rate entering the first-stage turbine is given, and the temperature rise of cooling water passing through the condenser is specified.

FIND: Determine (a) the net power, (b) the rate of heat transfer $\dot{Q}_{in}$, (c) the thermal efficiency, and (d) the mass flow rate of cooling water.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: Same as Example 8.5, except turbine stages and pumps operate in an internally reversible manner.

ANALYSIS: First, fix each principal state.

State 1: $p_1 = 160$ bar, $T_1 = 560°C \Rightarrow h_1 = 3465.4$ kJ/kg, $s_1 = 6.5132$ kJ/kg·K

State 2: $P_2 = 10$ bar, $s_2 = s_1 \Rightarrow x_2 = \frac{s_2 - s_{f2}}{s_{g2} - s_{f2}} = 0.9836$, $h_2 = 2745.1$ kJ/kg

State 3: $P_3 = 0.08$ bar, $s_3 = s_2 \Rightarrow x_3 = \frac{s_3 - s_{f3}}{s_{g3} - s_{f3}} = 0.7753$, $h_3 = 2037.0$ kJ/kg

State 4: $p_4 = 0.08$ bar, sat. liquid $\Rightarrow h_4 = 173.88$ kJ/kg

State 5: $h_5 \approx h_4 + v_4(p_5 - p_4)$

$$= 173.88 \frac{kJ}{kg} + (1.0084 \times 10^{-3}) \frac{m^3}{kg}(10 - 0.08)\,bar \left|\frac{10^5 N/m^2}{1\ bar}\right| \left|\frac{1\ kJ}{10^3 N\cdot m}\right|$$

$$= 173.88 + 1.00 = 174.88 \text{ kJ/kg}$$

State 6: $P_6 = 10$ bar, sat. liquid $\Rightarrow h_6 = 762.81$ kJ/kg

State 7: $h_7 \approx h_6 + v_6(p_7 - p_6) = 762.81 + (1.1273 \times 10^{-3})(160 - 10)\left|\frac{10^5}{10^3}\right| = 779.72 \frac{kJ}{kg}$

(a) For the control volume enclosing the turbine stages

$$\dot{W}_t = \dot{m}_1 [(h_1 - h_2) + (1 - y)(h_2 - h_3)]$$

To get y, apply mass and energy balances the the control volume enclosing the feedwater heater to get

$$y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{762.81 - 174.88}{2745.1 - 174.88} = 0.2287$$

Thus

$$\dot{W}_t = \left(120 \frac{kg}{s}\right)[(3465.4 - 2745.1) + (1 - 0.2287)(2745.1 - 2037.0)] \frac{kJ}{kg} \left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 1.519 \times 10^5 \text{ kW}$$

PROBLEM 8.46 (Cont'd.)

For the pumps

$$\dot{W}_p = \dot{W}_{p_1} + \dot{W}_{p_2} = \dot{m}_1\left[(1-y)(h_5-h_4)+(h_7-h_6)\right]$$

$$= (120)\left[(1-0.2287)(174.88-173.88)+(779.72-762.81)\right]\left|\frac{1}{1}\right|$$

$$= 2122 \text{ kW}$$

Thus, the net power developed is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p = 1.498\times10^5 \text{ kW} \longleftarrow \underline{\dot{W}_{cycle}}$$

(b) For the steam generator

$$\dot{Q}_{in} = \dot{m}_1(h_1-h_7) = (120)(3465.4-779.72)\left|\frac{1}{1}\right| = 3.223\times10^5 \text{ kW} \longleftarrow \underline{\dot{Q}_{in}}$$

(c) The thermal efficiency is $\eta = \dot{W}_{cycle}/\dot{Q}_{in} = 0.465\ (46.5\%) \longleftarrow \underline{\eta}$

(d) The rate of heat transfer from the working fluid to the cooling water passing through the condenser is

$$\dot{Q}_{out} = \dot{m}_1(1-y)(h_3-h_4) = (120)(1-0.2287)(2037.0-173.88)\left|\frac{1}{1}\right| = 1.724\times10^5 \text{ kW}$$

Thus, the mass flow rate of cooling water is

$$\dot{Q}_{out} = \dot{m}_{cw}(h_{out,cw}-h_{in,cw})$$

Assuming $h_{out,cw}-h_{in,cw} = c_{cw}\Delta T_{cw}$, and using $c_{cw} = 4.179\ \frac{kJ}{kg\,K}$, we get

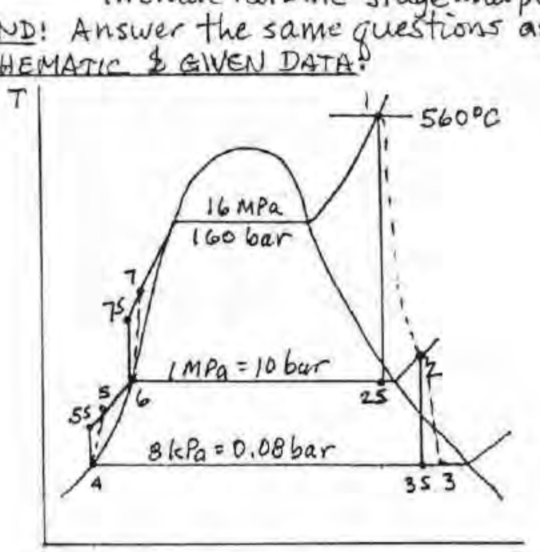
$$\dot{m}_{cw} = \frac{\dot{Q}_{out}}{c_{cw}\Delta T_{cw}} = \frac{1.724\times10^5 \text{ kW}}{(4.179)(18)\ \frac{kJ}{kg}}\left|\frac{1 \text{ kJ/s}}{1 \text{ kW}}\right| = 2292\ \frac{kg}{s} \longleftarrow \underline{\dot{m}_{cw}}$$

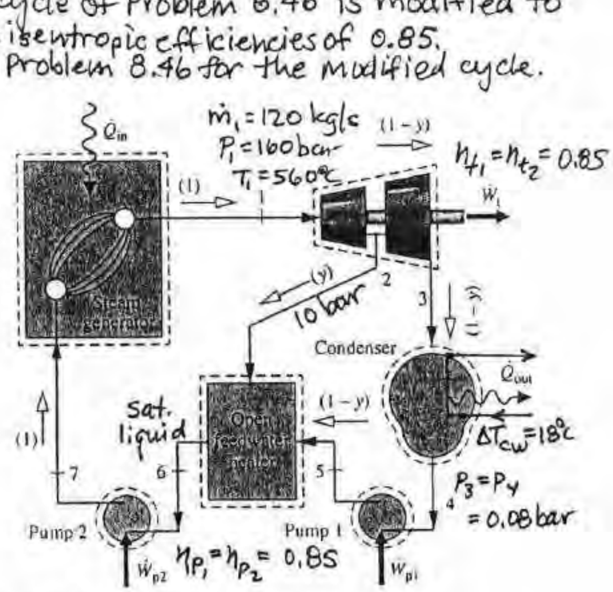
PROBLEM 8.47

KNOWN: The ideal regenerative Rankine cycle of Problem 8.46 is modified to include turbine stage and pump isentropic efficiencies of 0.85.

FIND: Answer the same questions as in Problem 8.46 for the modified cycle.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: Same as Example 8.5.

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 160$ bar, $T_1 = 560°C \Rightarrow h_1 = 3465.1$ kJ/kg, $s_1 = 6.5132$ kJ/kg·K

State 2: Using the isentropic efficiency of the first-stage turbine

$$\eta_{t_1} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_{t_1}(h_1 - h_{2s})$$

With $h_{2s} = 2745.1$ kJ/kg from Problem 8.46, $h_2 = 2853.1$ kJ/kg

State 3: $P_3 = 0.08$ bar, $s_{3s} = s_2 \Rightarrow$ Interpolating in Table A-4; $s_2 = 6.7451$ kJ/kg·K

Thus $x_{3s} = \dfrac{s_{3s} - s_{f3}}{s_{g3} - s_{f3}} = 0.8056$; $h_{3s} = 2109.8$ kJ/kg

Using the second-stage turbine isentropic efficiency

$$h_3 = h_2 - \eta_{t_2}(h_2 - h_{3s}) = 2221.3 \text{ kJ/kg}$$

State 4: $P_4 = 0.08$ bar, sat. liquid $\Rightarrow h_4 = 173.88$ kJ/kg

State 5: Using the isentropic pump efficiency; $\eta_{p_1} = (h_{5s} - h_4)/(h_5 - h_4)$

Thus $h_5 = h_4 + (h_{5s} - h_4)/\eta_{p_1} = 175.06$ kJ/kg

State 6: $P_6 = 10$ bar, sat. liquid $\Rightarrow h_6 = 762.81$ kJ/kg

State 7: With $\eta_{p_2} = (h_{7s} - h_6)/(h_7 - h_6)$; $h_7 = h_6 + (h_{7s} - h_6)/\eta_{p_2} = 782.7$ kJ/kg

(a) $y = \dfrac{h_6 - h_5}{h_2 - h_5} = 0.2195$ and $\dot{W}_t = \dot{m}_1[(h_1 - h_2) + (1 - y)(h_2 - h_3)] = 1.326 \times 10^5$ kW

For the pumps, $\dot{W}_p = \dot{m}_1[(1-y)(h_5 - h_4) + (h_7 - h_6)] = 2497$ kW

Thus $\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p = 1.301 \times 10^5$ kW ← $\dot{W}_{cycle}$

(b) $\dot{Q}_{in} = \dot{m}_1(h_1 - h_7) = 3.219 \times 10^5$ kW ← $\dot{Q}_{in}$

① (c) $\eta = \dot{W}_{cycle}/\dot{Q}_{in} = 0.404$ (40.4%) ← η

(d) $\dot{Q}_{out} = \dot{m}_1 (1-y)(h_3 - h_4) = 1.9176 \times 10^5$ kW

$\dot{m}_{cw} = \dot{Q}_{out}/c_{cw}\Delta T_{cw} = 2549$ kW ← $\dot{m}_{cw}$

1. These results can be compared with those of 8.46 to see some of the effects of turbine stage and pump irreversibilities on cycle performance.

PROBLEM 8.48

(a) Refer to Problem 8.47

IT Code

```
p1 = 160  // bar
T1 = 560  // °C
p2 = 10  // bar
p3 = 0.08 // bar
p4 = p3
p5 = p2
p6 = p5
p7 = p1
mdot1 = 120  // kg/s
Ccw = 4.179  // kJ/kg·K
delTcw = 18

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2s = h_Ps("Water/Steam", p2, s2s)
s2s = s1
s3s = s2s
h3s = h_Ps("Water/Steam", p3, s3s)
h4 = hsat_Px("Water/Steam", p4, 0)
v4 = vsat_Px("Water/Steam", p5, 0)
h5s = h4 + v4 * (p5 - p4) * 100
h6 = hsat_Px("Water/Steam", p6, 0)
v6 = vsat_Px("Water/Steam", p6, 0)
h7s = h6 + v6 * (p7 - p6) * 100

Qdotin = mdot1 * (h1 - h7s)
y = (h6 - h5s) / (h2s - h5s)
Wdott = mdot1 * ((h1 - h2s) + (1 - y) * (h2s - h3s))
Wdotp = mdot1 * ((1 - y) * (h5s - h4) + (h7s - h6))
Wdotcycle = Wdott - Wdotp
eta = Wdotcycle / Qdotin
Qdotout = mdot1 * (1 - y) * (h3s - h4)
mdotcw = Qdotout / (Ccw * delTcw)
```

IT Results (p_2 = 10 bar)

$\dot{Q}_{in}$ = 3.223 x 10^5 kW

$\dot{Q}_{out}$ = 1.724 x 10^5 kW

$\dot{W}_{cycle}$ = 1.498 x 10^5 kW

$\dot{m}_{cw}$ = 2292 kg/s

η = 0.4649

h_1 = 3465 kJ/kg

h_{2s} = 2744 kJ/kg

h_{3s} = 2037 kJ/kg

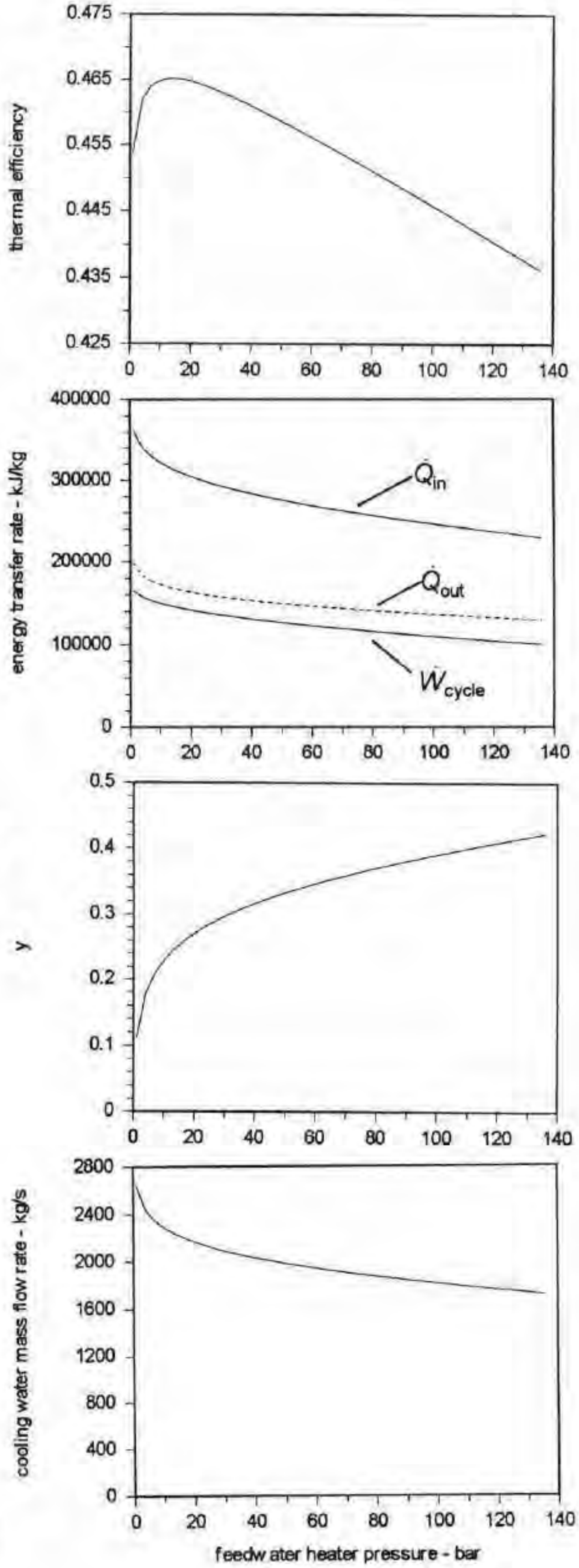
h_4 = 173.6 kJ/kg

h_{5s} = 174.8 kJ/kg

h_6 = 762.5 kJ/kg

h_{7s} = 779.4 kJ/kg

y = 0.2287



The thermal efficiency exhibits a maximum in the range of feedwater heater pressures studied.

PROBLEM 8.48 (Cont'd)

(b) Refer to Problem 8.47

IT Code

```
p1 = 160 // bar
T1 = 560 // °C
p2 = 10 // bar
p3 = 0.08 // bar
p4 = p3
p5 = p2
p6 = p5
p7 = p1
etat = 0.85
etap = etat
mdot1 = 120 // kg/s
Ccw = 4.179 // kJ/kg·K
delTcw = 18

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
s2s = s1
h2s = h_Ps("Water/Steam", p2, s2s)
h2 = h1 - etat * (h1 - h2s)
s2 = s_Ph("Water/Steam", p2, h2)
s3s = s2
h3s = h_Ps("Water/Steam", p3, s3s)
h3 = h2 - etat * (h2 - h3s)
h4 = hsat_Px("Water/Steam", p4, 0)
v4 = vsat_Px("Water/Steam", p5, 0)
h5s = h4 + v4 * (p5 - p4) * 100
h5 = h4 + (h5s - h4) / etap
h6 = hsat_Px("Water/Steam", p6, 0)
v6 = vsat_Px("Water/Steam", p6, 0)
h7s = h6 + v6 * (p7 - p6) * 100
h7 = h6 + (h7s - h6) / etap

Qdotin = mdot1 * (h1 - h7)
y = (h6 - h5s) / (h2 - h5)
Wdott = mdot1 * ((h1 - h2) + (1 - y) * (h2 - h3))
Wdotp = mdot1 * ((1 - y) * (h5 - h4) + (h7 - h6))
Wdotcycle = Wdott - Wdotp
eta = Wdotcycle / Qdotin
Qdotout = mdot1 * (1 - y) * (h3 - h4)
mdotcw = Qdotout / (Ccw * delTcw)
```

IT Results (p_2 = 10 bar)

$\dot{Q}_{in} = 3.219 \times 10^5$ kW

$\dot{Q}_{out} = 1.918 \times 10^5$ kW

$\dot{W}_{cycle} = 1.301 \times 10^5$ kW

$\dot{m}_{cw} = 2550$ kg/s

$\eta = 0.4041$

$h_1 = 3465$ kJ/kg

$h_2 = 2853$ kJ/kg

$h_3 = 2221$ kJ/kg

$h_{3s} = 2110$ kJ/kg

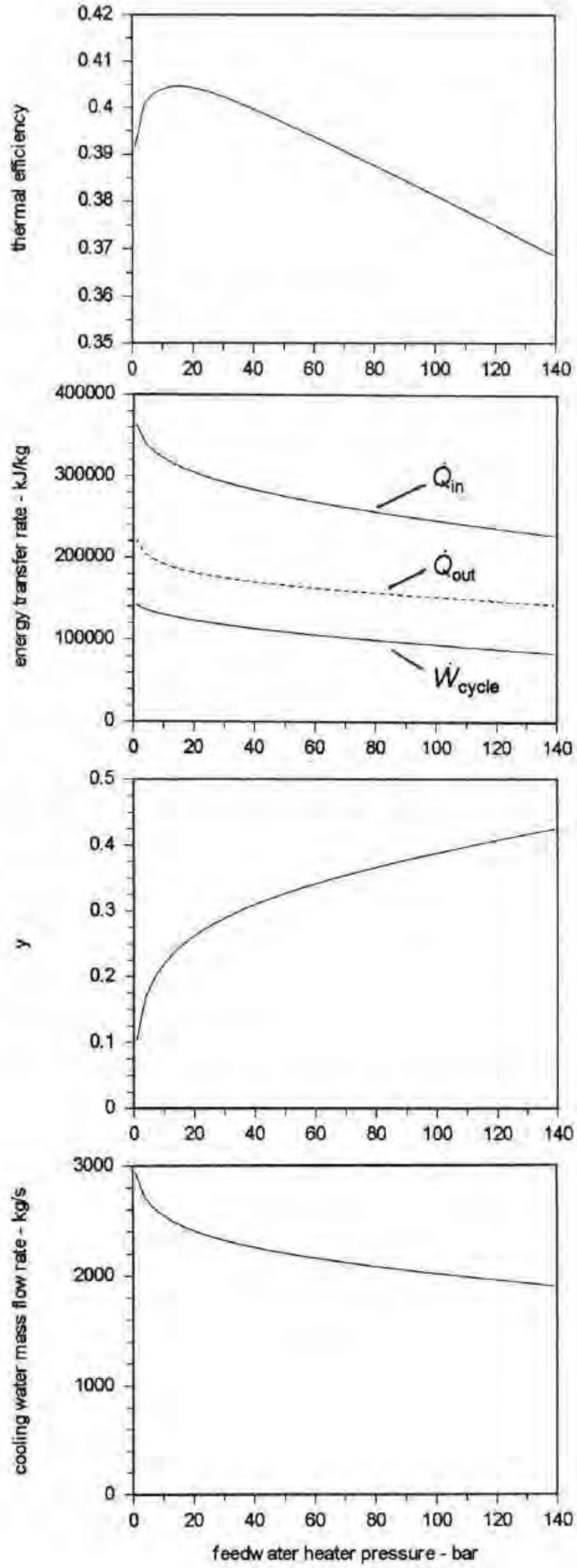
$h_4 = 173.6$ kJ/kg

$h_5 = 175$ kJ/kg

$h_6 = 762.5$ kJ/kg

$h_7 = 782.4$ kJ/kg

The thermal efficiency exhibits a maximum in the range of feedwater heater pressures studied.

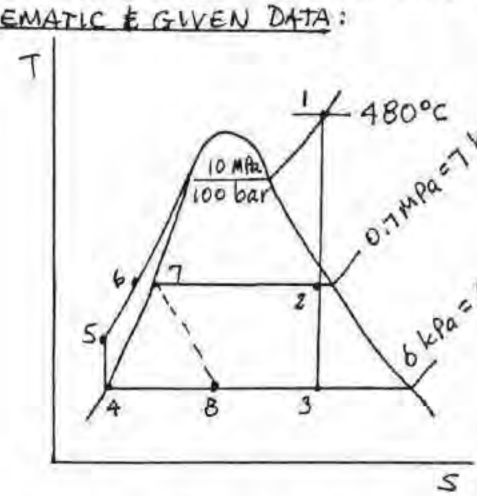


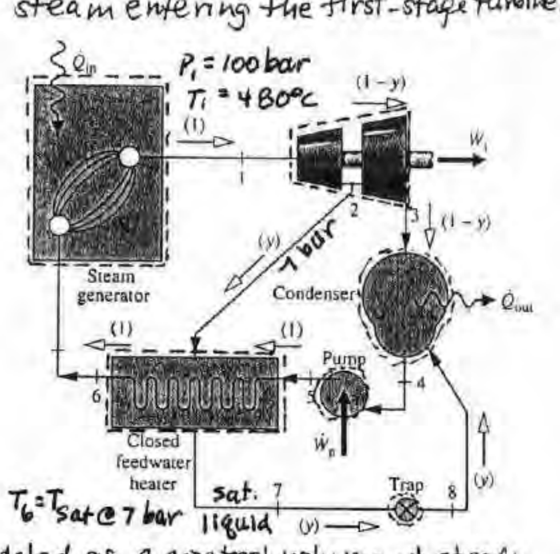
PROBLEM 8.49

KNOWN: Water is the working fluid in an ideal regenerative Rankine cycle with one closed feedwater heater. Data at various locations are known.

FIND: Determine (a) the rate of heat addition per kg of steam entering the first-stage turbine, (b) the thermal efficiency, and (c) the rate of heat transfer for the condenser per kg of steam entering the first-stage turbine.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) Each component is modeled as a control volume at steady state. (2) There are no stray heat transfers, (3) The working fluid undergoes an internally reversible process in passing through each component except the trap. (4) For the trap, $h_7 = h_8$ (throttling process). (5) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each principal state.

State 1: $p_1 = 100\text{bar}, T_1 = 480°C \Rightarrow h_1 = 3321.4 \text{ kJ/kg}, s_1 = 6.5282 \text{ kJ/kg}\cdot\text{K}$

State 2: $p_2 = 7\text{ bar}, s_2 = s_1 \Rightarrow x_2 = \dfrac{s_2 - s_{f2}}{s_{g2} - s_{f2}} = 0.9619, \; h_2 = 2684.8 \text{ kJ/kg}$

State 3: $p_3 = 0.06\text{ bar}, s_3 = s_2 \Rightarrow x_3 = 0.7692, \; h_3 = 2009.8 \text{ kJ/kg}$

State 4: $p_4 = 0.06\text{ bar}$, sat. liquid $\Rightarrow h_4 = 151.53 \text{ kJ/kg}$

State 5: $h_5 \approx h_4 + v_4(p_5 - p_4)$

$$= 151.53 + (1.0064\times10^{-3})\frac{m^3}{kg}(100 - 0.06)\text{bar}\left|\frac{10^5 \text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

$$= 151.53 + 10.06 = 161.59 \text{ kJ/kg}$$

State 6: T_{sat} @ 7 bar $= 165.0°C \Rightarrow h_6 \approx h_f(T_7) = 697.22 \text{ kJ/kg}$

State 7: $p_7 = 7$ bar, sat. liquid $\Rightarrow h_7 = 697.22 \text{ kJ/kg}$

State 8: $h_8 = h_7 = 697.22 \text{ kJ/kg}$

(a) For the steam generator

$$\dot{Q}_{in}/\dot{m}_1 = h_1 - h_6 = (3321.4 - 697.22) = 2624 \text{ kJ/kg} \leftarrow \dot{Q}_{in}/\dot{m}_1$$

(b) Applying mass and energy balances to the control volume enclosing the closed feed water heater

$$y = \frac{h_6 - h_5}{h_2 - h_7} = \frac{697.22 - 161.59}{2684.8 - 697.22} = 0.2695$$

For a control volume enclosing the turbine stages

$$\dot{W}_t/\dot{m}_1 = (h_1 - h_2) + (1-y)(h_2 - h_3)$$

$$= (3321.4 - 2684.8) + (1 - 0.2695)(2684.8 - 2009.8) = 1129.7 \text{ kJ/kg}$$

For the pump

$$\dot{W}_p/\dot{m}_1 = h_5 - h_4 = 161.59 - 151.53 = 10.06 \text{ kJ/kg}$$

PROBLEM 8.49 (Cont'd.)

The net power developed, per unit mass entering the first-stage turbine, is

$$\frac{\dot{W}_{cycle}}{\dot{m}_1} = \frac{\dot{W}_t}{\dot{m}_1} - \frac{\dot{W}_p}{\dot{m}_1} = 1129.7 - 10.06 = 1119.6 \text{ kJ/kg}$$

And the thermal efficiency is

$$\eta = \dot{W}_{cycle}/\dot{Q}_{in} = 1119.6 / 2624 = 0.427 \; (42.7\%) \qquad \eta$$

(c) For the condenser

$$\frac{\dot{Q}_{out}}{\dot{m}_1} = (1-y)\,h_3 + y\,h_8 - h_4$$

$$= (1-0.2695)(3321.4) + (0.2695)(697.22) - 151.53$$

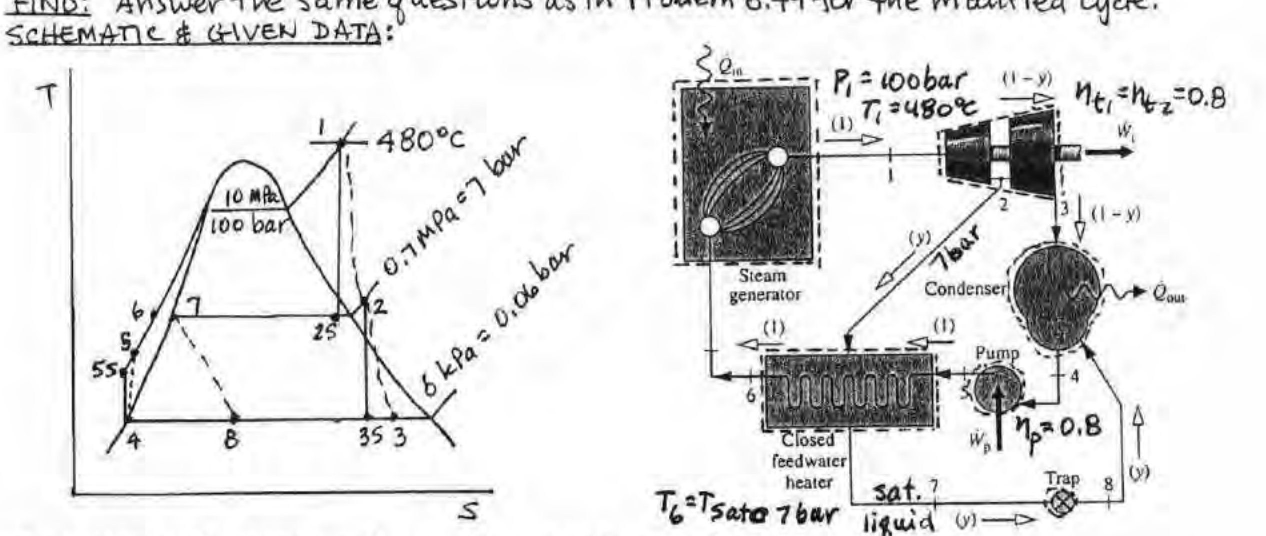
$$= 1504.1 \text{ kJ/kg} \qquad \dot{Q}_{out}/\dot{m}_1$$

PROBLEM 8.50

KNOWN: The ideal regenerative Rankine cycle of Problem 8.49 is modified to include turbine stage and pump isentropic efficiencies of 0.8.

FIND: Answer the same questions as in Problem 8.49 for the modified cycle.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Problem 8.49, except $\eta_{t_1} = \eta_{t_2} = \eta_p = 0.8$.

ANALYSIS: First, fix each principal state.

State 1: $p_1 = 100$ bar, $T_1 = 480°C \Rightarrow h_1 = 3321.4$ kJ/kg, $s_1 = 6.5282$ kJ/kg·K

State 2: Using $\eta_{t_1} = (h_1 - h_2)/(h_1 - h_{2s}) \Rightarrow h_2 = h_1 - \eta_{t_1}(h_1 - h_{2s})$

With $h_{2s} = 2684.8$ kJ/kg from Problem 8.49, $h_2 = 2812.1$ kJ/kg

State 3: $p_3 = 0.06$ bar, $s_{3s} = s_2$. To get s_2, interpolate in Table A-3: $s_2 = 6.810 \frac{kJ}{kg}$

Thus $x_{3s} = \dfrac{s_{3s} - s_{f3}}{s_{g3} - s_{f3}} = 0.8061$; $h_{3s} = 2099.0$ kJ/kg

Using η_{t_2}: $h_3 = h_2 - \eta_{t_2}(h_2 - h_{3s}) = 2241.6$ kJ/kg

State 4: $p_4 = 0.06$ bar, sat. liquid $\Rightarrow h_4 = 151.53$ kJ/kg

State 5: Using $\eta_p = (h_{5s} - h_4)/(h_5 - h_4)$ and $h_{5s} = 161.59 \frac{kJ}{kg}$ from Problem 8.49

$h_5 = h_4 + (h_{5s} - h_4)/\eta_p = 164.11$ kJ/kg

State 6: $h_6 \approx h_f(T_7) = 697.22$ kJ/kg

State 7: $p_7 = 7$ bar, sat. liquid $\Rightarrow h_7 = 697.22$ kJ/kg

State 8: $h_8 = h_7 = 697.22$ kJ/kg

(a) $\dot{Q}_{in}/\dot{m}_1 = h_1 - h_6 = 2624$ kJ/kg ← $\dot{Q}_{in}/\dot{m}_1$

(b) $y = \dfrac{h_6 - h_5}{h_2 - h_7} = \dfrac{697.22 - 164.11}{2812.1 - 697.22} = 0.2521$

$\dot{W}_t/\dot{m}_1 = (h_1 - h_2) + (1-y)(h_2 - h_3) = 935.98$ kJ/kg

$\dot{W}_p/\dot{m}_1 = h_5 - h_4 = 12.58$ kJ/kg

$\dot{W}_{cycle}/\dot{m}_1 = \dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1 = 923.4$ kJ/kg

(1) $\eta = \dfrac{\dot{W}_{cycle}/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \dfrac{923.4}{2624} = 0.352\ (35.2\%)$ ← η

(c) $\dot{Q}_{out}/\dot{m}_1 = (1-y)h_3 + y h_8 - h_4 = 1700.7$ kJ/kg ← $\dot{Q}_{out}/\dot{m}_1$

1. These results can be compared with those of Problem 8.49 to see some of the effects of turbine stage and pump irreversibilities on cycle performance.

PROBLEM 8.31

See Problem 8.50.

IT Code

```
p1 = 100  // bar
T1 = 480  // °C
p2 = 7  // bar
p3 = 0.06 // bar
p4 = p3
p5 = p1
p6 = p1
p7 = p2
p8 = p3
etat = 0.8
etap = etat
mdot1 = 1

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
s2s = s1
h2s = h_Ps("Water/Steam", p2, s2s)
h2 = h1 - etat * (h1 - h2s)
s2 = s_Ph("Water/Steam", p2, h2)
s3s = s2
h3s = h_Ps("Water/Steam", p3, s3s)
h3 = h2 - etat * (h2 - h3s)
h4 = hsat_Px("Water/Steam", p4, 0)
v4 = vsat_Px("Water/Steam", p4, 0)
h5s = h4 + v4 * (p5 - p4) * 100
h5 = h4 + (h5s - h4) / etap
h6 = hsat_Px("Water/Steam", p2, 0)
h7 = hsat_Px("Water/Steam", p7, 0)
h8 = h7

Qdotin = mdot1 * (h1 - h6)
y = (h6 - h5) / (h2 - h7)
Wdott = mdot1 * ((h1 - h2) + (1 - y) * (h2 - h3))
Wdotp = mdot1 * (h5 - h4)
Wdotcycle = Wdott - Wdotp
eta = Wdotcycle / Qdotin
Qdotout = mdot1 * ((1 - y) * h3  + y * h8 - h4)
```

IT Results (p_2 = 7 bar)

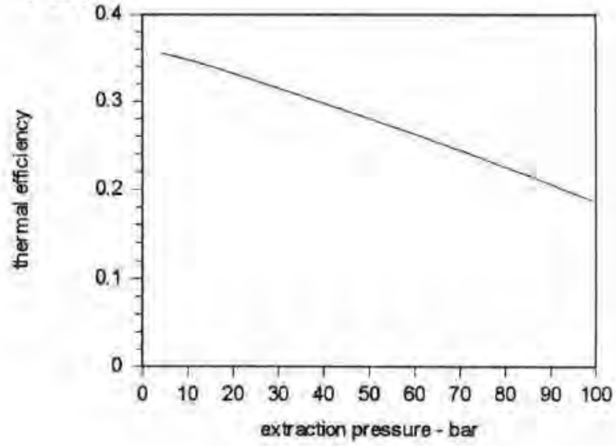
$\dot{Q}_{in} / \dot{m}_1$ = 2624 kJ/kg

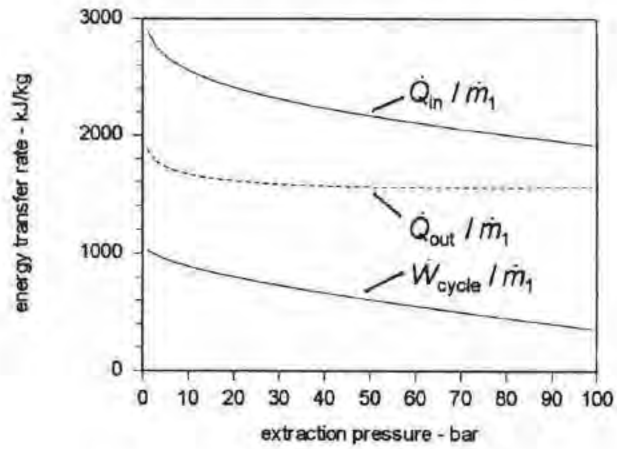
$\dot{Q}_{out} / \dot{m}_1$ = 1701 kJ/kg

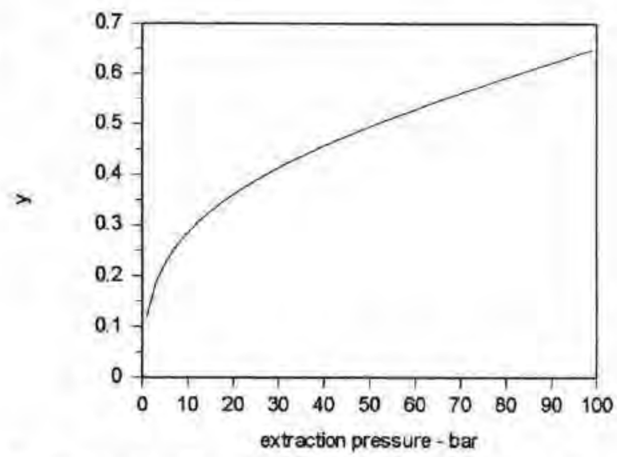
$\dot{W}_{cycle} / \dot{m}_1$ = 923.4 kJ/kg

η = 0.3519
h_1 = 3321 kJ/kg
h_2 = 2812 kJ/kg
h_3 = 2241 kJ/kg
h_4 = 151 kJ/kg
h_5 = 163.6 kJ/kg
h_6 = 696.8 kJ/kg
h_7 = 696.8 kJ/kg
h_8 = 696.8 kJ/kg
y = 0.2521

PLOTS







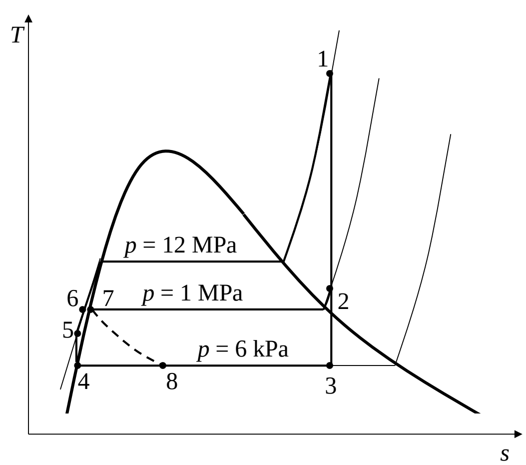
From the thermal efficiency plot, we see that η decreses considerably as the extraction pressure increases.

8.52 As indicated in Fig. P8.52, a power plant similar to that in Fig. 8.11 operates on a regenerative vapor power cycle with one closed feedwater heater. Steam enters the first turbine stage at state 1 where pressure is 12 MPa and temperature is 560°C. Steam expands to state 2 where pressure is 1 MPa and some of the steam is extracted and diverted to the closed feedwater heater. Condensate exits the feedwater heater at state 7 as saturated liquid at a pressure of 1 MPa, undergoes a throttling process through a trap to a pressure of 6 kPa at state 8, and then enters the condenser. The remaining steam expands through the second turbine stage to a pressure of 6 kPa at state 3 and then enters the condenser. Saturated liquid feedwater exiting the condenser at state 4 at a pressure of 6 kPa enters a pump and exits the pump at a pressure of 12 MPa. The feedwater then flows through the closed feedwater heater, exiting at state 6 with a pressure of 12 MPa. The net power output for the cycle is 330 MW. For isentropic processes in each turbine stage and the pump, determine
(a) the cycle thermal efficiency.
(b) the mass flow rate into the first turbine stage, in kg/s.
(c) the rate of entropy production in the closed feedwater heater, in kW/K.
(d) the rate of entropy production in the steam trap, in kW/K.

KNOWN: A regenerative vapor power cycle with one closed feedwater heater operates with steam as the working fluid. Operational data are provided.

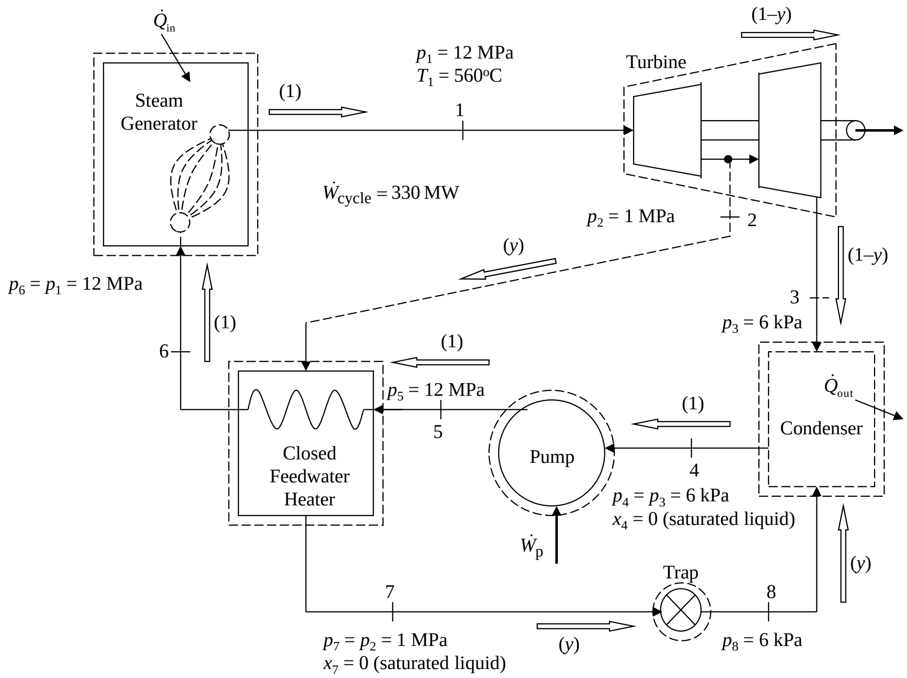
FIND: Determine (a) the cycle thermal efficiency, (b) the mass flow rate into the first turbine stage, in kg/s, (c) the rate of entropy production in the closed feedwater heater, in kW/K, and (d) the rate of entropy production in the steam trap, in kW/K.

SCHEMATIC AND GIVEN DATA:



State	p (kPa)	T (°C)	h (kJ/kg)	s (kJ/kg·K)	x
1	12,000	560	3506.2	6.6840	
2	1,000		2823.3	6.6840	
3	6		2058.2	6.6840	0.7892
4	6		151.53	0.5210	0
5	12,000		163.60	0.5210	
6	12,000		606.61	1.7808	
7	1,000		762.81	2.1387	0
8	6		762.81	2.4968	0.2530

P8.52



ENGINEERING MODEL:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. All processes of the working fluid are internally reversible except for heat transfer through a finite temperature difference in the closed feedwater heater and throttling through the trap.
3. The turbines, pump, closed feedwater heater, and steam trap operate adiabatically.
4. Kinetic and potential energy effects are negligible.
5. Saturated liquid exits the closed feedwater heater, and saturated liquid exits the condenser.

ANALYSIS:

(a) Applying energy and mass balances to the control volume enclosing the closed feedwater heater, the fraction of flow, y, extracted at location 2 is

$$y = \frac{h_6 - h_5}{h_2 - h_7} = \frac{(606.61 - 163.60)\text{ kJ/kg}}{(2823.3 - 762.81)\text{ kJ/kg}} = 0.2150$$

For the control volume surrounding the turbine stages

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1 - y)(h_2 - h_3)$$

$$\frac{\dot{W}_t}{\dot{m}_1} = (3506.2 - 2823.3)\frac{\text{kJ}}{\text{kg}} + (1 - 0.2150)(2823.3 - 2058.2)\frac{\text{kJ}}{\text{kg}} = 1283.5 \text{ kJ/kg}$$

For the pump

$$\frac{\dot{W}_{\mathrm{p}}}{\dot{m}_1} = (h_5 - h_4)$$

$$\frac{\dot{W}_{\mathrm{p}}}{\dot{m}_1} = (163.60 - 151.53)\frac{\mathrm{kJ}}{\mathrm{kg}} = 12.07 \text{ kJ/kg}$$

For the working fluid passing through the steam generator

$$\frac{\dot{Q}_{\mathrm{in}}}{\dot{m}_1} = h_1 - h_6 = (3506.2 - 606.61)\frac{\mathrm{kJ}}{\mathrm{kg}} = 2899.6 \text{ kJ/kg}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_{\mathrm{t}}/\dot{m}_1 - \dot{W}_{\mathrm{p}}/\dot{m}_1}{\dot{Q}_{\mathrm{in}}/\dot{m}_1} = \frac{(1283.5 - 12.07) \text{ kJ/kg}}{2899.6 \text{ kJ/kg}} = \underline{\mathbf{0.438\ (43.8\%)}}$$

(b) The ***net*** power developed is

$$\dot{W}_{\mathrm{cycle}} = \dot{m}_1(\dot{W}_{\mathrm{t}}/\dot{m}_1 - \dot{W}_{\mathrm{p}}/\dot{m}_1)$$

Thus,

$$\dot{m}_1 = \frac{\dot{W}_{\mathrm{cycle}}}{(\dot{W}_{\mathrm{t}}/\dot{m}_1 - \dot{W}_{\mathrm{p}}/\dot{m}_1)}$$

$$\dot{m}_1 = \frac{330 \text{ MW}}{(1283.5 - 12.07)\frac{\mathrm{kJ}}{\mathrm{kg}}}\left|\frac{1000\frac{\mathrm{kJ}}{\mathrm{s}}}{1 \text{ MW}}\right| = \underline{\mathbf{259.6\ kg/s}}$$

(c) The rate of entropy production in the closed feedwater heater is determined using the steady-state form of the entropy rate balance:

$$0 = \cancel{\sum_j \frac{\dot{Q}_j}{T_j}} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{\mathrm{cv}}$$

Since the feedwater heater is adiabatic, the heat transfer term drops. Thus,

$$\dot{\sigma}_{cv} = \sum_e \dot{m}_e s_e - \sum_i \dot{m}_i s_i = \dot{m}_6 s_6 + \dot{m}_7 s_7 - \dot{m}_5 s_5 - \dot{m}_2 s_2$$

$$\dot{\sigma}_{cv} = \dot{m}_1[s_6 - s_5 + y(s_7 - s_2)]$$

$$\dot{\sigma}_{cv} = 259.6\frac{\text{kg}}{\text{s}}[1.7808 - 0.5210 + (0.2150)(2.1387 - 6.6840)]\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right| = \mathbf{\underline{73.35\ kW/K}}$$

Heat transfer between a finite temperature difference within the closed feedwater heater is a source of irreversibility that produces entropy.

(d) The rate of entropy production in the steam trap is determined using the one-inlet, one-exit, steady-state form of the entropy rate balance:

$$0 = \cancel{\sum_j \frac{\dot{Q}_j}{T_j}} + \dot{m}(s_i - s_e) + \dot{\sigma}_{cv}$$

where $\dot{m}$ is the mass flow rate through the steam trap.

Since the steam trap is adiabatic, the heat transfer term drops. Thus,

$$\dot{\sigma}_{cv} = \dot{m}(s_e - s_i) = \dot{m}_7(s_8 - s_7) = y\dot{m}_1(s_8 - s_7)$$

$$\dot{\sigma}_{cv} = (0.2150)\left(259.6\frac{\text{kg}}{\text{s}}\right)(2.4968 - 2.1387)]\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right| = \mathbf{\underline{19.99\ kW/K}}$$

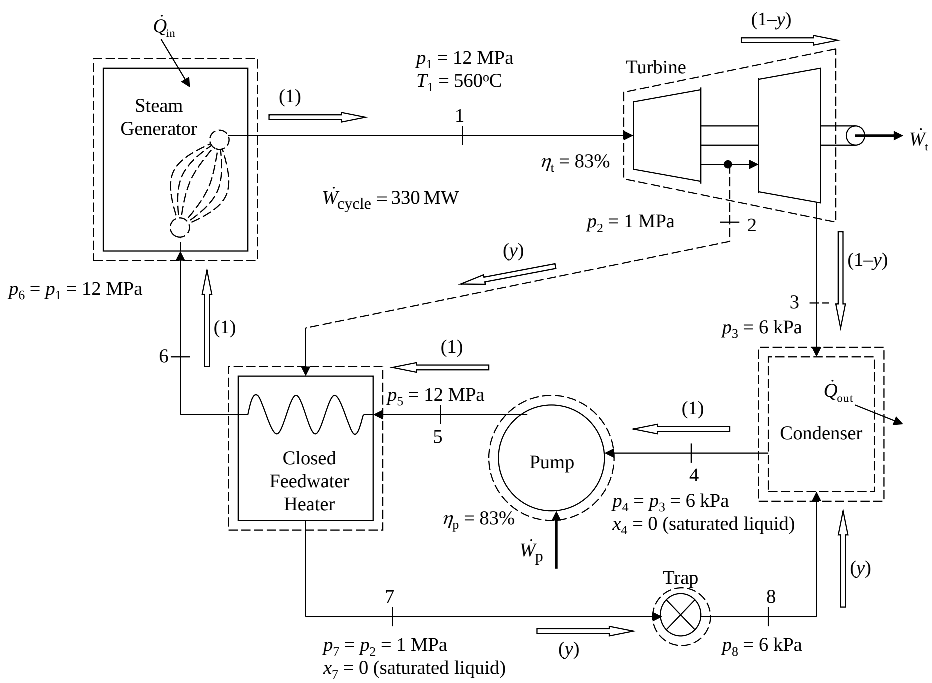
The throttling process in the steam trap is a source of irreversibility that produces entropy.

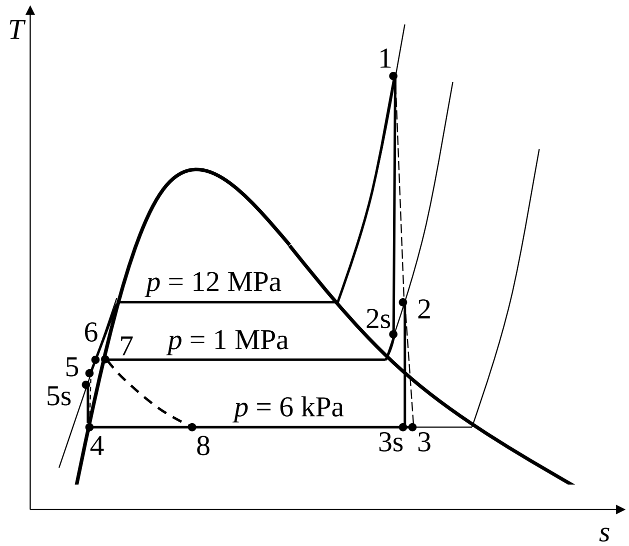
8.53 Reconsider the cycle of Problem 8.52, but include in the analysis that each turbine stage and the pump have isentropic efficiencies of 83%.

KNOWN: A regenerative vapor power cycle with one closed feedwater heater operates with steam as the working fluid. Operational data are provided.

FIND: Determine (a) the cycle thermal efficiency, (b) the mass flow rate into the first turbine stage, in kg/s, (c) the rate of entropy production in the closed feedwater heater, in kW/K, and (d) the rate of entropy production in the steam trap, in kW/K.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. All processes of the working fluid are internally reversible except for processes in the turbines and pumps, heat transfer through a finite temperature difference in the closed feedwater heater, and throttling in the steam trap.
3. The turbines, pump, closed feedwater heater, and steam trap operate adiabatically.
4. Kinetic and potential energy effects are negligible.
5. Saturated liquid exits the closed feedwater heater, and saturated liquid exits the condenser.

ANALYSIS: First fix each principal state.

State 1 (same as State 1 in problem 8.52): p_1 = 12 MPa (120 bar), T_1 = 560°C → h_1 = 3506.2 kJ/kg, s_1 = 6.6840 kJ/kg·K

State 2s (same as State 2 in problem 8.52) : p_{2s} = p_2 = 1 MPa (10 bar), s_{2s} = s_1 = 6.6840 kJ/kg·K → h_{2s} = 2823.3 kJ/kg

State 2: p_2 = 1 MPa (10 bar), h_2 = 2939.4 kJ/kg (see below) → s_2 = 6.9174 kJ/kg·K

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \rightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 3506.2\frac{\text{kJ}}{\text{kg}} - (0.83)(3506.2 - 2823.3)\frac{\text{kJ}}{\text{kg}} = 2939.4 \text{ kJ/kg}$$

State 3s: p_{3s} = p_3 = 6 kPa (0.06 bar), s_{3s} = s_2 = 6.9174 kJ/kg·K → x_{3s} = 0.8191, h_{3s} = 2130.4 kJ/kg

State 3: p_3 = 6 kPa (0.06 bar), h_3 = 2267.9 kJ/kg (see below) → x_3 = 0.8760, s_3 = 7.3620 kJ/kg·K

$$\eta_t = \frac{h_2 - h_3}{h_2 - h_{3s}} \rightarrow h_3 = h_2 - \eta_t(h_2 - h_{3s}) = 2939.4\frac{\text{kJ}}{\text{kg}} - (0.83)(2939.4 - 2130.4)\frac{\text{kJ}}{\text{kg}} = 2267.9 \text{ kJ/kg}$$

<u>State 4 (same as State 4 in problem 8.52)</u>: p_4 = 6 kPa (0.06 bar), saturated liquid → h_4 = 151.53 kJ/kg, v_4 = 0.0010064 m^3/kg, s_4 = 0.5210 kJ/kg·K

<u>State 5</u>: p_5 = 1 MPa (10 bar), h_5 = 166.07 kJ/kg (see below) → s_5 ≈ 0.5677 kJ/kg·K (***assuming the saturated liquid state corresponding to $h_5 = h_f$ in Table 2 and interpolating for $s_5 = s_f$***)

$$\eta_p = \frac{v_4(p_5 - p_4)}{h_5 - h_4} \rightarrow h_5 = h_4 + \frac{v_4(p_5 - p_4)}{\eta_p}$$

$$h_5 = 151.53\frac{\text{kJ}}{\text{kg}} + \frac{(0.0010064\frac{\text{m}^3}{\text{kg}})(12000 - 6)\text{ kPa}}{0.83}\left|\frac{1000\frac{\text{N}}{\text{m}^2}}{1\text{kPa}}\right|\left|\frac{1\text{ kJ}}{1000\text{ N}\cdot\text{m}}\right| = 166.07 \text{ kJ/kg}$$

<u>State 6 (same as State 6 in problem 8.52)</u>: p_6 = 12 MPa (120 bar), h_6 = 606.61 kJ/kg, s_6 = 1.7808 kJ/kg·K

<u>State 7 (same as State 7 in problem 8.52)</u>: p_7 = 1 MPa (10 bar), saturated liquid → h_7 = 762.81 kJ/kg, s_7 = 2.1387 kJ/kg·K

<u>State 8 (same as State 8 in problem 8.52)</u>: p_8 = 6 kPa (0.06 bar), h_8 =h_7 = 762.81 kJ/kg → s_8 = 2.4968 kJ/kg·K

(a) Applying energy and mass balances to the control volume enclosing the closed feedwater heater, the fraction of flow, y, extracted at location 2 is

$$y = \frac{h_6 - h_5}{h_2 - h_7} = \frac{(606.61 - 166.07)\text{ kJ/kg}}{(2939.4 - 762.81)\text{ kJ/kg}} = 0.2024$$

For the control volume surrounding the turbine stages

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1 - y)(h_2 - h_3)$$

$$\frac{\dot{W}_t}{\dot{m}_1} = (3506.2 - 2939.4)\frac{\text{kJ}}{\text{kg}} + (1 - 0.2024)(2939.4 - 2267.9)\frac{\text{kJ}}{\text{kg}} = 1102.4 \text{ kJ/kg}$$

For the pump

$$\frac{\dot{W}_p}{\dot{m}_1} = (h_5 - h_4)$$

$$\frac{\dot{W}_p}{\dot{m}_1} = (166.07 - 151.53)\frac{\text{kJ}}{\text{kg}} = 14.54 \text{ kJ/kg}$$

For the working fluid passing through the steam generator

$$\frac{\dot{Q}_{in}}{\dot{m}_1} = h_1 - h_6 = (3506.2 - 606.61)\frac{\text{kJ}}{\text{kg}} = 2899.6 \text{ kJ/kg}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{(1102.4 - 14.54)\text{ kJ/kg}}{2899.6 \text{ kJ/kg}} = \underline{\mathbf{0.375\ (37.5\%)}}$$

(b) The ***net*** power developed is

$$\dot{W}_{cycle} = \dot{m}_1(\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1)$$

Thus,

$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{(\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1)}$$

$$\dot{m}_1 = \frac{330\text{ MW}}{(1102.4 - 14.54)\frac{\text{kJ}}{\text{kg}}}\left|\frac{1000\frac{\text{kJ}}{\text{s}}}{1\text{ MW}}\right| = \underline{\mathbf{303.3\ kg/s}}$$

(c) The rate of entropy production in the closed feedwater heater is determined using the steady-state form of the entropy rate balance:

$$0 = \cancel{\sum_j \frac{\dot{Q}_j}{T_j}} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{cv}$$

Since the feedwater heater is adiabatic, the heat transfer term drops. Thus,

$$\dot{\sigma}_{cv} = \sum_e \dot{m}_e s_e - \sum_i \dot{m}_i s_i = \dot{m}_6 s_6 + \dot{m}_7 s_7 - \dot{m}_5 s_5 - \dot{m}_2 s_2$$

$$\dot{\sigma}_{cv} = \dot{m}_1[s_6 - s_5 + y(s_7 - s_2)]$$

$$\dot{\sigma}_{cv} = 303.3\frac{\text{kg}}{\text{s}}[1.7808 - 0.5677 + (0.2024)(2.1387 - 6.9174)]\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right| = \underline{\mathbf{74.58\ kW/K}}$$

(d) The rate of entropy production in the steam trap is determined using the one-inlet, one-exit, steady-state form of the entropy rate balance:

$$0 = \cancel{\sum_j \frac{\dot{Q}_j}{T_j}} + \dot{m}(s_i - s_e) + \dot{\sigma}_{cv}$$

where $\dot{m}$ is the mass flow rate through the steam trap.

Since the steam trap is adiabatic, the heat transfer term drops. Thus,

$$\dot{\sigma}_{cv} = \dot{m}(s_e - s_i) = \dot{m}_7(s_8 - s_7) = y\dot{m}_1(s_8 - s_7)$$

$$\dot{\sigma}_{cv} = (0.2024)\left(303.3\frac{\text{kg}}{\text{s}}\right)(2.4968 - 2.1387)]\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right| = \underline{\mathbf{21.98\ kW/K}}$$

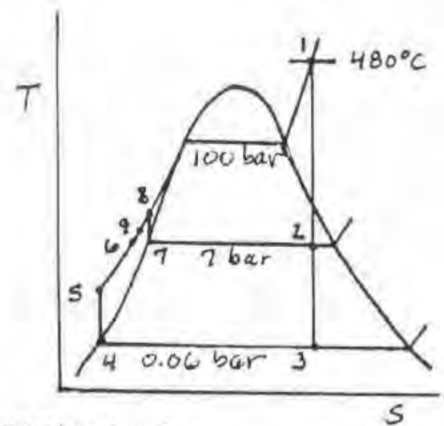
Compared to the ideal cycle in problem 8.52, the presence of internal irreversibilities in the turbine stages and the pump results in lower cycle thermal efficiency, higher required mass flow rate of steam entering the first-stage turbine, and greater rate of entropy production in the closed feedwater heater. Although the inlet and exit states for the steam trap are the same as those in Problem 8.52, the rate of entropy production during the throttling process is greater since the mass flow rate is higher.

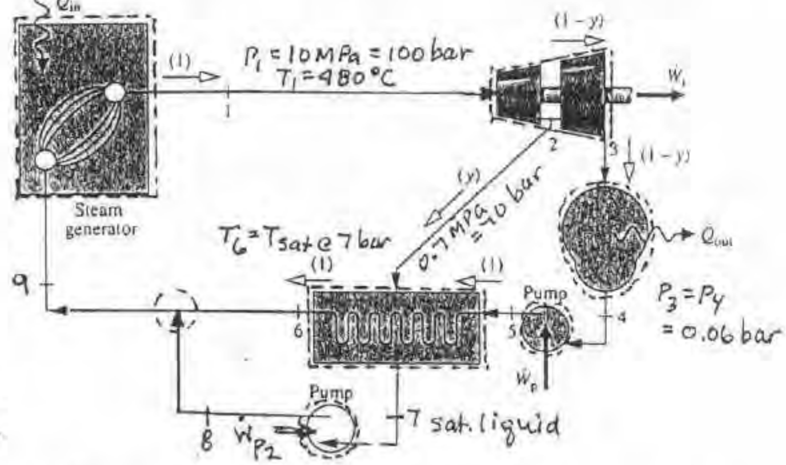
PROBLEM 8.54

KNOWN: The modified regenerative cycle in Problem 8.49 is reconsidered. The condensate is pumped into the boiler feedwater line.

FIND: Answer the same questions about the cycle with this further modification as in Problem 8.49. Compare the two condensate options.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the working fluid are internally reversible, except for the mixing of streams 8 and 6 to form 9. (3) Other assumptions are as in Problem 8.49.

ANALYSIS: First, fix each of the principal states. States 1-7 are the same as in Problem 8.49. Summarizing

$h_1 = 3321.4$ kJ/kg	$h_5 = 161.59$
$h_2 = 2684.8$	$h_6 = 697.22$
$h_3 = 2009.8$	$h_7 = 697.22$
$h_4 = 151.53$	

State 8: $h_8 \approx h_7 + v_7(p_8 - p_7)$

$$= 697.22\,\frac{kJ}{kg} + (1.108\times10^{-3})\,\frac{m^3}{kg}(100-7)\,bar\left|\frac{10^5\,N/m^2}{1\,bar}\right|\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|$$

$$= 697.22 + 10.30 = 707.52\ kJ/kg$$

State 9: First, applying mass and energy rate balances to the closed feedwater heater, the fraction of flow extracted at 2 is obtained as follows

$$0 = (1-y)(h_5 - h_6) + y(h_2 - h_7)$$

or

$$y = \frac{(h_6 - h_5)}{(h_2-h_7)+(h_6-h_5)} = \frac{(697.22-161.59)}{(2684.8-697.22)+(697.22-161.59)} = 0.2123$$

Now, for the control volume enclosing the mixing process

$$0 = (1-y)\,h_6 + y\,h_8 - h_9$$

or

$$h_9 = (1-y)\,h_6 + y\,h_8 = (0.7877)(697.22) + (.2123)(707.52)$$
$$= 699.41\ kJ/kg$$

(a) The heat transfer rate $\dot{Q}_{in}/\dot{m}_1$ is

$$\dot{Q}_{in}/\dot{m}_1 = h_1 - h_9 = (3321.4 - 699.41)\ kJ/kg = 2622\ kJ/kg \longleftarrow \dot{Q}_{in}/\dot{m}_1$$

PROBLEM 8.54 (Cont'd.)

(b) For the turbines

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1-y)(h_2 - h_3)$$
$$= (3321.4 - 2684.8) + (0.7877)(2684.8 - 2009.8) = 1168.3 \text{ kJ/kg}$$

And, for the pumps

$$\frac{\dot{W}_p}{\dot{m}_1} = (1-y)(h_5 - h_4) + y(h_8 - h_7)$$
$$= (0.7877)(161.59 - 151.53) + (0.2123)(707.52 - 697.22)$$
$$= 10.11 \text{ kJ/kg}$$

Thus $\frac{\dot{W}_{cycle}}{\dot{m}_1} = \frac{\dot{W}_t}{\dot{m}_1} - \frac{\dot{W}_p}{\dot{m}_1} = 1168.3 - 10.11 = 1158.2 \text{ kJ/kg}$

And $\eta = \frac{\dot{W}_{cycle}/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{1158.2}{2622} = 0.442 \ (44.2\%)$ ← η

(c) For the condenser

$$\frac{\dot{Q}_{out}}{\dot{m}_1} = (1-y)(h_3 - h_4) = (0.7877)(2009.8 - 151.53)$$
$$= 1463.8 \text{ kJ/kg}$$ ← $\dot{Q}_{out}/\dot{m}_1$

Discussion: Pumping the condensate into the boiler feedwater line has advantages over trapping it into the condenser. The fraction of steam y extrated at 2 is lower, resulting in greater work per unit mass flow through the turbine. Since the power is constant, less flow is required. This results in lower heat input and higher thermal efficiency. The pumping option also eliminates the irreversibility due to the trapping valve. Finally, the pumping option is somewhat more complex in terms of piping and requires an additional pump in the system with associated maintenance needs.

8.55 Water is the working fluid in an ideal regenerative Rankine cycle with one closed feedwater heater. Steam enters the turbine at 1400 lbf/in.^{2} and 1000°F and expands to 120 lbf/in.^{2}, where some of the steam is extracted and diverted to the closed feedwater heater. The remaining steam expands through the second-stage turbine to the condenser pressure of 2 lbf/in.^{2} Condensate exiting the feedwater heater as saturated liquid at 120 lbf/in.^{2} undergoes a throttling process as it passes through a trap into the condenser. The feedwater leaves the heater at 1400 lbf/in.^{2} and a temperature equal to the saturation temperature at 120 lbf/in.^{2} The net power output of the cycle is 1×10^9 Btu/h. Determine for the cycle

(a) the mass flow rate of steam entering the first stage of the turbine, in lb/h.

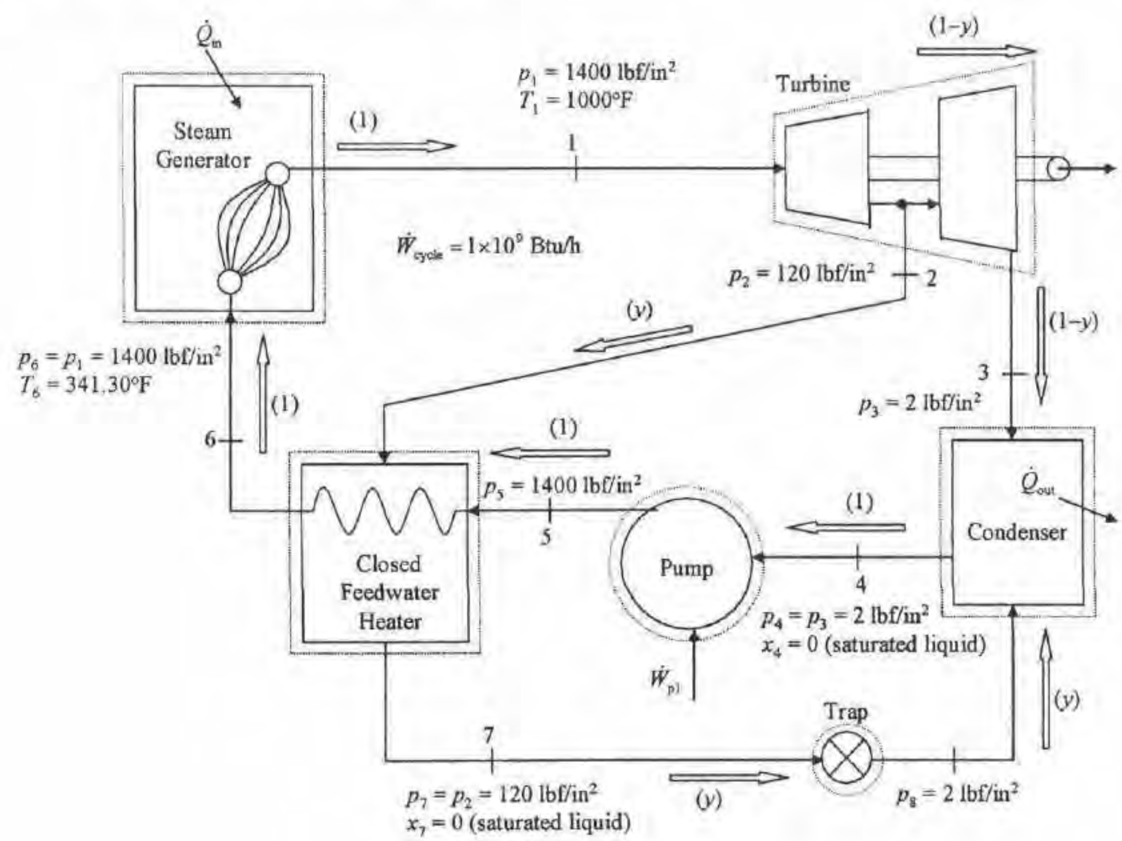
(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the steam generator.

(c) the thermal efficiency.

KNOWN: An ideal regenerative Rankine cycle with one closed feedwater heater and superheat operates with water as the working fluid. The net power output of the cycle is given.

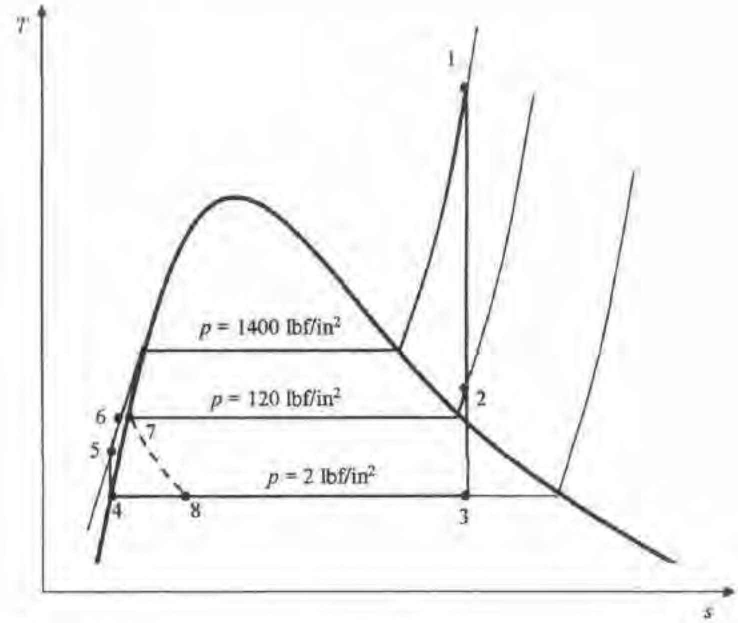
FIND: Determine the mass flow rate of steam entering the first stage of the turbine, the rate of heat transfer to the working fluid passing through the steam generator, and the thermal efficiency.

SCHEMATIC AND GIVEN DATA:



Problem 8.55 (Continued) – Page 2

T-s diagram



ENGINEERING MODEL:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. All processes of the working fluid are internally reversible.
3. The turbine and pump operate adiabatically.
4. Kinetic and potential energy effects are negligible.
5. Condensate exits the condenser as saturated liquid.
6. There is no heat transfer between the outside of the condenser and the surroundings.
7. Condensate exits the feedwater heater as saturated liquid.
8. There is no heat transfer between the outside of the closed feedwater heater and the surroundings.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 1400$ lbf/in.^{2}, $T_1 = 1000$°F $\rightarrow$ From Table A-4E: $h_1 = 1493.5$ Btu/lb and $s_1 = 1.6094$ Btu/(lb·°R)

State 2: $p_2 = 120$ lbf/in.^{2}, $s_2 = s_1 \rightarrow$ From Table A-4E (interpolated): $h_2 = 1208.21$ Btu/lb

State 3: $p_3 = 2$ lbf/in.^{2}, $s_3 = s_1 \rightarrow$ From Table A-3E: $x_3 = (1.6094 - 0.1750)/1.7448 = 0.8221$ and $h_3 = h_{f3} + x_3 h_{fg3} = 94.02 + (0.8221)(1022.1) = 934.29$ Btu/lb

State 4: $p_4 = p_3 = 2$ lbf/in.^{2}, sat liq. $\rightarrow$ From Table A-3E: $h_4 = h_{f4} = 94.02$ Btu/lb and $v_4 = v_{f4} = 0.01623$ ft^3/lb

State 5: $p_5 = p_2 = 120$ lbf/in.^{2}, $h_5 \approx h_4 + v_4(p_5 - p_4)$

$$h_5 = 94.02 \text{ Btu/lb} + 0.01623\left(\frac{\text{ft}^3}{\text{lb}}\right)(1400-2)\left(\frac{\text{lbf}}{\text{in}^2}\right)\left|\frac{144 \text{ in}^2}{\text{ft}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right| = 98.22 \text{ Btu/lb}$$

State 6: $p_6 = p_1 = 1400$ lbf/in.^{2}, $T_6 = T_{sat}$ @ $P = 120$ lbf/in^2 = 341.30°F. → From Table A-2E (compressed liquid) (interpolated): $h_6 \approx h_{f6} = 312.6$ Btu/lb

State 7: $p_7 = p_6 = 120$ lbf/in.^{2}, sat liq. → From Table A-3E: $h_7 = h_{f7} = 312.7$ Btu/lb

State 8: $p_8 = p_3 = 2$ lbf/in.^{2}, throttling process → $h_8 = h_7 = 312.7$ Btu/lb

(a) The mass flow rate of steam entering the first stage of the turbine can be determined from the net power output of the cycle

$$\dot{W}_{cycle} = \dot{W}_{t1} + \dot{W}_{t2} - \dot{W}_p$$

Let $\dot{m}_1$ be the mass flow rate of the steam entering the first stage of the turbine. Mass and energy balances for control volumes around the two turbine stages and the pump give, respectively

$$\dot{W}_{t1} = \dot{m}_1 (h_1 - h_2)$$
$$\dot{W}_{t2} = \dot{m}_1 (1-y)(h_2 - h_3)$$
$$\dot{W}_p = \dot{m}_1 (h_5 - h_4)$$

where y is the fraction of the flow entering the first-stage turbine extracted at 2.

Substituting for net power output of the cycle

$$\dot{W}_{cycle} = \dot{m}_1 [(h_1 - h_2) + (1-y)(h_2 - h_3) - (h_5 - h_4)]$$

Solving for $\dot{m}_1$ yields

$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{(h_1 - h_2) + (1-y)(h_2 - h_3) - (h_5 - h_4)} \tag{1}$$

The mass fraction of steam (y) extracted after the first stage of the turbine is unknown. Analyze the closed feedwater heater to determine y. Mass and energy balances for a control volume around the closed feedwater heater give

$$0 = \dot{Q} - \dot{W} + \dot{m}_1(h_5 - h_6) + \dot{m}_1(y)(h_2 - h_7)$$

Since there is no transfer of energy by heat or work, we can solve for y and substitute values for specific enthalpy to yield

$$y = \frac{h_6 - h_5}{h_2 - h_7} = \frac{312.6\dfrac{\text{Btu}}{\text{lb}} - 98.22\dfrac{\text{Btu}}{\text{lb}}}{1208.21\dfrac{\text{Btu}}{\text{lb}} - 312.7\dfrac{\text{Btu}}{\text{lb}}} = 0.239$$

Substituting values in (1)

$$\dot{m}_1 = \frac{1\times 10^9\,\dfrac{\text{Btu}}{\text{h}}}{(1493.5 - 1208.21)\dfrac{\text{Btu}}{\text{lb}} + (1-0.239)(1208.21 - 934.29)\dfrac{\text{Btu}}{\text{lb}} - (98.22 - 94.02)\dfrac{\text{Btu}}{\text{lb}}}$$

$$\dot{m}_1 = \underline{\mathbf{2.04 \times 10^6\ lb/h}}$$

(b) The rate of heat transfer to the working fluid passing through the steam generator can be determined by applying mass and energy balances to a control volume around the steam generator to give

$$\dot{Q}_{\text{in}} = \dot{m}_1(h_1 - h_6) = (2.04\times10^6\ \text{lb/h})(1493.5\ \text{Btu/lb} - 312.6\ \text{Btu/lb}) = \underline{\mathbf{2.41\times10^9\ Btu/h}}$$

(c) The thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = (1\times10^9\ \text{Btu/h})/(2.41\times10^9\ \text{Btu/h}) = \underline{\mathbf{0.4149\ (41.49\%)}}$$

The results of Problem 8.55 can be compared to the results of Problem 8.10 to see some of the effects of incorporating a closed feedwater heater on the performance of a Rankine cycle for cycles with the same net power output. In this case, use of a closed feedwater heater results in higher thermal efficiency and less heat addition, but the steam flow rate entering the first-stage turbine increases.

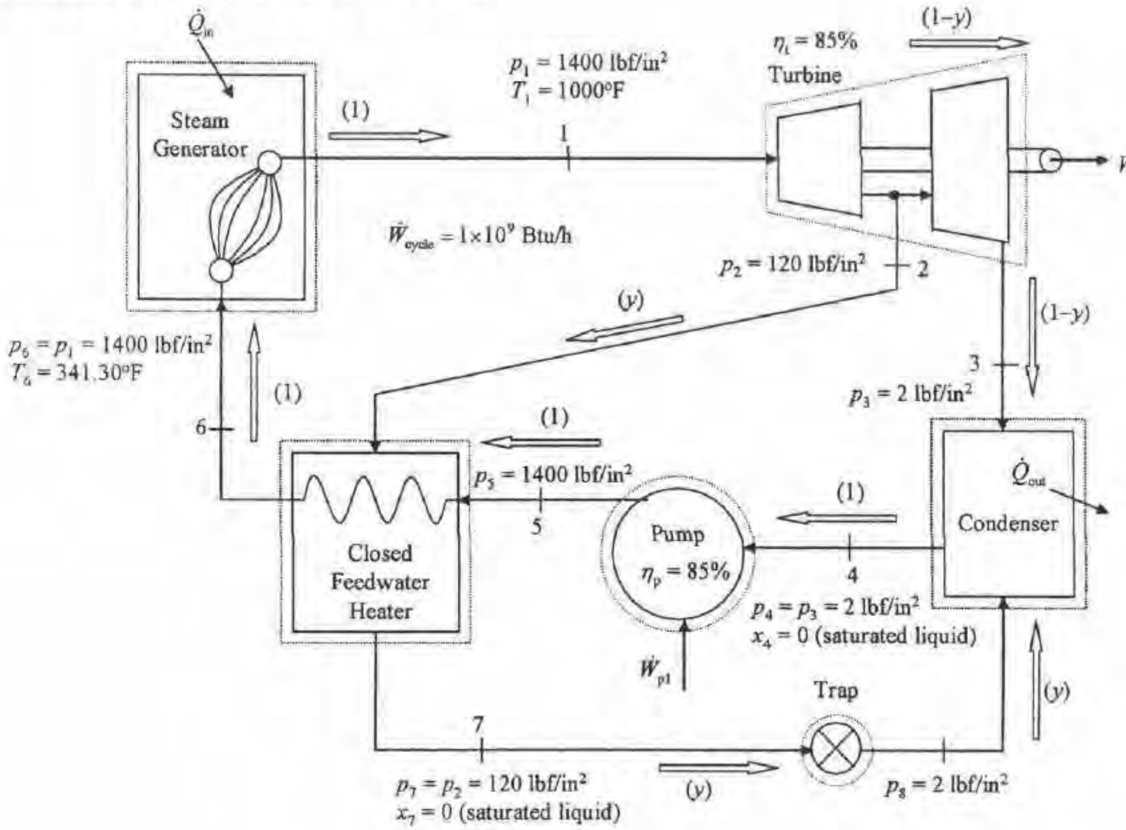
8.56 Water is the working fluid in a regenerative Rankine cycle with one closed feedwater heater. Steam enters the turbine at 1400 lbf/in.^{2} and 1000°F and expands to 120 lbf/in.^{2}, where some of the steam is extracted and diverted to the closed feedwater heater. The remaining steam expands through the second-stage turbine to the condenser pressure of 2 lbf/in.^{2} Each turbine stage and the pump have isentropic efficiencies of 85%. Flow through the condenser, closed feedwater heater, and steam generator is at constant pressure. Condensate exiting the feedwater heater as saturated liquid at 120 lbf/in.^{2} undergoes a throttling process as it passes through a trap into the condenser. The feedwater leaves the heater at 1400 lbf/in.^{2} and a temperature equal to the saturation temperature at 120 lbf/in.^{2} The net power output of the cycle is 1×10^9 Btu/h. Determine for the cycle

(a) the mass flow rate of steam entering the first stage of the turbine, in lb/h.
(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the steam generator.
(c) the thermal efficiency.

KNOWN: A regenerative Rankine cycle with one closed feedwater heater and superheat operates with water as the working fluid. The net power output of the cycle is given.

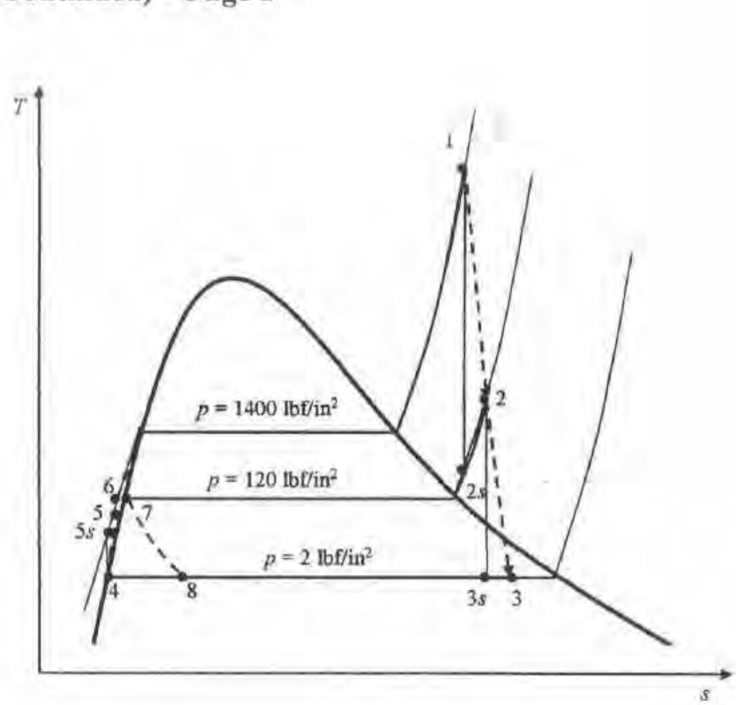
FIND: Determine the mass flow rate of steam entering the first stage of the turbine, the rate of heat transfer to the working fluid passing through the steam generator, and the thermal efficiency.

SCHEMATIC AND GIVEN DATA:



Problem 8.56 (Continued) – Page 2

T-s diagram



ENGINEERING MODEL:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
3. The turbine and pump each operate adiabatically with an isentropic efficiency of 85%.
4. Kinetic and potential energy effects are negligible.
5. Condensate exits the condenser as saturated liquid.
6. There is no heat transfer between the outside of the condenser and the surroundings.
7. Condensate exits the feedwater heater as saturated liquid.
8. There is no heat transfer between the outside of the closed feedwater heater and the surroundings.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 1400$ lbf/in.^{2}, $T_1 = 1000°F \rightarrow$ From Table A-4E: $h_1 = 1493.5$ Btu/lb and $s_1 = 1.6094$ Btu/(lb·°R)

State 2s: $p_{2s} = p_2 = 120$ lbf/in.^{2}, $s_{2s} = s_1 \rightarrow$ From Table A-4E (interpolated): $h_{2s} = 1208.21$ Btu/lb

State 2: $p_2 = 120$ lbf/in.^{2}, $h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 1493.5 - 0.85(1493.5 - 1208.21) = 1251.0$ Btu/lb and $\rightarrow$ From Table A-4E (interpolated): $s_2 = 1.6587$ Btu/(lb·°R)

State 3s: $p_{3s} = p_3 = 2$ lbf/in.^{2}, $s_{3s} = s_2 \rightarrow$ From Table A-3E: $x_{3s} = (1.6587 - 0.1750)/1.7448 = 0.8504$ and $h_{3s} = h_{f3s} + x_3 h_{fg3s} = 94.02 + (0.8504)(1022.1) = 963.21$ Btu/lb

State 3: $p_3 = 2$ lbf/in.^{2}, $h_3 = h_2 - \eta_t(h_2 - h_{3s}) = 1251.0 - 0.85(1251.0 - 963.21) = 1006.4$ Btu/lb

State 4: $p_4 = p_3 = 2$ lbf/in.^{2}, sat liq. → From Table A-3E: $h_4 = h_{f4} = 94.02$ Btu/lb and $v_4 = v_{f4} = 0.01623$ ft^3/lb

State 5: $p_5 = p_6 = 120$ lbf/in.^{2}, $h_5 = h_4 + \dfrac{v_4(p_5 - p_4)}{\eta_{p1}}$

$$h_5 = 94.02\text{ Btu/lb} + \frac{0.01623\left(\dfrac{\text{ft}^3}{\text{lb}}\right)(1400-2)\left(\dfrac{\text{lbf}}{\text{in}^2}\right)}{0.85}\left|\frac{144\text{ in}^2}{\text{ft}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right| = 120.71\text{ Btu/lb}$$

State 6: $p_6 = p_1 = 1400$ lbf/in.^{2}, $T_6 = T_{sat}$ @ $P = 120$ lbf/in^2 = 341.30°F. → From Table A-2E (compressed liquid) (interpolated): $h_6 \approx h_{f6} = 312.6$ Btu/lb

State 7: $p_7 = p_6 = 120$ lbf/in.^{2}, sat liq. → From Table A-3E: $h_7 = h_{f7} = 312.7$ Btu/lb

State 8: $p_8 = p_3 = 2$ lbf/in.^{2}, throttling process → $h_8 = h_7 = 312.7$ Btu/lb

(a) The mass flow rate of steam entering the first stage of the turbine can be determined from the net power output of the cycle

$$\dot{W}_{cycle} = \dot{W}_{t1} + \dot{W}_{t2} - \dot{W}_{p}$$

Let $\dot{m}_1$ be the mass flow rate of the steam entering the first stage of the turbine. Mass and energy balances for control volumes around the two turbine stages and the pump give, respectively

$$\dot{W}_{t1} = \dot{m}_1 (h_1 - h_2)$$
$$\dot{W}_{t2} = \dot{m}_1 (1-y)(h_2 - h_3)$$
$$\dot{W}_{p} = \dot{m}_1 (h_5 - h_4)$$

where y is the fraction of the flow entering the first-stage turbine that is extracted at 2.

Substituting for net power output of the cycle

$$\dot{W}_{cycle} = \dot{m}_1 [(h_1 - h_2) + (1-y)(h_2 - h_3) - (h_5 - h_4)]$$

Solving for $\dot{m}_1$ yields

$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{(h_1 - h_2) + (1-y)(h_2 - h_3) - (h_5 - h_4)} \quad (1)$$

The mass fraction of steam (y) extracted after the first stage of the turbine is unknown. Analyze the closed feedwater heater to determine y. Mass and energy balances for a control volume around the closed feedwater heater give

$$0 = \dot{Q} - \dot{W} + \dot{m}_1(h_5 - h_6) + \dot{m}_1(y)(h_2 - h_7)$$

Since there is no transfer of energy by heat or work, we can solve for y and substitute values for specific enthalpy to yield

$$y = \frac{h_6 - h_5}{h_2 - h_7} = \frac{312.6\dfrac{\text{Btu}}{\text{lb}} - 120.71\dfrac{\text{Btu}}{\text{lb}}}{1251.0\dfrac{\text{Btu}}{\text{lb}} - 312.7\dfrac{\text{Btu}}{\text{lb}}} = 0.205$$

Substituting values in (1) and solving for $\dot{m}_1$

$$\dot{m}_1 = \frac{1\times10^9\,\dfrac{\text{Btu}}{\text{h}}}{(1493.5 - 1251.0)\dfrac{\text{Btu}}{\text{lb}} + (1 - 0.205)(1251.0 - 1006.4)\dfrac{\text{Btu}}{\text{lb}} - (120.71 - 94.02)\dfrac{\text{Btu}}{\text{lb}}}$$

$$\dot{m}_1 = \underline{\mathbf{2.44\times10^6\ lb/h}}$$

(b) The rate of heat transfer to the working fluid passing through the steam generator can be determined by applying mass and energy balances to a control volume around the steam generator to give

$$\dot{Q}_{\text{in}} = \dot{m}_1(h_1 - h_6) = (2.44\times10^6\ \text{lb/h})(1493.5\ \text{Btu/lb} - 312.6\ \text{Btu/lb}) = \underline{\mathbf{2.88\times10^9\ Btu/h}}$$

(c) The thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = (1\times10^9\ \text{Btu/h})/(2.88\times10^9\ \text{Btu/h}) = \underline{\mathbf{0.3472\ (34.72\%)}}$$

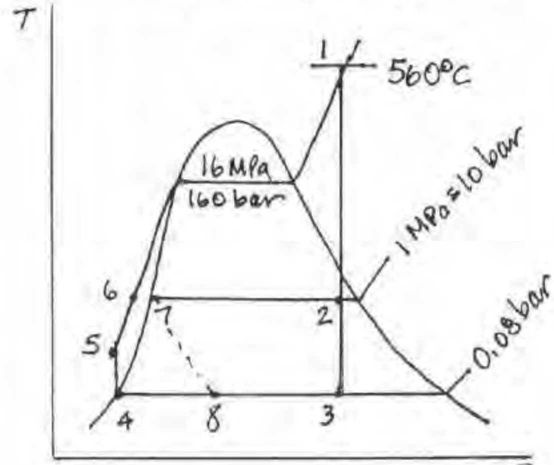
The results of Problem 8.56 can be compared to the results of Problem 8.19 to see some of the effects of incorporating a closed feedwater heater on the performance of an actual Rankine cycle for cycles with the same net power output. In this case, use of a closed feedwater heater results in higher thermal efficiency and less heat addition, but the steam flow rate into the first-stage turbine increases.

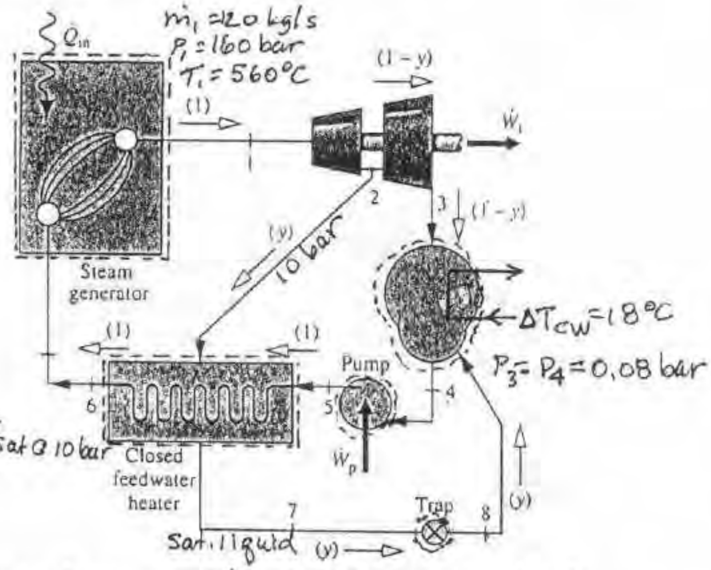
PROBLEM 8.57

KNOWN: Water is the working fluid in a regenerative vapor power cycle with one closed feedwater heater. Data are specified at various locations, and the mass flow rate of steam entering the first-stage turbine is given. The temperature of the cooling water passing through the condenser is known.

FIND: Determine (a) the net power developed, (b) the rate of heat addition, (c) the thermal efficiency, and (d) the mass flow rate of cooling water passing through the condenser.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the working fluid are internally reversible, except for the expansion through the trap (a throttling process) and in the closed feedwater heater. (3) The turbines, pump, and feedwater heater operate adiabatically. (4) Kinetic and potential energy effects are negligible. (5) Condensate exits the closed heater and the condenser as saturated liquid at the respective pressures.

ANALYSIS: First, fix each principal state.

State 1: $p_1 = 160$ bar, $T_1 = 560$°C $\Rightarrow h_1 = 3465.4$ kJ/kg, $s_1 = 6.5132$ kJ/kg

State 2: $p_2 = 10$ bar, $s_2 = s_1 \Rightarrow x_2 = \dfrac{s_2 - s_{f2}}{s_{g2} - s_{f2}} = 0.9836$, $h_2 = 2745.1$ kJ/kg

State 3: $p_3 = 0.08$ bar, $s_3 = s_2 \Rightarrow x_3 = \dfrac{s_3 - s_{f3}}{s_{g3} - s_{f3}} = 0.7753$, $h_3 = 2037.0$ kJ/kg

State 4: $p_4 = 0.08$ bar, sat. liquid $\Rightarrow h_4 = 173.88$ kJ/kg

State 5: $h_5 \approx h_4 + v_4(p_5 - p_4)$

$$= 173.88 + (1.0084\times10^{-3})\frac{m^3}{kg}(160 - 0.08)\,bar\left|\frac{10^5\,N/m^2}{1\,bar}\right|\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|$$

$$= 173.88 + 16.13 = 190.01\ kJ/kg$$

State 6: $p_6 = 160$ bar, $T_6 = T_{sat\,@\,10\,bar} \approx 180$°C. From Table A-5, $h_6 \approx 771$ kJ/kg

State 7: $p_7 = 10$ bar, Sat. liquid $\Rightarrow h_7 = 762.81$ kJ/kg

State 8: Throttling process $\Rightarrow h_8 = h_7 = 762.81$ kJ/kg

(a) For the control volume enclosing the closed feedwater heater

$$y = \frac{h_6 - h_5}{h_2 - h_7} = \frac{771 - 190.01}{2745.1 - 762.81} = 0.2931$$

Thus, for the control volume enclosing the turbine stages

$$\dot{W}_t = \dot{m}_1[(h_1 - h_2) + (1-y)(h_2 - h_3)]$$

$$= (120\ \tfrac{kg}{s})[(3465.4 - 2745.1) + (0.7069)(2745.1 - 2037.0)]\frac{kJ}{kg}\left|\frac{1\,kW}{1\,kJ/s}\right| = 1.465\times10^5\ kW$$

PROBLEM 8.57 (Cont'd)

For the pump

$$\dot{W}_p = \dot{m}_1 (h_5 - h_4) = (120 \tfrac{kg}{s})(190.01 - 173.88)\left|\frac{1\ kW}{1\ kJ/s}\right| = 1936\ kW$$

Thus $\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p = 1.4456 \times 10^5\ kW$ ← $\dot{W}_{cycle}$

(b) For the steam generator

$$\dot{Q}_{in} = \dot{m}_1 (h_1 - h_6) = (120)(3465.4 - 771)\left|\frac{1}{1}\right| = 3.233 \times 10^5\ kW$$ ← $\dot{Q}_{in}$

(c) The thermal efficiency is

$$\eta = \dot{W}_{cycle}/\dot{Q}_{in} = (1.445 \times 10^5)/(3.233 \times 10^5) = 0.447\ (44.7\%)$$ ← η

(d) For the control volume enclosing the condenser

$$0 = \dot{m}_1 [(1-y)h_3 + y h_8 - h_4] + \dot{m}_{cw}(h_{in,cw} - h_{out,cw})$$

or

$$\dot{m}_{cw} = \frac{\dot{m}_1 [(1-y)h_3 + y h_8 - h_4]}{(h_{out,cw} - h_{in,cw})}$$

With $(h_{out,cw} - h_{in,cw}) = c_{cw} \Delta T_{cw}$ and $c_{cw} = 4.179$ from Table A-19

$$\dot{m}_{cw} = \frac{(120\ kg/s)[(0.7069)(2037.0) + (0.2931)(762.81) - 173.88]\ kJ/kg}{(4.179)(18)\ kJ/kg}$$

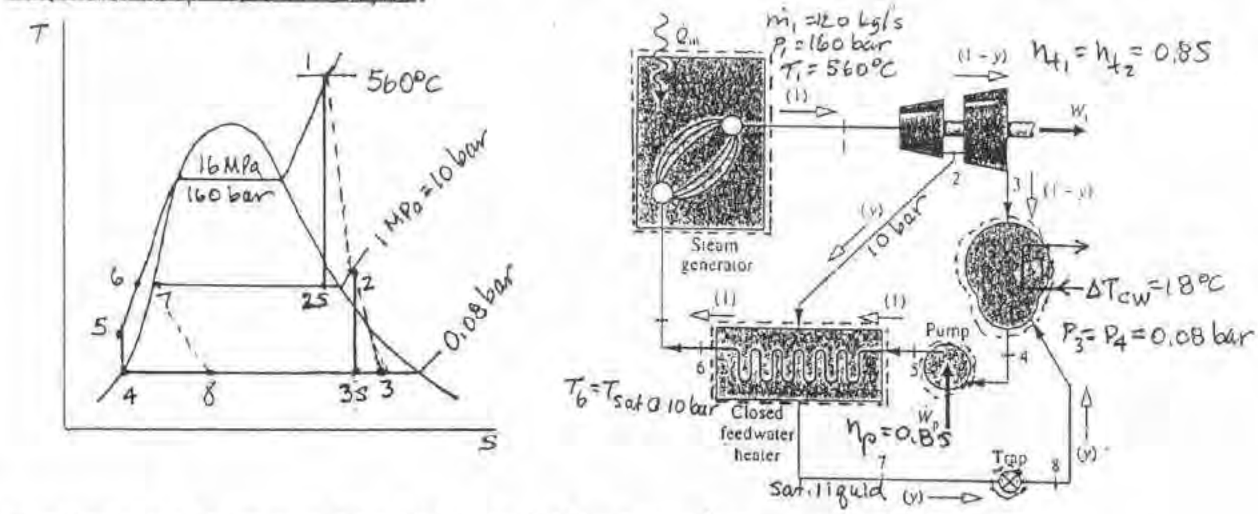
$$= \frac{1.788 \times 10^5}{75.222} = 2376\ kg/s$$ ← $\dot{m}_{cw}$

PROBLEM 8.58

KNOWN: The ideal regenerative Rankine cycle of Problem 8.57 is modified to include turbine stage and pump isentropic efficiencies of 0.85.

FIND: Answer the same questions as in Problem 8.57 for the modified cycle.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Problem 8.57, except $\eta_{t_1} = \eta_{t_2} = \eta_p = 0.85$.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 160$ bar, $T_1 = 560°C \Rightarrow h_1 = 3465.1$ kJ/kg, $s_1 = 6.5132$ kJ/kg·K

State 2: Using the isentropic efficiency of the first-stage turbine

$$\eta_{t_1} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_{t_1}(h_1 - h_{2s})$$

With $h_{2s} = 2745.1$ kJ/kg from Problem 8.57 $h_2 = 2853.1$ kJ/kg

State 3: $p_3 = 0.08$ bar, $s_{3s} = s_2 \Rightarrow$ Interpolating in Table A-4; $s_2 = 6.7451$ kJ/kg·K

Thus $$x_{3s} = \frac{s_{3s} - s_{f3}}{s_{g3} - s_{f3}} = 0.8056\ ;\ h_{3s} = 2109.8 \text{ kJ/kg}$$

Using the second-stage turbine isentropic efficiency

$$h_3 = h_2 - \eta_{t_2}(h_2 - h_{3s}) = 2221.3 \text{ kJ/kg}$$

State 4: $p_4 = 0.08$ bar, sat. liquid $\Rightarrow h_4 = 173.88$ kJ/kg

State 5: Using the isentropic pump efficiency; $\eta_p = (h_{5s} - h_4)/(h_5 - h_4)$

Thus, with $h_{5s} = 190.01$ kJ/kg from Problem 8.57

$$h_5 = h_4 + (h_{5s} - h_4)/\eta_p = 192.86 \text{ kJ/kg}$$

State 6: $p_6 = 160$ bar. From Problem 8.57, $h_6 = 771$ kJ/kg

State 7: $h_7 = 762.8$ kJ/kg

State 8: $h_8 = h_7 = 762.8$ kJ/kg

(a) $y = (h_6 - h_5)/(h_2 - h_7) = (771 - 192.86)/(2853.1 - 762.8) = 0.2766$

$$\left.\begin{array}{l}\dot{W}_t = \dot{m}_1[(h_1 - h_2) + (1-y)(h_2 - h_3)] = 1.2828 \text{ kW} \\ \dot{W}_p = \dot{m}_1(h_5 - h_4) = 2278 \text{ kW}\end{array}\right\} \dot{W}_{cycle} = \dot{W}_t - \dot{W}_p = 1.26 \times 10^5 \text{ kW}$$

$\dot{W}_{cycle}$

(b) $\dot{Q}_{in} = \dot{m}_1(h_1 - h_6) = 3.233 \times 10^5$ kW ← $\dot{Q}_{in}$

① (c) $\eta = \dot{W}_{cycle}/\dot{Q}_{in} = 0.390$ (39.0%) ← η

(d) $\dot{m}_{cw} = \dot{m}_1[(1-y)h_3 + yh_8 - h_4]/(c_{cw}\Delta T) = 2623$ kg/s ← $\dot{m}_{cw}$

1. These results can be compared to those of Problem 8.57 to see some of the effects of turbine stage and pump irreversibilities on cycle performance.

PROBLEM 8.59

KNOWN: The power plant layout of Fig. 8.12 is under consideration.

FIND: Determine expressions for the fractions of the total flow at states 8, 11, 17 in terms of the fractions y_2, y_3, y_6, and y_7.

SCHEMATIC & GIVEN DATA: See Fig. 8.12

ENGINEERING MODEL: Control volumes at steady state enclose each of the principal components.

ANALYSIS: Letting $\dot{m}_1$ denote the total flow entering the first turbine stage, we have

State 2: $y_2 = \frac{\dot{m}_2}{\dot{m}_1}$, state 3: $y_3 = \frac{\dot{m}_3}{\dot{m}_1}$, state 6: $y_6 = \frac{\dot{m}_6}{\dot{m}_1}$, state 7: $y_7 = \frac{\dot{m}_7}{\dot{m}_1}$

Applying a mass rate balance to a control volume enclosing the overall turbine,

$$\dot{m}_8 = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 - \dot{m}_6 - \dot{m}_7 + \underbrace{\dot{m}_5 - \dot{m}_4}_{= 0,\ \text{since } \dot{m}_4 = \dot{m}_5}$$

$$\therefore \quad \frac{\dot{m}_8}{\dot{m}_1} = 1 - \frac{\dot{m}_2}{\dot{m}_1} - \frac{\dot{m}_3}{\dot{m}_1} - \frac{\dot{m}_6}{\dot{m}_1} - \frac{\dot{m}_7}{\dot{m}_1} = 1 - y_2 - y_3 - y_6 - y_7 \quad \longleftarrow y_8$$

A mass balance for a control volume enclosing the condenser reads

$$\dot{m}_9 = \dot{m}_8 + \dot{m}_{20}$$

For the closed feedwater heater and valve, $\dot{m}_7 = \dot{m}_{19} = \dot{m}_{20}$. And for the closed feedwater heater and pump, $\dot{m}_{11} = \dot{m}_{10} = \dot{m}_9$. Collecting results

$$\dot{m}_{11} = \dot{m}_8 + \dot{m}_7$$

$$\Rightarrow \quad y_{11} = y_8 + y_7 \quad \text{or} \quad y_{11} = (1 - y_2 - y_3 - y_6 - y_7) + y_7$$

$$= 1 - y_2 - y_3 - y_6 \quad \longleftarrow y_{11}$$

For the intermediate-pressure closed feedwater heater, $\dot{m}_{17} = \dot{m}_3 + \dot{m}_{16}$. And for the high-pressure closed feedwater heater and valve, $\dot{m}_2 = \dot{m}_{15} = \dot{m}_{16}$. Collecting results

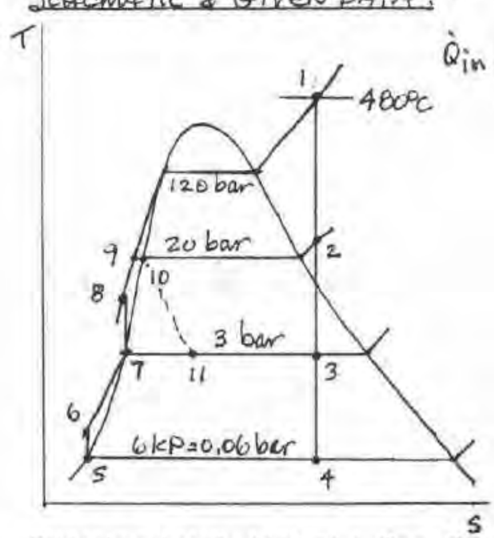
$$\dot{m}_{17} = \dot{m}_3 + \dot{m}_2 \Rightarrow y_{17} = y_3 + y_2 \quad \longleftarrow y_{17}$$

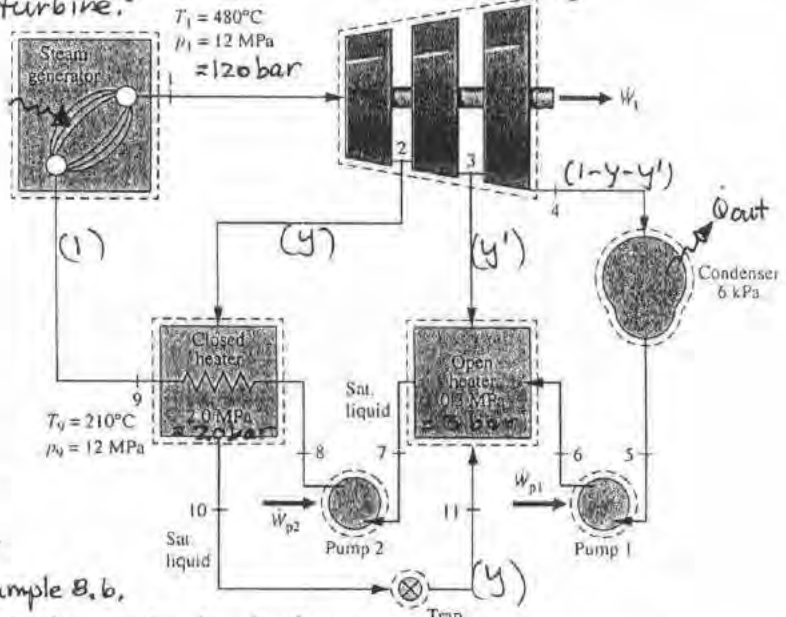
PROBLEM 8.60

KNOWN: Water is the working fluid in a regenerative vapor power cycle with one closed and one open feedwater heater. Data are known at various locations in the cycle.

FIND: Determine (a) the rate of heat addition per kg of steam entering the first-stage turbine, (b) the thermal efficiency, and (c) the rate of heat transfer to cooling water passing through the condenser, per kg of steam entering the first-stage turbine.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 8.6.

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 120$ bar, $T_1 = 480°C \Rightarrow h_1 = 3293.5$ kJ/kg, $s_1 = 6.4154$ kJ/kg·K

State 2: $P_2 = 20$ bar, $s_2 = s_1 \Rightarrow$ Interpolating in Table A-4 ; $h_2 = 2836.7$ kJ/kg

State 3: $P_3 = 3$ bar, $s_3 = s_1 \Rightarrow x_3 = \frac{s_3 - s_{f3}}{s_{g3} - s_{f3}} = 0.89164$, $h_3 = 2490.8$ kJ/kg

State 4: $P_4 = 0.06$ bar, $s_4 = s_1 \Rightarrow x_4 = \frac{s_4 - s_{f4}}{s_{g4} - s_{f4}} = 0.7548$, $h_4 = 1975.1$ kJ/kg

State 5: $P_5 = 0.06$ bar, sat. liquid $\Rightarrow h_5 = 151.53$ kJ/kg

State 6: $h_6 \approx h_5 + v_5(P_6 - P_5)$

$$\approx 151.53 \frac{kJ}{kg} + (1.0064 \times 10^{-3}) \frac{m^3}{kg} (3 - 0.06)\,bar \left| \frac{10^5 N/m^2}{1\,bar} \right| \left| \frac{1\,kJ}{10^3 N \cdot m} \right|$$

$$= 151.53 + 0.296 = 151.83 \text{ kJ/kg}$$

State 7: $P_7 = 3$ bar, sat. liquid $\Rightarrow h_7 = 561.47$ kJ/kg

State 8: $h_8 \approx h_7 + v_7(P_8 - P_7) = 561.47 + (1.0732 \times 10^{-3})(120 - 3) \left| \frac{10^5}{10^3} \right|$

$$= 561.47 + 12.56 = 574.03 \text{ kJ/kg}$$

State 9: Assume $h_9 \approx h_{f@\,210°C} = 897.76$ kJ/kg

State 10: $P_{10} = 20$ bar, sat. liquid $\Rightarrow 908.79$ kJ/kg

State 11: Throttling process $\Rightarrow h_{11} = h_{10} = 908.79$ kJ/kg

PROBLEM 8.60 (Cont'd.)

(a) For the control volume enclosing the steam generator

$$\dot{Q}_{in}/\dot{m}_1 = h_1 - h_9 = 3293.5 - 897.76 = 2395.7 \text{ kJ/kg} \longleftarrow \quad \dot{Q}_{in}/\dot{m}_1$$

(b) First, get the fractions extracted at 2 and 3 from energy and mass balances on the closed and open heater control volumes, respectively.

$$0 = y(h_2 - h_{10}) + (h_8 - h_9)$$

or

$$y = \frac{h_9 - h_8}{h_2 - h_{10}} = \frac{897.76 - 574.03}{2836.7 - 908.79} = 0.1679$$

Further

$$0 = y' h_3 + (1-y-y') h_6 + y h_{11} - h_7$$

or

$$y' = \frac{h_7 - h_6 + y(h_6 - h_{11})}{h_3 - h_6} = \frac{561.47 - 151.83 + (0.1679)(151.83 - 908.79)}{2490.8 - 151.83}$$

$$= 0.1208$$

For the control volume enclosing the turbine stages

$$\dot{W}_t = \dot{m}_1 [h_1 - y h_2 - y' h_3 - (1-y-y') h_4]$$

or

$$\dot{W}_t/\dot{m}_1 = h_1 - y h_2 - y' h_3 - (1-y-y') h_4)$$

$$= 3293.5 - (0.1679)(2836.7) - (0.1208)(2490.8) - (0.7113)(1975.1)$$

$$= 1111.44 \text{ kJ/kg}$$

For the pumps

$$\dot{W}_{P1}/\dot{m}_1 = (1-y-y')(h_6 - h_5) = (0.7113)(151.83 - 151.53) = 0.2134 \text{ kJ/kg}$$

$$\dot{W}_{P2}/\dot{m}_1 = h_8 - h_7 = 574.03 - 561.47 = 12.56 \text{ kJ/kg}$$

Thus

$$\dot{W}_{cycle}/\dot{m}_1 = \dot{W}_t/\dot{m}_1 - \dot{W}_{P1}/\dot{m}_1 - \dot{W}_{P2}/\dot{m}_2 = 1098.7 \text{ kJ/kg}$$

The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{1098.7}{2395.7} = 0.459 \; (45.9\%) \longleftarrow \quad \eta$$

(c) For the condenser

$$\dot{Q}_{out}/\dot{m}_1 = (1-y-y')(h_4 - h_5) = (0.7113)(1975.1 - 151.53)$$

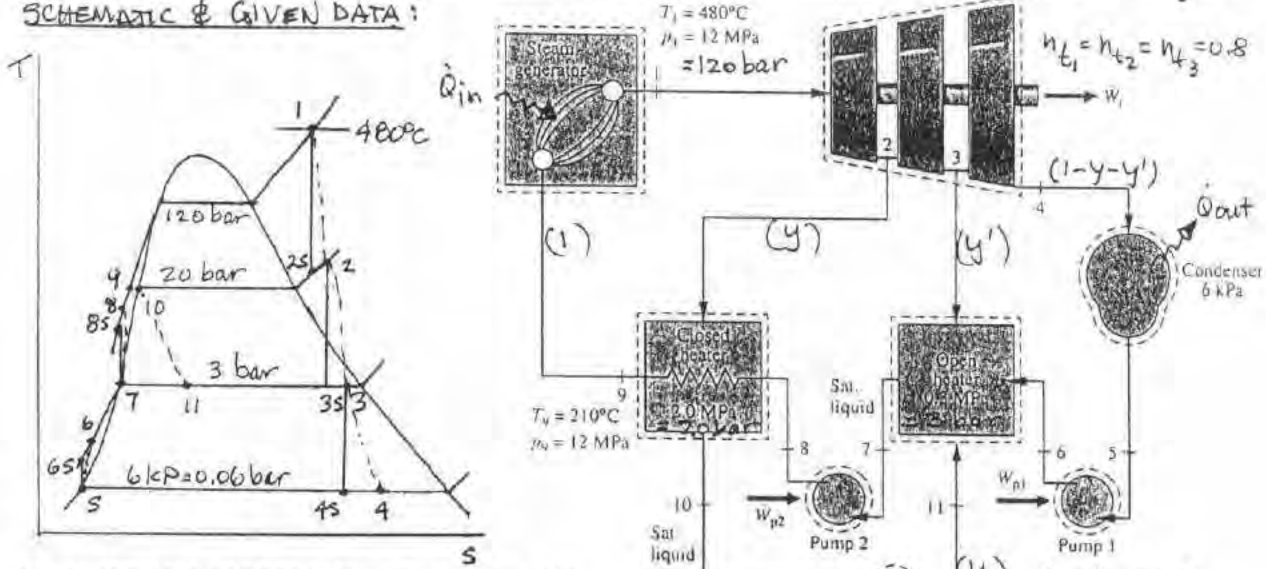
$$= 1297.1 \text{ kJ/kg} \longleftarrow \quad \dot{Q}_{out}/\dot{m}_1$$

PROBLEM 8.61

KNOWN: The regenerative vapor power cycle of Problem 8.60 is modified to include turbine stage and pump isentropic efficiencies of 0.8.

FIND: Answer the same questions as in Problem 8.60 for the modified cycle.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 8.6, except $\eta_{t_1} = \eta_{t_2} = \eta_{t_3} = \eta_{p_1} = \eta_{p_2} = 0.8$.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 120$ bar, $T_1 = 480°C \Rightarrow h_1 = 3293.5$ kJ/kg, $s_1 = 6.4154$ kJ/kg·K

State 2: Using the first-stage turbine efficiency: $\eta_{t_1} = (h_1 - h_2)/(h_1 - h_{2s})$

With $h_{2s} = 2836.7$ kJ/kg from Problem 8.60

$h_2 = h_1 - \eta_{t_1}(h_1 - h_{2s}) = 2928.1$ kJ/kg. From Table A-4; $s_2 = 6.5921 \frac{kJ}{kg K}$

State 3: $p_3 = 3$ bar, $s_{3s} = s_2 \Rightarrow x_{3s} = \dfrac{s_{3s} - s_{f3}}{s_{g3} - s_{f3}} = 0.92485$; $h_{3s} = 2562.7$ kJ/kg

Using the turbine stage efficiency

$h_3 = h_2 - \eta_{t_2}(h_2 - h_{3s}) = 2635.8$ kJ/kg ; $s_3 = 6.7722$ kJ/kg·K

$(x_3 = 0.9587)$

State 4: $p_4 = 0.06$ bar, $s_{4s} = s_3 \Rightarrow x_{4s} = \dfrac{s_{4s} - s_{f4}}{s_{g4} - s_{f4}} = 0.8005$; $h_{4s} = 2085.5$ kJ/kg

Using the turbine stage efficiency

$h_4 = h_3 - \eta_{t_3}(h_3 - h_{4s}) = 2195.6$ kJ/kg

State 5: $p_5 = 0.06$ bar, sat. liquid $\Rightarrow h_5 = 151.53$ kJ/kg

State 6: Using the pump efficiency with $h_{6s} = 151.83$ kJ/kg from Problem 8.60

$h_6 = h_5 + (h_{6s} - h_5)/\eta_p = 151.91$ kJ/kg

State 7: $p_7 = 3$ bar, sat. liquid $\Rightarrow h_7 = 561.47$ kJ/kg

State 8: Using the pump efficiency with $h_{8s} = 574.03$ kJ/kg from Problem 8.60

$h_8 = h_7 + (h_{8s} - h_7)/\eta_p = 577.17$ kJ/kg

State 9: Assume $h_9 \approx h_{f@210°C} = 897.76$ kJ/kg

State 10: $p_{10} = 20$ bar, sat. liquid $\Rightarrow h_{10} = 908.79$ kJ/kg

State 11: Throttling process $\Rightarrow h_{11} = h_{10} = 908.79$ kJ/kg

(a) $\dot{Q}_{in}/\dot{m}_1 = h_1 - h_9 = 3293.5 - 897.76 = 2395.7$ kJ/kg ← $\dot{Q}_{in}/\dot{m}_1$

(b) $$y = \frac{h_9 - h_8}{h_2 - h_{10}} = \frac{897.76 - 577.17}{2928.1 - 908.79} = 0.1588$$

$$y' = \frac{h_7 - h_6 + y(h_6 - h_{11})}{h_3 - h_6} = \frac{561.47 - 151.91 + (0.1588)(151.91 - 908.79)}{2635.8 - 151.91}$$
$$= 0.1165$$

$$\dot{W}_t/\dot{m}_1 = h_1 - yh_2 - y'h_3 - (1-y-y')h_4$$
$$= 3293.5 - (0.1588)(2928.1) - (0.1165)(2635.8) - (0.7247)(2195.6)$$
$$= 930.3 \text{ kJ/kg}$$

$$\dot{W}_p/\dot{m}_1 = \dot{W}_{P1}/\dot{m}_1 + \dot{W}_{P2}/\dot{m}_2$$
$$= (h_6 - h_5)(1-y-y') + (h_8 - h_7)$$
$$= (151.91 - 151.53)(0.7247) + (577.17 - 561.47) = 15.98 \text{ kJ/kg}$$

$$\frac{\dot{W}_{cycle}}{\dot{m}_1} = \frac{\dot{W}_t}{\dot{m}_1} - \frac{\dot{W}_p}{\dot{m}_1} = 914.3 \text{ kJ/kg}$$

① $$\eta = \frac{\dot{W}_{cycle}/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = 0.382 \,(38.2\%)$$ ← η

(c) $$\dot{Q}_{out}/\dot{m}_1 = (1-y-y')(h_4 - h_5) = (0.7247)(2195.6 - 151.53)$$
$$= 1481.3 \text{ kJ/kg}$$ ← $\dot{Q}_{out}/\dot{m}_1$

---

1. These results can be compared with those of Problem 8.60 to see some of the effects of turbine stage and pump irreversibilities on cycle performance.

PROBLEM 8.62

Refer to Problem 8.60.

IT Code

```
p1 = 120 // bar
T1 = 480 // °C
p2 = 20 // bar
p3 = 3 // bar
p4 = 0.06 // bar
p5 = p4
p6 = p3
p7 = p3
p8 = p1
p9 = p1
p10 = p2
p11 = p3
T9 = 210 // °C
mdot1 = 1

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
s2 = s1
h2 = h_Ps("Water/Steam", p2, s2)
s3 = s2
h3 = h_Ps("Water/Steam", p3, s3)
s4 = s3
h4 = h_Ps("Water/Steam", p4, s4)
h5 = hsat_Px("Water/Steam", p5, 0)
v5 = vsat_Px("Water/Steam", p5, 0)
h6 = h5 + v5 * (p5 - p4) * 100
h7 = hsat_Px("Water/Steam", p7, 0)
v7 = vsat_Px("Water/Steam", p7, 0)
h8 = h7 + v7 * (p8 - p7) * 100
psat = Psat_T("Water/Steam", T9)
h9 = hsat_Px("Water/Steam", psat, 0)
h10 = hsat_Px("Water/Steam", p10, 0)
h11 = h10

Qdotin = mdot1 * (h1 - h9)
y = (h9 - h8) / (h2 - h10)
y' = (h7 - h6 + y * (h6 - h11)) / (h3 - h6)
Wdott = mdot1 * (h1 - y * h2 - y' * h3 - (1 - y - y') * h4)
Wdotp1 = mdot1 * ((1 - y - y') * (h6 - h5))
Wdotp2 = mdot1 * (h8 - h7)
Wdotcycle = Wdott - Wdotp1 - Wdotp2
eta = Wdotcycle / Qdotin
Qdotout = mdot1 * ((1 - y - y') * (h4 - h5))
```

IT Results (p_2 = 20 bar)

$\dot{Q}_{in} / \dot{m}_1$ = 2395 kJ/kg

$\dot{Q}_{out} / \dot{m}_1$ = 1296 kJ/kg

$\dot{W}_{cycle} / \dot{m}_1$ = 1099 kJ/kg

η = 0.4587

h_1 = 3293 kJ/kg

h_2 = 2836 kJ/kg

h_3 = 2490 kJ/kg

h_4 = 1975 kJ/kg

h_5 = 151 kJ/kg

h_6 = 151 kJ/kg

h_7 = 561.2 kJ/kg

h_8 = 573.7 kJ/kg

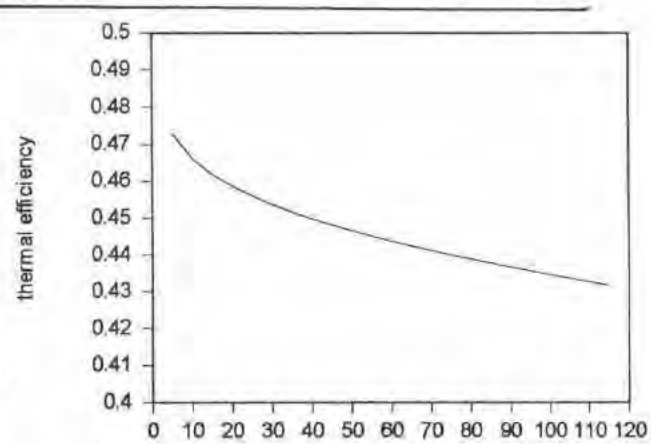
h_9 = 897.9 kJ/kg

h_{10} = 908.9 kJ/kg

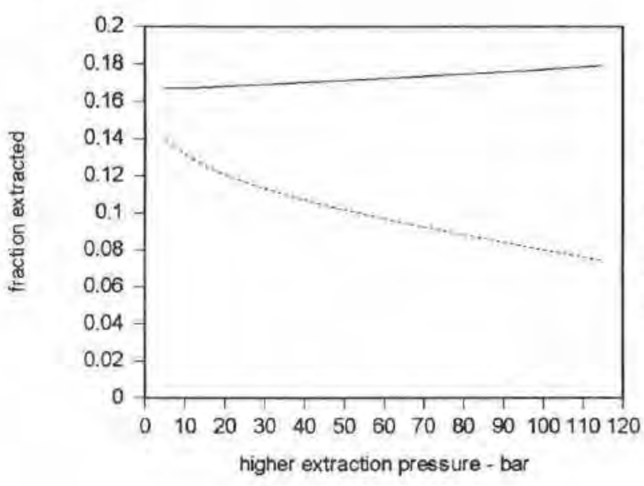
h_{11} = 908.9 kJ/kg

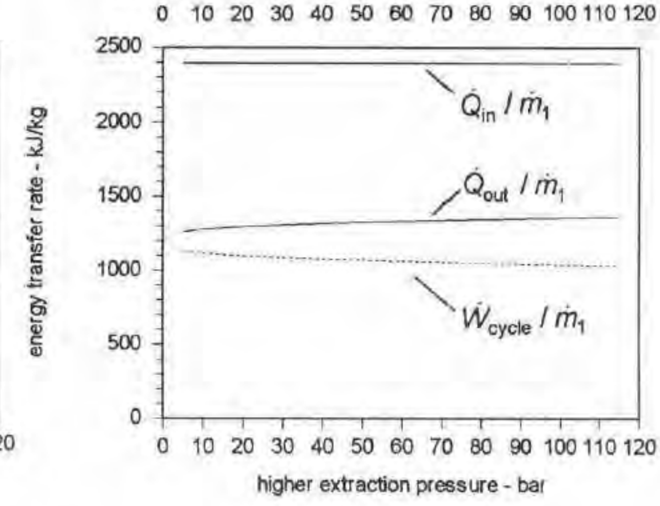
y = 0.1682

y' = 0.1208



PLOTS:





8.63 Data for a regenerative vapor power cycle using an open and a closed feedwater heater similar in design to that shown in Fig P8.60 are provided in the table below. Steam enters the turbine at 14 MPa, 560°C, state 1, and expands isentropically in three stages to a condenser pressure of 80 kPa, state 4. Saturated liquid exiting the condenser at state 5 is pumped isentropically to state 6 and enters the open feedwater heater. Between the first and second turbine stages, some steam is extracted at 1 MPa, state 2, and diverted to the closed feedwater heater. The diverted steam leaves the closed feedwater heater as saturated liquid at 1 MPa, state 10, undergoes a throttling process to 0.2 MPa, state 11, and enters the open feedwater heater. Steam is also extracted between the second and third turbine stages at 0.2 MPa, state 3, and diverted to the open feedwater heater. Saturated liquid at 0.2 MPa exiting the open feedwater heater at state 7 is pumped isentropically to state 8 and enters the closed feedwater heater. Feedwater exits the closed feedwater heater at 14 MPa, 170°C, state 9, and then enters the steam generator. If the net power developed by the cycle is 300 MW, determine
(a) the cycle thermal efficiency.
(b) the mass flow rate into the first turbine stage, in kg/s.
(c) the rate of heat transfer from the working fluid as it passes through the condenser, in MW.

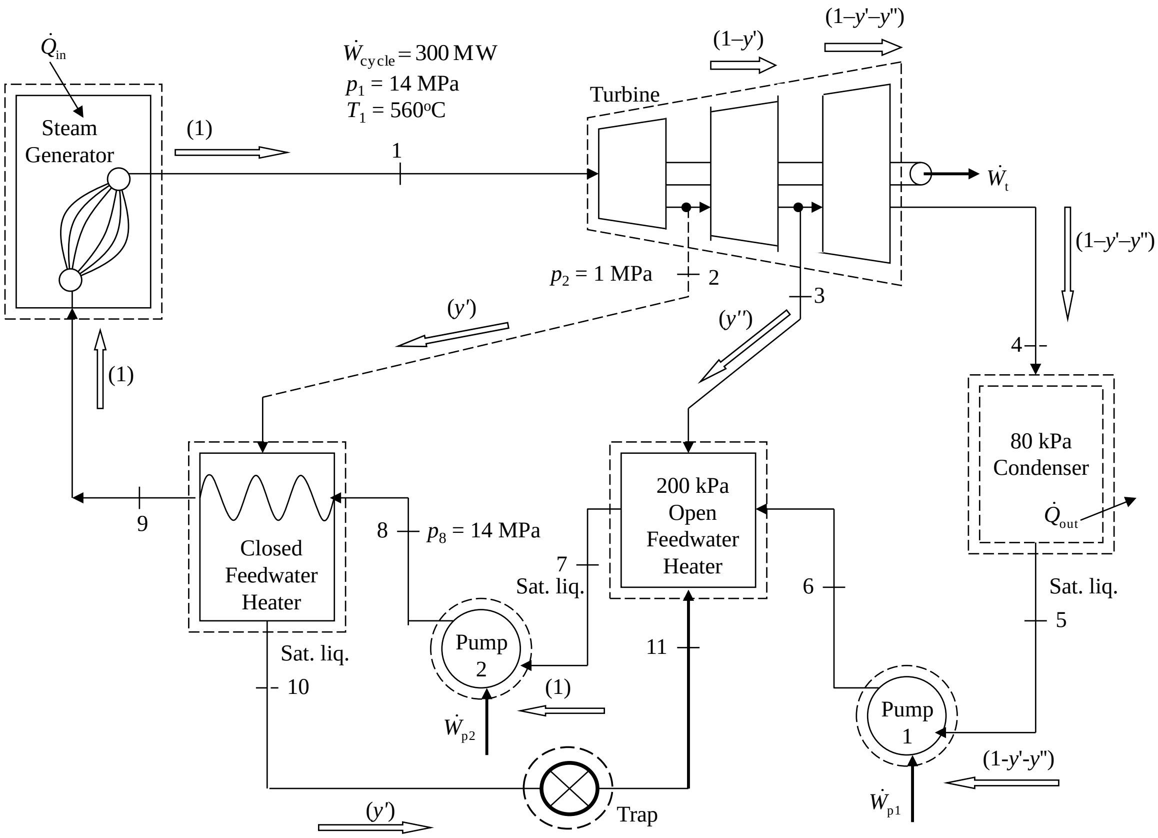
State	p (kPa)	T (°C)	h (kJ/kg)	s (kJ/kg·K)	x
1	14,000	560	3486.0	6.5941	--
2	1,000		2781.6	6.5941	--
3	200		2497.0	6.5941	0.9048
4	80	93.5	2357.6	6.5941	0.8645
5	80	93.5	391.66	1.2329	0
6	200		391.70	1.2329	--
7	200		504.70	1.5301	0
8	14,000		504.71	1.5301	--
9	14,000	170	719.21	2.0419	--
10	1,000		762.81	2.1387	0
11	200		762.81	2.1861	0.1172

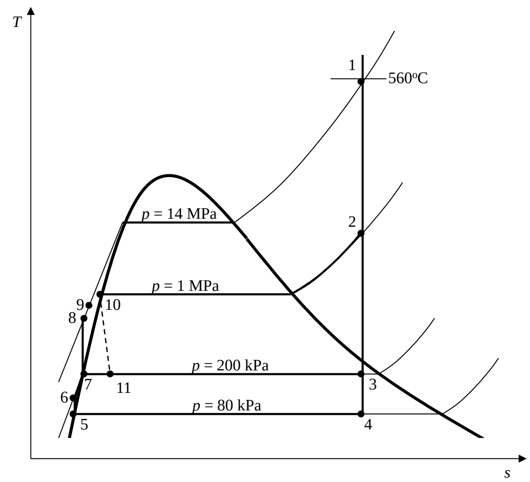
KNOWN: A regenerative vapor power cycle with one closed feedwater heater and one open feedwater heater operates with steam as the working fluid. Operational data are provided.

FIND: Determine (a) the cycle thermal efficiency, (b) the mass flow rate into the first turbine stage, in kg/s, (c) the rate of heat transfer from the working fluid as it passes through the condenser, in MW.

SCHEMATIC AND GIVEN DATA:

State	p (kPa)	T (°C)	h (kJ/kg)	s (kJ/kg·K)	x
1	14,000	560	3486.0	6.5941	--
2	1,000		2781.6	6.5941	--
3	200		2497.0	6.5941	0.9048
4	80	93.5	2357.6	6.5941	0.8645
5	80	93.5	391.66	1.2329	0
6	200		391.70	1.2329	--
7	200		504.70	1.5301	0
8	14,000		504.71	1.5301	--
9	14,000	170	719.21	2.0419	--
10	1,000		762.81	2.1387	0
11	200		762.81	2.1861	0.1172





ENGINEERING MODEL:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. All processes of the working fluid are internally reversible except for mixing in the open feedwater heater, heat transfer through a finite temperature difference in the closed feedwater heater, and throttling in the trap.
3. The turbines, pump, closed feedwater heater, open feedwater heater, and trap operate adiabatically.
4. Kinetic and potential energy effects are negligible.
5. Saturated liquid exits the open feedwater heater and condenser, and diverted steam exits the closed feedwater heater as saturated liquid.

ANALYSIS:

(a) Applying mass and energy rate balances to the control volume enclosing the closed feedwater heater, the fraction of flow, y', extracted at location 2 is

$$y' = \frac{h_9 - h_8}{h_2 - h_{10}} = \frac{(719.21 - 504.71)\text{ kJ/kg}}{(2781.6 - 762.81)\text{ kJ/kg}} = 0.1063$$

Applying mass and energy rate balances to the control volume enclosing the open feedwater heater, the fraction of flow, y'', extracted at location 3 is

$$y'' = \frac{h_7 - h_6 + y'(h_6 - h_{11})}{h_3 - h_6} = \frac{[504.70 - 391.70 + (0.1063)(391.70 - 762.81)]\text{ kJ/kg}}{(2497.0 - 391.70)\text{ kJ/kg}} = 0.0349$$

For the control volume surrounding the turbine stages

$$\frac{\dot{W}_t}{\dot{m}_1} = h_1 - y'h_2 - y''h_3 - (1 - y' - y'')h_4$$

$$\frac{\dot{W}_t}{\dot{m}_1} = [3486.0 - (0.1063)(2781.6) - (0.0349)(2497.0) - (1 - 0.1063 - 0.0349)(2357.6)]\frac{\text{kJ}}{\text{kg}}$$

$$\frac{\dot{W}_t}{\dot{m}_1} = 1078.5 \text{ kJ/kg}$$

For the pumps

$$\frac{\dot{W}_p}{\dot{m}_1} = h_8 - h_7 + (1 - y' - y'')(h_6 - h_5)$$

$$\frac{\dot{W}_p}{\dot{m}_1} = [504.71 - 504.70 + (1 - 0.1063 - 0.0349)(391.70 - 391.66)]\frac{\text{kJ}}{\text{kg}} = 0.044 \text{ kJ/kg}$$

For the working fluid passing through the steam generator

$$\frac{\dot{Q}_{in}}{\dot{m}_1} = h_1 - h_9 = (3486.0 - 719.21)\frac{\text{kJ}}{\text{kg}} = 2766.8 \text{ kJ/kg}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{(1078.5 - 0.044)\text{ kJ/kg}}{2766.8\text{ kJ/kg}} = \underline{\mathbf{0.3898\ (38.98\%)}}$$

(b) The ***net*** power developed is

$$\dot{W}_{cycle} = \dot{m}_1(\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1)$$

Thus,

$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{(\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1)}$$

$$\dot{m}_1 = \frac{300\text{ MW}}{(1078.5 - 0.044)\frac{\text{kJ}}{\text{kg}}}\left|\frac{1000\frac{\text{kJ}}{\text{s}}}{1\text{ MW}}\right| = \underline{\mathbf{278.2\ kg/s}}$$

(c) The rate of heat transfer from the working fluid as it passes through the condenser is determined by analyzing the steam side of the condenser. For the working fluid passing through the condenser

$$\dot{Q}_{\text{out}} = (1 - y' - y'')\dot{m}_1(h_4 - h_5)$$

$$\dot{Q}_{\text{out}} = (1 - 0.1063 - 0.0349)\left(278.2\,\frac{\text{kg}}{\text{s}}\right)(2357.6 - 391.66)\,\frac{\text{kJ}}{\text{kg}}\left|\frac{1\,\text{MW}}{1000\,\text{kJ/s}}\right| = \underline{\mathbf{470\ MW}}$$

Alternatively, the rate of heat transfer from the working fluid as it passes through the condenser can be determined by applying an overall energy balance to the cycle.

$$\dot{Q}_{\text{out}} = \dot{Q}_{\text{in}} - \dot{W}_{\text{cycle}}$$

where $\dot{Q}_{\text{in}} = \dfrac{\dot{W}_{\text{cycle}}}{\eta} = \dfrac{300\,\text{MW}}{0.3898} = 770\ \text{MW}$

Substituting values and solving give

$$\dot{Q}_{\text{out}} = 770\,\text{MW} - 300\,\text{MW} = \underline{\mathbf{470\ MW}}$$

8.64 Reconsider the cycle of Problem 8.63, but include in the analysis that each turbine stage and the pumps have an isentropic efficiency of 83%. Comparing calculated values with those obtained in Problem 8.63, respectively, what is the effect of irreversibilties within the turbines and pumps?

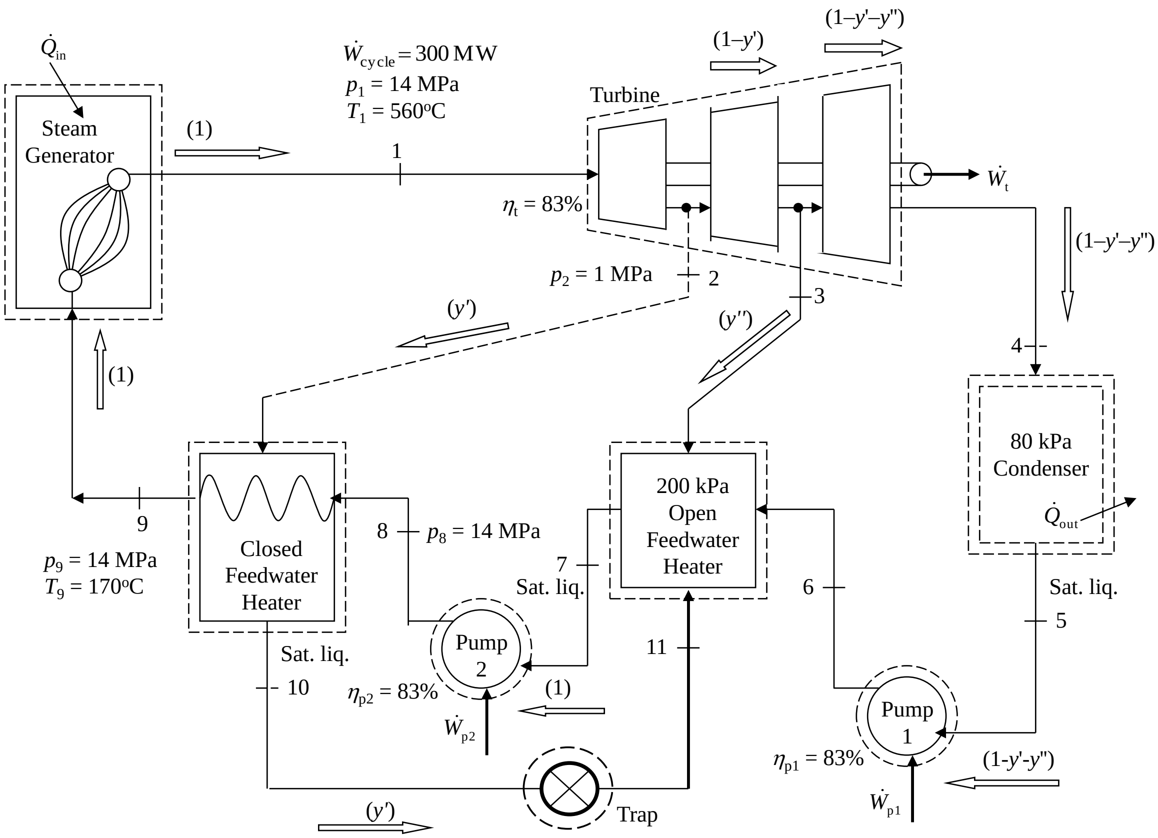
KNOWN: A regenerative vapor power cycle with one closed feedwater heater and one open feedwater heater operates with steam as the working fluid. Operational data are provided.

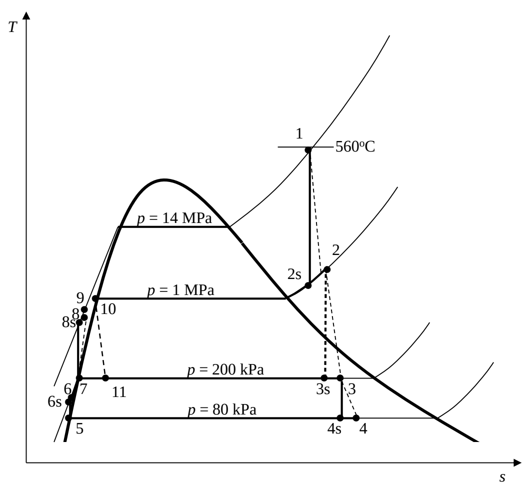
FIND: Determine (a) the cycle thermal efficiency, (b) the mass flow rate into the first turbine stage, in kg/s, (c) the rate of heat transfer from the working fluid as it passes through the condenser, in MW. Comparing calculated values with those obtained in Problem 8.63, respectively, comment on the effect of irreversibilties within the turbines and pumps.

SCHEMATIC AND GIVEN DATA:

State	p (kPa)	T (^{o}C)	h (kJ/kg)	s (kJ/kg·K)	x
1	14,000	560	3486.0	6.5941	--
2	1,000		2781.6	6.5941	--
3	200		2497.0	6.5941	0.9048
4	80	93.5	2357.6	6.5941	0.8645
5	80	93.5	391.66	1.2329	0
6	200		391.70	1.2329	--
7	200		504.70	1.5301	0
8	14,000		504.71	1.5301	--
9	14,000	170	719.21	2.0419	--
10	1,000		762.81	2.1387	0
11	200		762.81	2.1861	0.1172

From Problem 8.63





ENGINEERING MODEL:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The turbines, pumps, closed feedwater heater, open feedwater heater, and trap operate adiabatically.
3. Kinetic and potential energy effects are negligible.

4. Saturated liquid exits the open feedwater heater and condenser, and steam generator feedwater exits the closed feedwater heater as saturated liquid.

ANALYSIS: First fix each principal state.

<u>State 1 (same as State 1 in problem 8.63)</u>: p_1 = 14 MPa (140 bar), T_1 = 560°C → h_1 = 3486.0 kJ/kg, s_1 = 6.5941 kJ/kg·K

<u>State 2s (same as State 2 in problem 8.63)</u>: $p_{2s} = p_2$ = 1 MPa (10 bar), $s_{2s} = s_1$ = 6.5941 kJ/kg·K → h_{2s} = 2781.6 kJ/kg

<u>State 2</u>: p_2 = 1 MPa (10 bar), h_2 = 2901.3 kJ/kg (see below) → s_2 = 6.8429 kJ/kg·K

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \rightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 3486.0\frac{\text{kJ}}{\text{kg}} - (0.83)(3486.0 - 2781.6)\frac{\text{kJ}}{\text{kg}} = 2901.3 \text{ kJ/kg}$$

<u>State 3s</u>: $p_{3s} = p_3$ = 200 kPa (2 bar), $s_{3s} = s_2$ = 6.8429 kJ/kg·K → x_{3s} = 0.9492, h_{3s} = 2594.7 kJ/kg

<u>State 3</u>: p_3 = 200 kPa (2 bar), h_3 = 2646.8 kJ/kg (see below) → x_3 = 0.9728, s_3 = 6.9749 kJ/kg·K

$$\eta_t = \frac{h_2 - h_3}{h_2 - h_{3s}} \rightarrow h_3 = h_2 - \eta_t(h_2 - h_{3s}) = 2901.3\frac{\text{kJ}}{\text{kg}} - (0.83)(2901.3 - 2594.7)\frac{\text{kJ}}{\text{kg}} = 2646.8 \text{ kJ/kg}$$

<u>State 4s</u>: $p_{4s} = p_4$ = 80 kPa (0.8 bar), $s_{4s} = s_3$ = 6.9749 kJ/kg·K → x_{4s} = 0.9259, h_{4s} = 2497.2 kJ/kg

<u>State 4</u>: p_4 = 80 kPa (0.8 bar), h_4 = 2522.6 kJ/kg (see below) → x_4 = 0.9370, s_4 = 7.0439 kJ/kg·K

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} \rightarrow h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 2646.8\frac{\text{kJ}}{\text{kg}} - (0.83)(2646.8 - 2497.2)\frac{\text{kJ}}{\text{kg}} = 2522.6 \text{ kJ/kg}$$

<u>State 5 (same as State 5 in problem 8.63)</u>: p_5 = 80 kPa (0.8 bar), saturated liquid → h_5 = 391.66 kJ/kg, v_5 = 0.0010380 m^3/kg, s_5 = 1.2329 kJ/kg·K

<u>State 6</u>: p_6 = 200 kPa (2 bar), h_6 = 391.81 kJ/kg (see below) → $s_6 \approx$ 1.2332 kJ/kg·K (***assuming the saturated liquid state corresponding to $h_6 = h_f$ in Table 2 and interpolating for $s_6 = s_f$***)

$$\eta_{p1} = \frac{v_5(p_6 - p_5)}{h_6 - h_5} \rightarrow h_6 = h_5 + \frac{v_5(p_6 - p_5)}{\eta_{p1}}$$

$$h_6 = 391.66\,\frac{\text{kJ}}{\text{kg}} + \frac{(0.0010380\,\frac{\text{m}^3}{\text{kg}})(200-80)\text{ kPa}}{0.83}\left|\frac{1000\,\frac{\text{N}}{\text{m}^2}}{1\,\text{kPa}}\right|\left|\frac{1\,\text{kJ}}{1000\text{ N}\cdot\text{m}}\right| = 391.81\text{ kJ/kg}$$

<u>State 7 (same as State 7 in problem 8.63)</u>: p_7 = 200 kPa (2 bar), h_7 = 504.70 kJ/kg, s_7 = 1.5301 kJ/kg·K

<u>State 8</u>: p_8 = 14 MPa (140 bar), h_8 = 522.33 kJ/kg (see below) → $s_8 \approx$ 1.5743 kJ/kg·K (***assuming the saturated liquid state corresponding to $h_8 = h_f$ in Table 2 and interpolating for $s_8 = s_f$***)

$$\eta_{p2} = \frac{v_7(p_8 - p_7)}{h_8 - h_7} \rightarrow h_8 = h_7 + \frac{v_7(p_8 - p_7)}{\eta_{p2}}$$

$$h_8 = 504.70\,\frac{\text{kJ}}{\text{kg}} + \frac{(0.0010605\,\frac{\text{m}^3}{\text{kg}})(14{,}000-200)\text{ kPa}}{0.83}\left|\frac{1000\,\frac{\text{N}}{\text{m}^2}}{1\,\text{kPa}}\right|\left|\frac{1\,\text{kJ}}{1000\text{ N}\cdot\text{m}}\right| = 522.33\text{ kJ/kg}$$

<u>State 9 (same as State 9 in problem 8.63)</u>: p_9 = 14 MPa (140 bar), T_9 = 170°C → h_9 = 719.21 kJ/kg, s_9 = 2.0419 kJ/kg·K

<u>State 10 (same as State 10 in problem 8.63)</u>: p_{10} = 1 MPa (10 bar), saturated liquid → h_{10} = 762.81 kJ/kg, s_{10} = 2.1387 kJ/kg·K

<u>State 11 (same as State 11 in problem 8.63)</u>: p_{11} = 200 kPa (2 bar), $h_{11} = h_{10}$ = 762.81 kJ/kg → s_{11} = 2.1861 kJ/kg·K

(a) Applying mass and energy rate balances to the control volume enclosing the closed feedwater heater, the fraction of flow, y', extracted at location 2 is

$$y' = \frac{h_9 - h_8}{h_2 - h_{10}} = \frac{(719.21 - 522.33)\text{ kJ/kg}}{(2901.3 - 762.81)\text{ kJ/kg}} = 0.0921$$

Applying mass and energy rate balances to the control volume enclosing the open feedwater heater, the fraction of flow, y'', extracted at location 3 is

$$y'' = \frac{h_7 - h_6 + y'(h_6 - h_{11})}{h_3 - h_6} = \frac{[504.70 - 391.81 + (0.0921)(391.81 - 762.81)]\text{ kJ/kg}}{(2646.8 - 391.81)\text{ kJ/kg}} = 0.0349$$

For the control volume surrounding the turbine stages

$$\frac{\dot{W}_t}{\dot{m}_1} = h_1 - y'h_2 - y''h_3 - (1 - y' - y'')h_4$$

$$\frac{\dot{W}_t}{\dot{m}_1} = [3486.0 - (0.0921)(2901.3) - (0.0349)(2646.8) - (1 - 0.0921 - 0.0349)(2522.6)]\frac{\text{kJ}}{\text{kg}}$$

$$\frac{\dot{W}_t}{\dot{m}_1} = 924.19 \text{ kJ/kg}$$

For the pumps

$$\frac{\dot{W}_p}{\dot{m}_1} = h_8 - h_7 + (1 - y' - y'')(h_6 - h_5)$$

$$\frac{\dot{W}_p}{\dot{m}_1} = [522.33 - 504.70 + (1 - 0.0921 - 0.0349)(391.81 - 391.66)]\frac{\text{kJ}}{\text{kg}} = 17.76 \text{ kJ/kg}$$

For the working fluid passing through the steam generator

$$\frac{\dot{Q}_{in}}{\dot{m}_1} = h_1 - h_9 = (3486.0 - 719.21)\frac{\text{kJ}}{\text{kg}} = 2766.8 \text{ kJ/kg}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t / \dot{m}_1 - \dot{W}_p / \dot{m}_1}{\dot{Q}_{in} / \dot{m}_1} = \frac{(924.19 - 17.76) \text{ kJ/kg}}{2766.8 \text{ kJ/kg}} = \underline{\mathbf{0.3276\ (32.76\%)}}$$

(b) The ***net*** power developed is

$$\dot{W}_{cycle} = \dot{m}_1(\dot{W}_t / \dot{m}_1 - \dot{W}_p / \dot{m}_1)$$

Thus,

$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{(\dot{W}_t / \dot{m}_1 - \dot{W}_p / \dot{m}_1)}$$

$$\dot{m}_1 = \frac{300 \text{ MW}}{(924.19 - 17.76)\frac{\text{kJ}}{\text{kg}}} \left| \frac{1000 \frac{\text{kJ}}{\text{s}}}{1 \text{ MW}} \right| = \underline{\mathbf{331.0\ kg/s}}$$

(c) The rate of heat transfer from the working fluid as it passes through the condenser is determined by analyzing the steam side of the condenser. For the working fluid passing through the condenser:

$$\dot{Q}_{\text{out}} = (1 - y' - y'')\dot{m}_1(h_4 - h_5)$$

$$\dot{Q}_{\text{out}} = (1 - 0.0921 - 0.0349)\left(331.0\,\frac{\text{kg}}{\text{s}}\right)(2522.6 - 391.66)\,\frac{\text{kJ}}{\text{kg}}\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right| = \underline{\mathbf{616\ MW}}$$

Alternatively, the rate of heat transfer from the working fluid as it passes through the condenser can be determined by applying an overall energy balance to the cycle.

$$\dot{Q}_{\text{out}} = \dot{Q}_{\text{in}} - \dot{W}_{\text{cycle}}$$

where $\dot{Q}_{\text{in}} = \dfrac{\dot{W}_{\text{cycle}}}{\eta} = \dfrac{300\,\text{MW}}{0.3276} = 916\ \text{MW}$

Substituting values and solving give

$$\dot{Q}_{\text{out}} = 916\,\text{MW} - 300\,\text{MW} = \underline{\mathbf{616\ MW}}$$

Compared to the ideal cycle in problem 8.63, the presence of internal irreversibilities in the turbine stages and the pumps results in lower cycle thermal efficiency, higher required mass flow rate of steam entering the first-stage turbine, and greater rate of heat rejection from the working fluid in the condenser.

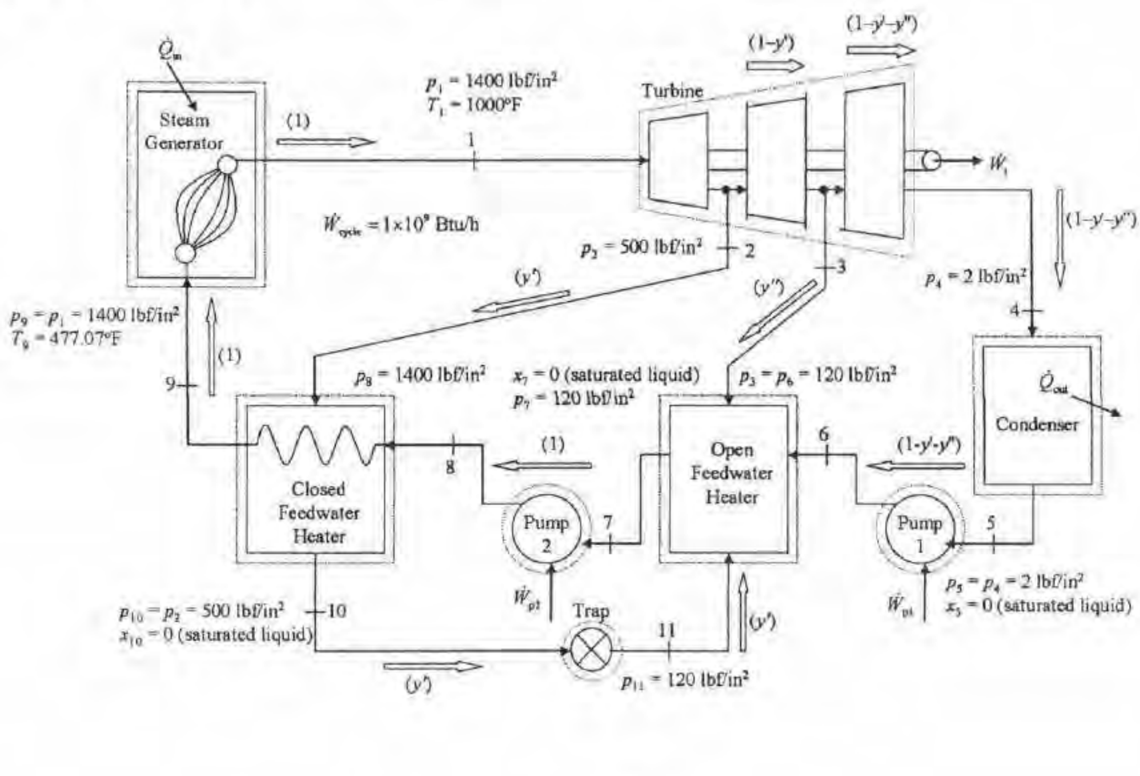
8.65 Water is the working fluid in a regenerative Rankine cycle with one closed feedwater heater and one open feedwater heater. Steam enters the turbine at 1400 $lbf/in.^2$ and 1000°F and expands to 500 $lbf/in.^2$, where some of the steam is extracted and diverted to the closed feedwater heater. Condensate exiting the closed feedwater heater as saturated liquid at 500 $lbf/in.^2$ undergoes a throttling process to 120 $lbf/in.^2$ as it passes through a trap into the open feedwater heater. The feedwater leaves the closed feedwater heater at 1400 $lbf/in.^2$ and a temperature equal to the saturation temperature at 500 $lbf/in.^2$ The remaining steam expands through the second-stage turbine to 120 $lbf/in.^2$, where some of the steam is extracted and diverted to the open feedwater heater operating at 120 $lbf/in.^2$ Saturated liquid exits the open feedwater heater at 120 $lbf/in.^2$ The remaining steam expands through the third-stage turbine to the condenser pressure of 2 $lbf/in.^2$ The net power output of the cycle is 1×10^9 Btu/h. All processes of the working fluid in the turbine stages and pumps are internally reversible. Determine for the cycle

(a) the mass flow rate of steam entering the first stage of the turbine, in lb/h.
(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the steam generator.
(c) the thermal efficiency.

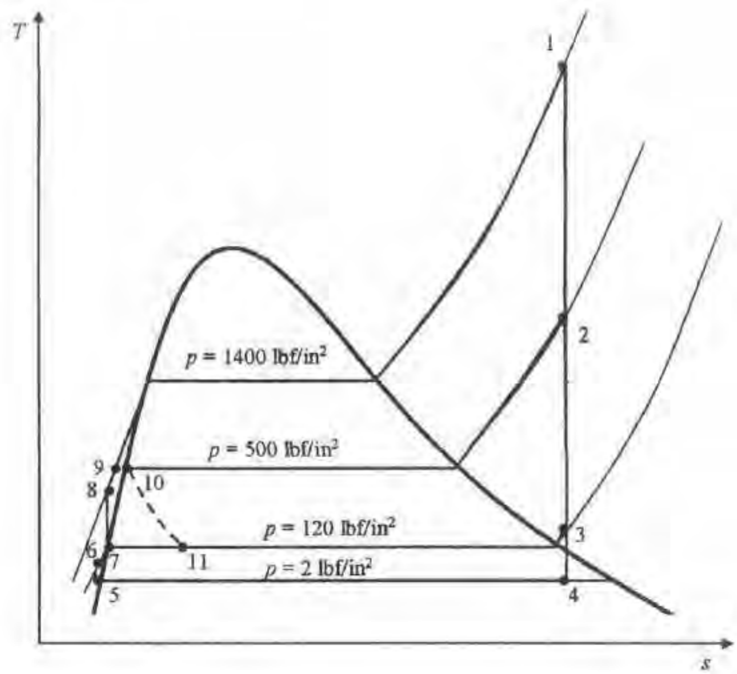
KNOWN: A regenerative Rankine cycle with one closed feedwater heater and one open feedwater heater operates with water as the working fluid. The net power output of the cycle is given.

FIND: Determine the mass flow rate of steam entering the first stage of the turbine, the rate of heat transfer to the working fluid passing through the steam generator, and the thermal efficiency.

SCHEMATIC AND GIVEN DATA:



T-s diagram



ENGINEERING MODEL:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The turbine and pumps operate adiabatically, and the working fluid undergoes an internally reversible process in passing through each device
3. Kinetic and potential energy effects are negligible.
4. Condensate exits the condenser, the closed feedwater heater, and the open feedwater heater as saturated liquid.
5. There is no heat transfer between the outside of the condenser and the surroundings.
6. There is no heat transfer between the outside of the closed feedwater heater and the surroundings.
7. There is no heat transfer between the outside of the open feedwater heater and the surroundings.

ANALYSIS: First, fix each of the principal states.

State 1: p_1 = 1400 lbf/in.^{2}, T_1 = 1000°F → From Table A-4E: h_1 = 1493.5 Btu/lb and s_1 = 1.6094 Btu/(lb·°R)

State 2: p_2 = 500 lbf/in.^{2}, $s_2 = s_1$ → From Table A-4E (interpolated): h_2 = 1354.71 Btu/lb

State 3: p_3 = 120 lbf/in.^{2}, $s_3 = s_1$ → From Table A-4E (interpolated): h_3 = 1208.21 Btu/lb

State 4: $p_4 = 2$ lbf/in.^{2}, $s_4 = s_1 \rightarrow$ From Table A-3E: $x_3 = (1.6094 - 0.1750)/1.7448 = 0.8221$ and $h_4 = h_{f4} + x_4 h_{fg4} = 934.29$ Btu/lb

State 5: $p_5 = p_4 = 2$ lbf/in.^{2}, sat liq. $\rightarrow$ From Table A-3E: $h_5 = h_{f5} = 94.02$ Btu/lb and $v_5 = v_{f5} = 0.01623$ ft^3/lb

State 6: $p_6 = p_3 = 120$ lbf/in.^{2}, $h_6 \approx h_5 + v_5(p_6 - p_5)$

$$h_6 = 94.02\text{ Btu/lb} + 0.01623\left(\frac{\text{ft}^3}{\text{lb}}\right)(120-2)\left(\frac{\text{lbf}}{\text{in}^2}\right)\left|\frac{144\text{ in}^2}{\text{ft}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right| = 94.37\text{ Btu/lb}$$

State 7: $p_7 = p_3 = 120$ lbf/in.^{2}, sat liq. $\rightarrow$ From Table A-3E: $h_7 = h_{f7} = 312.7$ Btu/lb and $v_7 = v_{f7} = 0.01789$ ft^3/lb

State 8: $p_8 = p_1 = 1400$ lbf/in.^{2}, $h_8 \approx h_7 + v_7(p_8 - p_7)$

$$h_8 = 312.7\text{ Btu/lb} + 0.01789\left(\frac{\text{ft}^3}{\text{lb}}\right)(1400-120)\left(\frac{\text{lbf}}{\text{in}^2}\right)\left|\frac{144\text{ in}^2}{\text{ft}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right| = 316.94\text{ Btu/lb}$$

State 9: $p_9 = p_1 = 1400$ lbf/in.^{2}, $T_9 = T_{sat}$ @ $P = 500$ lbf/in^2 = 477.07°F. $\rightarrow$ From Table A-2E (compressed liquid) (interpolated): $h_9 \approx h_{f9} = 460.93$ Btu/lb

State 10: $p_{10} = p_2 = 500$ lbf/in.^{2}, sat liq. $\rightarrow$ From Table A-3E: $h_{10} = h_{f10} = 449.5$ Btu/lb

State 11: $p_{11} = p_3 = 120$ lbf/in.^{2}, throttling process $\rightarrow h_{11} = h_{10} = 449.5$ Btu/lb

(a) The mass flow rate of steam entering the first stage of the turbine can be determined from the net power output of the cycle

$$\dot{W}_{cycle} = \dot{W}_{t1} + \dot{W}_{t2} + \dot{W}_{t3} - \dot{W}_{p1} - \dot{W}_{p2}$$

Let $\dot{m}_1$ be the mass flow rate of the steam entering the first stage of the turbine. Mass and energy balances for control volumes around the three turbine stages and the two pumps give, respectively

$$\dot{W}_{t1} = \dot{m}_1(h_1 - h_2)$$
$$\dot{W}_{t2} = \dot{m}_1(1-y')(h_2 - h_3)$$
$$\dot{W}_{t3} = \dot{m}_1(1-y'-y'')(h_3 - h_4)$$
$$\dot{W}_{p1} = \dot{m}_1(1-y'-y'')(h_6 - h_5)$$
$$\dot{W}_{p2} = \dot{m}_1(h_8 - h_7)$$

Problem 8.65 (Continued) – Page 4

where y' and y'' are the fractions of the steam flow entering the first turbine stage extracted at 2 and 3, respectively.

Substituting for net power output of the cycle

$$\dot{W}_{cycle} = \dot{m}_1(h_1 - h_2) + \dot{m}_1(1-y')(h_2 - h_3) + \dot{m}_1(1-y'-y'')(h_3 - h_4) - \dot{m}_1(1-y'-y'')(h_6 - h_5) - \dot{m}_1(h_8 - h_7)$$

Solving for $\dot{m}_1$

$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{(h_1 - h_2) + (1-y')(h_2 - h_3) + (1-y'-y'')(h_3 - h_4) - (1-y'-y'')(h_6 - h_5) - (h_8 - h_7)}$$

The mass fraction of steam (y') extracted after the first stage of the turbine and the mass fraction of steam (y'') extracted after the second stage of the turbine are both unknown. Analyze the closed feedwater heater to determine y'. Mass and energy balances for a control volume around the closed feedwater heater give

$$0 = \dot{Q} - \dot{W} + \dot{m}_1(1)(h_8 - h_9) + \dot{m}_1(y')(h_2 - h_{10})$$

Since there is no transfer of energy by heat or work, we can solve for y' and substitute values for specific enthalpy to get

$$y' = \frac{h_9 - h_8}{h_2 - h_{10}} = \frac{460.93\,\frac{\text{Btu}}{\text{lb}} - 316.94\,\frac{\text{Btu}}{\text{lb}}}{1354.71\,\frac{\text{Btu}}{\text{lb}} - 449.5\,\frac{\text{Btu}}{\text{lb}}} = 0.159$$

Analyze the open feedwater heater to determine y''. Mass and energy balances for a control volume around the open feedwater heater give

$$0 = \dot{Q} - \dot{W} + \dot{m}_1(y'')h_3 + \dot{m}_1(1-y'-y'')h_6 + \dot{m}_1(y')h_{11} - \dot{m}_1(1)h_7$$

Since there is no transfer of energy by heat or work, we can substitute for y' to get

$$0 = \dot{m}_1(y'')h_3 + \dot{m}_1(1-0.159-y'')h_6 + \dot{m}_1(0.159)h_{11} - \dot{m}_1(1)h_7$$

$$0 = \dot{m}_1(y'')h_3 + \dot{m}_1(0.841-y'')h_6 + \dot{m}_1(0.159)h_{11} - \dot{m}_1(1)h_7$$

Dividing by $\dot{m}_1$ and grouping terms

$$0 = 0.841h_6 + 0.159h_{11} + y''(h_3 - h_6) - h_7$$

Problem 8.65 (Continued) – Page 5

Solving for y'' and substituting values for specific enthalpy, we get

$$y'' = \frac{h_7 - 0.841h_6 - 0.159h_{11}}{h_3 - h_6} = \frac{312.7\frac{\text{Btu}}{\text{lb}} - 0.841\left(94.37\frac{\text{Btu}}{\text{lb}}\right) - 0.159\left(449.5\frac{\text{Btu}}{\text{lb}}\right)}{1208.21\frac{\text{Btu}}{\text{lb}} - 94.37\frac{\text{Btu}}{\text{lb}}} = 0.145$$

Thus

$$(1 - y') = 1 - 0.159 = 0.841 \quad \text{and} \quad (1 - y' - y'') = 1 - 0.159 - 0.145 = 0.696$$

Substituting values and solving for $\dot{m}_1$

$$\dot{m}_1 = \frac{1\times10^9\ \frac{\text{Btu}}{\text{h}}}{[(1493.5-1354.71)+(0.841)(1354.71-1208.21)+(0.696)(1208.21-934.29)-(0.696)(94.37-94.02)-(316.94-312.7)]\frac{\text{Btu}}{\text{lb}}}$$

$$\dot{m}_1 = \underline{\mathbf{2.23\times10^6\ lb/h}}$$

(b) The rate of heat transfer to the working fluid passing through the steam generator can be determined by applying mass and energy balances to a control volume around the steam generator to give

$$\dot{Q}_{\text{in}} = \dot{m}_1(h_1 - h_9) = (2.23\times10^6\ \text{lb/h})(1493.5\ \text{Btu/lb} - 460.93\ \text{Btu/lb}) = \underline{\mathbf{2.30\times10^9\ Btu/h}}$$

(c) The thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = (1\times10^9\ \text{Btu/h})/(2.30\times10^9\ \text{Btu/h}) = \underline{\mathbf{0.4348\ (43.48\%)}}$$

The results of Problem 8.65 can be compared to the results of Problem 8.10 to see some of the effects of incorporating a closed and open feedwater heater on the performance of a Rankine cycle with the same net power output. In this case, use of two feedwater heaters results in higher thermal efficiency and less heat addition, but the steam flow rate entering the first-stage turbine increases. Comparing with the results of Problem 8.55, we see that incorporating an open feedwater heater operating at 120 lbf/in.² results in higher thermal efficiency and less heat addition, but the flow rate entering the first-stage turbine increases.

8.66 Water is the working fluid in a regenerative Rankine cycle with one closed feedwater heater and one open feedwater heater. Steam enters the turbine at 1400 lbf/in.^{2} and 1000°F and expands to 500 lbf/in.^{2}, where some of the steam is extracted and diverted to the closed feedwater heater. Condensate exiting the closed feedwater heater as saturated liquid at 500 lbf/in.^{2} undergoes a throttling process to 120 lbf/in.^{2} as it passes through a trap into the open feedwater heater. The feedwater leaves the closed feedwater heater at 1400 lbf/in.^{2} and a temperature equal to the saturation temperature at 500 lbf/in.^{2} The remaining steam expands through the second-stage turbine to 120 lbf/in.^{2}, where some of the steam is extracted and diverted to the open feedwater heater operating at 120 lbf/in.^{2} Saturated liquid exits the open feedwater heater at 120 lbf/in.^{2} The remaining steam expands through the third-stage turbine to the condenser pressure of 2 lbf/in.^{2} The turbine stages and the pumps each operate adiabatically with isentropic efficiencies of 85%. Flow through the condenser, closed feedwater heater, open feedwater heater and steam generator is at constant pressure. The net power output of the cycle is 1×10^9 Btu/h. Determine for the cycle

(a) the mass flow rate of steam entering the first stage of the turbine, in lb/h.

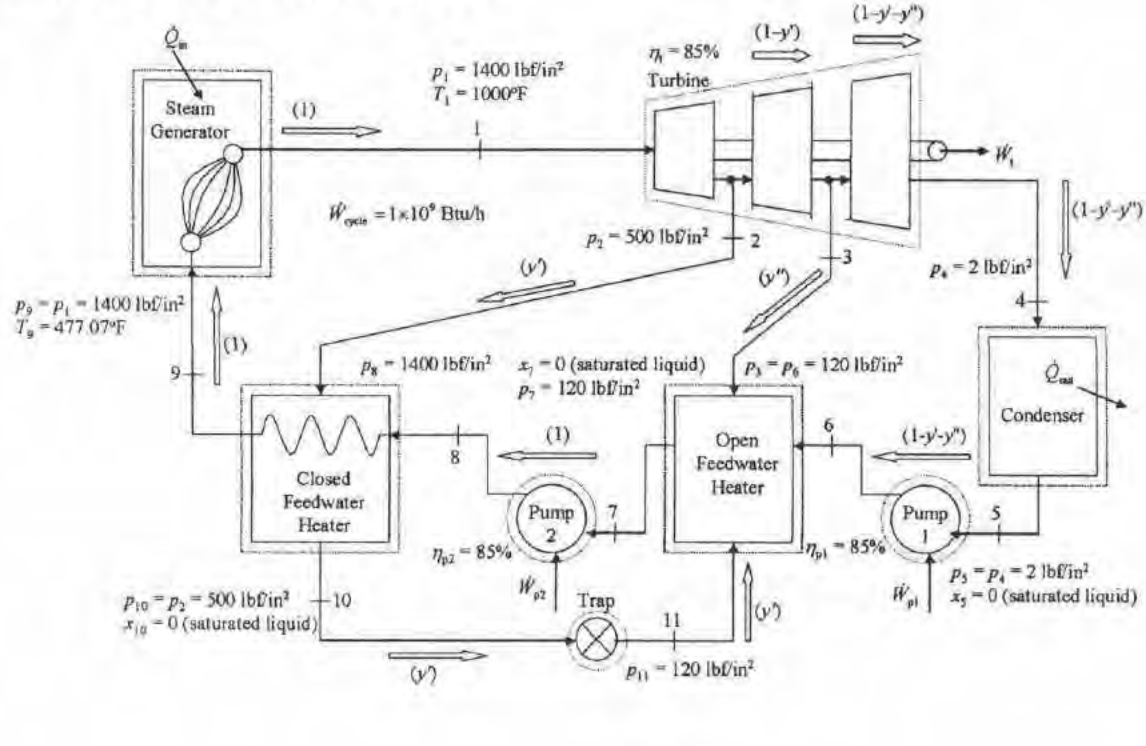
(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the steam generator.

(c) the thermal efficiency.

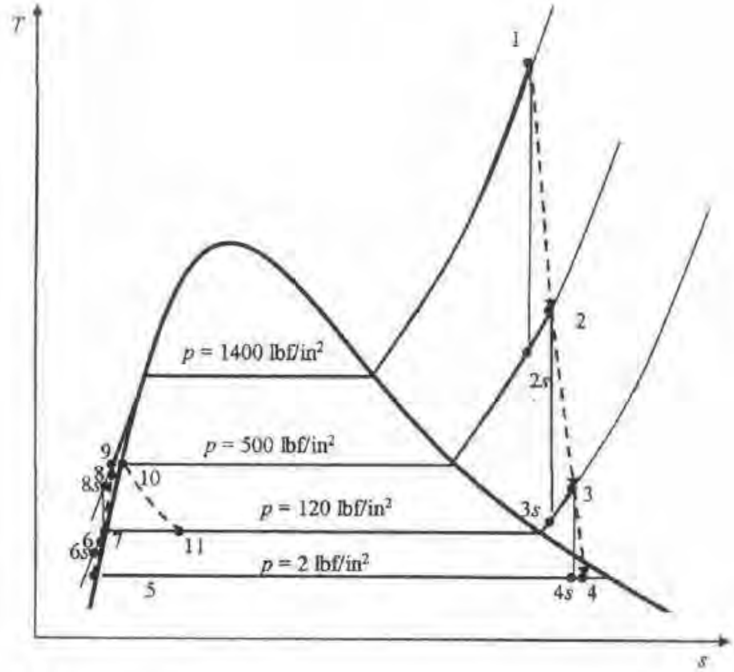
KNOWN: A regenerative Rankine cycle with one closed feedwater heater and one open feedwater heater operates with water as the working fluid. The net power output of the cycle is given.

FIND: Determine the mass flow rate of steam entering the first stage of the turbine, the rate of heat transfer to the working fluid passing through the steam generator, and the thermal efficiency.

SCHEMATIC AND GIVEN DATA:



T-s diagram



ENGINEERING MODEL:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The turbine stages and pumps each operate adiabatically with an isentropic efficiency of 85%.
3. Flow through the steam generator, closed feedwater heater, open feedwater heater, and condenser is at constant pressure.
4. Kinetic and potential energy effects are negligible.
5. Condensate exits the condenser, the closed feedwater heater, and the open feedwater heater as saturated liquid.
6. There is no heat transfer between the outside of the condenser and the surroundings.
7. There is no heat transfer between the outside of the closed feedwater heater and the surroundings.
8. There is no heat transfer between the outside of the open feedwater heater and the surroundings.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 1400$ lbf/in.^{2}, $T_1 = 1000$°F $\rightarrow$ From Table A-4E: $h_1 = 1493.5$ Btu/lb and $s_1 = 1.6094$ Btu/(lb·°R)

State 2*s*: $p_{2s} = p_2 = 500$ lbf/in.^{2}, $s_{2s} = s_1$ $\rightarrow$ From Table A-4E (interpolated): $h_{2s} = 1354.71$ Btu/lb

State 2: $p_2 = 500$ lbf/in.^{2}, $h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 1493.5 - 0.85(1493.5 - 1354.71) = 1375.55$ Btu/lb and $\rightarrow$ From Table A-4E (interpolated): $s_2 = 1.6268$ Btu/(lb·°R)

State 3*s*: $p_{3s} = p_3 = 120$ lbf/in.^{2}, $s_{3s} = s_2 \rightarrow$ From Table A-4E (interpolated): $h_{3s} = 1222.73$ Btu/lb

State 3: $p_3 = 120$ lbf/in.^{2}, $h_3 = h_2 - \eta_t(h_2 - h_{3s}) = 1375.55 - 0.85(1375.55 - 1222.73) = 1245.65$ Btu/lb and $\rightarrow$ From Table A-4E (interpolated): $s_3 = 1.6527$ Btu/(lb·°R)

State 4*s*: $p_{4s} = p_4 = 2$ lbf/in.^{2}, $s_{4s} = s_3 \rightarrow$ From Table A-3E: $x_{3s} = (1.6527 - 0.1750)/1.7448 = 0.8469$ and $h_{4s} = h_{f4s} + x_4 h_{fg4s} = 959.64$ Btu/lb

State 4: $p_4 = 2$ lbf/in.^{2}, $h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 1245.65 - 0.85(1245.65 - 959.64) = 1002.54$ Btu/lb

State 5: $p_5 = p_4 = 2$ lbf/in.^{2}, sat liq. $\rightarrow$ From Table A-3E: $h_5 = h_{f5} = 94.02$ Btu/lb and $v_5 = v_{f5} = 0.01623$ ft^3/lb

State 6: $p_6 = p_3 = 120$ lbf/in.^{2}, $h_6 = h_5 + \dfrac{v_5(p_6 - p_5)}{\eta_{p1}}$

$$h_6 = 94.02 \text{ Btu/lb} + \frac{0.01623\left(\frac{\text{ft}^3}{\text{lb}}\right)(120-2)\left(\frac{\text{lbf}}{\text{in}^2}\right)}{0.85}\left|\frac{144\text{ in}^2}{\text{ft}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right| = 94.44 \text{ Btu/lb}$$

State 7: $p_7 = p_3 = 120$ lbf/in.^{2}, sat liq. $\rightarrow$ From Table A-3E: $h_7 = h_{f7} = 312.7$ Btu/lb and $v_7 = v_{f7} = 0.01789$ ft^3/lb

State 8: $p_8 = p_1 = 1400$ lbf/in.^{2}, $h_8 = h_7 + \dfrac{v_7(p_8 - p_7)}{\eta_{p2}}$

$$h_8 = 312.7 \text{ Btu/lb} + \frac{0.01789\left(\frac{\text{ft}^3}{\text{lb}}\right)(1400-120)\left(\frac{\text{lbf}}{\text{in}^2}\right)}{0.85}\left|\frac{144\text{ in}^2}{\text{ft}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right| = 339.64 \text{ Btu/lb}$$

State 9: $p_9 = p_1 = 1400$ lbf/in.^{2}, $T_9 = T_{sat}$ @ $P = 500$ lbf/in^2 = 477.07°F. $\rightarrow$ From Table A-2E (compressed liquid) (interpolated): $h_9 \approx h_{f9} = 460.93$ Btu/lb

State 10: $p_{10} = p_2 = 500$ lbf/in.^{2}, sat liq. $\rightarrow$ From Table A-3E: $h_{10} = h_{f10} = 449.5$ Btu/lb

State 11: $p_{11} = p_3 = 120$ lbf/in.^{2}, throttling process $\rightarrow h_{11} = h_{10} = 449.5$ Btu/lb

(a) The mass flow rate of steam entering the first stage of the turbine can be determined from the net power output of the cycle

Problem 8.66 (Continued) – Page 4

$$\dot{W}_{cycle} = \dot{W}_{t1} + \dot{W}_{t2} + \dot{W}_{t3} - \dot{W}_{p1} - \dot{W}_{p2}$$

Let $\dot{m}_1$ be the mass flow rate of the steam entering the first stage of the turbine. Mass and energy balances for control volumes around the three turbine stages and the two pumps give, respectively

$$\dot{W}_{t1} = \dot{m}_1 (h_1 - h_2)$$
$$\dot{W}_{t2} = \dot{m}_1 (1 - y')(h_2 - h_3)$$
$$\dot{W}_{t3} = \dot{m}_1 (1 - y' - y'')(h_3 - h_4)$$
$$\dot{W}_{p1} = \dot{m}_1 (1 - y' - y'')(h_6 - h_5)$$
$$\dot{W}_{p2} = \dot{m}_1 (h_8 - h_7)$$

where y' and y'' are the fractions of steam extracted after the first and second turbine stages, respectively.

Substituting for net power output of the cycle

$$\dot{W}_{cycle} = \dot{m}_1 (h_1 - h_2) + \dot{m}_1 (1 - y')(h_2 - h_3) + \dot{m}_1 (1 - y' - y'')(h_3 - h_4) - \dot{m}_1 (1 - y' - y'')(h_6 - h_5) - \dot{m}_1 (h_8 - h_7)$$

Solving for $\dot{m}_1$

$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{(h_1 - h_2) + (1 - y')(h_2 - h_3) + (1 - y' - y'')(h_3 - h_4) - (1 - y' - y'')(h_6 - h_5) - (h_8 - h_7)}$$

The mass fractions y' and y'' are both unknown. First, analyze the closed feedwater heater to determine y'. Mass and energy balances for a control volume around the closed feedwater heater give

$$0 = \dot{Q} - \dot{W} + \dot{m}_1 (1)(h_8 - h_9) + \dot{m}_1 (y')(h_2 - h_{10})$$

Since there is no transfer of energy by heat or work, we can solve for y' and substitute values for specific enthalpy to get

$$y' = \frac{h_9 - h_8}{h_2 - h_{10}} = \frac{460.93 \frac{\text{Btu}}{\text{lb}} - 339.64 \frac{\text{Btu}}{\text{lb}}}{1375.55 \frac{\text{Btu}}{\text{lb}} - 449.5 \frac{\text{Btu}}{\text{lb}}} = 0.131$$

Now we can analyze the open feedwater heater to determine y''. Mass and energy balances for a control volume around the open feedwater heater give

$$0 = \dot{Q} - \dot{W} + \dot{m}_1(y'')h_3 + \dot{m}_1(1 - y' - y'')h_6 + \dot{m}_1(y')h_{11} - \dot{m}_1(1)h_7$$

Since there is no transfer of energy by heat or work, we can substitute for y' to get

$$0 = \dot{m}_1(y'')h_3 + \dot{m}_1(1 - 0.131 - y'')h_6 + \dot{m}_1(0.131)h_{11} - \dot{m}_1(1)h_7$$

$$0 = \dot{m}_1(y'')h_3 + \dot{m}_1(0.869 - y'')h_6 + \dot{m}_1(0.131)h_{11} - \dot{m}_1(1)h_7$$

Dividing by $\dot{m}_1$ and grouping terms

$$0 = 0.869h_6 + 0.131h_{11} + y''(h_3 - h_6) - h_7$$

Solving for y'' and substituting values for specific enthalpy, we get

$$y'' = \frac{h_7 - 0.869h_6 - 0.131h_{11}}{h_3 - h_6} = \frac{312.7\frac{\text{Btu}}{\text{lb}} - 0.869\left(94.44\frac{\text{Btu}}{\text{lb}}\right) - 0.131\left(449.5\frac{\text{Btu}}{\text{lb}}\right)}{1245.65\frac{\text{Btu}}{\text{lb}} - 94.44\frac{\text{Btu}}{\text{lb}}} = 0.149$$

Thus,

$$(1 - y') = 1 - 0.131 = 0.869 \quad \text{and} \quad (1 - y' - y'') = 1 - 0.131 - 0.149 = 0.720$$

Substituting values and solving for $\dot{m}_1$

$$\dot{m}_1 = \frac{1\times 10^9\,\frac{\text{Btu}}{\text{h}}}{[(1493.5 - 1375.55) + (0.869)(1375.55 - 1245.65) + (0.720)(1245.65 - 1002.54) - (0.720)(94.44 - 94.02) - (339.64 - 312.7)]\frac{\text{Btu}}{\text{lb}}}$$

$$\dot{m}_1 = \underline{\mathbf{2.64\times 10^6\ lb/h}}$$

(b) The rate of heat transfer to the working fluid passing through the boiler can be determined by applying mass and energy balances to a control volume around the steam generator to give

$$\dot{Q}_{\text{in}} = \dot{m}_1(h_1 - h_9) = (2.64\times 10^6\ \text{lb/h})(1493.5\ \text{Btu/lb} - 460.93\ \text{Btu/lb}) = \underline{\mathbf{2.73\times 10^9\ Btu/h}}$$

(c) The thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = (1\times 10^9\ \text{Btu/h})/(2.73\times 10^9\ \text{Btu/h}) = \underline{\mathbf{0.3663\ (36.63\%)}}$$

Problem 8.66 (Continued) – Page 6

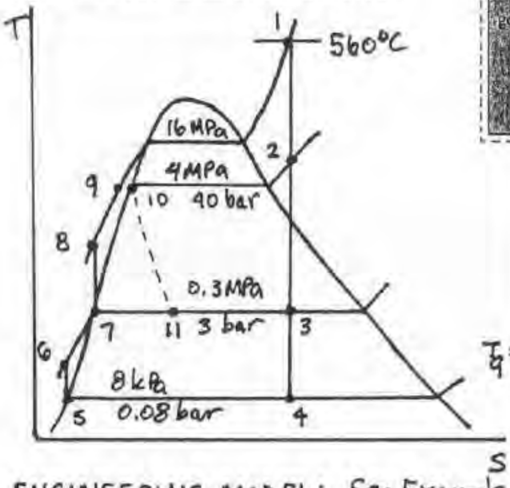
The results of Problem 8.66 can be compared to the results of Problem 8.19 to see some of the effects of incorporating a closed and an open feedwater heater on the performance of an actual Rankine cycle with the same net power output. In this case, the addition of the feedwater heaters results in slightly higher thermal efficiency and slightly less heat addition, but the mass flow rate entering the first-stage turbine is increased.

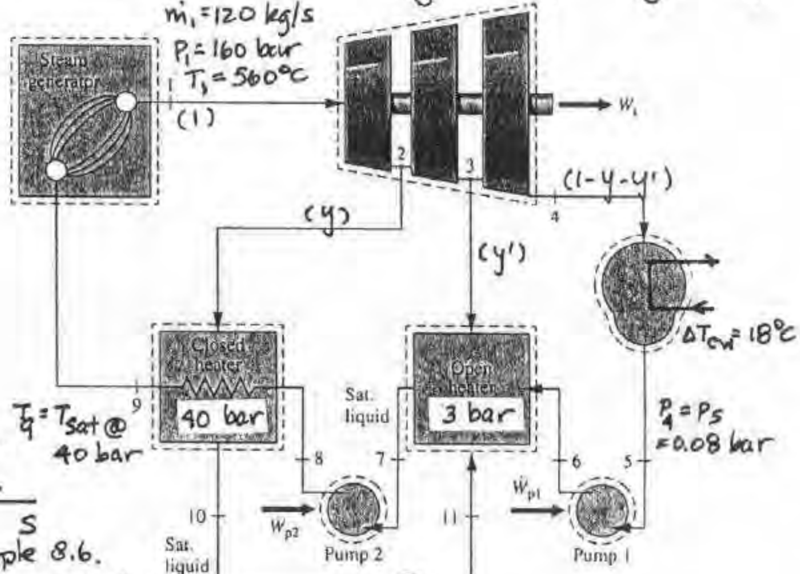
PROBLEM 8.67

KNOWN: Water is the working fluid in a regenerative vapor power cycle with one closed and one open feedwater heater. Data are specified at various locations for the cycle. The mass flow rate of steam entering the first-stage turbine is known.

FIND: Determine (a) the net power, (b) rate of heat addition, (c) the thermal efficiency, and (d) the mass flow rate of cooling water passing through the condenser.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 8.6.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 160$ bar, $T_1 = 560°C \Rightarrow h_1 = 3465.4$ kJ/kg, $s_1 = 6.5132$ kJ/kg·K

State 2: $p_2 = 40$ bar, $s_2 = s_1 \Rightarrow$ Interpolating in Table A-4: $h_2 = 3050.9$ kJ/kg

State 3: $p_3 = 3$ bar, $s_3 = s_2 \Rightarrow x_3 = \dfrac{s_3 - s_{f3}}{s_{g3} - s_{f3}} = 0.9100$, $h_3 = 2530.5$ kJ/kg

State 4: $p_4 = 0.08$ bar, $s_4 = s_3 \Rightarrow x_4 = \dfrac{s_4 - s_{f4}}{s_{g4} - s_{f4}} = 0.7753$, $h_4 = 2037.0$ kJ/kg

State 5: $p_5 = 0.08$ bar, sat. liquid $\Rightarrow h_5 = 173.88$ kJ/kg

State 6: $h_6 \approx h_5 + v_5(p_6 - p_5)$

$$= 173.88 \frac{kJ}{kg} + (1.0084 \times 10^{-3}) \frac{m^3}{kg}(3 - 0.08)\,bar \left|\frac{10^5 N/m^2}{1\,bar}\right|\left|\frac{1\,kJ}{10^3 N\cdot m}\right|$$

$$= 173.88 + 0.294 = 174.17 \text{ kJ/kg}$$

State 7: $p_7 = 3$ bar, sat. liquid $\Rightarrow h_7 = 561.47$ kJ/kg

State 8: $h_8 \approx h_7 + v_7(p_8 - p_7) = 561.47 + (1.0732 \times 10^{-3})(160 - 3)\left|\frac{10^5}{10^3}\right|$

$$= 561.47 + 16.85 = 578.32 \text{ kJ/kg}$$

State 9: Assume $h_9 \approx h_{f@T_{10}} = 1087.3$ kJ/kg

State 10: $p_{10} = 40$ bar, sat. liquid $\Rightarrow h_{10} = 1087.3$ kJ/kg

State 11: Throttling process $\Rightarrow h_{11} = h_{10} = 1087.3$ kJ/kg

(a) Apply mass and energy balances to the closed heater to get the fraction extracted y

$$0 = y(h_2 - h_{10}) + (h_8 - h_9)$$

or
$$y = \frac{h_9 - h_8}{h_2 - h_{10}} = \frac{1087.3 - 578.32}{3050.9 - 1087.3} = 0.2592$$

For the open feedwater heater
$$0 = y' h_3 + y h_{11} + (1-y-y') h_6 - h_7$$

Solving for y'
$$y' = \frac{h_7 - h_6 + y(h_6 - h_{11})}{(h_3 - h_6)} = \frac{561.47 - 174.17 + (0.2592)(174.17 - 1087.3)}{(2530.5 - 174.17)}$$
$$= 0.0639$$

For the control volume enclosing the turbine stages
$$\dot{W}_t = \dot{m}_1 [h_1 - y h_2 - y' h_3 - (1-y-y') h_4]$$
$$= \left(120 \frac{kg}{s}\right) [3465.4 - (.2592)(3050.9) - (0.0639)(2530.5) - (0.6769)(2037.0)] \frac{kJ}{kg} \left| \frac{1\ kW}{1\ kJ/s} \right|$$
$$= 1.3609 \times 10^5\ kW$$

For the pumps
$$\dot{W}_p = \dot{W}_{p1} + \dot{W}_{p2} = \dot{m}_1 [(1-y-y')(h_6 - h_5) + (h_8 - h_7)]$$
$$= (120)[(0.6769)(174.17 - 173.88) + (578.32 - 561.47)] = 2045.6\ kW$$

The net power developed is
$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p = 1.340 \times 10^5\ kW \longleftarrow \underline{\dot{W}_{cycle}}$$

(b) For the steam generator
$$\dot{Q}_{in} = \dot{m}_1 (h_1 - h_9) = (120)(3465.4 - 1087.3) = 2.854 \times 10^5\ kW \longleftarrow \underline{\dot{Q}_{in}}$$

(c) The thermal efficiency is
$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = 0.4695\ (46.95\%) \longleftarrow \underline{\eta}$$

(d) For a control volume enclosing the condenser
$$0 = \dot{m}_1 (1-y-y')(h_4 - h_5) + \dot{m}_{cw}(h_{cw,in} - h_{cw,out})$$

Solving for $\dot{m}_{cw}$
$$\dot{m}_{cw} = \frac{\dot{m}_1 (1-y-y')(h_4 - h_5)}{(h_{cw,out} - h_{cw,in})}$$

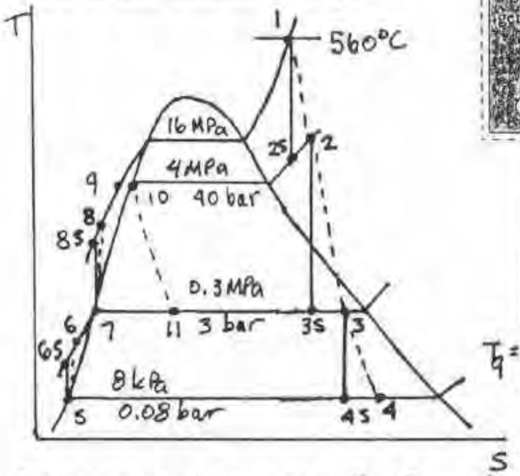
Using $h_{cw,out} - h_{cw,in} = c_{cw}(T_{cw,out} - T_{cw,in})$ with $c_{cw} = 4.179$ from Table A-19
$$\dot{m}_{cw} = \frac{(120\ kg/s)(0.6769)(2037.0 - 173.88)\ kJ/kg}{(4.179)(18)\ kJ/kg}$$
$$= \frac{1.5134 \times 10^5}{75.222} = 1354\ kg/s \longleftarrow \underline{\dot{m}_{cw}}$$

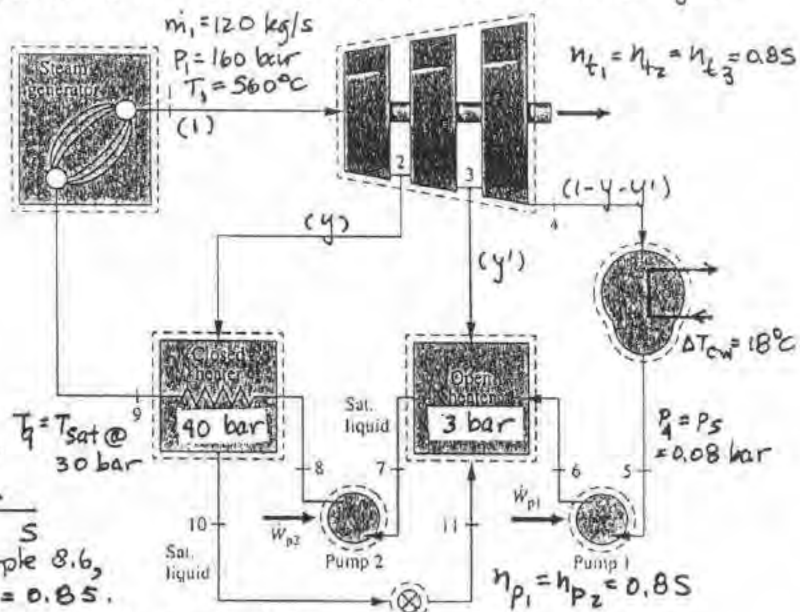
PROBLEM 8.68

KNOWN: The regenerative vapor power cycle of Problem 8.67 is modified to include turbine stage and pump isentropic efficiencies of 0.85.

FIND: Answer the same questions as in Problem 8.67 for the modified cycle.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 8.6, except $\eta_{t_1} = \eta_{t_2} = \eta_{t_3} = \eta_{p_1} = \eta_{p_2} = 0.85$.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 160$ bar, $T_1 = 560°C \Rightarrow h_1 = 3465.4$ kJ/kg, $s_1 = 6.5132$ kJ/kg·K

State 2: Using the first-stage turbine efficiency: $\eta_{t_1} = (h_1 - h_2)/(h_1 - h_{2s})$

With $h_{2s} = 3050.9$ kJ/kg from Problem 8.67

$$h_2 = h_1 - \eta_{t_1}(h_1 - h_{2s}) = 3113.1 \text{ kJ/kg}. \text{ From Table A-4; } s_2 = 6.6148 \text{ kJ/kg·K}$$

State 3: $p_3 = 3$ bar, $s_{3s} = s_2 \Rightarrow x_{3s} = \dfrac{s_{3s} - s_{f3}}{s_{g3} - s_{f3}} = 0.9291$, $h_{3s} = 2571.9$ kJ/kg

Using the second-stage turbine efficiency

$$h_3 = h_2 - \eta_{t_2}(h_2 - h_{3s}) = 2653.1 \text{ kJ/kg} \; ; \; s_3 = 6.8142 \; (x_3 = 0.9666)$$

State 4: $p_4 = 0.08$ bar, $s_{4s} = s_3 \Rightarrow x_{4s} = 0.8148$; $h_{4s} = 2131.9$ kJ/kg

Using the third-stage turbine efficiency

$$h_4 = h_3 - \eta_{t_3}(h_3 - h_{4s}) = 2210.1 \text{ kJ/kg}$$

State 5: $p_5 = 0.08$ bar, sat. liquid $\Rightarrow h_5 = 173.88$ kJ/kg

State 6: Using the pump efficiency with $h_{6s} = 174.17$ kJ/kg from Problem 8.67

$$h_6 = h_5 + (h_{6s} - h_5)/\eta_{p_1} = 174.22 \text{ kJ/kg}$$

State 7: $p_7 = 3$ bar, sat. liquid $\Rightarrow h_7 = 561.47$ kJ/kg

State 8: Using the pump efficiency with $h_{8s} = 578.32$ kJ/kg from Problem 8.67

$$h_8 = h_7 + (h_{8s} - h_7)/\eta_{p_2} = 581.29 \text{ kJ/kg}$$

State 9: Assume $h_9 \approx h_{f@T_{10}} = 1087.3$ kJ/kg

State 10: $p_{10} = 40$ bar, sat. liquid $\Rightarrow h_{10} = 1087.3$ kJ/kg

State 11: Throttling process $\Rightarrow h_{11} = h_{10} = 1087.3$ kJ/kg

PROBLEM 8.68

(a) $$y = \frac{h_9 - h_8}{h_2 - h_{10}} = \frac{1087.3 - 581.29}{3113.1 - 1087.3} = 0.2498$$

$$y = \frac{h_7 - h_6 + y(h_6 - h_{11})}{(h_3 - h_6)} = \frac{561.47 - 174.22 + (0.2498)(174.22 - 1087.3)}{(2653.1 - 174.22)} = 0.0642$$

$$\dot{W}_t = \dot{m}_1 [h_1 - y h_2 - y' h_3 - (1 - y - y') h_4]$$

$$= (120\ \tfrac{kg}{s})[3465.4 - (0.2498)(3113.1) - (0.0642)(2653.1) - (0.6860)(2210.1)]\ \frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 1.202 \times 10^5\ kW$$

$$\dot{W}_p = \dot{m}_1 [(1 - y - y')(h_6 - h_5) + (h_8 - h_7)]$$

$$= (120)[(0.6860)(174.22 - 173.88) + (581.29 - 561.47)] = 2406\ kW$$

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p = 1.178 \times 10^5\ kW \longleftarrow \dot{W}_{cycle}$$

(b) $$\dot{Q}_{in} = \dot{m}_1 (h_1 - h_9) = (120)(3465.4 - 1087.3) = 2.854 \times 10^5\ kW \longleftarrow \dot{Q}_{in}$$

(c) $$\eta = \dot{W}_{cycle} / \dot{Q}_{in} = 0.413\ (41.3\ \%) \longleftarrow \eta$$

(d)

1 $$\dot{m}_{cw} = \frac{\dot{m}_1 (1 - y - y')(h_4 - h_5)}{c_{cw}\, \Delta T_{cw}}$$

$$= \frac{(120\ kg/s)(0.6860)(2210.1 - 173.88)\ kJ/kg}{(4.179)(18)\ kJ/kg}$$

$$= 2228\ kg/s \longleftarrow \dot{m}_{cw}$$

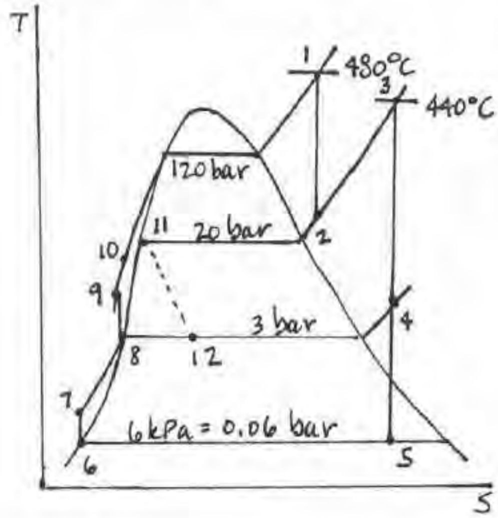
1. These results can be compared with those of Problem 8.67 to see some of the effects of turbine stage and pump isentropic efficiencies on cycle performance.

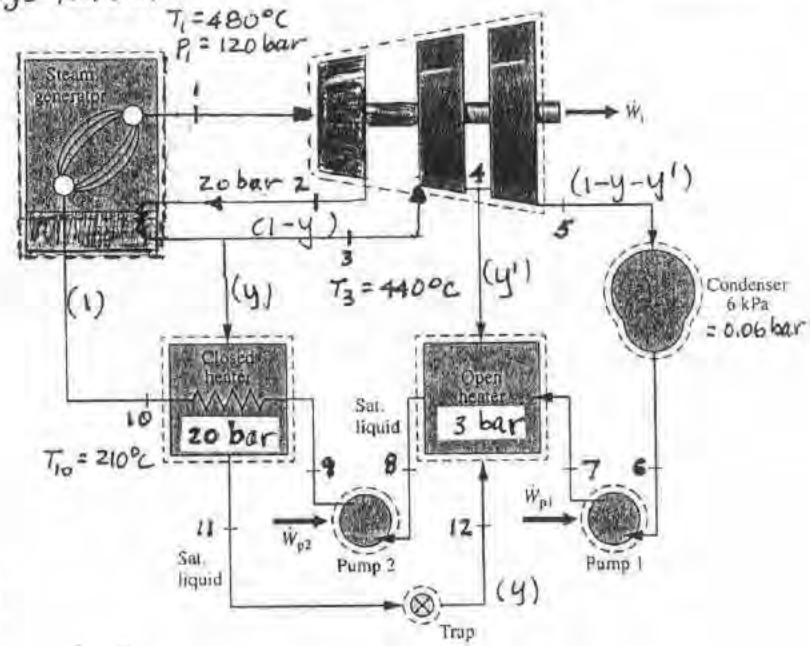
PROBLEM 8.69

KNOWN: Water is the working fluid in a regenerative vapor power cycle with one closed and one open feedwater heater and reheat. Data are known at various locations in the cycle.

FIND: Determine (a) the rate of heat addition per kg of steam entering the first-stage turbine, (b) the thermal efficiency, and (c) the rate of heat transfer to cooling water passing through the condenser, per kg of steam entering the first-stage turbine

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 8.6.

ANALYSIS: First, fix each of the principal states.

From the solution to problem 8.60: (Note - some states are renumbered)

$h_1 = 3293.5$ kJ/kg $\quad h_9 = 574.03$ kJ/kg

$h_2 = 2836.7$ kJ/kg $\quad h_{10} = 897.76$ kJ/kg

$h_6 = 151.53$ kJ/kg $\quad h_{11} = 908.79$ kJ/kg

$h_7 = 151.83$ kJ/kg $\quad h_{12} = 908.79$ kJ/kg

$h_8 = 561.47$ kJ/kg

State 3: $P_3 = 20$ bar, $T_3 = 440°C \Rightarrow h_3 = 3335.5$, $s_3 = 7.2540$ kJ/kg·K

State 4: $P_4 = 3$ bar, $s_4 = s_3 = 7.2540$ kJ/kg $\Rightarrow$ From Table A-4; $h_4 = 2839.5$ kJ/kg

State 5: $P_5 = 0.06$ bar, $s_5 = s_4 \Rightarrow x_5 = \dfrac{s_5 - s_{g5}}{s_{g5} - s_{f5}} = 0.8622$, $h_5 = 2234.4$ kJ/kg

(a) For the control volume enclosing the steam generator

$$\dot{Q}_{in}/\dot{m}_1 = (h_1 - h_{10}) + (h_3 - h_2)$$

$$= (3293.5 - 897.76) + (3335.5 - 2836.7) = 2894.5 \text{ kJ/kg} \longleftarrow \dot{Q}_{in}/\dot{m}_1$$

(b) First, get the fractions extracted at 3 and 4 from energy and mass balances on the closed and open heater control volumes, respectively.

$$0 = y(h_3 - h_{11}) + (h_9 - h_{10})$$

PROBLEM 8.69 (Cont'd.)

or
$$y = \frac{h_{10} - h_9}{h_3 - h_{11}} = \frac{897.76 - 574.03}{3335.5 - 908.79} = 0.1334$$

Further
$$0 = y' h_4 + (1-y-y') h_7 + y h_{12} - h_8$$

or
$$y' = \frac{h_8 - h_7 + y(h_7 - h_{12})}{h_4 - h_7} = \frac{561.47 - 151.83 + (0.1334)(151.83 - 908.79)}{2839.5 - 151.83}$$
$$= 0.1148$$

For the control volume enclosing the turbine stages
$$\dot{W}_t/\dot{m}_1 = h_1 - h_2 + (1-y) h_3 - y' h_4 - (1-y-y') h_5$$
$$= 3293.5 - 2836.7 + (0.8666)(3335.5) - (0.1148)(2839.5) - (0.7518)(2234.4)$$
$$= 1341.5 \text{ kJ/kg}$$

For the pumps
$$\dot{W}_p/\dot{m}_1 = \dot{W}_{P1}/\dot{m}_1 + \dot{W}_{P2}/\dot{m}_1 = (1-y-y')(h_7 - h_6) + (h_9 - h_8)$$
$$= (0.7518)(151.83 - 151.53) + (574.03 - 561.47)$$
$$= 12.78 \text{ kJ/kg}$$

$$\frac{\dot{W}_{cycle}}{\dot{m}_1} = \frac{\dot{W}_t}{\dot{m}_1} - \frac{\dot{W}_p}{\dot{m}_1} = 1328.7 \text{ kJ/kg}$$

The thermal efficiency is
$$\eta = \frac{\dot{W}_{cycle}/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{1328.7}{2894.5} = 0.459 \quad (45.9\%) \longleftarrow \quad \eta$$

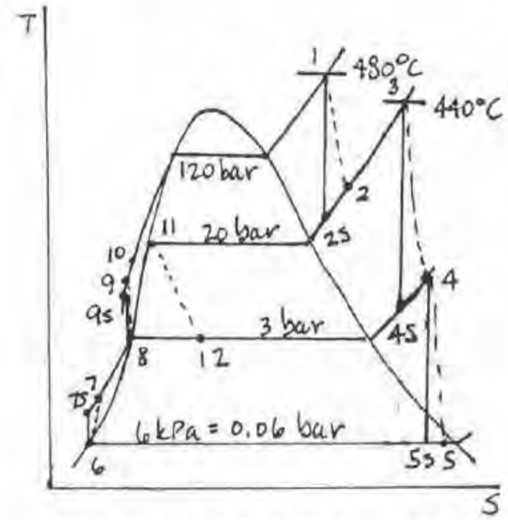
(c) For the condenser
$$\dot{Q}_{out}/\dot{m}_1 = (1-y-y')(h_5 - h_6)$$
$$= (0.7518)(2234.4 - 151.53) = 1565.9 \text{ kJ/kg} \longleftarrow \quad \dot{Q}_{out}/\dot{m}_1$$

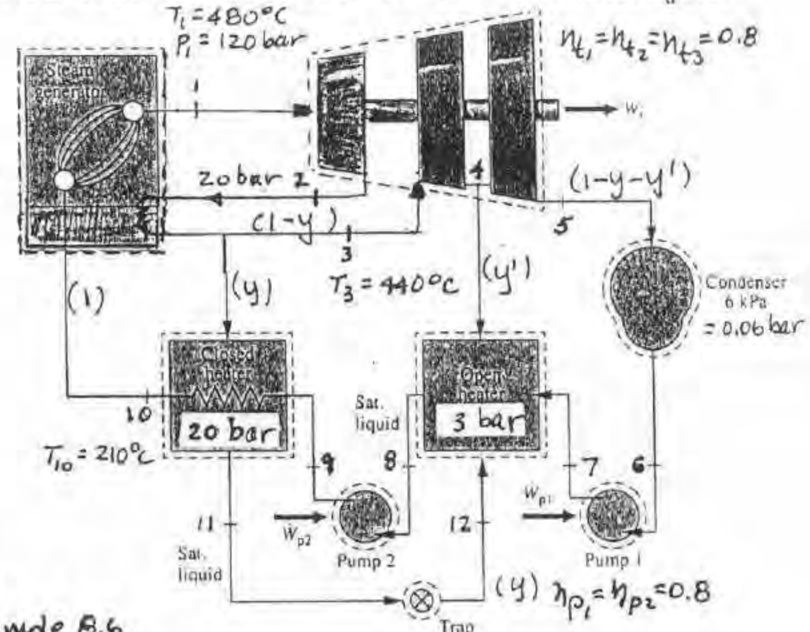
PROBLEM 8.70

KNOWN: The regenerative vapor power cycle of Problem 8.69 is modified to include turbine stage and pump isentropic efficiencies of 0.8.

FIND: Answer the same questions as in Problem 8.69 for the modified cycle.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 8.6, except $\eta_{t_1} = \eta_{t_2} = \eta_{t_3} = \eta_{p_1} = \eta_{p_2} = 0.8$.

ANALYSIS: First, fix each of the principal states. From the solution to Problem 8.69:

$h_1 = 3293.5$ kJ/kg $\qquad h_6 = 151.53$ kJ/kg, $h_{7s} = 151.83$

$h_{2s} = 2836.7 \qquad h_8 = 561.47$, $h_{9s} = 574.03$

$h_3 = 3335.5 \qquad h_{10} = 897.76$

$h_{4s} = 2837.5 \qquad h_{11} = h_{12} = 908.79$

State 2: Using the first-stage turbine efficiency; $\eta_{t_1} = (h_1 - h_2)/(h_1 - h_{2s})$

$$h_2 = h_1 - \eta_{t_1}(h_1 - h_{2s}) = 2928.1 \text{ kJ/kg}$$

State 4: For the second-stage turbine

$$h_4 = h_3 - \eta_{t_2}(h_3 - h_{4s}) = 2938.7 \text{ kJ/kg}; \; s_4 = 7.4600 \text{ kJ/kg}\cdot\text{K}$$

State 5: $P_5 = 0.06$ bar, $s_{5s} = s_4 \Rightarrow x_{5s} = 0.8885$; $h_{5s} = 2298.1$ kJ/kg

With $\eta_{t_3} = 0.8$; $h_5 = h_4 - (h_4 - h_{5s}) = 2426.2$ kJ/kg

State 7: Using the pump efficiency; $\eta_{p_1} = (h_{7s} - h_6)/(h_7 - h_6)$

$$h_7 = h_6 + (h_{7s} - h_6)/\eta_{p_1} = 151.91 \text{ kJ/kg}$$

State 9: For pump 2

$$h_9 = h_8 + (h_{9s} - h_8)/\eta_{p_2} = 577.17 \text{ kJ/kg}$$

(a) For the control volume enclosing the steam generator

$$\dot{Q}_{in}/\dot{m}_1 = (h_1 - h_{10}) + (h_3 - h_2)$$

$$= (3293.5 - 897.76) + (3335.5 - 2928.1) = 2803.1 \text{ kJ/kg} \longleftarrow \dot{Q}_{in}/\dot{m}_1$$

(b) For the closed feedwater heater

$$y = \frac{h_{10} - h_9}{h_3 - h_{11}} = \frac{897.76 - 577.17}{3335.5 - 908.79} = 0.1321$$

PROBLEM 8.70 (Cont'd.)

For the open feedwater heater

$$y' = \frac{h_8 - h_7 + y(h_7 - h_{12})}{h_4 - h_7}$$

$$= \frac{561.47 - 151.91 + (0.1321)(151.91 - 908.79)}{2938.7 - 151.91} = 0.1111$$

For the control volume enclosing the turbine stages

$$\dot{W}_t/\dot{m}_1 = h_1 - h_2 + (1-y)h_3 - y'h_4 - (1-y-y')h_5$$

$$= 3293.5 - 2928.1 + (0.8679)(3335.3) - (0.1111)(2938.7) - (0.7568)(2426.2)$$

$$= 1097.6 \text{ kJ/kg}$$

For the pumps

$$\frac{\dot{W}_p}{\dot{m}_1} = \frac{\dot{W}_{p1}}{\dot{m}_1} + \frac{\dot{W}_{p2}}{\dot{m}_1} = (1-y-y')(h_7 - h_6) + (h_9 - h_8)$$

$$= (0.7568)(151.91 - 151.53) + (577.17 - 561.47)$$

$$= 15.99 \text{ kJ/kg}$$

$$\frac{\dot{W}_{cycle}}{\dot{m}_1} = \frac{\dot{W}_t}{\dot{m}_1} - \frac{\dot{W}_p}{\dot{m}_1} = 1081.6 \text{ kJ/kg} \longleftarrow \underline{\dot{W}_{cycle}/\dot{m}_1}$$

The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{1081.6}{2803.1} = 0.386\ (38.6\%) \longleftarrow \underline{\eta}$$

(c) For the condenser

① $$\dot{Q}_{out}/\dot{m}_1 = (1-y-y')(h_5 - h_6)$$

$$= (0.7568)(2426.2 - 151.53) = 1721.5 \text{ kJ/kg} \longleftarrow \underline{\dot{Q}_{out}/\dot{m}_1}$$

---

1. These results can be compared with those of Problem 8.69 to see some of the effects of turbine stage and pump irreversibilities on cycle performance.

PROBLEM 8.71

Refer to Problem 8.70

IT Code

```
p1 = 120  // bar
T1 = 480  // °C
p2 = 20  // bar
p3 = p2
T3 = 440  // °C
p4 = 3  // bar
p5 = 0.06  // bar
p6 = p5
p7 = p4
p8 = p4
p9 = p1
p10 = p1
p11 = p2
p12 = p4
T10 = 210 // °C
etat = .8
etap = etat
mdot1 = 1
etatt = etat * 100

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
s2s = s1
h2s = h_Ps("Water/Steam", p2, s2s)
h2 = h1 - etat * (h1 - h2s)
h3 = h_PT("Water/Steam", p3, T3)
s3 = s_PT("Water/Steam", p3, T3)
s4s = s3
h4s = h_Ps("Water/Steam", p4, s4s)
h4 = h3 - etat * (h3 - h4s)
s4 = s_Ph("Water/Steam", p4, h4)
s5s = s4
h5s = h_Ps("Water/Steam", p5, s5s)
h5 = h4 - etat * (h4 - h5s)
h6 = hsat_Px("Water/Steam", p6, 0)
v6 = vsat_Px("Water/Steam", p6, 0)
h7s = h6 + v6 * (p6 - p5) * 100
h7 = h6 + (h7s - h6) / etap
h8 = hsat_Px("Water/Steam", p8, 0)
v8 = vsat_Px("Water/Steam", p8, 0)
h9s = h8 + v8 * (p9 - p8) * 100
h9 = h8 + (h9s - h8) / etap
psat = Psat_T("Water/Steam", T10)
h10 = hsat_Px("Water/Steam", psat, 0)
h11 = hsat_Px("Water/Steam", p11, 0)
h12 = h11

Qdotin = mdot1 * ((h1 - h10) + (h3 - h2))
y = (h10 - h9) / (h3 - h11)
y' = (h8 - h7 + y * (h7 - h12)) / (h4 - h7)
Wdott = mdot1 * (h1 - h2 + (1 - y) * h3 - y' * h4 - (1 - y - y') * h5)
Wdotp1 = mdot1 * ((1 - y - y') * (h7 - h6))
Wdotp2 = mdot1 * (h9 - h8)
Wdotcycle = Wdott - Wdotp1 - Wdotp2
eta = Wdotcycle / Qdotin
Qdotout = mdot1 * ((1 - y - y') * (h5 - h6))
```

IT Results ($\eta_{t1} = \eta_{t2} = \eta_{t3} = \eta_{p1} = \eta_{p2} = 0.8$)

$\dot{Q}_{in} / \dot{m}_1 = 2803$ kJ/kg

$\dot{Q}_{out} / \dot{m}_1 = 1721$ kJ/kg

$\dot{W}_{cycle} / \dot{m}_1 = 1082$ kJ/kg

$\eta = 0.3862$

$h_1 = 3293$ kJ/kg

$h_2 = 2927$ kJ/kg

$h_3 = 3335$ kJ/kg

$h_4 = 2938$ kJ/kg

$h_5 = 2426$ kJ/kg

$h_6 = 151$ kJ/kg

$h_7 = 151$ kJ/kg

$h_8 = 561.2$ kJ/kg

$h_9 = 576.9$ kJ/kg

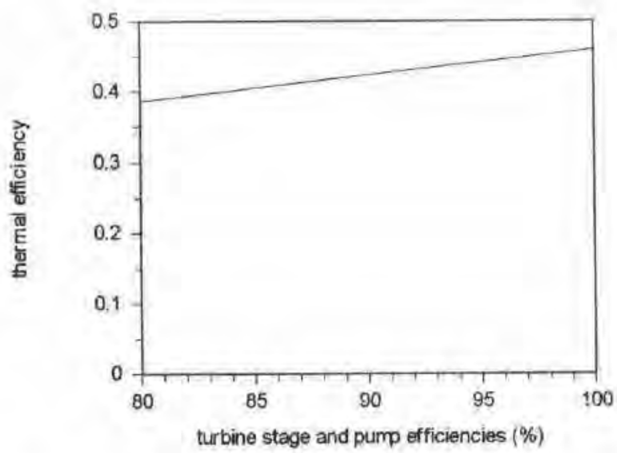
$h_{10} = 897.9$ kJ/kg

$h_{11} = 908.9$ kJ/kg

$h_{12} = 908.9$ kJ/kg

$y = 0.1323$

$y' = 0.1112$



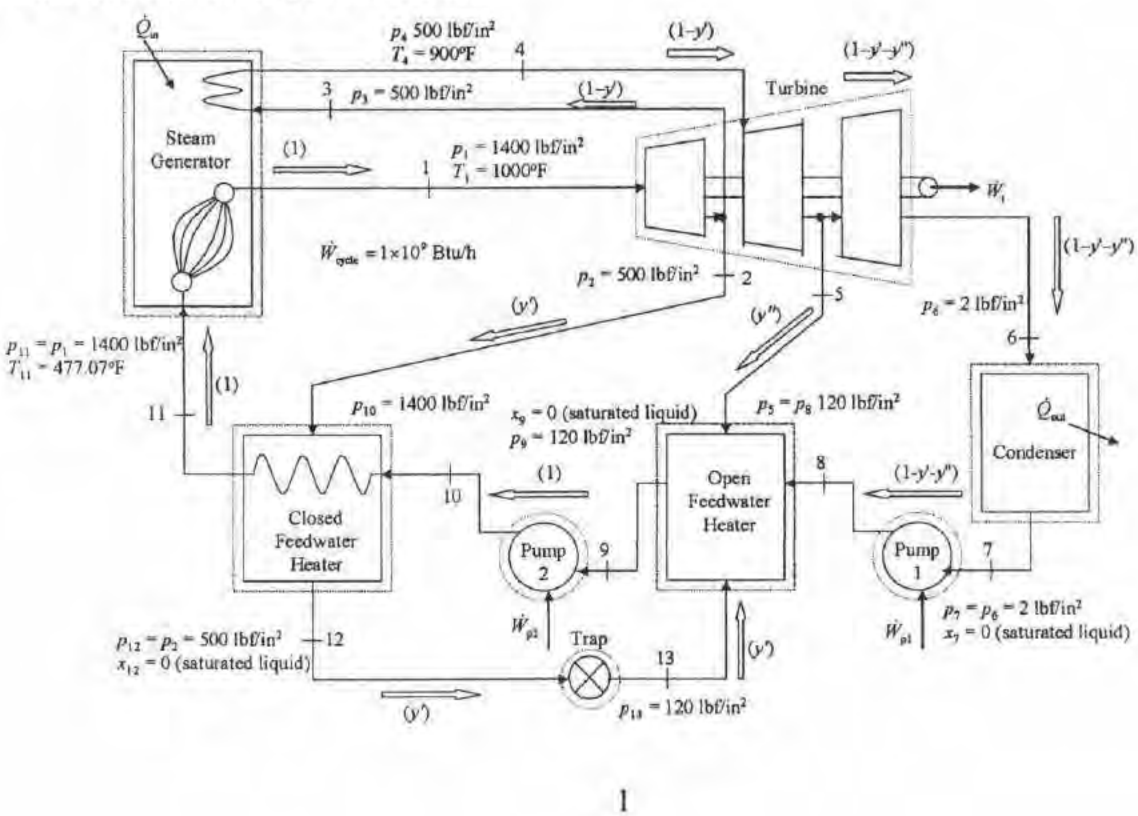
Increased irreversibilities in the turbine stages and pumps (lower isentropic efficiencies) result in lower thermal efficiency (as expected).

8.72 Water is the working fluid in a reheat-regenerative Rankine cycle with one closed feedwater heater and one open feedwater heater. Steam enters the turbine at 1400 lbf/in.^{2} and 1000°F and expands to 500 lbf/in.^{2}, where some of the steam is extracted and diverted to the closed feedwater heater. Condensate exiting the closed feedwater heater as saturated liquid at 500 lbf/in.^{2} undergoes a throttling process to 120 lbf/in.^{2} as it passes through a trap into the open feedwater heater. The feedwater leaves the closed feedwater heater at 1400 lbf/in.^{2} and a temperature equal to the saturation temperature at 500 lbf/in.^{2} The remaining steam is reheated to 900°F before entering the second-stage turbine, where it expands to 120 lbf/in.^{2} Some of the steam is extracted and diverted to the open feedwater heater operating at 120 lbf/in.^{2} Saturated liquid exits the open feedwater heater at 120 lbf/in.^{2} The rest expands through the third-stage turbine to the condenser pressure of 2 lbf/in.^{2} All processes of the working fluid in the turbine stages and pumps are internally reversible. The net power output of the cycle is 1×10^9 Btu/h. Determine for the cycle
(a) the mass flow rate of steam entering the first stage of the turbine, in lb/h.
(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the steam generator including the reheat section.
(c) the thermal efficiency.

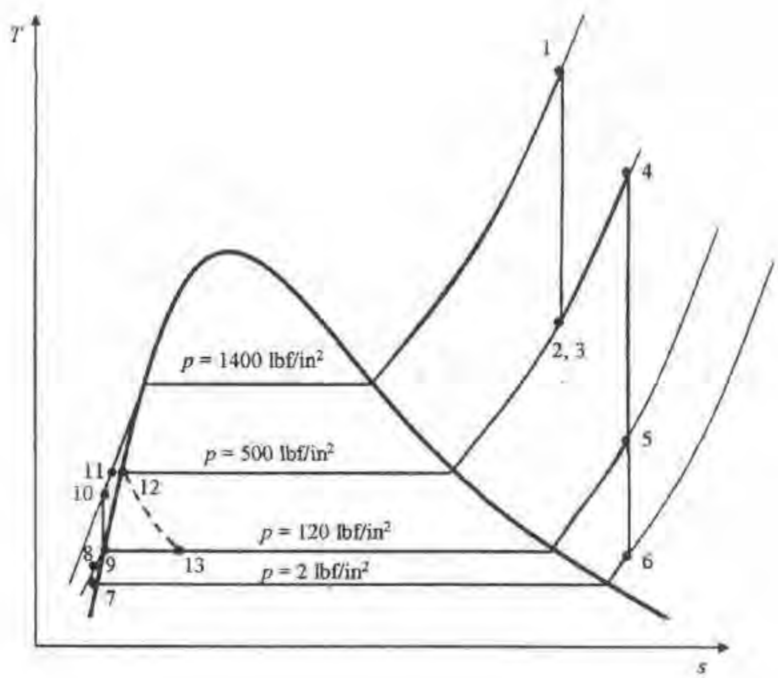
KNOWN: A reheat-regenerative Rankine cycle with one closed feedwater heater and one open feedwater heater operates with water as the working fluid. The net power output of the cycle is given.

FIND: Determine the mass flow rate of steam entering the first stage of the turbine, the rate of heat transfer to the working fluid passing through the steam generator and reheater, and the thermal efficiency.

SCHEMATIC AND GIVEN DATA:



T-s diagram



ENGINEERING MODEL:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The turbine and pumps operate adiabatically and the working fluid undergoes internally reversible processes in passing through each unit.
3. Kinetic and potential energy effects are negligible.
4. Condensate exits the condenser, the closed feedwater heater, and the open feedwater heater as saturated liquid.
5. There is no heat transfer between the outside of the condenser and the surroundings.
6. There is no heat transfer between the outside of the closed feedwater heater and the surroundings.
7. There is no heat transfer between the outside of the open feedwater heater and the surroundings.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 1400$ lbf/in.^{2}, $T_1 = 1000°F$ → From Table A-4E: $h_1 = 1493.5$ Btu/lb and $s_1 = 1.6094$ Btu/(lb·°R)

State 2: $p_2 = 500$ lbf/in.^{2}, $s_2 = s_1$ → From Table A-4E (interpolated): $h_2 = 1354.71$ Btu/lb

State 3: $p_3 = 500$ lbf/in.^{2}, $s_3 = s_1$ → From Table A-4E (interpolated): $h_3 = 1354.71$ Btu/lb

Problem 8.72 (Continued) – Page 3

State 4: $p_4 = 500$ lbf/in.^{2}, $T_4 = 900$°F → From Table A-4E: $h_4 = 1466.5$ Btu/lb and $s_4 = 1.6987$ Btu/(lb·°R)
State 5: $p_5 = 120$ lbf/in.^{2}, $s_5 = s_4$ → From Table A-4E (interpolated): $h_5 = 1289.09$ Btu/lb

State 6: $p_6 = 2$ lbf/in.^{2}, $s_6 = s_4$ → From Table A-3E: $x_6 = (1.6987 - 0.1750)/1.7448 = 0.8733$ and $h_4 = h_{f4} + x_4 h_{fg4} = 94.02 + (0.8733)(1022.1) = 986.62$ Btu/lb

State 7: $p_7 = p_6 = 2$ lbf/in.^{2}, sat liq. → From Table A-3E: $h_7 = h_{f7} = 94.02$ Btu/lb and $v_7 = v_{f7} = 0.01623$ ft^3/lb

State 8: $p_8 = p_5 = 120$ lbf/in.^{2}, $h_8 \approx h_7 + v_7(p_8 - p_7)$

$$h_8 = 94.02 \text{ Btu/lb} + 0.01623\left(\frac{\text{ft}^3}{\text{lb}}\right)(120-2)\left(\frac{\text{lbf}}{\text{in}^2}\right)\left|\frac{144\text{ in}^2}{\text{ft}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right| = 94.37 \text{ Btu/lb}$$

State 9: $p_9 = p_5 = 120$ lbf/in.^{2}, sat liq. → From Table A-3E: $h_9 = h_{f9} = 312.7$ Btu/lb and $v_9 = v_{f9} = 0.01789$ ft^3/lb

State 10: $p_{10} = p_1 = 1400$ lbf/in.^{2}, $h_{10} \approx h_9 + v_9(p_{10} - p_9)$

$$h_{10} = 312.7 \text{ Btu/lb} + 0.01789\left(\frac{\text{ft}^3}{\text{lb}}\right)(1400-120)\left(\frac{\text{lbf}}{\text{in}^2}\right)\left|\frac{144\text{ in}^2}{\text{ft}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right| = 316.94 \text{ Btu/lb}$$

State 11: $p_{11} = p_1 = 1400$ lbf/in.^{2}, $T_{11} = T_{sat}$ @ $P = 500$ lbf/in^2 = 477.07°F. → From Table A-2E (compressed liquid) (interpolated): $h_{11} \approx h_{f11} = 460.93$ Btu/lb

State 12: $p_{12} = p_2 = 500$ lbf/in.^{2}, sat liq. → From Table A-3E: $h_{12} = h_{f12} = 449.5$ Btu/lb

State 13: $p_{13} = p_5 = 120$ lbf/in.^{2}, throttling process → $h_{13} = h_{12} = 449.5$ Btu/lb

(a) The mass flow rate of steam entering the first stage of the turbine can be determined from the definition of the net power output of the cycle

$$\dot{W}_{cycle} = \dot{W}_{t1} + \dot{W}_{t2} + \dot{W}_{t3} - \dot{W}_{p1} - \dot{W}_{p2}$$

Let $\dot{m}_1$ be the mass flow rate of the steam entering the first stage of the turbine. Mass and energy balances for control volumes around the three turbine stages and the two pumps give, respectively

$$\dot{W}_{t1} = \dot{m}_1(h_1 - h_2)$$
$$\dot{W}_{t2} = \dot{m}_1(1-y')(h_4 - h_5)$$

$\dot{W}_{t3} = \dot{m}_1 (1-y'-y'')(h_5 - h_6)$
$\dot{W}_{p1} = \dot{m}_1 (1-y'-y'')(h_8 - h_7)$
$\dot{W}_{p2} = \dot{m}_1 (h_{10} - h_9)$
where y' and y'' are the fractions of the flow entering the first turbine stage that are extracted at 2 and 5, respectively.

Substituting for net power output of the cycle

$$\dot{W}_{cycle} = \dot{m}_1 (h_1 - h_2) + \dot{m}_1 (1-y')(h_4 - h_5) + \dot{m}_1 (1-y'-y'')(h_5 - h_6) - \dot{m}_1 (1-y'-y'')(h_8 - h_7) - \dot{m}_1 (h_{10} - h_9)$$

Solving for $\dot{m}_1$

$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{(h_1 - h_2) + (1-y')(h_4 - h_5) + (1-y'-y'')(h_5 - h_6) - (1-y'-y'')(h_8 - h_7) - (h_{10} - h_9)}$$

The mass fraction of steam (y') extracted after the first stage of the turbine and the mass fraction of steam (y'') extracted after the second stage of the turbine are both unknown. Analyze the closed feedwater heater to determine y'. Mass and energy balances for a control volume around the closed feedwater heater give

$$0 = \dot{Q} - \dot{W} + \dot{m}_1 (1)(h_{10} - h_{11}) + \dot{m}_1 (y')(h_2 - h_{12})$$

Since there is no transfer of energy by heat or work, we can solve for y' and substitute values for specific enthalpy to get

$$y' = \frac{h_{11} - h_{10}}{h_2 - h_{12}} = \frac{460.93\frac{\text{Btu}}{\text{lb}} - 316.94\frac{\text{Btu}}{\text{lb}}}{1354.71\frac{\text{Btu}}{\text{lb}} - 449.5\frac{\text{Btu}}{\text{lb}}} = 0.159$$

Analyze the open feedwater heater to determine y''. Mass and energy balances for a control volume around the open feedwater heater give

$$0 = \dot{Q} - \dot{W} + \dot{m}_1 (y'')h_5 + \dot{m}_1 (1-y'-y'')h_8 + \dot{m}_1 (y')h_{13} - \dot{m}_1 (1)h_9$$

Since there is no transfer of energy by heat or work, we can substitute for y' to get

$$0 = \dot{m}_1 (y'')h_5 + \dot{m}_1 (1 - 0.159 - y'')h_8 + \dot{m}_1 (0.159)h_{13} - \dot{m}_1 (1)h_9$$

$$0 = \dot{m}_1 (y'')h_5 + \dot{m}_1 (0.841 - y'')h_8 + \dot{m}_1 (0.159)h_{13} - \dot{m}_1 (1)h_9$$

Dividing by $\dot{m}_1$ and grouping terms

Problem 8.72 (Continued) – Page 5

$$0 = 0.841h_8 + 0.159h_{13} + y''(h_5 - h_8) - h_9$$

Solving for y'' and substituting values for specific enthalpy, we get

$$y'' = \frac{h_9 - 0.841h_8 - 0.159h_{13}}{h_5 - h_8} = \frac{312.7\frac{\text{Btu}}{\text{lb}} - 0.841\left(94.37\frac{\text{Btu}}{\text{lb}}\right) - 0.159\left(449.5\frac{\text{Btu}}{\text{lb}}\right)}{1289.09\frac{\text{Btu}}{\text{lb}} - 94.37\frac{\text{Btu}}{\text{lb}}} = 0.135$$

Thus,

$$(1 - y') = 1 - 0.159 = 0.841 \quad \text{and} \quad (1 - y' - y'') = 1 - 0.159 - 0.135 = 0.706$$

Substituting values and solving for $\dot{m}_1$

$\dot{m}_1 =$

$$\frac{1\times10^9\ \frac{\text{Btu}}{\text{h}}}{[(1493.5 - 1354.71) + (0.841)(1466.5 - 1289.09) + (0.706)(1289.09 - 986.62) - (0.706)(94.37 - 94.02) - (316.94 - 312.7)]\frac{\text{Btu}}{\text{lb}}}$$

$$\dot{m}_1 = \underline{\mathbf{2.01\times10^6\ lb/h}}$$

(b) The rate of heat transfer to the working fluid passing through the steam generator including reheater can be determined by applying mass and energy balances to a control volume around the steam generator to give

$$\dot{Q}_{in} = \dot{m}_1[(1)(h_1 - h_{11}) + (1 - y')(h_4 - h_3)]$$

$$\dot{Q}_{in} = (2.01\times10^6\ \text{lb/h})[(1)(1493.5 - 460.93) + (0.841)(1466.5 - 1354.71)]\ \text{Btu/lb}$$

$$\dot{Q}_{in} = \underline{\mathbf{2.26\times10^9\ Btu/h}}$$

(c) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = (1\times10^9\ \text{Btu/h})/(2.26\times10^9\ \text{Btu/h}) = \underline{\mathbf{0.4425\ (44.25\%)}}$$

The results of Problem 8.72 can be compared with the results of Problem 8.65 to see some of the effects of incorporating reheat into a regenerative Rankine cycle with two feedwater heaters developing the same net power output. In this case, the results indicate that thermal efficiency increases slightly and the rate heat addition decreases slightly, but the mass flow rate entering the first-stage turbine is increased.

8.73 Water is the working fluid in a reheat-regenerative Rankine cycle with one closed feedwater heater and one open feedwater heater. Steam enters the turbine at 1400 lbf/in.^{2} and 1000°F and expands to 500 lbf/in.^{2}, where some of the steam is extracted and diverted to the closed feedwater heater. Condensate exiting the closed feedwater heater as saturated liquid at 500 lbf/in.^{2} undergoes a throttling process to 120 lbf/in.^{2} as it passes through a trap into the open feedwater heater. The feedwater leaves the closed feedwater heater at 1400 lbf/in.^{2} and a temperature equal to the saturation temperature at 500 lbf/in.^{2} The remaining steam is reheated to 900°F before entering the second-stage turbine, where it expands to 120 lbf/in.^{2} Some of the steam is extracted and diverted to the open feedwater heater operating at 120 lbf/in.^{2} Saturated liquid exits the open feedwater heater at 120 lbf/in.^{2} The remaining steam expands through the third-stage turbine to the condenser pressure of 2 lbf/in.^{2} The turbine stages and the pumps each operate adiabatically with isentropic efficiencies of 85%. Flow through the condenser, closed feedwater heater, open feedwater heater, steam generator, and reheater is at constant pressure. The net power output of the cycle is 1×10^9 Btu/h. Determine for the cycle

(a) the mass flow rate of steam entering the first stage of the turbine, in lb/h.

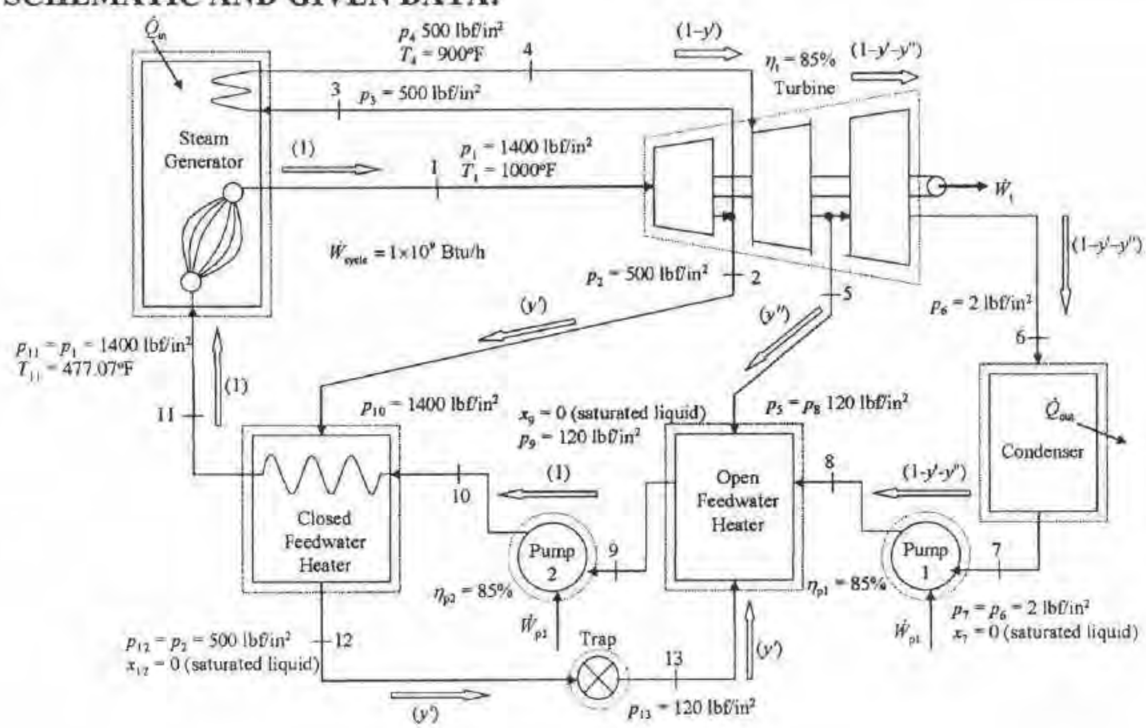
(b) the rate of heat transfer, in Btu/h, to the working fluid passing through the steam generator including the reheat section.

(c) the thermal efficiency.

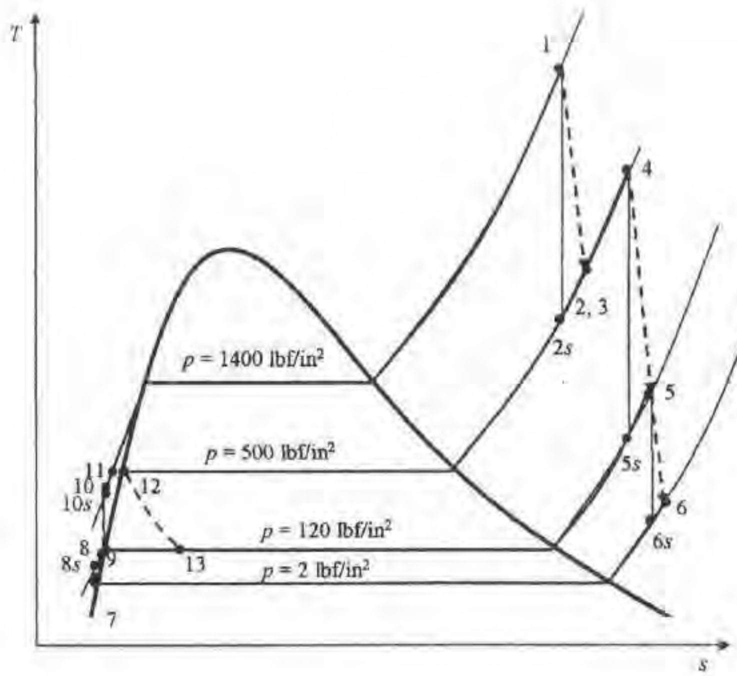
KNOWN: A reheat-regenerative Rankine cycle with one closed feedwater heater and one open feedwater heater operates with water as the working fluid. The net power output of the cycle is given.

FIND: Determine the mass flow rate of steam entering the first stage of the turbine, the rate of heat transfer to the working fluid passing through the steam generator including the reheater, and the thermal efficiency.

SCHEMATIC AND GIVEN DATA:



T-s diagram



ENGINEERING MODEL:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The turbine and pumps each operate adiabatically with an isentropic efficiency of 85%.
3. Flow through the boiler, reheater, closed feedwater heater, open feedwater heater, and condenser is at constant pressure.
4. Kinetic and potential energy effects are negligible.
5. Condensate exits the condenser, the closed feedwater heater, and the open feedwater heater as saturated liquid.
6. There is no heat transfer between the outside of the condenser and the surroundings.
7. There is no heat transfer between the outside of the closed feedwater heater and the surroundings.
8. There is no heat transfer between the outside of the open feedwater heater and the surroundings.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 1400$ lbf/in.^{2}, $T_1 = 1000$°F → From Table A-4E: $h_1 = 1493.5$ Btu/lb and $s_1 = 1.6094$ Btu/(lb·°R)

State 2s: $p_{2s} = p_2 = 500$ lbf/in.^{2}, $s_{2s} = s_1$ → From Table A-4E (interpolated): $h_{2s} = 1354.71$ Btu/lb

Problem 8.73 (Continued) – Page 3

State 2: $p_2 = 500$ lbf/in.^{2}, $h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 1493.5 - 0.85(1493.5 - 1354.71) = 1375.55$ Btu/lb and → From Table A-4E (interpolated): $s_2 = 1.6268$ Btu/(lb·°R)

State 3: $p_3 = 500$ lbf/in.^{2}, State 3 is the same state as State 2 → $h_3 = 1375.55$ Btu/lb

State 4: $p_4 = 500$ lbf/in.^{2}, $T_4 = 900$°F → From Table A-4E: $h_4 = 1466.5$ Btu/lb and $s_4 = 1.6987$ Btu/(lb·°R)

State 5s: $p_{5s} = p_5 = 120$ lbf/in.^{2}, $s_{5s} = s_4$ → From Table A-4E (interpolated): $h_{5s} = 1289.09$ Btu/lb

State 5: $p_5 = 120$ lbf/in.^{2}, $h_5 = h_4 - \eta_t(h_4 - h_{5s}) = 1466.5 - 0.85(1466.5 - 1289.09) = 1315.70$ Btu/lb and → From Table A-4E (interpolated): $s_5 = 1.7251$ Btu/(lb·°R)

State 6s: $p_{6s} = p_6 = 2$ lbf/in.^{2}, $s_{6s} = s_5$ → From Table A-3E: $x_{6s} = (1.7251 - 0.1750)/1.7448 = 0.8884$ and $h_{6s} = h_{f6s} + x_{6s}h_{fg6s} = 959.64$ Btu/lb $= 94.02 + (0.8884)(1022.1) = 1002.05$ Btu/lb

State 6: $p_6 = 2$ lbf/in^2, $h_6 = h_5 - \eta_t(h_5 - h_{6s}) = 1315.70 - 0.85(1315.70 - 1002.05) = 1049.10$ Btu/lb

State 7: $p_7 = p_6 = 2$ lbf/in.^{2}, sat liq. → From Table A-3E: $h_7 = h_{f7} = 94.02$ Btu/lb and $v_7 = v_{f7} = 0.01623$ ft^3/lb

State 8: $p_8 = p_5 = 120$ lbf/in.^{2}, $h_8 = h_7 + \dfrac{v_7(p_8 - p_7)}{\eta_{p1}}$

$$h_8 = 94.02\text{ Btu/lb} + \frac{0.01623\left(\frac{\text{ft}^3}{\text{lb}}\right)(120-2)\left(\frac{\text{lbf}}{\text{in}^2}\right)}{0.85}\left|\frac{144\text{ in}^2}{\text{ft}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right| = 94.44\text{ Btu/lb}$$

State 9: $p_9 = p_5 = 120$ lbf/in.^{2}, sat liq. → From Table A-3E: $h_9 = h_{f9} = 312.7$ Btu/lb and $v_9 = v_{f9} = 0.01789$ ft^3/lb

State 10: $p_{10} = p_1 = 1400$ lbf/in.^{2}, $h_{10} = h_9 + \dfrac{v_9(p10_8 - p_9)}{\eta_{p2}}$

$$h_{10} = 312.7\text{ Btu/lb} + \frac{0.01789\left(\frac{\text{ft}^3}{\text{lb}}\right)(1400-120)\left(\frac{\text{lbf}}{\text{in}^2}\right)}{0.85}\left|\frac{144\text{ in}^2}{\text{ft}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right| = 339.64\text{ Btu/lb}$$

State 11: $p_{11} = p_1 = 1400$ lbf/in.^{2}, $T_{11} = T_{sat}$ @ $P = 500$ lbf/in^2 = 477.07°F. → From Table A-2E (compressed liquid) (interpolated): $h_{11} \approx h_{f11} = 460.93$ Btu/lb

Problem 8.73 (Continued) – Page 4

State 12: $p_{12} = p_2 = 500$ lbf/in.^{2}, sat liq. $\rightarrow$ From Table A-3E: $h_{12} = h_{f12} = 449.5$ Btu/lb

State 13: $p_{13} = p_5 = 120$ lbf/in.^{2}, throttling process $\rightarrow h_{13} = h_{12} = 449.5$ Btu/lb

(a) The mass flow rate of steam entering the first stage of the turbine can be determined from the definition of the net power output of the cycle

$$\dot{W}_{cycle} = \dot{W}_{t1} + \dot{W}_{t2} + \dot{W}_{t3} - \dot{W}_{p1} - \dot{W}_{p2}$$

Let $\dot{m}_1$ be the mass flow rate of the steam entering the first stage of the turbine. Mass and energy balances for control volumes around the three turbine stages and the two pumps give, respectively

$$\begin{aligned}
\dot{W}_{t1} &= \dot{m}_1 (h_1 - h_2) \\
\dot{W}_{t2} &= \dot{m}_1 (1 - y')(h_4 - h_5) \\
\dot{W}_{t3} &= \dot{m}_1 (1 - y' - y'')(h_5 - h_6) \\
\dot{W}_{p1} &= \dot{m}_1 (1 - y' - y'')(h_8 - h_7) \\
\dot{W}_{p2} &= \dot{m}_1 (h_{10} - h_9)
\end{aligned}$$

where y' and y'' are the fractions of steam extracted after the first and second turbine stages, respectively.

Substituting for net power output of the cycle

$$\dot{W}_{cycle} = \dot{m}_1 (h_1 - h_2) + \dot{m}_1 (1 - y')(h_4 - h_5) + \dot{m}_1 (1 - y' - y'')(h_5 - h_6) - \dot{m}_1 (1 - y' - y'')(h_8 - h_7) - \dot{m}_1 (h_{10} - h_9)$$

Solving for $\dot{m}_1$

$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{(h_1 - h_2) + (1 - y')(h_4 - h_5) + (1 - y' - y'')(h_5 - h_6) - (1 - y' - y'')(h_8 - h_7) - (h_{10} - h_9)}$$

The mass fractions y' and y'' are both unknown. First, analyze the closed feedwater heater to determine y'. Mass and energy balances for a control volume around the closed feedwater heater give

$$0 = \dot{Q} - \dot{W} + \dot{m}_1 (1)(h_{10} - h_{11}) + \dot{m}_1 (y')(h_2 - h_{12})$$

Since there is no transfer of energy by heat or work, we can solve for y' and substitute values for specific enthalpy to get

$$y' = \frac{h_{11} - h_{10}}{h_2 - h_{12}} = \frac{460.93 \frac{\text{Btu}}{\text{lb}} - 339.64 \frac{\text{Btu}}{\text{lb}}}{1375.55 \frac{\text{Btu}}{\text{lb}} - 449.5 \frac{\text{Btu}}{\text{lb}}} = 0.131$$

Now we can analyze the open feedwater heater to determine y''. Mass and energy balances for a control volume around the open feedwater heater give

$$0 = \dot{Q} - \dot{W} + \dot{m}_1(y'')h_5 + \dot{m}_1(1 - y' - y'')h_8 + \dot{m}_1(y')h_{13} - \dot{m}_1(1)h_9$$

Since there is no transfer of energy by heat or work, we can substitute for y' to get

$$0 = \dot{m}_1(y'')h_5 + \dot{m}_1(1 - 0.131 - y'')h_8 + \dot{m}_1(0.131)h_{13} - \dot{m}_1(1)h_9$$

$$0 = \dot{m}_1(y'')h_5 + \dot{m}_1(0.869 - y'')h_8 + \dot{m}_1(0.131)h_{13} - \dot{m}_1(1)h_9$$

Dividing by $\dot{m}_1$ and grouping terms

$$0 = 0.869h_8 + 0.131h_{13} + y''(h_5 - h_8) - h_9$$

Solving for y'' and substituting values for specific enthalpy, we get

$$y'' = \frac{h_9 - 0.869h_8 - 0.131h_{13}}{h_5 - h_8} = \frac{312.7 \frac{\text{Btu}}{\text{lb}} - 0.869\left(94.44 \frac{\text{Btu}}{\text{lb}}\right) - 0.131\left(449.5 \frac{\text{Btu}}{\text{lb}}\right)}{1315.70 \frac{\text{Btu}}{\text{lb}} - 94.44 \frac{\text{Btu}}{\text{lb}}} = 0.141$$

Thus,

$$(1 - y') = 1 - 0.131 = 0.869 \quad \text{and} \quad (1 - y' - y'') = 1 - 0.131 - 0.141 = 0.728$$

Substituting values and solving for $\dot{m}_1$

$$\dot{m}_1 = \frac{1 \times 10^9 \frac{\text{Btu}}{\text{h}}}{[(1493.5 - 1375.55) + (0.869)(1466.5 - 1315.70) + (0.728)(1315.70 - 1049.10) - (0.728)(94.44 - 94.02) - (339.64 - 312.7)]\frac{\text{Btu}}{\text{lb}}}$$

$$\dot{m}_1 = \underline{\mathbf{2.40 \times 10^6\ lb/h}}$$

(b) The rate of heat transfer to the working fluid passing through the steam generator including reheater can be determined by applying mass and energy balances to a control volume around the steam generator to give

Problem 8.73 (Continued) – Page 6

$$\dot{Q}_{\text{in}} = \dot{m}_1\left[(1)(h_1 - h_{11}) + (1 - y')(h_4 - h_3)\right]$$

$$\dot{Q}_{\text{in}} = (2.40\times10^6\ \text{lb/h})[(1)(1493.5 - 460.93) + (0.869)(1466.5 - 1375.55)]\ \text{Btu/lb}$$

$$\dot{Q}_{\text{in}} = \underline{\mathbf{2.67\times10^9\ Btu/h}}$$

(c) The thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = (1\times10^9\ \text{Btu/h})/(2.67\times10^9\ \text{Btu/h}) = \underline{\mathbf{0.3745\ (37.45\%)}}$$

The results of Problem 8.73 can be compared with those of Problem 8.19 to see some of the effects of incorporating reheat and regeneration into an actual Rankine cycle with the same net power output. In this case, the results indicate that the thermal efficiency increases and the rate of heat addition is less, but the mass flow rate into the first-stage turbine increases.

8.74 Data for a power plant similar in design to that shown in Fig 8.12 are provided in the table below. The plant operates on a regenerative vapor power cycle with four feedwater heaters, three closed and one open, and reheat. Steam enters the turbine at 16,000 kPa, 600°C, expands in three stages to the reheat pressure of 2000 kPa, is reheated to 500°C, and then expands in three more stages to the condenser pressure of 10 kPa. Saturated liquid exits the condenser at 10 kPa. Between the first and second stages, some steam is diverted to a closed feedwater heater at 8000 kPa. Between the second and third stages, additional steam is diverted to a second closed feedwater heater at 4000 kPa. Steam is extracted between the fourth and fifth turbine stages at 800 kPa and fed into an open feedwater heater operating at that pressure. Saturated liquid at 800 kPa leaves the open feedwater heater. Between the fifth and sixth stages, some steam is diverted to a closed feedwater heater at 200 kPa. Condensate leaves each closed feedwater heater as saturated liquid at the respective extraction pressures. For isentropic processes in each turbine stage and adiabatic processes in the pumps, all closed feedwater heaters, all traps, and the open feedwater heater show that
(a) the fraction of the steam diverted between the first and second stage is 0.1000.
(b) the fraction of the steam diverted between the second and third stage is 0.1500.
(c) the fraction of the steam diverted between the fourth and fifth stage is 0.0009.
(d) the fraction of the steam diverted between the fifth and sixth stage is 0.1302.

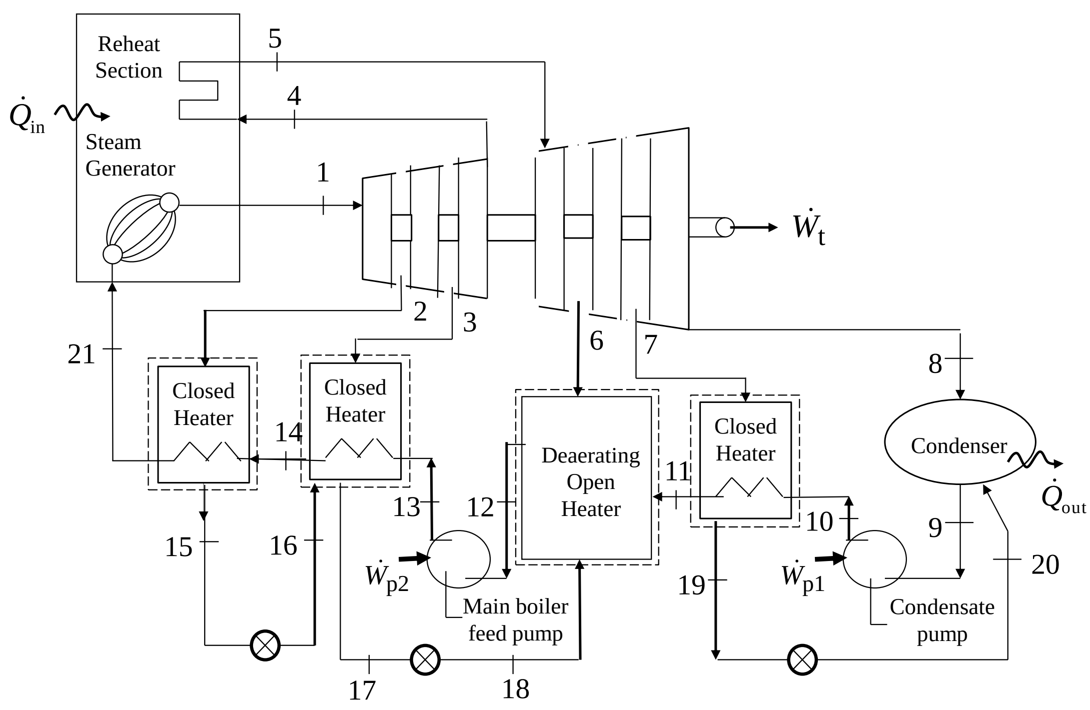
State	*p* (kPa)	*T* (°C)	*h* (kJ/kg)	*s* (kJ/kg·K)	*x*	**State**	*p* (kPa)	*T* (°C)	*h* (kJ/kg)	*s* (kJ/kg·K)	*x*
1	16,000	600	3573.5	6.6399	--	**12**	800		721.11	2.0462	0
2	8,000		3334.7	6.6399	--	**13**	16,000		738.05	2.0837	--
3	4,000		3129.2	6.6399	--	**14**	16,000		1067.3	2.7584	--
4	2,000		2953.6	6.6399	--	**15**	8,000		1316.6	3.2068	0
5	2,000	500	3467.6	7.4317	--	**16**	4,000		1316.6	3.2344	0.1338
6	800		3172.1	7.4317	--	**17**	4,000		1087.3	2.7964	0
7	200		2824.7	7.4317	--	**18**	800		1087.3	2.8716	0.1788
8	10		2355.4	7.4317	0.9042	**19**	200		504.70	1.5301	0
9	10		191.83	0.6493	0	**20**	10		504.70	1.6304	0.1308
10	800		192.63	0.6517	--	**21**	16,000		1269.1	3.1245	--
11	800		595.92	1.7553	--						

KNOWN: A regenerative vapor power cycle with three closed feedwater heaters, one open feedwater heater, and reheat operates with steam as the working fluid. Operational data are provided.

FIND: Show that (a) the fraction of the steam diverted between the first and second stage is 0.1000, (b) the fraction of the steam diverted between the second and third stage is 0.1500, (c) the fraction of the steam diverted between the fourth and fifth stage is 0.0009, and (d) the fraction of the steam diverted between the fifth and sixth stage is 0.1302.

SCHEMATIC AND GIVEN DATA:

State	p (kPa)	T (°C)	h (kJ/kg)	s (kJ/kg·K)	x	State	p (kPa)	T (°C)	h (kJ/kg)	s (kJ/kg·K)	x
1	16,000	600	3573.5	6.6399	--	**12**	800		721.11	2.0462	0
2	8,000		3334.7	6.6399	--	**13**	16,000		738.05	2.0837	--
3	4,000		3129.2	6.6399	--	**14**	16,000		1067.3	2.7584	--
4	2,000		2953.6	6.6399	--	**15**	8,000		1316.6	3.2068	0
5	2,000	500	3467.6	7.4317	--	**16**	4,000		1316.6	3.2344	0.1338
6	800		3172.1	7.4317	--	**17**	4,000		1087.3	2.7964	0
7	200		2824.7	7.4317	--	**18**	800		1087.3	2.8716	0.1788
8	10		2355.4	7.4317	0.9042	**19**	200		504.70	1.5301	0
9	10		191.83	0.6493	0	**20**	10		504.70	1.6304	0.1308
10	800		192.63	0.6517	--	**21**	16,000		1269.1	3.1245	--
11	800		595.92	1.7553	--						



ENGINEERING MODEL:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The expansions through the turbine stages occur isentropically.
3. The turbines, pumps, closed feedwater heaters, open feedwater heater, and traps operate adiabatically.

4. Kinetic and potential energy effects are negligible.

ANALYSIS:

(a) Applying mass and energy rate balances to the control volume enclosing the first closed feedwater heater, the fraction of flow, *y*', extracted at location 2 is

$$y' = \frac{h_{21} - h_{14}}{h_2 - h_{15}} = \frac{(1269.1 - 1067.3)\ \text{kJ/kg}}{(3334.7 - 1316.6)\ \text{kJ/kg}} = \underline{\mathbf{0.1000}}$$

(b) Applying mass and energy rate balances to the control volume enclosing the second closed feedwater heater, the fraction of flow, *y*'', extracted at location 3 is

$$y'' = \frac{h_{14} - h_{13} + y'(h_{17} - h_{16})}{h_3 - h_{17}} = \frac{[1067.3 - 738.05 + (0.1000)(1087.3 - 1316.6)]\ \text{kJ/kg}}{(3129.2 - 1087.3)\ \text{kJ/kg}} = \underline{\mathbf{0.1500}}$$

(c) Applying mass and energy rate balances to a control volume enclosing the open feedwater heater, the fraction of flow, *y*''', extracted at location 6 is

$$y''' = \frac{h_{12} - h_{11} + (y' + y'')(h_{11} - h_{18})}{h_6 - h_{11}}$$

$$y''' = \frac{[721.11 - 595.92 + (0.2500)(595.92 - 1087.3)]\ \text{kJ/kg}}{(3172.1 - 595.92)\ \text{kJ/kg}} = \underline{\mathbf{0.0009}}$$

(d) Applying mass and energy rate balances to the control volume enclosing the third closed feedwater heater, the fraction of flow, *y*'''', extracted at location 7 is

$$y'''' = \frac{(1 - y' - y'' - y''')(h_{11} - h_{10})}{h_7 - h_{19}}$$

$$y'''' = \frac{[(0.7491)(595.92 - 192.63)]\ \text{kJ/kg}}{(2824.7 - 504.70)\ \text{kJ/kg}} = \underline{\mathbf{0.1302}}$$

8.75 For the power plant in Problem 8.74 with extraction mass fractions as indicated determine the cycle thermal efficiency.

KNOWN: A regenerative vapor power cycle with three closed feedwater heaters, one open feedwater heater, and reheat operates with steam as the working fluid. Operational data are provided in Problem 8.74.

FIND: Determine the cycle thermal efficiency.

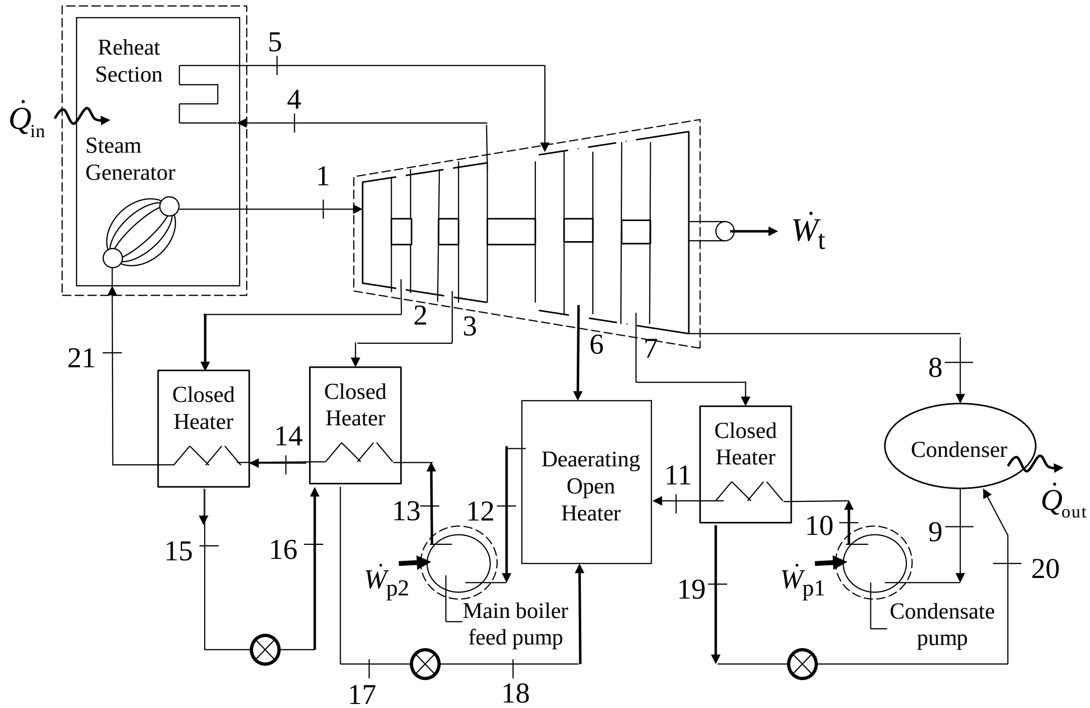
SCHEMATIC AND GIVEN DATA:

State	p (kPa)	T (°C)	h (kJ/kg)	s (kJ/kg·K)	x	State	p (kPa)	T (°C)	h (kJ/kg)	s (kJ/kg·K)	x
1	16,000	600	3573.5	6.6399	--	**12**	800		721.11	2.0462	0
2	8,000		3334.7	6.6399	--	**13**	16,000		738.05	2.0837	--
3	4,000		3129.2	6.6399	--	**14**	16,000		1067.3	2.7584	--
4	2,000		2953.6	6.6399	--	**15**	8,000		1316.6	3.2068	0
5	2,000	500	3467.6	7.4317	--	**16**	4,000		1316.6	3.2344	0.1338
6	800		3172.1	7.4317	--	**17**	4,000		1087.3	2.7964	0
7	200		2824.7	7.4317	--	**18**	800		1087.3	2.8716	0.1788
8	10		2355.4	7.4317	0.9042	**19**	200		504.70	1.5301	0
9	10		191.83	0.6493	0	**20**	10		504.70	1.6304	0.1308
10	800		192.63	0.6517	--	**21**	16,000		1269.1	3.1245	--
11	800		595.92	1.7553	--						

P8.74

Fraction of flow extracted at location 2: y' = 0.1000.
Fraction of flow extracted at location 3: y'' = 0.1500.
Fraction of flow extracted at location 6: y''' = 0.0009.
Fraction of flow extracted at location 7: y'''' = 0.1302.

Regenerative Vapor Power Cycle with Reheat



ENGINEERING MODEL:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The expansions through the turbine stages occur isentropically.
3. The turbines, pumps, closed feedwater heaters, open feedwater heater, and traps operate adiabatically.
4. Kinetic and potential energy effects are negligible.

ANALYSIS:

For an overall control volume enclosing the turbine stages, an energy balance on the basis of a unit of mass entering reads

$$\frac{\dot{W}_t}{\dot{m}_1} = h_1 + (1 - y' - y'')h_5 - y'h_2 - y''h_3 - (1 - y' - y'')h_4 - y'''h_6 - y''''h_7 - (1 - y' - y'' - y''' - y'''')h_8$$

Substituting enthalpy values and the following flow fractions gives

y' = 0.1000
y'' = 0.1500
$(1 - y' - y'')$ = 0.7500
y''' = 0.0009
$(1 - y' - y'' - y''')$ = 0.7491
y'''' = 0.1302

$(1 - y' - y'' - y''' - y'''') = 0.6189$

$$\frac{\dot{W}_t}{\dot{m}_1} = [3573.5 + (0.7500)(3467.6) - (0.1000)(3334.7) - (0.1500)(3129.2) - (0.7500)(2953.6)$$
$$- (0.0009)(3172.1) - (0.1302)(2824.7) - (0.6189)(2355.3)]\frac{\text{kJ}}{\text{kg}}$$

$$\frac{\dot{W}_t}{\dot{m}_1} = 1327.8 \text{ kJ/kg}$$

For the pumps

$$\frac{\dot{W}_p}{\dot{m}_1} = h_{13} - h_{12} + (1 - y' - y'' - y''')(h_{10} - h_9)$$

$$\frac{\dot{W}_p}{\dot{m}_1} = [738.05 - 721.11 + (0.7491)(192.63 - 191.83)]\frac{\text{kJ}}{\text{kg}} = 17.54 \text{ kJ/kg}$$

For the working fluid passing through the steam generator and reheat section

$$\frac{\dot{Q}_{in}}{\dot{m}_1} = h_1 - h_{21} + (1 - y' - y'')(h_5 - h_4)$$

$$\frac{\dot{Q}_{in}}{\dot{m}_1} = [3573.5 - 1269.1 + (0.7500)(3467.6 - 2953.6)]\frac{\text{kJ}}{\text{kg}} = 2689.9 \text{ kJ/kg}$$

Thus, the thermal efficiency is

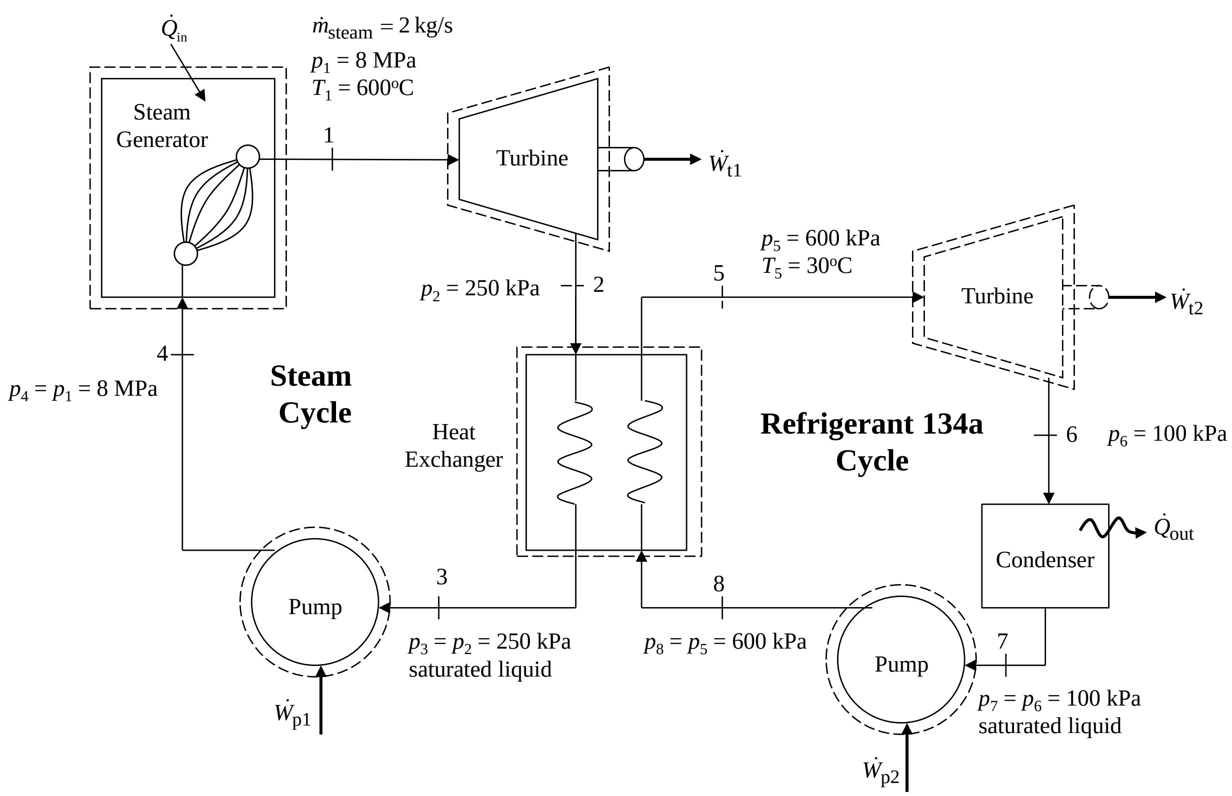
$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{(1327.8 - 17.54) \text{ kJ/kg}}{2689.9 \text{ kJ/kg}} = \underline{\mathbf{0.487\ (48.7\%)}}$$

8.76 A binary vapor power cycle consists of two ideal Rankine cycles with steam and Refrigerant 134a as the working fluids. The mass flow rate of steam is 2 kg/s. In the steam cycle, superheated vapor enters the turbine at 8 MPa, 600°C, and saturated liquid exits the condenser at 250 kPa. In the interconnecting heat exchanger, energy rejected by heat transfer from the steam cycle is provided to the Refrigerant 134a cycle. The heat exchanger experiences no stray heat transfer with its surroundings. Superheated Refrigerant 134a leaves the heat exchanger at 600 kPa, 30°C, which enters the Refrigerant 134a turbine. Saturated liquid leaves the Refrigerant 134a condenser at 100 kPa. Determine
(a) The net power developed by the binary cycle, in kW.
(b) The rate of heat addition to the binary cycle, in kW.
(c) The thermal efficiency of the binary cycle.
(d) The rate of entropy production in the interconnecting heat exchanger, in kW/K.

KNOWN: A binary power cycle operates with steam and Refrigerant 134a to produce power. Operational data are provided.

FIND: Determine (a) the net power developed by the binary cycle, in kW, (b) the rate of heat addition to the binary cycle, in kW, (c) the thermal efficiency of the binary cycle, and (d) the rate of entropy production in the interconnecting heat exchanger, in kW/K.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.

2. All processes of the working fluids are internally reversible, except in the interconnecting heat exchanger.
3. The turbine stages and pumps operate adiabatically.
4. Kinetic and potential energy effects are negligible.
5. There is no stray heat transfer from the interconnecting heat exchanger.

ANALYSIS: First fix each principal state.

For the steam (Tables A-3 and A-4)

State 1: p_1 = 8 MPa (80 bar), T_1 = 600°C → h_1 = 3642.0 kJ/kg, s_1 = 7.0206 kJ/kg·K

State 2: p_2 = 250 kPa (2.5 bar), $s_2 = s_1$ = 7.0206 kJ/kg·K → x_2 = 0.9941, h_2 = 2704.0 kJ/kg

State 3: p_3 = 250 kPa (2.5 bar), saturated liquid → h_3 = 535.37 kJ/kg, s_3 = 1.6072 kJ/kg·K, v_3 = 0.0010672 m³/kg

State 4: p_4 = 8 MPa (80 bar), $h_4 \approx h_3 + v_3(p_4 - p_3)$

$$h_4 = 535.37\frac{\text{kJ}}{\text{kg}} + \left(0.0010672\frac{\text{m}^3}{\text{kg}}\right)(8000-250)\text{kPa}\left|\frac{1000\ \text{N/m}^2}{1\,\text{kPa}}\right|\left|\frac{1\,\text{kJ}}{1000\ \text{N}\cdot\text{m}}\right| = 543.64\ \text{kJ/kg}$$

For the Refrigerant 134a (Tables A-10, A-11, and A-12)

State 5: p_5 = 600 kPa (6 bar), T_5 = 30°C → h_5 = 267.89 kJ/kg, s_5 = 0.9388 kJ/kg·K

State 6: p_6 = 100 kPa (1 bar), $s_6 = s_5$ = 0.9388 kJ/kg·K → x_6 = 0.9992, h_6 = 231.18 kJ/kg

State 7: p_7 = 100 kPa (1 bar), saturated liquid → h_7 = 16.29 kJ/kg, s_7 = 0.0678 kJ/kg·K, v_7 = 0.0007258 m³/kg

State 8: p_8 = 600 kPa (6 bar), $s_8 = s_7$ = 0.0678 kJ/kg·K, $h_8 \approx h_7 + v_7(p_8 - p_7)$

$$h_8 = 16.29\frac{\text{kJ}}{\text{kg}} + \left(0.0007258\frac{\text{m}^3}{\text{kg}}\right)(600-100)\text{kPa}\left|\frac{1000\ \text{N/m}^2}{1\,\text{kPa}}\right|\left|\frac{1\,\text{kJ}}{1000\ \text{N}\cdot\text{m}}\right| = 16.65\ \text{kJ/kg}$$

(a) The net power developed by the binary cycle is the sum of the net power developed by the steam cycle and the net power developed by the Refrigerant 134a cycle. For the control volume surrounding the steam turbine

$$\dot{W}_{\text{t1}} = \dot{m}_{\text{steam}}(h_1 - h_2)$$

$$\dot{W}_{\text{t1}} = \left(2\frac{\text{kg}}{\text{s}}\right)(3642.0-2704.0)\frac{\text{kJ}}{\text{kg}}\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right| = 1876\ \text{kW}$$

For the steam cycle pump

$$\dot{W}_{p1} = \dot{m}_{steam}(h_4 - h_3)$$

$$\dot{W}_{p1} = \left(2\frac{kg}{s}\right)(543.64 - 535.37)\frac{kJ}{kg}\left|\frac{1\,kW}{1\,kJ/s}\right| = 16.5\text{ kW}$$

The net power developed by the steam cycle is

$$\dot{W}_{\text{steam cycle}} = \dot{W}_{t1} - \dot{W}_{p1} = (1876 - 16.5)\text{ kW} = 1859.5\text{ kW}$$

To determine the mass flow rate of Refrigerant 134a, apply energy and mass balances to the control volume enclosing the interconnecting heat exchanger

$$\dot{m}_{R\text{-}134a} = \frac{\dot{m}_{steam}(h_2 - h_3)}{(h_5 - h_8)} = \frac{(2\text{ kg/s})(2704.0 - 535.37)\text{ kJ/kg}}{(267.89 - 16.65)\text{ kJ/kg}} = 17.3\text{ kg/s}$$

For the control volume surrounding the Refrigerant 134a turbine

$$\dot{W}_{t2} = \dot{m}_{R\text{-}134a}(h_5 - h_6)$$

$$\dot{W}_{t2} = \left(17.3\frac{kg}{s}\right)(267.89 - 231.18)\frac{kJ}{kg}\left|\frac{1\,kW}{1\,kJ/s}\right| = 635.1\text{ kW}$$

For the Refrigerant 134a cycle pump

$$\dot{W}_{p2} = \dot{m}_{R\text{-}134a}(h_8 - h_7)$$

$$\dot{W}_{p} = \left(17.3\frac{kg}{s}\right)(16.65 - 16.29)\frac{kJ}{kg}\left|\frac{1\,kW}{1\,kJ/s}\right| = 6.2\text{ kW}$$

The net power developed by the Refrigerant 134a cycle is

$$\dot{W}_{\text{R-134a cycle}} = \dot{W}_{t2} - \dot{W}_{p2} = (635.1 - 6.2)\text{ kW} = 628.9\text{ kW}$$

The net power developed by the binary cycle is

$$\dot{W}_{Net} = \dot{W}_{\text{steam cycle}} + \dot{W}_{\text{R-134a cycle}} = (1859.5 + 628.9)\text{ kW} = \underline{\mathbf{2488.4\text{ kW}}}$$

(b) The thermal efficiency of the binary cycle is given by

$$\eta = \frac{\dot{W}_{Net}}{\dot{Q}_{in}}$$

Applying energy and mass balances to the control volume enclosing the steam generator

$$\dot{Q}_{\text{in}} = \dot{m}_{\text{steam}}(h_1 - h_4)$$

$$\dot{Q}_{\text{in}} = \left(2\frac{\text{kg}}{\text{s}}\right)(3642.0 - 543.64)\frac{\text{kJ}}{\text{kg}}\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right| = 6196.7\text{ kW}$$

Substituting values gives

$$\eta = \frac{2488.4\text{ kW}}{6196.7\text{ kW}} = \underline{\mathbf{0.402\ (40.2\%)}}$$

(c) The rate of entropy production in the interconnecting heat exchanger is determined using the steady-state form of the entropy rate balance:

$$0 = \cancel{\sum_j \frac{\dot{Q}_j}{T_j}} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{\text{cv}}$$

Since the interconnecting heat exchanger has no stray heat transfer with the surroundings, the heat transfer term drops. Thus,

$$\dot{\sigma}_{\text{cv}} = \sum_e \dot{m}_e s_e - \sum_i \dot{m}_i s_i = \dot{m}_{\text{steam}}(s_3 - s_2) + \dot{m}_{\text{R-134a}}(s_5 - s_8)$$

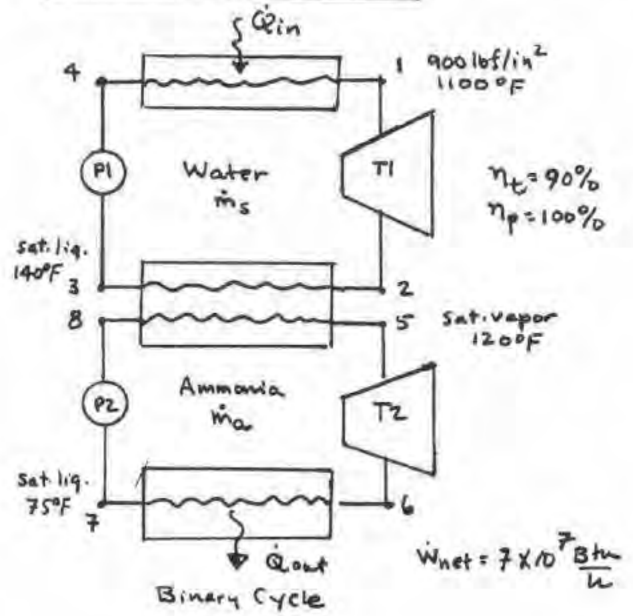
$$\dot{\sigma}_{\text{cv}} = \left[\left(2\frac{\text{kg}}{\text{s}}\right)(1.6072 - 7.0206)\frac{\text{kJ}}{\text{kg}\cdot\text{K}} + \left(17.3\frac{\text{kg}}{\text{s}}\right)(0.9388 - 0.0678)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right| = \underline{\mathbf{4.24\ kW/K}}$$

PROBLEM 8.77

KNOWN: Operating data are provided for a binary vapor cycle consisting of two Rankine cycles with steam and ammonia as the working fluids.

FIND: (a) For the binary cycle, determine the quality at the exit of each turbine, the mass flow rate of each working fluid, and the overall thermal efficiency. (b) Compare the performance to a single Rankine cycle using water as the working fluid, condensing at 75°F.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. Control volumes enclosing the principal components are at steady state.
2. The turbines and pumps operate adiabatically. There are no stray heat transfers, and kinetic and potential energy effects can be ignored.
3. For the heat exchangers, the pressure of the working fluid remains constant.

ANALYSIS: (a) Begin by fixing the numbered states on the schematic. From Table A-4E, $h_1 = 1565.4$ Btu/lb, $s_1 = 1.7036$ Btu/lb. With $s_{2s} = s_1$ and data from Table A-2E at 140°F

$$x_{2s} = \frac{1.7036 - 0.1985}{1.8892 - 0.1985} = 0.89$$

$\Rightarrow h_{2s} = 107.96 + 0.89(1014) = 1010.42 \frac{Btu}{lb}$. Then, with $\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}}$,

$h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 1565.4 - 0.9(1565.4 - 1010.42) = 1065.92$ Btu/lb. And

$$x_2 = \frac{h_2 - h_f}{h_g - h_f} = \frac{1065.92 - 107.96}{1014} = 0.945$$

For pump P1, Eq. 8.7b gives, $\frac{\dot{W}_{p1}}{\dot{m}_s} = v_3 \Delta p = \left(0.01629 \frac{ft^3}{lb}\right)(900 - 2.892)\frac{lbf}{in^2}\left|\frac{144\ in^2}{1\ ft^2}\right|\left|\frac{1\ Btu}{778\ ft\cdot lbf}\right| = 2.7$ Btu/lb

$\Rightarrow h_4 = h_3 + \frac{\dot{W}_{p1}}{\dot{m}_s} = 107.96 + 2.7 = 110.66$ Btu/lb.

Considering the ammonia cycle, Table A-13E gives $h_5 = 632.95$ Btu/lb, $s_5 = 1.1405$ Btu/lb·°R. With $s_{6s} = s_5$ and data from Table A-13E at 75°F

$x_{6s} = \frac{1.1405 - 0.2636}{1.2048 - 0.2636} = 0.932 \Rightarrow h_{6s} = 126.02 + 0.932(503.18) = 594.98$ Btu/lb. And

$h_6 = h_5 - \eta_t(h_5 - h_{6s}) = 632.95 - 0.9(632.95 - 594.98) = 598.78$ Btu/lb

$$\Rightarrow x_6 = \frac{598.78 - 126.02}{503.18} = 0.94$$

For pump P2, $\frac{\dot{W}_{p2}}{\dot{m}_a} = v_7 \Delta p = (0.02650)(286.4 - 140.6)\left|\frac{144}{778}\right| = 0.72 \frac{Btu}{lb}$

$\Rightarrow h_8 = h_7 + \frac{\dot{W}_{p2}}{\dot{m}_a} = 126.02 + 0.72 = 126.74$ Btu/lb

PROBLEM 8.77 (Contd.)

Next, an energy rate balance on the interconnecting heat exchanger gives,

$$\frac{\dot m_s}{\dot m_a} = \frac{h_5 - h_8}{h_2 - h_3} = \frac{632.95 - 126.74}{1065.92 - 107.96} = 0.528$$

Then,

$$\dot W_{net} = \dot m_s \left[h_1 - h_2 - \frac{\dot W_{p1}}{\dot m_s} \right] + \dot m_a \left[h_5 - h_6 - \frac{\dot W_{p2}}{\dot m_a} \right]$$

$$\left(7\times10^7 \frac{Btu}{h}\right) = 0.528\,\dot m_a \left[1565.4 - 1065.92 - 2.7\right] + \dot m_a \left[632.95 - 598.78 - 0.72\right]$$

$$\Rightarrow \dot m_a = 236{,}686 \frac{lb}{h}\ ,\ \dot m_s = 124{,}970 \frac{lb}{h}$$

The thermal efficiency is

$$\eta = \frac{\dot W_{net}}{\dot m_s (h_1 - h_4)} = \frac{7\times10^7\ Btu/h}{\left(124{,}970 \frac{lb}{h}\right)(1565.4 - 110.66)\frac{Btu}{lb}} = 0.385 \quad (38.5\%)$$

(b) For a single steam cycle condensing at 75°F, the exit condition for the steam turbine is fixed using $s_{2s} = s_1$. Then, with data from Table A-2E

$$x_{2s} = \frac{1.7036 - 0.08402}{2.04975 - 0.08402} = 0.824 \Rightarrow h_{2s} = 43.09 + 0.824(1051.15) = 909.24\ Btu/lb$$

$$\Rightarrow h_2 = h_1 - \eta_t (h_1 - h_{2s}) = 1565.4 - 0.9(1565.4 - 909.24) = 974.86\ Btu/lb$$

① $$\Rightarrow x_2 = \frac{974.86 - 43.09}{1051.15} = 0.886$$

The pump work per unit of steam flowing is

$$\frac{\dot W_p}{\dot m_s} = v_3 \Delta p = 0.01606\,(900 - 0.43)\left|\frac{144}{778}\right| = 2.67\ Btu/lb$$

$$\Rightarrow h_4 = h_3 + \frac{\dot W_p}{\dot m_s} = 43.09 + 2.67 = 45.76\ Btu/lb$$

The mass flow rate of the steam is

② $$\dot m_s = \frac{\dot W_{net}}{(h_1 - h_2) - \dot W_p/\dot m_s} = \frac{7\times10^7\ Btu/h}{[(1565.4 - 974.86) - 2.67]\ Btu/lb} = 119{,}074 \frac{lb}{h}$$

The thermal efficiency is

③ $$\eta = \frac{\dot W_{net}}{\dot m_s (h_1 - h_4)} = \frac{7\times10^7}{119{,}074(1565.4 - 45.76)} = 0.387 \quad (38.7\%)$$

---

1. Since the quality at the exit of a steam turbine should be at least 90%, the performance of the single Rankine cycle is less satisfactory than the binary cycle in this respect.

2. The mass flow rate of the steam in the single Rankine cycle is about 5% less than in the binary cycle.

3. Since the average temperatures of heat addition and heat rejection are the same in the two cycles, the thermal efficiencies are the same when roundoff is taken into account.

8.78 Figure P8.78 shows a vapor power cycle that provides process heat and produces power. The steam generator produces vapor at 500 lbf/in.^{2}, 800°F, at a rate of 8×10^4 lb/h. Eighty-eight percent of the steam expands through the turbine to 10 lbf/in.^{2} and the remainder is directed to the heat exchanger. Saturated liquid exits the heat exchanger at 500 lbf/in.^{2} and passes through a trap before entering the condenser at 10 lbf/in.^{2} Saturated liquid exits the condenser at 10 lbf/in.^{2} and is pumped to 500 lbf/in.^{2} before entering the steam generator. The turbine and pump have isentropic efficiencies of 85% and 89%, respectively. Determine
(a) The process heat production rate, in Btu/h.
(b) The thermal efficiency of the cycle.

KNOWN: A vapor power cycle operates with steam to produce process heat and power. Operational data are provided.

FIND: Determine (a) the production rate of process heat, in Btu/h, and (b) the thermal efficiency of the cycle.

SCHEMATIC AND GIVEN DATA:

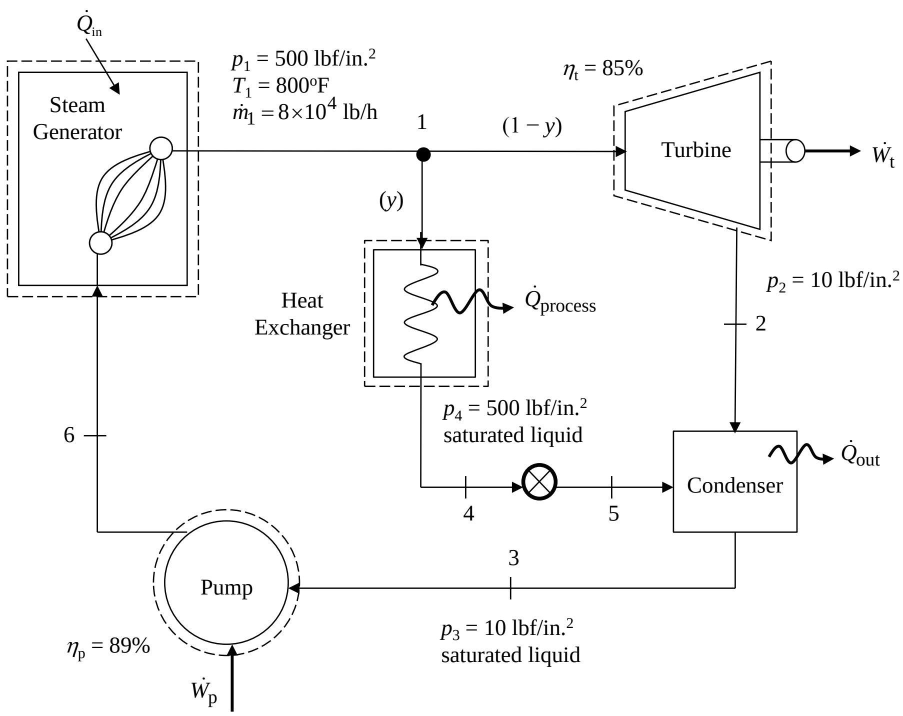


Fig. P8.78

ENGINEERING MODEL:
1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The turbine and pump operate adiabatically.
3. The expansion through the trap is a throttling process.
4. Flow through the heat exchanger and condenser occurs at constant pressure.
5. Kinetic and potential energy effects are negligible.

ANALYSIS: First fix each principal state.

State 1: p_1 = 500 lbf/in.^{2}, T_1 = 800°F → h_1 = 1412.1 Btu/lb, s_1 = 1.6571 Btu/lb·°R

State 2s: $p_{2s} = p_2$ = 10 lbf/in.^{2}, $s_{2s} = s_1$ = 1.6571 Btu/lb·°R → x_{2s} = 0.9132, h_{2s} = 1058.1 Btu/lb

State 2: p_2 = 10 lbf/in.^{2}, h_2 = 1111.2 Btu/lb (see below) → x_2 = 0.9673, s_2 = 1.7385 Btu/lb·°R

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \rightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 1412.1\frac{\text{Btu}}{\text{lb}} - (0.85)(1412.1 - 1058.1)\frac{\text{Btu}}{\text{lb}} = 1111.2 \text{ Btu/lb}$$

State 3: p_3 = 10 lbf/in.^{2}, saturated liquid → h_3 = 161.23 Btu/lb, v_3 = 0.01659 ft^3/lb

State 4: p_4 = 500 lbf/in.^{2}, saturated liquid → h_4 = 449.5 Btu/lb

State 5: p_5 = 10 lbf/in.^{2}, $h_5 = h_4$ =449.5 Btu/lb (throttling process)

State 6: p_6 = 500 lbf/in.^{2}, $h_6 \approx h_3 + \dfrac{v_3(p_6 - p_3)}{\eta_p}$

$$h_6 = 161.23\frac{\text{Btu}}{\text{lb}} + \frac{\left(0.01659\frac{\text{ft}^3}{\text{lb}}\right)(500-10)\frac{\text{lbf}}{\text{in.}^2}}{0.89}\left|\frac{144 \text{ in.}^2}{1 \text{ ft}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right| = 162.92 \text{ Btu/lb}$$

(a) Applying energy and mass balances to the control volume enclosing the heat exchanger

$$\dot{Q}_{\text{process}} = y\dot{m}_1(h_1 - h_4)$$

where y = 0.12 is the fraction of steam passing through the heat exchanger.

$$\dot{Q}_{\text{process}} = (0.12)\left(8\times10^4 \frac{\text{lb}}{\text{h}}\right)(1412.1 - 449.5)\frac{\text{Btu}}{\text{lb}} = 9.24 \times 10^6 \text{ Btu/h}$$

(b) The thermal efficiency of the cycle is given by

$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}}$$

where

$$\dot{W}_{\text{cycle}} = \dot{W}_t - \dot{W}_p$$

For the control volume surrounding the turbine

$$\dot{W}_t = (1-y)\dot{m}_1(h_1 - h_2)$$

$$\dot{W}_t = (0.88)\left(8\times10^4 \frac{\text{lb}}{\text{h}}\right)(1412.1-1111.2)\frac{\text{Btu}}{\text{lb}} = 21.18 \times 10^6 \text{ Btu/h}$$

For the pump

$$\dot{W}_p = \dot{m}_1(h_6 - h_3)$$

$$\dot{W}_p = \left(8\times10^4 \frac{\text{lb}}{\text{h}}\right)(162.92-161.23)\frac{\text{Btu}}{\text{lb}} = 0.14 \times 10^6 \text{ Btu/h}$$

Applying energy and mass balances to the control volume enclosing the steam generator

$$\dot{Q}_{in} = \dot{m}_1(h_1 - h_6)$$

$$\dot{Q}_{in} = \left(8\times10^4 \frac{\text{lb}}{\text{h}}\right)(1412.1-162.92)\frac{\text{Btu}}{\text{lb}} = 99.93 \times 10^6 \text{ Btu/h}$$

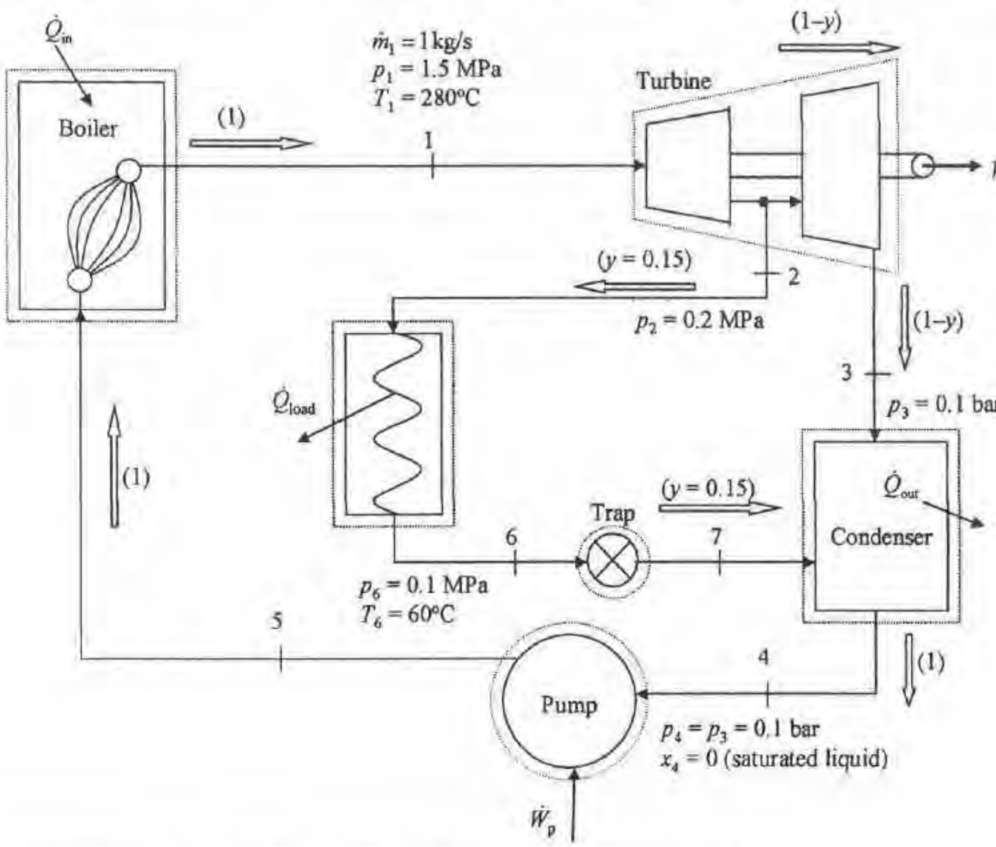
Substituting values gives

$$\eta = \frac{(21.16\times10^6 - 0.14\times10^6)\text{ Btu/h}}{99.93\times10^6\text{ Btu/h}} = \mathbf{0.210\ (21.0\%)}$$

The thermal efficiency is low in this case because only one of two valuable products is considered, net power out. The other product, process heat, is not included in the thermal efficiency. In such cases, an exergetic efficiency gives a more accurate picture of the performance.

8.79 Figure P8.79 provides steady-state operating data for a cogeneration cycle that generates electricity and provides heat for campus buildings. Steam at 1.5 MPa, 280°C, enters a two-stage turbine with a mass flow rate of 1 kg/s. A fraction of the total flow, 0.15, is extracted between the two stages at 0.2 MPa to provide for building heating, and the remainder expands through the second stage to the condenser pressure of 0.1 bar. Condensate returns from the campus buildings at 0.1 MPa, 60°C and passes through a trap into the condenser, where it is reunited with the main feedwater flow. Saturated liquid leaves the condenser at 0.1 bar. Determine

(a) the rate of heat transfer to the working fluid passing through the boiler, in kW.
(b) the net power developed, in kW.
(c) the rate of heat transfer for building heating, in kW.
(d) the rate of heat transfer to the cooling water passing through the condenser, in kW.



State	P	T (°C)	h (kJ/kg)
1	1.5 MPa	280	2992.7
2	0.2 MPa	sat	2652.9
3	0.1 bar	sat	2280.4
4	0.1 bar	sat	191.83
5	1.5 MPa		193.34
6	0.1 MPa	60	251.13
7	0.1 bar		251.13

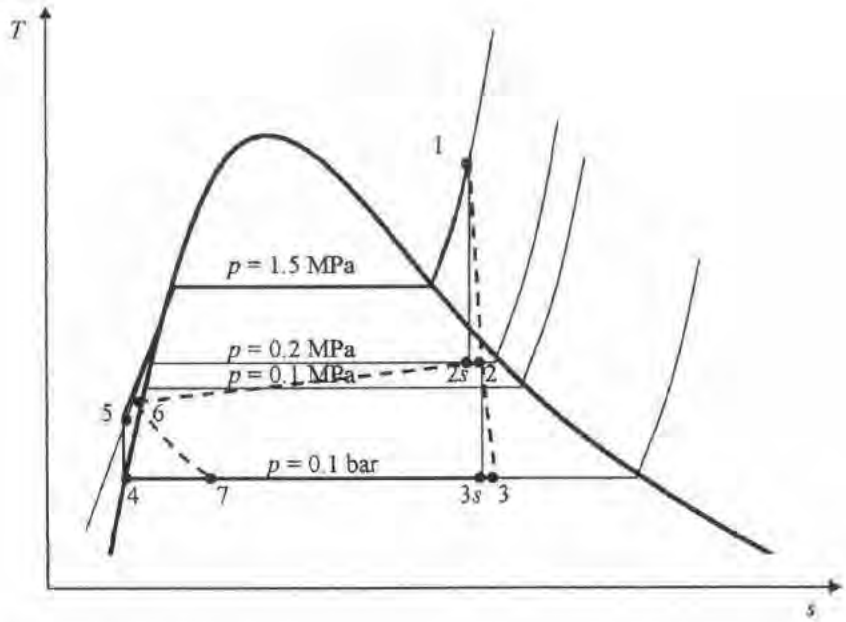
Fig P8.79

KNOWN: Water is the working fluid in a cogeneration cycle that generates electricity and provides heating for buildings.

FIND: Determine (a) the rate of heat transfer to the working fluid passing through the steam generator, (b) the net power developed, (c) the rate of heat transfer for building heating, and (d) the rate of heat transfer to the cooling water passing through the condenser.

SCHEMATIC AND GIVEN DATA: See problem statement above and *T-s* diagram below.

T-s diagram



ENGINEERING MODEL:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. Water exits the condenser as saturated liquid.
3. There is no heat transfer between the outside of the condenser and the surroundings.

ANALYSIS: (a) For a control volume enclosing the tubes carrying the working fluid through the boiler

$$\dot{Q}_{in} = \dot{m}_1(h_1 - h_5) = \left(1\frac{\text{kg}}{\text{s}}\right)\left[(2992.7 - 193.34)\frac{\text{kJ}}{\text{kg}}\right]\left|\frac{\text{kW}}{\frac{\text{kJ}}{\text{s}}}\right| = \underline{\mathbf{2799.4\ kW}}$$

(b) Since the fraction of the total flow extracted at state 2 is $y = 0.15$, the fraction of total flow expanded through the second stage of the turbine is $(1 - y) = 1 - 0.15 = 0.85$. The net power developed is obtained from analysis of the turbine and the pump. For a control volume enclosing the turbine

$$\dot{W}_t = \dot{m}_1(h_1 - h_2) + (1-y)\dot{m}_1(h_2 - h_3)$$

$$\dot{W}_t = \left(\left(1\frac{\text{kg}}{\text{s}}\right)\left[(2992.7 - 2652.9)\frac{\text{kJ}}{\text{kg}}\right] + (0.85)\left(1\frac{\text{kg}}{\text{s}}\right)\left[(2652.9 - 2280.4)\frac{\text{kJ}}{\text{kg}}\right]\right)\left|\frac{\text{kW}}{\frac{\text{kJ}}{\text{s}}}\right| = 656.43\ \text{kW}$$

For a control volume enclosing the pump

$$\dot{W}_p = \dot{m}_1(h_5 - h_4) = \left(1\frac{\text{kg}}{\text{s}}\right)\left[(193.34 - 191.83)\frac{\text{kJ}}{\text{kg}}\right]\left|\frac{\text{kW}}{\frac{\text{kJ}}{\text{s}}}\right| = 1.51\ \text{kW}$$

The net power developed by the cycle is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p = 656.43\ \text{kW} - 1.51\ \text{kW} = \underline{\mathbf{654.9\ kW}}$$

(c) For a control volume enclosing the heat exchanger providing thermal energy for the heating load

$$\dot{Q}_{load} = y\dot{m}_1(h_2 - h_6) = (0.15)\left(1\frac{\text{kg}}{\text{s}}\right)\left[(2652.9 - 251.13)\frac{\text{kJ}}{\text{kg}}\right]\left|\frac{\text{kW}}{\frac{\text{kJ}}{\text{s}}}\right| = \underline{\mathbf{360.3\ kW}}$$

For a control volume enclosing the condenser

$$\dot{Q}_{out} = y\dot{m}_1 h_7 + (1-y)\dot{m}_1 h_3 - m_1 h_4$$

$$\dot{Q}_{out} = \left[(0.15)\left(1\frac{\text{kg}}{\text{s}}\right)\left(251.13\frac{\text{kJ}}{\text{kg}}\right) + (0.85)\left(1\frac{\text{kg}}{\text{s}}\right)\left(2280.4\frac{\text{kJ}}{\text{kg}}\right) - \left(1\frac{\text{kg}}{\text{s}}\right)\left(191.83\frac{\text{kJ}}{\text{kg}}\right)\right]\left|\frac{\text{kW}}{\frac{\text{kJ}}{\text{s}}}\right|$$

$$\dot{Q}_{out} = \underline{\mathbf{1784.2\ kW}}$$

An overall energy rate balance reads:

• Rate energy is added by heat transfer in the boiler	2799.4 kW	
• Disposition		
- Net power developed	654.9 kW	(23.4%)
- Energy to campus buildings	360.3 kW	(12.9%)
- Energy to cooling water	1784.2 kW	(63.7%)
	2799.4 kW	(100%)

8.80 Consider a cogeneration system operating as shown in Fig. P8.80. Steam enters the first turbine stage at 6 MPa, 540°C. Between the first and second stages, 45% of the steam is extracted at 500 kPa and diverted to a process heating load of 5×10^8 kJ/h. Condensate exits the process heat exchanger at 450 kPa with specific enthalpy of 589.13 kJ/kg and is mixed with liquid exiting the lower pressure pump at 450 kPa. The entire flow is then pumped to the steam generator pressure. At the inlet to the steam generator the specific enthalpy is 469.91 kJ/kg. Saturated liquid at 60 kPa leaves the condenser. The turbine stages and the pumps operate with isentropic efficiencies of 82% and 88%, respectively. Determine
(a) the mass flow rate of steam entering the first turbine stage, in kg/s.
(b) the net power developed by the cycle, in MW.
(c) the rate of entropy production in the turbine, in kW/K.

KNOWN: A cogeneration system operates to produce power and process heat. Operational data are provided.

FIND: Determine (a) the mass flow rate of steam entering the first turbine stage, in kg/s, (b) the net power developed by the cycle, in MW, and (c) the rate of entropy production in the turbine, in kW/K.

SCHEMATIC AND GIVEN DATA:

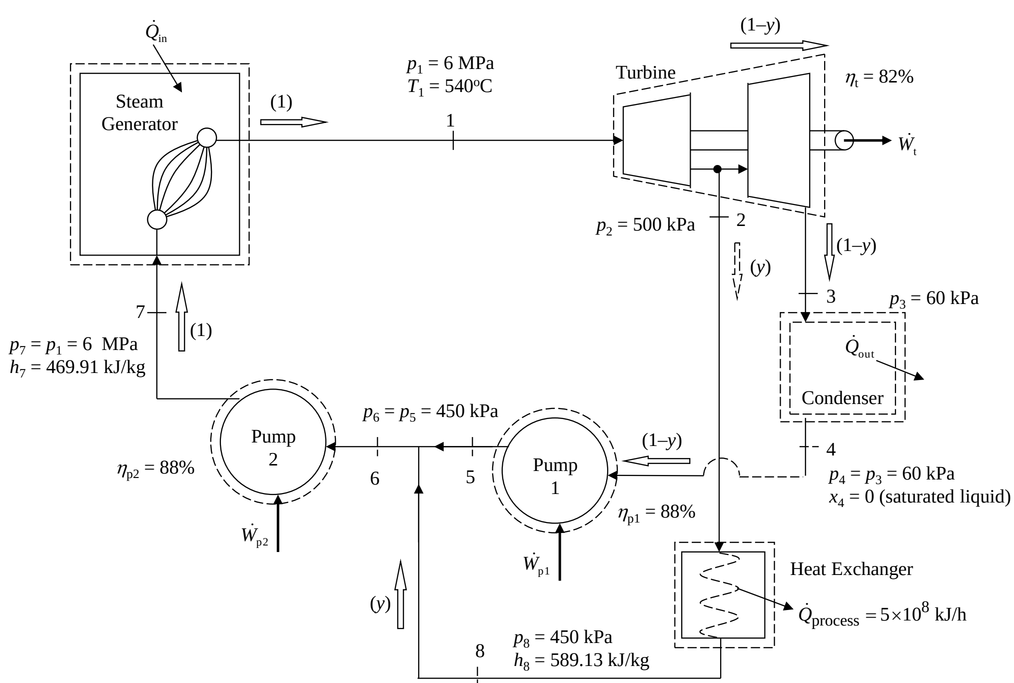


Fig. P8.80

ENGINEERING MODEL:
1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The turbine stages and pumps operate adiabatically.
3. Kinetic and potential energy effects are negligible.
4. Saturated liquid exits the condenser.

ANALYSIS: First fix each principal state.

State 1: p_1 = 6 MPa (60 bar), T_1 = 540°C → h_1 = 3157.0 kJ/kg, s_1 = 6.9999 kJ/kg·K

State 2s: $p_{2s} = p_2$ = 500 kPa (5 bar), $s_{2s} = s_1$ = 6. 9999 kJ/kg·K → h_{2s} = 2827.9 kJ/kg

State 2: p_2 = 500 kPa (5 bar), h_2 = 2951.9 kJ/kg (see below) → s_2 = 7.2532 kJ/kg·K

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \rightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 3517.0\frac{\text{kJ}}{\text{kg}} - (0.82)(3517.0 - 2827.9)\frac{\text{kJ}}{\text{kg}} = 2951.9 \text{ kJ/kg}$$

State 3s: $p_{3s} = p_3$ = 60 kPa (0.6 bar), $s_{3s} = s_2$ = 7.2532 kJ/kg·K → x_{3s} = 0.9563,
h_{3s} = 2553.2 kJ/kg

State 3: p_3 = 60 kPa (0.6 bar), h_3 = 2625.0 kJ/kg (see below) → x_3 = 0.9876,
s_3 = 7.4528 kJ/kg·K

$$\eta_t = \frac{h_2 - h_3}{h_2 - h_{3s}} \rightarrow h_3 = h_2 - \eta_t(h_2 - h_{3s}) = 2951.9\frac{\text{kJ}}{\text{kg}} - (0.82)(2951.9 - 2553.2)\frac{\text{kJ}}{\text{kg}} = 2625.0 \text{ kJ/kg}$$

State 4: p_4 = 60 kPa (0.6 bar), saturated liquid → h_4 = 359.86 kJ/kg, v_4 = 0.0010331 m³/kg

State 5: p_5 = 450 kPa (4.50 bar), $h_5 \approx h_4 + \dfrac{v_4(p_5 - p_4)}{\eta_{p1}}$

$$h_5 = 359.86\frac{\text{kJ}}{\text{kg}} + \frac{\left(0.0010331\dfrac{\text{m}^3}{\text{kg}}\right)(450-60)\text{kPa}}{0.88}\left|\frac{1000 \text{ N/m}^2}{1 \text{ kPa}}\right|\left|\frac{1 \text{ kJ}}{1000 \text{ N}\cdot\text{m}}\right| = 360.32 \text{ kJ/kg}$$

State 8: p_8 = 450 kPa (4.5 bar), h_8 = 589.13 kJ/kg

State 6: p_6 = 450 kPa (4.50 bar), h_6 = 463.28 kJ/kg (see below) → $v_6 \approx v_{f6}$ = 0.0010520 m³/kg (***assuming the saturated liquid state corresponding to $h_6 = h_f$ in Table A-2 and interpolating for $v_6 = v_f$***)

An energy balance at the junction of states 5, 6, and 8 gives

$$h_6 = (1-y)h_5 + yh_8 = [(0.55)(360.32) + (0.45)(589.13)]\frac{\text{kJ}}{\text{kg}} = 463.28 \text{ kJ/kg}$$

State 7: p_7 = 6 MPa (60 bar), h_7 = 469.91 kJ/kg

(a) Applying energy and mass balances to the control volume enclosing the process heat exchanger

$$\dot{m}_1 = \frac{\dot{Q}_{\text{process}}}{y(h_2 - h_8)} = \frac{(5\times10^8)\text{ kJ/h}}{(0.45)(2951.9 - 589.13)\text{ kJ/kg}}\left|\frac{1\text{ h}}{3600\text{ s}}\right| = \underline{\mathbf{130.6\ kg/s}}$$

(b) For the control volume surrounding the turbine stages

$$\dot{W}_\text{t} = \dot{m}_1[h_1 - yh_2 - (1-y)h_3]$$

$$\dot{W}_\text{t} = \left(130.6\frac{\text{kg}}{\text{s}}\right)[3517.0 - (0.45)(2951.9) - (0.55)(2625.0)]\frac{\text{kJ}}{\text{kg}}\left|\frac{1\text{ MW}}{1000\text{ kJ/s}}\right| = 97.28\text{ MW}$$

For the pumps

$$\dot{W}_\text{p} = \dot{m}_1[(h_7 - h_6) + (1-y)(h_5 - h_4)]$$

$$\dot{W}_\text{p} = \left(130.6\frac{\text{kg}}{\text{s}}\right)[(469.91 - 463.28) + (0.55)(360.32 - 359.86)]\frac{\text{kJ}}{\text{kg}}\left|\frac{1\text{ MW}}{1000\text{ kJ/s}}\right| = 0.90\text{ MW}$$

The net power developed by the cycle is

$$\dot{W}_\text{cycle} = \dot{W}_\text{t} - \dot{W}_\text{p} = (97.28 - 0.90)\text{ MW} = \underline{\mathbf{96.38\ MW}}$$

(c) The rate of entropy production in the turbine is determined using the steady-state form of the entropy rate balance:

$$0 = \cancel{\sum_j \frac{\dot{Q}_j}{T_j}} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_\text{cv}$$

Since the turbine is adiabatic, the heat transfer term drops. Thus,

$$\dot{\sigma}_\text{cv} = \sum_e \dot{m}_e s_e - \sum_i \dot{m}_i s_i = \dot{m}_1 y s_2 + \dot{m}_1(1-y)s_3 - \dot{m}_1 s_1$$

$$\dot{\sigma}_\text{cv} = \dot{m}_1[ys_2 + (1-y)s_3 - s_1]$$

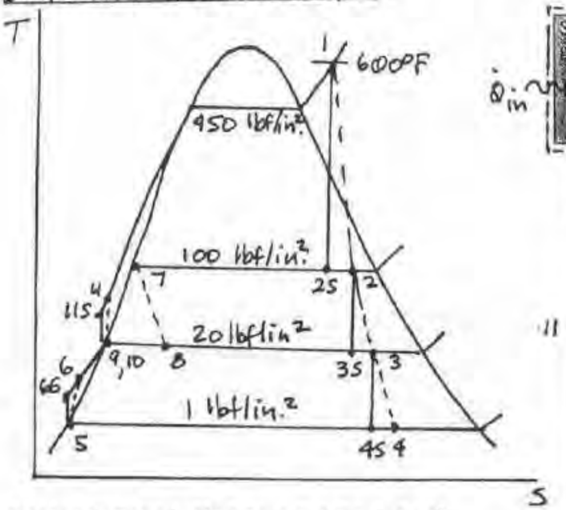
$$\dot{\sigma}_\text{cv} = 130.6\frac{\text{kg}}{\text{s}}[(0.45)(7.2532) + (0.55)(7.4528) - 6.9999]\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = \underline{\mathbf{47.4\ kW/K}}$$

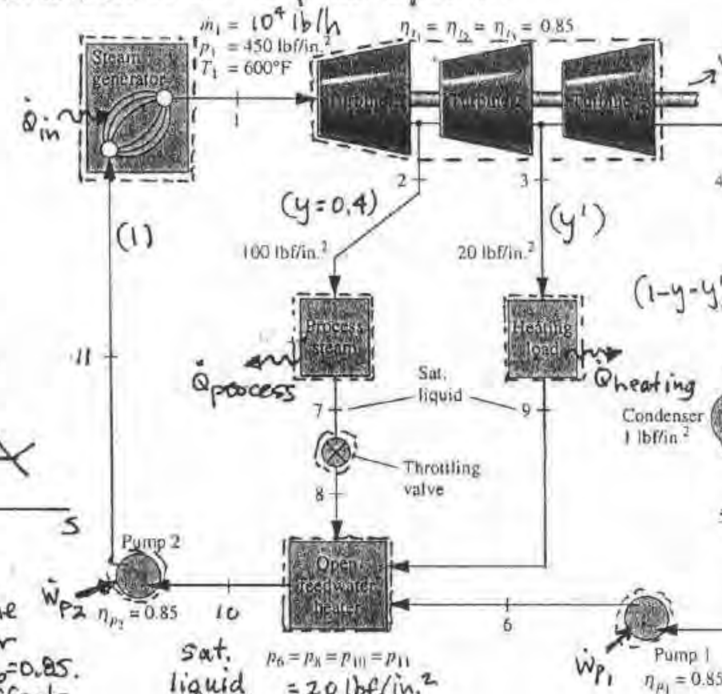
PROBLEM 8.81

KNOWN: A combined heat and power system provides turbine power, process Steam, and steam for a space heating load. Data are given at key states.

FIND: Determine (a) the rates steam is extracted, (b) the rates of heat transfer for each load, and (c) the net power. Devise and evaluate an overall energy-based efficiency for the combined heat and power system.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) Each component is modeled as a control volume at steady state. (2) The turbine stages, pumps and feedwater heater operate adiabatically, and $\eta_t = \eta_p = 0.85$. (3) Kinetic and potential energy effects are negligible. (4) Saturated liquid exits the condenser.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 450$ lbf/in.², $T_1 = 600$°F $\Rightarrow h_1 = 1302.5 \frac{\text{Btu}}{\text{lb}}$, $s_1 = 1.5732 \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$

State 2: $p_2 = 100$ lbf/in.², $s_{2s} = s_1 \Rightarrow x_{2s} = (s_{2s} - s_{f2})/s_{fg2} = 0.9783 \Rightarrow h_{2s} = 1164.1$ Btu/lb

With the turbine-stage efficiency $h_2 = h_1 - \eta_{t1}(h_1 - h_{2s}) = 1184.9$ Btu/lb

Using h_2, $x_2 = (h_2 - h_{f2})/h_{fg2} = 0.9967 \Rightarrow s_2 = 1.5997$ Btu/lb·°R

State 3: $p_3 = 20$ lbf/in.², $s_{3s} = s_2 \Rightarrow x_{3s} = 0.9052$, $h_{3s} = 1065.3$ Btu/lb

Thus $h_3 = h_2 - \eta_{t2}(h_2 - h_{3s}) = 1083.2$ Btu/lb. Also, $x_3 = 0.9238$, $s_3 = 1.6256 \frac{\text{Btu}}{\text{lb}°\text{R}}$

State 4: $p_4 = 1$ lbf/in.², $s_{4s} = s_3 \Rightarrow x_{4s} = 0.8090$, $h_{4s} = 907.9$ Btu/lb

Thus $h_4 = h_3 - \eta_{t3}(h_3 - h_{4s}) = 934.2$ Btu/lb

State 5: $p_1 = 1$ lbf/in.², sat. liquid $\Rightarrow h_5 = 69.74$ Btu/lb

State 6: $p_6 = 20$ lbf/in.²; $h_{6s} \approx h_5 + v_5(p_6 - p_5)$

$$= 69.74 \frac{\text{Btu}}{\text{lb}} + \left(0.01614 \frac{\text{ft}^3}{\text{lb}}\right)(20-1)\frac{\text{lbf}}{\text{in}^2}\left|\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right|$$
$$= 69.80 \text{ Btu/lb}$$

with the pump efficiency, $h_6 = h_5 + (h_{6s} - h_5)/\eta_{p1} = 69.81$ Btu/lb

State 7: $p_7 = 100$ lbf/in.², sat. liquid $\Rightarrow h_7 = 298.6$ Btu/lb, State 8: $h_8 = h_7 = 298.6$ Btu/lb

State 9, 10: $p_9 = p_{10} = 20$ lbf/in.², sat. liquid $\Rightarrow h_9 = h_{10} = 196.26$ Btu/lb

State 11: $h_{11s} \approx h_{10} + v_{10}(p_{11} - p_{10}) = 196.26 + (0.01683)(450-20)\left|\frac{144}{778}\right| = 197.60$ Btu/lb

$h_{11} = h_{10} + (h_{11s} - h_{10})/\eta_{p2} = 197.84$ Btu/lb

(a) For the steam extracted at state 2: $y = \dot{m}_2/\dot{m}_1 \Rightarrow \dot{m}_2 = y\,\dot{m}_1 = (0.4)(10^4\ \text{lb/h})$
$= 4\times10^3\ \text{lb/h}$ ← $\dot{m}_2$

To find $\dot{m}_3$: $\dot{m}_3 = y'\,\dot{m}_1$

For the open feedwater heater: $0 = y\,h_8 + y'\,h_9 + (1-y-y')h_6 - h_{10}$

Solving for y':

$$y' = \frac{y(h_6-h_8)+h_{10}-h_6}{(h_9-h_6)} = \frac{(0.4)(69.81-298.6)+196.26-69.81}{(196.26-69.81)} = 0.2763$$

Thus

$$\dot{m}_3 = (0.2763)(10^4\ \text{lb/h}) = 2.763\times10^3\ \text{lb/h}$$ ← $\dot{m}_3$

(b) $\dot{Q}_{\text{Process}} = \dot{m}_2(h_2-h_7) = (4\times10^3\ \text{lb/h})(1184.9-298.6)\ \frac{\text{Btu}}{\text{lb}} = 3.545\times10^6\ \text{Btu/h}$ ← $\dot{Q}_{\text{process}}$

$\dot{Q}_{\text{Heating}} = \dot{m}_3(h_3-h_9) = (2.763\times10^3)(1083.2-196.26) = 2.451\times10^6\ \text{Btu/h}$ ← $\dot{Q}_{\text{Heating}}$

(c) For the control volume enclosing the turbine stages

$$\dot{W}_t = \dot{m}_1\left[h_1 - y\,h_2 - y'h_3 - (1-y-y')h_4\right]$$
$$= \left(10^4\ \tfrac{\text{lb}}{\text{h}}\right)\left[1302.5-(0.4)(1184.9)-(0.2763)(1083.2)-(0.3237)(934.2)\right]\ \tfrac{\text{Btu}}{\text{lb}}$$
$$= 2.268\times10^6\ \text{Btu/h}$$

For the pumps

$$\dot{W}_p = \dot{W}_{P_1} + \dot{W}_{P_2} = \dot{m}_1\left[(1-y-y')(h_6-h_5)+(h_{11}-h_{10})\right]$$
$$= \left(10^4\ \tfrac{\text{lb}}{\text{h}}\right)\left[(0.3237)(69.80-69.74)+(197.84-196.26)\right]\ \tfrac{\text{Btu}}{\text{lb}} = 1.6\times10^4\ \text{Btu/h}$$

$\dot{W}_{\text{cycle}} = \dot{W}_t - \dot{W}_p = 2.252\times10^6\ \text{Btu/h}$ ← $\dot{W}_{\text{cycle}}$

<u>Energy-based Efficiency</u>: The energy transfers $\dot{Q}_{\text{Process}}$, $\dot{Q}_{\text{Heating}}$, and $\dot{W}_{\text{cycle}}$ all have economic value. Thus, a rational energy-based efficiency is

① $$\eta_{\text{overall}} = \frac{(\dot{Q}_{\text{Process}}+\dot{Q}_{\text{Heating}}+\dot{W}_{\text{cycle}})}{\dot{Q}_{\text{in}}} = \frac{(\dot{Q}_{\text{Process}}+\dot{Q}_{\text{Heating}}+\dot{W}_{\text{cycle}})}{\dot{m}_1(h_1-h_{11})}$$
$$= \frac{3.545\times10^6+2.451\times10^6+2.252\times10^6}{(10^4)(1302.5-197.84)}$$
$$= \frac{8.248\times10^6}{1.1047\times10^7} = 0.747\ (74.7\%)$$ ← η_{overall}

1. An exergetic efficiency would provide a more accurate picture of the economic value provided with the combined heating and power cycle.

8.82 Figure P8.82 shows a cogeneration cycle that provides power and process heat. In the steam cycle, superheated vapor enters the turbine at 40 bar, 440°C and expands isentropically to 1 bar. The steam passes through a heat exchanger, which serves as a boiler of the Refrigerant 134a cycle and the condenser of the steam cycle. The condensate leaves the heat exchanger as saturated liquid at 1 bar and is pumped isentropically to the steam generator pressure. The rate of heat transfer to the working fluid passing through the steam generator of the steam cycle is 13 MW. The Refrigerant 134a cycle is an ideal Rankine cycle with refrigerant entering the turbine at 16 bar, 100°C. The refrigerant passes through a heat exchanger, which provides process heat and acts as a condenser for the Refrigerant 134a cycle. Saturated liquid exits the heat exchanger at 9 bar. Determine

(a) The mass flow rate of steam entering the steam turbine, in kg/s.
(b) The mass flow rate of Refrigerant 134a entering the refrigerant turbine, in kg/s.
(c) The percent of total power provided by each cycle.
(d) The rate of heat transfer provided as process heat, in kW.

KNOWN: A cogeneration system operates to produce power and process heat. Operational data are provided.

FIND: Determine (a) the mass flow rate of steam entering the steam turbine, in kg/s, (b) the mass flow rate of Refrigerant 134a entering the refrigerant turbine, in kg/s, (c) the percent of total power provided by each cycle, and (d) the rate of heat transfer provided as process heat, in kW.

SCHEMATIC AND GIVEN DATA:

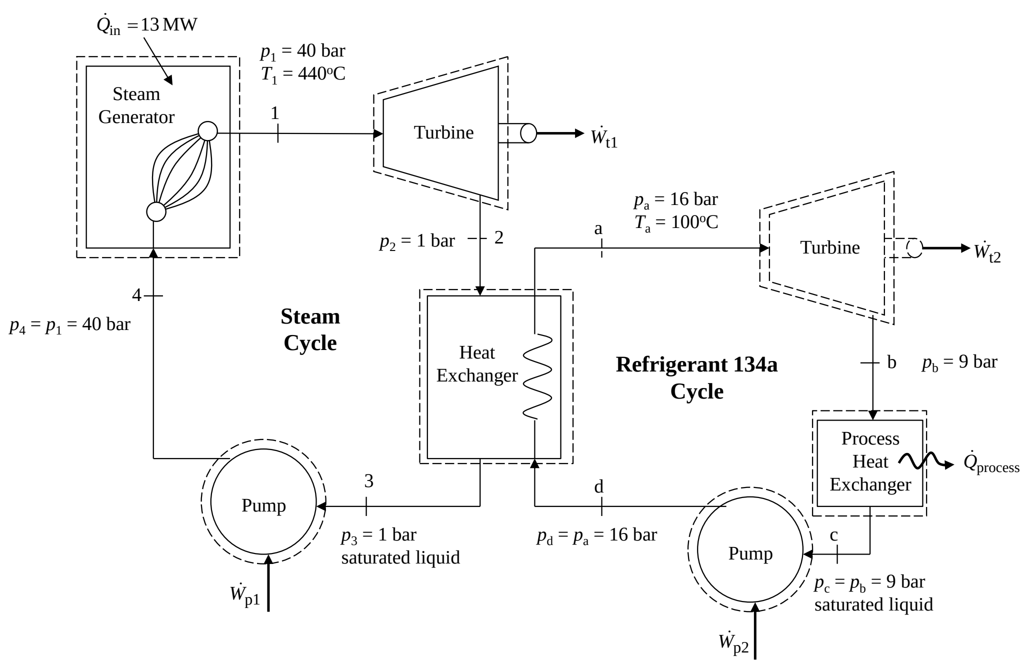


Fig. P8.82

ENGINEERING MODEL:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The turbines and pumps operate isentropically.

3. For the heat exchanger, no stray heat transfer occurs with the surroundings.
4. Kinetic and potential energy effects are negligible.
5. Saturated liquid exits the heat exchanger and the process heat exchanger.

ANALYSIS: First fix each principal state.

For the steam cycle:

State 1: p_1 = 40 bar, T_1 = 440°C → h_1 = 3307.1 kJ/kg, s_1 = 6.9041 kJ/kg·K

State 2: p_2 = 1 bar, $s_2 = s_1$ = 6.9041 kJ/kg·K → x_2 = 0.9248, h_2 = 2505.7 kJ/kg

State 3: p_3 = 1 bar, saturated liquid → $h_3 = h_{f3}$ = 417.46 kJ/kg, $v_3 = v_{f3}$ = 0.0010432 m^3/kg

State 4: $p_4 = p_1$ = 40 bar, $h_4 \approx h_3 + v_3(p_4 - p_3)$

$$h_4 \approx 417.46\frac{\text{kJ}}{\text{kg}} + \left(0.0010432\frac{\text{m}^3}{\text{kg}}\right)(40-1)\text{ bar}\left|\frac{100\text{ kPa}}{1\text{ bar}}\right|\left|\frac{1000\text{ N/m}^2}{1\text{ kPa}}\right|\left|\frac{1\text{ kJ}}{1000\text{ N}\cdot\text{m}}\right| = 421.53\text{ kJ/kg}$$

For the Refrigerant 134a cycle:

State a: p_a = 16 bar, T_a = 100°C → h_a = 327.46 kJ/kg, s_a = 1.0467 kJ/kg·K

State b: p_b = 9 bar, $s_b = s_a$ = 1.0467 kJ/kg·K → h_b = 312.74 kJ/kg

State c: p_c = 9 bar, saturated liquid → $h_c = h_{fc}$ = 99.56 kJ/kg, $v_c = v_{fc}$ = 0.0008576 m^3/kg

State d: $p_d = p_a$ = 16 bar, $h_d \approx h_c + v_c(p_d - p_c)$

$$h_d \approx 99.56\frac{\text{kJ}}{\text{kg}} + \left(0.0008576\frac{\text{m}^3}{\text{kg}}\right)(16-9)\text{ bar}\left|\frac{100\text{ kPa}}{1\text{ bar}}\right|\left|\frac{1000\text{ N/m}^2}{1\text{ kPa}}\right|\left|\frac{1\text{ kJ}}{1000\text{ N}\cdot\text{m}}\right| = 100.16\text{ kJ/kg}$$

(a) Applying energy and mass balances to the control volume enclosing the steam generator

$$\dot{m}_{\text{steam}} = \frac{\dot{Q}_{\text{in}}}{h_1 - h_4} = \frac{13{,}000\text{ kW}}{(3307.1-421.53)\text{ kJ/kg}}\left|\frac{1\text{ kJ/s}}{1\text{ kW}}\right| = \underline{\mathbf{4.505 kg/s}}$$

(b) Applying energy and mass balances to the control volume enclosing the heat exchanger

$$\dot{m}_{\text{R134a}} = \frac{\dot{m}_{\text{steam}}(h_2 - h_3)}{(h_a - h_d)} = \frac{(4.505\text{ kg/s})(2505.7-417.46)\text{ kJ/kg}}{(327.46-100.16)\text{ kJ/kg}} = \underline{\mathbf{41.39\text{ kg/s}}}$$

(c) Determine the net power developed by each cycle to calculate the percent power developed by each cycle. The net power developed by the steam cycle is

$$\dot{W}_{\text{steamcycle}} = \dot{W}_{\text{t1}} - \dot{W}_{\text{p1}}$$

$$\dot{W}_{\text{steamcycle}} = \dot{m}_{\text{steam}}[(h_1 - h_2) - (h_4 - h_3)]$$

$$\dot{W}_{\text{steamcycle}} = \left(4.505\,\frac{\text{kg}}{\text{s}}\right)[(3307.1 - 2505.7) - (421.53 - 417.46)]\frac{\text{kJ}}{\text{kg}}\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right| = 3592\text{ kW}$$

The net power developed by the Refrigerant 134a cycle is

$$\dot{W}_{\text{R134acycle}} = \dot{W}_{\text{t2}} - \dot{W}_{\text{p2}}$$

$$\dot{W}_{\text{R134acycle}} = \dot{m}_{\text{R134a}}[(h_a - h_b) - (h_d - h_c)]$$

$$\dot{W}_{\text{R134acycle}} = \left(41.39\,\frac{\text{kg}}{\text{s}}\right)[(327.46 - 312.74) - (100.16 - 99.56)]\frac{\text{kJ}}{\text{kg}}\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right| = 584\text{ kW}$$

The total net power developed by both cycles is

$$\dot{W}_{\text{cycle}} = \dot{W}_{\text{steamcycle}} + \dot{W}_{\text{R134acycle}}$$

$$\dot{W}_{\text{cycle}} = (3592 + 584)\text{ kW} = \underline{\mathbf{4176\ kW}}$$

Solving for the percent of power provided by each cycle gives

Steam Cycle: % Power = $\dfrac{\dot{W}_{\text{steamcycle}}}{\dot{W}_{\text{cycle}}} = \dfrac{3592\text{ kW}}{4176\text{ kW}} = \underline{\mathbf{0.860\ (86.0\%)}}$

Refrigerant 134a Cycle: % Power = $\dfrac{\dot{W}_{\text{R134acycle}}}{\dot{W}_{\text{cycle}}} = \dfrac{584\text{ kW}}{4176\text{ kW}} = \underline{\mathbf{0.140\ (14.0\%)}}$

(d) The rate of heat transfer for process heat is determined by writing a mass and energy rate balance for the Refrigerant 134a line through the process heat exchanger.

$$\dot{Q}_{\text{process}} = \dot{m}_{\text{R134a}}(h_b - h_c) = (41.39\text{ kg/s})(312.74 - 99.56)\text{ kJ/kg}\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right| = \underline{\mathbf{8824\ kW}}$$

Alternatively, the rate of heat transfer for process heat can be determined from an overall cycle energy balance.

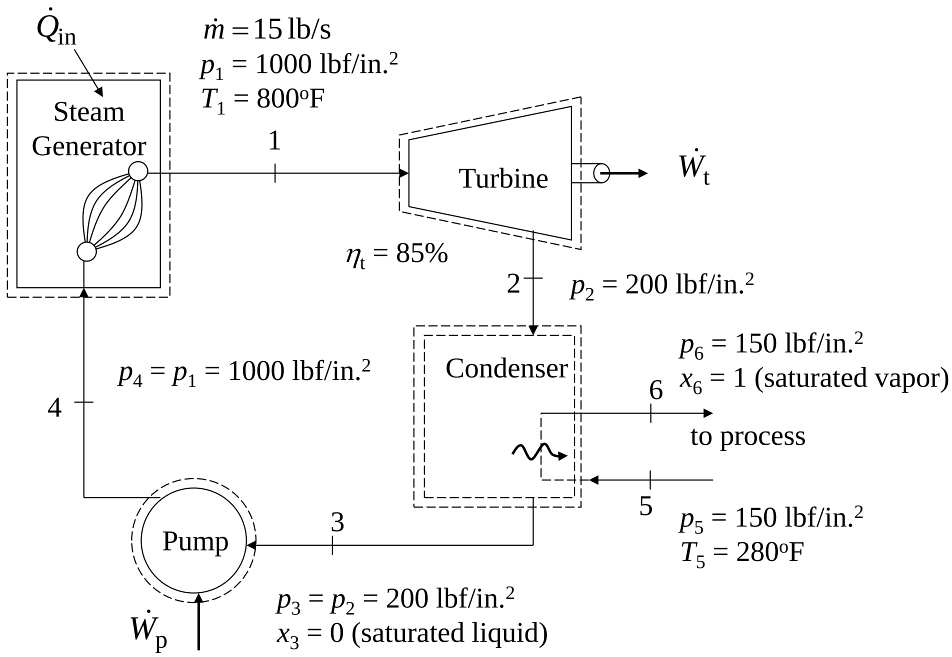
$$\dot{Q}_{\text{process}} = \dot{Q}_{\text{in}} - \dot{W}_{\text{cycle}} = (13{,}000 - 4176)\text{ kW} = \underline{\mathbf{8824\ kW}}$$

8.83 In a *cogeneration* system, a Rankine cycle operates with steam entering the turbine at a rate of 15 lb/s at 1000 lbf/in.^{2}, 800°F, and a condenser pressure of 200 lbf/in.^{2} The isentropic turbine efficiency is 85% while the pump operates isentropically. Energy rejected by the condensing steam is transferred to a separate process stream of water entering at 280°F, 150 lbf/in.^{2} and exiting as saturated vapor at 150 lbf/in.^{2} Determine the mass flow rate, in lb/s, for the process stream. Based on the increase in exergy of the steam passing through the steam generator, devise and evaluate an exergetic efficiency for the overall cogeneration system. Let T_0 = 70°F, p_0 = 14.7 lbf/in.^{2}

KNOWN: Water is the working fluid in a Rankine cycle used for cogeneration. The cycle produces power, and energy rejected from the condensing steam is transferred to a separate process stream. Data are known at various locations.

FIND: Determine the mass flow rate, in lb/s, for the process stream and devise and evaluate an exergetic efficiency for the overall cogeneration system.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The pump operates isentropically.
3. For the turbine and condenser, no stray heat transfer occurs with the surroundings.
4. Kinetic and potential energy effects are negligible.
5. Saturated liquid exits the condenser.
6. T_0 = 70°F = 530 $^\circ$R, p_0 = 14.7 lbf/in.^{2}

ANALYSIS: First fix each principal state.

State 1: p_1 = 1000 lbf/in.^{2}, T_1 = 800$^\text{o}$F → h_1 = 1388.5 Btu/lb, s_1 = 1.5665 Btu/lb·$^\text{o}$R

State 2s: p_{2s} = 200 lbf/in.^{2}, $s_{2s} = s_1$ = 1.5665 Btu/lb · $^\text{o}$R → h_{2s} = 1216.6 Btu/lb

State 2: p_2 = 200 lbf/in.^{2}, h_2 = 1242.4 Btu/lb (see below) → s_2 = 1.5956 Btu/lb·$^\text{o}$R

$$\eta_\text{t} = \frac{h_1 - h_2}{h_1 - h_{2s}} \rightarrow h_2 = h_1 - \eta_\text{t}(h_1 - h_{2s}) = 1388.5\frac{\text{Btu}}{\text{lb}} - (0.85)(1388.5 - 1216.6)\frac{\text{Btu}}{\text{lb}} = 1242.4 \text{ Btu/lb}$$

State 3: p_3 = 200 lbf/in.^{2}, saturated liquid → $h_3 = h_{f3}$ = 355.6 Btu/lb, $v_3 = v_{f3}$ = 0.01839 ft^3/lb,
$s_3 = s_{f3}$ = 0.5440 Btu/ lb·$^\text{o}$R

State 4: $p_4 = p_1$ = 1000 lbf/in.^{2}, $s_4 = s_3$ = 0.5440 Btu/ lb·$^\text{o}$R $\rightarrow h_4 \approx h_3 + v_3(p_4 - p_3)$

$$h_4 \approx 355.6\frac{\text{Btu}}{\text{lb}} + \left(0.01839\frac{\text{ft}^3}{\text{lb}}\right)(1000-200)\frac{\text{lbf}}{\text{in}^2}\left|\frac{144\,\text{in}^2}{1\,\text{ft}^2}\right|\left|\frac{1\,\text{Btu}}{778\,\text{ft}\cdot\text{lbf}}\right| = 358.3 \text{ Btu/lb}$$

State 5: p_5 = 150 lbf/in.^{2}, T_5 = 280$^\text{o}$F → sub-cooled liquid. Thus, with Eq. 3.14 we get

$h_5 \approx h_{f5}$ (@ 280$^\text{o}$F) = 249.2 Btu/lb and $s_5 \approx s_{f5}$ (@ 280$^\text{o}$F) = 0.4099 Btu/lb·$^\text{o}$R

State 6: p_6 = 150 lbf/in.^{2}, saturated vapor → $h_6 = h_{g6}$ = 1194.9 Btu/lb,
$s_6 = s_{g6}$ =1.5704 Btu/lb·$^\text{o}$R

The mass flow rate of the process stream can be determined by writing an energy balance for the condenser. With no stray heat transfer with the surroundings and no work, the energy balance for the condenser reduces to

$$0 = \dot{m}(h_2 - h_3) + \dot{m}_\text{process}(h_5 - h_6)$$

where $\dot{m}$ is the mass flow rate of the steam and $\dot{m}_\text{process}$ is the mass flow rate of the process stream. Rearranging to solve for the mass flow rate of the process stream gives

$$\dot{m}_\text{process} = \frac{\dot{m}(h_2 - h_3)}{(h_6 - h_5)}$$

Substituting values and solving give

$$\dot{m}_\text{process} = \frac{(15\,\text{lb/s})(1242.4 - 355.6)\text{ Btu/lb}}{(1194.9 - 249.2)\text{ Btu/lb}} = \underline{\mathbf{14.1\ lb/s}}$$

For the cogeneration system, the cycle net work and the exergy transferred to the process stream are the outputs, and the exergy increase of the working fluid passing through the steam generator is the input. The difference between the input and output represents exergy destroyed due to irreversibilities. Thus, a reasonable exergetic efficiency is

$$\epsilon = \frac{\text{net rate of exergy output}}{\text{rate of exergy input}}$$

$$\epsilon = \frac{\dot{W}_{\text{cycle}} + \dot{m}_{\text{process}}(\mathsf{e}_{f6} - \mathsf{e}_{f5})}{\dot{m}(\mathsf{e}_{f1} - \mathsf{e}_{f4})}$$

Evaluating the various quantities in this expression

$$\dot{W}_{\text{cycle}} = \dot{W}_t - \dot{W}_p = \dot{m}[(h_1 - h_2) - (h_4 - h_3)]$$

$$\dot{W}_{\text{cycle}} = \left(15\frac{\text{lb}}{\text{s}}\right)[(1388.5 - 1242.4) - (358.3 - 355.6)]\frac{\text{Btu}}{\text{lb}} = 2151 \text{ Btu/s}$$

$$\mathsf{e}_{f6} - \mathsf{e}_{f5} = [(h_6 - h_5) - T_0(s_6 - s_5)]$$

$$\mathsf{e}_{f6} - \mathsf{e}_{f5} = \left[(1194.9 - 249.2)\frac{\text{Btu}}{\text{lb}} - (530^\circ\text{R})(1.5704 - 0.4099)\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right] = 330.6 \text{ Btu/lb}$$

$$\mathsf{e}_{f1} - \mathsf{e}_{f4} = [(h_1 - h_4) - T_0(s_1 - s_4)]$$

$$\mathsf{e}_{f1} - \mathsf{e}_{f4} = \left[(1388.5 - 358.3)\frac{\text{Btu}}{\text{lb}} - (530^\circ\text{R})(1.5665 - 0.5440)\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right] = 488.3 \text{ Btu/lb}$$

Thus,

$$\epsilon = \frac{2151\frac{\text{Btu}}{\text{s}} + \left(14.1\frac{\text{lb}}{\text{s}}\right)\left(330.6\frac{\text{Btu}}{\text{lb}}\right)}{\left(15\frac{\text{lb}}{\text{s}}\right)\left(488.3\frac{\text{Btu}}{\text{lb}}\right)} = \underline{\mathbf{0.9301\ (93.01\%)}}$$

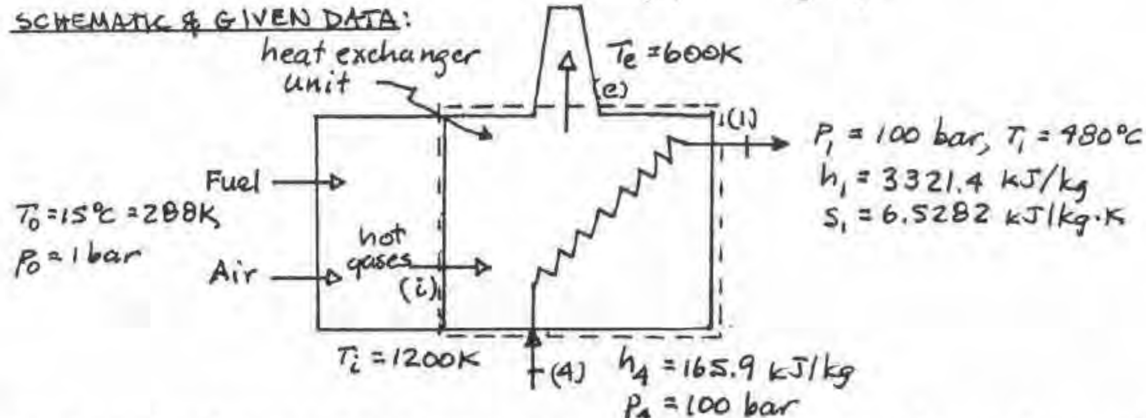
In this case the very significant exergy destruction accompanying combustion of fuel in the steam generator is not considered. As indicated in Table 8.4 approximately 30% of the exergy entering the steam generator with the fuel is destroyed during combustion. The exergy increase of the working fluid as it passes through the steam generator and reheater is the remaining 70% of exergy from the fuel. Thus the overall exergetic efficiency is much less than the value calculated.

PROBLEM 8.84

KNOWN: Hot combustion gases enter the heat exchanger unit of the steam generator of Problem 8.17. The inlet and exit temperatures of the gas stream are provided. Other data are from the solution of Problem 8.17.

FIND: Determine (a) the net rate exergy is carried in by the gas stream, (b) the net rate exergy is carried out by the waterstream, (c) the rate of exergy destruction, (d) the exergetic efficiency given by Eq. 7.27.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) For the steady-state control volume shown, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) Kinetic and potential energy effects are negligible. (3) The combustion gases are modeled as air as an ideal gas, (4) Each stream experiences negligible pressure drop. (5) $T_0 = 288$ K, $p_0 = 1$ bar.

ANLYSIS: From steady-state mass balances on the water and air streams, respectively; $\dot{m}_4 = \dot{m}_1 \equiv \dot{m}$ and $\dot{m}_i = \dot{m}_e \equiv \dot{m}_{air}$. Thus, the energy rate balance is

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_4 - h_1) + \dot{m}_{air}(h_i - h_e)$$

or

$$\frac{\dot{m}_{air}}{\dot{m}} = \frac{h_1 - h_4}{h_i - h_e}$$

From Table A-22, $h_i = 1277.79$ kJ/kg and $h_e = 607.02$ kJ/kg. Thus

$$\frac{\dot{m}_a}{\dot{m}} = \frac{3321.4 - 165.9}{1277.79 - 607.02} = 4.704$$

(a) The net rate exergy is carried in by the gas stream is

$$\frac{\dot{E}_{f_i} - \dot{E}_{f_e}}{\dot{m}} = \frac{\dot{m}_{air}}{\dot{m}}(e_{f_i} - e_{f_e}) = \frac{\dot{m}_{air}}{\dot{m}}\left[(h_i - h_e) - T_0(s_i - s_e)\right]$$

Since $p_i = p_e$; $s_i - s_e = s_i^\circ - s_e^\circ - \cancel{R \ln(p_i/p_e)}^{\,0} = s_i^\circ - s_e^\circ$. With data from Table A-22

$$\frac{\dot{E}_{f_i} - \dot{E}_{f_e}}{\dot{m}} = (4.704)\left[(1277.79 - 607.02) - (288)(3.17888 - 2.40902)\right]\frac{kJ}{kg}$$

$$= 2112 \text{ kJ/kg of steam flow} \longleftarrow$$ Net rate exergy carried in

(b) The net rate exergy is carried out by the water stream is

$$\frac{\dot{E}_{f_1} - \dot{E}_{f_4}}{\dot{m}} = (e_{f_1} - e_{f_4}) = (h_1 - h_4) - T_0(s_1 - s_4)$$

Assuming $s_4 \approx s_f(T_4)$ and interpolating in Table A-2 with h_4; $s_4 \approx 0.5672$

Thus $e_{f_1} - e_{f_4} = (3321.4 - 165.9) - (288)(6.5282 - 0.5672) = 1438.7\ \frac{kJ}{kg}$ ← net rate exergy carried out

(c) Applying the exergy rate balance to the heat exchanger unit

$$0 = \sum_j \cancel{\left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j}^{\,0} - \cancel{\dot{W}_{cv}}^{\,0} + \dot{m}(e_{f_4} - e_{f_1}) + \dot{m}_{air}(e_{f_i} - e_{f_e}) - \dot{E}_d$$

or

① $$\frac{\dot{E}_d}{\dot{m}} = \frac{\cancel{\dot{m}}(e_{f_4} - e_{f_1})}{\cancel{\dot{m}}} + \frac{\dot{m}_{air}(e_{f_i} - e_{f_e})}{\dot{m}}$$

$$= (-1438.7) + (2112) = 673.3\ kJ/kg$$ ← $\dot{E}_d/\dot{m}$

(d) Applying Eq. 7.27

$$\epsilon = \frac{\dot{m}(e_{f_1} - e_{f_4})}{\dot{m}_{air}(e_{f_i} - e_{f_e})} = \frac{1438.7}{2112} = 0.681\ (68.1\%)$$ ← ϵ

---

1. Exergy destruction in this case is due to heat transfer from the higher-temperature combustion gas stream to the vaporizing water.

PROBLEM 8.85

KNOWN: The vapor power cycle of Problem 8.17 is to be analyzed from the perspective of exergy accounting.

FIND: Determine the rate of input to the working fluid passing through the steam generator and account for all outputs, losses, and destructions of this exergy.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.17.

ENGINEERING MODEL: Same as Problem 8.17. Also, $T_0 = 15°C = 288 K$, $p_0 = 1$ bar.

ANALYSIS: With data from the solution to Problem 8.17

State 1: $h_1 = 3321.4$ kJ/kg, $s_1 = 6.5282$ kJ/kg·K

State 2: $h_2 = 2272.1$ kJ/kg, $s_2 = 7.8334$ kJ/kg·K $(x_2 = 0.8778)$

State 3: $h_3 = 151.53$ kJ/kg, $s_3 = 0.5210$ kJ/kg·K

State 4: $h_4 = 165.9$ kJ/kg, $s_4 \approx 0.5672$ kJ/kg·K (Assume $s_4 \approx s_f(T_4)$)

Input
$$\frac{\dot{E}_{f_1} - \dot{E}_{f_4}}{\dot{m}} = (e_{f_1} - e_{f_4}) = (h_1 - h_4) - T_0(s_1 - s_4)$$
$$= (3321.4 - 65.9) - (288)(6.5282 - 0.5672)$$
$$= 1438.7 \text{ kJ/kg} \leftarrow \text{Input}$$

Outputs

- Net Power: $\frac{\dot{W}_{cycle}}{\dot{m}} = 1034.9 \frac{kJ}{kg}$

- Condenser: $\frac{\dot{E}_{f_2} - \dot{E}_{f_3}}{\dot{m}} = e_{f_2} - e_{f_3} = (h_2 - h_3) - T_0(s_2 - s_3)$
$$= (2272.1 - 151.53) - (288)(7.8334 - 0.5210)$$
$$= 14.6 \text{ kJ/kg}$$

Destructions:

- Turbine: $\frac{\dot{E}_{d,turb}}{\dot{m}} = T_0(s_2 - s_1) = (288)(7.8334 - 6.5282)$
$$= 375.9 \text{ kJ/kg}$$

- Pump: $\frac{\dot{E}_{d,pump}}{\dot{m}} = T_0(s_4 - s_3) = (288)(0.5672 - 0.5210)$
$$= 13.31 \text{ kJ/kg}$$

Exergy Accounting Summary

Input	1438.7 kJ/kg
Outputs	1034.9
	14.6
	1049.5 kJ/kg
Destructions:	375.9
	13.3
	389.2 kJ/kg
Total:	1438.7 kJ/kg

8.86 In the steam generator of the cycle of Problem 8.19, the energy input to the working fluid is provided by heat transfer from hot gaseous products of combustion, which cool as a separate stream from 1490 to 380°F with a negligible pressure drop. The gas stream can be modeled as air as an ideal gas. Determine, in Btu/h, the rate of exergy destruction in the

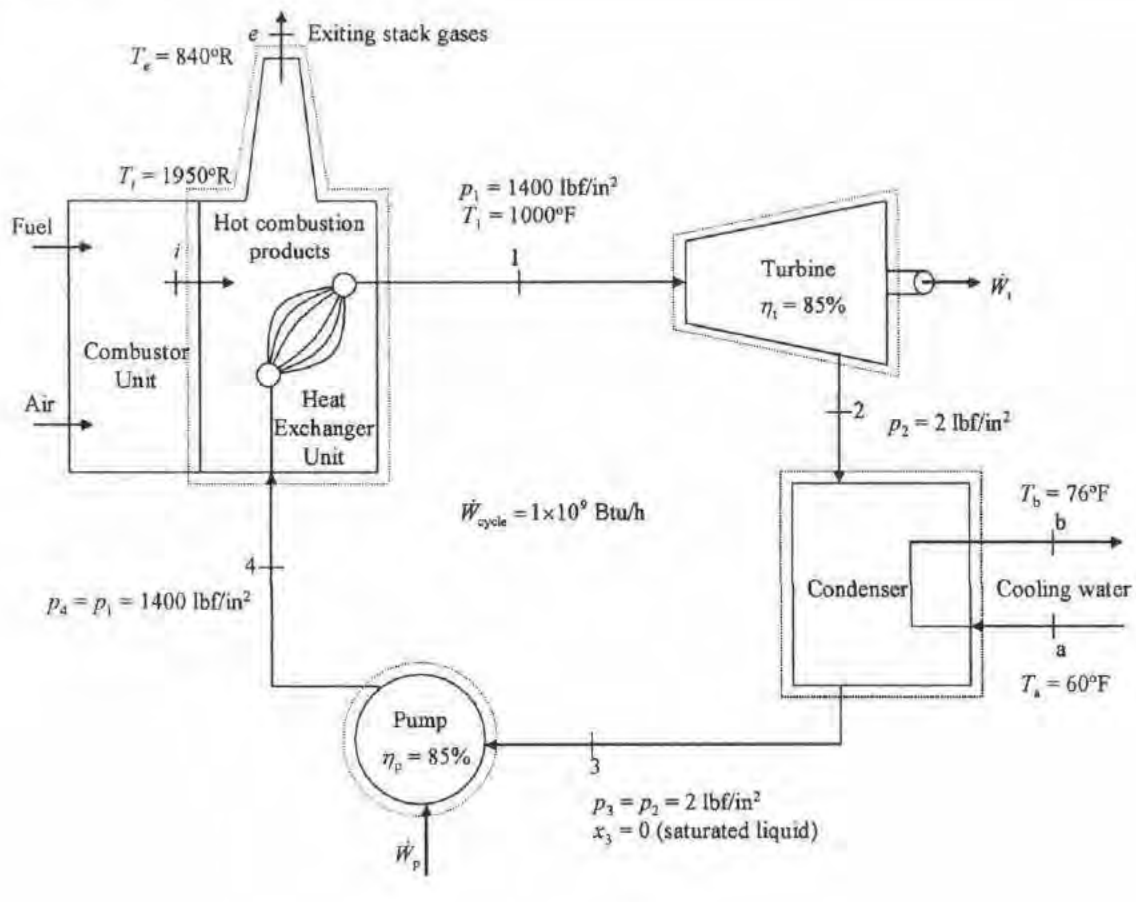
(a) heat exchanger unit of the steam generator.
(b) turbine and pump.
(c) condenser.

Also, calculate the net rate at which exergy is carried away by the cooling water passing through the condenser, in Btu/h. Let $T_0 = 60°F$, $p_0 = 14.7\ lbf/in.^2$

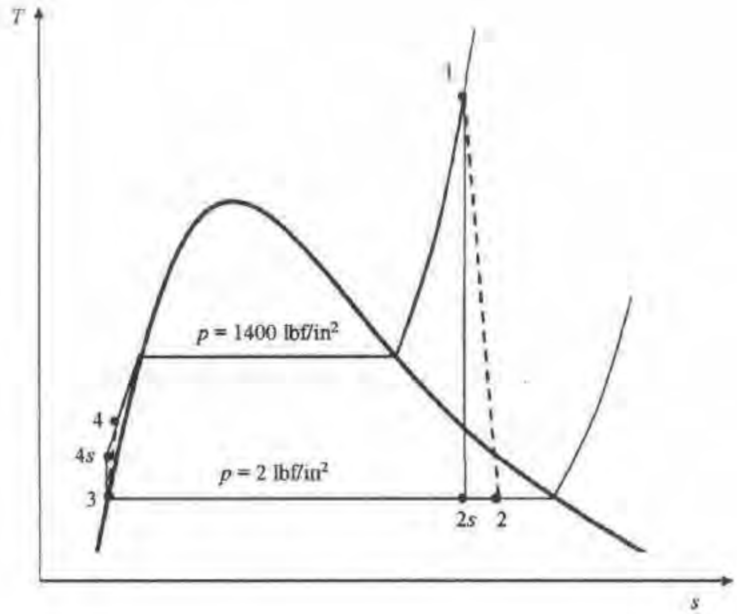
KNOWN: Hot combustion gases enter the heat exchanger unit of the steam generator of Problem 8.19. The inlet and exit temperatures of the gas stream are known. Other data are from Problem 8.19.

FIND: Determine the exergy destruction rates in (a) the heat exchanger unit, (b) the turbine and pump, and (c) the condenser. Also, calculate the net rate exergy is carried away by the cooling water passing through the condenser.

SCHEMATIC AND GIVEN DATA:



T-s diagram



ENGINEERING MODEL:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The turbine and pump each operate adiabatically with an isentropic efficiency of 85%.
3. Kinetic and potential energy effects are negligible.
4. Condensate exits the condenser as saturated liquid.
5. There is no heat transfer between the outside of the condenser and the surroundings.
6. $T_0 = 60°F$, $p_0 = 14.7$ lbf/in.^{2}
7. The combustion gases are modeled as air as an ideal gas.
8. The gas stream, the working fluid passing through the heat exchanger unit, the condensing steam in the condenser, and the condenser cooling water stream each experience no pressure drop.
9. There is no heat transfer between the heat exchanger unit and the surroundings.

ANALYSIS: A summary of results from Problem 8.19 is as follows:

State	T (°F)	p (lbf/in.^{2})	h (Btu/lb)	s (Btu/(lb·°R))	$\dot{m}$ (lb/h)	x
1	1000	1400	1493.5	1.6094	2.13×10^6	
2		2	1018.17		2.13×10^6	
3		2	94.02		2.13×10^6	0
4		1400	98.96		2.13×10^6	
a	60		28.08		1.23×10^8	
b	76		44.09		1.23×10^8	

(a) First, determine the mass flow rate of the air passing through the heat exchanger unit. For the control volume surrounding the heat exchanger unit, steady-state mass balances for the water and air stream reduce, respectively, to $\dot{m}_4 = \dot{m}_1 = \dot{m}$ and $\dot{m}_i = \dot{m}_e = \dot{m}_{\text{air}}$. Thus, the energy rate balance is

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_4 - h_1) + \dot{m}_{\text{air}}(h_i - h_e)$$

Since no heat transfer occurs with the surroundings and there is no work, the energy rate balance reduces to

$$\dot{m}_{\text{air}} = \frac{\dot{m}(h_1 - h_4)}{(h_i - h_e)}$$

From Table A-22E, h_i = 490.88 Btu/lb and h_e = 201.56 Btu/lb. Thus

$$\dot{m}_{\text{air}} = \frac{2.13\times10^6\,\frac{\text{lb}}{\text{h}}\left(1493.5\frac{\text{Btu}}{\text{lb}} - 98.96\frac{\text{Btu}}{\text{lb}}\right)}{\left(490.88\frac{\text{Btu}}{\text{lb}} - 201.56\frac{\text{Btu}}{\text{lb}}\right)} = 1.03\times10^7 \text{ lb/h}$$

The rate of exergy destruction can be found from $\dot{\mathsf{E}}_{\text{d}} = T_0\dot{\sigma}_{cv}$. The entropy rate balance for the control volume is

$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{m}(s_4 - s_1) + \dot{m}_{\text{air}}(s_i - s_e) + \dot{\sigma}_{cv}$$

Since no heat transfer occurs with the surroundings

$$\dot{\sigma}_{cv} = \dot{m}(s_1 - s_4) + \dot{m}_{\text{air}}(s_e - s_i)$$

Thus, for the **<u>heat exchanger unit</u>**

$$\dot{\mathsf{E}}_{\text{d}} = T_0\left[\dot{m}(s_1 - s_4) + \dot{m}_{\text{air}}(s_e - s_i)\right]$$

Since $p_i = p_e$, $s_e - s_i = s^{\circ}(T_e) - s^{\circ}(T_i) - R\ln\left(\frac{p_e}{p_i}\right) = s^{\circ}(T_e) - s^{\circ}(T_i)$. With values from Table A-22E, $s^{\circ}(T_e)$ = 0.70747 and $s^{\circ}(T_i)$ = 0.92505. From Problem 8.19, s_1 = 1.6094 Btu/(lb·°R). From Table A-5E (double-interpolated), s_4 = 0.17537 Btu/(lb·°R).

Problem 8.86 (Continued) – Page 4

$$\dot{E}_d = (520^\circ\text{R})\left[\begin{array}{l} 2.13\times10^6\,\dfrac{\text{lb}}{\text{h}}\left(1.6094\,\dfrac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} - 0.17537\,\dfrac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right) \\ +1.03\times10^7\,\dfrac{\text{lb}}{\text{h}}\left(0.70747\,\dfrac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} - 0.92505\,\dfrac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\end{array}\right] = \underline{\mathbf{4.23\times10^8\ Btu/h}}$$

(b) **For the turbine**

$$\dot{E}_d = T_0\dot{m}(s_2 - s_1)$$

Apply the quality relations to determine x_2 and then s_2. From Table A-3E: h_{f2} = 94.02 Btu/lb, h_{fg2} = 1022.1 Btu/lb, s_{f2} = 0.1750 Btu/(lb·°R), s_{fg2} = 1.7448 Btu/(lb·°R).

Substituting values

$$x_2 = \frac{1018.17\,\dfrac{\text{Btu}}{\text{lb}} - 94.02\,\dfrac{\text{Btu}}{\text{lb}}}{1022.1\,\dfrac{\text{Btu}}{\text{lb}}} = 0.9042$$

s_2 = 0.1750 Btu/(lb·°R) + (0.9042)(1.7448 Btu/(lb·°R)) = 1.7526 Btu/(lb·°R)

Substituting values to solve for rate of exergy destruction

$$\dot{E}_d = (520^\circ\text{R})\left[2.13\times10^6\,\frac{\text{lb}}{\text{h}}\left(1.7526\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} - 1.6094\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\right] = \underline{\mathbf{1.59\times10^8\ Btu/h}}$$

For the pump

$$\dot{E}_d = T_0\dot{m}(s_4 - s_3)$$

Since state 3 is saturated liquid, from Table A-3E $s_3 = s_{f3}$ = 0.1750 Btu/(lb·°R). Substituting values to solve for rate of exergy destruction

$$\dot{E}_d = (520^\circ\text{R})\left[2.13\times10^6\,\frac{\text{lb}}{\text{h}}\left(0.17537\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} - 0.1750\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\right] = \underline{\mathbf{4.10\times10^5\ Btu/h}}$$

(c) **For the condenser**

$$\dot{E}_d = T_0\left[\dot{m}(s_3 - s_2) + \dot{m}_{cw}(s_b - s_a)\right]$$

Problem 8.86 (Continued) – Page 5

Using Table 2E for the cooling water: $s_a \approx s_{fa}$ @ $T_a = 60°F = 0.05555$ Btu/(lb·°R) and $s_b \approx s_{fb}$ @ $T_b = 76°F = 0.08589$ Btu/(lb·°R). Substituting values

$$\dot{E}_d = (520°R)\left[\begin{array}{l} 2.13\times10^6 \dfrac{lb}{h}\left(0.1750\dfrac{Btu}{lb\cdot°R} - 1.7526\dfrac{Btu}{lb\cdot°R}\right) \\ +1.23\times10^8 \dfrac{lb}{h}\left(0.08589\dfrac{Btu}{lb\cdot°R} - 0.05555\dfrac{Btu}{lb\cdot°R}\right)\end{array}\right] = \underline{\mathbf{1.93 \times 10^8\ Btu/h}}$$

The ***net*** **rate exergy is carried out by the cooling water passing through the condenser** is

$$\dot{m}_{cw}(e_{fb} - e_{fa}) = \dot{m}_{cw}[(h_b - h_a) - T_0(s_b - s_a)]$$

$$\dot{m}_{cw}(e_{fb} - e_{fa}) = 1.23\times10^8 \frac{lb}{h}\left[(44.09 - 28.08)\frac{Btu}{lb} - (520°R)(0.08589 - 0.05555)\frac{Btu}{lb\cdot°R}\right]$$

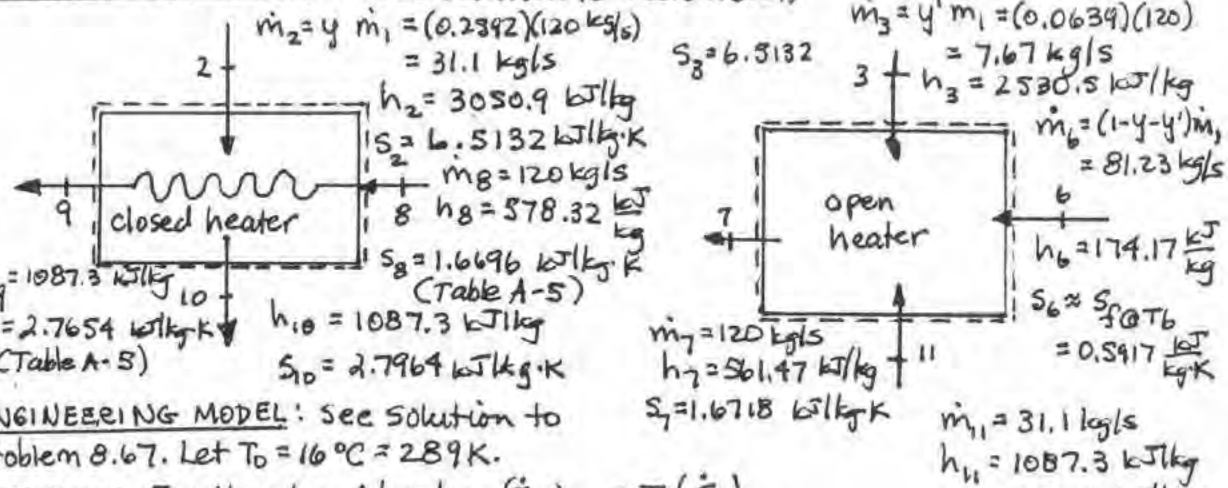
$$\dot{m}_{cw}(e_{fb} - e_{fa}) = \underline{\mathbf{2.87 \times 10^7\ Btu/h}}$$

PROBLEM 8.87

KNOWN: Feedwater heaters operate as in the regenerative vapor power cycle of Problem 8.67.

FIND: Determine the exergy destruction rates for each feedwater heater and express each as a percentage of the flow exergy increase of the working fluid passing through the steam generator.

SCHEMATIC & GIVEN DATA: (See solution to Problem 8.67)



ENGINEERING MODEL: See solution to Problem 8.67. Let $T_0 = 16\,°C = 289\,K$.

ANALYSIS: For the closed heater, $(\dot{E}_d)_{CH} = T_0(\dot{\sigma}_{cv})_{CH}$.

Applying an entropy balance

$$0 = \cancel{\sum_j \left(\frac{\dot{Q}}{T}\right)_j}^{\,0} + \dot{m}_2(s_2 - s_{10}) + \dot{m}_8(s_8 - s_9) + (\dot{\sigma}_{cv})_{CH}$$

Thus

$$(\dot{E}_d)_{CH} = T_0\left[\dot{m}_2(s_{10} - s_2) + \dot{m}_8(s_9 - s_8)\right]$$

$$= (289\,K)\left[(31.1)(2.7964 - 6.5132) + (120)(2.7654 - 1.6696)\right]\frac{kJ}{s\cdot K}\left|\frac{1\,kW}{1\,kJ/s}\right|$$

$$= 4596\ kW \qquad (\dot{E}_d)_{CH}$$

Similarly for the open feedwater heater

$$0 = \cancel{\sum_j \left(\frac{\dot{Q}}{T}\right)_j}^{\,0} + \dot{m}_3 s_3 + \dot{m}_6 s_6 + \dot{m}_{11} s_{11} - \dot{m}_7 s_7 + (\dot{\sigma}_{cv})_{OH}$$

And

$$(\dot{E}_d)_{OH} = T_0\left[\dot{m}_7 s_7 - \dot{m}_3 s_3 - \dot{m}_6 s_6 - \dot{m}_{11} s_{11}\right]$$

$$= (289)\left[(120)(1.6718) - (7.67)(6.5132) - (81.23)(0.5917) - (31.1)(2.9646)\right]$$

$$= 3005\ kW \qquad (\dot{E}_d)_{OH}$$

For the working fluid passing through the steam generator

$$\dot{E}_{f1} - \dot{E}_{f9} = \dot{m}_1(e_{f1} - e_{f9}) = \dot{m}_1\left[(h_1 - h_9) - T_0(s_1 - s_9)\right]$$

$$= (120)\left[(3465.4 - 1087.3) - (289)(6.5132 - 2.7654)\right] = 1.554 \times 10^5\ kW$$

Thus

$$\frac{(\dot{E}_d)_{CH}}{\dot{E}_{f1} - \dot{E}_{f9}} = \frac{4596}{1.554 \times 10^5} = 0.0296\ (2.96\%) \qquad \text{closed heater}$$

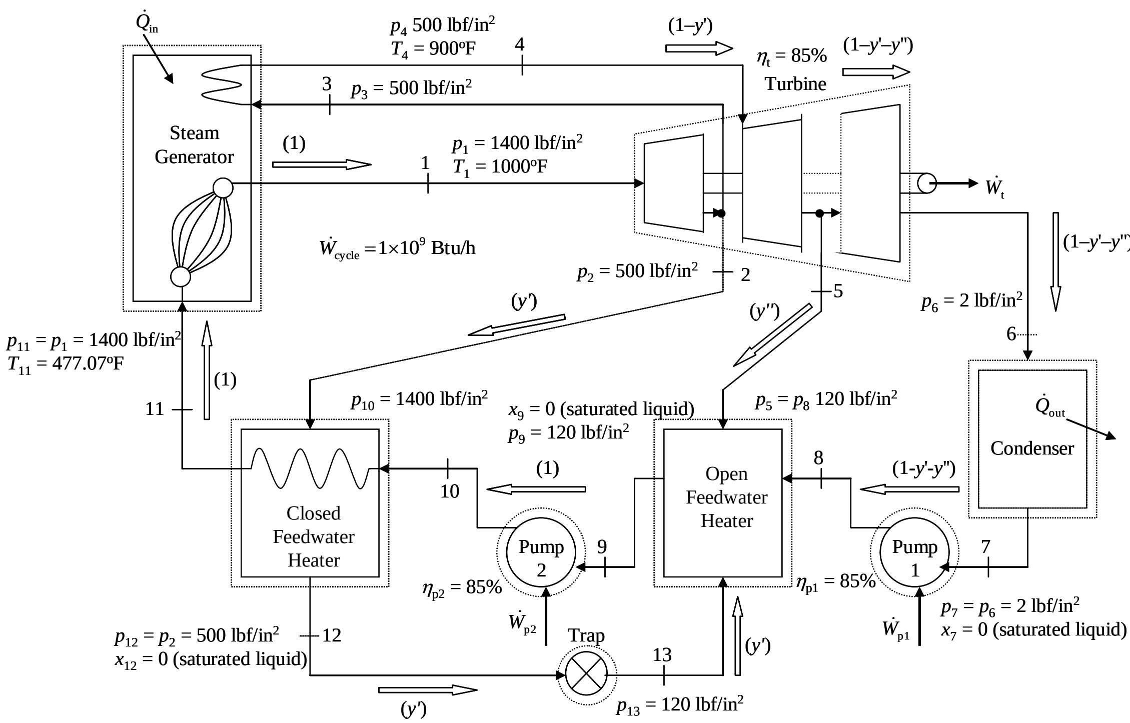
$$\frac{(\dot{E}_d)_{OH}}{\dot{E}_{f1} - \dot{E}_{f2}} = \frac{3005}{1.554 \times 10^5} = 0.0193\ (1.93\%) \qquad \text{open heater}$$

8.88 Determine the rate of exergy input, in Btu/s, to the working fluid passing through the steam generator in Problem 8.73. Perform calculations to account for all outputs, losses, and destructions of this exergy. Let $T_0 = 60^{o}F$, $p_0 = 14.7$ lbf/in.^{2}

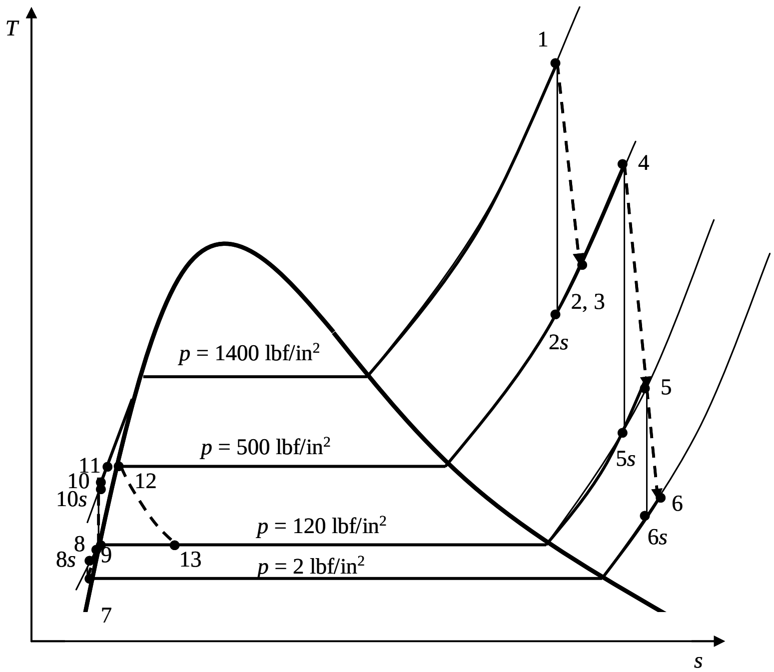
KNOWN: A reheat-regenerative Rankine cycle with one closed feedwater heater and one open feedwater heater operates with water as the working fluid. The net power output of the cycle is given.

FIND: Determine the rate of exergy input, in Btu/s, to the working fluid passing through the steam generator and account for the disposition of this exergy.

SCHEMATIC AND GIVEN DATA:



T-*s* diagram



ENGINEERING MODEL:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The turbine and pumps each operate adiabatically with an isentropic efficiency of 85%.
3. Flow through the boiler, reheater, closed feedwater heater, open feedwater heater, and condenser is at constant pressure.
4. Kinetic and potential energy effects are negligible.
5. Condensate exits the condenser, the closed feedwater heater, and the open feedwater heater as saturated liquid.
6. There is no heat transfer between the outside of the condenser and the surroundings.
7. There is no heat transfer between the outside of the closed feedwater heater and the surroundings.
8. There is no heat transfer between the outside of the open feedwater heater and the surroundings.
9. Let $T_0 = 60^{\circ}F$ and $p_0 = 14.7$ lbf/in.^{2}

ANALYSIS: First, fix each of the principal states (***from problem 8.73 solution***).

State 1: $p_1 = 1400$ lbf/in.^{2}, $T_1 = 1000^{\circ}F$ → From Table A-4E: $h_1 = 1493.5$ Btu/lb and $s_1 = 1.6094$ Btu/(lb·°R)

State 2*s*: $p_{2s} = p_2 = 500$ lbf/in.^{2}, $s_{2s} = s_1$ → From Table A-4E (interpolated): $h_{2s} = 1354.71$ Btu/lb

State 2: $p_2 = 500$ lbf/in.^{2}, $h_6 = 1375.55$ Btu/lb (see below) → From Table A-4E (interpolated): $s_2 = 1.6268$ Btu/(lb·°R)

$$h_2 = h_1 - \eta_t(h_1 - h_{2s})$$

$$h_2 = 1493.5 \text{ Btu/lb} - 0.85(1493.5 - 1354.71) \text{ Btu/lb} = 1375.55 \text{ Btu/lb}$$

State 3: $p_3 = 500$ lbf/in.^{2}, State 3 is the same state as State 2 → $h_3 = 1375.55$ Btu/lb, $s_3 = 1.6268$ Btu/(lb·^{o}R)

State 4: $p_4 = 500$ lbf/in.^{2}, $T_4 = 900^o$F → From Table A-4E: $h_4 = 1466.5$ Btu/lb and $s_4 = 1.6987$ Btu/(lb·^{o}R)

State 5*s*: $p_{5s} = p_5 = 120$ lbf/in.^{2}, $s_{5s} = s_4$ → From Table A-4E (interpolated): $h_{5s} = 1289.09$ Btu/lb

State 5: $p_5 = 120$ lbf/in.^{2}, $h_6 = 1315.70$ Btu/lb (see below) → From Table A-4E (interpolated): $s_5 = 1.7251$ Btu/(lb·^{o}R)

$$h_5 = h_4 - \eta_t(h_4 - h_{5s})$$

$$h_5 = 1466.5 \text{ Btu/lb} - 0.85(1466.5 - 1289.09) \text{ Btu/lb} = 1315.70 \text{ Btu/lb}$$

State 6*s*: $p_{6s} = p_6 = 2$ lbf/in.^{2}, $s_{6s} = s_5 = 1.7251$ Btu/(lb·^{o}R) → $x_{6s} = 0.8884$, $h_{6s} = 1002.05$ Btu/lb

State 6: $p_6 = 2$ lbf/in^2, $h_6 = 1049.10$ Btu/lb (see below) → $x_6 = 0.9344$, $s_6 = 1.9083$ Btu/(lb·^{o}R)

$$h_6 = h_5 - \eta_t(h_5 - h_{6s})$$

$$h_6 = 1315.70 \text{ Btu/lb} - 0.85(1315.70 - 1002.05) \text{ Btu/lb} = 1049.10 \text{ Btu/lb}$$

State 7: $p_7 = p_6 = 2$ lbf/in.^{2}, sat liq. → From Table A-3E: $h_7 = h_{f7} = 94.02$ Btu/lb, $v_7 = v_{f7} = 0.01623$ ft^3/lb, $s_7 = s_{f7} = 0.1750$ Btu/(lb·^{o}R)

State 8: $p_8 = p_5 = 120$ lbf/in.^{2}, $h_8 = 94.44$ Btu/lb (see below) → $s_8 \approx 0.1757$ Btu/(lb·^{o}R) (***assuming the saturated liquid state corresponding to $h_8 = h_f$ in Table A-2E and interpolating for $s_8 = s_f$***)

$$h_8 = h_7 + \frac{v_7(p_8 - p_7)}{\eta_{p1}}$$

$$h_8 = 94.02 \text{ Btu/lb} + \frac{0.01623\left(\frac{\text{ft}^3}{\text{lb}}\right)(120-2)\left(\frac{\text{lbf}}{\text{in}^2}\right)}{0.85}\left|\frac{144\,\text{in}^2}{\text{ft}^2}\right|\left|\frac{1\,\text{Btu}}{778\,\text{ft}\cdot\text{lbf}}\right| = 94.44 \text{ Btu/lb}$$

State 9: $p_9 = p_5 = 120$ lbf/in.^{2}, sat liq. → From Table A-3E: $h_9 = h_{f9} = 312.7$ Btu/lb, $v_9 = v_{f9} = 0.01789$ ft^3/lb, $s_9 = s_{f9} = 0.4920$ Btu/(lb·^{o}R)

State 10: $p_{10} = p_1 = 1400$ lbf/in.^{2}, $h_{10} = 338.64$ Btu/lb (see below) $\rightarrow s_{10} \approx 0.5237$ Btu/(lb·^{o}R) (***assuming the saturated liquid state corresponding to $h_{10} = h_f$ in Table A-2E and interpolating for $s_8 = s_f$***)

$$h_{10} = h_9 + \frac{v_9(p_{10} - p_9)}{\eta_{p2}}$$

$$h_{10} = 312.7 \text{ Btu/lb} + \frac{0.01789\left(\frac{\text{ft}^3}{\text{lb}}\right)(1400-120)\left(\frac{\text{lbf}}{\text{in}^2}\right)}{0.85}\left|\frac{144\,\text{in}^2}{\text{ft}^2}\right|\left|\frac{1\,\text{Btu}}{778\,\text{ft}\cdot\text{lbf}}\right| = 339.64 \text{ Btu/lb}$$

State 11: $p_{11} = p_1 = 1400$ lbf/in.^{2}, $T_{11} = T_{sat}$ @ $p = 500$ lbf/in.^{2} = 477.07^{o}F. $\rightarrow$ From Table A-2E (compressed liquid) (interpolated): $h_{11} \approx h_{f11} = 460.93$ Btu/lb, $s_{11} \approx s_{f11} = 0.6611$ Btu/(lb·^{o}R)

State 12: $p_{12} = p_2 = 500$ lbf/in.^{2}, sat liq. $\rightarrow$ From Table A-3E: $h_{12} = h_{f12} = 449.5$ Btu/lb, $s_{12} = s_{f12} = 0.6490$ Btu/(lb·^{o}R)

State 13: $p_{13} = p_5 = 120$ lbf/in.^{2}, throttling process ($h_{13} = h_{12} = 449.5$ Btu/lb) $\rightarrow x_{13} = 0.1557$, $s_{13} = 0.6627$ Btu/(lb·^{o}R)

(a) The mass flow rate of steam entering the first stage of the turbine can be determined from the definition of the net power output of the cycle

$$\dot{W}_{cycle} = \dot{W}_{t1} + \dot{W}_{t2} + \dot{W}_{t3} - \dot{W}_{p1} - \dot{W}_{p2}$$

Let $\dot{m}_1$ be the mass flow rate of the steam entering the first stage of the turbine. Mass and energy balances for control volumes around the three turbine stages and the two pumps give, respectively

$$\dot{W}_{t1} = \dot{m}_1 (h_1 - h_2)$$
$$\dot{W}_{t2} = \dot{m}_1 (1 - y')(h_4 - h_5)$$
$$\dot{W}_{t3} = \dot{m}_1 (1 - y' - y'')(h_5 - h_6)$$
$$\dot{W}_{p1} = \dot{m}_1 (1 - y' - y'')(h_8 - h_7)$$
$$\dot{W}_{p2} = \dot{m}_1 (h_{10} - h_9)$$

where y' and y'' are the fractions of steam extracted after the first and second turbine stages, respectively.

Substituting for net power output of the cycle

$$\dot{W}_{cycle} = \dot{m}_1 (h_1 - h_2) + \dot{m}_1 (1 - y')(h_4 - h_5) + \dot{m}_1 (1 - y' - y'')(h_5 - h_6) - \dot{m}_1 (1 - y' - y'')(h_8 - h_7) - \dot{m}_1 (h_{10} - h_9)$$

Solving for $\dot{m}_1$

$$\dot{m}_1 = \frac{\dot{W}_{\text{cycle}}}{(h_1 - h_2) + (1 - y')(h_4 - h_5) + (1 - y' - y'')(h_5 - h_6) - (1 - y' - y'')(h_8 - h_7) - (h_{10} - h_9)}$$

The mass fractions y' and y'' are both unknown. First, analyze the closed feedwater heater to determine y'. Mass and energy balances for a control volume around the closed feedwater heater give

$$y' = \frac{h_{11} - h_{10}}{h_2 - h_{12}} = \frac{460.93\,\frac{\text{Btu}}{\text{lb}} - 339.64\,\frac{\text{Btu}}{\text{lb}}}{1375.55\,\frac{\text{Btu}}{\text{lb}} - 449.5\,\frac{\text{Btu}}{\text{lb}}} = 0.131$$

Now we can analyze the open feedwater heater to determine y''. Mass and energy balances for a control volume around the open feedwater heater give

$$y'' = \frac{h_9 - h_8 + y'(h_8 - h_{13})}{h_5 - h_8}$$

$$y'' = \frac{312.7\,\frac{\text{Btu}}{\text{lb}} - 94.44\,\frac{\text{Btu}}{\text{lb}} + 0.131\left(94.44\,\frac{\text{Btu}}{\text{lb}} - 449.5\,\frac{\text{Btu}}{\text{lb}}\right)}{1315.70\,\frac{\text{Btu}}{\text{lb}} - 94.44\,\frac{\text{Btu}}{\text{lb}}} = 0.141$$

Thus,

$$(1 - y') = 1 - 0.131 = 0.869 \quad \text{and} \quad (1 - y' - y'') = 1 - 0.131 - 0.141 = 0.728$$

Substituting values and solving for $\dot{m}_1$

$$\dot{m}_1 = \frac{1\times 10^9\,\frac{\text{Btu}}{\text{h}}\left|\frac{1\,\text{h}}{3600\,\text{s}}\right|}{[(1493.5 - 1375.55) + (0.869)(1466.5 - 1315.70) + (0.728)(1315.70 - 1049.10) - (0.728)(94.44 - 94.02) - (339.64 - 312.7)]\,\frac{\text{Btu}}{\text{lb}}}$$

$$\dot{m}_1 = 668.0 \text{ lb/s}$$

(***End of solution from problem 8.73***)

To conduct an exergy balance, determine input, output, losses, and destruction of exergy for the cycle:

Input: Rate of exergy input to the working fluid in the steam generator/reheater is

$\dot{m}_1[(e_{f1} - e_{f11}) + (1 - y')(e_{f4} - e_{f3})] =$

$$\dot{m}_1\{[(h_1 - h_{11}) - T_0(s_1 - s_{11})] + (1 - y')[(h_4 - h_3) - T_0(s_4 - s_3)]\}$$

$\dot{m}_1[(e_{f1} - e_{f11}) + (1 - y')(e_{f4} - e_{f3})] =$

$$668.0\frac{\text{lb}}{\text{s}}\left\{\begin{array}{l}\left[(1493.5 - 460.93)\frac{\text{Btu}}{\text{lb}} - (520°\text{R})(1.6094 - 0.6611)\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right] + \\ (0.869)\left[(1466.5 - 1375.55)\frac{\text{Btu}}{\text{lb}} - (520°\text{R})(1.6987 - 1.6268)\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right]\end{array}\right\}$$

$$\dot{m}_1[(e_{f1} - e_{f11}) + (1 - y')(e_{f4} - e_{f3})] = \underline{\mathbf{3.91 \times 10^5\ Btu/s}}$$

Output: Exergy output from the cycle is the net cycle work given.

$\dot{W}_{\text{cycle}} = 1 \times 10^9$ Btu/h = $\underline{\mathbf{2.78 \times 10^5\ Btu/s}}$

Loss: Exergy loss from the cycle occurs in the condenser.

$$\text{Condenser loss} = \dot{m}_1(1 - y' - y'')(e_{f6} - e_{f7}) = \dot{m}_1(1 - y' - y'')[(h_6 - h_7) - T_0(s_6 - s_7)]$$

$$\text{Condenser loss} = \left(668.0\frac{\text{lb}}{\text{s}}\right)(0.728)\left[(1049.10 - 94.02)\frac{\text{Btu}}{\text{lb}} - (520°\text{R})(1.9083 - 0.1750)\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right]$$

$$\text{Condenser loss} = \underline{\mathbf{2.61 \times 10^4\ Btu/s}}$$

Destruction: Exergy is destroyed in the turbine, pumps, feedwater heaters, and trap.

<u>For the turbine</u>:

$$\dot{E}_d = T_0\dot{\sigma}_{cv} = T_0\dot{m}_1[y's_2 + (1 - y')s_3 + y''s_5 + (1 - y' - y'')s_6 - s_1 - (1 - y')s_4]$$

$$\dot{E}_d = (520°\text{R})\left(668.0\frac{\text{lb}}{\text{s}}\right)\left[\begin{array}{l}(0.131)(1.6268) + (0.869)(1.6268) + (0.141)(1.7251) + \\ (0.728)(1.9083) - 1.6094 - (0.869)(1.6987)\end{array}\right]\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$$

$$\dot{E}_d = \underline{\mathbf{6.03 \times 10^4\ Btu/s}}$$

<u>For the pumps:</u>

Pump 1: $\dot{E}_d = T_0\dot{\sigma}_{cv} = T_0\dot{m}_1(1 - y' - y'')(s_8 - s_7)$

$$\dot{E}_d = (520°\text{R})\left(668.0\frac{\text{lb}}{\text{s}}\right)(0.728)\left(0.1757\frac{\text{Btu}}{\text{lb}\cdot°\text{R}} - 0.1750\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right) = \underline{\mathbf{1.77 \times 10^2\ Btu/s}}$$

Pump 2: $\dot{E}_d = T_0\dot{\sigma}_{cv} = T_0\dot{m}_1(s_{10} - s_9)$

$$\dot{E}_d = (520^{o}R)\left(668.0\frac{lb}{s}\right)\left(0.5237\frac{Btu}{lb\cdot°R} - 0.4920\frac{Btu}{lb\cdot°R}\right) = \mathbf{\underline{1.10 \times 10^4\ Btu/s}}$$

<u>For the closed feedwater heater</u>:

$$\dot{E}_d = T_0\dot{\sigma}_{cv} = T_0\dot{m}_1[(s_{11} - s_{10}) + y'(s_{12} - s_2)]$$

$$\dot{E}_d = (520^{o}R)\left(668.0\frac{lb}{s}\right)\begin{bmatrix}\left(0.6611\frac{Btu}{lb\cdot°R} - 0.5237\frac{Btu}{lb\cdot°R}\right) + \\ (0.131)\left(0.6490\frac{Btu}{lb\cdot°R} - 1.6268\frac{Btu}{lb\cdot°R}\right)\end{bmatrix} = \mathbf{\underline{3.23 \times 10^3\ Btu/s}}$$

<u>For the open feedwater heater</u>:

$$\dot{E}_d = T_0\dot{\sigma}_{cv} = T_0\dot{m}_1[s_9 - y''s_5 - (1 - y' - y'')s_8 - y's_{13}]$$

$$\dot{E}_d = (520^{o}R)\left(668.0\frac{lb}{s}\right)\begin{bmatrix}0.4920 - (0.141)(1.7251) - \\ (0.728)(0.1757) - (0.131)(0.6627)\end{bmatrix}\frac{Btu}{lb\cdot°R} = \mathbf{\underline{1.18 \times 10^4\ Btu/s}}$$

<u>For the trap:</u>

$$\dot{E}_d = T_0\dot{\sigma}_{cv} = T_0\dot{m}_1 y'(s_{13} - s_{12})$$

$$\dot{E}_d = (520^{o}R)\left(668.0\frac{lb}{s}\right)(0.131)\left(0.6627\frac{Btu}{lb\cdot°R} - 0.6490\frac{Btu}{lb\cdot°R}\right) = \mathbf{\underline{6.23 \times 10^2\ Btu/s}}$$

Total Exergy Destroyed = 8.71 ×10^4 Btu/s

Summary:

Input:	3.91 ×10^5 Btu/s	Output:	2.78 ×10^5 Btu/s (71%)
		Loss:	2.61 ×10^4 Btu/s (7%)
		Destroyed:	<u>8.71 ×10^4 Btu/s (22%)</u>
			3.91 ×10^5 Btu/s (100%)

In this case the very significant exergy destruction accompanying combustion of fuel in the steam generator is not considered. As indicated in Table 8.4 approximately 30% of the exergy

entering the steam generator with the fuel is destroyed during combustion. The exergy increase of the working fluid as it passes through the steam generator and reheater is the remaining 70% of exergy from the fuel. Of this, most becomes useful output from the cycle, followed by exergy destroyed in various devices, and exergy loss from the condenser.

8.89 For the power plant in Problem 8.74, develop a full accounting, in MW, of the rate of exergy increase as the working fluid passes through the steam generator and reheater with a mass flow rate of 10 kg/s. Let T_0 = 20°C, p_0 = 1 bar.

KNOWN: A regenerative vapor power cycle with three closed feedwater heaters, one open feedwater heater, and reheat operates with steam as the working fluid. Operational data are provided in Problem 8.74.

FIND: Develop a full accounting, in MW, of the net rate of exergy increase as the working fluid passes through the steam generator and reheater.

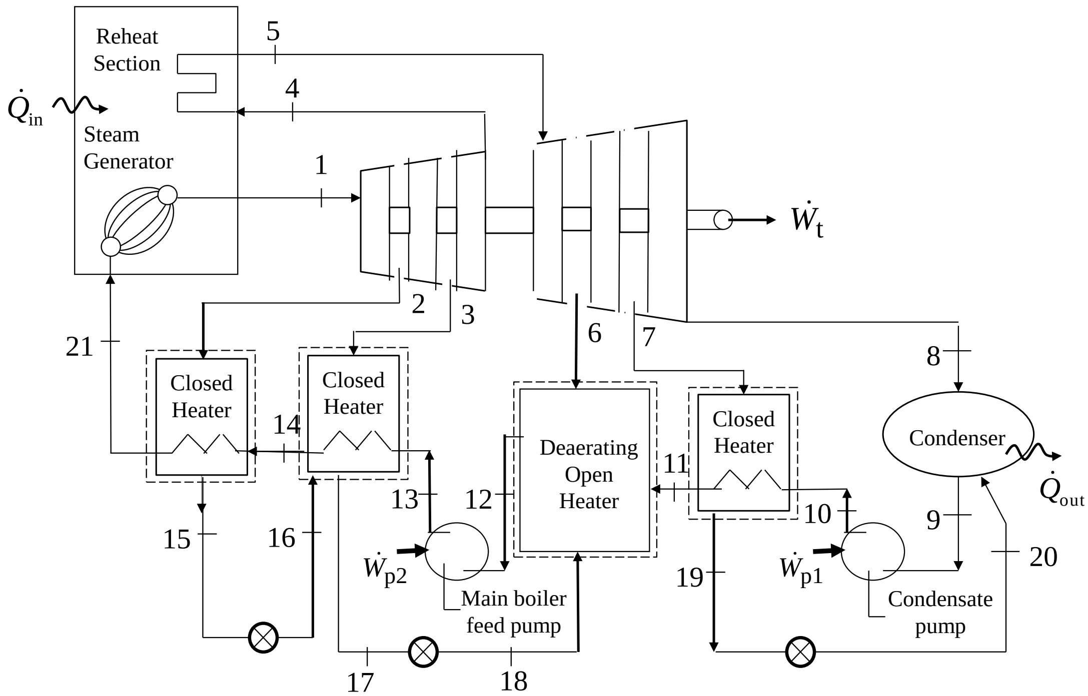
SCHEMATIC AND GIVEN DATA:

State	p (kPa)	T (°C)	h (kJ/kg)	s (kJ/kg·K)	x	State	p (kPa)	T (°C)	h (kJ/kg)	s (kJ/kg·K)	x
1	16,000	600	3573.5	6.6399	--	**12**	800		721.11	2.0462	0
2	8,000		3334.7	6.6399	--	**13**	16,000		738.05	2.0837	--
3	4,000		3129.2	6.6399	--	**14**	16,000		1067.3	2.7584	--
4	2,000		2953.6	6.6399	--	**15**	8,000		1316.6	3.2068	0
5	2,000	500	3467.6	7.4317	--	**16**	4,000		1316.6	3.2344	0.1338
6	800		3172.1	7.4317	--	**17**	4,000		1087.3	2.7964	0
7	200		2824.7	7.4317	--	**18**	800		1087.3	2.8716	0.1788
8	10		2355.4	7.4317	0.9042	**19**	200		504.70	1.5301	0
9	10		191.83	0.6493	0	**20**	10		504.70	1.6304	0.1308
10	800		192.63	0.6517	--	**21**	16,000		1269.1	3.1245	--
11	800		595.92	1.7553	--						

P8.74

Fraction of flow extracted at location 2: y' = 0.1000.
Fraction of flow extracted at location 3: y'' = 0.1500.
Fraction of flow extracted at location 6: y''' = 0.0009.
Fraction of flow extracted at location 7: y'''' = 0.1302.

Mass fraction summary:
y' = 0.1000
y'' = 0.1500
$y' + y''$ = 0.2500
$(1 - y' - y'')$ = 0.7500
y''' = 0.0009
$(1 - y' - y'' - y''')$ = 0.7491
y'''' = 0.1302
$(1 - y' - y'' - y''' - y'''')$ = 0.6189



ENGINEERING MODEL:

1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The expansions through the turbine stages occur isentropically.
3. The turbines, pumps, closed feedwater heaters, open feedwater heater, and traps operate adiabatically.
4. Kinetic and potential energy effects are negligible.
5. Saturated liquid exits the open feedwater heater, closed feedwater heaters at their extraction pressures, and condenser.
6. Let $T_0 = 20^{o}C = 293$ K, $p_0 = 1$ bar.

ANALYSIS:

Input: Rate of exergy input to the working fluid in the steam generator/reheater is

$$\dot{m}_1[(e_{f1} - e_{f21}) + (1 - y' - y'')(e_{f5} - e_{f4})]$$

Solving for differences in specific flow exergy

$e_{f1} - e_{f21} = [(h_1 - h_{21}) - T_0(s_1 - s_{21})]$

$$e_{f1} - e_{f21} = \left[(3573.5 - 1269.1)\frac{\text{kJ}}{\text{kg}} - (293\,\text{K})(6.63997 - 3.1245)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right] = \mathbf{1274.37\ kJ/kg}$$

$e_{f5} - e_{f4} = [(h_5 - h_4) - T_0(s_5 - s_4)]$

$$e_{f5} - e_{f4} = \left[(3467.6 - 2953.6)\frac{\text{kJ}}{\text{kg}} - (293\,\text{K})(7.4317 - 6.6399)\frac{\text{kJ}}{kg\cdot\text{K}}\right] = \mathbf{282.00\ kJ/kg}$$

Substituting values and solving for rate of exergy input give

$$\dot{m}_1[(e_{f1} - e_{f21}) + (1 - y' - y'')(e_{f5} - e_{f4})] = \left(10\frac{\text{kg}}{\text{s}}\right)[1274.37 + (0.7500)(282.00)]\frac{\text{kJ}}{\text{kg}}\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right|$$

$$\dot{m}_1[(e_{f1} - e_{f21}) + (1 - y' - y'')(e_{f5} - e_{f4})] = 14{,}859\ \text{kW} = \underline{\mathbf{14.859\ MW}}$$

To conduct an exergy balance, determine output, losses, and destruction of exergy for the cycle:

Output: Rate of exergy output from the cycle is the net cycle power.

For an overall control volume enclosing the turbine stages, an energy balance on the basis of a unit of mass entering reads

$$\frac{\dot{W}_t}{\dot{m}_1} = h_1 + (1 - y' - y'')h_5 - y'h_2 - y''h_3 - (1 - y' - y'')h_4 - y'''h_6 - y''''h_7 - (1 - y' - y'' - y''' - y'''')h_8$$

Substituting enthalpy values and appropriate flow mass fractions gives

$$\frac{\dot{W}_t}{\dot{m}_1} = [3573.5 + (0.7500)(3467.6) - (0.1000)(3334.7) - (0.1500)(3129.2) - (0.7500)(2953.6)$$

$$- (0.0009)(3172.1) - (0.1302)(2824.7) - (0.6189)(2355.4)]\frac{\text{kJ}}{\text{kg}}$$

$$\frac{\dot{W}_t}{\dot{m}_1} = 1327.8\ \text{kJ/kg}$$

For the pumps

$$\frac{\dot{W}_p}{\dot{m}_1} = h_{13} - h_{12} + (1 - y' - y'' - y''')(h_{10} - h_9)$$

$$\frac{\dot{W}_p}{\dot{m}_1} = [738.05 - 721.11 + (0.7491)(192.63 - 191.83)]\frac{\text{kJ}}{\text{kg}} = 17.54\ \text{kJ/kg}$$

Solving for the net power produced by the cycle gives

$$\dot{W}_{\text{cycle}} = \dot{m}_1\left(\frac{\dot{W}_t}{\dot{m}_1} - \frac{\dot{W}_p}{\dot{m}_1}\right) = \left(10\frac{\text{kg}}{\text{s}}\right)(1327.8 - 17.54)\frac{\text{kJ}}{\text{kg}}\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right| = 13{,}103\text{ kW} = \mathbf{13.103\ MW}$$

Loss: Exergy loss from the cycle occurs in the condenser.

$$\text{Condenser loss} = \dot{m}_1(1 - y' - y'' - y''' - y'''')\mathsf{e}_{f8} + \dot{m}_1\, y''''\mathsf{e}_{f20} - \dot{m}_1(1 - y' - y'' - y''')\mathsf{e}_{f9}$$

Rearranging gives

$$\text{Condenser loss} = \dot{m}_1[(1 - y' - y'' - y''')(\mathsf{e}_{f8} - \mathsf{e}_{f9}) + y''''(\mathsf{e}_{f20} - \mathsf{e}_{f8})]$$

Solving for differences in specific flow exergy

$$\mathsf{e}_{f8} - \mathsf{e}_{f9} = [(h_8 - h_9) - T_0(s_8 - s_9)]$$

$$\mathsf{e}_{f8} - \mathsf{e}_{f9} = \left[(2355.4 - 191.83)\frac{\text{kJ}}{\text{kg}} - (293\text{ K})(7.4317 - 0.6493)\frac{\text{kJ}}{kg\cdot\text{K}}\right] = 176.33\text{ kJ/kg}$$

$$\mathsf{e}_{f20} - \mathsf{e}_{f8} = [(h_{20} - h_8) - T_0(s_{20} - s_8)]$$

$$\mathsf{e}_{f20} - \mathsf{e}_{f8} = \left[(504.70 - 2355.4)\frac{\text{kJ}}{\text{kg}} - (293\text{ K})(1.6304 - 7.4317)\frac{\text{kJ}}{kg\cdot\text{K}}\right] = -150.92\text{ kJ/kg}$$

Substituting values and solving for rate of exergy loss in the condenser give

$$\text{Condenser loss} = \left(10\frac{\text{kg}}{\text{s}}\right)[(0.7491)(176.33) + (0.1302)(-150.92)]\frac{\text{kJ}}{\text{kg}}\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right| = 1124\text{ kW} = \mathbf{1.124\ MW}$$

Destruction: Since the turbine operates isentropically, no exergy is destroyed in the turbine. Exergy is destroyed in the pumps, closed feedwater heaters, open feedwater heater, and traps.

<u>For the pumps</u>:

Pump 1:

$$\dot{\mathsf{E}}_d = T_0\dot{\sigma}_{cv} = T_0\dot{m}_1[(1 - y' - y'' - y''')(s_{10} - s_9)]$$

$$\dot{\mathsf{E}}_d = (293\text{ K})\left(10\frac{\text{kg}}{\text{s}}\right)[(0.7491)(0.6517 - 0.6493)]\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right| = \mathbf{5.3\ kW}$$

Pump 2:

$$\dot{E}_d = T_0\dot{\sigma}_{cv} = T_0\dot{m}_1[s_{13} - s_{12}]$$

$$\dot{E}_d = (293\text{ K})\left(10\frac{\text{kg}}{\text{s}}\right)[2.0837 - 2.0462]\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$

$$\dot{E}_d = \mathbf{110\ kW}$$

<u>For the closed feedwater heater</u>:

Closed feedwater heater 1:

$$\dot{E}_d = T_0\dot{\sigma}_{cv} = T_0\dot{m}_1[(s_{21} - s_{14}) + y'(s_{15} - s_2)]$$

$$\dot{E}_d = (293\text{ K})\left(10\frac{\text{kg}}{\text{s}}\right)[(3.1245 - 2.7584) + (0.1000)(3.2068 - 6.6399)]\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = \mathbf{67\ kW}$$

Closed feedwater heater 2:

$$\dot{E}_d = T_0\dot{\sigma}_{cv} = T_0\dot{m}_1[s_{14} + (y'+y'')s_{17} - y''s_3 - s_{13} - y's_{16}]$$

$$\dot{E}_d = (293\text{ K})\left(10\frac{\text{kg}}{\text{s}}\right)\begin{bmatrix}2.7584 + (0.2500)(2.7964) - (0.1500)(6.6399) \\ - 2.0837 - (0.1000)(3.2344)\end{bmatrix}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$

$$\dot{E}_d = \mathbf{159\ kW}$$

Closed feedwater heater 3:

$$\dot{E}_d = T_0\dot{\sigma}_{cv} = T_0\dot{m}_1[y''''(s_{19} - s_7) + (1 - y' - y'' - y''')(s_{11} - s_{10})]$$

$$\dot{E}_d = (293\text{ K})\left(10\frac{\text{kg}}{\text{s}}\right)[(0.1302)(1.5301 - 7.4317) + (0.7491)(1.7553 - 0.6517)]\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$

$$\dot{E}_d = \mathbf{171\ kW}$$

<u>For the open feedwater heater</u>:

$$\dot{E}_d = T_0\dot{\sigma}_{cv} = T_0\dot{m}_1[s_{12} - y'''s_6 - (1 - y' - y'' - y''')s_{11} - (y'+y'')s_{18}]$$

$$\dot{E}_d = (293\text{ K})\left(10\frac{\text{kg}}{\text{s}}\right)\begin{bmatrix}2.0462-(0.0009)(7.4317)\\-(0.7491)(1.7553)-(0.2500)(2.8716)\end{bmatrix}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = \mathbf{20\ kW}$$

<u>For the traps:</u>

Trap 1:

$$\dot{E}_d = T_0\dot{\sigma}_{cv} = T_0\dot{m}_1 y'(s_{16}-s_{15})$$

$$\dot{E}_d = (293\text{ K})\left(10\frac{\text{kg}}{\text{s}}\right)(0.1000)(3.2344-3.2068)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = \mathbf{8.1\ kW}$$

Trap 2:

$$\dot{E}_d = T_0\dot{\sigma}_{cv} = T_0\dot{m}_1(y'+y'')(s_{18}-s_{17})$$

$$\dot{E}_d = (293\text{ K})\left(10\frac{\text{kg}}{\text{s}}\right)(0.2500)(2.8716-2.7964)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = \mathbf{55\ kW}$$

Trap 3:

$$\dot{E}_d = T_0\dot{\sigma}_{cv} = T_0\dot{m}_1 y''''(s_{20}-s_{19})$$

$$\dot{E}_d = (293\text{ K})\left(10\frac{\text{kg}}{\text{s}}\right)(0.1302)(1.6304-1.5301)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = \mathbf{38\ kW}$$

Total Exergy Destroyed = 633.4 kW = **0.633 MW**

Summary

Input:	14.859 MW	Output:	13.103 MW
		Loss:	1.124 MW
		Destroyed:	<u>0.633 MW</u>
			14.860 MW

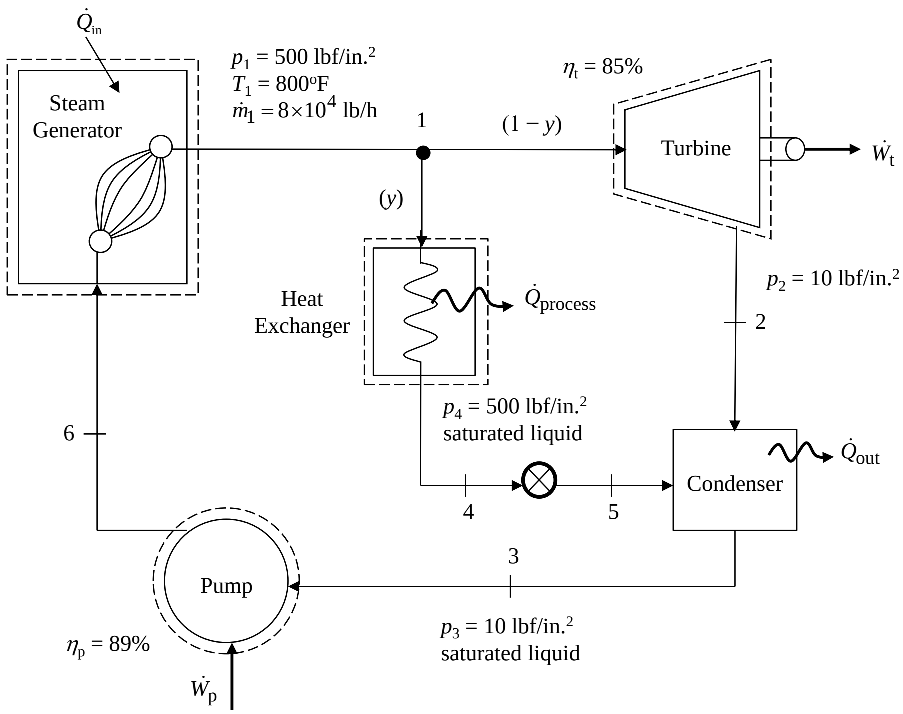
The slight difference in exergy input (exergy increase of the working fluid as it passes through the steam generator and reheater) and the sum of exergy output, loss, and destroyed is due to round-off in intermediate calculations. In this case the very significant exergy destruction accompanying combustion of fuel in the steam generator is not considered. As indicated in Table 8.4 approximately 30% of the exergy entering the steam generator with the fuel is destroyed during combustion. The exergy increase of the working fluid as it passes through the steam generator and reheater is the remaining 70% of exergy from the fuel. Of this, most becomes useful output from the cycle, followed by exergy loss from the condenser, and exergy destroyed in various devices. However, the significant exergy destruction that would occur in the turbines is not considered in the present analysis. As shown by the data of Table 8.4, the total exergy destroyed in the turbines could be on the order of 5%, or more, of the exergy increase.

8.90 Determine the rate of exergy input, in Btu/h, to the working fluid passing through the steam generator in Problem 8.78. Perform calculations to account for all outputs, losses, and destructions of this exergy. For the process heat exchanger, assume the temperature at which heat transfer occurs is 465°F. Let T_0 = 60°F, p_0 = 14.7 lbf/in.^{2}

KNOWN: A vapor power cycle operates with steam to produce process heat and power. Operational data are provided.

FIND: Determine the rate of exergy input, in Btu/h, to the working fluid passing through the steam generator and account for the disposition of this exergy.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Each component of the cycle is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The turbine and pump operate adiabatically.
3. Kinetic and potential energy effects are negligible.
4. For the process heat exchanger, the temperature at which heat transfer occurs is 465°F.
5. T_0 = 60°F, p_0 = 14.7 lbf/in.^{2}

ANALYSIS: (***Results from Problem 8.78 are as follows***):

<u>State 1</u>: p_1 = 500 lbf/in.^{2}, T_1 = 800°F → h_1 = 1412.1 Btu/lb, s_1 = 1.6571 Btu/lb·°R

<u>State 2s</u>: $p_{2s} = p_2$ = 10 lbf/in.^{2}, $s_{2s} = s_1$ = 1.6571 Btu/lb·°R → x_{2s} = 0.9132, h_{2s} = 1058.1 Btu/lb

State 2: p_2 = 10 lbf/in.^{2}, h_2 = 1111.2 Btu/lb (see below) → x_2 = 0.9673, s_2 = 1.7385 Btu/lb·°R

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \rightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 1412.1\frac{\text{Btu}}{\text{lb}} - (0.85)(1412.1 - 1058.1)\frac{\text{Btu}}{\text{lb}} = 1111.2 \text{ Btu/lb}$$

State 3: p_3 = 10 lbf/in.^{2}, saturated liquid → h_3 = 161.23 Btu/lb, s_3 = 0.2836 Btu/lb·°R,
v_3 = 0.01659 ft^3/lb

State 4: p_4 = 500 lbf/in.^{2}, saturated liquid → h_4 = 449.5 Btu/lb, s_4 = 0.6490 Btu/lb·°R,

State 5: p_5 = 10 lbf/in.^{2}, h_5 = h_4 =449.5 Btu/lb → x_5 = 0.2935, s_5 = 0.7251 Btu/lb·°R

State 6: p_6 = 500 lbf/in.^{2}, $h_6 \approx h_3 + \dfrac{v_3(p_6 - p_3)}{\eta_p}$ = 162.92 Btu/lb (see below) →
s_6 = 0.2862 Btu/lb·°R (***assuming the saturated liquid state corresponding to $h_6 = h_f$ in Table A-2E and interpolating for $s_6 = s_f$***)

$$h_6 = 161.23\frac{\text{Btu}}{\text{lb}} + \left(0.01659\frac{\text{ft}^3}{\text{lb}}\right)(500 - 10)\frac{\text{lbf}}{\text{in.}^2}\left|\frac{144\,\text{in.}^2}{1\,\text{ft}^2}\right|\left|\frac{1\,\text{Btu}}{778.17\,\text{ft}\cdot\text{lbf}}\right| = 162.92 \text{ Btu/lb}$$

(a) Applying energy and mass balances to the control volume enclosing the heat exchanger

$$\dot{Q}_{\text{process}} = y\dot{m}_1(h_1 - h_4)$$

where y = 0.12 is the fraction of steam passing through the heat exchanger.

$$\dot{Q}_{\text{process}} = (0.12)\left(8\times10^4\frac{\text{lb}}{\text{h}}\right)(1412.1 - 449.5)\frac{\text{Btu}}{\text{lb}} = 9.241 \times 10^6 \text{ Btu/h}$$

(b) The net power developed by the cycle is

$$\dot{W}_{\text{cycle}} = \dot{W}_t - \dot{W}_p$$

For the control volume surrounding the turbine

$$\dot{W}_t = (1 - y)\dot{m}_1(h_1 - h_2)$$

$$\dot{W}_t = (0.88)\left(8\times10^4\frac{\text{lb}}{\text{h}}\right)(1412.1 - 1111.2)\frac{\text{Btu}}{\text{lb}} = 21.183 \times 10^6 \text{ Btu/h}$$

For the pump

$$\dot{W}_p = \dot{m}_1(h_6 - h_3)$$

$$\dot{W}_p = \left(8\times10^4 \frac{\text{lb}}{\text{h}}\right)(162.92-161.23)\frac{\text{Btu}}{\text{lb}} = 0.135 \times 10^6 \text{ Btu/h}$$

Substituting values gives

$$\dot{W}_{\text{cycle}} = 21.183 \times 10^6 \text{ Btu/h} - 0.135 \times 10^6 \text{ Btu/h} = 21.048 \times 10^6 \text{ Btu/h}$$

Applying energy and mass balances to the control volume enclosing the steam generator

$$\dot{Q}_{\text{in}} = \dot{m}_1(h_1 - h_6)$$

$$\dot{Q}_{\text{in}} = \left(8\times10^4 \frac{\text{lb}}{\text{h}}\right)(1412.1-162.92)\frac{\text{Btu}}{\text{lb}} = 99.934 \times 10^6 \text{ Btu/h}$$

(***End of results from problem 8.78***)

Input: Rate of exergy input to the working fluid in the steam generator/reheater is

$$\dot{m}_1(\mathsf{e}_{f1} - \mathsf{e}_{f6}) = \dot{m}_1[(h_1 - h_6) - T_0(s_1 - s_6)]$$

$$\dot{m}_1\boldsymbol{(\mathsf{e}_{f1} - \mathsf{e}_{f6})} = \left(8\times10^4 \frac{\text{lb}}{\text{h}}\right)\left[(1412.1-162.92)\frac{\text{Btu}}{\text{lb}} - (520^\circ\text{R})(1.6571-0.2862)\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right]$$

$$\dot{m}_1(\mathsf{e}_{f1} - \mathsf{e}_{f6}) = \mathbf{42.905 \times 10^6 \text{ Btu/h}}$$

To conduct an exergy balance, determine outputs, losses, and destruction of exergy for the cycle:

Output: Rate of exergy output from the cycle is the net cycle power developed by the cycle and the rate of exergy associated with the process heat.

The **net cycle power** developed calculated above is

$$\dot{W}_{\text{cycle}} = 21.048 \times 10^6 \text{ Btu/h}$$

The **rate of exergy output with the process heat** is

$$\dot{\mathsf{E}}_q = \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j$$

Converting temperatures to absolute scale, T_0 = 520°R and T_j = 925°R and substituting the rate of heat transfer from the process heat exchanger determined above ($\dot{Q}_{\text{process}}$ = 9.241 × 10^6 Btu/h) give

$$\dot{\mathsf{E}}_q = \left(1 - \frac{520^\circ \mathrm{R}}{925^\circ \mathrm{R}}\right)\left(9.241 \times 10^6 \ \frac{\mathrm{Btu}}{\mathrm{h}}\right) = 4.046 \times 10^6 \ \mathrm{Btu/h}$$

The total rate of exergy output is 21.048×10^6 Btu/h + 4.046×10^6 Btu/h = **25.094×10^6 Btu/h**

Loss: Exergy loss from the cycle occurs in the condenser.

$$\text{Condenser loss} = \dot{m}_1[(1 - y)\mathsf{e}_{f2} + y\mathsf{e}_{f5} - \mathsf{e}_{f3}]$$

Rearranging gives

$$\text{Condenser loss} = \dot{m}_1[(\mathsf{e}_{f2} - \mathsf{e}_{f3}) + y(\mathsf{e}_{f5} - \mathsf{e}_{f2})]$$

Solving for differences in specific flow exergy

$$\mathsf{e}_{f2} - \mathsf{e}_{f3} = [(h_2 - h_3) - T_0(s_2 - s_3)]$$

$$\mathsf{e}_{f2} - \mathsf{e}_{f3} = \left[(1111.2 - 161.23)\frac{\mathrm{Btu}}{\mathrm{lb}} - (520^\circ \mathrm{R})(1.7385 - 0.2836)\frac{\mathrm{Btu}}{\mathrm{lb} \cdot ^\circ \mathrm{R}}\right] = 193.422 \ \mathrm{Btu/lb}$$

$$\mathsf{e}_{f5} - \mathsf{e}_{f2} = [(h_5 - h_2) - T_0(s_5 - s_2)]$$

$$\mathsf{e}_{f5} - \mathsf{e}_{f2} = \left[(449.5 - 1111.2)\frac{\mathrm{Btu}}{\mathrm{lb}} - (520^\circ \mathrm{R})(0.7251 - 1.7385)\frac{\mathrm{Btu}}{\mathrm{lb} \cdot ^\circ \mathrm{R}}\right] = -134.732 \ \mathrm{Btu/lb}$$

Substituting values and solving for rate of exergy loss in the condenser give

$$\text{Condenser loss} = \left(8 \times 10^4 \ \frac{\mathrm{lb}}{\mathrm{h}}\right)[193.422 + (0.12)(-134.732)]\frac{\mathrm{Btu}}{\mathrm{lb}} = 14.180 \times 10^6 \ \mathrm{Btu/h}$$

Destruction: Exergy is destroyed in the turbine, the heat exchanger, the trap, and the pump.

For the Turbine:

$$\dot{\mathsf{E}}_d = T_0\dot{\sigma}_{cv} = T_0\dot{m}_1(1 - y)(s_2 - s_1)$$

$$\dot{\mathsf{E}}_d = \left(8 \times 10^4 \ \frac{\mathrm{lb}}{\mathrm{h}}\right)\left[(520^\circ \mathrm{R})(0.88)(1.7385 - 1.6571)\frac{\mathrm{Btu}}{\mathrm{lb} \cdot ^\circ \mathrm{R}}\right] = 2.980 \times 10^6 \ \mathrm{Btu/h}$$

For the Heat Exchanger:

$$\dot{E}_d = T_0 \dot{\sigma}_{cv} = T_0 \left[y\dot{m}_1 (s_4 - s_1) - \frac{\dot{Q}}{T_j} \right]$$

$$\dot{E}_d = (520^\circ\text{R}) \left[\left(8 \times 10^4 \frac{\text{lb}}{\text{h}} \right) (0.12)(0.6490 - 1.6571) \frac{\text{Btu}}{\text{lb} \cdot {}^\circ\text{R}} - \frac{(-9.241 \times 10^6 \text{ Btu/h})}{925^\circ\text{R}} \right]$$

where a negative sign is included with the heat transfer value since heat transfer is from the working fluid.

$$\dot{E}_d = 0.163 \times 10^6 \text{ Btu/h}$$

For the trap:

$$\dot{E}_d = T_0 \dot{\sigma}_{cv} = T_0 \dot{m}_1 y (s_5 - s_4)$$

$$\dot{E}_d = (520^\circ\text{R}) \left(8 \times 10^4 \frac{\text{lb}}{\text{h}} \right) (0.12)(0.7251 - 0.6490) \frac{\text{Btu}}{\text{lb} \cdot {}^\circ\text{R}} = 0.380 \times 10^6 \text{ Btu/h}$$

For the pump:

$$\dot{E}_d = T_0 \dot{\sigma}_{cv} = T_0 \dot{m}_1 (s_6 - s_3)$$

$$\dot{E}_d = (520^\circ\text{R}) \left(8 \times 10^4 \frac{\text{lb}}{\text{h}} \right) (0.2862 - 0.2836) \frac{\text{Btu}}{\text{lb} \cdot {}^\circ\text{R}} = 0.108 \times 10^6 \text{ Btu/h}$$

Total Exergy Destroyed = 3.631×10^6 Btu/h

Summary

Input:	42.905×10^6 Btu/h	Output:	25.094×10^6 Btu/h	(58.5%)
		Loss:	14.180×10^6 Btu/h	(33.0%)
		Destroyed:	3.631×10^6 Btu/h	(8.5%)
			42.905×10^6 Btu/h	

In this case the very significant exergy destruction accompanying combustion of fuel in the steam generator is not considered. As indicated in Table 8.4 approximately 30% of the exergy entering the steam generator with the fuel is destroyed during combustion. The exergy increase of the working fluid as it passes through the steam generator and reheater is the remaining 70% of exergy from the fuel. Of this, most becomes useful output from the cycle, followed by exergy loss from the condenser, and exergy destroyed in various devices. The significant exergy loss in the condenser (33%) suggests the possibility for improved thermodynamic performance.

PROBLEM 8.91

KNOWN: The regenerative vapor power cycle of Problem 8.46 is analyzed from the perspective of exergy accounting.

FIND: Determine the rate of transfer to the working fluid passing through the steam generator per kg of steam entering the first-stage turbine. Perform a complete exergy accounting.

SCHEMATIC & GIVEN DATA: See the solution to Problem 8.46.

ENGINEERING MODEL: Same as in Problem 8.46. Also, let $T_0 = 15°C = 288K$ and $p_0 = 0.1$ MPa = 1 bar. Assume the cooling water enters at 288K and is an incompressible substance with $c_{cw} = 4.179$ kJ/kg·K.

ANALYSIS: From Problem 8.46, the principal states are fixed. The specific enthalpies and entropies are

State	h(kJ/kg)	s(kJ/kg·K)
1	3465.4	6.5132
2	2745.1	6.5132
3	2037.0	6.5132
4	173.88	0.5926
5	174.88	0.5926
6	762.81	2.1387
7	779.72	2.1387

$s_1 = s_2 = s_3$

$s_5 = s_4$

$s_7 = s_6$

$\dot{W}_{cycle} = 1.498 \times 10^5$ kW

$\dot{m}_1 = 120$ kg/s

$y = 0.2287$

$\dot{m}_{cw} = 2292$ kg/s

$T_0 = 288K$

$T_{cw,in} = 288K$

$T_{cw,out} = 288 + 18 = 306K$

Input

• Steam generator: $\frac{\dot{E}_{f1} - \dot{E}_{f7}}{\dot{m}_1} = e_{f1} - e_{f7} = (h_1 - h_7) - T_0(s_1 - s_7)$

$= (3465.4 - 779.72) - (288)(6.5132 - 2.1387) = 1425.8 \frac{kJ}{kg}$

Output

• Net work: $\dot{W}_{cycle}/\dot{m}_1 = \frac{1.498 \times 10^5}{120} = 1248.3$ kJ/kg

Loss to cooling water

$$\frac{(\dot{E}_{f,out} - \dot{E}_{f,in})_{cw}}{\dot{m}_1} = \frac{\dot{m}_{cw}}{\dot{m}_1}\left[(h_{out} - h_{in})_{cw} - T_0(s_{out} - s_{in})_{cw}\right]$$

$$= \frac{\dot{m}_{cw}}{\dot{m}_1}\left[c_{cw}\Delta T_{cw} - T_0 c_{cw} \ln\left(\frac{T_{out}}{T_{in}}\right)_{cw}\right]$$

$$= \frac{2292}{120}\left[(4.179)(18) - (288)(4.179)\ln\left(\frac{306}{288}\right)\right] = 43.1 \text{ kJ/kg}$$

DESTRUCTION

• Open Heater: $\left(\frac{\dot{E}_d}{\dot{m}_1}\right)_{OH} = T_0\left(\frac{\dot{\sigma}_{cv}}{\dot{m}_1}\right)_{OH} = T_0\left[s_6 - y s_2 - (1-y)s_5\right]$

$= (288)\left[(2.1387) - (0.2287)(6.5132) - (0.7713)(0.5926)\right]$

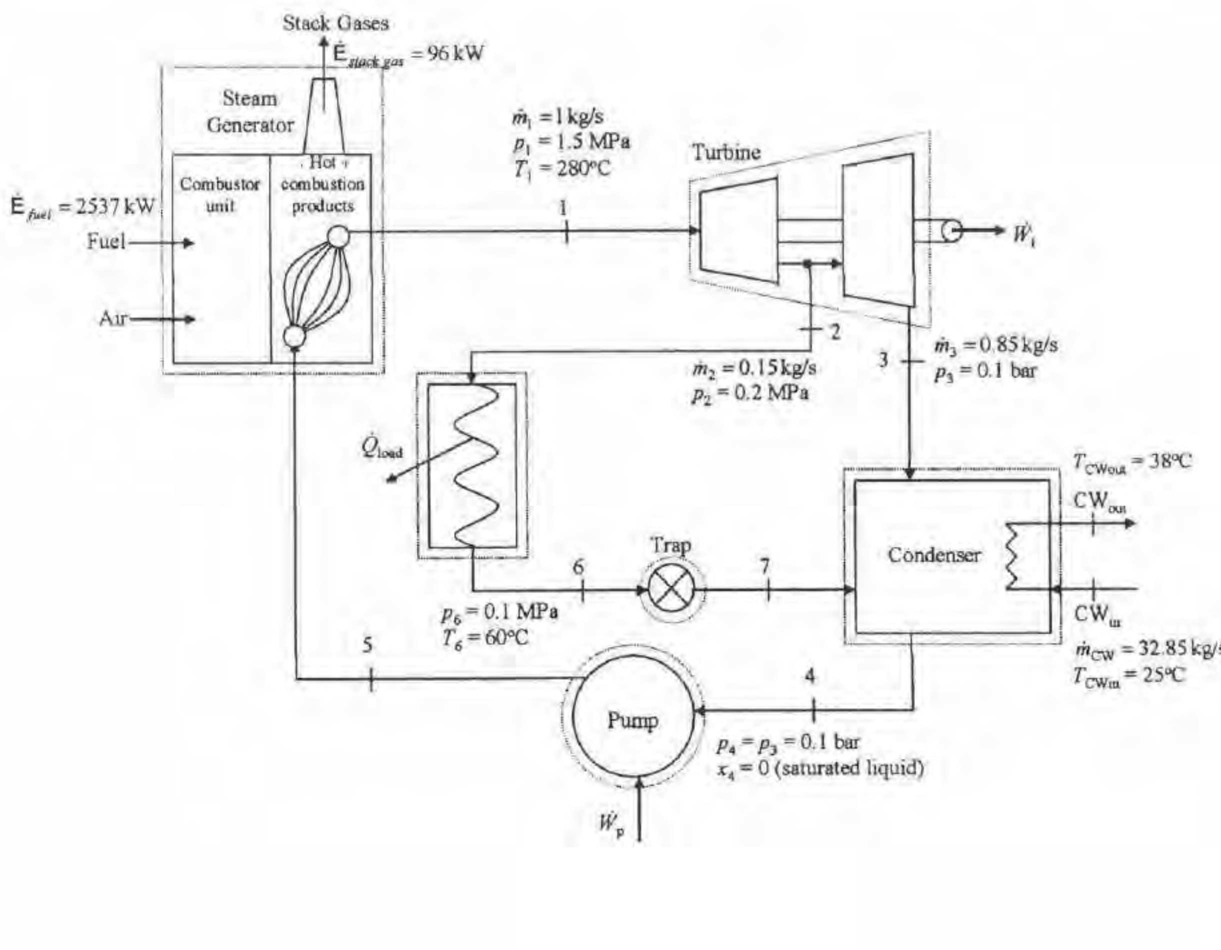
$= 55.3$ kJ/kg

• Condenser: $\left(\frac{\dot{E}_d}{\dot{m}_1}\right)_{cond} = T_0\left[(1-y)(s_4 - s_5) + \frac{\dot{m}_{cw}}{\dot{m}_1} c_w \ln\left(\frac{T_{out}}{T_{in}}\right)_{cw}\right]$

$= 288\left[(0.7713)(0.5926 - 6.5132) + \left(\frac{2292}{120}\right)(4.179)\ln\left(\frac{306}{288}\right)\right]$

$= 79.3$ kJ/kg

8.92 Figure P8.92 provides steady-state operating data for a cogeneration cycle that generates electricity and provides heat for campus buildings. Steam at 1.5 MPa, 280°C, enters a two-stage turbine with a mass flow rate of 1 kg/s. Steam is extracted between the two stages at 0.2 MPa with mass flow rate of 0.15 kg/s to provide for building heating, while the remainder expands through the second turbine stage to the condenser pressure of 0.1 bar with mass flow rate of 0.85 kg/s. The campus load heat exchanger in the schematic represents all of the heat transfer to the campus buildings. For the purposes of this analysis, we assume that the heat transfer in the campus load heat exchanger occurs at an average boundary temperature of 110°C. Condensate returns from the campus buildings at 0.1 MPa, 60°C and passes through a trap into the condenser, where it is reunited with the main feedwater flow. The cooling water has a mass flow rate of 32.85 kg/s entering the condenser at 25°C and exits the condenser at 38°C. The working fluid leaves the condenser as saturated liquid at 0.1 bar. The rate of exergy input with fuel entering the combustor unit of the steam generator is 2537 kW, and no exergy is carried in by the combustion air. The rate of exergy loss with the stack gases exiting the steam generator is 96 kW. Let $T_0 = 25°C$, $p_0 = 0.1$ MPa. Determine, as percentages of the rate of exergy input with fuel entering the combustor unit, all outputs, losses, and destructions of this exergy for the cogeneration cycle.



State	P	T (°C)	h (kJ/kg)	s (kJ/kg·K)
1	1.5 MPa	280	2992.7	6.8381
2	0.2 MPa	sat	2652.9	6.9906
3	0.1 bar	sat	2280.4	7.1965
4	0.1 bar	sat	191.83	0.6493
5	1.5 MPa		193.34	0.6539
6	0.1 MPa	60	251.13	0.8312
7	0.1 bar		251.13	0.8352
CWin		25	104.89	0.3674
CWout		38	159.21	0.5458

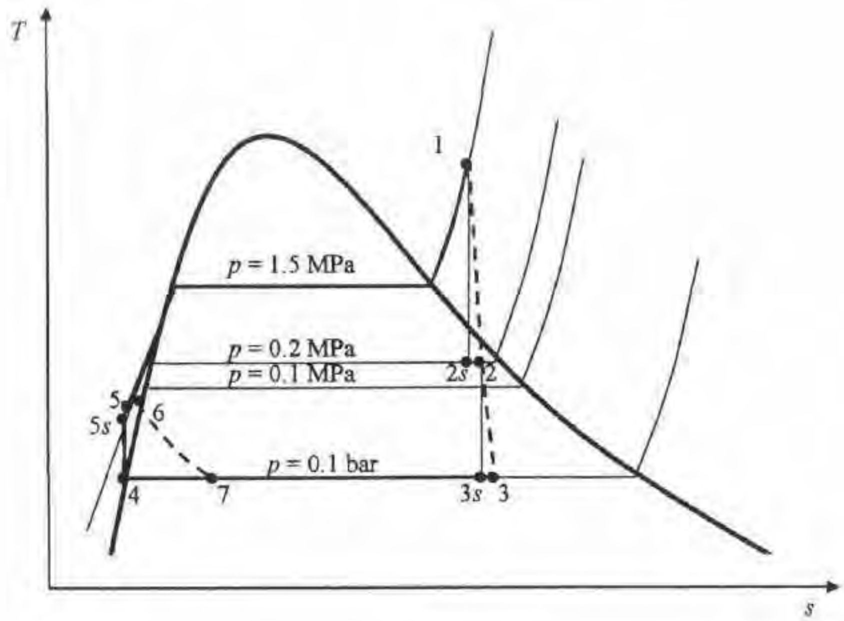
Fig P8.92

KNOWN: Water is the working fluid in a cogeneration cycle that generates electricity and provides heating for buildings.

FIND: Determine as percentages of the rate of exergy input with fuel entering the combustor unit, all outputs, losses, and destructions of this exergy.

SCHEMATIC AND GIVEN DATA: See problem statement above and T-s diagram below.

T-s diagram



ENGINEERING MODEL:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. Water exits the condenser as saturated liquid.

3. There is no heat transfer between the outside of the condenser and the surroundings.
4. Let $T_o = 25°C$ and $p_o = 0.1$ MPa.

ANALYSIS:

Stack Gases (Rate of Exergy Loss): The percentage of the rate of exergy input with fuel entering the combustor unit that exits with stack gases is

$$\% \text{ exiting (stack gases)} = \frac{\dot{E}_{stack\ gas}}{\dot{E}_{fuel}} = \frac{96\ \text{kW}}{2537\ \text{kW}} = \underline{\mathbf{3.784\%}}$$

Steam Generator (Rate of Exergy Destruction): Exergy and mass rate balances for a control volume enclosing the steam generator give

$$0 = \dot{E}_q - \dot{W}_{cv} + \dot{E}_{fuel} + \dot{E}_{air} - \dot{E}_{stack\ gas} + \dot{m}_1(e_{f5} - e_{f1}) - \dot{E}_d$$

For the steam generator, $\dot{E}_q$ and $\dot{W}_{cv}$ are zero. No exergy enters with the combustion air. Expanding the flow exergy term and solving for rate of exergy destruction

$$\dot{E}_d = \dot{E}_{fuel} - \dot{E}_{stack\ gas} + \dot{m}_1[(h_5 - h_1) - T_0(s_5 - s_1)]$$

$$\dot{E}_d = 2537\ \text{kW} - 96\ \text{kW} + \left(1\frac{\text{kg}}{\text{s}}\right)\left[(193.34 - 2992.7)\frac{\text{kJ}}{\text{kg}} - (298\ \text{K})(0.6539 - 6.8381)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right|$$

$$\dot{E}_d = 1484.5\ \text{kW}$$

Thus

$$\% \text{ destroyed (steam generator)} = \frac{\dot{E}_d}{\dot{E}_{fuel}} = \frac{1484.5\ \text{kW}}{2537\ \text{kW}} = \underline{\mathbf{58.514\%}}$$

Turbine and Pump (Net Power Developed): The net power developed is obtained from analyses of the turbine and the pump. For a control volume enclosing the turbine

$$\dot{W}_t = \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_2 - h_3)$$

$$\dot{W}_t = \left[\left(1\frac{\text{kg}}{\text{s}}\right)(2992.7 - 2652.9)\frac{\text{kJ}}{\text{kg}} + \left(0.85\frac{\text{kg}}{\text{s}}\right)(2652.9 - 2280.4)\frac{\text{kJ}}{\text{kg}}\right]\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right| = 656.4\ \text{kW}$$

For a control volume enclosing the pump

$$\dot{W}_p = \dot{m}_1(h_5 - h_4) = \left(1\frac{\text{kg}}{\text{s}}\right)\left[(193.34 - 191.83)\frac{\text{kJ}}{\text{kg}}\right]\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right| = 1.5\ \text{kW}$$

The net power developed by the cycle is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p = 656.4\text{ kW} - 1.5\text{ kW} = 654.9\text{ kW}$$

The percentage of the rate of exergy input with fuel entering the combustor unit developed as net cycle power is

$$\%\text{ net power} = \frac{\dot{W}_{cycle}}{\dot{\mathsf{E}}_{fuel}} = \frac{654.9\text{ kW}}{2537\text{ kW}} = \underline{\mathbf{25.814\%}}$$

Turbine (Rate of Exergy Destruction): Exergy and mass rate balances for a control volume enclosing the turbine gives

$$0 = \dot{\mathsf{E}}_q - \dot{W}_t + \dot{m}_1(\mathsf{e}_{f1} - \mathsf{e}_{f2}) + \dot{m}_3(\mathsf{e}_{f2} - \mathsf{e}_{f3}) - \dot{\mathsf{E}}_d$$

Expanding the flow exergy terms and solving for rate of exergy destruction give

$$\dot{\mathsf{E}}_d = -\dot{W}_t + \dot{m}_1[(h_1 - h_2) - T_0(s_1 - s_2)] + \dot{m}_3[(h_2 - h_3) - T_0(s_2 - s_3)]$$

$$\dot{\mathsf{E}}_d = -656.4\text{ kW} + \left(1\frac{\text{kg}}{\text{s}}\right)\left((2992.7 - 2652.9)\frac{\text{kJ}}{\text{kg}} - (298\text{ K})(6.8381 - 6.9906)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$
$$+\left(0.85\frac{\text{kg}}{\text{s}}\right)\left((2652.9 - 2280.4)\frac{\text{kJ}}{\text{kg}} - (298\text{ K})(6.9906 - 7.1965)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$
$$\dot{\mathsf{E}}_d = 97.6\text{ kW}$$

The percentage of the rate of exergy input with fuel entering the combustor unit destroyed in the turbine is

$$\%\text{ destroyed (turbine)} = \frac{\dot{\mathsf{E}}_d}{\dot{\mathsf{E}}_{fuel}} = \frac{97.6\text{ kW}}{2537\text{ kW}} = \underline{\mathbf{3.847\%}}$$

Pump (Rate of Exergy Destruction): Exergy and mass rate balances for a control volume enclosing the pump gives

$$0 = \dot{\mathsf{E}}_q - \dot{W}_p + \dot{m}_1(\mathsf{e}_{f4} - \mathsf{e}_{f5}) - \dot{\mathsf{E}}_d$$

Expanding the flow exergy terms and solving for rate of exergy destruction give

$$\dot{\mathsf{E}}_d = -\dot{W}_p + \dot{m}_1[(h_4 - h_5) - T_0(s_4 - s_5)]$$

$$\dot{\mathsf{E}}_\mathsf{d} = -(-1.5\,\text{kW}) + \left(1\frac{\text{kg}}{\text{s}}\right)\left((191.83 - 193.34)\frac{\text{kJ}}{\text{kg}} - (298\,\text{K})(0.6493 - 0.6539)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right|$$

$$\dot{\mathsf{E}}_\mathsf{d} = 1.4\ \text{kW}$$

The percentage of the rate of exergy input with fuel entering the combustor unit destroyed in the turbine and pump is

$$\%\ \text{destroyed (pump)} = \frac{\dot{\mathsf{E}}_\mathsf{d}}{\dot{\mathsf{E}}_{fuel}} = \frac{1.4\,\text{kW}}{2537\,\text{kW}} = \underline{\mathbf{0.055\%}}$$

<u>Campus Buildings Heat Exchanger (Rate of Exergy Loss)</u>: The rate of exergy transfer to campus buildings is

$$\dot{\mathsf{E}}_\mathsf{q} = \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j = \left(1 - \frac{T_0}{T_j}\right)\left[\dot{m}_2(h_6 - h_2)\right]$$

$$\dot{\mathsf{E}}_\mathsf{q} = \left(1 - \frac{298\,\text{K}}{383\,\text{K}}\right)\left[\left(0.15\frac{\text{kg}}{\text{s}}\right)(251.13 - 2652.9)\frac{\text{kJ}}{\text{kg}}\right]\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right| = -80.0\ \text{kW}$$

The negative sign associated with rate of exergy transfer with heat transfer indicates exergy is transferred from the working fluid in the heat exchanger.

The percentage of exergy input with fuel entering the combustor unit transferred to campus buildings is

$$\%\ \text{transferred (building load)} = \frac{\dot{\mathsf{E}}_\mathsf{q}}{\dot{\mathsf{E}}_{fuel}} = \frac{80.0\,\text{kW}}{2537\,\text{kW}} = \underline{\mathbf{3.153\%}}$$

<u>Campus Buildings Load Heat Exchanger (Rate of Exergy Destruction)</u>: Exergy and mass rate balances for a control volume enclosing the heat exchanger give

$$0 = \dot{\mathsf{E}}_\mathsf{q} - \dot{W}_{cv} + \dot{m}_2(\mathsf{e}_{f2} - \mathsf{e}_{f6}) - \dot{\mathsf{E}}_\mathsf{d}$$

Noting that $\dot{W}_{cv} = 0$ and expanding the flow exergy term, we can solve for rate of exergy destruction

$$\dot{\mathsf{E}}_\mathsf{d} = \dot{\mathsf{E}}_\mathsf{q} + \dot{m}_2\left[(h_2 - h_6) - T_0(s_2 - s_6)\right]$$

$$\dot{\mathsf{E}}_\mathsf{d} = -80.0\,\text{kW} + \left(0.15\frac{\text{kg}}{\text{s}}\right)\left[(2652.9 - 251.13)\frac{\text{kJ}}{\text{kg}} - (298\,\text{K})(6.9906 - 0.8312)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right|$$

$$\dot{E}_d = 4.9 \text{ kW}$$

The percentage of the rate of exergy input with fuel entering the combustor unit destroyed in the campus buildings load heat exchanger is

$$\% \text{ destroyed (building load heat exchanger)} = \frac{\dot{E}_d}{\dot{E}_{fuel}} = \frac{4.9 \text{ kW}}{2537 \text{ kW}} = \underline{\mathbf{0.193\%}}$$

Condenser (Rate of Exergy Loss): The rate of exergy exiting with cooling water from the condenser is

$$\begin{pmatrix} \text{net rate at which} \\ \text{exergy is carried out} \\ \text{by cooling water} \end{pmatrix} = \dot{m}_{CW}\left(e_{f\,CW_{out}} - e_{f\,CW_{in}}\right) = \dot{m}_{CW}\left[\left(h_{CW_{out}} - h_{CW_{in}}\right) - T_0\left(s_{CW_{out}} - s_{CW_{in}}\right)\right]$$

$$\begin{pmatrix} \text{net rate at which} \\ \text{exergy is carried out} \\ \text{by cooling water} \end{pmatrix} = \left(32.85\frac{\text{kg}}{\text{s}}\right)\left[(159.21 - 104.89)\frac{\text{kJ}}{\text{kg}} - (298\text{ K})(0.5458 - 0.3674)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$

$$\begin{pmatrix} \text{net rate at which} \\ \text{exergy is carried out} \\ \text{by cooling water} \end{pmatrix} = 38.0 \text{ kW}$$

The percentage of exergy input with fuel entering the combustor unit carried out by cooling water is

$$\% \text{ carried out (by cooling water)} = \frac{\dot{E}_{out}}{\dot{E}_{fuel}} = \frac{38.0 \text{ kW}}{2537 \text{ kW}} = \underline{\mathbf{1.498\%}}$$

Condenser (Rate of Exergy Destruction): Exergy and mass rate balances for a control volume enclosing the condenser give

$$0 = \dot{E}_q - \dot{W}_{cv} + \dot{m}_3 e_{f3} + \dot{m}_2 e_{f7} + \dot{m}_{CW} e_{fCW_{in}} - \dot{m}_1 e_{f4} - \dot{m}_{CW_{out}} e_{fCW_{out}} - \dot{E}_d$$

or

$$0 = \dot{E}_q - \dot{W}_{cv} + \dot{m}_3\left(e_{f3} - e_{f4}\right) + \dot{m}_2\left(e_{f7} - e_{f4}\right) + \dot{m}_{CW}\left(e_{fCW_{in}} - e_{fCW_{out}}\right) - \dot{E}_d$$

For the condenser $\dot{E}_q$ and $\dot{W}_{cv}$ are zero. Expanding the flow exergy terms and solving for rate of exergy destruction give

$$\dot{\mathsf{E}}_\mathsf{d} = \dot{m}_3\left[(h_3 - h_4) - T_0(s_3 - s_4)\right] + \dot{m}_2\left[(h_7 - h_4) - T_0(s_7 - s_4)\right] + \dot{m}_{CW}\left[\left(h_{CW_{int}} - h_{CW_{out}}\right) - T_0\left(s_{CW_{int}} - s_{CW_{out}}\right)\right]$$

$$\dot{\mathsf{E}}_\mathsf{d} = \left(0.85\frac{\text{kg}}{\text{s}}\right)\left[(2280.4 - 191.83)\frac{\text{kJ}}{\text{kg}} - (298\,\text{K})(7.1965 - 0.6493)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right|$$
$$+\left(0.15\frac{\text{kg}}{\text{s}}\right)\left[(251.13 - 191.83)\frac{\text{kJ}}{\text{kg}} - (298\,\text{K})(0.8352 - 0.6493)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right|$$
$$+\left(32.85\frac{\text{kg}}{\text{s}}\right)\left[(104.89 - 159.21)\frac{\text{kJ}}{\text{kg}} - (298\,\text{K})(0.3674 - 0.5458)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right|$$
$$\dot{\mathsf{E}}_\mathsf{d} = 79.5\ \text{kW}$$

The percentage of the rate of exergy input with fuel entering the combustor unit destroyed in the condenser is

$$\%\ \text{destroyed (condenser)} = \frac{\dot{\mathsf{E}}_\mathsf{d}}{\dot{\mathsf{E}}_{fuel}} = \frac{79.5\,\text{kW}}{2537\,\text{kW}} = \underline{\mathbf{3.134\%}}$$

Trap (Rate of Exergy Destruction): Exergy and mass rate balances for a control volume enclosing the trap give

$$0 = \dot{\mathsf{E}}_\mathsf{q} - \dot{W}_{cv} + \dot{m}_2\left(\mathsf{e}_{f6} - \mathsf{e}_{f7}\right) - \dot{\mathsf{E}}_\mathsf{d}$$

For the trap $\dot{\mathsf{E}}_\mathsf{q}$ and $\dot{W}_{cv}$ are zero. Expanding the flow exergy terms and solving for rate of exergy destruction give

$$\dot{\mathsf{E}}_\mathsf{d} = \dot{m}_2\left[(h_6 - h_7) - T_0(s_6 - s_7)\right]$$
$$\dot{\mathsf{E}}_\mathsf{d} = \left(0.15\frac{\text{kg}}{\text{s}}\right)\left[(251.13 - 251.13)\frac{\text{kJ}}{\text{kg}} - (298\,\text{K})(0.8312 - 0.8352)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right|$$
$$\dot{\mathsf{E}}_\mathsf{d} = 0.2\ \text{kW}$$

The percentage of the rate of exergy input with fuel entering the combustor unit destroyed in the trap is

$$\%\ \text{destroyed (trap)} = \frac{\dot{\mathsf{E}}_\mathsf{d}}{\dot{\mathsf{E}}_{fuel}} = \frac{0.2\,\text{kW}}{2537\,\text{kW}} = \underline{\mathbf{0.008\%}}$$

Problem 8.92 (Continued) – Page 8

Exergy Accounting Summary:

• Rate of exergy input with fuel entering the combustor unit	2537 kW	
• Exergy Out		
- Stack gas	96.0 kW	(3.78%)
- Net power developed	654.9 kW	(25.81%)
- Exergy to campus buildings	80.0 kW	(3.15%)
- Exergy to cooling water	38.0 kW	(1.50%)
• Exergy Destroyed		
- Steam Generator	1484.5 kW	(58.51%)
- Turbine	97.6 kW	(3.85%)
- Pump	1.4 kW	(0.06%)
- Campus Load Heat Exchanger	4.9 kW	(0.19%)
- Condenser	79.5 kW	(3.13%)
- Trap	0.2 kW	(0.01%)
	2537 kW	(≈100%) Slight difference due to round-off

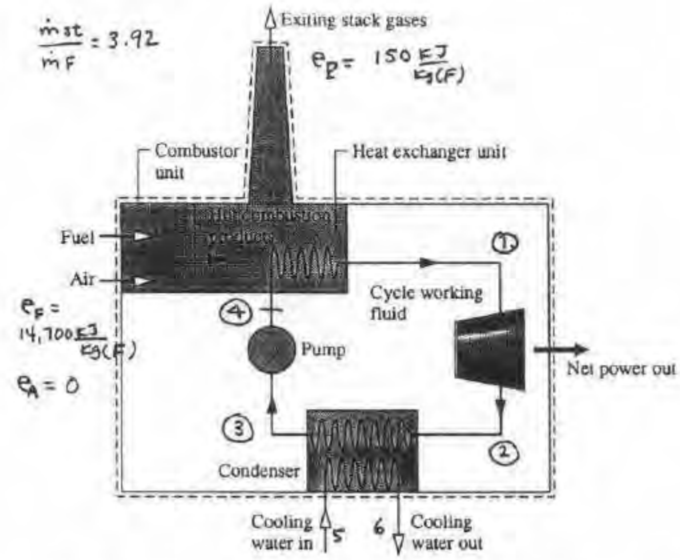
Note that the exergy destruction in the steam generator (due to the irreversible combustion process and heat transfer from the hot combustion products to the working fluid) is by far the biggest contributor to inefficiency in the cogeneration system. The losses due to the stack gases, condenser, and exergy destruction in the other components of the cycle are small by comparison.

PROBLEM 8.93

KNOWN: Water is the working fluid in a simple vapor power plant. Data are known at various locations. For the steam generator, specific exergy values are provided for the fuel, air, and stack gases.

FIND: Develop an exergy accounting of the exergy entering the plant with the fuel.

SCHEMATIC & GIVEN DATA:



State	T(°C)	p(bar)	h(kJ/kg)	s(kJ/kg·K)
1	520	100	3425.1	6.6622
2	(x=.90)	0.08	2336.7	7.4651
3	(sat. Liq)	0.08	173.9	0.5926
4	43°C	100	188.9	0.6061
5	20	—	84	0.2966
6	35	—	146.7	0.5053

ENGINEERING MODEL: 1. Control volumes enclosing each component are at steady state. 2. There are no stray heat transfers, and kinetic/potential energy effects can be ignored. 4. $T_0 = 20°C$, $p_0 = 1$ atm

ANALYSIS: Begin by fixing each state and obtaining h, s data: Table A-4 gives $h_1 = 3425.1$ kJ/kg, $s_1 = 6.6622$ kJ/kg·K. Then, with data from Table A-3

$h_2 = 173.88 + 0.9(2403.1) = 2336.7$, $s_2 = 0.5926 + 0.9(8.2287 - 0.5926) = 7.4651$,
$h_3 = h_f(0.08\text{ bar}) = 173.9$, $s_3 = s_f = 0.5926$. Then, from Table A-5, $h_4 = 188.9$,
$s_4 = 0.6061$. And with $h = h_f(T)$, $s = s_f(T)$ and data from Table A-2,
$h_5 = 84$, $s_5 = 0.2966$, $h_6 = 146.7$, $s_7 = 0.5053$

For the stack gases:

$$\frac{\dot{E}_P}{\dot{E}_F} = \frac{150\text{ kJ/kg(F)}}{14{,}700\text{ kJ/kg(F)}} = 0.01 \quad (1\%)$$

Exergy Destruction – Steam Generator: An exergy rate balance reduces to read

$$0 = \cancelto{0}{\dot{E}_{Air}} + \dot{E}_F - \dot{E}_P + \dot{m}_{st}[e_{f4} - e_{f1}] - \dot{E}_d$$

$$\Rightarrow \frac{\dot{E}_d}{\dot{m}_F} = e_F - e_P + \frac{\dot{m}_{st}}{\dot{m}_F}\left[(h_4 - h_1) - T_0(s_4 - s_1)\right]$$

$$= 14{,}700 - 150 + 3.92\left[(188.9 - 3425.1) - 293(0.6061 - 6.6622)\right] = 8820\text{ kJ/kg(F)}$$

Expressed as a percentage of the fuel exergy: $= \frac{8820}{14700} = 0.6 \quad (60\%)$

Net Work Developed:

$$\dot{W}_{net}/\dot{m}_F = (\dot{m}_{st}/\dot{m}_F)\left[(h_1 - h_2) - (h_4 - h_3)\right] = 3.92\left[(3425.1 - 2336.7) - (188.9 - 173.9)\right] = 4208 \frac{kJ}{kg(F)}$$

Expressed as a percentage of the fuel exergy: $= \frac{4208}{14700} = 0.286 \quad (28.6\%)$

PROBLEM 8.93 (Contd)

Exergy Destruction in Turbine/Pump: Using $\dot{E}_d = T_0 \dot{\sigma}$

$$\frac{\dot{E}_d}{\dot{m}_F} = \frac{\dot{m}_{st}}{\dot{m}_F}\left[T_0\left[s_2 - s_1\right] - T_0\left[s_4 - s_3\right]\right] = (3.92)(293)\left[(7.4651 - 6.6622) + (0.6061 - 0.5926)\right]$$

$= 938 \frac{kJ}{kg(F)}$. When expressed as a percentage of the fuel exergy: $= \frac{938}{14,700} = 0.064$ (6.4%)

Exergy exiting with the cooling water:

$$= \left(\frac{\dot{m}_{st}}{\dot{m}_F}\right)\left(\frac{\dot{m}_{cw}}{\dot{m}_{st}}\right)\left[e_{f,6} - e_{f,5}\right] = \left(\frac{\dot{m}_{st}}{\dot{m}_F}\right)\left(\frac{\dot{m}_{cw}}{\dot{m}_{st}}\right)\left[(h_6 - h_5) - T_0(s_6 - s_5)\right]$$

An energy rate balance on the condenser gives

$$\frac{\dot{m}_{cw}}{\dot{m}_{st}} = \frac{h_2 - h_3}{h_6 - h_5} = \frac{2336.7 - 173.9}{146.7 - 84} = 34.49$$

Thus,

$$\left(\begin{array}{c}\text{Exergy exiting}\\ \text{with c.w.}\end{array}\right) = (3.92)(34.49)\left[(146.7 - 84) - 293(0.5053 - 0.2966)\right] = 210 \text{ kJ/kg(F)}$$

when expressed as a percentage of the fuel exergy: $= \frac{210}{14,700} = 0.014$ (1.4%)

Exergy Destruction in the Condenser. Using $\dot{E}_d = T_0 \dot{\sigma}$

$$\frac{\dot{E}_d}{\dot{m}_F} = T_0\left[\frac{\dot{m}_{st}}{\dot{m}_F}(s_3 - s_2) + \left(\frac{\dot{m}_{st}}{\dot{m}_F}\right)\left(\frac{\dot{m}_{cw}}{\dot{m}_{st}}\right)(s_6 - s_5)\right]$$

$$= (293)(3.92)\left[(0.5926 - 7.4651) + 34.49(0.5053 - 0.2966)\right] = 374 \text{ kJ/kg(F)}$$

when expressed as a percentage of the fuel exergy $= \frac{374}{14,700} = 0.025$ (2.5%)

SUMMARY:

• Exergy entering with the fuel, per unit mass of fuel:	14,700 kJ/kg(F)	(100%)
• Exergy out		
– net work	4208	(28.6%)
– cooling water	210	(1.4%)
– stack gases	150	(1.0%)
• Exergy Destroyed		
– turbine/pump	938	(6.4%)
– condenser	374	(2.5%)
– Steam generator	8820	(60%)
	14,700	(100%)

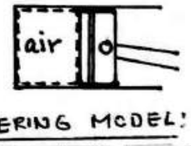
PROBLEM 9.1

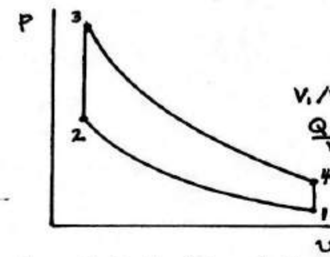
9.1 At the beginning of the compression process of an air-standard Otto cycle, $p_1 = 1$ bar and $T_1 = 300$ K. The compression ratio is 8.5 and the heat addition per unit mass of air is 1400 kJ/kg. Determine the net work, in kJ/kg, (b) the thermal efficiency of the cycle, (c) the mean effective pressure, in bar, (d) the maximum temperature of the cycle, in K.

KNOWN: An air-standard Otto cycle has a known compression ratio and a specified state at the beginning of compression. The heat addition per unit mass of air is given.

FIND: Determine (a) the net work per unit mass of air, (b) the thermal efficiency, (c) the mean effective pressure, (d) maximum temperature of the cycle.

SCHEMATIC & GIVEN DATA:





$V_1/V_2 = 8.5$

$\frac{Q_{23}}{m} = 1400$ kJ/kg

$p_1 = 100$ kPa

$T_1 = 300$K

ENGINEERING MODEL:

See Example 9.1

ANALYSIS: Begin by fixing each principal state of the cycle (Table A-22):

State 1 $p_1 = 100$ kPa, $T_1 = 300$K $\Rightarrow$ $u_1 = 214.07$ kJ/kg, $v_{r1} = 621.2$

State 2 For isentropic compression

$$v_{r2} = v_{r1}\frac{V_2}{V_1} = \frac{621.2}{8.5} = 73.082$$

Thus, $T_2 = 688.2$ K, $u_2 = 503.06$

State 3 The specific internal energy u_3 is found using the energy balance for process 2-3

$$m(u_3 - u_2) = Q_{23} - \cancel{W_{23}}^{\,0}$$

$$u_3 = \frac{Q_{23}}{m} + u_2 = 1400\,\frac{kJ}{kg} + 503.06\,\frac{kJ}{kg} = 1903.06$$

Thus, $T_3 = 2231.3$ K, $v_{r3} = 1.9192$ ← Tmax (part (d))

State 4 For the isentropic expansion

$$v_{r4} = v_{r3}\frac{V_4}{V_3} = v_{r3}\frac{V_1}{V_2} = (1.9192)(8.5) = 16.3132$$

Finally, $T_4 = 1154.3$ K, $u_4 = 892.95$ kJ/kg

(a) To find the net work, note that $W_{cycle} = Q_{cycle}$, so

$$\frac{W_{cycle}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = \frac{Q_{23}}{m} - (u_4 - u_1)$$

$$= 1400 - (892.95 - 214.07) = 721.12\ \frac{kJ}{kg} \longleftarrow W_{cycle}$$

(b) The thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = \frac{721.12}{1400} = 0.515\ (51.5\%) \longleftarrow \eta$$

(c) The displacement volume is $V_1 - V_2 = m(v_1 - v_2)$, so the mean effective pressure is given by

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}/m}{v_1 - v_2} = \frac{W_{cycle}/m}{v_1(1 - v_2/v_1)}$$

Evaluating v_1

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(\frac{8.314}{28.97}\ \frac{kJ}{kg \cdot K}\right)(300\ K)}{(100\ kPa)}\left(\frac{1\ kPa}{10^3\ N/m^2}\right)\left(\frac{10^3\ N \cdot m}{1\ kJ}\right)$$

$$= 0.861\ m^3/kg$$

Thus

$$mep = \frac{(721.12\ kJ/kg)}{(0.861\ \frac{m^3}{kg})(1 - \frac{1}{8.5})}\left(\frac{10^3\ N \cdot m}{1\ kJ}\right)\left(\frac{1\ kPa}{10^3\ N/m^2}\right)$$

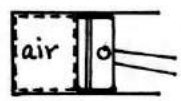
$$= 949.2\ kPa \longleftarrow \underline{\quad mep \quad}$$

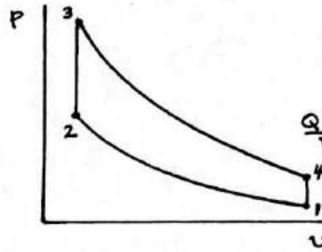
PROBLEM 9.2

KNOWN: An air-standard Otto cycle has a known compression ratio and a specified state at the beginning of compression. The heat addition per unit mass of air is given.

FIND: Determine (a) the net work per unit mass of air, (b) the thermal efficiency, (c) the mean effective pressure, and (d) the maximum cycle temperature. (e) plot each of these quantities versus compression ratio ranging from 1 to 12.

SCHEMATIC & GIVEN DATA:





$\frac{Q_{23}}{m} = 1350$ kJ/kg

$P_1 = 100$ kPa
$T_1 = 300$K

ENGINEERING MODEL: See Example 9.1.

ANALYSIS: Begin by fixing each principal state of the cycle (Table A-22) for the sample case $V_1/V_2 = 9$.

State 1 $p_1 = 100$ kPa, $T_1 = 300$K $\Rightarrow$ $u_1 = 214.07$ kJ/kg, $v_{r1} = 621.2$

State 2 For isentropic compression

$$v_{r2} = v_{r1}\frac{V_2}{V_1} = \frac{621.2}{9} = 69.022$$

Thus, $T_2 = 702.7$ K, $u_2 = 514.50$ kJ/kg

State 3 The specific internal energy u_3 is found using the energy balance for process 2-3

$$m(u_3 - u_2) = Q_{23} - \cancel{W_{23}}^{\,0}$$

$$u_3 = \frac{Q_{23}}{m} + u_2 = 1350\,\frac{kJ}{kg} + 514.50\,\frac{kJ}{kg} = 1864.5\,\frac{kJ}{kg}$$

Thus, $T_3 = 2191.9$ K, $v_{r3} = 2.0385$ (d) T_3

State 4 For the isentropic expansion

$$v_{r4} = v_{r3}\frac{V_4}{V_3} = v_{r3}\frac{V_1}{V_2} = (2.0385)(9) = 18.3465$$

Finally. $T_4 = 1110.9$ K, $u_4 = 854.81$ kJ/kg

(a) To find the net work, note that $W_{cycle} = Q_{cycle}$, so

$$\frac{W_{cycle}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = \frac{Q_{23}}{m} - (u_4 - u_1)$$

$$= 1350 - (854.81 - 214.07) = 709.26\,\frac{kJ}{kg} \longleftarrow \text{(a) } W_{cycle}$$

(b) The thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = \frac{709.26}{1350} = 0.525\ (52.5\%) \longleftarrow \text{(b) } \eta$$

(c) The displacement volume is $V_1 - V_2 = m(v_1 - v_2)$, so the mean effective pressure is given by

PROBLEM 9.1 (Contd.) – Page 2

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}/m}{v_1 - v_2} = \frac{W_{cycle}/m}{v_1(1 - v_2/v_1)}$$

Evaluating v_1

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(\frac{8.314}{28.97}\frac{kJ}{kg \cdot K}\right)(300\,K)}{(100\ kPa)}\left|\frac{1\ kPa}{10^3\ N/m^2}\right|\left|\frac{10^3\ N\cdot m}{1\ kJ}\right|$$

$$= 0.861\ m^3/kg$$

Thus

$$mep = \frac{(709.26\ kJ/kg)}{\left(0.861\ \frac{m^3}{kg}\right)\left(1 - \frac{1}{9}\right)}\left|\frac{10^3\ N\cdot m}{1\ kJ}\right|\left|\frac{1\ kPa}{10^3\ N/m^2}\right|$$

$$= 926.7\ kPa$$ (c) mep

The data for the required plots are obtained using *IT*, as follows:

IT Code

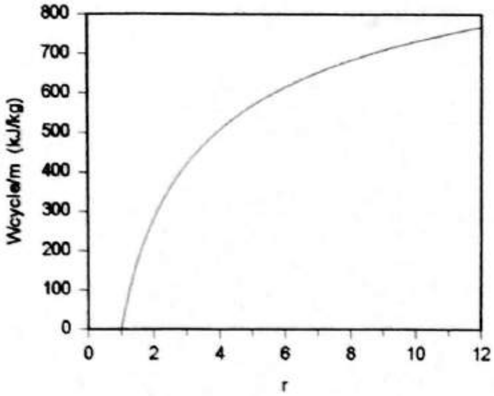
```
r = 9
p1 = 100   // kPa
T1 = 300   // K
Q23 / m = 1350   // kJ/kg
m = 1    // assume a unit mass of 1 kg.

v1 = v_TP("Air",T1,p1)
s1 = s_TP("Air", T1, p1)
u1 = u_T("Air", T1)
s2 = s1
v2 = v1 / r
s2 = s_TP("Air", T2, p2)
v2 = v_TP("Air",T2,p2)
u2 = u_T("Air", T2)
v3 = v2
m * (u3 - u2) = Q23 - W23
W23 = 0
u3 = u_T("Air", T3)
v3 = v_TP("Air",T3, p3)
s3 = s_TP("Air", T3, p3)
v4 = r * v3
s4 = s3
v4 = v_TP("Air",T4,p4)
s4 = s_TP("Air", T4, p4)
u4 = u_T("Air", T4)

Wcycle = Qcycle
Wcycle / m = Q23 / m - Q41 / m
Q41/ m = u4 - u1
eta = Wcycle / Q23
V1 = v1 * m
V2 = v2 * m
mep = Wcycle / (V1 - V2)
```

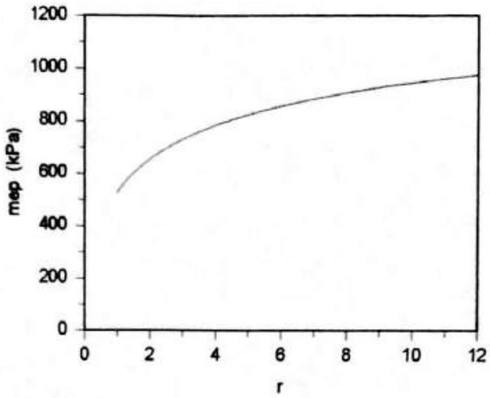
IT Results for $r = 9$

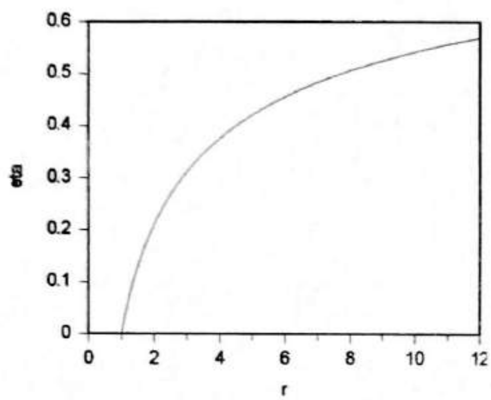
T_2 = 702.8 K
T_3 = 2194 K
T_4 = 1111 K
u_1 = 213.9 kJ.kg
u_2 = 514.3 kJ/kg
u_3 = 1864 kJ/kg
u_4 = 854.4 kJ/kg
v_1 = 0.861 m^3/kg
Q_{41}/m = 640.5 kJ/kg
W_{cycle}/m = 709.5 kJ/kg
η = 0.5256
mep = 927.1 kPa

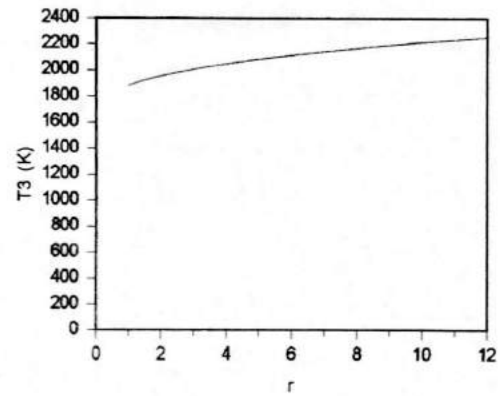
PROBLEM 9.2 (Cont'd.) – Page 3

PLOTS:









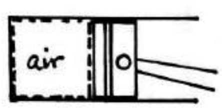
Thermal efficiency increases with increasing compression ratio. Since the heat addition is constant, the net work of the cycle increases, as expected. Also, the maximum cycle temperature increases, since the temperature at the end of compression, T_2, goes up with increasing r. The mep increases as well for similar reasons.

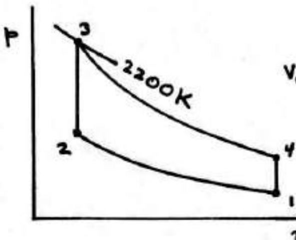
PROBLEM 9.3

KNOWN: An air-standard Otto cycle has a known compression ratio, a specified state at the beginning of compression, and a specified maximum temperature.

FIND: Determine (a) the heat added, (b) net work, (c) thermal efficiency, (d) mean effective pressure.

SCHEMATIC & GIVEN DATA:





$V_1/V_2 = 8$
$p_1 = 1$ bar
$T_1 = 290$K
$V_1 = 400$ cm^3

ENGINEERING MODEL: See Example 9.1.

ANALYSIS: First, fix each of the principal states of the cycle (Table A-22).

State 1: $p_1 = 1$ bar, $T_1 = 290$K $\Rightarrow u_1 = 206.91$ kJ/kg, $v_{r1} = 676.1$

State 2: For isentropic compression, $v_{r2} = v_{r1}(V_2/V_1) = \frac{676.1}{8} = 84.5125$

Thus, $T_2 = 652.4$ K, $u_2 = 475.11$ kJ/kg

State 3: $T_3 = 2200$K $\Rightarrow u_3 = 1872.4$ kJ/kg, $v_{r3} = 2.012$

State 4: For isentropic expansion, $v_{r4} = v_{r3}(V_4/V_3) = v_{r3}(V_1/V_2) = 16.096$

Thus, $T_4 = 1159.3$ K, $u_4 = 897.3$ kJ/kg

(a) To find the heat addition, first determine the mass of air

$$m = \frac{p_1V_1}{RT_1} = \frac{(1\text{ bar})(400\text{ cm}^3)}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(290\text{K})}\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|\left|\frac{1\text{ m}^3}{10^6\text{ cm}^3}\right|$$

$$= 4.806\times10^{-4}\text{ kg}$$

Thus

$$m(u_3-u_2) = Q_{23} - \cancelto{0}{W_{23}}$$

$$Q_{23} = m(u_3-u_2) = (4.806\times10^{-4})(1872.4-475.11)$$

$$= 0.6715\text{ kJ} \longleftarrow Q_{23}$$

(b) To find the net work, note that $W_{cycle} = Q_{cycle}$, so

$$W_{cycle} = Q_{23} - Q_{41} = Q_{23} - m(u_4-u_1)$$

$$= 0.6715 - (4.806\times10^{-4})(897.3-206.91) = 0.3397\text{ kJ} \longleftarrow W_{cycle}$$

(c) The thermal efficiency is

$$\eta = \frac{W_{cycle}}{Q_{23}} = \frac{.3397}{.6715} = 0.506\ (50.6\%) \longleftarrow \eta$$

(d) The mean effective pressure is given by

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}}{V_1(1 - V_2/V_1)}$$

Inserting values

$$mep = \frac{(0.3397\text{ kJ})}{(400\text{ cm}^3)(1-\frac{1}{8})}\left|\frac{10^3\text{N}\cdot\text{m}}{1\text{ kJ}}\right|\left|\frac{1\text{ bar}}{10^5\text{ N/m}^2}\right|\left|\frac{10^6\text{ cm}^3}{1\text{ m}^3}\right|$$

$$= 9.71\text{ bar} \longleftarrow mep$$

PROBLEM 9.4

The data for the required plots are obtained using IT, as follows. See Problem 9.3 for sample calculations for the case of r = 8.

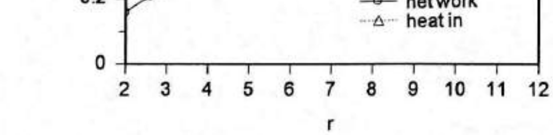
IT Code

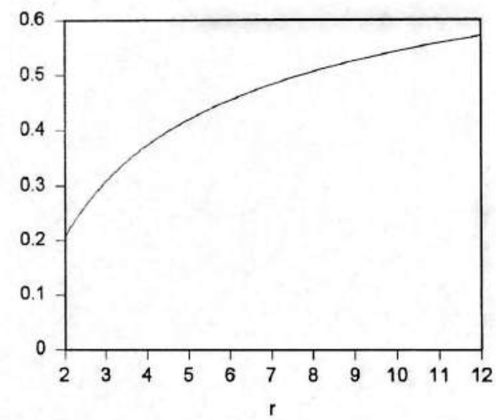
```
r = 8
p1 = 1   // bar
T1 = 290   // K
V1 = 400   // cm³
T3 = 2200   // K

v1 = v_TP("Air",T1,p1)
v1 = (V1 / 10^6) / m
s1 = s_TP("Air", T1, p1)
u1 = u_T("Air", T1)
s2 = s1
v2 = v1 / r
s2 = s_TP("Air", T2, p2)
v2 = v_TP("Air",T2, p2)
u2 = u_T("Air", T2)
v3 = v2
u3 = u_T("Air", T3)
v3 = v_TP("Air",T3,p3)
s3 = s_TP("Air", T3, p3)
v4 = r * v3
s4 = s3
v4 = v_TP("Air",T4,p4)
s4 = s_TP("Air", T4, p4)
u4 = u_T("Air", T4)

m * (u3 - u2) = Q23 - W23   // kJ
W23 = 0
Wcycle = Qcycle   // kJ
Wcycle / m = Q23 / m - Q41 / m
Q41/ m = u4 - u1
eta = Wcycle / Q23
mep = Wcycle * ( 10^3 / 10^5) / ((V1/ 10 ^6) * (1 - 1 / r))   // bar
```

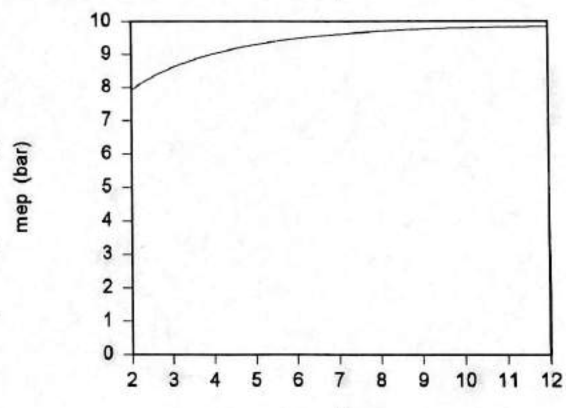
IT results for r = 8

Q_{41}= 0.3312 kJ
T_2 = 652.4 K
T_4 = 1158 K
u_1 = 206.8 kJ
u_2 = 474.9 kJ
u_3 = 1871 kJ
u4 = 896 kJ
T_3 = 2200 K
Q_{23} = 0.6707 kJ
W_{cycle} = 0.3395 kJ
η = 0.5062
mep = 9.7 bar

PLOTS:







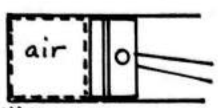
As seen in the accompanying plots, both η and mep increase with increasing compression ratio, r. Since T_3 is constant, the heat input decreases and the net work increases with r.

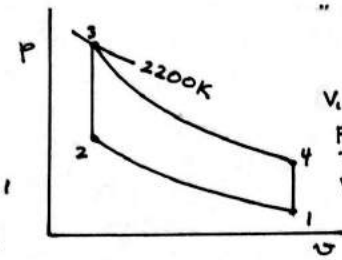
PROBLEM 9.5

KNOWN: A cold air-standard Otto cycle has a known compression ratio, a specified state at the beginning of compression, and a specified maximum temperature.

FIND: Determine (a) the heat added, (b) net work, (c) thermal efficiency, (d) mean effective pressure.

SCHEMATIC & GIVEN DATA:





$V_1/V_2 = 8$
$p_1 = 1\,\text{bar}$
$T_1 = 290\text{K}$
$V_1 = 400\,\text{cm}^3$

ENGINEERING MODEL: Same as Example 9.1. Also, assume constant specific heats evaluated at 300K

ANALYSIS: First, determine the temperature at each principal state of the cycle. $T_1 = 290\text{K}$ is given.

State 2: For isentropic compression, $T_2 = (V_1/V_2)^{k-1}T_1 = 666.2\text{ K}$ where $k = 1.4$ from Table A-20

State 3: $T_3 = 2200\text{K}$ is given.

State 4: For isentropic expansion,

$$T_4 = \left(\frac{V_3}{V_4}\right)^{k-1} T_3 = \left(\frac{V_2}{V_1}\right)^{k-1} T_3 = 957.6\text{ K}$$

(a) To find the heat addition, first determine the mass of air

$$m = \frac{p_1 V_1}{RT_1} = \frac{(1\text{ bar})(400\text{ cm}^3)}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(290\text{K})}\left|\frac{10^5\text{ N/m}^2}{1\text{bar}}\right|\left|\frac{1\text{kJ}}{10^3\text{ N}\cdot\text{m}}\right|\left|\frac{1\text{ m}^3}{10^6\text{ cm}^3}\right| = 4.806\times10^{-4}\text{ kg}$$

Then

$$m(u_3 - u_2) = Q_{23} - \cancelto{0}{W_{23}} \Rightarrow m c_v (T_3 - T_2) = Q_{23}$$

$$Q_{23} = (4.806\times10^{-4})(.718\tfrac{\text{kJ}}{\text{kg}\cdot\text{K}})(2200 - 666.2)\text{K}$$

$$= 0.5293\text{ kJ} \longleftarrow Q_{23}$$

(b) To find the net work, note that $W_{cycle} = Q_{cycle}$, so

$$W_{cycle} = Q_{23} - Q_{41} = Q_{23} - m c_v (T_4 - T_1)$$

$$= 0.5293 - (4.806\times10^{-4})(.718)(957.6 - 290)$$

$$= 0.2989\text{ kJ} \longleftarrow W_{cycle}$$

(c) The thermal efficiency is

$$\eta = \frac{W_{cycle}}{Q_{23}} = \frac{0.2989}{0.5293} = 0.565\ (56.5\%) \longleftarrow \eta$$

(d) The mean effective pressure is given by

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}}{V_1(1 - V_2/V_1)}$$

Inserting values

$$mep = \frac{(0.2989\text{ kJ})}{(400\text{ cm}^3)(1 - 1/8)}\left|\frac{10^3\text{ N}\cdot\text{m}}{1\text{kJ}}\right|\left|\frac{1\text{ bar}}{10^5\text{ N/m}^2}\right|\left|\frac{10^6\text{ cm}^3}{1\text{ m}^3}\right|$$

$$= 8.54\text{ bar} \longleftarrow mep$$

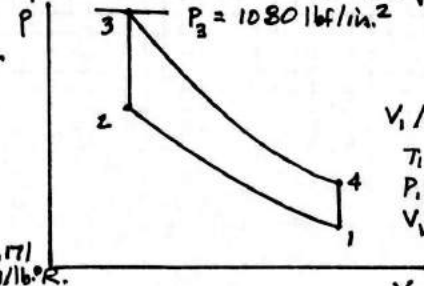
PROBLEM 9.6

KNOWN: A four-cylinder, four-stroke engine operates at a known RPM. The processes in each cylinder are modeled as a cold air-standard Otto cycle with a specified state at the beginning of compression, a known compression ratio, and a specified maximum cycle pressure.

FIND: Determine the power developed and the mean effective pressure.

SCHEMATIC & GIVEN DATA:

Four-cylinder
Four-stroke
(1) 2800 RPM



$V_1/V_2 = 10$
$T_1 = 80°F = 540°R$
$p_1 = 14.7\ \text{lbf/in.}^2$
$V_1 = 0.0196\ \text{ft}^3$

ENGINEERING MODEL: See Example 9.1. Also, $k = 1.4$ and $c_v = 0.171$ Btu/lb·°R.

ANALYSIS: First, determine each temperature in the cycle.

State 2: Using Eq. 9.6, $T_2 = (V_1/V_2)^{k-1} T_1 = (10)^{0.4}(540°R) = 1356.4°R$

Also, $p_2 = (V_1/V_2)^k p_1 = (10)^{1.4}(14.7) = 369.2\ \text{lbf/in.}^2$

State 3: $V_3 = V_2 \Rightarrow T_3 = (p_3/p_2)T_2 = (1080/369.2)(1356.4) = 3967.8°R$

State 4: $T_4 = (V_3/V_4)^{k-1} T_4 = \left(\frac{1}{10}\right)^{0.4}(3967.8) = 1579.6°R$

To get the power, first determine the net work per cycle. The mass in the cylinder is

$$m = \frac{p_1V_1}{RT_1} = \frac{(14.7\ \text{lbf/in.}^2)(0.0196\ \text{ft}^3)}{\left(\frac{1545}{28.97}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°R}\right)(540°R)}\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right| = 0.001441\ \text{lb}$$

$$W_{cycle} = W_{12} + W_{34}$$
$$= m\,c_v\left[(T_1 - T_2) + (T_3 - T_4)\right]$$
$$= (0.001441\ \text{lb})\left(0.171\ \frac{\text{Btu}}{\text{lb}\cdot°R}\right)\left[(540 - 1356.4) + (3967.8 - 1579.6)\right]°R$$
$$= 0.3873\ \text{Btu/cycle}$$

$$\dot{W}_{net} = (4\ \text{cyl})\left(\frac{2800}{2}\ \frac{\text{cycle}}{\text{min}}\right)(0.3872\ \text{Btu/cycle})\left|\frac{60\ \text{min}}{1\ \text{h}}\right|\left|\frac{1\ \text{hp}}{2545\ \text{Btu/h}}\right|$$
$$= 51.1\ \text{hp} \leftarrow \dot{W}_{net}$$

The mean effective pressure is

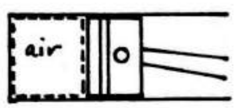
$$\text{mep} = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}}{V_1(1 - V_2/V_1)}$$
$$= \frac{0.3873\ \text{Btu}}{(0.0196\ \text{ft}^3)(1 - 0.1)}\left|\frac{778\ \text{ft}\cdot\text{lbf}}{1\ \text{Btu}}\right|\left|\frac{1\ \text{ft}^2}{144\ \text{in.}^2}\right| = 118.6\ \text{lbf/in.}^2 \leftarrow \text{mep}$$

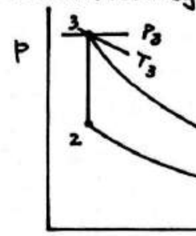
PROBLEM 9.7

KNOWN: An air-standard Otto cycle has a known compression ratio and a specified state at the beginning of compression. The heat addition per unit mass of air is given.

FIND: Determine (a) the maximum temperature, (b) the maximum pressure, (c) the thermal efficiency, (d) the mep.

SCHEMATIC & GIVEN DATA:





$\frac{Q_{23}}{m} = 600$ Btu/lb

$V_1/V_2 = 6$

$T_1 = 520\,°R$

$P_1 = 14.2$ lbf/in²

ENGR. Model: Same as Example 9.1

ANALYSIS: (a) To determine T_3, begin by fixing state 2. For the isentropic compression

$$v_{r2} = v_{r1}\left(\frac{V_2}{V_1}\right) = \frac{158.58}{6} = 26.43$$

Where data for air is obtained from Table A-22E. Thus, for $v_{r2} = 26.43$, $T_2 \approx 1051.4\,°R$, $u_2 \approx 181.74$ Btu/lb. The energy balance for process 2-3 is

$$m(u_3 - u_2) = Q_{23} - \cancel{W_{23}}^{\,0}$$

$$u_3 = \frac{Q_{23}}{m} + u_2 = 600\,\frac{\text{Btu}}{\text{lb}} + 181.74\,\frac{\text{Btu}}{\text{lb}} = 781.74\ \text{Btu/lb}$$

Thus, from Table A-13E; $T_3 \approx 3861\,°R$ ← T_3

(b) To find p_3, first determine p_2. For the isentropic compression

$$p_2 = p_1\,\frac{p_{r2}}{p_{r1}} = (14.2)\left(\frac{14.78}{1.2147}\right) = 172.78\ \text{lbf/in}^2$$

Now, since $V_3 = V_2$; $p_3 = \frac{T_3}{T_2}\cdot p_2 = 634.5$ lbf/in². ← P_3

(c) To determine the thermal efficiency, first find the net work. That is

$$\frac{W_{cycle}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = \frac{Q_{23}}{m} - (u_4 - u_1)$$

To fix state 4, consider the expansion process

$$v_{r4} = v_{r3}\left(\frac{V_4}{V_3}\right) = v_{r3}\left(\frac{V_1}{V_2}\right) = (.5095)(6) = 3.057$$

Thus, $T_4 \approx 2228\,°R$ and $u_4 \approx 415.69$ and the net work is

$$\frac{W_{cycle}}{m} = 600 - [(415.69) - (88.62)] = 272.93\ \text{Btu/lb}$$

Finally, the thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = \frac{272.93}{600} = 0.455\ (45.5\%)$$ ← η

(d) Applying the ideal gas model equation of state, $v_1 = RT_1/p_1 = 13.56$ ft³/lb.

$$\text{mep} = \frac{W_{cycle}/m}{v_1 - v_2} = \frac{W_{cycle}/m}{v_1[1 - v_2/v_1]} = \frac{(272.93\ \text{Btu/lb})}{(13.56\ \text{ft}^3/\text{lb})[1 - 1/6]}\left|\frac{778\ \text{ft·lbf}}{1\ \text{Btu}}\right|\left|\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right|$$

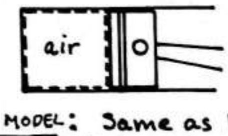
$$= 130.5\ \frac{\text{lbf}}{\text{in}^2}$$ ← mep

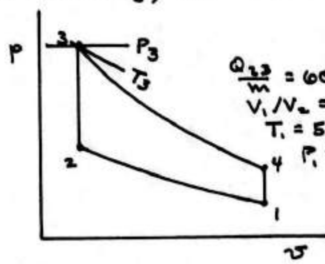
PROBLEM 9.8

KNOWN: A cold air-standard Otto cycle has a known compression ratio and a specified state at the beginning of compression. The heat addition per unit mass of air is given.

FIND: Determine (a) the maximum temperature, (b) the maximum pressure, (c) the thermal efficiency, and mep.

SCHEMATIC & GIVEN DATA:





$\frac{Q_{23}}{m} = 600$ Btu/lb
$V_1/V_2 = 6$
$T_1 = 520°R$
$P_1 = 14.2$ lbf/in.²

ENGR. MODEL: Same as Example 9.1. Also, assume constant specific heats evaluated at 520°R.

ANALYSIS: To determine T_3, begin by evaluating T_2. For the isentropic compression

$$T_2 = \left(\frac{V_1}{V_2}\right)^{k-1} T_1 = (6)^{.401}(520°R) = 1066.7\ °R$$

where k=1.401 from Table A-22E. The energy balance for process 2-3 is

$$m(u_3 - u_2) = Q_{23} - \cancel{W_{23}}^{0} \Rightarrow Q_{23} = m\,c_v(T_3 - T_2)$$

Thus

$$T_3 = \frac{Q_{23}}{m c_v} + T_2 = \frac{600\ \text{Btu/lb}}{.171\ \text{Btu/lb}\cdot°R} + 1066.7\ °R$$

$$= 4575.5\ °R \longleftarrow T_3$$

(b) To find p_3, first determine P_2. For the isentropic compression

$$P_2 = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} P_1 = 174.8\ \text{lbf/in}^2$$

Now, since $V_3 = V_2$; $P_3 = \frac{T_3}{T_2} P_2 = 749.8\ \text{lbf/in}^2 \longleftarrow P_3$

(c) The thermal efficiency can be determined using Eq. 9.8

① $$\eta = 1 - \frac{1}{r^{k-1}} = 1 - \frac{1}{(V_1/V_2)^{k-1}}$$

$$= 1 - \frac{1}{6^{(1.401-1)}} = 0.5125\ (51.25\%) \longleftarrow \eta$$

(d) Applying the ideal gas model equation of state, $v_1 = RT_1/p_1 = 13.56\ \text{ft}^3/\text{lb}$.

$$\text{mep} = \frac{W_{cycle}/m}{v_1 - v_2} = \frac{W_{cycle}/m}{v_1[1 - v_2/v_1]} \quad \text{where} \quad \frac{W_{cycle}}{m} = \eta\,\frac{Q_{23}}{m} = 307.5\ \frac{\text{Btu}}{\text{lb}}$$

$$\therefore \quad \text{mep} = \frac{307.5\ \text{Btu/lb}}{13.56\ \text{ft}^3/\text{lb}\,[1 - 1/6]} \left|\frac{778\ \text{ft}\cdot\text{lbf}}{1\ \text{Btu}}\right|\left|\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right| = 147\ \frac{\text{lbf}}{\text{in}^2} \longleftarrow \text{mep}$$

---

1. Alternatively, Eq 9.3 can be applied:

$$\eta = \frac{(u_3 - u_2) - (u_4 - u_1)}{(u_3 - u_2)} = 1 - \frac{\overbrace{(u_4 - u_1)}^{c_v[T_4 - T_1]}}{\underbrace{(u_3 - u_2)}_{= Q_{23}/m}}$$

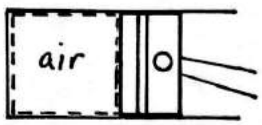
where T_4 is evaluated using Eq. 9.7.

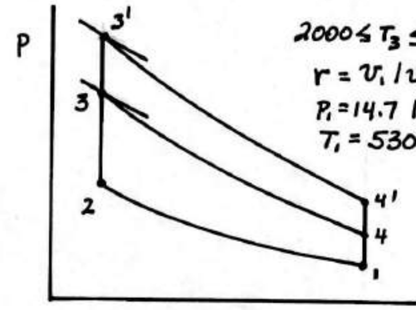
PROBLEM 9.9

KNOWN: An air-standard Otto cycle has known conditions at the beginning of compression. The maximum cycle temperature varies over a given range and the compression ratio takes on specified values.

FIND: Plot thermal efficiency and mean effective pressure versus maximum cycle temperature for each compression ratio.

SCHEMATIC & GIVEN DATA:





$2000 \le T_3 \le 5000°R$

$r = v_1/v_2 = 6, 8, 10$

$p_1 = 14.7$ lbf/in²

$T_1 = 530°R$

ENGINEERING MODEL:
Same as Example 9.1.

ANALYSIS: Sample calculation for $r = 8$, $T_3 = 2000°R$ using Table A-22E.
From Table A-22E, $u_1 = 90.33$ Btu/lb, $v_{r_1} = 151.38$. For the compression process

$$v_{r_2} = \left(\frac{v_2}{v_1}\right) v_{r_1} = \left(\frac{1}{8}\right)(151.38) = 18.9225$$

Thus, $T_2 = 1190.7°R$ and $u_2 = 207.33$ Btu/lb.

For various values of T_3, u_3 and v_{r3} can be determined from Table A-22E. Then, state 4 is fixed using the relation for the isentropic expansion

$$v_{r_4} = \left(\frac{v_4}{v_3}\right) v_{r_3} = \left(\frac{v_1}{v_2}\right) v_{r_3}$$

and u_4 can be evaluated from the tabular data. For $T_3 = 2000°R$

$$u_3 = 367.61 \text{ Btu/lb},\ v_{r_3} = 4.258,\ v_{r_4} = 34.064,\ u_4 = 164.23 \text{ Btu/lb}$$

Continuing for this case, the heat added to the cycle is

$$\frac{Q_{23}}{m} = u_3 - u_2 = 160.28 \text{ Btu/lb}$$

and the heat rejected is

$$\frac{Q_{41}}{m} = u_4 - u_1 = 73.9$$

Thus, the thermal efficiency is

$$\eta = 1 - \frac{Q_{41}/m}{Q_{32}/m} = 0.539\ (53.9\%) \longleftarrow \eta$$

The mean effective pressure is

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}/m}{v_1(1 - v_2/v_1)} = \frac{Q_{23}/m - Q_{41}/m}{v_1(1 - v_2/v_1)}$$

PROBLEM 9.9 (Cont'd) - Page 2

Evaluating v_1

$$v_1 = \frac{RT_1}{p_1} = \frac{\left(\frac{1545}{28.97}\ \frac{ft\cdot lbf}{lb\cdot {}^\circ R}\right)(530^\circ R)}{(14.7\ lbf/in^2)}\left|\frac{1\ ft^2}{144\ in^2}\right| = 13.35\ ft^3/lb$$

Thus

$$mep = \frac{(160.28 - 73.9)\ Btu/lb}{(13.35\ ft^3/lb)(1-\frac{1}{8})}\left|\frac{778\ ft\cdot lbf}{1\ Btu}\right|\left|\frac{1\ ft^2}{144\ in^2}\right| = 39.95\ lbf/in^2 \quad \underline{mep}$$

The data for the required plots are obtained using IT, as follows:

IT Code

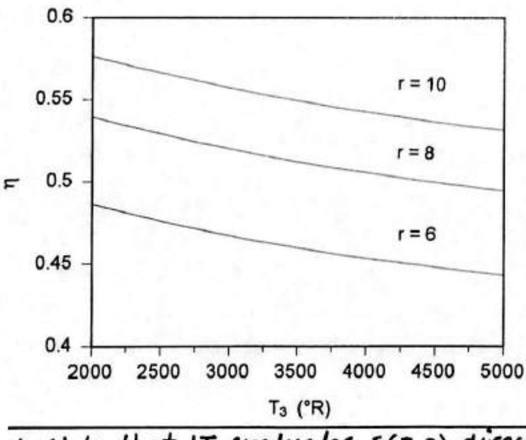
```
r = 8
p1 = 14.7   // lbf/in²
T1 = 530  // °R
T3 = 2000   // °R
m = 1    // assume a unit mass of 1 lb.

v1 = v_TP("Air",T1,p1)
s1 = s_TP("Air", T1, p1)
u1 = u_T("Air", T1)
s2 = s1
v2 = v1 / r
s2 = s_TP("Air", T2, p2)
v2 = v_TP("Air",T2,p2)
u2 = u_T("Air", T2)
v3 = v2
u3 = u_T("Air", T3)
v3 = v_TP("Air",T3,p3)
s3 = s_TP("Air", T3, p3)
v4 = r * v3
s4 = s3
v4 = v_TP("Air",T4,p4)
s4 = s_TP("Air", T4, p4)
u4 = u_T("Air", T4)

Wcycle = Q23 - Q41
Q23 = m * (u3 - u2)
Q41 = m * (u4 - u1)
eta = Wcycle / Q23
V1 = v1 * m
V2 = v2 * m
mep = Wcycle * (778 / 144) / (V1 - V2)
```

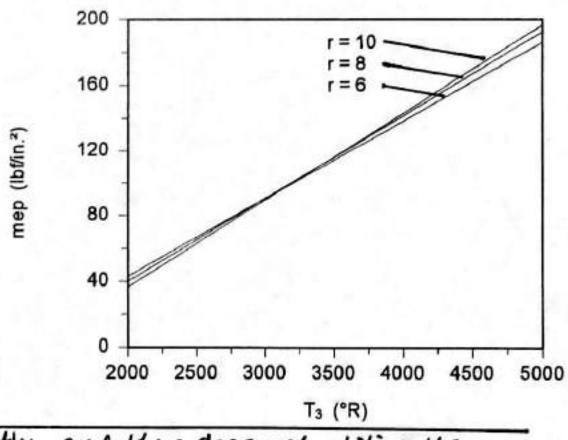
(1)

IT Results for r = 8, T_3 = 2000°R

T_2 = 1191°R
T_4 = 953.6°R
p_2 = 264.2 lbf/in.²
p_3 = 443.8 lbf/in.²
v1 = 13.35 ft³/lb
Q_{23}/m = 160 Btu/lb
Q_{41}/m = 73.73 Btu/lb
W_{cycle}/m = 86.29 Btu/lb
η = 0.5393
mep = 39.9 lbf/in.²

PLOTS:





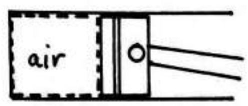
1. Note that IT evaluates s(T, p) directly, and thus does not utilize the special isentropic functions $v_r(T)$ and $p_r(T)$ as in Table A-22E.

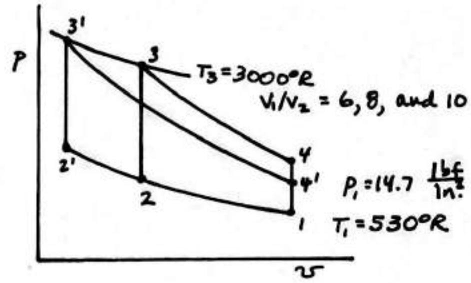
PROBLEM 9.10

KNOWN: A cold air-standard Otto cycle has a known state at the beginning of compression and a specified maximum cycle temperature.

FIND: Determine the thermal efficiency, net work per unit of mass, and mep for various compression ratios.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL: Same as in Example 9.1. Also, assume constant specific heats with $k = 1.4$.

ANALYSIS: The thermal efficiency is (eq. 9.8)

$$\eta = 1 - \frac{1}{r^{k-1}}$$

Thus,

$$\eta = \begin{cases} .512 & (r=6) \\ .565 & (r=8) \\ .602 & (r=10) \end{cases}$$

The mep is determined from

$$mep = \frac{W_{cycle}/m}{(v_1 - v_2)} = \frac{\eta(Q_{23}/m)}{v_1(1 - v_2/v_1)} = \frac{\eta\, c_v (T_3 - T_2)}{v_1(1 - v_2/v_1)}$$

For the isentropic compression process

$$T_2 = \left(\frac{V_1}{V_2}\right)^{k-1} T_1 \Rightarrow T_2 = \begin{cases} 1085.3°R & (r=6) \\ 1217.6°R & (r=8) \\ 1331.3°R & (r=10) \end{cases}$$

Further, $c_v = 0.171$ Btu/lb·°R and

$$v_1 = \frac{RT_1}{p_1} = \frac{\left(\frac{1545}{28.97}\ \frac{ft\cdot lbf}{lb\cdot °R}\right)(530°R)}{(14.7\ lbf/in^2)}\left(\frac{1\ ft^2}{144\ in^2}\right) = 13.35\ ft^3/lb$$

Thus, for $r=6$

$$\begin{cases} mep = \frac{\overbrace{(.512)(.171\ Btu/lb\cdot °R)(3000 - 1085.3)°R}^{W_{cycle}/m}}{(13.35\ ft^3/lb)(1 - 1/6)}\left(\frac{778\ ft\cdot lbf}{1\ Btu}\right)\left(\frac{1\ ft^2}{144\ in^2}\right) \\ \quad = 81.41\ lbf/in^2 \quad (r=6) \\ \frac{W_{cycle}}{m} = 167.6\ Btu/lb \end{cases}$$

Similarly, for $r = 8$ $\begin{cases} mep = 79.65\ lbf/in^2 \\ \frac{W_{cycle}}{m} = 172.2\ Btu/lb \end{cases}$

And for $r = 10$ $\begin{cases} mep = 77.24\ lbf/in^2 \\ \frac{W_{cycle}}{m} = 171.8\ Btu/lb \end{cases}$

9.11 Consider an air-standard Otto cycle. Operating data at principal states in the cycle are given in table below. The states are numbered as in Fig. 9.3. The mass of air is 0.002 kg. Determine

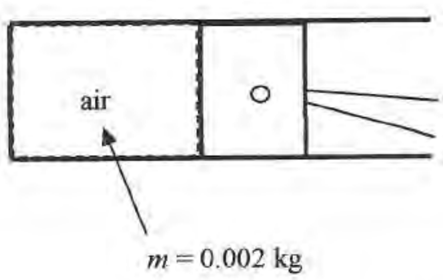
(a) the heat addition and heat rejection, each in kJ.
(b) the net work, in kJ.
(c) the thermal efficiency.
(d) the mean effective pressure, in kPa.

State	T (K)	p (kPa)	u (kJ/kg)
1	350	85	217.67
2	367.4	767.9	486.77
3	960	2005	725.20
4	458.7	127.8	329.01

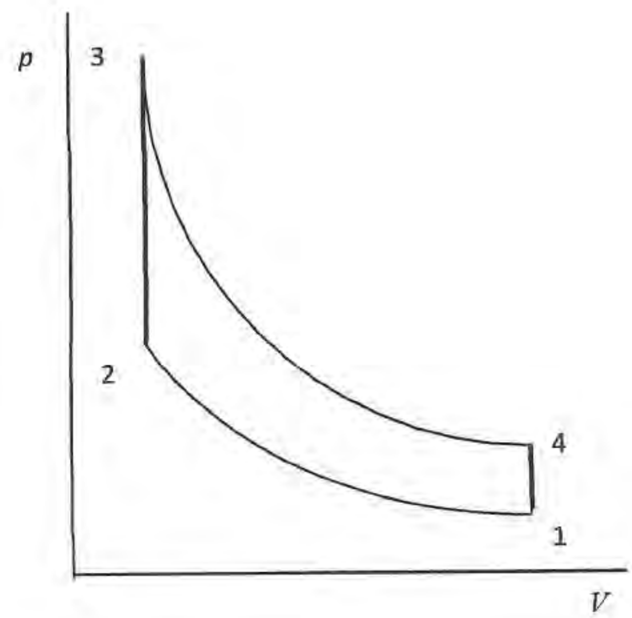
KNOWN: An air-standard Otto cycle operates with property data given at principal states.

FIND: Determine (a) the heat addition and heat rejection, (b) the net work, (c) the thermal efficiency, and (d) the mean effective pressure.

SCHEMATIC AND GIVEN DATA:



See table for other data.



ENGINEERING MODEL:

1. Air, modeled as an ideal gas, is the system.
2. The compression and expansion processes are adiabatic.
3. Kinetic and potential energy effects are negligible

Problem 9.11 (Continued) – Page 2

ANALYSIS:

(a) The heat addition is determined is determined by using an energy balance for process 2-3.

$$Q_{in} = Q_{23} = m(u_3 - u_2) = (0.002\text{ kg})(725.02 - 486.77)\text{ kJ/kg} = \mathbf{0.4765\ kJ}$$

Similarly, the heat rejection is determined by using an energy balance for process 4-1.

$$Q_{out} = |Q_{41}| = m(u_4 - u_4) = (0.002)(329.01 - 217.67) = \mathbf{0.2227\ kJ}$$

(b) The net work is

$$W_{cycle} = Q_{in} - Q_{out} = 0.4765 - 0.2227 = \mathbf{0.2538\ kJ}$$

(c) The thermal efficiency

$$\eta = W_{cycle}/Q_{in} = \mathbf{0.533\ (53.3\%)}$$

(d) To determine the mean effective pressure, first find V_1 and V_2.

$$V_1 = \frac{mRT_1}{p_1} = \left[\frac{(0.002\text{ kg})\left(\frac{8.314\text{ kJ}}{28.97\text{ kg}}\right)(305\text{ K})}{(85\text{ kPa})}\right]\left|\frac{1\text{ kPa}}{10^3\text{ N/m}^2}\right|\left|\frac{10^3\text{N}\cdot\text{m}}{1\text{ kJ}}\right| = 2.06 \times 10^{-3}\, m^3$$

Similarly

$$V_2 = \frac{mRT_2}{p_2} = \left[\frac{(0.002)\left(\frac{8.314}{28.97}\right)(367.4)}{(767.9)}\right] = 2.75 \times 10^{-4}\text{ m}^3$$

Thus

$$mep = \frac{W_{cycle}}{(V_1)(1-\frac{V_2}{V_1})} = \left[\frac{0.2538\text{ kJ}}{(2.06 \times 10^{-3} m^3)(1-\frac{0.000275}{0.00206})}\right]\left|\frac{10^3\text{N}\cdot\text{m}}{1\text{ kJ}}\right|\left|\frac{1\text{ kPa}}{10^3\text{ N/m}^2}\right| = \mathbf{142.2\ kPa}$$

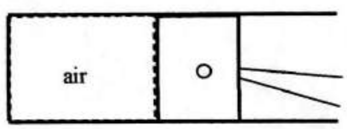
9.12 Consider a cold air-standard Otto cycle. Operating data at principal states in the cycle are given in table below. The states are numbered as in Fig. 9.3. The heat rejection from the cycle is 86 Btu per lb of air. Assuming $c_v = 0.172$ Btu/lb·°R, determine
(a) the compression ratio.
(b) the net work per unit mass of air, in Btu/lb.
(c) the thermal efficiency.
(d) the mean effective pressure, in lbf/in.^{2}

State	T (°R)	p (lbf/in.^{2})
1	500	47.50
2	1204.1	1030
3	2408.2	2060
4	1000	95

KNOWN: A cold air-standard Otto cycle operates with property data given at principal states.

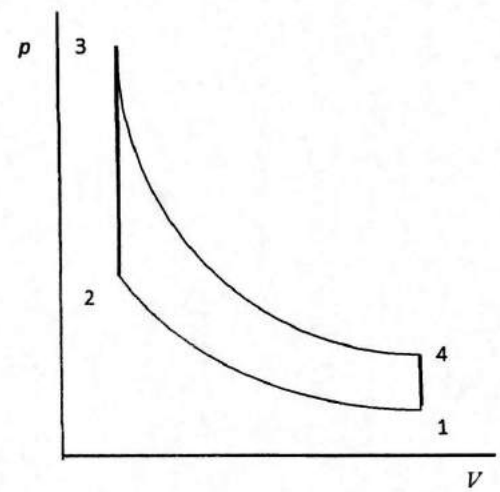
FIND: Determine (a) the compression ratio, (b) the net work per unit mass, (c) the thermal efficiency, and (d) the mean effective pressure.

SCHEMATIC AND GIVEN DATA:



$Q_{41}/m = 86$ Btu/lb

See table for other data.



ENGINEERING MODEL:
1. Air, modeled as an ideal gas, is the system.
2. The specific heats are constant, with $c_v = 0.172$ Btu/lb·°R.
3. The compression and expansion processes are adiabatic.
4. Kinetic and potential energy effects are negligible

Problem 9.12 (Continued) – Page 2

ANALYSIS:

(a) The compression ratio is $r = V_1/V_2$. With $V = mRT/p$ for an ideal gas

$$r = \frac{mRT_1/p_1}{mRT_2/p_2} = \frac{T_1}{T_2}\frac{p_2}{p_1} = \frac{500}{1204.1}\frac{1030}{47.50} = 9$$

(b) The net work is

$$W_{cycle}/m = Q_{23}/m - Q_{41}/m = c_v(T_3 - T_2) - Q_{41}/m$$

$$= (0.172\ \text{Btu/lb}\cdot{}^\circ\text{R})(2408.2 - 1204.1)\ {}^\circ\text{R} - 86\ \text{Btu/lb}$$

$$= 207.1 - 86 = \mathbf{121.1\ Btu}$$

(c) The thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = \frac{W_{cycle}/m}{c_v(T_3-T_2)} = (121.1)/(207.1) = \mathbf{0.585\ (58.5\%)}$$

(d) To determine the mean effective pressure, first find v_1.

$$v_1 = \frac{RT_1}{p_1} = \left[\frac{\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(500\ {}^\circ\text{R})}{(47.50\frac{\text{lbf}}{\text{in}^2})}\right]\left|\frac{1\ \text{ft}^2}{144\ \text{in}^2/\text{ft}^2}\right| = 3.898\ \text{ft}^3/\text{lb}$$

Thus

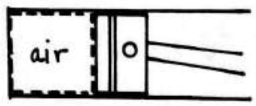
$$mep = \frac{W_{cycle}}{(V_1)(1-\frac{1}{r})} = \left[\frac{121.1\ \text{Btu/lb}}{(3.898\ \text{ft}^3/\text{lb})(1-\frac{1}{9})}\right]\left|\frac{1\ \text{ft}^2}{144\ \text{in}^2/\text{ft}^2}\right|\left|\frac{778\ \text{ft}\cdot\text{lbf}}{1\ \text{Btu}}\right| = \mathbf{188.8\ lbf/in.^2}$$

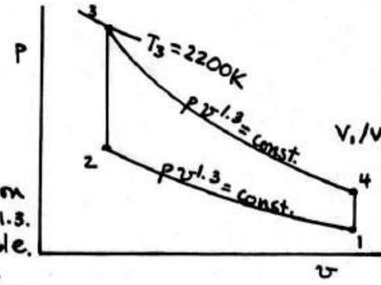
PROBLEM 9.13

KNOWN: An air-standard Otto cycle is modified by replacing the isentropic compression and expansion processes with polytropic processes having $n = 1.3$. The compression ratio, state at the beginning of compression, and the maximum cycle temperature are all known.

FIND: Determine (a) the heat transfer and work, per unit mass, for each process in the cycle, (b) the thermal efficiency, (c) the mep.

SCHEMATIC & GIVEN DATA:





$V_1/V_2 = 10$

$T_1 = 310K$

$P_1 = 1 bar$

ENGR. MODEL: (1) The air is the closed system. (2) The compression and expansion processes are polytropic, each with $n = 1.3$. (3) All processes are internally reversible. (4) The air behaves as an ideal gas. (5) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states. With $T_1 = 310K$, $u_1 = 221.25$ kJ/kg from Table A-22. Next, using Eq. 3.56 for the polytropic compression

$$T_2 = \left(\frac{V_1}{V_2}\right)^{n-1} T_1 = (10)^{.3}(310) = 618.53\ K$$

Thus, $u_2 = 448.96$ kJ/kg. Also, at $T_3 = 2200$ K; $u_3 = 1872.4$ kJ/kg. For the polytropic expansion process

$$T_4 = \left(\frac{V_3}{V_4}\right)^{n-1} T_3 = \left(\frac{V_2}{V_1}\right)^{n-1} T_3 = 1102.6\ K$$

Thus, $u_4 = 847.60$ kJ/kg

(1) (a) Analyzing each process, first determine W_{12}/m using Eq. 3.57

$$\frac{W_{12}}{m} = \int_1^2 p\,dv = \frac{R(T_2 - T_1)}{1-n}$$
$$= \frac{\left(\frac{8.314}{28.97}\frac{kJ}{kg \cdot K}\right)(618.53 - 310)K}{(1 - 1.3)} = -295.1\ kJ/kg \longleftarrow W_{12}/m$$

and, applying an energy balance

$$\frac{Q_{12}}{m} = (u_2 - u_1) + \frac{W_{12}}{m} = (448.96 - 221.25) + (-295.1) = -67.39\ kJ/kg \longleftarrow Q_{12}/m$$

Next, for process 2-3; $W_{23} = 0 \longleftarrow W_{23}/m$

$$\frac{Q_{23}}{m} = u_3 - u_2 = 1872.4 - 448.96 = 1423.4\ kJ/kg \longleftarrow Q_{23}/m$$

Again, using Eq. 3.57 for the polytropic expansion

PROBLEM 9.13 (Cont'd)

$$W_{34} = \int_3^4 p\,dV \Rightarrow \frac{W_{34}}{m} = \frac{R(T_4 - T_3)}{1-n}$$

$$\frac{W_{34}}{m} = \frac{\left(\frac{8.314}{28.97}\cdot\frac{kJ}{kg\cdot K}\right)(1102.6 - 2200)K}{(1-1.3)} = 1049.8 \text{ kJ/kg} \longleftarrow \quad W_{34}/m$$

And, applying an energy balance

$$\frac{Q_{34}}{m} = (u_4 - u_3) + \frac{W_{34}}{m}$$

$$= (847.60 - 1872.4) + 1049.8 = 25 \text{ kJ/kg} \longleftarrow \quad Q_{34}/m$$

Finally, for the heat rejection process; $W_{41} = 0$ ⟵ W_{41}/m
and

$$\frac{Q_{41}}{m} = (u_1 - u_4) = 221.25 - 847.60 = -626.35 \text{ kJ/kg} \longleftarrow \quad Q_{41}/m$$

(b) The thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{in}/m}$$

The net work per unit mass is

$$\frac{W_{cycle}}{m} = \frac{W_{12}}{m} + \frac{W_{34}}{m} = -295.1 + 1049.8 = 754.7 \text{ kJ/kg}$$

and the heat added is

$$\frac{Q_{in}}{m} = \frac{Q_{23}}{m} + \frac{Q_{34}}{m} = 1423.4 + 25 = 1448.4 \text{ kJ/kg}$$

Thus

$$\eta = \frac{754.7}{1448.4} = 0.521 \; (52.1\%) \longleftarrow \quad \eta$$

(c) $$\text{mep} = \frac{W_{cycle}/m}{v_1 - v_2} = \frac{W_{cycle}/m}{v_1[1 - v_2/v_1]}$$

Applying the ideal gas model equation of state: $v_1 = RT_1/p_1$, we get

$$v_1 = 0.89 \text{ m}^3/\text{kg}$$

Then

$$\text{mep} = \frac{754.7 \text{ kJ/kg}}{\left(0.89 \frac{m^3}{kg}\right)\left[1 - \frac{1}{10}\right]}\left|\frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}}\right|\left|\frac{1 \text{ bar}}{10^5 \text{ N/m}^2}\right| = 9.42 \text{ bar} \longleftarrow \quad \text{mep}$$

---

1. Here we use the sign convention for work and heat — not magnitudes.
As a check, note that

$$\frac{Q_{cycle}}{m} = -67.39 + 1423.4 + 25 - 626.35 = 754.7 \text{ kJ/kg}$$

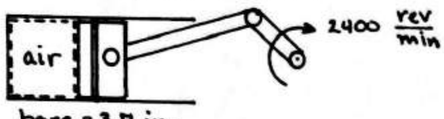
Thus, $Q_{cycle}/m = W_{cycle}/m$, as expected.

PROBLEM 9.14

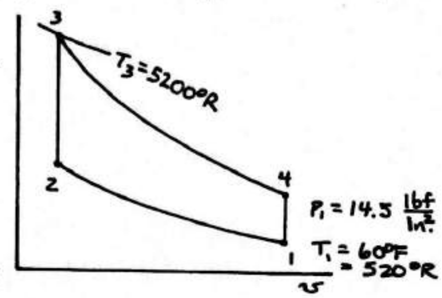
KNOWN: The bore, stroke, clearance volume, and speed of crankshaft rotation of a four-stroke, four-cylinder internal combustion engine are known. The processes within the cylinders are modeled by an Otto cycle with a known state at the beginning of compression and maximum cycle temperature.

FIND: Determine the net work per cycle and the power developed by the engine.

SCHEMATIC & GIVEN DATA:



bore = 3.7 in.
stroke = 3.4 in.
$V_1 = 0.16 V_2$



Engr. Model: Assumptions of Otto cycle are the same as in Example 9.1

ANALYSIS: Using the given data,

$$V_1 - V_2 = \frac{\pi\,(\text{bore})^2}{4} \times (\text{stroke}) = \frac{\pi\,(3.7)^2(3.4)}{4}\,in^3 \left(\frac{1\,ft^3}{12^3\,in^3}\right) = 0.02116\ ft^3$$

With $V_2 = 0.16 V_1$, $V_1 = 0.02519\ ft^3$. Thus, the mass of air is

$$m = \frac{P_1 V_1}{R T_1} = \frac{(14.5\ lbf/in^2)(0.02519\ ft^3)}{\left(\frac{1545}{28.97}\frac{ft\cdot lbf}{lb\cdot °R}\right)(520°R)}\left(\frac{144\ in^2}{1\ ft^2}\right) = 0.001896\ lb$$

Next, fix each of the principal states of the cycle. From Table A-22E at $T_1 = 520°R$; $u_1 = 88.62$ Btu/lb, $v_{r1} = 158.58$. Further, the compression is $V_1/V_2 = 6.25$. Thus, for the isentropic compression

$$v_{r2} = \frac{V_2}{V_1} v_{r1} = \left(\frac{1}{6.25}\right) 158.58 = 25.373$$

Thus, $T_2 = 1067.7\ °R$, $u_2 = 184.70$ Btu/lb. At $T_3 = 5200°R$; $u_3 = 1098.0$ Btu/lb and $v_{r3} = 0.1828$. For the isentropic expansion

$$v_{r4} = \frac{V_4}{V_3} v_{r3} = \frac{V_1}{V_2} v_{r3} = 1.1425$$

Thus, $T_4 = 3030.2\ °R$, $u_4 = 591.84$ Btu/lb.

The net work per cycle is

$$W_{cycle} = Q_{23} - Q_{41} = m\left[(u_3 - u_2) - (u_4 - u_1)\right]$$

$$= (.001896\ lb)\left[(1098.0 - 184.70) - (591.84 - 88.62)\right] \frac{Btu}{lb}$$

$$= 0.7775\ Btu \quad \leftarrow W_{cycle}$$

(1) For a four-stroke engine, the cycle is completed once for every two revolutions of the crankshaft. For four cylinders, the net power is

$$\dot{W}_{net} = (4)(1200\ cycles/min)(.7775\ Btu/cycle)\left(\frac{1\ hp}{42.42\ Btu/min}\right) = 88.0\ hp \quad \leftarrow \dot{W}_{net}$$

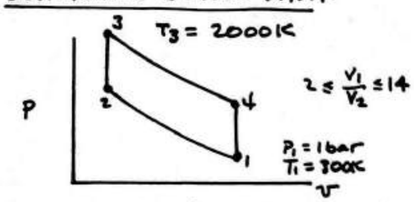
1. See the introductory discussion of Sec. 9.1.

PROBLEM 9.15

KNOWN: For an air-standard Otto cycle, the state at the beginning of compression and the maximum cycle temperature are known.

FIND: Plot the net work per unit of mass, the thermal efficiency, and the mean effective pressure versus compression ratio ranging from 2 to 14.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.1.

ANALYSIS: The following reasoning is used to solve the problem:

1. Given values of p_1, T_1, r, and T_3.
2. Fix state 2 using ideal gas relations and $s(T_1, p_1) = s(T_2, p_2)$. When using Table A-22, this is expressed in terms of $v_{r1}/v_{r2} = v_1/v_2$.
3. Use $v_2 = v_3$ and the ideal gas model to get p_3.
4. Fix state 4 as in step 2; use $s(T_3, p_3) = s(T_4, p_4)$, or $v_{r4}/v_{r3} = v_4/v_3$.

Now, using energy balances, and the fact that $W_{23} = W_{41} = 0$

$$Q_{23} = m(u_3 - u_2) \tag{1}$$

$$Q_{41} = m(u_1 - u_4) \tag{2}$$

Further, for the cycle

$$W_{cycle}/m = Q_{23}/m + Q_{41}/m \tag{3}$$

and

$$\eta = W_{cycle}/Q_{23} \tag{4}$$

Finally

$$v_1 = \frac{RT_1}{p_1} \; ; \; mep = \frac{W_{cycle}}{m\, v_1 (1 - \frac{1}{r})} \tag{5}$$

Sample calculation: $\boxed{r = 5}$ From Table A-22, at $p_1 = 1 bar$, $T_1 = 300K$; $u_1 = 214.07$ kJ/kg, $v_{r1} = 621.2$. Thus

$$v_{r2} = \frac{v_2}{v_1} v_{r1} = \left(\frac{1}{r}\right) v_{r1} = \left(\frac{1}{5}\right)(621.2) = 124.24 \Rightarrow \begin{cases} T_2 = 564.8K \\ u_2 = 408.0K \end{cases}$$

Now, $T_3 = 2000$ K $\Rightarrow u_3 = 1678.7$ kJ/kg, $v_{r3} = 2.776$. Thus, for state 4

$$\frac{v_{r4}}{v_{r3}} = \frac{v_4}{v_3} = \frac{v_1}{v_2} = 5 \Rightarrow v_{r4} = 13.88 \Rightarrow T_4 = 1216.32\text{ K},\; u_4 = 947.82 \text{ kJ/kg}.$$

Thus, with (1) – (5)

$$W_{cycle}/m = (u_3 - u_2) + (u_1 - u_4) = (1678.7 - 408.0) + (214.07 - 947.82)$$
$$= 1270.7 - 733.75 = 536.95 \text{ kJ/kg} \longleftarrow W_{cycle}/m$$

and

$$\eta = \frac{536.95}{1270.7} = 0.4226\ (42.26\%) \longleftarrow \eta$$

Finally

$$v_1 = \frac{\left(\frac{8.314}{28.97} \frac{kJ}{kg \cdot K}\right)(300K)}{(1\ bar)} \left|\frac{1\ bar}{10^5\ N/m^2}\right| \left|\frac{10^3\ N \cdot m}{1\ kJ}\right| = 0.861\ m^3/kg$$

$$mep = \frac{536.95\ kJ/kg}{(0.861\ m^3/kg)(1 - \frac{1}{5})} \left|\frac{10^3\ N \cdot m}{1\ kJ}\right| \left|\frac{1\ bar}{10^5\ N/m^2}\right| = 7.795\ bar \longleftarrow mep$$

PROBLEM 9.15 (Cont'd.) – Page 2

IT solution. The data for the required plots are obtained using IT, as follows:

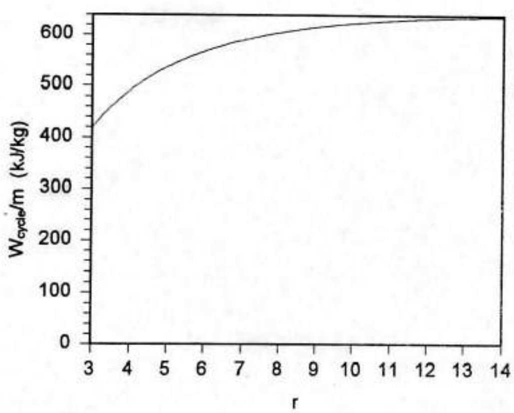
IT Code

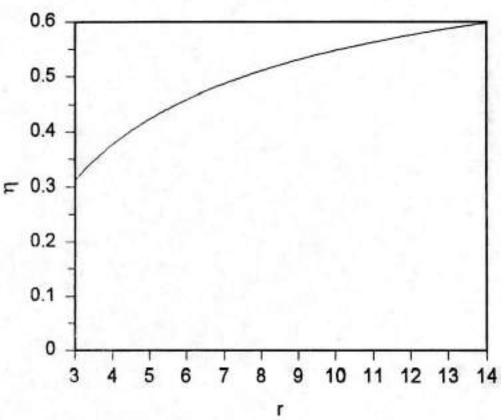
```
p1 = 1  // bar
T1 = 300  // K
r = 5
T3 = 2000  // K
m = 1  //  Assume a unit mass of 1 kg.

v1 = v_TP("Air",T1, p1)
s1 = s_TP("Air", T1, p1)
u1 = u_T("Air", T1)
v2 = v1 / r
v2 = v_TP("Air",T2,p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
u2 = u_T("Air", T2)
v3 = v_TP("Air",T3, p3)
v3 = v2
u3 = u_T("Air", T3)
s3 = s_TP("Air", T3, p3)
v4 = v1
v4 = v_TP("Air",T4, p4)
s4 = s_TP("Air", T4, p4)
s4 = s3
u4 = u_T("Air", T4)

Q23 = m * (u3 - u2)
Q41 = m * (u1 - u4)
Wcycle = Q23 + Q41
eta = Wcycle / Q23
mep = (Wcycle / (m * v1 * (1 - 1/r))) / 100  // bar
```

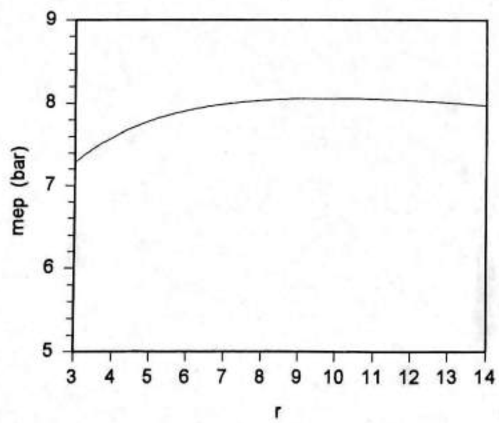
IT Results for r = 5

T_2 = 564.6 K
T_4 = 1215 K
p_3 = 33.33 bar
v1 = 0.861 m^3/kg
Q_{23}/m = 1269 kJ/kg
Q_{41}/m = -732.3 kJ/kg
W_{cycle}/m = 536.3 kJ/kg
η = 0.4227
mep = 7.787 bar

PLOTS:







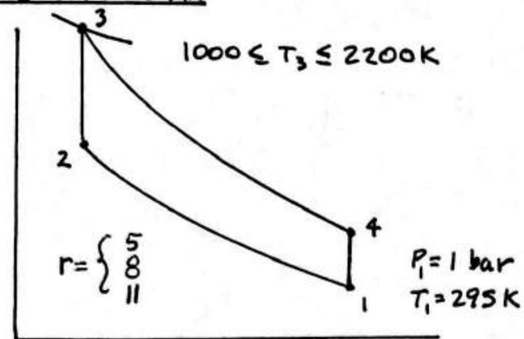
PROBLEM 9.16

KNOWN: An air-standard Otto cycle has known conditions at the beginning of compression. The maximum cycle temperature varies over a given range and the compression ratio takes on specified values.

FIND: Investigate the effect of maximum cycle temperature on the net work per unit mass of air.

SCHEMATIC & GIVEN DATA:

①



ENGINEERING MODEL: See Example 9.1.

ANALYSIS: Sample calculation for $r = 8$, $T_3 = 1000\text{K}$ using Table A-22.

From Table A-22, $u_1 = 210.49$ kJ/kg, $v_{r_1} = 647.9$. For the compression process

$$v_{r_2} = \left(\frac{v_2}{v_1}\right) v_{r_1} = \left(\frac{1}{8}\right)(647.9) = 80.99 \Rightarrow T_2 = 662.74 \text{ K}, u_2 = 483.15 \text{ kJ/kg}$$

Now, at $T_3 = 1000$ K; $u_3 = 758.94$ kJ/kg, $v_{r_3} = 25.17$. For the expansion

$$v_{r_4} = \left(\frac{v_4}{v_3}\right) v_{r_3} = \left(\frac{v_1}{v_2}\right) v_{r_3} = (8)(25.17) = 201.36 \Rightarrow u_4 = 336.50 \text{ kJ/kg}$$

Since $W_{23} = W_{41} = 0$; $Q_{23}/m = (u_3 - u_2)$ and $Q_{41}/m = (u_1 - u_4)$. For the cycle, $W_{cycle} = Q_{cycle}$. Thus

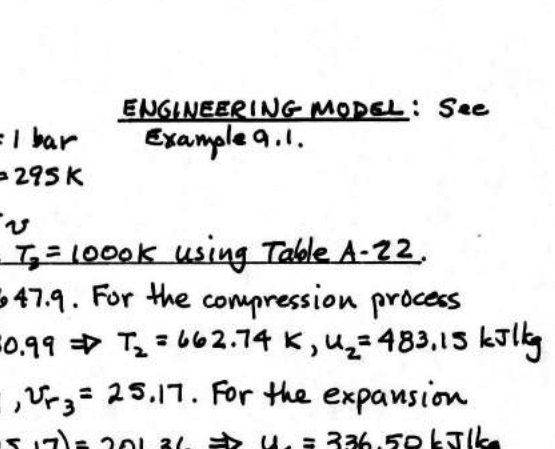
$$W_{cycle}/m = (u_3 - u_2) + (u_1 - u_4) = (758.94 - 483.15) + (210.49 - 336.50)$$
$$= 149.78 \text{ kJ/kg}$$

*IT*Code

```
p1 = 1  // bar
T1 = 295  // K
T3 = 1000  // K
r = 8
v1 = v_TP("Air",T1, p1)
s1 = s_TP("Air", T1, p1)
u1 = u_T("Air", T1)
v2 = v1 / r
v2 = v_TP("Air",T2, p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
u2 = u_T("Air", T2)
v3 = v_TP("Air",T3, p3)
s3 = s_TP("Air", T3, p3)
u3 = u_T("Air", T3)
v3 = v2
v4 = v1
v4 = v_TP("Air",T4, p4)
s4 = s_TP("Air", T4, p4)
s4 = s3
u4 = u_T("Air", T4)
Wcycle = (u3 - u2) + (u1 - u4)
```

IT Results for r = 8 and T3 = 1000K

u_1 = 210.4 kJ/kg
T_2 = 662.8 K
u_2 = 483 kJ/kg
u_3 = 758.6 kJ/kg
T_4 = 468.6 K
u_4 = 336.2 kJ/kg
W_{cycle}/m = 149.8 kJ/kg



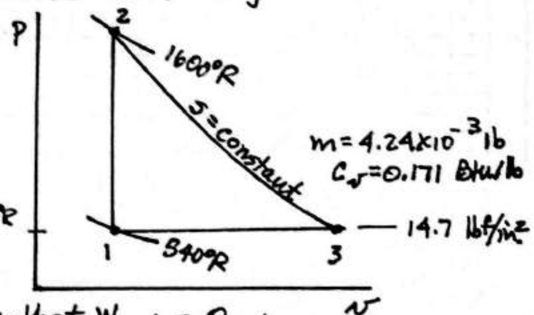
We see from the plot that the net work increases with T_3 at fixed compression ratio. Also, the plots cross each other, indicating the relation of Wcycle to r differs depending on T_3.

PROBLEM 9.17

KNOWN: An air-standard Lenoir cycle has a known state at the beginning of the constant volume heat addition and a known maximum temperature. The mass is given.

FIND: Determine (a) the net work, and (b) the thermal efficiency.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The cycle is modeled as processes in a closed system. (2) The expansion process is isentropic. (3) The air is modeled as an ideal gas with constant specific heats; $c_v = 0.171$ Btu/lb·°R and $c_p = 0.24$ Btu/lb·°R. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: (a) To find the net work, note that $W_{cycle} = Q_{cycle}$ and

$$W_{cycle} = Q_{12} + Q_{31}$$

For process 1-2: $m(u_2-u_1) = Q_{12} - \cancel{W_{12}}^{0}$

$$Q_{12} = m(u_2-u_1) = m c_v (T_2 - T_1)$$
$$= (4.24\times10^{-3}\,\text{lb})(0.171\,\tfrac{\text{Btu}}{\text{lb·°R}})(1600-540)\text{°R}$$
$$= 0.7685 \text{ Btu}$$

For Process 2-3: $T_3 = T_2(p_3/p_2)^{\frac{k-1}{k}}$. $p_2 = (T_2/T_1)p_1 = 43.56$ lbf/in.²

and $T_3 = (1600\text{°R})(14.7/43.56)^{0.2857} = 1173.1$ °R

For Process 3-1: $m(u_1-u_3) = Q_{31} - W_{31}$

$$W_{31} = m\int_3^1 p\,dv = mp(v_1 - v_3)$$

$$Q_{31} = m(u_1-u_3) + mp(v_1-v_3) = m c_p (T_1 - T_3)$$
$$= (4.24\times10^{-3})(0.24)(540-1173.1) = -0.6442 \text{ Btu}$$

Finally $W_{cycle} = Q_{12} + Q_{31} = 0.7685 + (-0.6442) = 0.1243$ Btu ← W_{cycle}

(b) The thermal efficiency is

$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{W_{cycle}}{Q_{12}} = \frac{0.1243}{0.7685} = 0.162\ (16.2\%)$$ ← η

PROBLEM 9.18

Figure P9.18 shows two cold air-standard cycles: 1–2–3–4–1 is an *Atkinson* cycle while 1–2–3–4′–1 is an Otto cycle. The compression ratio of these cycles is $r = v_1/v_2$.

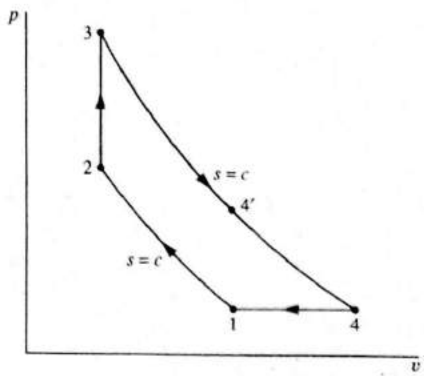


Fig. P9.18

(a) Showing all steps, develop the following **alternative** expressions for the thermal efficiency η_A of **the Atkinson** cycle:

$$\eta_A = 1 - \frac{k(r_p^{1/k} - 1)}{r^{k-1}(r_p - 1)} \qquad \text{(i)}$$

where $r_p = p_3/p_2$, and

$$\eta_A = 1 - k\left(\frac{r_e - r}{r_e^k - r^k}\right) \qquad \text{(ii)}$$

where $r_e = v_4/v_3$.

(b) Which cycle has the greater thermal efficiency, the Atkinson cycle or the Otto cycle? Explain.

ENGINEERING MODEL:

1. Air modeled as an ideal gas undergoes an Atkinson cycle.
2. For the air c_v and c_p are constant and thus the specific heat ratio k.
3. Volume change is the only work mode.
4. Kinetic and potential energy effects are negligible.

Analysis: An energy balance for process 2-3 reads

$$m(u_3 - u_2) = Q_{23} - \cancel{W_{23}}^{\,0} \Rightarrow \frac{Q_{23}}{m} = c_v(T_3 - T_2)$$

An energy balance for process 4-1 reads

$$m(u_1 - u_4) = Q_{41} - W_{41}, \quad \text{where } W_{41} = p(v_1 - v_4)$$

$$\Rightarrow \quad \frac{Q_{41}}{m} = (u_1 - u_4) + p(v_1 - v_4) = h_1 - h_4$$

$$= c_p(T_1 - T_4) \quad (<0)$$

Thus, the thermal efficiency is

$$\boxed{\eta_A = \frac{Q_{23}/m - |Q_{41}/m|}{Q_{23}/m} = 1 - k\frac{(T_4 - T_1)}{(T_3 - T_2)} = 1 - \frac{k\,T_1\left[T_4/T_1 - 1\right]}{T_2\left[T_3/T_2 - 1\right]}}$$

Process 1-2: Eq. 6.44 gives $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} = r^{k-1}$, where $r = V_1/V_2$. (1)

Eq. 6.45 gives $\left(\frac{V_2}{V_1}\right) = \left(\frac{P_1}{P_2}\right)^{1/k}$ (2)

Process 2-3 Since v = constant, the ideal gas equation of state gives

$$\frac{T_3}{T_2} = \frac{P_3}{P_2} = r_p, \quad \text{where } r_p = P_3/P_2. \qquad (3)$$

Process 3-4: Eq. 6.44 gives $\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{k-1} = r_e^{k-1}$, where $r_e = V_4/V_3$. (4)

Eq. 6.45 gives $\frac{V_4}{V_3} = \left(\frac{P_3}{P_4}\right)^{1/k}$ (5)

PROBLEM 9.18 (Continued)

○ Process 4-1: Since p is constant, the ideal gas equation of state gives

$$\frac{T_4}{T_1}=\frac{V_4}{V_1}=\frac{V_4}{V_3}\,\frac{\overset{=V_2}{V_3}}{V_1}=\frac{r_e}{r} \qquad (6)$$

Also, with Eqs. (2) and (5),

$$\frac{T_4}{T_1}=\frac{V_4}{V_3}\,\frac{\overset{=V_2}{V_3}}{V_1}=\frac{V_4}{V_3}\,\frac{V_2}{V_1}=\underbrace{\left(\frac{p_3}{p_4}\right)^{1/k}}_{\text{Eq. 5}}\underbrace{\left(\frac{\overset{=p_4}{p_1}}{p_2}\right)^{1/k}}_{\text{Eq. 2}}=\left(\frac{p_3}{p_4}\right)^{1/k}\left(\frac{p_4}{p_2}\right)^{1/k}$$

$$\frac{T_4}{T_1}=\left(\frac{p_3}{p_2}\right)^{1/k}=r_p^{1/k} \qquad (7)$$

Finally, with Eqs. (4) and (6)

$$T_3=T_4\,r_e^{k-1}=T_1\,\frac{r_e}{r}\,r_e^{k-1}=\frac{T_1\,r_e^{k}}{r} \qquad (8)$$

(a) With (1), (3), and (7), the boxed equation above becomes

$$\eta_A=1-k\left(\frac{T_1}{T_2}\right)\left[\frac{T_4/T_1-1}{T_3/T_2-1}\right]$$

$$=1-\frac{k}{r^{k-1}}\left[\frac{r_p^{1/k}-1}{r_p-1}\right] \longleftarrow (i)$$

With (1), (6), and (8), the boxed equation above becomes

$$\eta_A=1-k\left[\frac{T_4-T_1}{T_3-T_2}\right]$$

$$=1-k\left[\frac{\frac{r_e}{r}T_1-T_1}{\frac{r_e^k}{r}T_1-r^{k-1}T_1}\right]$$

$$=1-k\left[\frac{r_e-r}{r_e^k-r^k}\right] \longleftarrow (ii)$$

(b) Each cycle has the same heat addition: process 2-3. Thus, since the Atkinson cycle has the greater enclosed area, which represents the net work developed per unit of mass, the Atkinson cycle has the greater thermal efficiency.

PROBLEM 9.19

Referring again to Fig. P9.18, let $p_1 = 1$ bar, $T_1 = 300$ K, $r = 8.5$, $Q_{23}/m = 1400$ kJ/kg, and $k = 1.4$

(a) Evaluate the ratio of the thermal efficiency of the Atkinson cycle, 1–2–3–4–1, to the thermal efficiency of the Otto cycle, 1–2–3–4′–1.

(b) Evaluate the ratio of the mean effective pressure of the Atkinson cycle to the mean effective pressure of the Otto cycle.

SOLUTION

Known: A cold air standard Atkinson cycle is executed in a piston-cylinder assembly. Conditions are known at the beginning of the compression process. The heat addition per unit of mass and compression ratio are also known.

Find: Evaluate the ratio of the thermal efficiency of the Atkinson cycle to that of the corresponding Otto cycle. Repeat for the mean effective pressures of the two cycles.

Schematic and Given Data: See Fig. P9.18.

Engineering Model:

1. The air in the piston-cylinder assembly is the closed system.
2. The compression and expansion processes are adiabatic.
3. All processes are internally reversible.
4. The air is modeled as an ideal gas with $k=1.4$.
5. Kinetic and potential energy effects are negligible.

Analysis: (a) For the Atkinson cycle the thermal efficiency is the ratio

① $$\eta_A = \frac{W_{cycle}/m}{Q_{23}/m} = 1 - \frac{|Q_{41}/m|}{Q_{23}/m} = 1 - \frac{(h_4-h_1)}{Q_{23}/m} = 1 - \frac{c_p(T_4-T_1)}{Q_{23}/m} \qquad (1)$$

Using Eqs. 6.44, 6.45,

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{k-1} = (300\text{K})(8.5)^{0.4} = 706.14\text{ K}$$

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{k} = (1\text{ bar})(8.5)^{1.4} = 20.01\text{ bar}$$

An energy balance for process 2-3 reduces to read, $Q_{23}/m = u_3 - u_2$

② $$\therefore\quad T_3 - T_2 = \frac{Q_{23}/m}{c_v} \Rightarrow T_3 - T_2 = \frac{1400\text{ kJ/kg}}{0.717\text{ kJ/kg·K}} \Rightarrow T_3 = 2658.72\text{ K}$$

Since specific volume is constant, the ideal gas model equation of state gives

$$P_3 = P_2\left(\frac{T_3}{T_2}\right) = (20.01\text{ bar})\left(\frac{2658.72\text{K}}{706.14\text{K}}\right) = 75.34\text{ bar}$$

Using Eq. 6.43

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = T_3\left(\frac{P_1}{P_3}\right)^{(k-1)/k} = (2658.72\text{K})\left(\frac{1\text{ bar}}{75.34\text{ bar}}\right)^{0.286} = 772.4\text{ K}$$

PROBLEM 9.19 — Continued (p. 2)

Eq. (1) then gives

(3) $$\eta_A = 1 - \frac{(1.004\ kJ/kg\cdot K)(772.4 - 300)K}{1400\ kJ/kg} = 0.661\ (66.1\%)$$

Using Eq. 9.8, the thermal efficiency of the counterpart Otto cycle is

$$\eta_O = 1 - \frac{1}{r^{k-1}} = 1 - \frac{1}{(8.5)^{0.4}} = 0.575\ (57.5\%)$$

(4) $$\Rightarrow \frac{\eta_A}{\eta_O} = \frac{0.661}{0.575} = 1.15$$ ⟵

(b) The mean effective pressure (mep) is

$$mep = \frac{\begin{pmatrix}\text{net work developed}\\ \text{per unit of mass}\end{pmatrix}}{\begin{pmatrix}\Delta V \text{ for the power}\\ \text{stroke}\end{pmatrix}} = \frac{\eta(Q_{23}/m)}{\begin{pmatrix}\Delta V \text{ for the}\\ \text{power stroke}\end{pmatrix}}$$

$$\Rightarrow (mep)_A = \frac{\eta_A(Q_{23}/m)}{V_4 - \underbrace{V_3}_{=V_2}}, \quad (mep)_O = \frac{\eta_O(Q_{23}/m)}{\underbrace{V_4}_{=V_1} - \underbrace{V_3}_{=V_2}}$$

$$\Rightarrow \frac{(mep)_A}{(mep)_O} = \left(\frac{\eta_A}{\eta_O}\right)\left(\frac{V_1 - V_2}{V_4 - V_2}\right) \qquad (2)$$

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(\frac{8314}{28.97}\frac{N\cdot m}{kg\cdot K}\right)(300K)}{(10^5 N/m^2)} = 0.861\ \frac{m^3}{kg}$$

$$v_2 = v_1/8.5 = 0.101\ m^3/kg$$

$$v_4 = RT_4/P_4 = \frac{\left(\frac{8314}{28.97}\frac{N\cdot m}{kg\cdot K}\right)(772.4)}{(10^5 N/m^2)} = 2.217\ m^3/kg$$

Eq. (2) then gives

(5)(6) $$\frac{(mep)_A}{(mep)_O} = 1.15\left[\frac{\overbrace{0.861 - 0.101}^{0.76}}{\underbrace{2.217 - 0.101}_{2.116}}\right] = 0.413$$ ⟵

---

1. An energy balance for process 4-1 reads, $m[u_1 - u_4] = Q_{41} - W_{41}$ where $W_{41} = p[V_1 - V_4]$ $\Rightarrow \frac{Q_{41}}{m} = (u_1 - u_4) + p(v_1 - v_4)$

$$= h_1 - h_4$$

$$\Rightarrow |Q_{41}/m| = h_4 - h_1 = c_p(T_4 - T_1)$$

2. From Eq. 3.47b, $c_v = \frac{R}{k-1} = \frac{\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}}{(1.4-1)} = 0.717\ \frac{kJ}{kg\cdot K}$

3. $k = \frac{c_p}{c_v} \Rightarrow c_p = k\,c_v = (1.4)(0.717\ \frac{kJ}{kg\cdot K}) = 1.004\ \frac{kJ}{kg\cdot K}$

PROBLEM 9.19 – Continued (p.3)

4. Each cycle has the same heat addition: process 2-3. Thus, since the Atkinson cycle has the greater enclosed area, which represents the net work developed per unit of mass, the Atkinson cycle has the greater thermal efficiency.

5. As the Atkinson cycle has a much greater volume change during the power stroke than the Otto cycle, its mep is significantly less.

6. Using the result of Problem 9.18 to check the thermal efficiency of the Atkinson cycle, we have

$$r = 8.5, \quad r_p = \frac{P_3}{P_2} = 3.765, \quad r_e = \frac{V_4}{V_2} = 21.95$$

Then, with (i) of Problem 9.18

$$\eta_A = 1 - 1.4\frac{\left[(3.765)^{1/1.4} - 1\right]}{(8.5)^{0.4}\left[2.765\right]} = 0.661$$

With (ii) of Problem 9.18

$$\eta_A = 1 - 1.4\left[\frac{21.95 - 8.5}{(21.95)^{1.4} - (8.5)^{1.4}}\right] = 0.661$$

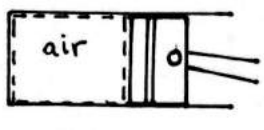
Those values agree, as expected.

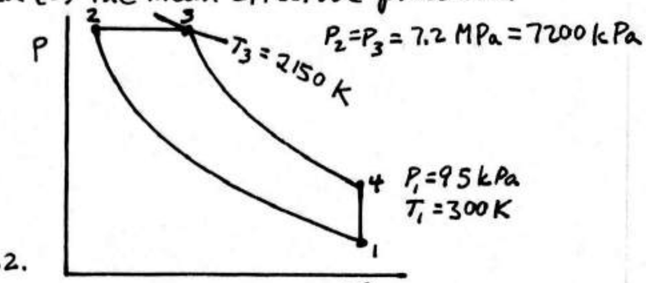
PROBLEM 9.20

KNOWN: An air-standard Diesel cycle has a specified state at the beginning of compression and a known pressure and temperature at the end of heat addition.

FIND: Determine (a) the compression ratio, (b) the cutoff ratio, (c) the thermal efficiency, and (d) the mean effective pressure.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 9.2.

ANALYSIS: Begin by fixing each principal state in the cycle (Table A-22).

State 1: $T_1 = 300$ K, $p_1 = 95$ kPa $\Rightarrow u_1 = 214.07$ kJ/kg, $v_{r1} = 621.2$, $p_{r1} = 1.3860$

State 2: For the isentropic compression

$$p_{r2} = p_{r1}\left(\frac{p_2}{p_1}\right) = (1.3860)\left(\frac{7200}{95}\right) = 105.04$$

Thus, $T_2 = 979.6$ K, $v_{r2} = 26.793$, $h_2 = 1022.82$ kJ/kg

State 3: $T_3 = 2150$ K, $p_3 = 7200$ kPa $\Rightarrow h_3 = 2440.3$ kJ/kg, $v_{r3} = 2.175$

State 4: For the isentropic expansion

$$\frac{v_4}{v_3} = \frac{v_1}{v_2}\cdot\frac{v_2}{v_3} = \frac{v_1}{v_2}\cdot\frac{T_2}{T_3} = \frac{v_{r1}}{v_{r2}}\cdot\frac{T_2}{T_3} = \frac{621.2}{26.793}\cdot\frac{979.6}{2150} = 10.56$$

$$v_{r4} = \frac{v_4}{v_3}\cdot v_{r3} = 22.98 \Rightarrow T_4 = 1031\text{ K},\ u_4 = 785.75\text{ kJ/kg}$$

(a) The compression ratio is

$$r = \frac{v_1}{v_2} = \frac{v_{r1}}{v_{r2}} = \frac{621.2}{26.793} = 23.19 \longleftarrow r$$

(b) The cutoff ratio is

$$r_c = \frac{v_3}{v_2} = \frac{T_3}{T_2} = \frac{2150}{979.6} = 2.19 \longleftarrow r_c$$

(c) The thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = \frac{(h_3 - h_2) - (u_4 - u_1)}{h_3 - h_2}$$

$$= \frac{(2440.3 - 1022.82) - (785.75 - 214.07)}{(2440.3 - 1022.82)}$$

$$= \frac{845.80}{1417.48} = 0.597\ (59.7\%) \longleftarrow \eta$$

PROBLEM 9.20 (Cont'd.) - Page 2

(d) The mean effective pressure is given as

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}/m}{v_1(1 - v_2/v_1)}$$

Evaluating v_1

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(\frac{8.314}{28.97} \frac{kJ}{kg \cdot K}\right)(300\ K)}{(95\ kPa)} \left|\frac{1\ kPa}{10^3\ N/m^2}\right| \left|\frac{10^3\ N \cdot m}{1\ kJ}\right|$$

$$= 0.9063\ m^3/kg$$

Thus

$$mep = \frac{(845.80\ kJ/kg)}{(0.9063\ \frac{m^3}{kg})(1 - 1/23.19)} \left|\frac{10^3\ N \cdot m}{1\ kJ}\right| \left|\frac{1\ kPa}{10^3\ N/m^2}\right|$$

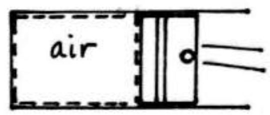
$$= 975\ kPa$$ mep

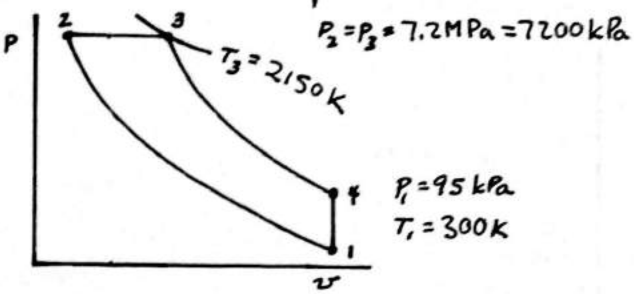
PROBLEM 9.21

KNOWN: An air-standard Diesel cycle has a specified state at the beginning of compression and a known pressure and temperature at the end of heat addition.

FIND: Determine (a) the compression ratio, (b) the cutoff ratio, (c) the thermal efficiency, and (d) the mean effective pressure.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 9.2. Also, assume constant specific heats evaluated at 300K.

ANALYSIS: First, determine the temperature at each principal state of the cycle. $T_1 = 300\ K$, $T_3 = 2150\ K$ are given.

State 2: For the isentropic compression, with $k = 1.4$ from Table A-20

$$T_2 = \left(\frac{P_2}{P_1}\right)^{k-1/k} T_1 = 1033.1\ K$$

State 4: Similarly, for the expansion

$$\frac{v_4}{v_3} = \frac{v_1}{v_2}\cdot\frac{v_2}{v_3} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{k-1}}\cdot\frac{T_2}{T_3} = 10.57$$

and

$$T_4 = \left(\frac{v_3}{v_4}\right)^{k-1} T_3 = 837.0\ K$$

(a) The compression ratio is: $r = \frac{v_1}{v_2} = \left(\frac{T_2}{T_1}\right)^{1/k-1} = 22.01$ ← r

(b) The cutoff ratio is: $r_c = \frac{v_3}{v_2} = \frac{T_3}{T_2} = 2.081$ ← r

(c) The thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = \frac{c_p(T_3 - T_2) - c_v(T_4 - T_1)}{c_p(T_3 - T_2)}$$

With $c_v = .718\ kJ/kg\cdot K$ and $c_p = 1.005\ kJ/kg\cdot K$ from Table A-20

$$\eta = \frac{(1.005)(2150 - 1033.1) - (.718)(837.0 - 300)}{(1.005)(2150 - 1033.1)} = \frac{736.9}{1122.5}$$

$$= 0.657\ (65.7\%)$$ ← η

(d) The mean effective pressure is given as

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}}{v_1(1 - v_2/v_1)}$$

Evaluating v_1

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(300\ K)}{(95\ kPa)}\left|\frac{1\ kPa}{10^3\ N/m^2}\right|\left|\frac{10^3\ N\cdot m}{1\ kJ}\right|$$

$$= 0.9063\ m^3/kg$$

PROBLEM 9.21 (Cont'd) - Page 2

Thus

$$mep = \frac{(763.9 \text{ kJ/kg})}{(0.9063 \text{ m}^3/\text{kg})(1 - 1/22.01)} \left| \frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}} \right| \left| \frac{1 \text{ kPa}}{10^3 \text{ N/m}^2} \right|$$

$$= 883.0 \text{ kPa} \longleftarrow \underline{\quad mep}$$

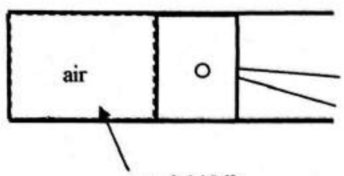
These results can be compared to those of Problem 9.20 to see some effects of the assumption of constant specific heats.

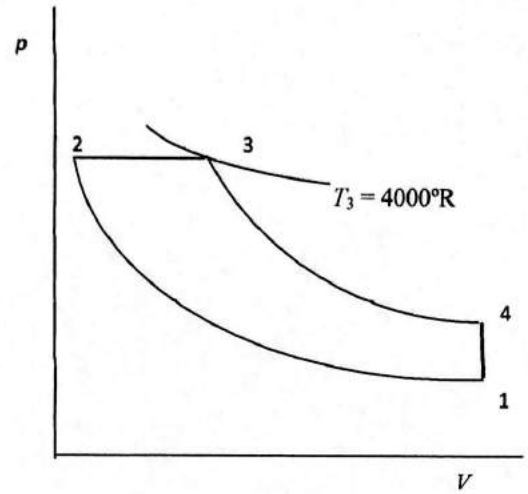
9.22 Consider a cold air-standard Diesel cycle. At the beginning of compression, $p_1 = 14.0$ lbf/in.^{2} and $T_1 = 520$°R. The mass of air is 0.145 lb and the compression ratio is 17. The maximum temperature in the cycle is 4000°R. Determine
(a) the heat addition, in Btu.
(b) the thermal efficiency.
(c) the cutoff ratio.

KNOWN: An air-standard Diesel cycle has a known compression ratio and a specified state at the beginning of compression. The mass and the maximum cycle temperature are given.

FIND: Determine (a) the heat addition, (b) the thermal efficiency, and (c) the cutoff ratio.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL: See Example 9.2.

ANALYSIS: Begin by fixing each principal state of the cycle (Table A-22E).

State 1: $T_1 = 520°\text{R} \rightarrow u_1 = 88.62$ Btu/lb, $v_{r2} = 158.58$

State 2: For the isentropic compression

$$v_{r2} = (V_2/V_1) \cdot v_{r1} = (1/17)(158.58) = 9.3282$$

Thus, interpolating in the table: $T_2 = 1534.5$°R, $h_2 = 378.32$ Btu/lb

State 3: $T_3 = 4000°\text{R} \rightarrow h_3 = 1088.3$ Btu/lb, $v_{r3} = 0.4518$

State 4: For the isentropic expansion

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$$(V_4/V_3) = (V_1/V_2)\cdot(V_2/V_3) = (V_1/V_2)\cdot(T_2/T_3) = (17)\cdot(1534.5/4000) = 6.522$$

and

$$v_{r4} = (V_4/V_3)\cdot v_{r4} = 2.9466$$

Thus, interpolating in the table: $T_4 = 2253.7°R$, $u_4 = 421.25$ Btu/lb

(a) The heat addition is determined from an energy balance on Process 2-3, as follows.

$$Q_{23} = m(u_3 - u_2) + W_{23} = m(u_3 - u_2) + mp_2(v_3 - v_2) = m(h_3 - h_2)$$

Inserting values $\quad Q_{23} = (0.145 \text{ lb})(1088.3 - 378.32) \text{ Btu/lb} = \mathbf{102.9\ Btu}$

(b) To determine the thermal efficiency, first evaluate the net work of the cycle.

$$W_{cycle} = Q_{cycle} = Q_{23} - Q_{41} = Q_{23} - m(u_4 - u_1)$$
$$= 102.9 - (0.145)(421.25 - 88.62) = 54.67 \text{ Btu}$$

Thus, the thermal efficiency is

$$\eta = W_{cycle}/Q_{23} = 54.67/102.9 = \mathbf{0.531\ (53.1\%)}$$

(c) Since $p_2 = p_3$, the cutoff ratio is

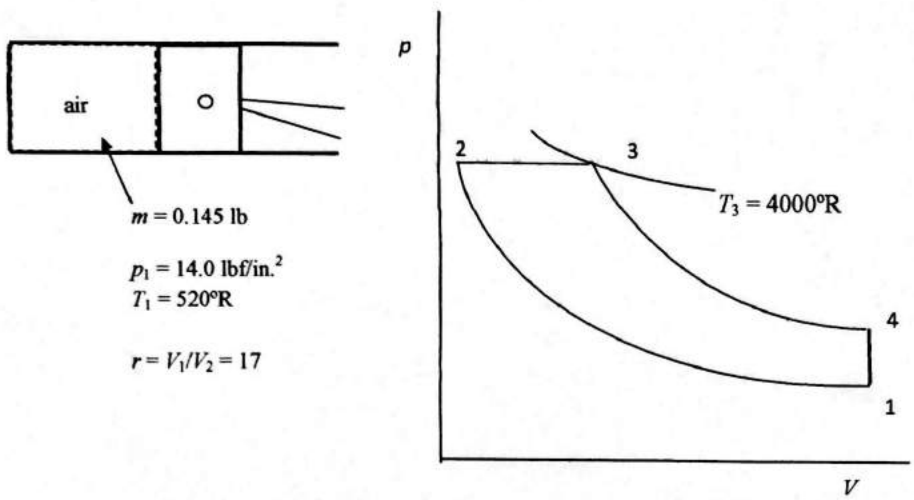
$$r_c = V_3/V_2 = T_3/T_2 = 4000/1534.5 = \mathbf{2.61}$$

9.23 Solve Problem 9.22 on a cold air-standard basis with specific heats evaluated at 520°R.

KNOWN: An air-standard Diesel cycle has a known compression ratio and a specified state at the beginning of compression. The mass and the maximum cycle temperature are given.

FIND: Determine (a) the heat addition, (b) the thermal efficiency, and (c) the cutoff ratio.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: See Example 9.2. Also, the specific heats are constant evaluated at 300K.

ANALYSIS: First, determine the temperature at each principal state of the cycle (Table A-22E). $T_1 = 520$°R and $T_3 = 4000$°R are given.

State 2: For the isentropic compression with $k = 1.401$ from Table A-20E

$$T_2 = (V_1/V_2)^{k-1}\, T_1 = (17)^{0.401}\,(520) = 1619.6\ \text{°R}$$

State 3: Similarly, for the isentropic expansion

$$V_4/V_3 = (V_1/V_2)(V_2/V_3) = (r)(T_2/T_3) = (17)(1619.6/4000) = 6.8833$$

and

$$T_4 = (V_3/V_4)^{k-1}\, T_3 = (1/6.8833)^{0.401}\,(4000) = 1845.5\text{°R}$$

(a) The heat addition is determined from an energy balance on Process 2-3, as follows.

$$Q_{23} = m(u_3 - u_2) + W_{23} = m(u_3 - u_2) + mp_2(v_3 - v_2) = m(h_3 - h_2) = mc_p(T_3 - T_2)$$

Problem 9.23 (Continued) – Page 2

With $c_p = 0.240$ from Table A-20E

$$Q_{23} = (0.145\text{ lb})(0.240\text{ Btu/lb}\cdot°\text{R})(4000 - 1619.6)\ °\text{R} = \mathbf{82.84\ Btu}$$

(b) To determine the thermal efficiency, first evaluate the net work of the cycle.

$$W_{\text{cycle}} = Q_{\text{cycle}} = Q_{23} - Q_{41} = Q_{23} - m(u_4 - u_1) = Q_{23} - mc_v(T_4 - T_1)$$

With $c_v = 0.171$ from Table A-20E

$$W_{\text{cycle}} = 82.84 - (0.145)(0.171)(1845.5 - 520) = 49.97\text{ Btu}$$

Thus, the thermal efficiency is

$$\eta = W_{\text{cycle}}/Q_{23} = 49.97/82.84 = \mathbf{0.603\ (60.3\%)}$$

(c) Since $p_2 = p_3$, the cutoff ratio is

$$r_c = V_3/V_2 = T_3/T_2 = 4000/1619.6 = \mathbf{2.47}$$

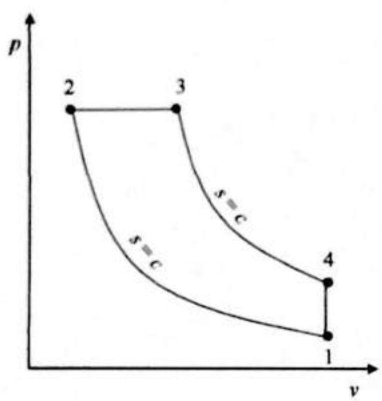
9.24 Consider an air-standard Diesel cycle. Operating data at principal states in the cycle are given in table below. The states are numbered as in Fig. 9.5. Determine
(a) the cut-off ratio.
(b) the heat addition per unit mass, in kJ/kg.
(c) the net work per unit mass, in kJ/kg.
(d) the thermal efficiency.

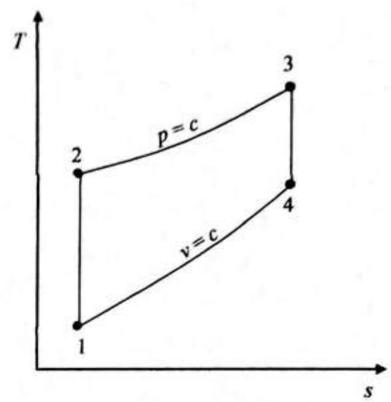
State	T (K)	p (kPa)	u (kJ/kg)	h (kJ/kg)
1	380	100	271.69	380.77
2	1096.6	5197.6	842.40	1157.18
3	1864.2	5197.6	1548.47	2082.96
4	875.2	230.1	654.02	905.26

KNOWN: An air-standard Diesel cycle operates with property data given at principal states.

FIND: Determine (a) the cut-off ratio, (b) the heat addition per unit mass, (c) the net work per unit mass, and (d) the thermal efficiency.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL:
1. Air, modeled as an ideal gas, is the system.
2. The compression and expansion processes are adiabatic.
3. Kinetic and potential energy effects are negligible.

ANALYSIS: (a) The cut-off ratio can be determined as follows. For the constant pressure process, $p_2 = p_3$. Noting that for an ideal gas, $p = RT/v$

$$\frac{RT_2}{v_2} = \frac{RT_3}{v_3}$$

Since $r_c = v_3/v_2$

$$r_c = \frac{v_3}{v_2} = \frac{T_3}{T_2} = \frac{1864.2\,\text{K}}{1096.6\,\text{K}} = \underline{\mathbf{1.70}}$$

(b) The heat addition occurs during process 2-3. Thus, noting that $W_{23} = mp(v_3 - v_2)$

$$m(u_3 - u_2) = Q_{23} - W_{23} = Q_{23} - mp(v_3 - v_2)$$

Thus

$$Q_{23}/m = (u_3 - u_2) + p(v_3 - v_2) = h_3 - h_2$$

Inserting values

$$\frac{Q_{23}}{m} = h_3 - h_2 = 2082.96\frac{\text{kJ}}{\text{kg}} - 1157.18\frac{\text{kJ}}{\text{kg}} = \underline{\mathbf{925.78\ kJ/kg}}$$

(c) The net work per unit mass can be determined from the net heat transfer per unit mass

$$\frac{W_{\text{cycle}}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m}$$

Applying the closed system energy balance to process 4-1

$$\frac{Q_{\text{out}}}{m} = \left|\frac{Q_{41}}{m}\right| = u_4 - u_1 = 654.02\frac{\text{kJ}}{\text{kg}} - 271.69\frac{\text{kJ}}{\text{kg}} = 382.33\ \text{kJ/kg}$$

Solving for net work per unit mass gives

$$\frac{W_{\text{cycle}}}{m} = 925.78\frac{\text{kJ}}{\text{kg}} - 382.33\frac{\text{kJ}}{\text{kg}} = \underline{\mathbf{543.45\ kJ/kg}}$$

(d) Thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}/m}{Q_{23}/m} = \frac{543.45\frac{\text{kJ}}{\text{kg}}}{925.78\frac{\text{kJ}}{\text{kg}}} = \underline{\mathbf{0.5870\ (58.70\%)}}$$

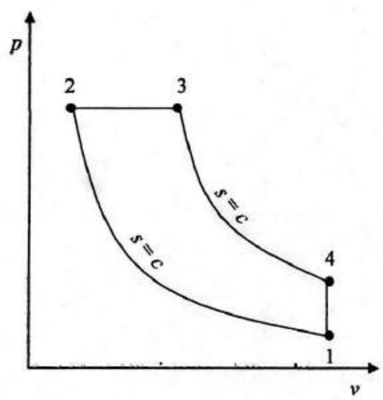
9.25 Consider a cold air-standard Diesel cycle. Operating data at principal states in the cycle are given in the table below. The states are numbered as in Fig. 9.5. For $k = 1.4$, $c_v = 0.718$ kJ/(kg·K), and $c_p = 1.005$ kJ/(kg·K), determine
(a) the heat transfer per unit mass and work per unit mass for each process, in kJ/kg, and the cycle thermal efficiency.
(b) the exergy transfers accompanying heat and work for each process, in kJ/kg. Define and evaluate an exergetic efficiency for the cycle. Let $T_0 = 300$ K and $p_0 = 100$ kPa.

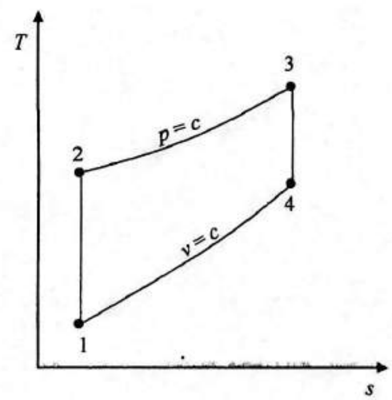
State	T (K)	p (kPa)	v (m^3/kg)
1	340	100	0.9758
2	1030.7	4850.3	0.06098
3	2061.4	4850.3	0.1220
4	897.3	263.9	0.9758

KNOWN: A cold air-standard Diesel cycle operates with property data given at principal states.

FIND: Determine (a) heat transfer per unit mass and work per unit mass for each process and the cycle thermal efficiency (b) exergy transfers accompanying heat and work for each process. Define and evaluate an exergetic efficiency for the cycle.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL:
1. Air, modeled as an ideal gas, is the system.
2. Specific heats are constant.
3. The compression and expansion processes are adiabatic.
4. All processes are internally reversible.
5. Kinetic and potential energy effects are negligible.
6. Let $T_0 = 300$ K and $p_0 = 100$ kPa.

Problem 9.25 (Continued) – Page 2

ANALYSIS: (a) The heat transfer per unit mass and work per unit mass for each process are determined from a closed system energy balance for each process.

Process 1-2 (isentropic compression)

$$\frac{Q_{12}}{m} = \underline{\mathbf{0}} \text{ (adiabatic process)}$$

$$\frac{W_{12}}{m} = u_1 - u_2 = c_v(T_1 - T_2) = \left(0.718\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(340\text{ K} - 1030.7\text{ K}) = \underline{\mathbf{-495.9\ kJ/kg}}$$

The negative sign indicates work is done on the system.

Process 2-3 (constant pressure heat addition)

$$\frac{Q_{23}}{m} = h_3 - h_2 = c_p(T_3 - T_2) = \left(1.005\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(2061.4\text{ K} - 1030.7\text{ K}) = \underline{\mathbf{1035.9\ kJ/kg}}$$

The positive sign indicates heat transfer is to the system.

$$\frac{W_{23}}{m} = \frac{Q_{23}}{m} + (u_2 - u_3) = \frac{Q_{23}}{m} + c_v(T_2 - T_3)$$

$$\frac{W_{23}}{m} = 1035.9\frac{\text{kJ}}{\text{kg}} + \left(0.718\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(1030.7\text{ K} - 2061.4\text{ K}) = \underline{\mathbf{295.9\ kJ/kg}}$$

The positive sign indicates work is done by the system.

Process 3-4 (isentropic expansion)

$$\frac{Q_{34}}{m} = \underline{\mathbf{0}} \text{ (adiabatic process)}$$

$$\frac{W_{34}}{m} = u_3 - u_4 = c_v(T_3 - T_4) = \left(0.718\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(2061.4\text{ K} - 897.3\text{ K}) = \underline{\mathbf{835.8\ kJ/kg}}$$

The positive sign indicates work is done by the system.

Problem 9.25 (Continued) – Page 3

Process 4-1 (constant volume heat rejection)

$$\frac{W_{41}}{m} = \underline{\mathbf{0}} \text{ (constant volume process)}$$

$$\frac{Q_{41}}{m} = u_1 - u_4 = c_v(T_1 - T_4) = \left(0.718 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(340\text{ K} - 897.3\text{ K}) = \underline{\mathbf{-400.1\ kJ/kg}}$$

The negative sign indicates heat transfer is from the system.

Cycle thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}/m}{Q_{\text{in}}/m} = \frac{295.9\frac{\text{kJ}}{\text{kg}} + 835.8\frac{\text{kJ}}{\text{kg}} - 495.9\frac{\text{kJ}}{\text{kg}}}{1035.9\frac{\text{kJ}}{\text{kg}}} = \underline{\mathbf{0.6138\ (61.38\%)}}$$

(b) The exergy transfers accompanying heat and work for each process are determined next.

Process 1-2 (isentropic compression)

Since $Q_{12} = 0$, $(\mathsf{E}_q)_{12} = 0$. Thus, $\frac{(\mathsf{E}_q)_{12}}{m} = \underline{\mathbf{0}}$.

$$\frac{(\mathsf{E}_w)_{12}}{m} = \frac{W_{12}}{m} - p_0(v_2 - v_1) = -495.9\frac{\text{kJ}}{\text{kg}} - (100\text{ kPa})(0.06098 - 0.9758)\frac{\text{m}^3}{\text{kg}}\left|\frac{10^3\ \frac{\text{N}}{\text{m}^2}}{1\text{ kPa}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

$$\frac{(\mathsf{E}_w)_{12}}{m} = \underline{\mathbf{-404.4\ kJ/kg}}$$

Process 2-3 (constant pressure heat addition)

$$(\mathsf{E}_q)_{23} = \int_2^3\left(1 - \frac{T_0}{T_b}\right)\delta Q = Q_{23} - T_0\int_2^3\left(\frac{\delta Q}{T}\right)_{\substack{\text{int}\\\text{rev}}} = Q_{23} - T_0 m(s_3 - s_2)$$

$$\frac{(\mathsf{E}_q)_{23}}{m} = \frac{Q_{23}}{m} - T_0(s_3 - s_2) = \frac{Q_{23}}{m} - T_0\left(c_p \ln\frac{T_3}{T_2} - R\ln\frac{p_3}{p_2}\right) = \frac{Q_{23}}{m} - T_0\left(c_p \ln\frac{T_3}{T_2} - 0\right)$$

$$\frac{(\mathsf{E}_q)_{23}}{m} = 1035.9\frac{\text{kJ}}{\text{kg}} - (300\text{ K})\left(1.005\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\ln\left(\frac{2061.4\text{ K}}{1030.7\text{ K}}\right) = \underline{\mathbf{826.9\ kJ/kg}}$$

$$\frac{(\mathsf{E}_w)_{23}}{m} = \frac{W_{23}}{m} - p_0(v_3 - v_2) = 295.9\frac{\text{kJ}}{\text{kg}} - (100\text{ kPa})(0.1220 - 0.06098)\frac{\text{m}^3}{\text{kg}}\left|\frac{10^3\ \frac{\text{N}}{\text{m}^2}}{1\text{ kPa}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

$$\frac{(\mathsf{E}_w)_{23}}{m} = \underline{\mathbf{289.8\ kJ/kg}}$$

Process 3-4 (isentropic expansion)

Since $Q_{34} = 0$, $(\mathsf{E}_q)_{34} = 0$. Thus, $\dfrac{(\mathsf{E}_q)_{34}}{m} = \underline{\mathbf{0}}$.

$$\frac{(\mathsf{E}_w)_{34}}{m} = \frac{W_{34}}{m} - p_0(v_4 - v_3) = 835.8\frac{\text{kJ}}{\text{kg}} - (100\text{ kPa})(0.9758 - 0.1220)\frac{\text{m}^3}{\text{kg}}\left|\frac{10^3\ \frac{\text{N}}{\text{m}^2}}{1\text{ kPa}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

$$\frac{(\mathsf{E}_w)_{34}}{m} = \underline{\mathbf{750.4\ kJ/kg}}$$

Process 4-1 (constant volume heat rejection)

$$(\mathsf{E}_q)_{41} = \int_4^1\left(1 - \frac{T_0}{T_b}\right)\delta Q = Q_{41} - T_0\int_4^1\left(\frac{\delta Q}{T}\right)_{\substack{\text{int}\\\text{rev}}} = Q_{41} - T_0 m(s_1 - s_4)$$

$$\frac{(\mathsf{E}_q)_{41}}{m} = \frac{Q_{41}}{m} - T_0(s_1 - s_4) = \frac{Q_{41}}{m} - T_0\left(c_v\ln\frac{T_1}{T_4} + R\ln\frac{v_1}{v_4}\right) = \frac{Q_{41}}{m} - T_0\left(c_v\ln\frac{T_1}{T_4} - 0\right)$$

$$\frac{(\mathsf{E}_q)_{41}}{m} = -400.1\frac{\text{kJ}}{\text{kg}} - (300\text{ K})\left(0.718\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\ln\left(\frac{340\text{ K}}{897.3\text{ K}}\right) = \underline{\mathbf{-191.1\ kJ/kg}}$$

Since $W_{41} = 0$ and $v_1 = v_4$, $(\mathsf{E}_w)_{41} = 0$. Thus, $\dfrac{(\mathsf{E}_w)_{41}}{m} = \underline{\mathbf{0}}$.

Exergetic efficiency can be defined as

$$\varepsilon = \frac{(\mathsf{E}_w)_{\text{cycle}}/m}{(\mathsf{E}_q)_{\text{in}}/m}$$

where

$$\frac{(E_w)_{cycle}}{m} = \frac{(E_w)_{12}}{m} + \frac{(E_w)_{23}}{m} + \frac{(E_w)_{34}}{m} + \frac{(E_w)_{41}}{m}$$

$$\frac{(E_w)_{cycle}}{m} = -404.4\frac{\text{kJ}}{\text{kg}} + 289.8\frac{\text{kJ}}{\text{kg}} + 750.4\frac{\text{kJ}}{\text{kg}} + 0 = 635.8 \text{ kJ/kg}$$

and

$$\frac{(E_q)_{in}}{m} = \frac{(E_q)_{23}}{m} = 826.9 \text{ kJ/kg}$$

Substituting values to solve for exergetic efficiency

$$\varepsilon = \frac{635.8\frac{\text{kJ}}{\text{kg}}}{826.9\frac{\text{kJ}}{\text{kg}}} = \underline{\mathbf{0.7689\ (76.89\%)}}$$

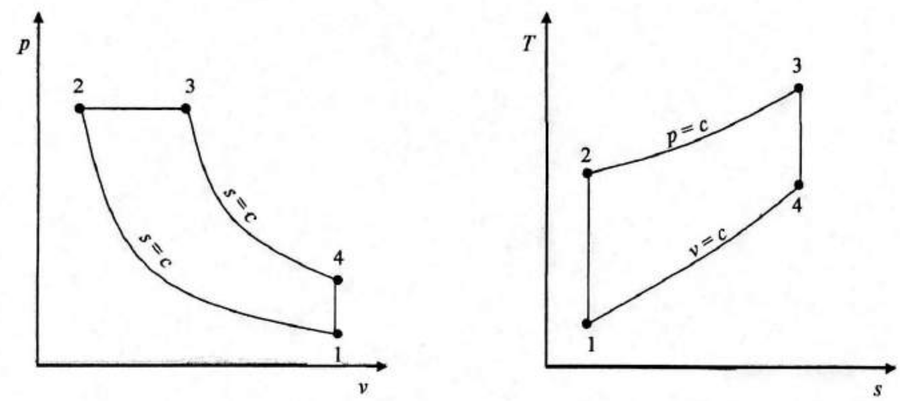
9.26 Consider an air-standard Diesel cycle. Operating data at principal states in the cycle are given in the table below. The states are numbered as in Fig. 9.5. Determine
(a) the cut-off ratio.
(b) the heat addition per unit mass, in Btu/lb.
(c) the net work per unit mass, in Btu/lb.
(d) the thermal efficiency.

State	T (°R)	p (lbf/in.^{2})	u (Btu/lb)	h (Btu/lb)
1	520	14.2	88.62	124.27
2	1502.5	657.8	266.84	369.84
3	3000	657.8	585.04	790.68
4	1527.1	41.8	271.66	376.36

KNOWN: An air-standard Diesel cycle operates with property data given at principal states.

FIND: Determine (a) the cut-off ratio, (b) the heat addition per unit mass, (c) the net work per unit mass, and (d) the thermal efficiency.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Air, modeled as an ideal gas, is the system.
2. The compression and expansion processes are adiabatic.
3. Kinetic and potential energy effects are negligible.

ANALYSIS: (a) The cut-off ratio can be determined as follows. For the constant pressure process, $p_2 = p_3$. Noting that for an ideal gas, $p = RT/v$

$$\frac{RT_2}{v_2} = \frac{RT_3}{v_3}$$

Problem 9.26 (Continued) – Page 2

Since $r_c = v_3/v_2$

$$r_c = \frac{v_3}{v_2} = \frac{T_3}{T_2} = \frac{3000°\text{R}}{1502.5°\text{R}} = \underline{\mathbf{2.00}}$$

(b) The heat addition occurs during process 2-3. Thus, noting that $W = mp(v_3 - v_2)$

$$m(u_3 - u_2) = Q_{23} - W_{23} = Q_{23} - mp(\,v_3 - v_2)$$

Thus

$$Q_{23}/m = (u_3 - u_2) + p(v_3 - v_2) = h_3 - h_2$$

Inserting values

$$\frac{Q_{23}}{m} = h_3 - h_2 = 790.68\frac{\text{Btu}}{\text{lb}} - 369.84\frac{\text{Btu}}{\text{lb}} = \underline{\mathbf{420.84\ Btu/lb}}$$

(c) The net work per unit mass can be determined from the net heat transfer per unit mass

$$\frac{W_{\text{cycle}}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m}$$

Applying the closed system energy balance to process 4-1

$$\frac{Q_{\text{out}}}{m} = \left|\frac{Q_{41}}{m}\right| = u_4 - u_1 = 271.66\frac{\text{Btu}}{\text{lb}} - 88.62\frac{\text{Btu}}{\text{lb}} = 183.04 \text{ Btu/lb}$$

Solving for net work per unit mass gives

$$\frac{W_{\text{cycle}}}{m} = 420.84\frac{\text{Btu}}{\text{lb}} - 183.04\frac{\text{Btu}}{\text{lb}} = \underline{\mathbf{237.80\ Btu/lb}}$$

(d) Thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}/m}{Q_{23}/m} = \frac{237.80\frac{\text{Btu}}{\text{lb}}}{420.84\frac{\text{Btu}}{\text{lb}}} = \underline{\mathbf{0.5651\ (56.51\%)}}.$$

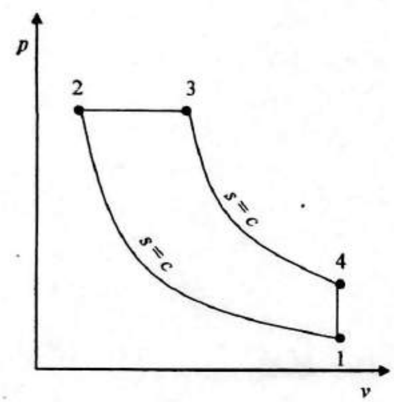
9.27 Consider a cold air-standard Diesel cycle. Operating data at principal states in the cycle are given in the table below. The states are numbered as in Fig. 9.5. For $k = 1.4$, $c_v = 0.172$ Btu/(lb·°R), and $c_p = 0.240$ Btu/(lb·°R), determine
(a) heat transfer per unit mass and work per unit mass for each process, in Btu/lb, and the cycle thermal efficiency.
(b) exergy transfers accompanying heat and work for each process, in Btu/lb. Define and evaluate an exergetic efficiency for the cycle. Let $T_0 = 540$ °R and $p_0 = 14.7$ lbf/in.^{2}

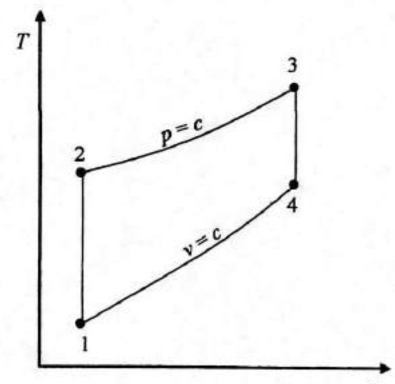
State	T (°R)	p (lbf/in.^{2})	v (ft^3/lb)
1	540	14.7	13.60
2	1637	713.0	0.85
3	3274	713.0	1.70
4	1425.1	38.8	13.60

KNOWN: A cold air-standard Diesel cycle operates with property data given at principal states.

FIND: Determine (a) heat transfer per unit mass and work per unit mass for each process and the cycle thermal efficiency (b) exergy.transfers accompanying heat and work for each process. Define and evaluate an exergetic efficiency for the cycle.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL:
1. Air, modeled as an ideal gas, is the system.
2. Specific heats are constant.
3. The compression and expansion processes are adiabatic.
4. All processes are internally reversible.
5. Kinetic and potential energy effects are negligible.
6. Let $T_0 = 540$ °R and $p_0 = 14.7$ lbf/in.^{2}

ANALYSIS: (a) The heat transfer per unit mass and work per unit mass for each process are determined from a closed system energy balance for each process.

Process 1-2 (isentropic compression)

$$\frac{Q_{12}}{m} = \underline{0} \text{ (adiabatic process)}$$

$$\frac{W_{12}}{m} = u_1 - u_2 = c_v(T_1 - T_2) = \left(0.172 \frac{\text{Btu}}{\text{lb} \cdot {}^\circ\text{R}}\right)(540^\circ\text{R} - 1637^\circ\text{R}) = \underline{\mathbf{-188.68\ Btu/lb}}$$

The negative sign indicates work is done on the system.

Process 2-3 (constant pressure heat addition)

$$\frac{Q_{23}}{m} = h_3 - h_2 = c_p(T_3 - T_2) = \left(0.240 \frac{\text{Btu}}{\text{lb} \cdot {}^\circ\text{R}}\right)(3274^\circ\text{R} - 1637^\circ\text{R}) = \underline{\mathbf{392.88\ Btu/lb}}$$

The positive sign indicates heat transfer is to the system.

$$\frac{W_{23}}{m} = \frac{Q_{23}}{m} + (u_2 - u_3) = \frac{Q_{23}}{m} + c_v(T_2 - T_3)$$

$$\frac{W_{23}}{m} = 392.9 \frac{\text{Btu}}{\text{lb}} + \left(0.172 \frac{\text{Btu}}{\text{lb} \cdot {}^\circ\text{R}}\right)(1637^\circ\text{R} - 3274^\circ\text{R}) = \underline{\mathbf{111.34\ Btu/lb}}$$

The positive sign indicates work is done by the system.

Process 3-4 (isentropic expansion)

$$\frac{Q_{34}}{m} = \underline{0} \text{ (adiabatic process)}$$

$$\frac{W_{34}}{m} = u_3 - u_4 = c_v(T_3 - T_4) = \left(0.172 \frac{\text{Btu}}{\text{lb} \cdot {}^\circ\text{R}}\right)(3274^\circ\text{R} - 1425.1^\circ\text{R}) = \underline{\mathbf{318.01\ Btu/lb}}$$

The positive indicates work is done by the system.

Process 4-1 (constant volume heat rejection)

$$\frac{W_{41}}{m} = \underline{0} \text{ (constant volume process)}$$

$$\frac{Q_{41}}{m} = u_1 - u_4 = c_v(T_1 - T_4) = \left(0.172\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)(540°\text{R} - 1425.1°\text{R}) = \underline{\mathbf{-152.24\ Btu/lb}}$$

The negative sign indicates heat transfer is from the system.

Cycle thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}/m}{Q_{\text{in}}/m} = \frac{111.34\frac{\text{Btu}}{\text{lb}} + 318.01\frac{\text{Btu}}{\text{lb}} - 188.68\frac{\text{Btu}}{\text{lb}}}{392.88\frac{\text{Btu}}{\text{lb}}} = \underline{\mathbf{0.6126\ (61.26\%)}}$$

(b) The exergy transfers accompanying heat and work for each process are determined next.

Process 1-2 (isentropic compression)

Since $Q_{12} = 0$, $(\mathsf{E}_q)_{12} = 0$. Thus, $\frac{(\mathsf{E}_q)_{12}}{m} = \underline{\mathbf{0}}$.

$$\frac{(\mathsf{E}_w)_{12}}{m} = \frac{W_{12}}{m} - p_0(v_2 - v_1) = -188.68\frac{\text{Btu}}{\text{lb}} - \left(14.7\frac{\text{lbf}}{\text{in}^2}\right)(0.85 - 13.60)\frac{\text{ft}^3}{\text{lb}}\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|\left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right|$$

$$\frac{(\mathsf{E}_w)_{12}}{m} = \underline{\mathbf{-153.99\ Btu/lb}}$$

Process 2-3 (constant pressure heat addition)

$$(\mathsf{E}_q)_{23} = \int_2^3\left(1 - \frac{T_0}{T_b}\right)\delta Q = Q_{23} - T_0\int_2^3\left(\frac{\delta Q}{T}\right)_{\substack{\text{int}\\\text{rev}}} = Q_{23} - T_0 m(s_3 - s_2)$$

$$\frac{(\mathsf{E}_q)_{23}}{m} = \frac{Q_{23}}{m} - T_0(s_3 - s_2) = \frac{Q_{23}}{m} - T_0\left(c_p\ln\frac{T_3}{T_2} - R\ln\frac{p_3}{p_2}\right) = \frac{Q_{23}}{m} - T_0\left(c_p\ln\frac{T_3}{T_2} - 0\right)$$

$$\frac{(\mathsf{E}_q)_{23}}{m} = 392.88\frac{\text{Btu}}{\text{lb}} - (540°\text{R})\left(0.240\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)\ln\left(\frac{3274°\text{R}}{1637°\text{R}}\right) = \underline{\mathbf{303.05\ Btu/lb}}$$

$$\frac{(\mathsf{E}_w)_{23}}{m}=\frac{W_{23}}{m}-p_0(v_3-v_2)=111.34\frac{\text{Btu}}{\text{lb}}-\left(14.7\frac{\text{lbf}}{\text{in}^2}\right)(1.70-0.85)\frac{\text{ft}^3}{\text{lb}}\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right|\left|\frac{144\text{ in}^2}{1\text{ ft}^2}\right|$$

$$\frac{(\mathsf{E}_w)_{23}}{m}=\underline{\mathbf{109.03\ Btu/lb}}$$

<u>Process 3-4 (isentropic expansion)</u>

Since $Q_{34}=0$, $(\mathsf{E}_q)_{34}=0$. Thus, $\frac{(\mathsf{E}_q)_{34}}{m}=\underline{\mathbf{0}}$.

$$\frac{(\mathsf{E}_w)_{34}}{m}=\frac{W_{34}}{m}-p_0(v_4-v_3)=318.01\frac{\text{Btu}}{\text{lb}}-\left(14.7\frac{\text{lbf}}{\text{in}^2}\right)(13.60-1.70)\frac{\text{ft}^3}{\text{lb}}\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right|\left|\frac{144\text{ in}^2}{1\text{ ft}^2}\right|$$

$$\frac{(\mathsf{E}_w)_{34}}{m}=\underline{\mathbf{285.63\ Btu/lb}}$$

<u>Process 4-1 (constant volume heat rejection)</u>

$$(\mathsf{E}_q)_{41}=\int_4^1\left(1-\frac{T_0}{T_b}\right)\delta Q=Q_{41}-T_0\int_4^1\left(\frac{\delta Q}{T}\right)_{\substack{\text{int}\\ \text{rev}}}=Q_{41}-T_0 m(s_1-s_4)$$

$$\frac{(\mathsf{E}_q)_{41}}{m}=\frac{Q_{41}}{m}-T_0(s_1-s_4)=\frac{Q_{41}}{m}-T_0\left(c_v\ln\frac{T_1}{T_4}+R\ln\frac{v_1}{v_4}\right)=\frac{Q_{41}}{m}-T_0\left(c_v\ln\frac{T_1}{T_4}-0\right)$$

$$\frac{(\mathsf{E}_q)_{41}}{m}=-152.24\frac{\text{Btu}}{\text{lb}}-(540°\text{R})\left(0.172\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)\ln\left(\frac{540°\text{R}}{1425.1°\text{R}}\right)=\underline{\mathbf{-62.11\ Btu/lb}}$$

Since $W_{41}=0$ and $v_1=v_4$, $(\mathsf{E}_w)_{41}=0$. Thus, $\frac{(\mathsf{E}_w)_{41}}{m}=\underline{\mathbf{0}}$.

Exergetic efficiency can be defined as

$$\varepsilon=\frac{(\mathsf{E}_w)_{\text{cycle}}/m}{(\mathsf{E}_q)_{\text{in}}/m}$$

where

$$\frac{(\mathsf{E}_w)_{\text{cycle}}}{m}=\frac{(\mathsf{E}_w)_{12}}{m}+\frac{(\mathsf{E}_w)_{23}}{m}+\frac{(\mathsf{E}_w)_{34}}{m}+\frac{(\mathsf{E}_w)_{41}}{m}$$

$$\frac{(\mathsf{E}_w)_{\text{cycle}}}{m}=-153.99\frac{\text{Btu}}{\text{lb}}+109.03\frac{\text{Btu}}{\text{lb}}+285.63\frac{\text{Btu}}{\text{lb}}+0=240.67\text{ Btu/lb}$$

and

$$\frac{(\mathsf{E}_q)_{\text{in}}}{m}=\frac{(\mathsf{E}_q)_{23}}{m}=303.05\text{ Btu/lb}$$

Substituting values to solve for exergetic efficiency

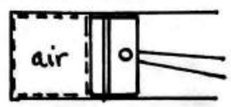
$$\varepsilon=\frac{240.67\frac{\text{Btu}}{\text{lb}}}{303.05\frac{\text{Btu}}{\text{lb}}}=\underline{\mathbf{0.7942\ (79.42\%)}}$$

PROBLEM 9.28

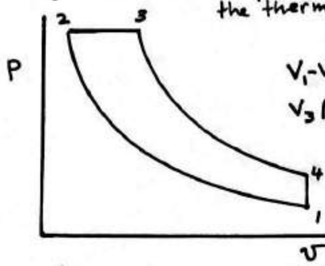
KNOWN: The displacement volume of and internal combustion engine is known. The processes within each cylinder are modeled by an air-standard Diesel cycle with a known cutoff ratio and a specified state at the beginning of compression. The number of cycles per minute is also given.

FIND: Determine the net work per cycle, the power developed, and the thermal efficiency.

SCHEMATIC & GIVEN DATA:



1000 cycles/min



$V_1 - V_2 = 3\ \ell$

$V_3/V_2 = 2.5$

$P_1 = 95$ kPa

$T_1 = 22°C$

$V_1 = 3.17\ \ell$

ENGR. MODEL: See Example 9.2

ANALYSIS: First, determine the compression ratio. From the given data

$$V_2 = V_1 - 3\ \ell = 0.17\ \ell \Rightarrow \frac{V_1}{V_2} = 18.65$$

State 1: $T_1 = 22°C = 295\text{ K} \Rightarrow u_1 = 210.49$ kJ/kg, $v_{r1} = 647.9$

State 2: For the isentropic compression, $v_{r2} = (V_2/V_1)\, v_{r1} = 34.740$.
Thus, $T_2 = 896.3$ K and $h_2 = 928.75$ kJ/kg

State 3: For the constant pressure process, $T_3 = (V_3/V_2)\, T_2 = 2240.7$ K.
Thus, $h_3 = 2554.6$ kJ/kg and $v_{r3} = 1.8915$

State 4: For the isentropic expansion, $v_{r4} = (V_4/V_3)\, v_{r3}$. However,

$$\frac{V_4}{V_3} = \frac{V_1}{V_2}\cdot\frac{V_2}{V_3} \Rightarrow v_{r4} = (18.65)\left(\frac{1}{2.5}\right) 1.8915 = 14.110$$

Thus, $T_4 = 1210.0$ K and $u_4 = 942.17$ kJ/kg

The net work per cycle is

$$W_{cycle} = Q_{23} - Q_{41} = m\left[(h_3 - h_2) - (u_4 - u_1)\right] \qquad (a)$$

Evaluating m

$$m = \frac{P_1 V_1}{R T_1} = \frac{(95\text{ kPa})(3.17\ \ell)}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(295\text{ K})}\left(\frac{10^3\text{ N/m}^2}{1\text{ kPa}}\right)\left(\frac{10^{-3}\text{ m}^3}{1\ \ell}\right)\left(\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right) = 3.56\times10^{-3}\text{ kg}$$

Thus

$$W_{cycle} = (3.56\times10^{-3})\left[(2554.6 - 928.75) - (942.17 - 210.49)\right]$$
$$= 3.183\text{ kJ/cycle} \longleftarrow W_{cycle}$$

The power developed is

$$\dot{W}_{net} = \left(1000\,\frac{\text{cycles}}{\text{min}}\right)\left(3.183\,\frac{\text{kJ}}{\text{cycle}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{1\text{ kW}}{1\text{ kJ/s}}\right)$$
$$= 53.05\text{ kW} \longleftarrow \dot{W}_{net}$$

The thermal efficiency is with Eq. (a)

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = \frac{(h_3 - h_2) - (u_4 - u_1)}{h_3 - h_2} = 1 - \frac{u_4 - u_1}{h_3 - h_2}$$

$$= 1 - \frac{942.17 - 210.49}{2554.6 - 928.75}$$

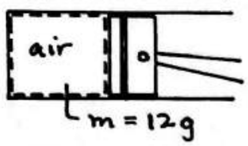
$$= 0.55\ (55\%) \longleftarrow \eta$$

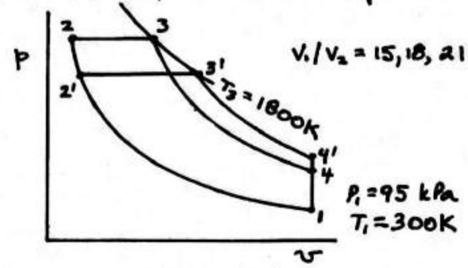
PROBLEM 9.29

KNOWN: An air-standard Diesel cycle has a known maximum temperature and a specified state at the beginning of compression. The mass of air is given.

FIND: Determine for various compression ratios (a) the net work, (b) the thermal efficiency, and (c) the mean effective pressure.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL: See Example 9.2

ANALYSIS: From Table A-22 at $T_1 = 300K$, $u_1 = 214.07$ kJ/kg, $v_{r1} = 621.2$. Also, at $T_3 = 1800K$: $h_3 = 2003.3$ kJ/kg and $v_{r3} = 3.944$.

For each compression ratio, V_1/V_2, v_{r2} is found using

$$v_{r2} = \left(\frac{V_2}{V_1}\right) v_{r1}$$

Thus, T_2 and h_2 can be found in Table A-22. For the isentropic expansion

$$v_{r4} = \left(\frac{V_4}{V_3}\right) v_{r3} = \left(\frac{V_1}{V_2}\right)\left(\frac{V_2}{V_3}\right) v_{r3} = \left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_3}\right) v_{r3}$$

Thus, T_4 and u_4 can be found in the Table as well. Key results for the various compression ratio are

V_1/V_2	v_{r2}	h_2 (kJ/kg)	u_4 (kJ/kg)
15	41.413	869.63	731.99
18	34.511	930.97	666.50
21	29.581	985.87	615.51

(a),(b) The net work and thermal efficiency are evaluated using

$$W_{cycle} = m\left[(h_3 - h_2) - (u_4 - u_1)\right], \quad \eta = \frac{W_{cycle}}{m(h_3 - h_2)}$$

(c) The mean effective pressure is given as

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}}{V_1(1 - V_2/V_1)}$$

Evaluating V_1

$$V_1 = \frac{mRT_1}{P_1} = \frac{(12g)\left(\frac{8.314}{28.97}\frac{kJ}{kg \cdot K}\right)(300K)}{(95\ kPa)}\left(\frac{1kg}{10^3 g}\right)\left(\frac{1\ kPa}{10^3\ N/m^2}\right)\left(\frac{10^3 N\cdot m}{1 kJ}\right)$$

$$= 0.01087\ m^3$$

The results of calculations for parts (a),(b), and (c) are tabulated below:

V_1/V_2	Q_{23}	Q_{41}	W_{cycle}	η (%)	mep (kPa)
15	13.604	6.215	7.389	54.3	728.3
18	12.868	5.429	7.439	57.8	724.6
21	12.209	4.817	7.392	60.5	714.0

① (a)(b)(c)

1. As compression ratio increases, η increases and mep decreases.

9.30 The thermal efficiency, η, of a cold air-standard diesel cycle can be expressed as

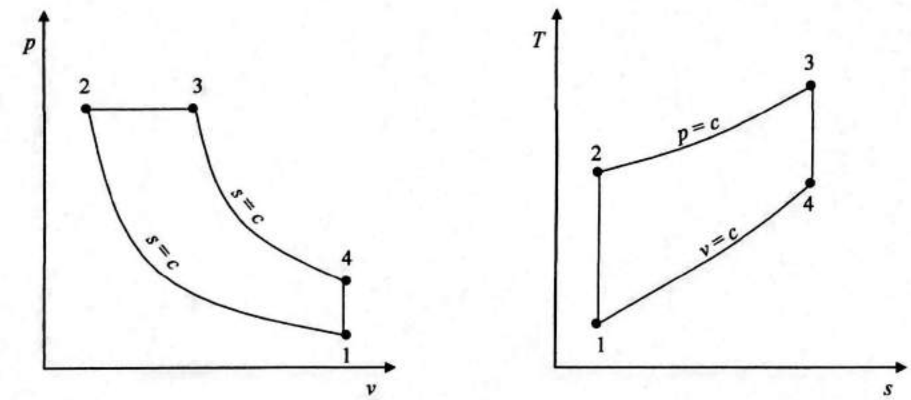
$$\eta = 1 - \frac{1}{r^{k-1}}\left[\frac{r_c^k - 1}{k(r_c - 1)}\right]$$

where r is compression ratio and r_c is cutoff ratio. Derive this expression.

KNOWN: Expression for cold air-standard Diesel cycle thermal efficiency.

FIND: Derive the expression for cold air-standard Diesel cycle thermal efficiency.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Air, modeled as an ideal gas, is the system.
2. The compression and expansion processes are adiabatic.
3. All processes are internally reversible.
4. Kinetic and potential energy effects are negligible.
5. Specific heats are constant.

ANALYSIS: The following relationships for state properties are applied in the derivation.

General relationships

(1) $r = \frac{v_1}{v2}$ (compression ratio)

(2) $r_c = \frac{v_3}{v2}$ (cutoff ratio)

(3) $k = \dfrac{c_p}{c_v}$ (specific heat ratio)

<u>Process 1-2 (isentropic compression)</u>

(4) $\dfrac{T_2}{T_1} = \left(\dfrac{v_1}{v_2}\right)^{k-1} = r^{k-1}$ (isentropic relation and Eq. (1))

(5) $\dfrac{p_2}{p_1} = \left(\dfrac{v_1}{v_2}\right)^{k} = r^{k}$ (isentropic relation and Eq. (1))

<u>Process 2-3 (constant pressure heat addition and expansion)</u>

(6) $p_2 = \dfrac{(\overline{R}/M)T_2}{v_2} = \dfrac{(\overline{R}/M)T_3}{v_3} = p_3 \rightarrow \dfrac{T_3}{T_2} = \dfrac{v_3}{v_2} = r_c$ (ideal gas equation of state and Eq. (2))

(7) $\dfrac{Q_{in}}{m} = \dfrac{Q_{23}}{m} = h_3 - h_2 = c_p(T_3 - T_2)$ (closed system energy balance)

<u>Process 3-4 (isentropic expansion)</u>

(8) $\dfrac{p_4}{p_3} = \left(\dfrac{v_3}{v_4}\right)^{k}$ (isentropic relation)

<u>Process 4-1 (constant volume heat rejection)</u>

(9) $v_4 = \dfrac{(\overline{R}/M)T_4}{p_4} = \dfrac{(\overline{R}/M)T_1}{p_1} = v_1 \rightarrow \dfrac{T_4}{T_1} = \dfrac{p_4}{p_1}$ (ideal gas equation of state)

(10) Introducing Eqs. (8), (5), and (2) into Eq. (9)

$$\frac{T_4}{T_1} = \frac{p_4}{p_1} = \frac{p_4/p_3}{p_1/p_3} = \frac{p_4/p_3}{p_1/p_2} = \frac{(v_3/v_4)^k}{(v_2/v_1)^k} = \left(\frac{v_3/v_4}{v_2/v_1}\right)^k = \left(\frac{v_3/v_4}{v_2/v_4}\right)^k = \left(\frac{v_3}{v_2}\right)^k = r_c^k$$

(11) $\dfrac{Q_{out}}{m} = \dfrac{Q_{41}}{m} = u_4 - u_1 = c_v(T_4 - T_1)$ (closed system energy balance – heat transfer out is expressed as a positive value)

55

<u>Derivation</u>

Cycle thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}/m}{Q_{\text{in}}/m} = 1 - \frac{Q_{\text{out}}/m}{Q_{\text{in}}/m}$$

Substituting Eqs. (11) and (7)

$$\eta = 1 - \frac{c_v(T_4 - T_1)}{c_p(T_3 - T_2)}$$

Substituting Eq. (3)

$$\eta = 1 - \frac{(T_4 - T_1)}{k(T_3 - T_2)}$$

Factoring T_1/T_2 from the second term

$$\eta = 1 - \frac{T_1}{T_2}\left[\frac{\left(\frac{T_4}{T_1} - 1\right)}{k\left(\frac{T_3}{T_2} - 1\right)}\right]$$

Substituting Eq. (4)

$$\eta = 1 - \frac{1}{r^{k-1}}\left[\frac{\left(\frac{T_4}{T_1} - 1\right)}{k\left(\frac{T_3}{T_2} - 1\right)}\right]$$

Substituting Eq. (10)

$$\eta = 1 - \frac{1}{r^{k-1}}\left[\frac{\left(r_c^k - 1\right)}{k\left(\frac{T_3}{T_2} - 1\right)}\right]$$

Substituting Eq. (6)

$$\eta = 1 - \frac{1}{r^{k-1}}\left[\frac{r_c^k - 1}{k(r_c - 1)}\right]$$

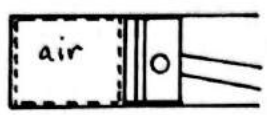
This completes the derivation.

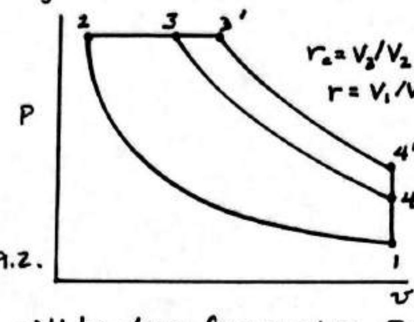
PROBLEM 9.31

KNOWN: An air-standard Diesel cycle has a known compression ratio and a specified state at the beginning of compression.

FIND: Plot (a) the maximum temperature, (b) the pressure at the end of expansion, (c) the net work per unit mass of air, and (d) the thermal efficiency versus cutoff ratio ranging from 1.5 to 2.5

SCHEMATIC & GIVEN DATA:





$r_c = V_3/V_2 = 1.5 - 2.5$

$r = V_1/V_2 = 15$

$P_1 = 100\,kPa$

$T_1 = 310\,K$

ENGINEERING MODEL: See Example 9.2.

ANALYSIS: A sample calculation will be done for $r_c = 1.5$. First, fix each of the principal states of the cycle:

(a) State 1: From Table A-22 at $T_1 = 310K$; $u_1 = 221.25$ kJ/kg and $v_{r_1} = 572.3$.

State 2: For the isentropic compression

$$v_{r_2} = \left(\frac{V_2}{V_1}\right) v_{r_1} = \left(\frac{1}{15}\right)(572.3) = 38.153$$

Interpolating in Table A-22; $h_2 = 896.86$ kJ/kg, $T_2 = 867.7\,K$

State 3: For each value of cutoff ratio, $r_c = V_3/V_2$, the value of T_3 can be determined from $T_3 = (V_3/V_2)T_2$. In this case

$$T_3 = 1301.5\,K;\ h_3 = 1397.7\ \text{kJ/kg},\ v_{r_3} = 11.235$$ Tmax

State 4: Note that for the isentropic expansion

$$v_{r_4} = \left(\frac{V_4}{V_3}\right) v_{r_3} = \left(\frac{V_1}{V_2}\cdot\frac{V_2}{V_3}\right) v_{r_3} = \left(r \cdot \frac{1}{r_c}\right) v_{r_3}$$

Thus $v_{r_4} = 112.35$; $T_4 = 586.6\,K$, $u_4 = 424.54$ kJ/kg

(b) The pressure at the end of the expansion is

$$P_4 = \left(\frac{T_4}{T_1}\right) P_1 = 189.2\,kPa$$ P4

(c) The net work per unit mass of air is

$$\frac{W_{cycle}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = (h_3 - h_2) - (u_4 - u_1) = 297.5\ \text{kJ/kg}$$ Wcycle/m

(d) The thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = 0.594\ (59.4\%)$$

The data for the required plots are obtained using IT, as follows:

PROBLEM 9.31 (Cont'd.) – Page 2

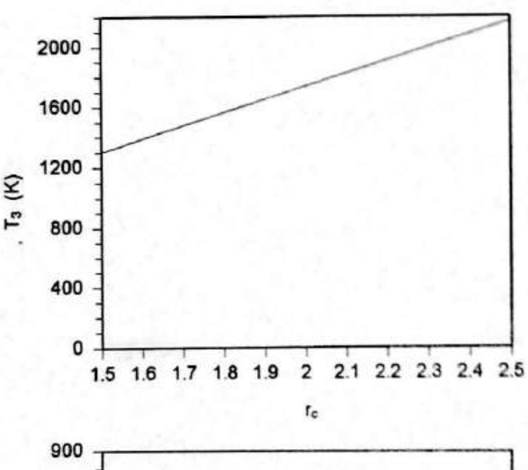
<u>*IT* Code</u>

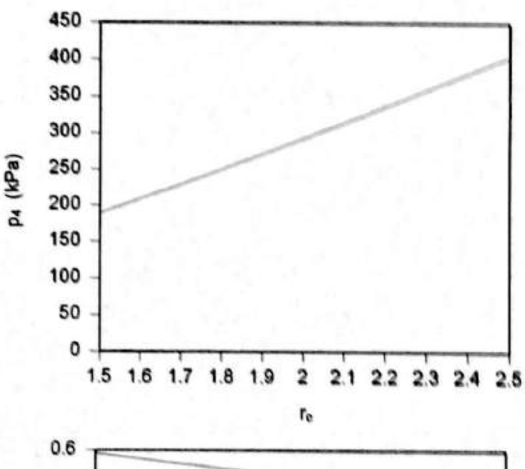
```
p1 = 100 // kPa
T1 = 310 // K
r = v1 / v2
r = 15
rc = v3 / v2
rc = 1.5
m = 1 // Assume a unit mass of 1 kg.

u1 = u_T("Air", T1)
v1 = v_TP("Air",T1, p1)
s1 = s_TP("Air", T1, p1)
v2 = v_TP("Air",T2, p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
h2 = h_T("Air", T2)
v3 = v_TP("Air",T3, p3)
p3 = p2
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
v4 = v_TP("Air",T4, p4)
s4 = s_TP("Air", T4, p4)
s4 = s3
v4 = v1
u4 = u_T("Air", T4)

Q23 = m * (h3 - h2)
Q41 = m * (u1 - u4)
Wcycle = Q23 + Q41
eta = Wcycle / Q23
```

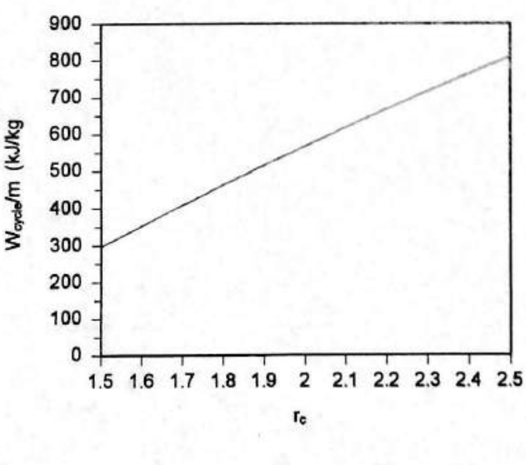
<u>*IT* Results for r_c = 1.5</u>

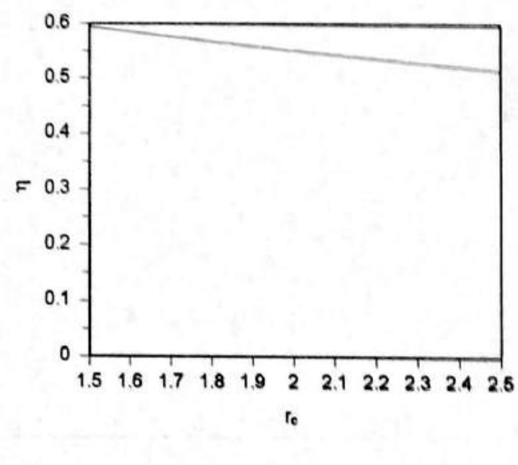
h_2 = 896.6 kJ/kg
p_2 = 4199 kPa
h_3 = 1397 kJ/kg
T_4 = 586.3 K
Q_{23}/m = 500.4 kJ/kg
Q_{41}/m = -203.1 kJ/kg
T_3 = 1302 K
p_4 = 189.1 kPa
W_{cycle}/m = 297.3 kJ/kg
η = 0.5941

PLOTS:







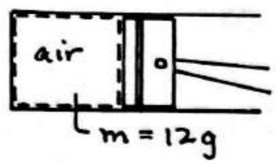


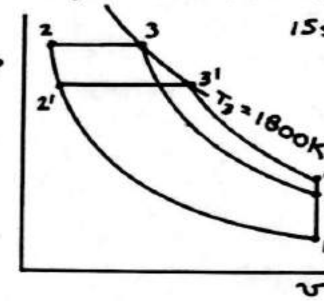
PROBLEM 9.32

<u>KNOWN</u>: An air-standard Diesel cycle has a known maximum temperature and a specified state at the beginning of compression. The mass of air is given.

<u>FIND</u>: Determine for various compression ratios (a) the net work, (b) the thermal efficiency, and (c) the mean effective pressure.

<u>SCHEMATIC & GIVEN DATA</u>:





$15 \le V_1/V_2 \le 25$

$p_1 = 95$ kPa
$T_1 = 300$K

<u>ENGINEERING MODEL</u>: See Example 9.2.

<u>ANALYSIS</u>: Consider first a <u>sample calculation</u> using data from Table A-22. for the case of $r = V_1/V_2 = 15$. From Table A-22 at $T_1 = 300K$; $u_1 = 214.07$ kJ/kg, $v_{r_1} = 621.2$. Also, at $T_3 = 1800K$; $h_3 = 2003.3$ kJ/kg, $v_{r_3} = 3.944$. For r = 15

$$v_{r_2} = (V_2/V_1)\, v_{r_1} = \left(\tfrac{1}{15}\right) 621.2 = 41.413$$

Interpolating in Table A-22; $h_2 = 869.63$ kJ/kg, $T_2 = 843.2$K. Now, for process 3-4

$$v_{r_4} = \left(\frac{V_4}{V_3}\right) v_{r_3} \overset{V_4 = V_1}{=} \left(\frac{V_1}{V_2}\right)\left(\frac{V_2}{V_3}\right) v_{r_3} \overset{\frac{V_2}{V_3} = \frac{T_2}{T_3}}{=} \left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_3}\right) v_{r_3} = (15)\left(\frac{843.2}{1800}\right)(3.944) = 27.713$$

Interpolating in Table A-22; $u_4 = 731.99$ kJ/kg.

(a) The net work is calculated from $W_{cycle} = Q_{cycle}$, where $Q_{cycle} = Q_{23} + Q_{34}$. Thus

$$W_{cycle} = m\,[(h_3 - h_2) + (u_1 - u_4)]$$

$$= (12\,g)\left|\frac{1\,kg}{10^3\,g}\right|[(2003.3 - 869.63) + (214.07 - 731.99)]\,\frac{kJ}{kg} = 7.389\ kJ \longleftarrow \frac{W_{cycle}}{(r=15)}$$

(b) The thermal efficiency is

$$\eta = \frac{W_{cycle}}{Q_{23}} = \frac{W_{cycle}}{m(h_3 - h_2)} = \frac{7.389}{(12\times10^{-3})(2003.3 - 869.63)} = 0.543\ (54.3\%) \longleftarrow \frac{\eta}{(r=15)}$$

(c) To get the mean effective pressure, first determine V_1.

$$V_1 = \frac{mRT_1}{p_1} = \frac{(12\,g)\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(300K)}{(95\ kPa)}\left|\frac{1\,kg}{10^3\,g}\right|\left|\frac{1\ kPa}{10^3\ N/m^2}\right|\left|\frac{10^3\ N\cdot m}{1\ kJ}\right| = 1.088\times10^{-2}\ m^3$$

Thus

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}}{V_1(1 - 1/r)}$$

$$= \frac{7.389\ kJ}{(1.088\times10^{-2}\ m^3)(1 - 1/15)}\left|\frac{10^3\ N\cdot m}{1\ kJ}\right|\left|\frac{1\ kPa}{10^3\ N/m^2}\right| = 727.6\ kPa \longleftarrow \frac{mep}{(r=15)}$$

PROBLEM 9.32 (Cont'd.) – Page 2

The data for the required plots are obtained using IT, as follows:

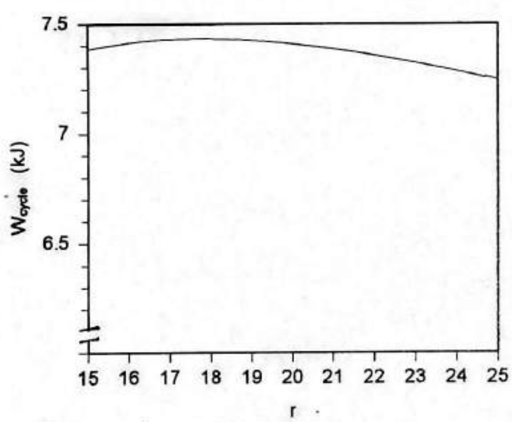
IT Code

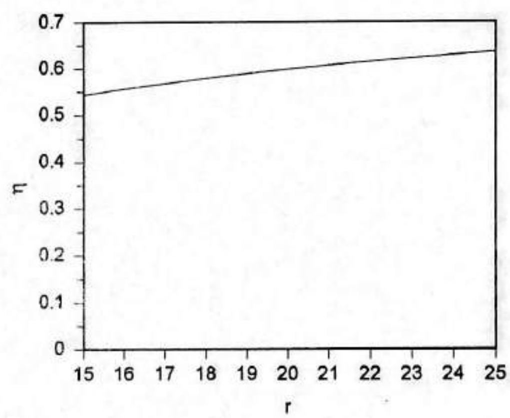
```
p1 = 95  // kPa
T1 = 300  // K
T3 = 1800  // K
r = 15
m  = 12E-03  // kg

u1 = u_T("Air", T1)
v1 = v_TP("Air",T1, p1)
s1 = s_TP("Air", T1, p1)
r = v1 / v2
v2 = v_TP("Air",T2, p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
h2 = h_T("Air", T2)
p3 = p2
h3 = h_T("Air", T3)
v3 = v_TP("Air",T3, p3)
s3 = s_TP("Air", T3, p3)
v4 = v_TP("Air",T4,p4)
s4 = s_TP("Air", T4, p4)
s4 = s3
v4 = v1
u4 = u_T("Air", T4)

Q23 = m * (h3 - h2)
Q41 = m * (u1 - u4)
Wcycle = Q23 + Q41
eta = Wcycle / (m * (h3 - h2))
V1 = v1 * m
mep = Wcycle / (V1 * (1 - 1 / r))  // kPa
```

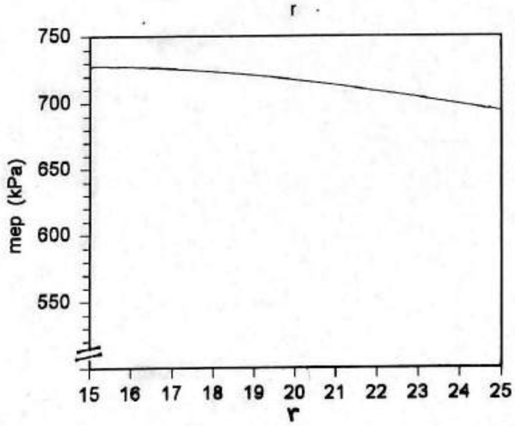
IT Results for r = 15

T_2 = 843.4 K
p_2 = 4006 kPa
h_2 = 869.4 kJ/kg
T_3 = 1800 K
h_3 = 2002 kJ/kg
T_4 = 967.2 K
p_4 = 306.3 kPa
Q_{23} = 13.59 kJ
Q_{41} = -6.202 kJ
W_{cycle} = 7.384 kJ
η = 0.5435
mep = 727.4 kPa

PLOTS:







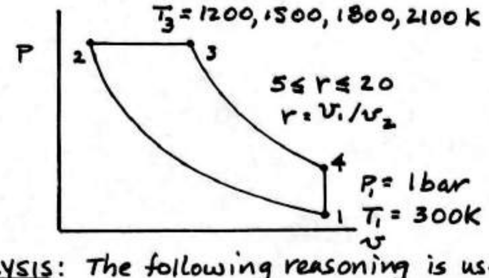
Note that the work doesn't vary greatly with r, but it does exhibit a maximum. Further, thermal efficiency increases with r because the average temperature of heat addition increases and the average temperature of heat rejection decreases, as can be verified by reference to the T-s diagram of the cycle. Also note that the mep decreases with increasing r.

PROBLEM 9.33

KNOWN: For an air-standard Diesel cycle, the state at the beginning of compression and several values for the maximum cycle temperature are known.

FIND: Plot the net work per unit mass, the mean effective pressure, and the thermal efficiency, each versus compression ratio ranging from 5 to 20.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.2.

ANALYSIS: The following reasoning is used to solve the problem:

1. Given values: p_1, T_1, r, and T_3.
2. Fix state 2 using ideal gas relations and $s(T_1,p_1) = s(T_2,p_2)$. When using Table A-22, this expressed in terms of $v_{r1}/v_{r2} = v_1/v_2$.
3. Using $p_3 = p_2$ and the ideal gas equation: $v_3 = (T_3/T_2)\,v_2$
4. Fix the state at 4 as in step 2: $s(T_3,p_3) = s(T_4,p_4)$; or $v_{r4}/v_{r3} = v_4/v_3$. Also, note that $v_4 = v_1$.

Now, using energy balances and $W_{41} = 0$; $Q_{23} = m(h_3 - h_2)$, $Q_{41} = m(u_1 - u_4)$. (1)

Thus
$$W_{cycle} = Q_{23} + Q_{41} = m[(h_3 - h_2) + (u_1 - u_4)] \quad (2)$$

Further
$$v_1 = \frac{RT_1}{P_1} \;;\; mep = \frac{W_{cycle}/m}{v_1(1 - 1/r)} \quad (3)$$

and
$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} \quad (4)$$

Sample calculation. $\boxed{r = 15,\ T_3 = 1200K}$ From Table A-22 at $T_1 = 300K$

$u_1 = 214.07$ kJ/kg, $v_{r1} = 621.2$. Thus $v_{r2} = (v_2/v_1)\,v_{r1} = (1/r)\,v_{r1} = 41.413$

Interpolating in Table A-22; $T_2 = 843.2$ K, $h_2 = 869.63$ kJ/kg. Now, at $T_3 = 1200$K, $h_3 = 1277.79$ kJ/kg, $v_{r3} = 14.470$. Further

$$v_{r4} = (v_4/v_3)\,v_{r3} = (v_4/v_2)(v_2/v_3)\,v_{r3} = (v_1/v_2)(T_2/T_3)\,v_{r3} = 152.514$$

Thus, $T_4 = 522.1$K, $u_4 = 375.96$ kJ/kg.

Now, from (1)-(4)

$$Q_{23}/m = (1277.79 - 869.63) = 408.2 \text{ kJ/kg}$$

$$W_{cycle} = (408.2) + (214.07 - 375.96) = 246.3 \text{ kJ/kg}$$

$$\eta = 246.3/408.2 = 0.6034 \ (60.34\%)$$

Finally
$$v_1 = \frac{\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(300K)}{(1\ bar)}\left|\frac{1\ bar}{10^5 N/m^2}\right|\left|\frac{10^3 N\cdot m}{1 kJ}\right| = 0.861\ m^3/kg$$

$$mep = \frac{(246.3\ kJ/kg)}{(0.861\ m^3/kg)(1 - 1/15)}\left|\frac{10^3\ N\cdot m}{1 kJ}\right|\left|\frac{1\ bar}{10^5\ N/m^2}\right| = 3.065\ bar$$

PROBLEM 9.33 (Cont'd.) – Page 2

The data for the required plots are obtained using IT, as follows:

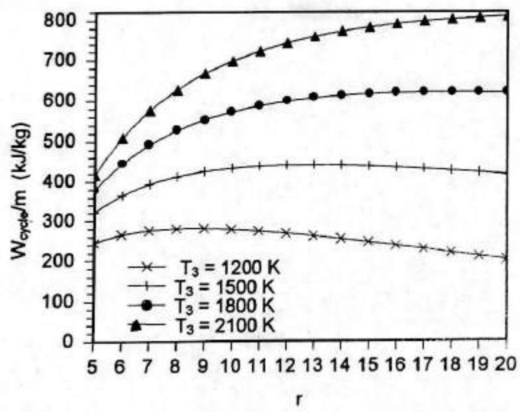
IT Code

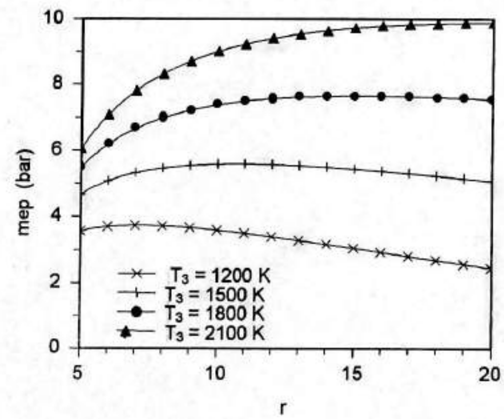
```
p1 = 1  // bar
T1 = 300  // K
T3 = 1200  // K
r = 15
m = 1  // Asume a unit mass of 1 kg.

v1 = v_TP("Air",T1, p1)
s1 = s_TP("Air", T1, p1)
r = v1 / v2
v2 = v_TP("Air",T2, p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
v3 /v2 = T3 / T2
v3 = v_TP("Air",T3, p3)
s3 = s_TP("Air", T3, p3)
v4 = v_TP("Air",T4, p4)
v4 = v1
s4 = s_TP("Air", T4, p4)
s4 = s3
u1 = u_T("Air", T1)
h2 = h_T("Air", T2)
h3 = h_T("Air", T3)
u4 = u_T("Air", T4)

Q23 = m * (h3 - h2)
Q41 = m * (u1 - u4)
Wcycle = Q23 + Q41
mep = (Wcycle / m) / (v1 * (1 - 1 / r)) / 100
eta = Wcycle / Q23
```

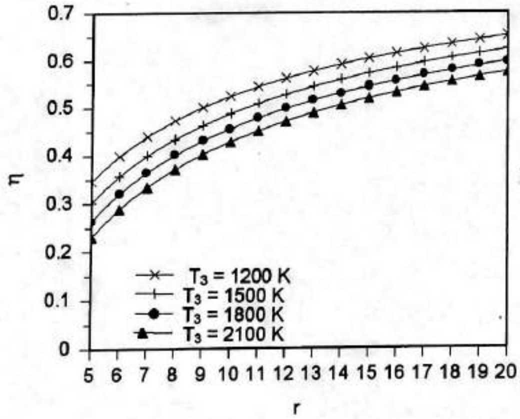
IT Results for r = 15 and T3 = 1200 K

T_2 = 843.4 K
p_2 = 42.17 bar
h_2 = 869.4 kJ/kg
h_3 = 1277 kJ/kg
T_4 = 521.9 K
p4 = 1.74 bar
Q_{23}/m = 407.7 kJ/kg
Q_{41}/m = -161.7 kJ/kg
W_{cycle}/m = 245.9 kJ/kg
mep = 3.06 bar
η = 0.6033

PLOTS:





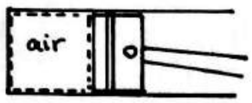


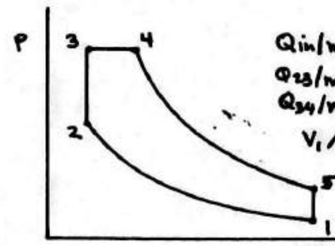
PROBLEM 9.34

KNOWN: An air-standard dual cycle has a known compression ratio and a specified state at the beginning of compression. The heat additions at constant volume and constant pressure are also given.

FIND: Determine (a) the temperatures at the end of each heat addition process, (b) the net work per unit mass, (c) the thermal efficiency, and (d) the mean effective pressure.

SCHEMATIC & GIVEN DATA:





$Q_{in}/m = 1400$ kJ/kg
$Q_{23}/m = 933.33$ kJ/kg
$Q_{34}/m = 466.7$ kJ/kg
$V_1/V_2 = 9$
$p_1 = 100$ kPa
$T_1 = 300$K

ENGR. MODEL: See Example 9.3

ANALYSIS: Begin by fixing each principle state of the cycle (Table A-22):

State 1: $T_1 = 300K \Rightarrow u_1 = 214.07$, $v_{r1} = 621.2$

State 2: For the isentropic compression, $v_{r2} = (V_2/V_1)v_{r1} = 69.022$

Thus, $T_2 = 702.7$ K and $u_2 = 514.49$ kJ/kg

State 3: For the heat addition process from 2 to 3:

$$m(u_3 - u_2) = Q_{23} - \cancel{W_{23}}^{0}$$

or

$$u_3 = Q_{23}/m + u_2 = 933.33 + 514.49 = 1447.82 \text{ kJ/kg}$$

(a) Thus, $T_3 = 1758.5$ K and $h_3 = 1952.04$ kJ/kg ← T_3

State 4: For the heat addition process from 3 to 4:

$$Q_{34}/m = h_4 - h_3$$

or

$$h_4 = h_3 + Q_{34}/m = 1952.04 + 466.7 = 2418.74 \text{ kJ/kg}$$

Thus, $T_4 = 2132.8$ K and $v_{r4} = 2.237$ ← T_4

State 5: For the isentropic expansion

$$v_{r5} = \left(\frac{V_5}{V_4}\right) v_{r4} = \left(\frac{V_1}{V_2} \cdot \frac{V_3}{V_4}\right) v_{r4} = \left(\frac{V_1}{V_2} \cdot \frac{T_3}{T_4}\right) v_{r4} = 16.5997$$

Thus, $T_5 = 1147.8$ K and $u_5 = 887.24$ kJ/kg

(b) For the cycle, $W_{cycle} = Q_{cycle}$. Thus

$$W_{cycle}/m = (Q_{23}/m + Q_{34}/m) - Q_{51}/m$$
$$= Q_{in}/m - (u_5 - u_1) = 1400 - (887.24 - 214.07)$$
$$= 726.83 \text{ kJ/kg}$$ ← W_{cycle}/m

(c) The thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{in}/m} = \frac{726.83}{1400} = 0.519 \; (51.9\%)$$ ← η

(d) The mean effective pressure is given by

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}/m}{v_1(1 - V_2/V_1)}$$

Evaluating v_1

$$v_1 = \frac{RT_1}{p_1} = \frac{\left(\frac{8.314}{28.97} \frac{kJ}{kg \cdot K}\right)(300K)}{(100 \text{ kPa})} \left(\frac{1 \text{ kPa}}{10^3 \text{ N/m}^2}\right)\left(\frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}}\right) = 0.86096 \frac{m^3}{kg}$$

Thus

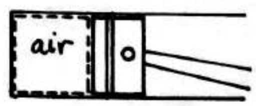
$$mep = \frac{(726.83 \text{ kJ/kg})}{(.86096 \text{ m}^3/\text{kg})(1 - 1/9)} \left(\frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}}\right)\left(\frac{1 \text{ kPa}}{10^3 \text{ N/m}^2}\right)$$
$$= 949.7 \text{ kPa}$$ ← mep

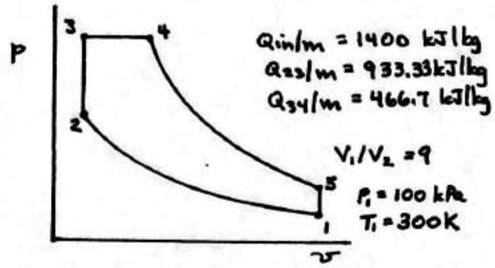
PROBLEM 9.35

KNOWN: A cold air-standard dual cycle has a known compression ratio and a specified state at the beginning of compression. The heat additions at constant volume and at constant pressure are also given.

FIND: Determine (a) the temperatures at the end of each heat addition process, (b) the net work per unit mass, (c) the thermal efficiency, and (d) the mean effective pressure.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL: See Example 9.3. Also, assume constant specific heats evaluated at 300K.

ANALYSIS: First, determine the temperatures at each of the principal states. From Table A-20, $k = 1.4$, $c_v = 0.718$ kJ/kg·K, and $c_p = 1.005$ kJ/kg·K. Also, $T_1 = 300$ K is given.

State 2: For the isentropic compression,

$$T_2 = (V_1/V_2)^{k-1} T_1 = 722.5 \text{ K}$$

(a) State 3: For the constant volume heat addition

$$Q_{23}/m = u_3 - u_2 = c_v(T_3 - T_2)$$

Thus

$$T_3 = \frac{Q_{23}/m}{c_v} + T_2 = \frac{(933.33)}{(0.718)} + 722.5 = 2022.4 \text{ K} \longleftarrow T_3$$

State 4: For the constant pressure process, $Q_{34}/m = h_4 - h_3 = c_p(T_4 - T_3)$.

Thus

$$T_4 = \frac{Q_{34}/m}{c_p} + T_3 = \frac{(466.7)}{(1.005)} + 2022.4 = 2486.8 \text{ K} \longleftarrow T_4$$

State 5: For the isentropic expansion

$$T_5 = \left(\frac{V_4}{V_5}\right)^{k-1} T_4 = \left(\frac{V_2}{V_1} \cdot \frac{V_4}{V_3}\right)^{k-1} T_4 = \left(\frac{V_2}{V_1} \cdot \frac{T_4}{T_3}\right)^{k-1} T_4 = 1121.6 \text{ K}$$

(b) For the cycle, $W_{cycle} = Q_{cycle}$. Thus

$$\frac{W_{cycle}}{m} = \frac{Q_{in}}{m} - \frac{Q_{51}}{m} = \frac{Q_{in}}{m} - (u_5 - u_1) = \frac{Q_{in}}{m} - c_v(T_5 - T_1)$$

$$= 1400 - (.718)(1121.6 - 300) = 810.1 \text{ kJ/kg} \longleftarrow W_{cycle}/m$$

(c) The thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{in}/m} = \frac{810.1}{1400} = 0.579 \; (57.9\%) \longleftarrow \eta$$

(d) The mean effective pressure is given by

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}/m}{v_1(1 - V_2/V_1)}$$

Evaluating v_1

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(\frac{8.314}{28.97} \cdot \frac{kJ}{kg \cdot K}\right)(300K)}{(100 \text{ kPa})} \left(\frac{1 \text{ kPa}}{10^3 \text{ N/m}^2}\right)\left(\frac{10^3 \text{ N·m}}{1 \text{ kJ}}\right) = 0.86096 \frac{m^3}{kg}$$

Thus

$$mep = \frac{(810.1 \text{ kJ/kg})}{(.86096 \text{ m}^3/\text{kg})(1 - 1/9)} \left(\frac{10^3 \text{ N·m}}{1 \text{ kJ}}\right)\left(\frac{1 \text{ kPa}}{10^3 \text{ N/m}^2}\right)$$

$$= 1058.5 \text{ kPa} \longleftarrow mep$$

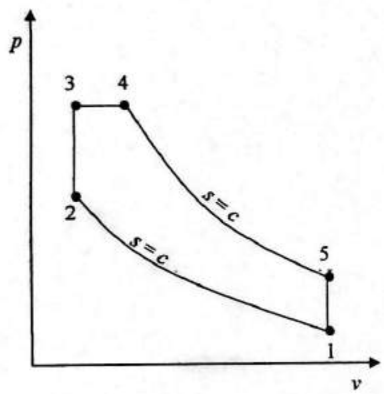
9.36 Consider an air-standard dual cycle. Operating data at principal states in the cycle are given in the table below. If the mass of air is 0.05 kg, determine
(a) the cut-off ratio.
(b) the heat addition to the cycle, in kJ.
(c) the heat rejection from the cycle, in kJ.
(d) the net work, in kJ.
(e) the thermal efficiency.

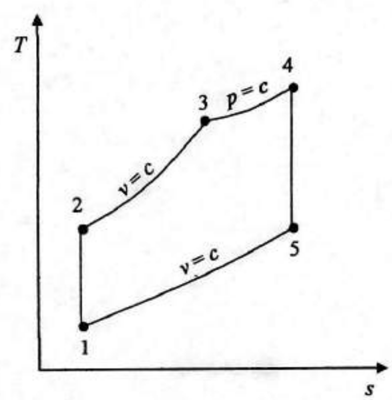
State	T (K)	p (kPa)	u (kJ/kg)	h (kJ/kg)
1	300	95	214.07	300.19
2	862.4	4372.8	643.35	890.89
3	1800	9126.9	1487.2	2003.3
4	1980	9126.9	1659.5	2227.1
5	840.3	265.7	625.19	866.41

KNOWN: An air-standard dual cycle operates with property data given at principal states and known mass.

FIND: Determine (a) the cut-off ratio, (b) the heat addition to the cycle, (c) the heat rejection from the cycle, (d) the net work, and (e) the thermal efficiency.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL:
1. Air, modeled as an ideal gas, is the system.
2. The compression and variable pressure expansion processes are adiabatic.
3. Kinetic and potential energy effects are negligible.

ANALYSIS: (a) The cut-off ratio can be determined as follows. For the constant pressure process, $p_3 = p_4$. Noting that for an ideal gas, $p = RT/v$

65

Problem 9.36 (Continued) – Page 2

$$\frac{RT_3}{v_3} = \frac{RT_4}{v_4}$$

Since $r_c = v_4/v_3$

$$r_c = \frac{v_4}{v_3} = \frac{T_4}{T_3} = \frac{1980\text{ K}}{1800\text{ K}} = \underline{\mathbf{1.1}}$$

(b) Heat addition to the cycle occurs during process 2-3 and process 3-4. For constant volume process 2-3

$$Q_{23} = m(u_3 - u_2) = (0.05\text{ kg})(1487.2 - 643.35)\frac{\text{kJ}}{\text{kg}} = 42.19\text{ kJ}$$

Noting that $W = mp(v_4 - v_3)$ for constant pressure process 3-4

$$m(u_4 - u_3) = Q_{34} - W_{34} = Q_{34} - mp(v_4 - v_3)$$

Thus

$$Q_{34} = m[(u_4 - u_3) + p(v_4 - v_3)] = m(h_4 - h_3)$$

Inserting values

$$Q_{34} = m(h_4 - h_3) = (0.05\text{ kg})(2227.1 - 2003.3)\frac{\text{kJ}}{\text{kg}} = 11.19\text{ kJ}$$

Total heat addition is

$$Q_{in} = 42.19\text{ kJ} + 11.19\text{ kJ} = \underline{\mathbf{53.38\text{ kJ}}}$$

(c) Heat rejection from the cycle occurs during process 5-1

$$Q_{51} = m(u_1 - u_5) = (0.05\text{ kg})(214.07 - 625.19)\frac{\text{kJ}}{\text{kg}} = -20.56\text{ kJ}$$

The negative sign indicates heat transfer is from the system.

The total heat rejection is

$$Q_{out} = \underline{\mathbf{20.56\text{ kJ}}}$$

(d) The net work can be determined from the net heat transfer for the cycle

$$W_{cycle} = Q_{in} - Q_{out} = 53.38\text{ kJ} - 20.56\text{ kJ} = \underline{\mathbf{32.82\text{ kJ}}}$$

(e) The thermal efficiency is

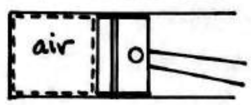
$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{32.82\text{ kJ}}{53.38\text{ kJ}} = \underline{\mathbf{0.6148\ (61.48\%)}}$$

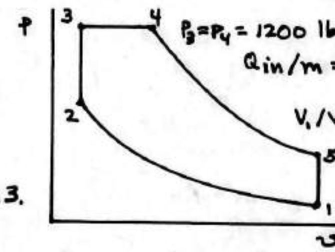
PROBLEM 9.37

KNOWN: An air-standard dual cycle has a known compression ratio and a specified state at the beginning of compression. The total heat addition per unit mass and the pressure at the end of the constant volume heat addition are also known.

FIND: Determine the net work and the heat rejection, each per unit mass of air. Also, the thermal efficiency and cutoff ratio. Plot those quantities versus compression ratio ranging from 10 to 28

SCHEMATIC & GIVEN DATA:





$p_3 = p_4 = 1200\ \text{lbf/in}^2$

$Q_{in}/m = 800$ Btu/lb

$V_1/V_2 = 15$

$p_1 = 14.0\ \frac{\text{lbf}}{\text{in}^2}$

$T_1 = 520°R$

ENGINEERING MODEL: See Example 9.3.

ANALYSIS: Begin by fixing each of the principal states of the cycle (Table A-16E).

State 1: $T_1 = 520°R \Rightarrow u_1 = 88.62$ Btu/lb, $v_{r_1} = 158.58$

State 2: For the isentropic compression, $v_{r_2} = (V_2/V_1)\, v_{r_1} = 10.572$.
Thus, $T_2 = 1468.8°R$, $u_2 = 260.26$ Btu/lb, and $p_{r_2} = 51.561$

State 3: Since $V_3 = V_2$, $T_3 = (p_3/p_2)\, T_2$. To get p_2, use

$$p_2 = \left(\frac{p_{r2}}{p_{r_1}}\right) p_1 = 594.26\ \text{lbf/in}^2 \Rightarrow T_3 = 2965.9\ °R$$

Thus, $u_3 = 577.4$ Btu/lb and $h_3 = 780.7$ Btu/lb

State 4: The total heat addition is known. Thus

$$\frac{Q_{in}}{m} = \frac{Q_{23}}{m} + \frac{Q_{34}}{m} = (u_3 - u_2) + (h_4 - h_3)$$

or

$$h_4 = \frac{Q_{in}}{m} - (u_3 - u_2) + h_3 = 1263.56\ \text{Btu/lb}$$

Thus, $T_4 = 4577.6\ °R$ and $v_{r4} = 0.2848$

State 5: For the isentropic expansion

$$\frac{V_5}{V_4} = \frac{V_1}{V_2} \cdot \frac{V_3}{V_4} = \frac{V_1}{V_2} \cdot \frac{T_3}{T_4} = 9.7187$$

and

$$v_{r5} = \left(\frac{V_5}{V_4}\right) v_{r4} = 2.768 \Rightarrow T_5 = 2299.2\ °R,\ u_5 = 431.0\ \text{Btu/lb}$$

Now, the heat rejection per unit mass is

(b) $$\frac{Q_{51}}{m} = u_5 - u_1 = 431.0 - 88.62 = 342.4\ \text{Btu/lb} \longleftarrow \quad Q_{51}/m$$

and for the cycle, $W_{cycle} = Q_{cycle}$. Thus

(a) $$\frac{W_{cycle}}{m} = \frac{Q_{in}}{m} - \frac{Q_{51}}{m} = 457.6\ \text{Btu/lb} \longleftarrow \quad W_{cycle}/m$$

(c) The thermal efficiency is

$$\eta = (W_{cycle}/m)/(Q_{in}/m) = (457.6)/(800) = 0.572\ (57.2\%) \longleftarrow \quad \eta$$

(d) The cutoff ratio is

$$r_c = v_4/v_3 = T_4/T_3 = 4577.6/2965.9 = 1.543 \longleftarrow \quad r_c$$

PROBLEM 9.37 (Cont'd.) - Page 2

(e) The data for the required plots are obtained using IT, as follows:

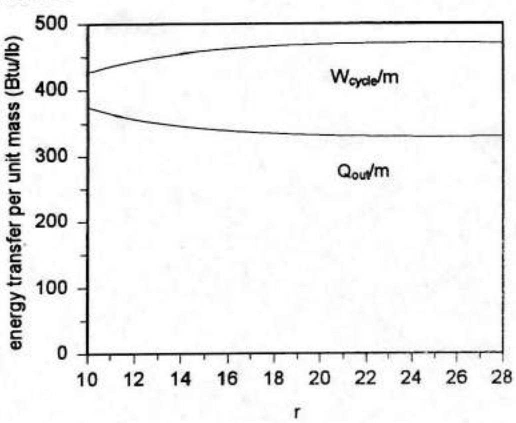
IT Code

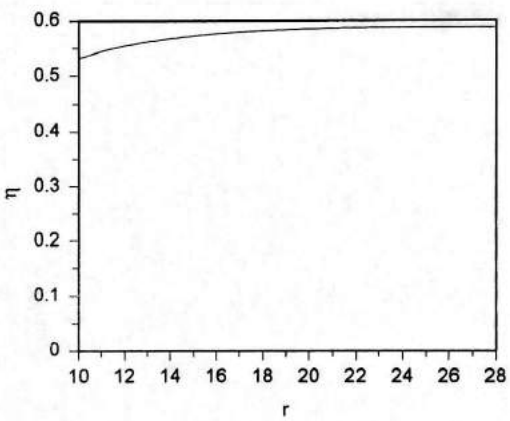
```
p1 = 14  // lbf/in.²
T1 = 520  // °R
r = v1 / v2
r = 15
p3 = 1200  // lbf/in.²
Qin / m = 800  // Btu/lb
m = 1  // Assume a unit mass of 1 lb.

u1 = u_T("Air", T1)
v1 = v_TP("Air",T1, p1)
s1 = s_TP("Air", T1, p1)
v2 = v_TP("Air",T2, p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
u2 = u_T("Air", T2)
T3 / T2 = p3 / p2
u3 = u_T("Air", T3)
h3 = h_T("Air", T3)
v3 = v2
Qin / m = (u3 - u2) + (h4 - h3)
h4 = h_T("Air", T4)
v4 = v_TP("Air",T4, p4)
p4 = p3
s4 = s_TP("Air", T4, p4)
v5 = v_TP("Air",T5, p5)
s5 = s_TP("Air", T5, p5)
s5 = s4
v5 = v1
u5 = u_T("Air", T5)

Qout / m = u5 - u1
Wcycle / m = Qin / m - Qout / m
eta = Wcycle / Qin
rc = v4 / v3
```

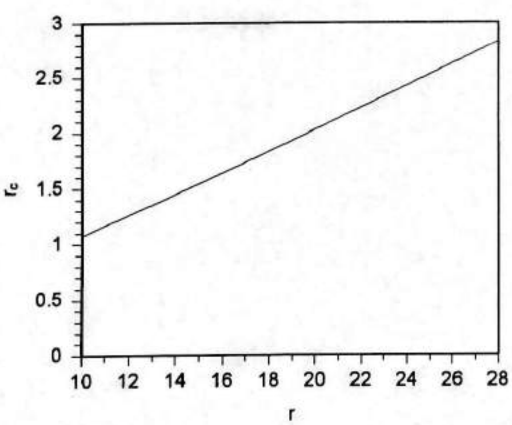
IT Results for r = 15

T_2 = 1469 °R
T_3 = 2971 °R
T_4 = 4582 °R
T_5 = 2301 °R
W_{cycle}/m = 457.9 Btu/lb
Q_{out}/m = 342.1 Btu/lb
η = 0.5724
r_c = 1.542

PLOTS:







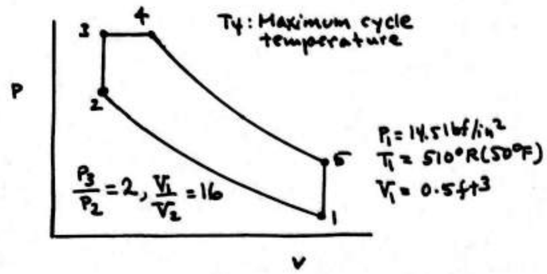
We see from the graphs that the thermal efficiency increases slightly with increasing r. Since Qin is constant, it follows that Wcycle would increase and Qout would decrease. Also, as r increases, the cutoff ratio increases, indication that of the heat input occurs at constant pressure.

PROBLEM 9.38

KNOWN: Operating data are provided for an air-standard dual cycle.

FIND: For a maximum cycle temperature of 3000°R determine (a) the heat added, (b) the net work, (c) the thermal efficiency, (d) the mean effective pressure. (e) Plot the quantities of parts (a)-(d) versus maximum cycle temperature varying from 3000 to 4000°R.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

See Example 9.3

ANALYSIS:

$T_4 = 3000°R$ ... begin by fixing the state of each principal state using data from Table A-22E.

State 1: $T_1 = 510°R$, $u_1 = 86.91$ Btu/lb, $v_{r1} = 166.74$.

State 2: For the isentropic compression, $v_{r2} = (V_2/V_1)v_{r1} = (166.74/16) = 10.421$
$\Rightarrow T_2 = 1476.1\ °R$, $u_2 = 261.68$ Btu/lb

State 3: For the constant volume heat addition, $T_3/T_2 = P_3/P_2$. Thus,
$T_3 = 2T_2 = 2952.2°R \Rightarrow u_3 = 574.33$ Btu/lb, $h_3 = 776.69$ Btu/lb.

State 4: With $T_4 = 3000°R$, $h_4 = 790.68$ Btu/lb, $v_{r4} = 1.18$

State 5: For the isentropic expansion $v_{r5} = (V_5/V_4)v_{r4}$, where

$$\frac{V_5}{V_4} = \frac{V_1}{V_2}\underbrace{\frac{V_3}{V_4}}_{\text{since } P_3=P_4,\ \frac{V_3}{V_4}=\frac{T_3}{T_4}} = \frac{V_1}{V_2}\cdot\frac{T_3}{T_4} = 16\left(\frac{2952.2}{3000}\right) = 15.745 \Rightarrow v_{r5} = 18.579$$
$\Rightarrow u_5 = 208.76$ Btu/lb

The mass of the system is obtained from

$$m = \frac{P_1V_1}{RT_1} = \frac{(14.5 \times 144\ lbf/ft^2)(0.5\ ft^3)}{\left(\frac{1545}{28.97}\ \frac{ft\cdot lbf}{lb\cdot °R}\right)(510°R)} = 0.03838\ lb$$

(a) The heat added is the sum $Q_{23} + Q_{34}$

$Q_{23} = m(u_3 - u_2) = 0.03838\,(574.33 - 261.68) = 12$ Btu
$Q_{34} = m(h_4 - h_3) = 0.03838\,(790.68 - 776.69) = 0.54$ Btu
$\Big\}\ Q_{in} = 12.54$ Btu ← Q_{in}

(b) Since $W_{cycle} = Q_{cycle}$, and processes 1-2, 4-5 are adiabatic

$W_{cycle} = Q_{in} - Q_{51} = Q_{in} - m(u_5 - u_1) = 12.54 - 0.03838(208.76 - 86.91) = 7.86$ Btu ← W_{cycle}

(d) $$\text{mep} = \frac{W_{cycle}}{(V_1 - V_2)} = \frac{W_{cycle}}{V_1[1 - V_2/V_1]} = \frac{7.86 \times 778\ ft{\cdot}lbf}{0.5\,ft^3\left[1 - \frac{1}{16}\right]}\left|\frac{1\,ft^2}{144\,in^2}\right| = 90.59\ \frac{lbf}{in^2}$$ ← mep

(c) $$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{7.86}{12.54} = 0.627\quad (62.7\%)$$ ← η

PROBLEM 9.38' (cont'd.) – Page 2

(e) The data for the required plots are obtained using IT, as follows:

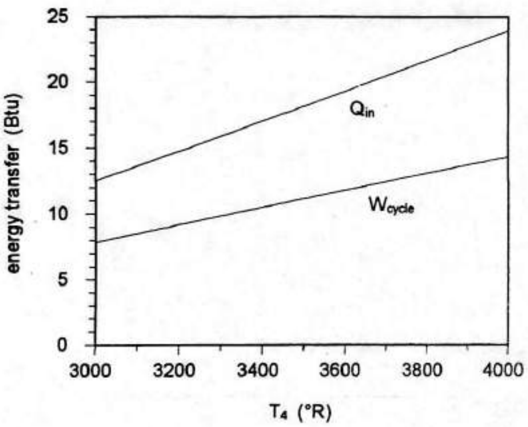
<u>*IT* Code</u>

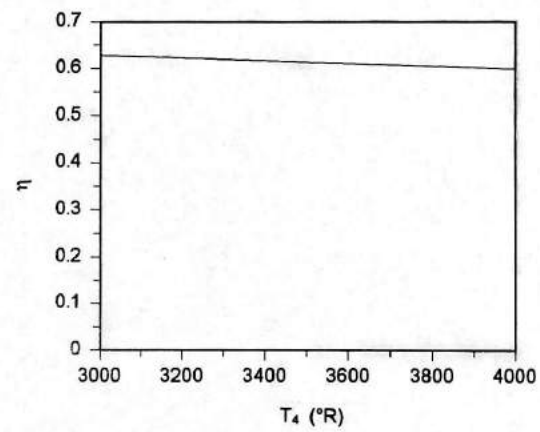
```
p1 = 14.5  // lbf/in.²
T1 = 510  // °R
V1 = 0.5  // ft³
V1 / V2 = 16
p3 / p2 = 2
T4 = 3000  // °R

v1 = v_TP("Air",T1, p1)
u1 = u_T("Air", T1)
s1 = s_TP("Air", T1, p1)
v2 = v_TP("Air",T2, p2)
v1 / v2 = 16
s2 = s_TP("Air", T2, p2)
s2 = s1
u2 = u_T("Air", T2)
T3 / T2 = p3 / p2
u3 = u_T("Air", T3)
h3 = h_T("Air", T3)
v3 = v2
h4 = h_T("Air", T4)
v4 = v_TP("Air",T4, p4)
p4 = p3
s4 = s_TP("Air", T4, p4)
v5 = v_TP("Air",T5, p5)
s5 = s_TP("Air", T5, p5)
s5 = s4
v5 = v1
u5 = u_T("Air", T5)

v1 = V1 / m
Qin = m * (u3 - u2) + m * (h4 - h3)
Q51 = m * (u1 - u5)
Wcycle = Qin + Q51
eta = Wcycle / Qin
mep = Wcycle / (V1 * (1 - V2/ V1)) * (778 / 144)
```

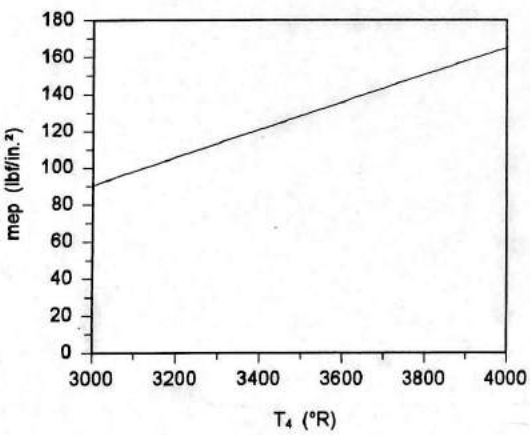
<u>*IT* Results for T_4 = 3000°R</u>

T_2 = 1477°R
T_3 = 2954°R
T_5 = 1196°R
m = 0.03838 lb
Q_{in} = 12.49 Btu
W_{cycle} = 7.837 Btu
η= 0.6273
mep = 90.33 lbf/in.²

PLOTS:







As the maximum cycle temperature increases, we see that the net work and mep both increase as well. However, the heat addition also goes up, and as a result the thermal efficiency decreases slightly.

PROBLEM 9.39

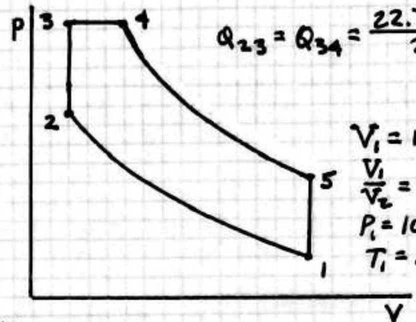
An air-standard dual cycle has a compression ratio of 9. At the beginning of compression, $p_1 = 100$ kPa, $T_1 = 300$ K, and $V_1 = 14$ L. The total amount of energy added by heat transfer is 22.7 kJ. Plot the temperatures at the end of each heat addition process, in K, the net work per unit of mass of air, in kJ/kg, the thermal efficiency, and the mean effective pressure, in kPa, versus the ratio of the constant-volume heat addition to total heat addition varying from 0 to 1.

KNOWN: Operating data are provided for an air-standard dual cycle.

FIND: Plot specified quantities versus the ratio of the constant-volume heat addition to total heat addition.

ENGINEERING MODEL: See Example 9.3.

ANALYSIS: Begin with calculations for the sample case: $Q_{23} = Q_{34} = \frac{22.7\text{ kJ}}{2} = 11.35$ kJ.



$$Q_{23} = Q_{34} = \frac{22.7\text{ kJ}}{2} = 11.35\text{ kJ}$$

$V_1 = 14\text{L} = 0.014\text{ m}^3$

$\frac{V_1}{V_2} = 9$

$P_1 = 100$ kPa

$T_1 = 300$ K

Using data from Table A-22, obtain data at each principal state.

State 1: $T_1 = 300\text{K}$, $p_1 = 100$ kPa

$\Rightarrow u_1 = 214.07$ kJ/kg, $v_{r1} = 621.2$

State 2: For the isentropic compression

$$v_{r2} = v_{r1}\left(\frac{V_2}{V_1}\right) = \frac{621.2}{9} = 69.022$$

$\Rightarrow T_2 = 702.7$ K, $u_2 = 514.49$ kJ/kg

State 3: For process 2-3; $Q_{23} = m(u_3 - u_2) \Rightarrow u_3 = \frac{Q_{23}}{m} + u_2$

The mass is

$$m = \frac{P_1V_1}{RT_1} = \frac{(100\text{ kPa})(0.014\text{ m}^3)}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(300\text{K})}\left|\frac{10^3\text{ N/m}^2}{1\text{ kPa}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| = 0.01626\text{ kg}$$

Thus $u_3 = \frac{(11.35)}{(0.01626)} + 514.49 = 1212.52$ kJ/kg $\Rightarrow \begin{cases} T_3 = 1507.7\text{K} \\ h_3 = 1645.31\ \frac{\text{kJ}}{\text{kg}} \end{cases}$ ← T_3

State 4: For the constant pressure heat addition, $Q_{34} = m(h_4 - h_3)$. Thus

$$h_4 = \frac{Q_{34}}{m} + h_3 = \frac{(11.35)}{(0.01626)} + 1645.31 = 2343.34\text{ kJ/kg}$$

Thus $\Rightarrow T_4 = 2072.9$K ← T_4

$v_{r4} = 2.4639$

State 5: $v_{r5} = v_{r4}\left(\frac{V_5}{V_4}\right) = v_{r4}\left(\frac{V_1}{V_4}\right) = v_{r4}\left(\frac{V_1}{V_2}\cdot\frac{V_3}{V_4}\right) = (2.4639)(9)\left(\frac{1507.7}{2072.9}\right)$

$= 16.1298$

$\Rightarrow T_5 = 1158.5$K, $u_5 = 896.6\ \frac{\text{kJ}}{\text{kg}}$

since $P_3 = P_4$; $T_3/T_4 = V_3/V_4$

(b) $W_{cycle} = Q_{cycle} = Q_{23} + Q_{34} - Q_{51} = Q_{23} + Q_{34} - m(u_5 - u_1)$

$= (22.7) - (0.01626)(896.6 - 214.07) = 22.7 - 11.098 = 11.60$ kJ

$W_{cycle}/m = 713.4$ kJ/kg ← W_{cycle}/m

(c) $\eta = W_{cycle}/Q_{in} = 11.60/22.7 = 0.511$ (51.1%) ← η

(d) The mean effective pressure is

$$mep = \frac{W_{cycle}}{V_1(1 - V_2/V_1)}$$

$$= \frac{(11.60\text{ kJ})}{(0.014\text{ m}^3)(1 - \frac{1}{9})}\left|\frac{10^3\text{ N}\cdot\text{m}}{1\text{ kJ}}\right|\left|\frac{1\text{ kPa}}{10^3\text{ N/m}^2}\right| = 932.1\text{ kPa}$$ ← mep

These values correspond to the ratio $Q_{23}/(Q_{23} + Q_{34}) = 0.5$. The plots corresponding to $0 \le \text{ratio} \le 1$ are considered next.

PROBLEM 9.39 (Continued)

Continuing, the required data are obtained using *IT*, as follows:

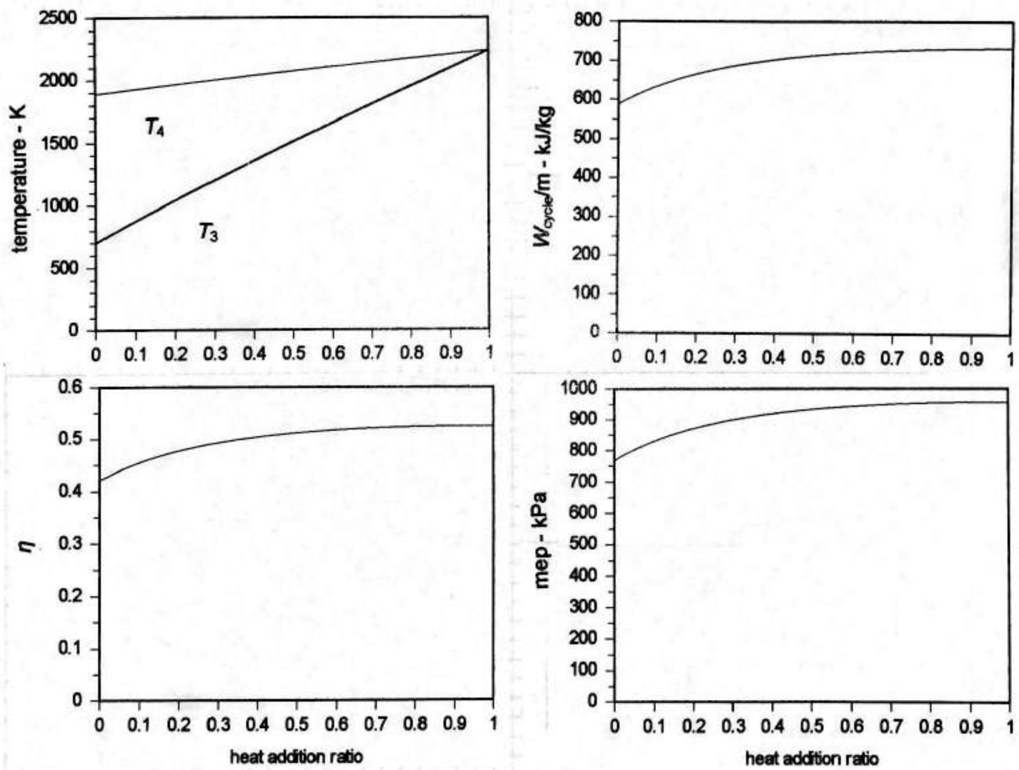
<u>*IT* Code</u>

```
p1 = 100  // kPa
T1 = 300  // K
v1 / v2 = 9
V1 = 14 /1000  // m³
Q23 + Q34 = 22.7  // kJ
Q23 / (Q23 + Q34) = ratio
ratio = .5
m = V1 / v1

u1 = u_T("Air", T1)
v1 = v_TP("Air",T1, p1)
s1 = s_TP("Air", T1, p1)
v2 = v_TP("Air",T2, p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
Q23 = m * (u3 - u2)
u2 = u_T("Air", T2)
u3 = u_T("Air", T3)
Q34 = m * (h4 - h3)
h3 = h_T("Air", T3)
h4 = h_T("Air", T4)
p3 / p2 = T3 / T2
p4 = p3
v4 = v_TP("Air",T4, p4)
s4 = s_TP("Air", T4, p4)
v5 = v1
v5 = v_TP("Air",T5, p5)
s5 = s_TP("Air", T5, p5)
s5 = s4
u5 = u_T("Air", T5)

Wcycle = Q23 + Q34 - Q51
Q51 = m * (u5 - u1)
wcycle = Wcycle / m
eta = Wcycle / (Q23 + Q34)
mep = (Wcycle / m) / (v1 - v2)
```

<u>*IT* Results for $Q_{23} = Q_{34} = 11.35$ kJ</u>

$T_2 = 702.8$ K
$T_3 = 1509$ K
$T_4 = 2074$ K
$T_5 = 1159$ K
$u_1 = 213.9$ kJ/kg
$u_2 = 514.3$ kJ/kg
$u_3 = 1212$ kJ/kg
$h_3 = 1645$ kJ/kg
$h_4 = 2343$ kJ/kg
$u_5 = 896.2$ kJ/kg
$Q_{51} = 11.09$ kJ
$w_{cycle}/m = 713.7$ kJ/kg
$\eta = 0.5113$
mep = 932.6 kPa

PLOTS:

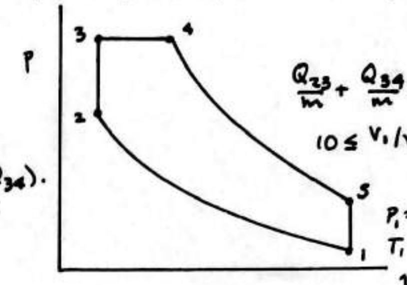


PROBLEM 9.40

KNOWN: An air-standard dual cycle has a known compression ratio and a specified state at the beginning of compression. The total heat addition is given.

FIND: Plot the net work per unit mass, the mean effective pressure, and the thermal efficiency versus compression ratio ranging from 10 to 20 for various values of the ratio of constant volume to total heat input.

SCHEMATIC & GIVEN DATA:



$\frac{Q_{23}}{m} + \frac{Q_{34}}{m} = 1000$ kJ/kg

$10 \le V_1/V_2 \le 20$

$p_1 = 1$ bar

$T_1 = 300$K

ENGINEERING MODEL: See Example 9.3.

ANALYSIS: The following reasoning is used to solve the problem:

1. Given values: p_1, T_1, r, and $Q_{23}/(Q_{23}+Q_{34})$.
2. Fix state 2 using ideal gas relations and $s(T_1, p_1) = s(T_2, p_2)$. When using Table A-22, this expressed in terms of $v_{r_1}/v_{r_2} = v_1/v_2$.
3. Use $Q_{23} = m(u_3 - u_2)$ to get T_3. Then $v_3 = v_2$ gives $p_3 = (T_3/T_2)\,p_2$.
4. For state 4, use $p_4 = p_3$ and $Q_{34} = m(h_4 - h_3)$ to get T_4, v_4.
5. Fix state 5 as in step 2 using $s(T_4, p_4) = s(T_5, p_5)$ or $v_{r4}/v_{r5} = v_4/v_5$. Also, note that $v_5 = v_1$.

Now, using energy balances: $W_{cycle} = Q_{cycle} \Rightarrow \frac{W_{cycle}}{m} = \frac{Q_{23}}{m} + \frac{Q_{34}}{m} - (u_5 - u_1)$

where $Q_{out}/m = u_5 - u_1$. Also

$$\eta = \frac{W_{cycle}/m}{(Q_{23}+Q_{34})/m} \quad \text{and} \quad mep = \frac{W_{cycle}/m}{v_1(1 - 1/r)}$$

► Sample calculation. $\boxed{r = 10,\ Q = Q_{23}/(Q_{23}+Q_{34}) = 0.5}$ From Table A-22 at $T_1 = 300$K ; $u_1 = 214.07$ kJ/kg , $v_{r_1} = 621.2$. Thus $v_{r_2} = (v_2/v_1)\,v_{r_1} = 62.12$. Interpolating in Table A-22 ; $T_2 = 730$ K, $u_2 = 536.1$ kJ/kg. Now

$$u_3 = Q_{23}/m + u_2 = 1036.1 \text{ kJ/kg} \Rightarrow T_3 = 1314.7\text{K},\ h_3 = 1413.46 \text{ kJ/kg}$$

Further, $h_4 = Q_{34}/m + h_3 = 1913.46$ kJ/kg $\Rightarrow T_4 = 1727.1$K , $v_{r_4} = 4.526$

$$v_{r5} = (v_5/v_4)\,v_{r4} = (v_1/v_2)(v_3/v_4)\,v_{r4} = (v_1/v_2)(T_3/T_4)\,v_{r4} = 34.453$$

Thus, $T_5 = 898.8$ K , $u_5 = 673.55$ kJ/kg.

Now

$$W_{cycle}/m = 1000 - (673.55 - 214.07) = 540.5 \text{ kJ/kg}$$

$$\eta = \frac{W_{cycle}/m}{(Q_{23}+Q_{34})/m} = 0.5405 \ (54.05\%)$$

$$v_1 = \frac{RT_1}{p_1} = \frac{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(300\text{K})}{(1\text{ bar})}\left|\frac{10^3 \text{N}\cdot\text{m}}{1 \text{kJ}}\right|\left|\frac{1 \text{bar}}{10^5 \text{N/m}^2}\right| = 0.861 \text{ m}^3\text{/kg}$$

$$mep = \frac{(540.5 \text{ kJ/kg})}{(0.861 \text{m}^3\text{/kg})(1 - 1/10)}\left|\frac{10^3 \text{N}\cdot\text{m}}{1\text{kJ}}\right|\left|\frac{1 \text{bar}}{10^5 \text{N/m}^2}\right| = 6.975 \text{ bar}$$

The data for the required plots are obtained using IT, as follows:

PROBLEM 9.40 (Cont'd.) - Page 2

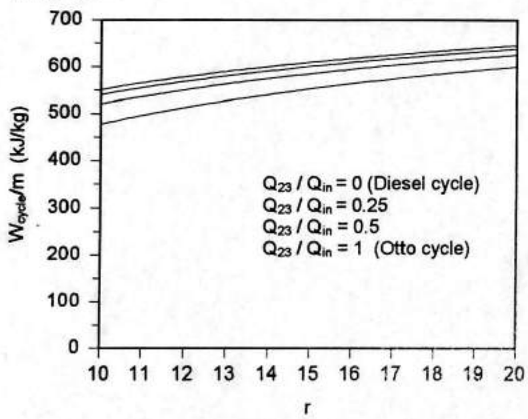
<u>IT Code</u>

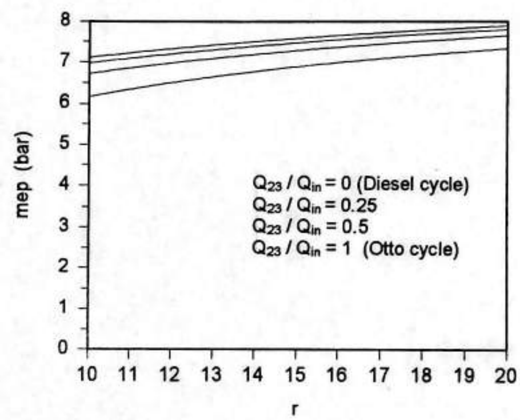
```
p1 = 1   // bar
T1 = 300   // K
(Q23 + Q34) / m = 1000   // kJ/kg
Q = Q23 / (Q23 + Q34)
Q = .5
r = 10
m = 1   // Assume a unit mass of 1
kg.

v1 = v_TP("Air",T1,p1)
s1 = s_TP("Air", T1, p1)
u1 = u_T("Air", T1)
s2 = s1
v2 = v1 / r
v2 = v_TP("Air",T2,p2)
s2 = s_TP("Air", T2, p2)
u2 = u_T("Air", T2)
m * (u3 - u2) = Q23
u3 = u_T("Air", T3)
v3 = v2
v3 = v_TP("Air",T3,p3)
s3 = s_TP("Air", T3, p3)
h3 = h_T("Air", T3)
Q34 / m = h4 - h3
h4 = h_T("Air", T4)
p4 = p3
v4 = v_TP("Air",T4,p4)
s4 = s_TP("Air", T4, p4)
u4 = u_T("Air", T4)
v5 = v1
s5 = s4
v5 = v_TP("Air",T5,p5)
s5 = s_TP("Air", T5, p5)
u5 = u_T("Air", T5)

Wcycle = Q23 / m + Q34 / m + Q51 / m
Q51 = m * (u1 - u5)
eta = Wcycle / (Q23 + Q34)
mep = (Wcycle / m) / (v1 * (1 - 1 / r)) / 100
```

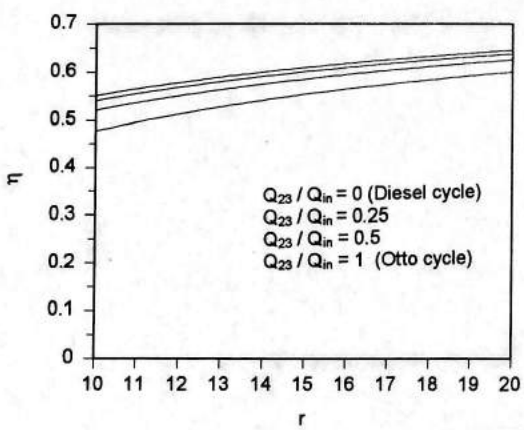
<u>IT Results for Q23 / Q1n = 0.5 and r = 10</u>

T_2 = 730.2 K
p_2 = 24.34 bar
T_3 = 1316 K
p_3 = 43.86 bar
T_4 = 1729 K
T_5 = 899.3 K
W_{cycle}/m = 540.2 kJ/kg
η = 0.5402
mep = 6.972 bar

PLOTS:







PROBLEM 9.41

KNOWN: An expression is given for the thermal efficiency of a cold air-standard dual cycle in terms of compression ratio, r, cutoff ratio, r_c, and pressure ratio for the constant volume heat addition, r_p.

FIND: Derive the expression.

ENGINEERING MODEL: See Example 9.3. Also, the specific heats are constant.

ANALYSIS: Begin with Eq. 9.14, which is numbered as in Fig. 9.7:

$$\eta = 1 - \frac{(u_5 - u_1)}{(u_3 - u_2) + (h_4 - h_3)}$$

$$= 1 - \frac{c_v (T_5 - T_1)}{c_v (T_3 - T_2) + c_p (T_4 - T_3)} = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + k (T_4 - T_3)} \qquad (*)$$

Note that $r = v_1/v_2$, $r_c = v_4/v_3$, and $r_p = p_3/p_2$. Therefore

$$T_1 = \left(\frac{v_2}{v_1}\right)^{k-1} T_2 = \frac{T_2}{r^{k-1}}$$

$$T_3 = \left(\frac{p_3}{p_2}\right) T_2 = r_p T_2$$

$$T_4 = \left(\frac{v_4}{v_3}\right) T_3 = r_c T_3 = r_c r_p T_2$$

$$T_5 = \left(\frac{v_4}{v_5}\right)^{k-1} T_4 = \left(\frac{v_4}{v_3} \cdot \frac{v_2}{v_1}\right)^{k-1} T_4 = \left(r_c \cdot \frac{1}{r}\right)^{k-1} T_4 = r_p \left(\frac{r_c^k}{r^{k-1}}\right) T_2$$

Substituting these relations into (*) gives

$$\eta = 1 - \frac{1}{r^{k-1}} \left[\frac{r_p r_c^k - 1}{(r_p - 1) + k\, r_p (r_c - 1)}\right] \longleftarrow \text{result}$$

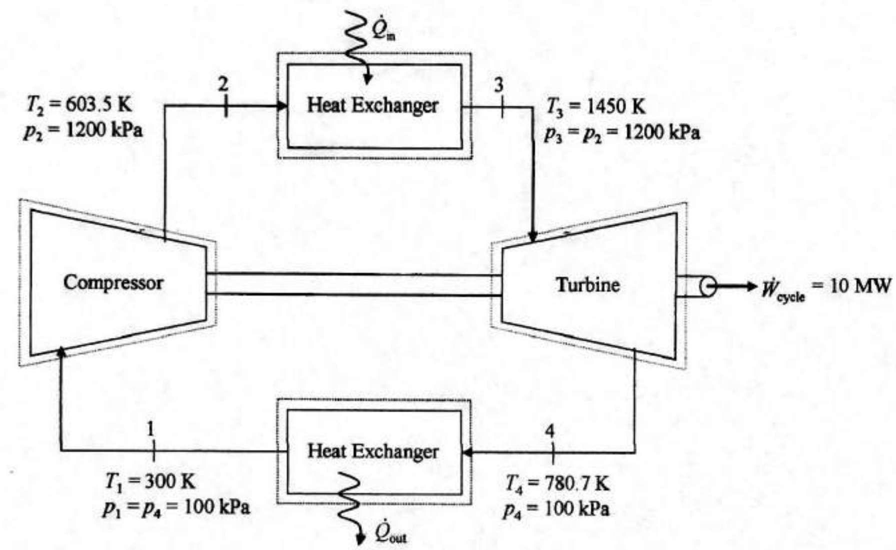
9.42 An ideal air-standard Brayton cycle produces 10 MW of power. Operating data at principal states in the cycle are given in the table below. The states are numbered as in Fig. 9.9. Sketch the *T-s* diagram for the cycle and determine
(a) the mass flow rate of air, in kg/s.
(b) the rate of heat transfer, in kW, to the working fluid passing through the heat exchanger.
(c) the thermal efficiency.

State	p (kPa)	T (K)	h (kJ/kg)
1	100	300	300.19
2	1200	603.5	610.65
3	1200	1450	1575.57
4	100	780.7	800.78

KNOWN: An ideal air-standard Brayton cycle operates with property data given at principal states. The net power output of the cycle is given.

FIND: Determine the mass flow rate of air, the rate of heat transfer to the working fluid passing through the heat exchanger, and the thermal efficiency.

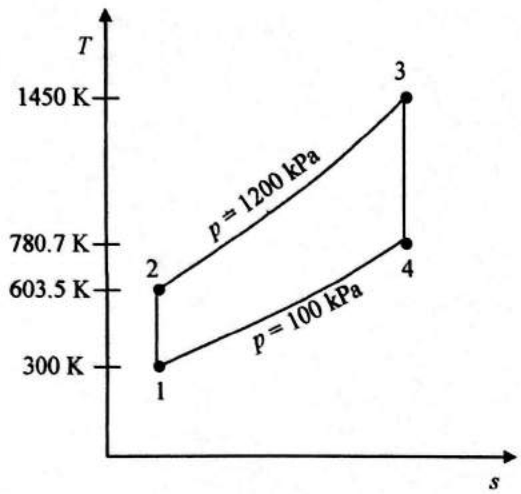
SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. All processes of the working fluid are internally reversible.
3. The turbine and compressor operate adiabatically.

4. There are no pressure drops for flow through the heat exchangers.
5. Kinetic and potential energy effects are negligible.
6. The working fluid is air modeled as an ideal gas.

ANALYSIS: The *T-s* diagram for the cycle is shown below.



(a) The mass flow rate of air is found as follows. Mass and energy rate balances for control volumes enclosing the turbine and compressor give

$$\dot{W}_t = \dot{m}(h_3 - h_4) \quad \text{and} \quad \dot{W}_c = \dot{m}(h_2 - h_1)$$

The net power of the cycle is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = \dot{m}[(h_3 - h_4) - (h_2 - h_1)]$$

Solving for $\dot{m}$

$$\dot{m} = \frac{\dot{W}_{cycle}}{[(h_3 - h_4) - (h_2 - h_1)]}$$

Inserting values

$$\dot{m} = \frac{10{,}000 \text{ kW}}{\left(1575.57\frac{\text{kJ}}{\text{kg}} - 800.78\frac{\text{kJ}}{\text{kg}}\right) - \left(610.65\frac{\text{kJ}}{\text{kg}} - 300.19\frac{\text{kJ}}{\text{kg}}\right)}\left|\frac{\frac{\text{kJ}}{\text{s}}}{\text{kW}}\right| = \underline{\mathbf{21.54\ kg/s}}$$

(b) The rate of heat transfer to the working fluid passing through the heat exchanger can be determined by applying mass and energy balances to a control volume around the heat exchanger to give

$$\dot{Q}_{in} = \dot{m}(h_3 - h_2) = \left(21.54\frac{\text{kg}}{\text{s}}\right)\left(1575.57\frac{\text{kJ}}{\text{kg}} - 610.65\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{\text{kW}}{\frac{\text{kJ}}{\text{s}}}\right| = \underline{\mathbf{20{,}784\ kW}}$$

(c) The thermal efficiency is

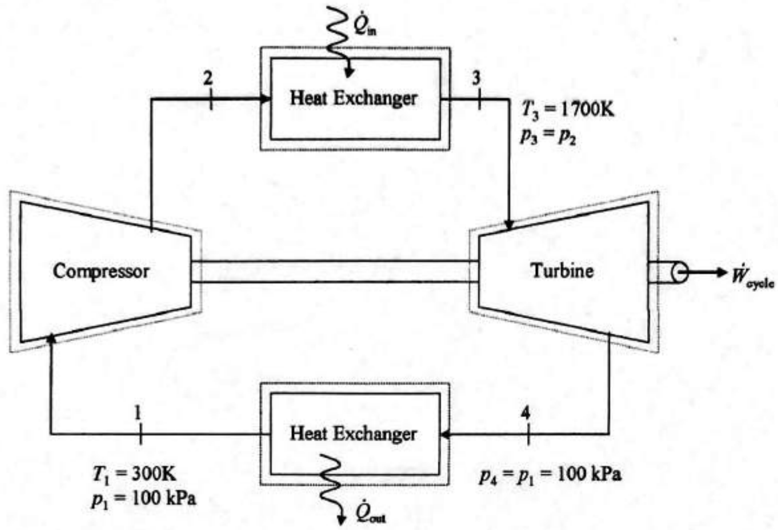
$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = (10{,}000 \text{ kW})/(20{,}784.4 \text{ kW}) = \underline{\mathbf{0.481\ (48.1\%)}}$$

9.43 An ideal air standard Brayton cycle operates at steady state with compressor inlet conditions of 300 K and 100 kPa and a fixed turbine inlet temperature of 1700 K. For the cycle,
(a) determine the net work per unit mass flowing, in kJ/kg, and the thermal efficiency for a compressor pressure ratio of 8.
(b) plot the net work per unit mass flowing, in kJ/kg, and the thermal efficiency, each versus compressor pressure ratio ranging from 2 to 50.

KNOWN: An ideal air standard Brayton cycle operates with fixed compressor inlet conditions of 300 K and 100 kPa and fixed turbine inlet temperature of 1700.

FIND: (a) the net power per unit mass flowing and the thermal efficiency for a compressor pressure ratio of 8 and (b) plot the net power per unit mass flowing and the thermal efficiency, each versus compressor pressure ratio ranging from 2 to 50.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. Air, modeled as an ideal gas, is the working fluid.
3. All processes of the working fluid are internally reversible.
4. The compressor and turbine operate adiabatically.
5. Kinetic and potential energy effects are negligible.

ANALYSIS: (a) The net work of the cycle per unit of mass flow using an air standard analysis

Problem 9.43 (Continued) – Page 2

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = (h_3 - h_4) - (h_2 - h_1)$$

State 1: $T_1 = 300$ K → From Table A-22: $h_1 = 300.19$ kJ/kg and $p_{r1} = 1.3860$.

Process 1-2 is an isentropic process. Thus

$$\frac{p_2}{p_1} = \frac{p_{r2}}{p_{r1}} \Rightarrow p_{r2} = p_{r1}\frac{p_2}{p_1} = 1.3860(8) = 11.088$$

From Table A-22 (interpolated): $h_2 = 544.18$ kJ/kg.

State 3: $T_3 = 1700$ K → From Table A-22: $h_3 = 1880.1$ kJ/kg and $p_{r3} = 1025$.

Process 3-4 is an isentropic process. Thus

$$\frac{p_4}{p_3} = \frac{p_1}{p_2} = \frac{p_{r4}}{p_{r3}} \Rightarrow p_{r4} = p_{r3}\frac{p_1}{p_2} = 1025\left(\frac{1}{8}\right) = 128.125$$

From Table A-22 (interpolated): $h_4 = 1079.85$ kJ/kg.

Solving for net work of the cycle per unit of mass flow

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = \left(1880.1\frac{\text{kJ}}{\text{kg}} - 1079.85\frac{\text{kJ}}{\text{kg}}\right) - \left(544.18\frac{\text{kJ}}{\text{kg}} - 300.19\frac{\text{kJ}}{\text{kg}}\right) = \underline{\mathbf{556.3\ kJ/kg}}$$

Thermal efficiency is

$$\eta = \frac{\dot{W}_{\text{cycle}}/\dot{m}}{\dot{Q}_{\text{in}}/\dot{m}} = \frac{\dot{W}_{\text{cycle}}/\dot{m}}{h_3 - h_2} = \frac{556.3\frac{\text{kJ}}{\text{kg}}}{1880.1\frac{\text{kJ}}{\text{kg}} - 544.18\frac{\text{kJ}}{\text{kg}}} = \underline{\mathbf{0.4164\ (41.64\%)}}$$

IT Code

```
/*ANALYSIS: Air Standard Analysis*/

rp = 8
mdot = 1 // kg/s

/* State 1 */
p1 = 100 // kPa
T1 = 300 // K
s1 = s_Tp("Air",T1,p1) // kJ/(kg-K)
h1 = h_T("Air",T1) // kJ/kg
```

```
/* State 2 */
s2 = s1 // kJ/(kg-K)
p2 = rp * p1 // kPa
s_Tp("Air",T1,p1) = s_Tp("Air",T2,p2) // Returns T2 in K
h2 = h_T("Air",T2) // kJ/kg

/* State 3 */
T3 = 1700 // K
p3 = p2 // kPa
s3 = s_Tp("Air",T3,p3) // kJ/(kg-K)
h3 = h_T("Air",T3) // kJ/kg

/* State 4 */
p4 = p1 // kPa
s_Tp("Air",T3,p3) = s_Tp("Air",T4,p4) // Returns T4 in K
h4 = h_T("Air",T4) // kJ/kg
s4 = s3 // kJ/(kg-K)

/* Energy Transfers and Cycle Performance */
Wdotcyclepermdot = (h3 - h4) - (h2 - h1) // kJ/kg
Qdotinpermdot = h3 - h2 // kJ/kg
eta = Wdotcyclepermdot / Qdotinpermdot
```

IT Results for pressure ratio of 8 (Note: The values in this table compare favorably to those calculated above using Air Table data. The specific heat functions used in *IT* are not **exactly** the ones used to generate the Air Tables. That and round-off account for the slight differences.)

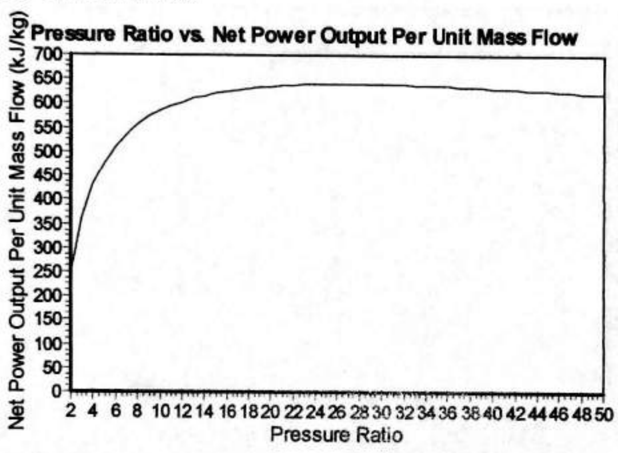
eta	0.4166
h1	300
h2	544.1
h3	1878
h4	1078
p2	800
p3	800
p4	100
Qdotinpermdot	1334
s1	1.706
s2	1.706
s3	3.004
s4	3.004
T2	539.8
T4	1029
Wdotcyclepermdot	555.8
mdot	1
p1	100
rp	8
T1	300
T3	1700

IT Results for rp = 25 through rp = 29

eta	0.5682	0.5727	0.577	0.581	0.5849
h1	300	300	300	300	300
h2	751.2	759.5	767.6	775.4	783.1
h3	1878	1878	1878	1878	1878
h4	786.7	778.1	769.9	762.1	754.6
p2	2500	2600	2700	2800	2900
p3	2500	2600	2700	2800	2900
p4	100	100	100	100	100
Qdotinpermdot	1127	1119	1111	1103	1095
s1	1.706	1.706	1.706	1.706	1.706
s2	1.706	1.706	1.706	1.706	1.706
s3	2.677	2.665	2.655	2.644	2.634
s4	2.677	2.665	2.655	2.644	2.634
T2	735.4	743.1	750.5	757.8	764.8
T4	768.1	760.2	752.6	745.4	738.6
Wdotcyclepermdot	640.5	640.7	640.9	640.8	640.6
mdot	1	1	1	1	1
p1	100	100	100	100	100
rp	25	26	27	28	29
T1	300	300	300	300	300
T3	1700	1700	1700	1700	1700

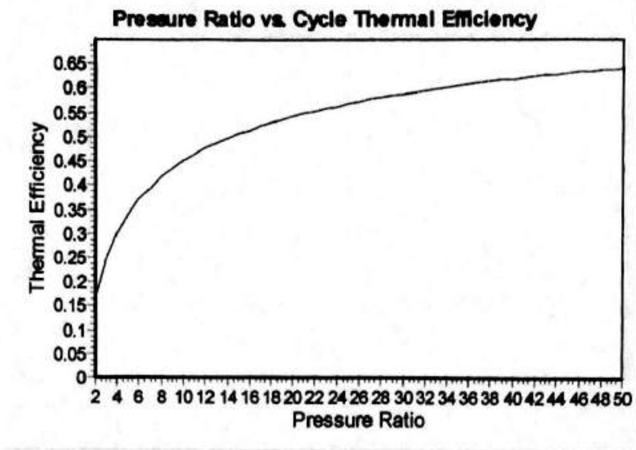
Plots

Power Output Per Unit Mass Flow



Problem 9.43 (Continued) – Page 5

Thermal Efficiency



Discussion

Maximum net work output per unit mass flow (640.9 kJ/kg) occurs at a pressure ratio of 27 using air standard analysis. Thermal efficiency continues to increase with increasing pressure ratio.

9.44 An ideal cold air standard Brayton cycle operates at steady state with compressor inlet conditions of 300 K and 100 kPa, fixed turbine inlet temperature of 1700 K, and $k = 1.4$. For the cycle,

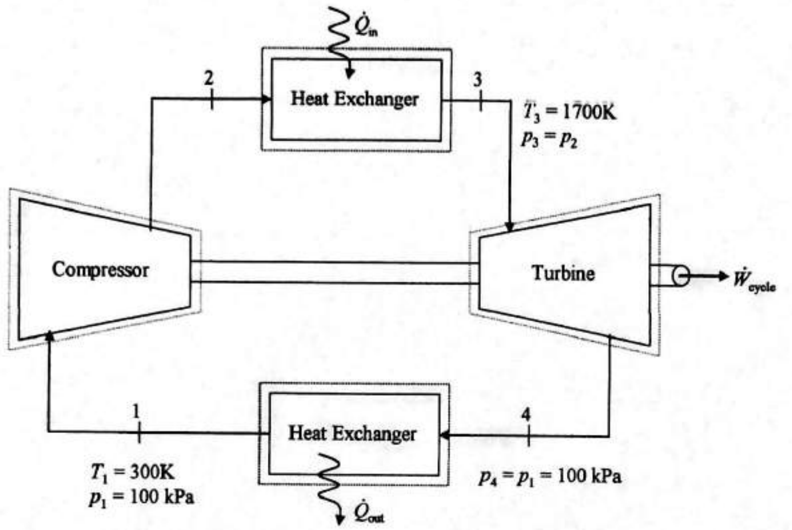
(a) determine the net power per unit mass flowing, in kJ/kg, and the thermal efficiency for a compressor pressure ratio of 8.

(b) plot the net power per unit mass flowing, in kJ/kg, and the thermal efficiency, each versus compressor pressure ratio ranging from 2 to 50.

KNOWN: An ideal cold air standard Brayton cycle operates with fixed compressor inlet conditions of 300 K and 100 kPa and fixed turbine inlet temperature of 1700.

FIND: (a) the net power per unit mass flowing and the thermal efficiency for a compressor pressure ratio of 8 and (b) plot the net power per unit mass flowing and the thermal efficiency, each versus compressor pressure ratio ranging from 2 to 50.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. Air, modeled as an ideal gas, is the working fluid.
3. All processes of the working fluid are internally reversible.
4. The compressor and turbine operate adiabatically.
5. Kinetic and potential energy effects are negligible.
6. Specific heats of air are constant with $k = 1.4$.

ANALYSIS: (a) The net work of the cycle per unit of mass flow using a cold air standard analysis

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = c_p[(T_3 - T_4) - (T_2 - T_1)]$$

<u>State 1</u>: $T_1 = 300$ K.

Process 1-2 is an isentropic process. Thus

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k} \Rightarrow T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(k-1)/k} = (300\text{ K})(8)^{(1.4-1)/1.4} = 543.4\text{ K}$$

<u>State 3</u>: $T_3 = 1700$ K.

Process 3-4 is an isentropic process. Thus

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{(k-1)/k} = \left(\frac{p_1}{p_2}\right)^{(k-1)/k} \Rightarrow T_4 = T_3\left(\frac{p_1}{p_2}\right)^{(k-1)/k} = (1700\text{ K})\left(\frac{1}{8}\right)^{(1.4-1)/1.4} = 938.5\text{ K}$$

From Table A-20 for air, $c_p = 1.005$ kJ/(kg·K). Solving for net work of the cycle per unit of mass flow

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = \left(1.005\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)[(1700\text{ K} - 938.5\text{ K}) - (543.4\text{ K} - 300\text{ K})] = \underline{\mathbf{520.7\ kJ/kg}}$$

Thermal efficiency is given by

$$\eta = \frac{\dot{W}_{\text{cycle}}/\dot{m}}{\dot{Q}_{\text{in}}/\dot{m}} = \frac{\dot{W}_{\text{cycle}}/\dot{m}}{c_p(T_3 - T_2)} = \frac{520.7\frac{\text{kJ}}{\text{kg}}}{\left(1.005\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(1700\text{ K} - 543.4\text{ K})} = \underline{\mathbf{0.4480\ (44.80\%)}}$$

(b) **<u>IT code</u>**

```
/* ANALYSIS:  Cold Air Standard Analysis*/

cp = 1.005 // kJ/(kg-K)
k = 1.4
rp = 8
mdot = 1 // kg/s

/* State 1 */
p1 = 100 // kPa
T1 = 300 // K

/* State 2 */
```

Problem 9.44 (Continued) – Page 3

```
p2 = rp * p1 // kPa
T2 = T1*rp^((k-1)/k) // K

/* State 3 */
T3 = 1700 // K
p3 = p2 // kPa

/* State 4 */
p4 = p1 // kPa
T4 = T3*(1/rp)^((k-1)/k) // K

/* Energy Transfers and Cycle Performance */
Wdotcyclepermdot = cp*((T3 - T4) - (T2 - T1)) // kJ/kg
Qdotinpermdot = cp*(T3 - T2) // kJ/kg
eta = Wdotcyclepermdot / Qdotinpermdot
```

IT Results for pressure ratio of 8

eta	0.448
p2	800
p3	800
p4	100
Qdotinpermdot	1162
T2	543.4
T4	938.5
Wdotcyclepermdot	520.7
cp	1.005
k	1.4
mdot	1
p1	100
rp	8
T1	300
T3	1700

These values compare favorably with the values calculated above.

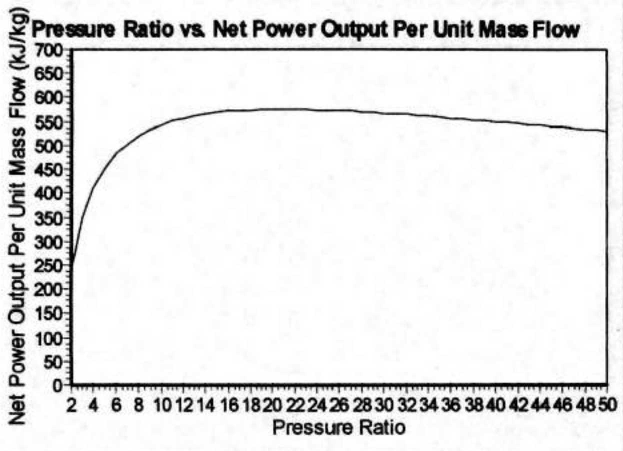
Problem 9.44 Continued) – Page 4

IT Results for rp = 19 through rp = 23

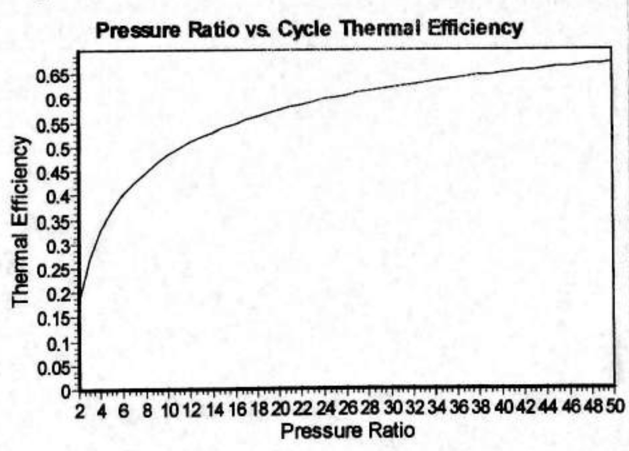
eta	0.5688	0.5751	0.581	0.5865	0.5917
p2	1900	2000	2100	2200	2300
p3	1900	2000	2100	2200	2300
p4	100	100	100	100	100
Qdotinpermdot	1009	998.9	988.9	979.3	970
T2	695.8	706.1	716	725.6	734.8
T4	733	722.3	712.3	702.9	694
Wdotcyclepermdot	574.1	574.5	574.6	574.4	574
cp	1.005	1.005	1.005	1.005	1.005
k	1.4	1.4	1.4	1.4	1.4
mdot	1	1	1	1	1
p1	100	100	100	100	100
rp	19	20	21	22	23
T1	300	300	300	300	300
T3	1700	1700	1700	1700	1700

Plots

Net Power Output Per Unit Mass Flow



Thermal Efficiency



Discussion

Maximum net work output per unit mass flow (574.6 kJ/kg) occurs at a pressure ratio of 21 using cold air standard analysis. Thermal efficiency continues to increase with increasing pressure ratio.

PROBLEM 9.45

9.45 For an ideal Brayton cycle on a cold air-standard basis show that

(a) the back work ratio is given by

$$\text{bwr} = T_1/T_4$$

where T_1 is the temperature at the compressor inlet and T_4 is the temperature at the turbine exit.

(b) the temperature at the compressor exit that maximizes the net work developed per unit of mass flowing is given by

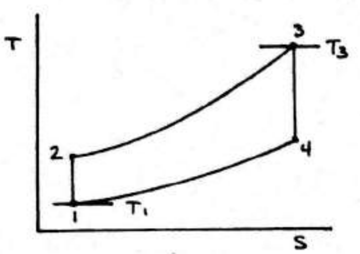
$$T_2 = (T_1 T_3)^{1/2}$$

where T_1 is the temperature at the compressor inlet and T_3 is the temperature at the turbine inlet.

KNOWN: An ideal cold air-standard Brayton cycle is under consideration.

FIND: (a) Show the back work ratio takes a specific form. (b) Show that the compressor edit temperature that maximizes the net work developed per unit of mass takes a specific form.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

See Example 9.5

(a)
$$\text{bwr} = \frac{h_2 - h_1}{h_3 - h_4} = \frac{c_p\,(T_2 - T_1)}{c_p\,(T_3 - T_4)}$$
$$= \frac{T_1\,(T_2/T_1 - 1)}{T_4\,(T_3/T_4 - 1)}$$

For the isentropic processes

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}, \quad \frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{k-1}{k}}$$

Then, since $P_3 = P_2$ and $P_4 = P_1$, we get $(T_2/T_1) = (T_3/T_4)$. Finally, $\text{bwr} = \frac{T_1}{T_4}$, as required.

(b) Using Eq. (a) of the solution to Example 9.5, the compressor pressure ratio for maximum net work developed per unit of mass flow is

$$\frac{P_2}{P_1} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}}$$

Further, as noted in part (a), for the isentropic compression,

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}}$$

Collecting results,

$$\left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}} \Rightarrow \frac{T_2}{T_1} = \left(\frac{T_3}{T_1}\right)^{\frac{1}{2}}$$

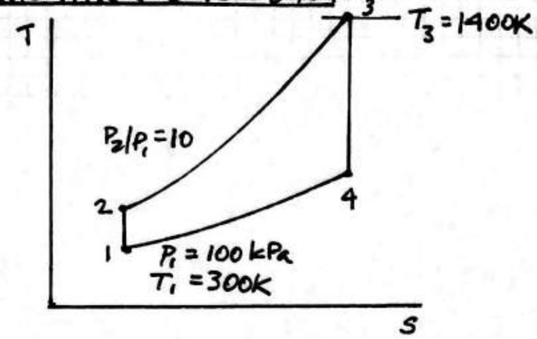
Finally, $T_2 = (T_1 T_3)^{1/2}$, as required.

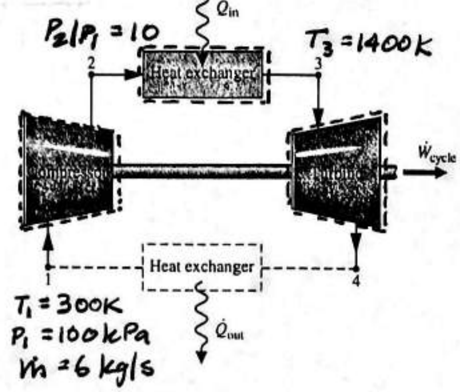
PROBLEM 9.46

KNOWN: Air enters a cold air-standard ideal Brayton cycle with a given flow rate and at a specified state. The compressor pressure ratio and maximum cycle temperature are known.

FIND: Determine (a) the thermal efficiency, (b) the back work ratio, and (c) the net power.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 9.4
Also, assume $k = 1.4$ and $c_p = 1.005$ kJ/kg·K.

ANALYSIS: First, determine T_2 and T_4, as follows:

(Eq. 9.23) $T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = (300)(10)^{\frac{1.4-1}{1.4}} = 579.2\text{ K}$

(Eq. 9.24) $T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = (1400)\left(\frac{1}{10}\right)^{\frac{1.4-1}{1.4}} = 725.13\text{ K}$

(a) To evaluate thermal efficiency, use

$$\eta = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}}$$

$\underline{\dot{Q}_{in}}$: $\dot{Q}_{in} = \dot{m}(h_3 - h_2) = \dot{m} c_p (T_3 - T_2)$

$= (6\text{ kg/s})(1.005 \frac{\text{kJ}}{\text{kg·K}})(1400 - 579.2)\text{K} = 4949.4\text{ kJ/s}$

$\underline{\dot{Q}_{out}}$: $\dot{Q}_{out} = \dot{m}(h_4 - h_1) = \dot{m} c_p (T_4 - T_1) = (6)(1.005)(725.13 - 300) = 2563.5 \frac{\text{kJ}}{\text{s}}$

Thus $\eta = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}} = 1 - \frac{2563.5}{4949.4} = 0.482\ (48.2\%) \longleftarrow \eta$

(b) The back work ratio is $bwr = \dot{W}_c / \dot{W}_t$

$\dot{W}_c = \dot{m}(h_2 - h_1) = \dot{m} c_p (T_2 - T_1) = (6)(1.005)(579.2 - 300)$
$= 1683.6\text{ kJ/s}$

$\dot{W}_t = \dot{m}(h_3 - h_4) = \dot{m} c_p (T_3 - T_4) = (6)(1.005)(1400 - 725.13) = 4069.5 \frac{\text{kJ}}{\text{s}}$

① and $bwr = \dot{W}_c / \dot{W}_t = \frac{1683.6}{4069.5} = 0.4137 \longleftarrow bwr$

(c) The net power developed is

$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = (4069.5 - 1683.6) \frac{\text{kJ}}{\text{s}} \left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = 2385.9\text{ kW} \longleftarrow \dot{W}_{cycle}$

---

1. Using the result of Problem 9.45(a)

$$bwr = \frac{T_1}{T_4} = \frac{300K}{725.13K} = 0.4137$$

which agrees with the value determined, as expected.

PROBLEM 9.47

See Problem 9.46

IT Code

```
T1 = 300 // K
p1 = 100 // kPa
rp = 10
p2 = p1 * rp
p3 = p2
T3 = 1400 // K
p4 = p1
k = 1.4
cp = 1.005 // kJ/kg·K
mdot = 6 // kg/s

T2 = T1 * (p2 / p1) ^ ((k-1)/k)
T4 = T3 * (p4 / p3) ^ ((k-1)/k)

Qdotin = mdot * cp * (T3 - T2)
Qdotout = mdot * cp * (T4 - T1)
eta = 1 - Qdotout / Qdotin
Wdotc = mdot * cp * (T2 - T1)
Wdott = mdot * cp * (T3 - T4)
bwr = Wdotc / Wdott
Wdotcycle = Wdott - Wdotc
```

IT Results for $r_p = 10$, $T_3 = 1400$ K

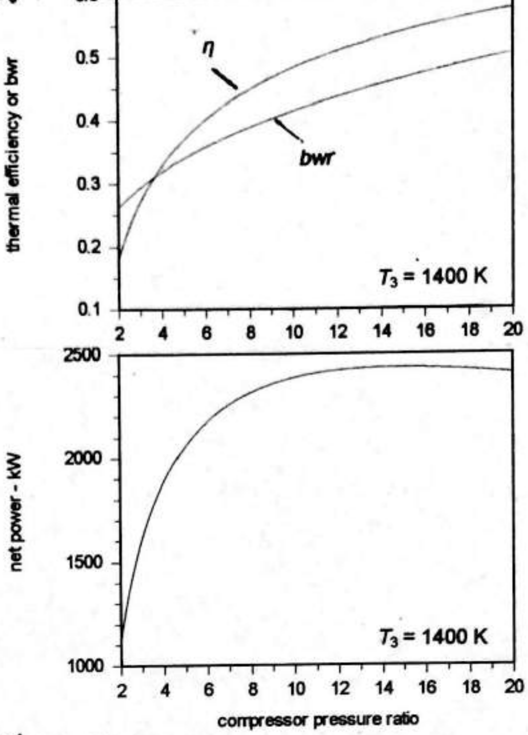
$T_2 = 579.2$ K
$T_4 = 725.1$ K
$bwr = 0.4137$
$\eta = 0.4821$
$\dot{Q}_{in} = 4949$ kW
$\dot{Q}_{out} = 2564$ kW
$\dot{W}_c = 1684$ kW
$\dot{W}_t = 4069$ kW
$\dot{W}_{cycle} = 2386$ kW

DISCUSSION:

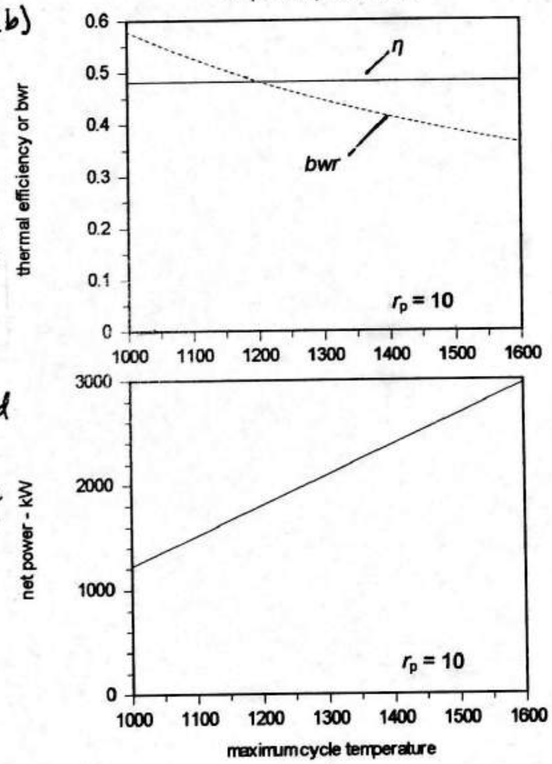
(a) Thermal efficiency and back work ratio both increase with increasing compressor pressure ratio at fixed T_3. The net power exhibits a maximum in the range of r_p studied. See the discussion of Sec. 9.6.2 and Ex. 9.5.

(b) For fixed compression ratio, the thermal efficiency of the cold air-standard cycle is constant with T_3, and the back work ratio increases. The net power increases as well as T_3 increases, since the compressor power is constant, but $\dot{W}_t$ increases.

(a)



(b)



PROBLEM 9.48

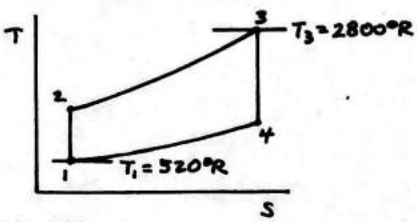
9.48 The rate of heat addition to an ideal air-standard Brayton cycle is 5.2×10^6 Btu/h. The pressure ratio for the cycle is 12 and the minimum and maximum temperatures are 520°R and 2800°R, respectively. Determine

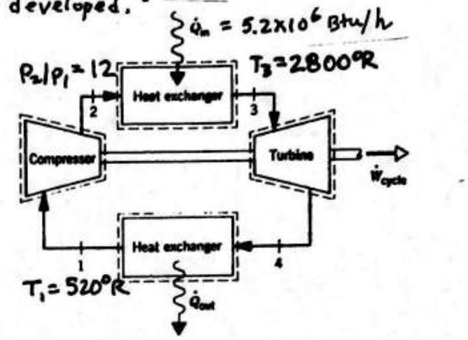
(a) the thermal efficiency of the cycle.
(b) the mass flow rate of air, in lb/h.
(c) the net power developed by the cycle, in hp.

KNOWN: The net power developed and the pressure ratio are known for an ideal air-standard Brayton cycle. The minimum and maximum temperatures are also known.

FIND: Determine (a) the thermal efficiency, (b) the mass flow rate of air, (c) the net power developed.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL: See Example 9.4

ANALYSIS: First, fix each of the principal states (Table A-22E).

State 1: $T_1 = 520°R \Rightarrow h_1 = 124.27$ Btu/lb, $p_{r1} = 1.2147$

State 2: For the isentropic compression, $p_{r2} = (P_2/P_1)\, p_{r1} = 14.5764$
Thus, $T_2 = 1047.6°R$ and $h_2 = 252.84$ Btu/lb

State 3: $T_3 = 2800°R \Rightarrow h_3 = 732.33$ Btu/lb, $p_{r3} = 702.0$

State 4: For the isentropic expansion, $p_{r4} = (P_4/P_3)\, p_{r3} = 58.5$
Thus, $T_4 = 1518°R$ and $h_4 = 373.95$ Btu/lb

(a) The thermal efficiency is evaluated using Eq. 9.19 in the form

$$\eta = 1 - \frac{\dot{Q}_{out}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = 1 - \frac{(h_4 - h_1)}{(h_3 - h_2)}$$

$$= 1 - \frac{(373.95 - 124.27)}{(732.33 - 252.84)} = 0.479 \; (47.9\%) \longleftarrow \eta$$

(b) To determine the mass flow rate of air, start with the rate of heat addition

$$\dot{m} = \frac{\dot{Q}_{in}}{(h_3 - h_2)} = \frac{(5.2 \times 10^6 \text{ Btu/h})}{(732.33 - 252.84) \text{ Btu/lb}}$$

$$= 1.084 \times 10^4 \text{ lb/h} \longleftarrow \dot{m}$$

(c) The net power is

$$\dot{W}_{cycle} = \dot{m}\left[(h_3 - h_4) - (h_2 - h_1)\right]$$

$$= (1.084 \times 10^4)\left[(732.33 - 373.95) - (252.84) - 124.27)\right]$$

$$= 2.49 \times 10^6 \text{ Btu/h} \longleftarrow \dot{W}_{cycle}$$

$$= 2.49 \times 10^6 \text{ Btu/h} \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| = 978 \text{ hp}$$

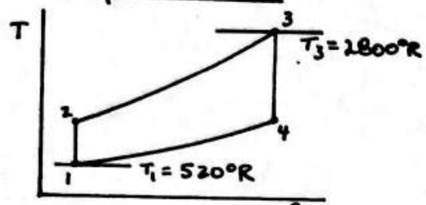
PROBLEM 9.49

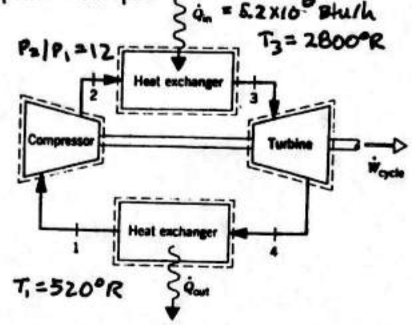
9.49 Solve Problem 9.48 on a cold air-standard basis with specific heats evaluated at 520°R.

KNOWN: The net power developed and the pressure ratio are known for a cold air-standard ideal Brayton cycle. The minimum and maximum temperatures are also known.

FIND: Determine (a) the thermal efficiency, (b) the mass flow rate of air, (c) the net power developed.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL: See Example 9.4
Also, assume constant specific heats evaluated at 520°R.

ANALYSIS: From Table A-20E, $k = 1.401$ and $c_p = 0.240$ Btu/lb·°R. $T_1 = 520°R$ and $T_3 = 2800°R$ are given. For the isentropic compression process

$$T_2 = \left(\frac{p_2}{p_1}\right)^{\frac{(k-1)}{k}} T_1 = 1059.0\ °R$$

Also, for the isentropic expansion

$$T_4 = \left(\frac{p_4}{p_3}\right)^{\frac{(k-1)}{k}} T_3 = 1374.9\ °R$$

(a) The thermal efficiency is given by Eq. 9.25

① $$\eta = 1 - \frac{1}{(p_2/p_1)^{(k-1)/k}} = 0.509\ (50.9\ \%) \longleftarrow \eta$$

(b) To determine the mass flow rate of air, start with the rate of heat addition

$$\dot{m} = \frac{\dot{Q}_{in}}{c_p(T_3 - T_2)} = \frac{(5.2\times10^6\ \text{Btu/h})}{(0.240\ \text{Btu/lb·°R})(2800 - 1059.0)°R}$$

$$= 1.244\times10^4\ \text{lb/h} \longleftarrow \dot{m}$$

(c) The net power is

$$\dot{W}_{cycle} = \dot{m}[(h_3 - h_4) - (h_2 - h_1)] = \dot{m}c_p[(T_3 - T_4) - (T_2 - T_1)]$$

$$= (1.244\times10^4)(0.240)[(2800 - 1374.9) - (1059.0 - 520)]$$

$$= 2.65\times10^6\ \text{Btu/h} \longleftarrow \dot{W}_{cycle}$$

$$= 2.65\times10^6\ \frac{\text{Btu}}{\text{h}}\left|\frac{1\ \text{hp}}{2545\ \text{Btu/h}}\right| = 1041\ \text{hp}$$

1. Alternatively, with Eq. 9.19

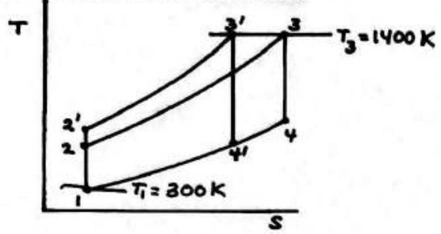
$$\eta = 1 - \frac{(h_4 - h_1)}{(h_3 - h_2)} = 1 - \frac{c_p(T_4 - T_1)}{c_p(T_3 - T_2)} \Rightarrow \eta = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} = 0.509\ (50.9\%)$$

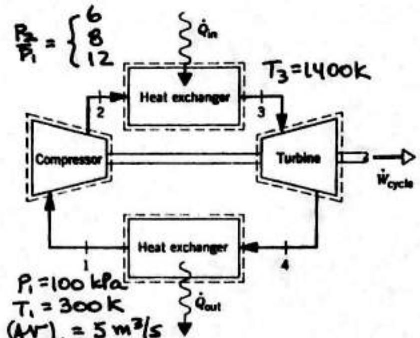
PROBLEM 9.50

KNOWN: Air enters the compressor of an ideal Brayton cycle with known conditions. The turbine inlet temperature is specified.

FIND: Determine for various compressor pressure ratios (a) the thermal efficiency, (b) the back work ratio, and (c) the net power developed.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL: See Example 9.4

ANALYSIS: First, fix each of the principal states (Table A-22)

State 1 $T_1 = 300$ K $\Rightarrow h_1 = 300.19$ kJ/kg, $p_{r1} = 1.3860$

State 2 For isentropic compression, $p_{r2} = (P_2/P_1)\, p_{r1} \Rightarrow T_2, h_2$

Thus,

$$P_2/P_1 = \begin{cases} 6 \Rightarrow p_{r2} = 8.316, & T_2 = 498.4 \text{ K}, h_2 = 501.36 \text{ kJ/kg} \\ 8 \Rightarrow p_{r2} = 11.088, & T_2 = 539.8 \text{ K}, h_2 = 544.18 \text{ kJ/kg} \\ 12 \Rightarrow p_{r3} = 16.632, & T_2 = 603.4 \text{K}, h_2 = 610.65 \text{ kJ/kg} \end{cases}$$

State 3 $T_3 = 1400$K $\Rightarrow h_3 = 1515.42$ kJ/kg, $p_{r3} = 450.5$

State 4 For the isentropic expansion, $p_{r4} = (P_4/P_3)\, p_{r3} \Rightarrow T_4, h_4$

Thus,

$$p_{r4} = \begin{cases} 75.083, & T_4 = 899.3 \text{ K}, & h_4 = 932.20 \text{ kJ/kg} \\ 56.312, & T_4 = 834.9 \text{K}, & h_4 = 860.40 \text{ kJ/kg} \\ 37.542, & T_4 = 751 \text{ K}, & h_4 = 768.38 \text{ kJ/kg} \end{cases}$$

(a) The thermal efficiency is

$$\eta = 1 - \frac{h_4 - h_1}{h_3 - h_2}$$

(b) The back work ratio is

$$bwr = \frac{h_2 - h_1}{h_3 - h_4}$$

(c) The net power developed is

$$\dot{W}_{net} = \dot{m}\,[(h_3 - h_4) - (h_2 - h_1)] = \frac{(AV)_1 P_1}{RT_1}[(h_3 - h_4) - (h_2 - h_1)]$$

① The results are summarized in the following table:

P_2/P_1	η (%)	bwr	$\dot{W}_{net}$ (kW)
6	37.67	.345	2218.6
8	42.32	.372	2386.8
12	48.25	.416	2535.2

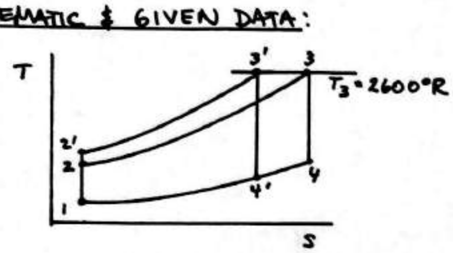
1. As the compressor ratio increases, η increases, bwr increases, and net power developed increases.

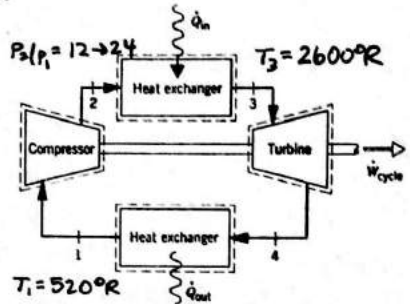
PROBLEM 9.51

KNOWN: An ideal air-standard Brayton cycle has a known compressor inlet temperature and a known maximum turbine inlet temperature.

FIND: Plot the net work per unit mass of air flow and the thermal efficiency versus compressor pressure ratio. Estimate the pressure ratio for maximum net work and the corresponding thermal efficiency and compare with a cold air-standard analysis.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 9.4.

ANALYSIS: Sample calculation. $P_2/P_1 = 12$ Using data from Table A-22E to fix each of the principal states:

State 1 $T_1 = 520°R \Rightarrow h_1 = 124.27$ Btu/lb, $p_{r1} = 1.2147$

State 2 For the isentropic compression, $p_{r2} = (P_2/P_1)p_{r1} = 14.5764$
Thus, $T_2 = 1047.5°R$ and $h_2 = 252.84$ Btu/lb

State 3 $T_3 = 2600°R \Rightarrow h_3 = 674.49$ Btu/lb, $p_{r3} = 513.5$

State 4 For the isentropic expansion, $p_{r4} = (P_4/P_3)p_{r3} = 42.792$
$T_4 = 1399.2°R$ and $h_4 = 342.69$ Btu/lb

The net work per unit mass of air flow is

$$\dot{W}_{cycle}/\dot{m} = (h_3 - h_4) - (h_2 - h_1) = 203.23 \text{ Btu/lb}$$

and the thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{\dot{W}_{cycle}/\dot{m}}{(h_3 - h_2)} = 0.4820 \ (48.20\%)$$

The data for the required plots are obtained using IT, as follows:

IT Code

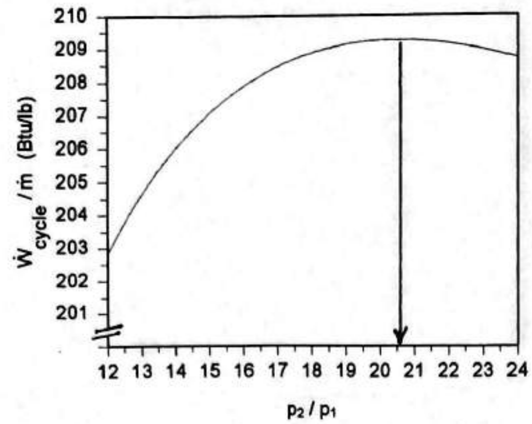
```
T1 = 520  // °R
T3 = 2600  // °R
rp = 12
mdot = 1  // Assume a mass flow rate of unity.
p1 = 1  // Assume a value for pressure
//        (any value would work).

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
s2 = s1
p2 = p1 * rp
s2 = s_hP("Air", h2, p2)
p3 = p2
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
s4 = s3
p4 = p1
s4 = s_hP("Air", h4, p4)
Wdotnet = mdot * ((h3 - h4) - (h2 - h1))
eta = Wdotnet / Qdotin
Qdotin = mdot * (h3 - h2)
```

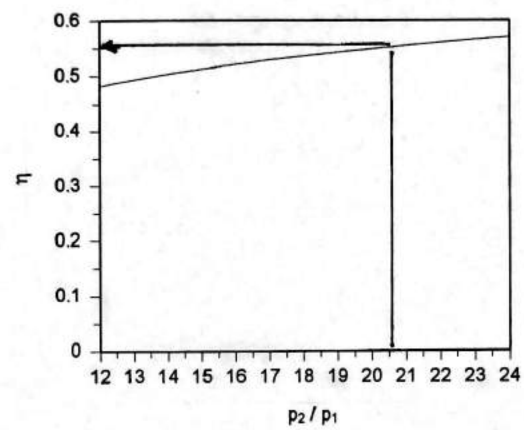
IT Results for $r_p = 12$

$h_1 = 124.3$ Btu/lb
$h_2 = 252.9$ Btu/lb
$h_3 = 673.6$ Btu/lb
$h_4 = 342.2$ Btu/lb
$\dot{W}_{cycle}/\dot{m} = 202.8$ Btu/lb
$\eta = 0.4821$

PROBLEM 9.51 (Cont'd.) - Page 2

PLOTS:





From the plots: $(p_2/p_1)_{\substack{max\\work}} \approx 20.6$

$(\eta)_{\substack{max\\work}} \approx 55.2\%$

Using the result of Example 9.5, on a cold air-standard basis, the pressure ratio is

$$\left(\frac{p_2}{p_1}\right)_{\substack{max\\work}} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}} = 16.72$$

and the corresponding thermal efficiency is

$$\eta = 1 - \frac{1}{(p_2/p_1)^{(k-1)/k}} = 0.553$$

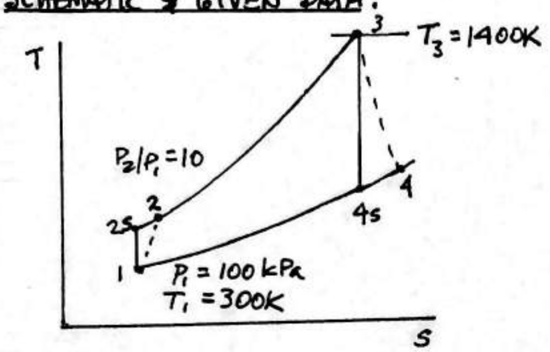
Discussion: The cold air-standard analysis predicts a pressure ratio for maximum net work that is quite different than the value obtained graphically. Interestingly, the corresponding thermal efficiencies are nearly equal.

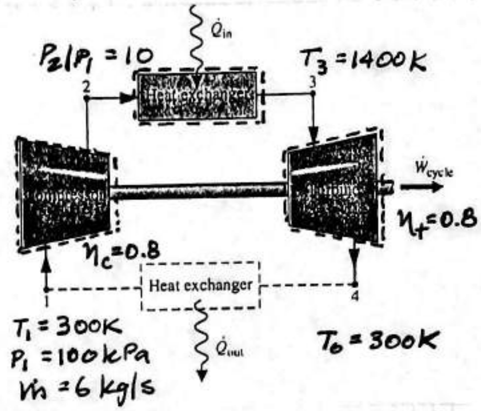
PROBLEM 9.52

KNOWN: Air enters a cold air-standard Brayton cycle at a specified state and with a given mass flow rate. The compressor pressure ratio and maximum cyle temperature are known. The compressor and turbine have isentropic efficiencies of 0.8.

FIND: Determine (a) the thermal efficiency, (b) the back work ratio, (c) the net power, and (d) the rates of exergy destruction in the compressor and turbine for $T_0 = 300K$. Plot these quantities vs. $\eta_c = \eta_t$ ranging from 70 to 100%.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 9.6. Also, assume $k = 1.4$ and $c_p = 1.005\ kJ/kg\cdot K$.

ANALYSIS: First, determine T_2 and T_4, as follows:

(Eq. 9.23) $T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = (300k)(10)^{\frac{1.4-1}{1.4}} = 579.2K$

Using the compressor efficiency: $\eta_c = (T_{2s}-T_1)/(T_2-T_1)$. Thus

$$T_2 = T_1 + (T_{2s}-T_1)/\eta_c = 300 + (579.2-300)/0.8 = 649\ K$$

Similarly

(9.24) $T_{4s} = T_3\left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = (1400)\left(\frac{1}{10}\right)^{\frac{1.4-1}{1.4}} = 725.13K$

and $T_4 = T_3 - (T_3 - T_{4s})\eta_t = 1400 - (1400-725.13)(0.8) = 860.1\,K$

(a) To evaluate the thermal efficiency, use $\eta = 1 - \dot{Q}_{out}/\dot{Q}_{in}$.

$$\dot{Q}_{in} = \dot{m}(h_3 - h_2) = \dot{m}c_p(T_3 - T_2) = \left(6\frac{kg}{s}\right)\left(1.005\frac{kJ}{kg\cdot K}\right)(1400-649)K = 4528.5\ kJ/s$$

$$\dot{Q}_{out} = \dot{m}(h_4 - h_1) = \dot{m}c_p(T_4 - T_1) = (6)(1.005)(860.1-300) = 3377.4K$$

Thus $\eta = 1 - \dot{Q}_{out}/\dot{Q}_{in} = 1 - 3377.4/4528.5 = 0.254\ (25.4\%) \longleftarrow \eta$

(b) The back work ratio is: $bwr = \dot{W}_c/\dot{W}_t$

$$\dot{W}_t = \dot{m}(h_3 - h_4) = \dot{m}c_p(T_3 - T_4) = (6)(1.005)(1400-860.1) = 3255.6\frac{kJ}{s}$$

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \dot{m}c_p(T_2 - T_1) = (6)(1.005)(649-300) = 2104.5\ kJ/s$$

and $bwr = \dot{W}_c/\dot{W}_t = 2104.5/3255.6 = 0.6464$

(c) The net power developed is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = (3255.6 - 2104.5)\frac{kJ}{s}\left|\frac{1\ kW}{1\ kJ/s}\right| = 1151.1\ kW \longleftarrow \dot{W}_{cycle}$$

PROBLEM 9.52 (Cont'd.)

(d) For the compressor, $(\dot{E}_d)_{comp} = T_0 (\dot{\sigma}_{cv})_{comp}$. Applying the steady-state entropy balance to the control volume enclosing the compressor

$$0 = \cancel{\sum_j \left(\frac{\dot{Q}}{T}\right)_j}^{\,0} + \dot{m}(s_1 - s_2) + (\dot{\sigma}_{cv})_{comp}$$

or

$$\dot{\sigma}_{comp} = \dot{m}(s_2 - s_1)$$

For an ideal gas with constant specific heats: $s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R \ln\frac{p_2}{p_1}$.
Thus

$$(\dot{E}_d)_{comp} = T_0 \dot{m}\left[c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}\right]$$

$$= (300\,K)\left(6\,\frac{kg}{s}\right)\left[\left(1.005\,\frac{kJ}{kg\cdot K}\right)\ln\left(\frac{649}{300}\right) - \left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)\ln\left(\frac{10}{1}\right)\right]\left|\frac{1\,kW}{1\,kJ/s}\right|$$

$$= 206.5\ kW \longleftarrow (\dot{E}_d)_{comp}$$

Similarly, for the turbine

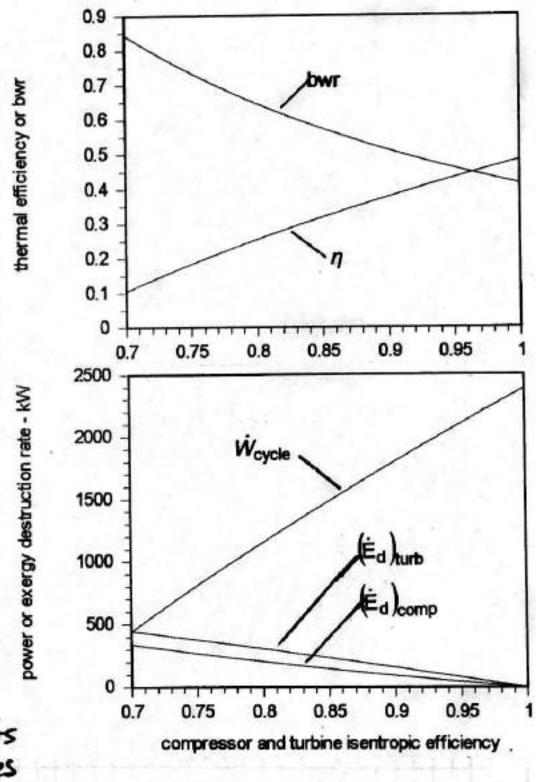
$$(\dot{E}_d)_{turb} = T_0 \dot{m}(s_4 - s_3) = T_0 \dot{m}\left[c_p \ln\frac{T_4}{T_3} - R\ln\frac{p_4}{p_3}\right]$$

$$= (300)(6)\left[(1.005)\ln\left(\frac{860.1}{1400}\right) - \left(\frac{8.314}{28.97}\right)\ln\left(\frac{1}{10}\right)\right] = 308.2\ kW \leftarrow (\dot{E}_d)_{turb}$$

(e) *IT* Code

```
T1 = 300  // K
p1 = 100  // kPa
rp = 10
p2 = p1 * rp
p3 = p2
T3 = 1400  // K
p4 = p1
k = 1.4
cp = 1.005  // kJ/kg·K
mdot = 6  // kg/s
To = 300  // K
Rair = 8.314 / 28.97  // kJ/kg·K
etac = 0.8
etat = etac

T2s = T1 * (p2 / p1) ^ ((k-1)/k)
T2 = T1 + (T2s - T1) / etac
T4s = T3 * (p4 / p3) ^ ((k-1)/k)
T4 = T3 - etat * (T3 - T4s)

Qdotin = mdot * cp * (T3 - T2)
Qdotout = mdot * cp * (T4 - T1)
eta = 1 - Qdotout / Qdotin
Wdotc = mdot * cp * (T2 - T1)
Wdott = mdot * cp * (T3 - T4)
bwr = Wdotc / Wdott
Wdotcycle = Wdott - Wdotc
Edotc = To * mdot * (cp * ln (T2 / T1) - Rair * ln (p2 / p1))
Edott = To * mdot * (cp * ln (T4 / T3) - Rair * ln (p4 / p3))
```

IT Results for $\eta_c = \eta_t = 0.8$
$T_2 = 649$ K
$T_4 = 860.1$ K
bwr = 0.6464
$\eta = 0.2542$
$\dot{Q}_{in} = 4528$ kW
$\dot{Q}_{out} = 3377$ kW
$\dot{W}_{cycle} = 1151$ kW
$(\dot{E}_d)_{comp} = 206.5$ kW
$(\dot{E}_d)_{turb} = 308.2$ kW



These curves illustrate some of the effects of compressor and turbine irreversibilities on the performance of the cold air-standard Brayton cycle.

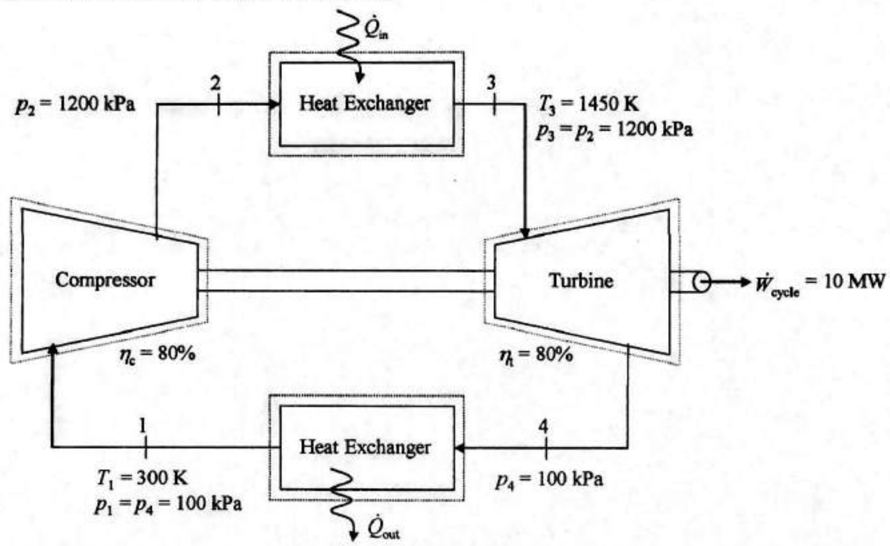
9.53 The cycle of Problem 9.42 is modified to include the effects of irreversibilities in the adiabatic expansion and compression processes. If the states at the compressor and turbine inlets remain unchanged, the cycle produces 10 MW of power, and the compressor and turbine isentropic efficiencies are both 80%, determine

(a) the pressure (kPa), temperature (K), and enthalpy (kJ/kg) at each principal state of the cycle and sketch the *T-s* diagram.
(b) the mass flow rate of air, in kg/s.
(c) the rate of heat transfer, in kW, to the working fluid passing through the heat exchanger.
(d) the thermal efficiency.

KNOWN: An air-standard Brayton cycle operates with known states at the turbine and compressor inlets and known compressor and turbine isentropic efficiencies. The net power output of the cycle is given.

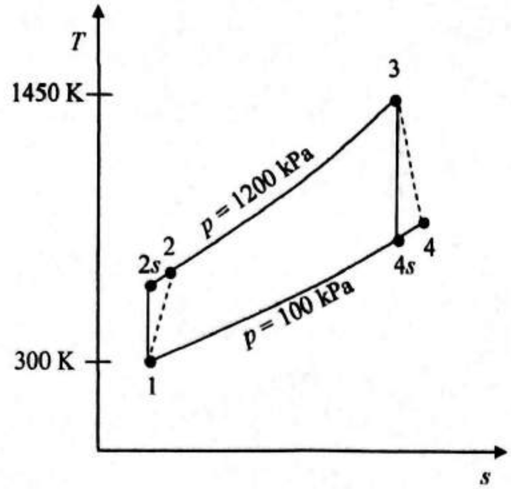
FIND: Determine the mass flow rate of air, the rate of heat transfer to the working fluid passing through the heat exchanger, and the thermal efficiency.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The turbine and compressor operate adiabatically.
3. There are no pressure drops for flow through the heat exchangers.
4. Kinetic and potential energy effects are negligible.
5. The working fluid is air modeled as an ideal gas.

ANALYSIS: (a) The *T-s* diagram for the cycle is shown below.



States 1 and 3 are the same as the corresponding states in Problem 9.42. Thus, $h_1 = 300.19$ kJ/kg and $h_3 = 1575.57$ kJ/kg. Furthermore, States 2 and 4 in Prob. 9.42 correspond to States 2s and 4s in the current problem, so $h_{2s} = 610.65$ kJ/kg and $h_{4s} = 800.78$ kJ/kg.

State 2 can be determined using the isentropic compressor efficiency

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1}$$

Solving for h_2 and inserting values

$$h_2 = h_1 + (h_{2s} - h_1)/\eta_c = 300.19 \text{ kJ/kg} + (610.65 \text{ kJ/kg} - 300.19 \text{ kJ/kg})/(0.80) = 688.27 \text{ kJ/kg}$$

Interpolating in Table A-22, $T_2 \approx 676.7$ K.

Similarly, State 4 can be determined using the isentropic turbine efficiency

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

$$h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 1575.57 \text{ kJ/kg} - (0.80)(1575.57 \text{ kJ/kg} - 800.78 \text{ kJ/kg}) = 955.74 \text{ kJ/kg}$$

From Table A-22, $T_4 \approx 920.3$ K.

Problem 9.53 (Continued) – Page 3

In summary

State	p (kPa)	T (K)	h (kJ/kg)
1	100	300	300.19
2s	1200	603.5	610.65
2	1200	676.7	688.27
3	1200	1450	1575.57
4s	100	780.7	800.78
4	100	920.3	955.74

(b) The mass flow rate of air is found as follows. Mass and energy rate balances for control volumes enclosing the turbine and compressor give

$$\dot{W}_t = \dot{m}(h_3 - h_4) \quad \text{and} \quad \dot{W}_c = \dot{m}(h_2 - h_1)$$

The net power of the cycle is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = \dot{m}[(h_3 - h_4) - (h_2 - h_1)]$$

Solving for $\dot{m}$

$$\dot{m} = \frac{\dot{W}_{cycle}}{[(h_3 - h_4) - (h_2 - h_1)]}$$

Inserting values

$$\dot{m} = \frac{10{,}000\ \text{kW}}{\left(1575.57\frac{\text{kJ}}{\text{kg}} - 955.74\frac{\text{kJ}}{\text{kg}}\right) - \left(688.27\frac{\text{kJ}}{\text{kg}} - 300.19\frac{\text{kJ}}{\text{kg}}\right)}\left|\frac{\frac{1\ \text{kJ}}{\text{s}}}{1\ \text{kW}}\right| = \underline{\mathbf{43.15\ kg/s}}$$

(c) The rate of heat transfer to the working fluid passing through the heat exchanger can be determined by applying mass and energy balances to a control volume around the heat exchanger to give

$$\dot{Q}_{in} = \dot{m}(h_3 - h_2) = \left(43.15\frac{\text{kg}}{\text{s}}\right)\left(1575.57\frac{\text{kJ}}{\text{kg}} - 688.27\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{1\ \text{kW}}{\frac{1\ \text{kJ}}{\text{s}}}\right| = \underline{\mathbf{38{,}287\ kW}}$$

(d) The thermal efficiency is

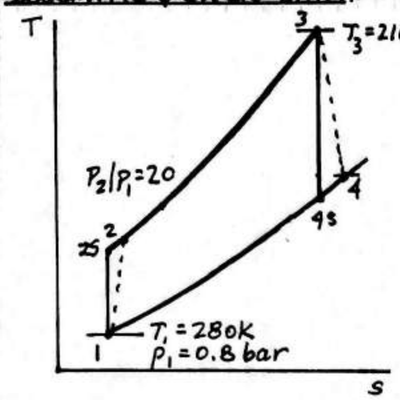
$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = (10{,}000\ \text{kW})/(38{,}287.0\ \text{kW}) = \underline{\mathbf{0.2612\ (26.12\%)}}$$

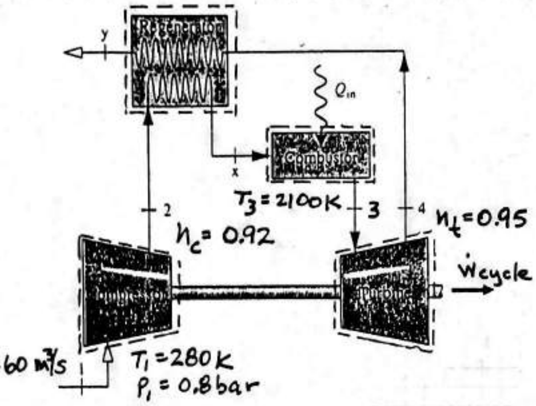
PROBLEM 9.54

KNOWN: Air enters the compressor of an air-standard Brayton cycle at a specified state and a given volumetric flow rate. The compressor pressure ratio and maximum cycle temperature are known. The compressor and turbine isentropic efficiencies are known.

FIND: Determine (a) the net power, (b) the rate of heat addition, and (c) the thermal efficiency.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 9.7. Also, $\eta_c = 0.92$, $\eta_t = 0.95$.

ANALYSIS: First, fix each of the principal states.

State 1: $T_1 = 280K \Rightarrow h_1 = 280.13\ kJ/kg$, $p_{r1} = 1.0889$

State 2: $p_{r2} = (p_2/p_1)\,p_{r1} = (20)(1.0889) = 21.778 \Rightarrow T_{2s} = 649.3K$, $h_{2s} = 659.13 \frac{kJ}{kg}$

Using the isentropic compressor efficiency; $\eta_c = (h_{2s} - h_1)/(h_2 - h_1)$

$$h_2 = h_1 + (h_{2s} - h_1)/\eta_c = 280.13 + (659.13 - 280.13)/(0.92) = 692.09\ kJ/kg$$

State 3: $T_3 = 2100K$; $h_3 = 2377.4\ kJ/kg$, $p_{r3} = 2559$

State 4: $p_{r4} = (p_4/p_3)\,p_{r3} = (\frac{1}{20})(2559) = 127.95 \Rightarrow T_{4s} = 1029.2K$, $h_{4s} = 1079.4 \frac{kJ}{kg}$

Using the isentropic turbine efficiency; $\eta_t = (h_3 - h_4)/(h_3 - h_{4s})$

$$h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 2377.4 - (0.95)(2377.4 - 1079.4) = 1144.3\ kJ/kg$$

Now, determine the mass flow rate.

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{(AV)_1\,p_1}{R\,T_1} = \frac{(60\ m^3/s)(0.8\ bar)}{\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(280K)}\left|\frac{10^5\ N/m^2}{1\ bar}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

$$= 59.73\ kg/s$$

(a) $\dot{W}_c = \dot{m}(h_2 - h_1) = (59.73 \frac{kg}{s})(692.09 - 280.13)\frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right| = 2.461 \times 10^4\ kW$

$\dot{W}_t = \dot{m}(h_3 - h_4) = (59.73)(2377.4 - 1144.3) = 7.365 \times 10^4\ kW$

$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = 4.904 \times 10^4\ kW$ ← $\dot{W}_{cycle}$

(b) $\dot{Q}_{in} = \dot{m}(h_3 - h_2) = (59.73)(2377.4 - 692.09) = 1.0066 \times 10^5\ kW$ ← $\dot{Q}_{in}$

(c) $\eta = \dot{W}_{cycle}/\dot{Q}_{in} = 4.904 \times 10^4 / 1.0066 \times 10^5 = 0.487\ (48.7\%)$ ← η

PROBLEM 9.55

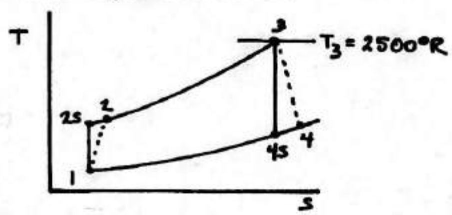
9.55 Air enters the compressor of a simple gas turbine at p_1 = 14 lbf/in.^{2}, T_1 = 520°R, and a volumetric flow rate of 10,000 ft^3/min. The isentropic efficiencies of the compressor and turbine are 83 and 87%, respectively. The compressor pressure ratio is 14 and the temperature at the turbine inlet is 2500°R. On the basis of an air-standard analysis, calculate

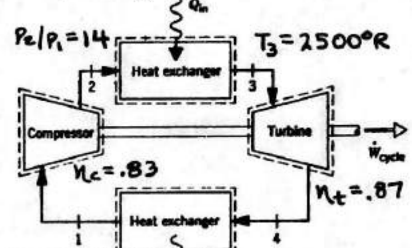
(a) the thermal efficiency of the cycle.
(b) the net power developed, in hp.
(c) the rates at which entropy is produced within the compressor and turbine, each in hp/°R.

KNOWN: A simple gas turbine is analyzed on an air-standard basis. The conditions entering the compressor and the compressor and turbine efficiencies are known. The compressor pressure ratio and the temperature at the turbine inlet are specified as well.

FIND: Determine (a) the thermal efficiency, (b) the net power developed, (c) the rates of entropy production in the compressor and turbine.

SCHEMATIC & GIVEN DATA:





ENGR. MODEL: See Example 9.6

ANALYSIS: First, fix each of the principal states (Table A-22E)

State 1 $T_1 = 520\ °R \Rightarrow h_1 = 124.27$ Btu/lb, $p_{r1} = 1.2147$

State 2 For an isentropic compression: $p_{r2s} = (p_2/p_1)p_{r1} = 17.006$
Thus, $h_{2s} = 264.12$ Btu/lb. Using the compressor efficiency,

$$\eta_c = \frac{h_{2s}-h_1}{h_2-h_1} \Rightarrow h_2 = h_1 + \frac{h_{2s}-h_1}{\eta_c} = 292.76 \text{ Btu/lb}$$

State 3 $T_3 = 2500°R \Rightarrow h_3 = 645.78$ Btu/lb, $p_{r3} = 435.7$

State 4 For an isentropic expansion, $p_{r4s} = (p_4/p_3)p_{r3} = 31.121$
Thus, $h_{4s} = 313.36$ Btu/lb. Using the turbine efficiency,

$$\eta_t = \frac{h_3-h_4}{h_3-h_{4s}} \Rightarrow h_4 = h_3 - \eta_t(h_3-h_{4s}) = 356.57 \text{ Btu/lb}$$

(a) The thermal efficiency is

$$\eta = 1 - \frac{h_4-h_1}{h_3-h_2} = 1 - \frac{356.57-124.27}{645.78-292.76} = 0.342 \ (34.2\%) \qquad \eta$$

(b) To find the net power developed, use

$$\dot{W}_{cycle} = \dot{m}\left[(h_3-h_4)-(h_2-h_1)\right]$$

Evaluating $\dot{m}$

$$\dot{m} = \frac{(AV)_1 P_1}{RT_1} = \frac{(10{,}000 \text{ ft}^3/\text{min})(14 \text{ lbf/in.}^2)}{\left(\frac{1545}{28.97}\cdot\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(520°\text{R})}\left(\frac{144 \text{ in.}^2}{\text{ft}^2}\right)\left(\frac{60 \text{ min}}{1 \text{ h}}\right) = 4.362\times10^4 \frac{\text{lb}}{\text{h}}$$

Thus

$$\dot{W}_{cycle} = (4.362\times10^4)\left[(645.78-356.57)-(292.76-124.27)\right]$$
$$= 5.266\times10^6 \text{ Btu/h} \qquad \dot{W}_{cycle}$$

PROBLEM 9.55 (Continued)

Converting units,

$$\dot{W}_{cycle} = 5.266 \times 10^6 \frac{Btu}{h} \left| \frac{1\ hp}{2545\ Btu/h} \right|$$
$$= 2069\ hp$$

(c) Reducing the entropy rate balance Eq. 6.36, the rate of entropy production for the compressor is

$$\dot{\sigma}_C = \dot{m}(s_2 - s_1) = \dot{m}\left[s_2^o - s_1^o - R \ln \frac{P_2}{P_1} \right]$$

where $P_2/P_1 = 14$. s^o_1 and s^o_2 are obtained from Table A-22 E using T_1 and h_2, respectively. Thus,

$$\dot{\sigma}_C = \left(4.362 \times 10^4 \frac{lb}{h}\right)\left[(0.79748 - 0.59172)\frac{Btu}{lb \cdot °R} - \frac{1.986}{28.97} \frac{Btu}{lb \cdot °R} \ln 14\right]$$
$$= 1084 \frac{Btu/h}{°R} \left| \frac{1\ hp}{2545\ Btu/h} \right| = 0.426 \frac{hp}{°R}$$

Similarly, for the turbine the rate of entropy production is

$$\dot{\sigma}_t = \dot{m}(s_4 - s_3) = \dot{m}\left[s_4^o - s_3^o - R \ln \frac{P_4}{P_3} \right]$$

where $P_4/P_3 = 1/14$. s_3^o and s^o_4 are obtained from Table A-22E using T_3 and h_4, respectively. Thus,

$$\dot{\sigma}_t = \left(4.362 \times 10^4 \frac{lb}{h}\right)\left[(0.84560 - 0.99497)\frac{Btu}{lb \cdot °R} - \frac{1.986}{28.97} \frac{Btu}{lb \cdot °R} \ln \frac{1}{14}\right]$$

(1) $$= 1376 \frac{Btu/h}{°R} \left| \frac{1\ hp}{2545\ Btu/h} \right| = 0.541 \frac{hp}{°R}$$

---

1. The rate of exergy destruction within the compressor and turbine is obtained using $\dot{E}_d = T_0 \dot{\sigma}$. Assuming $T_0 = 520°R$, we get, respectively,

$$(\dot{E}_d)_c = 520°R\left[0.426 \frac{hp}{°R}\right] = 221.5\ hp$$
$$(\dot{E}_d)_t = 520°R\left[0.541 \frac{hp}{°R}\right] = 281.3\ hp$$

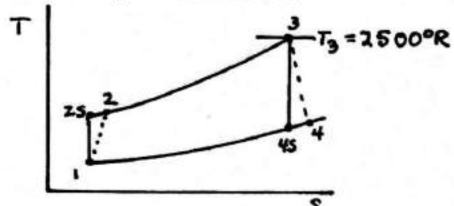
PROBLEM 9.56

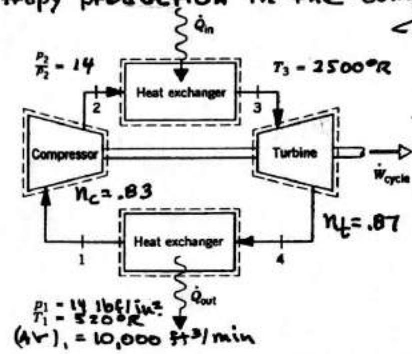
9.56 Solve Problem 9.55 on a cold air-standard basis with specific heats evaluated at 520°R.

KNOWN: A simple gas turbine is analyzed on a cold air-standard basis. The conditions entering the turbine and the turbine and compressor efficiencies are known. The compressor pressure ratio and the temperature at the turbine inlet are specified as well.

FIND: Determine (a) the thermal efficiency, (b) the net power developed, (c) the rates of entropy production in the compressor and turbine

SCHEMATIC & GIVEN DATA:





ENGR. MODEL: Same as in Example 9.6. Also, assume constant specific heats evaluated at 520°R.

ANALYSIS: From Table A-20E, $k = 1.401$, $c_p = 0.240$ Btu/lb·°R. $T_1 = 520°R$ is given and $T_3 = 2500°R$ is given.

State 2 For an isentropic compression, $T_{2s} = (P_2/P_1)^{\frac{k-1}{k}} T_1 = 1106.8°R$

Using the compressor efficiency,

$$\eta_c = \frac{h_{2s}-h_1}{h_2-h_1} = \frac{c_p(T_{2s}-T_1)}{c_p(T_2-T_1)} \Rightarrow T_2 = T_1 + \frac{T_{2s}-T_1}{\eta_c} = 1227.0°R$$

State 4 For an isentropic expansion, $T_{4s} = (P_4/P_3)^{\frac{k-1}{k}} T_3 = 1174.6°R$

Using the turbine efficiency,

$$\eta_t = \frac{h_3-h_4}{h_3-h_{4s}} = \frac{c_p(T_3-T_4)}{c_p(T_3-T_{4s})} \Rightarrow T_4 = T_3 - \eta_t(T_3-T_{4s}) = 1346.9°R$$

(a) The thermal efficiency is

$$\eta = 1 - \frac{h_4-h_1}{h_3-h_2} = 1 - \frac{T_4-T_1}{T_3-T_2} = 0.3504 \quad (35.04\%) \longleftarrow \eta$$

(b) The net power developed is

$$\dot{W}_{cycle} = \dot{m}\left[(h_3-h_4)-(h_2-h_1)\right] = \dot{m}c_p\left[(T_3-T_4)-(T_2-T_1)\right]$$

Evaluating $\dot{m}$

$$\dot{m} = \frac{(AV)_1 P_1}{RT_1} = \frac{(10{,}000 \text{ ft}^3/\text{min})(14. \text{ lbf/in.}^2)}{\left(\frac{1545}{28.97}\frac{\text{ft·lbf}}{\text{lb·°R}}\right)(520°R)}\left(\frac{144 \text{ in.}^2}{1 \text{ ft}^2}\right)\left(\frac{60 \text{ min}}{1 \text{ h}}\right) = 4.362\times10^4 \frac{\text{lb}}{\text{h}}$$

Thus

$$\dot{W}_{cycle} = (4.362\times10^4)(.240)\left[(2500-1346.9)-(1227-520)\right]$$

$$= 4.67\times10^6 \text{ Btu/h} \longleftarrow \dot{W}_{cycle}$$

PROBLEM 9.56 (Continued)

Converting units,

$$\dot{W}_{cycle} = 4.67 \times 10^6 \frac{Btu}{h} \left| \frac{1 hp}{2545 Btu/h} \right| = 1835 hp \longleftarrow$$

(c) Reducing the entropy balance Eq. 6.36 and using Eq. 6.22, the rate of entropy production for the compressor is

$$\dot{\sigma}_c = \dot{m}(s_2 - s_1) = \dot{m}\left[c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right]$$

$$= \left(4.362 \times 10^4 \frac{lb}{h}\right)\left[0.24 \frac{Btu}{lb \cdot °R} \ln \frac{1227}{520} - \frac{1.986}{28.97} \frac{Btu}{lb \cdot °R} \ln 14 \right]$$

$$= 1096 \frac{Btu/h}{°R} \left| \frac{1 hp}{2545 Btu/h} \right| = 0.431 \frac{hp}{°R} \longleftarrow$$

Similarly, the rate of entropy production for the turbine is

$$\dot{\sigma}_t = \dot{m}(s_4 - s_3) = \dot{m}\left[c_p \ln \frac{T_4}{T_3} - R \ln \frac{P_4}{P_3} \right]$$

$$= \left(4.362 \times 10^4 \frac{lb}{h}\right)\left[0.24 \ln \frac{1346.9}{2500} - \frac{1.986}{28.97} \ln \frac{1}{14} \right] \frac{Btu}{lb \cdot °R}$$

① $$= 1417 \frac{Btu/h}{°R} \left| \frac{1 hp}{2545 Btu/h} \right| = 0.557 \frac{hp}{°R} \longleftarrow$$

---

1 The rate of exergy destruction within the compressor and turbine is obtained using $\dot{E}_d = T_0 \dot{\sigma}$. Assuming $T_0 = 520°R$, we get, respectively

$$(\dot{E}_d)_c = 520°R \left(0.431 \frac{hp}{°R}\right) = 224.1 \text{ hp}$$

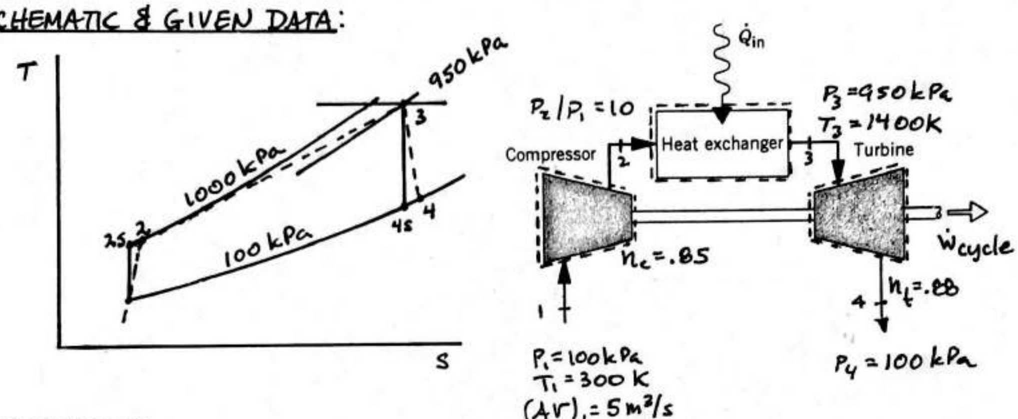
$$(\dot{E}_d)_t = 520°R \left(0.557 \frac{hp}{°R}\right) = 289.6 \text{ hp}$$

PROBLEM 9.57

KNOWN: A simple gas turbine is analyzed on an air-standard basis from an exergy viewpoint. Data are known at various locations.

FIND: (a) Develop a full exergy accounting of the net exergy increase of the air passing through the gas turbine combustor, (b) devise and evaluate an exergetic efficiency for the gas turbine cycle.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The compressor and turbine are adiabatic. (3) Kinetic and potential energy effects are negligible. (4) The working fluid is air modeled as an ideal gas. (5) Let $T_0 = 300$ K and $p_0 = 100$ kPa.

ANALYSIS: Data are obtained for each principal state from Table A-22:

State	T (K)	p (kPa)	h (kJ/kg)	s° (kJ/kg·K)
1	300	100	300.19	1.70203
2	621.1	1000	629.21	2.44539
3	1400	950	1515.42	3.36200
4	873.9	100	903.72	2.81558

The increase in flow exergy of the air passing through the heat exchanger is taken as the net input of exergy to the gas turbine.

Input: $\dot{m}(e_{f3} - e_{f2}) = \dot{m}\left[(h_3 - h_2) - T_0(s_3^\circ - s_2^\circ - R \ln p_3/p_2)\right]$

Evaluating $\dot{m}$:

$$\dot{m} = \frac{(AV)_1 P_1}{RT_1} = \frac{(5\ m^3/s)(100\ kPa)}{\left(\frac{8.314\ kJ}{28.97\ kg\cdot K}\right)(300\ k)}\left|\frac{10^3\ N/m^2}{1\ kPa}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right| = 5.807\ kg/s$$

Thus

$$\dot{m}(e_{f3} - e_{f2}) = \left(5.807\ \frac{kg}{s}\right)\Big[(1515.42 - 629.21)\ kJ/kg$$
$$- (300\ k)\left(3.36200 - 2.44539 - \frac{8.314}{28.97}\ln\frac{950}{1000}\right)\frac{kJ}{kg\cdot K}\Big]\left|\frac{1\ kW}{1\ kJ/s}\right|$$
$$= 3524\ kW\ \text{(Input)}$$

Exergy is destroyed due to irreversibilities in the compressor and turbine. Thus, using $\dot{E}_d = T_0 \dot{\sigma}$, we get

Destructions:

$$\dot{E}_{d,comp} = T_0 \dot{m}(s_2 - s_1) = T_0 \dot{m}\left[(s_2^\circ - s_1^\circ) - R \ln p_2/p_1\right] = 143.8 \text{ kW}$$

$$\dot{E}_{d,turb} = T_0 \dot{m}\left[(s_4^\circ - s_3^\circ) - R \ln p_4/p_3\right] = 173.6 \text{ kW}$$

The net power developed by the cycle represents the output of exergy from the cycle, or

$$\dot{W}_{cycle} = \dot{m}\left[(h_3 - h_4) - (h_2 - h_1)\right]$$
$$= 1641.5 \text{ kW (Output)}$$

Finally, the air enters the gas turbine at p_0 and T_0. Thus, the change in flow exergy from inlet to exit reprepresents the loss due to the hot air being discharged. Thus

Loss: $\dot{m}(e_{f4} - e_{f1}) = \dot{m}\left[(h_4 - h_1) - T_0(s_4^\circ - s_1^\circ - R \ln p_4/p_1)\right]$

$$= 1564.8 \text{ kW}$$

Summarizing

- Input: 3524 kW
- Disposition:
  - Net Power: 1641.5 kW (46.6%)
  - Destroyed: 317.4 (9 %)
  - ① - Loss: 1564.8 (44.4%)

  3523.7 kW

Since the objective of the gas turbine is to produce power, an exergetic efficiency expressed as the ratio of the desired output (net power) to the exergy input is, by inspection of the summary,

$$\epsilon = 46.6\%$$

---

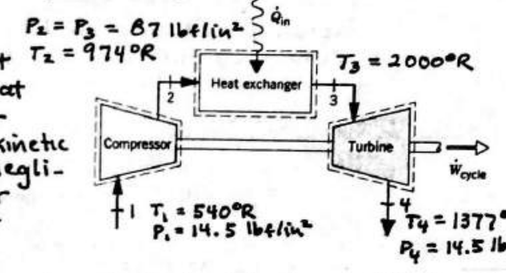
1. Considerable exergy is carried out by the air discharged at 4. This might be exploited by the regenerative approach discussed in Sec. 9.7 or by means of a combined cycle as discussed in Sec. 9.9.

PROBLEM 9.58

KNOWN: A simple gas turbine is analyzed on an air-standard basis from an exergy viewpoint. Data are known at various locations.

FIND: (a) Develop a full exergy accounting of the net exergy increase of the air passing through the gas turbine combustor, (b) devise and evaluate an exergetic efficiency for the gas turbine cycle.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The compressor and turbine are adiabatic. (3) Kinetic and potential energy effect are negligible. (4) The working fluid is air modeled as an ideal gas. (5) Let $T_0 = 540$°R, $p_0 = 14.5$ lbf/in².

ANALYSIS: Data are obtained at each principal state from Table A-22E:

State	T (°R)	p (lbf/in²)	h (Btu/lb)	s° (Btu/lb·°R)
1	540	14.5	129.06	0.60078
2	974	87	234.53	0.74387
3	2000	87	504.71	0.93205
4	1377	14.5	336.91	0.83169

The increase in flow exergy of the air passing through the heat exchanger is taken as the net input of exergy to the gas turbine.

Input: $(e_{f3} - e_{f2}) = (h_3 - h_2) - T_0[(s_3^\circ - s_2^\circ) - R \, \overbrace{\ln p_3/p_2}^{0}]$

$= 168.6$ Btu/lb

Exergy is destroyed due to irreversibilities in the compressor and turbine. Thus, with $\dot{E}_d = T_0\dot{\sigma}$

Destructions: $(\dot{E}_d/\dot{m})_{comp} = T_0[(s_2^\circ - s_1^\circ) - R\ln p_2/p_1] = 10.94$ Btu/lb

$(\dot{E}_d/\dot{m})_{turb} = T_0[(s_4^\circ - s_3^\circ) - R\ln p_4/p_3] = 12.13$ Btu/lb

Exergy is carried out via the net work:

$\dot{W}_{cycle}/\dot{m} = (h_3 - h_4) - (h_2 - h_1) = 62.33$ Btu/lb

Exergy also is carried out with the stream exiting at 4. The net exergy loss is then

$e_{f4} - e_{f3} = (h_4 - h_1) - T_0((s_4^\circ - s_1^\circ) - R \, \overbrace{\ln p_4/p_1}^{0})$

$= 83.16$ Btu/lb

Summary:

- Input : 168.6 Btu/lb
- Disposition:
  - net work: 62.33 Btu/lb (37 %)
  - destroyed: 23.07 Btu/lb (13.7%)
  - loss: 83.16 Btu/lb (49.3%) ①
  - 168.6 Btu/lb

Exergetic Efficiency: $\epsilon = \left(\dfrac{\text{net power output}}{\text{input}}\right) = 37\%$

1. Considerable exergy is carried out by the air discharged at 4. This might be exploited by the regenerative approach discussed in Sec. 9.7 or by means of a combined cycle as discussed in Sec. 9.9

9.59 An ideal air-standard regenerative Brayton cycle produces 10 MW of power. Operating data at principal states in the cycle are given in the table below. The states are numbered as in Fig. 9.14. Sketch the *T-s* diagram and determine

(a) the mass flow rate of air, in kg/s.

(b) the rate of heat transfer, in kW, to the working fluid passing through the combustor.

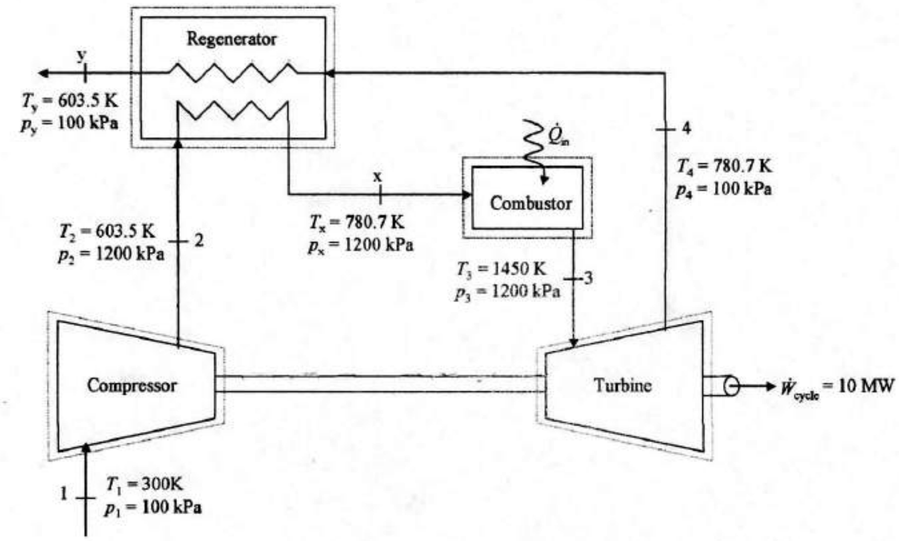
(c) the thermal efficiency.

State	p (kPa)	T (K)	h (kJ/kg)
1	100	300	300.19
2	1200	603.5	610.65
x	1200	780.7	800.78
3	1200	1450	1575.57
4	100	780.7	800.78
y	100	603.5	610.65

KNOWN: An ideal air-standard regenerative Brayton cycle operates with property data given at principal states. The net power output of the cycle is given.

FIND: Determine the mass flow rate of air, the rate of heat transfer to the working fluid passing through the combustor, and the thermal efficiency.

SCHEMATIC AND GIVEN DATA:

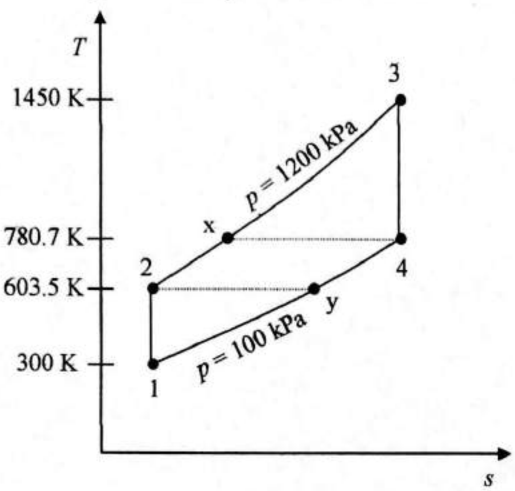


Problem 9.59 (Continued) – Page 2

ENGINEERING MODEL:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. All processes of the working fluid are internally reversible.
3. The turbine and compressor operate adiabatically.
4. There are no pressure drops for flow through the regenerator and combustor.
5. Kinetic and potential energy effects are negligible.
6. The working fluid is air modeled as an ideal gas.

ANALYSIS: The *T*-*s* diagram for the cycle is shown below.



(a) The mass flow rate of air is found as follows. Mass and energy rate balances for control volumes enclosing the turbine and compressor give

$$\dot{W}_t = \dot{m}(h_3 - h_4) \quad \text{and} \quad \dot{W}_c = \dot{m}(h_2 - h_1)$$

The net power of the cycle is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = \dot{m}[(h_3 - h_4) - (h_2 - h_1)]$$

Solving for $\dot{m}$

$$\dot{m} = \frac{\dot{W}_{cycle}}{[(h_3 - h_4) - (h_2 - h_1)]}$$

Inserting values

$$\dot{m} = \frac{10{,}000 \text{ kW}}{\left(1575.57\frac{\text{kJ}}{\text{kg}} - 800.78\frac{\text{kJ}}{\text{kg}}\right) - \left(610.65\frac{\text{kJ}}{\text{kg}} - 300.19\frac{\text{kJ}}{\text{kg}}\right)}\left|\frac{\frac{\text{kJ}}{\text{s}}}{\text{kW}}\right| = \underline{\mathbf{21.54\ kg/s}}$$

(b) The rate of heat transfer to the working fluid passing through the combustor can be determined by applying mass and energy balances to a control volume around the combustor to give

$$\dot{Q}_{in} = \dot{m}(h_3 - h_x) = \left(21.54\frac{\text{kg}}{\text{s}}\right)\left(1575.57\frac{\text{kJ}}{\text{kg}} - 800.78\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{\text{kW}}{\frac{\text{kJ}}{\text{s}}}\right| = \underline{\mathbf{16{,}689\ kW}}$$

(c) The thermal efficiency is

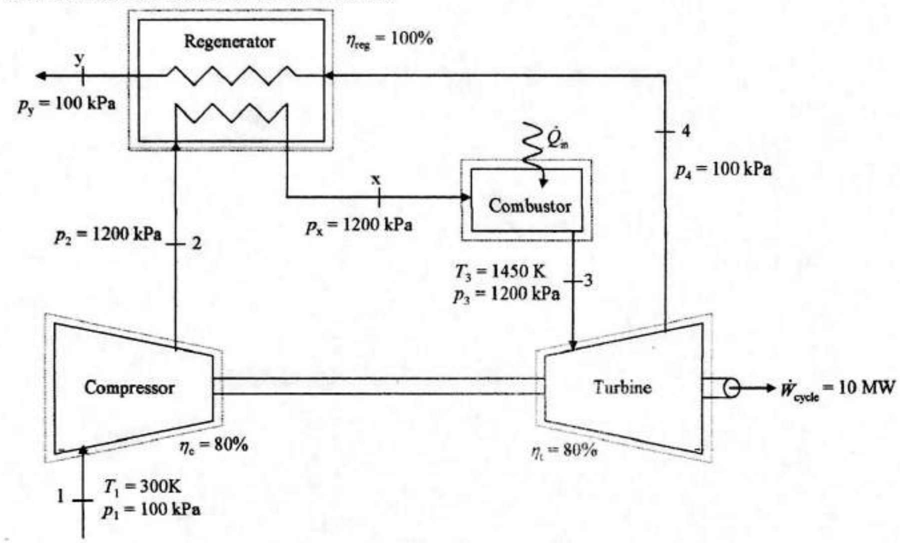
$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = (10{,}000 \text{ kW})/(16{,}689 \text{ kW}) = \underline{\mathbf{0.599\ (59.9\%)}}$$

9.60 The cycle of Problem 9.59 is modified to include the effects of irreversibilities in the adiabatic expansion and compression processes. The regenerator effectiveness is 100%. If the states at the compressor and turbine inlets remain unchanged, the cycle produces 10 MW of power, and the compressor and turbine isentropic efficiencies are both 80%, determine
(a) the pressure (kPa), temperature (K), and enthalpy (kJ/kg) at each principal state of the cycle and sketch the T-s diagram.
(b) the mass flow rate of air, in kg/s.
(c) the rate of heat transfer, in kW, to the working fluid passing through the combustor.
(d) the thermal efficiency.

KNOWN: An air-standard regenerative Brayton cycle operates with known states at the turbine and compressor inlets and known compressor and turbine isentropic efficiencies. The net power output of the cycle is given.

FIND: Determine the mass flow rate of air, the rate of heat transfer to the working fluid passing through the combustor, and the thermal efficiency. Provide a summary table of pressure, temperature, and enthalpy for each state.

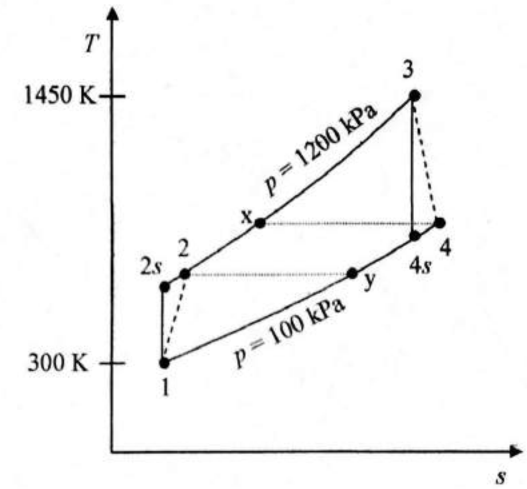
SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The turbine and compressor operate adiabatically.
3. The regenerator efficiency is 100%.
4. There are no pressure drops for flow through the regenerator and combustor.
5. Kinetic and potential energy effects are negligible.
6. The working fluid is air modeled as an ideal gas.

Problem 9.60 (Continued) – Page 2

ANALYSIS: (a) The T-s diagram for the cycle is shown below.



States 1 and 3 are the same as the corresponding states in Problem 9.59. Thus, $h_1 = 300.19$ kJ/kg and $h_3 = 1575.57$ kJ/kg. Furthermore, States 2 and 4 in Prob. 9.59 correspond to States 2s and 4s in the current problem, so $h_{2s} = 610.65$ kJ/kg and $h_{4s} = 800.78$ kJ/kg.

State 2 can be determined using the isentropic compressor efficiency

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1}$$

Solving for h_2 and inserting values

$$h_2 = h_1 + (h_{2s} - h_1)/\eta_c = 300.19 \text{ kJ/kg} + (610.65 \text{ kJ/kg} - 300.19 \text{ kJ/kg})/(0.80) = 688.27 \text{ kJ/kg}$$

Interpolating in Table A-22, $T_2 \approx 676.7$ K.

Similarly, State 4 can be determined using the isentropic turbine efficiency

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

$$h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 1575.57 \text{ kJ/kg} - (0.80)(1575.57 \text{ kJ/kg} - 800.78 \text{ kJ/kg}) = 955.74 \text{ kJ/kg}$$

From Table A-22 (interpolated), $T_4 \approx 920.3$ K.

Problem 9.60 (Continued) – Page 3

State x can be determined using the regenerator effectiveness.

$$\eta_{reg} = \frac{h_x - h_2}{h_4 - h_2}$$

Solving for h_x and inserting values

$$h_x = h_2 + \eta_{reg}(h_4 - h_2) = 688.27 \text{ kJ/kg} + (1.00)(955.74 \text{ kJ/kg} - 688.27 \text{ kJ/kg}) = 955.74 \text{ kJ/kg}$$

Interpolating in Table A-22, $T_x \approx 920.3$ K.

The enthalpy at State y can be determined from mass and energy balances for a control volume enclosing the regenerator.

$$0 = \dot{Q} - \dot{W} + \dot{m}(h_2 + h_4 - h_x - h_y)$$

With $\dot{Q} = 0$ and $\dot{W} = 0$, we can solve for h_y and insert values to get

$$h_y = h_2 + h_4 - h_x = 688.27 \text{ kJ/kg} + 955.74 \text{ kJ/kg} - 955.74 \text{ kJ/kg} = 688.27 \text{ kJ/kg}$$

From Table A-22, $T_y \approx 676.7$ K.

In summary

State	p (kPa)	T (K)	h (kJ/kg)
1	100	300	300.19
2*s*	1200	603.5	610.65
2	1200	676.7	688.27
x	1200	920.3	955.74
3	1200	1450	1575.57
4*s*	100	780.7	800.78
4	100	920.3	955.74
y	100	676.7	688.27

(b) The mass flow rate of air is found as follows. Mass and energy rate balances for control volumes enclosing the turbine and compressor give

$$\dot{W}_t = \dot{m}(h_3 - h_4) \quad \text{and} \quad \dot{W}_c = \dot{m}(h_2 - h_1)$$

The net power of the cycle is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = \dot{m}[(h_3 - h_4) - (h_2 - h_1)]$$

Solving for $\dot{m}$

$$\dot{m} = \frac{\dot{W}_{\text{cycle}}}{[(h_3 - h_4) - (h_2 - h_1)]}$$

Inserting values

$$\dot{m} = \frac{10{,}000\ \text{kW}}{\left(1575.57\frac{\text{kJ}}{\text{kg}} - 955.74\frac{\text{kJ}}{\text{kg}}\right) - \left(688.27\frac{\text{kJ}}{\text{kg}} - 300.19\frac{\text{kJ}}{\text{kg}}\right)}\left|\frac{\frac{\text{kJ}}{\text{s}}}{\text{kW}}\right| = \underline{\mathbf{43.15\ kg/s}}$$

(c) The rate of heat transfer to the working fluid passing through the combustor can be determined by applying mass and energy balances to a control volume around the combustor to give

$$\dot{Q}_{\text{in}} = \dot{m}(h_3 - h_x)$$

Inserting values

$$\dot{Q}_{\text{in}} = \dot{m}(h_3 - h_x) = \left(43.15\frac{\text{kg}}{\text{s}}\right)\left(1575.57\frac{\text{kJ}}{\text{kg}} - 955.74\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{\text{kW}}{\frac{\text{kJ}}{\text{s}}}\right| = \underline{\mathbf{26{,}746\ kW}}$$

(d) The thermal efficiency is

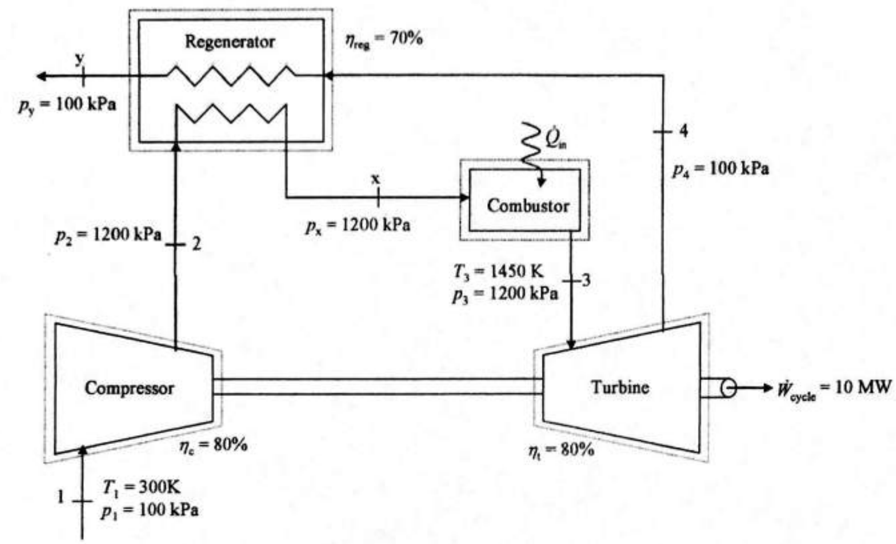
$$\eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = (10{,}000\ \text{kW})/(26{,}746\ \text{kW}) = \underline{\mathbf{0.374\ (37.4\%)}}$$

9.61 The cycle of Problem 9.60 is modified to include a regenerator with an effectiveness of 70%. Determine
(a) the specific enthalpy, in kJ/kg, and the temperature, in K, for each stream exiting the regenerator and sketch the *T-s* diagram.
(b) the mass flow rate of air, in kg/s.
(c) the rate of heat transfer, in kW, to the working fluid passing through the combustor.
(d) the thermal efficiency.

KNOWN: An air-standard regenerative Brayton cycle operates with known states at the turbine and compressor inlets, known compressor and turbine isentropic efficiencies, and known regenerator effectiveness. The net power output of the cycle is given.

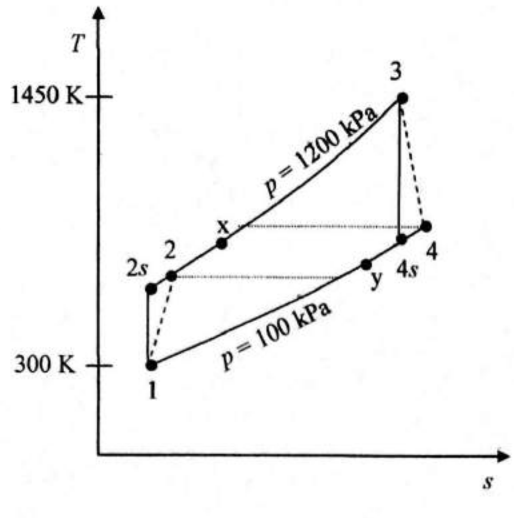
FIND: Determine the mass flow rate of air, the rate of heat transfer to the working fluid passing through the combustor, and the thermal efficiency. Provide a summary table of pressure, temperature, and enthalpy for each state.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The turbine and compressor operate adiabatically.
3. There are no pressure drops for flow through the regenerator and combustor.
4. Kinetic and potential energy effects are negligible.
5. The working fluid is air modeled as an ideal gas.

ANALYSIS: (a) The *T-s* diagram for the cycle is shown below.



For the cycle with a regenerator effectiveness of less than 100%, all principal states are unchanged from Problem 9.60 except for states *x* and *y*.

State x can be determined using the regenerator effectiveness

$$\eta_{reg} = \frac{h_x - h_2}{h_4 - h_2}$$

Solving for h_x and inserting values

$$h_x = h_2 + \eta_{reg}(h_4 - h_2) = 688.27 \text{ kJ/kg} + (0.70)(955.74 \text{ kJ/kg} - 688.27 \text{ kJ/kg}) = 875.50 \text{ kJ/kg}$$

Interpolating in Table A-22, $T_x \approx 848.5$ K.

The specific enthalpy at state y can be determined from a mass and energy balances for a control volume enclosing the regenerator. With $\dot{Q} = 0$ and $\dot{W} = 0$

$$0 = \dot{m}(h_2 + h_4 - h_x - h_y)$$

Solving for h_y and inserting values

$$h_y = h_2 + h_4 - h_x = 688.27 \text{ kJ/kg} + 955.74 \text{ kJ/kg} - 875.50 \text{ kJ/kg} = 768.51 \text{ kJ/kg}$$

From Table A-22, $T_y \approx 751.1$ K.

Problem 9.61 (Continued) – Page 3

In summary

State	p (kPa)	T (K)	h (kJ/kg)
1	100	300	300.19
2s	1200	603.5	610.65
2	1200	676.7	688.27
x	1200	848.5	875.50
3	1200	1450	1575.57
4s	100	780.7	800.78
4	100	920.3	955.74
y	100	751.1	768.51

(b) The mass flow rate of air is found as follows. Mass and energy rate balances for control volumes enclosing the turbine and compressor give

$$\dot{W}_t = \dot{m}(h_3 - h_4) \quad \text{and} \quad \dot{W}_c = \dot{m}(h_2 - h_1)$$

The net power of the cycle is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = \dot{m}[(h_3 - h_4) - (h_2 - h_1)]$$

Solving for $\dot{m}$

$$\dot{m} = \frac{\dot{W}_{cycle}}{[(h_3 - h_4) - (h_2 - h_1)]}$$

Inserting values

$$\dot{m} = \frac{10{,}000 \text{ kW}}{\left(1575.57\frac{\text{kJ}}{\text{kg}} - 955.74\frac{\text{kJ}}{\text{kg}}\right) - \left(688.27\frac{\text{kJ}}{\text{kg}} - 300.19\frac{\text{kJ}}{\text{kg}}\right)}\left|\frac{\frac{\text{kJ}}{\text{s}}}{\text{kW}}\right| = \underline{\mathbf{43.15\ kg/s}}$$

(c) The rate of heat transfer to the working fluid passing through the combustor can be determined by applying mass and energy balances to a control volume around the combustor to give

$$\dot{Q}_{in} = \dot{m}(h_3 - h_x)$$

$$\dot{Q}_{in} = \dot{m}(h_3 - h_x) = \left(43.15\frac{\text{kg}}{\text{s}}\right)\left(1575.57\frac{\text{kJ}}{\text{kg}} - 875.50\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{\text{kW}}{\frac{\text{kJ}}{\text{s}}}\right| = \underline{\mathbf{30{,}208\ kW}}$$

(d) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = (10{,}000 \text{ kW})/(30{,}208 \text{ kW}) = \underline{\mathbf{0.331\ (33.1\%)}}$$

PROBLEM 9.62

9.62 Air enters the compressor of a cold air-standard Brayton cycle with regeneration at 100 kPa, 300 K, with a mass flow rate of 6 kg/s. The compressor pressure ratio is 10, and the turbine inlet temperature is 1400 K. The turbine and compressor each have isentropic efficiencies of 80% and the regenerator effectiveness is 80%. For $k = 1.4$, calculate

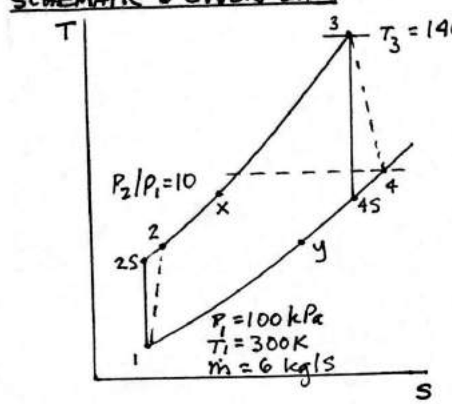
(a) the thermal efficiency of the cycle.
(b) the back work ratio.
(c) the net power developed, in kW.
(d) the rate of entropy production in the regenerator, in kW/K.

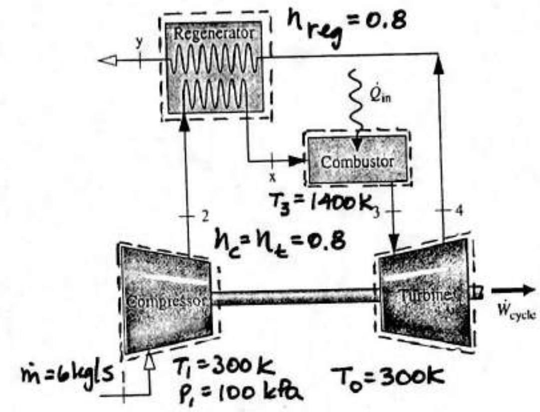
KNOWN: Steady-state operating data are provided for a cold air-standard Brayton cycle with regeneration.

FIND: (a) η, (b) bwr, (c) $\dot{W}_{cycle}$, (d) $\dot{\sigma}_{regen}$

ENGINEERING MODEL: See Example 9.7. Also $\eta_c = \eta_t = 0.8$ and $\eta_{regen} = 0.8$. The specific heats are constant, with $k = 1.4$ and $c_p = 1.005$ kJ/kg·K

SCHEMATIC & GIVEN DATA:





ANALYSIS: Applying the isentropic compressor and turbine efficiencies (Eqs. 6.48 and 6.46, respectively) on a cold air-standard basis, together with Eq. 6.43, we get $T_2 = 649$ K and $T_4 = 860.1$ K.

To find T_x, use the regenerator effectiveness;

$$\eta_{reg} \equiv \frac{h_x - h_2}{h_4 - h_2} = \frac{\cancel{c_p}(T_x - T_2)}{\cancel{c_p}(T_4 - T_2)} \Rightarrow T_x = T_2 + \eta_{reg}(T_4 - T_2)$$
$$= 649 + (.8)(860.1 - 649) = 817.9\text{K}$$

For the control volume enclosing the regenerator

$$0 = \dot{m}\left[(h_2 - h_x) + (h_4 - h_y)\right]$$

or

$$0 = c_p(T_2 - T_x) + c_p(T_4 - T_y) \Rightarrow T_y = T_4 - (T_x - T_2) = 691.2\text{K}$$

(a) $\dot{Q}_{in} = \dot{m}c_p(T_3 - T_x) = (6 \tfrac{kg}{s})(1.005 \tfrac{kJ}{kg\cdot K})(1400 - 817.9)\text{K} = 3510.1$ kJ/s

$\dot{Q}_{out} = \dot{m}c_p(T_y - T_1) = (6)(1.005)(691.2 - 300) = 2358.9$ kJ/s

$\eta = 1 - \dot{Q}_{out}/\dot{Q}_{in} = 1 - 2358.9/3510.1 = 0.328\ (32.8\%)$ ← η

(b) $\dot{W}_c = \dot{m}c_p(T_2 - T_1) = 2104.5$ kJ/s, $\dot{W}_t = \dot{m}c_p(T_3 - T_4) = 3255.6$ kJ/s

bwr $= \dot{W}_c/\dot{W}_t = 0.6464$ ← bwr

(c) $\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = 1151.1$ kW ← $\dot{W}_{cycle}$

(d) For the regenerator: $0 = \cancelto{0}{\sum_j \left(\frac{\dot{Q}}{T}\right)_j} + \dot{m}\left[(s_2 - s_x) + (s_4 - s_y)\right] + (\dot{\sigma}_{cv})_{regen}$

$$\rightarrow (\dot{\sigma}_{cv})_{regen} = \dot{m}\left[c_p \ln(T_x/T_2) - \cancelto{0}{R\ln(p_x/p_2)} + c_p \ln(T_y/T_4) - \cancelto{0}{R\ln(p_y/p_4)}\right]$$

$$\therefore\ (\dot{\sigma}_{cv})_{regen} = (6\ kg/s)(1.005 \tfrac{kJ}{kg\cdot K})\left[\ln\left(\frac{817.9}{649}\right) + \ln\left(\frac{691.2}{860.1}\right)\right]\left|\frac{1\ kW}{1\ kJ/s}\right|$$

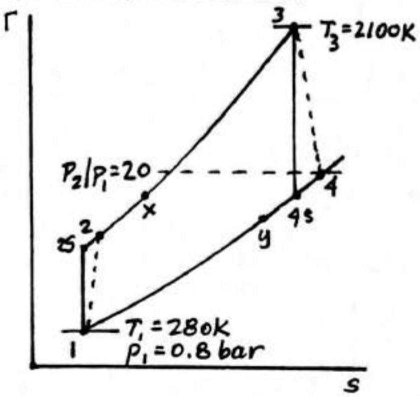
$= 0.0766$ kW/K ← $(\dot{\sigma}_{cv})_{regen}$

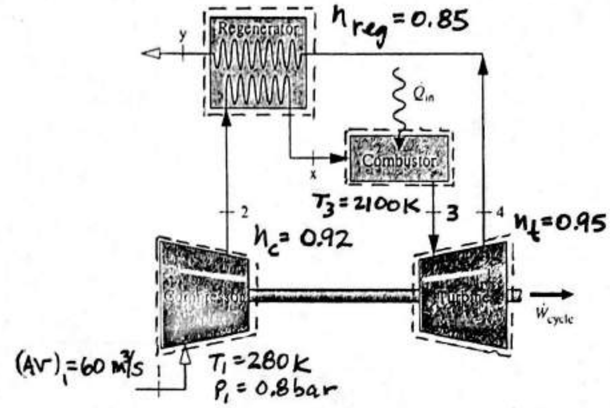
PROBLEM 9.63

KNOWN: Air enters the compressor of a regenerative air-standard Brayton cycle at a specified state and a given volumetric flow rate. The compressor pressure ratio and maximum cycle temperature are known. The compressor and turbine isentropic efficiencies and the regenerator effectiveness are known.

FIND: Determine (a) the net power, (b) the rate of heat addition, and (c) the thermal efficiency. Plot these quantities vs. regenerator effectiveness ranging from 0 to 100%. Discuss.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 9.7. Also, $\eta_c = 0.92$, $\eta_t = 0.95$, and $\eta_{reg} = 0.85$.

ANALYSIS: First, fix each of the principal states.

State 1: $T_1 = 280K \Rightarrow h_1 = 280.13$ kJ/kg, $p_{r1} = 1.0889$

State 2: $p_{r2} = (P_2/P_1)\,p_{r1} = (20)(1.0889) = 21.778 \Rightarrow T_{2s} = 649.3K,\ h_{2s} = 659.13\ \frac{kJ}{kg}$

Using the isentropic compressor efficiency; $\eta_c = (h_{2s} - h_1)/(h_2 - h_1)$

$$h_2 = h_1 + (h_{2s} - h_1)/\eta_c = 280.13 + (659.13 - 280.13)/(0.92) = 692.09\ kJ/kg$$

State 3: $T_3 = 2100K$; $h_3 = 2377.4$ kJ/kg, $p_{r3} = 2559$

State 4: $p_{r4} = (P_4/P_3)\,p_{r3} = (\frac{1}{20})(2559) = 127.95 \Rightarrow T_{4s} = 1029.2K,\ h_{4s} = 1079.4\ \frac{kJ}{kg}$

Using the isentropic turbine efficiency; $\eta_t = (h_3 - h_4)/(h_3 - h_{4s})$

$$h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 2377.4 - (0.95)(2377.4 - 1079.4) = 1144.3\ kJ/kg$$

State x: From the regenerator effectiveness; $\eta_{reg} = (h_x - h_2)/(h_4 - h_2)$

$$h_x = h_2 + \eta_{reg}(h_4 - h_2) = 692.09 + (0.85)(1144.3 - 692.09) = 1076.5\ \frac{kJ}{kg}$$

Now, determine the mass flow rate.

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{(AV)_1\,p_1}{RT_1} = \frac{(60\ m^3/s)(0.8\ bar)}{\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(280K)}\left|\frac{10^5\ N/m^2}{1\ bar}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

$$= 59.73\ kg/s$$

(a) $\dot{W}_t = \dot{m}(h_3 - h_4) = (59.73\ \frac{kg}{s})(2377.4 - 1144.3)\frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right| = 7.365\times10^4\ kW$

$\dot{W}_c = \dot{m}(h_2 - h_1) = (59.73)(692.09 - 280.13) = 2.461\times10^4\ kW$

$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = 4.904\times10^4\ kW$ ← $\dot{W}_{cycle}$

(b) $\dot{Q}_{in} = \dot{m}(h_3 - h_x) = (59.73)(2377.4 - 1076.5) = 7.770\times10^4\ kW$ ← $\dot{Q}_{in}$

PROBLEM 9.63 (cont'd.) - Page 2

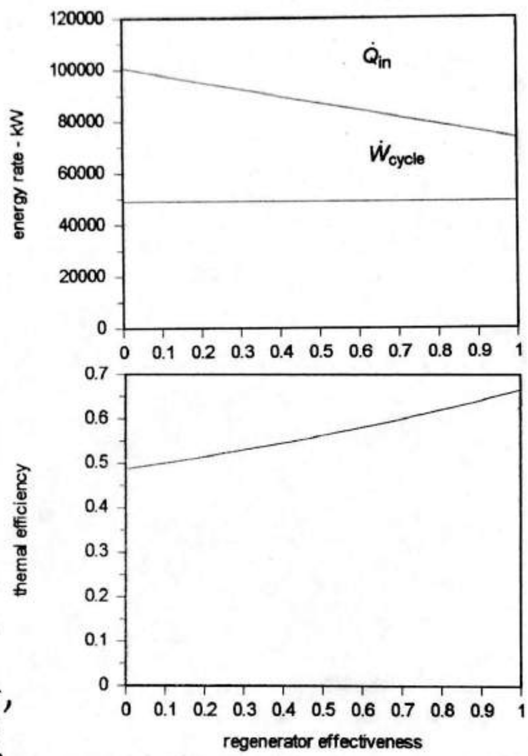
(c) The thermal efficiency is $\eta = \dot{W}_{cycle}/\dot{Q}_{in} = 0.631$ (63.1%) ◄ η

(d) *IT* Code

```
T1 = 280 // K
p1 = 0.8 // bar
r = 20
p2 = r * p1
p3 = p2
T3 = 2100 // K
p4 = p3 / r
etac = 0.92
etat = 0.95
etareg = 0.85
AV1 = 60 // m³/s

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
s2s = s1
s2s = s_hP("Air",h2s,p2)
h2 = h1 + (h2s - h1) / etac
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
s4s = s3
s4s = s_hP("Air",h4s,p4)
h4 = h3 - etat * (h3 - h4s)
hx = h2 + etareg * (h4 - h2)

v1 = v_TP("Air",T1,p1)
mdot = AV1 / v1
Wdott = mdot * (h3 - h4)
Wdotc = mdot * (h2 - h1)
Wdotcycle = Wdott - Wdotc
Qdotin = mdot * (h3 - hx)
eta = Wdotcycle / Qdotin
```

IT Results for $\eta_{reg} = 0.85$, $\eta_c = 0.92$, and $\eta_t = 0.95$

$\dot{m} = 59.73$ kg/s
$\dot{Q}_{in} = 7.768 \times 10^4$ kW
$\dot{W}_{cycle} = 4.903 \times 10^4$ kW
$\eta = 0.6312$
$h_1 = 280$ kJ/kg
$h_2 = 691.9$ kJ/kg
$h_3 = 2376$ kJ/kg
$h_4 = 1143$ kJ/kg
$h_x = 1075$ kJ/kg



Discussion

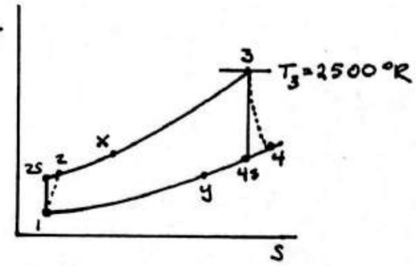
The states at the inlet and exit of the compressor and turbine, respectively, are not changed as the regenerator effectiveness changes. Hence, the power is constant. However, as the regenerator effectiveness increases, the specific enthalpy h_x increases and less heat addition is required in the combustor (external heat addition). The result is a substantial increase in the thermal efficiency.

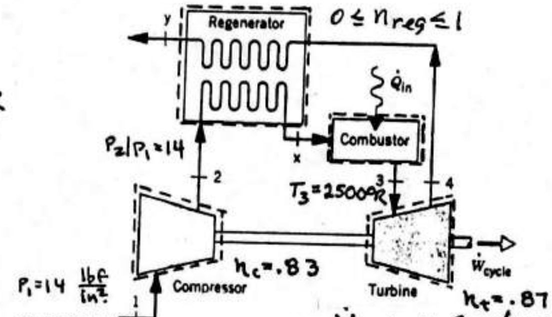
PROBLEM 9.64

KNOWN: Steady-state operating data are provided for a regenerative air-standard Brayton cycle.

FIND: Plot (a) the thermal efficiency and (b) the percent decrease in heat addition, each versus regenerator effectiveness ranging from 0 to 1.

SCHEMATIC & GIVEN DATA:





$\dot{W}_{cycle} = 5 \times 10^6$ Btu/h

ENGINEERING MODEL: See Example 9.7.

ANALYSIS: Consider the sample case of $\eta_{reg} = 0.78$. Using data from Table A-22E together with Eqs. 6.48 and 6.46 for the compressor and turbine isentropic efficiencies, respectively, we get

$$h_1 = 124.27\ \text{Btu/lb},\ h_2 = 292.76,\ h_3 = 645.78,\ h_4 = 356.57$$

The specific enthalpy h_x is found using the regenerator effectiveness;

$$\eta_{reg} = \frac{h_x - h_2}{h_4 - h_2} \Rightarrow h_x = \eta_{reg}(h_4 - h_2) + h_2 = 342.53\ \text{Btu/lb}$$

From an energy balance on the regenerator

$$h_y = (h_2 - h_x) + h_4 = 306.8\ \text{Btu/lb}$$

(a) The thermal efficiency is

$$\eta = 1 - \frac{h_y - h_1}{h_3 - h_x} = 0.398\ (39.8\%)$$

(b) The percent decrease in heat addition is

$$\%\,\text{decrease} = \frac{(h_3 - h_2) - (h_3 - h_x)}{(h_3 - h_2)} \times 100 = 14.1\%$$

The data for the required plots are obtained using IT, as follows:

IT Code

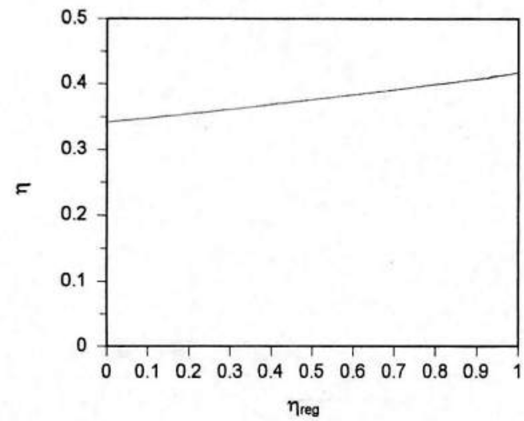
```
p1 = 14  // lbf/in.²
T1 = 520  // °R
p2 / p1 = 14
p3 = p2
T3 = 2500  // °R
p4 = p1
eta_c = 0.83
eta_t = 0.87
eta_reg = 0.78

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)

s2s = s_hP("Air", h2s, p2)
s2s = s1
h2 = h1 + (h2s - h1) / eta_c
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
s4s = s_hP("Air", h4s, p4)
s4s = s3
h4 = h3 - (h3 - h4s) * eta_t
hx = eta_reg * (h4 - h2) + h2
hy = h2 - hx + h4
eta = 1 - (hy - h1) / (h3 - hx)
pct = (((h3 - h2) - (h3 - hx)) / (h3 - h2)) * 100
```

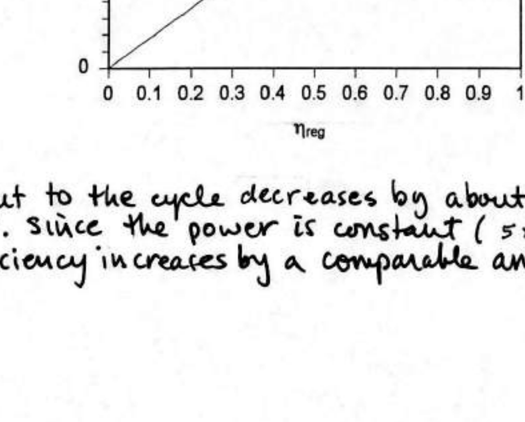
PROBLEM 9.64 (Cont'd.) - Page 2

IT Results for $\eta_{reg} = 0.78$

$h_1 = 124.3$ Btu/lb
$h_2 = 292.8$ Btu/lb
$h_{2s} = 264.2$ Btu/lb
$h_3 = 644.9$ Btu/lb
$h_4 = 356.2$ Btu/lb
$h_{4s} = 313$ Btu/lb
$h_x = 342.2$ Btu/lb
$h_y = 306.7$ Btu/lb
% difference = 14.04
$\eta = 0.3975$

PLOTS:





The heat input to the cycle decreases by about 18% with an ideal regenerator. Since the power is constant (5×10^6 Btu/h), the thermal efficiency increases by a comparable amount.

PROBLEM 9.65

9.65 On the basis of a cold air-standard analysis, show that the thermal efficiency of an ideal regenerative gas turbine can be expressed alternatively as

$$\eta = 1 - \left(\frac{T_1}{T_3}\right)(r)^{(k-1)/k} \qquad \text{(a)}$$

where r is the compressor pressure ratio, T_1 and T_3 denote the temperatures at the compressor and turbine inlets, respectively, and as

$$\eta = 1 - \frac{T_2}{T_3} \qquad \text{(b)}$$

where T_2 is the temperature at the compressor exit.

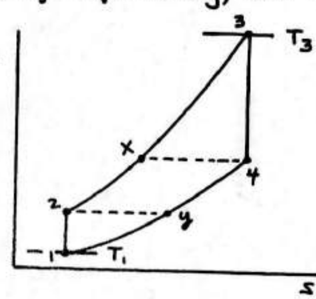
KNOWN: An ideal regenerative gas turbine is analyzed on a cold air-standard basis.

FIND: Show that the thermal efficiency can be expressed as

(a) $\eta = 1 - \left(\frac{T_1}{T_3}\right)(r)^{(k-1)/k}$ and (b) $\eta = 1 - \frac{T_2}{T_3}$

where r is pressure ratio, and T_1 and T_3 denote the compressor and turbine inlet temperatures, respectively, and T_2 is the compressor inlet temperature.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: Same as in Example 9.7 except $\eta_{reg} = 100\%$. Also, assume constant specific heats.

ANALYSIS: The thermal efficiency of the ideal cold air-standard cycle can be expressed as

$$\eta = 1 - \frac{\cancel{c_p}(T_y - T_1)}{\cancel{c_p}(T_3 - T_x)}$$

With $\eta_{reg} = 100\%$, $T_x = T_4$ and $T_y = T_2$. Thus

$$\eta = 1 - \frac{T_2 - T_1}{T_3 - T_4} = 1 - \frac{T_1\,(T_2/T_1 - 1)}{T_3\,(1 - T_4/T_3)}$$

If $r = P_2/P_1 = P_3/P_4$, the isentropic relations apply:

$$\frac{T_2}{T_1} = r^{\left(\frac{k-1}{k}\right)} \quad \text{and} \quad \frac{T_4}{T_3} = r^{-\left(\frac{k-1}{k}\right)} \qquad (1), (2)$$

Thus,

$$\eta = 1 - \frac{T_1}{T_3}\left[\frac{r^{(k-1/k)} - 1}{1 - r^{-(k-1/k)}}\right] \qquad (3)$$

(a) Taking $a = r^{(k-1/k)}$, the bracketed term is of the form

$$\left[\frac{a-1}{1-(1/a)}\right] = a$$

Thus, Eq. (1) reads

$$\eta = 1 - \frac{T_1}{T_3}\left(r\right)^{(k-1/k)}$$, which is the objective. ⟵ (a)

(b) Combining Eq. (1) and Eq. (a)

$$\eta = 1 - \frac{T_1}{T_3}\,\frac{T_2}{T_1}$$

$$\eta = 1 - \frac{T_2}{T_3}$$, which is the objective ⟵ (b)

PROBLEM 9.66

Air at 1 bar and 15°C enters the compressor of an ideal cold air-standard regenerative Brayton cycle. The pressure at the compressor exit is 10 bar and the maximum cycle temperature is 1100°C. For $k = 1.4$, determine

(a) the net work developed, in kJ per kg of air flowing.
(b) the energy added by heat transfer, in kJ per kg of air flowing.
(c) the thermal efficiency.
(d) Using Eq. (b) of Problem 9.65, check the value obtained in (c) for thermal efficiency.

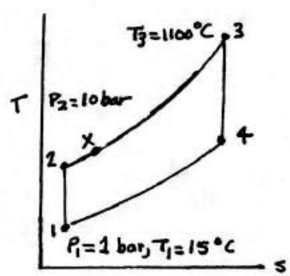
SOLUTION

Known: An ideal cold air-standard regenerative Brayton cycle operates with a specified state at the compressor inlet, pressure at the compressor exit, and turbine inlet temperature.

Find: Determine the net work developed and the energy added by heat transfer. Also determine the thermal efficiency and check the value using the result of Problem 9.65.

Engineering Model:

Schematic and Given Data



1. Each component is analyzed as a control volume at steady state. Kinetic and potential effects are ignored.
2. The turbine and compressor processes are isentropic.
3. There are no pressure drops for flow through the heat exchangers. $\eta_{regen} = 100\%$.
4. The working fluid is air modeled as an ideal gas with $k = 1.4$.

Analysis: Since $\eta_{regen} = 100\%$, $h_x = h_4$ and thus $T_x = T_4$.

Using Eq. 6.43,

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (288.15\,K)(10)^{0.286} = 556.7\,K, \quad T_4 = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (1373.15\,K)(0.1)^{0.286} = 710.7\,K$$

① (a) $$\frac{\dot{W}_{net}}{\dot{m}} = (h_3 - h_4) - (h_2 - h_1) = c_p\left[(T_3 - T_4) - (T_2 - T_1)\right] = 1.004\frac{kJ}{kg\cdot K}\left[662.45 - 268.55\right]K = 395.5\frac{kJ}{kg}$$

(b) $$\frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_x = c_p(T_3 - \overset{=T_4}{T_x}) = 1.004\frac{kJ}{kg\cdot K}(1373.15 - 710.7)K = 665.1\frac{kJ}{kg}$$

(c) $$\eta = \frac{\dot{W}_{net}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{395.5\ kJ/kg}{665.1\ kJ/kg} = 0.595\ (59.5\%)$$

(d) Using the result of Problem 9.65(b)

$$\eta = 1 - \frac{T_2}{T_3} = 1 - \frac{556.7K}{1373.15K} = 0.595\ (59.5\%)$$

---

1 $$c_p = \frac{kR}{k-1} = \left(\frac{1.4}{0.4}\right)\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right) = 1.004\frac{kJ}{kg\cdot K}$$

PROBLEM 9.67

An air-standard Brayton cycle has a compressor pressure ratio of 10. Air enters the compressor at $p_1 = 14.7$ lbf/in.^{2}, $T_1 = 70°F$ with a mass flow rate of 90,000 lb/h. The turbine inlet temperature is 2200°R. Calculate the thermal efficiency and the net power developed, in horsepower, if

(a) the turbine and compressor isentropic efficiencies are each 100%.
(b) the turbine and compressor isentropic efficiencies are 88 and 84%, respectively.
(c) the turbine and compressor isentropic efficiencies are 88 and 84%, respectively, and a regenerator with an effectiveness of 80% is incorporated.

KNOWN: Operating data are provided for an air-standard Brayton cycle.

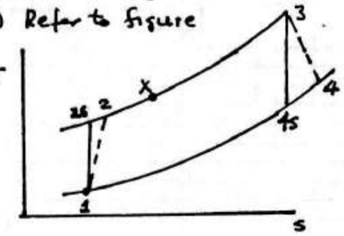
FIND: Determine thermal efficiency and net power developed in each of three cases.

SCHEMATIC & Given Data:

(a) Refer to Fig. 9.10b $\eta_t = \eta_c = 100\%$

(b) Refer to Fig 9.13 b $\eta_t = 88\%$, $\eta_c = 84\%$

(c) Refer to figure



$\eta_t = 88\%$
$\eta_c = 84\%$
$\eta_{regen} = 80\%$

In each case, $P_2/P_1 = 10$, $T_1 = 70°F$, $T_3 = 2200°R$, $\dot{m} = 9 \times 10^4$ lb/h

ENGINEERING MODEL:

See Examples 9.4, 9.6, 9.7

ANALYSIS: Using data from Table A-22E,

State 1. $T_1 = 530°R$, $h_1 = 126.66$ Btu/lb
$P_{r1} = 1.2998$.

State 2s. $P_{r,2s} = \frac{P_2}{P_1} P_{r1} = 10(1.2998) = 12.998$

$\Rightarrow h_{2s} = 244.68$ Btu/lb

State 3. $T_3 = 2200°R$, $h_3 = 560.59$ Btu/lb
$P_{r,3} = 256.6$

State 4s. $P_{r,4s} = \frac{P_4}{P_3} P_{r,3} = \frac{256.6}{10} = 25.66$ ($\frac{1}{10}$)

$\Rightarrow h_{4s} = 296.7$ Btu/lb

(a) $$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}} \stackrel{\text{see Eqs. 9.17, 9.18}}{=} 1 - \frac{(h_{4s} - h_1)}{(h_3 - h_{2s})} = 1 - \left(\frac{296.7 - 126.66}{560.59 - 244.68}\right) = 0.462\ (46.2\%) \quad (a)$$

The net power developed is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = \dot{m}((h_3 - h_{4s}) - (h_{2s} - h_1))$$
$$= \left(9 \times 10^4 \frac{lb}{h}\right)\left[(560.59 - 296.7) - (244.68 - 126.6)\right] \frac{Btu}{lb} \left| \frac{1\ hp}{2545\ Btu/h} \right| = 5158\ hp \quad (a)$$

(b) With η_t and η_c from Sec. 9.6.3 and previously obtained values,

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} \Rightarrow h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 328.4 \text{ Btu/lb}$$

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{(h_{2s} - h_1)}{\eta_c} = 267.16 \text{ Btu/lb}$$

Then

$$\eta = 1 - \frac{(h_4 - h_1)}{(h_3 - h_2)} = 1 - \frac{(328.4 - 126.66)}{(560.59 - 267.16)} = 0.312\ (31.2\%) \quad \leftarrow (b)$$

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = \dot{m}\left[(h_3 - h_4) - (h_2 - h_1)\right]$$
$$= \left(9 \times 10^4 \frac{lb}{h}\right)\underbrace{\left[(560.59 - 328.4) - (267.16 - 126.66)\right]}_{91.69} \frac{Btu}{lb} \left| \frac{1\ hp}{2545\ Btu/h} \right|$$
$$= 3242\ hp \quad \leftarrow (b)$$

Note that the values of $\dot{W}_{cycle}$ and η in part (b) are much less than the respective values of part (a).

(c) The specific enthalpy h_x at the cold-side exit of the regenerator is found using the regenerator effectiveness:

$$\eta_{regen} = \frac{h_x - h_2}{h_4 - h_2} \Rightarrow h_x = \eta_{regen}(h_4 - h_2) + h_2$$
$$= 0.8(328.4 - 267.16) + 267.16 = 316.15 \text{ Btu/lb}$$

The addition of a regenerator does not affect the net power developed. Thus the net power is the same as in part (b):

$$\dot{W}_{cycle} = 3242 \text{ hp} \quad \longleftarrow (c)$$

However, $\dot{Q}_{in} = \dot{m}(h_3 - h_x)$. Thus

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \frac{\dot{m}\left((h_3 - h_4) - (h_2 - h_1)\right)}{\dot{m}(h_3 - h_x)}$$

$$= \frac{(560.59 - 328.4) - (267.16 - 126.66)}{(560.59 - 316.15)}$$

$$= \frac{91.69 \text{ Btu/lb}}{244.44 \text{ Btu/lb}} = 0.375 \; (37.5\%) \quad \longleftarrow (c)$$

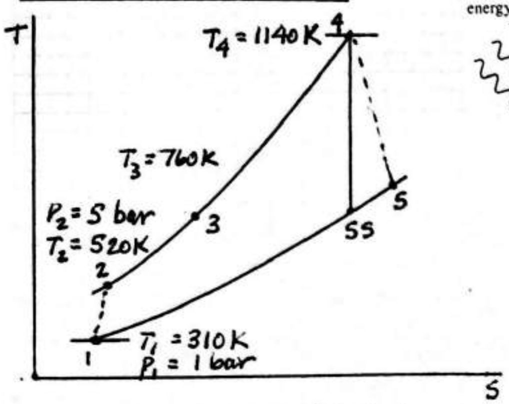
Note that with the addition of a regenerator the thermal efficiency in part (c) is greater than in part (b).

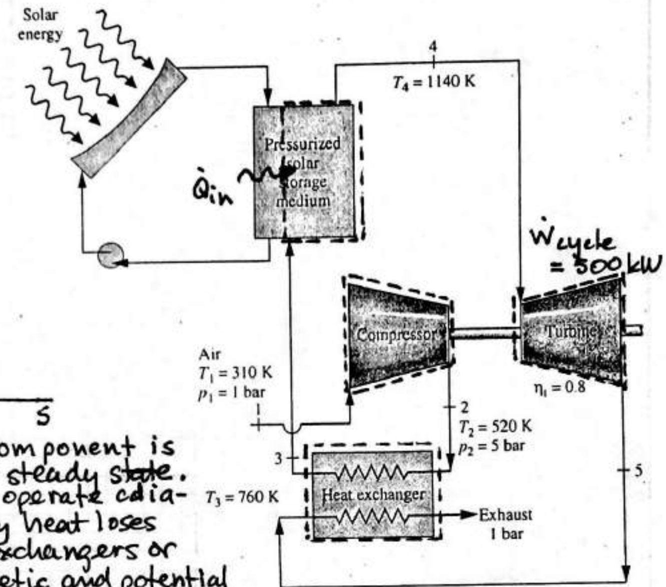
PROBLEM 9.68

KNOWN: Data are known for a regenerative gas turbine power plant using solar energy a the source of heat addition.

FIND: Determine (a) the thermal efficiency, and (b) the mass flow rate for a net power output of 500 kW.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) Each component is modeled as a control volume at steady state. (2) The compressor and turbine operate adiabatically. (3) There are no stray heat loses or pressure drops in the heat exchangers or interconnecting piping. (4) Kinetic and potential energy effects are negligible. (5) The working fluid is air modeled as an ideal gas.

ANALYSIS: For states 1-4, the temperatures are known. Thus

$T_1 = 310\text{K} \Rightarrow h_1 = 310.24 \text{ kJ/kg}$

$T_2 = 520\text{K} \Rightarrow h_2 = 523.63 \text{ kJ/kg}$

$T_3 = 760\text{K} \Rightarrow h_3 = 778.18 \text{ kJ/kg}$

$T_4 = 1140\text{K} \Rightarrow h_4 = 1207.57 \text{ kJ/kg}$

State 5: $p_{r5} = (p_5/p_4)\, p_{r4} = (1/5)(193.1) = 38.62 \Rightarrow h_{5s} = 774.49 \text{ kJ/kg}$

Using $\eta_t = (h_4 - h_5)/(h_4 - h_{5s}) \Rightarrow h_5 = h_4 - \eta_t(h_4 - h_{5s})$

$= 861.11 \text{ kJ/kg}$

(a) The thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m} - \dot{W}_c/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{(h_4-h_5)-(h_2-h_1)}{(h_4-h_3)} = \frac{(1207.57-861.11)-(523.63-310.24)}{(1207.57-778.18)}$$

$$= \frac{133.07}{429.39} = 0.31 \ (31.\%)$$

η

(b) For $\dot{W}_{cycle} = 500$ kW

$$\dot{m} = \frac{\dot{W}_{cycle}}{(h_4-h_5)-(h_2-h_1)} = \frac{500 \text{ kW}}{133.07 \text{ kJ/kg}}\left|\frac{1 \text{ kJ/s}}{1 \text{ kW}}\right|$$

$$= 3.76 \text{ kg/s}$$

$\dot{m}$

PROBLEM 9.69

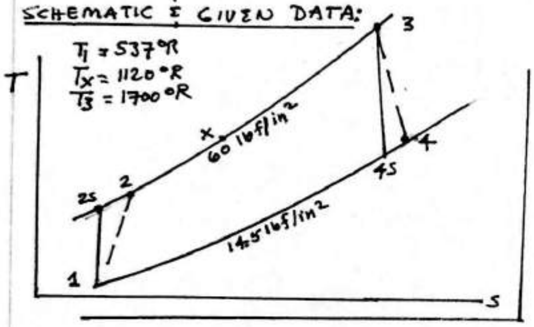
9.69 Air enters the compressor of a regenerative gas turbine at 14.5 lbf/in.^{2}, 77°F, and is compressed to 60 lbf/in.^{2} The air then passes through the regenerator and exits at 1120°R. The temperature at the turbine inlet is 1700°R. The compressor and turbine each has an isentropic efficiency of 84%. The net power developed is 1000 hp. Using an air-standard analysis, calculate

(a) the thermal efficiency of the cycle.
(b) the back work ratio.
(c) the regenerator effectiveness.
(d) the mass flow rate of the air, in lb/s.

KNOWN: A regenerative gas turbine is analyzed on an air-standard basis. Data are known at various states and isentropic efficiencies are specified. The net power developed is also specified.

FIND: Determine the (a) thermal efficiency, (b) bwr, (c) regenerator effectiveness, and (d) mass flow rate.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.7 except $\eta_c = \eta_t = 0.84$.

ANALYSIS: Using Table A-22E, fix each of the principal states:

State 1: $T_1 = 537°R \Rightarrow h_1 = 128.34$ Btu/lb, $p_{r1} = 1.3593$

State 2s: $p_{r2s} = \left(\frac{p_2}{p_1}\right) p_{r1} = 5.625$

$\Rightarrow h_{2s} = 192.76$ Btu/lb

State x: $T_x = 1120°R$, $h_x = 271.03$ Btu/lb

State 2: Using the compressor efficiency, $h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c}\right) = 205.03$ Btu/lb

State 3: $T_3 = 1700°R \Rightarrow h_3 = 422.59$ Btu/lb, $p_{r3} = 90.95$.

State 4s: $p_{r4s} = \left(\frac{p_4}{p_3}\right) p_{r3} = 21.98 \Rightarrow h_{4s} = 284.01$ Btu/lb.

State 5: Using the turbine efficiency, $h_4 = h_3 - \eta_t (h_3 - h_{4s}) = 306.18$ Btu/lb

(a) The thermal efficiency is $\eta = \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_x)} = \frac{116.41 - 76.69}{151.56} = 0.262$ (26.2%)

(b) $bwr = \frac{(h_2 - h_1)}{(h_3 - h_4)} = \frac{76.69}{116.41} = 0.659$

(c) The renerator effectiveness is $\eta_{reg} = \frac{h_x - h_2}{h_4 - h_2} = \frac{66}{101.15} = 0.652$ (65.2%)

(d) The net power developed is $\dot{W}_{cycle} = \dot{m}\left[(h_3 - h_4) - (h_2 - h_1)\right]$

$$\Rightarrow \dot{m} = \frac{\dot{W}_{cycle}}{(h_3 - h_4) - (h_2 - h_1)} = \frac{(1000\ \text{hp})\left|\frac{2545\ \text{Btu/h}}{1\ \text{hp}}\right|\left|\frac{1\ \text{h}}{3600\ \text{s}}\right|}{(116.41 - 76.69)\ \text{Btu/lb}}$$

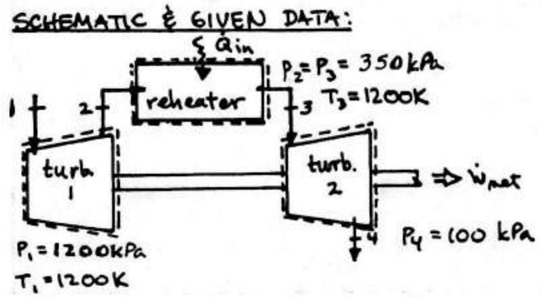
$$= 17.8\ \frac{\text{lb}}{\text{s}}$$

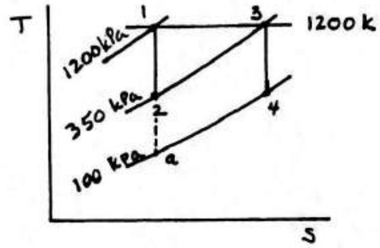
PROBLEM 9.70

KNOWN: Air expands in two stages through a turbine with reheat between the stages. The states are specified at the inlet and exit of each component.

FIND: Determine, per unit mass of air flowing, (a) the work developed by each stage, (b) the heat transfer for reheat, and (c) the increase in net work compared to a single stage of expansion with no reheat.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) Each control volume is at steady state. (2) The turbines operate isentropically. (3) Kinetic and potential energy effects are negligible. (4) The working fluid is air modeled as an ideal gas.

ANALYSIS: First, fix each of the principal states (Table A-22).

State 1 $T_1 = 1200\text{K} \Rightarrow h_1 = 1277.79$ kJ/kg, $p_{r1} = 238.0$

State 2 $p_{r2} = (P_2/P_1)\, p_{r1} = 69.417 \Rightarrow h_2 = 912.11$ kJ/kg

State 3 $T_3 = 1200\text{K} \Rightarrow h_3 = h_1 = 1277.79,\ p_{r3} = p_{r1} = 238.0$

State 4 $p_{r4} = (P_4/P_3)\, p_{r3} = 68 \Rightarrow h_4 = 906.85$ kJ/kg

(a) The work developed by each stage is

$\dot{W}_{t1}/\dot{m} = h_1 - h_2 = 365.68$ kJ/kg ← $\dot{W}_{t1}/\dot{m}$

$\dot{W}_{t2}/\dot{m} = h_3 - h_4 = 370.94$ kJ/kg ← $\dot{W}_{t2}/\dot{m}$

(b) For the reheater

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2 = (1277.79 - 912.11) = 365.7 \text{ kJ/kg}$$ ← $\dot{Q}_{in}/\dot{m}$

(c) To determine the work for a single stage of expansion, determine h_a, as follows.

$$p_{ra} = (P_a/P_1)\, p_{r1} = 19.833 \Rightarrow h_a = 638.58 \text{ kJ/kg}$$

Thus $\dot{W}/\dot{m} = (h_1 - h_a) = 639.21$ kJ/kg

and

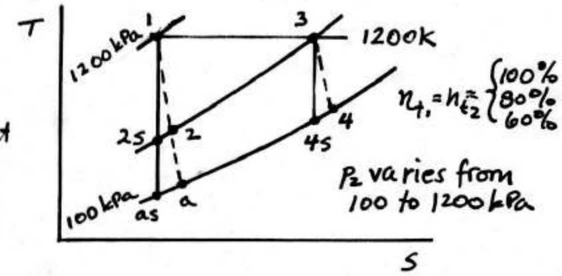
$$\% \text{ increase} = \frac{(365.68 + 370.94) - 639.21}{639.21} \times 100 = 15.2\%$$ ← % inc.

PROBLEM 9.71

KNOWN: The two-stage turbine with reheat in Problem 9.70 is reconsidered with turbine stage efficiencies less than 100%.

FIND: Plot (a) the work developed by each stage, (b) the heat transfer for the reheat process, and (c) the percent increase in work for the two-stage turbine with reheat compared to a single expansion, for each efficiency, and for interstage pressures ranging from 100 to 1200 kPa.

SCHEMATIC & GIVEN DATA:



See solution to Problem 9.70 for schematic.

ENGINEERING MODEL: (1) Each control volume is at steady state. (2) The turbines operate adiabatically. (3) Kinetic and potential energy effects can be neglected. (4) The working fluid is air as an ideal gas.

ANALYSIS: Consider the sample case of $P_2 = 350$ kPa and $\eta_{t_1} = \eta_{t_2} = 0.8$. Fixing each of the principal states:

State 1: From Table A-22, $T_1 = 1200$ K, $h_1 = 1277.79$ kJ/kg

State 2: With $h_{2s} = 912.11$ from Problem 9.70 and using the turbine efficiency

$$h_2 = h_1 - \eta_{t_1}(h_1 - h_{2s}) = 985.25 \text{ kJ/kg}$$

State 3: $h_3 = 1277.79$ kJ/kg

State 4: With $h_{4s} = 906.85$ kJ/kg (from Problem 9.70), $h_4 = h_3 - \eta_{t_2}(h_3 - h_{4s}) = 981.04$ kJ/kg

State a: $h_{as} = 638.58$ kJ/kg ; $h_a = h_1 - \eta_{t_1}(h_1 - h_{as}) = 766.42$ kJ/kg

(a) The work developed by each stage is, respectively

$$\dot{W}_{t_1}/\dot{m} = h_1 - h_2 = 292.54 \text{ kJ/kg}$$
$$\dot{W}_{t_2}/\dot{m} = h_3 - h_4 = 296.75 \text{ kJ/kg}$$

(b) For the reheater

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2 = 292.54 \text{ kJ/kg}$$

(c) For comparison to a single stage, the percent increase in work is

$$\dot{W}/\dot{m} = h_1 - h_a = 511.37 \text{ kJ/kg}$$

Thus $$\% \text{ increase} = \frac{(292.54 + 296.75) - 511.37}{511.37} \times 100 = 15.2\,\%$$

The plots on the next page show how each of these quantities vary with interstage pressure for each of three efficiency values. Note that only one curve is needed for the % increase in work, since the efficiency value cancels out if the same value is used for each stage.

The data for the required plots were obtained using IT, as follows:

PROBLEM 9.71 (Cont'd.) - Page 2

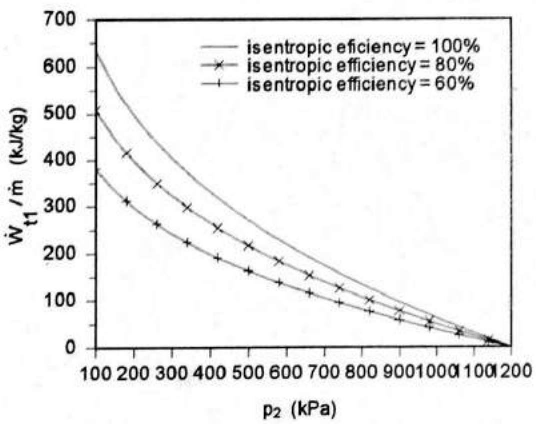
IT Code

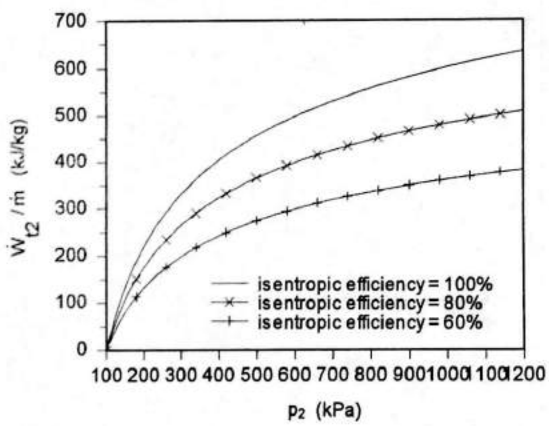
```
p1 = 1200 // kPa
T1 = 1200 // K
p2 = 350 // kPa
T3 = 1200 // K
p3 = p2
p4 = 100 // kPa
eta_t = 0.6

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
s2s = s_hP("Air", h2s, p2)
s2s = s1
h2 = h1 - (h1 - h2s) * eta_t
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
s4s = s_hP("Air", h4s, p4)
s4s = s3
h4 = h3 - (h3 - h4s) * eta_t

Wdot1 / mdot = (h1 - h2)
Wdot2 / mdot = (h3 - h4)
mdot = 1 // Assume a unit mass flow rate.
Qdot_in / mdot = h3 - h2

Wdots / mdot = h1 - ha
sas = s_hP("Air", has, p4)
sas = s1
ha = h1 - (h1 - has) * eta_t
pct = (((Wdot1 + Wdot2) - Wdots) / Wdots) * 100
```

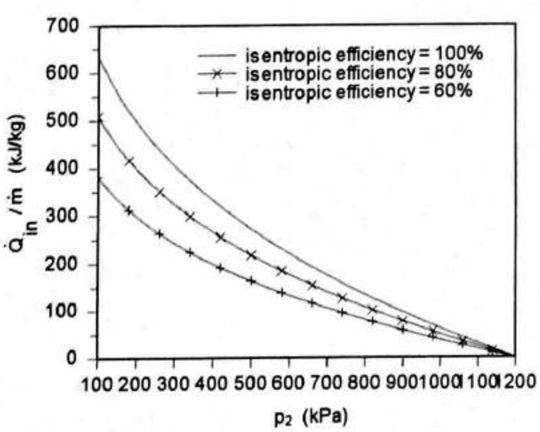
IT Results for $\eta_t = 0.8$

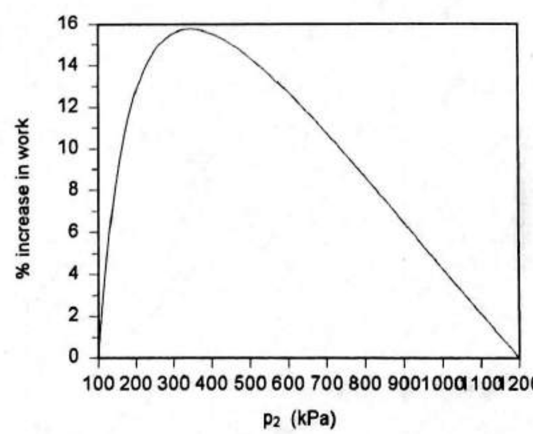
h_1 = 1277 kJ/kg
h_2 = 984.9 kJ/kg
h_{2s} = 911.9 kJ/kg
h_3 = 1277 kJ/kg
h_4 = 980.8 kJ/kg
h_{4s} = 906.7 kJ/kg
h_a = 768.7 kJ/kg
h_{as} = 641.7 kJ/kg
$\dot{W}_{t1}/\dot{m}$ = 292.2 kJ/kg
$\dot{W}_{t2}/\dot{m}$ = 296.3 kJ/kg
$\dot{Q}_{in}/\dot{m}$ = 292.2 kJ/kg
% increase = 15.77

PLOTS:









PROBLEM 9.72

The schematic and T–s diagram of a two-stage turbine operating at steady state with reheat at pressure p_i between the two stages is shown in Fig. P9.72. Values for p_1, T_1 and p_2 are known. The temperature at the inlet to each turbine stage is the same, the turbine expansions are isentropic, and kinetic and potential energy effects are negligible. Assuming the ideal gas model with constant k for the air, show that

(a) the maximum total work per unit of mass flowing is developed when the pressure ratio is the same across each stage.

(b) the temperature at the exit of each turbine stage is the same.

(c) the work developed per unit mass of air flowing is the same for each turbine stage.

(d) the heat transfer per unit of mass flowing equals the work value determined in part (c).

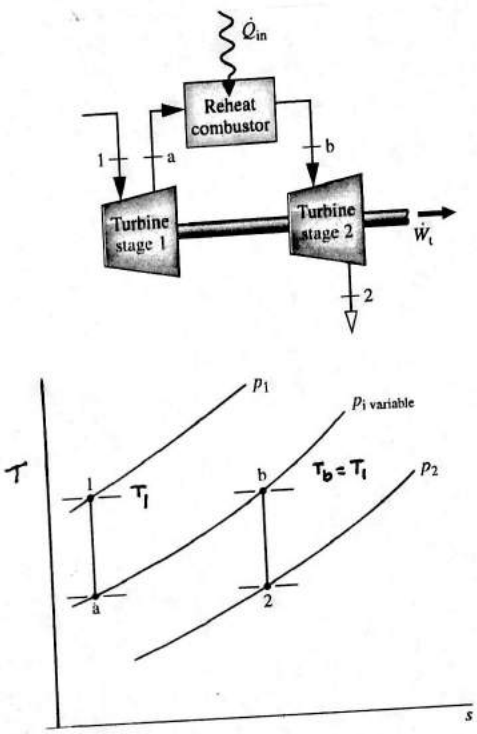


Fig. P9.72

KNOWN: A two-stage turbine with reheat operates at steady state under specified conditions.

FIND: On the basis of a cold air-standard analysis demonstrate four distinct performance features.

ENGINEERING MODEL: 1. Each component is analyzed as a control volume at steady state. 2. The turbine expansions are isentropic. 3. There is no pressure change through the reheater. 4. The temperatures at the turbine inlets are equal. 5. Kinetic and potential effects are negligible. 6. The working fluid is air modeled as an ideal gas with constant k and c_p.

ANALYSIS: (a) The turbine work developed per unit of mass flowing is

$$\frac{\dot{W}_t}{\dot{m}} = (h_1 - h_a) + (h_b - h_2) = c_p\left[(T_1 - T_a) + (T_b - T_2)\right]$$

With $T_1 = T_b$

$$\frac{\dot{W}_t}{\dot{m}} = c_p\left[2T_1 - T_a - T_2\right] = c_p T_1\left[2 - \frac{T_a}{T_1} - \frac{T_2}{T_1}\right]$$

For the isentropic expansions Eq. 6.43 gives

$$\frac{T_a}{T_1} = \left(\frac{p_i}{p_1}\right)^{\frac{k-1}{k}} \quad \text{and} \quad \frac{T_2}{T_b} = \frac{T_2}{T_1} = \left(\frac{p_2}{p_i}\right)^{\frac{k-1}{k}} \qquad (1)$$

Thus

$$\dot{W}_t/\dot{m} = c_p T_1\left[2 - \left(\frac{p_i}{p_1}\right)^{\frac{k-1}{k}} - \left(\frac{p_2}{p_i}\right)^{\frac{k-1}{k}}\right]$$

To determine the pressure p_i that maximizes the total, form the derivative

$$\frac{\partial(\dot{W}_t/\dot{m})}{\partial p_i} = -c_p T_1\left(\frac{k-1}{k}\right)\left[\left(\frac{p_i}{p_1}\right)^{-\frac{1}{k}}\left(\frac{1}{p_1}\right) + \left(\frac{p_2}{p_i}\right)^{-\frac{1}{k}}\left(-\frac{p_2}{p_i^2}\right)\right]$$

$$= -c_p T_1\left(\frac{k-1}{k}\right)\left(\frac{1}{p_i}\right)\left[\left(\frac{p_i}{p_1}\right)^{\frac{k-1}{k}} - \left(\frac{p_2}{p_i}\right)^{\frac{k-1}{k}}\right]$$

For $\frac{\partial(\dot{W}_t/\dot{m})}{\partial p_i} = 0 \Rightarrow \frac{p_i}{p_1} = \frac{p_2}{p_i}$ ←——— (a)

By checking the second derivative it can be verified that the total turbine work is a maximum.

PROBLEM 9.72 (continued)

(b) With $\frac{p_i}{p_1} = \frac{p_2}{p_i}$, Eqs.(1) give $T_a = T_2$, since $T_1 = T_b$ ← (b)

(c) $$\left(\frac{\dot{W}_t}{\dot{m}}\right)_1 = h_1 - h_a = c_p(T_1 - \underbrace{T_a}_{=T_2})$$

$$\left(\frac{\dot{W}_t}{\dot{m}}\right)_2 = h_b - h_2 = c_p(\underbrace{T_b}_{=T_1} - T_2)$$

$$\Rightarrow \left(\frac{\dot{W}_t}{\dot{m}}\right)_1 = \left(\frac{\dot{W}_t}{\dot{m}}\right)_2 = c_p(T_1 - T_2)$$ ← (c)

(d) For the reheater the energy balance reduces to give

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_b - h_a = c_p(\underbrace{T_b}_{=T_1} - \underbrace{T_a}_{=T_2})$$

$$= c_p(T_1 - T_2)$$

Thus the heat added per unit mass flowing has the same magnitude as the work developed per unit mass flowing of each turbine stage. ← (d)

PROBLEM 9.73

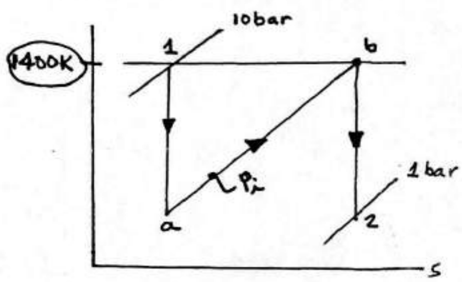
9.73 Air at 10 bar enters a two-stage turbine with reheat operating at steady state. The overall pressure ratio across the stages is 10. Reheat occurs at the pressure that maximizes total turbine work per unit mass of air flowing as determined in Problem 9.72(a). The temperature at the inlet of each turbine stage is 1400 K and each stage operates isentropically. Assuming the ideal gas model with $k = 1.4$ for the air, determine

(a) the reheater pressure, in bar.
(b) for each turbine stage the work developed per unit mass of air flowing, in kJ/kg.
(c) for the reheater the heat transfer per unit mass of air flowing, in kJ/kg.

Known: Steady-state data are provided for a two-stage turbine with reheat between the stages.

Find: Determine the reheat pressure, the work developed per unit of mass flowing for each turbine stage, and the heat transfer per unit of mass flowing.

Schematic and Given Data:



Engineering Model

1. Each component is modeled as a control volume at steady state.
2. The turbine expansions are isentropic.
3. Reheat occurs at the pressure that maximizes total turbine work.
4. The working fluid is air modeled as an ideal gas with $k=1.4$.
5. Kinetic and potential energy effects are ignored.

Analysis: (a) From Problem 9.72(a) maximum work is developed when the pressure ratio across each stage is the same. That is $P_1/P_a = P_b/P_2$. Since $P_a = P_b$, the reheat pressure, P_i, is $P_i = \sqrt{P_1 P_2} = \sqrt{(1\text{bar})(10\text{bar})} = 3.16\text{ bar}$ ← (a)

(b) With Eq. 6.43 $\frac{T_a}{T_1} = \left(\frac{P_i}{P_1}\right)^{(k-1)/k}$ and $\frac{T_2}{T_b} = \left(\frac{P_2}{P_i}\right)^{(k-1)/k} \Rightarrow \frac{T_a}{T_1} = \frac{T_2}{T_b} \Rightarrow T_a = T_2$. ($T_b = T_1$)

Then $T_a = T_1\left(\frac{P_i}{P_1}\right)^{(k-1)/k} = (1400\text{K})\left(\frac{3.16\text{ bar}}{10\text{ bar}}\right)^{0.286} = 1007\text{ K}$

Since $T_1 = T_b$ and $T_a = T_2$, the work per unit of mass flowing for each stage is the same: (b)

(1) $$\left(\dot{W}_t/\dot{m}\right) = c_p(T_1 - T_2) = \left(1.004\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(1400-1007)\text{K} = 394.6\frac{\text{kJ}}{\text{kg}}$$

(c) For the reheater

$$\dot{Q}_{in}/\dot{m} = h_b - h_a = c_p(\underbrace{T_b}_{=T_1} - \underbrace{T_a}_{=T_2}) = 394.6\frac{\text{kJ}}{\text{kg}} \quad \leftarrow \text{(c)}$$

---

1. With Eq. 3.47a,

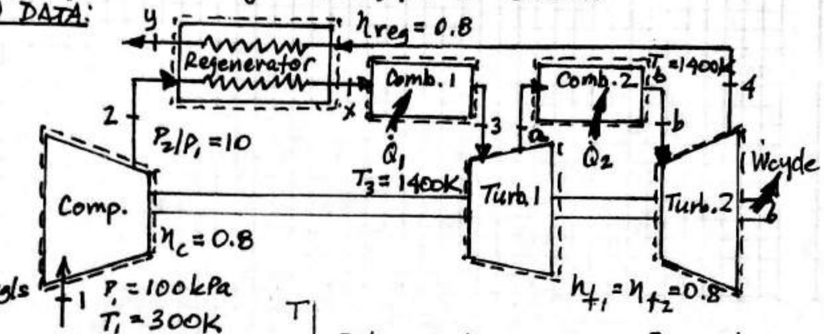
$$c_p = \frac{kR}{k-1} = \frac{1.4}{0.4}\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right) = 1.004\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

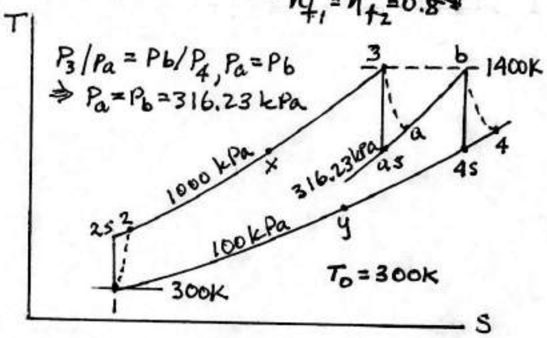
PROBLEM 9.74

<u>KNOWN</u>: Air enters a cold air-standard regenerative Brayton cycle with reheat at a specified state with a given mass flow rate. The compressor pressure ratio, maximum cycle temperature, and reheat temperature are known. The compressor and turbines each have isentropic efficiencies of 0.8 and the regenerator effectiveness is 0.8.

<u>FIND</u>: Determine (a) the thermal efficiency, (b) the back work ratio, (c) the net power, and (d) the rates of exergy destruction in the compressor, each turbine stage, and the regenerator, for $T_0 = 300$ K.

<u>SCHEMATIC & GIVEN DATA</u>:





<u>ENGINEERING MODEL</u>: See Example 9.11. Also, $\eta_c = \eta_{t_1} = \eta_{t_2} = 0.8$ and $\eta_{reg} = 0.8$. The specific heats are constant, with $k = 1.4$ and $c_p = 1.005$ kJ/kg·K. Let $T_0 = 300$K

<u>ANALYSIS</u>: From the solution to Problem 9.55 : $T_2 = 649$ K.

To find T_a, first find T_{as} as follows:

$$T_{as} = \left(\frac{P_a}{P_3}\right)^{\frac{k-1}{k}}(T_3) = \left(\frac{316.23}{1000}\right)^{\frac{1.4-1}{1.4}}(1400) = 1007.6\text{K}$$

Using the first-stage turbine efficiency: $\eta_{t_1} = \dfrac{h_3 - h_a}{h_3 - h_{as}} = \dfrac{c_p(T_3 - T_a)}{c_p(T_3 - T_{as})}$

Thus $T_a = T_3 - \eta_{t_1}(T_3 - T_{as}) = 1400 - (0.8)(1400 - 1007.6) = 1086.1$ K

Similarly, for the second-stage turbine $T_{4s} = \left(\dfrac{P_4}{P_b}\right)^{\frac{k-1}{k}}(T_b) = \left(\dfrac{100}{316.23}\right)^{\frac{1.4-1}{1.4}}(1400) = 1007.6$ K

$$T_4 = T_b - \eta_{t_2}(T_b - T_{4s}) = 1086.1 \text{ K}$$

The regenerator effectiveness is: $\eta_{reg} = \dfrac{h_x - h_2}{h_4 - h_2} = \dfrac{\cancel{c_p}(T_x - T_2)}{\cancel{c_p}(T_4 - T_2)}$

or $T_x = T_2 + \eta_{reg}(T_4 - T_2) = 998.7$ K

For the control volume enclosing the regenerator

$$0 = (h_2 - h_x) + (h_4 - h_y) \Rightarrow 0 = \cancel{c_p}(T_2 - T_x) + \cancel{c_p}(T_4 - T_y)$$

or $T_y = T_4 - (T_x - T_2) = 736.4$ K

(a) The rate of heat addition is

$$\dot{Q}_{in} = \dot{Q}_1 + \dot{Q}_2 = \dot{m}\left[c_p(T_3 - T_x) + c_p(T_b - T_a)\right]$$

$$= \left(6\,\frac{\text{kg}}{\text{s}}\right)\left(1.005\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left[(1400 - 998.7) + (1400 - 1086.1)\right] = 4312.7 \text{ kJ/s}$$

and $\dot{Q}_{out} = \dot{m}c_p(T_y - T_1)$

$$= (6)(1.005)(736.4 - 300) = 2631.5 \text{ kJ/s}$$

PROBLEM 9.74 (Cont'd.) – Page 2

The thermal efficiency is

$$\eta = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}} = 1 - \frac{2631.5}{4312.7} = 0.390\ (39.0\%) \longleftarrow \eta$$

(b) $\dot{W}_c = \dot{m} c_p (T_2 - T_1) = (6\ \frac{kg}{s})(1.005\ \frac{kJ}{kg \cdot K})(649 - 300)K = 2104.5\ kJ/s$

For turbine 1: $\dot{W}_{t_1} = \dot{m} c_p (T_3 - T_a) = (6)(1.005)(1400 - 1086.1) = 1892.8\ \frac{kJ}{s}$

Since $T_b = T_3$ and $T_a = T_4$; $\dot{W}_{t_2} = \dot{W}_{t_1} = 1892.8\ kJ/s$

Thus

$$\dot{W}_t = (2)(1892.8) = 3785.6\ kJ/s$$

and

$$bwr = \frac{\dot{W}_c}{\dot{W}_t} = \frac{2104.5}{3785.6} = 0.556 \longleftarrow bwr$$

(c) The net power is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = (3785.6 - 2104.5)\frac{kJ}{s}\left|\frac{1\ kW}{1\ kJ/s}\right| = 1681.1\ kW \longleftarrow \dot{W}_{cycle}$$

(d) For the compressor: $0 = \cancel{\sum_j (\dot{Q}/T)_j}^{\,0} + \dot{m}(s_1 - s_2) + (\dot{\sigma}_{cv})_{comp}$

$$(\dot{E}_d)_{comp} = T_0(\dot{\sigma}_{cv})_{comp} = T_0 \dot{m}\left[c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right)\right]$$

$$= (300K)(6\ \frac{kg}{s})\left[(1.005\ \frac{kJ}{kg \cdot K}) \ln\left(\frac{649}{300}\right) - \left(\frac{8.314}{28.97}\frac{kJ}{kg \cdot K}\right)\ln\left(\frac{1000}{100}\right)\right]$$

$$= 206.5\ kW \longleftarrow (\dot{E}_d)_{comp}$$

Turbine stage 1: $(\dot{E}_d)_{turb,1} = T_0(\dot{\sigma}_{cv})_{turb,1} = T_0 \dot{m}(s_a - s_3)$

$$(\dot{E}_d)_{turb,1} = T_0 \dot{m}\left[c_p \ln\left(\frac{T_a}{T_3}\right) - R \ln\left(\frac{p_a}{p_3}\right)\right]$$

$$= (300)(6)\left[(1.005)\ln\left(\frac{1086.1}{1400}\right) - \left(\frac{8.314}{28.97}\right)\ln\left(\frac{316.23}{1000}\right)\right]$$

$$= 135.5\ kW \longleftarrow (\dot{E}_d)_{turb,1}$$

Since $T_b = T_3$, $T_4 = T_a$ and $p_a/p_3 = p_4/p_b$; $(\dot{E}_d)_{turb,2} = (\dot{E}_d)_{turb,1}$

$$= 135.5\ kW \longleftarrow (\dot{E}_d)_{turb,2}$$

Finally, for the regenerator:

$$0 = \cancel{\sum_j (\dot{Q}/T)_j}^{\,0} + \dot{m}[(s_2 - s_x) + (s_4 - s_y)] + (\dot{\sigma}_{cv})_{reg}$$

and

$$(\dot{E}_d)_{reg} = T_0(\dot{\sigma}_{cv})_{reg} = T_0 \dot{m}\left[c_p \ln\left(\frac{T_x}{T_2}\right) - \cancel{R \ln\frac{p_x}{p_2}}^{\,0} + c_p \ln\left(\frac{T_y}{T_4}\right) - \cancel{R \ln\frac{p_y}{p_4}}^{\,0}\right]$$

$$= (300)(6)(1.005)\left[\ln\left(\frac{998.7}{649}\right) + \ln\left(\frac{736.4}{1086.1}\right)\right]$$

$$= 76.8\ kW \longleftarrow (\dot{E}_d)_{reg}$$

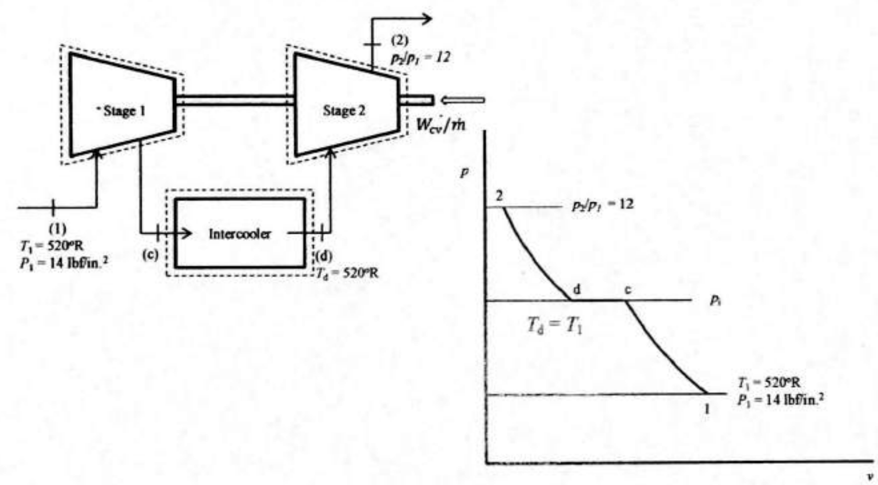
PROBLEM 9.75

Air enters a two-stage compressor operating at steady state at 520°R, 14 lbf/in.[2] The overall pressure ratio across the stages is 12, and each stage operates isentropically. Intercooling occurs at constant pressure at the value that minimizes compressor work input as determined in Example 9.10, with air exiting the intercooler at 520°R. Assuming ideal gas behavior, with k = 1.4, determine the work per unit mass of air flowing for the two-stage compressor. Kinetic and potential energies can be ignored.

KNOWN: Air enters a two-stage compressor operating at steady state with known temperature and pressure. Each stage operates isentropically, and ideal intercooling occurs between the stages at the pressure that minimizes compressor work input. The overall pressure ratio and the temperature of the air exiting the intercooler are given.

FIND: Determine the work per unit mass of air flowing for the two-stage compressor.

SCEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

1. The compressor stages and intercooler are analyzed as control volumes at steady state.
2. The compression processes are isentropic.
3. There is no pressure drop for flow through the intercooler.
4. Kinetic and potential energy effect are ignored.
5. The air is modeled as an ideal gas with $k = 1.4$.

ANALYSIS:
From Example 9.10, the intercooler pressure that corresponds to the minimum compressor work input is found from $p_i/p_1 = p_2/p_i$, with $p_2 = 12\,p_1$

$$p_i/p_1 = \sqrt{12} = 3.464$$

From Example 9.10, the work *input* per unit mass is

$$\frac{\dot{W}_c}{\dot{m}} = c_p T_1\left[\left(\frac{p_i}{p_1}\right)^{\frac{k-1}{k}} + \left(\frac{p_2}{p_i}\right)^{\frac{k-1}{k}} - 2\right] = 2c_p T_1\left[\left(\frac{p_i}{p_1}\right)^{\frac{k-1}{k}} - 1\right]$$

For $k = 1.4$, $c_p = kR/(k-1) = 0.24$ Btu/lb·°R. Thus

$$\frac{\dot{W}_c}{\dot{m}} = 2(0.24\frac{\text{Btu}}{\text{lb}}\cdot°\text{R}\ (520°R)\left[(3.464)^{\frac{1.4-1}{1.4}} - 1\right] = \underline{\mathbf{106.4\ Btu/lb}}$$

PROBLEM 9.76

Air enters a two-stage compressor operating at steady state at 1 bar, 290 K. The overall pressure ratio across the stages is 16 and each stage operates isentropically. Intercooling occurs at the pressure that minimizes total compressor work, as determined in Example 9.10. Air exits the intercooler at 290 K. Assuming ideal gas behavior with $k = 1.4$, determine

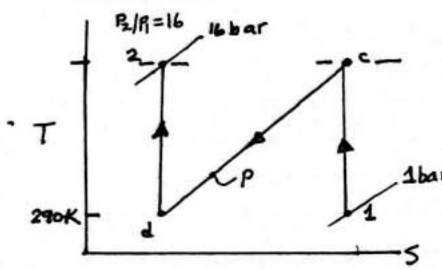
(a) the intercooler pressure, in bar, and the heat transfer, in kJ per kg of air flowing.

(b) the work required for each compressor stage, in kJ per kg of air flowing.

Known: Steady-state data are provided for a two-stage compressor with intercooling between the stages.

Find: Determine the intercooler pressure, the work input per unit of mass flowing for each compressor stage, and the heat transfer per unit of mass flowing.

Schematic and Given Data:



Engineering Model

1. Each component is modeled as a control volume at steady state.
2. The compression processes are isentropic.
3. Intercooling occurs at constant pressure: the pressure that minimizes total work input.
4. The working fluid is air modeled as an ideal gas with $k = 1.4$
5. Kinetic and potential energy effects are ignored.

ANALYSIS: (a) From Example 9.10, since $T_1 = T_d$, the minimum total work input is required when the pressure ratio is the same across each stage: $P_c/P_1 = P_2/P_d$. Since $P_c = P_d$, the intercooler pressure, P, is

$$P = \sqrt{P_1 P_2} = \sqrt{(1\,bar)(16\,bar)} = 4\,bar.$$

With Eq. 6.43, $\frac{T_c}{T_1} = \left(\frac{P_c}{P_1}\right)^{(k-1)/k}$ and $\frac{T_2}{T_d} = \left(\frac{P_2}{P_d}\right)^{(k-1)/k}$ $\Rightarrow \frac{T_c}{T_1} = \frac{T_2}{\underset{(=T_1)}{T_d}} \Rightarrow \underline{\underline{T_2 = T_c}}$ (as shown on the schematic)

where

$$T_c = T_1\left(\frac{P}{P_1}\right)^{(k-1)/k} = 290\,K\,(4)^{0.286} = 431\,K$$

An energy rate balance for the intercooler gives

① $$\frac{\dot{Q}}{\dot{m}} = h_d - h_c = c_p(T_d - T_c) = \left(1.004\,\frac{kJ}{kg \cdot K}\right)(290\,K - 431\,K) = -141.6\,\frac{kJ}{kg}$$

(b) The work input for each stage is the same:

② $$\left(\frac{\dot{W}_c}{\dot{m}}\right)_{1-c} = \left(\frac{\dot{W}_c}{\dot{m}}\right)_{d-2}$$

$$= c_p(T_c - T_1) = \left(1.004\,\frac{kJ}{kg \cdot K}\right)(431\,K - 290\,K) = +141.6\,\frac{kJ}{kg}$$

---

1. $c_p = \frac{kR}{k-1} = \frac{1.4}{0.4}\left(\frac{8.314}{28.97}\,\frac{kJ}{kg \cdot K}\right) = 1.004\,\frac{kJ}{kg \cdot K}$

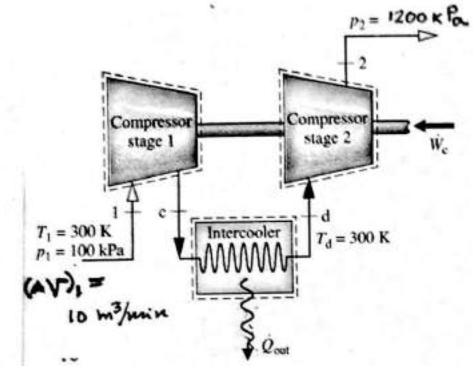
2. $\left(\frac{\dot{W}_c}{\dot{m}}\right)_{d-2} = c_p(\underset{(=T_c)}{T_2} - \underset{(=T_1)}{T_d})$

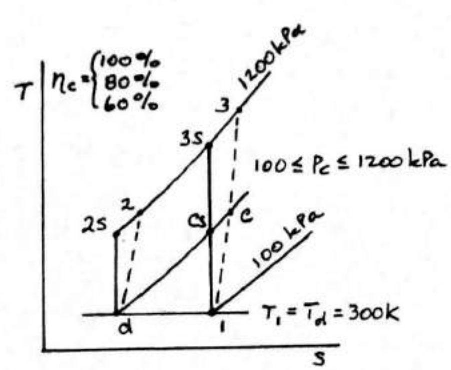
PROBLEM 9.77

KNOWN: Air is compressed in a two-stage compressor with intercooling between the stages. Operating temperatures, pressures, and compressor stage efficiencies are given.

FIND: Plot the (a) power input to each stage, (b) heat transfer rate for the intercooler, and (c) percent decrease in power for two-stage compression compared a single compression stage, for each efficiency value given, and for interstage pressures ranging from 100 to 1200 kPa.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: Same as in Example 9.9, except $\eta_{c1} = \eta_{c2} \le 100\%$

ANALYSIS: Considering the sample case of $P_c = P_d = 350$ kPa and $\eta_{c1} = \eta_{c2} = 0.8$, and using data from Table A-22, we get

State	1	cs	c	d	2s	2	3s	3
h (kJ/kg)	300.19	429.77	462.16	300.19	427.21	459.0	610.65	688.3

The mass flow rate is

$$\dot{m} = \frac{(AV)_1 P_1}{RT_1} = \frac{(10\ m^3/min)(100\ kPa)}{\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(300K)}\left|\frac{1\ min}{60\ s}\right|\left|\frac{10^3\ N/m^2}{1\ kPa}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right| = 0.1936\ kg/s$$

(a) The power input for each stage is, for $\dot{m} = 0.1936$ kg/s

$$\dot{W}_{c1} = \dot{m}(h_c - h_1) = 31.36\ kW$$

$$\dot{W}_{c2} = \dot{m}(h_2 - h_d) = 30.75\ kW$$

also $\dot{W}_c = \dot{W}_{c1} + \dot{W}_{c2} = 62.11$ kW

(b) For the intercooler

$$\dot{Q}_{out} = \dot{m}(h_c - h_d) = 31.36\ kW$$

(c) The power for a single stage is $\dot{W} = \dot{m}(h_3 - h_1) = 75.14$ kW

Finally

$$\%\ decrease = \frac{75.14 - 62.11}{75.14} \times 100 = 17.34\%$$

The plots on the next page show how each of these quantities varies with interstage pressure for each of the efficiency values. Note that only one curve is needed for the % decrease in power input, since the efficiency value cancels out if the same value is used for each stage.

PROBLEM 9.77 (Cont'd.) - Page 2

The data for the required plots are obtained using IT, as follows:

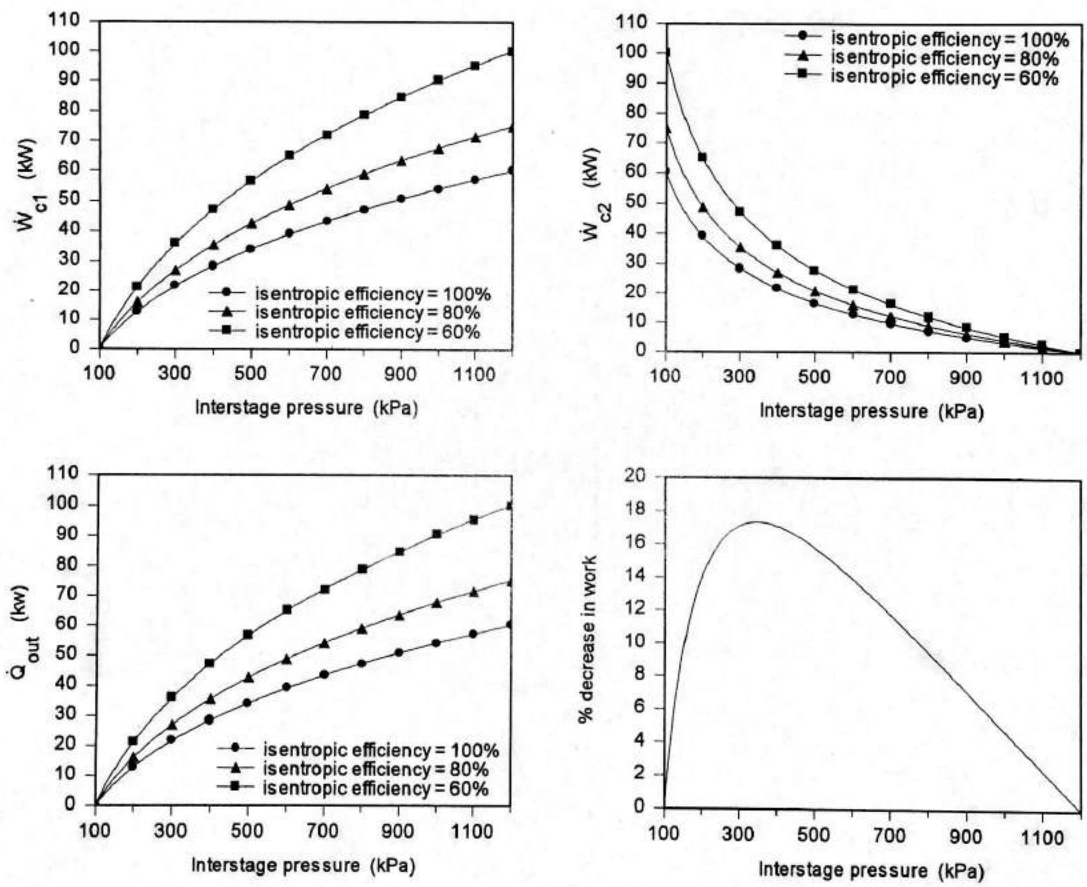
IT Code

```
p1 = 100  // kPa
T1 = 300  // K
pc = 350  // kPa
pd = pc
Td = T1
p2 = 1200  // kPa
p3 = p2
eta_c = 0.8
AV1 = 10  // m³/min

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
scs = s_hP("Air", hcs, pc)
scs = s1
hc = h1 + (hcs - h1) / eta_c
hd = h_T("Air", Td)
sd = s_TP("Air", Td, pd)
s2s = s_hP("Air", h2s, p2)

s2s = sd
h2 = hd + (h2s - hd) / eta_c
s3s = s_hP("Air", h3s, p3)
s3s = s1
h3 = h1 + (h3s - h1) / eta_c

mdot = (AV1 / v1) * (1 / 60)  // kg/s
v1 = v_TP("Air",T1, p1)
Wdotc1 = mdot * (hc - h1)
Wdotc2 = mdot * (h2 - hd)
Wdotc = Wdotc1 + Wdotc2
Qdotout = mdot * (hc - hd)
Wdot = mdot * (h3 - h1)
pct = ((Wdot - Wdotc) / Wdot) * 100
```

IT Results for $p_c = p_d = 350$ kPa, $\eta_{c1} = \eta_{c2} = 80\%$

$h_1 = 300$ kJ/kg
$h_2 = 458.9$ kJ/kg
$h_{2s} = 427.1$ kJ/kg
$h_3 = 688.2$ kJ/kg
$h_{3s} = 610.5$ kJ/kg
$h_c = 462.1$ kJ/kg
$h_{cs} = 429.7$ kJ/kg
$h_d = 300$ kJ/kg
$\dot{m} = 0.1936$ kg/s
$\dot{W}_{c1} = 31.37$ kW
$\dot{W}_{c2} = 30.76$ kW
$\dot{Q}_{out} = 31.37$ kW
% decrease = 17.32

PLOTS:



PROBLEM 9.78

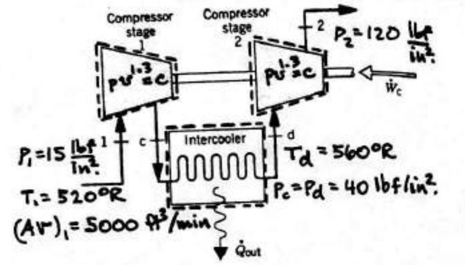
Air enters a compressor operating at steady state at 15 lbf/in.², 60°F, with a volumetric flow rate of 5000 ft³/min. The compression occurs in two stages, with each stage being a polytropic process with $n = 1.3$. The air is cooled to 100°F between the stages by an intercooler operating at 40 lbf/in.² Air exits the compressor at 120 lbf/in.² Determine, in Btu per min

(a) the power and heat transfer rate for each compressor stage.
(b) the heat transfer rate for the intercooler.

KNOWN: Air is compressed at steady state in a two-stage compressor with intercooling between the stages. Operating pressures and temperatures are known.

FIND: Determine (a) the power and heat transfer rate for each compressor stage and (b) the intercooler heat transfer rate.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. Each component is analyzed as a control volume at steady state.
2. Compressions are polytropic with $n = 1.3$.
3. There is no pressure drop through the intercooler.
4. Kinetic and potential energy are negligible.
5. The air behaves as an ideal gas.

ANALYSIS: (a) Using Eq. 3.56,

$$T_c = \left(\frac{P_c}{P_1}\right)^{(n-1)/n} T_1 = \left(\frac{40}{15}\right)^{0.231}(520°R) = 652.08\ °R$$

Then with Eq. 6.55a and the ideal gas model

① $$\frac{\dot{W}_{c1}}{\dot{m}} = \frac{nR}{(n-1)}(T_c - T_1) = \left(\frac{1.3}{0.3}\right)\left(\frac{1.986}{28.97}\ \frac{Btu}{lb \cdot °R}\right)(652.08 - 520)\ °R = 39.24\ \frac{Btu}{lb}$$

The mass flow rate is

$$\dot{m} = \frac{(AV)_1 P_1}{RT_1} = \frac{(5000\ ft^3/min)(15 \times 144\ lbf/ft^2)}{\left(\frac{1545}{28.97}\ \frac{ft \cdot lbf}{lb \cdot °R}\right)(520\ °R)} = 389.4\ \frac{lb}{min}$$

The power input for stage 1 is

$$\dot{W}_{c1} = \left(389.4\ \frac{lb}{min}\right)\left(39.24\right)\frac{Btu}{lb} = 15{,}280\ \frac{Btu}{min}$$ ←

Applying an energy rate balance to stage 1 using Table A-22E data

① $$\dot{Q}_{c1} = \dot{m}(h_c - h_1) - \dot{W}_{c1}$$
$$= \left(389.4\ \frac{lb}{min}\right)(156.01 - 124.27)\frac{Btu}{lb} - 15{,}280\ \frac{Btu}{min}$$
$$= -2920\ \frac{Btu}{min}$$ ←

Turning to stage 1, Eq. 3.56 gives

$$T_2 = \left(\frac{P_2}{P_d}\right)^{(n-1)/n} T_d = \left(\frac{120}{40}\right)^{0.231}(560\ °R) = 721.8\ °R$$

$$\dot{W}_{c2} = \dot{m}\left(\frac{nR}{n-1}\right)(T_2 - T_d) = 18{,}717\ \frac{Btu}{min}$$ ←

PROBLEM 9.78 (Continued)

With $h_d = 133.86$ Btu/lb and $h_2 = 172.8$ Btu/lb from Table A-22E, an energy rate balance for for stage 2 gives

$$\dot{Q}_{c2} = \dot{m}(h_2 - h_d) - \dot{W}_{c2}$$
$$= 389.4(172.8 - 133.86) - 18,717 = -3544 \frac{\text{Btu}}{\text{min}}$$

(b) Apply an energy rate balance to the intercooler,

② $$\dot{Q}_{out} = \dot{m}(h_c - h_d)$$
$$= \left(389.4 \frac{\text{lb}}{\text{min}}\right)(156.01 - 133.86) \frac{\text{Btu}}{\text{lb}}$$
$$= 8625 \frac{\text{Btu}}{\text{min}}$$

1. Here $\dot{W}_c$ denotes a positive value accounting for the power input. Also, h_1 and h_c are from Table A-22E.
2. Here $\dot{Q}_{out}$ denotes a positive value accounting for the heat transfer of energy from the intercooler.

PROBLEM 9.79

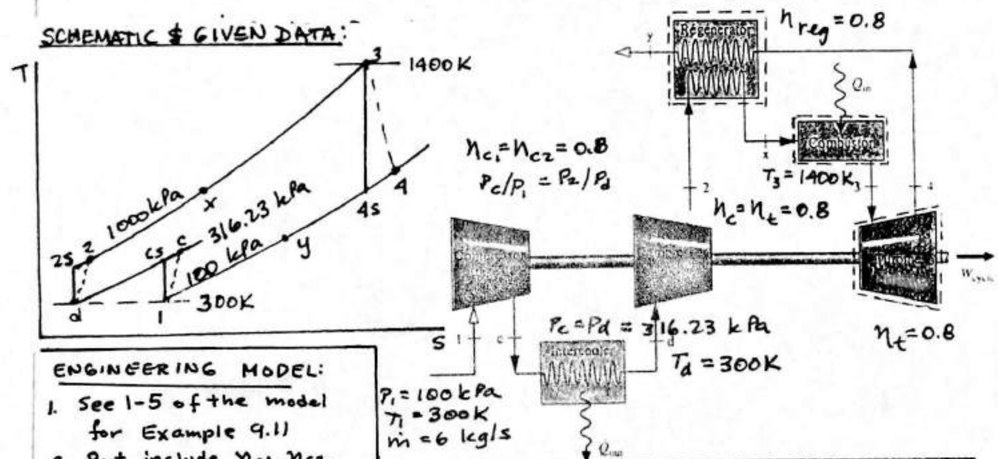
Air enters the first compressor stage of a cold air-standard Brayton cycle with regeneration and intercooling at 100 kPa, 300 K, with a mass flow rate of 6 kg/s. The overall compressor pressure ratio is 10, and the pressure ratios are the same across each compressor stage. The temperature at the inlet to the second compressor stage is 300 K. The temperature at the inlet to the turbine is 1400 K. The compressor stages and turbine each have isentropic efficiencies of 80% and the regenerator effectiveness is 80%. For $k = 1.4$, calculate

(a) the back work ratio.
(b) the net power developed, in kW.
(c) the thermal efficiency of the cycle.
(d) the rates of entropy production in each compressor stage, the turbine stage, and the regenerator, each in kW/K.

KNOWN: Steady-state operating data are provided for a cold air-standard Brayton cycle with regeneration and intercooling

FIND: Determine (a) the back work ratio, (b) net power developed, (c) thermal efficiency, and (d) rates of entropy production in each compressor stage, the turbine, and the regenerator.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:
1. See 1-5 of the model for Example 9.11
2. But include $\eta_{c1}, \eta_{c2}, \eta_t$, and η_{reg} in the present application.
3. For the air, $k = 1.4$ and $c_p = 1.005$ kJ/kg·K.

ANALYSIS: Observe that $P_c = P_d$ and $P_c/P_1 = P_2/P_d$. Accordingly, the intercooler pressure is $p = \sqrt{P_1 P_2}$. Since $P_2 = 10\,P_1 = 1000$ kPa, we get $p = 316.23$ kPa for the intercooler pressure. This is shown on the schematic above.

Using the first-stage compressor efficiency,

$$\eta_{c1} = \frac{h_{cs} - h_1}{h_c - h_1} = \frac{T_{cs} - T_1}{T_c - T_1} \Rightarrow T_c = T_1 + \frac{(T_{cs} - T_1)}{\eta_{c1}}$$

where $$T_{cs} = \left(\frac{P_c}{P_1}\right)^{\frac{(k-1)}{k}} T_1 = (3.1623)^{\frac{(1.4-1)}{1.4}}(300K) = 416.85\,K$$

Thus, $$T_c = 300 + \left(\frac{416.85 - 300}{0.8}\right) = 446.1\ K$$

Since the pressure ratio is the same for each compressor stage, $\eta_{c1} = \eta_{c2}$, and $T_d = T_1$, the same analysis gives $T_2 = 446.1$ K

Considering the turbine, we have

$$T_{4s} = T_3\left(\frac{P_{4s}}{P_3}\right)^{\frac{(k-1)}{k}} = 1400K\left(\frac{100\ kPa}{1000\ kPa}\right)^{\frac{(1.4-1)}{1.4}} = 725.1\ K$$

The turbine efficiency is

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{T_3 - T_4}{T_3 - T_{4s}} \Rightarrow T_4 = T_3 - \eta_t(T_3 - T_{4s})$$
$$= 1400K - 0.8(1400 - 725.1) \Rightarrow T_4 = 860.1\,K$$

PROBLEM 9.79 (Continued - p.2)

The regenerator effectiveness is

$$\eta_{reg} = \frac{h_x - h_2}{h_4 - h_2} = \frac{T_x - T_2}{T_4 - T_2} \Rightarrow T_x = T_2 + \eta_{reg}(T_4 - T_2)$$

$$= 446.1 + 0.8(860.1 - 446.1)$$

$$= 777.3 \text{ K}$$

Finally, an energy rate balance for the regenerator reads,

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}\left[\underbrace{(h_2 - h_x)}_{c_p(T_2 - T_x)} + \underbrace{(h_4 - h_y)}_{c_p(T_4 - T_4)}\right]$$

$$\Rightarrow \quad T_y = T_4 + (T_2 - T_x)$$

$$= 860.1 + 446.1 - 777.3$$

$$= 528.9 \text{ K}$$

(a) The turbine power is

$$\dot{W}_t = \dot{m}(h_3 - h_4) = \dot{m} c_p (T_3 - T_4)$$

$$= \left(6 \tfrac{kg}{s}\right)\left(1.005 \tfrac{kJ}{kg \cdot K}\right)(1400 - 860.1)\text{K} = 3255.6 \text{ kJ/s}$$

The total compressor power input is $\dot{W}_c = \dot{W}_{c1} + \dot{W}_{c2}$, where $\dot{W}_{c2} = \dot{W}_{c1}$ and

$$\dot{W}_{c1} = \dot{m}(h_c - h_1) = \dot{m} c_p (T_c - T_1)$$

$$= \left(6 \tfrac{kg}{s}\right)\left(1.005 \tfrac{kJ}{kg \cdot K}\right)(446.1 - 300)\text{K} = 881 \text{ kJ/s}$$

Thus, $\dot{W}_c = 1762$ kJ/s.

The back work ratio is

$$bwr = \frac{1762}{3255.6} = 0.541 \quad \longleftarrow$$

(b) The net power is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = (3255.6 - 1762)\tfrac{kJ}{s}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$

$$= 1493.6 \text{ kW} \quad \longleftarrow$$

(c) The thermal efficiency is $\eta = \dot{W}_{cycle}/\dot{Q}_{in}$, where

$$\dot{Q}_{in} = \dot{m}(h_3 - h_x) = \dot{m} c_p (T_3 - T_x)$$

$$= \left(6 \tfrac{kg}{s}\right)\left(1.005 \tfrac{kJ}{kg \cdot K}\right)(1400 - 777.3)\text{K}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$

$$= 3754.9 \text{ kW}$$

$$\Rightarrow \quad \eta = \frac{1493.6}{3754.9} = 0.398 \; (39.8\%) \quad \longleftarrow$$

(d) Reducing an entropy rate balance for the first compressor stage

$$(\dot{\sigma}_{cv})_{c1} = \dot{m}[s_c - s_1]$$

$$= \dot{m}\left[c_p \ln\frac{T_c}{T_1} - R \ln\frac{p_c}{p_1}\right]$$

$$= \left(6 \tfrac{kg}{s}\right)\left[\left(1.005 \tfrac{kJ}{kg \cdot K}\right)\ln\left(\frac{446.1}{300}\right) - \left(\frac{8.314}{28.97}\tfrac{kJ}{kg \cdot K}\right)\ln(3.1623)\right]\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$

$$= 0.410 \tfrac{kW}{K} \quad \longleftarrow$$

Since the pressure ratio is the same for each compressor stage, $T_d = T_1$, and $T_2 = T_c$, the same value is obtained:

$$(\dot{\sigma}_{cv})_{c2} = 0.410 \tfrac{kW}{K} \quad \longleftarrow$$

PROBLEM 9.79 (Continued - p.3)

Reducing an entropy rate balance for the turbine

$$(\dot{\sigma}_{cv})_t = \dot{m}[s_4 - s_3]$$

$$= \dot{m}\left[c_p \frac{T_4}{T_3} - R \ln \frac{p_4}{p_3}\right]$$

$$= 6\frac{kg}{s}\left[1.005 \ln\left(\frac{860.1}{1400}\right) - \frac{8.314}{28.97}\ln\left(\frac{1}{10}\right)\right]\frac{kJ}{kg\cdot K}\left|\frac{1\,kW}{1\,kJ/s}\right|$$

$$= 1.027\ kW/K \longleftarrow$$

Reducing an entropy rate balance for the regerator

$$(\dot{\sigma}_{cv})_{reg} = \dot{m}\left[(s_x - s_2) + (s_y - s_4)\right]$$

$$= \dot{m}\left[\left(c_p \ln \frac{T_x}{T_2} - R \ln \cancel{\frac{p_x}{p_2}}^{\,0}\right) + \left(c_p \ln \frac{T_y}{T_4} - R \ln \cancel{\frac{p_y}{p_4}}^{\,0}\right)\right]$$

$$(p_x = p_2) \qquad (p_y = p_4)$$

$$= \dot{m} c_p\left[\ln \frac{T_x}{T_2} + \ln \frac{T_y}{T_4}\right]$$

$$= \left(6\frac{kg}{s}\right)\left(1.005\frac{kJ}{kg\cdot K}\right)\left[\underbrace{\ln\left(\frac{777.3}{446.1}\right) + \ln\left(\frac{528.9}{860.1}\right)}_{\ln\left(\frac{(777.3)(528.9)}{(446.1)(860.1)}\right)}\right]\left|\frac{1\,kW}{1\,kJ/s}\right|$$

① $$= 0.416\ \frac{kW}{K}$$

---

1. Assuming $T_0 = 300K$, the following rates of exergy destruction are obtained using $\dot{E}_d = T_0 \dot{\sigma}_{cv}$.

   - Each compressor stage:
   $$(\dot{E}_d)_c = (300K)\left(0.410\ \frac{kW}{K}\right) = 123.0\ kW$$

   - Turbine:
   $$(\dot{E}_d)_t = (300K)\left(1.027\ \frac{kW}{K}\right) = 308.1\ kW$$

   - Regenerator:
   $$(\dot{E}_d)_{reg} = (300K)\left(0.416\ \frac{kW}{K}\right) = 124.8\ kW$$

Comments: (a) Exergy is also carried out of the gas turbine accompanying heat transfer from the intercooler and accompanying mass flow at the regenerator exit.

(b) By selecting a turbine having an isentropic efficiency greater than 80%, a significant source of exergy destruction can be reduced.

(c) The most significant site of exergy destruction by far is the combustor.

9.80 An air-standard regenerative Brayton cycle operating at steady state with intercooling and reheat produces 10 MW of power. Operating data at principal states in the cycle are given in the table below. The states are numbered as in Fig. 9.19. Sketch the *T*-*s* diagram for the cycle and determine

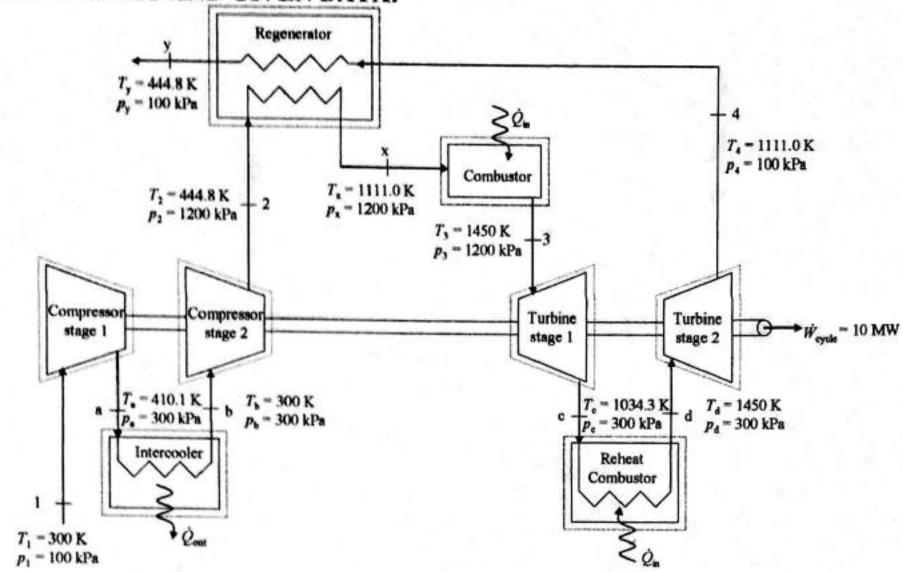
(a) the mass flow rate of air, in kg/s.
(b) the rate of heat transfer, in kW, to the working fluid passing through each combustor.
(c) the thermal efficiency.

State	*p* (kPa)	*T* (K)	*h* (kJ/kg)
1	100	300	300.19
a	300	410.1	411.22
b	300	300	300.19
2	1200	444.8	446.50
x	1200	1111.0	1173.84
3	1200	1450	1575.57
c	300	1034.3	1085.31
d	300	1450	1575.57
4	100	1111.0	1173.84
y	100	444.8	446.50

KNOWN: An ideal air-standard regenerative Brayton cycle operates with property data given at principal states. The net power output of the cycle is given.

FIND: Determine the mass flow rate of air, the rate of heat transfer to the working fluid passing through the combustor, and the thermal efficiency.

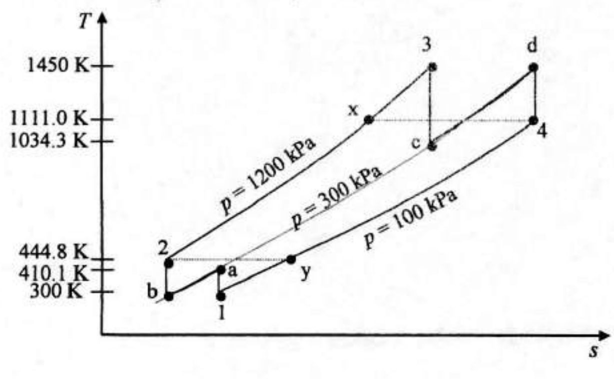
SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. All processes of the working fluid are internally reversible.
3. The turbine stages and compressor stages operate adiabatically.
4. There are no pressure drops for flow through the intercooler, regenerator, combustor, and reheat combustor.
5. Kinetic and potential energy effects are negligible.
6. The working fluid is air modeled as an ideal gas.

ANALYSIS: The *T-s* diagram for the cycle is shown below.



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Problem 9.80 (Continued) – Page 3

(a) The mass flow rate of air is found as follows. Mass and energy rate balances for control volumes enclosing the turbine stages and compressor stages give

$$\dot{W}_{t1} = \dot{m}(h_3 - h_c)$$
$$\dot{W}_{t2} = \dot{m}(h_d - h_4)$$
$$\dot{W}_{c1} = \dot{m}(h_a - h_1)$$
$$\dot{W}_{c2} = \dot{m}(h_2 - h_b)$$

The net power of the cycle is

$$\dot{W}_{cycle} = \dot{W}_{t1} + \dot{W}_{t2} - \dot{W}_{c1} - \dot{W}_{c2} = \dot{m}[(h_3 - h_c) + (h_d - h_4) - (h_a - h_1) - (h_2 - h_b)]$$

Solving for $\dot{m}$

$$\dot{m} = \frac{\dot{W}_{cycle}}{[(h_3 - h_c) + (h_d - h_4) - (h_a - h_1) - (h_2 - h_b)]}$$

Inserting values (Need to fix the conversion factors.)

$$\dot{m} = \frac{10{,}000\ \text{kW}}{[(1575.57 - 1085.31) + (1575.57 - 1173.84) - (411.22 - 300.19) - (446.50 - 300.19)]\frac{\text{kJ}}{\text{kg}}}\left|\frac{\frac{1\ \text{kJ}}{\text{s}}}{1\ \text{kW}}\right|$$

$\dot{m}$ = **15.76 kg/s**

(b) The rate of heat transfer to the working fluid passing through the combustor and reheat combustor can be determined by applying mass and energy balances to control volumes around the combustor and reheat combustor to give

$$\dot{Q}_{in} = \dot{m}[(h_3 - h_x) + (h_d - h_c)]$$

$$\dot{Q}_{in} = \left(15.76\frac{\text{kg}}{\text{s}}\right)[(1575.57 - 1173.84) + (1575.57 - 1085.31)]\frac{\text{kJ}}{\text{kg}}\left|\frac{1\ \text{kW}}{\frac{1\ \text{kJ}}{\text{s}}}\right| = \mathbf{14{,}058\ kW}$$

(c) The thermal efficiency is

$\eta = \dfrac{\dot{W}_{cycle}}{\dot{Q}_{in}}$ = (10,000 kW)/(14,058 kW) = **0.711 (71.1%)**

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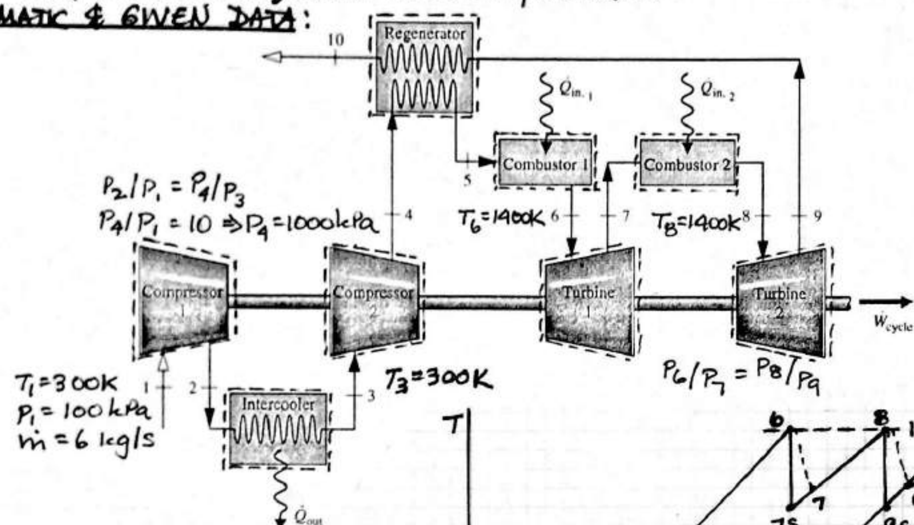
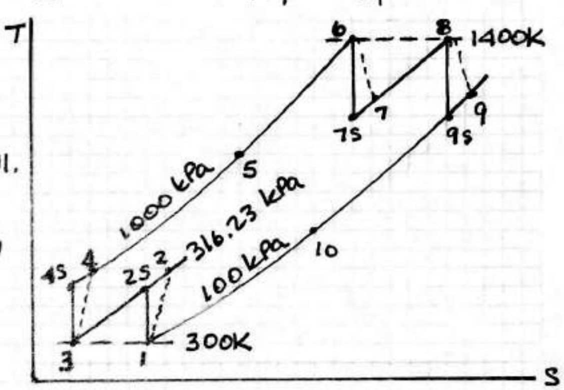
PROBLEM 9.81

KNOWN: Air enters a cold air-standard regenerative Brayton cycle with intercooling and reheat at a specified state and with a given mass flow rate. The overall compressor pressure ratio, maximum cycle temperature, and intercooler and reheat temperatures known. For the compressor and turbine stages, the isentropic efficiency is 0.8 and the regenerator effectiveness is 0.8.

FIND: Determine (a) the thermal efficiency, (b) the back work ratio, (c) the net power, and (d) exergy destruction rates in each compressor and turbine stage and in the regenerator.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 9.11. Also, $\eta_{c_1} = \eta_{c_2} = \eta_{t_1} = \eta_{t_2} = 0.8$ and $\eta_{reg} = 0.8$. The specific heats are assumed constant, with $k = 1.4$ and $c_p = 1.005\ kJ/kg\text{-}K$.

ANALYSIS: Determine the unknown temperatures.

$T_{2s} = (P_2/P_1)^{\frac{k-1}{k}}(T_1) = 416.85\,K$. With the compressor stage efficiency

$$T_2 = T_1 + (T_{2s} - T_1)/\eta_{c_1} = 446.1\,K$$

Since $T_3 = T_1$ and $P_4/P_3 = P_2/P_1$; $T_4 = T_2 = 446.1\,K$

$$T_{7s} = (P_7/P_6)^{\frac{k-1}{k}}(T_6) = 1007.6\,K$$

$$T_7 = T_6 - \eta_{t_1}(T_6 - T_{7s}) = 1086.1\,K$$

Since $T_8 = T_6$ and $P_7/P_6 = P_9/P_8$; $T_9 = T_7 = 1086.1\,K$

For the regenerator; $T_5 = T_4 + \eta_{reg}(T_9 - T_4) = 958.1\,K$

and $T_{10} = T_9 - (T_5 - T_4) = 574.1\,K$

(a) $\dot{Q}_{in} = \dot{Q}_{in,1} + \dot{Q}_{in,2} = \dot{m}c_p[(T_6 - T_5) + (T_8 - T_7)]$

$$= \left(6\,\frac{kg}{s}\right)\left(1.005\,\frac{kJ}{kg\cdot K}\right)[(1400 - 958.1) + (1400 - 1086.1)]\,K = 4557.5\,\frac{kJ}{s}$$

PROBLEM 9.81 (cont'd.)

and $\dot{Q}_{out,total} = \dot{m} c_p (T_{10} - T_1) + \dot{m} c_p (T_2 - T_3)$

$$= (6)(1.005)\left[(574.1-300) + (446.1-300)\right] = 2533.8 \text{ kJ/s}$$

The thermal efficiency is $\eta = 1 - \dot{Q}_{out}/\dot{Q}_{in} = 0.444$ (44.4%) $\quad \eta$

(b) The power for each turbine stage is equal. Thus

$$\dot{W}_t = 2 \cdot \dot{W}_t = 2\,\dot{m}\,c_p (T_6 - T_7) = (2)(6)(1.005)(1400-1086.1) = 3785.6 \ \frac{\text{kJ}}{\text{s}}$$

Similarly, for the compressor stages

$$\dot{W}_c = 2 \cdot \dot{W}_{c1} = 2\,\dot{m}\,c_p (T_2 - T_1) = (2)(6)(1.005)(446.1-300) = 1762.0 \ \frac{\text{kJ}}{\text{s}}$$

and $bwr = \dot{W}_c / \dot{W}_t = 1762 / 3785.6 = 0.4654$ $\quad$ bwr

(c) The net power is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = (3785.6 - 1762)\frac{\text{kJ}}{\text{s}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 2023.6 \text{ kW} \quad \dot{W}_{cycle}$$

(d) For the first compressor stage

$$(\dot{E}_d)_{comp,1} = T_0 \dot{m} (s_2 - s_1) = T_0 \dot{m} \left[c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right)\right]$$

$$= (300 \text{ K})\left(6 \frac{\text{kg}}{\text{s}}\right)\left[\left(1.005 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right) \ln\left(\frac{446.1}{300}\right) - \left(\frac{8.314}{28.97} \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right) \ln\left(\frac{316.23}{100}\right)\right] \left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$

$$= 123.0 \text{ kW}$$

$(\dot{E}_d)_{comp,2} = (\dot{E}_d)_{comp,1}$ $\quad$ $(\dot{E}_d)$ (compressor stages)

For turbine stage 1

$$(\dot{E}_d)_{turb,1} = T_0 \dot{m} (s_7 - s_6) = T_0 \dot{m} \left[c_p \ln\left(\frac{T_7}{T_6}\right) - R \ln\left(\frac{p_7}{p_6}\right)\right]$$

$$= (300)(6)\left[(1.005) \ln\left(\frac{1086.1}{1400}\right) - \frac{8.314}{28.97} \ln\left(\frac{316.23}{1000}\right)\right]$$

$$= 135.5 \text{ kW}$$

$(\dot{E}_d)_{turb,2} = (\dot{E}_d)_{turb,1}$ $\quad$ $(\dot{E}_d)$ (Turbine stages)

For the regenerator

$$(\dot{E}_d)_{reg} = T_0 \dot{m} \left[(s_5 - s_4) + (s_{10} - s_9)\right]$$

$$= T_0 \dot{m} \left[c_p \ln \frac{T_5}{T_4} - \cancel{R \ln \frac{p_5}{p_4}}^{\,0} + c_p \ln\left(\frac{T_{10}}{T_9}\right) - \cancel{R \ln \frac{p_{10}}{p_9}}^{\,0}\right]$$

$$= (300)(6)(1.005)\left[\ln\left(\frac{958.1}{446.1}\right) + \ln\left(\frac{574.1}{1086.1}\right)\right] = 229.5 \text{ kW} \quad (\dot{E}_d)_{reg}$$

9.82 An air-standard Brayton cycle produces 10 MW of power. The compressor and turbine isentropic efficiencies are both 80%. Operating data at principal states in the cycle are given in the table below. The states are numbered as in Fig. 9.9.

(a) Fill in the missing data in the table and sketch the *T-s* diagram for the cycle.

(b) Determine the mass flow rate of air, in kg/s.

(c) Perform a full accounting for the net rate of exergy increase as the air passes through the combustor.

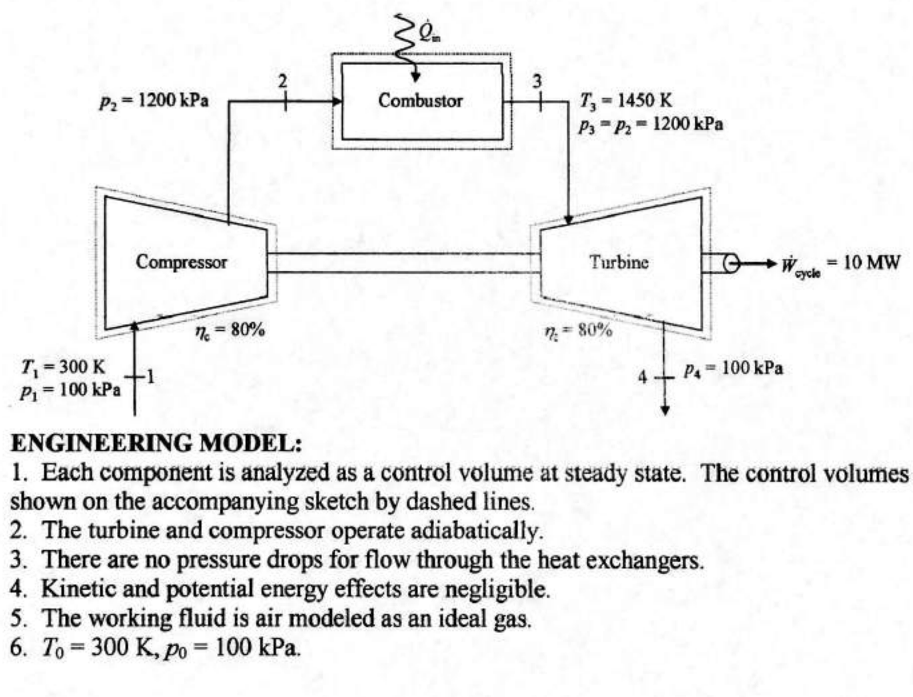
Let $T_0 = 300$ K, $p_0 = 100$ kPa.

State	p (kPa)	T (K)	h (kJ/kg)	$s°$ [kJ/(kg·K)]	p_r
1	100	300	300.19	1.70203	1.3860
2	1200				[redacted]
3	1200	1450	1575.57	3.40417	522
4	100				[redacted]

KNOWN: An air-standard Brayton cycle operates with property data given at principal states. The net power output of the cycle is given, and the isentropic efficiencies of the compressor and turbine are known.

FIND: Determine the mass flow rate of air and develop a full accounting of the net rate of exergy increase of the air passing through the combustor.

SCHEMATIC AND GIVEN DATA:

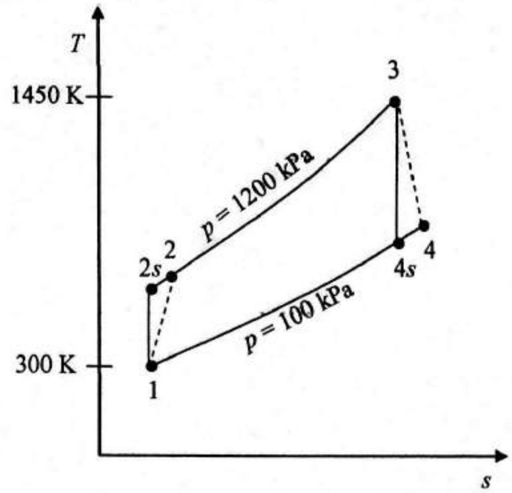


ENGINEERING MODEL:

1. Each component is analyzed as a control volume at steady state. The control volumes are shown on the accompanying sketch by dashed lines.
2. The turbine and compressor operate adiabatically.
3. There are no pressure drops for flow through the heat exchangers.
4. Kinetic and potential energy effects are negligible.
5. The working fluid is air modeled as an ideal gas.
6. $T_0 = 300$ K, $p_0 = 100$ kPa.

Problem 9.82 (Continued) – Page 2

ANALYSIS: (a) The *T-s* diagram for the cycle is shown below.



State 2 is found using the isentropic compressor efficiency.

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \rightarrow h_2 = h_1 + (h_{2s} - h_1)/\eta_c$$

To find h_{2s} for isentropic compression

$$\frac{p_{r2s}}{p_{r1}} = \frac{p_2}{p_1} \rightarrow p_{r2s} = p_{r1}\left(\frac{p_2}{p_1}\right) = (1.3860)\left(\frac{1200\text{ kPa}}{100\text{ kPa}}\right) = 16.632$$

Interpolating in Table A-22, $h_{2s} \approx 610.65$ kJ/kg.

Substituting values

$$h_2 = 300.19\text{ kJ/kg} + (610.65\text{ kJ/kg} - 300.19\text{ kJ/kg})/(0.80) = 688.27\text{ kJ/kg}$$

From Table A-22, $T_2 \approx 676.7$ K, $s_2^\circ \approx 2.53648$ kJ/(kg·K).

The specific enthalpy at state 4 can be determined using the isentropic turbine efficiency

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} \rightarrow h_4 = h_3 - \eta_t(h_3 - h_{4s})$$

Problem 9.82 (Continued) – Page 3

To get h_{4s}

$$\frac{p_{r4s}}{p_{r3}} = \frac{p_4}{p_3} \rightarrow p_{r4s} = p_{r3}\left(\frac{p_4}{p_3}\right) = (522)\left(\frac{100 \text{ kPa}}{1200 \text{ kPa}}\right) = 43.5$$

From Table A-22, $h_{4s} \approx 800.78$ kJ/kg.

Substituting

$$h_4 = 1575.57 \text{ kJ/kg} - (0.80)(1575.57 \text{ kJ/kg} - 800.78 \text{ kJ/kg}) = 955.74 \text{ kJ/kg}$$

From Table A-22, $T_4 \approx 920.3$ K, $s_4^\circ \approx 2.87363$ kJ/(kg·K).

In summary

State	p (kPa)	T (K)	h (kJ/kg)	s° [kJ/(kg·K)]	p_r
1	100	300	300.19	1.70203	1.3860
2	1200	676.7	688.27	2.53648	[redacted]
3	1200	1450	1575.57	3.40417	522
4	100	920.3	955.74	2.87363	[redacted]

(b) The mass flow rate of air is found as follows. Mass and energy rate balances for control volumes enclosing the turbine and compressor give

$$\dot{W}_t = \dot{m}(h_3 - h_4) \quad \text{and} \quad \dot{W}_c = \dot{m}(h_2 - h_1)$$

The net power of the cycle is

$$\dot{W}_{\text{cycle}} = \dot{W}_t - \dot{W}_c = \dot{m}[(h_3 - h_4) - (h_2 - h_1)]$$

Solving for $\dot{m}$

$$\dot{m} = \frac{\dot{W}_{\text{cycle}}}{[(h_3 - h_4) - (h_2 - h_1)]}$$

Inserting values

$$\dot{m} = \frac{10{,}000 \text{ kW}}{\left(1575.57\frac{\text{kJ}}{\text{kg}} - 955.74\frac{\text{kJ}}{\text{kg}}\right) - \left(688.27\frac{\text{kJ}}{\text{kg}} - 300.19\frac{\text{kJ}}{\text{kg}}\right)}\left|\frac{\frac{\text{kJ}}{\text{s}}}{\text{kW}}\right| = \underline{\mathbf{43.15\ kg/s}}$$

(c) The net rate of exergy increase of the air passing through the combustor is

$$\dot{E}_{f3} - \dot{E}_{f2} = \dot{m}\left[h_3 - h_2 - T_0(s_3 - s_2)\right]$$

$$\dot{E}_{f3} - \dot{E}_{f2} = \dot{m}\left[h_3 - h_2 - T_0\left(s_3^o - s_2^o - R\ln\frac{p_3}{p_2}\right)\right]$$

With assumption 4

$$\dot{E}_{f3} - \dot{E}_{f2} = \dot{m}\left[h_3 - h_2 - T_0\left(s_3^o - s_2^o - \overbrace{\cancel{R\ln\frac{p_3}{p_2}}}^{0}\right)\right]$$

$$\dot{E}_{f3} - \dot{E}_{f2} = \left(43.15\frac{\text{kg}}{\text{s}}\right)\left[(1575.57 - 688.27)\frac{\text{kJ}}{\text{kg}} - (300\text{ K})(3.40417 - 2.53648)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]\left|\frac{\text{MW}}{1000\frac{\text{kJ}}{\text{s}}}\right|$$

$$\dot{E}_{f3} - \dot{E}_{f2} = 27.05\text{ MW}$$

- The net rate exergy is carried out by the exhaust air stream at 4 is

$$\dot{E}_{f4} - \dot{E}_{f1} = \dot{m}\left[h_4 - h_1 - T_0\left(s_4^o - s_1^o - \overbrace{\cancel{R\ln\frac{p_4}{p_1}}}^{0}\right)\right]$$

$$\dot{E}_{f4} - \dot{E}_{f1} = \left(43.15\frac{\text{kg}}{\text{s}}\right)\left[(955.74 - 300.19)\frac{\text{kJ}}{\text{kg}} - (300\text{ K})(2.87363 - 1.70203)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]\left|\frac{\text{MW}}{1000\frac{\text{kJ}}{\text{s}}}\right|$$

$$\dot{E}_{f4} - \dot{E}_{f1} = 13.12\text{ MW}$$

- The rate of exergy destruction for the adiabatic compressor is

$$\dot{E}_d = \dot{m}T_0(s_2 - s_1) = \dot{m}T_0\left(s_2^o - s_1^o - R\ln\frac{p_2}{p_1}\right)$$

$$\dot{E}_d = \left(43.15\frac{\text{kg}}{\text{s}}\right)(300\text{ K})\left[(2.53648 - 1.70203)\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - \left(\frac{8.314\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}}{28.97\frac{\text{kg}}{\text{kmol}}}\right)\ln\left(\frac{1200\text{ kPa}}{100\text{ kPa}}\right)\right]\left|\frac{\text{MW}}{1000\frac{\text{kJ}}{\text{s}}}\right|$$

$$\dot{E}_d = 1.57\text{ MW}$$

- The rate of exergy destruction for the adiabatic turbine is

$$\dot{\mathsf{E}}_d = \dot{m}T_0(s_4 - s_3) = \dot{m}T_0\left(s_4^\circ - s_3^\circ - R\ln\frac{p_4}{p_3}\right)$$

$$\dot{\mathsf{E}}_d = \left(43.15\frac{\text{kg}}{\text{s}}\right)(300\ \text{K})\left[(2.87363 - 3.40417)\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - \left(\frac{8.314\dfrac{\text{kJ}}{\text{kmol}\cdot\text{K}}}{28.97\dfrac{\text{kg}}{\text{kmol}}}\right)\ln\left(\frac{100\ \text{kPa}}{1200\ \text{kPa}}\right)\right]\left|\frac{\text{MW}}{1000\dfrac{\text{kJ}}{\text{s}}}\right|$$

$$\dot{\mathsf{E}}_d = 2.36\ \text{MW}$$

The results are summarized by the following exergy rate balance sheet in terms of exergy magnitudes on a rate basis:

Net exergy increase of the gas passing through the combustor:	27.05 MW	100.0%
Disposition of the exergy:		
• Net power developed	10.00 MW	37.0%
• Net exergy lost with exhaust at state 4	13.12 MW	48.5%
• Exergy destruction		
o Compressor	1.57 MW	5.8%
o Turbine	2.36 MW	8.7%
Total:	27.05 MW	100.0%

Note that the exergy loss with exhaust is the biggest loss in the air-standard Brayton cycle. The rates of exergy destruction in the compressor and the turbine are small by comparison. Because of the high rate that exergy exits with the hot exhaust gases, the gases can be used for regeneration and/or for combined cycle applications.

PROBLEM 9.83

Air enters the compressor of an air-standard regenerative gas turbine at $p_1 = 14.7$ lbf/in.^{2}, $T_1 = 530°R$ with a mass flow rate of 90,000 lb/h. The compressor pressure ratio is 10, the turbine inlet temperature is 2200°R, and the regenerator effectiveness is 80%. Determine the thermal efficiency and net power developed, in hp, for each of the following configurations that build on Problem 9.67(c).

(a) Introduce two-stage compression with intercooling between the compressor stages at a pressure of 50 lbf/in.^{2} Each compressor stage has an isentropic efficiency of 84% and the temperature of the air entering the second stage is 530°R. The isentropic turbine efficiency is 88%.

(b) Introduce two-stage expansion with reheat between the turbine stages at a pressure of 50 lbf/in.^{2} Each turbine stage has an isentropic efficiency of 88% and the temperature of the air entering the second stage is 2000°R. The isentropic compressor efficiency is 84%.

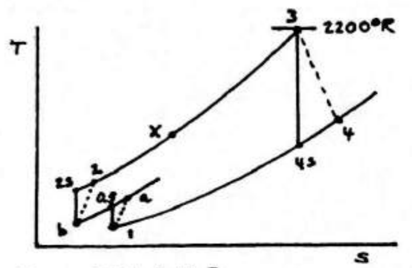
KNOWN: Steady-state operating are provided for an air-standard renenerative gas turbine

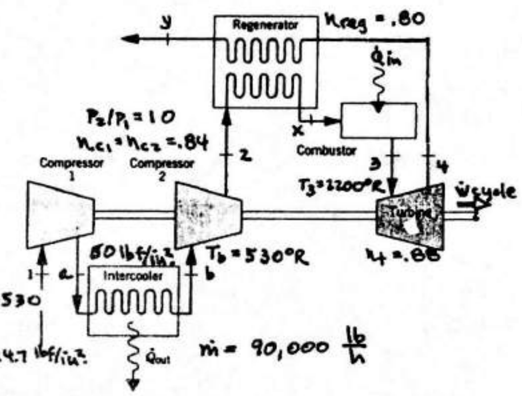
FIND: For each of two alternative configurations, determine the thermal efficiency and net power developed.

ENGINEERING MODEL: See Example 9.11, but note the specified isentropic compressor and turbine efficiencies.

ANALYSIS:

(a) SCHEMATIC & GIVEN DATA:





From Table A-22 E,

$h_1 = h_b = 126.66$ kJ/kg

$h_3 = 560.59$ kJ/kg

Using the isentropic turbine efficiency and data from Table A-22 E, $h_4 = 328.4$ Btu/lb

For the first compressor stage, $p_{r_{as}} = (p_a/p_1)\,p_{r1} = (50/14.7)(1.2997) = 4.4207$ and $h_{as} = 179.90$ Btu/lb. Thus, with the compressor stage efficiency

$$\eta_{c1} = \frac{h_{as}-h_1}{h_a-h_1} \Rightarrow h_a = h_1 + \left(\frac{h_{as}-h_1}{\eta_{c1}}\right) = 190.04 \text{ Btu/lb}$$

Similarly, for stage 2; $p_{r2s} = (p_2/p_b)\,p_{rb} = 3.8211 \Rightarrow h_{2s} = 172.58$ Btu/lb

and $$h_2 = h_b + \left(\frac{h_{2s}-h_b}{\eta_{c2}}\right) = 181.33 \text{ Btu/lb}$$

For state x use the regenerator effectiveness

$$\eta_{reg} = \frac{h_x-h_2}{h_4-h_2} \Rightarrow h_x = \eta_{reg}(h_4-h_2)+h_2 = 298.99 \text{ Btu/lb}$$

To find the thermal efficiency

$$\frac{\dot{W}_{cycle}}{\dot{m}} = (h_3-h_4) - (h_a-h_1) - (h_2-h_b) = 114.14 \text{ Btu/lb}$$

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_3-h_x = 261.6 \text{ Btu/lb}$$

Thus, $$\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{114.14}{261.6} = 0.436 \ (43.6\%) \longleftarrow \eta$$

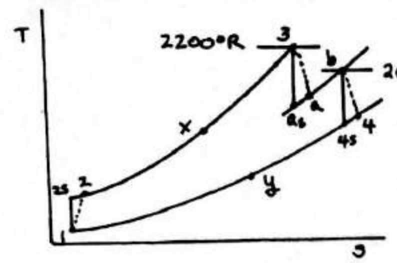
The net power developed is

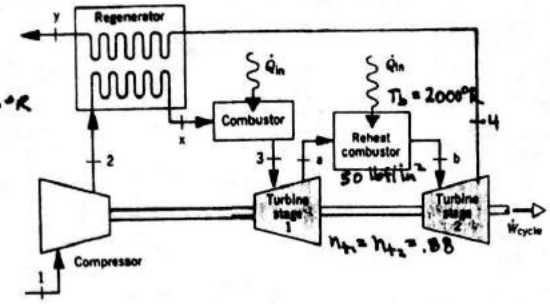
$$\dot{W}_{cycle} = \dot{m}\left(\frac{\dot{W}_{cycle}}{\dot{m}}\right) = \left(90{,}000 \frac{lb}{h}\right)\left(114.14\frac{Btu}{lb}\right)\left|\frac{1 \text{ hp}}{2545 \text{ Btu/h}}\right|$$

$$= 4036 \text{ hp} \longleftarrow \dot{W}_{cycle}$$

PROBLEM 9.83 (Continued)

(b) SCHEMATIC & GIVEN DATA:





$h_1 = 126.67$ Btu/lb. Using the isentropic compression efficiency,
$h_2 = 267.16$ Btu/lb. $h_3 = 560.59$ Btu/lb, $h_b = 504.71$ Btu/lb.

For turbine stage 1,

$$p_{ras} = (p_b/p_3)\,p_{r3} = (50/147)(256.6) = 87.279 \Rightarrow h_{as} = 417.68 \text{ Btu/lb}$$

Thus $\eta_{t_1} = \dfrac{h_3 - h_a}{h_3 - h_{as}} \Rightarrow h_a = h_3 - \eta_{t_1}(h_3 - h_{as}) = 434.83$ Btu/lb

Similarly, for turbine state 2; $p_{rb} = 174.0$ and $p_{r4s} = 51.156 \Rightarrow h_{4s} = 360.14$ Btu/lb
With $h_b = 504.71$ Btu/lb, $h_4 = h_b - \eta_{t_2}(h_b - h_{4s}) = 377.49$ Btu/lb

The specific enthalpy h_x is calculated from

$$\eta_{reg} = \frac{h_x - h_2}{h_4 - h_2} \Rightarrow h_x = \eta_{reg}(h_4 - h_2) + h_2 = 355.42 \text{ Btu/lb}$$

The thermal efficiency is found from

$$\frac{\dot{Q}_{in}}{\dot{m}} = (h_3 - h_x) + (h_b - h_a) = 275.05 \text{ Btu/lb}$$

and $$\frac{\dot{W}_{cycle}}{\dot{m}} = (h_3 - h_a) + (h_b - h_4) - (h_2 - h_1) = 112.48 \text{ Btu/lb}$$

Thus $$\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = 0.409 \; (40.9\,\%)$$ η

The net power is

$$\dot{W}_{cycle} = \left(90{,}000\ \frac{\text{lb}}{\text{h}}\right)(112.48)\frac{\text{Btu}}{\text{lb}}\left|\frac{1\text{ hp}}{2545\text{ Btu/h}}\right|$$

① $$= 3978 \text{ hp}$$ $\dot{W}_{cycle}$

1. The thermodynamic performance, as measured by thermal efficiency and net power, is better in configuration (a) than in (b).

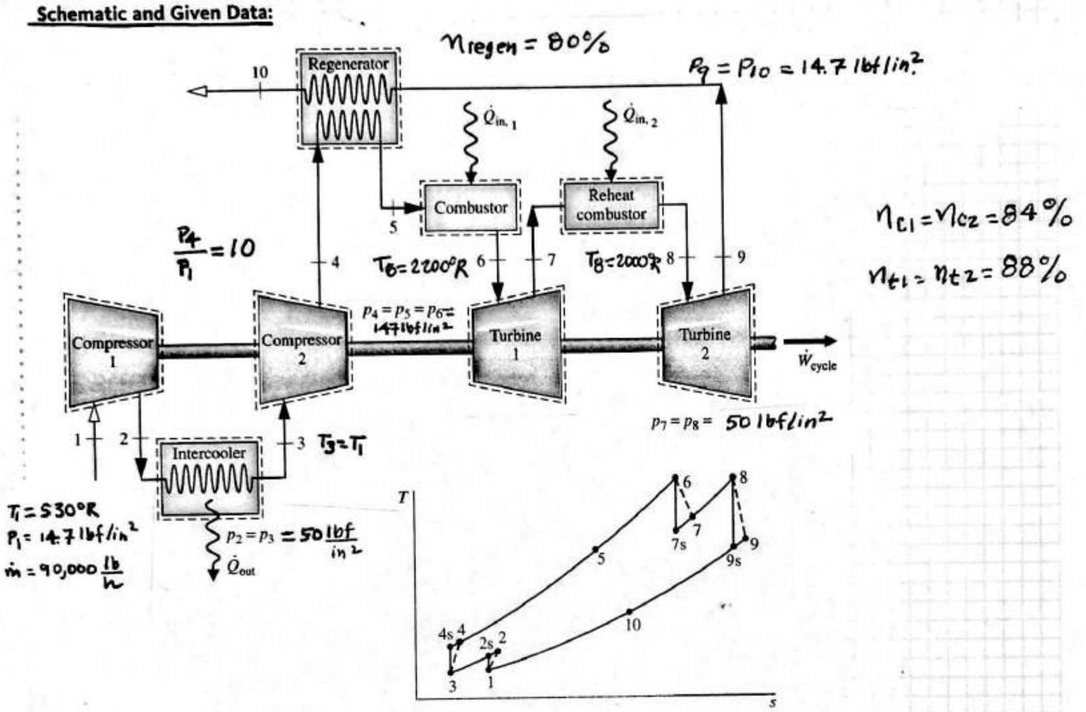
PROBLEM 9.84

Combining the features considered in Problem 9.83, air enters the compressor of an air-standard regenerative gas turbine at $p_1 = 14.7$ lb/in.^{2} and $T_1 = 530$°R with a mass flow rate of 90,000 lb/h. The compressor pressure ratio is 10. The turbine inlet temperature is 2200°R, and the regenerator effectiveness is 80%. Compression occurs in two stages with intercooling to 530°R between the two stages at 50 lbf/in.^{2} The turbine expansion also occurs in two stages, with reheat to 2000°R between the stages at 50 lbf/in.^{2} The isentropic efficiencies of the compressor and turbine stages are 84 and 88%, respectively. For the regenerative gas turbine, determine the thermal efficiency and net power developed, in hp.

For KNOWN and Engineering Model, see Example 9.11.

FIND: Determine the thermal efficiency and net power developed, in hp.

Schematic and Given Data:



ANALYSIS: Begin by getting specific enthalpy values at the key states.

State 1. From Table A-22E at 530°R, $h_1 = 126.66$ Btu/lb, $p_{r1} = 1.2998$.

State 2. First, get h_{2s}: $p_{r,2s} = \frac{p_2}{p_1} p_{r,1} = (50/14.7)(1.2998) = 4.421 \Rightarrow h_{2s} = 179.9 \frac{\text{Btu}}{\text{lb}}$

Then, with

$$\eta_{c1} = 0.84 = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{(h_{2s} - h_1)}{0.84} = 126.66 + \frac{(179.9 - 126.66)}{0.84} = 190.04 \frac{\text{Btu}}{\text{lb}}$$

State 3. $T_3 = T_1 \Rightarrow h_3 = 126.66$ Btu/lb, $p_{r3} = 1.2998$

State 4. First, get h_{4s}: $p_{r,4s} = \frac{p_4}{p_3} p_{r3}$, where $p_4 = 10 p_1 = 147$ lbf/in^2. Thus

$p_{r,4s} = (147/50)(1.2998) = 3.8214 \Rightarrow h_{4s} = 172.58$ Btu/lb

Then, $$h_4 = h_3 + \frac{(h_{4s} - h_3)}{0.84} = 126.66 + \frac{(172.58 - 126.66)}{0.84} = 181.33 \frac{\text{Btu}}{\text{lb}}$$

State 5. See below, after state 9 is considered.

PROBLEM 9.84 (Contd.)

State 6. $T_6 = 2200\,°R$. $h_6 = 560.59$ Btu/lb, $p_{r6} = 256.6$.

State 7. First, get h_{7s}: $p_{r,7s} = (p_7/p_6)\,p_{r6} = \left(\frac{50}{147}\right)(256.6) = 87.279$

$\Rightarrow h_{7s} = 417.68$ Btu/lb

Then, with

$$\eta_{t1} = \frac{h_6 - h_7}{h_6 - h_{7s}} \Rightarrow h_7 = h_6 - \eta_{t1}(h_6 - h_{7s})$$
$$= 560.59 - 0.88(560.59 - 417.68)$$
$$= 434.83 \text{ Btu/lb}$$

State 8. $T_8 = 2000\,°R$. $h_8 = 504.71$ Btu/lb, $p_{r8} = 174.0$

State 9. First, get h_{9s}: $p_{r,9s} = (p_9/p_8)\,p_{r8} = \left(\frac{14.7}{50}\right)(174.0) = 51.156$

$\Rightarrow h_{9s} = 360.14$ Btu/lb

$$\Rightarrow \quad h_9 = h_8 - \eta_{t2}(h_8 - h_{9s}) = 504.71 - 0.88(504.71 - 360.14)$$
$$= 377.49 \text{ Btu/lb}$$

State 5. Using the regenerator effectiveness,

$$\eta_{regen} = \frac{h_5 - h_4}{h_9 - h_4} \Rightarrow h_5 = h_4 + \eta_{regen}(h_9 - h_4)$$
$$= 181.33 + 0.8(377.49 - 181.33) = 338.26 \text{ Btu/lb}$$

THERMAL EFFICIENCY: $\eta = \dfrac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}}$

$$\frac{\dot{W}_{cycle}}{\dot{m}} = (h_6 - h_7) + (h_8 - h_9) - (h_2 - h_1) - (h_4 - h_3)$$
$$= (560.59 - 434.83) + (504.71 - 377.49) - (190.04 - 126.66) - (181.33 - 126.66)$$
$$= 134.93 \text{ Btu/lb}$$

$$\frac{\dot{Q}_{in}}{\dot{m}} = (h_6 - h_5) + (h_8 - h_7) = (560.59 - 338.26) + (504.71 - 434.83) = 292.21 \frac{\text{Btu}}{\text{lb}}$$

$$\eta = \frac{134.93}{292.21} = 0.462 \quad (46.2\%) \quad \longleftarrow \eta$$

NET POWER DEVELOPED:

(1) $$\dot{W}_{cycle} = \dot{m}\left(\dot{W}_{cycle}/\dot{m}\right) = \left(9\times10^4 \frac{\text{lb}}{\text{h}}\right)\left(134.93 \frac{\text{Btu}}{\text{lb}}\right)\left(\frac{1\text{ hp}}{2545\text{ Btu/h}}\right)$$
$$= 4772 \text{ hp} \quad \longleftarrow \dot{W}_{cycle}$$

---

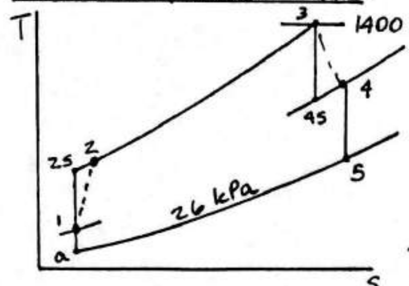
1. The thermodynamic performance, as measured by thermal efficiency and net power, in the present configuration is better than in either of those in Problem 9.83, as expected.

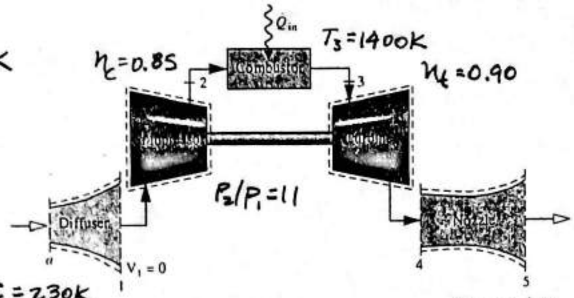
PROBLEM 9.85

KNOWN: A turbojet engine is analyzed on an air-standard basis. Data are known at various locations and the mass flow rate is specified.

FIND: Determine (a) the pressures and temperatures at each principal state, (b) the rate of heat addition, and (c) the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:





$T_a = 230K$
$P_a = 26kPa$
$V_a = 220 m/s$
$\dot{m} = 25 kg/s$

ENGINEERING MODEL: Same as in Example 9.13, except $\eta_c = 0.85$ and $\eta_t = 0.90$.

ANALYSIS: (a) Fix each of the principal states.

State a: $T_a = 230K$, $P_a = 26$ kPa $\Rightarrow h_a = 230.02$ kJ/kg, $Pr_a = 0.5477$

State 1: From an energy balance for the diffuser: $0 = h_a + V_a^2/2 - h_1$. Thus

$$h_1 = h_a + \frac{V_a^2}{2} = 230.02 + \frac{220^2 m^2/s^2}{2}\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|\left|\frac{1\ kJ}{10^3 N\cdot m}\right| = 254.22\frac{kJ}{kg}$$

$\Rightarrow T_1 = 254.15$ K (Table A-22).

Interpolating in Table A-22, $Pr_1 = 0.77759$. For the isentropic process from a to 1:

$$P_1 = (Pr_1/Pr_a)P_a = \left(\frac{0.77759}{0.5477}\right)(26\ kPa) = 36.91\ kPa$$

State 2: $Pr_{2s} = Pr_1\left(\underbrace{P_2/P_1}_{=P_{2s}}\right) = 0.77759(11) = 8.5535$. Then, $h_{2s} = 505.39$ kJ/kg.

With the compressor efficiency

$$h_2 = h_1 + \frac{(h_{2s} - h_1)}{\eta_c} = 254.22 + \frac{(505.39 - 254.22)}{0.85} = 549.71\ kJ/kg$$

$\Rightarrow T_2 = 545.21$ K

$P_2 = 11(36.91\ kPa) = 406.01\ kPa$

State 3: $T_3 = 1400K$. $h_3 = 1515.42$ kJ/kg. $Pr_3 = 450.5$,
$P_3 = P_2 = 406.01$ kPa.

State 4: The turbine is assumed to drive the compressor only: $\dot{W}_c = \dot{W}_t$

$$\Rightarrow (h_2 - h_1) = (h_3 - h_4) \Rightarrow h_4 = h_3 - (h_2 - h_1) = 1515.42 - (549.71 - 254.22) = 1219.93\ kJ/kg$$

From Table A-22, $T_4 = 1150.6K$, $Pr_4 = 200.564$

With the turbine efficiency, $h_{4s} = h_3 - \frac{(h_3 - h_4)}{\eta_t} = 1515.42 - \frac{(1515.42 - 1219.93)}{0.9} = 1187.10\ \frac{kJ}{kg}$

Interpolating, $Pr_{4s} = 181.32$.

Then

$$\underbrace{P_4}_{=P_{4s}} = P_3\left(\frac{Pr_{4s}}{Pr_3}\right) = (406.01\ kPa)\left(\frac{181.32}{450.5}\right) = 163.4\ kPa$$

PROBLEM 9.85 (Continued)

State 5 $\quad p_{r,5} = p_{r,4}\left(\frac{p_5}{p_4}\right) = 200.564\left(\frac{26}{163.4}\right) = 31.913$

Interpolating, $h_5 = 734.12$ kJ/kg

$T_5 = 719.3$ K

(a) Summary

State	p(kPa)	T(K)	h(kJ/kg)
1	36.91	254.15	254.22
2	406.01	545.2	549.71
3	406.01	1400	1515.42
4	163.4	1130.6	1219.93
5	26	719.3	734.12

p, T

(b) $\dot{Q}_{in} = \dot{m}(h_3 - h_2)$

$= (25 \text{ kg/s})(1515.42 - 549.71) \text{ kJ/kg} = 2.414 \times 10^4 \text{ kJ/s}$

$\dot{Q}_{in}$

(c) For the nozzle

$$0 = (h_4 - h_5) + \left(\frac{\cancel{V_4^2}^{\,0} - V_5^2}{2}\right)$$

Thus $V_5 = \sqrt{2(h_4 - h_5)}$

$$= \sqrt{2(1219.93 - 734.12)\frac{\text{kJ}}{\text{kg}}\left|\frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}}\right|\left|\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right|}$$

$= 985.7$ m/s

V_5

PROBLEM 9.86

See Problem 9.85

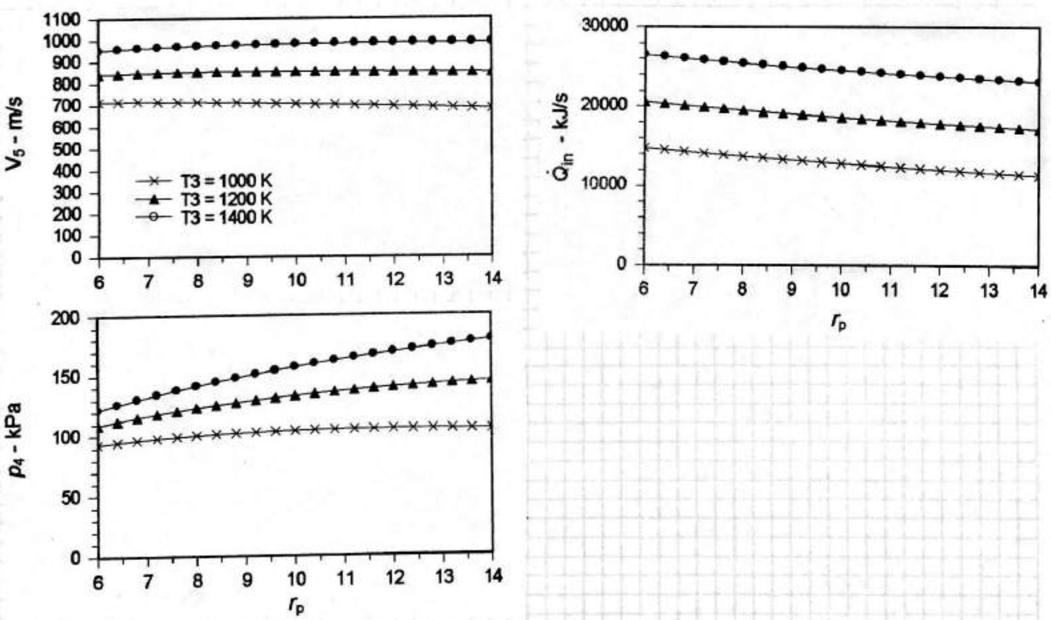
IT Code

```
pa = 26   // kPa
Ta = 230   // K
Va = 220  // m/s
rp = 11
T3 = 1400  // K
p5 = 26  // kPa
eta_t = 0.90
eta_c = 0.85
mdot = 25  // kg/s

ha = h_T("Air", Ta)
sa = s_TP("Air", Ta, pa)
h1 = ha + (Va^2 / 2) * (1 / 1000)
s1 = s_hP("Air", h1, p1)
s1 = sa
h1 = h_T("Air", T1)
p2 = p1 * rp
s2s = s_hP("Air", h2s, p2)
s2s = s1
h2 = h1 + (h2s - h1) / eta_c
h2 = h_T("Air", T2)
p3 = p2
h3 = h_T("Air", T3)
s3 = s_hP("Air",h3,p3)
s4s = s3
h2 - h1 = eta_t * (h3 - h4s)
s4s = s_hP("Air", h4s, p4)
h4 = h3 - (h3 - h4s) * eta_t
h4 = h_T("Air", T4)
s4 = s_hP("Air", h4, p4)

s5 = s4
s5 = s_hP("Air", h5, p5)
h5 = h_T("Air", T5)

h4 = h5 + (V5^2 / 2) * (1 / 1000)
Qdotin = mdot * (h3 - h2)
```

IT for T_3 = 1400 K, r_p = 11

T_a = 230 K
T_1 = 254.1 K
T_2 = 544.8 K
T_4 = 1150 K
T_5 = 719.3 K
h_a = 229.8 kJ/kg
h_1 = 254 kJ/kg
h_2 = 549.2 kJ/kg
h_{2s} = 504.9 kJ/kg
h_3 = 1514 kJ/kg
h_4 = 1219 kJ/kg
h_{4s} = 1186 kJ/kg
h_5 = 733.8 kJ/kg
p_a = 26 kPa
p_2 = 405.3 kPa
p_3 = 405.3 kPa
p_4 = 163.2 kPa
p_5 = 26 kPa
$\dot{Q}_{in}$ = 2.412 x 10^4 KJ/s
V_5 = 985 m/s

PLOTS:

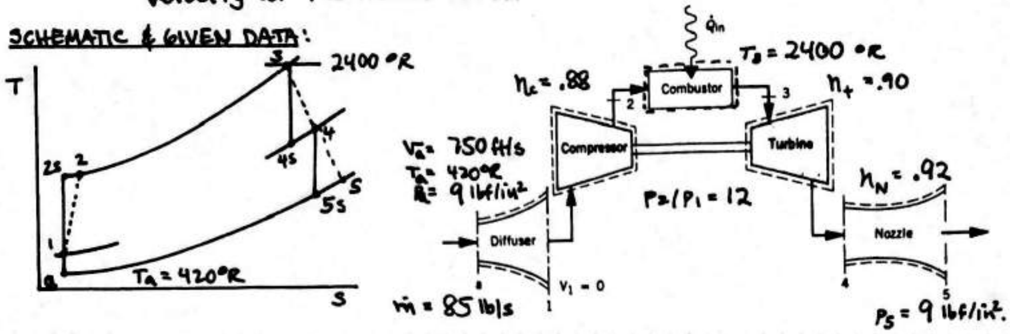


PROBLEM 9.87

KNOWN: A turbojet engine is analyzed on an air-standard basis. Data are know at various locations.

FIND: Determine (a) the rate of heat addition, (b) the pressure at the turbine exit, (c) the compressor power input, (d) the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Example 9.13, except $\eta_c = 0.88$ and $\eta_t = 0.90$.

ANALYSIS: First, fix each of the principal states (Table A-22E).

State a $T_a = 420°R \Rightarrow h_a = 100.32$ Btu/lb, $p_{ra} = 0.5760$

State 1 For the diffuser, $0 = (h_a + V_a^2/2) - h_1$. Thus

$$h_1 = 100.32 \frac{Btu}{lb} + \frac{750^2\ ft^2/s^2}{2}\left|\frac{1\ lbf}{32.2\ lb\cdot ft/s^2}\right|\left|\frac{1\ Btu}{778\ ft\cdot lbf}\right| = 111.55 \frac{Btu}{lb}$$

$$p_1 = \left(\frac{p_{r1}}{p_{ra}}\right)p_a = \left(\frac{.83492}{.5760}\right) 9 = 13.05\ lbf/in^2.$$

State 2 For isentropic compression, $p_{r2s} = p_{r1}(p_2/p_1) = 10.019$ and $h_{2s} = 227.29$ Btu/lb. With the compressor efficiency

$$h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c}\right) = 243.07\ Btu/lb, \quad p_2 = 12 p_1 = 156.6\ lbf/in^2.$$

State 3 $T_3 = 2400°R \Rightarrow h_3 = 617.22$ Btu/lb, $p_{r3} = 367.6$

State 4 For a turbojet, $\dot{W}_c/\dot{m} = \dot{W}_t/\dot{m}$. Thus $h_2 - h_1 = \eta_t(h_3 - h_{4s})$.

$$h_{4s} = h_3 - (h_2 - h_1)/\eta_t = 471.09\ Btu/lb. \Rightarrow p_{r4s} = 135.24$$

Thus, $p_4 = (p_{r4s}/p_{r3})p_3 = 57.61\ lbf/in^2.$ ← (b)

Using the turbine efficiency, $h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 485.70$ Btu/lb

State 5s $p_{r5} = p_{r4}(p_5/p_4) = (151.24)(9/57.61) = 23.627 \Rightarrow h_{5s} = 289.93 \frac{Btu}{lb}$

(a) The rate of heat addition is

$$\dot{Q}_{in} = \dot{m}(h_3 - h_2) = (85 \tfrac{lb}{s})(617.22 - 243.07) Btu/lb \left|\frac{3600 s}{1 h}\right| = 1.145 \times 10^8 \frac{Btu}{h} \leftarrow (a)$$

(c) The compressor power is $\dot{W}_c = \dot{m}(h_2 - h_1) = 85(243.07 - 111.55)|3600| = 4.025 \times 10^7 \frac{Btu}{h}$ ← (c)

(d) An energy rate balance for an isentropic expansion gives

$$(V_5^2)_s = 2(h_4 - h_{5s}) = 2(485.70 - 289.93)\frac{Btu}{lb}\left|\frac{778 ft\cdot lbf}{1 Btu}\right|\left|\frac{32.2\ lb\cdot ft/s^2}{1\ lbf}\right| = 9.809 \times 10^6 \frac{ft^2}{s^2}$$

Then, with the nozzle efficiency

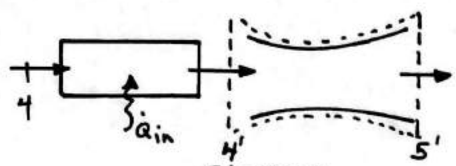
$$V_5 = (V_5)_s\sqrt{\eta_N} = 3004.0\ ft/s \qquad \leftarrow (d)$$

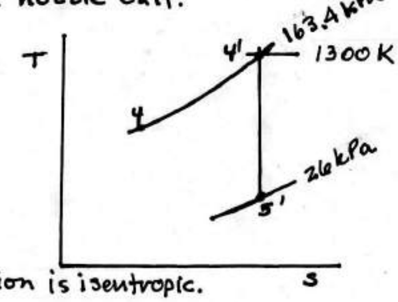
PROBLEM 9.88

KNOWN: An afterburner is added to the turbojet in Problem 9.85

FIND: Determine the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: The nozzle expansion is isentropic.

ANALYSIS: For the isentropic process

$$p_{r5'} = (p_5/p_4)\, p_{r4'} = (26/163.4)(330.9) = 52.652$$

$$h_{5'} = 844.25 \text{ kJ/kg}$$

Thus, the nozzle exit velocity is

$$V_5 = \sqrt{2(h_{4'} - h_{5'})}$$

① $$= \sqrt{2(1395.97 - 844.25)\ \frac{\text{kJ}}{\text{kg}} \left|\frac{10^3\ \text{N}\cdot\text{m}}{1\ \text{kJ}}\right| \left|\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right|} = 1050 \text{ m/s} \longleftarrow$$

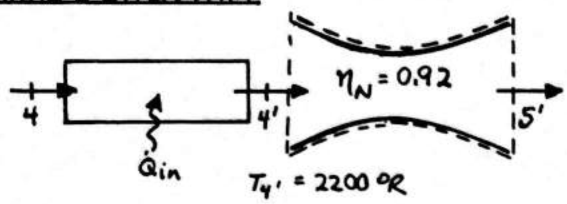
1. Comparing this result with that of Problem 9.85, we see that the use of an afterburner increases the exit velocity, and thus the engine thrust.

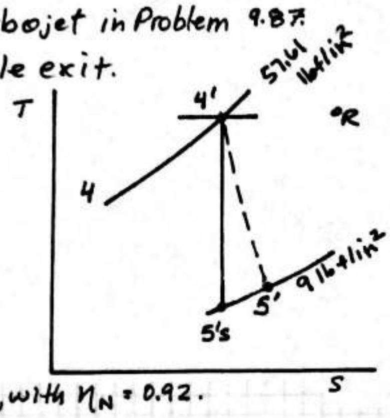
PROBLEM 9.89

KNOWN: An afterburner is added to the turbojet in Problem 9.87.

FIND: Determine the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: Nozzle operation is adiabatic, with $\eta_N = 0.92$.

ANALYSIS: For ideal expansion through the nozzle

$$p_{r5s'} = (p_5/p_4)\, p_{r4'}$$

From Table A-22E; $p_{r4'} = 256.6$. Thus

$$p_{r5s'} = 40.087 \Rightarrow h_{5s'} = 336.39 \text{ Btu/lb}$$

In the ideal case

$$0 = h_{4'} - \left(h_{5s'} + \left(\frac{V_5^2}{2}\right)_s\right)$$

$$\left(\frac{V_5^2}{2}\right)_s = h_{4'} - h_{5s'}$$

or

$$(V_5)_s = \sqrt{2(h_{4'} - h_{5s'})}$$

$$= \sqrt{2(560.59 - 336.39)\frac{\text{Btu}}{\text{lb}}\left|\frac{778 \text{ ft}\cdot\text{lbf}}{1 \text{ Btu}}\right|\left|\frac{32.2 \text{ lb}\cdot\text{ft/s}^2}{1 \text{ lbf}}\right|}$$

$$= 3352 \text{ ft/s}$$

Using the nozzle efficiency

$$\frac{V_5^2}{2} = \eta_N \left(\frac{V_5^2}{2}\right)_s$$

① $$V_5 = \sqrt{\eta_N}\,(V_5)_s = 3215 \text{ ft/s} \quad V_5$$

1. Comparing this result with that of Problem 9.87 we see that the use of an afterburner increases the exit velocity, and thus the engine thrust.

PROBLEM 9.90

Air enters the diffuser of a ramjet engine (Fig. 9.27c) at 6 lbf/in.^{2}, 420°R, with a velocity of 1600 ft/s, and decelerates essentially to zero velocity. After combustion, the gases reach a temperature of 2200°R before being discharged through the nozzle at 6 lbf/in.^{2} On the basis of an air-standard analysis, determine

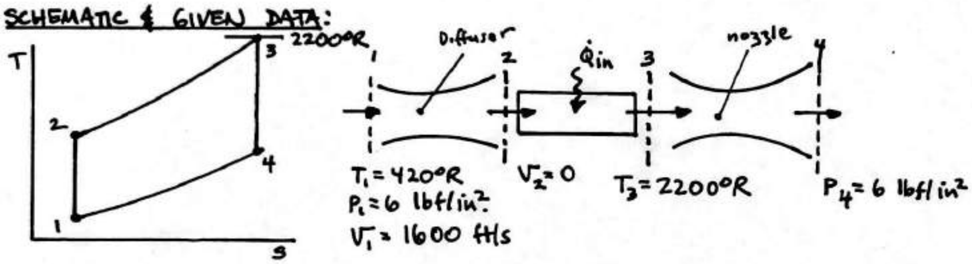
(a) the pressure at the diffuser exit, in lbf/in.^{2}
(b) the velocity at the nozzle exit, in ft/s.

Neglect kinetic energy except at the diffuser inlet and the nozzle exit. Assume combustion occurs at constant pressure and flow through the diffuser and nozzle is isentropic.

KNOWN: Air enters the diffuser of a ramjet with known conditions. The temperature after combustion is specified.

FIND: Using an air-standard analysis, determine (a) the pressure at the diffuser exit and (b) the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The combustion process is modeled as a constant pressure heat addition. (3) The diffuser and nozzle operate isentropically. (4) Neglect kinetic energy at locations 2 and 3. (5) The working fluid is air modeled as an ideal gas.

ANALYSIS: (a) For the diffuser; $0 = \left(h_1 + \frac{V_1^2}{2}\right) - h_2$. Thus,

$$h_2 = h_1 + \frac{V_1^2}{2} = 100.32 + \frac{1600^2 \text{ ft}^2/\text{s}^2}{2}\left(\frac{1 \text{ lbf}}{32.2 \text{ lb}\cdot\text{ft/s}^2}\right)\left(\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right)$$

$$= 151.41 \text{ Btu/lb}$$

With T_1 and h_2, Table A-22 E gives $p_{r1} = 0.5760$ and $p_{r2} = 2.4214$. Then, since the flow occurs isentropically

$$P_2 = \left(\frac{p_{r2}}{p_{r1}}\right) P_1 = \left(\frac{2.4214}{0.5760}\right)\left(6 \frac{\text{lbf}}{\text{in}^2}\right) = 25.22 \frac{\text{lbf}}{\text{in}^2} \longleftarrow P_2$$

(b) $T_3 = 2200°R \Rightarrow h_3 = 560.59$ Btu/lb, $p_{r3} = 256.6$. For isentropic expansion through the nozzle

$$p_{r4} = p_{r3}(P_4/P_3) = 256.6\,(6/25.22) = 61.047$$

$$h_4 = 378.34 \text{ Btu/lb}$$

The velocity is found from

$$0 = h_3 - \left(h_4 + \frac{V_4^2}{2}\right)$$

or

$$V_4 = \sqrt{2(h_3 - h_4)}$$

$$= \sqrt{2(560.59 - 378.34)\frac{\text{Btu}}{\text{lb}}\left(\frac{32.2 \text{ lb}\cdot\text{ft/s}^2}{1 \text{ lbf}}\right)\left(\frac{778 \text{ ft}\cdot\text{lbf}}{1 \text{ Btu}}\right)}$$

$$= 3022 \text{ ft/s} \longleftarrow V_4$$

PROBLEM 9.91

Air enters the diffuser of a ramjet engine (Fig. 9.27c) at 25 kPa, 220 K, with a velocity of 3080 km/h and decelerates to negligible velocity. On the basis of an air-standard analysis, the heat addition is 900 kJ per kg of air passing through the engine. Air exits the nozzle at 25 kPa. Determine

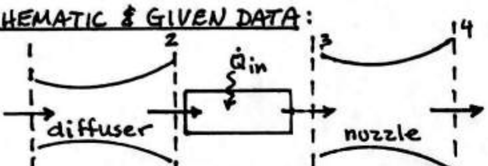
(a) the pressure at the diffuser exit, in kPa.
(b) the velocity at the nozzle exit, in m/s.

Neglect kinetic energy except at the diffuser inlet and the nozzle exit. Assume combustion occurs at constant pressure and flow through the diffuser and nozzle is isentropic.

KNOWN: Air enters the diffuser of a ramjet with known conditions. The heat addition per unit mass of air flowing is specified.

FIND: Using air-standard analysis, determine (a) the pressure at the diffuser exit and (b) the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:

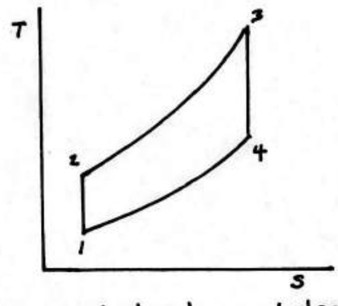


$P_1 = 25$ kPa
$T_1 = 220$ K
$V_1 = 3080$ km/h
$= 855.6$ m/s

$V_2 = 0$

$\frac{\dot{Q}_{in}}{\dot{m}} = 900$ kJ/kg

$P_4 = 25$ kPa



ENGR. MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The combustion process is modeled as a constant pressure heat addition. (3) The diffuser and nozzle operate isentropically. (4) Neglect kinetic energy at locations 2 & 3 and neglect potential energy throughout. (5) The working fluid is air modeled as an ideal gas.

ANALYSIS: (a) For the diffuser; $0 = (h_1 + \frac{V_1^2}{2}) - h_2$

Thus $h_2 = h_1 + \frac{V_1^2}{2} = 219.97 \frac{kJ}{kg} + \frac{(855.6)^2}{2} \frac{m^2}{s^2}\left(\frac{1\,N}{1\,kg\cdot m/s^2}\right)\left(\frac{1\,kJ}{10^3\,N\cdot m}\right)$

$= 586.0$ kJ/kg

With T_1, Table A-22 gives $p_{r1} = 0.4690$. Also, with h_2 we get $T_2 \approx 580$ K, $p_{r2} = 14.38$. Since the flow occurs isentropically

$$P_2 = \left(\frac{p_{r2}}{p_{r1}}\right) P_1 = \left(\frac{14.38}{0.4690}\right)(25\text{ kPa}) = 766.5\text{ kPa} \longleftarrow P_2$$

(b) State 3 is fixed by considering the heat addition process

$$0 = \dot{Q}_{in} + \dot{m}(h_2 - h_3)$$

or

$$h_3 = \frac{\dot{Q}_{in}}{\dot{m}} + h_2 = 900 + 586.0 = 1486 \text{ kJ/kg}$$

Again, from Table A-22, $T_3 \approx 1375.5$ K, $p_{r3} = 418.5$

For isentropic expansion through the nozzle

$$p_{r4} = (P_4/P_3)\, p_{r3} = (25/766.5)(418.5) = 13.65 \Rightarrow h_4 = 577.37 \frac{kJ}{kg}$$

The velocity is obtained from; $0 = h_3 - (h_4 + \frac{V_4^2}{2})$

or $V_4 = \sqrt{2(h_3 - h_4)}$

$$= \sqrt{2(1486 - 577.37)\frac{kJ}{kg}\left(\frac{10^3\,N\cdot m}{1\,kJ}\right)\left(\frac{1\,kg\cdot m/s^2}{1\,N}\right)}$$

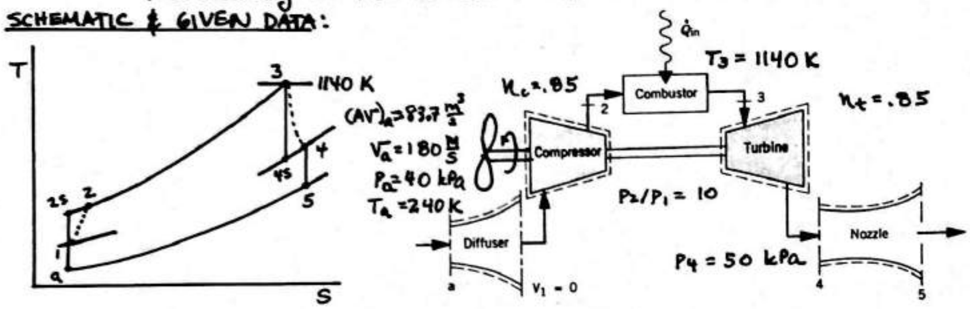
$= 1348$ m/s $\longleftarrow V_4$

PROBLEM 9.92

KNOWN: A turboprop engine is analyzed on an air-standard basis. Data are known at various locations.

FIND: Determine (a) the power delivered to the propeller, and (b) the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: Same as in Example 9.13, except $\eta_c = \eta_t = 0.85$.

ANALYSIS: First, fix each of the principal states (Table A-22).

State a: $T_a = 240\text{ K} \Rightarrow h_a = 240.02\text{ kJ/kg}$, $p_{ra} = 0.6355$

State 1: An energy balance for the diffuser gives; $0 = \left(h_a + \frac{V_a^2}{2}\right) - h_1$. Thus

$$h_1 = h_a + V_a^2/2 = 240.02\text{ kJ/kg} + \frac{180^2\text{ m}^2/\text{s}^2}{2}\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

$$= 256.22\text{ kJ/kg} \Rightarrow p_{r1} = 0.79902$$

$$p_1 = (p_{r1}/p_{ra})\,p_1 = 50.29\text{ kPa}$$

State 2: For an isentropic compression, $p_{r2s} = (p_2/p_1)\,p_{r1} = 7.9902 \Rightarrow h_{2s} = 495.65\ \frac{\text{kJ}}{\text{kg}}$

Using the compressor efficiency, $h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c}\right) = 537.9\text{ kJ/kg}$

State 3: $T_3 = 1140 \Rightarrow h_3 = 1207.57\text{ kJ/kg}$, $p_{r3} = 193.1$

State 4: For isentropic expansion, $p_{r4s} = (p_4/p_3)\,p_{r3} = \left(\frac{50}{502.9}\right)193.1 = 19.199$.

Thus, $h_{4s} = 634.06\text{ kJ/kg}$. Using the turbine efficiency,

$$h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 720.09\text{ kJ/kg},\quad p_{r4} = 29.80$$

State 5: $p_{r5} = (p_5/p_4)\,p_{r4} = 23.84 \Rightarrow h_5 = 676.17\text{ kJ/kg}$

(a) The power delivered to the propeller is

$$\dot{W}_p = \dot{W}_t - \dot{W}_c = \dot{m}\left[(h_3 - h_4) - (h_2 - h_1)\right]$$

where

$$\dot{m} = \frac{(AV)_a\,p_a}{RT_a} = \frac{(83.7\text{ m}^3/\text{s})(4\times10^4\text{ N/m}^2)}{\left(\frac{8314}{28.97}\ \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(240\text{ K})} = 48.6\ \frac{\text{kg}}{\text{s}}$$

$$\Rightarrow \dot{W}_p = \left(48.6\ \frac{\text{kg}}{\text{s}}\right)\left[(1207.57 - 720.09) - (537.9 - 256.22)\right]\frac{\text{kJ}}{\text{kg}}\left|\frac{1\text{ MW}}{10^3\text{ kJ/s}}\right| = 10\text{ MW} \leftarrow \dot{W}_p$$

(b) An energy balance for the nozzle gives

$$V_5 = \sqrt{2(h_4 - h_5)}$$

$$= \sqrt{2(720.09 - 676.17)\ \frac{\text{kJ}}{\text{kg}}\left|\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right|\left|\frac{10^3\text{ N}\cdot\text{m}}{1\text{ kJ}}\right|}$$

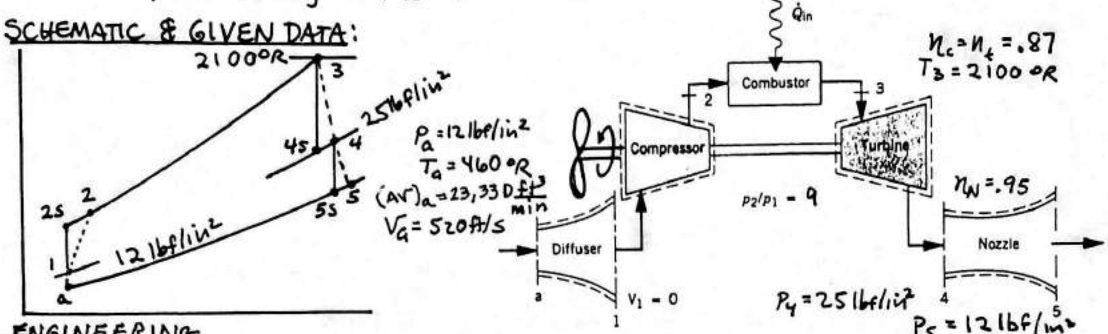
$$= 296.4\text{ m/s} \longleftarrow V_5$$

PROBLEM 9.93

KNOWN: A turboprop engine is analyzed on an air standard basis. Data are known at various locations.

FIND: Determine (a) the power delivered to the propeller, and (b) the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The diffuser process is isentropic and the turbine, compressor, and nozzle operate adiabatically. (3) Except at the inlet and exit of the engine, kinetic energy effects can be neglected. Potential energy effects are negligible throughout. (4) The working fluid is air modeled as an ideal gas.

ANALYSIS: First, fix each of the principal states (Table A-22E).

State a: $T_a = 460°R \Rightarrow h_a = 109.90$ Btu/lb, $p_{ra} = 0.7913$

State 1: An energy balance for the diffuser gives

$$h_1 = h_a + \frac{V_a^2}{2} = 109.90\,\frac{\text{Btu}}{\text{lb}} + \left(\frac{520^2\ \text{ft}^2/\text{s}^2}{2}\right)\left|\frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

$$= 115.30\ \text{Btu/lb} \Rightarrow p_{r1} = 0.93604$$

$$p_1 = (p_{r1}/p_{ra})\, p_a = 14.195\ \text{lbf/in}^2$$

State 2: For ideal compression; $p_{r2s} = (p_2/p_1)\, p_{r1} = 8.42436 \Rightarrow h_{2s} = 216.36\ \frac{\text{Btu}}{\text{lb}}$

Using the compressor efficiency

$$h_2 = h_1 + (h_{2s} - h_1)/\eta_c = 231.46\ \text{Btu/lb}$$

State 3: $T_3 = 2100\ °R \Rightarrow h_3 = 532.55$ Btu/lb, $p_{r3} = 212.1$

State 4: For ideal expansion; $p_{r4s} = (p_4/p_3)\, p_{r3} = 41.505 \Rightarrow h_{4s} = 339.7$ Btu/lb

Using the turbine efficiency

$$h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 364.77\ \text{Btu/lb} \Rightarrow p_{r4} = 53.515$$

State 5s: $p_{r5s} = (p_5/p_4)\, p_{r4} = 25.687 \Rightarrow h_{5s} = 296.79$ Btu/lb

(a) The power delivered to the propeller is

$$\dot{W}_p = \dot{W}_t - \dot{W}_c = \dot{m}\,[(h_3 - h_4) - (h_2 - h_1)]$$

Evaluating the mass flow rate

$$\dot{m} = \frac{(AV)_a\, p_a}{RT_a} = \frac{(23{,}330\ \text{ft}^3/\text{min})(12\ \text{lbf/in}^2)}{\left(\frac{1545}{28.97}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(460°\text{R})}\left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right| = 1643.3\ \text{lb/min}$$

PROBLEM 9.93 (Cont'd) - Page 2

Thus, the power is

$$\dot{W}_p = \left(1643.3\ \frac{lb}{min}\right)\left(\frac{60\ min}{h}\right)\left[(532.55-364.77)-(231.46-115.30)\right]\frac{Btu}{lb}\left|\frac{1\ hp}{2545\ Btu/h}\right|$$

$$= 2000\ hp \longleftarrow \dot{W}_p$$

(b) For an ideal expansion through the nozzle

$$0 = h_4 - \left(h_{5s} + \frac{V_{5s}^2}{2}\right)$$

or

$$V_{5s} = \sqrt{2(h_4 - h_{5s})}$$

$$= \sqrt{2(364.77-296.79)|32.2||778|} = 1845.5\ ft/s$$

From the definition of nozzle efficiency

$$V_5 = \sqrt{\eta_N}\, V_{5s} = \sqrt{(0.95)}\,(1845.5)$$

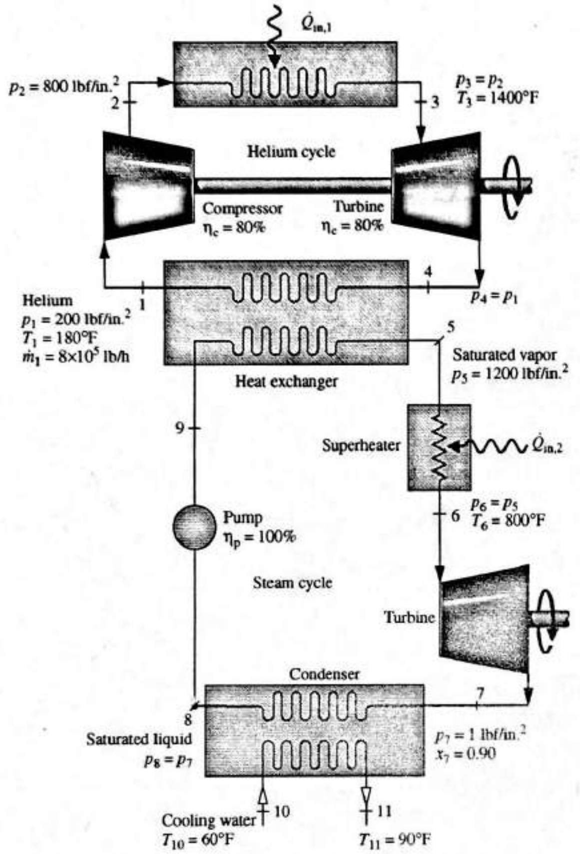
$$= 1798.8\ ft/s \longleftarrow V_5$$

PROBLEM 9.94

KNOWN: Steady-state operating data are provided for a combined cycle power plant using helium and water as the working fluids.

FIND: Determine (a) the mass flow rates of the steam and cooling water, (b) the net power developed by the gas turbine and vapor cycles, (c) the thermal efficiency of the combined cycle.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. Control volumes enclosing the principal components are at steady state.
2. There are no stray heat transfers, and kinetic and potential energy effects can be ignored.
3. Helium is modeled as an ideal gas with constant $c_p = 5/2R$, $k = 1.67$ (Figure 3.13).

ANALYSIS: Begin by evaluating data at principal states.

Helium: Using Eq. 6.45

$$T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 640K\left(\frac{800}{200}\right)^{\frac{1.67-1}{1.67}}$$

$$= 1116.16°R$$

$$T_{4s} = T_3\left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = 1860\left(\frac{200}{800}\right)^{\frac{1.67-1}{1.67}}$$

$$= 1066.51°R$$

$$\eta_t = \frac{h_3-h_4}{h_3-h_{4s}} = \frac{c_p(T_3-T_4)}{c_p(T_3-T_{4s})}$$

$$\Rightarrow T_4 = T_3 - \eta_t(T_3-T_{4s})$$

$$= 1860 - 0.8(1860-1066.51)$$

$$= 1225.2°R$$

$$\eta_c = \frac{h_{2s}-h_1}{h_2-h_1} = \frac{c_p(T_{2s}-T_1)}{c_p(T_2-T_1)}$$

$$\Rightarrow T_2 = T_1 + \frac{(T_{2s}-T_1)}{\eta_c}$$

$$= 640 + \frac{(1116.16-640)}{0.8}$$

$$= 1235.2°R$$

Steam: Table A-3E, $h_5 = 1183.9$ Btu/lb. Table A-4E, $h_6 = 1378.4$ Btu/lb. Table A-3E, $h_8 = 69.74$ Btu/lb

$h_7 = h_f + x_7 h_{fg} = 69.74 + 0.9(1036) = 1002.14$ Btu/lb.

Also

$$h_9 \approx h_8 + v_8\Delta p = 69.74\frac{Btu}{lb} + \left(.01614\frac{ft^3}{lb}\right)\left(1199\times144\frac{lbf}{ft^2}\right)\left|\frac{1\,Btu}{778\,ft\cdot lbf}\right| = 73.32\text{ Btu/lb}$$

Cooling water: $h_{10} \sim h_f(T_{10}) = 28.08$ Btu/lb, $h_{11} \approx h_f(T_{11}) = 58.07$ Btu/lb

(a) Evaluate the mass flow rate of the steam by study of a control volume enclosing the heat exchanger. Mass and energy rate balances reduce to give

$$\dot{m}_S = \frac{\dot{m}_g[h_4-h_1]}{[h_5-h_9]} = \frac{\dot{m}_g c_p(T_4-T_1)}{[h_5-h_9]}$$

PROBLEM 9.94 (Contd.) - Page 2

Inserting values, $c_p = 5/2\, R = \frac{5}{2}\left(\frac{1.986}{4.003}\frac{Btu}{lb \cdot °R}\right) = 1.24\ Btu/lb \cdot °R$

$$\dot{m}_s = \frac{(8 \times 10^5\ lb/h)(1.24\ Btu/lb \cdot °R)[1225.2 - 640]}{[1183.9 - 73.32]} = 5.23 \times 10^5\ \frac{lb}{h} \qquad \text{(a)}$$

For a control volume enclosing the condenser, mass and energy rate balances reduce to give

$$\dot{m}_{cw} = \frac{\dot{m}_s(h_7 - h_8)}{h_{11} - h_{10}} = \frac{(5.23 \times 10^5)[1002.14 - 69.74]}{(58.07 - 28.08)} = 1.63 \times 10^7\ \frac{lb}{h} \qquad \text{(a)}$$

(b) The net power developed by the gas turbine is

$$\dot{W}_g = \dot{W}_t - \dot{W}_c = \dot{m}_g[(h_3 - h_4) - (h_2 - h_1)] = \dot{m}_g c_p[(T_3 - T_4) - (T_2 - T_1)]$$

$$= (8 \times 10^5\ lb/h)(1.24\ \frac{Btu}{lb \cdot °R})[(1860 - 1225.2) - (1235.2 - 640)]\ °R = 3.93 \times 10^7\ \frac{Btu}{h} \qquad \text{(b)}$$

The net power developed by the vapor cycle is

$$\dot{W}_s = \dot{W}_t - \dot{W}_p = \dot{m}_s[(h_6 - h_7) - (h_9 - h_8)] = (5.23 \times 10^5)[(1378.4 - 1002.14) - (73.32 - 69.74)]$$

$$= 19.49 \times 10^7\ Btu/h \qquad \text{(b)}$$

(c) The thermal efficiency for the combined cycle is

$$\eta = \frac{\dot{W}_g + \dot{W}_s}{\dot{Q}_{in}} = \frac{\dot{W}_g + \dot{W}_s}{\dot{Q}_{in,1} + \dot{Q}_{in,2}}$$

where

$$\dot{Q}_{in,1} = \dot{m}_g(h_3 - h_2) = \dot{m}_g c_p(T_3 - T_2) = (8 \times 10^5\ \frac{lb}{h})(1.24\ \frac{Btu}{lb \cdot °R})(1860 - 1235.2)\ °R$$

$$= 61.98 \times 10^7\ Btu/h$$

$$\dot{Q}_{in,2} = \dot{m}_s(h_6 - h_5) = (5.23 \times 10^5)(1378.4 - 1183.9) = 10.17 \times 10^7\ Btu/h$$

Thus

① ② $$\eta = \frac{(3.93 + 19.49) \times 10^7}{(61.98 + 10.17) \times 10^7} = 0.325 \quad (32.5\%) \qquad \text{(c)}$$

---

1. For this combined cycle the thermal efficiency is low. This is due in part to the low isentropic turbine efficiencies: 80% for the gas turbine and 73% for the steam turbine (as can be verified).
2. Alternatively, applying an energy balance to the overall combined cycle,

$$\dot{W}_g + \dot{W}_s = \dot{Q}_{in} - \dot{m}_{cw}(h_{11} - h_{10})$$

$$\Rightarrow \quad \eta = \frac{\dot{W}_g + \dot{W}_s}{\dot{Q}_{in}} = 1 - \frac{\dot{m}_{cw}(h_{11} - h_{10})}{\dot{Q}_{in}}$$

$$= 0.323 \quad (32.3\%)$$

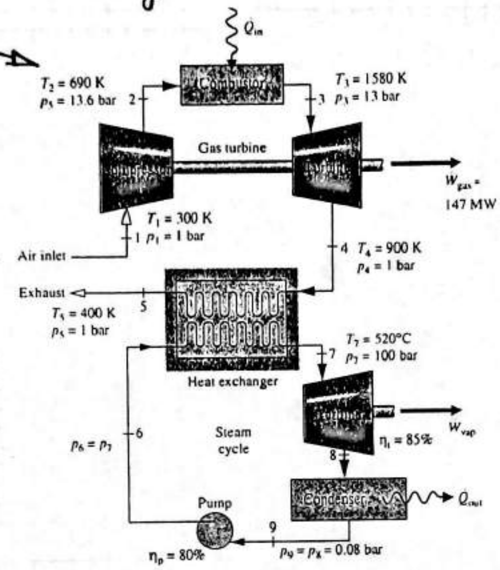
which agrees to within roundoff.

PROBLEM 9.95

KNOWN: Data are provided at principal states of a combined gas turbine–vapor power plant. For the gas turbine use an air standard analysis.

FIND: (a) Determine the net power developed by the overall plant.
(b) The overall thermal efficiency.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) Each component is a control volume at steady state. (2) The compressor, turbines, and pump operate adiabatically. (3) Kinetic and potential energy effects are negligible. (4) The air is modeled as an ideal gas. (5) There are no stray heat losses.

ANALYSIS: First, fix all of the principal states. For the air, use Table A-22.

Evaluating state data

$T_1 = 300K$; $h_1 = 300.19 \frac{kJ}{kg}$

$T_2 = 690K$; $h_2 = 702.52$,

$T_3 = 1580K$; $h_3 = 1733.17$,

$T_4 = 900K$; $h_4 = 932.93$,

$T_5 = 400K$; $h_5 = 400.98$.

Vapor cycle state data:

State 7: $P_7 = 100$ bar, $T_7 = 520°C$. Table A-4 gives $h_7 = 3425.1 \frac{kJ}{kg}$, $s_7 = 6.6622 \frac{kJ}{kg \cdot K}$.

State 8: $P_8 = 0.08$ bar. Then, with $s_{8s} = s_7$ and data from Table A-3

$$x_{8s} = \frac{6.6622 - 0.5926}{8.2287 - 0.5926} = 0.7949$$

$$h_{8s} = h_f + x_{8s}(h_g - h_f) = 173.88 + 0.7949(2403.1) = 2084.1 \text{ kJ/kg}$$

Using the isentropic turbine efficiency

$$h_8 = h_7 - \eta_t(h_7 - h_{8s}) = 3425.1 - 0.85(3425.1 - 2084.1) = 2285.3 \text{ kJ/kg}$$

State 9: Sat. liquid at 0.08 bar. Table A-3: $h_9 = 173.88$ kJ/kg

State 6: Using Eq. 8.7b, $h_{6s} \approx h_9 + v_9(\overset{= P_{6s}}{P_6} - P_9)$.

$$h_{6s} = 173.88 \frac{kJ}{kg} + \left(\frac{1.0084}{10^3}\frac{m^3}{kg}\right)(100 \text{ bar} - 0.08 \text{ bar})\left|\frac{10^5 N/m^2}{1 \text{ bar}}\right|\left|\frac{1 kJ}{10^3 N \cdot m}\right|$$

$$= 183.96 \text{ kJ/kg}$$

with the isentropic pump efficiency

$$h_6 = h_9 + \frac{(h_{6s} - h_9)}{\eta_p} = 173.88 + \frac{(183.96 - 173.88)}{0.8} = 186.48 \frac{kJ}{kg}$$

PROBLEM 9.95 Continued

Preliminary calculations to obtain the gas and steam mass flow rates:

○ The power developed by the gas turbine is 147 MW. Mass and energy rate balances give

$$\dot{W}_{gas} = \dot{m}_a\left[\frac{\dot{W}_t}{\dot{m}_a} - \frac{\dot{W}_c}{\dot{m}_a}\right] = \dot{m}_a\left[(h_3-h_4)-(h_2-h_1)\right]$$

Solving for the mass flow rate of the air

$$\dot{m}_a = \frac{\dot{W}_{gas}}{(h_3-h_4)-(h_2-h_1)} = \frac{147\,MW \left|10^3\,kJ/s / 1\,MW\right|}{(1733.17-932.93)-(702.52-300.19)} = 369.43\ \frac{kg(a)}{s}$$

○ Mass and energy rate balances for the inter-connecting heat exchanger give

$$0 = \dot{m}_a(h_4-h_5) + \dot{m}_{st}(h_6-h_7)$$

$$\therefore\quad \dot{m}_{st} = \frac{\dot{m}_a(h_4-h_5)}{(h_7-h_6)} = \frac{\left(369.43\frac{kg(a)}{s}\right)(932.93-400.98)\frac{kJ}{kg(a)}}{(3425.1-186.48)}$$

$$= 60.68\ \frac{kg}{s}$$

(a) Net power of the overall plant:

$$\dot{W}_{cycle} = \dot{W}_{gas} + \dot{W}_{vap}\ ,\ \text{where}\ \dot{W}_{gas} = 147\,MW\ \text{and}$$

$$\dot{W}_{VAP} = \dot{W}_t - \dot{W}_p = \dot{m}_{st}\left[(h_7-h_8)-(h_6-h_9)\right]$$

$$= 60.68\frac{kg}{s}\left[(3425.1-2285.3)-(186.48-173.88)\right]\frac{kJ}{kg}\left|\frac{1\,MW}{10^3\,kJ/s}\right|$$

$$= 68.4\ MW$$

$$\therefore\quad \dot{W}_{cycle} = (\underset{68.2\%}{147} + \underset{31.8\%}{68.4})\,MW = 215.4\,MW \qquad \text{(a)}$$

Here the gas turbine is the greater contributor to the total.

(b) The rate of heat addition is

$$\dot{Q}_{in} = \dot{m}_a(h_3-h_2) = 369.43\frac{kg(a)}{s}(1733.17-702.52)\frac{kJ}{kg(a)}\left|\frac{1\,MW}{10^3\,kJ/s}\right|$$

$$= 380.75\ MW$$

and

① $$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \frac{215.4\,MW}{380.75\,MW} = 0.566\ (56.6\%) \qquad \text{(b)}$$

---

1. Note that the combined-cycle thermal efficiency in this case is much greater than those of the stand-alone regenerative vapor and gas cycles considered in Examples 8.5 and 9.11, respectively.

PROBLEM 9.96

KNOWN: Operating data are provided for a combined gas turbine – vapor power plant operating as in Fig. 9.22.

FIND: Determine (a) the mass flow rates of the air, steam, and cooling water, (b) the net power developed by each cycle, (c) the thermal efficiency of the combined cycle.

SCHEMATIC & GIVEN DATA See Fig. 9.22. Table of data provided in the problem statement.

ENGINEERING MODEL: See 1-5 of Ex. 9.12.

ANALYSIS: Applying the energy rate balance to control volumes enclosing the several components we get

(a)

$$\dot{m}_{gas} = \frac{\dot{Q}_{in}}{h_3 - h_2} = \frac{50\times10^3 \text{ kJ/s}}{972.5 \text{ kJ/kg}} = 51.4 \text{ kg/s}$$

$$\dot{m}_{vap} = \dot{m}_g\left[\frac{h_4 - h_5}{h_7 - h_6}\right] = \left(51.4\,\frac{\text{kg}}{\text{s}}\right)\left[\frac{447.9}{3187.3}\right] = 7.34 \text{ kg/s}$$

$$\dot{m}_{cw} = \dot{m}_{vap}\left[\frac{h_8 - h_9}{h_{11} - h_{10}}\right] = \left(7.34\,\frac{\text{kg}}{\text{s}}\right)\left[\frac{1988.8}{62.7}\right] = 232.2 \text{ kg/s}$$

(b)

$$\dot{W}_{gas} = \dot{W}_t - \dot{W}_c = \dot{m}_{gas}\left[(h_3 - h_4) - (h_2 - h_1)\right]$$
$$= 51.4\,\frac{\text{kg}}{\text{s}}\left[740.7 - 393.2\right]\frac{\text{kJ}}{\text{kg}}\left|\frac{1 \text{ MW}}{10^3 \text{ kJ/s}}\right| = 17.86 \text{ MW}$$

$$\dot{W}_{vap} = \dot{W}_t - \dot{W}_p = \dot{m}_{vap}\left[(h_7 - h_8) - (h_6 - h_9)\right]$$
$$= 7.34\,\frac{\text{kg}}{\text{s}}\left[1166.2 - 12.7\right]\frac{\text{kJ}}{\text{kg}}\left|\frac{1 \text{ MW}}{10^3 \text{ kg/s}}\right| = 8.47 \text{ MW}$$

(1) (c)

$$\eta = \frac{\dot{W}_{gas} + \dot{W}_{vap}}{\dot{Q}_{in}} = \frac{17.86 + 8.47}{50} = 0.527 \; (52.7\%)$$

---

1. The combined-cycle thermal efficiency in this case is much greater than those of the stand-alone vapor and gas cycles considered in Examples 8.5 and 9.11, respectively.

PROBLEM 9.97

KNOWN: Operating data are provided for a combined gas turbine - vapor power plant operating as in Fig. 9.22.

FIND: Determine (a) the mass flow rates of the air and steam, (b) the thermal efficiency of the combined cycle, (c) a full accounting of net exergy increase of the air passing through the gas turbine combustor and the exergetic efficiency of the combined cycle.

SCHEMATIC & GIVEN DATA: See Fig. 9.22. Table of data provided in the problem statement.

ENGINEERING MODEL: See 1-6 of Ex. 9.12.

ANALYSIS: (a) Mass and energy rate balances for the heat-recovery steam generator give

$$\frac{\dot{m}_{vap}}{\dot{m}_{gas}} = \frac{h_4 - h_5}{h_7 - h_6} = \frac{858.02 - 482.49}{3138.3 - 183.96} = 0.1271$$

For the gas turbine, the net power developed is

$$\dot{W}_{gas} = \dot{m}_{gas}\left[(h_3 - h_4) - (h_2 - h_1)\right]$$
$$= \dot{m}_{gas}\left(287.8\ \frac{kJ}{kg}\right)$$

and for the vapor cycle

$$\dot{W}_{vap} = \dot{m}_{vap}\left[(h_7 - h_8) - (h_6 - h_9)\right]$$
$$= \dot{m}_{vap}\left(1023.5\ \frac{kJ}{kg}\right)$$

The net power developed is given as 100 MW. Collecting results

$$\underbrace{\dot{W}_{net}}_{=100\,MW} = \dot{m}_{gas}\left[287.8\ \frac{kJ}{kg} + \underbrace{\frac{\dot{m}_{vap}}{\dot{m}_{gas}}}_{=0.1271}\left(1023.5\ \frac{kJ}{kg}\right)\right]$$

$$\Rightarrow \quad \dot{m}_{gas} = \frac{100 \times 10^3\ kJ/s}{(287.8 + 0.1271\,(1023.5))\ kJ/kg} = 239.3\ \frac{kg}{s} \quad \longleftarrow$$

and

$$\dot{m}_{vap} = (0.1271)(239.3) = 30.4\ \frac{kg}{s} \quad \longleftarrow$$

(b) The thermal efficiency is

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}_{net}}{\dot{m}_{gas}(h_3 - h_2)} = \frac{100\ MW}{\left(239.3\ \frac{kg}{s}\right)\left(845.6\ \frac{kJ}{kg}\right)}\left|\frac{10^3\ kJ/s}{1\ MW}\right|$$

① $$= 0.494\ (49.4\%) \quad \longleftarrow$$

This completes the mass and energy balance portion. Next, exergy aspects are considered.

PROBLEM 9.97 (Continued - p. 2)

(c) The net exergy increase of the air passing through the gas turbine combustor is

$$(\dot{E}_{f3} - \dot{E}_{f2}) = \dot{m}_{gas}\left[(h_3 - h_2) - T_0(s_3^\circ - s_2^\circ) - R \ln \underbrace{P_3/P_2}_{=1} \right] \text{ since } P_2 = P_3$$

$$= \dot{m}_{gas}\left[(h_3 - h_2) - T_0(s_3^\circ - s_2^\circ) \right]$$

$$= \left(239.3 \tfrac{kg}{s}\right)\left[(1515.42 - 669.79) - 300\,(3.3620 - 2.5088) \right] \frac{kJ}{kg} \left| \frac{1\ MW}{10^3\ kJ/s} \right|$$

$$= 141.1\ MW$$

Exergy losses:

⊙ Air out:

$$(\dot{E}_{f5} - \dot{E}_{f1}) = \dot{m}_{gas}\left[(h_5 - h_1) - T_0(s_5^\circ - s_1^\circ) \right]$$

$$= (239.3)\left[(482.49 - 300.19) - 300\,(2.1776 - 1.7020) \right] \left| \frac{1}{10^3} \right|$$

$$= 9.48\ MW$$

⊙ From water passing through the condenser:

$$(\dot{E}_{f8} - \dot{E}_{f9}) = \dot{m}_{vap}\left[(h_8 - h_9) - T_0(s_8 - s_9) \right]$$

$$= (30.4)\left[(2104.74 - 173.88) - 300\,(6.7282 - 0.5926) \right] \left| \frac{1}{10^3} \right|$$

$$= 2.74\ MW$$

Total losses = 12.22 MW

Exergy Destructions: Use $\dot{E}_d = T_0 \dot{\sigma}_{cv} = T_0\, \dot{m}\,(\Delta s)$ for the turbines, compressor, and pump.

⊙ Gas turbine:

$$\dot{E}_d = \dot{m}_{gas} T_0 (s_4 - s_3) = \dot{m}_{gas}\, T_0\left(s_4^\circ - s_3^\circ - R \ln P_4/P_3 \right)$$

$$= \left(239.3 \tfrac{kg}{s}\right)(300K)\left(2.7620 - 3.3620 - \frac{8.314}{28.97} \ln \frac{1}{12} \right) \frac{kJ}{kg} \left| \frac{1}{10^3} \right|$$

$$= 8.12\ MW$$

⊙ Compressor:

$$\dot{E}_d = \dot{m}_{gas} T_0 (s_2 - s_1) = (239.3)(300)\left(2.5088 - 1.7020 - \frac{8.314}{28.97} \ln 12 \right) \left| \frac{1}{10^3} \right|$$

$$= 6.72\ MW$$

⊙ Steam turbine:

$$\dot{E}_d = \dot{m}_{vap} T_0 (s_8 - s_7) = 30.4 \tfrac{kg}{s}\,(300K)(6.7282 - 6.3634) \frac{kJ}{kg \cdot K} \left| \frac{1}{10^3} \right|$$

$$= 3.33\ MW$$

⊙ Pump:

$$\dot{E}_d = \dot{m}_{vap} T_0 (s_6 - s_9) = (30.4)(300)(0.5975 - 0.5926) \left| \frac{1}{10^3} \right| = 0.04\ MW$$

⊙ Heat-recovery steam generator:

$$\dot{E}_d = T_0\left[\dot{m}_{gas}(s_5^\circ - s_4^\circ) + \dot{m}_{vap}(s_7 - s_6) \right]$$

$$= (300)\left[239.3(-0.5844) + 30.4\,(5.7659) \right] \left| \frac{1}{10^3} \right| = 10.63\ MW$$

Collecting results, the total exergy destruction is 28.84 MW.

Summary:

Exergy Increase of the Air in the combustor:	141.1 MW	
Net Power developed:	100.0 MW	(70.9%)
Total exergy destruction:	28.84 MW	(20.4%)
Total exergy loss:	12.22 MW	(8.7%)
	141.1 MW	

Exergetic Efficiency:

② $$\epsilon = \frac{100\text{ MW}}{141.1\text{ MW}} = 0.709 \quad (70.9\%)$$

---

1. See note #1 of Example 9.12
2. See note #3 of Example 9.12

PROBLEM 9.98

KNOWN: Operating data are provided for the combined cycle power plant of Fig 9.22.

FIND: Determine (a) the mass flow rates of the air, steam, and cooling water, (b) the net power developed by the gas turbine and vapor cycle, (c) the thermal efficiency of the combined cycle, and (d) a full accounting of the net exergy increase of the air passing through the gas turbine combustor and the exergetic efficiency of the combined cycle.

SCHEMATIC & GIVEN DATA: See Fig. 9.22. Table of data provided in the problem statement.

ENGINEERING MODEL: See 1–6 of Ex. 9.12.

ANALYSIS: (a) The mass flow rate of the air is

$$\dot{m}_{air} = \frac{(AV)_1 P_1}{RT_1} = \frac{(40{,}000\ ft^3/min)(14.7 \times 144\ lbf/ft^2)}{\left(\frac{1545}{28.97}\ \frac{ft \cdot lbf}{lb \cdot °R}\right)(520°R)}\left|\frac{60\ min}{1\ h}\right|$$

$$= 1.832 \times 10^5\ \frac{lb}{h}$$

Mass and energy rate balances for the heat-recovery steam generator give

$$\dot{m}_{vap} = \dot{m}_{air}\left[\frac{h_4 - h_5}{h_7 - h_6}\right] = \left(1.832 \times 10^5 \frac{lb}{h}\right)\left[\frac{180.95}{1373.9}\right] = 2.413 \times 10^4\ \frac{lb}{h}$$

Mass and energy rate balances for the condenser give

$$\dot{m}_{cw} = \dot{m}_{vap}\left[\frac{h_8 - h_9}{h_{11} - h_{10}}\right] = \left(2.413 \times 10^4 \frac{lb}{h}\right)\left(\frac{885.3}{20}\right) = 1.07 \times 10^6\ \frac{lb}{h}$$

(b) The net power developed by the gas turbine is

$$\dot{W}_{gas} = \dot{m}_{air}\left[(h_3 - h_4) - (h_2 - h_1)\right] = \left(1.832 \times 10^5 \frac{lb}{h}\right)\left[291.98 - 146.1\right]\frac{Btu}{lb}$$

$$= 2.67 \times 10^7\ Btu/h$$

The net power developed by the vapor power cycle is

$$\dot{W}_{vap} = \dot{m}_{vap}\left[(h_7 - h_8) - (h_6 - h_9)\right] = (2.413 \times 10^4)(493.1 - 4.5)$$

$$= 1.179 \times 10^7\ Btu/h$$

(c) The rate of energy transfer by heat into the combined cycle is

$$\dot{Q}_{in} = \dot{m}_{air}\left[h_3 - h_2\right] = (1.832 \times 10^5)(404.12) = 7.403 \times 10^7\ Btu/h$$

The thermal efficiency is

① $$\eta = \frac{\dot{W}_{gas} + \dot{W}_{vap}}{\dot{Q}_{in}} = \frac{3.849 \times 10^7}{7.403 \times 10^7} = 0.52\ (52\%)$$

This completes the mass and energy balance portion. Next, exergy aspects are considered.

PROBLEM 9.98 (Continued - p. 2)

(d) The net exergy increase of the air passing through the gas turbine combustor is

$$(\dot{E}_{f3} - \dot{E}_{f2}) = \dot{m}_{air} \left[(h_3 - h_2) - T_0 (s_3^\circ - s_2^\circ - R \ln \underbrace{p_3/p_2}_{=1 \text{ since } p_2 = p_3} \right]$$

$$= \dot{m}_{air} \left[(h_3 - h_2) - T_0 (s_3^\circ - s_2^\circ) \right]$$

$$= \left(1.832 \times 10^5 \ \frac{lb}{h}\right) \left[404.12 - 520(0.22803) \right] \frac{Btu}{lb}$$

$$= 5.231 \times 10^7 \ Btu/lb$$

Exergy Losses:

- Air exiting at 5:

$$\dot{E}_{f5} - \dot{E}_{f1} = \dot{m}_{air} \left[(h_5 - h_1) - T_0 (s_5^\circ - s_1^\circ) \right]$$

$$= (1.832 \times 10^5) \left[77.29 - 60.19 \right] = 3.133 \times 10^6 \ \frac{Btu}{h}$$

- Cooling water exiting at 11:

$$\dot{E}_{f11} - \dot{E}_{f10} = \dot{m}_{cw} \left[h_{11} - h_{10} - T_0 (s_{11} - s_{10}) \right]$$

$$= (1.07 \times 10^6)(20 - 19.6) = 0.428 \times 10^6 \ \frac{Btu}{h}$$

Exergy Destructions: Use $\dot{E}_d = T_0 \sigma_{cv} = T_0 \dot{m} \Delta s$ for the turbines, compressor, and pump.

- Gas Turbine:

$$\dot{E}_d = \dot{m}_{air} T_0 \left[s_4 - s_3 \right] = \dot{m}_{air} T_0 \left[s_4^\circ - s_3^\circ - R \ln p_4/p_3 \right]$$

$$= (1.832 \times 10^5)(520) \left[-0.14334 - \frac{1.986}{28.97} \ln \frac{1}{12} \right) = 2.573 \times 10^6 \ Btu/h$$

- Compressor:

$$\dot{E}_d = \dot{m}_{air} T_0 \left[s_2^\circ - s_1^\circ - R \ln p_2/p_1 \right] = (1.832 \times 10^5)(520) \left[0.18648 - 0.17035 \right]$$

$$= 1.537 \times 10^6 \ Btu/h$$

- Steam Turbine:

$$\dot{E}_d = \dot{m}_{vap} T_0 \left[s_8 - s_7 \right] = (2.413 \times 10^4)(520) \left[0.0975 \right] = 1.223 \times 10^6 \ Btu/h$$

- Pump

$$\dot{E}_d = \dot{m}_{vap} T_0 \left[s_6 - s_9 \right] = (2.413 \times 10^4)(520) \left[0.0023 \right] = 0.029 \times 10^6 \ Btu/h$$

- Heat-recovery steam generator:

$$\dot{E}_d = T_0 \left[\dot{m}_{air} (s_5^\circ - s_4^\circ) + \dot{m}_{vap} (s_7 - s_6) \right]$$

$$= 520 \left[(1.832 \times 10^5)(-0.15542) + (2.413 \times 10^4)(1.477) \right]$$

$$= 3.727 \times 10^6 \ Btu/h$$

PROBLEM 9.98 Continued - p.3)

- Condenser

$$\dot{E}_d = T_0\left[\dot{m}_{vap}(s_9 - s_8) + \dot{m}_{cw}(s_{11} - s_{10})\right]$$

$$= 520\left[(2.413\times10^4)(-1.5768) + 1.07\times10^6(0.0377)\right]$$

$$= 1.192\times10^6 \text{ Btu/h}$$

Summary:

- Exergy Increase of the Air in the combustor:	5.231×10^7 Btu/h
- Net Power Developed	3.849×10^7 Btu/h (73.6%)
- Total Exergy Destruction	1.028×10^7 Btu/h (19.7%)
- Total Exergy Loss	0.356×10^7 Btu/h (6.8%)
	5.233×10^7 Btu/h

Exergetic efficiency:

(2) $$\epsilon = \frac{3.849\times10^7}{5.231\times10^7} = 0.736 \quad (73.6\%)$$

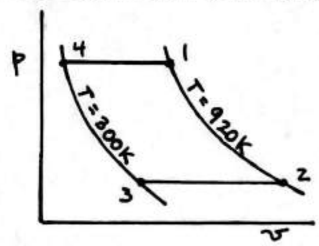
1. See note #1 of Example 9.12.
2. See note #3 of Example 9.12

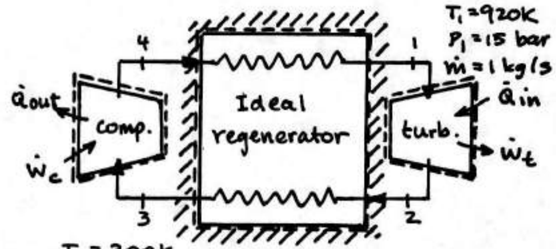
PROBLEM 9.99

KNOWN: Hydrogen is the working fluid in an Ericsson cycle with data known at various locations.

FIND: Determine (a) the net power developed, (b) the thermal efficiency, (c) the back work ratio.

SCHEMATIC & GIVEN DATA:





$T_3 = 300K$
$P_3 = 1.5$ bar

ENGINEERING MODEL: (1) Each component is analyzed as a control volume at steady state. (2) All processes are internally reversible. (3) The compression and expansion processes are isothermal. (4) Kinetic and potential energy effects are negligible. (5) The hydrogen behaves as an ideal gas.

ANALYSIS: (a) The turbine work is evaluated using Eq. 6.51a and the ideal gas equation of state

$$\dot{W}_t = -\dot{m}\int_1^2 v\,dp = -\dot{m}RT_1 \ln p_2/p_1$$

$$= -\left(1\frac{kg}{s}\right)\left(\frac{8.314}{2.018}\frac{kJ}{kg\cdot K}\right)(920K)\ln\left(\frac{1.5}{15}\right)\left(\frac{1\,kW}{1\,kJ/s}\right) = 8728\text{ kW}$$

and for the compressor

$$\dot{W}_c = \dot{m}RT_3 \ln p_4/p_3 = 2846\text{ kW}$$

Thus $\dot{W}_{cycle} = 8728 - 2846 = 5882$ kW ← $\dot{W}_{cycle}$

(b) Determining $\dot{Q}_{in}$ from an energy balance on the turbine

$$0 = \dot{Q}_{in} - \dot{W}_t + \dot{m}\cancel{(h_1 - h_2)}^{0}$$

thus $\dot{Q}_{in} = \dot{W}_t = 8728$ kW

and $\eta = \dfrac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \dfrac{5882}{8728} = 0.6739$ ← η

Alternatively, $\eta = 1 - \dfrac{T_3}{T_1} = 1 - \dfrac{300}{920} = 0.6739$

(c) The back work ratio is

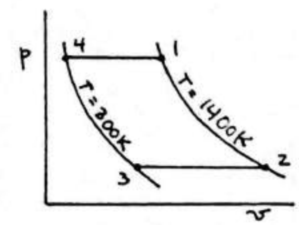
$bwr = \dfrac{\dot{W}_c}{\dot{W}_t} = 0.326$ ← bwr

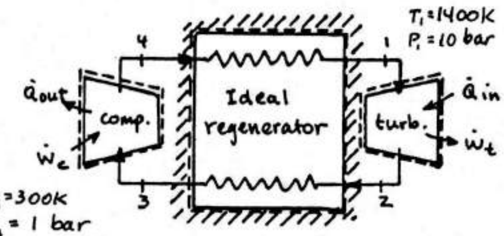
PROBLEM 9.100

KNOWN: Air is the working fluid in an Ericsson. Sample data are provided at one operating condition.

DETERMINE: Using the sample data, evaluate the net power developed, thermal efficiency, and bwr. Then, for specified turbine inlet temperatures, plot the net power developed versus compressor pressure ratio.

SCHEMATIC & GIVEN DATA:





$T_3 = 300K$
$P_3 = 1$ bar
$\dot{m} = 5$ kg/s

ENGINEERING MODEL: (1) Each component is analyzed as a control volume at steady state. (2) All processes are internally reversible. (3) The compression and expansion processes are isothermal. (4) Kinetic and potential energy effects are negligible. (5) The air behaves as an ideal gas.

ANALYSIS: (a) The turbine power is evaluated using Eq. 6.51a and the ideal gas equation of state:

$$\dot{W}_t = -\dot{m}\int_1^2 v\,dp = -\dot{m} R T_1 \ln(P_2/P_1)$$

$$= -(5\ \text{kg/s})\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(1400\text{K}) \ln\left(\frac{1}{10}\right)\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right| = 4626\ \text{kW}$$

and for the compressor

$$\dot{W}_c = \dot{m} R T_3 \ln P_4/P_3 = 991.2\ \text{kW}$$

Thus $\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = 3635\ \text{kW}$ ← $\dot{W}_{cycle}$

(b) The thermal efficiency is

$$\eta = 1 - \frac{T_3}{T_1} = 1 - \frac{300}{1400} = 0.786\ (78.6\%)$$ ← η

Alternatively, from an energy balance on the turbine; $\dot{Q}_{in} = \dot{W}_t$. Thus

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \frac{3635}{4626} = 0.786$$

(c) The back work ratio is

$$bwr = \frac{\dot{W}_c}{\dot{W}_t} = \frac{991.2}{4626} = 0.214$$ ← bwr

(d) Consider next the plots of $\dot{W}_{cycle}$ versus compressor pressure ratio ranging from 2 to 15 for turbine inlet temperatures of 1000K, 1200K, and 1400K. The calculations of parts (a)-(c) above correspond to pressure ratio = 10 and $T_1 = 1400$K.

PROBLEM 9.100 — Continued

IT Code

```
T1 = 1400  // K
T3 = 300  // K
p3 = 1  // bar
rp = 10
p4 = rp * p3
p1 = p4
p2 = p3
mdot = 5  // kg/s
R = 8.314 / 28.97  // kJ/kg·K

Wdot_t = - mdot * R * T1 * ln (p2 / p1)
Wdot_c = mdot * R * T3 * ln (p4 / p3)
Wdot_cycle = Wdot_t - Wdot_c
```

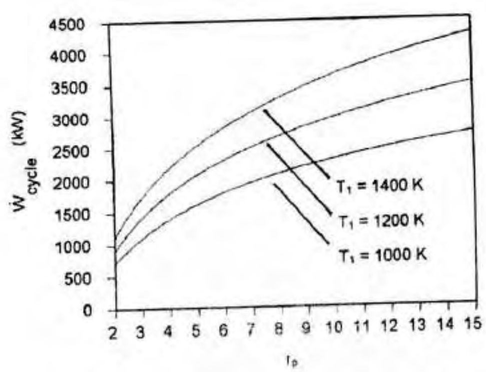
IT Results for r_p = 10, T_1 1400 K

$\dot{W}_t$ = 4626 kW

$\dot{W}_c$ = 991.2 kW

$\dot{W}_{cycle}$ = 3634 kW

PLOT:



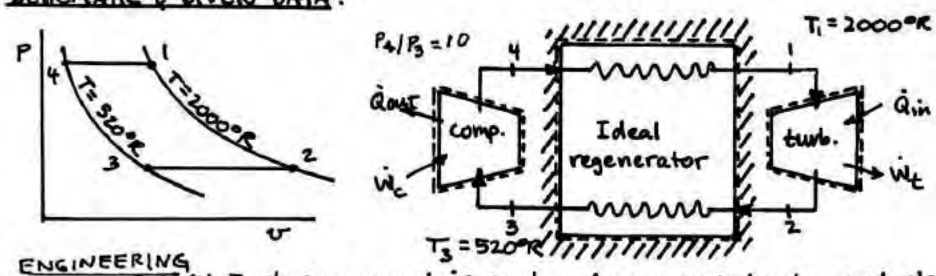
We see that the power increases significantly with compressor pressure ratio and with maximum cycle temperature for this idealized cycle.

PROBLEM 9.101

KNOWN: Air is the working fluid in an Ericsson cycle with data known at various locations.

FIND: Determine (a) the net work per unit mass of air flowing and (b) the thermal efficiency.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) Each component is analyzed as a control volume at steady state. (2) All processes are internally reversible. (3) The compression and expansion processes are isothermal. (4) Kinetic and potential energy effects are negligible. (5) The air behaves as an ideal gas.

ANALYSIS: (a) The turbine work is evaluated using Eq. 6.51a and the ideal gas equation of state

$$\frac{\dot{W}_t}{\dot{m}} = -\int_1^2 v\,dp = -RT_1 \ln \underbrace{\frac{P_2}{P_1}}_{= P_3/P_4}$$

$$= -\left(\frac{1.986}{28.97}\cdot\frac{\text{Btu}}{\text{lb}\cdot°R}\right)(2000°R)\ln(0.1) = 315.7 \text{ Btu/lb}$$

For the compressor

$$\frac{\dot{W}_c}{\dot{m}} = RT_3 \ln\left(\frac{P_4}{P_3}\right) = 82.08 \text{ Btu/lb}$$

Thus, the net work is

$$\frac{\dot{W}_{cycle}}{\dot{m}} = \frac{\dot{W}_t}{\dot{m}} - \frac{\dot{W}_c}{\dot{m}} = 233.6 \text{ Btu/lb}$$ $\dot{W}_{cycle}/\dot{m}$

(b) The thermal efficiency is

$$\eta = 1 - \frac{T_3}{T_1} = 0.74$$ η

Alternatively, $\dot{Q}_{in}/\dot{m} = \frac{\dot{W}_t}{\dot{m}} + \cancel{(h_2 - h_1)}^{\,0} = 315.7$ Btu/lb.

thus

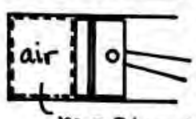
$$\eta = \frac{233.6}{315.7} = 0.74$$

PROBLEM 9.102

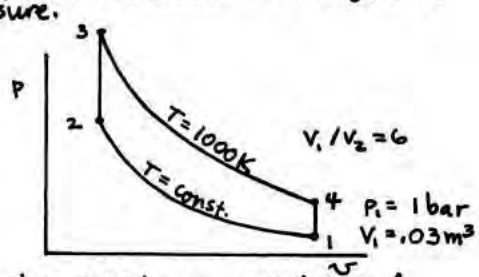
KNOWN: Air undergoes a Stirling cycle with a known compression ratio. Other data are also given for the cycle.

FIND: Determine (a) the net work, (b) the thermal efficiency, and (c) the mean effective pressure.

SCHEMATIC & GIVEN DATA:



$m = 36\,g = .036\,kg$



ENGINEERING MODEL: (1) The air is a closed system. (2) The compression and expansion are isothermal. (3) All processes are internally reversible. (4) An ideal regenerator is available to store the energy rejected at constant volume to be used as heat input during the constant volume heat transfer process. (5) The air behaves as an ideal gas.

ANALYSIS: (a) First, calculate the temperature during compression

$$T_1 = \frac{p_1V_1}{mR} = \frac{(1\,bar)(.03\,m^3)}{(.036\,kg)\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)}\left(\frac{10^5\,N/m^2}{1\,bar}\right)\left(\frac{1\,kJ}{10^3\,N\cdot m}\right) = 290.4\,K$$

The compression work is calculated using Eq. 2.17 and the ideal gas equation of state:

$$W_{12} = \int_1^2 p\,dV = mRT_1 \ln\frac{V_2}{V_1}$$

$$= (.036\,kg)\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(290.4\,K)\ln\left(\frac{1}{6}\right) = -5.376\,kJ$$

Similarly, for the expansion

$$W_{34} = mRT_3 \ln\frac{V_4}{V_3} = (.036)\left(\frac{8.314}{28.97}\right)(1000)\ln(6) = 18.51\,kJ$$

Thus, $W_{cycle} = W_{12} + W_{34} = 13.134\,kJ$ ← W_{cycle}

(b) The thermal efficiency is

$$\eta = 1 - \frac{T_1}{T_3} = 1 - \frac{290.4}{1000} = 0.71$$ ← η

Alternatively, $\eta = \frac{W_{cycle}}{Q_{34}}$, where from an energy balance

$$\underbrace{U_4 - U_3}_{=0} = Q_{34} - W_{34} \Rightarrow Q_{34} = W_{34} = 18.51\,kJ$$

so,

$$\eta = \frac{13.134}{18.51} = 0.71$$

(c) The mean effective pressure is

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}}{V_1(1 - V_2/V_1)}$$

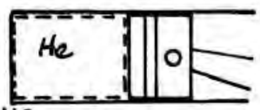
$$= \frac{(13.134\,kJ)}{(.03\,m^3)(1 - \frac{1}{6})}\left(\frac{10^3\,N\cdot m}{1\,kJ}\right)\left(\frac{1\,bar}{10^5\,N/m^2}\right) = 5.254\,bar$$ ← mep

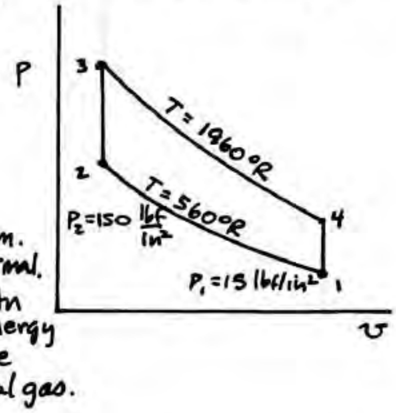
PROBLEM 9.103

KNOWN: Helium is the working fluid in a Stirling cycle. Data are given for various states in the cycle.

FIND: Determine (a) the work and heat transfer for each process and (b) the thermal efficiency.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) The helium is a closed system. (2) The compression and expansion are isothermal. (3) All processes are internally reversible. (4) An ideal regenerator is available to store the energy transferred by heat during constant volume processes. (5) The helium behaves as an ideal gas.

ANALYSIS: (a) Considering each process

Process 1-2: The work is calculated using Eq. 2.17 and the ideal gas equation of state

$$\frac{W_{12}}{m} = \int_1^2 p\,dv = RT_1 \ln\frac{v_2}{v_1} = RT_1 \ln\left(\frac{P_1}{P_2}\right)$$

$$= \left(\frac{1545}{4.003}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right|(560^\circ\text{R})\ln\left(\frac{15}{150}\right) = -639.7\ \frac{\text{Btu}}{\text{lb}} \qquad W_{12}/m$$

From an energy balance

$$\frac{Q_{12}}{m} = (\cancel{u_2 - u_1})^{0} + \frac{W_{12}}{m} = -639.7\ \frac{\text{Btu}}{\text{lb}} \longleftarrow \qquad Q_{12}/m$$

Process 3-4:

$$\frac{W_{34}}{m} = RT_3 \ln\frac{v_4}{v_3} = RT_3 \ln\frac{v_1}{v_2} = RT_3 \ln\frac{P_2}{P_1}$$

$$= \left(\frac{1545}{4.003}\right)\left|\frac{1}{778}\right|(1960)\ln\left(\frac{150}{15}\right) = 2238.9\text{ Btu/lb} \longleftarrow \qquad W_{34}/m$$

and $\frac{Q_{34}}{m} = (\cancel{u_4 - u_3})^{0} + \frac{W_{34}}{m} = 2238.9\text{ Btu/lb} \longleftarrow \qquad Q_{34}/m$

Processes 2-3 and 4-1: For each, $W = 0$.

$$\left.\begin{array}{l}\frac{Q_{23}}{m} = u_3 - u_2 \\ \frac{Q_{41}}{m} = u_1 - u_4\end{array}\right\}\ \frac{Q_{23}}{m} = -\frac{Q_{41}}{m}$$

Thus, if the regenerator is ideal, there is no net exchange of energy with the surroundings.

(b) The thermal efficiency is

$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{W_{12}/m + W_{34}/m}{Q_{34}/m} = 0.714\ (71.4\%)$$

Alternatively

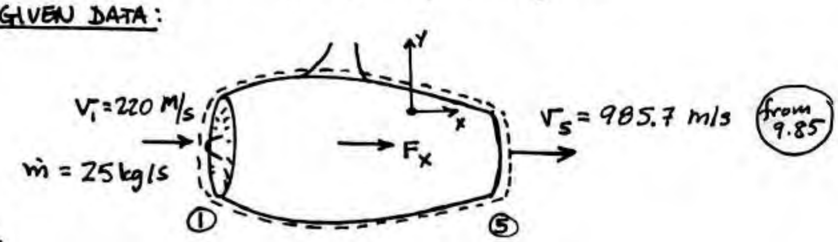
$$\eta = 1 - \frac{T_1}{T_3} = 1 - \frac{560}{1960} = 0.714$$

PROBLEM 9.104

KNOWN: Data are known for the turbojet engine of problem 9.85

FIND: Calculate the thrust developed by the engine.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume is at steady state. (2) The force
① F_x is the net force acting on the control volume.

ANALYSIS: Applying Eq. 9.31

$$F_x = \dot{m}(V_5 - V_1)$$

$$= \left(25 \frac{kg}{s}\right)(985.7 - 220)\frac{m}{s}\left|\frac{1 \text{ kN}}{10^3 \text{ kg}\cdot\text{m/s}^2}\right|$$

$$= 19.14 \text{ kN}$$

Thus, the thrust is the force developed by the control volume

$$F_{thrust} = -F_x = -19.14 \text{ kN}$$ ← thrust

to the left

---

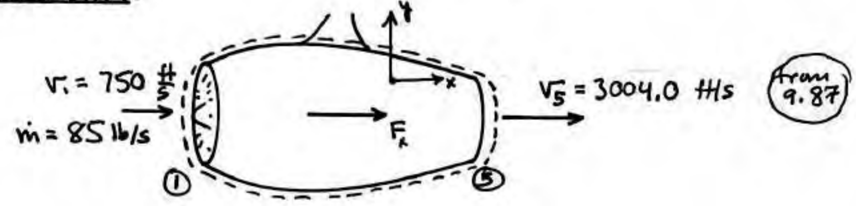
1. The effect of the surrounding pressure on the control volume boundary cancels.

PROBLEM 9.105

KNOWN: Conditions are known for the turbojet engine analyzed in Problem 9.87

FIND: Calculate the thrust developed by the engine.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume is at steady state. (2) The force
① F_x is the net force acting on the control volume.

ANALYSIS: Applying Eq. 9.31

$$F_x = \dot{m}\,(V_5 - V_1)$$

$$= \left(85\ \frac{lb}{s}\right)(3004.0 - 750)\ \frac{ft}{s}\left|\frac{1\ lbf}{32.2\ lb\cdot ft/s^2}\right|$$

$$= 5950\ \text{lbf}$$

The thrust is the force developed by the engine. Thus

$$F_{thrust} = -F_x = -5950\ \text{lbf}$$ ← thrust

to the left

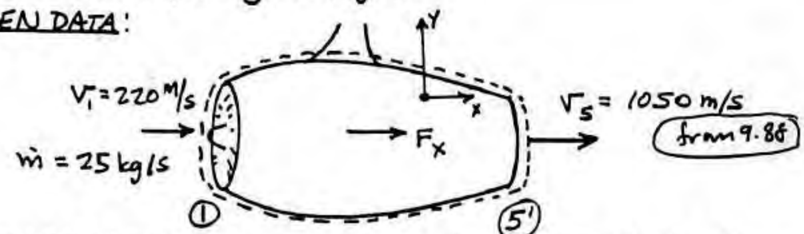
1. The effect of the surrounding pressure on the control volume cancels.

PROBLEM 9.106

KNOWN: Data are know for a turbojet with afterburner as in Problem 9.88

FIND: Calculate the thrust developed by the engine.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume is at steady state. (2) The force F_x is the net force acting on the control volume.

ANALYSIS: Applying Eq. 9.31

① $$F_x = \dot{m}(V_5 - V_4)$$

$$= \left(25 \frac{kg}{s}\right)(1050 - 220)\frac{m}{s} \left| \frac{1 \text{ kN}}{10^3 \text{ kg}\cdot\text{m/s}^2} \right| = 20.75 \text{ kN}$$

Thus, the thrust is the force developed by the control volume

② $$F_{thrust} = -F_x = -20.75 \text{ kN} \quad \longleftarrow \text{thrust}$$

to the left

1. The effect of the surrounding pressure on the control volume boundary cancels.

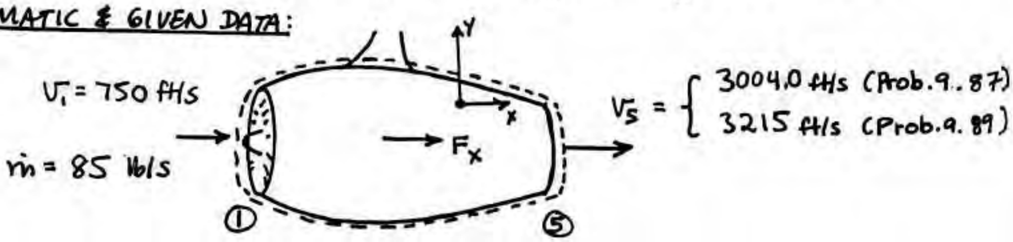
2. Comparing this result with that of Problem 9.103, we see that the use of an afterburner increases the engine thrust.

PROBLEM 9.107

KNOWN: The turbojet in Problem 9.87 and the modified turbojet in Problem 9.89 are reconsidered.

FIND: Calculate the thrust developed by each engine. Discuss.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) The force
① F_x is the net force acting on the control volume.

ANALYSIS: Applying Eq. 9.31

$$F_x = \dot{m}(V_5 - V_1)$$

The thrust is the force developed by the control volume. Thus

$$F_{thrust} = -F_x = -\dot{m}(V_5 - V_1)$$

Inserting values, first for Problem 9.87 (turbojet)

$$F_{thrust} = -\left(85\frac{lb}{s}\right)(3004.0 - 750)\frac{ft}{s}\left|\frac{1 \text{ lbf}}{32.2 \text{ lb}\cdot\text{ft/s}^2}\right|$$

$$= -5950 \text{ lbf (to the left)}$$ ← thrust (Prob. 9.87)

Similarly, for Problem 9.89 turbojet + afterburner)

$$F_{thrust} = -(85)(3215 - 750)\left|\frac{1}{32.2}\right|$$

$$= -6507 \text{ lbf (to the left)}$$ ← thrust (Prob. 9.89)

The addition of an afterburner increases the velocity of gases leaving the engine. In this case, the result is a 9.4% increase in thrust developed by the engine.

---

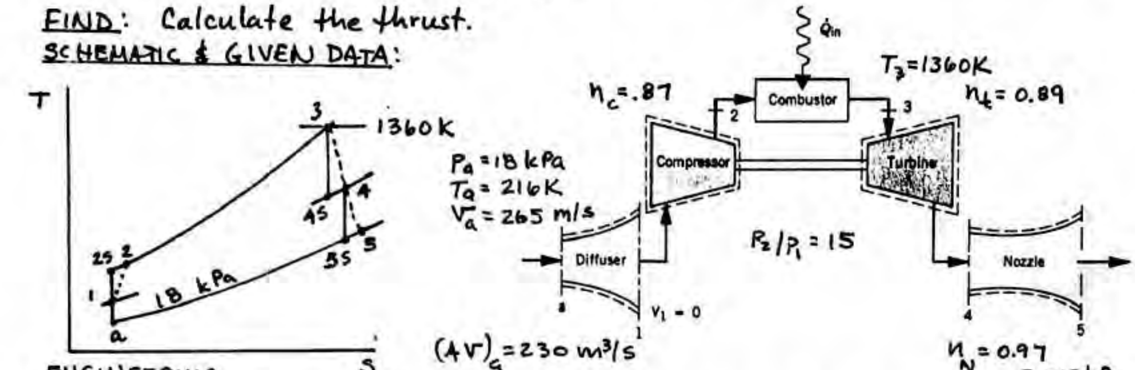
1. The effect of the surrounding pressure on the control volume boundary cancels.

PROBLEM 9.108

KNOWN: A turbojet engine is analyzed on an air-standard basis. Data are known at various locations.

FIND: Calculate the thrust.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: Same as in Example 9.13, except $\eta_c = 0.87$, $\eta_t = 0.89$, $\eta_N = 0.97$.

ANALYSIS: To determine the thrust, we begin by finding the velocity at the nozzle exit, V_5. First, fix each of the principal states (Table A-22).

State a $T_a = 216\text{K} \Rightarrow h_a = 215.97$ kJ/kg, $p_{ra} = 0.44088$

State 1 Applying the energy balance to the diffuser; $0 = h_a + \frac{V_a^2}{2} - h_1$. Thus

$$h_1 = h_a + \frac{V_a^2}{2} = 215.97\,\frac{\text{kJ}}{\text{kg}} + \frac{265^2\,\text{m}^2/\text{s}^2}{2}\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| = 251.08\text{ kJ/kg}$$

Since the diffuser is assumed to operate isentropically

$$p_1 = (p_{r1}/p_{ra})\,p_a = (0.74394/0.44088)(18\text{ kPa}) = 30.373\text{ kPa}$$

State 2 For isentropic compression, $p_{r2s} = p_{r1}(p_2/p_1) = 11.159$ and $h_{2s} = 545.16\,\frac{\text{kJ}}{\text{kg}}$ with the compressor efficiency

$$h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c}\right) = 589.10\text{ kJ/kg}, \quad p_2 = 15\cdot p_1 = 455.6\text{ kPa}$$

State 3 $T_3 = 1360\text{K} \Rightarrow h_3 = 1467.49$ kJ/kg, $p_{r3} = 399.1$

State 4 For a turbojet, $\dot{W}_c/\dot{m} = \dot{W}_t/\dot{m} \Rightarrow (h_2 - h_1) = \eta_t(h_3 - h_{4s})$. Thus

$h_{4s} = 1087.69$ kJ/kg $\Rightarrow p_{r4s} = 131.51$ and $p_4 = (p_{r4s}/p_{r3})\,p_3 = 150.13$ kPa

Now, with the turbine efficiency

$$h_4 = h_3 - (h_3 - h_{4s})\eta_t = 1129.47\text{ kJ/kg} \Rightarrow p_{r4} = 151.07$$

State 5 For isentropic expansion through the nozzle

$$p_{r5s} = (p_5/p_4)\,p_{r4} = 18.113 \Rightarrow h_{5s} = 625.61\text{ kJ/kg}$$

Now, with the isentropic nozzle efficiency

$$V_5 = \sqrt{\eta_N}\,V_{5s} = \sqrt{\eta_N\, 2(h_4 - h_{5s})}$$

$$= \sqrt{(0.97)\,2\,(1129.47 - 625.61)\frac{\text{kJ}}{\text{kg}}\left|\frac{10^3\text{ N}\cdot\text{m}}{1\text{ kJ}}\right|\left|\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right|} = 988.7\text{ m/s}$$

Now, applying Eq. 9.31 to a control volume enclosing the entire engine

$$F_x = \dot{m}\,(V_5 - V_a)$$

where F_x is the net force acting on the control volume.

PROBLEM 9.108 (Cont'd.) - Page 2

The mass flow rate is found from $\dot{m} = (AV)_a / v_a$, where

$$v_a = \frac{RT_a}{p_a} = \frac{\left(\frac{8.314}{28.97} \frac{kJ}{kg \cdot K}\right)(216\ K)}{(18\ kPa)} \left|\frac{10^3\ N \cdot m}{1\ kJ}\right| \left|\frac{1\ kPa}{10^3\ N/m^2}\right| = 3.444\ m^3/kg$$

and $$\dot{m} = \frac{230\ m^3/s}{3.444\ m^3/kg} = 66.78\ kg/s$$

Thus $$F_x = \left(66.78\ \frac{kg}{s}\right)(988.7\ m/s - 265\ m/s) \left|\frac{1\ N}{1\ kg \cdot m/s^2}\right| \left|\frac{1\ kN}{10^3\ N}\right| = 48.33\ kN$$

Finally, the thrust is the force developed by the engine, or

$$F_{thrust} = -F_x = -48.33\ kN \longleftarrow F_{thrust}$$

to the left

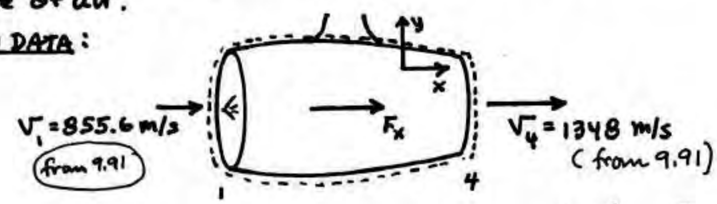
PROBLEM 9.109

Calculate the ratio of the thrust developed to the mass flow rate of air, in N per kg/s, for the ramjet engine in Problem 9.91.

KNOWN: Conditions are known for the ramjet analyzed in Problem 9.91

FIND: Determine the ratio of the thrust developed to the mass flow rate of air.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state. (2) The force F_x is the net force acting on the control volume.

ANALYSIS: Applying Eq. 9.31

$$F_x = \dot{m}(V_4 - V_1)$$

or

$$\frac{F_x}{\dot{m}} = V_4 - V_1$$

$$= (1348 - 855.6)\frac{m}{s}\left(\frac{1\ N}{1\ kg \cdot m/s^2}\right)$$

$$= 492.4 \frac{N}{kg/s}$$

The thrust is the force developed by the engine. Thus

$$\frac{F_{thrust}}{\dot{m}} = -\frac{F_x}{\dot{m}} = -492.4 \frac{N}{kg/s}$$ ← $F_{thrust}/\dot{m}$

to the left

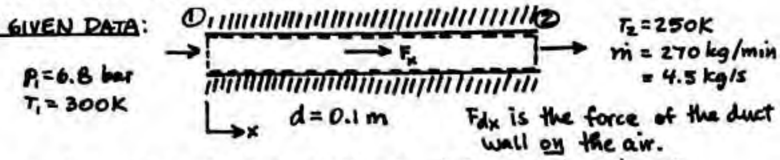
COMMENT: The effect of the surrounding pressure on the control volume boundary cancels.

PROBLEM 9.110

KNOWN: Air flows through a horizontal, well-insulated duct of constant cross-sectional area. Data are known at the inlet and exit.

FIND: Determine the magnitude and direction of the net horizontal force exerted by the duct wall on the air.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume is at steady state, and $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) Potential energy effects are negligible. (3) The air behaves as an ideal gas.

ANALYSIS: To find the force F_x, we need to evaluate the velocities at ① and ② and p_2. First

$$v_1 = \frac{RT_1}{p_1} = \frac{\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(300)}{(6.8\text{ bars})}\left(\frac{1\text{ bar}}{10^5\text{ N/m}^2}\right)\left(\frac{10^3\text{ N}\cdot\text{m}}{1\text{ kJ}}\right) = 0.1266\text{ m}^3/\text{kg}$$

$$A = \pi d^2/4 = 7.85\times10^{-3}\text{ m}^2$$

and
$$V_1 = \frac{\dot{m} v_1}{A} = \frac{(4.5\text{ kg/s})(0.1266\text{ m}^3/\text{kg})}{(7.85\times10^{-3}\text{ m}^2)} = 72.57\text{ m/s}$$

To find V_2, begin with the energy balance at steady state

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}\left[(h_1-h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\cancelto{0}{(z_1-z_2)}\right]$$

or
$$V_2 = \sqrt{2(h_1-h_2) + V_1^2}$$

From Table A-22; $h_1 = 300.19$ kJ/kg and $h_2 = 250.05$ kJ/kg. Thus

$$V_2 = \sqrt{2(300.19-250.05)\frac{kJ}{kg}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right)\left(\frac{10^3\text{ N}\cdot\text{m}}{1\text{ kJ}}\right) + 72.57^2\text{ m}^2/\text{s}^2}$$

$$= 324.9\text{ m/s}$$

Now, with $AV_1/v_1 = AV_2/v_2 = \dot{m}$

$$v_2 = \frac{AV_2}{\dot{m}} = \frac{(7.85\times10^{-3}\text{ m}^2)(324.9\text{ m/s})}{(4.5\text{ kg/s})} = 0.5668\text{ m}^3/\text{kg}$$

and
$$p_2 = \frac{RT_2}{v_2} = \frac{\left(\frac{8314}{28.97}\frac{N\cdot m}{kg\cdot K}\right)(250K)}{(0.5668\text{ m}^3/\text{kg})}\left|\frac{1\text{ bar}}{10^5\text{ N/m}^2}\right| = 1.266\text{ bar}$$

Finally, using Eq. 9.31 $\quad p_1A - p_2A + F_x = \dot{m}(V_2 - V_1)$. Thus

$$F_x = \dot{m}(V_2 - V_1) + (p_2 - p_1)A$$

$$= \left(4.5\frac{kg}{s}\right)(324.9 - 72.57)\frac{m}{s}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) + (1.266 - 6.8)\text{ bar}\left(\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right)(7.85\times10^{-3}\text{ m}^2)$$

$$= -3209\text{ N (to the left)} \xleftarrow{F_x}$$

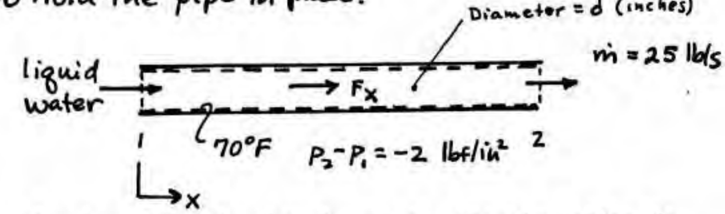
PROBLEM 9.111

Liquid water at 70°F flows through a horizontal, constant-diameter pipe at steady state. The mass flow rate is 25 kg/s and pressure decreases by 2 lbf/in.^{2} from pipe inlet to exit. Plot the magnitude, in lbf, of the horizontal force required to hold the pipe in place versus pipe diameter varying from 1 to 6 inches. Specify the direction of the horizontal force.

KNOWN: Water flows at steady state through a constant-diameter horizontal pipe. Data are known for flow through the pipe.

FIND: Determine the magnitude and direction of the horizontal force needed to hold the pipe in place.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume is at steady state. (2) The water is modeled as incompressible, with $v = v_{f @ 70°F}$. (3) The area of the pipe is constant.

ANALYSIS: Begin with the mass balance

$$\frac{\cancel{A_1} V_1}{\cancel{v_1}} = \frac{\cancel{A_2} V_2}{\cancel{v_2}} \Rightarrow V_1 = V_2$$

Now, applying Eq. 9.31

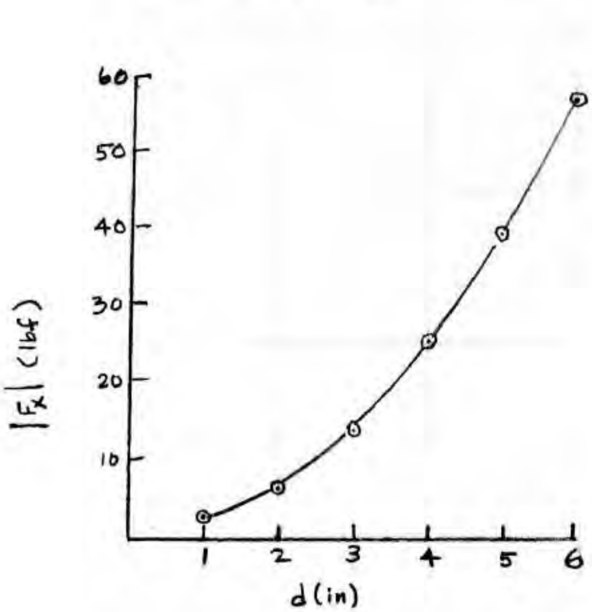
$$F_x + P_1 A - P_2 A = \dot{m}\,(\cancel{V_2 - V_1})^{0}$$

or

$$F_x = (P_2 - P_1)A = (P_2 - P_1)\left(\frac{\pi d^2}{4}\right)$$

$$\Rightarrow \quad F_x = \left(-2\ \frac{\text{lbf}}{\text{in}^2}\right)\left(\frac{\pi d^2}{4}\right)$$

$$= -1.57\ \frac{\text{lbf}}{\text{in}^2}\,(d)^2 \qquad \text{(Force acts to the left)}$$

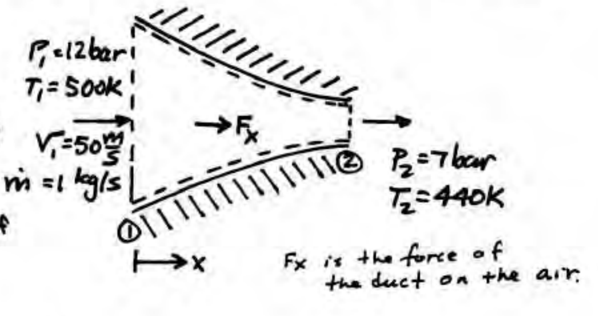


PROBLEM 9.112

KNOWN: Air flows through a well-insulated, horizontal duct. Data are known at the inlet and exit.

FIND: Determine the force exerted by the air on the duct in the direction of flow.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:
(1) The control volume is at steady state with $\dot{W}_{cv} = \dot{Q}_{cv} = 0$. (2) Potential energy effects are negligible. (3) The air is modeled as an ideal gas. (4) The force F_x is the force of the duct on the air.

ANALYSIS: Applying Eq. 9.31

$$F_x + p_1 A_1 - p_2 A_2 = \dot{m}(V_2 - V_1)$$

or

$$F_x = \dot{m}(V_2 - V_1) + p_2 A_2 - p_1 A_1 \qquad (*)$$

Now, find V_2 using an energy balance

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\cancelto{0}{(z_1 - z_2)}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Thus, with data from Table A-22

$$V_2 = \sqrt{2(h_1 - h_2) + V_1^2}$$

$$= \sqrt{2(503.02 - 441.61)\frac{kJ}{kg}\left|\frac{1\ kg\cdot m/s^2}{1\ N}\right|\left|\frac{10^3\ N\cdot m}{1\ kJ}\right| + 50^2\ m^2/s^2} = 354\ m/s$$

Next, calculate A_1 and A_2 using $\dot{m} = AV/v$ as follows:

$$A_1 = \frac{\dot{m} v_1}{V_1} = \frac{\dot{m} R T_1}{V_1 p_1} = \frac{(1\ kg/s)\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(500K)}{(12\ bar)(50\ m/s)}\left|\frac{1\ bar}{10^5\ N/m^2}\right|\left|\frac{10^3\ N\cdot m}{1\ kJ}\right|$$

$$= 2.392 \times 10^{-3}\ m^2$$

Similarly

$$A_2 = \frac{(1)\left(\frac{8.314}{28.97}\right)(440)}{(7)(354)}\left|\frac{10^3}{10^5}\right| = 5.1 \times 10^{-4}\ m^2$$

Inserting values in (*)

$$F_x = \left(1\ \frac{kg}{s}\right)(354 - 50)\frac{m}{s}\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right| + \left[(7\ bar)(5.1\times 10^{-4})m^2 - (12)(2.39\times 10^{-3})\right]\left|\frac{10^5\ N/m^2}{1\ bar}\right|$$

$$= -2009\ N$$

$F_{air/duct} = -F_x = 2009$ N (to the right) ← $F_{air/duct}$

PROBLEM 9.113

9.113 Using the ideal gas model, determine the sonic velocity, in m/s, of

(a) air at 1000 K.
(b) carbon dioxide at 500 K.
(c) helium at 300 K.

KNOWN: Ideal gas at known temperature.

FIND: Determine the sonic velocity.

ANALYSIS: Eq. 9.37 gives the sonic velocity.

(a) Air at 1000K ⇒ $k = 1.336$ (Table A-20)

$$c = \sqrt{kRT} = \sqrt{(1.336)\left(\frac{8.314}{28.97} \cdot \frac{kJ}{kg \cdot K}\right)(1000K)\left(\frac{10^3 N \cdot m}{1 kJ}\right)\left(\frac{1 kg \cdot m/s^2}{1N}\right)}$$
$$= 619.2 \text{ m/s}$$

(b) CO_2 at 500K ⇒ $k = 1.229$ (Table A-20)

$$c = \sqrt{(1.229)\left(\frac{8.314}{44.01}\right)(500)(10^3)} = 340.7 \text{ m/s}$$

(c) He at 300K ⇒ $c_p = \frac{5}{2}R$, $c_v = \frac{3}{2}R$ ⇒ $k = \frac{5/2}{3/2} = 1.667$ (See Fig. 3.13)

$$c = \sqrt{(1.667)\left(\frac{8.314}{4.003}\right)(300)(10^3)} = 1019.2 \text{ m/s}$$

PROBLEM 9.114

9.114 While attending a July 4 fireworks show, you see the flash of an explosion and about 2 seconds later hear the explosion. If the ambient temperature is 80°F, about how far are you, in feet, from the flash?

Known: A flash is sighted, and 2 seconds later the sound is heard.

Find: Estimate how far away the flash took place.

ENGR. MODEL:

1. Ambient air is modeled as an ideal gas at 80°F
2. Eq. 9.37 is applicable.

Analysis: From Table A-20E, $k = 1.4$. Then with Eq. 9.37

$$c = \sqrt{(1.4)\left(\frac{1545\ \text{ft}\cdot\text{lbf}}{28.97\ \text{lb}\cdot°\text{R}}\right)(540\,°\text{R})\left|\frac{32.2\ \text{lb}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right|}$$

$$= 1139.4\ \frac{\text{ft}}{\text{s}}$$

Estimating the distance, d,

$$d \cong c\cdot t = \left(1139.4\ \frac{\text{ft}}{\text{s}}\right)(2\ \text{s}) = 2279\ \text{ft} \quad (0.43\ \text{mi})$$

PROBLEM 9.115

KNOWN: Steam at a given state.

FIND: Estimate the sonic velocity using data from Table A-4 and using the ideal gas model. Compare.

ANALYSIS: Beginning with Eq. 9.36b

$$c = \sqrt{(-v^2)\left.\frac{\partial p}{\partial v}\right)_s} \approx \sqrt{(-v^2)\left.\frac{\Delta p}{\Delta v}\right)_s}$$

where the partial derivative is estimated by using differences in tabular data. At $T = 360°C$, $p = 60$ bar : $v = 0.04331$ m³/kg
$s = 6.3782$ kJ/kg·K

Interpolating in adjacent table columns at the same entropy

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	p = 40 bar = 4.0 MPa (T_{sat} = 250.4°C)			
Sat.	0.04978	2602.3	2801.4	6.0701
280	$\boxed{0.05546}$	2680.0	2901.8	6.2568
320	$\boxed{0.06199}$	2767.4	3015.4	6.4553
360	0.06788	2845.7	3117.2	6.6215

→ $s = 6.3782$; $v = 0.05945$ m³/kg

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	p = 80 bar = 8.0 MPa (T_{sat} = 295.06°C)			
Sat.	0.02352	2569.8	2758.0	5.7432
320	0.02682	2662.7	2877.2	5.9489
360	0.03089	2772.7	3019.8	6.1819
400	$\boxed{0.03432}$	2863.8	3138.3	6.3634
440	$\boxed{0.03742}$	2946.7	3246.1	6.5190

→ $s = 6.3782$; $v = 0.03461$ m³/kg

Using these values

$$c = \sqrt{-(0.04331^2)\frac{m^3}{kg}\frac{(80-40)\text{ bar}}{(.03461-.05945)}\left|\frac{10^5 N/m^2}{1\text{ bar}}\right|\left|\frac{1\text{ kg}\cdot m/s^2}{1 N}\right|}$$

$$= 549.6 \text{ m/s} \longleftarrow \qquad c$$

Using the function for H_2O in Table A-21 : $c_p = 2.035$ kJ/kg·K and with $c_p = kR/(k-1)$, $k = 1.293$. Thus, assuming the ideal gas model

$$c = kRT = \sqrt{(1.293)\left(\frac{8.314}{18.02}\right)(633)|10^3|} = 614.5 \text{ m/s}$$

① The ideal gas model appears to significantly over-estimate the sonic velocity at the given state.

1 Locating the state on Fig. A-1 (generalized compressibility chart), we see $Z \approx 0.91$, and thus the ideal gas model is at best approximate.

PROBLEM 9.116

9.116 Plot the Mach number of carbon dioxide at 1 bar, 460 m/s, as a function of temperature in the range 250 to 1000 K.

KNOWN: Carbon dioxide at 1 bar and a given velocity.

FIND: Plot the Mach number versus temperature ranging from 250 to 1000K.

ENGINEERING MODEL: The carbon dioxide is modeled as an ideal gas, as can be verified readily.

ANALYSIS: The Mach number is $M = V/c$, where c is obtained from Eq. 9.37: $c = \sqrt{kRT}$. Table A-20 gives k versus T.

Moreover,

$$c = \sqrt{k\left(\frac{8.314}{44.01}\frac{kJ}{kg \cdot K}\right) T \left|\frac{1\ kg \cdot m/s^2}{1\ N}\right|\left|\frac{10^3\ N \cdot m}{1\ kJ}\right|}$$

$$= \sqrt{188.91\left(\frac{m^2}{s^2 \cdot K}\right) k\, T} \quad \text{in K} \qquad (1)$$

As a sample calculation, at 250K, $k = 1.314$ (Table A-20). Eq. (1) then gives

$$c = \sqrt{188.91\left(\frac{m^2}{s^2 \cdot K}\right)(1.314)(250K)} = 249.1\ \frac{m}{s} \Rightarrow M = \frac{460\ m/s}{249.1\ m/s} = 1.847$$

Like calculations can be performed for selected temperature values in the interval from 250 to 1000K. The values of the Mach number obtained can be plotted by hand versus temperature.

1. Alternatively, the plot can be obtained using IT:

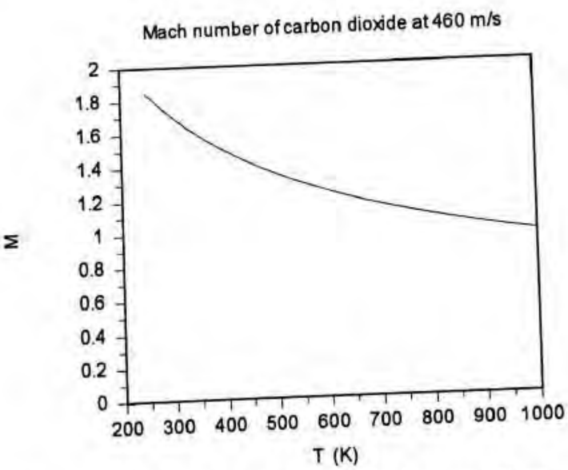
IT Code

```
T = 250  // K
V = 460  // m/s

cp = cp_T("CO2", T)
cv = cv_T("CO2", T)
k = cp / cv
R = 8.314 / 44.01
c = sqrt (k * R * T * 1000)
M = V / c
```

IT Results for T = 250 K

c_p = 0.7944 kJ/kg·K
c_v = 0.6054 kJ/kg·K
k = 1.312
c = 248.9 m/s
M = 1.848

PLOT:



Note that the sonic velocity increases with temperature. Thus, the Mach number decreases, as expected.

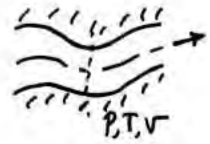
PROBLEM 9.117

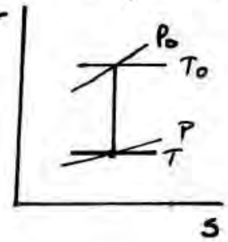
KNOWN: An ideal gas flows through a duct. At a particular location, p, T, and V are known.

FIND: Determine M, T_0, p_0 for three different gases at specified conditions.

SCHEMATIC & GIVEN DATA:

ENGINEERING MODEL: Each gas behaves as an ideal gas, as can be verified.





ANALYSIS:

(a) Air; $p = 100$ lbf/in.², $T = 310°F = 770°R$
$V = 1400$ ft/s

From Table A-20E at $T = 770°R$; $k = 1.3935$. The sonic velocity is

$$c = \sqrt{kRT} \quad \sqrt{(1.3935)\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°R}\right)(770°R)\left|\frac{32.2\ \text{lb}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right|} = 1357.4\ \text{ft/s}$$

and $M = V/c = 1400/1357.4 = 1.031$ (supersonic) ← (a) M

Using data from Table A-22E

$$h_0 = h + V^2/2 = 184.51 + \left(\frac{1400^2}{2}\frac{\text{ft}^2}{\text{s}^2}\right)\left|\frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

$$= 223.63\ \text{Btu/lb} \Rightarrow T_0 = 929.9\ °R \leftarrow \quad \text{(a) } T_0$$

and

$$p_0 = p\left(\frac{p_r(T_0)}{p_r(T)}\right) = (100)\left(\frac{9.4658}{4.829}\right) = 196.0\ \text{lbf/in.}^2 \leftarrow \quad \text{(a) } p_0$$

(b) Helium; $p = 20$ lbf/in.², $T = 520°R$, $V = 900$ ft/s

① For the monatomic gas; $k = 5/3 = 1.667$, $c_p = \frac{5}{2}R = \frac{5}{2}\left(\frac{1545}{4.003}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°R}\right) = 964.9$ ft·lbf/lb·°R

$$c = \sqrt{kRT} = \sqrt{(1.667)\left(\frac{1545}{4.003}\right)(520)(32.2)} = 3281.9\ \frac{\text{ft}}{\text{s}}$$

$$M = 900/3281.9 = 0.2742 \leftarrow \quad \text{(b) } M$$

$$0 = c_p(T_0 - T) - V^2/2$$

Thus

$$T_0 = T + \frac{V^2}{2c_p} = (520°R) + \frac{900^2\ \text{ft}^2/\text{s}^2}{(2)(964.9\ \text{ft}\cdot\text{lbf/lb}\cdot°R)}\left|\frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2}\right|$$

$$= 533\ °R \leftarrow \quad \text{(b) } T_0$$

$$p_0 = p\left(\frac{T_0}{T}\right)^{\frac{k}{k-1}} = (20)\left(\frac{533}{520}\right)^{2.5} = 21.27\ \text{lbf/in.}^2 \leftarrow \quad \text{(b) } p_0$$

(c) Nitrogen; 50 lbf/in.², 600°R, 500 ft/s ⇒ $k = 1.399$

$$c = \sqrt{1.399\left(\frac{1545}{28.01}\right)(600)\left|\frac{32.2}{1}\right|} = 1221.0\ \text{ft/s}\ ;\ M = 500/1221 = 0.4095 \leftarrow \quad \text{(c) } M$$

Using data from Table A-23E; $\bar{h}_0 = \bar{h} + \frac{V^2}{2}$ (Mol. wt.)

$$\bar{h}_0 = 4167.9\ \frac{\text{Btu}}{\text{lbmol}} + \frac{500^2\ \text{ft}^2/\text{s}^2}{2}\left(\frac{28.01\ \text{lb}}{\text{lbmol}}\right)\left|\frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right| = 4307.7\ \text{Btu/lbmol}$$

$$\Rightarrow T_0 = 620°R \leftarrow \quad \text{(c) } T_0$$

To get p_0: $0 = \bar{s}°(T_0) - \bar{s}°(T) - \bar{R}\ln(p_0/p)$

$$\ln p_0/p = \frac{1}{\bar{R}}\left[\bar{s}°(T_0) - \bar{s}°(T)\right] = \frac{(46.743 - 46.514)}{(1545/778)} = 0.1153 \Rightarrow p_0/p = 1.1222$$

$$\Rightarrow p_0 = 56.11\ \text{lbf/in.}^2 \leftarrow \quad \text{(c) } p_0$$

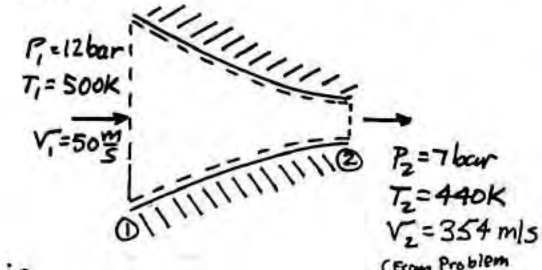
1 For Helium and other monatomic gases, see Figure 3.13.

PROBLEM 9.118

KNOWN: Air flows through a duct as in Problem 9.104.

FIND: Determine the Mach number, stagnation temperature, and stagnation pressure at the inlet and exit of the duct.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: See Solution to Problem 9.112.

ANALYSIS: At state 1: $k_1 = 1.387$ from Table A-20. The sonic velocity is

$$c_1 = \sqrt{k_1 R T_1} = \sqrt{(1.387)\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(500K)\left|\frac{1\ kg\cdot m/s^2}{1\ N}\right|\left|\frac{10^3 N\cdot m}{1\ kJ}\right|} = 446.1\ m/s$$

Thus $M_1 = V_1/c_1 = 50/446.1 = 0.112$ ← M_1

The stagnation enthalpy is

$$h_{o_1} = h_1 + V_1^2/2 = 503.02\frac{kJ}{kg} + \left(\frac{50^2\ m^2/s^2}{2}\right)\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|\left|\frac{1\ kJ}{10^3 N\cdot m}\right| = 504.27\ kJ/kg$$

Interpolating in Table A-22; $T_{o_1} = 501.2$ K ← T_{o_1}

$$P_{o_1} = P_1\left(\frac{P_r(T_{o_1})}{P_r(T_1)}\right) = (12\ bar)\left(\frac{8.4862}{8.411}\right) = 12.11\ bar$$ ← P_{o_1}

At state 2: $k_2 = 1.3918$

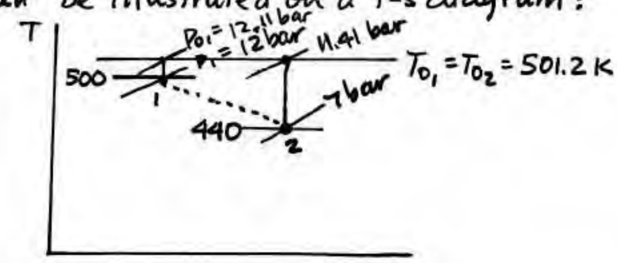
$$c_2 = \sqrt{(1.3918)\left(\frac{8.314}{28.97}\right)(440)\,|10^3|} = 419.2\ m/s$$

$M_2 = 354/419.2 = 0.845$ ← M_2

$h_{o_2} = h_{o_1} = 504.27$ kJ/kg $\Rightarrow T_{o_2} = T_{o_1} = 501.2$ K ← T_{o_2}

$$P_{o_2} = P_2\left(\frac{P_r(T_{o_2})}{P_r(T_2)}\right) = (7\ bar)\left(\frac{8.4862}{5.332}\right) = 11.14\ bar$$ ← P_{o_2}

These states can be illustrated on a T-s diagram:



PROBLEM 9.119

9.119 Using *Interactive Thermodynamics: IT*, determine for water vapor at 500 lbf/in.^{2}, 600°F, and 1000 ft/s,

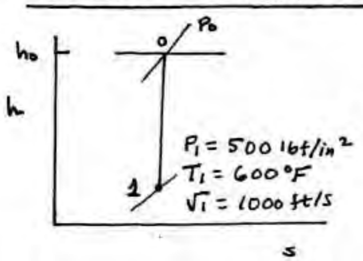
(a) the stagnation enthalpy, in Btu/lb.
(b) the stagnation temperature, in °F.
(c) the stagnation pressure, in lbf/in.^{2}

Check the values obtained by inspection of the Mollier diagram, Fig. A-8E.

Known: Water vapor flows with known pressure, temperature, and velocity.

Find: Using IT determine specified property data at the corresponding stagnation state: h_0, T_0, and p_0. Check the values by inspection of Fig. A-8E.

Schematic and Given Data:



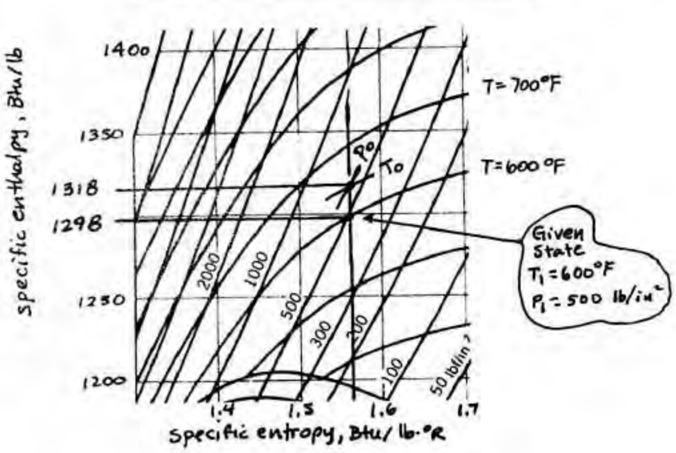
$$\begin{cases} h_0 = h_1 + \dfrac{V_1^2}{2} = h_1 + \dfrac{(1000\ \text{ft/s})^2}{2}\left|\dfrac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2}\right|\left|\dfrac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right| \\ \qquad = h_1 + 19.96\ \text{Btu/lb} \\ s_0 = s_1 \end{cases}$$

Analysis. Using IT:

```
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)

T1=600
p1=500
h0=h1+19.96

h0 = h_PT("Water/Steam", p0, T0)
s1 = s_PT("Water/Steam", p0, T0)
```

h1=1298 h0=1318	} Btu/lb
p0=600.1	lbf/in^2
T0=645.7	°F
s1=1.558	Btu/lb·°R

Check using the Mollier diagrams



$h_0 = 1298 + 19.96 = 1318$ Btu/lb, $s_0 = s_1$

$p_0 \approx 600$ lbf/in^2

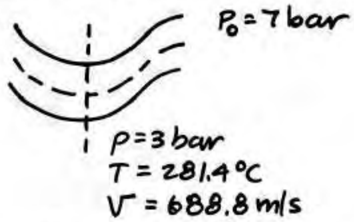
$T_0 \approx 645$°F

PROBLEM 9.120

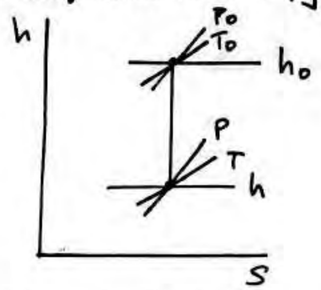
KNOWN: Conditions for steam flowing at a particular location in a passageway are known. The stagnation pressure is known.

FIND: Determine the corresponding specific stagnation enthalpy and stagnation temperature.

SCHEMATIC & GIVEN DATA:



①



ANALYSIS: $h_0 = h + \frac{V^2}{2}$ From Table A-4; $h = 3031.5$ kJ/kg

Thus $h_0 = 3031.5 \frac{kJ}{kg} + \left(\frac{688.8^2 \, m^2/s^2}{2}\right)\left|\frac{1 \, kg \cdot m/s^2}{1 \, N}\right|\left|\frac{1 \, kJ}{10^3 \, N \cdot m}\right|$

$= 3268.7$ kJ/kg ← h_0

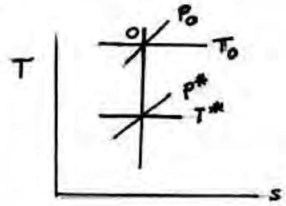
With $h_0 = 3268.7$ and $p_0 = 7$ bar $\Rightarrow T_0 = 400K$ ← T_0

PROBLEM 9.121

Consider isentropic flow of an ideal gas with constant k.

(a) Show that $\frac{T^*}{T_0} = \frac{2}{k+1}$ and $\frac{p^*}{p_0} = \left(\frac{2}{k+1}\right)^{k/(k-1)}$ where T^* and p^* are the temperature and pressure, respectively, at the state where Mach number is unity, and T_0 and p_0 are the temperature and pressure, respectively, at the stagnation state.

(b) Using the results of part (a), evaluate T^* and p^* for Example 9.14, in K and kPa, respectively.



KNOWN: Isentropic flow of an ideal gas with constant k.

FIND: Develop two specified relationships and apply them to the case of Example 9.14

ANALYSIS: (a) Begin with Eq. 9.50:

$$\frac{T_0}{T} = 1 + \left(\frac{k-1}{2}\right)M^2$$

When $M=1$, $T = T^*$. Thus

$$\frac{T_0}{T^*} = 1 + \left(\frac{k-1}{2}\right) = \frac{k+1}{2}$$

$$\Rightarrow \frac{T^*}{T_0} = \frac{2}{k+1} \qquad \longleftarrow (1)$$

Similarly, begin with Eq. 9.51:

$$\frac{p_0}{p} = \left(1 + \left(\frac{k-1}{2}\right)M^2\right)^{k/(k-1)}$$

When $M=1$, $p = p^*$. Thus

$$\frac{p_0}{p^*} = \Big(\underbrace{1 + \frac{(k-1)}{2}}_{\frac{k+1}{2}}\Big)^{k/(k-1)} = \left(\frac{k+1}{2}\right)^{k/(k-1)}$$

$$\Rightarrow \frac{p^*}{p_0} = \left(\frac{2}{k+1}\right)^{k/(k-1)} \qquad \longleftarrow (2)$$

Alternatively, combine Eq. (1) with the isentropic relationship,

$$\frac{p^*}{p_0} = \left(\frac{T^*}{T_0}\right)^{k/(k-1)}$$

(b) In Example 9.14, $k = 1.4$ and $T_0 = 360$ K. Thus, Eq. (1) gives

$$\frac{T^*}{360\,K} = \frac{2}{1.4+1} \Rightarrow T^* = 300K \qquad \longleftarrow$$

This agrees with the value determined in part (a) of the example.

In Example 9.14, $p_0 = 1$ MPa $= 1000$ kPa. Thus, Eq. (2) gives

$$\frac{p^*}{1000\,kPa} = \left(\frac{2}{2.4}\right)^{(1.4/0.4)} \Rightarrow p^* = 528\ kPa \qquad \longleftarrow$$

This agrees with the value shown on Fig. E9.14.

PROBLEM 9.122

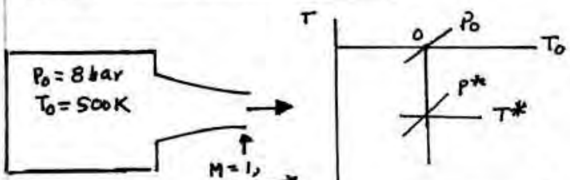
Consider isentropic flow of an ideal gas with constant k through a converging nozzle from a large tank at 500 K, 8 bar. Using the results of Problem 9.121(a) with k at 500 K, evaluate the temperature, in K, and pressure, in bar, at the state where Mach number is unity for

(a) air.
(b) oxygen, O_2.
(c) carbon dioxide, CO_2.

KNOWN: Isentropic flow of an ideal gas with constant k through a converging nozzle from a large tank at known temperature and pressure.

FIND: For each of three gases, use the results of Problem 9.121(a) to evaluate T^* and p^*

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. An ideal gas with constant k flows isentropically.
2. Take k at 500 K

ANALYSIS: (a) AIR. Table A-20 gives $k = 1.387$ (at 500K). Then, the expressions of Ex. 9.121(a) give

$$\frac{T^*}{T_0} = \frac{2}{k+1} \Rightarrow T^* = \left(\frac{2}{2.387}\right)(500\text{K}) = 418.9 \text{ K}$$

$$\frac{p^*}{p_0} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \Rightarrow p^* = \left(\frac{2}{2.387}\right)^{(3.584)}(8 \text{ bar}) = 4.24 \text{ bar}$$

(b) O_2. Table A-20 gives $k = 1.365$ (at 500K). Then

$$\frac{T^*}{T_0} = \frac{2}{k+1} \Rightarrow T^* = \left(\frac{2}{2.365}\right)(500\text{K}) = 422.8 \text{ K}$$

$$\frac{p^*}{p_0} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \Rightarrow p^* = \left(\frac{2}{2.365}\right)^{(3.7397)}(8 \text{ bar}) = 4.27 \text{ bar}$$

(c) CO_2. Table A-2 gives $k = 1.229$ (at 500K). Then

$$\frac{T^*}{T_0} = \frac{2}{k+1} \Rightarrow T^* = \left(\frac{2}{2.229}\right)(500\text{k}) = 448.6 \text{ K}$$

① $$\frac{p^*}{p_0} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \Rightarrow p^* = \left(\frac{2}{2.229}\right)^{(5.3668)}(8 \text{ bar}) = 4.47 \text{ bar}$$

---

1. T^* and p^* increase as k decreases.

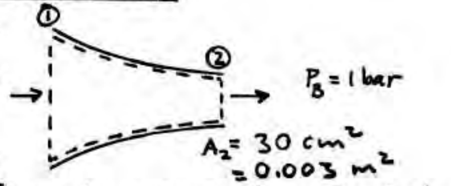
PROBLEM 9.123

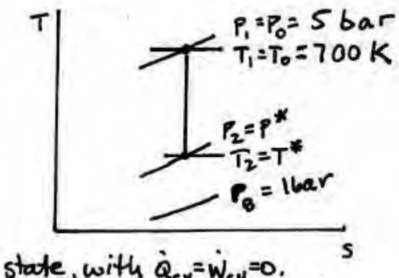
KNOWN: An ideal gas mixture flows isentropically through a converging nozzle. The inlet conditions, back pressure, and exit area are specified.

FIND: Determine (a) the exit temperature, (b) the exit velocity, and (c) the mass flow rate.

SCHEMATIC & GIVEN DATA:

$p_1 = p_0 = 5$ bar
$T_1 = T_0 = 700$ K
$k = 1.31$
Mol.wt. = 23





ENGINEERING MODEL: (1) The control volume is at steady state, with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) The gas expands isentropically within the nozzle. (3) The velocity at the inlet is negligible, and potential energy effects can be ignored. (4) The mixture behaves as an ideal gas with constant specific heats.

ANALYSIS: First, it is necessary to determine if the nozzle is choked. Using Eq. 9.51, with $M = 1$

$$\frac{p_0}{p^*} = \left[1 + \left(\frac{k-1}{2}\right)(1)^2\right]^{k/(k-1)} = \left(\frac{k+1}{2}\right)^{k/(k-1)} = 1.838$$

Thus, $p^* = p_0/1.838 = 2.720$ bar. $p_B < p^* \Rightarrow$ choked conditions $\Rightarrow p_2 = p^*$.

(a) $T_2 = T^*$. Using Eq. 9.50, with $M_2 = 1$

$$\frac{T^\circ}{T^*} = 1 + \left(\frac{k-1}{2}\right)(1)^2 = \frac{k+1}{2} = 1.155 \Rightarrow T^* = 606.1 \text{ K} \longleftarrow T_2$$

(b) Since $M_2 = 1$, $V_2 = c_2 = \sqrt{kRT_2}$

$$V_2 = \sqrt{(1.31)\left(\frac{8.314}{23}\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(606.1\text{ K})\left|\frac{10^3\text{ N}\cdot\text{m}}{1\text{ kJ}}\right|\left|\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right|}$$

$$= 535.7 \text{ m/s} \longleftarrow V_2$$

(c) The mass flow rate is

$$\dot{m} = \frac{A_2 V_2}{v_2} = \frac{p_2 A_2 V_2}{RT_2}$$

$$= \frac{(2.720\text{ bar})(0.003\text{ m}^2)(535.7\text{ m/s})}{\left(\frac{8.314}{23}\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(606.1\text{ K})}\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

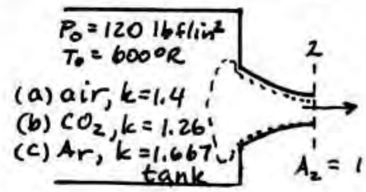
$$= 1.995 \text{ kg/s} \longleftarrow \dot{m}$$

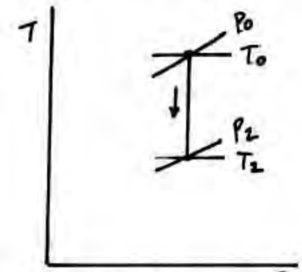
PROBLEM 9.124

KNOWN: An ideal gas expands isentropically through a converging nozzle from a large tank. The conditions in the tank, the pressure in the back, and the nozzle exit area are known.

FIND: Determine the mass flow rate if the gas is (a) air, (b) carbon dioxide, and (c) argon.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) The control volume is at steady state. (2) The flow is isentropic. (3) The gas behaves as an ideal gas with constant specific heats.

ANALYSIS: First, it is necessary to determine if the nozzle is choked. Using Eq. 9.51 with $M_2 = 1$

$$\frac{p_0}{p^*} = \left[1 + \left(\frac{k-1}{2}\right)(1)^2\right]^{k/k-1} = \left(\frac{k+1}{2}\right)^{k/k-1} \qquad (*)$$

(a) For air, with $k = 1.4$

$$\frac{p_0}{p^*} = 1.89293 \Rightarrow p^* = 63.39 \text{ lbf/in}^2$$

Since $p_b < p^*$, the nozzle is choked. Thus, $p_2 = p^* = 63.39$ lbf/in².

Now, for the isentropic expansion

$$\frac{T_2}{T_0} = \left(\frac{p_2}{p_0}\right)^{\frac{k-1}{k}} \Rightarrow T_2 = 500°R$$

Using the energy balance: $0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}\left[(h_0 - h_2) + \left(\frac{\cancel{V_0^2}^{0} - V_2^2}{2}\right) + g\,\cancel{(z_0 - z_2)}^{0}\right]$

$$V_2 = \sqrt{2(h_0 - h_2)} = \sqrt{2 c_p (T_0 - T_2)}$$

$$= \sqrt{(2)\left(0.24 \frac{\text{Btu}}{\text{lb}°R}\right)(600-500)°R \left|\frac{32.2 \text{ lb}\cdot\text{ft/s}^2}{1 \text{ lbf}}\right|\left|\frac{778 \text{ ft}\cdot\text{lbf}}{1 \text{ Btu}}\right|}$$

$$= 1096.6 \text{ ft/s}$$

All of the conditions are known at the exit. Thus

$$\dot{m} = \frac{A_2 V_2}{v_2} = \frac{A_2 V_2 p_2}{R T_2}$$

$$= \frac{(1 \text{ in}^2)(1096.6 \text{ ft/s})(63.39 \text{ lbf/in}^2)}{\left(\frac{1545}{28.97} \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°R}\right)(500°R)} = 2.61 \text{ lb/s} \longleftarrow \dot{m}$$

(b) For CO_2, with $k = 1.26$ ($c_p = 0.2187$ Btu/lb·°R)

$$\frac{p_0}{p^*} = 1.8081 \Rightarrow p^* = 66.37 \text{ lbf/in}^2$$

Again, $p_b < p^* \Rightarrow$ choked conditions, and $p_2 = p^* = 66.37$ lbf/in².

Thus
$$\frac{T_2}{T_0} = \left(\frac{p_2}{p_0}\right)^{\frac{k-1}{k}} \Rightarrow T_2 = 531.0\,°R$$

and
$$V_2 = \sqrt{(2)(0.2187)(600 - 531.0)|32.2|\,|778|} = 869.5 \text{ ft/s}$$

Finally
$$\dot{m} = \frac{(1)(869.5)(66.37)}{\left(\frac{1545}{44.01}\right)(531.0)} = 3.096 \text{ lb/s} \longleftarrow \dot{m}$$

(c) For Ar, $k = 1.667$ ($c_p = 0.1243$ Btu/lb·°R)

$$\frac{p_0}{p^*} = 2.053 \Rightarrow p^* = 58.45 \text{ lbf/in}^2$$

In this case, $p_b > p^*$, which means the nozzle is <u>not</u> choked. Thus, the exit plane pressure is equal to the back region pressure. That is

$$p_2 = p_b = 60 \text{ lbf/in}^2$$

and
$$\frac{T_2}{T_0} = \left(\frac{p_2}{p_0}\right)^{\frac{k-1}{k}} \Rightarrow T_2 = 454.7\,°R$$

The exit velocity is

$$V_2 = \sqrt{(2)(0.1243)(600 - 454.7)|32.2|\,|778|} = 951.3 \text{ ft/s}$$

Finally, the mass flow rate is

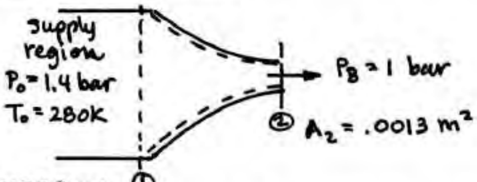
$$\dot{m} = \frac{(1)(951.3)(60)}{\left(\frac{1545}{39.94}\right)(454.7)} = 3.245 \text{ lb/s} \longleftarrow \dot{m}$$

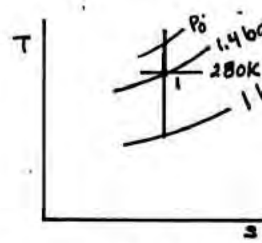
PROBLEM 9.125

KNOWN: Air expands isentropically through a converging nozzle and discharges to the atmosphere. The inlet conditions are specified and the exit plane area is given.

FIND: Determine the mass flow rate for each of two specified supply region pressures.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) The control volume is at steady state, with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) The air expands isentropically. (3) The air behaves as an ideal gas. (4) $k = 1.4$.

ANALYSIS: (a) First, determine if the nozzle is choked. Using Eq. 9.51 with $M = 1$ gives

$$\frac{p_0}{p^*} = \left(\frac{k+1}{2}\right)^{k/k-1} = 1.8929 \Rightarrow p^* = 0.7396 \text{ bar}$$

NOT CHOKED

$\therefore P_2 = 1$ bar

For the isentropic expansion

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 254.3 \text{ K}$$

The exit velocity is, with $c_p = 1.004$ kJ/kg·K

$$V_2 = \sqrt{2\, c_p(T_1 - T_2)}$$

$$= \sqrt{2\,(1.004 \tfrac{kJ}{kg\cdot K})(280 - 254.3)K \left|\frac{10^3 N\cdot m}{1\ kJ}\right| \left|\frac{1\ kg\cdot m/s^2}{1N}\right|}$$

$$= 227.2 \text{ m/s}$$

Thus, the mass flow rate is

$$\dot{m} = \frac{A_2 V_2 p_2}{R T_2} = \frac{(.0013\ m^2)(227.2\ m/s)(1\ bar)}{\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(254.3\ K)} \left|\frac{10^5 N/m^2}{1\ bar}\right| \left|\frac{1\ kJ}{10^3 N\cdot m}\right|$$

$$= 0.4047 \text{ kg/s} \longleftarrow \frac{\dot{m}}{(a)}$$

(b) Next, consider the case for which $p_0 = 2$ bar, $T_0 = 280$K and $p_B = 1$ bar. With Eq. 9.51

$$\frac{P_0}{p^*} = \left(\frac{k+1}{2}\right)^{k/k-1} = 1.8929 \Rightarrow p^* = 1.0566 \text{ bar}$$

$p_B < p^* \Rightarrow$ choked conditions $\Rightarrow p_2 = p^* = 1.0566$ bar

PROBLEM 9.125 (Cont'd.) - Page 2

Thus, $T_2 = T^*$. From Eq. 9.50 with $M = 1$

$$\frac{T_0}{T^*} = \frac{k+1}{2} = 1.2 \Rightarrow T^* = 233.33\,K$$

The exit velocity is

$$V_2 = \sqrt{2(1.004)(280 - 233.33)\left|10^3\right|} = 306.1 \text{ m/s}$$

Thus, the mass flow rate is

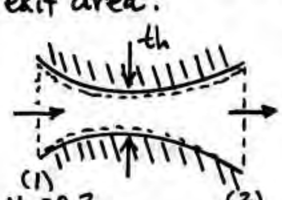
$$\dot{m} = \frac{(0.0013)(306.1)(1)}{\left(\frac{8.314}{28.97}\right)(233.33)}\left|\frac{10^5}{10^3}\right| = 0.5943 \text{ kg/s} \longleftarrow \underline{\dot{m}} \quad (b)$$

PROBLEM 9.126

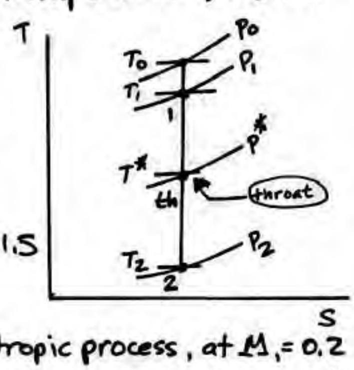
KNOWN: Air expands isentropically through a converging-diverging nozzle from know inlet conditions to a specified Mach number at the exit.

FIND: Determine (a) the stagnation pressure and temperature, (b) the throat area, and (c) the exit area.

SCHEMATIC & GIVEN DATA:



$M_1 = 0.2$
$T_1 = 595°R$
$p_1 = 77.8\ lbf/in^2$
$\dot{m} = 1\ lb/s$
$M_2 = 1.5$



ENGINEERING MODEL: (1) The control volume is at steady state, with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) The air undergoes an isentropic process. (3) The air behaves as an ideal gas with $k = 1.4$.

ANALYSIS: (a) Using data from Table 9.2 for the isentropic process, at $M_1 = 0.2$

$$p_1/p_o = 0.97250 \Rightarrow p_o = \frac{77.8}{0.97250} = 80\ lbf/in^2 \quad \longleftarrow p_o$$

$$T_1/T_o = 0.99206 \Rightarrow T_o = \frac{595}{0.99206} = 600°R \quad \longleftarrow T_o$$

(b) Since $M_1 < 1$ and $M_2 > 1$, the flow is sonic at the throat. Thus

$$T_{th} = T^* = T_o(0.83333) = 500°R$$
$$p_{th} = p^* = p_o(0.52828) = 42.26\ lbf/in^2$$

Thus
$$v_{th} = \frac{RT^*}{p^*} = \frac{\left(\frac{1545}{28.97}\frac{ft\cdot lbf}{lb\cdot°R}\right)(500°R)}{(42.26\ lbf/in^2)}\left|\frac{1\ ft^2}{144\ in^2}\right| = 4.382\ ft^3/lb$$

and
$$V_{th} = \sqrt{kRT^*}$$
$$= \sqrt{(1.4)\left(\frac{1545}{28.97}\frac{ft\cdot lbf}{lb\cdot°R}\right)(500°R)\left|\frac{32.2\ lb\cdot ft/s^2}{1\ lbf}\right|} = 1096.4\ ft/s$$

Finally
$$A_{th} = \frac{\dot{m}\,v_{th}}{V_{th}} = \frac{(1\ lb/s)(4.382\ ft^3/lb)}{(1096.4\ ft/s)}\left|\frac{144\ in^2}{1\ ft^2}\right| = 0.576\ in^2 \quad \longleftarrow A_{th}$$

(c) From Table 9-2, at $M_2 = 1.5$; $A_2/A^* = 1.1762$. Thus

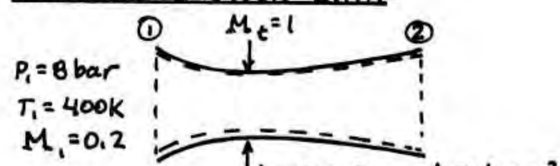
$$A_2 = (1.1762)(0.576) = 0.677\ in^2 \quad \longleftarrow A_2$$

PROBLEM 9.127

KNOWN: A converging-diverging nozzle operates at steady state with isentropic flow of air.

FIND: Determine specified conditions at the nozzle exit and throat.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The air behaves as an ideal gas with $k = 1.4$.

ANALYSIS: Use Table 9.2.

First, find the stagnation state.

$$M_1 = 0.2 \Rightarrow T_1/T_0 = 0.99206 \Rightarrow T_0 = 403.2K$$
$$p_1/p_0 = 0.97250 \Rightarrow p_0 = 8.226 \text{ bar}$$

If the nozzle flow is choked, $M_t = 1$, and

$$T_t = T_0(.83333) = 336.0K$$
$$p_t = p_0(.52828) = 4.3456 \text{ bar}$$

Further, $v_{th} = \dfrac{RT_t}{p_t} = 0.2219\ m^3/kg$

$$V_{th} = (1)\sqrt{kRT_{th}} = 367.4\ m/s$$

Thus, the mass flow rate is

$$\dot{m} = \frac{A_t V_t}{v_t} = 0.497\ kg/s \leftarrow \dot{m}$$

Now, with $A_2/A^* = 6\ cm^2/3\ cm^2 = 2$, there are two cases in Table 9.2.

Supersonic

$$M_2 \approx 2.20 \Rightarrow p_2 = (.09352)p_0 = 0.769 \text{ bar} \leftarrow p_2$$
$$T_2 = (.50813)T_0 = 204.9K \leftarrow T_2$$

Subsonic

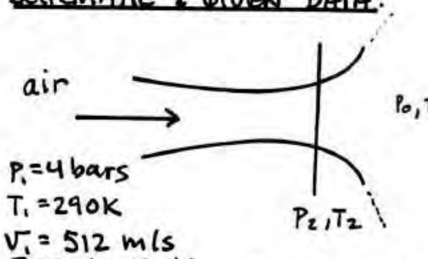
$$M_1 \approx 0.3079 \Rightarrow p_2 = (.93601)p_0 = 7.70 \text{ bar} \leftarrow p_2$$
$$T_2 = (.98127)T_0 = 395.6K \leftarrow T_2$$

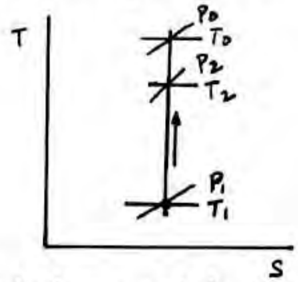
PROBLEM 9.128

KNOWN: Air with $k = 1.4$ flows isentropically through a diffuser.

FIND: Plot velocity, Mach number, and area ratio A/A^* for locations in the flow corresponding to a range of pressures.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) Isentropic flow. (2) Ideal gas model with $k = 1.4$.

ANALYSIS: The following expressions are used to calculate the required quantities:

1. $c = \sqrt{kRT}$
2. $M = V/c$
3. $p_0/p = \left[1 + \frac{k-1}{2} M^2\right]^{k/k-1}$ (Eq. 9.51)
4. $T_0/T = \left[p_0/p\right]^{k-1/k}$ (Eq. 9.50)
5. $A/A^* = \frac{1}{M}\left[\left(\frac{2}{k+1}\right)\left(1 + \frac{k-1}{2}\right) M^2\right]^{(k+1)/2(k-1)}$ (Eq. 9.52)

First, use given values of T_1, p_1, V_1 to get c_1, M_1, p_0, T_0. Then, vary p to get M, T, A/A^*, and V.

The data for the required plots are obtained using IT, as follows:

IT Code

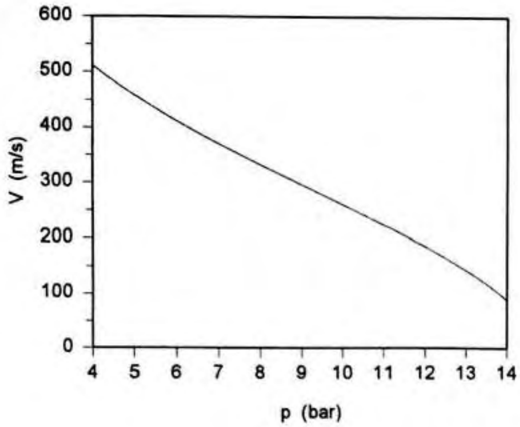
```
p1 = 4  // bar
T1 = 290  // K
V1 = 512  // m/s
k = 1.4
Rbar = 8.314  // kJ/kmol·K
MW = 28.97  // kg/kmol

c1 = sqrt(k * (Rbar / MW) * T1 * 1000)
M1 = V1 / c1
po / p1 = (1 + ((k - 1) / 2) * M1^2)^(k / (k - 1))
To / T1 = (po / p1)^((k - 1) / k)

p = 10
c = sqrt(k * (Rbar / MW) * T * 1000)
M = V / c
Aratio = (1 / M) * ((2 / (k+1)) * ((1 + ((k - 1) / 2) * M^2) ))^((k + 1) / (2 * (k - 1)))
po / p = (1 + ((k - 1) / 2) * M^2)^(k / (k - 1))
To / T = (po / p )^((k - 1) / k)
```

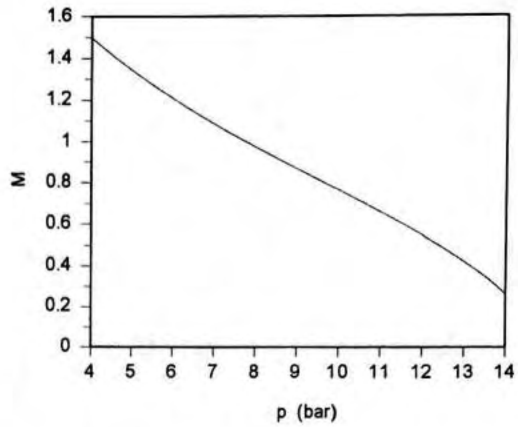
IT Results for p = 10 bar

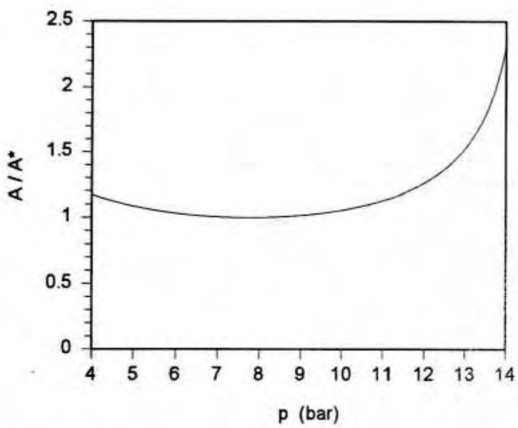
T_o = 420.5 K
p_o = 14.6 bar
T = 290 K
c = 341.3 m/s
V = 260 m/s
M = 0.7616
A/A* = 1.056

PROBLEM 9.128 (Cont'd.) - Page 2

PLOTS:







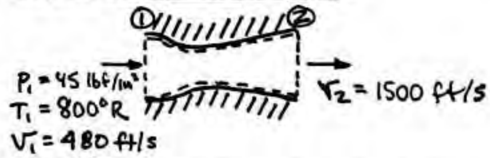
Note that the minimum value of A/A* occurs at a Mach number of unity, as expected.

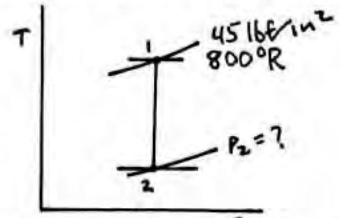
PROBLEM 9.129

KNOWN: Air expands isentropically through a nozzle from known inlet conditions to a known exit velocity.

FIND: Determine (a) the exit pressure, (b) the ratio of exit area to inlet area, (c) whether the nozzle is converging-diverging or converging only.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) The control volume is at steady state, with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) The air undergoes an isentropic process. (3) The air behaves as an ideal gas. Data from Table A-22E apply.

ANALYSIS: To fix the exit state, determine h_2 using an energy balance. Thus

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(\cancel{z_1 - z_2})^{0}\right]$$

or

$$h_2 = h_1 + \left(\frac{V_1^2 - V_2^2}{2}\right)$$

$$= 191.81\,\frac{\text{Btu}}{\text{lb}} + \left(\frac{480^2 - 1500^2}{2}\right)\frac{\text{ft}^2}{\text{s}^2}\left|\frac{1\ \text{lbf}\cdot\text{s}^2}{32.2\ \text{lb}\cdot\text{ft}}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

$$= 151.50\,\frac{\text{Btu}}{\text{lb}} \Rightarrow p_{r2} = 2.4265,\quad T_2 = 633.4\,°\text{R}$$

Finally

$$P_2 = P_1\left(\frac{p_{r2}}{p_{r1}}\right) = \left(45\,\frac{\text{lbf}}{\text{in}^2}\right)\left(\frac{2.4265}{5.526}\right) = 19.760\,\frac{\text{lbf}}{\text{in}^2} \longleftarrow P_2$$

(b) From the mass balance; $\frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2}$. With $v = RT/p$,

$$\frac{A_1 V_1 P_1}{RT_1} = \frac{A_2 V_2 P_2}{RT_2} \Rightarrow \frac{A_2}{A_1} = \frac{V_1}{V_2}\cdot\frac{P_1}{P_2}\cdot\frac{T_2}{T_1} = 0.577 \longleftarrow A_2/A_1$$

(c) To determine the nozzle type, calculate the inlet and exit Mach numbers.

$$M_1 = \frac{V_1}{\sqrt{kRT_1}} = \frac{(480\ \text{ft/s})}{\sqrt{(1.39)\left(\frac{1545}{28.97}\right)\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}(800°\text{R})\left|\frac{32.2\ \text{lb}\cdot\text{ft}}{1\ \text{lbf}\cdot\text{s}^2}\right|}}$$

$$= 0.347\ \text{(subsonic)}$$

$$M_2 = \frac{(1500)}{\sqrt{(1.4)\left(\frac{1545}{28.97}\right)(633.4)|32.2|}} = 1.216\ \text{(supersonic)}$$

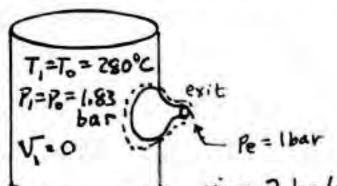
Thus, a <u>converging-diverging</u> nozzle is needed. ⟵ Nozzle type

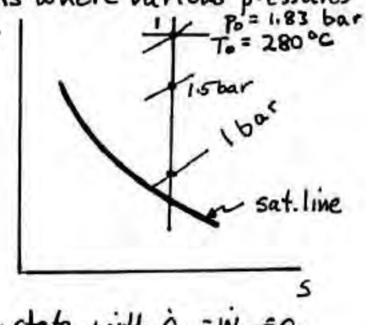
PROBLEM 9.130

KNOWN: Steam expands isentropically through a converging nozzle from a large tank. The flow is choked, and the mass flow rate and exit plane pressure are known.

FIND: Determine the nozzle diameter at locations where various pressures are specified.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) The control volume shown is at steady state, with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) The steam expands isentropically. (3) The nozzle is choked. (4) The velocity in the tank is negligible, and potential energy effects can be ignored.

ANALYSIS: For each case to be considered, $P_1 = P_0 = 1.83$ bar and $T_1 = T_0 = 280$ °C. Thus, from Table A-4, $h_1 = h_0 = 3031.9$ kJ/kg and $s_1 = s_0 = 7.8839$ kJ/kg·K. Further, the velocity is found from

$$V_2 = \sqrt{2(h_0 - h_2)} \qquad (1)$$

and

$$A_2 = \frac{\dot{m}\, v_2}{V_2} \Rightarrow d_2 = \sqrt{\frac{4A_2}{\pi}} \qquad (2)$$

$\underline{P_2 = 1.5 \text{ bar}}$ $\quad h_2 = 2994.6$ kJ/kg, $v_2 = 1.6355$ m³/kg

$$V_2 = \sqrt{2(3031.9 - 2994.6)\frac{\text{kJ}}{\text{kg}} \left|\frac{10^3 \text{ N·m}}{1 \text{ kJ}}\right| \left|\frac{1 \text{ kg·m/s}^2}{1 \text{ N}}\right|} = 273.13 \text{ m/s}$$

$$A_2 = \frac{(2 \text{ kg/s})(1.6355 \text{ m}^3/\text{kg})}{(273.13 \text{ m/s})} = 0.01198 \text{ m}^2$$

$$d_2 = \sqrt{\frac{(4)(0.01198) \text{ m}^2}{\pi} \left|\frac{10^4 \text{ cm}^2}{1 \text{ m}^2}\right|} = 12.348 \text{ cm} \longleftarrow \frac{d_2}{(1.5 \text{ bar})}$$

$\underline{P_2 = 1 \text{ bar}}$ $\quad h_2 = 2899.8$ kJ/kg, $v_2 = 2.2298$ m³/kg

$V_2 = 514.0$ m/s, $A_2 = 0.0086676$ m², $d_2 = 10.510$ cm $\longleftarrow \frac{d_2}{(1 \text{ bar})}$

PROBLEM 9.131

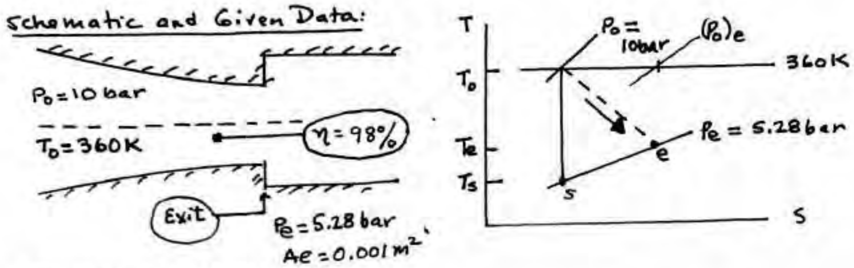
Air enters a converging nozzle operating at steady state with negligible velocity at 10 bar, 360 K and exits at 5.28 bar. The exit area is 0.001 m^2 and the isentropic nozzle efficiency is 98%. The air is modeled as an ideal gas with $k = 1.4$. Potential energy effects are negligible. Determine at the nozzle exit

(a) the velocity, in m/s.
(b) the temperature, in K.
(c) the Mach number.
(d) the stagnation pressure, in bar.
(e) Also evaluate the mass flow rate, in kg/s.

Known: Air expands through a converging nozzle. Data are provided at the inlet and exit.

Find: Determine velocity, temperature, Mach number, and stagnation pressure at the exit. Also determine the mass flow rate.

Schematic and Given Data:



Engineering Model:

1. A control volume at steady state encloses the nozzle.
2. For the control volume $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and potential energy effects are negligible.
3. The air behaves as an ideal gas with $k = 1.4$.
4. The isentropic nozzle efficiency is 98%.

Analysis: With Eq. 6.47, $\eta = (V_e^2/2)/(V_s^2/2)$, where V_e is the velocity at the nozzle exit and V_s is the exit velocity for an isentropic expansion to $P_e = 5.28$ bar, as shown in the schematic.

Accordingly, it's necessary to establish conditions at the exit for the isentropic expansion: We have $P_s/P_0 = 0.528$. Then, with Table 9.2 we get $M_s = 1$ and $T_s/T_0 = 0.83333$. Thus, $T_s = 300$K.

Since $M_s = 1$, Eq. 9.37 gives

$$V_s = \sqrt{kRT_s} = \sqrt{(1.4)\left(\frac{8314}{28.97}\frac{N\cdot m}{kg\cdot K}\right)(300K)\left(\frac{1 kg\cdot m/s^2}{1N}\right)} = 347.2 \text{ m/s}$$

Then, with $V_e = \sqrt{\eta}\, V_s$, we get

$$V_e = \sqrt{0.98}\,(347.2 \text{ m/s}) = 343.7 \text{ m/s} \longleftarrow (a)$$

For the adiabatic expansion, an energy balance gives

$$0 = (h_o - h_e) + \left(0 - \frac{V_e^2}{2}\right) \Rightarrow 0 = c_p(T_o - T_e) - \frac{V_e^2}{2}$$

$$\Rightarrow \quad T_e = T_o - \frac{V_e^2}{2c_p}$$

With $c_p = \frac{kR}{(k-1)} = 1.004$ kJ/kg·K

$$T_e = 360K - \frac{(343.7 \text{ m/s})^2}{(2(1.004 kJ/kg\cdot K)}\left|\frac{1 kJ}{10^3 N\cdot m}\right|\left|\frac{1N}{1 kg\cdot m/s^2}\right|$$

$$= 301.2K \longleftarrow (b)$$

The Mach number at the exit is

$$M_e = \frac{V_e}{\sqrt{kRT_e}} = \frac{343.7 \text{ m/s}}{\sqrt{(1.4)\left(\frac{8314}{28.97}\frac{N\cdot m}{kg\cdot K}\right)(301.2K)\left(\frac{1 kg\cdot m/s^2}{1N}\right)}} = 0.988 \longleftarrow (c)$$

Then, with Eq. 9.51 (or Table 9.2)

$$\frac{(p_o)_e}{p_e} = \left[1 + \left(\frac{1.4-1}{2}\right)(0.988)^2\right]^{3.5} = 1.867$$

$$\Rightarrow (p_o)_e = (1.867)(5.28 \text{ bar}) = 9.86 \text{ bar} \longleftarrow (d)$$

Finally

1 $$\dot{m} = \frac{p_e}{RT_e}A_eV_e = \frac{(5.28\times10^5 N/m^2)(0.001m^2)(343.7 m/s)}{\left(\frac{8314}{28.97}\frac{N\cdot m}{kg\cdot K}\right)(301.2K)} = 2.099\frac{kg}{s} \longleftarrow (e)$$

---

1. Compared to an isentropic expansion to $p_e = 5.28$ bar, the effect of friction in the actual flow is to give a lower exit velocity, a higher exit temperature, a lower exit Mach number, and a reduction in the exit stagnation pressure. Moreover, the mass flow rate is reduced.

PROBLEM 9.132

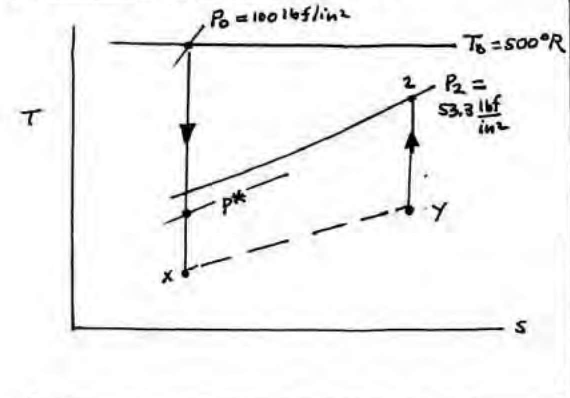
9.132 In part (e) of Example 9.15 a converging–diverging nozzle is considered that experiences a shock in the diverging section. At the nozzle exit for this case the pressure is 53.3 lbf/in.[2] Demonstrate that a converging nozzle also can achieve that exit pressure while flow is *isentropic throughout* from the same stagnation state as in part(e). Determine the Mach number at the exit of the converging nozzle.

KNOWN: Air flowing from specified stagnation conditions through a converging-diverging nozzle experiences a shock in the diverging section and exits at 53.3 lbf/in².

FIND: Demonstrate that a converging nozzle also can achieve that exit pressure while flowing isentropically from the same stagnation state.

Schematic and Given Data:

For Case e of Example 9.15:



Engineering Model: See Example 9.15

Analysis: Referring to the T-s diagram, the converging nozzle can achieve a pressure of 53.3 lbf/in² at its exit only if

$$p^* \leq 53.3 \text{ lbf/in}^2.$$

With Eq. 9.51, M = 1:

$$\frac{p_0}{p^*} = \left[1 + \left(\frac{1.4-1}{2}\right)(1)^2\right]^{3.5} = 1.893$$

$$\Rightarrow \quad p^* = \frac{100 \text{ lbf/in}^2}{1.893} = 52.83 \frac{\text{lbf}}{\text{in}^2}$$

Accordingly, the specified exit pressure can be attained by the converging nozzle.

To evaluate the Mach number at the exit, solve Eq. 9.51 (as in the solution to part (b) of Example 9.15) : as using $p_0/p = 100/53.3 = 1.876$:

(1) $$M = \left\{ \frac{2}{k-1}\left[\left(\frac{p_0}{p}\right)^{(k-1)/k} - 1\right]\right\}^{1/2} = \left\{5\left[(1.876)^{0.286} - 1\right]\right\}^{1/2} = 0.993$$

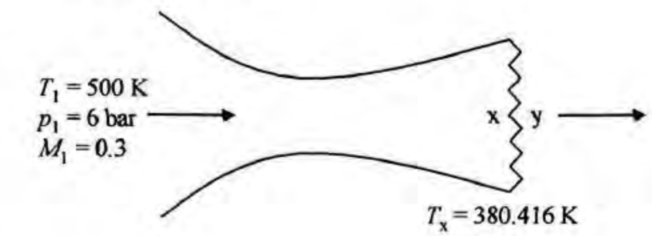
1. The values for p^* and M also can be obtained using Table 9.2 data:
   - when M = 1, the table gives $p^*/p_0 = 0.52828 \Rightarrow p^* = 52.83$ lbf/in².
   - when $p/p_0 = 53.3/100 = 0.533$, interpolation in the table gives M = 0.993.

9.133 A converging-diverging nozzle operates at steady state. Air as an ideal gas with $k = 1.4$ enters the nozzle at 500 K, 6 bar, and a Mach number of 0.3. The air flows isentropically to the exit plane, where its temperature is 380.416 K and a normal shock forms. Determine the back pressure, in bar.

KNOWN: Air flows through a converging-diverging nozzle. The flow is isentropic up to the exit plane where a normal shock stands.

FIND: Determine the back pressure.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Air is modeled as an ideal gas with constant $k = 1.4$.
2. Flow upstream of the shock is isentropic.

ANALYSIS: Since the normal shock forms at the exit plane, $p_B = p_y$. Use Table 9.2 to determine stagnation temperature, T_{o1}, and stagnation pressure, p_{o1}. For $M_1 = 0.3$

$$\frac{T_1}{T_{o1}} = 0.98232 \rightarrow T_{o1} = \frac{500\text{ K}}{0.68965} = 509.0\text{ K}$$

$$\frac{p_1}{p_{o1}} = 0.93947 \rightarrow p_{o1} = \frac{6\text{ bar}}{0.93947} = 6.39\text{ bar}$$

Since flow is isentropic through the nozzle, stagnation temperature and stagnation pressure remain constant up to the exit plane and $T_{ox} = T_{o1} = 509.0$ K and $p_{ox} = p_{o1} = 6.39$ bar. Use Table 9.2 to determine M_x.

$$\frac{T_x}{T_{ox}} = \frac{380.416\text{ K}}{500\text{ K}} = 0.74768 \rightarrow M_x = 1.30\ \text{(Table 9.3)}$$

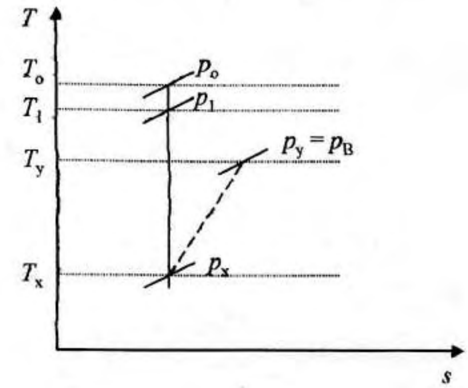
Use Table 9.2 to determine, p_x. For $M_x = 1.3$

$$\frac{p_x}{p_{ox}} = 0.36092 \rightarrow p_x = (6.39 \text{ bar})(0.36092) = 2.306 \text{ bar}$$

From Table 9.3 at $M_x = 1.30$, $\frac{p_y}{p_x} = 1.8050$.

$$p_B = p_y = (2.306 \text{ bar})(1.8050) = \underline{\mathbf{4.162 \text{ bar}}}$$

T-s diagram:

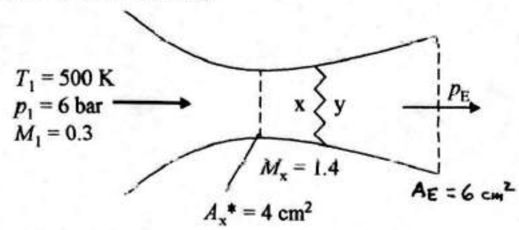


9.134 A converging-diverging nozzle operates at steady state. Air as an ideal gas with $k = 1.4$ enters the nozzle at 500 K, 6 bar, and a Mach number of 0.3. A normal shock stands in the diverging section at a location where the Mach number as 1.40. The cross-sectional areas of the throat and exit plane are 4 cm2 and 6 cm^2, respectively. The flow is isentropic, except where the shock stands. Determine the exit pressure, in bar, and the mass flow rate.

KNOWN: Air flows through a converging-diverging nozzle and forms a shock in the diverging section. The flow is isentropic, except where the shock stands.

FIND: Determine the exit pressure and the mass flow rate.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Air is modeled as an ideal gas with constant $k = 1.4$.
2. Flow is isentropic, except where the shock stands.

ANALYSIS: First, find the exit pressure. Since the normal shock forms before the exit plane, the Mach number at the exit plane can be determined using the identity

$$\frac{A_E}{A_y^*} = \left(\frac{A_E}{A_x^*}\right)\left(\frac{A_x^*}{A_y^*}\right)$$

Substituting Eq. 9.57

$$\frac{A_E}{A_y^*} = \left(\frac{A_E}{A_x^*}\right)\left(\frac{p_{oy}}{p_{ox}}\right)$$

Use Table 9.2 to determine stagnation pressure, p_{o1}. For $M_1 = 0.3$

$$\frac{p_1}{p_{o1}} = 0.93947 \rightarrow p_{o1} = \frac{6\text{ bar}}{0.93947} = 6.39\text{ bar}$$

Since flow is isentropic from state 1 through the nozzle until the shock at state x, stagnation pressure remains constant up to location x and $p_{ox} = p_{o1} = 6.39$ bar. From Table 9.3 at $M_x = 1.40$

$$\frac{p_{oy}}{p_{ox}} = 0.95819$$

Substituting values

$$\frac{A_E}{A_y^*} = \left(\frac{6\ \text{cm}^2}{4\ \text{cm}^2}\right)(0.95819) = 1.4373$$

Entering Table 9.2 at $\frac{A_E}{A_y^*} = 1.4373$ (subsonic flow) and interpolating

$$M_E = 0.461$$

$$\frac{p_E}{p_{oE}} = 0.86351$$

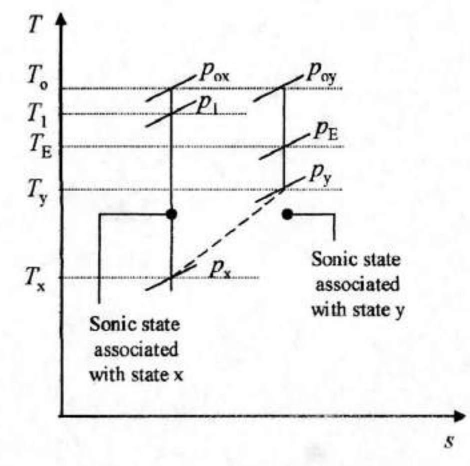
Since flow is isentropic from state y to the exit, $p_{oE} = p_{oy}$. Solving for p_{oy} from the ratio of stagnation pressures associated with y and x

$$\frac{p_{oy}}{p_{ox}} = 0.95819 \rightarrow p_{oy} = p_{oE} = (6.39\ \text{bar})(0.95819) = 6.12\ \text{bar}$$

Solving for the exit pressure

$$\frac{p_E}{p_{oE}} = 0.86351 \rightarrow p_E = (6.12\ \text{bar})(0.86351) = \underline{\mathbf{5.28\ bar}}$$

T-s diagram:



Next, find the mass flow rate using $\dot{m} = A_1V_1/v_1$. From Table 9.2 at $M_1 = 0.3$; $(A_1/A^*) = 2.0351$. Thus

$$A_1 = (2.0351)(4\ \text{cm}^2) = 8.1404\ \text{cm}^2$$

Next, using the ideal gas equation of state

$$v_1 = \frac{RT_1}{p_1} = \frac{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(500\ \text{K})}{(6\ \text{bar})}\left|\frac{10^3\text{N}\cdot\text{m}}{1\ \text{kJ}}\right|\left|\frac{1\ \text{bar}}{10^5\text{N/m}^2}\right| = 0.2392\ \text{m}^3/\text{kg}$$

Finally, from $M = V/\sqrt{kRT}$

$$V_1 = M_1\sqrt{kRT_1}$$

$$= (0.3)\sqrt{(1.4)\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(500\ \text{K})\left|\frac{10^3\text{N}\cdot\text{m}}{1\ \text{kJ}}\right|\left|\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right|} = 134.5\ \text{m/s}$$

Thus

$$\dot{m} = \frac{(8.1404\ \text{cm}^2)(134.5\frac{\text{m}}{\text{s}})}{(0.2392\frac{\text{kg}}{\text{m}^3})}\left|\frac{1\ \text{m}^2}{10^4\ \text{cm}^2}\right| = \underline{\mathbf{0.4675\ kg/s}}$$

PROBLEM 9.135

KNOWN: Air flows through a converging-diverging duct with known conditions at the inlet and a known exit pressure. A normal shock stands at a location in the converging section where $M_x = 1.5$.

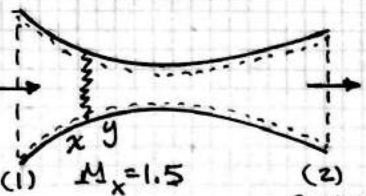
FIND: Determine the temperature and Mach number at the exit.

SCHEMATIC & GIVEN DATA:

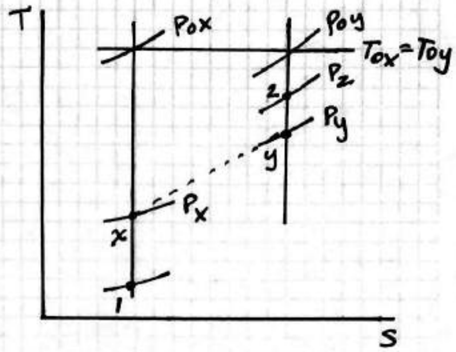
$M_1 = 2$

$P_1 = 26\ \frac{lbf}{in^2}$

$T_1 = 445\ °R$



$P_2 = 150\ lbf/in^2$



ENGINEERING MODEL: (1) The flow is isentropic everywhere except in the immediate vicinity of the shock. (2) The air behaves as an ideal gas with $k = 1.4$.

ANALYSIS: First, determine the upstream stagnation conditions using data from Table 9-2. At $M_1 = 2$

$$P_1/P_{0x} = 0.12780\ ;\quad P_{0x} = \frac{26}{0.12780} = 203.44\ lbf/in^2$$

$$T_1/T_{0x} = 0.55556\ ;\quad T_{0x} = \frac{445}{0.55556} = 801\ °R$$

Now, from Table 9-3 at $M_x = 1.5$; $P_{0y}/P_{0x} = 0.92978 \Rightarrow P_{0y} = 189.15\ \frac{lbf}{in^2}$

With $P_2 = 150$, we get

$$\frac{P_2}{P_{0y}} = \frac{150}{189.15} = 0.79302$$

Interpolating in Table 9-2 at $P/P_0 = 0.79302$

$$M_2 = 0.5847 \longleftarrow M_2$$

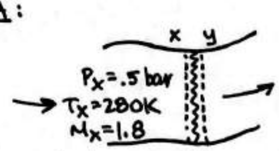
and $T_2/T_0 = 0.93583 \Rightarrow T_2 = 749.6\ °R \longleftarrow T_2$

PROBLEM 9.136

KNOWN: Air undergoes a normal shock. Conditions upstream are known.

FIND: Determine (a) p_y, (b) p_{ox}, (c) T_{ox}, and (d) the change in specific entropy across the shock, $s_y - s_x$.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The air is an ideal gas with constant specific heat ratio, k=1.4.

ANALYSIS: (a) Using Table 9.3, at $M_x = 1.8$;

$$p_y/p_x = 3.6133 \Rightarrow p_y = (3.6133)(.5\text{ bar}) = 1.8066\text{ bar} \leftarrow p_y$$

(b) Using Table 9.3, at $M_x = 1.8$;

$$p_x/p_{ox} = 0.17404 \Rightarrow p_{ox} = \frac{(.5\text{ bars})}{.17404} = 2.873\text{ bar} \leftarrow p_{ox}$$

(c) Also from Table 9.2, $T_x/T_{ox} = .60680 \Rightarrow T_{ox} = 461.4$ K $\leftarrow T_{ox}$

(d) To calculate $s_y - s_x$,

$$s_y - s_x = c_p \ln(T_y/T_x) - R \ln(p_y/p_x)$$

These ratios can be read directly from Table 9.3;

$$s_y - s_x = 1.004 \ln(1.5316) - \frac{8.314}{28.97}\ln(3.6133)$$
$$= 0.05935\text{ kJ/kg}\cdot\text{K} \leftarrow \Delta s$$

The data for the required plots are obtained using IT, as follows:

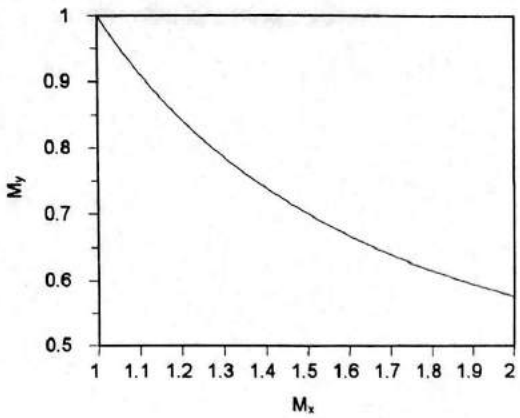
IT Code

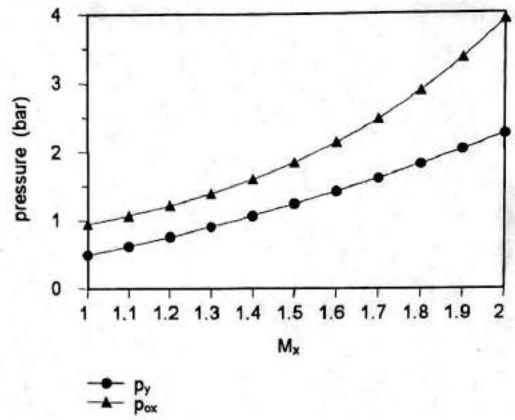
```
px = 0.5  // bar
Tx = 280  // K
Mx = 1.8
k = 1.4

My^2 = (Mx^2+(2/(k-1)))/(((2*k)/(k-1))*Mx^2-1)
py / px = (1+k*Mx^2)/(1+k*My^2)
Ty / Tx = (1+((k-1)/2)*Mx^2)/(1+((k-1)/2)*My^2)
pox / px = (1+((k-1)/2)*Mx^2)^(k/(k-1))
Tox / Tx = (pox / px)^((k-1)/k)
dels = 1.004 * ln(Ty/Tx) - (8.314 / 28.97) * ln(py/px)
```

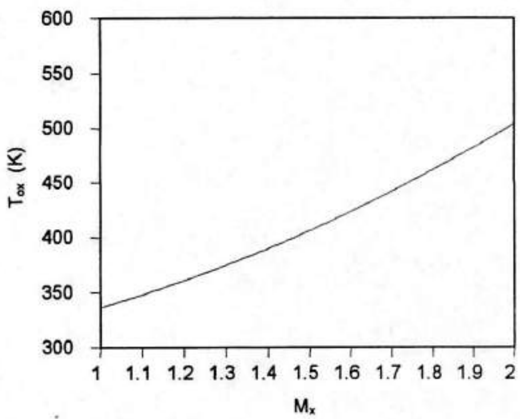
IT Results for Mx = 1.8

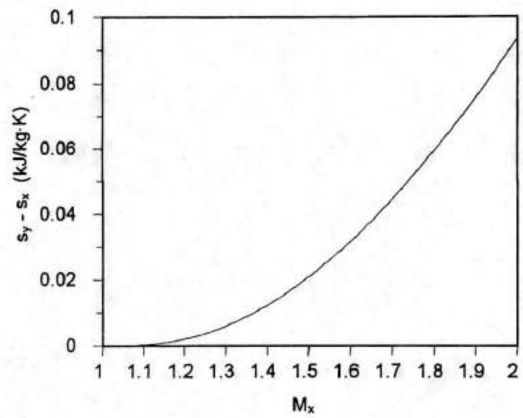
My = 0.6165
Tox = 461.4 K
Ty = 428.8 K
pox = 2.873 bar
py = 1.807 bar
$s_y - s_x = 0.05933$ kJ/kg·K

PROBLEM 9.136 (Cont'd.) - Page 2







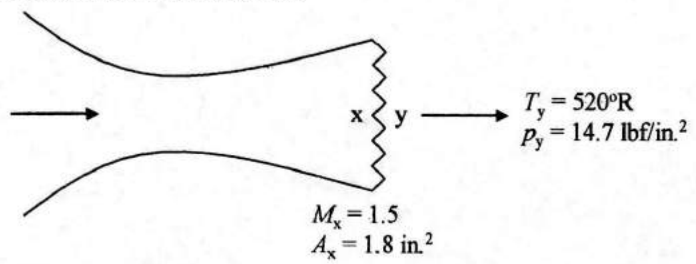


9.137 A converging-diverging nozzle operates at steady state. Air as an ideal gas with $k = 1.4$ flows through the nozzle, discharging to the atmosphere at 14.7 lbf/in.^{2} and 520°R. A normal shock stands at the exit plane with $M_x = 1.5$. The exit plane area is 1.8-in.^{2} Upstream of the shock, the flow is isentropic. Determine
(a) the stagnation pressure p_{ox}, in lbf/in.^{2}
(b) the stagnation temperature T_{ox} in °R.
(c) the mass flow rate, in lb/s.

KNOWN: Air flows through a converging-diverging nozzle and discharges to the atmosphere. The flow is isentropic up to the exit plane where a normal shock stands.

FIND: Determine (a) the stagnation pressure p_{ox}, (b) the stagnation temperature T_{ox}, and (c) the mass flow rate.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Air is modeled as an ideal gas with constant $k = 1.4$.
2. Flow up to the shock is isentropic.

ANALYSIS: (a) To determine stagnation pressure, first determine p_x. From Table 9.3, at $M_x =$ 1.5

$$\frac{p_y}{p_x} = 2.4583 \rightarrow p_x = \frac{14.7 \frac{\text{lbf}}{\text{in.}^2}}{2.4583} = 5.980 \text{ lbf/in.}^2$$

Use Table 9.2 to determine stagnation pressure, p_{ox}

$$\frac{p_x}{p_{ox}} = 0.27240 \rightarrow p_{ox} = \frac{5.980 \frac{\text{lbf}}{\text{in.}^2}}{0.27240} = \underline{\mathbf{21.95 \text{ lbf/in.}^2}}$$

(b) To determine stagnation temperature, first determine T_x. From Table 9.3, at $M_x = 1.5$

$$\frac{T_y}{T_x} = 1.3202 \rightarrow T_x = \frac{520°\text{R}}{1.3202} = 393.9°\text{R}$$

Problem 9.137 (Continued) – Page 2

Use Table 9.2 to determine stagnation temperature, T_{ox}

$$\frac{T_x}{T_{ox}} = 0.68965 \rightarrow T_{ox} = \frac{393.9°\text{R}}{0.68965} = \underline{\mathbf{571.2°R}}$$

(c) Mass flow rate is determined from

$$\dot{m} = \frac{A_x \mathsf{V}_x}{v_x} = \frac{A_x \mathsf{V}_x p_x}{RT_x}$$

Velocity is determined from the Mach number

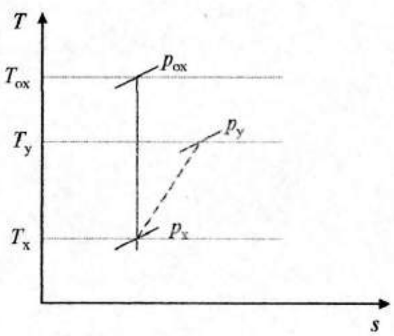
$$M_x = \frac{\mathsf{V}_x}{c_x} = \frac{\mathsf{V}_x}{\sqrt{kRT_x}} \rightarrow \mathsf{V}_x = M_x\sqrt{kRT_x}$$

$$\mathsf{V}_x = (1.5)\sqrt{(1.4)\left(\frac{1545\dfrac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot°\text{R}}}{28.97\dfrac{\text{lb}}{\text{lbmol}}}\right)(393.9°\text{R})\left|\frac{32.2\dfrac{\text{lb}\cdot\text{ft}}{\text{s}^2}}{1\ \text{lbf}}\right|} = 1460\ \text{ft/s}$$

Substituting values and solving for mass flow rate

$$\dot{m} = \frac{\left(1.8\ \text{in.}^2\right)\left(1460\dfrac{\text{ft}}{\text{s}}\right)\left(5.980\dfrac{\text{lbf}}{\text{in.}^2}\right)}{\left(\dfrac{1545\dfrac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot°\text{R}}}{28.97\dfrac{\text{lb}}{\text{lbmol}}}\right)(393.9°\text{R})} = \underline{\mathbf{0.75\ lb/s}}$$

T-s diagram:

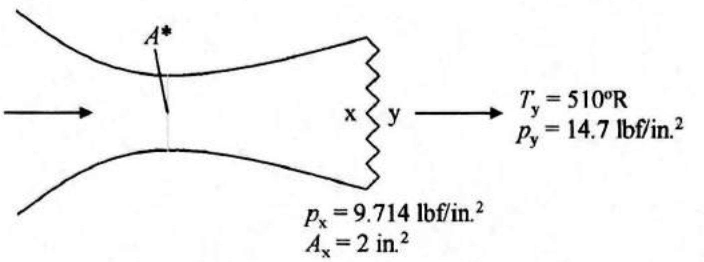


9.138 A converging-diverging nozzle operates at steady state. Air as an ideal gas with $k = 1.4$ flows through the nozzle, discharging to the atmosphere at 14.7 lbf/in.^{2} and 510°R. A normal shock stands at the exit plane with $p_x = 9.714$ lbf/in.^{2} The exit plane area is 2 in.^{2} Upstream of the shock, the flow is isentropic. Determine
(a) the throat area, in in.^{2}
(b) the entropy produced, in Btu/°R per lb of air flowing.

KNOWN: Air flows through a converging-diverging nozzle and discharges to the atmosphere. The flow is isentropic up to the exit plane where a normal shock stands.

FIND: Determine (a) the throat area and (b) the entropy produced.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Air is modeled as an ideal gas with constant $k = 1.4$.
2. Flow up to the shock is isentropic.
3. Flow across the normal shock is adiabatic.

ANALYSIS: (a) To determine the throat area, first determine M_x from the ratio p_y/p_x and Table 9.3

$$\frac{p_y}{p_x} = \frac{14.7\frac{\text{lbf}}{\text{in.}^2}}{9.714\frac{\text{lbf}}{\text{in.}^2}} = 1.51328 \rightarrow M_x = 1.2$$

Use Table 9.2 to determine throat area

$$\frac{A_x}{A^*} = 1.03044 \rightarrow A^* = \frac{2\ \text{in.}^2}{1.3044} = \underline{1.53\ \text{in.}^2}$$

(b) Since the flow upstream of the shock is isentropic, entropy is produced only across the shock. Further, there is no heat transfer associated with flow the shock, so the entropy produced is given by

$$\frac{\dot{\sigma}}{\dot{m}} = s_y - s_x = c_p \ln\left(\frac{T_y}{T_x}\right) - R\ln\left(\frac{p_y}{p_x}\right)$$

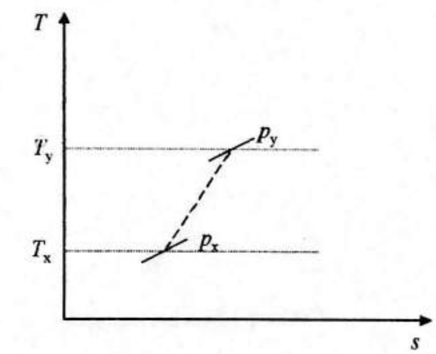
Use Table 9.3 to determine T_x. From Table 9.3, at $M_x = 1.2$

$$\frac{T_y}{T_x} = 1.1280 \rightarrow T_x = \frac{510°\text{R}}{1.1280} = 452.1°\text{R}$$

Substituting values and solving for entropy the rate of entropy production per unit mass of air flowing

$$\frac{\dot{\sigma}}{\dot{m}} = \left(0.240\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)\ln\left(\frac{510°\text{R}}{452.1°\text{R}}\right) - \left(\frac{1.986\dfrac{\text{Btu}}{\text{lbmol}\cdot°\text{R}}}{28.97\dfrac{\text{lb}}{\text{lbmol}}}\right)\ln\left(\frac{14.7\dfrac{\text{lbf}}{\text{in.}^2}}{9.714\dfrac{\text{lbf}}{\text{in.}^2}}\right) = \underline{\mathbf{0.00052\ Btu/(lb\cdot °R)}}$$

T-s diagram:



PROBLEM 9.139

9.139 Air at 3.4 bar, 530 K, and a Mach number of 0.4 enters a converging–diverging nozzle operating at steady state. A normal shock stands in the diverging section at a location where the Mach number is $M_x = 1.8$. The flow is isentropic, except where the shock stands. The air behaves as an ideal gas with $k = 1.4$. Determine

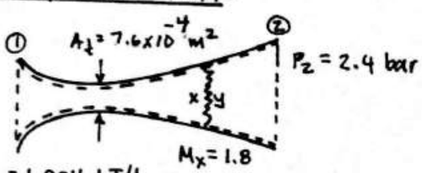
(a) the stagnation temperature T_{ox}, in K.
(b) the stagnation pressure p_{ox}, in bar.
(c) the pressure p_x, in bar.
(d) the pressure p_y, in bar.
(e) the stagnation pressure p_{oy}, in bar.
(f) the stagnation temperature T_{oy}, in K.
(g) If the throat area is 7.6×10^{-4} m^2, and the exit plane pressure is 2.4 bar, determine the mass flow rate, in kg/s, and the exit area, in m^2.

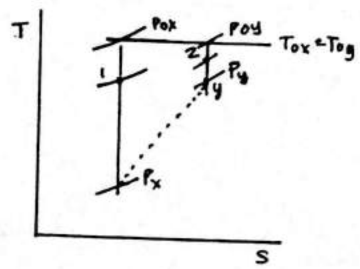
KNOWN: Air flows through a converging-diverging nozzle. A normal shock stands in the diverging section and the flow is isentropic elsewhere.

FIND: Determine T_{ox}, P_{ox}, P_x, P_y, P_{oy} and T_{oy}. For a specified exit plane pressure and throat area, determine the exit area and the mass flow rate.

SCHEMATIC & GIVEN DATA:

$P_1 = 3.4$ bar
$T_1 = 530$ K
$M_1 = 0.4$





$k = 1.4$, $c_p = 1.004$ kJ/kg·K

ENGINEERING MODEL: The air behaves as an ideal gas with constant specific heats ($k = 1.4$).

ANALYSIS:

(a) $M_1 = 0.4 \Rightarrow T_1/T_{ox} = 0.96899 \Rightarrow T_{ox} = 547.0$ K ← (Table 9.2) T_{ox}

(b) $P_1/P_{ox} = 0.89562 \Rightarrow P_{ox} = 3.796$ bar ← (Table 9.2) P_{ox}

(c) at $M_x = 1.8$; $P_x/P_{ox} = 0.17404 \Rightarrow P_x = 0.6607$ bar ← (Table 9.2) P_x

(d) at $M_x = 1.8$; $P_y/P_x = 3.6133 \Rightarrow P_y = 2.387$ bar ← (Table 9.3) P_y

(e) $M_y = 0.61650 \Rightarrow P_y/P_{oy} = 0.77359 \Rightarrow P_{oy} = 3.086$ bar ← (Tables 9.3, 9.2) P_{oy} ①

(f) $T_{oy} = T_{ox} = 547.0$ K ← T_{oy} ②

(g) Since $M = 1$ at the throat and the area is known there, the mass flow rate is $\dot{m} = A_t c^*/v^*$, or with the ideal gas equation of state:

$$\dot{m} = \frac{p^* A_t c^*}{R T^*}$$

From Table 9.2, $p^*/P_{ox} = 0.52828 \Rightarrow p^* = 2.005$ bar

From Table 9.2, $T^*/T_{ox} = 0.83333 \Rightarrow T^* = 455.8$ K

And $c^* = \sqrt{k R T^*}$

$$= \sqrt{(1.4)\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(455.8\text{ K})\left|\frac{10^3\text{ N}\cdot\text{m}}{1\text{ kJ}}\right|\left|\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right|} = 427.9\ \frac{\text{m}}{\text{s}}$$

Accordingly,

$$\dot{m} = \frac{(2.005\text{ bar})(7.6\times10^{-4}\text{ m}^2)(427.9\text{ m/s})}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(455.8\text{ K})}\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| = 0.498\ \frac{\text{kg}}{\text{s}} \quad \dot{m}$$

---

1. Alternatively, use Table 9.3 with $M_x = 1.8$: $P_{oy}/P_{ox} = 0.81268$, which gives $P_{oy} = 3.085$ bar.

2. Apply Eq. 9.47b.

PROBLEM 9.139 (Continued)

The exit area is

$$A_2 = \frac{\dot{m} R T_2}{p_2 V_2}$$

where $p_2 = 2.4$ bar. Since $p_{oy} = 3.086$ bar (part (e)), $p_2/p_{oy} = 0.7777$. Then from Table 9.2, $M_2 = 0.61$ and $T_2/T_{2oy} = 0.93063 \Rightarrow T_2 = 509.1\,K$.

Additionally,

$$V_2 = M_2\sqrt{kRT_2} = (0.61)\sqrt{(1.4)\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(509.1\,K)\left|\frac{10^3 N\cdot m}{1\,kJ}\right|\left|\frac{1\,kg\cdot m/s^2}{1N}\right|}$$
$$= 275.9\,m/s$$

Finally

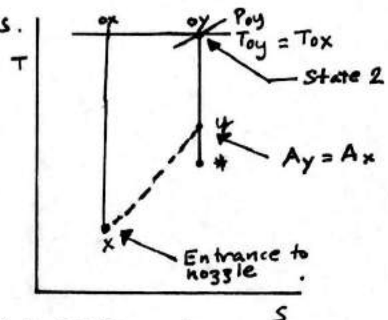
$$A_2 = \frac{(0.498\,kg/s)\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(509.1\,K)}{(2.4\,bar)(275.9\,m/s)}\left|\frac{1\,bar}{10^5\,N/m^2}\right|\left|\frac{10^3 N\cdot m}{1\,kJ}\right|$$
$$= 10.99 \times 10^{-4}\,m^2 \longleftarrow$$

PROBLEM 9.140

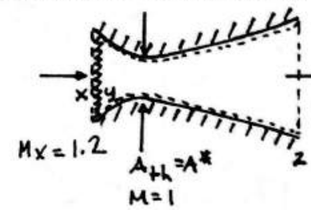
9.140 Air as an ideal gas with $k = 1.4$ enters a converging–diverging channel at a Mach number of 1.2. A normal shock stands at the inlet to the channel. Downstream of the shock the flow is isentropic; the Mach number is unity at the throat; and the air exits at 100 lbf/in.^{2}, 540°R, with negligible velocity. If the mass flow rate is 100 lb/s, determine the inlet and throat areas, in ft^2.

KNOWN: Air enters a converging-diverging channel at a supersonic Mach number. A normal shock stands at the inlet, and downstream of the shock, the flow is isentropic. The mass flow rate, and conditions at the exit are known.

FIND: Determine the inlet and throat areas.



SCHEMATIC & GIVEN DATA:



$p_2 = p_{oy} = 100$ lbf/in^2

$T_2 = T_{oy} = 540$°R

$\dot{m} = 100$ lb/s

$V_2 \approx 0$

ENGINEERING MODEL: (1) The control volume is at steady state. (2) Downstream of the normal shock, the flow is isentropic. (3) Potential energy effects and the exit kinetic energy can be neglected. (4) The air behaves as an ideal gas with k=1.4.

ANALYSIS: The area at the inlet is evaluated using the known mass flow rate and properties at state y:

$$A_y = \frac{\dot{m} R T_y}{p_y V_y} \qquad (A_y = A_x) \qquad (a)$$

Using $M_x = 1.2$, Table 9.3 gives $M_y = 0.84217$. Interpolating with M_y in Table 9.2, we get

$$T_y/T_{oy} = 0.87558 \Rightarrow T_y = 472.8\ °R$$

$$p_y/p_{oy} = 0.62871 \Rightarrow p_y = 62.871\ \text{lbf/in}^2$$

$$A_y/A^* = 1.02584 \qquad (b)$$

To find V_y, use

$$V_y = M_y\sqrt{kRT_y} = (0.84217)\sqrt{(1.4)\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(472.8\text{K})\left(\frac{32.2\ \text{lb}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right)}$$

$$= 897.9\ \text{ft/s}$$

Inserting values into Eq. (a)

$$A_y = \frac{(100\ \text{lb/s})\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(472.8\ °\text{R})}{(62.871 \times 144\ \text{lbf/ft}^2)(897.9\ \text{ft/s})} = 0.3102\ \text{ft}^2 \longleftarrow \quad (A_x = A_y)$$

Then with Eq(b)

$$A^* = \frac{A_y}{1.02584} = \frac{0.3102\ \text{ft}^2}{1.02584} = 0.3024\ \text{ft}^2 \longleftarrow$$

PROBLEMS 9.141 and 1.142

The computer programs to be written in these problems involve straightforward evaluations of equations in the text.

<u>**9.141**</u> Eqs. 9.50, 9.51, and 9.52

<u>**9.142**</u> Eqs. 9.53, 9.54, 9.55, and 9.56

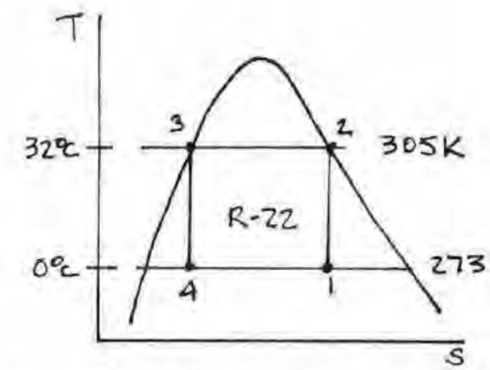
As computers and software vary, the details of the programs are left to the reader.

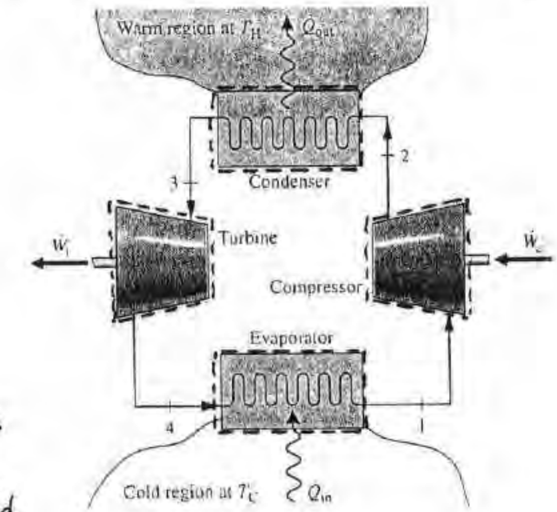
PROBLEM 10.1

KNOWN: R-22 is the working fluid in a Carnot refrigeration cycle. The states at the inlet and exit of the condenser and the evaporator temperature are specified.

FIND: Determine the coefficient of performance. Also, determine per unit mass of refrigerant flowing (a) the compressor work, (b) the turbine work, and (c) the heat transfer to the refrigerant passing through the evaporator.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the working fluid are internally reversible. (3) The compression and expansion are adiabatic. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 2 $T_2 = 32°C$, sat. vapor $\Rightarrow h_2 = 259.32$ kJ/kg, $s_2 = 0.8842$ kJ/kg·K

State 1 $T_1 = 0°C$, $s_1 = s_2 \Rightarrow x_1 = 0.9428$, $h_1 = 238.21$ kJ/kg

State 3 $T_3 = 32°C$, sat. liquid $\Rightarrow h_3 = 84.14$ kJ/kg, $s_3 = 0.3101$ kJ/kg·K

State 4 $T_4 = 0°C$, $s_4 = s_3 \Rightarrow x_4 = 0.1771$, $h_4 = 81.39$ kJ/kg

For the Carnot refrigeration cycle

① $$\beta = \frac{T_C}{T_H - T_C} = \frac{273}{305-273} = 8.53 \quad \beta$$

(a) For the compressor

$$\frac{\dot{W}_c}{\dot{m}} = (h_2 - h_1) = 21.11 \text{ kJ/kg} \quad \dot{W}_c/\dot{m}$$

(b) For the turbine

$$\frac{\dot{W}_t}{\dot{m}} = (h_3 - h_4) = 2.75 \text{ kJ/kg} \quad \dot{W}_t/\dot{m}$$

(c) For the evaporator

$$\frac{\dot{Q}_{in}}{\dot{m}} = (h_1 - h_4) = 156.82 \text{ kJ/kg} \quad \dot{Q}_{in}/\dot{m}$$

---

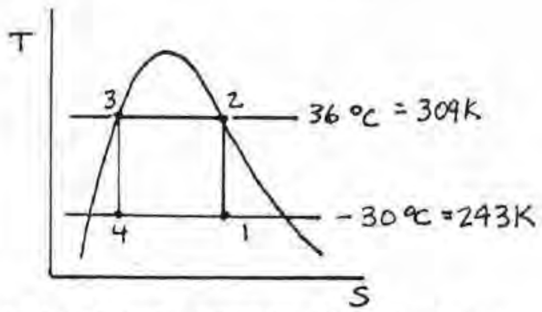
1. Alternatively
$$\beta = \frac{\dot{Q}_{in}/\dot{m}}{\dot{W}_c/\dot{m} - \dot{W}_t/\dot{m}} = \frac{156.82}{21.11 - 2.75} = 8.54$$
The difference is due to roundoff.

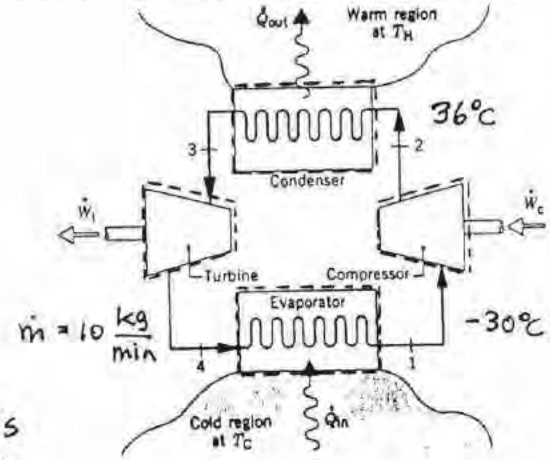
PROBLEM 10.2

KNOWN: Refrigerant 22 is the working fluid in a Carnot vapor refrigration cycle. Data are known at various locations and the mass flow rate is specified.

FIND: Determine (a) the rate of heat transfer to the R-22 passing through the evaporator, (b) the net power input, (c) the coefficient of performance, and (d) the refrigerating capacity.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the R-22 are internally reversible. (3) The compression and expansion are adiabatic. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states (Table A-7).

State 2 $T_2 = 36°C$, sat. vapor $\Rightarrow h_2 = 260.11$ kJ/kg, $s_2 = 0.8790$ kJ/kg·K

State 1 $T_1 = -30°C$, $s_2 = s_1 \Rightarrow x_1 = 0.8931$; $h_1 = 213.53$ kJ/kg

State 3 $T_3 = 36°C$, sat. liquid $\Rightarrow h_3 = 89.29$ kJ/kg, $s_3 = 0.3265$ kJ/kg·K

State 4 $T_4 = -30°C$, $s_4 = s_3 \Rightarrow x_4 = 0.3007$; $h_4 = 79.19$ kJ/kg

(a) For the evaporator

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = \left(10\ \frac{kg}{min}\right)(213.53 - 79.19)\frac{kJ}{kg}\left|\frac{1\ min}{60\ s}\right|\left|\frac{1\ kW}{1\ kJ/s}\right| = 22.39\ kW \longleftarrow \quad \dot{Q}_{in}$$

(b) The net power input is

$$\dot{W}_{cycle} = \dot{W}_c - \dot{W}_t = \dot{m}\,[(h_2 - h_1) - (h_3 - h_4)]$$
$$= (10)[(260.11 - 213.53) - (89.29 - 79.19)]\left|\frac{1}{60}\right| = 6.08\ kW \longleftarrow \quad \dot{W}_{cycle}$$

(c) The coefficient of performance is

① $$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{cycle}} = \frac{22.39}{6.08} = 3.68 \longleftarrow \quad \beta$$

(d) The refrigerating capacity is

$$\text{refrig. capacity} = \dot{Q}_{in} = 22.39\ kW\left|\frac{1\ kJ/s}{1\ kW}\right|\left|\frac{60\ s}{1\ min}\right|\left|\frac{1\ ton}{211\ kJ/min}\right|$$
$$= 6.37\ \text{tons} \longleftarrow \quad \text{refrigerating capacity}$$

1. Alternatively $\beta = T_C/(T_H - T_C) = 243/(309 - 243) = 3.68$

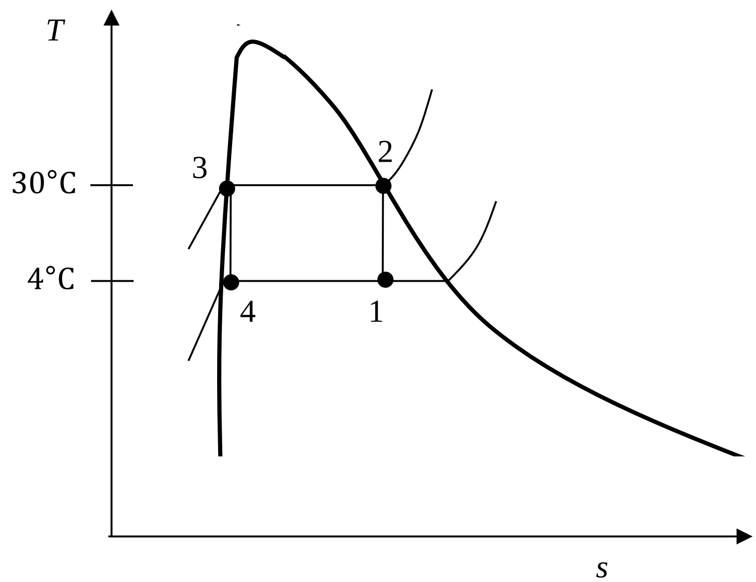
Problem 10.3

A Carnot vapor refrigeration cycle operates between thermal reservoirs at 4°C and 30°C. The working fluid is saturated vapor at the end of the compression process and saturated liquid at the beginning of the expansion process. For (a) Refrigerant 134a, (b) propane, (c) water, (d) ammonia, (e) CO_2 (using Fig. A-10), and (f) Refrigerant 410A (using Fig. A-11) as the working fluid, determine the operating pressures in the condenser and evaporator, in bar, and the coefficient of performance.

Known:
A Carnot vapor refrigeration cycle operates between known reservoir temperatures.

Find:
Determine operating pressures in the condenser and evaporator for (a) Refrigerant 134a, (b) propane, (c) water, (d) ammonia, (e) CO_2, and (f) Refrigerant 410A as the working fluid. Calculate the coefficient of performance.

Schematic and Known Data:



Engineering Model:
(1) Each component operates at steady state.
(2) All processes are internally reversible.
(3) The condenser and evaporator operate at the respective reservoir temperatures.
(4) The compression and expansion are adiabatic.

Analysis:
Determine the operating pressures in the condenser and evaporator at the respective temperatures.

(a) Pressure values for Refrigerant 134a from Table A-10

#1 $p_{\text{cond}} = p_{\text{sat}}@30°\text{C} = 7.7006\ \text{bar}$

$p_{\text{evap}} = p_{\text{sat}}@4°\text{C} = 3.3765\ \text{bar}$

(b) Pressure values for propane from Table A-16

$p_{\text{cond}} = p_{\text{sat}}@30°\text{C} = 10.8\ \text{bar}$

$p_{\text{evap}} = p_{\text{sat}}@4°\text{C} = 5.349\ \text{bar}$

(c) Pressure values for water from Table A-2

#2 $p_{\text{cond}} = p_{\text{sat}}@30°\text{C} = 0.04246\ \text{bar}$

$p_{\text{evap}} = p_{\text{sat}}@4°\text{C} = 0.00813\ \text{bar}$

(d) Pressure values for ammonia from Tab. A-13

$p_{\text{cond}} = p_{\text{sat}}@30°\text{C} = 11.6865\ \text{bar}$

$p_{\text{evap}} = p_{\text{sat}}@4°\text{C} = 4.9773\ \text{bar}$

(e) Pressure values for CO_2 from Fig. A-10

$p_{\text{cond}} = p_{\text{sat}}@30°\text{C} \approx 7.2\ \text{MPa} = 72\ \text{bar}$

$p_{\text{evap}} = p_{\text{sat}}@4°\text{C} \approx 3.9\ \text{MPa} = 3.9\ \text{bar}$

(f) Pressure values for Refrigerant 410A from Fig. A-11

$p_{\text{cond}} = p_{\text{sat}}@30°\text{C} \approx 1.95\ \text{MPa} = 19.5\ \text{bar}$

$p_{\text{evap}} = p_{\text{sat}}@4°\text{C} \approx 0.9\ \text{MPa} = 9\ \text{bar}$

The coefficient of performance is:

$$\beta_{\max} = \frac{T_C}{T_H - T_C} = \frac{277}{303-277} = 10.65$$

<u>Comments:</u>

1. Refrigerant 134a has lower operating pressures than propane, Refrigerant 410A, CO_2, and ammonia in this cycle.
2. Water operates at very low pressures and cannot be used to achieve temperatures lower than 0° C.

10.4 Consider a Carnot vapor refrigeration cycle with Refrigerant 134a as the working fluid. The cycle maintains a cold region at 40°F when the ambient temperature is 90°F. Data at principal states in the cycle are given in the table below. The states are numbered as in Fig. 10.1. Sketch the *T-s* diagram for the cycle and determine the

(a) temperatures in the evaporator and condenser, each in °R.

(b) compressor and turbine work, each in Btu per lb of refrigerant flowing.

(c) coefficient of performance.

(d) coefficient of performance for a Carnot cycle operating at the reservoir temperatures. Compare the coefficients of performance determined in (c) and (d) and comment.

State	p (lbf/in.^{2})	h (Btu/lb)	s (Btu/lb°R)
1	40	104.12	0.2161
2	140	114.95	0.2161
3	140	44.43	0.0902
4	40	42.57	0.0902

Fig. P10.4

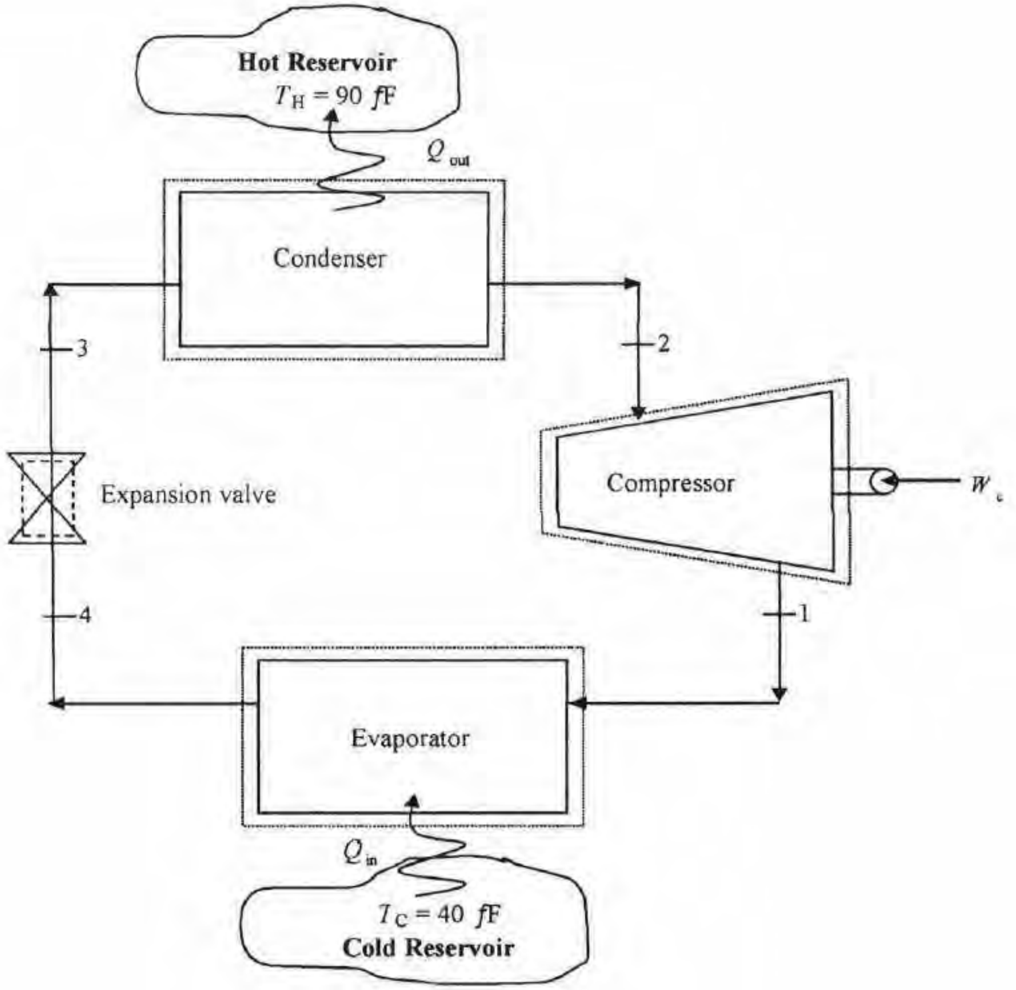
KNOWN: A Carnot vapor refrigeration cycle is used to maintain a cold region at a specified temperature for a specified ambient temperature. Operating data are provided at principal states of the cycle.

FIND: Determine the (a) the temperatures in the evaporator and condenser, (b) compressor and turbine work per unit mass of refrigerant flowing, (c) coefficient of performance, and (d) coefficient of performance for a Carnot cycle operating at the reservoir temperatures. Compare the results of (c) and (d) and comment.

SCHEMATIC AND GIVEN DATA:

State	p (lbf/in.^{2})	h (Btu/lb)	s (Btu/lb°R)
1	40	104.12	0.2161
2	140	114.95	0.2161
3	140	44.43	0.0902
4	40	42.57	0.0902

Problem 10.4 (Continued) – Page 2



ENGINEERING MODEL:

(1) Each component is analyzed as a control volume at steady state.
(2) The compression and expansion processes are adiabatic.
(3) Kinetic and potential energy effects are negligible.

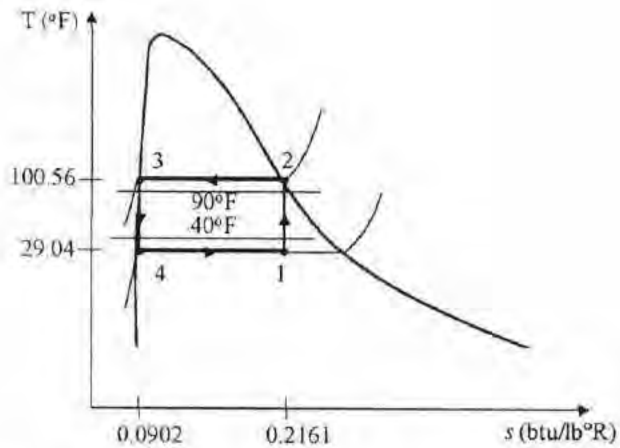
ANALYSIS:

(a) Using data from Table A-11E

For the evaporator, $p = 40$ lbf/in.^{2}, and $T_{evap} = T_{sat} = 29.04°F = 488.7°R$

For the condenser, $p = 140$ lbf.in.^{2}, and $T_{cond} = T_{sat} = 100.56°F = 560.2°R$

The T-s diagram for the cycle is



(b) From mass and energy rate balances for the compressor and turbine, respectively

$$\frac{\dot{W}_c}{\dot{m}} = (h_2 - h_1) \text{ and } \frac{\dot{W}_t}{\dot{m}} = (h_3 - h_4)$$

Inserting values

$$\frac{\dot{W}_c}{\dot{m}} = (114.95 - 104.12)\frac{\text{Btu}}{\text{lb}} = 10.83\frac{\text{Btu}}{\text{lb}}$$

$$\frac{\dot{W}_t}{\dot{m}} = (44.43 - 42.57)\frac{\text{Btu}}{\text{lb}} = 1.86\frac{\text{Btu}}{\text{lb}}$$

(c) The coefficient of performance (β) is

$$\beta = \frac{\frac{\dot{Q}_{in}}{\dot{m}}}{\frac{\dot{W}_c}{\dot{m}} - \frac{\dot{W}_t}{\dot{m}}} = \frac{(h_1 - h_4)}{(h_2 - h_1) - (h_3 - h_4)} = \frac{(104.12 - 42.57)\frac{\text{Btu}}{\text{lb}}}{(10.83 - 1.86)\frac{\text{Btu}}{\text{lb}}} = 6.86$$

(d) For a Carnot cycle operating at $T_c = 40$ °F = 500.67 °R and $T_H = 90$ °F = 550.67 °R

Problem 10.4 (Continued) – Page 4

$$\beta_{\text{Carnot}} = \frac{T_C}{T_H - T_C} = \frac{500.67}{550.67 - 500.67} = \mathbf{10.0}$$

The coefficient of performance for the Carnot cycle operating at the reservoir temperatures is higher than that of the original cycle. Referring to the T-s diagram, we see that the evaporator temperature is lower than T_C and the condenser temperature is higher than T_H. By operating the evaporator and condenser at these temperatures, there would be irreversibility for the heat transfer in the respective heat exchangers.

10.5 For the cycle in Problem 10.4, determine
(a) the rates of heat transfer , in Btu per unit mass of refrigerant flowing, for the evaporator and condenser, respectively.
(b) the rates and directions of exergy transfer accompanying each of these heat transfers, in Btu per lb of refrigerant flowing. Let $T_0 = 90°F$.

KNOWN: A Carnot vapor refrigeration cycle is used to maintain a cold region at a specified temperature for a specified ambient temperature. Operating data are provided at principal states of the cycle.

FIND: Determine (a) the rates of heat transfer for the refrigerant flowing through the evaporator and condenser, and (b) the rates and directions of exergy transfer accompanying each of these heat transfers, each per unit mass of refrigerant flowing.

SCHEMATIC AND GIVEN DATA:

State	p (lbf/in.^{2})	h (Btu/lb)	s (Btu/lb°R)
1	40	104.12	0.2161
2	140	114.95	0.2161
3	140	44.43	0.0902
4	40	42.57	0.0902

See Problem 10.4 for schematic.

$T_0 = 90°F = 550°R$

ENGINEERING MODEL:
(1) Each component is analyzed as a control volume at steady state.
(2) The compression and expansion processes are adiabatic.
(3) Kinetic and potential energy effects are negligible.

ANALYSIS:

(a) For the evaporator

$$\frac{\dot{Q}_{in}}{\dot{m}} = (h_1 - h_4) = (104.12 - 42.57) = 61.55 \text{ Btu/lb}$$

And, for the condenser

$$\frac{\dot{Q}_{cond}}{\dot{m}} = (h_3 - h_2) = 44.43 - 114.95 = -70.52 \text{ Btu/lb}$$

(b) To determine the exergy transfer accompanying heat transfer for the evaporator, we apply the exergy rate balance to the control volume enclosing the evaporator to get

$$0 = \dot{E}_q - \cancel{\dot{W}_{cv}} + \dot{m}(e_{f4} - e_{f1}) - \cancel{\dot{E}_d}$$

Problem 10.5 (Continued) – Page 2

With $e_f = h - T_0 s$

$$\left(\frac{\dot{E}_q}{\dot{m}}\right)_{\text{evap}} = (h_1 - h_4) - T_0(s_1 - s_4) = (104.12 - 42.57) - (550)(0.2161 - 0.0902)$$

$= -7.695$ Btu/lb (*from* the system *to* the surroundings)

Similarly, for the condenser

$$\left(\frac{\dot{E}_q}{\dot{m}}\right)_{\text{cond}} = (h_3 - h_2) - T_0(s_3 - s_2) = (44.43 - 114.95) - (550)(0.0902 - 0.2161)$$

$= -1.275$ Btu/lb (*from* the system *to* the surroundings)

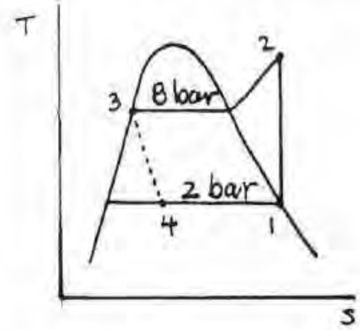
*Note that for the evaporator, the direction of heat transfer is **into** the working fluid (energy is removed from the cold reservoir). Yet, the exergy transfer accompanying the heat transfer is in the **opposite** direction (exergy is removed from the working fluid as the state of the working fluid moves closer to the dead state.gy transfer by heat and the exergy transfer accompanying heat transfer are both **from** the working fluid **to** the surroundings.*

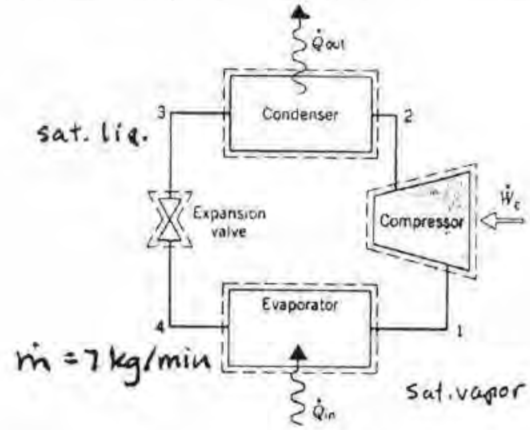
PROBLEM 10.6

KNOWN: Refrigerant 134a is the working fluid in an ideal vapor-compression refrigaration cycle. Operating data are known.

FIND: Determine (a) the compressor power, (b) the refrigerating capacity, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The expansion through the valve is a throttling process. All other processes are internally reversible. (3) The compressor and valve operate adiabatically. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 1 $p_1 = 2$ bar, sat. vapor $\Rightarrow h_1 = 241.30$ kJ/kg, $s_1 = 0.9253 \frac{kJ}{kg \cdot K}$

State 2 $p_2 = 8$ bar, $s_2 = s_1 \Rightarrow h_2 = 269.92$ kJ/kg

State 3 $p_3 = 8$ bar, sat. liquid $\Rightarrow h_3 = 93.42$ kJ/kg

State 4 Throttling process $\Rightarrow h_4 = h_3 = 93.42$ kJ/kg

(a) The compressor power is

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(7 \frac{kg}{min}\right)(269.92 - 241.30)\frac{kJ}{kg}\left|\frac{1\ min}{60\ s}\right|\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 3.34\ kW$$ $\dot{W}_c$

(b) The refrigerating capacity is

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = \left(7\frac{kg}{min}\right)(241.30 - 93.42)\frac{kJ}{kg}\left|\frac{1\ ton}{211\ kJ/min}\right|$$

$$= 4.91\ tons$$ $\dot{Q}_{in}$

(c) The coefficient of performance is

$$\beta = \frac{(h_1 - h_4)}{(h_2 - h_1)} = \frac{(241.30 - 93.42)}{(269.92 - 241.30)} = 5.17$$ β

10.7 Plot each of the quantities in Problem 10.6 versus evaporator temperature for evaporator pressures ranging from 0.6 to 4 bar, while the condenser pressure remains fixed at 8 bar.

IT Code

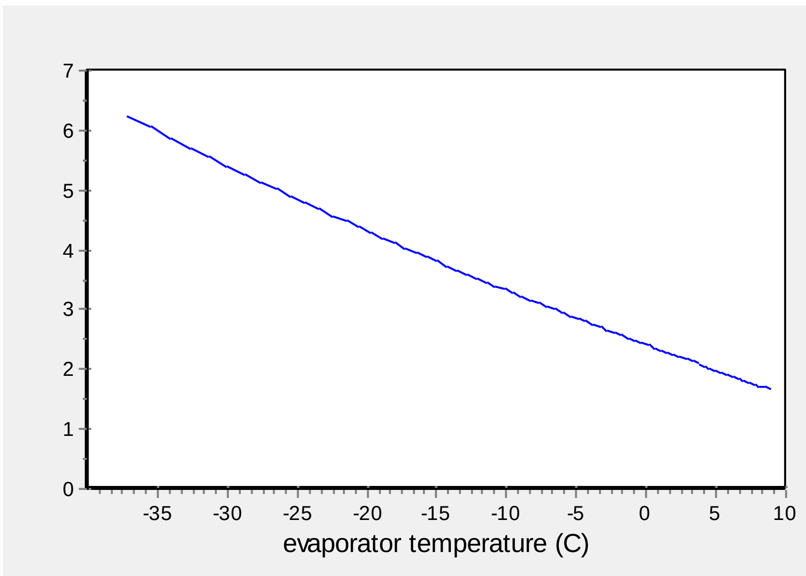
```
p1 = 2  // bar
p2 = 8  // bar
p3 = p2
mdot = 7  // kg/min

h1 = hsat_Px("R134A", p1, 1)
s1 = ssat_Px("R134A", p1, 1)
T1 = Tsat_P("R134A", p1)
s2 = s1
h2 = h_Ps("R134A", p2, s2)
h3 = hsat_Px("R134A", p3, 0)
h4 = h3

Wdotc = mdot * (h2 - h1) / (60)  // kW
Qdotin = mdot * (h1 - h4) / (211)  // tons
beta = (h1 - h4) / (h2 - h1)
```

IT Results for p_1 = 2 bar

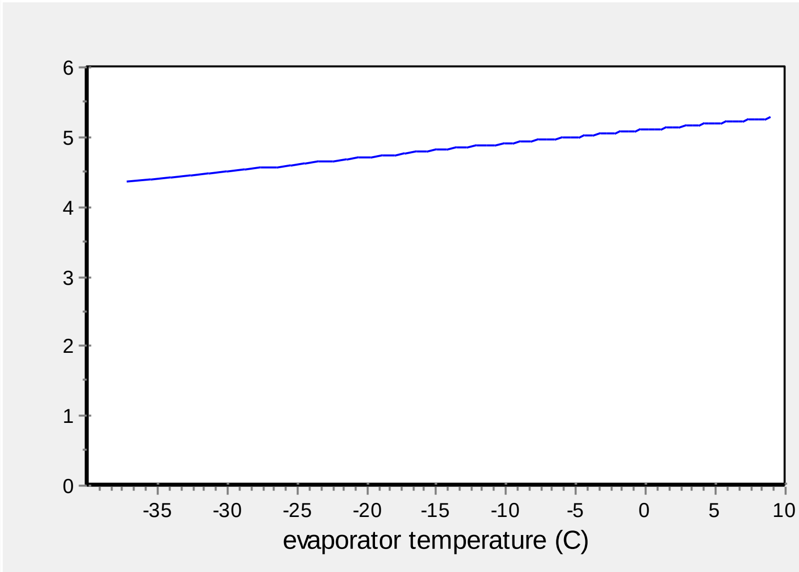
h_1 = 241.3 kJ/kg
h_2 = 269.9 kJ/kg
h_3 = 93.42 kJ/kg
h_4 = 93.42 kJ/kg

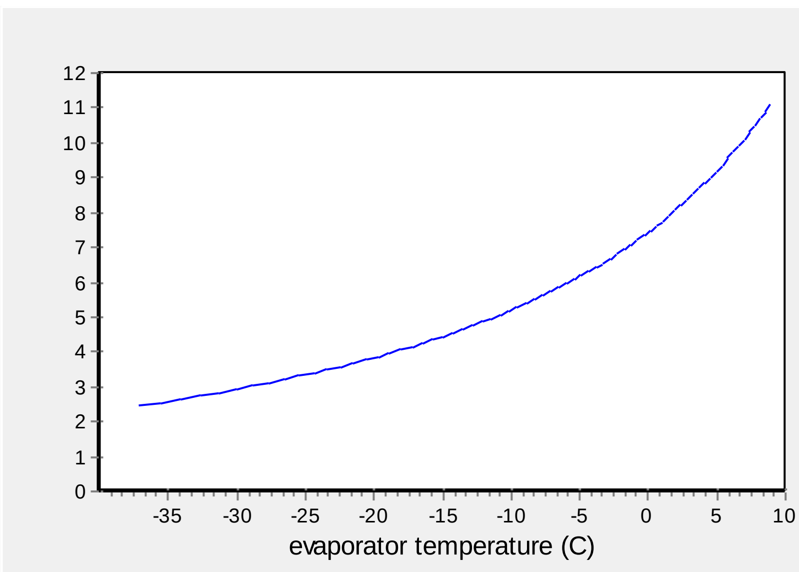
T1 = -10.09 °C
$\dot{W}_{cycle}$ = 3.337 kW
$\dot{Q}_{in}$ = 4.906 tons
β = 5.169

Plots:



Problem 10.7 (Continued) – Page 2



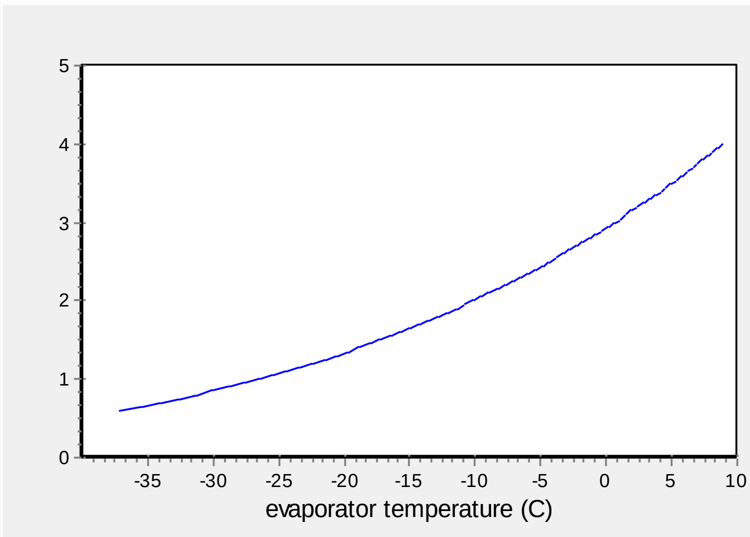


<u>Discussion</u>

Evaporator temperature increases as evaporator pressure increases. For reference, the relation between evaporator temperature and pressure is shown below:

As the evaporator pressure increases at fixed condenser pressure, significantly less work is required for compression. Further, the refrigerating capacity increases slightly. As a result, the coefficient of performance increases significantly.

Problem 10.7 (Continued) – Page 3

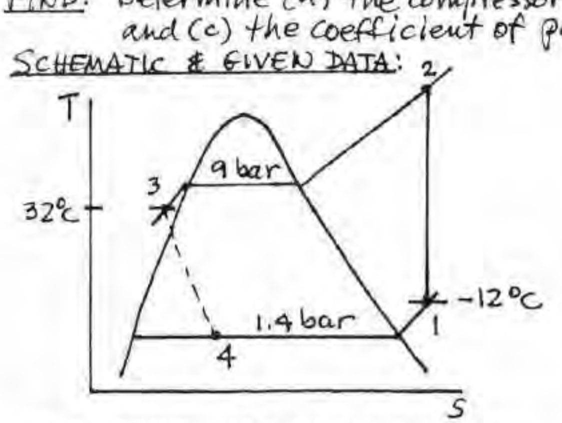


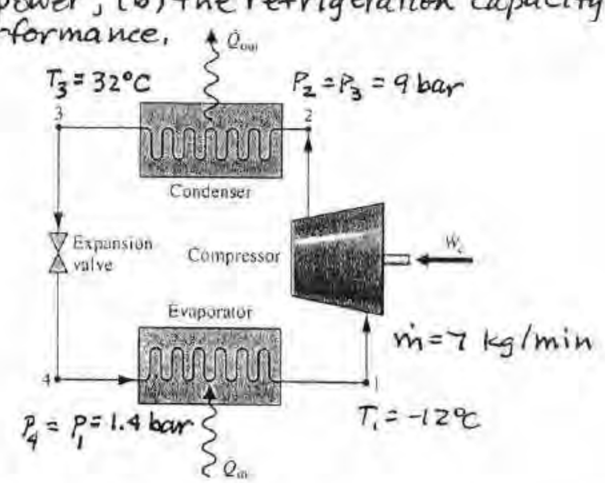
PROBLEM 10.8

KNOWN: R-134a is the working fluid in an ideal vapor compression refrigeration cycle. Operating data are known, and the refrigerant mass flow rate is given.

FIND: Determine (a) the compressor power, (b) the refrigeration capacity, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 10.1, items 1–4.

ANALYSIS: First, fix each of the principal states.

State 1 $P_1 = 1.4$ bar, $T_1 = -12°C \Rightarrow h_1 = 241.73$ kJ/kg, $s_1 = 0.95415$ kJ/kg·K

State 2 $P_2 = 9$ bar, $s_2 = s_1 \Rightarrow h_2 = 281.56$ kJ/kg

State 3 $P_3 = 9$ bar, $T_3 = 32°C \Rightarrow$ sub-cooled liquid. Thus, $h_3 \approx h_{f@T_3} = 94.39 \frac{kJ}{kg}$

State 4 Throttling process $\Rightarrow h_4 = h_3 = 94.39$ kJ/kg

(a) For the compressor

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(7\,\frac{kg}{min}\right)(281.56 - 241.73)\frac{kJ}{kg}\left|\frac{1\ min}{60\ s}\right|\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 4.65\ kW \longleftarrow \dot{W}_c$$

(b) For the evaporator

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = \left(7\,\frac{kg}{min}\right)(241.73 - 94.39)\frac{kJ}{kg}\left|\frac{1\ ton}{211\ kJ/min}\right|$$

$$= 4.89\ tons \longleftarrow \dot{Q}_{in}$$

(c) The coefficient of performance is

$$\beta = \frac{\dot{Q}_{in}/\dot{m}}{\dot{W}_c/\dot{m}} = \frac{(h_1 - h_4)}{(h_2 - h_1)} = \frac{(241.73 - 94.39)}{(281.56 - 241.73)} = 3.70 \longleftarrow \beta$$

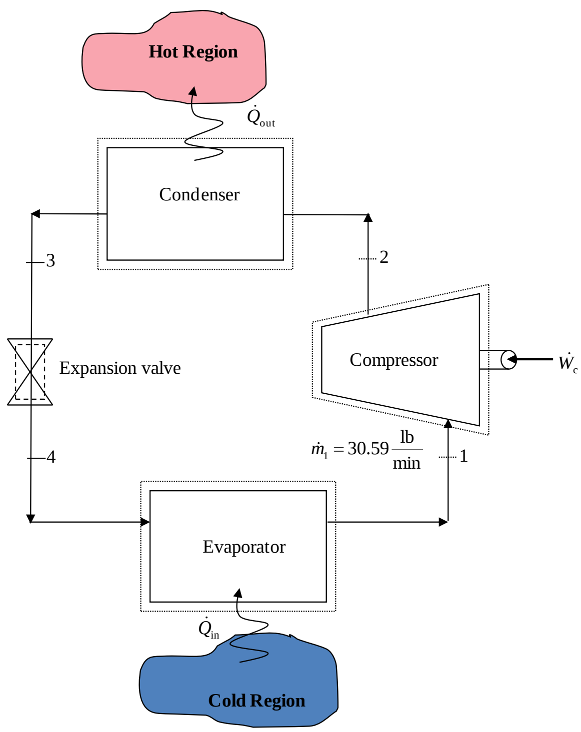
Problem 10.9

Figure P10.9 provides steady-state operating data for an ideal vapor-compression refrigeration cycle with Refrigerant 134a as the working fluid. The mass flow rate of refrigerant is 30.59 lb/min. Sketch the *T*-*s* diagram of the cycle and determine

(a) the compressor power, in horsepower.

(b) the rate of heat transfer, from the working fluid passing through the condenser, in Btu/min.

(c) the coefficient of performance.



State	*p* (lbf/in.^{2})	*T* (°F)	*h* (Btu/lb)	*s* (Btu/lb·°R)
1	10	0	102.94	0.2391
2	180	---	131.04	0.2391
3	180	Sat.	50.64	0.1009
4	10	Sat.	50.64	---

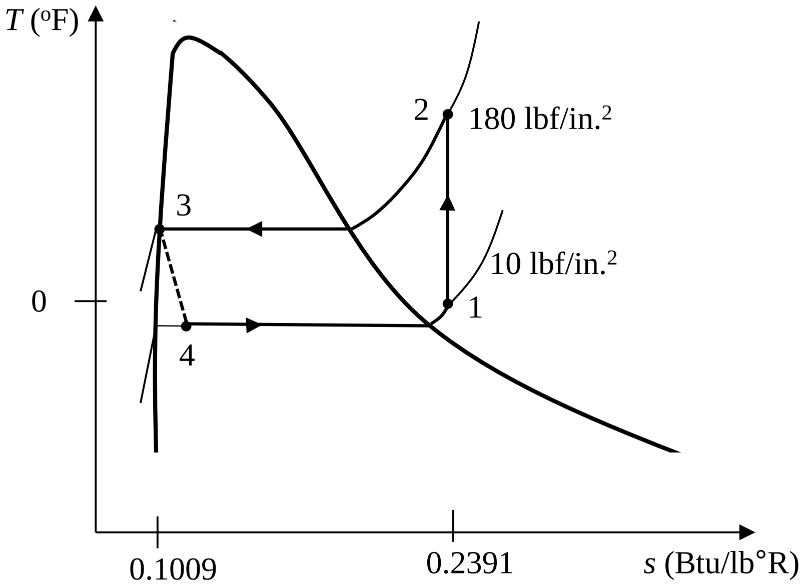
Fig. P10.9

Solution:

Known:
An ideal vapor-compression refrigeration cycle operates with Refrigerant 134a as its working fluid. The refrigerant mass flow rate and operating data at principal states of the cycle are provided.

Find:
Sketch the *T*-*s* diagram for the cycle and determine (a) the compressor power, (b) the rate of heat transfer from the working fluid passing through the condenser, and (c) the coefficient of performance.

Schematic and Known Data:
Refer to Fig. P10.9 and the following



<u>Engineering Model:</u>

(1) Each component is analyzed as a control volume at steady state. The control volumes are indicated by dashed lines on the accompanying figure.
(2) The compression and expansion processes are adiabatic.
(3) Kinetic and potential energy effects are negligible.

<u>Analysis:</u>

(a) The compressor power is $\dot{W}_c$

$$\dot{W}_c = \dot{m}(h_2 - h_1) = 30.59\,\frac{\text{lb}}{\text{min}}(131.04 - 102.94)\,\frac{\text{Btu}}{\text{lb}}\left|\frac{1\ \text{hp}}{2545\ \text{Btu/h}}\right|\left|\frac{60\ \text{min}}{1\ \text{h}}\right| = 20.27\ \text{hp} \longleftarrow$$

(b) The rate of heat transfer from the working fluid passing through the condenser is

$$\dot{Q}_{\text{out}} = \dot{m}(h_2 - h_3) = 30.59\,\frac{\text{lb}}{\text{min}}(131.04 - 50.64)\,\frac{\text{Btu}}{\text{lb}} = 2459.44\,\frac{\text{Btu}}{\text{min}} \longleftarrow$$

(c) The coefficient of performance is

$$\beta = \frac{\dot{Q}_{\text{in}}}{\dot{W}_c} = \frac{\dot{m}(h_1 - h_4)}{\dot{m}(h_2 - h_1)} = \frac{(102.94 - 50.64)\,\frac{\text{Btu}}{\text{lb}}}{(131.04 - 102.94)\,\frac{\text{Btu}}{\text{lb}}} = 1.86 \longleftarrow$$

Problem 10.10

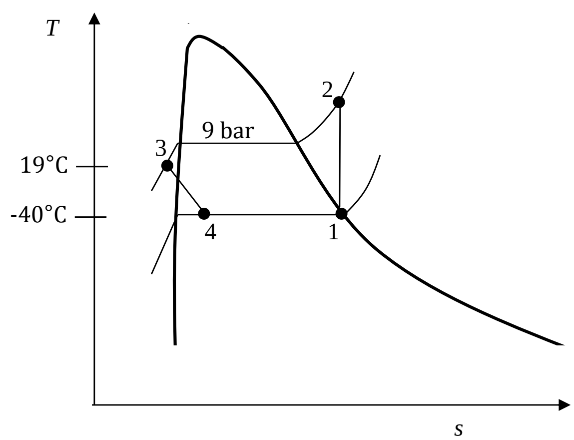
Refrigerant 22 enters the compressor of an ideal vapor compression refrigeration system as saturated vapor at -40°C with a volumetric flow rate of 15 m^3/min. The refrigeration leaves the condenser at 19°C, 9 bar. Determine

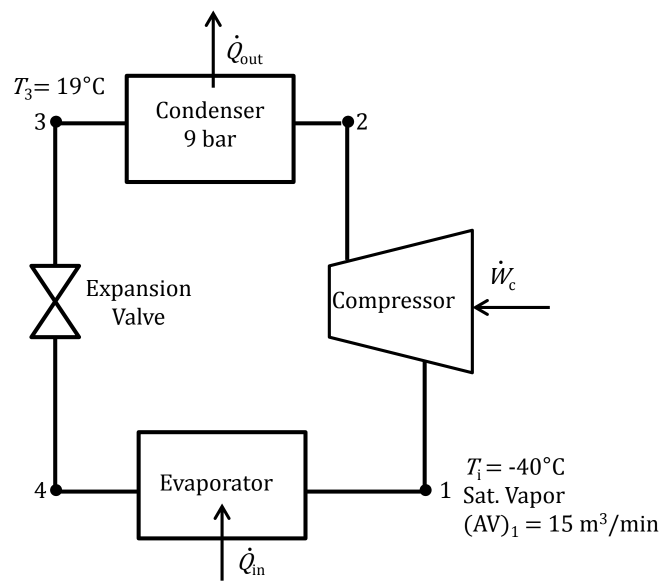
(a) the compressor power, in kW.
(b) the refrigerating capacity, in tons.
(c) the coefficient of performance.
(d) the rate of entropy production for the cycle, in kW/K.

<u>Known:</u>
Refrigerant 22 is the working fluid in an ideal vapor-compression refrigeration cycle. Operating data are known.

<u>Find:</u>
Determine (a) the compressor power, (b) the refrigerating capacity, (c) the coefficient of performance, and (d) the rate of entropy production.

<u>Schematic and Known Data:</u>





<u>Engineering Model:</u>
(1) Each component of the cycle is analyzed as a control volume at steady state.
(2) Except for the expansion through the valve, which is a throttling process, all processes of the refrigerant are internally reversible.
(3) Heat transfer between the refrigerant and each region occur with no temperature differences, there are no external irreversibilities.
(4) The compressor and expansion valve operate adiabatically.
(5) There are no pressure drops through the evaporator and the condenser.
(6) Kinetic and potential energy effects are negligible.

<u>Analysis:</u>

First, fix each of the principal states.

State 1: $T_1 = -40°\text{C}$, saturated vapor $\rightarrow h_1 = 233.27 \frac{\text{kJ}}{\text{kg}}, \quad s_1 = 1.0005 \frac{\text{kJ}}{\text{kg·K}}$

State 2: $p_2 = 9$ bar, $s_2 = s_1$ using Table A – 9 with interpolation $\rightarrow h_2 = 287.55 \frac{\text{kJ}}{\text{kg}}$

State 3: In compressed liquid region, using Table A – 7

$h_3 \approx h_f(19°C) = 67.87 \frac{\text{kJ}}{\text{kg}}, s_3 = 0.2566 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$

State 4: Throttling process $\rightarrow h_4 = h_3 = 67.87 \frac{\text{kJ}}{\text{kg}}$, using $T_4 = -40°\text{C}$ and Table A – 7 $\rightarrow$

$x_4 = 0.7091$ and $s_4 = 0.7095 \frac{\text{kJ}}{\text{kg·K}}$

(a) From Table A-7, $v_1 = 0.2052 \frac{\text{m}^3}{\text{kg}}$. the mass flow rate is:

$$\dot{m} = \frac{(\text{AV})_1}{v_1} = \frac{15 \frac{\text{m}^3}{\text{min}}}{0.2052 \frac{\text{m}^3}{\text{kg}}} = 73.10 \frac{\text{kg}}{\text{min}}$$

The compressor power is:

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(73.10 \frac{\text{kg}}{\text{min}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)(287.55 - 233.27)\left(\frac{\text{kJ}}{\text{kg}} \cdot \frac{\text{kW}}{\frac{\text{kJ}}{\text{s}}}\right) = 66.13 \text{ kW} \longleftarrow$$

(b) The refrigerating capacity is:

$$\dot{Q}_{\text{in}} = \dot{m}(h_1 - h_4) = \left(73.10 \frac{\text{kg}}{\text{min}}\right)(233.27 - 67.87)\left(\frac{\text{kJ}}{\text{kg}} \cdot \frac{1 \text{ ton}}{211 \frac{\text{kJ}}{\text{min}}}\right) = 57.3 \text{ tons} \longleftarrow$$

(c) The coefficient of performance is:

$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = \frac{233.27 - 67.87}{287.55 - 233.27} = 3.05 \longleftarrow$$

(d) To find the entropy production, it is first necessary to determine the rate of heat transfer to the surroundings.

$$\dot{Q}_{\text{out}} = \dot{m}(h_3 - h_2) = \left(73.10 \frac{\text{kg}}{\text{min}}\right)(67.87 - 287.55)\left(\frac{\text{kJ}}{\text{kg}}\right)\frac{1 \text{ min}}{60 \text{ s}} = -267.64 \text{ kW}$$

Recall from above $\dot{Q}_{\text{in}} = 57.3 \text{ tons} \left(\frac{211 \frac{\text{kJ}}{\text{min}}}{1 \text{ ton}} \cdot \frac{1 \text{ min}}{60 \text{ s}}\right) = 201.51 \text{ kW}$

With assumptions 1,3, and 4, the entropy production is:

$$\underbrace{\frac{\text{d}S}{dT}}_{=0} = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_{\text{e}} \dot{m}_{\text{e}} s_{\text{e}} + \dot{\sigma}_{cv} \rightarrow \dot{\sigma}_{cv} = \sum_e \dot{m}_{\text{e}} s_{\text{e}} - \sum_i \dot{m}_i s_i - \sum_j \frac{\dot{Q}_j}{T_j}$$

Taking into account each component separately:

$$\dot{\sigma}_{cv} = \dot{m}\left[(s_2 - s_1) + \left((s_3 - s_2) - \frac{Q_{\text{out}}}{T_3}\right) + (s_4 - s_3) + \left((s_1 - s_4) - \frac{Q_{\text{in}}}{T_1}\right)\right]$$

$$= \ -\frac{\dot{Q}_{\text{out}}}{T_3} - \frac{\dot{Q}_{\text{in}}}{T_1} = -\frac{(-267.64\text{ kW})}{292\text{ K}} - \frac{201.51\text{ kW}}{233\text{ K}} = 0.052\,\frac{\text{kW}}{\text{K}} \qquad \longleftarrow$$

Problem 10.11

Ammonia with a mass flow rate of 5 kg/min is the working fluid within an ideal vapor compression refrigeration cycle. Saturated vapor enters the compressor and saturated liquid exits the condenser. The evaporator temperature is -10°C and the condenser pressure is 10 bar. Determine

(a) the coefficient of performance.

(b) the refrigerating capacity, in tons.

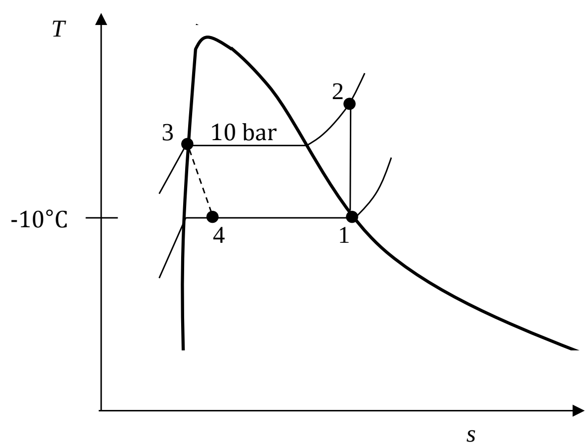
Known:

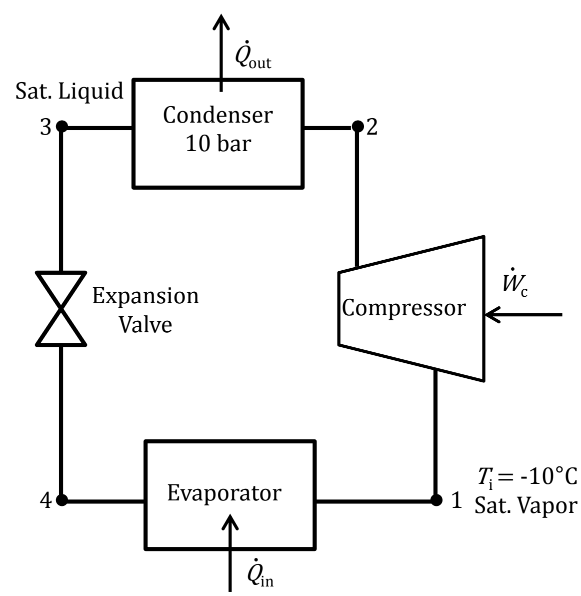
An ideal vapor-compression refrigeration cycle uses ammonia as the working fluid. Operating data are known.

Find:

Determine (a) the coefficient of performance, and (b) the refrigerating capacity.

Schematic and Known Data:





Engineering Model

(1) Each component of the cycle is analyzed as a control volume at steady state.

(2) Except for the expansion through the valve, which is a throttling process, all processes of the refrigerant are internally reversible.

(3) The compressor and expansion valve operate adiabatically.

(4) There are no pressure drops through the evaporator and the condenser.

(5) Kinetic and potential energy effects are negligible.

(6) Saturated vapor enters the compressor and saturated liquid exits the condenser.

Analysis:

First, fix each of the principal states.

State 1: $T_1 = -10°\text{C}$, saturated vapor $\rightarrow h_1 = 1430.55\,\frac{\text{kJ}}{\text{kg}}$, $s_1 = 5.4662\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

State 2: $p_2 = 10$ bar, $s_2 = s_1$ using Table A − 15 with interpolation $\rightarrow h_2 = 1604.09\,\frac{\text{kJ}}{\text{kg}}$

State 3: $p_2 = p_3 = 10$ bar, saturated liquid, using Table A − 14 $\rightarrow h_3 = 297.76\,\frac{\text{kJ}}{\text{kg}}$

State 4: Throttling process $\rightarrow h_4 = h_3 = 297.76\,\frac{\text{kJ}}{\text{kg}}$

(a) The coefficient of performance is:

$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = \frac{1430.55 - 297.76}{1604.09 - 1430.55} = 6.528$$

(b) The refrigerating capacity is:

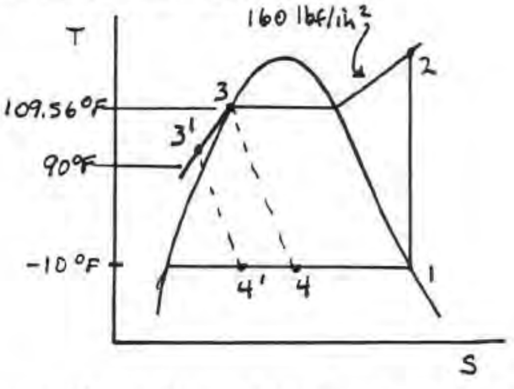
$$\dot{Q}_{\text{in}} = \dot{m}(h_1 - h_4) = \left(5\,\frac{\text{kg}}{\text{min}}\right)(1430.55 - 297.76)\left(\frac{\text{kJ}}{\text{kg}} \cdot \frac{1\text{ ton}}{211\,\frac{\text{kJ}}{\text{min}}}\right) = 26.84\text{ tons}$$

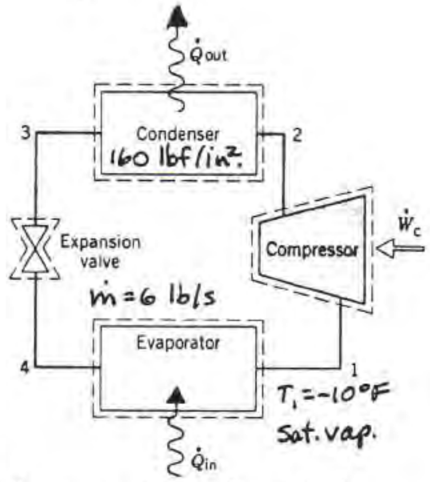
PROBLEM 10.12

KNOWN: An ideal vapor-compression refrigeration cycle uses refrigerant 134a as the working fluid. Operating data are known.

FIND: Plot the coefficient of performance and the refrigerating capacity for a range of condenser exit temperatures.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 10.1, items 1-4.

ANALYSIS: For a sample calculation, consider state 3 to be saturated liquid. Fixing each principal state:

State 1: $T_1 = -10°F$, sat. vapor $\Rightarrow h_1 = 100.29$ Btu/lb, $s_1 = 0.2236$ Btu/lb·°R

State 2: $p_2 = 160$ lbf/in², $s_2 = s_1 \Rightarrow h_2 = 120.48$ Btu/lb

State 3: $p_2 = 160$ lbf/in², sat. liquid $\Rightarrow h_3 = 47.65$ Btu/lb

State 4: Throttling process $\Rightarrow h_4 = h_3 = 47.65$ Btu/lb

The coefficient of performance is

$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = 2.61$$

The refrigerating capacity is

$$\dot{Q}_{in} = \dot{m}\,(h_1 - h_4) = \left(6\ \frac{\text{lb}}{\text{min}}\right)(100.29 - 47.65)\ \frac{\text{Btu}}{\text{lb}}\left|\frac{1\ \text{ton}}{200\ \text{Btu/min}}\right| = 1.58\ \text{tons}$$

The data for the required plots are obtained using IT, as follows:

IT Code

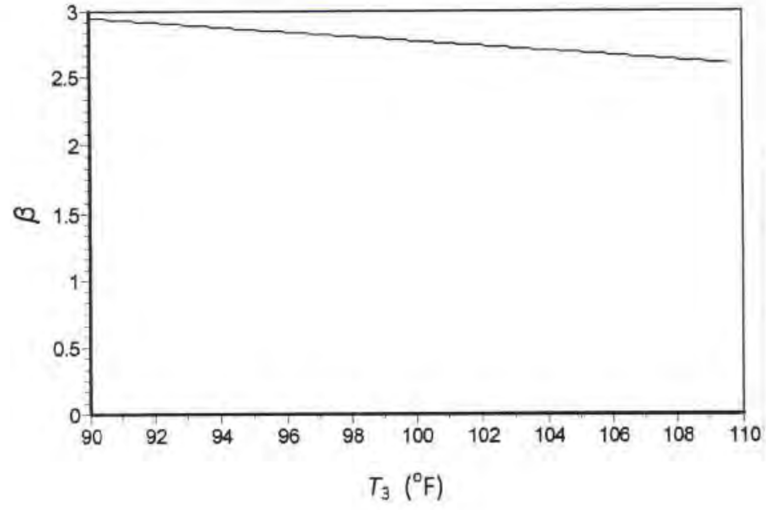
```
T1 = -10  // °F
p3 = 160  // lbf/in²
mdot = 6  // lb/min
T3 = 109.5  // °F

x1 = 1
p1 = Psat_T("R134A", T1)
h1 = hsat_Px("R134A", p1, x1)
s1 = ssat_Px("R134A", p1, x1)
s2 = s1
p2 = p3
h2 = h_Ps("R134A", p2, s2)
h3 = h_PT("R134A", p3, T3)
h4 = h3

beta = (h1 - h4) / (h2 - h1)
Qdotin = mdot * (h1 - h4) * (1 / 200)
```

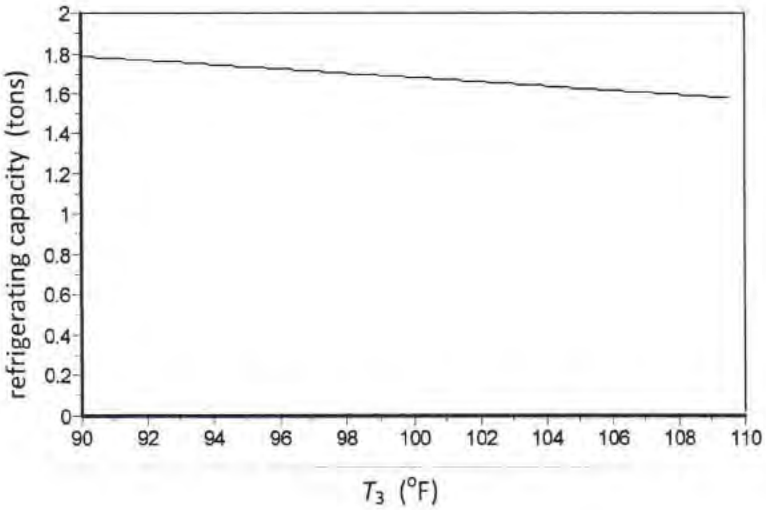
IT Results for T_3 = 109.5 °F

h_1 = 100.3 Btu/lb
h_2 = 120.5 Btu/lb
h_3 = 47.63 Btu/lb
h_4 = 47.63 Btu/lb
β = 2.608
$\dot{Q}_{in}$ = 1.58 tons

PROBLEM 10.12 (Continued) – Page 2

PLOTS:





As T_3 decreases (more subcooling) point 4 on the T-s diagram moves to the left, increasing the change in specific enthalpy across the evaporator. For constant mass flow rate, the refrigeration capacity increases accordingly. Also, since the compressor power is constant, the coefficient of performance is greater as subcooling increases.

Problem 10.13

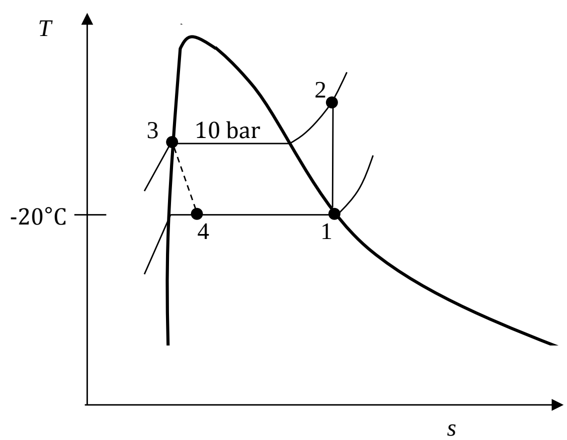
An ideal vapor compression refrigeration cycle with ammonia as the working fluid has an evaporator temperature of -20°C and a condenser pressure of 12 bar. Saturated vapor enters the compressor and saturated liquid exits the condenser. The mass flow rate of the refrigerant is 3 kg/min. Determine
(a) the coefficient of performance.
(b) the refrigerating capacity, in tons.

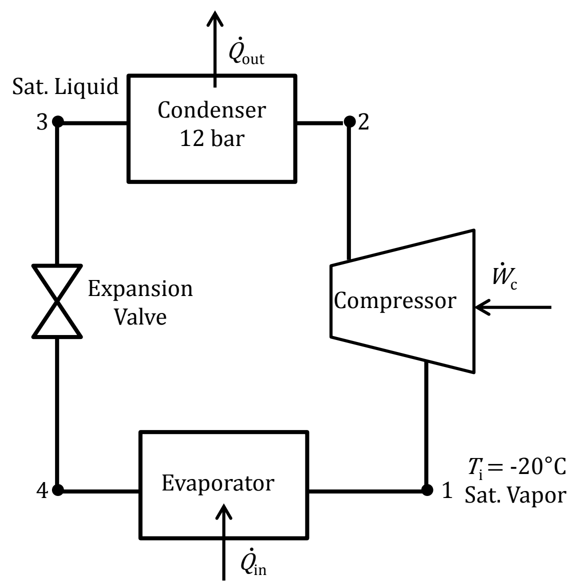
To determine the effect of changing the evaporator temperature on the cycle performance, plot the coefficient of performance and the refrigerating capacity, in tons, for saturated vapor entering the compressor at temperatures ranging from -40 to -10° C.

Known:
An ideal vapor-compression refrigeration cycle uses ammonia as the working fluid. Operating data are known.

Find:
Determine (a) the coefficient of performance, and (b) the refrigerating capacity.

Schematic and Known Data:





Engineering Model
(1) Each component of the cycle is analyzed as a control volume at steady state.
(2) Except for the expansion through the valve, which is a throttling process, all processes of the refrigerant are internally reversible.
(3) The compressor and expansion valve operate adiabatically.
(4) There are no pressure drops through the evaporator and the condenser.
(5) Kinetic and potential energy effects are negligible.
(6) Saturated vapor enters the compressor and saturated liquid exits the condenser.

Analysis:

First, fix each of the principal states.

State 1: $T_1 = -20℃$, saturated vapor $\rightarrow h_1 = 1417.79 \frac{\text{kJ}}{\text{kg}}, \quad s_1 = 5.6144 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

State 2: $p_2 = 12 \text{ bar}, s_2 = s_1 \rightarrow h_2 = 1689.5 \frac{\text{kJ}}{\text{kg}}$

State 3: $p_2 = p_3 = 12 \text{ bar}$, saturated liquid $\rightarrow h_3 = 327.01 \frac{\text{kJ}}{\text{kg}}$

State 4: Throttling process $\rightarrow h_4 = h_3 = 327.01 \frac{\text{kJ}}{\text{kg}}$

(a) The coefficient of performance is:

$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = \frac{1417.79 - 327.01}{1689.05 - 1417.79} = 4.015$$

(b) The refrigerating capacity is:

$$\dot{Q}_{\text{in}} = \dot{m}(h_1 - h_4) = \left(3 \frac{\text{kg}}{\text{min}}\right)(1417.79 - 327.01)\left(\frac{\text{kJ}}{\text{kg}} \cdot \frac{1 \text{ ton}}{211 \frac{\text{kJ}}{\text{min}}}\right) = 15.51 \text{ tons}$$

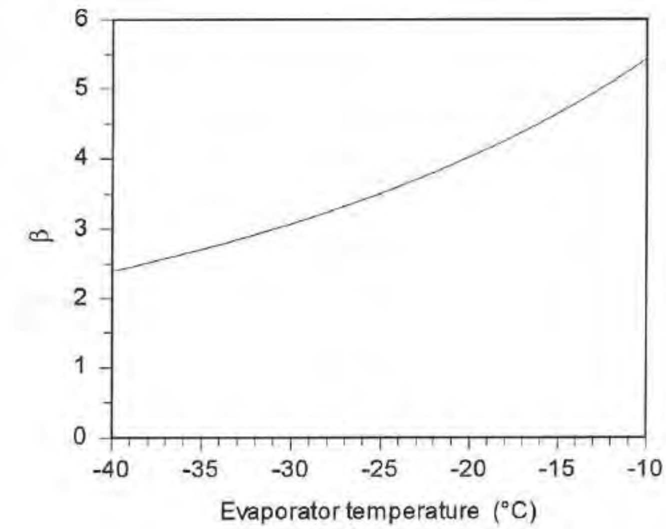
IT Code:

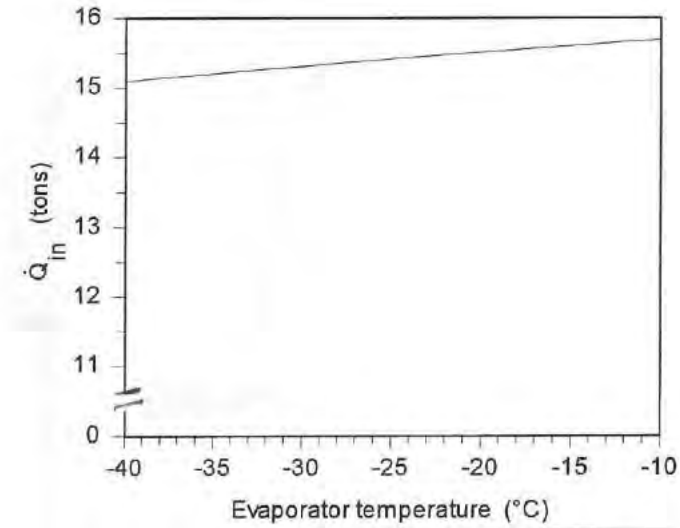
```
T1 = -20   // C
p3 = 12   // bar
mdot = 3   // kg/min

x1 = 1
p1 = Psat_T("Ammonia", T1)
h1 = hsat_Px("Ammonia", p1, x1)
s1 = ssat_Px("Ammonia", p1, x1)
s2 = s1
p2 = p3
s2 = s_PT("Ammonia", p2, T2)
h2 = h_PT("Ammonia", p2, T2)
x3 = 0
h3 = hsat_Px("Ammonia", p3, x3)
h4 = h3

beta = (h1 - h4) / (h2 - h1)
Qdotin = mdot * (h1 - h4) * (1/211)  // tons
```

IT Results:

```
h1 = 1418 kJ/kg
h2 = 1689 kJ/kg
h3 = 327 kJ/kg
h4 = 327 kJ/kg
Beta = 4.019
Qdotin = 15.51 tons
```

Plots:





In this case, the condenser pressure is constant. Thus, $h_3 = h_4$ remains unchanged as evaporator temperature changes. Since h_1 decreases as evaporator temperature decreases, the Δh across the evaporator decreases as T_1 goes down. Accordingly, the refrigerating capacity decreases as evaporator temperature decreases. Furthermore, the compressor power is greater as the evaporator temperature goes down, which contributes to the dramatic decrease in coefficient of performance shown on the graph.

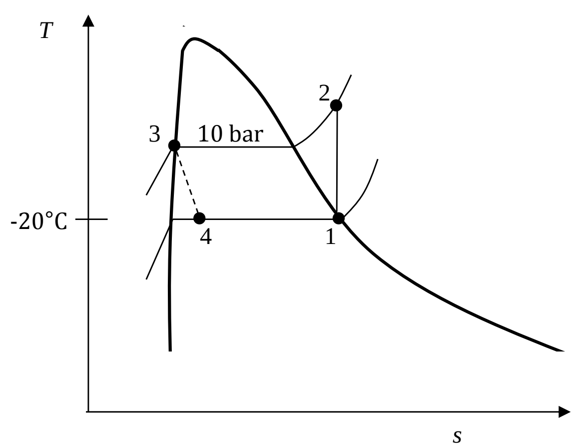
Problem 10.14

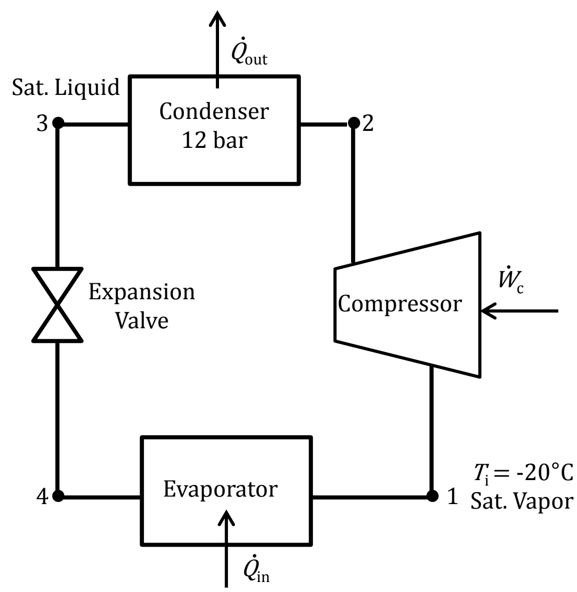
To determine the effect of changing condenser pressure on the performance of an ideal vapor-compression refrigeration cycle, plot the coefficient of performance and the refrigerating capacity, in tons, for the cycle in Problem 10.13 for condenser pressures ranging from 8 to 16 bar. All other conditions are the same as in Problem 10.13.

Known:
An ideal vapor-compression refrigeration cycle uses ammonia as the working fluid. Operating data are known.

Find:
Plot the coefficient of performance and the refrigerating capacity as the condenser pressure changes.

Schematic and Known Data:

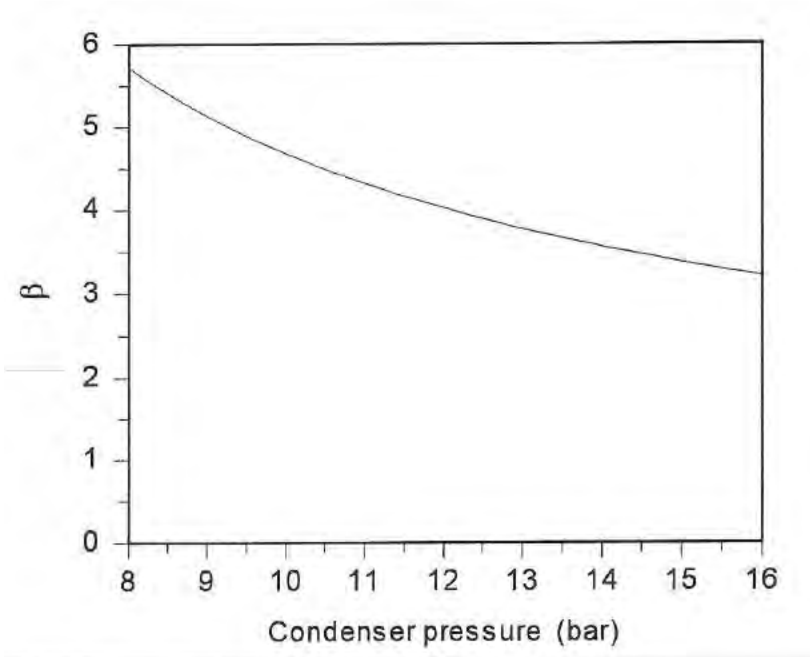


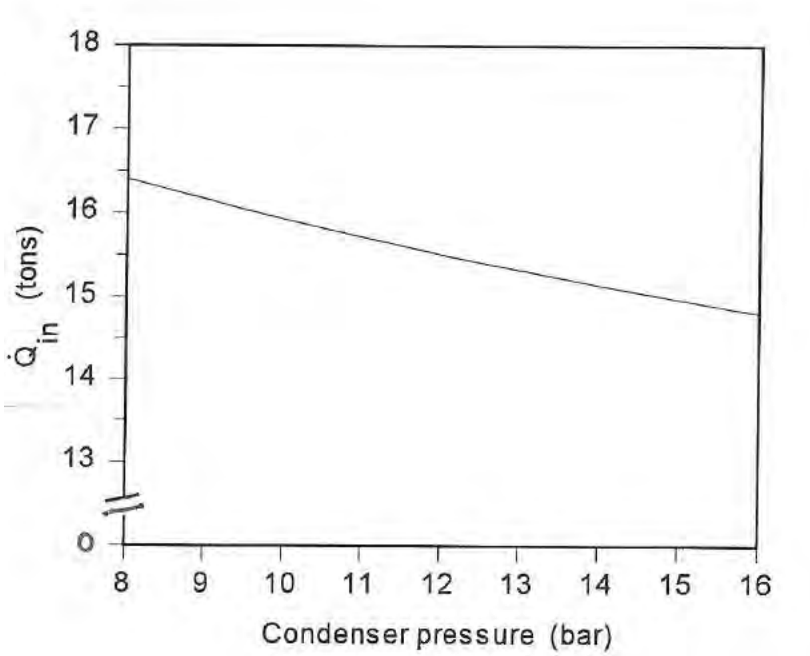


Engineering Model
(1) Each component of the cycle is analyzed as a control volume at steady state.
(2) Except for the expansion through the valve, which is a throttling process, all processes of the refrigerant are internally reversible.
(3) The compressor and expansion valve operate adiabatically.
(4) There are no pressure drops through the evaporator and the condenser.
(5) Kinetic and potential energy effects are negligible.
(6) Saturated vapor enters the compressor and saturated liquid exits the condenser.

Analysis:
The data for the required plots are obtained using the IT code given in the solution to Problem 10.13. In this case, the variable to sweep in p_3, the condenser pressure.

Plots:





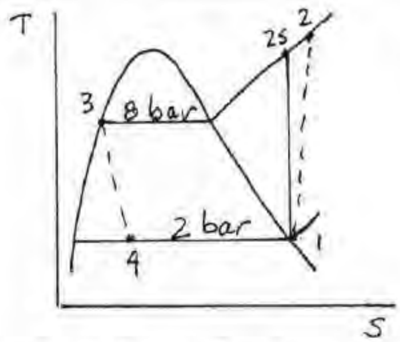
In this case, with the evaporator pressure constant and the compressor inlet state fixed, the specific enthalpy h_1 is constant. As condenser pressure is increased with saturated liquid condition at its exit, state 4 on the *T-s* diagram moves to the right, thereby decreasing the Δh across the evaporator. With mass flow rate fixed, the refrigerating capacity decreases accordingly. Also, the compressor power increases with increasing condenser pressure which contributes to the dramatic decrease in coefficient of performance shown on the graph.

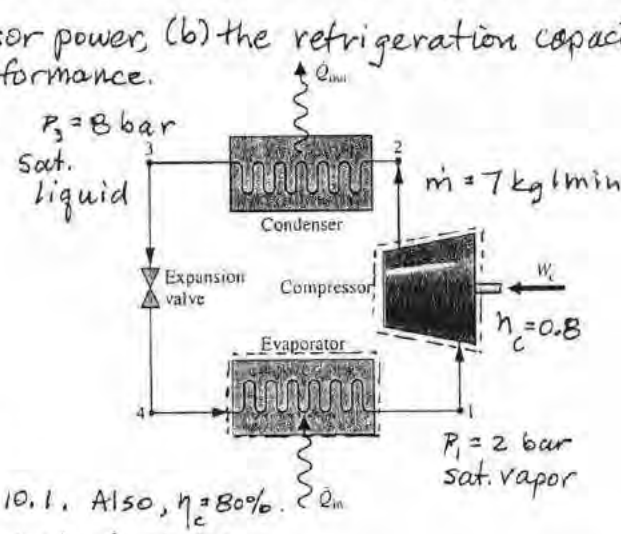
PROBLEM 10.15

KNOWN: Refrigerant 134a is the working fluid in a vapor-compression refrigeration cycle. Operating data are known, and the refrigerant mass flowrate is given.

FIND: Determine (a) the compressor power, (b) the refrigeration capacity, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 10.1. Also, $\eta_c = 80\%$.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 2$ bar, sat. vapor $\Rightarrow h_1 = 241.30$ kJ/kg, $s_1 = 0.9253$ kJ/kg·K

State 2: $p_2 = 8$ bar, $s_{2s} = s_1$ $\Rightarrow$ Interpolating in Table A-12; $h_{2s} = 269.92 \frac{kJ}{kg}$

Using the compressor efficiency

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c}\right) = 277.08 \text{ kJ/kg}$$

State 3: $p_3 = 8$ bar, sat. liquid $\Rightarrow h_3 = 93.42$ kJ/kg

State 4: Throttling process $\Rightarrow h_4 = h_3 = 93.42$ kJ/kg

(a) The compressor power is

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(7 \frac{kg}{min}\right)(277.08 - 241.30)\frac{kJ}{kg}\left|\frac{1 \text{ min}}{60 \text{ s}}\right|\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right| = 4.17 \text{ kW} \leftarrow \dot{W}_c$$

(b) The refrigerating capacity is

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = \left(7 \frac{kg}{min}\right)(241.3 - 93.42)\frac{kJ}{kg}\left|\frac{1 \text{ ton}}{211 \text{ kJ/min}}\right| = 4.906 \text{ tons} \leftarrow \dot{Q}_{in}$$

(c) The coefficient of performance is

$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = 4.13 \leftarrow \beta$$

Problem 10.16

Modify the cycle in problem 10.9 to have an isentropic compressor efficiency of 83% and let the temperature of the liquid leaving the condenser be 100°F. Determine, for the modified cycle,

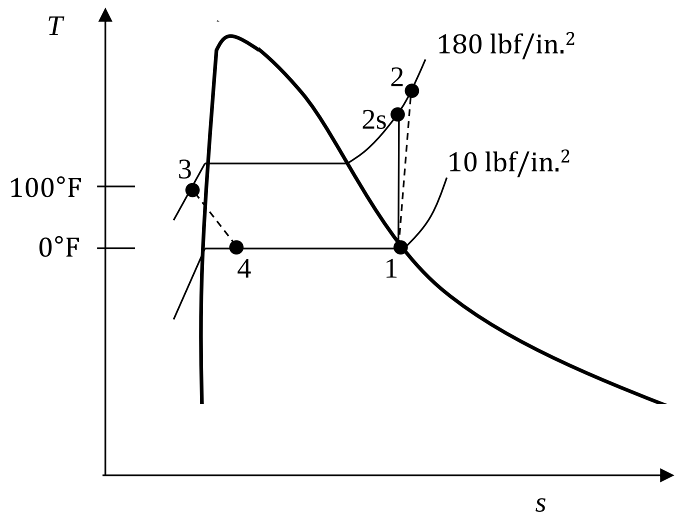
(a) the compressor power, in horsepower.
(b) the rate of heat transfer from the working fluid passing through the condenser, in Btu/min.
(c) the coefficient of performance.
(d) the rates of entropy production in the compressor and expansion valve, in Btu/min·°R.
(e) the rates of exergy destruction in the compressor and expansion valve, each in Btu/min. Let T_0 = 90°F.

Known:
The ideal vapor-compression refrigeration cycle of problem 10.9 is modified to have an isentropic compressor efficiency of 83% and subcooled liquid at a given temperature exiting the condenser.

Find:
Determine (a) the compressor power, (b) the rate of heat transfer from the working fluid passing through the condenser, (c) the coefficient of performance, (d) the rates of entropy production in the compressor and expansion valve, and (d) the rates of exergy destruction in the compressor and expansion valve for given environment temperature.

Schematic and Known Data:

State	***p* (lbf/in.^{2})**	***T* (°F)**	***h* (Btu/lb)**	***s* (Btu/lb·°R)**
1	10	0	102.94	0.2391
2	180	---	?	?
2s	180	---	131.04	0.2391
3	180	100	?	?
4	10	Sat.	?	?



Engineering Model:
(1) Each component of the cycle is analyzed as a control volume at steady state.
(2) There is no pressure drops through the evaporator and condenser.
(3) The compressor operates adiabatically with an efficiency of 83%. The expansion through the valve is a throttling process.
(4) Kinetic and potential energy effects are negligible.
(5) The environment temperature for calculating exergy is T_0 = 550°R (90°F).

Analysis:
Fix the unknown principal states using the isentropic compressor efficiency:

$$\eta_c = \left(h_{2s} - \frac{h_1}{h_2 - h_1}\right) \rightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c} = 136.76\,\frac{\text{Btu}}{\text{lb}}, s_2 = 0.2481\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$

For state 3, p_3 = 180 lbf/in.^{2}, T_3 = 100°F subcooled liquid. Therefore:

$$h_3 = h_f(100°F) = 44.23\,\frac{\text{Btu}}{\text{lb}}, s_3 = s_f(100°F) = 0.0898\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$

With the throttling process from states 3 to 4 where $h_3 = h_4$, find quality and specific entropy,

$$x_4 = \frac{h_4 - h_{f4}}{h_{fg4}} = \left(\frac{44.23 - 2.91}{94.45}\right) = 0.4375$$

$$s_4 = s_{f4} + x_4(s_{g4} - s_{f4}) = .0068\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} + .4375(0.2265 - .0068)\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} = 0.1029\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$

(a) The compressor power is:

$$\dot{W}_c = \dot{m}(h_2 - h_1) = 30.59\,\frac{\text{lb}}{\text{min}}(136.76 - 102.94)\,\frac{\text{Btu}}{\text{lb}}\left|\frac{1\text{ hp}}{2545\ \text{Btu}/\text{h}}\right|\left|\frac{60\text{ min}}{1\text{ h}}\right| = 24.39\text{ hp}$$ ⟵

(b) The rate of heat transfer from the working fluid passing through the condenser is:

$$\dot{Q}_{in} = \dot{m}(h_2 - h_3) = \left(30.59\,\frac{\text{lb}}{\text{min}}\right)(136.76 - 44.23)\left(\frac{\text{Btu}}{\text{lb}}\right) = 2830.49\,\frac{\text{Btu}}{\text{min}}$$ ⟵

(c) The coefficient of performance is:

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_c} = \frac{\dot{m}(h_1 - h_4)}{\dot{m}(h_2 - h_1)} = \frac{102.94 - 44.23}{136.76 - 102.94} = 1.74$$ ⟵

(d) The entropy production rate in the compressor is:

$$\underbrace{\frac{dS}{dT}}_{=0} = \underbrace{\sum_j \frac{\dot{Q}_j}{T_j}}_{=0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{comp} \rightarrow \dot{\sigma}_{comp} = \dot{m}(s_2 - s_1) = \left(30.59\,\frac{\text{lb}}{\text{min}}\right)(0.2481 - 0.2391)\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} = 0.27531\,\frac{\text{Btu}}{\text{min}\cdot{}^\circ\text{R}}$$ ⟵

The entropy production rate in the expansion valve is:

$$\underbrace{\frac{dS}{dT}}_{=0} = \underbrace{\sum_j \frac{\dot{Q}_j}{T_j}}_{=0} + \dot{m}(s_3 - s_4) + \dot{\sigma}_{\text{valve}} \rightarrow \dot{\sigma}_{\text{valve}} = \dot{m}(s_4 - s_3)$$

$$= \left(30.59 \frac{\text{lb}}{\text{min}}\right)(0.1029 - 0.0898)\frac{\text{Btu}}{\text{lb} \cdot {}^\circ\text{R}} = 0.40073 \frac{\text{Btu}}{\text{min} \cdot {}^\circ\text{R}} \longleftarrow$$

(e) The rate of exergy destruction in the compressor is:

$$\dot{E}_{\text{d}_{\text{comp}}} = T_0 \dot{\sigma}_{\text{comp}} = (550{}^\circ R)\left(0.27531 \frac{\text{Btu}}{\text{min}\cdot{}^\circ\text{R}}\right) = 151.42 \frac{\text{Btu}}{\text{min}} \longleftarrow$$

The rate of exergy destruction in the valve is:

$$\dot{E}_{\text{d}_{\text{valve}}} = T_0 \dot{\sigma}_{\text{valve}} = (550{}^\circ\text{R})\left(0.40073 \frac{\text{Btu}}{\text{min} \cdot {}^\circ\text{R}}\right) = 220.40 \frac{\text{Btu}}{\text{min}} \longleftarrow$$

10.17 Data for steady state operation of a vapor-compression refrigeration cycle with Refrigerant 134a as the working fluid are given in the table below. The states are numbered as in Fig. 10.3. The refrigeration capacity is 4.6 tons. Ignoring heat transfer between the compressor and its surroundings, sketch the T-s diagram of the cycle and determine

(a) the mass flow rate of the refrigerant, in kg/min.
(b) the isentropic compressor efficiency.
(c) the coefficient of performance .
(d) the rates of exergy destruction in the compressor and expansion valve, each in kW.
(e) the net changes in flow exergy rate of the refrigerant passing through the evaporator and condenser, respectively, each in kW.

Let $T_0 = 21°C$, $p_0 = 1$ bar.

State	p (bar)	T (°C)	h (kJ/kg)	s (kJ/kg·K)
1	1.4	-10	243.40	0.9606
2	7	58.5	295.13	1.0135
3	7	24	82.90	0.3113
4	1.4	-18.8	82.90	0.33011

KNOWN: A vapor-compression refrigeration cycle operates with Refrigerant 134a as its working fluid. The refrigeration capacity and operating data at principal states in the cycle are provided.

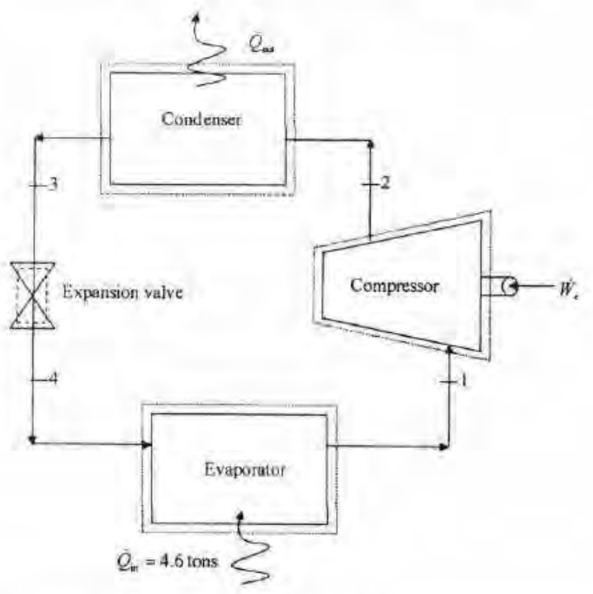
FIND: Sketch the T-s diagram of the cycle and determine (a) the mass flow rate of the refrigerant, (b) the isentropic compressor efficiency, (c) the coefficient of performance, (d) the rates of exergy destruction in the compressor and expansion valve, and (e) the net changes in flow exergy rate of the refrigerant passing through the evaporator and condenser.

SCHEMATIC AND GIVEN DATA:

State	p (bar)	T (°C)	h (kJ/kg)	s (kJ/kg·K)
1	1.4	-10	243.40	0.9606
2	7	58.5	295.13	1.0135
3	7	24	82.90	0.3113
4	1.4	-18.8	82.90	0.33011

$T_0 = 21°C = 294$ K
$p_0 = 1$ bar

Problem 10.17 (Continued) – Page 2

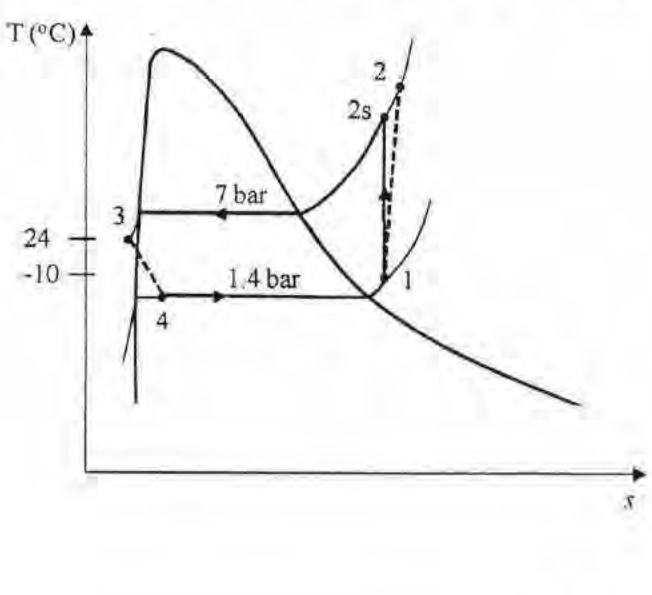


ENGINEERING MODEL:

(1) Each component is analyzed as a control volume at steady state. The control volumes are indicated by dashed lines on the accompanying figure.
(2) There are no pressure drops through the evaporator and condenser.
(3) The compressor operates adiabatically. The expansion through the valve is a throttling process.
(4) Kinetic and potential energy effects are negligible.
(5) T_0 = 294 K (21°C), p_0 = 1 bar.

ANALYSIS:

Referring to Table A-11 for R-134a, the T-s diagram is



Problem 10.17 (Continued) – Page 3

(a) The mass flow rate is

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) \text{ or } \dot{m} = \frac{\dot{Q}_{in}}{(h_1 - h_4)}$$

Therefore

$$\dot{m} = \frac{4.6 \text{ tons}\left|\dfrac{211\dfrac{\text{kJ}}{\text{min}}}{1\text{ ton}}\right|}{(243.40 - 82.90)\dfrac{\text{kJ}}{\text{kg}}} = 6\,\frac{\text{kg}}{\text{min}}$$

(b) For isentropic compression
$s_{2s} = s_1 = 0.9606$ kJ/kg·K

Interpolating in Table A-12 at $p_2 = 7$ bar and $s_{2s} = 0.9606$ kJ/kg·K; $h_{2s} = 278.06$ kJ/kg

Thus

$\eta_c = (h_{2s} - h_1)/(h_2 - h_1) = (278.06 - 243.40)/(295.13 - 243.40) =$ **0.671 (67.1%)**

(c) The coefficient of performance is

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_c} = \frac{\dot{m}(h_1 - h_4)}{\dot{m}(h_2 - h_1)} = \frac{(243.40 - 82.90)\dfrac{\text{kJ}}{\text{kg}}}{(295.13 - 243.40)\dfrac{\text{kJ}}{\text{kg}}} = 3.10$$

(d) The rate of exergy destruction in the compressor can be obtained by using an exergy rate balance or using

$$\dot{\text{E}}_d = \dot{\sigma} T_o$$

where $\dot{\sigma}$ is the rate of entropy production obtained from an entropy balance. Using the entropy approach, the entropy and mass balances for a control volume enclosing the compressor give

$$\dot{\sigma}_{comp} = \dot{m}(s_2 - s_1) = 6\frac{\text{kg}}{\text{min}}\left|\frac{1\text{ min}}{60\text{ s}}\right|\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|(1.0135 - 0.9606)\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 0.00529\frac{\text{kW}}{\text{K}}$$

The exergy destruction is

$$\left(\dot{\mathbf{E}}_d\right)_{comp} = T_0\dot{\sigma}_{comp} = 294\text{ K}\left(0.00529\frac{\text{kW}}{\text{K}}\right) = 1.56\text{ kW}$$

Similarly for the expansion valve

Problem 10.17 (Continued) – Page 4

$$\left(\dot{\mathbf{E}}_{\mathrm{d}}\right)_{\text{valve}} = T_0\dot{\sigma}_{\text{valve}}$$

$$= T_0\dot{m}(s_4 - s_3) = 294\,\mathrm{K}\left(6\frac{\mathrm{kg}}{\mathrm{min}}\left|\frac{1\,\mathrm{min}}{60\,\mathrm{s}}\right|\right)(0.33011 - 0.3113)\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}} = 0.553\,\mathrm{kW} \longleftarrow$$

(e) The net change in flow exergy rate for refrigerant passing through the evaporator is

$$\dot{m}(\mathrm{e}_{f1} - \mathrm{e}_{f4}) = \dot{m}\left[h_1 - h_4 - T_0(s_1 - s_4)\right]$$

$$= \left(6\frac{\mathrm{kg}}{\mathrm{min}}\left|\frac{1\,\mathrm{min}}{60s}\right|\left|\frac{1\,\mathrm{kW}}{1\,\mathrm{kJ/s}}\right|\right)\left[(243.40 - 82.90)\frac{\mathrm{kJ}}{\mathrm{kg}} - 294\,\mathrm{K}(0.9606 - 0.33011)\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\right]$$

#1 $$= -2.486\,\mathrm{kW} \longleftarrow$$

Similarly for the condenser

$$\dot{m}(\mathrm{e}_{f3} - \mathrm{e}_{f2}) = \dot{m}\left[h_3 - h_2 - T_0(s_3 - s_2)\right]$$

$$= \left(6\frac{\mathrm{kg}}{\mathrm{min}}\left|\frac{1\,\mathrm{min}}{60s}\right|\left|\frac{1\,\mathrm{kW}}{1\,\mathrm{kJ/s}}\right|\right)\left[(82.90 - 295.13)\frac{\mathrm{kJ}}{\mathrm{kg}} \longleftarrow - 294\,\mathrm{K}(0.3113 - 1.0135)\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\right]$$

$$= -0.578\,\mathrm{kW}$$

---

1. Although there is heat transfer to the refrigerant passing through the evaporator, the specific flow exergy decreases. This can be explained by noting that the state of the working fluid moves closer to the dead state as it is heated at a temperature below T_0.

Problem 10.18

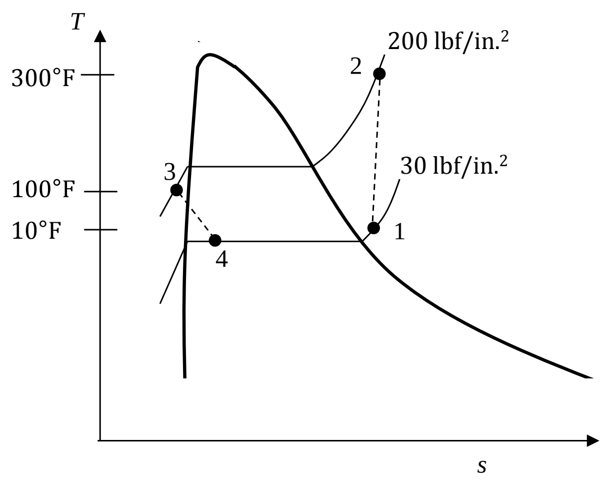
A vapor-compression refrigeration system uses ammonia as the working fluid. Data for the cycle are provided in the table below. The principal states are numbered as in Fig. 10.3. The heat transfer rate from the working fluid passing through the condenser is 50,000 Btu/h. If the compressor operates adiabatically, determine
(a) the compressor power input, in hp.
(b) the coefficient of performance of the cycle.

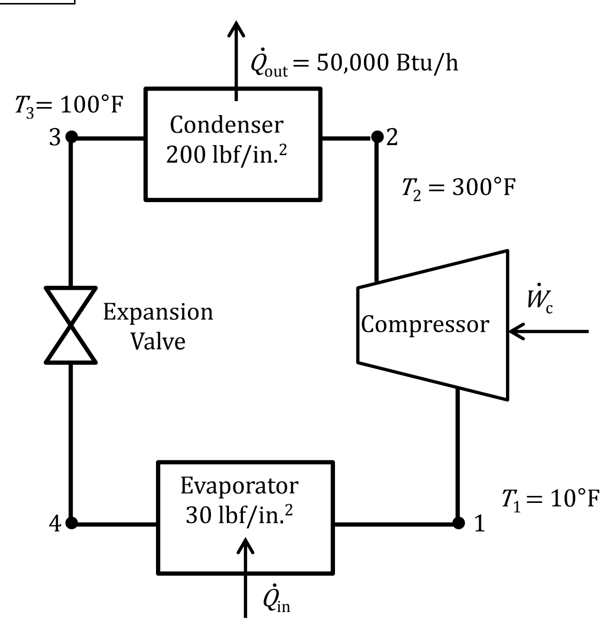
Known:
A vapor-compression refrigeration system uses ammonia as the working fluid. Data are known at various locations and the heat transfer rate from the working fluid passing through the condenser is specified.

Find:
Determine (a) the compressor power, and (b) the coefficient of performance.

Schematic and Known Data:

State	*p* (lbf/in.^{2})	*T* (°F)	*h* (Btu/lb)	*s* (Btu/lb·°R)
1	30	10	617.07	1.3479
2	200	300	763.74	1.3774
3	200	100	155.05	---
4	30	---	155.05	---





Engineering Model:
(1) Each component of the cycle is analyzed as a control volume at steady state.
(2) There are no pressure drops through the evaporator and condenser.

(3) The compressor operates adiabatically. The expansion through the valve is a throttling process.
(4) Kinetic and potential energy effects are negligible.

Analysis:

(a) The compressor power is:

$$\dot{W}_c = \dot{m}(h_2 - h_1)$$

Evaluating $\dot{m}$:

$$\dot{Q}_{\mathrm{out}} = \dot{m}(h_2 - h_3) \rightarrow \dot{m} = \frac{\dot{Q}_{\mathrm{out}}}{h_2 - h_3} = \frac{50{,}000\,\frac{\mathrm{Btu}}{\mathrm{h}}}{(763.74 - 155.05)\,\frac{\mathrm{Btu}}{\mathrm{lb}}} = 82.14\,\frac{\mathrm{lb}}{\mathrm{h}}$$

Thus:

$$\dot{W}_{\mathrm{c}} = \dot{m}(h_2 - h_1) = \left(82.14\,\frac{\mathrm{lb}}{\mathrm{h}}\right)(763.74 - 617.07)\left(\frac{\mathrm{Btu}}{\mathrm{lb}} \cdot \frac{1\ \mathrm{hp}}{2545\,\frac{\mathrm{Btu}}{\mathrm{h}}}\right) = 4.73\ \mathrm{hp} \longleftarrow$$

(b) The coefficient of performance is:

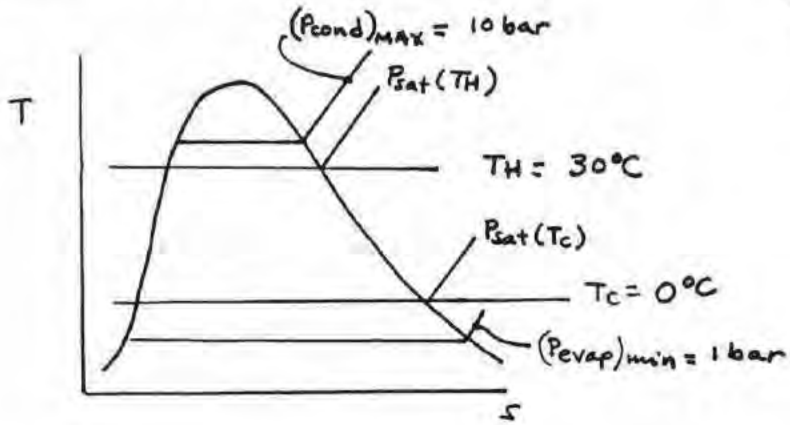
$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = 3.15 \longleftarrow$$

PROBLEM 10.19

KNOWN: For a vapor-compression refrigeration cycle, the minimum and maximum allowed refrigeration pressures are 1 and 10 bar, respectively. The cold and warm region temperatures are also specified.

FIND: Among several candidate working fluids, determine which (if any) can be used for this duty.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The refrigeration cycle adheres to the model presented for the vapor-compression cycle in Sec. 10.2

ANALYSIS: For heat transfer to the working fluid passing through the evaporator from the cold region, we must have

$$1 \text{ bar} \le P_{evap} \le P_{sat}(T_C)$$

For heat transfer from the working fluid passing through the condenser to the warm region, we must have

$$P_{sat}(T_H) \le P_{cond} \le 10 \text{ bar}$$

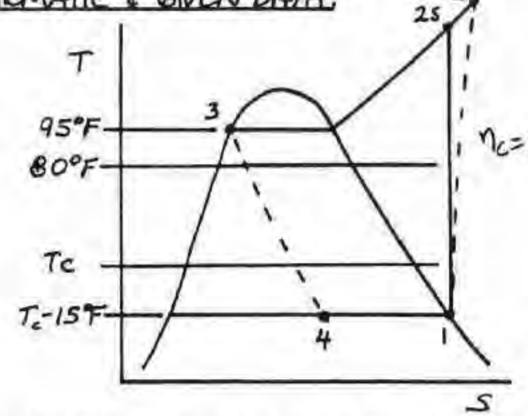
working fluid	table	$P_{sat}(0°C)$	$P_{sat}(30°C)$	feasible
R22	A-7	4.98 bar	11.93 bar	No - $P_{sat}(30°C)$ too high
R134a	A-10	2.93	7.7	yes
Ammonia	A-13	4.3	11.69	No - $P_{sat}(30°C)$ too high
Propane	A-16	4.74	10.8	No - $P_{sat}(30°C)$ too high

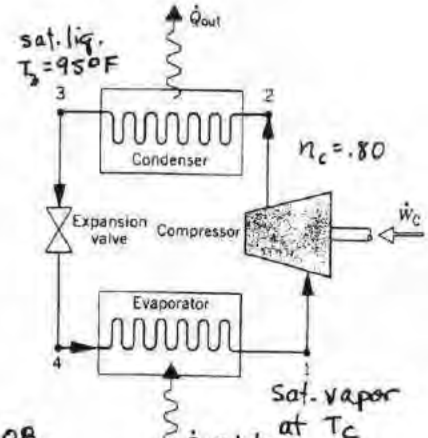
PROBLEM 10.20

KNOWN: A vapor-compression refrigeration cycle is used to maintain a cold region at temperature T_c. Refrigerant conditions are given at various points. The refrigerating capacity is specified.

FIND: Plot refrigerant mass flow rate, coefficient of performance, and refigerating efficiency versus a range of T_c values and for the refrigerants (a) R134a, (b) propane, (c) R22, (d) ammonia.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 10.2, except $\eta_c = 0.8$.

ANALYSIS: The procedure is the same for each refrigerant. The principal states are fixed as follows:

State 1: $T_1 = T_c - 15$, sat. vapor ; h_g and s_g at T_1

State 2: Using T_3, determine $p_3 = p_2 = p_{sat} @ T_3$. Then, with p_2 and $s_1 = s_{2s}$ determine h_{2s}. When using the superheated vapor table, double interpolation is likely. Next, use η_c to get h_2;

$$h_2 = h_1 + (h_{2s} - h_1)/\eta_c$$

State 3: $T_3 = 95\,°F$. $h_3 = h_f @ 95°F$

State 4: Throttling process: $h_4 = h_3$

To get the refrigerant mass flow rate

$$\dot{m} = \frac{\dot{Q}_{in}}{h_1 - h_4}$$

The coefficient of performance is

$$\beta = \frac{h_1 - h_4}{h_2 - h_1}$$

Finally; $$\beta_{carnot} = \frac{T_c(°R)}{540 - T_c(°R)}$$

and $$\eta_{ref} = \beta / \beta_{carnot}$$

PROBLEM 10.20 (Cont'd.) – Page 2

The data for the required plots are obtained using IT, as follows. Shown here is the code for ammonia. The code for other refrigerants is similar.

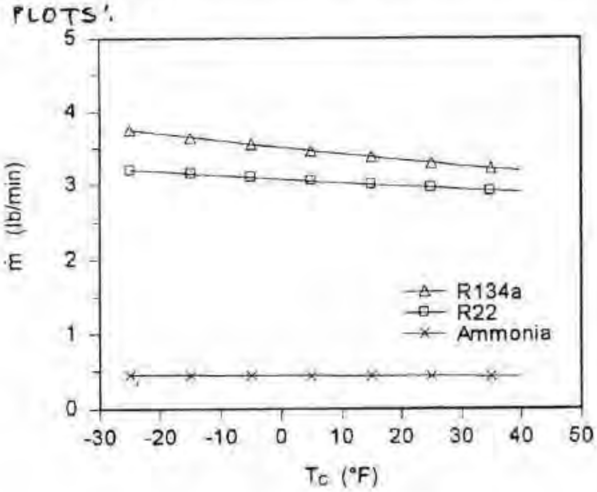
IT Code

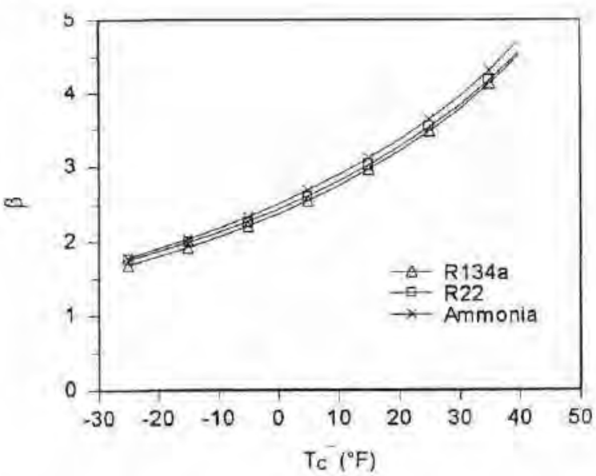
```
T1 = TC - 15   // °F
eta_c = 0.8
T3 = 95  // °F
Qdotin = 1   // ton
TC = -25    // °F
TH = 80   // °F

x1 = 1
p1 = Psat_T("Ammonia", T1)
h1 = hsat_Px("Ammonia", p1, x1)
s1 = ssat_Px("Ammonia", p1, x1)
s2s = s1
p2 = p3
s2s = s_PT("Ammonia", p2, T2s)
h2s = h_PT("Ammonia", p2, T2s)
h2 = h1 + (h2s - h1) / eta_c
p3 = Psat_T("Ammonia", T3)
x3 = 0
h3 = hsat_Px("Ammonia", p3, x3)
h4 = h3

mdot = (Qdotin * 200) / (h1 - h4)
beta = (h1 - h4) / (h2 - h1)
betacarnot = (TC + 460) / ((TH + 460) - (TC + 460))
eff = beta / betacarnot
```

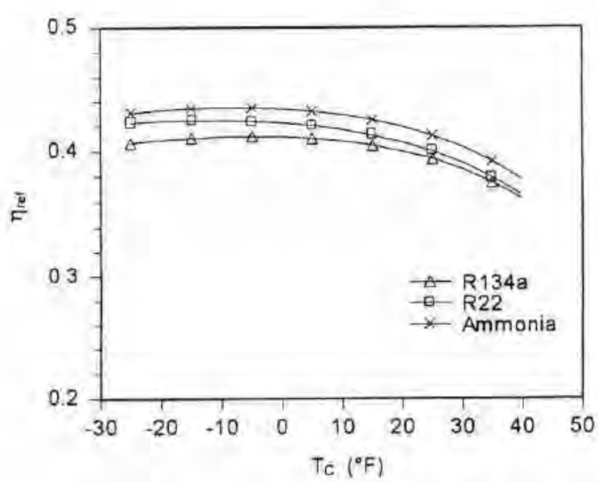
IT Results for Ammonia, T_C = -25 °F

h_1 = 597 Btu/lb
h_2 = 847.8 Btu/lb
h_{2s} = 797.7 Btu/lb
h_3 = 149.1 Btu/lb
h_4 = 149.1 Btu/lb
β_{carnot} = 4.143
$\dot{m}$ = 0.4465 lb/min
β = 1.785
η_{ref} = 0.431

PLOTS:







Property data for propane are not available in IT. Using data from Tables A-16E, A-17E, and A-18E, we get the following resuts for a sample case:

$\underline{T_C = -25°F = 435°F}$

$\dot{m} = 1.985$ lb/min

$\beta = 2.07$

$\eta_{ref} = 0.5$

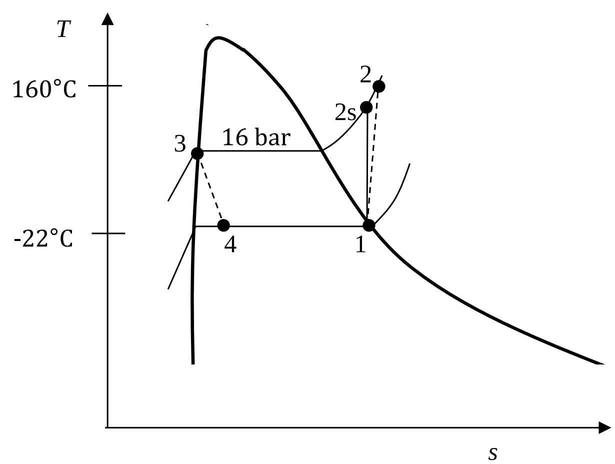
Problem 10.21

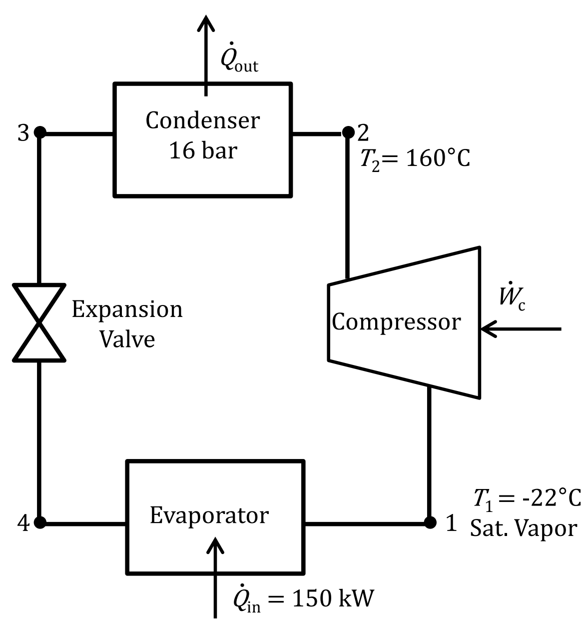
In a vapor-compression refrigeration cycle, ammonia exits the evaporator as saturated vapor at -22°C. The refrigerant enters the condenser at 16 bar and 160°C, and saturated liquid exits at 16 bar. There is no significant heat transfer between the compressor and its surroundings, and the refrigerant passes through the evaporator with a negligible change in pressure. If the refrigerating capacity is 150 kW, determine
(a) the mass flow rate of the refrigerant, in kg/s.
(b) the power input to the compressor, in kW.
(c) the coefficient of performance.
(d) the isentropic compressor efficiency.
(e) the rate of entropy production, in kW/K, for the compressor.

Known:
Ammonia is the working fluid in a vapor-compression refrigeration system. Data are known at various locations and the refrigerating capacity is given.

Find:
Determine (a) the mass flow rate of the refrigerant, (b) the compressor power, (c) the coefficient of performance, (d) the isentropic compressor efficiency, and (e) the rate of entropy production for the compressor.

Schematic and Known Data:





Engineering model:
(1) Each component of the cycle is analyzed as a control volume at steady state.
(2) There are no pressure drops through the evaporator and condenser.
(3) The compressor operates adiabatically. The expansion through the valve is a throttling process.
(4) Kinetic and potential energy effects are negligible.

Analysis:
First, fix each of the principal states:

State 1: $T_1 = -22°\text{C}, \text{saturated vapor} \rightarrow h_1 = 1415.08\,\frac{\text{kJ}}{\text{kg}},\quad s_1 = 5.6457\,\frac{\text{kJ}}{\text{kg·K}}$

State 2: $p_2 = 16\text{ bar}, T_2 = 160°\text{C} \rightarrow h_2 = 1798.45\,\frac{\text{kJ}}{\text{kg}}, s_2 = 5.7475\,\frac{\text{kJ}}{\text{kg·K}}$

State 3: $p_3 = 16\text{ bar}, \text{saturated liquid} \rightarrow h_3 = 376.46\,\frac{\text{kJ}}{\text{kg}}$

State 4: $\text{Throttling process} \rightarrow h_4 = h_3 = 376.46\,\frac{\text{kJ}}{\text{kg}}$

(a) The mass flow rate is determined using the refrigerating capacity

$$\dot{Q}_{\text{in}} = \dot{m}(h_1 - h_4) \rightarrow \dot{m} = \frac{\dot{Q}_{\text{in}}}{h_1 - h - 4} = \frac{150\text{ kW}}{(1415.08 - 376.46)\,\frac{\text{kJ}}{\text{kg}}}\left|\frac{1\,\frac{\text{kJ}}{\text{s}}}{1\text{ kW}}\right| = 0.1444\,\frac{\text{kg}}{\text{s}} \longleftarrow$$

(b) The compressor power is:

$$\dot{W}_{\text{c}} = \dot{m}(h_2 - h_4) = \left(0.1444\,\frac{\text{kg}}{\text{s}}\right)(1798.45 - 1415.08)\,\frac{\text{kJ}}{\text{kg}}\left|\frac{1\text{ kW}}{1\,\frac{\text{kJ}}{\text{s}}}\right| = 55.36\text{ kW} \longleftarrow$$

(c) The coefficient of performance is:

$$\beta = \frac{\dot{Q}_{\text{in}}}{\dot{W}_{\text{c}}} = \frac{150\text{ kW}}{55.36\text{ kW}} = 2.71 \longleftarrow$$

(d) For isentropic compression, $p_2 = 16\text{ bar}, s_{2s} = s_1$. With this information, $h_{2s} = 1755.38\,\frac{kJ}{kg}$. Thus, the isentropic efficiency is: $\longleftarrow$

$$\eta_c = \frac{h_{s2} - h_1}{h_2 - h_1} = \frac{1755.38 - 1415.08}{1798.45 - 1415.08} = 0.888 = 88.8\%$$

(e) The entropy production for the compressor is:

$$\underbrace{\frac{dS}{dT}}_{=0} = \underbrace{\sum_j \frac{\dot{Q}_j}{T_j}}_{=0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{\text{comp}} \rightarrow \dot{\sigma}_{\text{comp}} = \dot{m}(s_2 - s_1)$$

$$= \left(\left(0.1444\,\frac{\text{kg}}{\text{s}}\right)(5.7475 - 5.6457)\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left|\frac{1\text{ kW}}{1\,\frac{\text{kJ}}{\text{s}}}\right| = 0.01470\,\frac{\text{kW}}{\text{K}}$$

$\longleftarrow$

Problem 10.22

A vapor-compression refrigeration system with a capacity of 10 tons has Refrigerant 134a as the working fluid. Information and data for the cycle are provided in Fig. P10.22 and in the table below. The compression process is internally reversible and can be modeled by $pv^{1.01}$ = *constant*. The condenser is water cooled, with water entering and leaving with a negligible change in pressure. Heat transfer from the outside of the condenser can be neglected. Determine

(a) the mass flow rate of refrigerant, in kg/s.
(b) the power input and the heat transfer rate for the compressor, each in kW.
(c) the coefficient of performance.
(d) the mass flow rate of the cooling water, in kg/s.
(e) the rates of entropy production in the condenser and expansion valve, in kW/K.
(f) the rates of exergy destruction in the condenser and expansion valve, each expressed as a percentage of the compressor power input. Let T_0 = 20°C.

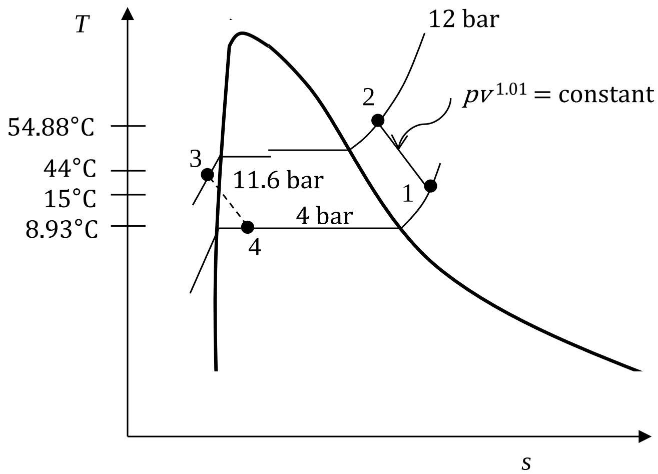
Known:
R134a is the working fluid in a vapor-compression refrigeration system having a water cooled condenser. The capacity is known and the compression process is described by $pv^{1.01}$ = *constant*.

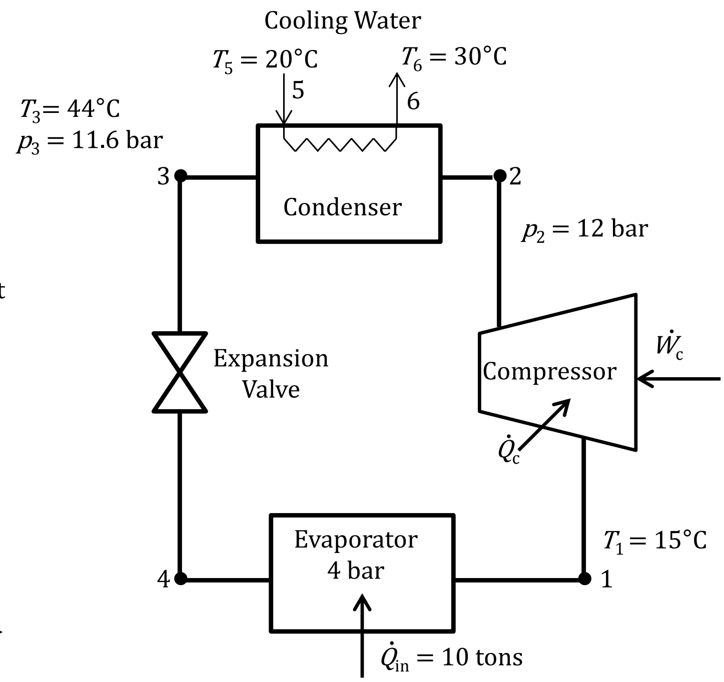
Find:
Determine (a) the mass flow rate of R134a, (b) power input and heat transfer rate for the compressor, (c) coefficient of performance, (d) cooling water mass flow rate, (e) the rate of entropy production in the condenser and expansion valve, and (f) rates of exergy destruction in the condenser and expansion valve, each expressed as a percentage of the power input.

Schematic and Known Data:

State	***p* (bar)**	***T* (°C)**	***v* (m^3/kg)**	***h* (kJ/kg)**	***s* (kJ/kg·K)**
1	4	15	0.05258	258.15	0.9348
2	12	54.88	0.01772	281.33	0.9341
3	11.6	44	0.0008847	112.22	0.4054
4	4	8.93	0.01401	112.22	0.4179
5	-	20	-	83.96	0.2966
6	-	30	-	125.79	0.4369

Engineering Model:
(1) Each component of the cycle is analyzed as a control volume at steady state.
(2) The compression is polytropic with n = 1.01.
(3) The expansion through the valve is a throttling process.
(4) Heat transfer from the outside of the condenser can be neglected.
(5) Kinetic and potential energy effects are negligible.
(6) Let T_0 = 293 K.





Analysis:

(a) The mass flow rate is found using the refrigerating capacity:

$$\dot{m} = \frac{\dot{Q}_{in}}{h_1 - h_4} = \frac{10 \text{ tons}}{(258.15 - 112.22)\frac{\text{kJ}}{\text{kg}}}\left(\frac{211\frac{\text{kJ}}{\text{min}}}{1 \text{ ton}} \cdot \frac{1 \text{ min}}{60 \text{ s}}\right) = 0.241\frac{\text{kg}}{\text{s}}$$

(b) Equation 6.55 is used to determine the compressor work input:

$$\frac{\dot{W}_c}{\dot{m}} = \int_1^2 v dp = \left(\frac{n}{n-1}\right)(p_2 v_2 - p_1 v_1)$$

$$= \left(\frac{1.01}{0.01}\right)\left[(12 \text{ bar})\left(0.01772\frac{\text{m}^3}{\text{kg}}\right) - (4 \text{ bar})\left(0.05258\frac{\text{m}^3}{\text{kg}}\right)\right]\left|\frac{10^5\frac{\text{N}}{\text{m}^2}}{1 \text{ bar}}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}}\right| = 23.432\frac{\text{kJ}}{\text{kg}}$$

Thus, the compressor power input is:

$$\dot{W}_c = \dot{m}\left(\frac{\dot{W}_c}{\dot{m}}\right) = \left(0.241\frac{\text{kg}}{\text{s}}\right)\left(23.432\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{1 \text{ kW}}{1\frac{\text{kJ}}{\text{s}}}\right| = 5.647 \text{ kW}$$

The heat transfer rate is:

$$\dot{Q}_{cv} = -\dot{W}_{cv} + \dot{m}(h_2 - h_1) = -(5.647 \text{ kW}) + \left(0.241\frac{\text{kg}}{\text{s}}\right)(281.33 - 258.15)\frac{\text{kJ}}{\text{kg}}$$

$$= -0.0606 \text{ kW}$$

(c) The coefficient of performance is:

$$\beta = \frac{\dot{Q}_{\text{in}}}{\dot{W}_{\text{c}}} = \frac{10 \text{ tons}}{5.647 \text{ kW}}\left|\frac{211\,\frac{\text{kJ}}{\text{min}}}{1 \text{ ton}}\right|\left|\frac{1 \text{ min}}{60 \text{ s}}\right|\left|\frac{1 \text{ kW}}{1\,\frac{\text{kJ}}{\text{s}}}\right| = 6.227$$

(d) Evaluating $\dot{m}_{\text{cw}}$ from an energy balance:

$$0 = \dot{m}(h_2 - h_3) + \dot{m}_{\text{cw}}(h_5 - h_6)$$

$$\dot{m}_{\text{cw}} = \dot{m}\left(\frac{h_2 - h_3}{h_6 - h_5}\right) = \left(0.241\,\frac{\text{kg}}{\text{s}}\right)\left(\frac{281.33 - 112.22}{125.79 - 83.96}\right) = 0.9743\,\frac{\text{kg}}{\text{s}}$$

(e) The rate of entropy production for the condenser can be found through the following:

$$0 = \sum_j \underbrace{\frac{\dot{Q}_j}{T_j}}_{=0} + \dot{m}(s_2 - s_3) + \dot{m}_{\text{w}}(s_5 - s_6) + \dot{\sigma}_{\text{cond}}$$

$$\dot{\sigma}_{\text{cond}} = \dot{m}(s_3 - s_2) + \dot{m}_{\text{cw}}(s_6 - s_5)$$

$$= \left(\left(0.241\,\frac{\text{kg}}{\text{s}}\right)(0.4054 - 0.9341)\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}} + \left(0.9743\,\frac{\text{kg}}{\text{s}}\right)(0.4369 - 0.2966)\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left|\frac{1 \text{ kW}}{1\,\frac{\text{kJ}}{\text{s}}}\right| = 0.00924\,\frac{\text{kW}}{\text{K}}$$

For the valve:

$$\dot{\sigma}_{\text{valve}} = \dot{m}(s_4 - s_3) = \left(0.241\,\frac{\text{kg}}{\text{s}}\right)(0.4179 - 0.4054)\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1 \text{ kW}}{1\,\frac{\text{kJ}}{\text{s}}}\right| = 0.003003\,\frac{\text{kW}}{\text{K}}$$

(f) The exergy destruction for the condenser is:

$$\dot{E}_{\text{d}_{\text{comp}}} = T_0\dot{\sigma}_{\text{comp}} = (293 \text{ K})\left(0.00924\,\frac{\text{kW}}{\text{K}}\right) = 2.718 \text{ kW}$$

Expressed as a percentage of the compressor power:

$$\% = \frac{\dot{E}_{\text{d}_{\text{comp}}}}{\dot{W}_{\text{c}}} = \frac{2.178}{5.647} = 0.48 = 48\%$$

The exergy destruction for the valve is:

$$\dot{E}_{\text{d}_{\text{valve}}} = T_0\dot{\sigma}_{\text{valve}} = (293 \text{ K})\left(0.003003\,\frac{\text{kW}}{\text{K}}\right) = 0.88 \text{ kW}$$

Expressed as a percentage of the compressor power:

$$\% = \frac{\dot{E}_{\text{d}_{\text{valve}}}}{\dot{W}_{\text{c}}} = \frac{0.88}{5.647} = 0.16 = 16\%$$

10.23 Data for steady state operation of a vapor-compression refrigeration cycle with propane as the working fluid are given in the table below. The sates are numbered as in Fig. 10.3. The mass flow rate of refrigerant is 8.42 lb/min. Heat transfer from the compressor to its surroundings occurs at a rate of 3.5 Btu per lb of refrigerant passing through the compressor. The condenser is water-cooled, with water entering at 65°F and leaving at 80°F with negligible change in pressure. Sketch the *T-s* diagram of the cycle and determine

(a) the refrigeration capacity, in tons.
(b) the compressor power, in horsepower.
(c) the mass flow rate of the condenser cooling water, in lb/min.
(d) the coefficient of performance.

State	p (lbf/in.^{2})	T (°F)	h (Btu/lb)
1	38.4	0	193.2
2	180	120	229.8
3	180	85	74.41
4	38.4	0	74.41

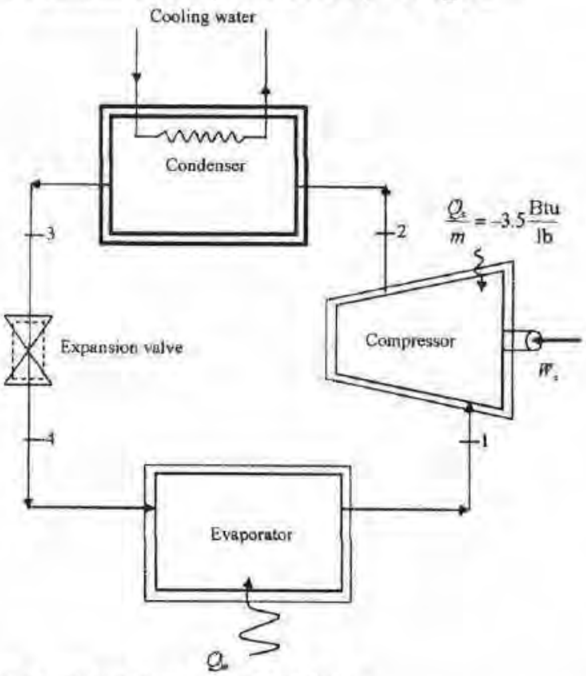
KNOWN: A vapor-compression refrigeration cycle operates with propane as its working fluid. The refrigerant mass flow rate and operating data at principal states in the cycle are provided. The condenser is water-cooled.

FIND: Sketch the *T-s* diagram of the cycle and determine (a) the refrigeration capacity, (b) the compressor power, (c) the mass flow rate of the condenser cooling water, and (d) the coefficient of performance.

SCHEMATIC AND GIVEN DATA:

State	p (lbf/in.^{2})	T (°F)	h (Btu/lb)
1	38.4	0	193.2
2	180	120	229.8
3	180	85	74.41
4	38.4	0	74.41

Problem 10.23 (Continued) – Page 2



ENGINEERING MODEL:

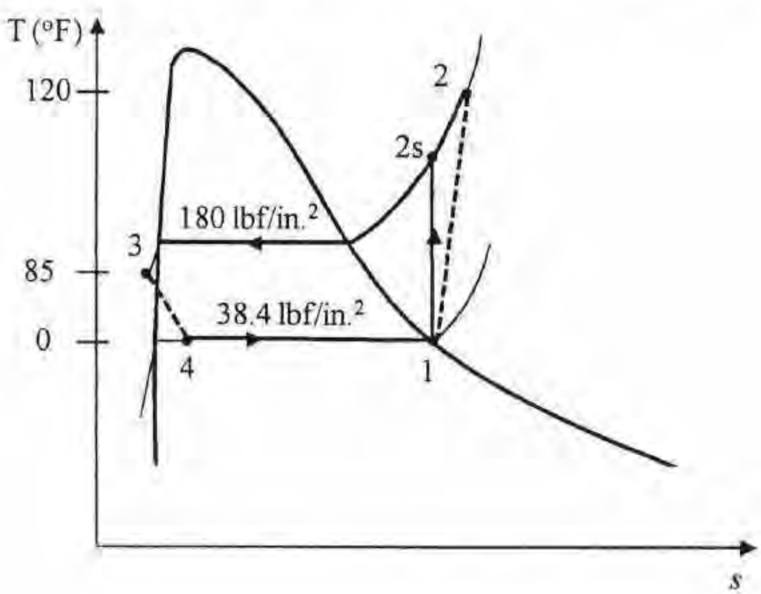
(1) Each component is analyzed as a control volume at steady state. The control volumes are indicated by dashed lines on the accompanying figure. There are no pressure drops through the evaporator and condenser.

(2) The expansion through the valve is a throttling process.

(3) Kinetic and potential energy effects are negligible.

(4) Heat transfer from the outside of the condenser can be neglected.

ANALYSIS: Referring to data from Table A-17E, the *T-s* diagram is



Problem 10.23 (Continued) – Page 3

(a) The refrigeration capacity is

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = \left(8.42\frac{\text{lb}}{\text{min}}\right)(193.2 - 74.41)\frac{\text{Btu}}{\text{lb}}\left|\frac{1\text{ ton}}{200\frac{\text{Btu}}{\text{min}}}\right| = 5\text{ tons}$$

(b) Applying energy and mass balances to the control volume enclosing the compressor, the compressor work is

$$\dot{W}_c = \dot{Q}_{c,out} + \dot{m}(h_2 - h_1) = \dot{m}\left[\left(\frac{\dot{Q}_{c,out}}{\dot{m}}\right) + (h_2 - h_1)\right]$$

Noting that $\dot{Q}_{c,out}/\dot{m} = 3.5$ Btu/lb, and inserting values

$$\dot{W}_c = \left(8.42\ \frac{\text{Btu}}{\text{lb}}\right)\left[\left(3.5\ \frac{\text{Btu}}{\text{lb}}\right) + (229.8 - 193.2)\frac{\text{Btu}}{\text{lb}}\right]\left|\frac{60\text{ min}}{1\text{ h}}\right|\left|\frac{1\text{ hp}}{2545\frac{\text{Btu}}{\text{h}}}\right| = 7.96\text{ hp}$$

Energy and mass balances on a control volume enclosing the condenser give

$$0 = \dot{m}(h_2 - h_3) + \dot{m}_{cw}(h_{cw,in} - h_{cw,out})$$

Using $h_{cw} = h_f(T)$, and solving for $\dot{m}_{cw}$

$$\dot{m}_{cw} = \dot{m}\left[\frac{(h_2 - h_3)}{h_f(T_{cw,out}) - h_f(T_{cw,in})}\right]$$

With data for $h_f(T)$ from Table A-2E

$$\dot{m}_{cw} = \left(8.42\frac{\text{lb}}{\text{min}}\right)\left[\frac{(229.8 - 74.41)}{(48.09 - 33.09)}\right] = 87.23\frac{\text{lb}}{\text{min}}$$

(c) The coefficient of performance is

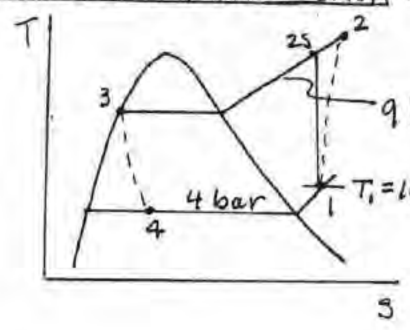
$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_c} = \frac{5\text{ tons}\left|\frac{200\frac{\text{Btu}}{\text{min}}}{1\text{ ton}}\right|}{7.96\text{ hp}\left|\frac{2545\frac{\text{Btu}}{\text{h}}}{1\text{ hp}}\right|\left|\frac{1\text{ h}}{60\text{ min}}\right|} = 2.96$$

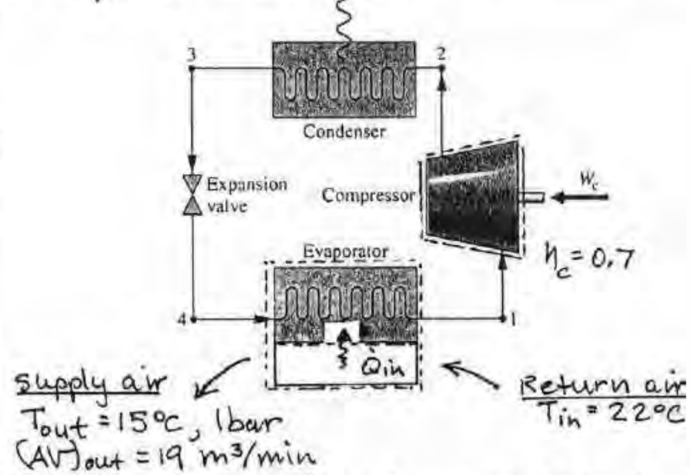
PROBLEM 10.24

KNOWN: Data are known for a window-mounted air conditioner. The air conditioner operates on a vapor-compression refrigeration cycle with R-22 as the working fluid. Operating data for the cycle are known, and conditions of the air entering and leaving the unit are specified.

FIND: Determine (a) the compressor power, (b) the refrigeration capacity, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) Each component is analyzed as a control volume at steady state. (2) There are no pressure drops through the evaporator and condenser. (3) The compressor operates adiabatically, with $\eta_c = 70\%$, and the expansion through the valve is a throttling process. (4) Saturated liquid leaves the condenser. (5) The air behaves as an ideal gas with $c_{p_{air}} = 1.005\ \frac{kJ}{kg \cdot K}$.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 4$ bar, $T_1 = 10°C \Rightarrow h_1 = 259.18$ kJ/kg, $s_1 = 0.9795$

State 2: $p_2 = 9$ bar, $s_{2s} = s_1 \Rightarrow h_{2s} = 280.68$ kJ/kg

Using the compressor efficiency

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c}\right) = 289.89 \text{ kJ/kg}$$

State 3: $p_3 = 9$ bar, sat. liquid $\Rightarrow h_3 = 68.59$ kJ/kg

State 4: Throttling process $\Rightarrow h_4 = h_3 = 68.59$ kJ/kg

To get the mass flowrate of R-22, analyze the evaporator. First

$$\dot{m}_{air} = \frac{(AV)_{air}\, p}{R\, T_{out}} = \frac{(19\ \frac{m^3}{min})(1\ bar)}{\left(\frac{8.314}{28.97}\ \frac{kJ}{kg \cdot K}\right)(288\ K)} \left|\frac{10^5\ N/m^2}{1\ bar}\right| \left|\frac{1\ kJ}{10^3\ N \cdot m}\right|$$

$$= 22.99\ kg/min$$

For the control volume enclosing the evaporator

$$0 = \dot{m}_R (h_4 - h_1) + \dot{m}_{air} (h_{in} - h_{out})$$

With $h_{in} - h_{out} = c_p (T_{in} - T_{out})$ $(c_{p_{air}} = 1.005\ kJ/kg \cdot K)$

PROBLEM 10.24 (Cont'd) - Page 2

$$\dot{m}_R = \frac{\dot{m}_{air}\, c_{p_{air}}(T_{in} - T_{out})}{(h_1 - h_4)}$$

$$= \frac{(22.99\ \text{kg/min})(1.005\ \text{kJ/kg}\cdot\text{K})(22-15)\text{K}}{(259.18 - 68.59)\ \text{kJ/kg}} = 0.8486\ \text{kg/min}$$

(a) the compessor power is

$$\dot{W}_c = \dot{m}_R(h_2 - h_1) = \left(0.8486\frac{\text{kg}}{\text{min}}\right)\left|\frac{1\ \text{min}}{60\ \text{s}}\right|(289.89 - 259.18)\frac{\text{kJ}}{\text{kg}}\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right|$$

$$= 0.434\ \text{kW} \longleftarrow \dot{W}_c$$

(b) the refrigeration capacity is

$$\dot{Q}_{in} = \dot{m}_R(h_1 - h_4)$$

$$= \left(0.8486\frac{\text{kg}}{\text{min}}\right)(259.18 - 68.59)\frac{\text{kJ}}{\text{kg}}\left|\frac{1\ \text{ton}}{211\ \text{kJ/min}}\right|$$

$$= 0.767\ \text{tons} \longleftarrow \dot{Q}_{in}$$

(c) The coefficient of performance is

$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = 6.21 \longleftarrow \beta$$

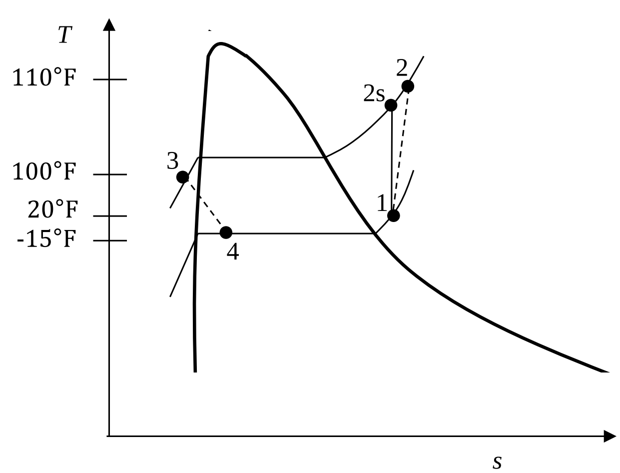
Problem 10.25

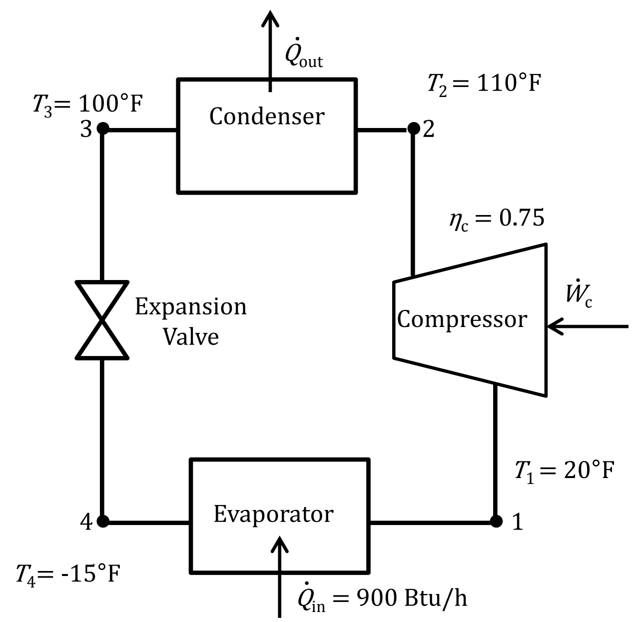
In a vapor-compression refrigeration system for a household refrigerator has a refrigerating capacity of 900 Btu/h. Refrigerant enters the evaporator at -15°F and exits at 20°F. The isentropic compressor efficiency is 75%. The refrigerant condenses at 110°F and exits the condenser subcooled at 100°F. There are no significant pressure drops in the flows through the evaporator and condenser. Determine the evaporator and condenser pressures, each in lbf/in.^{2}, the mass flow rate of refrigerant, in lb/min, the compressor power input, in horsepower, and the coefficient of performance for working fluids: (a) Refrigerant 134a, (b) propane, ~~and (c) CO_2~~
#1 ~~(using data from Fig. A-10E)~~.

Known:
Data are known at various locations in a vapor-compression refrigeration cycle. The refrigerating capacity is 900 Btu/h.

Find:
Determine the evaporator and condenser pressures, mass flow rate of refrigerant, compressor power input, and coefficient of performance for (a) Refrigerant 134a and (b) propane.

Schematic and Known Data:





Engineering Model:
(1) Each component of the cycle is analyzed as a control volume at steady state.
(2) There are no pressure drops through the evaporator and condenser.
(3) Kinetic and potential energy effects are negligible.
(4) The compressor and expansion valve operate adiabatically.

Analysis:
(a) First, fix each principal state for **Refrigerant 134a**:

State 1: $p_1 = p_{\text{sat}}@-15°\text{F} = 14.718\,\frac{\text{lbf}}{\text{in.}^2}$

Interpolating in Table A-12E at T_1 = 20°F and p_1: $h_1 = 106.37\,\frac{\text{Btu}}{\text{lb}},\ s_1 = 0.2391\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$

State 2: $p_2 = p_{\text{sat}}@110°\text{F} = 161.04\,\frac{\text{lbf}}{\text{in.}^2}$

For isentropic compression,

$p_{2s} = p_2$ and $s_{2s} = s_1 = 0.2391\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$, using Tab. A − 12E with double interpolation →

$h_{2s} = 129.85\,\frac{\text{Btu}}{\text{lb}}$

$$h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c} = 106.37 + \frac{129.85 - 106.37}{0.75} = 137.68\,\frac{\text{Btu}}{\text{lb}}$$

State 3: Using Table A-10E, $h_3 \approx h_f(T_3) = 44.23\,\frac{\text{Btu}}{\text{lb}}$

State 4: $h_4 = h_3 = 44.23\,\frac{\text{Btu}}{\text{lb}}$

The refrigerating capacity is used to find the mass flow rate:

$$\dot{m}_{\text{R134a}} = \frac{\dot{Q}_{\text{in}}}{h_1 - h_4} = \frac{900\frac{\text{Btu}}{\text{h}}}{(106.37 - 44.23)\frac{\text{Btu}}{\text{lb}}}\left|\frac{1\text{ h}}{60\text{ min}}\right| = 0.241\,\frac{\text{lb}}{\text{min}}$$

To find the compressor power input:

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(0.241\,\frac{\text{lb}}{\text{min}}\right)(137.68 - 106.37)\,\frac{\text{Btu}}{\text{lb}}\left|\frac{60\text{ min}}{1\text{ h}}\right|\left|\frac{1\text{ hp}}{2545\,\frac{\text{Btu}}{\text{h}}}\right| = 0.178\text{ hp}$$

To calculate the coefficient of performance:

$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = \frac{106.37 - 44.23}{137.68 - 106.37} = 1.985$$

(b) First, fix each principal state for **propane**:

State 1: $p_1 = p_{\text{sat}}@-15°\text{F} = 28.65\,\frac{\text{lbf}}{\text{in.}^2}$

Interpolating in Table A-18E at T_1 = 20°F and p_1: $h_1 = 202.2\,\frac{\text{Btu}}{\text{lb}},\ s_1 = 0.456\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$

#2 State 2: $p_2 = p_{\text{sat}}@110°\text{F} = 214.3\,\frac{\text{lbf}}{\text{in.}^2}$, $p_{2s} = p_2$ and $s_{2s} = s_1 = 0.456\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$, using Table A − 18E with double interpolation → $h_{2s} = 248.55\,\frac{\text{Btu}}{\text{lb}}$, $h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c} = 264\,\frac{\text{Btu}}{\text{lb}}$

State 3: Using Table A-16E, $h_3 \approx h_f(T_3) = 84.56\,\frac{\text{Btu}}{\text{lb}}$

State 4: $h_4 = h_3 = 84.56\,\frac{\text{Btu}}{\text{lb}}$

The refrigerating capacity is used to find the mass flow rate:

$$\dot{m}_{\text{propane}} = \frac{\dot{Q}_{\text{in}}}{h_1 - h_4} = \frac{900\,\frac{\text{Btu}}{\text{h}}}{(202.2 - 84.56)\,\frac{\text{Btu}}{\text{lb}}}\left|\frac{1\text{ h}}{60\text{ min}}\right| = 0.1275\,\frac{\text{lb}}{\text{min}}$$

To find the compressor power input:

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(0.1275\,\frac{\text{lb}}{\text{min}}\right)(264 - 202.2)\,\frac{\text{Btu}}{\text{lb}}\left|\frac{60\text{ min}}{1\text{ h}}\right|\left|\frac{1\text{ hp}}{2545\,\frac{\text{Btu}}{\text{h}}}\right| = 0.186\text{ hp}$$

To calculate the coefficient of performance:

#3 $\beta = \frac{h_1 - h_4}{h_2 - h_1} = \frac{202.2 - 84.56}{264 - 202.2} = 1.90$

Comments:

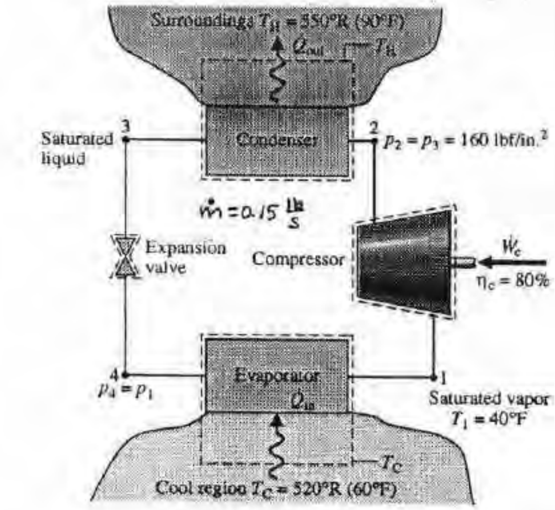
1. In the first printing of the 8^{th} Edition, part (c) using carbon dioxide was included in error. Please delete part (c) from the problem statement.
2. Note that the propane cycle operates at higher pressures than the R-134a cycle.
3. The propane cycle has a much lower mass flow rate than the R-134a cycle. However, the power and coefficient of performance are nearly the same for the two working fluids in this case.

PROBLEM 10.26

KNOWN: Steady-state operating data are provided for a vapor-compression air conditioning system.

FIND: Determine (a) the power required by the compressor, (b) the coefficient of performance, (c) the rates of exergy destruction in the compressor and valve, and the rates of exergy destruction and exergy transfer accompanying heat transfer for a control volumes enclosing (d) the evaporator and a portion of the cool region, (e) the condenser and a portion of the surroundings.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:
1. The control volumes shown in the schematic are at steady state.
2. Kinetic and potential energy changes are negligible as are pressure changes for flow through the evaporator and condenser.
3. The expansion across the valve is a throttling process.
4. There is no stray heat transfer for any control volume.
5. $T_0 = 550°R$.

ANALYSIS: For the principal states, essential property data can be obtained from the tables for R134a

state	T(°F)	p(lbf/in.^{2})	h (Btu/lb)	s (Btu/lb·°R)	
1	40	49.7	107.39	0.2189	(Table A-10E)
2	–	160	120.33	0.2234	
3	–	160	47.65	0.0958	(Table A-10E)
4	–	49.7	47.65	0.0994	

Using $s_{2s} = s_1$ and interpolating in Table A-12E gives h_{2s} = 117.74 Btu/lb. Then, using the isentropic efficiency $h_2 = h_1 + \frac{(h_{2s} - h_1)}{\eta_c} = 107.39 + \frac{(117.74 - 107.39)}{0.8} = 120.33$ Btu/lb

Interpolation in Table A-12E then yields s_2 = 0.2234 Btu/lb·°R.

Since $h_4 = h_3$, $x_4 = \frac{47.65 - 24.05}{83.34} = 0.2832 \Rightarrow s_4 = 0.0522 + 0.2832(0.2189 - 0.0522) = 0.0994 \frac{Btu}{lb \cdot °R}$

(a) An energy rate balance for the compressor gives the compressor power input:

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(0.15 \frac{lb}{s}\right)(120.33 - 107.39)\frac{Btu}{lb} = 1.94 \frac{Btu}{s} \qquad \text{(a)}$$

(b) The coefficient of performance is

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_c} = \frac{\dot{m}(h_1 - h_4)}{\dot{W}_c} = \frac{(0.15)(107.39 - 47.65)\ Btu/s}{1.94\ Btu/s} = \frac{8.96\ Btu/s}{1.94\ Btu/s} = 4.62 \qquad \text{(b)}$$

To evaluate exergy destruction rates, we use $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production obtained from an entropy rate balance. Thus

(c) For the compressor

$$(\dot{E}_d)_{comp} = T_0 \dot{m}(s_2 - s_1) = (550°R)(0.15 \tfrac{lb}{s})(0.2234 - 0.2189)\frac{Btu}{lb\cdot °R} = 0.371 \frac{Btu}{s}$$

For the valve

$$(\dot{E}_d)_{valve} = T_0 \dot{m}(s_4 - s_3) = (550)(0.15)(0.0994 - 0.0958) = 0.297 \frac{Btu}{s}$$

(d) For a control volume enclosing the evaporator, as shown in the schematic, an entropy rate balance reads

$$0 = \frac{\dot{Q}_{in}}{T_C} + \dot{m}(s_4 - s_1) + \dot{\sigma}_{cv}$$

$$\Rightarrow \quad (\dot{E}_d)_{evap} = T_0\left[-\frac{\dot{Q}_{in}}{T_C} + \dot{m}(s_1 - s_4)\right] = 550°R\left[-\frac{8.96\ Btu/s}{520°R} + (0.15 \tfrac{lb}{s})(0.2189 - 0.0994)\frac{Btu}{lb\cdot °R}\right]$$

$$= 0.382 \frac{Btu}{s}$$

The transfer of exergy accompanying heat transfer is

① $\dot{E}_q = \left[1 - \frac{T_0}{T_C}\right]\dot{Q}_{in} = \left[1 - \frac{550}{520}\right](8.96) = -0.517 \frac{Btu}{s}$. The exergy transfer is opposite to the direction of the heat transfer because heat transfer occurs at $T_C < T_0$.

(e) For a control volume enclosing the condenser, as shown in the schematic, an entropy rate balance reads

$$0 = -\frac{\dot{Q}_{out}}{T_H} + \dot{m}(s_2 - s_3) + \dot{\sigma}_{cv}$$

$$\Rightarrow \quad (\dot{E}_d)_{COND} = T_0\left[\frac{\dot{Q}_{out}}{T_H} + \dot{m}(s_3 - s_2)\right] = 550\left[\frac{10.9}{550} + 0.15(0.0958 - 0.2234)\right]$$

$$\left(\dot{Q}_{out} = \dot{m}(h_2 - h_3) = 0.15(120.33 - 47.65) = 10.9\ Btu/s\right)$$

$$= 0.373\ Btu/s$$

The transfer of exergy accompanying heat transfer is

$$\dot{E}_q = \left[1 - \frac{T_0}{T_H}\right]\dot{Q}_{out} = 0, \text{ since } T_0 = T_H.$$

---

1. The exergy evaluations can be summarized as follows:

- Rate of exergy in:
  - compressor power input — 1.94 $\frac{Btu}{s}$

- Disposition of the exergy:

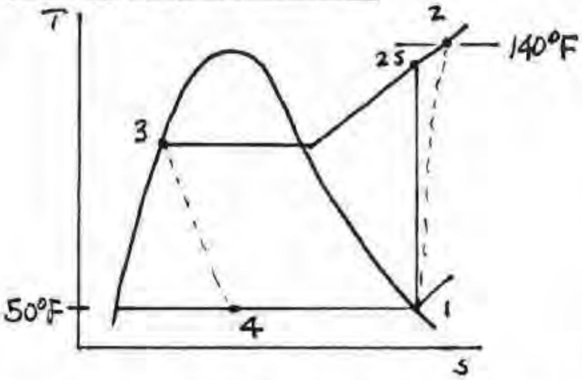
– Transfer to cool region	0.517 Btu/s		(26.6%)
– Transfer to warm region	0		
– Destructions			
compressor	0.371	"	(19.1%)
valve	0.297	"	(15.3%)
evaporator	0.382	"	(19.7%)
condenser	0.373	"	(19.2%)
	1.94 Btu/s		

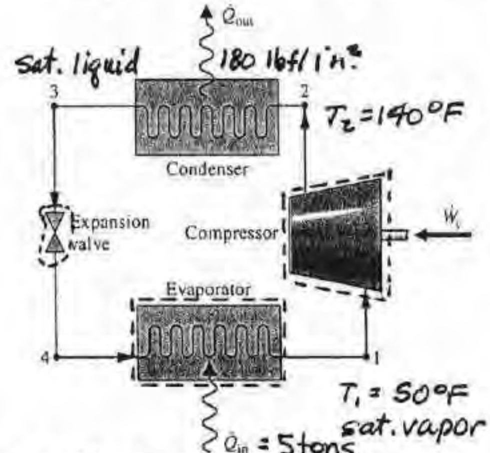
PROBLEM 10.27

KNOWN: Data are known at various locations in a vapor-compression refrigeration cycle. The refrigerating capacity is 5 tons.

FIND: Determine (a) the refrigerant mass flow rate, (b) the compressor isentropic efficiency, (c) the compressor power, and (d) the coefficient of performance. Plot each of these quantities for compressor exit temperatures ranging from 130°F to 140°F.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 10.3, items 1–4, except the compressor efficiency is not specified.

ANALYSIS: First, fix each of the principal states.

State 1 $T_1 = 50°F$, saturated vapor $\Rightarrow h_1 = 108.74$ Btu/lb, $s_1 = 0.2183$ Btu/lb·°R

State 2 $p_2 = 180$ lbf/in.², $T_2 = 140°F \Rightarrow h_2 = 123.21$ Btu/lb

State 3 $p_3 = 180$ lbf/in.², Sat. liquid $\Rightarrow h_3 = 50.64$ Btu/lb

State 4 Throttling process $\Rightarrow h_4 = h_3 = 50.64$ Btu/lb

(a) With the refrigerating capacity

$$\dot{m} = \frac{\dot{Q}_{in}}{(h_1-h_4)} = \frac{5 \text{ tons}}{(108.74-50.64)\text{ Btu/lb}}\left|\frac{200 \text{ Btu/min}}{1 \text{ ton}}\right| = 17.21 \text{ lb/min} \quad \dot{m}$$

(b) $p_2 = 180$ lbf/in.², $s_{2s} = s_1 \Rightarrow h_{2s} = 118.42$ Btu/lb

$$\eta_c = \frac{h_{2s}-h_1}{h_2-h_1} = \frac{118.42-108.74}{123.21-108.74} = 0.669 \ (66.9\%) \quad \eta_c$$

(c) $$\dot{W}_c = \dot{m}(h_2-h_1) = \left(17.21\frac{\text{lb}}{\text{min}}\right)(123.21-108.74)\frac{\text{Btu}}{\text{lb}}\left|\frac{60 \text{ min}}{1 \text{ h}}\right|\left|\frac{1 \text{ hp}}{2545 \text{ Btu/h}}\right|$$

$$= 5.87 \text{ hp} \quad \dot{W}_c$$

(d) $$\beta = \frac{\dot{Q}_{in}}{\dot{W}_c} = \left(\frac{5 \text{ tons}}{5.87 \text{ hp}}\right)\left|\frac{200 \text{ Btu/min}}{1 \text{ ton}}\right|\left|\frac{60 \text{ min}}{1 \text{ h}}\right|\left|\frac{1 \text{ hp}}{2545 \text{ Btu/h}}\right|$$

$$= 4.02 \quad \beta$$

PROBLEM 10.27 (Cont'd.) - Page 2

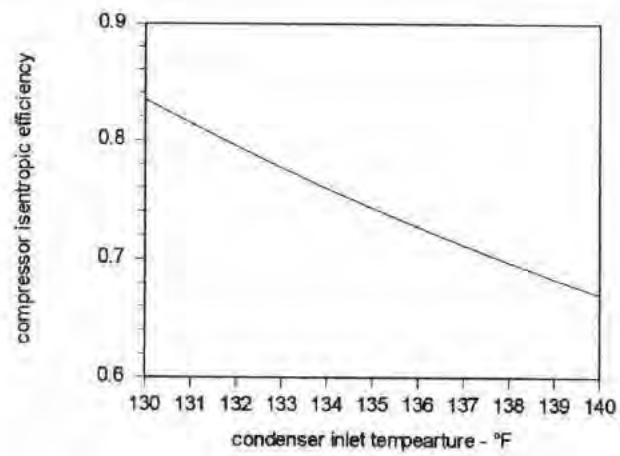
The required data for the plots are obtained using IT, as follows.

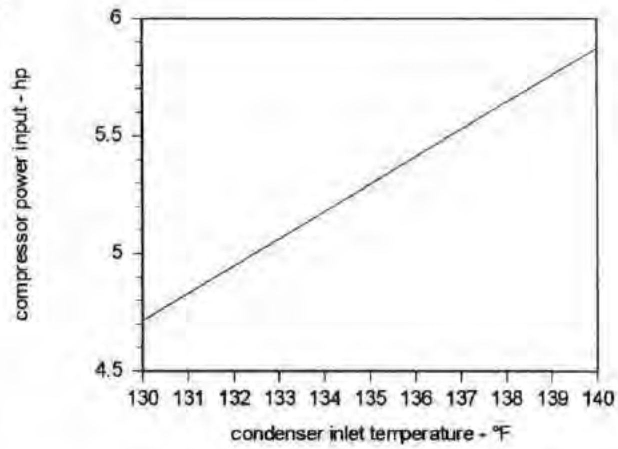
IT Code

```
T1 = 50  // °F
T2 = 140  // °F
p2 = 180  // lbf/in.²
p3 = p2
Qdotin = 5  // tons

p1 = Psat_T("R134A", T1)
h1 = hsat_Px("R134A", p1, 1)
s1 = ssat_Px("R134A", p1, 1)
h2 = h_PT("R134A", p2, T2)
s2s = s1
h2s = h_Ps("R134A", p2, s2s)
h3 = hsat_Px("R134A", p3, 0)
h4 = h3

mdot = Qdotin / (h1 - h4) * 200  // conversion to tons
etac = (h2s - h1) / (h2 - h1)
Wdotc = mdot * (h2 - h1 ) * (60 / 2545)  // conversion to hp
beta = (Qdotin / Wdotc) * (200 * 60 / 2545)
```

IT Results (for T_2 = 140 °F)

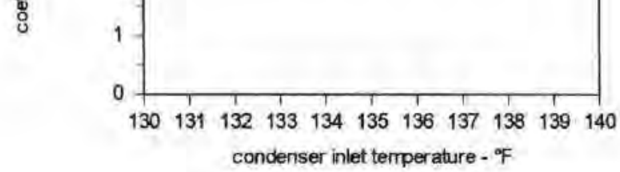
$\dot{m}$ = 17.21 lb/min
$\dot{W}_c$ = 5.873 hp
β = 4.014
η_c = 0.6699
h_1 = 108.7 Btu/lb
h_2 = 123.2 Btu/lb
h_{2s} = 118.4 Btu/lb
h_3 = 50.64 Btu/lb
h_4 = 50.64 Btu/lb





<u>Note</u>: $\dot{m}$ doesn't vary with T_2. Thus, no plot is included.

As T_2 increases, the compressor efficiency decreases and more input power is required. For fixed $\dot{Q}_{in}$, this results in a decrease in coefficient of performance.

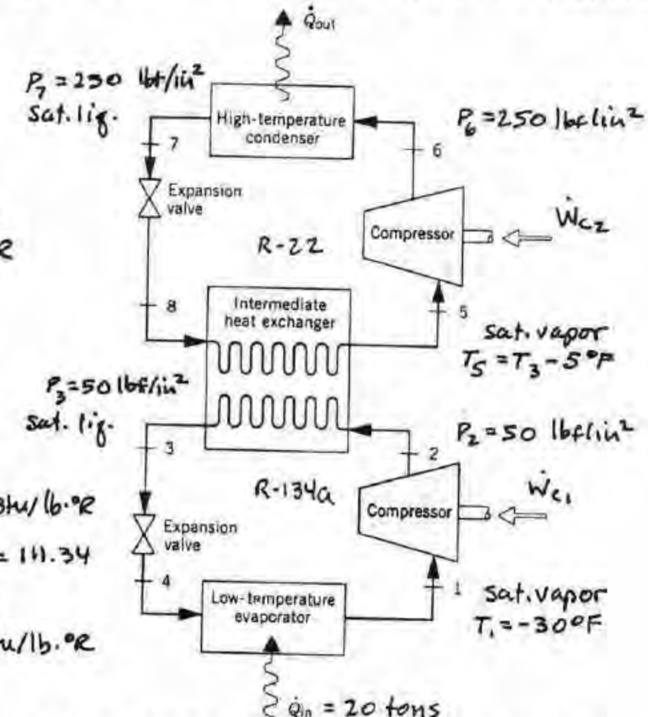


PROBLEM 10.28

KNOWN: A cascade refrigeration system is composed of two ideal vapor-compression cycles with an inter-connecting heat exchanger. Data are known at various locations in each cycle, and the refrigerating capacity is given.

FIND: Determine (a) the power input to each compressor, (b) the coefficient of performance, and (c) the rate of exergy destruction in the intermediate heat exchanger.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The items in Example 10.1 apply for each cycle. (2) There is no heat transfer from the outside of the intermediate heat exchanger. (3) Let $T_0 = 80°F = 540°R$ and $p_0 = 14.7$ lbf/in².

ANALYSIS: First, fix each of the principal states.

For the R-134a cycle:

State 1: $T_1 = -30°F$, sat. vapor
$\Rightarrow h_1 = 97.32$ Btu/lb, $s_1 = 0.2266$ Btu/lb·°R

State 2: $p_2 = 50$ lbf/in², $s_2 = s_1 \Rightarrow h_2 = 111.34$

State 3: $p_3 = 50$ lbf/in², sat. liquid
$\Rightarrow h_3 = 24.14$ Btu/lb, $s_3 = 0.0523$ Btu/lb·°R
$T_3 = 40.27°F$

State 4: $P_4 = P_1$, Throttling process; $h_4 = h_3 = 24.14 \Rightarrow s_4 = 0.0563$ Btu/lb·°R

For the R-22 cycle:

State 5: $T_5 = T_3 - 5 = 35.27°F$, sat. vapor $\Rightarrow h_5 = 107.8$ Btu/lb, $s_5 = 0.22095 \frac{\text{Btu}}{\text{lb·°R}}$

State 6: $P_6 = 250$ lbf/in², $s_6 = s_5 \Rightarrow h_6 = 120.51$ Btu/lb

State 7: $P_7 = 250$ lbf/in², Sat. liquid $\Rightarrow h_7 = 43.46$ Btu/lb

State 8: $T_8 = 35.27°F$; Throttling process $\Rightarrow h_8 = h_7 = 43.46$ Btu/lb and $s_8 = 0.09088$ Btu/lb·°R

(a) The mass flow rate of R-134a is determined using the given refrigerating capacity

$$\dot{Q}_{in} = \dot{m}_{R\text{-}134a}(h_1 - h_4) \Rightarrow \dot{m}_{R\text{-}134a} = \frac{\dot{Q}_{in}}{(h_1 - h_4)} = \frac{(20 \text{ tons})}{(97.32 - 24.14)\frac{\text{Btu}}{\text{lb}}}\left|\frac{200 \text{ Btu/min}}{1 \text{ ton}}\right|$$
$$= 54.66 \text{ lb/min}$$

Thus, for the R-134a compressor

$$\dot{W}_{c1} = \dot{m}_{R\text{-}134a}(h_2 - h_1) = \left(54.66 \frac{\text{lb}}{\text{min}}\right)(111.34 - 97.32)\frac{\text{Btu}}{\text{lb}}$$
$$= 766.3 \text{ Btu/min}$$ $\dot{W}_{c1}$

To find the refrigerant 22 mass flow rate, analyze the intermediate heat exchanger.

$$0 = \dot{m}_{R\text{-}134a}(h_2 - h_3) + \dot{m}_{R\text{-}22}(h_8 - h_5)$$

or

$$\dot{m}_{R\text{-}22} = \dot{m}_{R\text{-}134a}\left(\frac{h_2 - h_3}{h_5 - h_8}\right) = \left(54.66\ \frac{lb}{min}\right)\left(\frac{111.34 - 24.14}{107.8 - 43.46}\right)$$

$$= 74.08\ lb/min$$

Thus, for the R-22 compressor

$$\dot{W}_{c2} = \dot{m}_{R\text{-}22}(h_6 - h_5) = \left(74.08\ \frac{lb}{min}\right)(120.51 - 107.8)\ \frac{Btu}{lb} = 941.6\ Btu/min \longleftarrow \dot{W}_{c2}$$

(b) The overall coefficient of performance is

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{c1} + \dot{W}_{c2}} = \frac{(20\ tons)}{(766.3 + 941.6)\ Btu/min}\left|\frac{200\ Btu/min}{1\ ton}\right|$$

$$= 2.34 \longleftarrow \beta$$

(c) For the heat exchanger

$$0 = \cancel{\sum_j\left(\frac{\dot{Q}}{T}\right)_j}^{0} + \dot{m}_{R\text{-}134a}(s_2 - s_3) + \dot{m}_{R\text{-}22}(s_8 - s_5) + \dot{\sigma}_{HX}$$

Thus

$$\dot{E}_d = T_0\dot{\sigma}_{HX} = T_0\left[\dot{m}_{R\text{-}134a}(s_3 - s_2) + \dot{m}_{R\text{-}22}(s_5 - s_8)\right]$$

$$= (540°R)\left[(54.66)(.0523 - .2266)\frac{Btu}{lb\cdot°R} + (74.08)(.22095 - .09088)\right]$$

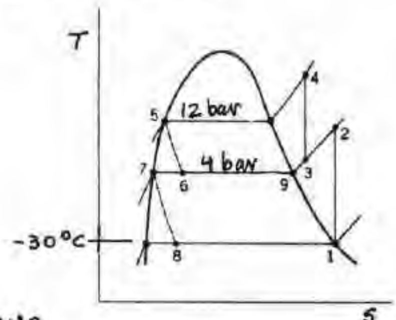
$$= 58.51\ Btu/min$$

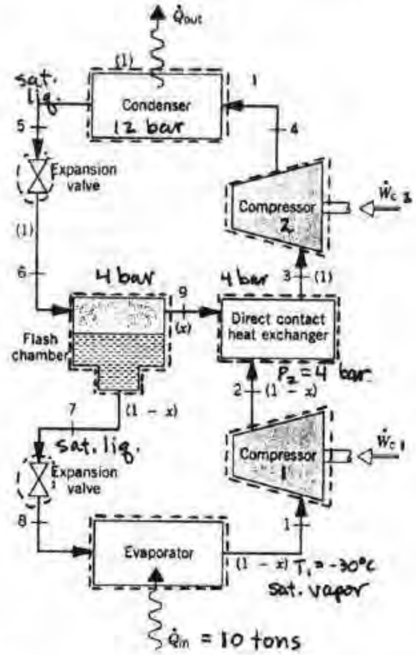
PROBLEM 10.29

KNOWN: Refrigerant 134a is the working fluid in a vapor-compression refrigeration system using two-stage compression with intercooling between the stages. The refrigerating capacity is given, and data at various locations are known.

FIND: Determine (a) the power input to each compressor, and (b) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The compressors operate isentropically. (3) There are no pressure drops for flow through any of the heat exchangers or the flash chamber. (4) The flash chamber and direct contact heat exchanger operate adiabatically. (5) The expansion in the valve is a throttling process. (6) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 1 $T_1 = -30\,°C$, sat. vapor $\Rightarrow h_1 = 229.14$ kJ/kg, $s_1 = 0.9434$ kJ/kg·K.

State 2 $p_2 = 4$ bar, $s_2 = s_1 \Rightarrow h_2 = 260.62$ kJ/kg

State 5 $p_5 = 12$ bar, sat. liquid $\Rightarrow h_5 = 115.76$ kJ/kg

State 6 Throttling process $\Rightarrow h_6 = h_5 = 115.76$ kJ/kg, $x_6 = 0.2825$

State 7 $p_7 = 4$ bar, sat. liquid $\Rightarrow h_7 = 62.00$ kJ/kg

State 8 Throttling process $\Rightarrow h_8 = h_7 = 62.00$ kJ/kg

State 9 $p_9 = 4$ bar, sat. vapor $\Rightarrow h_9 = 252.32$ kJ/kg

State 3 The fraction of the flow into the flash chamber at 6 that exits as saturated vapor at 9 is equal to the quality at 6. The liquid leaving the flash chamber at 7 is the fraction $1-x_6$. With these flow rate ratios

$$0 = (1-x_6)h_2 + x_6 h_9 - 1\,h_3$$

or

$$h_3 = (1-x_6)h_2 + x_6 h_9$$

$$= (1-0.2825)(260.62) + 0.2825(252.32) = 258.27 \text{ kJ/kg}$$

and by interpolation in Table A-12 with h_3 we get $s_3 = 0.9352$ kJ/kg·K.

State 4. $p_4 = 12$ bar, $s_4 = s_3 \Rightarrow h_4 = 281.69$ kJ/kg.

PROBLEM 10.29 (Contd.) - Page 2

(a) To determine the compressor power, first determine the mass flow rates. For the evaporator

$$\dot{Q}_{in} = \dot{m}_1(h_1 - h_8)$$

or

$$\dot{m}_1 = \frac{\dot{Q}_{in}}{(h_1-h_8)} = \frac{(10 \text{ tons})}{(229.14 - 62.00)\frac{kJ}{kg}} \left|\frac{211 \text{ kJ/min}}{1 \text{ ton}}\right| \left|\frac{1 \text{ min}}{60 \text{ s}}\right|$$

$$= 0.21 \text{ kg/s}$$

Also, since $1-x_6$ is the fraction of the total flow passing through the evaporator

$$\frac{\dot{m}_1}{\dot{m}_3} = 1 - x_6 \Rightarrow \dot{m}_3 = \frac{\dot{m}_1}{1-x_6} = 0.293 \text{ kg/s}$$

Thus,

$$\dot{W}_{c1} = \dot{m}_1(h_2 - h_1) = (0.21 \tfrac{kg}{s})(260.62 - 229.14)\frac{kJ}{kg}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$

$$= 6.61 \text{ kW} \longleftarrow \dot{W}_{c1}$$

and

$$\dot{W}_{c2} = \dot{m}_3(h_4 - h_3) = (0.293)(281.69 - 258.27) = 6.86 \text{ kW} \longleftarrow \dot{W}_{c2}$$

(b) The coefficient of performance is

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{c1}+\dot{W}_{c2}} = \frac{(10 \text{ tons})}{(6.61+6.86)\text{ kW}}\left|\frac{211 \text{ kJ/min}}{1 \text{ ton}}\right|\left|\frac{1 \text{ min}}{60 \text{ s}}\right|\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$

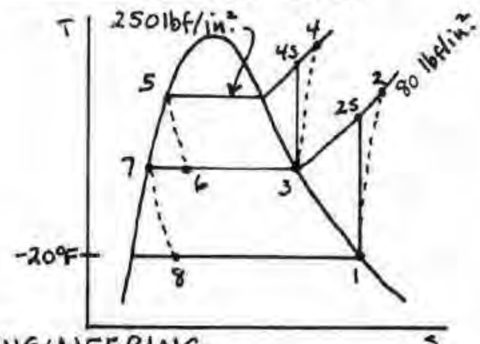
$$= 2.61 \longleftarrow \beta$$

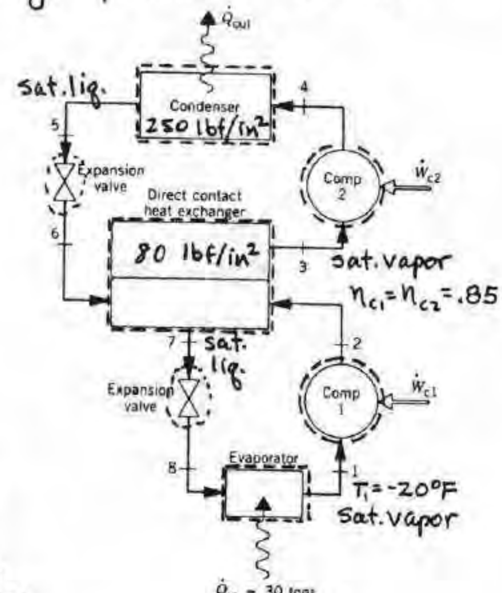
PROBLEM 10.30

KNOWN: Ammonia is the working fluid in a two-stage vapor-compression refrigeration system using a direct contact heat exchanger to achieve intercooling. Data are known at various locations, and the refrigerating capacity is given.

FIND: Determine (a) the ratio of the mass flow rates $\dot{m}_3/\dot{m}_1$ (see figure), (b) the power input to each compressor stage, and (c) the coefficient of performance. (d) Plot each of the quantities in parts (a)-(c) versus the direct-contact heat exchanger pressure. Discuss.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) Each component is analyzed as a control volume at steady state. (2) The compressors, valves, and direct contact heat exchanger each operate adiabatically. (3) The expansions through the valves are throttling processes. (4) There are no significant pressure drops for flow through the heat exchangers. (5) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 1 $T_1 = -20\,°F$, sat. vapor $\Rightarrow h_1 = 604.61$ Btu/lb, $s_1 = 1.3762$ Btu/lb·°R

State 2 For isentropic compression, $p_2 = 80$ lbf/in², $s_{2s} = s_1 \Rightarrow h_{2s} = 692.14$ $\frac{Btu}{lb}$
Using the compressor efficiency,

$$\eta_c = \frac{h_{2s}-h_1}{h_2-h_1} \Rightarrow h_2 = h_1 + \left(\frac{h_{2s}-h_1}{\eta_c}\right) = 707.59 \text{ Btu/lb}$$

State 3 $p_3 = 80$ lbf/in², sat. vapor $\Rightarrow h_3 = 623.32$ Btu/lb, $s_3 = 1.2529$ Btu/lb·°R

State 4 $p_4 = 250$ lbf/in², $s_{4s} = s_3 \Rightarrow h_{4s} = 693.53$ Btu/lb. thus

$$h_4 = h_3 + \left(\frac{h_{4s}-h_3}{\eta_c}\right) = 705.92 \text{ Btu/lb}$$

State 5 $p_5 = 250$ lbf/in², sat. liquid $\Rightarrow h_5 = 167.77$ Btu/lb

State 6 Throttling process $\Rightarrow h_6 = h_5 = 167.77$ Btu/lb

State 7 $p_7 = 80$ lbf/in², sat. liquid $\Rightarrow h_7 = 91.22$ Btu/lb

State 8 Throttling process $\Rightarrow h_8 = h_7 = 91.22$ Btu/lb

(a) The mass flow rate through the evaporator is

$$\dot{m}_1 = \frac{\dot{Q}_{in}}{h_1 - h_8} = \frac{(30 \text{ tons})}{(604.61 - 91.22)\frac{Btu}{lb}}\left|\frac{200 \text{ Btu/min}}{1 \text{ ton}}\right| = 11.69 \frac{lb}{min}$$

The mass flow rate into the second turbine stage is determined by using an energy balance on the direct contact heat exchanger

$$0 = \dot{m}_6 h_6 + \dot{m}_2 h_2 - \dot{m}_3 h_3 - \dot{m}_7 h_7$$

But $\quad \dot{m}_6 = \dot{m}_3$ and $\dot{m}_7 = \dot{m}_2 = \dot{m}_1$

Thus $\quad 0 = \dot{m}_3 (h_6 - h_3) + \dot{m}_1 (h_2 - h_7)$

or $\quad \dot{m}_3 = \dot{m}_1 \left(\frac{h_7 - h_2}{h_6 - h3}\right) = (11.69)\left(\frac{91.22 - 707.59}{167.77 - 623.32}\right) = 15.82 \text{ lb/min}$

The mass flow rate ratio is

$\dot{m}_3/\dot{m}_1 = 1.353 \quad \dot{m}_3/\dot{m}_1$

(b) The power input to the first compressor stage is

$$\dot{W}_{c1} = \dot{m}_1 (h_2 - h_1)$$
$$= \left(11.69 \frac{\text{lb}}{\text{min}}\right)\left|\frac{60 \text{ min}}{1 \text{ h}}\right|(707.59 - 604.61)\frac{\text{Btu}}{\text{lb}}\left|\frac{1 \text{ hp}}{2545 \text{ Btu/h}}\right|$$
$$= 28.38 \text{ hp} \quad \dot{W}_{c1}$$

And, the power input to the second compressor stage is

$$\dot{W}_{c2} = \dot{m}_3 (h_4 - h_3)$$
$$= (15.82)(705.92 - 623.32)\left|\frac{60}{2545}\right| = 30.81 \text{ hp} \quad \dot{W}_{c2}$$

(c) The coefficient of performance is

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{c1} + \dot{W}_{c2}} = \frac{(30 \text{ tons})}{(28.38 + 30.81)\text{hp}}\left|\frac{1 \text{ hp}}{2545 \text{ Btu/h}}\right|\left|\frac{200 \text{ Btu/min}}{1 \text{ ton}}\right|\left|\frac{60 \text{ min}}{1 \text{ h}}\right|$$
$$= 2.390 \quad \beta$$

(d) The data for the required plots are obtained using IT, as follows:

<u>IT Code</u>

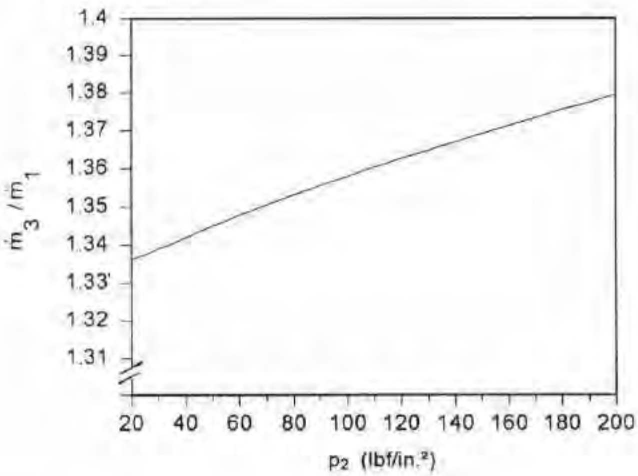
```
Qdotin = 30   // tons
T1 = -20   // °F
p2 = 80   // lbf/in²
p3 = p2
p4 = 250   // lbf/in²
eta_c1 = 0.85
eta_c2 = 0.85

x1 = 1
p1 = Psat_T("Ammonia", T1)
h1 = hsat_Px("Ammonia", p1, x1)
s1 = ssat_Px("Ammonia", p1, x1)
s2s = s1
h2s = h_Ps("Ammonia", p2, s2s)
h2 = h1 + (h2s - h1) / eta_c1
x3 = 1
h3 = hsat_Px("Ammonia", p3, x3)
s3 = ssat_Px("Ammonia", p3, x3)
s4s = s3
h4s = h_Ps("Ammonia", p4, s4s)
h4 = h3 + (h4s - h3) / eta_c2
p5 = p4
h5 = hsat_Px("Ammonia", p5, 0)
h6 = h5
p7 = p3
h7 = hsat_Px("Ammonia", p7, 0)
h8 = h7

mdot1 = (Qdotin * 200) / (h1 - h8)  // Btu/min
0 = mdot3 * (h6 - h3) + mdot1 * (h2 - h7)
mratio = mdot3 / mdot1
Wdotc1 = mdot1 * (h2 - h1) * (60 / 2545)
Wdotc2 = mdot3 * (h4 - h3) * (60 / 2545)
Wdotc = Wdotc1 + Wdotc2
beta = ((Qdotin*200)/(Wdotc1+Wdotc2))* (60/2545)
```

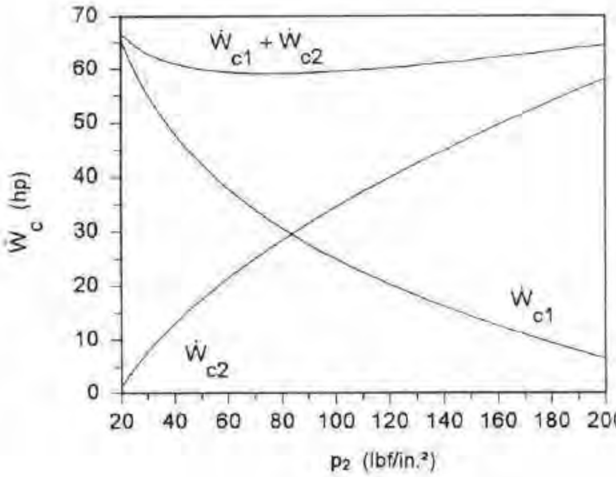
<u>IT Results for p_2 = 80 lbf/in.²</u>

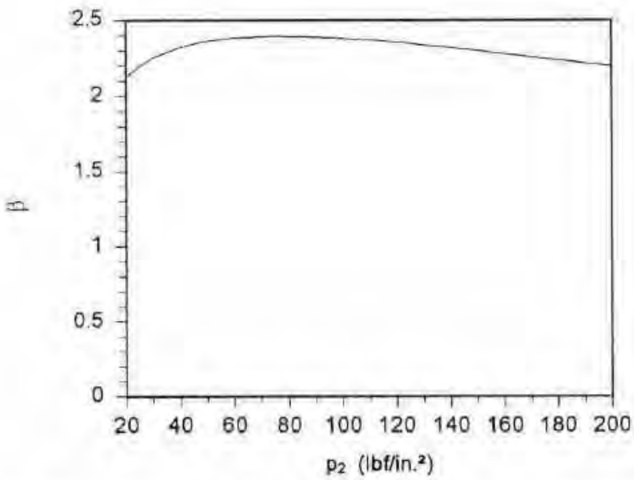
h_1 = 604.2 Btu/lb
h_2 = 707.1 Btu/lb
h_{2s} = 691.7 Btu/lb
h_3 = 622.9 Btu/lb
h_4 = 705.2 Btu/lb
h_{4s} = 692.9 Btu/lb
h_5 = 167.7 Btu/lb
h_6 = 167.7 Btu/lb
h_7 = 91.16 Btu/lb
h_8 = 91.16 Btu/lb
$\dot{m}_3/\dot{m}_1$ = 1.353
$\dot{W}_{c1}$ = 28.37 hp
$\dot{W}_{c2}$ = 30.71 hp
$\dot{W}_c$ = 59.08 hp
β = 2.394

PROBLEM 10.30 (Cont'd.) - page 3

PLOTS:







From the plots we see that as the interstage pressure increases, the fraction of the flow entering the first compressor stage that enters the second stage increases. Also, the power required by the first stage decreases and that required by the second stage increases. The curve that shows the total compressor power exhibits a minimum value. Accordingly, the coefficient of performance exhibits a maximum value.

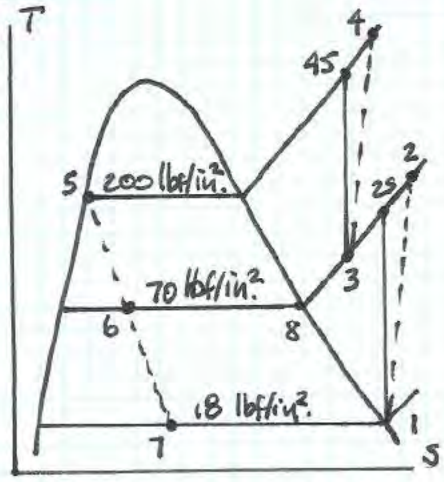
PROBLEM 10.31

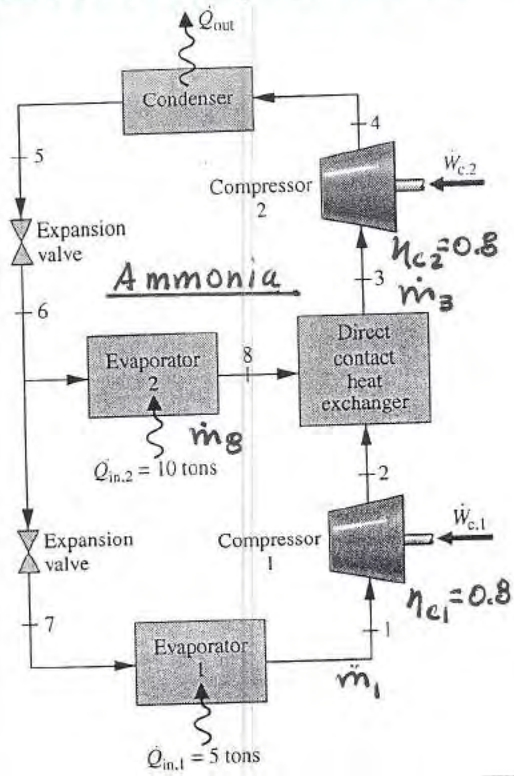
KNOWN: Ammonia is the working fluid in a two-stage, vapor-compression refrigeration system with two evaporators and a direct contact heat exchanger. Data are given at various locations, and the capacities of both evaporators are specified.

①

FIND: Determine (a) the evaporator temperatures, (b) the power input to each compressor stage, and (c) the overall coefficient of performance.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL:
(1) Each component operates as a control volume at steady state. (2) There are no pressure drops through the evaporators, condenser, or direct contact heat exchanger. (3) The compressors and heat exchanger operate adiabatically. The expansion through the valve is a throttling process. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 1 $p_1 = 18\ \text{lbf/in.}^2$, sat. vapor $\Rightarrow$ $h_1 = 604.40\ \frac{\text{Btu}}{\text{lb}}$

$s_1 = 1.3775\ \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$

State 2 $p_2 = 70\ \text{lbf/in.}^2$, $s_{2s} = s_1 \Rightarrow h_{2s} = 683.72\ \text{Btu/lb}$

With the compressor stage efficiency; $\eta_c = (h_{2s} - h_1)/(h_2 - h_1)$

$$h_2 = h_1 + (h_{2s} - h_1)/\eta_{c_1} = 604.40 + (683.72 - 604.40)/0.8 = 703.55\ \text{Btu/lb}$$

State 3 First, get $\dot{m}_1$ and $\dot{m}_8$. $\boxed{\text{Note: } h_5 = h_6 = h_7 = 150.73\ \text{Btu/lb}}$

$$\dot{m}_1 = \frac{\dot{Q}_{in,1}}{(h_1 - h_7)} = \frac{5\ \text{tons}}{(604.40 - 150.73)\frac{\text{Btu}}{\text{lb}}}\left|\frac{200\ \text{Btu/min}}{1\ \text{ton}}\right| = 2.204\ \text{lb/min} = \dot{m}_2$$

$$\dot{m}_8 = \frac{\dot{Q}_{in,2}}{(h_8 - h_6)} = \frac{10\ |200|}{(621.74 - 150.73)} = 4.246\ \text{lb/min}$$

$h_8 = h_g$ @ 70 lbf/in.²

For the direct contact heat exchanger: $0 = \dot{m}_2 h_2 + \dot{m}_8 h_8 - (\dot{m}_2 + \dot{m}_8) h_3$

$$h_3 = \frac{\dot{m}_2 h_2 + \dot{m}_8 h_8}{(\dot{m}_2 + \dot{m}_8)} = \frac{(2.204)(703.55) + (4.246)(621.74)}{(2.204 + 4.246)} = 649.69\ \frac{\text{Btu}}{\text{lb}}$$

State 4 $p_3 = 70\ \text{lbf/in.}^2$, $h_3 = 649.69\ \text{Btu/lb} \Rightarrow s_3 = 1.3179\ \text{Btu/lb}\cdot°\text{R}$

$p_4 = 200\ \text{lbf/in.}^2$, $s_{4s} = s_3 \Rightarrow h_{4s} = 720.82\ \text{Btu/lb}$. $\eta_{c_2} = (h_{4s} - h_3)/(h_4 - h_3)$

$$h_4 = h_3 + (h_{4s} - h_3)/\eta_{c_2} = 649.69 + (720.82 - 649.69)/0.8 = 738.6\ \text{Btu/lb}$$

States 5, 6, 7, 8 $h_5 = h_6 = h_7 = 150.73\ \text{Btu/lb}$, $h_8 = 621.74\ \text{Btu/lb}$

PROBLEM 10.31 (Cont'd.) - Pag2

(a) From Table A-14E: $T_1 = -20.60\ ^\circ F$, $T_8 = 37.67$ T_1, T_8

(b) $\dot{W}_{c_1} = \dot{m}_1 (h_2 - h_1)$

$= (2.204 \frac{lb}{min})(703.55 - 604.40) \frac{Btu}{lb} \left| \frac{60\ min}{1\ h} \right| \left| \frac{1\ hp}{2545\ Btu/h} \right|$

$= 5.15\ hp$ $\dot{W}_{c1}$

$\dot{W}_{c_2} = (\underbrace{6.45}_{\dot{m}_3 = \dot{m}_2 + \dot{m}_8})(738.6 - 649.69) \left| \frac{60}{2545} \right| = 13.5\ hp$ $\dot{W}_{c2}$

(c) $\beta = \frac{(\dot{Q}_{in,1} + \dot{Q}_{in,2})}{(\dot{W}_{c1} + \dot{W}_{c2})} = \frac{(10 + 5)\ tons}{(5.15 + 13.5)\ hp} \left| \frac{12000\ Btu/h}{1\ ton} \right| \left| \frac{1\ hp}{2545\ Btu/h} \right|$

$= 3.79$ β

Problem 10.32

Figure P10.32 shows the schematic diagram of a vapor-compression refrigeration system with two evaporators using Refrigerant 134a as the working fluid. This arrangement is used to achieve refrigeration at two different temperatures with a single compressor and a single condenser. The lower temperature evaporator has a refrigerating capacity of 3 tons while the higher-temperature evaporator has a refrigerating capacity of 2 tons. Operating data are provided in the accompanying table. Calculate:

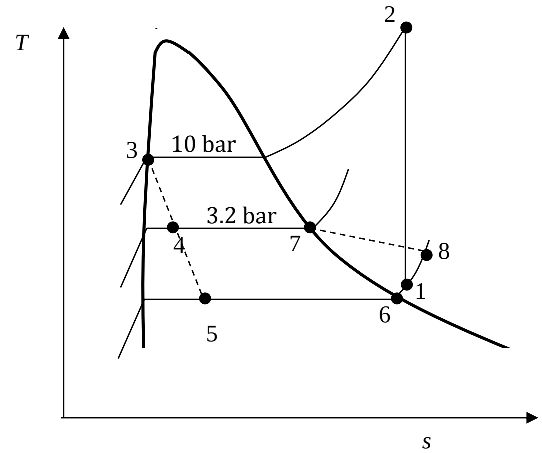
(a) the mass flow rate of refrigerant through each evaporator, in kg/min.
(b) the compressor power input, in kW.
(c) the rate of heat transfer from the refrigerant passing through the condenser, in kW.

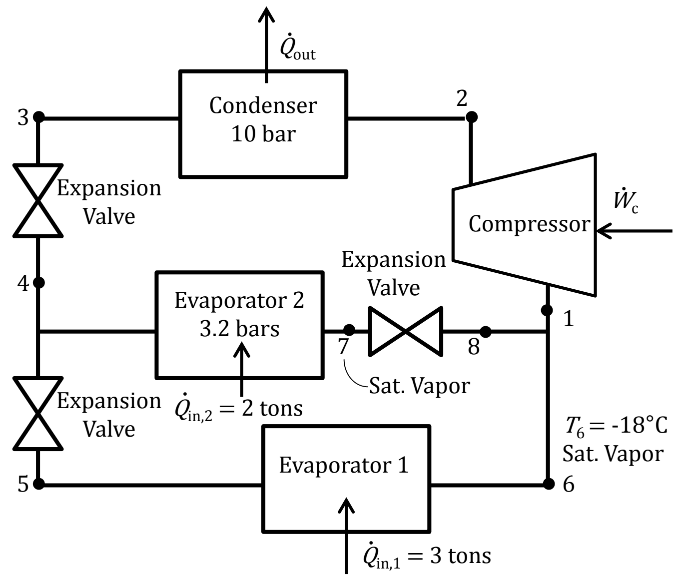
Known:
Refrigerant 134a is the working fluid in a vapor-compression refrigeration system with two evaporators. Data are known at various locations and the refrigerating capacity of each evaporator is specified. The system uses only one compressor.

Find:
Determine (a) the mass flow rates through each evaporator, (b) the compressor power input, and (c) the heat transfer from the refrigerant passing through the condenser.

Schematic and Known Data:

State	***p*** **(bar)**	***T*** **(°C)**	***h*** **(kJ/kg)**	***s*** **(kJ/kg·K)**
1	1.4483	-12.52	241.13	0.9493
2	10	51.89	282.3	0.9493
3	10	39.39	105.29	0.3838
4	3.2	2.48	105.29	0.3975
5	1.4483	-18	105.29	0.4171
6	1.4483	-18	236.53	0.9315
7	3.2	2.48	248.66	0.9177
8	1.4483	-3.61	248.66	0.9779





Engineering Model:
(1) Each component of the cycle is analyzed as a control volume at steady state.
(2) All processes of the working fluid are internally reversible except for the expansion through each valve, which is a throttling process.
(3) Kinetic and potential energy effects are negligible.
(4) The compressor operates adiabatically.

Analysis:
(a) The mass flow rates through each evaporator can be found as follows:

$$\dot{Q}_{\text{in}_1} = \dot{m}_6(h_6 - h_5) \rightarrow \dot{m}_6 = \frac{\dot{Q}_{\text{in}_1}}{h_6 - h_5} = \frac{3 \text{ tons}}{(236.53 - 105.29)\,\frac{\text{kJ}}{\text{kg}}}\left|\frac{211\,\frac{\text{kJ}}{\text{min}}}{1 \text{ ton}}\right| = 4.823\,\frac{\text{kg}}{\text{min}}$$

$$\dot{m}_8 = \frac{\dot{Q}_{\text{in}_2}}{h_7 - h_4} = \frac{2 \text{ tons}}{(248.66 - 105.29)\frac{\text{kJ}}{\text{kg}}} \cdot \frac{211\frac{\text{kJ}}{\text{min}}}{1 \text{ ton}} = 2.943\,\frac{\text{kg}}{\text{min}}$$

(b) Recognizing $\dot{m}_6 + \dot{m}_8 = \dot{m}_1$, the compressor power is: $\dot{W}_c = (\dot{m}_6 + \dot{m}_8)(h_2 - h_1) =$

$$\left[(4.823 + 2.943)\left|\frac{\text{kg}}{\text{min}}\right|\left|\frac{1 \text{ min}}{60 \text{ s}}\right|\right]\left[(282.3 - 241.13)\,\frac{\text{kJ}}{\text{kg}}\right]\left|\frac{1 \text{ kW}}{1\,\frac{\text{kJ}}{\text{s}}}\right| = 5.329 \text{ kW}$$

(c) For the condenser:

$$\dot{Q}_{\text{out}} = (\dot{m}_6 + \dot{m}_8)(h_2 - h_3) =$$

$$\left[(4.823 + 2.943)\left|\frac{\text{kg}}{\text{min}}\right|\left|\frac{1 \text{ min}}{60 \text{ s}}\right|\right]\left[(282.3 - 105.29)\,\frac{\text{kJ}}{\text{kg}}\right]\left|\frac{1 \text{ kW}}{1\,\frac{\text{kJ}}{\text{s}}}\right| = 22.91 \text{ kW}$$

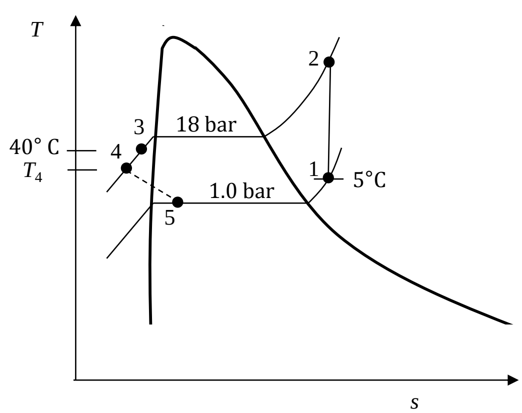
Problem 10.33

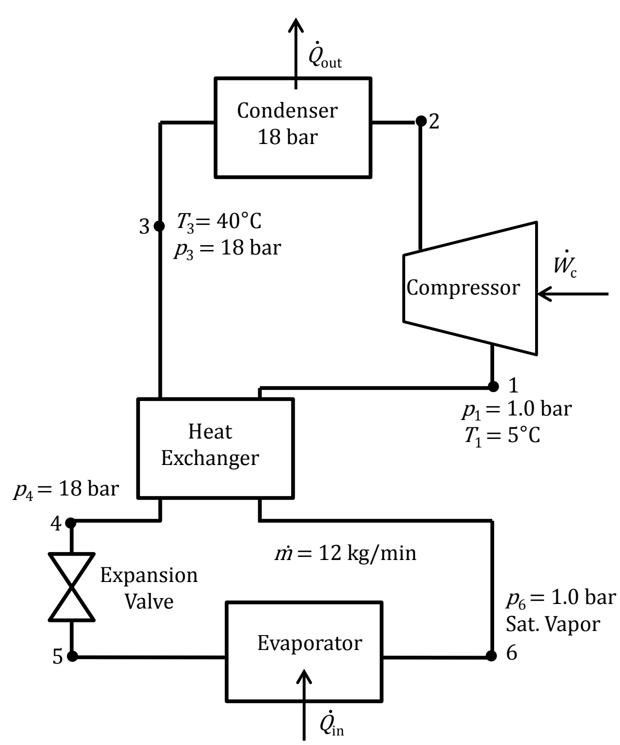
An ideal vapor-compression refrigeration cycle is modified to include a counterflow heat exchanger, as shown in Fig. P10.33. Ammonia leaves the evaporator as saturated vapor at 1.0 bar and is heated at constant pressure to 5°C before entering the compressor. Following isentropic compression to 18 bar, the refrigerant passes through the condenser, exiting at 40°C, 18 bar. The liquid then passes through the heat exchanger, entering the expansion valve at 18 bar. If the mass flow rate of the refrigerant is 12 kg/min, determine:
(a) the refrigeration capacity, in tons of refrigeration.
(b) the compressor power input, in kW.
(c) the coefficient of performance.
(d) the rate of entropy production in the compressor, in kW/K.
(e) the rate of exergy destruction in the compressor, in kW. Let T_0 = 20°C.
Discuss advantages and disadvantages of this arrangement.

Known:
An ideal vapor-compression refrigeration cycle is modified to include a counterflow heat exchanger between the streams exiting the condenser and evaporator. Ammonia is the working fluid. Data are known at various locations, and the mass flow rate is given.

Find:
Determine (a) the refrigerating capacity, (b) the compressor power input, (c) the coefficient of performance, (d) the rate of entropy production for the compressor, and (e) the rate of exergy destruction for the compressor.

Schematic and Known Data:





Engineering Model:

(1) Each component of the cycle is analyzed as a control volume at steady state.
(2) Except for the expansion through the valve, which is a throttling process, all processes of the refrigerant are internally reversible.
(3) The compressor and expansion valve operate adiabatically.
(4) Kinetic and potential energy effects are negligible.
(5) Heat transfer from the outside of the heat exchanger can be neglected.
(6) Let T_0 = 293 K.

Analysis:

First, fix each of the principal states.

State 1: $p_1 = 1\text{ bar}, T_1 = 5℃ \to h_1 = 1483.25\,\frac{\text{kJ}}{\text{kg}}, s_1 = 6.1676\,\frac{\text{kJ}}{\text{kg·K}}$

State 2: $p_2 = 18\text{ bar},\ s_2 = s_1 \to h_2 = 2025.54\,\frac{\text{kJ}}{\text{kg}}$

State 3: $p_3 = 18\text{ bar},\ T_3 = 40℃ \to h_3 \approx h_f(40℃) = 371.35\,\frac{\text{kJ}}{\text{kg}}$

State 4: Using an energy balance on the heat exchanger control volume:

$$0 = \dot{m}(h_6 - h_1) + \dot{m}(h_3 - h_4) \to h_4 = h_6 - h_1 + h_3 = 286.51\,\frac{\text{kJ}}{\text{kg}}$$

State 5: Throttling process $\to h_5 = h_4 = 286.51\,\frac{\text{kJ}}{\text{kg}}$

State 6: $p_6 = 1.0\text{ bar}, \text{saturated vapor} \to h_6 = 1398.41\,\frac{\text{kJ}}{\text{kg}}$

(a) The refrigerating capacity is:

$$\dot{Q}_{in} = \dot{m}(h_6 - h_5) = \left(12\,\frac{\text{kg}}{\text{min}}\right)(1398.41 - 286.5)\,\frac{\text{kJ}}{\text{kg}}\left|\frac{1\text{ ton}}{211\frac{\text{kJ}}{\text{min}}}\right| = 63.24\text{ tons}$$

(b) The compressor power is:

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(12\,\frac{\text{kg}}{\text{min}}\right)(2025.54 - 1483.25)\,\frac{\text{kJ}}{\text{kg}}\left|\frac{1\text{ min}}{60\text{ s}}\right|\left|\frac{1\text{ kW}}{1\,\frac{\text{kJ}}{\text{s}}}\right| = 108.5\text{ kW}$$

(c) The coefficient of performance is:

$$\beta = \frac{h_6 - h_5}{h_2 - h_1} = 2.05$$

(d) As expected with an ideal cycle, the rate of entropy production for the compressor is:

$$0 = \sum_j \underbrace{\frac{\dot{Q}_j}{T_j}}_{=0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{\text{comp}} \to \dot{\sigma}_{\text{comp}} = \dot{m}(s_2 - s_1) = 0\,\frac{\text{kW}}{\text{K}}$$

(e) The rate of exergy destruction for the compressor follows as $\dot{\text{E}}_\text{d} = T_0\dot{\sigma}_{\text{comp}} = 0\text{ kW}$

Discussion: The heat exchanger (1) tends to increase the capacity, (2) helps ensure superheated vapor enters the compressor, and (3) increases the average specific volume in the compressor, thereby increasing the power required.

10.34 Figure P10.34 gives data for an ideal vapor-compression heat pump cycle *operating at steady state with Refrigerant 134a as the working fluid. The heat pump* provides heating at a rate of 15 kW to maintain the interior of a building at 20°C when the outside temperature is 5°C. Sketch the *T-s* diagram for the cycle and determine the

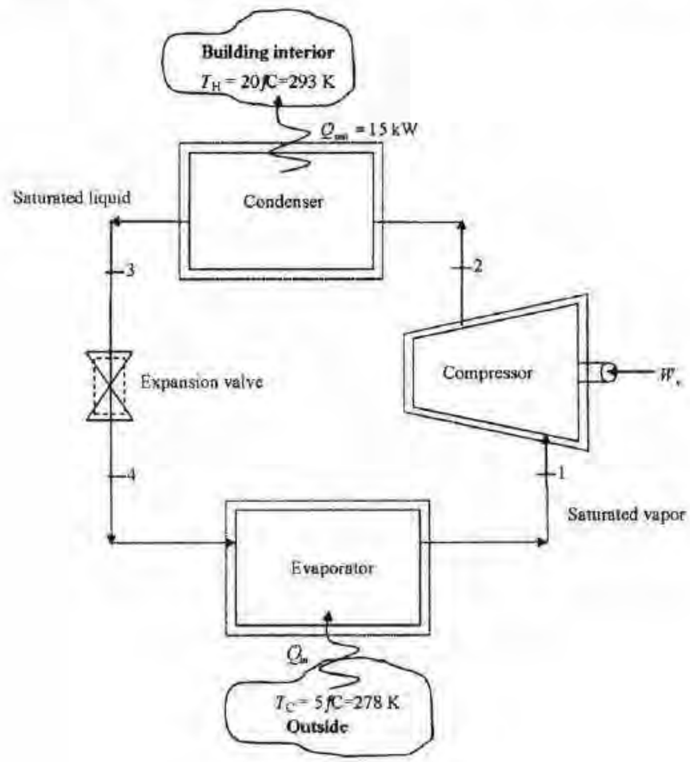
(a) temperatures at the principal states of the cycle, each in °C.

(b) the power input to the compressor, in kW.

(c) the coefficient of performance.

(d) the coefficient of performance for a Carnot heat pump cycle operating between reservoirs at the building interior and outside temperatures, respectively.

Compare the coefficients of performance determined in (c) and (d). Discuss.



State	p (bar)	h (kJ/kg)
1	2.4	244.09
2	8	268.97
3	8	93.42
4	2.4	93.42

Fig. P10.34

KNOWN: An ideal vapor-compression heat pump cycle uses Refrigerant 134a as the working fluid and provides a known energy output to heat a building. Operating data are provided at principal states of the cycle.

Problem 10.34 (Continued) – Page 2

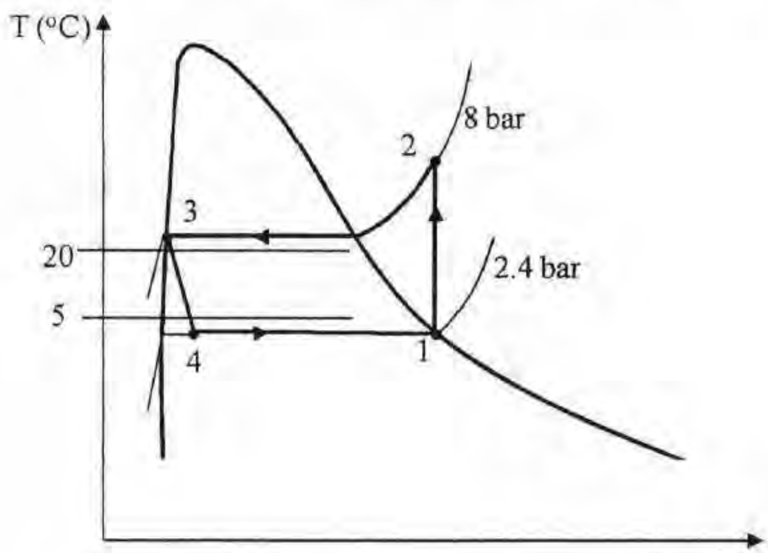
FIND: Sketch the *T-s* diagram for the cycle and determine (a) temperatures at principal states, (b) the power input to the compressor, (c) the coefficient of performance, and (d) the coefficient of performance for a Carnot heat pump cycle operating at the reservoir temperatures and discuss.

SCHEMATIC AND GIVEN DATA: See Fig. 10.34 for the schematic.

State	p (bar)	h (kJ/kg)
1	2.4	244.09
2	8	268.97
3	8	93.42
4	2.4	93.42

ENGINEERING MODEL:
(1) Each component is analyzed as a control volume at steady state.
(2) The compression and expansion processes are adiabatic.
(3) Kinetic and potential energy effects are negligible.

ANALYSIS: Referring to data from Table A-11, the *T-s* diagram is



(a) Using given information and data from Tables A-11 and A-12

State	p (bar)	h (kJ/kg)	Comment	T (°C)
1	2.4	244.09	Saturation temperature at 2.4 bar	-5.37
2	8	268.97	Interpolate in Table A-12 with $s_2 = s_1$	35.72
3	8	93.42	Saturation temperature at 8 bar	31.33
4	2.4	93.42	Saturation temperature at 2.4 bar	-5.37

Problem 10.34 (Continued) – Page 3

(b) To determine the power input to the compressor, first find the refrigerant mass flow rate from a energy and mass rate balances for a control volume enclosing the condenser

$$\dot{Q}_{\text{out}} = \dot{m}(h_2 - h_3) \text{ or } \dot{m} = \frac{\dot{Q}_{\text{out}}}{(h_2 - h_3)} = \frac{15\,\text{kW}\left|\frac{1\frac{\text{kJ}}{\text{s}}}{1\,\text{kW}}\right|}{(268.97 - 93.42)\frac{\text{kJ}}{\text{kg}}} = 0.08545\frac{\text{kg}}{\text{s}}$$

Therefore

$$\dot{W}_c = \dot{m}(h_2 - h_1) = 0.08545\frac{\text{kg}}{\text{s}}(268.97 - 244.09)\frac{\text{kJ}}{\text{kg}}\left|\frac{1\,\text{kW}}{1\frac{\text{kJ}}{\text{s}}}\right| = 2.126\,\text{kW}$$ ←

(c) The coefficient of performance (β) is

$$\gamma = \frac{\dot{Q}_{\text{out}}}{\dot{W}_c} = \frac{15}{2.126} = 7.056$$ ←

(d) For a Carnot cycle operating between $T_C = 5\ °\text{C} = 278$ K and $T_H = 20\ °\text{C} = 293$ K

$$\gamma_{\text{Carnot}} = \frac{T_H}{T_H - T_C} = \frac{293\,\text{K}}{293\,\text{K} - 278\,\text{K}} = 19.53$$ ←

The coefficient of performance for the Carnot heat pump cycle operating at the reservoir temperatures is higher than that of the specified ideal vapor-compression cycle. Referring to the T-s diagram, we see that the evaporator temperature is lower than T_C and the condenser temperature is higher than T_H. By operating the evaporator and condenser at these temperatures, there would be irreversibility associated with the heat transfer between the working fluid passing through each heat exchanger and the respective reservoir with which it exchanges energy.

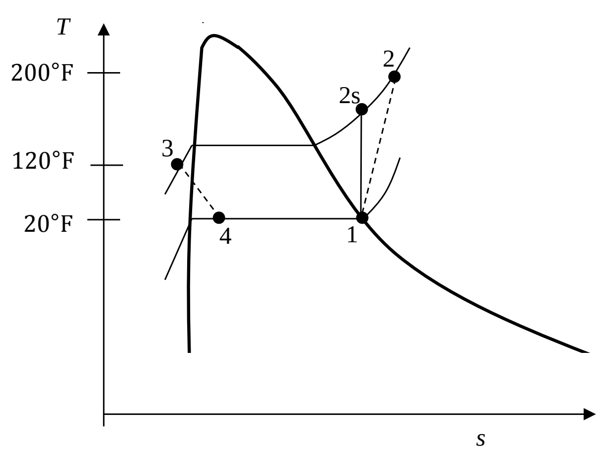
Problem 10.35

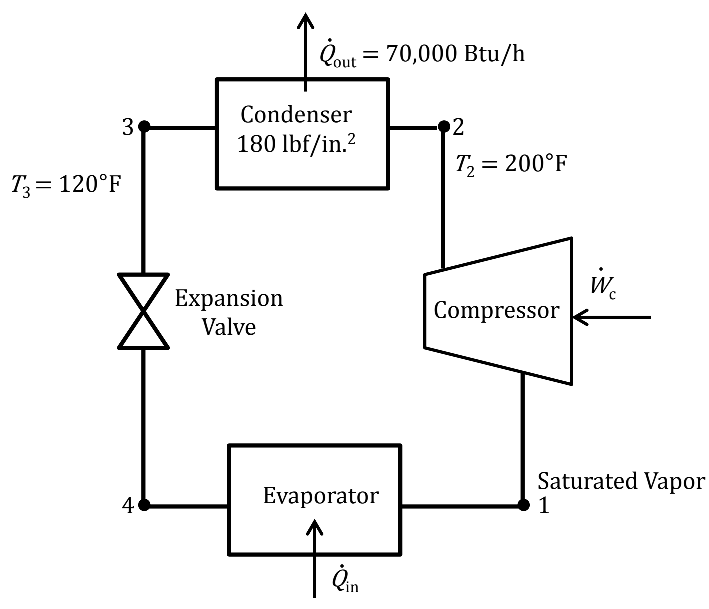
Refrigerant 134a is the working fluid in a vapor-compression heat pump system with a heating capacity of 70,000 Btu/h. The condenser operates at 180 lbf/in.^{2}, and the evaporator temperature is 20°F. The refrigerant is a saturated vapor at the evaporator exit and exits the condenser at 120°F. Pressure drops in the flows through the evaporator and condenser are negligible. The compression process is adiabatic, and the temperature at the compressor exit is 200°F. Determine
(a) the mass flow rate of the refrigerant, in lb/min.
(b) the compressor power input, in horsepower.
(c) the isentropic compressor efficiency.
(d) the coefficient of performance.

Known:
Data are known at various locations in a vapor-compression heat pump cycle with known heating capacity. Refrigerant-134A is the working fluid.

Find:
Determine the (a) mass flow rate of the refrigerant, (b) compressor power input, (c) isentropic compressor efficiency, and (d) coefficient of performance.

Schematic and Known Data:





Engineering Model:
(1) Each component of the cycle is analyzed as a control volume at steady state.
(2) There are no pressure drops through the evaporator and condenser.
(3) Kinetic and potential energy effects are negligible.
(4) The compressor and expansion valve operate adiabatically.

Analysis:
First, fix each principal state:

State 1: $T_1 = 20°\text{F}$, saturated vapor $\rightarrow h_1 = 104.61\,\frac{\text{Btu}}{\text{lb}},\, s_1 = 0.2205\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$

State 2: $p_2 = 180\,\frac{\text{lbf}}{\text{in.}^2}, T_2 = 200°\text{F} \rightarrow h_2 = 139.53\,\frac{\text{Btu}}{\text{lb}},\, s_{2s} = s_1 \rightarrow h_{2s} = 119.72\,\frac{\text{Btu}}{\text{lb}}$

State 3: $p_3 = 180\,\frac{\text{lbf}}{\text{in.}^2}, T_3 = 120°\text{F} \rightarrow h_3 \approx h_f(120°\text{F}) = 51.47\,\frac{\text{Btu}}{\text{lb}}$

State 4: Throttling process $\rightarrow h_4 = h_3 = 51.47\,\frac{\text{Btu}}{\text{lb}}$

(a) The mass flow rate is:

$$\dot{Q}_{\text{out}} = \dot{m}(h_2 - h_3) \rightarrow \dot{m} = \frac{\dot{Q}_{\text{out}}}{h_2 - h_3} = \frac{70{,}000\,\frac{\text{Btu}}{\text{h}}}{(139.53 - 51.47)\,\frac{\text{Btu}}{\text{lb}}}\left|\frac{1\text{ h}}{60\text{ min}}\right| = 13.25\,\frac{\text{lb}}{\text{min}}$$

(b) Thus, the compressor power is:

$$\dot{W}_{\text{c}} = \dot{m}(h_2 - h_1) = \left(13.25\,\frac{\text{lb}}{\text{min}}\right)(139.53 - 104.61)\,\frac{\text{Btu}}{\text{lb}}\left|\frac{60\text{ min}}{1\text{ h}}\right|\left|\frac{1\text{ hp}}{2545\,\frac{\text{Btu}}{\text{h}}}\right| = 10.91\text{ hp}$$

(c) The compressor efficiency is found as follows:

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{119.72 - 104.61}{139.53 - 104.61} = 0.4327 = 43.27\%$$

(d) The coefficient of performance is:

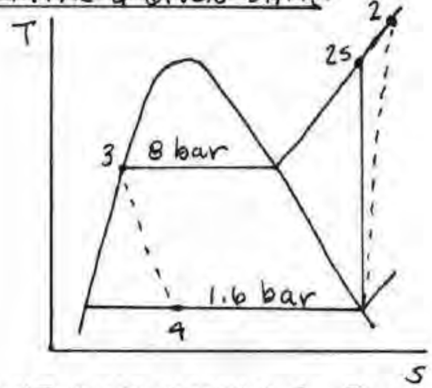
$$\gamma = \frac{h_2 - h_3}{h_2 - h_1} = \frac{139.53 - 51.47}{139.53 - 104.61} = 2.522$$

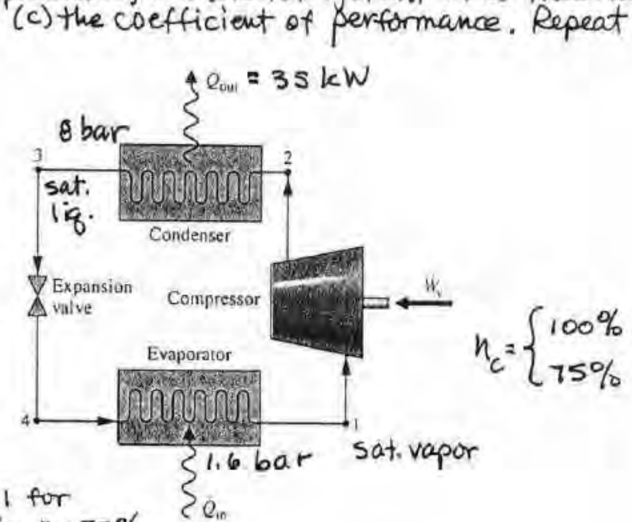
PROBLEM 10.36

KNOWN: Data are given for a vapor-compression heat pump using R-134a as the working fluid. The rate of heat transfer to a dwelling is specified.

FIND: Determine, for isentropic compression, (a) the refrigerant mass flow rate, (b) the compressor power, and (c) the coefficient of performance. Repeat for $\eta_c = 0.75$.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: See Example 10.1 for $\eta_c = 100\%$. See Example 10.3, items 1–4, for $\eta_c = 75\%$.

ANALYSIS: First, fix each of the principal states for $\eta_c = 100\%$.

State 1 $p_1 = 1.6$ bar, sat. vapor $\Rightarrow h_1 = 237.97$ kJ/kg, $s_1 = 0.9295$ kJ/kg·K

State 2s $p_2 = 8$ bar, $s_{2s} = s_1 \Rightarrow h_{2s} = 271.22$ kJ/kg

State 3 $p_3 = 8$ bar, sat. liquid $\Rightarrow h_3 = 93.42$ kJ/kg

State 4 Throttling process $\Rightarrow h_4 = h_3 = 93.42$ kJ/kg

(a) Using the given value of $\dot{Q}_{out}$ to get $\dot{m}$

$$\dot{m} = \frac{\dot{Q}_{out}}{(h_{2s} - h_3)} = \frac{35 \text{ kW}}{(271.22 - 93.42) \text{ kJ/kg}} \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| = 0.197 \frac{\text{kg}}{\text{s}} \longleftarrow \frac{\dot{m}}{(\eta_c = 100\%)}$$

(b) The compressor power is

$$\dot{W}_c = \dot{m}(h_{2s} - h_1) = (0.197)(271.22 - 237.97) = 6.55 \text{ kW} \longleftarrow \frac{\dot{W}_c}{(\eta_c = 100\%)}$$

(c) the coefficient of performance for the heat pump is

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_c} = \frac{35}{6.55} = 5.34 \longleftarrow \frac{\gamma}{(\eta_c = 100\%)}$$

For $\eta_c = 75\%$; $h_2 = h_1 + (h_{2s} - h_1)/\eta_c = 237.97 + (271.22 - 237.97)/(0.75)$
$= 282.30$ kJ/kg

$$\dot{m} = \frac{35}{(282.30 - 93.42)} = 0.1853 \text{ kg/s}$$

$$\dot{W}_c = \dot{m}(h_2 - h_1) = (0.1853)(282.30 - 237.97) = 8.214 \text{ kW} \longleftarrow \frac{\dot{W}_c}{(\eta_c = 75\%)}$$

$$\gamma = \frac{35}{8.214} = 4.26 \longleftarrow \frac{\gamma}{(\eta_c = 75\%)}$$

The presence of irreversibilitis in the compressor increases the power input by over 25%.

Problem 10.37

An office building requires a heat transfer rate of 20 kW to maintain the inside temperature at 21° C when the outside temperature is 0° C. A vapor-compression heat pump with Refrigerant 134a as the working fluid is to be used to provide the necessary heating. Assume the compressor's isentropic efficiency is 82%. Specify appropriate evaporator and condenser pressures of a cycle for this purpose assuming the temperature at $\Delta T_{cond} = \Delta T_{evap} = 10°$ C as shown in Figure P10.37. The states are numbered as in Fig. 10.11. The refrigerant exist the evaporator as saturated vapor and exits the condenser as saturated liquid at the respective pressures. Determine the

(a) mass flow rate of the refrigerant, in kg/s.
(b) compressor power, in kW.
(c) coefficient of performance and compare with the coefficient of performance for a Carnot heat pump cycle operating between the reservoir temperatures.

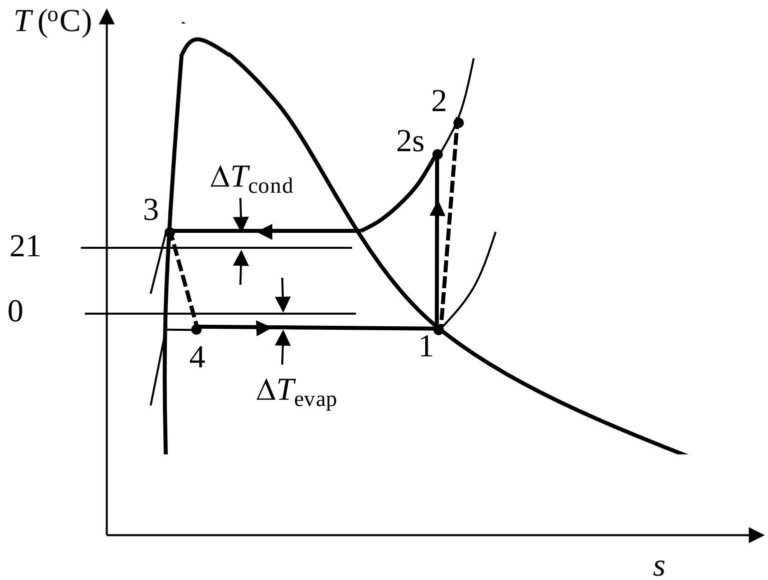


Fig. P10.37

Solution:

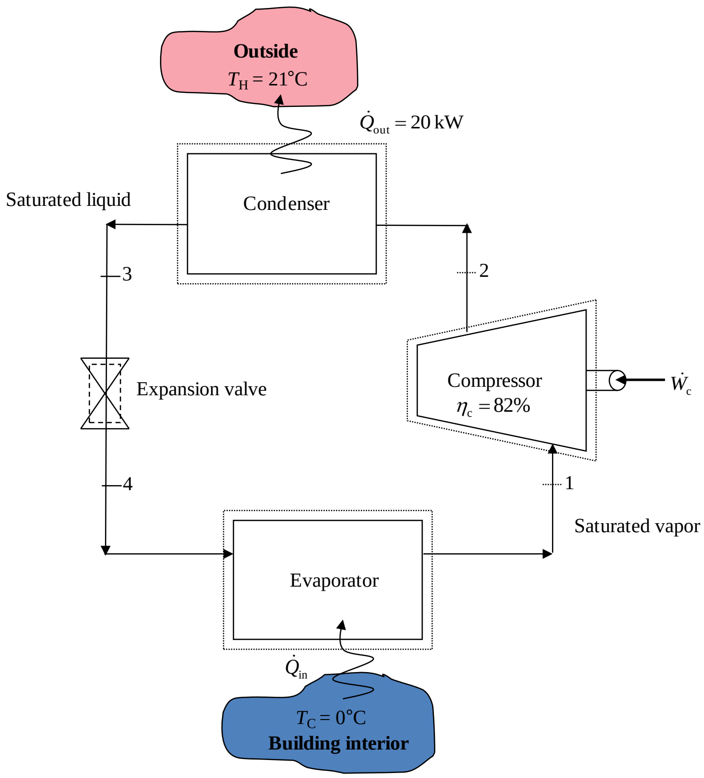
Known:

A Refrigerant 134a vapor-compression heat pump is proposed to develop process heat at a specified rate and temperature from a source at a known temperature.

Find:

Determine (a) the mass flow rate of the refrigerant, (b) the compressor power, and (c) the actual and Carnot coefficients of performance and compare.

Schematic and Known Data:



Engineering Model:

(1) Each component is analyzed as a control volume at steady state. The control volumes are indicated by dashed lines on the accompanying figure.
(2) There are no pressure drops through the evaporator and condenser.
(3) The compressor has an isentropic efficiency of 82% and operates adiabatically. The expansion through the valve is a throttling process.
(4) Kinetic and potential energy effects are negligible.
(5) Saturated vapor enters the compressor and saturated liquid leaves the condenser.

Analysis:

Fix the principal states for the cycle. Using $\Delta T_{cond} = \Delta T_{evap} = 10°C$ as follows

$\Delta T_{cond} = 10^\circ\text{C} \approx T_3 - 21\ ^\circ\text{C therefore } T_3 \approx 31^\circ\text{ C}$

Using Table A-11 with $T_3 = T_{sat}$, $p_{cond} = p_3 = 8$ bar ($T_{sat} = 31.33^\circ$ C)

Using the same approach for the evaporator, $T_1 \approx -10^\circ$ C and $p_{evap} = p_1 = 2$ bar ($T_{sat} = -10.09^\circ$ C)

For state 1, using Table A-11 with saturated vapor at $p_1 = 2$ bar:
$h_1 = 241.30$ kJ/kg, $s_1 = 0.9253$ kJ/kg·K

For state 2, using Table A-12 with $p_2 = 8$ bar, $\eta_c = 82\%$, and $s_{2s} = s_1$: $h_{2s} = 269.92$ kJ/kg

$$\eta_c = \frac{(h_{2s} - h_1)}{(h_2 - h_1)} \quad \text{or} \quad h_2 = \frac{(h_{2s} - h_1)}{\eta_c} + h_1 = 276.20\ \frac{\text{kJ}}{\text{kg}}$$

For state 3, using Table A-11 with saturated liquid at $p_3 = 8$ bar: $h_3 = 93.42$ kJ/kg

For state 4, this is a throttling process and therefore $h_4 = h_3$ =93.42 kJ/kg

(a) Find the refrigerant mass flow rate from an energy rate balance at the condenser

$$\dot{Q}_{\text{out}} = \dot{m}(h_2 - h_3) \text{ or } \dot{m} = \frac{\dot{Q}_{\text{out}}}{(h_2 - h_3)} = \frac{20\,\text{kW}}{(276.20 - 93.42)\,\frac{\text{kJ}}{\text{kg}}}\left|\frac{1\,\frac{\text{kJ}}{\text{s}}}{1\,\text{kW}}\right| = 0.1094\,\frac{\text{kg}}{\text{s}}$$

(b) The compressor power follows:

$$\dot{W}_{\text{c}} = \dot{m}(h_2 - h_1) = 0.1094\,\frac{\text{kg}}{\text{s}}(276.20 - 241.30)\,\frac{\text{kJ}}{\text{kg}} = 3.82\,\text{kW}$$

(c) The coefficient of performance (γ) is

$$\gamma = \frac{\dot{Q}_{\text{out}}}{\dot{W}_{\text{c}}} = \frac{20\,\text{kW}}{3.82\,\text{kW}} = 5.24$$

For a Carnot heat pump cycle operating between T_{C} = 0 ºC = 273 K and T_{H} = 21 ºC = 294 K

$$\gamma_{\text{Carnot}} = \frac{T_{\text{H}}}{T_{\text{H}} - T_{\text{C}}} = \frac{294\,\text{K}}{294\,\text{K} - 273\,\text{K}} = 14$$

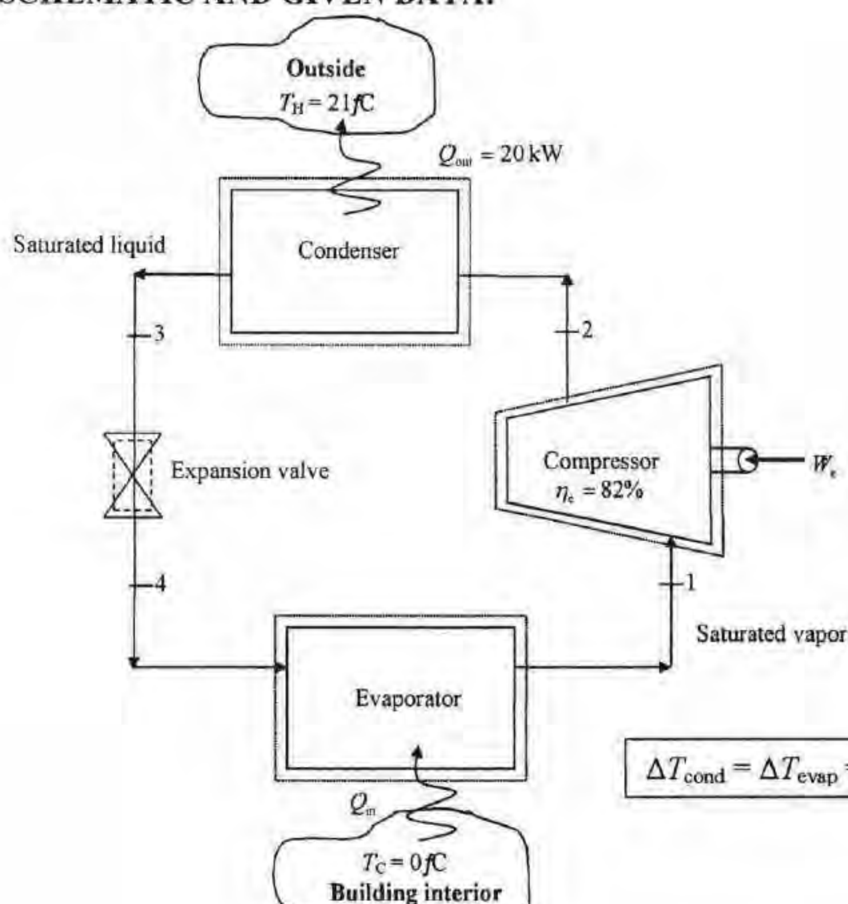
The coefficient of performance for the Carnot heat pump cycle operating at the reservoir temperatures is higher than that of the original cycle. Referring to the *T*-*s* diagram, we see that the evaporator temperature is lower than T_{C} and the condenser temperature is higher than T_{H}. By operating the evaporator and condenser at these temperatures, there would be irreversibility associated with the heat transfer between the working fluid passing through each heat exchanger and the respective reservoir.

10.38 Repeat the calculations of Problem 10.37 for Refrigerant 22 as the working fluid. Compare the results with those of Problem 10.37 and discuss.

KNOWN: A Refrigerant 22 vapor-compression heat pump provides heat transfer at a specified rate to maintain the interior of a building at a specified temperature for a specified outside temperature. The skeleton of the cycle *T-s* diagram is provided the compressor isentropic efficiency is given.

FIND: Specify the evaporator and condenser pressures and determine the (a) mass flow rate of refrigerant, (b) compressor power, and (c) actual and Carnot coefficients of performance and compare.

SCHEMATIC AND GIVEN DATA:



$\Delta T_{cond} = \Delta T_{evap} = 10°C$

Problem 10.38 (Continued) – Page 3

(a) Find the refrigerant mass flow rate from energy and mass rate balances for a control volume enclosing the condenser

$$\dot{Q}_{out} = \dot{m}(h_2 - h_3) \text{ or } \dot{m} = \frac{\dot{Q}_{out}}{(h_2 - h_3)} = \frac{20\,\text{kW}}{(283.48 - 81.91)\dfrac{\text{kJ}}{\text{kg}}}\left|\frac{1\dfrac{\text{kJ}}{\text{s}}}{1\,\text{kW}}\right| = 0.0987\,\frac{\text{kg}}{\text{s}}$$

(b) The compressor power is

$$\dot{W}_c = \dot{m}(h_2 - h_1) = 0.0987\,\frac{\text{kg}}{\text{s}}(283.48 - 246.00)\frac{\text{kJ}}{\text{kg}} = 3.70\,\text{kW}$$

(c) The coefficient of performance (γ) is

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_c} = \frac{20\,\text{kW}}{3.70\,\text{kW}} = 5.41$$

For a Carnot heat pump cycle operating between $T_C = 0\ ^\circ\text{C} = 273$ K and $T_H = 21\ ^\circ\text{C} = 294$ K.

$$\gamma_{Carnot} = \frac{T_H}{T_H - T_C} = \frac{294\,\text{K}}{294\,\text{K} - 273\,\text{K}} = 14$$

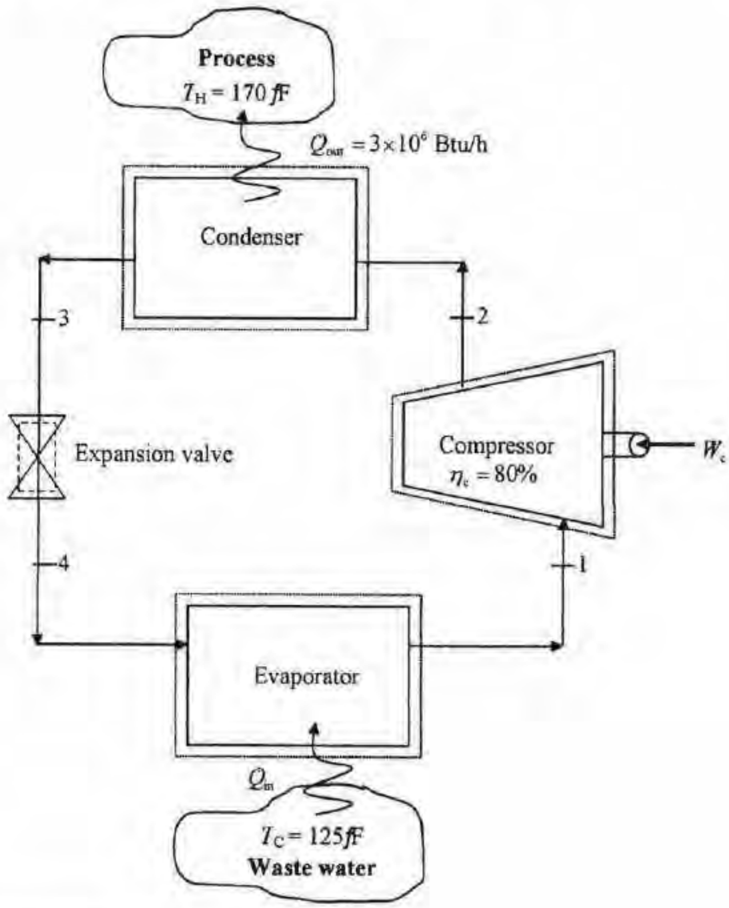
The coefficient of performance for the Carnot heat pump cycle operating at the reservoir temperatures is higher than that of the original cycle. Referring to the T-s diagram, we see that the evaporator temperature is lower than T_C and the condenser temperature is higher than T_H. By operating the evaporator and condenser at these temperatures, there would be irreversibility associated with the heat transfer between the working fluid passing through each heat exchanger and the respective reservoir.

Comparing the calculations of Problem 10.38 using Refrigerant 134a with those of Problem 10.37, using Refrigerant 22 results in

- *Somewhat higher evaporator and condenser pressures.*
- *Slightly lower mass flow rate and input power required.*
- *Slightly higher coefficient of performance.*

10.39 A process requires a heat transfer rate of 3×10^6 Btu/h at 170°F. It is proposed that a Refrigerant 134a vapor-compression heat pump be used to develop the process heat using a waste water stream at 125°F as the lower-temperature source. Figure P10.39 provides data for this cycle operating at steady state. The compressor isentropic efficiency is 80%. Sketch the *T-s* diagram for the cycle and determine the

(a) the specific enthalpy at the compressor exit, in Btu/lb.
(b) the temperatures at each of the principal states, in °F.
(c) the mass flow rate of the refrigerant, in lb/h.
(d) the compressor power, in Btu/h.
(e) the coefficient of performance and compare with the coefficient of performance for a Carnot heat pump cycle operating between reservoirs at the process temperature and the waste water temperature, respectively.



State	p (lbf/in.^{2})	h (Btu/lb)
1	180	116.74
2	400	?
3	400	76.11
4	180	76.11

Fig. P10.39

Problem 10.39 (Continued) – Page 2

KNOWN: A Refrigerant 134a vapor-compression heat pump is proposed to develop process heat at a specified rate and temperature from a waste water stream at a known temperature. Operating data are provided at principal states of the cycle.

FIND: Sketch the *T-s* diagram for the cycle and determine (a) the specific enthalpy at the compressor exit, (b) temperatures at each principal state, (c) the mass flow rate of the refrigerant, (d) the compressor power, and (e) the actual and Carnot coefficients of performance and compare.

SCHEMATIC AND GIVEN DATA: For schematic, refer to Fig. P10.39.

State	p (lbf/in.^{2})	h (Btu/lb)
1	180	116.74
2	400	?
3	400	76.11
4	180	76.11

ENGINEERING MODEL:

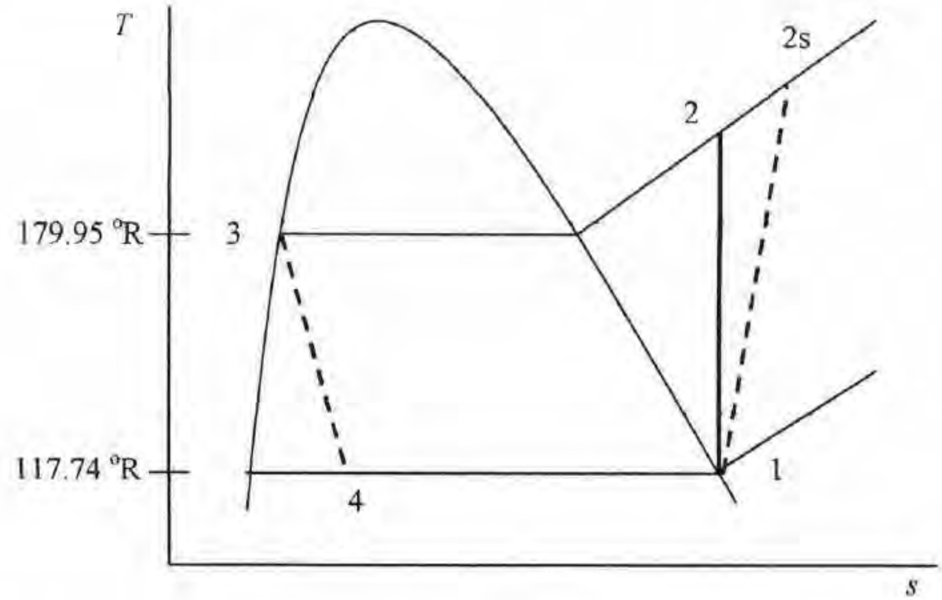
(1) Each component is analyzed as a control volume at steady state. The control volumes are indicated by dashed lines on the accompanying figure.

(2) There are no pressure drops through the evaporator and condenser.

(3) The compressor has an isentropic efficiency of 80% and operates adiabatically. The expansion through the valve is a throttling process.

(4) Kinetic and potential energy effects are negligible.

ANALYSIS: Referring to Tables A11-E and A12-E, the *T-s* diagram is



(a) Determine the enthalpy at the compressor exit, h_2, by using the compressor isentropic efficiency

$$\eta_c = \frac{(h_{2s} - h_1)}{(h_2 - h_1)} \quad \text{or} \quad h_2 = \frac{(h_{2s} - h_1)}{\eta_c} + h_1 \text{ therefore find } h_{2s}$$

From Table A - 11E, $s_1 = 0.2154$ Btu/lb·°R $= s_{2s}$

Interpolating in Table A - 12E with p_2 and s_{2s}, $h_{2s} = 123.32 \dfrac{\text{Btu}}{\text{lb}}$

$$h_2 = \frac{(123.32 - 116.74)\dfrac{\text{Btu}}{\text{lb}}}{0.80} + 116.74 = 124.97 \frac{\text{Btu}}{\text{lb}}$$

(b) Using given information and data from Table A-11E, and interpolating in A-12E to get T_2

State	p (lbf/in.^{2})	h (Btu/lb)	T (°F)
1	180	116.74	117.74
2	400	124.97	191.63
3	400	76.11	179.95
4	180	76.11	117.74

Problem 10.39 (Continued) – Page 4

(c) Find the refrigerant mass flow rate from an energy rate balance on a control volume enclosing the condenser

$$\dot{Q}_{out} = \dot{m}(h_2 - h_3) \text{ or } \dot{m} = \frac{\dot{Q}_{out}}{(h_2 - h_3)} = \frac{3\times10^6 \dfrac{\text{Btu}}{\text{h}}}{(124.97 - 76.11)\dfrac{\text{Btu}}{\text{lb}}} = 61340\frac{\text{lb}}{\text{h}}$$

(d) The compressor power is

$$\dot{W}_c = \dot{m}(h_2 - h_1) = 61340\frac{\text{lb}}{\text{h}}(124.97\text{-}116.74)\frac{\text{Btu}}{\text{lb}} = 5.048\times10^5\ \frac{\text{Btu}}{\text{h}}$$

(e) The coefficient of performance (γ) is

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_c} = \frac{3\times10^6 \dfrac{\text{Btu}}{\text{h}}}{5.048\times10^5 \dfrac{\text{Btu}}{\text{h}}} = 5.95$$

For a Carnot cycle operating between $T_C = 125$ °F = 584.67 °R and $T_H = 170$ °F = 629.67 °R

$$\gamma_{\text{Carnot}} = \frac{T_H}{T_H - T_C} = \frac{629.67}{629.67 - 584.67} = 13.99$$

The coefficient of performance for the Carnot heat pump cycle operating at the reservoir temperatures is higher than that of the original vapor compression cycle. Referring to the T-s diagram, we see that the evaporator temperature is lower than T_C and the condenser temperature is higher than T_H. By operating the evaporator and condenser at these temperatures, there would be irreversibility associated with the heat transfer between the working fluid passing through each heat exchanger and the respective reservoir. Irreversibilities within the vapor compression heat pump cycle further reduce the coefficient of performance compared to the Carnot cycle.

Problem 10.40

A vapor-compression heat pump has a heating capacity of 500 kJ/min and uses Refrigerant 134a as the working fluid. The isentropic compressor efficiency is 80%. The heat pump is driven by a power cycle with a thermal efficiency of 25%. For the power cycle, 80% of the heat rejected is transferred to the heated space. Data for the cycle are provided in the table below. The principal states are numbered as in Fig. 10.3.

(a) Determine the power input to the heat pump compressor, in kW.

(b) Evaluate the ratio of the total rate that heat is delivered to the heated space to the rate of heat input to the power cycle. Discuss.

<u>Known:</u>

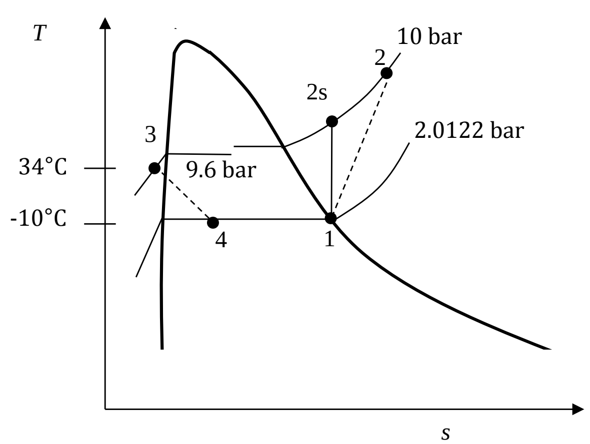
Refrigerant 134a is the working fluid in a vapor-compression heat pump driven by a power cycle. Operating data are specified for the heat pump and the power cycle.

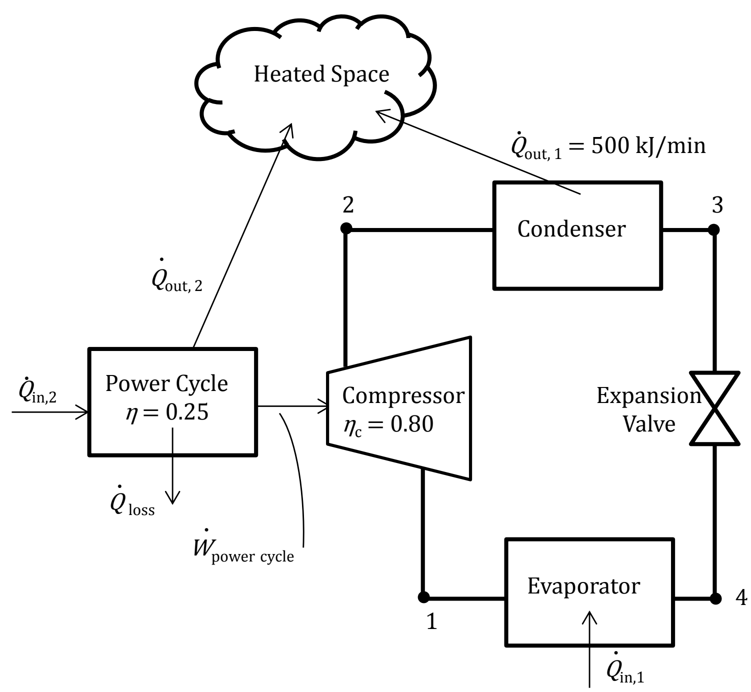
<u>Find:</u>

Determine (a) the heat pump compressor power and (b) the ratio of the total rate heat is delivered to the heat space to the rate of heat input to the power cycle. Discuss.

<u>Schematic and Known Data:</u>

State	***p*** **(bar)**	***T*** **(°C)**	***h*** **(kJ/kg)**	***s*** **(kJ/kg·K)**
1	2.0122	-10	241.34	0.9253
2s	10	45.17	274.63	0.9253
2	10	52.47	282.95	0.9512
3	9.6	34	97.31	0.3584
4	2.0122	-10	97.31	0.3779





Engineering Model:
(1) Each component of the cycle is analyzed as a control volume at steady state.
(2) There are no pressure drops through the evaporator and the condenser.
(3) The compressor operates adiabatically with an isentropic efficiency of 80%. The expansion through the valve is a throttling process.
(4) Kinetic and potential energy effects are negligible.

Analysis:
(a) The mass flow rate of the refrigerant is:

$$\dot{m} = \frac{\dot{Q}_{\text{out}_1}}{h_2 - h_3} = \frac{500\,\frac{\text{kJ}}{\text{min}}}{(282.95 - 97.31)\,\frac{\text{kJ}}{\text{kg}}}\left|\frac{1\text{ min}}{60\text{ s}}\right| = 0.04489\,\frac{\text{kg}}{\text{s}}$$

The compressor power becomes:

$$\dot{W}_{\text{c}} = \dot{m}(h_2 - h_1) = \left(0.04489\,\frac{\text{kg}}{\text{s}}\right)(282.95 - 241.34)\,\frac{\text{kJ}}{\text{kg}}\left|\frac{1\text{ kW}}{1\,\frac{\text{kJ}}{\text{s}}}\right| = 1.868\text{ kW}$$

(b) For the power cycle, $\eta = 0.25$. With $\dot{W}_{\text{power cycle}} = \dot{W}_{\text{heat pump}} = \dot{W}_{\text{c}} = 1.868\text{ kW}$:

$$\dot{Q}_{\text{in}_2} = \frac{\dot{W}_{\text{power cycle}}}{\eta} = \frac{1.868}{0.25} = 7.472\text{ kW}$$

The total heat rejected from the power cycle is $\dot{Q}_{\text{rej}} = \dot{Q}_{\text{loss}} + \dot{Q}_{\text{out,2}}$ and can also be stated as $\dot{Q}_{\text{rej}} = \dot{Q}_{\text{in}_2} - \dot{W}_{\text{power cycle}} = 7.472 - 1.868 = 5.604\text{ kW}$

Thus: $\dot{Q}_{\text{out}_2} = (0.80)\dot{Q}_{\text{loss}} = (0.80)(5.604) = 4.483\text{ kW}$

Finally: $\frac{\dot{Q}_{\text{out}_1} + \dot{Q}_{\text{out}_2}}{\dot{Q}_{\text{in}_2}} = \frac{\left(500\frac{\text{kJ}}{\text{min}}\cdot\frac{1\text{ min}}{60\text{ s}}\right) + 4.483\text{ kW}}{7.472\text{ kW}} = 1.715$

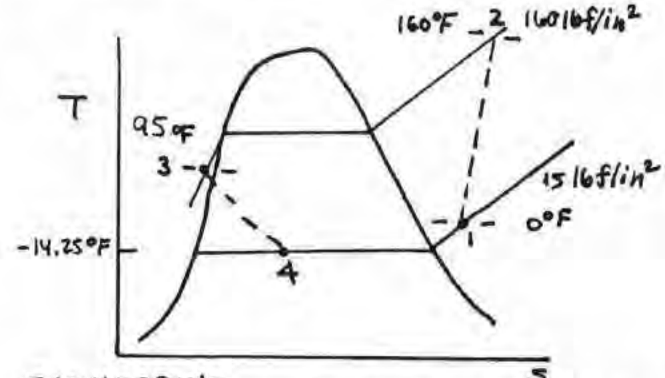
Discussion:
The engine-driven heat pump delivers more energy to the heated space than could be obtained by burning the fuel directly.

PROBLEM 10.41

KNOWN: Operating data are provided for a vapor compression heat pump using R134a as the working fluid.

FIND: Determine (a) the isentropic compressor efficiency, and (b) the coefficient of performance. (c) Perform an exergy accounting of the compressor power input.

SCHEMATIC & GIVEN DATA:



State	h (Btu/lb)	s (Btu/lb·°R)
1	102.42	0.2303
2	129.78	0.2391
3	42.47	0.0867
4	42.47	0.0958

ENGINEERING MODEL: 1. Control volumes enclosing each of the principal components are at steady state. 2. Kinetic and potential energy effects can be ignored as can stray heat transfers and pressure drops for flow through the evaporator and condenser. 3. The expansion across the valve is a throttling process. 4. $T_0 = 480°R$

ANALYSIS: First, fix each of the principal states and obtain the data shown in the table above.

State 1: $p_1 = 15\ lbf/in^2$, $T_1 = 0\ °F$. Table A-12E gives $h_1 = 102.42\ Btu/lb$, $s_1 = 0.2303\ \frac{Btu}{lb \cdot °R}$

State 2: $p_2 = 160\ lbf/in^2$, $T_2 = 160°F$, Table A-12E gives $h_2 = 129.78\ Btu/lb$, $s_2 = 0.2391\ \frac{Btu}{lb \cdot °R}$

Also, $s_{2s} = s_1 \Rightarrow h_{2s} = 124.41\ Btu/lb$

State 3: $h_3 = h_f(95°F) = 42.47\ Btu/lb$, $s_3 = s_f(95°F) = 0.0867\ Btu/lb \cdot °R$ from Table A-10E.

State 4: $h_4 = h_3 = 42.47\ Btu/lb$

$$\Rightarrow x_4 = \frac{42.47 - 7.40}{92.27} = 0.380 \Rightarrow s_4 = 0.0171 + 0.380[0.2242 - 0.0171] = 0.0958\ \frac{Btu}{lb\,°R}$$

(a) The isentropic compressor efficiency is

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{124.41 - 102.42}{129.78 - 102.42} = 0.804\ (80.4\%)$$

(b) The coefficient of performance is

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_c} = \frac{\cancel{\dot{m}}(h_2 - h_3)}{\cancel{\dot{m}}(h_2 - h_1)} = \frac{129.78 - 42.47}{129.78 - 102.42} = 3.19$$

(c) The compressor work input per unit of mass flowing is

$$\dot{W}_c/\dot{m} = h_2 - h_1 = 27.36\ Btu/lb$$

PROBLEM 10.41 (Contd.) – Page 2

The rates of exergy destruction for the compressor and valve can be found using $\dot{E}_d = T_0 \sigma_{cv}$. That is,

- Compressor

$$(\dot{E}_d/\dot{m})_{comp} = T_0(s_2 - s_1) = 480°R(0.2391 - 0.2303)\frac{Btu}{lb\cdot °R} = 4.22\ Btu/lb$$

- Valve

$$(\dot{E}_d/\dot{m})_{valve} = T_0(s_4 - s_3) = (480°R)(0.0958 - 0.0867)\frac{Btu}{lb\cdot °R} = 4.37\ Btu/lb$$

① The change in flow exergy as the refrigerant passes through the evaporator is

$$e_{f1} - e_{f4} = h_1 - h_4 - T_0(s_1 - s_4)$$

$$= (102.42 - 42.47) - 480(0.2303 - 0.0958) = -4.61\ Btu/lb$$

The change in flow exergy as the refrigerant passes through the condenser is

$$e_{f3} - e_{f2} = h_3 - h_2 - T_0(s_3 - s_2)$$

$$= (42.47 - 129.78) - 480(0.0867 - 0.2391) = -14.16\ Btu/lb$$

Summary:

- Exergy input – compressor work: 27.36 Btu/lb
- Disposition of the exergy input:

– Exergy destruction	compressor	4.22 Btu/lb	(15.4 %)
	valve	4.37 Btu/lb	(16.0 %)
– Exergy transfer out	evaporator	4.61 Btu/lb	(16.8 %)
	condenser	14.16 Btu/lb	(51.8 %)
		27.36 Btu/lb	

The summary indicates that there are two significant sources of irreversibility: the compression process and the expansion process. The compressor irreversibility can be reduced by means of a compressor having a higher isentropic efficiency than found in part (a). Economic consequences should be carefully considered, however.

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1. Since there is no work (except flow work) and no internal irreversibilities (pressure is constant), an exergy rate balance for the refrigerant side of the evaporator, and of the condenser, reads

$$0 = \dot{E}_q - \cancel{\dot{W}_{cv}} + \dot{m}(e_{f,in} - e_{f,out}) - \cancel{\dot{E}_d} \Rightarrow \dot{E}_q = \dot{m}(e_{f,out} - e_{f,in}).$$

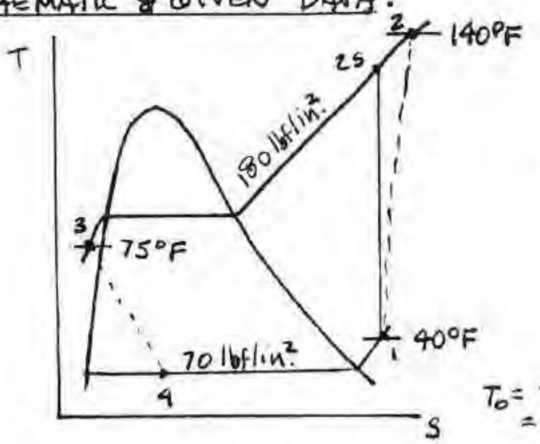
Thus, the change in flow exergy equals the exergy transfer accompanying heat transfer. Since heat transfer occurs at $T < T_0$ in the evaporator case, the exergy transfer accompanying heat is opposite to the direction of the heat transfer: from the refrigerant to the cold space. For the condenser case, heat transfer and exergy transfer are in the same direction: from the refrigerant to the heated space.

PROBLEM 10.42

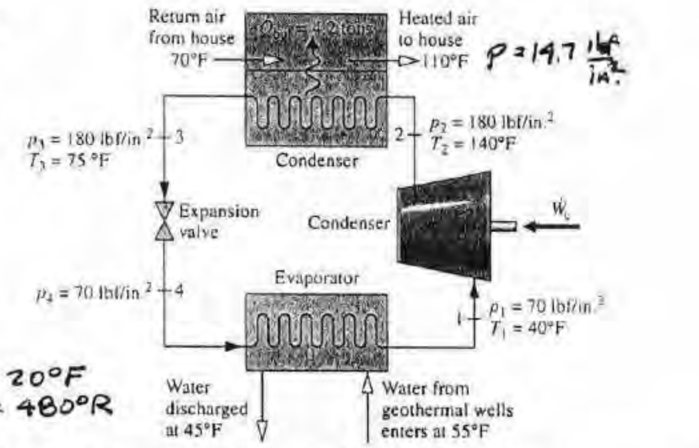
KNOWN: Data are provided for a geothermal heat pump providing energy to heat a house. The heating capacity is known, and the working fluid is R-22.

FIND: Determine (a) the volumetric flow rate of air to heat the house, (b) the isentropic compressor efficiency, (c) the compressor power, (d) the coefficient of performance, and (e) the gal/min of water from the geothermal wells. Perform a full exergy accounting of the compressor power input and devise and evaluate a second law efficiency.

SCHEMATIC & GIVEN DATA:



$T_0 = 20°F = 480°R$



ENGINEERING MODEL: See Example 10.3, items 1-4, except the compressor efficiency is not specified. Also, assume the air behaves as an ideal gas with $c_{p_a} = 0.24\ \frac{Btu}{lb \cdot °R}$.

ANALYSIS: First, fix all of the principal states in the R-22 cycle.

State 1 $p_1 = 70$ lbf/in.², $T_1 = 40°F \Rightarrow h_1 = 109.07$ Btu/lb, $s_1 = 0.2254$ Btu/lb·°R

State 2 $p_2 = 180$ lbf/in.², $T_2 = 140°F \Rightarrow h_2 = 122.11$ Btu/lb, $s_2 = 0.2299$ Btu/lb·°R

State 3 $p_3 = 180$ lbf/in.², $T_3 = 75°F \Rightarrow$ compressed liquid. Thus

$$h_3 \approx h_{f@75°F} = 31.79 \text{ Btu/lb},\ s_3 \approx s_{f@75°F} = 0.0661 \text{ Btu/lb·°R}$$

State 4 Throttling process $\Rightarrow h_4 = h_3 = 31.79$ Btu/lb. Also, $x_4 = 0.14551$ and $s_4 = 0.06766$ Btu/lb·°R

(a) Using the given value of $\dot{Q}_{out}$ to get $\dot{m}_{air}$

$$\dot{m}_{air} = \frac{\dot{Q}_{out}}{(h_{out} - h_{in})_{air}} = \frac{\dot{Q}_{out}}{c_{p_a}(T_{out} - T_{in})_{air}} = \frac{4.2 \text{ tons}}{0.24(110-70) \text{ Btu/lb}} \left| \frac{200 \text{ Btu/min}}{1 \text{ ton}} \right| = 87.5 \text{ lb/min}$$

The volumetric flow rate of the air delivered is

$$(AV)_{\substack{air\\out}} = \dot{m}_{air} v_{\substack{air\\out}} = \frac{\dot{m}_{air} R T_{out}}{P}$$

$$= \frac{(87.5 \text{ lb/min})\left(\frac{1545}{28.97} \frac{ft \cdot lbf}{lb \cdot °R}\right)(570°R)}{14.7 \text{ lbf/in}^2} \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| = 1257 \frac{ft^3}{min} \leftarrow (AV)_{air}$$

(b) $p_2 = 180$ lbf/in.², $s_{2s} = s_1 \Rightarrow h_{2s} = 119.46$ Btu/lb. The compressor isentropic efficiency is

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{119.46 - 109.07}{122.11 - 109.07} = 0.797\ (79.7\%) \leftarrow \eta_c$$

(c) The mass flow rate of R-22 is

$$\dot{m}_R = \frac{\dot{Q}_{out}}{h_2 - h_3} = \frac{4.2 \text{ tons}}{(122.11 - 31.79)\text{Btu/lb}} \left| \frac{200 \text{ Btu/min}}{1 \text{ ton}} \right| = 9.3 \frac{\text{lb}}{\text{min}}$$

And the compressor power is

$$\dot{W}_c = \dot{m}_R(h_2 - h_1) = \left(9.3 \frac{\text{lb}}{\text{min}}\right)(122.11 - 109.07)\frac{\text{Btu}}{\text{lb}} \left| \frac{60 \text{ min}}{1 \text{ h}} \right| \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right|$$

$$= 2.86 \text{ hp} \longleftarrow \quad \dot{W}_c$$

(d) The coefficient of performance is

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_c} = \frac{4.2 \text{ tons}}{2.86 \text{ hp}} \left| \frac{12000 \text{ Btu/h}}{1 \text{ ton}} \right| \left| \frac{1 \text{ hp}}{2545 \text{ Btu/h}} \right| = 6.92 \longleftarrow \quad \gamma$$

(e) For the evaporator: $0 = \dot{m}_R(h_4 - h_1) + \dot{m}_w(h_{in} - h_{out})_w$

With $\Delta h_w = c_w \Delta T$, and $c_w = 1$ Btu/lb·°R, we get

$$\dot{m}_w = \frac{\dot{m}_R(h_1 - h_4)}{c_w(T_{in} - T_{out})_w} = \frac{(9.3 \text{ lb/min})(109.07 - 31.79)\text{Btu/lb}}{(1)(55 - 45)\text{ Btu/lb}} = 71.87 \frac{\text{lb}}{\text{min}}$$

Noting the $v_{f @ 55°F} = 0.01602$ ft³/lb, the volumetric flow rate of water is

$$(AV)_w = \dot{m}_w v_f = \left(71.87 \frac{\text{lb}}{\text{min}}\right)\left(0.01602 \frac{\text{ft}^3}{\text{lb}}\right) \left| \frac{1 \text{ gal}}{0.13368 \text{ ft}^3} \right|$$

$$= 8.6 \text{ gal/min} \longleftarrow \quad (AV)_w$$

<u>Exergy Accounting</u>

<u>Power Input</u>: $\dot{W}_c = (9.3)(122.11 - 109.07) = 121.27$ Btu/min

<u>Exergy Input from well water</u>: $(\dot{E}_{f\,in} - \dot{E}_{f\,out})_w$

$$(\dot{E}_{f\,in} - \dot{E}_{f\,out})_w = \left[(h_{in} - h_{out})_w - T_0(s_{in} - s_{out})_w\right]\dot{m}_w$$

$$= \left\{c_{p_w}(T_{in} - T_{out})_w - T_0\left[c_{p_w} \ln\left(\frac{T_{in}}{T_{out}}\right)_w - \cancel{R \ln\left(\frac{p_{in}}{p_{out}}\right)_w}^{0}\right]\right\}\dot{m}_w$$

$$= \left\{(1)(55 - 45) - (480)(1)\ln\left(\frac{515}{505}\right)\right\}(71.87) = 42.25 \text{ Btu/min}$$

<u>Destructions</u> (using Eq. 6.13 for water)

• Evaporator: $0 = \dot{m}_R(s_4 - s_1) + \dot{m}_w\left(c_w \ln\left(\frac{T_{in}}{T_{out}}\right)_w + (\dot{\sigma}_{cv})_{evap}\right.$

$$(\dot{E}_d)_{evap} = T_0(\dot{\sigma}_{cv})_{evap} = T_0\left[\dot{m}_R(s_1 - s_4) + \dot{m}_w c_w \ln\left(\frac{T_{out}}{T_{in}}\right)_w\right]$$

$$= (480)\left[(9.3)(0.2254 - 0.06766) + (71.87)(1)\ln\left(\frac{505}{515}\right)\right] = 27.71 \frac{\text{Btu}}{\text{min}}$$

• Compressor: $(\dot{E}_d)_c = T_0(\dot{\sigma}_{cv})_{comp} = T_0 \dot{m}(s_2 - s_1) = (480)(9.3)(0.2299 - 0.2254)$

$$= 20.09 \text{ Btu/lb}$$

• Condenser: $0 = \dot{m}_R(s_2 - s_3) + \dot{m}_{air}\left[c_{p_a} \ln\left(\frac{T_{in}}{T_{out}}\right)_{air} - \cancel{R \ln\left(\frac{p_{in}}{p_{out}}\right)_{air}}^{0}\right] + (\dot{\sigma}_{cv})_{cond}$

$$(\dot{E}_d)_{cond} = T_0(\dot{\sigma}_{cv})_{cond} = T_0\left[\dot{m}_R(s_3 - s_2) + \dot{m}_{air} c_{p_a} \ln\left(\frac{T_{out}}{T_{in}}\right)_{air}\right]$$

$$= (480)\left[(9.3)(0.0661 - 0.2299) + (87.5)(0.24)\ln\left(\frac{570}{530}\right)\right] = 2.21 \text{ Btu/min}$$

• Valve: $0 (\dot{E}_d)_{valve} = T_0 (\dot{\sigma}_{cv})_{valve} = T_0 \dot{m}_R (s_4 - s_3)$

$$= (480)(9.3)(0.06766 - 0.0661) = 6.96 \text{ Btu/min}$$

<u>Exergy delivered to air</u>: $(\dot{E}_{f\,out} - \dot{E}_{f\,in})_{air}$

$$(\dot{E}_{f\,out} - \dot{E}_{f\,in})_{air} = \dot{m}_{air} [(h_{out} - h_{in})_{air} - T_0 (s_{out} - s_{in})_{air}]$$

$$= \dot{m}_{air} \left[c_{pa}(T_{out} - T_{in}) - (480) \left\{ c_{pa} \ln\left(\frac{T_{out}}{T_{in}}\right)_{air} - \cancel{R \ln\left(\frac{P_{out}}{P_{in}}\right)_{air}}^{\,0} \right\} \right]$$

$$= (87.5) \left[(0.24)(110 - 70) - (480)(0.24) \ln\left(\frac{570}{530}\right) \right] = 106.59 \text{ Btu/min}$$

<u>Summary</u>

<u>Inputs</u>

Compressor	121.27 Btu/min
Evaporator	42.25
TOTAL	163.52 Btu/min

<u>Destructions</u>

Evaporator	27.71 Btu/min
Compressor	20.09
Condenser	2.21
Valve	6.96
SUB-TOTAL	56.97 Btu/min

<u>Output</u>

Heated air	106.59 Btu/min
TOTAL	163.56 Btu/min

Exergy Accounting

The output of exergy is the exergy transferred to the heated air. The input that we pay for is the compressor power. Thus

$$\varepsilon = \frac{\text{Output to heated air}}{\text{Compressor power}} = \frac{106.59}{121.27} = 0.879$$

Exergetic efficiency

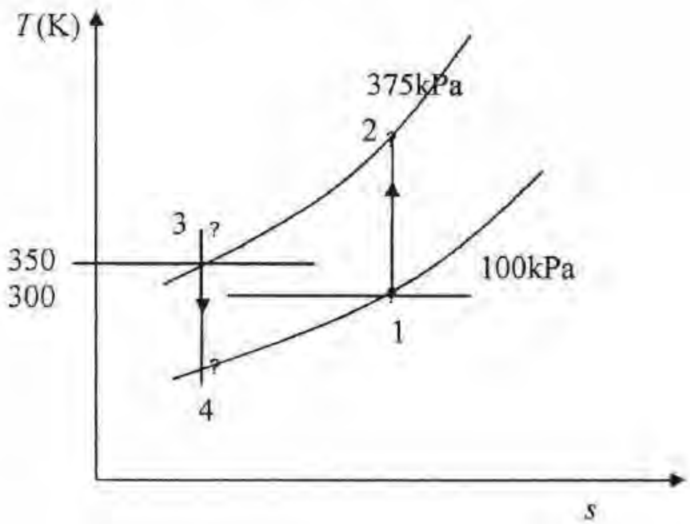
10.43 Air enters the compressor of an ideal Brayton refrigeration cycle at 100 kPa, 300 K. The compressor pressure ratio is 3.75, and the temperature at the turbine inlet is 350 K. Determine the

(a) net work input, per unit mass of air flow, in kJ/kg.
(b) refrigeration capacity, per unit mass of air flow, in kJ/kg.
(c) coefficient of performance.
(d) coefficient of performance of a Carnot refrigeration cycle operating between thermal reservoirs at $T_C = 300$ K and $T_H = 350$ K, respectively.

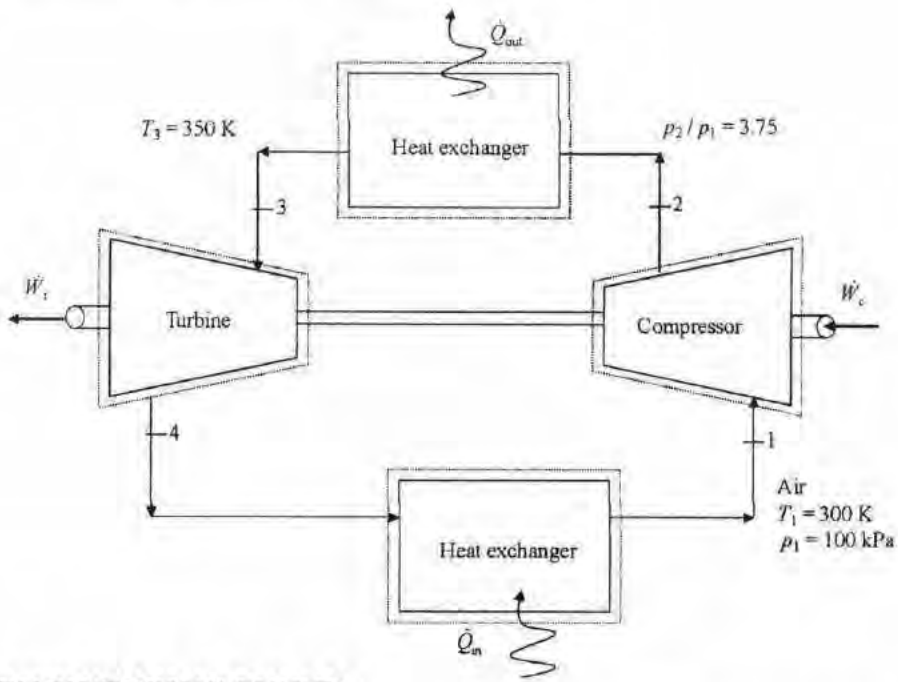
KNOWN: An ideal Brayton refrigeration cycle uses air as the working fluid. The compressor pressure ratio is given. Inlet conditions are known at the compressor and turbine.

FIND: Determine the (a) net work input, (b) refrigeration capacity, (c) coefficient of performance, and (d) coefficient of performance for a Carnot cycle.

SCHEMATIC AND GIVEN DATA:



Problem 3.43 (Continued) – Page 2



ENGINEERING MODEL:

(1) Each component is analyzed as a control volume at steady state.
(2) The compression and expansion processes are isentropic.
(3) There are no pressure drops through the heat exchangers.
(4) Kinetic and potential energy effects are negligible.
(5) The working fluid is air modeled as an ideal gas.

ANALYSIS: First, fix the principal states of the cycle (See Table A-22).

State 1 $T_1 = 300$ K: $h_1 = 300.19$ kJ/kg, $p_{r1} = 1.3860$

State 2 $p_{r2} = p_{r1}\,(p_2/p_1) = 1.3860(3.75) = 5.1975$: $h_2 = 438.33$ kJ/kg

State 3 $T_3 = 350$ K: $h_3 = 350.49$ kJ/kg, $p_{r3} = 2.379$

State 4 $p_{r4} = p_{r3}\,(p_4/p_3) = 2.379(1/3.75) = 0.6344$: $h_4 = 239.9$ kJ/kg

(a) The net work input per unit mass of air flow is

$$\frac{\dot{W}_{cycle}}{\dot{m}} = \frac{\dot{W}_c}{\dot{m}} - \frac{\dot{W}_t}{\dot{m}} = (h_2 - h_1) - (h_3 - h_4)$$

$$= \left[(438.33 - 300.19) - (350.49 - 239.9)\right]\frac{\text{kJ}}{\text{kg}} = 27.55\,\frac{\text{kJ}}{\text{kg}} \quad \longleftarrow$$

(b) The refrigeration capacity is

$$\frac{\dot{Q}_{\text{in}}}{\dot{m}} = (h_1 - h_4) = (300.19 - 239.9)\frac{\text{kJ}}{\text{kg}} = 60.29\,\frac{\text{kJ}}{\text{kg}}$$

(c) The coefficient of performance (β) is

$$\beta = \frac{\dfrac{\dot{Q}_{\text{in}}}{\dot{m}}}{\dfrac{\dot{W}_{\text{cycle}}}{\dot{m}}} = \frac{60.29}{27.55} = 2.19$$

(d) For a Carnot refrigeration cycle with $T_C = 300$ K and $T_H = 350$ K

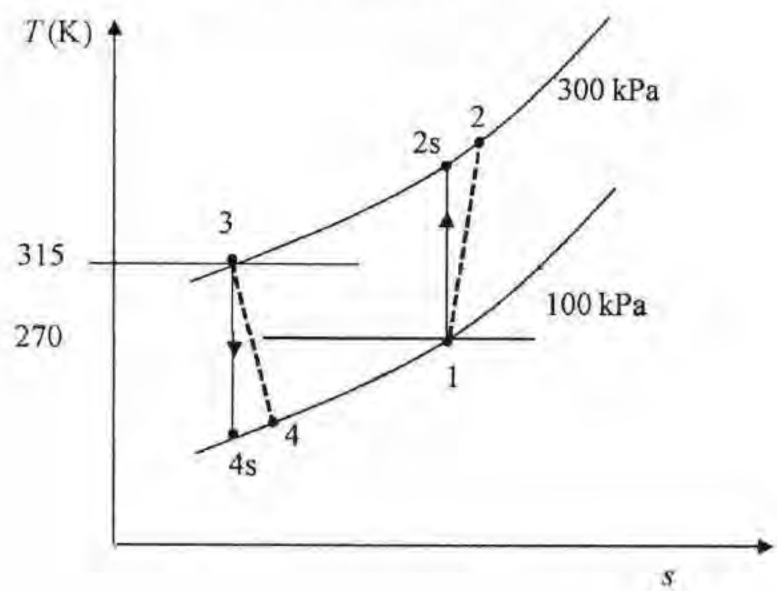
$$\beta_{Carnot} = \frac{T_C}{T_H - T_C} = \frac{300}{350 - 300} = 6$$

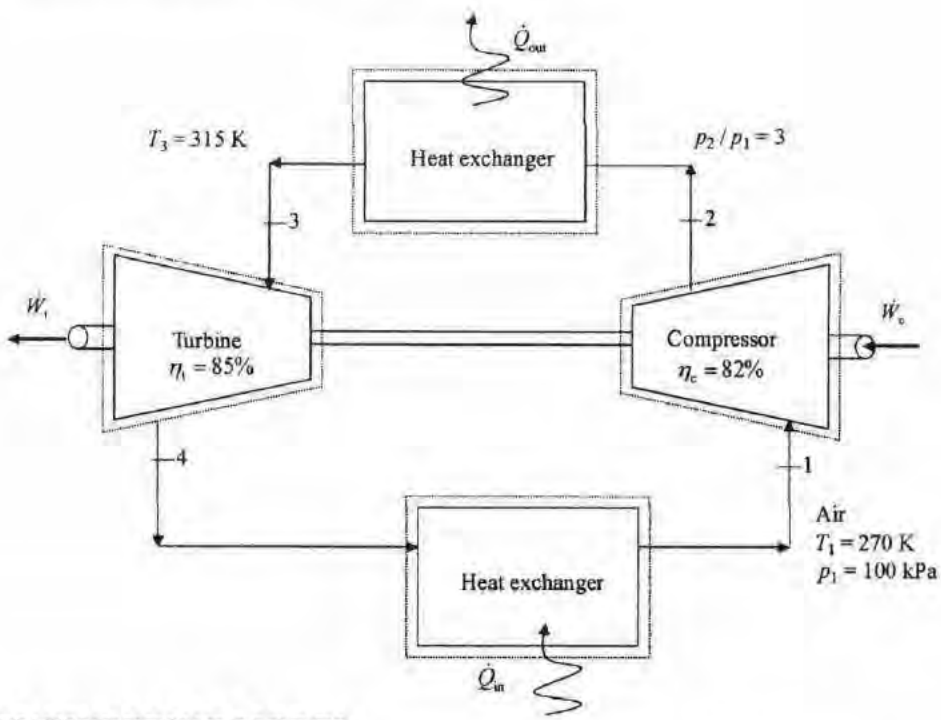
10.44 Air enters the compressor of a Brayton refrigeration cycle at 100 kPa, 270 K. The compressor pressure ratio is 3, and the temperature at the turbine inlet is 315 K. The compressor and turbine have isentropic efficiencies of 82 and 85%, respectively. Determine the
(a) net work input, per unit mass of air flow, in kJ/kg.
(b) exergy accounting of the net power input, in kJ per kg of air flowing. Discuss.
Let $T_0 = 315$ K.

KNOWN: A Brayton refrigeration cycle uses air as the working fluid. Inlet conditions are known for the compressor and turbine, and the compressor and turbine isentropic efficiencies are given. The compressor pressure ratio is also known.

FIND: Determine the (a) net work per unit mass of air flowing, and (b) exergy accounting of the net power input. Discuss.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL:
(1) Each component is analyzed as a control volume at steady state.
(2) The compression and expansion processes are adiabatic.
(3) There are no pressure drops through the heat exchangers.
(4) Kinetic and potential energy effects are negligible.
(5) The working fluid is air modeled as an ideal gas.
(6) Let $T_o = 315$ K.

ANALYSIS: First, fix each of the principal states of the cycle (See Table A-22).

State 1 $T_1 = 270$ K: $h_1 = 270.11$ kJ/kg, $p_{r1} = 0.9590$

State 2

$$p_{r2} = p_{r1}\frac{(p_2)}{(p_1)} = 0.9590\,(3) = 2.877$$

Interpolating in Table A - 22, $h_{2s} = 370.10$ kJ/kg

Determine the enthalpy at the compressor exit, h_2, by using the compressor isentropic efficiency

$$\eta_c = \frac{(h_{2s} - h_1)}{(h_2 - h_1)} \quad \text{or} \quad h_2 = \frac{(h_{2s} - h_1)}{\eta_c} + h_1$$

$$h_2 = \frac{(370.10 - 270.11)\dfrac{\text{kJ}}{\text{kg}}}{0.82} + 270.11\frac{\text{kJ}}{\text{kg}} = 392.05\frac{\text{kJ}}{\text{kg}}$$

Problem 10.44 (Continued) – Page3

State 3 $T_3 = 315$ K: $h_3 = 315.27$ kJ/kg, $p_{r3} = 1.6442$

State 4

$$p_{r4} = p_{r3}\frac{(p_4)}{(p_3)} = 1.6442\,(3) = 0.5481$$

Interpolating in Table A - 22, $h_{4s} = 230.07$ kJ/kg

Determine the enthalpy at the turbine exit, h_4, by using the turbine isentropic efficiency

$$\eta_t = \frac{(h_4 - h_3)}{(h_{4s} - h_3)} \quad \text{or} \quad h_4 = \eta_t(h_{4s} - h_3) + h_3$$

$$h_4 = 0.85(230.07 - 315.27)\frac{\text{kJ}}{\text{kg}} + 315.27\frac{\text{kJ}}{\text{kg}} = 242.85\frac{\text{kJ}}{\text{kg}}$$

(a) The coefficient of performance (β) is

$$\beta = \frac{\dfrac{\dot{Q}_{\text{in}}}{\dot{m}}}{\dfrac{\dot{W}_{\text{cycle}}}{\dot{m}}} = \frac{(h_1 - h_4)}{(h_2 - h_1) - (h_3 - h_4)}$$

$$\beta = \frac{(270.11 - 242.85)}{(392.05 - 270.11) - (315.27 - 242.85)} = \frac{27.26}{49.52} = 0.55$$

(b) The exergy destruction rate for the compressor is

$$\left(\frac{\dot{\text{E}}_{\text{d}}}{\dot{m}}\right)_{\text{c}} = T_{\text{o}}\left(\frac{\dot{\sigma}_{\text{c}}}{\dot{m}}\right) = T_{\text{o}}\left[\left(s_2^{\circ} - s_1^{\circ}\right) - R\ln\left(\frac{p_2}{p_1}\right)\right]$$

From Table A - 22, $s_1^{\circ} = 1.59634$ kJ/kg·K and with interpolating, $s_2^{\circ} = 1.96930$ kJ/kg·K

Therefore

$$\left(\frac{\dot{\text{E}}_{\text{d}}}{\dot{m}}\right)_{\text{c}} = 315\text{ K}\left[(1.96930 - 1.59634)\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - \left(\frac{8.314}{28.97}\right)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\ln\left(\frac{300}{100}\right)\right] = 18.17\frac{\text{kJ}}{\text{kg}}$$

Similarly for the turbine

$$\left(\frac{\dot{\text{E}}_{\text{d}}}{\dot{m}}\right)_{\text{t}} = T_{\text{o}}\left(\frac{\dot{\sigma}_{\text{t}}}{\dot{m}}\right) = T_{\text{o}}\left[\left(s_4^{\circ} - s_3^{\circ}\right) - R\ln\left(\frac{p_4}{p_3}\right)\right]$$

From Table A - 22, $s_3^{\circ} = 1.75106$ kJ/kg·K and with interpolating, $s_4^{\circ} = 1.48979$ kJ/kg·K

Therefore

$$\left(\frac{\dot{\text{E}}_{\text{d}}}{\dot{m}}\right)_{\text{t}} = 315\text{ K}\left[(1.48979 - 1.75106)\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - \left(\frac{8.314}{28.97}\right)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\ln\left(\frac{100}{300}\right)\right] = 17.02\frac{\text{kJ}}{\text{kg}}$$

Problem 10.44 (Continued) – Page 4

\#1 The net change in exergy as air passes through the high-temperature heat exchanger is

$$e_{f3} - e_{f2} = (h_3 - h_2) - T_o(s_3 - s_2) = (h_3 - h_2) - T_o\left[(s_3^o - s_2^o) - R\ln\left(\frac{p_3}{p_2}\right)\right]$$

$$= (315.27 - 392.05)\frac{\text{kJ}}{\text{kg}} - 315\text{ K}\left[(1.75106 - 1.96930)\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 0\right] = -8.03\frac{\text{kJ}}{\text{kg}}$$

The net change in exergy as air passes through the low-temperature heat exchanger is

$$e_{f1} - e_{f4} = (h_1 - h_4) - T_o(s_1 - s_4) = (h_1 - h_4) - T_o\left[(s_1^o - s_4^o) - R\ln\left(\frac{p_1}{p_4}\right)\right]$$

$$= (270.11 - 242.85)\frac{\text{kJ}}{\text{kg}} - 315\text{ K}\left[(1.59634 - 1.48979)\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 0\right] = -6.30\frac{\text{kJ}}{\text{kg}}$$

From the coefficient of performance calculation, net work input is 49.52 kJ/kg.

Exergy accounting of the net power input:

Net exergy input by work:	49.52 kJ/kg	
Disposition of the net net exergy input		
• Exergy transfer out		
o High-temperature heat exchanger:	8.03 kJ/kg	(16.2%)
o Low-temperature heat exchanger:	6.30 kJ/kg	(12.7%)
• Exergy destruction:		
o Compressor:	18.17 kJ/kg	(36.7%)
o Turbine:	17.02 kJ/kg	(34.4%)
	49.52 kJ/kg	100%

In this case, over 2/3 of the net exergy input to the cycle is destroyed while achieving a modest refrigeration effect as shown by the coefficient of performance.

---

1. For the heat exchangers there is no work other than flow work. Also, there are no internal irreversibilties as the pressure is constant. Thus, an energy rate balance for the air side of each heat exchanger takes the form

$$0 = \dot{E}_q - \dot{W}_{cv} + \dot{m}(e_{f,in} - e_{f,out}) - \dot{E}_d$$

$$\dot{m}(e_{f,out} - e_{f,in}) = \dot{E}_q$$

where the change in flow exergy is equal to the exergy transfer accompanying heat transfer. For the low-temperature heat exchganger, heat transfer occurs at $T < T_0$, and thus the direction of exergy transfer is *opposite* the direction of heat transfer; i.e from the air to the cold space. For the high-tempoerature heat exchanger, heat transfer and exergy transfer are in the same direction; from the air to the warm space.

10.45 Plot the quantities calculated in parts (a) through (c) of Problem 10.43 versus the compressor pressure ratio ranging from 3 to 6. Repeat for compressor and turbine isentropic efficiencies of 90%, 85%, and 80%.

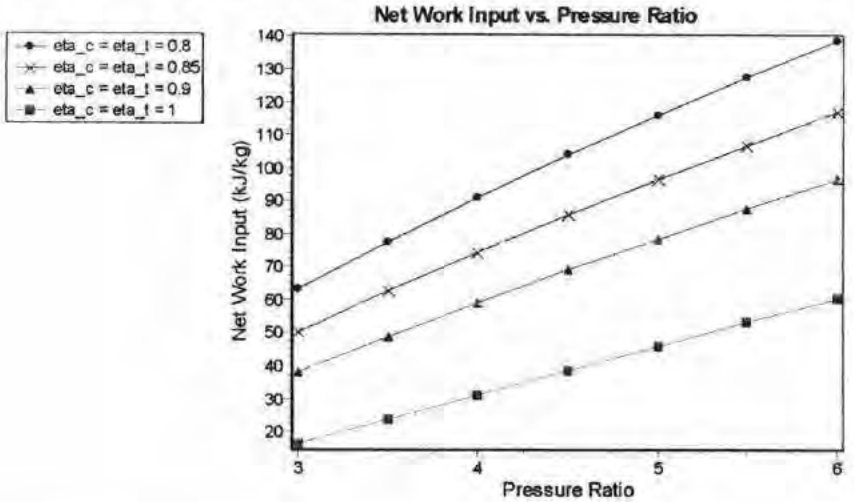
IT CODE:

```
T1 = 300 //K
p1 = 100 //kPa
rp = 3.75
T3 = 350 //K
eta_t = 1
eta_c = 1
mdot = 1 //Assume a unit mass flow rate of 1 kg/s.

h1 = h_T("Air", T1)
s1 = s_hP("Air", h1, p1)
s2s = s1
p2 = rp * p1
s2s = s_hP("Air", h2s, p2)
h2 = h1 + (h2s - h1) / eta_c
h3 = h_T("Air", T3)
p3 = p2
s3 = s_hP("Air", h3, p3)
s4s = s3
p4 = p1
s4s = s_hP("Air", h4s, p4)
h4 = h3 - (h3-h4s)*eta_t

Wdotcycle/mdot = Wdotc / mdot - Wdott/mdot
Wdott/mdot = (h3 - h4)
Wdotc/mdot = (h2 - h1)
Qdotin/mdot = (h1 - h4)
beta = (h1 - h4) / ((h2 - h1) - (h3 - h4))
```

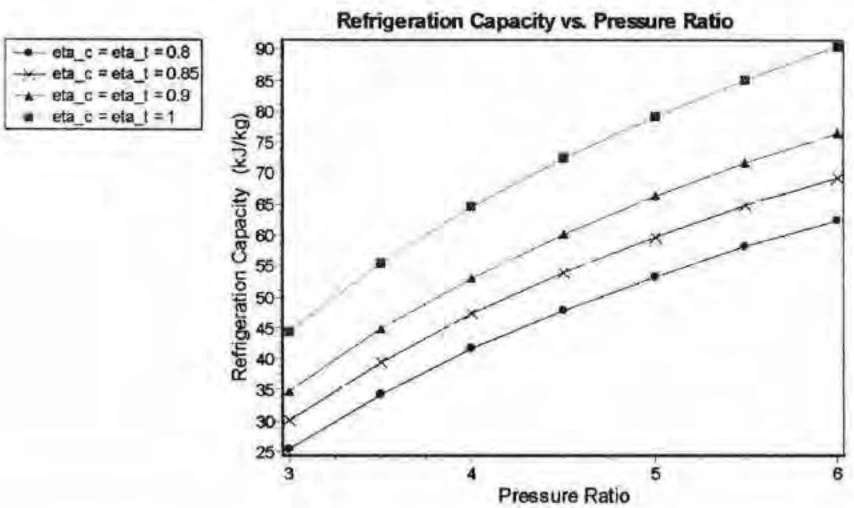
These result compare favorably with the results from the solution of Problem 10.43. The slight differences are due to small differences in the data stored in *IT* compared to Table A-22.

β	2.185
h_1	300 kJ/kg
h_{2s}	438.2 kJ/kg
h_3	350.3 kJ/kg
h_{4s}	239.7 kJ/kg
$\dot{Q}_{in}/\dot{m}$	60.31 kJ/kg
$\dot{W}_{cycle}/\dot{m}$	27.6 kJ/kg
η_c	1
η_t	1
r_p	3.75

Problem 10.45 (Continued) – Page 2

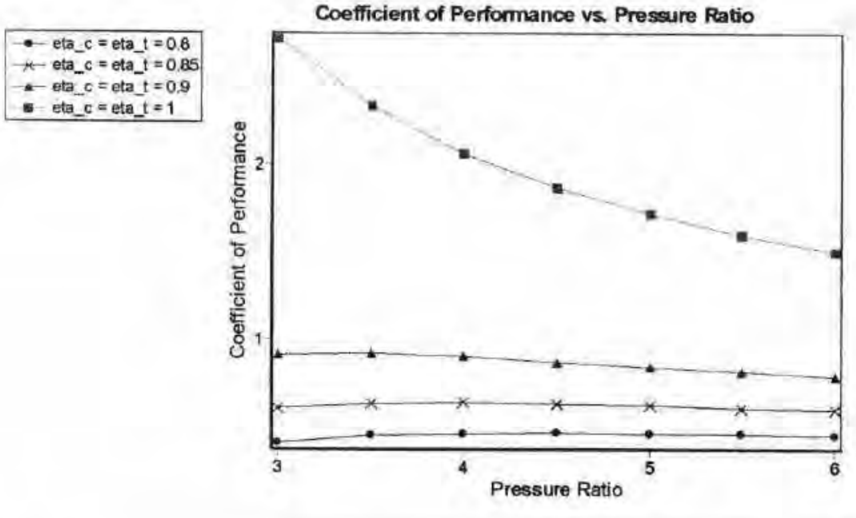


As pressure ratio increases, the required net work input increases. Also, as the turbine and compressor efficiencies decrease, the turbine produces less work and the compressor requires more work. Hence the net work input also increases.



As the pressure ratio increases, the refrigeration capacity increases for fixed T_3. Further, the refrigeration capacity decreases as the turbine and compressor efficiencies decrease.

Problem 10.45 (Continued) – Page 3



For the isentropic case, the coefficient of performance decreases continuously with increasing pressure ratio. However, the curves for lower efficiencies exhibit a slight maximum in the pressure ratio range studied.

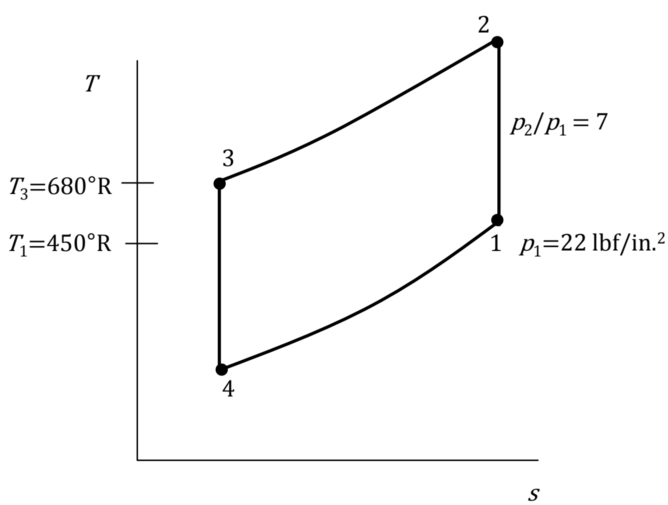
Problem 10.46

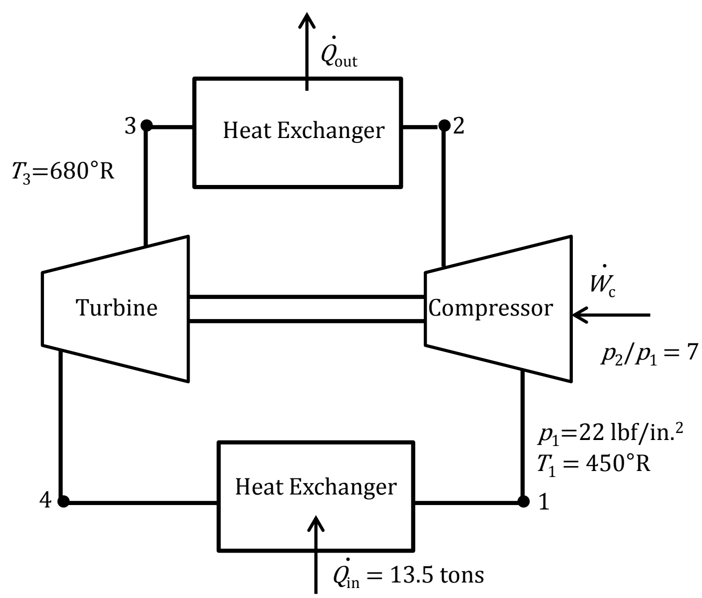
An ideal Brayton refrigeration cycle has a compressor pressure ratio of 7. At the compressor inlet, the pressure and temperature of the entering air are 22 lbf/in.^{2} and 450°R. The temperature at the inlet of the turbine is 680°R. For a refrigerating capacity of 13.5 tons, determine:
(a) the mass flow rate of the refrigerant, in lb/min.
(b) the net power input, in Btu/min.
(c) the coefficient of performance.

Known:
Air is the working fluid in an ideal Brayton refrigeration cycle. Operating data and the refrigerating capacity are specified.

Find:
Determine (a) the air mass flow rate, (b) the net power input, and (c) the coefficient of performance.

Schematic and Known Data:





Engineering Model:
(1) Each component of the cycle is analyzed as a control volume at steady state.
(2) The turbine and compressor processes are isentropic.
(3) There are no pressure drops through the heat exchangers.
(4) Kinetic and potential energy effects are negligible.
(5) The working fluid is air modeled as an ideal gas.

Analysis:

First, fix each principal state (Table A-22E):
State 1: $T_1 = 450°\text{R}$ with interpolation $\rightarrow h_1 = 107.51 \frac{\text{Btu}}{\text{lb}}, p_{r_1} = 0.7345$

State 2: $s_2 = s_1,\ \ p_{r_2} = p_{r_1}\frac{p_2}{p_1} = 0.7345(7) = \ 5.1415$ with interpolation $\rightarrow h_2 = 187.87\,\frac{\text{Btu}}{\text{lb}}$

State 3: $T_3 = 680°\text{R} \rightarrow h_3 = 162.73\,\frac{\text{Btu}}{\text{lb}},\ \ p_{r_3} = 3.111$

State 4: $s_4 = s_3,\ \ p_{r_4} = p_{r_3}\frac{P_4}{P_3} = \frac{3.111}{7} = 0.4444$ with interpolation $\rightarrow h_4 = 93.05\,\frac{\text{Btu}}{\text{lb}}$

(a) The mass flow rate is:

$$\dot{m} = \frac{\dot{Q}_{\text{in}}}{h_1 - h_4} = \frac{13.5\ \text{tons}}{(107.51 - 93.05)\,\frac{\text{Btu}}{\text{lb}}} \cdot \left|\frac{200\,\frac{\text{Btu}}{\text{min}}}{1\ \text{ton}}\right| = 186.72\,\frac{\text{lb}}{\text{min}} \quad \longleftarrow$$

(b) Thus, the net power input is:

$$\begin{aligned}\dot{W}_{\text{c}} &= \dot{m}[(h_2 - h_1) - (h_3 - h_4)] \\ &= \left(186.72\,\frac{\text{lb}}{\text{min}}\right)[(187.87 - 107.51) - (162.73 - 93.05)]\,\frac{\text{Btu}}{\text{lb}} \\ &= 1994.2\,\frac{\text{Btu}}{\text{min}} \quad \longleftarrow\end{aligned}$$

(c) The coefficient of performance is:

$$\beta = \frac{\dot{Q}_{\text{in}}}{\dot{W}_{\text{c}}} = \frac{13.5\ \text{tons}}{1994.2\,\frac{\text{Btu}}{\text{min}}} \cdot \left|\frac{200\,\frac{\text{Btu}}{\text{min}}}{1\ \text{ton}}\right| = 1.35 \quad \longleftarrow$$

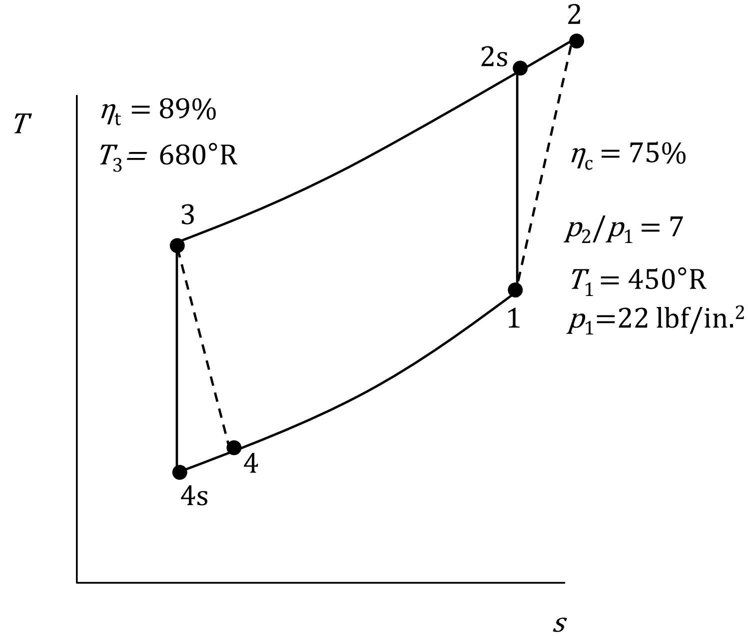
Problem 10.47

Reconsider Problem 10.46, but include in the analysis that the compressor and turbine have isentropic efficiencies of 75% and 89%, respectively. Answer the same questions as in Problem 10.46 and determine the rate of entropy production within the compressor and turbine, each in Btu/min·°R.

<u>Known:</u>
The Brayton refrigeration cycle of Problem 10.46 is modified to include that the compressor and turbine have isentropic efficiencies of 75% and 89%, respectively.

<u>Find:</u>
Determine (a) the air mass flow rate, (b) the net power input, (c) the coefficient of performance and (d) the entropy production within the compressor and turbine.

<u>Schematic and Known Data:</u>



<u>Engineering Model:</u>
(1) Each component of the cycle is analyzed as a control volume at steady state.
(2) The turbine and compressor processes are adiabatic and operate with isentropic efficiencies given.
(3) There are no pressure drops through the heat exchangers.
(4) Kinetic and potential energy effects are negligible.
(5) The working fluid is air modeled as an ideal gas.

<u>Analysis:</u>
First, fix each principal state (Table A-22E). From the solution of problem 10 .46:

$$h_1 = 107.51\frac{\text{Btu}}{\text{lb}},\ \ h_{2s} = 187.87\frac{\text{Btu}}{\text{lb}},\ \ h_3 = 162.73\frac{\text{Btu}}{\text{lb}},\ \ h_{4s} = 93.05\frac{\text{Btu}}{\text{lb}}$$

<u>State 1:</u> $T_1 = 450°\text{R}$ with interpolation $\rightarrow s_1^o = 0.55704 \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$

<u>State 2:</u> $\eta_c = \frac{h_{2s}-h_1}{h_2-h_1} \rightarrow h_2 = h_1 + \frac{h_{2s}-h_1}{\eta_c} = 107.51 + \frac{187.87-107.51}{0.75} = 214.66 \frac{\text{Btu}}{\text{lb}}$

and with interpolation $\rightarrow s_2^o = 0.72258 \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$

<u>State 3:</u> $T_3 = 680°\text{R} \rightarrow s_3^o = 0.65621 \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$

<u>State 4:</u>

$\eta_t = \frac{h_3-h_4}{h_3-h_{4s}} \rightarrow h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 162.73 - 0.89(162.73 - 93.05) = 100.715 \frac{\text{Btu}}{\text{lb}}$

and with interpolation $\rightarrow s_4^o = 0.54149 \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$

(a) The mass flow rate is:

#1 $$\dot{m} = \frac{\dot{Q}_{\text{in}}}{h_1-h_4} = \frac{13.5 \text{ tons}}{(107.51-100.715)\frac{\text{Btu}}{\text{lb}}}\left|\frac{200\frac{\text{Btu}}{\text{min}}}{1 \text{ ton}}\right| = 397.35 \frac{\text{lb}}{\text{min}}$$

#1 (b) Thus, the net power input is:

$$\begin{aligned}\dot{W}_c &= \dot{m}[(h_2 - h_1) - (h_3 - h_4)] \\ &= \left(397.35 \frac{\text{lb}}{\text{min}}\right)[(214.66 - 107.51) - (162.73 - 100.715)] \frac{\text{Btu}}{\text{lb}} \\ &= 17{,}934.4 \frac{\text{Btu}}{\text{min}}\end{aligned}$$

#1 (c) The coefficient of performance is:

$$\beta = \frac{\dot{Q}_{\text{in}}}{\dot{W}_{\text{c}}} = \frac{13.5 \text{ tons}}{17934.4 \frac{\text{Btu}}{\text{min}}}\left|\frac{200 \frac{\text{Btu}}{\text{min}}}{1 \text{ ton}}\right| = 0.15055$$

(d) The entropy production within the compressor is:

#2 $$\dot{\sigma}_{\text{comp}} = \dot{m}(s_2 - s_1) - \underbrace{\frac{\dot{Q}_{\text{comp}}}{T_{\text{b}}}}_{=0} = \dot{m}\left(s_2^o - s_1^o - R\ln\frac{p_2}{p_1}\right) = \left(397.35 \frac{\text{lb}}{\text{min}}\right)\Bigg[(0.72258 - 0.55704) \frac{\text{Btu}}{\text{lb}\cdot°\text{R}} - \left(\frac{1.986\frac{\text{Btu}}{\text{lbmol}\cdot°\text{R}}}{28.97\frac{\text{lb}}{\text{lbmol}}}\right)\ln 7\Bigg] = 12.77 \frac{\text{Btu}}{\text{min}\cdot°\text{R}}$$

The entropy production within the turbine is:

#2 $$\dot{\sigma}_{\text{turbine}} = \dot{m}(s_4 - s_3) - \underbrace{\frac{\dot{Q}_{\text{turbine}}}{T_{\text{b}}}}_{=0} = \dot{m}\left(s_4^o - s_3^o - R\ln\frac{p_4}{p_3}\right) = \left(397.35 \frac{\text{lb}}{\text{min}}\right)\Bigg[(0.54149 - 0.65621) \frac{\text{Btu}}{\text{lb}\cdot°\text{R}} - \left(\frac{1.986\frac{\text{Btu}}{\text{lbmol}\cdot°\text{R}}}{28.97\frac{\text{lb}}{\text{lbmol}}}\right)\ln\left(\frac{1}{7}\right)\Bigg] = 7.42 \frac{\text{Btu}}{\text{min}\cdot°\text{R}}$$

<u>Comments:</u>

1. Note, the mass flow rate and net power input are higher and the coefficient of performance is lower than in Problem 10.46 due to irreversibilities in the compressor and turbine.
2. The entropy production in the compressor is slightly higher than in the turbine due in part to the lower isentropic efficiency of the compressor compared to the turbine.

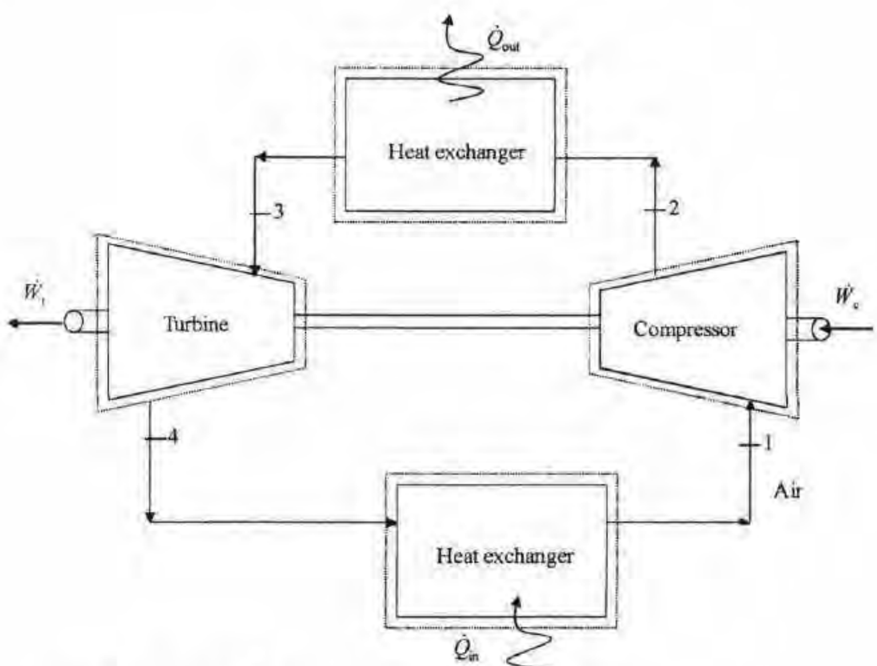
10.48 The table below provides steady state operating data for an ideal Brayton refrigeration cycle with air as the working fluid. The principal states are numbered as in Fig. 10.13. The volumetric flow rate at the turbine inlet is 0.4 m^3/s. Sketch the *T-s* diagram for the cycle and determine the

(a) specific enthalpy, in kJ/kg, at the turbine exit.
(b) mass flow rate, in kg/s.
(c) net power input, in kW.
(d) refrigeration capacity, in kW.
(a) coefficient of performance.

State	p (kPa)	T (K)	h (kJ/kg)	p_r
1	140	270	270.11	0.9590
2	420	-	370.10	2.877
3	420	320	320.29	1.7375
4	140	-	?	-

KNOWN: An ideal Brayton refrigeration cycle uses air as the working fluid. Operating data are provided at principal states of the cycle. The volumetric flow rate at the turbine inlet is known.

FIND: Sketch the *T-s* diagram for the cycle and determine (a) the specific enthalpy at the turbine exit, (b) the mass flow rate, (c) the net power input, (d) the refrigeration capacity, and (e) the coefficient of performance.



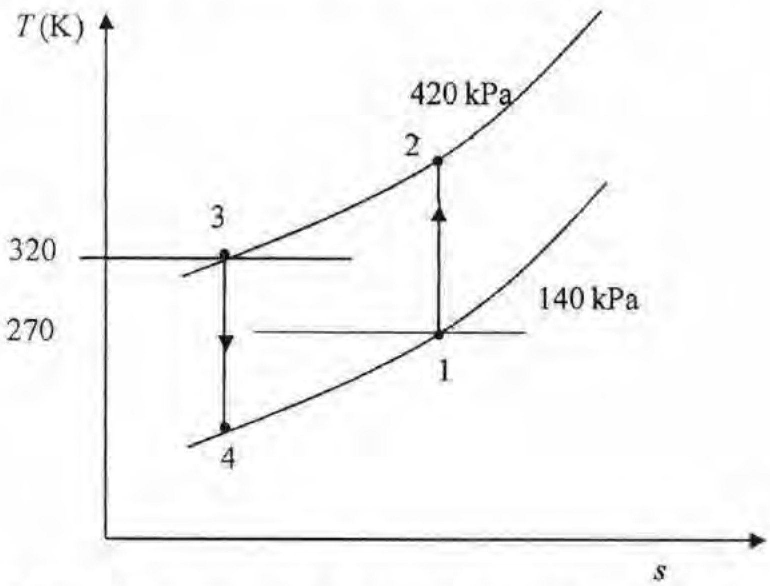
$(AV)_3 = 0.4\ m^3/s$

Problem 10.48 (Continued) – Page 2

ENGINEERING MODEL:

(1) Each component is analyzed as a control volume at steady state.
(2) The compression and expansion processes are isentropic.
(3) There are no pressure drops through the heat exchangers.
(4) Kinetic and potential energy effects are negligible.
(5) The working fluid is air modeled as an ideal gas.

Analysis: The T-s diagram for the ideal cycle is



(a) Since the compression process is isentropic

$$p_{r4} = p_{r3}\frac{(p_4)}{(p_3)} = 1.7375\frac{(140\text{ kPa})}{(420\text{ kPa})} = 0.5792$$

Interpolating in Table A - 22 ; $h_4 = 233.61$ kJ/kg

Therefore

State	p (kPa)	T (K)	h (kJ/kg)	p_r
1	140	270	270.11	0.9590
2	420	-	370.10	2.877
3	420	320	320.29	1.7375
4	140	-	233.61	0.5792

(b) The mass flow rate is

$$\dot{m} = \frac{(\text{AV})_3}{(v_3)} = \frac{(\text{AV})_3(p_3)}{\left(\frac{\overline{R}}{M}\right)(T_3)} = \frac{\left(0.4\frac{\text{m}^3}{\text{s}}\right)(420\text{ kPa})}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(320\text{ K})}\left|\frac{10^3\frac{\text{N}}{\text{m}^2}}{1\text{ kPa}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| = 1.829\frac{\text{kg}}{\text{s}}$$

Problem 10.48 (Continued) – Page 3

(c) Therefore, the net power input is

$$\dot{W}_{cycle} = \dot{W}_c - \dot{W}_t = \dot{m}[(h_2 - h_1) - (h_3 - h_4)]$$

$$= 1.829\,\frac{kg}{s}\left[(370.10 - 270.11) - (320.29 - 233.61)\right]\frac{kJ}{kg}\left|\frac{1\,kW}{1\,\frac{kJ}{s}}\right| = 24.35\,kW$$

(d) The refrigeration capacity is

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = 1.829\,\frac{kg}{s}(270.11 - 233.61)\frac{kJ}{kg}\left|\frac{1\,kW}{1\,\frac{kJ}{s}}\right| = 66.76\,kW$$

(e) The coefficient of performance (β) is

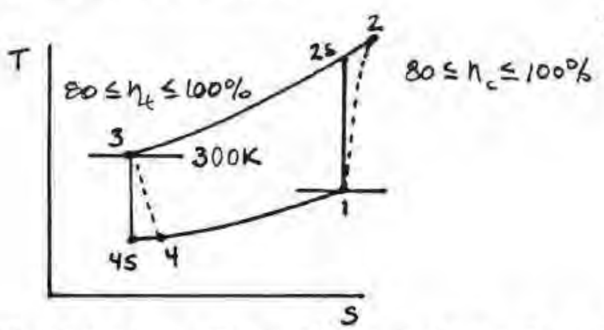
$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{cycle}} = \frac{66.76\,kW}{24.35\,kW} = 2.742$$

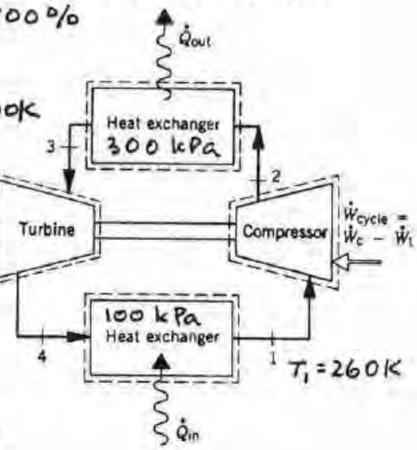
PROBLEM 10.49

KNOWN: Air is the working fluid in a Brayton refrigeration cycle. Operating data are specified.

FIND: (a) Determine the net work per unit mass of air flow and the coefficient of performance for $\eta_c = \eta_t = 100\%$. (b) Plot the same quantities for $\eta_c = \eta_t$ ranging from 80% to 100%

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: For part (a), see Example 10.4. For part (b), see Example 10.5.

ANALYSIS: Sample calculations using data from Table A-22:

State 1: $T_1 = 260K \Rightarrow h_1 = 260.09$ kJ/kg, $p_{r1} = 0.8405$

State 2: $p_{r2s} = p_{r1}(p_2/p_1) = 2.5215 \Rightarrow h_{2s} = 356.31$ kJ/kg

Using the isentropic compressor efficiency

$$h_2 = h_1 + (h_{2s} - h_1)/\eta_c \ ; \ \eta_c = 100\% \Rightarrow h_2 = 356.31 \text{ kJ/kg} \quad (a)$$

$$\eta_c = 80\% \Rightarrow h_2 = 380.37 \text{ kJ/kg} \quad (b)$$

State 3: $T_3 = 300K \Rightarrow h_3 = 300.19$ kJ/kg, $p_{r3} = 1.3860$

State 4: $p_{r4s} = p_{r3}(p_4/p_3) = 0.462 \Rightarrow h_{4s} = 218.97$ kJ/kg

Using the isentropic turbine efficiency

$$h_4 = h_3 - (h_3 - h_{4s})\eta_t \ ; \ \eta_t = 100\% \Rightarrow h_4 = 218.97 \text{ kJ/kg} \quad (a)$$

$$\eta_t = 80\% \Rightarrow h_4 = 235.21 \text{ kJ/kg} \quad (b)$$

The net work is

$$\frac{\dot{W}_{cycle}}{\dot{m}} = \frac{\dot{W}_c}{\dot{m}} - \frac{\dot{W}_t}{\dot{m}} = (h_2 - h_1) - (h_3 - h_4)$$

$\eta_c = \eta_t = 100\% \Rightarrow \dot{W}_{cycle}/\dot{m} = 15$ kJ/kg ⟵ $\dot{W}_{cycle}/\dot{m}$ (part a)

$\eta_c = \eta_t = 80\% \Rightarrow \dot{W}_{cycle}/\dot{m} = 55.3$ kJ/kg

Now, the coefficient of performance is

$$\beta = \frac{\dot{Q}_{in}/\dot{m}}{\dot{W}_{cycle}/\dot{m}} = \frac{(h_1 - h_4)}{\dot{W}_{cycle}/\dot{m}}$$

$\eta_c = \eta_t = 100\% \Rightarrow \beta = 2.74$ ⟵ β (part a)

$\eta_c = \eta_t = 80\% \Rightarrow \beta = 0.45$

PROBLEM 10.49 (Cont'd.) - Page 2

(b) The data for the required plots are obtained using IT, as follows:

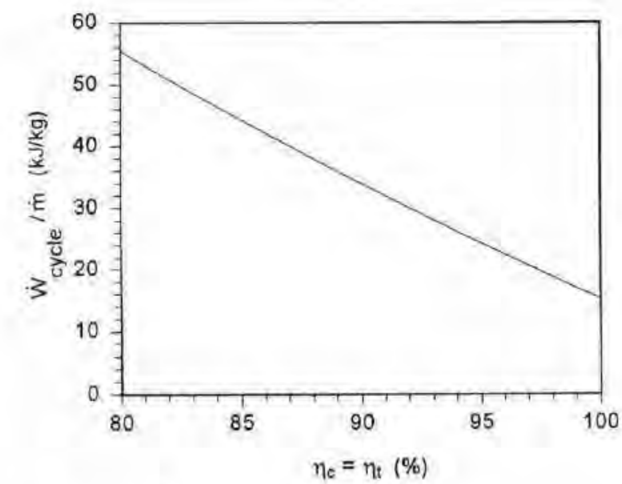
IT Code

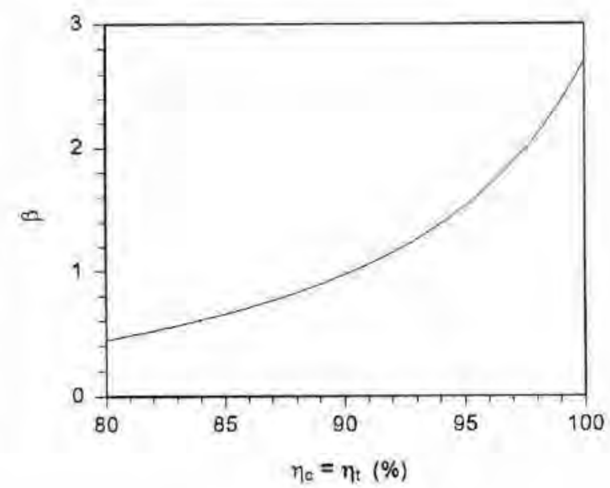
```
T1 = 260 // K
p1 = 100 // kPa
p2 = 300 // kPa
T3 = 300 // K
p3 = p2
p4 = p1
eta_c = 80 // %
eta_t = eta_c
mdot = 1 // Assume a unit mass
//            flow rate of 1 kg/s.

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
s2s = s_hP("Air", h2s, p2)
s2s = s1
h2 = h1 + (h2s - h1) / (eta_c / 100)
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
s4s = s_hP("Air", h4s, p4)
s4s = s3
h4 = h3 - (h3 - h4s) * (eta_t / 100)

Wdot_c / mdot = (h2 - h1)
Wdot_t / mdot = (h3 - h4)
Wdotcycle / mdot = Wdot_c / mdot - Wdot_t / mdot
beta = (h1 - h4) / (Wdotcycle / mdot)
```

IT Results for $\eta_c = \eta_t = 80\%$

$h_1 = 259.9$ kJ/kg
$h_2 = 380.3$ kJ/kg
$h_{2s} = 356.2$ kJ/kg
$h_3 = 300$ kJ/kg
$h_4 = 235.1$ kJ/kg
$h_{4s} = 218.9$ kJ/kg
$\dot{W}_c / \dot{m} = 120.4$ kJ/kg
$\dot{W}_t / \dot{m} = 64.94$ kJ/kg
$\dot{W}_{cycle} / \dot{m} = 55.45$ kJ/kg
$\beta = 0.4471$

PLOTS:





Discussion:

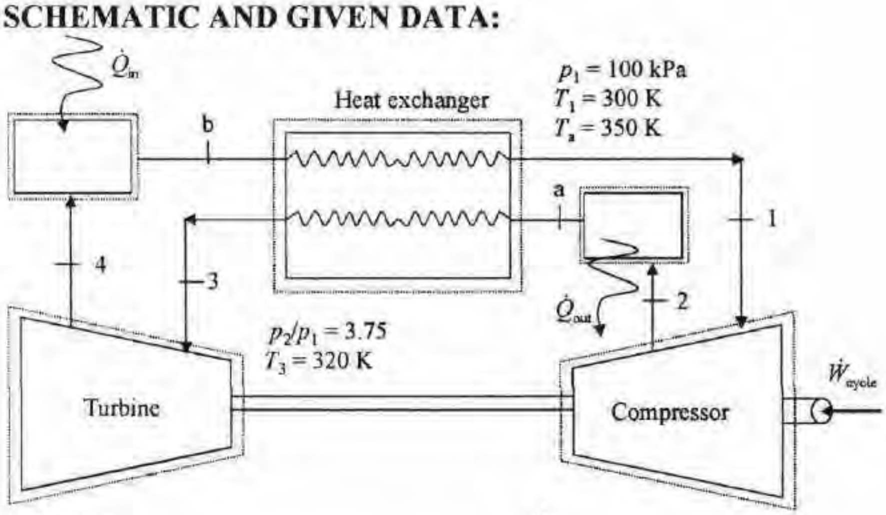
- As the isentropic compressor and turbine efficiencies decrease, the net work input to the cycle increases.
- As turbine efficiency decreases, point 4 on the accompanying T-s diagram moves to the right, thereby decreasing the capacity. The decrease in capacity, coupled with the increase in net work, result in a dramatic drop in coefficient of performance compared to the ideal case (isentropic compression and expansion.)

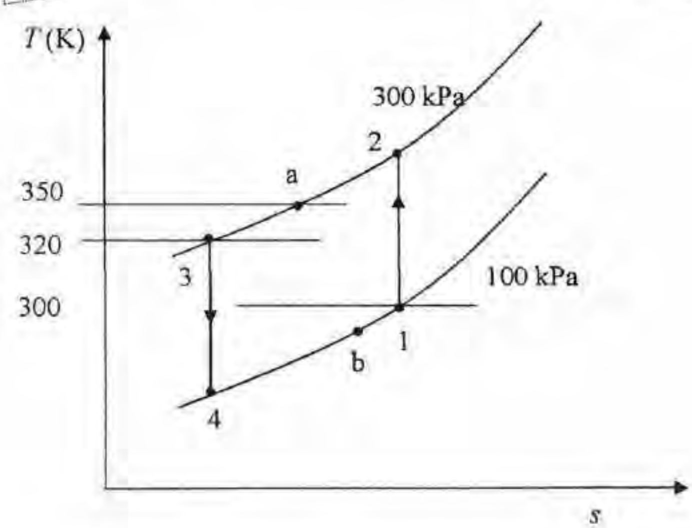
10.50 The Brayton refrigeration cycle of Problem 10.43 is modified by the introduction of a regenerative heat exchanger. In the modified cycle, compressed air enters the regenerative heat exchanger at 350 K and is cooled to 320 K before entering the turbine. Determine for the cycle,
(a) the lowest temperature, in K.
(b) the net work input, per unit mass of air flow, in kJ/kg.
(c) the refrigeration capacity, per unit mass of air flow, in kJ/kg.
(d) the coefficient of performance.

KNOWN: The ideal Brayton refrigeration cycle of Problem 10.43 is modified to include a regenerative heat exchanger.

FIND: Determine (a) the lowest temperature, (b) the net work input per unit mass of air flow, (c) the refrigeration capacity, per unit mass of air flow, and (d) the coefficient of performance.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL:

(1) Each component is analyzed as a control volume at steady state. The control volumes are indicated by dashed lines on the accompanying sketch.
(2) The compression and expansion processes are isentropic.
(3) There are no pressure drops through the heat exchangers.
(4) Kinetic and potential energy effects are negligible.
(5) The working fluid is air modeled as an ideal gas.
(6) There is no heat transfer from the heat exchanger to its surroundings.

ANALYSIS:

Fix the principal states for the cycle (Table A-22). From the solution of Problem 10.43;
$h_1 = 300.19$ kJ/kg,
$h_2 = 438.33$ kJ/kg,
$h_a = 350.49$ kJ/kg (corresponds with state 3 from Problem 10.43)

<u>State 3</u> $T_3 = 320$ K: $h_3 = 320.29$ kJ/kg, $p_{r3} = 1.7375$

<u>State 4</u> $p_{r4} = p_{r3}(p_4/p_3) = p_{r3}(p_1/p_2) = 1.7375(1/3.75) = 0.46333$: $h_4 = 219.16$ kJ/kg

Applying an energy balance to the control volume enclosing the heat exchanger, with assumption 6, we get

$$0 = (h_b - h_1) + (h_a - h_3)$$

Thus

$$h_b = h_1 - h_a + h_3 = 269.99 \text{ kJ/kg}$$

(a) The lowest temperature in the cycle corresponds to state 4. Interpolating in Table A-22 with h_4; $T_4 = 219.2$ K. ←

(b) The net work is

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = \frac{\dot{W}_c}{\dot{m}} - \frac{\dot{W}_t}{\dot{m}} = (h_2 - h_1) - (h_3 - h_4)$$

$$= [(438.33 - 300.19) - (320.29 - 219.16)]\frac{\text{kJ}}{\text{kg}} = 37.01\frac{\text{kJ}}{\text{kg}} \quad \leftarrow$$

(c) The refrigeration capacity becomes

$$\frac{\dot{Q}_{\text{in}}}{\dot{m}} = (h_b - h_4) = (269.99 - 219.16)\frac{\text{kJ}}{\text{kg}} = 50.83\frac{\text{kJ}}{\text{kg}} \quad \leftarrow$$

(d) The coefficient of performance (β) is

Problem 10.50 (Continued) – Page 3

#1 $$\beta = \frac{\dfrac{\dot{Q}_{\text{in}}}{\dot{m}}}{\dfrac{\dot{W}_{\text{cycle}}}{\dot{m}}} = \frac{50.83}{37.01} = 1.373$$

---

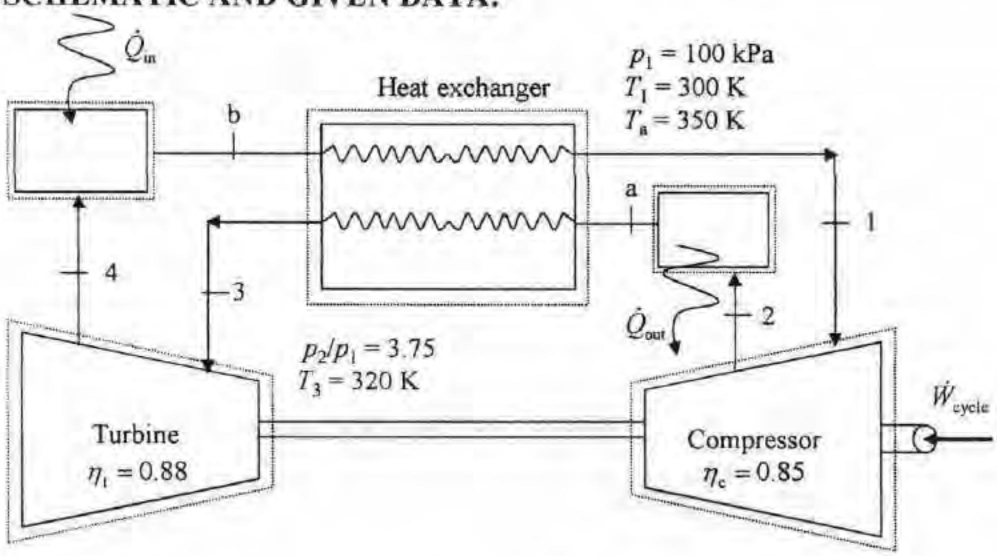
1. Comparing the results with those of Problem 10.43, we see that introducing a regenerator has reduced T_4 significantly. However, comparing the other quantities indicates that the net work has increased, the capacity has decreased, and the coeficient of performance has decreased.

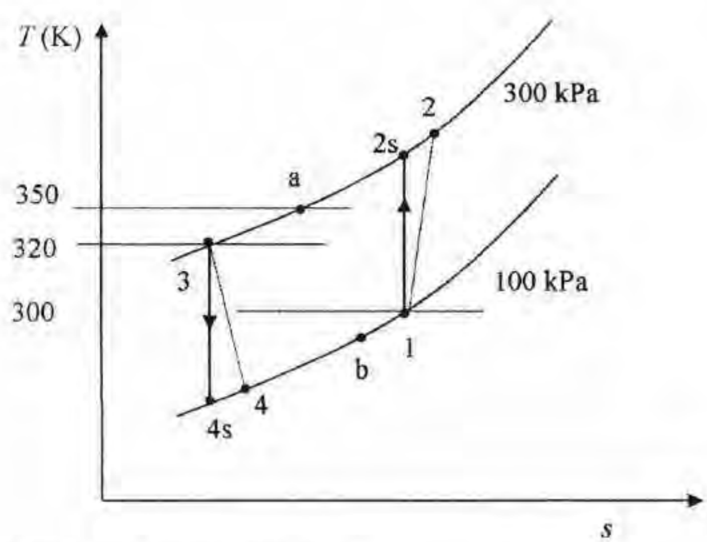
10.51 Reconsider Problem 10.50, but include in the analysis that the compressor and turbine have isentropic efficiencies of 85 and 88% respectively. Answer the same questions as in Problem 10.50.

KNOWN: Reconsider the cycle of Problem 10.50 and include that the compressor and turbine have isentropic efficiencies of 85 and 88%, respectively.

FIND: Determine (a) the lowest temperature, (b) the net work input per unit mass of air flow, (c) the refrigeration capacity, per unit mass of air flow, and (d) the coefficient of performance.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL:

(1) Each component is analyzed as a control volume at steady state. The control volumes are indicated by dashed lines on the accompanying sketch.

(2) The compressor and turbine are adiabatic.

Problem 10.51 (Continued) – Page 2

(3) There are no pressure drops through the heat exchangers.
(4) Kinetic and potential energy effects are negligible.
(5) The working fluid is air modeled as an ideal gas.
(6) There is no heat transfer from the heat exchanger to its surroundings.

ANALYSIS:

Fix the principal states for the cycle (Table A-22). From the solutions of Problems 10.43 and 10.50;

h_1 = 300.19 kJ/kg
h_{2s} = 438.33 kJ/kg (corresponds to state 2 in Problem 10.50)
h_a = 350.49 kJ/kg
h_3 = 320.29 kJ/kg
h_{4s} = 219.16 kJ/kg (corresponds to state 4 in Problem 10.50)
h_b = 269.99 kJ/kg

For state 2, using the compressor efficiency

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1}$$

$$h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c}\right) = 300.19\,\frac{\text{kJ}}{\text{kg}} + \left(\frac{438.33 - 300.19}{0.85}\right)\frac{\text{kJ}}{\text{kg}} = 462.71\,\frac{\text{kJ}}{\text{kg}}$$

For state 4, using the turbine efficiency

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

$$h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 320.29\,\frac{\text{kJ}}{\text{kg}} - 0.88(320.29 - 219.16)\frac{\text{kJ}}{\text{kg}} = 231.3\,\frac{\text{kJ}}{\text{kg}}$$

(a) The lowest temperature in the cycle corresponds to state 4. Interpolating in Table A-22 with h_4; T_4 = 231.3 K. ←

(b) The net work is

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = \frac{\dot{W}_c}{\dot{m}} - \frac{\dot{W}_t}{\dot{m}} = (h_2 - h_1) - (h_3 - h_4)$$

$$= \left[(462.71 - 300.19) - (320.29 - 231.3)\right]\frac{\text{kJ}}{\text{kg}} = 73.53\,\frac{\text{kJ}}{\text{kg}}$$ ←

(c) The refrigeration capacity is

$$\frac{\dot{Q}_{\text{in}}}{\dot{m}} = (h_b - h_4) = (269.99 - 231.3)\frac{\text{kJ}}{\text{kg}} = 38.69\,\frac{\text{kJ}}{\text{kg}}$$ ←

Problem 10.51 (Continued) – Page 3

(d) The coefficient of performance (β) is

#1 $$\beta = \frac{\dfrac{\dot{Q}_{\text{in}}}{\dot{m}}}{\dfrac{\dot{W}_{\text{cycle}}}{\dot{m}}} = \frac{38.69}{73.53} = 0.526$$

---

1. Comparing the results with those of Problem 10.50, we see the penalties associated with irreversibilities in the compressor and tubine include: increased $T_{\min}$, increased net work, decreased capacity, and decreased coeficient of performance.

10.52 Plot the quantities calculated in parts (a) through (d) of Problem 10.50 versus compressor pressure ratio ranging from 4 to 7. Repeat for equal compressor and turbine isentropic efficiencies of 95%, 90%, and 80%.

See the solutions to Problems 10.50 and 10.43 for more detail.

IT Code

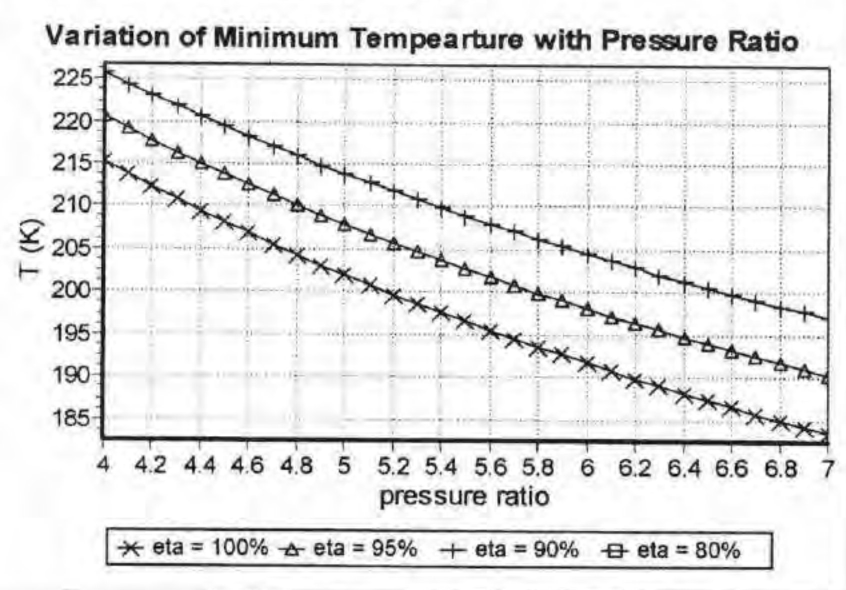
```
// Given Data
T1 = 300  //K
p1 = 100  //kPa
rp = 3.75
Ta = 350
T3 = 320
pa = p2
p3 = p2
p4 = p1
pb = p1
p2 = rp*p1
etac = 1
etat = etac
mdot = 1

// Fix the states
h1 = h_T("Air",T1)
s1 = s_Tp("Air",T1,p1)
s2s = s1
s2s = s_hp("Air",h2s,p2)
h2 = h1 + (h2s - h1)/etac
ha = h_T("Air",Ta)
h3 = h_T("Air",T3)
s3 = s_Tp("Air",T3,p3)
s4s = s3
s4s = s_hp("Air",h4s,p4)
h4 = h3 - (h3 - h4s)*etat
(ha - h3) = (h1 - hb)

// part (a)
T4 = T_h("Air",h4)

// part (b)
Wdot_cycle/mdot = (h2 - h1) - (h3 - h4)

// part (c)
Qdot_in/mdot = (hb - h4)

// part (d)
beta = Qdot_in/Wdot_cycle
```

Results for $\eta_c = \eta_t$ = 100%, rp = 3.75 (Base case from Problem 10.50)

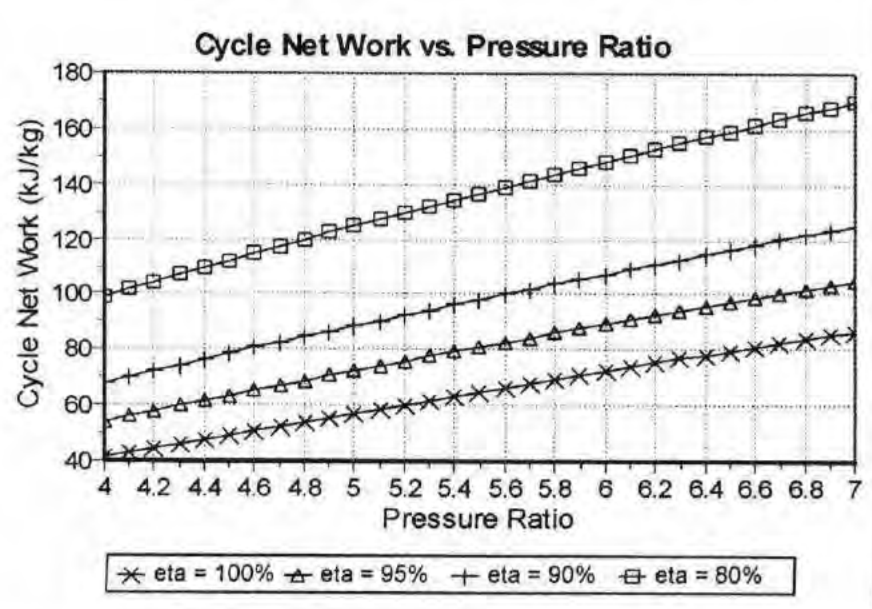
T4 = 219.3 K
Wdot_cycle = 3.71 kJ/kg
Qdot_in = 50.81 kJ/kg
beta = 1.369
etat = 100%
h1 = 300 kJ/kg
h2 = 438.2 kJ/kg
h3 = 320.1 kJ/kg
h4 = 219 kJ/kg
ha = 350.3 kJ/kg
hb = 269.8 kJ/kg

Plots:

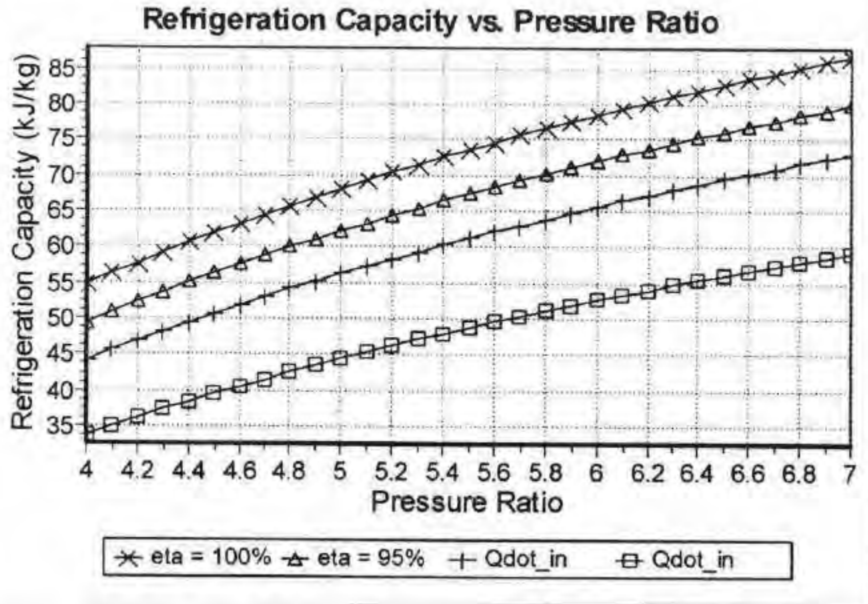
Problem 10.52 (Continued) – Page 2



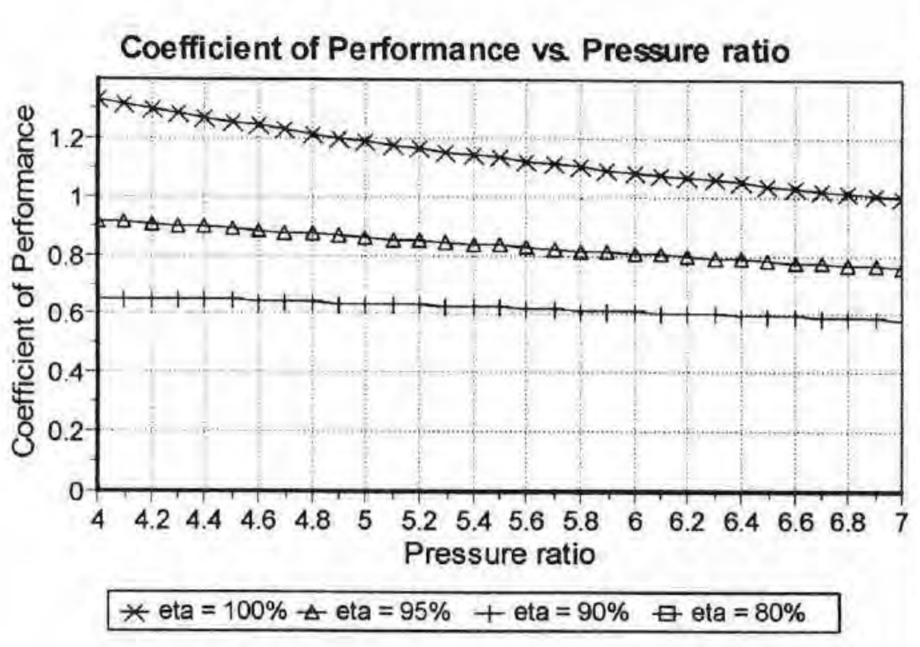
(a) The lowest cycle temperature, T_4, decreases with increasing pressure ratio. The lowest temperatures are reached when the compressor and turbine operate isentropically. As compressor and turbine isentropic efficiencies decrease, the lowest temperatures increase.



(b) The required work input to the cycle increases with increasing pressure ratio for all isentropic efficiencies. As the isentropic efficiencies of the compressor and turbine decrease, more net work input is required at each pressure ratio.



(c) The plots show that the refrigeration capacity increases with increasing compressor pressure ratio. As shown in part (b), the required compressor work input also increases in order to achieve increased refrigeration capacity. Further, as compressor and turbine isentropic efficiencies decrease, the refrigeration capacity also decreases for fixed compressor pressure ratio.



The coefficients of performance decrease as pressure ratio increases, as refrigeration capacity decreases and cycle net work input increases. Further, the coefficients of performance decrease as compressor and turbine isentropic efficiencies decrease.

Problem 10.53

Consider a Brayton refrigeration cycle with a regenerative heat exchanger. Air enters the compressor at 500°R, 16 lbf/in.^{2} and is compressed isentropically to 45 lbf/in.^{2} Compressed air enters the regenerative heat exchanger at 550°R and is cooled to 490°R before entering the turbine. The expansion through the turbine is isentropic. If the refrigeration capacity is 14 tons, calculate:

(a) the volumetric flow rate at the compressor inlet, in ft^3/min.

(b) the coefficient of performance.

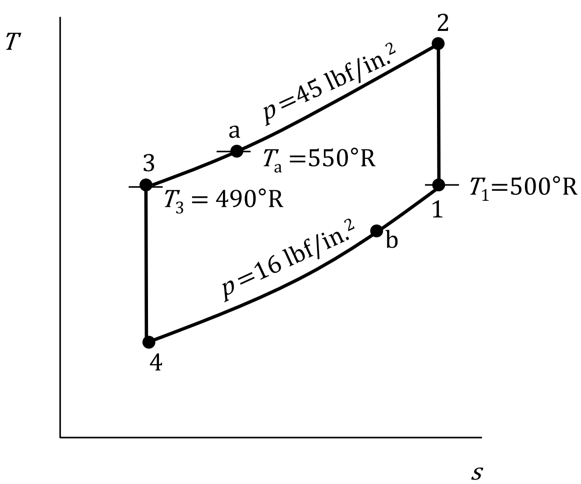
Known:

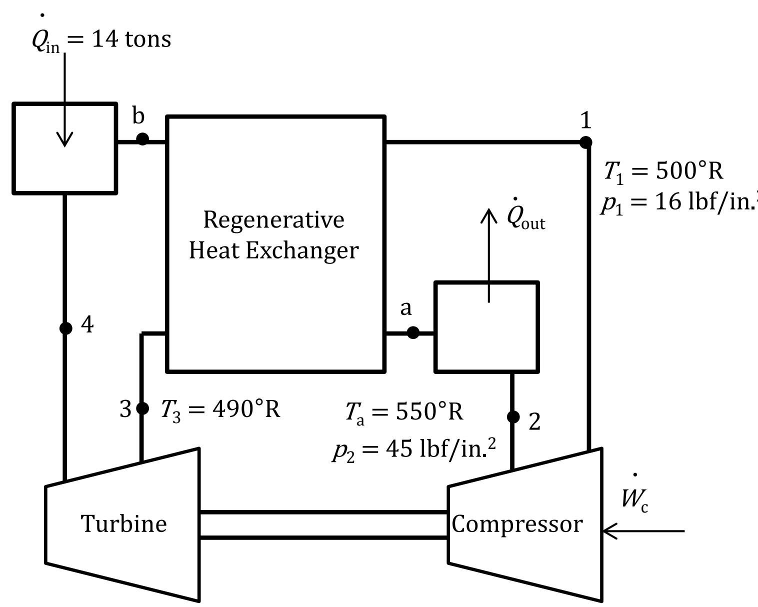
Air is the working fluid in an ideal Brayton refrigeration cycle with a regenerative heat exchanger. Data are known at various locations and the refrigeration capacity is given.

Find:

Determine (a) the air mass flow rate, and (b) the coefficient of performance.

Schematic and Known Data:





Engineering Model:

(1) Each component of the cycle is analyzed as a control volume at steady state.

(2) The turbine and compressor are isntropic.

(3) There are no pressure drops through the heat exchangers.

(4) Kinetic and potential energy effects are negligible.

(5) The working fluid is air modeled as an ideal gas.

(6) There is no heat transfer from the regenerative heat exchanger to its surroundings.

Analysis:

First, fix each principal state (Table A-22E):

State 1: $T_1 = 500°\text{R} \rightarrow h_1 = 119.48\,\frac{\text{Btu}}{\text{lb}}, p_{\text{r}_1} = 1.0590$

State 2: $s_2 = s_1,\;\; p_{\text{r}_2} = p_{\text{r}_1}\frac{p_2}{p_1} = (1.0590)\frac{45}{16} = 2.9784$, interpolating $\rightarrow h_2 = 160.67\,\frac{\text{Btu}}{\text{lb}}$

State a: $T_\text{a} = 550°\text{R}$ with interpolation $\rightarrow h_\text{a} = 131.46\,\frac{\text{Btu}}{\text{lb}}$

State 3: $T_3 = 490°\text{R} \rightarrow h_3 = 117.09\,\frac{\text{Btu}}{\text{lb}}, p_{\text{r}_3} = 0.9886$

State 4: $s_4 = s_3,\;\; p_{\text{r}_4} = p_{\text{r}_3}\frac{p_4}{p_3} = (0.9886)\frac{16}{45} = 0.3515$, interpolating $\rightarrow h_4 = 87.01\,\frac{\text{Btu}}{\text{lb}}$

State b: $0 = (h_b - h_1) + (h_a - h_3) \rightarrow h_b = h_1 - h_a + h_3 = 105.11\,\frac{\text{Btu}}{\text{lb}}$

(a) The mass flow rate is:

$$\dot{m} = \frac{\dot{Q}_\text{in}}{h_\text{b} - h_4} = \frac{14\text{ tons}}{(105.11 - 87.01)\,\frac{\text{Btu}}{\text{lb}}} \cdot \left|\frac{200\,\frac{\text{Btu}}{\text{min}}}{1\text{ ton}}\right| = 154.7\,\frac{\text{lb}}{\text{min}}$$

And the volumetric flow rate at the compressor inlet is:

$$\text{AV}_1 = \frac{\dot{m}RT_1}{p_1} = \frac{\left(154.7\,\frac{\text{lb}}{\text{min}}\right)\left(\frac{1545}{28.97}\,\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(500°\text{R})}{16\,\frac{\text{lbf}}{\text{in.}^2}\cdot\left|\frac{144\text{ in.}^2}{1\text{ ft}^2}\right|} = 1790\,\frac{\text{ft}^3}{\text{min}} \longleftarrow$$

(b) The coefficient of performance is:

$$\beta = \frac{h_b - h_4}{(h_2 - h_1) - (h_3 - h_4)} = \frac{(105.11 - 87.01)}{(160.67 - 119.48) - (117.09 - 87.01)} = 1.63 \longleftarrow$$

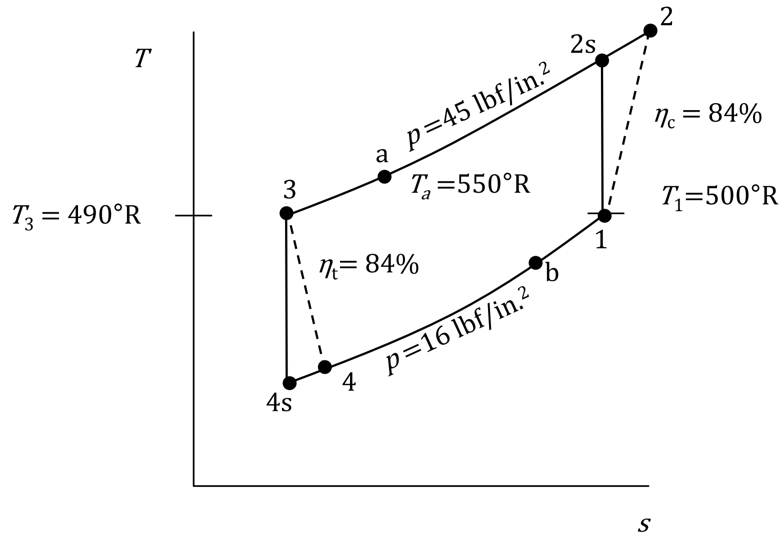
Problem 10.54

Reconsider Problem 10.53, but include in the analysis that the compressor and turbine each have isentropic efficiencies of 84%. Answer the same questions for the modified cycle in Problem 10.53 and determine the rate of entropy production within the compressor and turbine, each in Btu/min·°R.

Known:
Reconsider the cycle of Problem 10.53, but include compressor and turbine efficiencies of 84% each in the analysis.

Find:
Determine (a) the air mass flow rate, (b) the coefficient of performance, and (c) the entropy production within the compressor and turbine.

Schematic and Known Data:



Engineering Model:
(1) Each component of the cycle is analyzed as a control volume at steady state.
(2) There are no pressure drops through the evaporator and condenser.
(3) The compressor and turbine operate adiabatically at the given isentropic efficiencies.
(4) Kinetic and potential energy effects are negligible.
(5) The working fluid is air modeled as an ideal gas.
(6) There is no heat transfer from the regenerative heat exchanger to its surroundings.

Analysis:
From the solution to Problem 10.53:
$h_1 = 119.48 \frac{\text{Btu}}{\text{lb}}$, $h_{2s} = 160.67 \frac{\text{Btu}}{\text{lb}}$, $h_a = 131.46 \frac{\text{Btu}}{\text{lb}}$, $h_3 = 117.09 \frac{\text{Btu}}{\text{lb}}$, $h_{4s} = 87.01 \frac{\text{Btu}}{\text{lb}}$,
$$h_b = 105.11 \frac{\text{Btu}}{\text{lb}}$$

Fix each principal state (Table A-22E).

State 1: $T_1 = 500°\mathrm{R} \rightarrow s_1^o = 0.58233\,\frac{\mathrm{Btu}}{\mathrm{lb\cdot°R}}$

State 2: $\eta_\mathrm{c} = \frac{h_{2s}-h_1}{h_2-h_1} \rightarrow h_2 = h_1 + \frac{h_{2s}-h_1}{\eta_c} = 119.48 + \frac{160.67-119.48}{0.84} = 168.52\,\frac{\mathrm{Btu}}{\mathrm{lb}}$

Interpolating from Table A-22E, $s_2^o = 0.66456\,\frac{\mathrm{Btu}}{\mathrm{lb\cdot°R}}$

State 3: $T_3 = 490°\mathrm{R} \rightarrow s_3^o = 0.57744\,\frac{\mathrm{Btu}}{\mathrm{lb\cdot°R}}$

State 4: $\eta_\mathrm{t} = \frac{h_3-h_4}{h_3-h_{4s}} \rightarrow h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 117.09 - 0.84(117.09 - 87.01) =$ $91.82\,\frac{\mathrm{Btu}}{\mathrm{lb}}$, Interpolating from Table A-22E, $s_4^o = 0.51938\,\frac{\mathrm{Btu}}{\mathrm{lb\cdot°R}}$

(a) The mass flow rate is:

$$\dot{m} = \frac{\dot{Q}_\mathrm{in}}{h_\mathrm{b} - h_4} = \frac{14\ \mathrm{tons}}{(105.11 - 91.82)\,\frac{\mathrm{Btu}}{\mathrm{lb}}} \cdot \left|\frac{200\,\frac{\mathrm{Btu}}{\mathrm{min}}}{1\ \mathrm{ton}}\right| = 210.7\,\frac{\mathrm{lb}}{\mathrm{min}}$$

And the volumetric flow rate at the compressor inlet is:

#1 $$\mathrm{AV}_1 = \frac{\dot{m}RT_1}{p_1} = \frac{\left(210.7\frac{\mathrm{lb}}{\mathrm{min}}\right)\left(\frac{1545\,\mathrm{ft\cdot lbf}}{28.97\,\mathrm{lb\cdot°R}}\right)(500°\mathrm{R})}{16\frac{\mathrm{lbf}}{\mathrm{in.}^2}\cdot\left|\frac{144\ \mathrm{in.}^2}{1\ \mathrm{ft}^2}\right|} = 2438.6\,\frac{\mathrm{ft}^3}{\mathrm{min}}$$

(b) The coefficient of performance is:

$$\beta = \frac{h_\mathrm{b} - h_4}{(h_2 - h_1) - (h_3 - h_4)} = \frac{(105.11 - 91.82)}{(168.52 - 119.48) - (117.09 - 91.82)} = 0.56$$

#1

(c) The entropy production within the compressor is:

#2 $$\dot{\sigma}_\mathrm{comp} = \dot{m}(s_2 - s_1) - \underbrace{\frac{\dot{Q}_\mathrm{comp}}{T_\mathrm{b}}}_{=0} = \dot{m}\left(s_2^o - s_1^o - R\ln\frac{p_2}{p_1}\right) = \left(210.7\,\frac{\mathrm{lb}}{\mathrm{min}}\right)\left[(0.66456 - 0.58233)\,\frac{\mathrm{Btu}}{\mathrm{lb\cdot°R}} - \left(\frac{1.986\frac{\mathrm{Btu}}{\mathrm{lbmol\cdot°R}}}{28.97\frac{\mathrm{lb}}{\mathrm{lbmol}}}\right)\ln\left(\frac{45}{16}\right)\right] = 2.39\,\frac{\mathrm{Btu}}{\mathrm{min\cdot°R}}$$

The entropy production within the turbine is:

#2 $$\dot{\sigma}_\mathrm{turbine} = \dot{m}(s_4 - s_3) - \underbrace{\frac{\dot{Q}_\mathrm{turbine}}{T_\mathrm{b}}}_{=0} = \dot{m}\left(s_4^o - s_3^o - R\ln\frac{p_4}{p_3}\right) = \left(210.7\,\frac{\mathrm{lb}}{\mathrm{min}}\right)\left[(0.51938 - 0.57744)\,\frac{\mathrm{Btu}}{\mathrm{lb\cdot°R}} - \left(\frac{1.986\frac{\mathrm{Btu}}{\mathrm{lbmol\cdot°R}}}{28.97\frac{\mathrm{lb}}{\mathrm{lbmol}}}\right)\ln\left(\frac{16}{45}\right)\right] = 2.70\,\frac{\mathrm{Btu}}{\mathrm{min\cdot°R}}$$

Comments:

1. Note, the volumetric flow rate is higher and the coefficient of performance is lower than in Problem 10.53 due to irreversibilities in the compressor and turbine.
2. The isentropic efficiency of the compressor and turbine are equivalent and the corresponding entropy production rates differ by approximately 15%.

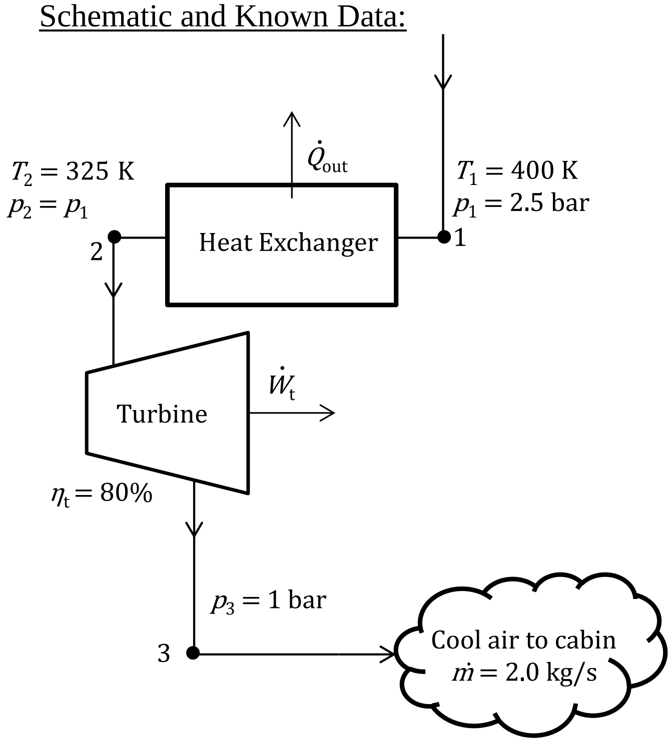
Problem 10.55

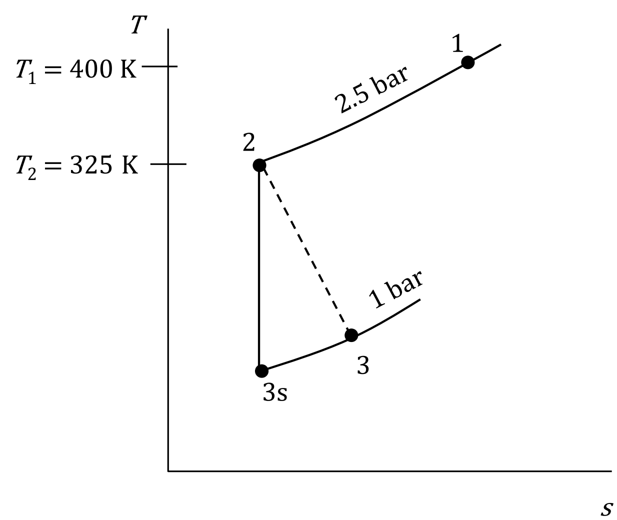
Air at 2.5 bar, 400 K is extracted from a main jet engine compressor for cabin cooling. The extracted air enters a heat exchanger where it is cooled at constant pressure to 325 K through heat transfer with the ambient. It then expands adiabatically to 1.0 bar through a turbine and is discharged into the cabin. The turbine has an isentropic efficiency of 80%. If the mass flow rate of the air is 2.0 kg/s, determine:
(a) the power developed by the turbine, in kW.
(b) the rate of heat transfer from the air to the ambient, in kW.

Known:
Air is extracted from a main jet engine for cabin cooling. The air passes through a heat exchanger and turbine before being discharged into the cabin.

Find:
Determine (a) the power developed by the turbine, and (b) the heat transfer rate from the air passing through the heat exchanger.

Schematic and Known Data:





Engineering Model:
(1) Each component of the cycle is analyzed as a control volume at steady state.
(2) The air experiences no pressure drop in passing through the heat exchanger.
(3) Kinetic and potential energy effects are negligible.
(4) The turbine operates adiabatically.
(5) The air is modeled as an ideal gas.

Analysis:

From Table A-22:

State 1: $h_1 = 400.98\,\frac{\text{kJ}}{\text{kg}}$

State 2: $h_2 = 325.31\,\frac{\text{kJ}}{\text{kg}}$, $p_{r_2} = 1.8345$

State 3s: $p_{r_{3s}} = p_{r_2}\left(\frac{p_3}{p_2}\right) = 1.8345\left(\frac{1}{2.5}\right) = 0.7338$, interpolating $\rightarrow$ $h_{3s} = 250.13\,\frac{\text{kJ}}{\text{kg}}$

(a) Using the turbine efficiency:

$$\dot{W}_t = \dot{m}\eta_t(h_2 - h_{3s}) = \left(2\,\frac{\text{kg}}{\text{s}}\right)(0.80)(325.31 - 250.13)\,\frac{\text{kJ}}{\text{kg}}\left|\frac{1\text{ kW}}{1\,\frac{\text{kJ}}{\text{s}}}\right| = 120.3\text{ kW}$$

(b) Using an energy balance around the heat exchanger:

$$0 = \dot{Q}_{cv} + \dot{m}(h_1 - h_2) \rightarrow \dot{Q}_{cv} = \dot{m}(h_2 - h_1) = \left(2\,\frac{\text{kg}}{\text{s}}\right)(325.31 - 400.98)\,\frac{\text{kJ}}{\text{kg}}\left|\frac{1\text{ kW}}{1\,\frac{\text{kJ}}{\text{s}}}\right|$$

$$= -151.34\text{ kW}$$

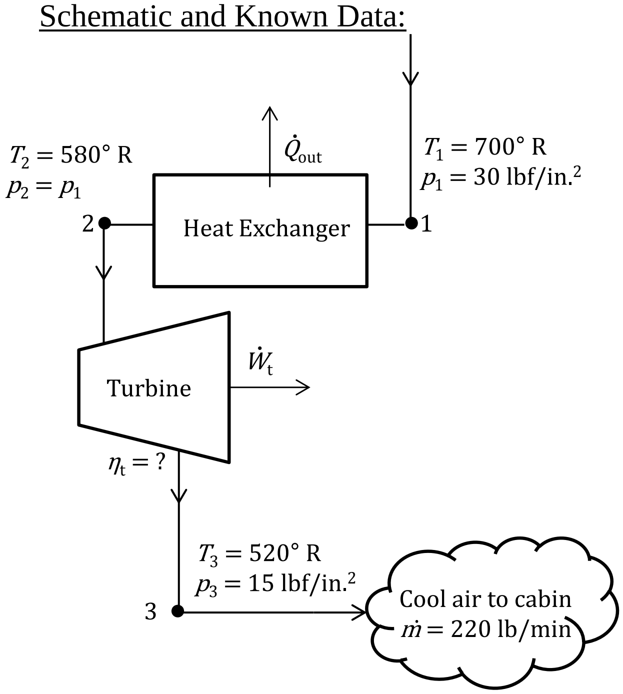
Problem 10.56

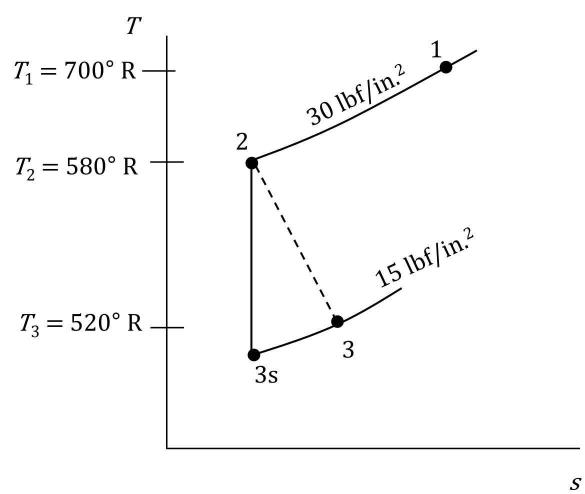
Air at 30 lbf/in.^{2}, 700°R is extracted from a main jet engine compressor for cabin cooling. The extracted air enters a heat exchanger where it is cooled at constant pressure to 580°R through heat transfer with the ambient. It then expands adiabatically to 15 lbf/in.^{2} through a turbine and is discharged into the cabin at 520°R with a mass flow rate of 220 lb/min. Determine
(a) the power developed by the turbine, in horsepower.
(b) the isentropic turbine efficiency.
(b) the rate of heat transfer from the air to the ambient, in Btu/min.

Known:
Air is extracted from a main jet engine for cabin cooling. The air passes through a heat exchanger and turbine before being discharged into the cabin.

Find:
Determine (a) the power developed by the turbine, (b) the isentropic turbine efficiency, and (c) the heat transfer rate from the air passing through the heat exchanger.

Schematic and Known Data:





Engineering Model:
(1) Each component of the cycle is analyzed as a control volume at steady state.
(2) The air experiences no pressure drop in passing through the heat exchanger.
(3) Kinetic and potential energy effects are negligible.
(4) The turbine operates adiabatically.
(5) The air is modeled as an ideal gas.

Analysis:
From Table A-22E:
State 1: $h_1 = 167.56 \frac{\text{Btu}}{\text{lb}}$
State 2: $h_2 = 138.66 \frac{\text{Btu}}{\text{lb}}$, $p_{r_2} = 1.7800$
State 3s: For isentropic expansion, $p_{r_{3s}} = p_{r_2} \frac{p_3}{p_2} = (1.7800)\left(\frac{15}{30}\right) = 0.8900$, interpolating
$\rightarrow h_{3s} = 113.63 \frac{\text{Btu}}{\text{lb}}$
State 3: $h_3 = 124.27 \frac{\text{Btu}}{\text{lb}}$

(a) Using above values, the power developed by the turbine is:

$$\dot{W}_\text{t} = \dot{m}(h_2 - h_3) = \left(220 \frac{\text{lb}}{\text{min}}\left|\frac{60 \text{ min}}{1 \text{ h}}\right|\right)(138.66 - 124.27)\left(\frac{\text{Btu}}{\text{lb}} \cdot \left|\frac{1 \text{ hp}}{2545 \frac{\text{Btu}}{\text{h}}}\right|\right)$$
$$= 74.64 \text{ hp} \longleftarrow$$

(b) The turbine isentropic efficiency is:

$$\eta_\text{t} = \frac{h_2 - h_3}{h_2 - h_{3s}} = \frac{138.66 - 124.27}{138.66 - 113.63} = 0.575 = 57.5\% \longleftarrow$$

(c) The heat transfer rate from the air passing through the heat exchanger follows:

$$\dot{Q}_\text{out} = \dot{m}(h_1 - h_2) = \left(220 \frac{\text{lb}}{\text{min}}\right)(167.56 - 138.66)\frac{\text{Btu}}{\text{lb}} = 6358 \frac{\text{Btu}}{\text{min}} \longleftarrow$$

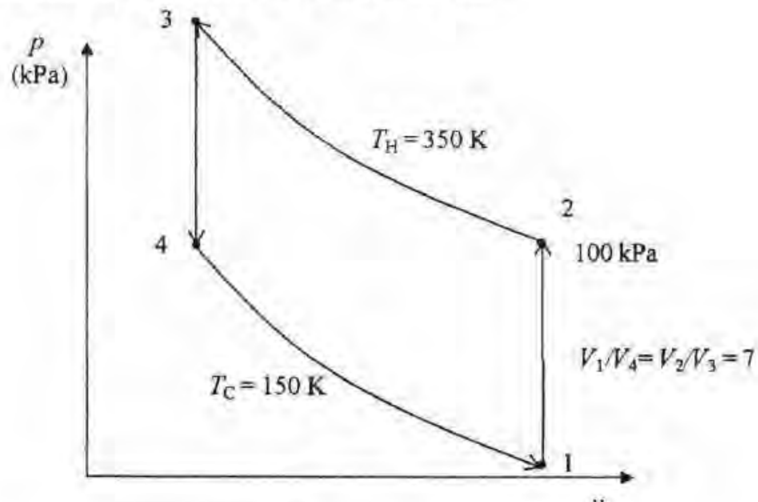
10.57 Air within a piston-cylinder assembly undergoes a *Stirling refrigeration cycle*, which is the reverse of the Stirling power cycle introduced in Chapter 9. At the beginning of the isothermal compression, the pressure and temperature are 100 kPa and 350 K, respectively. The compression ratio is 7, and the temperature during the isothermal expansion is 150 K. Determine the

(a) heat transfer for the isothermal compression, in kJ per kg of air.
(b) net work for the cycle, in kJ per kg of air.
(c) coefficient of performance.

KNOWN: Air is the working fluid in a Stirling refrigeration cycle with a known compression ratio. Other operating data are also provided for the cycle.

FIND: Determine (a) the heat transfer for the isothermal compression, (b) the net work, and (c) the coefficient of performance.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
(1) The air is a closed system.
(2) The compression and expansion processes are isothermal.
(3) All processes are internally reversible.
(4) An ideal regenerator is available to store the energy rejected during process 3-4 to be used as the energy input during process 1-2.
(5) Kinetic and potential energy effects are negligible.
(6) The working fluid is air modeled as an ideal gas.

Problem 10.57 (Continued) – Page 2

ANALYSIS:

(a) Since there is no change in specific internal energy for an ideal gas undergoing an isothermal process, the energy balance for process 2-3 reduces to

$$\frac{Q_{23}}{m} = \frac{W_{23}}{m}$$

The work for the reversible isothermal process is calculated as follows

$$\frac{W_{23}}{m} = \int_2^3 p dv = \left(\frac{\overline{R}}{M}\right) T_C \ln\left(\frac{v_3}{v_2}\right) = \left(\frac{8.314}{28.97} \frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right)(350\,\text{K})\ln\left(\frac{1}{7}\right) = -195.46 \frac{\text{kJ}}{\text{kg}} = \frac{Q_{23}}{m}$$

(b) For net work of the cycle, find the expansion work. Again, for the reversible isothermal process

$$\frac{W_{41}}{m} = \int_4^1 p dv = \left(\frac{\overline{R}}{M}\right) T_C \ln\left(\frac{v_1}{v_4}\right) = \left(\frac{8.314}{28.97} \frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right)(150\,\text{K})\ln(7) = 83.77 \frac{\text{kJ}}{\text{kg}}$$

Therefore

$$\frac{W_{\text{cycle}}}{m} = \frac{W_c}{m} - \frac{W_t}{m} = \left|\frac{W_{23}}{m}\right| - \frac{W_{41}}{m} = 111.69 \frac{\text{kJ}}{\text{kg}}$$

(c) The coefficient of performance (β) is

$$\beta = \frac{\left(\frac{Q_{\text{in}}}{m}\right)}{\left(\frac{W_{\text{cycle}}}{m}\right)} = \frac{\left(\frac{Q_{41}}{m}\right)}{\left(\frac{W_{\text{cycle}}}{m}\right)}$$

The energy balance for process 4-1 reduces to

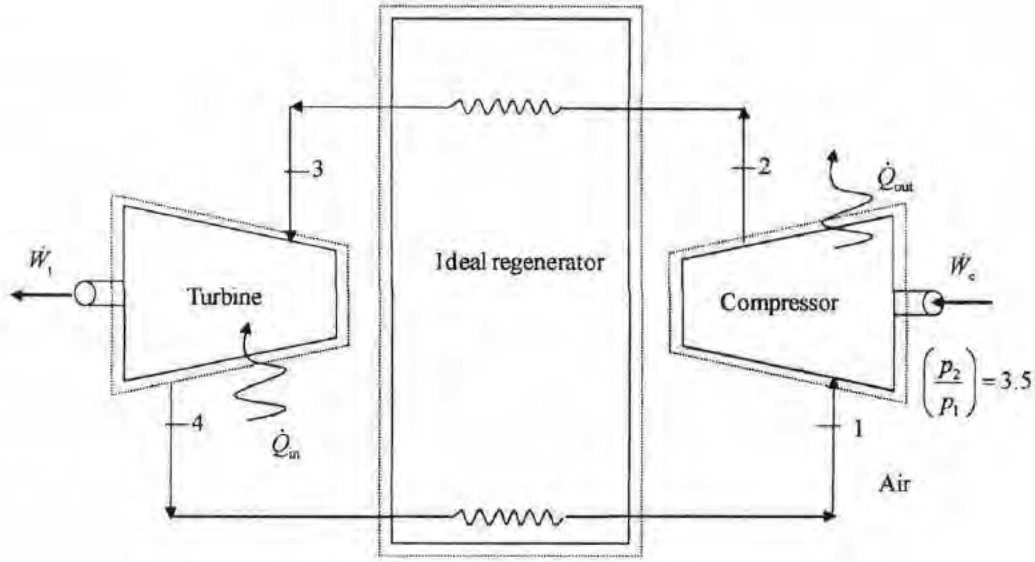
$\frac{Q_{41}}{m} = \frac{W_{41}}{m} = 83.77 \frac{\text{kJ}}{\text{kg}}$, therefore

#1 $$\beta = \frac{(83.77)}{(111.69)} = 0.75$$

---

1. Alternatively, for the reversible refrigeration cycle; $\beta = \beta_{\max} = T_C/(T_H - T_C) = 0.75$.

10.58 Air undergoes an *Ericsson refrigeration cycle*, which is the reverse of the Ericsson power cycle introduced in Chapter 9. Figure P10.58 provides data for the cycle operating at steady state. Sketch the p-v diagram for the cycle and determine the
(a) the heat transfer for the isothermal expansion, per unit mass of air flow, in kJ/kg.
(b) the net work, per unit mass of air flow, in kJ/kg.
(c) the coefficient of performance.



State	p (kPa)	T (K)
1	100	310
2	350	310
3	350	270
4	100	270

Fig. P10.58

KNOWN: Air is the working fluid in an Ericsson refrigeration cycle. Operating data are provided at principal states of the cycle.

FIND: Sketch the p-v diagram for the cycle and determine (a) the heat transfer for the isothermal expansion, (b) the net work, and (c) the coefficient of performance.

SCHEMATIC AND GIVEN DATA: For the schematic, refer to Fig. P10.58.

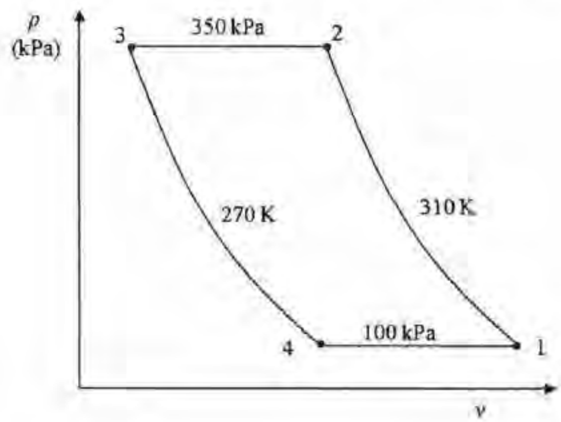
State	p (kPa)	T (K)
1	100	310
2	350	310
3	350	270
4	100	270

Problem 10.58 (Continued) – Page 2

ENGINEERING MODEL:

(1) Each component is analyzed as a control volume at steady state.
(2) All processes are internally reversible.
(3) The compression and expansion processes are isothermal.
(4) Kinetic and potential energy effects are negligible.
(5) The working fluid is air modeled as an ideal gas.

ANALYSIS: the p-v diagram for the cycle is



(a) From an energy balance on a control volume enclosing the turbine

$$0 = \dot{Q}_{in} - \dot{W}_t + \dot{m}(h_3 - h_4)$$

Since

$$T_3 = T_4; h_3 = h_4, \text{and therefore} \frac{\dot{Q}_{in}}{\dot{m}} = \frac{\dot{W}_t}{\dot{m}}$$

We see that the heat transfer per unit mass of air flow for the isothermal expansion is equal to the work per unit mass of air flow. For an internally reversible process at steady-state, in a one-inlet, one-exit control volume, the work per unit mass flowing is

$$\frac{\dot{W}_t}{\dot{m}} = -\int_3^4 v dp = -\left(\frac{\overline{R}}{M}\right) T_3 \ln\left(\frac{p_4}{p_3}\right) = \left(\frac{\overline{R}}{M}\right) T_3 \ln\left(\frac{p_3}{p_4}\right)$$

$$= \left(\frac{8.314}{28.97} \frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right)(270\,\text{K})\ln(3.5) = 95.07 \frac{\text{kJ}}{\text{kg}} = \frac{\dot{Q}_{in}}{\dot{m}}$$

(b) For net work of the cycle, first find compressor work

$$\frac{\dot{W}_c}{\dot{m}} = \left(\frac{\overline{R}}{M}\right) T_1 \ln\left(\frac{p_2}{p_1}\right) = \left(\frac{8.314}{28.97} \frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right)(310\,\text{K}) \ln(3.5) = 111.45 \frac{\text{kJ}}{\text{kg}}$$

Therefore

$$\frac{\dot{W}_{\text{cycle}}}{\dot{m}} = \frac{\dot{W}_c}{\dot{m}} - \frac{\dot{W}_t}{\dot{m}} = 14.38 \frac{\text{kJ}}{\text{kg}}$$

(c) The coefficient of performance (β) is

$$\beta = \frac{\left(\frac{\dot{Q}_{\text{in}}}{\dot{m}}\right)}{\left(\frac{\dot{W}_{\text{cycle}}}{\dot{m}}\right)} = \frac{97.07}{14.38} = 6.75$$

PROBLEM 11.1

KNOWN: 100 lb of CO_2 is at 212°F in a 19.3-ft^3 cylinder.

FIND: Determine the pressure using (a) the van der Waals equation, (b) the compressibility chart, (c) the ideal gas model.

ANALYSIS: Using the given data $v = 0.193\ ft^3/lb$.

(a) The van der Waals equation is given by Eq. 11.2

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}^2}$$

From Table A-24E

$$a = 926\ atm\left(\frac{ft^3}{lbmol}\right)^2,\quad b = 0.686\ \frac{ft^3}{lbmol}$$

$$p = \frac{\left(1545\ \frac{ft\cdot lbf}{lbmol\cdot °R}\right)(672°R)\left|\frac{ft^2}{144\ in^2}\right|\left|\frac{atm}{14.696\ lbf/in^2}\right|}{\left(0.193\ \frac{ft^3}{lb}\right)\left(44.01\ \frac{lb}{lbmol}\right) - 0.686\ \frac{ft^3}{lbmol}} - \frac{926\ atm\ (ft^3/lbmol)^2}{\left[(0.193)(44.01)\ \frac{ft^3}{lbmol}\right]^2} = 50\ atm \longleftarrow \text{vdW}$$

(b) Compressibility Chart

From Table A-1E, $T_c = 548\ °R$, $P_c = 72.9\ atm$. Thus, $T_R = 672/548 = 1.226$ and

$$v_R' = \frac{\bar{v}P_c}{\bar{R}T_c} = \frac{\left[(0.193)(44.01)\ ft^3/lbmol\right]\left[72.9\ atm\right]\left|14.696\ \frac{lbf/in^2}{atm}\right|\left|\frac{144\ in^2}{ft^2}\right|}{\left(1545\ \frac{ft\cdot lbf}{lbmol\cdot °R}\right)(548°R)} = 1.548$$

Then Fig. A-1 gives $P_R \approx 0.685 \Rightarrow p = 49.94\ atm$. $\longleftarrow$ Z chart

(c) With the ideal gas equation of state

$$p = \frac{\bar{R}T}{\bar{v}} = \frac{(1545/(144)(14.696))\,672}{(0.193)(44.01)} = 57.76\ atm \longleftarrow \text{ideal gas}$$

Discussion: Methods (a), (b) suggest that the pressure level would be safe, but at the high end of the allowed range. The ideal gas model suggests that the pressure would not be satisfactory.

PROBLEM 11.2

① KNOWN: Ten lb of C_3H_8 has a volume of 2 ft³ and a pressure of 600 lbf/in.²

FIND: Determine the temperature using (a) the Van der Waals equation, (b) the compressibility chart, (c) the ideal gas equation of state, and (d) the propane tables.

ANALYSIS: $p = \left(600 \frac{lbf}{in^2}\right)\left|\frac{1\ atm}{14.696\ lbf/in.^2}\right| = 40.83\ atm$, $\bar{v} = \left(\frac{2\ ft^3}{10\ lb}\right)\left|\frac{44.09\ lb}{1\ lb{\cdot}mol}\right| = 8.818\ \frac{ft^3}{lb{\cdot}mol}$

(a) With a and b from Table A-24E, the Van der Waals Equation gives

$$T = \frac{(p + a/\bar{v}^2)(\bar{v} - b)}{\bar{R}}$$

$$= \frac{\left[40.83\ atm + \frac{2369\ (ft^3/lbmol)^2}{(8.818\ ft^3/lbmol)^2}\right](8.818 - 1.444)\frac{ft^3}{lbmol}}{\left(1545\ \frac{ft\cdot lbf}{lbmol\cdot °R}\right)}\left|\frac{14.696\ lbf/in^2}{1\ atm}\right|\left|\frac{144\ in.^2}{1\ ft^2}\right|$$

$= 720.1\ °R$ ← (a)

(b) From Table A-1E; $T_c = 666\ °R$, $p_c = 42.1$ atm. Thus $p_R = 40.83/42.1 = 0.97$, and

$$v_R' = \frac{\bar{v}\,p_c}{\bar{R}T_c} = \frac{(8.818\ ft^3/lbmol)(42.1\ atm)}{\left(1545\ \frac{ft\cdot lbf}{lbmol\cdot °R}\right)(666\ °R)}\left|\frac{14.696\ lbf/in.^2}{1\ atm}\right|\left|\frac{144\ in.^2}{1\ ft^2}\right| = 0.764$$

Then, Fig A-1 gives $T_R \approx 1.08 \Rightarrow T = 719.3\ °R$ ← (b)

(c) The ideal gas equation gives

$$T = \frac{p\bar{v}}{\bar{R}} = \frac{(600\ lbf/in.^2)(8.818\ ft^3/lbmol)}{\left(1545\ \frac{ft\cdot lbf}{lbmol\ °R}\right)}\left|\frac{144\ in.^2}{1\ ft^2}\right| = 493\ °R$$ ← (c)

(d) $p = 600$ lbf/in.², $\bar{v} = 0.2$ ft³/lb. Interpolating in Table A-18E, we get

$T = 261.5\ °F = 721.5\ °R$ ← (d)

Discussion. The answers for parts (a), (b) and (d) agree well. The ideal gas value deviates significantly from the others.

PROBLEM 11.3

KNOWN: 1000 kg of water vapor fills a 23.3-m^3 tank at 360°F.

FIND: Estimate the pressure using (a) the ideal gas equation, (b) the van der Waals equation, (c) the Redlich-Kwong equation, (d) the compressibility chart, (e) the steam tables.

ANALYSIS:

(a) ideal gas equation.

$$p = \frac{RT}{v} = \frac{(8314/18.02)\left(\frac{N\cdot m}{kg\cdot K}\right)(633\ K)}{(23.31/10^3)\ (m^3/kg)}\left|\frac{1\ bar}{10^5 N/m^2}\right| = 125.29\ bar \quad (25\%\ high) \longleftarrow (a)$$

(b) van der Waals equation. With a and b from Table A-24

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}^2} = \left[\frac{\left(8314\ \frac{N\cdot m}{kmol\cdot K}\right)(633\ K)}{(0.42 - 0.0305)\ m^3/kmol}\right]\left|\frac{1\ bar}{10^5 N/m^2}\right| - \frac{5.531\ bar\,(m^3/kmol)^2}{(0.42\ m^3/kmol)^2} = 103.76\ bar \quad (\sim 4\%\ high) \longleftarrow (b)$$

where $\bar{v} = vM = \left(\frac{23.31}{10^3}\right)\left(\frac{m^3}{kg}\right)\left(18.02\ \frac{kg}{kmol}\right) = 0.42\ \frac{m^3}{kmol}$.

(c) Redlich-Kwong equation. with a and b from Table A-24

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}(\bar{v}+b)T^{1/2}} = \frac{(8314)(633)}{(0.42-0.02111)}\left|\frac{1}{10^5}\right| - \frac{142.59}{(0.42)(0.42+0.02111)(633)^{1/2}} = 101.34\ bar \quad (\sim 1\%\ high) \longleftarrow (c)$$

(d) Compressibility Chart. From Table A-1, $T_c = 647.3\ K$, $p_c = 220.9\ bar$. Thus, $T_R = 633/647.3 = 0.978$. And

$$v_R' = \frac{\bar{v}\,p_c}{\bar{R}T_c} = \frac{(0.42\ m^3/kmol)(220.9\times10^5\ N/m^2)}{\left(8314\ \frac{N\cdot m}{kmol\cdot K}\right)(647.3\ K)} = 1.724$$

Then, Fig A-1 gives $Z \approx 0.81$, so

$$p = Z\frac{RT}{v} = 0.81(125.29) = 101.48\ bar \quad (\sim 1.5\%\ high) \longleftarrow (d)$$

(e) Steam tables. Table A-4 gives 100 bar.

Discussion: All but the ideal gas model suggest that the pressure would be in the safe range. Using the steam table value, 100 bar, as the standard, the various methods depart by the following percentages:

ideal gas: 25% high
van der Waals: 4% high
Redlich-Kwong: 10% high
Compressibility chart: 1.5% high

PROBLEM 11.4

KNOWN: Water vapor is at $T = 500°C$, $\rho = 24\ kg/m^3$.

FIND: Determine the pressure using (a) the steam tables, (b) the compressibility chart, (c) the Redlich – Kwong equation, (d) the van der Waals equation, (e) the ideal gas model.

ANALYSIS: (a) At 500°C and $v = 1/\rho = 0.0417\ m^3/kg$; Steam Table data gives $p = 80.1$ bar. ← (a)

(b) Compressibility chart $T_R = \frac{773}{647} = 1.19$, $\bar{v}_R' = \frac{\bar{v}\,p_c}{\bar{R}\,T_c} = \frac{\left[(0.0417)(18.02)\frac{m^3}{kmol}\right]\left[220.9\times10^5\frac{N}{m^2}\right]}{\left(8314\ \frac{N\cdot m}{kmol\cdot K}\right)(647K)}$

or $\bar{v}_R' = 3.08$. Then, Figure A-1 gives $Z \sim 0.93$. So

$$p = \frac{ZRT}{v} = \frac{0.93\left[\frac{8314}{18.02}\ \frac{N\cdot m}{kg\cdot K}\right]\left[773K\right]}{\left(0.0417\frac{m^3}{kg}\right)\left|10^5 N/m^2/bar\right|} = 79.5\ \text{bar} \quad (1\%\ \text{low}) \leftarrow (b)$$

(c) the Redlich Kwong Equation with data from Table A-24

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}(\bar{v}+b)\sqrt{T}} = \frac{\left(8314\ \frac{N\cdot m}{kmol\cdot K}\right)(773K)\left|1bar/10^5 N/m^2\right|}{\left[\underbrace{(0.0417)(18.02)}_{0.7514\ m^3/kmol} - 0.02111\right]\frac{m^3}{kmol}} - \frac{142.59\ bar\left(\frac{m^3}{kmol}\right)^2(K)^{1/2}}{\left(0.7514\frac{m^3}{kmol}\right)\left(0.77251\frac{m^3}{kmol}\right)(773K)^{1/2}}$$

$$= 88.0 - 8.84 = 79.2\ \text{bar} \quad (\sim 1\%\ \text{low}) \leftarrow (c)$$

(d) the van der Waals Equation with data from Table A-24

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}^2} = \frac{\left(8314\frac{N\cdot m}{kmol\cdot K}\right)(773K)}{\left(0.7514\frac{m^3}{kmol} - 0.0305\frac{m^3}{kmol}\right)}\left|\frac{1bar}{10^5 N/m^2}\right| - \frac{5.531\ bar\left(m^3/kmol\right)^2}{\left(0.7514\ \frac{m^3}{kmol}\right)^2}$$

$$= 89.15 - 9.8 = 79.35\ \text{bar} \quad (\sim 1\%\ \text{low}) \leftarrow (d)$$

(e) ideal gas model

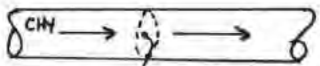
$$p = \frac{RT}{v} = \frac{\left(8314/18.02\ \frac{N\cdot m}{kg\cdot K}\right)(773K)}{\left(0.0417\ m^3/kg\right)}\left|\frac{1bar}{10^5 N/m^2}\right| = 85.53\ \text{bar} \quad (\sim 7\%\ \text{high}) \leftarrow (e)$$

PROBLEM 11.5

KNOWN: CH_4 flows through a pipeline at a pressure of 183 atm, a temperature of 56°F, and a volumetric flow rate of 11 ft³/s.

FIND: Determine the mass flow rate using (a) the ideal gas equation, (b) the van der Waals equation, (c) the compressibility chart.

SCHEMATIC & GIVEN DATA:



$p = 183$ atm
$T = 56°F = 516°R$
$(AV) = 11$ ft³/s

ANALYSIS: The mass flow rate and volumetric flow rate are related by

$$\dot{m} = \frac{(AV)}{v} \qquad (1)$$

(a) Ideal Gas Equation:

$$v = \frac{RT}{p} = \frac{(1545/16.04)\left(\frac{ft\cdot lbf}{lb\cdot °R}\right)(516°R)}{(183\ atm)\left|14.696 \times 144 \frac{lbf/ft^2}{atm}\right|} = 0.128\ \frac{ft^3}{lb}$$

Thus, Eq. (1) gives

$$\dot{m} = \frac{11\ ft^3/s}{0.128\ ft^3/lb} = 85.9\ lb/s \qquad (a)$$

(b) van der Waals Equation: With a and b from Table A-24E

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}^2} \Rightarrow 183\ atm = \left[\frac{\left(1545\frac{ft\cdot lbf}{lbmol\cdot °R}\right)(516°R)}{(\bar{v} - 0.685)\ ft^3/lbmol}\right]\left|\frac{atm}{(14.696)(144)\frac{lbf}{ft^2}}\right| - \frac{581}{\bar{v}^2}$$

or

$$183 = \frac{376.718}{\bar{v} - 0.685} - \frac{581}{\bar{v}^2}$$

Using an equation solver, $\bar{v} = 1.611$ ft³/lbmol. Thus, $v = \bar{v}/M = (1.611/16.04) = 0.1004$ ft³/lb, and Eq. (1) gives

$$\dot{m} = \frac{11\ ft^3/s}{0.1004\ ft^3/lb} = 109.6\ lb/s \qquad (b)$$

(c) Generalized Chart. From Table A-1E $T_c = 344°R$, $P_c = 45.8$ atm. Thus

$$T_R = \frac{516}{344} = 1.5, \quad P_R = 4.0$$

Then, from Fig. A-2, $Z = 0.81$. Accordingly

$$v = Z\frac{RT}{p} = (0.81)\underbrace{(0.128)}_{\text{part(a)}} = 0.1037\ \frac{ft^3}{lb}$$

Finally, with Eq. (1)

$$\dot{m} = \frac{11\ ft^3/s}{0.1037\ ft^3/lb} = 106.1\ lb/s \qquad (c)$$

PROBLEM 11.6

KNOWN: Water vapor is at 20 MPa, 400°C.

FIND: v in m^3/kg using (a) steam tables, (b) compressibility chart, (c) Redlich-Kwong equation, (d) van der Waals equation, (e) ideal gas model.

ANALYSIS: (a) Steam Tables. Table A-4 gives, $v = 0.00994\ m^3/kg$. ◄—— (a)

(b) Compressibility Chart. With p_c, T_c from Table A-1

$T_R = \frac{673}{647} = 1.04$, $P_R = \frac{20}{22.09} = 0.91$. Figure A-1 gives $Z \approx 0.65$. Then

$$v = \frac{ZRT}{p} = 0.65\left[\underbrace{\frac{(8314/18.02)\frac{N\cdot m}{kg\cdot K}\cdot 673K}{20\times 10^6 N/m^2}}_{0.0155\ m^3/kg}\right] = 0.0101\ m^3/kg \quad \text{◄—— (b)}$$

$0.0155\ m^3/kg$ ◄—— (e)

(c) Redlich-Kwong Equation. With a, b from Table A-24

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}(\bar{v}+b)\sqrt{T}} \Rightarrow 200\,bar = \frac{(8314\frac{N\cdot m}{kmol\cdot K})(673K)}{(\bar{v}-0.02111)\frac{m^3}{kmol}}\left|\frac{1\,bar}{10^5 N/m^2}\right| - \frac{142.59\ bar}{\bar{v}(\bar{v}+0.02111)(673)^{1/2}}$$

or

$$200 = \frac{55.953}{(\bar{v}-0.02111)} - \frac{5.496}{\bar{v}(\bar{v}+0.0211)}$$. Using an equation solver, $\bar{v} = 0.18\ m^3/kmol$.

$$\Rightarrow \quad v = \frac{0.18\ m^3/kmol}{18.02\ kg/kmol} = 0.00999\ m^3/kg \quad \text{◄—— (c)}$$

(d) van der Waals Equation. With a, b from Table A-24

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}^2} \Rightarrow 200\,bar = \frac{(8314)(673)}{(\bar{v}-0.0305)}\left|\frac{1}{10^5}\right| - \frac{5.531}{\bar{v}^2}$$

or

$$200 = \frac{55.953}{\bar{v}-0.0305} - \frac{5.531}{\bar{v}^2}$$. Using an equation solver, $\bar{v} = 0.186\ m^3/kmol$.

$$\Rightarrow \quad v = \frac{0.186}{18.02} = 0.01032\ \frac{m^3}{kg} \quad \text{◄——}$$

Discussion: Comparing the calculated values with the Steam Table result,

compressibility chart ~ 2% high
Redlich-Kwong equation ~ 1/2% high
van der Waals equation ~ 4% high
ideal gas model ~ 6% high

PROBLEM 11.7

KNOWN: A vessel whose volume is $1\ m^3$ contains 4 kmol of CH_4 at 100°C.

FIND: Estimate the pressure using (a) the ideal gas model, (b) the Redlich–Kwong equation, (c) the Benedict-Webb-Rubin equation.

ANALYSIS: (a) Ideal Gas Model

$$p = \frac{\bar{R}T}{\bar{v}} = \frac{\left(8314\ \frac{N\cdot m}{kmol\cdot K}\right)(373K)}{(1 m^3/4 kmol)}\left|\frac{1 bar}{10^5 \frac{N}{m^2}}\right| = 124.04 bar. \longleftarrow \quad (a)$$

(b) Redlich-Kwong. With data from Table A-24

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}(\bar{v}+b)(T)^{1/2}} = \frac{\left(8314\ \frac{N\cdot m}{kmol\cdot K}\right)(373K)}{(0.25-0.02965)\frac{m^3}{kmol}}\left|\frac{1 bar}{10^5 N/m^2}\right| - \frac{32.11\ bar}{0.25(0.25+0.02965)(373)^{1/2}}$$

$$= 140.74 - 23.78 = 116.96\ bar \longleftarrow \quad (b)$$

(c) Benedict-Webb-Rubin. With data from Table A-24

$$p = \frac{\bar{R}T}{\bar{v}} + \left[B\bar{R}T - A - \frac{C}{T^2}\right]\frac{1}{\bar{v}^2} + \left[\frac{b\bar{R}T - a}{\bar{v}^3}\right] + \frac{a\alpha}{\bar{v}^6} + \frac{c}{\bar{v}^3T^2}\left(1+\frac{\gamma}{\bar{v}^2}\right)\exp\left(-\frac{\gamma}{\bar{v}^2}\right)$$

$$p = \frac{\left(0.08314\ \frac{bar\cdot m^3}{kmol\cdot K}\right)(373K)}{0.25\ m^3/kmol} + \left[(0.04260)(0.08314)(373) - 1.8796 - \frac{2.287\times10^4}{(373)^2}\right]\frac{1}{(0.25)^2}$$

$$+\left[\frac{(0.00338)(0.08314)(373) - 0.0501}{(0.25)^3}\right] + \frac{(0.0501)(1.244\times10^{-4})}{(0.25)^6}$$

$$+\left[\frac{(2.579\times10^3)}{(0.25)^3(373)^2}\left(1+\frac{0.0060}{(0.25)^2}\right)\exp\left(-\frac{0.0060}{(0.25)^2}\right)\right]$$

$$= 124.04 - 11.57 + 3.50 + 0.03 + 1.18 = 117.18\ bar \longleftarrow \quad (c)$$

Discussion: The Redlich-Kwong and Benedict-Webb-Rubin equations suggest that the pressure is in the safe range. Using p_c and T_c from Table A-1,

$$p_R = \frac{120\ bar}{46.4\ bar} = 2.59,\quad T_R = \frac{373K}{191K} = 1.95$$

Figure A-2 gives $Z \approx 0.96$, and so $p = Z\bar{R}T/\bar{v} = (0.96)(\underbrace{124.04}_{part(a)})\ bar =$ 119.1 bar, which is also in the safe range

PROBLEM 11.8

KNOWN: A 10-m^3 tank contains CH_4 at 100 atm and -18°C.

FIND: Determine the mass contained in the tank using (a) the ideal gas model, (b) the van der Waals equation, the Benedict-Webb-Rubin equation.

ANALYSIS: (a) Ideal Gas Model

$$\bar{v} = \frac{\bar{R}T}{p} = \frac{\left(8314 \frac{N\cdot m}{kmol\cdot K}\right)(255 K)}{(100\,atm)\left|1.01325\times10^5\,N/m^2/1\,atm\right|} = 0.209 \frac{m^3}{kmol}$$

Thus

$$m = \frac{V}{v} = \frac{V}{\bar{v}/M} = \left(\frac{10\,m^3}{0.209 \frac{m^3}{kmol}}\right)\left(\frac{16.04\,kg}{kmol}\right) = 767.5\,kg \longleftarrow (a)$$

(b) van der Waals equation. With data from Table A-24

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}^2} \Rightarrow 1.01325\times10^7 \frac{N}{m^2} = \frac{\left(8314 \frac{N\cdot m}{kmol\cdot K}\right)(255K)}{(\bar{v}-0.0428)\left(\frac{m^3}{kmol}\right)} - \frac{2.293\times10^5\,N/m^2}{\bar{v}^2}$$

Using an equation solver, $\bar{v} = 0.14\,m^3/kmol$.

$$\Rightarrow m = \frac{VM}{\bar{v}} = \frac{(10)(16.04)}{0.14} = 1145.7\,kg \longleftarrow (b)$$

(c) Benedict-Webb-Rubin equation. With data from Table A-24

$$101.325\,bar = \frac{(0.08314)(255)}{\bar{v}} + \left[\frac{(0.0426)(0.08314)(255) - 1.8796 - \frac{(2.287)(10^4)}{(255)^2}}{\bar{v}^2}\right] +$$

$$\left[\frac{(0.00338)(0.08314)(255) - 0.0501}{\bar{v}^3}\right] + \frac{(0.0501)(1.244\times10^{-4})}{\bar{v}^6} +$$

$$\left[\frac{2.579\times10^3}{\bar{v}^3\,(255)^2}\left(1 + \frac{0.0060}{\bar{v}^2}\right)\exp\left(-\frac{0.0060}{\bar{v}^2}\right)\right]$$

Using an equation solver, $\bar{v} = 0.149\,m^3/kmol$.

1 $$\Rightarrow m = \frac{VM}{\bar{v}} = \frac{(10)(16.04)}{0.149} = 1076.5\,kg \longleftarrow (c)$$

---

1. With p_c, T_c from Table A-1

$$p_R = \frac{100\,atm}{45.8\,atm} = 2.18, \quad T_R = \frac{255\,K}{191\,K} = 1.34$$

Figure A-2 gives $Z \sim 0.72$. Thus

$$\bar{v} = Z\frac{\bar{R}T}{p} = 0.72\,\overbrace{(0.209)}^{part\,(a)} = 0.15\,m^3/kmol$$

$$\Rightarrow m = \frac{VM}{\bar{v}} = \frac{(10)(16.04)}{0.15} = 1069.3\,kg$$

This agrees closely with the result of part (c).

PROBLEM 11.9

KNOWN: 165 kg of CH_4 is at 200 atm, 400K.

FIND: Using the Benedict-Webb-Rubin equation, determine the volume in m^3. Compare with the results obtained from the ideal gas model and the compressibility chart.

ANALYSIS: With the ideal gas equation of state

$$\bar{v} = \frac{\bar{R}T}{p} = \frac{(8314\ \frac{N\cdot m}{kmol\cdot K})(400\ K)}{(200\ atm)\left|1.01325\times10^5\ N/m^2/atm\right|} = 0.164\ \frac{m^3}{kmol}$$

Thus,

$$V = m\frac{\bar{v}}{M} = (165\ kg)\left(\frac{0.164\ m^3/kmol}{16.04\ kg/kmol}\right) = 1.69\ m^3 \longleftarrow \text{ideal gas.}$$

With T_c, P_c from Table A-1, $T_R = 400/191 = 2.09$, $P_R = 200/45.8 = 4.37$. Then, Fig A-2 gives $Z \approx 0.98$. Thus

$$v = Z\frac{\bar{R}T}{p} = 0.98(0.164) = 0.161\ \frac{m^3}{kmol}$$

So

$$V = m\frac{\bar{v}}{M} = \frac{(165)(0.161)}{(16.04)} = 1.66\ m^3 \longleftarrow \text{compressibility chart}$$

With constants from Table A-24 and $\bar{R} = 0.08314\ bar\cdot m^3/kmol\cdot K$, Eq. 11.12 gives with $p = (200)(1.01325) = 202.65$ bar

$$p = \frac{\bar{R}T}{\bar{v}} + \left[B\bar{R}T - A - \frac{C}{T^2}\right]\frac{1}{\bar{v}^2} + \frac{(b\bar{R}T - a)}{\bar{v}^3} + \frac{a\alpha}{\bar{v}^6} + \frac{c}{\bar{v}^3T^2}\left(1 + \frac{\gamma}{\bar{v}^2}\right)\exp\left(-\frac{\gamma}{\bar{v}^2}\right)$$

$$202.65 = \frac{(0.08314)(400)}{\bar{v}} + \left[(0.04260)(0.08314)(400) - 1.8796 - \frac{2.287\times10^4}{(400)^2}\right]\frac{1}{\bar{v}^2} +$$
$$\left[\frac{(0.00338)(0.08314)(400) - 0.0501}{\bar{v}^3}\right] + \frac{(0.0501)(1.244\times10^{-4})}{\bar{v}^6} +$$
$$\frac{2.579\times10^3}{\bar{v}^3(400)^2}\left(1 + \frac{0.0060}{\bar{v}^2}\right)\exp\left(-\frac{0.0060}{\bar{v}^2}\right)$$

$$202.65 = \frac{33.256}{\bar{v}} - \frac{0.6058}{\bar{v}^2} + \frac{0.0623}{\bar{v}^3} + \frac{(0.0623\times10^{-4})}{\bar{v}^6} + \frac{0.0161\left(1 + \frac{0.0060}{\bar{v}^2}\right)\exp\left(-\frac{0.0060}{\bar{v}^2}\right)}{\bar{v}^3}$$

Using an equation solver, $\bar{v} = 0.161\ m^3/kmol$, giving $V = 1.66\ m^3$. $\longleftarrow$ B-W-R equation

Discussion: The B-W-R equation and compressibility chart values agree. Since $Z \sim 0.98$, the ideal gas value gives a reasonable approximation as well.

PROBLEM 11.10

KNOWN: A rigid tank contains 1 kg of O_2 at $p_1 = 40$ bar, $T_1 = 180$K. The gas is cooled to a final state where $T_2 = 150$K.

FIND: Determine the tank volume and p_2 using (a) the ideal gas equation, (b) the Redlich-Kwong equation, (c) the compressibility chart.

SCHEMATIC & GIVEN DATA:

$p_1 = 40$ bar, $T_1 = 180$K
$T_2 = 150$K

ENGINEERING MODEL: As shown above, the O_2 is the system.

ANALYSIS: (a) Ideal Gas. The ideal gas equation of state gives

$$V = \frac{mRT}{p} = \frac{(1\,kg)(8314/32)\left(\frac{N\cdot m}{kg\cdot K}\right)(180K)}{(40\times10^5\,N/m^2)} = 5.846\times10^{-3}\,m^3 \quad \text{(a)}$$

Since V is constant

$$\left.\begin{matrix} p_1 V = mRT_1 \\ p_2 V = mRT_2 \end{matrix}\right\} \quad \frac{T_2}{T_1} = \frac{p_2}{p_1} \Rightarrow p_2 = \frac{T_2}{T_1} p_1 = \frac{150}{180}(40\text{ bar}) = 33.33\text{ bar}$$

(b) Redlich-Kwong. With values for a and b from Table A-24 and $\bar{R} = 0.08314 \frac{bar\cdot m^3}{kmol\cdot K}$

$$p_1 = \frac{\bar{R}T_1}{\bar{v}_1 - b} - \frac{a}{\bar{v}_1(\bar{v}_1+b)(T_1)^{1/2}} \Rightarrow 40\text{ bar} = \frac{\left(0.08314\frac{bar\cdot m^3}{kmol\cdot K}\right)(180K)}{(\bar{v}_1 - 0.02197)\,m^3/kmol} - \frac{17.22}{\bar{v}_1(\bar{v}_1+0.02197)(180)^{1/2}}$$

Simplifying
$$40 = \frac{14.9652}{\bar{v}_1 - 0.02197} - \frac{1.2835}{\bar{v}_1(\bar{v}_1 + 0.02197)} \quad (1)$$

Since Eq(1) is implicit in $\bar{v}_1$, an iterative solution is required if the result is pursued using hand calculations. Alternatively, an equation-solver such as IT can be used. The result is $\bar{v}_1 = 0.3015\ m^3/kmol$. Thus

$$V = mv_1 = \frac{m\bar{v}_1}{M} = \frac{(1)(0.3015)}{32} = 9.422\times10^{-3}\,m^3 \quad \text{(b)}$$

To find p_2 write, using $\bar{v}_2 = \bar{v}_1$

$$p_2 = \frac{\bar{R}T_2}{\bar{v}_2 - b} - \frac{a}{\bar{v}_2(\bar{v}_2+b)(T_2)^{1/2}} \Rightarrow p_2 = \frac{(0.08314)(150)}{(0.3015-0.02197)} - \frac{17.22}{(0.3015)(.3015+.02197)(150)^{1/2}}$$

$$= 30.2\text{ bar}$$

(c) Compressibility Chart. With T_c and p_c from Table A-1

$$(T_R)_1 = 180/154 = 1.169, \quad (P_R)_1 = 40/50.5 = 0.792$$

Then Fig. A-1 gives $(v_R')_1 \approx 1.22$. Thus

$$v_1 = (v_R')_1\left(\frac{RT_c}{p_c}\right) = 1.22\left[\frac{\left(\frac{8314}{32}\frac{N\cdot m}{kg\cdot K}\right)(154K)}{50.5\times10^5\,N/m^2}\right] = 9.666\times10^{-3}\,\frac{m^3}{kg}$$

Accordingly, $V = mv_1 = 9.666\times10^{-3}\,m^3$ (c)

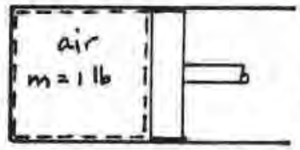
To find p_2, use $v_{R2}' = v_{R1}'$ and $T_{R2} = 150/154 = 0.97$ to locate state 2 on Fig. A-1, giving $P_{R2} \approx 0.6$. Accordingly $p_2 = p_c P_{R2} = (50.5)(0.6) = 30.3$ bar.

PROBLEM 11.11

KNOWN: One lb of air initially at $V_1 = 0.4\ ft^3$ and $1000\ \frac{lbf}{in^2}$ expands isothermally until $V_2 = 2\ ft^3$.

FIND: Using the Van der Waals equation of state, determine (a) T, (b) p_2, and (c) the work.

SCHEMATIC & GIVEN DATA:



$V_1 = 0.4\ ft^3$
$P_1 = 1000\ lbf/in^2$
$V_2 = 2\ ft^3$

ENGINEERING MODEL: (1) The closed system consists of 1 lb of air. (2) The expansion takes place without irreversibilities. (3) The air is modeled by the Van der Waals equation of state.

ANALYSIS: $\bar{v}_1 = \frac{(0.4\ ft^3)}{(1\ lb)}\left(\frac{28.97\ ft \cdot lbf}{lb \cdot mol \cdot °R}\right) = 11.588\ \frac{ft^3}{lbmol}$, $\bar{v}_2 = \frac{(2)(28.97)}{(1)} = 57.94\ \frac{ft^3}{lbmol}$

$$P_1 = (1000\ lbf/in^2)\left|\frac{1\ atm}{14.696\ lbf/in^2}\right| = 68.05\ atm$$

(a) With constants from Table A-24E and $\bar{R} = 0.73\ \frac{atm \cdot ft^3}{lbmol \cdot °R}$

$$T_1 = \left(\frac{\bar{v}_1 - b}{\bar{R}}\right)\left(P_1 + \frac{a}{\bar{v}_1^2}\right) = \frac{(11.588 - 0.586)\ ft^3/lbmol}{\left(0.73\ \frac{atm \cdot ft^3}{lbmol \cdot °R}\right)}\left[68.05\ atm + \frac{345\ \frac{atm \cdot lbmol^2}{ft^6}}{(11.588\ ft^3/lbmol)^2}\right]$$

$$= 1064.3\ °R \longleftarrow T_1$$

(b) Since $T_2 = T_1$

$$P_2 = \frac{\bar{R}T_2}{\bar{v}_2 - b} - \frac{a}{\bar{v}_2^2} = \frac{(0.73)(1064.3)}{(11.588 - 0.586)} - \frac{345}{11.588^2} = 68.05\ atm$$

$$P_2 = (68.05\ atm)\left|\frac{14.696\ lbf/in^2}{1\ atm}\right| = 1000\ lbf/in^2 \longleftarrow P_2$$

(c) The work can be evaluated from

$$W = \int_1^2 p\,dV = n\int_1^2\left[\frac{\bar{R}T}{\bar{v} - b} - \frac{a}{\bar{v}^2}\right]d\bar{v} = n\left[\bar{R}T \ln\left(\frac{\bar{v}_2 - b}{\bar{v}_1 - b}\right) + a\left(\frac{1}{\bar{v}_2} - \frac{1}{\bar{v}_1}\right)\right]$$

Thus

$$\frac{W}{n} = \left(1.986\ \frac{Btu}{lbmol \cdot °R}\right)(1064.3\ °R)\left[\ln\left(\frac{57.94 - 0.586}{11.588 - 0.586}\right)\right]$$
$$+ \left(345\ \frac{atm \cdot ft^3}{lbmol}\right)\left[\frac{1}{57.94} - \frac{1}{11.588}\right]\left|\frac{14.696\ lbf/in^2}{1\ atm}\right|\left|\frac{1\ Btu}{778\ ft \cdot lbf}\right|\left|\frac{144\ in^2}{1\ ft^2}\right|$$

$$= 3425.2\ Btu/lbmol$$

or

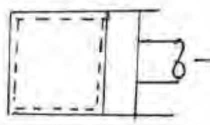
$$W = \frac{(1\ lb)}{\left(28.97\ \frac{lb}{lbmol}\right)}\left(3425.2\ \frac{Btu}{lbmol}\right) = 118.2\ Btu \longleftarrow W$$

PROBLEM 11.12

KNOWN: Water vapor expands isothermally and without irreversibilities from 240°C, 1 MPa to 0.1 MPa. A truncated virial equation of state is specified.

FIND: Determine the work.

SCHEMATIC & GIVEN DATA:



$T = 240°C$
$P_1 = 1\ MPa$
$P_2 = 0.1\ MPa$

$$Z = 1 + \frac{B}{v} + \frac{C}{v^2}$$

ENGINEERING MODEL: (1) The water vapor is the closed system. (2) The process is isothermal with no irreversibilities. (3) Volume change is the only work mode. (4) The p-v-T relation is given by the truncated virial series above.

ANALYSIS: The work is given by

$$\frac{W}{m} = \int_{v_1}^{v_2} p\,dv$$

Then, with

$$\frac{pv}{RT} = 1 + \frac{B}{v} + \frac{C}{v^2} \Rightarrow p = RT\left(\frac{1}{v} + \frac{B}{v^2} + \frac{C}{v^3}\right) \tag{1}$$

$$\frac{W}{m} = RT\left[\ln\frac{v_2}{v_1} - B\left(\frac{1}{v_2} - \frac{1}{v_1}\right) - \frac{C}{2}\left[\frac{1}{v_2^2} - \frac{1}{v_1^2}\right]\right] \tag{2}$$

where from Table A-4, $v_1 = 0.2275\ m^3/kg$, $v_2 = 2.359\ m^3/kg$.

The coefficients B, C can be evaluated using steam table data by noting that

$$Z = 1 + \frac{B}{v} + \frac{C}{v^2} \Rightarrow (Z-1)v = B + C(1/v)$$

① Thus, B and C are the intercept and slope, respectively, of a plot of $(Z-1)v$ vs $(1/v)$. The outcome of this procedure using steam table data is

$$B = -84.0 \times 10^{-4}\ m^3/kg, \quad C = -1.25 \times 10^{-4}\ m^6/kg^2.$$

as can be verified by using Eq. (1) to check steam Table pressure values at 240°C.

Calculating

$$\frac{W}{m} = \left(\frac{8.314}{18.02}\frac{kJ}{kg \cdot K}\right)(513.15\ K)\left[\ln\left(\frac{2.359}{0.2275}\right) + \frac{84}{10^4}\left[\frac{1}{2.359} - \frac{1}{0.2275}\right] + \frac{1.25}{(2)(10^4)}\left[\frac{1}{(2.359)^2} - \frac{1}{(0.2275)^2}\right]\right]$$

$$= 545.6\ \frac{kJ}{kg}$$

1. The use of appropriate computer software facilitates this step. Alternatively, table data can be plotted by hand, and the values of B and C determined graphically.

PROBLEM 11.13

KNOWN: The compressibility factor Z can be expressed in the forms

$$Z = 1 + \hat{B}(T)p + \hat{C}(T)p^2 + \hat{D}(T)p^3 + \cdots \qquad (3.29)$$

$$Z = 1 + \frac{B(T)}{\bar{v}} + \frac{C(T)}{\bar{v}^2} + \frac{D(T)}{\bar{v}^3} + \cdots \qquad (3.30)$$

FIND: Show that

$$\hat{B} = B/\bar{R}T, \quad \hat{C} = (C - B^2)/\bar{R}^2T^2$$

ANALYSIS: With $Z = p\bar{v}/\bar{R}T$, Eq. 3.30 gives

$$p = \frac{\bar{R}T}{\bar{v}} + \frac{\bar{R}TB}{\bar{v}^2} + \frac{\bar{R}TC}{\bar{v}^3} + \cdots$$

Inserting this into Eq. 3.29

$$Z = 1 + \hat{B}\left[\frac{\bar{R}T}{\bar{v}} + \frac{\bar{R}TB}{\bar{v}^2} + \frac{\bar{R}TC}{\bar{v}^3} + \cdots\right] + \hat{C}\left[\frac{\bar{R}T}{\bar{v}} + \frac{\bar{R}TB}{\bar{v}^2} + \cdots\right]^2 + \cdots$$

$$= 1 + \frac{\hat{B}\bar{R}T}{\bar{v}} + \frac{\hat{B}\bar{R}TB + \hat{C}(\bar{R}T)^2}{\bar{v}^2} + \underbrace{\cdots}_{\text{terms in } \bar{v}^3 \text{ or higher}}$$

Comparing this with Eq. 3.30

$$B = \hat{B}\bar{R}T \Rightarrow \hat{B} = B/\bar{R}T$$

and

$$\hat{B}\bar{R}TB + \hat{C}(\bar{R}T)^2 = C(T)$$

or

$$\left(\frac{B}{\bar{R}T}\right)\bar{R}TB + \hat{C}(\bar{R}T)^2 = C(T)$$

① $$B^2 + \hat{C}(\bar{R}T)^2 = C(T) \Rightarrow \hat{C} = \frac{C(T) - (B(T))^2}{\bar{R}^2T^2}$$

1. This procedure can be contined to obtain expressions for $\hat{D}$ and higher terms.

PROBLEM 11.14

KNOWN: The van der Waals equation expressed in terms of Z is

$$Z = \frac{v_R'}{v_R' - 1/8} - \frac{27/64}{T_R v_R'}$$

FIND: Express this (a) as a virial series in v_R', (b) as a virial series in p_R. (c) Dropping higher terms in the expression of part (b), obtain an approximate form, and (d) compare with data from the literature.

ANALYSIS: (a) The term $1/(v_R' - 1/8)$ can be written as

$$\frac{1}{(v_R' - 1/8)} = \frac{1}{v_R'} + \frac{1}{8(v_R')^2} + \frac{1}{(8)^2(v_R')^3} + \cdots$$

Accordingly

$$Z = v_R'\left[\frac{1}{v_R'} + \frac{1}{8(v_R')^2} + \frac{1}{(8)^2(v_R')^3} + \cdots\right] - \frac{27/64}{T_R v_R'}$$

$$= 1 + \left[\frac{1}{8} - \frac{27/64}{T_R}\right]\frac{1}{v_R'} + \frac{1}{(8)^2(v_R')^2} + \underbrace{\cdots\cdots}_{\text{terms in } (v_R')^3 \text{ and higher}}$$

This is the desired form:

$$Z = 1 + \frac{B}{v_R'} + \frac{C}{(v_R')^2} + \cdots\cdots \longleftarrow (a)$$

(b) A virial series in p_R has the form

$$Z = 1 + \hat{B}(T)\, p_R + \hat{C}(T)\,(p_R)^2 + \cdots$$

With $Z = p_R v_R'/T_R$, the result of part (a) becomes

$$\frac{p_R v_R'}{T_R} = 1 + \left[\frac{1}{8} - \frac{27/64}{T_R}\right]\frac{1}{v_R'} + \frac{1}{(8)^2(v_R')^2} + \cdots\cdots$$

or

$$p_R = \frac{T_R}{v_R'} + \left[\frac{1}{8} - \frac{27/64}{T_R}\right]\frac{T_R}{(v_R')^2} + \frac{T_R}{(8)^2(v_R')^3} + \cdots$$

Inserting this into the series in p_R

$$Z = 1 + \hat{B}\left[\frac{T_R}{v_R'} + \left[\frac{1}{8} - \frac{27/64}{T_R}\right]\frac{T_R}{(v_R')^2} + \frac{T_R}{(8)^2(v_R')^3} + \cdots\right] + \hat{C}\left[\frac{T_R}{v_R'} + \cdots\right]^2$$

$$= 1 + \frac{\hat{B}\, T_R}{v_R'} + \left[\frac{\hat{B}\left[\frac{1}{8} - \frac{27/64}{T_R}\right]T_R + \hat{C}\, T_R^2}{(v_R')^2}\right] + \underbrace{\cdots\cdots}_{\text{terms in } (v_R')^3 \text{ or higher}}$$

Comparing this with the result of part (a)

$$\hat{B}\, T_R = \left[\frac{1}{8} - \frac{27/64}{T_R}\right] \Rightarrow \hat{B} = \frac{1}{T_R}\left[\frac{1}{8} - \frac{27/64}{T_R}\right]$$

$$\hat{B}\left[\frac{1}{8} - \frac{27/64}{T_R}\right]T_R + \hat{C}\, T_R^2 = \frac{1}{(8)^2} \Rightarrow \left[\frac{1}{8} - \frac{27/64}{T_R}\right]^2 + \hat{C}\, T_R^2 = \frac{1}{(8)^2}$$

$$\Rightarrow \hat{C} = \left[1/(8)^2 - \left[\frac{1}{8} - \frac{27/64}{T_R}\right]^2\right]\frac{1}{(T_R)^2}$$

PROBLEM 11.14 (Cont'd.) - Page 2

(c) The series obtained in part (b) has the form

$$Z = 1 + \hat{B}\, P_R + \underbrace{\hat{C}\,(P_R)^2 + \cdots}_{\text{Terms in } (P_R)^2,\ (P_R)^3,\ \cdots}$$

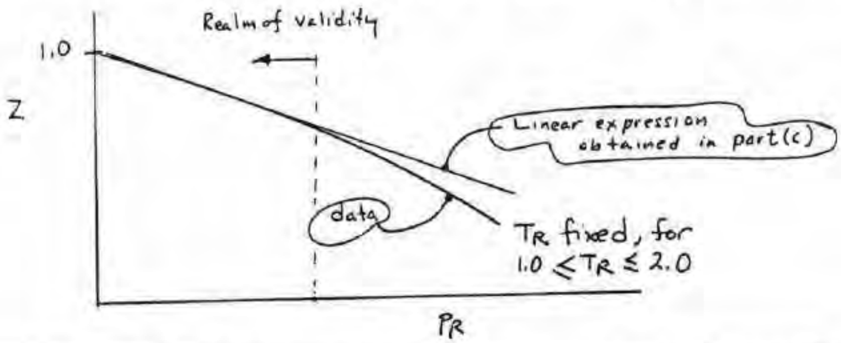
Dropping terms involving $(P_R)^2$ and higher

$$Z = 1 + \hat{B}\, P_R$$

or

$$Z = 1 + \left[\frac{1}{8} - \frac{27/64}{T_R} \right] \frac{P_R}{T_R}$$

(d) Calculating compressibility factors from the result of part (c) and comparing them with values obtained from the
1 literature, the realm of validity can be determined. This procedure is indicated by the following sketch:



1. See, for example, R.C. Reid and T.K. Sherwood, The Properties of Gases and Liquids, 2nd ed., McGraw-Hill, New York, 1966, pp. 587-595.

PROBLEM 11.15

KNOWN: The Berthelot equation of state has the form

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{T\bar{v}^2}$$

FIND: (a) Using Eqs. 11.3, evaluate a and b in terms of T_c and p_c, (b) Express the equation in terms of Z, T_R, and v_R'.

ANALYSIS: (a) Differentiating

$$\left(\frac{\partial p}{\partial \bar{v}}\right)_T = -\frac{\bar{R}T}{(\bar{v}-b)^2} + \frac{2a}{T\bar{v}^3}$$

At the critical point: $(\partial p/\partial \bar{v})_T = 0$. Thus

$$-\frac{\bar{R}T_c}{(\bar{v}_c-b)^2} + \frac{2a}{T_c\bar{v}_c^3} = 0 \Rightarrow -\bar{R}T_c^2\bar{v}_c^3 + 2a(\bar{v}_c-b)^2 = 0 \qquad (1)$$

Differentiating

$$\left(\frac{\partial^2 p}{\partial \bar{v}^2}\right)_T = +\frac{2\bar{R}T}{(\bar{v}-b)^3} - \frac{6a}{T\bar{v}^4}$$

At the critical point: $(\partial^2 p/\partial \bar{v}^2)_T = 0$. Thus

$$\frac{2\bar{R}T_c}{(\bar{v}_c-b)^3} - \frac{6a}{T_c\bar{v}_c^4} = 0 \Rightarrow 2\bar{R}T_c^2\bar{v}_c^4 - 6a(\bar{v}_c-b)^3 = 0 \qquad (2)$$

At the critical point:

$$p_c = \frac{\bar{R}T_c}{\bar{v}_c-b} - \frac{a}{T_c\bar{v}_c^2} \qquad (3)$$

Combining (1) and (2), $b = \bar{v}_c/3$. Inserting this into Eq. (1)

$$-\bar{R}T_c^2\bar{v}_c^3 + 2a\left(\bar{v}_c - \frac{\bar{v}_c}{3}\right)^2 = 0 \Rightarrow a = 9\bar{R}T_c^2\bar{v}_c/8$$

Inserting the expressions for a and b into Eq. (3), $(p_c v_c/\bar{R}T_c) = 3/8$, or $v_c = \frac{3}{8}\frac{\bar{R}T_c}{p_c}$. Finally

$$a = \frac{9\bar{R}T_c^2}{8}\left[\frac{3}{8}\frac{\bar{R}T_c}{p_c}\right] = \frac{27}{64}\frac{\bar{R}^2T_c^3}{p_c}, \quad b = \frac{1}{8}\frac{\bar{R}T_c}{p_c} \longleftarrow (a)$$

(b) With $Z = p\bar{v}/\bar{R}T$ and the above expressions for a and b

$$\frac{p\bar{v}}{\bar{R}T} = \frac{\bar{v}}{\bar{v}-b} - \frac{a}{\bar{R}T^2\bar{v}} = \frac{\bar{v}}{\bar{v}-\left[\frac{\bar{R}T_c}{8p_c}\right]} - \frac{(27/64)(\bar{R}^2T_c^3/p_c)}{\bar{R}T^2\bar{v}}$$

or

$$Z = \frac{1}{1-\frac{1}{8}\left[\frac{\bar{R}T_c}{\bar{v}p_c}\right]} - \frac{(27/64)(\bar{R}T_c/p_c\bar{v})}{T_R^2}$$

Thus

$$Z = \frac{v_R'}{v_R' - \frac{1}{8}} - \frac{27/64}{v_R' T_R^2} \longleftarrow (b)$$

PROBLEM 11.16

KNOWN: The Beattie-Bridgeman equation of state can be expressed as

$$p = \frac{RT(1-\epsilon)(v+B)}{v^2} - \frac{A}{v^2}$$

where

$$A = A_0\left(1-\frac{a}{v}\right), \quad B = B_0\left(1-\frac{b}{v}\right), \quad \epsilon = \frac{c}{vT^3}$$

FIND: Express the equation in terms of P_R, T_R, v_R', and dimensionless constants.

ANALYSIS: With $T = T_R T_c$, $p = P_R P_c$, and $v = v_R'(RT_c/P_c)$, the equation becomes

$$P_R P_c = \frac{RT_R T_c(1-\epsilon)(v_R' RT_c/P_c + B)}{(v_R')^2(RT_c/P_c)^2} - \frac{A}{(v_R')^2(RT_c/P_c)^2}$$

where

$$A = A_0\left(1-\frac{a}{v_R'(RT_c/P_c)}\right), \quad B = B_0\left(1-\frac{b}{v_R'(RT_c/P_c)}\right), \quad \epsilon = \frac{c}{v_R' T_R^3 (RT_c/P_c)(T_c)^3}$$

Define the following dimensionless quantities

$$a' \equiv \frac{a}{RT_c/P_c}, \quad b' \equiv \frac{b}{RT_c/P_c}, \quad c' \equiv \frac{c}{RT_c^4/P_c}$$

$$A_0' \equiv \frac{A_0}{R^2T_c^2/P_c}, \quad B_0' \equiv \frac{B_0}{RT_c/P_c}, \quad A' \equiv \frac{A}{R^2T_c^2/P_c}, \quad B' \equiv \frac{B}{RT_c/P_c}$$

Then

$$A' = A_0'\left(1-\frac{a'}{v_R'}\right), \quad B' = B_0'\left(1-\frac{b'}{v_R'}\right), \quad \epsilon = \frac{c'}{v_R' T_R^3}$$

Upon simplification, the equation of state becomes

$$P_R = \frac{RT_R T_c\left(\frac{RT_c}{P_c}\right)(1-\epsilon)\left(v_R' + \frac{BP_c}{RT_c}\right)}{P_c(RT_c/P_c)^2(v_R')^2} - \frac{A}{P_c(v_R')^2(RT_c/P_c)^2}$$

or

$$P_R = \frac{T_R(1-\epsilon)(v_R' + B')}{(v_R')^2} - \frac{A'}{(v_R')^2}$$

which is the required result.

PROBLEM 11.17

KNOWN: The Dieterici equation of state is

$$p = \left(\frac{RT}{v-b}\right) \exp\left(\frac{-a}{RTv}\right)$$

FIND: (a) Evaluate a and b in terms of T_c and p_c using Eqs. 11.3. (b) Express the equation in terms of Z, T_R, and v_R'. (c) Convert the result of part (b) into a virial series in v_R'.

ANALYSIS: (a) Differentiating twice with respect to v, we get

$$\left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT}{(v-b)^2}\exp\left(-\frac{a}{RTv}\right) + \left(\frac{RT}{v-b}\right)\exp\left(-\frac{a}{RTv}\right)\cdot\left(+\frac{a}{RTv^2}\right)$$

$$= -\frac{RT}{(v-b)^2}\exp\left(-\frac{a}{RTv}\right) + \frac{a}{v^2(v-b)}\exp\left(-\frac{a}{RTv}\right) \qquad (1)$$

$$\left(\frac{\partial^2 p}{\partial v^2}\right)_T = \frac{2RT}{(v-b)^3}\exp\left(-\frac{a}{RTv}\right) - \frac{a}{v^2(v-b)^2}\exp\left(-\frac{a}{RTv}\right) -$$

$$\frac{a(3v-2b)}{v^3(v-b)^2}\exp\left(-\frac{a}{RTv}\right) + \frac{a^2}{RTv^4(v-b)}\exp\left(-\frac{a}{RTv}\right) \qquad (2)$$

With $(\partial p/\partial v)_T = 0$ at the critical point, Eq. (1) gives

$$0 = -\frac{RT_c}{(v_c-b)} + \frac{a}{v_c^2} \Rightarrow a = \frac{RT_c(v_c)^2}{(v_c-b)} \qquad (3)$$

With $(\partial^2 p/\partial v^2)_T = 0$ at the critical point, Eq. (2) gives

$$0 = \frac{2RT_c}{(v_c-b)^2} - \frac{a}{v_c^2(v_c-b)} - \frac{a(3v_c-2b)}{v_c^3(v_c-b)} + \frac{a^2}{RT_c v_c^4} \qquad (4)$$

Inserting Eq. (3) into Eq. (4) and simplifying, $b = v_c/2$. Thus, $a = 2RT_c v_c$.

At the critical point, the equation of state reads

$$p_c = \frac{RT_c}{v_c-b}\exp\left(-\frac{a}{RT_c v_c}\right) = \frac{RT_c}{\left(v_c-\frac{v_c}{2}\right)}\exp\left(-\frac{2RT_c v_c}{RT_c v_c}\right)$$

Upon reduction, $v_c = 2RT_c/p_c e^2$, giving

$$a = \frac{4R^2T_c^2}{p_c e^2}, \quad b = \frac{RT_c}{p_c e^2}$$

(b) With the given equation of state, the results of part (a), and $T = T_R T_c$, $v = v_R' RT_c/p_c$, we get

$$\frac{pv}{RT} = \frac{v}{v-b}\exp\left(-\frac{a}{RTv}\right)$$

PROBLEM 11.17 (Cont'd)-Page2

Thus

$$Z = \frac{V_R' \, RT_c/P_c}{[V_R' \, RT_c/P_c - RT_c/P_c e^2]} \exp\left(\frac{\frac{4R^2T_c^2}{P_c e^2}}{R(T_R T_c)(V_R' \, RT_c/P_c)} \right)$$

$$Z = \left[\frac{V_R'}{V_R' - 1/e^2}\right] \exp\left(-\frac{4}{T_R V_R' e^2}\right) \longleftarrow \quad (b)$$

(c) The term $1/(V_R' - 1/e^2)$ can be written as

$$\frac{1}{V_R' - 1/e^2} = \frac{1}{V_R'} + \frac{1}{e^2 (V_R')^2} + \frac{1}{e^4 (V_R')^3} + \cdots$$

The exponential term can also be written as a series. Collect results and reduce to read

$$Z = 1 + \frac{1}{e^2}\left[1 - \frac{4}{T_R}\right]\frac{1}{V_R'} + \frac{1}{e^4}\left[1 - \frac{4}{T_R} - \frac{8}{T_R^2}\right]\frac{1}{(V_R')^2} + \cdots \longleftarrow \quad (c)$$

PROBLEM 11.18

KNOWN: The Peng-Robinson equation of state is specified.

FIND: Evaluate the constants a, b, c in terms of P_c, T_c, Z_c, using Eqs. 11.3

ANALYSIS: Evaluating the Peng-Robinson equation of state at the critical point and using Eqs. 11.3 gives the following three expressions

$$P_c = \frac{RT_c}{v_c - b} - \frac{a}{v_c^2 - c^2} \qquad (1)$$

$$\frac{-RT_c}{(v_c-b)^2} + \frac{2a v_c}{(v_c^2-c^2)^2} = 0 \qquad (2)$$

$$\frac{2RT_c}{(v_c-b)^3} + \frac{2a(v_c^2-c^2) - 8a v_c^2}{(v_c^2-c^2)^3} = 0 \qquad (3)$$

Solving these expressions simultaneously

①
$$a = \frac{8 v_c^3 P_c^2}{RT_c} = 8 v_c^2 P_c Z_c \qquad (4)$$

$$b = 3v_c - \frac{RT_c}{P_c} = v_c\left[3 - \frac{1}{Z_c}\right] \qquad (5)$$

$$c^2 = v_c^2\left[8\frac{P_c v_c}{RT_c} - 3\right] = v_c^2\left[8Z_c - 3\right] \qquad (6)$$

as can be verified by substitution into Eqs. (1)-(3) and reduction of the resulting expressions.

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1. Note that when $Z_c = 3/8$, Eqs. (4, 5) reduce to Eqs. (11.4a, 4b), respectively. The equation of state reduces to the van der Waals equation.

PROBLEM 11.19

<u>KNOWN</u>: The Carnhan - Starling - De Santis equation of state is provided, together with values of the constants in the equation for the cases of Refrigerants 12 and 13.

<u>FIND</u>: Specify which of the two refrigerants would allow the smaller amount of mass to be stored in a 10-m^3 vessel at 0.2 MPa, 80°C. Determine the mass.

<u>ANALYSIS</u>:

For the case of R12, using the values of the constants provided,

$$b = (0.15376) + (-1.84195 \times 10^{-4})(353) + (-5.03644 \times 10^{-8})(353)^2$$
$$= 8.2463 \times 10^{-2} \ \frac{L}{mol} = 8.2463 \times 10^{-2} \ \frac{m^3}{kmol}$$

$$a = (3.52412 \times 10^3) \exp\left((-2.7723 \times 10^{-3})(353) + (-0.67318 \times 10^{-6})(353)^2\right)$$
$$= 1217.896 \ \frac{J \cdot L}{(mol)^2} = 1.2179 \times 10^6 \ \frac{J \cdot m^3}{(kmol)^2}$$

Inserting values into the equation of state

$$\frac{(0.2 \times 10^6 \ N/m^2)\,\bar{v}}{(8314 \ \frac{J}{kmol \cdot K})(353 K)} = \frac{1 + \beta + \beta^2 - \beta^3}{(1+\beta)^3} - \frac{1.2179 \times 10^6 \ \frac{J \cdot m^3}{(kmol)^2}}{(8314 \ \frac{J}{kmol \cdot K})(353 K)(\bar{v} + 8.2463 \times 10^{-2}) \frac{m^3}{kmol}}$$

where $\beta = 8.2463 \times 10^{-2} / 4\bar{v}$.

Using an equation solver, we get $\bar{v} = 14.21 \ m^3/kmol$. Then, with M = 120.92 for R12,

$$v = \frac{14.21}{120.92} = 0.1175 \ m^3/kg$$

① This value is within 1% of the value obtained from the literature: v = 0.1187 m^3/kg.

Similarly, for R13 at the same state, v = 0.1391 m^3/kg. Thus, a smaller mass of R13 can be stored:

$$m = \frac{V}{v} = \frac{10 \ m^3}{0.1391 \ m^3/kg} = 71.89 \ kg$$

---

1. For example, R12 and R13 data can be obtained from C. Borgnakke and R. E. Sonntag, <u>Thermodynamic and Transport Properties</u>, Wiley, New York, 1997, Tables B.12, B.13.

PROBLEM 11.20

KNOWN: Two expressions for dp in terms of v and T are provided.

FIND: Determine which of the two expressions is the differential of an equation of state: $p = p(v, T)$, and obtain the equation.

ANALYSIS: For the first of the two equations, let

$$M = \frac{2(v-b)}{RT} \quad , \quad N = \frac{(v-b)^2}{RT^2}$$

If this is the differential of an equation of state, the test for exactness must be satisfied: $\partial M/\partial T)_v = \partial N/\partial v)_T$. Checking

$$\frac{\partial M}{\partial T}\Big)_v = \frac{-2(v-b)}{RT^2} \quad , \quad \frac{\partial N}{\partial v}\Big)_T = \frac{2(v-b)}{RT^2}$$

As these are unequal, the first expression given cannot be the differential of an equation of state.

For the second of the two equations, let

$$M = -\frac{RT}{(v-b)^2} \quad , \quad N = \frac{R}{v-b}$$

Applying the test for exactness: $(\partial M/\partial T)_v = -R/(v-b)^2$, $(\partial N/\partial v)_T = -R/(v-b)^2$. As these are equal, a function $p = p(T, v)$ exists for which the second differential form is the exact differential. To obtain the function, write

$$dp = \left(\frac{\partial p}{\partial v}\right)_T dv + \left(\frac{\partial p}{\partial T}\right)_v dT$$

Comparing with the given differential form

$$\left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT}{(v-b)^2} \quad , \quad \left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v-b}$$

Integrating the second of these

$$p = \frac{RT}{v-b} + f(v)$$

Inserting this trial form into the first gives

$$\frac{-RT}{(v-b)^2} + \frac{df}{dv} = -\frac{RT}{(v-b)^2} \Rightarrow \frac{df}{dv} = 0 \Rightarrow f = \text{constant}.$$

Accordingly, the equation of state has the form: $p = \frac{RT}{(v-b)} + \text{constant}$. ◄

PROBLEM 11.21

KNOWN: $\delta Q_{\substack{int\\rev}} = dU + p\,dV$

FIND: Use the given expression together with the test for exactness to demonstrate that $Q_{\substack{int\\rev}}$ is not a property.

ENGINEERING MODEL: (1) A simple compressible system of fixed mass is under consideration. (2) The system undergoes an internally reversible process.

ANALYSIS: For a simple compressible system of fixed mass, $U = U(V, T)$. The differential of this is

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV = M\,dT + N\,dV \qquad (1)$$

As U, V, and T are properties, this is an exact differential. Applying the test for exactness:

$$\frac{\partial}{\partial V}\Big[\underbrace{\left(\frac{\partial U}{\partial T}\right)_V}_{M}\Big]_T = \frac{\partial}{\partial T}\Big[\underbrace{\left(\frac{\partial U}{\partial V}\right)_T}_{N}\Big]_V \qquad (2)$$

Inserting Eq. (1) into the given expression for $\delta Q_{\substack{int\\rev}}$

$$\delta Q_{\substack{int\\rev}} = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV + p\,dV$$

$$= \left(\frac{\partial U}{\partial T}\right)_V dT + \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right] dV \qquad (3)$$

If $Q_{\substack{int\\rev}}$ is a property, the last expression would be an exact differential. Applying the test for exactness to Eq. (3)

$$M = \left(\frac{\partial U}{\partial T}\right)_V \Rightarrow \left(\frac{\partial M}{\partial V}\right)_T = \frac{\partial}{\partial V}\left[\left(\frac{\partial U}{\partial T}\right)_V\right]_T$$

$$N = \left(\frac{\partial U}{\partial V}\right)_T + p \Rightarrow \left(\frac{\partial N}{\partial T}\right)_V = \frac{\partial}{\partial T}\left[\left(\frac{\partial U}{\partial V}\right)_T\right]_V + \left(\frac{\partial p}{\partial T}\right)_V$$

Accordingly

$$\left(\frac{\partial M}{\partial V}\right)_T = \left(\frac{\partial N}{\partial T}\right)_V \Rightarrow \frac{\partial}{\partial V}\left[\left(\frac{\partial U}{\partial T}\right)_V\right]_T = \frac{\partial}{\partial T}\left[\left(\frac{\partial U}{\partial V}\right)_T\right]_V + \left(\frac{\partial p}{\partial T}\right)_V$$

equal by Eq. (2)

Using Eq. (2) in this, as indicated, the test for exactness is satisfied only when

$$\left(\frac{\partial p}{\partial T}\right)_V \equiv 0$$

However, reference to Fig. 11.1 shows that this expression is not generally satisfied. It can be concluded therefore that the test for exactness is not satisfied and $Q_{\substack{int\\rev}}$ cannot be a property. The use of the symbol δ with $Q_{\substack{int\\rev}}$ is to signal this fact. ←

PROBLEM 11.22

KNOWN: An equation of state has the form $p = [RT/(v-b)] + a$

FIND: Show that Eq. 11.16 is satisfied.

ANALYSIS: Eq. 11.16 has the form

$$\left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z = -1$$

Identifying $p: x$, $v: y$, $z: T$, then

$$\left(\frac{\partial y}{\partial z}\right)_x = \left(\frac{\partial v}{\partial T}\right)_p$$

$$\left(\frac{\partial z}{\partial x}\right)_y = \left(\frac{\partial T}{\partial p}\right)_v$$

$$\left(\frac{\partial x}{\partial y}\right)_z = \left(\frac{\partial p}{\partial v}\right)_T$$

Using the equation of state, $(\partial p/\partial v)_T = -RT(v-b)^{2}$ (1)

Solving,

$$v = b + \frac{RT}{p-a} \Rightarrow \left.\frac{\partial v}{\partial T}\right)_p = \frac{R}{p-a} \Rightarrow \left.\frac{\partial v}{\partial T}\right)_p = \frac{v-b}{T}$$ (2)

Solving

$$T = \frac{(p-a)(v-b)}{R} \Rightarrow \left.\frac{\partial T}{\partial p}\right)_v = \frac{v-b}{R}$$ (3)

With (1)-(3), Eq. 11.16 takes the form

$$\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_v \left(\frac{\partial p}{\partial v}\right)_T = \left(\frac{v-b}{T}\right)\left(\frac{v-b}{R}\right)\left(-\frac{RT}{(v-b)^2}\right) = -1$$

Thus, Eq. 11.16 is satisfied.

PROBLEM 11.23

KNOWN: $x = x(y, w)$, $y = y(z, w)$, $z = z(x, w)$.

FIND: Show that

$$\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial x}\right)_w = 1$$

ANALYSIS: Forming differentials

$$dx = \left(\frac{\partial x}{\partial y}\right)_w dy + \left(\frac{\partial x}{\partial w}\right)_y dw \qquad (1)$$

$$dy = \left(\frac{\partial y}{\partial z}\right)_w dz + \left(\frac{\partial y}{\partial w}\right)_z dw \qquad (2)$$

$$dz = \left(\frac{\partial z}{\partial x}\right)_w dx + \left(\frac{\partial z}{\partial w}\right)_x dw \qquad (3)$$

Replacing dy in Eq. (1) with Eq. (2), and then replacing dz in the resulting expression by Eq. (3):

$$dx = \left(\frac{\partial x}{\partial y}\right)_w \left[\left(\frac{\partial y}{\partial z}\right)_w dz + \left(\frac{\partial y}{\partial w}\right)_z dw\right] + \left(\frac{\partial x}{\partial w}\right)_y dw = \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w dz + \left[\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial w}\right)_z + \left(\frac{\partial x}{\partial w}\right)_y\right] dw$$

$$= \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left[\left(\frac{\partial z}{\partial x}\right)_w dx + \left(\frac{\partial z}{\partial w}\right)_x dw\right] + \left[\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial w}\right)_z + \left(\frac{\partial x}{\partial w}\right)_y\right] dw$$

$$= \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial x}\right)_w dx + \left[\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial w}\right)_x + \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial w}\right)_z + \left(\frac{\partial x}{\partial w}\right)_y\right] dw$$

On rearrangement

$$\left[1 - \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial x}\right)_w\right] dx = \left[\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial w}\right)_x + \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial w}\right)_z + \left(\frac{\partial x}{\partial w}\right)_y\right] dw$$

Since x and w are independent, hold w constant and vary x. That is, $dw = 0$ and $dx \neq 0$. Then

$$\left[1 - \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial x}\right)_w\right] dx = 0 \Rightarrow \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial x}\right)_w = 1. \longleftarrow$$

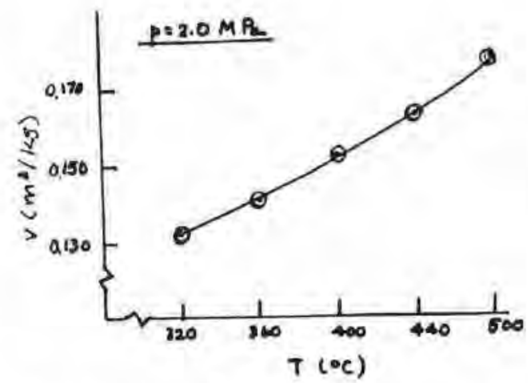
PROBLEM 11.24

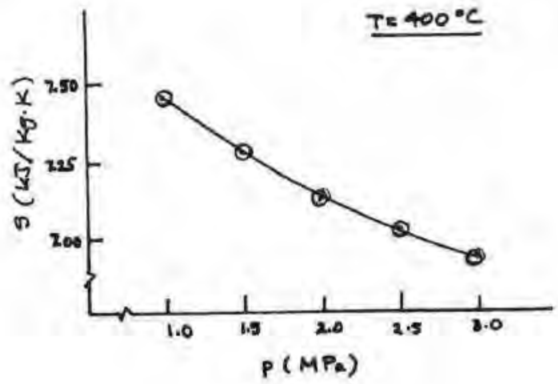
KNOWN:

$$\left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial s}{\partial p}\right)_T \qquad (11.35)$$

FIND: Using Eq. (11.35), check the consistency of (a) the steam tables at 2 MPa, 400°C, (b) the R134a tables at 2 bar, 50°C.

ANALYSIS: We select a graphical approach here. (a) Using data from Table A-4





From the v vs. T plot, at 400°C

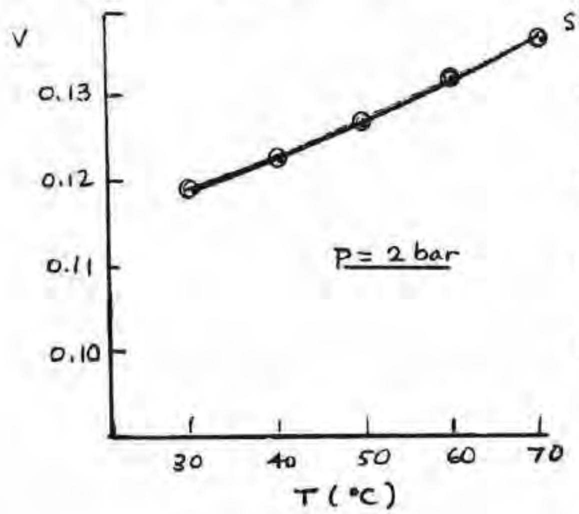
$$\left(\frac{\partial v}{\partial T}\right)_p \approx 0.25 \times 10^{-3}\ \frac{m^3}{kg\cdot K}$$

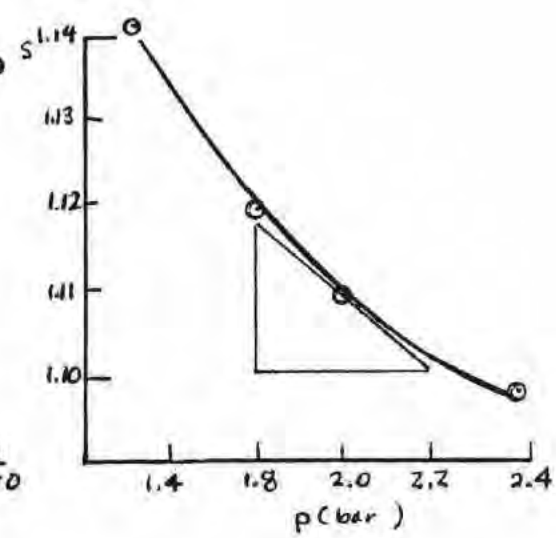
From the s vs p plot, at 2 MPa

$$\left(\frac{\partial s}{\partial p}\right)_T \approx -0.249\ \frac{kJ/kg\cdot K}{MPa} = -0.249\ \frac{(10^3\ N\cdot m)}{kg\cdot K}\left(\frac{1}{10^6\ N/m^2}\right) = -0.249\times 10^{-3}\ \frac{m^3}{kg\cdot K}$$

These values are in good agreement, as required by 11.35.

(b)
Using data from Table A-12





From the v vs. T plot, at 50°C

$$\left(\frac{\partial v}{\partial T}\right)_p \approx 4.45 \times 10^{-4}\ m^3/kg\cdot K$$

From the s vs p plot, at 2 bar

$$\left(\frac{\partial s}{\partial p}\right)_T \approx 4.45 \times 10^{-4}\ m^3/kg\cdot K$$

In accordance with Eq. 11.35, the data are in agreement.

PROBLEM 11.24 (Cont'd.) - Page2

An alternative solution using *IT* follows:

IT Code - part (a)

```
// (a) steam at 2 MPa, 400°C
p = 2000 // kPa
T = 400 // °C
dT = 0.001
dp = 0.001
T1 = T - dT
T2 = T + dT
p1 = p - dp
p2 = p + dp
v1 = v_PT("Water/Steam", p, T1)
v2 = v_PT("Water/Steam", p, T2)
dvdT_a = (v2 - v1) / (T2 - T1)
s1 = s_PT("Water/Steam", p1, T)
s2 = s_PT("Water/Steam", p2, T)
dsdp = (s2 - s1) / (p2 - p1)
```

IT Results - part (a)

$(\partial v/\partial T)_p = 0.2491 \times 10^{-3}$ m³/kg·K
$(\partial s/\partial p)_T = - 0.2492 \times 10^{-3}$ m³/kg·K

IT Code - part (b)

```
(b) R-134a at 2 bar, 50°C
p_b = 200 // kPa
T_b = 50 // °C
v = v_PT("R134A", p_b, T_b)
T1_b = T_b - dT
T2_b = T_b + dT
p1_b = p_b - dp
p2_b = p_b + dp
v1_b = v_PT("R134A", p_b, T1_b)
v2_b = v_PT("R134A", p_b, T2_b)
dvdT_b = (v2_b - v1_b) / (T2_b - T1_b)
s2_b = s_PT("R134A", p2_b, T_b)
s1_b = s_PT("R134A", p1_b, T_b)
dsdp_b = (s2_b - s1_b) / (p2_b - p1_b)
```

IT Results - part (b)

$(\partial v/\partial T)_p = 4.449 \times 10^{-4}$ m³/kg·K
$(\partial s/\partial p)_T = - 4.449 \times 10^{-4}$ m³/kg·K

The results of the IT solution compare favorably with the graphical results.

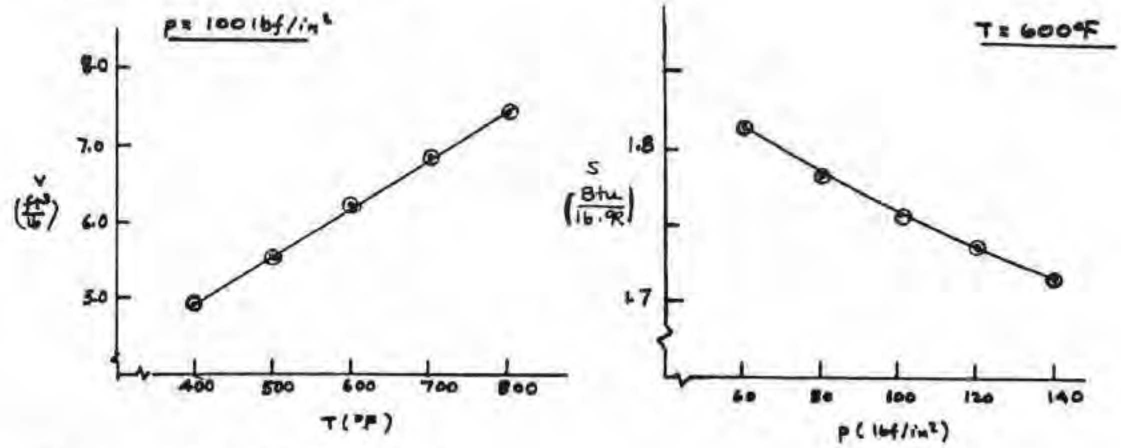
PROBLEM 11.25

KNOWN:

$$\left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial s}{\partial p}\right)_T \qquad (11.35)$$

FIND: Using Eq. 11.35, check the consistency of (a) the steam tables at 100 lbf/in², 600°F, (b) the R134a tables at 40 lbf/in², 100°F

ANALYSIS: We select a graphical approach here. (a) Using data from Table A-4E



From the v vs T plot at 600°F

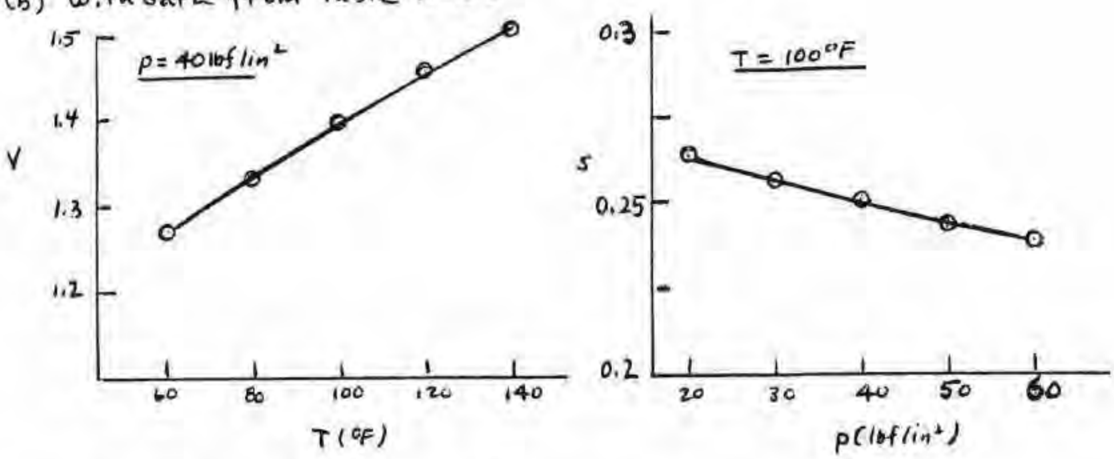
$$\left(\frac{\partial v}{\partial T}\right)_p = 6.24 \times 10^{-3} \ \frac{ft^3}{lb\cdot°R}$$

From the s vs p plot at 100 lbf/in²

$$\left(\frac{\partial s}{\partial T}\right)_p = -1.168\times10^{-3} \ \frac{Btu/lb\cdot°R}{lbf/in^2}\left(\frac{778 ft\cdot lbf}{Btu}\right)\left(\frac{ft^2}{144 in^2}\right) = -6.31\times10^{-3} \ \frac{ft^3}{lb\cdot°R}$$

These values are in good agreement, as required by Eq. 11.35.

(b) With data from Table A-12E



From the v vs. T plot, at 100°F:

$$\left(\frac{\partial v}{\partial T}\right)_p \approx 3.05\times10^{-3} \ \frac{ft^3}{lb\cdot°R}$$

From the s vs. p plot, at 40 lbf/in²:

$$\left(\frac{\partial s}{\partial p}\right)_T \approx -3.11\times10^{-3} \ \frac{ft^3}{lb\cdot°R}$$

again, we have good agreement, in accordance with Eq. 11.35.

PROBLEM 11.25 (Cont'd.) - Page 2

An alternative solution using IT follows:

IT Code - part (a)

```
// (a)  steam at 100 lbf/in², 600°F
pa = 100  //  lbf/in²
Ta = 600  //  °F
dTa = 0.1
T1a = Ta - dTa
T2a = Ta + dTa
dpa = 0.1
p1a = pa - dpa
p2a = pa + dpa
v1a = v_PT("Water/Steam", pa, T1a)
v2a = v_PT("Water/Steam", pa, T2a)
dvdTa = (v2a - v1a) / (T2a - T1a)
s1a = s_PT("Water/Steam", p1a, Ta)
s2a = s_PT("Water/Steam", p2a, Ta)
dsdpa = ((s2a - s1a) / (p2a - p1a)) * (778.17 / 144)
```

IT Results - part (a)

$(\partial v/\partial t)_p = 6.225 \times 10^{-3}$ ft³/lb·°R
$(\partial s/\partial p)_T = -6.226 \times 10^{-3}$ ft³/lb·°R

IT Code - part (b)

```
// (b)  R134a at 40 lbf/in², 100°F
pb = 40  //  lbf/in²
Tb = 100  //  °F
dTb = 0.1
T1b = Tb - dTb
T2b = Tb + dTb
dpb = 0.1
p1b = pb - dpb
p2b = pb + dpb
v1b = v_PT("R134A", pb, T1b)
v2b = v_PT("R134A", pb, T2b)
dvdTb = (v2b - v1b) / (T2b - T1b)
s1b = s_PT("R134A", p1b, Tb)
s2b = s_PT("R134A", p2b, Tb)
dsdpb = ((s2b - s1b) / (p2b - p1b)) * (778.17 / 144)
```

IT Results - part (b)

$(\partial v/\partial t)_p = 3.042 \times 10^{-3}$ ft³/lb·°R
$(\partial s/\partial p)_T = -3.041 \times 10^{-3}$ ft³/lb·°R

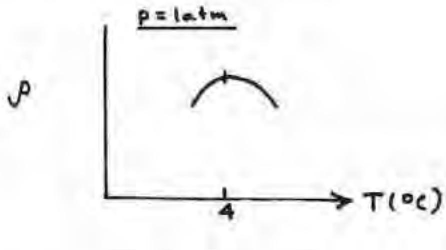
The results of the IT solution compare favorably with the graphical results.

PROBLEM 11.26

KNOWN: At 1atm, liquid water has a state of maximum density at about 4°C.

FIND: Determine what can be concluded about $(\partial s/\partial p)_T$ at (a) 3°C, (b) 4°C, (c) 5°C?

SCHEMATIC & GIVEN DATA:



ANALYSIS: From Eq. 11.35

$$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$$

Since $\left(\frac{\partial \rho}{\partial T}\right)_p = \left(\frac{\partial v}{\partial T}\right)_p \frac{d\rho}{dv} = -\frac{1}{v^2}\left(\frac{\partial v}{\partial T}\right)_p \Rightarrow -\left(\frac{\partial v}{\partial T}\right)_p = \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial T}\right)_p$

Thus

$$\left(\frac{\partial s}{\partial p}\right)_T = \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial T}\right)_p \Rightarrow \begin{array}{ll} (\partial s/\partial p)_T > 0 & \text{for } T < 4°C \\ (\partial s/\partial p)_T = 0 & \text{for } T = 4°C \\ (\partial s/\partial p)_T < 0 & \text{for } T > 4°C \end{array}$$

PROBLEM 11.27

KNOWN: A gas enters a compressor and is compressed isentropically.

FIND: Determine if the specific enthalpy increases or decreases as the gas passes from inlet to exit.

ENGINEERING MODEL: The compression is isentropic.

ANALYSIS: From Eq. 11.27

$$\left.\frac{\partial h}{\partial p}\right)_s = v$$

Since v is a positive number and pressure p increases (compression), the specific enthalpy must also increase.

PROBLEM 11.28

KNOWN: A fundamental thermodynamic function $u = u(s,v)$ is provided.

FIND: Show that T, p, h, ψ, and g can each be determined using $u(s,v)$.

ANALYSIS: From Eqs. 11.24 and 11.25 T and p can be found by differentiation

$$T = \left(\frac{\partial u}{\partial s}\right)_v \qquad \longleftarrow T$$

$$p = -\left(\frac{\partial u}{\partial v}\right)_s \qquad \longleftarrow p$$

Then, by definition $h = u + pv$, giving

$$h = u(s,v) + \left(-\left(\frac{\partial u}{\partial v}\right)_s\right) v \qquad \longleftarrow h$$

Also, by definition $\psi = u - Ts$, giving

$$\psi = u(s,v) - \left(\frac{\partial u}{\partial s}\right)_v s \qquad \longleftarrow \psi$$

Too, $g = h - Ts$, giving $g = u(s,v) - v(\partial u/\partial v)_s - s(\partial u/\partial s)_v \qquad \longleftarrow g$

PROBLEM 11.29

KNOWN: The Helmholtz function is known for a certain substance.

FIND: Evaluate p, s, u, h, c_v, and c_p.

ANALYSIS: The Helmholtz function has the form

$$\psi = -RT \ln \frac{v}{v'} - cT'\left[1 - \frac{T}{T'} + \frac{T}{T'} \ln \frac{T}{T'}\right]$$

where v', T' define the reference state and c is a constant. Note that at the reference state: when $v = v'$ and $T = T'$, $\psi = 0$.

With Eq. 11.28

$$p = -\left(\frac{\partial \psi}{\partial v}\right)_T = -\left(-\frac{RT}{v}\right) \Rightarrow \quad p = RT/v \quad \longleftarrow p$$

With Eq. 11.29

$$s = -\left(\frac{\partial \psi}{\partial T}\right)_v = -\left(-R\ln\frac{v}{v'} - cT'\left[-\frac{1}{T'} + \frac{1}{T'}\ln\frac{T}{T'} + \frac{T}{T'}\left(\frac{1}{T}\right)\right]\right)$$

$$= R\ln\frac{v}{v'} + cT'\left[-\frac{1}{T'} + \frac{1}{T'}\ln\frac{T}{T'} + \frac{1}{T'}\right]$$

$$= R\ln\frac{v}{v'} + c\ln\frac{T}{T'} \quad \longleftarrow s$$

Note that $s = 0$ when $v = v'$, $T = T'$.

Then, since $\psi = u - Ts$, $u = \psi + Ts$

$$u = \left[-RT\ln\frac{v}{v'} - cT'\left[1 - \frac{T}{T'} + \frac{T}{T'}\ln\frac{T}{T'}\right]\right] + T\left[R\ln\frac{v}{v'} + c\ln\frac{T}{T'}\right]$$

$$= \cancel{-RT\ln\frac{v}{v'}} - cT' + cT - \cancel{cT\ln\frac{T}{T'}} + \cancel{RT\ln\frac{v}{v'}} + \cancel{cT\ln\frac{T}{T'}}$$

$$u = c[T - T'] \quad \longleftarrow u$$

So, $u = 0$ when $T = T'$.

With $h = u + pv$,

$$h = c[T - T'] + RT \quad \longleftarrow h$$

Further, $c_v = \partial u/\partial T)_v$, so

$$c_v = c \quad \longleftarrow c_v$$

and $c_p = \partial h/\partial T)_p$, so

① $$c_p = c + R \quad \longleftarrow c_p$$

---

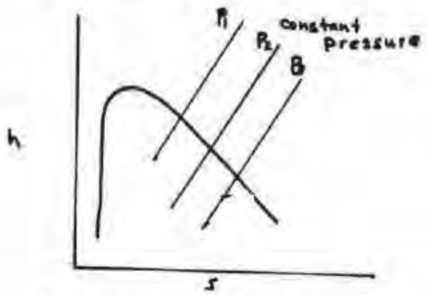
1. This substance adheres to the ideal gas model with constant c_v.

PROBLEM 11.30

KNOWN: The Mollier diagram gives a fundamental thermodynamic function $h = h(s, p)$.

FIND: Show that at any state fixed by s and p, the properties T, v, u, ψ, and g can be evaluated from the diagram.

SCHEMATIC & GIVEN DATA



ANALYSIS: From Eqs. 11.26 and 11.27 T and v can be found by differentiation (which can be performed graphically)

$$T = \left(\frac{\partial h}{\partial s}\right)_p \quad , \quad v = \left(\frac{\partial h}{\partial p}\right)_s$$

Then, since $h = u + pv$

$$u = h - pv = h(s,p) - p\left(\frac{\partial h}{\partial p}\right)_s$$

And with $\psi = u - Ts$

$$\psi = h(s,p) - p\left(\frac{\partial h}{\partial s}\right)_p - s\left(\frac{\partial h}{\partial s}\right)_p$$

Finally, using $g = h - Ts$

$$g = h(s,p) - s\left(\frac{\partial h}{\partial s}\right)_p$$

PROBLEM 11.31

KNOWN:
$$c_p = -T\left(\frac{\partial^2 g}{\partial T^2}\right)_p$$

FIND: Derive the above expression.

ANALYSIS: Since T and p are regarded as independent, the following functions can be considered: $u(T,p)$, $h(T,p)$, $s(T,p)$, etc. As $c_p = \partial h/\partial T)_p$, form the differential of $h(T,p)$

$$dh = \left(\frac{\partial h}{\partial T}\right)_p dT + \left(\frac{\partial h}{\partial p}\right)_T dp$$
$$= c_p\,dT + \left(\frac{\partial h}{\partial p}\right)_T dp \qquad (1)$$

Also, forming the differential of $s(T,p)$

$$ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp \qquad (2)$$

Substituting Eqs. (1) and (2) into Eq 11.19 : $dh = T ds + v dp$

$$\left[c_p\,dT + \left(\frac{\partial h}{\partial p}\right)_T dp\right] = T\left[\left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp\right] + v\,dp$$

Collecting terms

$$\left[c_p - T\left(\frac{\partial s}{\partial T}\right)_p\right] dT = \left[T\left(\frac{\partial s}{\partial p}\right)_T + v - \left(\frac{\partial h}{\partial p}\right)_T\right] dp$$

As T and p are independent, fix p while varying T: $dp = 0$ and $dT \neq 0$; then

$$\left[c_p - T\left(\frac{\partial s}{\partial T}\right)_p\right] dT = 0 \implies \left[c_p - T\left(\frac{\partial s}{\partial T}\right)_p\right] \equiv 0$$

or

$$c_p = T\left(\frac{\partial s}{\partial T}\right)_p \qquad (3)$$

Next, introduce Eq. 11.31 : $-s = (\partial g/\partial T)_p$ to obtain on differentiation

$$-\left(\frac{\partial s}{\partial T}\right)_p = \left(\frac{\partial^2 g}{\partial T^2}\right)_p \qquad (4)$$

Finally, combining Eqs. (3) and (4)

$$c_p = -T\left(\frac{\partial^2 g}{\partial T^2}\right)_p$$

which is the result to be obtained.

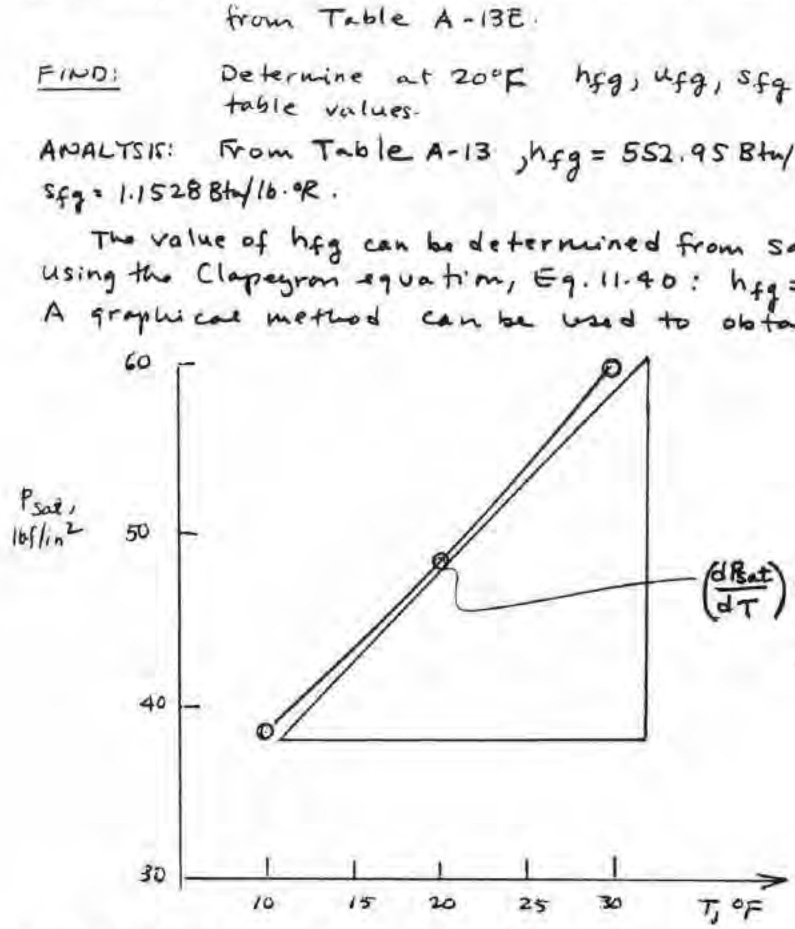
PROBLEM 11.32

KNOWN: p-v-T data for saturated ammonia are available from Table A-13E.

FIND: Determine at 20°F h_{fg}, u_{fg}, s_{fg}, and compare with table values.

ANALYSIS: From Table A-13, $h_{fg} = 552.95$ Btu/lb, $u_{fg} = 500.46$ Btu/lb, $s_{fg} = 1.1528$ Btu/lb·°R.

The value of h_{fg} can be determined from saturated p-v-T data using the Clapeyron equation, Eq. 11.40: $h_{fg} = T\, v_{fg} \left(\frac{dp}{dT}\right)_{sat}$. (1)
A graphical method can be used to obtain $(dp/dT)_{sat}$, as follows:



$$\left(\frac{dP_{sat}}{dT}\right) \approx \frac{(60-38)|144|}{(32-11)} \left(\frac{\text{lbf/ft}^2}{°R}\right) = 150.9 \left(\frac{\text{lbf/ft}^2}{°R}\right)$$

Inserting values into Eq. (1)

$$h_{fg} = \frac{(479.67°R)\left(5.878\ \frac{ft^3}{lb}\right)\left(150.9\ \frac{lbf}{°R\, ft^2}\right)}{|778.17\ ft \cdot lbf/Btu|} = 546.7 \text{ Btu/lb} \longleftarrow h_{fg}$$

Then, with Eq. 11.38

$$s_{fg} = \frac{h_{fg}}{T} = \frac{546.7 \text{ Btu/lb}}{479.67°R} = 1.1397 \frac{Btu}{lb \cdot °R} \longleftarrow s_{fg}$$

Finally, with $h = u + pv$

$$u_{fg} = h_{fg} - P_{sat}\, v_{fg} = 546.7 - \frac{(48.224)|144|(5.878)}{|778.17\ ft \cdot lbf/Btu|} = 499.2 \frac{Btu}{lb} \longleftarrow u_{fg}$$

Each of these is about 1% less than the corresponding table value. The values obtained using the graphical approach are sensitive to the accuracy of the slope $(dP/dT)_{sat}$ determined graphically, as above.
Alternatively, an IT solution like that of Example 11.4 can be employed:

PROBLEM 11.32 (Cont'd.) - Page 2

IT Code

```
T = 20 // °F
dT = 0.01
T1 = T - dT
T2 = T + dT

p2 = Psat_T("Ammonia", T2)
p1 = Psat_T("Ammonia", T1)
dpdT_sat = ((p2 - p1) / (T2 - T1)) * 144
p = Psat_T("Ammonia", T)
vg = vsat_Px("Ammonia", p, 1)
vf = vsat_Px("Ammonia", p, 0)

hfg = ((T + 459.67) * (vg - vf) * dpdT_sat) / 778.17
sg = ssat_Px("Ammonia", p, 1)
sf = ssat_Px("Ammonia", p, 0)
sfg = hfg / (T + 459.67)
ufg = hfg - p * (vg - vf) * 144 / 778.67
```

IT Results

$(\partial p/\partial T)_{sat} = 152.5$ lbf/ft²·°R
$h_{fg} = 552.6$ Btu/lb
$s_{fg} = 1.152$ Btu/lb·°R
$u_{fg} = 500.2$ Btu/lb

PROBLEM 11.33

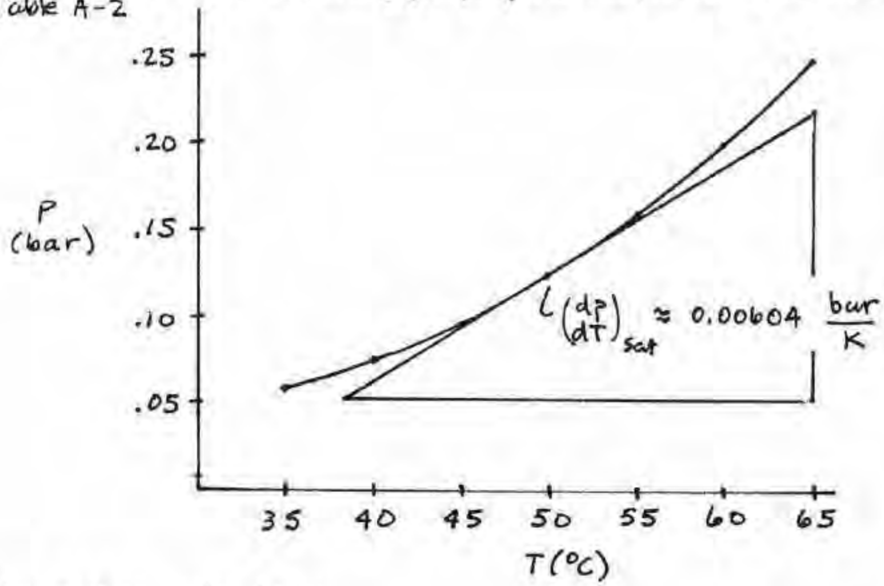
KNOWN: p-v-T data for water are available from the steam tables.

FIND: Determine at 50°C, (a) $h_g - h_f$, (b) $u_g - u_f$, and (c) $s_g - s_f$, and compare with table values.

ANALYSIS: The value of $(h_g - h_f)$ can be calculated using saturation data and the Clapeyron equation, Eq 11.40

$$h_g - h_f = T(v_g - v_f)\left(\frac{dp}{dT}\right)_{sat} \qquad (1)$$

To obtain the value for $(dp/dT)_{sat}$, prepare the plot shown below using data from Table A-2



Inserting values into Eq.(1)

$$h_g - h_f = (50+273.15)K(12.032 - 1.0121\times10^{-3})\frac{m^3}{kg}\left(0.00604\frac{bar}{K}\right)\left|\frac{10^5 N/m^2}{1 bar}\right|\left|\frac{1 kJ}{10^3 N\cdot m}\right|$$
$$= 2348 \text{ kJ/kg}$$

With Eq. 11.38

$$s_g - s_f = \frac{h_g - h_f}{T} = \frac{2348}{323.15} = 7.266 \text{ kJ/kg}\cdot K$$

Further, using $u = h - pv$

$$u_g - u_f = (h_g - h_f) - p(v_g - v_f)$$
$$= \left(2348\frac{kJ}{kg}\right) - (0.1235 \text{ bar})(12.032 - 1.021\times10^{-3})\frac{m^3}{kg}\left|\frac{10^5 N/m^2}{1 bar}\right|\left|\frac{1 kJ}{10^3 N\cdot m}\right|$$
$$= 2199.4 \text{ kJ/kg}$$

The respective table values are $h_g - h_f = 2382.7$ kJ/kg, $s_g - s_f = 7.3725$ kJ/kg·K, and $u_g - u_f = 2234.2$ kJ/kg. The percent difference between calculated and table values is about 1.5% for all quantities.

An altenative solution using IT follows:

<u>*IT* Code</u>

```
T = 50 // °C
dT = 0.001
T1 = T - dT
T2 = T + dT

p1 = Psat_T("Water/Steam", T1)
p2 = Psat_T("Water/Steam", T2)
dpdT_sat = ((p2 - p1) / (T2 - T1)) // bar/K
p = Psat_T("Water/Steam", T)
vg = vsat_Px("Water/Steam", p, 1)
vf = vsat_Px("Water/Steam", p, 0)

hfg = (T + 273.15) * (vg - vf) * dpdT_sat * 100
sg = ssat_Px("Water/Steam", p, 1)
sf = ssat_Px("Water/Steam", p, 0)
sfg = sg - sf
ufg = hfg - p * (vg - vf) * 100
```

<u>*IT* Results</u>

$(\partial p/\partial T)_{sat}$ = 0.006127 bar
h_{fg} = 2382 kJ/kg
s_{fg} = 7.372 kJ/kg·K
u_{fg} = 2234 kJ/kg

The IT results compare very favorably with the table data.

PROBLEM 11.34

KNOWN: h_{fg}, v_{fg}, and P_{sat} at 10°F are known from the R134a tables.

FIND: Estimate P_{sat} at 20°F. Comment on the accuracy of this estimate.

ENGINEERING MODEL: (1) The ratio h_{fg}/v_{fg} does not change significantly with temperature over the interval from 10° to 20°F. (2) v_f can be ignored relative to v_g, and v_g can be evaluated using the ideal gas model.

ANALYSIS: Table A-10E at 20°F gives $P_{sat} = 33.137$ lbf/in².

Method 1: Assuming h_{fg}/v_{fg} is constant, Eq. 11.40 gives upon integration

$$(\Delta P)_{sat} = \frac{h_{fg}}{v_{fg}} \ln \frac{T_2}{T_1}$$

With data from Table A-10E at 10°F

$$(\Delta P)_{sat} = \left[\frac{88.53 \text{ Btu/lb}}{(1.7251 - 0.012)\text{ ft}^3/\text{lb}}\right]\left|\frac{778 \text{ ft}\cdot\text{lbf}}{1 \text{ Btu}}\right|\left|\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right| \ln\left(\frac{480}{470}\right)$$

$$= 5.878 \text{ lbf/in}^2$$

At 10°F, $P_{sat} = 26.651$ lbf/in². So

$$P_{sat}(20°F) \approx 26.651 + 5.878 = 32.529 \text{ lbf/in}^2$$

This value is about 1.8% less than the table value.

Method 2: With assumption 2, Eq. 11.40 can be converted to the Clausius-Clapeyron equation; Eq. 11.42, which upon integration gives

$$\ln \frac{P_2}{P_1} = -\frac{h_{fg}}{R}\left[\frac{1}{T_2} - \frac{1}{T_1}\right]$$

Thus

$$\ln \frac{P_2}{P_1} = -\frac{(88.53 \text{ Btu/lb})}{\left(\frac{1.986}{120.92} \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)}\left[\frac{1}{480} - \frac{1}{470}\right]\left(\frac{1}{°\text{R}}\right)$$

$$\Rightarrow \quad \frac{P_2}{P_1} = 1.27 \Rightarrow P_2 = 33.85 \text{ lbf/in}^2$$

This is about 2% above the table value.

Discussion. Even though the assumptions are only satisfied approximately, satisfactory results are obtained with both methods. Excellent agreement with the table value is achieved with the following IT approach based on integrating Eq. 11.40 without making the above assumptions:

IT Code

```
hg = hsat_Px("R134A", p, 1)
hf = hsat_Px("R134A", p, 0)
vf = vsat_Px("R134A", p, 0)
vg = vsat_Px("R134A", p, 1)
p = Psat_T("R134A", T)

// Clapeyron Eq. 11.40
dpdT_sat = ((hg - hf) / ((vg - vf) * (T + 459.67))) * (778.17 / 144)
psat = Integral(dpdT_sat,T) + 26.651
```

IT Result (Sweep T from 10 to 20°F in steps of 0.1.)

$p_{sat}(20°F) = 33.14$ lbf/in.²

PROBLEM 11.35

KNOWN: h_{fg}, v_{fg}, and p_{sat} at 26°C are known from the ammonia tables.

FIND: Estimate p_{sat} at 30°C. Comment on the accuracy of this estimate.

ENGINEERING MODEL: (1) The ratio h_{fg}/v_{fg} does not change significantly with temperature over the interval from 26°C to 30°C. (2) v_f can be ignored relative to v_g, and v_g can be evaluated using the ideal gas model.

ANALYSIS: Table A-13 gives at 26°C,

$$p_{sat} = 10.3602 \text{ bar}, \quad v_{fg} = 0.1229 \frac{m^3}{kg}, \quad h_{fg} = 1160.7 \text{ kJ/kg}$$

Method #1. With assumption (1), Eq. 11.40 gives on integration

$$(\Delta p)_{sat} = \frac{h_{fg}}{v_{fg}} \ln \frac{T_2}{T_1} = \left(\frac{1160.7 \text{ kJ/kg}}{0.1229 \text{ m}^3\text{/kg}}\right) \left|\frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}}\right| \left|\frac{1 \text{ bar}}{10^5 \text{ N/m}^2}\right| \ln\left(\frac{303}{299}\right)$$

$$= 1.255 \text{ bar}$$

$$\Rightarrow \quad p_{sat}(30°C) \simeq 10.3602 + 1.255 = 11.615 \text{ bar}.$$

Comparing with the table value: 11.6865 bar, this is about 0.6% low.

Method #2. With assumption (2), Eq. 11.40 can be converted to the Clausius-Clapeyron equation: Eq. 11.42, which on integration gives

$$\ln \frac{p_2}{p_1} = -\frac{h_{fg}}{R}\left[\frac{1}{T_2} - \frac{1}{T_1}\right] = -\left(\frac{1160.7 \text{ kJ/kg}}{\frac{8314}{17.04} \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}}\right)\left(\frac{1}{303\text{K}} - \frac{1}{299\text{K}}\right)\left|\frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}}\right|$$

$$\Rightarrow \frac{p_2}{p_1} = 1.11 \Rightarrow p_2 = 11.5 \text{ bar}$$

This is about 1.6% below the table value.

Discussion. Even though the assumptions are only satisfied approximately, satisfactory results are obtained with both methods. Excellent agreement with the table value is achieved with the following IT approach based on integrating Eq. 11.40 without making the above assumptions.

IT Code

```
hg = hsat_Px("Ammonia", p, 1)
hf = hsat_Px("Ammonia", p, 0)
vg = vsat_Px("Ammonia", p, 1)
vf = vsat_Px("Ammonia", p, 0)
p = Psat_T("Ammonia", T)

// Clapeyron Eq. 11.40
dpdT_sat = ((hg - hf) / ((vg - vf) * (T + 273.15))) / 100
psat = Integral(dpdT_sat,T) + 10.3602
```

IT Result (Sweep T from 26 to 30 in steps of 0.1.)

$p_{sat}(30°C) = 11.69$ bar

PROBLEM 11.36

KNOWN: Triple point data for water are available from Table A-6E.

FIND: Estimate P_{sat} at $-40°F$ and compare with the table value, which is 0.0019 lbf/in² from Table A-6E.

ENGINEERING MODEL: (1) The value of h_{ig} is constant independent of temperature over the interval from the triple point temperature to $-40°F$, (2) v_i can be ignored relative to v_g and v_g can be evaluated using the ideal gas model.

ANALYSIS: With the assumptions listed, Eq. 11.41 can be converted to give

$$\left(\frac{d \ln P}{dT}\right)_{sat} = \frac{h_{ig}}{RT^2}$$

which gives upon integration

$$\ln \frac{P_2}{P_1} = \frac{h_{ig}}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

With data from Table A-6E: $h_{ig} = 1218.7$ Btu/lb, $P_1 = 0.0887$ lbf/in²

$$\ln \frac{P_2}{P_1} = \frac{(1218.7 \text{ Btu/lb})}{\left(\frac{1.986}{18.02} \frac{\text{Btu}}{\text{lb} \cdot °\text{R}}\right)}\left[\frac{1}{492°\text{R}} - \frac{1}{420°\text{R}}\right]$$

$$\Rightarrow \quad \frac{P_2}{P_1} = 0.02122$$

$$\Rightarrow \quad P_2 = 0.00188 \text{ lbf/in}^2$$

Discussion: The calculated value agrees closely with the table value. This is an expected outcome, for by reference to Table A-6E data the two assumptions are closely adhered to in this case.

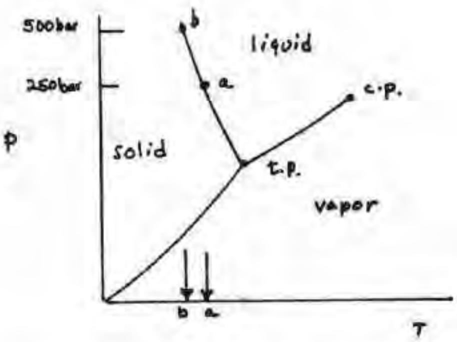
PROBLEM 11.37

KNOWN: For water at 0°C

$$v_i = 1.0911 \times 10^{-3}\ m^3/kg$$
$$v_f = 1.0002 \times 10^{-3}\ m^3/kg$$
$$h_{if} = 333.4\ kJ/kg$$

FIND: Estimate the melting temperature of ice at (a) 250 bar, (b) 500 bar.

SCHEMATIC & GIVEN DATA:



Not to scale

ENGINEERING MODEL: h_{if} and v_{if} do not vary significantly with temperature.

ANALYSIS: With the assumptions listed, Eq 11.41 gives upon integration

$$\Delta p = \left(\frac{h_f - h_i}{v_f - v_i}\right) \ln \frac{T_2}{T_1}$$

$$= \left[\frac{333.4\ kJ/kg}{\left(\frac{1.0002 - 1.0911}{10^3}\right)\frac{m^3}{kg}}\right]\left|\frac{10^3\ N\cdot m}{kJ}\right|\left|\frac{bar}{10^5\ N/m^2}\right| \ln \frac{T_2}{T_1}$$

or

$$\Delta p = -(36{,}677.7\ bar) \ln \frac{T_2}{T_1}$$

At 0°C, the pressure is negligible relative to p_2: $\Delta p \approx p_2$.

(a) $p_2 = 250$ bar

$$\ln \frac{T_2}{273.15} = -\frac{250}{36{,}677.7} \Rightarrow T_2 = 271.29\ K\ (-1.86\ °C) \quad \text{(a)}$$

(b) $p_2 = 500$ bar

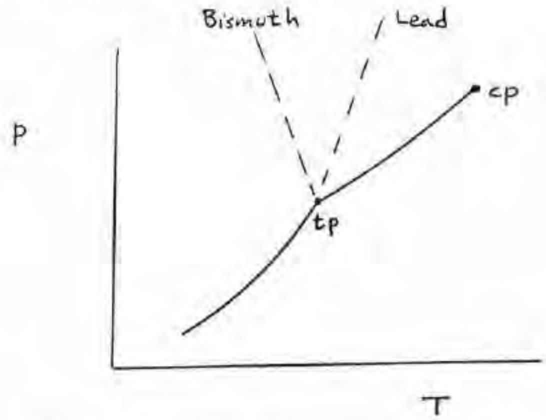
$$\ln \frac{T_2}{273.15} = \frac{-500}{36{,}677.7} \Rightarrow T_2 = 269.45\ K\ (-3.7\ °C) \quad \text{(b)}$$

PROBLEM 11.38

KNOWN: Bismuth and lead are the substances under consideration.

FIND: Confirm that the line representing the two-phase solid-liquid region on the phase diagram slopes to the left for bismuth and to the right for lead.

SCHEMATIC & GIVEN DATA:



ANALYSIS: The key expression is Eq 11.41

$$\left(\frac{dP}{dT}\right)_{sat} = \frac{h'' - h'}{T(v'' - v')}$$

where ' denotes solid and " denotes liquid. As $(h''-h')$ would be positive, the slope is determined by the difference $(v''-v')$.

The CRC Handbook of Chemistry and Physics, 37th Ed, 1955-1956, gives a table of the change in volume due to fusion for selected substances, p. 2138. For the substances under consideration

Bismuth: change in volume = $-0.0034\ cm^3/g$
Lead: " " " = $+0.0034\ cm^3/g$

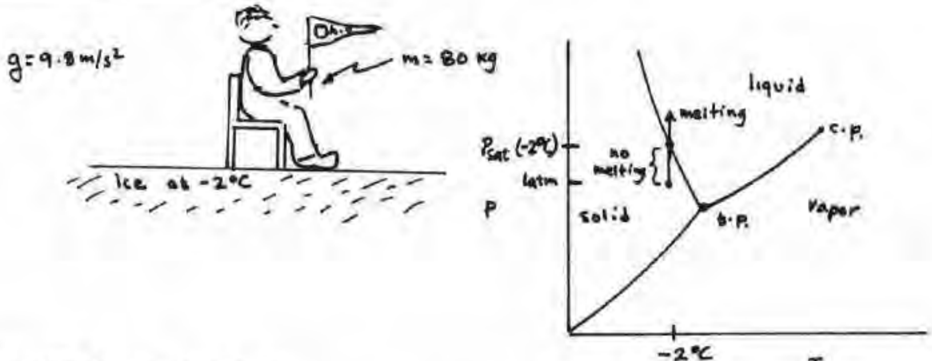
Accordingly, $(dP/dT)_{sat} < 0$ for bismuth and $(dP/dT)_{sat} > 0$ for lead.

PROBLEM 11.39

KNOWN: An occupied chair of total mass equal to 80 kg is at rest on an ice rink. The ice temperature is $-2°C$.

FIND: Determine the minimum total area the tips of the chair legs can have before the ice in contact with the legs would melt.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

(1) The ice temperature remains at $-2°C$. (2) The local acceleration of gravity is $9.8\ m/s^2$. (3) At $0°C$, $h_{if} = 333.4\ kJ/kg$, $v_i = 1.0911 \times 10^{-3}\ m^3/kg$, $v_f = 1.0002 \times 10^{-3}\ m^3/kg$, h_{if} and v_{if} remain constant with temperature.

ANALYSIS: As shown by the phase diagram, melting would take place if the pressure exerted on the ice (at $-2°C$) would exceed $p_{sat}(-2°C)$.

The pressure $p_{sat}(-2°C)$ can be estimated using Eq. 11.41, together with the assumption that h_{if} and v_{if} remain constant with temperature, for then the equation becomes on integration

$$\Delta p = \left(\frac{h_f - h_i}{v_f - v_i}\right) \ln \frac{T_2}{T_1}$$

Inserting values for h_{if}, v_i, v_f and taking $T_1 = 273.15\,K$ ($0°C$), $T_2 = 271.15\,K$ ($-2°C$)

$$\Delta p = \left[\frac{\left(333.4\ \frac{kJ}{kg}\right)\left|\frac{1000\ N\cdot m}{kJ}\right|}{\left(\frac{1.0002 - 1.0911}{1000}\right)\left(\frac{m^3}{kg}\right)}\right]\left|\frac{bar}{10^5\ N/m^2}\right| \ln \frac{271.15}{273.15} = 269.54\ bar$$

Since $p_{sat}(0°C) = 0.61\ kPa$, $\quad p_{sat}(-2°C) = 269.55\ bar$

The pressure exerted on the ice by the chair/occupant and the atmosphere is

$$p = p_{atm} + \frac{mg}{A}$$

where A is the total area of the tips of the 4 chair legs. The pressure p can be no greater than 269.54 bar. Thus the minimum total area A is

$$269.55\ bar = 1.01325\ bar + \frac{(80\ kg)(9.8\ m/s^2)}{A\ (m^2)}\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|\left|\frac{bar}{10^5\ N/m^2}\right|$$

or

$$A_{min} = (2.92 \times 10^{-5}\ m^2)\left|\frac{100\ cm}{m}\right|^2$$

$$= 0.292\ cm^2 \longleftarrow A_{min}$$

PROBLEM 11.40

KNOWN: Over a certain temperature interval the saturation pressure-temperature curve of a substance is represented by

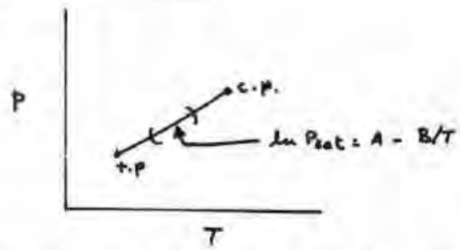
$$\ln P_{sat} = A - B/T$$

where A and B are constants.

FIND: (a) Obtain expressions for h_{fg} and s_{fg} in terms of p-v-T data and B.

(b) Using the result of (a), determine h_{fg} and s_{fg} for water vapor at 25°C and compare with steam table data.

SCHEMATIC & GIVEN DATA:



ANALYSIS: (a) The Clapeyron equation takes the form

$$\left(\frac{dp}{dT}\right)_{sat} = \frac{h_{fg}}{T\,v_{fg}} \qquad (1)$$

To obtain $(dp/dT)_{sat}$, differentiate the given expression for $\ln P_{sat}$

$$\frac{d}{dT}(\ln P_{sat}) = +\frac{B}{T^2}$$

$$\frac{1}{P_{sat}}\frac{dP_{sat}}{dT} = \frac{B}{T^2}$$

or

$$\left(\frac{dp}{dT}\right)_{sat} = \frac{P_{sat}(T)\,B}{T^2} \qquad (2)$$

Combining Eqs. (1) and (2)

$$h_{fg} = \frac{P_{sat}(T)\,v_{fg}(T)\,B}{T} \qquad (3) \longleftarrow h_{fg}$$

Then, with Eq. 11.38: $s_{fg} = h_{fg}/T$

$$s_{fg} = \frac{P_{sat}(T)\,v_{fg}(T)\,B}{T^2} \qquad (4) \longleftarrow s_{fg}$$

(b) Using standard computer routines, the values of A and B are found such that

$$\ln P_{sat} = \overbrace{28.83593}^{Ⓐ} - \frac{\overbrace{5299.62}^{Ⓑ}}{T} \qquad (P_{sat}: \text{Pa},\ T: \text{K}) \qquad (5)$$

Then, with Eq. (3), B from Eq. (5), and P_{sat}, v_{fg} from Table A-2 at 25°C

① $$h_{fg} = \frac{(0.03169\times10^5 \frac{N}{m^2})(43.359 \frac{m^3}{kg})(5299.62\,K)}{298.15\,K}\left|\frac{kJ}{10^3\,N\cdot m}\right| = 2442.4\ \frac{kJ}{kg} \quad (\text{table value: } 2442.3\ kJ/kg)$$

Similarly, Eq (4) gives

$$s_{fg} = \frac{(0.03169\times10^5)(43.359)(5299.62)}{(298.15)^2}\left|\frac{1}{10^3}\right| = 8.192\ \frac{kJ}{kg\cdot K} \quad (\text{table value: } 8.1906\ kJ/kg\cdot K)$$

---

1. In this case, Excellent agreement is realized with steam table data.

PROBLEM 11.41

KNOWN: Data for water from Table A-2 is available for use.

FIND: Determine the constants A and B to give the best fit in a least-squares sense in the interval from 20° to 30°C by the equation $\ln p_{sat} = A - B/T$. Using this equation determine dp_{sat}/dT at 25°C. Calculate h_{fg} at 25°C and compare with the steam table value.

ANALYSIS: Using standard computer routines, the values of A and B are found so that

$$\ln p_{sat} = -1.999728 - \frac{35.767963}{T} \quad (p_{sat}: \text{bar},\ T: {}^\circ\text{C}) \qquad (1)$$

or

$$\ln p_{sat} = 25.83593 - \frac{5299.62}{T} \quad (p_{sat}: \text{Pa},\ T: \text{K}) \qquad (2)$$

Using Eq. (2),

$$p_{sat} = \exp\left(25.83593 - \frac{5299.62}{T}\right)$$

Thus

$$\frac{dp_{sat}}{dT} = \frac{5299.62}{T^2}\exp\left(25.83593 - \frac{5299.62}{T}\right)$$

At 298.15 K, $dp_{sat}/dT = 188.88$ Pa/K.

Using Eq. 11.40 and v_{fg} from Table A-2 at 25°C

$$h_{fg} = T v_{fg}\left(\frac{dp}{dT}\right)_{sat}$$

$$= (298.15\,\text{K})\left(\frac{43360 - 1.0029}{10^3}\right)\left(\frac{\text{m}^3}{\text{kg}}\right)\left(\frac{188.88\ \text{N/m}^2}{\text{K}}\right)\left|\frac{\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right|$$

$$= 2441.7\ \text{kJ/kg}$$

Table A-2 gives at 25°C, 2442.3 kJ/kg. Accordingly the calculated and table values of h_{fg} agree extremely well.

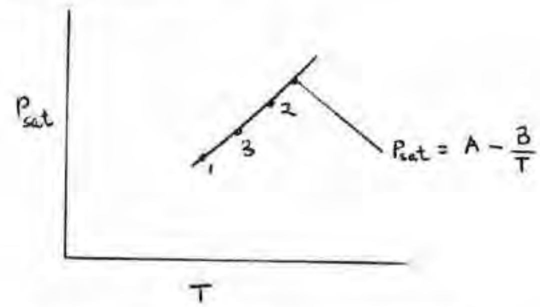
PROBLEM 11.42

KNOWN: The saturation pressure - temperature curve for two-phase liquid-vapor states is represented by $\ln P_{sat} = A - B/T$ where A, B are constants.

FIND: Show that any three points on the curve are related by

$$\frac{P_{sat,3}}{P_{sat,1}} = \left[\frac{P_{sat,2}}{P_{sat,1}}\right]^{\tau} \quad \text{where} \quad \tau = \frac{T_2[T_3 - T_1]}{T_3[T_2 - T_1]}$$

SCHEMATIC & GIVEN DATA:



ANALYSIS: Using the given expression

$$\ln(P_{sat,1}) = A - \frac{B}{T_1} \Rightarrow A = \ln(P_{sat,1}) + \frac{B}{T_1}$$

$$\ln(P_{sat,2}) = A - \frac{B}{T_2} \Rightarrow \ln(P_{sat,2}) = \ln(P_{sat,1}) + \frac{B}{T_1} - \frac{B}{T_2}$$

$$\Rightarrow \ln\left(\frac{P_{sat,2}}{P_{sat,1}}\right) = B\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\ln(P_{sat,3}) = A - \frac{B}{T_3} \Rightarrow \ln\left(\frac{P_{sat,3}}{P_{sat,1}}\right) = B\left[\frac{1}{T_1} - \frac{1}{T_3}\right]$$

Combining,

$$\frac{\ln\left(\frac{P_{sat,2}}{P_{sat,1}}\right)}{\ln\left(\frac{P_{sat,3}}{P_{sat,1}}\right)} = \frac{\frac{1}{T_1} - \frac{1}{T_2}}{\frac{1}{T_1} - \frac{1}{T_3}} = \frac{\frac{T_2 - T_1}{T_1 T_2}}{\frac{T_3 - T_1}{T_1 T_3}} = \frac{T_3}{T_2} \cdot \frac{T_2 - T_1}{T_3 - T_1} = \frac{1}{\tau}$$

where $\tau \equiv \frac{T_2}{T_3} \cdot \frac{T_3 - T_1}{T_2 - T_1}$. Then

$$\ln\left(\frac{P_{sat,3}}{P_{sat,1}}\right) = \tau \ln\left(\frac{P_{sat,2}}{P_{sat,1}}\right)$$

$$\Rightarrow \quad \frac{P_{sat,3}}{P_{sat,1}} = \left[\frac{P_{sat,2}}{P_{sat,1}}\right]^{\tau}$$

PROBLEM 11.43

KNOWN: The result of Problem 11.42 is applicable.

FIND: Using saturation pressure temperature data from Table A-2
(a) evaluate P_{sat} at 30°C and compare with the tabulated value.
(b) evaluate T_{sat} at 0.006 MPa and compare with the tabulated value.

ANALYSIS: The result of Problem 11.42 is

$$\frac{P_{sat,3}}{P_{sat,1}} = \left[\frac{P_{sat,2}}{P_{sat,1}}\right]^{\tau} \quad \text{where} \quad \tau = \frac{T_2}{T_3}\left[\frac{T_3 - T_1}{T_2 - T_1}\right]$$

(a) $T_1 = 20°C$, $T_2 = 30°C$, $T_3 = 40°C$. From Table A-2,

$$P_{sat,1} = 0.02339 \text{ bar}$$
$$P_{sat,3} = 0.07384 \text{ bar}$$

$$\tau = \frac{T_2}{T_3}\left(\frac{T_3 - T_1}{T_2 - T_1}\right) = \frac{303.15}{313.15}\left(\frac{20}{10}\right) = 1.9361$$

Accordingly

$$\ln\frac{P_{sat,2}}{P_{sat,1}} = \frac{1}{\tau}\ln\frac{P_{sat,3}}{P_{sat,1}} = \frac{1}{1.9361}\ln\left(\frac{0.07384}{0.02339}\right)$$

$$\Rightarrow \quad \frac{P_{sat,2}}{P_{sat,1}} = 1.8108 \Rightarrow P_{sat,2} = 0.04235 \text{ bar}$$

Table A-2 gives $P_{sat,2} = 0.04246$ bar. Accordingly the percent difference is

$$\% = \left(\frac{0.04235 - 0.04246}{0.04246}\right)(100) = -0.26$$

(b) $T_1 = 20°C$, $P_2 = 0.006$ MPa, $T_3 = 40°C$. $P_{sat,1}$ and $P_{sat,3}$ are given in part (a). Accordingly

$$\tau = \frac{\ln\left(\frac{P_{sat,3}}{P_{sat,1}}\right)}{\ln\left(\frac{P_{sat,2}}{P_{sat,1}}\right)} = \frac{\ln\left(\frac{0.07384}{0.02339}\right)}{\ln\left(\frac{0.06}{0.02339}\right)} = 1.22033$$

And

$$\tau = \frac{T_2\,[20]}{313.15\,[T_2 - 293.15]}$$

So

$$20\,T_2 = 1.22033\left[313.15\,[T_2 - 293.15]\right]$$

$$\Rightarrow T_2 = 309.34\text{ K} \; (36.19°C)$$

Table A-3 gives $T = 36.16°C$. The percent difference is

$$\% = \left(\frac{36.19 - 36.16}{36.16}\right)(100) = 0.08$$

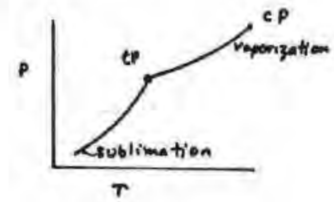
PROBLEM 11.44

KNOWN: Four exercises dealing with slopes are described.

FIND: (a) Using steam table data, evaluate the ratio of the slope of the vaporization line to the slope of the sublimation line. (b) On a T-s diagram, show that the slope of a constant specific volume line is greater than the slope of a constant pressure line through the same superheated vapor state. (c) For an h-s diagram, obtain an expression for the slope of a constant pressure line in terms of p-v-T data only. (d) For a p-h diagram, obtain an expression for the slope of an isentropic line in terms of p-v-T data only.

ANALYSIS: (a) Using the Clapeyron equation, Eq. 11.40

$$\left(\frac{dp}{dT}\right)_{sat} = \frac{1}{T}\frac{\Delta h}{\Delta v}$$



Accordingly at the triple point

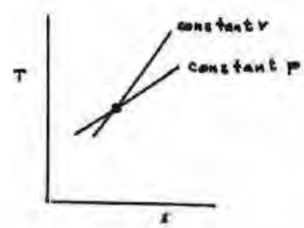
$$\frac{(dp/dT)_{sat}]_{subl.}}{(dp/dT)_{sat}]_{vap}} = \frac{[h_g - h_i]/[v_g - v_i]}{[h_g - h_f]/[v_g - v_f]}$$

With data from Tables A-2 and A-6 at 0.01 °C

$$\frac{(dp/dT)_{sat}]_{subl}}{(dp/dT)_{sat}]_{vap}} = \frac{[2834.8]/(206100 - 1.0908)}{[2501.3]/(206136 - 1.0002)} = 1.134 \longleftarrow (a)$$

(b) With Eqs. 11.46 and 11.55

$$\left(\frac{\partial s}{\partial T}\right)_v = \frac{c_v}{T}\ , \quad \left(\frac{\partial s}{\partial T}\right)_p = \frac{c_p}{T}$$



Or, using Eq. 11.15

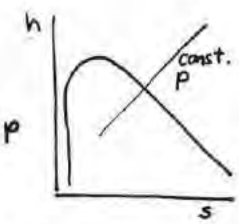
$$\left(\frac{\partial T}{\partial s}\right)_v = \frac{T}{c_v}\ , \quad \left(\frac{\partial T}{\partial s}\right)_p = \frac{T}{c_p}$$

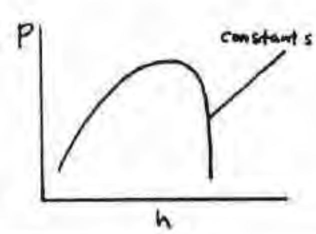
Forming a ratio

$$\frac{(\partial T/\partial s)_v}{(\partial T/\partial s)_p} = \frac{T/c_v}{T/c_p} = \frac{c_p}{c_v} = k > 1 \longleftarrow (b)$$

(c) and (d) Eq. 11.19 reads

$$dh = T ds + v dp$$





The function h(s,p) gives

$$dh = \left(\frac{\partial h}{\partial s}\right)_p ds + \left(\frac{\partial h}{\partial p}\right)_s dp$$

By comparison

(11.26): $$\left(\frac{\partial h}{\partial s}\right)_p = T \longleftarrow (c)$$

(11.27): $$\left(\frac{\partial h}{\partial p}\right)_s = v$$

With Eq. 11.15, the second of these gives

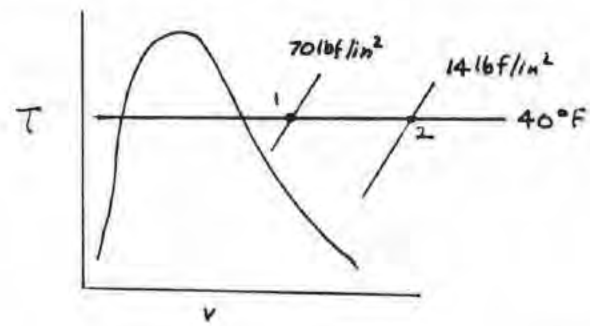
$$\left(\frac{\partial p}{\partial h}\right)_s = \frac{1}{v} \longleftarrow (d)$$

PROBLEM 11.45

KNOWN: NH_3 undergoes a process from 70 lbf/in², 40°F to 14 lbf/in², 40°F.

FIND: Using p-v-T data, evaluate (h_2-h_1) and (s_2-s_1), and compare with table values

SCHEMATIC & GIVEN DATA:



ANALYSIS: Since the states under consideration are in the superheated vapor region, temperature and pressure can be selected as the independent properties that fix the state. Then, Eqs. 11.59 and 11.60 give, respectively

① $$s(T, p_2) - s(T, p_1) = \int_{p_1}^{p_2} \left(\frac{\partial v}{\partial T}\right)_p (T, p)\, dp \qquad (1)$$

$$h(T, p_2) - h(T, p_1) = \int_{p_1}^{p_2} \left[v(T,p) - T\left(\frac{\partial v}{\partial T}\right)_p (T,p) \right] dp \qquad (2)$$

Consequently, the analysis reduces to the evaluation of $(\partial v/\partial T)_p$ as a function of pressure at 40°F, and the subsequent use of these data to obtain the required integrals. A numerical procedure using data obtained from the source listed for Tables A-13 is suggested. The values of Δh and Δs obtained directly from table data are, respectively

$$h_2 - h_1 = 14.46 \text{ Btu/lb}$$
$$s_2 - s_1 = 0.2103 \text{ Btu/lb}\cdot°R$$

---

1. Since the integrations are at fixed temperature, the terms of Eqs. 11.59 and 11.60 involving the specific heat c_p drop out of consideration, thereby allowing the required property changes to be found using only p-v-T data. These integrations require appropriate software.

PROBLEM 11.46

KNOWN: One kmol of Ar initially occupies one side of a container divided by a partition. The other side is evacuated. The partition is removed and the Ar expands to fill the entire volume.

FIND: Find the final temperature of the argon using (1) the van der Waals equation of state, (2) the ideal gas model.

SCHEMATIC & GIVEN DATA:

ENGINEERING MODEL: (1) The system is shown in the figure. (2) For the system, $Q = W = 0$, kinetic and potential energy effects are absent, and the partition can be ignored. (3) The argon is modeled by the van der Waals equation of state and then by the ideal gas model.

ANALYSIS: Using indicated assumptions, the energy balance reduces to give $\Delta U = 0$, or since mass is fixed $\Delta u = 0$. Invoking Eq. 11.51

$$\Delta u = \int_{①}^{②} c_v\, dT + \int_{①}^{②} \left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right] dv = 0 \qquad (1)$$

• van der Waals Equation. Since $p = \frac{RT}{v-b} - \frac{a}{v^2}$

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v-b}, \quad \left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right] = \frac{RT}{v-b} - \left[\frac{RT}{v-b} - \frac{a}{v^2}\right] = \frac{a}{v^2}$$

To investigate c_v, the test for exactness applied to Eq 11.48 gives

$$\left.\frac{\partial(c_v/T)}{\partial v}\right)_T = \frac{\partial}{\partial T}\left(\frac{\partial p}{\partial T}\right)_v\Big)_v \Rightarrow \left.\frac{\partial(c_v/T)}{\partial v}\right)_T = \frac{\partial}{\partial T}\left(\frac{R}{v-b}\right) = 0 \Rightarrow \left.\frac{\partial c_v}{\partial v}\right)_T \equiv 0.$$ This implies

c_v is independent of v and can depend only on T. Accordingly, it can be concluded that $c_v = 3/2\, R$, which is the limiting value for a monatomic gas such as Ar. Inserting results into Eq (1), and integrating

$$\frac{3}{2} R\,[T_2 - T_1] - a\left[\frac{1}{v_2} - \frac{1}{v_1}\right] = 0$$

$$\Rightarrow \quad T_2 - T_1 = \frac{2a}{3R}\left[\frac{1}{v_2} - \frac{1}{v_1}\right]$$

$$= \frac{2\left(\frac{27}{64}\frac{\bar{R}^2 T_c^2}{p_c}\right)}{3\bar{R}}\left[\frac{1}{\bar{v}_2} - \frac{1}{\bar{v}_1}\right] = \frac{9}{32}\left[\frac{\bar{R}T_c^2}{p_c}\right]\left[\frac{1}{\bar{v}_2} - \frac{1}{\bar{v}_1}\right]$$

With $\bar{v}_1 = 0.2\ m^3/kmol$, $\bar{v}_2 = 0.4\ m^3/kmol$, and data from Table A-1

$$T_2 - T_1 = \frac{9}{32}\left[8314\frac{N\cdot m}{kmol\cdot K}\right]\left[151\,K\right]^2\left[\frac{1}{48.6\times10^5\ N/m^2}\right]\left[\frac{1}{0.4} - \frac{1}{0.2}\right]\frac{kmol}{m^3}$$

$$= -27.43\ K \Rightarrow T_2 = 272.6\ K$$

• ideal gas model. Since $p = RT/v$, $(\partial p/\partial T)_v = R/v$ and $[T(\partial p/\partial T)_v - p] \equiv 0$. Thus, with $c_v = 5/2\, R$, Eq. (1) becomes

$$\frac{3}{2} R[T_2 - T_1] = 0 \quad \Rightarrow \quad T_2 = T_1 = 300\,K$$

PROBLEM 11.47

KNOWN: A gas obeys the equation of state $p(v-b) = RT$

FIND: Obtain the relationship between c_p and c_v for this gas.

ANALYSIS:

Method 1. Using Eq. 11.47

$$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - p \tag{1}$$

The given equation of state can be rearranged to read

$$p = \frac{RT}{v-b}$$

Then

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v-b} \tag{2}$$

Combining Eqs. (1) and (2)

$$\left(\frac{\partial u}{\partial v}\right)_T = \frac{RT}{v-b} - p \equiv 0 \Rightarrow u = u(T) \quad (u \text{ depends on } T \text{ only})$$

Since $h = u + pv$, the above equation indicates that

$$h(T,p) = u(T) + pv$$

Then, since $c_p = (\partial h/\partial T)_p$ and $c_v = (\partial u/\partial T)_v = du/dT$

$$c_p = \frac{du}{dT} + \left(\frac{\partial (pv)}{\partial T}\right)_p = c_v + p\left(\frac{\partial v}{\partial T}\right)_p \tag{3}$$

The equation of state can be expressed as

$$v = \frac{RT}{p} + b \Rightarrow \left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p}$$

Inserting this into Eq. (3)

1 $$c_p = c_v + p\left(\frac{R}{p}\right) = c_v + R$$

Method 2. Using Eq. 11.68

$$c_p - c_v = -T\left(\frac{\partial v}{\partial T}\right)_p^2 \left(\frac{\partial p}{\partial v}\right)_T$$

From above, $(\partial v/\partial T)_p = R/p$, and using $p = RT/(v-b)$

$$\left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT}{(v-b)^2}$$

Collecting results

$$c_p - c_v = -T\left(\frac{R}{p}\right)^2\left(-\frac{RT}{(v-b)^2}\right)$$

$$= -T\left(\frac{R}{RT/(v-b)}\right)^2\left(\frac{-RT}{(v-b)^2}\right) = R$$

or

$$c_p = c_v + R$$

---

1. Method 1 also brings out the conclusion that the specific internal energy, u, can only vary with temperature. Thus, c_v (and c_p) can only vary with temperature as well.

PROBLEM 11.48

KNOWN: The p-v-T relation for a certain gas is

$$v = \frac{RT}{p} + B - \frac{A}{RT}$$

where A and B are constants.

FIND: Obtain expressions for $[h(p_2,T) - h(p_1,T)]$, $[u(p_2,T) - u(p_1,T)]$, and $[s(p_2,T) - s(p_1,T)]$.

ANALYSIS: As T and p are the independent properties that fix the state and an equation of state explicit in v is known, Eqs. 11.59 and 11.60 are convenient to use here. Each of these requires $(\partial v/\partial T)_p$. Thus, with the given equation of state

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} + \frac{A}{RT^2}$$

Then, Eq. 11.59 reads

1 $$s(p_2,T) - s(p_1,T) = -\int_{p_1}^{p_2} \left(\frac{\partial v}{\partial T}\right)_p dp = -\int_{p_1}^{p_2} \left[\frac{R}{p} + \frac{A}{RT^2}\right] dp$$

$$= -R \ln\frac{p_2}{p_1} - \frac{A}{RT^2}[p_2 - p_1] \longleftarrow \Delta s$$

Similarly, Eq. 11.60 gives

$$h(p_2,T) - h(p_1,T) = \int_{p_1}^{p_2} \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] dp$$

$$= \int_{p_1}^{p_2} \left[\left(\frac{RT}{p} + B - \frac{A}{RT}\right) - T\left(\frac{R}{p} + \frac{A}{RT^2}\right)\right] dp$$

$$= \int_{p_1}^{p_2} \left(B - \frac{2A}{RT}\right) dp = \left[B - \frac{2A}{RT}\right](p_2 - p_1) \longleftarrow \Delta h$$

Then, with $h = u + pv$

$$h(p_2,T) - h(p_1,T) = [u(p_2,T) - u(p_1,T)] + [p_2 v(p_2,T) - p_1 v(p_1,T)]$$

$$\left[B - \frac{2A}{RT}\right](p_2 - p_1) = [u(p_2,T) - u(p_1,T)] + p_2\left[\frac{RT}{p_2} + B - \frac{A}{RT}\right] - p_1\left[\frac{RT}{p_1} + B - \frac{A}{RT}\right]$$

$$\left(B - \frac{2A}{RT}\right)(p_2 - p_1) = [u(p_2,T) - u(p_1,T)] + (p_2 - p_1)\left[B - \frac{A}{RT}\right]$$

$$\therefore\ u(p_2,T) - u(p_1,T) = (p_2 - p_1)\left[\left(B - \frac{2A}{RT}\right) - \left(B - \frac{A}{RT}\right)\right]$$

$$= -\frac{A}{RT}(p_2 - p_1) \longleftarrow \Delta u$$

---

1. Since the integrations are at fixed temperature, the terms of Eqs. 11.59 and 11.60 involving c_p drop out of consideration, thereby allowing the required property changes to be evaluated in terms of T_1, T_2, p_1, and p_2.

PROBLEM 11.49

KNOWN: The equations of state are the van der Waals equation and the Redlich-Kwong equation.

FIND: For each equation of state, determine $[h(v_2,T)-h(v_1,T)]$, $[u(v_2,T)-u(v_1,T)]$, $[s(v_2,T)-s(v_1,T)]$.

ANALYSIS: Since the equations of state are explicit in pressure, and temperature is fixed, it is convenient to use Eqs. 11.50 and 11.51, which reduce to read

$$u(v_2,T)-u(v_1,T)=\int_{v_1}^{v_2}\left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right]dv \qquad (1)$$

$$s(v_2,T)-s(v_1,T)=\int_{v_1}^{v_2}\left(\frac{\partial p}{\partial T}\right)_v dv \qquad (2)$$

Also, Δh can be found using Eq. 11.61:

$$h_2-h_1=(u_2-u_1)+(p_2v_2-p_1v_1) \qquad (3)$$

(a) van der Waals. This equation is explicit in pressure. So $(\partial p/\partial T)_v = R/(v-b)$. Eq.(1) then becomes

$$u(v_2,T)-u(v_1,T)=\int_{v_1}^{v_2}\left(\frac{RT}{v-b}-p\right)dv = \int_{v_1}^{v_2}\left(\frac{RT}{v-b}-\left(\frac{RT}{v-b}-\frac{a}{v^2}\right)\right)dv = \int_{v_1}^{v_2}\frac{a}{v^2}dv$$

$$= -a\left(\frac{1}{v_2}-\frac{1}{v_1}\right) \longleftarrow \Delta u$$

Thus

$$h(v_2,T)-h(v_1,T) = p(v_2,T)v_2 - p(v_1,T)v_1 - a\left(\frac{1}{v_2}-\frac{1}{v_1}\right) \longleftarrow \Delta h$$

Also, Eq. (2) becomes

$$s(v_2,T)-s(v_1,T)=\int_{v_1}^{v_2}\left(\frac{R}{v-b}\right)dv = R\ln\left(\frac{v_2-b}{v_1-b}\right) \longleftarrow \Delta s$$

(b) Redlich-Kwong. This equation is explicit in pressure. So, $(\partial p/\partial T)_v = \frac{R}{v-b}+\frac{a}{2v(v+b)T^{3/2}}$. Eq.(1) then becomes

$$u(v_2,T)-u(v_1,T)=\int_{v_1}^{v_2}\left(\left(\frac{RT}{v-b}+\frac{a}{2v(v+b)T^{1/2}}\right)-\left(\frac{RT}{v-b}-\frac{a}{v(v+b)T^{1/2}}\right)\right)dv$$

$$=\frac{3}{2}\int_{v_1}^{v_2}\frac{a}{v(v+b)T^{1/2}}dv = \frac{-3a}{2bT^{1/2}}\ln\left[\frac{v_2+b}{v_1+b}\cdot\frac{v_1}{v_2}\right] \longleftarrow \Delta u$$

Thus

$$h(v_2,T)-h(v_1,T) = p(v_2,T)v_2 - p(v_1,T)v_1 - \frac{3a}{2bT^{1/2}}\ln\left[\frac{v_2+b}{v_1+b}\cdot\frac{v_1}{v_2}\right] \longleftarrow \Delta h$$

Also, Eq. (2) becomes

$$s(v_2,T)-s(v_1,T)=\int_{v_1}^{v_2}\left(\frac{R}{v-b}+\frac{a}{2v(v+b)T^{3/2}}\right)dv$$

$$= R\ln\left(\frac{v_2-b}{v_1-b}\right)-\frac{a}{2bT^{3/2}}\ln\left[\frac{v_2+b}{v_1+b}\cdot\frac{v_1}{v_2}\right] \longleftarrow \Delta s$$

PROBLEM 11.50

KNOWN: At certain states the p-v-T data of a gas can be described as

$$Z = 1 - \frac{Ap}{T^4}$$

where A is a constant.

FIND: Obtain an expression for (a) $(\partial p/\partial T)_v$ in terms of p, T, A and R, (b) the quantity $[s(p_2,T) - s(p_1,T)]$, and (c) $[h(p_2,T) - h(p_1,T)]$.

ANALYSIS: Since $Z = pv/RT$, the equation can be expressed as

$$\frac{pv}{RT} = 1 - \frac{Ap}{T^4} \Rightarrow v = \frac{RT}{p} - \frac{AR}{T^3} \qquad (1)$$

(a) To obtain $(\partial p/\partial T)_v$, Eq. (1) can be solved for p and then differentiated:

$$p = \frac{RT}{v + AR/T^3}$$

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R\left[v + \frac{AR}{T^3}\right] - RT\left[-\frac{3AR}{T^4}\right]}{\left(v + \frac{AR}{T^3}\right)^2} = \frac{Rv + \frac{4AR^2}{T^3}}{\left(v + \frac{AR}{T^3}\right)^2} = \frac{R\left[\frac{RT}{p} - \frac{AR}{T^3}\right] + \frac{4AR^2}{T^3}}{(RT/p)^2}$$

$$= \frac{p^2}{R^2T^2}\left[\frac{R^2T}{p} + \frac{3AR^2}{T^3}\right] = \frac{p^2}{RT}\left[\frac{R}{p} + \frac{3AR}{T^4}\right] \qquad (a)$$

Alternatively, Eq. 11.16 can be used:

$$\left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_v = -1$$

$$\left(\frac{\partial p}{\partial v}\right)_T = \frac{-RT}{\left[v + \frac{AR}{T^3}\right]^2} = -\frac{p^2}{RT}$$

Then using Eq. 11.15

$$\left(\frac{\partial p}{\partial T}\right)_v = -\left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p$$

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} + \frac{3AR}{T^4}$$

$$= -\left[-\frac{p^2}{RT}\right]\left[\frac{R}{p} + \frac{3AR}{T^4}\right] = \frac{p^2}{RT}\left[\frac{R}{p} + \frac{3AR}{T^4}\right]$$

(b) Using Eq. 11.59

$$s(p_2,T) - s(p_1,T) = \int_{p_1}^{p_2} -\left(\frac{\partial v}{\partial T}\right)_p dp = -\int_{p_1}^{p_2} \left[\frac{R}{p} + \frac{3AR}{T^4}\right] dp$$

$$= -R\ln\frac{p_2}{p_1} + \frac{3AR}{T^4}[p_2 - p_1] \qquad (b)$$

(c) Using Eq. 11.60

$$h(p_2,T) - h(p_1,T) = \int_{p_1}^{p_2} \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] dp$$

From (a)

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} + \frac{3AR}{T^4} \Rightarrow T\left(\frac{\partial v}{\partial T}\right)_p = \frac{RT}{p} + \frac{3AR}{T^3} \Rightarrow \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] = \left[\frac{RT}{p} - \frac{AR}{T^3}\right] - \left[\frac{RT}{p} + \frac{3AR}{T^3}\right] = -\frac{4AR}{T^3}$$

So

$$h(p_2,T) - h(p_1,T) = \int_{p_1}^{p_2} \left(-\frac{4AR}{T^3}\right) dp = -\frac{4AR}{T^3}(p_2 - p_1) \qquad (c)$$

PROBLEM 11.51

KNOWN: The p-v-T behavior of a gas is described by

$$Z = 1 + \frac{B(T)p}{RT}$$

FIND: Obtain expressions for (a) $[h(p_2,T) - h(p_1,T)]$, (b) $[u(p_2,T) - u(p_1,T)]$ (c) $[s(p_2,T) - s(p_1,T)]$.

ANALYSIS: Since $Z = pv/RT$, the equation becomes

$$\frac{pv}{RT} = 1 + \frac{B(T)p}{RT}$$

$$v = \frac{RT}{p} + B(T) \qquad (1)$$

As T and p are the independent properties that fix the state and an equation of state explicit in v is known, Eqs. 11.59 and 11.60 are convenient to use here. Each of these requires $(\partial v/\partial T)_p$. Thus, from Eq.(1)

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} + \frac{dB}{dT}$$

Eq. 11.59 then gives

$$s(p_2,T) - s(p_1,T) = -\int_{p_1}^{p_2} \left(\frac{\partial v}{\partial T}\right)_p dp = -\int_{p_1}^{p_2} \left[\frac{R}{p} + \frac{dB}{dT}\right] dp$$

$$= -R \ln \frac{p_2}{p_1} - \frac{dB}{dT}[p_2 - p_1] \longleftarrow \Delta s$$

Similarly, using Eq. 11.60

$$h(p_2,T) - h(p_1,T) = \int_{p_1}^{p_2} \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] dp$$

$$= \int_{p_1}^{p_2} \left[\left(\frac{RT}{p} + B(T)\right) - T\left(\frac{R}{p} + \frac{dB}{dT}\right)\right] dp$$

$$= \int_{p_1}^{p_2} \left[B - T\frac{dB}{dT}\right] dp$$

$$= \left[B - T\frac{dB}{dT}\right][p_2 - p_1] \longleftarrow \Delta h$$

Then

$$\Delta h = \Delta u + \Delta(pv) = \Delta u + [p_2 v(p_2,T) - p_1 v(p_1,T)]$$

$$= \Delta u + p_2\left[\frac{RT}{p_2} + B\right] - p_1\left[\frac{RT}{p_1} + B\right]$$

$$= \Delta u + B(p_2 - p_1)$$

$$\Rightarrow \Delta u = \underbrace{\left[B - T\frac{dB}{dT}\right](p_2 - p_1)}_{\Delta h} - B(p_2 - p_1) \Rightarrow \Delta u = -T\frac{dB}{dT}(p_2 - p_1) \longleftarrow \Delta u$$

PROBLEM 11.52

KNOWN: The p-v-T behavior of a gas is described by

$$Z = 1 + \frac{B(T)}{v} + \frac{C(T)}{v^2}$$

FIND: Obtain an expression for $[s(v_2,T) - s(v_1,T)]$.

ANALYSIS: Since $Z = pv/RT$, the equation becomes

$$p = \frac{RT}{v} + \frac{RT\,B(T)}{v^2} + \frac{RT\,C(T)}{v^3} \qquad (1)$$

As T and v are the independent properties that fix the state and an equation of state explicit in p is known, Eq. 11.50 is convenient to use here. This expression requires $(\partial p/\partial T)_v$. Thus, from Eq. (1)

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v} + \frac{R\,B(T) + RT\,B'(T)}{v^2} + \frac{RC(T) + RTC'(T)}{v^3}$$

Then, with Eq. 11.50

$$s(v_2,T) - s(v_1,T) = \int_{v_1}^{v_2} \left(\frac{\partial p}{\partial T}\right)_v dv$$

$$= \int_{v_1}^{v_2} \left[\frac{R}{v} + \frac{[RB + RTB']}{v^2} + \frac{[RC + RTC']}{v^3} \right] dv$$

$$= R \ln\frac{v_2}{v_1} - [RB + RTB']\left[\frac{1}{v_2} - \frac{1}{v_1}\right] - \frac{[RC + RTC']}{2}\left[\frac{1}{v_2^2} - \frac{1}{v_1^2}\right]$$

$$= R \ln\frac{v_2}{v_1} - R[B + TB']\left[\frac{1}{v_2} - \frac{1}{v_1}\right] - \frac{R}{2}[C + TC']\left[\frac{1}{v_2^2} - \frac{1}{v_1^2}\right] \longleftarrow \Delta s$$

or

$$s(v_2,T) - s(v_1,T) = R \ln\frac{v_2}{v_1} + R\left[\frac{1}{v_1} - \frac{1}{v_2}\right]\left\{ \frac{d(BT)}{dT} + \frac{1}{2}\frac{d(CT)}{dT}\left[\frac{1}{v_1} + \frac{1}{v_2}\right]\right\}$$

PROBLEM 11.53

KNOWN: A 1-kg copper sphere is at 300K and an initial pressure of 10 bar.

FIND: If the volume of the sphere is not allowed to vary by more than 0.1 %, determine the maximum allowed pressure the sphere can attain, in bar.

SCHEMATIC & GIVEN DATA:

$m = 1\,kg$
$T = 300\,K$
$p_1 = 10\,bar$
$\rho = 8888\,kg/m^3$
$\beta = 49.2\times10^{-6}(K)^{-1}$
$\kappa = 0.776\times10^{-11}\,m^2/N$

ENGINEERING MODEL: (1) The copper sphere is the system. (2) Temperature remains constant at 300K. (3) ρ, β, κ do not vary as pressure changes.

ANALYSIS: As the volume would decrease as pressure increases, the restriction on volume change reads

$$\frac{(\Delta V)_{MAX}}{V} = -0.001 \qquad (1)$$

With Eq. 11.63

① $$\left(\frac{\partial v}{\partial p}\right)_T = -\kappa v = -\kappa/\rho$$

and at fixed T

$$\Delta v = -\int_{p_1}^{p_2}\frac{\kappa}{\rho}\,dp = -\frac{\kappa}{\rho}(p_2-p_1)$$

and

$$\Delta V = \left[\frac{-\kappa}{\rho}(p_2-p_1)\right]m = -\kappa\Delta p\, V$$

$$\rightarrow \frac{\Delta V}{V} = -\kappa\Delta p \qquad (2)$$

Combining (1), (2)

$$-\kappa(\Delta p)_{MAX} = -0.001$$

or

$$(\Delta p)_{MAX} = \left(\frac{0.001}{0.776\times10^{-11}\,m^2/N}\right)\left|\frac{1\,bar}{10^5\,N/m^2}\right| = 1289\,bar$$

② $$\rightarrow (p_2)_{MAX} = 1299\,bar$$

---

1. Alternatively, this equation can be expressed as

$$\left(\frac{\partial \ln V}{\partial p}\right)_T = -\kappa$$

Since κ is constant with p (assumption 3), integration gives

$$\frac{V_2}{V_1} = \exp\left(-\kappa(p_2-p_1)\right)$$

So

$$\Delta V = V_2 - V_1 = V_1\left[\exp\left(-\kappa(p_2-p_1)\right) - 1\right]$$

2. β is not required for the solution.

PROBLEM 11.54

KNOWN: A 1-lb copper sphere is at 80°F and an initial pressure of 1 atm.

FIND: If the volume of the sphere is not allowed to vary by more than 0.1%, determine the maximum allowed pressure the sphere can attain, in atm.

SCHEMATIC & GIVEN DATA:

$m = 1\ lb$
$T = 80°F$
$p_1 = 1\ atm$
$\rho = 555\ lb/ft^3$
$\beta = 2.75\times10^{-5}\ (°R)^{-1}$
$\kappa = 3.72\times10^{-10}\ ft^2/lbf$

ENGINEERING MODEL: (1) The copper sphere is the system. (2) Temperature remains constant at 80°F. (3) ρ, β, κ do not vary as pressure changes.

ANALYSIS: As the volume would decrease as pressure increases, the restriction on volume change reads

$$\frac{(\Delta V)_{MAX}}{V} = -0.001 \qquad (1)$$

With Eq. 11.63

① $$\left(\frac{\partial v}{\partial p}\right)_T = -\kappa v = -\kappa/\rho$$

and at fixed T

$$\Delta v = -\int_{p_1}^{p_2} \frac{\kappa}{\rho}\, dp = -\frac{\kappa}{\rho}\Delta p$$

and

$$\Delta V = \left[-\frac{\kappa}{\rho}\Delta p\right] m = -\kappa\, \Delta p\, V$$

$$\Rightarrow \frac{\Delta V}{V} = -\kappa\, \Delta p \qquad (2)$$

Combining (1), (2)

$$-\kappa(\Delta p)_{MAX} = -0.001$$

$$(\Delta p)_{MAX} = \left[\frac{0.001}{3.72\times10^{-10}\ ft^2/lbf}\right]\left|\frac{1\ atm}{2116\ lbf/ft^2}\right|$$

$$= 1270\ atm$$

② $$\Rightarrow (p_2)_{MAX} = 1271\ atm$$

---

1. Alternatively, this equation can be expressed as

$$\left(\frac{\partial \ln v}{\partial p}\right)_T = -\kappa$$

Since κ is constant with pressure (assumption 3), integration gives

$$\frac{v_2}{v_1} = \exp(-\kappa(p_2-p_1)) \Rightarrow \Delta v = v_2 - v_1 = v_1\left[\exp(-\kappa(p_2-p_1)) - 1\right]$$

2. β is not required for the solution

PROBLEM 11.55

KNOWN: Three cases are under consideration: (a) an ideal gas, (b) a gas whose equation of state is $p(v-b) = RT$, (c) a gas obeying the van der Waals equation.

FIND: Derive expressions for β and κ for each case.

ANALYSIS: From Eqs. 11.62 and 11.63

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p \quad \text{and} \quad \kappa = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$$

(a) Ideal Gas: $v = RT/p$

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} \quad \text{and} \quad \left(\frac{\partial v}{\partial p}\right)_T = -\frac{RT}{p^2}$$

Thus

$$\beta = \frac{1}{v}\left[\frac{R}{p}\right] = \frac{1}{T} \qquad \text{(a)}$$

$$\kappa = -\frac{1}{v}\left[-\frac{RT}{p^2}\right] = \frac{1}{p}\left[\frac{RT}{pv}\right] = \frac{1}{p}$$

(b) $v = (RT/p) + b$

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p}, \quad \left(\frac{\partial v}{\partial p}\right)_T = -\frac{RT}{p^2}$$

Thus

$$\beta = \frac{1}{v}\left[\frac{R}{p}\right] = \frac{R}{v}\left[\frac{v-b}{RT}\right] = \frac{1}{T}\left[\frac{v-b}{v}\right] \qquad \text{(b)}$$

$$\kappa = -\frac{1}{v}\left[-\frac{RT}{p^2}\right] = \frac{1}{p}\left[\frac{RT}{pv}\right] = \frac{1}{p}\left[\frac{RT/v}{RT/(v-b)}\right] = \frac{1}{p}\left[\frac{v-b}{v}\right]$$

Note: When $b=0$ these expressions reduce to those of part (a).

(c) van der Waals: $p = \frac{RT}{(v-b)} - \frac{a}{v^2}$.

As the van der Waals equation is not explicit in v, the required partial derivatives are not so easily found as in parts (a) and (b). Thus, following the procedure explained in Example 11.2, an expression for $(\partial v/\partial T)_p$ is obtained as

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{-R/(v-b)}{\left[2a/v^3 - RT/(v-b)^2\right]}$$

So

$$\beta = \frac{-R(v-b)v^2}{2a(v-b)^2 - RTv^3}$$

Using Eqs. 11.15 and 11.16

$$\left(\frac{\partial v}{\partial p}\right)_T = \frac{-(\partial v/\partial T)_p}{(\partial p/\partial T)_v} = \frac{\frac{R/(v-b)}{\left[2a/v^3 - RT/(v-b)^2\right]}}{R/(v-b)} = \frac{1}{\left[2a/v^3 - RT/(v-b)^2\right]} \qquad \text{(c)}$$

Thus

$$\kappa = -\frac{1}{v\left[2a/v^3 - RT(v-b)^2\right]} = \frac{-v^2(v-b)^2}{\left[2a(v-b)^2 - RTv^3\right]}$$

PROBLEM 11.56

FIND: Derive expressions for β and κ in terms of $T, p, Z, (\partial Z/\partial T)_p, (\partial Z/\partial p)_T$. For gas states with $p_R < 3$, $T_R < 2$, determine the sign of κ.

ANALYSIS: From Eqs. 11.62 and 11.63

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p \quad , \quad \kappa = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$$

The compressibility factor is

$$Z(T,p) = \frac{pv}{RT}$$

$$\Rightarrow \quad v = \frac{RT}{p} \cdot Z(T,p)$$

Then

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} Z + \frac{RT}{p}\left(\frac{\partial Z}{\partial T}\right)_p$$

and

$$\left(\frac{\partial v}{\partial p}\right)_T = -\frac{RT}{p^2} Z + \frac{RT}{p}\left(\frac{\partial Z}{\partial p}\right)_T$$

Accordingly

$$\beta = \frac{1}{v}\left[\frac{R}{p} Z + \frac{RT}{p}\left(\frac{\partial Z}{\partial T}\right)_p\right]$$

$$= \frac{R}{pv}\left(\frac{pv}{RT}\right) + \frac{1}{Z}\left(\frac{\partial Z}{\partial T}\right)_p$$

$$= \frac{1}{T} + \frac{1}{Z}\left(\frac{\partial Z}{\partial T}\right)_p \longleftarrow \beta$$

And

$$\kappa = -\frac{1}{v}\left[-\frac{RT}{p^2} Z + \frac{RT}{p}\left(\frac{\partial Z}{\partial p}\right)_T\right]$$

$$= \left[\frac{1}{p} \cdot \frac{RT}{pv} \cdot Z - \frac{RT}{pv}\left(\frac{\partial Z}{\partial p}\right)_T\right]$$

$$= \frac{1}{p} - \frac{1}{Z}\left(\frac{\partial Z}{\partial p}\right)_T \longleftarrow \kappa$$

Referring to Figure A-2, for $p_R < 3$, $T_R < 0$ all isotherms have a negative slope: $(\partial Z/\partial p)_T < 0$. Thus, it is evident that κ must be positive at all such states.

PROBLEM 11.57

FIND: Show that $\kappa > \alpha$.

ANALYSIS: From the development of Sec. 11.5.2, the following expressions are obtained

$$c_p - c_v = v \frac{T\beta^2}{\kappa} \qquad (11.69)$$

$$k = \frac{\kappa}{\alpha} \qquad (11.73)$$

where $k = c_p/c_v$. Following the discussion of Eq. 11.69 $c_p > c_v$ except when $\beta = 0$ and as $T \to 0$. Accordingly, except for such special circumstances, $k > 1$ and so $\kappa/\alpha > 1$, as was to be established.

PROBLEM 11.58

FIND: Prove that $(\partial\beta/\partial p)_T = -(\partial\kappa/\partial T)_p$.

ANALYSIS: Consider $v = v(T, p)$ and form the differential

$$dv = \left(\frac{\partial v}{\partial T}\right)_p dT + \left(\frac{\partial v}{\partial p}\right)_T dp$$

With Eqs. 11.62 and 11.63 this becomes

$$dv = \underbrace{(v\beta)}_{M} dT + \underbrace{(-v\kappa)}_{N} dp \qquad (1)$$

Since this is an exact differential, the test for exactness: Eq. 11.14 must be satisfied. That is

$$\left(\frac{\partial M}{\partial p}\right)_T = \left(\frac{\partial N}{\partial T}\right)_p$$

Thus

$$\left.\frac{\partial(v\beta)}{\partial p}\right]_T = \left.\frac{\partial(-v\kappa)}{\partial T}\right)_p$$

$$\beta\left(\frac{\partial v}{\partial p}\right)_T + v\left(\frac{\partial\beta}{\partial p}\right)_T = -\kappa\left(\frac{\partial v}{\partial T}\right)_p - v\left(\frac{\partial\kappa}{\partial T}\right)_p$$

$$\underline{\frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p\left(\frac{\partial v}{\partial p}\right)_T} + v\left(\frac{\partial\beta}{\partial p}\right)_T = +\underline{\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T\left(\frac{\partial v}{\partial T}\right)_p} - v\left(\frac{\partial\kappa}{\partial T}\right)_p$$

The underlined terms cancel, leaving

1
$$\left(\frac{\partial\beta}{\partial p}\right)_T = -\left(\frac{\partial\kappa}{\partial T}\right)_p$$

---

1. The desired result is obtained more directly by writing Eq. (1) as

$$d\ln v = \underbrace{\beta}_{M} dT + \underbrace{(-\kappa)}_{N} dp$$

and then applying the test for exactness.

PROBLEM 11.59

KNOWN: Data are provided for aluminum at 0°C.

FIND: Determine the percent error in c_v that would result if it were assumed that $c_p = c_v$.

SCHEMATIC & GIVEN DATA:

$T = 0°C$ $\quad$ $\beta = 7.14\times10^{-8}\ K^{-1}$

$\rho = 2700\ kg/m^3$ $\quad$ $\kappa = 1.34\times10^{-13}\ m^2/N$

$c_p = 0.9211\ kJ/kg\cdot K$

ANALYSIS: Eq 11.69 can be used to find $c_p - c_v$:

$$c_p - c_v = v\,\frac{T\beta^2}{\kappa} = \frac{T\beta^2}{\rho\kappa}$$

$$= \frac{(273\ K)(7.14\times10^{-8}\ K^{-1})^2}{(2700\ kg/m^3)(1.34\times10^{-13}\ m^2/N)}\left|\frac{1\ kJ}{10^3\ N\cdot m}\right| = 3.846\times10^{-6}\ kJ/kg\cdot K$$

Thus, $c_p \doteq c_v$. That is, the difference is negligible. Specifically

$$\%\ \text{error} = \left(\frac{3.846\times10^{-6}}{0.9211}\right)(100) = 4.2\times10^{-4}\ \% \longleftarrow \text{error}$$

Note that specific heat data reported for solids and liquids are normally c_p values, since these are more easily determined for solids and liquids than c_v.

PROBLEM 11.60

KNOWN: Data are provided for mercury at 0°C, 1 bar.

FIND: Estimate the temperature rise for an isentropic process to 1000 bar.

SCHEMATIC & GIVEN DATA:

$T_1 = 0°C$ $\quad$ $\bar{c}_p = 28.0$ kJ/kmol·K

$p_1 = 1$ bar $\quad$ $\bar{v} = 0.0147$ m³/kmol

$p_2 = 1000$ bar $\quad$ $\beta = 17.8 \times 10^{-5}$ K^{-1}

$s_1 = s_2$

ENGINEERING MODEL: The temperature change is small, and $\bar{c}_p$, $\bar{v}$, and β can be taken as constant.

ANALYSIS: Begin with Eq. 11.33: $\partial T/\partial p)_s = \frac{\partial \bar{v}}{\partial \bar{s}}\Big)_p$. The derivative on the right hand side of this expression can be expanded as follows:

$$\frac{\partial T}{\partial p}\Big)_s = \frac{\partial \bar{v}}{\partial T}\Big)_p \frac{\partial T}{\partial \bar{s}}\Big)_p$$

$$\frac{\partial \bar{v}}{\partial T}\Big)_p = \bar{v}\beta \qquad \frac{\partial T}{\partial \bar{s}}\Big)_p = \frac{T}{\bar{c}_p}$$

(from Eq. 11.62) $\qquad$ (from Eq. 11.55)

Thus

$$\frac{\partial T}{\partial p}\Big)_s = \frac{\bar{v}\beta T}{\bar{c}_p}$$

Using the assumption, we can integrate

$$\int_{T_1}^{T_2} dT = \int_{p_1}^{p_2} \frac{\bar{v}\beta T}{\bar{c}_p} dp \Rightarrow \Delta T = \left(\frac{\bar{v}\beta T}{\bar{c}_p}\right)\Delta p$$

Inserting values

$$\Delta T = \frac{(0.0147 \text{ m}^3/\text{kmol})(17.8\times10^{-5} \text{ K}^{-1})(273.15 \text{ K})(999 \text{ bar})}{(28.0 \text{ kJ/kmol}\cdot\text{K})}\left|\frac{10^5 \text{ N/m}^2}{1 \text{ bar}}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}}\right|$$

$$= 2.55 \text{ K} \longleftarrow \Delta T$$

PROBLEM 11.61

KNOWN: The p-v-T data for a certain gas can be represented as

$$Z = 1 - \frac{Ap}{T^4}$$

where A is a constant.

FIND: Obtain an expression for c_p and verify that the expression reduces to Eq. 3.47a when $Z = 1$.

ANALYSIS: Using Eq. 11.68

$$c_p - c_v = -T\left(\frac{\partial v}{\partial T}\right)_p^2 \left(\frac{\partial p}{\partial v}\right)_T$$

With Eq. 11.15

$$\left(\frac{\partial p}{\partial v}\right)_T = \frac{1}{(\partial v/\partial p)_T}$$

Accordingly

$$c_p - c_v = -T\,\frac{(\partial v/\partial T)_p^2}{(\partial v/\partial p)_T} \qquad (1)$$

This requires an equation of state explicit in v. With $Z = pv/RT$, the given equation becomes

$$v = \frac{RT}{p} - \frac{AR}{T^3}$$

Thus

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} + \frac{3AR}{T^4} \qquad (2)$$

$$\left(\frac{\partial v}{\partial p}\right)_T = -\frac{RT}{p^2} \qquad (3)$$

Combining Eqs. (1)-(3)

$$c_p - c_v = \frac{T\left[\frac{R}{p} + \frac{3AR}{T^4}\right]^2}{RT/p^2}$$

$$= R\left[1 + \frac{3Ap}{T^4}\right]^2$$

With $(Ap/T^4) = 1 - Z$

$$c_p - c_v = R\left[1 + 3(1-Z)\right]^2 = R(4 - 3Z)^2$$

With $k = c_p/c_v$, $c_v = c_p/k$ and this expression can be written as

$$c_p = \frac{kR}{k-1}\left[4 - 3Z\right]^2 \quad \leftarrow$$

If $Z = 1$, this becomes Eq. 3.47a:

$$c_p = \frac{kR}{k-1} \quad \leftarrow$$

PROBLEM 11.62

KNOWN: A gas obeys the van der Waals equation of state.

FIND: (a) Show that $(\partial c_v/\partial v)_T = 0$. (b) Develop an expression for $(c_p - c_v)$.

(c) Develop expressions for $[u(T_2, v_2) - u(T_1, v_1)]$, $[s(T_2, v_2) - s(T_1, v_1)]$.

(d) Complete the results of (c) if $c_v = a + bT$ (Note: a, b are constants, but do not correspond to the constants of the Equation of State.)

ANALYSIS: (a) $c_v = (\partial u/\partial T)_v$. Then

$$\left(\frac{\partial c_v}{\partial v}\right)_T = \frac{\partial}{\partial v}\left[\left(\frac{\partial u}{\partial T}\right)_v\right]_T = \frac{\partial}{\partial T}\left[\left(\frac{\partial u}{\partial v}\right)_T\right]_v$$

Inserting Eq. 11.47

$$\left(\frac{\partial c_v}{\partial v}\right)_T = \frac{\partial}{\partial T}\left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right]_v = \left(\frac{\partial p}{\partial T}\right)_v + T\left(\frac{\partial^2 p}{\partial T^2}\right)_v - \left(\frac{\partial p}{\partial T}\right)_v = T\left(\frac{\partial^2 p}{\partial T^2}\right)_v$$

Using the van der Waals equation

$$\frac{\partial p}{\partial T} = \frac{R}{v-b}, \quad \frac{\partial^2 p}{\partial T^2} = 0 \Rightarrow \left(\frac{\partial c_v}{\partial v}\right)_T = 0. \qquad \longleftarrow \text{(a)}$$

That is, c_v is independent of v and depends only on T: $c_v = c_v(T)$.

(b) Begin with Eq. 11.68, and note that $(\partial v/\partial T)_p$ is evaluated for the van der Waals equation in Ex 11.2. Further,

$$\left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT}{(v-b)^2} + \frac{2a}{v^3}$$

Thus

$$c_p - c_v = -T\left[\frac{R/(v-b)}{2a/v^3 - RT/(v-b)^2}\right]^2\left[\frac{2a}{v^3} - \frac{RT}{(v-b)^2}\right]$$

$$= \frac{-R^2 T}{(v-b)^2\left[\frac{2a}{v^3} - \frac{RT}{(v-b)^2}\right]}$$

$$= \frac{R}{1 - 2a(v-b)^2/RTv^3} \qquad \longleftarrow (c_p - c_v)$$

(c) Using the result of part (a), Eqs. 11.50 and 11.51 give

$$s(T_2,v_2) - s(T_1,v_1) = \int_{T_1}^{T_2} \frac{c_v(T)}{T}\,dT + \int_{v_1}^{v_2}\left(\frac{R}{v-b}\right)dv = \int_{T_1}^{T_2}\frac{c_v(T)}{T}\,dT + R\ln\left(\frac{v_2-b}{v_1-b}\right) \qquad \longleftarrow \Delta s$$

$$u(T_2,v_2) - u(T_1,v_1) = \int_{T_1}^{T_2} c_v(T)\,dT + \int_{v_1}^{v_2}\left[T\left[\frac{R}{v-b}\right] - \left(\frac{RT}{v-b} - \frac{a}{v^2}\right)\right]dv$$

$$= \int_{T_1}^{T_2} c_v(T)\,dT - a\left[\frac{1}{v_2} - \frac{1}{v_1}\right] \qquad \longleftarrow \Delta u$$

To evaluate these expressions requires only the function $c_v(T)$.

(d) If $c_v = a + bT$, then

$$\Delta s = \int_{T_1}^{T_2}\left(\frac{a+bT}{T}\right)dT + R\ln\left(\frac{v_2-b}{v_1-b}\right) = a\ln\frac{T_2}{T_1} + b(T_2-T_1) + R\ln\left(\frac{v_2-b}{v_1-b}\right)$$

$$\Delta u = \int_{T_1}^{T_2}(a+bT)\,dT - a\left[\frac{1}{v_2} - \frac{1}{v_1}\right] = a(T_2-T_1) + \frac{b}{2}(T_2^2 - T_1^2) - a\left[\frac{1}{v_2} - \frac{1}{v_1}\right]$$

PROBLEM 11.63

KNOWN: For air $c_v = 0.1965$ Btu/lb·°R at $T_1 = 1000°F$, $v_1 = 36.8$ ft³/lb.

FIND: Using the Berthelot equation of state, determine c_v at $T_2 = 1000°F$, $v_2 = 0.0555$ ft³/lb.

ENGINEERING MODEL: The Berthelot Eqn. describes the behavior of air at the specified states:

$$p = \frac{RT}{v-b} - \frac{a}{Tv^2} \quad \text{where} \quad a = \frac{27}{64}\frac{R^2T_c^3}{p_c}, \; b = \frac{1}{8}\frac{RT_c}{p_c}$$

ANALYSIS: $c_v = \partial u/\partial T)_v$. Also, Eq. 11.47 gives $(\partial u/\partial v)_T = T(\partial p/\partial T)_v - p$. Thus

$$\frac{\partial}{\partial T}\left[\left(\frac{\partial u}{\partial v}\right)_T\right]_v = \frac{\partial}{\partial T}\left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right]$$

$$\frac{\partial}{\partial v}\left[\left(\frac{\partial u}{\partial T}\right)_v\right]_T = T\left(\frac{\partial^2 p}{\partial T^2}\right)_v + \left(\frac{\partial p}{\partial T}\right)_v - \left(\frac{\partial p}{\partial T}\right)_v$$

or

$$\left(\frac{\partial c_v}{\partial v}\right)_T = T\left(\frac{\partial^2 p}{\partial T^2}\right)_v \qquad (1)$$

Differentiation of the equation of state gives

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v-b} + \frac{a}{T^2v^2} \quad , \left(\frac{\partial^2 p}{\partial T^2}\right)_v = -\frac{2a}{T^3v^2}$$

Inserting this into Eq. (1) and integrating

$$c_v(T, v_2) - c_v(T, v_1) = \left.\frac{2a}{T^2 v}\right]_{v_1}^{v_2}$$

$$= \frac{2a}{T^2}\left[\frac{1}{v_2} - \frac{1}{v_1}\right]$$

The value of a is determined using T_c and p_c from Table A-1E

$$a = \frac{27}{64}\frac{R^2T_c^3}{p_c} = \frac{27}{64}\frac{\left(1545/28.97 \; \frac{ft\cdot lbf}{lb\cdot °R}\right)^2 (239°R)^3}{(37.2)\left|14.696 \times 144\right| lbf/ft^2}$$

$$= 208{,}081 \; \frac{lbf\cdot ft^4\cdot(°R)^2}{(lb)^2}\left|\frac{Btu}{778 \; ft\cdot lbf}\right|$$

$$= 267.46 \; \frac{ft^3\cdot(°R)^2\cdot Btu}{(lb)^2}$$

Accordingly, for $T = 1460°R$

$$c_v(T, 0.0555 \; ft^3/lb) = 0.1965 + \frac{2(267.46)}{(1460)^2}\left[\frac{1}{0.0555} - \frac{1}{36.8}\right]$$

$$= 0.1965 + 0.0045$$

$$= 0.2010 \; Btu/lb\cdot°R$$

PROBLEM 11.64

KNOWN: The specific heat ratio can be expressed as

$$k = \frac{c_p \kappa}{c_p \kappa - T v \beta^2}$$

FIND: Derive this expression and use it together with steam table data to evaluate k at 200 lbf/in², 500°F.

ANALYSIS: Begin with Eq. 11.69, divide each side by c_p, giving

$$1 - \frac{c_v}{c_p} = \frac{vT\beta^2}{c_p \kappa} \Rightarrow 1 - \frac{1}{k} = \frac{vT\beta^2}{c_p \kappa}$$

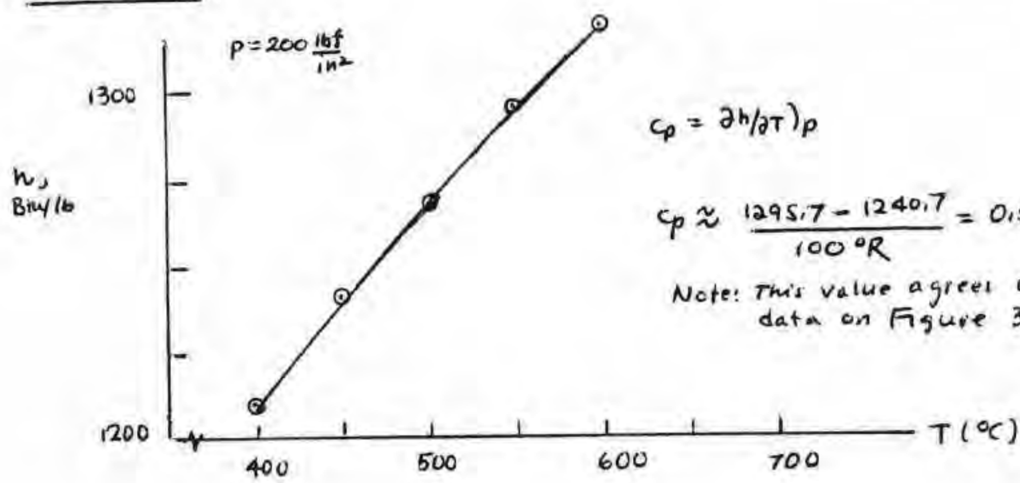
Then

$$1 - \frac{vT\beta^2}{c_p \kappa} = \frac{1}{k} \Rightarrow k = \frac{c_p \kappa}{c_p \kappa - vT\beta^2} = \frac{1}{1 - (vT\beta^2/c_p \kappa)} \quad (1)$$

To evaluate k requires c_p, κ, β, or alternatively c_p, $\left.\frac{\partial v}{\partial T}\right)_p$, $\left.\frac{\partial v}{\partial p}\right)_T$

GRAPHICAL SOLUTION:

To find c_p

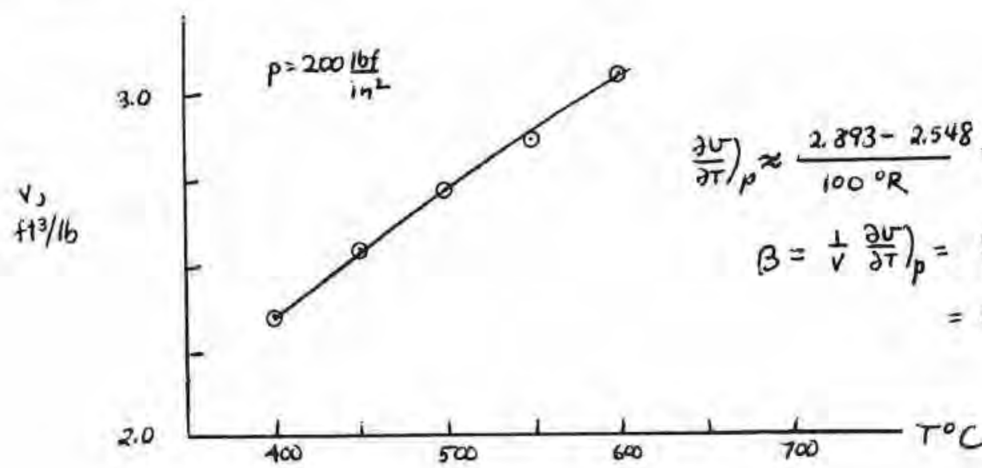


$c_p = \partial h / \partial T)_p$

$$c_p \approx \frac{1295.7 - 1240.7}{100\ °R} = 0.55\ \frac{Btu}{lb \cdot °R}$$

Note: This value agrees with data on Figure 3.9

To find $\partial v/\partial T)_p$

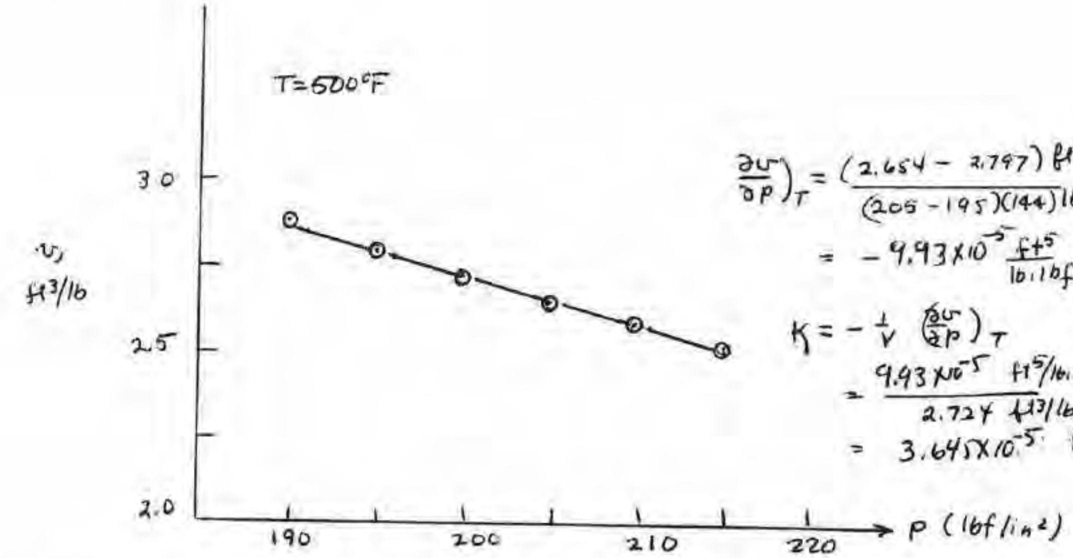


$$\left.\frac{\partial v}{\partial T}\right)_p \approx \frac{2.893 - 2.548}{100\ °R} = 0.00345\ \frac{ft^3}{lb \cdot °R}$$

$$\beta = \frac{1}{v}\left.\frac{\partial v}{\partial T}\right)_p = \frac{0.00345}{2.724} = 1.267 \times 10^{-3}\ (°R)^{-1}$$

PROBLEM 11.64 (Cont'd) - Page 2

To find $(\partial v/\partial p)_T$



$$\left(\frac{\partial v}{\partial p}\right)_T = \frac{(2.654 - 2.797)\ \text{ft}^3/\text{lb}}{(205-195)(144)\ \text{lbf/ft}^2} = -9.93\times10^{-5}\ \frac{\text{ft}^5}{\text{lb}\cdot\text{lbf}}$$

$$\kappa = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T = \frac{9.93\times10^{-5}\ \text{ft}^5/\text{lb}\cdot\text{lbf}}{2.724\ \text{ft}^3/\text{lb}} = 3.645\times10^{-5}\ \text{ft}^2/\text{lbf}$$

With these values

$$\frac{vT\beta^2}{c_p\kappa} = \frac{(2.724\ \text{ft}^3/\text{lb})(959.67\,°\text{R})(1.267\times10^{-3}\ (°\text{R})^{-1})^2}{\left(0.55\ \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)\left(778.17\ \frac{\text{ft}\cdot\text{lbf}}{\text{Btu}}\right)\left(3.645\times10^{-5}\ \frac{\text{ft}^2}{\text{lbf}}\right)} = 0.269$$

and Eq. (1) gives $k = 1.37$, which agrees with the value determined from Fig. 6 of *Steam Tables* (English Units) by Keenan et al., Wiley, New York, 1969.

IT Code

```
p = 200 // lbf/in²
T = 500 // °F

dT = 0.001
dp = 0.001
T1 = T - dT
T2 = T + dT
p1 = p - dp
p2 = p + dp
h1 = h_PT("Water/Steam", p, T1)
h2 = h_PT("Water/Steam", p, T2)
c_p = (h2 - h1) / (T2 - T1)
v2 = v_PT("Water/Steam", p, T2)
v1 = v_PT("Water/Steam", p, T1)
v = v_PT("Water/Steam", p, T)
beta = (1 / v) * ((v2 - v1) / (T2 - T1))
v22 = v_PT("Water/Steam", p2, T)
v11 = v_PT("Water/Steam", p1, T)
kappa = - (1 / v) * ((v22 - v11) / (p2 - p1)) / 144
k = 1 / (1 - (v * (T + 459.67) * beta^2) / (c_p * kappa * 778.17))
```

IT Results

$\beta = 0.001265\ (1/°\text{R})$
$c_p = 0.548\ \text{Btu/lb}\cdot°\text{R}$
$\kappa = 3.656\times10^{-5}\ \text{ft}^2/\text{lbf}$
$k = 1.366$

PROBLEM 11.65

KNOWN: Liquid water at 40 °C, 1atm is under consideration. Data are available from Table 11.2.

FIND: Determine (a) c_v, (b) velocity of sound.

ANALYSIS: (a) Using Eq. 11.69

$$c_v = c_p - v\frac{T\beta^2}{\kappa} = c_p - \frac{T\beta^2}{\rho\kappa} \qquad (1)$$

From Table 11.2 at 40°C

$\rho = 992.22\ \frac{kg}{m^3}$, $\beta = \frac{385.4}{10^6}(K)^{-1}$, $\kappa = \frac{44.24}{10^6}\ (bar)^{-1}$

① From Table A.19, $c_p = 4.18\ kJ/kg\cdot K$.

Inserting values in Eq. (1)

$$c_v = \left(4.18\ \frac{kJ}{kg\cdot K}\right) - \frac{(313.15K)(385.4\times10^{-6})^2\,(bar)}{\left(992.22\ \frac{kg}{m^3}\right)(K)^2\left(44.24\times10^{-6}\right)}\left|\frac{10^5 N/m^2}{1 bar}\right|\left|\frac{1\ kJ}{10^3 N\cdot m}\right|$$

② $= (4.18 - 0.11) = 4.07\ kJ/kg\cdot K$ ◄

(b) With Eqs. 11.65, 11.73

$$c = \sqrt{\frac{k}{\alpha}} = \sqrt{\frac{k}{\rho\kappa}}$$

with the result of (a), $k = \frac{c_p}{c_v} = \frac{4.18}{4.07} = 1.027$

$$c = \sqrt{\frac{(1.027)\ (bar)}{\left(992.22\ \frac{kg}{m^3}\right)(44.24\times10^{-6})}\left|\frac{10^5 N/m^2}{1 bar}\right|\left|\frac{1 kg\cdot m/s^2}{1N}\right|}$$

$= 1530\ m/s$ ◄

---

1. Following the discussion of Sec. 3.3.6, we take c_p at 1 atm, 40°C as the saturated liquid value at 40°C.

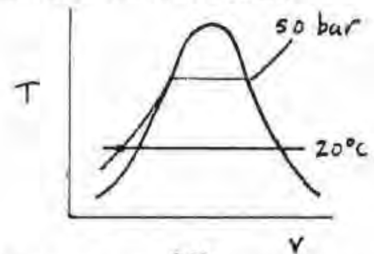
2. The result of part (a) shows that for liquid water at the given state, c_p and c_v are closely equal.

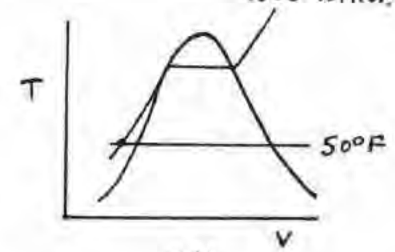
PROBLEM 11.66

KNOWN: Liquid water is at specified states.

FIND: Using steam table data, estimate the velocity of sound at (a) 20°C, 50 bar, (b) 50°F, 1500 lbf/in.^{2}

SCHEMATIC & GIVEN DATA:



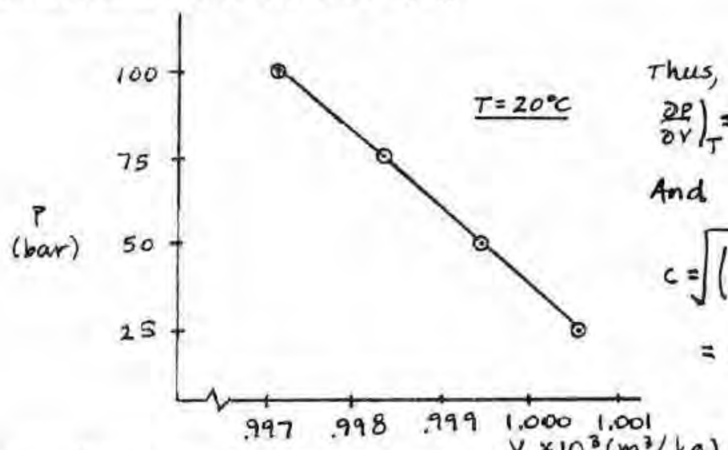


ENGINEERING MODEL: For liquid water in the range of temperatures under consideration, the ratio of specific heats is $k \approx 1.0$. (see discussion in Sec. 11.52)

ANALYSIS: With Eq. 11.74 and the above assumption

$$c = \sqrt{-k\, v^2 \left.\frac{\partial p}{\partial v}\right)_T} \approx \sqrt{-v^2 \left.\frac{\partial p}{\partial v}\right)_T}$$

(a) With data from Table A-5



Thus,

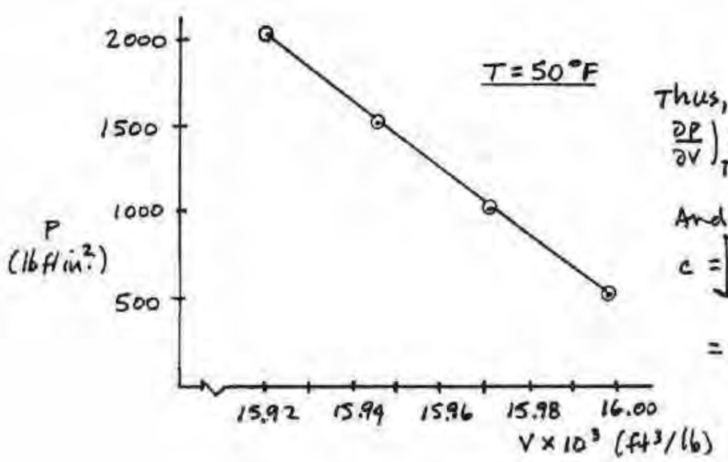
$$\left.\frac{\partial p}{\partial v}\right)_T \approx \frac{(75-25)\,10^5\,\text{N/m}^2}{\dfrac{(.9984-1.0006)}{10^3}\dfrac{\text{m}^3}{\text{kg}}} = -22727\times10^8\ \frac{\text{N}\cdot\text{kg}}{\text{m}^5}$$

And

$$c = \sqrt{\left(\frac{.9995}{10^3}\,\frac{\text{m}^3}{\text{kg}}\right)^2 (22727\times10^8)\,\frac{\text{N}\cdot\text{kg}}{\text{m}^5}\left|\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\,\text{N}}\right|}$$

$= 1507$ m/s ← (a)

(b) With data from Table A-5E



Thus,

$$\left.\frac{\partial p}{\partial v}\right)_T \approx \frac{(2000-1000)\,144\ \text{lbf/ft}^2}{(0.015920-0.015972)\ \text{ft}^3/\text{lb}} = -2.7692\times10^9\ \frac{\text{lbf}\cdot\text{lb}}{\text{ft}^5}$$

And

$$c = \sqrt{\left(\frac{15.946}{10^3}\,\frac{\text{ft}^3}{\text{lb}}\right)^2 (2.7692\times10^9)\,\frac{\text{lbf}\cdot\text{lb}}{\text{ft}^5}\left|\frac{32.2\ \text{lb}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right|}$$

$= 4762$ ft/s ← (b)

PROBLEM 11.67

KNOWN: At a point within a stream of air $T = 500°F$, $p = 1 atm$, $V = 2115 ft/s$.

FIND: Determine the Mach number.

ENGINEERING MODEL: Air is modeled as an ideal gas.

ANALYSIS: The Mach number is $M = V/c$, where c is the velocity of sound. For an ideal gas, c is given by Eq. 9.37, obtained by reduction of Eq. 11.74. Then

$$M = \frac{V}{c} = \frac{V}{\sqrt{kRT}} = \frac{2115 ft/s}{\sqrt{(1.383)\left(\frac{1545}{28.97}\frac{ft\cdot lbf}{lb\cdot °R}\right)(960°R)\left|32.2\frac{lb\cdot ft}{lbf\cdot s^2}\right|}}$$

$$= \frac{2115}{1510} = 1.4$$

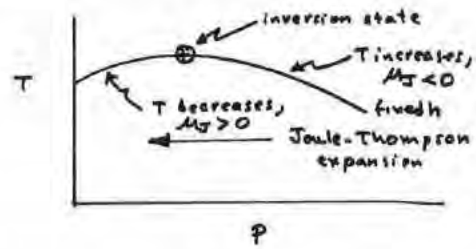
where k is obtained from Table A-20E.

PROBLEM 11.68

KNOWN: A gas obeys the equation of state, $p(v-b) = RT$, where b is a positive constant.

FIND: Determine if the temperature can be reduced in a Joule–Thompson expansion.

SCHEMATIC & GIVEN DATA



$$\mu_J = \left(\frac{\partial T}{\partial p}\right)_h$$

ANALYSIS: Using Eq. 11.77

$$\mu_J = \frac{1}{c_p}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right]$$

With the given equation of state $(\partial v/\partial T)_p = R/p$. Thus

$$\mu_J = \frac{1}{c_p}\left[\frac{RT}{p} - v\right]$$

$$= \frac{1}{c_p}\left[\frac{RT}{p} - \left(\frac{RT}{p} + b\right)\right] = -\frac{b}{c_p} < 0$$

Accordingly, the Joule – Thompson coefficient is negative, and so the temperature can only increase in a Joule – Thompson expansion. ◄

PROBLEM 11.69

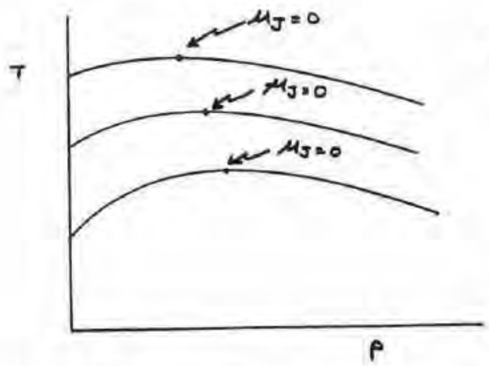
KNOWN: A gas is described by the equation of state

$$v = \frac{RT}{p} - \frac{A}{T} + B$$

where A and B are constants.

FIND: (a) Obtain an expression for the temperatures at the Joule-Thompson inversion states. (b) Obtain an expression for $(c_p - c_v)$.

SCHEMATIC & GIVEN DATA:



At Joule-Thompson inversion states, $\mu_J = 0$

ANALYSIS: (a) At the Joule-Thompson inversion states, the Joule-Thompson coefficient vanishes. Thus, Eq. 11.77 becomes

$$0 = \frac{1}{c_p}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] \Rightarrow T\left(\frac{\partial v}{\partial T}\right)_p - v \equiv 0$$

With the given equation of state

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} + \frac{A}{T^2} \qquad (1)$$

$$T\left(\frac{\partial v}{\partial T}\right)_p = \frac{RT}{p} + \frac{A}{T}$$

Accordingly

$$\left(\frac{RT}{p} + \frac{A}{T}\right) - \left(\frac{RT}{p} - \frac{A}{T} + B\right) = 0 \Rightarrow \frac{2A}{T} - B \Rightarrow T = \frac{2A}{B} \longleftarrow$$

① (b) To find $c_p - c_v$, employ Eq. 11.66. This requires $(\partial v/\partial T)_p$ and $(\partial p/\partial T)_v$. $(\partial v/\partial T)_p$ is given by Eq. (1). Rewriting the equation of state

$$p = RT\left[v + \frac{A}{T} - B\right]^{-1}, \quad \left(\frac{\partial p}{\partial T}\right)_v = R\left[v + \frac{A}{T} - B\right]^{-1} - RT\left[v + \frac{A}{T} - B\right]^{-2}\left[-\frac{A}{T^2}\right]$$

$$= \frac{R(v-B) + 2RA/T}{\left[v + \frac{A}{T} - B\right]^2}$$

② $$\Rightarrow \quad c_p - c_v = T\left[\frac{R}{p} + \frac{A}{T^2}\right]\left[\frac{R(v-B) + 2RA/T}{\left(v + \frac{A}{T} - B\right)^2}\right] \longleftarrow$$

---

1. Alternatively, $(c_p - c_v)$ can be evaluated from Eq. 11.68.
2. For an ideal gas $A = B = 0$. Then, this expression for $(c_p - c_v)$ reduces to $(c_p - c_v) = R$, which is Eq. 3.44.

PROBLEM 11.70

KNOWN: Three equations of state are under consideration: (a) van der Waals equation, (b) Redlich-Kwong equation, (c) Dieterici equation.

FIND: Determine the maximum Joule-Thompson inversion temperature in terms of the critical temperature, T_c, predicted by each of these equations.

ANALYSIS: Eq. 11.77 can be rearranged to give

$$\mu_J = \frac{\left[T(\partial v/\partial T)_p - v\right]}{c_p}$$

The maximum inversion temperature is determined in accordance with $\lim_{p\to 0} \mu_J = 0$. Since $\lim_{p\to 0} c_p(T,p) \to c_p(T)$ (ideal gas), this reduces to a consideration of

$$\lim_{p\to 0}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = 0$$

Also, note that the limit as pressure tends to zero corresponds to the limit as $v \to \infty$. Accordingly, the condition to be considered is

$$\lim_{v\to\infty}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = 0$$

(a) van der Waals. The partial derivative $(\partial v/\partial T)_p$ is evaluated in Example 11.2 as

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{-R/(v-b)}{\left[\frac{2a}{v^3} - \frac{RT}{(v-b)^2}\right]} = \frac{Rv^3(v-b)}{RTv^3 - 2a(v-b)^2}$$

Thus

$$\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = \frac{RTv^3(v-b)}{RTv^3 - 2a(v-b)^2} - v$$

$$= \frac{+RTv^4 - bRTv^3 - vRTv^3 + 2av(v-b)^2}{RTv^3 - 2a(v-b)^2}$$

$$= \frac{2av(v-b)^2 - bRTv^3}{RTv^3 - 2a(v-b)^2} = \frac{2a - \frac{bRT}{(1-(b/v))^2}}{\frac{RT}{(1-(b/v))^2} - \frac{2a}{v}}$$

Considering the limit as $v \to \infty$

$$\lim_{v\to\infty}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = \frac{2a - bRT}{RT} = 0 \quad \Rightarrow \quad T = \frac{2a}{Rb}$$

Introducing Eqs. 11.4a and 11.4b

$$T = \frac{2\left(\frac{27}{64}\frac{R^2T_c^2}{p_c}\right)}{\left(\frac{1}{8}\frac{RT_c}{p_c}\right)} = \frac{27}{4}T_c = 6.75\,T_c \longleftarrow \text{(a)}$$

(b) Redlich-Kwong. To evaluate $(\partial v/\partial T)_p$, use the relation

$$\left(\frac{\partial v}{\partial T}\right)_p\left(\frac{\partial p}{\partial v}\right)_T\left(\frac{\partial T}{\partial p}\right)_v = -1 \quad \Rightarrow \quad \left(\frac{\partial v}{\partial T}\right)_p = -\frac{(\partial p/\partial T)_v}{(\partial p/\partial v)_T}$$

For the Redlich-Kwong equation, the partial derivatives are

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{(v-b)} + \frac{a}{2v(v+b)T^{3/2}}\ , \quad \left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT}{(v-b)^2} + \frac{a(2v+b)}{v^2(v+b)^2T^{1/2}}$$

So

$$\left[T\frac{\partial v}{\partial T}\right)_p - v\Big] = -T\left[\frac{\frac{R}{(v-b)} + \frac{a}{2v(v+b)T^{3/2}}}{\frac{a(2v+b)}{v^2(v+b)^2T^{1/2}} - \frac{RT}{(v-b)^2}}\right] - v$$

PROBLEM 11.70 (Contd.) - Page 2

Upon rearrangement

$$\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = \frac{\frac{-RT}{(v-b)} - \frac{a}{2v(v+b)T^{1/2}} - \frac{a(2v+b)}{v(v+b)^2T^{1/2}} + \frac{vRT}{(v-b)^2}}{\left(\frac{a(2v+b)}{v^2(v+b)^2T^{1/2}} - \frac{RT}{(v-b)^2}\right)}$$

$$= \frac{\frac{bRT}{(v-b)^2} - \frac{a}{2(v+b)^2T^{1/2}}\left[5+3\frac{b}{v}\right]}{\left(\frac{a(2v+b)}{v^2(v+b)^2T^{1/2}} - \frac{RT}{(v-b)^2}\right)} = \left[\frac{\frac{bRT}{(1-\frac{b}{v})^2} - \frac{a\left[5+3\frac{b}{v}\right]}{2(1+\frac{b}{v})^2T^{1/2}}}{\frac{a(2+\frac{b}{v})}{v(v+b)^2T^{1/2}} - \frac{RT}{(1-\frac{b}{v})^2}}\right]$$

Considering the limit as $v \to \infty$

$$\lim_{v\to\infty}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = \frac{bRT - \frac{5a}{2T^{1/2}}}{-RT} = 0 \Rightarrow \quad T = \left(\frac{5a}{2bR}\right)^{2/3}$$

With Eqs. 11.8

① $$T = \left(\frac{5\,(0.42748)\,R^2T_c^{5/2}/p_c}{(2)(0.08664)\left(\frac{RT_c}{p_c}\right)R}\right)^{2/3} = 5.34\ T_c \longleftarrow \quad \text{(b)}$$

(c) Dieterici. As above

$$\left(\frac{\partial v}{\partial T}\right)_p = -\frac{(\partial p/\partial T)_v}{(\partial p/\partial v)_T}$$

For the Dieterici equation, the partial derivatives are

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{(v-b)}\exp\left(-\frac{a}{RTv}\right)\left[1+\frac{a}{RTv}\right], \quad \left(\frac{\partial p}{\partial v}\right)_T = \frac{RT}{(v-b)}\exp\left(-\frac{a}{RTv}\right)\left[\frac{a}{RTv^2} - \frac{1}{(v-b)}\right]$$

Thus

$$\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = \frac{\left[1+\frac{a}{RTv}\right]}{\left[\frac{a}{RTv^2} - \frac{1}{(v-b)}\right]} - v = \frac{\frac{v}{v-b} - \frac{2a}{RTv} - 1}{\frac{a}{RTv^2} - \frac{1}{(v-b)}}$$

As $\lim_{v\to\infty}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = \frac{0}{0}$, apply L'Hospital's rule to obtain

$$\lim_{v\to\infty}\left[\frac{\frac{-b}{(v-b)^2} + \frac{2a}{RTv^2}}{\frac{-2a}{RTv^3} + \frac{1}{(v-b)^2}}\right] = \lim_{v\to\infty}\left[\frac{\frac{-b}{(1-\frac{b}{v})^2} + \frac{2a}{RT}}{\frac{-2a}{RTv} + \frac{1}{(1-\frac{b}{v})^2}}\right] = -b + \frac{2a}{RT}$$

$$\Rightarrow \quad T = \frac{2a}{bR}$$

Introducing the expressions for a and b given in Problem 11.17

$$T = \frac{2\left(\frac{4R^2T_c^2}{p_c e^2}\right)}{R\left(\frac{RT_c}{p_c e^2}\right)} = 8\ T_c \longleftarrow \quad \text{(c)}$$

---

1. The maximum inversion temperature for a wide range of gases is about $5T_c$. Of the three equations of state considered here, only the Redlich-Kwong equation predicts a maximum inversion temperature in the neighborhood of $5T_c$.

PROBLEM 11.71

KNOWN: A certain gas obeys the van der Waals equation of state and has a specific heat c_v

(1)

$$c_v = A + BT + CT^2$$

where A, B, and C are constants.

FIND: Derive an equation for μ_J as a function of v and T. Evaluate the Joule-Thompson inversion state temperatures in terms of v, R, and the van der Waals constants a and b.

ANALTSIS: Using Eq. 11.77: $\mu_J = [T(\partial v/\partial T)_p - v]/c_p$. This requires $(\partial v/\partial T)_p$ which is evaluated for the van der Waals equation in Example 11.2 as

$$\left(\frac{\partial v}{\partial T}\right)_p = -\frac{R/(v-b)}{[2a/v^3 - RT/(v-b)^2]} = \frac{-Rv^3(v-b)}{2a(v-b)^2 - RTv^3}$$

Thus

$$\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = \frac{-RTv^3(v-b)}{(2a(v-b)^2 - RTv^3)} - v = \frac{2av(v-b)^2 - bRTv^3}{RTv^3 - 2a(v-b)^2}$$

An expression for c_p is also required. This can be found using Eq. 11.66

$$c_p - c_v = T\left(\frac{\partial v}{\partial T}\right)_p\left(\frac{\partial p}{\partial T}\right)_v$$

From the van der Waals equation, $(\partial p/\partial T)_v = R/(v-b)$. Thus

$$c_p = c_v + T\left[\frac{-Rv^3(v-b)}{2a(v-b)^2 - RTv^3}\right]\frac{R}{(v-b)}$$

$$= (A + BT + C^2) - \frac{R^2 T v^3}{2a(v-b)^2 - RTv^3}$$

$$= \frac{(A + BT + C^2)[RTv^3 - 2a(v-b)^2] + R^2Tv^3}{RTv^3 - 2a(v-b)^2}$$

Collecting results

$$\mu_J = \left\{\frac{\left(\frac{2av(v-b)^2 - bRTv^3}{RTv^3 - 2a(v-b)^2}\right)}{\frac{(A+BT+C^2)[RTv^3 - 2a(v-b)^2] + R^2Tv^3}{RTv^3 - 2a(v-b)^2}}\right\}$$

$$= \frac{2av(v-b)^2 - bRTv^3}{(A+BT+C^2)(RTv^3 - 2a(v-b)^2) + R^2Tv^3} \longleftarrow \mu_J$$

At Joule-Thompson inversion states, $\mu_J = 0$. Accordingly, the inversion temperatures T_i are determined by

(2) $0 = 2av(v-b)^2 - bRT_iv^3 \Rightarrow T_i = \frac{2a(v-b)^2}{bRv^2} = \frac{2a}{bR}\left[1 - \left(\frac{b}{v}\right)\right]^2 \longleftarrow T_i$

---

1. For a van der Waals gas, $c_v = f(T)$. See Problem 11.62a.
2. In the limit as $p \to 0$, $v \to \infty$. Thus, the maximum inversion temperature is $(T_i)_{MAX} = \frac{2a}{bR}$, which is in agreement with the result of Problem 11.70(a).

PROBLEM 11.72

KNOWN: Equation 11.77 is written in an alternative form.

FIND: Verify the alternative form. (a) Using this result, obtain an expression for μ_J for a gas obeying

$$v = \frac{RT}{p} - \frac{Ap}{T^2} \quad \text{where A is a constant.}$$

(b) Using the expression obtained in (b), determine c_p for CO_2 at a state where $T = 400\,K$, $p = 1\,atm$, $\mu_J = 0.57\,K/atm$.

ANALYSIS: The alternative expression for μ_J is

$$\mu_J = \frac{T^2}{c_p}\left(\frac{\partial(v/T)}{\partial T}\right)_p \Rightarrow \mu_J = \frac{T^2}{c_p}\left[\frac{1}{T}\left(\frac{\partial v}{\partial T}\right)_p - \frac{v}{T^2}\right] = \frac{1}{c_p}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right]$$

which agrees with Eq. 11.77.

(b) Using the given relationship, we get $(v/T) = (R/p) - (Ap/T^3)$. Thus

$$\left.\frac{\partial(v/T)}{\partial T}\right)_p = \frac{3Ap}{T^4} \Rightarrow \mu_J = \frac{T^2}{c_p}\left[\frac{3Ap}{T^4}\right] = \frac{3Ap}{c_p T^2} \longleftarrow$$

(c) Solving for c_p, and using $A = 2.78 \times 10^{-3}\,m^5 \cdot K^2/kg \cdot N$ and other known data

$$c_p = \frac{3AP}{\mu_J T^2} = \frac{3(2.78\times10^{-3}\,m^5\cdot K^2/kg\cdot N)(1.01325\times10^5\,N/m^2)}{\left(0.57\,\frac{K}{atm}\right)\left|\frac{1\,atm}{1.01325\times10^5\,N/m^2}\right|(400\,K)^2}\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|$$

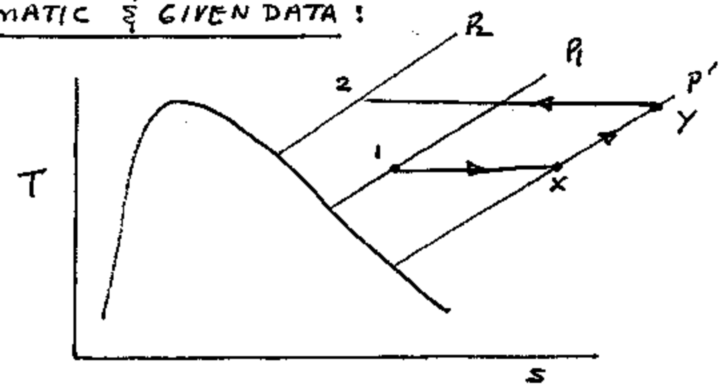
$$= 0.939\,\frac{kJ}{kg\cdot K} \longleftarrow$$

PROBLEM 11.73

KNOWN: For a gas obeying the van der Waals equation, the specific heat c_v is given at pressure p' by $c_v = A + BT$, where A and B are constants.

FIND: Develop an expression for $[s(T_2, P_2) - s(T_1, P_1)]$

SCHEMATIC & GIVEN DATA:



At 1, $v_1 = v(T_1, P_1)$
At x, $v_x = v(T_1, p')$
At y, $v_y = v(T_2, p')$
At 2, $v_2 = v(T_2, P_2)$
When $p = p'$, $c_v = A + BT$

MODEL: At the states under consideration, the gas is described by the van der Waals equation and given c_v expression.

ANALYSIS: For any choice of states 1, 2 for which the model applies,

$$(s_2 - s_1) = (s_2 - s_y) + (s_y - s_x) + (s_x - s_1)$$

$$[s(T_2,P_2) - s(T_1,P_1)] = \underbrace{[s(T_2,P_2) - s(T_2,p')]}_{(y \to 2)} + \underbrace{[s(T_2,p') - s(T_1,p')]}_{(x \to y)} + \underbrace{[s(T_1,p') - s(T_1,P_1)]}_{(1 \to x)} \quad (1)$$

The specific volumes v_1, v_x, v_y, v_2 can be found by solving the van der Waals equation at the respective temperature and pressure.

From the van der Waals equation, $p = \frac{RT}{(v-b)} - \frac{a}{v^2}$, we get

$$(\partial p/\partial T)_v = \frac{R}{(v-b)}$$

Thus, with Eq. 11.50

$$s_x - s_1 = \cancel{\int_1^x \frac{c_v}{T} dT}^{\,0} + \int_1^x \left(\frac{\partial p}{\partial T}\right)_v dv = \int_{v_1}^{v_x} \frac{R}{(v-b)} = R \ln\left(\frac{v_x - b}{v_1 - b}\right) \quad (2)$$

$$s_y - s_x = \int_x^y \frac{c_v}{T} dT + \int_x^y \left(\frac{\partial p}{\partial T}\right)_v dv = \int_{T_1}^{T_2} \left(\frac{A + BT}{T}\right) dT + \int_{v_x}^{v_y} \frac{R}{(v-b)} dv$$

$$= A \ln \frac{T_2}{T_1} + B(T_2 - T_1) + R \ln\left(\frac{v_y - b}{v_x - b}\right) \quad (3)$$

$$s_2 - s_y = \cancel{\int_y^2 \frac{c_v}{T} dT}^{\,0} + \int_y^2 \left(\frac{\partial p}{\partial T}\right)_v dv = \int_{v_y}^{v_2} \frac{R}{(v-b)} = R \ln\left(\frac{v_2 - b}{v_y - b}\right) \quad (4)$$

Equation (1) then reads

$$[s(T_2,P_2) - s(T_1,P_1)] = R \ln\left(\frac{v_2 - b}{v_y - b}\right) + \left\{A \ln \frac{T_2}{T_1} + B(T_2 - T_1) + R \ln\left(\frac{v_y - b}{v_x - b}\right)\right\} + R \ln\left(\frac{v_x - b}{v_1 - b}\right)$$

$$= R \ln\left(\frac{v_2 - b}{v_1 - b}\right) + A \ln \frac{T_2}{T_1} + B(T_2 - T_1)$$

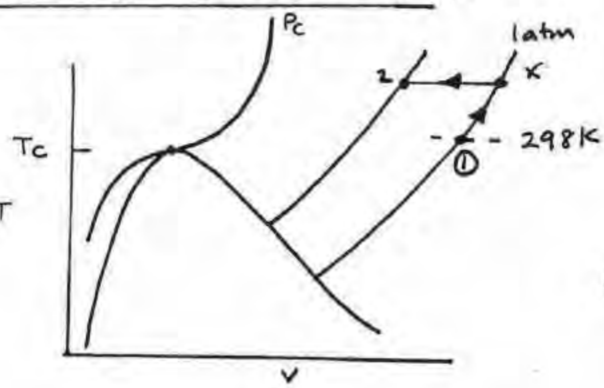
where A and B are keyed to pressure p' (process $x \to y$).

PROBLEM 11.74

KNOWN: Air is modeled by the van der Waals equation and $c_p(T)$ from Table A-21.

FIND: Write a computer program evaluating Δh between a state where ($T_1 = 298K$, $P_1 = 1 atm$) and (T_2, P_2).

SCHEMATIC & GIVEN DATA:



From Table A-1, $P_c = 37.2$ atm, $T_c = 133K$

ENGINEERING MODEL: The ideal gas model is valid for the path 1−x (check Fig. A-1) and the specific heat function of Table A-21 is valid for $298 \le T \le 1000K$ ①

ANALYSIS: The evaluation of Δh is conveniently accomplished by considering two paths: 1−x and x−2. As noted, the ideal gas model applies for path 1−x, and so

$$h_x - h_1 = \int_{T_1}^{T_2 = T_x} c_p(T)\,dT = R\left[\alpha(T_2 - T_1) + \frac{\beta}{2}(T_2^2 - T_1^2) + \frac{\gamma}{3}(T_2^3 - T_1^3) + \frac{\delta}{4}(T_2^4 - T_1^4) + \frac{\epsilon}{5}(T_2^5 - T_1^5)\right] \qquad (1)$$

where $\alpha, \beta, \gamma, \delta$, and ϵ are obtained from Table A-21

Since the van der Waals equation is explicit in pressure, it is convenient to evaluate $h_2 - h_x$ using

$$h_2 - h_x = (u_2 - u_x) + P_2 v_2 - P_x v_x \qquad (2)$$

where $(u_2 - u_x)$ is evaluated from Eq. 11.51, which reduces to give

$$u_2 - u_x = \int_{v_x}^{v_2} \left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right] dv = \int_{v_x}^{v_2} \left(T\left(\frac{R}{v-b}\right) - p\right) dv = \int_{v_x}^{v_2} \frac{a}{v^2} = -a\left[\frac{1}{v_2} - \frac{1}{v_x}\right] \qquad (3)$$

With Eqs. (2) and (3)

$$h_2 - h_x = P_2 v_2 - P_x v_x - a\left[\frac{1}{v_2} - \frac{1}{v_x}\right] \qquad (4)$$

The value of v_x can be obtained using the ideal gas equation of state: $v_x = RT_x/P_x = RT_2/P_1$. The value of v_2 can be obtained iteratively from the van der Waals equation using the specified values for P_2, T_2.

The overall change $(h_2 - h_1)$ is then obtained from Eqs. (1) and (4):

$$(h_2 - h_1) = \underbrace{(h_2 - h_x)}_{\text{Eq.(4)}} + \underbrace{(h_x - h_1)}_{\text{Eq.(1)}}$$

This provides the required relations for finding Δh. Computer program details are left to the reader.

---

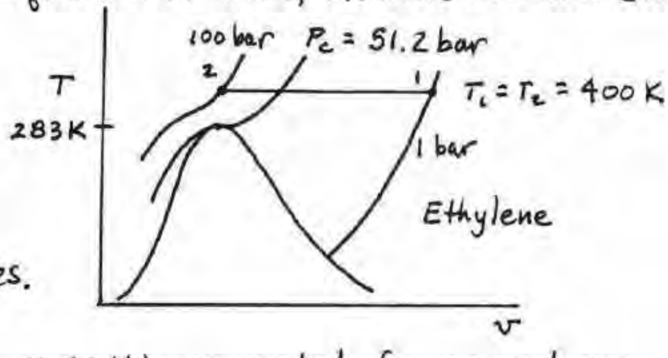
1. For temperatures out of this range, other representations of $c_p(T)$ may be required.

PROBLEM 11.75

KNOWN: Two states of ethylene (C_2H_4) are under consideration: $T_1 = T_2 = 400K$, $P_1 = 1$ bar, $P_2 = 100$ bar.

FIND: Using the Redlich-Kwong equation of state, evaluate $\Delta\bar{s}$ and $\Delta\bar{h}$.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The Redlich-Kwong equation of state is applicable at these states.

ANALYSIS: The results of Problem 11.49(b) are quoted for present use:

$$\bar{h}_2 - \bar{h}_1 = p(\bar{v}_2, T)\bar{v}_2 - p(\bar{v}_1, T)\bar{v}_1 - \frac{3a}{2bT^{1/2}} \ln\left[\frac{\bar{v}_2 + b}{\bar{v}_1 + b} \cdot \frac{\bar{v}_1}{\bar{v}_2}\right] \quad (1)$$

$$\bar{s}_2 - \bar{s}_1 = \bar{R} \ln\left(\frac{\bar{v}_2 - b}{\bar{v}_1 - b}\right) - \frac{a}{2bT^{3/2}} \ln\left[\frac{\bar{v}_2 + b}{\bar{v}_1 + b} \cdot \frac{\bar{v}_1}{\bar{v}_2}\right] \quad (2)$$

where the values of a and b are obtained using Eq. 11.8 as

$$a = 77.76 \text{ bar} \left(\frac{m^3}{kmol}\right)^2 K^{1/2}, \quad b = 0.03981 \frac{m^3}{kmol}$$

The values of $\bar{v}_1$ and $\bar{v}_2$ can be found using IT to solve the Redlich-Kwong equation knowing the pressure and temperature at each state. The results are

$$\bar{v}_1 = 33.18 \text{ m}^3/\text{kmol}, \quad \bar{v}_2 = 0.2636 \text{ m}^3/\text{kmol}$$

Inserting numerical values into Eq. (2)

$$\bar{s}_2 - \bar{s}_1 = \left(8.314 \frac{kJ}{kmol \cdot K}\right) \ln\left(\frac{.22379}{32.9164}\right) - \frac{77.76 \text{ bar}\left(\frac{m^3}{kmol}\right)^2 K^{1/2}}{2\left(.03981 \frac{m^3}{kmol}\right)(400 K)^{3/2}} \left|\frac{10^5 N/m^2}{1 \text{ bar}}\right| \left|\frac{1 kJ}{10^3 N \cdot m}\right| \ln\left[\frac{0.30341}{33.21981} \cdot \frac{33.18}{0.2636}\right]$$

$$= -43.20 \text{ kJ/kmol} \cdot K \longleftarrow \Delta\bar{s}$$

$$\bar{h}_2 - \bar{h}_1 = \left[(100 \text{ bar})(0.2636 \text{ m}^3/\text{kmol}) - (1)(33.18)\right] \left|\frac{10^5 N/m^2}{1 \text{ bar}}\right| \left|\frac{1 kJ}{10^3 N \cdot m}\right|$$

$$- \frac{(3)\left(77.76 \text{ bar}\left(\frac{m^3}{kmol}\right)^2 K^{1/2}\right)}{(2)(0.03981 \text{ m}^3/\text{kmol})(400 K)^{1/2}} \ln\left[\frac{0.30341}{33.21981} \cdot \frac{33.18}{0.2636}\right]$$

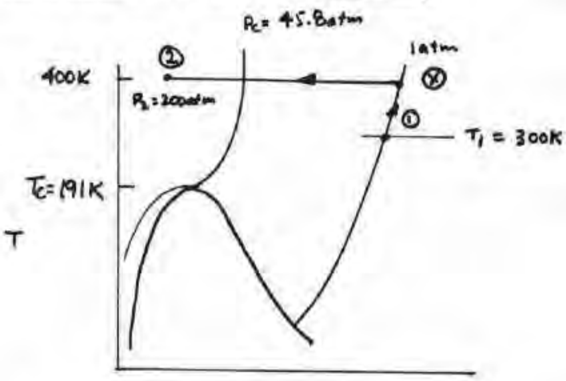
$$= -2270.6 \text{ kJ/kmol} \longleftarrow \Delta\bar{h}$$

PROBLEM 11.76

KNOWN: Two states of CH_4 are under consideration: $T_1 = 300\,K$, $p_1 = 1\,atm$, $T_2 = 400\,K$, $p_2 = 200\,atm$.

FIND: Using the Benedict-Webb-Rubin equation of state together with an appropriate specific heat relation, determine $\Delta\bar{h}$.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The ideal gas model applies along path 1–x.

ANALYSIS: The evaluation of $\Delta\bar{h}$ can be accomplished conveniently by considering two paths: 1–x followed by x–2. For $p = 1\,atm$, $p_R = 1/45.8 = 0.022$. Reference to Fig A-1 suggests that at this pressure the ideal gas model is applicable. Thus, since T_1 and $T_x = T_2$ are in the range of Table A-21

$$\bar{h}_x - \bar{h}_1 = \int_{T_1}^{T_x = T_2} \bar{c}_p(T)\,dT = \bar{R}\left[\alpha(T_2 - T_1) + \frac{\beta}{2}(T_2^2 - T_1^2) + \frac{\gamma}{3}(T_2^3 - T_1^3) + \frac{\delta}{4}(T_2^4 - T_1^4) + \frac{\epsilon}{5}(T_2^5 - T_1^5)\right] \quad (1)$$

where $\alpha, \beta, \gamma, \delta$, and ϵ are given in the table.

Since the Benedict-Webb-Rubin equation is explicit in pressure, it is convenient to evaluate $h_2 - h_x$ using

$$\bar{h}_2 - \bar{h}_x = (\bar{u}_2 - \bar{u}_x) + p_2\bar{v}_2 - p_x\bar{v}_x \quad (2)$$

where $(\bar{u}_2 - \bar{u}_x)$ is evaluated from Eq. 11.51, which reduces to give

$$\bar{u}_2 - \bar{u}_x = \int_{\bar{v}_x}^{\bar{v}_2}\left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right]d\bar{v}$$

Differentiating Eq 11.12

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{\bar{R}}{\bar{v}} + \left[B\bar{R} + \frac{2C}{T^3}\right]\frac{1}{\bar{v}^2} + \frac{b\bar{R}}{\bar{v}^3} - \frac{2c}{\bar{v}^3T^3}\left(1 + \frac{\gamma}{\bar{v}^2}\right)\exp\left(-\frac{\gamma}{\bar{v}^2}\right)$$

$$T\left(\frac{\partial p}{\partial T}\right)_v = \frac{\bar{R}T}{\bar{v}} + \left[B\bar{R}T + \frac{2C}{T^2}\right]\frac{1}{\bar{v}^2} + \frac{b\bar{R}T}{\bar{v}^3} - \frac{2c}{\bar{v}^3T^2}\left(1 + \frac{\gamma}{\bar{v}^2}\right)\exp\left(-\frac{\gamma}{\bar{v}^2}\right)$$

$$\left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right] = \left[A + \frac{3C}{T^2}\right]\frac{1}{\bar{v}^2} + \frac{a}{\bar{v}^3} - \frac{a\alpha}{\bar{v}^6} - \frac{3c}{\bar{v}^3T^2}\left(1 + \frac{\gamma}{\bar{v}^2}\right)\exp\left(-\frac{\gamma}{\bar{v}^2}\right)$$

Thus

$$\bar{u}_2 - \bar{u}_x = \left[A + \frac{3C}{T_2^2}\right]\left[\frac{1}{\bar{v}_x} - \frac{1}{\bar{v}_2}\right] + \frac{a}{2}\left[\frac{1}{\bar{v}_x^2} - \frac{1}{\bar{v}_2^2}\right] + \frac{a\alpha}{5}\left[\frac{1}{\bar{v}_2^5} - \frac{1}{\bar{v}_x^5}\right]$$
$$- \frac{3c}{T_2^2}\int_{\bar{v}_x}^{\bar{v}_2}\frac{1}{\bar{v}^3}\exp\left(-\frac{\gamma}{\bar{v}^2}\right)d\bar{v} - \frac{3c\gamma}{T_2^2}\int_{\bar{v}_x}^{\bar{v}_2}\frac{1}{\bar{v}^5}\exp\left(-\frac{\gamma}{\bar{v}^2}\right)d\bar{v} \quad (3)$$

In Eq. (3)

$$\int_{\bar{v}_x}^{\bar{v}_2} \frac{1}{\bar{v}^3}\exp\left(-\frac{\gamma}{\bar{v}^2}\right)d\bar{v} = \frac{1}{2\gamma}\left[\exp\left(-\frac{\gamma}{\bar{v}_2^2}\right) - \exp\left(-\frac{\gamma}{\bar{v}_x^2}\right)\right]$$

$$\int_{\bar{v}_x}^{\bar{v}_2} \frac{1}{\bar{v}^5}\exp\left(-\frac{\gamma}{\bar{v}^2}\right)d\bar{v} = \frac{1}{2\gamma^2}\left(\frac{\gamma}{\bar{v}_2^2}+1\right)\exp\left(-\frac{\gamma}{\bar{v}_2^2}\right) - \frac{1}{2\gamma^2}\left(\frac{\gamma}{\bar{v}_x^2}+1\right)\exp\left(-\frac{\gamma}{\bar{v}_x^2}\right)$$

Collecting results

$$\bar{u}_2-\bar{u}_x = \left[A+\frac{3C}{T_2^2}\right]\left[\frac{1}{\bar{v}_x}-\frac{1}{\bar{v}_2}\right] + \frac{a}{2}\left[\frac{1}{\bar{v}_x^2}-\frac{1}{\bar{v}_2^2}\right] + \frac{a\alpha}{5}\left[\frac{1}{\bar{v}_2^5}-\frac{1}{\bar{v}_x^5}\right] - \frac{3c}{2\gamma T_2^2}\left[\exp\left(-\frac{\gamma}{\bar{v}_2^2}\right)-\exp\left(-\frac{\gamma}{\bar{v}_x^2}\right)\right] - \frac{3c}{2\gamma T_2^2}\left[\left(\frac{\gamma}{\bar{v}_2^2}+1\right)\exp\left(-\frac{\gamma}{\bar{v}_2^2}\right) - \left(\frac{\gamma}{\bar{v}_x^2}+1\right)\exp\left(-\frac{\gamma}{\bar{v}_x^2}\right)\right]$$

Finally

$$\bar{u}_2-\bar{u}_x = \left[A+\frac{3C}{T_2^2}\right]\left[\frac{1}{\bar{v}_x}-\frac{1}{\bar{v}_2}\right] + \frac{a}{2}\left[\frac{1}{\bar{v}_x^2}-\frac{1}{\bar{v}_2^2}\right] + \frac{a\alpha}{5}\left[\frac{1}{\bar{v}_2^5}-\frac{1}{\bar{v}_x^5}\right] - \frac{3c}{2\gamma T_2^2}\left(\frac{\gamma}{\bar{v}_2^2}+2\right)\exp\left(-\frac{\gamma}{\bar{v}_2^2}\right) + \frac{3c}{2\gamma T_2^2}\left(\frac{\gamma}{\bar{v}_x^2}+2\right)\exp\left(-\frac{\gamma}{\bar{v}_x^2}\right) \quad (4)$$

The value of v_x can be found using the ideal gas equation of state: $v_x = RT_x/P_x = RT_2/P_1$. The value of v_2 can be obtained by solving the Benedict-Webb-Rubin equation. The results are

$$v_x = 32.807\ \frac{m^3}{kmol}, \quad v_2 = 0.1607\ \frac{m^3}{kmol}$$

The values of the Benedict-Webb-Rubin constants are obtained from Table A-24. Accordingly, all values that are required to evaluate Eqs. (1), (2), and (4) are known. The final result is

$$\bar{h}_2-\bar{h}_1 \approx 2305\ \frac{kJ}{kmol} \quad \left(\approx 144\ \frac{kJ}{kg}\right) \longleftarrow \Delta\bar{h}$$

PROBLEM 11.77

KNOWN: A substance has the property relations

(vapor phase) $v = \frac{RT}{p} - \frac{Bp}{T^2}$

(sat. p vs. T) $\ln P_{sat} = 12 - \frac{2400}{T}$

(ideal gas) c_{p0} = constant $0 < T < 300°F$

(Joule-Thomson) $\mu_J(10\ lbf/in^2, 200°F) = 0.004\ °R \cdot ft^2/lbf$

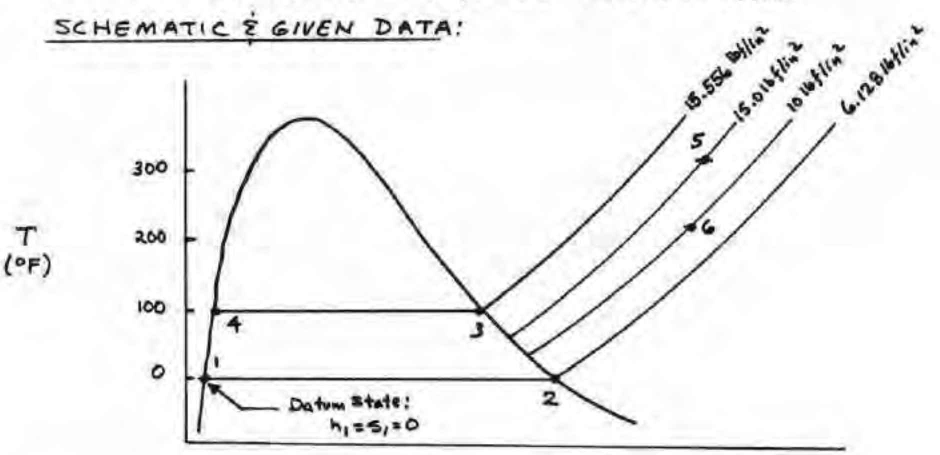
where $v: ft^3/lb$, $T: °R$, $p: lbf/ft^2$, $R = 50\ ft \cdot lbf/lb \cdot °R$, $B = 100\ ft^5(°R)^2/lb \cdot lbf$

FIND: (a) Complete the table

T(°F)	p(lbf/in²)	v (ft³/lb)		h (Btu/lb)		s (Btu/lb·°R)	
–	–	v_f	v_g	h_f	h_g	s_f	s_g
0		0.03		0		0	
100		0.03					

(b) Evaluate v, h, s at 300°F, 15 lbf/in²

SCHEMATIC & GIVEN DATA:



State 5: 300°F, 15 lbf/in²
State 6: 200°F, 10 lbf/in²

ANALYSIS: (a) Using the given expression for ln P_{sat}

at 0°F: $\ln P_{sat} = 12 - \frac{2400}{460} \Rightarrow P_2 = P_1 = 882.37\ lbf/ft^2$ (6.128 lbf/in²)

at 100°F: $\ln P_{sat} = 12 - \frac{2400}{560} \Rightarrow P_3 = P_4 = 2240.12\ lbf/ft^2$ (15.556 lbf/in²)

State 2: With the above pressure value, and the vapor phase equation of state,

$$v_2 = \frac{RT_2}{P_2} - \frac{BP_2}{T_2^2} = \frac{(50\ ft\cdot lbf/lb\cdot °R)(460°R)}{882.37\ lbf/ft^2} - \frac{(100\ ft^5(°R)^2/lb\cdot lbf)(882.37\ lbf/ft^2)}{(460°R)^2}$$

$$= 25.65\ ft^3/lb$$

Similarly

$$v_3 = \frac{RT_3}{P_3} - \frac{BP_3}{T_3^2} = \frac{(50)(560)}{2240.12} - \frac{(100)(2240.12)}{(560)^2} = 11.79\ ft^3/lb$$

$$v_5 = \frac{RT_5}{P_5} - \frac{BP_5}{T_5^2} = \frac{(50)(760)}{2160} - \frac{(100)(2240.12)}{(760)^2} = 17.20\ ft^3/lb$$

PROBLEM 11.77 (Cont'd) - page 2

Using the Clapeyron equation, Eq. 11.40,

$$h_2 = \cancel{h_f}^{\,0} + T_2(v_g - v_f)\left(\frac{dp}{dT}\right)_{sat}$$

Since

$$p_{sat} = \exp\left(12 - \frac{2400}{T}\right)$$

$$\frac{dp_{sat}}{dT} = \frac{2400}{T^2}\exp\left(12 - \frac{2400}{T}\right) = \frac{2400\,p_{sat}}{T^2}$$

So

$$h_2 = T_2(v_g - v_f)\left[\frac{2400\,p_2}{T_2^2}\right] = \frac{(2400)(v_g - v_f)\,p_2}{T_2}$$

$$= \frac{(2400°R)(25.65 - 0.08)\left(\frac{ft^3}{lb}\right)(882.37\ lbf/ft^2)}{460°R}\left|\frac{Btu}{778\ ft{\cdot}lbf}\right| = 151.60\ \frac{Btu}{lb}$$

Then, with Eq. 11.38

$$s_2 - \cancel{s_f}^{\,0} = \frac{h_2 - \cancel{h_f}^{\,0}}{T} \Rightarrow s_2 = \frac{151.60}{460} = 0.330\ \frac{Btu}{lb\cdot°R}$$

State 3. With Eq. 11.82

$$h(T_3, p_3) = h^*(T_3) + \int_0^{p_3}\left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right]dp \qquad (1)$$

$$h(T_2, p_2) = h^*(T_2) + \int_0^{p_2}\left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right]dp \qquad (2)$$

where with the given equation of state

$$\left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] = \left(\frac{RT}{p} - \frac{Bp}{T^2}\right) - T\left(\frac{R}{p} + \frac{2Bp}{T^3}\right) = -\frac{3Bp}{T^2} \qquad (3)$$

Accordingly, with Eqs. (1)-(3)

$$h_3 - h_2 = \bar{h}^*(T_3) - \bar{h}^*(T_2) + \int_0^{p_3}\left(-\frac{3Bp}{T_3^2}\right)dp - \int_0^{p_2}\left(-\frac{3Bp}{T_2^2}\right)dp$$

$$= \int_{T_2}^{T_3} c_{po}\,dT - \frac{3Bp_3^2}{2T_3^2} + \frac{3Bp_2^2}{2T_2^2} = c_{po}[T_3 - T_2] - \frac{3B}{2}\left[\left(\frac{p_3}{T_3}\right)^2 - \left(\frac{p_2}{T_2}\right)^2\right] \qquad (4)$$

where c_{po} is given in the problem statement as constant.

To calculate $h_3 - h_2$ requires the value of c_{po}. This can be determined as follows: Applying the test for exactness to Eq. 11.57 gives

$$\left(\frac{\partial c_p}{\partial p}\right)_T = -T\left(\frac{\partial^2 v}{\partial T^2}\right)_p$$

Then, paralleling the development of Eq 11.82

$$c_p(T, p) - c_{po}(T) = -T\int_0^p\left(\frac{\partial^2 v}{\partial T^2}\right)_p dp$$

From the given equation of state $(\partial^2 v/\partial T^2)_p = -6Bp/T^4$. Thus,

$$c_p(T, p) - c_{po}(T) = -T\int_0^p\left(-\frac{6Bp}{T^4}\right)dp = \frac{3Bp^2}{T^3}$$

Rearranging this

$$c_{po} = c_p(T, p) - \frac{3Bp^2}{T^3} \qquad (5)$$

Thus c_{po} can be found from the value of c_p at any state. The value of c_p at state 6 can be found using the Joule-Thomson coefficient. Thus, with Eq. 11.77

$$c_p = \frac{1}{\mu_J}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = \frac{3Bp}{\mu_J T^2}$$

At $T = 660°R$, $p = 1440\ lbf/ft^2$ this gives

$$c_p = \frac{3\,(100\ ft^5\cdot(°R)^2/lb\cdot lbf)(1440\ lbf/ft^2)}{(0.004\ °R\cdot ft^2/lbf)(660°R)^2}\left|\frac{Btu}{778\ ft\cdot lbf}\right| = 0.3187\ \frac{Btu}{lb\cdot°R}$$

Then, with Eq.(5)

$$c_{p0} = 0.3187 - \frac{(3)(100)(1440)^2}{(660)^3}\left|\frac{1}{778}\right| = 0.3159\ \frac{Btu}{lb\cdot°R}$$

Returning to Eq.(4)

$$h_3 - h_2 = 0.3159(100) - \frac{3(100\ ft^3\cdot(°R)^2/lb\cdot lbf)}{2}\left[\left(\frac{2240.12}{560}\right)^2 - \left(\frac{882.37}{460}\right)^2\right]\left(\frac{lbf/ft^2}{°R}\right)^2\left|\frac{Btu}{778\ ft\cdot lbf}\right|$$

$$= 31.59 - 2.38 = 29.21\ Btu/lb$$

Then, with the previously determined value for h_2, $h_3 = 180.81\ Btu/lb$.

With Eq. 11.90

$$s(T,p) - s^*(T,p) = \int_0^p \left[\frac{R}{p} - \left(\frac{\partial v}{\partial T}\right)_p\right] dp$$

Using the given equation of state

$$s(T,p) - s^*(T,p) = \int_0^p \left[\frac{R}{p} - \left(\frac{R}{p} + \frac{2Bp}{T^3}\right)\right] dp = -\frac{Bp^2}{T^3} \qquad (6)$$

Using Eq. (6),

$$s_3 - s_3^* = -\frac{Bp_3^2}{T_3^3}, \quad s_2 - s_2^* = -\frac{Bp_2^2}{T_2^3}$$

Accordingly

$$s_3 - s_2 = s_3^* - s_2^* - B\left[\frac{p_3^2}{T_3^3} - \frac{p_2^2}{T_2^3}\right]$$

$$= c_{p0}\ln\frac{T_3}{T_2} - R\ln\frac{p_3}{p_2} - B\left[\frac{p_3^2}{T_3^3} - \frac{p_2^2}{T_2^3}\right]$$

Substituting values

$$s_3 - s_2 = 0.3159\ln\left[\frac{560}{460}\right] - \left(\frac{50}{778}\right)\ln\frac{2240.12}{882.37} - \left(\frac{100}{778}\right)\left[\frac{(2240.12)^2}{(560)^3} - \frac{(882.37)^2}{(460)^3}\right]$$

$$= -0.00038\ \frac{Btu}{lb\cdot°R}$$

Thus, with the previously determined value for s_2, $s_3 = 0.3296\ Btu/lb\cdot°R$.

PROBLEM 11.77 (Contd.) - Page 4

State 4. Using the Clapeyron equation

$$h_3 - h_4 = T_3 (v_3 - v_4) \left(\frac{dP}{dT}\right)_{sat}$$

Thus

$$h_4 = h_3 - \frac{2400(v_3 - v_4) P_3}{T_3} = 180.81 - \frac{2400(11.79-0.03)(2240.12)}{(560)|778|}$$

$$= 35.69 \text{ Btu/lb}$$

And with Eq. 11.38

$$s_3 - s_4 = \frac{h_3 - h_4}{T_3} = 0.259 \Rightarrow s_4 = 0.0705 \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$

SUMMARY:

T(°F)	p (lbf/in²)	v_f	v_g	h_f	h_g	s_f	s_g
0	6.128	0.03	25.65	0	151.6	0	0.330
100	15.556	0.03	11.79	35.69	180.81	0.0705	0.3296

(b) To evaluate v_5, h_5, s_5.

With the equation of state, $v_5 = 17.20$ ft³/lb (see page 1).
Following the same procedures as used to evaluate h_3 and s_3

$$h_5 = h_2 + c_{p0}(T_5 - T_2) - \frac{3B}{2}\left[\left(\frac{P_5}{T_5}\right)^2 - \left(\frac{P_2}{T_2}\right)^2\right]$$

$$= 151.6 + 0.3159(300) - \frac{(3)(100)}{(2)(778)}\left[\left(\frac{2160}{760}\right)^2 - \left(\frac{882.37}{460}\right)^2\right]$$

$$= 245.52 \frac{\text{Btu}}{\text{lb}}$$ h_5

$$s_5 = s_2 + c_{p0} \ln \frac{T_5}{T_2} - R \ln \frac{P_5}{P_2} - B\left[\frac{P_5^2}{T_5^3} - \frac{P_2^2}{T_2^3}\right]$$

$$= 0.3296 + 0.3159 \ln\left(\frac{760}{460}\right) - \left(\frac{50}{778}\right) \ln \frac{2160}{882.37} - \left(\frac{100}{778}\right)\left[\frac{(2160)^2}{(760)^3} - \frac{(882.37)^2}{(460)^3}\right]$$

$$= 0.4303 \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$ s_5

11.78 In Table A-2, at temperatures up to 50°C, the values of u_f and h_f differ in most cases by 0.01 kJ/kg. Yet at each of these temperatures the product $p_{sat}v_f$ is small enough to be neglected and the table values of u_f and h_f should be the same. Comment.

ANALYSIS: At 0°C the product $p_{sat}v_f$ is calculated based on data from Table A-2 as follows:

$$p_{sat}v_f = (0.00611\text{ bar})\left(1.0002 \times 10^{-3}\frac{\text{m}^3}{\text{kg}}\right)\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N·m}}\right| = 0.000611\text{ kJ}$$

Similarly, for a range of temperatures

$\underline{T_{sat} = 30°\text{C}}$

$$p_{sat}v_f = (0.04246\text{ bar})\left(1.0043 \times 10^{-3}\frac{\text{m}^3}{\text{kg}}\right)\left|\frac{10^5}{10^3}\right| = 0.00426\text{ kJ}$$

$\underline{T_{sat} = 40°\text{C}}$

$$p_{sat}v_f = (0.07384\text{ bar})\left(1.0078 \times 10^{-3}\frac{\text{m}^3}{\text{kg}}\right)\left|\frac{10^5}{10^3}\right|\text{ kJ} = 0.00744\text{ kJ}$$

$\underline{T_{sat} = 50°\text{C}}$

$$p_{sat}v_f = (0.1235\text{ bar})\left(1.0121 \times 10^{-3}\frac{\text{m}^3}{\text{kg}}\right)\left|\frac{10^5}{10^3}\right| = 0.0125\text{ kJ}$$

The fundamental thermodynamic function used to calculate the data in Table A-2 is given in Table 11-3 of the text. The methods of calculation are discussed in the original source of the data (J.H Keenan, F. G. Keyes, P.G. Hill, and J.G. Moore, *Steam Tables*, Wiley, New York, 1969). The fundamental function used was validated extensively using experimental data, as discussed in the original work.

Based on the calculation methods available at that time, the disparity is mainly due to round-off errors. Also contributing were experimental error associated with the evaluation of the constants in the fundamental function and other calculation issues.

Nevertheless, the fundamental function in Keenen, et al, is still used extensively in software to generate data for steam and the original work has passed the test of time.

PROBLEM 11.79

FIND: Beginning with Eq. 11.90 derive Eq. 11.91.

ANALYSIS:

$$s(T,p) - s^*(T,p) = \int_0^p \left[\frac{R}{p} - \left(\frac{\partial v}{\partial T}\right)_p \right] dp$$

With $v = ZRT/p$

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{RZ}{p} + \frac{RT}{p}\left(\frac{\partial Z}{\partial T}\right)_p$$

Thus

$$\left[\frac{R}{p} - \left(\frac{\partial v}{\partial T}\right)_p \right] = \frac{R}{p}(1-Z) - \frac{RT}{p}\left(\frac{\partial Z}{\partial T}\right)_p$$

With $T = T_c T_R$, $p = p_c p_R$ $\quad (\partial Z/\partial T)_p = \frac{1}{T_c}(\partial Z/\partial T_R)_{p_R}$

$$\left[\frac{R}{p} - \left(\frac{\partial v}{\partial T}\right)_p \right] = \frac{R}{p_R p_c}(1-Z) - \frac{R T_R T_c}{p_R p_c} \cdot \frac{1}{T_c}\left(\frac{\partial Z}{\partial T_R}\right)_{p_R}$$

collecting results

$$s(T,p) - s^*(T,p) = \int_0^p \frac{R}{p_R p_c}(1-Z)\,dp - \int_0^p \frac{R T_R}{p_R p_c}\left(\frac{\partial Z}{\partial T_R}\right)_{p_R} dp$$

$$= R\left[\int_0^{p_R} (1-Z)\frac{dp_R}{p_R} - \underbrace{\int_0^{p_R} T_R \left(\frac{\partial Z}{\partial T_R}\right)_{p_R} \frac{dp_R}{p_R}}_{\frac{h^*(T) - h(T,p)}{R T_c T_R}} \right] \quad \text{(Eq. 11.84)}$$

So

$$\frac{s^*(T,p) - s(T,p)}{R} = \frac{h^*(T) - h(T,p)}{R T_R T_c} + \int_0^{p_R} (Z-1)\frac{dp_R}{p_R}$$

or on a per mole basis

$$\frac{\bar{s}^*(T,p) - \bar{s}(T,p)}{\bar{R}} = \frac{\bar{h}^*(T) - \bar{h}(T,p)}{\bar{R} T_c T_R} + \int_0^{p_R} (Z-1)\frac{dp_R}{p_R} \longleftarrow$$

PROBLEM 11.80

FIND: (a) Derive an expression giving $[u(T,v) - u^*(T)]$. (b) Derive an expression giving $[s(T,v) - s^*(T,v)]$.

ANALYSIS: (a) Eq. 11.47 gives

$$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - p$$

Integrating from specific volume v' to v at fixed T

$$u(T,v) - u(T,v') = \int_{v'}^{v}\left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right]dv$$

Adding and subtracting $u^*(T)$ on the left side

$$[u(T,v) - u^*(T)] - [u(T,v') - u^*(T)] = \int_{v'}^{v}\left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right]dv \qquad (1)$$

As pressure tends to zero at fixed temperature, the internal energy of a substance approaches that of its ideal gas model. However, $\lim_{p\to 0}$ (at fixed T) is equivalent to $\lim_{v\to\infty}$ (fixed T). Accordingly, Eq. (1) becomes in the limit as $v' \to \infty$ (at fixed T)

$$\lim_{v'\to\infty}\left\{[u(T,v) - u^*(T)] - [u(T,v') - u^*(T)]\right\} = \int_{\infty}^{v}\left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right]dv$$

or

$$u(T,v) - u^*(T) = \int_{\infty}^{v}\left[T\left[\frac{\partial p}{\partial T}\right]_v - p\right]dv \qquad (a)$$

(b) Eq. 11.34 gives

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v$$

Integrating from specific volume v' to v at fixed T

$$s(T,v) - s(T,v') = \int_{v'}^{v}\left(\frac{\partial p}{\partial T}\right)_v dv \qquad (2)$$

For an ideal gas $p = RT/v$, so $(\partial p/\partial T)_v = R/v$. Using this, Eq. (2) gives for an ideal gas

$$s^*(T,v) - s^*(T,v') = \int_{v'}^{v}\frac{R}{v}\,dv \qquad (3)$$

Subtracting Eq. (3) from Eq. (2)

$$[s(T,v) - s^*(T,v)] - [s(T,v') - s^*(T,v')] = \int_{v'}^{v}\left[\left(\frac{\partial p}{\partial T}\right)_v - \frac{R}{v}\right]dv \qquad (4)$$

As noted in part (a), as specific volume tends to infinity at fixed temperature, the specific entropy of a substance approaches that of its ideal gas model. Thus, in the limit as $v' \to \infty$ (at fixed T), Eq. (4) gives

$$s(T,v) - s^*(T,v) = \int_{\infty}^{v}\left[\left(\frac{\partial p}{\partial T}\right)_v - \frac{R}{v}\right]dv \qquad (b)$$

PROBLEM 11.81

KNOWN: The equation of state is

$$Z = 1 + B(T_R)\, p_R$$

FIND: Derive expressions for the enthalpy and entropy departures.

ANALYSIS: Beginning with Eq. 11.84

$$\frac{\bar{h}^*(T) - \bar{h}(T,p)}{\bar{R}T_c} = T_R^2 \int_0^{p_R} \left(\frac{\partial Z}{\partial T_R}\right)_{p_R} \frac{dp_R}{p_R}$$

where

$$\left(\frac{\partial Z}{\partial T_R}\right)_{p_R} = \frac{dB}{dT_R}\, p_R$$

Thus

$$\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c} = T_R^2 \int_0^{p_R} \left(\frac{dB}{dT_R}\, p_R\right) \frac{dp_R}{p_R}$$

$$= T_R^2 \left(\frac{dB}{dT_R}\right) p_R$$

Beginning with Eq. 11.91

$$\frac{\bar{s}^*(T,p) - \bar{s}(T,p)}{\bar{R}} = \frac{\bar{h}^* - \bar{h}}{\bar{R}T_c T_R} + \underbrace{\int_0^{p_R} (Z-1)\frac{dp_R}{p_R}}_{B p_R}$$

$$= \frac{T_R^2 \left(\frac{dB}{dT_R}\right) p_R}{T_R} + B p_R$$

$$= p_R T_R \frac{dB}{dT_R} + B p_R$$

$$= p_R \frac{d[T_R B]}{dT_R}$$

PROBLEM 11.82

KNOWN: An expression is provided for the enthalpy departure for use when an equation of state explicit in pressure is in hand.

FIND: (a) Derive the expression. (b) Using (a), obtain an expression for the enthalpy departure for a gas obeying the Redlich-Kwong equation. (c) Using (b), evaluate $\Delta\bar{h}$ for CO_2 in a process at 300K from 50 to 20 bar.

ANALYSIS: (a) Beginning with $h = h(T,v)$, write $dh = \left.\frac{\partial h}{\partial T}\right)_v dT + \left.\frac{\partial h}{\partial v}\right)_T dv$. Then, at fixed T

$$dh]_T = \left.\frac{\partial h}{\partial v}\right)_T dv \Rightarrow h(T,v) - h(T,v') = \int_{v'}^{v} \left(\frac{\partial h}{\partial v}\right)_T dv \tag{1}$$

With $h = u + pv$, $(\partial h/\partial v)_T = (\partial u/\partial v)_T + (\partial(pv)/\partial v)_T$. Introducing Eq. 11.47, this becomes

$$\left.\frac{\partial h}{\partial v}\right)_T = \left[T\left.\frac{\partial p}{\partial T}\right)_v - p\right] + \left.\frac{\partial(pv)}{\partial v}\right)_T \tag{2}$$

Inserting Eq. (2) into Eq. (1)

$$h(T,v) - h(T,v') = \int_{v'}^{v} \left[T\left.\frac{\partial p}{\partial T}\right)_v - p\right] dv + [pv](T,v) - [pv](T,v') \tag{3}$$

In the limit as $p \to 0$, $v' \to \infty$ and $[pv](T,v') \to RT$, $h(T,v') \to \bar{h}^*(T)$; so

$$h(T,v) - h^*(T) = \int_{\infty}^{v} \left[T\left.\frac{\partial p}{\partial T}\right)_v - p\right] dv + pv - RT$$

or

$$h^*(T) - h(T,v) = RT\left[1 - \underbrace{\frac{pv}{RT}}_{Z} - \frac{1}{RT}\int_{\infty}^{v}\left[T\left.\frac{\partial p}{\partial T}\right)_v - p\right]dv\right]$$

With $T = T_R T_c$, and expressing this on a molar basis

$$\frac{\bar{h}^*(T) - \bar{h}(T,\bar{v})}{\bar{R}\,T_c} = T_R\left[1 - Z - \frac{1}{\bar{R}T}\int_{\infty}^{\bar{v}}\left[T\frac{\partial p}{\partial T})_v - p\right]d\bar{v}\right] \longleftarrow \text{(a)}$$

(b) Using the Redlich-Kwong equation of state,

$$T\left(\frac{\partial p}{\partial T}\right)_v = \frac{\bar{R}T}{\bar{v}-b} + \frac{1}{2}\frac{a}{\bar{v}(\bar{v}+b)\sqrt{T}} = p + \frac{3}{2}\frac{a}{\bar{v}(\bar{v}+b)\sqrt{T}} \Rightarrow T\left.\frac{\partial p}{\partial T}\right)_v - p = \frac{3}{2}\frac{a}{\bar{v}(\bar{v}+b)\sqrt{T}}$$

$$\therefore \quad \frac{\bar{h}^* - \bar{h}(T,\bar{v})}{\bar{R}T_c} = T_R\left[1 - Z - \frac{3/2\,a}{\bar{R}\,T^{3/2}}\underbrace{\int_{\infty}^{\bar{v}}\frac{d\bar{v}}{\bar{v}(\bar{v}+b)}}_{-\frac{1}{b}\ln\left(1+\frac{b}{\bar{v}}\right)_{\infty}^{\bar{v}} = -\frac{1}{b}\ln\left(1+\frac{b}{\bar{v}}\right)}\right]$$

So

$$\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c} = T_R\left[1 - Z + \frac{3a}{2b\bar{R}T^{3/2}}\ln\left(1+\frac{b}{\bar{v}}\right)\right] \tag{4} \longleftarrow \text{(b)}$$

(c) At 300 K, $p_1 = 50$ bar, $p_2 = 20$ bar. Obtaining a, b from Table A-24, the Redlich-Kwong equation can be solved to give $\bar{v}_1 = 0.3475$ m³/kmol, $\bar{v}_2 = 1.119$ m³/kmol. The corresponding values of Z are $Z_1 = 0.697$, $Z_2 = 0.897$. With Eq. (4)

$$\bar{h}_2 - \bar{h}_1 = \bar{R}T\left[Z_2 - Z_1 + \frac{3a}{2b\bar{R}T^{3/2}}\ln\left[\frac{1 + b/\bar{v}_1}{1 + b/\bar{v}_2}\right]\right] \tag{5}$$

Inserting values

$$\bar{h}_2 - \bar{h}_1 = \left(8.314\frac{kJ}{kmol\cdot K}\right)(300K)\left[0.897 - 0.697 + \frac{3\left(64.43\,\text{bar}\left(\frac{m^3}{kmol}\right)^2(K)^{1/2}\right)\left|10^5 N/m^2/\text{bar}\right|}{2\left(0.02963\frac{m^3}{kmol}\right)\left(8314\frac{N\cdot m}{kmol\cdot K}\right)(300)^{3/2}}\ln\left[\frac{1 + \frac{0.02963}{0.3475}}{1 + \frac{0.02963}{1.119}}\right]\right]$$

$$= 1548\ \frac{kJ}{kmol} \longleftarrow \text{(c)}$$

PROBLEM 11.83

KNOWN: The equation of state has the form

$$Z = 1 + \left[\frac{1}{8} - \frac{27/64}{T_R}\right]\frac{P_R}{T_R}$$

FIND: For water vapor at 550°C, 20 MPa, evaluate v and c_p. Compare with data from Table A-4 and Fig. 3.9, respectively.

ANALYSIS: At 550°C, 20 MPa, Table A-4 gives $v = 0.01655$ m³/kg. Fig. 3.9 gives $c_p \approx 2.95$ kJ/kg·K.

The equation of state can be expressed alternatively as

$$\frac{pv}{RT} = 1 + \left[\frac{1}{8} - \frac{27/64}{T/T_c}\right]\left(\frac{p/p_c}{T/T_c}\right) \Rightarrow v = \frac{RT}{p} + \left[\frac{1}{8} - \frac{(27/64)T_c}{T}\right]\left[\frac{RT_c}{p_c}\right]$$

Substituting values from Table A-1

$$v = \left(\frac{8314}{18.02}\,\frac{N\cdot m}{kg\cdot K}\right)\left(\frac{823.15\,K}{200\times10^5\,N/m^2}\right) + \left(\frac{1}{8} - \frac{(27/64)(647.3)}{823.15}\right)\left(\frac{8314}{18.02}\,\frac{N\cdot m}{kg\cdot K}\right)\left(\frac{647.3\,K}{220.9\times10^5\,N/m^2}\right)$$

$$= 0.0162 \text{ m}^3/\text{kg}$$

Comparing with the steam table value

$$\% = \left(\frac{0.0162 - 0.01655}{0.01655}\right)(100) = -2.1$$

Differentiating the given equation of state

$$\left(\frac{\partial Z}{\partial T_R}\right)_{P_R} = -\frac{1}{8}P_R T_R^{-2} + \frac{27}{32}P_R T_R^{-3} \Rightarrow \frac{1}{P_R}\left(\frac{\partial Z}{\partial T_R}\right)_{P_R} = -\frac{1}{8}T_R^{-2} + \frac{27}{32}T_R^{-3}$$

Substituting this into Eq. 11.84

$$\frac{h^*(T) - h(T,p)}{RT_c} = T_R^2\int_0^{P_R}\left[-\frac{1}{8}T_R^{-2} + \frac{27}{32}T_R^{-3}\right]dP_R = \left[-\frac{1}{8} + \frac{27}{32}T_R^{-1}\right]P_R$$

or

$$h(T,p) - h^*(T) = RT_c\left[\frac{1}{8} - \frac{27}{32}\frac{1}{T_R}\right]P_R$$

But with $c_p = \partial h/\partial T)_p$, $c_{po} = dh^*/dT$, this yields upon differentiation

$$c_p - c_{po} = \frac{\partial}{\partial T_R}\left[RT_c\left[\frac{1}{8} - \frac{27}{32}\frac{1}{T_R}\right]P_R\right]_{P_R}\frac{dT_R}{dT}$$

$$= \left[\frac{27}{32}R\right]\left[\frac{P_R}{T_R^2}\right]$$

Using p_c, T_c from Table A-1, $P_R = (200/220.9) = 0.905$, $T_R = (823.15/647.3) = 1.272$. So

$$c_p - c_{po} = \left(\frac{27}{32}\right)\left(\frac{8.314}{18.02}\,\frac{kJ}{kg\cdot K}\right)\left(\frac{.905}{(1.272)^2}\right) = 0.218\ \frac{kJ}{kg\cdot K}$$

Evaluating c_{po} from the appropriate expression given in Table A-21, $c_{po} = 2.163$ kJ/kg·K. Finally

$$c_p = 2.163 + 0.218 = 2.381\ \frac{kJ}{kg\cdot K}$$

Comparing with the value from Fig. 3.9

① $$\% = \left(\frac{2.381 - 2.95}{2.95}\right)(100) = -19.3$$

---

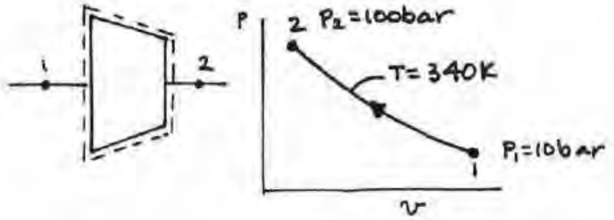
1. The simple equation of state used does a fair job in representing the p-v-T variation but does not suffice to yield other properties, such as c_p, accurately.

PROBLEM 11.84

KNOWN: Steady-state operating data are provided for a compressor for which ethylene is the working fluid.

FIND: Evaluate per kg of ethylene flowing the (a) work required, (b) heat transfer.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:
1. The control volume is at steady state.
2. Compression occurs without internal irreversibilities at fixed temperature.
3. Kinetic and potential energy effects can be ignored.

ANALYSIS: For this case, Eq. 6.51 is applicable. Since temperature is constant, $(\dot{Q}_{cv}/\dot{m})_{\substack{int\\rev}} = T(s_2 - s_1)$, where $(s_2 - s_1)$ is obtained from Eq. 11.92.

From Table A-1, $p_c = 51.2$ bar, $T_c = 283$ K. Thus, $T_R = (340/283) = 1.2$, $P_{R1} = (10/51.2) = 0.2$, $P_{R2} = (100/51.2) = 1.95$. Then, from Fig. A-5, we get $\left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_1 = 0.08$, $\left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_2 = 1.25$.

Thus

$$\bar{s}_2 - \bar{s}_1 = \underbrace{\bar{s}^\circ(T_2) - \bar{s}^\circ(T_1)}_{=0 \;\; T_1 = T_2} - \bar{R}\ln\frac{p_2}{p_1} - \bar{R}\left[\left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_2 - \left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_1\right] = -\bar{R}\left[\ln\frac{p_2}{p_1} + \left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_2 - \left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_1\right]$$

$$= -\left(8.314 \frac{kJ}{kg\cdot K}\right)\left[\ln 10 + 1.25 - 0.08\right] = -28.87 \frac{kJ}{kmol\cdot K}$$

$$\Rightarrow \left(\frac{\dot{Q}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = 340\,K\left[-28.87 \frac{kJ}{kmol\cdot K}\right]\left[\frac{1}{28.05\,kg/kmol}\right] = -350 \text{ kJ/kg} \qquad \text{(b)}$$

Reduction of mass and energy rate balances gives $(\dot{W}_{cv}/\dot{m}) = (\dot{Q}_{cv}/\dot{m}) + (h_1 - h_2)$. From Fig. A-4, we get $\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1 = 0.15$, $\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2 = 2$. Then, with Eq. 11.85

$$\bar{h}_2 - \bar{h}_1 = \underbrace{\bar{h}_2^* - \bar{h}_1^*}_{=0 \;\; T_1 = T_2} - \bar{R}T_c\left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1\right] = -(8.314)(283)\left[2 - 0.15\right] = -4352.8 \frac{kJ}{kmol}$$

$$\Rightarrow h_1 - h_2 = \frac{4352.8}{28.05} = 155 \frac{kJ}{kg}$$

Finally,

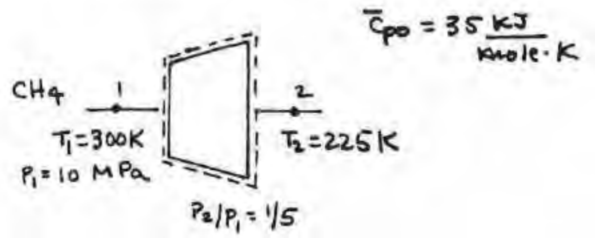
$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = -350 + 155 = -195 \text{ kJ/kg} \qquad \text{(a)}$$

PROBLEM 11.85

KNOWN: Steady-state operating data are provided for the adiabatic expansion of CH_4 through a turbine.

FIND: Determine the work developed per unit of mass flowing and compare with the value obtained using the ideal gas model only.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:
1. The control volume is at steady state.
2. $\dot{Q}_{cv} = 0$, kinetic & potential energy effects can be ignored.
3. The ideal gas specific heat $\bar{c}_{po}$ is constant.

ANALYSIS: Reducing mass and energy rate balances, we get

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right) = \cancel{\left(\frac{\dot{Q}_{cv}}{\dot{m}}\right)}^{0} + (h_1 - h_2) \Rightarrow \left(\frac{\dot{W}_{cv}}{\dot{m}}\right) = h_1 - h_2$$

, where $(h_1 - h_2)$ can be found using Eq. 11.85:

$$h_1 - h_2 = \frac{1}{M}\left[\underbrace{(\bar{h}_1^* - \bar{h}_2^*)}_{\bar{c}_{po}(T_1 - T_2)} - \bar{R}T_c\left(\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2\right)\right]$$

With data from Table A-1, $M = 16.04$ kg/kmole, $T_c = 191$ K, $p_c = 46.4$ bar. Thus

$$T_{R1} = \frac{300}{191} = 1.57, \quad T_{R2} = \frac{225}{191} = 1.18, \quad p_{R1} = \frac{100}{46.4} = 2.16, \quad p_{R2} = \frac{20}{46.4} = 0.43.$$

Then, from Fig. A-4

$$\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1 = 1, \quad \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2 = 0.34$$

So

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right) = \left[\frac{1}{16.04 \text{ kg/kmol}}\right]\left[\left(35\frac{kJ}{kmol \cdot K}\right)(300 - 225)K - \left(8.314\frac{kJ}{kmol \cdot K}\right)(191 K)\left[1 - 0.34\right]\right]$$

$$= \left(\frac{1}{16.04}\right)\left[2625 - 1048\right] = 98.3 \text{ kJ/kg}$$

The ideal gas value is

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right) = \frac{1}{M}\left[\bar{h}_1^* - \bar{h}_2^*\right] = \frac{1}{M}\left[\underbrace{\bar{c}_{po}(T_1 - T_2)}_{2625}\right] = 163.7 \frac{kJ}{kg}$$

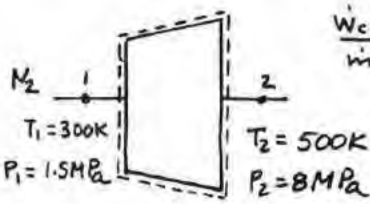
which is 67% greater than the value found using Eq. 11.85.

PROBLEM 11.86

KNOWN: Steady-state operating data is provided for a compressor for which N_2 is the working fluid.

FIND: Determine the heat transfer per unit mass of N_2 flowing.

SCHEMATIC & GIVEN DATA:



$\frac{\dot{W}_{cv}}{\dot{m}} = -240\ \frac{kJ}{kg}$

ENGINEERING MODEL:

1. The control volume is at steady state.
2. Kinetic & potential energy changes can be ignored.

ANALYSIS: Reducing mass and energy rate balances, we get

$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{\dot{W}_{cv}}{\dot{m}} + (h_2 - h_1)$, where $(h_2 - h_1)$ can be found using Eq. 11.85.

With data from Table A-1, $M = 28.01$, $T_c = 126K$, $p_c = 33.9$ bar,

$T_{R1} = \frac{300}{126} = 2.38$, $T_{R2} = \frac{500}{126} = 3.97$, $p_{R1} = \frac{15}{33.9} = 0.44$, $p_{R2} = \frac{80}{33.9} = 2.36$.

From Figure A-4

$$\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1 = 0.12, \quad \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2 = 0.09$$

From Table A-23, $\bar{h}_1^* = 8723$ kJ/kmol, $\bar{h}_2^* = 14{,}581$ kJ/kmol.

Thus

$$h_2 - h_1 = \frac{1}{M}\left[\bar{h}_2^* - \bar{h}_1^* - \bar{R}T_c\left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1\right]\right]$$

① $$= \frac{1}{28.01}\left[\underbrace{(14{,}581 - 8723)}_{5858} - \underbrace{(8.314)(126)\left[0.09 - 0.12\right]}_{-31.4}\right]$$

$$= 210.3\ kJ/kg$$

Finally,

$$\frac{\dot{Q}_{cv}}{\dot{m}} = -240\ \frac{kJ}{kg} + 210.3\ \frac{kJ}{kg} = -29.7\ \frac{kJ}{kg}$$ ◀

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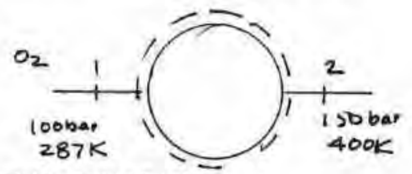
1. In this case, the ideal gas model contribution dominates. As can be seen from Fig A-2, the large T_R values give $Z \sim 1$ without regard for P_R.

PROBLEM 11.87

KNOWN: O_2 enters a control volume at steady state with a mass flow rate of 9 kg/min at 100 bar, 287K and is compressed adiabatically to 150 bar, 400K

FIND: Determine the power required and the rate of entropy production.

SCHEMATIC & GIVEN DATA:



$\dot{m} = 9 \text{ kg/min}$

ENGINEERING MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible.

ANALYSIS: Reducing mass, energy, and entropy balances

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 \quad , \quad \frac{\dot{\sigma}_{cv}}{\dot{m}} = s_1 - s_2$$

Using Eq. 11.85 and Eq. 11.92 to evaluate these differences via the generalized plots,

$$T_{R1} = \frac{287K}{154K} = 1.86 \; , \; P_{R1} = \frac{100 \text{ bar}}{50.5 \text{ bar}} = 1.98 \Rightarrow \left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right]_1 \approx 0.63 \; , \; \left[\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right]_1 \approx 0.27$$

$$T_{R2} = \frac{400}{154} = 2.60 \; , \; P_{R2} = \frac{150}{50.5} = 2.97 \Rightarrow \left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right]_2 \approx 0.43 \; , \; \left[\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right]_2 \approx 0.17$$

For O_2, Table A-23 gives $\bar{h}_1^* - \bar{h}_2^* = 8355 - 11{,}711 = -3356$ kJ/kmol. Then

$$\frac{\dot{W}_{cv}}{\dot{m}} = \frac{\bar{h}_1^* - \bar{h}_2^* - \bar{R}T_c\left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2\right]}{M} = -\frac{3356}{32} - \left(\frac{8.314}{32}\right)(154)\left[0.63 - 0.43\right]$$

$$= -104.9 - 8 = -112.9 \text{ kJ/kg}$$

Thus

$$\dot{W}_{cv} = \left(9 \frac{\text{kg}}{\text{min}}\right)\left(-112.9 \frac{\text{kJ}}{\text{kg}}\right)\left|\frac{\text{min}}{60\text{s}}\right|\left|\frac{1 \text{kW}}{1 \text{kJ/s}}\right| = -16.94 \text{ kW} \longleftarrow \dot{W}_{cv}$$

With $\Delta\bar{s}^* = \bar{s}^\circ(T_2) - \bar{s}^\circ(T_1) - \bar{R}\ln P_2/P_1$ and data from Table A-23

$$\Delta\bar{s}^* = 213.765 - 203.91 - 8.314 \ln\frac{150}{100} = 6.484 \frac{\text{kJ}}{\text{kmol}\cdot\text{K}}$$

Accordingly

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = \frac{\Delta\bar{s}^* - \bar{R}\left[\left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_2 - \left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_1\right]}{M} = \frac{6.484 - 8.314(0.17 - 0.27)}{32} = 0.2286 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

Finally

① $$\dot{\sigma}_{cv} = \left(9 \frac{\text{kg}}{\text{min}}\right)\left(0.2286 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left|\frac{\text{min}}{60\text{s}}\right|\left|\frac{1 \text{kW}}{1 \text{kJ/s}}\right| = 0.034 \frac{\text{kW}}{\text{K}} \longleftarrow \dot{\sigma}_{cv}$$

---

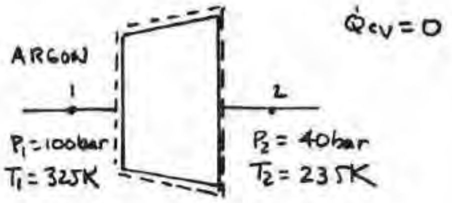
1. Owing to the approximate nature of retrieving data from the generalized charts, extreme accuracy in the calculated results should not be expected. This applies with greater effect for entropy evaluations, which are generally sensitive to roundoff.

PROBLEM 11.88

KNOWN: Steady-state operating data are provided for a turbine in which argon is the working fluid.

FIND: Determine per unit mass of argon flowing (a) the work, (b) the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The control volume is at steady state.
2. $\dot{Q}_{cv} = 0$, and kinetic & potential energy changes can be ignored.

ANALYSIS: Reducing mass, energy and entropy balances, we get

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 \quad (1) \quad , \quad \frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1 \quad (2)$$

where Eqs. 11.85, 11.92 are used to find $(h_1 - h_2)$, $(s_2 - s_1)$. With data from Table A-1, $M = 39.94$ kg/kmol, $T_c = 151$ K, $p_c = 48.6$ bar. Then,

$$T_{R1} = \frac{325}{151} = 2.15, \quad T_{R2} = \frac{235}{151} = 1.56, \quad P_{R1} = \frac{100}{48.6} = 2.06, \quad P_{R2} = \frac{40}{48.6} = 0.82$$

Also, from Table A-21, $\bar{c}_p^* = \frac{5}{2}\bar{R}$, and from Figs. A-4, A-5

$$\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1 = 0.4, \quad \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2 = 0.38, \quad \left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_1 = 0.2, \quad \left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_2 = 0.18$$

Then,

$$\frac{\dot{W}_{cv}}{\dot{m}} = \frac{1}{M}\left[\underbrace{(\bar{h}_1^* - \bar{h}_2^*)}_{2.5\bar{R}(T_1 - T_2)} - \bar{R}T_c\left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1\right]\right] = \frac{8.314}{39.94}\left[2.5(90) - 151\left[0.4 - 0.38\right]\right]$$

$$= 46.2 \text{ kJ/kg} \qquad \text{(a)}$$

And with Eq. 6.23

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = \frac{1}{M}\left[\left(\overset{2.5\bar{R}}{\bar{c}_p^*} \ln\frac{T_2}{T_1} - \bar{R}\ln\frac{P_2}{P_1}\right) - \bar{R}\left[\left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_2 - \left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_1\right]\right]$$

① $$= \frac{8.314}{39.94}\left[\left(2.5\ln\frac{235}{325} - \ln\frac{40}{100}\right) + \left[0.2 - 0.18\right]\right] = 0.026 \ \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \qquad \text{(b)}$$

---

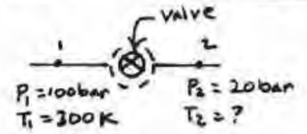
1. Owing to the approximate nature of retrieving data from the generalized charts, extreme accuracy in the calculated results should not be expected. This applies with greater effect for entropy evaluations, which are generally sensitive to roundoff.

PROBLEM 11.89

KNOWN: O_2 undergoes a throttling process from 100 bar, 300K to 20 bar.

FIND: Determine the temperature after throttling, and compare with the value obtained using the ideal gas model.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

The expansion across the valve adheres to the throttling process model of Section 4.10, for which $h_2 = h_1$.

ANALYSIS: Using Eq. 11.85 with $h_2 = h_1$, we get

$$\bar{h}^*(T_2) - \bar{R}T_c\left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right]_2 = \bar{h}^*(T_1) - \bar{R}T_c\left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right]_1 \qquad (1)$$

with data from Table A-1, $T_c = 154\text{K}$, $P_c = 50.5\text{ bar}$,

$$P_{R1} = \frac{100}{50.5} = 1.98 \;,\; P_{R2} = \frac{20}{50.5} = 0.4 \;,\; T_{R1} = \frac{300}{154} = 1.95$$

Fig. A-4 gives $\left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right]_1 = 0.5$. Table A-23 gives $\bar{h}_1^* = 8736\text{ kJ/kmol}$.

Thus, Eq. (1) becomes

$$\bar{h}^*(T_2) - (8.314)(154)\left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right]_2 = 8736\frac{\text{kJ}}{\text{kmol}} - \underbrace{\left(8.314\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(154\text{K})(0.5)}_{640.2}$$

$$= 8096\frac{\text{kJ}}{\text{kmol}} \qquad (2)$$

Equation (2) can then be used in a trial procedure with data from
① Table A-23: $\bar{h}_2^*$ and from Fig. A-4: $\left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right]_2$. Iterating, $T_2 = 280\text{K}$. ← T_2

---

1. At $T_2 = 280\text{K}$, Table A-23 gives $\bar{h}_2^* = 8150\text{ kJ/kmol}$. Using $T_{R2} = (280/154) = 1.82$ and $P_{R2} = 0.4$, Fig. A-4 gives $\left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right]_2 = 0.1$.

   With these values the left side of Eq. 2 is

$$\bar{h}^*(T_2) - (8.314)(154)\left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right]_2 = 8150 - (8.314)(154)(0.1) = 8022\frac{\text{kJ}}{\text{kmol}}$$

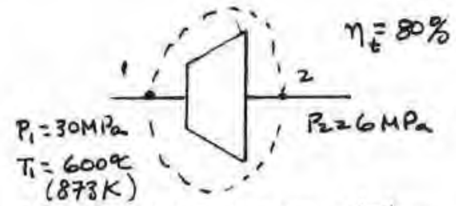
   which is within 1% of the value 8096 kJ/kmol on the right side of Eq. (2). The approximate nature of obtaining data from the generalized charts does not justify additional iteration.

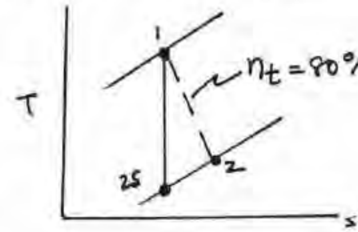
PROBLEM 11.90

KNOWN: Steady state operating data are provided for a steam turbine.

FIND: Using the generalized charts, determine the work developed per kg of steam flowing, and compare with the steam table result.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible.

ANALYSIS: Reducing mass and energy rate balances, $\dot{W}_{cv}/\dot{m} = h_1 - h_2$. Then, with the isentropic turbine efficiency

$$\frac{\dot{W}_{cv}}{\dot{m}} = \eta_t (h_1 - h_{2s})$$

STEAM TABLE SOLUTION. With steam table data, $h_1 = 3443.9\,kJ/kg$, $s_1 = 6.2331\,kJ/kg\cdot K$. Then, with $s_{2s} = s_1$, $h_{2s} = 2981.6\,kJ/kg$, $T_{2s} = 329\,°C$. Finally

$$\frac{\dot{W}_{cv}}{\dot{m}} = 0.8(3443.9 - 2981.6) = 369.8\ kJ/kg$$ ◄ steam tables

GENERALIZED CHART SOLUTION.

$$T_{R1} = \frac{873K}{647.3K} = 1.35, \quad P_{R1} = \frac{300\,bar}{220.9\,bar} = 1.36 \Rightarrow \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1 \approx 0.91 \ \text{(Fig A-4)}, \quad \left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_1 \approx 0.5 \ \text{(Fig. A-5)}$$

$$P_{R2} = \frac{60}{220.9} = 0.27$$

To fix state 2s, use $\bar{s}_2 - \bar{s}_{2s} = 0$. That is, with Eq. 11.92

$$0 = \left[\bar{s}^\circ(T_{2s}) - \bar{s}^\circ(T_1) - \bar{R}\ln\frac{P_2}{P_1}\right] - \bar{R}\left[\left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_{2s} - \left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_1\right]$$

with $\bar{s}^\circ(T_1)$ from Table A-23, this takes the form

$$\bar{s}^\circ(T_{2s}) - \bar{R}\left[\left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_{2s}\right] = 227.11 + 8.314\ln\frac{6}{30} - 8.314(0.5) = 209.57\ \frac{kJ}{kg\cdot K} \quad (1)$$

① Eq. (1) can be used iteratively to determine state 2s. We get $T_{2s} = 600K$. Thus, $(T_R)_{2s} = 0.93$. Table A-23 gives $\bar{h}_1^* = 30754\,kJ/kmol$, $\bar{h}_{2s}^* = 20402$; and Fig. A-4 gives $\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_{2s} = 0.45$. Thus, Eq. 11.85 yields

$$\bar{h}_1 - \bar{h}_{2s} = \bar{h}_1^* - \bar{h}_{2s}^* - \bar{R}T_c\left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_{2s}\right] = 30754 - 20402 - \underbrace{8.314(647.3)[0.91 - 0.45]}_{2476}$$

$$= 7876\ kJ/kmol$$

And $$\frac{\dot{W}_{cv}}{\dot{m}} = \frac{0.8}{18.02}(7876) = 349.7\,\frac{kJ}{kg}$$ ◄ generalized charts

which is about 5% less than the steam table result.

---

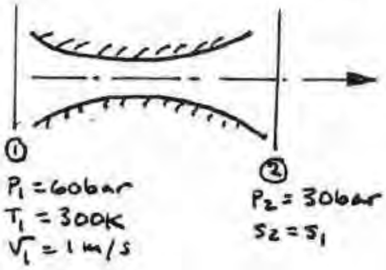
1. Using $T_{2s} = 600K$, Table A-23 gives $\bar{s}^\circ_{2s} = 212.92$. Figure A-5 gives $\left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_{2s} = 0.3$. Thus, the left side of Eq. (1) is $[(212.92) - (8.314)(0.3)] = 210.4\,\frac{kJ}{kg\cdot K}$, which is about 0.4% greater than the value on the right side of Eq. (1).

PROBLEM 11.91

KNOWN: O_2 expands isentropically through a nozzle. Steady-state operating data are provided.

FIND: Determine the velocity at the exit.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. A control volume enclosing the nozzle is at steady state.
2. The expansion is isentropic.
3. Potential energy change can be ignored. $\dot{W}_{cv} = \dot{Q}_{cv} = 0$.

ANALYSIS: Reducing mass and energy rate balances $V_2 = \sqrt{V_1^2 + 2(h_1 - h_2)}$, (1) where $(h_1 - h_2)$ can be evaluated using Eq. 11.85. First, state 2 is fixed using $s_2 = s_1$ and Eq. 11.92.

From Table A-1, $M = 32$, $T_c = 154$ K, $p_c = 50.5$ bar. Thus,

$$P_{r1} = \frac{60}{50.5} = 1.19, \quad P_{r2} = \frac{30}{50.5} = 0.59, \quad T_{r1} = \frac{300}{154} = 1.95$$

When $s_2 = s_1$, Eq. 11.92 reduces to read

$$\bar{s}^\circ(T_2) - \bar{R}\left[\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right]_2 = \underbrace{\bar{s}^\circ(T_1)}_{\substack{\text{Table A-23}\\205.213}} + \bar{R}\ln\frac{p_2}{p_1} - \underbrace{\bar{R}\left[\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right]_1}_{\substack{\text{Fig A-5}\\=0.14}} = 205.213 + 8.314(\ln 0.5 - 0.14) = 198.29 \frac{\text{kJ}}{\text{kmol}\cdot\text{K}} \quad (2)$$

① This result can be used to determine T_2 using trial with data from Table A-23, Fig A-5. We get $T_2 = 245$ K.

Then, to find $(h_1 - h_2)$ use the following expression with data from Table A-23, Fig A-4,

$$\bar{h}_1 - \bar{h}_2 = \bar{h}_1^* - \bar{h}_2^* - \bar{R}T_c\left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2\right] = 8736 - 7130 - (8.314)(154)\left[0.35 - 0.26\right]$$

$$= 1491 \frac{\text{kJ}}{\text{kmol}}$$

Then, returning to Eq. (1)

$$V_2 = \sqrt{\left(1\frac{\text{m}}{\text{s}}\right)^2 + 2\left[\frac{1491 \text{ kJ/kmol}}{32 \text{ kg/kmol}}\right]\left|\frac{1 \text{ kg}\cdot\text{m/s}^2}{1 \text{ N}}\right|\left|\frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}}\right|} = 305.3 \text{ m/s}$$

---

1. At 245 K, $T_{R2} = 1.59$. Then, with $P_{R2} = 0.59$, Fig. A-5 gives $\left[\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right]_2 = 0.12$. Table A-23 gives $\bar{s}^\circ(T_2) = 199.29$. Thus, the left-side of Eq (2) reads

$$\bar{s}^\circ(T_2) - \bar{R}\left[\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right]_2 = 199.29 - 8.314(0.12) = 198.29 \text{ kJ/kmol}\cdot\text{K},$$

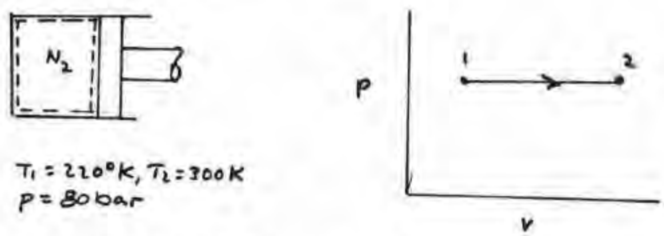
which agrees with the term on the right side of Eq. (2).

PROBLEM 11.92

KNOWN: A quantity of N_2 expands at a constant pressure of 80 bar from $T_1 = 220\,K$ to $T_2 = 300\,K$.

FIND: Determine W/n and Q/n.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The system shown in the accompanying figure experiences no change in kinetic or potential energy.

ANALYSIS: As pressure is fixed the work is given by

$$\frac{W}{n} = \int_1^2 p\,d\bar{v} = p[\bar{v}_2 - \bar{v}_1]$$

Then, since $Z = p\bar{v}/\bar{R}T$, this equation becomes

$$\frac{W}{n} = \bar{R}\,[T_2 Z_2 - T_1 Z_1] \qquad (1)$$

where Z is the compressibility factor. From Table A-1, $T_c = 126\,K$, $p_c = 33.9\,bar$, so

$p_R = \frac{80}{33.9} = 2.36$, $T_{R1} = \frac{220}{126} = 1.75$, $T_{R2} = 2.38$. From Fig. A-2, $Z_1 = 0.91$ and $Z_2 = 0.99$. Then, Eq. (1) gives

$$\frac{W}{n} = \left(8.314\,\frac{kJ}{kJ\cdot K}\right)\left[(300)(.99) - (220)(0.91)\right]K = 805\ \frac{kJ}{kmol} \qquad \longleftarrow \quad W/n$$

An energy balance reduces to $\Delta U = Q - W$ or $Q = \Delta U + W$. Thus

$$\frac{Q}{n} = (\bar{u}_2 - \bar{u}_1) + \frac{W}{n} = (\bar{u}_2 - \bar{u}_1) + (p\bar{v}_2 - p\bar{v}_1) = \bar{h}_2 - \bar{h}_1$$

Then, with Eq. 11.85

$$\frac{Q}{n} = \bar{h}^*(T_2) - \bar{h}^*(T_1) - \bar{R}T_c\left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1\right]$$

with data from Table A-23 and Fig. A-4

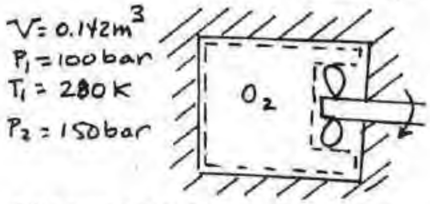
$$\frac{Q}{n} = 8723 - 6391 - (8.314)(126)\left[0.43 - 0.86\right] = 2782\,\frac{kJ}{kmol} \qquad \longleftarrow \quad Q/n$$

PROBLEM 11.93

KNOWN: O_2 contained in a closed, rigid, insulated vessel is stirred by paddle wheel. State data are provided.

FIND: Determine (a) the final temperature, (b) the work, and (c) the amount of exergy destroyed.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The system shown in the accompanying figure experiences no changes in kinetic and potential energy between the initial and final states. (2) The generalized property charts provide data with sufficient accuracy. (3) $T_0 = 7°C\ (280K)$

ANALYSIS: From Table A-1, $T_c = 154 K$, $p_c = 50.5 bar$. Thus, $p_{R1} = \frac{100}{50.5} = 1.98$, $p_{R2} = \frac{150}{50.5} = 2.97$, $T_{R1} = 1.82$. The gen. compressibility chart gives $Z_1 = 0.932, Z_2 = 1.01$

(a) Since volume is constant, $v_2 = v_1 \Rightarrow \left(\frac{ZRT}{p}\right)_2 = \left(\frac{ZRT}{p}\right)_1 \Rightarrow$

$$T_2 = \left(\frac{Z_1}{Z_2}\right)\left(\frac{P_2}{P_1}\right)T_1 = \left(\frac{0.932}{1.01}\right)\left(\frac{150}{100}\right)(280) = 387.6 K \longleftarrow T_2$$

(b) An energy balance reduces to give $\Delta U = \cancel{Q} - W \Rightarrow W = -\Delta U$, or

$W = -m[u_2 - u_1] = -m[(h - pv)_2 - (h - pv)_1] = -m[(h_2 - h_1) - v(p_2 - p_1)]$. or

since $pv = ZRT$, $W = -m[(h_2 - h_1) - R(Z_2T_2 - Z_1T_1)]$. To find m, use

$$Z_1 = \frac{p_1 V}{mRT_1} \Rightarrow m = \frac{p_1 V}{Z_1 R T_1} = \frac{(100 \times 10^5 N/m^2)(0.142 m^3)}{(0.932)\left(\frac{8314}{32}\frac{N \cdot m}{kg K}\right)(280K)} = 20.94 kg$$

$(h_2 - h_1)$ is evaluated using Eq. 11.85 with data from Table A-23, Fig. A-4:

$$h_2 - h_1 = \frac{1}{M}\left[\bar{h}_2^* - \bar{h}_1^* - \bar{R}T_c\left[\left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right]_2 - \left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right]_1\right]\right] = \frac{1}{32}\left[11{,}337 - 8150 - (8.314)(154)[.45 - .66]\right]$$
$$= 108 \ kJ/kg$$

Then

$$W = -20.94\left[108 - \frac{8.314}{32}\left((1.01)(387.6) - (0.932)(280)\right)\right] = -1551 \ kJ \longleftarrow W$$

(c) Using $E_d = T_0\sigma$, where from an entropy balance $\sigma = m(s_2 - s_1)$, together with Eq. 11.92 and data from Table A.23, Fig. A-5, we get

$$E_d = mT_0(s_2 - s_1) = \frac{mT_0}{M}\left[\left(\bar{s}^\circ(T_2) - \bar{s}^\circ(T_1) - \bar{R}\ln\frac{P_2}{P_1}\right) - \bar{R}\left[\left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_2 - \left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_1\right]\right]$$

$$= \frac{(20.94)(280)}{32}\left[212.82 - 203.19 - 8.314 \ln 1.5 - 8.314[0.18 - 0.29]\right]$$

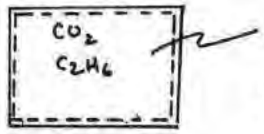
$$= 1314.4 \ kJ \longleftarrow E_d$$

PROBLEM 11.94

KNOWN: A 1 kmol mixture of CO_2 and C_2H_6 occupies a volume of $0.15 m^3$ at $T = 400 K$. $y_{CO_2} = 0.3$. Pressure should not exceed 180 bar.

FIND: Determine p using (a) the ideal gas equation of state, (b) Kay's rule and the compressibility chart, (c) the additive pressure rule and the compressibility chart.

SCHEMATIC & GIVEN DATA:



$T = 400 K$
$V = 0.15 m^3$
$n = 1$ kmol
$y_{CO_2} = 0.3$

ENGINEERING MODEL: The mixture is a closed system.

ANALYSIS: (a) Ideal Gas.

$$p = \frac{n\bar{R}T}{V} = \frac{(1\,kmol)(8314\,\frac{N\cdot m}{kmol\cdot K})(400K)}{0.15\,m^3}\left|\frac{bar}{10^5\,N/m^2}\right| = 221.7\,bar \longleftarrow (a)$$

(b) Kay's Rule plus Z chart. From Table A-1

		T_c (K)	P_c (bar)
1.	CO_2	304	73.9
2.	C_2H_6	305	48.8

Then, with Eqs. 11.97

$$T_c = y_1 T_{c1} + y_2 T_{c2} = (0.3)(304) + (0.7)(305) = 304.7\,K$$
$$p_c = y_1 p_{c1} + y_2 p_{c2} = (0.3)(73.9) + (0.7)(48.8) = 56.33\,bar$$

Accordingly, $T_R = 400/304.7 = 1.313$ and

$$v_R' = \frac{\bar{v}\,p_c}{\bar{R}T_c} = \frac{(0.15\,m^3/kmol)(56.33\times10^5\,N/m^2)}{(8314\,\frac{N\cdot m}{kmol\cdot K})(304.7K)} = 0.334$$

With these values, Fig. A-2 gives $Z \approx 0.67$, $p_R \approx 2.65$. With the Z value

$$p = Z\frac{n\bar{R}T}{V} = Z\,p_{ideal} = 0.67(221.7) = 148.5\,bar \longleftarrow (b)$$

With the p_R value

$$p = p_c\,p_R = 56.33(2.65) = 149.3\,bar$$

(c) Additive Pressure Rule plus Z chart. Selecting Eq. 11.99b

$$Z = y_1 Z_1(T,V) + y_2 Z_2(T,V)$$

$$T_{R1} = 400/304 = 1.32,\quad v_{R1}' = \frac{(0.15\,m^3/0.3\,kmol)(73.9\times10^5\,N/m^2)}{(8314\,\frac{N\cdot m}{kmol\cdot K})(304K)} = 1.46 \Rightarrow Z_1 \approx 0.9.$$

$$T_{R2} = 400/305 = 1.31,\quad v_{R2}' = \frac{(0.15\,m^3/0.7\,kmol)(48.8\times10^5\,N/m^2)}{(8314\,\frac{N\cdot m}{kmol\cdot K})(305K)} = 0.412 \Rightarrow Z_2 \approx 0.69$$

Then, $Z = (0.3)(0.9) + (0.7)(0.69) = 0.753$

and

$$p = Z\frac{n\bar{R}T}{V} = Z\,p_{ideal} = 0.753(221.7\,bar) = 166.9\,bar \longleftarrow$$

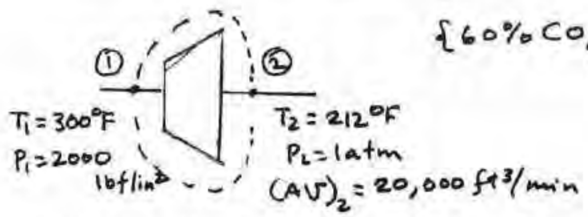
Discussion: At this state, where $Z = 0.67$, the ideal gas model is not applicable. Although methods (b), (c) yield different predictions, the methods do suggest that operation in the safe range might be realized.

PROBLEM 11.95

KNOWN: A gaseous mixture with the molar composition {60% CO, 40% H_2} enters a turbine operating at steady state at 300°F, 2000 lbf/in² and exits at 212°F, 1 atm with a volumetric flow rate of 20,000 ft³/min.

FIND: Using Kay's rule estimate the volumetric flow rate at the turbine inlet, and compare with the value yielded by the ideal gas model.

SCHEMATIC & GIVEN DATA:



{60% CO, 40% H_2}

ENGINEERING MODEL: The control volume shown in the figure is at steady state.

ANALYSIS: At steady state a mass rate balance reduces to give

$$\frac{(AV)_1}{v_1} = \frac{(AV)_2}{v_2} \Rightarrow (AV)_1 = \frac{v_1}{v_2}(AV)_2 \qquad (1)$$

Ideal Gas Model. With $v = RT/p$, Eq(1) gives

$$(AV)_1 = \left(\frac{T_1}{T_2}\right)\left(\frac{P_2}{P_1}\right)(AV)_2 = \left(\frac{760°R}{672°R}\right)\left(\frac{14.7\ lbf/in^2}{2000\ lbf/in^2}\right)\left(20,000\ \frac{ft^3}{min}\right) = 166.3\ \frac{ft^3}{min}$$

Kay's Rule. With $Z = pv/RT$, $v = ZRT/p$, and Eq.(1) becomes

$$(AV)_1 = \frac{Z_1}{Z_2}\underbrace{\left(\frac{T_1}{T_2}\right)\left(\frac{P_2}{P_1}\right)(AV)_2}_{\text{ideal gas value}} = \frac{Z_1}{Z_2}\underbrace{\left(166.3\ \frac{ft^3}{min}\right)}_{\text{ideal gas value}} \qquad (2)$$

With T_c, P_c data from Tables A-1 using the corrections for H_2 footnoted in Sec. 3.4

CO: $T_c = 133\ K$, $P_c = 34.5\ atm$ $\qquad$ H_2: $T_c = 33.2 + 8 = 41.2\ K$, $P_c = 12.8 + 8 = 20.8\ atm$

Then with Kay's rule

$$T_c = (0.6)(133) + (0.4)(41.2) = 96.3\ K$$
$$P_c = (0.6)(34.5) + (0.4)(20.8) = 29\ atm$$

Thus,

$$T_{R1} = \frac{760/1.8}{96.3} = 4.38,\quad P_{R1} = \frac{2000/14.7}{29} = 4.69 \Rightarrow Z_1 \approx 1.08 \qquad \text{(Fig A-2)}$$

$$T_{R2} = \frac{672/1.8}{96.3} = 3.88,\quad P_{R2} = 1/29 = 0.03 \Rightarrow Z_2 \approx 1.0 \qquad \text{(Fig. A-1)}$$

Finally, using Eq. (2)

$$(AV)_1 = \left(\frac{1.08}{1.0}\right)\left(166.3\ \frac{ft^3}{min}\right) = 179.6\ \frac{ft^3}{min} \longleftarrow$$

The ideal gas value is about 8% less than the volumetric flow rate predicted by Kay's rule.

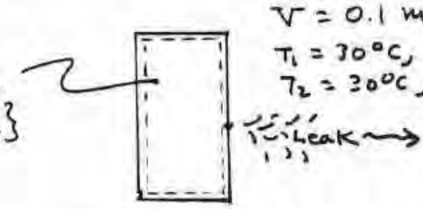
PROBLEM 11.96

KNOWN: A 0.1-m^3 cylinder contains a gaseous mixture with the molar composition {97% CO, 3% CO_2} initially at 30°C and 138 bar. Mixture leaks from the cylinder, eventually the cylinder contents are at 129 bar, 30°C.

FIND: Using Kay's rule, estimate the amount of mixture that leaks from the cylinder.

SCHEMATIC & GIVEN DATA:

{97% CO, 3% CO_2}



$V = 0.1\ m^3$
$T_1 = 30°C$, $p_1 = 138$ bar
$T_2 = 30°C$, $p_2 = 129$ bar

ENGINEERING MODEL: The control volume shown on the accompanying sketch is the system.

ANALYSIS: With $Z = pV/n\bar{R}T$; $n = pV/Z\bar{R}T$. For the control volume, the amount of mixture that leaks out equals Δn. Thus

$$n_1 - n_2 = \frac{p_1 V}{Z_1 \bar{R} T} - \frac{p_2 V}{Z_2 \bar{R} T} = \frac{V}{\bar{R} T}\left[\frac{p_1}{Z_1} - \frac{p_2}{Z_2}\right] \quad (1)$$

With data from Table A-1

CO: $T_c = 133$ K, $p_c = 35$ bar CO_2: $T_c = 304$ K, $p_c = 73.9$ bar

Kay's Rule

$$T_c = 0.97(133) + 0.03(304) = 138.1\ K$$
$$p_c = 0.97(35) + 0.03(73.9) = 36.2\ bar$$

Thus

$$p_{R1} = \frac{138}{36.2} = 3.81, \quad T_{R1} = \frac{303}{138.1} = 2.19, \quad Z_1 \approx 0.98$$

$$p_{R2} = \frac{129}{36.2} = 3.56, \quad T_{R2} = T_{R1} = 2.19, \quad Z_2 \approx 0.97$$

Accordingly, Eq. (1) gives

$$n_1 - n_2 = \frac{0.1\ m^3}{\left(8314 \frac{N\cdot m}{kmol\cdot K}\right)(303 K)}\left[\frac{138}{0.98} - \frac{129}{0.97}\right] bar \left|\frac{10^5\ N/m^2}{1\ bar}\right|$$

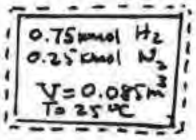
$$= 0.031\ kmol \longleftarrow \text{leak}$$

PROBLEM 11.97

KNOWN: A gaseous mixture consisting of 0.75 kmol H_2 and 0.25 kmol N_2 occupies 0.085 m³ at 25°C.

FIND: Estimate the pressure using (a) the ideal gas equation of state, (b) Kay's rule with the gen. compressibility chart, (c) van der Waals equation, (d) rule of additive pressure, using the gen. compressibility chart.

SCHEMATIC & GIVEN DATA:



ANALYSIS:

(a) Ideal gas equation of state

$$p = \frac{n\bar{R}T}{V} = \frac{(1\,\text{kmol})(8314\,\frac{N\cdot m}{kg\cdot K})(298K)}{0.085\,m^3}\left|\frac{1\,bar}{10^5\frac{N}{m^2}}\right| = 291.5\ \text{bar} \quad (a)$$

(b) Kay's rule plus gen. compressibility chart. With data from Table A-1 and using the corrections for H_2 indicated in the footnote of Sec. 3.4.

H_2: $T_{c1}' = 41.2K$ N_2: $T_{c2} = 126K$

$p_{c1}' = 21.08\,bar$: $p_{c2} = 33.9\,bar$

$$T_c = y_1 T_{c1}' + y_2 T_{c2} = (0.75)(41.2) + (0.25)(126) = 62.4\,K$$

$$p_c = y_1 p_{c1}' + y_2 p_{c2} = (0.75)(21.08) + (0.25)(33.9) = 24.3\,bar$$

$$\therefore T_R = \frac{298}{62.4} = 4.78, \quad v_R' = \frac{(V/n)p_c}{\bar{R}T_c} = \frac{(0.085\,m^3/kmol)(24.3\times10^5 N/m^2)}{(8314\frac{N\cdot m}{kmol\cdot K})(62.4K)} = 0.398$$

Figure A-3 gives $Z \cong 1.3$. Then, $p = Z\left[\frac{n\bar{R}T}{V}\right] = Z\,p_{id} = 1.3\,(291.5) = 379\,bar$ ← (b)

(c) van der Waals equation. From Table A-24, for N_2

$a_2 = 1.366\,bar\left(\frac{m^3}{kmol}\right)^2$, $b_2 = 0.0386\,m^3/kmol$. Using Eqs. 11.4 for H_2

$a_1 = 0.247$, $b_1 = 0.0265$. Then, with Eqs. 11.96

$$a = [y_1 a_1^{1/2} + y_2 a_2^{1/2}]^2 = [0.75(0.247)^{1/2} + 0.25(1.366)^{1/2}] = 0.442$$

$$b = [y_1 b_1 + y_2 b_2] = [0.25(0.0265) + 0.75(0.0386)] = 0.0296$$

Then, Eq. 11.2 gives

$$p = \frac{(8314\frac{N\cdot m}{kmol\cdot K})(298K)}{(0.085-0.0296)\frac{m^3}{kmol}}\left|\frac{1\,bar}{10^5 N/m^2}\right| - \frac{0.442\,bar(m^3/kmol)^2}{(0.085\,m^3/kmol)^2} = 386\,bar \quad (c)$$

(d) Additive pressure rule plus gen. compressibility chart. Using T_{c1}', p_{c1}' for H_2 from part (b), $T_{R1} = 298/41.2 = 7.23$,

$$v_{R1}' = \frac{\bar{v}\,p_{c1}'}{\bar{R}T_{c1}'} = \left(\frac{0.085}{0.75}\frac{m^3}{kmol}\right)\frac{(21.08\times10^5 N/m^2)}{(8314\frac{N\cdot m}{kmol\cdot K})(41.2K)} = 0.697 \quad \Big\}\ \text{Fig. A-2},\ Z_1 \sim 1.05$$

For N_2, $T_{R2} = 298/126 = 2.37$

$$v_{R2}' = \left(\frac{0.085}{0.25}\right)\frac{(33.9\times10^5)}{(8314)(126)} = 1.1 \quad \Big\}\ \text{Fig. A-2},\ Z_2 \sim 0.99$$

Then, with Eq. 11.99b,

$$Z = 0.75(1.05) + (0.25)(0.99) = 1.04$$

① $\Rightarrow$ $p = Z\frac{n\bar{R}T}{V} = Z\,p_{id} = 1.04\,(291.5) = 303\,bar$ ← (d)

1. Parts (a) and (d) are roughly in agreement. Also, parts (c) and (d) agree fairly well. A conservative choice dictated by the application at hand would govern the value that would be used for the estimated pressure.

PROBLEM 11.98

KNOWN: A gaseous mixture of 0.5 lbmol CH_4 and 0.5 lbmol of C_3H_8 occupies a volume of 7.65 ft³ at 194 °F. The measured pressure is 50 atm.

FIND: Estimate the pressure using each of six specified procedures and compare with the measured pressure.

SCHEMATIC & GIVEN DATA

$V = 7.65\,ft^3$
$T = 654\,°R$
$p = 50\,atm$

		y
1:	CH_4	0.5
2.	C_3H_8	0.5

ENGINEERING MODEL: The mixture is a closed system.

ANALYSIS: (a) Ideal Gas Equation of State.

$$p = \frac{n\bar{R}T}{V} = \frac{(1.0\,lbmol)(1545\,ft\cdot lbf/lbmol\cdot°R)(654°R)}{7.65\,ft^3}\left|\frac{atm}{14.696\times 144\,lbf/ft^2}\right| = 62.4\,atm \qquad (a)$$

(b) Kay's Rule plus Z chart. With data from Table A-1E and Eqs. 11.97

$$T_c = y_1 T_{c1} + y_2 T_{c2} = 0.5(344 + 666) = 505°R$$

$$p_c = y_1 p_{c1} + y_2 p_{c2} = 0.5(45.8 + 42.1) = 43.95\,atm$$

Then

$$T_R = 654/505 = 1.3$$

$$v_R' = \frac{\bar{v}\,p_c}{\bar{R}T_c} = \frac{(7.65/1\,\frac{ft^3}{lbmol})(43.95\times 14.696\times 144)\frac{lbf}{ft^2}}{(1545\,\frac{ft\cdot lbf}{lbmol\cdot °R})(505°R)} = 0.911$$

Fig. A-2, $Z \cong .825$. Thus

$$p = Z\frac{n\bar{R}T}{V} = Z\,p_{ideal} = (.825)(62.4) = 51.48\,atm \qquad (b)$$

(c) van der Waals Equation of State. With data from Table A-24E and Eqs. 11.96

$$a = [y_1(a_1)^{1/2} + y_2(a_2)^{1/2}]^2 = 0.25[(581)^{1/2} + (2369)^{1/2}]^2 = 1323.9\,atm\left(\frac{ft^3}{lbmol}\right)^2$$

$$b = y_1 b_1 + y_2 b_2 = 0.5[0.685 + 1.444] = 1.065\,\frac{ft^3}{lbmol}$$

Then, Eq. 11.2 gives

$$p = \frac{(1545\,\frac{ft\cdot lbf}{lbmol\cdot °R})(654°R)}{(7.65 - 1.065)\frac{ft^3}{lbmol}}\left|\frac{atm}{14.696\times 144\,\frac{lbf}{ft^2}}\right| - \frac{1323.9\,atm}{(7.65)^2} = 49.9\,atm \qquad (c)$$

(d) Additive Pressure Rule plus van der Waals Equation. Selecting Eq. 11.99a

$$p = p_1\left(T, \frac{V}{n_1}\right) + p_2\left(T, \frac{V}{n_2}\right)$$

with data from Table A-24E and Eq. 11.2

$$p_1 = \frac{(1545)(654)}{\left[\left(\frac{7.65}{0.5}\right) - 0.685\right]|14.696\times 144|} - \frac{581}{(7.65/0.5)^2} = 30.19\,atm$$

$$p_2 = \frac{(1545)(654)}{\left[\left(\frac{7.65}{0.5}\right) - 1.444\right]|14.696\times 144|} - \frac{2369}{(7.65/0.5)^2} = 24.34\,atm$$

Then

$$p = 30.19 + 24.34 = 54.5\,atm \qquad (d)$$

PROBLEM 11.98 (Contd.) - Page 2

(e) Additive Pressure Rule plus Z chart. Selecting Eq. 11.99a

$$Z = y_1 Z_1(T,V) + y_2 Z_2(T,V) = 0.5[Z_1 + Z_2]$$

$$T_{R1} = \frac{654}{344} = 1.90,\quad v_{R1}' = \frac{\left(\frac{7.65}{0.5}\right)(45.8)(14.696)(144)}{(1545)(344)} = 2.79 \Rightarrow Z_1 \approx 0.98$$

$$T_{R2} = \frac{654}{666} = 0.982,\quad v_{R2}' = \frac{\left(\frac{7.65}{0.5}\right)(42.1)(14.696)(144)}{(1545)(666)} = 1.325 \Rightarrow Z_2 \approx 0.77$$

Then
$$Z = 0.5(0.98 + 0.77) = 0.875 \quad \Rightarrow p = Z\frac{n\bar{R}T}{V} = Z\,p_{ideal} = (0.875)(62.4)$$
$$= 54.6 \text{ atm} \longleftarrow (e)$$

(f) Additive Volume Rule plus Vander Waals Equation. Selecting Eq. 11.100a

$$V = V_1(p,T) + V_2(p,T) \qquad (1)$$

With data from Table A-24E Eq 11.2 gives

$$p = \frac{(1545)(654)}{\left(\frac{V_1}{0.5} - 0.685\right)(14.696)(144)} - \frac{581}{(V_1/0.5)^2} \Rightarrow p = \frac{238.73}{V_1 - 0.3425} - \frac{145.25}{V_1^2} \qquad (2)$$

$$p = \frac{(1545)(654)}{\left(\frac{V_2}{0.5} - 1.444\right)(14.696)(144)} - \frac{2369}{(V_2/0.5)^2} \Rightarrow p = \frac{238.73}{V_2 - 0.722} - \frac{592.25}{V_2^2} \qquad (3)$$

Setting Eqs. (2) and (3) equal

$$\frac{238.73}{V_1 - 0.3425} - \frac{145.25}{V_1^2} = \frac{238.73}{V_2 - 0.722} - \frac{592.25}{V_2^2}$$

Introducing Eq. (1): $V_1 = V - V_2 = 7.65 - V_2$

$$\frac{238.73}{7.3075 - V_2} - \frac{145.25}{(7.65 - V_2)^2} = \frac{238.73}{V_2 - 0.722} - \frac{592.25}{V_2^2} \qquad (4)$$

Using an equation solver, $V_2 = 1.76\ ft^3$. Eq. (3) then gives, $p = 38.8$ atm $\longleftarrow$

Discussion: Comparing the calculated values with the measured value, 50 atm, the result is summarized as follows.

(a)	Ideal Gas Equation	62.4 atm	+ 24.8 %
(b)	Kay's Rule / Z chart	51.48 atm	+ 3.0 %
(c)	van der Waals Equation	49.9 atm	—
(d)	Additive pressure / van der Waals	54.5 atm	+ 9.0 %
(e)	Additive pressure / Z chart	54.6 atm	+ 9.0 %
(f)	Additive volume / van der Waals	38.8 atm	− 22.4 %

PROBLEM 11.99

KNOWN: One lbmol of {69.5% CO_2, 30.5 C_2H_4} occupies 1.78 ft^3 at 212°F.

FIND: Estimate the pressure using (a) the ideal gas equation of state, (b) Kay's rule with gen. compressibility chart, (c) additive pressure rule with gen. compressibility chart, (d) van der Waals equation.

SCHEMATIC & GIVEN DATA:

V = 1.78 ft3, T = 672 °R

CO_2: $y_1 = 0.695$

C_2H_4: $y_2 = 0.305$

ENGINEERING MODEL: The mixture is the closed system, as shown.

ANALYSIS: (a) Ideal Gas Model

$$p = \frac{n\bar{R}T}{V} = \frac{(1\text{ lbmol})(1545\ \frac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot°R})(672°R)}{(1.78\text{ ft}^3)\left|(144)\times(14.7)\frac{\text{lbf}}{\text{ft}^2}/\text{atm}\right|}$$

$= 275.5$ atm (a)

(b) Kay's rule with gen. compressibility chart. With data from Table A-1E, Eqs. 11.97 give

$T_c = y_1 T_{c1} + y_2 T_{c2} = (0.695)(548°R) + (0.305)(510°R) = 536.4°R$

$p_c = y_1 p_{c1} + y_2 p_{c2} = (0.695)(72.9\text{ atm}) + (0.305)(50.5\text{ atm}) = 66.07\text{ atm}$

Then, $T_R = (672/536.4) = 1.25$

$$v_R' = \frac{\bar{v}\,p_c}{\bar{R}\,T_c} = \frac{(1.78\text{ ft}^3/\text{lbmol})(66.07\times14.7\times144\text{ lbf/ft}^2)}{(1545\ \frac{\text{ft lbf}}{\text{lbmol}\,°R})(536.4°R)} = 0.3$$

Fig A-2: $Z \sim 0.61$

$\Rightarrow\quad p = Z\frac{n\bar{R}T}{V} = Z\,p_{id} = 0.61(275.5\text{ atm}) = 168.1\text{ atm}$ (b)

(c) Additive pressure rule with gen. compressibility chart.

$T_{R1} = 672/548 = 1.23$

$$v_{R1}' = \left(\frac{1.78}{0.695}\right)\frac{(72.9\times14.7\times144)}{(1545)(548)} = 0.47$$

Fig. A-2: $Z_1 \approx 0.68$

$T_{R2} = 672/510 = 1.32$

$$v_{R2}' = \left(\frac{1.78}{0.305}\right)\frac{(50.5\times14.7\times144)}{(1545)(510)} = 0.79$$

Fig. A-2: $Z_2 \approx 0.81$

With Eq. 11.99b,

$Z = (0.695)(0.68) + (0.305)(0.81) = 0.72$

$\therefore\ p = Z\,p_{id} = (0.72)(275.5\text{ atm}) = 198.4\text{ atm}$ (c)

(d) van der Waals equation. From Table A-24E, for CO_2

$a_1 = 926\text{ atm}(\text{ft}^3/\text{lbmol})^2$, $b_1 = 0.686\text{ ft}^3/\text{lbmol}$. Then, using Eqs. 11.4 for C_2H_4,

$a_2 = 1158$, $b_2 = 0.921$. With these values, Eqs. 11.96 give

$a = [(0.695)(926)^{1/2} + (0.305)(1158)^{1/2}] = 994\text{ atm}(\text{ft}^3/\text{lbmol})^2$

$b = [(0.695)(0.686) + (0.305)(0.921)] = 0.758\text{ ft}^3/\text{lbmol}$

Eq. 11.2 then gives

① $$p = \frac{(1545\text{ ft}\cdot\text{lbf/lbmol}\cdot°R)(672°R)}{(1.78-0.758)(\text{ft}^3/\text{lbmol})}\left|\frac{1\text{ atm}}{(144\times14.7)\text{ lbf/ft}^2}\right| - \frac{994\text{ atm}(\text{ft}^3/\text{lbmol})^2}{(1.78\text{ ft}^3/\text{lbmol})^2}$$

$= 166.2$ atm (d)

1. Although the results of (b) and (d) are in fair agreement, the results obtained with the four methods are scattered. A conservative choice dictated by the application at hand would govern the value that would be used for the estimated pressure.

PROBLEM 11.100

KNOWN: 100 kg of air with the molar composition {79% N_2, 21% O_2} fills a 0.36-m^3 vessel at a measured pressure and temperature of 101 bar, 180K, respectively.

FIND: Compare the measured pressure with the pressure predicted by (a) the ideal gas model, (b) Kay's rule, (c) additive pressure rule with the Redlich-Kwong equation, (d) additive volume rule with the Redlich-Kwong equation.

SCHEMATIC & GIVEN DATA:

ENGINEERING MODEL: The closed system is the 100kg of air, as shown.

① ANALYSIS: (a) Ideal gas model, $p = m\bar{R}T/MV$, where $M = y_1 M_1 + y_2 M_2$ or

$M = (0.79)(28.01) + (0.21)(32) = 28.85$. Thus

$$p = \frac{(100\,kg)\left(\frac{8314}{28.85}\,\frac{N\cdot m}{kg\cdot K}\right)(180K)}{0.36\,m^3}\left|\frac{1\,bar}{10^5 N/m^2}\right| = 144.1\,bar \quad (\sim 43\%\ \text{high})$$

(b) With data from Table A-1

N_2: $T_c = 126\,K$, $p_c = 33.9\,bar$ $\quad$ O_2: $T_c = 154\,K$, $p_c = 50.5\,bar$ $\Rightarrow$ $T_c = (0.79)(126) + (0.21)(154) = 131.9\,K$; $p_c = (0.79)(33.9) + (0.21)(50.5) = 37.39\,bar$

Then, $T_R = 180/131.9 = 1.36$

$$v_R' = \frac{v\,p_c}{\bar{R}T_c} = \left[\frac{0.36\,m^3}{(100/28.85)\,kmol}\right]\left[\frac{37.39\times10^5 N/m^2}{\left(8314\,\frac{N\cdot m}{kmol\cdot K}\right)(131.9K)}\right] = 0.354$$

Figure A-2: $Z \sim 0.7$

With $Z = pV/mRT \Rightarrow$

$$p = Z\underbrace{\left(\frac{mRT}{V}\right)}_{\text{ideal gas value}} = 0.7(144.1\,bar) = 100.9\,bar \quad \left(\begin{array}{c}\text{within 0.1\% of}\\ \text{the measured}\\ \text{value}\end{array}\right)$$

(c) Additive pressure model: $p = p_1 + p_2$, where p_1 denotes the pressure of N_2 if it occupied the full volume at 180K and p_2 denotes the pressure of O_2 if it occupied the full volume at 180K.

With $n = \frac{m}{M} = \frac{100\,kg}{28.85\,kg/kmol} = 3.466\,kmol$. Then

$n_{N_2} = 0.79(3.466) = 2.738\,kmol$, $n_{O_2} = 0.728\,kmol$

Then, with data from Table A-24 for the Redlich-Kwong equation

Nitrogen: $\bar{v} = 0.36\,m^3/2.738\,kmol = 0.131\,m^3/kmol$

$$p_1 = \frac{\left(8314\,\frac{N\cdot m}{kmol\cdot K}\right)(180K)}{(0.131 - 0.02677)\,m^3/kmol}\left|\frac{bar}{10^5 N/m^2}\right| - \frac{(15.53)\,bar}{(0.131)(0.131+0.02677)(180)^{1/2}}$$

$= 143.58\,bar - 56.01\,bar = 87.57\,bar$

PROBLEM 11.100 (Continued) - Page 2

Oxygen: $\bar{v} = 0.36/0.728 = 0.495\ m^3/kmol$

$$p_2 = \frac{(8314)(180)}{(0.495-0.02197)}\left|\frac{1}{10^5}\right| - \frac{17.22}{(0.495)(0.495+0.02197)(180)^{1/2}}$$

$$= 31.64 - 5.02 = 26.62\ bar$$

Then, $p = p_1 + p_2 = 87.57 + 26.62 = 114.2\ bar$ (13% higher than the measured value)

(d) Additive volume model: $V = V_1 + V_2$, where V_1 is the volume occupied by N_2 if it were at the mixture temperature and pressure and V_2 is the volume occupied by O_2 if it were at the mixture temperature and pressure. Thus, $V = n_1\bar{v}_1(T,p) + n_2\bar{v}_2(T,p)$, or

$$\frac{V}{n} = y_1\bar{v}_1(T,p) + y_2\bar{v}_2(T,p) \qquad (1)$$

Also, with the Redlich-Kwong equation, when each gas is at the same T, p

$$p = \frac{\bar{R}T}{\bar{v}_1 - b_1} - \frac{a_1}{\bar{v}_1(\bar{v}_1+b_1)\sqrt{T}} = \frac{\bar{R}T}{\bar{v}_2 - b_2} - \frac{a_2}{\bar{v}_2(\bar{v}_2+b_2)\sqrt{T}} \qquad (2)$$

Eqs. (1), (2) are simultaneous equations for $\bar{v}_1, \bar{v}_2$. Inserting values

$$\begin{cases} 0.1039 = 0.79\,\bar{v}_1 + 0.21\,\bar{v}_2 & (1)' \\ \dfrac{(8314)(180)}{(\bar{v}_1 - 0.02677)(10^5)} - \dfrac{15.53}{\bar{v}_1(\bar{v}_1+0.02677)(180)^{1/2}} = \dfrac{(8314)(180)}{(\bar{v}_2-0.02197)(10^5)} - \dfrac{17.22}{\bar{v}_2(\bar{v}_2+0.02197)(180)^{1/2}} & (2)' \end{cases}$$

With an equation solver, $\bar{v}_1 = 0.11$, $\bar{v}_2 = 0.078$. Using $\bar{v}_1$, Eq (2) gives $p = 102.86$ bar. Using $\bar{v}_2$, Eq. (2) gives $p = 102.49$ bar. Thus, $p \approx 102.7$ bar, which is about 1.7% higher than the measured value.

---

1. $M_{mix} = \dfrac{m_{mix}}{n_{mix}} = \dfrac{m_1 + m_2}{n_{mix}} = \dfrac{M_1 n_1 + M_2 n_2}{n_{mix}} = y_1 M_1 + y_2 M_2$

PROBLEM 11.101

KNOWN: A gaseous mixture with the molar analysis below is contained in a tank at 20 atm, 320°R.

1. Argon : $y_1 = .50$
2. N_2 : $y_2 = .50$

FIND: Estimate the specific volume using each of four specified procedures.

SCHEMATIC & GIVEN DATA:

$T = 320°R$
$p = 20\ atm$

$y_1 = y_2 = 0.5$

ENGINEERING MODEL: The mixture is the closed system, as shown.

ANALYSIS: (a) Ideal Gas Model

$$v = \frac{(\bar{R}/M)T}{p} = \frac{\left(\frac{1545}{33.975}\ \frac{ft \cdot lbf}{lb \cdot °R}\right)(320°R)}{(20)(14.696)(144)\ lbf/ft^2} = 0.344\ ft^3/lb$$

where the mixture molecular weight is found using

$$M_{mix} = \frac{m_{mix}}{n_{mix}} = \frac{m_1 + m_2}{n_{mix}} = \frac{M_1 n_1 + M_2 n_2}{n_{mix}} = y_1 M_1 + y_2 M_2$$
$$= 0.5(39.94 + 28.01) = 33.975$$

(b) Kay's Rule plus Z chart. With data from Table A-1E and Eqs. 11.97

$$T_c = y_1 T_{c1} + y_2 T_{c2} = 0.5[272 + 227] = 249.5°R$$
$$p_c = y_1 p_{c1} + y_2 p_{c2} = 0.5[47.97 + 33.5] = 40.74\ atm$$

Then

$$\left.\begin{aligned} T_R &= \frac{320}{249.5} = 1.283 \\ p_R &= \frac{20}{40.74} = 0.49 \end{aligned}\right\} : \quad Z \approx 0.92, \quad v_R' \approx 2.40$$

With $Z = 0.92$

$$V = Z\ \frac{n R T}{p} = Z\ V_{ideal} = 0.92(1.03) = 0.95\ ft^3$$

(b)

With $v_R' = 2.4$

$$v = v_R' \frac{R T_c}{p_c} = \frac{2.4\ (1545/33.975)(249.5)}{(40.74)(14.696)(144)} = 0.316\ \frac{ft^3}{lb}$$

(c). Redlich - Kwong Equation. Using data from Table A-24E for N_2

$a_2 = 5280\ atm\left(\frac{ft^3}{lbmol}\right)^2 (°R)^{1/2}$, $b_2 = 0.4286\ ft^3/lbmol$

For argon use Eqs. 11.8 together with critical data from Table A-1E

$$a_1 = \frac{0.42748\left[0.73\ \frac{atm\, ft^3}{lbmol \cdot °R}\right]^2 [272°R]^{5/2}}{47.97\ atm} = 5794.5\ atm\left(\frac{ft^3}{lbmol}\right)^2 (°R)^{1/2}$$

$$b_1 = 0.08664\left(\frac{(0.73)(272)}{47.97}\right) = 0.3586\ ft^3/lbmol$$

Then, using Eqs. 11.96

$$a = \left[y_1 a_1^{1/2} + y_2 a_2^{1/2}\right]^2 = 0.25\left[(5280)^{1/2} + (5794.5)^{1/2}\right]^2 = 5534.3\ atm\left(\frac{ft^3}{lbmol}\right)^2 (°R)^{1/2}$$

$$b = y_1 b_1 + y_2 b_2 = 0.5[0.4286 + 0.3586] = 0.3936\ ft^3/lbmol$$

PROBLEM 11.101 (Contd.) - Page 2

Then, the Redlich - Kwong equation of state becomes

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}(\bar{v}+b)T^{1/2}}$$

$$20\,\text{atm} = \frac{\left(0.73\,\frac{\text{atm}\cdot\text{ft}^3}{\text{lbmol}\cdot{}^\circ\text{R}}\right)(320^\circ\text{R})}{(\bar{v}-0.3936)\,\text{ft}^3/\text{lbmol}} - \frac{5534.3\ \text{atm}\left(\frac{\text{ft}^3}{\text{lbmol}}\right)({}^\circ\text{R})^{1/2}}{\bar{v}(\bar{v}+0.3936)\left(\frac{\text{ft}^3}{\text{lbmol}}\right)^2(320^\circ\text{R})^{1/2}}$$

or

$$20 = \frac{233.6}{\bar{v}-0.3936} - \frac{309.37}{\bar{v}(\bar{v}+0.3936)}$$

With an equation solver, $\bar{v} = 10.73$ ft³/lbmol. Finally

$$v = \frac{10.73}{33.975} = 0.316\ \text{ft}^3/\text{lb} \qquad \longleftarrow \text{(c)}$$

(d) Additive volume rule with gen. compressibility chart. With data from Table A-1E

ARGON: $T_{R1} = \frac{320}{272} = 1.18$, $p_{R1} = \frac{20}{47.97} = 0.42$ $\Big\}\ Z_1 = 0.92$ $\qquad$ N_2: $T_{R2} = \frac{320}{227} = 1.41$, $p_{R2} = \frac{20}{33.5} = 0.6$ $\Big\}\ Z_2 = 0.94$

Then, with Eq. 11.100b, $Z = (0.5)(.92) + (0.5)(0.94) = 0.93$. Since $Z = p\bar{V}/mRT$

① $$v = \frac{ZRT}{p} = Z\,v_{id} = 0.93\left(0.344\,\frac{\text{ft}^3}{\text{lb}}\right) = 0.32\ \text{ft}^3/\text{lb} \qquad \longleftarrow \text{(d)}$$

---

1. Methods (b)-(d) give estimates for specific volume of about 0.32 ft³/lb.

PROBLEM 11.102

KNOWN: A mixture rule is provided for the Carnahan-Starling-DeSantis equation of state

FIND: Evaluate the pressure, in kPa, for a mixture of R12 and R13, in which R12 is 40% by mass, at $v = 0.005\ m^3/kg$, $T = 180°C$

ANALYSIS: Rearranging the equation of state

$$p = \frac{\bar{R}T}{\bar{v}}\left\{\frac{1+\beta+\beta^2-\beta^3}{(1+\beta)^3} - \frac{a}{\bar{R}T(\bar{v}+b)}\right\}$$

The term in brackets {...} is seen to "correct" the pressure that an ideal gas would exhibit at the same state: $P_{id} = \bar{R}T/\bar{v}$. Begin by calculating this pressure.

The molar masses are $M_{12} = 120.92$, $M_{13} = 104.5$. Considering a typical 1 kg of mixture, then, $m_{12} = 0.4$ kg, $m_{13} = 0.6$ kg, and so

$$n_{12} = \frac{0.4}{120.92},\ n_{13} = \frac{0.6}{104.5} \Rightarrow y_{12} = \frac{\left(\frac{0.4}{120.92}\right)}{\left(\frac{0.4}{120.92}\right)+\left(\frac{0.6}{104.5}\right)} = 0.3655$$

$$y_{13} = 1 - y_{12} = 0.6345$$

Accordingly, for the mixture

(1) $$M = y_{12}M_{12} + y_{13}M_{13} = 110.5\ \frac{kg}{kmol}$$

Then $\bar{v} = Mv = \left(110.5\ \frac{kg}{kmol}\right)\left(0.005\ \frac{m^3}{kg}\right) = 0.5525\ m^3/kmol$. Using this

$$P_{id} = \frac{\bar{R}T}{\bar{v}} = \left(\frac{8314\ \frac{N\cdot m}{kmol\cdot K}}{0.5525\ m^3/kmol}\right)(453\,K)\left|\frac{1\ kPa}{10^3\ N/m^2}\right| = 6817\ kPa$$

Turning next to the term in brackets {...}, the individual values of a, b must be determined for each component. That is, for R12

$$a_{12} = a_0 \exp(a_1 T + a_2 T^2) = 3524.12 \exp\left(-2.77230\left(\frac{453}{10^3}\right) - 0.67318\left(\frac{453}{10^3}\right)^2\right)$$
$$= 3524.12 \exp(-1.25585 - 0.13814) = 874.276\ \frac{J\cdot L}{(mol)^2}$$
$$= 874{,}276\ \frac{J\cdot m^3}{(kmol)^2}$$

$$b_{12} = b_0 + b_1 T + b_2 T^2 = 0.15376 - \frac{1.84195}{10}\left(\frac{453}{10^3}\right) - \frac{5.03644}{10^2}\left(\frac{453}{10^3}\right)^2$$
$$= 0.15376 - 0.08344 - 0.01034 = 0.05998\ \frac{m^3}{kmol}$$

And for R13

$$a_{13} = 2298.13 \exp\left(-3.41828(.453) - 1.52430(.453)^2\right)$$
$$= 2298.13 \exp(-1.54848 - 0.3128) = 357.298\ \frac{J\cdot L}{(mol)^2} = 357{,}298\ \frac{J\ m^3}{(kmol)^2}$$

$$b_{13} = 0.12814 - \frac{1.84474}{10}(.453) - \frac{10.7851}{10^2}(.453)^2$$
$$= 0.12814 - 0.08357 - 0.02215 = 0.0224\ \frac{m^3}{kmol}$$

Next, using the given mixture rule, mixture values for a & b are calculated as follows:

$$b = y_{12}\, b_{12} + y_{13}\, b_{13} = (0.3655)(0.05998) + (0.6345)(0.0224)$$
$$= 0.03614 \ \frac{m^3}{kmol}$$

$$a = y_{12}^2 a_{12} + 2y_{12}y_{13}(1-f)(a_{12}a_{13})^{1/2} + y_{13}^2\, a_{13}$$

$$= \left[(.3655)^2(.874276) + 2(.3655)(.6345)(1-.035)\left[(.874276)(.357298)\right]^{1/2} + (0.6345)^2(.357298)\right] \times 10^6$$

$$= \left[0.116795 + 0.250159 + 0.143845\right] \times 10^6 = 510{,}799 \ \frac{J \cdot m^3}{(kmol)^2}$$

Also,

$$\beta = \frac{b}{4\bar{v}} = \frac{0.03614}{4(0.5525)} = 0.01635$$

Accordingly

$$\left\{ \frac{1+\beta+\beta^2-\beta^3}{(1+\beta)^3} - \frac{a}{\bar{R}T(\bar{v}+b)} \right\} = 0.96831 - \frac{510{,}799 \ \frac{J \cdot m^3}{(kmol)^2}}{\left(8314 \frac{J}{kmol \cdot K}\right)(453K)\left(0.58864 \frac{m^3}{kmol}\right)}$$

$$= 0.7379$$

Collecting results

$$p = (6817 \text{ kPa})(0.7379)$$

② $$= 5030 \text{ kPa} \longleftarrow p$$

---

1. See Sec. 12.1, Eq. 12.9.

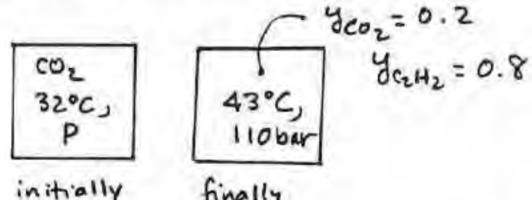
2. The calculated value is within 9% of the value read from a thermodynamic diagram for the mixture: ≈5500 kPa. See "Thermodynamic Diagrams for Refrigerant Mixtures" by J.S. Gallagher et. al. ASHRAE TRANS., 1988 (Part 2), 2119-2135.

PROBLEM 11.103

KNOWN: A rigid vessel initially contains CO_2 at 32°C and pressure p. C_2H_4 enters the tank until a mixture of 20% CO_2, 80% C_2H_4 (molar basis) is contained at 43°C, 110 bar.

FIND: Determine the pressure p.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. Kay's rule together with the gen. compressibility chart suffices.

ANALYSIS: The pressure p can be found, in principle, from the gen. compressibility chart using $p = p_R p_c$, where $p_c = 73.9$ bar from Table A-1. p_R is fixed by $T_R = T/T_c = 305/304 = 1.0$ and

$$v_R' = \frac{\bar{v}\, p_c}{\bar{R}\, T_c} = \frac{\bar{v}\,(73.9 \times 10^5\ \mathrm{N/m^2})}{(8314\ \mathrm{N\cdot m/kmol\cdot K})(304\ \mathrm{K})} = 2.924\, \bar{v} \qquad (1)$$

where $\bar{v}$ is the molar specific volume of CO_2 initially. The specific volume $\bar{v}$ is determined next.

For the final mixture, $\dfrac{V}{n_{mix}} = Z_{mix} \dfrac{\bar{R}\, T_{mix}}{p_{mix}} = Z_{mix}\left(\dfrac{(8314)(316)}{110 \times 10^5}\right) = 0.239\, Z_{mix}$

Since the amount of CO_2 present initially and finally is the same, we can write $n_{CO_2} = 0.2\, n_{mix}$. Thus

$$\bar{v} = \frac{V}{n_{CO_2}} = \frac{V}{0.2\, n_{mix}} = 5\left(\frac{V}{n_{mix}}\right).$$

Combining the last two expressions, $\bar{v} = 5(0.239)\, Z_{mix}$. (2)

To find Z_{mix}, use Kay's rule with Table A-1 data:

$$T_c = (yT_c)_{CO_2} + (yT_c)_{C_2H_4} = (0.2)(304) + (0.8)(283) = 287.2\ \mathrm{K}$$

$$p_c = (yp_c)_{CO_2} + (yp_c)_{C_2H_4} = (0.2)(73.9) + (0.8)(51.2) = 55.74\ \mathrm{bar}$$

Then, for the final mixture

$$(T_R)_{mix} = \frac{316}{287.2} = 1.1, \quad (p_R)_{mix} = \frac{110}{55.74} = 1.97 \Longrightarrow \underset{(\text{Fig A-2})}{Z_{mix} = 0.4}$$

Accordingly, Eq. (2) gives $\bar{v} = 5(0.239)(0.4) = 0.478\ \mathrm{m^3/kmol}\,(CO_2)$. And Eq. (1) gives

$$v_R' = (2.924)(0.478) = 1.4$$

With $T_R = 1$, Fig. A-1 gives $p_R = 0.57$. Then $p = (0.57)(73.9\ \mathrm{bar}) = 42.1\ \mathrm{bar}$

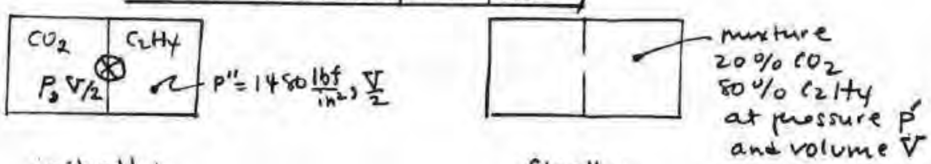
PROBLEM 11.104

KNOWN: Two equal-volume tanks are connected by a valve. Initially, one tank contains CO_2 at 560°R, pressure p. The other tank contains C_2H_4 at 560°R, 1480 lbf/in². The gases are allowed to mix until a final mixture of 20% CO_2, 80% C_2H_4 is achieved at 560°R and pressure p′.

FIND: Determine the pressures p and p′.

SCHEMATIC & GIVEN DATA:

T = 560°R initially and finally



initially　　　　　finally

ANALYSIS: Let's begin by evaluating the specific volume of the ethylene in the initial condition. With data from Table A-1E for C_2H_2, $T_R = (560°R/510°R) = 1.1$, $P_R = \frac{(1480/14.7)}{50.5} = 1.99$. Then, Fig A-2 gives $Z = 0.4$. Using this

$$\bar{v}_{C_2H_4} = \frac{Z\bar{R}T}{p''} = \frac{(0.4)(1545\ ft\cdot lbf/lbmol\cdot°R)(560°R)}{(1480\times 144\ lbf/ft^2)} = 1.62\ \frac{ft^3}{lbmol}$$

Since the amount of C_2H_4 present initially and finally is the same, we can write $n_{C_2H_4} = 0.8\,n$, where n is the total amount of mixture at the final condition. Accordingly, the specific volume $\bar{v}_{C_2H_4}$ is

$$\bar{v}_{C_2H_4} = \frac{V/2}{n_{C_2H_4}} = \frac{V/2}{0.8n} = \frac{1}{1.6}\left(\frac{V}{n}\right) \Rightarrow \frac{V}{n} = 1.6\,\bar{v}_{C_2H_4} = 1.6\left(1.62\ \frac{ft^3}{lbmol}\right) = 2.59\ \frac{ft^3}{lbmol}$$

where V/n is the specific volume of the mixture at the final condition. Next, using Kay's rule

$$T_c = (yT_c)_{CO_2} + (yT_c)_{C_2H_4} = (0.2)(548) + (0.8)(510) = 518°R$$
$$P_c = (yP_c)_{CO_2} + (yP_c)_{C_2H_4} = (0.2)(72.9) + (0.8)(50.5) = 55\ atm$$

Thus, for the final condition, $T_R = (560/518) = 1.08$ and

$$v_R' = \frac{\bar{v}\,P_c}{\bar{R}T_c} = \frac{(2.59\ ft^3/lbmol)(55\times 14.7\times 144\ lbf/ft^2)}{(1545\ ft\cdot lbf/lbmol\cdot°R)(518°R)} = 0.377$$

Referring again to Fig. A-2, we get $P_R = 1.4$, and thus $p' = P_R P_c = 1.4\,(55\,atm) = 77\,atm$ or $p' = (77\times 14.7) = 1132\ lbf/in^2$. ◀ p

Considering next CO_2 in the initial condition, its specific volume is

$$\bar{v}_{CO_2} = \frac{V/2}{n_{CO_2}} = \frac{V/2}{0.2n} = \left(\frac{1}{0.4}\right)\left(\frac{V}{n}\right) = \left(\frac{1}{0.4}\right)(2.59) = 6.48\ ft^3/lbmol.$$

Accordingly, for the CO_2, $T_R = (560/548) = 1.02$ and

$$v_R' = \frac{(6.48)(72.9\times 14.7\times 144)}{(1545)(560)} = 1.16$$

Reffering to Fig. A-1, $P_R = 0.67$. Then, $p = P_R P_c = 0.67(72.9)(14.7) = 718\ lbf/in^2$ ◀ p′

PROBLEM 11.105

KNOWN: A binary solution at 25°C consists of 59 kg of C_2H_5OH and 41 kg of water. The partial molal volumes are 0.0573 and 0.0172 $m^3/kmol$, respectively.

FIND: Determine the total volume using (a) the partial molal volumes and (b) the molar specific volumes of the pure components, each a liquid at 25°C.

ENGINEERING MODEL: The solution is the closed system.

ANALYSIS: (a) With Eq. 11.104

$$V = n_1 \bar{V}_1 + n_2 \bar{V}_2$$

$$= \left(\frac{59\text{ kg}}{46.07\text{ kg/kmol}}\right)\left(0.0573\frac{m^3}{kmol}\right) + \left(\frac{41\text{ kg}}{18.02\text{ kg/kmol}}\right)\left(0.0172\frac{m^3}{kmol}\right)$$

$$= 0.0734 + 0.0391 = 0.1125\ m^3 \longleftarrow$$

(b) Table A-2 gives for liquid water at 25°C, $v = \frac{1.0029}{10^3}\frac{m^3}{kg}$. A handbook value for the specific volume of liquid C_2H_5OH at 25°C is $\frac{1.2738}{10^3}\frac{m^3}{kg}$. Thus

$$V = m_1 v_1 + m_2 v_2$$

$$= (59\text{ kg})\left(\frac{1.2738}{10^3}\frac{m^3}{kg}\right) + (41\text{ kg})\left(\frac{1.0029}{10^3}\frac{m^3}{kg}\right)$$

$$= 0.0752 + 0.0411 = 0.1163\ m^3$$ (about 3% above the value obtained previously)

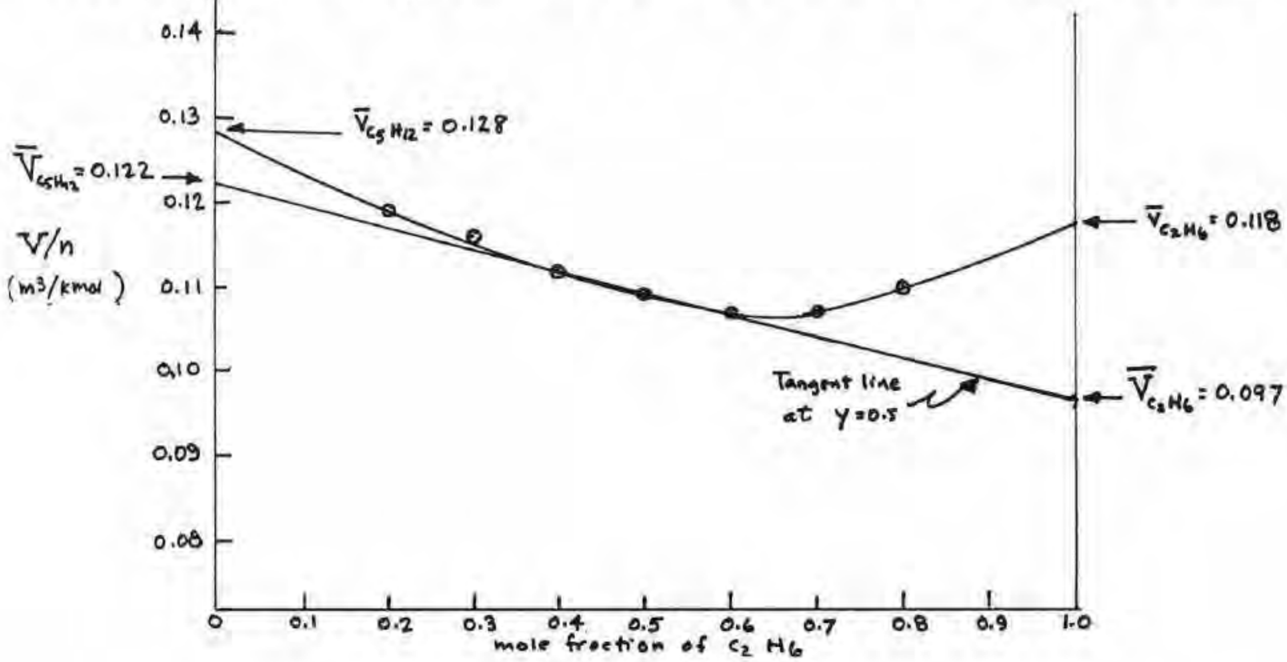
PROBLEM 11.106

KNOWN: Data are provided for a solution of C_2H_6 and C_5H_{12}.

FIND: Estimate (a) the specific volumes of C_2H_6 and C_5H_{12}, (b) the partial molal volumes of C_2H_6 and C_5H_{12} for an equimolar solution.

ENGINEERING MODEL: The solution is the closed system.

ANALYSIS: Using the data provided, the following plot can be developed. The method of intercepts (Sec. 11.9.1) allows the partial molal volumes to be found.



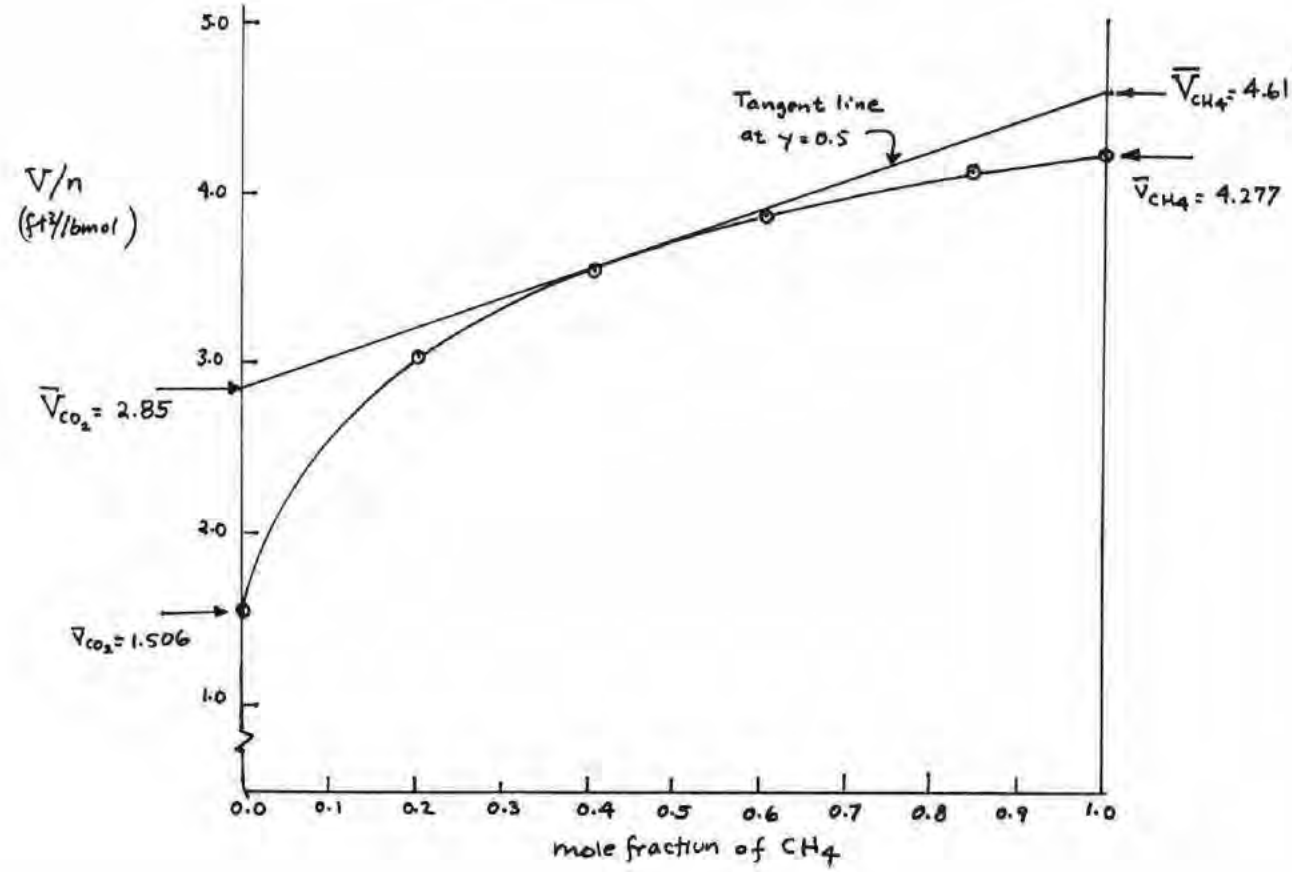
PROBLEM 11.107

KNOWN: Data are provided for a mixture of CO_2 and CH_4.

FIND: Estimate (a) the specific volumes of CO_2 and CH_4, (b) the partial molal volumes of CO_2 and CH_4 for an equimolar mixture.

ENGINEERING MODEL: The mixture is the closed system.

ANALYSIS: Using the data provided, the following plot can be developed. The method of intercepts (Sec. 11.9.1) allows the partial molal volumes to be found.



PROBLEM 11.108

KNOWN: Water as a saturated vapor at (a) 280°C, (b) 500°F is under consideration.

FIND: Using p-v-T data from the steam tables determine the fugacity at each state and compare with the value yielded by the generalized fugacity chart.

ANALYSIS: (a) Saturated vapor at 280°C.

From Table A-1 $T_c = 647.3$ K, $P_c = 220.9$ bar. Then $T_R = 553/647.3 = 0.854$ ($P_R = 64.12/220.9 = 0.29$) Then, Fig. A-6 gives $f/p \approx 0.825$. Thus

$$f = 0.825(64.12) = 52.9 \text{ bar} \qquad \longleftarrow \text{chart}$$

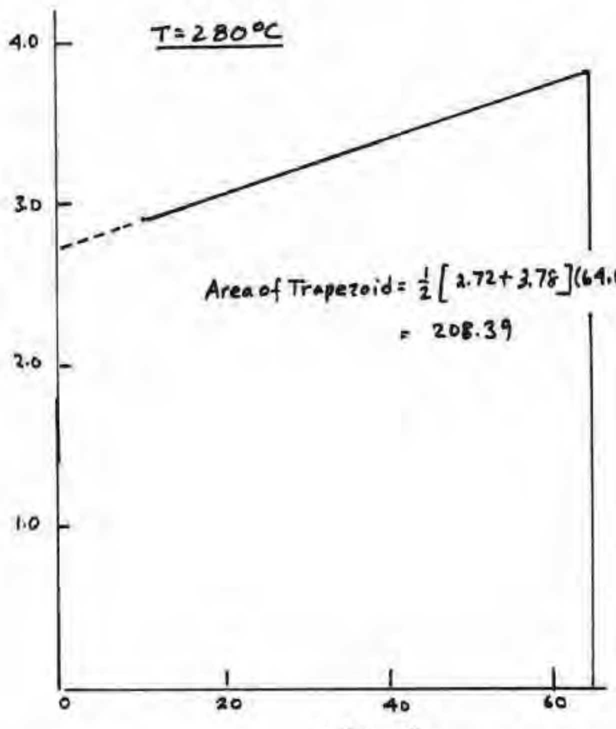
From the discussion of Sec. 11.9.4

$$\ln \frac{f}{p} = \int_0^p \left(\frac{Z-1}{p}\right) dp$$

Graphical Solution:

Using p-v-T data from the steam tables the following plot can be drawn at T = 280°C

$\left(\frac{1-Z}{p}\right) \times 10^3$



Area of Trapezoid $= \frac{1}{2}\left[2.72 + 3.78\right](64.12)$
$= 208.39$

$$\Rightarrow \ln \frac{f}{p} = -208.39 \times 10^{-3}$$

$$\Rightarrow \frac{f}{p} = 0.812$$

$$\Rightarrow f = 52.1 \text{ bar} \qquad \longleftarrow \text{table data}$$

The answers obtained using these two graphical methods agree to within 2%.

(b) Saturated vapor at 500°F

From Table A-1E $T_c = 1165$°R, $P_c = 218$ atm. Then, $T_R = 960/1165 = 0.82$ ($P_R = \frac{680/14.696}{218} = 0.212$). Fig A-6 gives $f/p \approx 0.87$ Thus

$$f = 0.87\,(680) = 592 \text{ lbf/in}^2 \qquad \longleftarrow \text{chart}$$

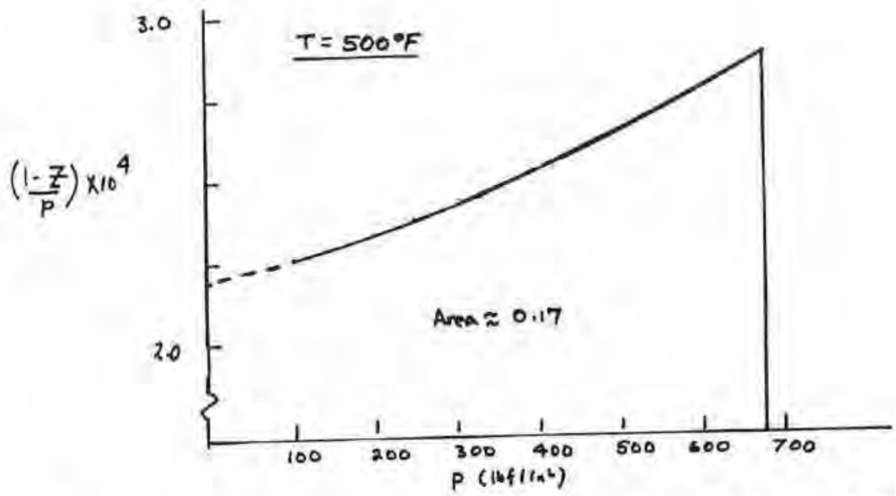
From the discussion of Sec. 11.9.4

$$\ln \frac{f}{p} = \int_0^p \left(\frac{Z-1}{p}\right) dp$$

PROBLEM 11.108 Continued) - Page 2

Graphical Solution:

Using p-v-T data from the steam tables the following plot can be drawn at T=500°F



Thus $\quad \ln \frac{f}{p} = -0.17 \Rightarrow \frac{f}{p} = 0.844 \Rightarrow f = 574 \frac{\text{lbf}}{\text{in}^2}$

The answers obtained using these two graphical methods agree to within 3%.

The following IT code provides an alternative to using Steam Table data for part(a). A similar approach can be used for part(b).

IT Code

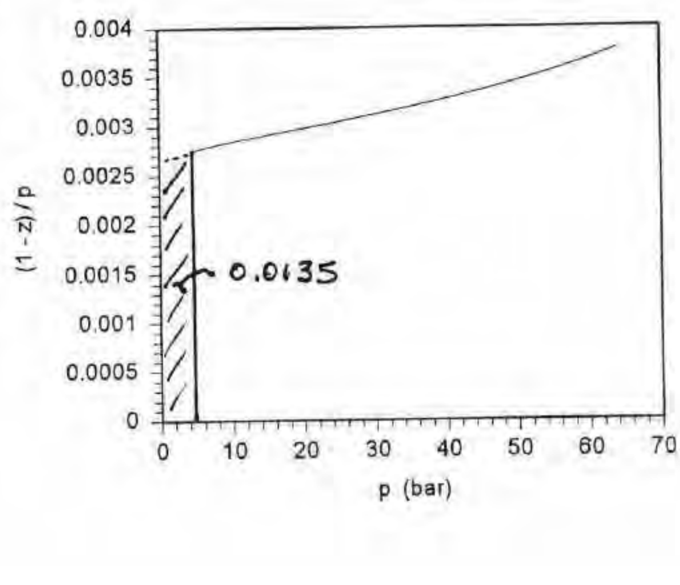
```
T = 280  // °C
psat = 64.12  // bar

R = 8.314/18.02  // kJ/kg·K
Z = (p * v * 100) / (R * (T + 273.15))
v = v_PT("Water/Steam",p,T)
Int = - (Z - 1) / p
Inf_p = Integral(Int,p)
f_p = exp(Inf_p + 0.0135)
f = f_p * p
```

/* The constant 0.0135 is added to account for the area from 0 to 5 bar. The computer cannot evaluate Int for that range because the pressure is too low. The value was determined by extrapolating the data from the BROWSE table to low pressure. */

IT Results

ln f/p = - 0.1898
f/p = 0.816
f = 52.31 bar



PROBLEM 11.109

KNOWN: Systems at specified states are given.

FIND: For each case determine the fugacity, in atm.

ANALYSIS:

(a) Butane 555 K, 150 bar

For Butane $T_c = 425\,K$, $p_c = 38\,bar$. Then

$$T_R = \frac{555}{425} = 1.31, \quad P_R = \frac{150}{38} = 3.95$$

Figure A-6, $\frac{f}{p} = 0.62 \Rightarrow f = 0.62(150\,bar)\left|\frac{1\,atm}{1.01325\,bar}\right| = 91.8\,atm$ ← (a)

(b) Methane 120°F, 800 lbf/in²

For Methane $T_c = 344\,°R$, $P_c = 45.8\,atm$. Then

$$T_R = \frac{580}{344} = 1.69, \quad P_R = \frac{\left(800\,lbf/in^2 / 14.7\,\frac{lbf/in^2}{atm}\right)}{45.8\,atm} = 1.19$$

Figure A-6, $\frac{f}{p} = 0.95 \Rightarrow f = 51.7\,atm$ ← (b)

(c) Benzene 890°R, 135 atm

For Benzene $T_c = 1013\,°R$, $P_c = 48.7\,atm$. Then

$$T_R = \frac{890}{1013} = 0.88, \quad P_R = \frac{135\,atm}{48.7\,atm} = 2.77$$

Figure A-6, $\frac{f}{p} = 0.18 \Rightarrow f = 24.3\,atm$ ← (c)

PROBLEM 11.110

KNOWN: The equation of state has the form

$$Z = 1 + \left[\frac{1}{8} - \frac{27/64}{T_R}\right]\frac{P_R}{T_R}$$

FIND: Using the equation of state evaluate the fugacity of ammonia at 750K, 100 atm am compare with the value obtained from Fig. A-6.

ANALYSIS: Eq. 11.124 is the applicable expression

$$\ln\frac{f}{p} = \int_0^{P_R}(Z-1)\,d\ln P_R = \int_0^{P_R}\left(\frac{Z-1}{P_R}\right)dP_R$$

The equation of state gives

$$\frac{Z-1}{P_R} = \left[\frac{1}{8} - \frac{27/64}{T_R}\right]\frac{1}{T_R}$$

Combining these two results

$$\ln\frac{f}{p} = \int_0^{P_R}\frac{1}{T_R}\left[\frac{1}{8} - \frac{27/64}{T_R}\right]dP_R = \frac{P_R}{T_R}\left[\frac{1}{8} - \frac{27/64}{T_R}\right] \qquad (1)$$

For ammonia, Table A-1 gives $T_c = 406\,K$, $P_c = 112.8\,bar$. Thus,

$$P_R = \frac{(100\,atm)(1.01325\,bar/atm)}{112.8\,bar} = 0.898$$

$$T_R = \frac{750K}{406\,K} = 1.847$$

Inserting values into Eq.(1)

$$\ln\frac{f}{p} = \frac{0.898}{1.847}\left[\frac{1}{8} - \frac{27/64}{1.847}\right] \Rightarrow \frac{f}{p} = 0.951$$

Thus, $f = 95.1$ atm.

By inspection of Fig. A-6, $f/p = 0.97$, giving $f = 97$atm.

$$\%\ \text{difference} = \left(\frac{95.1 - 97}{97}\right)(100) = -1.96$$

PROBLEM 11.111

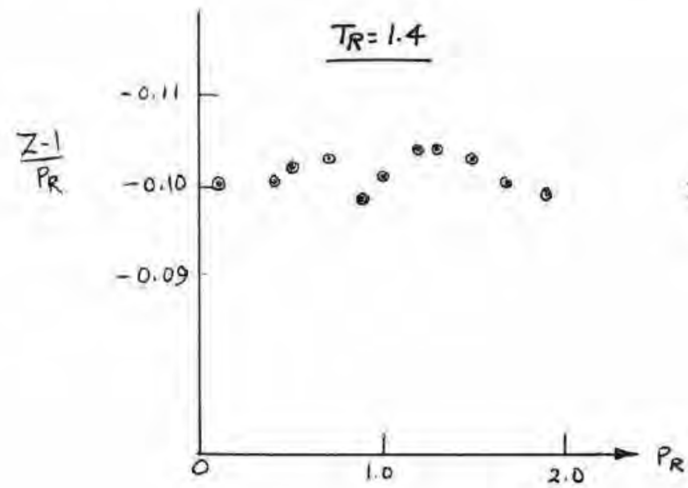
KNOWN: Tabulated compressibility data from the literature is to be used.

FIND: Evaluate f/p at $T_R = 1.4$, $P_R = 2.0$, and compare with the value from Fig A-6.

ANALYSIS: From Fig A-6, $f/p = 0.824$. For calculation, Eq. 11.124 is the applicable expression:

$$\ln \frac{f}{p} = \int_0^{P_R} (Z-1)\, d\ln P_R = \int_0^{P_R} \left(\frac{Z-1}{P_R}\right) dP_R$$

The plan is to evaluate $(Z-1)/P_R$ at $T_R = 1.4$ in the range $0 < P_R \le 2.0$, and then plot vs P_R. (Data from "The Properties of Gases and Liquids" 2nd ed., by R.C. Reid and T.K. Sherwood, McGraw-Hill, 1966, pp. 587–595.)



$$\Rightarrow \quad \frac{Z-1}{P_R} \approx -0.102$$

Accordingly

$$\ln \frac{f}{p} \approx (-0.102)(2.0) = -0.204$$

$$\Rightarrow \frac{f}{p} \approx 0.815$$

Comparison

$$\% = \left(\frac{0.815 - 0.824}{0.824}\right)(100) = -1.1$$

PROBLEM 11.112

KNOWN: A truncated virial expansion is specified:

$$Z = 1 + \hat{B}(T_R)\,p_R + \hat{C}(T_R)\,p_R^2 + \hat{D}(T_R)\,p_R^3$$

FIND: (a) Using tabulated compressibility data, evaluate $\hat{B}, \hat{C}, \hat{D}$ for $0 < p_R < 1.0$ and $T_R = 1.0, 1.2, 1.4, 1.6, 1.8, 2.0$. (b) Obtain an expression for $\ln f/p$ in terms of T_R, p_R. Then, using the coefficients of part (a), evaluate f/p at selected states and compare with tabulated values.

ANALYSIS: (a) Eq. 11.124 is the applicable expression:

$$\ln \frac{f}{p} = \int_0^{p_R} (Z-1)\, d\ln p_R = \int_0^{p_R} \left(\frac{Z-1}{p_R}\right) dp_R$$

Using the given equation of state

$$\ln \frac{f}{p} = \int_0^{p_R} \left(\hat{B}(T_R) + \hat{C}(T_R)\,p_R + \hat{D}(T_R)\,p_R^2\right) dp_R$$

$$= \hat{B}(T_R)\,p_R + \hat{C}(T_R)\frac{p_R^2}{2} + \hat{D}(T_R)\frac{p_R^3}{3} \qquad (1)$$

where the coefficients $\hat{B}, \hat{C}, \hat{D}$ are obtained for selected T_R values and p_R ranging from 0 to 1.0 by linear regression using tabulated compressibility data. The coefficients evaluated this way are

TR	$\hat{B}$	$\hat{C}$	$\hat{D}$
1	-0.66404	1.274314	-1.30854
1.0	-0.32199	0.081732	-0.14687
1.1	-0.24781	-0.02425	-0.02729
1.2	-0.17771	0.012029	-0.04160
1.3	-0.12188	-0.05410	0.034815
1.4	-0.09219	-0.02849	0.021567
1.5	-0.07307	-0.00706	0.010698
1.6	-0.06488	0.028939	-0.01516
1.8	-0.05442	0.068857	-0.04020
2	-0.03951	0.070563	-0.04350

(a)

(b) As a sample calculation, at $p_R = 0.5$, $T_R = 1.2$, $f/p = 0.92$. Inserting values into Eq. (1)

$$\ln \frac{f}{p} = -0.17771(0.5) + 0.012029\frac{(.5)^2}{2} - 0.04160\frac{(.5)^3}{3}$$

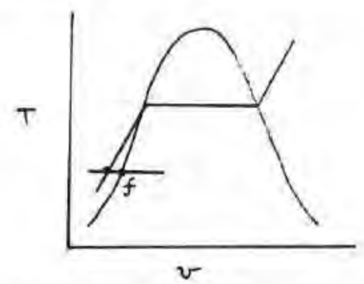
$$\Rightarrow \frac{f}{p} = 0.915 \qquad (b)$$

The % difference is

$$\% = \left(\frac{0.915 - 0.92}{0.92}\right)(100) = -0.54$$

PROBLEM 11.113

KNOWN: Approximate expressions for the fugacity of a liquid are specified.
FIND: Derive the expressions and comment.
SCHEMATIC & GIVEN DATA:



ANALYSIS: Beginning with Eq 11.122

$$RT\left(\frac{\partial \ln f}{\partial p}\right)_T = v$$

Then, approximating v as $v \approx v_f(T)$

$$\left(\frac{\partial \ln f}{\partial p}\right)_T \approx \frac{v_f(T)}{RT} \Rightarrow \ln\frac{f}{f^L_{sat}} \approx \frac{v_f(T)}{RT}[p - p_{sat}]$$

or

$$f(T,p) \approx f^L_{sat}(T)\exp\left\{\frac{v_f(T)}{RT}(p - p_{sat})\right\}$$

Letting $x = \frac{v_f(T)}{RT}[p - p_{sat}]$ and expanding $\exp(x)$

$$\frac{f(T,p)}{f^L_{sat}(T)} \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

Accordingly, $f(T,p) \approx f^L_{sat}$ when the terms in powers of x are small enough to ignore. For example, considering the case of water at 27°C, Table A-2 gives $p_{sat} = 0.03567$ bar, $v_f = 1.0035 \times 10^{-3}$ m³/kg

$$x = \frac{v_f(T)}{RT}[p - p_{sat}] = \frac{1.0035\times10^{-3}\ \text{m}^3/\text{kg}\ [p - 0.03567]\ \text{bar}}{\left(\frac{8314}{18.02}\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(300.15\text{K})\left|\frac{1\text{ bar}}{10^5\text{ N/m}^2}\right|}$$

$$= (7.25\times10^{-4})[p - 0.03567]$$

The pressure range for which $x < 10^{-n}$, where n is an integer, is

$$7.25\times10^{-4}[p - 0.03567] < 10^{-n}$$

$$\Rightarrow [p - 0.03567] < 0.13793\ 10^{4-n}$$

n	p (bar)
2	< 13.83
3	< 1.41

PROBLEM 11.114

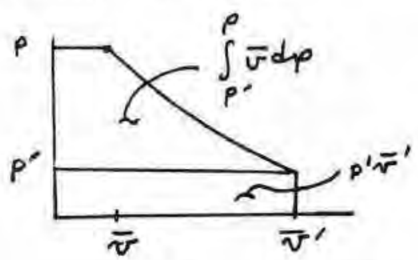
KNOWN: The Redlich-Kwong equation of state describes the p-$\bar{v}$-T behavior of a gas.

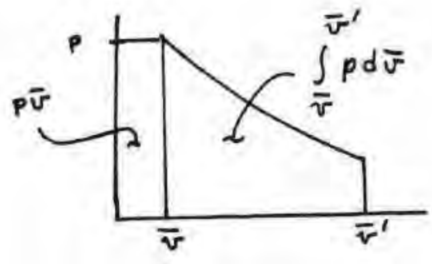
FIND: (a) Evaluate $\ln f$ for the gas. (b) Using the result of part (a), evaluate f, in bar, for Refrigerant 134a at 90°C, 10 bar, and compare with the value obtained from Fig. A-6, the generalized fugacity chart.

ANALYSIS: (a) Beginning with Eq. 11.122, integrate at fixed T from $(p', \bar{v}')$ to $(p, \bar{v})$

$$\bar{R}T \ln f \Big]_{p'}^{p} = \int_{p'}^{p} \bar{v}\, dp \qquad (1)$$

Considering areas,





$$\int_{p'}^{p} \bar{v}\,dp + p'\bar{v}' = \int_{\bar{v}}^{\bar{v}'} p\,dv + p\bar{v}$$

$$\Rightarrow \int_{p'}^{p} \bar{v}\,dp = p\bar{v} - p'\bar{v}' + \int_{\bar{v}}^{\bar{v}'} p\,d\bar{v}$$

$$= p\bar{v} - p'\bar{v}' - \int_{\bar{v}'}^{\bar{v}} p\,d\bar{v} \qquad (2)$$

Combining Eqs. (1) and (2)

$$\bar{R}T \ln f \Big]_{p'}^{p} = p\bar{v} - p'\bar{v}' - \int_{\bar{v}'}^{\bar{v}} p\,d\bar{v} \qquad (3)$$

Introducing the Redlich-Kwong equation in the integral

$$\bar{R}T \ln f \Big]_{p'}^{p} = p\bar{v} - p'\bar{v}' - \int_{\bar{v}'}^{\bar{v}} \left[\frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\sqrt{T}\,\bar{v}(\bar{v}+b)} \right] d\bar{v}$$

$$\Rightarrow \quad \ln \frac{f}{f'} = \frac{p\bar{v}}{\bar{R}T} - \frac{p'\bar{v}'}{\bar{R}T} - \ln\left[\frac{\bar{v}-b}{\bar{v}'-b}\right] - \frac{a}{b\bar{R}T^{3/2}} \ln\left[\frac{\bar{v}+b}{\bar{v}} \left(\frac{1}{1+b/\bar{v}'} \right) \right]$$

or

$$\ln f = \frac{p\bar{v}}{\bar{R}T} - \frac{p'\bar{v}'}{\bar{R}T} + \ln\left[f' \frac{(\bar{v}'-b)}{\bar{v}-b} \right] - \frac{a}{b\bar{R}T^{3/2}} \ln\left[\frac{\bar{v}+b}{\bar{v}} \left(\frac{1}{1+b/\bar{v}'} \right) \right]$$

Next, consider the limit as $p' \to 0$ ($\bar{v}' \to \infty$) at fixed T:

$f' \to p' \to 0$, $p'\bar{v}' \to \bar{R}T$, $p'(\bar{v}'-b) \to \bar{R}T$, $1 + \frac{b}{\bar{v}'} \to 1$.

Thus

$$\ln f = Z - 1 + \ln\left[\frac{\bar{R}T}{\bar{v}-b}\right] - \frac{a}{b\bar{R}T^{3/2}} \ln\left(\frac{\bar{v}+b}{\bar{v}}\right) \qquad (4)$$

Eq (4) can be rewritten using the Redlich-Kwong equation:

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}(\bar{v}+b)T^{1/2}} \Rightarrow \underbrace{\frac{p\bar{v}}{\bar{R}T}}_{Z} = \frac{\bar{v}}{\bar{v}-b} - \frac{a}{(\bar{v}+b)\bar{R}T^{3/2}}$$

$$\Rightarrow Z = \frac{\bar{v}}{\bar{v}-b} - \frac{a}{(\bar{v}+b)\bar{R}T^{3/2}} \Rightarrow Z - 1 = \frac{b}{\bar{v}-b} - \frac{a}{(\bar{v}+b)\bar{R}T^{3/2}}$$

Thus

$$\ln f = \frac{b}{\bar{v}-b} + \ln\left[\frac{\bar{R}T}{\bar{v}-b}\right] - \frac{a}{\bar{R}T^{3/2}}\left[\frac{1}{(\bar{v}+b)} + \frac{1}{b}\ln\left(\frac{\bar{v}+b}{\bar{v}}\right)\right] \quad (5)$$

(b) From Table A-1, for R134a, $T_c = 374\,K$, $p_c = 40.7\,bar$. Thus

$T_R = \frac{363\,K}{374\,K} = 0.97$, $p_R = \frac{10\,bar}{40.7\,bar} = 0.25$

Figure A-6 gives, $\frac{f}{p} \sim 0.92 \Rightarrow f = 9.2\,bar$

Evaluating a and b for R134a via Eqs. 11.8

$a = 196.39\ bar\left(\frac{m^3}{kmol}\right)^2(K)^{1/2}$, $b = 0.0662\ m^3/kmol$

Solving the Redlich-Kwong equation for $\bar{v}$ when $T = 363\,K$ and $p = 10\,bar$ gives $\bar{v} = 2.724\ m^3/kmol$.

Finally, inserting values into Eq. (5), using $\bar{R} = 0.08314\ \frac{bar \cdot m^3}{kmol \cdot K}$

$$\ln f = \frac{0.0662}{2.6578} + \ln\left[\frac{(0.08314)(363)}{2.6578}\right] - \frac{196.39}{(0.08314)(363)^{3/2}}\left[\frac{1}{2.7902} + \frac{1}{0.0662}\ln\left(\frac{2.7902}{2.724}\right)\right]$$

$$= 2.208 \Rightarrow f = 9.1\,bar$$

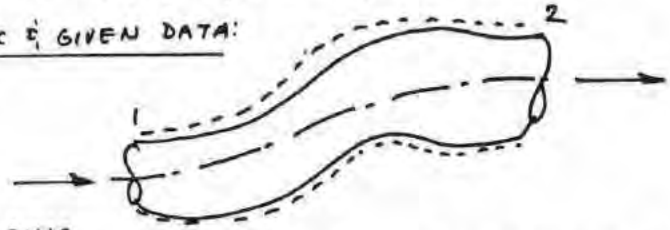
The calculated and chart values for f agree to within $\approx 1\%$.

PROBLEM 11.115

KNOWN: Under consideration is a one-inlet one-exit control volume at steady state through which the flow is internally reversible and isothermal.

FIND: Derive an expression for the work per unit of mass flowing in terms of the fugacity and other relevant quantities.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying sketch is at steady state. (2) The flow is internally reversible and isothermal. (3) A single component (or a mixture with known properties such as air) is flowing.

ANALYSIS:

mass rate balance: $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$

energy rate balance: $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)\right]$

entropy rate balance (with assumption 2):

$$0 = \frac{\dot{Q}_{cv}}{T} + \dot{m}(s_1 - s_2) + \cancelto{0}{\dot{\sigma}_{cv}}$$

Eliminating $\dot{Q}_{cv}$ between the energy and entropy rate balances

$$\frac{\dot{W}_{cv}}{\dot{m}} = T(s_2 - s_1) + (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)$$

The Gibbs function is $g = h - Ts$, thus

$$\frac{\dot{W}_{cv}}{\dot{m}} = (g_1 - g_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)$$

For a single component, the chemical potential equals the Gibbs function per mole. Then, with Eq. 11.121

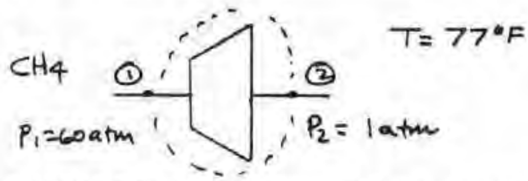
$$\frac{\dot{W}_{cv}}{\dot{m}} = -RT \ln \frac{f_2}{f_1} + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)$$

PROBLEM 11.116

KNOWN: Methane expands isothermally and without irreversibilities through a turbine operating at steady state.

FIND: Using the generalized fugacity chart, determine the work developed per lb of CH_4 flowing.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The control volume shown in the figure is at steady state. (2) The expansion is isothermal and internally reversible. (3) Kinetic and potential energy effects can be ignored

ANALYSIS: Using the result of Problem 11.115 and data from Table A-1E and Figure A-6, $P_{R1} = \frac{60\text{ atm}}{45.8\text{ atm}} = 1.31$, $T_{R1} = T_{R2} = \frac{537°R}{344°R} = 1.56$

$$P_{R2} = \frac{1}{45.8} = 0.02$$

$$\left(\frac{f}{p}\right)_1 \approx 0.92,\ \left(\frac{f}{p}\right)_2 \approx 1.0 \Rightarrow \left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = -RT\ln\frac{f_2}{f_1} = -\left(\frac{1.986}{16.04}\frac{Btu}{lb \cdot °R}\right)(537°R)\ln\left(\frac{1.0 \times 1\text{ atm}}{0.92 \times 60\text{ atm}}\right)$$

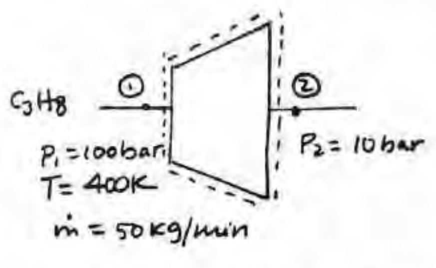
$$= 266.7\text{ Btu/lb}$$

PROBLEM 11.117

KNOWN: Propane expands isothermally and without irreversibilities through a turbine operating at steady state.

FIND: Using the generalized fugacity chart, determine the power developed.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The control volume shown operates at steady state.
2. The expansion is isothermal and internally reversible
3. Kinetic & potential energy effects can be ignored.

ANALYSIS: Using the result of Problem 11.115 and data from Table A-1 and Fig A-6, $p_{R1} = \frac{100\text{ bar}}{42.7\text{ bar}} = 2.34$, $p_{R2} = \frac{10}{42.7} = 0.23$, $T_R = \frac{400}{370} = 1.08$

$\left(\frac{f}{p}\right)_1 = 0.48$, $\left(\frac{f}{p}\right)_2 = 0.96$. Then

$$\dot{W}_{cv} = -\dot{m} RT \ln\left(\frac{f_2}{f_1}\right) = -\dot{m}\left(\frac{\bar{R}}{M}\right) T \ln\left(\left(\frac{f_2}{p_2}\right)\left(\frac{p_2}{p_1}\right)\left(\frac{p_1}{f_1}\right)\right)$$

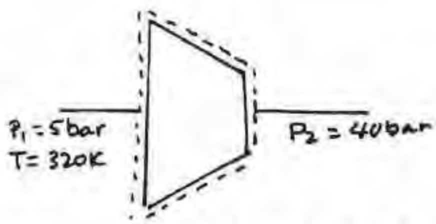
$$= -\left(\frac{50}{60}\frac{\text{kg}}{\text{s}}\right)\left(\frac{8.314}{44.09}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(400\text{K}) \ln\left((0.96)(0.1)\left(\frac{1}{0.48}\right)\right)\left|\frac{\text{kW}}{\text{kJ/s}}\right| = 101.1 \text{ kW} \leftarrow \dot{W}_{cv}$$

PROBLEM 11.118

KNOWN: C_2H_6 is compressed isothermally without internal irreversibilities.

FIND: Determine the work and heat transfer, each per unit mass of C_2H_6 flowing.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:
1. The control volume is at steady state.
2. Kinetic & potential energy effects can be ignored.
3. The process occurs isothermally and without internal irreversibilities.

ANALYSIS: Using the result of Prob. 11.115

$$\frac{\dot{W}_{cv}}{\dot{m}} = -RT \ln\left(\left(\frac{f}{p}\right)_2 \left(\frac{P_2}{P_1}\right)\left(\frac{p}{f}\right)_1\right)$$

With data from Table A-1 and Fig. A-6,

$$T_R = \frac{320\,K}{305\,K} = 1.05 \quad P_{R1} = \frac{5\text{ bar}}{48.8\text{ bar}} = 0.1,\; P_{R2} = \frac{40}{48.8} = 0.82,\; \left(\frac{f}{p}\right)_1 = 0.98,\; \left(\frac{f}{p}\right)_2 = 0.79$$

Thus

$$\frac{\dot{W}_{cv}}{\dot{m}} = -\left(\frac{8.314}{30.07}\frac{kJ}{kg\cdot K}\right)(320K)\ln\left((0.79)\left(\frac{40}{5}\right)\left(\frac{1}{0.98}\right)\right) = -164.9\,\frac{kJ}{kg} \longleftarrow \dot{W}_{cv}/\dot{m}$$

Reducing mass and energy rate balances gives $\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}(h_2 - h_1)$. Invoking Eq. 11.85 and Fig A-4 data

$$\bar{h}_2 - \bar{h}_1 = \underbrace{\bar{h}_2^* - \bar{h}_1^*}_{\substack{=0 \\ \text{since } T \\ \text{is const}}} - \bar{R}T_c\left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1\right]$$

$$\Rightarrow h_2 - h_1 = -\left(\frac{8.314}{30.07}\right)(305)\left[(1.0) - (0.09)\right] = -76.7\,\frac{kJ}{kg}$$

Then

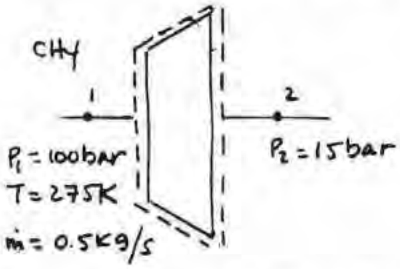
$$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{\dot{W}_{cv}}{\dot{m}} + (h_2 - h_1) = -164.9\,\frac{kJ}{kg} + (-76.7)\,\frac{kJ}{kg} = -241.6\,\frac{kJ}{kg} \longleftarrow \dot{Q}_{cv}/\dot{m}$$

PROBLEM 11.119

KNOWN: Steady-state operating data are provided for methane expanding isothermally and without internal irreversibilities through a turbine.

FIND: Determine the power developed and the rate of heat transfer.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:
1. The control volume is at steady state.
2. Kinetic & potential energy effects can be ignored.

ANALYSIS: Using the result of Prob. 11.115 together with data from Table A-1 and Fig A-6: $M = 16.04$, $T_R = \frac{275K}{191K} = 1.44$, $P_{R1} = \frac{100\,bar}{46.4\,bar} = 2.16$, $P_{R2} = \frac{15}{46.4} = 0.32$, $\left(\frac{f}{p}\right)_1 = 0.82$, $\left(\frac{f}{p}\right)_2 = 0.97$.

Then
$$\dot{W}_{cv} = -\dot{m}\, R\, T \ln\left(\frac{f_2}{f_1}\right) = -\dot{m}\left(\frac{\bar{R}}{M}\right) T \ln\left(\left(\frac{f}{p}\right)_2\left(\frac{P_2}{P_1}\right)\left(\frac{p}{f}\right)_1\right)$$

$$= -\left(0.5\,\frac{kg}{s}\right)\left(\frac{8.314}{16.04}\right)(275)\ln\left((0.97)\left(\frac{15}{100}\right)\left(\frac{1}{0.82}\right)\right) = 123.3\,kW \quad \longleftarrow \dot{W}_{cv}$$

Mass and energy rate balances reduce to give $\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}(h_2 - h_1)$. $(h_2 - h_1)$ can be found using Eq. 11.85 and data from Fig A-4

$$\bar{h}_2 - \bar{h}_1 = \underbrace{\bar{h}_2^* - \bar{h}_1^*}_{=0 \text{ since } T \text{ is const}} - \bar{R}\, T_c\left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}\, T_c}\right)_2 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R} T_c}\right)_1\right]$$

$$\Rightarrow \quad h_2 - h_1 = -\left(\frac{8.314}{16.04}\right)(191)\left[(0.17) - (1.18)\right] = 100\,\frac{kJ}{kg}$$

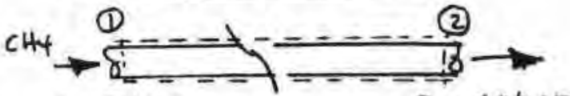
Thus
$$\dot{Q}_{cv} = (123.3\,kW) + \left(0.5\,\frac{kg}{s}\right)\left(100\,\frac{kJ}{kg}\right)\left|\frac{kW}{kJ/s}\right| = 173.3\,kW \quad \longleftarrow \dot{Q}_{cv}$$

PROBLEM 11.120

KNOWN: CH_4 flows isothermally and without irreversibilities through a horizontal pipe. Steady-state data are provided.

FIND: Determine the exit velocity.

SCHEMATIC & GIVEN DATA:



$P_1 = 50\,bar$ $\quad$ $P_2 = 40\,bar$

$T = 300K$

$V = 10\,m/s$

ENGINEERING MODEL: The control volume enclosing the pipe is at steady state.
2. For the control volume, potential energy effects can be ignored. and $\dot{W}_{cv} = 0$.

ANALYSIS: Using the result of Problem 11.115

$$\cancel{\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}}}^{0} = -RT\ln\left(\frac{f_2}{f_1}\right) + \frac{V_1^2 - V_2^2}{2} + \cancel{g(z_1 - z_2)}^{0}$$

$$\Rightarrow \quad V_2^2 = V_1^2 - 2RT\ln\left(\frac{f_2}{f_1}\right) \qquad (1)$$

With data from Table A-1 and Fig A-6

$M = 16.04$, $T_R = \frac{300K}{191K} = 1.57$, $P_{R1} = \frac{50}{46.4} = 1.08$, $P_{R2} = \frac{40}{46.4} = 0.86$

$\left(\frac{f}{p}\right)_1 = 0.93$, $\left(\frac{f}{p}\right)_2 = 0.95$

Thus, Eq. (1) gives

$$V_2^2 = (10\,m/s)^2 - 2\left(\frac{8314}{16.04}\frac{N\cdot m}{kg\cdot K}\right)(300K)\left|\frac{1\,kg\cdot m/s^2}{1\,N}\right| \ln\left((0.95)\left(\frac{40}{50}\right)\left(\frac{1}{0.93}\right)\right)$$

$$= (10\,m/s)^2 - (-62820\,m^2/s^2) = 162.8\,m^2/s^2$$

$\therefore V_2 = 12.76\,m/s \longleftarrow V_2$

PROBLEM 11.121

KNOWN: Ethane (C_2H_6) is (a) pure at 310 K, 20.4 atm., (b) a component of an ideal solution at 310 K, 20.4 atm with a mole fraction of 0.35.

FIND: In each case, evaluate the fugacity.

ANALYSIS: (a) From Table A-1, $T_c = 305$ K, $p_c = 48.8$ bar. Then, $T_R = 310/305 = 1.02$,

$$p_R = \frac{20.4 \text{ atm}}{48.8 \text{ bar}}\left|\frac{1.01325 \text{ bar}}{1 \text{ atm}}\right| = 0.42. \quad \text{Fig. A-6 gives } \frac{f}{p} = 0.89 \Rightarrow f = 18.2 \text{ atm} \longleftarrow \text{(a)}$$

(b) For ethane as a member of an ideal solution, Eq. 11.134 is applicable:

$\bar{f}_i = y_i f_i$, where f_i is the fugacity of pure i at the same T, p. Thus,

$$\bar{f}_i = 0.35(18.2 \text{ atm}) = 6.37 \text{ atm}. \longleftarrow \text{(b)}$$

PROBLEM 11.122

KNOWN: The fugacity of the solute (denoted by 2) in a dilute binary liquid solution at T, p is proportional to its mole fraction in the solution: $\bar{f}_2 = K y_2$, where K is a constant.

FIND: Show that the fugacity of the solvent (denoted by 1) is $\bar{f}_1 = y_1 f_1$, where y_1 is the mole fraction of the solvent and f_1 is the fugacity of pure 1 at T, p

ANALYSIS: The intensive state of the solution is fixed by T, p, y_1 and y_2. But with T, p assumed known, either y_1 or y_2 can be viewed as the independent variable since $y_1 + y_2 = 1$. Accordingly, the Gibbs-Duhem equation (Eq. 11.113) reduces at fixed T, p and y_2 as the independent variable to read

$$n_1 \frac{d\mu_1}{dy_2} + n_2 \frac{d\mu_2}{dy_2} = 0 \Rightarrow y_1 \frac{d\mu_1}{dy_2} + y_2 \frac{d\mu_2}{dy_2} = 0$$

Introducing $\mu_i = \bar{R}T \ln \bar{f}_i + C_i(T)$ (Eq 11.125)

$$y_1 \frac{d \ln \bar{f}_1}{dy_2} + y_2 \frac{d \ln \bar{f}_2}{dy_2} = 0 \qquad (1)$$

Since $\bar{f}_2 = K y_2 \Rightarrow \ln \bar{f}_2 = \ln y_2 + \ln K$, and so $\frac{d \ln \bar{f}_2}{dy_2} = \frac{1}{y_2}$ (2)

Combining Eqs. (1), (2)

$$y_1 \frac{d \ln \bar{f}_1}{dy_2} + 1 = 0 \Rightarrow \frac{d \ln \bar{f}_1}{dy_2} = -\frac{1}{y_1} \leftarrow (1 = y_1 + y_2)$$
$$= -\frac{1}{(1-y_2)}$$

Accordingly

$$\ln \bar{f}_1 = \ln(1-y_2) + \ln K' \Rightarrow \bar{f}_1 = K'(1-y_2)$$
$$\bar{f}_1 = K' y_1 \qquad (3)$$

Since $\bar{f}_1 \to f_1$ as $y_1 \to 1$, $K' = f_1$, and thus Eq. (3) becomes

$$\bar{f}_1 = y_1 f_1$$

PROBLEM 11.123

KNOWN: A tank contains 310 kg of a gaseous mixture with the molar analysis {70% C_2H_6, 30% N_2} at 311 K, 170 atm.
FIND: Determine the tank volume using the compressibility chart and (a) Kay's rule, (b) the ideal solution model. Compare with the measured value: 1 m³.

SCHEMATIC & GIVEN DATA:

T = 311 K
p = 170 atm
m = 310 kg

1. C_2H_6 $y_1 = 0.7$
2. N_2 $y_2 = 0.3$

ENGINEERING MODEL: The mixture is the closed system.

ANALYSIS: (a) Kay's Rule plus Z chart. With data from Table A-1, Eqs. 11.97 give

$$T_c = y_1 T_{c1} + y_2 T_{c2} = (0.7)(305) + (0.3)(126) = 251.3\text{K}$$
$$p_c = y_1 p_{c1} + y_2 p_{c2} = (0.7)(48.2) + (0.3)(33.5) = 43.79 \text{ atm}$$

Thus, $T_R = 311/251.3 = 1.24$, $p_R = 170/43.79 = 3.88$. Then, Fig. A-2 gives, $Z \approx 0.65$, so

$$V = Z\left[\frac{mRT}{p}\right] = 0.65\left[\frac{(310\text{ kg})(8314/29.45)(\text{N}\cdot\text{m/kg}\cdot\text{K})(311\text{K})}{170\times 1.01325\times 10^5 \text{ N/m}^2}\right] = 1.03\text{ m}^3 \quad \text{(a)}$$

① where $M = y_1 M_1 + y_2 M_2 = (0.7)(30.07) + (0.3)(28.01) = 29.45$ kg/kmol of mixture.

(b) Ideal Solution Model plus Z chart. Using Eqs. 11.136 for an ideal solution

$$V = n_1 \bar{V}_1 + n_2 \bar{V}_2 = n_1 \bar{v}_1 + n_2 \bar{v}_2$$

where $\bar{v}_i$ is the molar specific volume of pure i at the temperature and pressure of the solution. Dividing by n, the number of moles of solution gives

$$\frac{V}{n} = y_1 \bar{v}_1 + y_2 \bar{v}_2 \quad (1)$$

To find $\bar{v}_1$, calculate $T_{R1} = 311/305 = 1.02$, $p_{R1} = 170/48.2 = 3.53$. Then, from Fig. A-2 $Z_1 \approx 0.51$. Accordingly

$$\bar{v}_1 = \frac{Z_1 \bar{R} T}{p} = \frac{(0.51)(8314\text{ N}\cdot\text{m/kmol}\cdot\text{K})(311\text{K})}{(170\times 1.01325\times 10^5 \text{ N/m}^2)} = 0.0766 \frac{\text{m}^3}{\text{kmol}}$$

To find $\bar{v}_2$, calculate $T_{R2} = 311/126 = 2.47$, $p_{R2} = 170/33.5 = 5.07$. From Fig. A-2, $Z_2 \approx 1.05$. Accordingly

$$\bar{v}_2 = \frac{Z_2 \bar{R} T}{p} = \frac{(1.05)(8314)(311)}{(170\times 1.01325\times 10^5)} = 0.1576 \frac{\text{m}^3}{\text{kmol}}$$

Then, with Eq.(1), $V = n[y_1 \bar{v}_1 + y_2 \bar{v}_2]$, or

$$V = \left(\frac{310\text{ kg}}{29.45\text{ kg/kmol}}\right)\left[(0.7)(0.0766) + (0.3)(0.1576)\right]\frac{\text{m}^3}{\text{kmol}} = 1.06\text{ m}^3 \quad \text{(b)}$$

---

1. Using Eq. 12.9 of Sec. 12.1

PROBLEM 11.124

KNOWN: A tank contains 157 lb of a mixture of 75% Ar and 25% C_2H_2 (molar basis) at 77°F, 81.42 atm.

FIND: Estimate the tank volume using (a) the ideal gas equation of state, (b) Kay's rule with the gen. compressibility chart, (c) ideal solution model with the compressibility chart.

SCHEMATIC & GIVEN DATA:

$T = 537°R$
$p = 81.42\ atm$
$m = 157\ lb$

1. Argon $y_1 = 0.75$
2. C_2H_4 $y_2 = 0.25$

ENGINEERING MODEL: The mixture is the closed system.

(a) Ideal gas model.

$$V_{id} = \frac{m\bar{R}T}{\bar{M}p} = \frac{(157\ lb)(1545/36.97)(ft\cdot lbf/lb\cdot°R)(537°R)}{(81.42 \times 14.7 \times 144)\ lbf/ft^2} = 20.44\ ft^3 \qquad (a)$$

① where $M = y_1M_1 + y_2M_2 = (0.75)(39.94) + (0.25)(28.05) = 36.97\ lb/lbmol$.

(b) Kay's rule with gen. compressibility chart. With data from Table A-1E, Eqs. 11.97 give

$T_c = y_1T_{c1} + y_2T_{c2} = (0.75)(272) + (0.25)(510) = 331.5°R$

$P_c = y_1P_{c1} + y_2P_{c2} = (0.75)(47.97) + (0.25)(50.5) = 48.61\ atm$

Thus, $T_R = 537/331.5 = 1.62$, $P_R = 1.67$. Fig. A-2 gives $Z \approx 0.91$, so

$$V = Z\frac{mRT}{P} = Z\,V_{id} = 0.91(20.44\ ft^3) = 18.6\ ft^3 \qquad (b)$$

(c) Ideal solution with gen. compressibility chart. Using Eq. 11.136

$$V = n_1\bar{V}_1 + n_2\bar{V}_2 = n_1\bar{v}_1 + n_2\bar{v}_2$$

where $\bar{v}_i$ is the molar specific volume of pure i at the temperature and pressure of the solution. Dividing by n, the number of moles of solution gives

$$\frac{V}{n} = y_1\bar{v}_1 + y_2\bar{v}_2 \qquad (1)$$

To find $\bar{v}_1$, calculate $T_{R1} = 537/272 = 1.97$, $P_{R1} = 81.42/47.97 = 1.7$. From Fig. A-2 $Z_1 \approx 0.97$. Accordingly

$$\bar{v}_1 = \frac{Z_1\bar{R}T}{p} = \frac{(0.97)(1545)(537)}{[(81.42)(14.696)(144)]} = 4.671\ \frac{ft^3}{lbmol}$$

To find $\bar{v}_2$, calculate $T_{R2} = 537/510 = 1.053$, $P_{R2} = 81.42/50.5 = 1.612$. From Fig. A-2, $Z_2 \approx 0.32$. Accordingly

$$\bar{v}_2 = \frac{Z_2\bar{R}T}{p} = \frac{(0.32)(1545)(537)}{[(81.42)(14.696)(144)]} = 1.541\ \frac{ft^3}{lbmol}$$

With Eq. (1)

$$V = n[y_1\bar{v}_1 + y_2\bar{v}_2] = \left(\frac{157}{36.97}\right)\left[0.75(4.671) + 0.25(1.541)\right] = 16.51\ ft^3 \qquad (c)$$

---

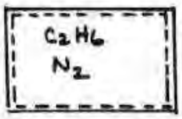
1. Using Eq. 12.9 of Sec. 12.1

PROBLEM 11.125

KNOWN: A tank contains 2130 kg of a mixture of 70% C_2H_6, 30% N_2 at 400K, 200 atm.

FIND: Estimate the tank volume using (a) the ideal gas equation of state, (b) Kay's rule with the gen. compressibility chart, (c) the ideal solution model with the gen. compressibility chart.

SCHEMATIC & GIVEN DATA:



T = 400K
p = 200 atm
m = 2130 kg

1. C_2H_6, $y_1 = 0.7$
2. N_2, $y_2 = 0.3$

ENGINEERING MODEL: The mixture is the closed system.

ANALYSIS: (a) Ideal Gas Model.

$$V_{id} = \frac{m\,RT}{p} = \frac{(2130\,kg)(8314/29.45)(N\cdot m/kg\cdot K)(400K)}{(200 \times 1.01325 \times 10^5\ N/m^2)} = 11.87\,m^3 \quad \longleftarrow (a)$$

① where $M = y_1M_1 + y_2M_2 = (0.7)(30.07) + (0.3)(28.01) = 29.45$ kg/kmol.

(b) Kay's rule with gen. compressibility chart. With Eqs. 11.97

$T_c = y_1 T_{c1} + y_2 T_{c2} = (0.7)(305) + (0.3)(126) = 251.3\,K$

$p_c = y_1 p_{c1} + y_2 p_{c2} = (0.7)(48.2) + (0.3)(33.5) = 43.79$ atm

Then, $T_R = 400/251.3 = 1.59$, $P_R = 200/43.79 = 4.57$. Fig. A-2 gives $Z \approx 0.87$. So

$$V = Z\left[\frac{mRT}{p}\right] = Z\,V_{id} = (0.87)(11.87) = 10.33\,m^3 \quad \longleftarrow (b)$$

(c) Ideal Solution plus Z chart. Using Eqs. 11.136 for an ideal solution

$$V = n_1\bar{V}_1 + n_2\bar{V}_2 = n_1\bar{v}_1 + n_2\bar{v}_2$$

where $\bar{v}_i$ is the molar specific volume of pure i at the temperature and pressure of the solution. Dividing by n, the number of moles of solution gives

$$\frac{V}{n} = y_1\bar{v}_1 + y_2\bar{v}_2 \qquad (1)$$

To find $\bar{v}_1$, calculate $T_{R1} = 400/305 = 1.31$, $P_{R1} = 200/48.2 = 4.15$. Then, Fig. A-2 gives $Z_1 \approx 0.7$. Accordingly

$$\bar{v}_1 = \frac{Z_1\bar{R}T}{p} = \frac{(0.7)(8314\ \frac{N\cdot m}{kmol\cdot K})(400K)}{[(200)(1.01325 \times 10^5\ N/m^2)]} = 0.1149\ \frac{m^3}{kmol}$$

To find $\bar{v}_2$, calculate $T_{R2} = 400/126 = 3.175$, $P_{R2} = 200/33.5 = 5.97$. Fig. A-2 gives $Z_2 \approx 1.08$. Accordingly

$$\bar{v}_2 = \frac{Z_2\bar{R}T}{p} = \frac{(1.08)(8314)(400)}{[(200)(1.01325 \times 10^5)]} = 0.1772\ \frac{m^3}{kmol}$$

with Eq. (1)

$$V = n\left[y_1\bar{v}_1 + y_2\bar{v}_2\right] = \left(\frac{2130}{29.45}\right)\left[(0.7)(0.1149) + (0.3)(0.1772)\right] = 9.66\,m^3 \quad \longleftarrow (c)$$

---

1. Using 12.9 of Sec. 12.1

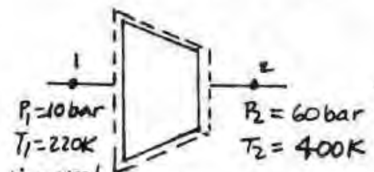
PROBLEM 11.126

KNOWN: Steady-state operating data are provided for a compressor handling an equimolar mixture of O_2 and N_2.

FIND: Determine (a) the power required, (b) the rate of entropy production.

SCHEMATIC & GIVEN DATA:

Mixture: $y_{O_2} = y_{N_2} = 0.5$



For the pure components

	10 bar, 220K		60 bar, 400K	
	h (kJ/kg)	s (kJ/kg·K)	h (kJ/kg)	s (kJ/kg·K)
O_2	195.6	5.521	358.2	5.601
N_2	224.1	5.826	409.8	5.911

ENGINEERING MODEL: The control volume shown is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$, kinetic & potential energy effects can be ignored. (3) The mixture adheres to the ideal solution model.

ANALYSIS: Reducing mass, energy, and entropy balances, we get

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2) = \frac{\dot{m}}{M_{mix}}\left[\bar{h}_1 - \bar{h}_2\right], \quad \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1) = \frac{\dot{m}}{M_{mix}}(\bar{s}_2 - \bar{s}_1)$$

① where $M_{mix} = y_{O_2} M_{O_2} + y_{N_2} M_{N_2} = 30.01$.

(a) With Eq. 11.138

$$\dot{W}_{cv} = \frac{\dot{m}}{M_{mix}}\left[y_{O_2}(h_1 - h_2)_{O_2} M_{O_2} + y_{N_2}(h_1 - h_2)_{N_2} M_{N_2}\right]$$

$$= \left(\frac{1\ \text{kg/s}}{30.01\ \text{kg/kmol}}\right)\left[(0.5)\left[(\underbrace{195.6 - 358.2}_{-162.6})(32) + (\underbrace{224.1 - 409.8}_{-185.7})(28.01)\right]\right]\frac{\text{kJ}}{\text{kmol}}\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right|$$

$$= 173.4\ \text{kW}$$

$\dot{W}_{cv}$

(b) Similarly

② $$\dot{\sigma}_{cv} = \left(\frac{1}{30.01}\right)\left[(0.5)\left[(\underbrace{5.601 - 5.521}_{0.08})(32) + (\underbrace{5.911 - 5.826}_{0.085})(28.01)\right]\right]$$

$$= 0.082\ \frac{\text{kW}}{\text{K}}$$

$\dot{\sigma}_{cv}$

---

1. Using Eq. 12.9 of Sec. 12.1

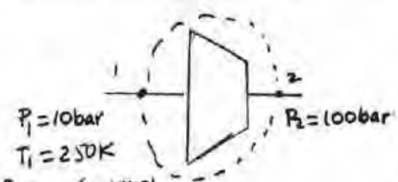
2. Since composition remains constant, this approach suffices. For the special case of ideal gas mixture processes at constant composition, see the discussion of Sec. 12.4.1 leading to Eq. 12.36.

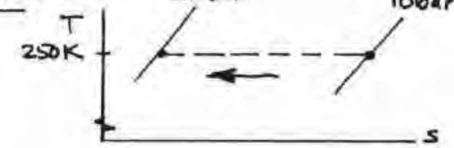
PROBLEM 11.127

KNOWN: A gaseous mixture with the molar analysis {70% CH_4, 30% N_2} enters a compressor operating at steady state and exits at a higher pressure. During compression the temperature of the mixture departs by no more than 0.1 K from a temperature of 250 K. The power required is claimed to be 6 kW.

FIND: Determine if the claimed power requirement can be correct.

SCHEMATIC & GIVEN DATA:





{70% CH_4, 30% N_2}

$\dot{W}_{cv} = -6$ kW (claim)

At 250 K	h (kJ/kg)		s (kJ/kg·K)	
	10 bar	100 bar	10 bar	100 bar
Methane	506.0	358.6	10.003	8.3716
Nitrogen	256.18	229.68	5.962	5.188

ENGINEERING MODEL: (1) The control volume shown in the figure is at steady state. (2) Kinetic and potential energy changes can be ignored. (3) The mixture is modeled as an ideal solution. (4) The temperature at which heat transfer takes place is 250 K.

ANALYSIS: Reducing mass, energy, and entropy balances

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{n}(\bar{h}_1 - \bar{h}_2)\ , \quad 0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{n}(\bar{s}_1 - \bar{s}_2) + \dot{\sigma}_{cv}$$

Combining these expressions, we get

$$(-\dot{W}_{cv}) = \dot{n}\left[(\bar{h}_2 - \bar{h}_1) - T_b(\bar{s}_2 - \bar{s}_1)\right] + T_b\dot{\sigma}_{cv}$$

Since states 1 and 2 are fixed, the minimum theoretical power input corresponds to $\dot{\sigma}_{cv} = 0$: $(-\dot{W}_{cv})_{min} = \dot{n}\left[(\bar{h}_2 - \bar{h}_1) - T_b(\bar{s}_2 - \bar{s}_1)\right]$, where T_b is the temperature at which heat transfer occurs. Since the mixture is modeled as an ideal solution, use of Eq. 11.138 gives

$$(\bar{h}_2 - \bar{h}_1) = y_{CH_4}[\bar{h}_2 - \bar{h}_1]_{CH_4} + y_{N_2}[\bar{h}_2 - \bar{h}_1]_{N_2}$$
$$= (0.7)[358.6 - 506.0](16.04) + (0.3)[229.68 - 256.18](28.01) = -1877.7 \frac{kJ}{kmol}$$

Similarly,

① $$(\bar{s}_2 - \bar{s}_1) = y_{CH_4}[\bar{s}_2 - \bar{s}_1]_{CH_4} + y_{N_2}[\bar{s}_2 - \bar{s}_1]_{N_2}$$
$$= (0.7)[8.3716 - 10.003](16.04) + (0.3)[5.188 - 5.962](28.01) = -24.82 \frac{kJ}{kmol \cdot K}$$

Collecting results

$$(-\dot{W}_{cv})_{min} = \left(6\,\frac{kmol}{h}\right)\left|\frac{1\,h}{3600\,s}\right|\left[-1877.7 - 250(-24.82)\right]\left(\frac{kJ}{kmol \cdot K}\right)\left|\frac{kW}{kJ/s}\right|$$
$$= 7.2\ kW$$

Since the claimed value for $(-\dot{W}_{cv})$ is less than the minimum theoretical value, the claim cannot be valid. ◄

---

1. Since composition remains constant, this approach suffices. For the special case of ideal gas mixture processes at constant composition, see the discussion of Sec. 12.4.1 leading to Eq. 12.36.

PROBLEM 11.128

Known: The departure of a solution from ideal behavior is represented by the activity coefficient $\gamma_i = a_i / y_i$, or with Eq. 11.140

$$\gamma_i = \frac{\bar{f}_i}{y_i f_i^0}$$

Find: Using the Gibbs - Duhem equation, obtain the following expression for a binary solution, $i = 1, 2$, relating the activity coefficients and the composition at temperature T and pressure p

$$\left[y_1 \frac{d \ln \gamma_1}{d y_1} \right]_{p,T} = \left[y_2 \frac{d \ln \gamma_2}{d y_2} \right]_{p,T}$$

How might this expression be useful ?

Analysis:

The Gibbs - Duhem equation is given by

$$\sum_{i=1}^{j} n_i \, d\mu_i = V \, dp - S \, dT$$

where $j = 2$ for a binary mixture.

Considering T and p to be the independent thermodynamic properties

$$d\mu_i = \left.\frac{\partial \mu_i}{\partial y_i}\right|_{T,p,y_\ell} dy_i + \left.\frac{\partial \mu_i}{\partial T}\right|_{p,y} dT + \left.\frac{\partial \mu_i}{\partial p}\right|_{T,y} dp$$

For $p =$ constant and $T =$ constant, $dT = dp = 0$

$$d\mu_i = \left.\frac{\partial \mu_i}{\partial y_i}\right|_{T,p,y_\ell} dy_i \qquad (1)$$

Also, for $dp = dT = 0$, the Gibbs-Duhem equation reduces to

$$\sum_{i=1}^{2} n_i \, d\mu_i = 0 \qquad (2)$$

for a binary mixture.

Also for a binary mixture,

$$y_1 + y_2 = 1$$

$$y_1 = 1 - y_2$$

$$\Rightarrow \; dy_1 = -dy_2 \qquad (3)$$

The difference in the chemical potential of i between a specified state of the mixture and the reference state is given by Eq. 11.139

$$\mu_i - \mu_i^{\circ} = RT \ln\left(\frac{\bar{f}_i}{f_i^{\circ}}\right)$$

Inserting the activity coefficient and rearranging

$$\mu_i = \mu_i^{\circ} + RT \ln\left(\gamma_i y_i\right)$$

Performing the differentiation indicated in (1)

$$\frac{d\mu_i}{dy_i} = RT \left.\frac{d \ln \gamma_i}{d y_i}\right|_{T,p} + \frac{RT}{y_i}$$

$$\Rightarrow \; d\mu_i = RT\left[dy_i \left.\frac{d \ln \gamma_i}{dy_i}\right|_{T,p} + \frac{dy_i}{y_i}\right]$$

Substituting this result into (2)

$$n_1 RT\left[dy_1 \left.\frac{d \ln \gamma_1}{dy_1}\right|_{T,p} + \frac{dy_1}{y_1}\right] + n_2 RT\left[dy_2 \left.\frac{d \ln \gamma_2}{dy_2}\right|_{T,p} + \frac{dy_2}{y_2}\right] = 0$$

Recalling that $dy_2 = -dy_1$

$$n_1 RT\left[dy_1 \left.\frac{\partial \ln \gamma_1}{\partial y_1}\right|_{T,p} + \frac{dy_1}{y_1}\right] + n_2 RT\left[-dy_1 \left.\frac{\partial \ln \gamma_2}{\partial y_2}\right|_{T,p} - \frac{dy_1}{y_2}\right] = 0$$

$$n_1\left[dy_1 \left.\frac{\partial \ln \gamma_1}{\partial y_1}\right|_{T,p} + \frac{dy_1}{y_1}\right] = n_2\left[dy_1 \left.\frac{\partial \ln \gamma_2}{\partial y_2}\right|_{T,p} + \frac{dy_1}{y_2}\right]$$

$$n_1\left[\left.\frac{\partial \ln \gamma_1}{\partial y_1}\right|_{T,p} + \frac{1}{y_1}\right] = n_2\left[\left.\frac{\partial \ln \gamma_2}{\partial y_2}\right|_{T,p} + \frac{1}{y_2}\right]$$

$$y_i = \frac{n_i}{n} \Rightarrow \frac{n_i}{y_i} = n$$

$$\Rightarrow n_1\left[\frac{\partial \ln \gamma_1}{\partial y_1}\right]_{T,p} + n = n_2\left[\frac{\partial \ln \gamma_2}{\partial y_2}\right]_{T,p} + n$$

$$n_1\left[\frac{\partial \ln \gamma_1}{\partial y_1}\right]_{T,p} = n_2\left[\frac{\partial \ln \gamma_2}{\partial y_2}\right]_{T,p}$$

Dividing through by n gives the desired relation

$$y_1\left[\frac{\partial \ln \gamma_1}{\partial y_1}\right]_{T,p} = y_2\left[\frac{\partial \ln \gamma_2}{\partial y_2}\right]_{T,p}$$

This expression is of particular value in minimizing the number of experimental data necessary to evaluate the properties of a system and for detecting inconsistent or erroneous measurements.

Corrected October, 2011

Problem 12.1

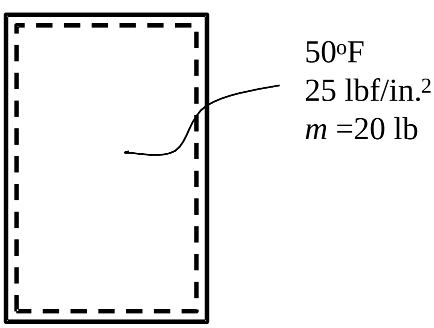
The analysis on a mass basis of an ideal gas mixture at 50°F, 25 lbf/in.^{2} is 60% CO_2, 25% SO_2, and 15% N_2. Determine
(a) the analysis in terms of mole fractions.
(b) the apparent molecular weight of the mixture.
(c) the partial pressure of each component, in lbf/in.^{2}
(d) the volume occupied by 20 lb of the mixture, in ft^3.

Solution:

Known:
An analysis on a mass basis is specified for a gas mixture.

Find:
Determine the analysis in terms of mole fractions, the apparent molecular weight of the mixture, the partial pressure of each component, and the volume occupied by 20 lb of mixture.

Schematic and Known Data:



i	m_{fi}
CO_2	0.60
SO_2	0.25
N_2	0.15

Engineering Model:
(1) The mixture acts as an ideal gas.

Analysis:

(a) Considering 1 lb of mixture

	m_i	M_i	$n_i = m_i / M_i$	$y_i = n_i / n$
CO_2	0.60	44.01	0.0136	0.5939
SO_2	0.25	64.06	0.0039	0.1703
N_2	0.15	28.01	0.0054	0.2358

$$\boxed{n = 0.0229}$$

(b) The apparent molecular weight of the mixture is

$$M = y_{CO_2} M_{CO_2} + y_{SO_2} M_{SO_2} + y_{N_2} M_{N_2}$$

$$M = (0.5939)(44.01) + (0.1703)(64.06) + (0.2358)(28.01)$$

$$M = 43.65 \frac{\text{lb}}{\text{lbmol}}$$

(c) The partial pressures are

$$p_i = y_i p$$

$$p_{CO_2} = (0.5939)\left(25 \frac{\text{lbf}}{\text{in.}^2}\right) = 14.85 \frac{\text{lbf}}{\text{in.}^2}$$

$$p_{SO_2} = (0.1703)\left(25 \frac{\text{lbf}}{\text{in.}^2}\right) = 4.26 \frac{\text{lbf}}{\text{in.}^2}$$

$$p_{N_2} = (0.2358)\left(25 \frac{\text{lbf}}{\text{in.}^2}\right) = 5.9 \frac{\text{lbf}}{\text{in.}^2}$$

(d) Using the ideal gas equation of state for the overall mixture

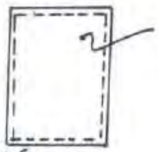
$$V = \frac{mRT}{p} = \frac{(20\text{lb})\left(\frac{1545}{43.67} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot {}^\circ\text{R}}\right)(510^\circ \text{R})}{\left(25 \frac{\text{lbf}}{\text{in.}^2}\right)\left(144 \frac{\text{in.}^2}{\text{ft.}^2}\right)} = 100.24 \text{ ft}^3$$

PROBLEM 12.2

KNOWN: The molar analysis of a gas mixture is specified.

FIND: Determine the analysis in terms of mass fractions, the partial pressure of each component, and the volume occupied by 10 kg of mixture.

SCHEMATIC & GIVEN DATA:



30°C, 2 bar

10 kg

i	y_i
N_2	0.4
CO_2	0.5
CH_4	0.1

ENGINEERING MODEL: (1) The overall mixture acts as an ideal gas. (2) Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature. (3) Calculations are based on 1 kmol of mixture in part (a).

ANALYSIS (a) Considering a typical 1 kmol of mixture

①

i	n_i	M_i	$m_i = n_i M_i$	mf_i
N_2	0.4	28.01	11.204	0.3218
CO_2	0.5	44.01	22.005	0.6321
CH_4	0.1	16.04	1.604	0.0461
			$34.813 \frac{kg}{kmol}$	1.0000

(a)

(b) With Eq. 12.12

$$p_{N_2} = y_{N_2}\, p = (0.4)(2\,\text{bar}) = 0.8\,\text{bar}$$
$$p_{CO_2} = y_{CO_2}\, p = (0.5)(2\,\text{bar}) = 1.0\,\text{bar} \quad \text{(b)}$$
$$p_{CH_4} = y_{CH_4}\, p = (0.1)(2\,\text{bar}) = 0.2\,\text{bar}$$

(c) With the ideal gas equation of state applied to the overall mixture.

$$V = \frac{m(\bar{R}/M)\,T}{p}$$
$$= \frac{(10\,\text{kg})\left(\frac{8314}{34.813}\,\frac{N\cdot m}{kg\cdot K}\right)(303\,K)}{2\times 10^5\,N/m^2} = 3.618\ m^3 \quad \text{(c)}$$

---

1. The apparent molecular weight of the mixture is obtained in the calculations of part (a). Equivalently, Eq. 12.9 can be used: $M = \sum y_i M_i$

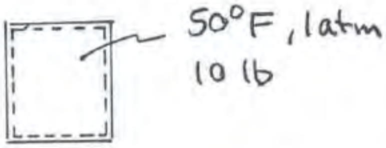
$$M = y_{N_2} M_{N_2} + y_{CO_2} M_{CO_2} + y_{CH_4} M_{CH_4}$$
$$= (0.4)(28.01) + (0.5)(44.01) + (0.1)(16.04) = 34.813\ \frac{kg}{kmol}$$

PROBLEM 12.3

KNOWN: The molar analysis of a gas mixture is specified.

FIND: Determine the analysis in terms of mass fractions, the partial pressure of each component, the volume occupied by 10 lb of mixture.

SCHEMATIC & GIVEN DATA:



i	y_i
Ar	0.2
CO_2	0.35
O_2	0.45

ENGINEERING MODEL: (1) The overall mixture acts as an ideal gas. (2) Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature. (3) Calculations are based on 1 lbmol of mixture in part (a).

ANALYSIS: (a) Considering a typical 1 lbmol of mixture

1

i	n_i	M_i	$m_i = n_i M_i$	mf_i
Ar	0.2	39.94	7.988	0.2114
CO_2	0.35	44.01	15.404	0.4076
O_2	0.45	32.00	14.400	0.3810
			$37.792 \frac{lb}{lbmol}$	

(a)

(b)
$$P_{Ar} = y_{Ar} P = (0.2)(14.7) = 2.940 \text{ lbf/in}^2$$
$$P_{CO_2} = y_{CO_2} P = (0.35)(14.7) = 5.145 \text{ lbf/in}^2$$
$$P_{O_2} = y_{O_2} P = (0.45)(14.7) = 6.615 \text{ lbf/in}^2$$

(b)

(c) Using the ideal gas equation of state

$$V = \frac{m \bar{R}/M\, T}{P} = \frac{(10 \text{ lb})\left(\frac{1545}{37.792} \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(510^\circ\text{R})}{(14.7)(144 \text{ lbf/ft}^2)}$$
$$= 98.50 \text{ ft}^3$$

(c)

---

1. The apparent molecular weight of the mixture is obtained in the calculations of part (a). Equivalently, Eq. 12.9 can be used: $M = \sum y_i M_i$

$$M = y_{Ar} M_{Ar} + y_{CO_2} M_{CO_2} + y_{O_2} M_{O_2}$$
$$= (0.2)(39.94) + (0.35)(44.01) + (0.45)(32.00) = 37.792 \frac{lb}{lbmol}$$

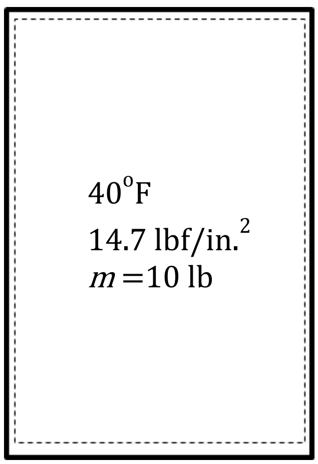
Problem 12.4

The analysis on a mass basis of a gas mixture at 40°F, 14.7 lbf/in.^{2} is 60% CO_2, 25% CO, 15% O_2. Determine:
(a) the analysis in terms of mole fractions.
(b) the partial pressure of each component, in lbf/in.^{2}
(c) the volume occupied by 10 lb of the mixture, in ft^3.

Known:
An analysis on a mass basis is specified for a gas mixture.

Find:
Determine the analysis in terms of mole fractions, the partial pressure of each component, the volume occupied by 10 lb of mixture.

Schematic and Known Data:



i	mf_i
CO_2	0.6
CO	0.25
O_2	0.15

Engineering Model:
(1) The overall mixture acts as an ideal gas.
(2) Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature.
(3) Calculations based on 1 lb of mixture in part (a).

Analysis:
(a) Considering a typical 1 lb mixture:

i	mf_i	M_i	$n_i = m_i/M_i$	y_i
CO_2	0.6	44.01	0.01363	0.5002
CO	0.25	28.01	0.00893	0.3277
O_2	0.15	32.00	0.00469	0.1721
			0.02725	

#1 $M = m/n = 1 \text{ lb} / 0.02725 \text{ lbmol} = 36.697 \text{ lb/lbmol}$

(b)

$$p_{CO_2} = y_{CO_2}p = (0.5002)\left(14.7\,\frac{\text{lbf}}{\text{in.}^2}\right) = 7.353\,\frac{\text{lbf}}{\text{in.}^2}$$
$$p_{CO} = y_{CO}p = (0.3277)\left(14.7\,\frac{\text{lbf}}{\text{in.}^2}\right) = 4.817\,\frac{\text{lbf}}{\text{in.}^2} \longleftarrow$$
$$p_{O_2} = y_{O_2}p = (0.1721)\left(14.7\,\frac{\text{lbf}}{\text{in.}^2}\right) = 2.530\,\frac{\text{lbf}}{\text{in.}^2}$$

(c) Using ideal gas law:

$$V = \frac{mRT}{p} = \frac{(10\text{ lb})\left(\frac{1545}{36.697}\,\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(500^\circ\text{R})}{\left(14.7\,\frac{\text{lbf}}{\text{in.}^2}\right)\left|144\,\frac{\text{in.}^2}{\text{ft}^2}\right|} = 99.446\text{ ft}^3 \longleftarrow$$

Comment:

1. The apparent molecular weight of the mixture is obtained in the calculations of part (a). Eq. 12.9 can also be used:

$$M = \sum_{i=1}^{j} y_i M_i = y_{CO_2}M_{CO_2} + y_{CO}M_{CO} + y_{O_2}M_{O_2}$$
$$= (0.5002)(44.01) + (0.3277)(28.01) + (0.1721)(32) = 36.7\,\frac{\text{lb}}{\text{lbmol}}$$

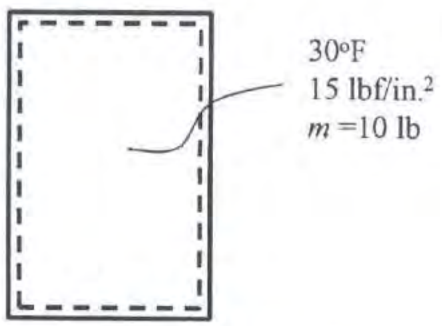
   The difference between this value and the one obtained in part (a) is due to round off.

12.5 The analysis on a mass basis of an ideal gas mixture at 30°F, 15 lbf/in.^{2} is 55% CO_2, 30% CO, 15% O_2. Determine
(a) the analysis in terms of mole fractions.
(b) the apparent molecular weight of the mixture.
(c) the partial pressure of each component, in lbf/in.^{2}.
(d) the volume occupied by 10 lb of the mixture, in ft^3.

KNOWN: An analysis on a mass basis is specified for an ideal gas mixture.

FIND: Determine (a) the analysis in terms of mole fractions, (b) the apparent molecular weight of the mixture, (c) the partial pressure of each component, and (d) the volume occupied by 10 lb of mixture.

SCHEMATIC AND GIVEN DATA:



i	mf_i
CO_2	0.55
CO	0.30
O_2	0.15

ENGINEERING MODEL:
(1) The mixture acts as an ideal gas. Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature.
(2) Calculations are based on 1 lb of mixture in parts (a) and (b).

<u>Analysis:</u>

(a) Considering a typical 1 lb of mixture

i	m_i	M_i	$n_i = m_i / M_i$	$y_i = n_i / n$
CO_2	0.55	44.01	0.0125	0.4480
CO	0.30	28.01	0.0107	0.3835
O_2	0.15	32.00	0.0047	0.1685
			$n = 0.0279$	

(b) The apparent molecular weight of the mixture is

Problem 12.5 (Continued) – Page 2

$$M = y_{CO_2} M_{CO_2} + y_{CO} M_{CO} + y_{O_2} M_{O_2}$$

$$M = (0.4480)(44.01) + (0.3835)(28.01) + (0.1685)(32.00)$$

$$M = 35.85 \frac{\text{lb}}{\text{lbmol}}$$

(c) The partial pressures are

$$p_i = y_i p$$

$$p_{CO_2} = (0.4480)\left(15 \frac{\text{lbf}}{\text{in.}^2}\right) = 6.72 \frac{\text{lbf}}{\text{in.}^2}$$

$$p_{CO} = (0.3835)\left(15 \frac{\text{lbf}}{\text{in.}^2}\right) = 5.75 \frac{\text{lbf}}{\text{in.}^2}$$

$$p_{O_2} = (0.1685)\left(15 \frac{\text{lbf}}{\text{in.}^2}\right) = 2.53 \frac{\text{lbf}}{\text{in.}^2}$$

(d) Using the ideal gas equation of state

$$V = \frac{mRT}{p} = \frac{(10\text{lb})\left(\frac{1545}{35.84} \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot {}^\circ\text{R}}\right)(490^\circ\text{R})}{\left(15 \frac{\text{lbf}}{\text{lb}^2}\right)\left(144 \frac{\text{in.}^2}{\text{ft}^2}\right)} = 97.8\,\text{ft}^3$$

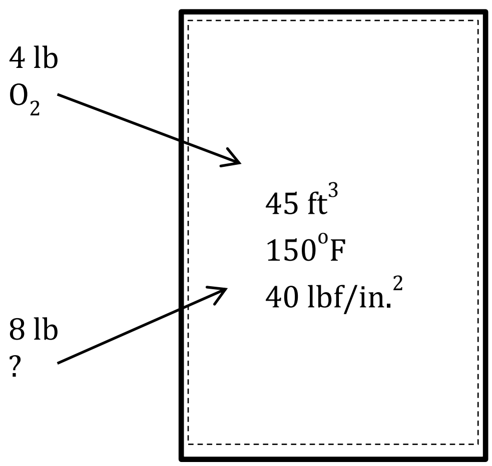
Problem 12.6

A 4-lb mass of oxygen (O_2) is mixed with 8 lb of another gas to form a mixture that occupies 45 ft^3 at 150^{o}F, 40 lbf/in.^{2} Applying ideal gas mixture principles, determine:
(a) the molecular weight of the unspecified gas.
(b) the analysis of the mixture in terms of mole fractions.

Known:
Four lb of O_2 is mixed with 8 lb of another gas to form a mixture that occupies 45 ft^3 at 150^{o}F, 40 lbf/in.^{2}.

Find:
Determine the molecular weight of the unspecified gas and the molar analysis of the mixture.

Schematic & Given Data:



Engineering Model:
(1) The overall mixture acts as an ideal gas.
(2) Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature.

Analysis:
(a) Considering the ideal gas equation of state:

$$n = \frac{pV}{\bar{R}T} \tag{1}$$

Letting *M* denote the molecular weight of the unspecified gas, the total number of moles of mixture *n* is:

$$n = n_{O_2} + n_? = \frac{m_{O_2}}{M_{O_2}} + \frac{m_?}{M_?} = \frac{4}{32} + \frac{8}{M} \tag{2}$$

Combining Eqs. (1) and (2), and inserting known values:

$$\frac{4\text{ lb}}{32\ \frac{\text{lb}}{\text{lbmol}}} + \frac{8\text{ lb}}{M} = \frac{\left(40\ \frac{\text{lbf}}{\text{in.}^2}\right)\left|\frac{144\text{ in.}^2}{\text{ft}^2}\right|(45\text{ft}^3)}{(610\text{ °R})\left(1545\,\frac{\text{ft}\cdot\text{ lbf}}{\text{lbmol}\cdot\text{ °R}}\right)} = 0.27503$$

Solving for *M*:

$$M = 53.323\,\frac{\text{lb}}{\text{lbmol}} \longleftarrow$$

(b) The composition is given by:

$$n_{O_2} = \frac{4\text{ lb}}{32\ \frac{\text{lb}}{\text{lbmol}}} = 0.125\text{ lbmol}$$

$$n_? = \frac{8\text{ lb}}{53.323\ \frac{\text{lb}}{\text{lbmol}}} = 0.15003\text{ lbmol}$$

$$n = n_{O_2} + n_? = 0.27503\text{ lbmol}$$

Finally, the analysis of the mixture in terms of molar analysis:

$$y_{O_2} = \frac{n_{O_2}}{n} = \frac{0.125}{0.27503} = 0.4545$$

$$y_? = \frac{n_?}{n} = \frac{0.15003}{0.27503} = 0.5455$$

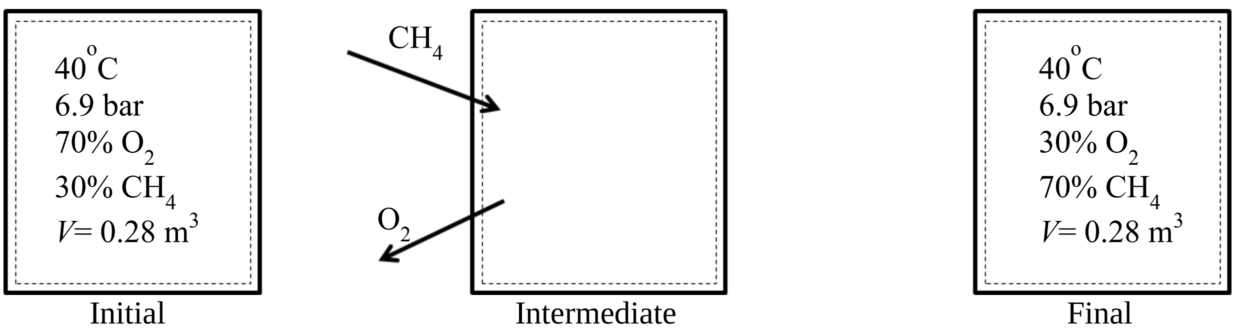
Problem 12.7

A vessel having a volume of 0.28 m^3 contains a mixture at 40°C, 6.9 bar with a molar analysis of 70% O_2, 30% CH_4. Determine the mass of methane that would have to be added and the mass of oxygen that would have to be removed, each in kg, to obtain a mixture having a molar analysis of 30% O_2, 70% CH_4 at the same temperature and pressure.

Known:
A vessel of known volume contains a mixture of O_2, CH_4 at a specified temperature, pressure and molar analysis.

Find:
Determine the mass of CH_4 that would have to be added, and the mass of O_2 that would have to be removed, to produce a mixture at the same temperature and pressure, but with a different specified molar analysis.

Schematic & Given Data:



Engineering Model:
(1) The overall mixture acts as an ideal gas.
(2) Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature.

Analysis:
As the initial and final mixtures are at the same temperature and pressure, and occupy the same total volume, the total number of moles present initially and finally is the same. Using the ideal gas equation of state:

$$n = \frac{\mathrm{m}}{M} = \frac{pV}{\bar{R}T} = \frac{\left(6.9 \cdot 10^5 \ \frac{\mathrm{N}}{\mathrm{m}^2}\right)(0.28\ \mathrm{m}^3)}{(313\ \mathrm{K})\left(8314 \frac{\mathrm{N \cdot m}}{\mathrm{kmol \cdot K}}\right)} = 0.0742\ \mathrm{kmol(mixture)}$$

Thus, the changes in the number of moles of O_2 and CH_4 present are

$$\Delta n_{\mathrm{O_2}} = \left[\left(y_{\mathrm{O_2}}\right)_{final} - \left(y_{\mathrm{O_2}}\right)_{initial}\right] n = (0.3 - 0.7)(0.0742) = -0.02968\ \mathrm{kmol}$$

$$\Delta n_{\mathrm{CH_4}} = \left[\left(y_{\mathrm{CH_4}}\right)_{final} - \left(y_{\mathrm{CH_4}}\right)_{intial}\right] n = (0.7 - 0.3)(0.0742) = 0.02968\ \mathrm{kmol}$$

Then, with molecular weights from Table A-1:

$$\Delta m_{\mathrm{O_2}} = -0.9498\ \mathrm{kg}$$

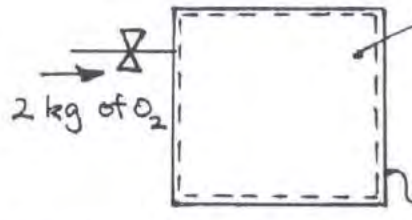
$$\Delta m_{\mathrm{CH_4}} = 0.4761\ \mathrm{kg}$$

PROBLEM 12.8

KNOWN: A known amount of oxygen (O_2) is added to Nitrogen (N_2) in a rigid tank. The initial state of the nitrogen is given, and the final temperature is given as equal to the initial temperature.

FIND: Determine the final molar analysis and the final pressure.

SCHEMATIC & GIVEN DATA:



Initially
$P_{1,N_2} = 150$ kPa
$T_{1,N_2} = 313$ K
$T_2 = T_1 = 313$ K

ENGINEERING MODEL:
1. The final contents of the tank constitute the system.
2. $T_2 = T_1$
3. The gases behave as ideal gases.

ANALYSIS: First, determine the moles of N_2 present

$$n_{N_2} = \frac{P_{1,N_2} V}{\bar{R} T} = \frac{(150\ kPa)(1\ m^3)}{(8.314\ \frac{kJ}{kmol \cdot K})(313 K)} \left| \frac{10^3\ N/m^2}{1\ kPa} \right| \left| \frac{1\ kJ}{10^3\ N \cdot m} \right|$$

$$= 0.0576\ kmol$$

For the O_2; $n_{O_2} = (2\ kg)\left(\frac{1\ kmol}{32.00\ kg}\right) = 0.0625$ kmol

Thus

$$n_{tot} = 0.0576 + 0.0625 = 0.1201\ kmol$$

$$y_{N_2} = \frac{0.0576}{0.1201} = 0.48, \quad y_{O_2} = \frac{0.0625}{0.1201} = 0.52 \longleftarrow y_{N_2}, y_{O_2}$$

The final pressure is

$$P_2 = \frac{n_{tot} \bar{R} T}{V} = \frac{(0.1201)(8.314)(313)}{(1)} \left| \frac{10^3}{10^3} \right| = 312.5\ kPa \longleftarrow P_2$$

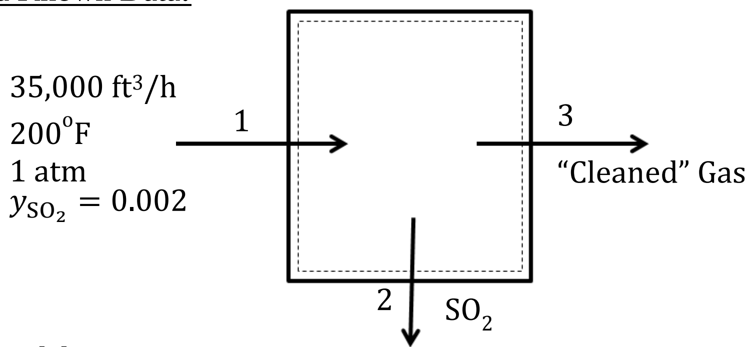
Problem 12.9

A flue gas in which the mole fraction of SO_2 is 0.002 enters a *packed bed wet scrubber* operating at steady state at 200° F, 1 atm with a volumetric flow rate of 35,000 ft^3/h. If the scrubber removes 90% (molar basis) of the entering SO_2, determine the rate at which SO_2 is removed, in lb/h.

Known:
A flue gas containing SO_2 enters a packed bed wet scrubber at a specified state and volumetric flow rate. The scrubber removes 90% (molar basis) of the SO_2.

Find:
Determine the rate SO_2 is removed.

Schematic and Known Data**:**



Engineering Model:
(1) The control volume shown in the figure is at steady state.

Analysis:
The molar flow rate of the entering mixture, $\dot{n}$, can be found as follows

$$\dot{n}_1 = \frac{(\mathrm{AV})_1}{\bar{v}_1} = \frac{(\mathrm{AV})_1 p_1}{\bar{R}T_1} = \frac{\left(35{,}000\,\frac{\mathrm{ft}^3}{\mathrm{h}}\right)\left(14.7\,\frac{\mathrm{lbf}}{\mathrm{in.}^2}\cdot\frac{144\mathrm{in.}^2}{\mathrm{ft}^2}\right)}{\left(1545\,\frac{\mathrm{ft}\cdot\mathrm{lbf}}{\mathrm{lbmol}\cdot{}^\circ\mathrm{R}}\right)(660^\circ\mathrm{R})} = 72.657\,\frac{\mathrm{lbmol\ (mixture)}}{\mathrm{h}}$$

The rate SO_2 enters is then

$$\dot{n}_{1_{SO_2}} = (0.002)\left(72.657\,\frac{\mathrm{lbmol}}{\mathrm{h}}\right) = 0.1453\,\frac{\mathrm{lbmol\ (SO_2)}}{\mathrm{h}}$$

If 90% is removed by the packed bed wet scrubber

$$\dot{n}_{2_{SO_2}} = (0.90)\left(0.1453\,\frac{\mathrm{lbmol\ (SO_2)}}{\mathrm{h}}\right) = 0.1308\,\frac{\mathrm{lbmol\ (SO_2)}}{\mathrm{h}}$$

Then from Table A-1, $M = 64.06\,\frac{\mathrm{lb}}{\mathrm{lbmol}}$ for SO_2

$$\dot{m}_{2_{SO_2}} = \left(64.06\,\frac{\mathrm{lb}}{\mathrm{lbmol}}\right)\left(0.1308\,\frac{\mathrm{lbmol}_{SO_2}}{\mathrm{h}}\right) = 8.38\,\frac{\mathrm{lb}_{SO_2}}{\mathrm{h}} \longleftarrow$$

12.10 A gas mixture with a molar analysis 20% C_3H_8 (propane) and 80% air enters a control volume operating at steady state at location 1 with a mass flow rate of 5 kg/min, as shown in Fig. 12.10 A stream of pure air enters as a separate stream at 2 and dilutes the mixture. A single stream exits with a mole fraction of propane of 3%. Assuming air has a molar analysis of 21% O_2 and 79% N_2, determine

(a) the molar flow rate of the entering air at 2, in kmol/min.

(b) the mass flow rate of oxygen in the exiting stream, in kg/min.

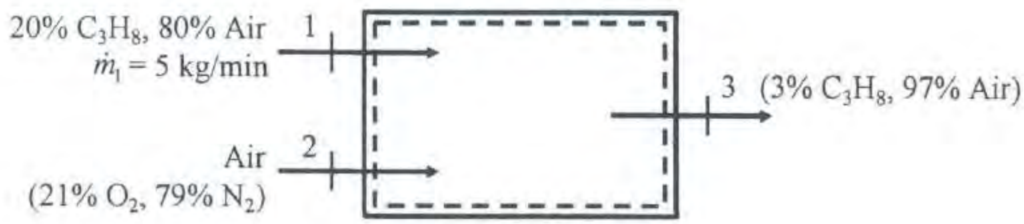
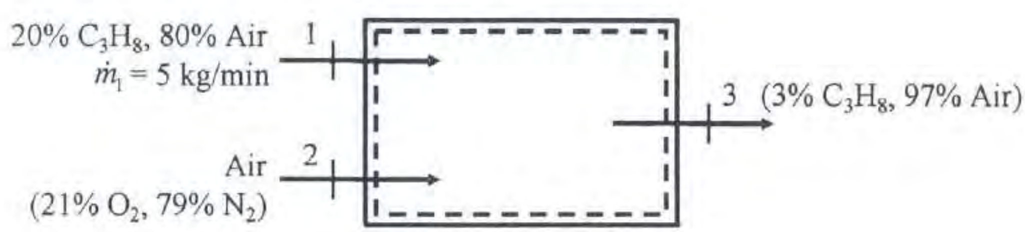


Fig. P12.10

KNOWN: A control volume at steady state has two entering streams and a single exiting stream. The molar analysis of each entering stream is provided. The mole fraction of C_3H_8 in the exiting stream is also provided.

FIND: Determine (a) the molar flow rate of the entering air stream at 2, (b) the mass flow rate of the O_2 in the exiting stream.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

(1) The control volume shown is at steady state.

(2) Air has a molar analysis of 21% O_2, 79% N_2.

Problem 12.10 (Continued) – Page 2

ANALYSIS:

(a) For the control volume, a mass rate balance is $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$, or $\dot{m}_2 = \dot{m}_3 - \dot{m}_1$. Also $\dot{n} = \dot{m}/M$

So

$$\dot{n}_2 = \frac{\dot{m}_2}{M_2} = \frac{\dot{m}_3 - \dot{m}_1}{M_2} \qquad (1)$$

The molecular weight of air is $M_2 = (0.21)(32.00) + (0.79)(28.01) = 28.85\,\frac{\text{kg}}{\text{kmol}}$

Since the molar flow rates of C_3H_8 at 1 and 3 are equal,

$0.20\dot{n}_1 = 0.03\dot{n}_3$. With $\dot{n} = \dot{m}/M$

$$0.20\frac{\dot{m}_1}{M_1} = 0.03\frac{\dot{m}_3}{M_3} \Rightarrow \dot{m}_3 = \left(\frac{0.20}{0.03}\right)\frac{M_3}{M_1}\dot{m}_1 \qquad (2)$$

where

$$M_1 = 0.20\left(44.09\frac{\text{kg}}{\text{kmol}}\right) + 0.80\left(28.85\frac{\text{kg}}{\text{kmol}}\right) = 31.9\frac{\text{kg}}{\text{kmol}}$$

$$M_3 = 0.03\left(44.09\frac{\text{kg}}{\text{kmol}}\right) + 0.97\left(28.85\frac{\text{kg}}{\text{kmol}}\right) = 29.31\frac{\text{kg}}{\text{kmol}}$$

Substituting into Eq. (2)

$$\dot{m}_3 = \left(\frac{0.20}{0.03}\right)\frac{29.31}{31.9}\left(5\frac{\text{kg}}{\text{min}}\right) = 30.63\frac{\text{kg}}{\text{min}}$$

Finally, with Eq. (1)

$$\dot{n}_2 = \frac{(30.63-5)\frac{\text{kg}}{\text{min}}}{28.85\frac{\text{kg}}{\text{kmol}}} = 0.89\frac{\text{kmol}}{\text{min}} \longleftarrow$$

(b) The molar flow rate of O_2 in the exiting stream is

$\dot{n}_{O_2} = y_{O_2}\dot{n}_3$

with $\dot{m}_{O_2} = \dot{n}_{O_2}M_{O_2}$, this becomes

$$\dot{m}_{O_2} = M_{O_2}\left(y_{O_2}\dot{n}_3\right) = M_{O_2}y_{O_2}\left(\frac{\dot{m}_3}{M_3}\right)$$

Problem 12.10 (Continued) – Page 3

To determine y_{O_2} note that the exiting mixture is

$$(0.03\,C_3H_8,\ 0.97\,\text{Air}) \Rightarrow (0.03\,C_3H_8,\ ((0.97)(0.21\,O_2),(0.97)(0.79\,N_2)))$$

$$(0.03\,C_3H_8,\ 0.97\,\text{Air}) \Rightarrow (0.03\,C_3H_8,\ ((0.2037\,O_2),(0.7663\,N_2)))$$

Thus

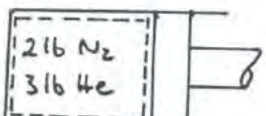
$$\dot{m}_{O_2} = \left(32.00\frac{\text{kg}}{\text{kmol}}\right)(0.2037)\left(\frac{30.63\dfrac{\text{kg}}{\text{min}}}{29.31\dfrac{\text{kg}}{\text{kmol}}}\right) = 6.81\frac{\text{kg}}{\text{min}} \longleftarrow$$

PROBLEM 12.11

KNOWN: A gas mixture of 2 lb of N_2 and 3 lb of He is maintained at a constant pressure.

FIND: Determine the mixture analysis in terms of mass fractions and mole fractions, the heat transfer required to increase the mixture temperature from 70 to 150 °F, and the change in entropy of the mixture for this process.

SCHEMATIC & GIVEN DATA:



$T_1 = 70$ °F
$T_2 = 150$ °F
$p = \text{constant}$

ENGINEERING MODEL: (1) For the system shown in the accompanying figure, changes in kinetic and potential energy are negligible. (2) The overall mixture acts as an ideal gas. Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature. (3) The specific heats of the gases are constants.

ANALYSIS: (a) The analysis of the mixture in terms of mass fractions is determined with $(mf)_i = m_i/m$

$$(mf)_{N_2} = \frac{2}{2+3} = 0.40 \quad , \quad (mf)_{He} = \frac{3}{2+3} = 0.6 \quad \longleftarrow mf$$

(b) The molar analysis of the mixture is determined with $n_i = m_i/M_i$

$$\left.\begin{aligned} n_{N_2} &= \frac{2}{28.01} = 0.0714 \text{ lbmol} \\ n_{He} &= \frac{3}{4.003} = 0.7494 \text{ lbmol} \end{aligned}\right\} \Rightarrow \begin{aligned} y_{N_2} &= \frac{0.0714}{0.0714+0.7494} = 0.0870 \\ y_{He} &= \frac{0.7494}{0.0714+0.7494} = 0.9130 \end{aligned} \quad \longleftarrow y$$

(c) With assumption 1 an energy balance reduces to $\Delta U = Q - W$ where $W = \int p\,dV = p\Delta V$. Thus,

$$Q = \Delta U + p\Delta V = (U_2 - U_1) + p(V_2 - V_1) = (U_2 + pV_2) - (U_1 + pV_1) = \Delta H$$

The change in enthalpy of the mixture is the sum of the enthalpy changes of the components:

$$\Delta H = m_{N_2}\Delta h_{N_2} + m_{He}\Delta h_{He}$$

With assumption 3, $\Delta h_{N_2} = c_{p,N_2}(T_2 - T_1)$ and $\Delta h_{He} = c_{p,He}(T_2 - T_1)$. Thus, with c_p data for N_2 from Table A-20E and for He, according to Table A-21E, $c_p = (2.5)\left(\frac{1.986}{4.003}\right) = 1.240$ Btu/lb·°R

$$Q = [m_{N_2}c_{p,N_2} + m_{He}c_{p,He}](T_2 - T_1) = \left[(2\text{ lb})\left(0.248\tfrac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right) + (3\text{ lb})\left(1.240\tfrac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)\right](80\,°\text{R})$$

$$= 337.3 \text{ Btu} \quad \longleftarrow Q$$

(d) Since pressure remains constant, the entropy change for the mixture is

$$\Delta S = m\left[c_p \ln\frac{T_2}{T_1} - R\ln\overset{0}{\cancel{\frac{p_2}{p_1}}}\right] = mc_p \ln T_2/T_1$$

where $c_p = (m_{N_2}c_{p,N_2} + m_{He}c_{p,He})/m = [(2)(.248)+(3)(1.240)]/5 = 0.8432$ Btu/lb°R

Thus

$$\Delta S = (5\text{ lb})\left(0.8432\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)\ln\frac{610}{530} = 0.593\,\frac{\text{Btu}}{°\text{R}} \quad \longleftarrow \Delta S$$

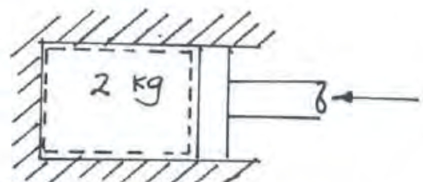
PROBLEM 12.12

KNOWN: A mixture having a specified analysis on a mass basis is compressed adiabatically between specified states.

FIND: Determine the work and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:

$mf_{N_2} = 0.3$
$mf_{CO_2} = 0.4$
$mf_{O_2} = 0.3$



$P_1 = 1\text{ bar},\ T_1 = 300\text{ K}$
$P_2 = 4\text{ bar},\ T_2 = 500\text{ K}$

ENGINEERING MODEL: (1) For the system shown in the accompanying figure, $Q = 0$ and changes in kinetic and potential energy are negligible. (2) The overall mixture acts as an ideal gas. Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature.

ANALYSIS: An energy balance reduces with assumption 1 to give $\Delta U = \cancel{Q} - W$. The internal energy change of the mixture, ΔU, equals the sum of the internal energy changes of the components. Thus, on a unit mass of mixture basis

$$-\frac{W}{m} = (mf)_{N_2}\,\Delta u_{N_2} + (mf)_{CO_2}\,\Delta u_{CO_2} + (mf)_{O_2}\,\Delta u_{O_2}$$

With data from the ideal gas tables and molecular weights from Table A-1

$$-\frac{W}{m} = 0.3\left[\frac{10423-6229}{28.01}\right] + 0.4\left[\frac{13521-6939}{44.01}\right] + 0.3\left[\frac{10614-6242}{32}\right]$$

Thus

$$\frac{W}{m} = -(44.92 + 59.82 + 40.99) = -145.73\ \text{kJ/kg}$$

Then, with m = 2 kg

$$W = -291.46\ \text{kJ} \longleftarrow W$$

In this calculation, the relationship $\Delta u = \Delta\bar{u}/M$ is used for each of the three components.

An entropy rate balance reduces as follows

$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{\,0} + \sigma \Rightarrow \sigma = \Delta S$$

The entropy change of the mixture, ΔS, equals the sum of the entropy changes of the components:

$$\Delta S = (\Delta S)_{N_2} + (\Delta S)_{CO_2} + (\Delta S)_{O_2} \Rightarrow \frac{\Delta S}{m} = (mf)_{N_2}\left(\frac{\Delta\bar{s}}{M}\right)_{N_2} + (mf)_{CO_2}\left(\frac{\Delta\bar{s}}{M}\right)_{CO_2} + (mf)_{O_2}\left(\frac{\Delta\bar{s}}{M}\right)_{O_2}$$

where each of the $\Delta\bar{s}$ terms is determined using Eq. 12.36. Thus, with $\bar{s}^{\circ}$ data

$$\frac{\sigma}{m} = 0.3\left[\frac{206.630-191.682-8.314\ln 4/1}{28.01}\right] + 0.4\left[\frac{234.814-213.915-8.314\ln 4/1}{44.01}\right] + 0.3\left[\frac{220.589-205.213-8.314\ln 4/1}{32}\right]$$

$$= (0.03665 + 0.08519 + 0.03610)\ \frac{\text{kJ/K}}{\text{kg}}$$

$$= 0.15794\ \frac{\text{kJ/K}}{\text{kg}}$$

or

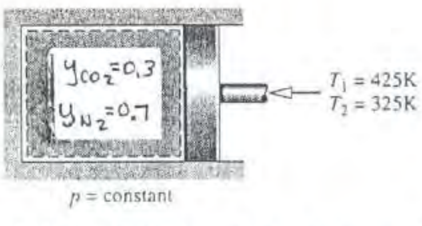
$$\sigma = 0.3159\ \frac{\text{kJ}}{\text{K}} \longleftarrow \sigma$$

PROBLEM 12.13

KNOWN: An ideal gas mixture with known molar analysis is compressed at constant pressure in a piston-cylinder assemby.

FIND: Determine the heat transfer and work, each per kg of mixture.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:
(1) The mixture is the closed system. (2) Pressure is constant. (3) The gases follow the ideal gas model with specific heats evaluated at 375 K (4) $\Delta KE = \Delta PE = 0$.

ANALYSIS: First, determine the properties of the mixture. Interpolating in Table A-20: $c_{p\,CO_2} = 1.045$ kJ/kg·K and $c_{p\,N_2} = 1.0425$ kJ/kg·K

Using the method of Example 12.1 to get the mass fractions, for n = 1 kmol

Component	$n_i \times M_i = m_i$	mf_i
CO_2	$0.3 \times 44.01 = 13.203$	0.4024
N_2	$0.7 \times 28.01 = 19.607$	0.5976
	1.0 32.810	
	$M_{mix} = 32.810$ kg/kmol	

Finally $c_{p\,mix} = mf_{CO_2} c_{p\,CO_2} + mf_{N_2} c_{p\,N_2} = 0.9920$ kJ/kg·K

From the energy balance

$$m(u_2-u_1) = Q - \underbrace{W}_{p(V_2-V_1) = mp(v_2-v_1)}$$

so

$$Q = m(u_2-u_1) + mp(v_2-v_1) = m(h_2-h_1)$$

With $h_2-h_1 = c_{p\,mix}(T_2-T_1)$

$$Q/m = c_{p\,mix}(T_2-T_1) = \left(0.9920 \frac{kJ}{kg \cdot K}\right)(325-425)K$$

$$= -99.2 \ kJ/kg \quad \text{Q/m}$$

For the work

① $$\frac{W}{m} = p(v_2-v_1) = R_{mix}(T_2-T_1)$$

$$= \left(\frac{8.314}{32.81} \frac{kJ}{kg \cdot K}\right)(325-425)K = -25.34 \ kJ \quad \text{W/m}$$

1. Aternatively, from the energy balance

$$W/m = Q/m - (u_2-u_1) = Q/m - c_v(T_2-T_1)$$

with $c_{v\,mix} = 0.7385$ kJ/kg·K (as can be verified)

$$W/m = -99.2 - (0.7385)(-100) = -25.35 \ kJ/kg$$

which agrees within round off with the value calculated above.

Problem 12.14

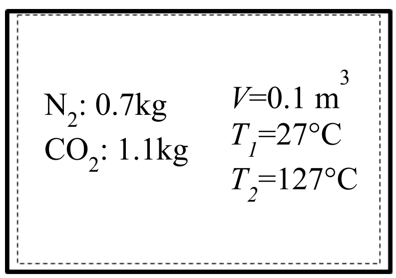
A closed, rigid tank having a volume of 0.1 m^3 contains 0.7 kg of N_2 and 1.1 kg of CO_2 at 27°C. Determine:

(a) the analysis of the mixture in terms of mass fractions.
(b) the analysis of the mixture in terms of mole fractions.
(c) the partial pressure of each component, in bar.
(d) the mixture pressure, in bar.
(e) the heat transfer, in kJ, required to bring the mixture to 127°C.
(f) the entropy change of the mixture for the process of part (e), in kJ/K.

Known:
A closed, rigid tank having a volume of 0.1 m^3 contains 0.7 kg of N_2 and 1.1 kg of CO_2 at 27°C.

Find:
Determine the mixture analysis in terms of mass and mole fractions, the partial pressure of each component and the mixture pressure, the heat transfer required to bring the mixture to 127°C and the entropy change of the mixture for this process.

Schematic & Given Data:



Engineering Model:
(1) For the system shown in the accompanying figure, changes in kinetic and potential energy are negligible.
(2) The overall mixture behaves as an ideal gas.
(3) Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature.

Analysis:
(a) The analysis of the mixture in terms of mass fractions

$$mf_{N_2} = \frac{m_{N_2}}{m} = \frac{0.7 \text{ kg}}{(1.1 + 0.7)\text{kg}} = 0.389$$

$$mf_{CO_2} = \frac{m_{CO_2}}{m} = \frac{1.1 \text{ kg}}{(1.1 + 0.7)\text{kg}} = 0.611$$

(b) The molar analysis of the mixture

$$n_{N_2} = \frac{m_{N_2}}{M_{N_2}} = \frac{0.7\text{ kg}}{28.01\frac{\text{kg}}{\text{kmol}}} = 0.025\text{ kmol}$$

$$n_{CO_2} = \frac{m_{CO_2}}{M_{CO_2}} = \frac{1.1}{44.01\frac{\text{kg}}{\text{kmol}}} = 0.025\text{ kmol}$$

Thus, $n = 0.05\text{ kmol}$ and $y_{CO_2} = y_{N_2} = 0.5$ ⟵

(c) The partial pressure of the N_2 is obtained using the ideal gas equation of state

$$p_{N_2} = \frac{n_{N_2}\overline{R}T}{V} = \frac{(0.025\text{ kmol})\left(8.314\ \frac{\text{KJ}}{\text{kmol}\cdot\text{K}}\right)\left|\frac{1000\text{ N}\cdot\text{m}}{1\text{ KJ}}\right|(300\text{ K})}{(0.1\text{ m}^3)}\left|\frac{1\text{ bar}}{10^5\frac{\text{N}}{\text{m}^2}}\right|$$

$$= 6.2355\text{ bar}$$ ⟵

Similarly, since $n_{CO_2} = n_{N_2}$, $p_{CO_2} = 6.2355\text{ bar}$. ⟵

(d) The mixture pressure is the sum of the partial pressure

$p = p_{N_2} + p_{CO_2} = 6.2355 + 6.2355 = 12.471\text{ bar}$ ⟵

(e) With assumption 1 an energy balance reduces to give

$\Delta U = Q - \cancel{W} = Q$

The change in internal energy of the mixture, ΔU, equals the sum of the internal energy changes of the components

$Q = n_{N_2}\left[\overline{u}_{N_2}(T_2) - \overline{u}_{N_2}(T_1)\right] + n_{CO_2}\left[\overline{u}_{CO_2}(T_2) - \overline{u}_{CO_2}(T_1)\right]$

With data from Tables A-23

$$Q = (0.025\text{ kmol})[8314 - 6229]\left(\frac{\text{KJ}}{\text{kmol}}\right) + (0.025\text{ kmol})[10{,}046 - 6939]\left(\frac{\text{kJ}}{\text{kmol}}\right)$$

$$= 129.8\text{ KJ}$$ ⟵

(f) The change in entropy of the mixture equals the sum of the entropy changes of the components, that is

$\Delta S = n_{N_2}\Delta\overline{s}_{N_2} + n_{CO_2}\Delta\overline{s}_{CO_2}$

Where $\Delta\overline{s}_{N_2}$ and $\Delta\overline{s}_{CO_2}$ are evaluated with Eq. 12.36 and values of $\overline{s}^o$ come from Table A-23

$$\Delta S = (0.025\text{ kmol})\left[200.071 - 191.682 - 8.314\ln\frac{p_2}{p_1}\right]\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}$$

$$+ (0.025\text{ kmol})\left[225.225 - 213.915 - 8.314\ln\frac{p_2}{p_1}\right]\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}$$

To find p_2/p_1, use the ideal gas equation of state to write

$$p_2 = \frac{nRT_2}{V};\ \ p_1 = \frac{nRT_1}{V};\ \ \rightarrow \frac{p_2}{p_1} = \frac{T_2}{T_1} = \frac{400\text{ K}}{300\text{ K}} = \frac{4}{3}$$

Thus

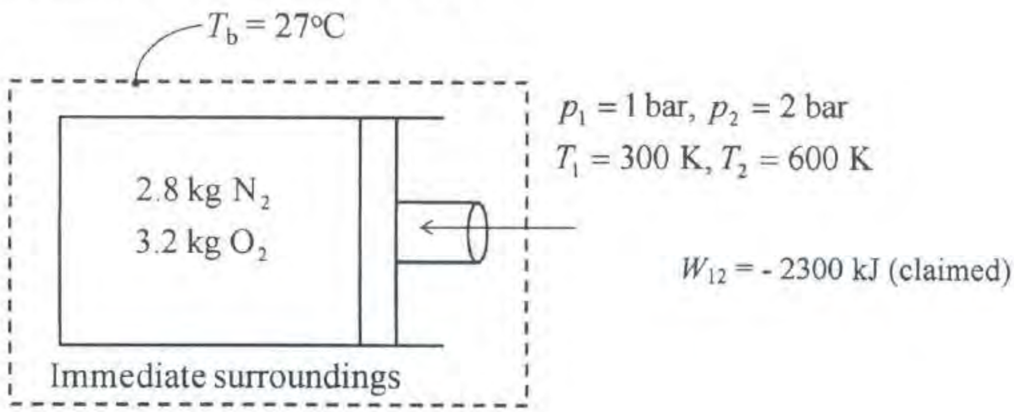
$$\Delta S = (0.025\text{ kmol})[5.9972]\frac{\text{kJ}}{\text{kmol}\cdot\text{K}} + (0.025\text{ kmol})[8.9182]\frac{\text{kJ}}{\text{kmol}\cdot\text{K}} = 0.3729\frac{\text{kJ}}{\text{K}} \longleftarrow$$

12.15 A mixture consisting of 2.8 kg of N_2 and 3.2 kg of O_2 is compressed from 1 bar, 300 K to 2 bar, 600 K. During the process there is heat transfer from the mixture to the surroundings, which are at 27°C. The work done on the mixture is claimed to be 2300 kJ. Can this value be corrected?

KNOWN: An ideal gas mixture of known composition is compressed between two specified states. Known heat transfer occurs from the mixture to the surroundings at a specified temperature.

FIND:
Determine if the work done on the mixture during the process can be 2300 kJ.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
(1) As shown in the figure, the system consists of the mixture and a portion of its immediate surroundings.
(2) There are no significant kinetic or potential energy effects. There is no net change in the state of the immediate surroundings.
(3) The overall mixture acts as an ideal gas. Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature.

ANALYSIS:
The plan is to determine the entropy production and use this as a basis for deciding if $W = -2300$ kJ is possible. However, as heat transfer occurs from the mixture at temperatures ranging from 300 K to 600 K, this objective can be best approached by considering an enlarged system as shown n the accompanying figure. Then an entropy balance reads

Problem 12.15 (Continued) – Page 2

$$\Delta S = \frac{Q}{T_b} + \sigma \quad (1)$$

where σ accounts for entropy production within the enlarged system. However, in light of assumption 2, ΔS is just the change in entropy of the mixture and Q is the heat transfer for a system consisting of the mixture alone. An energy balance for the mixture reads $\Delta U = Q - W \Rightarrow Q = \Delta U + W = n_{N_2}(\Delta\overline{u})_{N_2} + n_{O_2}(\Delta\overline{u})_{O_2} + W$

With $W = -2300$ kJ and data from Tables A-1 and A-23

$$Q = \left(\frac{2.8\,\text{kg}}{28.01\,\text{kg/kmol}}\right)\left[(12574 - 6229)\frac{\text{kJ}}{\text{kmol}}\right] + \left(\frac{3.2\,\text{kg}}{32.00\,\text{kg/kmol}}\right)\left[(12940 - 6242)\frac{\text{kJ}}{\text{kmol}}\right] - 2300\,\text{kJ}$$
$$= -995.93\,\text{kJ}$$

Then, with Eq. (1), incorporating text Eq 12.34 and inserting data from Table A-23

$$\sigma = \left[n_{N_2}(\Delta\overline{s})_{N_2} + n_{O_2}(\Delta\overline{s})_{O_2}\right] - \frac{Q}{T_b}$$
$$= \left\{n_{N_2}\left[\left(\overline{s}_2^\circ - \overline{s}_1^\circ\right) - \overline{R}\ln\left(\frac{p_2}{p_1}\right)\right]_{N_2} + n_{O_2}\left[\left(\overline{s}_2^\circ - \overline{s}_1^\circ\right) - \overline{R}\ln\left(\frac{p_2}{p_1}\right)\right]_{O_2}\right\} - \frac{Q}{T_b}$$
$$= \left(\frac{2.8\,\text{kg}}{28.01\,\text{kg/kmol}}\right)\left[(212.066 - 191.682)\frac{\text{kJ}}{\text{kmol}\cdot\text{K}} - 8.314\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\ln\left(\frac{2}{1}\right)\right]$$
$$+ \left(\frac{3.2\,\text{kg}}{32.00\,\text{kg/kmol}}\right)\left[(226.346 - 205.213)\frac{\text{kJ}}{\text{kmol}\cdot\text{K}} - 8.314\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\ln\left(\frac{2}{1}\right)\right] - \left(\frac{-995.93\,\text{kJ}}{300\,\text{K}}\right)$$
$$= 1.4616\frac{\text{kJ}}{\text{K}} + 1.537\frac{\text{kJ}}{\text{K}} + 3.3198\frac{\text{kJ}}{\text{K}} = 6.3184\frac{\text{kJ}}{\text{K}}$$

Since σ is positive, and there is no apparent inconsistency with any other thermodynamic principle, the given work value must be admitted as ***possible***.

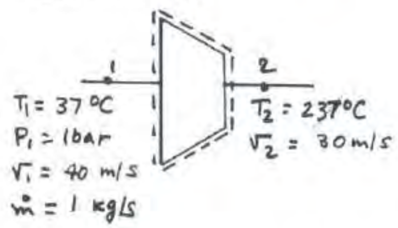
PROBLEM 12.16

KNOWN: A mixture with a specified molar analysis enters a compressor operating at steady state at a specified mass flow rate and state and exits at a specified temperature and velocity. The rate of heat transfer from the compressor to its surroundings is also specified.

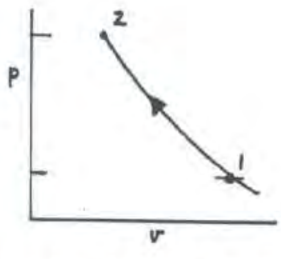
FIND: (a) Determine the power input to the compressor. (b) If the compression were polytropic, evaluate the polytropic exponent n and the exit pressure.

SCHEMATIC & GIVEN DATA:

① $\dot{Q}_{cv} = 0.5\,\dot{W}_{cv}$



i	y_i
CO_2	0.5
CO	0.333
O_2	0.167



ENGINEERING MODEL: (1) The control volume is steady state. (2) Potential energy effects are negligible. (3) The mixture adheres to the idealizations of the Dalton model. (4) The mixture composition remains constant.

ANALYSIS: (a) At steady state, mass and energy rate balances reduce to give

$$\frac{\dot{W}_{cv}}{\dot{m}} = \frac{\dot{Q}_{cv}}{\dot{m}} + h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}$$

Since $\dot{Q}_{cv} = 0.5\,\dot{W}_{cv}$, this becomes

$$0.95\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} \qquad (1)$$

In this equation $(h_1 - h_2)$ represents a change in the specific enthalpy of the mixture, per unit of mass of mixture. This can be evaluated as follows

$$h_1 - h_2 = \frac{\bar{h}_1 - \bar{h}_2}{M} = \frac{1}{M}\left[y_{CO_2}(\bar{h}_1 - \bar{h}_2)_{CO_2} + y_{CO}(\bar{h}_1 - \bar{h}_2)_{CO} + y_{O_2}(\bar{h}_1 - \bar{h}_2)_{O_2} \right]$$

where M is the mixture molecular weight:

$$M = y_{CO_2}M_{CO_2} + y_{CO}M_{CO} + y_{O_2}M_{O_2} = 0.5(44.01) + 0.333(28.01) + 0.167(32) = 36.68$$

With data from the ideal gas tables

$$h_1 - h_2 = \frac{1}{36.68\,\frac{kg(mix)}{kmol(mix)}}\left[0.5\,\frac{kmol(CO_2)}{kmol(mix)}(9807 - 18126)\frac{kJ}{kmol(CO_2)} + 0.333(9014 - 14898) + 0.167(9030 - 15082) \right]$$

$$= -194.37\ kJ/kg(mix)$$

Substituting values into Eq. (1)

$$0.95\frac{\dot{W}_{cv}}{\dot{m}} = (-194.37)\frac{kJ}{kg(mix)} + \underbrace{\frac{[(40\,m/s)^2 - (30\,m/s)^2]}{2}\left|\frac{1\,N}{1\,kg\cdot\frac{m}{s^2}}\right|\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|}_{0.35\ kJ/kg} = -194.02\frac{kJ}{kg\cdot mix}$$

or

$$\frac{\dot{W}_{cv}}{\dot{m}} = -204.23\ kJ/kg(mix) \longleftarrow \frac{\dot{W}_{cv}}{\dot{m}}$$

(b) If the process adheres to pv^n = constant, the work required would be given by Eq. 6.55a:

$$\frac{\dot{W}_{cv}}{\dot{m}} = -\frac{nR}{n-1}(T_2 - T_1) \quad \Rightarrow \quad \frac{n-1}{n} = \frac{R(T_2 - T_1)}{(-\dot{W}_{cv}/\dot{m})}$$

PROBLEM 12.16 (Contd.) - Page 2

Inserting known values

$$\frac{n-1}{n} = \frac{(8.314/36.68)(200)}{204.23} \Rightarrow n = 1.285 \longleftarrow n$$

Then, with Eq. 3.56

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{n/(n-1)} \Rightarrow P_2 = P_1 \left(\frac{T_2}{T_1}\right)^{n/(n-1)}$$

or, with n = 1.285

$$P_2 = (1 \text{ bar})\left(\frac{510K}{310K}\right)^{n/(n-1)} = 9.4 \text{ bar} \longleftarrow P_2$$

---

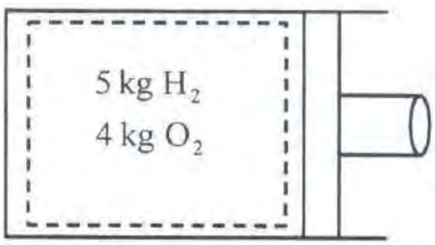
1. Since a power <u>input</u> is required $\dot{W}_{cv}$ takes on a negative value. Thus, in this expression $\dot{Q}_{cv}$ is negative, which is in accord with a heat <u>loss</u> during the compression.

12.17 A mixture of 5 kg of H_2 and 4 kg O_2 is compressed in a piston-cylinder assembly in a polytropic process for which $n = 1.6$. The temperature increases from 40 to 250°C. Using constant values for the specific heats, determine
(a) the heat transfer, in kJ.
(b) the entropy change, in kJ/K.

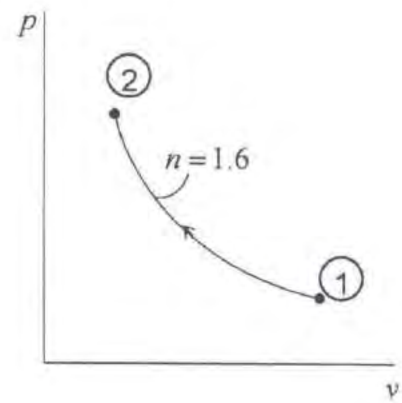
KNOWN: A specified mixture of H_2 and O_2 and known mass is compressed in a polytropic process. The polytropic exponent is known, and the initial and final temperatures are given.

FIND: Determine the (a) heat transfer, and (b) entropy change.

SCHEMATIC AND GIVEN DATA:



$pv^n = \text{const.}, n = 1.6$
$T_1 = 40°\text{C}, T_2 = 250°\text{C}$



ENGINEERING MODEL:
(1) System is a closed system, as shown in figure above.
(2) For the system, the composition remains constant and kinetic/potential energy effects are negligible.
(3) Each component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature. The overall mixture acts as an ideal gas.
(4) The process is polytropic with $pv^n = \text{const.}$
(5) Specific heats are constant at the mean temperature.

Analysis:

(a) An energy balance reduces to give $Q = \Delta U + W$, where

$$W = \int_1^2 p dv = \frac{m\left(\overline{R}/M\right)(T_2 - T_1)}{1-n} \tag{1}$$

Problem 12.17 (Continued) – Page 2

From the given data; $m = 5 + 4 = 9\,\text{kg}$

The molecular weight of the mixture is determined from

$$M = y_1 M_1 + y_2 M_2 \tag{2}$$

$$\text{H}_2\text{:}\;\; n_1 = \frac{m_1}{M_1} = \frac{5}{2.016} = 2.480\,\text{kmol} \quad \text{O}_2\text{:}\;\; n_2 = \frac{4}{32.00} = 0.125\,\text{kmol}$$

Thus

$$n = n_1 + n_2 = 2.605$$

and the mole fraction are

$$y_1 = 2.480/2.605 = 0.952$$

$$y_2 = 1 - y_1 = 0.048$$

Thus, the molecular weight is

$$M = (0.952)(2.016) + (0.048)(32.00)$$

$$M = 3.455\,\frac{\text{kg}}{\text{kmol}}$$

Substituting values into Eq. (1)

$$W = \frac{(9\text{kg})\left(\dfrac{8.314}{3.455}\dfrac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(210\,\text{K})}{1-1.6} = -7580.1\,\text{kJ}$$

Next determine ΔU

$$\Delta U = m_1 c_{v1}[T_2 - T_1] + m_2 c_{v2}[T_2 - T_1] = \underbrace{(m_1 c_{v1} + m_2 c_{v2})}_{mc_v}[T_2 - T_1]$$

The specific heats c_v are evaluated by interpolating in Table A-20 at the mean temperature 418 K to get :

$$c_{v1} = 10.361\,\text{kJ}/\text{kg}\cdot\text{K}$$

$$c_{v2} = 0.686\,\text{kJ}/\text{kg}\cdot\text{K}$$

Then

Problem 12.17 (Continued) – Page 3

$$\Delta U = [(5)(10.361)+(4)(0.686)]\left(\frac{\text{kJ}}{\text{kg}}\right)[210\ \text{K}]$$

$$= \left(54.549\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(210\ \text{K})$$

$$= 11455.3\ \text{kJ}$$

Collecting results

$$Q = \Delta U + W = 11455.3\text{kJ} - 7580.1\text{kJ} = 3875\ \text{kJ}$$

(b) To find ΔS

$$\Delta S = m\left[c_v \ln\frac{T_2}{T_1} + \frac{\overline{R}}{M}\ln\frac{V_2}{V_1}\right]$$

$$= m\left[c_v \ln\frac{T_2}{T_1} + \frac{\overline{R}}{M}\ln\left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}\right]$$

$$= m\left[c_v \ln\frac{T_2}{T_1} - \frac{\overline{R}/M}{n-1}\ln\frac{T_2}{T_1}\right]$$

$$= m\left[c_v - \frac{\overline{R}/M}{n-1}\right]\ln\frac{T_2}{T_1} \text{ where } c_v = \frac{m_1c_{v1} + m_2c_{v2}}{m}$$

$$= (9\ \text{kg})\left[\frac{54.549}{9}\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - \frac{8.314/3.455}{1.6-1}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]\ln\frac{523}{313}$$

$$= (9\ \text{kg})\left[6.061\frac{\text{kJ}}{\text{kg}\cdot\text{K}} - 4.011\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]\left(\ln\frac{523}{313}\right)$$

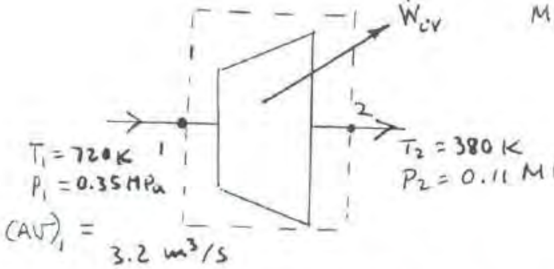
$$= 9.472\frac{\text{kJ}}{\text{K}}$$

PROBLEM 12.18

KNOWN: A specified mixture expands through a turbine for which steady state operating data are provided.

FIND: Determine the power developed.

SCHEMATIC & GIVEN DATA:



Mixture: 10% CO_2, 19% H_2O, 71% N_2

ENGINEERING MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) The mixture adheres to the idealizations of the Dalton model and the composition remains constant.

ANALYSIS: Reducing mass and energy balances at steady state

$$\dot{W}_{cv} = \dot{m}[h_1 - h_2] = \dot{m}\left[\frac{\bar{h}_1 - \bar{h}_2}{M_{mix}}\right] = \dot{m}\left[\frac{y_{CO_2}(\bar{h}_1-\bar{h}_2)_{CO_2} + y_{H_2O}(\bar{h}_1-\bar{h}_2)_{H_2O} + y_{N_2}(\bar{h}_1-\bar{h}_2)_{N_2}}{M_{mix}}\right]$$

And

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{(AV)_1 P_1}{(\bar{R}/M_{mix})T_1} \Rightarrow \frac{\dot{m}}{M_{mix}} = \frac{(AV)_1 P_1}{\bar{R}\,T_1} = \frac{(3.2\,kg/s)(0.35\times10^6\,\frac{N}{m^2})}{(8314\,\frac{N\cdot m}{kmol\cdot K})(720K)} = 0.1871\,\frac{kmol(mix)}{s}$$

Then, with data from the ideal gas tables

$$\dot{W}_{cv} = 0.1871\left[0.10(28{,}121-12{,}552) + 0.19(24{,}840-12{,}672) + 0.71(21{,}220-11{,}055)\right]\frac{kJ}{s}\left|\frac{1\,kW}{1\,kJ/s}\right|$$

$$= 2074.2\,kW$$

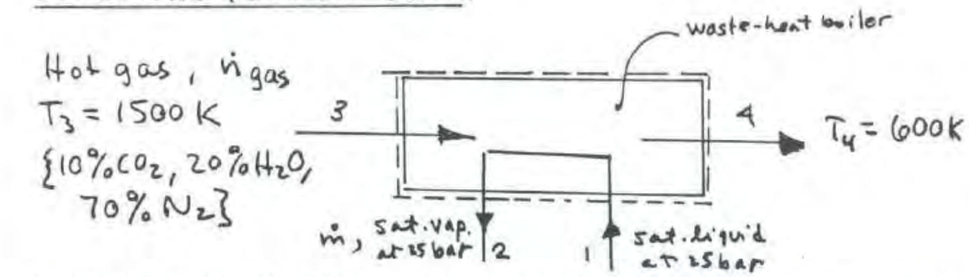
$\dot{W}_{cv}$

PROBLEM 12.19

KNOWN: Operating data are provided for a waste-heat boiler operating at steady state.

FIND: Determine the mass flow rate of the steam generated, in kg per kmol of entering hot gas.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$, and kinetic/potential energy effects are negligible. (3) The entering and exiting gas mixtures are modeled as ideal gas mixtures.

ANALYSIS: With the indicated assumptions, the mass and energy rate balances reduce to read

$$0 = \dot{m}[h_1 - h_2] + \dot{n}_{gas}\Big[y_{CO_2}(\bar{h}_{CO_2}(T_3) - \bar{h}_{CO_2}(T_4)) + y_{H_2O}(\bar{h}_{H_2O}(T_3) - \bar{h}_{H_2O}(T_4)) + y_{N_2}(\bar{h}_{N_2}(T_3) - \bar{h}_{N_2}(T_4))\Big]$$

or

$$\frac{\dot{m}}{\dot{n}_{gas}} = \frac{y_{CO_2}[\bar{h}_{CO_2}(T_3) - \bar{h}_{CO_2}(T_4)] + y_{H_2O}[\bar{h}_{H_2O}(T_3) - \bar{h}_{H_2O}(T_4)] + y_{N_2}[\bar{h}_{N_2}(T_3) - \bar{h}_{N_2}(T_4)]}{h_2 - h_1}$$

With data from Table A-23 for the numerator and from Table A-3 for the denominator

$$\frac{\dot{m}}{\dot{n}_{gas}} = \frac{0.1[71{,}078 - 22{,}280] + 0.2[57{,}999 - 20{,}402] + 0.7[47{,}073 - 17{,}563]\ \frac{kJ}{kmol(mix)}}{1841.0\ kJ/kg(steam)}$$

$$= 17.96\ \frac{kg(steam)}{kmol(mix)} \longleftarrow \frac{\dot{m}}{\dot{n}_{gas}}$$

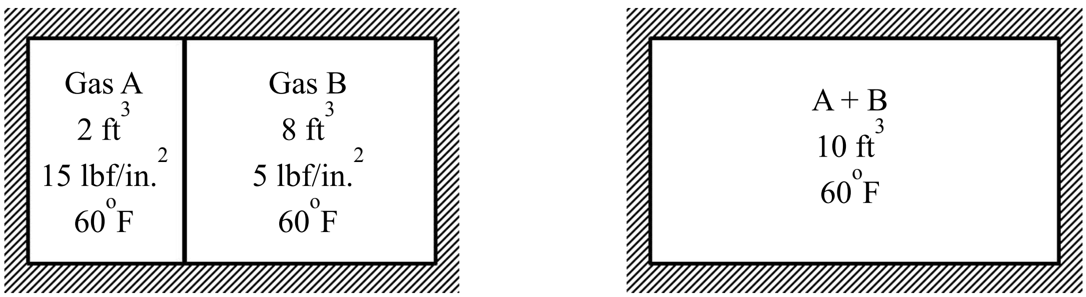
Problem 12.20

Two cubic feet of gas A initially at 60°F, 15 lbf/in.^{2} is allowed to mix adiabatically with 8 ft^3 of gas B initially at 60°F, 5 lbf/in.^{2} Assuming that the total volume remains constant and applying ideal gas mixture principles, determine
(a) the final mixture pressure, in lbf/in.^{2}
(b) the entropy change of each gas in Btu/lbmol·°R.

Known:
Two ft^3 of gas A initially at 60°F, 15 lbf/in.^{2} is allowed to mix adiabatically with 8 ft^3 of gas B initially at 60°F, 5 lbf/in.^{2}.

Find:
Determine the final pressure and the entropy change of each gas.

Schematic & Given Data:



Initial Final

Engineering Model:
(1) The individual gases and the overall mixture behave as an ideal gas.
(2) The Dalton model applies to the mixture.

Analysis:
As illustrated in the accompanying figure, the final temperature is 60°F. This can be confirmed by application of an energy balance to a system consisting of the two gases.

(a) The final pressure can be determined using the ideal gas equation of state:

$$p = \frac{n\bar{R}T}{V}$$

Where $n = n_\mathrm{A} + n_\mathrm{B}$ and $V = V_\mathrm{A} + V_\mathrm{B}$. The amount of each gas present can also be found using the ideal gas equation of state:

$$n = n_\mathrm{A} + n_\mathrm{B} = \frac{p_\mathrm{A}V_\mathrm{A}}{\bar{R}T_\mathrm{A}} + \frac{p_B V_\mathrm{B}}{\bar{R}T_\mathrm{B}}$$

Accordingly:

$$p = \frac{\left(\frac{p_A V_A}{\bar{R} T_A} + \frac{p_B V_B}{\bar{R} T_B}\right) \bar{R} T}{V} = \left[\left(\frac{V_A}{V}\right) p_A + \left(\frac{V_B}{V}\right) p_B\right] = \left(\frac{2}{10}\right)(15) + \left(\frac{8}{10}\right)(5) = 7 \frac{\text{lbf}}{\text{in.}^2} \longleftarrow$$

(b) The change in entropy of each component can be determined using either Eq. 6.21 or Eq. 6.22. Since the initial volume of each is known and each gas is assumed to occupy the full mixture volume, Eq. 6.21 is convenient:

$$\Delta \bar{s}_A = c_v \ln \frac{T}{T_A} + \bar{R} \ln \frac{V}{V_A} = 0 + \left(1.986 \frac{\text{Btu}}{\text{lbmol} \cdot {}^\circ\text{R}}\right) \ln \frac{10}{2} = 3.1963 \frac{\text{Btu}}{\text{lbmol} \cdot {}^\circ\text{R}} \longleftarrow$$

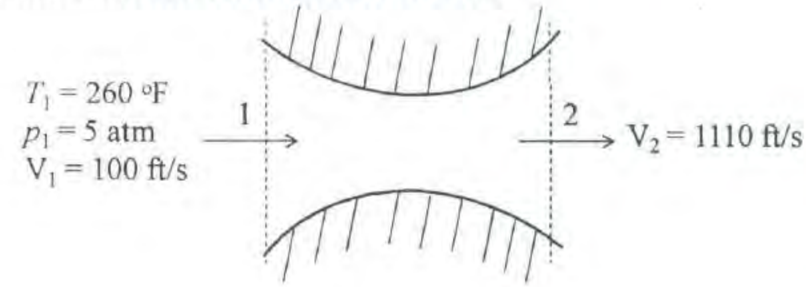
$$\Delta \bar{s}_B = c_v \ln \frac{T}{T_B} + \bar{R} \ln \frac{V}{V_B} = 0 + \left(1.986 \frac{\text{Btu}}{\text{lbmol} \cdot {}^\circ\text{R}}\right) \ln \frac{10}{8} = 0.4432 \frac{\text{Btu}}{\text{lbmol} \cdot {}^\circ\text{R}} \longleftarrow$$

12.21 An equimolar mixture of helium (He) and carbon dioxide (CO_2) enters an insulated nozzle at 260°F, 5 atm, 100 ft/s and expands isentropically to a velocity of 1110 ft/s. Determine the temperature, in °F, and the pressure, in atm, at the nozzle exit. Neglect potential energy effects.

KNOWN: An equimolar mixture of He and CO_2 expands isentropically through a nozzle from a specified inlet state to a specified exit velocity.

FIND: Determine the exit temperature and pressure.

SCHEMATIC AND GIVEN DATA:



i	y_i	M_i (lb/lbmol)
He	0.5	4.003
CO_2	0.5	44.01

ENGINEERING MODEL:
(1) The nozzle operates at steady state and the expansion is isentropic.
(2) Potential energy effects are negligible.
(3) The mixture adheres to the idealizations of the Dalton model and the composition remains constant.

ANALYSIS:
Reduction of mass and energy rate balances at steady state result in

$$0 = \underbrace{\dot{Q}_{cv}}_{=0} - \underbrace{\dot{W}_{cv}}_{=0} + \dot{m}\left((h_1 - h_2) + \frac{V_2^2 - V_1^2}{2} + g\underbrace{(z_1 - z_2)}_{=0}\right)$$

Thus

$$(h_1 - h_2) + \frac{V_2^2 - V_1^2}{2} = 0 \Rightarrow \frac{(\bar{h}_1 - \bar{h}_2)}{M} + \frac{V_2^2 - V_1^2}{2} = 0 \quad (1)$$

where M is the mixture molecular weight:

$$M = y_{He}M_{He} + y_{CO_2}M_{CO_2}$$
$$= (0.5)\left(4.003\frac{\text{lb}}{\text{lbmol}}\right) + (0.5)\left(44.01\frac{\text{lb}}{\text{lbmol}}\right) = 24.01\frac{\text{lb}}{\text{lbmol}}$$

and

Problem 12.21 (Continued) – Page 2

$$\bar{h}_2 - \bar{h}_1 = y_{\mathrm{He}}\left(\bar{h}_2 - \bar{h}_1\right)_{\mathrm{He}} + y_{\mathrm{CO_2}}\left(\bar{h}_2 - \bar{h}_1\right)_{\mathrm{CO_2}} = y_{\mathrm{He}} c_{\mathrm{p,He}}\left(T_2 - T_1\right)_{\mathrm{He}} + y_{\mathrm{CO_2}}\left(\bar{h}_2 - \bar{h}_1\right)_{\mathrm{CO_2}}$$

where $c_{\mathrm{p,He}} = (5/2)\bar{R} = (5/2)(1.986\ \mathrm{Btu/lbmol\cdot{}^\circ R}) = 4.965\ \mathrm{Btu/lbmol\cdot{}^\circ R}$ from note (a) in Table A-21E.

Using $\bar{h}$ values for CO_2 from Table A-23E and substituting into Eq. (1)

$$\frac{(0.5)\left(4.965\dfrac{\mathrm{Btu}}{\mathrm{lbmol\cdot{}^\circ R}}\right)\left(T_2 - 720^\circ\mathrm{R}\right) + (0.5)\left[\bar{h}_{\mathrm{CO_2}}(T_2) - 5748.4\dfrac{\mathrm{Btu}}{\mathrm{lbmol\cdot{}^\circ R}}\right]}{24.01\ \mathrm{lb/lbmol}}$$

$$+\frac{(1110\ \mathrm{ft/s})^2 - (100\ \mathrm{ft/s})^2}{2}\left|\frac{\mathrm{lbf}}{32.2\ \mathrm{lb\cdot ft/s^2}}\right|\left|\frac{\mathrm{Btu}}{778\ \mathrm{ft\cdot lbf}}\right| = 0$$

$$\left(2.4825\frac{\mathrm{Btu}}{\mathrm{lbmol\cdot{}^\circ R}}\right)T_2 + 0.5\bar{h}_{\mathrm{CO_2}}(T_2) = 4075.96\frac{\mathrm{Btu}}{\mathrm{lb\,(mixture)}}$$

Solving this equation by trial gives $T_2 = 640^\circ\mathrm{R}$. ←

The expansion is isentropic: $\bar{s}_2 - \bar{s}_1 = 0$. This can be expressed as

$$y_{\mathrm{He}}\left(\bar{s}_2 - \bar{s}_1\right)_{\mathrm{He}} + y_{\mathrm{CO_2}}\left(\bar{s}_2 - \bar{s}_1\right)_{\mathrm{CO_2}} = 0$$

or

$$y_{\mathrm{He}}\left[c_{\mathrm{p,He}}\ln\left(\frac{T_2}{T_1}\right) - \bar{R}\ln\left(\frac{p_2}{p_1}\right)\right] + y_{\mathrm{CO_2}}\left[\bar{s}^\circ_{\mathrm{CO_2}}(T_2) - \bar{s}^\circ_{\mathrm{CO_2}}(T_1) - \bar{R}\ln\left(\frac{p_2}{p_1}\right)\right] = 0$$

Solving and inserting values from Table A-23

$$\ln\frac{p_2}{p_1} = \frac{y_{\mathrm{He}} c_{\mathrm{p,He}}\ln\dfrac{T_2}{T_1} + y_{\mathrm{CO_2}}\left(\bar{s}_{\mathrm{CO_2}}(T_2) - \bar{s}_{\mathrm{CO_2}}(T_1)\right)}{\bar{R}}$$

$$= 0.5\left[\frac{(5/2)\bar{R}\ln(640/720) + (52.641 - 53.780)}{\bar{R}}\right]$$

$$= -0.43399$$

Thus,

$$\frac{p_2}{p_1} = 0.6479$$

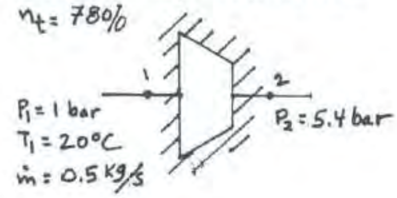
giving $p_2 = 3.24$ atm. ←

PROBLEM 12.22

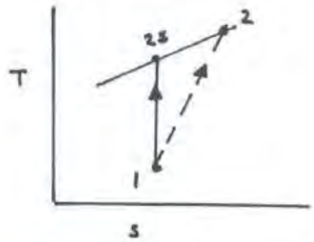
KNOWN: A gas mixture with a specified molar analysis enters a compressor at a specified mass flow rate and state and exits at a specified pressure. The isentropic compressor efficiency is also known.

FIND: Determine the temperature at the exit, the power required, and the rate of entropy production.

SCHEMATIC & GIVEN DATA:



i	y_i	M_i
O_2	0.6	32
N_2	0.4	28.01



ENGINEERING MODEL: (1) The compressor is well insulated and operates at steady state. (2) Kinetic and potential energy effects can be ignored. (3) The mixture adheres to the idealizations of the Dalton model and composition remains constant.

ANALYSIS: The temperature T_2 can be determined using the isentropic compressor efficiency, but first the temperature at state 2s, T_{2s}, must be found. Since $\bar{s}_{2s} - \bar{s}_1 = 0$, Eq. 12.36 can be invoked to write

$$y_{O_2}(\bar{s}_{2s} - \bar{s}_1)_{O_2} + y_{N_2}(\bar{s}_{2s} - \bar{s}_1)_{N_2} = y_{O_2}\left(\bar{s}^\circ_{2s} - \bar{s}^\circ_1 - \bar{R}\ln\frac{P_2}{P_1}\right)_{O_2} + y_{N_2}\left(\bar{s}^\circ_{2s} - \bar{s}^\circ_1 - \bar{R}\ln\frac{P_2}{P_1}\right)_{N_2} = 0$$

Accordingly, with $\bar{s}^\circ$ data from the ideal gas tables

$$y_{O_2}(\bar{s}^\circ_{2s})_{O_2} + y_{N_2}(\bar{s}^\circ_{2s})_{N_2} = y_{O_2}(\bar{s}^\circ_1)_{O_2} + y_{N_2}(\bar{s}^\circ_1)_{N_2} + \bar{R}\ln\frac{P_2}{P_1}$$

$$= (0.6)(204.524) + (0.4)(190.998) + 8.314\ln 5.4 = 213.13\ \frac{\text{kJ}}{\text{kmol}\cdot\text{K}}$$

① Solving iteratively using table data, $T_{2s} = 470\text{ K}$.

In accord with listed assumptions, the isentropic compressor efficiency is

$$\eta_c = \frac{\bar{h}_{2s} - \bar{h}_1}{\bar{h}_2 - \bar{h}_1}$$

Thus

$$\bar{h}_2 - \bar{h}_1 = \frac{\bar{h}_{2s} - \bar{h}_1}{\eta_c} = \frac{y_{O_2}(\bar{h}_{2s} - \bar{h}_1)_{O_2} + y_{N_2}(\bar{h}_{2s} - \bar{h}_1)_{N_2}}{\eta_c} = \frac{0.6(13842 - 8533) + 0.4(13693 - 8521)}{0.78}$$

$$= 6736\text{ kJ/kmol}$$

Accordingly

$$y_{O_2}(\bar{h}_2 - \bar{h}_1)_{O_2} + y_{N_2}(\bar{h}_2 - \bar{h}_1)_{N_2} = 6736\text{ kJ/kmol}$$

or

$$y_{O_2}(\bar{h}_2)_{O_2} + y_{N_2}(\bar{h}_2)_{N_2} = y_{O_2}(\bar{h}_1)_{O_2} + y_{N_2}(\bar{h}_1)_{N_2} + 6736$$

$$= (0.6)(8533) + (0.4)(8521) + 6736 = 15{,}264\text{ kJ/kmol}$$

Solving, $T_2 = 520\text{K}\ (247°\text{C})$ ◄—— T_2

The power required is

$$\dot{W}_c = -\dot{m}(h_2 - h_1) = -\dot{m}\left(\frac{\bar{h}_2 - \bar{h}_1}{M}\right)$$

where M is the mixture molecular weight: $M = y_{O_2}M_{O_2} + y_{N_2}M_{N_2} = (0.6)(32) + (0.4)(28.01) = 30.4$. Therefore

PROBLEM 12.22 (Cont'd.) - Page 2

$$\dot{W}_c = -\left(0.5\frac{kg}{s}\right)\left[\frac{6736\ kJ/kmol}{30.4\ \ kg/kmol}\right]\left|\frac{1\ kW}{1\ kJ/s}\right| = -110.8\ kW \quad \dot{W}_c$$

Mass and entropy rate balances reduce at steady state to give

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{\,0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

Thus

$$\dot{\sigma}_{cv} = \dot{m}(s_2 - s_1)$$

$$= \dot{m}\left(\frac{\bar{s}_2 - \bar{s}_1}{M}\right) = \dot{m}\left[\frac{y_{O_2}(\bar{s}_2 - \bar{s}_1)_{O_2} + y_{N_2}(\bar{s}_2 - \bar{s}_1)_{N_2}}{M}\right]$$

With $\bar{s}^\circ$ data from Table A-23

$$\dot{\sigma}_{cv} = \left(0.5\frac{kg}{s}\right)\left[\frac{0.6(221.812 - 204.524) + 0.4(207.792 - 190.998) - 8.314\ln 5.4}{30.4}\right]\left(\frac{kJ}{kg\cdot K}\right)\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 0.0505\ \frac{kW}{K} \quad \dot{\sigma}_{cv}$$

---

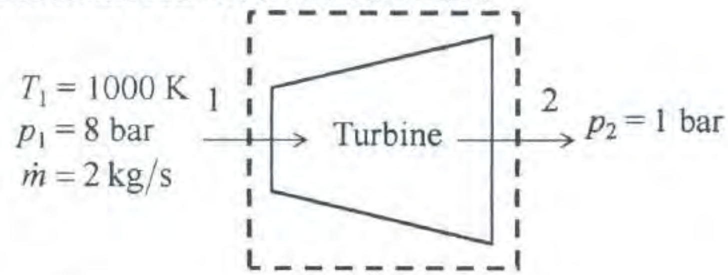
1. An iterative solution with table data can be avoided by using IT. The resuts for T_{2s}, T_2, $\dot{W}_{cv}$, and $\dot{\sigma}_{cv}$ obtained using IT agree with the values given here.

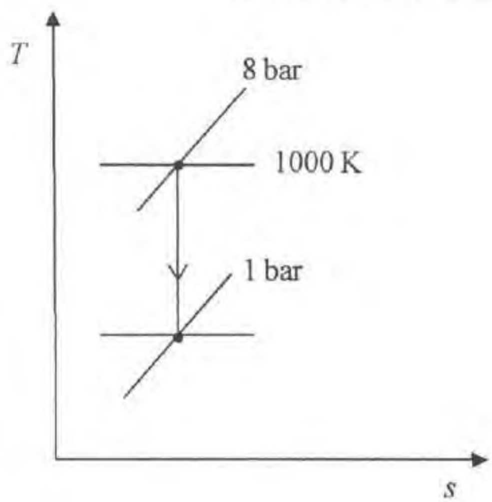
12.23 A mixture having a molar analysis of 60% N_2, 17% CO_2, and 17% H_2O enters a turbine at 1000 K, 8 bar, with a mass flow rate of 2 kg/s and expands isentropically to a pressure of 1 bar. Ignoring kinetic and potential energy effects, determine for steady-state operation
(a) the temperature at the exit, in K.
(b) the power developed by the turbine, in kW.

KNOWN: A mixture with a specified molar analysis enters a turbine operating at steady state at a specified mass flow rate and state and expands isentropically to a specified pressure.

FIND: Determine (a) the exit temperature and (b) the power developed.

SCHEMATIC AND GIVEN DATA:





i	y_i
N_2	0.66
CO_2	0.17
H_2O	0.17

ENGINEERING MODEL:
(1) The turbine operates at steady state and the expansion is isentropic.
(2) Kinetic and potential energy effects are negligible and there is no heat transfer.
(3) The mixture adheres to the idealizations of the Dalton model and the composition remains constant.

Problem 12.23 (Continued) – Page 2

ANALYSIS:

(a) As the expansion is isentropic; $\bar{s}_2 - \bar{s}_1 = 0$, where

$$\bar{s}_2 - \bar{s}_1 = y_{N_2}(\bar{s}_2 - \bar{s}_1)_{N_2} + y_{CO_2}(\bar{s}_2 - \bar{s}_1)_{CO_2} + y_{H_2O}(\bar{s}_2 - \bar{s}_1)_{H_2O}$$

and each of the $y_{N_2}(\bar{s}_2 - \bar{s}_1)_{N_2}$, $y_{CO_2}(\bar{s}_2 - \bar{s}_1)_{CO_2}$, and $y_{H_2O}(\bar{s}_2 - \bar{s}_1)_{H_2O}$ terms is evaluated as follows:

$$\bar{s}_2 - \bar{s}_1 = y_{N_2}\left[\bar{s}^{\circ}_{N_2}(T_2) - \bar{s}^{\circ}_{N_2}(T_1) - \bar{R}\ln\left(\frac{p_2}{p_1}\right)\right] + y_{CO_2}\left[\bar{s}^{\circ}_{CO_2}(T_2) - \bar{s}^{\circ}_{CO_2}(T_1) - \bar{R}\ln\left(\frac{p_2}{p_1}\right)\right]$$
$$+ y_{H_2O}\left[\bar{s}^{\circ}_{H_2O}(T_2) - \bar{s}^{\circ}_{H_2O}(T_1) - \bar{R}\ln\left(\frac{p_2}{p_1}\right)\right]$$

Then, since $\bar{s}_2 - \bar{s}_1 = 0$ and $y_{N_2} + y_{CO_2} + y_{H_2O} = 1$

$$y_{N_2}\bar{s}^{\circ}_{N_2}(T_2) + y_{CO_2}\bar{s}^{\circ}_{CO_2}(T_2) + y_{H_2O}\bar{s}^{\circ}_{H_2O}(T_2)$$
$$= y_{N_2}\bar{s}^{\circ}_{N_2}(T_1) + y_{CO_2}\bar{s}^{\circ}_{CO_2}(T_1) + y_{H_2O}\bar{s}^{\circ}_{H_2O}(T_1) - \bar{R}\ln\left(\frac{p_2}{p_1}\right)$$

Introducing $\bar{s}^{\circ}$ data at T_1 from Tables A-23

$$y_{N_2}\bar{s}^{\circ}_{N_2}(T_2) + y_{CO_2}\bar{s}^{\circ}_{CO_2}(T_2) + y_{H_2O}\bar{s}^{\circ}_{H_2O}(T_2)$$
$$= (0.66)\left(228.057\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right) + (0.17)\left(269.215\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)$$
$$+ (0.17)\left(232.597\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right) + 8.314\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\ln\left(\frac{1}{8}\right)$$
$$= \left(150.52\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right) + \left(45.77\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right) + \left(39.54\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right) + \left(-17.29\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)$$
$$= 218.54\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}$$

The exit temperature T_2 can be determined in an iterative procedure with the above equation and data from Table A-23. The result is $T_2 = 618$ K. ⟵

(b) Using mass and energy balances for the control volume at steady state, the power developed by the turbine is

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2) = \dot{m}\frac{(\bar{h}_1 - \bar{h}_2)}{M} = \dot{m}\frac{y_{N_2}(\bar{h}_1 - \bar{h}_2)_{N_2} + y_{CO_2}(\bar{h}_1 - \bar{h}_2)_{CO_2} + y_{H_2O}(\bar{h}_1 - \bar{h}_2)_{H_2O}}{M}$$

where $(\bar{h}_1 - \bar{h}_2)$ is the change in specific enthalpy for the mixture and M is the mixture molecular weight. M is found with data from Table A-1

Problem 12.23 (Continued) – Page 3

$$M = y_{N_2} M_{N_2} + y_{CO_2} M_{CO_2} + y_{H_2O} M_{H_2O}$$
$$= (0.66)\left(28.01\frac{\text{kg}}{\text{kmol}}\right) + (0.17)\left(44.01\frac{\text{kg}}{\text{kmol}}\right) + (0.66)\left(18.02\frac{\text{kg}}{\text{kmol}}\right)$$
$$= 29.03\frac{\text{kg (mixture)}}{\text{kmol (mixture)}}$$

Accordingly, with $\bar{h}$ data from Table A-23

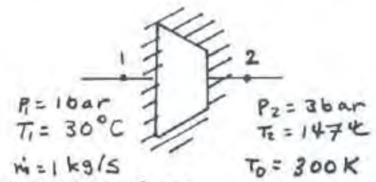
$$\dot{W}_{cv} = (2\,\text{kg/s})\left[\frac{0.66(30129 - 18106) + 0.17(42769 - 23136) + 0.17(35882 - 21057)}{29.03}\right]\frac{\text{kJ}}{\text{kg}}$$
$$= 950.3\,\text{kW} \longleftarrow$$

PROBLEM 12.24

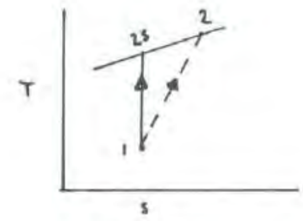
KNOWN: A mixture with a specified molar analysis enters a compressor operating at steady state at a specified mass flow rate and state and exits at a specified temperature and pressure.

FIND: Determine the power required, the isentropic compressor efficiency, and the rate of exergy destruction.

SCHEMATIC & GIVEN DATA



i	y_i	M_i
N_2	0.6	28.01
CO_2	0.4	44.01



ENGINEERING MODEL: (1) The compressor is well insulated and operates at steady state. (2) Kinetic and potential energy effects can be ignored. (3) The mixture adheres to the idealizations of the Dalton model and the composition remains constant. (4) For the environment, $T_0 = 300K$.

ANALYSIS: Reduction of the mass and energy rate balances at steady state gives

$$\dot{W}_c = \dot{m}\left[h_1 - h_2\right] = \dot{m}\left[\frac{y_{N_2}(\bar{h}_1-\bar{h}_2)_{N_2} + y_{CO_2}(\bar{h}_1-\bar{h}_2)_{CO_2}}{M}\right]$$

where M is the mixture molecular weight: $M = y_{N_2}M_{N_2} + y_{CO_2}M_{CO_2} = 0.6(28.01) + 0.4(44.01) = 34.41$. Then, with data from Table A-23

$$\dot{W}_c = (1\ kg/s)\left[\frac{0.6(8810.3 - 12225) + 0.4(9543.8 - 14206)}{34.41}\right]\left|\frac{kJ}{kg}\right|\left|\frac{kW}{kJ/s}\right| = -113.7\ kW \longleftarrow \dot{W}_c$$

To determine the isentropic compressor efficiency requires the power for an isentropic compression from state 1 to the pressure p_2 — that is, from 1 to state 2s. For this process, $\bar{s}_{2s} - \bar{s}_1 = 0$. Or

$$y_{N_2}[\bar{s}_{2s} - \bar{s}_1]_{N_2} + y_{CO_2}[\bar{s}_{2s} - \bar{s}_1]_{CO_2} = 0 \Rightarrow y_{N_2}\left[\bar{s}^\circ_{2s} - \bar{s}^\circ_1 - \bar{R}\ln\frac{p_2}{p_1}\right]_{N_2} + y_{CO_2}\left[\bar{s}^\circ_{2s} - \bar{s}^\circ_1 - \bar{R}\ln\frac{p_2}{p_1}\right]_{CO_2} = 0$$

Then, with data from Table A-23

$$y_{N_2}(\bar{s}^\circ_{2s})_{N_2} + y_{CO_2}(\bar{s}^\circ_{2s})_{CO_2} = (0.6)(191.969) + (0.4)(214.288) + 8.314\ln 3 = 210.029\ \frac{kJ}{kmol\cdot K}$$

(1) Solving this equation by iteration with table data; $T_{2s} = 399$ K. Accordingly

$$(\dot{W}_c)_s = \dot{m}(h_1 - h_{2s}) = \dot{m}\left[\frac{y_{N_2}(\bar{h}_1-\bar{h}_{2s})_{N_2} + y_{CO_2}(\bar{h}_1-\bar{h}_{2s})_{CO_2}}{M}\right]$$

$$= (1)\left[\frac{0.6(8810.3 - 11610.7) + 0.4(9543.8 - 13330.8)}{34.41}\right] = -92.85\ kW$$

The isentropic compressor efficiency is then

$$\eta_c = \frac{(\dot{W}_c)_s}{(\dot{W}_c)} = \frac{(-92.85)}{(-113.7)} = 0.817\ (81.7\%) \longleftarrow \eta_c$$

The rate of exergy destruction can be found using $\dot{E}_d = T_0\dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production obtained from an entropy rate balance, which reduces at steady state as follows:

$$0 = \sum_j \frac{\cancel{\dot{Q}_j}^{\,0}}{T_j} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1)$$

$$\Rightarrow \quad \dot{\sigma}_{cv} = \dot{m}\left[\frac{y_{N_2}(\bar{s}_2-\bar{s}_1)_{N_2} + y_{CO_2}(\bar{s}_2-\bar{s}_1)_{CO_2}}{M}\right]$$

PROBLEM 12.24 (Contd.)–Page 2

The terms $(\bar{s}_2-\bar{s}_1)_{N_2}$ and $(\bar{s}_2-\bar{s}_1)_{CO_2}$ are evaluated using Eq. 12.36, so

$$\dot{\sigma}_{cv} = \dot{m}\left[\frac{y_{N_2}\left(\bar{s}^\circ_{N_2}(T_2) - \bar{s}^\circ_{N_2}(T_1) - \bar{R}\ln\frac{p_2}{p_1}\right) + y_{CO_2}\left(\bar{s}^\circ_{CO_2}(T_2) - \bar{s}^\circ_{CO_2}(T_1) - \bar{R}\ln p_2/p_1\right)}{M}\right]$$

$$= \dot{m}\left[\frac{y_{N_2}\left(\bar{s}^\circ_{N_2}(T_2) - \bar{s}^\circ_{N_2}(T_1)\right) + y_{CO_2}\left(\bar{s}^\circ_{CO_2}(T_2) - \bar{s}^\circ_{CO_2}(T_1)\right) - \bar{R}\ln p_2/p_1}{M}\right]$$

Then, with $\bar{s}^\circ$ data from Tables A-23

$$\dot{\sigma}_{cv} = \left(1\frac{kg}{s}\right)\left[\frac{0.6(201.499-191.969)+0.4(227.258-214.284)-8.314\ln 3}{34.41}\right]\left|\frac{kJ}{kg\cdot K}\right|\left|\frac{1\,kW}{1\,kJ/s}\right|$$

$$= 0.0516\ kW/K$$

Multiplying this by T_0

(2) $$\dot{E}_d = T_0\dot{\sigma}_{cv} = (300K)\left(0.0516\frac{kW}{K}\right) = 15.48\,kW \longleftarrow \underline{\dot{E}_d}$$

---

1. An iterative solution using table data can be avoided by using IT. The results for $\dot{W}_c$, T_{2s}, η_c, and $\dot{E}_d$ obtained using IT agree with the values given here.
2. When expressed as a percentage of the power input to the compressor

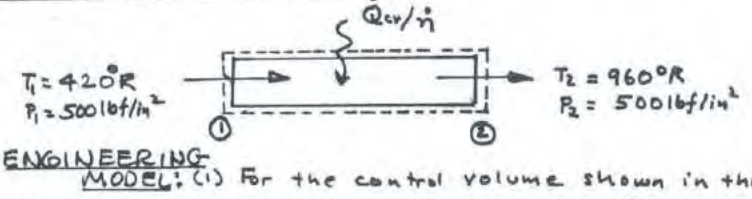
$$\% = [\dot{E}_d/(-\dot{W}_{cv})](100) = (15.48/113.7)(100) = 13.6\%$$

PROBLEM 12.25

KNOWN: An equimolar mixture of N_2 and CO_2 enters a heat exchanger at a known temperature and pressure and exits with no change in composition at a known temperature and pressure.

FIND: Determine the rate of heat transfer to the mixture per lbmol of mixture flowing using (a) ideal gas mixture principles, (b) the generalized enthalpy chart together with Kay's rule.

SCHEMATIC & GIVEN DATA:



i	y_i
N_2	0.5
CO_2	0.5

ENGINEERING MODEL: (1) For the control volume shown in the accompanying figure $\dot{W}_{cv}=0$ and kinetic and potential energy effects are negligible. (2) The control volume is at steady state. (3) The composition of the mixture remains constant. (4) In part (a), the mixture adheres to the Dalton model. (5) In Part (b), Kay's rule applies.

ANALYSIS: At steady state, mass and energy rate balances reduce to give

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}}^{0} + \cancel{g(z_1-z_2)}^{0}\right] \Rightarrow \dot{Q}_{cv} = \dot{m}(h_2 - h_1)$$

or, when expressed in terms of the molar flow rate of the mixture

$$\dot{Q}_{cv} = \dot{n}(\bar{h}_2 - \bar{h}_1) \Rightarrow \frac{\dot{Q}_{cv}}{\dot{n}} = \bar{h}_2 - \bar{h}_1 \qquad (1)$$

(a) When the ideal gas mixture principles of Chap. 12 are used, the change in specific enthalpy of the mixture can be expressed in terms of the enthalpy changes of the components evaluated at the inlet and exit temperatures:

$$\frac{\dot{Q}_{cv}}{\dot{n}} = y_{N_2}\left[\bar{h}_{N_2}(T_2) - \bar{h}_{N_2}(T_1)\right] + y_{CO_2}\left[\bar{h}_{CO_2}(T_2) - \bar{h}_{CO_2}(T_1)\right]$$

With data from Table A-23

$$\frac{\dot{Q}_{cv}}{\dot{n}} = 0.5\left[6693.1 - 2916.1\right] + 0.5\left[8243.8 - 3035.7\right] = 4492.6\ \text{Btu/lbmol} \longleftarrow \text{(a)}$$

(b) Invoking Eq. 11.85 for the mixture:

$$\frac{\dot{Q}_{cv}}{\dot{n}} = \underline{\bar{h}_2^* - \bar{h}_1^*} - \bar{R}T_c\left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1\right] \qquad (2)$$

The underlined term is the enthalpy change for the mixture evaluated in part (a) using ideal gas mixture principles. The remaining terms can be evaluated from Figure A-4 using T_c and P_c for the mixture calculated via Kay's rule:

$$T_c = y_{N_2}(T_c)_{N_2} + y_{CO_2}(T_c)_{CO_2} = 0.5(227°R + 548°R) = 388°R$$

$$P_c = y_{N_2}(P_c)_{N_2} + y_{CO_2}(P_c)_{CO_2} = 0.5(33.5\ atm + 72.9\ atm) = 53.2\ atm$$

Thus

$$(T_R)_1 = \frac{420}{388} = 1.08, \quad (T_R)_2 = \frac{960}{388} = 2.47, \quad P_{R1} = P_{R2} = \left(\frac{500/14.7}{53.2}\right) = 0.64$$

Fig. A-4 gives $\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1 = 0.7$, $\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2 \approx 0.1$. And from Eq. (2) we get

1 $$(\dot{Q}_{cv}/\dot{n}) = 4492.6\ \text{Btu/lbmol} - (1.986\ \text{Btu/lbmol}\cdot°R)(388°R)(0.1 - 0.7) = 4954.9\ \text{Btu/lbmol}$$

---

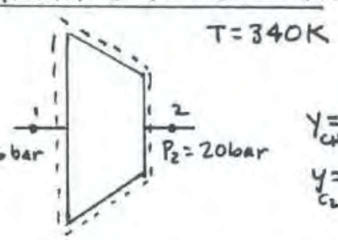
1. The result of part (b) is about 10% greater than that of part (a).

PROBLEM 12.26

KNOWN: Natural gas with a specified composition is compressed isothermally without internal irreversibilities from 6 to 20 bar. The compressor operates at steady state.

FIND: Determine the work and heat transfer per unit mole of mixture flowing (a) using ideal gas mixture principles, (b) using ideal solution principles with given data for the pure components.

SCHEMATIC & GIVEN DATA:



$y_{CH_4}=0.6$

$y_{C_2H_6}=0.4$

	h (kJ/kg)		s (kJ/kg·K)	
	6bar	20bar	6bar	20bar
CH_4	715.33	704.40	10.9763	10.3275
C_2H_6	469.39	439.13	7.3493	6.9680

ENGINEERING MODEL: (1) The control volume operates at steady state with negligible kinetic and potential energy changes. (2) The mixture composition remains constant. (3) In part (a) the mixture adheres to the Dalton model. The ideal solution model applies in part (b).

ANALYSIS: At steady state, mass and energy rate balances reduce to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left(h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}}^{\,0} + \cancel{g(z_1 - z_2)}^{\,0}\right) \Rightarrow \dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}\,(h_1 - h_2)$$

Mass and entropy rate balances reduce to give

$$0 = \frac{\dot{Q}_{cv}}{T} + \dot{m}(s_1 - s_2) + \cancel{\dot{\sigma}_{cv}} \Rightarrow \dot{Q}_{cv} = \dot{m}\,T(s_2 - s_1) = \dot{n}\,T(\bar{s}_2 - \bar{s}_1) \qquad (1)$$

Combining these results

$$\dot{W}_{cv} = \dot{m}\left[h_1 - h_2 - T(s_1 - s_2)\right]$$

or, in terms of the molar flow rate of the mixture $\dot{n}$

$$\dot{W}_{cv} = \dot{n}\left[\bar{h}_1 - \bar{h}_2 - T(\bar{s}_1 - \bar{s}_2)\right] \Rightarrow \frac{\dot{W}_{cv}}{\dot{n}} = \bar{h}_1 - \bar{h}_2 - T(\bar{s}_1 - \bar{s}_2) \qquad (2)$$

(a) When the ideal gas mixture principles of Chap. 12 are used, the enthalpy change of the mixture vanishes since temperature remains constant. The specific entropy change of the mixture can be expressed in terms of the specific entropy changes of the components using Eq. 12.36:

① $$\bar{s}_1 - \bar{s}_2 = y_{CH_4}\Big[\underbrace{\bar{s}_1^\circ - \bar{s}_2^\circ}_{\substack{=0 \text{ since} \\ T_1 = T_2}} - \bar{R}\ln\frac{P_1}{P_2}\Big]_{CH_4} + y_{C_2H_6}\Big[\underbrace{\bar{s}_1^\circ - \bar{s}_2^\circ}_{\substack{=0 \text{ since} \\ T_1 = T_2}} - \bar{R}\ln\frac{P_1}{P_2}\Big]_{C_2H_6} = -\bar{R}\ln\frac{P_1}{P_2}$$

Collecting results, Eq. (1) becomes

$$\frac{\dot{W}_{cv}}{\dot{n}} = \bar{R}\,T\ln\frac{P_1}{P_2} = \left(8.314\,\frac{kJ}{kmol\cdot K}\right)(340\,K)\ln\frac{6}{20} = -3403\,\frac{kJ}{kmol}$$ (a)

Since $(h_1 - h_2) = 0$, $(\dot{Q}_{cv}/\dot{n}) = (\dot{W}_{cv}/\dot{n}) = -3403\,\frac{kJ}{kmol}$

(b) Using the ideal solution model together with the data above for the pure components, we get

$$\bar{h}_1 - \bar{h}_2 = 0.6\left[(16.04)[715.3 - 704.4]\right] + 0.4\left[(30.07)[462.39 - 439.13]\right] = 385\,\frac{kJ}{kmol}$$

PROBLEM 12.26 (Contd) - Page 2

① $\bar{s}_1 - \bar{s}_2 = 0.6\left[(16.04)(10.9763 - 10.3275)\right] + 0.4\left[(30.07)[7.3493 - 6.9680]\right] = 10.83 \frac{kJ}{kmol \cdot K}$

Then, with Eq. (2)

② $$\left(\frac{\dot{W}_{cv}}{\dot{n}}\right) = \bar{h}_1 - \bar{h}_2 - T(\bar{s}_1 - \bar{s}_2) = 385 - 340(10.83) = -3297 \frac{kJ}{kmol}$$

and with Eq. (1)

$$\left(\frac{\dot{Q}_{cv}}{\dot{n}}\right) = T(s_2 - s_1) = 340(-10.83) = -3682 \frac{kJ}{kmol}$$

---

1. Since composition remains constant, this approach suffices. See discussion leading up to Eq. 12.36.

2. Comparing the results of part (a) in terms of <u>magnitudes</u>,
$\left(\frac{-\dot{W}_{cv}}{\dot{n}}\right)$ in part (a) is about 3% greater than the part (b) result.
$\left(\frac{-\dot{Q}_{cv}}{\dot{n}}\right)$ in part (a) is about 8% less than the part (b) result.

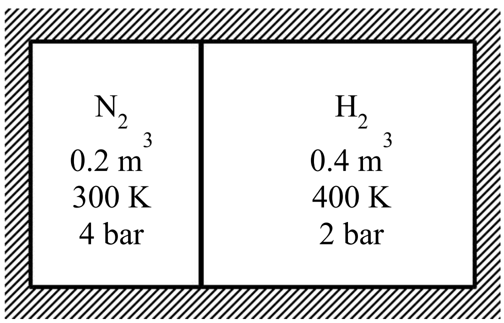
Problem 12.27

An insulated tank having a total volume of 0.6 m^3 is divided into two compartments. Initially one compartment contains 0.4 m^3 of hydrogen (H_2) at 127°C, 2 bar and the other contains nitrogen (N_2) at 27°C, 4 bar. The gases are allowed to mix until an equilibrium state is attained. Assuming the ideal gas model with constant specific heats, determine
(a) the final temperature, in °C.
(b) the final pressure, in bar.
(c) the amount of entropy produced, in kJ/K.

Known:
Hydrogen and nitrogen, initially separate at different temperatures and pressures, are allowed to mix adiabatically to equilibrium.

Find:
Determine the final temperature and pressure, and the amount of entropy produced.

Schematic & Given Data:



Engineering Model:
(1) The system shown in the accompanying figure experiences no change in kinetic or potential energy and there is no energy transfer by heat transfer or work.
(2) The individual gases and the overall mixture behave as an ideal gas.
(3) The Dalton model applies to the mixture.
(4) Specific heats are constant.

Analysis:
(a) The final temperature can be determined from an energy balance which reduces to give:
$\Delta U = \cancel{Q} - \cancel{W} = 0$
The change in internal energy equals the sum of the internal energy changes of the two gases

$$\Delta U = \Delta U_{\mathrm{H_2}} + \Delta U_{\mathrm{N_2}} = n_{\mathrm{H_2}}\bar{c}_{v,\mathrm{H_2}}\left[T_f - T_{\mathrm{H_2}}\right] + n_{\mathrm{N_2}}\bar{c}_{v,\mathrm{N_2}}\left[T_f - T_{\mathrm{N_2}}\right] = 0 \quad (1)$$

Where T_f is the final temperature and T_{H2} and T_{N2} denote, respectively, the initial temperatures of the H_2 and N_2. In accord with assumption 4, the specific heats are taken as constants. Solving Eq. (1)

$$T_f = \frac{n_{\mathrm{H_2}}\bar{c}_{v,\mathrm{H_2}}T_{\mathrm{H_2}} + n_{\mathrm{N_2}}\bar{c}_{v,\mathrm{N_2}}T_{\mathrm{N_2}}}{n_{\mathrm{H_2}}\bar{c}_{v,\mathrm{H_2}} + n_{\mathrm{N_2}}\bar{c}_{v,\mathrm{N_2}}} \quad (2)$$

Evaluating specific heats at 350 K from Table A-20, $c_{v,\mathrm{H_2}} = 10.302\,\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}$ and $c_{v,\mathrm{N_2}} = 0.744\,\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}$ Then, with the ideal gas equation of state

$$n_{\mathrm{H_2}} = \frac{p_{\mathrm{H_2}}V}{\overline{R}T_{\mathrm{H_2}}} = \frac{\left(2\cdot 10^5\,\frac{\mathrm{N}}{\mathrm{m^2}}\right)(0.4\ \mathrm{m^3})}{\left(8314\,\frac{\mathrm{N\cdot m}}{\mathrm{kmol\cdot K}}\right)(400\ \mathrm{K})} = 0.02406\ \mathrm{kmol}$$

$$n_{\mathrm{N_2}} = \frac{p_{\mathrm{N_2}}V}{\overline{R}T_{\mathrm{N_2}}} = \frac{\left(4\cdot 10^5\,\frac{\mathrm{N}}{\mathrm{m^2}}\right)(0.2\ \mathrm{m^3})}{\left(8314\,\frac{\mathrm{N\cdot m}}{\mathrm{kmol\cdot K}}\right)(300\ \mathrm{K})} = 0.03207\ \mathrm{kmol}$$

Where p_{H2} and p_{N2} denote the initial pressures of H_2 and N_2 respectively, and **not** partial pressures. Inserting values into Eq. (2) together with $\overline{c}_v = Mc_v$ using Table A-1

$$T_f = \frac{(0.02406\ \mathrm{kmol})\left(2.016\,\frac{\mathrm{kg}}{\mathrm{kmol}}\cdot 10.302\,\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\right)(400\ \mathrm{K}) + (0.03207\ \mathrm{kmol})\left(28.01\,\frac{\mathrm{kg}}{\mathrm{kmol}}\cdot 0.744\,\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\right)(300\ \mathrm{K})}{(0.02406\ \mathrm{kmol})\left(2.016\,\frac{\mathrm{kg}}{\mathrm{kmol}}\cdot 10.302\,\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\right) + (0.03207\ \mathrm{kmol})\left(28.01\,\frac{\mathrm{kg}}{\mathrm{kmol}}\cdot 0.744\,\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\right)}$$

$$= \frac{199.879 + 200.496}{1.168} = 342.8\ \mathrm{K} \cong 70°\mathrm{C} \quad \longleftarrow$$

(b) Using the ideal gas equation of state:

$$p_f = \frac{n\overline{R}T_f}{V}$$

$$= \left[\frac{[(0.02406 + 0.03207)\mathrm{kmol}]\left(8.314\,\frac{\mathrm{kJ}}{\mathrm{kmol\cdot K}}\right)\left|\frac{1000\ \mathrm{N\cdot m}}{1\ \mathrm{KJ}}\right|(342.8\ \mathrm{K})}{0.6\ \mathrm{m^3}}\right]\left|\frac{1\ \mathrm{bar}}{10^5\,\frac{\mathrm{N}}{\mathrm{m^2}}}\right|$$

$$= 2.67\ \mathrm{bar} \quad \longleftarrow$$

(c) An entropy balance reduces with $Q = 0$ to give

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma \Longrightarrow \sigma = \Delta S = \Delta S_{\mathrm{H_2}} + \Delta S_{\mathrm{N_2}} \qquad (3)$$

The H_2 undergoes a process from 400 K, 2 bar to 342.8 K with the partial pressure:

$$y_{\mathrm{H_2}}p_f = \left(\frac{0.02406\ \mathrm{kmol}}{0.05613\ \mathrm{kmol}}\right)(2.67\ \mathrm{bar}) = 1.144\ \mathrm{bar}$$

Thus, with $\overline{c}_P = \overline{c}_v + \overline{R}$

$$\overline{c}_{p,\mathrm{H_2}} = \left(2.018\,\frac{\mathrm{kg}}{\mathrm{kmol}}\cdot 10.302\,\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\right) + 8.314\,\frac{\mathrm{kJ}}{\mathrm{kmol\cdot K}} = 29.103\,\frac{\mathrm{kJ}}{\mathrm{kmol\cdot K}}$$

Using Eq. 6.22 with p_2 corresponding to the final partial pressure for each component

$$\Delta S_{H_2} = (0.02406 \text{ kmol})\left[\left(29.103 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}\right)\left(\ln \frac{342.8}{400}\right) - \left(8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}\right)\left(\ln \frac{1.144}{2}\right)\right]$$

$$= 0.00369 \frac{\text{kJ}}{\text{K}}$$

The N_2 undergoes a process from 300 K, 4 bar to 342.8 K and the partial pressure

$$y_{N_2} p_f = \left(\frac{0.03207 \text{ kmol}}{0.05613 \text{ kmol}}\right)(2.67 \text{ bar}) = 1.526 \text{ bar}$$

Thus, with $\bar{c}_P = \bar{c}_v + \bar{R}$:

$$\bar{c}_{p,N_2} = \left(28.01 \frac{\text{kg}}{\text{kmol}} \cdot 0.744 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right) + 8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}} = 29.153 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$

$$\Delta S_{N_2} = (0.03207 \text{ kmol})\left[\left(29.153 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}\right)\left(\ln \frac{342.8}{300}\right) - \left(8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}\right)\left(\ln \frac{1.526}{4}\right)\right]$$

$$= 0.38162 \frac{\text{kJ}}{\text{K}}$$

Substituting values into Eq. (3):

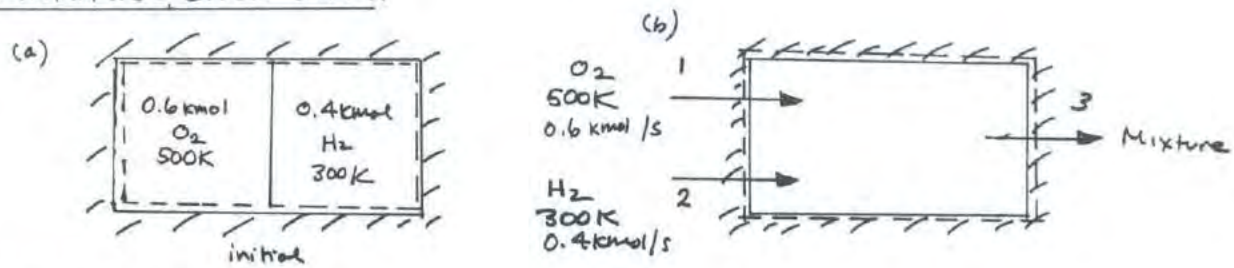
$$\sigma = (0.00369 + 0.38162) \frac{\text{kJ}}{\text{K}} = 0.3853 \frac{\text{kJ}}{\text{K}} \longleftarrow$$

PROBLEM 12.28

KNOWN: Two cases involving the mixing of O_2 and H_2 are specified.

FIND: For each case determine the mixture temperature.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) For case (a), a closed system consisting of the O_2 and N_2 is the system. For case (b), a control volume at steady state is the system. (2) For each system, $\dot{Q}=0$, $\dot{W}=0$ (except for flow work in (b)), and there are no significant kinetic/potential energy effects. (3) Each gas and the overall mixture behaves as an ideal gas. The Dalton mixture model applies. (4) Appropriate constant specific heats are employed.

ANALYSIS: (a) An energy balance reduces to read $\Delta U = \cancel{Q}^0 - \cancel{W}^0$, or $\Delta U = 0$. That is

$$[n\,\Delta\bar{u}]_{O_2} + [n\Delta\bar{u}]_{H_2} = 0 \qquad (1)$$

Since the initial temperature difference is just 200K, and c_v does not vary significantly over such an interval, an appropriate constant c_v can be used for each gas. Thus, with data from Table A-20 at 400K, Eq. (1) reads

$$[n(c_v M)[T_f - T_i]]_{O_2} + [n(c_v M)[T_f - T_i]]_{H_2} = 0$$

$$\Rightarrow \quad (0.6)(0.681\times 32)[T_f - 500] + (0.4)(10.352\times 2.016)[T_f - 300] = 0 \Rightarrow T_f = 422\text{K}$$

(b) An energy rate balance reduces to read

$$0 = \cancel{\dot{Q}_{cv}}^0 - \cancel{\dot{W}_{cv}}^0 + \dot{n}_{O_2}\bar{h}_{O_2}(T_1) + \dot{n}_{H_2}\bar{h}_{H_2}(T_2) - [\dot{n}_{O_2}\bar{h}_{O_2}(T_3) + \dot{n}_{H_2}\bar{h}_{H_2}(T_3)]$$

or

$$0 = \dot{n}_{O_2}[\bar{h}_{O_2}(T_1) - \bar{h}_{O_2}(T_3)] + \dot{n}_{H_2}[\bar{h}_{H_2}(T_2) - \bar{h}_{H_2}(T_3)] \qquad (2)$$

Then, with cp for each gas from Table A-20 at 400K we get

$$0 = \dot{n}_{O_2}[(c_p M)_{O_2}[T_1 - T_3]] + \dot{n}_{H_2}[(c_p M)_{H_2}[T_2 - T_3]]$$

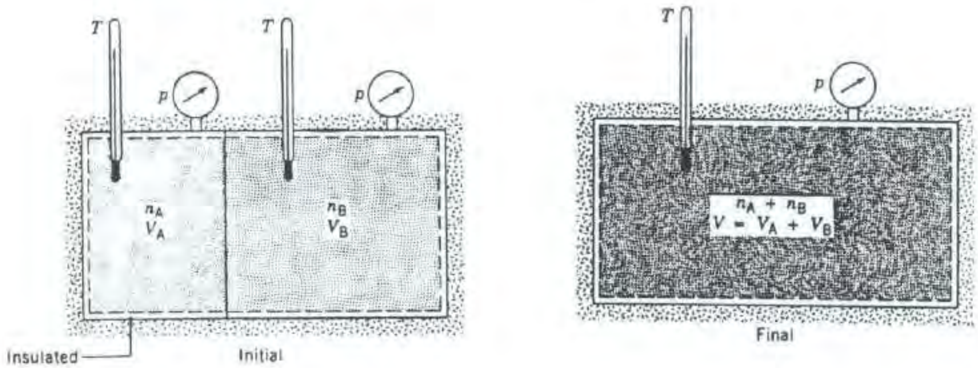
$$0 = (0.6)[(0.941\times 32)[500 - T_3]] + (0.4)[(14.476)(2.016)[300 - T_3]] \Rightarrow T_f = 422\text{K}$$

PROBLEM 12.29

KNOWN: A system consists initially of n_A moles of gas A at pressure p and temperature T and n_B moles of gas B separate from gas A but at the same pressure and temperature. The gases are allowed to mix at fixed total volume. The final temperature is T and the final pressure is p.

FIND: (a) Obtain an expression for the entropy produced in terms of $\bar{R}$, n_A and n_B. (b) Using the result of part (a) demonstrate that $\sigma > 0$. (c) Would entropy be produced if the gases were identical?

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The system consists of gases A and B together.
2. Each gas behaves as an ideal gas. The final mixture also acts as an ideal gas with each mixture component occupying the total volume and exhibiting the mixture temperature.
3. No heat or work interactions with the surroundings occur.

Analysis: An entropy balance for the closed system reduces to

$$S_2 - S_1 = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{\,0} + \sigma$$

where the entropy transfer term drops out in the adiabatic process. S_1 and S_2 denote the initial and final entropies of the system, respectively.

The initial entropy of the system is the sum of the entropies of the gases *when separate*. That is

$$S_1 = n_A\bar{s}_A(T, p) + n_B\bar{s}_B(T, p)$$

Since each gas initially exists separately at T and p, each of the specific entropies appearing in this expression is evaluated at T, p.

The entropy of the system at the final state equals the sum of the entropies of the gases A and B evaluated at the conditions at which they exist in the mixture. That is

$$S_2 = n_A\bar{s}_A(T, y_A p) + n_B\bar{s}_B(T, y_B p)$$

In this expression, the specific entropy of each gas is evaluated at its partial pressure in the mixture and at the mixture temperature.

Combining the last three equations and solving for the entropy production

$$\sigma = n_A[\bar{s}_A(T, y_A p) - \bar{s}_A(T, p)] + n_B[\bar{s}_B(T, y_B p) - \bar{s}_B(T, p)]$$

The specific entropy changes can be evaluated using Eq. 6.25b. Since there is no change in temperature

$$\bar{s}_A(T, y_A p) - \bar{s}_A(T, p) = -\bar{R}\ln\frac{y_A p}{p} = -\bar{R}\ln y_A$$

$$\bar{s}_B(T, y_B p) - \bar{s}_B(T, p) = -\bar{R}\ln\frac{y_B p}{p} = -\bar{R}\ln y_B$$

Introducing the foregoing into the entropy production expression gives

① ② $$\sigma = -\bar{R}(n_A \ln y_A + n_B \ln y_B)$$ ← (a)

Since y_A and y_B are each less than unity, σ would be positive, in accordance with the second law. ← (b)

PROBLEM 12.29 (Contd.) - Page 2

If the gases were identical, the initial entropy of the system would be

$$S_1 = n_A s(T, p) + n_B s(T, p)$$

where n_A and n_B denote the amounts of the gas present initially in the two tanks. The final entropy of the system would be

$$S_2 = (n_A + n_B) s(T, p)$$

Clearly, $\sigma = 0$ for this case. No entropy would be produced. ← (c)

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1. The irreversibility in this case is due to the *free expansion* each individual gas would undergo from its initial volume, V_A or V_B, and initial pressure p to the total volume V and its partial pressure in the mixture, p_A or p_B. The mixture would be formed spontaneously; however, once formed, energy would be required from the surroundings to separate the gases and return them to their respective initial conditions.

2. For the case of mixing several components, each initially at the same temperature and pressure, the expression for the entropy production takes the form

$$\sigma = -\bar{R} \sum_{i=1}^{j} n_i \ln y_i$$

where n_i and y_i are the number of moles and the mole fraction, respectively, of component i. The result obtained in the solution is a special case of this equation.

Problem 12.30

Carbon dioxide (CO_2) at 197°C, 2 bar enters a chamber at steady state with a molar flow rate of 2 kmol/s and mixes with nitrogen (N_2) entering at 27°C, 2 bar with a molar flow rate of 1 kmol/s. Heat transfer from the mixing chamber occurs at an average surface temperature of 127°C. A single stream exits the mixing chamber at 127°C, 2 bar and passes through a duct, where it cools at constant pressure to 42°C through heat transfer with the surroundings at 27°C. Kinetic and potential energy effects can be ignored. Determine the rates of heat transfer and exergy destruction, each in kW, for control volumes enclosing

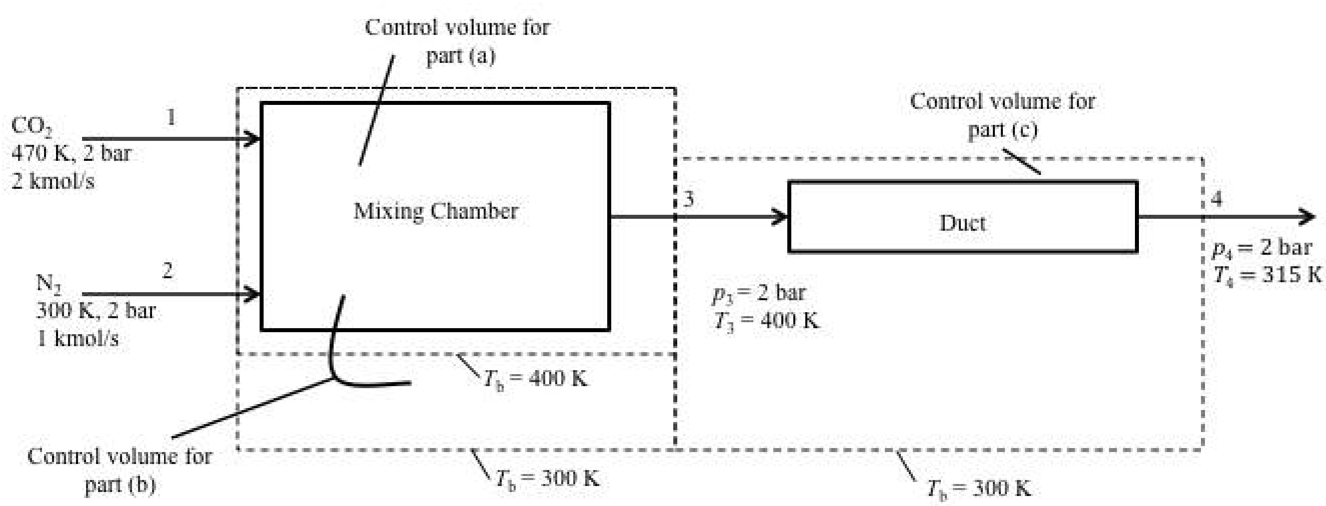
(a) the mixing chamber only.
(b) the mixing chamber and enough of the nearby surroundings that heat transfer occurs at 27°C.
(c) the duct and enough of the nearby surroundings that heat transfer occurs at 27°C.

Let T_0 = 27°C.

Known:
CO_2 and N_2 at specified temperatures, pressures, and water flow rates enter a chamber. A mixture exits at a specified temperature and pressure and passes through a duct where it cools at constant pressure to a known temperature.

Find:
Determine the rates of heat transfer and exergy destroyed for each of three alternative control volumes.

Schematic & Given Data:



Engineering Model:
(1) The control volumes shown in the accompanying figure are at steady state, with $\dot{W}_{cv} = 0$, and negligible effects of kinetic and potential energy.
(2) Each gas behaves as an ideal gas and the mixture adheres to the Dalton model
(3) The dead state reference environment is at T_o = 300 K.

Analysis:

The molar flow rate of the mixture at 3 is: $\dot{n}_3 = \dot{n}_1 + \dot{n}_2 = 3\,\frac{\text{kmol}}{\text{s}}$. The mole fractions of the mixture components at 3 are

$$y_{\mathrm{CO_2}} = \frac{2}{3}\,, \qquad y_{\mathrm{N_2}} = \frac{1}{3}$$

(a) For a control colume enclosing the mixing chamber only, an energy rate balance reduces with assumption 1 to give

$$0 = \dot{Q}_{\mathrm{cv}} - \cancel{\dot{W}_{\mathrm{cv}}} + \dot{n}_1\bar{h}_{\mathrm{CO_2}}(T_1) + \dot{n}_2\bar{h}_{\mathrm{N_2}}(T_2) - \dot{n}\bar{h}(T_3)$$

Where $\bar{h}(T_3) = y_{\mathrm{CO_2}}\bar{h}_{\mathrm{CO_2}}(T_3) + y_{\mathrm{N_2}}\bar{h}_{\mathrm{N_2}}(T_3)$. Thus, collecting results

$$\dot{Q}_{\mathrm{cv}} = \dot{n}_1\left[\bar{h}_{\mathrm{CO_2}}(T_3) - \bar{h}_{\mathrm{CO_2}}(T_1)\right] + \dot{n}_2\left[\bar{h}_{\mathrm{N_2}}(T_3) - \bar{h}_{\mathrm{N_2}}(T_2)\right]$$

With $\bar{h}$ data from Table A-23

$$\dot{Q}_{\mathrm{cv}} = \left[\left(2\,\frac{\text{kmol}}{\text{s}}\right)\left[(13{,}372 - 16{,}351)\,\frac{\text{kJ}}{\text{kmol}}\right] + \left(1\,\frac{\text{kmol}}{\text{s}}\right)\left[(11{,}640 - 8723)\,\frac{\text{kJ}}{\text{kmol}}\right]\right]\left(\frac{\text{kW}}{\text{kJ/s}}\right) \longleftarrow$$
$$= -3041\ \text{kW}$$

For heat transfer taking place at T_{b} = 400 K, an entropy rate balance reads

$$0 = \frac{\dot{Q}_{\mathrm{cv}}}{T_b} + \dot{n}_1\bar{s}_{\mathrm{CO_2}}(T_1, p_1) + \dot{n}_2\bar{s}_{\mathrm{N_2}}(T_2, p_2) - \dot{n}_3\bar{s}_3 + \dot{\sigma}_{\mathrm{cv}} \qquad (1)$$

Where $\bar{s}_3 = y_{\mathrm{CO_2}}\bar{s}_{\mathrm{CO_2}}\left(T_3, y_{\mathrm{CO_2}}p_3\right) + y_{N_2}\bar{s}_{\mathrm{N_2}}\left(T_3, y_{\mathrm{N_2}}p_3\right)$. Collecting results and solving for $\dot{\sigma}_{\mathrm{cv}}$

$$\begin{aligned}\dot{\sigma}_{\mathrm{cv}} &= -\frac{\dot{Q}_{\mathrm{cv}}}{T_{\mathrm{b}}} + \dot{n}_1\left[\bar{s}_{CO_2}\left(T_3, y_{\mathrm{CO_2}}p_3\right) - \bar{s}_{\mathrm{CO_2}}(T_1, p_1)\right] + \dot{n}_2\left[\bar{s}_{\mathrm{N_2}}\left(T_3, y_{\mathrm{N_2}}p_3\right) - \bar{s}_{\mathrm{N_2}}(T_2, p_2)\right]\\ &= -\frac{\dot{Q}_{\mathrm{cv}}}{T_{\mathrm{b}}} + \dot{n}_1\left[\bar{s}^o_{\mathrm{CO_2}}(T_3) - \bar{s}^o_{\mathrm{CO_2}}(T_1) - \bar{R}\ln y_{\mathrm{CO_2}}\right] + \dot{n}_2\left[\bar{s}^o_{\mathrm{N_2}}(T_3) - \bar{s}^o_{\mathrm{N_2}}(T_2) - \bar{R}\ln y_{\mathrm{N_2}}\right] \qquad (2)\end{aligned}$$

With $\bar{s}^o$ data from Table A-23

$$\begin{aligned}\dot{\sigma}_{\mathrm{cv}} &= \frac{-\left(-3041\,\frac{\text{kJ}}{\text{s}}\right)}{400\ \text{K}} + \left(2\,\frac{\text{kmol}}{\text{s}}\right)\left[225.225 - 232.080 - 8.314\ln\frac{2}{3}\right]\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\\ &\quad + \left(1\,\frac{\text{kmol}}{\text{s}}\right)\left[200.071 - 191.682 - 8.314\ln\frac{1}{3}\right]\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\\ &= \left(7.6025\,\frac{\text{kJ/s}}{\text{K}}\right) + \left(-6.9679\,\frac{\text{kJ/s}}{\text{K}}\right) + \left(17.5229\,\frac{\text{kJ/s}}{\text{K}}\right) = 18.1575\,\frac{\text{kJ/s}}{\text{K}}\end{aligned}$$

The exergy destruction rate is:

$$\dot{\mathrm{E}}_{\mathrm{d}} = T_o \dot{\sigma}_{\mathrm{cv}} = (300\ \mathrm{K})\left(18.1575\,\frac{\mathrm{kJ/s}}{\mathrm{K}}\right) = 5447\ \mathrm{kW}$$

(b) The rate of heat transfer is the same in part (a) for an enlarged control volume enclosing the mixing chamber and enough of the nearby surroundings that heat transfer occurs at T_{b} = 300 K. The rate of entropy production is given by Eq. (2), except now T_{b} = 300 K

$$\dot{\sigma}_{\mathrm{cv}} = \left(\frac{-\left(-3041\,\frac{\mathrm{kJ}}{\mathrm{s}}\right)}{300\ \mathrm{K}}\right) + \left(-6.9679\,\frac{\mathrm{kJ/s}}{\mathrm{K}}\right) + \left(17.5229\,\frac{\mathrm{kJ/s}}{\mathrm{K}}\right) = 20.6917\,\frac{\mathrm{kJ/s}}{\mathrm{K}}$$

And the exergy destruction rate is

$$\dot{\mathrm{E}}_{\mathrm{d}} = T_0 \dot{\sigma}_{\mathrm{cv}} = (300\ \mathrm{K})\left(20.6917\,\frac{\mathrm{kJ/s}}{\mathrm{K}}\right) = 6208\ \mathrm{kW}$$

(c) For a control volume enclosing the duct and enough of the nearby surroundings that heat transfer occurs at T_{b} = 300K, an energy rate balance reduces to:

$$0 = \dot{Q}_{\mathrm{cv}} - \cancel{\dot{W}_{cv}} + \dot{n}_3 \bar{h}_3 - \dot{n}_4 \bar{h}_4 \implies \dot{Q}_{\mathrm{cv}} = \dot{n}_4 \bar{h}_4 - \dot{n}_3 \bar{h}_3 = \dot{n}\left(\bar{h}_4 - \bar{h}_3\right)$$

The molar flow rates are equal and each enthalpy has the form $\bar{h} = y_{\mathrm{CO_2}} \bar{h}_{\mathrm{CO_2}} + y_{\mathrm{N_2}} \bar{h}_{\mathrm{N_2}}$. Accordingly and using Table A-23 with interpolation

$$\begin{aligned}\dot{Q}_{\mathrm{cv}} &= \dot{n}_1\left[\bar{h}_{\mathrm{CO_2}}(T_4) - \bar{h}_{\mathrm{CO_2}}(T_3)\right] + \dot{n}_2\left[\bar{h}_{\mathrm{N_2}}(T_4) - \bar{h}_{\mathrm{N_2}}(T_3)\right] \\ &= \left(2\,\frac{\mathrm{kmol}}{\mathrm{s}}\right)\left[(9996.5 - 13{,}372)\,\frac{\mathrm{kJ}}{\mathrm{kmol}}\right] + \left(1\,\frac{\mathrm{kmol}}{\mathrm{s}}\right)\left[(9160 - 11{,}640)\,\frac{\mathrm{kJ}}{\mathrm{kmol}}\right] \\ &= -9231\ \mathrm{kW}\end{aligned}$$

An entropy rate balance takes the form:

$$0 = \frac{\dot{Q}_{\mathrm{cv}}}{T_{\mathrm{b}}} + \dot{n}(\bar{s}_3 - \bar{s}_4) + \dot{\sigma}_{\mathrm{cv}} \implies \dot{\sigma}_{\mathrm{cv}} = -\frac{\dot{Q}_{\mathrm{cv}}}{T_{\mathrm{b}}} + \dot{n}(\bar{s}_4 - \bar{s}_3)$$

Each entropy term has the form $\bar{s} = y_{\mathrm{CO_2}} \bar{s}_{\mathrm{CO_2}} + y_{\mathrm{N_2}} \bar{s}_{\mathrm{N_2}}$, where $\bar{s}_{\mathrm{CO_2}}$ and $\bar{s}_{\mathrm{N_2}}$ are evaluated at the mixture temperature and the partial pressure of CO_2 and N_2 respectively. Accordingly and using Table A-23 with interpolation

$$\dot{\sigma}_{\mathrm{cv}} = -\frac{\dot{Q}_{\mathrm{cv}}}{T_{\mathrm{b}}} + \dot{n}_1\left[\bar{s}^o_{\mathrm{CO_2}}(T_4) - \bar{s}^o_{\mathrm{CO_2}}(T_3) - \cancel{\bar{R}\ln\frac{y_{\mathrm{CO_2}} p_4}{y_{\mathrm{CO_2}} p_3}}\right] + \dot{n}_2\left[\bar{s}^o_{\mathrm{N_2}}(T_4) - \bar{s}^o_{\mathrm{N_2}}(T_3) - \cancel{\bar{R}\ln\frac{y_{\mathrm{CO_2}} p_4}{y_{\mathrm{CO_2}} p_3}}\right]$$

Thus:

$$\dot{\sigma}_{\text{cv}} = -\frac{\dot{Q}_{\text{cv}}}{T_{\text{b}}} + \dot{n}_1\left[\bar{s}^o_{\text{CO}_2}(T_4) - \bar{s}^o_{\text{CO}_2}(T_3)\right] + \dot{n}_2\left[\bar{s}^o_{\text{N}_2}(T_4) - \bar{s}^o_{\text{N}_2}(T_3)\right]$$

$$= \frac{-\left(-9231\,\frac{\text{kJ}}{\text{s}}\right)}{300\text{ K}} + \left(2\,\frac{\text{kmol}}{\text{s}}\right)[215.749 - 225.225]\,\frac{\frac{\text{kJ}}{\text{s}}}{\text{kmol}\cdot\text{K}}$$

$$+ \left(1\,\frac{\text{kmol}}{\text{s}}\right)[193.1 - 200.071]\,\frac{\frac{\text{kJ}}{\text{s}}}{\text{kmol}\cdot\text{K}}$$

$$= 30.77\,\frac{\frac{\text{kJ}}{\text{s}}}{\text{K}} + \left(-18.952\,\frac{\frac{\text{kJ}}{\text{s}}}{\text{K}}\right) + \left(-6.971\,\frac{\frac{\text{kJ}}{\text{s}}}{\text{K}}\right) = 4.847\,\frac{\text{kW}}{\text{K}}$$

The exergy destruction rate is:

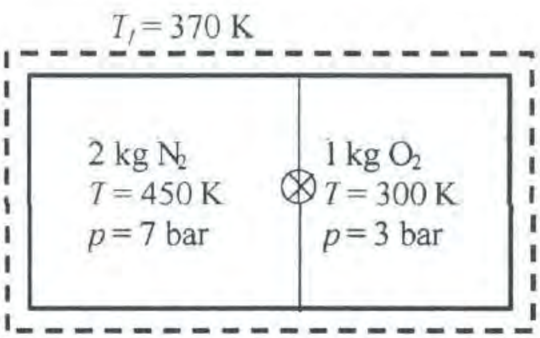
$$\dot{\text{E}}_{\text{d}} = T_0\dot{\sigma}_{\text{cv}} = (300\text{ K})\left(4.847\,\frac{\text{kJ/s}}{\text{K}}\right) = 1454\text{ kW} \longleftarrow$$

12.31 Two kg of N_2 at 450 K, 7 bar is contained in a rigid tank connected by a valve to another rigid tank holding 1 kg of O_2 at 300 K, 3 bar. The valve is opened and gases are allowed to mix, achieving an equilibrium state at 370 K. Determine
(a) the volume of each tank, in m^3.
(b) the final pressure, in bar.
(c) the heat transfer to or from the gases during the process, in kJ.
(d) the entropy change of each gas, in kJ/K.

KNOWN: Nitrogen and oxygen are contained initially in separate tanks, each at specified states. A valve between the two tanks is opened and the gases mix to form an equilibrium mixture at a known temperature.

Find:
Determine (a) the volume of each gas initially, (b) the final pressure, (c) the heat transfer, and (d) the entropy change of each gas.

Schematic and Known Data:



Engineering Model:
(1) For the system shown in the accompanying figure, there is no change in kinetic or potential energy between the initial and final states.
(2) The tanks are rigid, and $W = 0$ for the process.
(3) Each gas and the overall mixture behave as ideal gases. The Dalton mixture model applies.

Analysis:

(a) The volume of each tank can be determined using the ideal gas equation of state together with known initial data for each gas:

$$V_{N_2} = \frac{m_{N_2}\left(\overline{R}/M_{N_2}\right)T_{N_2}}{p_{N_2}} = \frac{(2\text{ kg})\left(\frac{8314}{28.01}\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(450\text{ K})}{(7\text{ bar})\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|} = 0.3816\text{ m}^3 \longleftarrow$$

$$V_{O_2} = \frac{m_{O_2}\left(\overline{R}/M_{O_2}\right)T_{O_2}}{p_{O_2}} = \frac{(1\text{ kg})\left(\frac{8314}{32.00}\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(300\text{ K})}{(3\text{ bar})\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|} = 0.2598\text{ m}^3 \longleftarrow$$

where p_{N_2} and p_{O_2} represent the initial pressures of the N_2 and O_2, respectively, and are *not* partial pressures since the gases are in separate tanks.

(b) The final pressure, p_f, can also be determined using the ideal gas equation of state:

$$p_f = \frac{n\overline{R}T_f}{V}$$

where

$$V = 0.3816\text{ m}^3 + 0.2598\text{ m}^3 = 0.6414\text{ m}^3$$

and

$$n = n_{N_2} + n_{O_2} = \frac{2\text{ kg}}{28.01\text{ kg/kmol}} + \frac{1\text{ kg}}{32.00\text{ kg/kmol}} = 0.0714\text{ kmol} + 0.0313\text{ kmol} = 0.1027\text{ kmol}$$

Finally

$$p_f = \frac{(0.1027\text{ kmol})(8314\text{ N}\cdot\text{m/kmol}\cdot\text{K})(370\text{ K})}{0.6414\text{ m}^3}\left|\frac{1\text{ bar}}{10^5\text{ N/m}^3}\right| = 4.93\text{ bar} \longleftarrow$$

(c) Considering the contents of both tanks to be a closed system, as shown on the system sketch, the energy balance reduces to give $\Delta U = Q - \underbrace{W}_{=0}$, or

$$Q = \Delta U = \Delta U\big|_{N_2} + \Delta U\big|_{O_2} = n_{N_2}\left[\overline{u}(T_f) - \overline{u}(T_{N_2})\right]_{N_2} + n_{O_2}\left[\overline{u}(T_f) - \overline{u}(T_{O_2})\right]_{O_2}$$

with $\overline{u}$ data from Table A-23

$$Q = (0.0741\text{ kmol})[7687 - 9363]\frac{\text{kJ}}{\text{kmol}} + (0.0313\text{ kmol})[7733 - 6242]\frac{\text{kJ}}{\text{kmol}} = -73\text{ kJ} \longleftarrow$$

(d) The N_2 is initially at 450 K, 7 bar, and finally at 370 K. The final partial pressure is

$$y_{N_2}p_f = \left(n_{N_2}/n\right)p_f = (0.0714/0.1027)(4.93\text{ bar}) = 3.4275\text{ bar}$$

Thus, with data from Table A-23 and the ideal gas model

$$(\Delta S)_{N_2} = n_{N_2}\left[\bar{s}_2^\circ - \bar{s}_1^\circ - \bar{R}\ln\frac{p_{f,N_2}}{p_{1,N_2}}\right]$$

$$= (0.0741\,\text{kmol})\left[197.794 - 204.523 - 8.314\ln\frac{3.4275}{7}\right]\frac{\text{kJ}}{\text{kmol}\cdot\text{K}} = 0.01484\,\frac{\text{kJ}}{\text{K}} \longleftarrow$$

The O_2 is initially at 300 K, 3 bar, and finally at 370 K and the final partial pressure is

$$y_{O_2}p_f = (n_{O_2}/n)p_f = (0.0313/0.1027)(4.93\,\text{bar}) = 1.5025\,\text{bar}$$

Thus, with data from Table A-23 and the ideal gas model

$$(\Delta S)_{O_2} = n_{O_2}\left[\bar{s}_2^\circ - \bar{s}_1^\circ - \bar{R}\ln\frac{p_{f,O_2}}{p_{1,O_2}}\right]$$

$$= (0.0313\,\text{kmol})\left[211.423 - 205.213 - 8.314\ln\frac{1.5025}{3}\right]\frac{\text{kJ}}{\text{kmol}\cdot\text{K}} = 0.37432\,\frac{\text{kJ}}{\text{K}} \longleftarrow$$

Note: Even though the entropy transfer accompanying heat transfer is out of the system, the entropy increases because of entropy production due to irreversibilties during the mixing process.

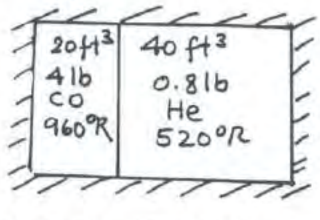
PROBLEM 12.32

KNOWN: Carbon monoxide and helium, initially separate at different states, are allowed to mix adiabatically to equilibrium.

FIND: Determine (a) the final temperature, (b) the final pressure, (c) the exergy destruction.

SCHEMATIC & GIVEN DATA:

Initially,



ENGINEERING MODEL:

1. The system is the total quantity of gas present.
2. For the system, $Q=0$, and kinetic and potential energy effects can be ignored.
3. Each gas behaves as an ideal gas. The mixture adheres to the Dalton model.
4. $T_0 = 520°R$.

ANALYSIS: To begin, the number of moles of each gas and initial pressures are found...

$$n_{CO} = \frac{4}{28.01} = 0.1428, \quad n_{He} = \frac{0.8}{4.003} = 0.1999 \Bigg\} n = 0.3427 \Rightarrow \quad y_{CO} = \frac{0.1428}{0.3427} = 0.4167, \quad y_{He} = \frac{0.1999}{0.3427} = 0.5833$$

Using the ideal gas model equation of state the initial pressures are

$$p_{CO} = \frac{n_{CO}\bar{R}T_{CO}}{V_{CO}} = \frac{(0.1428)(1545)(960)}{(20)}\left|\frac{1\ ft^2}{144\ in^2}\right| = 73.542\ \frac{lbf}{in^2}$$

$$p_{He} = \frac{n_{He}\bar{R}T_{He}}{V_{He}} = \frac{(0.1999)(1545)(520)}{(40)|144|} = 27.882\ \frac{lbf}{in^2}$$

note: these are the initial pressures, not partial pressures.

(a) An energy balance reduces to $\Delta U = \cancel{Q}^{0} - \cancel{W}^{0} = 0$, where ΔU is

$$\Delta U = \Delta U]_{CO} + \Delta U]_{He} = n_{CO}[\bar{u}_{CO}(T_f) - \bar{u}_{CO}(T_{CO})] + n_{He}\bar{c}_{v,He}[T_f - T_{He}] = 0$$

Since helium is monatomic, Table A-21 gives $\bar{c}_p = 5/2\,\bar{R}$. So, with $\bar{c}_p = \bar{c}_v + \bar{R}$, we get $\bar{c}_{v,He} = 1.5\bar{R}$. For CO, $\bar{u}$ is obtained from Table A-23. Rearranging

$$n_{CO}\bar{u}_{CO}(T_f) + n_{He}\bar{c}_{v,He}T_f = n_{CO}\bar{u}_{CO}(T_{CO}) + n_{He}\bar{c}_{v,He}T_{He}$$

$$= (0.1428)(4798.5) + (0.1999)(1.5 \times 1.986)(520) = 994.89$$

① Solving iteratively using table data, $T_f = 762°R$ (302 °F) ◄ T_f

(b) Using the ideal gas equation of state applied to the final mixture

$$p_f = \frac{n\bar{R}T_f}{V} = \frac{(0.3427)(1545)(762)}{60|144|} = 46.696\ \frac{lbf}{in^2}$$ ◄ p_f

(c) The exergy destruction can be found using $E_d = T_0\sigma$, where σ is the amount of entropy produced, obtained from an entropy balance: $\sigma = \Delta S$. Thus

$$E_d = T_0\Delta S = T_0[\Delta S)_{CO} + \Delta S)_{He}], \text{ where}$$

$$\Delta S)_{CO} = n_{CO}\left[\bar{s}^{\circ}_{CO}(T_f) - \bar{s}^{\circ}_{CO}(T_{CO}) - \bar{R}\ln\frac{y_{CO}p_f}{p_{CO}}\right] = 0.1428\left[49.715 - 51.353 - 1.986\ln\frac{19.458}{73.542}\right] = 0.1432$$

$$\Delta S)_{He} = n_{He}\left[\bar{c}_{p,He}\ln\frac{T_f}{T_{He}} - \bar{R}\ln\frac{y_{He}p_f}{p_{He}}\right] = (0.1999)[1.986]\left(2.5\ln\frac{762}{520} - \ln\frac{27.238}{27.882}\right) = 0.3885$$

Btu/°R

Thus, $E_d = (520\ °R)[0.1432 + 0.3885]\ \frac{Btu}{°R} = 276.5\ Btu$ ◄ E_d

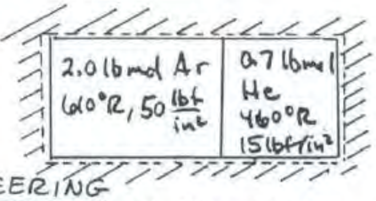
1. An iterative solution using table data can be avoided using IT. The results agree with those given here.

PROBLEM 12.33

KNOWN: Argon and helium, each initially at different temperatures and pressures are allowed to mix adiabatically to a final equilibrium state.

FIND: Determine the final temperature and pressure, and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The system shown in the accompanying figure experiences no change in kinetic or potential energy, and $Q = 0$. (2) Each gas behaves as an ideal gas. The final mixture adheres to the Dalton model.

ANALYSIS: (a) An energy balance reduces as follows, $\Delta U = \cancel{Q}^{0} - \cancel{W}^{0}$. Thus

$$\Delta U = \Delta U)_{Ar} + \Delta U)_{He} = n_{Ar}\,\bar{c}_{v,Ar}\left[T_f - T_{Ar}\right] + n_{He}\,\bar{c}_{v,He}\left[T_f - T_{He}\right] = 0 \qquad (1)$$

where T_{Ar}, T_{He} denote the initial temperatures of the argon and helium, respectively. The final temperature is denoted by T_f. From Table A-2/E for monatomic gases $\bar{c}_p = 5/2\,\bar{R}$. Then, since $\bar{c}_p = \bar{c}_v + \bar{R}$ (Eq. 3.45), $\bar{c}_v = \frac{3}{2}\bar{R}$, and Eq. (1) reduces to give

$$T_f = \frac{n_{Ar}T_{Ar} + n_{He}T_{He}}{n_{Ar} + n_{He}} = \frac{(2.0)(610) + (0.7)(460)}{2.7} = 571.11\ °R\ \ (111\ °F) \longleftarrow T_f$$

(b) The ideal gas equation of state gives $p_f = n\bar{R}T_f/V$, where V is the total volume:

$$V = \frac{n_{Ar}\bar{R}T_{Ar}}{p_{Ar}} + \frac{n_{He}\bar{R}T_{He}}{p_{He}}$$

where p_{Ar}, p_{He} denote the initial pressures of the argon and helium, and not partial pressures. Thus

$$p_f = \frac{(n_{Ar} + n_{He})\bar{R}T_f}{\left[\frac{n_{Ar}\bar{R}T_{Ar}}{p_{Ar}} + \frac{n_{He}\bar{R}T_{He}}{p_{He}}\right]} = \frac{(2.7)(571.11)}{\left[\frac{(2)(610)}{(50)} + \frac{(0.7)(460)}{(15)}\right]} = 33.62\ \frac{\text{lbf}}{\text{in}^2}$$

$$= 2.29\ \text{atm} \longleftarrow p_f$$

(c) An entropy balance reduces as follows

$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{0} + \sigma \Rightarrow \sigma = \Delta S = \Delta S)_{Ar} + \Delta S)_{He} \qquad (2)$$

The argon undergoes a process from 610 °R, 50 lbf/in² to 571.11 °R and the partial pressure of argon in the final mixture: $y_{Ar}\,p_f = \left(\frac{2}{2.7}\right)(33.62) = 24.90$ lbf/in². Thus, with $\bar{c}_p = \frac{5}{2}\bar{R}$

$$\Delta S)_{Ar} = (2\ \text{lbmol})\left(1.986\ \frac{\text{Btu}}{\text{lbmol}\cdot °R}\right)\left[\frac{5}{2}\ln\frac{571.11}{610} - \ln\frac{24.90}{50}\right] = 2.115\ \frac{\text{Btu}}{°R}$$

The helium undergoes a process from 460 °R, 15 lbf/in² to 571.11 °R and the partial pressure of helium in the final mixture: $y_{He}\,p_f = \left(\frac{0.7}{2.7}\right)(33.62) = 8.72$ lbf/in². Thus

$$\Delta S)_{He} = (0.7)(1.986)\left[\frac{5}{2}\ln\frac{571.11}{460} - \ln\frac{8.72}{15}\right] = 1.506\ \frac{\text{Btu}}{°R}$$

Inserting values into Eq. (2)

$$\sigma = 2.115 + 1.506 = 3.621\ \frac{\text{Btu}}{°R} \longleftarrow \sigma$$

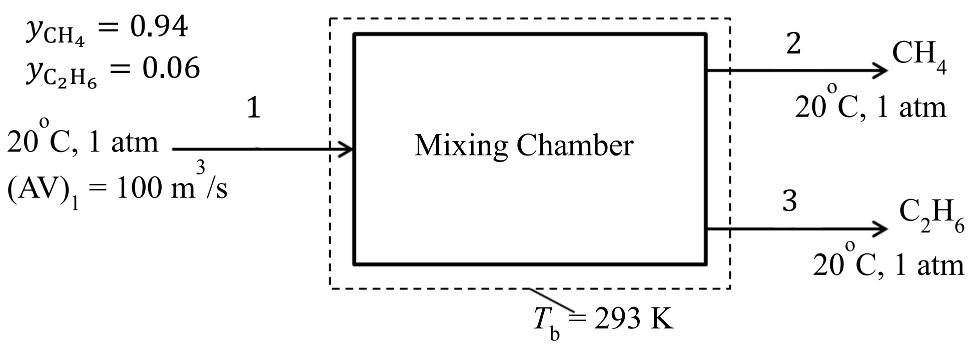
Problem 12.34

A device is being designed to separate a natural gas having a molar analysis of 94% CH_4 and 6% C_2H_6 into components. The device will receive natural gas at 20°C, 1 atm with a volumetric flow rate of 100 m^3/s. Separate streams of CH_4 and C_2H_6 will exit, each at 20°C, 1 atm. The device will operate isothermally at 20°C. Ignoring kinetic and potential energy effects and assuming ideal gas behavior, determine the minimum theoretical work input required at steady state, in kW.

Known:
Data are provided for a device operating at steady state that separates a natural gas into components.

Find:
Determine the minimum theoretical power input required.

Schematic & Given Data:



Engineering Model:
(1) The control volumes shown in the accompanying figure operates isothermally and at steady state.
(2) Kinetic and potential energy effects can be ignored.
(3) Ideal gas principles apply for the pure components.
(4) The mixture adheres to the Dalton model.

Analysis:
The mass flow rate at 1 is obtained by using the given volumetric flow rate and the ideal gas equation of state:

$$\dot{m}_1 = \frac{(\mathrm{AV})_1}{v_1} = \frac{p_1(\mathrm{AV})_1}{(\bar{R}/M)T_1}$$

Where $M = y_{CH_4}M_{CH_4} + y_{C_2H_6}M_{C_2H_6} = (0.94)(16.04) + (0.06)(30.07) = 16.88\ \frac{\mathrm{kg}}{\mathrm{kmol}}$. Thus

$$\dot{m}_1 = \frac{\left(1.01325 \cdot 10^5\,\frac{\mathrm{N}}{\mathrm{m}^2}\right)\left(100\,\frac{\mathrm{m}^3}{\mathrm{s}}\right)}{\left(\frac{8314}{16.88}\,\frac{\mathrm{N}\cdot\mathrm{m}}{\mathrm{kg}\cdot\mathrm{K}}\right)(293\ \mathrm{K})} = 70.2\,\frac{\mathrm{kg}}{\mathrm{s}}$$

The molar flow rate is

$$\dot{n}_1 = \frac{\dot{m}_1}{M} = \frac{70.2\,\frac{\text{kg}}{\text{s}}}{16.88\,\frac{\text{kg}}{\text{kmol}}} = 4.1588\,\frac{\text{kmol}}{\text{s}}$$

Then:

$$\dot{n}_{\text{CH}_4} = y_{\text{CH}_4}\dot{n}_1 = (0.94)(4.1588) = 3.9093\,\frac{\text{kmol}}{\text{s}}$$

$$\dot{n}_{\text{C}_2\text{H}_6} = y_{\text{C}_2\text{H}_6}\dot{n}_1 = (0.06)(4.1588) = 0.2495\,\frac{\text{kmol}}{\text{s}}$$

With assumptions 1 and 2, an energy rate balance reduces to

$$0 = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \left[\dot{n}_{\text{CH}_4}\bar{h}_{\text{CH}_4}(T_1) + \dot{n}_{\text{C}_2\text{H}_6}\bar{h}_{\text{C}_2\text{H}_6}(T_1)\right] - \left[\dot{n}_{\text{CH}_4}\bar{h}_{\text{CH}_4}(T_2) + \dot{n}_{\text{C}_2\text{H}_6}\bar{h}_{\text{C}_2\text{H}_6}(T_3)\right]$$

Since T_1=T_2=T_3, the only remaining terms are $\dot{W}_{\text{cv}} = \dot{Q}_{\text{cv}}$.

An entropy rate balance reduces at steady state to:

$$\begin{aligned}0 = \frac{\dot{Q}_{\text{cv}}}{T_{\text{b}}} &+ \left[\dot{n}_{\text{CH}_4}\bar{s}_{\text{CH}_4}\left(T_1, y_{\text{CH}_4}p_1\right) + \dot{n}_{\text{C}_2\text{H}_6}\bar{s}_{\text{C}_2\text{H}_6}\left(T_1, y_{\text{C}_2\text{H}_6}p_1\right)\right] \\ &- \left[\dot{n}_{\text{CH}_4}\bar{s}_{\text{CH}_4}(T_2, p_2) + \dot{n}_{\text{C}_2\text{H}_6}\bar{s}_{\text{C}_2\text{H}_6}(T_3, p_3)\right] + \dot{\sigma}_{\text{cv}}\end{aligned}$$

Or, upon rearrangement and inserting $\dot{W}_{\text{cv}} = \dot{Q}_{\text{cv}}$:

$$\begin{aligned}-\dot{W}_{\text{cv}} = T_b\Big\{&\dot{n}_{\text{CH}_4}\left[\bar{s}_{\text{CH}_4}\left(T_1, y_{\text{CH}_4}p_1\right) - \bar{s}_{\text{CH}_4}(T_2, p_2)\right] \\ &+ \dot{n}_{\text{C}_2\text{H}_6}\left[\bar{s}_{\text{C}_2\text{H}_6}\left(T_1, y_{\text{C}_2\text{H}_6}p_1\right) - \bar{s}_{\text{C}_2\text{H}_6}(T_3, p_3)\right] + \dot{\sigma}_{\text{cv}}\Big\}\end{aligned}$$

In this equation $\left(-\dot{W}_{\text{cv}}\right)$ is the work input. Since $\dot{\sigma}_{\text{cv}} \geq 0$, it follows that $\left(-\dot{W}_{\text{cv}}\right)$ is a minimum when $\dot{\sigma}_{\text{cv}} = 0$. Accordingly:

$$\begin{aligned}\left(-\dot{W}_{\text{cv}}\right)_{min} = T_b\Big\{&\dot{n}_{\text{CH}_4}\left[\bar{s}^o_{\text{CH}_4}(T_1) - \bar{s}^o_{\text{CH}_4}(T_2) - \bar{R}\ln y_{\text{CH}_4}\right] \\ &+ \dot{n}_{\text{C}_2\text{H}_6}\left[\bar{s}^o_{\text{C}_2\text{H}_6}(T_1) - \bar{s}^o_{\text{C}_2\text{H}_6}(T_3) - \bar{R}\ln y_{\text{C}_2\text{H}_6}\right]\Big\}\end{aligned}$$

Again, since T_1=T_2=T_3, the $\bar{s}^o$ terms cancel each other out, leaving:

$$\begin{aligned}\left(\dot{W}_{\text{cv}}\right)_{min} &= -\bar{R}T_{\text{b}}\left\{\dot{n}_{\text{CH}_4}\ln y_{\text{CH}_4} + \dot{n}_{\text{C}_2\text{H}_6}\ln y_{\text{C}_2\text{H}_6}\right\} \\ &= -\left(8.314\,\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(293\text{ K})\left\{\left(3.9093\,\frac{\text{kmol}}{\text{s}}\right)\ln 0.94\right. \\ &\quad \left.+ \left(0.2495\,\frac{\text{kmol}}{\text{s}}\right)\ln 0.06\right\} = -2299.2\,\frac{\text{kJ}}{\text{s}}\left|\frac{1\text{ kW}}{1\,\frac{\text{kJ}}{\text{s}}}\right| = -2299.2\text{ kW}\end{aligned}$$

$\longleftarrow$

Problem 12.35

Air at 50°C, 1 atm and a volumetric flow rate of 60 m^3/min enters an insulated control volume operating at steady state and mixes with helium entering as a separate stream at 120°C, 1 atm and a volumetric flow rate of 25 m^3/min. A single mixed stream exits at 1 atm. Ignoring kinetic and potential energy effects, determine for the control volume

(a) the temperature of the exiting mixture, in °C.
(b) the rate of entropy production, in kW/K.
(c) the rate of exergy destruction, in kW, for T_0 = 295 K.

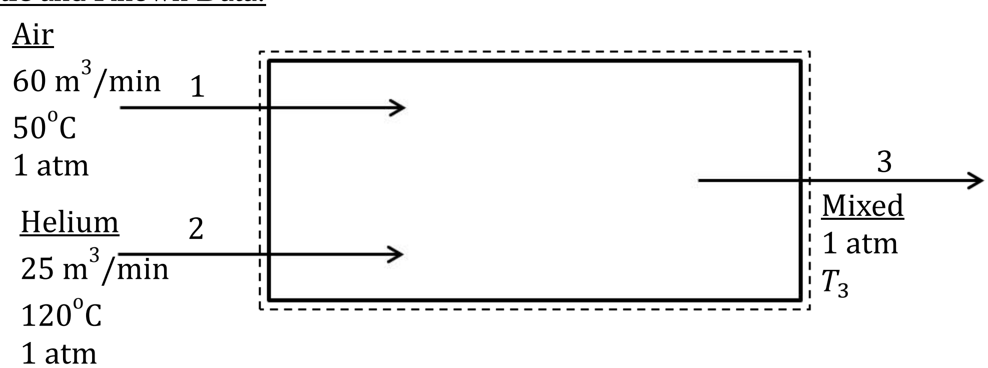
Known:

60 m^3/min of air at 50°C, 1 atm is mixed adiabatically and at steady state with 25 m^3/min of helium at 100°C, 1 atm to form a mixed stream at 1 atm.

Find:

Determine (a) the temperature of the exiting mixture, (b) the rate of entropy production, and (c) the rate of exergy destruction.

Schematic and Known Data:



Engineering Model:

(1) The control volume shown in the figure is at steady state.
(2) For the control volume, $\dot{Q}_{\text{cv}} = \dot{W}_{\text{cv}} = 0$, and the effects of kinetic and potential energy can be ignored.
(3) The entering gases each can be modeled as an ideal gas and the exiting mixture adheres to the Dalton model.
(4) The specific heat $c_{p,a}$ is taken as a constant for air at 350 K.

Analysis:

(a) Mass rate balances at steady state indicate that

$\dot{m}_{\text{a}_1} = \dot{m}_{\text{a}_3} \equiv \dot{m}_{\text{a}}$

$\dot{m}_{\text{h}_2} = \dot{m}_{\text{h}_3} \equiv \dot{m}_{\text{h}}$

An energy rate balance at steady state reduces to read

$0 = \left[\dot{m}_{\text{a}_1} h_a(T_1) + \dot{m}_{\text{h}_2} h_h(T_2)\right] - \left[\dot{m}_{\text{a}_3} h_{\text{a}}(T_3) + \dot{m}_{\text{h}_3} h_{\text{h}}(T_3)\right]$, or

$$0 = \dot{m}_{\text{a}}[h_{\text{a}}(T_3) - h_{\text{a}}(T_1)] - \dot{m}_{\text{h}}[h_{\text{h}}(T_3) - h_{\text{h}}(T_2)] \quad (1)$$

For helium, Table A-21 (see note a) indicates that $c_{p_h} = \frac{5}{2}R$. Since the final temperature will fall between T_1 and T_2, and the difference is relatively narrow, c_p for air is also taken as constant. The value at 350 K is used: $c_{p_a} = 1.008 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$ (Table A-20). Thus, Eq. (1) gives:
$0 = \dot{m}_\text{a} c_{p_a}(T_3 - T_1) + \dot{m}_\text{h} c_{p_h}(T_3 - T_2)$, rearranging and solving for T_3

$$T_3 = \frac{\dot{m}_\text{a} c_{p_a} T_1 + \dot{m}_\text{h} c_{p_h} T_2}{\left(\dot{m}_\text{h} c_{p_h} + \dot{m}_\text{a} c_{p_a}\right)} \qquad (2)$$

The mass flow rates are found from the volumetric flow rates

$$\dot{m}_\text{a} = \frac{(\text{AV})_1 p_1}{RT_1} = \frac{\left(60\,\frac{\text{m}^3}{\text{min}}\cdot\frac{1\text{ min}}{60\text{ s}}\right)\left(1.013\cdot 10^5\,\frac{\text{N}}{\text{m}^2}\right)}{\left(\frac{8314}{28.97}\,\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(323\text{ K})} = 1.093\,\frac{\text{kg}}{\text{s}}$$

$$\dot{m}_\text{h} = \frac{(\text{AV})_2 p_2}{RT_2} = \frac{\left(25\,\frac{\text{m}^3}{\text{min}}\cdot\frac{1\text{ min}}{60\text{ s}}\right)\left(1.013\cdot 10^5\,\frac{\text{N}}{\text{m}^2}\right)}{\left(\frac{8314}{4.003}\,\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(393\text{ K})} = 0.0517\,\frac{\text{kg}}{\text{s}}$$

Solving Eq. (2):

$$T_3 = \frac{\dot{m}_\text{a} c_{p_a} T_1 + \dot{m}_\text{h} c_{p_h} T_2}{\left(\dot{m}_\text{h} c_{p_h} + \dot{m}_\text{a} c_{p_a}\right)}$$

$$= \frac{\left(1.093\,\frac{\text{kg}}{\text{s}}\right)\left(1.008\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(323\text{ K}) + \left(0.0517\,\frac{\text{kg}}{\text{s}}\right)\left(\frac{5}{2}\left(\frac{8.314\,\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}}{4.003\,\frac{\text{kg}}{\text{kmol}}}\right)\right)(393\text{ K})}{\left(\left(0.0517\,\frac{\text{kg}}{\text{s}}\right)\left(\frac{5}{2}\left(\frac{8.314\,\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}}{4.003\,\frac{\text{kg}}{\text{kmol}}}\right)\right) + \left(1.093\,\frac{\text{kg}}{\text{s}}\right)\left(1.008\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\right)}$$

$$= \frac{(355.86\text{ kW}) + (105.5\text{ kW})}{\left(0.268\,\frac{\text{kW}}{\text{K}}\right) + \left(1.102\,\frac{\text{kW}}{\text{K}}\right)} = 336.8\text{ K} = 63.6°\text{C}$$

$\longleftarrow$

(b) Reducing an entropy rate balance

$$0 = \dot{m}_\text{a} s_\text{a}(T_1, p) + \dot{m}_\text{h} s_h(T_2, p) - [\dot{m}_\text{a} s_\text{a}(T_3, y_\text{a} p) + \dot{m}_\text{h} s_\text{h}(T_3, y_\text{h} p)] + \dot{\sigma}$$

Rearranging and simplifying

$$\dot{\sigma} = \dot{m}_\text{a}\left[c_{p_a}\ln\frac{T_3}{T_1} - \frac{\bar{R}}{M_a}\ln y_\text{a}\right] + \dot{m}_\text{h}\left[c_{p_h}\ln\frac{T_3}{T_2} - \frac{\bar{R}}{M_\text{h}}\ln y_\text{h}\right]$$

The mole fractions are evaluated as follows

$$\dot{n}_\text{a} = \frac{\dot{m}_\text{a}}{M_\text{a}} = \frac{1.093\,\frac{\text{kg}}{\text{s}}}{28.97\,\frac{\text{kg}}{\text{kmol}}} = 0.03773\,\frac{\text{kmol}}{\text{s}}$$

$$\dot{n}_\text{h} = \frac{\dot{m}_\text{h}}{M_\text{h}} = \frac{0.05167\,\frac{\text{kg}}{\text{s}}}{4.003\,\frac{\text{kg}}{\text{kmol}}} = 0.0129\,\frac{\text{kmol}}{\text{s}}$$

Using molar flow rates to determine mole fraction:

$$y_\text{a} = \frac{\dot{n}_\text{a}}{\dot{n}} = \frac{0.03773}{0.03773 + 0.0129} = 0.745, \qquad y_\text{h} = \frac{\dot{n}_\text{h}}{\dot{n}} = \frac{0.0129}{0.03773 + 0.0129} = 0.255$$

Solving for $\dot{\sigma}$:

$$\begin{aligned}\dot{\sigma} = {} & \left(1.093\,\frac{\text{kg}}{\text{s}}\right)\left[\left(1.008\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\ln\frac{336.8}{323} - \left(\frac{8.314\,\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}}{28.97\,\frac{\text{kg}}{\text{kmol}}}\right)\ln 0.745\right] \\ & + \left(0.0517\,\frac{\text{kg}}{\text{s}}\right)\left[\frac{5}{2}\left(\frac{8.314\,\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}}{4.003\,\frac{\text{kg}}{\text{kmol}}}\right)\ln\frac{336.8}{393} - \left(\frac{8.314\,\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}}{4.003\,\frac{\text{kg}}{\text{kmol}}}\right)\ln 0.255\right] \\ & = 0.2437\,\frac{\text{kW}}{\text{K}}\end{aligned}$$

$\longleftarrow$

(c) The exergy destruction rate is:

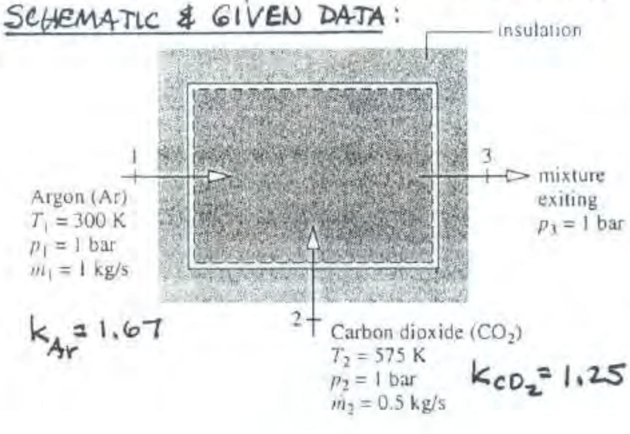
$\dot{\text{E}}_\text{d} = T_0\dot{\sigma}_\text{cv} = (295\ \text{K})\left(0.2437\,\frac{\text{kW}}{\text{K}}\right) = 72\ \text{kW}$ $\longleftarrow$

PROBLEM 12.36

KNOWN: Argon and carbon dioxide enter an insulated mixing chamber operating at steady state. Data are known at the inlets and exit.

FIND: Determine (a) the molar analysis of the exiting mixture, (2) the exit temperature, and (c) the rate of entropy production.

SCHEMATIC & GIVEN DATA:



$k_{Ar} = 1.67$ $\qquad k_{CO_2} = 1.25$

ENGINEERING MODEL: (1) The control volume is at steady state, with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) Kinetic and potential energy effects are negligible. (3) The ideal gas model applies, and the specific heats are constant.

Using Eq. 3.47a

$$c_{p_{Ar}} = \frac{kR}{k-1} = \frac{(1.67)\left(\frac{8.314}{39.94}\right)}{(0.67)} = 0.5189 \frac{kJ}{kg \cdot K}$$

$$c_{p\,CO_2} = \frac{(1.25)\left(\frac{8.314}{44.01}\right)}{(0.25)} = 0.9446 \frac{kJ}{kg \cdot K}$$

ANALYSIS: Based on the entering mass flow rates, the gravimetric analysis of the exiting mixture is

$$mf_{Ar} = \frac{\dot{m}_1}{\dot{m}_3} = \frac{1}{1.5} = 0.6667, \quad mf_{CO_2} = \frac{\dot{m}_2}{\dot{m}_3} = \frac{0.5}{1.5} = 0.3333$$

Following the method of Example 12.2 for 1 kg/s of mixture

Component	$mf_i \div M_i$ =	n_i	y_i
Ar	$0.6667 \div 39.94$ =	0.01669	0.6880
CO_2	$0.3333 \div 44.01$ =	0.00757	0.3120
		0.02426	1.0000

molar analysis

(b) The mass and energy rate balances reduce to

$$0 = \dot{m}_1 h_{Ar,1} + \dot{m}_2 h_{CO_2,2} - \dot{m}_3 h_{mix}$$

With $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$ and $h_{mix} = mf_{Ar} h_{Ar,3} + mf_{CO_2} h_{CO_2,3}$ we get

$$0 = \dot{m}_1 h_{ar,1} + \dot{m}_2 h_{ar,2} - \underbrace{\dot{m}_3 mf_{Ar}}_{=\dot{m}_1} h_{Ar,3} - \underbrace{\dot{m}_3 mf_{CO_2}}_{=\dot{m}_2} h_{CO_2,3}$$

$$0 = \dot{m}_1 \underbrace{(h_{ar,1} - h_{ar,3})}_{c_{p_{Ar}}(T_1 - T_3)} + \dot{m}_2 \underbrace{(h_{CO_2,2} - h_{CO_2,3})}_{c_{p_{CO_2}}(T_2 - T_3)}$$

Solving T_3 and inserting values

$$T_3 = \frac{\dot{m}_1 c_{p_{Ar}} T_1 + \dot{m}_2 c_{p_{CO_2}} T_2}{\dot{m}_1 c_{p_{Ar}} + \dot{m}_2 c_{p_{CO_2}}}$$

$$= \frac{(1)(0.5189)(300) + (0.5)(0.9446)(575)}{(1)(0.5189) + (0.5)(0.9446)} = 431 \text{ K} \longleftarrow T_3$$

PROBLEM 12.36 (Cont'd.) - Page 2

(c) Reducing the entropy balance

$$0 = \cancel{\sum_j \left(\frac{\dot{Q}}{T}\right)_j}^{0} + \dot{m}_1 s_{Ar,1} + \dot{m}_2 s_{CO_2,2} - mf_{Ar}\dot{m}_3 s_{Ar,3} - mf_{CO_2}\dot{m}_3 s_{CO_2,3} + \dot{\sigma}_{cv}$$

$$= \dot{m}_1 \underbrace{(s_{Ar,1} - s_{Ar,3})}_{= c_{p_{Ar}} \ln \frac{T_1}{T_3} - R_{Ar} \ln \frac{P_1}{P_{Ar,3}}} + \dot{m}_2 \underbrace{(s_{CO_2,2} - s_{CO_2,3})}_{= c_{p_{CO_2}} \ln \frac{T_2}{T_3} - R_{CO_2} \ln \frac{P_2}{P_{CO_2,3}}} + \dot{\sigma}_{cv}$$

Solving for $\dot{\sigma}_{cv}$, and introducing $P_{Ar,3} = y_{Ar} P_3$, $P_{CO_2,3} = y_{CO_2} P_3$

$$\dot{\sigma}_{cv} = \dot{m}_1 \left[c_{p_{Ar}} \ln \frac{T_3}{T_1} - R_{Ar} \ln y_{Ar} \right] + \dot{m}_1 \left[c_{p_{CO_2}} \ln \frac{T_3}{T_2} - R_{CO_2} \ln y_{CO_2} \right]$$

$$= (1\ \mathrm{kg/s}) \left[\left(0.5189 \frac{\mathrm{kJ}}{\mathrm{kg \cdot K}}\right) \ln\left(\frac{431}{300}\right) - \frac{8.314}{39.94} \frac{\mathrm{kJ}}{\mathrm{kg \cdot K}} \ln(0.6888) \right]$$

$$+ (0.5) \left[(0.9446) \ln\left(\frac{431}{575}\right) - \frac{8.314}{44.01} \ln(0.3120) \right]$$

$$= 0.2397\ \mathrm{kJ/s \cdot K} \left| \frac{1\ \mathrm{kW}}{1\ \mathrm{kJ/s}} \right| = 0.2397 \frac{\mathrm{kW}}{\mathrm{K}} \longleftarrow \dot{\sigma}_{cv}$$

Problem 12.37

Carbon dioxide (CO_2) at 100°F, 18 lbf/in.^{2} and a volumetric flow rate of 250 ft^3/min enters an insulated control volume operating at steady state and mixes with oxygen (O_2) entering as a separate stream at 190°F, 18 lbf/in.^{2} and a mass flow rate of 60 lb/min. A single mixed stream exits at 15 lbf/in.^{2} Kinetic and potential energy effects can be ignored. Using the ideal gas model with constant specific heats, determine for the control volume

(a) the temperature of the exiting mixture, in °F.
(b) the rate of entropy production, in Btu/min°R.
(c) the rate of exergy destruction, in Btu/min, for T_0 = 40°F.

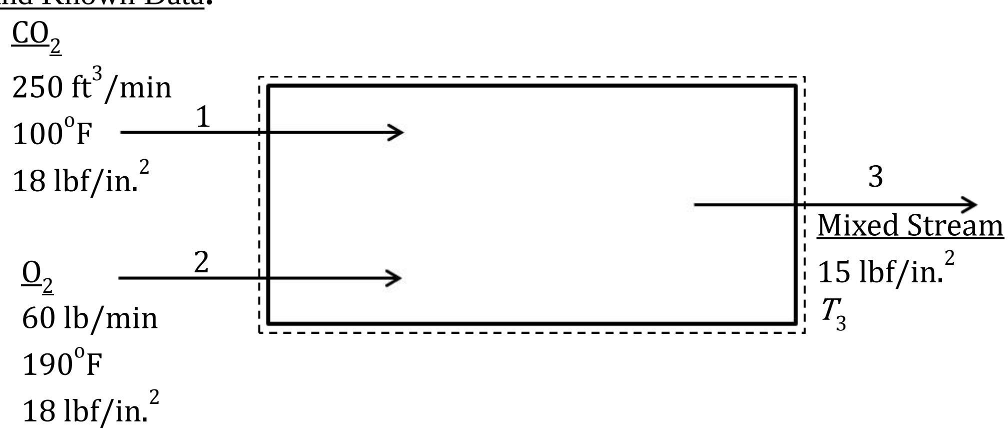
Known:

250 ft^3/min of CO_2 at 100°F, 18 lbf/in.^{2} is mixed adiabatically at steady state with 60 lb/min of O_2 at 190°F, 18 lbf/in.^{2} to form a mixed stream at 15 lbf/in.^{2}

Find:

Determine (a) the temperature of the exiting mixture, (b) the rate of entropy production, and (c) the rate of exergy destruction.

Schematic and Known Data**:**



Engineering Model:

(1) The control volume shown in the figure is at steady state.
(2) For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$, and the effects of kinetic and potential energy can be ignored.
(3) The entering gases each can be modeled as an ideal gas and the exiting mixture adheres to the Dalton model.
(4) The specific heats c_p can be taken as constants.
(5) For the exergy reference environment, T_0 = 500° R.

Analysis:

(a) Mass rate balances at steady state indicate that:

$$\dot{m}_{CO_2,1} = \dot{m}_{CO_2,3} \equiv \dot{m}_{CO_2}$$
$$\dot{m}_{O_2,2} = \dot{m}_{O_2,3} \equiv \dot{m}_{O_2}$$

An energy rate balance at steady state reduces to read

$$0 = \left[\dot{m}_{CO_2} h_{CO_2}(T_1) + \dot{m}_{O_2} h_{O_2}(T_2)\right] - \left[\dot{m}_{CO_2} h_{CO_2}(T_3) + \dot{m}_{O_2} h_{O_2}(T_3)\right]$$

or on rearrangement with assumption (4)

$0 = \dot{m}_{CO_2} c_{p_{CO_2}}(T_3 - T_1) + \dot{m}_{O_2} c_{p_{O_2}}(T_3 - T_2)$ and solving for T_3

$$T_3 = \frac{\dot{m}_{CO_2} c_{p_{CO_2}} T_1 + \dot{m}_{O_2} c_{p_{O_2}} T_2}{\left(\dot{m}_{CO_2} c_{p_{CO_2}} + \dot{m}_{O_2} c_{p_{O_2}}\right)} \qquad (1)$$

As T_3 will fall between T_1 and T_2, and the temperature difference is narrow, c_p for each gas is then taken as constant at the average temperature of 145°F. With data from Table A-20E, $c_{p_{CO_2}} = 0.2104 \frac{\text{Btu}}{\text{lb°R}}$, $c_{p_{O_2}} = 0.2214 \frac{\text{Btu}}{\text{lb°R}}$. The mass flow rate of CO_2 is found as follows:

$$\dot{m}_{CO_2} = \frac{(\text{AV})_1 p_1}{RT_1} = \frac{\left(250 \frac{\text{ft}^3}{\text{min}}\right)\left(18 \frac{\text{lbf}}{\text{in.}^2} \cdot \frac{144\text{in.}^2}{\text{ft}^2}\right)}{\left(\frac{1545}{44.01} \frac{\text{ft} \cdot \text{lbf}}{\text{lb°R}}\right)(560°\text{R})} = 33 \frac{\text{lb}}{\text{min}}$$

Substituting values in Eq. (1) and solving:

$$T_3 = \frac{\left(33 \frac{\text{lb}}{\text{min}}\right)\left(0.2104 \frac{\text{Btu}}{\text{lb} \cdot °\text{R}}\right)(560°\text{R}) + \left(60 \frac{\text{lb}}{\text{min}}\right)\left(0.2214 \frac{\text{Btu}}{\text{lb} \cdot °\text{R}}\right)(650°\text{R})}{\left(\left(33 \frac{\text{lb}}{\text{min}}\right)\left(0.2104 \frac{\text{Btu}}{\text{lb} \cdot °\text{R}}\right) + \left(60 \frac{\text{lb}}{\text{min}}\right)\left(0.2214 \frac{\text{Btu}}{\text{lb} \cdot °\text{R}}\right)\right)}$$

$$T_3 = 619°\text{R} = 159°\text{F} \qquad \longleftarrow$$

(b) An entropy balance reads:

$$0 = \dot{m}_{CO_2} s_{CO_2}(T_1, p_1) + \dot{m}_{O_2} s_{O_2}(T_2, p_2) - [\dot{m}_{CO_2} s_{CO_2}\left(T_3, y_{CO_2} p_3\right) + \dot{m}_{O_2} s_{O_2}\left(T_3, y_{O_2} p_3\right) + \dot{\sigma}$$

or

$$\dot{\sigma} = \dot{m}_{CO_2}\left[s_{CO_2}\left(T_3, y_{CO_2} p_3\right) - s_{CO_2}(T_1, p_1)\right] + \dot{m}_{O_2}\left[s_{O_2}\left(T_3, y_{O_2} p_3\right) - s_{O_2}(T_2, p_2)\right]$$

$$\dot{\sigma} = \dot{m}_{CO_2}\left[c_{p_{CO_2}} \ln \frac{T_3}{T_1} - \frac{\bar{R}}{M_{CO_2}} \ln y_{CO_2} \frac{p_3}{p_1}\right] + \dot{m}_{O_2}\left[c_{p_{O_2}} \ln \frac{T_3}{T_2} - \frac{\bar{R}}{M_{O_2}} \ln y_{O_2} \frac{p_3}{p_2}\right]$$

To obtain the mole fractions

$$\dot{n}_{CO_2} = \frac{\dot{m}_{CO_2}}{M_{CO_2}} = \frac{33 \frac{\text{lb}}{\text{min}}}{44.01 \frac{\text{lb}}{\text{lbmol}}} = 0.7498 \frac{\text{lbmol}}{\text{min}}$$

$$\dot{n}_{O_2} = \frac{\dot{m}_{O_2}}{M_{O_2}} = \frac{60 \frac{\text{lb}}{\text{min}}}{32 \frac{\text{lb}}{\text{lbmol}}} = 1.875 \frac{\text{lbmol}}{\text{min}}$$

therefore

$$y_{CO_2} = \frac{\dot{n}_{CO_2}}{\dot{n}} = \frac{0.7498}{0.7498 + 1.875} = 0.286, \quad y_{O_2} = \frac{\dot{n}_{O_2}}{\dot{n}} = \frac{1.875}{0.7498 + 1.875} = 0.714$$

Thus

$$\begin{aligned}\dot{\sigma} = &\left(33\,\frac{\text{lb}}{\text{min}}\right)\left[\left(0.2104\,\frac{\text{Btu}}{\text{lb°R}}\right)\ln\frac{619}{560} - \left(\frac{1.986\,\frac{\text{Btu}}{\text{lbmol°R}}}{44.01\,\frac{\text{lb}}{\text{lbmol}}}\right)\ln\left(0.286\left(\frac{15}{18}\right)\right)\right] \\ &+ \left(60\,\frac{\text{lb}}{\text{min}}\right)\left[\left(0.2214\,\frac{\text{Btu}}{\text{lb°R}}\right)\ln\frac{619}{650} - \left(\frac{1.986\,\frac{\text{Btu}}{\text{lbmol°R}}}{32\,\frac{\text{lb}}{\text{lbmol}}}\right)\ln\left(0.714\left(\frac{15}{18}\right)\right)\right] \\ &= 4.12\,\frac{\text{Btu}}{\text{min}\cdot\text{°R}}\end{aligned}$$

(c) The rate of exergy destruction can be found as follows:

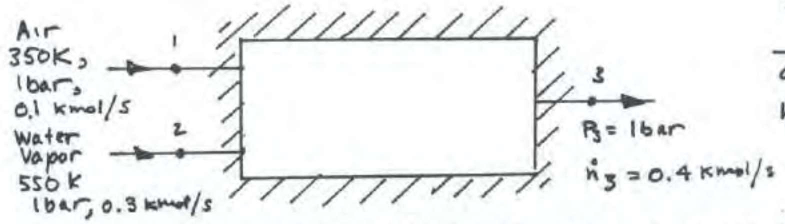
$$\dot{\text{E}}_{\text{d}} = T_0\dot{\sigma} = (500\text{°R})\left(4.12\,\frac{\text{Btu}}{\text{min}\cdot\text{°R}}\right) = 2060\,\frac{\text{Btu}}{\text{min}}$$

PROBLEM 12.38

KNOWN: Air at a specified temperature and pressure enters an insulated chamber at steady state and mixes with water vapor entering at a specified temperature and pressure. The mixture exits at a known pressure and molar analysis.

FIND: Determine the temperature of the exiting mixture and the rate entropy is produced.

SCHEMATIC & GIVEN DATA:



At 3, based on the given molar flow rates,

i	y_i
air	0.25
H_2O	0.75

ENGINEERING MODEL: (1) The chamber is well insulated and at steady state. (2) Kinetic and potential energy effects can be ignored. (3) Each gas can be modeled as an ideal gas and the exiting mixture adheres to the Dalton model.

ANALYSIS: (a) An energy rate balance at steady state, expressed on a molar basis, reduces with assumptions 1 and 2 to give

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{n}_1 \bar{h}_1 + \dot{n}_2 \bar{h}_2 - \dot{n}_3 \bar{h}_3 \Rightarrow \bar{h}_3 = \left(\frac{\dot{n}_1}{\dot{n}_3}\right)\bar{h}_1 + \left(\frac{\dot{n}_2}{\dot{n}_3}\right)\bar{h}_2$$

where $\dot{n}_1$, $\dot{n}_2$, and $\dot{n}_3$ are the molar flow rates of the air, water vapor, and mixture, respectively. Since $\dot{n}_1/\dot{n}_3 = 0.25$ and $\dot{n}_2/\dot{n}_3 = 0.75$, this becomes

$$\bar{h}_3 = 0.25\,\bar{h}_1 + 0.75\,\bar{h}_2 = 0.25\,\bar{h}_{air}(T_1) + 0.75\,\bar{h}_{H_2O}(T_2)$$

The enthalpy of the mixture, per kmol of mixture, is $\bar{h}_3 = 0.25\,\bar{h}_{air}(T_3) + 0.75\,\bar{h}_{H_2O}(T_3)$.

① Accordingly, with data from Table A-23,

$$0.25\,[28.97\,h_{air}(T_3)] + 0.75\,\bar{h}_{H_2O} = (0.25)(28.97)(350.49) + (0.75)(18601) = 16489 \text{ kJ}$$

where $\bar{h}_{air} = M_{air} h_{air}$.

An iterative solution using data from Table A-23 gives

② $T_3 = 507$ K (234°C). ← T_3

(b) An entropy rate balance at steady state, expressed on a molar basis, reads

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{0} + \dot{n}_1 \bar{s}_1 + \dot{n}_2 \bar{s}_2 - \dot{n}_3 \bar{s}_3 + \dot{\sigma}_{cv}$$

Thus

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_3} = \bar{s}_3 - \left(\overbrace{\frac{\dot{n}_1}{\dot{n}_3}}^{0.25} \bar{s}_1 + \overbrace{\frac{\dot{n}_2}{\dot{n}_3}}^{0.75} \bar{s}_2 \right)$$

The specific entropy of the mixture is

$$\bar{s}_3 = 0.25\,\bar{s}_{Air}(T_3, y_{air} P_3) + 0.75\,\bar{s}_{H_2O}(T_3, y_{H_2O} P_3)$$

Combining the last two equations and using s^o data from Table A-23

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_3} = 0.25\left[\bar{s}_{Air}(T_3, y_{Air}P_3) - \bar{s}_{Air}(T_1, P_1)\right] + 0.75\left[\bar{s}_{H_2O}(T_3, y_{H_2O}P_3) - \bar{s}_{H_2O}(T_2, P_2)\right]$$

$$= (0.25)(28.97)\left[s^o_{Air}(T_3) - s^o_{Air}(T_1) - \frac{\bar{R}}{M_{air}}\ln\frac{y_{air}P_3}{P_1}\right] +$$

$$0.75\left[\bar{s}^o_{H_2O}(T_3) - \bar{s}^o_{H_2O}(T_2) - \bar{R}\ln\frac{y_{H_2O}P_3}{P_2}\right]$$

Continuing the calculation

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_3} = (0.25)(28.97)\left[2.2338 - 1.85708 - \frac{8.314}{28.97}\ln\frac{(0.25)(1)}{(1)}\right] +$$

$$0.75\left[206.902 - 209.795 - 8.314\ln\frac{(0.75)(1)}{(1)}\right]$$

$$= 5.6098 + (-0.3759) = 5.2339\ \frac{kJ}{kmol\cdot K}$$

or

$$\dot{\sigma}_{cv} = \left(0.4\ \frac{kmol}{s}\right)\left(5.2339\ \frac{kJ}{kmol\cdot K}\right)\left|\frac{1\ kW}{1\ kJ/s}\right| = 2.094\ \frac{kW}{K} \longleftarrow \dot{\sigma}_{cv}$$

---

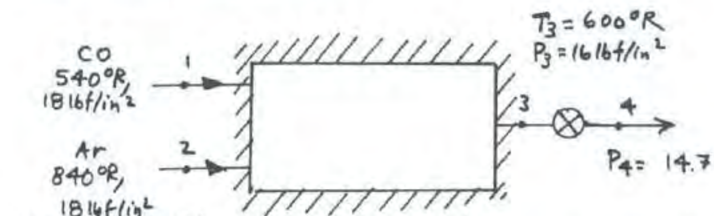
1. Table A-23 can be used here for water vapor because water vapor is modeled as an ideal gas at each of the three states.
2. An iterative solution using table data can be avoided in this case by using constant cp for dry air and water vapor evaluated at the average of the inlet temperatures: 450 K. Alternatively, IT could be used.

PROBLEM 12.39

KNOWN: Streams of CO and Ar, each at known states, form a mixture that expands across a valve to a specified pressure.

FIND: Determine (a) the mass and molar analyses of the mixture, (b) the temperature at the valve exit, (c) the rates of exergy destruction for the mixing chamber and the valve, each per unit mass of mixture.

SCHEMATIC & GIVEN DATA:



$T_0 = 500°R$

ENGINEERING MODEL: (1) The mixing chamber is well insulated and at steady state. (2) Kinetic and potential energy effects can be ignored. (3) The expansion across the valve is a throttling process. (4) Each gas can be modeled as an ideal gas and the mixture adheres to the Dalton model. (5) $T_0 = 500°R$.

ANALYSIS: (a) Reducing mass and energy rate balances for a control volume enclosing the mixing chamber, we get $0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$, or expressed alternatively

$$0 = \dot{m}_{co} h_{co}(540°R) + \dot{m}_{Ar} h_{Ar}(840°R) - [\dot{m}_{co} h_{co}(600°R) + \dot{m}_{Ar} h_{Ar}(600°R)]$$

$$\Rightarrow \dot{m}_{co}\left[\frac{\bar{h}_{co}(600°R) - \bar{h}_{co}(540°R)}{M_{CO}}\right] = \dot{m}_{Ar}\left[\frac{\bar{h}_{Ar}(840) - \bar{h}_{Ar}(600)}{M_{Ar}}\right]$$

For Ar from Table A-21, $\bar{c}_p = 2.5\bar{R}$. Data for CO is obtained from Table A-23. Thus

$$\dot{m}_{co}\left[\frac{4168 - 3750.3}{28.01}\right] = \dot{m}_{AR}\left[\frac{(2.5)(1.986)[240]}{39.94}\right] \Rightarrow \frac{\dot{m}_{co}}{\dot{m}_{Ar}} = 2$$

Accordingly, $(mf)_{co} = 2/3$, $(mf)_{Ar} = 1/3$

Considering a typical unit mass of mixture

$$n_{co} = \frac{2/3 \text{ lb}}{28.01 \text{ lb/lbmol}} = 0.0238, \quad n_{Ar} = \frac{1/3}{39.94} = 0.0083 \Rightarrow \quad y_{co} = \frac{0.0238}{0.0321} = 0.741, \quad y_{Ar} = 0.259$$

(b) Since the expansion across the valve is a throttling process, $h_4 = h_3$. And since the enthalpy of an ideal gas depends only on temperature, $T_4 = T_3 = 600°R$.

(c) The rate of exergy destruction can be found using $\dot{E}_d = T_0 \dot{\sigma}$, where $\dot{\sigma}$ is the rate of entropy production. For a control volume enclosing the mixing chamber

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{\,0} + \dot{m}_1 s_{co}(T_1, P_1) + \dot{m}_2 s_{Ar}(T_2, P_2) - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$$

Thus, with $\bar{s}°$ data from Table A-23

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_3} = s_3 - \left(\frac{\dot{m}_1}{\dot{m}_3}\right) s_{co}(T_1, P_1) - \left(\frac{\dot{m}_2}{\dot{m}_3}\right) s_{Ar}(T_2, P_2)$$

$$= [(mf)_{co} s_{co}(T_3, y_{co} P_3) + (mf)_{Ar} s_{Ar}(T_3, y_{Ar} P_3)] - \left(\frac{\dot{m}_1}{\dot{m}_3}\right) s_{co}(T_1, P_1) - \left(\frac{\dot{m}_2}{\dot{m}_3}\right) s_{Ar}(T_2, P_2)$$

$$= \frac{2}{3}[s_{co}(T_3, y_{co} P_3) - s_{co}(T_1, P_1)] + \frac{1}{3}[s_{Ar}(T_3, y_{Ar} P_3) - s_{Ar}(T_2, P_2)]$$

PROBLEM 12.39 (Contd.) - Page 2

That is

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_3} = \frac{2}{3}\left[\frac{\bar{s}^o_{co}(T_3) - \bar{s}^o_{co}(T_1) - \bar{R}\ln\frac{y_{co}P_3}{P_1}}{M_{co}}\right] + \frac{1}{3}\left[\frac{\bar{c}_{p,Ar}\ln\frac{T_3}{T_2} - \bar{R}\ln\frac{y_{Ar}P_3}{P_2}}{M_{Ar}}\right]$$

$$= \frac{2}{3}\left[\frac{48.044 - 47.310 - 1.986\ln\left(\frac{0.741 \times 16}{18}\right)}{28.01}\right] + \frac{1}{3}\left[\frac{(1.986)\left[2.5\ln\frac{600}{840} - \ln\left(\frac{0.259 \times 16}{18}\right)\right]}{39.94}\right]$$

$$= 0.0372 + 0.0104 = 0.0476 \ \frac{\text{Btu}/{}^oR}{\text{lb(mix)}}$$

Thus, for the mixing chamber

$$(\dot{E}_d)_{mix} = T_0\frac{\dot{\sigma}_{cv}}{\dot{m}_3} = (500^oR)\left(0.0476\frac{\text{Btu}/{}^oR}{\text{lb(mix)}}\right) = 23.8\ \frac{\text{Btu}}{\text{lbmix}}$$

For a control volume enclosing the valve, an entropy rate balance at steady state reduces to

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_3} = s_4 - s_3$$

$$= \left[(mf)_{co}\, s_{co}(T_4, y_{co}P_4) + (mf)_{Ar}\, s_{Ar}(T_4, y_{Ar}P_4)\right]$$

$$- \left[(mf)_{co}\, s_{co}(T_3, y_{co}P_3) + (mf)_{Ar}\, s_{Ar}(T_3, y_{Ar}P_3)\right]$$

$$= (mf)_{co}\left[\frac{\bar{s}^o_{co}(T_4) - \bar{s}^o_{co}(T_3) - \bar{R}\ln\frac{y_{co}P_4}{y_{co}P_3}}{M_{co}}\right] + (mf)_{Ar}\left[\frac{\bar{c}_{p,Ar}\ln\frac{T_4}{T_3} - \bar{R}\ln\frac{y_{Ar}P_4}{y_{Ar}P_3}}{M_{Ar}}\right]$$

Since $T_3 = T_4$, the terms involving temperature vanish, leaving

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_3} = -\bar{R}\left[\frac{(mf)_{co}}{M_{co}} + \frac{(mf)_{Ar}}{M_{Ar}}\right]\ln\frac{P_4}{P_3}$$

$$= -1.986\left[0.0238 + 0.0083\right]\ln\frac{14.7}{16} = 0.0054 \ \frac{\text{Btu}/{}^oR}{\text{lb(mix)}}$$

Then

1 $$(\dot{E}_d)_{valve} = (500)(0.0054) = 2.7\ \frac{\text{Btu}}{\text{lbmix}}$$

---

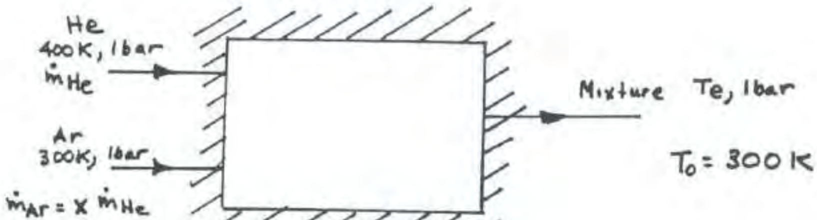
1. Significant exergy destruction accompanies the mixing of different substances initially at different states and the expansion of substances across valves.

PROBLEM 12.40

KNOWN: Helium at 400 K, 1 bar enters an insulated mixing chamber where it mixes with argon entering at 300 K, 1 bar. The mixture exits at a pressure of 1 bar. The argon mass flow rate is x times that of helium.

FIND: Plot versus x the exit temperature and the rate of exergy destruction per unit mass of He entering.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The mixing chamber is well insulated and at steady state. (2) Kinetic and potential energy effects can be ignored. (3) Each gas can be modeled as an ideal gas and the mixture adheres to the Dalton model.

ANALYSIS: At steady state the mass flow rate of the mixture exiting equals the sum of the incoming flow rates:

$$\dot{m}_{mix} = \dot{m}_{He} + \dot{m}_{Ar} = (1+x)\,\dot{m}_{He} \qquad (1)$$

The energy rate balance reduces at steady state to give

$$0 = \cancel{\dot{Q}_{cv}}^{\,0} - \cancel{\dot{W}_{cv}}^{\,0} + \dot{m}_{He}\,h_{He}(400\text{K}) + \dot{m}_{Ar}\,h_{Ar}(300\text{K}) - \left[\dot{m}_{He}\,h_{He}(T_e) + \dot{m}_{Ar}\,h_{Ar}(T_e)\right]$$

Thus

$$\dot{m}_{He}\left[h_{He}(T_e) - h_{He}(400\text{K})\right] + \dot{m}_{Ar}\left[h_{Ar}(T_e) - h_{Ar}(300\text{K})\right] = 0$$

For the monatomic gases He and Ar, $c_p = 5/2\,\bar{R}/M$ (Table A-21). Accordingly,

$$\dot{m}_{He}\left[\tfrac{5}{2}\tfrac{\bar{R}}{M_{He}}(T_e - 400)\right] + x\,\dot{m}_{He}\left[\tfrac{5}{2}\tfrac{\bar{R}}{M_{Ar}}(T_e - 300)\right] = 0$$

or

$$\frac{T_e - 400}{M_{He}} + \frac{x}{M_{Ar}}\left[T_e - 300\right] = 0$$

giving

$$T_e = \frac{\dfrac{400}{M_{He}} + \dfrac{300x}{M_{Ar}}}{\dfrac{1}{M_{He}} + \dfrac{x}{M_{Ar}}} = \frac{400\left(\dfrac{M_{Ar}}{M_{He}}\right) + 300x}{\left(\dfrac{M_{Ar}}{M_{He}}\right) + x} \qquad (2)$$

where $M_{Ar} = 39.94$, $M_{He} = 4.003$;

The entropy rate balance reduces at steady state to give

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{\,0} + \dot{m}_{He}\,s_{He}(400\text{K}, 1\text{bar}) + \dot{m}_{Ar}\,s_{Ar}(300\text{K}, 1\text{bar})$$
$$- \left[\dot{m}_{He}\,s_{He}(T_e, y_{He}\cdot 1\text{bar}) + \dot{m}_{Ar}\,s_{Ar}(T_e, y_{Ar}\cdot 1\text{bar})\right] + \dot{\sigma}_{cv}$$

PROBLEM 12.40 (Cont'd.) – Page 2

Since $y_{Ar} + y_{He} = 1$, $y_{He} = 1 - y_{Ar}$, and $\dot{E}d/\dot{m}_{He} = T_o \dot{\sigma}_{cv}/\dot{m}_{He}$. Thus

$$\frac{\dot{E}d}{\dot{m}_{He}} = T_o \left\{ \frac{\bar{R}}{M_{He}} \left[\frac{5}{2} \ln \frac{T_e}{400} - \ln(1 - y_{Ar}) \right] + x \, \frac{\bar{R}}{M_{Ar}} \left[\frac{5}{2} \ln \frac{T_e}{300} - \ln y_{Ar} \right] \right\} \tag{3}$$

To find y_{Ar}, notice that the mass fractions of He and Ar in the exiting mixture are, respectively

$$(mf)_{He} = \frac{\dot{m}_{He}}{\dot{m}_{mix}} = \frac{\dot{m}_{He}}{(1+x)\dot{m}_{He}} = \frac{1}{1+x}$$

$$(mf)_{Ar} = \frac{\dot{m}_{Ar}}{\dot{m}_{mix}} = \frac{x\,\dot{m}_{He}}{(1+x)\dot{m}_{He}} = \frac{x}{1+x}$$

The number of moles of He and Ar in a typical unit mass of mixture is

$$n_{He} = \frac{1/(1+x)}{M_{He}} \, , \quad n_{Ar} = \frac{x/(1+x)}{M_{Ar}}$$

Thus

$$y_{Ar} = \frac{n_{Ar}}{n_{He} + n_{Ar}} = \frac{\left(\frac{x}{(1+x)M_{Ar}}\right)}{\left[\frac{1}{(1+x)M_{He}} + \frac{x}{(1+x)M_{Ar}}\right]} = \frac{x}{\left[\frac{M_{Ar}}{M_{He}} + x\right]} \tag{4}$$

The data for the required plots are obtained using *IT*, as follows:

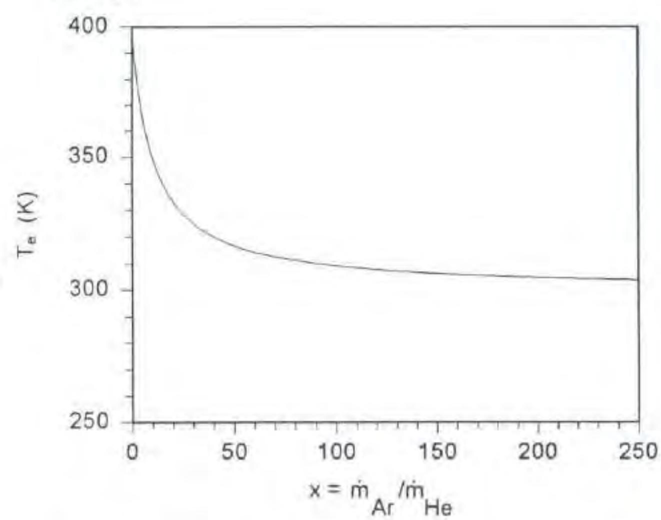
IT Code

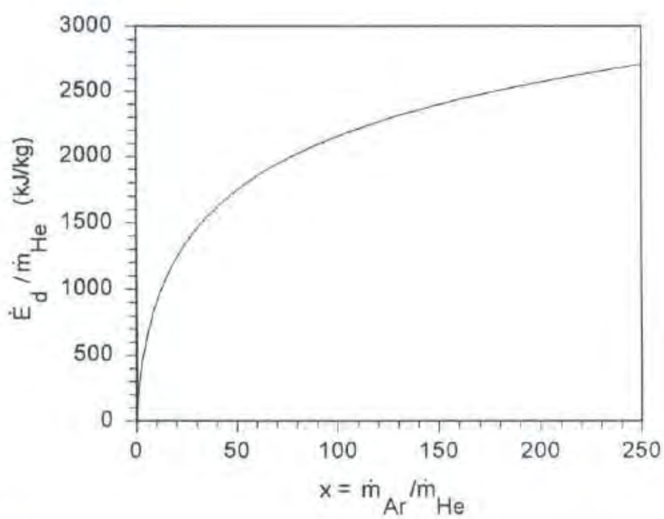
```
mdotHe = 1
MHe = 4.003  // kg/kmol
MAr = 39.94  // kg/kmol
To = 300  // K
Rbar = 8.314  // kJ/kmol·K
x = 1

mdotHe + mdotAr = (1 +x) * mdotHe
Te = (400 * (MAr / MHe) + 300 * x) / ((MAr / MHe) + x)

Edotd / mdotHe = To * (A + B)
A = (Rbar / MHe) * (2.5 * ln(Te / 400) -  ln(1 - yAr))
B = x * (Rbar / MAr) * (2.5 * ln(Te / 300) - ln(yAr))
yAr = x / ((MAr / MHe) + x)
```

IT Results for x = 1

$y_{Ar} = 0.0911$
$T_e = 390.9$ K
$\dot{E}_d / \dot{m}_{He} = 214.6$ kJ/kg

PLOTS:





As the fraction of argon increases, the exit temperature approaches 300K, which is the temperature of the entering argon stream, as expected. Also, as the argon fraction increases, the exergy destruction increases due to the temperature difference between the two streams.

12.41 Hydrogen (H_2) at 77°C, 4 bar enters an insulated chamber at steady state where it mixes with nitrogen (N_2) entering as a separate stream at 277°C, 4 bar. The mixture exits at 3.8 bar with the molar analysis 75% H_2, 25% N_2. Kinetic and potential energy effects can be ignored. Determine

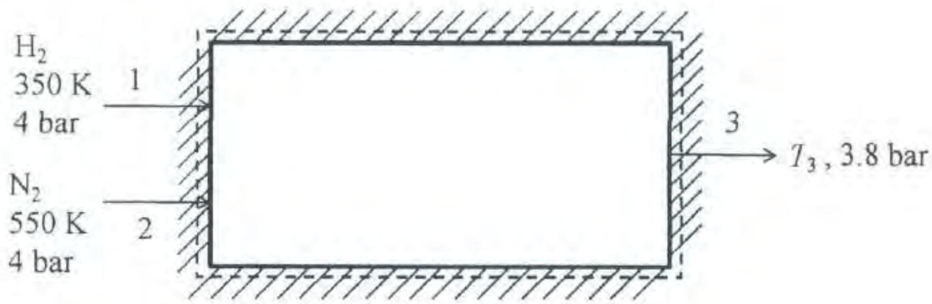
(a) the temperature of the exiting mixture, in °C.

(b) the rate at which entropy is produced, in kJ/K per kmol of mixture exiting.

KNOWN: A stream of H_2 at a specified temperature and pressure enters an insulated chamber at steady state and mixes with a separate stream of N_2 entering at a specified temperature and pressure. A mixture exits at a known pressure and molar analysis.

FIND: Determine (a) the temperature of the exiting mixture and (b) the rate of entropy production per kmol of mixture exiting.

SCHEMATIC AND GIVEN DATA:



i	y_i
H_2	0.75
N_2	0.25

ENGINEERING MODEL:

(1) The control volume is at steady state.

(2) For the control volume, $\dot{W}_{cv} = 0$ and $\dot{Q}_{cv} = 0$.

(3) Kinetic and potential energy effects can be ignored.

(4) Each gas can be modeled as an ideal gas and the exiting mixture adheres to the Dalton model.

ANALYSIS:

(a) An energy rate balance at steady state, expressed on a molar basis, reduces with assumptions 1, 2, and 3 to give

$$0 = \underbrace{\dot{Q}_{cv}}_{=0} - \underbrace{\dot{W}_{cv}}_{=0} + \dot{n}_1\bar{h}_1 + \dot{n}_2\bar{h}_2 - \dot{n}_3\bar{h}_3 \Rightarrow \bar{h}_3 = \left(\frac{\dot{n}_1}{\dot{n}_3}\right)\bar{h}_1 + \left(\frac{\dot{n}_2}{\dot{n}_3}\right)\bar{h}_2 \qquad (1)$$

where $\dot{n}_1$, $\dot{n}_2$, $\dot{n}_3$ and are the molar flow rates of H_2, N_2 and mixture, respectively. Since $\dot{n}_1/\dot{n}_3 = 0.75$ and $\dot{n}_2/\dot{n}_3 = 0.25$, Eq. (1) becomes

Problem 12.41 (Continuing) – Page 2

$$\bar{h}_3 = 0.75\bar{h}_1 + 0.25\bar{h}_2 = 0.75\bar{h}_{H_2}(T_1) + 0.25\bar{h}_{N_2}(T_2) \qquad (2)$$

The enthalpy of the mixture per kmol of mixture, is

$$\bar{h}_3 = 0.75\bar{h}_{H_2}(T_3) + 0.25\bar{h}_{N_2}(T_3) \qquad (3)$$

Substitute Eq. (3) into Eq. (2) and use data from Table A-23

$$0.75\bar{h}_{H_2}(T_3) + 0.25\bar{h}_{N_2}(T_3) = 0.75\bar{h}_{H_2}(T_1) + 0.25\bar{h}_{N_2}(T_2)$$
$$= (0.75)\left(9971\,\frac{\text{kJ}}{\text{kmol}}\right) + (0.25)\left(16064\,\frac{\text{kJ}}{\text{kmol}}\right) = 11494.25\,\frac{\text{kJ}}{\text{kmol}}$$

Solving this iteratively with data from Table A-23; $T_3 \approx 400.5$ K ←

(b) An entropy rate balance at steady state, expressed on a molar basis, reads

$$0 = \underbrace{\sum \frac{\dot{Q}_j}{T_j}}_{=0} + \dot{n}_1\bar{s}_1 + \dot{n}_2\bar{s}_2 - \dot{n}_3\bar{s}_3 + \dot{\sigma}_{cv}$$

Thus

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_3} = \bar{s}_3 - \left[\left(\frac{\dot{n}_1}{\dot{n}_3}\right)\bar{s}_1 + \left(\frac{\dot{n}_2}{\dot{n}_3}\right)\bar{s}_2\right] \qquad (4)$$

From the given data, $\dot{n}_1/\dot{n}_3 = 0.75$ and $\dot{n}_2/\dot{n}_2 = 0.25$. The specific entropy of the exiting mixture on a molar basis is

$$\bar{s}_3 = (0.75)\bar{s}_{H_2}(T_3, y_{H_2}p_3) + (0.25)\bar{s}_{N_2}(T_3, y_{N_2}p_3) \qquad (5)$$

where $y_{H_2} = 0.75$ and $y_{N_2} = 0.25$. Substituting Eq. (5) into Eq. (4) and rearranging

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_3} = 0.75\left[\bar{s}_{H_2}(T_3, y_{H_2}p_3) - \bar{s}_{H_2}(T_1, p_1)\right] + 0.25\left[\bar{s}_{N_2}(T_3, y_{N_2}p_3) - \bar{s}_{N_2}(T_2, p_2)\right]$$

Using the ideal gas model and data from Table A-23

Problem 12.41 (Continued) – Page 3

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_3} = 0.75\left[\bar{s}^{\circ}_{H_2}(T_3) - \bar{s}_{H_2}(T_1) - \bar{R}\ln\frac{y_{H_2}p_3}{p_1}\right] + 0.25\left[\bar{s}^{\circ}_{N_2}(T_3) - \bar{s}_{N_2}(T_2) - \bar{R}\ln\frac{y_{N_2}p_3}{p_1}\right]$$

$$= 0.75\left[(139.1416 - 135.2085)\frac{\text{kJ}}{\text{kmol}\cdot\text{K}} - \left(8.314\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)\ln\frac{(0.75)(3.8\text{ bar})}{4\text{ bar}}\right]$$

$$+ 0.25\left[(200.1072 - 209.461)\frac{\text{kJ}}{\text{kmol}\cdot\text{K}} - \left(8.314\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)\ln\frac{(0.25)(3.8\text{ bar})}{4\text{ bar}}\right]$$

$$= 5.7131\frac{\text{kJ}}{\text{kmol}} \longleftarrow$$

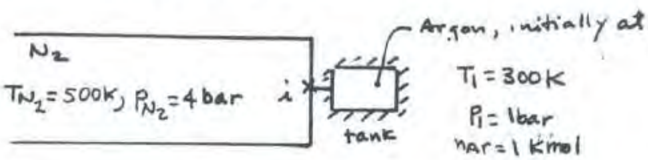
The rate of entropy production per kmol of exiting mixture is positive for the adiabatic mixing process, as expected.

PROBLEM 12.42

KNOWN: An insulated, rigid tank initially containing Ar at a known temperature and pressure is connected to a large vessel containing N_2 at a known temperature and pressure. Nitrogen is allowed to flow into the tank, forming an Ar-N_2 mixture at T, p.

FIND: Plot T and p versus the amount of N_2 in the tank.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. For a control volume enclosing the tank, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. Kinetic & potential energy can be ignored.
2. At the control volume inlet, N_2 enters at 500K, 4bar.
3. Each pure component behaves as an ideal gas. The mixture adheres to the Dalton model

ANALYSIS: Since the tank volume remains constant

$$V = \frac{n_{Ar}\bar{R}T_1}{p_1} = \frac{[n_{Ar} + \overset{\text{amount of } N_2 \text{ in tank finally}}{n_{N_2}}]\bar{R}T}{p} \Rightarrow p = p_1\left[1 + \frac{n_{N_2}}{n_{Ar}}\right]\left[\frac{T}{T_1}\right] \qquad (1)$$

where $p \le 4$bar.

Mass and energy rate balances reduce to give (see Sec. 4.4)

$$\Delta U_{cv} = \bar{h}_{N_2}(T_{N_2})\, n_{N_2}$$

or

$$\left\{\left[n_{N_2}\bar{u}_{N_2}(T) + n_{Ar}\bar{u}_{Ar}(T)\right] - \left[n_{Ar}\bar{u}_{Ar}(T_1)\right]\right\} = \bar{h}_{N_2}(T_{N_2})\, n_{N_2}$$

$$\Rightarrow \quad n_{N_2}\bar{u}_{N_2}(T) + n_{Ar}\underbrace{\left[\bar{u}_{Ar}(T) - \bar{u}_{Ar}(T_1)\right]}_{\substack{\text{From Table A-21}\\ \bar{c}_v = 1.5\bar{R}}} = \bar{h}_{N_2}(T_{N_2})\, n_{N_2}$$

$$\Rightarrow \quad n_{N_2}\bar{u}_{N_2}(T) + n_{Ar}\left[1.5\bar{R}(T - T_1)\right] = \bar{h}_{N_2}(T_{N_2})\, n_{N_2} \qquad (2)$$

Sample Calculation: when $n_{N_2} = 0.2$ kmol, Eq.(2) reads with data from Table A-23

$$(0.2\text{kmol})\,\bar{u}_{N_2}(T) + (1\text{kmol})(1.5 \times 8.314 \tfrac{kJ}{kmol\cdot K})(T - 300)K = (0.2\text{kmol})(14581 \tfrac{kJ}{kmol})$$

$$(0.2)\bar{u}_{N_2}(T) + 12.471\,T = 6657.5 \text{ kJ}$$

Solving iteratively with Table A-23 data, $T = 400K$. Then, Eq.(1) gives

$$p = (1\text{ bar})\left[1 + \frac{0.2}{1.0}\right]\left[\frac{400}{300}\right] = 1.6 \text{ bar}.$$

The data for the required plots are obtained using the following simple IT code:

IT Code

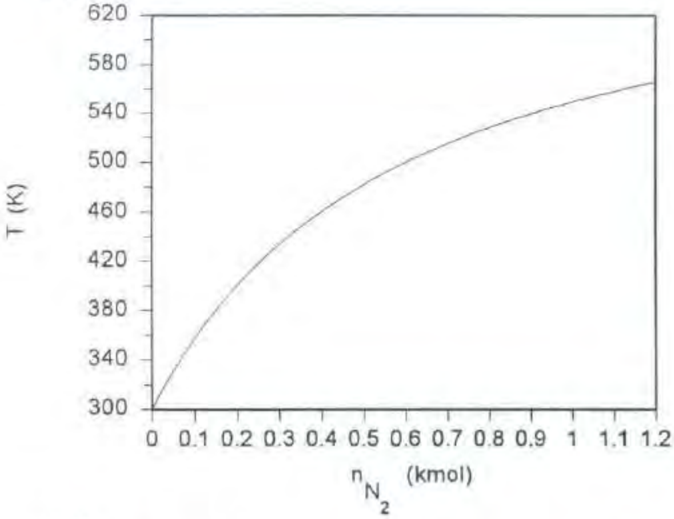
```
p1 = 1  // bar
T1 = 300  // K
TN2 = 500  // K
n_N2 = 0.2  // kmol
Rbar = 8.314  // kJ/kmol·K

p = p1 * (1 + n_N2) * ( T / T1)
n_Ar = 1  // kmol
n_N2 * u_T("N2",T) + n_Ar * 1.5 * Rbar *(T - T1) = n_N2 * h_T("N2",TN2)
```

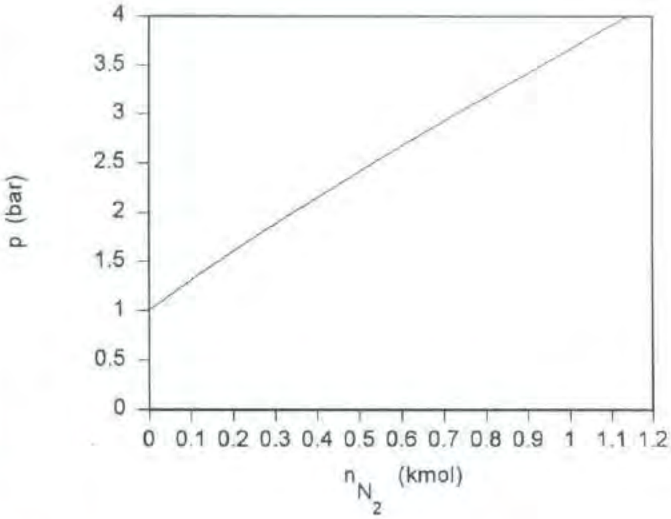
IT Results for n_{N2} = 0.2 kmol

T = 400.4 K
p = 1.602 bar

PROBLEM 12.42 (Cont'd.) - Page 2

PLOTS:





Note ....

- As more N_2 is introduced, the pressure and temperature both increase.
- The tank temperature increases to values greater than the temperature of the nitrogen entering, due to flow work.
- When n_{N_2} reaches 1.13 kmol, the pressure in the tank reaches 4 bar and the process ceases.

12.43 A stream of oxygen (O_2) at 100°F, 2 atm enters an insulated chamber at steady state with a mass flow rate of 1 lb/min and mixes with a stream of air entering separately at 200°F, 1.5 atm with a mass flow rate of 2 lb/min. The mixture exits at a pressure of 1 atm. Kinetic and potential energy effects can be ignored. On the basis of constant specific heats, determine

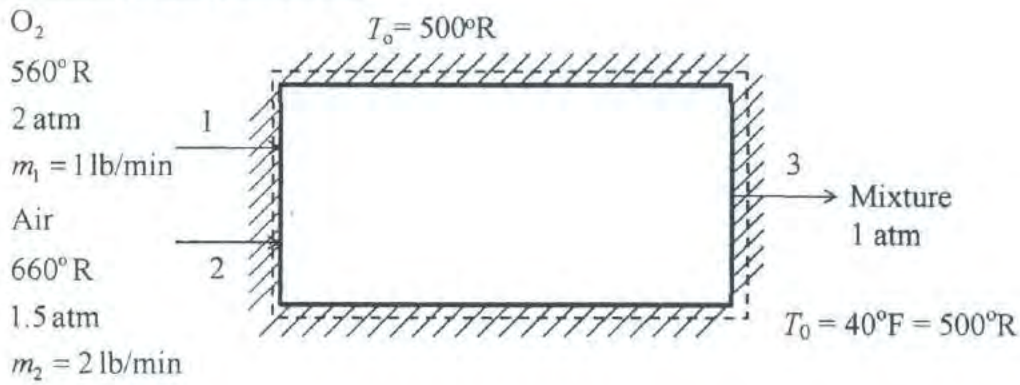
(a) the temperature of the exiting mixture, in °F.

(b) the rate of exergy destruction, in Btu/min, for T_o = 40°F.

KNOWN: A stream of O_2 at known temperature, pressure, and mass flow rate enters an insulated mixing chamber at a steady state and mixes with a separate stream of air entering at a known temperature, pressure, and mass flow rate. The mixture exits at a pressure of 1 atm.

FIND: Determine (a) the temperature of the exiting mixture and (b) the rate of exergy destruction.

Schematic and Known Data:



ENGINEERING MODEL:

(1) The control volume is at steady state.

(2) For the control volume, $\dot{W}_{cv} = 0$ and $\dot{Q}_{cv} = 0$.

(3) Kinetic and potential energy effects can be ignored.

(4) Each gas can be modeled as an ideal gas and the exiting mixture adheres to the Dalton model.

(5) For each gas, the specific heat is constant at its value at 610°R.

Analysis:

(a) At steady state, the rate O_2 enters the chamber equals the rate O_2 exits the chamber. Similarly, the rate air enters equals the rate air exits. Accordingly, for the exiting mixture

Problem 12.43 (Continued) – Page 2

$(mf)_{O_2} = 1/3$, $(mf)_{air} = 2/3$

An energy rate balance reduces at steady state to give

$$0 = \underbrace{\dot{Q}_{cv}}_{=0} - \underbrace{\dot{W}_{cv}}_{=0} + \dot{m}_1 h_{O_2}(T_1) + \dot{m}_2 h_{air}(T_2) - \dot{m}_3 h_{mix}(T_3)$$

$$\Rightarrow 0 = h_{mix}(T_3) - \left(\frac{\dot{m}_1}{\dot{m}_3} h_{O_2}(T_1) + \frac{\dot{m}_2}{\dot{m}_3} h_{air}(T_2)\right)$$

where $h_{mix} = (mf)_{O_2} h_{O_2} + (mf)_{air} h_{air}$. Accordingly, with assumption (5)

$$0 = (1/3)c_{p,O_2}[T_3 - T_1] + (2/3)c_{p,air}[T_3 - T_2]$$

Or, upon rearrangement and with c_p values at 610°R from Table A-20E

$$T_3 = \frac{(1/3)c_{p,O_2} T_1 + (2/3)c_{p,air} T_2}{(1/3)c_{p,O_2} + (2/3)c_{p,air}}$$

$$= \frac{(1)\left(0.2215 \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)(560^\circ\text{R}) + (2)\left(0.2405 \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)(660^\circ\text{R})}{(1)\left(0.2215 \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right) + (2)\left(0.2405 \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)}$$

$$= 628.5\ {}^\circ\text{R}\ (169^\circ\text{F})$$ ←

(b) The exergy destruction rate can be evaluated via $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production from an entropy rate balance at steady state:

$$0 = \underbrace{\sum \frac{\dot{Q}_{cj}}{T_j}}_{=0} + \dot{m}_1 s_{O_2}(T_1, p_1) + \dot{m}_2 s_{air}(T_2, p_2) - \dot{m}_3 s_{mix} + \dot{\sigma}_{cv}$$

with $s_{mix} = (mf)_{O_2} s_{O_2}(T_3, y_{O_2} p_3) + (mf)_{air} s_{air}(T_3, y_{air} p_3)$, this gives

$$\dot{\sigma}_{cv} = \dot{m}_3 \left[(mf)_{O_2} s_{O_2}(T_3, y_{O_2} p_3) + (mf)_{air} s_{air}(T_3, y_{air} p_3)\right] - \dot{m}_1 s_{O_2}(T_1, p_1) - \dot{m}_{air} s_{air}(T_1, p_2)$$

Using appropriate thermodynamic relations for the specific entropy changes and rearranging

$$\dot{\sigma}_{cv} = \dot{m}_1 \left[c_{p,O_2} \ln\frac{T_3}{T_1} - \frac{\overline{R}}{M_{O_2}} \ln\frac{y_{O_2} p_3}{p_1}\right] + \dot{m}_2 \left[c_{p,air} \ln\frac{T_3}{T_2} - \frac{\overline{R}}{M_{air}} \ln\frac{y_{air} p_3}{p_2}\right] \qquad (1)$$

Problem 12.43 (Continued) – Page 3

To find y_{O_2} and y_{air}, first obtain the respective molar flow rates:

$$\dot{n}_{O_2} = \frac{\dot{m}_1}{M_{O_2}} = \frac{(1\ \text{lb/min})}{(32.00\ \text{lb/lbmol})} = 0.03125\ \frac{\text{lbmol}}{\text{min}}$$

$$\dot{n}_{air} = \frac{\dot{m}_2}{M_{air}} = \frac{(2\ \text{lb/min})}{(28.97\ \text{lb/lbmol})} = 0.06904\ \frac{\text{lbmol}}{\text{min}}$$

$$y_{O_2} = \frac{\dot{n}_{O_2}}{\dot{n}_3} = \frac{0.03125}{0.10029} = 0.3116$$

$$y_{air} = \frac{\dot{n}_{air}}{\dot{n}_3} = \frac{0.06904}{0.10029} = 0.6884$$

Returning to Eq. (1) THIS IS NOT CORRECT. THE UNITS SHOULD BE BTU/MIN. PUT UNITS ON THE MASS FLOW RATES AND CARRY THEM THROUGH.

$$\begin{aligned}\dot{\sigma}_{cv} &= \left(1\frac{\text{Btu}}{\text{min}}\right)\left[\left(0.2215\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln\frac{628.5}{560} - \frac{1.986}{32.00}\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\ln\frac{(0.3116)(1)}{(2)}\right] \\ &\quad + \left(2\frac{\text{Btu}}{\text{min}}\right)\left[\left(0.2405\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln\frac{628.5}{660} - \frac{1.986}{28.97}\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\ln\frac{(0.6884)(1)}{(1.5)}\right] \\ &= 0.14095 + 0.08326 = 0.22421\frac{\text{Btu}/{}^\circ\text{R}}{\text{min}}\end{aligned}$$

Finally

$$\begin{aligned}\dot{\text{E}}_{cv} &= T_o\dot{\sigma}_{cv} \\ &= (500^\circ\,\text{R})\left(0.22421\frac{\text{Btu}/{}^\circ\text{R}}{\text{min}}\right) = 112.1\ \text{Btu/min}\end{aligned}$$

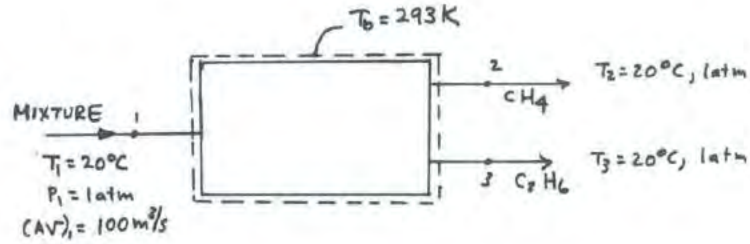
PROBLEM 12.44

KNOWN: Data are provided for a device operating at steady state that separates a natural gas into components.

FIND: Plot the minimum theoretical power input versus y, the mole fraction of C_2H_6

SCHEMATIC & GIVEN DATA:

Mixture	
component	mole fraction
C_2H_6	y
CH_4	$1-y$
$0.05 \le y \le 0.15$	



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates isothermally and at steady state. (2) Kinetic and potential energy effects can be ignored. (3) Ideal gas principles apply for the pure components. The mixture adheres to the Dalton model.

ANALYSIS: The mass flow rate at 1 is obtained using the given volumetric flow rate and the ideal gas equation of state:

$$\dot{m}_1 = \frac{(AV)_1}{v_1} = \frac{P_1 (AV)_1}{(\bar{R}/M)T_1}$$

where $M = y_{CH_4} M_{CH_4} + y_{C_2H_6} M_{C_2H_6} = (0.94)(16.04) + (0.06)(30.07) = 16.88$. Thus

$$\dot{m}_1 = \frac{(1.01325 \times 10^5\,N/m^2)(100\,m^3/s)}{\left(\frac{8314}{16.88}\,\frac{N\cdot m}{kg\cdot K}\right)(293\,K)} = 70.2\,kg/s$$

The molar flow rate is

$$\dot{n}_1 = \frac{70.2}{16.88} = 4.1588\,\frac{kmol(mix)}{s}$$

Then

$$\begin{cases} \dot{n}_{C_2H_6} = y(4.1588) \\ \dot{n}_{CH_4} = (1-y)(4.1588) \end{cases} \qquad (1)$$

With assumptions 1 and 2, an energy rate balance reduces to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \underline{\left[\dot{n}_{CH_4}\bar{h}_{CH_4}(T_1) + \dot{n}_{C_2H_6}\bar{h}_{C_2H_6}(T_1)\right] - \dot{n}_{CH_4}\bar{h}_{CH_4}(T_2) - \dot{n}_{C_2H_6}\bar{h}_{C_2H_6}(T_3)}$$

Since $T_1 = T_2 = T_3$, the underlined term vanishes, leaving $\dot{W}_{cv} = \dot{Q}_{cv}$.

An entropy rate balance reduces at steady state to

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \left[\dot{n}_{CH_4}\bar{s}_{CH_4}(T_1, y_{CH_4}P_1) + \dot{n}_{C_2H_6}\bar{s}_{C_2H_6}(T_1, y_{C_2H_6}P_1)\right] - \dot{n}_{CH_4}\bar{s}_{CH_4}(T_2, P_2) - \dot{n}_{C_2H_6}\bar{s}_{C_2H_6} + \dot{\sigma}_{cv}$$

Or, upon rearrangement, and inserting $\dot{W}_{cv} = \dot{Q}_{cv}$

$$-\dot{W}_{cv} = T_b\left\{\dot{n}_{CH_4}\left(\bar{s}_{CH_4}(T_1, y_{CH_4}P_1) - \bar{s}_{CH_4}(T_2, P_2)\right) + \dot{n}_{C_2H_6}\left(\bar{s}_{C_2H_6}(T_1, y_{C_2H_6}P_1) - \bar{s}_{C_2H_6}(T_3, P_3)\right) + \dot{\sigma}_{cv}\right\}$$

In this equation $(-\dot{W}_{cv})$ is the work input. Since $\dot{\sigma}_{cv} \ge 0$, it follows that $(-\dot{W}_{cv})$ is a minimum when $\dot{\sigma}_{cv} = 0$. Accordingly,

PROBLEM 12.44 (Continued) - Page 2

$$(-\dot{W}_{cv})_{MIN} = T_b \left\{ \dot{n}_{CH_4} \left(\underbrace{\bar{s}^\circ_{CH_4}(T_1) - \bar{s}^\circ_{CH_4}(T_2)}_{=0 \text{ since } T_1 = T_2} - \bar{R} \ln \frac{y_{CH_4} \cancel{p_1}}{\cancel{p_2}} \right) + \dot{n}_{C_2H_6} \left(\underbrace{\bar{s}^\circ_{C_2H_6}(T_1) - \bar{s}^\circ_{C_2H_6}(T_2)}_{=0 \text{ since } T_1 = T_2} - \bar{R} \ln \frac{y_{C_2H_6} \cancel{p_1}}{\cancel{p_2}} \right) \right\}$$

$$= -\bar{R} T_b \left\{ \dot{n}_{CH_4} \ln y_{CH_4} + \dot{n}_{C_2H_6} \ln y_{C_2H_6} \right\} \qquad (2)$$

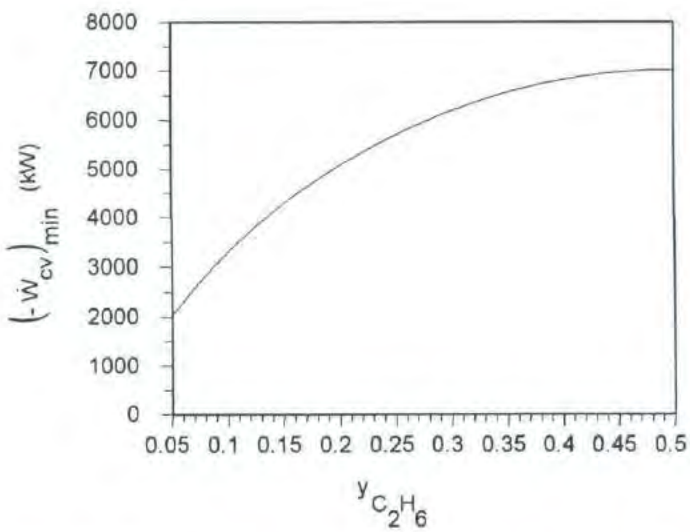
The governing equations are Equations (1) and (2). That is

$$(-\dot{W}_{cv})_{min} = \left(-8.314 \frac{kJ}{kmol \cdot K}\right)(293 K)\left(4.1588 \frac{kmol}{s}\right)\left\{(1-y)\ln(1-y) + y \ln y\right\} \left| \frac{kW}{kJ/s} \right|$$

① $$= -(10{,}131 \text{ kW}) \left\{(1-y)\ln(1-y) + y \ln y\right\} \qquad (3)$$

Eq (3) can be plotted using available plotting software. The result using IT is presented below.

PLOT:



We see from the plot that more power is required as the mole fraction of C_2H_6 increases.

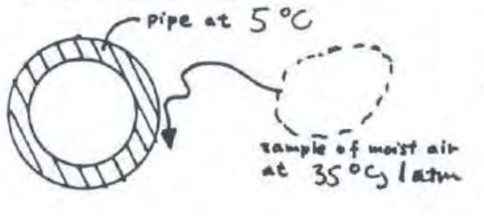
1. Owing to inevitable irreversibilities, the actual power required would exceed the minimum theoretical value obtained from Eq. (3).

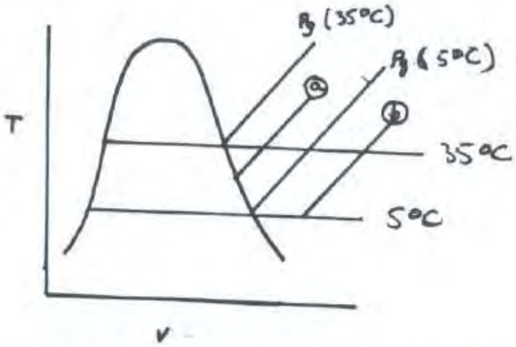
PROBLEM 12.45

KNOWN: A water pipe at 5°C runs between buildings through air at 35°C

FIND: Determine the maximum relative humidity the air can have before condensation occurs on the wall.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) The system consists of a sample of moist air initially at 35°C. (2) As the system comes close to the pipe at 5 °C, the system undergoes a cooling process at fixed total pressure from 35°C to 5°C.

ANALYSIS: As the sample of moist air is cooled at fixed total pressure, the partial pressure of the water vapor remains constant as long as no condensation occurs, for $P_v = y_v p$ and y_v remains constant.

Accordingly, if the initial pressure is less than P_g (5 °C), such as ⓑ shown on the T-v diagram, the sample would be cooled to 5 °C without condensation. However, if the initial partial pressure is greater than P_g (5 °C), such as ⓐ shown on the T-v diagram, the system would be cooled until a saturated mixture is attained. Subsequent cooling to 5 °C would involve condensation. It can be concluded, therefore, that the partial pressure must be less than, or equal to, P_g (5 °C). Thus

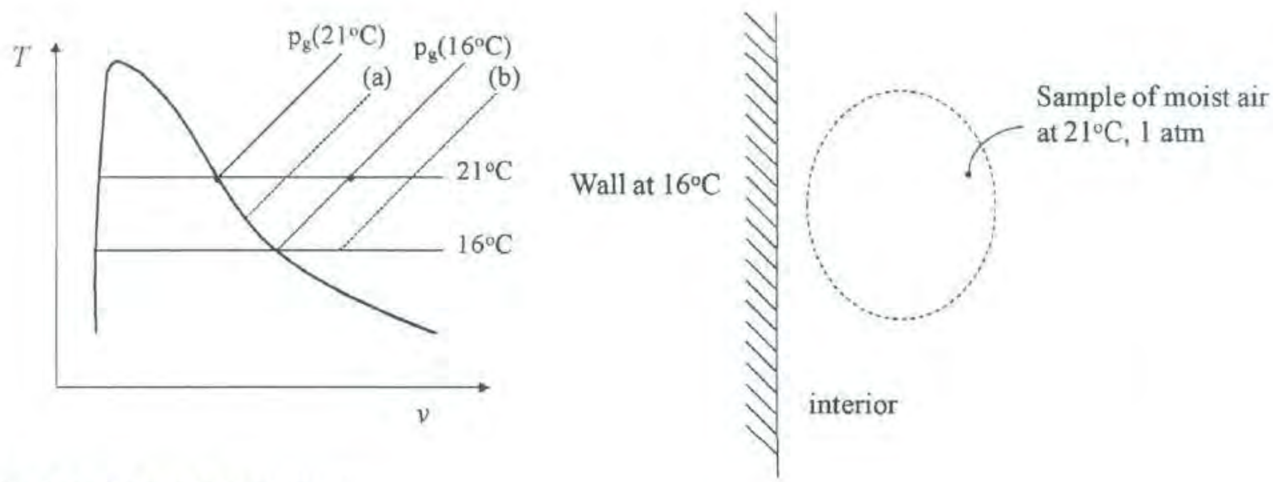
$$\phi = \frac{P_v}{P_g(35°C)} \leq \frac{P_g(5°C)}{P_g(35°C)} = \frac{0.00872 \text{ bar}}{0.05628 \text{ bar}} = 0.155 \ (15.5\%) \text{ ——— } \phi_{max}$$

12.46 The temperature of a wall in a dwelling is 16°C. If the air within the room is at 21°C, what is the maximum relative humidity the air can have before condensation occurs on the wall?

KNOWN: The temperature of a wall in a building is 16°C. The air within the room is at 21°C.

FIND: Determine the maximum relative humidity the air can have before condensation occurs on the wall.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

(1) The system consists of a sample of moist air initially at 21°C.
(2) As the system comes close to the wall at 16°C, the system undergoes a cooling process at fixed total pressure from 21°C to 16°C.

ANALYSIS:

As the sample of moist air is cooled at fixed total pressure, the partial pressure of the water vapor remains constant as long as no condensation occurs, for $p_v = y_v p$ and y_v remains constant.

Accordingly, if the initial partial pressure is less than $p_g(16°C)$, such as line (b) shown on the T-v diagram, the sample would be cooled to 16°C without condensation. However, if the initial partial pressure is greater than $p_g(16°C)$ such as line (a) shown on the T-v diagram, the system would be cooled until a saturated mixture is attained. Subsequent cooling to 16°C would involve condensation. It can be concluded, therefore, that the partial pressure must be less than, or equal to, $p_g(16°C)$. Thus, using data from Table A-2

Problem 12.46 (Continued) – Page 2

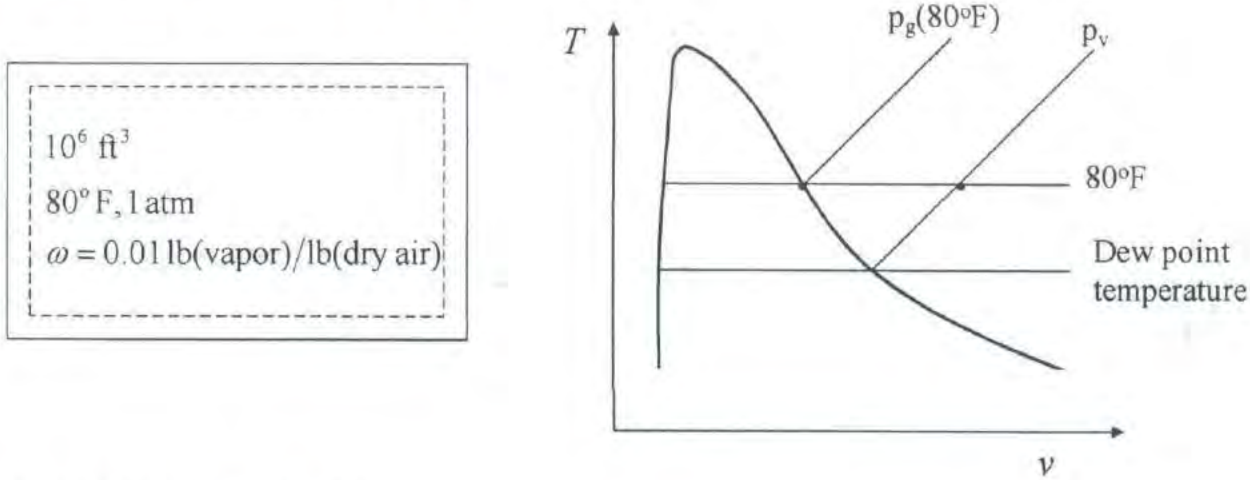
$$\phi = \frac{p_v}{p_g(21^\circ\text{C})} \le \frac{p_g(16^\circ\text{C})}{p_g(21^\circ\text{C})} = \frac{0.01818\text{ bar}}{0.02487\text{ bar}} = 0.731 \quad (73.1\%) \qquad \longleftarrow$$

12.47 A lecture hall having a volume of 10^6 ft^3 contains air at 80°F, 1 atm, and a humidity ratio of 0.01 lb of water vapor per lb of dry air. Determine
(a) the relative humidity.
(b) the dew point temperature, in °F.
(c) the mass of water vapor contained in the room, in lb

KNOWN: A hall of known volume contains moist air. The temperature, pressure, and humidity ratio are known.

FIND: Determine (a) the relative humidity, (b) the dew point temperature, and (c) the mass of water vapor present.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
(1) The contents of the given volume are taken as a closed system.
(2) The moist air present acts as an ideal gas mixture with each component adhering to the Dalton model.

ANALYSIS:
(a) The relative humidity is $\phi = p_v / p_g(80°\text{F})$. To find p_v

$$\omega = 0.622 \frac{p_v}{p - p_v} \Rightarrow p_v = \frac{(\omega)(p)}{\omega + 0.622} = \frac{(0.01)(14.7\ \text{lbf/in.}^2)}{0.01 + 0.622} = 0.2326 \frac{\text{lbf}}{\text{in.}^2}$$

The relative humidity is, then, with p_g from Table A-2E

$$\phi = \frac{0.2326\ \text{lbf/in.}^2}{0.5073\ \text{lbf/in.}^2} = 0.459\ (45.9\%)$$

(b) The dew point temperature is the saturation temperature corresponding to p_v. Thus, with Table A-2E, $T_{db} = 57.3°\text{F}$.

(c) Using the ideal gas equation of state

#1

$$m_v = \frac{p_v V}{\left(\overline{R}/M_v\right)T} = \frac{\left(0.2326\ \text{lbf/in.}^2\right)\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right|\left(10^6\ \text{ft}^3\right)}{\left(\frac{1545}{18.02}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)\left(540°\text{R}\right)} = 723.4\ \text{lb(vapor)}$$

Comment: Using $\omega = 0.01\ \text{lb(v)/lb(da)}$, the mass of dry air present is

$$m_a = \frac{m_v}{\omega} = \frac{723.4\ \text{lb(vapor)}}{0.01\ \text{lb(vapor)/lb(dry air)}} = 72340\ \text{lb(air)}$$

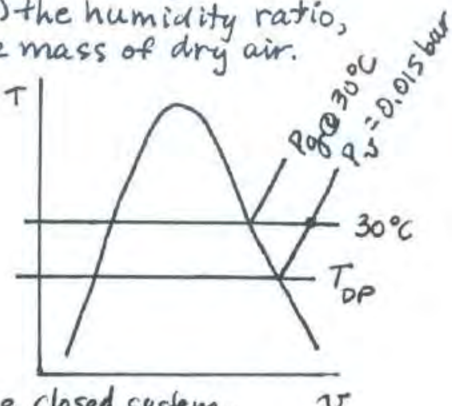
PROBLEM 12.48

KNOWN: Conditions are known for a large room containing moist air

FIND: Determine (a) the relative humidity, (b) the humidity ratio, (c) the dew point temperature, and (d) the mass of dry air.

SCHEMATIC & GIVEN DATA:

$T = 30°C$
$p = 102$ kPa $= 1.02$ bar
$p_v = 1.5$ kPa $= 0.015$ bar
$m_v = 10$ kg



ENGINEERING MODEL: (1) The air in the room is the closed system. (2) The moist air is treated as an ideal gas mixture.

ANALYSIS:

(a) The relative humidity is $\phi = p_v / p_{g@30°C}$. With data from Table A-2

$$\phi = \frac{0.015 \text{ bar}}{0.04246 \text{ bar}} = 0.353 \; (35.3\%) \longleftarrow \phi$$

(b) The specific humidity is

$$\omega = 0.622\left(\frac{p_v}{p - p_v}\right) = 0.622\left(\frac{1.5}{102 - 1.5}\right) = 0.009284 \frac{\text{kg(v)}}{\text{kg(a)}} \longleftarrow \omega$$

(c) The dew point temperature is the saturation temperature at $p_v = 0.015$ bar. Thus

$$T_{DP} \approx 13°C \longleftarrow T_{DP}$$

(d) To get the mass of dry air, use $\omega = m_v / m_a$.

① $$m_a = \frac{m_v}{\omega} = \frac{10 \text{ kg}}{0.009284} = 1077 \text{ kg(a)} \longleftarrow m_a$$

---

1. The volume of the room can be found as follows:

$$V = \frac{m_a R_a T}{p_a}$$

$$= \frac{(1077 \text{ kg})\left(\frac{8.314}{28.97} \frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right)(303 \text{ K})}{(102 - 1.5) \text{ kPa}} \left|\frac{1 \text{ kPa}}{10^3 \text{ N/m}^2}\right| \left|\frac{10^3 \text{ N} \cdot \text{m}}{1 \text{ kJ}}\right| = 932 \text{ m}^3$$

Approximate dimensions: Assume 3 m ceiling height

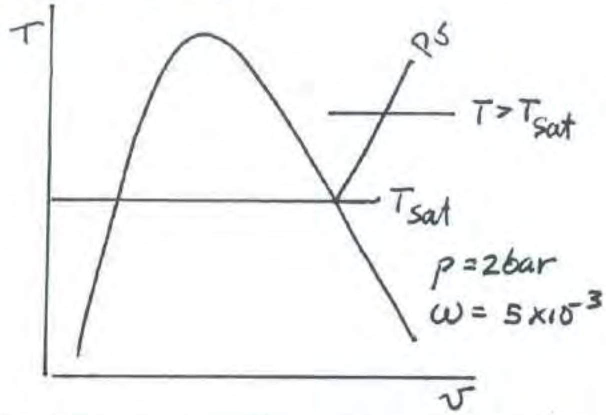
3 m × 17.5 m × 17.5 m

PROBLEM 12.49

KNOWN: moist air with known ω is cooled at constant pressure.

FIND: At what temperature is the moist air saturated?

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL

The moist air behaves as an ideal gas mixture.

ANALYSIS: Using $\omega = 0.622(p_v/p-p_v)$ and solving for p_v

$$p_v = \frac{\omega p}{(.622+\omega)}$$

$$= \frac{(5\times10^{-3})(2 \text{ bar})}{(0.622+5\times10^{-3})} = 0.01595 \text{ bar}$$

From Table A-2 at $p_v = 0.01598$ bar; $T_{sat} \approx 14\,°C$ ← T_{sat}

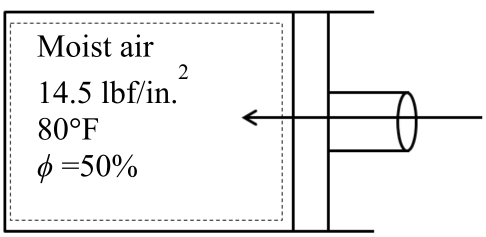
Problem 12.50

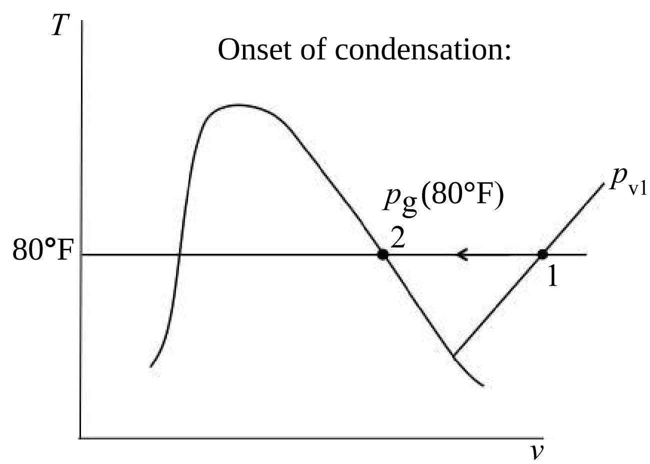
A fixed amount of air initially at 14.5 lbf/in.^{2}, 80°F, and a relative humidity of 50% is compressed isothermally until condensation of water begins. Determine the pressure of the mixture at the onset of condensation, in lbf/in.^{2}

<u>Known:</u>
A fixed amount of air initially at 14.5 lbf/in.^{2}, 80°F, and ϕ = 50% is compressed isothermally until the onset of condensation.

<u>Find:</u>
Determine the mixture pressure when condensation begins.

<u>Schematic & Given Data:</u>





<u>Engineering Model:</u>
(1) The system consists of a fixed amount of moist air, as illustrated in the accompanying figure.
(2) The moist air acts as an ideal gas mixture with each component adhering to the Dalton model.

<u>Analysis:</u>
As long as there is no condensation, the mole fraction of the water vapor, y_v remains constant. Thus, the partial pressure of the water vapor at each state visited during the isothermal compression is $p_v=y_v p$, where p is the corresponding mixture pressure. For example, $p_{v1}=y_v p_1$, where p_1=14.5 lbf/in.^{2}. The onset of condensation occurs when the mixture becomes saturated: state 2. Accordingly, p_g(80°F)=$y_v p_2$, where p_2 is the mixture pressure. Forming the ratio of partial pressures

$$\frac{p_{v_1}}{p_g(80°F)} = \frac{p_{v_1}}{\phi_1} = \frac{y_v p_1}{y_v p_2} = \frac{p_1}{p_2}$$

Accordingly

$$p_2 = \frac{p_1}{\phi_1} = \frac{14.5\,\frac{\text{lbf}}{\text{in.}^2}}{0.5} = 29\,\frac{\text{lbf}}{\text{in.}^2}$$

Corrected October, 2011

Problem 12.51

As seen in Fig. P12.51, moist air at 30°C, 2 bar, and 50% relative humidity enters a heat exchanger operating at steady state with a mass flow rate of 600 kg/h and is cooled at essentially constant pressure to 20°C. Ignoring kinetic and potential energy effects, determine the rate of heat transfer from the moist air stream, in kJ/h.

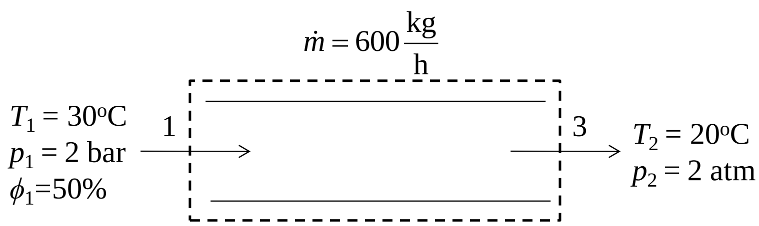


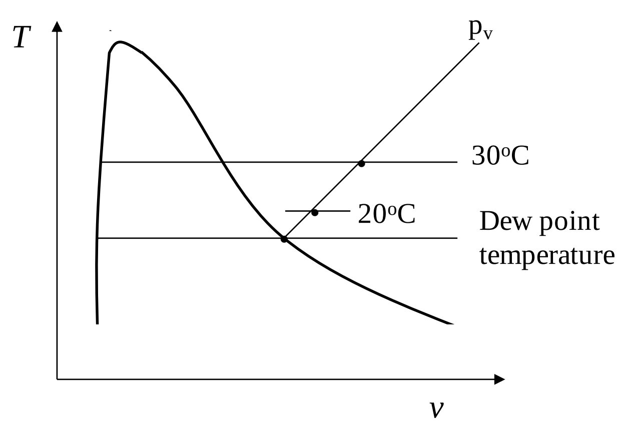
Fig. P12.51

Solution:

Known:
Moist air at known conditions enters a heat exchanger operating at steady state and is cooled.

Find:
Determine the rate of heat transfer.

Schematic and Known Data:
Refer to Fig. P12.51 and below.



Engineering Model:
(1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy.

(2) Moist air acts as an ideal gas, with each component adhering to the Dalton model.
(3) Pressure remains constant through the cooling process.

Analysis:

The first step is to determine whether condensation occurs. As long as there is no condensation the mole fraction of the water vapor, y_v, remains constant. Thus, as cooling occurs at fixed mixture pressure (assumption 3), the partial pressure of the water vapor remains constant until a saturated mixture would be attained: $p_v = y_v p$. The onset of condensation in this case corresponds, therefore, to the dew point temperature. Using given data

$$p_v = \phi_1 p_g(30^\circ\text{C}) = (0.50)(0.04246\text{ bar}) = 0.2123\text{ bar} \Rightarrow T_{dp} = 18.4^\circ\text{C}$$

Accordingly, condensation does not take place.

At steady state, mass rate balances result in

air: $\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a$

water: $\dot{m}_{v1} = \dot{m}_{v2} = \dot{m}_v$

therefore, $\omega = \dfrac{\dot{m}_v}{\dot{m}_a}$

The total mass flow rate is the sum: $\dot{m} = \dot{m}_v + \dot{m}_a = \dot{m}_a(1+\omega)$, where

$$\omega = 0.622\frac{p_v}{p\text{-}p_v} = 0.622\frac{0.2123\text{ bar}}{2-0.2123\text{ bar}} = 0.00667\frac{\text{kg (vapor)}}{\text{kg(air)}}$$

Accordingly,

$$\dot{m}_a = \frac{\dot{m}}{1+\omega} = \frac{600\text{ kg/h}}{(1.00667)} = 596.02\frac{\text{kg(air)}}{\text{h}}$$

With assumptions 1 and 2 an energy rate balance reduces to give

$$0 = \dot{Q}_{cv} - \underbrace{\dot{W}_{cv}}_{=0} + [\dot{m}_a h_a + \dot{m}_v h_v]_1 - [\dot{m}_a h_a + \dot{m}_v h_v]_2$$

or

$$\begin{aligned}\dot{Q}_{cv} &= \dot{m}_a[h_a(T_2) - h_a(T_1)] + \dot{m}_v[h_v(T_2) - h_v(T_1)] \\ &= \dot{m}_a\{[h_a(T_2) - h_a(T_1)] + \omega[h_g(T_2) - h_g(T_1)]\}\end{aligned}$$

where in accordance with $h_v \approx h_g(T)$ and with values from Tables A-2 and A-22

$$\begin{aligned}\dot{Q}_{cv} &= \left(596.02\frac{\text{kg(air)}}{\text{h}}\right)\Bigg\{[293.17-303.21]\frac{\text{kJ}}{\text{kg(air)}} \\ &\qquad + \left(0.00667\frac{\text{kg(vapor)}}{\text{kg(air)}}\right)[2538.1-2556.3]\frac{\text{kJ}}{\text{kg(vapor)}}\Bigg\} \\ &= (596.02)\{-10.04-0.12\}\frac{\text{kJ}}{\text{h}} \\ &= -6055.6\frac{\text{kJ}}{\text{h}} \quad \longleftarrow\end{aligned}$$

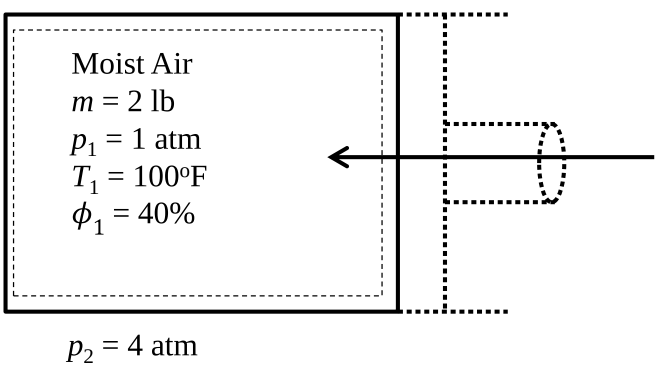
Problem 12.52

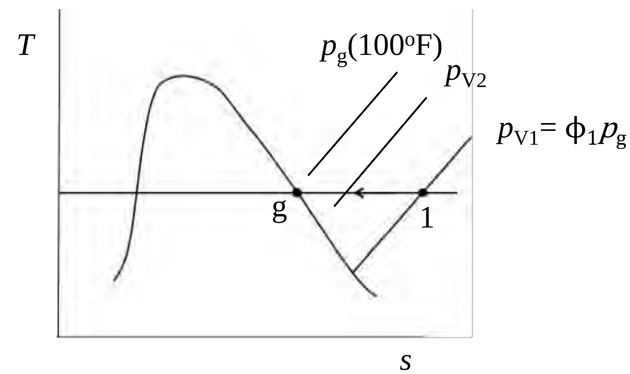
Two pounds of moist air initially at 100°F, 1 atm, 40% relative humidity is compressed isothermally to 4 atm. If condensation occurs, determine the amount of water condensed, in lb. If there is no condensation, determine the final relative humidity.

<u>Known:</u>
Two pounds of moist air initially at 100°F, 1 atm, ϕ_1 = 40% is compressed isothermally to 4 atm.

<u>Find:</u>
Determine if condensation occurs. If so, find the amount condensed. If not, find ϕ_2.

<u>Schematic and Known Data</u>**:**





<u>Engineering Model:</u>
(1) The system consists of a 2 lb sample of dry air and water.
(2) For the vapor phase, ideal gas mixture principles apply.

<u>Analysis:</u>
If no condensation occurs, $p_{v_2} \le p_g(100°\text{F})$. To check this

$$\omega_1 = \omega_2 \Rightarrow 0.622\frac{p_{v_1}}{p_1 - p_{v_1}} = 0.622\frac{p_{v_2}}{p_2 - p_{v_2}} \Rightarrow \frac{p_{v_1}}{p_1} = \frac{p_{v_2}}{p_2} \Rightarrow p_{v_2} = \frac{p_2}{p_1}p_{v_1}$$

Then, with $p_{v_1} = \phi_1 p_g(100°\text{F})$, this expression gives:

$$p_{v_2} = \left(\frac{4\text{ atm}}{1\text{ atm}}\right)\left(0.40p_g(100°\text{F})\right) = 1.6p_g(100°\text{F})$$

However, for no condensation $p_{v_2} \le p_g$, and so condensation occurs in this case. ⟵

At the final state, the system consists of a gas phase (dry air plus saturated water vapor) and a liquid phase (saturated liquid). The amount condensed is:

$$\text{Amount condensed} = m_{v_1} - m_{v_2} = m_a(\omega_1 - \omega_2)$$

Using values from Table A-2E

$$\omega_1 = 0.622\frac{p_{v_1}}{p_1 - p_{v_1}} = \frac{0.622(0.40)p_g}{p_1 - (0.40)p_g} = \frac{(0.622)(0.40)\left(0.9503\,\frac{\text{lbf}}{\text{in.}^2}\right)}{[14.7 - (0.40)(0.9503)]\,\frac{\text{lbf}}{\text{in.}^2}} = 0.01651\,\frac{\text{lb (vapor)}}{\text{lb (air)}}$$

$$\omega_2 = 0.622\frac{p_g}{p_2 - p_g} = \frac{(0.622)\left(0.9503\,\frac{\text{lbf}}{\text{in.}^2}\right)}{[4 \times 14.7 - (0.9503)]\,\frac{\text{lbf}}{\text{in.}^2}} = 0.01022\,\frac{\text{lb (vapor)}}{\text{lb (air)}}$$

The mass of dry air is found from $m_{\text{mix}} = m_a + m_{v_1} = m_a(1 + \omega_1)$, giving:

$$m_a = \frac{m_{\text{mix}}}{1 + \omega_1} = \frac{2\text{ lb}}{1.01651\,\frac{\text{lb (vapor)}}{\text{lb (air)}}} = 1.968\text{ lb (air)}$$

Collecting results:

Amount condensed $= m_a(\omega_1 - \omega_2) = 1.968(0.01651 - 0.01022) = 0.01238$ lb (vapor) $\longleftarrow$

Problem 12.53

A closed, rigid tank having a volume of 3 m^3 contains moist air in equilibrium with liquid water at 80°C. The respective masses present initially are 10.4 kg of dry air, 0.88 kg of water vapor, and 0.17 kg of liquid water. If the tank contents are heated to 160°C, determine

(a) the final pressure, in bar.

(b) the heat transfer, in kJ.

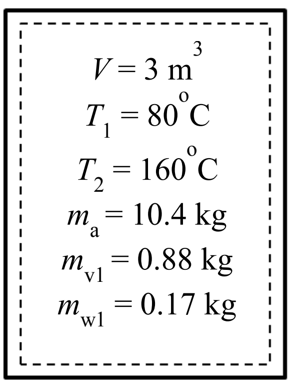
Known:

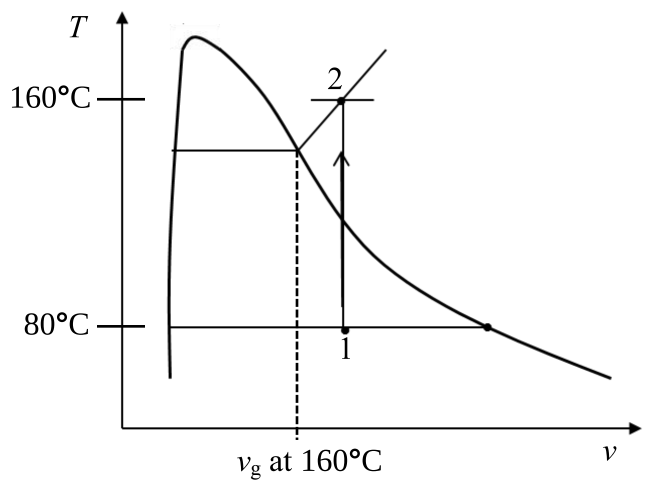
A tank having a fixed volume initially contains moist air in equilibrium with water. The tank contents are heated from 80°C to 160°C.

Find:

Determine (a) the final pressure and (b) the heat transfer.

Schematic & Given Data:





Engineering Model:

(1) As shown by the accompanying figure, the system is the tank contents.

(2) Changes in kinetic and potential energy are zero value.

(3) The moist air acts as an ideal gas, with each component adhering to the Dalton model.

Analysis:

The first step is to establish the condition of the system when at the final temperature. As the volume of the tank and the total mass of water remain constant, the water undergoes a constant specific volume process (see *T*-*v* diagram). The specific volume is found using v_f and v_g at 80°C from Table A-2:

#1

$$v_f = 1.0291 \times 10^{-3} \frac{\text{m}^3}{\text{kg}}; \quad v_g = 3.407 \frac{\text{m}^3}{\text{kg}}$$

$$v = \frac{m_{v_1} v_g + m_{w_1} v_f}{m_{v_1} + m_{w_1}} = \frac{(0.88 \text{ kg})\left(3.407 \frac{\text{m}^3}{\text{kg}}\right) + (0.17 \text{ kg})\left(1.0291 \times 10^{-3} \frac{\text{m}^3}{\text{kg}}\right)}{(0.88 + 0.17)\text{kg}} = 2.8556 \frac{\text{m}^3}{\text{kg}}$$

From Table A-2, since v is greater than $v_{\mathrm{g}}(160^{o}\mathrm{C}) = 0.3071\,\frac{\mathrm{m}^3}{\mathrm{kg}}$, all if the water present is vapor when the system is at 160°C.

(a) The final pressure of the moist air within the tank can be obtained using the ideal gas equation of state:

#1

$$p_2 = \frac{n\bar{R}T_2}{V} = \frac{\left(\frac{m_a}{M_a} + \frac{m_v}{M_v}\right)\bar{R}T_2}{V}$$

$$= \frac{\left[\left(\frac{10.4}{28.97} + \frac{1.05}{18.02}\right)\mathrm{kmol}\right]\left(8314\,\frac{\mathrm{N\cdot m}}{\mathrm{kmol\cdot K}}\right)(433\ \mathrm{K})}{(3\ \mathrm{m}^3)}\left|\frac{1\mathrm{bar}}{10^5\ \mathrm{N/m}^2}\right| = 5.01\ \mathrm{bar}$$

#2

(b) With assumptions 2 and 3, an energy balance gives $\Delta U = Q - \cancel{W}$ or:

$$Q = \Delta U = \left[m_{\mathrm{a}}u_{\mathrm{a}}(T_2) + m_{\mathrm{v}}u_{\mathrm{g}}(T_2)\right] - \left[m_{\mathrm{a}}u_{\mathrm{a}}(T_1) + m_{\mathrm{v1}}u_{\mathrm{g}}(T_1) + m_{\mathrm{w1}}u_{\mathrm{f}}(T_1)\right] =$$
$$m_{\mathrm{a}}[u_{\mathrm{a}}(T_2) - u_{\mathrm{a}}(T_1)] + m_{\mathrm{v}}u_{\mathrm{g}}(T_2) - m_{\mathrm{v1}}u_{\mathrm{g}}(T_1) - m_{\mathrm{w1}}u_{\mathrm{f}}(T_1)$$

With data from Tables A-2 and A-22

$$Q = (10.4\ \mathrm{kg})\left[(310.18 - 252.19)\frac{\mathrm{kJ}}{\mathrm{kg}}\right] + (1.05\ \mathrm{kg})\left(2568.4\,\frac{\mathrm{kJ}}{\mathrm{kg}}\right) - (0.88\ \mathrm{kg})\left(2482.2\,\frac{\mathrm{kJ}}{\mathrm{kg}}\right) -$$
$$(0.17\ \mathrm{kg})\left(334.86\,\frac{\mathrm{kJ}}{\mathrm{kg}}\right) = 1058.65\ \mathrm{kJ}$$

<u>Comments:</u>

#1 The numerator of this expression represents the rigid tank's volume given as 3m^3. When calculated here, the V= 2.9983m^3 which is rounded to 3 m^3 as given for use in part (a).

#2 Since water is present both as a liquid and vapor, it is necessary to use steam table data when evaluating Q, and not data from the ideal gas table for water (Table A-23).

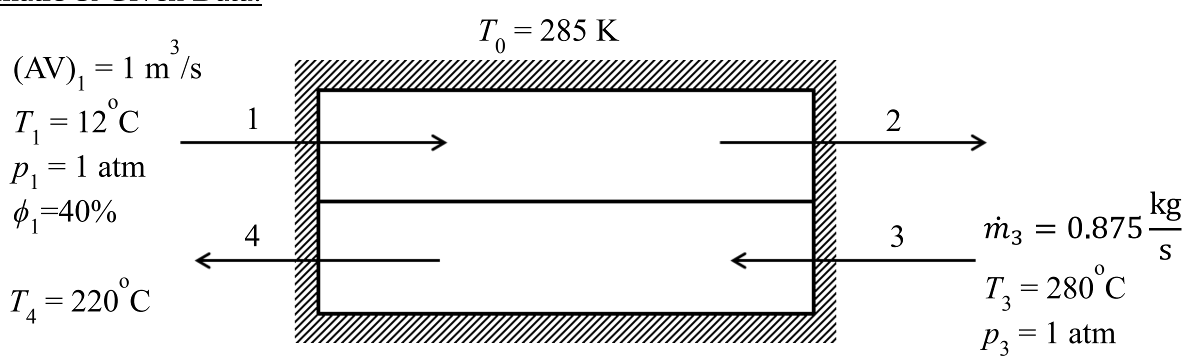
Problem 12.54

Air at 12°C, 1 atm, and 40% relative humidity enters a heat exchanger with a volumetric flow rate of 1 m^3/s. A separate stream of dry air enters at 280°C, 1 atm with a mass flow rate of 0.875 kg/s and exits at 220°C. Neglecting heat transfer between the heat exchanger and its surroundings, pressure drops of each stream, and kinetic and potential energy effects, determine
(a) the temperature of the exiting moist air, in °C.
(b) the rate of exergy destruction, in kW, for T_0 = 12°C.

Known:
Operating data are provided for a heat exchanger at steady state involving a stream of moist air and dry air stream.

Find:
Determine the temperature of the exiting moist air and the exergy destruction rate.

Schematic & Given Data:



Engineering Model:
(1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{\text{cv}} = \dot{W}_{\text{cv}} = 0$ and negligible effects of kinetic and potential energy.
(2) There is no pressure drop in either stream.
(3) The moist air acts as an ideal gas as does the dry air stream.

Analysis:
Mass rate balances give $\dot{m}_4 = \dot{m}_3$; $\dot{m}_{a2} = \dot{m}_1 = \dot{m}_a$; $\dot{m}_{v_1} = \dot{m}_{v_2}$. Thus, $\omega_1 = \omega_2 = \omega$. To find ω, use:

$$p_{v_1} = \phi_1 p_{g_1} = (0.4)(0.01402 \text{ bar}) = 0.00561 \text{ bar}$$

$$\omega = 0.622 \left[\frac{0.00561}{1.01325 - 0.0056}\right] = 0.003462 \ \frac{\text{kg (vapor)}}{\text{kg (dry air)}}$$

The mass flow rate of dry air is evaluated using the volumetric flow rate at 1 with the ideal gas equation of state and with $p_{a1} = p - p_{v1}$ = 1.01325 - 0.00561 = 1.00764 bar

$$\dot{m}_{\mathrm{a}} = \frac{(AV)_1}{v_{\mathrm{a}_1}} = \frac{p_{\mathrm{a}_1}(\mathrm{AV})_1}{\left(\frac{\bar{R}}{M_{\mathrm{a}}}\right)T_1} = \frac{\left(1.00764\ \times\ 10^5\,\frac{\mathrm{N}}{\mathrm{m}^2}\right)\left(1\,\frac{\mathrm{m}^3}{\mathrm{s}}\right)}{\left(\frac{8314}{28.97}\,\frac{\mathrm{N\cdot m}}{\mathrm{kg\cdot K}}\right)(285\ \mathrm{K})} = 1.232\,\frac{\mathrm{kg\ (air)}}{\mathrm{s}}$$

(a) An energy rate balance at steady state reduces to:

$$0 = \cancel{\dot{Q}}_{\mathrm{cv}} - \cancel{\dot{W}}_{\mathrm{cv}} + \left[\dot{m}_{\mathrm{a}}h_{\mathrm{a}_1} + \dot{m}_{\mathrm{v}_1}h_{\mathrm{v}_1}\right] - \left[\dot{m}_{\mathrm{a}}h_{\mathrm{a}_2} + \dot{m}_{\mathrm{v}_2}h_{\mathrm{v}_2}\right] + \dot{m}_3h_3 - \dot{m}_4h_4$$

Or, using the above mass rate balances and rearranging

$$h_{\mathrm{a}}(T_2) + \omega h_{\mathrm{v}}(T_2) = h_{\mathrm{a}}(T_1) + \omega h_{\mathrm{v}}(T_1) + \frac{\dot{m}_3}{\dot{m}_{\mathrm{a}}}(h_3 - h_4)$$

As water at state 2 is present only in the vapor phase, Table A-23 is used to evaluate h_{v}. Data for h_{a} is obtained from Table A-22 with interpolation

$$\begin{aligned} h_a(T_2) + \omega h_{\mathrm{v}}(T_2) \\ &= 285.14\,\frac{\mathrm{kJ}}{\mathrm{kg}} + (0.003462)\left(9463.5\,\frac{\mathrm{kJ}}{\mathrm{kmol}}\right)\left(\frac{1\ \mathrm{kmol}}{18.02\ \mathrm{kg}}\right) \\ &+ \left(\frac{0.875}{1.232}\right)\left[(557.87 - 495.82)\,\frac{\mathrm{kJ}}{\mathrm{kg}}\right] = 331\,\frac{\mathrm{kJ}}{\mathrm{kg_a}} \end{aligned}$$

Solving iteratively, $T_2 \approx 329\ \mathrm{K} = 56°\mathrm{C}$ ←

(b) The exergy destruction rate is given by $\dot{\mathrm{E}}_{\mathrm{d}} = T_0\dot{\sigma}_{\mathrm{cv}}$, where $\dot{\sigma}_{\mathrm{cv}}$ is the rate of entropy production obtained from an entropy rate balance

$$0 = \sum_{\mathrm{j}} \frac{\dot{Q}_{\mathrm{j}}}{T_{\mathrm{j}}} + \left[\dot{m}_{\mathrm{a}}s_{\mathrm{a}_1} + \dot{m}_{\mathrm{v}_1}s_{\mathrm{v}_1}\right] - \left[\dot{m}_{\mathrm{a}}s_{\mathrm{a}_2} + \dot{m}_{\mathrm{v}_2}s_{\mathrm{v}_2}\right] + \dot{m}_3[s_3 - s_4] + \dot{\sigma}_{\mathrm{cv}}$$

Or:

$$\dot{\sigma}_{\mathrm{cv}} = \dot{m}_{\mathrm{a}}\left[\left(s_{\mathrm{a}_2} - s_{\mathrm{a}_1}\right) + \omega\left(s_{\mathrm{v}_2} - s_{\mathrm{v}_1}\right)\right] + \dot{m}_3[s_4 - s_3]$$

Since $p_{\mathrm{a}_2} = p_{\mathrm{a}_1}$; $p_{\mathrm{v}_2} = p_{\mathrm{v}_1}$, and $p_3 = p_4$, this becomes

$$\begin{aligned} \dot{\sigma}_{\mathrm{cv}} &= \dot{m}_{\mathrm{a}}\left\{s_{\mathrm{a}}^o(T_2) - s_{\mathrm{a}}^o(T_1) + \frac{\omega}{M_{\mathrm{v}}}\left[\bar{s}_{\mathrm{v}}^o(T_2) - \bar{s}_{\mathrm{v}}^o(T_1)\right]\right\} + \dot{m}_3\left[s_{\mathrm{a}}^o(T_4) - s_{\mathrm{a}}^o(T_3)\right] \\ &= \left(1.232\,\frac{\mathrm{kg}}{\mathrm{s}}\right)\left\{\left[(1.79476 - 1.65055)\,\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\right] + \left(\frac{0.00346}{18.02\,\frac{\mathrm{kg}}{\mathrm{kmol}}}\right)\left[\frac{(192.032 - 187.204)\mathrm{kJ}}{\mathrm{kmol\cdot K}}\right]\right\} \\ &+ \left(0.875\,\frac{\mathrm{kg}}{\mathrm{s}}\right)\left[(2.20499 - 2.32372)\,\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\right] = 0.0749\,\frac{\mathrm{kJ}}{\mathrm{s\cdot K}} \end{aligned}$$

Finally, solving for the exergy destruction rate:

$\dot{\mathrm{E}}_{\mathrm{d}} = T_0\dot{\sigma}_{\mathrm{cv}} = (285\ \mathrm{K})\left(0.0749\,\frac{\mathrm{kJ}}{\mathrm{s\cdot K}}\right) = 21.35\ \mathrm{kW}$ ←

Problem 12.55

Solve Problem 12.47 using the psychrometric chart, Fig. A-9E.

Known:
A volume of 10^6 ft^3 contains air at 80°F, 1 atm, ω = 0.01 lb(v)/lb(a).

Find:
Determine (a) the relative humidity, (b) dew point temperature, and (c) the mass of water vapor present using Fig. A-9E.

Schematic & Given Data:
See solution to Prob. 12.47.

Engineering Model:
See solution to Prob. 12.47.

Analysis:
By inspection of Fig. A-9E

$\phi = 46\%$ ←

$T_{\mathrm{dp}} = 57°\mathrm{F}$ ←

$$v_{\mathrm{a}} = 13.82 \frac{\mathrm{ft}^3}{\mathrm{lb\ (air)}}$$

Thus

$$v_{\mathrm{v}} = \frac{v_{\mathrm{a}}}{\omega} = \frac{13.82 \frac{\mathrm{ft}^3}{\mathrm{lb_a}}}{0.01 \frac{\mathrm{lb_v}}{\mathrm{lb_a}}} = 1382 \frac{\mathrm{ft}^3}{\mathrm{lb\ (vapor)}}$$

So

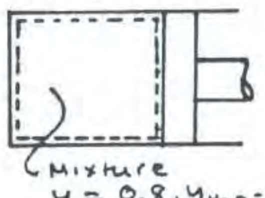
$$m_{\mathrm{v}} = \frac{V}{v_{\mathrm{v}}} = \frac{10^6\ \mathrm{ft}^3}{1382 \frac{\mathrm{ft}^3}{\mathrm{lb\ (vapor)}}} = 723.6\ \mathrm{lb\ (vapor)}$$ ←

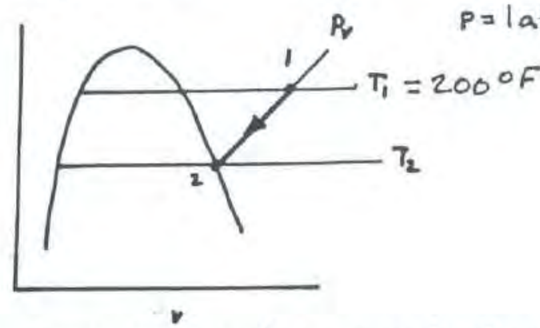
PROBLEM 12.56

KNOWN: A mixture of nitrogen and water vapor having a molar analysis 80% N_2, 20% water vapor is at 200°F, 1atm.

FIND: If the mixture is cooled at constant pressure, determine the temperature at which water vapor begins to condense.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) The system consists of a fixed amount of a nitrogen/water vapor mixture, as illustrated in the accompanying figure. (2) The mixture acts like an ideal gas, with each component adhering to the Dalton model.

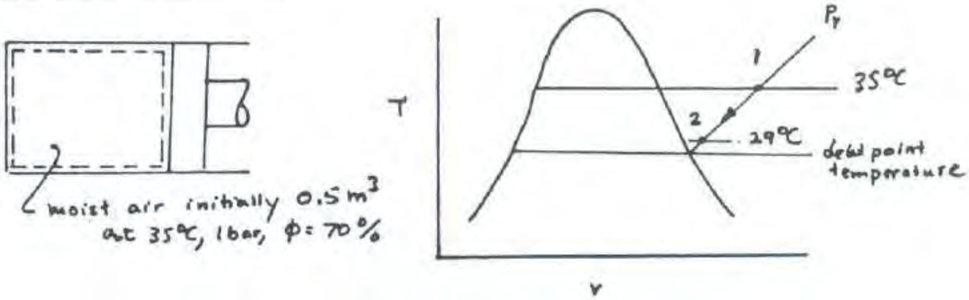
ANALYSIS: As long as there is no condensation, the mole fraction of the water vapor, y_v, remains constant. Moreover, as cooling occurs at fixed mixture pressure, the partial pressure of the water vapor remains constant: $p_v = y_v p$. Thus, cooling takes place at constant p_v, and the onset of condensation occurs at state 2 shown on the accompanying figure. The corresponding temperature is the saturation temperature, T_2. Therefore, $p_v = y_{H_2O} p = (0.2)(14.7\ lbf/in^2) = 2.94\ lbf/in^2$. Interpolating in Table A-3E gives $T_2 = 140.5°F$ ← T_2

PROBLEM 12.57

KNOWN: A system consisting initially of $0.5\,m^3$ of air at 35°C, 1 bar, $\phi = 70\%$ is cooled at constant pressure to 29°C.

FIND: Determine the work and heat transfer for the process.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) As shown in the figure, the system consists of the specified quantity of moist air. (2) The mixture acts as an ideal gas, with each component adhering to the Dalton model.

ANALYSIS: The first step is to determine whether condensation occurs. As long as there is no condensation the mole fraction of the water vapor, y_v, remains constant. Thus, as the cooling occurs at fixed mixture pressure, the partial pressure of the water vapor remains constant until a saturated mixture would be attained: $p_v = y_v p$. The onset of condensation in this case corresponds, therefore, to the dew point temperature. Using given data

$$p_v = \phi_1\, p_g(35°C) = (0.7)(0.05628\text{ bars}) = 0.0394\text{ bar} \implies T_{dp} = 28.7°C$$

Accordingly, condensation does not take place.

Since the mixture undergoes a constant pressure process

$$W = \int_1^2 p\,dV = p\,(V_2 - V_1) = pV_1\left[\frac{V_2}{V_1} - 1\right]$$

The total amount of mixture remains constant, so the ideal gas equation of state gives

$$\left.\begin{matrix} pV_1 = n\bar{R}T_1 \\ pV_2 = n\bar{R}T_2 \end{matrix}\right\} \quad \frac{V_2}{V_1} = \frac{T_2}{T_1}$$

Thus

$$W = p_1 V_1\left[\frac{T_2}{T_1} - 1\right] = \left(10^5\,\frac{N}{m^2}\right)(1\,m^3)\left[\frac{302}{308} - 1\right]\left|\frac{kJ}{10^3\,N\cdot m}\right| = -1.948\text{ kJ} \longleftarrow W$$

With assumption 3, an energy balance gives $\Delta U = Q - W$, or $Q = \Delta U + W$. The change in internal energy of the system is the sum of the internal energy changes of the dry air and water vapor: $\Delta U = (\Delta U)_a + (\Delta U)_v$. Thus

$$Q = \left\{m_a\left[u_a(T_2) - u_a(T_1)\right] + m_v\left[u_v(T_2) - u_v(T_1)\right]\right\} + W$$

PROBLEM 12.57 (Cont'd) - Page 2

In accordance with the discussion of Sec. 12.5.2, $u_v \approx u_g(T)$ from the steam tables. For air u_a is obtained from Table A-22. The mass amounts m_a and m_v are obtained from the ideal gas equation of state:

$$m_v = \frac{p_{v1} V_1}{\bar{R}/M_v T_1} = \frac{(0.0394 \times 10^5 \, N/m^2)(0.5 m^3)}{(8314/18.02 \; N \cdot m/kg \cdot K)(308 K)} = 0.01387 \text{ kg(v)}$$

$$m_a = \frac{p_{a1} V_1}{\bar{R}/M_a T_1} = \frac{(p_1 - p_{v1}) V_1}{\bar{R}/M_a T_1} = \frac{(0.9606 \times 10^5)(0.5)}{(8314/28.97)(308)} = 0.54338 \text{ kg(a)}$$

Substituting values into the expression for Q

$$Q = \left\{ 0.54338 \text{ kg(a)} \left[215.51 - 219.8\right] \frac{kJ}{kg(a)} + (0.01387)\left[2415.2 - 2423.4\right] \frac{kJ}{kg(v)} \right\} + (-1.948 \text{ kJ})$$

$$= \{-2.331 - 0.114\} + (-1.948) = -4.393 \text{ kJ}$$

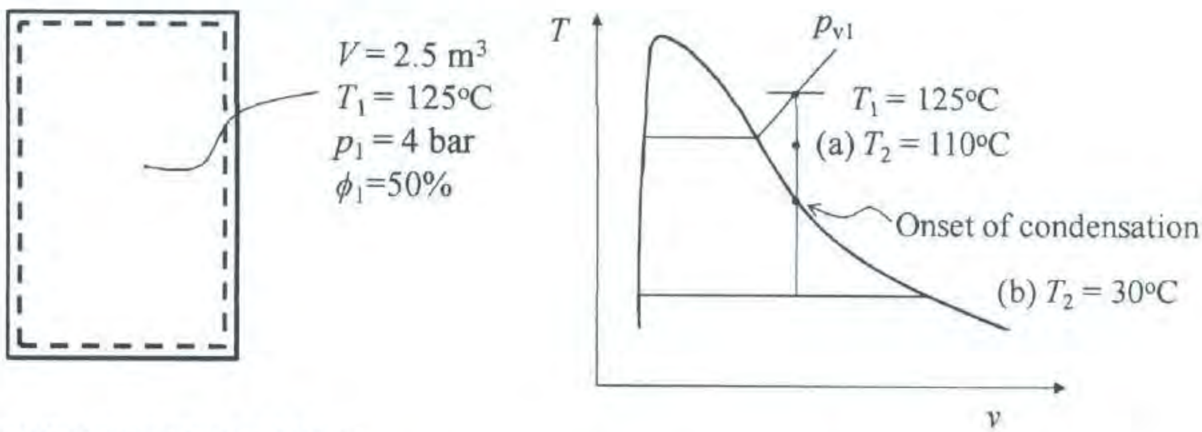
Q

12.58 Moist air initially at 125°C, 4 bar, and 50% relative humidity is contained in a 2.5-m^3 closed, rigid tank. . The tank contents are cooled. Determine the heat transfer, in kJ, if the final temperature in the tank is (a) 110°C, (b) 30°C.

KNOWN: A tank with known volume initially contains moist air at known conditions.

FIND: Determine the heat transfer when the tank contents are cooled to (a) 110°C, (b) 30°C.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

(1) As shown by figure, the system consists of the tank contents. There are no changes in kinetic and potential energy.
(2) If condensation occurs, the volume occupied by the condensate is ignored.
(3) The gas mixture behaves as an ideal gas, with each component adhering to the Dalton model.

ANALYSIS:

Begin by determining if condensation occurs during cooling.

As the total volume of the tank and the total mass of the water present are constant, the water undergoes a constant specific volume process as the tank contents are cooled (assumption 2). This is illustrated in the above T-v diagram. Using the ideal gas equation of state for the water vapor

$$v_{v1} = \frac{T_1\left(\overline{R}/M_v\right)}{p_{v1}}$$

Further, with data from Table A-2

Problem 12.58 (Continued) - Page 2

$$\phi_1 = \frac{p_{v1}}{p_{g1}} \text{ or } p_{v1} = \phi_1 p_{g1} = (0.50)(2.343 \text{ bar}) = 1.1715 \text{ bar}$$

Thus

$$v_{v1} = \frac{(398\text{ K})\left(\frac{8.314}{18.02}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1000\text{ J}}{1\text{ kJ}}\right|\left|\frac{1\text{ N}\cdot\text{m}}{1\text{ J}}\right|\right)}{(1.1715\text{ bar})\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|} = \frac{1.8363\times 10^5 \frac{\text{N}\cdot\text{m}}{\text{kg}}}{1.1715\times 10^5 \frac{\text{N}}{\text{m}^2}} = 1.567\frac{\text{m}^3}{\text{kg}}$$

Interpolating in Table A-2 with $v_{v1} = v_g$ gives T = 102.3°C as the temperature at which condensation would begin.

Next, determine the mass of the air and the water in the tank, each of which remains constant throughout the process. Use the ideal gas equation of state at the initial condition.

$$m_a = \frac{(p_{a1}V)}{\left(\frac{\overline{R}}{M_a}T_1\right)} = \frac{V(p_1 - p_{v1})}{\left(\frac{\overline{R}}{M_a}T_1\right)} = \frac{(2.5\text{ m}^3)(4 - 1.1715)\text{bar}\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1000\text{ J}}{1\text{ kJ}}\right|\left|\frac{1\text{ N}\cdot\text{m}}{1\text{ J}}\right|\right)(398\text{ K})}$$

$$= \frac{707125\text{ N}\cdot\text{m}}{114220.64\frac{\text{N}\cdot\text{m}}{\text{kg}}} = 6.191\text{ kg (mass of air)}$$

$$m_{v1} = \frac{V}{v_{v1}} = \frac{2.5\text{ m}^3}{1.567\text{ m}^3/\text{kg}} = 1.5954\text{ kg (mass of vapor)}$$

(a) $\underline{T_2 = 110^\circ\text{C}}$. In this case there is no condensation (condensation doesn't occur until the temperature reaches 102.3°C) .

The energy balance $Q = \Delta U + \underbrace{W}_{=0}$ reduces to $Q = \Delta U$

and

$$U_1 = m_a u_{a1} + m_{v1}u_{v1} = m_a u_{a1} + m_{v1}u_{g1}$$

$$U_2 = m_a u_{a2} + m_{v2}u_{v2} = m_a u_{a2} + m_{v2}u_{g2}$$

Because there is no condensation, $m_{v1} = m_{v2} = m_v$

(In these equations, the subscripts a and v denote dry air and water vapor, respectively).

The specific internal energy of the water vapor at the initial state and state 2 can be approximated at the saturated vapor value (u_g) at the respective temperatures. Then, with data from Tables A-2, A-22 and using interpolation:

$$
\begin{aligned}
Q &= m_a(u_{a2} - u_{a1}) + m_v(u_{g2} - u_{g1}) \\
&= (6.191\,\text{kg})(273.86 - 284.71)\frac{\text{kJ}}{\text{kg}} + (1.5954\,\text{kg})(2518.1 - 2534.6)\frac{\text{kJ}}{\text{kg}} \\
&= (-67.172\,\text{kJ}) + (-26.324\,\text{kJ}) = -93.5\,\text{kJ} \quad \longleftarrow
\end{aligned}
$$

(b) $\underline{T_2 = 30^\circ\text{C}}$. In this case condensation does occur. The energy balance $Q = \Delta U + \underbrace{W}_{=0}$, reduces to $Q = U_2 + U_1$

U_1 is the same as in (a) and $U_2 = m_a u_{a2} + m_{v2}u_{v2} + m_w u_{w2} = m_a u_{a2} + m_{v2}u_{g2} + m_{w2}u_{f2}$

where m_{w2} is the mass of water that condenses. The liquid water at the final state is saturated, so its specific internal energy is u_f at T_2. Collecting results

$$
Q = m_a(u_{a2} - u_{a1}) + (m_{v2}u_{g2} + m_{w2}u_{f2} - m_{v1}u_{g1}) \tag{1}
$$

Since the water within the system undergoes a constant specific volume process $v_{v1} = v_{v2} = 1.567\,\text{m}^3/\text{kg}$

The quality of the two-phase liquid-vapor mixture at T_2 follows

$$
x_2 = \frac{(v_{v2} - v_f)}{(v_g - v_f)} = \frac{(1.567 - 1.0043\times10^{-3})\,\text{m}^3/\text{kg}}{(32.894 - 1.0043\times10^{-3})\,\text{m}^3/\text{kg}} = 0.0476
$$

Thus, the amount of water vapor present in the final mixture is

$$
m_{v2} = x_2 m_{v1} = (0.0476)(1.5954\,\text{kg}) = 0.076\,\text{kg (mass of vapor)}
$$

The mass of condensate is

$$
m_{w2} = m_{v1} - m_{v2} = 1.5954 - 0.076 = 1.52\,\text{kg}
$$

Substitute all known values in Eq. (1) with data from Tables A-2, A-22

Problem 12.58 (Continued) – Page 4

$$Q = m_a\left(u_{a2} - u_{a1}\right) + \left(m_{v2}u_{g2} + m_{w2}u_{f2} - m_{v1}u_{g1}\right)$$

$$= (6.191\,\text{kg})(216.23 - 284.71)\frac{\text{kJ}}{\text{kg}} + \left[(0.076\,\text{kg})\left(2416.6\frac{\text{kJ}}{\text{kg}}\right)\right.$$

$$\left. + (1.52\,\text{kg})\left(125.78\frac{\text{kJ}}{\text{kg}}\right) - (1.5954\text{kg})\left(2534.6\frac{\text{kJ}}{\text{kg}}\right)\right]$$

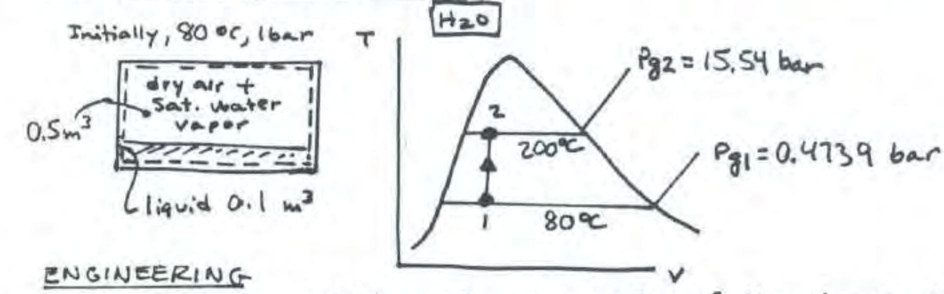
$$= (-423.96\,\text{kJ}) + [183.66 + 191.19 - 4043.70]\,\text{kJ} = -4092.8\,\text{kJ} \quad \longleftarrow$$

PROBLEM 12.59

KNOWN: A closed, rigid tank, initially containing moist air in equilibrium with liquid water, is heated.

FIND: Determine (a) the final pressure, (b) the heat transfer.

SCHEMATIC & GIVEN DATA



ENGINEERING MODEL: (1) The system consists of the tank contents. (2) The gas phase adheres to ideal gas principles. (3) W = 0 and kinetic & potential energy effects are absent.

ANALYSIS: (a) Since total volume is constant, the water present as liquid and water vapor, undergoes a constant specific volume process — as shown in the schematic. In stating this, the Dalton model is used: the water vapor occupies the full volume filled with water vapor and dry air. The initial amounts of liquid and vapor are

$$m_{vap} = \frac{V_{vap}}{v_g(80°C)} = \frac{0.5\ m^3}{3.407\ m^3/kg} = 0.1468\ kg\ ,\quad m_{liq} = \frac{0.1\ m^3}{(1.0291/10^3)\ m^3/kg} = 97.1723\ kg$$

The total mass of water is then $m_{water} = 97.3191$ kg. The final specific volume is

$$v = \frac{0.60\ m^3}{97.3191\ kg} = 6.165\times10^{-3}\frac{m^3}{kg} \Rightarrow$$ at state 2 the water is a two-phase liquid-vapor mixture with quality

$$x = \left[\frac{(6.165\times10^{-3}) - (1.1565\times10^{-3})}{0.1274 - (1.1565\times10^{-3})}\right] = 0.03967 \Rightarrow \quad m_{v2} = x\cdot m_{water} = (0.03967)(97.1723) = 3.855\ kg$$

The mass of dry air is

$$m_a = \frac{p_{a1} V_{gas}}{R\, T_1} = \frac{[[1-0.4739]\times10^5 N/m^2][0.5\ m^3]}{\left(\frac{8314}{28.97}\right)(353\ K)} = 0.2597\ kg \Rightarrow \omega_2 = \frac{m_{v2}}{m_a} = \frac{3.855}{0.259} = 14.846$$

Rearranging Eq. 12.43

$$p_2 = p_{v2}\left[1 + \frac{0.622}{\omega_2}\right] = 15.54\ bar)\left[1 + \frac{0.622}{14.846}\right] = 16.19\ bar = 1.62\ MPa \quad \leftarrow p_2$$

(b) An energy balance reduces to read $\Delta U = Q - \cancel{W}^{\,0}$, where

$$U_1 = m_{liq}\, u_f(80°C) + m_{vap}\, u_g(80°C) + m_a\, u_a(80°C)$$

$$U_2 = m_{water}[u_f(200°C) + x\, u_g(200°C)] + m_a\, u_a(200°C)$$

or

$$Q = m_{water}[u_f(200°C) + x\, u_g(200°C)] - [m_{liq}\, u_f(80°C) + m_{vap}\, u_g(80°C)] + m_a[u_a(200°C) - u_a(80°C)]$$

$$Q = (97.3191)[850.65 + (0.03967)(2595.3)] - [(97.1723)(334.86) + (0.1468)(2482.2)] + (0.2597)[339.5 - 252.2]$$

$$= 92{,}804 - 32{,}904 + 23$$

$$= 59{,}923\ kJ \quad \leftarrow Q$$

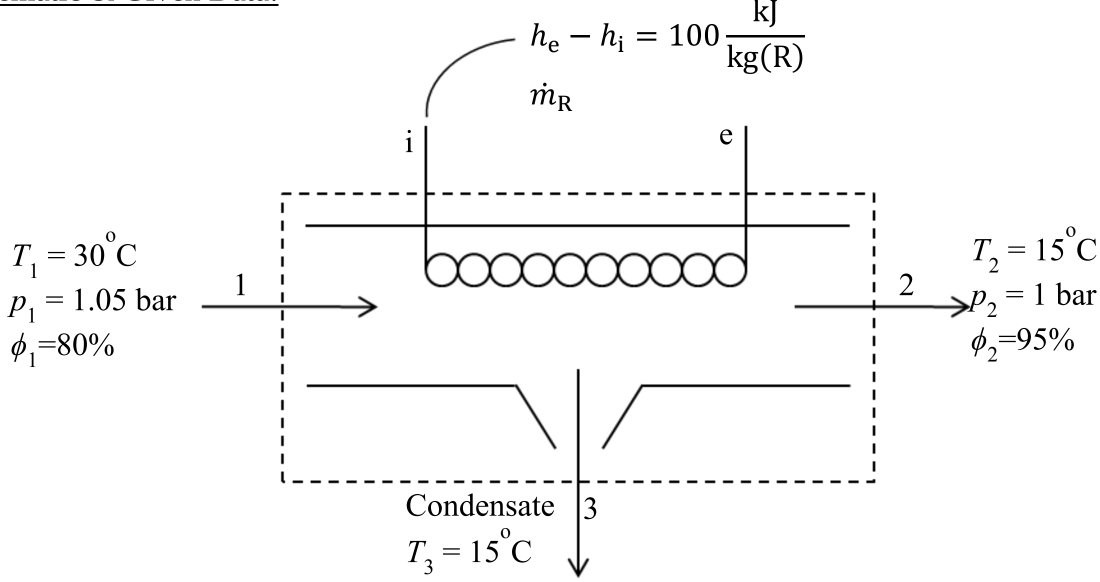
Problem 12.60

Air at 30°C, 1.05 bar, and 80% relative humidity enters a dehumidifier operating at steady state. Moist air exits at 15°C, 1 bar and 95% relative humidity. Condensate exits in a separate stream at 15°C. A refrigerant flows through the cooling coil of the dehumidifier with an increase in its specific enthalpy of 100 kJ per kg of refrigerant flowing. Heat transfer between the humidifier and its surroundings, and kinetic and potential energy effects can be ignored. Determine the refrigerant flow rate, in kg per kg of dry air.

<u>Known:</u>
Operating data are provided for a dehumidifier at steady state.

<u>Find:</u>
Determine the refrigerant flow rate, in kg per kg of dry air.

<u>Schematic & Given Data:</u>



<u>Engineering Model:</u>
(1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy.
(2) Condensate exits as a saturated liquid at 15°C.
(3) Assume constant specific heat for the air at 300 K.

Analysis:
At steady state mass rate balances give $\dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a$ and $\dot{m}_{v_1} = \dot{m}_{v_2} + \dot{m}_w$. Thus, the rate water condensed per unit mass of dry air is

$$\frac{\dot{m}_w}{\dot{m}_a} = \omega_1 - \omega_2 \qquad (1)$$

An energy rate balance at steady state reduces to:

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_R(h_i - h_e) + [\dot{m}_a h_{a_1} + \dot{m}_{v_1} h_{v_1}] - [\dot{m}_a h_{a_2} + \dot{m}_{v_2} h_{v_2}] - \dot{m}_w h_3$$

Or, with Eq. (1) rearranging and using assumption (3)

$$\frac{\dot{m}_{\mathrm{R}}}{\dot{m}_{\mathrm{a}}}=\frac{\left(h_{\mathrm{a}_2}-h_{\mathrm{a}_1}\right)+\omega_2 h_{\mathrm{g}_2}-\omega_1 h_{\mathrm{g}_1}+(\omega_1-\omega_2)h_{\mathrm{f}_3}}{(h_{\mathrm{i}}-h_{\mathrm{e}})_{\mathrm{R}}}$$
$$=\frac{c_{p_{\mathrm{a}}}(T_2-T_1)+\omega_2 h_{\mathrm{g}_2}-\omega_1 h_{\mathrm{g}_1}+(\omega_1-\omega_2)h_{\mathrm{f}_3}}{(\mathrm{h}_{\mathrm{i}}-h_{\mathrm{e}})_{\mathrm{R}}}$$

To find ω_1 and ω_2 and using values from Table A-2, $p_{\mathrm{v}_1}=\phi_1 p_{\mathrm{g}}(T_1)=0.8(0.04246)=$ 0.03397 bar and $p_2=\phi_2 p_{\mathrm{g}}(T_2)=0.95(0.01705)=0.0162$ bar. Using Eq. 12.43

$$\omega_1=0.622\frac{p_{\mathrm{v}_1}}{p_1-p_{\mathrm{v}_1}}=0.622\left[\frac{0.03397}{1.05-0.03397}\right]=0.0208\frac{\text{kg (vapor)}}{\text{kg (air)}}$$
$$\omega_2=0.622\frac{p_{\mathrm{v}_2}}{p_2-p_{\mathrm{v}_2}}=0.622\left[\frac{0.0162}{1-0.0162}\right]=0.0102\frac{\text{kg (vapor)}}{\text{kg (air)}}$$

Then, with $c_{p_a}=1.005\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}$ from Table A-20 and enthalpy values from Table A-2

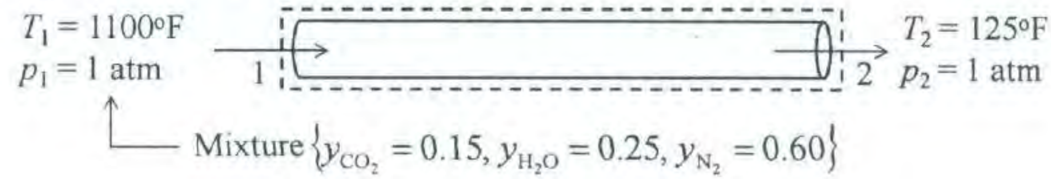
$$\frac{\dot{m}_{\mathrm{R}}}{\dot{m}_{\mathrm{a}}}$$
$$=\frac{\left(1.005\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\right)(15-30)\mathrm{K}+0.0102\left(2528.9\frac{\mathrm{kJ}}{\mathrm{kg}}\right)-0.0208\left(2556.3\frac{\mathrm{kJ}}{\mathrm{kg}}\right)+(0.0208-0.0102)\left(62.99\frac{\mathrm{kJ}}{\mathrm{kg}}\right)}{\left(-100\frac{\mathrm{kJ}}{\text{kg (ref)}}\right)}$$
$$=0.418\frac{\text{kg (ref)}}{\text{kg (air)}} \longleftarrow$$

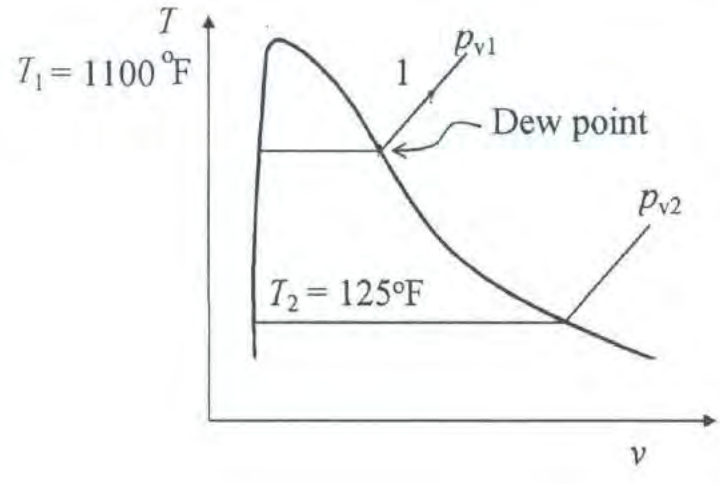
12.61 Products of combustion with a molar analysis of 15% CO_2, 25% H_2O, 60% N_2 enter an engine's exhaust pipe at 1100°F, 1 atm. The products are cooled as they pass through the pipe to 125°F, 1 atm. Determine the heat transfer at steady state, in Btu per lb of entering mixture.

KNOWN: A gaseous mixture including water vapor enters and exits an ENGINE exhaust pipe at known conditions.

FIND: Determine the heat transfer at steady state, in Btu per lb of entering mixture.

SCHEMATIC AND GIVEN DATA:





ENGINEERING MODEL:

(1) The control volume shown is at steady state.
(2) For the control volume there is no net work.
(3) The kinetic/potential energy effects are negligible.
(4) The gas phase can be treated as an ideal gas mixture.

ANALYSIS:

To begin, check if condensation would occur by determining the dew point temperature of the mixture entering the pipe. For the water vapor entering

$p_{v1} = y_v p_1 = (0.25) * (1\,\text{atm}) = 0.25\,\text{atm} = 3.67\,\text{lbf/in.}^2$

Problem 12.61 (Continued) – Page2

The corresponding dew point temperature from Table A-3E is ~150°F. Since the exiting mixtures os cooled to 125°F, condensation occurs. Therefore, at the exit, there is H_2O in a saturated gas phase and as a saturated liquid.

Next, consider the gas phase exiting. The partial pressure of the water vapor is p_{v2} and the mole fraction of the water vapor is

$$y_v = \left(\frac{n_v}{n_{dry} + n_v} \right)$$

where n_v is the amount of water vapor exiting per lbmol of entering mixture and n_{dry} is the amount of CO_2 and N_2 exiting per lbmol of entering mixture or n_{dry}=0.75 . Accordingly

$$p_{v2} = y_v p_2 = \left(\frac{n_v}{0.75 + n_v} \right) p_2$$

From Table A-2E at T_2,; p_{v2}=1.96 lbf/in.^{2} Therefore

$$1.96 \frac{\text{lbf}}{\text{in.}^2} = \left(\frac{n_v}{0.75 + n_v} \right) 14.7 \frac{\text{lbf}}{\text{in.}^2} \Rightarrow n_v = 0.1154 \frac{\text{lbmol vapor exiting}}{\text{lbmol entering mixture}}$$

The amount of condensate at the exit would be

$$0.25 - 0.1154 = 0.1346 \frac{\text{lbmol condensate}}{\text{lbmol entering mixture}}$$

Reducing energy and mass balances at steady state

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{mix,1}} + \left[0.15\bar{h}_{CO_2} + 0.25\bar{h}_{H_2O(v)} + 0.60\bar{h}_{N_2} \right]_1$$
$$- \left[0.15\bar{h}_{CO_2} + 0.1154\bar{h}_{H_2O(v)} + 0.60\bar{h}_{N_2} \right]_2 - \left[0.1346\bar{h}_{H_2O(l)} \right]_2$$

Rearrange and solve using $h_{H_2O(v)} = h_g$ and $h_{H_2O(l)} = h_f$ at the exit; $\bar{h}_{H_2O(v)}$ at the inlet from
#1 Table A-2E and A-4E; data for CO_2 and N_2 from Table A-23E; and the relationship $h = \bar{h}/M$

Problem 12.61 (Continued) – Page 3

$$\frac{\dot{Q}_{cv}}{\dot{n}_{mix,1}} = \left[0.15\bar{h}_{CO_2} + 0.1154\bar{h}_{H_2O(v)} + 0.60\bar{h}_{N_2}\right]_2 + \left[0.1346\bar{h}_{H_2O(l)}\right]_2 - \left[0.15\bar{h}_{CO_2} + 0.25\bar{h}_{H_2O(v)} + 0.60\bar{h}_{N_2}\right]_1$$

$$\frac{\dot{Q}_{cv}}{\dot{n}_{mix,1}} = 0.15\left[\bar{h}_{CO_2}(T_2) - \bar{h}_{CO_2}(T_1)\right] + 0.60\left[\bar{h}_{N_2}(T_2) - \bar{h}_{N_2}(T_1)\right] + M_{H_2O}\left[0.1154h_g(T_2) + 0.1346h_f(T_2) - 0.25h_g(T_1)\right]$$

$$= 0.15[4460.1 - 15325.3]\frac{\text{Btu}}{\text{lbmol}} + 0.60[4061.2 - 11104.3]\frac{\text{Btu}}{\text{lbmol}} + 18.02\frac{\text{lb}}{\text{lbmol}}[0.1154(1115.65) + 0.1346(92.99) - 0.25(1586.4)]\frac{\text{Btu}}{\text{lb}}$$

$$= (-1629.8 - 4225.9 - 4601.2)\frac{\text{Btu}}{\text{lbmol}}$$

$$= -10{,}456.9\frac{\text{Btu}}{\text{lbmol mixture}}$$

For the exiting mixture,

$$M_{mix} = \sum_{i=1}^{j} y_i M_i = (0.15)(44.01) + (0.25)(18.02) + (0.6)(28.01) = 27.9\frac{\text{lb}}{\text{lbmol mixture}}$$

$$\frac{\dot{Q}_{cv}}{\dot{m}_{mix,1}} = \left(\frac{\dot{Q}_{cv}}{\dot{n}_{mix,1}}\right)\frac{1}{M_{mix}} = -10{,}456.9\frac{\text{Btu}}{\text{lbmol(mix)}}\left(\frac{1}{27.9\frac{\text{lb}}{\text{lbmol(mix)}}}\right) = -374.8\frac{\text{Btu}}{\text{lb mixture}} \longleftarrow$$

---

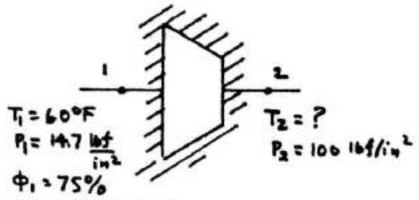
1. For the entering mixture, $p_{v1} = 3.67$ lbf/in.^{2}. At this pressure, the enthalpy of water vapor varies little with pressure and is determined mainly by temperature.

PROBLEM 12.62

KNOWN: Steady-state operating data are provided for a compressor.

FIND: (a) For $\eta_c = 0.8$, determine the exit temperature, the work and exergy destruction per unit mass of dry air flowing. (b) Plot the quantities of part (a) versus η_c ranging from 0.7 to 1.0.

SCHEMATIC & GIVEN DATA:



Note: Since T_2 would be greater than T_1, as a consequence of compression, there is no possibility of condensation.

ENGINEERING MODEL: (1) The compressor operates at steady state and is well insulated. (2) Changes in kinetic and potential energy effects from inlet to exit can be ignored. (3) The moist air acts as an ideal gas and each component adheres to the Dalton model. (4) $T_0 = 520°R$.

ANALYSIS: At steady state mass rate balances give $\dot{m}_{a1} = \dot{m}_{a2} \equiv \dot{m}_a$ and $\dot{m}_{v1} = \dot{m}_{v2} \equiv \dot{m}_v$. Using assumptions 1 – 3 an energy rate balance reduces to

$$0 = \cancel{\dot{Q}_{cv}}^{\,0} - \dot{W}_{cv} + [\dot{m}_a h_a(T_1) + \dot{m}_v h_v(T_1)] - [\dot{m}_a h_a(T_2) + \dot{m}_v h_v(T_2)]$$

or

$$\dot{W}_{cv} = \dot{m}_a [h_a(T_1) - h_a(T_2)] + \dot{m}_v [h_v(T_1) - h_v(T_2)]$$

Noting that $\omega = \dot{m}_v/\dot{m}_a$ this becomes

$$\frac{\dot{W}_{cv}}{\dot{m}_a} = [h_a(T_1) - h_a(T_2)] + \omega [h_v(T_1) - h_v(T_2)] \qquad (1)$$

This applies both for the actual compression and an isentropic compression from the same initial state. For the isentropic compression

$$\left(\frac{\dot{W}_{cv}}{\dot{m}_a}\right)_s = [h_a(T_1) - h_a(T_{2s})] + \omega [h_v(T_1) - h_v(T_{2s})] \qquad (2)$$

where T_{2s} is the temperature at the exit for an isentropic compression.

The actual work requirement can be found using the given isentropic compression efficiency together with Eq. (2):

$$\frac{\dot{W}_{cv}}{\dot{m}_a} = \frac{(\dot{W}_{cv}/\dot{m}_a)_s}{\eta_c} \qquad (3)$$

To determine the numerator of Eq. (3) requires T_{2s} which can be found in an iterative procedure using the requirement that the specific entropy of the mixture does not change from inlet to exit in an isentropic compression. When expressed per unit mass of dry air, this condition takes the form

$$[s_a(T_1, p_{a1}) + \omega s_v(T_1, p_{v1})] = [s_a(T_{2s}, p_{a2}) + \omega s_v(T_{2s}, p_{v2})]$$

or

$$[s_a(T_{2s}, p_{a2}) - s_a(T_1, p_{a1})] + \omega [s_v(T_{2s}, p_{v2}) - s_v(T_1, p_{v1})] = 0$$

$$\left[s_a^\circ(T_{2s}) - s_a^\circ(T_1) - \frac{\bar{R}}{M_a} \ln \frac{y_a p_2}{y_a p_1}\right] + \omega \left[\frac{\bar{s}_v^\circ(T_{2s}) - \bar{s}_v^\circ(T_1) - \bar{R} \ln \frac{y_v p_2}{y_v p_1}}{M_v}\right] = 0$$

With values from Tables A-22E, 23E at 520°R

$$0 = \left[s_a^\circ(T_{2s}) - 0.59172 - \frac{1.986}{28.97} \ln \frac{100}{14.7}\right] + \omega \left[\frac{\bar{s}_v^\circ(T_{2s})}{M_v} - \frac{(44.921 + 1.986 \ln \frac{100}{14.7})}{18.02}\right] \qquad (4)$$

PROBLEM 12.62 (Contd.) - page 2

ω is found from Eq. 12.43 using $p_v = \phi_1 p_g(T_1) = 0.75(0.2563) = 0.1922$ lbf/in^2

Thus

$$\omega = 0.622\left(\frac{0.1922}{14.7 - 0.1922}\right) = 0.622\left(\frac{0.1922}{14.5078}\right) = 0.0082 \frac{lb(v)}{lb(a)}$$

With this Eq. (4) gives upon rearrangement

$$s_a^\circ(T_{2s}) + \left(\frac{0.0082}{18.02}\right)\bar{s}_v^\circ(T_{2s}) = 0.7453 \qquad (5)$$

Solving Eq (5) iteratively, $T_{2s} \approx 895°R$.

With T_{2s} known, Eq. (2) yields

$$\left(\frac{\dot{W}_{cv}}{\dot{m}_a}\right)_s = (124.27 - 215.03) + 0.0082\left(\frac{4122 - 7188.9}{18.02}\right)$$
$$= -90.76 - 1.40 = -92.16 \text{ Btu/lb(a)}$$

Equation (3) then gives

$$\left(\frac{\dot{W}_{cv}}{\dot{m}_a}\right) = \frac{-92.16}{0.8} = -115.2 \frac{\text{Btu}}{\text{lb(a)}} \longleftarrow \frac{\dot{W}_{cv}}{\dot{m}_a}$$

Finally, with $(\dot{W}_{cv}/\dot{m}_a)$ known, Eq. (1) can be used iteratively to find T_2. Thus, upon rearrangemt

$$h_a(T_2) + \frac{\omega}{M_v}\bar{h}_v(T_2) = h_a(T_1) + \frac{\omega}{M_v}\bar{h}_v(T_1) - \left(\frac{\dot{W}_{cv}}{\dot{m}_a}\right)$$
$$= 124.27 + \frac{0.0082}{18.02}(4122) + 115.2$$
$$= 241.35 \text{ Btu/lb(a)}$$

Solving this expression iteratively gives $T_2 \approx 987°R \longleftarrow T_2$

The exergy destruction can be obtained using $(\dot{E}_d/\dot{m}_a) = T_0(\dot{\sigma}_{cv}/\dot{m}_a)$, where $\dot{\sigma}_{cv}$ is obtained from an entropy balance.

$$(\dot{\sigma}_{cv}/\dot{m}_a) = \left[s_a^\circ(T_2) - s_a^\circ(T_1) - \frac{\bar{R}}{M_a}\ln\frac{p_2}{p_1}\right] + \omega\left[\frac{\bar{s}_v^\circ(T_2) - \bar{s}_v^\circ(T_1) - \bar{R}\ln\frac{p_2}{p_1}}{M_v}\right]$$

$$= \left[0.7472 - 0.59172 - \frac{1.986}{28.97}\ln\frac{100}{14.7}\right] + (0.0082)\left[\frac{50.079 - 44.821 - 1.986\ln(100/14.7)}{18.02}\right]$$

$$= 0.0240 + 0.0007 = 0.0247 \text{ Btu/lb(a)·°R}$$

Then, $(\dot{E}_d/\dot{m}_a) = 520°R\left(0.0247 \frac{\text{Btu}}{\text{lb(a)·°R}}\right) = 12.84 \frac{\text{Btu}}{\text{lb(a)}} \longleftarrow \frac{\dot{E}_d}{\dot{m}_a}$

b) The data for the required plots are obtained using IT, as follows:

PROBLEM 12.62 (Cont'd.) - page 3

IT Code

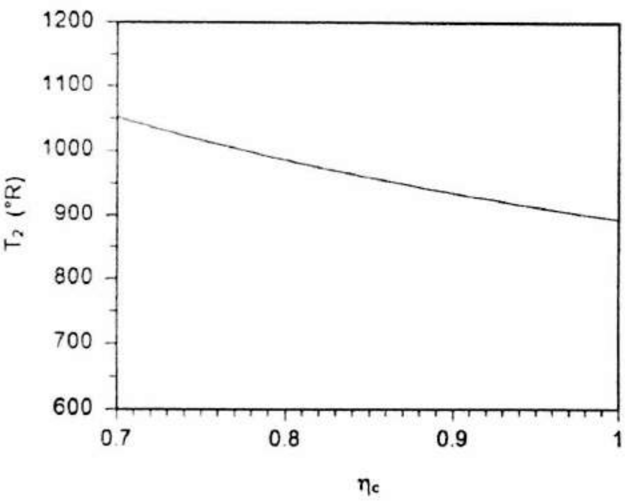
```
T1 = 520
phi1 = 0.75
p1 = 14.7
p2 = 100
w = w_Tphi(T1,phi1,p1)
mdota = 1  // Assume a unit mass flow rate.

Wdot / mdota = (ha1 - ha2) + w * (hv1 - hv2)
Wdots / mdota = (ha1 - ha2s) + w * (hv1 - hv2s)
Wdot / mdota = (Wdots / mdota) / eta_c
eta_c = 0.8

ha1 = h_T("Air", T1)
ha2 = h_T("Air", T2)
hv1 = h_T("H2O", T1)
hv2 = h_T("H2O", T2)
ha2s = h_T("Air", T2s)
hv2s = h_T("H2O", T2s)

pg1 = Psat_T("Water/Steam", T1)
pv1 = phi1 * pg1
yv = pv1 / p1
ya = 1 - yv
pa1 = p1 - pv1
pv2 = yv * p2
pa2 = p2 - pv2

sa1 = s_TP("Air", T1, pa1)
sa2s = s_TP("Air", T2s, pa2)
sv1 = s_TP("H2O", T1, pv1)
sv2s = s_TP("H2O", T2s, pv2)
(sa1 - sa2s) + w * (sv1 - sv2s) = 0
sa2 = s_TP("Air", T2, pa2)
sv2 = s_TP("H2O", T2, pv2)

Edot_d / mdota = To * ((sa2 - sa1) + w * (sv2 - sv1))
To = 520
```

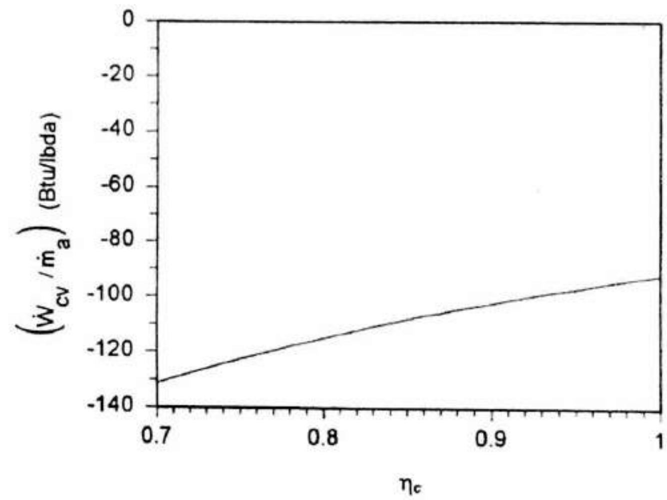
IT Results for $\eta_c = 0.8$

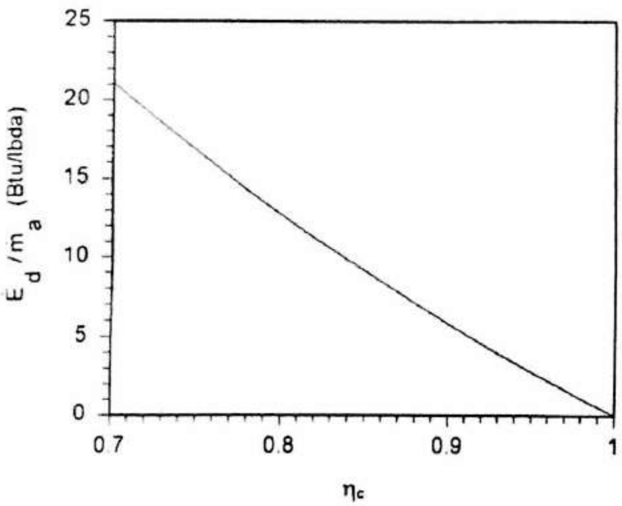
$p_{a1} = 14.51$ lbf/in.^{2}
$p_{a2} = 98.68$ lbf/in.^{2}
$p_{v1} = 0.1945$ lbf/in.^{2}
$p_{v2} = 1.323$ lbf/in.^{2}
$T_{2s} = 894.4$ °R
$w = 0.008341$
$\left(\dot{W}_{cv}/\dot{m}_a\right)_s = -91.98$ Btu / lb(da)
$T_2 = 986.1$ °R
$\left(\dot{W}_{cv}/\dot{m}_a\right) = -115$ Btu / lb(da)

$\dot{E}_d/\dot{m}_a = 12.73$ Btu / lb(da)

PLOTS:







From the plots, we see that lower isentropic compressor efficiency corresponds to increased exit temperature, power input, and exergy destruction, as expected.

Problem 12.63

Dry air enters a device operating at steady state at 27°C, 2 bar with a volumetric flow rate of 300 m^3/min. Liquid water is injected and a moist air stream exits at 15°C, 2 bar, and 91% relative humidity. Determine

(a) the mass flow rate at the exit, in kg/min.

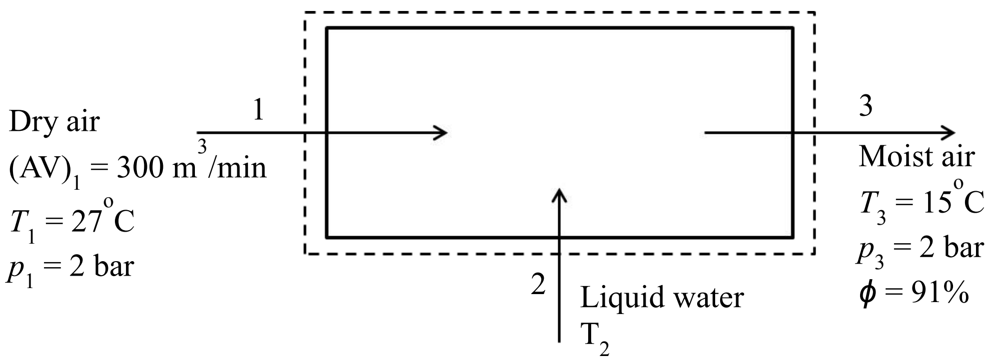
(b) the temperature, in °C, of the liquid water injected into the air stream.

Ignore heat transfer between the device and its surroundings and ignore kinetic and potential energy effects.

Known:
Dry air at a specified temperature, pressure, and volumetric flow rate enters a device operating at steady state. Liquid water is injected and a moist air stream exits at a known temperature, pressure and relative humidity.

Find:
Determine (a) the mass flow rate at the exit and (b) the temperature of the injected liquid water.

Schematic & Given Data:



Engineering Model:

(1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy.

(2) The liquid enters as saturated liquid.

(3) The dry air and moist air streams act as ideal gases.

Analysis:

(a) At steady state mass rate balances give $\dot{m}_{a_1} = \dot{m}_{a_3} = \dot{m}_a$ and $\dot{m}_{v_2} = \dot{m}_w$. The mass flow rate at 1 is obtained using the volumetric flow rate and the ideal gas equation of state:

$$\dot{m}_{a_1} = \frac{(AV)_1}{v_1} = \frac{p_1(AV)_1}{(\bar{R}/M_a)T_1} = \frac{\left(2 \times 10^5 \frac{N}{m^2}\right)\left(300 \frac{m^3}{min}\right)}{\left(\frac{8314}{28.97} \frac{N \cdot m}{kg \cdot K}\right)(300\ K)} = 696.897 \frac{kg}{min}$$

The mass flow rate at 3 is:

$$\dot{m}_3 = \dot{m}_{a_3} + \dot{m}_{v_3} = \dot{m}_a(1 + \omega_3) \quad (1)$$

To find ω_3 and using Table A-2, $p_{v_3} = \phi_3 p_g(T_3) = (0.91)(0.01705 \text{ bar}) = 0.01552 \text{ bar}$, and so with Eq. 12.43

$$\omega_3 = 0.662\frac{p_{v_3}}{p_3 - p_{v_3}} = 0.622\frac{0.01552}{2 - 0.01552} = 0.00486\frac{\text{kg (vapor)}}{\text{kg (air)}} \longleftarrow$$

Accordingly, Eq. (1) gives

$$\dot{m}_3 = \left(696.897\frac{\text{kg}}{\text{min}}\right)(1 + 0.00486) = 700.284\frac{\text{kg}}{\text{min}}$$

The mass flow rate of the incoming liquid is then

$$\dot{m}_w = \dot{m}_3 - \dot{m}_{a_1} = 700.284 - 696.897 = 3.387\frac{\text{kg}}{\text{min}}$$

(b) An energy rate balance at steady state reads

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_a h_a(T_1) + \dot{m}_w h_f(T_2) - \left[\dot{m}_a h_a(T_3) + \dot{m}_{v_3} h_g(T_3)\right]$$

Thus

$$h_f(T_2) = \frac{\dot{m}_a\left[h_a(T_3) - h_a(T_1) + \omega_3 h_g(T_3)\right]}{\dot{m}_w}$$

Using data from Tables A-2 and A-22:

$$h_f(T_2) = \frac{\left(696.897\frac{\text{kg}}{\text{min}}\right)[288.15 - 300.19 + 0.00486(2528.9)]\frac{\text{kJ}}{\text{kg}}}{3.387\frac{\text{kg}}{\text{min}}} = 51.53\frac{\text{kJ}}{\text{kg}}$$

#1 Interpolation in Table A-2 gives

$$T_2 \approx 12°\text{C} \longleftarrow$$

<u>Comment:</u>

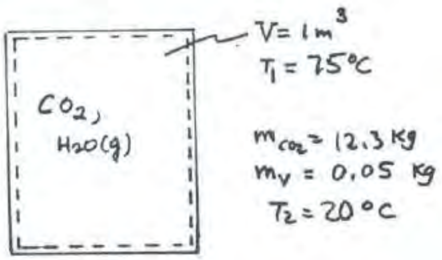
#1 Owing to the small amount of water injected, the calculated value for T_2 is relatively sensitive to round off of the intermediate values.

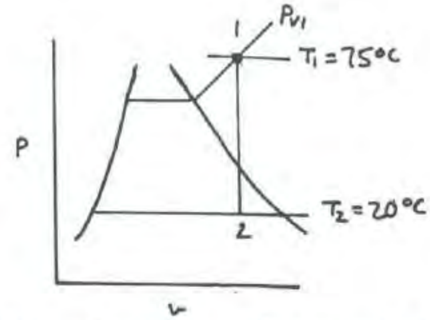
PROBLEM 12.64

KNOWN: A tank having a known volume initially contains 12.3 kg of CO_2 and 0.05 kg of water vapor at 75°C, 15 bar. The tank contents are cooled to 20°C.

FIND: Determine the heat transfer.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) As shown on the figure, the system is the tank contents. (2) There are no changes in kinetic or potential energy. (3) The gas mixture behaves as an ideal gas, with each component adhering to the Dalton model.

ANALYSIS: The first step is to determine if condensation occurs. As the total volume of the tank and the total mass of water present are constant, the water undergoes a constant specific volume process as the tank contents are cooled (see T-v diagram). The specific volume of the water is $v = 1\,m^3/0.05\,kg = 20.00\ m^3/kg$. By inspection of v_g data in Table A-2, condensation would occur at a temperature between 38 and 40°C. Accordingly, there is condensation in this case.

An energy balance reduces to give $\Delta U = Q - \cancel{W}$, or $Q = \Delta U$:

$$Q = [m_{CO_2} u_{CO_2}(T_2) + m_{v2} u_g(T_2) + m_{w2} u_f(T_2)] - [m_{CO_2} u_{CO_2}(T_1) + m_{v1} u_g(T_1)]$$

$$= \frac{m_{CO_2}[\bar{u}_{CO_2}(T_2) - \bar{u}_{CO_2}(T_1)]}{M_{CO_2}} + [m_{v2} u_g(T_2) - m_{v1} u_g(T_1) + m_{w2} u_f(T_2)] \qquad (1)$$

To find m_{v2} and m_{w2}, note that the quality of the two-phase liquid-vapor mixture of water at the final state is

$$x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{20.00 - 0.0010018}{57.791 - 0.0010018} = 0.3461$$

Accordingly, $m_{v2} = x_2 m_{v1} = (0.3461)(0.05) = 0.01731$ kg. So, $m_{w2} = 0.03269$ kg.

Returning to Eq.(1) and using data from Tables A-2 and A-23

$$Q = \left(\frac{12.3}{44.01}\right)(6739 - 8377) + [(0.01731)(2402.9) - (0.05)(2475.9) + (0.03269)(83.95)]$$

$$= -457.79 + [41.59 - 123.80 + 2.74] = -537.3\ kJ \longleftarrow Q$$

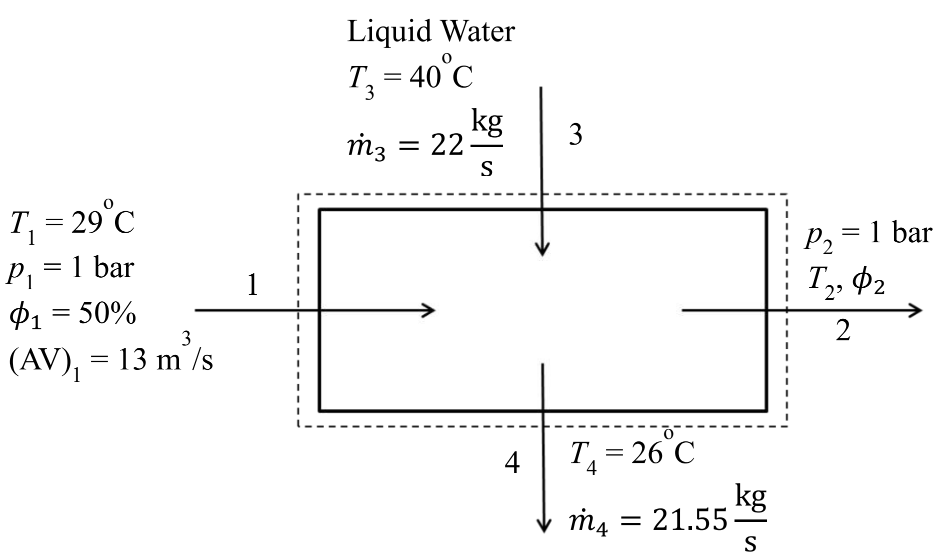
Problem 12.65

At steady state, moist air at 29°C, 1 bar, and 50% relative humidity enters a device with a volumetric flow rate of 13 m^3/s. Liquid water at 40°C is sprayed into the moist air with a mass flow rate of 22 kg/s. The liquid water that does not evaporate into the moist air stream is drained and flows to another device at 26°C with a mass flow rate of 21.55 kg/s. A single moist air stream exits at 1 bar. Determine the temperature and relative humidity of the moist air stream exiting. Ignore heat transfer between the device and its surroundings and kinetic and potential energy effects.

Known:
Operating data are provided for a device at steady state for which there are incoming and exiting moist air and liquid streams.

Find:
Determine the temperature and relative humidity of the exiting moist air stream.

Schematic & Given Data:



Engineering Model:
(1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{\text{cv}} = \dot{W}_{\text{cv}} = 0$ and negligible effects of kinetic and potential energy.
(2) The liquid streams enter and exit as saturated liquid.
(3) The moist air streams adhere to the ideal gas model.

Analysis:
At steady state mass rate balances give $\dot{m}_{\text{a}_1} = \dot{m}_{\text{a}_2} = \dot{m}_{\text{a}}$ and $\dot{m}_{\text{v}_1} + \dot{m}_3 = \dot{m}_{\text{v}_2} + \dot{m}_4$. Thus

$$\dot{m}_{\text{v}_2} = \dot{m}_{\text{v}_1} + \dot{m}_3 - \dot{m}_4 \quad (1)$$

Using the given volumetric flow rate and the ideal gas equation of state with values from Tables A-1 and A-2

$$\dot{m}_{v_1} = \frac{(AV)_1}{v_{v_1}} = \frac{p_{v_1}(AV)_1}{(\bar{R}/M_v)T_1} = \frac{\left(\phi_1 p_g(T_1)\right)(AV)_1}{(\bar{R}/M_v)T_1} = \frac{(0.5)\left(0.04008 \times 10^5 \frac{N}{m^2}\right)\left(13 \frac{m^3}{s}\right)}{\left(\frac{8314}{18.02} \frac{N \cdot m}{kg \cdot K}\right)(302\ K)}$$

$$= 0.187 \frac{\text{kg (vapor)}}{\text{s}}$$

$$\dot{m}_{a_1} = \frac{p_{a_1}(AV)_1}{(\bar{R}/M_a)T_1} = \frac{\left[\left(1 - (0.5)(0.04008)\right) * 10^5 \frac{N}{m^2}\right]\left(13 \frac{m^3}{s}\right)}{\left(\frac{8314}{28.97} \frac{N \cdot m}{kg \cdot K}\right)(302\ K)} = 14.7 \frac{\text{kg (air)}}{\text{s}}$$

Then, from Eq. (1), $\dot{m}_{v_2} = 0.187 + 22 - 21.55 = 0.637 \frac{\text{kg (vapor)}}{\text{s}}$. The humidity ratios are

$$\omega_1 = \frac{\dot{m}_{v_1}}{\dot{m}_a} = \frac{0.187}{14.7} = 0.01272 \frac{\text{kg (vapor)}}{\text{kg (air)}}, \omega_2 = \frac{0.637}{14.7} = 0.04333 \frac{\text{kg (vapor)}}{\text{kg (air)}}$$

The temperature T_2 can be determined using an energy rate balance which reduces to give

$$0 = \cancel{\dot{Q}}_{cv} - \cancel{\dot{W}}_{cv} + \dot{m}_a h_{a_1} + \dot{m}_3 h_3 - \dot{m}_4 h_4 - \left[\dot{m}_a h_{a_2} + \dot{m}_{v_2} h_{v_2}\right]$$

Rearranging and inserting values from Tables A-2 and A-22

$$h_a(T_2) + \omega_2 h_{g_2} = \left[h_a(T_1) + \omega_1 h_{g_1}\right] + \frac{\dot{m}_3}{\dot{m}_a} h_{f_3} - \frac{\dot{m}_4}{\dot{m}_a} h_{f_4}$$

$$= [302.2 + (0.01272)(2554.5)] \frac{kJ}{kg}$$

$$+ \frac{\left(22 \frac{kg}{s}\right)\left(167.57 \frac{kJ}{kg}\right) - \left(21.55 \frac{kg}{s}\right)\left(109.07 \frac{kJ}{kg}\right)}{14.7 \frac{kg}{s}} = 425.58 \frac{kJ}{kg_a}$$

Solving this iteratively using data from Tables A-2 and A-22

$T_2 \approx 41°C$ $\longleftarrow$

To find ϕ_2, p_{v2}, solve Eq. 12.43

$$p_{v_2} = \frac{p_2 \omega_2}{0.622 + \omega_2} = \frac{(1\ \text{bar})(0.0433)}{0.622 + 0.0433} = 0.0651\ \text{bar}$$

Using a value from Table A-2 with interpolation and Eq. 12.44

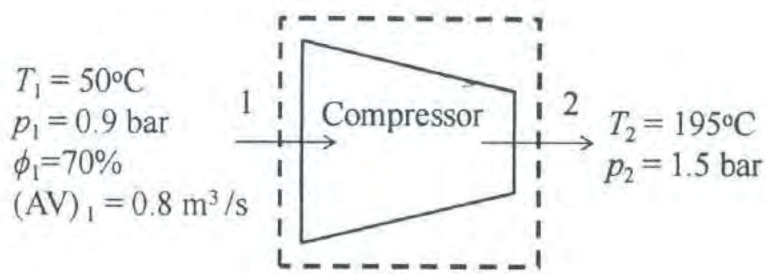
$$\phi_2 = \frac{p_{v_2}}{p_g(T_2)} = \frac{0.0651}{0.0783} = 0.831 = 83.1\%$$ $\longleftarrow$

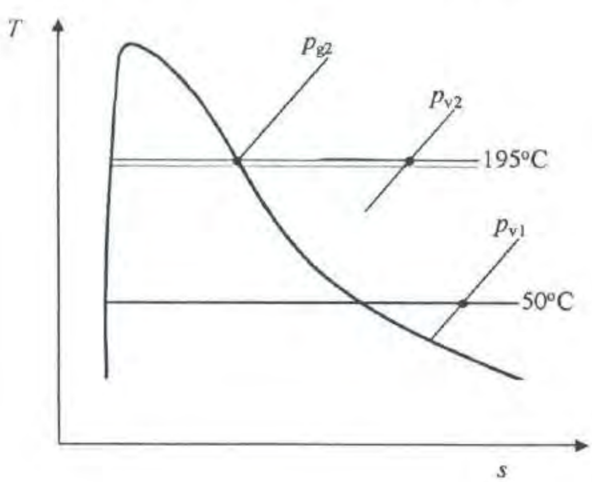
12.66 Air enters a compressor operating at steady state at 50°C, 0.9 bar, 70% relative humidity with a volumetric flow rate of 0.8 m^3/s. The moist air exits the compressor at 195°C, 1.5 bar. Assuming the compressor is well insulated, determine
(a) the relative humidity at the exit.
(b) the power input, in kW.
(c) the rate of entropy production, in kW/K.

KNOWN: Steady state operating data are provided for a compressor handling moist air. The volumetric flow rate at the inlet is known.

FIND: Determine (a) the relative humidity at the exit, (b) the power input, and (c) the rate of entropy production.

Schematic and Known Data:





ENGINEERING MODEL:
(1) The control volume is at steady state.
(2) For the control volume $\dot{Q}_{cv} = 0$ and kinetic and potential effects are negligible.
(3) The moist air behaves as an ideal gas mixture.

Problem 12.66 (Continued) – Page 2

ANALYSIS:

First, we must check to see if condensation occurs at the exit condition. Referring to Table A-2 at $T2$ = 195°C, the corresponding saturation pressure is approximately 14 bar, denoted as p_{g2} on the T-v diagram above. Since the exit pressure of the *mixture* is 1.5 bar, and p_{v2} is even less than 1.5 bar, condensation does not occur.

(a) Mass rate balances reduce to $\dot{m}_{a1} = \dot{m}_{a2}$, $\dot{m}_{v1} = \dot{m}_{v2}$, or $\omega_2 = \omega_1$. Thus

$$(0.622)\frac{p_{v1}}{p_1 - p_{v1}} = (0.622)\frac{p_{v2}}{p_2 - p_{v2}} \Rightarrow \frac{p_{v2}}{p_2} = \frac{p_{v1}}{p_1} \Rightarrow p_{v2} = p_{v1}\left(\frac{p_2}{p_1}\right)$$

Using the definition of relative humidity and Table A-2 for the p_{g1} value

$$\begin{aligned} p_{v1} &= \phi_1 p_{g1} \\ &= (0.7)(0.1235\text{ bar}) \\ &= 0.08645\text{ bar} \end{aligned}$$

$$p_{v2} = (0.08645\text{ bar})\left(\frac{1.5\text{ bar}}{0.9\text{ bar}}\right) = 0.1441\text{ bar}$$

Accordingly, with the p_{g2} value from Table A-2 using interpolation:

$$\phi_2 = \frac{p_{v2}}{p_{g2}} = \frac{0.1441\text{ bar}}{14.04\text{ bar}} = 0.01026\,(1.03\%) \longleftarrow$$

(b) Reducing an energy rate balance gives after rearrangement

#1 $$\dot{W}_{cv} = \dot{m}_a\left[(h_a + \omega h_v)_1 - (h_a + \omega h_v)_2\right] = \dot{m}_a\left[h_{a1} - h_{a2} + \omega(h_{v1} - h_{v2})\right]$$

To solve, determine the mass flow rate of dry air as $\dot{m}_a = \dfrac{(\mathrm{AV})_1}{v_{a1}}$.

The specific volume of the dry air, v_{a1}, is found as follows:

Problem 12.66 (Continued) – Page 3

$$
\begin{aligned}
p_{a1} &= p_1 - p_{v1} \\
&= (0.9\ \text{bar}) - (0.08645\ \text{bar}) \\
&= 0.8136\ \text{bar}
\end{aligned}
$$

$$
\begin{aligned}
v_{a1} &= \left(\frac{(\bar{R}/M)T_1}{p_{a1}} \right) \\
&= \left[\frac{\left(\dfrac{8.314}{28.97} \dfrac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) 323\ \text{K}}{0.8136\ \text{bar}} \right] \left| \frac{1000\ \text{J}}{1\ \text{kJ}} \right| \left| \frac{1\ \text{N} \cdot \text{m}}{1\ \text{J}} \right| \left| \frac{1\ \text{bar}}{10^5\ \dfrac{\text{N}}{\text{m}^3}} \right| \\
&= 1.139\ \frac{\text{m}^3}{\text{kg}}
\end{aligned}
$$

Thus

$$
\dot{m}_a = \frac{(0.8\ \text{m}^3/\text{s})}{1.139\ \text{m}^3/\text{kg}} = 0.702\ \frac{\text{kg (air)}}{\text{s}}
$$

Determine the humidity ratio with p_{v1} from above. Because there is no condensation: $\omega_1 = \omega_2 = \omega$. Thus

$$
\omega = 0.622 \frac{p_{v1}}{p_1 - p_{v1}} = (0.622) \frac{0.08645\ \text{bar}}{0.9\ \text{bar} - 0.08645\ \text{bar}} = 0.066 \frac{\text{kg (vapor)}}{\text{kg (air)}}
$$

Specific enthalpy values are obtained from Table A-23 for water vapor and A-22 for dry air. Inserting values

$$
\begin{aligned}
\dot{W}_{cv} &= \left(0.702 \frac{\text{kg}}{\text{s}} \right) \left[\left(323.30 \frac{\text{kJ}}{\text{kg}} - 470.20 \frac{\text{kJ}}{\text{kg}} \right) + (0.066) \left(\frac{(10740 - 15707) \dfrac{\text{kJ}}{\text{kmol}}}{18.02 \dfrac{\text{kg}}{\text{kmol}}} \right) \right] \\
&= \left(0.702 \frac{\text{kg}}{\text{s}} \right) \left[\left(-146.9 \frac{\text{kJ}}{\text{kg}} \right) + \left(-18.19 \frac{\text{kJ}}{\text{kg}} \right) \right] \\
&= -115.9\ \text{kW}
\end{aligned}
$$

The negative sign denotes a power input, as expected.

(c) An entropy rate balance reduces to give

$$\dot{\sigma}_{cv} = \dot{m}_a \left[(s_{a2} - s_{a1}) + \omega (s_{v2} - s_{v1}) \right]$$

$$= \dot{m}_a \left[\left(s^o_{a2} - s^o_{a1} - \frac{\overline{R}}{M_a} \ln\left(\frac{p_{a2}}{p_{a1}}\right) \right) + \omega \left(s^o_{v2} - s^o_{v1} - \frac{\overline{R}}{M_v} \ln\left(\frac{p_{v2}}{p_{v1}}\right) \right) \right]$$

Using $p_{a2}/p_{a1} = p_{v2}/p_{v1} = p_2/p_1$, s^o data from Table A-22, and $\bar{s}^o$ data from Table A-23

$$\dot{\sigma}_{cv} = 0.702 \frac{\text{kg}}{\text{s}} \left\{ \left[(2.15165 - 1.77625) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} - \frac{8.314}{28.97} \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \ln\left(\frac{1.5 \text{ bar}}{0.9 \text{ bar}} \right) \right] \right.$$

$$\left. + 0.066 \left[\frac{(204.097 - 191.409) \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}}{18.02 \frac{\text{kg}}{\text{kmol}}} - \frac{8.314}{18.02} \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \ln\left(\frac{1.5 \text{ bar}}{0.9 \text{ bar}} \right) \right] \right\}$$

$$= 0.702 \frac{\text{kg}}{\text{s}} \{ [0.2288] + 0.066 [0.4684] \} \frac{\text{kJ}}{\text{kg} \cdot \text{K}} = 0.182 \frac{\text{kW}}{\text{K}} \longleftarrow$$

---

1. Alternatively, parts (b) and (c) could be solved on the basis of constant specific heats c_{pa}, c_{pv} evaluated at the mean temperatures, for example.

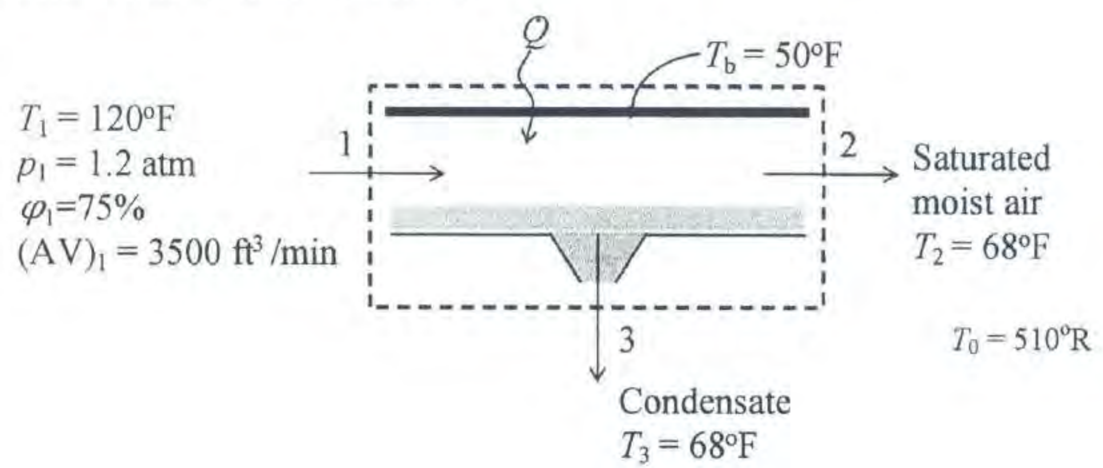
12.67 Moist air enters a control volume operating at steady state with a volumetric flow rate of 3500 ft^3/min. The moist air enters at 120°F, 1.2 atm, and 75% relative humidity. Heat transfer occurs through a surface maintained at 50°F. Saturated moist air and condensate exit the control volume at 68°F. Assuming $\dot{W}_{cv} = 0$, and kinetic and potential energy effects are negligible, determine

(a) the mass flow rate of condensate, in lb/min.
(b) the rate of heat transfer, in Btu/min.
(c) the rate of entropy production, in Btu/°R · min.
(d) the rate of exergy destruction, in Btu/min, for T_o = 50°F.

KNOWN: Operating data are provided for a control volume at steady state.

FIND: Determine (a) the mass flow rate of the condensate, (b) the heat transfer rate, (c) entropy production rate, and (d) the rate of exergy destruction.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

(1) The control volume is at steady state.
(2) For the control volume $\dot{W}_{cv} = 0$, and kinetic and potential energy effects are negligible.
(3) The moist air adheres to ideal gas principles.
(4) The total pressure remains constant from inlet to exit.
(5) Use $c_p\Delta T$ to determine changes in specific enthalpy of dry air.
(6) The temperature of the reference environment is 50°F: T_o = 510°R.

Problem 12.67 (Continued) – Page 2

ANALYSIS:

(a) Mass rate balances reduce to

$$\dot{m}_{a1} = \dot{m}_{a2} \equiv \dot{m}_a$$

$$\dot{m}_{v1} = \dot{m}_{v2} + \dot{m}_3$$

Thus

$$\dot{m}_3 = \dot{m}_a(\omega_1 - \omega_2) \quad (1)$$

From Table A-2E: p_{g1}=1.695 lbf/in.^{2} and p_{g2} = 0.3391 lbf/in.^{2} and p_{v1} is found using

$$p_{v1} = \phi_1 p_{g1}$$
$$= (0.75)(1.695 \text{ lbf/in.}^2) = 1.2713 \text{ lbf/in.}^2$$

Calculate humidity ratios using

$$p_1 = 1.2 \text{ atm}\left|\frac{14.696 \text{ lbf/in.}^2}{1 \text{ atm}}\right| = 17.8 \text{ lbf/in.}^2$$

$$\omega_1 = 0.622\left[\frac{p_{v1}}{p_1\text{-}p_{v1}}\right] = 0.622\left[\frac{1.2713 \text{ lbf/in.}^2}{(17.8 - 1.2713)\text{lbf/in.}^2}\right] = 0.0478\frac{\text{lb (vapor)}}{\text{lb (air)}}$$

$$\omega_2 = 0.622\left[\frac{p_{g2}}{p_1\text{-}p_{g2}}\right] = 0.622\left[\frac{0.3391 \text{ lbf/in.}^2}{(17.8 - 0.3391)\text{lbf/in.}^2}\right] = 0.0121\frac{\text{lb (vapor)}}{\text{lb (air)}}$$

Determine the mass flow rate of the air as $\dot{m}_a = \frac{(\text{AV})_1}{v_{a1}}$. The specific volume of dry air is found using the ideal gas equation of state and the partial pressure of the air at 1.

$$p_{a1} = p_1 - p_{v1} = (17.8 \text{ lbf/in.}^2) - (1.2713 \text{ lbf/in.}^2) = 16.529 \text{ lbf/in.}^2$$

$$v_{a1} = \left(\frac{\overline{R}}{M_a}\frac{T_1}{p_{a1}}\right) = \left(\frac{1545 \frac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot{}^\circ\text{R}}}{28.97 \frac{\text{lb}}{\text{lbmol}}}\right)\left(\frac{580\ {}^\circ\text{R}}{16.329 \text{ lbf/in.}^2}\right)\left|\frac{1 \text{ ft}^2}{144 \text{ in.}^2}\right| = 13.155\frac{\text{ft}^3}{\text{lb}}$$

The mass flow rate of dry air is

$$\dot{m}_a = \frac{(3500 \text{ ft}^3/\text{min})}{13.155 \text{ ft}^3/\text{lb}} = 266.06\frac{\text{lb (air)}}{\text{min}}$$

The mass flow of the condensate is determined using Eq. (1)

$$\dot{m}_3 = \left(266.06\frac{\text{lb (air)}}{\text{min}}\right)(0.0478 - 0.0121)\frac{\text{lb (vapor)}}{\text{lb (air)}} = 9.5\frac{\text{lb}}{\text{min}}$$

Problem 12.67 (Continued) – Page 3

(b) The energy rate balance reduces to

$$0 = \dot{Q}_{cv} - \underbrace{\dot{W}_{cv}}_{=0} + \dot{m}_a\left[(h_a - \omega h_v)_1 - (h_a - \omega h_v)_2\right] - \dot{m}_3 h_{f3}$$

where the specific enthalpies of the vapor at 1 and 2 are evaluated at the saturated vapor values for T_1 and T_2, respectively, and the specific enthalpy of the exiting condensate is evaluated as h_f at T_2. For water vapor, $h = h_g(T)$ from Table A-2E. For dry air, $c_{pa} = 0.24$ Btu/lb·°R.

$$\begin{aligned}
\dot{Q}_{cv} &= \dot{m}_a\left\{c_{pa}(T_2 - T_1) + (\omega_2 h_{g2} - \omega_1 h_{g1}) + (\omega_1 - \omega_2)h_{f3}\right\} \\
&= \left(266.06\,\frac{\text{lb (air)}}{\text{min}}\right)\left\{(0.24)(50 - 120)\frac{\text{Btu}}{\text{lb}} + [(0.0121)(1091.2) - (0.0478)(1113.5)]\frac{\text{Btu}}{\text{lb}}\right. \\
&\qquad \left. + (0.0478 - 0.0121)(36.09)\frac{\text{Btu}}{\text{lb}}\right\} \\
&= \left(266.06\,\frac{\text{lb (air)}}{\text{min}}\right)\{-16.8 + (-40.02) + (1.29)\}\frac{\text{Btu}}{\text{lb}} \\
&= -14{,}774\ \text{Btu/min}
\end{aligned}$$

(c) The entropy rate balance reduces to read

$$0 = \frac{\dot{Q}_{cv}}{T_b} + [\dot{m}_a s_{a1} + \dot{m}_{v1} s_{v1}] - [\dot{m}_a s_{a2} + \dot{m}_{v2} s_{v2}] - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$$

or

$$\dot{\sigma}_{cv} = -\frac{\dot{Q}_{cv}}{T_b} + \dot{m}_a[s_{a2} - s_{a1}] + \dot{m}_{v2} s_{v2} - \dot{m}_{v1} s_{v1} + \dot{m}_3 s_3$$

where

$$\begin{aligned}
\dot{m}_{v2} &= \omega_2 \dot{m}_a \\
\dot{m}_{v1} &= \omega_1 \dot{m}_a \\
\dot{m}_3 &= (\omega_1 - \omega_2)\dot{m}_a
\end{aligned}$$

Then

$$\dot{\sigma}_{cv} = -\frac{\dot{Q}_{cv}}{T_b} + \dot{m}_a\left[\left(c_{pa}\ln\frac{T_2}{T_1} - \frac{\overline{R}}{M}\ln\frac{p_{a2}}{p_{a1}}\right) + \omega_2 s_{g2} - \omega_1\left(s_{g1} - \frac{\overline{R}}{M}\ln\phi_1\right) + (\omega_1 - \omega_2)s_{f3}\right]$$

At 3 the liquid is saturated, $s_3 = s_f(T_2) = 0.07084\ \text{Btu/lb}\cdot{}^\circ\text{R}$. At 2 the vapor is saturated, $s_{v2} = s_{g2} = 2.0701\ \text{Btu/lb}\cdot{}^\circ\text{R}$. As discussed in Sec. 12.5.2, at 1

$$s_{v1} = s_{g1} - \frac{\overline{R}}{M}\ln(\phi_1) = 1.9336\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} - \frac{1.986\ \text{Btu/lbmol}\cdot{}^\circ\text{R}}{18.02\ \text{lb/lbmol}}\ln(0.75) = 1.965\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$

Problem 12.67 (Continued) – Page 4

The change in the entropy of the dry air is found with the ideal gas model and

$c_{pa} = 0.24 \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$ from Table A-20E

$$p_{a2} = p_1 - p_{v2} = (17.8 - 0.3391)\,\text{lbf/in.}^2 = 17.4609\,\text{lbf/in.}^2$$

$$p_{a1} = p_1 - p_{v1} = (17.8 - 1.2713)\,\text{lbf/in.}^2 = 16.5287\,\text{lbf/in.}^2$$

$$s_{a2} - s_{a1} = \left(c_{pa} \ln\frac{T_2}{T_1} - \frac{\overline{R}}{M}\ln\frac{p_{a2}}{p_{a1}} \right)$$

$$= 0.24 \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} \ln\left(\frac{528}{580}\right) - \frac{1.986\,\text{Btu/lbmol}\cdot{}^\circ\text{R}}{28.97\,\text{lb/lbmol}} \ln\left(\frac{17.4609}{16.5287}\right) = -0.0263 \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$

Inserting values in the entropy balance

$$\dot{\sigma}_{cv} = -\frac{-14774\,\text{Btu/min}}{510\,{}^\circ\text{R}} + \left(266.06\frac{\text{lb (air)}}{\text{min}}\right)\Big[(-0.0263) + (0.0121)(2.0701)$$

$$- (0.0478)(1.956) + (0.0478 + 0.0121)(0.07084)\Big]\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$

$$= 28.97\frac{\text{Btu/min}}{{}^\circ\text{R}} + \left(266.06\frac{\text{lb (air)}}{\text{min}}\right)\left[-0.0263 + 0.0250 - 0.0935 + 0.0042\right]$$

$$= 4.864\frac{\text{Btu/min}}{{}^\circ\text{R}} \longleftarrow$$

(d) Exergy destruction is found using $\dot{E}_{d,cv} = T_o\dot{\sigma}_{cv}$. Substituting previously found values

$$\dot{E}_{d,cv} = (510^\circ\text{R})\left(4.864\frac{\text{Btu/min}}{{}^\circ\text{R}}\right) = 2480.64\frac{\text{Btu}}{\text{min}} \longleftarrow$$

12.68 Moist air at 15°C, 1.3 atm, 63% relative humidity and a volumetric flow rate of 770 m^3/h enters a control volume at steady state and flows along a surface maintained at 187°C, through which heat transfer occurs. Liquid water at 15°C is injected at a rate of 7 kg/h and evaporates into the flowing stream. For the control volume, $\dot{W}_{cv} = 0$, and kinetic and potential energy effects are negligible. Moist air exits at 45°C, 1.3 atm. Determine

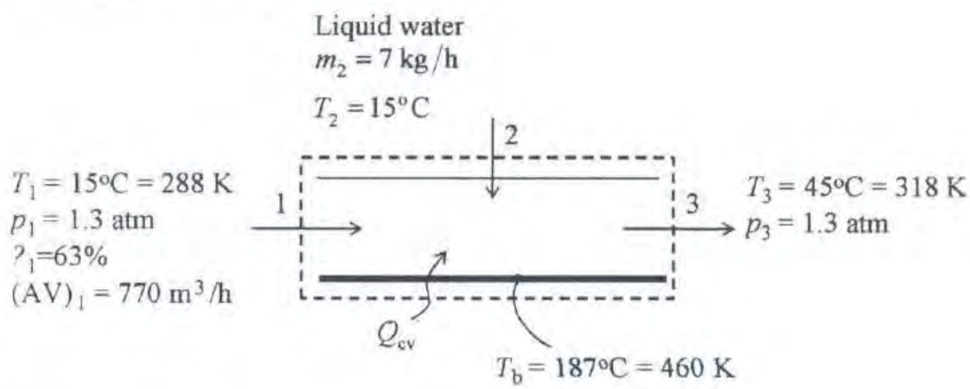
(a) the rate of heat transfer, in kW.

(b) the rate of entropy production, in kW/K.

KNOWN: Operating data are provided for a control volume at steady state.

FIND: Determine (a) the heat transfer rate, (b) entropy production rate.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

(1) The control volume is at steady state.

(2) For the control volume, $\dot{W}_{cv} = 0$, and kinetic and potential energy effects are negligible.

(3) The moist air adheres to ideal gas principles.

ANALYSIS:

(a) Using data from Table A-2, $p_{v1} = 0.01705$ bar. Thus

$$p_{v1} = \phi_1 p_{g1} = (0.63)(0.01705\text{ bar}) = 0.010742\text{ bar}$$

$$p_{a1} = p_1 - p_{v1} = (1.3\text{ atm})\left|\frac{1.01325\text{ bar}}{1\text{ atm}}\right| - 0.010742\text{ bar} = 1.3065\text{ bar}$$

Mass rate balances reduce to give $\dot{m}_{a1} = \dot{m}_{a3} \equiv \dot{m}_a$, $\dot{m}_{v1} + \dot{m}_2 = \dot{m}_{v3}$. The mass flow rate of the air is found using $\dot{m}_a = \dfrac{(AV)_1}{v_{a1}}$. The specific volume for the air at 1 is found using the ideal gas model.

Problem 12.68 (Continued) – Page 2

$$v_{a1} = \left(\frac{\bar{R}}{M_a}\frac{T_1}{p_{a1}}\right)$$
$$= \left(\frac{8.314\,\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}}{28.97\,\frac{\text{kg}}{\text{kmol}}}\right)\left(\frac{288\,\text{K}}{1.3065\,\text{bar}}\right)\left|\frac{1000\,\text{J}}{1\,\text{kJ}}\right|\left|\frac{1\,\text{N}\cdot\text{m}}{1\,\text{J}}\right|\left|\frac{1\,\text{bar}}{10^5\,\frac{\text{N}}{\text{m}^3}}\right|$$
$$= 0.6326\,\frac{\text{m}^3}{\text{kg}}$$

The mass flow rate is then found

$$\dot{m}_a = \frac{\left(770\,\text{m}^3/\text{h}\right)}{0.6326\,\text{m}^3/\text{kg}} = 1217.20\,\frac{\text{kg (air)}}{\text{h}}$$

The ratio between the mass flow rates is determined using

$$\frac{\dot{m}_2}{\dot{m}_a} = \frac{7\,\text{kg/h}}{1217.20\,\text{kg/h}} = 0.00575\,\frac{\text{kg (liquid)}}{\text{kg (air)}}$$

The humidity ratios at 1 and 3 are found as

$$\omega_1 = 0.622\frac{p_{v1}}{p_{a1}} = 0.622\frac{0.010742\,\text{bar}}{1.3065\,\text{bar}} = 0.00511\,\frac{\text{kg (liquid)}}{\text{kg (air)}}$$
$$\omega_3 = \frac{\dot{m}_2}{\dot{m}_a} + \omega_1 = 0.00575 + 0.00511 = 0.01086\,\frac{\text{kg (liquid)}}{\text{kg (air)}}$$

The partial pressure of the vapor at 3 is found from the humidity ratio at 3

$$\omega_3 = 0.622\frac{p_{v3}}{p_3 - p_{v3}} \Rightarrow \omega_3\left(p_3 - p_{v3}\right) = 0.622\,p_{v3} \Rightarrow p_{v3} = \frac{\omega_3 p_3}{0.622 + \omega_3}$$
$$p_{v3} = \frac{(0.01086)\left((1.3\,\text{atm})\left|\frac{1.01325\,\text{bar}}{1\,\text{atm}}\right|\right)}{0.622 + 0.01086} = 0.02260\,\text{bar}$$

The relative humidity at 3 is then found using Table A-2

$$\phi_3 = \frac{p_{v3}}{p_g(T_3)} = \frac{0.02260\,\text{bar}}{0.09593\,\text{bar}} = 0.23559$$

The energy and mass rate balances reduce to

$$0 = \dot{Q}_{cv} + \dot{m}_a\left[\left(h_a - \omega h_v\right)_1 + \frac{\dot{m}_2}{\dot{m}_a}h_2 - \left(h_a - \omega h_v\right)_3\right]$$

Rearranging and recognizing that $h_2 = h_{f2}$ and $h_v \approx h_g(T)$, we get

Problem 12.68 (Continued) – Page 3

$$\dot{Q}_{cv} = \dot{m}_a\left[(h_{a3}-h_{a1})+\omega_3 h_{g3}-\omega_1 h_{g1}-\frac{\dot{m}_2}{\dot{m}_a}h_{f2}\right]$$

Using enthalpy values for wnd liquid water ater vapor from Table A-2 and enthalpy values from A-22 for the air, the heat transfer rate is

$$\dot{Q}_{cv} = \left(1217.20\,\frac{kg}{h}\right)\left[(318.28-288.15)\frac{kJ}{kg}+(0.01086)(2583.2)\frac{kJ}{kg}\right.$$

$$\left.-(0.00511)(2528.9)\frac{kJ}{kg}-(0.00575)(62.99)\frac{kJ}{kg}\right]$$

$$=\left(1218.4\,\frac{kg}{h}\right)[30.13+28.054-12.923-0.362]\frac{kJ}{kg}$$

$$=54704.94\,\frac{kJ}{h}\left|\frac{1\,h}{3600\,s}\right|\left|\frac{1\,kW}{1\,kJ/s}\right| = 15.196\,kW$$

(b) The entropy rate balance reduces to read

$$0 = -\frac{\dot{Q}_{cv}}{T_b}+[\dot{m}_a s_{a1}+\dot{m}_{v1}s_{v1}]+\dot{m}_2 s_2-[\dot{m}_a s_{a3}+\dot{m}_{v3}s_{v3}]+\dot{\sigma}_{cv}$$

or

$$\dot{\sigma}_{cv} = -\frac{\dot{Q}_{cv}}{T_b}+\dot{m}_a[s_{a3}-s_{a1}]+\dot{m}_{v3}s_{v3}-\dot{m}_{v1}s_{v1}-\dot{m}_2 s_2$$

The mass flow rate values are determined from

$$\dot{m}_{v3} = \omega_3\dot{m}_a$$

$$\dot{m}_{v1} = \omega_1\dot{m}_a$$

Then, the entropy balance becomes

$$\dot{\sigma}_{cv} = -\frac{\dot{Q}_{cv}}{T_b}+\dot{m}_a\left[(s_{a3}-s_{a1})+\omega_3 s_{v3}-\omega_1 s_{v1}-\frac{\dot{m}_2}{\dot{m}_a}s_{f2}\right]$$

The specific entropy values are found using data from Table A-2 and the methods of Sec. 12.5.2.:

$$s_{v3} = s_{g3}-\frac{\overline{R}}{M_v}\ln\phi_3 = 8.1648\,\frac{kJ}{kg\cdot K}-\frac{8.314\,kJ/kmol\cdot K}{18.02\,kg/kmol}\ln(0.23559) = 8.8318\,\frac{kJ}{kg\cdot K}$$

$$s_{v1} = s_{g1}-\frac{\overline{R}}{M_v}\ln\phi_1 = 8.7814\,\frac{kJ}{kg\cdot K}-\frac{8.314\,kJ/kmol\cdot K}{18.02\,kg/kmol}\ln(0.63) = 8.9946\,\frac{kJ}{kg\cdot K}$$

$$s_2 = s_{f2} = 0.2245\,\frac{kJ}{kg\cdot K}$$

Also, for the dry air

$$p_{a3} = p_3 - p_{v3} = \left((1.3\ \text{atm}) \left| \frac{1.01325\ \text{bar}}{1\ \text{atm}} \right| \right) - 0.02260\ \text{bar} = 1.2946\ \text{bar}$$

$$s_{a3} - s_{a1} = c_{pa} \ln \frac{T_3}{T_1} - \frac{\overline{R}}{M} \ln \frac{p_{a3}}{p_{a1}}$$

$$= 1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \ln\left(\frac{318\ \text{K}}{288\ \text{K}}\right) - \frac{8.314\ \text{kJ/kmol} \cdot \text{K}}{28.97\ \text{kg/kmol}} \ln\left(\frac{1.2946\ \text{bar}}{1.3065\ \text{bar}}\right) = 0.1022 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

The entropy production rate is

$$\dot{\sigma}_{cv} = -\frac{15.196}{460} \frac{\text{kW}}{\text{K}} + \left(1218.4 \frac{\text{kg}}{\text{h}} \left| \frac{1\ \text{h}}{3600\ \text{s}} \right| \right) \left[0.1022 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} + (0.01086)(8.8318) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right.$$

$$\left. - (0.00511)(8.9946) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} - (0.00575)(0.2245) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right]$$

$$= -0.03303 \frac{\text{kW}}{\text{K}} + \left(0.33844 \frac{\text{kg}}{s} \right) [(0.1022) + (0.09591) - (0.04596) - (0.00129)] \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$= 0.0180 \frac{\text{kW}}{\text{K}}$$

PROBLEM 12.69

KNOWN: The dry-bulb and wet-bulb temperatures are provided for each of two cases.

FIND: Using Eq. 12.48, determine ω and ϕ for each case.

ENGINEERING MODEL: (1) The wet bulb temperature can be used in place of the adiabatic saturation temperature. (2) $p = 1\,atm$.

ANALYSIS: (a) $T_{db} = 24°C$, $T_{wb} = 16°C$, $p = 1\,atm$

ω can be found from Eq. 12.48. The value of ω' in this expression is found using p_g at the wet bulb temperature from Table A-2

$$\omega' = 0.622\left[\frac{0.01818}{1.01325 - 0.01818}\right] = 0.0114\ \frac{kg(v)}{kg(a)}$$

Equation 12.48 then gives with $c_{pa} = 1.005\ kJ/kg\cdot K$ and data from Table A-2

$$\omega = \frac{c_{pa}(T_{wb} - T) + \omega'[h_g(T_{wb}) - h_f(T_{wb})]}{h_g(T) - h_f(T_{wb})}$$

$$= \frac{(1.005)(16-24) + 0.0114[2463.6]}{(2545.4 - 67.2)} = \frac{-8.04 + 28.09}{2478.2} = 0.0081\ \frac{kg(v)}{kg(a)}$$

ω

Solving Eq. 12.43 for p_v

$$p_v = \frac{\omega p}{0.622 + \omega} = \frac{(0.0081)(1.01325\ bar)}{0.622 + 0.0081} = 0.013\ bar$$

Then

$$\phi = \frac{p_v}{p_g(T)} = \frac{0.013}{0.02985} = 0.436 \quad (43.6\%)$$

(b) $T_{db} = 75°F$, $T_{wb} = 60°F$, $p = 1\,atm$

ω can be found from Eq. 12.48. The value of ω' in this expression is found using p_g at the wet bulb temperature from Table A-2E

$$\omega' = 0.622\left[\frac{0.2563}{14.7 - 0.2563}\right] = 0.01104\ \frac{lb(v)}{lb(a)}$$

Equation 12.48 then gives with $c_{pa} = 0.24\ Btu/lb\cdot°R$ and data from Table A-2E

$$\omega = \frac{c_{pa}(T_{wb} - T) + \omega'[h_g(T_{wb}) - h_f(T_{wb})]}{h_g(T) - h_f(T_{wb})}$$

$$= \frac{0.24[60-75] + 0.01104[1059.6]}{1094.25 - 28.08} = \frac{-3.6 + 11.698}{1066.17} = 0.0076\ \frac{lb(v)}{lb(a)}$$

Solving Eq 12.43 for p_v

$$p_v = \frac{\omega p}{0.622 + \omega} = \frac{(0.0076)(14.696)}{0.622 + 0.0076}\ \ 0.1774\ lbf/in^2$$

Then

$$\phi = \frac{p_v}{p_g(T)} = \frac{0.1774}{0.4302} = 0.412 \quad (41.2\%)$$

PROBLEM 12.69 (Contd.) - Page 2

(c) Referring to Figure A-9 with the data of part (a), $\phi = 44\%$, $\omega = 0.0081$ kg(v)/kg(a).
Referring to Figure A-9E with the data of part (b), $\phi = 42\%$, $\omega = 0.0078$ lb(v)/lb(a).

(d) <u>*IT* Solution</u>

```
T_a = 24  // °C
Twb_a = 16  // °C
p = 1.01325  // bar
w_a = w_TTwb(T_a,Twb_a, p)
phi_a = phi_Tw(T_a,w_a, p)
```

<u>*IT* Results - Part (a)</u>

$\phi = 0.435$
$\omega = 0.00807$

```
T_b = 75  // °F
Twb_b = 60  // °F
p = 1  // atm
w_b = w_TTwb(T_b,Twb_b, p)
phi_b = phi_Tw(T_b,w_b, p)
```

<u>*IT* Results - Part (b)</u>

$\phi = 0.4124$
$\omega = 0.00759$

PROBLEM 12.70

FIND: Using the psychrometric chart, determine

(a) ϕ, ω, $h_a + \omega h_g$ when $T_{db} = 30°C$, $T_{wb} = 25°C$.

(b) ω, $h_a + \omega h_g$, T_{wb} when $T_{db} = 30°C$, $\phi = 60\%$

(c) $T_{\text{dew point}}$ for $T_{db} = 30°C$, $T_{wb} = 20°C$

(d) Repeat (a)-(c) using IT.

ANALYSIS: Using the psychrometric chart ($p = 1atm$)

(a) $T_{db} = 30°C$, $T_{wb} = 25°C$

$$\phi = 68\%, \quad \omega = .0182, \quad h_a + \omega h_g = 76.3 \text{ kJ/kg(a)}$$

(b) $T_{db} = 30°C$, $\phi = 60\%$

$$\omega = 0.0163, \quad h_a + \omega h_g = 71.5 \text{ kJ/kg(a)}, \quad T_{wb} \approx 24°C$$

(c) $T_{db} = 30°C$, $T_{wb} = 20°C$, $T_{\text{dew pt}} \approx 15°C$

(d) Using IT for parts (a), (b), (c)

```
p = 1.01325 // bar

T_a = 30 // °C
Twb_a = 25 // °C
Twb_a = Twb_Tphi(T_a, phi_a, p)
w_a = w_TTwb(T_a,Twb_a, p)
h_a = ha_Tw(T_a, w_a)
```

IT Results for part (a)

$h_a + \omega h_v = 76.13$ kJ / kg(a)
$\phi = 0.6706$
$\omega = 0.018$

```
T_b = 30 // °C
phi_b = 0.6
w_b = w_Tphi(T_b, phi_b, p)
h_b = ha_Tphi(T_b, phi_b, p)
Twb_b = Twb_Tphi(T_b, phi_b, p)
```

IT Results for part (b)

$T_{wb} = 23.83$ °C
$h_a + \omega h_v = 71.15$ kJ / kg(a)
$\omega = 0.01605$

```
T_c = 30 // °C
Twb_c = 20 // °C
Twb_c = Twb_Tphi(T_c, phi_c, p)
pg = Psat_T("Water/Steam", T_c)
pv = phi_c * pg
T_dewpt = Tsat_P("Water/Steam", pv)
```

IT Results for part (c)

$p_g = 0.04246$ bar
$p_v = 0.0169$ bar
$\phi = 0.398$
$T_{\text{dew point}} = 14.86$ °C

PROBLEM 12.71

FIND: Using the psychrometric chart, determine

(a) $T_{dew\,pt}$ for $T_{db} = 80°F$, $T_{wb} = 70°F$

(b) ω, $h_a + \omega h_g$, T_{wb} for $T_{db} = 80°F$, $\phi = 70\%$

(c) ϕ, ω, $h_a + \omega h_g$ for $T_{db} = 80°F$, $T_{wb} = 65°F$

(d) Repeat (a)-(c) using IT

ANALYSIS: Using the psychrometric chart ($p = 1$ atm)

(a) $T_{db} = 80°F$, $T_{wb} = 70°F$

$$T_{dew\,pt} = 65.5°F$$

(b) $T_{db} = 80°F$, $\phi = 70\%$

$$\omega = 0.0155$$

$$h_a + \omega h_g = 36.1 \text{ Btu/lb(a)}$$

$$T_{wb} = 72.5°F$$

(c) $T_{db} = 80°F$, $T_{wb} = 65°F$

$$\phi = 45\%$$

$$\omega = 0.0099$$

$$h_a + \omega h_g = 30 \text{ Btu/lb(a)}$$

(d) Using IT for parts (a), (b), (c)

```
p = 1  // atm

T_a = 80  // °F
Twb_a = 70  // °F
Twb_a = Twb_Tphi(T_a, phi_a, p)
pg = Psat_T("Water/Steam", T_a)
pv = phi_a * pg
Tdewpt = Tsat_P("Water/Steam", pv)
```

IT Results for part (a)

$p_g = 0.03452$ lbf/in.^{2}
$p_v = 0.02114$ lbf/in.^{2}
$\phi = 0.6124$
$T_{dew\,point} = 65.46°F$

```
T_b = 80  // °F
phi_b = 0.7
w_b = w_Tphi(T_b, phi_b, p)
h_b = ha_Tphi(T_b, phi_b, p)
Twb_b = Twb_Tphi(T_b, phi_b, p)
```

IT Results for part (b)

$\omega = 0.01539$
$h_a + \omega h_v = 36.03$ Btu/lb(a)
$T_{wb} = 72.39°F$

```
T_c = 80  // °F
Twb_c = 65  // °F
Twb_c = Twb_Tphi(T_c, phi_c, p)
w_c = w_TTwb(T_c, Twb_c, p)
h_c = ha_Tphi(T_c, phi_c, p)
```

IT Results for part (c)

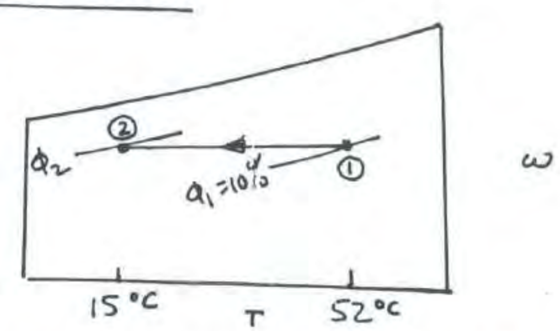
$\phi = 0.447$
$\omega = 0.009739$
$h_a + \omega h_v = 29.84$ Btu/lb(a)

PROBLEM 12.72

KNOWN: Moist air at 52°C, 1 atm, $\phi = 10\%$ is cooled at constant pressure to 15 °C.

FIND: Using the psychrometric chart, determine whether condensation occurs. If so, find the amount condensed per kg of dry air. If not, determine ϕ at the final state.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: For moist air, ideal gas principles apply.

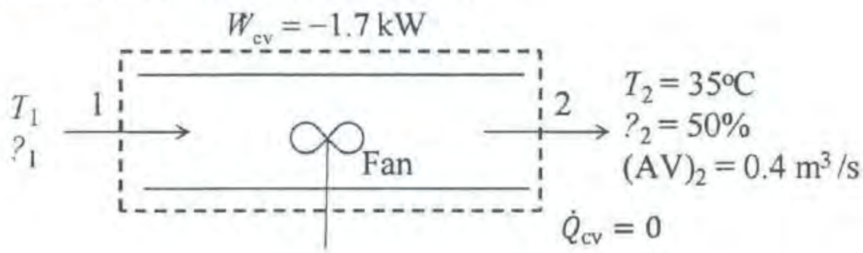
ANALYSIS: As long as there is no condensation the mole fraction of water vapor, and thus p_v remains constant during constant-pressure cooling: $p_v = y_v p$. Since both p_v and p are constant, Eq. 12.43 shows ω remains constant, as shown on the schematic. With the given data, Fig. A-9 indicates that no condensation occurs and $\phi_2 = 80\%$.

12.73 A fan in an insulated duct delivers moist air at the duct exit at 35°C, 50% relative humidity, and a volumetric flow rate of 0.4 m^3/s. At steady state, the power input to the fan is 1.7 kW. The pressure in the duct is nearly 1 atm throughout. Using the psychrometric chart, determine the temperature and relative humidity at the duct inlet.

KNOWN: Steady state operating data are given for a fan in an insulated duct. The volumetric flow rate is given at the inlet and the power input is known.

FIND: Determine the temperature and relative humidity at the duct inlet using data from the psychrometric chart.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

(1) The control volume shown in the figure is at steady state.

(2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible.

(3) The moist air adheres to ideal gas principles.

(4) The pressure is nearly 1 atm throughout, so the psychrometric chart can be used.

ANALYSIS:

(a) From the mass rate balances for the dry air and water vapor, $\omega_1 = \omega_2$. Reducing the energy balance

$$0 = \underbrace{\dot{Q}_{cv}}_{=0} - \dot{W}_{cv} + \dot{m}_a\left[(h_a - \omega h_v)_1 - (h_a - \omega h_v)_2\right]$$

Thus, solving for the mixture enthalpy per kg of dry air entering

$$(h_a - \omega h_v)_1 = (h_a - \omega h_v)_2 + \frac{\dot{W}_{cv}}{\dot{m}_a} \tag{1}$$

Using ϕ_2 and T_2, the psychrometric chart (Fig. A-9) gives

Problem 12.73 (Continued) – Page 2

$$\omega_2 = 0.018\frac{\text{kg (vapor)}}{\text{kg (air)}}$$

$$(h_a - \omega h_v)_2 = 81.5\frac{\text{kJ}}{\text{kg (air)}}$$

$$v_{a2} = 0.898\frac{\text{m}^3}{\text{kg (air)}}$$

Then

$$\dot{m}_a = \frac{(\text{AV})_1}{v_{a1}} = \frac{0.4\frac{\text{m}^3}{\text{s}}}{0.898\frac{\text{m}^3}{\text{kg (air)}}} = 0.4454\frac{\text{kg (air)}}{\text{s}}$$

Inserting values

$$(h_a - \omega h_v)_1 = 81.5\frac{\text{kJ}}{\text{kg (air)}} + \frac{(-1.7\,\text{kW})}{(0.4454\,\text{kg(air)/s})} = 77.68\frac{\text{kJ}}{\text{kg (air)}}$$

Then, using $(h_a - \omega h_v)_1 = 77.68$ kJ/kg (air) and $\omega_1 = \omega_2 = 0.018$, Fig. A-9 gives

$T_1 \approx 31.5^\circ\text{C}$

$\phi_1 \approx 62\%$

PROBLEM 12.74

KNOWN: Moist air adheres to ideal gas principles.

FIND: Derive expressions in SI and English units for the mixture enthalpy per unit of mass as used on Figs A-9, A-9E.

ANALYSIS:

SI CASE.

Turning to the discussion of Fig. A-9 in Sec. 12.9, note that the enthalpy of the dry air is determined relative to a datum of 0°C. That is

$$h_a(T) = \cancelto{0}{h_a(T_{ref})} + \underbrace{c_{pa}}_{1.005\,\frac{kJ}{kg(a)\cdot K}} [\underbrace{T - T_{ref}}_{T(°C)}]$$

The enthalpy of the water vapor is evaluated as $h_v \approx h_g$ at the dry-bulb temperature. A plot of h_g vs $T(°C)$ using data from Tables A-2, 6 shows a nearly linear variation; with h_g being given closely by

$$h_g \approx 2507.1 + \underbrace{1.82}_{\approx c_{pv}} T(°C)$$

ENGLISH CASE.

The enthalpy of the dry air is determined relative to a datum of 0°F. That is

$$h_a(T) = \cancelto{0}{h_a(T_{ref})} + \underbrace{c_{pa}}_{0.24\,\frac{Btu}{lb\cdot °R}} [\underbrace{T - T_{ref}}_{T(°F)}]$$

Plotting h_g vs $T(°F)$ using data from Tables A-2E, 6E shows a nearly linear variation, with h_g being given closely by

$$h_g \approx 1061 + \underbrace{0.444}_{\approx c_{pv}} T(°F)$$

Problem 12.75

Each case listed gives the dry-bulb temperature and relative humidity of the moist air stream entering an air-conditioning system:

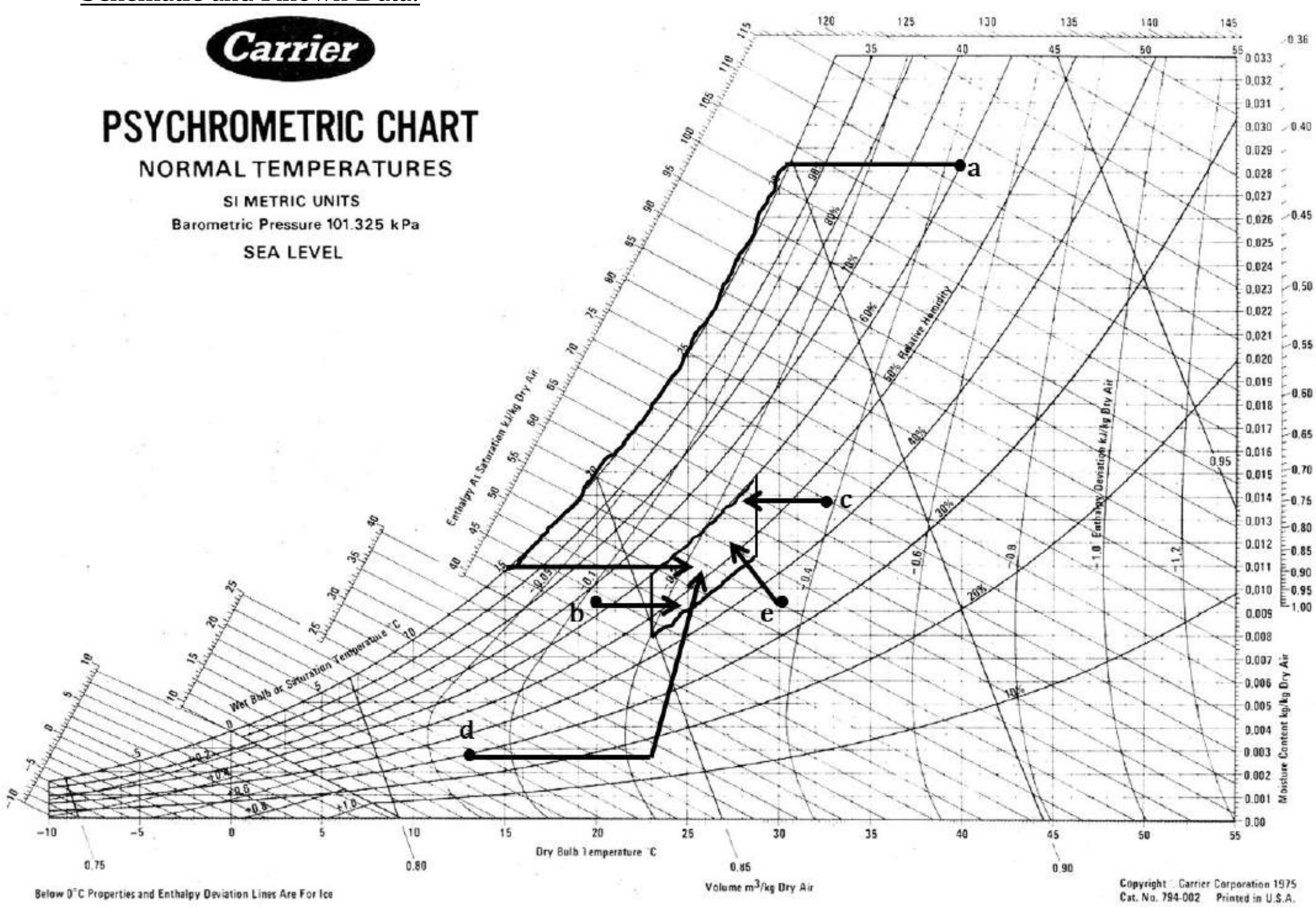
(a) 40°C, 60%
(b) 20°C, 65%
(c) 32°C, 45%
(d) 13°C, 30%
(e) 30°C, 35%

The condition of the moist air stream exiting the system must satisfy these *constraints*: $23 \leq T_{db} \leq 28°C$, $45 \leq \phi \leq 60\%$. In each case, develop a schematic of equipment and process from Sec. 12.8 that would achieve the desired result. The processes might include combinations of cooling, dehumidification, heating, and humidification. Sketch the process on a psychrometric chart.

Known:
Five cases are provided for the condition of the moist-air stream entering an air-conditioning system, together with the constraints that the exiting moist-air stream must satisfy.

Find:
In each case develop a schematic of equipment and processes from Sec. 12.8 that would achieve the desired result.

Schematic and Known Data:



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Engineering Model:

(1) The principles of Sec. 12.8 apply.

Analysis:

(a) Dehumidifying followed by reheating, see Figure 12.11.
(b) Heating at fixed moisture.
(c) Cooling at fixed moisture.
(d) Heating and humidifying, see Figure 12.12.
(e) Evaporative cooling, see Figure 12.13.

Note: The alternatives presented are not the only ones that might be considered. Depending on the condition at the exit, other processes can be proposed.

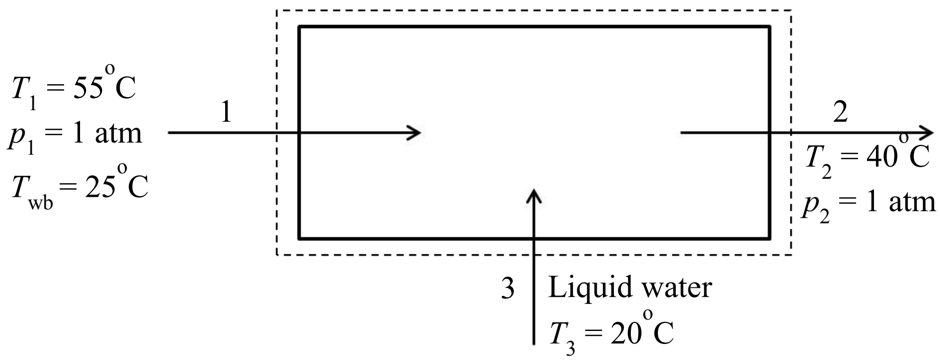
Problem 12.76

Moist air enters a device operating at steady state at 1 atm with a dry-bulb temperature of 55°C and a wet-bulb temperature of 25°C. Liquid water at 20°C is sprayed into the air stream, bringing it to 40°C, 1 atm at the exit. Determine
(a) the relative humidities at the inlet and exit.
(b) the rate that liquid water is sprayed into the air stream, in kg per kg of dry air.

Known:
Operating data are provided for an air conditioner at steady state.

Find:
Determine (a) the relative humidities at the inlet and exit, and (b) the rate of liquid water sprayed into the air stream.

Schematic & Given Data:



Engineering Model:
(1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy.
(2) The liquid spray is saturated liquid.
(3) The moist air streams adhere to the ideal gas model.
(4) The wet bulb temperature is used in place of the adiabatic saturation temperature.

Analysis:
At steady state mass rate balances give $\dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a$ and $\dot{m}_{v_1} + \dot{m}_w = \dot{m}_{v_2}$. Thus:

$$\dot{m}_w = \dot{m}_{v_2} - \dot{m}_{v_1} = \dot{m}_a(\omega_2 - \omega_1) \qquad (1)$$

(a) The relative humidity ϕ_1 can be obtained using ω_1, found using Eqs. 12.49 and 12.48 with constant specific heat at 300 K and values from Table A-2

$$\omega' = 0.622\frac{p_g(T_{wb})}{p - p_g(T_{wb})} = 0.622\left[\frac{0.03169}{1.01325 - 0.03169}\right] = 0.02008\frac{\text{kg (vapor)}}{\text{kg (air)}}$$

$$\omega_1 = \frac{h_a(T_{wb}) - h_a(T_1) + \omega'[h_g(T_{wb}) - h_f(T_{wb})]}{h_g(T_1) - h_f(T_{wb})} = \frac{c_p(T_{wb} - T_1) + \omega'[h_{fg}(T_{wb})]}{h_g(T_1) - h_f(T_{wb})}$$

$$= \frac{\left(1.005\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(25-55)\text{K} + (0.02008)\left(2442.3\,\frac{\text{kJ}}{\text{kg}}\right)}{(2600.9 - 104.89)\,\frac{\text{kJ}}{\text{kg}}}$$

$$= 0.00757\,\frac{\text{kg (vapor)}}{\text{kg (air)}}$$

Solving Eq. 12.43 for p_v

$$p_{v_1} = \frac{\omega_1 p_1}{0.622 + \omega_1} = \frac{(0.00757)(1.01325\text{ bar})}{0.622 + 0.00757} = 0.0122\text{ bar}$$

Solving for ϕ_1

$$\phi_1 = \frac{p_{v_1}}{p_g(T_1)} = \frac{0.0122}{0.1576} = 0.077 = 7.7\%$$

The relative humidity ϕ_2 can be obtained using ω_2 found from an energy rate balance. At steady state

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_a h_{a_1} + \dot{m}_{v_1} h_{v_1} + \dot{m}_w h_3 - [\dot{m}_a h_{a_2} + \dot{m}_{v_2} h_{v_2}]$$

Thus, with Eq. (1)

$$0 = \dot{m}_a[(h_{a_1} - h_{a_2}) + \omega_1 h_{g_1} - \omega_2 h_{g_2} + (\omega_2 - \omega_1)h_{f_3}]$$

$$0 = (h_{a_1} - h_{a_2}) + \omega_1(h_{g_1} - h_{f_3}) - \omega_2(h_{g_2} - h_{f_3})$$

Solving for ω_2

$$\omega_2 = \frac{(h_{a_1} - h_{a_2}) + \omega_1(h_{g_1} - h_{f_3})}{(h_{g_2} - h_{f_3})} = \frac{c_p(T_1 - T_2) + \omega_1(h_{g_1} - h_{f_3})}{(h_{g_2} - h_{f_3})}$$

$$= \frac{\left(1.005\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(55-40)\text{K} + (0.00757)(2600.9 - 83.96)\,\frac{\text{kJ}}{\text{kg}}}{(2574.3 - 83.96)\,\frac{\text{kJ}}{\text{kg}}}$$

$$= 0.0137\,\frac{\text{kg (vapor)}}{\text{kg (air)}}$$

Solving for p_{v_2}

$$p_{v_2} = \frac{\omega_2 p_2}{0.622 + \omega_2} = \frac{0.0137(1.01325\text{ bar})}{0.622 + 0.0137} = 0.02184\text{ bar}$$

Solving for ϕ_2

$$\phi_2 = \frac{p_{v_2}}{p_g(T_2)} = \frac{0.02184}{0.07384} = 0.296 = 29.6\%$$

(b) Finally, the amount of liquid water sprayed per kg of dry air is

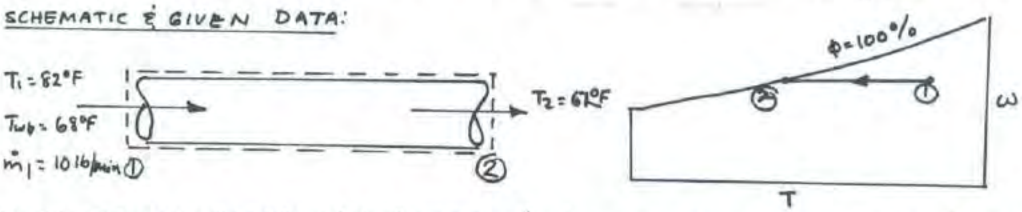
$$\frac{\dot{m}_\mathrm{w}}{\dot{m}_\mathrm{a}} = \omega_2 - \omega_1 = 0.0137 - 0.00757 = 0.00613\,\frac{\text{kg (liquid)}}{\text{kg (air)}}$$

PROBLEM 12.77

KNOWN: Moist air enters a duct with a known mass flow rate and is cooled at constant pressure to a specified temperature.

FIND: Using table data, determine (a) the relative humidity at the duct inlet and (b) the rate of heat transfer. (c) Check the results of parts (a), (b) using the Psychrometric Chart. (d) Check the results of parts (a), (b) using IT.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume in the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) The moist air adheres to ideal gas mixture principles. (3) The wet-bulb temperature can be used in place of T_{as} to evaluate ω.

ANALYSIS: (a) To find ϕ_1, first evaluate ω_1 using Eqs. 12.52 and 12.53. Thus

$$\omega' = 0.622 \frac{p_g(T_{wb})}{p - p_g(T_{wb})} = 0.622\left(\frac{0.3391}{14.696 - 0.3391}\right) = 0.01469 \frac{lb(v)}{lb(a)}$$

and

$$\omega_1 = \frac{0.24[68 - 82] + 0.01469[1055.1]}{1097.3 - 36.09} = 0.0114 \frac{lb(v)}{lb(a)}$$

Solving Eq. 12.43

$$p_{v1} = \frac{\omega_1 \, p}{0.622 + \omega_1} = \frac{(0.0114)(14.696)}{0.622 + 0.0114} = 0.2645 \frac{lbf}{in^2}$$

Then

$$\phi_1 = \frac{p_{v1}}{p_g(82°F)} = \frac{0.2645}{0.5415} = .489 \quad (48.9\%) \longleftarrow \phi_1$$

(b) Before applying an energy balance it is necessary to determine if condensation occurs upon cooling. Using p_{v1}, the dew point temperature is ≈ 61°F. Accordingly, no condensation takes place. At steady state an energy rate balance reduces to

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}}^{\,0} + [\dot{m}_a h_{a1} + \dot{m}_v h_{v1}] - [\dot{m}_a h_{a2} + \dot{m}_v h_{v2}]$$

or

$$\dot{Q}_{cv} = \dot{m}_a [(h_{a2} - h_{a1}) + \omega(h_{g2} - h_{g1})]$$

The total mass flow rate is $\dot{m} = \dot{m}_a + \dot{m}_v = \dot{m}_a(1+\omega)$. Thus, $\dot{m}_a = \dot{m}/(1+\omega)$, so

$$\dot{Q}_{cv} = \frac{\dot{m}}{1+\omega}[(h_{a2} - h_{a1}) + \omega(h_{g2} - h_{g1})]$$

$$= \left(\frac{10}{1.0114} \frac{lb(a)}{min}\right)\left[0.24(62 - 82) + 0.0114(1088.6 - 1097.3)\right] \frac{Btu}{lb(a)}$$

$$= \left(\frac{10}{1.0114}\right)(-4.9) = -48.4 \frac{Btu}{min} \longleftarrow \dot{Q}_{cv}$$

PROBLEM 12.77 (Cont'd.) - Page 2

(c) In the psychrometric chart solution, inspection of Fig. A-9E using T_1 and T_{wb}, $\phi_1 \approx 50\%$. Also, the energy rate balance takes the form ← ϕ_1

$$\dot{Q}_{cv} = \frac{\dot{m}}{1+\omega}\left[(h_a+\omega h_g)_2 - (h_a+\omega h_g)_1\right]$$

Fig. A-9E gives $(h_a+\omega h_g)_2 \approx 27.3$ Btu/lb(a) and $(h_a+\omega h_g)_1 \approx 32.2$ Btu/lb(a). Accordingly

$$\therefore \dot{Q}_{cv} = \left(\frac{10}{1.0114}\right)\left[27.3 - 32.2\right] = -48.4 \ \frac{\text{Btu}}{\text{lb}}$$ ← $\dot{Q}_{cv}$

(d) *IT* Code

```
T1 = 82  // °F
Twb1 = 68  // °F
mdot1 = 10  // lb/min
T2 = 62  // °F
p = 1 // atm

// Check for condensation.
Twb1 = Twb_Tphi(T1, phi1, p)
pg1 = Psat_T("Water/Steam", T1)
pv1 = phi1 * pg1
Tdp1 = Tsat_P("Water/Steam", pv1)
// Result:  Tdp = 61.03°F
// No condenstaion.

w = w_TTwb(T1, Twb1, p)
h1 = ha_Tw(T1, w)
h2 = ha_Tw(T2, w)
mdota1 = mdot1 / (1 + w)
Qdot = mdota1 * (h2 - h1)
```

IT Results

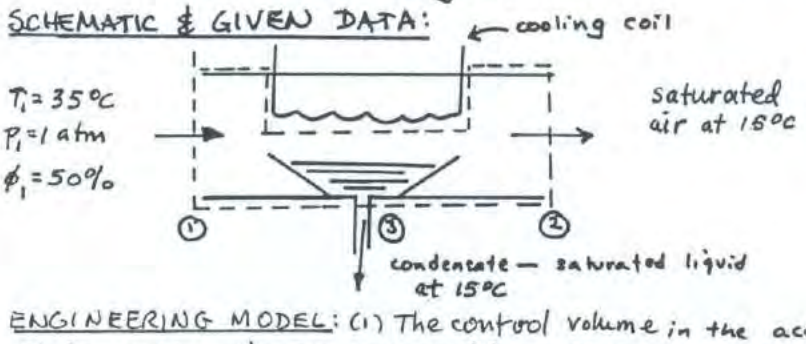
h_1 = 32.24 Btu/lb(a)
h_2 = 27.34 Btu/lb(a)
ω = 0.01148
T_{dp1} = 61.03°F
$\dot{m}_a$ = 9.886 lb/min
ϕ_1 = 0.491 ← ϕ_1
$\dot{Q}_{cv}$ = -48.39 Btu/min ← $\dot{Q}_{cv}$

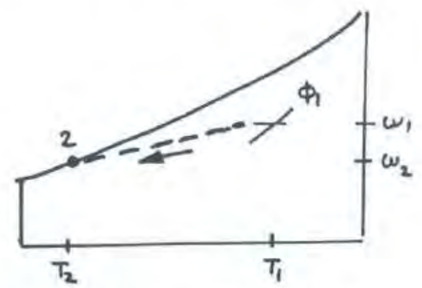
PROBLEM 12.78

KNOWN: Operating data are provided for a dehumidifier at steady state.

FIND: Using table data, determine (a) the heat transfer from the moist air, and (b) the amount of water condensed, each per kg of dry air flowing. Check the results of parts (a) and (b) using (c) the psychrometric chart, and (d) using IT.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) The control volume in the accompanying figure is at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The moist air at 2 is saturated. The condensate at 3 is a saturated liquid. (3) Pressure remains constant throughout at 1 atm.

ANALYSIS: (a) At steady state mass rate balances give $\dot{m}_{a1} = \dot{m}_{a2} \equiv \dot{m}_a$ and $\dot{m}_{v1} = \dot{m}_{v2} + \dot{m}_w$. Thus, the rate water is condensed per unit mass of dry air is

$$\frac{\dot{m}_w}{\dot{m}_a} = \omega_1 - \omega_2 . \tag{1}$$

To find ω_1, use $p_{v1} = \phi_1 p_g(T_1) = (0.5)(0.05628) = 0.02814$ bar. Then

$$\omega_1 = .622\left[\frac{0.02814}{1.01325 - 0.02814}\right] = 0.01777 \ \frac{kg(v)}{kg(a)}$$

Since the moist air is saturated at 2, $p_{v2} = p_g(T_2) = 0.01705$ bar, and

$$\omega_2 = .622\left[\frac{0.01705}{1.01325 - 0.01705}\right] = 0.01065 \ \frac{kg(v)}{kg(a)}$$

These values are closely the same as obtained from Fig A-9. Substituting into Eq. (1)

$$\frac{\dot{m}_w}{\dot{m}_a} = 0.01777 - 0.01065 = 0.00717 \ \frac{kg(v)}{kg(a)} \longleftarrow \frac{\dot{m}_w}{\dot{m}_a}$$

At steady state an energy rate balance reduces to

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}}^{0} + [\dot{m}_a h_{a1} + \dot{m}_{v1} h_{v1}] - [\dot{m}_a h_{a2} + \dot{m}_{v2} h_{v2}] - \dot{m}_w h_f$$

Or, with Eq. (1)

$$\frac{\dot{Q}_{cv}}{\dot{m}_a} = (h_{a2} - h_{a1}) + \omega_2 h_{g2} - \omega_1 h_{g1} + (\omega_1 - \omega_2) h_{f3} \tag{2}$$

With $c_{pa} = 1.005$ kJ/kg·K and data from Table A-2 for water vapor

$$\frac{\dot{Q}_{cv}}{\dot{m}_a} = 1.005[15 - 35] + 0.01065(2528.9) - 0.01777(2565.3) + 0.00717(62.99)$$

$$= -20.1 + 26.93 - 45.59 + 0.452 = -38.31 \ \frac{kJ}{kg(a)} \longleftarrow \frac{\dot{Q}_{cv}}{\dot{m}_a}$$

PROBLEM 12.78 (Cont'd.) - Page2

(c) In the psychrometric chart solution, the energy rate equation, Eq.(2), takes the form

$$\frac{\dot{Q}_{cv}}{\dot{m}_a} = (h_a + \omega h_g)_2 - (h_a + \omega h_g)_1 + (\omega_1 - \omega_2) h_{f3}$$

From Fig A-9; $(h_a + \omega h_g)_1 \approx 80.6$ kJ/kg(a) and $(h_a + \omega h_g)_2 \approx 42$ kJ/kg(a). Thus

$$\frac{\dot{Q}_{cv}}{\dot{m}_a} = 42 - 80.6 + 0.452 = -38.15 \longleftarrow \frac{\dot{Q}_{cv}}{\dot{m}_a}$$

(d) *IT* Code

```
T1 = 35  // °C
phi1 = 0.5
T2 = 15 // °C
phi2 = 1
T3 = 15  // °C
p = 1.01325  // bar
mdota = 1  // Assume a unit mass flow rate.

w1 = w_Tphi(T1, phi1, p)
w2 = w_Tphi(T2, phi2, p)
h1 = ha_Tphi(T1, phi1, p)
h2 = ha_Tphi(T2, phi2, p)
psat = Psat_T("Water/Steam", T3)
h3 = hsat_Px("Water/Steam", psat, 0)

mdotw / mdota = w1 - w2
Qdotcv / mdota = h2 - h1 + (w1 - w2) * h3
```

IT Results

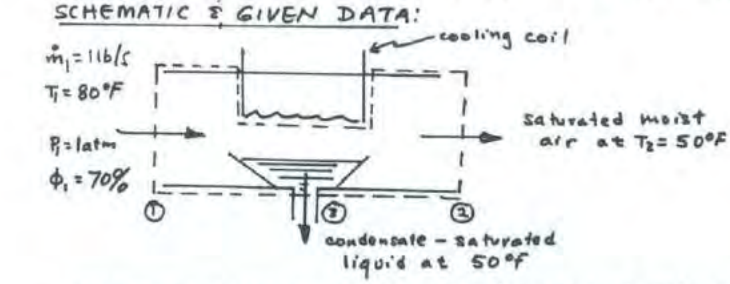
w1 = 0.01775 kg(v)/kg(a)
w2 = 0.01066 kg(v)/kg(a)
h1 = 80.66 kJ/kg(a)
h2 = 42.02 kJ/kg(a)
h3 = 61.93 kJ/kg(a)
$\dot{m}_w / \dot{m}_a$ = 0.007085 kg/kg(a) $\longleftarrow$
$\dot{Q}_{cv} / \dot{m}_a$ = -38.21 kJ/kg(a) $\longleftarrow$

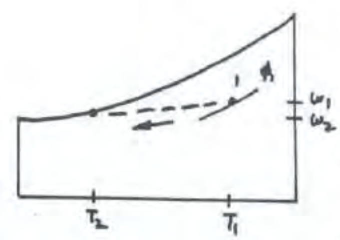
PROBLEM 12.79

KNOWN: Steady-state operating data are provided for a dehumidifier.
FIND: Using table data, determine (a) the rate of heat transfer from the moist air, (b) the rate water is condensed. (c) Check results using the psychrometric chart. (d) Check results using IT.

KNOWN: Operating data are provided for a dehumidifier at steady state.
FIND: Determine the heat transfer from the moist air and the rate water is condensed, in tons and lb/s, respectively.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) The control volume in the accompanying figure is at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The moist air at 2 is saturated. The condensate at 3 is saturated liquid. (3) Pressure remains constant throughout at 1atm.

ANALYSIS: At steady state mass rate balances give $\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a$ and $\dot{m}_{v1} = \dot{m}_{v2} + \dot{m}_w$. Thus, the rate water is condensed per unit mass of dry air is

$$\frac{\dot{m}_w}{\dot{m}_a} = \omega_1 - \omega_2 \tag{1}$$

To find ω_1, use $p_{v1} = \phi_1 p_g(T_1) = 0.7(0.5073) = 0.3551$ lbf/in². Then

$$\omega_1 = 0.622\left[\frac{0.3551}{14.696 - 0.3551}\right] = 0.0154 \ \frac{\text{lb(v)}}{\text{lb(a)}}$$

Since the moist air is saturated at 2, $p_{v2} = p_g(T_2) = 0.1780$ lbf/in², and

$$\omega_2 = 0.622\left[\frac{0.1780}{14.696 - 0.1780}\right] = 0.0076 \ \frac{\text{lb(v)}}{\text{lb(a)}}$$

These values for ω_1 and ω_2 are closely the same as obtained from Fig. A-9E. Substituting into Eq. (1)

$$\frac{\dot{m}_w}{\dot{m}_a} = 0.0154 - 0.0076 = 0.0078 \ \frac{\text{lb(v)}}{\text{lb(a)}}$$

At the inlet $\dot{m}_1 = \dot{m}_{v1} + \dot{m}_{a1} = \dot{m}_{a1}(1+\omega_1)$. Thus

$$\dot{m}_{a1} = \frac{\dot{m}_1}{1+\omega} = \frac{1 \text{ lb/s}}{1.0154} = 0.985 \ \frac{\text{lb(a)}}{\text{s}}$$

Thus

$$\dot{m}_w = \left[0.0078 \ \frac{\text{lb(v)}}{\text{lb(a)}}\right]\left(0.985 \ \frac{\text{lb(a)}}{\text{s}}\right] = 0.0077 \ \frac{\text{lb}}{\text{s}} \longleftarrow (b)$$

At steady state an energy rate balance reduces with Eq. (1) to give

$$\frac{\dot{Q}_{cv}}{\dot{m}_a} = (h_{a2} - h_{a1}) + \omega_2 h_{g2} - \omega_1 h_{g1} + (\omega_1 - \omega_2) h_{f3} \tag{2}$$

with $c_p = 0.24$ Btu/lb·°R

$$\frac{\dot{Q}_{cv}}{\dot{m}_a} = 0.24(50-80) + 0.0076(1083.3) - 0.0154(1096.4) + 0.0078(18.06)$$
$$= -7.2 + 8.23 - 16.88 + 0.14 = -15.71 \text{ Btu/lb(a)}$$

Thus
$$\dot{Q}_{cv} = \left[-15.71 \frac{\text{Btu}}{\text{lb(a)}}\right]\left[0.985 \frac{\text{lb(a)}}{\text{s}}\right]\left|\frac{1 \text{ ton}}{200 \text{ Btu/min}}\right|\left|\frac{60 \text{ s}}{\text{min}}\right| = -4.64 \text{ tons} \longleftarrow (a)$$

PROBLEM 12.79 (Contd.) -Page 2

(c) By inspection of Fig. A-9E, $\omega_1 = 0.0155$, $\omega_2 = 0.0076$, $(h_a + \omega h_g)_1 = 36.1$ Btu/lb(a), $(h_a + \omega h_g)_2 = 20.2$ Btu/lb(a). Thus, Eq. (1) gives $\frac{\dot{m}_w}{\dot{m}_a} = \omega_1 - \omega_2 = 0.0078 \frac{lb(v)}{lb(a)}$, which agrees well with the table result. Rearranging Eq. (2) to read

$$(\dot{Q}_{cv}/\dot{m}_a) = (h_a + \omega h_g)_2 - (h_a + \omega h_g)_1 + (\omega_1 - \omega_2)\, h_{f3}$$

$$= 20.2 - 36.1 + 0.14 = -15.76 \text{ Btu/lb(a)}$$

This also agrees with the table result, as expected.

(d) *IT* Code

```
T1 = 80  // °F
phi1 = 0.7
T2 = 50  // °F
phi2 = 1
T3 = 50  // °F
p = 1  // atm
mdot1 = 1  // lb/s

w1 = w_Tphi(T1, phi1, p)
w2 = w_Tphi(T2, phi2, p)
h1 = ha_Tphi(T1, phi1, p)
h2 = ha_Tphi(T2, phi2, p)
psat = Psat_T("Water/Steam", T3)
h3 = hsat_Px("Water/Steam", psat, 0)

mdota = mdot1 / ( 1 + w1)
mdotw / mdota = w1 - w2
Qdotcv / mdota = (h2 - h1 + (w1 - w2) * h3) * (60 / 200)  // tons
```

It Results

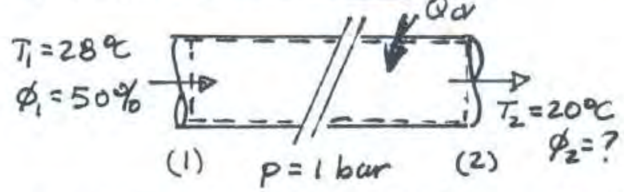
$\omega_1 = 0.01539$ lb(v)/lb(a)
$\omega_2 = 0.007624$ lb(v)/lb(a)
$h_1 = 36.03$ Btu/lb(a)
$h_2 = 20.23$ Btu/lb(a)
$h_3 = 17.67$ Btu/lb(a)
$\dot{m}_a = 0.9848$ lb/s
$\dot{m}_w = 0.007645$ lb/s
$\dot{Q}_{cv} = -4.628$ tons

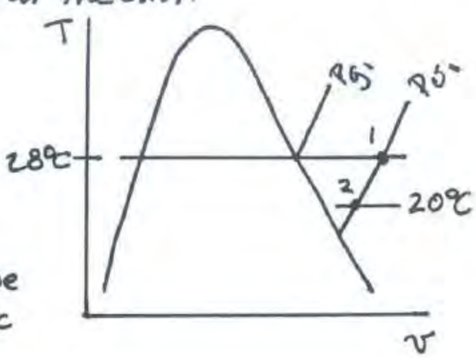
PROBLEM 12.80

KNOWN: Moist air is cooled as it flows through a duct.

FIND: Determine the rate of heat transfer per unit mass of dry air flowing and the relative humidity at the exit.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) The control volume is at steady state, and $\dot{W}_{cv} = 0$. (2) Kinetic and potential energy effects are negligible. (3) The moist air behaves as an ideal gas mixture. (4) Let $c_{pa} = 1.005$ kJ/kg·K.

ANALYSIS: First, check for condensation: $\phi_1 = 0.5 = \dfrac{p_{v_1}}{p_{g@T_1}}$

$$p_{v_1} = (0.5)(0.03782 \text{ bar}) = 0.01891 \text{ bar}$$

$\therefore T_{DP_1} \approx 17°C \Rightarrow$ no condensation $\Rightarrow \omega_1 = \omega_2$ and $p_{v_1} = p_{v_2}$

The mass and energy rate balances reduce to

$$0 = \dot{Q}_{cv} + \dot{m}_a\left[(h_{a_1} - h_{a_2}) + \omega(h_{v_1} - h_{v_2})\right]$$

or

$$\frac{\dot{Q}_{cv}}{\dot{m}_a} = c_{pa}(T_2 - T_1) + \omega(h_{v_2} - h_{v_1})$$

Using p_v calculated above; $\omega = .622 \dfrac{0.01891}{(1 - 0.01891)} = 0.01199$

Thus

$$\frac{\dot{Q}_{cv}}{\dot{m}_a} = (1.005)(20-28)\frac{kJ}{kg} + (0.01199)(2538.1 - 2552.6)\frac{kJ}{kg}$$

$$= -8.214 \text{ kJ/kg(a)} \longleftarrow \quad \dot{Q}_{cv}/\dot{m}_a$$

The relative humidity at the exit is

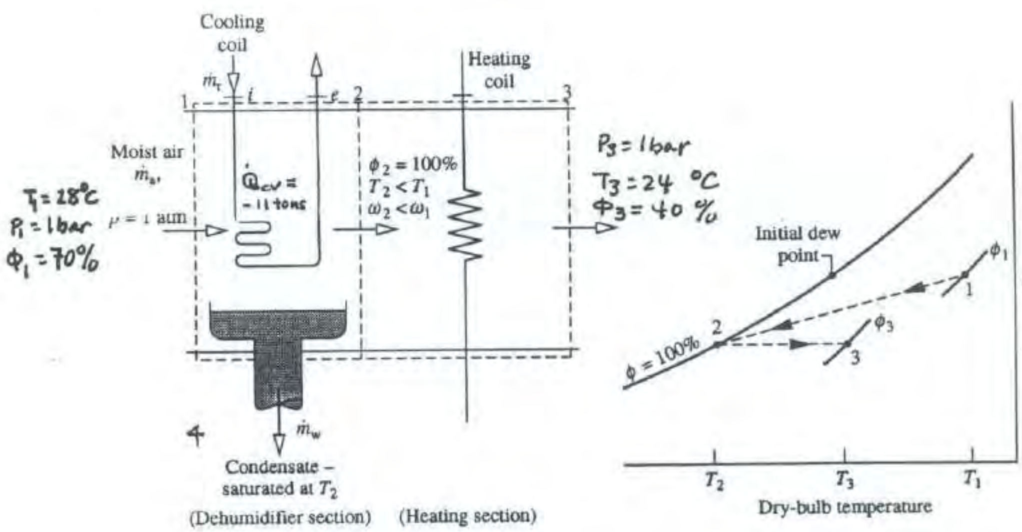
$$\phi_2 = \frac{p_v}{p_{g@T_2}} = \frac{0.01891}{0.02339} = 0.808\ (80.8\%) \longleftarrow \quad \phi_2$$

PROBLEM 12.81

KNOWN: Steady-state operating data are provided for an air conditioner wherein moisture is removed from the moist air, which is then heated.

FIND: Determine (a) the temperature of the air exiting the dehumidifier, (b) the volumetric flow rate at the air conditioner inlet, (c) the rate water is condensed, and (d) the rate of heat transfer to the air passing through the heating unit.

SCHEMATIC & GIVEN DATA



ENGINEERING MODEL: (1) The control volumes shown are at steady state. The coils are not included in the control volumes enclosing the two individual sections. (2) For each control volume, $\dot{W}_{cv} = 0$ and kinetic and potential energy effects can be ignored. (3) At 2, the moist air is saturated. At 4, the liquid is saturated at T_2.

ANALYSIS: At steady state, mass rate balances give $\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_{a3} \equiv \dot{m}_a$, $\dot{m}_{v1} = \dot{m}_w + \dot{m}_{v2}$, $\dot{m}_{v2} = \dot{m}_{v3}$. Accordingly, $\omega_2 = \omega_3$, and

$$\dot{m}_w = \dot{m}_a(\omega_1 - \omega_2) = \dot{m}_a(\omega_1 - \omega_3) \qquad (1)$$

Using $\omega_2 = \omega_3$,

$$0.622\left[\frac{p_g(T_2)}{p - p_g(T_2)}\right] = 0.622\left[\frac{p_{v3}}{p - p_{v3}}\right] \Rightarrow p_g(T_2) = p_{v3}$$

where $p_{v3} = \phi_3 p_{g3} = 0.4\,(0.02985\text{ bar}) = 0.01194$ bar. Interpolating in Table A-2 with $p_g(T_2) = 0.01194$ bar gives $T_2 \approx 9.6°C$ ← (a)

Then, with $p_{v1} = \phi_1 p_{g1} = (0.7)(0.03782) = 0.02647$ bar

$$\omega_1 = 0.622\left[\frac{0.02647}{1 - 0.02647}\right] = 0.01691\ \frac{kg(v)}{kg(a)}, \quad \omega_2 = \omega_3 = 0.622\left[\frac{0.01194}{1 - 0.01194}\right] = 0.007516$$

An energy rate balance on the dehumidifier reads

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}}^{\,0} + [\dot{m}_a h_{a1} + \dot{m}_{v1} h_{v1}] - [\dot{m}_a h_{a2} + \dot{m}_{v2} h_{v2}] - \dot{m}_w h_w$$

$$0 = \dot{Q}_{cv} + \dot{m}_a[h_{a1} + \omega_1 \underbrace{h_{v1}}_{h_{g1}}] - \dot{m}_a[h_{a2} + \omega_2 \underbrace{h_{v2}}_{h_{g2}}] - \dot{m}_a(\omega_1 - \omega_2)\underbrace{h_w}_{h_{f2}}$$

$$0 = \dot{Q}_{cv} + \dot{m}_a\Big[\underbrace{(h_{a1} - h_{a2})}_{c_{pa}(T_1 - T_2)} + \omega_1 h_{g1} - \omega_2 h_{g2} - (\omega_1 - \omega_2) h_{f2}\Big]$$

PROBLEM 12.81 (Contd.) - Page 2

Solving

$$\dot{m}_a = \frac{\dot{Q}_{cv}}{c_{pa}(T_2 - T_1) + \omega_2 h_{g2} - \omega_1 h_{g1} + (\omega_1 - \omega_2) h_{f2}}$$

$$= \frac{(-11 \text{ tons}) \left| \frac{211 \text{ kJ/min}}{\text{ton}} \right|}{\left[1.005[9.6 - 28] + 0.007516(2519.06) - 0.01691(2552.6) + (0.0094)(40.33)\right] \frac{\text{kJ}}{\text{kg(a)}}}$$

$$= 54.81 \text{ kg(a)/min}$$

The volumetric flow rate at 1 is then

$$(AV)_1 = \dot{m}_a v_{a1} = \dot{m}_a \frac{RT_1}{(P_1 - P_{v1})} = \left(54.81 \frac{\text{kg(a)}}{\text{min}}\right) \frac{\left(\frac{8314}{28.97} \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(301 \text{ K})}{(0.97353 \times 10^5 \text{ N/m}^2)} = 48.63 \frac{\text{m}^3}{\text{min}} \leftarrow$$

Using Eq. (1), the rate water is condensed is

$$\dot{m}_w = (54.81)(0.01691 - 0.007516) = 0.515 \frac{\text{kg(l)}}{\text{min}}$$

Finally, an energy rate balance on the heating section reduces to give

$$\dot{Q}_{cv} = \dot{m}_a \left[(h_{a3} - h_{a2}) + \omega_3 (h_{g3} - h_{g2})\right]$$

$$= 54.81 \left[(1.005)(24 - 9.6) + 0.007516(2545.4 - 2519.06)\right]$$

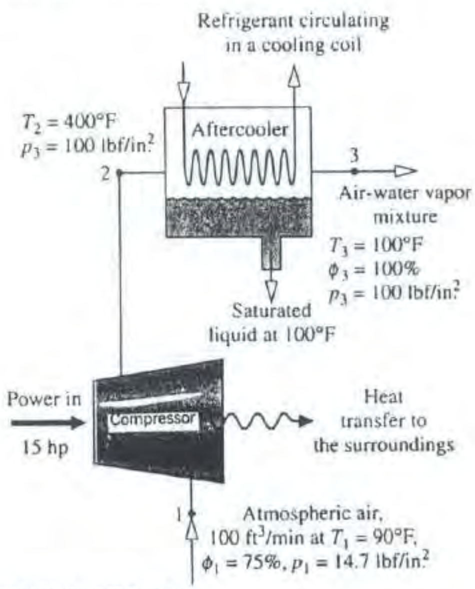
$$= 804.1 \frac{\text{kJ}}{\text{min}} \leftarrow$$

PROBLEM 12.82

KNOWN: Steady-state operating data are provided for a compressor followed by an aftercooler.

FIND: Determine (a) the rate of heat transfer from the compressor to its surroundings, (b) the mass flow rate of the condensate, (c) the rate of heat transfer from the moist air to the refrigerant.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The compressor and aftercooler operate at steady state with negligible changes in kinetic and potential energy. (2) There is no stray heat transfer between the aftercooler and its surroundings.

ANALYSIS: At steady state mass rate balances on control volumes enclosing the compressor and aftercooler give $\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_{a3} = \dot{m}_a$, $\dot{m}_{v1} = \dot{m}_{v2}$, $\dot{m}_{v2} = \dot{m}_{v3} + \dot{m}_w$. Accordingly, $\omega_1 = \omega_2$ and $\dot{m}_w/\dot{m}_a = (\omega_2 - \omega_3) = (\omega_1 - \omega_3)$.

To find ω_1 and ω_3, $p_{v1} = \phi_1 p_{g1} = (0.75)(0.6988) = 0.5241$ lbf/in² and $p_{v3} = \phi_3 p_{g3} = p_{g3} = 0.9503$ lbf/in². Thus

$$\omega_1 = 0.622\left[\frac{0.5241}{14.7-0.5241}\right] = 0.023\ \frac{lb(v)}{lb(a)}\ ,\quad \omega_3 = 0.622\left[\frac{0.9503}{100-0.9503}\right] = 0.00597\ \frac{lb(v)}{lb(a)}$$

The mass flow rate of the dry air can be evaluated using the volumetric flow rate at 1 and the ideal gas equation of state:

$$\dot{m}_a = \frac{(AV)_1}{v_{a1}} = \frac{p_{a1}(AV)_1}{RT_1} = \frac{[(14.7-0.5241)\times 144\ lbf/ft^2](100\ ft^3/min)}{(1545/28.97\ ft\cdot lbf/lb\cdot°R)(550°R)} = 6.96\ \frac{lb(a)}{min}$$

The mass flow rate of condensate is then

$$\dot{m}_w = \dot{m}_a(\omega_1 - \omega_3) = 6.96(0.023 - 0.00597) = 0.1185\ \frac{lb}{min} \quad (b)$$

An energy rate balance at steady state for a control volume enclosing the compressor gives

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{m}_a h_{a1} + \dot{m}_{v1} h_{v1}] - [\dot{m}_a h_{a2} + \dot{m}_{v2} h_{v2}]$$

or $\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}_a[(h_{a2} - h_{a1}) + \omega_1(\underbrace{h_{v2}}_{h_{g2}} - \underbrace{h_{v1}}_{h_{g1}})]$

with data from Tables A-2E, 22E

$$\dot{Q}_{cv} = (-15\,hp)\left|\frac{2545\ Btu/h}{1\ hp}\right|\left|\frac{1\ h}{60\ min}\right| + \left(6.96\frac{lb(a)}{min}\right)\left[(206.46 - 131.46) + 0.023(1202 - 1100.7)\right]\frac{Btu}{lb(a)}$$

$$= -636.25 + 538.22 = -98.03\ Btu/min \qquad (a)$$

An energy rate balance at steady state for a control volume enclosing the aftercooler, but excluding the refrigerant line

$$0 = \dot{Q}_{cv} - \cancelto{0}{\dot{W}_{cv}} + [\dot{m}_a h_{a2} + \dot{m}_{v2} h_{v2}] - [\dot{m}_a h_{a3} + \dot{m}_{v3} h_{v3}] - \dot{m}_w h_4$$

Thus, with $h_v \approx h_g$

$$-\dot{Q}_{cv} = \dot{m}_a\{(h_{a2} - h_{a3}) + \omega_2 h_{g2} - \omega_3 h_{g3}\} - \dot{m}_w h_{f4}$$

$$= 6.96\{(206.46 - 133.86) + (0.023)(1202) - (0.00597)(1105)\} - (0.1185)(68.05)$$

$$= 6.96\{72.6 + 27.65 - 6.60\} - 8.06$$

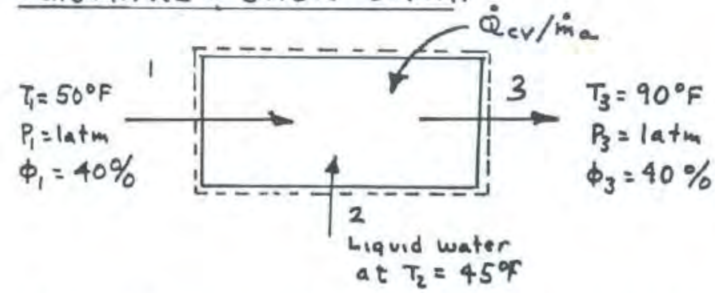
$$= 643.74\ \frac{Btu}{min}$$

1 ton of refrigeration = 200 Btu/min (Sec. 10.2.1). Thus

$$-\dot{Q}_{cv} = \left(643.74\frac{Btu}{min}\right)\left|\frac{1\ ton}{200\ Btu/min}\right| = 3.22\ tons \qquad (c)$$

PROBLEM 12.83

KNOWN: Operating data are provided for an air conditioner at steady state.
FIND: Determine the amount of liquid water injected and the heat transfer rate, each per lb of dry air.
SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The liquid water enters as saturated liquid and the moist air streams adhere to ideal gas mixture principles.

ANALYSIS: (a) At steady state, mass rate balances give $\dot{m}_{a1} = \dot{m}_{a2} \equiv \dot{m}_a$, $\dot{m}_{v1} + \dot{m}_w = \dot{m}_{v2}$. Accordingly, the rate liquid is injected is

$$\dot{m}_w = \dot{m}_{v3} - \dot{m}_{v1} = \dot{m}_a(\omega_3 - \omega_1) \qquad (1)$$

To find ω_1 and ω_3, $p_{v1} = \phi_1 p_g(T_1) = 0.4(0.1781\ lbf/in^2) = 0.0712\ lbf/in^2$ and $p_{v3} = \phi_3 p_g(T_3) = 0.4(0.6988) = 0.2795\ lbf/in.^2$ Then

$$\omega_1 = 0.622\left[\frac{0.0712}{14.696 - 0.0712}\right] = 0.00303\ \frac{lb(v)}{lb(a)},\ \omega_3 = 0.622\left[\frac{0.2795}{14.696 - 0.2795}\right] = 0.01206$$

To get $\dot{m}_a$, use $\dot{m}_a = (AV)_3/v_{a3}$ with the ideal gas equation

$$\dot{m}_a = \frac{(p - p_{v3})(AV)_3}{R_a T_3} = \frac{(14.696 - 0.2795)\frac{lbf}{in.^2}(1000\ ft^3/min)}{\left(\frac{1545}{28.97}\cdot\frac{ft\cdot lbf}{lb\cdot °R}\right)(90+460)°R}\left|\frac{144\ in.^2}{1\ ft^2}\right|$$

$$= 70.77\ lb/min$$

Inserting values into (1)

$$\dot{m}_w = 70.77(0.01206 - 0.00303) = 0.639\ lb/min \longleftarrow \dot{m}_w$$

(b) An energy balance at steady state reduces to give

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}}^{0} + [\dot{m}_a h_{a1} + \dot{m}_{v1} h_{v1}] + \dot{m}_w h_2 - [\dot{m}_a h_{a3} + \dot{m}_{v3} h_{v3}]$$

or

$$\dot{Q}_{cv} = \dot{m}_a[(h_{a3} - h_{a1}) + \omega_3 h_{g3} - \omega_1 h_{g1}] - \dot{m}_w h_2$$

With data from Tables A-2E and A-22E

$$\dot{Q}_{cv} = \left(70.77\ \frac{lb}{min}\right)[(131.46 - 121.88) + (0.01206)(1100.7) - (0.00303)(1083.3)]\ \frac{Btu}{lb} - (.639)(13.04)\ \frac{Btu}{min}$$

$$= \left(1376.8\ \frac{Btu}{min}\right)\left|\frac{60\ min}{1\ h}\right| = 8.26 \times 10^4\ Btu/h \longleftarrow \dot{Q}_{cv}$$

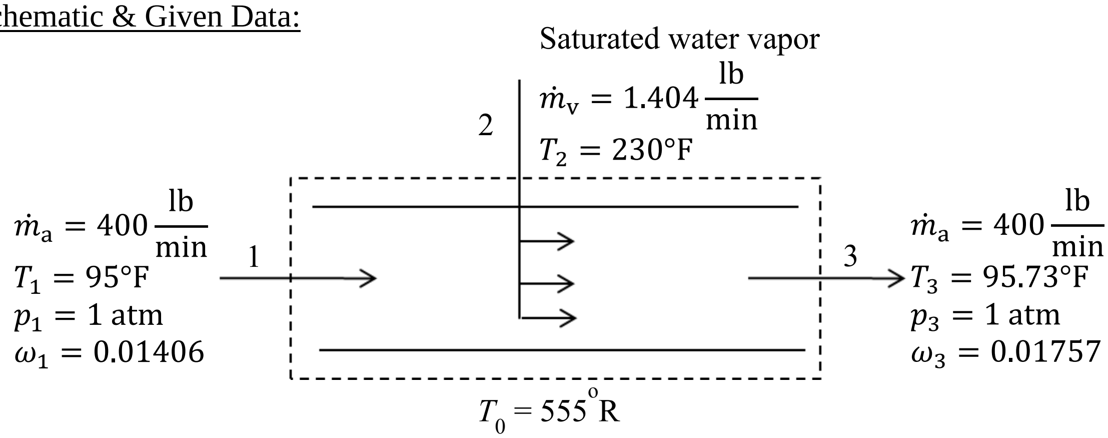
Problem 12.84

Figure P12.84 shows a steam-spray humidification device at steady state. Heat transfer between the device and its surroundings can be ignored, as can kinetic and potential energy effects. Determine the rate of exergy destruction, in Btu/min, for T_0=95°F.

Known:
Operating data are provided for a steam-spray humidification device at steady state.

Find:
Determine the rate of energy destruction.

Schematic & Given Data:



Engineering Model:
(1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy.
(2) The moist air streams adhere to the ideal gas model.

Analysis:
As a preliminary, the mole fractions for dry air and water vapor are evaluated. At 1, the mass flow rate of vapor is $\dot{m}_{v_1} = \omega_1 \dot{m}_{a_1} = (0.01406)(400) = 5.624 \frac{\text{lb}}{\text{min}}$. Thus, the molar flow rates at 1 are

$$\dot{n}_{a_1} = \frac{\dot{m}_{a_1}}{M_a} = \frac{400 \frac{\text{lb}}{\text{min}}}{28.97 \frac{\text{lb}}{\text{kmol}}} = 13.8074 \frac{\text{kmol}}{\text{min}}; \quad \dot{n}_{v_1} = \frac{\dot{m}_{v_1}}{M_v} = \frac{5.624 \frac{\text{lb}}{\text{min}}}{18.02 \frac{\text{lb}}{\text{kmol}}} = 0.3121 \frac{\text{kmol}}{\text{min}}$$

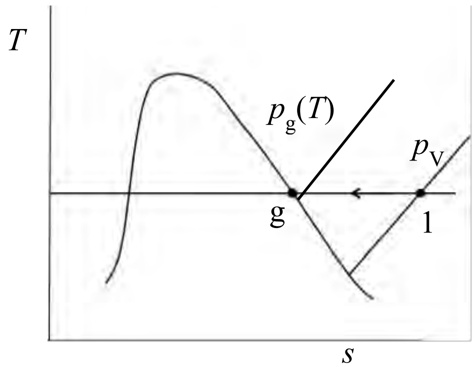
$$y_{a_1} = \frac{\dot{n}_{a_1}}{\dot{n}_{a_1} + \dot{n}_{v_1}} = \frac{13.8074}{13.8074 + 0.3121} = 0.9779; \quad y_{v_1} = \frac{\dot{n}_{v_1}}{\dot{n}_{a_1} + \dot{n}_{v_1}} = \frac{0.3121}{13.8074 + 0.3121} = 0.0221$$

At 3, the mass flow rate of vapor is $\dot{m}_{v_3} = \dot{m}_{v_1} + \dot{m}_v = 5.624 + 1.404 = 7.028 \frac{\text{lb}}{\text{min}}$. The molar flow rates at 3 are

$$\dot{n}_{a_3} = \dot{n}_{a_1} = 13.8074\,\frac{\text{kmol}}{\text{min}};\;\; \dot{n}_{v_3} = \frac{\dot{m}_{v_3}}{M_v} = \frac{7.028\,\frac{\text{lb}}{\text{min}}}{18.02\,\frac{\text{lb}}{\text{kmol}}} = 0.39\,\frac{\text{kmol}}{\text{min}}$$

$$y_{a_3} = \frac{\dot{n}_{a_3}}{\dot{n}_{a_3} + \dot{n}_{v_3}} = \frac{13.8074}{13.8074 + 0.39} = 0.9725;\;\; y_{v_3} = \frac{\dot{n}_{v_3}}{\dot{n}_{a_3} + \dot{n}_{v_3}} = \frac{0.39}{13.8074 + 0.39} = 0.0275$$

Also, it may be recalled that at states of water vapor where the ideal gas model is applicable, Eq. 6.18 can be used



$$s(T, p_v) - s(T, p_g) = \int \cancel{c_p}\,\frac{dT}{T} - R\ln\frac{p_v}{p_g}$$

or

$$s(T, p_v) = s_g(T) - \frac{\bar{R}}{M_v}\ln\phi \qquad (1)$$

With these preliminaries in hand, the exergy destruction rate can be evaluated using $\dot{E}_d = T_0\dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production from an entropy rate balance

$$0 = \cancel{\sum\frac{\dot{Q}_j}{T_j}} + \left[\dot{m}_a s_a(T_1, y_{a_1}p) + \dot{m}_{v_1} s_v(T_1, y_{v_1}p)\right] + \dot{m}_v s_g(T_2)$$
$$- \left[\dot{m}_a s_a(T_3, y_{a_3}p) + \dot{m}_{v_3} s_v(T_3, y_{v_3}p)\right] + \dot{\sigma}_{cv}$$

Thus

$$\dot{\sigma}_{cv} = \dot{m}_a\left[s_a(T_3, y_{a_3}p) - s_a(T_1, y_{a_1}p)\right] + \dot{m}_{v_3} s_v(T_3, y_{v_3}p) - \dot{m}_{v_1} s_v(T_1, y_{v_1}p) - \dot{m}_v s_g(T_2)$$

Using Eq. 6.22 and Eq. (1) from above:

$$\dot{\sigma}_{cv} = \dot{m}_a\left[c_{p_a}\ln\frac{T_3}{T_1} - \frac{\bar{R}}{M_a}\ln\frac{y_{a_3}}{y_{a_1}}\right] + \dot{m}_{v_3}\left[s_g(T_3) - \frac{\bar{R}}{M_v}\ln\frac{y_{v_3}p}{p_g(T_3)}\right] - \dot{m}_{v_1}\left[s_g(T_1) - \frac{\bar{R}}{M_v}\ln\frac{y_{v_1}p}{p_g(T_1)}\right]$$
$$- \dot{m}_v s_g(T_2)$$
$$= \left(400\,\frac{\text{lb}}{\text{min}}\right)\left[\left(0.24\,\frac{\text{Btu}}{\text{lb°R}}\right)\ln\frac{555.73\text{°R}}{555\text{°R}} - \left(\frac{1.986}{28.97}\,\frac{\text{Btu}}{\text{lb°R}}\right)\ln\frac{0.9725}{0.9779}\right]$$
$$+ \left(7.028\,\frac{\text{lb}}{\text{min}}\right)\left[1.9932\,\frac{\text{Btu}}{\text{lb°R}} - \left(\frac{1.986}{18.02}\,\frac{\text{Btu}}{\text{lb°R}}\right)\ln\frac{(0.0275)(14.696)}{0.8348}\right]$$
$$- \left(5.624\,\frac{\text{lb}}{\text{min}}\right)\left[1.9951\,\frac{\text{Btu}}{\text{lb°R}} - \left(\frac{1.986}{18.02}\,\frac{\text{Btu}}{\text{lb°R}}\right)\ln\frac{(0.0221)(14.696)}{0.8165}\right]$$
$$- \left(1.404\,\frac{\text{lb}}{\text{min}}\right)\left(1.7289\,\frac{\text{Btu}}{\text{lb°R}}\right) = 0.6289\;\frac{\text{Btu}}{\text{min°R}}$$

Finally ⟵

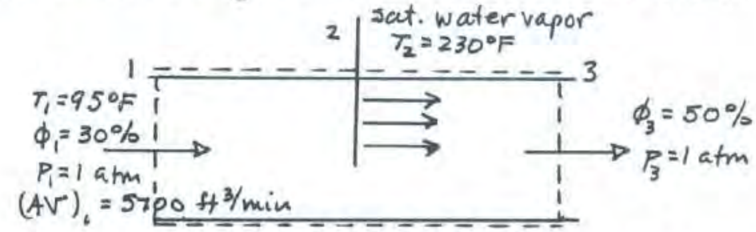
$$\dot{E}_d = T_0\dot{\sigma}_{cv} = (555\text{°R})\left(0.6289\,\frac{\text{Btu}}{\text{min°R}}\right) = 349.04\,\frac{\text{Btu}}{\text{min}}$$

PROBLEM 12.85

KNOWN: Operating data are provided for a steam-spray humidification device at steady state.

FIND: Determine the temperature of the exiting moist air stream and the rate of steam injection.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The moist air streams adhere to the ideal gas model.

ANALYSIS: Governing Equations. The state at 3 will be determined using mass and energy rate balances. For mass, $\dot{m}_{a_1} = \dot{m}_{a_3} \equiv \dot{m}_a$ and $\dot{m}_{v_1} + \dot{m}_2 = \dot{m}_{v_3}$. Thus

$$\dot{m}_2 = \dot{m}_a(\omega_3 - \omega_1) \qquad (1)$$

To get $\dot{m}_a$, use

$$\dot{m}_a = \frac{(AV)_1}{v_{a_1}} = \frac{(p - p_{v_1})(AV)_1}{R_a T_1} \qquad (2)$$

The energy balance at steady state reduces as follows:

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}_a\left[(h_{a_1} + \omega_1 h_{v_1}) + (\omega_3 - \omega_1)h_2 - (h_{a_3} + \omega_3 h_{v_3})\right]$$

or

$$(h_{a_3} + \omega_3 h_{g_3}) = (h_{a_1} + \omega_1 h_{g_1}) - (\omega_3 - \omega_1) h_{g_2} \qquad (3)$$

Evaluating Properties. To find ω_1, use $p_{v_1} = \phi_1 p_g(T_1)$ and $\omega_1 = 0.622\,(p_{v_1}/p - p_{v_1})$. With T_1 and p_1 known, ω_1, h_{a_1}, and h_{g_1} can be determined. Also, h_{g_2} can be determined since T_2 is known.

The value of ω_3 is related to T_3 using $p_{v_3} = \phi_3\, p_g(T_3)$ and $\omega_3 = 0.622\,(p_{v_3}/p - p_{v_3})$. Also, h_{a_3} and h_{g_3} are related directly to T_3.

Solution Procedure. We see from (3) and the discussion of property evaluation that T_3 can be found using an iterative procedure with data from Tables A-22E and A-2E. The following IT code provides an alternative to the iterative procedure:

IT Code

```
T1 = 95 // °F
p = 14.696
phi1 = 0.3
AV1 = 5700 // ft³/min
T2 = 230 // °F
x2 = 1
phi3 = 0.5

w1 = w_Tphi(T1,phi1,p)
h1 = ha_Tw(T1,w1)
va1 = va_Tw(T1,w1,p)
psat = Psat_T("Water/Steam", T2)
h2 = hsat_Px("Water/Steam", psat, x2)
w3 = w_Tphi(T3, phi3, p)
h3 = ha_Tphi(T3, phi3, p)

mdota = AV1 / va1
mdotv = mdota * (w3 - w1)
0 = mdota * (h1 - h3) + mdotv * h2
```

IT Results

$\omega_1 = 0.01053$ lb(v)/lb(a)
$\omega_3 = 0.01877$ lb(v)/lb(a)
$h_1 = 34.37$ Btu/lb(a)
$h_2 = 1157$ Btu/lb(a)
$h_3 = 43.9$ Btu/lb(a)
$\dot{m}_a = 400.9$ lb/min
$\dot{m}_2 = 3.304$ lb/min ← $\dot{m}_2$
$T_3 = 96.79°F$ ← T_3

To check the validity of the IT results, Fig A-9E gives $h_1 = 34.4$, $\omega_1 = 0.0106$, which agree closely with the respective IT answers. Also, referring to Fig A-9E with ω_3 and h_3 from IT, we get $T_3 \approx 96.5°F$ which also agrees well with the IT value.

PROBLEM 12.86

KNOWN: Consider the steam-spray humidifier of Problem 12.85.

FIND: Determine the exergy destruction rate.

SCHEMATIC & GIVEN DATA: See solution to Problem 12.85 for the schematic.

State 1	State 2	State 3
$T_1 = 95°F$	$T_2 = 230°F$	$T_3 = 96.79°F$
$p_1 = 14.696$ lbf/in.^{2}	Sat. vapor	$p_3 = 14.696$ lbf/in.^{2}
$\omega_1 = 0.01053$ lb(v)/lb(a)	$\dot m_2 = 3.304$ lb/min	$\omega_3 = 0.01877$ lb(v)/lb(a)
$\dot m_a = 400.9$ lb/min		

ENGINEERING MODEL: See Problem 12.85. Also let $T_0 = 95°F = 550°R$.

ANALYSIS: The exergy destruction rate can be evaluated using $\dot E_d = T_0 \dot\sigma_{cv}$, where $\dot\sigma_{cv}$ is found from an entropy rate balance

$$0 = \sum_j \cancel{\frac{\dot Q_j}{T_j}}^{\,0} + \dot m_a[s_a(T_1, p_{a1}) + \omega_1 s_v(T_1, p_{v1})] + \dot m_2 s_g(T_2) - \dot m_a[s_a(T_3, p_{a3}) + \omega_3 s_v(T_3, p_{v3})] + \dot\sigma_{cv}$$

or

$$\dot\sigma_{cv} = \dot m_a \underbrace{[s_a(T_3, p_{a3}) + \omega_3 s_v(T_3, p_{v3})]}_{s(T_3,\,\omega_3,\,p_3)} - \dot m_a \underbrace{[s_a(T_1, p_{a1}) + \omega_1 s_v(T_1, p_{v1})]}_{s(T_1,\,\omega_1,\,p_3)} - \dot m_2 s_g(T_2) \qquad (1)$$

As noted, the terms in brackets are each uniquely determined from the given data. ① These functions are provided by IT. Thus, $\dot E_d$ is determined using IT as follows:

IT Code

```
T1 = 95  // °F
p1 = 14.696  // lbf/in.²
w1 = 0.01053  // lb(v)/lb(a)
mdota = 400.9  // lb/min
T2 = 230  // °F
x2 = 1
T3 = 96.79  // °F
p3 = 14.696  // lbf/in.²
w3 = 0.01877  // lb(v)/lb(a)
To = 555 // °R

s1 = sa_Tw(T1,w1,p1)
s3 = sa_Tw(T3,w3,p3)
psat = Psat_T("Water/Steam", T2)
s2 = ssat_Px("Water/Steam", psat, x2)

sigmadot = mdota * (s3 - s1 - (w3 - w1) * s2)
Edotd = To * sigmadot
```

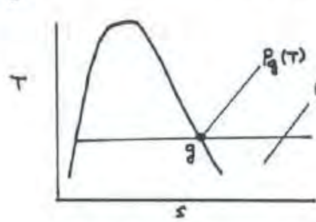
IT Results

$\dot\sigma_{cv}$ = 1.526 Btu/min·°R

$\dot E_d$ = 846.8 Btu/min ⟵ $\dot E_d$

---

1. For states of water vapor where the ideal gas model applies, Eq. 6.18 can be invoked to write



$$s(T, p_v) - s(T, p_g) = \cancel{\int c_p \frac{dT}{T}}^{\,0} - R \ln \frac{p_v}{p_g}$$

or

$$s(T, p_v) = s_g(T) - \frac{\bar R}{M_v} \ln \underbrace{\frac{p_v}{p_g}}_{=\,\phi}$$

Further, applying Eq. 6.20b to the dry air

$$s(T, p_a) - s(T_{ref}, p_{ref})^{0} = s°(T) - s°(T_{ref}) - R_a \ln \frac{p_a}{p_{ref}}$$

$$s(T, p_a) = s°(T) - s°(T_{ref}) - \frac{\bar R}{M_a} \ln \frac{(p - p_v)}{p_{ref}}$$

Thus

$$s(T, \omega, p) = \left[s°(T) - s°(T_{ref}) - \frac{\bar R}{M_a} \ln \frac{(p - p_v)}{p_{ref}}\right] + \omega\left[s_g(T) - \frac{\bar R}{M_v} \ln \phi\right]$$

The functions in IT are based on this relationship. The development of a hand calculation to verify the IT result is left to the reader.

Problem 12.87

Atmospheric air having dry-bulb and wet-bulb temperatures of 33 and 29°C, respectively, enters a well-insulated chamber operating at steady state and mixes with air entering with dry-bulb and wet-bulb temperatures of 16 and 12°C, respectively. The volumetric flow rate of the lower temperature stream is twice that of the other stream. A single mixed stream exits. Determine for the exiting stream

(a) the relative humidity.

(b) the temperature, in °C.

Pressure is uniform throughout at 1 atm. Neglect kinetic and potential energy effects.

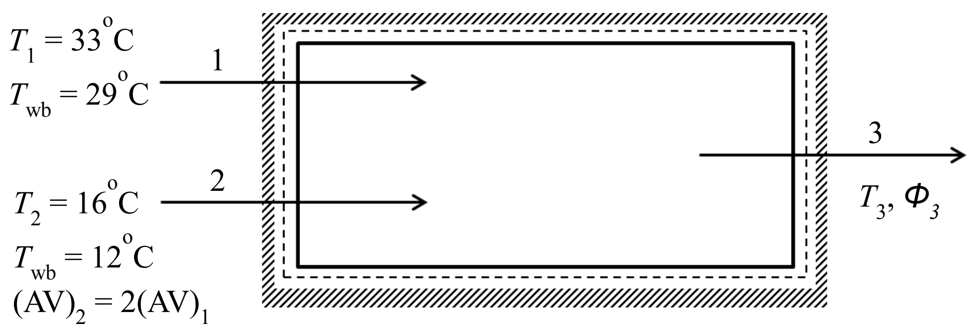
Known:

Atmosphere air and moist air with given conditions mix adiabatically in a well-insulated chamber.

Find:

For the exiting stream, determine (a) the relative humidity and (b) temperature.

Schematic & Given Data:



Engineering Model:

(1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy.

(2) The pressure is uniform throughout at 1 atm.

(3) The wet-bulb temperature can be used in place of the adiabatic saturation temperature.

Analysis:

a.) At steady state, mass rate balances give $\dot{m}_{a_1} + \dot{m}_{a_2} = \dot{m}_{a_3}$ and $\dot{m}_{v_1} + \dot{m}_{v_2} = \dot{m}_{v_3}$. Thus $\omega_1 \dot{m}_{a_1} + \omega_2 \dot{m}_{a_2} = \omega_3 \dot{m}_{a_3}$

Combining

$$\omega_3 = \frac{\omega_1 \dot{m}_{a_1} + \omega_2 \dot{m}_{a_2}}{\dot{m}_{a_1} + \dot{m}_{a_2}} = \frac{\omega_1 + \left(\frac{\dot{m}_{a_2}}{\dot{m}_{a_1}}\right)\omega_2}{1 + \left(\frac{\dot{m}_{a_2}}{\dot{m}_{a_1}}\right)} \tag{1}$$

The mass flow rate ratio is obtained using the given volumetric flow rate relation

$$\dot{m}_{\mathrm{a}_1} = \frac{(\mathrm{AV})_1}{v_{\mathrm{a}_1}} = \frac{p_{\mathrm{a}_1}(\mathrm{AV})_1}{\frac{\bar{R}}{M_\mathrm{a}}T_1};\ \ \dot{m}_{\mathrm{a}_2} = \frac{p_{\mathrm{a}_2}(\mathrm{AV})_2}{\frac{\bar{R}}{M_\mathrm{a}}T_2} \Longrightarrow \frac{\dot{m}_{\mathrm{a}_2}}{\dot{m}_{\mathrm{a}_1}} = \frac{p_{\mathrm{a}_2}}{p_{\mathrm{a}_1}}\frac{T_1}{T_2}\frac{(\mathrm{AV})_2}{(\mathrm{AV})_1} = 2\frac{p_{\mathrm{a}_2}}{p_{\mathrm{a}_1}}\frac{T_1}{T_2} \qquad (2)$$

The terms ω_1 and ω_2 can be calculated using Eq. 12.48 and 12.49 and by assuming constant specific heat. First, to find ω_1

$$\omega' = 0.622\frac{p_\mathrm{g}(T_\mathrm{wb})}{p - p_\mathrm{g}(T_\mathrm{wb})} = 0.622\frac{0.04008\ \mathrm{bar}}{(1.01325 - 0.04008)\mathrm{bar}} = 0.0256$$

$$\omega_1 = \frac{c_p(T_\mathrm{wb} - T_1) + \omega' h_\mathrm{fg}(T_\mathrm{wb})}{h_\mathrm{g}(T_1) - h_\mathrm{f}(T_\mathrm{wb})} = \frac{\left(1.005\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\right)(29 - 33)\mathrm{K} + (0.0256)\left(2432.8\frac{\mathrm{kJ}}{\mathrm{kg}}\right)}{(2561.7 - 121.61)\frac{\mathrm{kJ}}{\mathrm{kg}}}$$

$$= 0.0239\frac{\mathrm{kg\ (vapor)}}{\mathrm{kg\ (air)}}$$

To find ω_2

$$\omega' = 0.622\frac{p_\mathrm{g}(T_\mathrm{wb})}{p - p_\mathrm{g}(T_\mathrm{wb})} = 0.622\frac{0.01402\ \mathrm{bar}}{(1.01325 - 0.01402)\mathrm{bar}} = 0.0087$$

$$\omega_2 = \frac{c_p(T_\mathrm{wb} - T_2) + \omega' h_\mathrm{fg}(T_\mathrm{wb})}{h_\mathrm{g}(T_2) - h_\mathrm{f}(T_\mathrm{wb})} = \frac{\left(1.005\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\right)(12 - 16)\mathrm{K} + (0.0087)\left(2523.4\frac{\mathrm{kJ}}{\mathrm{kg}}\right)}{(2530.8 - 50.41)\frac{\mathrm{kJ}}{\mathrm{kg}}}$$

$$= 0.0072\frac{\mathrm{kg\ (vapor)}}{\mathrm{kg\ (air)}}$$

Solving Eq. 12.43

$$p_{\mathrm{v}_1} = \frac{\omega_1 p}{0.622 + \omega_1} = \frac{(0.0239)(1.01325)}{0.622 + 0.0239} = 0.0375\ \mathrm{bar}\ \Longrightarrow$$

$$\mathrm{p}_{\mathrm{a}_1} = p - p_{\mathrm{v}_1} = 1.01325 - 0.0375 = 0.97576\ \mathrm{bar}$$

$$p_{\mathrm{v}_2} = \frac{\omega_2 p}{0.622 + \omega_2} = \frac{(0.0072)(1.01325)}{0.622 + 0.0072} = 0.01159\ \mathrm{bar}\ \Longrightarrow$$

$$p_{\mathrm{a}_2} = p - p_{\mathrm{v}_2} = 1.01325 - 0.0116 = 1.00166\ \mathrm{bar}$$

Inserting values into Eq. (2) gives

$$\dot{m}_{a_1} = 2\frac{p_{\mathrm{a}_2}}{p_{\mathrm{a}_1}}\frac{T_1}{T_2} = 2\left(\frac{1.00166}{0.97576}\right)\left(\frac{306}{289}\right) = 2.174$$

Then, with known values, Eq. (1) yields

$$\omega_3 = \frac{0.0239 + (2.174)(0.0072)}{1 + 2.174} = 0.01246\frac{\mathrm{kg\ (vapor)}}{\mathrm{kg\ (air)}}$$

Accordingly

$$p_{v_3} = \frac{\omega_3 p}{0.622 + \omega_3} = \frac{(0.01246)(1.01325)}{0.622 + 0.01246} = 0.0199 \text{ bar}$$

Then

$$\phi_3 = \frac{p_{v_3}}{p_g(T_3)} = \frac{0.0199}{p_g(T_3)} \tag{3}$$

b.) To find T_3 write an energy balance for the control volume with assumption (1):

$$0 = \cancel{\dot{Q}}_{cv} - \cancel{\dot{W}}_{cv} + \dot{m}_{a_1} h_{a_1} + \dot{m}_{v_1} h_{v_1} + \dot{m}_{a_2} h_{a_2} + \dot{m}_{v_2} h_{v_2} - \left[\dot{m}_{a_3} h_{a_3} + \dot{m}_{v_3} h_{v_3}\right]$$

Thus

$$0 = \dot{m}_{a_1}\left[h_{a_1} + \omega_1 h_{g_1}\right] + \dot{m}_{a_2}\left[h_{a_2} + \omega_2 h_{g_2}\right] - \dot{m}_{a_3}\left[h_{a_3} + \omega_3 h_{g_3}\right]$$

Or

$$h_{a_3} + \omega_3 h_{g_3} = \frac{\dot{m}_{a_1}\left[h_{a_1} + \omega_1 h_{g_1}\right] + \dot{m}_{a_2}\left[h_{a_2} + \omega_2 h_{g_2}\right]}{\dot{m}_{a_1} + \dot{m}_{a_2}}$$

$$= \frac{\left[h_{a_1} + \omega_1 h_{g_1}\right] + \left(\frac{\dot{m}_{a_2}}{\dot{m}_{a_1}}\right)\left[h_{a_2} + \omega_2 h_{g_2}\right]}{1 + \left(\frac{\dot{m}_{a_2}}{\dot{m}_{a_1}}\right)}$$

With values from Tables A-2 and A-22

$$h_{a_3} + \omega_3 h_{g_3} = \frac{[306.2 + (0.0239)(2561.7)]\frac{\text{kJ}}{\text{kg}} + (2.174)[289.2 + (0.0072)(2530.8)]\frac{\text{kJ}}{\text{kg}}}{1 + 2.174}$$

$$= 326.3 \frac{\text{kJ}}{\text{kg (air)}}$$

(b) Solving iteratively

$$T_3 \approx 21.5°\text{C} \longleftarrow$$

(a) Returning to Eq. (3)

$$\phi_3 = \frac{0.0199}{0.0257} = 0.774 = 77.4\% \longleftarrow$$

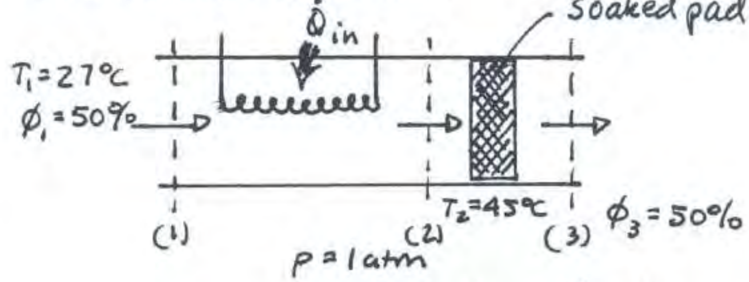
Comment: The solution of this problem can be facilitated by use of a psychrometric chart. However, note that the specific enthalpy of moist air appearing on the chart is calculated as $\left[c_{pa} T(°\text{C}) + \omega h_g\right]$, and not as determined in the above solution.

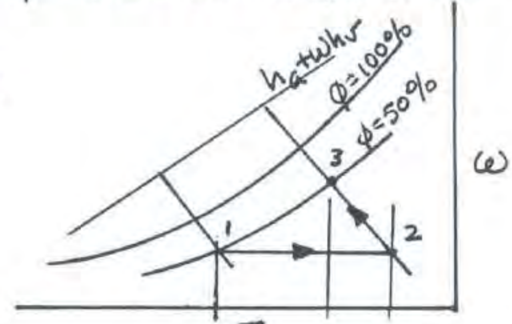
PROBLEM 12.88

KNOWN: Moist air is heated and then passes through a soaked pad evaporative cooling unit. Data are known at various locations.

FIND: Determine (a) the entering humidity ratio, (b) the rate of heat transfer in the heating section per unit mass of moist air flowing, and (c) the humidity ratio and temperature at the exit of the evaporative cooling section.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) Each section is considered a control volume at steady state, with $\dot{W}_{cv} = 0$. (2) Kinetic and potential energy effects are negligible. (3) The moist air mixture is modeled as an ideal gas mixture. (4) For the cooling section, $\dot{Q}_{cv} = 0$ and the process is modeled as occurring at constant wet bulb temperature. Hence, $h_2 \approx h_3$ on the psychrometric chart.

ANALYSIS: (a) From Fig. A-9; $\omega_1 = 0.0112$ kg(v)/kg(a) ← ω_1

(b) For the heating section, $\omega_1 = \omega_2 \equiv \omega$. Thus

$$0 = \dot{Q}_{in} + \dot{m}_a[(h_a + \omega h_v)_1 - (h_a + \omega h_v)_2]$$

Further, at location (1): $\dot{m}_1 = \dot{m}_a + \dot{m}_v = \dot{m}_a(1+\omega)$

Thus $\dot{m}_a = \dot{m}_1/(1+\omega)$

$$\dot{Q}_{in} = [\dot{m}_1/(1+\omega)][(h_a + \omega h_v)_2 - (h_a + \omega h_v)_1]$$

$$\frac{\dot{Q}_{in}}{\dot{m}_1} = \left(\frac{1}{1+\omega}\right)[(h_a + \omega h_v)_2 - (h_a + \omega h_v)]$$

$$= \left(\frac{1}{1+0.0112}\right)[(74.3) - (55.5)] = 18.59 \frac{\text{kJ}}{\text{kg (mixture)}}$$ ← $\dot{Q}_{in}/\dot{m}_1$

(c) From Fig. A-9; $\omega_3 = 0.016$ ← ω_3

$T_3 = 33°C$ ← T_3

Problem 12.89

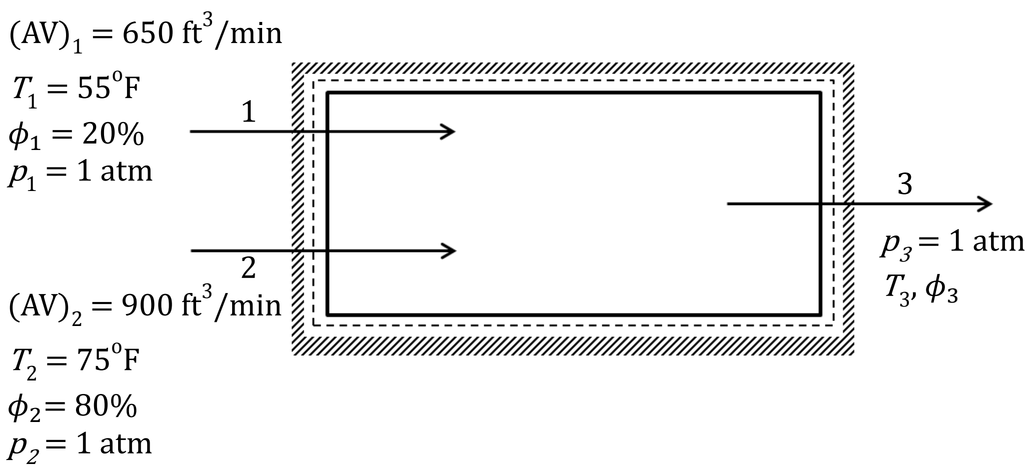
At steady state, a stream consisting of 650 ft^3/min of air at 55°F, 1 atm, 20% relative humidity is mixed adiabatically with a stream consisting of 900 ft^3/min of air at 75°F, 1 atm, 80% relative humidity. A single mixed stream exits at 1 atm. Neglect kinetic and potential energy effects. Determine for the exiting stream
(a) the relative humidity.
(b) the temperature, in $^\circ$F.

Solution:

Known:
Two air streams with given conditions mix adiabatically.

Find:
Determine for the exiting stream, (a) the relative humidity and (b) the temperature.

Schematic and Known Data:



Engineering Model:
(1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy.

Analysis:
At steady state mass rate balances give $\dot{m}_{a_1} + \dot{m}_{a_2} = \dot{m}_{a_3}$ and $\dot{m}_{v_1} + \dot{m}_{v_2} = \dot{m}_{v_3}$. Thus

$$\omega_1\dot{m}_{a_1} + \omega_2\dot{m}_{a_2} = \omega_3\dot{m}_{a_3}; \implies \omega_3 = \frac{\omega_1\dot{m}_{a_1} + \omega_2\dot{m}_{a_2}}{\dot{m}_{a_1} + \dot{m}_{a_2}} \qquad (1)$$

The mass flow rates are obtained using the given volumetric flow rates and $p_{v_1} = \phi_1 p_{g_1} = (0.2)(0.21415) = 0.0428\,\frac{\text{lbf}}{\text{in.}^2}$; $p_{v_2} = \phi_2 p_{g_2} = (0.8)(0.4302) = 0.3442\,\frac{\text{lbf}}{\text{in.}^2}$

$$\dot{m}_{a_1} = \frac{(\mathrm{AV})_1}{v_{a_1}} = \frac{p_{a_1}(\mathrm{AV})_1}{\frac{\bar{R}}{M_a}T_1} = \frac{\left[(14.696-0.0428)\,\frac{\mathrm{lbf}}{\mathrm{in.}^2}\cdot\frac{144\ \mathrm{in.}^2}{\mathrm{ft}^2}\right]\left(650\,\frac{\mathrm{ft}^3}{\mathrm{min}}\right)}{\left(\frac{1545}{28.97}\,\frac{\mathrm{ft}\cdot\mathrm{lbf}}{\mathrm{lb}°\mathrm{R}}\right)(515°\mathrm{R})} = 49.94\,\frac{\mathrm{lb\ (air)}}{\mathrm{min}}$$

$$\dot{m}_{a_2} = \frac{p_{a_2}(\mathrm{AV})_2}{\frac{\bar{R}}{M_a}T_2} = \frac{\left[(14.696-0.3442)\,\frac{\mathrm{lbf}}{\mathrm{in.}^2}\cdot\frac{144\ \mathrm{in.}^2}{\mathrm{ft}^2}\right]\left(900\,\frac{\mathrm{ft}^3}{\mathrm{min}}\right)}{\left(\frac{1545}{28.97}\,\frac{\mathrm{ft}\cdot\mathrm{lbf}}{\mathrm{lb}°\mathrm{R}}\right)(535°\mathrm{R})} = 65.19\,\frac{\mathrm{lb\ (air)}}{\mathrm{min}}$$

Solving Eq. 12.43, the values of ω_1 and ω_2 are:

$$\omega_1 = 0.622\,\frac{p_{v_1}}{p-p_{v_1}} = 0.622\,\frac{0.0428}{14.696-0.0428} = 0.0018\,\frac{\mathrm{lb\ (vapor)}}{\mathrm{lb\ (air)}}$$

$$\omega_2 = 0.622\,\frac{p_{v_2}}{p-p_{v_2}} = 0.622\,\frac{0.3442}{14.696-0.3442} = 0.0149\,\frac{\mathrm{lb\ (vapor)}}{\mathrm{lb\ (air)}}$$

Accordingly, Eq. (1) gives:

$$\omega_3 = \frac{\omega_1\dot{m}_{a_1}+\omega_2\dot{m}_{a_2}}{\dot{m}_{a_1}+\dot{m}_{a_2}} = \frac{(0.0018)\left(49.94\,\frac{\mathrm{lb}}{\mathrm{min}}\right)+(0.0149)\left(65.19\,\frac{\mathrm{lb}}{\mathrm{min}}\right)}{(49.94+65.19)\,\frac{\mathrm{lb}}{\mathrm{min}}}$$

$$= 0.0092\,\frac{\mathrm{lb\ (vapor)}}{\mathrm{lb\ (air)}}$$

Solving Eq. 12.43

$$p_{v_3} = \frac{\omega_3 p_3}{0.622+\omega_3} = \frac{(0.0092)(14.696)}{0.622+0.0092} = 0.2142\,\frac{\mathrm{lbf}}{\mathrm{in.}^2} \implies \phi_3 = \frac{p_{v_3}}{p_g(T_3)}$$

$$= \frac{0.2142}{p_g(T_3)} \qquad (2)$$

The temperature T_3 can be obtained from an energy rate balance which reduces with the assumptions listed above to

$$0 = \cancel{\dot{Q}}_{cv} - \cancel{\dot{W}}_{cv} + \dot{m}_{a_1}h_{a_1} + \dot{m}_{v_1}h_{v_1} + \dot{m}_{a_2}h_{a_2} + \dot{m}_{v_2}h_{v_2} - \left[\dot{m}_{a_3}h_{a_3} + \dot{m}_{v_3}h_{v_3}\right]$$

Thus, $0 = \dot{m}_{a_1}\left[h_{a_1}+\omega_1 h_{g_1}\right] + \dot{m}_{a_2}\left[h_{a_2}+\omega_2 h_{g_2}\right] - \dot{m}_{a_3}\left[h_{a_3}+\omega_3 h_{g_3}\right]$

or

$$h_{a_3}+\omega_3 h_{g_3} = \frac{\dot{m}_{a_1}\left[h_{a_1}+\omega_1 h_{g_1}\right] + \dot{m}_{a_2}\left[h_{a_2}+\omega_3 h_{g_2}\right]}{\dot{m}_{a_1}+\dot{m}_{a_2}}$$

With values from Tables A-2 and A-22:

$$h_{a_3}+\omega_3 h_{g_3}$$

$$= \frac{\left(44.94\,\frac{\mathrm{lb}}{\mathrm{min}}\right)[123.07+(0.0018)(1085.5)]\,\frac{\mathrm{Btu}}{\mathrm{lb}} + \left(65.19\,\frac{\mathrm{lb}}{\mathrm{min}}\right)[127.86+(0.0149)(1094.25)]\,\frac{\mathrm{Btu}}{\mathrm{lb}}}{(49.94+65.19)\,\frac{\mathrm{lb}}{\mathrm{min}}}$$

$$= 135.86\,\frac{\mathrm{Btu}}{\mathrm{lb\ (air)}}$$

(b) Solving iteratively

$T_3 \approx 68°\text{F}$

(a) Returning to Eq. (2)

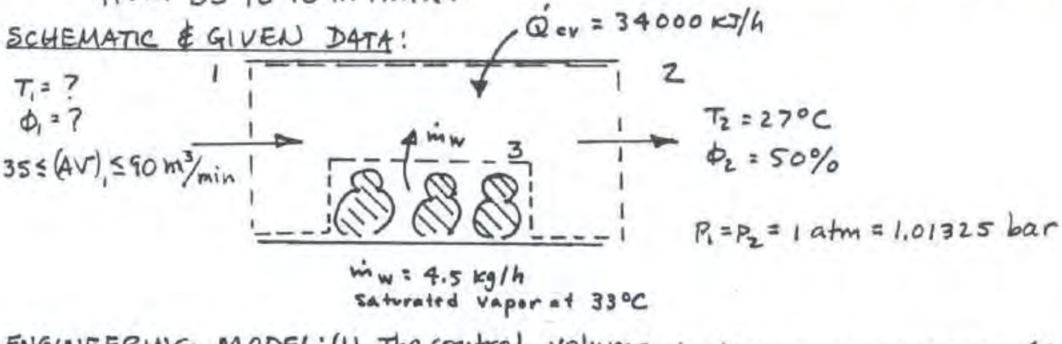
$$\phi_3 = \frac{0.2142}{0.3391} = 0.632 = 63.2\%$$

PROBLEM 12.90

KNOWN: Data are provided for moist air being supplied and removed from an occupied class room.

FIND: (a) For a specified supply air volumetric flow rate, determine the supply air temperature and relative humidity. (b) Plot supply air temperature and relative humidity versus supply air volumetric flow rate ranging from 35 to 90 m³/min.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume in the accompanying figure is at steady state with $\dot{W}_{cv}=0$ and negligible effects of kinetic and potential energy. (2) The moisture entering from the occupants is saturated vapor at 33°C. The moist air streams adhere to the ideal gas model. Pressure is uniform throughout at 1 atm.

ANALYSIS: At steady state, mass rate balances give: $\dot{m}_{a1} = \dot{m}_{a2} \equiv \dot{m}_a$; $\dot{m}_{v1} + \dot{m}_w = \dot{m}_{v2}$. Thus $\dot{m}_w = \dot{m}_a(\omega_2 - \omega_1)$, or

$$\dot{m}_a = \frac{\dot{m}_w}{(\omega_2 - \omega_1)} \qquad (1)$$

The humidity ratio ω_2 is found using $p_{v2} = \phi_2\, p_g(T_2)$ and $\omega_2 = .622\left[p_{v2}/(p - p_{v2})\right]$. To determine ω_1 and T_1 we use an energy rate balance, which reduces at steady state to

$$0 = \dot{Q}_{cv} - \cancelto{0}{\dot{W}_{cv}} + [\dot{m}_a h_{a1} + \dot{m}_{v1} h_{v1}] + \dot{m}_w h_3 - [\dot{m}_a h_{a2} + \dot{m}_{v2} h_{v2}]$$

or

$$0 = \dot{Q}_{cv} + \dot{m}_a\left[(h_{a1} - h_{a2}) + \omega_1 h_{g1} - \omega_2 h_{g2}\right] + \dot{m}_w h_g(T_3) \qquad (2)$$

With T_2 and T_3 known, the specific enthalpies h_{g2} and h_{g3} can be determined. Thus Eq. (1) has two unknowns: ω_1 and $\dot{m}_a$. Equation (2) has the same two unknowns and also has T_1 as an unknown.

Another expression relating these quantities is obtained from $(AV)_1 = \dot{m}_a v_{a1}$, as follows:

$$(AV)_1 = \frac{\dot{m}_w}{(\omega_2 - \omega_1)}\left[\frac{(\bar{R}/M_a)T_1}{p_{a1}}\right] = \frac{\dot{m}_w(\bar{R}/M_a)T_1}{(\omega_2 - \omega_1)(p - p_{v1})} \qquad (3)$$

where $\omega_1 = 0.622\left[p_{v1}/(p - p_{v1})\right]$. (4)

(a) Sample calculation using table data, $(AV)_1$ = 40 m³/min. To evaluate ω_2

$$p_{v2} = \phi_2\, p_g(T_2) = 0.5(0.03567) = 0.01784 \text{ bar. Thus}$$

$$\omega_2 = 0.622\left[\frac{(0.01784)}{(1.01325 - 0.01784)}\right] = 0.0111 \frac{\text{kg(v)}}{\text{kg(a)}}$$

Thus

$$\dot{m}_a = \frac{4.5 \text{ kg(v)/h}}{(0.0111 - \omega_1)\frac{\text{kg(v)}}{\text{kg(a)}}} \qquad (a)$$

Solving Eq. (2) for ω_1

$$\omega_1 = \frac{\omega_2\left[\dot{Q}_{cv}/\dot{m}_w + h_{g3} - h_{g2}\right] + (h_{a1} - h_{a2})}{(\dot{Q}_{cv}/\dot{m}_w + h_{g3} - h_{g1})}$$

PROBLEM 12.90 (Cont'd.) - Page 2

From Table A-2E, $h_{g2} = 2550.8$ and $h_{g3} = 2561.7$ kJ/kg. With $h_{a1} - h_{a2} = c_{pa}(T_1 - T_2)$ and $c_{pa} = 1.005$ kJ/kg·K, we get

$$\omega_1 = \frac{(0.0111)\left[(34000/4.5) + 2561.7 - 2550.8\right] + 1.005(T_1 - 27)}{\left[(34000/4.5) + 2561.7 - h_{g1}\right]} \qquad (b)$$

From Eq. (3)

$$40\,\frac{m^3}{min} = \frac{(4.5 \text{ kg/h})\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(T_1 + 273)K}{(0.0111 - \omega_1)(1.01325 - p_{v1})\text{ bar}}\left|\frac{1\ h}{60\ min}\right|\left|\frac{1\ bar}{10^5\ N/m^2}\right|\left|\frac{10^3\ N\cdot m}{1\ kJ}\right| \qquad (c)$$

From Eq. (4)

$$p_{v1} = \frac{\omega_1 p}{0.622 + \omega_1} = \frac{\omega_1 (1.01325)}{0.622 + \omega_1} \qquad (d)$$

Solving (a) - (d) simultaneously using an iterative procedure with data for $h_g(T_1)$ from Table A-2: $T_1 = 15.1$ °C ← T_1

$\omega_1 = 0.009546$

$p_{v1} = 0.01532$ bar

To get ϕ_1,

$$\phi_1 = p_{v1}/p_{g(T_1)} = 0.01532/0.01762 = 0.869 \ (86.9\%) \quad \leftarrow \phi_1$$

(b) The data for the required plots are obtained using IT, as follows:

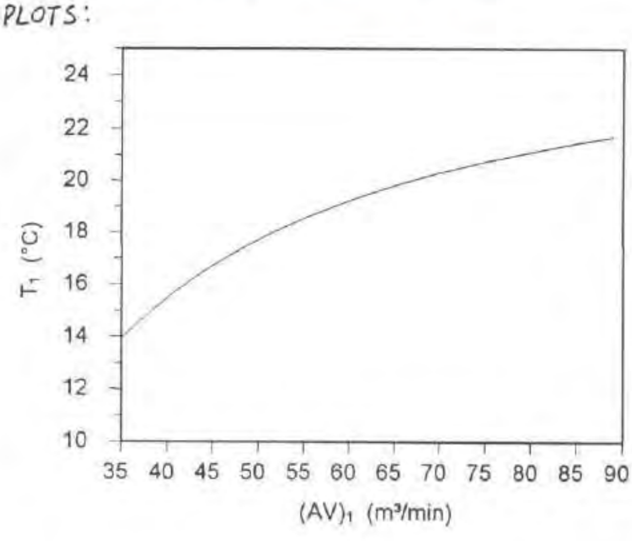
IT Code

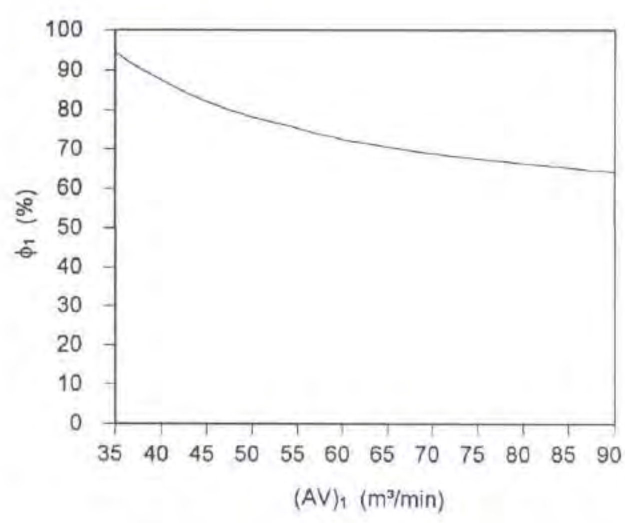
```
p = 1.01325  // bar
T2 = 27
phi2 = 0.5
T3 = 33  // °C
Qdot = 34000  // kJ/h
mdotw = 4.5  // kg/h
AV1 = 40  // m³/min

phi1 = phi_Tw(T1, w1, p)
h1 = ha_Tw(T1, w1)
h2 = ha_Tphi(T2, phi2, p)
w2 = w_Tphi(T2, phi2, p)
psat = Psat_T("Water/Steam", T3)
h3 = hsat_Px("Water/Steam", psat, 1)
va1 = va_Tw(T1,w1,p)
mdota = (AV1 * 60) / va1
mdota * w1 + mdotw = mdota * w2
0 = Qdot + mdota * (h1 - h2) + mdotw * h3
```

IT Results for $(AV)_1 = 40$ m³/min

$\omega_2 = 0.01115$ kg(v)/kg(a)
$h_2 = 55.54$ kJ/kg
$h_3 = 2561$ kJ/kg
$\dot{m}_a = 2891$ kg(a)/min
$h_1 = 39.79$ kJ/kg
$\omega_1 = 0.009592$ kg(v)/kg(a)
$T_1 = 15.47$°C
$\phi_1 = 0.8758$ (87.58%)

PLOTS:





Problem 12.91

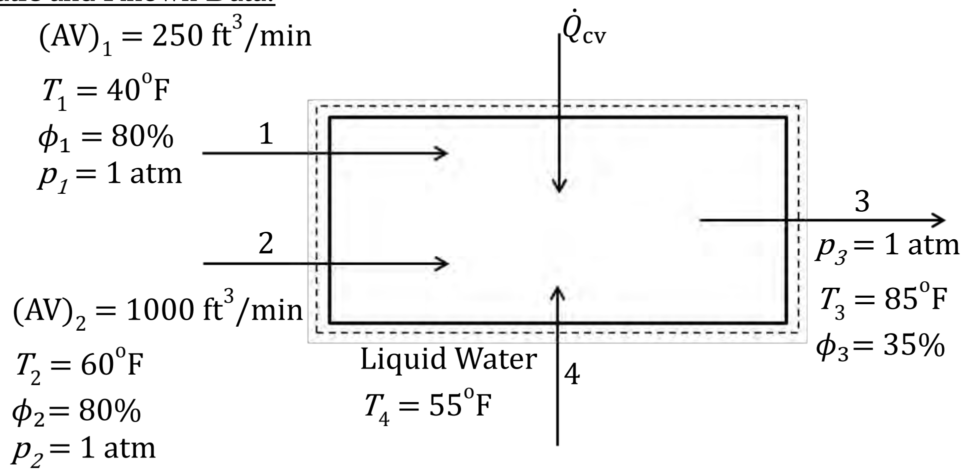
At steady state, a device for heating and humidifying air has 250 ft^3/min of air at 40°F, 1 atm, and 80% relative humidity entering at one location, 1000 ft^3/min of air at 60°F, 1 atm, and 80% relative humidity entering at another location, and liquid water injected at 55°F. A single moist air stream exits at 85°F, 1 atm, and 35% relative humidity. Using data from the psychrometric chart, Fig. A-9E, determine
(a) the rate of heat transfer to the device, in Btu/min.
(b) the rate at which liquid water is injected, in lb/min.

Neglect kinetic and potential energy effects.

Known:
Data are provided for a device heating and humidifying air.

Find:
Determine the rate of heat transfer to the device and the rate liquid water is injected.

Schematic and Known Data:



Engineering Model:
(1) The control volume in the accompanying figure is at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy.
(2) The liquid enters as saturated liquid. The moist air streams adhere to the ideal gas model.

Analysis:
(a) An energy rate equation at steady state gives:

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \left[\dot{m}_{a_1}h_{a_1} + \dot{m}_{v_1}h_{v_1}\right] + \left[\dot{m}_{a_2}h_{a_2} + \dot{m}_{v_s}h_{v_2}\right] + \dot{m}_w h_w - \left[\dot{m}_{a_3}h_{a_3} + \dot{m}_{v_3}h_{v_3}\right]$$

Thus

$$\dot{Q}_{cv} = \dot{m}_{a_3}\left[h_a(T_3) + \omega_3 h_g(T_3)\right] - \dot{m}_{a_1}\left[h_a(T_1) + \omega_1 h_g(T_1)\right] - \dot{m}_{a_2}\left[h_a(T_2) + \omega_2 h_g(T_2)\right] - \dot{m}_w h_f(T_4)$$

Substituting $h_1 = h_a(T_1) + \omega_1 h_g(T_1)$, $h_2 = h_a(T_2) + \omega_2 h_g(T_2)$, and $h_3 = h_a(T_3) + \omega_3 h_g(T_3)$

$$\dot{Q}_{cv} = \dot{m}_{a_3}[h_3] - \dot{m}_{a_1}[h_1] - \dot{m}_{a_2}[h_2] - \dot{m}_w h_f(T_4) \qquad (1)$$

Where h_1, h_2, and h_3 are found from the psychrometric chart (Figure A-9E) and the h_f value is from Table A-2E at T_4:

$$h_1 = 14\,\frac{\text{Btu}}{\text{lb dry air}}, \quad h_2 = 24\,\frac{\text{Btu}}{\text{lb dry air}}, \quad h_3 = 30.5\,\frac{\text{Btu}}{\text{lb dry air}}, h_f = 23.075\,\frac{\text{Btu}}{\text{lb vapor}}$$

Revisit part (a) after completing part (b).

(b) At steady state, mass rate balances give:

$$\dot{m}_{a_1} + \dot{m}_{a_2} = \dot{m}_{a_3}$$

$$\dot{m}_{v_1} + \dot{m}_{v_2} + \dot{m}_w = \dot{m}_{v_3} \Longrightarrow \dot{m}_w = \dot{m}_{v_3} - \dot{m}_{v_1} - \dot{m}_{v_2} = \omega_3 \dot{m}_{a_3} - \omega_1 \dot{m}_{a_1} - \omega_2 \dot{m}_{a_2}$$

Combining these results:

$$\dot{m}_w = \omega_3\left(\dot{m}_{a_1} + \dot{m}_{a_2}\right) - \omega_1 \dot{m}_{a_1} - \omega_2 \dot{m}_{a_2} = \dot{m}_{a_1}(\omega_3 - \omega_1) + \dot{m}_{a_2}(\omega_3 - \omega_2) \qquad (2)$$

To find ω_1, ω_2, ω_3 use the psychrometric chart (Figure A-9E):

$$\omega_1 = 0.0041\,\frac{\text{lb vapor}}{\text{lb dry air}}, \quad \omega_2 = 0.0089\,\frac{\text{lb vapor}}{\text{lb dry air}}, \quad \omega_3 = 0.0090\,\frac{\text{lb vapor}}{\text{lb dry air}}$$

To find $\dot{m}_{a_1}$ and $\dot{m}_{a_2}$ use the psychrometric chart (Figure A-9E) for specific volume and the given volumetric flow rates at 1 and 2.

$v_{a_1} = 12.68\,\frac{\text{ft}^3}{\text{lb dry air}}$ and $v_{a_2} = 13.3\,\frac{\text{ft}^3}{\text{lb dry air}}$

$$\dot{m}_{a_1} = \frac{(\text{AV})_1}{v_{a_1}} = \frac{250\,\frac{\text{ft}^3}{\text{min}}}{12.68\,\frac{\text{ft}^3}{\text{lb dry air}}} = 19.7\,\frac{\text{lb dry air}}{\text{min}}$$

$$\dot{m}_{a_2} = \frac{(\text{AV})_2}{v_{a_2}} = \frac{1000\,\frac{\text{ft}^3}{\text{min}}}{13.3\,\frac{\text{ft}^3}{\text{lb dry air}}} = 75.2\,\frac{\text{lb dry air}}{\text{min}}$$

$$\dot{m}_{a_1} + \dot{m}_{a_2} = \dot{m}_{a_3} = 19.7\,\frac{\text{lb dry air}}{\text{min}} + 75.2\,\frac{\text{lb dry air}}{\text{min}} = 94.9\,\frac{\text{lb dry air}}{\text{min}}$$

Substituting values into Eq. (2), the rate liquid is injected is:

$$\dot{m}_w = \dot{m}_{a_1}(\omega_3 - \omega_1) + \dot{m}_{a_2}(\omega_3 - \omega_2)$$

$$\dot{m}_{\mathrm{w}} = \left(19.7\,\frac{\text{lb dry air}}{\text{min}}\right)(0.009 - 0.0041)\,\frac{\text{lb vapor}}{\text{lb dry air}} + \left(75.2\,\frac{\text{lb}_{\mathrm{a}}}{\text{min}}\right)(0.009 - 0.0089)\,\frac{\text{lb vapor}}{\text{lb dry air}}$$
$$= 0.104\,\frac{\text{lb}}{\text{min}} \longleftarrow$$

Revisiting part (a) while substituting values into Eq. (1), the rate of heat transfer to the device is:

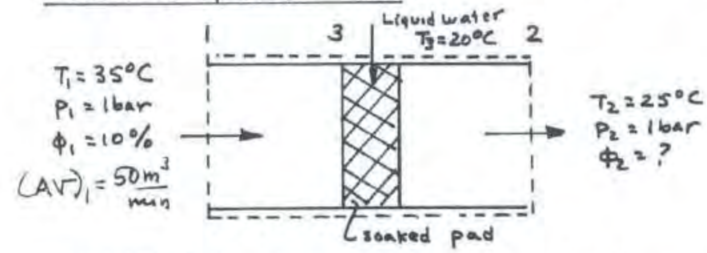
$$\dot{Q}_{\mathrm{cv}} = \dot{m}_{\mathrm{a}_3}[h_3] - \dot{m}_{\mathrm{a}_1}[h_1] - \dot{m}_{\mathrm{a}_2}[h_2] - \dot{m}_{\mathrm{w}} h_{\mathrm{f}}(T_4) = \left(94.9\,\frac{\text{lb}}{\text{min}}\right)\left[30.5\,\frac{\text{Btu}}{\text{lb dry air}}\right] - \left(19.7\,\frac{\text{lb dry air}}{\text{min}}\right)\left[14\,\frac{\text{Btu}}{\text{lb dry air}}\right] - \left(75.2\,\frac{\text{lb dry air}}{\text{min}}\right)\left[24\,\frac{\text{Btu}}{\text{lb dry air}}\right] - \left(0.104\,\frac{\text{lb}}{\text{min}}\right)\left(23.075\,\frac{\text{Btu}}{\text{lb}}\right) = 811.5\,\frac{\text{Btu}}{\text{min}} \longleftarrow$$

PROBLEM 12.92

KNOWN: Steady-state operating data are provided for an evaporative cooler.

FIND: Determine (a) the rate at which liquid enters, (b) the relative humidity at the exit, (c) the rate of exergy destruction.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) All of the entering liquid evaporates into the moist air stream. (3). The liquid enters as saturated liquid. (4) $T_0 = 293$.

ANALYSIS: At steady state mass rate balances give $\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a$, $\dot{m}_{v1} + \dot{m}_w = \dot{m}_{v2}$. Thus $\dot{m}_w = \dot{m}_a(\omega_2 - \omega_1)$ or $\dot{m}_w/\dot{m}_a = \omega_2 - \omega_1$. To find ω_1, use ϕ_1 to obtain $p_{v1} = \phi_1 p_g(T_1) = (0.10)(0.05628) = 0.005628$ bar. Thus

$$\omega_1 = 0.622\left[\frac{0.005628}{1 - 0.005628}\right] = 0.00352 \frac{kg(v)}{kg(a)}$$

The value of ω_2 can be obtained using an energy rate balance:

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + [\dot{m}_a h_{a1} + \dot{m}_{v1} h_{v1}] + \dot{m}_w h_3 - [\dot{m}_a h_{a2} + \dot{m}_{v2} h_{v2}]$$

or

$$0 = \dot{m}_a\left[(h_{a1} - h_{a2}) + \omega_1 h_{g1} + (\omega_2 - \omega_1) h_{f3} - \omega_2 h_{g2}\right]$$

Solving for ω_2

$$\omega_2 = \frac{(h_{a1} - h_{a2}) + \omega_1(h_{g1} - h_{f3})}{h_{g2} - h_{f3}} = \frac{1.005(35-25) + 0.00352(2565.3 - 83.96)}{2547.2 - 83.96} = 0.00763 \frac{kg(v)}{kg(a)}$$

The mass flow rate of dry air is

$$\dot{m}_a = \frac{(AV)_1 (p_{a1})}{RT_1} = \frac{(50\ m^3/min)[0.994372 \times 10^5 N/m^2]}{\left(\frac{8314}{28.97}\frac{N \cdot m}{kg \cdot K}\right)(308 K)} = 56.25 \frac{kg(a)}{min}$$

Thus, $\dot{m}_w = \dot{m}_a(\omega_2 - \omega_1) = (56.25)(0.00763 - 0.00352) = 0.231 \frac{kg(\ell)}{min}$ ← (a)

Solving Eq 12.43

$$p_{v2} = \frac{\omega_2 p}{0.622 + \omega_2} = \frac{(0.00763)(1)}{0.62963} = 0.01212 \text{ bars}, \Rightarrow \phi_2 = \frac{p_v}{p_g(T_2)} = \frac{0.01212}{0.03169} = 0.382 \ (38.2\%)$$ ← (b)

The rate of exergy destruction is found from $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production obtained from an entropy balance:

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_a} = [s_a(T_2, p_{a2}) - s_a(T_1, p_{a1})] + \omega_2 s_v(T_2, p_{v2}) - \omega_1 s_v(T_1, p_{v1}) - (\omega_2 - \omega_1) s_{f3}$$

Using Eq. 6.19, $s_v(T, p_v) = s_g(T) - (\bar{R}/M_v)\ln\phi$ and applying Eq. 6.27 to the dry air

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_a} = \left[c_{pa}\ln\frac{T_2}{T_1} - \frac{\bar{R}}{M_a}\ln\frac{p_{a2}}{p_{a1}}\right] + \omega_2\left[s_g(T_2) - \frac{\bar{R}}{M_v}\ln\phi_2\right] - \omega_1\left[s_g(T_1) - \frac{\bar{R}}{M_v}\ln\phi_1\right] - (\omega_2 - \omega_1)s_{f3}$$

$$= c_{pa}\ln\frac{T_2}{T_1} - \frac{\bar{R}}{M_a}\ln\frac{p_{a2}}{p_{a1}} + \omega_2(s_g(T_2) - s_f(T_3)) - \omega_1(s_g(T_1) - s_f(T_3)) + \frac{\bar{R}}{M_v}\left[\omega_1 \ln\phi_1 - \omega_2 \ln\phi_2\right]$$

$$= 1.005 \ln\frac{298}{308} - \frac{8.314}{28.97}\ln\frac{0.98788}{0.99437} + 0.00763[8.558 - 0.2966] - 0.00352[8.3531 - 0.2966] + \frac{8.314}{18.02}[0.00352 \ln 0.1 - 0.00763 \ln .382]$$

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_a} = 0.003 \frac{kJ/kg(a)}{°K}$$

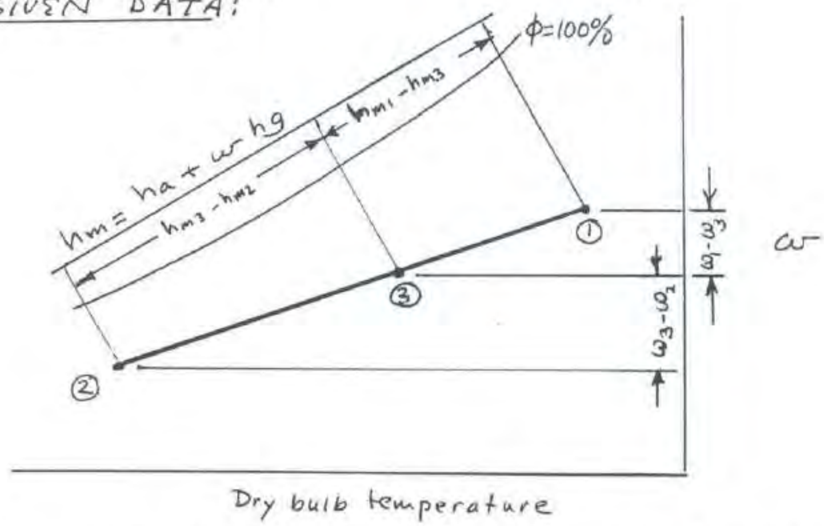
$$\Rightarrow \dot{E}_d = (56.25)(293)(0.003) = 49.44\ kJ/min$$ ← (c)

PROBLEM 12.93

KNOWN: The case of adiabatic mixing of two moist air streams is under consideration.

FIND: Show on a psychrometric chart that exit state 3 is on a straight line connecting inlet states 1 and 2.

SCHEMATIC & GIVEN DATA:



ANALYSIS: From Sec. 12.8.6, and writing for the mixture $h_m = h_a + \omega h_g$,

(12.56b) $\quad \omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2} = \omega_3 \dot{m}_{a3}$

(12.56c) $\quad \dot{m}_{a1} h_{m1} + \dot{m}_{a2} h_{m2} = \dot{m}_{a3} h_{m3}$ where $h_m = h_a + \omega h_g$

Since $\dot{m}_{a3} = \dot{m}_{a1} + \dot{m}_{a2}$, Eq 12.56b can be rearranged as

$$\omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2} = \omega_3 (\dot{m}_{a1} + \dot{m}_{a2})$$

$$\Rightarrow \quad (\omega_1 - \omega_3)\dot{m}_{a1} + (\omega_2 - \omega_3)\dot{m}_{a2} = 0$$

$$\Rightarrow \quad \frac{\dot{m}_{a1}}{\dot{m}_{a2}} = \frac{\omega_3 - \omega_2}{\omega_1 - \omega_3} \qquad (1)$$

Similarly, Eq. 12.56c gives

$$\frac{\dot{m}_{a1}}{\dot{m}_{a2}} = \frac{h_{m3} - h_{m2}}{h_{m1} - h_{m3}} \qquad (2)$$

Combining Eqs (1), (2)

$$\frac{\dot{m}_{a1}}{\dot{m}_{a2}} = \frac{\omega_3 - \omega_2}{\omega_1 - \omega_3} = \frac{h_{m3} - h_{m2}}{h_{m1} - h_{m3}} \qquad (3)$$

Since the ω and h_m scales are linear, the following proportionality applies for the sketch above

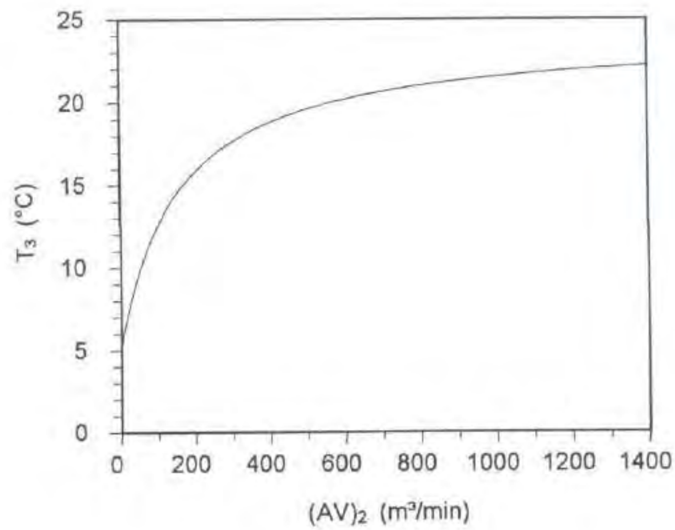
$$\frac{\text{Line } 2\text{-}3}{\text{Line } 3\text{-}1} = \frac{h_{m3} - h_{m2}}{h_{m1} - h_{m3}} = \frac{\omega_3 - \omega_2}{\omega_1 - \omega_3}$$

which corresponds to Eq. (3).

PROBLEM 12.94

See Example 12.14 for the IT code. To obtain data for the plot, use the **Explore** button to sweep AV2 from 0 to 1400 m^3/min in steps of 10.

PLOT:



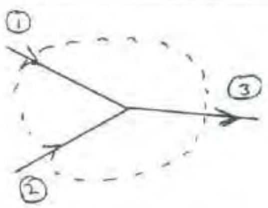
Discussion: When $(AV)_2$ approaches 0, there is no flow in stream 2, and the exit temperature approaches the value of T_1. When $(AV)_2$ becomes large, the effect of stream 1 is reduced and the exit temperature approaches the value of T_2

PROBLEM 12.95

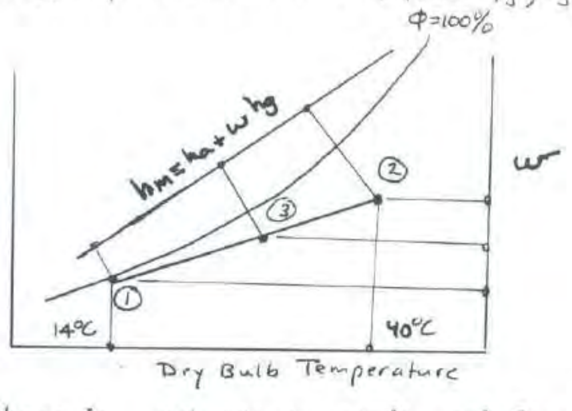
KNOWN: Steady state operating data are provided for the adiabatic mixing of two moist air streams.

FIND: Using the graphical procedure of Problem 12.93, find ϕ_3, T_3.

SCHEMATIC & GIVEN DATA:



$(AV)_1 = 35\ m^3/min$, $T_1 = 14°C$, $\phi_1 = 80\%$
$(AV)_2 = 80\ m^3/min$, $T_2 = 40°C$, $\phi_2 = 40\%$



ENGINEERING MODEL: (1) The control volume shown is at steady state. (2) For the control volume $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. Changes in kinetic and potential energy can be neglected. (3) The pressure remains constant at 1 atm.

ANALYSIS: Inspection of Fig A-9 gives

$h_{m1} = 34.0$ kJ/kg(a) $\qquad h_{m2} = 88.2$
$\omega_1 = 0.008$ kg(v)/kg(a) $\qquad \omega_2 = 0.0188$
$v_{a1} = 0.824$ m³/kg(a) $\qquad v_{a2} = 0.913$

Then

$$m_{a1} = \frac{(AV)_1}{v_{a1}} = \frac{35\ m^3/min}{0.824\ m^3/kg(a)} = 42.48\ \frac{m^3}{kg(a)}$$

$$m_{a2} = \frac{80}{0.913} = 87.62\ \frac{m^3}{kg(a)}$$

Therefore, the relation of Problem 12.93 becomes

$$\frac{42.48}{87.62} = \frac{\omega_3 - 0.0188}{0.008 - \omega_3} = \frac{h_{m3} - 88.2}{34.0 - h_{m3}}$$

Solving, $\omega_3 = 0.0153$ kg(v)/kg(a), $h_{m3} = 70.50$ kJ/kg(a). Using these values, Fig. A-9 gives

$\phi_3 \approx 52\%$ ← ϕ_3

$T_3 \approx 32°C$ ← T_3

Problem 12.96

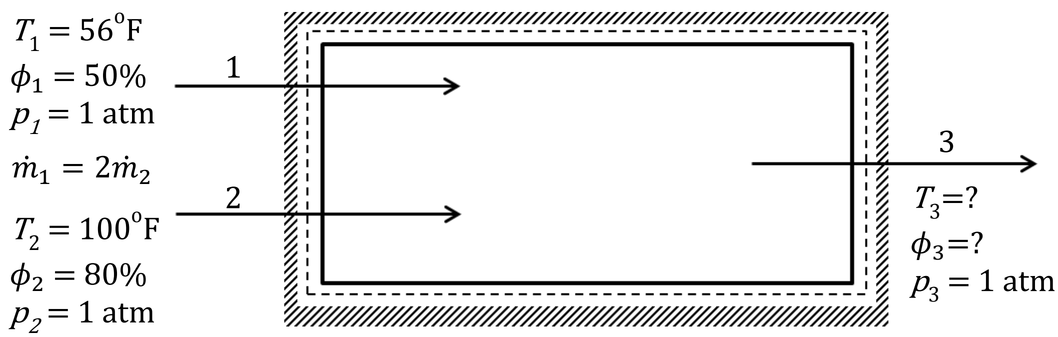
At steady state, a stream of air at 56°F, 1 atm, 50% relative humidity is mixed adiabatically with a stream of air at 100°F, 1 atm, 80% relative humidity. The mass flow rate of the higher-temperature stream is twice that of the other stream. A single mixed stream exits at 1 atm. Using the result of problem 12.74, determine for the exiting stream
(a) the temperature, in °F.
(b) the relative humidity.

Neglect kinetic and potential energy effects.

Known:
A moist air stream at 56°F, 1 atm, ϕ_1=50% is mixed adiabatically with a stream of air at 100°F, 1 atm, ϕ_2=80%. The mass flow rate of the 56°F stream is twice that of the 100°F stream.

Find:
Using the result of problem 12.74, determine the temperature and relative humidity of the exiting stream.

Schematic and Known Data:



Engineering Model:
(1) The control volume in the accompanying figure is at steady state with $\dot{W}_{\text{cv}} = \dot{Q}_{\text{cv}} = 0$ and negligible effects of kinetic and potential energy.

Analysis:
(a) At steady state, mass rate balances give:

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 \Rightarrow \dot{m}_3 = 1.5\dot{m}_1 \quad (1)$$

$$\dot{m}_{a_1} + \dot{m}_{a_2} = \dot{m}_{a_3}$$

$$\dot{m}_{v_1} + \dot{m}_{v_2} = \dot{m}_{v_3} \Rightarrow \dot{m}_{a_1}\omega_1 + \dot{m}_{a_2}\omega_2 = \dot{m}_{a_3}\omega_3 \quad (2)$$

Also, at each location:

$$\dot{m} = \dot{m}_a + \dot{m}_v = \dot{m}_a(1+\omega) \Rightarrow \dot{m}_a = \frac{\dot{m}}{1+\omega} \quad (3)$$

Combining Eqs. (1), (2), and (3) together with $\dot{m}_1 = 2\dot{m}_2$:

$$\frac{\dot{m}_1}{1+\omega_1}\omega_1 + \frac{\dot{m}_2}{1+\omega_2}\omega_2 = \frac{\dot{m}_3}{1+\omega_3}\omega_3 \Longrightarrow$$
$$\frac{\omega_1}{1+\omega_1} + (0.5)\frac{\omega_2}{1+\omega_2} = (1.5)\frac{\omega_3}{1+\omega_3} \qquad (4)$$

To find ω_1 and ω_2, first solve for p_{v_1} and p_{v_2} using values from Table A-2E

$$p_{v_1} = \phi_1 p_g(T_1) = 0.5(0.2219) = 0.11095\,\frac{\text{lbf}}{\text{in.}^2}$$
$$p_{v_2} = \phi_2 p_g(T_2) = 0.8(0.9503) = 0.76024\,\frac{\text{lbf}}{\text{in.}^2}$$
$$\omega_1 = 0.622\left(\frac{0.11095}{14.696 - 0.11095}\right) = 0.00473\,\frac{\text{lb}_\text{v}}{\text{lb}_\text{a}}$$
$$\omega_2 = 0.622\left(\frac{0.76024}{14.696 - 0.76024}\right) = 0.0339\,\frac{\text{lb}_\text{v}}{\text{lb}_\text{a}}$$

Substituting values into Eq. (4):

$$\frac{0.00473}{1.00473} + (0.5)\frac{0.0339}{1.0339} = (1.5)\frac{\omega_3}{1+\omega_3} \Longrightarrow \omega_3 = 0.01427\,\frac{\text{lb}_\text{v}}{\text{lb}_\text{a}}$$

Solving Eq. 12.43:

$$p_{v_3} = \frac{\omega_3 p}{0.622+\omega_3} = \frac{(0.01427)(14.696)}{0.622+0.01427} = 0.3296\,\frac{lbf}{in.^2} \Longrightarrow$$
$$\phi_3 = \frac{p_{v_3}}{p_g(T_3)} = \frac{0.3296}{p_g(T_3)} \qquad (5)$$

To find T_3, write an energy rate balance for the control volume. With assumption (1), this becomes:

$$0 = \cancel{\dot{Q}}_{\text{cv}} - \cancel{\dot{W}}_{\text{cv}} + \left[\dot{m}_{a_1}h_{a_1} + \dot{m}_{v_1}h_{v_1}\right] + \left[\dot{m}_{a_2}h_{a_2} + \dot{m}_{v_2}h_{v_2}\right] - \left[\dot{m}_{a_3}h_{a_3} + \dot{m}_{v_3}h_{v_3}\right]$$

or

$$\dot{m}_{a_3}\left[h_{a_3} + \omega_3 h_{g_3}\right] = \dot{m}_{a_1}\left[h_{a_1} + \omega_1 h_{g_1}\right] + \dot{m}_{a_2}\left[h_{a_2} + \omega_2 h_{g_2}\right]$$
$$\frac{\dot{m}_3}{1+\omega_3}\left[h_{a_3} + \omega_3 h_{g_3}\right] = \frac{\dot{m}_1}{1+\omega_1}\left[h_{a_1} + \omega_1 h_{g_1}\right] + \frac{\dot{m}_2}{1+\omega_2}\left[h_{a_2} + \omega_2 h_{g_2}\right] \qquad (6)$$

With the result of Problem 12.74:

$$\left[h_{a_1} + \omega_1 h_{g_1}\right] = 0.24(56) + 0.00473\big(1061 + 0.444(56)\big) = 18.58\,\frac{\text{Btu}}{\text{lb}_\text{a}}$$
$$\left[h_{a_2} + \omega_2 h_{g_2}\right] = 0.24(100) + 0.0339\big(1061 + 0.444(100)\big) = 61.47\,\frac{\text{Btu}}{\text{lb}_\text{a}}$$
$$\left[h_{a_3} + \omega_3 h_{g_3}\right] = 0.24(T_3) + 0.01427\big(1061 + 0.444(T_3)\big)$$

Thus, with $\dot{m}_3 = 1.5\dot{m}_1,\ \dot{m}_2 = 0.5\dot{m}_1$, Eq. (6) becomes:

$$0.24(T_3) + 0.01427\big(1061 + 0.444(T_3)\big)$$
$$= \frac{\left(\frac{1.01427}{1.00473}\right)\left(18.58\,\frac{\text{Btu}}{\text{lb}}\right) + 0.5\left(\frac{1.01427}{1.0339}\right)\left(61.47\,\frac{\text{Btu}}{\text{lb}}\right)}{1.5} = 32.6$$
$$0.2463T_3 = 17.46 \Longrightarrow T_3 = 70.9°\text{F}$$

(b) Then, with Eq. (5):

$$\phi_3 = \frac{p_{v_3}}{p_g(T_3)} = \frac{0.3296}{0.3747} = 0.8796 = 88\%$$

12.97 At steady state, air at 42°C, 1 atm, 30% relative humidity is mixed adiabatically with a second moist air stream entering at 1 atm. The mass flow rates of the two streams are the same. A single mixed stream exits at 29°C, 1 atm, 40% relative humidity with a mass flow rate of 2 kg/s. Kinetic and potential energy effects are negligible. For the second entering moist air stream, determine using data from the psychrometric chart

(a) the relative humidity.

(b) the temperature, in °C.

KNOWN: Moist Air with known temperature and relative humidity is mixed adiabatically with a second moist air stream with known pressure. The mass flow rates of the two streams are equal. The mixed stream exits at known conditions and mass flow rate.

FIND: Determine (a) the relative humidity and (b) the temperature of the second entering air stream. Use data from the psychrometric chart.

SCHEMATIC AND GIVEN DATA:

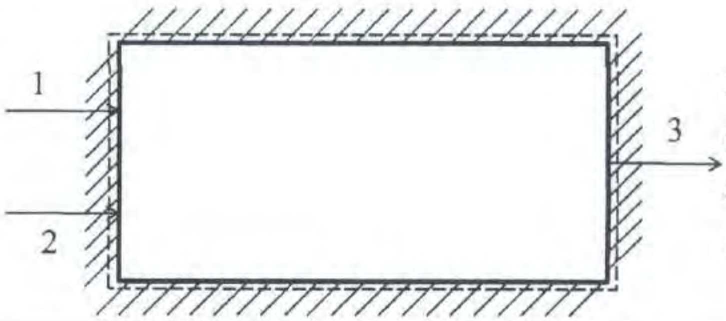
$T_1 = 42^\circ C$

$\phi_1 = 30\%$

$m_1 = m_2$

$T_2 = ?$

$\phi_2 = ?$



$T_3 = 29^\circ C$

$\phi_3 = 40\%$

$m_3 = 2\ \text{kg/s}$

$p_3 = p_1 = p_2 = 1\ \text{atm}$

ENGINEERING MODEL:

(1) The control volume shown in the accompanying figure operates at steady state.

(2) $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and the effects of kinetic and potential energy are negligible.

ANALYSIS:

At steady state, the mass rate balance gives

$$\dot{m}_1 + \dot{m}_2 - \dot{m}_3$$

With the given data, and with the fact that the mass flow rates at 1 and 2 are equal

$$\dot{m}_2 - \dot{m}_2 - \dot{m}_3/2 = 1\ \text{kg/s}$$

Problem 12.97 (Continued) – Page 2

Mass rate balances for the dry air reduce to give

$$\dot{m}_{a1} + \dot{m}_{a2} = \dot{m}_{a3} \rightarrow \dot{m}_{a2} = \dot{m}_{a3} - \dot{m}_{a1}$$

Also, for each stream

$$\dot{m} = \dot{m}_a + \dot{m}_v = \dot{m}_a(1+\omega) \qquad (1)$$

For stream 1, from Fig. A-9 at 42°C and 30% relative humidity; $\omega_1 = 0.0157$. Thus

$$\dot{m}_{a1} = \dot{m}_1/(1+\omega_1) = (1\tfrac{kg}{s})/(1 + 0.0157) = 0.9845 \text{ kg/s}$$

For stream 3, from Fig. A-9 at 30°C and 40% relative humidity; $\omega_3 = 0.012$. Thus

$$\dot{m}_{a3} = \dot{m}_3/(1+\omega_3) = (2\tfrac{kg}{s})/(1 + 0.012) = 1.98 \text{ kg/s}$$

And

$$\dot{m}_{a2} = 1.98 - 0.9845 = 0.9955 \text{ kg/s}$$

Solving (1) for ω we get

$$\omega_2 = \frac{\dot{m}_2}{\dot{m}_{a2}} - 1 = \frac{1}{0.9955} - 1 = 0.0045$$

Now, the energy rate balance reduces to

$$0 = \underbrace{\dot{Q}_{cv}}_{=0} - \underbrace{\dot{W}_{cv}}_{=0} + [\dot{m}_{a1}h_{a1} - \dot{m}_{v1}h_{v1}] + [\dot{m}_{a2}h_{a2} - \dot{m}_{v2}h_{v2}] - [\dot{m}_{a3}h_{a3} - \dot{m}_{v3}h_{v3}]$$

or

$$0 = \dot{m}_{a1}[h_{a1} - \omega_1 h_{g1}] + \dot{m}_{a2}[h_{a2} - \omega_2 h_{g2}] - \dot{m}_{a3}[h_{a3} - \omega_3 h_{g3}]$$

Each of the quantities in brackets can be found using Fig. A-9. Solving for the specific enthalpy per unit mass of dry air at 2

$$[h_{a2} - \omega_2 h_{g2}] = \frac{\dot{m}_{a3}[h_{a3} - \omega_3 h_{g3}] - \dot{m}_{a1}[h_{a1} - \omega_1 h_{g1}]}{\dot{m}_{a3} - \dot{m}_{a1}}$$

From Fig. A-9: $[h_{a1}+\omega_1 h_{v1}]$ = 82 kJ/kg(dry air) and $[h_{a3}+\omega_3 h_{v3}]$ = 54.5 kJ/kg(dry air)

Problem 12.97 (Continued) – Page 3

Inserting values

$$[h_{a2} + \omega_2 h_{v2}] = \frac{\dot{m}_{a3}[h_{a3}+\omega_3 h_{v3}]-\dot{m}_{a1}[h_{a1}+\omega_1 h_{v1}]}{\dot{m}_{a2}}$$

$$= \frac{(1.98)(54.2)-(0.985)(82)}{0.995}$$

$$= 26.7 \text{ kJ/kg(dry air)}$$

With $[h_{a2} + \omega_2 h_{v2}] = 26.7$ and $\omega_2 = 0.0045$ from above, the temperature and relative humidity are

$$T_2 \approx 16^\circ\text{C}$$
$$\varphi_2 \approx 40\%$$

Note: This problem can be solved using steam table and air table data, but the solution process would be iterative. Using the Psychrometric Chart facilitates the solution considerably.

12.98 Figure P12.98 shows two options for conditioning atmospheric air at steady state. In each case, air enters at 15°C, 1 atm, and 20% relative humidity with a volumetric flow rate of 150 m^3/min and exits at 30°C, 1 atm, and 40% relative humidity. One method conditions the air by injecting saturated water vapor at 1 atm. The other method allows the entering air to pass through a soaked pad replenished by liquid water entering at 20°C. The moist air stream is then heated by an electric resistor. For $T_o = 288$ K, which of the two options is preferable from the standpoint of minimal exergy destruction? Discuss.

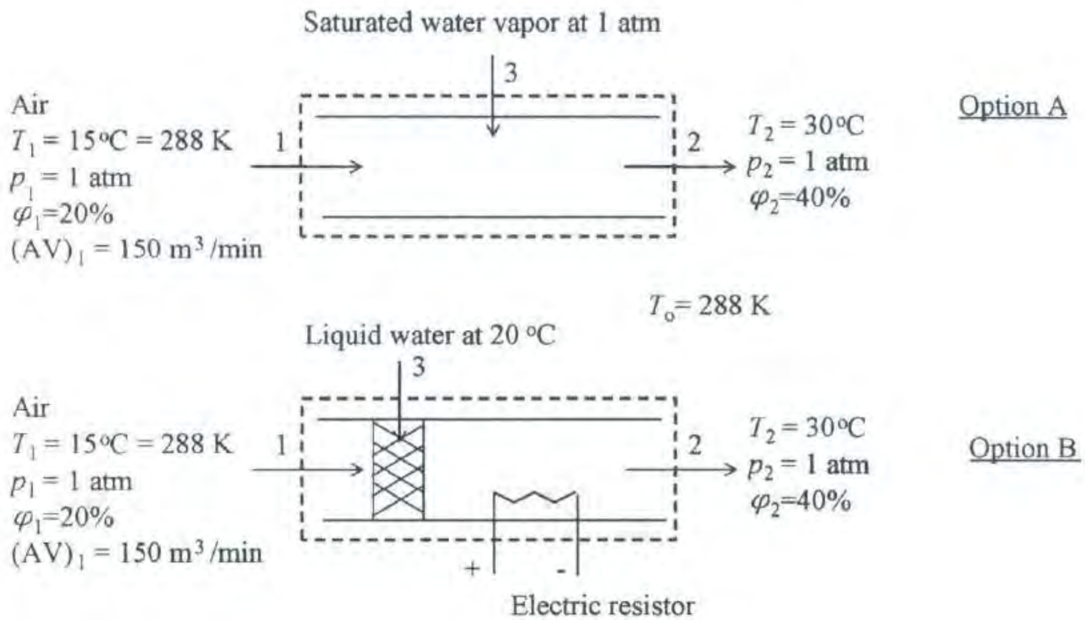


Fig. P12.81

KNOWN: Operating data are provided for each of two options for conditioning atmospheric air at steady state. Option A involves injecting steam into a moist air stream and Option B involves passing a moist air stream over a soaked pad followed by heating with an electric resistor. The entering and exiting states are the same for both options.

FIND: Determine which of the two options is preferable from the standpoint of minimal exergy destruction.

SCHEMATIC AND GIVEN DATA:

Refer to Fig. P12.81.

ENGINEERING MODEL:

(1) Each of the control volumes shown in the accompanying figure is at steady state with and negligible effects of kinetic and potential energy.

(2) The water vapor entering is saturated vapor in option A. The liquid entering in option B is saturated liquid.

(3) The temperature of the reference environment is $T_o = 288$ K.

ANALYSIS:

Compare the exergy destruction rates for the two options. For both options, the entropy rate balance is given by

$$\begin{aligned} 0 &= \dot{m}_1 s_1 + \dot{m}_3 s_3 - \dot{m}_2 s_2 + \dot{\sigma}_{cv} \\ 0 &= [\dot{m}_{a1} + \dot{m}_{v1}]s_1 + [\dot{m}_{a3} + \dot{m}_{v3}]s_3 - [\dot{m}_{a2} + \dot{m}_{v2}]s_2 + \dot{\sigma}_{cv} \end{aligned} \tag{1}$$

Realizing that $\dot{m}_{a1} = \dot{m}_{a2}$ and $\dot{m}_{a3} = 0$, Eq. (1) simplifies to

$$\dot{\sigma}_{cv} = \dot{m}_a[s_{a2} - s_{a1}] + \dot{m}_{v2}s_{v2} - \dot{m}_{v1}s_{v1} - \dot{m}_3 s_{v3}$$

Dividing through by $\dot{m}_a$ gives

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_a} = [s_{a2} - s_{a1}] + [\omega_2 s_{v2} - \omega_1 s_{v1}] - \frac{\dot{m}_3}{\dot{m}_a} s_{v3}$$

For Option A: $s_{v3} = s_g(1\text{ atm})$ and for Option B: $s_{v3} = s_g(20°\text{C})$ to give

Option A:

$$\left(\frac{\dot{\sigma}_{cv}}{\dot{m}_a}\right)_A = \underline{[s_{a2}(T_2, p_{a2}) - s_{a1}(T_1, p_{a1})] + [\omega_2 s_{v2}(T_2, p_{v2}) - \omega_1 s_{v1}(T_1, p_{v1})]} - \frac{\dot{m}_3}{\dot{m}_a} s_g(1\text{ atm})$$

Option B:

$$\left(\frac{\dot{\sigma}_{cv}}{\dot{m}_a}\right)_B = \underline{[s_{a2}(T_2, p_{a2}) - s_{a1}(T_1, p_{a1})] + [\omega_2 s_{v2}(T_2, p_{v2}) - \omega_1 s_{v1}(T_1, p_{v1})]} - \frac{\dot{m}_3}{\dot{m}_a} s_f(20°\text{C})$$

The underlined term is the same for each option. Also, from mass rate balances, the ratio $\dot{m}_3/\dot{m}_a = \omega_2 - \omega_1$ and thus is the same for both options. Subtracting the two expressions

$$\left(\frac{\dot{\sigma}_{cv}}{\dot{m}_a}\right)_B - \left(\frac{\dot{\sigma}_{cv}}{\dot{m}_a}\right)_A = (\omega_2 - \omega_1)\left[s_g(1\,\text{atm}) - s_g(20°\text{C})\right] \tag{2}$$

Using data from Table A-2

$$p_{v1} = \phi_1 p_g(T_1) = (0.2)(0.01705\,\text{bar}) = 0.00341\,\text{bar}$$
$$p_{a1} = p - p_{v1} = 1.01325\,\text{bar} - 0.00341\,\text{bar} = 1.00984\,\text{bar}$$
$$p_{v2} = \phi_2 p_g(T_2) = (0.4)(0.04246\,\text{bar}) = 0.01698\,\text{bar}$$

Then

$$\omega_1 = 0.622\frac{p_{v1}}{p - p_{v1}} = 0.622\frac{0.00341\,\text{bar}}{1.01325\,\text{bar} - 0.00341\,\text{bar}} = 0.0021\frac{\text{kg(vapor)}}{\text{kg(air)}}$$

$$\omega_2 = 0.622\frac{p_{v2}}{p - p_{v2}} = 0.622\frac{0.01698\,\text{bar}}{1.01325\,\text{bar} - 0.01698\,\text{bar}} = 0.0106\frac{\text{kg(vapor)}}{\text{kg(air)}}$$

$$\omega_2 - \omega_1 = (0.0106 - 0.0021)\frac{\text{kg(vapor)}}{\text{kg(air)}} = 0.0085\frac{\text{kg(vapor)}}{\text{kg(air)}}$$

also

$$\dot{m}_a = \frac{(\text{AV})_1}{v_{a1}} = \frac{(\text{AV})_1}{\left(\dfrac{\bar{R}T_1}{M_a p_{a1}}\right)} = \frac{(150\,\text{m}^3/\text{min})\left|1\,\text{min}/60\,\text{sec}\right|}{\left(\dfrac{8314}{28.97}\dfrac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)\left(\dfrac{288\,\text{K}}{1.00984\,\text{bar}\left|10^5\,\text{N/m}^2/1\,\text{bar}\right|}\right)} = 3.054\frac{\text{kg(air)}}{\text{s}}$$

Then, with $\dot{E}_d = T_o\dot{\sigma}_{cv}$, Table A-2, and known values, Eq. (2) gives

$$(\dot{E}_d)_B - (\dot{E}_d)_A = \dot{m}_a(\omega_2 - \omega_1)T_o\left[s_g(1\,\text{atm}) - s_g(20°\text{C})\right]$$
$$= \left(3.054\frac{\text{kg(air)}}{\text{s}}\right)\left[0.0085\frac{\text{kg(vapor)}}{\text{kg(air)}}\right](288\,\text{K})\left[(7.3558 - 0.2966)\frac{\text{kJ}}{\text{kg(vapor)}}\right]\left|\frac{1\,\text{kW}}{\text{kJ/s}}\right|$$
$$= 52.78\,\text{kW}$$

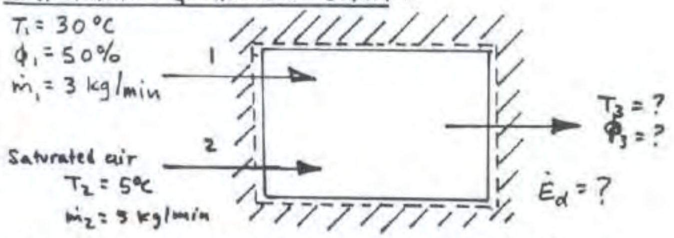
On this basis, Option A is preferable. On a cost basis, the cost of Option A would take into account the cost to produce and deliver saturated water vapor at 1 atm ***plus*** *the cost attributed to the exergy destroyed. For Option B the cost would take into account the cost to produce and deliver the electrical input* ***plus*** *the cost attributed to the exergy destroyed. Accordingly, Option A looks preferable.*

PROBLEM 12.99

KNOWN: Moist air at $T_1 = 30°C$, $p_1 = 1$ bar, $\phi_1 = 50\%$, $\dot{m}_1 = 3$ kg/min mixes adiabatically with saturated air at $T_2 = 5°C$, $p_2 = 1$ bar, $\dot{m}_2 = 5$ kg/min to produce a single mixed stream at $p_3 = 1$ bar.

FIND: Determine ϕ_3, T_3, and the exergy destruction rate.

SCHEMATIC & GIVEN DATA:



$p_1 = p_2 = p_3 = 1$ bar

$T_0 = 293$ K

ENGINEERING MODEL: (1) The control volume in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) Each stream can be modeled as an ideal gas mixture.

ANALYSIS: At steady state, mass rate balances reduce to give $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$ and $\dot{m}_{a3} = \dot{m}_{a1} + \dot{m}_{a2}$, $\dot{m}_{v3} = \dot{m}_{v1} + \dot{m}_{v2}$. Also, for each stream $\dot{m} = \dot{m}_a + \dot{m}_v = \dot{m}_a(1+\omega)$. Accordingly, $\dot{m}_{a3}\omega_3 = \dot{m}_{a1}\omega_1 + \dot{m}_{a2}\omega_2$, or

$$\omega_3 = \frac{\dot{m}_{a1}\omega_1 + \dot{m}_{a2}\omega_2}{\dot{m}_{a1} + \dot{m}_{a2}} = \frac{\left(\frac{\omega_1}{1+\omega_1}\right)\dot{m}_1 + \left(\frac{\omega_2}{1+\omega_2}\right)\dot{m}_2}{\left(\frac{\dot{m}_1}{1+\omega_1}\right) + \left(\frac{\dot{m}_2}{1+\omega_2}\right)}$$

To find ω_1 and ω_2, $p_{v1} = \phi_1 p_g(T_1) = 0.5(0.04246) = 0.02123$ bar, $p_{v2} = \phi_2 p_g(T_2) = 0.00872$ bar. Then

$$\omega_1 = 0.622\left(\frac{0.02123}{1-0.02123}\right) = 0.01349\ \frac{\text{kg(v)}}{\text{kg(a)}},\quad \omega_2 = 0.622\left(\frac{0.00872}{1-0.00872}\right) = 0.00547\ \frac{\text{kg(v)}}{\text{kg(a)}}$$

Accordingly

$$\omega_3 = \frac{\left(\frac{0.01349}{1.01349}\right)(3) + \left(\frac{0.00547}{1.00547}\right)(5)}{\left(\frac{3}{1.01349}\right) + \left(\frac{5}{1.00547}\right)} = 0.008463\ \frac{\text{kg(v)}}{\text{kg(a)}}$$

Solving Eq. 12.43

$$p_{v3} = \frac{p_3\omega_3}{0.622+\omega_3} = \frac{(1)(0.008463)}{0.63046} = 0.01342 \Rightarrow \phi_3 = \frac{p_{v3}}{p_g(T_3)} = \frac{0.01342}{p_g(T_3)} \qquad (1)$$

To find T_3, an energy rate balance reduces to

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + [\dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1}] + [\dot{m}_{a2}h_{a2} + \dot{m}_{v2}h_{v2}] - [\dot{m}_{a3}h_{a3} + \dot{m}_{v3}h_{v3}]$$

or

$$0 = \dot{m}_{a1}[h_{a1} + \omega_1 h_{g1}] + \dot{m}_{a2}[h_{a2} + \omega_2 h_{g2}] - \dot{m}_{a3}[h_{a3} + \omega_3 h_{g3}]$$

Then

$$h_{a3} + \omega_3 h_{g3} = \frac{\dot{m}_{a1}[h_{a1} + \omega_1 h_{g1}] + \dot{m}_{a2}[h_{a2} + \omega_2 h_{g2}]}{\dot{m}_{a1} + \dot{m}_{a2}}$$

$\dot{m}_{a1} = \dot{m}_1/(1+\omega_1) = 2.96$ kg(a)/min, $\dot{m}_{a2} = \dot{m}_2/(1+\omega_2) = 4.9728$ kg(a)/min. With data from Tables A-2, A-22

$$h_{a3} + \omega_3 h_{g3} = \frac{(2.96)[303.21 + (0.01349)(2556.3)] + 4.9728[(278.1 + (0.00547)(2510.6)]}{(2.96 + 4.9728)}$$

$$= 308.95\ \text{kJ/kg(a)}$$

(1) This equation has a single unknown, T_3. Solving iteratively using table data gives

$T_3 = 14.4$ °C ← T_3

Returning to Eq. (1), $\phi_3 = \frac{0.01342}{0.01641} = 0.818\ (81.8\%) \longleftarrow \phi_3$

The exergy destruction rate is obtained from $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production from an entropy rate balance which at steady state reduces to

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{\,0} + [\dot{m}_{a1} s_a(T_1, p_{a1}) + \dot{m}_{v1} s_v(T_1, p_{v1})] + [\dot{m}_{a2} s_a(T_2, p_{a2}) + \dot{m}_{v2} s_g(T_2)] - [\dot{m}_{a3} s_a(T_3, p_{a3}) + \dot{m}_{v3} s_v(T_3, p_{v3})] + \dot{\sigma}_{cv}$$

or

$$\dot{\sigma}_{cv} = \dot{m}_{a3}[s_a(T_3, p_{a3}) + \omega_3 s_v(T_3, p_{v3})] - \dot{m}_{a1}[s_a(T_1, p_{a1}) + \omega_1 s_v(T_1, p_{v1})] - \dot{m}_{a2}[s_a(T_2, p_{a2}) + \omega_2 s_g(T_2)]$$

$$= \dot{m}_{a1}[s_a(T_3, p_{a3}) - s_a(T_1, p_{a1})] + \dot{m}_{a2}[s_a(T_3, p_{a3}) - s_a(T_2, p_{a2})] + \dot{m}_{a3}\omega_3 s_v(T_3, p_{v3})$$

$$- \dot{m}_{a1}\omega_1 s_v(T_1, p_{v1}) - \dot{m}_{a2}\omega_2 s_g(T_2)$$

From Eq. 6.19, $s_v(T, p_v) = s_g(T) - \bar{R}/M_v \ln\phi$, and using Eq. 6.23 for the dry air

$$\dot{\sigma}_{cv} = \dot{m}_{a1}\left[c_{pa} \ln\frac{T_3}{T_1} - \frac{\bar{R}}{M_a}\ln\frac{p_{a3}}{p_{a1}}\right] + \dot{m}_{a2}\left[c_{pa} \ln\frac{T_3}{T_2} - \frac{\bar{R}}{M_a}\ln\frac{p_{a3}}{p_{a2}}\right]$$

$$+ \dot{m}_{a3}\omega_3\left[s_g(T_3) - \frac{\bar{R}}{M_v}\ln\phi_3\right] - \dot{m}_{a1}\omega_1\left[s_g(T_1) - \frac{\bar{R}}{M_v}\ln\phi_1\right] - \dot{m}_{a2}\omega_2 s_g(T_2)$$

From previous calculations: $p_{a1} = 0.97877$ bar, $p_{a2} = 0.99128$ bar, and $p_{a3} = 1 - 0.01342 = 0.98658$ bar. Evaluating $\dot{\sigma}_{cv}$ with $c_{pa} = 1.005$ kJ/kg·K

$$\dot{\sigma}_{cv} = 2.96\left[1.005 \ln\frac{287.4}{303} - \frac{8.314}{28.97}\ln\frac{0.98658}{0.97877}\right] + 4.9728\left[1.005 \ln\frac{287.4}{278} - \frac{8.314}{28.97}\ln\frac{0.98658}{0.99128}\right] +$$

$$(7.9323)(0.008463)\left[8.79544 - \frac{8.314}{18.02}\ln 0.818\right] - (2.96)(0.01349)\left[8.4533 - \frac{8.314}{18.02}\ln 0.5\right] -$$

$$(4.9728)(0.00547)(9.0257)$$

$$= 0.009828\ \frac{\text{kJ/min}}{\text{K}}$$

The exergy destruction rate is then

$$\dot{E}_d = (293\text{ K})\left(0.009828\ \frac{\text{kJ/min}}{\text{K}}\right)\left|\frac{1\text{ min}}{60\text{ s}}\right|\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = 0.048\text{ kW} \longleftarrow \dot{E}_d$$

---

1. An alternative solution that avoids iteration with table data is obtained using IT, as follows:

IT Code

```
T1 = 30 // °C
p = 1 // bar
phi1 = 0.5
mdot1 = 3 // kg/min
T2 = 5 // °C
phi2 = 1
mdot2 = 5 // kg/min

w1 = w_Tphi(T1,phi1,p)
w2 = w_Tphi(T2,phi2,p)
mdota1 = mdot1 / (1 + w1)
mdota2 = mdot2 / (1 + w2)
mdota3 = mdota1 + mdota2
mdota3 * w3 = mdota1* w1 + mdota2 * w2
mdota3 * h3 = mdota1* h1 + mdota2 * h2
h1 = ha_Tw(T1,w1)
h2 = ha_Tw(T2,w2)
h3 = ha_Tw(T3,w3)
w3 = w_Tphi(T3,phi3,p)

0 = mdota1* s1 + mdota2 * s2 - mdota3 * s3 + sigmadot
Edotd = To * sigmadot / 60
To = 293
s1 = sa_Tw(T1,w1,p)
s2 = sa_Tw(T2,w2,p)
s3 = sa_Tw(T3,w3,p)
```

IT Results

$\omega_1 = 0.0135$ kg(v)/kg(a)
$\omega_2 = 0.005486$ kg(v)/kg(a)
$\omega_3 = 0.008477$ kg(v)/kg(a)
$\dot{m}_{a1} = 2.96$ kg(a)/min
$\dot{m}_{a2} = 4.973$ kg(a)/min
$\dot{m}_{a3} = 7.933$ kg(a)/min
$\dot{\sigma}_{cv} = 0.01009$ kJ/min·K
$T_3 = 14.41$°C
$\phi_3 = 0.8167$ (81.67%)
$\dot{E}_d = 0.04926$ kW

Problem 12.100

Figure P12.100 shows a device for conditioning moist air entering at 5°C, 1 atm, 90% relative humidity, and a volumetric flow rate of 60 m^3/min. The incoming air is first heated at essentially constant pressure to 24°C. Superheated steam at 1 atm is then injected, bringing the moist air stream to 25°C, 1 atm, and 45% relative humidity. Determine for steady state operation

(a) the rate of heat transfer to the air passing through the heating section, in kJ/min.

(b) the mass flow rate of the injected steam, in kg/min.

(c) If the injected steam expands through a valve from a saturated vapor condition at the valve inlet, determine the inlet pressure, in bar.

Neglect kinetic and potential energy effects.

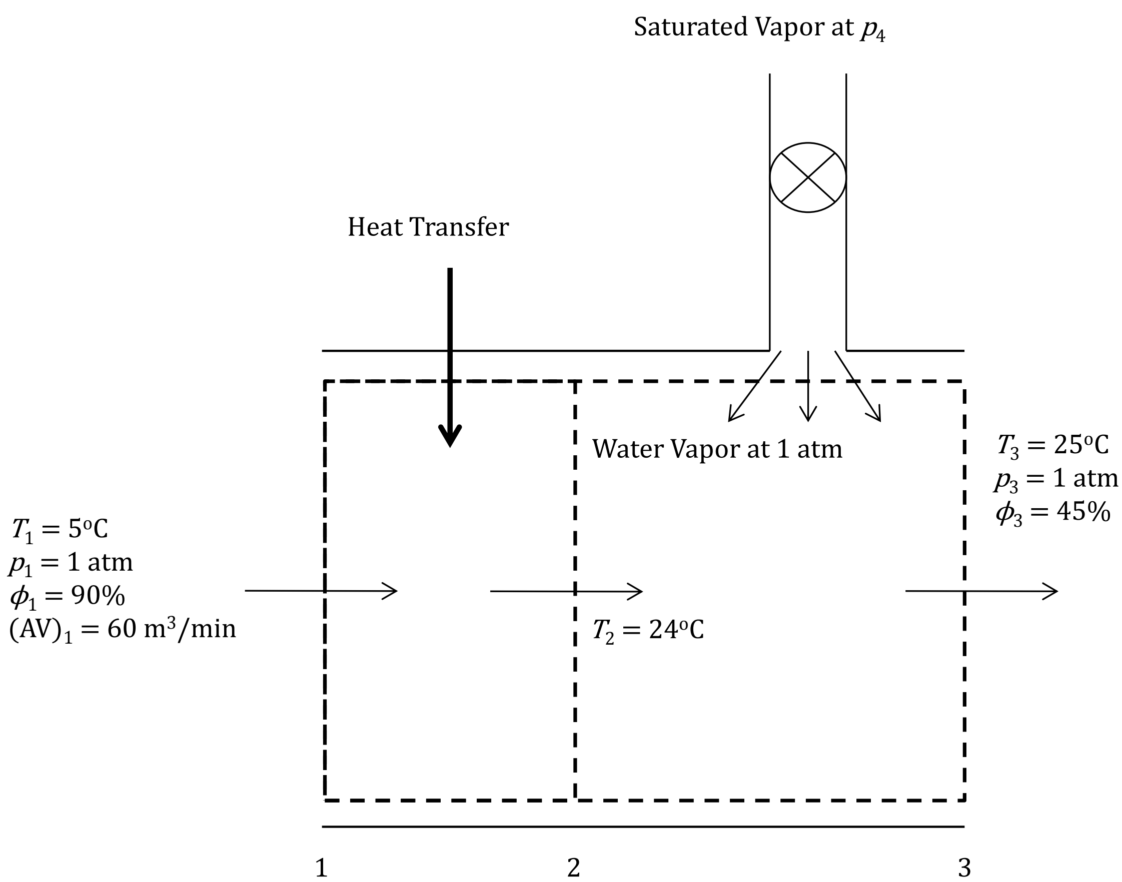
Known:

Operating data are provided for an air conditioner in which moist air is first heated and then superheated steam entering through a valve is injected into the air.

Find:

Determine (a) the rate of heat transfer, (b) the mass flow rate of the injected steam, and (c) the pressure at the valve inlet.

Schematic and Known Data:



Engineering Model:

(1) The control volumes shown in the accompanying figure are at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy.

(2) The control volume including the valve also has $\dot{Q}_{\text{cv}} = 0$.

Analysis:

(a) Start analysis with part (b).

(b) Steady state mass rate balances give $\dot{m}_{\text{a}_1} = \dot{m}_{\text{a}_2} = \dot{m}_{\text{a}_3} = \dot{m}_{\text{a}}$; $\dot{m}_{\text{v}_1} = \dot{m}_{\text{v}_2}$; $\dot{m}_{\text{v}_2} + \dot{m}_{\text{v}_4} = \dot{m}_{\text{v}_3}$. Thus $\omega_1 = \omega_2$ and:

$$\dot{m}_4 = \dot{m}_{\text{v}_3} - \dot{m}_{\text{v}_2} = \dot{m}_{\text{a}}(\omega_3 - \omega_2) = \dot{m}_{\text{a}}(\omega_3 - \omega_1) \qquad (1)$$

The mass flow rate is obtained using the given volumetric flow at 1, the ideal gas equation of state and $p_{\text{v}_1} = \phi_1 p_{\text{g}}(T_1) = (0.9)(0.00872\text{ bar}) = 0.00785\text{ bar}$:

$$\dot{m}_{\text{a}} = \frac{(\text{AV})_1}{v_{\text{a}_1}} = \frac{p_{\text{a}_1}(\text{AV})_1}{\frac{\bar{R}}{M_{\text{a}}}T_1} = \frac{\left(p_1 - p_{\text{v}_1}\right)(\text{AV})_1}{\frac{\bar{R}}{M_{\text{a}}}T_1} = \frac{\left[(1.01325 - 0.00785)\cdot 10^5\,\frac{\text{N}}{\text{m}^2}\right]\left(60\,\frac{\text{m}^3}{\text{min}}\right)}{\left(\frac{8314}{28.97}\,\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(278\text{ K})}$$
$$= 75.61\,\frac{\text{kg}}{\text{min}}$$

Solving Eq. 12.43 with $p_{\text{v}_3} = \phi_3 p_{\text{g}}(T_3) = (0.45)(0.03169\text{ bar}) = 0.01426\text{ bar}$, the values of ω_1 and ω_3 are

$$\omega_1 = 0.622\,\frac{p_{\text{v}_1}}{p - p_{\text{v}_1}} = 0.622\,\frac{0.00785}{1.01325 - 0.00785} = 0.00486\,\frac{\text{kg (vapor)}}{\text{kg (air)}}$$
$$\omega_3 = 0.622\,\frac{p_{\text{v}_3}}{p - p_{\text{v}_3}} = 0.622\,\frac{0.01426}{1.01325 - 0.01426} = 0.00888\,\frac{\text{kg (vapor)}}{\text{kg (air)}}$$

Substituting into Eq. (1) gives:

$$\dot{m}_4 = \dot{m}_a(\omega_3 - \omega_1) = \left(75.61\,\frac{\text{kg}}{\text{min}}\right)(0.00888 - 0.00486) = 0.304\,\frac{\text{kg}}{\text{min}} \longleftarrow$$

(a) Now revisit part (a) using an energy rate balance for the heating unit assumong constant specific heat which reduces to give

$$\dot{Q}_{\text{cv}} = \dot{m}_{\text{a}} h_{\text{a}_2} + \dot{m}_{\text{v}_2} h_{\text{v}_2} - \left[\dot{m}_{\text{a}} h_{\text{a}_1} + \dot{m}_{\text{v}_1} h_{\text{v}_1}\right] = \dot{m}_{\text{a}}\left[h_{\text{a}_2} - h_{\text{a}_1} + \omega_1\left(h_{\text{g}_2} - h_{\text{g}_1}\right)\right]$$
$$= \left(75.61\,\frac{\text{kg}}{\text{min}}\right)\left[\left(1.005\,\frac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{R}}\right)(24 - 5)^\circ\text{R}\right.$$
$$\left. + (0.00486)(2545.4 - 2510.6)\,\frac{\text{kJ}}{\text{kg}}\right] = 1456.6\,\frac{\text{kJ}}{\text{min}} \longleftarrow$$

(c) An energy rate balance on the control volume including the valve reads:

$$0 = \cancel{\dot{Q}}_{\text{cv}} - \cancel{\dot{W}}_{\text{cv}} + \dot{m}_{\text{a}} h_{\text{a}_2} + \dot{m}_{\text{v}_2} h_{\text{v}_2} + \dot{m}_4 h_4 - \left[\dot{m}_{\text{a}} h_{\text{a}_3} + \dot{m}_{\text{v}_3} h_{\text{v}_3}\right]$$

or

$$h_4 = \frac{\dot{m}_\mathrm{a}\left[h_{\mathrm{a}_3} - h_{\mathrm{a}_2} + \omega_3 h_{\mathrm{g}_3} - \omega_2 h_{\mathrm{g}_2}\right]}{\dot{m}_4}$$

$$= \frac{\left(75.61\,\frac{\mathrm{kg}}{\mathrm{min}}\right)\left[\left(1.005\,\frac{\mathrm{kJ}}{\mathrm{kg}\cdot{}^\circ\mathrm{R}}\right)(25-24)^\circ\mathrm{R} + (0.00888)\left(2547.2\,\frac{\mathrm{kJ}}{\mathrm{kg}}\right) - (.00486)\left(2545.4\,\frac{\mathrm{kJ}}{\mathrm{kg}}\right)\right]}{0.304\,\frac{\mathrm{kg}}{\mathrm{min}}}$$

$$= 2798.8\,\frac{\mathrm{kJ}}{\mathrm{kg}}$$

Interpolating in Table A-3 with $h_\mathrm{g} = h_4 = 2798.8\,\frac{\mathrm{kJ}}{\mathrm{kg}}$:

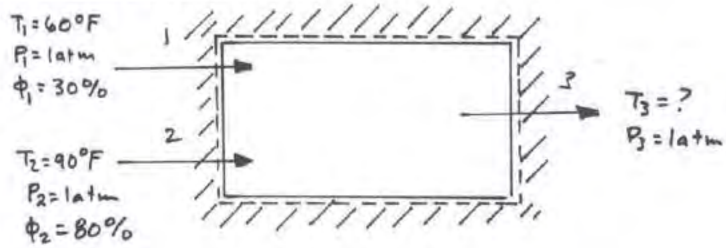
$p_4 = 19.52$ bar

PROBLEM 12.101

KNOWN: A stream of air at 60°F, 1atm, $\phi = 30\%$ is mixed adiabatically with a stream at 90°F, 1atm, $\phi = 80\%$. A single mixed stream exits.

FIND: Plot the temperature of the exiting stream versus the ratio of the mass flow rates of the dry air present in the two incoming streams.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy.

ANALYSIS: (a) At steady state mass rate balances reduce to give

air: $\dot{m}_{a1} + \dot{m}_{a2} = \dot{m}_{a3}$

water: $\dot{m}_{v1} + \dot{m}_{v2} = \dot{m}_{v3} \Rightarrow \omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2} = \omega_3 \dot{m}_{a3}$

Thus

$$\omega_3 = \frac{\omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2}}{\dot{m}_{a1} + \dot{m}_{a2}} = \frac{\left(\frac{\dot{m}_{a1}}{\dot{m}_{a2}}\right)\omega_1 + \omega_2}{\left(\frac{\dot{m}_{a1}}{\dot{m}_{a2}}\right) + 1} = \frac{r\omega_1 + \omega_2}{r+1} \quad (1)$$

where $r = \dot{m}_{a1}/\dot{m}_{a2}$. To find ω_1 and ω_2, $p_{v1} = \phi_1 \, p_g(T_1) = 0.3(0.2563) = 0.07689 \text{ lbf/in}^2$, $p_{v2} = \phi_2 \, p_g(T_2) = 0.8(0.6988) = 0.55904 \text{ lbf/in}^2$. Then

$$\omega_1 = 0.622 \frac{p_{v1}}{p_1 - p_{v1}} = \frac{0.622(0.07689)}{14.696 - 0.07689} = 0.00327 \frac{\text{lb(a)}}{\text{lb(v)}}$$

$$\omega_2 = 0.622 \frac{p_{v2}}{p_2 - p_{v2}} = \frac{(0.622)(0.55904)}{14.13696} = 0.024597 \frac{\text{lb(a)}}{\text{lb(v)}}$$

At steady state an energy rate balance reduces to

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + [\dot{m}_{a1} h_{a1} + \dot{m}_{v1} h_{g1}] + (\dot{m}_{a2} h_{a2} + \dot{m}_{v2} h_{v2}] - [\dot{m}_{a3} h_{a3} + \dot{m}_{v3} h_{v3}]$$

Thus

$$0 = \dot{m}_{a1}[h_{a1} + \omega_1 h_{g1}] + \dot{m}_{a2}[h_{a2} + \omega_2 h_{g2}] - \dot{m}_{a3}[h_{a3} + \omega_3 h_{g3}]$$

or

$$h_{a3} + \omega_3 h_{g3} = \frac{\dot{m}_{a1}[h_{a1} + \omega_1 h_{g1}] + \dot{m}_{a2}[h_{a2} + \omega_2 h_{g2}]}{\dot{m}_{a1} + \dot{m}_{a2}}$$

$$= \frac{\left(\frac{\dot{m}_{a1}}{\dot{m}_{a2}}\right)[h_{a1} + \omega_1 h_{g1}] + [h_{a2} + \omega_2 h_{g2}]}{(\dot{m}_{a1}/\dot{m}_{a2}) + 1}$$

$$= \frac{r[h_{a1} + \omega_1 h_{g1}] + [h_{a2} + \omega_2 h_{g2}]}{r+1} \quad (2)$$

Combining Eqs. (1) and (2)

$$h_{a3} + \left[\frac{r\omega_1 + \omega_2}{r+1}\right] h_{g3} = \frac{r[h_{a1} + \omega_1 h_{g1}] + [h_{a2} + \omega_2 h_{g2}]}{r+1} \quad (3)$$

Using the result of Problem 12.74, $(h_a + \omega h_g) = 0.24\,T(°F) + \omega(1061 + 0.444\,T(°F))$

PROBLEM 12.101 (Cont'd) - Page 2

Accordingly

$h_{a1} = 0.24(60) = 14.4$ Btu/lb(a) , $h_{a2} = 0.24(90) = 21.6$ Btu/lb(a)

$h_{g1} = 1061 + 0.444(60) = 1087.64$ Btu/lb(v)

$h_{g2} = 1061 + 0.444(90) = 1100.96$ Btu/lb(v)

Equation (3) becomes

$$0.24\,T_3 + \left[\frac{r\omega_1 + \omega_2}{r+1}\right]\left[1061 + 0.444\,T_3\right] = \frac{17.957\,r + 48.6803}{r+1}$$

or

$$\left\{0.24 + 0.444\left[\frac{r\omega_1 + \omega_2}{r+1}\right]\right\} T_3 = \frac{14.488\,r + 22.583}{r+1}$$

where $\omega_1 = 0.00327$ and $\omega_2 = 0.024597$. For $r = 2$, $T_3 = 70.26$ °F ← T_3

(b) The data for the required plot are obtained using IT, as follows:

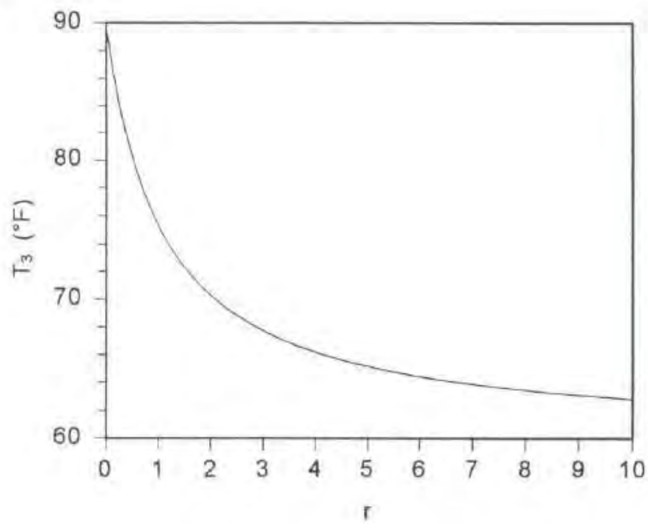
IT Code

```
T1 = 60 // °F
p = 1 // atm
phi1 = 0.3
T2 = 90 // °F
phi2 = 0.8
r = 2

w1 = w_Tphi(T1,phi1,p)
w2 = w_Tphi(T2,phi2,p)
w3 = w_Tphi(T3,phi3,p)
h1 = ha_Tw(T1,w1)
h2 = ha_Tw(T2,w2)
h3 = ha_Tw(T3,w3)

r * w1 + w2 = (1 + r) * w3
r * h1 + h2 = (1 + r) * h3
```

IT Results

$\omega_1 = 0.003269$ lb(v)/lb(a)
$\omega_2 = 0.02457$ lb(v)/lb(a)
$\omega_3 = 0.01037$ lb(v)/lb(a)
$\phi_3 = 0.6577$ (65.77%)
$T_3 = 70.26$°F

PLOT:



As $r \to \infty$, $T_3 \to T_1 = 60$°F, as expected.

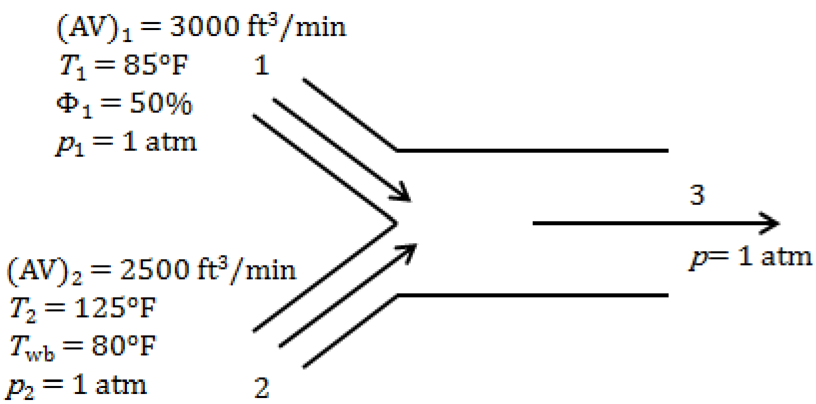
Problem 12.102

Figure P12.102 shows the adiabatic mixing of two moist air streams at steady state. Kinetic and potential energy effects are negligible. Determine the rate of exergy destruction, in Btu/min, for T_0 = 95°F.

Known:
Operating data are provided for two moist air streams mixing adiabatically at steady state.

Find:
Determine the exergy destruction rate.

Schematic & Known Data:



Engineering Model:
(1) The control volume shown operates at steady state with $\dot{W}_{cv} = \dot{Q}_{cv} = 0$ and negligible effects of kinetic and potential energy.
(2) Each stream is modeled as an ideal gas mixture.
(3) Let T_0 = 555°R.

Analysis:
To determine the exergy destruction rate, we first fix state 3 using mass and energy balances. At steady state:

air: $\dot{m}_{a_1} + \dot{m}_{a_2} = \dot{m}_{a_3}$

water: $\dot{m}_{v_1} + \dot{m}_{v_2} = \dot{m}_{v_3} \rightarrow \omega_1\dot{m}_{a_1} + \omega_2\dot{m}_{a_2} = \omega_3\dot{m}_{a_3}$

Thus:

$$\omega_3 = \frac{\omega_1\dot{m}_{a_1} + \omega_2\dot{m}_{a_2}}{\dot{m}_{a_1} + \dot{m}_{a_2}} \qquad (1)$$

At state 1 with data from Table A-2E,

$$p_{v_1} = \phi_1 p_g(T_1) = 0.5(0.5967) = 0.29835\,\frac{\text{lbf}}{\text{in.}^2}$$

$$\omega_1 = 0.622\,\frac{p_{v_1}}{p - p_{v_1}} = 0.622\left(\frac{0.29835}{14.696 - 0.29835}\right) = 0.01289\,\frac{\text{lb}_\text{v}}{\text{lb}_\text{a}}$$

To find ω_2, use Eqs. 12.53 and 12.52, respectively with $T_{\text{wb}_2} = T_{\text{a}_\text{s}}$:

$$\omega_2{}' = 0.622\left[\frac{p_\text{g}(T_{\text{wb}_2})}{p - p_\text{g}(T_{\text{wb}_2})}\right] = 0.622\left[\frac{0.5073}{14.696 - 0.5073}\right] = 0.0222$$

$$\omega_2 = \frac{c_{p_a}(T_{\text{wb}_2} - T_2) + \omega' h_{\text{fg}}(T_{\text{wb}})}{h_\text{g}(T_2) - h_\text{f}(T_{\text{wb}})} = \frac{\left(0.24\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)(80 - 125)\,{}^\circ\text{R} + (0.0222)\left(1048.3\,\frac{\text{Btu}}{\text{lb}}\right)}{(1115.6 - 48.09)\,\frac{\text{Btu}}{\text{lb}}}$$

$$= 0.01168\,\frac{\text{lb}_\text{v}}{\text{lb}_\text{a}}$$

Also:

$$p_{v_2} = \frac{\omega_2 p}{0.622 + \omega_2} = \frac{(0.01168)(14.696)}{0.622 + 0.01168} = 0.271\,\frac{\text{lbf}}{\text{in.}^2}$$

The mass flow rates are:

$$\dot{m}_{\text{a}_1} = \frac{(p - p_{\text{v}_1})(\text{AV})_1}{R_a T_1} = \frac{\left[(14.696 - 0.29835)\,\frac{\text{lbf}}{\text{in.}^2}\left|\frac{144\text{in.}^2}{\text{ft}^2}\right|\right]\left(3000\,\frac{\text{ft}^3}{\text{min}}\right)}{\left(\frac{1545}{28.97}\,\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(545{}^\circ\text{R})} = 213.99\,\frac{\text{lb}}{\text{min}}$$

$$\dot{m}_{\text{a}_2} = \frac{(p - p_{\text{v}_2})(\text{AV})_2}{R_\text{a} T_2} = \frac{\left[(14.696 - 0.271)\,\frac{\text{lbf}}{\text{in.}^2}\left|\frac{144\text{in.}^2}{\text{ft}^2}\right|\right]\left(2500\,\frac{\text{ft}^3}{\text{min}}\right)}{\left(\frac{1545}{28.97}\,\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(585{}^\circ\text{R})} = 166.45\ \ \frac{\text{lb}}{\text{min}}$$

$$\dot{m}_{\text{a}_3} = \dot{m}_{\text{a}_1} + \dot{m}_{\text{a}_2} = 380.44\,\frac{\text{lb}}{\text{min}}$$

At steady state the energy rate balance reduces to:

$$0 = \cancel{\dot{Q}}_{\text{cv}} - \cancel{\dot{W}}_{\text{cv}} + \left[\dot{m}_{\text{a}_1} h_{\text{a}_1} + \dot{m}_{\text{v}_1} h_{\text{v}_1}\right] + \left[\dot{m}_{\text{a}_2} h_{\text{a}_2} + \dot{m}_{\text{v}_2} h_{\text{v}_2}\right] - \left[\dot{m}_{\text{a}_3} h_{\text{a}_3} + \dot{m}_{\text{v}_3} h_{\text{v}_3}\right]$$

Thus:

$$0 = \dot{m}_{\text{a}_1}\left[h_{\text{a}_1} + \omega_1 h_{\text{g}_1}\right] + \dot{m}_{\text{a}_2}\left[h_{\text{a}_2} + \omega_2 h_{\text{g}_2}\right] - \dot{m}_{\text{a}_3}\left[h_{\text{a}_3} + \omega_3 h_{\text{g}_3}\right]$$

Or

$$h_{\text{a}_3} + \omega_3 h_{\text{g}_3} = \frac{\dot{m}_{\text{a}_1}}{\dot{m}_{\text{a}_3}}\left[h_{\text{a}_1} + \omega_1 h_{\text{g}_1}\right] + \frac{\dot{m}_{\text{a}_2}}{\dot{m}_{\text{a}_3}}\left[h_{\text{a}_2} + \omega_2 h_{\text{g}_2}\right] \qquad (2)$$

Using the result from Problem 12.74, $h_a + \omega h_g = 0.24T + \omega(1061 + 0.444T)$ where T is in °F.

$$h_{a_1} + \omega_1 h_{g_1} = 0.24(85) + (0.01289)\big(1061 + 0.444(85)\big) = 34.56\,\frac{\text{Btu}}{\text{lb}}$$

$$h_{a_2} + \omega_2 h_{g_2} = 0.24(125) + (0.01168)\big(1061 + 0.444(125)\big) = 43.04\,\frac{\text{Btu}}{\text{lb}}$$

Using Eq. (1)

$$\omega_3 = \frac{\omega_1 \dot{m}_{a_1} + \omega_2 \dot{m}_{a_2}}{\dot{m}_{a_1} + \dot{m}_{a_2}} = \frac{(0.01289)(213.99) + (0.01168)(166.45)}{(213.99) + (166.45)}$$
$$= 0.01236\,\frac{\text{lb}_\text{v}}{\text{lb}_\text{a}}$$

From Eq. (2):

$$\frac{\dot{m}_{a_1}}{\dot{m}_{a_3}}\left[h_{a_1} + \omega_1 h_{g_1}\right] + \frac{\dot{m}_{a_2}}{\dot{m}_{a_3}}\left[h_{a_2} + \omega_2 h_{g_2}\right] = \left(\frac{213.99}{380.44}\right)(34.56) + \left(\frac{166.45}{380.44}\right)(43.04) = 38.27$$

Using the results again from Problem 12.74, solve for T_3:

$$h_{a_3} + \omega_3 h_{g_3} = 0.24T_3 + \omega_3(1061 + 0.444T_3) =$$
$$38.27 = 0.24T_3 + 0.01236(1061 + .444T_3) \Longrightarrow \qquad T_3 = 102.5\text{°F}$$

The exergy destruction rate is $\dot{\text{E}}_\text{d} = T_0\dot{\sigma}_\text{cv}$, where $\dot{\sigma}_\text{cv}$ is the entropy production rate from an entropy balance which at steady state takes the form:

$$0 = \overbrace{\sum_\text{j}\frac{\dot{Q}_\text{j}}{T_\text{j}}}^{0} + \left[\dot{m}_{a_1}s_a(T_1, p_{a_1}) + \dot{m}_{v_1}s_v(T_1, p_{v_1})\right] + \left[\dot{m}_{a_2}s_a(T_2, p_{a_2}) + \dot{m}_{v_2}s_v(T_2, p_{v_2})\right]$$
$$- \left[\dot{m}_{a_3}s_a(T_3, p_{a_3}) + \dot{m}_{v_3}s_v(T_3, p_{v_3})\right] + \dot{\sigma}_\text{cv}$$

Or:

$$\dot{\sigma}_\text{cv} = \dot{m}_{a_3}\left[s_a(T_3, p_{a_3}) + \omega_3 s_v(T_3, p_{v_3})\right] - \dot{m}_{a_1}\left[s_a(T_1, p_{a_1}) + \omega_1 s_v(T_1, p_{v_1})\right]$$
$$- \dot{m}_{a_2}\left[s_a(T_2, p_{a_2}) + \omega_2 s_v(T_2, p_{v_2})\right]$$

Using $\dot{m}_{a_3} = \dot{m}_{a_1} + \dot{m}_{a_2}$ to rearrange:

$$\dot{\sigma}_\text{cv} = \dot{m}_{a_1}\left[s_a(T_3, p_{a_3}) - s_a(T_1, p_{a_1})\right] + \dot{m}_{a_2}\left[s_a(T_3, p_{a_3}) - s_a(T_2, p_{a_2})\right] + \dot{m}_{a_3}\omega_3 s_v(T_3, p_{v_3})$$
$$- \dot{m}_{a_1}\omega_1 s_v(T_1, p_{v_1}) - \dot{m}_{a_2}\omega_2 s_v(T_2, p_{v_2})$$

Using Eq. 6.18, the specific entropy for the water vapor is given by:

$$s_v(T, p_v) = s_g(T) - \frac{\bar{R}}{M_v}\ln\frac{p_v}{p_g} = s_g(T) - \frac{\bar{R}}{M_v}\ln\phi$$

Combining above with Eq. 6.22 for the dry air:

$$\dot{\sigma}_{\text{cv}} = \dot{m}_{\text{a}_1}\left[c_{\text{p}_\text{a}}\ln\frac{T_3}{T_1} - \frac{\bar{R}}{M_\text{a}}\ln\frac{p_{\text{a}_3}}{p_{\text{a}_1}}\right] + \dot{m}_{\text{a}_2}\left[c_{\text{p}_\text{a}}\ln\frac{T_3}{T_2} - \frac{\bar{R}}{M_\text{a}}\ln\frac{p_{\text{a}_3}}{p_{\text{a}_2}}\right] + \dot{m}_{\text{a}_3}\omega_3\left[s_\text{g}(T_3) - \frac{\bar{R}}{M_\text{v}}\ln\phi_3\right] - \dot{m}_{\text{a}_1}\omega_1\left[s_\text{g}(T_1) - \frac{\bar{R}}{M_\text{v}}\ln\phi_1\right] - \dot{m}_{\text{a}_2}\omega_2\left[s_\text{g}(T_2) - \frac{\bar{R}}{M_\text{v}}\ln\phi_2\right] \tag{3}$$

From above,

$p_{\text{a}_1} = p - p_{\text{v}_1} = 14.696 - 0.29835 = 14.398\,\frac{\text{lbf}}{\text{in.}^2}$

$p_{\text{a}_2} = p - p_{\text{v}_2} = 14.696 - 0.271 = 14.425\,\frac{\text{lbf}}{\text{in.}^2}$.

Now:

$$p_{\text{v}_3} = \frac{\omega_3 p}{0.622 + \omega_3} = \frac{(0.01236)(14.696)}{0.622 + (0.01236)} = 0.2863\,\frac{\text{lbf}}{\text{in.}^2} \Longrightarrow p_{\text{a}_3} = p - p_{\text{v}_3} = 14.41\,\frac{\text{lbf}}{\text{in.}^2}$$

Inserting values into Eq. (3) and knowing

$\phi_2 = \frac{p_{\text{v}_2}}{p_\text{g}(T_2)} = \frac{0.271}{1.96} = 0.1383$ and $\phi_3 = \frac{p_{\text{v}_3}}{p_\text{g}(T_3)} = \frac{0.2863}{1.0317} = 0.2775$

$$\begin{aligned}
\dot{\sigma}_{\text{cv}} = &\left(213.99\,\frac{\text{lb}}{\text{min}}\right)\left[\left(0.24\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln\frac{562.5}{545} - \left(\frac{1.986}{28.97}\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln\frac{14.41}{14.398}\right] \\
&+ \left(166.45\,\frac{\text{lb}}{\text{min}}\right)\left[\left(0.24\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln\frac{562.5}{585} - \left(\frac{1.986}{28.97}\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln\frac{14.41}{14.425}\right] \\
&+ \left(380.44\,\frac{\text{lb}}{\text{min}}\right)(0.01236)\left[1.976\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} - \left(\frac{1.986}{18.02}\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln 0.2775\right] \\
&- \left(213.99\,\frac{\text{lb}}{\text{min}}\right)(0.01289)\left[2.02175\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} - \left(\frac{1.986}{18.02}\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln 0.5\right] \\
&- \left(166.45\,\frac{\text{lb}}{\text{min}}\right)(0.01168)\left[1.92225\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} - \left(\frac{1.986}{18.02}\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln 0.1383\right] \\
&= 1.611 - 1.555 + 9.956 - 5.787 - 4.161 = 0.064\,\frac{\text{Btu}}{\text{min}\cdot{}^\circ\text{R}}
\end{aligned}$$

Finally:

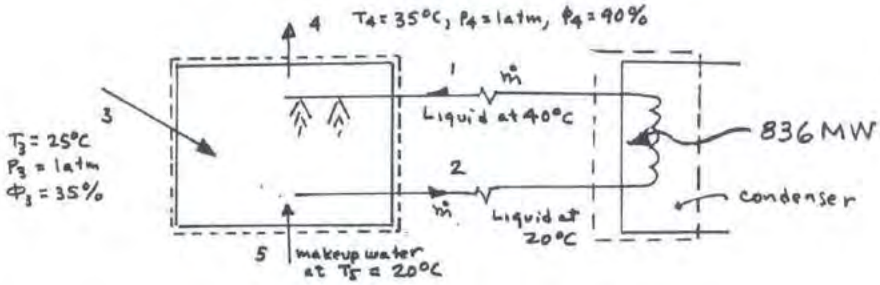
$$\dot{\text{E}}_\text{d} = T_0\dot{\sigma}_{\text{cv}} = (555^\circ\text{R})\left(0.064\,\frac{\text{Btu}}{\text{min}\cdot{}^\circ\text{R}}\right) = 35.52\,\frac{\text{Btu}}{\text{min}} \quad \longleftarrow$$

PROBLEM 12.103

KNOWN: Operating data are provided for a cooling tower that services a power plant.

FIND: Determine the mass flow rates of the entering atmospheric air and the make up water.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volumes shown in the figure are at steady state with negligible $\dot{Q}_{cv}$, $\dot{W}_{cv}$, and kinetic/potential energy effects. (2) For the streams 1, 2, 5, $h \approx h_f(T)$.

ANALYSIS: The mass flow rates at 1, 2 are equal, $\dot{m}$. An energy balance on the cooling water side of the condenser is $\dot{Q}_{cv} = \dot{m}[h_1 - h_2]$, or

$$\dot{m} = \frac{\dot{Q}_{cv}}{h_1 - h_2} = \frac{836 \times 10^3 \text{ kJ/s}}{(167.6 - 84)\text{ kJ/kg}} = 10^4 \text{ kg/s}$$

Next, for a control volume enclosing the cooling tower, mass rate balances give $\dot{m}_{a3} = \dot{m}_{a4} \equiv \dot{m}_a$ and

$$\dot{m}_1 + \dot{m}_{v3} + \dot{m}_5 = \dot{m}_2 + \dot{m}_{v4}$$

Since $\dot{m}_1 = \dot{m}_2$

$$\dot{m}_5 = \dot{m}_{v4} - \dot{m}_{v3} = \dot{m}_a(\omega_4 - \omega_3) \qquad (1)$$

The mass flow rate of the entering atmospheric air is

$$\dot{m}_3 = \dot{m}_a + \dot{m}_{v3} = \dot{m}_a(1 + \omega_3) \qquad (2)$$

From Eqs (1) and (2) it is clear that $\dot{m}_a$, ω_3, and ω_4 must be evaluated.

To find ω_3 and ω_4, write $p_{v3} = \phi_3 p_g(T_3) = (0.35)(0.03169) = 0.01109$ bars, $p_{v4} = \phi_4 p_g(T_4) = (0.9)(0.05628) = 0.05065$ bar. Then

$$\omega_3 = 0.622\left[\frac{0.01109}{1.01325 - 0.01109}\right] = 0.00688 \frac{\text{kg(v)}}{\text{kg(a)}}, \quad \omega_4 = 0.622\left[\frac{0.05065}{1.01325 - 0.05065}\right] = 0.0327 \frac{\text{kg(v)}}{\text{kg(a)}}$$

The mass flow rate $\dot{m}_a$ can be found from an energy rate balance which at steady state reduces to

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + [\dot{m}_a h_{a3} + \dot{m}_{v3} h_{v3}] + \dot{m}_1 h_1 + \dot{m}_5 h_5 - \dot{m}_2 h_2 - [\dot{m}_a h_{a4} + \dot{m}_{v4} h_{v4}]$$

Introducing Eq. (1)

$$0 = \dot{m}_a[h_{a3} + \omega_3 h_{g3}] + \dot{m}_1[h_f(T_1) - h_f(T_2)] + \dot{m}_a(\omega_4 - \omega_3)h_f(T_5) - \dot{m}_a[h_{a4} + \omega_4 h_{g4}]$$

Solving for $\dot{m}_a$

$$\dot{m}_a = \frac{\dot{m}_1[h_f(T_2) - h_f(T_1)]}{(h_{a3} - h_{a4}) + \omega_3 h_{g3} - \omega_4 h_{g4} + (\omega_4 - \omega_3)h_f(T_5)}$$

$$= \frac{10{,}000[84 - 167.6]}{[298.18 - 308.23] + (0.00688)(2547.2) - (0.0327)(2565.3) + (0.0327 - 0.00688)(83.96)} = 11{,}261 \text{ kg(a)/s}$$

With ω_3, ω_4, and $\dot{m}_a$, Eq. (1) gives

$$\dot{m}_5 = \dot{m}_a(\omega_4 - \omega_3) = 11{,}261(0.0327 - 0.00688) = 290.8 \text{ kg/s}$$ ← Makeup

And Eq. (2) gives

$$\dot{m}_3 = \dot{m}_a(1 + \omega_3) = 11{,}261[1 + .00688] = 11{,}338.5 \text{ kg/s}$$ ← Atm. air

12.104 Liquid water at 100° F enters a cooling tower operating at steady state and cooled water exits the tower at 80° F. Data for the various streams entering and exiting the tower are shown in Fig. P12.104. No makeup water is provided. Determine

(a) the mass flow rate of the entering atmospheric air, in lb/h.

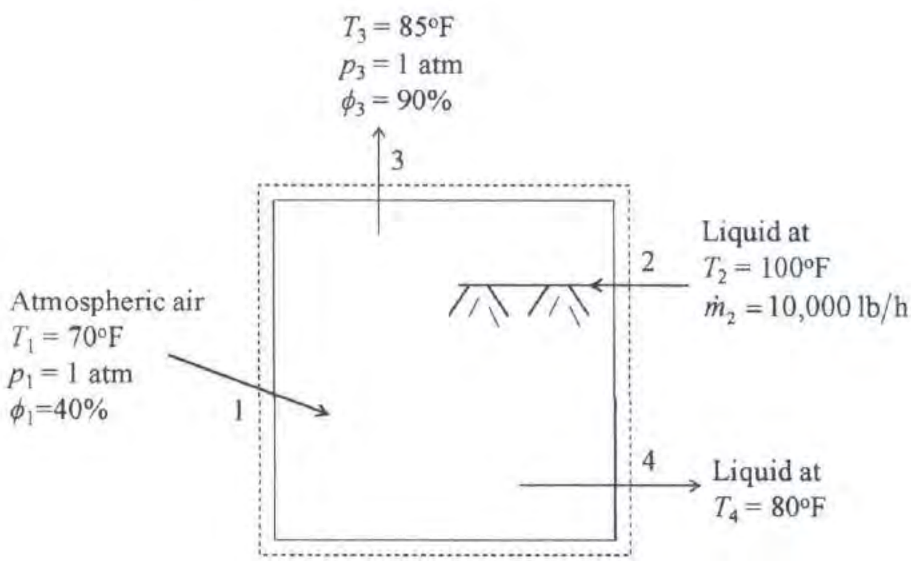
(b) the rate at which water evaporates, in lb/h.

(c) the mass flow rate of the exiting liquid stream, in lb/h.

KNOWN: Operating data are provided for a cooling tower at steady state.

FIND: Determine (a) the mass flow rate of the entering atmospheric air, (b)the rate at which water evaporates, and (c) the mass flow rate of the exiting liquid.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

(1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy.

(2) For streams 2 and 4, $h = h_f(T)$.

ANALYSIS:

At steady state mass rate balances reduce to give $\dot{m}_{a1} = \dot{m}_{a3} \equiv \dot{m}_a$ and

$$0 = \dot{m}_{v1} + \dot{m}_2 - \dot{m}_{v3} - \dot{m}_4 \Rightarrow \dot{m}_4 = \dot{m}_{v1} + \dot{m}_2 - \dot{m}_{v3} = \dot{m}_2 + \dot{m}_a(\omega_1 + \omega_3) \qquad (1)$$

Problem 12.104 (Continued) – Page 2

The mass flow rate of the entering atmospheric air is $\dot{m}_1 = \dot{m}_a + \dot{m}_{v1}$ or

$$\dot{m}_1 = \dot{m}_a\left(1+\omega_1\right) \tag{2}$$

The rate liquid evaporates into the moist air stream is

$$\left(\text{rate liquid evaporates}\right) = \dot{m}_{v3} - \dot{m}_{v1} = \dot{m}_a\left(\omega_3 - \omega_1\right) \tag{3}$$

From Eqs. (1), (2), and (3), $\dot{m}_a$, ω_3, and ω_1 must be found. To find ω_3 and ω_1

$$p_{v1} = \phi_1 p_g(T_1) = (0.4)(0.3632\ \text{lbf/in.}^2) = 0.14528\ \text{lbf/in.}^2$$
$$p_{v3} = \phi_3 p_g(T_3) = (0.9)(0.5967\ \text{lbf/in.}^2) = 0.53703\ \text{lbf/in.}^2$$

Then

$$\omega_1 = 0.622\frac{p_{v1}}{p - p_{v1}} = 0.622\frac{0.14528}{14.696 - 0.14528} = 0.0062\frac{\text{lb(vapor)}}{\text{lb(air)}}$$
$$\omega_3 = 0.622\frac{p_{v3}}{p - p_{v3}} = 0.622\frac{0.53703}{14.696 - 0.53703} = 0.0236\frac{\text{lb(vapor)}}{\text{lb(air)}}$$

The mass flow rate $\dot{m}_a$ can be found from an energy rate balance which at steady state reduces to

$$0 = \underbrace{\dot{Q}_{cv}}_{=0} - \underbrace{\dot{W}_{cv}}_{=0} + \left[\dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1}\right] + \dot{m}_2 h_2 - \left[\dot{m}_a h_{a3} - \dot{m}_{v3}h_{v3}\right] - \dot{m}_4 h_4$$

Introducing Eq. (1)

$$0 = \dot{m}_a\left[h_{a1} + \omega_1 h_{g1}\right] + \dot{m}_2 h_{f2}(T_2) - \dot{m}_a\left[h_{a3} + \omega_3 h_{g3}\right] - \left[\dot{m}_2 + \dot{m}_a(\omega_1 - \omega_3)\right]h_f(T_4)$$

Solving for $\dot{m}_a$ using Tables A-2E and A-22E

$$\dot{m}_a = \frac{\dot{m}_2\left[h_f(T_4) - h_f(T_2)\right]}{(h_{a1} - h_{a3}) + \omega_1 h_{g1} - \omega_3 h_{g3} - (\omega_1 - \omega_3)h_f(T_4)}$$
$$= \frac{(10000\ \text{lb/h})[43.09 - 68.05]\text{Btu/lb}}{[(126.66 - 130.26) + 0.0062(1092) - 0.0236(1098.55) - (0.0062 - 0.0236)(48.09)]\text{Btu/lb}}$$
$$= 9106\frac{\text{lb(air)}}{\text{h}}$$

Problem 12.104 (Continued) – Page 3

Inserting values into Eq. (2) the mass flow rate of the entering air is

$$\dot{m}_1 = 9106\,\frac{\text{lb(air)}}{\text{h}}\left(1 + 0.0062\right) = 9162\,\frac{\text{lb(air)}}{\text{h}} \qquad \longleftarrow$$

From Eq. (3)

$$\left(\text{rate liquid evaporates}\right) = 9106\,\frac{\text{lb(air)}}{\text{h}}\left(0.0236 - 0.0062\right) = 158.4\,\frac{\text{lb(air)}}{\text{h}} \qquad \longleftarrow$$

Finally, from Eq. (1) the mass flow rate of the exiting liquid is

$$\dot{m}_4 = 10000\,\frac{\text{lb(air)}}{\text{h}} - 158.4\,\frac{\text{lb(air)}}{\text{h}} = 9842\,\frac{\text{lb(air)}}{\text{h}} \qquad \longleftarrow$$

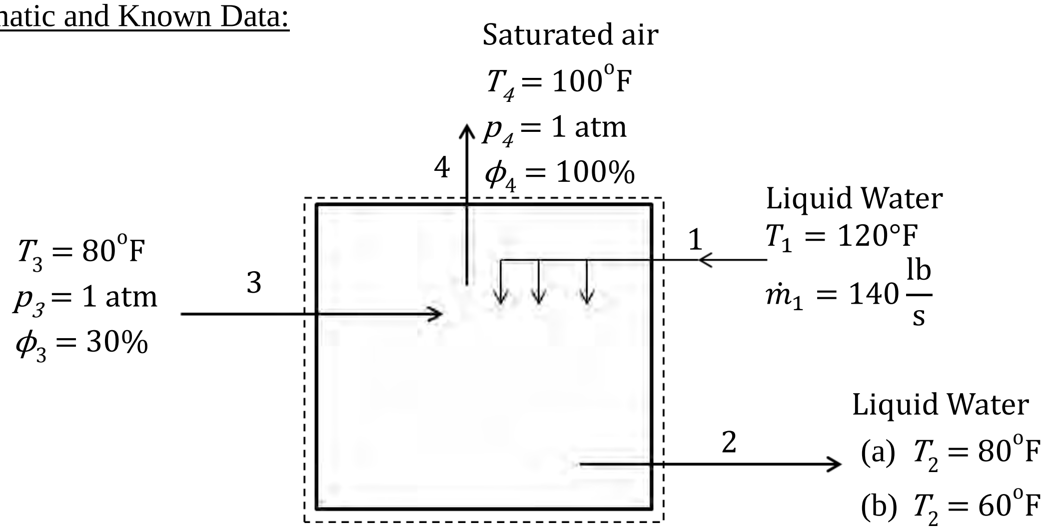
Problem 12.105

Liquid water at 120° F enters a cooling tower operating at steady state with a mass flow rate of 140 lb/s. Atmospheric air enters at 80° F, 1 atm, 30% relative humidity. Saturated air exits at 100° F, 1 atm. Makeup water is not provided. Determine the mass flow rate of dry air required, in lb/h, if cooled water exits the tower at (a) 80° F and (b) 60° F. Ignore kinetic and potential energy effects.

Known:
Operating data are provided for a cooling tower operating at steady state

Find:
Determine the mass flow rate of dry air for each of two cases

Schematic and Known Data:



Engineering Model:
(1) The control volumes shown in the accompanying figure are at steady state with $\dot{Q}_{\text{cv}} = \dot{W}_{\text{cv}} = 0$ and negligible effects of kinetic and potential energy.
(2) For streams 1 and 2, $h = h_f(T)$.

Analysis:
At steady state, mass rate balances give $\dot{m}_{a_3} = \dot{m}_{a_4} = \dot{m}_a$; $\dot{m}_{v_1} = \dot{m}_{v_2}$; $\dot{m}_{v_2} + \dot{m}_{v_4} = \dot{m}_{v_3}$ and
$\dot{m}_1 + \dot{m}_{v_3} = \dot{m}_2 + \dot{m}_{v_4} \Longrightarrow \dot{m}_2 = \dot{m}_1 + \dot{m}_a(\omega_3 - \omega_4)$ (1)

To find ω_3 and ω_4, write $p_{v_3} = \phi_3 p_g(T_3) = (0.3)(0.5073) = 0.15219\,\frac{\text{lbf}}{\text{in.}^2}$; $p_{v_4} = p_g(T_4) = 0.9503\,\frac{\text{lbf}}{\text{in.}^2}$. Then, using equation 12.43

$$\omega_3 = 0.622\,\frac{p_{v_3}}{p - p_{v_3}} = 0.622\,\frac{0.15219}{14.696 - 0.15219} = 0.0065\,\frac{\text{lb (vapor)}}{\text{lb (air)}}$$

$$\omega_4 = 0.622\frac{p_{v_4}}{p - p_{v_4}} = 0.622\frac{0.9503}{14.696 - 0.9503} = 0.0430\frac{\text{lb (vapor)}}{\text{lb (air)}}$$

The mass flow rate $\dot{m}_\mathrm{a}$ can be found from an energy rate balance which at steady state reduces to

$$0 = \cancel{\dot{Q}_\mathrm{cv}} - \cancel{\dot{W}_\mathrm{cv}} + \left[\dot{m}_\mathrm{a}h_{\mathrm{a}_3} + \dot{m}_{\mathrm{v}_3}h_{\mathrm{v}_3}\right] + \dot{m}_1 h_\mathrm{f}(T_1) + \dot{m}_1 h_1 - \dot{m}_2 h_2 - \left[\dot{m}_\mathrm{a}h_{\mathrm{a}_4} + \dot{m}_{\mathrm{v}_4}h_{\mathrm{v}_4}\right]$$

Introducing Eq. (1)

$$0 = \dot{m}_\mathrm{a}\left[h_{\mathrm{a}_3} + \omega_3 h_{\mathrm{g}_3}\right] + \dot{m}_1 h_\mathrm{f}(T_1) - [\dot{m}_1 + \dot{m}_\mathrm{a}(\omega_3 - \omega_4)]h_\mathrm{f}(T_2) - \dot{m}_\mathrm{a}\left(h_{\mathrm{a}_4} + \omega_4 h_{\mathrm{g}_4}\right)$$

Solving for $\dot{m}_\mathrm{a}$

$$\dot{m}_\mathrm{a} = \frac{\dot{m}_1[h_\mathrm{f}(T_1) - h_\mathrm{f}(T_2)]}{h_{\mathrm{a}_4} - h_{\mathrm{a}_3} + \omega_4 h_{\mathrm{g}_4} - \omega_3 h_{\mathrm{g}_3} + (\omega_3 - \omega_4)h_\mathrm{f}(T_2)}$$

$$= \frac{\left(140\,\frac{\text{lb}}{\text{s}}\right)\left[88\,\frac{Btu}{lb} - h_\mathrm{f}(T_2)\right]}{0.24\,\frac{\text{Btu}}{\text{lb°R}}(100 - 80)\text{°R} + (0.043)\left(1105\,\frac{\text{Btu}}{\text{lb}}\right) - (0.0065)\left(1096.4\,\frac{\text{Btu}}{\text{lb}}\right) + (0.043 - 0.0065)h_\mathrm{f}(T_2)}$$

$$= \frac{\left(140\,\frac{\text{lb}}{\text{s}}\right)\left[88\,\frac{\text{Btu}}{\text{lb}} - h_\mathrm{f}(T_2)\right]}{45.188\,\frac{\text{Btu}}{\text{lb}} - (0.0365)h_\mathrm{f}(T_2)}$$

(a) Substituting $h_\mathrm{f}(80\text{°F}) = 48.08\,\frac{\text{Btu}}{\text{lb}}$ into the equation above:

$$\dot{m}_\mathrm{a} = \frac{\left(140\,\frac{\text{lb}}{\text{s}}\right)[88 - 48.09]\,\frac{\text{Btu}}{\text{lb}}}{[45.188 - (0.0365)(48.09)]\,\frac{\text{Btu}}{\text{lb}}} \cdot \frac{3600\text{s}}{\text{h}} = 4.631 \cdot 10^5\,\frac{\text{lb}}{\text{h}} \qquad \longleftarrow$$

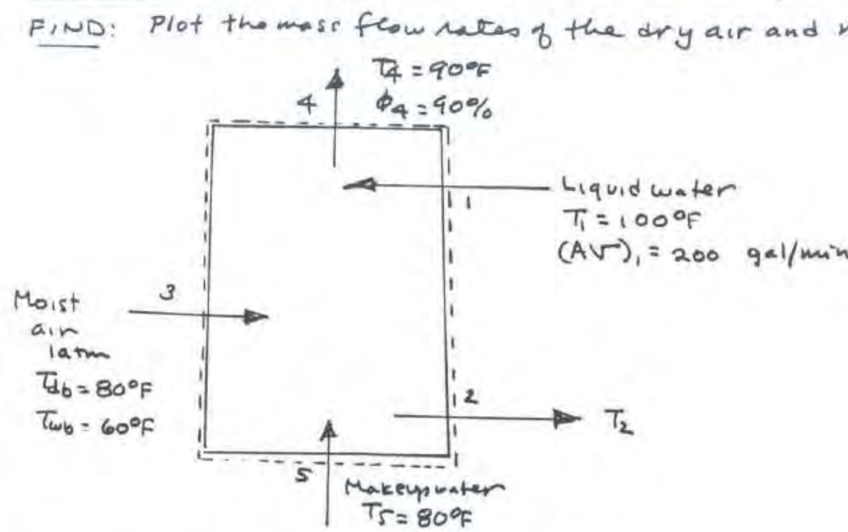
(b) Substituting $h_\mathrm{f}(60\text{°F}) = 28.08\,\frac{\text{Btu}}{\text{lb}}$ into the equation above:

$$\dot{m}_\mathrm{a} = \frac{\left(140\,\frac{\text{lb}}{\text{s}}\right)[88 - 28.08]\,\frac{\text{Btu}}{\text{lb}}}{[45.188 - (0.0365)(28.08)]\,\frac{\text{Btu}}{\text{lb}}} \cdot \frac{3600\text{s}}{\text{h}} = 6.838 \cdot 10^5\,\frac{\text{lb}}{\text{h}} \qquad \longleftarrow$$

PROBLEM 12.106

KNOWN: Steady state operating data are provided for a cooling tower.

FIND: Plot the mass flow rates of the dry air and makeup water versus T_2.



ENGINEERING MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume $\dot{Q}_{cv}$, $\dot{W}_{cv}$, and kinetic/potential energy effects are negligible. (3) Each liquid stream is regarded as a saturated liquid at the corresponding temperature (4) The moist air streams are modeled as ideal gas mixtures at 1atm. (5) The temperature T_2 is treated as a parameter, ranging $80°F \le T_2 < 100°F$.

ANALYSIS: Following the analysis of Example 12.17, the basic relations are

$$\dot{m}_5 = \dot{m}_a(\omega_4 - \omega_3) \tag{1}$$

$$\dot{m}_a = \frac{\dot{m}_1[h_{f1} - h_{f2}]}{(h_a + \omega h_g)_4 - (h_a + \omega h_g)_3 - (\omega_4 - \omega_3)h_{f5}} \tag{2}$$

Using T_{wb}, T_{db} at 3, the psychrometric chart gives $\omega_3 \approx .0066$, $(h_a + \omega h_g)_3 \approx 26.4$ Btu/lb(a). With T_4, ϕ_4 the chart gives $\omega_4 \approx 0.028$. The value of $(h_a + \omega h_g)_4 = (0.24)(90) + (0.028)(1100.7) = 52.4$ Btu/lb(a).

With $(AV)_1$ and $v_f(100°F)$,

$$\dot{m}_1 = \frac{\left(200\,\frac{gal}{min}\right)\left(0.13368\,\frac{ft^3}{gal}\right)}{0.01613\ ft^3/lb} = 1657.5\ \frac{lb}{min}$$

With these data, Eq. (1) becomes $\dot{m}_5 = \dot{m}_a(0.028 - 0.0066) = 0.0214\,\dot{m}_a$ (3)

and Eq. (2) becomes

$$\dot{m}_a = \frac{\left(1657.5\,\frac{lb}{min}\right)[68.05 - h_{f2}]\ Btu/lb}{(52.4 - 26.4) - (0.028 - 0.0066)(48.1)} = 66.38[68.05 - h_{f2}] \tag{4}$$

Sample Calculation: $T_2 = 80°F$. Then, $h_{f2} = 48.09$ Btu/lb. Eq. (4) gives $\dot{m}_a = 66.38(68.05 - 48.09) = 1324.9$ lb(a)/min. Eq. (3) gives $\dot{m}_5 = 28.35$ lb/min.

Data for the required plots are obtained using IT, as follows:

PROBLEM 12.106 (Cont'd.) - Page 2

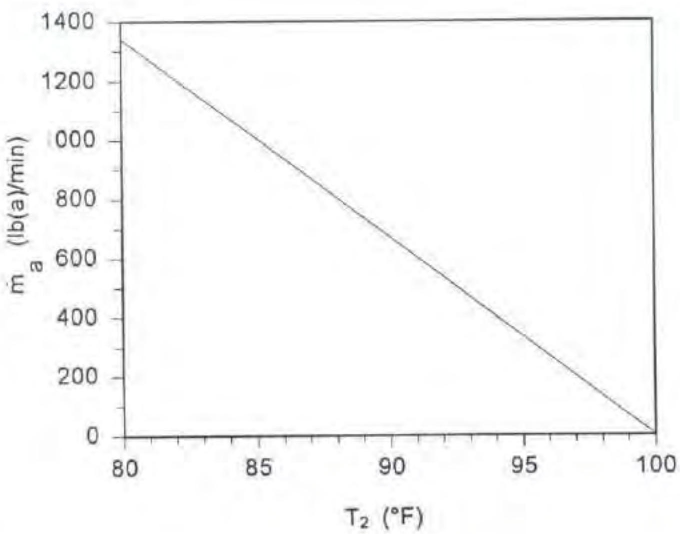
IT Code

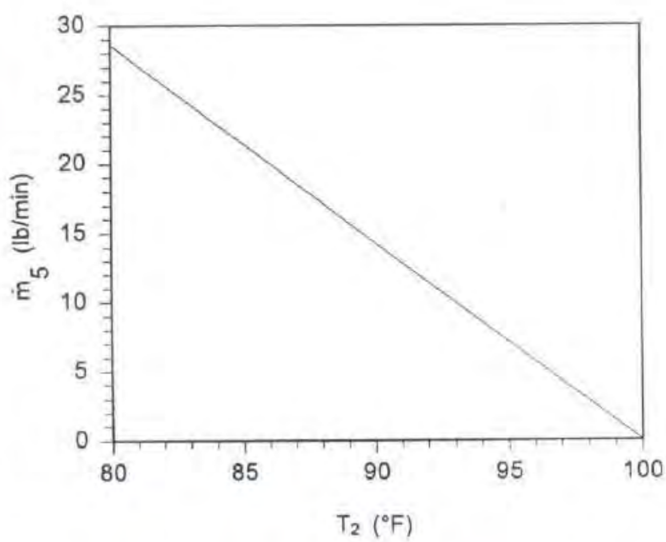
```
T1 = 100  // °F
AV1 = 200  // gal/min
T2 = 80  // °F
T3 = 80  // °F
Twb3 = 60  // °F
T4 = 90  // °F
phi4 = 0.9
T5 = 80  // °F
p = 14.696  // lbf/in²

h1 = hsat_Px("Water/Steam", psat1, 0)
psat1 = Psat_T("Water/Steam", T1)
v1 = vsat_Px("Water/Steam", psat1, 0)
h2 = hsat_Px("Water/Steam", psat2, 0)
psat2 = Psat_T("Water/Steam", T2)
w3 = w_TTwb(T3,Twb3,p)
h3 = ha_Tw(T3,w3)
w4 = w_Tphi(T4,phi4,p)
h4 = ha_Tw(T4,w4)
h5 = hsat_Px("Water/Steam", psat5, 0)
psat5 = Psat_T("Water/Steam", T5)

mdot5 = mdota * (w4 - w3)
mdot1 = AV1 * 0.13368 / v1  // lb/min
0 = mdota * (h3 - h4) + mdot5 * h5 + mdot1 * (h1 - h2)
```

IT Results for $T_2 = 80°F$

$\omega_3 = 0.006453$ lb(v)/lb(a)
$\omega_4 = 0.02778$ lb(v)/lb(a)
$T_2 = 80°F$
$\dot{m}_a = 1343$ lb(a)/min
$\dot{m}_5 = 28.64$ lb/min

PLOTS:





From the plots, we see that to achieve lower liquid temperature exiting the tower, we need increased air flow. Also, since there is more evaporation, greater make-up water flow is required. Both of these results are as expected.

P1.107 liquid water enters a cooling tower operating at steady state at 40°C with a mass flow rate of 10^5 kg/h. Cooled water at 25°C exits the cooling tower at the same mass flow rate. Makeup water is supplied at 23°C. Atmospheric air enters the tower at 30°C, 1 bar, 35% relative humidity. A saturated moist air stream exits at 34°C, 1 bar. Determine

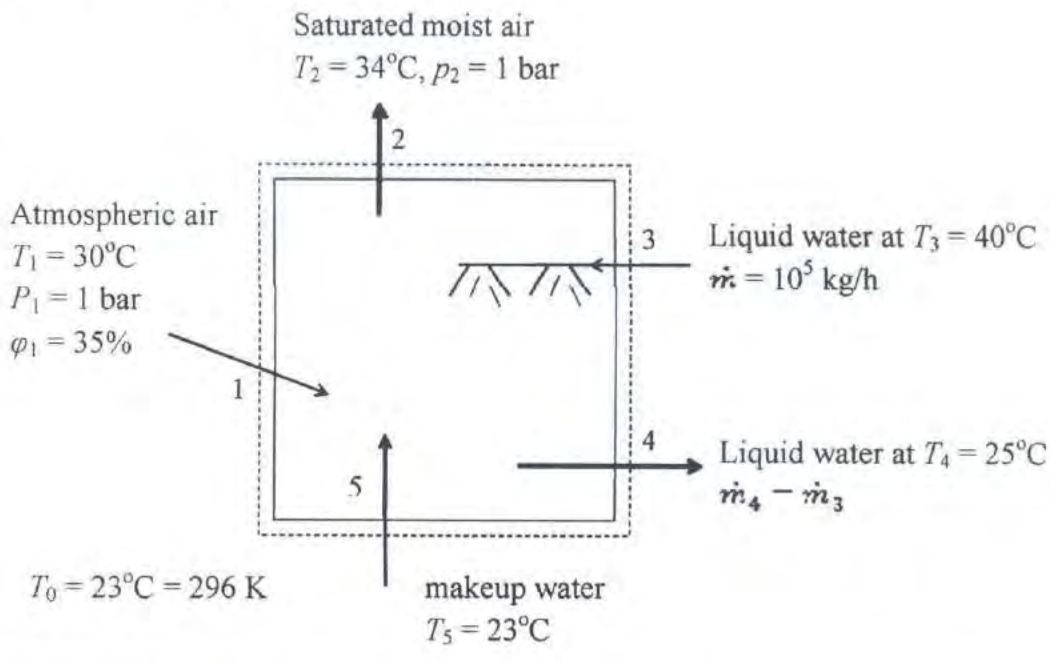
(a) The mass flow rates of dry air and makeup water, each in kg/h.

(b) The rate of exergy destruction, in kW, for T_0 = 23°C.

KNOWN: Operating data are provided for a cooling tower operating at steady state. The mass flow rate of cooling water entering is given.

FIND: Determine (a) the mass flow rates of dry air and makeup water and (b) the rate of exergy destruction.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

(1) The control volume is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energies.

(2) At 3, 4, an 5, $h = h_f(T)$ and $s = s_f(T)$.

ANALYSIS: (a) At steady state, mass balances give

Dry Air $\qquad \dot{m}_{a1} = \dot{m}_{a2} \equiv \dot{m}_a$

Water $\qquad \dot{m}_{v1} + \dot{m}_3 + \dot{m}_5 = \dot{m}_{v2} + \dot{m}_4$

Problem 12.107 (Continued) – Page 2

Since $\dot{m}_4 = \dot{m}_3$

$$\dot{m}_5 = \dot{m}_{v2} - \dot{m}_{v1} = \dot{m}_a(\omega_2 - \omega_1) \tag{1}$$

To find ω_1 and ω_2, use $p_v = \varphi p_g(T)$. With data from Table A-3, p_{v1} = (0.35)(0.04246) = 0.01486 bar and p_{v2} = (1)(0.05324) = 0.05234 bar. Then

$$\omega_1 = 0.622\left[\frac{0.01486}{1-0.01486}\right] = 0.00938 \text{ kg(v)/kg(a)},\ \omega_2 = 0.622\left[\frac{0.05234}{1-0.05234}\right] = 0.03498$$

To find $\dot{m}_a$, use an energy rate balance which reduces at with assumption (1) to

$$0 = [\dot{m}_a h_{a1} + \dot{m}_{v1} h_{v1}] - [\dot{m}_a h_{a2} + \dot{m}_{v2} h_{v2}] + \dot{m}_3 h_3 - \dot{m}_4 h_4 + \dot{m}_5 h_5$$

Introducing (1)

$$0 = \dot{m}_a[h_{a1} + \omega_1 h_{v1}] - \dot{m}_a[h_{a2} + \omega_2 h_{v2}] + \dot{m}_3[h_3 - h_4] + \dot{m}_a(\omega_2 - \omega_1)h_5$$

Substituting for water vapor $h_v = h_g(T)$, and with assumption (2), we solve for $\dot{m}_a$ to get

$$\dot{m}_a = \frac{\dot{m}_3[h_{f3} - h_{f4}]}{[h_{a2} - h_{a1}] + [\omega_2 h_{g2} - \omega_1 h_{g1}] - (\omega_2 - \omega_1)h_{f5}}$$

$$= \frac{(10^5)(167.37-104.89)}{1.005(34-30)+(0.03498)(2563.5)-(0.00938)(2556.3)-(0.0349-0.00938)(96.52)}$$

$= 9.32 \times 10^4$ kg/h ⟵

Then, using Eq. (1)

$\dot{m}_5 = \dot{m}_a(\omega_2 - \omega_1) = (9.32 \times 10^4)(0.0349-0.00938) = 2378.5$ kg/h ⟵

(b) The exergy destruction rate is best found using $\dot{E}_d = T_0\dot{\sigma}_{cv}$. To find $\dot{\sigma}_{cv}$ use an entropy balance as follows. With $\dot{Q}_{cv} = 0$

$$0 = [\dot{m}_a s_a(T_1, p_{a1}) + \dot{m}_{v1} s_v(T_1, p_{v1})] - [\dot{m}_a s_a(T_2, p_{a2}) + \dot{m}_{v2} s_g(T_2)] + \dot{m}_3 s_3 - \dot{m}_4 s_4 + \dot{m}_5 s_5 + \dot{\sigma}_{cv}$$

or

Problem 12.107 (Continued) – Page 3

$$\dot{\sigma}_{cv} = \dot{m}_a[s_a(T_2, p_{a2}) - s_a(T_1, p_{a1})] + \omega_2 s_g(T_2) - \omega_1 s_v(T_1, p_{v1}) + \dot{m}_3[s_f(T_4) - s_f(T_3)] - \dot{m}_5 s_f(T_5)$$

Noting that

$s_v(T, p_v) = s_g(T) - R_v \ln\emptyset$ and that for the dry air $s_a(T_2, p_{a2}) - s_a(T_1, p_{a1}) = c_p \ln\frac{T_2}{T_1} - R_a \ln\frac{p_{a2}}{p_{a1}}$

$$\dot{\sigma}_{cv} = \dot{m}_a\left[c_p \ln\frac{T_2}{T_1} - \frac{\bar{R}}{M_a}\ln\frac{p_{a2}}{p_{a1}}\right] + \omega_2 s_g(T_2) - \omega_1\left\{s_g(T_1) - \frac{\bar{R}}{M_v}\ln\emptyset_1\right\} + \dot{m}_3[s_f(T_4) - s_f(T_3)] - \dot{m}_5 s_f(T_5)$$

$$= (9.32 \times 10^4)[(1.005 \ln\frac{307}{302} - \frac{8.314}{28097}\ln\frac{0.94676}{0.98514}) + (0.03498)(8.3278)$$

$$- (0.00938)(8.4533 - \frac{8.314}{18.02}\ln 0.35)] - 10^5(0.5725 - 0.3674) - (2378.5)(0.3393)$$

$$= 475.24\ \frac{\text{kJ/h}}{\text{K}}$$

Finally

$$\dot{E}_d = T_0\dot{\sigma}_{cv} = (246\text{ K})(457.24\ \frac{\text{kJ/h}}{\text{K}})\left|\frac{1\text{ h}}{3600\text{ s}}\right|\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = 37.6\text{ kW}$$

Problem 12.108

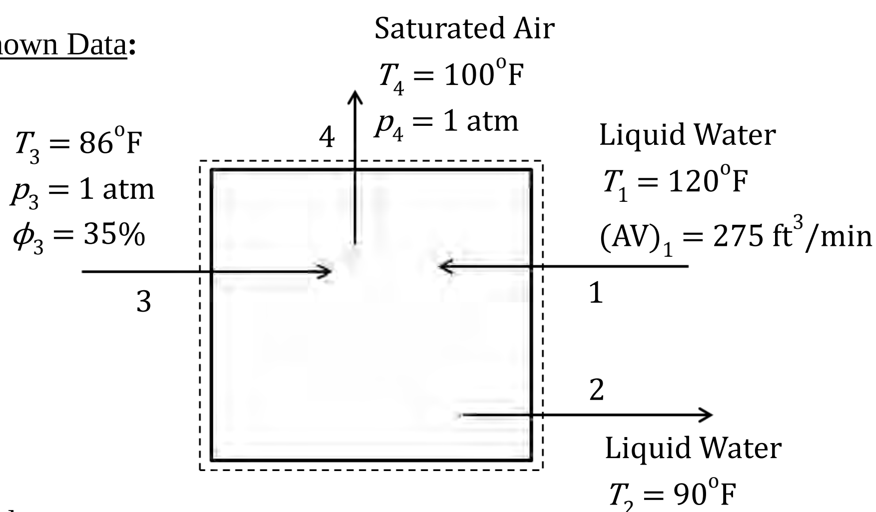
Liquid water at 120°F and a volumetric flow rate of 275 ft^3/min enters a cooling tower operating at steady state. Cooled water exits the cooling tower at 90°F. Atmospheric air enters the tower at 86°F, 1 atm, 35% relative humidity, and saturated moist air at 100°F, 1 atm exits the cooling tower. Determine
(a) the mass flow rates of the dry air and the cooled water, each in lb/min.
(b) the rate of exergy destruction within the cooling tower, in Btu/s, for T_0 = 77°F.

Ignore kinetic and potential energy effects.

Known:
Steady state operating data are provided for a cooling tower.

Find:
Determine (a) the mass flow rates of the dry air and cooled water, and (b) the tower rate of exergy destruction.

Schematic and Known Data:



Engineering Model:
(1) The control volume shown operates at steady state.
(2) $\dot{W}_{cv} = \dot{Q}_{cv} = 0$ and the effects of kinetic and potential energy are negligible.
(3) Each liquid stream is regarded as a saturated liquid at the corresponding temperature.
(4) The moist air streams are treated as ideal gas mixtures.
(5) Let T_0 = 537°R.

Analysis:
(a) Mass rate balances at steady state follow

$\dot{m}_{a_3} = \dot{m}_{a_4} \equiv \dot{m}_a$

$\dot{m}_2 = \dot{m}_1 + \dot{m}_{v_3} - \dot{m}_{v_4} = \dot{m}_1 + \dot{m}_a(\omega_3 - \omega_4)$ (1)

An energy rate balance reduces to

$0 = \dot{m}_1 h_{f_1} - \dot{m}_2 h_{f_2} + \dot{m}_a\left(h_{a_3} + \omega_3 h_{g_3}\right) - m_a\left(h_{a_4} + \omega_4 h_{g_4}\right)$

Or, with Eq. (1)

$$0 = \dot{m}_1 h_{f_1} - [\dot{m}_1 + \dot{m}_a(\omega_3 - \omega_4)]h_{f_2} + \dot{m}_a\left(h_{a_3} + \omega_3 h_{g_3}\right) - m_a\left(h_{a_4} + \omega_4 h_{g_4}\right)$$

Solving for $\dot{m}_a$

$$\dot{m}_a = \frac{\dot{m}_1\left(h_{f_1} - h_{f_2}\right)}{\left(h_{a_4} + \omega_4 h_{g_4}\right) - \left(h_{a_3} + \omega_3 h_{g_3}\right) + (\omega_3 - \omega_4)h_{f_2}}$$

$$\dot{m}_a = \frac{\dot{m}_1\left(h_{f_1} - h_{f_2}\right)}{c_{p_a}(T_4 - T_3) + \omega_4 h_{g_4} - \omega_3 h_{g_3} + (\omega_3 - \omega_4)h_{f_2}} \qquad (2)$$

Where

$$\omega_3 = 0.622\frac{\phi_3 p_{g_3}}{p - \phi_3 p_{g_3}} = \frac{0.622(0.35)(0.6158)}{14.7 - (0.35)(0.6158)} = 0.009255\,\frac{\text{lb}_\text{v}}{\text{lb}_\text{a}}$$

$$\omega_4 = 0.622\frac{\phi_4 p_{g_4}}{p - \phi_4 p_{g_4}} = \frac{0.622(1)(0.9503)}{14.7 - (1)(0.9503)} = 0.04299\,\frac{\text{lb}_\text{v}}{\text{lb}_\text{a}}$$

$$\dot{m}_1 = \frac{(\text{AV})_1}{v_{f_1}} = \frac{275\,\frac{\text{ft}^3}{\text{min}}}{0.01621\,\frac{\text{ft}^3}{\text{lb}}} = 16965\,\frac{\text{lb}}{\text{min}}$$

Inserting values into Eq. (2)

$$\dot{m}_a = \frac{\left(16965\,\frac{\text{lb}}{\text{min}}\right)(88.00 - 58.07)\,\frac{\text{Btu}}{\text{lb}}}{\left(0.24\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)(100 - 86)^\circ\text{R} + (0.04299)\left(1105.0\,\frac{\text{Btu}}{\text{lb}}\right) - (0.009255)\left(1099.0\,\frac{\text{Btu}}{\text{lb}}\right) + (0.009255 - 0.04299)\left(58.07\,\frac{\text{Btu}}{\text{lb}}\right)}$$

$$\dot{m}_a = 13109\,\frac{\text{lb}}{\text{min}} \longleftarrow$$

Then, with Eq. (1)

$$\dot{m}_2 = 16965 + 13109(0.009255 - 0.04299) = 16523\,\frac{\text{lb}}{\text{min}} \longleftarrow$$

(b) The rate of exergy destruction can be evaluated in terms of entropy production rate: $\dot{\text{E}}_\text{d} = T_0\sigma_{\text{cv}}$, where from an entropy rate balance

$$\dot{\sigma}_{\text{cv}} = \dot{m}_2 s_f(T_2) - \dot{m}_1 s_f(T_1) + \dot{m}_a\left[s_a\left(T_4, p_{a_4}\right) + \omega_4 s_g(T_4)\right] - \dot{m}_a\left[s_a\left(T_3, p_{a_3}\right) + \omega_3 s_v\left(T_3, p_{v_3}\right)\right]$$

Further rearranging

$$\dot{\sigma}_{\text{cv}} = \dot{m}_2 s_f(T_2) - \dot{m}_1 s_f(T_1) + \dot{m}_a\left[\left(s_a\left(T_4, p_{a_4}\right) - s_a\left(T_3, p_{a_3}\right)\right) + \omega_4 s_g(T_4) - \omega_3 s_v\left(T_3, p_{v_3}\right)\right]$$

Recall

$p_{a_4} = p_4 - p_{g_4}$ and $s_v\left(T_3, p_{v_3}\right) = s_g(T_3) - \frac{\bar{R}}{M}(\ln\phi_3)$ (see Sec. 12.5.2)

Using steam table data $\left(\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)$ from Table A-2E

$s_{\text{f}}(T_1) = 0.1647$ $\quad s_{\text{f}}(T_2) = 0.1117$ $\quad s_{\text{g}}(T_3) = 2.0190$ $\quad s_{\text{g}}(T_4) = 1.9822$

Substituting values into $s_{\text{v}}\left(T_3, p_{\text{v}_3}\right) = s_{\text{g}}(T_3) - \frac{\bar{R}}{M}(\ln \phi_3)$

$$s_{\text{v}}\left(T_3, p_{\text{v}_3}\right) = s_{\text{g}}(T_3) - \frac{\bar{R}}{M}\ln \phi_3 = 2.0190 - \frac{1.986}{18.02}\ln 0.35 = 2.1347$$

Also

$$s_{\text{a}}\left(T_4, p_{\text{a}_4}\right) - s_{\text{a}}\left(T_3, p_{\text{a}_3}\right) = c_p \ln\frac{T_4}{T_3} - \frac{\bar{R}}{M_{\text{a}}}\ln\frac{p_{\text{a}_4}}{p_{\text{a}_3}}$$
$$= 0.24\left(\ln\frac{560}{546}\right) - \left(\frac{1.986}{28.97}\right)\ln\frac{14.7 - 0.9503}{14.7 - (0.35)(0.6158)} = 0.009645\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$$

Then

$$\dot{\sigma}_{\text{cv}} = \dot{m}_2 s_{\text{f}}(T_2) - \dot{m}_1 s_{\text{f}}(T_1) + \dot{m}_{\text{a}}\left[\left(s_a\left(T_4, p_{\text{a}_4}\right) - s_{\text{a}}\left(T_3, p_{\text{a}_3}\right)\right) + \omega_4 s_{\text{g}}(T_4) - \omega_3 s_{\text{v}}\left(T_3, p_{\text{v}_3}\right)\right]$$
$$\dot{\sigma}_{\text{cv}} = \left(16523\,\frac{\text{lb}}{\text{min}}\right)\left(0.1117\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right) - \left(16965\,\frac{\text{lb}}{\text{min}}\right)\left(0.1647\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)$$
$$+ \left(13109\,\frac{\text{lb}}{\text{min}}\right)\left[0.009645\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}} + (0.04299)\left(1.9822\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)\right.$$
$$\left. - (0.009255)\left(2.1347\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)\right] = 36.01\,\frac{\text{Btu}}{\text{min}\cdot°\text{R}}$$

Finally,

$$\dot{\text{E}}_{\text{d}} = T_0\dot{\sigma}_{\text{cv}} = (537°\text{R})\left(36.01\,\frac{\text{Btu}}{\text{min}\cdot°\text{R}}\right) = 19337\,\frac{\text{Btu}}{\text{min}}$$

PROBLEM 13.1

KNOWN: Ten grams of C_3H_8 burns with just enough O_2 for complete combustion.

FIND: Determine the mass of oxygen and the mass of products formed.

ANALYSIS: For complete combustion of C_3H_8 with the theoretical amount of O_2: $C_3H_8 + 5O_2 \rightarrow 3CO_2 + 8H_2O$

To determine the mass of O_2:

$$\left(\frac{5 \text{ mol } O_2}{1 \text{ mol } C_3H_8}\right)\left(\frac{32 \text{ g/mol}}{44.09 \text{ g/mol}}\right) = 3.629 \frac{g(O_2)}{g(C_3H_8)}$$

For $m_{C_3H_8} = 10g$

$$\left(3.629 \frac{g(O_2)}{g(C_3H_8)}\right) \times (10 \text{ g } C_3H_8) = 36.29 \text{ g}(O_2) \leftarrow m_{O_2}$$

For the combustion reaction

① $$m_{prod} = m_{C_3H_8} + m_{O_2} = 10 + 36.29 = 46.29 \text{ g(prod)} \leftarrow m_{prod}$$

---

1. Alternatively

$$\frac{3(44.01) + 8(18.02)}{1(44.09)} = 4.629 \frac{g(prod)}{g(C_3H_8)}$$

thus

$$m_{prod} = \left(4.629 \frac{g(prod)}{g(C_3H_8)}\right) \times (10 g\ C_3H_8) = 46.29 \text{ g (prod)}$$

PROBLEM 13.2

KNOWN: C_2H_6 burns completely with theoretical air.

FIND: Determine the air-fuel ratio on a (a) molar basis, and (b) mass basis.

ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) N_2 is inert.

ANALYSIS: For complete combustion of ethane with theoretical air

$$C_2H_6 + \alpha(O_2 + 3.76N_2) \rightarrow \beta CO_2 + \gamma H_2O + \delta N_2$$

Carbon : $2 = \beta$

Hydrogen, H_2: $3 = \gamma$

Oxygen, O_2: $\alpha = \beta + \frac{\gamma}{2} = 3.5$

Nitrogen, N_2: $\delta = \alpha(3.76) = 13.16$

Thus

$$C_2H_6 + 3.5(O_2 + 3.76N_2) \rightarrow 2CO_2 + 3H_2O + 13.16N_2$$

(a) $$\overline{AF} = \frac{(3.5)(4.76)\text{ kmol (air)}}{1\text{ kmol }(C_2H_6)} = 16.66 \longleftarrow \overline{AF}$$

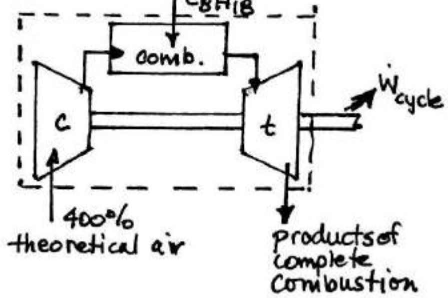
(b) $$AF = \overline{AF}\left(\frac{M_{air}}{M_{fuel}}\right) = 16.66\left(\frac{28.97}{30.07}\right) = 16.05 \longleftarrow AF$$

PROBLEM 13.3

KNOWN: A gas turbine burns C_8H_{18} completely with 400% of theoretical air.

FIND: Determine the amount of N_2 in the products, in kmol per kmol of fuel.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) The N_2 is inert.

ANALYSIS: For theoretical combustion

$$C_8H_{18} + \alpha_{th} O_2 \rightarrow 8CO_2 + 9H_2O \Rightarrow \alpha_{th} = 12.5$$

For $\alpha_{act} = 4\,\alpha_{th}$ (400% of theoretical air)

$$C_8H_{18} + 4(12.5)(O_2 + 3.76N_2) \rightarrow 8CO_2 + 9H_2O + 3(12.5)O_2 + \underbrace{4(12.5)(3.76)}_{188}N_2$$

$$n_{N_2} = 188 \frac{\text{kmol}(N_2)}{\text{kmol (fuel)}} \longleftarrow n_{N_2}$$

PROBLEM 13.4

A closed, rigid vessel initially contains a mixture of 40% CO and 60% O_2 on a mass basis. These substances react giving a final mixture of CO_2 and O_2. Determine the balanced reaction equation.

KNOWN: A mixture of 40% CO and 60% O_2 on a mass basis reacts to give a final mixture of CO_2 and O_2.

FIND: Determine the balanced reaction equation.

ANALYSIS: For 1 kg of mixture initially

	mf	m (kg)	M	n = m/M (kmol)
O_2	0.6	0.6	32	0.01875
CO	0.4	0.4	28.01	0.01428

$$\frac{n_{O_2}}{n_{CO}} = \frac{0.01875}{0.01428} = 1.313 \ \frac{\text{kmol}(O_2)}{\text{kmol}(CO)}$$

$$\Rightarrow \quad CO + 1.313 O_2 \rightarrow \alpha\, CO_2 + \beta O_2$$

$$C: \ 1 = \alpha$$
$$O: \ 1 + 2(1.313) = 2\alpha + 2\beta$$
$$\Rightarrow \beta = 0.813$$

Finally

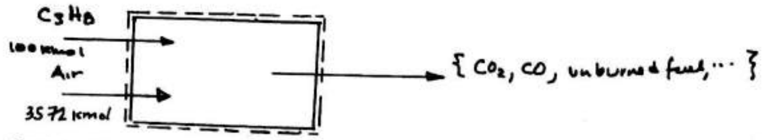
$$CO + 1.313 O_2 \rightarrow CO_2 + 0.813 O_2$$

PROBLEM 13.5

KNOWN: 100 kmole of C_3H_8 and 3572 kmol of air enter a furnace. CO_2, CO and unburned fuel are in the products of combustion.

FIND: Determine the percent excess or percent deficiency of air, as appropriate.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: 3.76 kmol of N_2 accompany each kmol of O_2 in air. N_2 is inert.

ANALYSIS: The data provided are for actual operation. Thus

$$\overline{AF} = \frac{3572}{100} = 35.72 \frac{\text{kmol (a)}}{\text{kmol (fuel)}}$$

The balanced equation for complete combustion with the theoretical amount of air is

$$C_3H_8 + 5(O_2 + 3.76 N_2) \rightarrow 3CO_2 + 4H_2O + 18.8 N_2$$

The theoretical air/fuel ratio is

$$(\overline{AF})_{theo} = \frac{5(4.76)}{1} = 23.8 \frac{\text{kmol (a)}}{\text{kmol (fuel)}}$$

Accordingly

$$\% \text{excess} = \left(\frac{35.72 - 23.8}{23.8}\right)(100) = 50\ \% \longleftarrow$$

PROBLEM 13.6

KNOWN: Three cases where C_3H_8 is burned with air.

FIND: Obtain the balanced chemical reaction for complete combustion.

ENGINEERING MODEL: 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert.

ANALYSIS: (a) Complete combustion with the theoretical amount of air.

$$C_3H_8 + 5(O_2+3.76N_2) \rightarrow 3CO_2 + 4H_2O + 18.8N_2$$

(b) Complete combustion with 20% excess air.

$$C_3H_8 + (5)(1.2)(O_2+3.76N_2) \rightarrow 3CO_2 + 4H_2O + \alpha O_2 + (5)(1.2)(3.76N_2)$$

$$O: (6)(2) = 6 + 4 + 2\alpha \Rightarrow \alpha = 1$$

$$\therefore\ C_3H_8 + 6(O_2+3.76N_2) \rightarrow 3CO_2 + 4H_2O + 1O_2 + 22.56N_2$$

(c) Combustion with 20% excess air. 90% of the fuel is burned.

$$C_3H_8 + 6(O_2+3.76N_2) \rightarrow 0.1C_3H_8 + \alpha CO_2 + \beta H_2O + \gamma O_2 + 22.56N_2$$

$$C: 3 = 0.3 + \alpha \Rightarrow \alpha = 2.7 \qquad O: 12 = 2(2.7) + (3.6) + 2\gamma \Rightarrow \gamma = 1.5$$

$$H: 8 = 0.8 + 2\beta \Rightarrow \beta = 3.6$$

$$\therefore\ C_3H_8 + 6(O_2+3.76N_2) \rightarrow 0.1C_3H_8 + 2.7CO_2 + 3.6H_2O + 1.5O_2 + 22.56N_2$$

PROBLEM 13.7

KNOWN: C_4H_{10} burns completely with air. The equivalence ratio is known.

FIND: Determine (a) the balanced reaction equation, and (b) the percent excess air.

ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) The N_2 is inert.

ANALYSIS:

(a) Theoretical combustion: $C_4H_{10} + \alpha_{th} O_2 \rightarrow 4CO_2 + 5H_2O \Rightarrow \alpha_{th} = 6.5$

The equivalence ratio is

$$\frac{(\overline{FA})_{actual}}{(\overline{FA})_{theoretical}} = 0.9 \Rightarrow (\overline{FA})_{actual} = 0.9(\overline{FA})_{theoretical}$$

$$(\overline{FA})_{theoretical} = \frac{1 \text{ kmol}(C_4H_{10})}{(6.5)(4.76) \text{ kmol(air)}} = 0.03232$$

$$(\overline{FA})_{actual} = (0.9)(0.03232) = 0.02909$$

$$(\overline{AF})_{actual} = \frac{1}{(\overline{FA})_{actual}} = 34.376 \frac{\text{kmol (air)}}{\text{kmol } (C_4H_{10})}$$

$$\therefore \alpha_{act} = \frac{34.376}{4.76} = 7.222$$

balanced equation

$$C_4H_{10} + 7.222(O_2 + 3.76N_2) \rightarrow 4CO_2 + 5H_2O + 0.722 O_2 + 27.15 N_2$$

(b) $$\% \text{ excess air} = \left(\frac{\alpha_{act} - \alpha_{th}}{\alpha_{th}}\right) \times 100 = 11.1\%$$

excess air

13.8 A natural gas mixture having a molar analysis 60% CH_4, 30% C_2H_6, 10% N_2 is supplied to a furnace where it burns completely with 20% excess air, Determine
(a) the balanced reaction equation.
(b) the air-fuel ratio, both on a molar and a mass basis.

KNOWN: A fuel mixture with a specified molar analysis burns completely with 20% excess air.

FIND: Determine (a) the balance reaction equation, (b) the air-fuel ratio on a molar and mass basis.

ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) N_2 is inert.

ANALYSIS: (a) On the basis of 1 mole of fuel mixture, the reaction equation for complete combustion with the theoretical amount of air is

$$(0.6\ CH_4 + 0.3\ C_2H_6 + 0.1\ N_2) + \alpha_{th}(O_2 + 3.76\ N_2) \rightarrow \beta\ CO_2 + \gamma\ H_2O + (\alpha_{th}3.76 + 0.1)\ N_2$$

C: $0.6 + (2)(0.3) = \beta$; $\beta = 1.2$
H_2: $(0.6)(2) + (0.3)(3) = \gamma$; $\gamma = 2.1$
O_2: $\alpha_{th} = \beta + \gamma/2 = 1.2 + (2.1)/2 = 2.25$

With 20% excess air, $\alpha_{act} = (1.2)\ \alpha_{th} = 2.7$. Thus, for the actual combustion

$$(0.6\ CH_4 + 0.3\ C_2H_6 + 0.1\ N_2) + 2.7(O_2 + 3.76\ N_2)$$
$$\rightarrow 1.2\ CO_2 + 2.1\ H_2O + (2.7 - 2.25)\ O_2 + [(2.7)(3.76) + 0.1]\ N_2$$

Thus, the balanced reaction equation for complete combustion with 20% excess air is

$$(0.6\ CH_4 + 0.3\ C_2H_6 + 0.1\ N_2) + 2.7(O_2 + 3.76\ N_2)$$

$$\rightarrow 1.2\ CO_2 + 2.1\ H_2O + 0.45\ O_2 + 10.252\ N_2 \qquad \leftarrow$$

(b) The air-fuel ratio on a molar basis is

$$\overline{AF} = \frac{(2.7)(4.76)}{1} = 12.852\ \frac{\text{kmol (air)}}{\text{kmol (fuel)}} \qquad \leftarrow$$

The air-fuel ratio on a mass basis is

$$M_{fuel} = (0.6)(16.04) + (0.3)(30.07) + (0.1)(28.01) = 21.45$$

So

$$\overline{AF} = (\overline{AF})\left({}^{M_{air}}/_{M_{fuel}}\right) = (12.852)\left({}^{(28.97)}/_{(21.45)}\right) = 17.368\frac{\text{kg (air)}}{\text{kg (fuel)}} \qquad \leftarrow$$

PROBLEM 13.9

A fuel mixture with the molar analysis 70% CH_4, 20% CO, 5% O_2, and 5% N_2 burns completely with 20% excess air. Determine

(a) the balanced reaction equation.
(b) the air–fuel ratio, both on a molar and mass basis.

KNOWN: A fuel mixture with a specified molar analysis burns completely with 20% excess air.

FIND: Determine (a) the balanced reaction equation, (b) the air-fuel ratio on a molar and a mass basis.

ENGR. MODEL: 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert.

ANALYSIS: On the basis of 1 mole of fuel mixture, the reaction equation for complete combustion with the theoretical amount of air is

$$(0.7\,CH_4 + 0.20\,CO + 0.05\,O_2 + 0.05\,N_2) + \alpha(O_2 + 3.76\,N_2) \rightarrow \beta\,CO_2 + \gamma\,H_2O + (3.76\alpha + 0.05)N_2$$

C: $0.7 + 0.2 = \beta$, $\beta = 0.9$ O: $0.2 + 0.10 + 2\alpha = 2(.9) + 1.4$, $\alpha = 1.45$

H: $2.8 = 2\gamma$, $\gamma = 1.4$

Thus

$$(0.7CH_4 + 0.2\,CO + 0.05\,O_2 + 0.05\,N_2) + 1.45(O_2 + 3.76N_2) \rightarrow 0.9CO_2 + 1.4\,H_2O + 5.502\,N_2$$

Thus, the reaction for complete combustion with 20% excess air is

$$(0.7\,CH_4 + 0.2\,CO + 0.05O_2 + 0.05N_2) + (1.2)(1.45)(O_2 + 3.76N_2) \rightarrow .9CO_2 + 1.4H_2O + \alpha\,O_2 + [(1.74)(3.76) + 0.05]N_2$$

O: $0.2 + 0.1 + 2(1.74) = 1.8 + 1.4 + 2\alpha$, $\alpha = 0.29$

Accordingly

$$(0.7CH_4 + 0.2\,CO + 0.05\,O_2 + 0.05N_2) + 1.74(O_2 + 3.76N_2) \rightarrow 0.9\,CO_2 + 1.4\,H_2O + 0.29\,O_2 + 6.59\,N_2$$

(b) The air-fuel ratio is

$$\overline{AF} = \frac{(1.74)(4.76)}{1} = 8.28\ \frac{\text{kmol (air)}}{\text{kmol (fuel)}}$$

The fuel molecular weight is

$$M_f = (0.7)(16.04) + 0.2(28.01) + 0.05(32) + 0.05(28.01) = 19.831$$

So

$$AF = (\overline{AF})\left(\frac{M_a}{M_f}\right) = (8.28)\left(\frac{28.97}{19.831}\right) = 12.1\ \frac{\text{kg(air)}}{\text{kg(fuel)}}$$

PROBLEM 13.10

KNOWN: A gas mixture with a known molar analysis reacts to form specified products.

FIND: Determine the amount of each product in kg per kg of mixture.

SCHEMATIC & GIVEN DATA:

Initially: 0.25 H_2, 0.25 CO, 0.50 O_2

Finally: α CO_2, β H_2O, γ O_2

ANALYSIS: $0.25H_2 + 0.25CO + 0.5\,O_2 \rightarrow \alpha CO_2 + \beta H_2O + \gamma O_2$

H_2: $0.25 = \beta$

C : $0.25 = \alpha$

O_2: $\frac{0.25}{2} + 0.5 = \alpha + \frac{\beta}{2} + \gamma \Rightarrow \gamma = 0.25$

$$0.25H_2 + 0.25CO + 0.5O_2 \rightarrow 0.25CO_2 + 0.25H_2O + 0.25O_2$$

Converting the analysis of the initial mixture to a mass basis

	y_i	$\times$	M_i	=	m_i	mf_i
H_2	0.25	$\times$	2.016	=	0.5040	0.0214
CO	0.25	$\times$	28.01	=	7.0025	0.2979
O_2	0.5	$\times$	32	=	16.0000	0.6807
	1.0 kmol				$\underbrace{23.5065\ \frac{kg}{kmol\,(mix)}}$	1.0000

$$\frac{1}{23.5065} = 0.04254\ \frac{kmol\,(mix)}{kg\,(mix)}$$

$$CO_2:\ \left(0.25\ \frac{kmol\,(CO_2)}{kmol\,(mix)}\right)\left(0.04254\ \frac{kmol\,(mix)}{kg\,(mix)}\right)\left(44.01\ \frac{kg\,(CO_2)}{kmol\,(CO_2)}\right)$$

$$= 0.468\ kg \longleftarrow m_{CO_2}$$

$$H_2O:\ (0.25)(0.04254)(18.02) = 0.1916\ kg \longleftarrow m_{H_2O}$$

$$O_2:\ (0.25)(0.04254)(32) = 0.3403\ kg \longleftarrow m_{O_2}$$

PROBLEM 13.11

13.11 A natural gas with the molar analysis 94.4% CH_4, 3.4% C_2H_6, 0.6% C_3H_8, 0.5% C_4H_{10}, 1.1% N_2 burns completely with 20% excess air in a reactor operating at steady state. If the molar flow rate of the fuel is 0.1 kmol/h, determine the molar flow rate of the air, in kmol/h.

KNOWN: A specified fuel mixture burns completely with 20% excess air.

FIND: Determine the molar flow rate of the air, if the molar flow rate of the fuel is 0.1 kmol/h.

ENGR. MODEL: 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert.

ANALYSIS: On the basis of 1 mole of fuel mixture, complete combustion with the theoretical amount of air is

$$[0.944\,CH_4 + 0.034\,C_2H_6 + 0.006\,C_3H_8 + 0.005\,C_4H_{10} + 0.011\,N_2] +$$
$$\alpha[O_2 + 3.76N_2] \rightarrow \beta CO_2 + \gamma H_2O + [3.76\alpha + 0.011]N_2$$

C: $0.944 + 2(0.034) + 3(0.006) + 4(0.005) = \beta \;,\; \beta = 1.05$

H: $4(0.944) + 6(0.034) + 8(0.006) + 10(0.005) = 2\gamma \;,\; \gamma = 2.039$

O: $2\alpha = 2(1.05) + 2.039 \;,\; \alpha = 2.0695$

Combustion with 20% excess air means that the molar flow rate is

$$\overline{AF} = \frac{1.2(2.0695)(4.76)}{1} = 11.82 \frac{\text{kmol (air)}}{\text{kmol (fuel)}}$$

Thus, with $\dot{n}_{fuel} = 0.1$ kmol/h

$$\dot{n}_{air} = 1.182 \text{ kmol (air)/h}$$

13.12 A natural gas fuel mixture has the molar analysis shown below. Determine the molar analysis of the products for complete combustion with 70% excess dry air.

Fuel	CH_4	H_2	NH_3
y_i	25%	30%	45%

KNOWN: A fuel mixture with known composition burns completely with 70% excess air.

FIND: Determine the mole fraction of each product.

SCHEMATIC AND GIVEN DATA:
Fuel Composition: 25% CH_4, 30% H_2, and 45% NH_3
Excess Air = 70%

ENGINEERING MODEL:
1. Each mole of oxygen in the combustion air is accompanied by 3.76 moles of nitrogen, which is inert.
2. Products of complete combustion are CO_2, H_2O, O_2, and N_2.

ANALYSIS: To determine the mole fraction of each product when fuel is burned with 70% excess air, *first determine the amount of air required for complete combustion with theoretical* air. For 1 kmol of fuel

$$(0.25CH_4 + 0.30H_2 + 0.45NH_3) + a(O_2 + 3.76N_2) \rightarrow bCO_2 + cH_2O + dN_2$$

Applying conservation of mass to carbon, hydrogen, oxygen, and nitrogen, respectively

C: $0.25 = b$

H: $4(0.25) + 2(0.30) + 3(0.45) = 2c \rightarrow c = 1.475$

O: $2a = 2b + c = 2(0.25) + 1.475 \rightarrow a = 0.9875$

N: $0.45 + 2(3.76)a = 2d \rightarrow d = 3.938$

The balanced chemical equation for complete combustion of the fuel with *theoretical air* is

$$0.25CH_4 + 0.30H_2 + 0.45NH_3 + 0.9875(O_2 + 3.76N_2) \rightarrow 0.25CO_2 + 1.475H_2O + 3.938N_2$$

For complete combustion of fuel with 70% excess air

$$0.25CH_4 + .30H_2 + .45NH_3 + (1.70)(0.9875)(O_2 + 3.76N_2) \rightarrow aCO_2 + bH_2O + cO_2 + dN_2$$

Applying conservation of mass to carbon, hydrogen, oxygen, and nitrogen, respectively

C: $0.25 = a$

H: $4(0.25) + 2(0.30) + 3(0.45) = 2b \rightarrow b = 1.475$

O: $2(0.9875)(1.70) = 2a + b + 2c \rightarrow c = 0.69125$

N: $0.45 + 2(3.76)(0.9875)(1.70) = 2d \rightarrow d = 6.5371$

The balanced chemical equation for complete combustion of the fuel with *70% excess air* is

$$0.25CH_4 + 0.30H_2 + 0.45NH_3 + 1.67875(O_2 + 3.76N_2) \rightarrow 0.25CO_2 + 1.475H_2O + 0.69125O_2 + 6.5371N_2$$

The calculation for the mole fraction (y_i) for each product is given in the following table

Product	n_i	$y_i = n_i/n_{total}$
CO_2	0.25	0.25/8.95338 = 0.028 = **2.8%**
H_2O	1.475	1.475/8.95338 = 0.165 = **16.5%**
O_2	0.69125	0.69125/8.95338 = 0.077 = **7.7%**
N_2	6.5371	6.5371/8.95338 = 0.730 = **73.0%**
$n_{total} = \sum n_i =$	**8.95335**	

PROBLEM 13.13

Coal with the mass analysis 79.2% C, 5.7% H_2, 10% O_2, 1.5% N_2, 0.6% S, 3% noncombustible ash burns completely with the theoretical amount of air. Determine

(a) the air–fuel ratio on a mass basis.
(b) the amount of SO_2 produced, in kg per kg of coal.

KNOWN: Coal with a specified mass analysis burns completely with the theoretical amount of air.

FIND: Determine the air-fuel ratio and the amount of SO_2 produced, each on a kg per kg of coal basis.

ENGR. MODEL: 1. 3.76 moles of N_2 accompany each mole of O_2 in the air. 2. N_2 is inert. 3. Molecular weights from Table A-1 are rounded. 4. The ash is noncombustible.

Analysis: On the basis of 100 kg of coal,

①

	m_i (kg)	M	n_i (kmol, per 100 kg of coal)
C	79.2	12	6.60
H_2	5.7	2	2.85
O_2	10.0	32	0.31
N_2	1.5	28	0.05
S	0.6	32	0.02
Ash	3.0		
	100.0		

(a)

$$6.6\,C + 2.85 H_2 + 0.31\,O_2 + 0.05\,N_2 + 0.02\,S + \omega[O_2 + 3.76 N_2] \rightarrow \alpha\,CO_2 + \beta H_2O + \gamma SO_2 + \delta N_2$$

C: $6.6 = \alpha$ S: $0.02 = \gamma$

H_2: $2.85 = \beta$ O: $2(0.31) + 2\omega = 2(6.6) + 2.85 + 2(0.2)$, $\omega = 7.74$

$$AF = \frac{[(7.74)(4.76)\,\text{kmol(air)}]\left[28.97 \frac{\text{kg}}{\text{kmol}}\right]}{100\ \text{kg (coal)}} = 10.67\ \frac{\text{kg(air)}}{\text{kg (coal)}}$$

(b)

$$m_{SO_2} = \frac{0.02\ \text{kmol}(SO_2)\left[64 \frac{\text{kg}(SO_2)}{\text{kmol}(SO_2)}\right]}{100\ \text{kg(coal)}} = 0.013\ \frac{\text{kg}(SO_2)}{\text{kg (coal)}}$$

---

1. Molecular weight values have been rounded.

PROBLEM 13.14

A coal sample has a mass analysis of 80.4% carbon, 3.9% hydrogen (H_2), 5.0% oxygen (O_2), 1.1% nitrogen (N_2), 1.1% sulfur, and the rest is noncombustible ash. For complete combustion with 120% of the theoretical amount of air, determine (a) the air–fuel ratio on a mass basis, (b) the amount of SO_2 produced, in kg per kg of coal.

KNOWN: Coal having a specified mass analysis burns completely with 120% theoretical air.

FIND: Determine the AF ratio on a mass basis and the amount of SO_2 produced, in kg per kg of coal.

ENGINEERING MODEL:
1. 3.76 moles of N_2 accompany each mole of O_2 in the air.
2. N_2 is inert.
3. The ash is noncombustible

ANALYSIS: On the basis of 100 kg of coal, the amounts present, in kmol, are

	m_i	M_i	$n_i = m_i/M_i$
C	80.4	12.01	6.694
H_2	3.9	2.016	1.935
O_2	5.0	32.00	0.156
N_2	1.1	28.01	0.039
S	1.1	32.06	0.034
Ash	8.5	---	---

For complete combustion with theoretical air

$$[6.694\,C + 1.935\,H_2 + 0.156\,O_2 + 0.039\,N_2 + 0.034\,S + \text{ASH}] + \alpha[O_2 + 3.76 N_2]$$
$$\longrightarrow \beta CO_2 + \gamma SO_2 + \omega H_2O + \lambda N_2 + \text{ASH}$$

Balancing:

$C: \quad 6.694 = \beta$

$H_2: \quad 1.935 = \omega$

$S: \quad 0.034 = \gamma$

$O: \quad 2(0.156) + 2\alpha = 2\beta + 2\gamma + \omega$

$\Rightarrow 2\alpha = 2(6.694) + 2(0.034) + 1.935 - 0.312$

$\Rightarrow \alpha = 7.54$

(a) On the above basis, the reaction requires (7.54×4.76) kmol of air per 100 kg of coal. Thus

$$(AF)_{theo} = \frac{(7.54 \times 4.76)(28.97)\,kg}{100\,kg} = 10.4\ \frac{kg(a)}{kg(coal)}$$

For 120% theoretical air

$$(AF) = (1.2)(10.4) = 12.48\ \frac{kg(a)}{kg(coal)}$$

(b) Excess air does not affect the sulfur balance. Accordingly

$$\frac{m_{SO_2}}{m_{coal}} = \frac{0.034\ kmol(SO_2)}{100\ kg(coal)}\left|\frac{64.06\ kg(SO_2)}{kmol(SO_2)}\right|$$

$$= 0.022\ kg/kg(coal)$$

PROBLEM 13.15

KNOWN: Dried feedlot manure with the mass analysis below burns completely with 120% of theoretical air.

{42.7% C, 5.5% H_2, 31.3% O_2, 2.4% N_2, 0.3% S, 17.8% Ash}

FIND: Determine (a) the balanced reaction equation, and (b) the air-fuel ratio on a mass basis.

ENGINEERING MODEL: (1) 3.76 kmol of N_2 accompany each kmol of O_2 in the air, and the N_2 is inert. (2) The ash is non-combustible.

ANALYSIS: (a) On the basis of 100 kg of manure, the amounts of the substances present, are respectively

	m_i (kg)	M_i	$n_i = m_i/M_i$ (kmol)
C:	42.7	12	3.558
H_2:	5.5	2.016	2.728
O_2:	31.3	32	0.978
N_2:	2.4	28.01	0.0857
S:	0.3	32.06	0.00936
Ash:	17.8	---	---
	100.0 kg		

For complete combustion with the theoretical amount of air

$$(3.558\,C + 2.728\,H_2 + 0.978\,O_2 + 0.0857\,N_2 + 0.00936\,S + Ash) + \alpha_{th}(O_2 + 3.76\,N_2)$$
$$\rightarrow \beta\,CO_2 + \gamma\,H_2O + \delta\,SO_2 + \epsilon\,N_2 + Ash$$

C: $3.558 = \beta$

H: $2(2.728) = 2\gamma \Rightarrow \gamma = 2.728$

S: $0.00936 = \delta$

O: $2(0.978) + 2\alpha_{th} = 2(3.558) + 2.728 + 2(0.00936) \Rightarrow \alpha_{th} = 3.958$

Then, for complete combustion with 120% of theoretical air

$$(3.558\,C + 2.728\,H_2 + 0.978\,O_2 + 0.0857\,N_2 + 0.00936\,S + Ash) + (1.2)(3.958)(O_2 + 3.76\,N_2)$$
$$\rightarrow 3.558\,CO_2 + 2.728\,H_2O + 0.00936\,SO_2 + 17.944\,N_2 + Ash \quad \longleftarrow \text{(a)}$$

(b) The actual reaction requires (1.2)(3.958)(4.76) kmol of air per 100 kg of manure. Thus

$$AF = \frac{(1.2)(3.958)(4.76)(28.97)}{100} = 6.55\ \frac{\text{kg(air)}}{\text{kg(manure)}} \quad \longleftarrow \text{AF}$$

PROBLEM 13.16

KNOWN: Coal with the mass analysis below burns completely with the theoretical amount of air.

$\{71.1\%\,C, 5.1\%\,H_2, 9.0\%\,O_2, 1.4\%\,N_2, 5.8\%\,S, 7.6\%\,Ash\}$

FIND: Determine (a) the amount of SO_2 produced, in kg per kg of coal, and (b) the air-fuel ratio on a mass basis.

ENGINEERING MODEL: (1) 3.76 kmol of N_2 accompany each kmol of O_2 in the air, and the N_2 is inert. (2) The ash is non-combustible.

ANALYSIS: On the basis of 100 kg of coal, the amounts of the substances present are, respectively

	m_i (kg)	M_i	$n_i = m_i/M_i$ (kmol)
C:	71.1	12	5.925
H_2:	5.1	2.016	2.530
O_2:	9.0	32	0.281
N_2:	1.4	28.01	0.050
S:	5.8	32.06	0.181
Ash:	7.6	----	----
	100 kg		

For complete combustion with the theoretical amount of air

$$(5.925C + 2.530H_2 + 0.281O_2 + 0.050N_2 + 0.181S + Ash) + \alpha_{th}(O_2 + 3.76N_2)$$
$$\rightarrow \beta CO_2 + \gamma H_2O + \delta SO_2 + \epsilon N_2$$

C: $5.925 = \beta$

H_2: $\gamma = 2.530$

S: $0.181 = \delta$

O: $(0.281)2 + \alpha_{th}(2) = (5.925)2 + 2.530 + (0.181)2 \Rightarrow \alpha_{th} = 7.09$

N: $(0.05)2 + (7.09)(3.76)(2) = \epsilon \cdot 2 \Rightarrow \epsilon = 26.71$

(a) The amount of SO_2 produced is 0.181 kmol per 100 kg of coal. Thus

$$m_{SO_2} = \frac{(0.181)(64.06)}{100} = 0.116 \ \frac{kg\,(SO_2)}{kg\,(coal)} \longleftarrow m_{SO_2}$$

(b) The amount of air required is (7.09)(4.76) kmol per 100 kg of coal. Thus

$$AF = \frac{(7.09)(4.76)(28.97)}{100} = 9.777 \ \frac{kg\,(air)}{kg\,(coal)} \longleftarrow AF$$

PROBLEM 13.17

Dodecane ($C_{12}H_{26}$) burns completely with 150% of theoretical air. Determine

(a) the air–fuel ratio on a molar and mass basis.

(b) the dew point temperature of the combustion products, in °C, when cooled at 1 atm.

KNOWN: $C_{12}H_{26}$ burns completely with 150% of theoretical air.

FIND: Determine (a) the air-fuel ratio on a molar and mass basis.
(b) the dew point temperature at 1atm.

ENGR. MODEL: 3.76 kmol of N_2 accompany each kmol of O_2 in the combustion air.
N_2 is inert.

ANALYSIS: (a) For complete combustion with the theoretical amount of air

$$C_{12}H_{26} + \alpha(O_2 + 3.76N_2) \rightarrow \beta CO_2 + \gamma H_2O + 3.76\alpha N_2$$

C: $12 = \beta$ O: $2\alpha = 2(12) + 13$, $\alpha = 18.5$

H: $26 = 2\gamma$, $\gamma = 13$

Then, complete combustion with 150% of the theoretical amount of air

$$C_{12}H_{26} + (1.5)(18.5)[O_2 + 3.76N_2] \rightarrow 12CO_2 + 13H_2O + \gamma O_2 + \underbrace{(1.5)(18.5)(3.76)}_{104.34}N_2$$

O: $2(1.5)(18.5) = 24 + 13 + 2\gamma$, $\gamma = 9.25$

Accordingly,

$$\overline{AF} = \frac{(1.5)(18.5)(4.76)}{1} = 132.09 \frac{\text{kmol (air)}}{\text{kmol (fuel)}} \quad \text{(a)}$$

$$AF = \overline{AF}\left(\frac{M_{air}}{M_{fuel}}\right) = \frac{(132.09)(28.97)}{\underbrace{(12)(12.01) + 26(1.008)}_{170.328}} = 22.47 \frac{\text{kg (air)}}{\text{kg (fuel)}}$$

(b) The partial pressure of the water in the combustion products is $p_v = y_v p$, where

$$y_v = \frac{13}{12+13+9.25+104.34} = \frac{13}{138.59} = 0.0938.$$

So,

$$p_v = (0.0938)(1.01325 \text{ bar}) = 0.09504 \text{ bar}$$

Then, from Table A-2

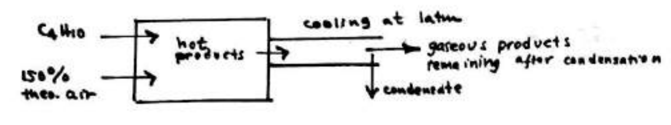
$T_{dew\ pt} \approx 44.8°C$ (b)

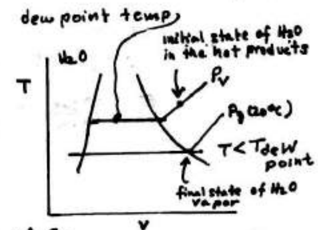
PROBLEM 13.18

KNOWN: C_4H_{10} burns completely with 150% of theoretical air. The products of combustion are cooled to temperature T at 1 atm, $20 \le T \le 60°C$.

FIND: Plot the amount of water vapor condensed, in kmol per kmol of fuel.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) The gaseous products of combustion can be modeled as an ideal gas.

ANALYSIS: Complete combustion of C_4H_{10} with the theoretical amount of air is described by

$$C_4H_{10} + 6.5(O_2 + 3.76N_2) \rightarrow 4CO_2 + 5H_2O + (6.5)(3.76)N_2$$

Complete combustion with 150% of theoretical air is then

$$C_4H_{10} + (1.5)(6.5)(O_2 + 3.76N_2) \rightarrow 4CO_2 + 5H_2O + 3.25O_2 + \underbrace{(1.5)(6.5)(3.76)}_{36.66}N_2$$

Condensation occurs when the combustion products are cooled below the dew point temperature. First, evaluate the partial pressure of water in the combustion products:

$$p_v = y_v p = \left(\frac{5}{4+5+3.25+36.66}\right)(1.01325\text{ bar}) = 0.10358\text{ bar}$$

Interpolation in Table A-2 gives $T_{dew\ pt} \approx 46.5°C$. Accordingly, the amount of condensate is zero until the temperature T is less than 46.5°C.

According to the model introduced in Chap. 12, the gaseous products remaining after the products have been cooled to $T < T_{dew\ pt.}$ would have saturated water vapor at temperature T. That is, the partial pressure of water vapor in this mixture would equal $p_g(T)$. The partial pressure is given by

$$p_v = \left(\frac{n_v}{n_v + n_{dry}}\right)p \qquad (1)$$

where n_{dry} accounts for the moles of "dry" products: CO_2, O_2, and N_2. That is, $n_{dry} = 4 + 3.25 + 36.66 = 43.91$. Accordingly, with p = 1 atm and $p_v = p_g(T)$, Eq. (1) becomes

$$p_g(T) = \left[\frac{n_v}{n_v + 43.91}\right](1.01325) \Rightarrow n_v = \frac{p_g(T)(43.91)}{1.01325 - p_g(T)} \text{ . Amount condensed} = 5 - n_v. \qquad (2)$$

Sample Calculation: $T = 20°C$, $p_g = 0.02339$ bar, $n_v = 1.038$, Amt. Condensed = 3.962,

The data for the required plots are obtained using IT, as follows:

PROBLEM 13.18 (Cont'd.) - Page 2

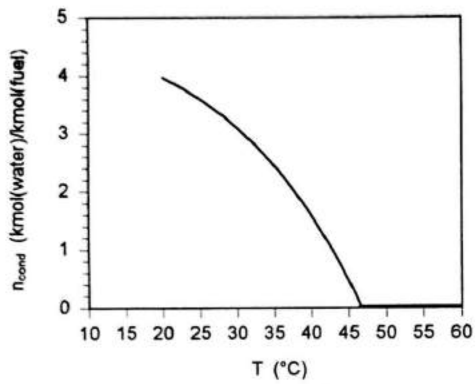
IT Code

```
T = 20  // °C
pv = 0.10358  // bar
Tdp = Tsat_P("Water/Steam", pv)
ncond = 5 - nv
nv  = pg * 43.41 / (1.01325 - pg)
pg = Psat_T("Water/Steam", T)
```

IT Results for T = 20°C

T_{dp} = 46.5°C
n_v = 1.026 bar
pg = 0.02339 bar
n_{cond} = 3.974 kmol(water)/kmol(fuel)

PLOT:



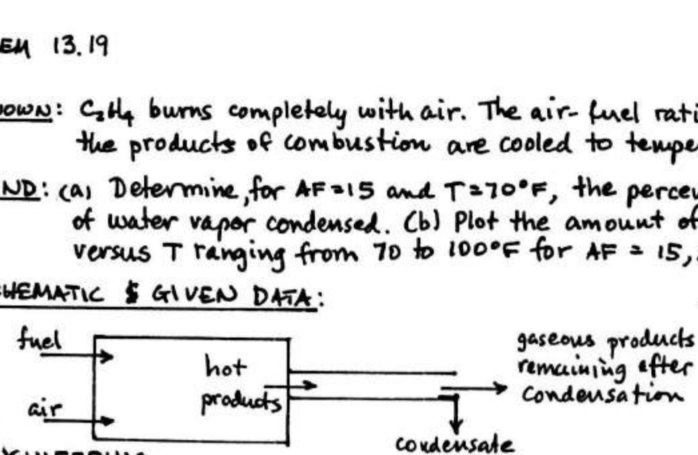
Note that condensation begins at the dew point temperature and increases as temperature is lowered further, as expected.

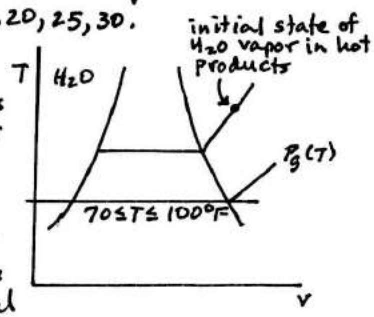
PROBLEM 13.19

KNOWN: C_2H_4 burns completely with air. The air-fuel ratio on a mass basis is AF, and the products of combustion are cooled to temperature T at 1 atm.

FIND: (a) Determine, for AF = 15 and T = 70°F, the percent excess air and the amount of water vapor condensed. (b) Plot the amount of water vapor condensed versus T ranging from 70 to 100°F for AF = 15, 20, 25, 30.

SCHEMATIC & GIVEN DATA:





ENGINEERING MODEL: (1) 3.76 lbmol of N_2 accompany each lbmol of O_2 in the air, and the N_2 is inert. (2) The gaseous products of combustion can be modeled as an ideal gas mixture.

ANALYSIS: (a) Complete combustion with the theoretical amount of air is described by

$$C_2H_4 + \alpha_{th}(O_2 + 3.76\ N_2) \rightarrow 2\,CO_2 + 2H_2O + \alpha_{th}(3.76)N_2 \Rightarrow \alpha_{th} = 3 \qquad (1)$$

For complete combustion with excess air

$$C_2H_4 + \alpha(O_2 + 3.76N_2) \rightarrow 2CO_2 + 2H_2O + (\alpha - \alpha_{th})O_2 + \alpha(3.76)N_2 \qquad (2)$$

For AF = 15

$$\overline{AF} = AF\left(\frac{M_{fuel}}{M_{air}}\right) = (15)\left(\frac{28.05}{28.97}\right) = 14.524\ \frac{lbmol(air)}{lbmol(fuel)}$$

Thus

$$\alpha = \frac{\overline{AF}}{4.76} = \frac{14.524}{4.76} = 3.0513$$

Accordingly

$$\%\ excess = \left(\frac{\alpha - \alpha_{th}}{\alpha_{th}}\right)100 = \left(\frac{3.0153 - 3}{3}\right)100 = 1.71\% \longleftarrow \%\ \text{Excess air}$$

Now, the actual reaction equation is

$$C_2H_4 + 3.0513(O_2 + 3.76N_2) \rightarrow 2CO_2 + 2H_2O + 0.0513\,O_2 + 11.473\,N_2$$

According to the model introduced in Chap. 12, the gaseous products remaining after the products have been below their dew point would have saturated vapor at T present. In this case, the partial pressure of the water vapor remaining would be $p_g(T) = p_g(70°F) = 0.3632\ lbf/in^2$. This pressure can be expressed as

$$p_v = \left(\frac{n_v}{n_v + n_{dry}}\right)p \qquad (3)$$

where "dry" refers to the products CO_2, O_2, and N_2. That is $n_{dry} = 2 + 0.0513 + 11.473 = 13.524$. With these results, Eq. (3) becomes

$$0.3632 = \left(\frac{n_v}{n_v + 13.524}\right)(14.696) \Rightarrow n_v = 0.3427\ lbmol\,(vapor)/lbmol\,(fuel)$$

Since 2 lbmol of H_2O are formed on combustion, the amount condensed is

$$m_{cond} = (2 - 0.3427)\,lbmol\ \frac{18.02\ lb}{lb\ mol} = 29.86\ lb\,(water)\ per\ lbmol\ of\ fuel \longleftarrow m_{cond}$$

PROBLEM 13.19 (Cont'd.) - Page 2

(b) The data for the required plot are obtained using IT, as follows:

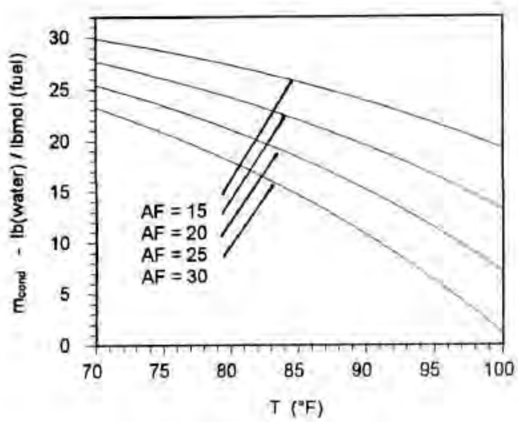
IT Code

```
T = 70 // °F
AF = 15 // kg(air) / kg(fuel)
alpha = AF * (28.05 / 28.97) / 4.76
pctXS = ((alpha - 3) / 3) * 100
ndry = 2 + (alpha - 3) + alpha * 3.76
pv = Psat_T("Water/Steam", T)
pv = (nv / (nv + ndry)) * 14.696
mcond = (2 - nv) * 18.02 // lb(water) / lbmol(fuel)
```

IT Results for AF = 15, T = 70°F

α = 3.051
% excess air = 1.706 %
n_{dry} = 13.52 lbmol(dry prod.)/ lbmol(fuel)
p_v = 0.3632 lbf/in.²
n_v 0.3427 lbmol(water vap.)/lbmol(fuel)
m_{cond} = 29.86 lb(water)/lbmol(fuel)

PLOT:



With higher AF (more air), there is more "dry" gas in the products. By Eq. (3) we see that n_v must increase as well. Thus, for fixed T there is less condensate formed as AF increases. For fixed AF, cooling to lower values of T causes more condensate to be formed, as expected.

13.20 A gaseous fuel mixture with a molar analysis of 70% CH_4, 10% H_2, 12% N_2, 3% O_2, and 5% CO_2 burns completely with moist air to form gaseous products at 1 atm consisting of CO_2, H_2O, and N_2 only. The humidity ratio of the moist air is 0.01 kg H_2O/kg dry air. Determine the dew point temperature of the products, in °C.

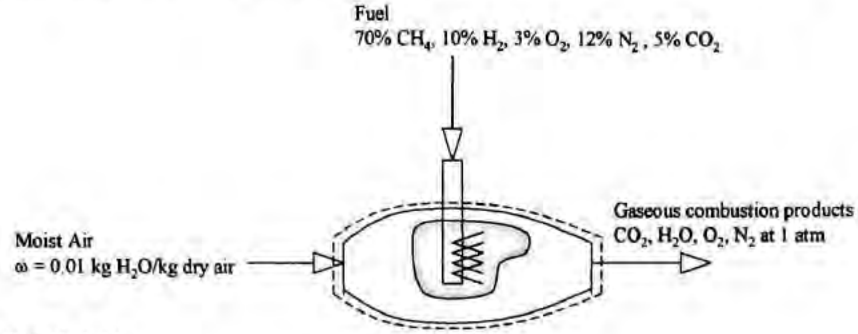
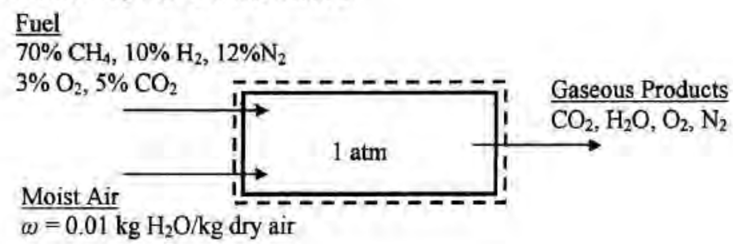


Fig. P13.20

KNOWN: A fuel mixture with known composition burns completely in moist air with known humidity ratio.

FIND: Determine the dew point temperature of the products.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

1. Each mole of oxygen in the combustion dry air is accompanied by 3.76 moles of nitrogen, which is inert.

ANALYSIS: The dew point temperature is the saturation temperature that corresponds to the partial pressure of the water vapor in the products

$$T_{dp} = T_{sat} @ p_v$$

Problem 13.20 (Continued) – Page 2

To determine the partial pressure of the water vapor in the products, the number of moles of each product is required from the balanced chemical equation. From the humidity ratio, determine the number of moles of water vapor for each 4.76 kmol of dry air in the reactant moist air.

$$n_{H_2O} = \frac{m_{H_2O}}{M_{H_2O}} = \frac{\omega\, m_{air}}{M_{H_2O}} = \frac{\omega\left(n_{air} M_{air}\right)}{M_{H_2O}}$$

$$n_{H_2O} = \frac{\left(0.01\,\frac{\text{kg H}_2\text{O}}{\text{kg air}}\right)\left(28.97\,\frac{\text{kg air}}{\text{kmol air}}\right)(4.76\text{ kmol air})}{18.02\,\frac{\text{kg H}_2\text{O}}{\text{kmol H}_2\text{O}}} = 0.0765\text{ kmol H}_2\text{O}$$

For 1 kmol of fuel

$$0.70CH_4 + 0.10H_2 + 0.12N_2 + 0.03O_2 + 0.05CO_2 + a(O_2 + 3.76N_2 + 0.0765H_2O)$$

$$\rightarrow bCO_2 + cH_2O + dN_2$$

Applying conservation of mass to carbon, hydrogen, oxygen, and nitrogen, respectively

C: $0.70 + 0.05 = b \rightarrow b = 0.75$

H: $4(0.70) + 2(0.10) + 2(0.0765)a = 2c \rightarrow c = 1.50 + 0.0765a$

O: $2(0.03) + 2(0.05) + 2a + 0.0765a = 2b + c$
$0.16 + 2.0765a = 2(0.75) + c \rightarrow a = 0.64532 + 0.48158c$

Substituting for c from the H balance:
$a = 0.64532 + 0.48158(1.50 + 0.0765a) \rightarrow a = 1.42$

Solving for c from the H balance:
$c = 1.50 + 0.0765(1.42) = 1.60863$

N: $2(0.12) + 2(3.76)a = 2d = \rightarrow d = 5.4592$

The balanced chemical equation for complete combustion of the fuel in *moist air* is

$$(0.70CH_4 + 0.10H_2 + 0.12N_2 + 0.03O_2 + 0.05CO_2) + 1.42(O_2 + 3.76N_2 + 0.0765H_2O)$$

$$\rightarrow 0.75CO_2 + 1.60863H_2O + 5.4592N_2$$

The calculation for the mole fraction (y_{H_2O}) for water in the products is given in the following table

Product	n_i	$y_i = n_i/n_{total}$
CO_2	0.75	
H_2O	1.60863	1.60863/7.81783 = 0.206
N_2	5.4592	
$n_{total} = \sum n_i =$	**7.81783**	

The partial pressure of the water vapor in the products is

$$p_{H_2O} = y_{H_2O}\,p = 0.206(1\text{ atm}) = (0.206\text{ atm})[1.01325\text{ bar}/1\text{ atm}] = 0.209\text{ bar}$$

Interpolating in Table A-3 for the saturation temperature that corresponds to $p_{H_2O} = 0.209$ bar

$$T_{sat} @ p_{H_2O} = T_{dp} \approx \underline{\mathbf{60.9^\circ C}}$$

PROBLEM 13.21

KNOWN: The volumetric analysis of a producer gas is provided.

FIND: If the gas burns completely with theoretical amount of air determine (a) the molar analysis of the dry products, (b) the amount of water vapor condensed if the products are cooled to 70°F at 1atm.

ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air, and N_2 is inert. (2) Ideal gas mixture principles apply to the products.

ANALYSIS: Basing the calculation on 100 moles of fuel mixture

$$[3.8CH_4 + 0.1\,C_2H_6 + 4.8CO_2 + 11.7H_2 + 0.6\,O_2 + 23.2CO + 55.8\,N_2] + \alpha[O_2 + 3.76N_2] \rightarrow \beta CO_2 + \gamma H_2O + \omega N_2$$

C: $3.8 + 2(0.1) + 4.8 + 23.2 = \beta$, $\beta = 32$

H: $(3.8)(4) + 6(0.1) + 2(11.7) = 2\gamma$, $\gamma = 19.6$

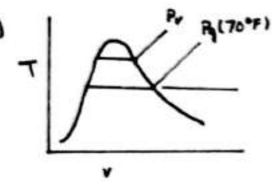
O: $(4.8)(2) + (0.6)(2) + 23.2 + 2\alpha = 2(32) + 19.6$, $\alpha = 24.8$

N_2: $(55.8) + (24.8)(3.76) = \omega$, $\omega = 149.05$

(a) The total amount of dry products is $32 + 149.05 = 181.05$. Thus, the molar analysis of the dry products is

$$\%\,CO_2 = \frac{32}{181.05} = 0.177\ (17.7\%)\ ,\ \%\,N_2 = \frac{149.05}{181.05} = 0.823\ (82.3\%) \leftarrow (a)$$

(b)



The partial pressure of the water vapor in the products is $p_v = \left(\frac{19.6}{32+149.05+19.6}\right)\left(14.7\frac{lbf}{in^2}\right) = 1.44\ lbf/in^2$

Since $p_v > p_g(70°F) = 0.3632\ lbf/in^2$, condensation would occur as the products are cooled to 70°F at p = 1atm.

The gas phase would consist of 181.05 moles of dry products plus n_v, which is the water vapor present. The partial pressure of the water vapor would be $p_g(70°F)$. Thus,

$$0.3632\frac{lbf}{in^2} = \left[\frac{n_v}{181.05+n_v}\right]\left(14.7\frac{lbf}{in^2}\right) \Rightarrow n_v = 4.587$$

Accordingly, the amount of water that condenses per kmol of fuel is

$$\frac{(19.6-4.587)\,lbmol(H_2O)}{100\,lbmol(fuel)} = 0.15\ \frac{lbmol(H_2O)}{lbmol(fuel)} \leftarrow (b)$$

PROBLEM 13.22

Propane (C_3H_8) burns completely with 180% of theoretical air entering at 40°C, 1 atm, 60% relative humidity. Obtain the balanced reaction equation, and determine the dew point temperature of the products, in °C, when cooled at 1 atm.

KNOWN: C_3H_8 burns completely with 180% of theoretical air entering at 40°C, 1atm, and $\phi = 60\%$

FIND: Obtain the balanced reaction equation and determine the dew point temperature when the products are cooled at 1 atm.

ENG. MODEL:
1. 3.76 moles of N_2 accompany each mole of O_2 in the air.
2. The N_2 and H_2O in the air are inert. 3. The moist air in the reactants and the combustion products are modeled as ideal gas mixtures.

ANALYSIS: Complete combustion of C_3H_8 with the theoretical amount of dry air is described by

$$C_3H_8 + 5(O_2 + 3.76N_2) \rightarrow 3CO_2 + 4H_2O + 18.8N_2$$

Complete combustion with 180% of theoretical air is then

$$C_3H_8 + 9(O_2 + 3.76N_2) \rightarrow 3CO_2 + 4H_2O + 4O_2 + 33.84N_2 \quad (a)$$

For moist air at 40°C, 1atm, $\phi = 60\%$ in which there is $(9)(4.76) = 42.84$ kmol of dry air and n_v kmol of water vapor, the partial pressure of the water vapor is

$$p_v = \left(\frac{n_v}{42.84 + n_v}\right)(1.01325 \text{ bar})$$

Also, the partial pressure p_v is given by $p_v = \phi\, p_g(40°C) = (0.60)(0.07384 \text{ bar}) = 0.0443$ bar. Collecting those expressions for p_v and solving for n_v, we get $n_v = 1.959$ kmol.

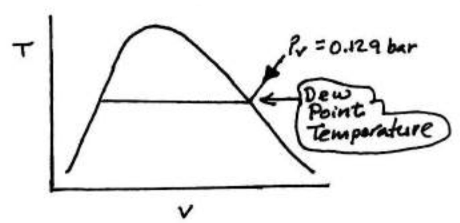
Accordingly, when the water vapor in the air is considered, the balanced reaction equation takes the form

$$C_3H_8 + 9(O_2 + 3.76N_2) + \underbrace{1.959H_2O}_{\text{water vapor in moist air}} \rightarrow 3CO_2 + \underbrace{5.959H_2O}_{\text{water in products}} + 4O_2 + 33.84N_2 \quad (b)$$

If the combustion products are cooled at 1atm, the mole fraction of the water vapor is

$$y_v = \frac{5.959}{46.799} = 0.1273$$

The partial pressure is then $p_v = 0.1273(1.01325 \text{ bar}) = 0.129$ bar.



Interpolating in Table A-2, we get

$$T_{dp} = 50.8\,°C$$

PROBLEM 13.23

KNOWN: C_4H_{10} burns completely with 160% of theoretical air at 20°C, 1 atm, and $\phi = 90\%$.

FIND: Determine (a) the balanced reaction equation, and (b) the dew point temperature of the products.

ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) N_2 and H_2O in the air supply are inert. (3) Air and the combustion products, each at 1 atm, are modeled as ideal gas mixtures.

ANALYSIS: Complete combustion of C_4H_{10} with the theoretical amount of dry air is

$$C_4H_{10} + 6.5(O_2 + 3.76N_2) \rightarrow 4CO_2 + 5H_2O + 24.44N_2 \qquad (1)$$

With 160% of theoretical dry air

$$C_4H_{10} + 10.4(O_2 + 3.76N_2) \rightarrow 4CO_2 + 5H_2O + 3.9O_2 + 39.104N_2$$

For moist air at 20°C, 1 atm, $\phi = 90\%$ in which there is $(10.4)(3.76) = 49.504$ kmol of dry air and n_v kmol of water vapor, the partial pressure p_v is

$$p_v = \left(\frac{n_v}{49.504 + n_v}\right)p = \left(\frac{n_v}{49.504 + n_v}\right)(1.01325 \text{ bar}) \qquad (2)$$

Using $\phi = 90\%$ and data from Table A-2: $p_v = \phi p_g = (0.9)(0.02339) = 0.02105$ bar

Solving (2) for n_v we get $n_v = 1.007$ kmol. Thus

$$C_4H_{10} + 10.4(O_2 + 3.76N_2) + 1.007H_2O$$
$$\rightarrow 4CO_2 + 6.007H_2O + 3.9O_2 + 39.104N_2 \longleftarrow \text{balanced equation}$$

Considering the combustion products, the mole fraction of water vapor is

$$y_v = \frac{6.007}{53.011} = 0.11332 \Rightarrow p_v = (0.11332)(1.01325) = 0.11482 \text{ bar}$$

Interpolating in Table A-3; $T_{\text{Dew point}} \approx 48°C \longleftarrow T_{DP}$

PROBLEM 13.24

Methane (CH_4) enters a furnace and burns completely with 150% of theoretical air entering at 25°C, 0.945 bar, 75% relative humidity. Determine

(a) the balanced reaction equation.
(b) the dew point temperature of the combustion products, in °C, when cooled at 0.945 bar.

KNOWN: CH_4 burns completely with 150% of theoretical air entering at 25°C, 0.945 bar, and $\phi = 75\%$

FIND: Obtain the balanced reaction equation and determine the dew point temperature when the products are cooled at 0.945 bar.

ENGR. MODEL:
1. 3.76 moles of N_2 accompany each mole of O_2 in the air.
2. The N_2 and H_2O in the air are inert. 3. The moist air in the reactants and the combustion products are modeled as ideal gases.

ANALYSIS: (a) Complete combustion of CH_4 with the theoretical amount of dry air is described by

$$CH_4 + 2(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 7.52N_2$$

Complete combustion with 150% of theoretical air is then

$$CH_4 + 3(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + O_2 + 11.28N_2$$

For moist air at 25°C, 0.945 bar, $\phi = 75\%$ in which there is $3 \times 4.76 = 14.28$ kmol of dry air and n_v kmol of water vapor, the partial pressure of the water vapor is

$$p_v = \left(\frac{n_v}{14.28 + n_v}\right)(0.945\,\text{bar})$$

Also, the partial pressure is given by $p_v = \phi\, p_g(25°C) = (0.75)(0.03169\,\text{bar}) = 0.0238$ bar. Collecting these expressions for p_v and solving for n_v, we get $n_v = 0.367$ kmol.

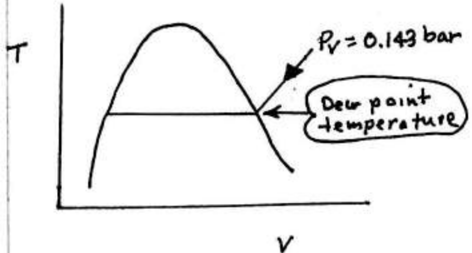
Accordingly, when the water vapor in the air is considered, the balanced reaction equation is (a)

$$CH_4 + 3(O_2 + 3.76N_2) + \underline{0.367\,H_2O} \rightarrow CO_2 + \underline{2.367\,H_2O} + O_2 + 11.28N_2$$

(b) If the combustion products are cooled at 0.945 bar, the mole fraction of the water vapor is

$$y_v = \frac{2.367}{15.647} = 0.1513$$

The partial press is then $p_v = (0.1513)(0.945\,\text{bar}) = 0.143$ bar



Interpolating in Table A-2, we get $T_{dp} = 52.9$°C (b)

Problem 13.25

Propane (C_3H_8) burns completely with the theoretical amount of air at 60°F, 1 atm, 90% relative humidity. Determine

(a) the balanced reaction equation.

(b) the dew point temperature, in °F, of the combustion products at 1 atm.

(c) the amount of water vapor condensed, in lbmol per lbmol of fuel, if the combustion products are cooled to 60°F, 1 atm.

KNOWN: Propane burns completely with the theoretical amount of air at known temperature, pressure, and relative humidity.

FIND: Determine (a) the balanced reaction equation, (b) the dew point temperature, and (c) the amount of water condensed if the products are cooled at constant pressure to a given temperature.

ENGINEERING MODEL: (1) 3.76 moles on N_2 accompany each mole of O_2 in the air. (2) The N_2 and H_2O in the air are inert. (3) The air and products are modeled as ideal gases.

ANALYSIS: (a) Complete combustion of C_3H_8 with the theoretical amount of dry air is described by

$$C_3H_8 + 5\,(O_2 + 3.76\,N_2) \rightarrow 3\,CO_2 + 4\,H_2O + 18.8\,N_2 \tag{1}$$

Consider moist air at 60°F, 1 atm, $\varphi = 90\%$ in which there are 5 (4.76) = 23.8 lbmol of dry air and n_v lbmol of water vapor. The partial pressure of the water vapor is

$$p_v = \left(\frac{n_v}{n_a + n_v}\right) p = \left(\frac{n_v}{23.8 + n_v}\right) (14.696\ \text{lbf/in}^2)$$

Also, $p_v = \varphi\, p_g(60°\text{F}) = (0.9)\,(0.2563) = 0.2307$ lbf/in.^{2} Collecting results

$$0.2307 = \left(\frac{n_v}{23.8 + n_v}\right)(14.696) \;\Rightarrow\; n_v = 0.3796\ \text{lbmol}$$

Including the water vapor in the combustion air, Eq. (1) reads

$$C_3H_8 + 5\,(O_2 + 3.76\,N_2) + 0.3796\,H_2O \rightarrow 3\,CO_2 + 4.3796\,H_2O + 18.8\,N_2 \qquad \longleftarrow \text{(a)}$$

(b) The partial pressure of the water vapor in the products is

$$p_v = \frac{4.3796}{(3 + 4.3796 + 18.8)}(14.696) = 2.4585\ \text{lbf/in}^2$$

Interpolating in Table A-3E with $p_v = p_g$, the dew point temperature of the products is

$$T_{\text{dp products}} \approx 133°\text{F} \qquad \longleftarrow \text{(b)}$$

(c) Since the final temperature of the products is 60°F, which is below the dew point temperature, condensation occurs. At 60°F, the vapor phase contains $n_{v,\,\text{final}}$ lbmol of water vapor and
$n_{\text{dry}} = 3 + 18.8 = 21.8$ lbmol of dry products. The partial pressure of the water vapor equals $p_g(60°\text{F})$, or 0.2563 lbf/in.^{2} from Table A-2E. Thus

$$0.2563 = \frac{n_{v,\,\text{final}}}{(21.8 + n_{v,\,\text{final}})}(14.696) \;\Rightarrow\; n_{v,\,\text{final}} = 0.3869\ \text{lbf/in.}^2$$

The amount of water vapor condensed per lbmol of fuel is

$$\begin{bmatrix}\text{amount of water}\\ \text{condensed per}\\ \text{lbmol of fuel}\end{bmatrix} = 4.3796 - 0.3869 = 3.9927\ \frac{\text{lbmol (water)}}{\text{lbmol (fuel)}} \qquad \longleftarrow \text{(c)}$$

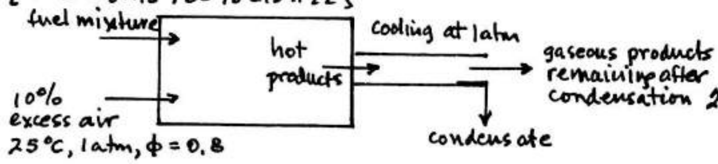
PROBLEM 13.26

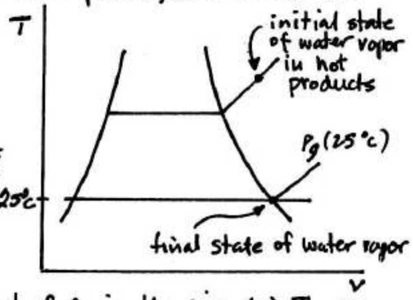
KNOWN: A liquid fuel mixture with a known mass analysis is burned completely with excess air at a specified temperature and relative humidity.

FIND: (a) Determine the equivalent hydrocarbon composition, C_aH_b, with the same carbon-hydrogen ratio on a mass basis as the mixture.
(b) If the products are cooled below their dew point, determine the amount of water vapor condensed.

SCHEMATIC & GIVEN DATA:

{40% C_8H_{18}, 60% $C_{10}H_{22}$}





ENGINEERING MODEL: (1) 3.76 kmol of N_2 accompany each kmol of O_2 in the air. (2) The N_2 and H_2O in the supply air are inert. (3) The air and products are both modeled as ideal gas mixtures.

ANALYSIS: (a) Assuming 100 kg of liquid fuel

	m_i (kg)	M_i	$n_i = m_i/M_i$ (kmol)
C_8H_{18}:	40	114	0.3509
$C_{10}H_{20}$:	60	142	0.4225

Thus, in 100 kg of fuel, there are the following amounts of C and H, respectively

$$C: [0.3509 \text{ kmol} (C_8H_{18})]\left[8 \frac{\text{kmol}(C)}{\text{kmol}(C_8H_{18})}\right] + [0.4225][10] = 7.0322 \text{ kmol}(C)$$

$$H: [0.3509][18] + [0.4225][22] = 15.61 \text{ kmol}(H)$$

The "equivalent" hydrocarbon composition is

$$C_{7.0322}H_{15.61} \quad \text{(a)}$$

(b) Complete combustion with the theoretical amount of air is described by

$$(0.3509\, C_8H_{18} + 0.4225\, C_{10}H_{22}) + \alpha_{th}(O_2 + 3.76 N_2) \rightarrow 7.0322\, CO_2 + 7.805\, H_2O + \alpha_{th} 3.76 N_2$$

$$\alpha_{th} = 7.0322 + \frac{7.805}{2} = 10.935$$

Complete combustion with 10% excess is then

$$(0.3509\, C_8H_{18} + 0.4225\, C_{10}H_{22}) + 12.0285(O_2 + 3.76 N_2)$$
$$\rightarrow 7.0322\, CO_2 + 7.805\, H_2O + 1.0935\, O_2 + 45.227\, N_2 \quad (1)$$

Consider next a moist air mixture at 25°C, 1 atm, ϕ = 80% in which there is 12.0285(4.76) = 57.256 kmol of dry air and n_v kmol of water vapor. The partial pressure of water vapor is

$$p_v = \left(\frac{n_v}{n_a + n_v}\right) p = \left(\frac{n_v}{57.256 + n_v}\right)(1.01325 \text{ bar})$$

Also, $p_v = \phi\, p_g(25°C) = 0.8(0.03169) = 0.02535$ bar. Collecting results

$$0.02535 = \left(\frac{n_v}{57.256 + n_v}\right)(1.01325) \Rightarrow n_v = 1.4693 \text{ kmol}$$

PROBLEM 13.26 (Cont'd.) - Page 2

When water vapor is present in the combustion air, Eq. (1) reads

$$(0.3509\, C_8H_{18} + 0.4225\, C_{10}H_{22}) + 12.0285\,(O_2 + 3.76N_2) + 1.4693\, H_2O$$

$$\rightarrow 7.0322\, CO_2 + 9.2743\, H_2O + 1.0935\, O_2 + 45.227\, N_2$$

According to the model introduced in chap. 12, the gaseous products remaining after the products have been cooled to 25°C would contain saturated water vapor at 25°C. That is, the partial pressure of the water vapor in this mixture would be $p_g(25°C) = 0.03169$ bar. The partial pressure is expressed as

$$p_v = \left(\frac{n_v'}{n_{dry} + n_v'}\right) p \qquad (2)$$

where $n_{dry} = 7.0322 + 1.0935 + 45.227 = 53.353$. Accordingly, from Eq. (2)

$$0.03169 = \left(\frac{n_v'}{53.353 + n_v'}\right)(1.01325) \Rightarrow n_v' = 1.7225 \text{ kmol}$$

Since there is 9.2743 kmol of H_2O per 100 kg of fuel in the products, the amount of water that condenses is $9.2743 - 1.7225 = 7.5518$ kmol (H_2O) per 100 kg of fuel. Thus

$$m_{cond} = \left[7.5518 \frac{\text{kmol}(H_2O)}{100\text{ kg(fuel)}}\right]\left[\frac{18.02\text{ kg}(H_2O)}{1\text{ kmol}(H_2O)}\right] = 1.361 \frac{\text{kg}(H_2O)}{\text{kg(fuel)}} \longleftarrow \text{(b)}$$

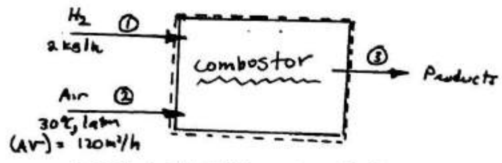
PROBLEM 13.27

Hydrogen (H_2) enters a combustor with a mass flow rate of 2 kg/h and burns with air entering at 30°C, 1 atm with a volumetric flow rate of 120 m³/h. Determine the percent of theoretical air used.

KNOWN: H_2 enters a combustor with a mass flow rate of 2 kg/h and burns with air entering at 30°C, 1 atm and a volumetric flow rate of 120 m³/h.

FIND: Determine the percent of theoretical air used.

SCHEMATIC & GIVEN DATA



ENGR. MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the entering air. N_2 is inert and the air is dry. (2) The air can be modeled as an ideal gas.

ANALYSIS: The percent theoretical air can be found from

$$\% \text{ theo air} = \frac{(\overline{AF})}{(\overline{AF})_{theo}} \times 100 \qquad (1)$$

For the complete combustion of H_2 with the theoretical amount of air,

$$H_2 + \tfrac{1}{2}(O_2 + 3.76 N_2) \rightarrow H_2O + 1.88 N_2$$

So, $(\overline{AF})_{THEO} = 2.38$

For the actual reaction, the mass flow rate of the air is

$$\dot{m}_{AIR} = \frac{(AV)_2}{v_2} = \frac{(AV)_2 P_2}{(\bar{R}/M) T_2} = \frac{(120 \text{ m}^3/h)(1.01325 \times 10^5 \text{ N/m}^2)}{\left(\frac{8314}{28.97} \frac{N \cdot m}{kg \cdot K}\right)(303 K)} = 139.83 \frac{kg(air)}{h}$$

Thus,

$$AF = \frac{139.83 \text{ kg(air)}/h}{2 \text{ kg (fuel)}/h} = 69.92 \frac{kg(air)}{kg(fuel)}$$

on a molar basis

$$\overline{AF} = 69.92 \frac{kg(air)}{kg(fuel)} \left[\frac{2.018 \text{ kg(fuel)}/\text{kmol(fuel)}}{28.97 \text{ kg(air)}/\text{kmol(air)}} \right]$$

$$= 4.87 \frac{kmol(air)}{kmol(fuel)}$$

Finally, returning to Eq. (1)

$$\% \text{ theo air} = \left(\frac{4.87}{2.38}\right)(100) = 204.6$$

13.28 Methyl alcohol (CH_3OH) burns with 200% theoretical air, yielding CO_2, H_2O, O_2, and N_2. Determine the
(a) balanced reaction equation.
(b) air-fuel ratio on a mass basis.
(c) molar analysis of the products.

KNOWN: Methyl alcohol burns with excess air yielding known products.

FIND: (a) The balanced reaction equation, (b) the air-fuel ratio on a mass basis, and (c) the molar analysis of the products.

SCHEMATIC AND GIVEN DATA:
Fuel Composition: 100% CH_3OH
Percent theoretical air = 200%
Products of Combustion: CO_2, H_2O, O_2, and N_2

ENGINEERING MODEL:
1. Each mole of oxygen in the combustion air is accompanied by 3.76 moles of nitrogen, which is inert.

ANALYSIS: (a) To determine the balanced reaction equation, first determine the balanced reaction equation for combustion with theoretical air

$$CH_3OH + a(O_2 + 3.76N_2) \rightarrow bCO_2 + cH_2O + dN_2$$

Applying conservation of mass to carbon, hydrogen, oxygen, and nitrogen, respectively

C: $1 = b$

H: $4 = 2c \rightarrow c = 2$

O: $1 + 2a = 2b + c = 2(1) + 2 \rightarrow a = 1.5$

N: $2(3.76)a = 2d = 2(3.76)(1.5) \rightarrow d = 5.64$

The balanced chemical equation for complete combustion of the fuel with *theoretical air* is

$$CH_3OH + 1.5(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 5.64N_2$$

For combustion of fuel with 200% theoretical air

$$CH_3OH + (2.00)1.5(O_2 + 3.76N_2) \rightarrow aCO_2 + bH_2O + cO_2 + dN_2$$

Applying conservation of mass to carbon, hydrogen, oxygen, and nitrogen, respectively

Problem 12.28 (Continued) – Page 2

C: $1 = a$

H: $3 + 1 = 2b \rightarrow b = 2$

O: $1 + 2(1.5)(2.00) = 2a + b + 2c = 2(1) + 2 + 2c \rightarrow c = 1.5$

N: $2(3.76)(1.5)(2.00) = 2d \rightarrow d = 11.28$

The balanced chemical equation for complete combustion of the fuel with *200% theoretical air* is

$$\underline{\mathbf{CH_3OH + 3(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 1.5O_2 + 11.28N_2}}$$

(b) The air-fuel ratio on a mass basis is

$$AF = \overline{AF}\left(\frac{M_{\text{air}}}{M_{\text{fuel}}}\right) = \frac{(3)(4.76\text{ kmol air})}{1\text{ kmol CH}_3\text{OH}}\left(\frac{28.97\frac{\text{kg air}}{\text{kmol air}}}{32.04\frac{\text{kg CH}_3\text{OH}}{\text{kmol CH}_3\text{OH}}}\right) = \underline{\mathbf{12.91\text{ kg air/kg CH}_3\text{OH}}}$$

(c) The calculation for the mole fraction (y_i) for each product is given in the following table

Product	n_i	$y_i = n_i/n_{\text{total}}$
CO_2	1	1/15.78 = 0.0634 = **6.34%**
H_2O	2	2/15.78 = 0.1267 = **12.67%**
O_2	1.5	1.5/15.78 = 0.0951 = **9.51%**
N_2	11.28	11.28/15.78 = 0.7148 = **71.48%**
$n_{\text{total}} = \sum n_i =$	**15.78**	

13.29 Octane (C_8H_{18}) is burned 20% excess air, yielding CO_2, CO, O_2, H_2O, and N_2 only. If 5% of the dry products (molar basis) is O_2, determine
(a) the balanced reaction equation.
(b) the analysis of the products on a dry molar basis.

KNOWN: C_8H_{18} is burned with 20% excess air yielding CO_2, CO, O_2, H_2O, and N_2 only. Five percent of the dry products (molar basis) is O_2.

FIND: Determine (a) the balanced reaction equation, and (b) the analysis of the products on a dry molar basis.

ENGINEERING MODEL: 3.76 moles of N_2 accompany each mole of O_2 in the air, and N_2 is inert.

ANALYSIS: (a) Complete combustion of C_8H_{18} with the theoretical amount of air is described by

$$C_8H_{18} + 12.5(O_2 + 3.76N_2) \rightarrow 8CO_2 + 9H_2O + 47N_2$$

Combustion with 20% excess air is then

C: $8 = a + b \Rightarrow b = 8 - a$
H: $18 = 2c \Rightarrow c = 9$
O: $30 = 2a + b + 9 + 2d = 2a + (8\text{-}a) + 9 + 2d \Rightarrow 13 = a + 2d \Rightarrow a = 13 - 2d$

The total moles of dry product is: $n_{\text{dry products}} = a + b + d + 56.4 = a + (8\text{-}a) + d + 56.4$. Thus, with the given data for oxygen in the dry products

$$0.05 = \frac{d}{d+64.4} \Rightarrow d = 3.389$$

Accordingly

$$C_8H_{18} + 15(O_2 + 3.76N_2) \rightarrow 6.222CO_2 + 1.778CO + 9H_2O + 3.389O_2 + 56.4N_2 \quad \leftarrow \text{(a)}$$

(b) The total moles of dry products is $n_{\text{dry products}} = d + 64.4 = 67.789$. Therefore, the mole fractions of the components of the dry product gas are

$$y_{CO_2} = \frac{6.222}{67.789} = 0.092\ (9.2\%)$$

$$y_{co} = \frac{1.778}{67.789} = 0.026\ (2.6\%)$$

$$y_{CO_2} = \frac{3.389}{67.789} = 0.05\ (5.0\%)$$

$$y_{CO_2} = \frac{56.4}{67.789} = 0.832\ (83.2\%)$$

$\leftarrow$ (b)

PROBLEM 13.30

13.30 Hexane (C_6H_{14}) burns with dry air to give products with the dry molar analysis 8.5% CO_2, 5.2% CO, 3% O_2, 83.3% N_2. Determine

(a) the balanced reaction equation.
(b) the percent of theoretical air.
(c) the dew point temperature, in °C, of the products at 1 atm.

KNOWN: C_6H_{14} burns with dry air. The dry molar analysis of the products is given.

FIND: Determine (a) the balanced reaction equation, (2) the percent theoretical air, (c) the dew point temperature of the products at 1 atm.

ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) The N_2 is inert. (3) The combustion products are modeled as an ideal gas mixture.

ANALYSIS: (a) Balancing the reaction equation for 1 kmol of hexane

$$C_6H_{14} + \alpha(O_2 + 3.76\,N_2) \rightarrow \beta(0.085\,CO_2 + 0.052\,CO + 0.03\,O_2 + 0.833\,N_2) + \gamma\,H_2O$$

$$C:\ 6 = \beta(0.085 + 0.052) \Rightarrow \beta = 43.796$$

$$H_2:\ \frac{14}{2} = \gamma \Rightarrow \gamma = 7$$

① $$N_2:\ \alpha(3.76) = \beta(0.833) \Rightarrow \alpha = 9.703$$

balanced equation

Thus

$$C_6H_{14} + 9.703(O_2 + 3.76N_2) \rightarrow 3.723\,CO_2 + 2.277\,CO + 1.314\,O_2 + 36.482\,N_2 + 7H_2O$$

(b) Theoretical combustion: $C_6H_{14} + \alpha_{th}O_2 \rightarrow 6CO_2 + 7H_2O \Rightarrow \alpha_{th} = 9.5$

$$\%\text{ theoretical air} = \left(\frac{\alpha}{\alpha_{th}}\right) \times 100 = (9.703/9.5) \times 100 = 102.1\,\%$$

% theo. air

(c) Considering the combustion products, the total number of moles is 50.796 kmol. The fraction of water vapor is

$$y_v = \frac{7}{50.796} = 0.1378 \Rightarrow p_v = (0.1378)(1.01325\text{ bar}) = 0.1396\text{ bar}$$

From data in Table A-3; $T_{\text{dew point}} \approx 52.4°C$

T_{DP}

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1. Alternatively, for O_2: $\alpha = \beta(0.085 + 0.052/2 + 0.03) + \gamma/2$

$$\Rightarrow \alpha = 9.675$$

The difference between this number and the value calculated above is experimental error and round-off.

PROBLEM 13.31

13.31 The components of the exhaust gas of a spark-ignition engine using a fuel mixture represented as C_8H_{17} have a dry molar analysis of 8.7% CO_2, 8.9% CO, 0.3% O_2, 3.7% H_2, 0.3% CH_4, and 78.1% N_2. Determine the equivalence ratio.

KNOWN: The products of combustion of C_8H_{17} have the following dry molar analysis: 8.7% CO_2, 8.9% CO, 0.3% O_2, 3.7% H_2, 0.3% CH_4, 78.1% N_2.

FIND: Determine the equivalence ratio

ENGINEERING MODEL: 3.76 moles of N_2 accompany each mole of O_2 in the air, and N_2 is inert.

ANALYSIS: Basing the calculation on 100 moles of dry products

$$\alpha\, C_8H_{17} + \beta(O_2 + 3.76N_2) \rightarrow 8.7\,CO_2 + 8.9\,CO + 0.3\,O_2 + 3.7\,H_2 + 0.3\,CH_4 + 78.1\,N_2 + \gamma\, H_2O$$

C: $8\alpha = 8.7 + 8.9 + 0.3 \Rightarrow \alpha = 2.2375$

H: $17(2.2375) = 2(3.7) + 4(0.3) + 2\gamma \Rightarrow \gamma = 14.7188$

N_2: $3.76\beta = 78.1 \Rightarrow \beta = 20.7713$

① Check

O: $(2)(20.7713) \overset{?}{=} (8.7)(2) + 8.9 + 2(.3) + 14.7188$

$41.54 \doteq 41.62$

The fuel air ratio is $FA = \dfrac{2.2375}{(20.7713)(4.76)} = 0.0226 \; \dfrac{\text{kmole(fuel)}}{\text{kmole (air)}}$

The reaction equation for complete combustion with the theoretical amount of air is

$$C_8H_{17} + \alpha(O_2 + 3.76N_2) \rightarrow \beta\, CO_2 + \gamma\, H_2O + 3.76\alpha\, N_2$$

C: $8 = \beta$

H: $17 = 2\gamma,\ \gamma = 8.5$

O: $2\alpha = 2\beta + \gamma = 16 + 8.5,\ \alpha = 12.25$

The fuel air ratio is $(FA)_{theo} = \dfrac{1}{(12.25)(4.76)} = 0.0171 \; \dfrac{\text{kmol (fuel)}}{\text{kmol (air)}}$

The equivalence ratio:

② $\left(\begin{matrix}\text{equivalence}\\ \text{ratio}\end{matrix}\right) = \dfrac{FA}{(FA)_{theo}} = \dfrac{0.0226}{0.0171} = 1.32$

1. When using experimental data, an exact closure cannot be expected.
2. The reactants form a rich mixture.

PROBLEM 13.32

13.32 The combustion of a hydrocarbon fuel, represented as C_aH_b, results in products with the dry molar analysis 11% CO_2, 0.5% CO, 2% CH_4, 1.5% H_2, 6% O_2, and 79% N_2. Determine the air–fuel ratio on (a) a molar basis, (b) a mass basis.

KNOWN: A hydrocarbon represented as C_aH_b burns with dry air. The dry molar analysis of the products is given.

FIND: Determine the air-fuel ratio an a (a) molar basis, (b) mass basis.

ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) The N_2 is inert, (3) Assume 100 kmol of dry product gas.

ANALYSIS: First, obtain the balanced reaction equation.

$$C_aH_b + \alpha(O_2 + 3.76N_2) \rightarrow 11.0\,CO_2 + 0.5\,CO + 2\,CH_4 + 1.5H_2 + 6\,O_2 + 79N_2 + \gamma H_2O$$

$$C: a = 11.0 + 0.5 + 2 = 13.5$$

$$H: b = (2)(4) + (1.5)(2) + (2)\gamma$$

$$N_2: \alpha(3.76) = 79 \Rightarrow \alpha = 21.01$$

$$O_2: 21.01 = 11.0 + \frac{0.5}{2} + 6 + \frac{\gamma}{2} \Rightarrow \gamma = 7.52$$

With $\gamma = 7.52$; $b = 26.04$

$$C_{13.5}H_{26.04} + 21.01(O_2 + 3.76N_2) \longrightarrow 11\,CO_2 + 0.5\,CO + 2CH_4 + 1.5H_2 + 6\,O_2 + 79N_2 + 7.52\,H_2O$$

(a) $$\overline{AF} = \frac{(21.01)(4.76)}{(1)} = 100 \quad \overline{AF}$$

(b) $$AF = \left(100\,\frac{\text{kmol air}}{\text{kmol fuel}}\right)\frac{(28.97)}{[(13.5)(12.01) + (26.04)(1.008)]}\frac{(\text{kg/kmol})\,\text{air}}{(\text{kg/kmol})\,\text{fuel}}$$

$$= 15.38 \quad AF$$

13.33 Decane ($C_{10}H_{22}$) burns completely in dry air. The air-fuel ratio on a mass basis is 33 kg. Determine the
(a) analysis of the products on a dry molar basis.
(b) percent of theoretical air.

KNOWN: Decane burns completely in air with known air-fuel ratio on a mass basis.

FIND: (a) The analysis of the products on a dry molar basis and (b) the per cent of theoretical air.

SCHEMATIC AND GIVEN DATA:
Fuel Composition: 100% $C_{10}H_{22}$
Air-fuel Ratio: $AF = 33$ kg air/kg $C_{10}H_{22}$
Products of Combustion: CO_2, H_2O, O_2, and N_2

ENGINEERING MODEL:
1. Each mole of oxygen in the combustion air is accompanied by 3.76 moles of nitrogen, which is inert.

ANALYSIS: (a) To determine the analysis of the product on a dry molar basis, the balanced chemical equation is required. The molar air-fuel ratio is

$$AF = \overline{AF}\left(\frac{M_{\text{air}}}{M_{\text{fuel}}}\right) \rightarrow \overline{AF} = AF\left(\frac{M_{\text{fuel}}}{M_{\text{air}}}\right)$$

$$\overline{AF} = \frac{33\,\text{kg air}}{1\,\text{kg}\,C_{10}H_{22}}\left(\frac{(10)\left(12.01\frac{\text{kg C}}{\text{kmol C}}\right)+(11)\left(2.016\frac{\text{kg}\,H_2}{\text{kmol}\,H_2}\right)}{28.97\frac{\text{kg air}}{\text{kmol air}}}\right) = 162.1 \text{ kmol air/kmol } C_{10}H_{22}$$

Each 4.76 kmol of dry air includes 1 kmol of O_2. Thus, the balanced chemical equation is

$$C_{10}H_{22} + (162.1/4.76)(O_2 + 3.76N_2) \rightarrow aCO_2 + bH_2O + cO_2 + dN_2$$

or

$$C_{10}H_{22} + (34.05)(O_2 + 3.76N_2) \rightarrow aCO_2 + bH_2O + cO_2 + dN_2$$

Applying conservation of mass to carbon, hydrogen, oxygen, and nitrogen, respectively

C: $10 = a$

H: $22 = 2b \rightarrow b = 11$

O: $2(34.05) = 2a + b + 2c = 2(10) + 11 + 2c \rightarrow c = 18.55$

N: $2(3.76)(34.05) = 2d \rightarrow d = 128.028$

The balanced chemical equation for combustion with an air-fuel ratio of 33 kg air/kg $C_{10}H_{22}$ is

$$C_{10}H_{22} + 34.05(O_2 + 3.76N_2) \rightarrow 10CO_2 + 11H_2O + 18.55O_2 + 128.028N_2$$

For an analysis of the products on a dry molar basis, do not consider water in the products. The mole fraction (y_i) for each dry product is given in the following table

Product	n_i	$y_i = n_i/n_{total}$
CO_2	10	10/156.578 = 0.06387 = **6.39%**
O_2	18.55	18.55/156. 578 = 0.11847 = **11.85%**
N_2	128.028	128.028/156. 578 = 0.81766 = **81.77%**
$n_{total} = \sum n_i =$	**156.578**	

(b) For combustion with theoretical air

$$C_{10}H_{22} + a(O_2 + 3.76N_2) \rightarrow bCO_2 + cH_2O + dN_2$$

Applying conservation of mass to carbon, hydrogen, oxygen, and nitrogen, respectively

C: $10 = b$

H: $22 = 2c \rightarrow c = 11$

O: $2a = 2b + c = 2(10) + 11 \rightarrow a = 15.5$

N: $2(3.76)a = 2d = 2(3.76)(15.5) \rightarrow d = 58.28$

The balanced chemical equation for combustion with theoretical air is

$$C_{10}H_{22} + 15.5(O_2 + 3.76N_2) \rightarrow 10CO_2 + 11H_2O + 58.28N_2$$

Thus, the percent theoretical air is

% theoretical air = 34.05/15.5 = **2.20 (220%)**

Since the per cent theoretical air is greater than 100%, the reactants form a ***lean*** *mixture.*

PROBLEM 13.34

13.34 Butane (C_4H_{10}) burns with air, giving products having the dry molar analysis 11.0% CO_2, 1.0% CO, 3.5% O_2, 84.5% N_2. Determine

(a) the percent of theoretical air.
(b) the dew point temperature of the combustion products, in °C, at 1 bar.

KNOWN: C_4H_{10} burns with air to form products with the dry molar analysis:
$\{11\%\ CO,\ 1\%\ CO,\ 3.5\%\ O_2,\ 84.5\%\ N_2\}$

FIND: Determine (a) the percent theoretical air, (b) the dewpoint temperature.

ENGINEERING MODEL: Each mole of O_2 in the combustion air is accompanied by 3.76 moles of N_2. N_2 is inert.

ANALYSIS: (a) On the basis of 100 moles of dry products, the reaction is

$$a\, C_4H_{10} + b(O_2 + 3.76 N_2) \rightarrow 11\, CO_2 + 1\, CO + 3.5\, O_2 + 84.5\, N_2 + c\, H_2O$$

C: $4a = 11 + 1, \quad a = 3$
H: $(10)(3) = 2c, \quad c = 15$
O: $2b = 22 + 1 + 7.0 + 15, \quad b = 22.5$
Check N: $(3.76)(22.5) \stackrel{?}{=} 84.5$
$84.6 \neq 84.5$ ok, close

For this reaction,

$$\overline{AF} = \frac{22.5(4.76)}{3} = 35.7$$

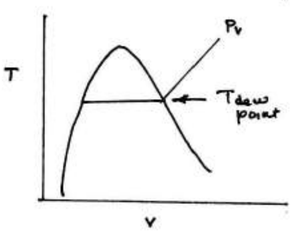
For complete combustion with the theoretical amount of air

$$C_4H_{10} + 6.5(O_2 + 3.76 N_2) \rightarrow 4\, CO_2 + 5\, H_2O + (3.76)(6.5)\, N_2$$

For this reaction $\quad (\overline{AF})_{theo} = \dfrac{(6.5)(4.76)}{1} = 30.94$

Then $\quad \%\ \text{theoretical air} = \left(\dfrac{35.7}{30.94}\right)(100) = 115\% \quad \longleftarrow$ (a)

(b) To find the dew point temperature for p = 1 bar



$$P_v = \left(\frac{n_v}{n_{prod}}\right) P$$
$$= \left(\frac{15}{115}\right)(1\ \text{bar})$$
$$= 0.1304\ \text{bar}$$

Table 2E; $\quad T_{dew\ pt} \approx 51\ °C \quad \longleftarrow$ (b)

PROBLEM 13.35

13.35 A natural gas with the volumetric analysis 97.3% CH_4, 2.3% CO_2, 0.4% N_2 is burned with air in a furnace to give products having a dry molar analysis of 9.20% CO_2, 3.84% O_2, 0.64% CO, and the remainder N_2. Determine

(a) the percent theoretical air.
(b) the dew point temperature, in °F, of the combustion products at 1 atm.

KNOWN: A natural gas with a known volumetric analysis burns with air to give products for which a dry molar analysis is provided.

FIND: (a) Determine the percent of theoretical air used, (b) the dew point temperature.

ENGINEERING MODEL: 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert.

ANALYSIS: Basing the calculation on one mole of fuel mixture, complete combustion with the theoretical amount of air is

$$[0.973\,CH_4 + 0.023\,CO_2 + 0.004\,N_2] + \alpha[O_2 + 3.76N_2] \rightarrow \beta CO_2 + \gamma H_2O + \lambda N_2$$

$$C:\ 0.973 + 0.023 = \beta \Rightarrow \beta = 0.996$$
$$H:\ 4(0.973) = 2\gamma,\quad \gamma = 1.946$$
$$O:\ 2(.023) + 2\alpha = 2(0.996) + 1.946,\quad \alpha = 1.946$$

The actual reaction equation takes the form

$$[0.973\,CH_4 + 0.023\,CO_2 + 0.004\,N_2] + \alpha[O_2 + 3.76N_2] \rightarrow \beta[9.2CO_2 + 3.84O_2 + 0.64\,CO + 86.32N_2] + \gamma H_2O$$

$$C:\ (0.973 + 0.023) = \beta[9.2 + 0.64],\quad \beta = 0.1012$$
$$H:\ 4(0.973) = 2\gamma,\quad \gamma = 1.946$$
$$O:\ 2[0.023] + 2\alpha = \beta[(9.2)(2) + (3.84)(2) + 0.64] + 1.946,\quad \alpha = 2.302$$

CHECK:

$$N_2:\quad (0.004) + (2.302)(3.76) = (0.1012)(86.32)$$
$$8.66 \overset{?}{=} 8.74$$

①

(a) The actual $\overline{AF}$ value is obtained as

$$\overline{AF} = \frac{(2.302)(4.76)}{1}$$

whereas the theoretical value is

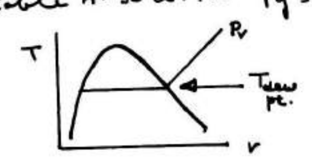
$$(\overline{AF})_{theo} = \frac{1.946(4.76)}{1}$$

$$\%\text{ theo. air} = \left[\frac{\overline{AF}}{(\overline{AF})_{theo}}\right]100 = \left(\frac{2.302}{1.946}\right)(100) = 118 \leftarrow$$

(b) The partial pressure of the water vapor is $p_v = y_v\,p$, where

$$y_v = \frac{1.946}{0.1012(100) + 1.946} = 0.1613$$

Then, with $p = 14.7$ lbf/in², $p_v = 2.37$ lbf/in². Interpolation in Table A-3E with $p_g = 2.37$ lbf/in² gives $T_{dew\ point} = 132°F \leftarrow$ TDP



PROBLEM 13.36

13.36 A fuel oil having an analysis on a mass basis of 85.7% C, 14.2% H, 0.1% inert matter burns with air to give products with a dry molar analysis of 12.29% CO_2, 3.76% O_2, 83.95% N_2. Determine the air–fuel ratio on a mass basis.

KNOWN: A fuel oil having the mass analysis {85.7% C, 14.2% H, 0.1% inert} burns with air to give products with a dry molar analysis {12.29% CO_2, 3.76% O_2, 83.95% N_2}.

FIND: Determine the air-fuel ratio on a mass basis.

ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air, and the N_2 is inert. (2) The inert matter in the fuel is ignored in writing the reaction equation below.

ANALYSIS: Based on 1 kg of fuel oil, the mass analysis of the fuel allows the molar amounts of the combustibles to be evaluated:

$$C = \frac{0.857\ kg(C)/kg(fuel)}{12.01\ kg(C)/kmol(C)} = 0.07136\ \frac{kmol(C)}{kg(fuel)}$$

$$H = \frac{0.142\ kg(H)/kg(fuel)}{1.009\ kg(H)/kmol(H)} = 0.14073\ \frac{kmol(H)}{kg(fuel)}$$

On the basis of 1 kg of fuel oil, the reaction equation reads

$$\{0.07136\ C + 0.14073\ H\} + a[O_2 + 3.76N_2] \rightarrow b[0.1229\ CO_2 + 0.0376\ O_2 + 0.8395N_2] + c\,H_2O$$

$$C: \quad 0.07136 = 0.1229\,b \Rightarrow b = 0.5806$$

$$H: \quad 0.14073 = 2c \Rightarrow c = 0.0704$$

$$N_2: \quad 3.76a = 0.8395\,b \Rightarrow a = 0.1296$$

Check for closure:

$$O: \quad 2a \stackrel{?}{=} b[(2)(0.1229) + 2(0.0376)] + 2c$$

$$2(0.1296) \stackrel{?}{=} 2(0.5806)[0.1229 + 0.0376] + (0.0704)$$

$$0.2592 \stackrel{?}{=} 0.2568 \quad (good)$$

The air-fuel ratio is then

$$AF = \frac{(0.1296)(4.76)(28.97)}{1}$$

$$= 17.87 \quad \longleftarrow AF$$

PROBLEM 13.37

Methanol (CH_3OH) burns with air. The product gas is analyzed and the laboratory report gives only the following percentages on a dry molar basis: 7.1% CO_2, 2.4% CO, 0.84% CH_3OH. Assuming the remaining components consist of O_2 and N_2, determine

(a) the percentages of O_2 and N_2 in the dry molar analysis.
(b) the percent excess air.

KNOWN: CH_3OH burns with air. A laboratory report gives only these percentages on a dry molar basis: 7.1% CO_2, 2.4% CO, 0.84% CH_3OH.

FIND: Determine (a) the percentages of O_2 and N_2 in the dry molar analysis, (b) percent excess air.

ENGR. MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the combustion air. N_2 is inert. (2) The dry products include only CO_2, CO, CH_3OH, O_2 and N_2.

ANALYSIS: (a) On the basis of 100 moles of dry products

$$y\, CH_3OH + \alpha(O_2 + 3.76 N_2) \rightarrow 7.1\, CO_2 + 2.4\, CO + 0.84\, CH_3OH$$ ①
$$+ x N_2 + (89.66 - x) O_2 + w H_2O$$

Balancing...

C: $y = 7.1 + 2.4 + 0.84$, $y = 10.34$

H: $4(10.34) = (0.84)(4) + 2w$, $w = 19$

O: $(10.34) + 2\alpha = (7.1)(2) + 2.4 + 0.84 + 2(89.66 - x) + 19$

$\Rightarrow \alpha = 102.71 - x$

N: $(\alpha)(2)(3.76) = 2x$

$\Rightarrow 3.76\alpha = x$

$\Rightarrow \alpha = 102.71 - 3.76\alpha \Rightarrow \alpha = 21.578$ and $x = 81.13$

Accordingly, the percentages of O_2 and N_2 are $\%N_2 = 81.13$, $\%O_2 = 8.53$ ← (a)

(b) For complete combustion with the theoretical amount of air

$$CH_3OH + \alpha(O_2 + 3.76 N_2) \rightarrow \beta CO_2 + \gamma H_2O + 3.76\alpha N_2$$

C: $1 = \beta$

H: $4 = 2\gamma$, $\gamma = 2$

O: $1 + 2\alpha = 2(1) + 2$

$\alpha = 3/2$

$$\left(\%\ \text{Theoretical Air}\right) = \left(\frac{21.578(4.76)/10.34}{1.5(4.76)/1}\right)(100) = 139$$

% excess air = 39 ← (b)

---

1. For the dry products, $100 = 7.1 + 2.4 + 0.84 + \underset{N_2}{x} + \underset{O_2}{z}$

$\Rightarrow x + z = 89.66$

$\Rightarrow z = 89.66 - x$

PROBLEM 13.38

A fuel oil with the mass analysis 87% C, 11% H, 1.4% S, 0.6% inert matter burns with 120% of theoretical air. The hydrogen and sulfur are completely oxidized, but 95% of the carbon is oxidized to CO_2 and the remainder to CO.

(a) Determine the balanced reaction equation.
(b) For the CO and SO_2, determine the amount, in kmol per 10^6 kmol of combustion products (that is, the amount in *parts per million*).

KNOWN: A fuel oil with the mass analysis {87% C, 11% H, 1.4% S, 0.6% inert} burns with 120% of theoretical air. H and S are fully oxidized, but 95% of the Carbon is oxidized to CO_2 and the balance to CO.

FIND: Determine (a) the balanced reaction equation, (b) for CO and SO_2 the amounts in kmol per 10^6 kmol of combustion products.

ENGINEERING MODEL: There are 3.76 kmoles of N_2 accompanying each kmol of O_2 in the combustion air. N_2 is inert.

ANALYSIS: (a) Based on 100 kg of fuel oil, the molar analysis is

$$C: \frac{87}{12.01} = 7.24, \quad H: \frac{11}{1.01} = 10.89, \quad S: \frac{1.4}{32.06} = 0.04 \text{ kmol}$$

To find the theoretical amount of air, the reaction equation for 100 kg of fuel oil is then

① $$(7.24\,C + 10.89\,H + 0.04\,S) + \alpha(O_2 + 3.76N_2) \rightarrow \beta\, CO_2 + \gamma H_2O + \omega SO_2 + 3.76\alpha N_2$$

C: $7.24 = \beta$
H: $10.89 = 2\gamma$, $\gamma = 5.45$
S: $0.04 = \omega$
O: $2\alpha = 2(7.24) + 5.45 + 2(0.04)$, $\alpha = 10.01$

Then, for the actual reaction

$$(7.24C + 10.89H + 0.04S) + (1.2)(10.01)[O_2 + 3.76N_2] \rightarrow (0.95)(7.24)\,CO_2 + (0.05)(7.24)\,CO + \alpha O_2 + \gamma H_2O + \omega SO_2 + (1.2)(10.01)(3.76)\,N_2$$

H: $10.89 = 2\gamma$, $\gamma = 5.45$ (as above)
S: $0.04 = \omega$ (as above)
O: $\underbrace{(1.2)(10.01)(2)}_{24.024} = 13.756 + 0.362 + 2\alpha + 5.45 + 2(0.04)$, $\alpha = 2.19$

The balanced reaction equation for 100 kg of fuel oil is (a)

$$(7.24C + 10.89H + 0.04S) + 12.01[O_2 + 3.76N_2] \rightarrow 6.88CO_2 + 0.36\,CO + 2.19\,O_2 + 5.45H_2O + 0.04\,SO_2 + 45.17N_2$$

(b) Total moles of products = 60.09 kmols/100 kg of fuel oil.

$$\text{ppm CO} = \left(\frac{0.36}{60.09}\right)(10^6) = 5991 \qquad \text{(b)}$$

$$\text{ppm } SO_2 = \left(\frac{0.04}{60.09}\right)(10^6) = 666$$

---

1. The presence of the inert matter is not shown in the reaction equation of part (a). Also, in part (b) only the gaseous products are considered in the ppm calculations.

PROBLEM 13.39

13.39 Pentane (C_5H_{12}) burns with air so that a fraction x of the carbon is converted to CO_2. The remaining carbon appears as CO. There is no free O_2 in the products. Develop plots of the air–fuel ratio and the percent of theoretical air versus x, for x ranging from zero to unity.

KNOWN: C_5H_{12} burns with air so that a fraction x of the carbon is converted to CO_2 and the rest appears as CO. No free O_2 appears in the products.

FIND: Plot $\overline{AF}$ and % theoretical air versus x.

ENGINEERING MODEL: 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert.

ANALYSIS: The reaction equation takes the form

$$C_5H_{12} + a(O_2 + 3.76N_2) \rightarrow 5x\,CO_2 + 5(1-x)\,CO + b\,H_2O + 3.76a\,N_2$$

H: $12 = 2b$, $b = 6$

O: $2a = 10x + 5(1-x) + 6 \Rightarrow 2a = 5x + 11 \Rightarrow a = (5x+11)/2$.

The complete combustion of C_5H_{12} with the theoretical amount of air is described by

$$C_5H_{12} + 8(O_2 + 3.76N_2) \rightarrow 5CO_2 + 6H_2O + (8)(3.76)N_2$$

Accordingly, the percent of theoretical air required is

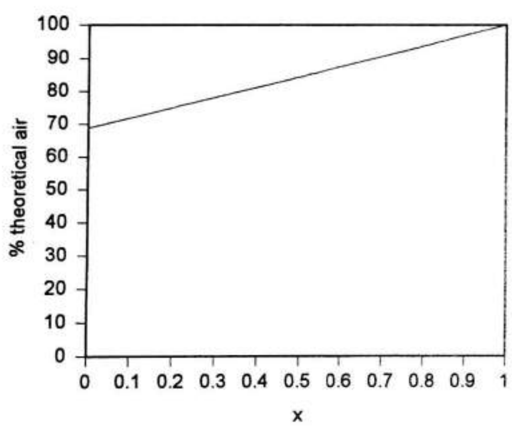
$$\%\text{ theo. air} = \left(\frac{\overline{AF}}{(\overline{AF})_{theo}}\right)(100) = \left(\frac{(5x+11)/2}{8}\right)100 = \left(\frac{5x+11}{16}\right)(100)$$

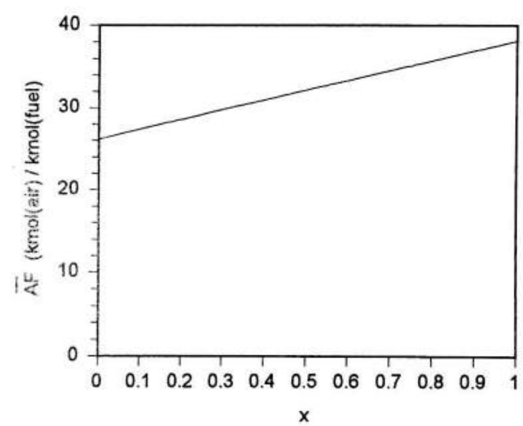
$$= 31.25x + 68.75 \qquad (1)$$

The air-fuel ratio is

$$\overline{AF} = \frac{[(5x+11)/2](4.76)}{1} = (2.5x + 5.5)\,4.76 \quad \text{or} \quad \overline{AF} = 11.9x + 26.18 \qquad (2)$$

Eqs.(1) and (2) are simple linear relations that can be plotted readily by hand or using computer software. The following plots are obtained using IT:





Note that with a deficiency of combustion air, the combustion is less complete, i.e. less CO_2 and more CO produced.

PROBLEM 13.40

For each of the following mixtures, determine the equivalence ratio and indicate if the mixture is lean or rich:

(a) 1 lbmol of methane (CH_4) and 8 lbmol of air.
(b) 1 kg of ethane (C_2H_6) and 17.2 kg of air.

KNOWN: Fuel-air mixtures are specified.

FIND: Determine the equivalence ratio for each case and indicate if the mixture is lean or rich

ENGR. MODEL: Accompanying each mole of O_2 in the air is 3.76 moles of N_2 which is inert.

ANALYSIS: (a) {1 lbmol CH_4, 8 lbmol air}

The reaction equation for the theoretical amount of air reads

$$CH_4 + 2[O_2 + 3.76 N_2] \rightarrow CO_2 + 2H_2O + (2)(3.76) N_2$$

$$\Rightarrow \quad (\overline{AF})_{THEO} = \frac{2(4.76)}{1} = 9.52$$

Then, the equivalence ratio is

(1) $$\text{Equivalence ratio} = \frac{(\overline{AF})_{THEO}}{(\overline{AF})_{ACT}} = \frac{9.52}{(8/1)} = 1.19 \quad (\text{Rich})$$

(b) {1 kg C_2H_6, 17.2 kg air}

The reaction equation for the theoretical amount of air reads

$$C_2H_6 + 3.5[O_2 + 3.76 N_2] \rightarrow 2 CO_2 + 3 H_2O + 13.16 N_2$$

$$\Rightarrow \quad (AF)_{THEO} = \frac{(3.5)(4.76)}{1}\left[\frac{28.97}{30.07}\right] = 16.05$$

Then, the equivalence ratio is

(2) $$\text{Equivalence ratio} = \frac{(AF)_{THEO}}{(AF)_{ACT}} = \frac{16.05}{(17.2/1)} = 0.933 \quad (\text{Lean})$$

1. Molar basis
2. Mass basis

13.41 Methyl alcohol (CH_3OH) burns in dry air according to the reaction

$$CH_3OH + 3.3(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 1.8O_2 + 12.408N_2$$

Determine the
(a) air-fuel ratio on a mass basis.
(b) equivalence ratio.
(c) percent excess air.

KNOWN: Methyl alcohol burns in dry air with known products.

FIND: (a) The air-fuel ratio on a mass basis and (b) the equivalence ratio, (c) the percent excess air.

SCHEMATIC AND GIVEN DATA:
Fuel Composition: 100% CH_3OH
Products of combustion: CO_2, H_2O, O_2, and N_2
Balanced reaction equation:

$$CH_3OH + 3.3(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 1.8O_2 + 12.408N_2$$

ENGINEERING MODEL:
1. Each mole of oxygen in the combustion air is accompanied by 3.76 moles of nitrogen, which is inert.

ANALYSIS: (a) The air-fuel ratio on a mass basis is

$$AF = \overline{AF}\left(\frac{M_{air}}{M_{fuel}}\right) = \frac{(3.3)(4.76\,\text{kmol air})}{1\,\text{kmol CH}_3\text{OH}}\left(\frac{28.97\dfrac{\text{kg air}}{\text{kmol air}}}{32.04\dfrac{\text{kg CH}_3\text{OH}}{\text{kmol CH}_3\text{OH}}}\right) = \underline{\mathbf{14.20\ kg\ air/kg\ CH_3OH}}$$

(b) Equivalence ratio is

$$Equivalence\ ratio = \frac{FA}{FA_{\text{theoretical air}}} = \frac{AF_{\text{theoretical air}}}{AF}$$

For complete combustion of the fuel with *theoretical air*

$$CH_3OH + a(O_2 + 3.76N_2) \rightarrow bCO_2 + cH_2O + dN_2$$

Applying conservation of mass to carbon, hydrogen, oxygen, and nitrogen, respectively

$$C:\ 1 = b$$

Problem 13.41 (Continued) – Page 2

H: $4 = 2c \rightarrow c = 2$

O: $1 + 2a = 2b + c = 2(1) + 2 \rightarrow a = 1.5$

N: $2(3.76)a = 2d = 2(3.76)(1.5) \rightarrow d = 5.64$

The balanced chemical equation for complete combustion of the fuel with *theoretical air* is

$$CH_3OH + 1.5(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 5.64N_2$$

The air-fuel ratio on a mass basis for combustion with theoretical air is

$$AF_{\text{theoretical air}} = \overline{AF}_{\text{theoretical air}}\left(\frac{M_{\text{air}}}{M_{\text{fuel}}}\right) = \frac{(1.5)(4.76\text{ kmol air})}{1\text{ kmol CH}_3\text{OH}}\left(\frac{28.97\dfrac{\text{kg air}}{\text{kmol air}}}{32.04\dfrac{\text{kg CH}_3\text{OH}}{\text{kmol CH}_3\text{OH}}}\right)$$

$$AF_{\text{theoretical air}} = 6.46\text{ kg air/kg CH}_3\text{OH}$$

Substituting values to solve for the equivalence ratio

$$\text{Equivalence ratio} = \frac{\left(6.46\dfrac{\text{kg air}}{\text{kg CH}_3\text{OH}}\right)}{\left(14.20\dfrac{\text{kg air}}{\text{kg CH}_3\text{OH}}\right)} = \underline{\mathbf{0.45}}$$

*Since the equivalence ratio is less than one, the reactants form a **lean** mixture.*

(c) The percent excess air is

$$\%\,Excess\ Air = \left[\left(\frac{3.3}{1.5}\right) - 1\right] \text{X } 100 = \underline{\mathbf{120\%}}$$

13.42 Ethyl alcohol (C_2H_5OH) burns in dry air according to the reaction

$$C_2H_5OH + 2.16(O_2 + 3.76N_2) \rightarrow 0.32CO_2 + 1.68CO + 3H_2O + 8.1216N_2$$

Determine the
(a) air-fuel ratio on a mass basis.
(b) equivalence ratio.
(c) percent theoretical air.

KNOWN: Methyl alcohol burns in dry air according to a specified reaction equation.

FIND: (a) The air-fuel ratio on a mass basis and (b) the equivalence ratio, (c) the percent theoretical air.

SCHEMATIC AND GIVEN DATA:
Fuel Composition: 100% C_2H_5OH
Products of combustion: CO_2, H_2O, O_2, and N_2
Balanced reaction equation:

$$C_2H_5OH + 2.16(O_2 + 3.76N_2) \rightarrow 0.32CO_2 + 1.68CO + 3H_2O + 8.1216N_2$$

ENGINEERING MODEL:
1. Each mole of oxygen in the combustion air is accompanied by 3.76 moles of nitrogen, which is inert.

ANALYSIS: (a) The air-fuel ratio on a mass basis is

$$AF = \overline{AF}\left(\frac{M_{\text{air}}}{M_{\text{fuel}}}\right) = \frac{(2.16)(4.76\ \text{kmol air})}{1\ \text{kmol}\ C_2H_5OH}\left(\frac{28.97\dfrac{\text{kg air}}{\text{kmol air}}}{46.07\dfrac{\text{kg}\ C_2H_5OH}{\text{kmol}\ C_2H_5OH}}\right) = \underline{\mathbf{6.47\ kg\ air/kg\ C_2H_5OH}}$$

(b) Equivalence ratio is

$$Equivalence\ ratio = \frac{FA}{FA_{\text{theoretical air}}} = \frac{AF_{\text{theoretical air}}}{AF}$$

For complete combustion of the fuel with *theoretical air*

$$C_2H_5OH + a(O_2 + 3.76N_2) \rightarrow bCO_2 + cH_2O + dN_2$$

Applying conservation of mass to carbon, hydrogen, oxygen, and nitrogen, respectively

C: $2 = b$

Problem 13.42 (Continued) – Page 2

H: $5 + 1 = 2c \rightarrow c = 3$

O: $1 + 2a = 2b + c = 2(2) + 3 \rightarrow a = 3$

N: $2(3.76)a = 2d = 2(3.76)(3) \rightarrow d = 11.28$

The balanced chemical equation for complete combustion of the fuel with *theoretical air* is

$$CH_3OH + 3(O_2 + 3.76N_2) \rightarrow 2CO_2 + 3H_2O + 11.28N_2$$

The air-fuel ratio on a mass basis for combustion with theoretical air is

$$AF_{\text{theoretical air}} = \overline{AF}_{\text{theoretical air}}\left(\frac{M_{\text{air}}}{M_{\text{fuel}}}\right) = \frac{(3)(4.76\ \text{kmol air})}{1\ \text{kmol}\ C_2H_5OH}\left(\frac{28.97\dfrac{\text{kg air}}{\text{kmol air}}}{46.07\dfrac{\text{kg}\ C_2H_5OH}{\text{kmol}\ C_2H_5OH}}\right)$$

$$AF_{\text{theoretical air}} = 8.98\ \text{kg air/kg}\ C_2H_5OH$$

Substituting values to solve for the equivalence ratio

$$\text{Equivalence ratio} = \frac{\left(8.98\dfrac{\text{kg air}}{\text{kg}\ C_2H_5OH}\right)}{\left(6.47\dfrac{\text{kg air}}{\text{kg}\ C_2H_5OH}\right)} = \underline{\mathbf{1.39}}$$

Since the equivalence ratio is greater than one, the reactants form a ***rich*** *mixture.*

(c) The percent theoretical air is

$$\%\ Theoretical\ Air = \left[\frac{(2.16)(4.76)\text{kmol air per kmol fuel}}{(3)(4.76)\text{kmol air per kmol fuel}}\right] X\ 100 = \underline{\mathbf{72\%}}$$

50

PROBLEM 13.43

13.43 Octane (C_8H_{18}) enters an engine and burns with air to give products with the dry molar analysis of CO_2, 10.5%; CO, 5.8%; CH_4, 0.9%; H_2, 2.6%; O_2, 0.3%; N_2, 79.9%. Determine the equivalence ratio.

KNOWN: C_8H_{18} burns with air to give products with the dry molar analysis: {10.5% CO_2, 5.8% CO, 0.9% CH_4, 2.6% H_2, 0.3% O_2, 79.9% N_2}

FIND: Determine the equivalence ratio.

ENGINEERING MODEL: Each mole of O_2 in the combustion air is accompanied by 3.76 moles of N_2 which is inert.

ANALYSIS: On the basis of 100 moles of dry products, the reaction equation is

$$x\, C_8H_{18} + y\,[O_2 + 3.76N_2] \rightarrow [10.5\,CO_2 + 5.8\,CO + 0.9\,CH_4 + 2.6\,H_2 + 0.3\,O_2 + 79.9\,N_2] + z\,H_2O$$

C: $8x = 10.5 + 5.8 + 0.9$, $x = 2.15$

H: $(18)(2.15) = 4(0.9) + 2(2.6) + 2z$, $z = 14.95$

N_2: $3.76y = 79.9$, $y = 21.25$

Check for closure

O: $2y = 2(10.5) + 5.8 + 2(.3) + 14.95$, $y = 21.18$ (good)

The air-fuel ratio is then

$$\overline{AF} = \frac{(21.18)(4.76)}{2.15} = 46.89$$

The balanced equation for the theoretical amount of air reads

$$C_8H_{18} + 12.5[O_2 + 3.76N_2] \rightarrow 8CO_2 + 9H_2O + 47N_2$$

Thus

$$(\overline{AF})_{THEO} = \frac{(12.5)(4.76)}{1} = 59.5$$

The equivalence ratio is

① $$\text{Equivalence ratio} = \frac{(\overline{AF})_{THEO}}{(\overline{AF})_{ACT}} = \frac{59.5}{46.89} = 1.269 \quad \text{(Rich)}$$

1. Alternatively

$$\left(\text{equivalence ratio}\right) = \frac{(\overline{FA})_{act}}{(\overline{FA})_{theo}}$$

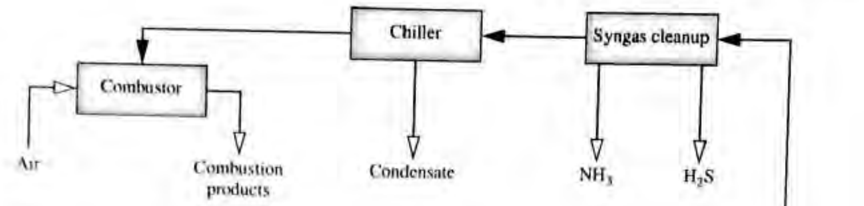
PROBLEM 13.44

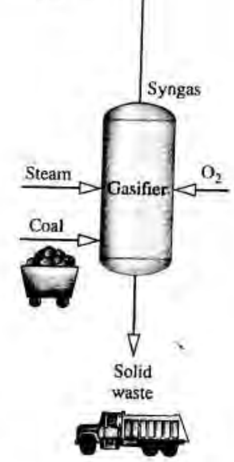
Figure P13.44 shows four components in series. Coal, oxygen (O_2), and steam are fed to the gasifier, which produces syngas (synthesis gas) with the following molar analysis:

CH_4, 0.3%; H_2, 29.6%; CO_2, 10.0%; CO, 41.0%; N_2, 0.8%; H_2O, 17.0%, H_2S, 1.1%, NH_3, 0.2%

The H_2S and NH_3 are removed and the mixture then passes through a chiller that condenses 98% of the water present in the syngas stream. The condensate is removed and the resulting gas stream is fed to the combustor, where its burns completely with 400% of theoretical air. For the combustor, determine the amount of air required, in kmol per kmol of syngas.

Figure P13.44:





KNOWN: Data are provided for components involving syngas.

FIND: For the combustor, determine the amount of air required, in kmol per kmol of syngas.

ANALYSIS: The H_2S, NH_3, and 98% of the water in the syngas are removed from the gas stream before it enters the combustor. Thus, based on 100 kmol of syngas, the gas entering the combustor has this composition: $0.3\,CH_4$; $29.6\,H_2$; $10\,CO_2$; $41\,CO$; $0.8\,N_2$; $0.34\,H_2O$

For complete combustion with the theoretical amount of air

$$[0.3CH_4 + 29.6H_2 + 10CO_2 + 41CO + 0.8N_2 + 0.34H_2O] + \alpha[O_2 + 3.76N_2]$$
$$\rightarrow \beta CO_2 + \gamma H_2O + \omega N_2$$

C: $0.3 + 41 + 10 = \beta$, $\beta = 51.3$

H: $4(0.3) + 2(29.6) + 2(0.34) = 2\gamma$, $\gamma = 30.54$

O: $41 + 20 + 0.34 + 2\alpha = 2(51.3) + 30.54$, $\alpha = 35.9 \Leftarrow$

N_2: $0.8 + 3.76(35.9) = \omega$, $\omega = 135.78$

For complete combustion with 400% theoretical air, the amount of air, based on 100 kmol of syngas, is $(4)(35.9)(4.76)$ kmol (air). Thus, the amount of air required is

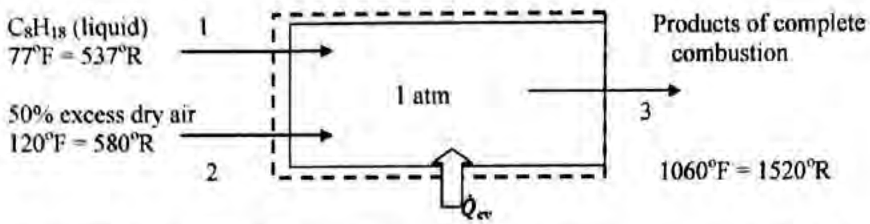
$$\frac{(4)(35.9)(4.76)\text{ kmol(air)}}{100\text{ kmol (syngas)}} = 6.84\frac{\text{kmol (air)}}{\text{kmol (syngas)}}$$

13.45 Liquid octane (C_8H_{18}) at 77°F, 1 atm enters a combustion chamber operating at steady state and burns completely with 50% excess dry air entering at 120°F. The products exit at 1060°F, 1 atm. Determine the rate of heat transfer between the combustions chamber and its surroundings, in Btu per lbmol of fuel entering. Kinetic and potential energy effects are negligible.

KNOWN: Liquid octane burns completely in a combustion chamber operating at steady state. The temperatures of the entering fuel, dry air, and exiting products are specified. The percent excess air is also known.

FIND: Determine the rate of heat transfer between the combustion and its surroundings, per lbmol of fuel entering.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying sketch is at steady state with $\dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) 3.76 moles of N_2 accompany each moles of O_2 in the dry air and the nitrogen is inert. (3) the ideal gas model is applicable to the combustion air and the products of combustion.

ANALYSIS: Complete combustion of the C_8H_{18} with the theoretical amount of dry air is described by

$$C_8H_{18} + 12.5(O_2 + 3.76N_2) \rightarrow 8CO_2 + 9H_2O + 47N_2$$

Thus, with 50% excess air

$$C_8H_{18} + 18.75\,(O_2 + 3.76N_2) \rightarrow 8CO_2 + 9H_2O + 6.25O_2 + 70.5N_2$$

An energy rate balance reduces to give

$$0 = \dot{Q}_{cv}/\dot{n}_{fuel} - \cancelto{0}{\dot{W}_{cv}/\dot{n}_{fuel}} + (\bar{h}_{fuel})_1 + [18.75\bar{h}_{O_2} + 70.5\bar{h}_{N_2}]_2$$
$$- [8\bar{h}_{CO_2} + 9\bar{h}_{H_2O}\ 6.25\bar{h}_{O_2} + 70.5\bar{h}_{N_2}]_3$$

Using $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$

$$\dot{Q}_{cv}/\dot{n}_{fuel} = \{8[\bar{h}_f^\circ + \bar{h}(1520) - \bar{h}(537)]_{CO2} + 9[\bar{h}_f^\circ + \bar{h}(1520) - \bar{h}(537)]_{H2O}$$
$$+ 6.25[\cancelto{0}{\bar{h}_f^\circ} + \bar{h}(1520) - \bar{h}(537)]_{O2} + 70.5[\cancelto{0}{\bar{h}_f^\circ} + \bar{h}(1520) - \bar{h}(537)]_{N2}\}$$
$$- (\bar{h}_f^\circ)_{liquid\ C8H18} - 18.75[\cancelto{0}{\bar{h}_f^\circ} + \bar{h}(580) - \bar{h}(537)]_{O2} - 70.5[\cancelto{0}{\bar{h}_f^\circ} + \bar{h}(580) - \bar{h}(537)]_{N2}\}$$

With data from Tables A-23E and A-25E

$$\dot{Q}_{cv}/\dot{n}_{fuel} = 8[-169{,}300 + 14824.9 - 4027.5] + 9[104{,}040 + 12738.8 - 4258.0]$$
$$+ 6.25[11179.6 - 3725.1] + 70.5[10800.4 - 3729.5] - (-107{,}530)$$
$$- 18.75[4027.3 - 3725.1] - 70.5[4028.7 - 3729.1]$$
$$= \underline{-1.582 \times 10^6 \text{ Btu/lbmol } (C_8H_{18})}$$

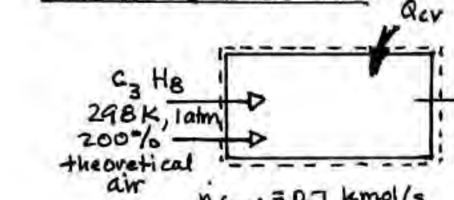
The negative sign indicates that the heat transfer is ***from*** *the control volume* ***to*** *the surroundings, as expected.*

PROBLEM 13.46

KNOWN: Propane enters a combustion chamber with know temperature and pressure and a given molar flow rate, where it burns completely with 200% theoretical air. The temperature of the exiting combustion products is specified.

FIND: Determine the heat transfer rate for each of two exit temperatures.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) The N_2 is inert. (3) The products are modeled as an ideal gas mixture. (4) The control volume is at steady state with $\dot{W}_{cv} = 0$. (5) Kinetic and potential energy effects are negligible.

ANALYSIS: Theoretical combustion: $C_3H_8 + 5(O_2 + 3.76N_2) \rightarrow 3CO_2 + 4H_2O + 18.8N_2$

Actual Combustion: $C_3H_8 + 10(O_2 + 3.76N_2) \rightarrow 3CO_2 + 4H_2O + 5O_2 + 37.6N_2$

$T_P = 560K$ The energy rate balance can be written as

$$0 = \dot{Q}_{cv} + \dot{n}_{fuel}\{(\bar{h}_f^\circ + \cancel{\Delta \bar{h}}^{\,0})_{fuel} + 10(\cancel{\bar{h}_f^\circ}^{\,0} + \cancel{\Delta \bar{h}}^{\,0})_{O_2,in} + 37.6(\cancel{\bar{h}_f^\circ}^{\,0} + \cancel{\Delta \bar{h}}^{\,0})_{N_2,in}$$
$$- 3(\bar{h}_f^\circ + \Delta \bar{h})_{CO_2,out} - 4(\bar{h}_f^\circ + \Delta \bar{h})_{H_2O,out} - 5(\cancel{\bar{h}_f^\circ}^{\,0} + \Delta \bar{h})_{O_2,out} - 37.6(\cancel{\bar{h}_f^\circ}^{\,0} + \Delta \bar{h})_{N_2,out}\}$$

$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = 3\left[\bar{h}^\circ_{f_{CO_2}} + \bar{h}_{CO_2}(560) - \bar{h}_{CO_2}(298)\right] + 4\left[\bar{h}^\circ_{f_{H_2O(g)}} + \bar{h}_{H_2O}(560) - \bar{h}_{H_2O}(298)\right]$$
$$+ 5\left[\bar{h}_{O_2}(560) - \bar{h}_{O_2}(298)\right] + 37.6\left[\bar{h}_{N_2}(560) - \bar{h}_{N_2}(298)\right] - (\bar{h}^\circ_{f\,C_3H_8})$$

With data from Table A-23

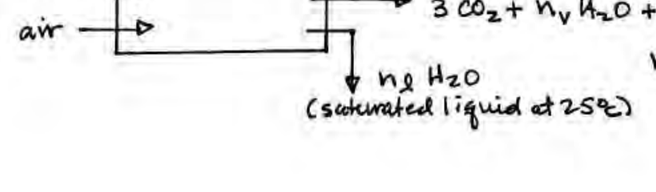
$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = 3\left[-393{,}520 + (20{,}407 - 9{,}364)\right] + 4\left[(-241{,}820 + (18{,}959 - 9{,}904)\right]$$
$$+ 5\left[16{,}654 - 8{,}682\right] + 37.6\left[16{,}363 - 8{,}669\right] - (-103{,}850)$$

$$= 3[-382{,}477] + 4[-232{,}765] + 5[7972] + 37.6[7694] - (-103{,}850)$$
$$= -1{,}645{,}299 \text{ kJ/kmol}$$

$$\dot{Q}_{cv} = \left(0.7\,\frac{\text{kmol}}{\text{s}}\right)\left(-1.645\times10^6\,\frac{\text{kJ}}{\text{kmol}}\right)\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = 1.152\times10^6\text{ kW} \longleftarrow \frac{\dot{Q}_{cv}}{(T_P = 560K)}$$

$T_P = 298K$ In this case, condensation might occur in the product gas mixture. To check this, determine the dew point temperature of the products, as follows: $n_v = 4$, $n_{tot} = 49.6 \Rightarrow y_v = 4/49.6 = 0.08065$
$p_v = (0.08065)(1.01325 \text{ bar}) = 0.08172$ bar. From Table A-2 $T_{dew\ point} \approx 42°C$

Since $298K = 25°C$ is less than the dew point, there is condensation. The following model will be used:



product gases saturated ($p_v = p_g @ 25°C = 0.03169$ bar)

$3CO_2 + n_v H_2O + 5O_2 + 37.6N_2$

$n_l = 4 - n_v$, $n_{tot} = 45.6 + n_v$ (in prod. gas)

PROBLEM 13.46 (Cont'd.) - Page 2

To find n_v and n_ℓ

$$y_v = \frac{p_v}{p} = \frac{0.03169}{1.01325} = 0.03128$$

and

$$y_v = \frac{n_v}{45.6 + n_v} \Rightarrow n_v = 1.4724$$

Finally, collecting results

$$C_3H_8 + 10(O_2 + 3.76N_2) \rightarrow (3\,O_2 + 1.4724\,H_2O(g) + 5O_2 + 37.6N_2) + 2.5276\,H_2O(\ell)$$

The energy balance becomes

$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = 3[\bar{h}^\circ_{f\,CO_2}] + 1.4724[\bar{h}^\circ_{f\,H_2O(g)}] + 2.5276[\bar{h}^\circ_{f\,H_2O(\ell)}] - [\bar{h}^\circ_{f\,C_3H_8}]$$

(Table A-25)

$$= 3(-393{,}520) + 1.4724(-241{,}820) + 2.5276(-285{,}830) - (-103{,}850)$$

$$= -2.155 \times 10^6 \text{ kJ/kmol fuel}$$

$$\dot{Q}_{cv} = (0.7)(-2.155 \times 10^6) = -1.5085 \times 10^6 \text{ kW} \longleftarrow \dot{Q}_{cv}\ (T_p = 298\text{K})$$

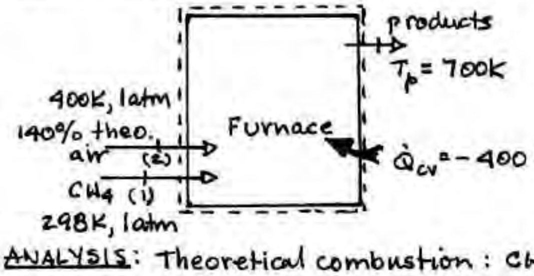
Note that cooling the gases to 298K and condensing some of the water vapor increases the heat transfer rate substantially.

PROBLEM 13.47

KNOWN: Methane enters a furnace with known conditions and burns completely with 140% of theoretical air entering at an elevated temperature. The temperature of the exiting products and the heat transfer rate to the surroundings are known.

FIND: Determine the mass flow rate of fuel.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of oxygen in the air. (2) The N_2 is inert. (3) The control volume is at steady state. (4) $\dot{W}_{cv} = 0$ and kinetic and potential energy effects are negligible. (5) The ideal gas model is used for all streams.

ANALYSIS: Theoretical combustion: $CH_4 + 2(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 7.52N_2$

Actual combustion: $CH_4 + 2.8(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 0.8O_2 + 10.528N_2$

The energy rate balance reduces at steady state to

$$0 = \dot{Q}_{cv} + \dot{n}_{fuel}\left[(\bar{h}_{CH_4})_1 + 2.8(\bar{h}_{O_2})_2 + 10.528(\bar{h}_{N_2})_2 - (\bar{h}_{CO_2})_p - 2(\bar{h}_{H_2O})_p - 0.8(\bar{h}_{O_2})_p - 10.528(\bar{h}_{N_2})_p\right]$$

or

$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = [\bar{h}^\circ_{f_{CO_2}} + \bar{h}_{CO_2}(700) - \bar{h}_{CO_2}(298)] + 2[\bar{h}^\circ_{f_{H_2O(g)}} + \bar{h}_{H_2O}(700) - \bar{h}_{H_2O}(298)]$$
$$+ 0.8[\cancel{\bar{h}^\circ_{f_{O_2}}}^{0} + \bar{h}_{O_2}(700) - \bar{h}_{O_2}(298)] + 10.528[\cancel{\bar{h}^\circ_{f_{N_2}}}^{0} + \bar{h}_{N_2}(700) - \bar{h}_{N_2}(298)]$$
$$- [\bar{h}^\circ_{f_{CH_4}}] - 2.8[\cancel{\bar{h}^\circ_{f_{O_2}}}^{0} + \bar{h}_{O_2}(400) - \bar{h}_{O_2}(298)]$$
$$- 10.528[\cancel{\bar{h}^\circ_{f_{N_2}}}^{0} + \bar{h}_{N_2}(400) - \bar{h}_{N_2}(298)]$$

$$= [-393{,}520 + (27{,}125 - 9{,}364)] + 2[-241{,}820 + (24{,}088 - 9{,}904)]$$
$$+ 0.8[21{,}184 - 8{,}682] + 10.528[20{,}604 - 8{,}669] - [-74{,}850]$$
$$- 2.8[11{,}711 - 8{,}682] - 10.528[11{,}640 - 8{,}669]$$

$$= -597{,}676 \text{ kJ/kmol fuel}$$

$$\dot{n}_{fuel} = \frac{-400 \text{ kW}}{-597{,}676 \text{ kJ/kmol fuel}}\left|\frac{1 \text{ kJ/s}}{1 \text{ kW}}\right| = 6.693 \times 10^{-4} \frac{\text{kmol}}{\text{s}}$$

$$\dot{m}_{fuel} = \dot{n}_{fuel} M_{fuel} = (6.693 \times 10^{-4})(16.04) = 0.0107 \text{ kg/s} \leftarrow \dot{m}_{fuel}$$

PROBLEM 13.48

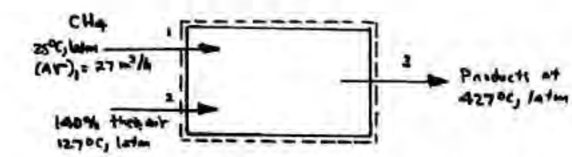
... Methane gas (CH_4) at 25°C, 1 atm and a volumetric flow rate of 27 m^3/h enters a heat-treating furnace operating at steady state. The methane burns completely with 140% of theoretical air entering at 127°C, 1 atm. Products of combustion exit at 427°C, 1 atm. Determine

(a) the volumetric flow rate of the air, in m^3/h.
(b) the rate of heat transfer from the furnace, in kJ/h.

KNOWN: CH_4 at 25°C, 1atm and a volumetric flow rate of 27 m^3/h enters a furnace and burns completely with 140% of theoretical air entering at 127°C, 1atm. Products of combustion exit at 427°C, 1atm.

FIND: Determine (a) the volumetric flow rate of the air and (b) the rate of heat transfer from the furnace.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The ideal gas model is applicable to the combustion air and the products of combustion.

ANALYSIS: Complete combustion of CH_4 with 140% of theoretical air is described by

$$CH_4 + 2.8(O_2 + 3.76 N_2) \rightarrow CO_2 + 2H_2O + 0.8\, O_2 + 10.528\, N_2 \qquad (1)$$

(a) The molar flow rate of the fuel is

$$\dot{n}_{CH_4} = \frac{(AV)_1}{\bar{v}_1} = \frac{(AV)_1}{\bar{R}T_1/P_1} = \frac{(27\,m^3/h)(1.01325\times10^5\,N/m^2)}{(8314\,\frac{N\cdot m}{kmol\cdot K})(298K)} = 1.104\,\frac{kmol(CH_4)}{h}$$

From Eq (1), (2.8)(4.76) kmol of air are required for each kmol of fuel. Thus, the molar flow rate of the air is

$$\dot{n}_{Air} = (2.8)(4.76)(1.104) = 14.714\,\frac{kmol(air)}{h}$$

And so

$$(AV)_2 = (\dot{n}_{Air})\frac{\bar{R}T_2}{P_2} = \frac{(14.717)(8314)(400)}{1.01325\times10^5} = 483.03\,\frac{m^3}{h} \longleftarrow (AV)_2$$

(b) An energy rate balance reduces to give

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{CH_4}}}^{\,0} + (\bar{h}_{CH_4})_1 + [2.8\bar{h}_{O_2} + 10.528\,\bar{h}_{N_2}] - [\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 0.8\bar{h}_{O_2} + 10.528\,\bar{h}_{N_2}]_2$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$

$$\frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}} = [\bar{h}_f^\circ + \bar{h}(700) - \bar{h}(298)]_{CO_2} + 2[\bar{h}_f^\circ + \bar{h}(700) - \bar{h}(298)]_{H_2O(g)} + 0.8[\bar{h}(700) - \bar{h}(298)]_{O_2} + 10.528\,[\bar{h}(700) - \bar{h}(298)]_{N_2} - (\bar{h}_f^\circ)_{CH_4} - 2.8[\bar{h}(400) - \bar{h}(298)]_{O_2} - 10.528\,[\bar{h}(400) - \bar{h}(298)]_{N_2}$$

Inserting data from Tables A-23 and A-25

$$\frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}} = [-393{,}520 + 27{,}125 - 9{,}364] + 2[-241{,}820 + 24{,}088 - 9{,}904] + 0.8[21{,}184 - 8{,}682] + 10.528\,[20{,}604 - 11{,}640] - (-74{,}850) - 2.8[11{,}711 - 8{,}682]$$

① $$= -660{,}287\ kJ/kmol(CH_4)$$

Thus

$$\dot{Q}_{cv} = \underbrace{(1.104)}_{\frac{kmol(CH_4)}{h}}\underbrace{(-660{,}287)}_{\frac{kJ}{kmol(CH_4)}} = -728{,}957\ kJ/h \longleftarrow \dot{Q}_{cv}$$

---

1. The N_2 terms have been expressed as $[\underbrace{\bar{h}(700K)}_{(20{,}604)} - \underbrace{\bar{h}(400K)}_{(11{,}640)}]$, each in kJ/kmol.

13.49 Liquid ethanol (C_2H_5OH), at 77°F, 1 atm enters a combustion chamber operating at steady state and burns completely with dry air entering at 340°F, 1 atm. The fuel flow rate is 50 lb /s and the equivalence ratio is 0.8. Products of combustion exit at 2000°F, 1 atm. Ignoring kinetic and potential energy effects, determine

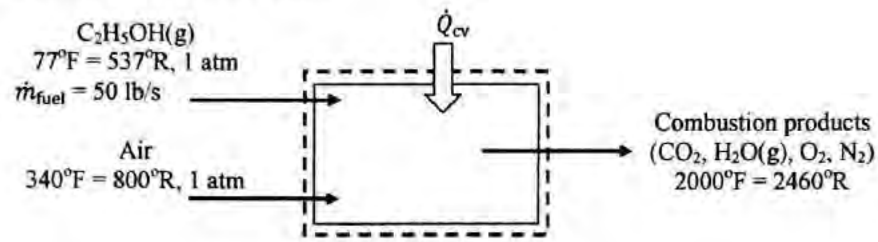
(a) the air-fuel ratio on a mass basis.

(b) the rate of heat transfer, in Btu/s.

KNOWN: Streams of liquid ethanol and air with specified equivalence ratio, temperature, and pressure enter and react in a combustion chamber at steady state. The mass flow rate of fuel is given. The products exit the chamber at 2000°F, 1 atm.

FIND: (a) the air-fuel ratio on a mass basis and (b) the rate of heat transfer

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

1. The control volume identified by a dashed line on the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$.
2. Kinetic and potential energy effects can be ignored.
3. Each mole of oxygen in the combustion air is accompanied by 3.76 moles of nitrogen, which is inert.
4. The combustion air and the products of combustion each form ideal gas mixtures.

ANALYSIS: (a) The balanced chemical equation for complete combustion of liquid ethanol with *theoretical* air is

$$C_2H_5OH(l) + 3(O_2 + 3.76N_2) \rightarrow 2CO_2 + 3H_2O + 11.28N_2$$

The molar air-fuel ratio with theoretical air is

$$\overline{AF}_{\text{theoretical}} = \frac{3(4.76\ \text{lbmol air})}{1\ \text{lbmol}\ C_2H_5OH} = 14.28\ \text{lbmol(air)/lbmol}(C_2H_5OH)$$

Equivalence ratio can be expressed in terms of molar air-fuel ratio by

$$Equivalence\ Ratio = \frac{\overline{FA}}{\overline{FA}_{\text{theoretical}}} = \frac{\dfrac{1}{\overline{AF}}}{\dfrac{1}{\overline{AF}_{\text{theoretical}}}} = \frac{\overline{AF}_{\text{theoretical}}}{\overline{AF}}$$

The actual molar air-fuel ratio is

$$\overline{AF} = \frac{\overline{AF}_{\text{theoretical air}}}{Equivalence\ Ratio} = \frac{14.28\dfrac{\text{lbmol air}}{\text{lbmol } C_2H_5OH}}{0.8} = 17.85 \text{ lbmol(air)/lbmol}(C_2H_5OH)$$

Thus, the air fuel ratio on a mass basis is

$$AF = \overline{AF}_{\text{theoretical air}}\left(\frac{M_{\text{air}}}{M_{\text{fuel}}}\right) = \left(17.85\frac{\text{lbmol air}}{\text{lbmol } C_2H_5OH}\right)\left(\frac{28.97\dfrac{\text{lb air}}{\text{lbmol air}}}{46.07\dfrac{\text{lb } C_2H_5OH}{\text{lbmol } C_2H_5OH}}\right)$$

$$AF = \underline{\mathbf{11.22 \text{ lb air/lb } C_2H_5OH}}$$

(b) Using the molar air-fuel ratio from Part (a), the balanced chemical equation is

$$C_2H_5OH(l) + (17.85/4.76)(O_2 + 3.76N_2) \rightarrow aCO_2 + bH_2O + cO_2 + dN_2$$

or

$$C_2H_5OH(l) + 3.75(O_2 + 3.76N_2) \rightarrow aCO_2 + bH_2O + cO_2 + dN_2$$

Applying conservation of mass to carbon, hydrogen, oxygen, and nitrogen, respectively

C: $2 = a$

H: $6 = 2b \rightarrow b = 3$

O: $1 + 2(3.75) = 2a + b + 2c = 2(2) + 3 + 2c \rightarrow c = 0.75$

N: $2(3.76)(3.75) = 2d \rightarrow d = 14.1$

The balanced chemical equation for actual combustion is

$$C_2H_5OH(l) + 3.75(O_2 + 3.76N_2) \rightarrow 2CO_2 + 3H_2O + 0.75O_2 + 14.1N_2$$

The energy rate balance at steady state reduces to

$$\frac{\dot{Q}_{cv}}{\dot{n}_F} = \bar{h}_P - \bar{h}_R$$

where

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Problem 13.49 (Continued) – Page 3

$$\bar{h}_R = \left(\bar{h}_f^\circ + \cancelto{0}{\Delta\bar{h}}\right)_{C_2H_5OH} + 3.75\left(\cancelto{0}{\bar{h}_f^\circ} + \Delta\bar{h}\right)_{O_2} + 14.1\left(\cancelto{0}{\bar{h}_f^\circ} + \Delta\bar{h}\right)_{N_2}$$

and

$$\bar{h}_P = 2\left(\bar{h}_f^\circ + \Delta\bar{h}\right)_{CO_2} + 3\left(\bar{h}_f^\circ + \Delta\bar{h}\right)_{H_2O} + 0.75\left(\cancelto{0}{\bar{h}_f^\circ} + \Delta\bar{h}\right)_{O_2} + 14.1\left(\cancelto{0}{\bar{h}_f^\circ} + \Delta\bar{h}\right)_{N_2}$$

With the enthalpy of formation for $C_2H_5OH(l)$ from Table A-25E and enthalpy values for O_2 and N_2 from Table A-23E

$$\bar{h}_R = [\left(\bar{h}_f^\circ\right)_{C_2H_5OH(g)} + 3.75\left(\Delta\bar{h}\right)_{O_2} + 14.1\left(\Delta\bar{h}\right)_{N_2}]$$

$$= [-119,470] + 3.75[5,602.0 - 3,725.1] + 14.1[5,564.4 - 3729.5]$$

$$= -86,560 \text{ Btu/lbmol}(C_2H_5OH)$$

With enthalpy of formation values for CO_2 and $H_2O(g)$ from Table A-25E, and enthalpy values for CO_2, H_2O, O_2, and N_2 from Table A-23E

$$\bar{h}_P = [2\left(\bar{h}_f^\circ + \Delta\bar{h}\right)_{CO_2} + 3\left(\bar{h}_f^\circ + \Delta\bar{h}\right)_{H_2O} + 0.75\left(\Delta\bar{h}\right)_{O_2} + 14.1\left(\Delta\bar{h}\right)_{N_2}]$$

$$= 2[-169,300 + 27,249 - 4027.5] + 3[-104,040 + 22,298 - 4,258.0]$$

$$+0.75[19,097 - 3,725.1] + 14.1[18,260 - 3,729.5]$$

$$= -333,748 \text{ Btu/lbmol}(C_2H_5OH)$$

Thus, the heat transfer rate per lbmol of fuel entering is

$$\frac{\dot{Q}_{cv}}{\dot{n}_F} = -333,748 - (-86,560) = -247,\ 188 \text{ Btu/lbmol}(C_2H_5OH)$$

The fuel molar flow rate is

$$\dot{n}_F = \frac{\dot{m}_F}{M_F} = \frac{50\dfrac{\text{lb}\,C_2H_5OH}{\text{s}}}{46.07\dfrac{\text{lb}\,C_2H_5OH}{\text{lbmol}\,C_2H_5OH}} = 1.09 \text{ lbmol } C_2H_5OH/\text{s}$$

Finally, the rate of heat transfer is

$$\dot{Q}_{cv} = \left(\frac{\dot{Q}_{cv}}{\dot{n}_F}\right)\dot{n}_F = \left(-247,188\frac{\text{Btu}}{\text{lbmol}\,C_2H_5OH}\right)\left(1.09\frac{\text{lbmol}\,C_2H_5OH}{\text{s}}\right) = \underline{\mathbf{-269,435\ Btu/s}}$$

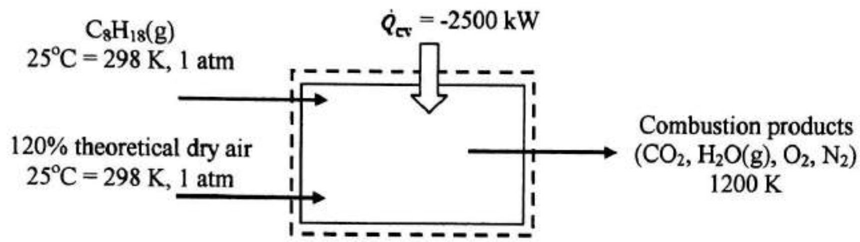
The negative sign indicates heat transfer is ***from*** *the control volume* ***to*** *the surroundings, as expected.*

13.50 Octane gas (C_8H_{18}) at 25°C, 1 atm enters a combustion chamber operating at steady state and burns with 120% theoretical air entering at 25°C, 1 atm. The combustion products exit at 1200 K and include only CO_2, H_2O, O_2, and N_2. If the rate of het transfer from the combustion chamber to the surroundings is 2500 kW, determine the mass flow rate of the fuel in kg/s.

KNOWN: Octane gas burns with 120% theoretical air with known products of combustion and a specified heat transfer rate.

FIND: Determine the mass flow rate of the fuel.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:

1. The control volume identified by a dashed line on the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$.
2. Kinetic and potential energy effects can be ignored.
3. The octane gas is modeled as an ideal gas. The combustion air and the products of combustion each form ideal gas mixtures.
4. Each mole of oxygen in the combustion air is accompanied by 3.76 moles of nitrogen, which is inert.

ANALYSIS: For combustion with *theoretical* air, the balanced chemical equation is

$$C_8H_{18} + 12.5(O_2 + 3.76N_2) \rightarrow 8CO_2 + 9H_2O + 47N_2$$

The balanced chemical equation for *120% theoretical* air is

$$C_8H_{18} + (1.20)(12.5)(O_2 + 3.76N_2) \rightarrow aCO_2 + bH_2O + cO_2 + dN_2$$

Applying conservation of mass to carbon, hydrogen, oxygen, and nitrogen, respectively

C: $8 = a$

H: $18 = 2b \rightarrow b = 9$

O: $2(12.5)(1.20) = 2a + b + 2c = 2(8) + 9 + 2c \rightarrow c = 2.5$

N: $2(3.76)(12.5)(1.20) = 2d \rightarrow d = 56.4$

The balanced chemical equation for combustion with *120% theoretical air* is

$$C_8H_{18} + 15(O_2 + 3.76N_2) \rightarrow 8CO_2 + 9H_2O + 2.5O_2 + 56.4N_2$$

The energy rate balance reduces with assumptions 1-3 to give

$$\frac{\dot{Q}_{cv}}{\dot{n}_F} = \bar{h}_P - \bar{h}_R$$

where

$$\bar{h}_R = \left(\bar{h}_f^\circ + \Delta\bar{h}\right)_{C_8H_{18}} + 15\left(\cancelto{0}{\bar{h}_f^\circ} + \cancelto{0}{\Delta\bar{h}}\right)_{O_2} + 56.4\left(\cancelto{0}{\bar{h}_f^\circ} + \cancelto{0}{\Delta\bar{h}}\right)_{N_2}$$

and

$$\bar{h}_P = 8\left(\bar{h}_f^\circ + \Delta\bar{h}\right)_{CO_2} + 9\left(\bar{h}_f^\circ + \Delta\bar{h}\right)_{H_2O} + 2.5\left(\cancelto{0}{\bar{h}_f^\circ} + \Delta\bar{h}\right)_{O_2} + 56.4\left(\cancelto{0}{\bar{h}_f^\circ} + \Delta\bar{h}\right)_{N_2}$$

With the enthalpy of formation for $C_8H_{18}(g)$ from Table A-25

$$\bar{h}_R = [\left(\bar{h}_f^\circ\right)_{C_8H_{18}(g)}]$$

$$= -208{,}450 \text{ kJ/kmol } (C_8H_{18})$$

With enthalpy of formation values for CO_2 and $H_2O(g)$ from Table A-25, and enthalpy values for CO_2, H_2O, O_2, and N_2 from Table A-23

$$\begin{aligned}
\bar{h}_P &= [8\left(\bar{h}_f^\circ + \Delta\bar{h}\right)_{CO_2} + 9\left(\bar{h}_f^\circ + \Delta\bar{h}\right)_{H_2O} + 2.5\left(\Delta\bar{h}\right)_{O_2} + 56.4\left(\Delta\bar{h}\right)_{N_2}] \\
&= [8\left(-393{,}520 + 53{,}848 - 9{,}364\right) + 9\left(-241{,}820 + 44{,}380 - 9{,}904\right) \\
&\quad + 2.5\left(38{,}447 - 8{,}682\right) + 56.4\left(36{,}777 - 8{,}669\right)] \\
&= -2{,}998{,}680 \text{ kJ/kmol}(C_8H_{18})
\end{aligned}$$

Solving for the molar flow rate of fuel

$$\dot{n}_F = \frac{\dot{Q}_{cv}}{\bar{h}_P - \bar{h}_R} = \frac{-2500 \text{ kW}}{[-2{,}998{,}680 - (-208{,}450)]\text{kJ/kmol}(C_8H_{18})}\left|\frac{1\frac{\text{kJ}}{\text{s}}}{1\text{ kW}}\right| = 8.96 \times 10^{-4} \text{ kmol}(C_8H_{18})\text{/s}$$

Multiplying the molar flow rate by the molecular weight of octane, we obtain

$$\dot{m}_F = M_F\dot{n}_F = \left(114.22\frac{\text{kg}(C_8H_{18})}{\text{kmol}(C_8H_{18})}\right)\left(8.96 \times 10^{-4}\frac{\text{kmol}(C_8H_{18})}{\text{s}}\right) = \underline{\mathbf{0.102\ kg(C_8H_{18})/s}}$$

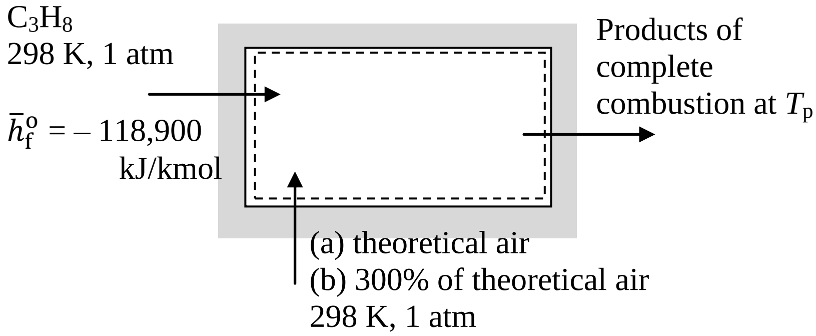
<u>Problem 13.51</u> (Corrected 12/14)

Liquid propane (C_3H_8) at 25^{o}C, 1 atm, enters a well-insulated reactor operating at steady state. Air enters at the same temperature and pressure. For liquid propane, $\bar{h}_f^o$ = – 118,900 kJ/kmol. Determine the temperature of the combustion products, in K, for complete combustion with
(a) the theoretical amount of air.
(b) 300% of theoretical air.

KNOWN: Liquid C_3H_8 and air, each at known temperature and pressure, enter a well-insulated reactor operating at steady state.

FIND: Determine the temperature of the products of complete combustion for (a) theoretical air, and (b) 300% of theoretical air.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The control volume is at steady state and $\dot{W}_{cv} = \dot{Q}_{cv}$ = 0. (2) 3.76 moles of N_2 accompany each kmol of O_2 in the air. (3) The N_2 is inert and combustion is complete. (4) Kinetic and potential energy effects can be neglected. (5) All gaseous streams are modeled as ideal gas mixtures.

ANALYSIS:

For complete combustion with the theoretical amount of air

$$C_3H_8 + 5(O_2 + 3.76\ N_2) \longrightarrow 3\ CO_2 + 4\ H_2O + 18.8\ N_2$$

For complete combustion with n times the theoretical amount of air

$$C_3H_8 + 5n\,(O_2 + 3.76\ N_2) \longrightarrow 3\ CO_2 + 4\ H_2O + 5(n-1)\ O_2 + 18.8n\ N_2$$

The energy balance reduces to

$$0 = [\bar{h}^o_{f_{C_3H_8(l)}} + \cancel{\Delta\bar{h}}] + 5n\,[\cancel{\bar{h}_f^o} + \cancel{\Delta\bar{h}}]_{O_2} + 18.8n\,[\cancel{\bar{h}_f^o} + \cancel{\Delta\bar{h}}]_{N_2} - 3\,[\bar{h}^o_{f_{CO_2}} + \bar{h}_{CO_2}(T_p) - \bar{h}_{CO_2}(298)]$$

$$-\,4\,[\bar{h}^o_{f_{H_2O}} + \bar{h}_{H_2O}(T_p) - \bar{h}_{H_2O}(298)] - 5(n-1)\,[\cancel{\bar{h}^o_{f_{O_2}}} + \bar{h}_{O_2}(T_p) - \bar{h}_{O_2}(298)]$$

$$-\,18.8n\,[\cancel{\bar{h}^o_{f_{N_2}}} + \bar{h}_{N_2}(T_p) - \bar{h}_{N_2}(298)] \qquad (*)$$

Rearranging

$$3\,\bar{h}_{CO_2}(T_p) + 4\,\bar{h}_{H_2O}(T_p) + 5(n-1)\,\bar{h}_{O_2}(T_p) + 18.8n\;\bar{h}_{N_2}(T_p)$$

$$= \bar{h}^o_{f_{C_3H_8(l)}} - 3\,[\bar{h}^o_{f_{CO_2}} - \bar{h}_{CO_2}(298)] - 4\,[\bar{h}^o_{f_{H_2O}} - \bar{h}_{H_2O}(298)] - 5(n-1)\,[-\bar{h}_{O_2}(298)]$$

$$- 18.8n\;[-\bar{h}_{N_2}(298)]$$

Inserting data from Tables A-23 and A-25

$$3\,\bar{h}_{CO_2}(T_p) + 4\,\bar{h}_{H_2O}(T_p) + 5(n-1)\,\bar{h}_{O_2}(T_p) + 18.8n\;\bar{h}_{N_2}(T_p)$$

$$= (-118{,}900) - 3\,[-393{,}520 - 9{,}364] - 4\,[-241{,}820 - 9{,}904] - 5(n-1)\,[-8{,}682] - 18.8n\,[-8{,}669]$$

$$= 206{,}387\,n + 2{,}053{,}238 \qquad (**)$$

Using data from Table A-23 with Eq. (**) and iterating to find T_p, we get

(a) for $n = 1$; $T_p \approx 2380$ K (theoretical air) ←

(b) for $n = 3$; $T_p \approx 1150$ K (300% of theoretical air) ←

Alternative Solution

An iterative solution can be avoided by using *Interactive Thermodynamics:IT* to solve Eq. (*) for $n = 1$, as follows:

```
h_C3H8 = -118900   //kJ/kmol
h_CO2_out = h_T("CO2",Tp)
h_H2O_out = h_T("H2O",Tp)
h_O2_out = h_T("O2",Tp)
h_N2_out = h_T("N2",Tp)
n = 1

// Energy Balance based on Eq. (*):
0 = h_C3H8  - 3*h_CO2_out - 4*h_H2O_out -5*(n-1)*h_O2_out - 18.8*n*h_N2_out
```

Solving for the cases of $n = 1$ and $n = 3$, *IT* gives

(a) $T_p = 2380$ K

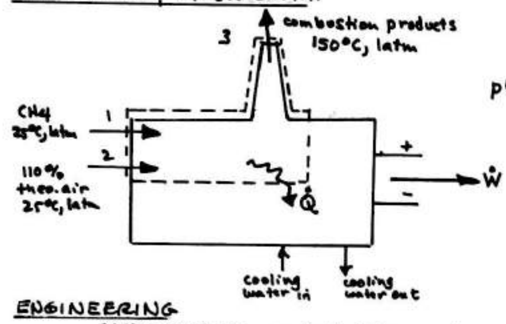
(b) $T_p = 1151$ K

PROBLEM 13.52

KNOWN: Steady state operating data are provided for a simple vapor plant.

FIND: Plot the mass flow rate of fuel required, in kg/h per MW of power developed, versus the plant thermal efficiency, η.

SCHEMATIC & GIVEN DATA:



plant thermal efficiency: $30 \le \eta \le 40\%$

ENGINEERING MODEL: (1) The control volume shown in the accompanying figure by the dashed line operates at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The combustion air and products of combustion are modeled as ideal gases.

ANALYSIS: By definition, $\eta = \dot{W}/\dot{Q} \Rightarrow \dot{Q} = \dot{W}/\eta$. For N MW of power developed generated,

$$\dot{Q} = \frac{N \times 10^3 \ \text{kJ/s}}{\eta} \qquad (1)$$

where η is expressed as a decimal.

To evaluate the mass flow rate of the fuel requires an energy rate balance, the chemical reaction equation, and this value for $\dot{Q}$.

Complete combustion of CH_4 with the theoretical amount of air is described by Eq. 13.4. Complete combustion with 110% of theoretical air is then

$$CH_4 + 2.2(O_2 + 3.76 N_2) \rightarrow CO_2 + 2H_2O + 0.2 O_2 + 8.272 N_2$$

Noting that $\dot{Q}$ is positive in the direction of the arrow on the above figure, an energy rate balance at steady state reduces to give

$$\frac{\dot{Q}}{\dot{n}_{CH_4}} = (\bar{h}_{CH_4})_1 + (2.2\bar{h}_{O_2} + 8.272\bar{h}_{N_2})_2 - (\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 0.2\bar{h}_{O_2} + 8.272\bar{h}_{N_2})_3$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$

$$\frac{\dot{Q}}{\dot{n}_{CH_4}} = (\bar{h}_f^\circ)_{CH_4} + (2.2\cancel{\bar{h}_f^\circ})_{O_2} + 8.272(\cancel{\bar{h}_f^\circ})_{N_2}) - [(\bar{h}_f^\circ + \bar{h}(423) - \bar{h}(298))_{CO_2} + 2(\bar{h}_f^\circ + \bar{h}(423) - \bar{h}(298))_{H_2O(g)}$$
$$+ 0.2[\bar{h}(423) - \bar{h}(298)]_{O_2} + 8.272(\bar{h}(423) - \bar{h}(298))_{N_2}]$$

Then, with data from the ideal gas tables

$$\frac{\dot{Q}}{\dot{n}_{CH_4}} = (-74,850) - [-393,520 + 14333 - 9364] - 2[-241,820 + 14,147 - 9,904] - 0.2[12,405 - 8,682]$$
$$- 8.272[12,313 - 8,669]$$
$$= 757,467 \ \text{kJ/kmol}(CH_4)$$

PROBLEM 13.52 (Cont'd.) - Page 2

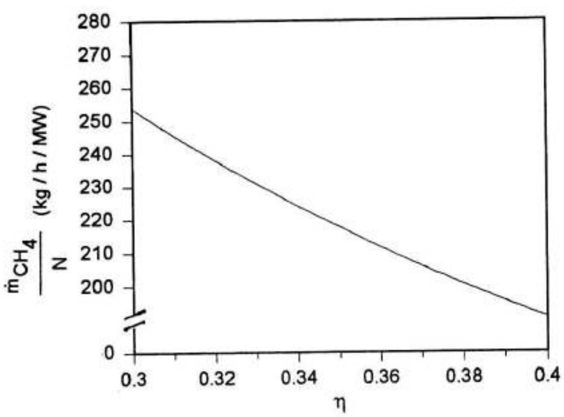
Combining this with (1)

$$\dot{n}_{CH_4} = \frac{N \times 10^3 \ kJ/s}{\eta\,(757{,}967 \ kJ/kmol)}$$

$$\dot{m}_{CH_4} = \frac{N \times 10^3 \ kJ/s}{\eta\,[757{,}967 \ kJ/kmol]} \left[\frac{16.04 \ kg}{kmol}\right] \left|\frac{3600 \ s}{h}\right|$$

$$\frac{\dot{m}_{CH_4}}{N} = \frac{76.18}{\eta} \left(\frac{kg/h}{MW}\right) \qquad (2)$$

Eq. (2) is a simple mathematical relation that can be plotted readily by hand or using computer software. The following plot is obtained using IT:



Note that as the thermal efficiency increases, less fuel is needed per MW of power developed, as expected.

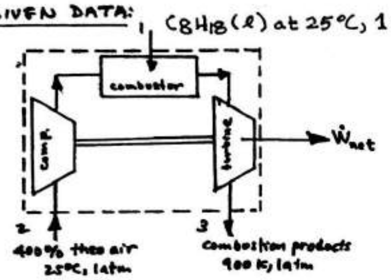
PROBLEM 13.53

Liquid octane (C_8H_{18}) at 25°C, 1 atm enters the combustor of a simple open gas turbine power plant and burns completely with 400% of theoretical air entering the compressor at 25°C, 1 atm. Products of combustion exit the turbine at 627°C, 1 atm. The rate of heat transfer from the gas turbine is estimated as 15% of the net power developed. Determine the net power developed, in kJ per kmol of fuel. Kinetic and potential energy effects are negligible.

KNOWN: C_8H_{18} at 25°C, 1atm enters the combustor of a simple open gas turbine power plant, and burns completely with 400% of theoretical air entering the compressor at 25°C, 1atm. Products of combustion exit the turbine at 900 K, 1atm. The rate of heat transfer from the power plant is 15% of the net power developed.

FIND: Determine the net power developed in kJ/kmol (fuel).

SCHEMATIC & GIVEN DATA:



$\dot{Q}_{cv} = -0.15\,\dot{W}_{net}$

ENGR. MODEL: (1) The control volume shown in the accompanying figure operates at steady state with negligible effects of kinetic and potential energy. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The ideal gas model is applicable to the combustion air and the products of combustion.

ANALYSIS: Complete combustion of C_8H_{18} with the theoretical amount of air is described by

$$C_8H_{18} + 12.5(O_2+3.76N_2) \rightarrow 8CO_2 + 9H_2O + 47N_2$$

Complete combustion with 400% of theoretical air is then

$$C_8H_{18} + 50(O_2+3.76N_2) \rightarrow 8CO_2 + 9H_2O + 37.5O_2 + 188N_2$$

An energy rate balance reduces at steady state to

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} - \frac{\dot{W}_{cv}}{\dot{n}_{fuel}} + (\bar{h}_{C_8H_{18}})_1 + (50\bar{h}_{O_2} + 188\bar{h}_{N_2})_2 - (8\bar{h}_{CO_2} + 9\bar{h}_{H_2O} + 37.5\bar{h}_{O_2} + 188\bar{h}_{N_2})_3$$

Then, with $\dot{Q}_{cv} = -0.15\dot{W}_{cv}$ and $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$

$$1.15\frac{\dot{W}_{cv}}{\dot{n}_{fuel}} = (\bar{h}_f^\circ)_{C_8H_{18}(l)} + (50\cancel{(\bar{h}_f^\circ)_{O_2}} + 188\cancel{(\bar{h}_f^\circ)_{N_2}}) - 8[\bar{h}_f^\circ + \bar{h}(900) - \bar{h}(298)]_{CO_2} - 9[\bar{h}_f^\circ + \bar{h}(900) - \bar{h}(298)]_{H_2O}$$
$$-37.5[\bar{h}(900) - \bar{h}(298)]_{O_2} - 188[\bar{h}(900) - \bar{h}(298)]_{N_2}$$

With data from Tables A-23 and A-25

$$1.15\frac{\dot{W}_{cv}}{\dot{n}_{fuel}} = -249{,}910 - 8[-393{,}520 + 37{,}405 - 9{,}364] - 9[-241{,}820 + 31{,}828 - 9{,}904]$$
$$-37.5[27{,}928 - 8{,}682] - 188(26{,}890 - 8{,}669)$$

$$= 505{,}713\ \frac{kJ}{kmol}$$

$\Rightarrow$ $\dfrac{\dot{W}_{cv}}{\dot{n}_{fuel}} = 439{,}750\ \dfrac{kJ}{kmol(fuel)}$ ← $\dfrac{\dot{W}_{cv}}{\dot{n}_{fuel}}$

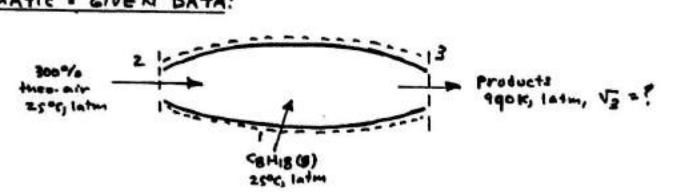
PROBLEM 13.54

Octane gas C_8H_{18} at 25°C enters a jet engine and burns completely with 300% of theoretical air entering at 25°C, 1 atm with a volumetric flow rate of 42 m^3/s. Products of combustion exit at 990 K, 1 atm. If the fuel and air enter with negligible velocities, determine the velocity of the exiting combustion products, in m/s. Neglect heat transfer between the engine and its surroundings.

KNOWN: $C_8H_{18}(g)$ at 25°C, 1atm enters a jet engine and burns completely with 300% of theoretical air entering at 25°C, 1atm. Products exit at 990 K, 1atm.

FIND: Determine the velocity of the exiting combustion products.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) The kinetic energies of the incoming air and fuel can be neglected. Potential energy can be neglected throughout. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The ideal gas model is applicable to the combustion air and the products of combustion.

ANALYSIS: Complete combustion of C_8H_{18} with the theoretical amount of air is described by

$$C_8H_{18} + 12.5(O_2 + 3.76N_2) \rightarrow 8CO_2 + 9H_2O + 47N_2$$

Complete combustion with 300% of theoretical air is then

$$C_8H_{18} + 37.5O_2 + 141N_2 \rightarrow 8CO_2 + 9H_2O + 25O_2 + 141N_2 \quad (1)$$

An energy rate balance at steady state reduces to give

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \underbrace{[\dot{n}_{C_8H_{18}}\bar{h}_{C_8H_{18}}]_1}_{(fuel)} + \underbrace{[\dot{n}_{O_2}\bar{h}_{O_2} + \dot{n}_{N_2}\bar{h}_{N_2}]_2}_{(air)} - \underbrace{\left\{[\dot{n}_{CO_2}\bar{h}_{CO_2} + \dot{n}_{H_2O}\bar{h}_{H_2O} + \dot{n}_{O_2}\bar{h}_{O_2} + \dot{n}_{N_2}\bar{h}_{N_2}] + \dot{m}_3\frac{V_3^2}{2}\right\}_3}_{(products)} \quad (2)$$

where $\dot{n}$ denotes a molar flow rate and $\dot{m}_3$ is the mass flow rate of the exiting products:

$$\dot{m}_3 = (\dot{m}_{CO_2} + \dot{m}_{H_2O} + \dot{m}_{O_2} + \dot{m}_{N_2})_3 = (M_{CO_2}\dot{n}_{CO_2} + M_{H_2O}\dot{n}_{H_2O} + M_{O_2}\dot{n}_{O_2} + M_{N_2}\dot{n}_{N_2})_3 \quad (3)$$

Dividing throughout by the molar flow rate of the fuel and solving for $V_3^2/2$, Eqs. (1) - (3) give

$$\frac{V_3^2}{2} = \frac{(\bar{h}_{C_8H_{18}})_1 + [37.5\bar{h}_{O_2} + 141\bar{h}_{N_2}]_2 - [8\bar{h}_{CO_2} + 9\bar{h}_{H_2O} + 25\bar{h}_{O_2} + 141\bar{h}_{N_2}]_3}{(8M_{CO_2} + 9M_{H_2O} + 25M_{O_2} + 141M_{N_2})}$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$ and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$\frac{V_3^2}{2} = \frac{(\bar{h}_f^\circ)_{C_8H_{18}(g)} - 8[\bar{h}_f^\circ + \bar{h}(990) - \bar{h}(298)]_{CO_2} - 9[\bar{h}_f^\circ + \bar{h}(990) - \bar{h}(298)]_{H_2O} - 25[\bar{h}(990) - \bar{h}(298)]_{O_2} - 141[\bar{h}(990) - \bar{h}(298)]_{N_2}}{(8M_{CO_2} + 9M_{H_2O} + 25M_{O_2} + 141M_{N_2})}$$

With data from Tables A-1, A-23, and A-25

$$\frac{V_3^2}{2} = \frac{(-208{,}450) - 8[-393{,}520 + 42{,}226 - 9{,}364] - 9[-241{,}820 + 35{,}472 - 9{,}904] - 25[31{,}041 - 8{,}682] - 141[29{,}803 - 8{,}669]}{(8(44.01) + 9(18.02) + 25(32.00) + 141(28.01))}$$

$$= \frac{-208{,}450 + 2{,}885{,}264 + 1{,}946{,}268 - 558{,}975 - 2{,}979{,}894}{5263.67} = 205.98 \frac{kJ}{kg(prod)}$$

Finally,

$$V_3 = \sqrt{2\left(205.98\frac{kJ}{kg}\right)\left|\frac{10^3 N\cdot m}{1 kJ}\right|\left|\frac{1 kg\cdot m/s^2}{1 N}\right|}$$

$$= 641.84 \text{ m/s}$$

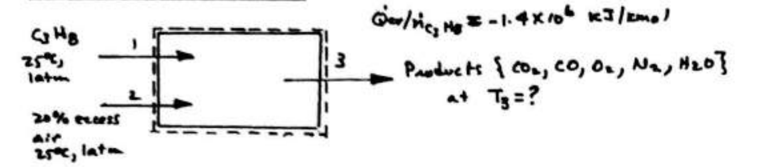
PROBLEM 13.55

Propane gas (C_3H_8) at 25°C, 1 atm enters a reactor operating at steady state and burns with 20% excess air entering at 25°C, 1 atm. Of the carbon entering with the fuel, 94% (molar basis) appears in the products as CO_2 and the rest as CO. Heat transfer from the reactor occurs at a rate of 1.4×10^6 kJ per kmol of propane. Ignoring kinetic and potential energy effects, determine the temperature of the combustion products exiting the reactor, in K.

KNOWN: C_3H_8 at 25°C, 1atm enters a furnace and burns with 20% excess air entering at 25°C, 1atm. Of the C entering with the fuel, 94% appears as CO_2 and 6% appears as CO. Heat transfer from the reactor occurs at the rate 1.4×10^6 kJ/kmol (C_3H_8).

FIND: Determine the temperature of the combustion products.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv}=0$ and negligible kinetic and potential energy effects. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The ideal gas model is applicable to the combustion air and the products of combustion.

ANALYSIS: Complete combustion of C_3H_8 with the theoretical amount of air is described by

$$C_3H_8 + 5[O_2 + 3.76 N_2] \rightarrow 3CO_2 + 4H_2O + 18.8 N_2$$

Combustion with 20% excess air in which 94% of the C entering goes to CO_2 and the rest to CO is described by

$$C_3H_8 + 6(O_2 + 3.76N_2) \rightarrow 2.82 CO_2 + 0.18 CO + 4H_2O + 1.09 O_2 + 22.56 N_2$$

At steady state an energy rate balance reduces to

$$\frac{\dot{Q}_{cv}}{\dot{n}_{C_3H_8}} = 2.82[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO_2} + 0.18[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO} + 4[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{H_2O}$$
$$+ 1.09[\bar{h}(T_3) - \bar{h}(298)]_{O_2} + 22.56[\bar{h}(T_3) - \bar{h}(298)]_{N_2} - (\bar{h}_f^\circ)_{C_3H_8} - 6\cancel{\bar{h}_{O_2}}^{\,0} - 22.56\cancel{\bar{h}_{N_2}}^{\,0}$$

With data from Tables A-23 and A-25

$$-1.4\times10^6 = 2.82[-393{,}520 + \bar{h}_{CO_2}(T_3) - 9364] + 0.18[-110{,}530 + \bar{h}_{CO}(T_3) - 8669] +$$
$$4[-241{,}820 + \bar{h}_{H_2O}(T_3) - 9904] + 1.09[\bar{h}_{O_2}(T_3) - 8682] + 22.56[\bar{h}_{N_2}(T_3) - 8669]$$
$$-(-103{,}850)$$

This gives

$$2.82\,\bar{h}_{CO_2}(T_3) + 0.18\,\bar{h}_{CO}(T_3) + 4\bar{h}_{H_2O}(T_3) + 1.09\,\bar{h}_{O_2}(T_3) + 22.56\,\bar{h}_{N_2}(T_3) = 865{,}671 \qquad (a)$$

1 Solving iteratively with Table A-23 data, $T_3 \approx 892$ K

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1. A first trial value can be obtained by assuming all products are N_2. That is, Eq. (a) becomes

$$30.65\,\bar{h}_{N_2}(T_3) = 865{,}671 \Rightarrow \bar{h}_{N_2}(T_3) = 28{,}244 \text{ kJ/kmol}$$

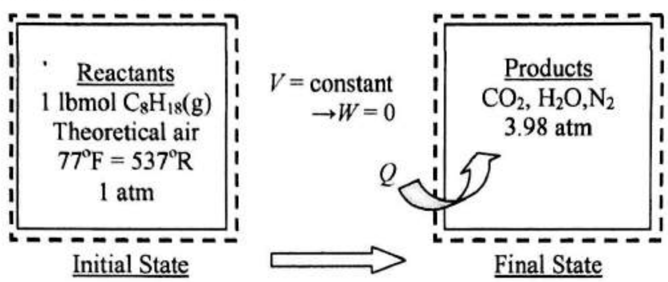
By inspection in Table A-23, $T_3 \approx 940$K

13.56 One lbmol of octane gas (C_8H_{18}) reacts with the theoretical amount of air in a closed, rigid tank. Initially, the reactants are at 77°F, 1 atm. After complete combustion the pressure in the tank is 3.98 atm. Determine the heat transfer, in Btu.

KNOWN: A rigid tank initially contains octane gas and air at known conditions. Complete combustion occurs and the final pressure is specified.

FIND: The heat transfer.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The contents of the closed, rigid tank are taken as the system (hence, the volume is constant).
2. Kinetic and potential energy effects are absent, and $W = 0$.
3. Combustion is complete.
4. The initial mixture and the products of combustion each form ideal gas mixtures.
5. Each mole of oxygen in the combustion air is accompanied by 3.76 moles of nitrogen, which is inert.
6. The initial and final states are equilibrium states.

ANALYSIS: For the closed system an energy balance reduces to

$$Q = \Delta U = U_P - U_R$$

To evaluate ΔU the final temperature is required. Using the ideal gas equation of state

$$pV = n\bar{R}\,T \rightarrow V = \frac{n\bar{R}T}{p}$$

Since the volume is constant

$$\frac{n_R\bar{R}T_R}{p_R} = \frac{n_P\bar{R}T_P}{p_P} \rightarrow T_P = T_R\left(\frac{n_R}{n_P}\right)\left(\frac{p_P}{p_R}\right) \qquad (1)$$

Problem 13.56 (Continued) – Page 2

The balanced chemical equation for the reaction is required to determine the numbers of moles of products and reactants. For complete combustion of one lbmol of octane gas with theoretical air

$$C_8H_{18} + a(O_2 + 3.76N_2) \rightarrow bCO_2 + cH_2O + dN_2$$

Applying conservation of mass to carbon, hydrogen, oxygen, and nitrogen, respectively

C: $8 = b$

H: $18 = 2c \rightarrow c = 9$

O: $2a = 2b + c = 2(8) + 9 \rightarrow a = 12.5$

N: $2(3.76)a = 2d = 2(3.76)(12.5) \rightarrow d = 47$

The balanced chemical equation for complete combustion with theoretical air is

$$C_8H_{18} + 12.5(O_2 + 3.76N_2) \rightarrow 8CO_2 + 9H_2O + 47N_2$$

Thus, $n_R = 1 + 12.5(4.76) = 60.5$ lbmol and $n_P = 8 + 9 + 47 = 64$ lbmol.

Substituting values into (1)

$$T_P = (537^\circ\mathrm{R})\left(\frac{60.5\ \mathrm{lbmol}}{64\ \mathrm{lbmol}}\right)\left(\frac{3.98\ \mathrm{atm}}{1\ \mathrm{atm}}\right) = 2020^\circ\mathrm{R}$$

The energy balance reduces with assumptions 1-3 to give

$$Q = U_P - U_R$$

where

$$U_R = [\bar{u}_{C_8H_{18}} + 12.5\bar{u}_{O_2} + 47\bar{u}_{N_2}]_R$$

and

$$U_P = [8\bar{u}_{CO_2} + 9\bar{u}_{H_2O} + 47\bar{u}_{N_2}]_P$$

With $\bar{u} = \bar{h} - \bar{R}T$ and $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$

$$U_R = [(\bar{h}_f^\circ + \overset{0}{\cancel{\Delta\bar{h}}} - \bar{R}T)_{C_8H_{18}} + 12.5(\overset{0}{\cancel{\bar{h}_f^\circ}} + \overset{0}{\cancel{\Delta\bar{h}}} - \bar{R}T)_{O_2} + 47(\overset{0}{\cancel{\bar{h}_f^\circ}} + \overset{0}{\cancel{\Delta\bar{h}}} - \bar{R}T)_{N_2}]_R$$

and

$$U_P = [8(\bar{h}_f^\circ + \Delta\bar{h} - \bar{R}T)_{CO_2} + 9(\bar{h}_f^\circ + \Delta\bar{h} - \bar{R}T)_{H_2O} + 47(\overset{0}{\cancel{\bar{h}_f^\circ}} + \Delta\bar{h} - \bar{R}T)_{N_2}]_P$$

The energy balance reduces to

$$Q = [8\left(\bar{h}_f^\circ + \Delta\bar{h}\right)_{CO_2} + 9\left(\bar{h}_f^\circ + \Delta\bar{h}\right)_{H_2O} + 47\left(\Delta\bar{h}\right)_{N_2} - 64\bar{R}T_P] - [\left(\bar{h}_f^\circ\right)_{C_8H_{18}} - 60.5\bar{R}T_R]$$

With data for $C_8H_{18}(g)$ from Table A-25E

$$U_R = \left(1\ \text{lbmol}\ C_8H_{18}\right)\left(-89{,}680\frac{\text{Btu}}{\text{lbmol}(C_8H_{18})}\right) - (60.5\ \text{lbmol})\left(1.986\frac{\text{Btu}}{\text{lbmol}\cdot{}^\circ\text{R}}\right)(537^\circ\text{R})$$

$$= -154{,}202\ \text{Btu}$$

With enthalpy of formation values for CO_2 and $H_2O(g)$ from Table A-25E, and enthalpy values for CO_2, H_2O, and N_2 from Table A-23E

$$U_P = [(8\ \text{lbmol}\ CO_2)(-169{,}300 + 21{,}284 - 4{,}027.5)\frac{\text{Btu}}{\text{lbmol}\ CO_2}$$

$$+ (9\ \text{lbmol}\ H_2O)(-104{,}040 + 17{,}643 - 4{,}258.0)\frac{\text{Btu}}{\text{lbmol}\ H_2O}$$

$$+ (47\ \text{lbmol}\ N_2)(14{,}694 - 3{,}729.5)\frac{\text{Btu}}{\text{lbmol}\ N_2} - (64\ \text{lbmol})\left(1.986\frac{\text{Btu}}{\text{lbmol}\cdot{}^\circ\text{R}}\right)(2020^\circ\text{R})]$$

$$= -1{,}773{,}662\ \text{Btu}$$

Solving for heat transfer

$$Q = -1,773{,}662\ \text{Btu} - (-154{,}202\ \text{Btu}) = \underline{\mathbf{-1{,}619,460\ Btu}}$$

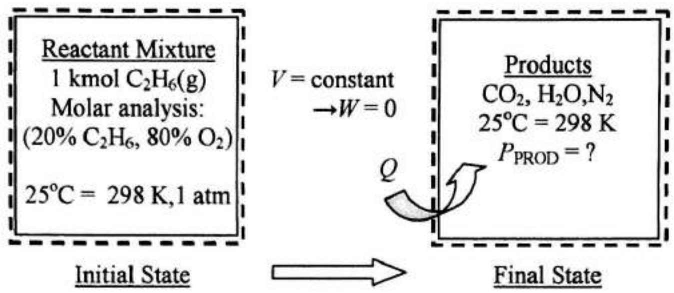
*The negative sign associated with heat transfer indicates heat transfer is **from** the system, as expected.*

13.57 A closed, rigid vessel initially contains a gaseous mixture at 25°C, 1 atm with the molar analysis of 20% ethane (C_2H_6), 80% oxygen (O_2). The initial mixture contains one kmol of ethane. Complete combustion occurs, and the products are cooled to 25°C. Determine the heat transfer, in kJ, and the final pressure, in atm.

KNOWN: A closed, rigid vessel initially contains ethane gas and oxygen at known conditions and with a known molar analysis. The mixture initially contains 1 kmol of ethane. The mixture burns completely and products are cooled to the initial temperature.

FIND: Determine the heat transfer and the final pressure.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The contents of the closed, rigid vessel are taken as the system.
2. Kinetic and potential energy effects are absent, and $W = 0$.
3. Combustion is complete.
4. The initial mixture and the products of combustion each form ideal gas mixtures.
5. The initial and final states are equilibrium states.

ANALYSIS: An energy balance reduces to

$$Q = \Delta U = U_P - U_R$$

The balanced chemical equation for the reaction is required to determine the numbers of moles of products and reactants. Since combustion is complete, there must be at least the theoretical amount of oxygen present. To determine if excess oxygen is present, we begin with the balanced chemical equation for complete combustion in oxygen.

$$C_2H_6 + aO_2 \rightarrow bCO_2 + cH_2O$$

Applying conservation of mass to carbon, hydrogen, and oxygen, respectively

C: $2 = b$

Problem 13.57 (Continued) – Page 2

H: $6 = 2c \rightarrow c = 3$

O: $2a = 2b + c = 2(2) + 3 \rightarrow a = 3.5$

The balanced chemical equation for complete combustion with oxygen is

$$C_2H_6 + 3.5O_2 \rightarrow 2CO_2 + 3H_2O$$

Based on the molar analysis for ethane and oxygen in the vessel, for the one kmol of ethane gas, four kmol of oxygen are present. Since more oxygen than that required for complete combustion is present in the initial mixture, excess oxygen will also be present in the products of combustion. The chemical equation for the actual reaction is

$$C_2H_6 + 4O_2 \rightarrow aCO_2 + bH_2O + cO_2$$

Applying conservation of mass to carbon, hydrogen, and oxygen, respectively

C: $2 = a$

H: $6 = 2b \rightarrow c = 3$

O: $2(4) = 2a + b + 2c = 2(2) + 3 + 2c \rightarrow c = 0.5$

The balanced chemical equation for the actual combustion is

$$C_2H_6 + 4O_2 \rightarrow 2CO_2 + 3H_2O + 0.5O_2$$

The energy balance reduces with assumptions 1-3 to give

$$Q = U_P - U_R$$

where

$$U_R = [\bar{u}_{C_2H_6} + 4\bar{u}_{O_2}]_R$$

and

$$U_P = [2\bar{u}_{CO_2} + 3\bar{u}_{H_2O} + 0.5\bar{u}_{O_2}]_P$$

With $\bar{u} = \bar{h} - \bar{R}T$ and $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$

$$U_R = [(\bar{h}_f^\circ + \cancelto{0}{\Delta\bar{h}} - \bar{R}T)_{C_2H_6} + 4(\cancelto{0}{\bar{h}_f^\circ} + \cancelto{0}{\Delta\bar{h}} - \bar{R}T)_{O_2}]_R$$

and

$$U_P = [2(\bar{h}_f^\circ + \cancelto{0}{\Delta\bar{h}} - \bar{R}T)_{CO_2} + 3(\cancelto{0}{\bar{h}_f^\circ} + \cancelto{0}{\Delta\bar{h}} - \bar{R}T)_{H_2O} + 0.5(\cancelto{0}{\bar{h}_f^\circ} + \Delta\bar{h} - \bar{R}T)_{O_2}]_P$$

Thus

$$U_R = \left(\bar{h}_f^o\right)_{C_2H_6} - 5\bar{R}T_R$$

With enthalpy of formation value for C_2H_6 from Table A-25

$$U_R = (1\text{ kmol } C_2H_6)\left(-84{,}680\frac{\text{kJ}}{\text{kmol } C_2H_6}\right) - (5\text{ kmol})\left(8.314\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(298\text{ K})$$

$$U_R = -97{,}068\text{ kJ}$$

Similarly for the products

$$U_P = [2\left(\bar{h}_f^o\right)_{CO_2} + 3\left(\bar{h}_f^o\right)_{H_2O} - 5.5\bar{R}T_P]$$

And, with enthalpy of formation values for CO_2 and $H_2O(g)$ from Table A-25

$$U_P = [(2\text{ kmol } CO_2)\left(-393{,}520\frac{\text{kJ}}{\text{kmol } CO_2}\right) + (3\text{ kmol } H_2O)\left(-241{,}820\frac{\text{kJ}}{\text{kmol } H_2O}\right)$$
$$-(5.5\text{ kmol})\left(8.314\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(298\text{ K})]$$

$$U_P = -1{,}526{,}127\text{ kJ}$$

Solving for heat transfer

$$Q = -1{,}526{,}127\text{ kJ} - (-97{,}068\text{ kJ}) = \underline{\mathbf{-1{,}429{,}059\text{ kJ}}}$$

*The negative sign associated with heat transfer indicates heat transfer is **from** the system, as expected.*

Using the ideal gas equation of state to determine the final pressure

$$pV = n\bar{R}T \rightarrow V = \frac{n\bar{R}T}{p}$$

Since volume is constant during the reaction and temperature of the reactants and the products is the same

$$\frac{n_R\bar{R}T_R}{p_R} = \frac{n_P\bar{R}T_P}{p_P} \rightarrow p_P = p_R\left(\frac{n_P}{n_R}\right)$$

From the balanced chemical equation for the reaction

$n_R = 1 + 4 = 5$ kmol reactants
$n_P = 2 + 3 + 0.5 = 5.5$ kmol products

Substituting values and solving for pressure of the products

$$p_P = (1\text{ atm})\left(\frac{5.5\text{ kmol}}{5\text{ kmol}}\right) = \underline{\mathbf{1.1\text{ atm}}}$$

PROBLEM 13.58

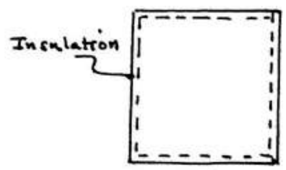
A mixture of 1 kmol of hydrogen (H_2) and x kmol of oxygen (O_2), initially at 25°C and 1 atm, burns completely in a closed, rigid, insulated container. The container finally holds a mixture of water vapor and O_2 at 3000 K. The ideal gas model applies to each mixture and there is no change in kinetic or potential energy between the initial and final states. Determine

(a) the value of x.
(b) the final pressure, in atm.

KNOWN: A mixture of hydrogen and oxygen, initially at 25°C, 1 atm, burns completely in closed insulated container. The products are at 3000K.

FIND: Determine the amount of oxygen present initially and the pressure of the products.

SCHEMATIC & GIVEN DATA:



$H_2 + x O_2 \rightarrow$
$H_2O + (x-0.5)\, O_2$
Initially: 25°C, 1 atm
Finally: 3000 K, p_P

Engineering Model:

1. The closed system shown by the dashed line initially contains H_2 and O_2 and finally $H_2O(g)$ and O_2
2. $Q = W = 0$. For the system there is no change in kinetic or potential energy.
3. The ideal gas model applies to the initial and final mixtures.

Analysis: For complete combustion, $1 H_2 + x O_2 \rightarrow \alpha H_2O + \beta O_2$.

Balancing: Hydrogen, $1 = \alpha$. Oxygen, $2x = 1 + 2\beta$, $\beta = (x - 0.5)$.

$$\Rightarrow H_2 + x O_2 \rightarrow H_2O + [x-0.5]\, O_2$$

(a) With assumptions 1 and 2, the closed system energy balance reads

$$U_P - U_R = \cancel{Q} - \cancel{W} \Rightarrow \left[1\, \bar{u}_{H_2O(g)} + (x-0.5)\, \bar{u}_{O_2}\right]_P - \left[\bar{u}_{H_2} + x\, \bar{u}_{O_2}\right]_R = 0$$

Then, with assumption 3, $\bar{u} = \bar{h} - \bar{R}T$, we get

$$\left[1\left(\bar{h}_{H_2O(g)} - \bar{R}T_P\right) + (x-0.5)\left(\bar{h}_{O_2} - \bar{R}T_P\right)\right] - \left[\left(\bar{h}_{H_2} - \bar{R}T_R\right) + x\left(\bar{h}_{O_2} - \bar{R}T_R\right)\right] = 0$$

(1) $$\Rightarrow \left[\bar{h}_{H_2O(g)} + (x-0.5)\,\bar{h}_{O_2}\right] - \left[\bar{h}_{H_2} + x\,\bar{h}_{O_2}\right] - (x+0.5)\bar{R}T_P + (1+x)\bar{R}T_R = 0$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, this becomes

$$\left[\bar{h}_f^\circ + \Delta\bar{h}\right]_{H_2O(g)} + (x-0.5)\left[\cancel{\bar{h}_f^\circ}^{\,0} + \Delta\bar{h}\right]_{O_2} - \left[\cancel{\bar{h}_f^\circ}^{\,0} + \cancel{\Delta\bar{h}}^{\,0}\right]_{H_2} - x\left[\cancel{\bar{h}_f^\circ} + \cancel{\Delta\bar{h}}\right]_{O_2}$$
$$- (x+0.5)\bar{R}T_P + (1+x)\bar{R}T_R = 0$$

With data from Table A-23,

$$\left[-241{,}820 + (136{,}264 - 9904)\right] + (x-0.5)\left[106{,}780 - 8682\right]$$
$$- (x+0.5)\bar{R}(3000\,K) + (1+x)\bar{R}(298.15\,K) = 0$$

Collecting terms and solving, $x = 2.31$.

(b) Applying the ideal gas model equation of state,

$$P_R V = n_R \bar{R} T_R$$

$$P_P V = n_P \bar{R} T_P$$

$$\Rightarrow \quad P_P = P_R \left[\frac{n_P}{n_R}\right]\left[\frac{T_P}{T_R}\right]$$

$$= 1\,\text{atm}\left[\frac{1 + 1.81}{1 + 2.31}\right]\left[\frac{3000\,\text{K}}{298.15\,\text{K}}\right]$$

$$= 8.54\,\text{atm}$$

---

1. This expression corresponds to Eq. 13.17b.

13.59 Calculate the enthalpy of combustion of gaseous pentane (C_5H_{12}), in kJ per kmol of fuel, with water vapor in the products at 298K.

KNOWN: Gaseous pentane reacts completely with air. The reactants and products are at 298 K, with water vapor in the products.

FIND: The enthalpy of combustion.

ENGINEERING MODEL:
1. Combustion is with the theoretical amount of air.
2. Each mole of oxygen in the combustion air is accompanied by 3.76 moles of nitrogen, which is inert.
3. The combustion air and the gaseous products of combustion each form ideal gas mixtures at 1 atm.

ANALYSIS: The balanced chemical equation for complete combustion of gaseous pentane with the theoretical amount of air is

$$C_5H_{12}(g) + 8(O_2 + 3.76N_2) \rightarrow 5CO_2 + 6H_2O + 30.08N_2$$

For water vapor in the products at 298 K, the enthalpy of combustion per kmol of C_5H_{12} at 298 K, 1 atm is

$$\bar{h}^{\circ}_{RP} = 5\left(\bar{h}^{\circ}_{f}\right)_{CO_2} + 6\left(\bar{h}^{\circ}_{f}\right)_{H_2O(g)} - \left(\bar{h}^{\circ}_{f}\right)_{C_5H_{12}(g)}$$

With data for the enthalpies of formation from Table A-25

$$\bar{h}^{\circ}_{RP} = (5)(-393,520) + (6)(-241,820) - (-146,440)$$

$$= \underline{\mathbf{-3.272 \times 10^6\ kJ/kmol(C_5H_{12})}}$$

PROBLEM 13.60

Plot the enthalpy of combustion for gaseous propane (C_3H_8), in Btu per lbmol of fuel, at 1 atm versus temperature in the interval 77 to 500°F. Assume water vapor in the products. For propane, let c_p = 0.41 Btu/lb · °R.

KNOWN: $C_3H_8(g)$ reacts completely with air. The reactants and products are each at 1 atm and temperature T. For $C_3H_8(g)$, $c_p = 0.41$ Btu/lb·°R.

FIND: Plot the enthalpy of combustion versus T for $77 \le T \le 500$ °F.

ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (2) Combustion is with theoretical amount of air. (3) In the products, H_2O is a vapor. (4) The combustion air and combustion products can be modeled as
1 ideal gases. (5) For $C_3H_8(g)$, c_p is constant.

ANALYSIS: Complete combustion of C_3H_8 with the theoretical amount of air is described by

$$C_3H_8(g) + 5(O_2 + 3.76 N_2) \rightarrow 3CO_2 + 4H_2O(g) + 18.8 N_2$$

The enthalpy of combustion is

$$\bar{h}_{RP} = 3\bar{h}_{CO_2} + 4\bar{h}_{H_2O(g)} - \bar{h}_{C_3H_8(g)} - 5\bar{h}_{O_2} \qquad (1)$$

$$= 3[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{CO_2} + 4[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{H_2O(g)}$$

$$- [\bar{h}_f^\circ + \Delta\bar{h}]_{C_3H_8(g)} - 5[\cancel{\bar{h}_f^\circ}^{\,0} + \bar{h}(T) - \bar{h}(537)]_{O_2}$$

$$= \left\{3(\bar{h}_f^\circ)_{CO_2} + 4(\bar{h}_f^\circ)_{H_2O(g)} - (\bar{h}_f^\circ)_{C_3H_8(g)}\right\} + 3[\bar{h}(T) - \bar{h}(537)]_{CO_2}$$

$$+ 4[\bar{h}(T) - \bar{h}(537)]_{H_2O(g)} - [\bar{c}_p(T - 537)]_{C_3H_8(g)} - 5[\bar{h}(T) - \bar{h}(537)]_{O_2}$$

Sample Calculation: T = 960°R (500°F). With data from Table A-23E and A-25E

$$\bar{h}_{RP} = \{3(-169{,}300) + 4(-104{,}040) - (-44680)\} + 3[\bar{h}(T) - 4028]_{CO_2} +$$

$$4[\bar{h}(T) - 4258]_{H_2O(g)} - (0.41)(44.09)[T - 537] - 5[\bar{h}(T) - 3725]_{O_2}$$

$$= -879{,}830 \frac{kJ}{kmol(fuel)} + 3[\bar{h}(T) - 4028]_{CO_2} + 4[\bar{h}(T) - 4258]_{H_2O(g)}$$

$$- \underbrace{(0.41)(44.04)}_{18.08}[T - 537] - 5[\bar{h}(T) - 3725]_{O_2}$$

$$= -879{,}830 + 3[8244 - 4028] + 4[7738 - 4258] - 18.08[960 - 537] - 5[6786 - 3725]$$

$$= -875{,}165 \text{ Btu/lbmol (fuel)}$$

The data for the required plot are obtained using IT, as follows:

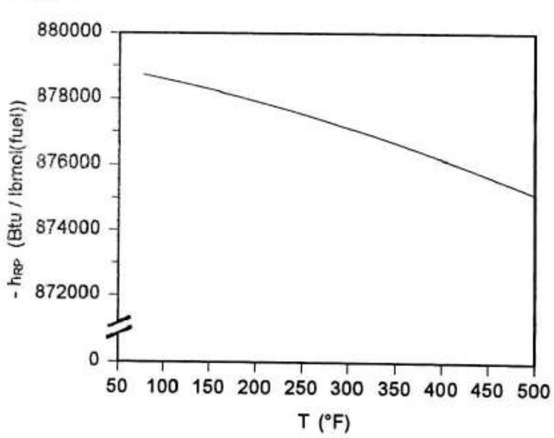
IT Code

```
T = 500  // °F

hCO2 = h_T("CO2",T)
hH2O = h_T("H2O",T)
hC3H8 = - 44680 + (0.41) * (44.04) * (T - 77)
hO2 = h_T("O2",T)

-hRP = 3 * hCO2 + 4 * hH2O - hC3H8 - 5 * hO2
```

IT Reults for T = 500°F

$- h_{RP} = 8.751 \times 10^5$ Btu / lbmol (fuel)

PLOT:



Note that h_{RP} varies only slightly with temperature in the range shown.

1. The results would differ only slightly if the variation of c_p with temperature were considered. IT includes a function for C_3H_8 that accounts for such variation.

13.61 Plot the enthalpy of combustion for gaseous methane (CH_4), in Btu per lbmol of fuel, at 1 atm versus temperature in the interval from 537 to 1800°R. Assume water vapor in the products. For methane, let $\bar{c}_p = 4.52 + 7.37(T/1000)$ Btu/lbmol · °R, where T is in °R.

KNOWN: $CH_4(g)$ reacts completly with air. The reactants and products are each at 1 atm and temperature T. For CH_4,

$$\bar{c}_p = 4.52 + 7.37(T/1000) \text{ Btu/lbmol}\cdot{}^\circ R, \quad T \text{ in } {}^\circ R.$$

FIND: Plot $\bar{h}_{RP}$ versus T for $537 \le T \le 1800°R$.

ENGINEERING MODEL: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (2) Combustion is with the theoretical amount of air. N_2 is inert. (3) In the products, H_2O is a vapor. (4) The CH_4, combustion air, and combustion products can be modeled as ideal gases.

ANALYSIS: Complete combustion of CH_4 with the theoretical amount of air is described by

$$CH_4 + 2O_2 + 7.52N_2 \rightarrow CO_2 + 2H_2O(g) + 7.52N_2$$

The enthalpy of combustion is

$$\begin{aligned}\bar{h}_{RP} &= \bar{h}_{CO_2} + 2\bar{h}_{H_2O(g)} - \bar{h}_{CH_4} - 2\bar{h}_{O_2} \\ &= [\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{CO_2} + 2[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{H_2O(g)} - [\bar{h}_f^\circ + \Delta\bar{h}]_{CH_4} - 2[\cancelto{0}{\bar{h}_f^\circ} + \bar{h}(T) - \bar{h}(298)]_{O_2} \\ &= \{(\bar{h}_f^\circ)_{CO_2} + 2(\bar{h}_f^\circ)_{H_2O(g)} - (\bar{h}_f^\circ)_{CH_4}\} + [\bar{h}(T) - \bar{h}(537)]_{CO_2} + 2[\bar{h}(T) - \bar{h}(537)]_{H_2O(g)} \\ &\quad - (\Delta\bar{h})_{CH_4} - 2[\bar{h}(T) - \bar{h}(298)]_{O_2} \qquad (1)\end{aligned}$$

where

$$(\Delta\bar{h})_{CH_4} = \int_{537°R}^{T}\left[4.52 + 7.37\left(\frac{T}{1000}\right)\right]dT = 4.52[T - 537] + \frac{7.37}{2}\left[\frac{T^2 - (537)^2}{1000}\right] \qquad (2)$$

Sample Calculation: $T = 900°R$. With data for ideal gases from Table A-23E and $(\bar{h}_f^\circ)_{CH_4}$ from Table A-25E

$$\begin{aligned}\bar{h}_{RP} &= \{(-169{,}300) + 2(-104{,}040) - (-32{,}210)\} + [7598 - 4028] + 2[7231 - 4258] \\ &\quad - \left\{4.52[900 - 537] + \frac{7.37}{2}\left[\frac{900^2 - 537^2}{1000}\right]\right\} - 2[6338 - 3725] \\ &= -344{,}446 \text{ Btu/lbmol (fuel)}\end{aligned}$$

The data for the required plot are obtained using IT, as follows:

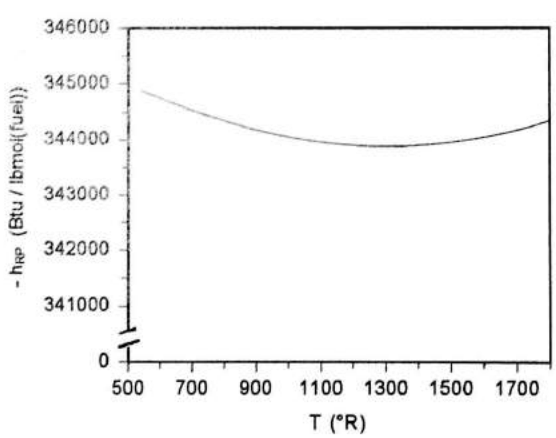
IT Code

```
T = 900 // °R

hCO2 = h_T("CO2", T)
hH2O = h_T("H2O", T)
hCH4 = -32210 + 4.52 * (T - 537) +
      (7.37 / 2000) * (T^2 - 537^2)
hO2 = h_T("O2", T)

-hRP = hCO2 + 2 * hH2O - hCH4 - 2 * hO2
```

IT result for T = 900°R

$-\bar{h}_{RP} = 3.442 \times 10^5$ Btu / lbmol(fuel)

PLOT:



From the plot we see that $\bar{h}_{RP}$ varies only slightly with temperature over the range from 537 to 1800°R. Interestingly, the curve exhibits a minimum within this range.

13.62 Determine the lower heating value, in kJ per kmol of fuel and in kJ per kg of fuel, at 25°C, 1 atm for

(a) gaseous ethane (C_2H_6)
(b) liquid ethanol (C_2H_5OH)
(c) gaseous propane (C_3H_8)
(d) liquid octane (C_8H_{18})

KNOWN: Specified fuels react completely with the theoretical amount of air. The reactants and products are both at 25°C, 1 atm.

FIND: The lower heating value for (a) gaseous ethane (C_2H_6), (b) liquid ethanol (C_2H_5OH), (c) gaseous propane (C_3H_8), and (d) liquid octane (C_8H_{18}). Compare with Table A-25 data.

ENGINEERING MODEL:
1. Combustion is with the theoretical amount of air.
2. Each mole of oxygen in the combustion air is accompanied by 3.76 moles of nitrogen, which is inert.
3. The combustion air and the products of combustion each form ideal gas mixtures.
4. The water formed during combustion is in the vapor phase in the products.

ANALYSIS: The *lower heating value* (LHV) is a positive number equal to the magnitude of the enthalpy of combustion at 298 K, 1 atm, when all the water formed by combustion is a vapor.

(a) Gaseous Ethane (C_2H_6): The balanced chemical equation for complete combustion of ethane with the theoretical amount of air is

$$C_2H_6 + 3.5(O_2 + 3.76N_2) \rightarrow 2CO_2 + 3H_2O(g) + 13.16N_2$$

The enthalpy of combustion per kmol of C_2H_6 at 25°C, 1 atm is

$$\bar{h}_{RP}^{o} = 2\left(\bar{h}_f^o\right)_{CO_2} + 3\left(\bar{h}_f^o\right)_{H_2O(g)} - \left(\bar{h}_f^o\right)_{C_2H_6}$$

$$\bar{h}_{RP}^{o} = \left(\frac{2\text{ kmol CO}_2}{\text{kmol C}_2\text{H}_6}\right)\left(-393{,}520\frac{\text{kJ}}{\text{kmol CO}_2}\right) + \left(\frac{3\text{ kmol H}_2\text{O}}{\text{kmol C}_2\text{H}_6}\right)\left(-241{,}820\frac{\text{kJ}}{\text{kmol H}_2\text{O}}\right)$$
$$-\left(-84{,}680\frac{\text{kJ}}{\text{kmol C}_2\text{H}_6}\right) = -1{,}427{,}820\text{ kJ/kmol(C}_2\text{H}_6)$$

Accordingly

$$\overline{\text{LHV}} = \underline{\mathbf{1{,}427{,}820\text{ kJ/kmol C}_2\text{H}_6}}$$

Dividing molar lower heating value by molecular weight of ethane, lower heating value on a mass basis is

$$\text{LHV} = \frac{1{,}427{,}820 \dfrac{\text{kJ}}{\text{kmol}\, C_2H_6}}{30.07 \dfrac{\text{kg}\, C_2H_6}{\text{kmol}\, C_2H_6}} = \underline{\mathbf{47{,}483\ kJ/kg\ C_2H_6}}$$

This value compares favorably with that in Table A-25.

(b) Liquid Ethanol (C_2H_5OH): The balanced chemical equation for complete combustion of liquid ethanol with the theoretical amount of air is

$$C_2H_5OH(l) + 3(O_2 + 3.76N_2) \rightarrow 2CO_2 + 3H_2O(g) + 11.28N_2$$

The enthalpy of combustion per kmol of C_2H_5OH at 25°C, 1 atm is

$$\bar{h}^{\circ}_{RP} = 2\left(\bar{h}^{\circ}_{f}\right)_{CO_2} + 3\left(\bar{h}^{\circ}_{f}\right)_{H_2O(g)} - \left(\bar{h}^{\circ}_{f}\right)_{C_2H_5OH(l)}$$

$$\bar{h}^{\circ}_{RP} = \left(\frac{2\ \text{kmol}\, CO_2}{\text{kmol}\, C_2H_5OH}\right)\left(-393{,}520 \frac{\text{kJ}}{\text{kmol}\, CO_2}\right) + \left(\frac{3\ \text{kmol}\, H_2O}{\text{kmol}\, C_2H_5OH}\right)\left(-241{,}820 \frac{\text{kJ}}{\text{kmol}\, H_2O}\right)$$
$$- \left(-277{,}690 \frac{\text{kJ}}{\text{kmol}\, C_2H_5OH}\right) = -1{,}234{,}810\ \text{kJ/kmol}\,(C_2H_5OH)$$

Accordingly

$$\overline{\text{LHV}} = \underline{\mathbf{1{,}234{,}810\ kJ/kmol\ C_2H_5OH}}$$

Dividing molar lower heating value by molecular weight of ethanol, the lower heating value on a mass basis is

$$\text{LHV} = \frac{1{,}234{,}810 \dfrac{\text{kJ}}{\text{kmol}\, C_2H_5OH}}{46.07 \dfrac{\text{kg}\, C_2H_5OH}{\text{kmol}\, C_2H_5OH}} = \underline{\mathbf{26{,}803\ kJ/kg\ C_2H_5OH}}$$

This value compares favorably with that in Table A-25.

(c) Gaseous Propane (C_3H_8): The balanced chemical equation for complete combustion of propane with the theoretical amount of air is

$$C_3H_8 + 5(O_2 + 3.76N_2) \rightarrow 3CO_2 + 4H_2O(g) + 18.8N_2$$

The enthalpy of combustion per kmol of C_3H_8 at 25°C, 1 atm is

$$\bar{h}^{o}_{RP} = 3\left(\bar{h}^{o}_{f}\right)_{CO_2} + 4\left(\bar{h}^{o}_{f}\right)_{H_2O(g)} - \left(\bar{h}^{o}_{f}\right)_{C_3H_8}$$

$$\bar{h}^{o}_{RP} = \left(\frac{3\,\text{kmol}\,CO_2}{\text{kmol}\,C_3H_8}\right)\left(-393{,}520\frac{\text{kJ}}{\text{kmol}\,CO_2}\right) + \left(\frac{4\,\text{kmol}\,H_2O}{\text{kmol}\,C_3H_8}\right)\left(-241{,}820\frac{\text{kJ}}{\text{kmol}\,H_2O}\right)$$

$$-\left(-103{,}850\frac{\text{kJ}}{\text{kmol}\,C_3H_8}\right) = -2{,}0437{,}990\ \text{kJ/kmol}\,C_3H_8$$

Accordingly

$$\overline{\text{LHV}} = \underline{\mathbf{2{,}043{,}990\ kJ/kmol\ C_3H_8}}$$

Dividing molar lower heating value by molecular weight of ethane, lower heating value on a mass basis is

$$\text{LHV} = \frac{2{,}043{,}990\dfrac{\text{kJ}}{\text{kmol}\,C_3H_8}}{44.09\dfrac{\text{kg}\,C_3H_8}{\text{kmol}\,C_3H_8}} = \underline{\mathbf{46{,}359\ kJ/kg\ C_3H_8}}$$

This value compares favorably with that in Table A-25.

(d) Liquid Octane (C_8H_{18}): The balanced chemical equation for complete combustion of octane with the theoretical amount of air is

$$C_8H_{18} + 12.5(O_2 + 3.76N_2) \rightarrow 8CO_2 + 9H_2O(g) + 47N_2$$

The enthalpy of combustion per kmol of C_8H_{18} at 25°C, 1 atm is

$$\bar{h}^{o}_{RP} = 8\left(\bar{h}^{o}_{f}\right)_{CO_2} + 9\left(\bar{h}^{o}_{f}\right)_{H_2O(g)} - \left(\bar{h}^{o}_{f}\right)_{C_8H_{18}}$$

$$\bar{h}^{o}_{RP} = \left(\frac{8\,\text{kmol}\,CO_2}{\text{kmol}\,C_8H_{18}}\right)\left(-393{,}520\frac{\text{kJ}}{\text{kmol}\,CO_2}\right) + \left(\frac{9\,\text{kmol}\,H_2O}{\text{kmol}\,C_8H_{18}}\right)\left(-241{,}820\frac{\text{kJ}}{\text{kmol}\,H_2O}\right)$$

$$-\left(-249{,}910\frac{\text{kJ}}{\text{kmol}\,C_8H_{18}}\right) = -5{,}074{,}630\ \text{kJ/kmol}\,C_8H_{18}$$

Accordingly

$$\overline{\text{LHV}} = \underline{\mathbf{5{,}074{,}630\ kJ/kmol\ C_8H_{18}}}$$

Dividing molar lower heating value by molecular weight of ethane, lower heating value on a mass basis is

$$\text{LHV} = \frac{5{,}074{,}630\dfrac{\text{kJ}}{\text{kmol}\,C_8H_{18}}}{114.22\dfrac{\text{kg}\,C_8H_{18}}{\text{kmol}\,C_8H_{18}}} = \underline{\mathbf{44{,}429\ kJ/kg\ C_8H_{18}}}$$

This value compares favorably with that in Table A-25.

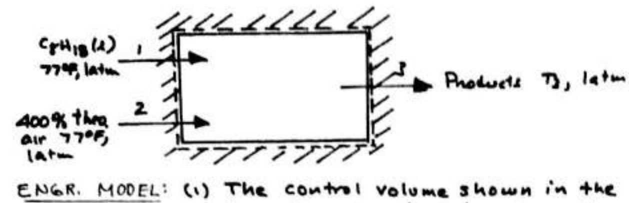
PROBLEM 13.63

Liquid octane (C_8H_{18}) at 77°F, 1 atm enters an insulated reactor operating at steady state and burns completely with 400% of theoretical air entering at 77°F, 1 atm. Determine the temperature of the exiting combustion products, in °R. Neglect kinetic and potential energy effects.

KNOWN: Liquid octane at 77°F, 1atm enters an insulated reactor and burns completely with 400% of theoretical air entering at 77°F, 1atm.

FIND: Determine the adiabatic flame temperature.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot Q_{cv} = \dot W_{cv} = 0$ and negligible kinetic and potential energy effects. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The combustion air and products of combustion can be modeled as ideal gases.

ANALYSIS: Complete combustion of C_8H_{18} with the theoretical amount of air is described by

$$C_8H_{18} + 12.5(O_2 + 3.76 N_2) \rightarrow 8CO_2 + 9H_2O + 47N_2$$

Complete combustion with 400% of theoretical air is then

$$C_8H_{18} + 50(O_2 + 3.76 N_2) \rightarrow 8CO_2 + 9H_2O + 37.5 O_2 + 188 N_2$$

An energy rate balance at steady state reduces to give

$$0 = \cancel{\frac{\dot Q_{cv}}{\dot n_{fuel}}}^{0} - \cancel{\frac{\dot W_{cv}}{\dot n_{fuel}}}^{0} + (\bar h_{C_8H_{18}})_1 + [50\bar h_{O_2} + 188\bar h_{N_2}]_2 - [8\bar h_{CO_2} + 9\bar h_{H_2O} + 37.5\bar h_{O_2} + 188\bar h_{N_2}]_3$$

With $\bar h = \bar h_f^\circ + \Delta\bar h$ and noting that $\bar h_f^\circ = 0$ for O_2 and N_2

$$0 = (\bar h_f^\circ)_{C_8H_{18}} - 8[\bar h_f^\circ + \bar h(T_3) - \bar h(537)]_{CO_2} - 9[\bar h_f^\circ + \bar h(T_3) - \bar h(537)]_{H_2O} - 37.5[\bar h(T_3) - \bar h(537)]_{O_2} - 188[\bar h(T_3) - \bar h(537)]_{N_2}$$

Accordingly, with data from Tables A-23E and A-25E

$$8\bar h_{CO_2}(T_3) + 9\bar h_{H_2O}(T_3) + 37.5\bar h_{O_2}(T_3) + 188\bar h_{N_2}(T_3) = (-107{,}530) - 8[-169{,}300 - 4027.5] - 9[-104{,}040 - 4258] + 37.5(3725.1) + 188(3729.5) \quad (a)$$

$$= 3{,}094{,}609$$

① Solving iteratively, $T_3 \approx 1732$ °R.

1. A first trial value can be obtained by assuming all products are N_2. That is, Eq. (a) becomes

$$242.5\,\bar h_{N_2}(T_3) = 3{,}094{,}609$$

$$\Rightarrow \bar h_{N_2}(T_3) = 12761 \text{ Btu/lbmol}$$

By inspection in Table A-23E, $T_3 \approx 1760$ °R.

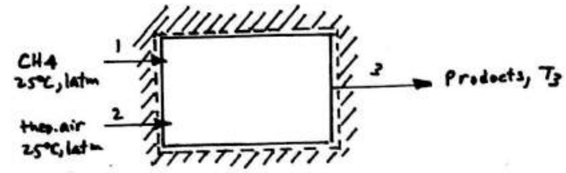
PROBLEM 13.64

Methane (CH_4) at 25°C, 1 atm enters an insulated reactor operating at steady state and burns with the theoretical amount of air entering at 25°C, 1 atm. Determine the temperature of the exiting combustion products, in K, if 90% of the carbon in the fuel burns to CO_2 and the rest to CO. Neglect kinetic and potential energy effects.

KNOWN: CH_4 at 25°C, 1atm enters an insulated reactor and burns completely with the theoretical amount of air entering at 25°C, 1atm.

FIND: Determine the temperature of the combustion products if 90% of the carbon in the fuel burns to CO_2 and the rest to CO.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) Combustion is with the theoretical amount of air. 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The combustion air and combustion products can be modeled as ideal gases.

ANALYSIS:

Combustion of CH_4 with 90% of the carbon in the fuel going to CO_2 is described by

$$CH_4 + 2O_2 + 7.52N_2 \rightarrow .9CO_2 + 0.1CO + 2H_2O + 0.05O_2 + 7.52N_2$$

At steady state an energy rate balance reduces to give

$$0 = \cancel{\frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}}}^{\,0} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{CH_4}}}^{\,0} + (\bar{h}_{CH_4})_1 + (2\bar{h}_{O_2} + 7.52\bar{h}_{N_2})_2 - (.9\bar{h}_{CO_2} + 0.1\bar{h}_{CO} + 2\bar{h}_{H_2O} + 0.05\bar{h}_{O_2} + 7.52\bar{h}_{N_2})_3$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$ and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2, this becomes

$$0 = (\bar{h}_f^\circ)_{CH_4} + (0) - 0.9(\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298))_{CO_2} - 0.1(\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298))_{CO} - 2(\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298))_{H_2O}$$
$$- 0.05[\bar{h}(T_3) - \bar{h}(298)]_{O_2} - 7.52[\bar{h}(T_3) - \bar{h}(298)]_{N_2}$$

Solving

$$0.9\bar{h}_{CO_2}(T_3) + 0.1\bar{h}_{CO}(T_3) + 2\bar{h}_{H_2O}(T_3) + 0.05\bar{h}_{O_2}(T_3) + 7.52\bar{h}_{N_2}(T_3) = (\bar{h}_f^\circ)_{CH_4}$$
$$- 0.9[\bar{h}_f^\circ - \bar{h}(298)]_{CO_2} - 0.1[\bar{h}_f^\circ - \bar{h}(298)]_{CO} - 2(\bar{h}_f^\circ - \bar{h}(298)]_{H_2O}$$
$$+ 0.05\,\bar{h}_{O_2}(298) + 7.52\,\bar{h}_{N_2}(298)$$

With data from Tables A-23 and A-25

$$0.9\bar{h}_{CO_2}(T_3) + 0.1\bar{h}_{CO}(T_3) + 2\bar{h}_{H_2O}(T_3) + 0.05\bar{h}_{O_2}(T_3) + 7.52\bar{h}_{N_2}(T_3) = -74{,}850 - 0.9[-393{,}520 - 9364]$$
$$- 0.1[-110{,}530 - 8669] - 2[-241{,}820 - 9904]$$
$$+ 0.05(8682) + 7.52(8669)$$
$$= 868739 \qquad \text{(a)}$$

① Solving iteratively $T_3 \approx 2265\,K$.

1. A first trial value can be obtained by assuming all products are N_2. that is, Eq. (a) becomes

$$10.57\,\bar{h}_{N_2}(T_3) = 868739 \Rightarrow \bar{h}_{N_2}(T_3) = 82{,}189 \frac{kJ}{kmol}$$

By inspection of Table A-23, $T_3 \approx 2450\,K$.

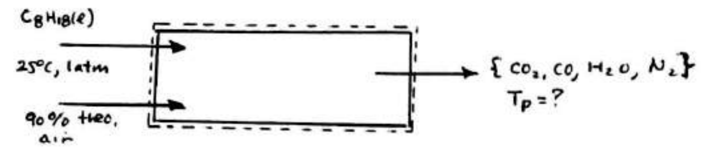
PROBLEM 13.65

Liquid octane (C_8H_{18}) at 25°C, 1 atm enters an insulated reactor operating at steady state and burns with 90% of theoretical air at 25°C, 1 atm to form products consisting of CO_2, CO, H_2O, and N_2 only. Determine the temperature of the exiting products, in K. Compare with the results of Example 13.8 and comment.

KNOWN: Operating data are provided for an insulated reactor in which $C_8H_{18}(l)$ is burned with 90% of theoretical air.

FIND: Determine the temperature of the exiting products.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and kinetic/potential energy effects can be ignored. (3) The ideal gas model is applicable to the combustion air and the combustion products. (4) 3.76 moles of inert N_2 accompany each mole of O_2 in the combustion air. N_2 is inert.

ANALYSIS: The balanced reaction equation for complete combustion with the theoretical amount of air is

$$C_8H_{18} + 12.5\,[O_2 + 3.76N_2] \rightarrow 8CO_2 + 9H_2O + 47N_2$$

For combustion with 90% of theoretical air

$$C_8H_{18} + 11.25\,O_2 + 42.3\,N_2 \rightarrow 5.5\,CO_2 + 2.5\,CO + 9H_2O + 42.3\,N_2$$

An energy rate balance reduces to read $\bar{h}_P = \bar{h}_R$. With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, this takes the form

$$5.5\,[\bar{h}_f^\circ + \Delta\bar{h}]_{CO_2} + 2.5\,[\bar{h}_f^\circ + \Delta\bar{h}]_{CO} + 9\,[\bar{h}_f^\circ + \Delta\bar{h}]_{H_2O} + 42.3\,[\Delta\bar{h}]_{N_2} = [\bar{h}_f^\circ]_{C_8H_{18}(l)}$$

or

$$5.5[\Delta\bar{h}]_{CO_2} + 2.5[\Delta\bar{h}]_{CO} + 9[\Delta\bar{h}]_{H_2O} + 42.3[\Delta\bar{h}]_{N_2} = [\bar{h}_f^\circ]_{C_8H_{18}(l)} - 5.5[\bar{h}_f^\circ]_{CO_2} - 2.5[\bar{h}_f^\circ]_{CO} - 9[\bar{h}_f^\circ]_{H_2O}$$

With data from the ideal gas tables

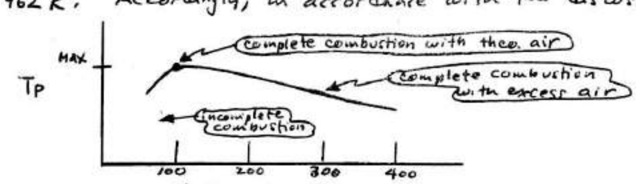
$$5.5\,[\bar{h}(T_P) - 9364]_{CO_2} + 2.5\,[\bar{h}(T_P) - 8669]_{CO} + 9[\bar{h}(T_P) - 9904]_{H_2O} + 42.3\,[\bar{h}(T_P) - 8669]_{N_2}$$

$$= [-249{,}910] - 5.5[-393{,}520] - 2.5[-110{,}530] - 9[-241{,}820]$$

$$= 4{,}367{,}155 \text{ kJ/kmol(fuel)} \qquad (a)$$

① The above expression is a single equation w/ just one unknown: T_P. Solving iteratively using table data, $T_P = 2286$ K.

Discussion. In Example 13.8, for the case of complete combustion with the theoretical amount of air T_P = 2395 K, and for complete combustion with 400% theoretical air T_P = 962 K. Accordingly, in accordance with the discussion of Sec. 13.3.3,



1. Iteration using table data can be avoided by using IT. Still, when using table data a first trial can be obtained assuming the products consist only of N_2. Then Eq. (a) reads $59.3\,\bar{h}_{N_2} = 4{,}367{,}155$

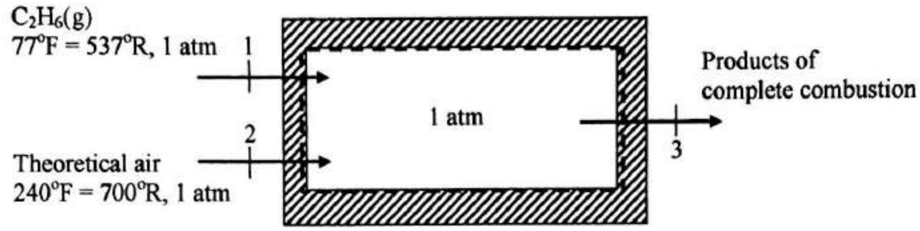
Solving gives $\bar{h}_{N_2}$ = 73,645 kJ/kmol. Inspection of Table A-23 then gives $T_P \approx 2200$ K.

13.66 Ethane (C_2H_6) gas at 77°F, 1 atm enters a well-insulated reactor operating at steady state and burns completely with theoretical air entering at 240°F, 1 atm. Determine the temperature of the products, in °F. Neglect kinetic and potential energy effects.

KNOWN: Ethane gas burns completely with theoretical air in a well-insulated reactor operating at steady state. The conditions of the entering fuel and air are specified.

FIND: Determine the temperature of the products exiting the reactor.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{W}_{cv} = \dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible. (3) Combustion is complete, and 3.76 moles of N_2 accompany each mole of O_2 in the air. (4) The theoretical amount of air is supplied. (5) The nitrogen is inert and the ideal gas model applies to the fuel, combustion air, and combustion products.

ANALYSIS: Complete combustion of ethane with the theoretical amount of air is described by

$$C_2H_6 + 3.5(O_2 + 3.76N_2) \rightarrow 2CO_2 + 3H_2O + 13.16N_2$$

The energy and mass rate balances reduces with the listed assumptions to read

$$0 = \left(\bar{h}_f^o + \cancel{\Delta\bar{h}}\right)_{C_2H_6} + 3.5\left[\cancel{\bar{h}_f^o} + \bar{h}(700) - \bar{h}(537)\right]_{O_2} + 13.16\left[\cancel{\bar{h}_f^o} + \bar{h}(700) - \bar{h}(537)\right]_{N_2}$$

$$-2\left[\bar{h}_f^o + \bar{h}(T_3) - \bar{h}(537)\right]_{CO_2} - 3\left[\bar{h}_f^o + \bar{h}(T_3) - \bar{h}(537)\right]_{H_2O} - 13.16\left[\bar{h}_f^o + \bar{h}(T_3) - \bar{h}(537)\right]_{N_2}$$

Or

$$2\left[\bar{h}(T_3)\right]_{N_2} + 3\left[\bar{h}(T_3)\right]_{H_2O} + 13.16\left[\bar{h}(T_3)\right]_{N_2}$$

$$= \left(\bar{h}_f^o\right)_{C_2H_6} + 3.5\left[(700) - \bar{h}(537)\right]_{O_2} + 13.16\left[(700) - \bar{h}(537)\right]_{N_2}$$

$$- 2\left[\bar{h}_f^o - \bar{h}(537)\right]_{CO_2} - 3\left[\bar{h}_f^o - \bar{h}(537)\right]_{H_2O} - 13.16\left[\bar{h}_f^o - \bar{h}(537)\right]_{N_2}$$

With specific enthalpy data from Table A-23E and the enthalpy of formation of the fuel from Table A-25E

$$2\left[\bar{h}(T_3)\right]_{N_2} + 3\left[\bar{h}(T_3)\right]_{H_2O} + 13.16\left[\bar{h}(T_3)\right]_{N_2}$$

$$= (-36{,}420) + 3[4879.3 - 3725.1] + 13.16[4864.9 - 3729.5] - 2[-169{,}300 - 4027.5]$$

$$- 3[-104{,}040 - 4258.0] + 13.16[-3729.5]$$

$$= 703{,}191 \text{ Btu/lbmol}(C_2H_6)$$

To find T_3, use an iterative procedure with data from Table A-23E to find the temperature that results in the left side taking on the value of 703,191 Btu/lbmol(C_2H_6).

The value is **$\underline{T_3 \approx 4390°R}$**

Note that the need for iteration can be eliminated by using IT:Interactive Thermodynamics.

PROBLEM 13.67

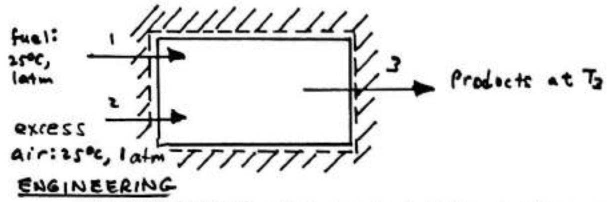
13.67 For each of the following fuels, plot the adiabatic flame temperature, in K, versus percent excess air for complete combustion in a combustor operating at steady state. The reactants enter at 25°C, 1 atm.

(a) carbon.
(b) hydrogen (H_2).
(c) liquid octane (C_8H_{18}).

KNOWN: Fuel at 25°C, 1 atm enters a reactor and burns completely with x percent excess air entering at 25°C, 1atm.

FIND: Plot the adiabatic flame temperature versus x for (a) C, (b) H_2, and (c) $C_8H_{18}(\ell)$.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) For the control volume shown in the accompanying figure $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and the effects of kinetic and potential energy are negligible. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The ideal gas model is applicable to the combustion air and products.

ANALYSIS: To find the adiabatic flame temperature, use Eq. 13.21a

$$\sum_P n_e \bar{h}_e = \sum_R n_i \bar{h}_i \qquad (1)$$

When using IT, the enthalpies are evaluated directly at the respective inlet and exit temperatures. When using table data, Eq. (1) takes the form

$$\sum_P n_e(\bar{h}_f^\circ + \Delta\bar{h})_e = \sum_R n_i(\bar{h}_f^\circ + \Delta\bar{h})_i$$

or

$$\sum_P n_e(\Delta\bar{h})_e = \sum_R n_i(\Delta\bar{h})_i + \sum_R n_i \bar{h}_{fi}^\circ - \sum_P n_e \bar{h}_{fe}^\circ \qquad (2)$$

(a) Fuel is C: For complete combustion with x percent excess air

$$C + (1 + x/100)[O_2 + 3.76N_2] \rightarrow CO_2 + (x/100)O_2 + (1 + x/100)(3.76)N_2$$

Applying Eq. (2) with $\Delta\bar{h}_i = 0$ and $\bar{h}_f^\circ = 0$ for O_2 and N_2 and C.

$$[\bar{h}_{CO_2}(T_3) - \bar{h}_{CO_2}(298)] + (x/100)[\bar{h}_{O_2}(T_3) - \bar{h}_{O_2}(298)] + (1 + x/100)(3.76)[\bar{h}_{N_2}(T_3) - \bar{h}_{N_2}(298)] = 0$$

For x = 100% ; we can solve iteratively using table data to get $T_3 = 1506$ K.

(b) Fuel is H_2. $H_2 + \frac{1}{2}(1 + x/100)[O_2 + 3.76N_2] \rightarrow H_2O + \frac{x}{200}O_2 + \frac{1}{2}(1 + x/100)(3.76)N_2$

and

$$[\bar{h}(T_3) - \bar{h}(298)]_{H_2O} + \left(\frac{x}{200}\right)[\bar{h}(T_3) - \bar{h}(298)]_{O_2} + \tfrac{1}{2}(1 + x/100)(3.76)[\bar{h}(T_3) - \bar{h}(298)]_{N_2} = -(\bar{h}_f^\circ)_{H_2O(g)}$$

For x = 100%, we get $T_3 = 1647$ K

(c) Fuel is $C_8H_{18}(\ell)$: $C_8H_{18} + 12.5(1 + \frac{x}{100})[O_2 + 3.76N_2] \rightarrow 8CO_2 + 9H_2O + \frac{12.5x}{100}O_2 + 12.5(1 + \frac{x}{100})(3.76)N_2$

and

$$8[\bar{h}(T_3) - \bar{h}(298)]_{CO_2} + 9[\bar{h}(T_3) - \bar{h}(298)]_{H_2O} + \frac{12.5x}{100}[\bar{h}(T_3) - \bar{h}(298)]_{O_2}$$
$$+ 12.5(1 + \frac{x}{100})(3.76)[\bar{h}(T_3) - \bar{h}(298)]_{N_2} = (\bar{h}_f^\circ)_{C_8H_{18}(\ell)} - 8(\bar{h}_f^\circ)_{CO_2} - 9(\bar{h}_f^\circ)_{H_2O(g)}$$

Solving for x = 100% ; $T_3 = 1507$ K.

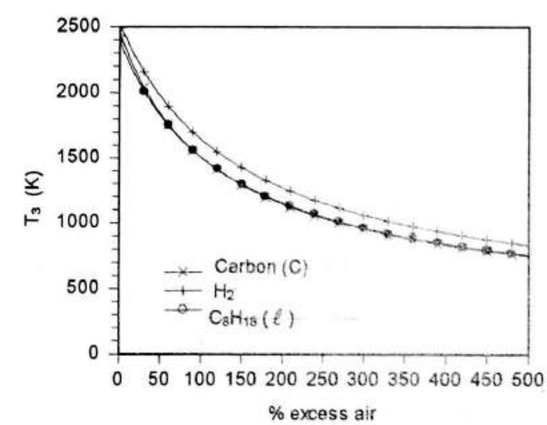
PROBLEM 13.67 (Cont'd.) – Page 2

The data for the required plots are obtained using IT, as follows:

<u>*IT* Code</u>

```
T = 25 + 273.15 // K
x = 1
PCT = x * 100

Part (a)
0 = - hCO2_3 - x * hO2_3 - (1+x) * 3.76 * hN2_3
hCO2_3 = h_T("CO2",Tp)
hO2_3 = h_T("O2", Tp)
hN2_3 = h_T("N2", Tp)

Result for x = 100%: Tp = 1507 K

Part (b)
0 = - hH2O_3 - (x/2) * hO2_3 - ((1+x) / 2) * 3.76 * hN2_3
hH2O_3 = h_T("H2O",Tp)
hO2_3 = h_T("O2",Tp)
hN2_3 = h_T("N2",Tp)

Result for x = 100%:  Tp = 1647 K  part (b)

Part (c)
0 = hC8H18_1 - 8 * hCO2_3 - 9 * hH2O_3 - 12.5 * x * hO2_3 - 12.5 * (1+x) * 3.76 * hN2_3
hC8H18_1 = -249910  // Table A-25
hCO2_3 = h_T("CO2",Tp)
hH2O_3 = h_T("H2O",Tp)
hO2_3 = h_T("O2",Tp)
hN2_3 = h_T("N2",Tp)

Result for x = 100%:  Tp = 1507 K  part(c)
```

PLOTS:



The excess air leads to more O_2 and N_2 in the products, thereby lowering the adiabatic flame temperature, as expected.

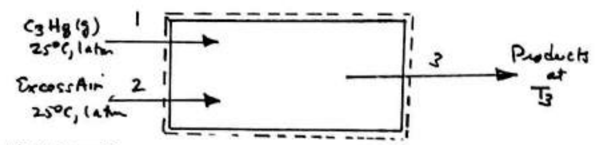
PROBLEM 13.68

13.68 Propane gas (C_3H_8) at 25°C, 1 atm enters an insulated reactor operating at steady state and burns completely with air entering at 25°C, 1 atm. Plot the adiabatic flame temperature versus percent of theoretical air ranging from 100 to 400%. Why does the adiabatic flame temperature vary with increasing combustion air?

KNOWN: $C_3H_8(g)$ at 25°C, 1atm enters an insulated reactor and burns completely with air entering at 25°C, 1atm.

FIND: Plot the adiabatic flame temperature versus the percent of theoretical air varying from 100 to 400%

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the figure is at steady state. (2) For the control volume $\dot{W}_{cv} = 0$, $\dot{Q}_{cv}$ and kinetic/potential energy effects are negligible. (3) Combustion is complete. 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. The combustion air and combustion products can each be modeled as ideal gas mixtures

ANALYSIS: Complete combustion of C_3H_8 with the theoretical amount of air is described by

$$C_3H_8 + 5[O_2 + 3.76] \rightarrow 3\,CO_2 + 4H_2O + (5)(3.76)\,N_2$$

For n times the theoretical amount of air

$$C_3H_8 + 5n[O_2 + 3.76\,N_2] \rightarrow 3CO_2 + 4H_2O + 5(n-1)O_2 + 18.8n\,N_2$$

An energy rate balance reduces with listed assumptions to read

$$0 = (\bar{h}^\circ_f)_{C_3H_8} - 3[\bar{h}^\circ_f + \Delta\bar{h}]_{CO_2} - 4[\bar{h}^\circ_f + \Delta\bar{h}]_{H_2O} - 5(n-1)[\Delta\bar{h}]_{O_2} - [\Delta\bar{h}]_{N_2} \qquad (1)$$

or

$$0 = (\bar{h}^\circ_f)_{C_3H_8} - 3[\bar{h}^\circ_f + \bar{h}(T_3) - \bar{h}(298)]_{CO_2} - 4[\bar{h}^\circ_f + \bar{h}(T_3) - \bar{h}(298)]_{H_2O}$$
$$- 5(n-1)[\bar{h}(T_3) - \bar{h}(298)]_{O_2} - 18.8n[\bar{h}(T_3) - \bar{h}(298)]_{N_2}$$

or

$$3\bar{h}_{CO_2}(T_3) + 4\bar{h}_{H_2O}(T_3) + 5(n-1)\bar{h}_{O_2}(T_3) + 18.8n\bar{h}_{N_2}(T_3) = (\bar{h}^\circ_f)_{C_3H_8} - 3[\bar{h}^\circ_f - \bar{h}(298)]_{CO_2}$$
$$- 4[\bar{h}^\circ_f - \bar{h}(298)]_{H_2O} +$$
$$5(n-1)\bar{h}_{O_2}(298) + 18.8n\,\bar{h}_{N_2}(298)$$

With ideal gas table data and $(\bar{h}^\circ_f)_{C_3H_8}$ from Table A-25

$$3\bar{h}_{CO_2}(T_3) + 4\bar{h}_{H_2O}(T_3) + 5(n-1)\bar{h}_{O_2}(T_3) + 18.8n\,\bar{h}_{N_2}(T_3) = -103{,}850 - 3[-393{,}520 - 9364]$$
$$- 4[-241{,}820 - 9904] -$$
$$5(n-1)(8682) - 18.8n(8669)$$

$$3\bar{h}_{CO_2}(T_3) + 4\bar{h}_{H_2O}(T_3) + 5(n-1)\bar{h}_{O_2}(T_3) + 18.8n\bar{h}_{N_2}(T_3) = 2{,}111{,}698 + 43{,}410(n-1) + 162{,}977n$$

<u>Sample Calculation</u>. Using Table A-23 data for the case $n = 1.2$, we get $T_3 = 2125$ K.

The data for the required plot are obtained using IT, as follows:

PROBLEM 13.68 (Cont'd.)

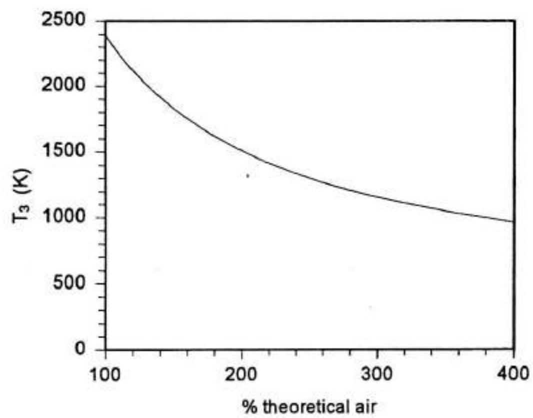
<u>*IT* Code</u>

```
n = 120  // % theoretical air

0 = hfo_C3H8 - 3 * hCO2_3 - 4 * hH2O_3 - 5*(n/100 - 1) * hO2_3 - 18.8 * n/100 * hN2_3
hfo_C3H8 = -103850  // Table A-25
hCO2_3 = h_T("CO2", T3)
hH2O_3 = h_T("H2O",T3)
hO2_3 = h_T("O2",T3)
hN2_3 = h_T("N2",T3)
```

<u>*IT* Result for n = 120%</u>

T_3 = 2124 K

PLOT:



The excess air leads to more O_2 and N_2 in the products, thereby lowering the adiabatic flame temperature, as expected.

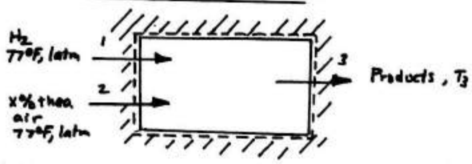
PROBLEM 13.69

13.69 Hydrogen (H_2) at 77°F, 1 atm enters an insulated reactor operating at steady state and burns completely with x% of theoretical air entering at 77°F 1 atm. Plot the adiabatic flame temperature for x ranging from 100 to 400%.

<u>KNOWN:</u> H_2 at 77°F, 1atm enters an insulated reactor and burns completely with x% of theoretical air entering at 77°F, 1atm.

<u>FIND:</u> Plot the adiabatic flame temperature for x ranging from 100 to 400 %.

<u>SCHEMATIC & GIVEN DATA:</u>



<u>ENGINEERING MODEL:</u> (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The combustion air and combustion products can be modeled as ideal gases.

<u>ANALYSIS:</u> Complete combustion of H_2 with the theoretical amount of air is described by

$$H_2 + 0.5\,O_2 + 1.88\,N_2 \rightarrow H_2O + 1.88\,N_2$$

Complete combustion with x percent of the theoretical amount of air is then

$$H_2 + 0.5x\,O_2 + 1.88x\,N_2 \rightarrow H_2O + 0.5(x-1)\,O_2 + 1.88x\,N_2$$

An energy rate balance at steady state reduces to

$$0 = \frac{\cancel{\dot{Q}_{cv}}^{0}}{\dot{n}_{H_2}} - \frac{\cancel{\dot{W}_{cv}}^{0}}{\dot{n}_{H_2}} + (\bar{h}_{H_2})_1 + (0.5x\,\bar{h}_{O_2} + 1.88x\,\bar{h}_{N_2})_2 - (\bar{h}_{H_2O} + 0.5(x-1)\bar{h}_{O_2} + 1.88x\,N_2)_3$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for H_2, O_2, and N_2, this becomes

$$0 = [\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{H_2O} + 0.5(x-1)[\bar{h}(T_3) - \bar{h}(537)]_{O_2} + 1.88x[\bar{h}(T_3) - \bar{h}(537)]_{N_2}$$

with data from the ideal gas tables

$$0 = [-104{,}040 + \bar{h}_{H_2O}(T_3) - 4258] + 0.5(x-1)[\bar{h}_{O_2}(T_3) - 3725.1] + 1.88x[\bar{h}_{N_2}(T_3) - 3729.5]$$

or

$$\bar{h}_{H_2O}(T_3) + 0.5(x-1)\bar{h}_{O_2}(T_3) + 1.88x\,\bar{h}_{N_2}(T_3) = 108298 + (x-1)(1862.55) + x(7011.46) \qquad (1)$$

<u>Sample Calculation:</u> With data from Table A-23 for the case x = 2, we get $T_3 \approx 2965$°R

The data for the required plot are obtained using IT, as follows:

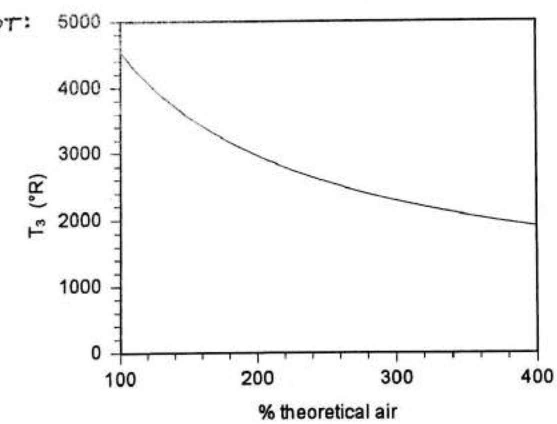
<u>*IT* Code</u>

```
x = 200  // % theoretical air

0 = hH2O_3 + 0.5 * (x/100 - 1) * hO2_3
     + 1.88 * x/100 * hN2_3
hH2O_3 = h_T("H2O",T3)
hO2_3 = h_T("O2",T3)
hN2_3 = h_T("N2",T3)
```

<u>*IT* Result for x = 200%</u>

T_3 = 2964 °R

PLOT:



The excess air leads to more O_2 and N_2 in the products, thereby lowering the adiabatic flame temperature, as expected.

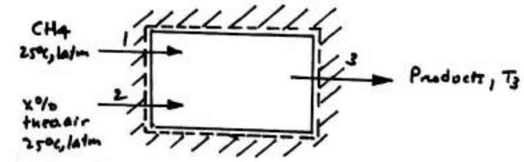
PROBLEM 13.70

13.70 Methane gas (CH_4) at 25°C, 1 atm enters an insulated reactor operating at steady state and burns completely with x% of theoretical air entering at 25°C, 1 atm. Plot the adiabatic flame temperature for x ranging from 100 to 400%.

KNOWN: CH_4 at 25°C, 1 atm enters an insulated reactor and burns completely with x % of theoretical air entering at 25°C, 1 atm.

FIND: Plot the adiabatic flame temperature for x ranging from 100 to 400 %

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The combustion air and combustion products can be modeled as ideal gases.

ANALYSIS: Complete combustion of CH_4 with the theoretical amount of air is given by Eq. 13.4. Complete combustion with x % of the theoretical amount of air is then

$$CH_4 + 2x\,O_2 + 7.52x\,N_2 \rightarrow CO_2 + 2H_2O + 2(x-1)O_2 + 7.52x\,N_2$$

An energy rate balance at steady state reduces to

$$0 = \cancel{\frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}}}^{\,0} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{CH_4}}}^{\,0} + (\bar{h}_{CH_4})_1 + (2x\,\bar{h}_{O_2} + 7.52x\,\bar{h}_{N_2})_2 - (\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 2(x-1)\bar{h}_{O_2} + 7.52x\,\bar{h}_{N_2}]_3$$

With $\bar{h} = \bar{h}^\circ_f + \Delta\bar{h}$, and noting that $\bar{h}^\circ_f = 0$ for O_2 and N_2, this becomes

$$0 = (\bar{h}^\circ_f)_{CH_4} + (0) - \Big[[\bar{h}^\circ_f + \bar{h}(T_3) - \bar{h}(298)]_{CO_2} + 2[\bar{h}^\circ_f + \bar{h}(T_3) - \bar{h}(298)]_{H_2O} +$$
$$2(x-1)[\bar{h}(T_3) - \bar{h}(298)]_{O_2} + 7.52x[\bar{h}(T_3) - \bar{h}(298)]_{N_2}\Big]$$

or

$$\bar{h}_{CO_2}(T_3) + 2\,\bar{h}_{H_2O}(T_3) + 2(x-1)\,\bar{h}_{O_2}(T_3) + 7.52x\,\bar{h}_{N_2}(T_3) = (\bar{h}^\circ_f)_{CH_4} - [\bar{h}^\circ_f - \bar{h}(298)]_{CO_2}$$
$$- 2[\bar{h}^\circ_f - \bar{h}(298)]_{H_2O} +$$
$$2(x-1)\bar{h}_{O_2}(298) + 7.52x\,\bar{h}_{N_2}(298)$$

With data from Tables A-23, A-25

$$\bar{h}_{CO_2}(T_3) + 2\bar{h}_{H_2O}(T_3) + 2(x-1)\bar{h}_{O_2}(T_3) + 7.52x\,\bar{h}_{N_2}(T_3) = -74{,}850 - [-393{,}520 - 9{,}364] -$$
$$2[-241{,}820 - 9{,}904] +$$
$$2(x-1)[8682] + 7.52x[8669]$$
$$= 831{,}482 + 17364(x-1) + 65191x$$

Sample calculation: when $x = 2$, $T_3 \approx 1481$ K.

The data for the required plot are obtained using IT, as follows:

PROBLEM 13.70 (Cont'd.) – Page 2

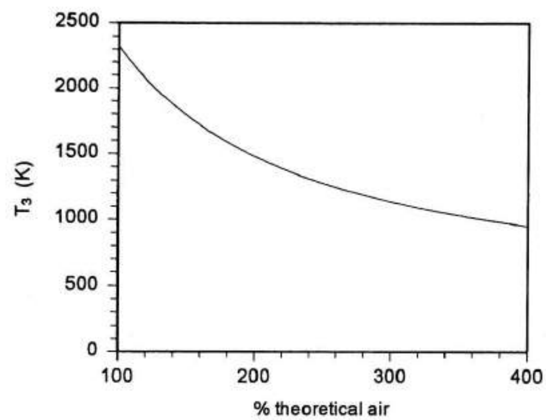
<u>*IT* Code</u>

```
T1 = 25 + 273.15  // K
x = 200  // % theoretical air

0 = hCH4_1 - (hCO2_3 + 2 * hH2O_3 + 2 * (x/100 - 1) * hO2_3 + 7.52 * x/100 * hN2_3)
hCH4_1 = h_T("CH4",T1)
hCO2_3 = h_T("CO2",T3)
hH2O_3 = h_T("H2O",T3)
hO2_3 = h_T("O2",T3)
hN2_3 = h_T("N2",T3)
```

<u>*IT* Results for x = 200%</u>

T_3 = 1481 K

PLOT:



The excess air lead to more O_2 and N_2 in the products, thereby lowering the adiabatic flame temperature, as expected.

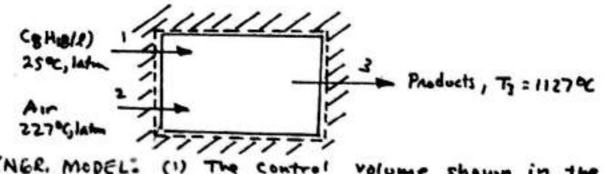
PROBLEM 13.71

Liquid octane (C_8H_{18}) at 25°C, 1 atm enters an insulated reactor operating at steady state and burns completely with air entering at 227°C, 1 atm. The combustion products exit the reactor at 1127°C. Determine the percent excess air used. Neglect kinetic and potential energy effects.

KNOWN: $C_8H_{18}(l)$ at 25°C, 1atm enters an insulated reactor and burns completely with air entering at 227°C, 1atm. The combustion products exit at 1127°C.

FIND: Determine the percent excess air used.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) Complete combustion occurs. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The combustion air and combustion products can be modeled as ideal gases.

ANALYSIS: The complete combustion of $C_8H_{18}(l)$ with excess air is described

$$C_8H_{18}(l) + x\,(O_2 + 3.76N_2) \rightarrow 8CO_2 + 9H_2O + (x-12.5)O_2 + 3.76x\,N_2$$

For combustion with the theoretical amount of air $x = 12.5$.

An energy rate balance at steady state reduces to

$$0 = \cancel{\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}}}^{\,0} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{fuel}}}^{\,0} + (\bar{h}_{C_8H_{18}(l)})_1 + (x\bar{h}_{O_2} + 3.76x\bar{h}_{N_2})_2 - (8\bar{h}_{CO_2} + 9\bar{h}_{H_2O} + (x-12.5)\bar{h}_{O_2} + 3.76x\bar{h}_{N_2})_3$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2, this becomes

$$0 = (\bar{h}_f^\circ)_{C_8H_{18}} + x[\bar{h}(500) - \bar{h}(298)]_{O_2} + 3.76x[\bar{h}(500) - \bar{h}(1400)]_{N_2} - 8[\bar{h}_f^\circ + \bar{h}(1400) - \bar{h}(298)]_{CO_2}$$
$$- 9[\bar{h}_f^\circ + \bar{h}(1400) - \bar{h}(298)]_{H_2O} - (x-12.5)[\bar{h}(1400) - \bar{h}(298)]_{O_2}$$

Solving for x and using data from Tables A-23 and A-25,

$$x = \frac{(\bar{h}_f^\circ)_{C_8H_{18}} - 8(\bar{h}_f^\circ + \bar{h}(1400) - \bar{h}(298))_{CO_2} - 9(\bar{h}_f^\circ + \bar{h}(1400) - \bar{h}(298))_{H_2O} + 12.5(\bar{h}(1400) - \bar{h}(298))_{O_2}}{3.76(\bar{h}(1400) - \bar{h}(500))_{N_2} + (\bar{h}(1400) - \bar{h}(500))_{O_2}}$$

$$= \frac{-249,910 - 8(-393,520 + 65,271 - 9364) - 9(-241,820 + 53,351 - 9904) + 12.5(45,648 - 8682)}{3.76(43,605 - 14,581) + (45,648 - 14,770)}$$

$$= 33.56$$

The percent excess air is then

$$\%\ \text{excess air} = \left(\frac{33.56 - 12.5}{12.5}\right)(100) = 168.5\% \longleftarrow \%$$

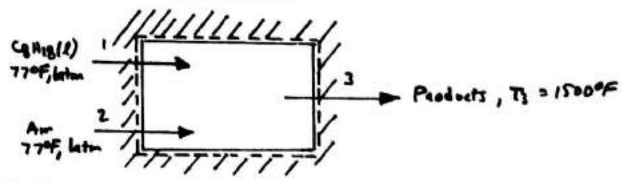
PROBLEM 13.72

Repeat Problem 13.71 if the fuel and air enter at 77°F, 1 atm and the products exit at 1500°F.

KNOWN: $C_8H_{18}(l)$ at 77°F, 1 atm enters an insulated reactor and burns completely with air entering at 77°F, 1 atm. Combustion products exit at 1500°F.

FIND: Determine the percent excess air used.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) Complete combustion occurs. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The combustion air and combustion products can be modeled as ideal gases.

ANALYSIS: The complete combustion of $C_8H_{18}(l)$ with excess air is described by

$$C_8H_{18}(l) + x(O_2 + 3.76N_2) \rightarrow 8CO_2 + 9H_2O + (x-12.5)O_2 + 3.76x\,N_2$$

For combustion with the theoretical amount of air $x = 12.5$.

An energy rate balance at steady state reduces to

$$0 = \frac{\cancel{\dot{Q}_{cv}}^{\,0}}{\cancel{\dot{n}_{fuel}}} - \frac{\cancel{\dot{W}_{cv}}^{\,0}}{\cancel{\dot{n}_{fuel}}} + (\bar{h}_{C_8H_{18}(l)})_1 + (x\bar{h}_{O_2} + 3.76x\bar{h}_{N_2})_2 - (8\bar{h}_{CO_2} + 9\bar{h}_{H_2O} + (x-12.5)\bar{h}_{O_2} + 3.76x\,\bar{h}_{N_2})_3$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2, this becomes

$$0 = (\bar{h}_f^\circ)_{C_8H_{18}(l)} + (0) - 8[\bar{h}_f^\circ + \bar{h}(1960) - \bar{h}(537)]_{CO_2} - 9[\bar{h}_f^\circ + \bar{h}(1960) - \bar{h}(537)]_{H_2O}$$
$$-(x-12.5)[\bar{h}(1960) - \bar{h}(537)]_{O_2} - 3.76x[\bar{h}(1960) - \bar{h}(537)]_{N_2}$$

Solving for x, and using data from Tables A-23E and A-25E

$$x = \frac{(\bar{h}_f^\circ)_{C_8H_{18}(l)} - 8(\bar{h}_f^\circ + \bar{h}(1960) - \bar{h}(537))_{CO_2} - 9(\bar{h}_f^\circ + \bar{h}(1960) - \bar{h}(537))_{H_2O} + 12.5(\bar{h}(1960) - \bar{h}(537))]_{O_2}}{3.76[\bar{h}(1960) - \bar{h}(537)]_{N_2} + [\bar{h}(1960) - \bar{h}(537)]_{O_2}}$$

$$= \frac{-107{,}530 + 8(-169{,}300 + 20{,}489 - 4028) - 9(-104{,}040 + 17{,}033 - 4258) + 12.5(14{,}827 - 3725)}{3.76(14{,}217 - 3730) + (14{,}827 - 3725)}$$

$$= 41.07$$

The percent excess air is then

$$\%\text{ excess air} = \left(\frac{41.07 - 12.5}{12.5}\right)(100) = 228.6\,\% \longleftarrow \%$$

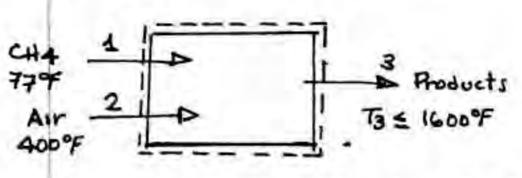
PROBLEM 13.73

Methane (CH_4) at 77°F, 1 atm enters the combustor of a gas turbine power plant operating at steady state and burns completely with air entering at 400°F. Owing to metallurgical limitations, the temperature of the combustion products exiting the combustor to the turbine can be no higher than 1600°F. Determine the percent excess air that allows this constraint to be met. Neglect heat transfer from the combustor and kinetic and potential energy effects.

KNOWN: CH_4 at 77°F, 1 atm enters a combustor and burns completely with air entering at 400°F

FIND: Determine the percent excess air for which th exiting products can be no greater than 1600°F

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The control volume shown in the figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy.
2. 3.76 moles of N_2 accompany each mole of O_2 with the air.
3. Complete combustion occurs.
4. The air and combustion products can be modeled as ideal gases.

ANALYSIS: For complete combustion of CH4 with excess air the balanced reaction equation is

$$CH_4 + 2x(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 2(x-1)O_2 + 7.52x\,N_2 \qquad (a)$$

where x=1 represents the case where combustion is with the theoretical amount of air.

Using Eq.(a), an energy rate balance at steady state reads

$$0 = \frac{\cancel{\dot{Q}_{cv}}^{\,0}}{\dot{n}_{CH4}} - \frac{\cancel{\dot{W}_{cv}}^{\,0}}{\dot{n}_{CH4}} + (\bar{h}_{CH4})_1 + (2x\bar{h}_{O_2} + 7.52x\bar{h}_{N_2})_2 - (\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 2(x-1)\bar{h}_{O_2} + 7.52x\bar{h}_{N_2})_3 \qquad (b)$$

Here T_1 and T_2 are specified and $T_3 \leq 2060°R$. According to the discussion of Sec. 13.3.3, T_3 decreases as x increases. The plan then is to assume $T_3 = 2060°R$ and solve for x. This is the minimum value that allows the constraint to be met.

With $\bar{h} = \bar{h}_f^o + \Delta\bar{h}$ and noting that $\bar{h}_f^o = 0$ for O_2 and N_2, Eq.(b) becomes

$$0 = (\bar{h}_f^o)_{CH4} + 2x[\bar{h}(860) - \bar{h}(537)]_{O_2} + 7.52x[\bar{h}(860) - \bar{h}(537)]_{N_2} - [\bar{h}_f^o + \bar{h}(2060) - \bar{h}(537)]_{CO_2} - 2[\bar{h}_f^o + \bar{h}(2060) - \bar{h}(537)]_{H_2O} - 2(x-1)[\bar{h}(2060) - \bar{h}(537)]_{O_2} - 7.52x(\bar{h}(2060) - \bar{h}(537)]_{N_2}$$

Alternatively, the term involving 2(x-1) can be expanded giving

$$0 = (\bar{h}_f^o)_{CH4} + \underline{2x[\bar{h}(860) - \bar{h}(537)]_{O_2} + 7.52x[\bar{h}(860) - \bar{h}(537)]_{N_2}} - [\bar{h}_f^o + \bar{h}(2060) - \bar{h}(537)]_{CO_2} - 2[\bar{h}_f^o + \bar{h}(2060) - \bar{h}(537)]_{H_2O} - \underline{2x[\bar{h}(2060) - \bar{h}(537)]_{O_2}} + 2[\bar{h}(2060) - \bar{h}(537)]_{O_2} - \underline{7.52x[\bar{h}(2060) - \bar{h}(537)]_{N_2}}$$

Collecting the underlined terms on the left and simplifying,

$$2x[\bar{h}(2060) - \bar{h}(860)]_{O_2} + 7.52x[\bar{h}(2060) - \bar{h}(860)]_{N_2} = (\bar{h}_f^o)_{CH4} - [\bar{h}_f^o + \bar{h}(2060) - \bar{h}(537)]_{CO_2} - 2[\bar{h}_f^o + \bar{h}(2060) - \bar{h}(537)]_{H_2O} + 2[\bar{h}(2060) - \bar{h}(537)]_{O_2}$$

PROBLEM 13.73 (Continued)

Solving for x and using data from Table A-23E and A-25E

$$x = \frac{-32{,}210 - (-169{,}300 + 21{,}818 - 4028) - 2(-104{,}040 + 18{,}054 - 4258) + 2(15{,}672 - 3725)}{2(15{,}672 - 6042) + 7.52(15{,}013 - 5986)}$$

$$x = 3.71$$

Accordingly, the minimum percent excess air that allows the constraint on T_3 to be met is

$$\begin{bmatrix} \text{MIN \%} \\ \text{excess} \\ \text{air} \end{bmatrix} = \left[\frac{\overline{AF} - (\overline{AF})_{theo}}{(\overline{AF})_{theo}}\right](100)$$

$$= \left[\frac{(2)(3.71)(4.76)/1}{(2)(4.76)/1} - 1\right](100)$$

$$= 271\%$$

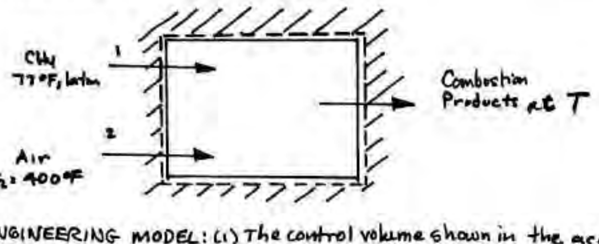
PROBLEM 13.74

13.74 Methane (CH_4) at 77°F enters the combustor of a gas turbine power plant operating at steady state and burns completely with air entering at 400°F. The temperature of the products of combustion flowing from the combustor to the turbine depends on the percent excess air for combustion. Plot the percent excess air versus combustion product temperatures ranging from 1400 to 1800°F. There is no significant heat transfer between the combustor and its surroundings, and kinetic and potential energy effects can be ignored.

KNOWN: Steady state operating data are provided for the combustor of a gas turbine power plant.

FIND: Plot the percent excess air versus the combustion product temperature for $1400 \le T \le 1800$ °F.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) Complete combustion with excess air occurs. (3). 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The combustion air and combustion products can be modeled as ideal gases.

ANALYSIS: Complete combustion of CH_4 with excess air is described by

$$CH_4 + 2x(O_2 + 3.76) \rightarrow CO_2 + 2H_2O + 2(x-1)O_2 + 7.52x\,N_2$$

For combustion with the theoretical amount of air, $x=1$.

An energy rate balance at steady state reduces to

$$0 = \frac{\cancel{\dot{Q}_{cv}}^{\,0}}{\dot{n}_{CH_4}} - \frac{\cancel{\dot{W}_{cv}}^{\,0}}{\dot{n}_{CH_4}} + (\bar{h}_{CH_4})_1 + (2x\bar{h}_{O_2} + 7.52x\bar{h}_{N_2})_2 - (\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 2(x-1)\bar{h}_{O_2} + 7.52x\bar{h}_{N_2})_3$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2, this becomes

$$0 = (\bar{h}_f^\circ)_{CH_4} + 2x[\bar{h}(860) - \bar{h}(537)]_{O_2} + 7.52x[\bar{h}(860) - \bar{h}(537)]_{N_2} - (\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537))_{CO_2}$$
$$-2[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{H_2O} - 2(x-1)[\bar{h}(T) - \bar{h}(537)]_{O_2} - 7.52x(\bar{h}(T) - \bar{h}(537))_{N_2}$$
$$= (\bar{h}_f^\circ)_{CH_4} - (\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537))_{CO_2} - 2(\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537))_{H_2O} - 2x[\bar{h}(T) - \bar{h}(860)]_{O_2}$$
$$+2[\bar{h}(T) - \bar{h}(537)]_{O_2} - 7.52x[\bar{h}(T) - \bar{h}(860)]$$

Introducing data from the ideal gas tables and Table A-25, and solving for x

$$x = \frac{(\bar{h}_f^\circ)_{CH_4} - [\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{CO_2} - 2[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{H_2O} + 2[\bar{h}(T) - \bar{h}(537)]_{O_2}}{2[\bar{h}(T) - \bar{h}(860)]_{O_2} + 7.52[\bar{h}(T) - \bar{h}(860)]_{N_2}}$$

$$= \frac{(-32{,}210) - [-169{,}300 + \bar{h}(T) - 4028]_{CO_2} - 2[-104{,}040 + \bar{h}(T) - 4258] + 2[\bar{h}(T) - 3725]_{O_2}}{2[\bar{h}(T) - 6042]_{O_2} + 7.52[\bar{h}(T) - 5986]_{N_2}}$$

$$x = \frac{-\bar{h}_{CO_2}(T) - 2\bar{h}_{H_2O}(T) + 2\bar{h}_{O_2}(T) + 350{,}264}{2\bar{h}_{O_2}(T) + 7.52\bar{h}_{N_2}(T) - 57099}$$

Sample Calculation: $T = 1860$°R (1400°F)

$$x = \frac{-19173 - 2(16028) + 2(13987) + 350{,}264}{2(13987) + 7.52(13427) - 57099} = 4.55 \qquad (355\%\ \text{excess air})$$

PROBLEM 13.74 (Cont'd.) - Page 2

The data for the required plot are obtained using IT, as follows:

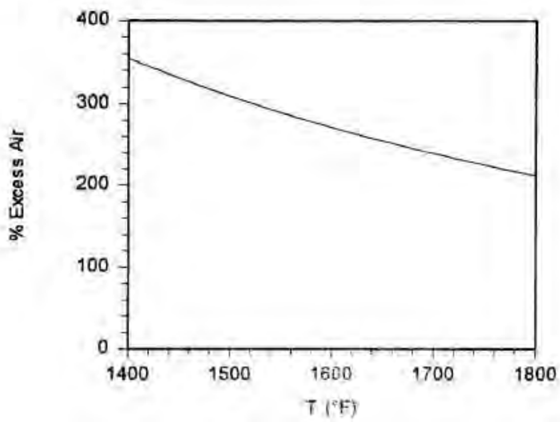
IT Code

```
T1 = 77 // °F
T2 = 400 // °F
T = 1400 // °F

0 = hCH4_1 + 2 *x*(hO2_2 + 3.76 * hN2_2) -
        (hCO2_3 + 2 * hH2O_3 + 2*(x - 1)*hO2_3 + 7.52*x*hN2_3)
hCH4_1 = h_T("CH4",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T)
hH2O_3 = h_T("H2O",T)
hO2_3 = h_T("O2",T)
hN2_3 = h_T("N2",T)
PCTexcess = (x - 1) * 100
```

IT Results for T = 1400°F

% Excess = 354.7%

PLOT:



Notice that excess air leads to a decrease in the temperature of the combustion products, as expected.

PROBLEM 13.75

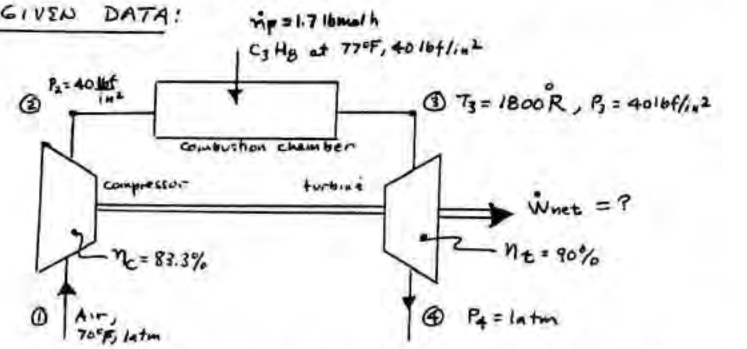
13.75 Air enters the compressor of a simple gas turbine power plant at 70°F, 1 atm, is compressed adiabatically to 40 lbf/in.^{2}, and then enters the combustion chamber where it burns completely with propane gas (C_3H_8) entering at 77°F, 40 lbf/in.^{2} and a molar flow rate of 1.7 lbmol/h. The combustion products at 1340°F, 40 lbf/in.^{2} enter the turbine and expand adiabatically to a pressure of 1 atm. The isentropic compressor efficiency is 83.3% and the isentropic turbine efficiency is 90%. Determine at steady state

(a) the percent of theoretical air required.
(b) the net power developed, in horsepower.

KNOWN: Operating data are provided for a simple gas turbine operating at steady state. C_3H_8 burns completely in the combustion chamber.

FIND: For a fuel consumption rate of 1.7 lbmol/h, determine (a) the percent of theoretical air, (b) the power developed, in hp.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) Control volumes enclosing various components, as required, are at steady state. (2) The compressor, turbine, and combustion chamber are adiabatic. (3) The fuel, air, and combustion products are modeled as ideal gases. (4) Kinetic and potential energy effects are negligible. (5) 3.76 moles of inert N_2 accompany each mole of O_2 in the combustion air. (6) Combustion is complete.

ANALYSIS: (a) Letting n denote the multiple of the theoretical amount of air, the balanced reaction equation reads

$$C_3H_8 + 5n\,[O_2 + 3.76N_2] \rightarrow 3CO_2 + 4H_2O + 5(n-1)O_2 + 18.8\,n\,N_2 \qquad (1)$$

An energy rate balance for the combustion chamber reads $\bar{h}_P = \bar{h}_R$. Solving this gives n, but T_2 is required. To find T_2, use the given compressor efficiency: $\eta_c = 0.833 = (h_{2s}-h_1)/(h_2-h_1)$. To determine T_{2s}, consider an isentropic compression of the assumed air mixture from $T_1 = 530°R$, $P_1 = 1$ atm to T_{2s}, 40 lbf/in^2. Then,

$$0 = \bar{s}_2 - \bar{s}_1 = 1\left[\bar{s}^\circ_{O_2}(T_{2s}) - \bar{s}^\circ_{O_2}(T_1) - \bar{R}\ln\frac{P_2}{P_1}\right] + 3.76\left[\bar{s}^\circ_{N_2}(T_{2s}) - \bar{s}^\circ_{N_2}(T_1) - \bar{R}\ln\frac{P_2}{P_1}\right]$$

$$\Rightarrow \quad \bar{s}^\circ_{O_2}(T_{2s}) + 3.76\,\bar{s}^\circ(T_{2s}) = \bar{s}^\circ_{O_2}(T_1) + 3.76\,\bar{s}^\circ_{N_2}(T_1) + 4.76\,\bar{R}\ln\frac{P_2}{P_1}$$
$$= 48.889 + 3.76(45.65) + (4.76)(1.986)\ln\frac{40}{14.7}$$
$$= 230 \text{ Btu/lb}\cdot°R$$

① Solving iteratively, $T_{2s} = 705°R$. Accordingly

②
$$\bar{h}_2 - \bar{h}_1 = \frac{\bar{h}_{2s} - \bar{h}_1}{\eta_c} = \frac{1(\bar{h}_{O_2}(705) - \bar{h}_{O_2}(530)) + 3.76\,(\bar{h}_{N_2}(705) - \bar{h}_{N_2}(530))}{0.833}$$
$$= \frac{1(4915 - 3676) + 3.76(4900 - 3681)}{0.833} = 6990 \frac{\text{Btu}}{\text{lbmol}(O_2)}$$

note

Then, with $\bar{h}_2 - \bar{h}_1 = 6990$, or

$$1[\bar{h}_{O_2}(T_2) - h_{O_2}(T_1)] + 3.76[\bar{h}_{N_2}(T_2) - \bar{h}_{N_2}(T_1)] = 6990$$

$$\bar{h}_{O_2}(T_2) + 3.76\,h_{N_2}(T_2) = 6990 + 3676 + 3.76(3681) = 24507 \frac{\text{Btu}}{\text{lbmol}(O_2)}$$

② Solving for T_2, $\quad T_2 \approx 740°R$.

Returning to the energy rate balance $\bar{h}_P = \bar{h}_R$, and using $\bar{h} = \bar{h}^\circ_f + \Delta\bar{h}$

$$3[\bar{h}^\circ_f + \Delta\bar{h}]_{CO_2} + 4[\bar{h}^\circ_f + \Delta\bar{h}]_{H_2O} + 5(n-1)[\Delta\bar{h}]_{O_2} + 18.8n\,[\Delta\bar{h}]_{N_2} = [\bar{h}^\circ_f]_{C_3H_8} + 5n[\Delta\bar{h}]_{O_2} + 18.8n[\Delta\bar{h}]_{N_2}$$

Then with data from the ideal gas tables and Table A-25E

$$3[-169,300+[18392-4028]]+4[-104,040+[15433-4258]]+5(n-1)[13486-3725]+18.8n[12956-3730]$$

$$=-44,680+5n[5167-3725]+18.8n[5144-3730]$$

Solving, $n = 4.46 \Rightarrow 446\%$ of theoretical air. ←(a)

(b) The net power developed is the difference between the turbine power output and the compressor power input. To find the turbine output

$$\eta_t = \frac{\dot{W}_t/\dot{n}_{fuel}}{(\dot{W}_t/\dot{n}_{fuel})_s} \Rightarrow \frac{\dot{W}_t}{\dot{n}_{fuel}} = \eta_t\left(\frac{\dot{W}_t}{\dot{n}_{fuel}}\right)_s \qquad (2)$$

Accordingly, consider next an isentropic expansion of the combustion products from (1800°R, 40 lbf/in²) to (T_{4s}, 1atm): $\bar{s}_3 - \bar{s}_{4s} = 0$. That is

$$0 = 3[\bar{s}^\circ(1800) - \bar{s}^\circ(T_{4s}) - \bar{R}\ln\tfrac{40}{14.7}]_{CO_2} + 4[\bar{s}^\circ(1800) - \bar{s}^\circ(T_{4s}) - \bar{R}\ln\tfrac{40}{14.7}] +$$
$$17.3[\bar{s}^\circ(1800) - \bar{s}^\circ(T_{4s}) - \bar{R}\ln\tfrac{40}{14.7}]_{O_2} + 83.848[\bar{s}^\circ(1800) - \bar{s}^\circ(T_{4s}) - \bar{R}\ln\tfrac{40}{14.7}]_{N_2}$$

Or, on rewriting this as

$$3\bar{s}^\circ_{CO_2}(T_{4s}) + 4\bar{s}^\circ_{H_2O}(T_{4s}) + 17.3\bar{s}^\circ_{O_2}(T_{4s}) + 83.848\bar{s}^\circ_{N_2}(T_{4s})$$
$$= 3\bar{s}^\circ_{CO_2}(1800) + 4\bar{s}^\circ_{H_2O}(1800) + 17.3\bar{s}^\circ_{O_2}(1800) + 83.848\bar{s}^\circ_{N_2}(1800) - 108.148\bar{R}\ln\frac{40}{14.7}$$
$$= 3(64.292) + 4(55.559) + 17.3(58.155) + 83.848(54.472) - 215$$
$$= 5773.6 \ \frac{Btu}{lbmol \cdot °R}$$

(2) Solving $T_{4s} \approx 1400°R$. The power that would be developed, per mole of fuel consumed, in an isentropic expansion is then

$$\left[\frac{\dot{W}_t}{\dot{n}_{fuel}}\right]_s = 3[\bar{h}(1800) - \bar{h}(1400)]_{CO_2} + 4[\bar{h}(1800) - \bar{h}(1400)]_{H_2O} + 17.3[\bar{h}(1800) - \bar{h}(1400)]_{O_2} + 83.848[\bar{h}(1800) - \bar{h}(1400)]_{N_2}$$
$$= 3[18392 - 13345] + 4[15433 - 11625] + 17.3[13486 - 10210] + 83.848[12956 - 9877]$$
$$= 343,539 \ \frac{Btu}{lbmol(fuel)}$$

Eq. (2) gives

$$\left(\frac{\dot{W}_t}{\dot{n}_{fuel}}\right) = 0.9(343,539) = 309,185 \ \frac{Btu}{lbmol(fuel)}$$

For the compressor

$$\frac{\dot{W}_c}{\dot{n}_{fuel}} = (\bar{h}_2 - \bar{h}_1)\left(\frac{lbmol(O_2)}{lbmol(fuel)}\right) = (6990)\left(\frac{(5)(4.46)}{1}\right) = 155,877 \ \frac{Btu}{lbmol(fuel)}$$

Finally, the net power output is

$$\dot{W}_{net} = \dot{n}_{fuel}\left[\frac{\dot{W}_t}{\dot{n}_{fuel}} - \frac{\dot{W}_c}{\dot{n}_{fuel}}\right]$$
$$= 1.7\ \frac{lbmol(fuel)}{h}\left[309,185 - 155,877\right]\left(\frac{Btu}{lbmol(fuel)}\right)\left|\frac{1\ hp}{2545\ Btu/h}\right| = 102.4\ hp \leftarrow (b)$$

1. By ignoring the slight difference between the combustion air model and the air of Table A-22E, table data can be used to analyze the compressor. Thus, for example, $p_{r,2s} = (p_2/p_1)p_{r1} = (40/14.7)(1.3) = 3.537 \Rightarrow T_{2s} = 705°R$.

2. An iterative solution using table data can be avoided by using IT.

PROBLEM 13.76

13.76 A mixture of gaseous octane (C_8H_{18}) and 200% of theoretical air, initially at 25°C, 1 atm, reacts completely in a rigid vessel.

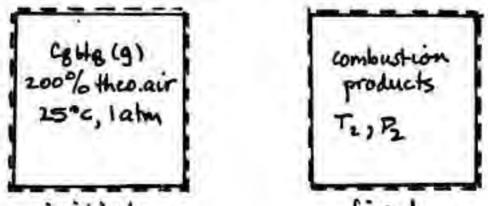
(a) If the vessel were well-insulated, determine the temperature, in °C, and the pressure, in atm, of the combustion products.

(b) If the combustion products were cooled at constant volume to 25°C, determine the final pressure, in atm, and the heat transfer, in kJ per kmol of fuel.

KNOWN: A mixture of gaseous C_8H_{18} and 200% of theoretical air, initially at 25°C, 1 atm, reacts completely in a rigid vessel.

FIND: (a) If the vessel is well-insulated, determine the temperature and pressure of the combustion products. (b) If the combustion products are cooled to 25°C, determine the final pressure and the heat transfer per kmol of fuel.

SCHEMATIC & GIVEN DATA:



(a) $Q = 0$; $T_2 = ?$, $P_2 = ?$
(b) $T_2 = T_1$, $P_2 = ?$, $Q/n_F = ?$

ENGINEERING MODEL: (1) For the closed system shown, (a) $Q = 0$, (b) $T_2 = T_1$, and $W = 0$. There are no effects of kinetic or potential energy. (2) Combustion is complete. (3) 3.76 kmol of N_2 accompany each kmol of O_2, and the nitrogen is inert. (4) The initial fuel-air mixture and the gaseous products of combustion can be modeled as ideal gas mixtures.

ANALYSIS: (a) Assuming 1 kmol of fuel initially present, the reaction is described by

$$C_8H_{18} + 25\,(O_2 + 3.76N_2) \rightarrow 8CO_2 + 9H_2O + 12.5O_2 + 94N_2 \qquad (1)$$

The energy balance reduces to $\Delta U + \cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0} = \cancel{Q}^{0} - \cancel{W}^{0} \Rightarrow \Delta U = 0$. Thus

$$0 = (8\bar{u}_{CO_2} + 9\bar{u}_{H_2O} + 12.5\bar{u}_{O_2} + 94\bar{u}_{N_2})_2 - (\bar{u}_{C_8H_{18}} + 25\bar{u}_{O_2} + 94\bar{u}_{N_2})_1$$

With $\bar{u} = \bar{h} - \bar{R}T$ this becomes

$$0 = (8\bar{h}_{CO_2} + 9\bar{h}_{H_2O} + 12.5\bar{h}_{O_2} + 94\bar{h}_{N_2})_2 - (\bar{h}_{C_8H_{18}} + 25\bar{h}_{O_2} + 94\bar{h}_{N_2})_1 - (8+9+12.5+94)\bar{R}T_2 + (1+25+94)\bar{R}T_1$$

Introducing $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$0 = 8\left[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(298)\right]_{CO_2} + 9\left[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(298)\right]_{H_2O(g)} + 12.5\left[\bar{h}(T_2) - \bar{h}(298)\right]_{O_2}$$
$$+ 94\left[\bar{h}(T_2) - \bar{h}(298)\right]_{N_2} - (\bar{h}_f^\circ)_{C_8H_{18}(g)} - \bar{R}\left[123.5\,T_2 - 120\,T_1\right]$$

With data from Tables A-23 and A-25

$$8\bar{h}_{CO_2}(T_2) + 9\bar{h}_{H_2O}(T_2) + 12.5\bar{h}_{O_2}(T_2) + 94\bar{h}_{N_2}(T_2) - (123.5)(8.314)T_2 = (-8)(-393{,}520 - 9364)$$
$$-9(-241{,}820 - 9904) + 12.5(8682) + 94(8669) - 208{,}450 - (120)(8.314)(298)$$
$$= 5{,}906{,}240$$

① Solving iteratively for T_2, $T_2 = 1859$ K. ⟵ (a) T_2

Using the ideal gas equation of state

$$\left.\begin{matrix} P_1V = n_1\bar{R}T_1 \\ P_2V = n_2\bar{R}T_2 \end{matrix}\right\} \frac{P_2}{P_1} = \frac{n_2}{n_1}\cdot\frac{T_2}{T_1} \Rightarrow P_2 = \frac{n_2}{n_1}\cdot\frac{T_2}{T_1}\cdot P_1 = \left(\frac{123.5}{120}\right)\left(\frac{1859}{298}\right)(1\text{ atm})$$
$$= 6.42 \text{ atm}$$ ⟵ (a) P_2

(b) The reaction is described by Eq. (1). For 1 kmol of fuel, the products contain 9 kmol of H_2O and $n_{dry} = 114.5$ kmol. When the products cool to 25°C, condensation occurs. Thus, if n_v is the amount of water vapor and n_ℓ the amount of saturated liquid water at the final state

$$n_v + n_\ell = 9 \text{ kmol}$$

Further, the partial pressure of the water vapor is

$$p_v = p_g(25°C) = 0.03169 \text{ bar} = \left(\frac{n_v}{114.5 + n_v}\right) p_2 \qquad (2)$$

Both p_2 and n_v are unknown. Another relation is obtained based on the assumption that the volume occupied by the liquid phase is negligible. Then

$$\frac{n_1 \bar{R} T}{p_1} = \frac{(n_{dry} + n_v)\bar{R}T}{p_2} \Rightarrow p_2 = \left(\frac{n_{dry} + n_v}{n_1}\right) p_1$$

$$= \left(\frac{114.5 + n_v}{120}\right)(1.01325 \text{ bar}) \qquad (3)$$

Solving (2) and (3) simultaneously

$$n_v = 3.753 \text{ kmol}, \quad p_2 = 0.9985 \text{ bar} \quad \text{(b) } p_2$$

$$n_\ell = 9 - n_v = 5.247 \text{ kmol}$$

Therefore

$$C_8H_{18(g)} + 25(O_2 + 3.76N_2) \rightarrow 8CO_2 + (3.753\, H_2O(g) + 5.247\, H_2O(\ell)) + 12.5O_2 + 94N_2$$

The energy balance reduces to $\Delta U = Q$. Thus

$$Q = (8\bar{u}_{CO_2} + 3.753\bar{u}_{H_2O(g)} + 5.247\bar{u}_{H_2O(\ell)} + 12.5\bar{u}_{O_2} + 94\bar{u}_{N_2})_2 - (\bar{u}_{C_8H_{18}} + 25\bar{u}_{O_2} + 94\bar{u}_{N_2})_1$$

With $\bar{u} = \bar{h} - \bar{R}T$ for the gas components and $\bar{u} = \bar{h} - p\bar{v}^{\,0}$ for the liquid

$$Q = (8\bar{h}_{CO_2} + 3.753\bar{h}_{H_2O(g)} + 12.5\bar{h}_{O_2} + 94\bar{h}_{N_2})_2 - (\bar{h}_{C_8H_{18}} + 25\bar{h}_{O_2} + 94\bar{h}_{N_2})_1$$
$$+ 5.247\bar{h}_{H_2O(\ell)} - (123.5 - 5.247 - 120)(8.314)(298)$$

Introducing $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2, and setting the $\Delta\bar{h}$ terms to zero since $T_2 = T_1$

$$Q = 8(\bar{h}_f^\circ)_{CO_2} + 3.753(\bar{h}_f^\circ)_{H_2O(g)} - (\bar{h}_f^\circ)_{C_8H_{18}(g)} + 5.247(\bar{h}_f^\circ)_{H_2O(\ell)} - (-1.747)(8.314)(298)$$

$$= 8(-393{,}520) + 3.753(-241{,}820) - (-208{,}450) + 5.247(-285{,}830) + 4328.3$$

$$= -5.343 \times 10^6 \text{ kJ/kmol (fuel)} \quad \text{(b) } Q$$

1. Iteration using table data can be avoided by using IT.

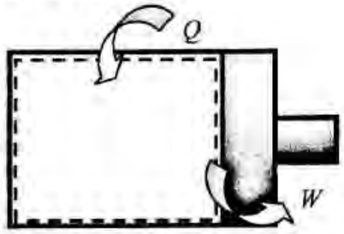
13.77 Methane gas (CH_4) reacts completely with the theoretical amount of oxygen (O_2) in a piston-cylinder assembly. Initially, the mixture is at 77°F, 1 atm. If the process occurs at constant pressure and the final volume is 1.9 times the initial volume, determine the work and the heat transfer, each in Btu per lbmol of fuel.

KNOWN: Methane gas reacts completely at constant pressure with theoretical oxygen in a piston-cylinder assembly. The initial state is known and the volume ratio for the process is specified.

FIND: Determine the work and heat transfer, each per lbmol of fuel.

SCHEMATIC AND GIVEN DATA:

Initial State
537°R,1 atm
CH_4 + theoretical
amount of O_2



Final State
$p_2 = p_1 = 1$ atm
$V_2/V_1 = 1.9$

ENGINEERING MODEL: (1) The system is a closed system. (2) The methane burns completely with the theoretical amount of oxygen. (3) The pressure is constant. (4) The reactants and products are modeled as ideal gas mixtures. (5) Kinetic and potential energy effects are negligible.

ANALYSIS: First, the balanced reaction equation for theoretical combustion of methane and oxygen is

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

The energy balance for the process of the closed system is $\Delta U = Q - W$. With $W = p(V_2 - V_1)$ for the constant pressure process

$$Q = \Delta U + p(V_2 - V_3)$$

Noting that $H = U + pV$

$$Q = \Delta H$$

Therefore

$$Q/n_{\text{fuel}} = [\bar{h}_f^o + \bar{h}(T_2) - \bar{h}(537)]_{CO_2} + 2[\bar{h}_f^o + \bar{h}(T_2) - \bar{h}(537)]_{H_2O}$$

$$- [\bar{h}_f^o + \cancel{\Delta \bar{h}}]_{CH_4} - 2[\cancel{\bar{h}_f^o} + \cancel{\Delta \bar{h}}]_{O_2}$$

Problem 13.77 (Continued) – Page 2

To get T_2, note that for the ideal gas mixture with $p_2 = p_1$ and $pV = n\bar{R}T$

$$\frac{n_{\text{products}}\bar{R}T_2}{V_2} = \frac{n_{\text{reactants}}\bar{R}T_1}{V_1} \quad \text{-or-} \quad \frac{T_2}{T_1} = \frac{n_{\text{reactants}}}{n_{\text{products}}}\frac{V_2}{V_1}$$

From the balanced reaction equation, $n_{\text{reactants}} = 3$ and $n_{\text{products}} = 3$. Thus

$$T_2 = (V_2/V_1)\,T_1 = (1.9)\,(537^{\circ}\text{R}) = 1020^{\circ}\text{R}$$

With values for specific enthalpy from Table A-23E and the enthalpy of formation of the fuel from Table A-25E, the heat transfer is

$$Q/n_{\text{fuel}} = [-169{,}300 + 8903.4 - 4027.5] + 2[-104{,}040 + 8250.4 - 4258.0] - [-32{,}210]$$

$$= \underline{\mathbf{-\ 332{,}310\ Btu/lbmol(CH4)}}$$

*The negative sign denotes that the heat transfer is **from** the system to the surroundings.*

Since $n_{\text{reactants}} = n_{\text{products}} = 3$ lbmol, the work is

$$W = p(V_2 - V_1) = n\bar{R}(T_2 - T_1) = (3\text{ lbmol})(1.986\frac{\text{Btu}}{\text{lbmol oR}})(1020 - 537)^{\circ}\text{R} = 2878\text{ Btu}$$

Finally, the work per lbmol of fuel is

$$W/n_{\text{fuel}} = (2877.7\text{ Btu})/(1\text{ lbmol }CH_4) = \underline{\mathbf{2878\ Btu/lbmol(CH_4)}}$$

*The positive sign denotes that the work is **from** the system to the surroundings for the expansion, as expected.*

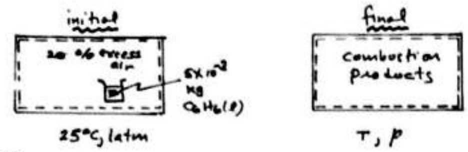
PROBLEM 13.78

13.78 A 5×10^{-3} kg sample of liquid benzene (C_6H_6) together with 20% excess air, initially at 25°C and 1 atm, reacts completely in a rigid, insulated vessel. Determine the temperature, in °C, and the pressure, in atm, of the combustion products.

KNOWN: 5×10^{-3} kg of liquid C_6H_6 and 20% excess air, initially at 25°C, 1 atm, reacts completely in a rigid insulated vessel.

FIND: Determine the final temperature and final pressure.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The system is shown in the figure above. (2) For the system, Q=0, W=0, and kinetic/potential energy effects are absent. (3) The reaction of the fuel and air is complete. 3.76 moles of N_2 accompany each mole of O_2 in the combustion air. N_2 is inert. (4) The volume occupied by the liquid fuel initially is much less than the volume of the gas phase. (5) The air and the combustion products can each be modeled as an ideal gas.

ANALYSIS: The reaction is described by

$$C_6H_6(l) + 9\,(O_2+3.76N_2) \rightarrow 6\,CO_2 + 3H_2O + 1.5\,O_2 + 33.84\,N_2 \qquad (1)$$

With assumption (1), the energy balance reduces to $\Delta U = 0$. Or

$$0 = (6\bar{u}_{CO_2} + 3\bar{u}_{H_2O} + 1.5\bar{u}_{O_2} + 33.84\,\bar{u}_{N_2}) - [\bar{u}_{C_6H_6} + 9\,\bar{u}_{O_2} + 33.84\,\bar{u}_{N_2}] \qquad (2)$$

For the gases, the ideal gas model gives $\bar{u} = \bar{h} - \bar{R}T$. For the liquid fuel, $pv << u$; so $\bar{h} \approx \bar{u}$. Thus Eq.(2) becomes

$$0 = 6[\bar{h}-\bar{R}T]_{CO_2} + 3[\bar{h}-\bar{R}T]_{H_2O} + 1.5[\bar{h}-\bar{R}T]_{O_2} + 33.84[\bar{h}-\bar{R}T]_{N_2}$$
$$- \bar{h}_{C_6H_6} - 9[\bar{h}-\bar{R}T_1]_{O_2} - 33.84[\bar{h}-\bar{R}T_1]_{N_2}$$

or

$$0 = 6[\bar{h}_f^\circ + \Delta\bar{h}]_{CO_2} + 3[\bar{h}_f^\circ + \Delta\bar{h}]_{H_2O} + 1.5[\Delta\bar{h}]_{O_2} + 33.84[\Delta\bar{h}]_{N_2}$$
$$- (\bar{h}_f^\circ)_{C_6H_6(l)} - 9\,[\cancel{\bar{h}_f^\circ + \Delta\bar{h}}]_{O_2} - 33.84\,[\cancel{\bar{h}_f^\circ + \Delta\bar{h}}]_{N_2}$$
$$+ \; 42.84\,\bar{R}T_1 - 44.34\,\bar{R}T$$

With ideal gas table data and data from the literature for $(\bar{h}_f^\circ)_{C_6H_6(l)} = 49{,}100$ kJ/kmol

$$0 = 6[-393{,}520 + \bar{h}_{CO_2}(T) - 9364] + 3[-241{,}820 + \bar{h}_{H_2O}(T) - 9904]$$
$$+ 1.5[\bar{h}_{O_2}(T) - 8682] + 33.84[\bar{h}_{N_2}(T) - 8669] - 49{,}100 +$$
$$(42.84)(8.314)(298) - (44.34)(8.314)\,T$$

$$\Rightarrow 6\bar{h}_{CO_2}(T) + 3\bar{h}_{H_2O}(T) + 1.5\bar{h}_{O_2}(T) + 33.84\,\bar{h}_{N_2}(T) - 368.6\,T = 3{,}421{,}819 \text{ kJ/kmol (fuel)}$$

① Solving iteratively with table data $T \approx 2700$K ($T = 2427$°C)

Using the ideal gas equation of state for the gas phase only

$pV = n_P \bar{R} T$

$p_1 V = n_R \bar{R} T_1$ ← gas phase only

② ← ignore volume occupied by the liquid

$$\Rightarrow \frac{p}{p_1} = \left(\frac{n_P}{n_R}\right)\left(\frac{T}{T_1}\right) = \left(\frac{44.34}{42.84}\right)\left(\frac{2700}{298}\right) = 9.38$$

$\Rightarrow p = 9.38$ atm

1. Iteration with table data can be avoided by using IT.
2. See (4) of the Engineering Model.

PROBLEM 13.79

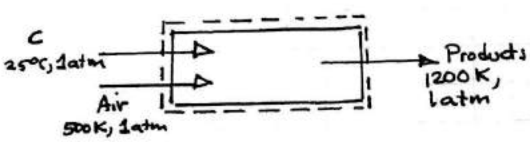
Carbon enters a well-insulated reactor at 25°C, 1 atm and reacts completely with excess air entering at 500 K, 1 atm. The products exit at 1200 K, 1 atm. For operation at steady state and ignoring kinetic and potential energy effects, determine (a) the percent excess air, (b) the rate of entropy production, in kJ/K per kmol of carbon.

SOLUTION

Known: Steady-state data are provided for a well-insulated reactor in which carbon reacts completely with excess air.

Find: Determine the percent excess air and the rate of entropy production within the reactor, in kJ/K per kmol of carbon.

Schematic and Given Data:



Engineering Model:

1. The control volume shown in the schematic is at steady state.
2. Each mole of O_2 in the air is accompanied by 3.76 moles of N_2 (inert)
3. The air and combustion products form ideal gas mixtures.
4. Combustion is complete.
5. For the control volume $\dot{Q}_{cv} = \dot{W}_{cv} = 0$, and kinetic and potential energy effects are ignored.

Analysis: Combustion of carbon with excess air:

$$C + \alpha(O_2 + 3.76N_2) \rightarrow \beta CO_2 + \gamma O_2 + 3.76\alpha N_2$$

$C:\ 1 = \beta$

$O:\ 2\alpha = 2 + 2\gamma,\ \gamma = \alpha - 1$

$$\Rightarrow\ C + \alpha(O_2 + 3.76N_2) \rightarrow CO_2 + (\alpha-1)O_2 + 3.76\alpha N_2$$

(a) An energy rate balance reads $0 = \cancel{\frac{\dot{Q}_{cv}}{\dot{n}_C}}^{0} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_C}}^{0} + H_R - H_P$, where with $\bar{h} = \bar{h}_f^o + \Delta\bar{h}$,

Table A-25

$$H_R = 0 + \alpha[0 + (14{,}770 - 8682)] + 3.76\alpha[0 + (14{,}581 - 8669)]$$

$$= 28{,}317\alpha$$

$$H_P = [-393{,}520 + (53848 - 9364)] + (\alpha-1)[38{,}447 - 8682] + 3.76\alpha[36{,}777 - 8669]$$

$$= -349{,}036 + (\alpha-1)(29{,}765) + 3.76\alpha(28{,}108)$$

$$= -378{,}801 + 135{,}451\alpha$$

$H_R = H_P \Rightarrow 28{,}317\alpha = -378{,}801 + 135{,}451\alpha \Rightarrow \alpha = 3.536$

For complete combustion with theoretical air, $\alpha = 1$.

$$\%\ \text{excess air} = \left(\frac{3.536 - 1}{1}\right)(100\%) = 253.6\% \leftarrow (a)$$

(b) An entropy rate balance on a per kmol of C basis reads (Eq. 13.24),

$$0 = \bar{s}_C + \underbrace{\alpha}_{\text{part (a)}}[\bar{s}_{O_2} + 3.76\,\bar{s}_{N_2}] - [\bar{s}_{CO_2} + (\alpha-1)\,\bar{s}_{O_2} + 3.76\alpha\,\bar{s}_{N_2}] + \dot{\sigma}_{cv}/\dot{n}_C$$

Applying Eq 13.23 with data from Table A-23 for O_2 and N_2 in the air,

$$\bar{s}_{O_2} = \bar{s}^\circ_{O_2}(500K) - \bar{R}\ln\frac{(y_{O_2})(1\,atm)}{(1\,atm)}$$

$$= 220.589 - 8.314\ln(0.21) = 233.564\ kJ/kmol\cdot K$$

$$\bar{s}_{N_2} = 206.630 - 8.314\ln(0.79) = 208.590 \quad ''$$

For the combustion products the mole fractions needed to apply Eq. 13.23 are $y_{CO_2} = 1/16.831 = 0.0594$, $y_{O_2} = 2.536/16.831 = 0.1507$, $y_{N_2} = 0.7899$.

Then

$$\bar{s}_{CO_2} = \bar{s}^\circ(1200K) - \bar{R}\ln\frac{(y_{CO_2})(1\,atm)}{(1\,atm)}$$

$$= 279.307 - 8.314\ln(0.0594) = 302.781\ kJ/kmol\cdot K$$

$$\bar{s}_{O_2} = 249.906 - 8.314\ln(0.1507) = 265.640 \quad ''$$

$$\bar{s}_{N_2} = 234.115 - 8.314\ln(0.7899) = 236.076 \quad ''$$

$$\Rightarrow \frac{\dot{\sigma}_{cv}}{\dot{n}_C} = [(302.781) + (2.536)(265.640) + (13.295)(236.076)]$$
$$-[\underbrace{5.74}_{\text{Table A-25 for C}} + (3.536)(233.564) + (13.295)(208.590)]$$

$$= 510.2\ kJ/K \text{ per kmol of Carbon.} \qquad \text{(b)}$$

PROBLEM 13.80

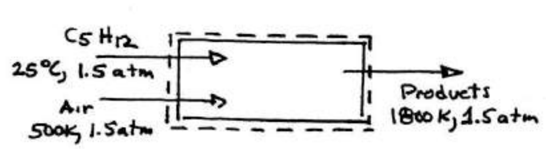
Pentane (C_5H_{12}) gas enters a well-insulated reactor at 25°C, 1.5 atm and reacts completely with excess air entering at 500 K, 1.5 atm. The products exit at 1800 K, 1.5 atm. For operation at steady state and ignoring kinetic and potential energy effects, determine (a) the percent excess air, (b) the rate of entropy production, in kJ/K per kmol of pentane.

SOLUTION

Known: Steady-state data are provided for a well-insulated reactor in which pentane reacts completely with excess air.

Find: Determine the percent excess air and rate of entropy production within the reactor, in kJ/K per kmol of pentane.

Schematic and Given Data:



Engineering Model:

1. The control volume shown in the schematic is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects.
2. Each mole of O_2 in the air is accompanied by 3.76 moles of N_2 (inert).
3. The air and combustion products form ideal gas mixtures. Pentane is also modeled as an ideal gas.
4. Combustion is complete

Analysis: Combustion of C_5H_{12} with the theoretical amount of air is given by

$$C_5H_{12} + 8[O_2 + 3.76N_2] \rightarrow 5CO_2 + 6H_2O + (8)(3.76)N_2$$

Then, complete combustion with excess air reads ($x > 1.0$)

$$C_5H_{12} + 8x(O_2 + 3.76N_2) \rightarrow 5CO_2 + 6H_2O + 8(x-1)O_2 + 30.08x\, N_2$$

(a) An energy rate balance reads $0 = \cancel{\frac{\dot{Q}_{cv}}{\dot{n}_F}}^{0} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_F}}^{0} + H_R - H_P$, where $\bar{h} = \bar{h}_f^{\circ} + \Delta\bar{h}$ and

$$H_R = \underbrace{-146{,}400}_{\text{Table A-25}} + 8x[0 + (14770 - 8682)] + 30.08x[0 + (14{,}581 - 8669)]$$

$$= -146{,}400 + 226{,}537x$$

$$H_P = 5[-393{,}520 + (88{,}806 - 9364)] + 6[-241{,}820 + (72{,}513 - 9904)] + 8(x-1)[0 + (60{,}371 - 8682)] + 30.08x[0 + (57{,}651 - 8669)]$$

$$= 1{,}886{,}891x - 3{,}059{,}168$$

$$H_P = H_R \Rightarrow 1{,}886{,}891x - 3{,}059{,}168 = -146{,}400 + 226{,}537x$$

$$\Rightarrow x = 1.754$$

Accordingly,

$$\%\ \text{Excess Air} = \left[\frac{(8)(\overset{x}{1.754}) - 8}{8}\right](100)$$

$$= 75.4\%$$

(b) An entropy rate balance on a per kmol of fuel basis reads (Eq. 13.24),

$$0 = \bar{s}_F + 8\underbrace{x}_{=1.754}[\bar{s}_{O_2} + 3.76\,\bar{s}_{N_2}] - [5\,\bar{s}_{CO_2} + 6\,\bar{s}_{H_2O} + 8(x-1)\,\bar{s}_{O_2} + 30.08x\,\bar{s}_{N_2}] + \frac{\dot{\sigma}_{cv}}{\dot{n}_F}$$

The mole fractions of the products are, $y_{CO_2} = 5/69.792 = 0.0716$, $y_{H_2O} = 6/69.792 = 0.0860$, $y_{O_2} = 0.0864$, $y_{N_2} = 0.7560$.

For the fuel, use Eq. 13.22: $\bar{s}_F = \bar{s}^\circ(298.15K) - \bar{R}\ln\frac{p}{p_{ref}}$, where $\bar{s}^\circ$ is from Table A-25:

$$\bar{s}_F = 348.40 - 8.314\ln\left(\frac{1.5}{1}\right) = 345.029 \text{ kJ/kmol}\cdot\text{K}$$

Applying Eq. 13.23 with data from Table A-23 for the air:

$$\bar{s}_{O_2} = \bar{s}^\circ_{O_2}(500K) - 8.314\ln\left[\frac{(0.21)(1.5\text{atm})}{1\text{atm}}\right]$$

$$= 220.589 - 8.314\ln(0.315) = 230.193 \text{ kJ/kmol}\cdot\text{K}$$

$$\bar{s}_{N_2} = 206.630 - 8.314\ln\left[\frac{(0.79)(1.5)}{1}\right] = 205.219 \text{ ''}$$

For the combustion products:

$$\bar{s}_{CO_2} = \bar{s}^\circ_{CO_2}(1800K) - 8.314\ln\left[\frac{(0.0716)(1.5\text{atm})}{1\text{atm}}\right]$$

$$= 302.884 - 8.314\ln(0.1074) = 321.434 \text{ kJ/kmol}\cdot\text{K}$$

$$\bar{s}_{H_2O} = 259.262 - 8.314\ln[(0.0860)(1.5)] = 276.289 \text{ ''}$$

$$\bar{s}_{O_2} = 264.701 - 8.314\ln[(0.0864)(1.5)] = 281.689 \text{ ''}$$

$$\bar{s}_{N_2} = 248.195 - 8.314\ln[(0.7560)(1.5)] = 247.150 \text{ ''}$$

Collecting results,

$$S_R = (345.029) + (14.032)(230.193) + (52.760)(205.219)$$
$$= 14{,}402.45 \text{ kJ/kmol(F)}\cdot\text{K}$$

$$S_P = 5(321.434) + 6(276.289) + 6.032(281.689) + 52.760(247.150)$$
$$= 18{,}003.69 \text{ kJ/kmol(F)}\cdot\text{K}$$

$$\Rightarrow \quad \frac{\dot{\sigma}_{cv}}{\dot{n}_F} = S_P - S_R = 3601.24 \text{ kJ/kmol(F)}\cdot\text{K}$$

PROBLEM 13.81

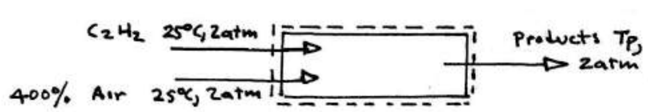
Ethylene (C_2H_4) gas enters a well-insulated reactor and reacts completely with 400% of theoretical air, each at 25°C, 2 atm. The products exit the reactor at 2 atm. For operation at steady state and ignoring kinetic and potential energy effects, determine (a) the balanced reaction equation, (b) the temperature, in K, at which the products exit, (c) the rate of entropy production, in kJ/K per kmol of ethylene.

SOLUTION

Known: Steady-state data are provided for a well-insulated reactor in which C_2H_4 reacts completely with 400% theoretical air.

Find: Determine the balanced reaction equation, the temperature at which products exit the reactor, and the rate of entropy production within the reactor, in kJ/K per kmol of ethylene.

Schematic and Given Data:



Engineering Model:

1. The control volume shown in the schematic is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects.
2. Each mole of O_2 in the air is accompanied by 3.76 moles of N_2 (inert).
3. The air and combustion products form ideal gas mixtures. C_2H_4 is also modeled as an ideal gas.
4. Combustion is complete.

Analysis: (a) Complete combustion of C_2H_4 with the theoretical amount of air is given by

$$C_2H_4 + 3(O_2 + 3.76N_2) \rightarrow 2CO_2 + 2H_2O + (3)(3.76)N_2$$

Complete combustion with 400% theoretical air is then

$$C_2H_4 + 12(O_2 + 3.76N_2) \rightarrow 2CO_2 + 2H_2O + 9O_2 + 45.12\,N_2 \quad \longleftarrow$$

(b) An energy rate balance reads $0 = \cancel{\frac{\dot{Q}_{cv}}{\dot{n}_F}}^{0} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_F}}^{0} + H_R - H_P$ where $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$

$$H_R = 52{,}280 + 0 + 0 = 52{,}280 \text{ kJ/kmol(F)}$$

$$H_P = 2[-393{,}520 + \bar{h}_{CO_2}(T_P) - 9364] + 2[-241{,}820 + \bar{h}_{H_2O}(T_P) - 9904] + 9[\bar{h}_{O_2}(T_P) - 8682] + 45.12[\bar{h}_{N_2}(T_P) - 8669]$$

$H_R = H_P \Rightarrow$

$$2\bar{h}_{CO_2}(T_P) + 2\bar{h}_{H_2O}(T_P) + 9\bar{h}_{O_2}(T_P) + 45.12\bar{h}_{N_2}(T_P) = 1{,}830{,}779 \quad (a)$$

1 Solving, $T_P = 1016$ K $\longleftarrow$

(c) An entropy rate balance on a per kmol of fuel basis reads (Eq. 13.24)

$$0 = \bar{s}_F + 12[\bar{s}_{O_2} + 3.76\,\bar{s}_{N_2}] - [2\bar{s}_{CO_2} + 2\bar{s}_{H_2O} + 9\bar{s}_{O_2} + 45.12\,\bar{s}_{N_2}] + \frac{\dot{\sigma}_{cv}}{\dot{n}_F}$$

The mole fractions of the products are $y_{CO_2} = 2/58.12 = 0.0344$, $y_{H_2O} = 0.0344$, $y_{O_2} = 9/58.12 = 0.1549$, $y_{N_2} = 45.12/58.12 = 0.7763$.

PROBLEM 13.81 (Continued)

For the fuel, use Eq. 13.22: $\bar{s}_F = \bar{s}^\circ(T) - \bar{R}\ln\frac{p}{p_{ref}}$, where $\bar{s}^\circ$ is from Table A-25:

$$\bar{s}_F = 219.83 - 8.314\ln\left(\frac{2\,atm}{1\,atm}\right) = 214.067\ kJ/kmol\cdot K$$

Applying Eq. 13.23 with data from Table A-23 for the air:

$$\bar{s}_{O_2} = \bar{s}^\circ_{O_2}(25°C) - 8.314\ln\left(\frac{(0.21)(2\,atm)}{1\,atm}\right)$$
$$= 205.033 - 8.314\ln(0.42) = 212.245\ kJ/kmol\cdot K$$

$$\bar{s}_{N_2} = 191.502 - 8.314\ln((0.79)(2)) = 187.699 \quad "$$

For the combustion products:

$$\bar{s}_{CO_2} = \bar{s}^\circ_{CO_2}(1016K) - 8.314\ln\left(\frac{(0.0344)(2\,atm)}{1\,atm}\right)$$
$$= 270.077 - 8.314\ln(0.0688) = 292.330\ kJ/kmol\cdot K$$

$$\bar{s}_{H_2O} = 233.251 - 8.314\ln(0.0688) = 255.504 \quad "$$

$$\bar{s}_{O_2} = 244.025 - 8.314\ln(0.3098) = 253.768 \quad "$$

$$\bar{s}_{N_2} = 228.576 - 8.314\ln(1.5526) = 224.918 \quad "$$

Collecting results,

$$S_R = (214.067) + 12\,(212.245) + 45.12\,(187.699) = 11{,}229.99\ kJ/kmol(F)\cdot K$$

$$S_P = 2\,(292.330) + 2\,(255.504) + 9\,(253.768) + 45.12\,(224.918)$$
$$= 13527.88\ kJ/kmol(F)\cdot K$$

$$\Rightarrow \frac{\dot{\sigma}_{cv}}{\dot{n}_F} = S_P - S_R = 2297.89\ \frac{kJ}{kmol(F)\cdot K} \quad \longleftarrow$$

---

1. When solving iteratively using Table A-23 data, a first trial can be obtained by assuming all products are nitrogen. Then Eq (a) reads

$$58.12\,\bar{h}_{N_2}(T_P) = 1{,}830{,}779$$
$$\Rightarrow \bar{h}_{N_2}(T_P) = 31{,}500\ kJ/kmol$$

By inspection of Table A-23, T_P falls between 1040K and 1060K. Interpolation gives $T_P = 1042K$, which then serves as a first trial value.

PROBLEM 13.82

Methane (CH_4) at 77°F, 1 atm enters an insulated reactor operating at steady state and burns completely with air entering in a separate stream at 77°F, 1 atm. The products of combustion exit as a mixture at 1 atm. For the reactor, determine the rate of entropy production, in Btu/°R per lbmol of methane entering, for combustion with

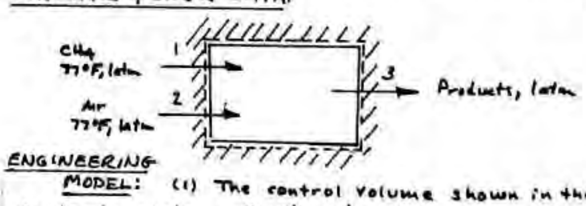
(a) the theoretical amount of air.
(b) 200% of theoretical air.

Neglect kinetic and potential energy effects.

KNOWN: CH_4 at 77°F, 1atm enters an insulated reactor and burns completely with air entering in a separate stream at 77°F, 1atm. The products exit at 1atm.

FIND: Determine the rate of entropy production, per lbmol of CH_4, for combustion with (a) the theoretical amount of air, (b) 200% of theoretical air.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) The reaction is complete. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The combustion air and products of combustion can be modeled as ideal gases.

ANALYSIS: (a) Complete combustion of CH_4 with the theoretical amount of air is described by

$$CH_4 + (2O_2 + 7.52N_2) \rightarrow CO_2 + 2H_2O + 7.52\,N_2$$

To determine T_3, write an energy rate balance at steady state:

$$0 = \cancel{\frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}}}^{\,0} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{CH_4}}}^{\,0} + (\bar{h}_{CH_4})_1 + (2\bar{h}_{O_2} + 7.52\bar{h}_{N_2})_2 - (\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 7.52\bar{h}_{N_2})_3$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$ and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$0 = (\bar{h}_f^\circ)_{CH_4} + (0) - \{[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{CO_2} + 2[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{H_2O} + 7.52[\bar{h}(T_3) - \bar{h}(537)]_{N_2}\}$$

With data from the ideal gas tables and Table A-25E

$$\bar{h}_{CO_2}(T_3) + 2\bar{h}_{H_2O}(T_3) + 7.52\bar{h}_{N_2}(T_3) = (\bar{h}_f^\circ)_{CH_4} - [\bar{h}_f^\circ - \bar{h}(537)]_{CO_2} - 2[\bar{h}_f^\circ - \bar{h}(537)]_{H_2O} + 7.52\bar{h}_{N_2}(537)$$

$$= -32{,}210 - [-169{,}300 - 4028] - 2[-104{,}040 - 4258] + 7.52[3730]$$

$$= 385{,}764$$

① Solving for T_3, $T_3 \approx 4187$°R.

An entropy rate balance at steady state takes the form

$$0 = \cancel{\sum \frac{\dot{Q}_j/T_j}{\dot{n}_{CH_4}}}^{\,0} + (\bar{s}_{CH_4})_1 + (2\bar{s}_{O_2} + 7.52\bar{s}_{N_2})_2 - (\bar{s}_{CO_2} + 2\bar{s}_{H_2O} + 7.52\,\bar{s}_{N_2})_3 + \dot{\sigma}_{cv}/\dot{n}_{CH_4}$$

or

$$\dot{\sigma}_{cv}/\dot{n}_{CH_4} = (\bar{s}_{CO_2} + 2\bar{s}_{H_2O} + 7.52\,\bar{s}_{N_2})_3 - (\bar{s}_{CH_4})_1 - (2\bar{s}_{O_2} + 7.52\bar{s}_{N_2})_2 \qquad (1)$$

The CH_4 enters separately at 537°R, 1atm, so from Table A-25E $\bar{s}_{CH_4} = 44.49$ Btu/lbmol·°R. The O_2 and N_2 enters as a mixture at $P_{ref} = 1$atm. With Eq. 13.23 and $\bar{s}^\circ$ from Table A-23E

$$\bar{s}_{O_2} = \bar{s}^\circ_{O_2}(537) - \bar{R}\ln\frac{y_{O_2}P_{ref}}{P_{ref}} = 48.98 - 1.986\ln 0.21 = 52.079 \text{ Btu/lbmol·°R}$$

$$\bar{s}_{N_2} = \bar{s}^\circ_{N_2}(537) - \bar{R}\ln\frac{y_{N_2}P_{ref}}{P_{ref}} = 45.74 - 1.986\ln 0.79 = 46.208 \text{ Btu/lbmol·°R}$$

The products exit as an ideal gas mixture at 1atm, 4187°R with the following composition $y_{CO_2} = 1/10.52$, $y_{H_2O} = 2/10.52$, $y_{N_2} = 7.52/10.52$. Then, with $\bar{s}^\circ$ data from the ideal gas tables

$$\bar{s}_{CO_2} = \bar{s}^\circ_{CO_2}(4187) - \bar{R}\ln\frac{y_{CO_2}P_{ref}}{P_{ref}} = 76.673 - 1.986\ln\frac{1}{10.52} = 80.747 \text{ Btu/lbmol·°R}$$

$$\bar{s}_{H_2O} = \bar{s}^\circ_{H_2O}(4187) - \bar{R}\ln y_{H_2O} = 65.105 - 1.986\ln\frac{2}{10.52} = 68.402 \text{ Btu/lbmol·°R}$$

$$\bar{s}_{N_2} = \bar{s}^\circ_{N_2}(4187) - \bar{R}\ln y_{N_2} = 61.494 - 1.986\ln\frac{7.52}{10.52} = 62.161 \text{ Btu/lbmol·°R}$$

PROBLEM 13.82 (Contd.) - Page 2

Inserting values into Eq. (1)

$$\dot{\sigma}_{cv}/\dot{n}_{CH_4} = [80.747 + 2(63.402) + 7.52(62.101)] - 44.44 - (2(52.079) + 7.52(46.208))$$
$$= 188.87 \text{ Btu/lbmol}(CH_4)\cdot{}^\circ R \longleftarrow \text{(a)}$$

(b) Complete combustion of CH_4 with 200 percent of the theoretical amount of air is

$$CH_4 + (4O_2 + 15.04 N_2) \rightarrow CO_2 + 2H_2O + 2O_2 + 15.04 N_2$$

To determine T_3, write an energy rate balance at steady state:

$$0 = \cancel{\frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}}}^{0} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{CH_4}}}^{0} + (\bar{h}_{CH_4})_1 + (4\bar{h}_{O_2} + 15.04\bar{h}_{N_2})_2 - (\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 2\bar{h}_{O_2} + 15.04\bar{h}_{N_2})_3$$

with $\bar{h} = \bar{h}^\circ_f + \Delta\bar{h}$

$$0 = (\bar{h}^\circ_f)_{CH_4} + (0) - \{[\bar{h}^\circ_f + \bar{h}(T_3) - \bar{h}(537)]_{CO_2} + 2[\bar{h}^\circ_f + \bar{h}(T_3) - \bar{h}(537)]_{H_2O} + 2[\bar{h}(T_3) - \bar{h}(537)]_{O_2} + 15.04[\bar{h}(T_3) - \bar{h}(537)]_{N_2}$$

With data from the ideal gas tables and Table A-25E

$$\bar{h}_{CO_2}(T_3) + 2\bar{h}_{H_2O}(T_3) + 2\bar{h}_{O_2}(T_3) + 15.04\bar{h}_{N_2}(T_3) = (\bar{h}^\circ_f)_{CH_4} - [\bar{h}^\circ_f - \bar{h}(537)]_{CO_2} - 2[\bar{h}^\circ_f - \bar{h}(537)]_{H_2O} + 2\bar{h}_{O_2}(537) + 15.04\,\bar{h}_{N_2}(537)$$

$$= -32{,}210 - [-169{,}300 - 4028] - 2[-104{,}040 - 4258] + 2(3725) + 15.04(3730)$$

$$= 421{,}264$$

① Solving for T_3, $T_3 \approx 2666^\circ R$.

At steady state an entropy rate balance reduces to give

$$\dot{\sigma}_{cv}/\dot{n}_{CH_4} = (\bar{s}_{CO_2} + 2\bar{s}_{H_2O} + 2\bar{s}_{O_2} + 15.04\bar{s}_{N_2}) - \bar{s}_{CH_4} - (4\bar{s}_{O_2} + 15.04\bar{s}_{N_2}) \qquad (2)$$

The values of $\bar{s}_{CH_4}$, $\bar{s}_{O_2}$ and $\bar{s}_{N_2}$ of the reactants are the same as in part (a). The products exit as an ideal gas mixture at 1 atm, 2666°R with the following composition $y_{CO_2} = 1/20.04$, $y_{H_2O} = y_{O_2} = 2/20.04$, $y_{N_2} = 15.04/20.04$. Then with $\bar{s}^\circ$ from the ideal gas tables

$$\bar{s}_{CO_2} = \bar{s}^\circ_{CO_2}(2666) - \bar{R}\ln y_{CO_2} = 69.594 - 1.986\ln\frac{1}{20.04} = 75.547 \text{ Btu/lbmol}\cdot{}^\circ R$$
$$\bar{s}_{H_2O} = \bar{s}^\circ_{H_2O}(2666) - \bar{R}\ln y_{H_2O} = 59.694 - 1.986\ln\frac{2}{20.04} = 64.271 \text{ Btu/lbmol}\cdot{}^\circ R$$
$$\bar{s}_{O_2} = \bar{s}^\circ_{O_2}(2666) - \bar{R}\ln y_{O_2} = 61.506 - 1.986\ln\frac{2}{20.04} = 66.083 \text{ Btu/lbmol}\cdot{}^\circ R$$
$$\bar{s}_{N_2} = \bar{s}^\circ_{N_2}(2666) - \bar{R}\ln y_{N_2} = 57.644 - 1.986\ln\frac{15.04}{20.04} = 58.214 \text{ Btu/lbmol}\cdot{}^\circ R$$

Inserting values into Eq. (2)

$$\dot{\sigma}_{cv}/\dot{n}_{CH_4} = 75.547 + 2(64.271) + 2(66.083) + 15.04(58.214) - 44.49 - 4(52.079) - 15.04(46.208)$$
$$= 263.97 \text{ Btu/lbmol}\cdot{}^\circ R \longleftarrow \text{(b)}$$

---

1. Iteration with table data can be avoided by using IT.
When using table data, a first trial value can be obtained by assuming that the products consist only of N_2. Then the energy balance for part (a) reads

$$10.52\,\bar{h}_{N_2}(T_3) = 385{,}764 \Rightarrow \bar{h}_{N_2}(T_3) = 36670 \text{ kJ/kmol}$$

Inspection of Table A-23E then gives $T_3 \approx 4580^\circ R$. A like approach can be applied to part (b).

PROBLEM 13.83

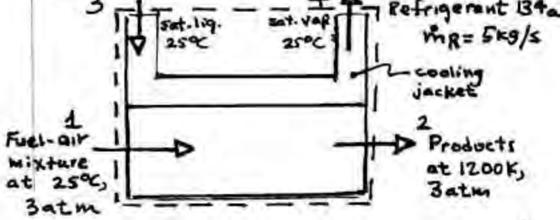
A gaseous mixture of butane (C_4H_{10}) and 80% excess air at 25°C, 3 atm enters a reactor operating at steady state. Complete combustion occurs and the products exit as a mixture at 1200 K, 3 atm. Refrigerant 134a with a mass flow rate of 5 kg/s enters an outer cooling jacket as saturated liquid and exits the jacket as saturated vapor, each at 25°C. No stray heat transfer occurs from the outside of the jacket, and kinetic and potential energy effects are negligible. Determine for the jacketed reactor

(a) the molar flow rate of the fuel, in kmol/s.
(b) the rate of entropy production, in kW/K.
(c) the rate of exergy destruction, in kW, for $T_0 = 25°C$.

KNOWN: Steady-state data are provided for a reactor equipped with an outer cooling jacket.

FIND: Determine the molar flow rate of the fuel entering the reactor, the rate of entropy production within the reactor, and the rate of exergy destruction within the reactor.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The control volume in the accompanying figure is at steady state.
2. For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and the effects of kinetic and potential energy are negligible.
3. Combustion is complete and 3.76 kmol of N_2 accompany each kmol of O_2 in the air.
4. The incoming fuel-air mixture and exiting combustion products are each modeled as ideal gases.
5. $T_0 = 25°C$.

ANALYSIS:

The complete combustion of C_4H_{10} with the theoretical amount of air is

$$C_4H_{10} + 6.5(O_2 + 3.76N_2) \rightarrow 4CO_2 + 5H_2O + 24.44\ N_2$$

Complete combustion with 80% excess air is

$$C_4H_{10} + 11.7(O_2 + 3.76N_2) \rightarrow 4CO_2 + 5H_2O + 5.2O_2 + 43.992\ N_2$$

(a) An energy rate balance at steady state reduces to

$$0 = \cancel{\dot{Q}_{cv}}^{\,0} - \cancel{\dot{W}_{cv}}^{\,0} + \dot{m}_R(h_3 - h_4) + \dot{n}_{fuel}\Big[\big[\bar{h}_{fuel} + 11.7\bar{h}_{O_2} + 43.992\bar{h}_{N_2}\big]_1 - \big(4\bar{h}_{CO_2} + 5\bar{h}_{H_2O} + 5.2\bar{h}_{O_2} + 43.992\bar{h}_{N_2}\big)\Big]_2$$

where $\dot{m}_R$ is the mass flow rate of the refrigerant and $\dot{n}_{fuel}$ is the molar flow rate of the butane. Solving

$$\frac{\dot{m}_R}{\dot{n}_{fuel}} = \frac{[\bar{h}_{fuel} + 11.7\bar{h}_{O_2} + 43.992\bar{h}_{N_2}]_1 - [4\bar{h}_{CO_2} + 5\bar{h}_{H_2O} + 5.2\bar{h}_{O_2} + 43.992\bar{h}_{N_2}]_2}{(h_4 - h_3)}$$

With $\bar{h} = \bar{h}_f^o + \Delta\bar{h}$, noting that $\bar{h}_f^o = 0$ for O_2 and N_2, and with $h_4 - h_3 = h_{fg}(25°C)$ for Refrigerant 134a from Table A-10,

$$\frac{\dot{m}_R}{\dot{n}_{fuel}} = \frac{(\bar{h}_f^o)_{fuel} - \{4[\bar{h}_f^o + \overbrace{\Delta\bar{h}}^{[\bar{h}(1200K) - \bar{h}(298)]}]_{CO_2} + 5[\bar{h}_f^o + \Delta\bar{h}]_{H_2O} + 5.2[\Delta\bar{h}]_{O_2} + 43.992[\Delta\bar{h}]_{N_2}\}}{h_{fg}}$$

$$= \frac{-126{,}150 - 4[-393{,}520 + 44484] + 5[-241{,}820 + 34476] - 5.2[29765] - 43.992[28108]}{176.6}$$

$$= \frac{915{,}409\ \text{KJ/kmol}}{176.6\ \text{kJ/kg}} = 5183.5\ \frac{\text{kg}}{\text{kmol}}$$

PROBLEM 13.83(Continued)

Then, with the known mass flow rate for the refrigerant,

$$\dot{n}_{fuel} = \frac{5\,kg/s}{5183.5\,kg/Kmol} = 0.001\ \frac{Kmol}{s} \quad \longleftarrow \text{(a)}$$

(b) An entropy rate balance at steady state reduces to

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{fuel}} = (4\bar{s}_{CO_2} + 5\bar{s}_{H_2O} + 5.2\bar{s}_{O_2} + 43.992\bar{s}_{N_2}) - (\bar{s}_{C_4H_{10}} + 11.7\bar{s}_{O_2} + 43.992\bar{s}_{N_2}) + \frac{\dot{m}_R}{\dot{n}_{fuel}} s_{fg} \qquad (1)$$

The fuel and air enter as a mixture at 25°C, 3atm with the composition, $y_{C_4H_{10}} = 1/56.692$, $y_{O_2} = 11.7/56.692$, $y_{N_2} = 43.992/56.692$. Accordingly

$$\bar{s}_{C_4H_{10}} = \bar{s}^\circ_{C_4H_{10}}(298) - \bar{R}\ln\frac{y_{C_4H_{10}}P}{P_{ref}} = 310.03 - 8.314\ln\frac{3}{56.692} = 334.46\ \ KJ/Kmol\cdot K$$

$$\bar{s}_{O_2} = \bar{s}^\circ_{O_2}(298) - \bar{R}\ln\frac{y_{O_2}P}{P_{ref}} = 205.03 - 8.314\ln\frac{(11.7)(3)}{56.692} = 209.02\ \ KJ/Kmol\cdot K$$

$$\bar{s}_{N_2} = \bar{s}^\circ_{N_2}(298) - \bar{R}\ln\frac{y_{N_2}P}{P_{ref}} = 191.5 - 8.314\ln\frac{(43.992)(3)}{56.692} = 184.47\ \ KJ/Kmol\cdot K$$

The products exit as a mixture at 1200K, 3 atm with the composition, $y_{CO_2} = 4/58.192$, $y_{H_2O} = 5/58.192$, $y_{O_2} = 5.2/58.192$, $y_{N_2} = 43.992/58.192$. Accordingly

$$\bar{s}_{CO_2} = \bar{s}^\circ_{CO_2}(1200) - \bar{R}\ln\frac{y_{CO_2}P}{P_{ref}} = 279.307 - 8.314\ln\frac{(4)(3)}{58.192} = 292.43\ KJ/Kmol\cdot K$$

$$\bar{s}_{H_2O} = \bar{s}^\circ_{H_2O}(1200) - \bar{R}\ln\frac{y_{H_2O}P}{P_{ref}} = 240.333 - 8.314\ln\frac{(5)(3)}{58.192} = 251.60\ KJ/Kmol\cdot K$$

$$\bar{s}_{O_2} = \bar{s}^\circ_{O_2}(1200) - \bar{R}\ln\frac{y_{O_2}P}{P_{ref}} = 249.906 - 8.314\ln\frac{(5.2)(3)}{58.192} = 260.85\ KJ/Kmol\cdot K$$

$$\bar{s}_{N_2} = \bar{s}^\circ_{N_2}(1200) - \bar{R}\ln\frac{y_{N_2}P}{P_{ref}} = 234.115 - 8.314\ln\frac{(43.992)(3)}{58.192} = 227.31\ KJ/Kmol\cdot K$$

From Table A-10; $s_{fg} = 0.5925\ \frac{KJ/K}{Kmol}$. Substituting results in Eq.(1)

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{fuel}} = 4(292.43) + 5(251.60) + 5.2(260.85) + 43.992(227.31) - 334.46 - 11.7(209.02) - 43.992(184.47) + 5183.5(0.5925)$$

$$= 5960\ \frac{KJ/K}{Kmol}$$

Finally

$$\dot{\sigma}_{cv} = \left(0.001\ \frac{Kmol}{s}\right)\left(5960\ \frac{KJ/K}{Kmol}\right) = 5.96\ \frac{KJ}{s}\left|\frac{1\,kW}{1\,KJ/s}\right| = 5.96\ \frac{kW}{K} \quad \longleftarrow \text{(b)}$$

(c) With $\dot{E}_d = T_0\,\dot{\sigma}_{cv}$, we get

$$\dot{E}_d = (298.15K)\left(5.96\ \frac{kW}{K}\right) = 1777\ kW \quad \longleftarrow \text{(c)}$$

PROBLEM 13.84

Liquid ethanol (C_2H_5OH) at 25°C, 1 atm enters a reactor operating at steady state and burns completely with 130% of theoretical air entering in a separate stream at 25°C, 1 atm. Combustion products exit at 227°C, 1 atm. Heat transfer from the reactor takes place at an average surface temperature of 127°C. Determine

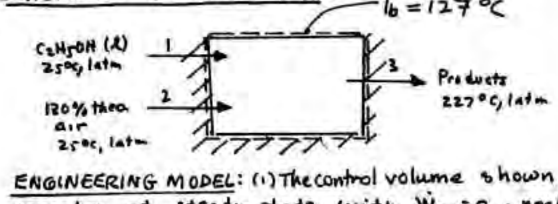
(a) the rate of entropy production within the reactor, in kJ/K per kmol of fuel,

(b) the rate of exergy destruction within the reactor, in kJ per kmol of fuel. Kinetic and potential energy effects are negligible. Let T_0 = 25°C.

KNOWN: Data are provided for a liquid ethanol-fueled reactor.

FIND: Determine the rate of entropy production and the rate of exergy destruction, each within the reactor.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$, negligible kinetic and potential energy effects, and heat transfer occuring at temperature T_b (2) Combustion is complete. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The combustion air and products of combustion can be modeled as ideal gases. (5) For the exergy reference environment, $T_0 = 25°C$ (298K)

ANALYSIS: Complete combustion of C_2H_5OH with theoretical amount of air is described by

$$C_2H_5OH + 3(O_2 + 3.76 N_2) \rightarrow 2CO_2 + 3H_2O + 11.28 N_2$$

Complete combustion with 180% of the theoretical amount of air is thus

$$C_2H_5OH + 3.9(O_2 + 3.76 N_2) \rightarrow 2CO_2 + 3H_2O + 0.9 O_2 + 14.664 N_2$$

(a) With assumption 1, an entropy rate balance at steady state reduces to

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_{fuel}}{T_b} + (\bar{s}_{fuel})_1 + (3.9\bar{s}_{O_2} + 14.664\bar{s}_{N_2})_2 - (2\bar{s}_{CO_2} + 3H_2O + 0.9\bar{s}_{O_2} + 14.664\bar{s}_{N_2})_3 + \frac{\dot{\sigma}_{cv}}{\dot{n}_{fuel}}$$

Accordingly

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{fuel}} = -\frac{\dot{Q}_{cv}/\dot{n}_{fuel}}{T_b} + (2\bar{s}_{CO_2} + 3\bar{s}_{H_2O} + 0.9\bar{s}_{O_2} + 14.664\bar{s}_{N_2})_3 - (\bar{s}_{fuel})_1 - (3.9\bar{s}_{O_2} + 14.664\bar{s}_{N_2})_2 \quad (1)$$

The term $\dot{Q}_{cv}/\dot{n}_{fuel}$ can be obtained from an energy rate balance which reduces with assumption 1 to give

$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = (2\bar{h}_{CO_2} + 3\bar{h}_{H_2O} + 0.9\bar{h}_{O_2} + 14.664\bar{h}_{N_2})_3 - (\bar{h}_{fuel})_1 - (3.9\bar{h}_{O_2} + 14.664\bar{h}_{N_2})_2$$

$$= 2[\bar{h}_f^\circ + \bar{h}(500) - \bar{h}(298)]_{CO_2} + 3[\bar{h}_f^\circ + \bar{h}(500) - \bar{h}(298)]_{H_2O} + 0.9[\bar{h}(500) - \bar{h}(298)]_{O_2}$$
$$+ 14.664[\bar{h}(500) - \bar{h}(298)]_{N_2} - (\bar{h}_f^\circ)_{fuel} - (0)$$

With data from the ideal gas tables and Table A-25

$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = 2[-393{,}520 + 17{,}678 - 9364] + 3[-241{,}820 + 16{,}828 - 9904] + 0.9[14770 - 8682]$$
$$+ 14.664[14{,}581 - 8669] - (-277{,}690)$$

$$= -1{,}105{,}237 \text{ kJ/kmol (fuel)}$$

PROBLEM 13.84 (Continued)

Next, the specific entropies appearing in Eq. (1) are evaluated. Since the fuel enters at 25°C, 1 atm, $\bar{s}_{fuel}$ is obtained from Table A-25: $\bar{s}_{fuel} = 160.7$ kJ/kmol·K. The combustion air enters as a mixture with $y_{O_2} = 0.21$, $y_{N_2} = 0.79$. Thus, with $p = p_{ref}$ and $T = 298$K

$$\bar{s}_{O_2} = \bar{s}^\circ_{O_2}(298) - \bar{R} \ln \frac{y_{O_2} p_{ref}}{p_{ref}} = 205.03 - 8.314 \ln 0.21 = 218.01 \text{ kJ/kmol·K}$$

$$\bar{s}_{N_2} = \bar{s}^\circ_{N_2}(298) - \bar{R} \ln \frac{y_{N_2} p_{ref}}{p_{ref}} = 191.502 - 8.314 \ln 0.79 = 193.462 \text{ kJ/kmol·K}$$

The combustion products exit as a mixture at 500K, 1atm with the composition $y_{CO_2} = 2/20.564$, $y_{H_2O} = 3/20.564$, $y_{O_2} = .9/20.564$, $y_{N_2} = 14.664/20.564$. Thus, with $\bar{s}^\circ$ data from the ideal gas tables

$$\bar{s}_{CO_2} = \bar{s}^\circ_{CO_2}(500) - \bar{R} \ln \frac{y_{CO_2} p_{ref}}{p_{ref}} = 234.81 - 8.314 \ln \frac{2}{20.564} = 254.185 \text{ kJ/kmol·K}$$

$$\bar{s}_{H_2O} = \bar{s}^\circ_{H_2O}(500) - \bar{R} \ln \frac{y_{H_2O} p_{ref}}{p_{ref}} = 206.413 - 8.314 \ln \frac{3}{20.564} = 222.417 \text{ kJ/kmol·K}$$

$$\bar{s}_{O_2} = \bar{s}^\circ_{O_2}(500) - \bar{R} \ln \frac{y_{O_2} p_{ref}}{p_{ref}} = 220.589 - 8.314 \ln \frac{0.9}{20.564} = 246.603 \text{ kJ/kmol·K}$$

$$\bar{s}_{N_2} = \bar{s}^\circ_{N_2}(500) - \bar{R} \ln \frac{y_{N_2} p_{ref}}{p_{ref}} = 206.680 - 8.314 \ln \frac{14.664}{20.564} = 209.441 \text{ kJ/kmol·K}$$

Collecting results

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{fuel}} = \frac{-(-1{,}105{,}237)}{T_b} + 2(254.185) + 3(222.417) + 0.9(246.603) + 14.664(209.441) - 160.7 - 3.9(218.01) - 14.664(193.402)$$

$$= \frac{1{,}105{,}237}{T_b} + 620.9 \frac{\text{kJ}}{\text{kmol(fuel)}\cdot\text{K}}$$

With $T_b = 400$K, we get

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{fuel}} = (2763.1 + 620.9) \frac{\text{kJ}}{\text{kmol(fuel)}\cdot\text{K}} = 3384 \frac{\text{kJ}}{\text{kmol(fuel)}\cdot\text{K}} \longleftarrow \text{(a)}$$

① (b) $$\frac{\dot{E}_d}{\dot{n}_{fuel}} = T_0 \frac{\dot{\sigma}_{cv}}{\dot{n}_{fuel}}$$

$$= (298\text{K})\left(3384 \frac{\text{kJ}}{\text{kmol(fuel)}\cdot\text{K}}\right) = 1.01 \times 10^6 \frac{\text{kJ}}{\text{kmol(fuel)}} \longleftarrow \text{(b)}$$

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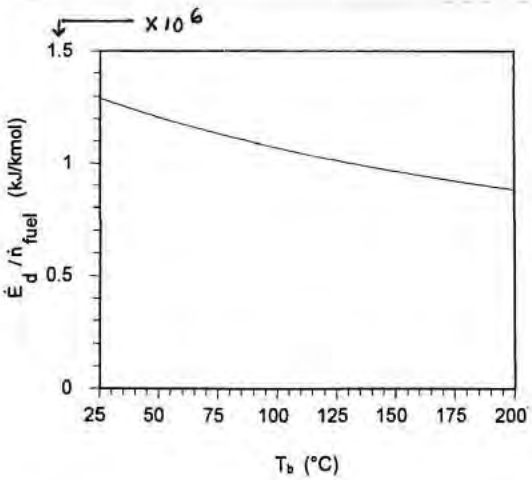
1. The following plot gives the exergy destruction versus temperature T_b, in °C.



PROBLEM 13.85

A gaseous mixture of ethane (C_2H_6) and the theoretical amount of air at 25°C, 1 atm enters a reactor operating at steady state and burns completely. Combustion products exit at 627°C, 1 atm. Heat transfer from the reactor takes place at an average surface temperature of 327°C. Determine

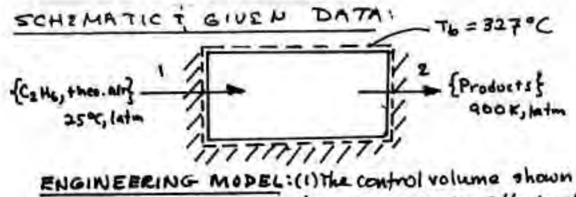
(a) the rate of entropy production within the reactor, in kJ/K per kmol of fuel,

(b) the rate of exergy destruction within the reactor, in kJ per kmol of fuel. Kinetic and potential energy effects are negligible. Let T_0 = 25°C.

KNOWN: Data are provided for a reactor fueled by a gas mixture.

FIND: Determine the rate of entropy production and the rate of exergy destruction, each within the reactor.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$, negligible effects of kinetic and potential energy, and heat transfer occuring at temperature T_b. (2) Combustion is complete. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The incoming and exiting mixtures can be modeled as ideal gases. (5) For the exergy reference environment, T_0 = 298K

ANALYSIS: Complete combustion of the gaseous mixture is described by

$$C_2H_6 + 3.5O_2 + 13.16N_2 \rightarrow 2CO_2 + 3H_2O + 13.16N_2$$

(a) With assumption 1, an entropy rate balance at steady state reduces to

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_{fuel}}{T_b} + (\bar{s}_{fuel} + 3.5\bar{s}_{O_2} + 13.16\bar{s}_{N_2})_1 - (2\bar{s}_{CO_2} + 3\bar{s}_{H_2O} + 13.16\bar{s}_{N_2})_2 + \frac{\dot{\sigma}_{cv}}{\dot{n}_{fuel}}$$

Accordingly

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{fuel}} = -\frac{\dot{Q}_{cv}/\dot{n}_{fuel}}{T_b} + (2\bar{s}_{CO_2} + 3\bar{s}_{H_2O} + 13.16\bar{s}_{N_2})_2 - (\bar{s}_{fuel} + 3.5\bar{s}_{O_2} + 13.16\bar{s}_{N_2})_1 \qquad (1)$$

The term $\dot{Q}_{cv}/\dot{n}_{fuel}$ can be obtained from an energy rate balance which reduces with assumption 1 to give

$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = (2\bar{h}_{CO_2} + 3\bar{h}_{H_2O} + 13.16\bar{h}_{N_2})_2 - (\bar{h}_{fuel} + 3.5\bar{h}_{O_2} + 13.16\bar{h}_{N_2})_1$$

$$= 2[\bar{h}_f^\circ + \bar{h}(900) - \bar{h}(298)]_{CO_2} + 3[\bar{h}_f^\circ + \bar{h}(900) - \bar{h}(298)]_{H_2O} + 13.16[\bar{h}(900) - \bar{h}(298)]_{N_2} - (\bar{h}_f^\circ)_{fuel}$$

With data from the ideal gas tables and Table A-25

$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = 2[-393,520 + 37,405 - 9364] + 3[-241,820 + 31,828 - 9904] + 13.16[26,890 - 8669] - (-84,680)$$

$$= -1,066,178 \text{ kJ/kmol } (C_2H_6)$$

Next, the specific entropies appearing in Eq. (1) are evaluated. The fuel and air enter as a mixture at $p = p_{ref}$ and $T = 298$ K with the composition $y_{fuel} = 1/17.66$, $y_{O_2} = 3.5/17.66$, $y_{N_2} = 13.16/17.66$. Thus

$$\bar{s}_{fuel} = \bar{s}^\circ_{fuel}(298) - \bar{R}\ln\frac{y_{fuel}\,p_{ref}}{p_{ref}} = 229.49 - 8.314\ln\frac{1}{17.66} = 253.36 \text{ kJ/kmol·K}$$

$$\bar{s}_{O_2} = \bar{s}^\circ_{O_2}(298) - \bar{R}\ln y_{O_2} = 205.03 - 8.314\ln\frac{3.5}{17.66} = 218.49 \text{ kJ/kmol·K}$$

$$\bar{s}_{N_2} = \bar{s}^\circ_{N_2}(298) - \bar{R}\ln y_{N_2} = 191.5 - 8.314\ln\frac{13.16}{17.66} = 193.95 \text{ kJ/kmol·K}$$

PROBLEM 13.85 (Continued)

The combustion products exit as a mixture at 900 K, 1 atm with the composition $y_{CO_2} = 2/18.16$, $y_{H_2O} = 3/18.16$, $y_{N_2} = 13.16/18.16$. Thus

$$\bar{s}_{CO_2} = \bar{s}^\circ_{CO_2}(900) - \bar{R} \ln \frac{y_{CO_2} p_{ref}}{p_{ref}} = 263.559 - 8.314 \ln \frac{2}{18.16} = 281.90 \text{ kJ/kmol·K}$$

$$\bar{s}_{H_2O} = \bar{s}^\circ_{H_2O}(900) - \bar{R} \ln y_{H_2O} = 228.321 - 8.314 \ln \frac{3}{18.16} = 243.29 \text{ kJ/kmol·K}$$

$$\bar{s}_{N_2} = \bar{s}^\circ_{N_2}(900) - \bar{R} \ln y_{N_2} = 224.647 - 8.314 \ln \frac{13.16}{18.16} = 227.32 \text{ kJ/kmol·K}$$

Collecting results

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{fuel}} = -\frac{(-1,066,178)}{T_b} + 2(281.90) + 3(243.29) + 13.16(227.32) - 257.36 - 3.5(218.49) - 13.16(193.95)$$

$$= \left(\frac{1,066,178}{T_b} + 714.7\right) \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$ ← (C_2H_6)

With $T_b = 600$ K

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{fuel}} = (1777 + 714.7) = 2491.7 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$ ← (C_2H_6) ← (a)

(b) $$\frac{\dot{E}_d}{\dot{n}_{fuel}} = T_0 \frac{\dot{\sigma}_{cv}}{\dot{n}_{fuel}}$$

1 $$= (298 \text{ K})(2491.7) \frac{\text{kJ}}{\text{kmol} \cdot \text{K}} = 0.74 \times 10^6 \frac{\text{kJ}}{\text{kmol}\,(C_2H_6)}$$ ← (b)

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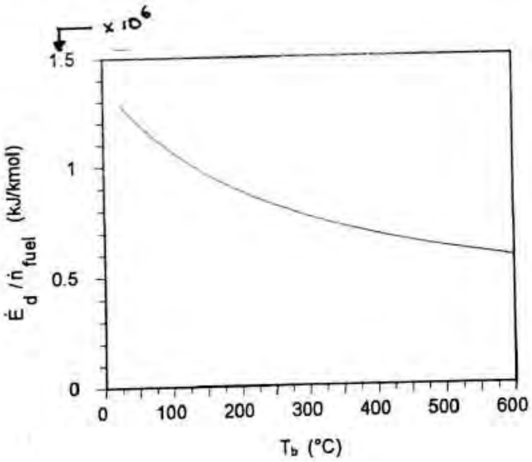
1. The following plot gives the exergy destruction versus temperature T_b, in °C.



PROBLEM 13.86

Determine the change in the Gibbs function, in kJ per kmol of methane, at 25°C, 1 atm for $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$, using

(a) Gibbs function of formation data.
(b) enthalpy of formation data, together with absolute entropy data.

KNOWN: The reaction is $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$. Reactants and products are at 25°C, 1 atm.

FIND: Determine ΔG using (a) Gibbs function of formation data, (b) enthalpy of formation and absolute entropy data.

ANALYSIS: For $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$, ΔG is

$$\Delta G = \bar{g}_{CO_2} + 2\bar{g}_{H_2O} - \bar{g}_{CH_4} - 2\bar{g}_{O_2} \qquad (1)$$

where each $\bar{g}$ is at 25°C, 1 atm

(a) With $\bar{g}_f^\circ$ data from Table A-25, for H_2O as a gas, Eq. (1) gives

$$\Delta G = -394{,}380 + (2)(-228{,}590) - (-50{,}790) - (2)(0) = -800{,}770 \text{ kJ/kmol } CH_4 \qquad \text{(a)}$$

(b) With $\bar{g} = \bar{h} - T\bar{s}$, Eq. (1) becomes

$$\Delta G = (\bar{h} - T\bar{s})_{CO_2} + 2(\bar{h} - T\bar{s})_{H_2O} - (\bar{h} - T\bar{s})_{CH_4} - 2(\bar{h} - T\bar{s})_{O_2}$$

$$= (\bar{h}_{CO_2} + 2\bar{h}_{H_2O} - \bar{h}_{CH_4} - 2\bar{h}_{O_2}) - T(\bar{s}_{CO_2} + 2\bar{s}_{H_2O} - \bar{s}_{CH_4} - 2\bar{s}_{O_2})$$

Inserting $\bar{h}_f^\circ$ and $\bar{s}^\circ$ data from Table A-25

$$\Delta G = [-393{,}520 + (2)(-241{,}820) - (-74{,}850) - (2)(0)]$$
$$- 298.15[213.69 + (2)(188.72) - 186.16 - (2)(205.03)]$$
$$= -802{,}310 - 298.15(-5.09)$$

① $$= \underline{-800{,}792 \text{ kJ/kmol } CH_4} \qquad \text{(b)}$$

1. This is in agreement with the answer of part (a). The difference is due to round off.

PROBLEM 13.87

Determine the change in the Gibbs function, in Btu per lbmol of hydrogen, at 77°F, 1 atm for $H_2 + \frac{1}{2}O_2 \rightarrow H_2O(g)$, using

(a) Gibbs function of formation data.

(b) enthalpy of formation data, together with absolute entropy data.

KNOWN: The reaction is $H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(g)$. Reactants and products are at 77°F, 1 atm.

FIND: Determine ΔG using (a) Gibbs function of formation data, (b) enthapy of formation and absolute entropy data.

ANALYSIS: For $H_2 + \frac{1}{2}O_2 \rightarrow H_2O$, ΔG is

$$\Delta G = \bar{g}_{H_2O(g)} - \bar{g}_{H_2} - \frac{1}{2}\bar{g}_{O_2} \qquad (1)$$

where each of the $\bar{g}$'s is eva uated at 77°F, 1 atm.

(a) With $\bar{g}_f^\circ$ data from Table A-25E, Eq.(1) becomes

$$\Delta G = (\bar{g}_f^\circ)_{H_2O(g)} - (\bar{g}_f^\circ)_{H_2} - \frac{1}{2}(\bar{g}_f^\circ)_{O_2}$$

$$= -98{,}350 - (0) - \tfrac{1}{2}(0) = -98{,}350 \text{ Btu/lbmol} \qquad (a)$$

(b) With $\bar{g} = \bar{h} - T\bar{s}$, Eq(1) becomes

$$\Delta G = (\bar{h} - T\bar{s})_{H_2O(g)} - (\bar{h} - T\bar{s})_{H_2} - \frac{1}{2}(\bar{h} - T\bar{s})_{O_2}$$

$$= (\bar{h}_{H_2O(g)} - \bar{h}_{H_2} - \tfrac{1}{2}\bar{h}_{O_2}) - T(\bar{s}_{H_2O(g)} - \bar{s}_{H_2} - \tfrac{1}{2}\bar{s}_{O_2})$$

Using $\bar{h}_f^\circ$ and $\bar{s}^\circ$ data from Table A-25E

$$\Delta G = [-104{,}040 - 0 - \tfrac{1}{2}(0)] - 537[45.08 - 31.19 - \tfrac{1}{2}(49.981)$$

① $$= -98{,}348 \text{ Btu/lbmol} \qquad (b)$$

1. This agrees very closely with the result of part(a). The slight difference is due to round off.

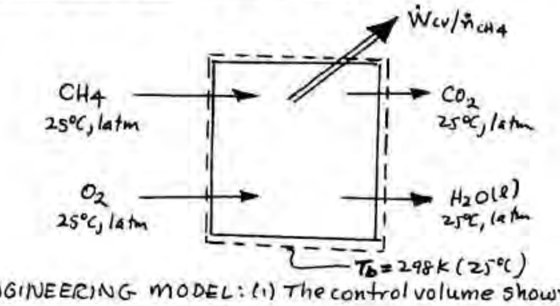
PROBLEM 13.88

Streams of methane (CH_4) and oxygen (O_2), each at 25°C, 1 atm, enter a fuel cell operating at steady state. Streams of carbon dioxide and water exit separately at 25°C, 1 atm. If the fuel cell operates isothermally at 25°C, 1 atm, determine the maximum theoretical work that it can develop, in kJ per kmol of methane. Ignore kinetic and potential energy effects.

KNOWN: Operating data are provided for a CH_4/O_2 fuel cell at steady state.

FIND: Determine the maximum theoretical work that can be developed.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown is at steady state. (2) Kinetic and potential energy effects are negligible. (3) Heat transfer takes place at $T_b = 298K\ (25°C)$. (4) At the condition specified, H_2O exits as a liquid.

ANALYSIS: The cell reaction is

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

Energy and entropy balances at steady state read, respectively.

(energy) $$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{CH4}} - \frac{\dot{W}_{cv}}{\dot{n}_{CH4}} + \left(\bar{h}_{CH4} + 2\bar{h}_{O_2} - \bar{h}_{CO_2} - 2\bar{h}_{H_2O}\right)(298K, 1atm)$$

(entropy) $$0 = \frac{\dot{Q}_{cv}/\dot{n}_{CH4}}{T_b} + \left(\bar{s}_{CH4} + 2\bar{s}_{O_2} - \bar{s}_{CO_2} - 2\bar{s}_{H_2O}\right)(298K, 1atm) + \dot{\sigma}_{cv}/\dot{n}_{CH4}$$

Eliminating the heat transfer term between these gives

$$\frac{\dot{W}_{cv}}{\dot{n}_{CH4}} = \left(\bar{h}_{CH4} + 2\bar{h}_{O_2} - \bar{h}_{CO_2} - 2\bar{h}_{H_2O}\right)(298K, 1atm) - T_b\left[\bar{s}_{CH4} + 2\bar{s}_{O_2} - \bar{s}_{CO_2} - 2\bar{s}_{H_2O}\right](298K, 1atm) - T_b\,\dot{\sigma}_{cv}/\dot{n}_{CH4}$$

In terms of the Gibbs function,

$$\frac{\dot{W}_{cv}}{\dot{n}_{CH4}} = \left[\bar{g}_{CH4} + 2\bar{g}_{O_2} - \bar{g}_{CO_2} - 2\bar{g}_{H_2O(l)}\right](298, 1atm) - T_b\,\dot{\sigma}_{cv}/\dot{n}_{CH4}$$

Accordingly, with data from Table A-25, and noting that the maximum value corresponds to the case of no entropy production,

$$\left(\dot{W}_{cv}/\dot{n}_{CH4}\right)_{MAX} = -50{,}790 + 2(0) - (-394{,}380) - 2(-237{,}180)$$

$$= 817{,}950\ \frac{kJ}{kmol(CH_4)}$$

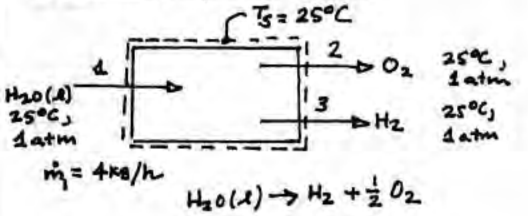
PROBLEM 13.89

An inventor has developed a device that at steady state takes in liquid water at 25°C, 1 atm with a mass flow rate of 4 kg/h and produces separate streams of hydrogen (H_2) and oxygen (O_2), each at 25°C, 1 atm. When the device operates isothermally at 25°C, the inventor says it requires an electricity input of 237,180 kJ per kmol of hydrogen produced. Heat transfer with the surroundings occurs, but kinetic and potential energy effects can be ignored. Evaluate the inventor's claim.

KNOWN: Steady-state operating data are provided for a device, including a claim about the electricity input required.

FIND: Evaluate the claim.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL:

1. The control volume operates at steady state with negligible effects of kinetic and potential energy.
2. The device operates isothermally at 25°C.

ANALYSIS: In this case we apply entropy and energy rate balances, as follows: At steady-state an entropy rate balance reduces to read

$$0 = \frac{\dot Q_{cv}/\dot n_{H_2O}}{T_s} + [\bar s_{H_2O(l)}]_1 - [\bar s_{H_2}]_3 - \tfrac{1}{2}[\bar s_{O_2}]_2 + \dot\sigma_{cv}/\dot n_{H_2O} \qquad (1)$$

The heat transfer rate can be evaluated from an energy rate balance at steady state:

$$0 = \frac{\dot Q_{cv}}{\dot n_{H_2O}} - \frac{\dot W_{cv}}{\dot n_{H_2O}} + [\bar h_{H_2O(l)}]_1 - [\bar h_{H_2}]_3 - \tfrac{1}{2}[\bar h_{O_2}]_2 \qquad (2)$$

Combining Eqs. (1) and (2)

$$T_s\left(\frac{\dot\sigma_{cv}}{\dot n_{H_2O}}\right) = -\frac{\dot W_{cv}}{\dot n_{H_2O}} - \left\{[\bar h_{H_2}]_3 + \tfrac{1}{2}[\bar h_{O_2}]_2 - [\bar h_{H_2O(l)}]_1\right\} + T_s\left\{[\bar s_{H_2}]_3 + \tfrac{1}{2}[\bar s_{O_2}]_2 - [\bar s_{H_2O(l)}]_1\right\}$$

Each of the $\bar h$ and $\bar s$ terms in this expression is evaluated at 25°C, 1 atm. Then, since $T_s = 25°C$ and $\bar g = \bar h - T\bar s$, it can be rewritten in terms of Gibbs functions as

$$T_s\left(\frac{\dot\sigma_{cv}}{\dot n_{H_2O}}\right) = -\frac{\dot W_{cv}}{\dot n_{H_2O}} - \left[(\bar g_{H_2})_3 + \tfrac{1}{2}(\bar g_{O_2})_2 - (\bar g_{H_2O(l)})_1\right](25°C, 1\,atm) \qquad (3)$$

Inserting the claimed value for electricity input and Gibbs function values from Table A-25, we get

① $$T_s\left(\frac{\dot\sigma_{cv}}{\dot n_{H_2O}}\right) = \left\{-\left[\overset{\text{electricity input}}{-237{,}180}\right] - \left[0 + \tfrac{1}{2}(0) - (-237{,}180)\right]\right\}\frac{kJ}{kmol(H_2O)}$$

$$\Rightarrow \left(\frac{\dot\sigma_{cv}}{\dot n_{H_2O}}\right) = 0$$

$\Rightarrow$ The inventor's claim requires the device to be free of internal irreversibilities, and thus cannot be allowed.

---

1 From the given reaction equation, values here are, equivalently, on a per kmol of water or hydrogen basis.

PROBLEM 13.90

As shown in Fig. P13.90, coal with a mass analysis of 88% C, 6% H, 4% O, 1% N, 1% S enters a reactor where it burns with the theoretical amount of air to give a gas stream consisting of CO_2, H_2O, N_2, SO_2. After the gas stream provides heating in an industrial furnace, it is directed at 25°C, 1 atm to a cleanup device operating at steady state that removes the CO_2 and SO_2, each in a separate stream. The remainder is discharged to the atmosphere. Each of these three streams exits the device at 25°C, 1 atm, heat transfer to the surroundings occurs at 25°C, and the effects of kinetic and potential energy are negligible. Determine the minimum theoretical work input required by any such cleanup device, in kJ per kg of coal entering the reactor. Why is a work input required?

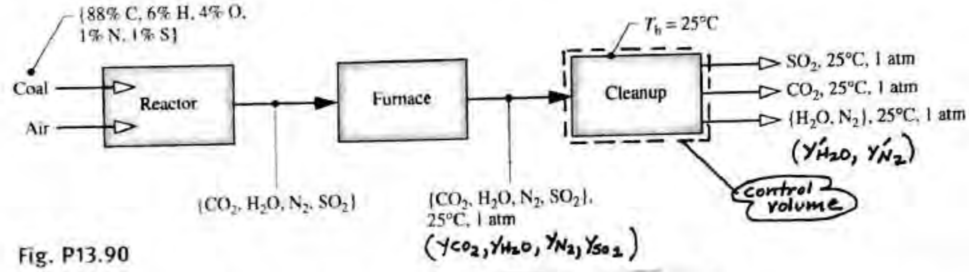


Fig. P13.90

KNOWN: Steady-state data are provided for a cleanup device.

FIND: Determine the minimum theoretical work input required by any such cleanup device.

Engineering Model:

1. The control volume shown in the schematic is at steady state.
2. The ideal gas model applies to each entering and exiting stream.
3. Each mole of air in the air is accompanied by 3.76 moles of N_2 (inert).
4. Kinetic and potential energy effects are negligible, and heat transfer occurs at 25°C.

Analysis: On the basis of 1 kg of coal feeding the reactor,

	m_i	M_i	n_i (kmol per kg coal)
C	0.88	12	0.0733
H	0.06	1	0.0600
O	0.04	16	0.0025
N	0.01	14	0.0007
S	0.01	32	0.0003

Reaction equation:

$$[0.0733\,C + 0.0600\,H + 0.0025\,O + 0.0007\,N + 0.0003\,S]$$
$$+ \alpha[O_2 + 3.76N_2] \rightarrow \beta CO_2 + \gamma H_2O + \omega SO_2 + \delta N_2$$

C: $0.0733 = \beta$, H: $0.0600 = 2\gamma$, $\gamma = 0.0300$. S: $0.0003 = \omega$

O: $0.0025 + 2\alpha = 2(0.0733) + (0.0300) + 2(0.0003) \Rightarrow \alpha = 0.08735$

N: $0.0007 + 3.76(2)(0.08735) = 2\delta$, $\delta = 0.32879$

Product analysis per kg of coal of the mixture entering the clean-up device:		y
CO_2	0.07330	0.1695
H_2O	0.03000	0.0694
N_2	0.32879	0.7604
SO_2	0.00030	0.0007
	0.43239	

Product analysis per kg of coal of the mixture exiting the clean-up device:		y'
H_2O	0.03000	0.08361
N_2	0.32879	0.91639
	0.35879	

An energy rate balance for the clean-up device reads, $T = 298.15\text{K}$:

$$0 = \frac{\dot{Q}_{cv}}{\dot{m}_{coal}} - \frac{\dot{W}_{cv}}{\dot{m}_{coal}} + \underbrace{\Big[\overset{\text{kmol per kg of coal}}{0.0733}\,\bar{h}_{CO_2}(T) + 0.03\,\bar{h}_{H_2O}(T) + \cdots \Big] - 0.07330\,\bar{h}_{CO_2}(T) - 0.00030\,\bar{h}_{SO_2}(T) - \big[0.03\,\bar{h}_{H_2O}(T) + 0.32879\,\bar{h}_{N_2}(T)\big]}_{\text{(These terms cancel)}} \qquad (a)$$

$$\Rightarrow \frac{\dot{Q}_{cv}}{\dot{m}_{coal}} = \frac{\dot{W}_{cv}}{\dot{m}_{coal}}$$

An entropy rate balance for the clean-up device reads, $T = 298.15\text{K}$, $p = 1\text{atm}$:

$$0 = \frac{\dot{Q}_{cv}/\dot{m}_{coal}}{T_b} + \big[0.0733\,\bar{s}_{CO_2}(T, y_{CO_2}p) + 0.03\,\bar{s}_{H_2O}(T, y_{H_2O}p) + 0.032879\,\bar{s}_{N_2}(T, y_{N_2}p) + 0.0003\,\bar{s}_{SO_2}(T, y_{SO_2}p)\big] - 0.0733\,\bar{s}_{CO_2}(T, p) - 0.0003\,\bar{s}_{SO_2}(T, p) - \big[0.03\,\bar{s}_{H_2O}(T, y'_{H_2O}p) + 0.32879\,\bar{s}_{N_2}(T, y'_{N_2}p)\big] + \dot{\sigma}_{cv}/\dot{m}_{coal}$$

Then, with Eq.(a) and Eqs. 6.20b and 13.23 (as needed),

$$\frac{\dot{W}_{cv}}{\dot{m}_{coal}} = T_b\left[0.0733\left[-\bar{R}\ln\frac{1}{y_{CO_2}}\right] + 0.0003\left[-\bar{R}\ln\frac{1}{y_{SO_2}}\right] + 0.03\left[-\bar{R}\ln\frac{y'_{H_2O}}{y_{H_2O}}\right] + 0.32879\left[-\bar{R}\ln\frac{y'_{N_2}}{y_{N_2}}\right]\right] - \frac{\dot{\sigma}_{cv}}{\dot{m}_{coal}}\Bigg]$$

The minimum theoretical value for the work corresponds to $\dot{\sigma}_{cv} = 0$.

$$\Rightarrow \left(\frac{\dot{W}_{cv}}{\dot{m}_{coal}}\right)_{min} = \left(-8.314\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(298.15\text{K})\left[0.0733\ln\frac{1}{0.1695} + 0.0003\ln\frac{1}{0.0007} + 0.03\ln\left(\frac{0.08361}{0.0694}\right) + 0.32879\ln\left(\frac{0.91639}{0.7604}\right)\right]$$

(1) $$= \underset{\text{input}}{-}\,494\ \text{kJ/kg(coal)}$$

1. A work input is required because the partial pressures associated with the substances entering the cleanup device are increased to their exiting pressure values — to 1atm for CO_2 and SO_2, for example.

PROBLEM 13.91

13.91 Applying Eq. 13.36 for (a) carbon, (b) hydrogen (H_2), (c) methane, (d) carbon monoxide, (e) nitrogen (N_2), (f) oxygen (O_2), and (g) carbon dioxide, determine the chemical exergy, in kJ/kg, relative to the following environment in which the gas phase obeys the ideal gas model:

Environment

$T_0 = 298.15$ K (25°C), $p_0 = 1$ atm

Gas Phase:	Component	y^e(%)
	N_2	75.67
	O_2	20.35
	$H_2O(g)$	3.12
	CO_2	0.03
	Other	0.83

(a) Carbon: $a=1, b=0, c=0$. Eq. 13.36 reads

$$\bar{e}_C^{CH} = \left[\bar{g}_C + \bar{g}_{O_2} - \bar{g}_{CO_2}\right] + \bar{R}T_0 \ln\left[\frac{y^e_{O_2}}{y^e_{CO_2}}\right]$$

With $\bar{g}_f^\circ$ data from Table A-25,

$$\bar{e}_C^{CH} = \left[0 + 0 - (-394,380 \text{ kJ/kmol})\right] + \left(8.314 \frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(298.15\text{K}) \ln\left[\frac{0.2035}{0.0003}\right]$$

$$= \left[394,380 + 16,161\right]\frac{\text{kJ}}{\text{kmol}} = 410,541 \frac{\text{kJ}}{\text{kmol}}$$

With $M = 12.01$ kg/kmol from Table A-1,

$$e_C^{CH} = \frac{410,541 \text{ kJ/kmol}}{12.01 \text{ kg/kmol}} = 34,183 \text{ kJ/kg} \longleftarrow C$$

(b) Hydrogen, H_2: $a=0, b=2, c=0$. Eq. 13.36 reads

$$\bar{e}_{H_2}^{CH} = \left[\bar{g}_{H_2} + \tfrac{1}{2}\bar{g}_{O_2} - \bar{g}_{H_2O(g)}\right] + \bar{R}T_0 \ln\left[\frac{[y^e_{O_2}]^{1/2}}{y^e_{H_2O}}\right]$$

With $\bar{g}_f^\circ$ data from Table A-25,

$$\bar{e}_{H_2}^{CH} = \left[0 + \tfrac{1}{2}(0) - \left(-228,590 \frac{\text{kJ}}{\text{kmol}}\right)\right] + \left(8.314 \frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(298.15\text{K}) \ln\left[\frac{(0.2035)^{\frac{1}{2}}}{(0.0312)}\right]$$

$$= \left[228,590 + 6622\right]\frac{\text{kJ}}{\text{kmol}} = 235,212 \frac{\text{kJ}}{\text{kmol}}$$

With $M = 2.016$ kg/kmol from Table A-1

$$e_{H_2}^{CH} = \frac{235,212 \text{ kJ/kmol}}{2.016 \text{ kg/kmol}} = 116,673 \frac{\text{kJ}}{\text{kg}} \longleftarrow H_2$$

PROBLEM 13.91 (Cont'd) - Page 2

(c) Methane: $a=1$, $b=4$, $c=0$. Eq. 13.36 reads

$$\bar{e}^{CH}_{CH_4} = \left[\bar{g}_{CH_4} + 2\bar{g}_{O_2} - \bar{g}_{CO_2} - 2\bar{g}_{H_2O(g)} \right] + \bar{R}T_0 \ln\left[\frac{(y^e_{O_2})^2}{(y^e_{CO_2})(y^e_{H_2O})^2} \right]$$

With $\bar{g}^\circ_f$ data from Table A-25

$$\bar{e}^{CH}_{CH_4} = \left[-50,790 + 2(0) - (-394,380) - 2(-228,590) \right] \frac{kJ}{kmol} + \left(8.314 \frac{kJ}{kmol \cdot K}\right)(298.15 K) \ln\left[\frac{(0.2035)^2}{(0.0003)(0.0312)^2} \right]$$

$$= [800,770 + 29,404] \frac{kJ}{kmol}$$

$$= 830,174 \frac{kJ}{kmol}$$

With $M = 16.04$ kg/kmol

$$e^{CH}_{CH_4} = \frac{830,174 \text{ kJ/kmol}}{16.04 \text{ kg/kmol}} = 51,756 \frac{kJ}{kg}$$ ← CH_4

(d) Carbon Monoxide, CO: $a=1$, $b=0$, $c=1$. Eq. 13.36 reads

$$\bar{e}^{CH}_{CO} = \left[\bar{g}_{CO} + \frac{1}{2}\bar{g}_{O_2} - \bar{g}_{CO_2} \right] + \bar{R}T_0 \ln\left[\frac{(y^e_{O_2})^{1/2}}{y^e_{CO_2}} \right]$$

with $\bar{g}^\circ_f$ data from Table A-25

$$\bar{e}^{CH}_{CO} = \left[(-137,150) + \frac{1}{2}(0) - (-394,380) \right] \frac{kJ}{kmol} + \left(8.314 \frac{kJ}{kmol \cdot K}\right)(298.15K) \ln\left[\frac{(0.2035)^{1/2}}{0.0003} \right]$$

$$= [257,230 + 18,134] \frac{kJ}{kmol} = 275,364 \text{ kJ/kmol}$$

With $M = 28.01$ kg/kmol

$$e^{CH}_{CO} = \frac{275,364 \text{ kJ/kmol}}{28.01 \text{ kg/kmol}} = 9831 \text{ kJ/kg}$$

(e), (f), (g). Each of these cases involves a substance appearing in the environment. For them, the chemical exergy is obtained using an expression of the form $\bar{e}^{CH} = \bar{R}T_0 \ln(1/y^e)$, which is an extension of Eq. 13.40 obtained from Eq. 13.36. The results are

	M kg/kmol	y^e (%)	$\bar{e}^{CH}$ kJ/kmol	e^{CH} kJ/kg
N_2	28.01	75.67	691.1	24.7
O_2	32.00	20.35	3946.5	123.3
CO_2	44.01	0.03	20,107.5	456.9

PROBLEM 13.92

13.92 The accompanying table shows an environment consisting of a gas phase and a condensed water phase. The gas phase forms an ideal gas mixture.

Environment		
$T_0 = 298.15$ K (25°C), $p_0 = 1$ atm		
Condensed Phase: $H_2O(l)$ at T_0, p_0		
Gas Phase:	Component	y^e(%)
	N_2	75.67
	O_2	20.35
	$H_2O(g)$	3.12
	CO_2	0.03
	Other	0.83

(a) Show that the chemical exergy of the hydrocarbon C_aH_b can be determined as

$$\bar{e}^{ch} = \left[\bar{g}_F + \left(a + \frac{b}{4}\right)\bar{g}_{O_2} - a\bar{g}_{CO_2} - \frac{b}{2}\bar{g}_{H_2O(l)}\right] + \bar{R}T_0 \ln\left[\frac{(y^e_{O_2})^{a+b/4}}{(y^e_{CO_2})^a}\right]$$

liquid water

← note: $\bar{g}_{H_2O}(l)$. Also, in the log term there is no term involving water.

(b) Using the result of part (a), repeat parts (a) through (c) of Problem 13.91.

(a) The derivation in this case follows that for Eq. 13.36, except for the following:

- In Fig. 13.6, instead of water vapor exiting the control volume, we now think of liquid water at T_0, P_0. Consequently, the Gibbs function for water in this case is for liquid water at T_0, P_0 rather than water vapor, as in Eq. 13.36. Moreover, no term involving water will appear in the natural log term.
- In the present development, $c = 0$. That is C_aH_b is a special case of $C_aH_bO_c$ considered in the derivation of Eq. 13.36.

For comparison, when $c=0$ Eq. 13.36 reads

$$\bar{e}^{CH} = \left[\bar{g}_F + \left(a+\frac{b}{4}\right)\bar{g}_{O_2} - a\,\bar{g}_{CO_2} - \frac{b}{2}\,\bar{g}_{H_2O}(g)^{*}\right] + \bar{R}T_0 \ln\left[\frac{(y^e_{O_2})^{a+\frac{b}{4}}}{(y^e_{CO_2})^a (y^e_{H_2O})^{b/2}}\right]^{*}$$

For this expression to give the same value for chemical exergy as the expression of part (a) above, it is necessary for $y^e_{H_2O}$ to be specified so that

$$\bar{g}_{H_2O}(g) + \bar{R}T_0 \ln y^e_{H_2O} = \bar{g}_{H_2O}(l)$$

where each of the $\bar{g}$'s is evaluated at T_0, P_0. Since $T_0 = 298.15K$, $P_0 = 1$ atm, Gibbs function values from Table A-25 apply, and thus

$$\ln y^e_{H_2O} = \frac{\bar{g}_{H_2O}(l) - \bar{g}_{H_2O}(g)}{\bar{R}T_0} = \frac{[(-237,180) - (-228,590)]\ kJ/kmol}{\left(8.314\ \frac{kJ}{kmol\cdot K}\right)(298.15\ K)}$$

$$\Rightarrow \quad y^e_{H_2O} = 0.03126$$

which agrees with the gas phase mole fraction of water in the table above to within round-off.

(b)

⊙ Carbon: $a=1$, $b=0$. Eq. (1) of the problem statement above reduces to

$$\bar{e}_C^{CH} = [\bar{g}_C + \bar{g}_{O_2} - \bar{g}_{CO_2}] + \bar{R}T_0 \ln\left[\frac{y_{O_2}^e}{y_{CO_2}^e}\right]$$

With $\bar{g}_f^o$ data from Table A-25

$$\bar{e}_C^{CH} = [0 + 0 - (-394{,}380 \text{ kJ/kmol})] + \left(8.314 \frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(298.15\text{K}) \ln\left[\frac{0.2035}{0.0003}\right]$$

$$= 410{,}541 \text{ kJ/kmol} \longleftarrow$$

With $M = 12.01$ kg/kmol from Table A-1, we get $e_C^{CH} = 34{,}183$ kJ/kg ⟵

⊙ Hydrogen, H_2: $a=0$, $b=2$. Eq. (1) reduces to

$$\bar{e}_{H_2}^{CH} = [\bar{g}_{H_2} + \tfrac{1}{2}\bar{g}_{O_2} - \bar{g}_{H_2O(\ell)}^{\,*}] + \bar{R}T_0 \ln[(y_{O_2}^e)^{1/2}]$$

no term for water here

With $\bar{g}_f^o$ data from Table A-25,

$$\bar{e}_{H_2}^{CH} = [0 + \tfrac{1}{2}(0) - (-237{,}180)]\frac{\text{kJ}}{\text{kmol}} + \left(8.314 \frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(298.15\text{K}) \ln((0.2035)^{1/2})$$

$$= [237{,}180 - 1973]\frac{\text{kJ}}{\text{kmol}} = 235{,}207 \text{ kJ/kmol} \longleftarrow$$

With $M = 2.016$ kg/kmol from Table A-1, we get $e_{H_2}^{CH} = 116{,}670$ kJ/kg. ⟵

⊙ Methane: $a=1$, $b=4$. Eq. (1) reduces to

$$\bar{e}_{CH_4}^{CH} = [\bar{g}_{CH_4} + 2\bar{g}_{O_2} - \bar{g}_{CO_2} - 2\bar{g}_{H_2O(\ell)}^{\,*}] + \bar{R}T_0 \ln\left[\frac{(y_{O_2}^e)^2}{(y_{CO_2}^e)}\right]$$

no term for water here

With $\bar{g}_f^o$ data from Table A-25

$$\bar{e}_{CH_4}^{CH} = [(-50{,}790) + 2(0) - (-394{,}380) - 2(-237{,}180)]\frac{\text{kJ}}{\text{kmol}} + \left(8.314 \frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(298.15\text{K}) \ln\left[\frac{(.2035)^2}{(0.0003)}\right]$$

$$= [817{,}950 + 12{,}215]\frac{\text{kJ}}{\text{kmol}}$$

$$= 830{,}165 \text{ kJ/kmol} \longleftarrow$$

With $M = 16.04$ kg/kmol, we get $e_{CH_4}^{CH} = 51{,}756$ kJ/kg ⟵

CLOSING COMMENT

The results calculated here are in excellent agreement with those of parts (a)-(c) of Problem 13.91, as expected.

PRORLEM 13.93

13.93 Justify the use of Eq. 13.36 for liquid methanol, CH_3OH, and liquid ethanol, C_2H_5OH, and apply it to evaluate the chemical exergy, in kJ/kmol, of each substance relative to the environment of Prob. 13.91. Compare with the respective standard chemical exergy values from Table A-26 (Model II).

ANALYSIS:

Consider the complete chemical reaction of CH_3OH with O_2 to form CO_2 and H_2O,

$$CH_3OH + \alpha O_2 \rightarrow \beta CO_2 + \gamma H_2O$$

C: $1 = \beta$

H: $4 = 2\gamma$, $\gamma = 2$

O: $1 + 2\alpha = 2\beta + \gamma = 4 \Rightarrow \alpha = 3/2$

$$CH_3OH + \tfrac{3}{2} O_2 \rightarrow CO_2 + 2H_2O$$

Thinking of CH_3OH in the form $C_aH_bO_c$, we have $a = 1, b = 4, c = 1$.
Eq. 13.30 reads,

$$CH_4O + [1 + \tfrac{4}{4} - \tfrac{1}{2}]O_2 \rightarrow 1\,CO_2 + \tfrac{4}{2} H_2O$$

or

$$CH_4O + \tfrac{3}{2} O_2 \rightarrow CO_2 + 2H_2O$$

which agrees with the previous result, as expected. With such considerations, Eq. 13.36 remains valid for methanol and ethanol.

Applying Eq. 13.36 to liquid methanol: $a = 1, b = 4, c = 1$

$$\bar{e}^{CH} = [\bar{g}_{CH_3OH} + \tfrac{3}{2}\bar{g}_{O_2} - \bar{g}_{CO_2} - 2\bar{g}_{H_2O(g)}] + \bar{R}T_0 \ln\left[\frac{(y^e_{O_2})^{3/2}}{(y^e_{CO_2})(y^e_{H_2O})^2}\right]$$

With $\bar{g}^\circ_f$ data from Table A-25

$$\bar{e}^{CH} = [(-166290) + \tfrac{3}{2}(0) - (-394,380) - 2(-228,590)]\ \text{kJ/kmol} + (8.314 \tfrac{kJ}{kmol \cdot K})(298.15 K) \ln\left[\frac{(0.2035)^{3/2}}{(0.0003)(0.0312)^2}\right]$$

$$= [685,270 + 31,378]\ \text{kJ/kmol}$$

$$= 716,648 \tfrac{kJ}{kmol} \quad (\text{Table A-26 (Model II)}: \bar{e}^{CH} = 718,000 \tfrac{kJ}{kmol})$$

Applying Eq. 13.36 to liquid ethanol: $a = 2, b = 6, c = 1$

$$\bar{e}^{CH} = [\bar{g}_{C_2H_5OH} + 3\bar{g}_{O_2} - 2\bar{g}_{CO_2} - 3\bar{g}_{H_2O(g)}] + \bar{R}T_0 \ln\left[\frac{(y^e_{O_2})^3}{(y^e_{CO_2})^2(y^e_{H_2O})^3}\right]$$

With $\bar{g}^\circ_f$ data from Table A-25

$$\bar{e}^{CH} = [-174,890 + 3(0) - 2(-394,380) - 3(-228,590)]\tfrac{kJ}{kmol} + (8.314\tfrac{kJ}{kmol \cdot K})(298.15 K) \ln\left[\frac{(0.2035)^3}{(0.0003)^2(0.0312)^3}\right]$$

$$= [1,299,640 + 54,160]\tfrac{kJ}{kmol}$$

$$= 1,353,800\ \text{kJ/kg}$$

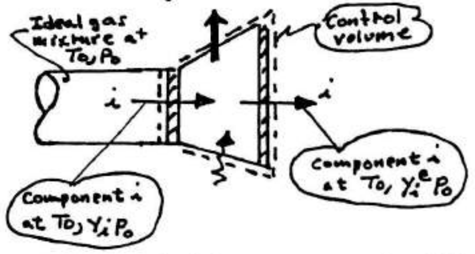
$$(\text{Table A-26 (Model II)}: \bar{e}^{CH} = 1,357,700\ \text{kJ/kmol})$$

Closing comment: For each case, very good agreement exists between the calculated chemical exergy value using Eq. 13.36 and the value from Table A-26 (Model II).

PROBLEM 13.94

Showing all important steps, derive (a) Eqs. 13.41a, b
(b) Eqs. 13.44 a, b.

(a) Ideal gas mixture at T_0, P_0 consisting only of substances in the environment.



Energy rate balance per mole of i:

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_i} - \frac{\dot{W}_{cv}}{\dot{n}_i} + \underbrace{[\bar{h}_i]_{in} - [\bar{h}_i]_{out}}_{=0}$$

$=0$ (since $\bar{h}$ depends on temperature alone for an ideal gas. Note gas i enters and exits at T_0)

$$\Rightarrow \frac{\dot{W}_{cv}}{\dot{n}_i} = \frac{\dot{Q}_{cv}}{\dot{n}_i} \quad (1)$$

Entropy rate balance per mole of i:

$0 = \frac{\dot{Q}_{cv}/\dot{n}_i}{T_0} + \bar{s}_i(T_0, y_i P_0) - \bar{s}_i(T_0, y_i^e P_0) + \dot{\sigma}_{cv}/\dot{n}_i$. When no internal irreversibilities are present, $\dot{\sigma}_{cv} = 0$, and the maximum value for $\dot{Q}_{cv}/\dot{n}_i$ is

$$\left(\frac{\dot{Q}_{cv}}{\dot{n}_i}\right)_{MAX} = -T_0\left[\bar{s}_i(T_0, y_i P_0) - \bar{s}_i(T_0, y_i^e P_0)\right] \quad (2)$$

Combining (1) and (2)

$$\left(\frac{\dot{W}_{cv}}{\dot{n}_i}\right)_{MAX} = -T_0\left[\underbrace{\bar{s}_i(T_0, y_i P_0)}_{\substack{\bar{s}_i(T_0,P_0) - \bar{R}\ln y_i \\ (\text{Eq. } 13.34)}} - \underbrace{\bar{s}_i(T_0, y_i^e P_0)}_{\substack{\bar{s}_i(T_0,P_0) - \bar{R}\ln y_i^e \\ (\text{Eq. } 13.34)}}\right] = \bar{R}T_0 \ln\left(\frac{y_i}{y_i^e}\right)$$

{ for component i of the mixture at T_0, P_0

Then, summing over all such expressions for the gases present, we get

$$\bar{e}^{CH} = \bar{R}T_0 \sum_i y_i \ln\left(\frac{y_i}{y_i^e}\right) \qquad (13.41a)$$

Continuing,

$$\bar{e}^{CH} = \bar{R}T_0 \sum_i y_i \ln\left(\frac{1}{y_i^e}\right) + \bar{R}T_0 \sum_i y_i \ln y_i$$

With Eq. 13.40, $\bar{e}_i^{CH} = \bar{R}T_0 \ln\left(\frac{1}{y_i^e}\right)$, we get

$$\bar{e}^{CH} = \bar{R}T_0 \sum_i y_i \bar{e}_i^{CH} + \bar{R}T_0 \sum_i y_i \ln y_i \qquad (13.41b)$$

(b) As explained in Sec. 13.7.1, the underlined term in Eq. 13.43 is the molar higher heating value at T_0, P_0. Introducing $\overline{HHV}(T_0, P_0)$ in Eq. 13.43 and substituting the resulting expression into Eq. 13.42 gives Eq. 13.44a.

Continuing, note that Eq. 13.43 can be expressed in terms of Gibbs functions. For example, $\bar{g}_F = \bar{h}_F - T_0\bar{s}_F$, all at T_0, P_0. This applies to the O_2, CO_2, and $H_2O(\ell)$ terms as well. Equation 13.44b results when Eq. 13.43 is fully expressed in terms of Gibbs functions and then used to rewrite Eq. 13.42.

PROBLEM 13.95

Using data from Tables A-25 and A-26, together with Eq. 13.44b, determine the standard molar chemical exergy, in kJ/kmol, of propane $C_3H_8(g)$. Compare this value with the standard chemical exergy from Table A-26 (Model II).

Propane: $C_3H_8(g)$ $a=3$, $b=8$. Eq. 13.44b reads

$$\bar{e}^{CH} = \left[\bar{g}_{C_3H_8(g)} + 5\bar{g}_{O_2} - 3\bar{g}_{CO_2} - 4\bar{g}_{H_2O(l)}\right] + 3\bar{e}^{CH}_{CO_2} + 4\bar{e}^{CH}_{H_2O(l)} - 5\bar{e}^{CH}_{O_2}$$

With $\bar{g}^\circ_f$ values from Table A-25 and $\bar{e}^{CH}$ values from Table A-26 (Model II),

$$\bar{e}^{CH} = \left[(-23{,}490) + 5(0) - 3(-394{,}380) - 4(-237{,}180)\right] \text{ kJ/kmol}$$
$$\left[+3(19{,}870) + 4(900) - 5(3970)\right] \text{ kJ/kmol}$$
$$= 2{,}151{,}730 \text{ kJ/kmol}$$

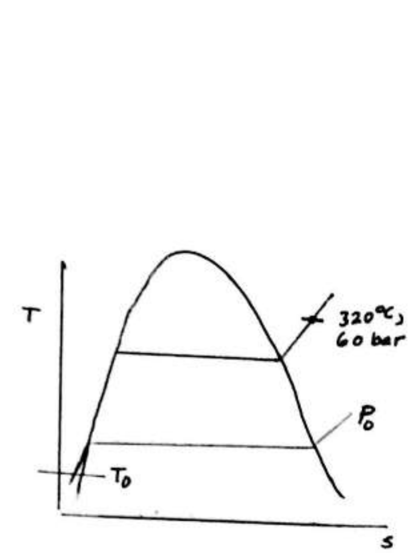
Table A-26 (Model II) gives $\bar{e}^{CH} = 2{,}154{,}000$ kJ/kmol, which very closely agrees with the calculated value, as expected.

PROBLEM 13.96

Evaluate the total specific flow exergy of water vapor, in kJ/kg, at 320°C, 60 bar. Neglect the effects of motion and gravity. Perform calculations relative to the environment of Table A-26 (Model II).

KNOWN: Water vapor at a specified T, p.

FIND: Evaluate the specific flow exergy relative to the environment of Table A-26 (Model II).



Engineering Model

1. The effects of motion and gravity are ignored.
2. The exergy reference of Table A-26 (Model II) applies.
3. At T_0, p_0 water is a liquid, and $h_0 \approx h_f(T_0)$, $s_0 \approx s_f(T_0)$.

Analysis: The total specific flow exergy is given by Eq. 13.47

$$e_f = \underline{(h - h_0) - T_0(s - s_0) + \frac{V^2}{2} + gz} + e^{CH}$$

where the underlined term is the thermomechanical contribution and e^{CH} is the chemical contribution. With assumption 1, the thermomechanical contribution is

$$e^{TM} = (h - h_0) - T_0(s - s_0)$$

With steam table data and assumption 3

$$e^{TM} = (2952.6 - 104.89) - 298.15(6.1846 - 0.3674)$$

$$= 1113.3 \text{ kJ/kg}$$

The chemical exergy is read from Table A-26. The liquid value applies:

$$e^{CH} = \left(900 \frac{kJ}{kmol}\right)\left(\frac{1}{18.02 \text{ kg/kmol}}\right) = 49.9 \frac{kJ}{kg}$$

Collecting results

(1) $$e_f = 1113.3 \frac{kJ}{kg} + 49.9 \frac{kJ}{kg} = 1163.2 \frac{kJ}{kg}$$

1. As expected for high-pressure, high-temperature steam, the thermomechanical contribution dominates — in this case accounting for 96% of the total.

PROBLEM 13.97

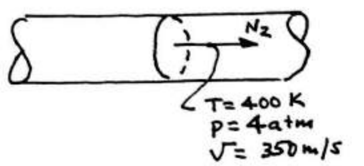
Nitrogen (N_2) flows through a duct. At a particular location the temperature is 400 K, the pressure is 4 atm, and the velocity is 350 m/s. Assuming the ideal gas model and ignoring the effect of gravity, determine the total specific flow exergy, in kJ/kmol. Perform calculations relative to the environment of Table A-26 (Model II).

KNOWN: Data are provided for N_2 flowing through a duct.

FIND: Determine the total specific flow exergy relative to the environment of Table A-26.

SOLUTION

Schematic and Given Data:



Engineering Model:

1. The ideal gas model applies.
2. The effect of gravity is ignored.
3. The exergy reference of Table A-26 (Model II) applies.

Analysis: The total specific flow exergy is given by Eq. 13.47, expressed here on a molar basis:

① $$e_f = \underline{\bar{h} - \bar{h}_0 - T_0(\bar{s} - \bar{s}_0) + M\frac{V^2}{2} + Mgz} + \bar{e}^{CH}$$

where $\bar{e}^{CH}$ is the chemical contribution and the underlined term is the thermomechanical contribution. With assumption 1, Eq. 6.20b, and data from Table A-23

$$\bar{e}^{TM} = \bar{h}(T) - \bar{h}(T_0) - T_0\left[\bar{s}^\circ(T) - \bar{s}^\circ(T_0) - \bar{R}\ln\frac{p}{p_0}\right] + M\frac{V^2}{2}$$

$$= \left\{(11{,}640 - 8{,}669) - 298.15\left[200.071 - 191.502 - 8.314\ln\frac{4}{1}\right]\right\}\frac{kJ}{kmol}$$

$$+ \left(28.01\ \frac{kg}{kmol}\right)\left(\frac{(350\ m/s)^2}{2}\right)\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

$$= \left\{2971 + 881.6\right\}\frac{kJ}{kmol} + 1715.6\ \frac{kJ}{kmol}$$

$$= 5568\ kJ/kmol$$

The chemical contribution is read from Table A-26: $\bar{e}^{CH} = 720$ kJ/kmol.

Collecting results

② $$\bar{e}_f = 5568 + 720 = 6288\ kJ/kmol \longleftarrow$$

---

1. The molecular weight M is included to give the specific kinetic energy on a per kmol basis. See the subsequent calculation of $\bar{e}^{TM}$.

2. As expected for a high-temperature, high-velocity N_2 gas stream, the thermomechanical contribution dominates — in this case accounting for 89% of the total.

PROBLEM 13.98

13.98 Evaluate the total specific flow exergy of an equimolar mixture of oxygen (O_2) and nitrogen (N_2), in kJ/kg, at 227°C, 1 atm. Neglect the effects of motion and gravity. Perform calculations

(a) relative to the environment of Problem 13.91.
(b) using data from Table A-26 (Model II).

ENGINEERING MODEL: (1) The ideal gas model is applicable. (2) The effects of motion and gravity are ignored. (3) Chemical exergy is relative to the environment of Problem 13.91 and Model II of Table A-26.

ANALYSIS: With assumption (2), Eq. 13.47 on a molar basis reads

$$\bar{e}_f = (\bar{h}-\bar{h}_o) - T_o(\bar{s}-\bar{s}_o) + \bar{e}^{CH} \qquad (1)$$

Using ideal gas mixture principles

$$\bar{h}-\bar{h}_o - T_o(\bar{s}-\bar{s}_o) = \left[y_{O_2}(\bar{h}_{O_2}(T) - \bar{h}_{O_2}(T_o)) + y_{N_2}(\bar{h}_{N_2}(T) - \bar{h}_{N_2}(T_o)) \right]$$
$$- T_o\left[y_{O_2}\left(\bar{s}^\circ_{O_2}(T) - \bar{s}^\circ_{O_2}(T_o) - \bar{R}\ln\frac{y_{O_2}p}{y_{O_2}p_o} \right) + y_{N_2}\left(\bar{s}^\circ_{N_2}(T) - \bar{s}^\circ_{N_2}(T_o) - \bar{R}\ln\frac{y_{N_2}p}{y_{N_2}p_o} \right) \right]$$

Since $p = p_o$, the ln terms drop out, leaving

$$\bar{h}-\bar{h}_o - T_o(\bar{s}-\bar{s}_o) = y_{O_2}(\bar{h}_{O_2}(T) - \bar{h}_{O_2}(T_o)) + y_{N_2}(\bar{h}_{N_2}(T) - \bar{h}_{N_2}(T_o))$$
$$- T_o\left[y_{O_2}(\bar{s}^\circ_{O_2}(T) - \bar{s}^\circ_{O_2}(T_o)) + y_{N_2}(\bar{s}^\circ_{N_2}(T) - \bar{s}^\circ_{N_2}(T_o)) \right]$$

Then, with $y_{O_2} = y_{N_2} = 0.5$ and data from Table A-23 at 500K and 298K

$$\left[\bar{h}-\bar{h}_o - T_o(\bar{s}-\bar{s}_o)\right] = 0.5\left[(14{,}770 - 8682) - 298(220.589 - 205.33)\right] +$$
$$0.5\left[(14{,}581 - 8669) - 298(206.630 - 191.502)\right] = 1472.3 \ \frac{kJ}{kmol(mix)}$$

The mixture molecular weight is $M = y_{O_2}M_{O_2} + y_{N_2}M_{N_2} = 0.5[32 + 28.01] = 30.01$

Evaluating e^{CH} relative to the environment of Prob. 13.91, we use Eq. 13.41a.

$$\bar{e}^{CH} = \bar{R}T_o\left[y_{O_2}\ln\frac{y_{O_2}}{y^e_{O_2}} + y_{N_2}\ln\frac{y_{N_2}}{y^e_{N_2}} \right]$$
$$= \left(8.314\frac{kJ}{kmol\cdot K}\right)(298\,K)(0.5)\left[\ln\left[\frac{0.5}{0.2035}\right] + \ln\left[\frac{0.5}{0.7567}\right] \right] = 600.3 \ \frac{kJ}{kmol(mix)}$$

Then, with Eq. (1), we get

$$e_f = \frac{(1472.3 + 600.3)\ kJ/kmol(mix)}{30.01 \ kg/kmol(mix)} = 69.06\ \frac{kJ}{kg} \qquad \longleftarrow (a)$$

Evaluating e^{CH} relative to Table A-26 (Model II), we use Eq. 13.41b

$$\bar{e}^{CH} = y_{O_2}\bar{e}^{CH}_{O_2} + y_{N_2}\bar{e}^{CH}_{N_2} + \bar{R}T_o(y_{O_2}\ln y_{O_2} + y_{N_2}\ln y_{N_2})$$
$$= 0.5[3970 + 720] + 0.5(8.314)(298)[2\ln 0.5] = 627.7 \ \frac{kJ}{kmol(mix)}$$

Then, with Eq. (1) we get

(1) $$e_f = \frac{1472.3 + 627.7}{30.01} = 69.98\ \frac{kJ}{kg} \qquad \longleftarrow (b)$$

1. The thermomechanical exergy dominates in this case. The results obtained for e^f are in good agreement. The chemical exergy contributions differ by about 4%.

PROBLEM 13.99

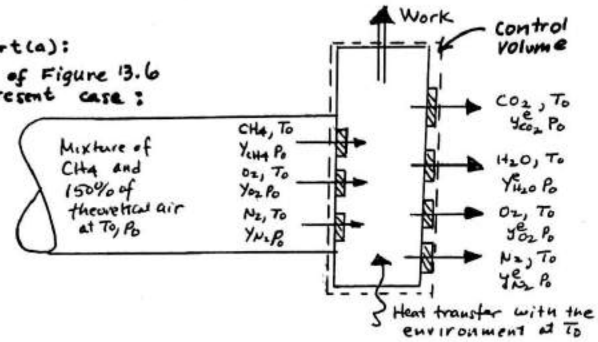
13.100 A mixture of methane gas (CH_4) and 150% of theoretical air enters a combustion chamber at 77°F, 1 atm. Determine the total specific flow exergy of the entering mixture, in Btu per lbmol of methane. Ignore the effects of motion and gravity. Perform calculations

(a) relative to the environment of Problem 13.91.
(b) using data from Table A-26 (Model II).

ENGINEERING MODEL: (1) The ideal gas model is applicable. (2) Effects of motion and gravity are ignored. (3) Chemical exergy is relative to the environment of Problem 13.91, and Model II of Table A-26.

SCHEMATIC & GIVEN DATA:

Figure for Part (a): Counterpart of Figure 13.6 for the present case:



ANALYSIS: Since the mixture enters the combustion chamber at $T_0 = 537°R$ and $P_0 = 1$ atm, which apply for both parts (a) and (b), and the effects of motion and gravity are ignored, the total flow exergy is just the chemical exergy. This follows from Eq. 13.47.

(a) To fully explain aspects mentioned in passing during the presentations of Sec. 13.6, the solution for part (a) will parallel the development leading to Eq. 13.36.
Complete reaction of CH_4 with 150% of theoretical air to yield CO_2, H_2O, O_2, and N_2 is given by Eq. 13.5:

$$CH_4 + 3O_2 + 11.28N_2 \rightarrow CO_2 + 2H_2O(g) + O_2 + 11.28N_2$$

This is the counterpart of Eq. 13.30 for the present discussion.
The mole fractions of the mixture of CH_4, O_2, and N_2 are

$$y_{CH_4} = \frac{1}{15.28} = 0.0654, \quad y_{O_2} = \frac{3}{15.28} = 0.1963, \quad y_{N_2} = \frac{11.28}{15.28} = 0.7382$$

An energy rate balance for the control volume above reduces to

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}} - \frac{\dot{W}_{cv}}{\dot{n}_{CH_4}} + \underbrace{\left[\bar{h}_{CH_4} + 3\bar{h}_{O_2} + 11.28\bar{h}_{N_2}\right]}_{\text{(entering c.v.)}} - \underbrace{\left[\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + \bar{h}_{O_2} + 11.28\bar{h}_{N_2}\right]}_{\text{(exiting c.v.)}}$$

or

$$\frac{\dot{W}_{cv}}{\dot{n}_{CH_4}} = \frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}} + \left(\bar{h}_{CH_4} + 3\bar{h}_{O_2} + 11.28\bar{h}_{N_2}\right) - \left(\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + \bar{h}_{O_2} + 11.28\bar{h}_{N_2}\right) \quad (1)$$

An entropy rate balance reads

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_{CH_4}}{T_0} + \left[\bar{s}_{CH_4}(T_0, y_{CH_4}P_0) + 3\bar{s}_{O_2}(T_0, y_{O_2}P_0) + 11.28\bar{s}_{N_2}(T_0, y_{N_2}P_0)\right] - \left[\bar{s}_{CO_2}(T_0, y^e_{CO_2}P_0) + 2\bar{s}_{H_2O}(T_0, y^e_{H_2O}P_0) + \bar{s}_{O_2}(T_0, y^e_{O_2}P_0) + 11.28\bar{s}_{N_2}(T_0, y^e_{N_2}P_0)\right] + \dot{\sigma}_{cv}/\dot{n}_{CH_4}$$

PROBLEM 13.99 (Continued) - Page 2

Using expressions having the form of Eq. 13.34, the entropy terms can be written in more convenient forms. Then, solving for $\dot{Q}_{cv}/\dot{n}_{CH4}$, we get

$$\frac{\dot{Q}_{cv}}{\dot{n}_{CH4}} = -T_0 \Big[\left(\bar{s}_{CH_4}(T_0,P_0) - \bar{R}\ln y_{CH4}\right) + 3\left(\bar{s}_{O_2}(T_0,P_0) - \bar{R}\ln y_{O_2}\right) + 11.28\left(\bar{s}_{N_2}(T_0,P_0) - \bar{R}\ln y_{N_2}\right)$$
$$- \left[\bar{s}_{CO_2}(T_0,P_0) - \bar{R}\ln y^e_{CO_2}\right] - 2\left[\bar{s}_{H_2O}(T_0,P_0) - \bar{R}\ln y^e_{H_2O}\right] - \left[\bar{s}_{O_2}(T_0,P_0) - \bar{R}\ln y^e_{O_2}\right]$$
$$- 11.28\left[\bar{s}_{N_2}(T_0,P_0) - \bar{R}\ln y^e_{N_2}\right]\Big] - T_0\,\dot{\sigma}_{cv}/\dot{n}_{CH4} \qquad (2)$$

Finally, combining (1) and (2) we get the counterpart of Eq. 13.36 in the limit as entropy production vanishes:

$$\bar{e}^{CH} = \left[\bar{g}_{CH_4} + 3\bar{g}_{O_2} + \cancel{11.28\,\bar{g}_{N_2}} - \bar{g}_{CO_2} - 2\bar{g}_{H_2O} - \bar{g}_{O_2} - \cancel{11.28\,\bar{g}_{N_2}}\right](T_0,P_0)$$
$$+ \bar{R}T_0 \ln\left\{\frac{y_{CH4}\,(y_{O_2})^3}{y^e_{CO_2}\,(y^e_{H_2O})^2\,y^e_{O_2}}\left[\frac{y_{N_2}}{y^e_{N_2}}\right]^{11.28}\right\}$$

Since $T_0 = 77°F$, $P_0 = 1$ atm, the $\bar{g}$'s can be read directly from Table A-25E. Accordingly

$$\bar{e}^{CH} = \left[-21,860 + 3(0) - (-169,680) - 2(-98,350) - (0)\right] +$$
$$(1.986)(537)\ln\left\{\frac{(0.0654)(0.1963)^3}{(0.0003)(0.0312)^2(0.2035)}\left[\frac{0.7382}{0.7567}\right]^{11.28}\right\}$$
$$= 344520 + 9329 = 353,849 \text{ Btu/lbmol(CH}_4\text{)}. \qquad (a)$$

(b) For use with data from Table A-26, Eq. 13.41(b) is invoked. In the present case, this equation takes the form

$$\bar{e}^{CH} = y_{CH4}\,\bar{e}^{CH}_{CH_4} + y_{O_2}\,\bar{e}^{CH}_{O_2} + y_{N_2}\,\bar{e}^{CH}_{N_2} + \underline{\bar{R}T_0\left[y_{CH4}\ln y_{CH4} + y_{O_2}\ln y_{O_2} + y_{N_2}\ln y_{N_2}\right]}$$

The underlined term is

$$= (8.314)(298.15)\left[0.0654\ln(0.0654) + 0.1963\ln(0.1963) + 0.7382\ln(0.7382)\right]$$
$$= -1789.8 \text{ kJ/kmol(mix)}$$ [Note the units here.]

So

② $$\bar{e}^{CH} = (0.0654)\,\bar{e}^{CH}_{CH_4} + (0.1963)\,\bar{e}^{CH}_{O_2} + (0.7382)\,\bar{e}^{CH}_{N_2} - 1789.8 \frac{\text{kJ}}{\text{kmol(mix)}}$$

With Table A-26 data

$$\bar{e}^{CH} = \left[(0.0654)(831,650) + (0.1963)(3970) + 0.7382(720) - 1789.8\right] \text{kJ/kmol(mix)}$$
$$= 53,910.9 \text{ kJ/kmol(mix)} \qquad (b)$$

Converting this to a Btu per lbmol of methane basis, we get

$$\bar{e}^{CH} = \frac{53,910.9 \text{ kJ/kmol(mix)}}{0.0654 \text{ kmol(CH4)/kmol(mix)}}\left|\frac{1 \text{ kg}}{2.2046 \text{ lb}}\right|\left|\frac{0.9478 \text{ Btu}}{1 \text{ kJ}}\right| = 354,393 \text{ Btu/lbmol (CH}_4\text{)}$$

The results of parts (a) and (b) are in very good agreement.

PROBLEM 13.100

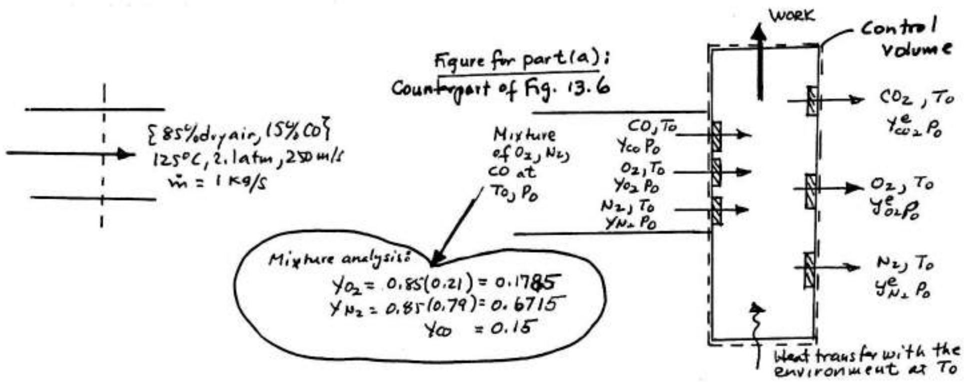
13.100 A mixture having an analysis on a molar basis of 85% dry air, 15% CO enters a device at 125°C, 2.1 atm, and a velocity of 250 m/s. If the mass flow rate is 1.0 kg/s, determine the rate exergy enters, in MW. Neglect the effect of gravity. Perform calculations

(a) relative to the environment of Problem 13.91.
(b) using data from Table A-26 (Model II).

ENGINEERING MODEL:

1. The ideal gas model is applicable.
2. Neglect the effect of gravity.
3. Chemical exergy is relative to the environment of Problem 13.91, and Model II of Table A-26.
4. Dry air is modeled on a molar basis as 21% O_2 and 79% N_2.

SCHEMATIC & GIVEN DATA:



ANALYSIS: With assumption 2, Eq. 13.47 reduces to

$$e_f = \underline{(h-h_0) - T_0(s-s_0) + \frac{V^2}{2}} + e^{CH}$$

where the underlined term is the thermomechanical contribution: e_f^{TM}. On a per kmol of mixture basis with $T = 398\,K$, $T_0 = 298\,K$, $p = 2.1\,atm$, $p_0 = 1\,atm$,

$$\bar{e}_f^{TM} = \Big[0.15[\bar{h}_{CO}(398K) - \bar{h}_{CO}(298K)] + 0.6715[\bar{h}_{N_2}(398) - \bar{h}_{N_2}(298)] + 0.1785[\bar{h}_{O_2}(398) - \bar{h}_{O_2}(298)]\Big]$$
$$- 298\Big[0.15\big(\bar{s}^\circ_{CO}(398) - \bar{s}^\circ_{CO}(298) - \bar{R}\ln\frac{p}{p_0}\big) + 0.6715\big(\bar{s}^\circ_{N_2}(398) - \bar{s}^\circ_{N_2}(298) - \bar{R}\ln\frac{p}{p_0}\big)$$
$$+ 0.1785\big(\bar{s}^\circ_{O_2}(398) - \bar{s}^\circ_{O_2}(298) - \bar{R}\ln\frac{p}{p_0}\big)\Big] + M\frac{V^2}{2} \qquad (1)$$

where M denotes the mixture molecular weight:

$$M = (0.1785)(32) + (0.6715)(28.01) + (0.15)(28.01) = 28.72\ \text{kg(mix)/kmol(mix)}$$

Then, with data from the ideal gas tables

$$\bar{e}_f^{TM} = \Big[0.15[11585 - 8669] + 0.6715[11581 - 8669] + 0.1785[11651 - 8682]\Big]$$
$$- 298\Big[0.15\big(205.98 - 197.54 - 8.314\ln\frac{2.1}{1}\big) + 0.6715[199.92 - 191.50 - 8.314\ln\frac{2.1}{1}]$$
$$+ 0.1785[213.61 - 205.03 - 8.314\ln\frac{2.1}{1}]\Big] + \left(\frac{28.72\ \frac{kg}{kmol}}{2}\right)\left(250\ \frac{m}{s}\right)^2\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

$$= \Big[0.15[2916] + 0.6715[2912] + 0.1785[2969]\Big] - 298\Big[0.15(2.2715) + 0.6715(2.2515)$$
$$+ 0.1785(2.4115)\Big] + 897.5$$

$$= 3140\ \frac{kJ}{kmol(mix)} \Rightarrow e_f^{TM} = \frac{3140\ kJ/kmol(mix)}{28.72\ kg(mix)/kmol(mix)} = 109.3\ kJ/kg(mix)$$

PROBLEM 13.100 (Continued) - Page 2

○ Solution using Table A-26 (Model II).

For use with data from Table A-26, Eq. 13.41(b) takes the form

$$\bar{e}^{CH} = y_{CO}\bar{e}^{CH}_{CO} + y_{O_2}\bar{e}^{CH}_{O_2} + y_{N_2}\bar{e}^{CH}_{N_2} + \underline{\bar{R}T_0[y_{CO}\ln y_{CO} + y_{O_2}\ln y_{O_2} + y_{N_2}\ln y_{N_2}]}$$

The underlined term is

$$= (8.314)(298.15)[0.15 \ln 0.15 + 0.1785 \ln 0.1785 + 0.6715 \ln 0.6715]$$

$$= -2130.8 \frac{kJ}{kmol(mix)}$$

So

$$\bar{e}^{CH} = \left[0.15\,\bar{e}^{CH}_{CO} + 0.1785\,\bar{e}^{CH}_{O_2} + 0.6715\,\bar{e}^{CH}_{N_2} - 2130.8\right]\frac{kJ}{kmol(mix)}$$

With data from Table A-26,

$$\bar{e}^{CH} = 0.15(275{,}100) + 0.1785(3970) + 0.6715(720) - 2130.8$$
$$= 40{,}326\ kJ/kmol(mix)$$

or $$e^{CH} = \frac{40{,}326\ kJ/kmol(mix)}{28.72\ kg(mix)/kmol(mix)} = 1404.1 \frac{kJ}{kg(mix)}$$

The sum is

$$e_f = (109.3 + 1404.1) = 1513.4 \frac{kJ}{kg(mix)}$$

For a mass flow rate of 1 kg/s

$$\dot{E}_f = \dot{m}\,e_f = \left(1 \frac{kg(mix)}{s}\right)\left(1513.4 \frac{kJ}{kg(mix)}\right)\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 1513.4\ kW \qquad \text{(b)}$$

○ Solution using the environment of Problem 13.91

To fully explain aspects mentioned in passing during the presentations of Sec. 13.6, the solution of part (a) will parallel the development leading to Eq. 13.36: For the control volume shown above

$$0.15\,CO + 0.1785\,O_2 + 0.6715\,N_2 \rightarrow 0.15\,CO_2 + 0.1035\,O_2 + 0.6715\,N_2$$

This is the counterpart of Eq. 13.30. An energy rate balance then reads

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{MIX}} - \frac{\dot{W}_{cv}}{\dot{n}_{MIX}} + [0.15\,\bar{h}_{CO} + 0.1785\,\bar{h}_{O_2} + \cancel{0.6715\,\bar{h}_{N_2}}] - [0.15\,\bar{h}_{CO_2} + 0.1035\,\bar{h}_{O_2} + \cancel{0.6715\,\bar{h}_{N_2}}]$$

An entropy balance reads

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_{MIX}}{T_0} + [0.15\,\bar{s}_{CO}(T_0, y_{CO}p_0) + 0.1785\,\bar{s}_{O_2}(T_0, y_{O_2}p_0) + 0.6715\,\bar{s}_{N_2}(T_0, y_{N_2}p_0)]$$
$$- [0.15\,\bar{s}_{CO_2}(T_0, y^e_{CO_2}p_0) + 0.1035\,\bar{s}_{O_2}(T_0, y^e_{O_2}p_0) + 0.6715\,\bar{s}_{N_2}(T_0, y^e_{N_2}p_0)] + \frac{\dot{\sigma}_{cv}}{\dot{n}_{MIX}}$$

Using expressions having the form of Eq. 13.34, the entropy terms are written in more convenient forms. Then, solving for $(Q_{cv}/\dot{n}_{mix})/T_0$

$$-\frac{\dot{Q}_{cv}/\dot{n}_{mix}}{T_0} = 0.15\left[\bar{s}_{CO}(T_0,p_0) - \bar{R}\ln y_{CO}\right] + 0.1785\left(\bar{s}_{O_2}(T_0,p_0) - \bar{R}\ln y_{O_2}\right) + 0.6715\left(\bar{s}_{N_2}(T_0,p_0) - \bar{R}\ln y_{N_2}\right)\Big]$$

$$-\Big[0.15\left[\bar{s}_{CO_2}(T_0,p_0) - \bar{R}\ln y^e_{CO_2}\right] + 0.1035\left(\bar{s}_{O_2}(T_0,p_0) - \bar{R}\ln y^e_{O_2}\right) + 0.6715\left(\bar{s}_{N_2}(T_0,p_0) - \bar{R}\ln y^e_{N_2}\right)\Big] + \dot{\sigma}_{cv}/\dot{n}_{mix}$$

Combining these results gives the counterpart to Eq 13.36 in the limit as entropy production vanishes:

$$\bar{e}^{CH} = 0.15\left[\bar{g}_{CO} + \tfrac{1}{2}\bar{g}_{O_2} - \bar{g}_{CO_2}\right](T_0,p_0) + \bar{R}T_0\left[0.15\ln y_{CO} + 0.1785\ln y_{O_2} - 0.15\ln y^e_{CO_2} - 0.1035\ln y^e_{O_2} + 0.6715\ln\frac{y_{N_2}}{y^e_{N_2}}\right]$$

$$= 0.15\left[\bar{g}_{CO} + \tfrac{1}{2}\bar{g}_{O_2} - \bar{g}_{CO_2}\right](T_0,p_0) + \bar{R}T_0\left[0.15\ln\left(\frac{y_{CO}}{y^e_{CO_2}}\right) + 0.6715\ln\frac{y_{N_2}}{y^e_{N_2}} + 0.1785\ln y_{O_2} - 0.1035\ln y^e_{O_2}\right]$$

Since $T_0 = 25°C$, $p_0 = 1\,atm$, the $\bar{g}$'s can be read directly from Table A-25. Accordingly, the chemical exergy per kmole of mixture is

$$\bar{e}^{CH} = 0.15\left[(-137{,}150) + \tfrac{1}{2}(0) - (-394{,}380)\right] + (8.314)(298)\left[0.15\ln\left(\frac{0.15}{0.0003}\right) + 0.6715\ln\left(\frac{0.6715}{0.7567}\right) + 0.1785\ln(0.1785) - 0.1035\ln(0.2035)\right]$$

$$= 40{,}342\ \text{kJ/kmol(mix)}$$

or on a unit mass of mixture basis

$$e^{CH} = \frac{40{,}342}{28.72} = 1404.7\ \frac{kJ}{kg(mix)}$$

Adding the e_f^{TM} and e^{CH} contributions

$$e_f = 109.3 + 1404.7 = 1514\ \text{kJ/kg(mix)}$$

For a mass flow rate of 1 kg/s

$$\dot{E}_f = \left(1\frac{kg}{s}\right)\left(1514\frac{kJ}{kg}\right)\left(\frac{1\,kW}{1\,kJ/s}\right) = 1514\,kW \longleftarrow (a)$$

The results of parts (a) and (b) are in very good agreement.

---

1. The mole fractions appearing the logarithmic entropy terms cancel.

PROBLEM 13.101

The following flow rates in lb/h are reported for the exiting syngas (synthesis gas) stream in a certain process for producing syngas from bituminous coal:

CH_4	429,684 lb/h
CO_2	9,093 lb/h
N_2	3,741 lb/h
H_2	576 lb/h
CO	204 lb/h
H_2O	60 lb/h

If the syngas stream is at 77°F, 1 atm, determine the rate at which exergy exits, in MW. Perform calculations relative to the environment of Table A-26 (Model II). Neglect the effects of motion and gravity.

KNOWN: Data are provided for a syngas stream.

FIND: Determine the rate at which exergy exits, relative to the environment of Table A-26 (Model II).

ENGINEERING MODEL: 1. The syngas stream is modeled as an ideal gas mixture. 2. The effects of motion and gravity are ignored. 3. Calculations are performed relative to the environment of Table A-26 (Model II)

ANALYSIS: Since the syngas stream exits at T_0, P_0, the thermomechanical contribution vanishes, leaving only the chemical exergy contribution.

Using the given table data, the mole fractions of the syngas are

i	M_i	$\dot{n}_i$ (lbmol/h)	$y_i = \dot{n}_i/\dot{n}$
CH_4	16.04	26,788.28	0.97680
CO_2	44.01	206.61	0.00753
N_2	28.01	133.56	0.00487
H_2	2.016	285.71	0.01041
CO	28.01	7.28	0.00027
$H_2O(g)$	18.02	3.33	0.00012

① The chemical exergy of the syngas can be determined using Eq. 13.41b together with data from Table A-26 (Model II). and the mole fractions above.

$$\bar{e}^{CH} = 0.97680\,(831,650) + 0.00753(19,870) + 0.00487(720) + 0.01041\,(236,100) + 0.00027\,(275,100) + 0.00012\,(9,500) + \bar{R}\,T_0\,[\,0.97680\ln(0.97680) + 0.00753\ln(0.00753) + 0.00487\ln(0.00487) + 0.01041\ln(0.01041) + 0.00027\ln(0.00027) + 0.00012\ln(0.00012)\,]$$

② $\bar{e}^{CH} = 814,705$ kJ/kmol

The total molar flow rate is 27,425 lbmol/h. Converting units

$$\dot{n} = 27,425\ \text{lbmol/h}\left|\frac{1\ \text{h}}{3600\ \text{s}}\right|\left|\frac{1\ \text{kmol}}{2.2046\ \text{lbmol}}\right|$$

$= 3.46$ kmol/s

PROBLEM 13.101 (Continued)

Accordingly,

$$\dot{E}^{CH} = \left(3.46 \frac{kmol}{s}\right)\left(814,705 \frac{kJ}{kmol}\right)\left|\frac{1 MW}{10^3 kJ/s}\right|$$

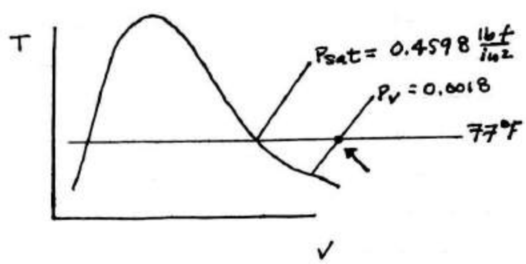
$$= 2819 \text{ MW}$$

---

1. Checking the condition of the water present in the mixture at 77°F, 1 atm:

$$P_V = y_V P = (0.00012)(14.7 \frac{lbf}{in^2})$$
$$= 0.0018 \frac{lbf}{in^2}$$

⇒ The water is a vapor. Thus, the $H_2O(g)$ value for the chemical exergy of the water is used here.



2. The value of the chemical exergy is dominated by the contribution from CH_4, as expected in this case.

PROBLEM 13.102

Propane (C_3H_8) gas at 25°C, 1 atm and a mass flow rate of 0.67 kg/min enters an internal combustion engine operating at steady state. The fuel burns with air entering at 25°C, 1 atm according to

$$C_3H_8 + 4.5[O_2 + 3.76N_2] \rightarrow 2.7CO_2 + 0.3CO + 3.3H_2O + 0.7H_2 + 16.92N_2$$

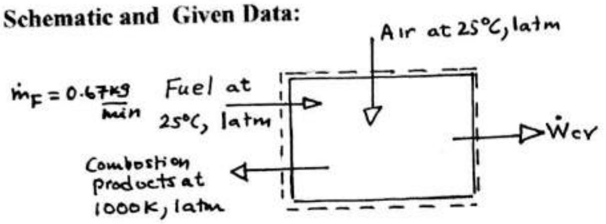
The combustion products exit at 1000 K, 1 atm and the rate of energy transfer by heat from the engine is 100 kW. For the hydrogen, $\bar{c}_p = 29.5$ kJ/kmol · K. The effects of motion and gravity can be ignored. Using the environment of Table A-26 (Model II), evaluate an exergetic efficiency for the engine.

SOLUTION

Known: Steady-state operating data are provided for an internal combustion engine fueled by propane.

Find: Evaluate an exergetic efficiency for the engine.

Schematic and Given Data:



$\dot{Q}_{cv} = -100$ kW

Engineering Model:

1. The control volume shown in the schematic operates at steady state. The effects of motion and gravity can be ignored.

2. The air and combustion products each form ideal gas mixtures. For the air, 3.76 moles of N2 (inert) accompany each mole of O2.

3. For the H2 in the combustion products, $\bar{c}_p$ = 29.5 kJ/kmol - K.

4. The exergy reference environment corresponds to that of Table A-26 (Model II).

(1) **5. The exergy entering with the air is neglected.**

Analysis. Referring to the solution of Example 13.16, an exergetic efficiency takes the form

$$\epsilon = \frac{\dot{W}_{cv}}{\dot{n}_F \bar{e}^{CH}} \quad (1)$$

Observe that the exergy of air is neglected (assumption 5) and that the fuel enters at T_0, p_0 and thus with only chemical exergy.

With the given fuel mass flow rate

$$\dot{n}_F = \frac{0.67 \text{ kg/min}}{44.09 \text{ kg/kmol}} = 0.015 \frac{\text{kmol}}{\text{min}}$$

Moreover, Table A-26 (Model II) gives $\bar{e}^{cH} = 2,154,1000$ kJ/kmol

An energy rate balance at steady state gives:

$$\frac{\dot{W}_{cv}}{\dot{n}_F} = \frac{\dot{Q}_{cv}}{\dot{n}_F} + (H_R - H_P)$$

where with data from Tables A-23, A-25, and assumption 3,

$$H_R = \bar{h}_f^{\circ})_{C_3H_8} = -103{,}850 \frac{kJ}{kmol(fuel)}$$

$$\begin{aligned} H_P = \; & 2.7[-393{,}520 + (42769 - 9364)]_{CO_2} + \\ & 0.3[-110{,}530 + (30{,}355 - 8669)]_{CO} + \\ & 3.3[-241{,}820 + (35{,}882 - 9904)]_{H_2O(g)} + \\ & 0.7[0 + (29.5 \tfrac{kJ}{kmol \cdot K})(1000 - 298.15)K]_{H_2} + \\ & 16.92[0 + (30{,}129 - 8669)]_{N_2} = -1{,}333{,}646 \frac{kJ}{kmol(fuel)} \end{aligned}$$

Then

$$\dot{W}_{cv} = -100\,kW + 0.015 \frac{kmol}{min} \left| \frac{1\,min}{60\,s} \right| \left[-103{,}850 + 1{,}333{,}646 \right] \frac{kJ}{kmol}$$

$$= 207.4 \frac{kJ}{s}$$

Substituting values into Eq. (1):

$$\epsilon = \frac{207.4\,kJ/s}{\left(\frac{0.015}{60} \frac{kmol}{s} \right)\left(2{,}154{,}000 \frac{kJ}{kmol} \right)}$$

② $$= 0.385 \quad (38.5\%)$$

---

1. **The air enters the control volume at T0, p0 with the composition 21% O2, 79 % N2. While there is no thermomechanical contribution to the total flow exergy of the air, a chemical contribution can be calculated from Eq. 13.41b, using the foregoing mole fractions together with chemical exergy data for O2 and N2 from Table A-26. The result is**

   $\bar{e}^{CH}$ = 129 kJ per kmol of air

   In the present analysis this relatively small value is neglected.

2. **For internal combustion engines, significant exergy destruction occurs within the engine mainly due to the combustion there. Exergy is also carried out of the engine accompanying the combustion products.**

PROBLEM 13.103

Liquid octane (C_8H_{18}) at 25°C, 1 atm and a mass flow rate of 0.57 kg/h enters an internal combustion engine operating at steady state. The fuel burns with air entering the engine in a separate stream at 25°C, 1 atm. Combustion products exit at 670 K, 1 atm with a dry molar analysis of 11.4% CO_2, 2.9% CO, 1.6% O_2, and 84.1% N_2. The engine develops power at the rate of 3 kW. Determine

(a) the balanced reaction equation.
(b) the rate of heat transfer from the engine, in kW.
(c) an exergetic efficiency for the engine.

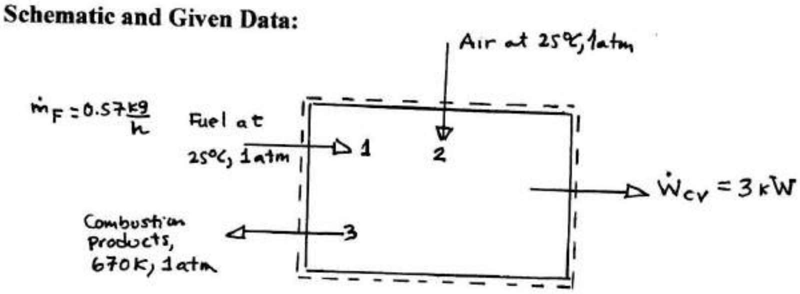
Use the environment of Table A-26 (Model II) and neglect the effects of motion and gravity.

SOLUTION

Known: Steady-state operating data are provided for an internal combustion engine fueled by octane.

Find: Obtain the balanced reaction equation. For the engine determine the rate of heat transfer and an exergetic efficiency.

Schematic and Given Data:



Engineering Model:

1. The control volume shown in the schematic operates at steady state. The effects of motion and gravity can be ignored.

2. The air and combustion products each form ideal gas mixtures. For the air, 3.76 moles of N2 (inert) accompany each mole of O2.

3. The exergy reference environment corresponds to that of Table A-26 (Model II).

1 **4. The exergy entering with the air is neglected.**

Analysis.

(a) The reaction equation takes the form

$$C_8H_{18}(l) + a[O_2 + 3.76N_2] \rightarrow b[0.114CO_2 + 0.029CO + 0.016O_2 + 0.841N_2] + cH_2O$$

C: $8 = b[0.114 + 0.029] \Rightarrow b = 55.9$

H: $18 = 2c \Rightarrow c = 9.$

O: $2a = 55.9[(2)(0.114) + 0.029 + (2)(0.016)] + 9 \Rightarrow a = 12.58$

N_2: $3.76a \approx 0.841b$

$\Rightarrow$ Reaction equation:

(a)

$$C_8H_{18}(l) + 12.58(O_2 + 3.76N_2) \rightarrow 55.9[0.114\,CO_2 + 0.029CO + 0.016O_2 + 0.841N_2] + 9H_2O$$

PROBLEM 13.103 (Continued)

(b) From an energy rate balance at steady state

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} - \frac{\dot{W}_{cv}}{\dot{n}_{fuel}} + (\bar{h}_{fuel})_1 + [12.58\,\bar{h}_{O_2} + (12.58)(3.76)\,\bar{h}_{N_2}]_2 - \{55.9\,(0.114\,\bar{h}_{CO_2} + 0.029\,\bar{h}_{CO} + 0.016\,\bar{h}_{O_2} + 0.841\,\bar{h}_{N_2})\}_3 + 9(\bar{h}_{H_2O})_3\}$$

Introducing $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = \frac{\dot{W}_{cv}}{\dot{n}_{fuel}} - (\bar{h}_f^\circ)_{C_8H_{18}(l)} + 55.9\,[0.114\,[\bar{h}_f^\circ + \bar{h}(670) - \bar{h}(298)]_{CO_2} + 0.029\,[\bar{h}_f^\circ + \bar{h}(670) - \bar{h}(298)]_{CO} + 0.016\,[\bar{h}(670) - \bar{h}(298)]_{O_2} + 0.841\,[\bar{h}(670) - \bar{h}(298)]_{N_2}] + 9\,[\bar{h}_f^\circ + \bar{h}(670) - \bar{h}(298)]_{H_2O}$$

$$= \frac{\dot{W}_{cv}}{\dot{n}_{fuel}} - (-249{,}910) + 55.9\,[0.114\,[-393{,}520 + 25648 - 9364] + 0.029\,[-110{,}530 + 19758 - 8669] + 0.016\,[20{,}197 - 8682] + 0.841\,[19{,}685 - 8669]] + 9\,[-241{,}820 + 22{,}970 - 9904]$$

$$= \frac{\dot{W}_{cv}}{\dot{n}_{fuel}} - 3{,}845{,}872\ \frac{kJ}{kmol\ (C_8H_{18})}$$

The molar flow rate of the fuel is $\dot{n}_{fuel} = \dot{m}_{fuel}/M_{fuel}$. Thus

$$\dot{Q}_{cv} = 3\,kW - \left(\frac{0.57\ kg/h}{114.22\ kg/kmol}\right)\left|\frac{1\,h}{3600\,s}\right|\left(3{,}845{,}872\ \frac{kJ}{kmol}\right)\left|\frac{1\,kW}{1\,kJ/s}\right|$$

$$= -2.33\ kW \qquad \longleftarrow \text{(b)}$$

(c) Referring to the solution of Example 13.16, the exergetic efficiency is

$$\epsilon = \frac{\dot{W}_{cv}}{\dot{n}_{fuel}\,\bar{e}^{CH}}$$

Observe that the exergy of the air is neglected (assumption 4) and that the fuel enters at T_0, P_0 and thus with only chemical exergy. From Table A-26, $\bar{e}^{CH} = 5{,}413{,}100$ kJ/kmol. Then

② $$\epsilon = \frac{3\,kW\left|\frac{1\,kJ/s}{1\,kW}\right|}{\left(\frac{0.57}{114.22}\ \frac{kmol}{h}\right)\left|\frac{1\,h}{3600\,s}\right|\left(5{,}413{,}100\ \frac{kJ}{kmol}\right)} = 0.4\ (40\%) \qquad \longleftarrow \text{(c)}$$

1. **The air enters the control volume at T0, p0 with the composition 21% O2, 79 % N2. While there is no thermomechanical contribution to the total flow exergy of the air, a chemical contribution can be calculated from Eq. 13.41b, using the foregoing mole fractions together with chemical exergy data for O2 and N2 from Table A-26. The result is**

   $$\bar{e}^{CH} = 129 \text{ kJ per kmol of air}$$

   In the present analysis this relatively small value is neglected.

2. **For internal combustion engines, significant exergy destruction occurs within the engine mainly due to the combustion there. Exergy is also carried out of the engine accompanying the combustion products.**

PROBLEM 13.104

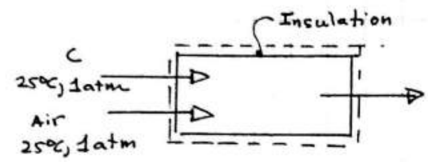
Carbon at 25°C, 1 atm enters an insulated reactor operating at steady state and reacts completely with the theoretical amount of air entering separately at 25°C, 1 atm. Combustion products exit at 2460 K, 1 atm. For the reactor, (a) determine the rate of exergy destruction, in kJ per kmol of carbon, and (b) evaluate an exergetic efficiency. Perform calculations relative to the environment of Table A-26 (Model II). Neglect the effects of motion and gravity.

SOLUTION

Known: Steady-state operating data are provided for an insulated reactor.

Find: Determine the rate of exergy destruction and evaluate an exergetic efficiency.

Schematic and Given Data:



Reaction equation:
$C + O_2 + 3.76N_2 \rightarrow CO_2 + 3.76N_2$

Engineering Model:

1. The control volume shown in the schematic operates at steady state.

2. $\dot{Q}_{cv}=0$, $\dot{W}_{cv}=0$. The effects of motion and gravity can be ignored.

3. The air and combustion products each form ideal gas mixtures. For the air, 3.76 moles of N2 (inert) accompany each mole of O2.

4. The exergy reference environment corresponds to that of Table A-26 (Model II).

(1) **5. The exergy entering with the air is neglected.**

(a) In this case where $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and the exergy of the air is neglected (assumption 5), the rate of exergy destruction is the difference between the exergy entering with the carbon and exiting with the combustion products. From Table A-26, the chemical exergy of the carbon is 410,260 kJ/kmol. Since carbon enters at T_0, P_0 there is no thermomechanical contribution.

For the combustion products the thermomechanical contribution to the total flow exergy on a per kmol of C basis is

$$\bar{e}^{TM} = (\bar{h}-\bar{h}_0) - T_0(\bar{s}-\bar{s}_0) = \left\{1\left[\bar{h}(2460)-\bar{h}(298)\right]_{CO_2} + 3.76\left[\bar{h}(2460)-\bar{h}(298)\right]_{N_2}\right\} -$$
$$T_0\left\{1\left[\bar{s}^\circ(2460)-\bar{s}^\circ(298) - \bar{R}\ln\left(\frac{y_{CO_2}P}{y_{CO_2}P_0}\right)\right]_{CO_2} + 3.76\left[\bar{s}^\circ(2460)-\bar{s}^\circ(298) - \bar{R}\ln\left(\frac{y_{N_2}P}{y_{N_2}P_0}\right)\right]_{N_2}\right\}$$

Since $P = P_0$, the log terms drop out of the entropy quantities. Then with data from Table A-23

PROBLEM 13.104 (Continued)

$$\bar{e}^{TM} = \{[1(128{,}833 - 9364) + 3.76(81{,}515 - 8669)\} -$$
$$298.15\{1(321.814 - 213.685) + 3.76(257.480 - 191.502)\}$$
$$= 284{,}925 \text{ kJ/kmol(C)}$$

Using data from Table A-26 and the mole fractions of the products: $y_{CO_2} = 1/4.76 = 0.21$, $y_{N_2} = 3.76/4.76 = 0.79$, each in kmol per kmol of products, Eq. 13.41b gives the chemical exergy contribution:

$$\bar{e}^{CH} = \{0.21(19870) + 0.79(720) + \bar{R}T_0[0.21 \ln 0.21 + 0.79 \ln 0.79]\}\left(\frac{4.76 \text{ kmol(prods.)}}{\text{kmol(C)}}\right)$$
$$= \left\{3468 \frac{\text{kJ}}{\text{kmol(products)}}\right\}\left(4.76 \frac{\text{kmol(products)}}{\text{kmol(C)}}\right) = 16{,}508 \frac{\text{kJ}}{\text{kmol(C)}}$$

For the combustion products the total flow exergy is

$$\bar{e}_f = \bar{e}^{TM} + \bar{e}^{CH} = 301{,}433 \text{ kJ/kmol(C)}$$

The rate of exergy destruction within the reactor is

2 $$\frac{\dot{E}_d}{\dot{n}_C} = 410{,}260 - 301{,}433 = 108827 \text{ kJ/kmol(C)} \longleftarrow \text{(a)}$$

(c) Following the analysis of Example 13.17

$$\epsilon = \frac{\dot{E}_{prods}/\dot{n}_C}{\dot{E}_F/\dot{n}_C} = \frac{301{,}433}{410{,}260} = 0.734 \quad (73.4\%) \longleftarrow \text{(b)}$$

---

1. **The air enters the control volume at T0, p0 with the composition 21% O2, 79 % N2. While there is no thermomechanical contribution to the total flow exergy of the air, a chemical contribution can be calculated from Eq. 13.41b, using the foregoing mole fractions together with chemical exergy data for O2 and N2 from Table A-26. The result is**

$$\bar{e}^{ch} = 129 \text{ kJ per kmol of air}$$

In the present analysis this relatively small value is neglected.

2. **For a reactor such as this, significant exergy destruction occurs due to the highly-irreversible combustion occurring within it.**

PROBLEM 13.105

Carbon monoxide (CO) at 25°C, 1 atm enters an insulated reactor operating at steady state and reacts completely with the theoretical amount of air entering in a separate stream at 25°C, 1 atm. The products exit as a mixture at 2665 K, 1 atm. Determine in kJ per kmol of CO

(a) the exergy entering with the carbon monoxide.

(b) the exergy exiting with the products.

(c) the rate of exergy destruction.

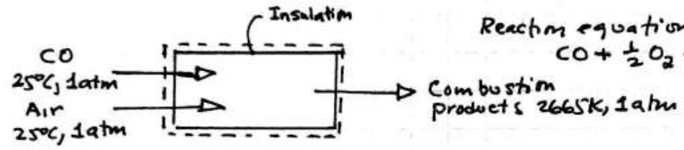
Also evaluate an exergetic efficiency for the reactor. Perform calculations relative to the environment of Table A-26 (Model II). Neglect the effects of motion and gravity.

SOLUTION:

Known: Steady-state operating data are provided for an insulated reactor.

Find: Determine the rate exergy enters the reactor with the CO and the rate exergy exits the reactor with the combustion products. Also, determine the rate exergy is destroyed within the reactor and evaluate an exergetic efficiency for the reactor.

Schematic and Given Data:



Reaction equation:

$$CO + \frac{1}{2}O_2 + 1.88N_2 \rightarrow CO_2 + 1.88N_2$$

Engineering Model:

1. The control volume shown in the schematic operates at steady state.

2. $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$. The effects of motion and gravity can be ignored.

3. The air and combustion products each form ideal gas mixtures. For the air, 3.76 moles of N2 (inert) accompany each mole of O2.

4. The exergy reference environment corresponds to that of Table A-26 (Model II).

① **5. The exergy entering with the air is neglected.**

(a) The CO enters at T_0, p_0 and thus with no thermomechanical contribution to the total flow exergy. The chemical contribution is is read from Table A-26 as 275,100 kJ/kmol (CO). ← (a)

(b) For the combustion products the thermomechanical contribution to the total flow exergy is

$$\bar{e}^{TM} = (\bar{h}-\bar{h}_0) - T_0(\bar{s}-\bar{s}_0) = \left\{1[\bar{h}(2665)-\bar{h}(298)]_{CO_2} + 1.88[\bar{h}(2665)-\bar{h}(298)]_{N_2}\right\} -$$

$$T_0\left\{1\left[\bar{s}^\circ(2665)-\bar{s}^\circ(298) - \bar{R}\ln\left(\frac{y_{CO_2}p}{y_{CO_2}p_0}\right)\right]_{CO_2} + 1.88\left[\bar{s}^\circ(2665) - \bar{s}^\circ(298) - \bar{R}\ln\left(\frac{y_{N_2}p}{y_{N_2}p_0}\right)\right]_{N_2}\right\}$$

Since $p = p_0$, the log terms drop out of the entropy quantities. Then with data from Table A-23

PROBLEM 13.105 (Continued)

$$\bar{e}^{TM} = \{1(141,459-9364) + 1.88(89,040-8669)\} - 298.15\{1(326.742-213.685) + 1.88[262.420-191.502]\}$$

$$= 209,733 \text{ kJ/kmol (CO)}$$

Using data from Table A-26 and the mole fractions of the products:
$y_{CO_2} = 1/2.88 = 0.3472$, $y_{N_2} = 1.88/2.88 = 0.6528$, each in kmol per kmol of products, Eq. 13.41b gives

$$\bar{e}^{CH} = \{0.3472(19870) + 0.6528(720) + (8.314)(298.15)[0.3472 \ln(0.3472) + 0.6528 \ln(0.6528)]\}\left(\frac{2.88 \text{ kmol (prods)}}{\text{kmol (CO)}}\right)$$

$$= \left\{5768 \frac{\text{kJ}}{\text{kmol (products)}}\right\}\left(2.88 \frac{\text{kmol (products)}}{\text{kmol (CO)}}\right) = 16,612 \frac{\text{kJ}}{\text{kmol (CO)}}$$

For the combustion products the total flow exergy is

$$\bar{e}_f = \bar{e}^{TM} + \bar{e}^{CH} = 226,345 \text{ kJ/kmol (CO)} \longleftarrow \text{(b)}$$

(c) In this case where $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and the exergy of the air is neglected (assumption 5), the rate of exergy destruction is the difference between the exergy entering with the CO and exiting with the combustion products. Thus

② $$\frac{\dot{E}_d}{\dot{n}_{CO}} = (275,100 - 226,345) \text{ kJ/kmol (CO)} = 48,755 \frac{\text{kJ}}{\text{kmol (CO)}} \longleftarrow \text{(c)}$$

Finally, following the analysis of Example 13.17

$$\epsilon = \frac{\dot{E}_{prod}/\dot{n}_C}{\dot{E}_F/\dot{n}_C} = \frac{226,345}{275,100} = 0.823 \ (82.3\%) \longleftarrow \text{(d)}$$

---

1. **The air enters the control volume at T0, p0 with the composition 21% O2, 79 % N2. While there is no thermomechanical contribution to the total flow exergy of the air, a chemical contribution can be calculated from Eq. 13.41b, using the foregoing mole fractions together with chemical exergy data for O2 and N2 from Table A-26. The result is**

$$\bar{e}^{ch} = 129 \text{ kJ per kmol of air}$$

In the present analysis this relatively small value is neglected.

2. **For a reactor such as this, significant exergy destruction occurs due to the highly-irreversible combustion occurring within it.**

PROBLEM 13.106

Propane gas (C_3H_8) at 25°C, 1 atm and a volumetric flow rate of 0.03 m^3/min enters a furnace operating at steady state and burns completely with 200% of theoretical air entering at 25°C, 1 atm. The furnace provides energy by heat transfer at 227°C for an industrial process and combustion products at 227°C, 1 atm for cogeneration of hot water. For the furnace, determine

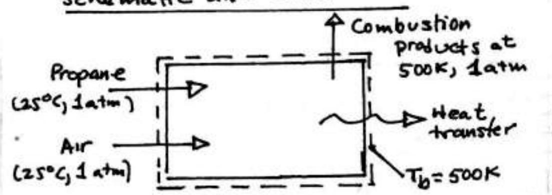
(a) the rate of heat transfer, in kJ/min, and
(b) the rate of entropy production, in kJ/K · min.
(c) Also devise and evaluate an exergetic efficiency for the furnace relative to the environment of Table A-26 (Model II). Ignore the effects of motion and gravity.

SOLUTION

Known: Steady-state data are provided for a furnace.

Find: For the furnace evaluate the rates of heat transfer and entropy production. Devise and evaluate an exergetic efficiency

Schematic and Given Data:



ENGINEERING MODEL:

1. The control volume shown in the schematic operates at steady state with $\dot{W}_{cv} = 0$ and negligible effects of motion and gravity.
2. Combustion is complete.
3. For the air, 3.76 moles of N2 (inert) accompany each mole of O2. The ideal gas model applies.
4. The air and combustion products are each modeled as an ideal gas mixture.
5. The exergy reference environment corresponds to that of Table A-26 (Model II).
6. The exergy entering with the air is neglected. 1

ANALYSIS: The reaction equation for C_3H_8 burning completely with 200% theoretical air is

$$C_3H_8(g) + 10[O_2 + 3.76N_2] \rightarrow 3\,CO_2 + 4H_2O + 5O_2 + 37.6\,N_2$$

(a) An energy rate balance for the furnace reads, $\frac{\dot{Q}_{cv}}{\dot{n}_F} = H_P - H_R$. With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$ and data from Table A-23

$$\frac{\dot{Q}_{cv}}{\dot{n}_F} = 3[\bar{h}_f^\circ + \Delta\bar{h}]_{CO_2} + 4[\bar{h}_f^\circ + \Delta\bar{h}]_{H_2O} + 5[\cancel{\bar{h}_f^\circ} + \Delta\bar{h}]_{O_2} + 37.6[\cancel{\bar{h}_f^\circ} + \Delta\bar{h}]_{N_2}$$
$$- (\bar{h}_f^\circ)_{C_3H_8} - 10(\cancel{\bar{h}_f^\circ})_{O_2} - 37.6(\cancel{\bar{h}_f^\circ})_{N_2}$$

With data from Tables A-23 and A-25

$$\frac{\dot{Q}_{cv}}{\dot{n}_F} = 3[-393{,}520 + (17678 - 9364)] + 4[-241{,}820 + (16828 - 9904)] + 5[14770 - 8682] + 37.6[14{,}581 - 8669] - [-103{,}850]$$

$$= -1{,}738{,}621 \text{ kJ/kmol(fuel)}$$

Using the ideal gas equation of state

$$\dot{n}_F = \frac{(AV)}{\bar{v}} = \frac{(0.03\ m^3/min)(1.01325\times10^5\ N/m^2)}{\left(8314\ \frac{N\cdot m}{kmol\cdot K}\right)(298.15\ K)} = 1.23\times10^{-3}\ \frac{kmol(fuel)}{min}$$

Thus,

$$\dot{Q}_{cv} = \left(1.23\times10^{-3}\ \frac{kmol(fuel)}{min}\right)\left(-1{,}738{,}621\ \frac{kJ}{kmol(fuel)}\right)$$

$$= -2138.5\ \frac{kJ}{min} \qquad \leftarrow (a)$$

(b) Applying an entropy rate balance, where p = 1 atm

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_F}{T_b} + \bar{s}_F(298, \underset{1atm}{p}) + [10\,\bar{s}_{O_2}(298, 0.21p) + 37.6\,\bar{s}_{N_2}(298, 0.79p)] -$$
$$[3\,\bar{s}_{CO_2}(500, y_{CO_2}p) + 4\,\bar{s}_{H_2O}(500, y_{H_2O}p) + 5\,\bar{s}_{O_2}(500K, y_{O_2}p) +$$
$$37.6\,\bar{s}_{N_2}(500, y_{N_2}p)] + \frac{\dot{\sigma}_{cv}}{\dot{n}_F} \qquad (a)$$

Here, $y_{CO_2} = \frac{3}{49.6} = 0.0605$, $y_{H_2O} = \frac{4}{49.6} = 0.0806$, $y_{O_2} = \frac{5}{49.6} = 0.1008$, $y_{N_2} = \frac{37.6}{49.6} = 0.7581$

From Table A-25, $\bar{s}_F = 269.91$ kJ/kmol·K

Applying Eq. 13.23 with $\bar{s}^\circ$ data from Table A-23,

Air:

$$\bar{s}_{O_2} = \bar{s}^\circ_{O_2}(298) - \bar{R}\ln 0.21 = 205.03 - 8.314\ln 0.21 = 218.01$$
$$\bar{s}_{N_2} = \bar{s}^\circ_{N_2}(298) - \bar{R}\ln 0.79 = 191.5 - 8.314\ln 0.79 = 193.46$$

Products:

$$\bar{s}_{CO_2} = \bar{s}_{CO_2}(500) - \bar{R}\ln(0.0605) = 234.814 - 8.314\ln(0.0605) = 258.14$$
$$\bar{s}_{H_2O} = \bar{s}_{H_2O}(500) - \bar{R}\ln(0.0806) = 206.413 - 8.314\ln(0.0806) = 227.35$$
$$\bar{s}_{O_2} = \bar{s}_{O_2}(500) - \bar{R}\ln(0.1008) = 220.589 - 8.314\ln(0.1008) = 239.67$$
$$\bar{s}_{N_2} = \bar{s}_{N_2}(500) - \bar{R}\ln(0.7581) = 206.63 - 8.314\ln(0.7581) = 208.93$$

Substituting into Eq. (a)

$$0 = \frac{-1,738,621\ \text{kJ/kmol(fuel)}}{500K} + [269.91 + 10(218.01) + 37.6(193.46)] -$$
$$[3(258.14) + 4(227.35) + 5(239.67) + 37.6(208.93)] + \frac{\dot{\sigma}_{cv}}{\dot{n}_F}$$

$$\Rightarrow \frac{\dot{\sigma}_{cv}}{\dot{n}_F} = 4491\ \frac{kJ}{K\cdot kmol(fuel)} \Rightarrow \dot{\sigma}_{cv} = (1.23\times10^{-3})(4491) = 5.52\ \frac{kJ}{K\cdot min} \longleftarrow (b)$$

Exergetic Efficiency: The valuable products are the exergy accompanying heat transfer and the exergy of the combustion products.

- Exergy accompanying heat transfer: $\dot{E}_q = \left[1 - \frac{T_0}{T_b}\right]\dot{Q}_{cv} = \left[1 - \frac{298.15}{500}\right](-2138.5) = -863.3\ \frac{kJ}{min}$
- At steady state the exergy of the combustion products can be found from an exergy rate balance:

$$\dot{E}_{prods} = \dot{E}_F + \dot{E}_{Air} + \dot{E}_q - \dot{E}_d \qquad (b)$$

where $\dot{E}_{Air}$ is ignored (assumption 6) and with data from Table A-26

$$\dot{E}_F = \dot{n}_F\,\epsilon_F^{CH} = (1.23\times10^{-3})(2,154,000) = 2649.4\ \frac{kJ}{min}$$

The rate of exergy destruction is determined using

$$\dot{E}_d = T_0 \dot{\sigma}_{cv}$$
$$= (298.15\ K)\left(5.52 \frac{kJ}{K \cdot min}\right) = 1645.8 \frac{kJ}{min}$$

(2)

Inserting values, Eq. (6) gives

$$\dot{E}_{prods} = 2649.4 + 0 + (-863.3) - 1645.8$$
$$= 140.3 \frac{kJ}{min}$$

Finally

$$\epsilon = \frac{|\dot{E}_q| + \dot{E}_{prods}}{\dot{E}_{fuel}}$$

(3)

$$= \frac{863.3 + 140.3}{2649.4} = 0.379\ (37.9\%) \longleftarrow (c)$$

---

1. **The air enters the control volume at T0, p0 with the composition 21% O2, 79 % N2. While there is no thermomechanical contribution to the total flow exergy of the air, a chemical contribution can be calculated from Eq. 13.41b, using the foregoing mole fractions together with chemical exergy data for O2 and N2 from Table A-26. The result is**

   $\bar{e}^{CH}$ **= 129 kJ per kmol of air**

   In the present analysis this relatively small value is neglected.

2. **For a furnace such as this, significant exergy destruction occurs due to the highly-irreversible combustion occurring within it.**

3. If the products are discarded without use, the exergetic efficiency is 32.6%.

PROBLEM 13.107

Complete the solution of Example 13.15 by providing details left to the reader for each of the following:

(a) Evaluation of h_4 and s_4, each in the units given in the table.
(b) Evaluation of the total flow exergy at state 4 assuming the *hypothetical* dead state, introduced in note 5 of the solution, where all water formed by combustion is in vapor form only.

(a) At state 4, $T = 1520\,K$, $p = 9.14$ bar. Using the molar analysis for the products, the mixture molecular weight is

For the combustion products at state 4 where $T = 1520\,K$, $p = 9.14$ bar

$$\bar{h} = 0.7507[\cancel{\bar{h}_f^\circ} + \Delta\bar{h}]_{N_2} + 0.1372[\cancel{\bar{h}_f^\circ} + \Delta\bar{h}]_{O_2} + 0.0314[\bar{h}_f^\circ + \Delta\bar{h}]_{CO_2} + 0.0807[\bar{h}_f^\circ + \Delta\bar{h}]_{}$$

$$= 0.7507[47{,}771 - 8669] + 0.1372[50{,}024 - 8682] + 0.0314[-393{,}520 + (72{,}246 - 9364)]$$
$$+ 0.0807[-241{,}820 + (58942 - 9904)]$$
$$= 9086 \text{ kJ/kmol}$$

$$\Rightarrow h = \frac{\bar{h}}{M} = \frac{9086 \text{ kJ/kmol}}{28.25 \text{ kg/kmol}} = 322 \frac{\text{kJ}}{\text{kg}}$$

For entropy use Eq. 13.23: $\bar{s}_i = \bar{s}_i^\circ - \bar{R} \ln(y_i p/p_{ref})$. Then for $T = 1520\,K$, $p = 9.14$ bar,

N_2: $\bar{s}_{N_2} = 242.228 - 8.314 \ln\left[\frac{(0.7507)(9.14)}{1.01325}\right] = 226.33 \frac{\text{kJ}}{\text{kmol}\cdot K}$

O_2: $\bar{s}_{O_2} = 258.450 - 8.314 \ln\left[\frac{(0.1372)(9.14)}{1.01325}\right] = 256.68$ "

CO_2: $\bar{s}_{CO_2} = 292.888 - 8.314 \ln\left[\frac{(0.0314)(9.14)}{1.01325}\right] = 303.38$ "

H_2O: $\bar{s}_{H_2O} = 251.074 - 8.314 \ln\left[\frac{(0.0807)(9.14)}{1.01325}\right] = 253.71$ "

Then,

$$\bar{s} = (0.7507)(226.33) + (0.1372)(256.68) + (0.0314)(303.38) + (0.0807)(253.71)$$
$$= 235.12 \frac{\text{kJ}}{\text{kmol}\cdot K}$$

$$\Rightarrow s = \frac{\bar{s}}{M} = \frac{235.12 \text{ kJ/kmol}\cdot K}{28.25 \text{ kg/kmol}} = 8.32 \frac{\text{kJ}}{\text{kg}\cdot K}$$

The calculated h and s values correspond to the table of Example 13.15.

PROBLEM 13.107 (Continued)

(b) At the dead state assume a hypothetical state where all of the water vapor formed on combustion is a vapor — that is, assume a molar analysis reading

N$_2$, 75.07%; O$_2$, 13.72%; CO$_2$, 3.14%; H$_2$O(g), 8.07%

The specific enthalpy $\bar{h}_0$ is then

$$\bar{h}_0 = 0.7507\left[\cancel{\bar{h}_f^\circ} + \cancel{\Delta\bar{h}}\right]_{N_2} + 0.1372\left[\cancel{\bar{h}_f^\circ} + \cancel{\Delta\bar{h}}\right]_{N_2} + 0.0314\left[\bar{h}_f^\circ + \cancel{\Delta\bar{h}}\right]_{CO_2} + 0.0807\left[\bar{h}_f^\circ + \cancel{\Delta\bar{h}}\right]_{H2O}$$

$$\bar{h}_0 = 0.0314\left[-393{,}520\right] + 0.0807(-241{,}820) = -31{,}871 \text{ kJ/kmol}.$$

To find s_0, apply 13.23, which for $p = p_0 = p_{ref} = 1$ atm reduces to $\bar{s}_i = \bar{s}_i^\circ - \bar{R}\ln y_i$. Thus

N_2: $\bar{s}_{N_2} = 191.502 - 8.314 \ln 0.7507 = 193.89 \frac{kJ}{kmol\cdot K}$

O_2: $\bar{s}_{O_2} = 205.033 - 8.314 \ln 0.1372 = 221.55$ "

CO_2: $\bar{s}_{CO_2} = 213.685 - 8.314 \ln 0.0314 = 242.46$ "

H_2O: $\bar{s}_{H_2O} = 188.720 - 8.314 \ln 0.0807 = 209.65$ "

$$\bar{s}_0 = (0.7507)(193.89) + (0.1372)(221.55) + (0.0314)(242.46) + (0.0807)(209.65)$$
$$= 200.48 \frac{kJ}{kmol\cdot K}$$

On a mass basis

$$h_0 = \frac{\bar{h}_0}{M} = \frac{-31{,}871}{28.25} = -1128 \frac{kJ}{kg}, \quad s_0 = \frac{\bar{s}_0}{M} = \frac{200.48}{28.25} = 7.1 \frac{kJ}{kg\cdot K}$$

The thermomechanical contribution to the total flow exergy is

$$\Rightarrow \quad e^{TM} = (h-h_0) - T_0(s-s_0) = [822 - (-1128)] - 298.15(8.32 - 7.1) = 1086 \frac{kJ}{kg}$$

The chemical exergy contribution is obtained from Eq. 13.41b and data from Table A-26:

$$\bar{e}^{CH} = (0.7507)(720) + (0.1372)(3{,}970) + 0.0314(19{,}870) + (0.0807)(9500) + (8.314)(298.15)\left[0.7507\ln(0.7507) + 0.1372\ln(0.1372) + 0.0314\ln(0.0314) + 0.0807\ln(0.0807)\right]$$

$$= 494 \frac{kJ}{kmol}$$

$$\Rightarrow \quad e^{CH} = \frac{494}{28.25} = 17 \frac{kJ}{kg}$$

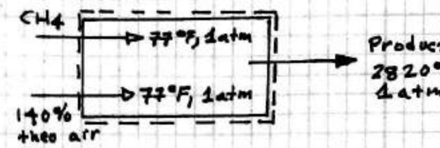
Finally, to total flow exergy is

$$e = e^{TM} + e^{CH} = 1103 \text{ kJ/kg}$$

PROBLEM 13.108

Methane gas enters a reactor and burns completely with 140% of theoretical air, each at 77°F, 1 atm. Combustion products exit at 2820°R, 1 atm. Assuming all water present in the combustion products is a vapor at the dead state and ignoring the effects of motion and gravity, evaluate the total specific flow exergy of the combustion products. Perform calculations relative to the environment of Table A-26 (Model II).

SCHEMATIC & GIVEN DATA:



SOLUTION

Known: Methane reacts completely with 140% theoretical air to form combustion products at a specified temperature. The environment is also specified.

Find: Evaluate the total flow exergy of the combustion products, in Btu per lbmol of methane.

Engineering Model

1. Combustion is complete.
2. For the air, 3.76 moles of N2 (inert) accompany each mole of O2.
3. The combustion products form an ideal gas mixture at each state considered.
1 **4. At the dead state, all water present in the combustion products is a vapor.**
5. The effects of motion and gravity are ignored.

Analysis: For 140% theoretical air, the reaction equation for complete combustion of methane is

$$CH_4 + 2.8(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 10.53N_2 + 0.8O_2$$

The total flow exergy is given by Eq. 13.47 in terms of thermomechanical and chemical contributions. The thermomechanical contribution, in Btu per mole of methane is

$$\begin{aligned}\bar{h} - \bar{h}_0 - T_0(\bar{s} - \bar{s}_0) = {} & [\bar{h}(T) - \bar{h}(T_0) - T_0(\bar{s}^\circ(T) - \bar{s}^\circ(T_0) - \bar{R}\ln(y_{CO_2}p/y_{CO_2}p_0))]_{CO_2} \\ & + 2[\bar{h}(T) - \bar{h}(T_0) - T_0(\bar{s}^\circ(T) - \bar{s}^\circ(T_0) - \bar{R}\ln(y_{H_2O}p/y_{H_2O}p_0))]_{H_2O} \\ & + 10.53[\bar{h}(T) - \bar{h}(T_0) - T_0(\bar{s}^\circ(T) - \bar{s}^\circ(T_0) - \bar{R}\ln(y_{N_2}p/y_{N_2}p_0))]_{N_2} \\ & + 0.8[\bar{h}(T) - \bar{h}(T_0) - T_0(\bar{s}^\circ(T) - \bar{s}^\circ(T_0) - \bar{R}\ln(y_{O_2}p/y_{O_2}p_0))]_{O_2}\end{aligned}$$

Since $p = p_0$, each of the logarithm terms drop out, and with $\bar{h}$ and $\bar{s}^\circ$ data at T_0 from Table A-23E, the thermomechanical contribution reads

$$\begin{aligned}\bar{h} - \bar{h}_0 - T_0(\bar{s} - \bar{s}_0) = {} & [\bar{h}(T) - 4027.5 - 537(\bar{s}^\circ(T) - 51.032)]_{CO_2} \\ & + 2[\bar{h}(T) - 4258 - 537(\bar{s}^\circ(T) - 45.079)]_{H_2O} \\ & + 10.53[\bar{h}(T) - 3729.5 - 537(\bar{s}^\circ(T) - 45.743)]_{N_2} \\ & + 0.8[\bar{h}(T) - 3725.1 - 537(\bar{s}^\circ(T) - 48.982)]_{O_2}\end{aligned}$$

Then, with $\bar{h}$ and $\bar{s}^\circ$ from Table A-23E at $T = 2820°R$,

	$\bar{h}$	$\bar{s}^\circ$
CO_2	32,264	76.382
H_2O	26,316	60.330
N_2	21,248	58.113
O_2	22,232	61.996

we get

$$\bar{h} - \bar{h}_0 - T_0(\bar{s} - \bar{s}_0) = 169,319 \text{ Btu per lbmol of } CH_4$$

PROBLEM 13.108 (Continued)

The chemical exergy contribution is evaluated from Eq. 13.41b using the molar analysis of the combustion products and data from Table A-26:

$$y_{CO_2} = \frac{1}{14.33} = 0.0698,\ y_{H_2O} = \frac{2}{14.33} = 0.1396,\ y_{N_2} = \frac{10.53}{14.33} = 0.7348,$$

$$y_{O_2} = \frac{0.8}{14.33} = 0.0558$$

That is, on a per kmol of products basis

$$\bar{e}^{CH} = 0.0698(19,870) + 0.1396(9,500) + 0.7348(720) + 0.0558(3,970) + (8.314)(298.15)[0.0698\ln(0.0698) + 0.1396\ln(0.1396) + 0.7348\ln(0.7348) + 0.0558\ln(0.0558)] = 1361.43 \frac{kJ}{kmol(products)}$$

$$\Rightarrow \bar{e}^{CH} = 1361.43 \frac{kJ}{kmol}\left|\frac{0.42992\ Btu/lbmol}{1\ kJ/kmol}\right| = 585.3\ \text{Btu per lbmol(products)}$$

When expressed on a per lbmol of fuel basis

$$\bar{e}^{CH} = 585.3 \frac{Btu}{lbmol(prods)}\left(\frac{14.33\ lbmol(prods)}{lbmol\ (CH_4)}\right) = 8387 \frac{Btu}{lbmol\ (CH_4)}$$

The total specific flow exergy is the sum

2 $$\bar{e} = 169,319 + 8387 = 177,706 \frac{Btu}{lbmol\ (CH_4)} \longleftarrow$$

---

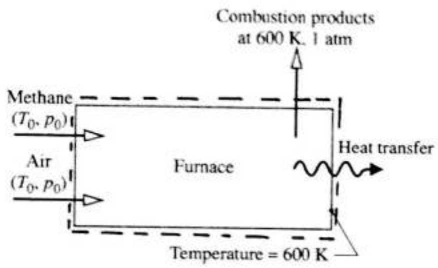
1. A <u>hypothetical</u> dead state is employed for simplicity. See note 5 of Example 13.15.
2. As expected for these high-temperature combustion products, the total flow exergy value is dominated by the thermomechanical contribution.

PROBLEM 13.109

Consider a furnace operating at steady state idealized as shown in Fig. P13.109. The fuel is methane, which enters at 25°C, 1 atm and burns completely with 200% theoretical air entering at the same temperature and pressure. The furnace delivers energy by heat transfer at 600 K. Combustion products at 600 K, 1 atm are provided to the surroundings for cogeneration of steam. There are no stray heat transfers, and the effects of motion and gravity can be ignored. Assuming all water present in the combustion products is a vapor at the dead state, determine in kJ per kmol of fuel

(a) the exergy entering the furnace with the fuel.
(b) the exergy exiting with the products.
(c) the rate of exergy destruction.

Also devise and evaluate an exergetic efficiency for the furnace and comment. Perform calculations relative to the environment of Table A-26 (Model II).



SOLUTION

Find: For the furnace, determine the rate exergy enters with the fuel, exits with the combustion products, and is destroyed. Also evaluate an exergetic efficiency for the furnace.

ENGINEERING MODEL:

1. The control volume shown in the schematic operates at steady state with $\dot{W}_{cv} = 0$ and negligible effects of motion and gravity.
2. Combustion is complete.
3. For the air, 3.76 moles of N2 (inert) accompany each mole of O2. The ideal gas model applies.
4. The combustion products are modeled as an ideal gas mixture. When the products are at the dead state, all water present is assumed to be a vapor. (1)
5. The exergy reference environment corresponds to that of Table A-26 (Model II).
(2) **6. The exergy entering with the air is neglected.**

Analysis: The balanced reaction equation for complete combustion of methane with 200% theoretical air is

$$CH_4 + 4(O_2 + 3.76 N_2) \rightarrow CO_2 + 2H_2O + 15.04 N_2 + 2 O_2$$

(a) Methane enters at T_0, P_0 and thus with no thermomechanical exergy. The chemical exergy is read from Table A-26 as 831,650 kJ/kmol. ← (a)

(b) The exiting combustion products have thermomechanical and chemical contributions to its total flow exergy. The thermomechanical contributio is given by.

$$\begin{aligned}\bar{h} - \bar{h}_0 - T_0(\bar{s} - \bar{s}_0) = {} & [\bar{h}(T) - \bar{h}(T_0) - T_0(\bar{s}^\circ(T) - \bar{s}^\circ(T_0) - \bar{R}\ln(y_{CO_2}p/y_{CO_2}p_0))]_{CO_2} \\ & + 2[\bar{h}(T) - \bar{h}(T_0) - T_0(\bar{s}^\circ(T) - \bar{s}^\circ(T_0) - \bar{R}\ln(y_{H_2O}p/y_{H_2O}p_0))]_{H_2O} \\ & + 15.04[\bar{h}(T) - \bar{h}(T_0) - T_0(\bar{s}^\circ(T) - \bar{s}^\circ(T_0) - \bar{R}\ln(y_{N_2}p/y_{N_2}p_0))]_{N_2} \\ & + 2[\bar{h}(T) - \bar{h}(T_0) - T_0(\bar{s}^\circ(T) - \bar{s}^\circ(T_0) - \bar{R}\ln(y_{O_2}p/y_{O_2}p_0))]_{O_2}\end{aligned}$$

Since $p = p_0$, each of the logarithm terms drop out, and with $\bar{h}$ and $\bar{s}^\circ$ data at T_0 from Table A-23, the thermomechanical contribution reads

$$\begin{aligned}\bar{h} - \bar{h}_0 - T_0(\bar{s} - \bar{s}_0) = {} & [22{,}280 - 9364 - 298.15(243.199 - 213.685)]_{CO_2} + \\ & 2[20{,}402 - 9904 - 298.15(212.920 - 188.720)]_{H_2O} + \\ & 15.04[17{,}563 - 8669 - 298.15(212.066 - 191.502)]_{N_2} + \\ & 2[17{,}929 - 8682 - 298.15(226.346 - 205.033)]_{O_2} \\ = {} & 58{,}020 \text{ kJ per kmol(CH}_4\text{)}\end{aligned}$$

The chemical contribution is evaluated from Eq. 13.41b using mole fractions for the combustion products and data from Table A-26.

$$y_{CO_2} = \frac{1}{20.04} = 0.05, \quad y_{H_2O} = \frac{2}{20.04} = 0.10, \quad y_{O_2} = 0.10, \quad y_{N_2} = \frac{15.04}{20.04} = 0.75$$

Thus

$$\bar{e}^{CH} = 0.05(19,870) + 0.10(9,500) + 0.1(3,970) + 0.75(720) + (8.314)(298.15)\left[0.05\ln(0.05) + 0.1\ln(0.1) + 0.1\ln(0.1) + 0.75\ln(0.75)\right]$$

$$= 832.8 \text{ kJ per kmol of products.}$$

Expressing this on a per kmol of CH_4 basis,

$$\bar{e}^{CH} = \left(832.8 \frac{kJ}{kmol(prods)}\right)\left(20.04 \frac{kmol(prods)}{kmol(CH_4)}\right) = 16689 \frac{kJ}{kmol(CH_4)}$$

Adding these contributions

$$\bar{e}_f = (58,020 + 16689)\frac{kJ}{kmol(CH_4)} = 74,709 \frac{kJ}{kmol(CH_4)} \quad \text{(b)}$$

(c) At steady state the rate exergy is destroyed equals the difference between the rates exergy enter and exit the control volume, respectively. Exergy exits with the combustion products, as determined in part (b), and with the heat transfer. The heat transfer is evaluated using an energy rate balance:

$$\frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}} = H_P - H_R$$

$$= \left\{[\bar{h}^\circ_f + \Delta\bar{h}]_{CO_2} + 2[\bar{h}^\circ_f + \Delta\bar{h}]_{H_2O} + 2[\Delta\bar{h}]_{O_2} + 15.04[\Delta\bar{h}]_{N_2}\right\} - \bar{h}^\circ_f\Big)_{CH_4}$$

$$= \{[-393,520 + 22280 - 9364] + 2[-241,820 + 20,402 - 9904] + 2[17929 - 8682] + 15.04[17,563 - 8669]\} - [-74,850]$$

$$= -616,138 \text{ kJ per kmol of } CH_4$$

The accompanying rate of exergy transfer is

$$\frac{\dot{E}_q}{\dot{n}_{CH_4}} = \left[1 - \frac{T_0}{T_b}\right]\left[\frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}}\right] = \left[1 - \frac{298.15}{600}\right](-616,138)$$

$$= -309,969 \text{ kJ per kmol of } CH_4$$

Then

$$\frac{\dot{E}_d}{\dot{n}_{CH_4}} = \begin{pmatrix}\text{exergy in}\\ \text{with } CH_4\end{pmatrix} - \begin{pmatrix}\text{exergy out}\\ \text{via heat}\\ \text{transfer}\end{pmatrix} - \begin{pmatrix}\text{exergy out}\\ \text{with combustion}\\ \text{products}\end{pmatrix} \quad \text{(c)}$$

③ $$= (831,650 - 309,969 - 74,709) = 446,972 \text{ kJ per kmol of } CH_4.$$

PROBLEM 13.109 (Continued) – Page 3

EXERGETIC EFFICIENCY

In this application the furnace provides two valuable products: relatively high-temperature combustion products and heating. Accordingly, an exergetic efficiency that takes such dual use into account is

$$\epsilon = \frac{\begin{pmatrix}\text{exergy out} \\ \text{via heat transfer}\end{pmatrix} + \begin{pmatrix}\text{exergy out with} \\ \text{combustion products}\end{pmatrix}}{(\text{exergy in with } CH_4)}$$

4 $$= \frac{309{,}969 + 74{,}709}{831{,}650} = 0.463\ (46.3\%)$$

1. A hypothetical dead state is employed for simplicity. See note 5 of Example 13.15

2. The air enters the control volume at T0, p0 with the composition 21% O2, 79 % N2. While there is no thermomechanical contribution to the total flow exergy of the air, a chemical contribution can be calculated from Eq. 13.41b, using the foregoing mole fractions together with chemical exergy data for O2 and N2 from Table A-26. The result is

$$\bar{e}^{CH} = 129 \text{ kJ per kmol of air}$$

In the present analysis this relatively small value is neglected.

3. For a furnace such as this, significant exergy destruction occurs due to the highly-irreversible combustion occurring within it.

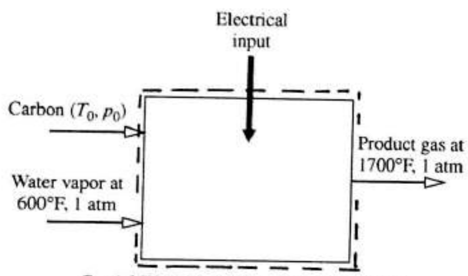
4. If the combustion products are discharged without use, the exergetic efficiency is 37.3%.

PROBLEM 13.110

Figure P13.110 shows a coal gasification reactor making use of the *carbon–steam* process. The energy required for the endothermic reaction is supplied electrically at a rate of 7.85×10^4 Btu per lbmol of carbon entering. The reactor operates at steady state, with no stray heat transfers and negligible effects of motion and gravity. Evaluate in Btu per lbmol of carbon entering

(a) the exergy entering with the carbon.
(b) the exergy entering with the steam.
(c) the exergy exiting with the product gas.
(d) the exergy destruction within the reactor.

Also devise and evaluate an exergetic efficiency for the reactor. Perform calculations relative to the environment of Table A-26 (Model II). Assume all water vapor present in the combustion products is a vapor at the dead state. For the hydrogen produced, $\bar{c}_p = 7.1$ Btu/lbmol · °R.



$C + 1.25H_2O(g) \rightarrow CO + H_2 + 0.25H_2O(g)$

SOLUTION

Known: Steady-state data are provided for a coal gasification reactor.

Find: Evaluate the rates exergy enter and exit the reactor and the rate exergy is destroyed within the reactor. Devise and evaluate an exergetic efficiency.

ENGINEERING MODEL:

1. The control volume shown in the schematic operates at steady state with $\dot{Q}_{cv} = 0$ and negligible effects of motion and gravity.
2. Combustion is complete.
3. The combustion products are modeled as an ideal gas mixture. For the h2 in the combustion products $\bar{c}p$ = 7.1 Btu/lbmol.°R.
1 **4. When the products are at the dead state, all water present is assumed to be a vapor.**
5. For liquid water at the dead state $h0 \approx hf(T0)$ $s0 \approx sf(T0)$.
6. The exergy reference environment corresponds to that of Table A-26 (Model II).

(a) The carbon enters at T_0, p_0 and thus with no thermomechanical contribution to exergy. The chemical contribution is read from Table A-26 as 410,260 kJ/kmol. Converting units,

$$\bar{e}^{CH} = \left(410{,}260 \frac{kJ}{kmol}\right)\left|\frac{0.42992 \text{ Btu/lbmol}}{1 \text{ kJ/kmol}}\right| = 176{,}379 \text{ Btu/lbmol (C)} \quad \leftarrow \text{(a)}$$

(b) For the steam entering the reactor the thermomechanical contribution to its exergy is

$$e^{TM} = (h - h_0) - T_0(s - s_0)$$

with assumption 5 and steam table data,

$$e^{TM} = (1335.2 - 45.1) - 537(1.9737 - 0.08775) = 277.3 \text{ Btu per lb of steam.}$$

Expressing this on a per lbmol of carbon basis

$$\bar{e}^{TM} = \left(277.3 \frac{Btu}{lb}\right)\left(\frac{18.02 \text{ lb}}{lbmol}\right)\left(\frac{1.25 \text{ lbmol(steam)}}{1 \text{ lbmol (C)}}\right) = 6246 \frac{Btu}{lbmol(C)}$$

The chemical exergy contribution from Table A-26 is 900 kJ per kmol. Converting to a per lbmol (C) basis,

$$\bar{e}^{CH} = \left[900 \frac{kJ}{kmol}\right]\left|\frac{0.42992 \text{ Btu/lbmol}}{1 \text{ kJ/kmol}}\right|\left(\frac{1.25 \text{ lbmol(steam)}}{1 \text{ lbmol(C)}}\right) = 484 \frac{Btu}{lbmol(C)}$$

The total exergy is the sum: $6246 + 484 = 6730 \frac{Btu}{lbmol(C)}$ $\leftarrow$ (b)

PROBLEM 13.110 (Continued)

(c) Since the combustion products exit at $P = P_0$, and invoking assumption 3, the thermechanical contribution to the total flow exergy is

$$[\bar{h} - \bar{h}_0 - T_0(\bar{s} - \bar{s}_0)] = [\bar{h}(2160) - \bar{h}(537) - 537(\bar{s}^\circ(2160) - \bar{s}^\circ(537)]_{CO} +$$
$$0.25[\bar{h}(2160) - \bar{h}(537) - 537(\bar{s}^\circ(2160) - \bar{s}^\circ(537))]_{H_2O} +$$
$$[\bar{c}_p(2160 - 537) - 537\,\bar{c}_p \ln(2160/537)]_{H_2}$$
$$= [15953 - 3725 - 537(57.546 - 47.272)]_{CO} +$$
$$0.25[19092 - 4258 - 537(57.412 - 45.079)]_{H_2O} +$$
$$[7.1(2160 - 537) - (537)(7.1)\ln(2160/537)]_{H_2}$$
$$= 14980 \text{ Btu/lbmol (C)}$$

The chemical exergy contribution is evaluated from Eq. 13.41b using the mole fractions of the products and data from Table A-26

$$y_{CO} = \frac{1}{2.25} = 0.4444,\ y_{H_2} = \frac{1}{2.25} = 0.4444,\ y_{H_2O} = \frac{0.25}{2.25} = 0.1111$$

$$\bar{e}^{CH} = 0.1111(9500) + 0.4444(275{,}100) + 0.4444(236{,}100) +$$
$$8.314(298.15)[0.1111\ln(0.1111) + (2)(0.4444)\ln(0.4444)]$$
$$= \left(225{,}841 \frac{kJ}{kmol(prods)}\right)\left(\frac{2.25 \text{ kmol(prods)}}{1 \text{ kmol (C)}}\right)\left|\frac{0.42992 \text{ Btu/lbmol}}{1 \text{ kJ/kmol}}\right|$$
$$= 218{,}461 \text{ Btu/lbmol (C)}$$

The sum is, $\bar{e}_f = \bar{e}^{TM} + \bar{e}^{CH} = 233{,}441$ Btu/lbmol (C) ◄ (c)

(d) At steady state, the rate of exergy destruction within the reactor equals the difference between the total exergy entering and the total exergy exiting:

$$\frac{\dot{E}_d}{\dot{n}_C} = \underbrace{\underset{(electricity)}{78{,}500} + \underset{(steam)}{6730} + \underset{(carbon)}{176{,}379}}_{(261{,}609 \text{ Btu/lbmol (C)})} - \underset{(products)}{233{,}441} \text{ Btu/lbmol (C)}$$
$$= 28{,}168 \text{ Btu/lbmol (C)}$$ ◄ (d)

EXERGETIC EFFICIENCY: Using the above values, an exergetic efficiency can be expressed as,

② $$\epsilon = \frac{\text{exergy exiting with the product gas}}{\left(\text{exergy entering via electricity, carbon and water vapor}\right)} = \frac{233{,}441}{261{,}609} = 0.892\ (89.2\%)$$

1. A hypothetical dead state is assumed for simplicity. See note 5 of Example 13.15

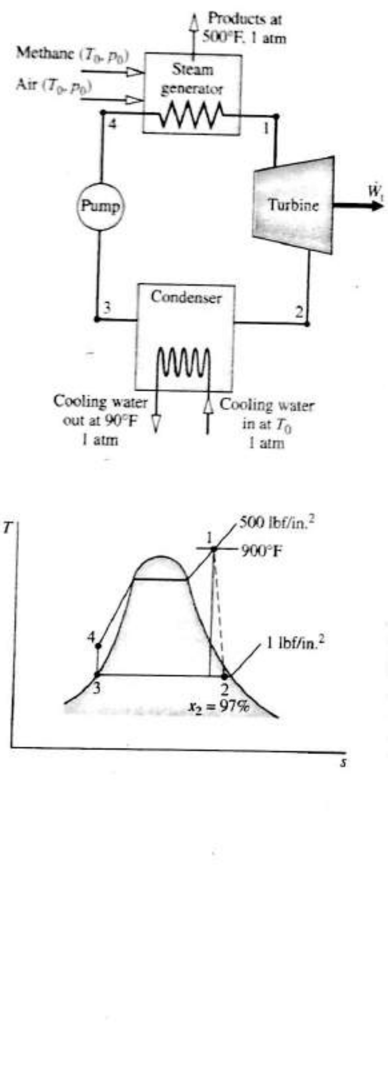
2. While the carbon-steam process is effective in converting the inputs: carbon, steam, and electricity into a combustible gas mixture, the inputs themselves must be produced. Accordingly, an overall exergetic efficiency will be less than the calculated value.

PROBLEM 13.111

Figure P13.111 shows a simple vapor power plant. The fuel is methane that enters at 77°F, 1 atm and burns completely with 200% theoretical air entering at 77°F, 1 atm. Steam exits the steam generator at 900°F, 500 lbf/in.^{2} The vapor expands through the turbine and exits at 1 lbf/in.^{2}, and a quality of 97%. At the condenser exit, the pressure is 1 lbf/in.^{2} and the water is a saturated liquid. The plant operates at steady state with no stray heat transfers from any plant component. Pump work and the effects of motion and gravity are negligible. Determine

(a) the balanced reaction equation.
(b) the vapor mass flow rate, in lb per lbmol of fuel.
(c) the cooling water mass flow rate, in lb per lbmol of fuel.
(d) each of the following, expressed as a percent of the exergy entering the steam generator with the fuel, (i) the exergy exiting with the stack gases, (ii) the exergy destroyed in the steam generator, (iii) the power developed by the turbine, (iv) the exergy destroyed in the turbine, (v) the exergy exiting with the cooling water, (vi) the exergy destroyed in the condenser.

Base exergy values on the environment of Table A-26 (Model II) and assume all water present in the combustion products is a vapor at the dead state.



SOLUTION

KNOWN: Steady-state data are provided for a simple vapor power plant.

FIND: Determine the balanced reaction equation and the vapor and cooling water mass flow rates. Also develop an exergy accounting of the exergy entering with the fuel.

ENGINEERING MODEL:

1. Control volumes enclosing the steam generator, turbine, and condenser are at steady state with no stray heat transfer and no significant effects of motion and gravity. Pump work is negligible.
2. Combustion is complete.
3. For the air, 3.76 moles of N2 (inert) accompany each mole of O2. The ideal gas model applies.
4. The combustion products are modeled as an ideal gas mixture. When the products are at the dead state, all water present is assumed to be a vapor. (1)
5. The exergy reference environment corresponds to that of Table A-26 (Model II).
(2) **6. The exergy entering with the air is neglected.**

ANALYSIS:

(a) The balanced reaction equation is

$$CH_4 + 4(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 2O_2 + 15.04N_2$$

(b) An energy rate balance for the steam generator reads

$$0 = \cancel{\frac{\dot{Q}_{cv}}{\dot{n}_F}}^{0} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_F}}^{0} + \bar{h}_F + [4\bar{h}_{O_2} + 15.04\bar{h}_{N_2}]_{AIR}$$
$$- [\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 2\bar{h}_{O_2} + 15.04\bar{h}_{N_2}]_{PRODUCTS}$$
$$+ \frac{\dot{m}_v}{\dot{n}_F}(h_4 - h_1)$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$ and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2, this becomes

$$0 = (\bar{h}_f^\circ)_{CH_4} - [\bar{h}_f^\circ + \Delta\bar{h}]_{CO_2} - 2[\bar{h}_f^\circ + \Delta\bar{h}]_{H_2O}$$
$$- 2[\Delta\bar{h}]_{O_2} - 15.04[\Delta\bar{h}]_{N_2} + \frac{\dot{m}_v}{\dot{n}_F}(h_4 - h_1)$$

Solving

$$\frac{\dot{m}_V}{\dot{n}_F} = \frac{(\bar{h}_f^\circ)_{CH_4} - [\bar{h}_f^\circ + \Delta\bar{h}]_{CO_2} - 2[\bar{h}_f^\circ + \Delta\bar{h}]_{H_2O} - 2[\Delta\bar{h}]_{O_2} - 15.04[\Delta\bar{h}]_{N_2}}{(h_1 - h_4)}$$

- From Table A-4E, $h_1 = 1466.5\ \frac{Btu}{lb}$, $s_1 = 1.6987\ \frac{Btu}{lb\cdot °R}$
- From Table A-3E, $h_3 = 69.74\ \frac{Btu}{lb}$, $s_3 = 0.1327\ \frac{Btu}{lb\cdot °R}$
  Then, with $s_4 = s_3$ and $p_4 = 500\ \frac{lbf}{in^2}$, Table A-5E gives $h_4 = 71.33$ Btu/lb ✓
- From Table A-25E, $(\bar{h}_f^\circ)_{CH_4} = -32{,}210$ Btu/lbmol
- From Table A-23E

$$[\bar{h}_f^\circ + \Delta\bar{h}]_{CO_2} = [-169{,}300 + 8243.8 - 4027.5] = -165{,}084$$
$$2[\bar{h}_f^\circ + \Delta\bar{h}]_{H_2O} = 2[-104{,}040 + 7738.0 - 4258.0] = -201{,}120$$
$$2[\Delta\bar{h}]_{O_2} = 2[6786.0 - 3725.1] = 6122$$
$$15.04[\Delta\bar{h}]_{N_2} = 15.04[6693.1 - 3729.5] = 44573$$

$$\Rightarrow \frac{\dot{m}_V}{\dot{n}_F} = \frac{283299\ Btu/lbmol}{1395.17\ Btu/lb} = 203.1\ \frac{lb(vapor)}{lbmol(CH_4)}$$

(c) An energy rate balance for the condenser reads

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_V[h_2 - h_3] + \dot{m}_{cw}[h_{cw,in} - h_{cw,out}]$$

$$\Rightarrow \quad \dot{m}_{cw} = \frac{\dot{m}_V[h_2 - h_3]}{[h_{cw,out} - h_{cw,in}]}$$

Introducing the result of part (b),

$$\frac{\dot{m}_{cw}}{\dot{n}_F} = 203.1\left[\frac{h_2 - h_3}{h_{cw,out} - h_{cw,in}}\right]$$

where
$$h_2 = h_f + x_2(h_g - h_f) = 69.74 + 0.97(1036) = 1074.66\ Btu/lb$$

$$h_{cw,in} \approx h_f(77°F) = 45.09\ Btu/lb$$
$$h_{cw,out} \approx h_f(90°F) = 58.07\ Btu/lb$$

Then
$$\frac{\dot{m}_{cv}}{\dot{n}_F} = 203.1\ \frac{lb(vapor)}{lbmol(CH_4)}\left[\frac{1074.66 - 69.74}{58.07 - 45.09}\right]\frac{Btu/lb(vapor)}{Btu/lb(cw)}$$

$$= 15{,}724\ \frac{lb(cw)}{lbmol(CH_4)}$$

Note that at state 2,

$$s_2 = s_f + x_2(s_g - s_f) = 0.1327 + 0.97(1.8453)$$
$$= 1.9226\ Btu/lb\cdot °R$$

which is required in part d(iv).

PROBLEM 13.111 Continued – p. 3

(d) The fuel enters at T_0, P_0 and thus has a chemical exergy contribution only. With the value for CH_4 from Table A-26

$$\bar{e}_F^{CH} = 831{,}650 \frac{kJ}{kmol} \left| \frac{0.42992 \text{ Btu/lbmol}}{1 \text{ kJ/kmol}} \right| = 357{,}543 \frac{Btu}{lbmol(CH_4)}$$

(i) Exergy exiting with the stack gases. With assumption 4, the thermomechanical contribution is (note the gases exit at $p = P_0 = 1\,atm$)

$$\bar{h} - \bar{h}_0 - T_0(s - \bar{s}_0) = [\bar{h}(960°R) - \bar{h}(537°R) - 537°R(\bar{s}°(960) - \bar{s}°(537))]_{CO_2} +$$
$$2[\bar{h}(960) - \bar{h}(537) - 537(\bar{s}°(960) - \bar{s}°(537))]_{H_2O} +$$
$$2[\bar{h}(960) - \bar{h}(537) - 537(\bar{s}°(960) - \bar{s}°(537))]_{O_2} +$$
$$15.04[\bar{h}(960) - \bar{h}(537) - 537(\bar{s}°(960) - \bar{s}°(537))]_{N_2}$$

$$= [8243.8 - 4027.5 - 537(56.765 - 51.032)]_{CO_2} +$$
$$2[7738 - 4258 - 537(49.843 - 45.079)]_{H_2O} +$$
$$2[6786 - 3725.1 - 537(53.170 - 48.982)]_{O_2} +$$
$$15.04[6698.1 - 3729.5 - 537(49.808 - 45.743)]_{N_2}$$

$$= 16{,}047 \text{ Btu per lbmol } (CH_4)$$

The chemical contribution is evaluated from Eq. 13.41b using the mole fractions for the combustion products: $y_{CO_2} = 0.05$, $y_{H_2O} = 0.10$, $y_{O_2} = 0.10$, $y_{N_2} = 0.75$, together with data from Table A-26.

$$\bar{e}_{CH} = 0.05(19{,}870) + 0.10(9{,}500) + 0.1(3970) + 0.75(720) +$$
$$(8.314)(298.15)[0.05 \ln(0.05) + 0.1 \ln(0.10) + 0.1 \ln(0.1) + 0.75 \ln(0.75)]$$
$$= 832.5 \text{ kJ per kmol of products.}$$

Expressing this on a per lbmol of CH_4 basis:

$$\bar{e}^{CH} = \left(832.5 \frac{kJ}{kmol\,(prods)}\right)\left(20.04 \frac{kmol\,(prods)}{kmol\,(CH_4)}\right)\left| \frac{0.42992 \text{ Btu/lbmol}}{1 \text{ kJ/kmol}} \right| = 7172 \frac{Btu}{lbmol\,(CH_4)}$$

Adding these contributions,

$$\frac{\dot{E}_{SG}}{\dot{n}_F} = 16{,}047 + 7172 = 23{,}219 \frac{Btu}{lbmol(CH_4)}$$

Expressing this as a percent of the exergy carried in with the fuel

$$\% = \left(\frac{23{,}219}{357{,}543}\right)(100) = 6.49\% \longleftarrow$$

(ii) Exergy destroyed in the steam generator.

Net Exergy is carried out of the steam generator by the stream flowing from 4 to 1. This is

$$[h_1 - h_4] - T_0[s_1 - s_4] = [1466.5 - 71.33] - 537[1.6987 - 0.1327]$$
$$= 554.2 \ \frac{\text{Btu}}{\text{lb(vapor)}}$$

Expressing this on a per lbmol of CH_4 basis,

$$\begin{bmatrix}\text{Net exergy} \\ \text{carried out of} \\ \text{the steam generator}\end{bmatrix} = \left(554.2 \ \frac{\text{Btu}}{\text{lb(vapor)}}\right)\left(203.1 \ \frac{\text{lb(vapor)}}{\text{lbmol (CH}_4\text{)}}\right)$$
$$= 112,558 \ \frac{\text{Btu}}{\text{lbmol (CH}_4\text{)}}$$

The exergy destroyed in the steam generator is the difference between the exergy carried in with the CH_4 and the exergy carried out by the combustion products and the stream from 4 - 1:

$$\frac{\dot{E}_d}{\dot{n}_{CH4}} = 357,543 - 23,219 - 112,558 = 221,766 \ \frac{\text{Btu}}{\text{lbmol (CH}_4\text{)}}$$

Expressing this as a percent of the exergy carried in with the fuel

3 $$\% = \left(\frac{221,766}{357,543}\right)(100) = 62\% \quad \longleftarrow$$

(iii) Power developed by the turbine.

An energy rate balance at steady state for the turbine gives

$$\frac{\dot{W}_{cv}}{\dot{n}_F} = \left(\frac{\dot{m}_v}{\dot{n}_F}\right)(h_1 - h_2) = \left[203.1 \ \frac{\text{lb(vapor)}}{\text{lbmol (CH}_4\text{)}}\right]\left[1466.5 - 1074.66\right] \frac{\text{Btu}}{\text{lb(vapor)}}$$
$$= 79,583 \ \frac{\text{Btu}}{\text{lbmol (CH}_4\text{)}}$$

Expressing this as a percent of the exergy carried in with the fuel

$$\% = \left(\frac{79583}{357,543}\right)(100) = 22.26\% \quad \longleftarrow$$

PROBLEM 13.111 – p. 5

(iv) Exergy destroyed in the turbine.

$$\begin{bmatrix}\text{The rate of}\\ \text{exergy destruction}\\ \text{per lbmole of } CH_4\end{bmatrix} = T_0\left[\dot{\sigma}_{cv}/\dot{n}_F\right]$$

$$= \left(\frac{\dot{m}_v}{\dot{n}_F}\right) T_0\,(s_2 - s_1)$$

$$= \left(203.1\ \frac{\text{lb(vapor)}}{\text{lbmol}(CH_4)}\right)(537\,°R)\left[1.9226 - 1.6987\right]\frac{\text{Btu}}{\text{lb(vapor)}\cdot °R}$$

$$= 24{,}420\ \frac{\text{Btu}}{\text{lbmol}(CH_4)}$$

Expressing this as a percent of the exergy carried in with the fuel

$$\% = \left(\frac{24{,}420}{357{,}543}\right)(100) = 6.83\%$$

(v) Exergy exiting with the cooling water.

The net exergy exiting with the cooling water per lbmol of CH_4

$$= \frac{\dot{m}_{cw}}{\dot{n}_F}\left[(h_{cw,out} - h_{cw,in}) - T_0\,(s_{cw,out} - s_{cw,in})\right]$$

$s_{cw,out} \approx s_f(90°F)$, $s_{cw,in} \approx s_f(77°F)$

$$= \left(15{,}724\ \frac{\text{lb(cw)}}{\text{lbmol}(CH_4)}\right)\left[(58.07 - 45.09) - 537(0.1117 - 0.08775)\right]\frac{\text{Btu}}{\text{lb(cw)}}$$

$$= 1869\ \frac{\text{Btu}}{\text{lbmol}(CH_4)}$$ ✓

Expressing this as a percent of the exergy carried in with the fuel

$$\% = \left(\frac{1869}{357{,}543}\right)(100) = 0.52\%$$ ✓

(vi) Exergy destroyed in the condenser.

$$\begin{bmatrix}\text{The rate of exergy}\\ \text{destruction per lbmol}\\ \text{of } CH_4\end{bmatrix} = T_0\left[\dot{\sigma}_{cv}/\dot{n}_F\right] = T_0\left[\frac{\dot{m}_{cw}}{\dot{n}_F}(s_{cw,out} - s_{cw,in}) + \frac{\dot{m}_v}{\dot{n}_F}(s_3 - s_2)\right]$$

$$= 537\left[15{,}724(0.1117 - 0.08775) + 203.1(0.1327 - 1.9226)\right]$$

$$= 7014\ \frac{\text{Btu}}{\text{lbmol}(CH_4)}$$

Expressing this as a percent of the exergy carried in with the fuel

$$\% = \left(\frac{7014}{357{,}543}\right)(100) = 1.96\%$$

PROBLEM 13.111 – p. 6

In summary, an exergy accounting in terms of the percentages of the fuel exergy determined in the solution:

Exergy Out

Power	22.26 %
Stack Gases	6.49 %
Cooling Water	0.52 %

Exergy Destroyed

Steam Generator	62.00 %
Turbine	6.83 %
Condenser	1.96 %

Comparing this exergy accounting with the exergy accounting of Table 8.4, the power developed in the present case is lower **and the exergy exiting with the combustion products is** higher, **suggesting a significant scope for improvement in the present case.**

---

1. **A hypothetical dead state is employed for simplicity. See note 5 of Example 13.15.**

2. **The air enters the control volume at T0, p0 with the composition 21% O2, 79 % N2. While there is no thermomechanical contribution to the total flow exergy of the air, a chemical contribution can be calculated from Eq. 13.41b, using the foregoing mole fractions together with chemical exergy data for O2 and N2 from Table A-26. The result is**

$$\bar{e}^{CH} = 129 \text{ kJ per kmol of air} = 55 \text{ Btu per lbmol of air}$$

In the present analysis this relatively small value is neglected.

3. **For Rankine power plants the steam generator is the most significant site of exergy destruction owing to highly-irreversible combustion and heat transfer within it.**

PROBLEM 13.112

For psychrometric applications such as those considered in Chap. 12, the environment often can be modeled simply as an ideal gas mixture of water vapor and dry air at temperature T_0 and pressure p_0. The composition of the environment is defined by the dry air and water vapor mole fractions y_a^e, y_v^e, respectively.

(a) Show that relative to such an environment the total specific flow exergy of a moist air stream at temperature T and pressure p with dry air and water vapor mole fractions y_a and y_v, respectively, can be expressed on a molar basis as

$$\bar{e}_f = T_0\left\{(y_a\bar{c}_{pa} + y_v\bar{c}_{pv})\left[\left(\frac{T}{T_0}\right) - 1 - \ln\left(\frac{T}{T_0}\right)\right] + \bar{R}\ln\left(\frac{p}{p_0}\right)\right\} + \bar{R}T_0\left[y_a\ln\left(\frac{y_a}{y_a^e}\right) + y_v\ln\left(\frac{y_v}{y_v^e}\right)\right]$$

where $\bar{c}_{pa}$ and $\bar{c}_{pv}$ denote the molar specific heats of dry air and water vapor, respectively. Neglect the effects of motion and gravity.

(b) Express the result of part (a) on a *per unit mass of dry air basis* as

$$e_f = T_0\left\{(c_{pa} + \omega c_{pv})\left[\frac{T}{T_0} - 1 - \ln\left(\frac{T}{T_0}\right)\right] + (1 + \tilde{\omega})R_a\ln(p/p_0)\right\} + R_aT_0\left\{(1 + \tilde{\omega})\ln\left(\frac{1 + \tilde{\omega}^e}{1 + \tilde{\omega}}\right) + \tilde{\omega}\ln\left(\frac{\tilde{\omega}}{\tilde{\omega}^e}\right)\right\}$$

where $R_a = \bar{R}/M_a$ and $\tilde{\omega} = \omega M_a/M_v = y_v/y_a$.

Part (a) For an ideal gas mixture at T_0, P_0 consisting only of substances present as gases in the environment, Eq. 13.41a is applicable. In the present case the gases are dry air and water vapor. Accordingly, Eq. 13.41a reads,

$$\bar{e}^{CH} = \bar{R}T_0\left[y_a\ln\left(\frac{y_a}{y_a^e}\right) + y_v\ln\left(\frac{y_v}{y_v^e}\right)\right] \qquad (1)$$

The thermomechanical contribution is

$$\bar{h} - \bar{h}_0 - T_0(\bar{s} - \bar{s}_0) = y_a\left[\bar{h}_a(T) - \bar{h}_a(T_0) - T_0\left(\bar{s}_a^\circ(T) - \bar{s}_a^\circ(T_0) - \bar{R}\ln\frac{\cancel{y_a}P}{\cancel{y_a}P_0}\right)\right] +$$
$$y_v\left[\bar{h}_v(T) - \bar{h}_v(T_0) - T_0\left(\bar{s}_v^\circ(T) - \bar{s}_v^\circ(T_0) - \bar{R}\ln\frac{\cancel{y_v}P}{\cancel{y_v}P_0}\right)\right]$$

Evaluating the enthalpy and entropy terms on the basis of mean values for the specific heats $\bar{c}_{pa}$ and $\bar{c}_{pv}$, this expression becomes

$$\bar{h} - \bar{h}_0 - T_0(\bar{s} - \bar{s}_0) = y_a\left[\bar{c}_{pa}[T - T_0] - T_0\left(\bar{c}_{pa}\ln T/T_0 - \bar{R}\ln P/P_0\right)\right] +$$
$$y_v\left[\bar{c}_{pv}[T - T_0] - T_0\left(\bar{c}_{pv}\ln T/T_0 - \bar{R}\ln P/P_0\right)\right]$$

$$= T_0\left\{(y_a\bar{c}_{pa} + y_v\bar{c}_{pv})\left[\frac{T}{T_0} - 1 - \ln\frac{T}{T_0}\right] + \bar{R}\underbrace{(y_a + y_v)}_{=1}\ln P/P_0\right\} \qquad (2)$$

Adding these contributions

$$\bar{e}_f = T_0\left\{(y_a\bar{c}_{pa} + y_v\bar{c}_{pv})\left[\frac{T}{T_0} - 1 - \ln\frac{T}{T_0}\right] + \bar{R}\ln\frac{P}{P_0}\right\} + \bar{R}T_0\left[y_a\ln\frac{y_a}{y_a^e} + y_v\ln\frac{y_v}{y_v^e}\right]$$

which is the result to be obtained.

PROBLEM 13.112 (Continued) - Page 2

Part (b) $\bar{e}_f$ found in part(a) is expressed on a per mole of mixture basis. Thus, this can be converted to a per unit mass of dry air basis, e_f, as follows: $e_f = \dfrac{\bar{e}_f\, n_{mix}}{m_a}$. With $m_a = n_a M_a$, this becomes

$e_f = \dfrac{\bar{e}_f\, n_{mix}}{n_a M_a}$ Then, since $\dfrac{n_a}{n_{mix}} = y_a$, we get $\underline{\underline{e_f = \dfrac{\bar{e}_f}{y_a M_a}}}$

Using this conversion, each of the three terms of part(a) is placed on a per unit mass of dry air basis, as follows:

First term: With $\bar{c}_{pa} = c_{pa}M_a$ and $\bar{c}_{pv} = c_{pv}M_v$

$$\frac{y_a\bar{c}_{pa} + y_v\bar{c}_{pv}}{y_a M_a} = \left[\frac{y_a c_{pa} M_a + y_v c_{pv} M_v}{y_a M_a}\right] = \left[c_{pa} + c_{pv}\underbrace{\left[\frac{y_v}{y_a}\frac{M_v}{M_a}\right]}_{\omega = \frac{M_v P_v}{M_a P_a} = \frac{M_v y_v \cancel{p}}{M_a y_a \cancel{p}}}\right]$$

$$= [c_{pa} + \omega c_{pv}] \quad \bigstar$$

Second term: $\bar{R}\ln(P/P_0)$

$$\frac{\bar{R}}{y_a M_a} = \underbrace{\frac{\bar{R}}{M_a}}_{R_a}\cdot\frac{1}{y_a} . \quad \text{since } y_a + y_v = 1 \Rightarrow 1 + \overbrace{\frac{y_v}{y_a}}^{\tilde{\omega}} = \frac{1}{y_a} \Rightarrow \frac{1}{y_a} = (1+\tilde{\omega})$$

$$= R_a(1+\tilde{\omega}) \quad \bigstar$$

$$\Rightarrow y_a = \frac{1}{(1+\tilde{\omega})} \quad (a)$$

$$y_v = \frac{\tilde{\omega}}{(1+\tilde{\omega})} \quad (b)$$

Third term:

$$\frac{\bar{R}T_0}{y_a M_a}\left[y_a\ln\left(\frac{y_a}{y_a^e}\right) + y_v\ln\left(\frac{y_v}{y_v^e}\right)\right] = R_a T_0\left[\ln\left(\frac{y_a}{y_a^e}\right) + \frac{y_v}{y_a}\ln\left(\frac{y_v}{y_v^e}\right)\right]$$

With Eq.(a) $= \ln\left[\dfrac{1+\tilde{\omega}^e}{1+\tilde{\omega}}\right]$ $\quad \dfrac{y_v}{y_a} = \tilde{\omega}$ $\quad$ With Eq.(b)

$$= R_a T_0\left[\ln\left[\frac{1+\tilde{\omega}^e}{1+\tilde{\omega}}\right] + \tilde{\omega}\ln\left[\frac{\tilde{\omega}}{1+\tilde{\omega}}\cdot\frac{1+\tilde{\omega}^e}{\tilde{\omega}^e}\right]\right]$$

$$= R_a T_0\left[\ln\left[\frac{1+\tilde{\omega}^e}{1+\tilde{\omega}}\right] + \tilde{\omega}\ln\left[\frac{1+\tilde{\omega}^e}{1+\tilde{\omega}}\right] + \tilde{\omega}\ln\left[\frac{\tilde{\omega}}{\tilde{\omega}^e}\right]\right]$$

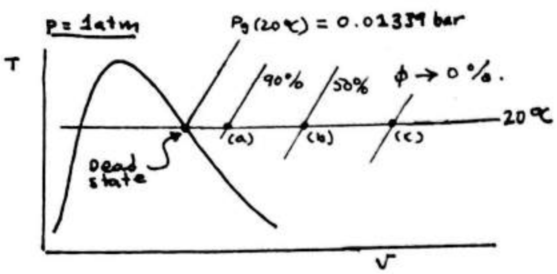
$$= R_a T_0\left[(1+\tilde{\omega})\ln\left[\frac{1+\tilde{\omega}^e}{1+\tilde{\omega}}\right] + \tilde{\omega}\ln\left[\frac{\tilde{\omega}}{\tilde{\omega}^e}\right]\right] \quad \bigstar$$

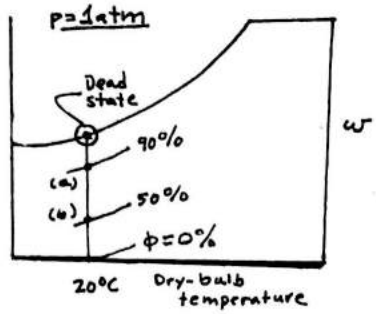
Collecting results, we get the expression for the specific flow exergy on a per unit mass of dry air basis.

PROBLEM 13.113

For each of the following, use the result of Problem 13.112(a) to determine the total specific flow exergy, in kJ/kg, relative to an environment consisting of moist air at 20°C, 1 atm, ϕ = 100%

(a) moist air at 20°C, 1 atm, ϕ = 90%.
(b) moist air at 20°C, 1 atm, ϕ = 50%.
(c) moist air at 20°C, 1 atm, ϕ = 10%.
(d) moist air at 20°C, 1 atm, ϕ = 0%.





ANALYSIS:

Since $T = T_0$ and $p = p_0$, the result of Problem 13.112(a) reduces to

$$\bar{e}_f = \bar{R} T_0 \left[y_a \ln\left(\frac{y_a}{y_a^e}\right) + y_v \ln\left(\frac{y_v}{y_v^e}\right) \right] \quad (1)$$

Observe that $p = p_a + p_v$ and $p_v = \phi\, p_g \Rightarrow p_a = p - p_v$ and $p_a = p - \phi p_g$.
Too,

$$y_v = \frac{p_v}{p} = \phi \left[\frac{p_g}{p}\right] \quad (2)$$

$$y_a = 1 - y_v = 1 - \phi\left[\frac{p_g}{p}\right] \quad (3)$$

At $\phi = 100\%$, $T_0 = 20°C$, $p = 1$ atm, $y_a = y_a^e$, $y_v = y_v^e$.

$$e_f = 0 \begin{cases} y_v^e = \dfrac{p_g}{p} = \dfrac{0.02339 \text{ bar}}{1.01325 \text{ bar}} = 0.0231 \\ y_a^e = 1 - y_v^e = 0.9769 \end{cases}$$

(a) $\phi = 90\%$ $y_v = 0.9(0.0231) = 0.02079$, $y_a = 0.97921$

Eq.(1) gives

$$\bar{e}_f = \left(8.314 \frac{kJ}{kmol(mix)\,K}\right)(293\,K)\left[0.97921 \ln\left[\frac{0.97921}{0.9769}\right] + 0.02079 \ln\left[\frac{0.02079}{0.0231}\right]\right]$$

$$= 0.3 \frac{kJ}{kmol(mix)}$$

For the mixture, $M = y_a M_a + y_v M_v = (0.97921)(28.97) + 0.02079(18.02) = 28.74 \frac{kg(mix)}{kmol(mix)}$

Then, on a mass basis

$$e_f = \frac{\bar{e}_f}{M} = \frac{0.3 \text{ kJ/kmol(mix)}}{28.74 \text{ kg(mix)/kmol(mix)}} = 0.01 \text{ kJ/kg(mix)}$$

(b) $\underline{\phi = 50\%}$. $y_v = 0.5(0.0231) = 0.01155$, $y_a = 0.98845$

$$\bar{e}_f = (8.314)(293)\left[0.98845 \ln\left(\frac{0.98845}{0.9769}\right) + 0.01155 \ln\left(\frac{0.01155}{0.0231}\right)\right]$$

$$= 8.8 \frac{kJ}{kmol(mix)}$$

.011754

$$M = y_a M_a + y_v M_v = (0.98845)(28.97) + 0.01155(18.02) = 28.84 \frac{kg(mix)}{kmol(mix)}$$

$$\therefore \quad e_f = \frac{\bar{e}_f}{M} = \frac{8.8\, kJ/kmol(mix)}{28.84\, kg(mix)/kmol(mix)} = 0.31 \frac{kJ}{kg(mix)}$$

(c) $\underline{\phi = 10\%}$ $y_v = 0.1(0.0231) = 0.00231$, $y_a = 0.99769$

$$\bar{e}_f = (8.314)(293)\left[0.99769 \ln\left(\frac{0.99769}{0.9769}\right) + 0.00231 \ln\left(\frac{0.00231}{0.0231}\right)\right]$$

$$= 38.2 \frac{kJ}{kmol(mix)}$$

$$M = y_a M_a + y_v M_v = (0.99769)(28.97) + 0.00231(18.02) = 28.94$$

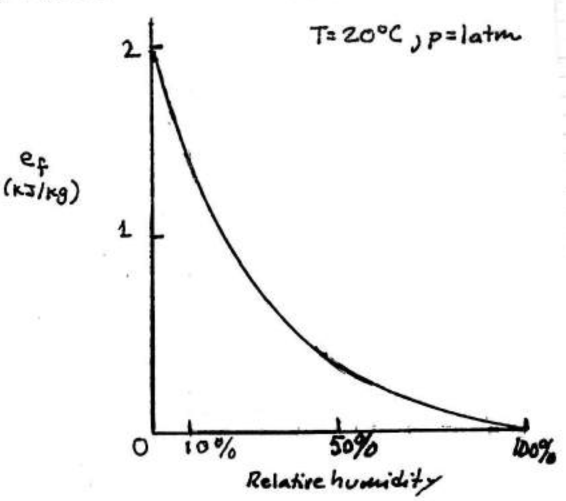
$$e_f = \frac{\bar{e}_f}{M} = \frac{38.2}{28.94} = 1.32 \frac{kJ}{kg(mix)}$$

(d) As $\phi \to 0\%$, Eqs (2) and (3) give $y_v = 0$, $y_a = 1$, and Eq. (1) gives (with $M = 28.97$)

① $$e_f = RT_0 \ln\left(\frac{1}{y_a^e}\right) = \left(\frac{8.314}{28.97}\right)(293) \ln\left(\frac{1}{0.9769}\right) = 1.97 \frac{kJ}{kg(da)}$$

---

1. Collecting results



PROBLEM 14.1

KNOWN: The reaction is

$$CH_4(g) + 2\,O_2 \rightleftarrows CO_2 + 2\,H_2O(g)$$

FIND: Determine the change in Gibbs function ΔG° at 25°C using (a) Gibbs function of formation data, (b) enthalpy of formation and absolute entropy data.

ANALYSIS: ΔG° is given by

$$\Delta G^\circ = \bar{g}^\circ_{CO_2} + 2\,\bar{g}^\circ_{H_2O(g)} - \bar{g}^\circ_{CH_4(g)} - 2\,\bar{g}^\circ_{O_2} \qquad (1)$$

(a) With data from Table A-25

$$\Delta G^\circ = [(-394{,}380) + 2(-228{,}590) - (-50{,}790) - 2(0)]\ \text{kJ/kmol}$$

$$= -800{,}770\ \text{kJ/kmol} \longleftarrow (a)$$

(b) Using $\bar{g} = \bar{h} - T\bar{s}$ Eq. (1) becomes

$$\Delta G^\circ = [\bar{h} - T\bar{s}]^\circ_{CO_2} + 2[\bar{h} - T\bar{s}]^\circ_{H_2O(g)} - [\bar{h} - T\bar{s}]^\circ_{CH_4(g)} - 2[\bar{h} - T\bar{s}^\circ]_{O_2}$$

$$= \left[\bar{h}^\circ_{CO_2} + 2\bar{h}^\circ_{H_2O(g)} - \bar{h}^\circ_{CH_4(g)} - 2\bar{h}^\circ_{O_2}\right] - (298.15\,\text{K})\left[\bar{s}^\circ_{CO_2} + 2\,\bar{s}^\circ_{H_2O(g)} - \bar{s}^\circ_{CH_4(g)} - 2\bar{s}^\circ_{O_2}\right]$$

With data from Table A-25

$$\Delta G^\circ = [(-393{,}520) + 2(-241{,}820) - (-74{,}850) - 2(0)]$$
$$-(298.15)(213.69 + 2(188.72) - 186.16 - 2(205.03))$$

① $$= -800{,}792\ \text{kJ/kmol} \longleftarrow (b)$$

---

1. The slight difference in values calculated results from round-off in data values listed in Table A-25.

PROBLEM 14.2

KNOWN: The reaction is

$$CO_2 \rightleftarrows CO + \tfrac{1}{2} O_2$$

FIND: Determine $\log_{10} K$ at (a) 500 K (b) 1800°R

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS: From Eq. 14.31 $\ln K = -\Delta G^\circ / \bar{R}T$ where

$$\Delta G^\circ = \bar{g}^\circ_{CO} + \tfrac{1}{2}\bar{g}^\circ_{O_2} - \bar{g}^\circ_{CO_2}$$

with $\bar{g} = \bar{h} - T\bar{s}$ where $\bar{h} = \bar{h}^\circ_f + \Delta\bar{h}$

$$\Delta G^\circ = \{[\bar{h}^\circ_f + \Delta\bar{h}]_{CO} + \tfrac{1}{2}[\bar{h}^\circ_f + \Delta\bar{h}]_{O_2} - [\bar{h}^\circ_f + \Delta\bar{h}]_{CO_2}\} - T[\bar{s}^\circ_{CO} + \tfrac{1}{2}\bar{s}^\circ_{O_2} - \bar{s}^\circ_{CO_2}] \quad (1)$$

(a) $\underline{T = 500K}$ With data from ideal gas tables

$$\Delta G^\circ = \{[(-110,530) + 14,600 - 8669] + \tfrac{1}{2}[0 + 14,770 - 8682] - [(-393,520) + 17,678 - 9364]$$
$$- 500[212.719 + \tfrac{1}{2}(220.589) - 254.814]\} = 239,551 \text{ kJ/kmol}$$

Then

$$\ln K = -239551/(8.314)(500) \Rightarrow \log_{10} K = -25.027$$

Table Value: $\log_{10} K = -25.025$

(b) $\underline{T = 1800°R = 1000K}$. With data from ideal gas tables

$$\Delta G^\circ = \{[(-110,530) + [30,355 - 8669]] + \tfrac{1}{2}[0 + [31,389 - 8682]] - [(-393,520) + [42,769 - 9364]]\}$$
$$- 1000[234.421 + \tfrac{1}{2}(243.471) - 269.215] = 195,683 \text{ kJ/kmol}$$

Then

$$\ln K = \frac{-195,683}{(8.314)(1000)} \Rightarrow \log_{10} K = -10.222$$

Table Value: $\log_{10} K = -10.221$.

PROBLEM 14.3

KNOWN: The reaction is

$$CO + H_2O(g) \rightleftarrows CO_2 + H_2 \quad (1)$$

FIND: Determine $\log_{10} K$ at (a) 298 K, (b) 1000 K. Compare with Table A-27 data.

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS: From Eq. 14.31 $\ln K = -\Delta G°/\bar{R}T$ where

$$\Delta G° = \bar{g}°_{CO_2} + \bar{g}°_{H_2} - \bar{g}°_{CO} - \bar{g}°_{H_2O(g)}$$

(a) 298 K With data from Table A-25

$$\Delta G° = (-394{,}380) + (0) - (-137{,}150) - (-228{,}590)$$

$$= -28{,}640 \text{ kJ/kmol}$$

Then

$$\ln K = \frac{28{,}640}{(8.314)(298.15)} \Rightarrow \log_{10} K = 5.018$$

Table A-27 gives $\log_{10} K$ for the reaction

$$CO_2 + H_2 \rightleftarrows CO + H_2O(g)$$

as $\log_{10} K = -5.018$, which in light of Eq. 14.34 and the accompanying discussion is in agreement with the calculated value.

(b) 1000 K With data from the ideal gas tables

1 $$\Delta G° = \left[[\bar{h}°_f + \Delta\bar{h}]_{CO_2} + [\bar{h}°_f + \Delta\bar{h}]_{H_2} - [\bar{h}°_f + \Delta\bar{h}]_{CO} - [\bar{h}°_f + \Delta\bar{h}]_{H_2O(g)} - T[\bar{s}°_{CO_2} + \bar{s}°_{H_2} - \bar{s}°_{CO} - \bar{s}°_{H_2O(g)}]\right]$$

$$= \left[[-393{,}520 + 42{,}769 - 9364] + [0 + 29{,}154 - 8468] - [-110\,530 + 30{,}355 - 8669] - [-241{,}820 + 35882 - 9904] - 1000[269.215 + 166.114 - 234.421 - 232.597] \right]$$

$$= -3054 \text{ kJ/kmol}$$

Then

$$\ln K = \frac{3054}{(8.314)(1000)} \Rightarrow \log_{10} K = 0.1595$$

Table A-27 at 1000 K for $CO_2 + H_2 \rightleftarrows CO + H_2O$, $\log_{10} K = -0.159$.
Thus, for $CO + H_2O \rightleftarrows CO_2 + H_2$, $\log_{10} K = +0.159$.

---

1. IT can be used alternatively.

PROBLEM 14.4

KNOWN: The reaction is

$$H_2O \rightleftarrows H_2 + \tfrac{1}{2} O_2$$

FIND: Calculate $\log_{10} K$ at (a) 298 K, (b) 3600°R and compare with values from Table A-27.

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS: From Eq. 14.31 $\ln K = -\Delta G^\circ / \bar{R} T$ where

$$\Delta G^\circ = \bar{g}^\circ_{H_2} + \tfrac{1}{2} \bar{g}^\circ_{O_2} - \bar{g}^\circ_{H_2O(g)} \qquad (1)$$

(a) At 298.15 K, Table A-25 data give

$$\Delta G^\circ = 0 + \tfrac{1}{2}(0) - (-228{,}590) = 228{,}590 \text{ kJ/kmol}$$

Then

$$\ln K = \frac{-228\,590}{(8.314)(298.15)} \Rightarrow \log_{10} K = -40.05 \qquad (a)$$

Table A-27 gives $\log_{10} K = -40.048$.

(b) At 3600°R = 2000 K

$$\Delta G^\circ = \{(\bar{h}^\circ_f + \Delta\bar{h})_{H_2} + \tfrac{1}{2}(\bar{h}^\circ_f + \Delta\bar{h})_{O_2} - (\bar{h}^\circ_f + \Delta\bar{h})_{H_2O(g)}\} - T\,[\bar{s}^\circ_{H_2} + \tfrac{1}{2}\bar{s}^\circ_{O_2} - \bar{s}^\circ_{H_2O(g)}]$$

Then, with ideal gas table data (or using IT)

$$\Delta G^\circ = \{[0 + (61{,}400 - 8468)] + \tfrac{1}{2}[0 + (67{,}881 - 8682)] - [-241{,}820 + (82{,}593 - 9904)]\} - 2000\,[188.297 + \tfrac{1}{2}(268.655) - 264.571]$$

$$= 135{,}555.5 \text{ kJ/kmol}$$

Then

$$\ln K = \frac{-135{,}555.5}{(8.314)(2000)} \Rightarrow \log_{10} K = -3.5405 \qquad (b)$$

which checks the value from Table A-27.

PROBLEM 14.5

KNOWN: Reactions are

(a) $H_2O \rightleftarrows H_2 + \frac{1}{2} O_2$

(b) $H_2 + \frac{1}{2}O_2 \rightleftarrows H_2O$

(c) $2H_2O \rightleftarrows 2H_2 + O_2$

FIND: Using Table A-27, determine $\log_{10} K$ at 2500K

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS: For reaction (a) Table A-27 gives $\log_{10} K = -2.224$. ← (a)

Using Eq. 14.34, the accompany discussion indicates that for reaction (b)

$$\log_{10} K_{(b)} = -\log_{10} K_{(a)} = +2.224. \quad \leftarrow (b)$$

For reaction (c), $(\Delta G°)_{(c)} = 2(\Delta G°)_{(a)}$. Then, with Eq. 14.31

$$\ln K_{(c)} = -\frac{\Delta G°_{(c)}}{\bar{R}T} = -\frac{2(\Delta G°)_{(a)}}{\bar{R}T}$$

or

$$\ln K_{(c)} = 2 \ln K_{(a)} \Rightarrow \log_{10} K_{(c)} = 2 \log_{10} K_{(a)} = -4.448 \quad \leftarrow (c)$$

PROBLEM 14.6

KNOWN: For interpolation in Table A-27, $\log_{10} K$ is nearly linear in $1/T$:

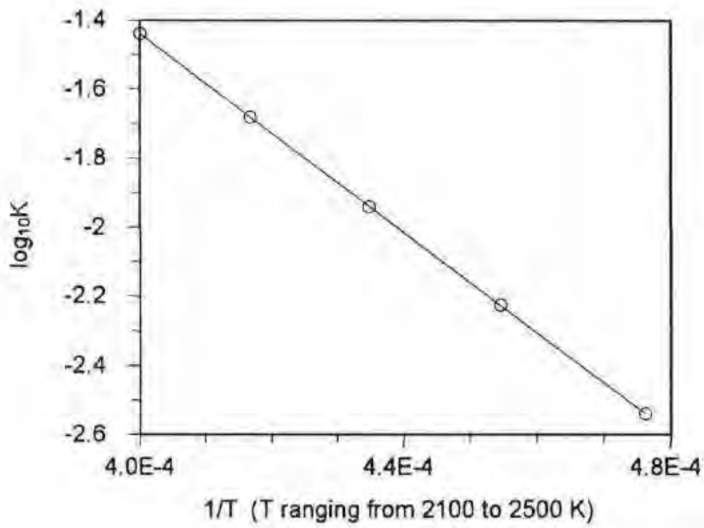
$$\log_{10} K = C_1 + C_2/T$$

where C_1 and C_2 are constants.

FIND: (a) Verify the interpolation rule by plotting $\log_{10}K$ vs $1/T$ for $2000 \le T \le 2500$ K.

(b) Evaluate C_1, C_2 for any pair of adjacent table entries in the temperature interval of (a)

ANALYSIS: (a) For $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$ as an example



(b) The linearity of $\log_{10}K$ vs. $1/T$ suggests a more accurate scheme for interpolating in Table A-27 than simple linear interpolation. First, let us determine C_1 and C_2 based on $T = 2400$ and $T = 2500$K and the corresponding $\log_{10}K$ data for $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$ from Table A-27:

$$\left.\begin{array}{ll} T = 2400\text{K}: & -1.679 = C_1 + C_2/2400 \\ T = 2500\text{K}: & -1.440 = C_1 + C_2/2500 \end{array}\right\} \Rightarrow \begin{array}{l} C_1 = 4.296 \\ C_2 = -14340 \end{array}$$

Thus, $\log_{10}K = 4.296 - 14340/T \quad (2400 \le T \le 2500\text{K})$ ← (b)

Let us use this result to estimate $\log_{10}K$ at $T = 2450$K. That is

$$\log_{10}K = 4.296 - 14340/2450 = -1.5571$$

For comparison, the value obtain by direct linear interpolation is -1.5595, which differs from the curve fit value by about 0.15%.

PROBLEM 14.7

KNOWN: Two alternative ways of expressing the ammonia synthesis reaction:

1. $\frac{1}{2} N_2 + \frac{3}{2} H_2 \rightleftarrows NH_3$
2. $N_2 + 3 H_2 \rightleftarrows 2 NH_3$

FIND: Determine the relationship between the ideal gas equilibrium constants K_1 and K_2.

ENGINEERING MODEL: (1) Ideal gas principles apply. (2) The temperature is T.

ANALYSIS: For the two reactions the corresponding ΔG° expressions are

$$(\Delta G^\circ)_{(1)} = \bar{g}^\circ_{NH_3} - \frac{1}{2}\bar{g}^\circ_{N_2} - \frac{3}{2}\bar{g}^\circ_{H_2}$$

$$(\Delta G^\circ)_{(2)} = 2\bar{g}^\circ_{NH_3} - \bar{g}^\circ_{N_2} - 3\bar{g}^\circ_{H_2}$$

That is,

$$(\Delta G^\circ)_{(2)} = 2(\Delta G^\circ)_{(1)} \qquad (1)$$

Using Eq. 14.31

$$\ln K_{(1)} = -\frac{(\Delta G^\circ)_{(1)}}{\bar{R}T} \qquad (2)$$

$$\ln K_{(2)} = -\frac{(\Delta G^\circ)_{(2)}}{\bar{R}T} \qquad (3)$$

Inserting Eq. (1) into Eq. (3) and using Eq. (2)

$$\ln K_{(2)} = -\frac{2(\Delta G^\circ)_{(1)}}{\bar{R}T}$$

$$= 2 \ln K_{(1)}$$

$$\Rightarrow \qquad K_{(2)} = K_{(1)}^2$$

PROBLEM 14.8

KNOWN: Reactions are specified as

1. $CO + H_2O \rightleftarrows H_2 + CO_2$
2. $2CO_2 \rightleftarrows 2CO + O_2$
3. $2H_2O \rightleftarrows 2H_2 + O_2$

FIND: Show that the equilibrium constants are related by the expression $K_1 = (K_3/K_2)^{1/2}$.

ENGINEERING MODEL: (1) Ideal gas principles apply. (2) The temperature is T.

ANALYSIS: Using Eq. 14.31 the equilibrium constants corresponding to the three reactions have the forms

$$\ln K_1 = -\frac{(\Delta G^\circ)_{(1)}}{\bar{R}T}, \quad \ln K_2 = -\frac{(\Delta G^\circ)_{(2)}}{\bar{R}T}, \quad \ln K_3 = -\frac{(\Delta G^\circ)_{(3)}}{\bar{R}T}$$

where

$$\left\{\begin{aligned} (\Delta G^\circ)_{(1)} &= \bar{g}^\circ_{H_2} + \bar{g}^\circ_{CO_2} - \bar{g}^\circ_{CO} - \bar{g}^\circ_{H_2O} \\ (\Delta G^\circ)_{(2)} &= 2\bar{g}^\circ_{CO} + \bar{g}^\circ_{O_2} - 2\bar{g}^\circ_{CO_2} \\ (\Delta G^\circ)_{(3)} &= 2\bar{g}^\circ_{H_2} + \bar{g}^\circ_{O_2} - 2\bar{g}^\circ_{H_2O} \end{aligned}\right. \qquad (1)$$

From Eqs. (1) it follows that

$$(\Delta G^\circ)_{(1)} = \frac{1}{2}\left[(\Delta G^\circ)_{(3)} - (\Delta G^\circ)_{(2)}\right]$$

Accordingly

$$\frac{(\Delta G^\circ)_{(1)}}{\bar{R}T} = \frac{1}{2}\left[\frac{(\Delta G^\circ)_{(3)}}{\bar{R}T} - \frac{(\Delta G^\circ)_{(2)}}{\bar{R}T}\right]$$

or

$$\ln K_1 = \frac{1}{2}\left[\ln K_3 - \ln K_2\right]$$

Finally

$$K_1 = \left(\frac{K_3}{K_2}\right)^{1/2}$$

PROBLEM 14.9

KNOWN: Reactions are specified as

1. $CO_2 + H_2 \rightleftarrows CO + H_2O$
2. $CO_2 \rightleftarrows CO + \frac{1}{2} O_2$
3. $H_2O \rightleftarrows H_2 + \frac{1}{2} O_2$

FIND: (a) Show that $K_1 = K_2 / K_3$. (b) Use the result from part (a) and data from Table A-27 to evaluate $\log_{10} K_1$ at 298K. (c) Check the result of part (b) using Eq. 14.31 together with $\bar{g}_f^\circ$ data from Table A-25.

ENGINEERING MODEL: (1) Ideal gas principles apply. (2) The temperature is T.

ANALYSIS: (a) For these reactions the respective ΔG° expressions are

$$(\Delta G^\circ)_{(1)} = \bar{g}^\circ_{CO} + \bar{g}^\circ_{H_2O(g)} \; \bar{g}^\circ_{CO_2} - \bar{g}^\circ_{H_2}$$

$$(\Delta G^\circ)_{(2)} = \bar{g}^\circ_{CO} + \frac{1}{2} \bar{g}^\circ_{O_2} - \bar{g}^\circ_{CO_2}$$

$$(\Delta G^\circ)_{(3)} = \bar{g}^\circ_{H_2} + \frac{1}{2} \bar{g}^\circ_{O_2} - \bar{g}^\circ_{H_2O(g)}$$

Thus

$$(\Delta G^\circ)_{(1)} = (\Delta G^\circ)_{(2)} - (\Delta G^\circ)_{(3)}$$

$$\frac{(\Delta G^\circ)_{(1)}}{\bar{R}T} = \frac{(\Delta G^\circ)_{(2)}}{\bar{R}T} - \frac{(\Delta G^\circ)_{(3)}}{\bar{R}T} \Rightarrow -\frac{(\Delta G^\circ)_{(1)}}{\bar{R}T} = -\left[\frac{\Delta G^\circ_{(2)}}{\bar{R}T} - \frac{(\Delta G^\circ)_{(3)}}{\bar{R}T} \right]$$

With Eq. 14.31

$$\ln K_1 = \ln K_2 - \ln K_3 \Rightarrow K_1 = \frac{K_2}{K_3} \quad \text{(a)}$$

(b) From part (a)

$$\log_{10} K_1 = \log_{10} K_2 - \log_{10} K_3$$

With values from Table A-27 at 298 K for reactions 2 and 3

$$\log_{10} K_1 = (-45.066) - (-40.048) = -5.018 \quad \text{(b)}$$

(c) With $\bar{g}_f^\circ$ data from Table A-25.

$$(\Delta G^\circ)_{(1)} = (-137,150) + (-228,590) - (-394,380) - (0)$$

$$= 28,640 \text{ kJ/kmol}$$

Then

$$\ln K_1 = -\frac{(\Delta G^\circ)_{(1)}}{\bar{R}T} = \frac{-28,640}{(8.314)(298.15)} \Rightarrow \log_{10} K_1 = -5.018 \quad \text{(c)}$$

PROBLEM 14.10

KNOWN: At 2000 K

(1) $C + \frac{1}{2} O_2 \rightleftarrows CO$, $\log_{10} K_1 = 7.469$

(2) $C + 2H_2 \rightleftarrows CH_4$, $\log_{10} K_2 = -3.408$

FIND: Evaluate K at 2000 K for

(3) $CH_4 + H_2O \rightleftarrows 3H_2 + CO$ $\left(\Delta G^\circ = 3\bar{g}^\circ_{H_2} + \bar{g}^\circ_{CO} - \bar{g}^\circ_{CH_4} - \bar{g}^\circ_{H_2O}\right)$

ENGINEERING MODEL: The principles of Sec. 14.3 apply.

ANALYSIS: For (1), (2)

(1)′ $(\Delta G^\circ)_{(1)} = \bar{g}^\circ_{CO} - \bar{g}^\circ_{C} - \frac{1}{2}\bar{g}^\circ_{O_2}$

(2)′ $(\Delta G^\circ)_{(2)} = \bar{g}^\circ_{CH_4} - \bar{g}^\circ_{C} - 2\bar{g}^\circ_{H_2}$

Also, for the following reaction Table A-27 gives

(4) $H_2O \rightleftarrows H_2 + \frac{1}{2} O_2$, $\log_{10} K_4 = -3.54$

and for (4)

(4)′ $(\Delta G^\circ)_{(4)} = \bar{g}^\circ_{H_2} + \frac{1}{2}\bar{g}^\circ_{O_2} - \bar{g}^\circ_{H_2O}$

Subtracting (2)′ from (1)′

$$(\Delta G^\circ)_{(1)} - (\Delta G^\circ)_{(2)} = \bar{g}^\circ_{CO} - \frac{1}{2}\bar{g}^\circ_{O_2} - \bar{g}^\circ_{CH_4} + 2\bar{g}^\circ_{H_2}$$

adding (4)′

$$(\Delta G^\circ)_{(1)} - (\Delta G^\circ)_{(2)} + (\Delta G^\circ)_{(4)} = \underbrace{\bar{g}^\circ_{CO} + 3\bar{g}^\circ_{H_2} - \bar{g}^\circ_{CH_4} - \bar{g}^\circ_{H_2O}}_{= \Delta G^\circ_{(3)}}$$

Accordingly,

$$\frac{\Delta G^\circ_{(3)}}{\bar{R}T} = \frac{\Delta G^\circ_{(1)}}{\bar{R}T} - \frac{\Delta G^\circ_{(2)}}{\bar{R}T} + \frac{\Delta G^\circ_{(4)}}{\bar{R}T}$$

$$\Rightarrow \quad \ln K_3 = \ln K_1 - \ln K_2 + \ln K_4$$

or

$$\log_{10} K_3 = \log_{10} K_1 - \log_{10} K_2 + \log_{10} K_4$$

$$= 7.469 - (-3.408) + (-3.54) = 7.337$$

$$\Rightarrow \quad K_3 = 2.173 \times 10^7 \longleftarrow$$

PROBLEM 14.11

KNOWN (a) One kmol of N_2O_4 dissociates to form an equilibrium mixture of N_2O_4 and NO_2 at 25°C, 2atm. For $N_2O_4 \rightleftarrows 2NO_2$, $\Delta G° = 5400$ kJ/kmol at 25°C.

(b) One kmol of CH_4 dissociates to form an equilibrium mixture at 1000K, 5 atm. For $C + 2H_2 \rightleftarrows CH_4$, $\log_{10} K = 1.011$ at 1000 K.

FIND: In each case determine the equilibrium composition.

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS: (a) Applying conservation of mass

$1 N_2O_4 \rightarrow (1-x) N_2O_4 + y NO_2 \Rightarrow N: 2(1-x) + y = 2$, or $y = 2x$ where $(1-x)$ is the amount of N_2O_4 present in the mixture. The total number of moles of mixture is $n = (1-x) + 2x = 1 + x$. At equilibrium $N_2O_4 \rightleftarrows 2NO_2$, so Eq. 14.35 takes the form

$$K = \left[\frac{2x}{1-x}\right]^2 \left[\frac{P/P_{ref}}{1+x}\right]^{2-1}$$

$$= \frac{2[2x]^2}{(1-x)(1+x)} = \frac{8x^2}{1-x^2} \qquad (1)$$

Using Eq. 14.31 and the known $\Delta G°$ value

$$\ln K = -\frac{5400}{(8.314)(298.15)} \Rightarrow K = 0.11322 \qquad (2)$$

Combining Eqs. (1) and (2)

$$\frac{8x^2}{1-x^2} = 0.11322$$

$$\Rightarrow \quad 8.11322\,x^2 = 0.11322 \quad \Rightarrow x = 0.118$$

Finally, the equilibrium mixture is $\{0.882\ N_2O_4, 0.236\ NO_2\}$ ← (a)

(b) Mass balance: $1 CH_4 \rightarrow (1-x) CH_4 + 2x H_2 + x C$, where $n = 1 + 2x$. Then Eq 14.35 reads for $C + 2H_2 \rightleftarrows CH_4$

$$K = \frac{(1-x)}{(x)(2x)^2}\left(\frac{P/P_{ref}}{n}\right)^{1-1-2} = \frac{(1-x)}{(x)(2x)^2}\left(\frac{5}{1+2x}\right)^{-2}$$

or

$$K = \frac{(1-x)}{x}\left(\frac{1+2x}{2x}\right)^2 \frac{1}{25} \qquad \text{where } \log_{10} K = 1.011$$

$$\Rightarrow \quad 256.413 = \left(\frac{1-x}{x}\right)\left(\frac{1+2x}{2x}\right)^2$$

Using an equation solver or iteration with a hand calculator, $x = 0.1088$, and the equilibrium mixture is

$\{0.8912\ CH_4, 0.2176\ H_2, 0.1088\ C\}$ ← (b)

PROBLEM 14.12

KNOWN (a) One lbmol of H_2O dissociates to form an equilibrium mixture of H_2O, H_2, O_2 at 5200°R, 1.25 atm.
(b) One lbmol of CO_2 dissociates to form an equilibrium mixture of CO_2, CO, O_2 at 5200°R, 1.25 atm.

FIND: For each case determine the extent to which dissociation occurs.

ENGINEERING MODEL: Ideal gas mixture principles apply.

(a) $$1\,H_2O \rightarrow (1-x)\,H_2O + y\,H_2 + z\,O_2$$

$$H:\quad 2 = 2(1-x) + 2y \Rightarrow y = x$$

$$O:\quad 1 = (1-x) + 2z \Rightarrow z = x/2$$

Thus

$$1\,H_2O \rightarrow (1-x)\,H_2O + x\,H_2 + x/2\,O_2$$

where $1-x$ is the amount of H_2O, in kmol, present in the mixture. The amount of mixture is $n = (1-x) + x + x/2 = (2+x)/2$.

At equilibrium $H_2O \rightleftarrows H_2 + 1/2\,O_2$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[x][x/2]^{\frac{1}{2}}}{[1-x]}\left[\frac{p/p_{ref}}{(2+x)/2}\right]^{1+\frac{1}{2}-1}$$

$$= \left(\frac{x}{1-x}\right)\left(\frac{x}{2+x}\right)^{1/2}\left(p/p_{ref}\right)^{1/2}$$

or

$$K^2 = \left(\frac{x}{1-x}\right)^2\left(\frac{x}{2+x}\right)\left(p/p_{ref}\right) \qquad (1)$$

① At 5200°R, Table A-27 gives for $H_2O \rightleftarrows H_2 + \frac{1}{2}O_2$, $\log_{10}K = -1.51311$ or $K = 0.030682$. Thus, Eq.(1) becomes with $p/p_{ref} = 1.25$

$$\Rightarrow \quad 7.5313\times10^{-4} = \left(\frac{x}{1-x}\right)^2\left(\frac{x}{2+x}\right).$$ Using an equation solver or iteration with a hand calculator, $x = 0.108$. Thus, the extent of dissociation is 0.892. ← (a)

(b) $$CO_2 \rightarrow (1-x)\,CO_2 + y\,CO + z\,O_2$$

$$C:\quad 1 = (1-x) + y \Rightarrow y = x$$

$$O:\quad 2 = 2(1-x) + y + 2z \Rightarrow z = x/2$$

Thus

$$CO_2 \rightarrow (1-x)\,CO_2 + x\,CO + \tfrac{x}{2}\,O_2$$

where $1-x$ is the amount of CO_2, in kmol, present in the mixture. The number of moles of mixture is $n = (1-x) + x + \frac{x}{2} = (2+x)/2$

At equilibrium, $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[x][\frac{x}{2}]^{\frac{1}{2}}}{[1-x]}\left[\frac{p/p_{ref}}{(2+x)/2}\right]^{1+\frac{1}{2}-1}$$

$$= \left(\frac{x}{1-x}\right)\left(\frac{x}{2+x}\right)^{1/2}\left(p/p_{ref}\right)^{1/2} \Rightarrow K^2 = \left(\frac{x}{1-x}\right)^2\left(\frac{x}{2+x}\right)\left(p/p_{ref}\right) \qquad (1)$$

At 5200°R, Table A-27 gives for $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$, $\log_{10}K = -0.668556$. Thus Eq.(1) gives

① $$3.6811\times10^{-2} = \left(\frac{x}{1-x}\right)^2\left(\frac{x}{2+x}\right).$$ Using an equation solver or iteration with a hand calculator, $x = 0.336$. The extent of dissociation is 0.664. ← (b)

---

1. $\log_{10}K$ is obtained from Table A-27 using linear interpolation. Alternatively, the interpolation procedure of Problem 14.6 can be used.

PROBLEM 14.13

KNOWN: One lbmol of C reacts with 2 lbmol of O_2 to form an equilibrium mixture of CO_2, CO, and O_2 at 5400 °R, 1 atm.

FIND: Determine the equilibrium composition.

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS: The reaction takes the form

$$C + 2O_2 \rightarrow x\,CO + y\,CO_2 + z\,O_2$$

Balancing

$$C: \quad 1 = x + y \Rightarrow y = 1 - x$$

$$O: \quad 4 = x + 2y + 2z \Rightarrow 4 = x + 2(1-x) + 2z \Rightarrow z = 1 + \frac{x}{2} = \frac{2+x}{2}$$

Thus

$$C + 2O_2 \rightarrow x\,CO + (1-x)CO_2 + \left(1 + \frac{x}{2}\right)O_2$$

The amount of mixture, in lbmol, is $n = x + (1-x) + \left(1 + \frac{x}{2}\right) = (4+x)/2$.
At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[x]\left[\frac{2+x}{2}\right]^{1/2}}{[1-x]}\left[\frac{P/P_{ref}}{(4+x)/2}\right]^{1+1/2-1}$$

$$= \frac{x}{1-x}\left[\frac{2+x}{4+x}\right]^{1/2}$$

At 5400°R, Table A-27 gives $\log_{10} K = -0.485 \Rightarrow K = 0.32734$. Thus

$$0.32734 = \frac{x}{1-x}\left[\frac{2+x}{4+x}\right]^{1/2}$$

Using an equation solver or iteration with a hand calculator, $x = 0.309$.
The equilibrium mixture is

$$\{0.309\,CO,\ 0.691\,CO_2,\ 1.1545\,O_2\}$$

PROBLEM 14.14

KNOWN: Three cases are provided involving the formation of oxides of nitrogen.

FIND: Provide information about the equilibrium mixture in each case.

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS: (a) One kmol of N_2O_4 dissociates at 25°C, 1atm to form an equilibrium mixture of $\{N_2O_4, NO_2\}$ in which the amount of N_2O_4 present is 0.8154. Determine the amount of N_2O_4 that would be present in an equilibrium mixture at 25°C, 0.5 atm.

$$N_2O_4 \rightarrow (1-x)N_2O_4 + 2x\,NO_2$$

The amount of mixture is $n = (1-x) + 2x = 1+x$.

At equilibrium $N_2O_4 \rightleftarrows 2NO_2$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[2x]^2}{[1-x]}\left[\frac{P/P_{ref}}{1+x}\right]^{2-1}$$

$$K = \frac{4x^2}{1-x^2}\left(P/P_{ref}\right)$$

As K depends on T which is 25°C in both cases, $K_1 = K_2$ or

$$\frac{4x_1^2}{1-x_1^2}\left[\frac{P_1}{P_{ref}}\right] = \frac{4x_2^2}{1-x_2^2}\left[\frac{P_2}{P_{ref}}\right]$$

where 1 denotes the case $P_1 = 1$atm and 2 denotes the case $P_2 = 0.5$. Thus

$$\frac{x_1^2}{1-x_1^2} = \frac{0.5x_2^2}{1-x_2^2}$$

From the given information, $1-x_1 = 0.8154 \Rightarrow x_1 = 0.1846$. Accordingly

$$\frac{x_2^2}{1-x_2^2} = 0.070559$$. Solving this quadratic equation, $x_2 = 0.2567$.

The amount of N_2O_4 present is then 0.7433 kmoles. ⟵ (a)

(b) 1 kmol NO, 10 kmol O_2, 40 kmol N_2 react to form an equilibrium mixture of $\{NO_2, NO, O_2\}$ at 500K, 0.1atm. If for $NO + \frac{1}{2}O_2 \rightleftarrows NO_2$ K=120 at 500 K, determine the equilibrium mixture composition. The reaction is described by

$$1\,NO + 10\,O_2 + 40N_2 \rightarrow (1-x)\,NO + x\,NO_2 + (10-0.5x)\,O_2 + 40\,N_2$$

where $n = 51 - 0.5x$. Equation 14.35 takes the form

$$K = \frac{x}{(1-x)[10-0.5x]^{1/2}}\left[\frac{P/P_{ref}}{51-0.5x}\right]^{1-1-1/2} = \left(\frac{x}{1-x}\right)\left[\frac{51-0.5x}{10-0.5x}\right]^{1/2}\left(\frac{1}{0.1}\right)^{1/2}$$

The value of K is given as 120. So

$$37.9473 = \left(\frac{x}{1-x}\right)\left(\frac{51-0.5x}{10-0.5x}\right)^{1/2}$$. Using an equation solver or iteration

with a hand calculator, $x = 0.943$. The equilibrium mixture is $\{0.057\,NO, 0.943\,NO_2, 9.5285\,O_2, 40\,N_2\}$

In this analysis, N_2 has been regarded as inert. If N_2 also dissociated, the procedures of Section 14.4.4 would be applied.

PROBLEM 14.14 (Continued)-Page2

(c) An equimolar mixture of O_2 and N_2 reacts to form an equilibrium mixture of O_2, N_2, NO. Plot the mole fraction of NO versus T ranging from 1200 to 2000 K.

The reaction is described by

$$1\,N_2 + 1\,O_2 \rightarrow 2x\,NO + (1-x)N_2 + (1-x)\,O_2$$

The amount of mixture is $n = 2x + (1-x) + (1-x) = 2$.

Thus, the mole fraction of NO in the equilibrium mixture is

$$y_{NO} = \frac{2x}{2} = x$$

For $\frac{1}{2}N_2 + \frac{1}{2}O_2 \rightleftarrows NO$, Equation 14.35 takes the form

$$K = \frac{2x}{[(1-x)(1-x)]^{1/2}}\left(\frac{p/p_{ref}}{n}\right)^{1-\frac{1}{2}-\frac{1}{2}} = \frac{2x}{(1-x)}$$

Here the term $\left(\frac{p/p_{ref}}{n}\right)$ drops out of the evaluation since $1-\frac{1}{2}-\frac{1}{2}=0$.

Solving,

$$x = \frac{K(T)}{2+K(T)}$$

<u>Sample calculation</u>: $T = 1200$ K. From Table A-27, $\log_{10}K = -3.275 \Rightarrow K = 0.0005309$ and $x = 2.654 \times 10^{-4}$.

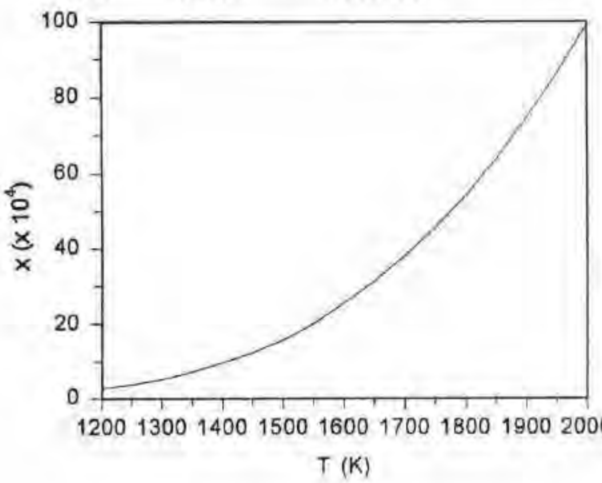
To obtain data for the required plot, use IT, as follows:

IT Code

```
T = 1200 // K
p = 1 // atm

// 1 N2 + 1 O2 ===> 2x NO + (1 - x) N2 + (1 - x) O2
nNO = 2 * x
nN2 = 1 - x
nO2 = 1 - x
ntot = 2 * x + 2 * (1 - x)
yNO = nNO / ntot
yN2 = nN2 / ntot
yO2 = nO2 / ntot
pref = 1 // atm

// For the reaction 1/2 N2 + 1/2 O2 <===> NO
K = (yNO / (yN2 * yO2)^0.5) * (p / pref)^0
// Data from Table A-27 are stored in EQNO.LUT.
log(K) = LOOKUPVAL(EQNO,1,T,3)
```

IT Results

T (K)	K	x
1200	.0005309	0.0002654
1300	.001015	0.0005073
1400	.001941	0.0009695
1500	.003155	0.001575
1600	.005129	0.002558
1700	.007656	0.003813
1800	.01091	0.005428
1900	.01503	0.007460
2000	.02000	0.009900



Note that at T=1200K, very little NO is formed, whereas at T=2000K, nearly all of the N_2 has combined with oxygen to form NO.

PROBLEM 14.15

KNOWN: One kmol of CO dissociates to form an equilibrium mixture of $\{CO_2, CO, O_2\}$ at T, p

FIND: (a) For $T = 3000K$, plot the amount of CO present, in kmol, versus pressure for $1 \le p \le 10$ atm.
(b) For $p = 1$atm, plot the amount of CO present, in kmol, versus temperature for $2000 \le T \le 3500K$

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS: The reaction takes the form

$$1\,CO_2 \rightarrow x\,CO + (1-x)\,CO_2 + \frac{x}{2}\,O_2$$

The amount of mixture is $n = x + (1-x) + \frac{x}{2} = 1 + \frac{x}{2}$

At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$, so

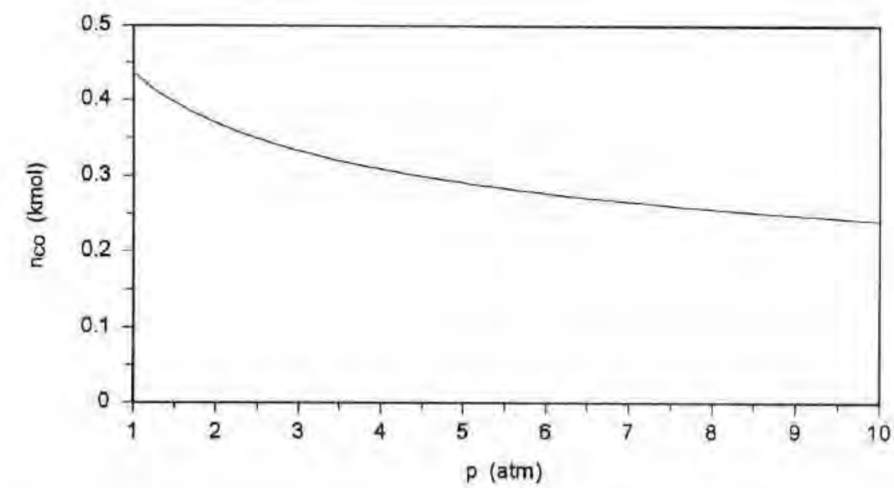
① $$K = \frac{x\,(x/2)^{1/2}}{1-x}\left[\frac{p/p_{ref}}{1+x/2}\right]^{1+\frac{1}{2}-1} = \frac{x}{1-x}\left[\frac{x}{2+x}\right]^{1/2}(p/p_{ref})^{1/2} \qquad (1)$$

(a) $T = 3000K$. Table A-27 gives $\log_{10}K = -0.485$ or $K = 0.32734$. Then Eq.(1) becomes

$$\frac{x}{1-x}\left[\frac{x}{2+x}\right]^{1/2} = \frac{0.32734}{[p/p_{ref}]^{1/2}}$$

Solving and plotting, we get

p(atm)	n_{CO}
1.000	0.436
2.000	0.370
3.000	0.333
4.000	0.309
5.000	0.291
6.000	0.277
7.000	0.265
8.000	0.256
9.000	0.247
10.000	0.240



Thus, at fixed temperature the amount of CO in the equilibrium mixture decreases as pressure increases.

PROBLEM 14.15 (Cont'd.) - Page 2

(b) <u>p = 1 atm</u>. Eq. (1) becomes $\frac{x}{1-x}\left[\frac{x}{2+x}\right]^{1/2} = K(T)$.

<u>Sample calculation</u>. From Table A-27 at T = 3000 K, $\log_{10} K = -0.485$. Thus $K = 0.32734$. Solving, $x = 0.4362$.

The data for the required plot are obtained using IT, as follows:

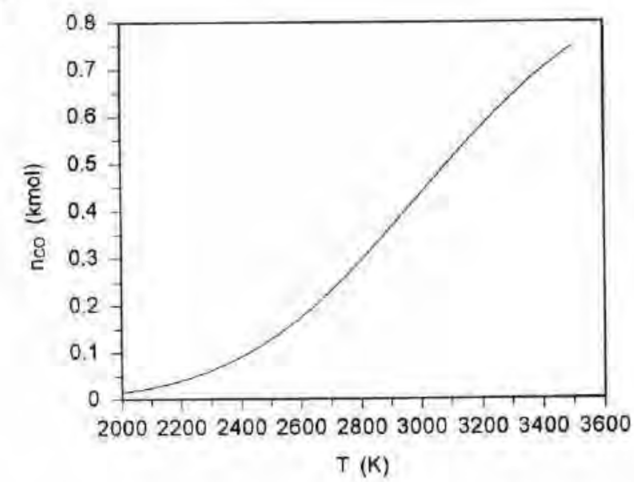
IT Code

```
T = 3000  // K
p = 1  // atm

// CO2 ===> x CO + (1 - x) CO2 + x/2 O2
nCO = x
nCO2 = 1 - x
nO2 = x / 2
ntot = x + (1 - x) + x / 2
yCO = nCO / ntot
yCO2 = nCO2 / ntot
yO2 = nO2 / ntot
pref = 1  // atm

K = ((yCO * yO2^0.5) / yCO2) * (p / pref)^0.5
// Data from Table A-27 are stored in
EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,1,T,3)
```

IT Results

T(K)	n_{CO} (kmol)
2000	0.01494
2100	0.02524
2200	0.04048
2300	0.06207
2400	0.09116
2500	0.1287
2600	0.1754
2700	0.2309
2800	0.2946
2900	0.3638
3000	0.4362
3100	0.5084
3200	0.5775
3300	0.6418
3400	0.6983
3500	0.7480

PLOT:



When pressure is held constant, the amount of CO present at equilibrium increases greatly with T over the range of values in this problem.

---

1. Eq. (1) can be written as

$$\underbrace{\frac{K^2}{P/P_{ref}}}_{\equiv \alpha} = \left(\frac{x}{1-x}\right)^2\left(\frac{x}{2+x}\right) \Rightarrow x^3 + 3\underbrace{\left[\frac{\alpha}{1-\alpha}\right]}_{\equiv r} x - 2\left[\frac{\alpha}{1-\alpha}\right] = 0$$

The real root of this cubic equation is

$$x = \left[r + r[1+r]^{1/2}\right]^{1/3} + \left[r - r[1+r]^{1/2}\right]^{1/3} \quad \text{where } r \equiv \frac{\alpha}{1-\alpha}$$

PROBLEM 14.16

KNOWN: 1 lbmol of H_2O dissociates to form an equilibrium mixture of H_2O, H_2, O_2 at T, p.

FIND: (a) For $T = 5400°R$, plot the amount of H_2 present in the mixture versus pressure for $1 \le p \le 10$ atm.

(b) For $p = 1$ atm, plot the amount of H_2 present versus Temperature, $3600°R \le T \le 6300°R$.

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS:

The reaction takes the form

$$1\,H_2O \rightarrow (1-x)H_2O + x\,H_2 + x/2\,O_2$$

The amount of mixture is $n = (1-x) + x + x/2 = (2+x)/2$.

At equilibrium $H_2O \rightleftarrows H_2 + \frac{1}{2}O_2$. Then Eq. 14.35 takes the form

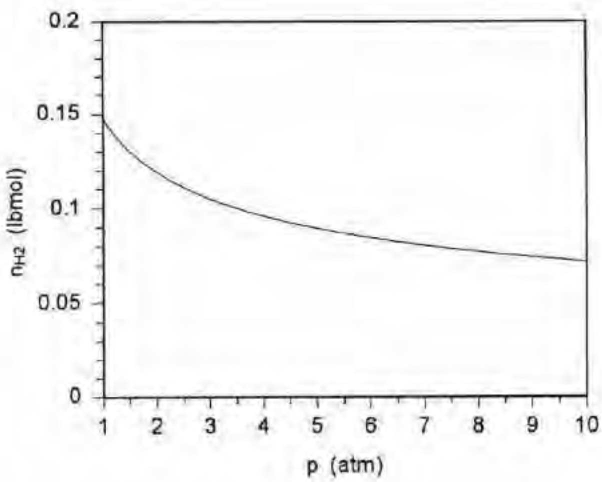
$$① \qquad K = \frac{x\,(x/2)^{1/2}}{(1-x)}\left[\frac{p/p_{ref}}{(2+x)/2}\right]^{1+1/2-1} = \left(\frac{x}{1-x}\right)\left[\frac{x}{2+x}\right]^{1/2}\left[p/p_{ref}\right]^{1/2} \qquad (1)$$

(a) $T = 5400°R$. Table A-27 gives $\log_{10} K = -1.343$, $K = 0.04539$. So, Eq (1) becomes

$$\left(\frac{x}{1-x}\right)\left(\frac{x}{2+x}\right)^{1/2} = \frac{0.04539}{[p/p_{ref}]^{1/2}}$$

Solving and plotting

p (atm)	n_{H2} (lbmol)
1	0.1476
2	0.1192
3	0.1050
4	0.09592
5	0.08938
6	0.08435
7	0.08031
8	0.07696
9	0.07411
10	0.07165



Thus, at fixed temperature the amount of H_2 in the equilibrium mixture decreases as pressure increases.

1. See note 1 of the solution to problem 14.15

PROBLEM 14.16 (Continued)-Page 2

(b) $\underline{p = 1\text{ atm}}$

Equation (1) takes the form

$$K(T) = \frac{x}{1-x}\left[\frac{x}{2+x}\right]^{1/2}$$

Sample calculation. From Table A-27 at T = 5400 °R, $\log_{10} K = -1.343 \Rightarrow K = 0.04539$.
Solving, x = 0.1476.

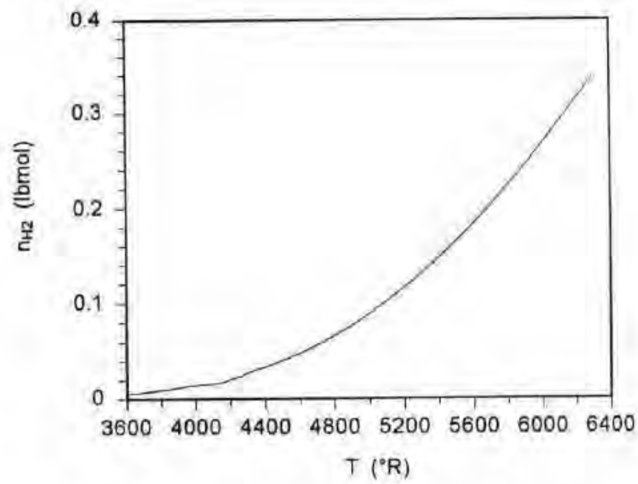
The data for the required plot are obtained using IT, as follows:

IT Code

```
T = 5400 // °R
p = 1 // atm

// H2O ↔ (1 - x) H2O + x H2 + x/2 O2
nH2O = 1 - x
nH2 = x
nO2 = x / 2
ntot = (1 - x) + x + x/2
yH2O = nH2O / ntot
yH2 = nH2 / ntot
yO2 = nO2 / ntot
pref = 1 // atm

// For the reaction H2O ↔ H2 + 1/2 O2
K = ((yH2 * yO2^0.5) / yH2O) * (p/pref)^0.5
//Data from Table A-27 are stored in EQH2O.LUT.
log(K) = LOOKUPVAL(EQH2O,2,T,3)
```

T (°R)	K	n_{H2} (lbmol)
3600	0.0002884	0.005485
3750	0.0005258	0.008174
3900	0.0009183	0.01183
4050	0.001332	0.01514
4200	0.002056	0.02016
4350	0.003922	0.03084
4500	0.005970	0.04061
4650	0.008814	0.05232
4800	0.01272	0.06629
4950	0.01797	0.08271
5100	0.02491	0.1017
5250	0.03391	0.1233
5400	0.04539	0.1476
5550	0.05961	0.1740
5700	0.07733	0.2030
5850	0.09897	0.2341
6000	0.1251	0.2671
6150	0.1566	0.3018
6300	0.1941	0.3380

PLOT:



Thus, at fixed pressure the amount of H2 present in the equilibrium mixture increases as temperature increases.

PROBLEM 14.17

KNOWN: 1 lbmol of H_2O plus x lbmol of N_2 forms an equilibrium mixture at 5400°R and 1 atm consisting of H_2O, H_2, O_2 and N_2

FIND: Plot the amount of H_2 present in the mixture versus x, $0 \le x \le 2$.

ENGINEERING MODEL: (1) Ideal gas principles apply. (2) N_2 is inert.

ANALYSIS: The reaction is

$$1\, H_2O + x\, N_2 \rightarrow (1-\alpha) H_2O + \alpha H_2 + \frac{\alpha}{2} O_2 + x\, N_2$$

The amount of mixture is $n = (1-\alpha) + \alpha + \alpha/2 + x = 1 + \frac{\alpha}{2} + x$

For $H_2O \rightleftarrows H_2 + \frac{1}{2} O_2$, Eq. 14.35 takes the form

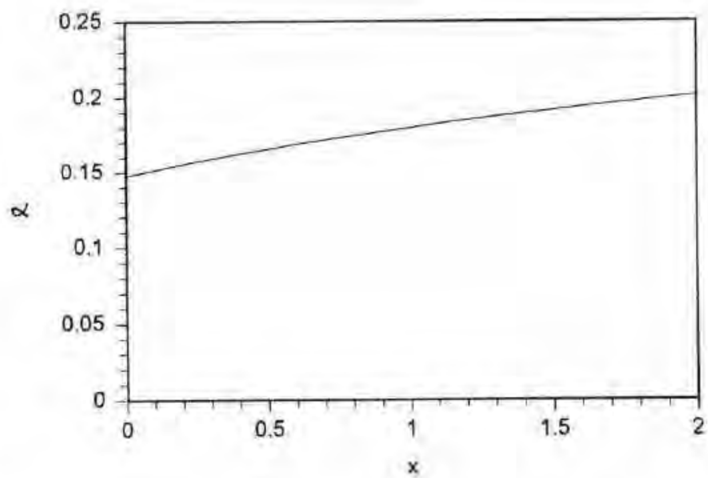
$$K = \frac{\alpha [\alpha/2]^{1/2}}{1-\alpha} \left[\frac{P/P_{ref}}{1+\frac{\alpha}{2}+x} \right]^{1+1/2-1} = \frac{\alpha}{1-\alpha} \left[\frac{\alpha/2}{1+\frac{\alpha}{2}+x} \right]^{1/2} \underbrace{(P/P_{ref})^{1/2}}_{=1}$$

From Table A-27 at 5400°R, $\log_{10} K = -1.343$, $K = 0.04539$. So

$$\frac{\alpha}{1-\alpha} \left[\frac{\alpha/2}{1+\frac{\alpha}{2}+x} \right]^{1/2} = 0.04539$$

Solving and plotting

x	α
0.000	0.148
0.100	0.152
0.200	0.155
0.300	0.159
0.400	0.162
0.500	0.166
0.600	0.169
0.700	0.172
0.800	0.174
0.900	0.177
1.000	0.180
1.100	0.182
1.200	0.185
1.300	0.187
1.400	0.189
1.500	0.191
1.600	0.193
1.700	0.195
1.800	0.197
1.900	0.199
2.000	0.201



Thus, at 5400°R, 1 atm the amount of H_2 in the equilibrium mixture increases as the amount of the inert N_2 increases.

PROBLEM 14.18

KNOWN: An equimolar mixture of CO and O_2 reacts to form an equilibrium mixture of CO_2, CO, and O_2 at 3000 K and pressure p.

FIND: Determine the effect of pressure on the composition of the mixture.

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS: The reaction takes the form

$$CO + O_2 \rightarrow x\,CO_2 + \alpha\,CO + \beta\,O_2$$

Balancing

$$C: \quad 1 = x + \alpha \quad , \; \alpha = 1 - x$$

$$O: \quad 3 = 2x + (1-x) + 2\beta \quad , \; \beta = 1 - \frac{x}{2} = \frac{2-x}{2}$$

Thus

$$CO + O_2 \rightarrow x\,CO_2 + (1-x)\,CO + \left(1 - \frac{x}{2}\right) O_2$$

The amount of mixture is $n = x + (1-x) + \left(1 - \frac{x}{2}\right) = 2 - \frac{x}{2} = \frac{4-x}{2}$

At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly, Eq. 14.35 takes the form

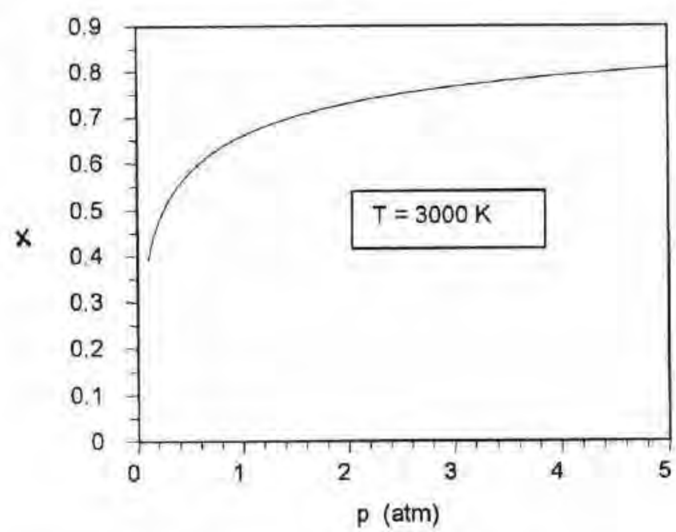
$$K = \frac{[1-x]\left[\frac{2-x}{2}\right]^{\frac{1}{2}}}{x}\left[\frac{p/p_{ref}}{\frac{4-x}{2}}\right]^{1+\frac{1}{2}-1}$$

$$K = \left[\frac{1-x}{x}\right]\left[\frac{2-x}{4-x}\right]^{1/2}\left[\frac{p}{p_{ref}}\right]^{1/2}$$

At 3000 K, Table A-27 gives $\log_{10} K = -0.485 \Rightarrow K = 0.32724$. Thus

$$0.32724 = \left[\frac{1-x}{x}\right]\left[\frac{2-x}{4-x}\right]^{1/2}\left[\frac{p}{p_{ref}}\right]^{1/2} \qquad (1)$$

The following plot is obtained using IT, with $p_{ref} = 1$ atm:



From the plot we conclude that the amount of CO_2 present at equilibrium decrease as pressure decreases at T = 3000 K.

PROBLEM 14.19

KNOWN: An equimolar mixture of CO and $H_2O(g)$ reacts to form an equilibrium mixture of CO_2, CO, H_2O, and H_2 at 2000 K, 1 atm.

FIND: Determine whether the amount of H_2 formed increases or decreases as temperature decreases. Also determine the effect of pressure on the amount of H_2 formed.

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS: The reaction takes the form

$$CO + H_2O \rightarrow \alpha CO_2 + \beta H_2O + \gamma CO + x H_2$$

$$C: \quad 1 = \alpha + \gamma \quad \Rightarrow \gamma = 1 - \alpha$$

$$O: \quad 2 = 2\alpha + \beta + \gamma$$

$$H: \quad 2 = 2\beta + 2x \quad \Rightarrow \beta = 1 - x$$

Finally, $\alpha = x$, $\gamma = 1 - x$, $\beta = 1 - x$. Thus

$$CO + H_2O \rightarrow x\,CO_2 + (1-x)H_2O + (1-x)CO + x\,H_2$$

The amount of mixture is $n = x + (1-x) + (1-x) + x = 2$

At equilibrium $CO_2 + H_2 \rightleftarrows CO + H_2O$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[1-x][1-x]}{[x][x]}\left[\frac{p/p_{ref}}{n}\right]^{1+1-1-1}$$

or

$$K = \left(\frac{1-x}{x}\right)^2$$

Thus

$$\sqrt{K} = \frac{1-x}{x} \qquad (1)$$

Solving Eq. (1)

$$x = \frac{1}{1+\sqrt{K}} \qquad (2)$$

Observe that pressure drops out of the final expression. Accordingly, pressure has no effect on the amount of H_2 formed. ← (b)

Referring to Table A-27 at 2000 K, $\log_{10}K = 0.656 \Rightarrow K = 4.52898$, whereas at 1900 K $\log_{10}K = 0.619 \Rightarrow 4.15911$. It follows from Eq. (2), therefore, that x increases as T decreases. ← (a)

PROBLEM 14.20

KNOWN: Nine percent of H_2 dissociates into H at p = 10 atm, T.

FIND: Determine T.

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS: The reaction is

$$1\,H_2 \rightarrow \alpha\,H_2 + \beta\,H$$

where $\alpha = 0.91$. Then

$$H: \quad 2 = 2(0.91) + \beta \Rightarrow \beta = 0.18$$

So

$$1\,H_2 \rightarrow 0.91\,H_2 + 0.18\,H$$

The amount of mixture is $n = 0.91 + 0.18 = 1.09$

At equilibrium $H_2 \rightleftarrows 2H$, so Eq. 14.35 takes the form

$$K = \frac{(0.18)^2}{(0.91)}\left[\frac{10}{1.09}\right]^{2-1} = 0.3266$$

① Then $\log_{10} K = -0.486$. Interpolating in Table A-27, T = 3488 K ←

Consider the case of 10% dissociation: $1\,H_2 \rightarrow 0.9\,H_2 + 0.2\,H$. Then

$$K = \frac{(0.2)^2}{(0.9)}\left[\frac{10}{1.10}\right]^{2-1} = 0.404 \Rightarrow \log_{10} K = -0.394 \Rightarrow T > 3488\,K$$

By inspection of Table A-27, T increases as the percentage of H_2 dissociating increases (i.e., as the amount of H_2 in the equilibrium mixture decreases).

---

1. Linear interpolation is used here. Alternatively, the approach of Problem 14.6 can be invoked.

PROBLEM 14.21

KNOWN: 2 kmol of CO_2 dissociate to form an equilibrium mixture of CO_2, CO, and O_2 in which 1.8 kmol is present at temperature T and pressure p.

FIND: plot T versus p for $0.5 \le p \le 10$ atm.

ENGINEERING MODEL: Ideal gas mixture principles apply.

ANALYSIS: The reaction takes the form

$$2\ CO_2 \rightarrow 1.8 CO_2 + \alpha\ CO + \beta\ O_2$$

C: $2 = 1.8 + \alpha \Rightarrow \alpha = 0.2$

O: $4 = (1.8)(2) + 0.2 + 2\beta \Rightarrow \beta = 0.1$

Thus

$$2 CO_2 \rightarrow 1.8 CO_2 + 0.2 CO + 0.1 O_2$$

The amount of mixture is $n = 2.1$.

At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2} O_2$. Accordingly, Eq 14.35 takes the form

$$K = \frac{[0.2][0.1]^{1/2}}{1.8}\left[\frac{p/p_{ref}}{2.1}\right]^{1+1/2-1} \qquad (1)$$

Sample calculation. T = 2400 K. From Table A-27, $\log_{10} K = -1.679 \Rightarrow K = 0.02094$.

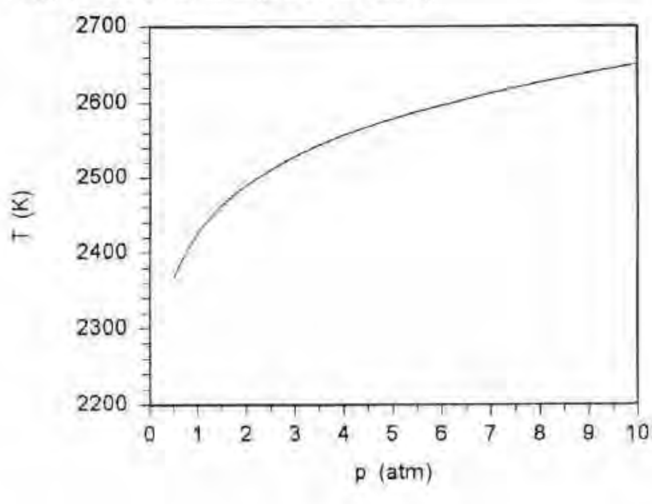
Soving for p, with $p_{ref} = 1$

$$p = \left(\frac{2.1}{0.1}\right)\left[\frac{(0.02094)(1.8)}{0.2}\right]^2 = 0.7459 \text{ atm}$$

The data for the required plot are obtained using IT, as follows:

IT Code

```
p = 0.5  // atm

// 2 CO2 → 1.8 CO2 + .2 CO + 0.1 O2
nCO2 = 1.8
nCO = 2 - nCO2
nO2 = 2 - nCO2 - nCO / 2
ntot = nCO2 + .2 + 0.1
yCO2 = nCO2 / ntot
yCO = nCO / ntot
yO2 = nO2 / ntot
pref = 1  // atm

//  For the reaction CO2 ↔ CO + 1/2 O2
K = ((yCO * yO2^0.5 ) /  yCO2) * (p / pref)^0.5
//Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,1,T,3)
```



IT Results

K	T (K)	p (atm)
0.01714	2367	0.5
0.02425	2427	1.0
0.02970	2463	1.5
0.03429	2490	2.0
0.03834	2511	2.5
0.04200	2529	3.0
0.04536	2544	3.5
0.04849	2557	4.0
0.05143	2568	4.5
0.05422	2579	5.0
0.05686	2588	5.5
0.05939	2597	6.0
0.06182	2605	6.5
0.06415	2613	7.0
0.06640	2620	7.5
0.06858	2627	8.0
0.07069	2634	8.5
0.07274	2640	9.0
0.07473	2645	9.5
0.07667	2651	10.0

For fixed composition, temperature increases as pressure increases.

PROBLEM 14.22

KNOWN: One kmol of $H_2O(g)$ dissociates to form an equilibrium mixture of $H_2O(g)$, H_2, and O_2 in which the amount of water vapor present is 0.95 kmol at temperature T and pressure p.

FIND: Plot T versus p for $1 \le p \le 10$ atm.

ENGINEERING MODEL: The equilibrium mixture is modeled as an ideal gas mixture.

ANALYSIS: The reaction takes the form

$$1\,H_2O \rightarrow 0.95\,H_2O + \alpha\,H_2 + \beta\,O_2$$

$$H:\ 2 = 1.9 + 2\alpha\ ,\ \alpha = 0.05$$

$$O:\ 1 = 0.95 + 2\beta\ ,\ \beta = 0.025$$

Thus

$$1\,H_2O \rightarrow 0.95\,H_2O + 0.05\,H_2 + 0.025\,O_2$$

The amount of mixture is 1.025 kmol.

At equilibrium $H_2O \rightleftarrows H_2 + \frac{1}{2}O_2$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[0.05][0.025]^{1/2}}{0.95}\left[\frac{p/p_{ref}}{1.025}\right]^{1+1/2-1}$$

Sample calculation. T = 2600 K. From Table A-27, $\log_{10} K = -2.038 \Rightarrow K = 0.0091622$.

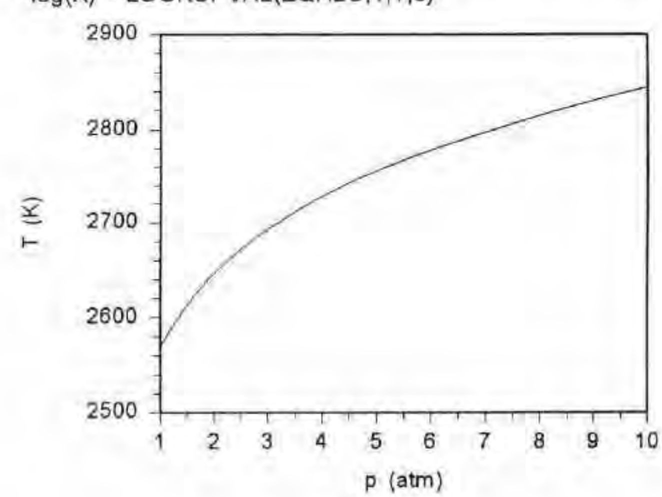
Solving for p, with $p_{ref} = 1$ atm

$$p = \left(\frac{1.025}{0.025}\right)\left[\frac{(0.0091622)(0.95)}{0.05}\right]^2 = 1.242 \text{ atm}$$

The data for the required plot are obtained using IT, as follows:

IT Code

```
p = 1 // atm

// H2O → 0.95 H2O + (1 - 0.95) H2 + ((1 - 0.95) / 2) O2
nH2O = 0.95
nH2 = 1 - nH2O
nO2 = (1 - nH2O) / 2
ntot = 0.95 + (1 - 0.95) + (1 - 0.95)/2
yH2O = nH2O / ntot
yH2 = nH2 / ntot
yO2 = nO2 / ntot
pref = 1 // atm

// For the reaction H2O ↔ H2 + 1/2 O2
K = ((yH2 * yO2^0.5) / yH2O) * (p / pref)^0.5
//Data from Table A-27 are stored in EQH2O.LUT.
log(K) = LOOKUPVAL(EQH2O,1,T,3)
```



IT Results

T (K)	K	p (atm)
2568	0.00822	1.0
2613	0.01007	1.5
2646	0.01162	2.0
2672	0.01300	2.5
2693	0.01424	3.0
2711	0.01538	3.5
2728	0.01644	4.0
2743	0.01744	4.5
2756	0.01838	5.0
2767	0.01928	5.5
2778	0.02013	6.0
2788	0.02096	6.5
2797	0.02175	7.0
2806	0.02251	7.5
2815	0.02325	8.0
2823	0.02396	8.5
2831	0.02466	9.0
2838	0.02533	9.5
2845	0.02599	10.0

For fixed composition, temperature increases as pressure increases.

PROBLEM 14.23

KNOWN: A vessel initially containing 1 kmol of $H_2O(g)$ and x kmol of N_2 forms an equilibrium mixture at 1 atm and temperature T consisting of $H_2O(g)$, H_2, O_2, N_2 in which 0.5 kmol of $H_2O(g)$ is present.

FIND: Plot x versus T for $3000 \le T \le 3500$ K.

ENGINEERING MODEL: (1) The equilibrium mixture is modeled as an ideal gas mixture. (2) N_2 is inert.

ANALYSIS: The reaction takes the form

$$1\ H_2O(g) + x\ N_2 \rightarrow 0.5\ H_2O(g) + \alpha H_2 + \beta O_2 + x N_2$$

H: $2 = (0.5)(2) + 2\alpha \Rightarrow \alpha = 0.5$

O: $1 = 0.5 + 2\beta \Rightarrow \beta = 0.25$

Thus

$$1\ H_2O(g) + x\ N_2 \rightarrow 0.5 H_2O(g) + 0.5 H_2 + 0.25 O_2 + x N_2$$

The amount of mixture is $n = 0.5 + 0.5 + 0.25 + x = 1.25 + x$.

At equilibrium $H_2O \rightleftarrows H_2 + \frac{1}{2} O_2$. Accordingly Eq 14.35 takes the form

$$K = \frac{[0.5][0.25]^{1/2}}{[0.5]}\left[\frac{p/p_{ref}}{1.25+x}\right]^{1/2}$$

Since $p/p_{ref} = 1$, this reduces to

$$K = \frac{0.5}{(1.25+x)^{1/2}} \Rightarrow x = \left[\frac{0.25}{\{K(T)\}^2} - 1.25\right]$$

Sample calculation. From Table A-27 at T = 3000 K, $\log_{10} K = -1.343 \Rightarrow K = 0.045394$. Thus x = 120.07.

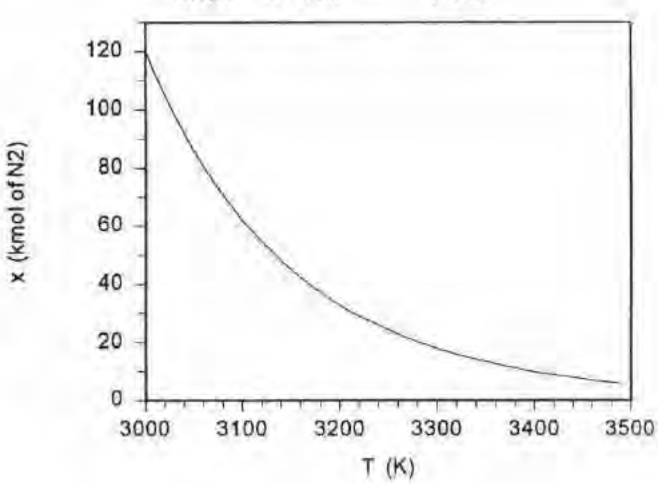
The data for the required plot are obtained using IT, as follows:

IT Code

```
p = 1  // atm
T = 3000  // K

// H2O + x N2 → 0.5 H2O + alpha H2 + beta O2 + x N2
nH2O = 0.5
alpha = 1 - 0.5
nH2 = alpha
beta = (1 - 0.5) / 2
nO2 = beta
ntot = 0.5 + alpha + beta + x
yH2O = nH2O / ntot
yH2 = nH2 / ntot
yO2 = nO2/ntot
pref = 1  // atm

// For the reaction H2O ↔ H2 + 1/2 O2
K = ((yH2 * yO2^0.5) / yH2O) * (p / pref)^0.5
// Data from Table A-27 are stored in EQH2O.LUT.
log(K) = LOOKUPVAL(EQH2O,1,T,3)
```

IT Results

T (K)	K	x (kmol of N_2)
3000	0.04539	120.1
3050	0.05346	86.24
3100	0.06295	61.84
3150	0.07345	45.09
3200	0.08570	32.79
3250	0.09897	24.27
3300	0.1143	17.89
3350	0.1309	13.34
3400	0.1500	9.866
3450	0.1706	7.339



For fixed composition in terms of H_2O, H_2, and O_2, the amount of inert N_2 required decreases as the temperature increases at fixed pressure.

PROBLEM 14.24

KNOWN: A vessel initially contains 2 lbmol of N_2 and 1 lbmol of O_2. An equilibrium mixture of N_2, O_2 and NO forms at 1 atm, T.

FIND: Plot the amount of NO formed versus T.

ENGINEERING MODEL: The ideal gas model applies.

ANALYSIS: The reaction takes the form

$$2N_2 + 1O_2 \rightarrow x\,N_2 + y\,O_2 + z\,NO$$

N: $4 = 2x + z \Rightarrow z = 4 - 2x$

O: $2 = 2y + z \Rightarrow 2 = 2y + (4-2x) \Rightarrow y = x - 1$

At equilibrium $\frac{1}{2}O_2 + \frac{1}{2}N_2 \rightleftarrows NO$. Accordingly

$$K = \frac{[4-2x]}{[x-1]^{1/2}[x]^{1/2}}\left[\frac{p/p_{ref}}{n}\right]^{\overbrace{1-\frac{1}{2}-\frac{1}{2}}^{=0}} \Rightarrow K(T) = \frac{[4-2x]}{[x(x-1)]^{1/2}} \qquad (1)$$

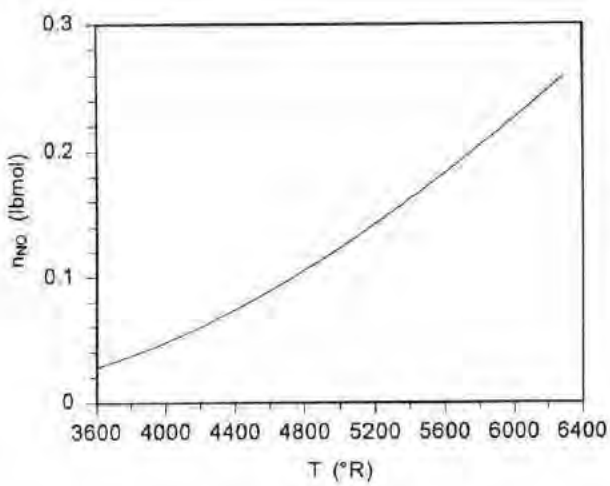
This expression is a quadratic equation that can be solved analytically for x as a function of K(T). Then, x can be calculated for given values of T using data for K(T) from Table A-27. Alternatively, data for the required plot are obtained using IT, as follows:

IT Code

```
p = 1  // atm
T = 5400  // °R

// 2 N2 + 1 O2 ===> x N2 + (x - 1) O2 + 2 * (2 - x) NO
nN2 = x
nO2 = x - 1
nNO = 2 * (2 - x)
ntot = x + (x - 1) + 2 * (2 - x)
yN2 = nN2 / ntot
yO2 = nO2 / ntot
yNO = nNO / ntot
pref = 1  // atm

// For the reaction 1/2 N2 + 1/2 O2 <===> NO
K = (yNO / (yN2^0.5 * yO2^0.5)) * (p / pref)^0
// Data from Table A-27 are stored in EQNO.LUT.
log(K) = LOOKUPVAL(EQNO,2,T,3)
```

IT results

T (°R)	K	n_{NO} (lbmol
3600	0.02000	0.02799
3800	0.02663	0.03713
4000	0.03441	0.04779
4200	0.04342	0.06002
4400	0.05366	0.07379
4600	0.06510	0.08898
4800	0.07774	0.1056
5000	0.09146	0.1233
5200	0.1063	0.1423
5400	0.1222	0.1622
5600	0.1388	0.1828
5800	0.1566	0.2044
6000	0.1750	0.2264
6200	0.1942	0.2489



As temperature increases at fixed pressure, the amount of NO in the equilibrium mixture increases.

PROBLEM 14.25

KNOWN: A vessel initially containing 1 kmol of CO and 4.76 kmols of dry air forms an equilibrium mixture of CO_2, CO, O_2, and N_2 at 3000K, 1atm.

FIND: Determine the equilibrium composition.

ENGINEERING MODEL: (1) The equilibrium mixture is modeled as an ideal gas mixture. (2) The N_2 is inert.

ANALYSIS: The reaction takes the form

$$1\,CO + 1O_2 + 3.76\,N_2 \rightarrow \alpha\,CO_2 + \beta CO + \gamma O_2 + 3.76 N_2$$

C: $1 = \alpha + \beta \Rightarrow \beta = 1-\alpha$

O: $3 = 2\alpha + \cancel{\beta} + 2\gamma \Rightarrow 3 = 2\alpha + (1-\alpha) + 2\gamma \Rightarrow \gamma = 1 - \frac{\alpha}{2}$

Thus

$$1\,CO + 1\,O_2 + 3.76 N_2 \rightarrow \alpha\,CO_2 + (1-\alpha)CO + \left(1-\frac{\alpha}{2}\right)O_2 + 3.76\,N_2$$

The amount of mixture is $n = \alpha + (1-\alpha) + (1-\frac{\alpha}{2}) + 3.76 = 5.76 - \frac{\alpha}{2}$.

At equilibrium $CO_2 \rightleftarrows CO + 1/2\,O_2$. Accordingly Eq 14.35 takes the form

$$K = \frac{[1-\alpha]\left[\frac{2-\alpha}{2}\right]^{1/2}}{\alpha}\left[\frac{P/P_{ref}}{(11.52-\alpha)/2}\right]^{1/2} \Rightarrow K = \frac{1-\alpha}{\alpha}\left[\frac{2-\alpha}{11.52-\alpha}\right]^{1/2}$$

From Table A-27 at 3000K, $\log_{10} K = -0.485 \Rightarrow K = 0.32734$. Thus

$$0.32734 = \frac{1-\alpha}{\alpha}\left[\frac{2-\alpha}{11.52-\alpha}\right]^{1/2}$$

Using an equation solver or iteration with a hand calculator, $\alpha = 0.528$.

The equilibrium composition is

$$\{0.528\,CO_2,\ 0.472\,CO,\ 0.736\,O_2,\ 3.76\,N_2\}$$

PROBLEM 14.26

KNOWN: A vessel initially containing 1 kmol O_2, 2 kmol N_2, 1 kmol Ar forms an equilibrium mixture of O_2, N_2, NO, and Ar at 3000K, 1atm.

FIND: Determine the equilibrium composition.

ENGINEERING MODEL: (1) The equilibrium mixture is modeled as an ideal gas. (2) Argon is inert.

ANALYSIS: The reaction takes the form

$$1\,O_2 + 2\,N_2 + 1\,Ar \rightarrow \alpha\,O_2 + \beta\,N_2 + x\,NO + 1\,Ar$$

O: $2 = 2\alpha + x$, $\alpha = 1 - x/2$

N: $4 = 2\beta + x$, $\beta = 2 - x/2$

Thus

$$1\,O_2 + 2\,N_2 + 1\,Ar \rightarrow \left(1-\frac{x}{2}\right)O_2 + \left(2-\frac{x}{2}\right)N_2 + x\,NO + 1\,Ar$$

The amount of mixture is $n = \left(1-\frac{x}{2}\right) + \left(2-\frac{x}{2}\right) + x + 1 = 4$.

At equilibrium $\frac{1}{2}O_2 + \frac{1}{2}N_2 \rightleftarrows NO$. Accordingly, Eq. 14.35 takes the form

(1) $$K = \frac{x}{\left[1-\frac{x}{2}\right]^{1/2}\left[2-\frac{x}{2}\right]^{1/2}}\left[\frac{P/P_{ref}}{4}\right]^{1-1/2-1/2} \Rightarrow K = \frac{x}{\left[1-\frac{x}{2}\right]^{1/2}\left[2-\frac{x}{2}\right]^{1/2}}$$

At 3000 K, Table A-27 gives $\log_{10} K = -0.913 \Rightarrow K = 0.12218$. Thus

$$0.12218 = \frac{x}{\left[\left(1-\frac{x}{2}\right)\left(2-\frac{x}{2}\right)\right]^{1/2}}$$

Using an equation solver or iteration with a hand calculator, $x = 0.162$. The equilibrium composition is

$$\{0.919\,O_2,\ 1.919\,N_2,\ 0.162\,NO,\ 1\,Ar\}$$

1. Note that (P/P_{ref}) drops out of this expression.

PROBLEM 14.27

KNOWN: One kmol CO and 0.5 kmol O_2 react to form {CO_2, O_2, CO} at T, p. When p = 1 atm, 0.35 kmol of CO is present in the equilibrium mixture.

FIND: Determine the amount of CO present in an equilibrium mixture at T, p = 10 atm.

ENGINEERING MODEL: The equilibrium mixture is modeled as an ideal gas.

ANALYSIS: The reaction takes the form

$$CO + 0.5\ O_2 \rightarrow \alpha\, CO_2 + \beta\, CO + \gamma\, O_2$$

C: $1 = \alpha + \beta \Rightarrow \alpha = 1 - \beta$

O: $2 = 2(1-\beta) + \beta + 2\gamma$

$$\Rightarrow \gamma = \beta/2$$

For this case, $n = (1-\beta) + \beta + \beta/2 = 1 + (\beta/2)$. For $CO_2 \rightleftarrows CO + \frac{1}{2} O_2$ Eq. 14.35 reads

$$K = \frac{\beta\,(\beta/2)^{1/2}}{(1-\beta)}\left[\frac{p/p_{ref}}{1+(\beta/2)}\right]^{1/2} \qquad (1)$$

Thus, when $\beta = 0.35$, and $p = p_{ref}$, we get

$$K = \frac{(0.35)(0.175)^{1/2}}{(0.65)}\left[\frac{1}{1.175}\right]^{1/2} = 0.2078$$

Since T remains unchanged, K remains unchanged. Then, with p = 10 atm, Eq. (1) reads

$$0.2078 = \frac{\beta\,(\beta/2)^{1/2}}{1-\beta}\left[\frac{10}{1+(\beta/2)}\right]^{1/2}$$

$$= \frac{\beta}{1-\beta}\left[\frac{10\beta}{2+\beta}\right]^{1/2}$$

Using an equation solver or iteration with a hand calculator,

① $\beta = 0.1844$ ◄

---

1. Keeping temperature constant and increasing pressure results in a greater amount of CO_2 formed.

PROBLEM 14.28

KNOWN: A vessel initially contains 1 kmol H_2 and 4 kmol N_2. An equilibrium mixture of H_2, H, and N_2 forms at 3000K, 1atm.

FIND: Determine the equilibrium composition. If pressure is increased while keeping temperature fixed, determine if the amount of H would increase or decrease.

ENGINEERING MODEL: (1) The equilibrium mixture is modeled as an ideal gas. (2) N_2 is inert.

ANALYSIS: The reaction takes the form

$$1\,H_2 + 4\,N_2 \rightarrow x\,H + \alpha\,H_2 + 4\,N_2$$

$$H:\; 2 = x + 2\alpha \Rightarrow \alpha = 1 - \frac{x}{2}$$

Thus

$$1\,H_2 + 4\,N_2 \rightarrow x\,H + \left(1-\frac{x}{2}\right)H_2 + 4\,N_2$$

The amount of mixture is $n = x + (1-\frac{x}{2}) + 4 = 5 + \frac{x}{2}$.

At equilibrium $H_2 \rightleftarrows 2H$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[x]^2}{[1-\frac{x}{2}]}\left[\frac{P/P_{ref}}{\frac{10+x}{2}}\right]^{2-1} \Rightarrow K = \frac{4x^2\,(P/P_{ref})}{(2-x)(10+x)} \qquad (1)$$

Eq (1) gives upon rearrangement

$$(\gamma+1)x^2 + 8x - 20 = 0$$

where $\gamma = [4(P/P_{ref})/K]$. Solving this quadratic equation

$$x = \frac{-8 + \sqrt{64 + 80(\gamma+1)}}{2(\gamma+1)} \qquad (2)$$

From Table A-27 at 3000 K $\log_{10} K = -1.606 \Rightarrow K = 0.024774$. With $P/P_{ref} = 1$, $\gamma = 161.46$. Thus, Eq. (2) gives $x = 0.3271$. The equilibrium mixture is then

$$\{\,0.3271\,H,\; 0.8365\,H_2,\; 4\,N_2\,\} \longleftarrow$$

To determine the effect on x owing to an increase in pressure at fixed temperature, consider $P/P_{ref} = 1.1$. Then $\gamma = 177.61$ and Eq. (2) gives $x = 0.313$. Accordingly x tends to decrease as p increases. $\longleftarrow$

PROBLEM 14.29

KNOWN: An equilibrium mixture of N_2, O_2, and NO at 4342 °R, 1 atm forms from dry air.

FIND: Determine the mole fraction of NO in the equilibrium mixture. Determine if the amount of NO increases or decreases as T decreases at fixed p.

ENGINEERING MODEL: (1) The equilibrium mixture is modeled as an ideal gas mixture. (2) The analysis is based on 1 lbmol of air initially.

ANALYSIS: The reaction takes the form

$$0.79\, N_2 + 0.21\, O_2 \rightarrow x\, NO + \alpha\, N_2 + \beta\, O_2$$

N: $(0.79)(2) = x + 2\alpha \Rightarrow \alpha = (1.58 - x)/2$

O: $(0.21)(2) = x + 2\beta \Rightarrow \beta = (0.42 - x)/2$

Thus

$$0.79\, N_2 + 0.21\, O_2 \rightarrow x\, NO + \left[\frac{1.58 - x}{2}\right] N_2 + \left[\frac{0.42 - x}{2}\right] O_2$$

The amount of mixture is $n = x + \left[\frac{1.58 - x}{2}\right] + \left[\frac{0.42 - x}{2}\right] = 1$

At equilibrium $\frac{1}{2} O_2 + \frac{1}{2} N_2 \rightleftarrows NO$. According Eq. 14.35 takes the form

1 $$K = \frac{x}{\left[\frac{1.58 - x}{2}\right]^{1/2}\left[\frac{0.42 - x}{2}\right]^{1/2}}\left[\frac{p/p_{ref}}{1}\right]^{1 - 1/2 - 1/2} = \frac{2x}{[1.58 - x]^{1/2}[0.42 - x]^{1/2}}$$

or

$$K^2 = \frac{4x^2}{(1.58 - x)(0.42 - x)} \Rightarrow \gamma x^2 + 2x - 0.6636 = 0 \qquad (1)$$

where $\gamma = \frac{4}{K^2} - 1$. Solving this quadratic equation

$$x = \frac{-2 + \sqrt{4 + 2.6544\gamma}}{2\gamma} \qquad (2)$$

At 4342 °R, Table A-27 gives $\log_{10} K = -1.2956 \Rightarrow K = 0.05062$. Thus $\gamma = 1560$ and Eq. (2) gives $x = 0.02$. The mole fraction of NO in the equilibrium mixture is then

$$y_{NO} = \frac{0.02}{1} = 0.02$$

To determine the effect of a higher temperature on x, consider T = 4500 °R. Then Table A-27 gives $\log_{10} K = -1.227 \Rightarrow K = 0.05929$. Thus $\gamma = 1136.9$ and Eq. (2) gives $x = 0.0233$. Thus, as temperature decreases the amount of NO also decreases.

---

1. Note that (p/p_{ref}) drops out of this expression, and thus of the calculation of the equilibrium composition.

PROBLEM 14.30

KNOWN: A gaseous mixture with a specified molar analysis is heated at a pressure of 1.5 bar and forms an equilibrium mixture of CO_2, CO, and O_2 at 3000 K.

FIND: Determine the molar analysis of the equilibrium mixture.

ENGINEERING MODEL: (1) The analysis is based on 1 kmol of initial mixture. (2) The equilibrium mixture is modeled as an ideal gas mixture.

ANALYSIS: On the basis of 1 kmol of initial mixture, the reaction is

$$0.2\,CO_2 + 0.4\,CO + 0.4\,O_2 \rightarrow \alpha\,CO_2 + \beta\,CO + \gamma\,O_2$$

$$C:\quad 0.2 + 0.4 = \alpha + \beta \Rightarrow \beta = .6 - \alpha$$

$$O:\quad 2(0.2) + 0.4 + 2(0.4) = 2\alpha + \beta + 2\gamma$$

$$1.6 = 2\alpha + (.6 - \alpha) + 2\gamma \Rightarrow \gamma = \frac{1-\alpha}{2}$$

Then

$$0.2\,CO_2 + 0.4\,CO + 0.4\,O_2 \rightarrow \alpha\,CO_2 + (0.6-\alpha)\,CO + \left(\frac{1-\alpha}{2}\right) O_2$$

The amount of mixture is $n = \alpha + (0.6-\alpha) + \left(\frac{1-\alpha}{2}\right) = (2.2-\alpha)/2$.

At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly Eq 14.35 takes the form

$$K = \frac{[0.6-\alpha]\left[\frac{1-\alpha}{2}\right]^{1/2}}{[\alpha]}\left[\frac{P/P_{ref}}{(2.2-\alpha)/2}\right]^{1+\frac{1}{2}-1}$$

$P/P_{ref} = 1.5/1.01325 \Rightarrow$

$$K = \left[\frac{0.6-\alpha}{\alpha}\right]\left[\frac{1-\alpha}{2.2-\alpha}\right]^{1/2}[1.21671]$$

From Table A-27 at 3000 K, $\log_{10} K = -0.485 \Rightarrow K = 0.32734$. Thus

$$0.27127 = \left[\frac{0.6-\alpha}{\alpha}\right]\left[\frac{1-\alpha}{2.2-\alpha}\right]^{1/2}$$

Using an equation solver or iteration with a hand calculator, $\alpha = 0.408$.

The analysis of the equilibrium mixture in terms of mole fractions is

$$\begin{cases} y_{CO_2} = \dfrac{0.408}{0.896} = 0.4554 & (45.54\%) \\ y_{CO} = \dfrac{0.192}{0.896} = 0.2143 & (21.43\%) \\ y_{O_2} = \dfrac{0.296}{0.896} = 0.3304 & (33.04\%) \end{cases}$$

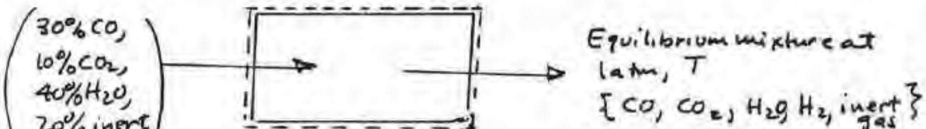
PROBLEM 14.31

KNOWN: An ideal gas mixture with a molar analysis {30% CO, 10% CO_2, 40% H_2O, 20% inert} enters a reactor operating at steady state. An equilibrium mixture of CO, CO_2, H_2O, H_2, and inert gas exits the reactor at 1 atm.

FIND: (a) If the exiting mixture is at 1200 K, determine the ratio of moles of H_2 exiting to moles of H_2O entering.

(b) If $y_{CO} = 0.075$ in the exiting mixture, determine the mixture temperature.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown is at steady state. (2) The exiting mixture is modeled as an ideal gas mixture.

ANALYSIS: Based on 100 kmol of entering gas, at steady state

$$30\,CO + 10\,CO_2 + 40\,H_2O + 20\,\text{inert} \rightarrow (30-x)CO + (10+x)CO_2 + (40-x)H_2O + x\,H_2 + 20\,\text{inert}$$

For the exiting mixture, $n = (30-x) + (10+x) + (40-x) + x + 20 = 100$ kmol

At equilibrium for $CO + H_2O \rightleftharpoons CO_2 + H_2$, and so

① $$K = \frac{(10+x)(x)}{(30-x)(40-x)}\left[\frac{P/P_{ref}}{n}\right]^{\overbrace{1+1-1-1}^{=0}} = \frac{(10+x)(x)}{(30-x)(40-x)} \qquad (1)$$

(a) When T = 1200 K, Table A-27 gives $\log_{10} K = -0.135 \Rightarrow K = 0.7328$.
Rearranging Eq. (1), it can be expressed as a quadratic equation:
$x^2 + 229.401x - 3291.02 = 0$. Solving with the quadratic formula, $x = 13.547$

Thus, $$\frac{13.547 \text{ kmol } H_2 \text{ (produced)}}{40 \text{ kmol } H_2O \text{ (feed)}} = 0.339 \longleftarrow \quad (a)$$

(b) When $y_{CO} = 0.075$, then

$y_{CO} = \left(\frac{30-x}{100}\right) = 0.075 \Rightarrow x = 22.5 \xrightarrow{\text{Eq.(1)}} K = \frac{(32.5)(22.5)}{(7.5)(17.5)} \Rightarrow \log_{10} K = 0.74597$

For use with Table A-27, $\log_{10} K = -0.74597$. Inspection shows that $500 < T < 1000$ K. Using the interpolation rule of Problem 14.6:

$$\log_{10} K = C_1 + \frac{C_2}{T}$$

500 K: $-2.139 = C_1 + \frac{C_2}{500}$
1000 K: $-0.159 = C_1 + \frac{C_2}{1000}$
$\Big\}$ $C_1 = 1.821$, $C_2 = -1980$ K

Thus, $$-0.74597 = 1.821 - \frac{1980}{T} \Rightarrow \frac{1980}{T} = 2.56697 \Rightarrow T = 771\text{ K} \longleftarrow \quad (b)$$

---

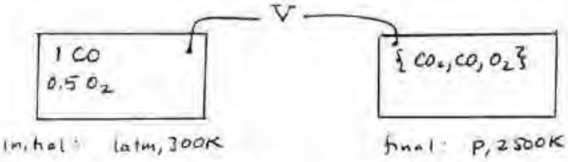
1. Note that (P/P_{ref}) drops out of this expression, and thus plays no role in the subsequent evaluations.

PROBLEM 14.32

KNOWN One kmol CO and 0.5 kmol O_2, initially at 1atm and 300K, reacts to form an equilibrium mixture of $\{CO_2, CO, O_2\}$ at 2500K and the same total volume

FIND Determine the final pressure

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: The ideal gas model applies.

ANALYSIS: The reaction has the form

$$1\,CO + 0.5\,O_2 \rightarrow \alpha\, CO_2 + \beta\, CO + \gamma\, O_2$$

C: $1 = \alpha + \beta \Rightarrow \alpha = 1-\beta$, O: $2 = 2(1-\beta) + \beta + 2\gamma$, $\gamma = \beta/2$

The amount of mixture is $n = (1-\beta) + \beta + \beta/2 = 1 + \beta/2$.

The ideal gas equation of state for the initial mixture gives $p'V = n'\bar{R}T'$, where the prime indicates the initial values. For the final mixture $pV = n\bar{R}T$. Accordingly

$$\frac{p}{p'} = \frac{nT}{n'T'} \Rightarrow \frac{p}{n} = \frac{p'}{n'}\frac{T}{T'} = \left(\frac{1\text{ atm}}{1.5\text{ kmol}}\right)\left(\frac{2500\text{ K}}{300\text{ K}}\right) = 5.556\ \frac{\text{atm}}{\text{kmol}} \qquad (1)$$

For $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$, Eq. 14.35 takes the form

$$K = \frac{\beta\,[\beta/2]^{1/2}}{1-\beta}\left[\underbrace{\frac{p}{n}}_{5.556\ \frac{\text{atm}}{\text{kmol}}\ (\text{Eq. }(1))}\right]^{1/2}$$

With $K = 0.0363$ from Table A-27 at 2500K

$$0.0363 = \frac{\beta}{1-\beta}\left[\frac{5.556}{2}\beta\right]^{1/2}$$

Using an equation solver or iteration with a hand calculator, $\beta = 0.074$. Thus

$$n = 1 + \frac{0.074}{2} = 1.037\text{ kmol}$$

Returning to Eq.(1)

$$p = (5.556)(1.037)$$
$$= 5.761\text{ atm}$$

PROBLEM 14.33

KNOWN: CH_4 burns with 90% of theoretical air to form an equilibrium mixture of CO_2, CO, $H_2O(g)$, H_2, N_2 at 1000K, 1atm

FIND: Determine the composition of the equilibrium mixture, per kmol of mixture

ENGINEERING MODEL: (1) The ideal gas model applies. (2) N_2 is inert.

ANALYSIS: The reaction takes the form

$$CH_4 + (0.9)(2)[O_2 + 3.76N_2] \rightarrow \alpha CO_2 + \beta CO + \gamma H_2O + \omega H_2 + 6.768N_2$$

C: $1 = \alpha + \beta \Rightarrow \alpha = 1 - \beta$

H: $4 = 2\gamma + 2\omega$, $2 = \gamma + \omega$

O: $3.6 = 2(1-\beta) + \beta + \gamma \Rightarrow \gamma = 1.6 + \beta \Rightarrow \omega = 0.4 - \beta$

At equilibrium, $CO_2 + H_2 \rightleftarrows CO + H_2O$. Table A-27 gives for $T = 1000K$, $K = 0.693$. And therefore

① $$0.693 = \frac{[\beta][1.6+\beta]}{[1-\beta][0.4-\beta]}\left[\frac{P/P_{ref}}{n}\right]^{\overbrace{1+1-1-1}^{0}}$$

Using an equation solver or iteration with a hand calculator, $\beta = 0.1065$. So, the composition of the equilibrium mixture per kmol of CH_4 is

$$\{0.8935\,CO_2,\ 0.1065\,CO,\ 1.7065\,H_2O,\ 0.2935\,H_2,\ 6.768\,N_2\}$$

The amount of mixture per kmol of CH_4 is 9.768 kmol. So, the molar analysis of the equilibrium mixture is

9.15% CO_2, 1.09% CO, 17.47% $H_2O(g)$, 3% H_2, 69.29% N_2 ◄

---

1. Note that (P/P_{ref}) drops out of this expression and thus out of the subsequent evaluations.

PROBLEM 14.34

KNOWN: C_8H_{18} burns with air to form an equilibrium mixture of {CO_2, H_2, CO, $H_2O(g)$, N_2} at 1700 K, 1 atm.

FIND: If the equivalence ratio is 1.2, determine the composition of the equilibrium mixture in kmol per kmol of fuel.

ENGINEERING MODEL: (1) The ideal gas model applies. (2) N_2 is inert

ANALYSIS: With $(\overline{AF})_{theo} = 12.5(4.76)$ from Example 13.4

$$\text{Equivalence ratio} = \frac{(\overline{AF})_{theo}}{\overline{AF}} \Rightarrow AF = \frac{12.5(4.76)}{1.2} = (10.417)(4.76)$$

Thus

$$C_8H_{18} + 10.417\,[O_2 + 3.76N_2] \rightarrow a\,CO_2 + b\,H_2O + c\,CO + d\,H_2 + 39.168\,N_2$$

$$C:\quad 8 = a + c \Rightarrow a = 8 - c$$

$$H:\quad 18 = 2b + 2d \Rightarrow d = 9 - b$$

$$O:\quad 20.834 = 2a + b + c$$
$$= 2(8-c) + b + C \Rightarrow b = 4.834 + C \quad \text{and} \quad d = 9 - [4.834 + c]$$
$$= 4.166 - C$$

At equilibrium, $CO_2 + H_2 \rightleftarrows CO + H_2O$. Table A-27 gives for $T = 1700\,K$, $K = 3.388$. And therefore

① $$3.388 = \frac{[c][4.834+c]}{[8-c][4.166-c]}\left[\frac{p/p_{ref}}{n}\right]^{\overbrace{1+1-1-1}^{0}}$$

This can be solved analytically by expressing it as a quadratic equation and using the quadratic formula. Alternatively, an equation solver can be used, or iteration with a hand calculator can be employed. We get $c = 2.883$.

The composition of the equilibrium mixture, per kmol of C_8H_{18}, is then

$$\{5.117\,CO_2,\ 7.717\,H_2O(g),\ 2.883\,CO,\ 1.283\,H_2,\ 39.168\,N_2\} \leftarrow$$

---

1. Note that (p/p_{ref}) drops out of this expression and thus from the following evaluations.

PROBLEM 14.35

<u>KNOWN:</u> C_2H_2 at 25°C, 1atm burns with 40% excess air at 25°C, 1atm, $\phi = 80\%$. An equilibrium mixture of $\{CO_2, H_2O, O_2, NO, N_2\}$ is formed at 2200K, 0.9atm

<u>FIND:</u> Determine the composition of the equilibrium mixture, per kmol of C_2H_2.

<u>ENGINEERING MODEL:</u> The ideal gas model applies.

<u>ANALYSIS</u> The balanced reaction equation for complete combustion of C_2H_2 with the theoretical amount of dry air is $C_2H_2 + \frac{5}{2}[O_2 + 3.76N_2] \rightarrow 2CO_2 + H_2O + (\frac{5}{2})3.76\,N_2$. The amount of dry air for combustion with 40% excess air is, then $(1.4)(2.5)(4.76)$ per kmol of C_2H_2. That is, $(3.5)(4.76) = 16.66$ kmol(air)/kmol(fuel). The amount of water vapor accompanying the dry air is found using Chap. 12 concepts: $p_v = \phi\, p_g(25°C) = (0.8)(0.03169\text{ bar}) = 0.025352$ bar. $p_a = p - p_v = 1.01325 - 0.02535 = 0.987898$ bar. Then, referring to the development of Eqs. 12.41, we get

$$\frac{n_v}{n_a} = \frac{p_v}{p_a} \Rightarrow n_v = \left(\frac{0.025352}{0.987898}\right)\left(16.66\,\frac{\text{kmol (dry air)}}{\text{kmol (fuel)}}\right) = 0.428\ \frac{\text{kmol}(H_2O)}{\text{kmol (fuel)}}$$

The reaction equation then has the form

$$C_2H_2 + 3.5(O_2 + 3.76N_2) + 0.428H_2O \rightarrow 2CO_2 + 1.428H_2O + \gamma NO + [1 - \tfrac{\gamma}{2}]O_2 + [13.16 - \tfrac{\gamma}{2}]N_2$$

At equilibrium, $\frac{1}{2}O_2 + \frac{1}{2}N_2 \rightleftarrows NO$. Table A-27 gives for 2200 K, K = 0.0328. And therefore

$$① \quad 0.0328 = \frac{\gamma}{[1-\frac{\gamma}{2}]^{1/2}[13.16 - \frac{\gamma}{2}]^{1/2}}\left[\frac{p/p_{ref}}{n}\right]^{\overbrace{1-\frac{1}{2}-\frac{1}{2}}^{0}}$$

$$0.0328 = \frac{\gamma}{\left[(1-\frac{\gamma}{2})(13.16-\frac{\gamma}{2})\right]^{1/2}}$$

Squaring both sides this can be expressed as a quadratic equation and solved using the quadratic formula. Alternatively, an equation solver can be used, or iteration with a hand calculator employed. We get $\gamma = 0.1134$.

The composition of the equilibrium mixture, per kmol of C_2H_2, is

$$\{2CO_2, 1.428H_2O, 0.1134NO, 0.9433O_2, 13.1033N_2\}$$

---

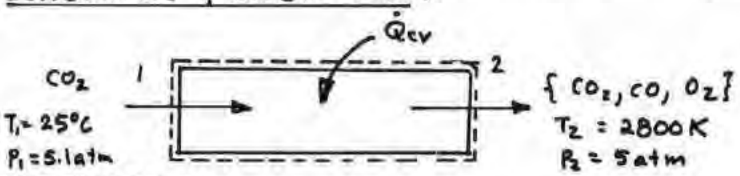
1. Note that (P/P_{ref}) drops out of this expression.

PROBLEM 14.36

KNOWN: CO_2 gas at 25°C, 5.1atm enters a heat exchanger operating at steady state. An equilibrium mixture of CO_2, CO, and O_2 exits at 2800 K, 5 atm.

FIND: Determine the composition of the exiting mixture and the heat transfer, each per kmol of CO_2 entering.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure is at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The ideal gas model applies to the incoming CO_2 and the exiting equilibrium mixture.

ANALYSIS: The reaction has the form

$$1\,CO_2 \rightarrow (1-x)\,CO_2 + x\,CO + \frac{x}{2}\,O_2$$

The amount of mixture is $n = (1-x) + x + \frac{x}{2} = (2+x)/2$. At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[x][x/2]^{1/2}}{[1-x]}\left[\frac{P/P_{ref}}{(x+2)/2}\right]^{1/2} = \left[\frac{x}{1-x}\right]\left[\frac{5x}{x+2}\right]^{1/2}$$

From Table A-27 at 2800 K, $\log_{10} K = -0.825 \Rightarrow K = 0.14962$. Solving, $x = 0.1867$. The equilibrium mixture has the following composition in kmol per kmol of CO_2 entering

$$\{0.8133\ CO_2,\ 0.1867\ CO,\ 0.09335\ O_2\}$$

An energy rate balance reduces at steady state to read

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{CO_2}} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{CO_2}}} + 1\cdot\bar{h}_{CO_2}(T_1) - \left[0.8133\,\bar{h}_{CO_2}(T_2) + 0.1867\,\bar{h}_{CO}(T_2) + 0.09335\,\bar{h}_{O_2}(T_2)\right]$$

or

$$\frac{\dot{Q}_{cv}}{\dot{n}_{CO_2}} = 0.8133\left[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(298)\right]_{CO_2} + 0.1867\left[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(298)\right]_{CO} + 0.09335\left[\cancel{\bar{h}_f^\circ} + \bar{h}(T_2) - \bar{h}(298)\right]_{O_2} - \left[\bar{h}_f^\circ\right]_{CO_2}$$

With data from the ideal gas tables

$$\frac{\dot{Q}_{cv}}{\dot{n}_{CO_2}} = 0.8133\left[-393520 + 149{,}808 - 9{,}364\right] + 0.1867\left[-110{,}530 + 94{,}784 - 8{,}669\right] + 0.09335\left[98{,}826 - 8{,}682\right] - \left[-393{,}520\right]$$

$$= 191{,}550\ \text{kJ/kmol}(CO_2)$$

IT can be used as an alternative to iteration using a hand calculator and to using table data in the energy balance. The IT program follows.

IT Code

```
T1 = 25 + 273.15  // K
p1 = 5.1  // atm
T2 = 2800  // K
p2 = 5  // atm

// CO2 ===> (1 - x) CO2 + x CO + x/2 O2
ndot1 = 1  // per kmol entering
ndotCO2_2 = 1 - x
ndotCO_2 = x
ndotO2_2 = x/2
ndot2 = ndotCO2_2 + ndotCO_2 + ndotO2_2
yCO2 = ndotCO2_2 / ndot2
yCO = ndotCO_2 / ndot2
yO2 = ndotO2_2 / ndot2
pref = 1  // atm

// For the reaction CO2 <===> CO + 1/2 O2
K = ((yCO * yO2^0.5) / yCO2) * (p2 / pref)^0.5
//Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,1,T2,3)

0 = Qdot + ndot1*hCO2_1 - ndotCO2_2*hCO2_2 - ndotCO_2*hCO_2 -
ndotO2_2*hO2_2
hCO2_1 = h_T("CO2",T1)
hCO2_2 = h_T("CO2",T2)
hCO_2 = h_T("CO",T2)
hO2_2 = h_T("O2",T2)
```

IT Results

$K = 0.1496$

$\dot{n}_{CO_2,2} / \dot{n}_{CO_2,1}$ = 0.8136 kmol / kmol (CO_2 entering)

$\dot{n}_{CO,2} / \dot{n}_{CO_2,1}$ = 0.1864 kmol / kmol (CO_2 entering)

$\dot{n}_{O_2,2} / \dot{n}_{CO_2,1}$ = 0.09322 kmol / kmol (CO_2 entering)

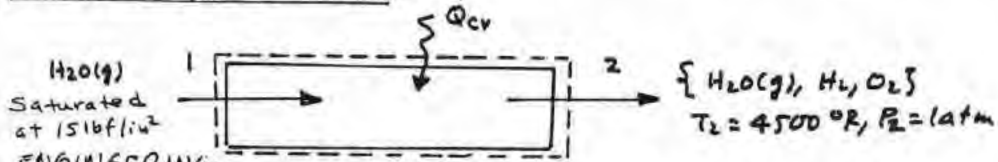
$\dot{Q}_{cv} / \dot{n}_{CO_2,1}$ = 1.916E5 kJ/kmol (CO_2 entering)

PROBLEM 14.37

KNOWN: Saturated water vapor at 15 lbf/in² enters a heat exchanger operating at steady state. An equilibrium mixture of $H_2O(g)$, H_2, and O_2 exits at 4500°R, 1 atm

FIND: Determine the composition of the exiting mixture and the heat transfer, each per lbmol of steam entering.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure is at steady state with $\dot{W}_{cv}=0$ and negligible effects of kinetic and potential energy. (2) The incoming steam and exiting equilibrium mixture can be modeled as ideal gases.

ANALYSIS: The reaction has the form

$$1\,H_2O \longrightarrow (1-x)\,H_2O + x\,H_2 + \frac{x}{2}\,O_2$$

The amount of mixture is $n = (1-x) + x + \frac{x}{2} = (x+2)/2$. At equilibrium $H_2O \rightleftarrows H_2 + \frac{1}{2}O_2$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{(x)\left[\frac{x}{2}\right]^{1/2}}{[1-x]}\left[\frac{P/P_{ref}}{(x+2)/2}\right]^{1/2} = \left[\frac{x}{1-x}\right]\left[\frac{x}{x+2}\right]^{1/2}$$

From Table A-27 at 4500°R $\log_{10}K = -2.224 \Rightarrow K = 0.00597$. Solving $x = 0.0406$. The equilibrium mixture has the following composition in lbmol per lbmol of steam entering

$$\{0.9594\ H_2O,\ 0.0406\ H_2,\ 0.0203\ O_2\} \longleftarrow$$

An energy rate balance reduces at steady state to read

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{H_2O}} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{H_2O}}}^{0} + \bar{h}_{H_2O}(T_1) - \left[0.9594\,\bar{h}_{H_2O}(T_2) + 0.0406\,\bar{h}_{H_2}(T_2) + 0.0203\,\bar{h}_{O_2}(T_2)\right]$$

Thus

$$\frac{\dot{Q}_{cv}}{\dot{n}_{H_2O}} = 0.9594\left[\bar{h}^{\circ}_f + \bar{h}(T_2) - \bar{h}(537)\right]_{H_2O} + 0.0406\left[\cancel{\bar{h}^{\circ}_f}^{0} + \bar{h}(T_2) - \bar{h}(537)\right]_{H_2} +$$
$$0.0203\left[\cancel{\bar{h}^{\circ}_f}^{0} + \bar{h}(T_2) - \bar{h}(537)\right]_{O_2} - \left[\bar{h}^{\circ}_f + \bar{h}(T_1) - \bar{h}(537)\right]_{H_2O}$$

With ideal gas table data

$T_1 = T_{sat}$ at 15 lbf/in² = 213 °F = 213°F (673°R)

$$\frac{\dot{Q}_{cv}}{\dot{n}_{H_2O}} = 0.9594\left[-104{,}040 + 46{,}836 - 4258\right] + 0.0406\left[33921.6 - 3640.3\right] +$$
$$0.0203\left[37{,}412 - 3725.1\right] - \left[-104{,}040 + 5356 - 4258\right]$$

$$= 45{,}888\ \text{Btu/lbmol}\,(H_2O) \longleftarrow$$

PROBLEM 14.37 (Cont'd.) Alternative IT solution - Page 2

IT Code

```
p1 = 15  // lbf/in², saturated vapor
T1 = Tsat_P("Steam",p1)
T2 = 4500  // °R
p2 = 1  // atm

// H2O → (1 - x) H2O + x H2 + x/2 O2
ndot1 = 1  // Do calculations on the basis of 1 lbmol of H2O entering.
ndotH2O_2 = 1 - x
ndotH2_2 = x
ndotO2_2 = x/2
ndot2 = ndotH2O_2 + ndotH2_2 + ndotO2_2
yH2O = ndotH2O_2 / ndot2
yH2 = ndotH2_2 / ndot2
yO2 = ndotO2_2 / ndot2
pref = 1  // atm

// For the reaction H2O ↔ H2 + 1/2 O2
K = ((yH2 * yO2^0.5) / yH2O) * (p2 / pref)^0.5
//Data from Table A-27are stored in EQH2O.LUT.
log(K) = LOOKUPVAL(EQH2O,2,T2,3)

0 = Qdot + ndot1 * hH2O_1 - (ndotH2O_2 * hH2O_2 + ndotH2_2 * hH2_2 + ndotO2_2 * hO2_2)
// Note: Use the ideal gas function for H2O in order to get the correct
// reference value for H2O in the calculations. This results in only a slight error.
hH2O_1 = h_T("H2O", T1)
hH2O_2 = h_T("H2O",T2)
hH2_2 = h_T("H2",T2)
hO2_2 = h_T("O2",T2)
```

IT Results

$K = 0.00597$

$\dot{n}_{H_2O,2} / \dot{n}_{H_2O,1} = 0.9594$ lbmol/lbmol(H_2O entering)

$\dot{n}_{H_2,2} / \dot{n}_{H_2O,1} = 0.04061$ lbmol/lbmol(H_2O entering)

$\dot{n}_{O_2,2} / \dot{n}_{H_2O,1} = 0.0203$ lbmol/lbmol(H_2O entering)

$\dot{Q}_{cv} / \dot{n}_{H_2O,1} = 4.582 \times 10^4$ Btu/lbmol(H_2O entering)

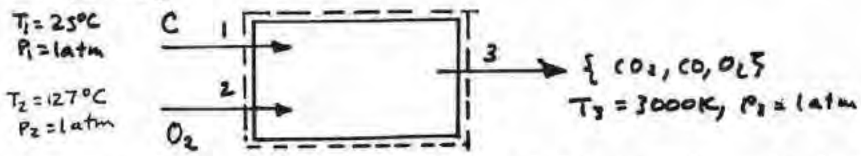
PROBLEM 14.38

KNOWN: Carbon at 25°C, 1atm and O_2 at 127°C, 1atm enter a reactor operating at steady state with equal molar flow rates. An equilibrium mixture of CO_2, CO, and O_2 exits at 3000K, 1atm.

FIND: Determine the composition of the exiting mixture and the heat transfer, each per kmol of carbon entering.

SCHEMATIC & GIVEN DATA:

$\dot{n}_C = \dot{n}_{O_2}$



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure is at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The ideal gas model applies to the equilibrium mixture.

ANALYSIS: The reaction takes the form

$$1C + 1O_2 \rightarrow x\,O_2 + 2x\,CO + (1-2x)\,CO_2$$

The amount of mixture is $n = x + 2x + (1-2x) = 1+x$. At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[2x][x]^{1/2}}{[1-2x]}\left[\frac{P/P_{ref}}{1+x}\right]^{1/2} = \frac{2x}{1-2x}\left[\frac{x}{1+x}\right]^{1/2}$$

At 3000K, Table A-27 gives $\log_{10}K = -0.485 \Rightarrow K = 0.3273$. Solving $x = 0.218$. The equilibrium mixture has the following composition in kmol per kmol of carbon entering

$$\{0.218\,O_2,\ 0.436\,CO,\ 0.564\,CO_2\}$$

An energy rate balance reduces at steady state to read

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_C} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_C}}^{\,0} + \cancel{\bar{h}_C(T_1,P_1)}^{\,0} + \bar{h}_{O_2}(T_2,P_2) - [0.218\,\bar{h}_{O_2}(T_3) + 0.436\,\bar{h}_{CO}(T_3) + 0.564\,\bar{h}_{CO_2}(T_3)]$$

Thus

$$\frac{\dot{Q}_{cv}}{\dot{n}_C} = 0.218[\cancel{\bar{h}_f^\circ}^{\,0} + \bar{h}(T_3) - \bar{h}(298)]_{O_2} + 0.436[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO} + 0.564[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO_2} - [\bar{h}(T_2) - \bar{h}(298)]_{O_2}$$

With data from the ideal gas tables

$$\frac{\dot{Q}_{cv}}{\dot{n}_C} = 0.218[106{,}780 - 8682] + 0.436[-110{,}530 + 102{,}210 - 8669] + 0.564[-393{,}520 + 162{,}226 - 9364] - [11{,}711 - 8682]$$

$$= -124{,}782\ \text{kJ/kmol(C)}$$

IT can be used as an alternative to iteration using a hand calculator and to using table data in the energy balance. The IT program follows.

PROBLEM 14.38 (Cont'd.)-Page2 Alternative IT solution.

IT Code

```
p1 = 1  // atm
T1 = 25 + 273.15 // K
p2 = 1  // atm
T2 = 127 + 273.15  // K
ndot1 = 1  //  Do all calculations on the basis of 1 kmol of carbon entering.
ndot2 = ndot1  //  Entering streams have equal molar flow rates.
T3 = 2727 + 273.15 // K
p3 = 1  // atm

//  C + O2  →  x O2 + 2x CO + (1 - 2x) CO2
ndotO2_3 = x
ndotCO_3 = 2*x
ndotCO2_3 = 1 - 2 * x
ndot3 = ndotO2_3 + ndotCO_3 + ndotCO2_3
yO2 = ndotO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yCO2 = ndotCO2_3/ ndot3
pref = 1  // atm

// For the reaction CO2  ↔  CO + 1/2 O2
K = ((yCO * yO2^0.5) / yCO2) * (p3 / pref)^0.5
// Data from Table A-27 are stored in EQCO2a.LUT.
log(K) = LOOKUPVAL(EQCO2A,1,T3,3)

0 = Qdot + ndot1*hC + ndot2*hO2_2 - (ndotCO2_3*hCO2_3 +
ndotCO_3*hCO_3 + ndotO2_3*hO2_3)
hC = 0  //  See Table A-25.
hO2_2 = h_T("O2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hO2_3 = h_T("O2",T3)
```

IT Results

$K = 0.3275$

$\dot{n}_{CO_2,3} / \dot{n}_1 = 0.5637$ kmol/kmol(C)

$\dot{n}_{CO,3} / \dot{n}_1 = 0.4363$ kmol/kmol(C)

$\dot{n}_{O_2,3} / \dot{n}_1 = 0.2181$ kmol/kmol(C)

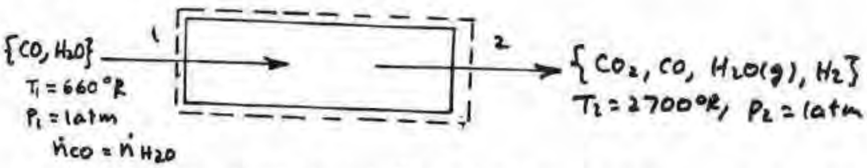
$\dot{Q}_{cv} / \dot{n}_1 = -1.246 \times 10^5$ kJ/kmol(C)

PROBLEM 14.39

KNOWN: An equimolar mixture of CO and $H_2O(g)$ at 200°F, 1atm enters a reactor operating at steady state. An equilibrium mixture of CO_2, CO, $H_2O(g)$, and H_2 exits at 2700°R, 1atm.

FIND: Determine the heat transfer per lbmol of CO entering.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure is at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The ideal gas model applies to the incoming and outgoing mixtures.

ANALYSIS: The reaction takes the form

$$1\,CO + 1\,H_2O \rightarrow x\,CO_2 + (1-x)\,CO + (1-x)\,H_2O + x\,H_2$$

The amount of mixture is $n = x + (1-x) + (1-x) + x = 2$. At equilibrium $CO_2 + H_2 \rightleftarrows CO + H_2O$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[1-x][1-x]}{[x][x]}\left[\frac{P/P_{ref}}{2}\right]^{1+1-1-1} = \left(\frac{1-x}{x}\right)^2 \Rightarrow x = \frac{1}{1+\sqrt{K}}$$

At 2700°R, Table A-27 gives $\log_{10} K = 0.4035 \Rightarrow K = 2.5322$. Solving, $x = 0.386$.

At steady state an energy rate balance reduces to read

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{co}} - \frac{\cancel{\dot{W}_{cv}}^{\,0}}{\dot{n}_{co}} + \bar{h}_{co}(T_1) + \bar{h}_{H_2O}(T_1) - \left[0.386\,\bar{h}_{CO_2}(T_2) + 0.614\,\bar{h}_{co}(T_2) + 0.614\,\bar{h}_{H_2O}(T_2) + 0.386\,\bar{h}_{H_2}(T_2)\right]$$

Thus

$$\frac{\dot{Q}_{cv}}{\dot{n}_{co}} = 0.386\left[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(537)\right]_{CO_2} + 0.614\left[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(537)\right]_{CO} + 0.614\left[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(537)\right]_{H_2O} + 0.386\left[\cancel{\bar{h}_f^\circ}^{\,0} + \bar{h}(T_2) - \bar{h}(537)\right]_{H_2} - \left\{1\left[\bar{h}_f^\circ + \bar{h}(T_1) - \bar{h}(537)\right]_{CO} + 1\left[\bar{h}_f^\circ + \bar{h}(T_1) - \bar{h}(537)\right]_{H_2O}\right\}$$

With data from the ideal gas tables

$$\frac{\dot{Q}_{cv}}{\dot{n}_{co}} = 0.386\left[-169{,}300 + 30{,}581 - 4027.5\right] + 0.614\left[-47{,}540 + 20{,}434 - 3725.1\right] + 0.614\left[-104{,}040 + 24{,}957 - 4258\right] + 0.386\left[19237.8 - 3640.3\right] - \left\{1\left[-47{,}540 + 4586.6 - 3725.1\right] + 1\left[-104{,}040 + 5250 - 4258\right]\right\}$$

$$= 30{,}547 \text{ Btu/lbmol(CO)}$$

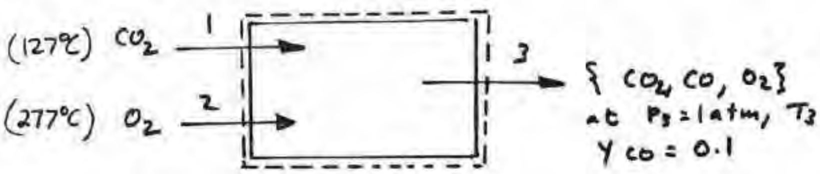
Alternatively, a solution using IT can be developed.

PROBLEM 14.40

KNOWN: CO_2 and O_2 in a 1:2 molar ratio enter a reactor operating at steady state in separate streams at 1 atm and 127°C, 277°C respectively. An equilibrium mixture of CO_2, CO, and O_2 exits at 1 atm. The mole fraction of CO in the exiting mixture is 0.1

FIND: Determine the heat transfer per kmol of CO_2 entering.

SCHEMATIC & GIVEN DATA:



$P_1 = P_2 = 1$ atm

$\dot{n}_{O_2}/\dot{n}_{CO_2} = 2$

ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The equilibrium mixture is modeled as an ideal gas.

ANALYSIS: The first step is to determine T_3 using equilibrium principles. The reaction takes the form

$$1\,CO_2 + 2O_2 \rightarrow x\,CO + (1-x)\,CO_2 + \left(\frac{4+x}{2}\right)O_2$$

The amount of mixture is $n = x + (1-x) + \left(\frac{4+x}{2}\right) = \frac{6+x}{2}$. From the given data $y_{CO} = 0.1$. Thus

$$0.1 = \frac{x}{(6+x)/2} \Rightarrow x = 0.316$$

Thus

$$1\,CO_2 + 2O_2 \rightarrow 0.316\,CO + 0.684\,CO_2 + 2.158\,O_2$$

At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly, Eq 14.35 takes the form

$$K = \frac{[0.316][2.158]^{1/2}}{0.684}\left[\frac{P/P_{ref}}{3.158}\right]^{1/2} = \left(\frac{0.316}{0.684}\right)\left(\frac{2.158}{3.158}\right)^{1/2} = 0.3819$$

Thus $\log_{10} K = -0.418$. Interpolation in Table A-27 gives $T_3 = 3044$ K.

At steady state an energy rate balance reduces to give

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{CO_2}} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{CO_2}}}^{\,0} + \bar{h}_{CO_2}(T_1) + 2\bar{h}_{O_2}(T_1) - \left[0.316\,\bar{h}_{CO}(T_3) + 0.684\,\bar{h}_{CO_2}(T_3) + 2.158\,\bar{h}_{O_2}(T_3)\right]$$

or

$$\frac{\dot{Q}_{cv}}{\dot{n}_{CO_2}} = 0.316\left[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)\right]_{CO} + 0.684\left[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)\right]_{CO_2} + 2.158\left[\cancel{\bar{h}_f^\circ}^{\,0} + \bar{h}(T_3) - \bar{h}(298)\right]_{O_2} - \left[\bar{h}_f^\circ + \bar{h}(T_1) - \bar{h}(298)\right]_{CO_2} - 2\left[\bar{h}(T_2) - \bar{h}(298)\right]_{O_2}$$

With data from ~~the ideal gas~~ tables

$$\frac{\dot{Q}_{cv}}{\dot{n}_{CO_2}} = 0.316\left[-110{,}530 + 103{,}849 - 8669\right] + 0.684\left[-393{,}520 + 164{,}967 - 9364\right] + 2.158\left[108{,}538 - 8682\right] - \left[(-393{,}520) + 13{,}372 - 9364\right] - 2\left[14{,}770 - 8682\right]$$

$$= 425{,}239 \text{ kJ/kmol}(CO_2)$$

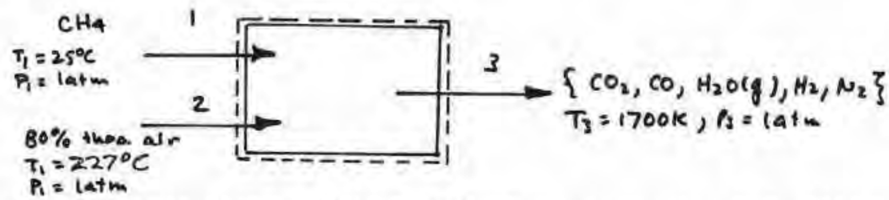
$\dot{Q}_{cv}/\dot{n}_{CO_2}$

PROBLEM 14.41

KNOWN: CH_4 at 25°C, 1atm enters a reactor operating at steady state and burns with 80% of theoretical air entering at 227°C, 1atm. An equilibrium mixture of CO_2, CO, $H_2O(g)$, H_2 and N_2 exits at 1700K, 1atm.

FIND: Determine the composition of the exiting mixture and the heat transfer, each per kmol of CH_4 entering.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}=0$ and negligible effects of kinetic and potential energy. (2) The equilibrium mixture is modeled as an ideal gas. (3) N_2 is inert.

ANALYSIS: The balanced reaction equation for complete combustion of CH_4 with the theoretical amount of air is given by Eq. 13.4. Accordingly, combustion with 80% of theoretical air to form CO_2, CO, $H_2O(g)$, H_2, and N_2 takes the form

$$CH_4 + (0.8)2[O_2 + 3.76N_2] \rightarrow x\,CO_2 + (1-x)CO + (2.2-x)H_2O + (x-0.2)H_2 + 6.016N_2$$

The amount of mixture is $n = x + (1-x) + (2.2-x) + (x-0.2) + 6.016 = 9.016$.

At equilibrium $CO_2 + H_2 \rightleftarrows CO + H_2O$. Accordingly, Eq. 14.35 takes the form

① $$K = \frac{[1-x][2.2-x]}{[x][x-0.2]}\left[\frac{P/P_{ref}}{9.016}\right]^{1+1-1-1} = \left(\frac{1-x}{x}\right)\left(\frac{2.2-x}{x-0.2}\right)$$

At 1700K, Table A-27 gives $\log_{10}K = 0.530 \Rightarrow K = 3.3884$. Solving, $x = 0.5674$. Thus, the composition of the exiting equilibrium mixture in kmol per kmol of CH_4 is

$$\{0.5674\,CO_2,\ 0.4326\,CO,\ 1.6326\,H_2O,\ 0.3674\,H_2,\ 6.016\,N_2\}$$

At steady state an energy rate balance reduces to read

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{CH_4}}}^{\,0} + \bar{h}_{CH_4}(T_1) + [1.6\bar{h}_{O_2}(T_2) + 6.016\bar{h}_{N_2}(T_2)] - [0.5674\bar{h}_{CO_2} + 0.4326\bar{h}_{CO} + 1.6326\bar{h}_{H_2O} + 0.3674\bar{h}_{H_2} + 6.016\bar{h}_{N_2}]$$

Thus

$$\frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}} = 0.5674[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO_2} + 0.4326[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO} + 1.6326[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{H_2O} + 0.3674[\cancel{\bar{h}_f^\circ}^{\,0} + \bar{h}(T_3) - \bar{h}(298)]_{H_2} + 6.016[\cancel{\bar{h}_f^\circ}^{\,0} + \bar{h}(T_3) - \bar{h}(298)]_{N_2} - [\bar{h}_f^\circ]_{CH_4} - 1.6[\bar{h}(T_2) - \bar{h}(298)]_{O_2} - 6.016[\bar{h}(T_2) - \bar{h}(298)]_{N_2}$$

With data from the ideal gas tables

$$\frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}} = 0.5674[-393{,}520 + 82{,}856 - 9364] + 0.4326[-110{,}530 + 54{,}609 - 8669] + 1.6326[-241{,}820 + 67{,}589 - 9904] + 0.3674[51805 - 8468] + 6.016[54{,}099 - 8669] - [-74{,}850] - 1.6[14{,}770 - 8682] - 6.016[14{,}581 - 8669]$$

$$= -191{,}558 \text{ kJ/(kmol } CH_4)$$

1. (P/P_{ref}) drops out of this expression.

PROBLEM 14.41 (Cont'd.) Page 2 **Alternative IT solution.**

IT Code

```
T1 = 25 + 273.15  // K
p1 = 1  // atm
T2 = 227 + 273.15  // K
p2 = 1  // atm
T3 = 1427 + 273.15  // K
p3 = 1  // atm
ndotCH4 = 1  //  Do calculations on the basis of 1 kmol of CH4 entering.

//  CH4 + 1.6 (O2 + 3.76 N2)  →  x CO2 + (1 - x) CO + (2.2 - x) H2O + (x - 0.2) H2 + 6.016 N2
ndotCO2_3 = x
ndotCO_3 = 1 - x
ndotH2O_3 = 2.2 - x
ndotH2_3 = x - 0.2
ndotN2_3 = 6.016
ndot3 = ndotCO2_3 + ndotCO_3 + ndotH2O_3 + ndotH2_3 + ndotN2_3
yCO2 = ndotCO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yH2O = ndotH2O_3 / ndot3
yH2 = ndotH2_3 / ndot3
pref = 1  // atm

// For the reaction CO2 + H2  ↔  CO + H2O
K = (yCO * yH2O) / (yCO2 * yH2) * (p3 / pref)^0
//Data from Table A-27are stored in EQWATGAS.LUT.
log(K) = LOOKUPVAL(EQWATGAS,1,T3,3)

0 = Qdot + ndotCH4*hCH4_1 + 1.6*hO2_2 + 6.016*hN2_2 - (ndotCO2_3*hCO2_3 +
          ndotCO_3*hCO_3 + ndotH2O_3*hH2O_3 + ndotH2_3*hH2_3 + 6.016*hN2_3)
hCH4_1 = h_T("CH4",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hH2O_3 = h_T("H2O",T3)
hH2_3 = h_T("H2",T3)
hN2_3 = h_T("N2",T3)
```

IT Results

K = 3.389

$\dot{n}_{CO,3}/\dot{n}_{CH_4}$ = 0.4326 kmol/kmol(CH_4)

$\dot{n}_{CO_2,3}/\dot{n}_{CH_4}$ = 0.5674 kmol/kmol(CH_4)

$\dot{n}_{H_2,3}/\dot{n}_{CH_4}$ = 0.3674 kmol/kmol(CH_4)

$\dot{n}_{H_2O,3}/\dot{n}_{CH_4}$ = 1.633 kmol/kmol(CH_4)

$\dot{n}_{N_2,3}/\dot{n}_{CH_4}$ = 6.016 kmol/kmol(CH_4)

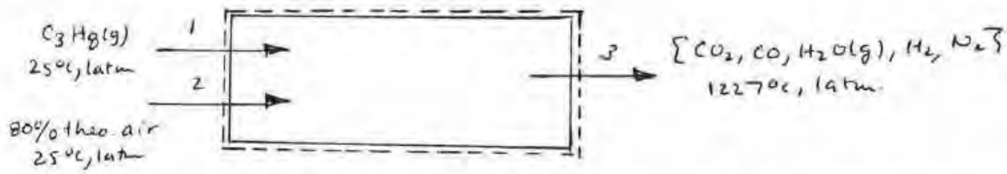
Qdot = -1.917×10^5 kJ/kmol(CH_4)

PROBLEM 14.42

KNOWN: C_3H_8 at 25°C, 1atm enters a reactor at steady state and burns with 80% of theoretical air entering separately at 25°C, 1atm. An equilibrium mixture of $\{CO_2, CO, H_2O(g), H_2, N_2\}$ exits at 1227°C, 1atm

FIND: Determine the heat transfer, in kJ per kmol of C_3H_8

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure is at steady state with $\dot{W}_{cv}=0$ and negligible effects of kinetic and potential energy. (2) The exiting equilibrium mixture is modeled as an ideal gas mixture. (3) N_2 is inert.

ANALYSIS: The complete combustion of C_3H_8 with the theoretical amount of air is described by

$$C_3H_8 + 5[O_2+3.76N_2] \rightarrow 3CO_2 + 4H_2O + 18.8N_2$$

Accordingly, the reaction of C_3H_8 with 80% of theoretical air to form the specified mixture is

$$C_3H_8 + 4[O_2+3.76N_2] \rightarrow x\,CO_2 + a\,CO + b\,H_2O + c\,H_2 + 15.04N_2$$

C: $3 = x + a \Rightarrow a = 3 - x$

H: $8 = 2b + 2c$

O: $8 = 2x + a + b$

$= 2x + (3-x) + b \Rightarrow b = 5 - x$

$\Rightarrow c = 4 - b = 4-(5-x) = x - 1$

That is

$$C_3H_8 + 4[O_2+3.76N_2] \rightarrow x\,CO_2 + (3-x)CO + (5-x)H_2O + (x-1)H_2 + 15.04N_2$$

At equilibrium $CO_2 + H_2 \rightleftarrows CO + H_2O$. Accordingly, Eq 14.35 takes the form

$$K = \frac{[3-x][5-x]}{[x][x-1]}\left[\frac{p/p_{ref}}{n}\right]^0$$

At 1500 K, Table A-27 gives $\log_{10}K = 0.4035 \Rightarrow K = 2.5322$. Solving $x = 1.818$. Accordingly

$$C_3H_8 + 4[O_2+3.76N_2] \rightarrow 1.818\,CO_2 + 1.182\,CO + 3.182\,H_2O + 0.818\,H_2 + 15.04N_2$$

An energy rate balance at steady state reduces to read

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{C_3H_8}} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{C_3H_8}}}^{\,0} + \bar{h}_{C_3H_8}(T_1) + [4\bar{h}_{O_2}(T_2) + 15.04\bar{h}_{N_2}(T_2)] - [1.818\bar{h}_{CO_2}(T_3) + 1.182\bar{h}_{CO}(T_3) + 3.182\bar{h}_{H_2O}(T_3) + 0.818\bar{h}_{H_2}(T_3) + 15.04\bar{h}_{N_2}]$$

Rearranging and noting that $\bar{h}_f^\circ = 0$ for H_2 and N_2

$$\frac{\dot{Q}_{cv}}{\dot{n}_{C_3H_8}} = 1.818[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO_2} + 1.182[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO} + 3.182[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{H_2O} + 0.818[\bar{h}(T_3) - \bar{h}(298)]_{H_2} + 15.04[\bar{h}(T_3) - \bar{h}(298)]_{N_2} - [\bar{h}_f^\circ]_{C_3H_8}$$

With data from the ideal gas tables

$$\frac{\dot{Q}_{cv}}{\dot{n}_{C_3H_8}} = 1.818[-393,520 + 71,078 - 9364] + 1.182[-110,530 + 47,517 - 8669] + 3.182[-241,820 + 57,999 - 9904] + 0.818[44,738 - 8468] + 15.04[47,073 - 8669] - (-103,850)$$

$$= -593,269 \frac{kJ}{kmol(C_3H_8)}$$

PROBLEM 14.42 (Cont'd.) Page 2 **Alternative IT solution**

IT can be used as an alternative to iteration with a hand calculator and to using table data in the energy balance. The IT program follows.

IT Code

```
T1 = 25 + 273.15  // K
T2 = 25 + 273.15  // K
T3 = 1227 + 273.15  // K
p3 = 1 // atm
ndotfuel = 1  //  Do calculations on the basis of 1 kmol of C3H8 entering.

// C3H8 + 4 (O2 + 3.76 N2) → x CO2 + (3 - x) CO + (5 - x) H2O + (x - 1) H2 + 15.04 N2
ndotCO2_3 = x
ndotCO_3 = 3 - x
ndotH2O_3 = 5 - x
ndotH2_3 = x - 1
ndotN2_3 = 15.04
ndot3 = ndotCO2_3 + ndotCO_3 + ndotH2O_3 + ndotH2_3 + ndotN2_3
yCO2 = ndotCO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yH2O = ndotH2O_3 / ndot3
yH2 = ndotH2_3 / ndot3
pref = 1 // atm

// For the reaction CO2 + H2 ↔ CO + H2O
K = ((yCO * yH2O) / (yCO2 * yH2)) * (p3/ pref)^0
// Data from Table A-27 are stored in EQWATGAS.LUT.
log(K) = LOOKUPVAL(EQWATGAS,1,T3,3)

0 = Qdot + ndotfuel*hC3H8 + 4*hO2_2 + 15.04*hN2_2 - (ndotCO2_3*hCO2_3 +
ndotCO_3*hCO_3 + ndotH2O_3*hH2O_3 + ndotH2_3*hH2_3 + ndotN2_3*hN2_3)
hC3H8 = h_T("C3H8",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hH2O_3 = h_T("H2O",T3)
hH2_3 = h_T("H2",T3)
hN2_3 = h_T("N2",T3)
```

IT Results

$K = 2.533$

$\dot{n}_{CO,3}/\dot{n}_{fuel} = 1.182$ kmol/lmol(C_3H_8)

$\dot{n}_{CO_2,3}/\dot{n}_{fuel} = 1.818$ kmol/lmol(C_3H_8)

$\dot{n}_{H_2,3}/\dot{n}_{fuel} = 0.8175$ kmol/lmol(C_3H_8)

$\dot{n}_{H_2O,3}/\dot{n}_{fuel} = 3.182$ kmol/lmol(C_3H_8)

$\dot{n}_{N_2,3}/\dot{n}_{fuel} = 15.04$ kmol/lmol(C_3H_8)

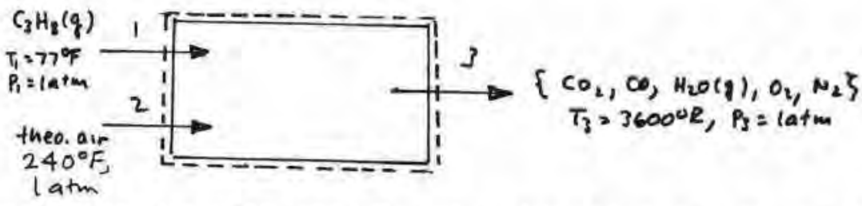
$\dot{Q}_{cv}/\dot{n}_{fuel} = -5.933 \times 10^5$ kJ/kmol(C_3H_8)

PROBLEM 14.43

KNOWN: $C_3H_8(g)$ at 77°F, 1atm enters a reactor operating at steady state and burns with the theoretical amount of air entering at 240°F, 1atm. An equilibrium mixture of CO_2, CO, $H_2O(g)$, O_2, and N_2 exits at 3600°R, 1atm.

FIND: Determine the heat transfer per lbmol of fuel entering.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure is at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The exiting equilibrium mixture is modeled as an ideal gas mixture. (3) N_2 is inert.

ANALYSIS: The reaction of $C_3H_8(g)$ with the theoretical amount of air to form CO_2, CO, $H_2O(g)$, O_2, and N_2 is given by

$$C_3H_8 + 5O_2 + 18.8N_2 \rightarrow (3-x)CO_2 + xCO + \frac{x}{2}O_2 + 4H_2O + 18.8N_2 \qquad (1)$$

The amount of mixture is $n = (3-x) + x + \frac{x}{2} + 4 + 18.8 = 25.8 + \frac{x}{2}$

At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly, Eq 14.35 takes the form

$$K = \frac{[x][\frac{x}{2}]^{1/2}}{[3-x]}\left[\frac{P/P_{ref}}{51.6+\frac{x}{2}}\right]^{1/2} = \frac{x}{3-x}\left[\frac{x}{51.6+x}\right]^{1/2}$$

At 3600°R, Table A-27 gives $\log_{10}K = -2.884 \Rightarrow K = 0.001306$. Solving, $x = 0.091$. Accordingly, Eq (1) becomes

$$C_3H_8 + 5O_2 + 18.8N_2 \rightarrow 2.909CO_2 + 0.091CO + 0.0455O_2 + 4H_2O + 18.8N_2$$

An energy rate balance at steady state reduces to read

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{C_3H_8}} - \frac{\dot{W}_{cv}^{\,0}}{\dot{n}_{C_3H_8}} + \bar{h}_{C_3H_8}(T_1) + [5\bar{h}_{O_2}(T_1) + 18.8\bar{h}_{N_2}(T_1)] - [2.909\,\bar{h}_{CO_2}(T_3) + 0.091\,\bar{h}_{CO}(T_3) + 0.0455\,\bar{h}_{O_2}(T_3) + 4\,\bar{h}_{H_2O}(T_3) + 18.8\,\bar{h}_{N_2}(T_3)]$$

Rearranging and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$\frac{\dot{Q}_{cv}}{\dot{n}_{C_3H_8}} = 2.909[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{CO_2} + 0.091[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{CO} + 0.0455[\bar{h}(T_3) - \bar{h}(537)]_{O_2}$$
$$+ 4[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{H_2O} + 18.8[\bar{h}(T_3) - \bar{h}(537)]_{N_2} - [\bar{h}_f^\circ]_{C_3H_8}$$
$$- 5[\bar{h}(T_2) - \bar{h}(537)]_{O_2} - 18.8[\bar{h}(T_2) - \bar{h}(537)]_{N_2}$$

With data from the ideal gas tables

$$\frac{\dot{Q}_{cv}}{\dot{n}_{C_3H_8}} = 2.909[-169{,}300 + 43{,}411 - 4027.5] + 0.091[-47{,}540 + 28127 - 3725.1] +$$
$$0.0455[29{,}174 - 3725.1] + 4[-104{,}040 + 35541 - 4258] +$$
$$18.8[27875 - 3729.5] - [-44{,}680] - 5[4879 - 3725] - 18.8[4865 - 3730]$$

$$= -198{,}396 \text{ Btu/lbmol}(C_3H_8)$$

PROBLEM 14.43 (Cont'd.)-Page 2 Alternative IT solution

IT can be used as an alternative to iteration with a hand calculator and to using table data in the energy balance. The IT program follows.

IT Code

```
T1 = 77 + 459.67 // °R
T2 = 240 + 459.67 // °R
T3 = 3140 + 460 // °R
p3 = 1 // atm
ndotfuel = 1 // Do calculations on the basis of 1 lbmol of C3H8 entering.

// C3H8 + 5 (O2 + 3.76 N2) → (3 - x) CO2 + x CO + x/2 O2 + 4 H2O + 18.8 N2
ndot_CO2 = 3 - x
ndot_CO = x
ndot_O2 = x / 2
ndot_H2O = 4
ndot_N2 = 18.8
ndot3 = ndot_CO2 + ndot_CO + ndot_O2 + ndot_H2O + ndot_N2
yCO2 = ndot_CO2 / ndot3
yCO = ndot_CO / ndot3
yO2 = ndot_O2 / ndot3
pref = 1 // atm

// For the reaction CO2 ↔ CO + 1/2 O2
K = ((yCO * yO2^0.5) / yCO2) * (p3 / pref)^.5
// Data from Table A-27are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,2,T3,3)

0 = Qdot + ndotfuel*hC3H8 + 5*hO2_2 + 18.8*hN2_2 - (ndot_CO2*hCO2_3 +
ndot_CO*hCO_3 + ndot_O2*hO2_3 + ndot_H2O*hH2O_3 + 18.8*hN2_3)
hC3H8 = h_T("C3H8",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hO2_3 = h_T("O2",T3)
hH2O_3 = h_T("H2O",T3)
hN2_3 = h_T("N2",T3)
```

IT Results

K = 0.001306

$\dot{n}_{CO,3} / \dot{n}_{fuel}$ = 0.09071 lbmol/lbmol(C_3H_8)

$\dot{n}_{CO_2,3} / \dot{n}_{fuel}$ = 2.909 lbmol/lbmol(C_3H_8)

$\dot{n}_{O_2,3} / \dot{n}_{fuel}$ = 0.04536 lbmol/lbmol(C_3H_8)

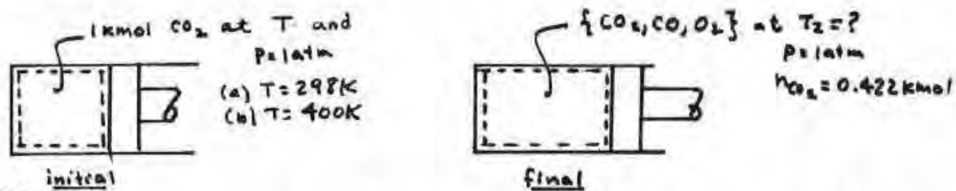
$\dot{Q}_{cv} / \dot{n}_{fuel}$ = -1.985E5 Btu/lbmol(C_3H_8)

PROBLEM 14.44

KNOWN: One kmol of CO_2 initially at T, 1 atm is heated at constant pressure until a final state is attained consisting of an equilibrium mixture of CO_2, CO, an O_2 in which 0.422 kmol of CO_2 is present.

FIND: Determine the heat transfer and the work for (a) T = 298K, (b) T = 400K.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The system is shown in the accompanying figure by the dashed line. (2) Changes in kinetic and potential energy are negligible. (3) The final mixture is an equilibrium mixture. (4) Pressure remains constant during the process. (4) Ideal gas model applies to the CO_2 initially present and to the final equilibrium mixture.

ANALYSIS: The first step is to determine T_2: The reaction takes the form

$$1\,CO_2 \rightarrow 0.422\,CO_2 + \alpha\,CO + \beta\,O_2$$

C: $1 = 0.422 + \alpha \Rightarrow \alpha = 0.578$

O: $2 = 2(0.422) + 0.578 + 2\beta \Rightarrow \beta = 0.289$

Thus

$$1\,CO_2 \rightarrow 0.422\,CO_2 + 0.578\,CO + 0.289\,O_2$$

At equilibrium $CO_2 \rightleftarrows CO + 1/2\,O_2$. Accordingly Eq. 14.35 takes the form

$$K = \frac{(0.578)(0.289)^{1/2}}{0.422}\left[\frac{p/p_{ref}}{1.299}\right]^{1/2} \Rightarrow K = \left[\frac{0.578}{0.422}\right]\left[\frac{0.289}{1.289}\right]^{1/2} \Rightarrow \log_{10} K = -0.1881$$

Interpolating in Table A-27, $T_2 \approx 3200K$.

(a) T = 298K

The work is given by $W = \int_1^2 p\,dV = p(V_2 - V_1)$. Using the ideal gas equation of state, $pV_1 = n_1\bar{R}T_1$, $pV_2 = n_2\bar{R}T_2$. Thus

$$W = \bar{R}[n_2T_2 - n_1T_1] = \left(8.314\frac{kJ}{kmol\cdot K}\right)\left((1.289)(3200) - (1)(298)\right)kmol\cdot K$$

$$= 31\,816\ kJ \longleftarrow W$$

An energy balance gives $\Delta U = Q - W$. With $W = p(V_2 - V_1)$ this yields

$$Q = (U_2 - U_1) + p(V_2 - V_1) = (U_2 + p_2V_2) - (U_1 + p_1V_1) = H_2 - H_1$$

Accordingly

$$Q = [0.422\,\bar{h}_{CO_2}(T_2) + 0.578\,\bar{h}_{CO}(T_2) + 0.289\,\bar{h}_{O_2}] - 1\,\bar{h}_{CO_2}(T_1)$$

$$= 4.22[\bar{h}_f^o + \bar{h}(T_2) - \bar{h}(298)]_{CO_2} + 0.578[\bar{h}_f^o + \bar{h}(T_2) - \bar{h}(298)]_{CO} +$$

$$0.289[\cancel{\bar{h}_f^o} + \bar{h}(T_2) - \bar{h}(298)]_{O_2} - (\bar{h}_f^o)_{CO_2}$$

with data from the ideal gas tables

$$Q = 0.422[-393{,}520 + 174{,}695 - 9364] + 0.578[-110{,}530 + 109{,}667 - 8669] +$$

$$0.289[114{,}809 - 8682] - [-393{,}520] = 322{,}386\ kJ \longleftarrow Q$$

PROBLEM 14.44 (Continued)-Page 2

(b) $T = 400K$

As in part (a)

$$W = \bar{R}\,(n_2 T_2 - n_1 T_1)$$
$$= (8314)((1.289)(3200) - (1)(400))$$
$$= 30{,}968 \text{ kJ} \longleftarrow W$$

And as in part (a), $Q = H_2 - H_1$. The only difference is the evaluation of the initial CO_2 enthalpy:

$$\left[\bar{h}_f^{\circ} + \bar{h}(400) - \bar{h}(298)\right]_{CO_2} = \left[-393{,}520 + 13{,}372 - 9364\right]$$
$$= -389{,}512 \text{ kJ/kmol}(CO_2)$$

Then

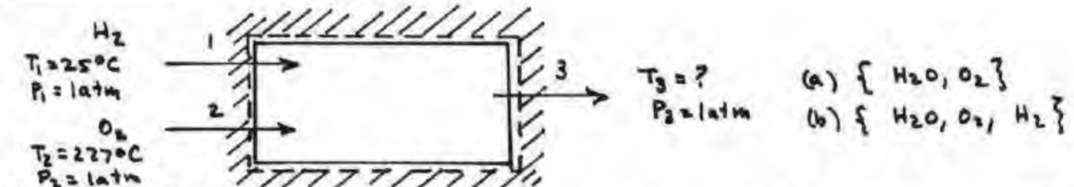
$$Q = 318{,}378 \text{ kJ} \longleftarrow Q$$

PROBLEM 14.45

KNOWN: H_2 at 25°C, 1atm enters an insulated reactor operating at steady state and reacts with 250% excess oxygen entering at 227°C, 1atm. Products of combustion exit at 1 atm.

FIND: Determine the temperature if (a) combustion is complete, (b) an equilibrium mixture of H_2O, H_2, and O_2 exits.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) In part (a), combustion is complete. In part (b), an equilibrium mixture exits. (3) The ideal gas model is applicable.

ANALYSIS: (a) For complete combustion with 250% excess O_2, the reaction equation is

$$H_2 + 1.75O_2 \rightarrow H_2O + 1.25O_2$$

An energy rate balance at steady state reduces to give

$$0 = \underbrace{\cancel{\dot{Q}_{cv}}^{0}}_{\dot{n}_{H_2}} - \underbrace{\cancel{\dot{W}_{cv}}^{0}}_{\dot{n}_{H_2}} + [\bar{h}_{H_2}]_1 + [1.75\,\bar{h}_{O_2}]_2 - [\bar{h}_{H_2O} + 1.25\,\bar{h}_{O_2}]_3$$

Thus

$$0 = [\cancel{\bar{h}_f^o}^{0}]_{H_2} + 1.75[\cancel{\bar{h}_f^o}^{0} + \bar{h}(T_2) - \bar{h}(298)]_{O_2} - [\bar{h}_f^o + \bar{h}(T_3) - \bar{h}(298)]_{H_2O} - 1.25[\cancel{\bar{h}_f^o}^{0} + \bar{h}(T_3) - \bar{h}(298)]_{O_2}$$

or

$$\bar{h}_{H_2O}(T_3) + 1.25\,\bar{h}_{O_2}(T_3) = -[\bar{h}_f^o - \bar{h}(298)]_{H_2O} + 1.75[\bar{h}(T_2) - \bar{h}(298)]_{O_2} + 1.25\,\bar{h}_{O_2}(298)$$

$$= -[-241{,}820 - 9904] + 1.75[14{,}770 - 8682] + 1.25(8682)$$

$$= 273{,}231 \text{ kJ/kmol}$$

Solving iteratively with data from Table A-23, $T_3 = 3033$ K. ← (a) T_3

(b) For this case the reaction takes the form

$$H_2 + 1.75O_2 \rightarrow x\,H_2O + (1-x)H_2 + \left(\frac{3.5-x}{2}\right)O_2$$

The amount of product mixture is $n = x + (1-x) + \left(\frac{3.5-x}{2}\right) = \left(\frac{5.5-x}{2}\right)$. At equilibrium $H_2O \rightleftarrows H_2 + \frac{1}{2}O_2$. Accordingly, Eq. 14.35 takes the form

$$K(T_3) = \frac{[1-x]\left[\frac{3.5-x}{2}\right]^{1/2}}{x}\left[\frac{p/p_{ref}}{(5.5-x)/2}\right]^{1/2} = \frac{1-x}{x}\left[\frac{3.5-x}{5.5-x}\right]^{1/2} \qquad (1)$$

Another equation relating x and T_3 is obtained from an energy rate equation at steady state which reduces to give

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + [\bar{h}_{H_2}]_1 + 1.75[\bar{h}_{O_2}]_1 - \left[x\,\bar{h}_{H_2O} + (1-x)\bar{h}_{H_2} + \left(\frac{3.5-x}{2}\right)\bar{h}_{O_2}\right]_3$$

Noting that $\bar{h}_f^o = 0$ for H_2 and O_2 this becomes

$$0 = 1.75[\bar{h}_{O_2}(500) - \bar{h}_{O_2}(298)] - \Big\{x[(\bar{h}_f^o)_{H_2O} + \bar{h}_{H_2O}(T_3) - \bar{h}_{H_2O}(298)] + (1-x)[\bar{h}_{H_2}(T_3) - \bar{h}_{H_2}(298)] + \left(\frac{3.5-x}{2}\right)[\bar{h}_{O_2}(T_3) - \bar{h}_{O_2}(298)]\Big\}$$

or

$$x[-241{,}820 + \bar{h}_{H_2O}(T_3) - 9904] + (1-x)[\bar{h}_{H_2}(T_3) - 8468] + \left(\frac{3.5-x}{2}\right)[\bar{h}_{O_2}(T_3) - 8682] = \underbrace{1.75[14{,}770 - 8682]}_{10{,}654} \qquad (2)$$

Eqs. (1), (2) are simultaneous equations in x and T_3. Solving iteratively with table data we get

$$x = 0.9933, \; T_3 = 2926 \text{ K}$$ (b) x, T_3

ALTERNATIVE IT SOLUTION

The following IT solution provides an alternative to iteration using table data by hand, as shown previously:

IT Code

```
T1 = 25 + 273.15  // K
T2 = 227 + 273.15  // K
p3 = 1  // atm
ndot1 = 1  //  Do calculations on the basis of 1 kmol of H2 entering.

/* Use this section for part (a), comment out part (b).
// For part (a):   H2 + 1.75 O2  →  H2O + 1.25 O2
hr = h_T("H2",T1) + 1.75 * h_T("O2",T2)
hp = h_T("H2O",T3) + 1.25 * h_T("O2",T3)
hp = hr
*/

// For part (b)  H2 + 1.75 O2  →  x H2O + (1 - x) H2 + ((3.5 - x)/2) O2
ndotH2O_3 = x
ndotH2_3 = 1 - x
ndotO2_3 = (3.5 - x) / 2
ndot3 = (5.5 - x) / 2
yH2O = ndotH2O_3 / ndot3
yH2 = ndotH2_3 / ndot3
yO2 = ndotO2_3 / ndot3
pref = 1  // atm

// For the reaction  H2O  ↔  H2 + 1/2 O2
K = ((yH2 * yO2^0.5) / yH2O) * (p3 / pref)^0.5
//  Data from Table A-27are stored in EQH2O.LUT.
log(K) = LOOKUPVAL(EQH2O,1,T3,3)

hr = h_T("H2",T1) + 1.75 * h_T("O2",T2)
hp = ndotH2O_3 * hH2O_3 + ndotH2_3 * hH2_3 + ndotO2_3 * hO2_3
hp = hr
hH2O_3 = h_T("H2O",T3)
hH2_3 = h_T("H2",T3)
hO2_3 = h_T("O2",T3)
```

IT Results

Part (a): $T_3 = 3033$ K

Part (b): $x = 0.9533$, $T_3 = 2926$ K

PROBLEM 14.46

KNOWN: H_2 at 25°C, 1atm enters an insulated reactor operating at steady state and reacts with 250% excess oxygen entering at 227°C, 1atm. Products of combustion exit at 1atm.

FIND: Determine the rate of entropy production per kmol of H_2 entering if (a) combustion is complete, (b) an equilibrium mixture of H_2O, H_2, and O_2 exits.

SCHEMATIC & GIVEN DATA: See Problem 14.45 solution.

ENGINEERING MODEL: See Problem 14.45 solution.

ANALYSIS: (a) Complete combustion of H_2 with 250% excess O_2 is described by

$$H_2 + 1.75\,O_2 \rightarrow H_2O + 1.25\,O_2$$

The solution to Problem 14.45(a) gives 3033K as the temperature of the products.

At steady state an entropy rate balance reduces to give

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{H_2}} = (\bar{s}_{H_2O} + 1.25\,\bar{s}_{O_2})_2 - (\bar{s}_{H_2})_1 - 1.75(\bar{s}_{O_2})_2 \qquad (1)$$

As H_2 enters at 25°C, 1atm, Table A-23 gives $\bar{s}_{H_2} = 130.57$ kJ/kmol·K. O_2 enters at 500 K, 1atm. Thus, with Eq. 13.22 and data from Table A-23 $\bar{s}_{O_2} = \bar{s}^o_{O_2}(500) = 220.589$ kJ/kmol·K. The products exit as a mixture at 3033K, 1atm with composition $y_{O_2} = 1.25/2.25$, $y_{H_2O} = 1/2.25$. Thus, with Eq. 13.23 and data from Table A-23

$$\bar{s}_{H_2O} = \bar{s}^o_{H_2O}(3033) - \bar{R}\ln\frac{y_{H_2O}\,p_{ref}}{p_{ref}} = 286.88 - 8.314\ln\frac{1}{2.25} = 293.62 \text{ kJ/kmol·K}$$

$$\bar{s}_{O_2} = \bar{s}^o_{O_2}(3033) - \bar{R}\ln y_{O_2} = 284.84 - 8.314\ln\frac{1.25}{2.25} = 289.73 \text{ kJ/kmol·K}$$

Inserting values into Eq. (1)

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{H_2}} = (293.62) + (1.25)(289.73) - 130.57 - 1.75(220.589) = 139.18 \frac{\text{kJ}}{\text{kmol}(H_2)\cdot\text{K}} \longleftarrow (a)$$

(b) The reaction is described by

$$H_2 + 1.75\,O_2 \longrightarrow 0.955\,H_2O + 0.045\,H_2 + 1.2725\,O_2$$

The temperature of the products is obtained in the solution to Problem 14.45(b) as 2926 K.

At steady state an entropy rate balance reduces to give

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{H_2}} = (0.955\,\bar{s}_{H_2O} + 0.045\,\bar{s}_{H_2} + 1.2725\,\bar{s}_{O_2})_3 - (\bar{s}_{H_2})_1 - (\bar{s}_{O_2})_2 \qquad (2)$$

The values of $(\bar{s}_{H_2})_1$ and $(\bar{s}_{O_2})_2$ are the same as in part (a) above. The products exit as a mixture at 2926 K, 1atm with the composition $y_{H_2O} = 0.955/2.2725$, $y_{H_2} = 0.045/2.2725$, $y_{O_2} = 1.2725/2.2725$. Thus, with Eq. 13.23 and data from the ideal gas tables

$$\bar{s}_{H_2O} = \bar{s}^o_{H_2O}(2926) - \bar{R}\ln y_{H_2O} = 284.88 - 8.314\ln\frac{0.955}{2.2725} = 292.09 \text{ kJ/kmol·K}$$

$$\bar{s}_{H_2} = \bar{s}^o_{H_2}(2926) - \bar{R}\ln y_{H_2} = 201.85 - 8.314\ln\frac{0.045}{2.2725} = 234.46 \text{ kJ/kmol·K}$$

$$\bar{s}_{O_2} = \bar{s}^o_{O_2}(2926) - \bar{R}\ln y_{O_2} = 283.40 - 8.314\ln\frac{1.2725}{2.2725} = 288.22 \text{ kJ/kmol·K}$$

Inserting values into Eq. (2)

1 $$\frac{\dot{\sigma}_{cv}}{\dot{n}_{H_2}} = (.955)(292.09) + (0.045)(234.46) + (1.2725)(288.22) - 130.57 - 1.75(220.589) = 139.66 \frac{\text{kJ}}{\text{kmol}(H_2)\cdot\text{K}} \longleftarrow (b)$$

---

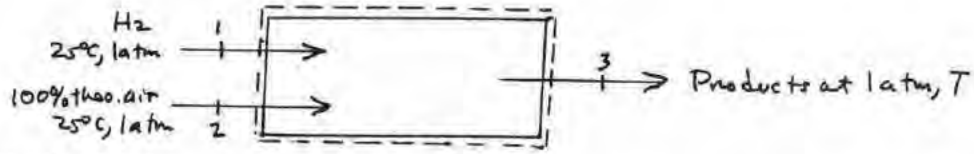
1. The greater entropy production of part (b) suggests that complete combustion would not occur under the specified conditions.

PROBLEM 14.47

KNOWN: H_2 at 25°C, 1atm enters an insulated reactor operating at steady state and reacts with 100% of theoretical air entering at 25°C, 1atm. The products of combustion exit at 1atm and temperature T.

FIND: Determine T if (a) combustion is complete, (b) an equilibrium mixture of H_2O, H_2, O_2, and N_2 exits.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown above is at steady state. (2) For the control volume, $\dot{Q}_{cv}=0$ and kinetic/potential energy effects are negligible. (3) The ideal gas model is applicable. (4) In part (a) combustion is complete. In part (b) an equilibrium mixture is formed.

ANALYSIS: (a) The complete reaction with 100% theoretical air is

$$H_2 + 0.5O_2 + 1.88N_2 \rightarrow H_2O + 1.88N_2$$

An energy rate balance reduces with given assumptions to read $H_R = H_P$, or

$$0 = [\bar{h}_f^o + \Delta\bar{h}]_{H_2O} + 1.88[\cancel{\bar{h}_f^o}^{0} + \Delta\bar{h}]_{N_2} - [\cancel{\bar{h}_f^o}]^{0}_{H_2} - 1.88[\cancel{\bar{h}_f^o}]^{0}$$

Inserting ideal gas table data

$$0 = [-241,820 + \bar{h}_{H_2O}(T) - 9904] + 1.88[\bar{h}_{N_2}(T) - 8669] = 0$$

or

$\bar{h}_{H_2O}(T) + 1.88\bar{h}_{N_2}(T) = 268,022$ kJ/kmol. Solving iteratively with table data, $T = 2525$ K.

(b) If an equilibrium mixture is formed, the reaction equation reads

$$H_2 + 0.5O_2 + 1.88N_2 \rightarrow (1-x)H_2O + xH_2 + \frac{x}{2}O_2 + 1.88N_2$$

Where $n = (1-x) + x + x/2 + 1.88 = 2.88 + x/2$. For $H_2O \rightleftarrows \frac{1}{2}O_2 + H_2$ and $p/p_{ref} = 1$

$$K = \frac{(x/2)^{1/2}(x)}{(1-x)}\left[\frac{p/p_{ref}}{n}\right]^{\frac{1}{2}+1-1} = \left(\frac{x}{1-x}\right)\left(\frac{x/2}{2.88+x/2}\right)^{1/2} \qquad (1)$$

An energy rate balance reduces to $H_R = H_P$, or

$$0 = (1-x)[\bar{h}_f^o + h(T) - h(298)]_{H_2O} + x[\bar{h}(T) - \bar{h}(298)]_{H_2} + \frac{x}{2}[\bar{h}(T) - \bar{h}(298)]_{O_2} + 1.88[\bar{h}(T) - \bar{h}(298)]_{N_2}$$

with ideal gas table data

$$0 = (1-x)[-241,820 + \bar{h}_{H_2O}(T) - 9904] + x[\bar{h}_{H_2}(T) - 8468] + \frac{x}{2}[\bar{h}_{O_2}(T) - 8682] + 1.88[\bar{h}_{N_2}(T) - 8669] \qquad (2)$$

Equations (1), (2) are simultaneous equations for x, T. Solving iteratively

1 with table data, $x = 0.045$, $T = 2433$K.

---

1. Because of dissociation — case (b) — the temperature of the products is less than in the complete combustion case (a).

PROBLEM 14.47 (Cont'd.) - Page 2

Alternative Solution Using IT

IT Code

```
T1 = 25 + 273.15  // K
T2 = 25 + 273.15  // K
p3 = 1  // atm
ndot1 = 1  //  Do all calcualtions on the basis of l kmol of H2 entering.


//  For part (a):  H2 + 0.5 O2 + 1.88 N2  →  H2O + 1.88 N2
hr = h_T("H2",T1) + 0.5 * h_T("O2",T2) + 1.88 * h_T("N2",T2)
hp = h_T("H2O",Tp) + 1.88 * h_T("N2",Tp)
hp =hr

/* Use this section fcr part (b) and comment out part (a).
//  For part (b):  H2 + 0.5 O2 + 1.88 N2  →  (1 - x) H2O + x H2 + x/2 O2 +
1.88 N2
ndotH2O_3 = 1 - x
ndotH2_3 = x
ndotO2_3 = x/2
ndotN2_3 = 1.88
ndot3 = ndotH2O_3 + ndotH2_3 + ndotO2_3 + ndotN2_3
yH2O = ndotH2O_3 / ndot3
yH2 = ndotH2_3 / ndot3
yO2 = ndotO2_3 / ndot3
pref = 1  // atm

// For the reaction H2O  ↔  H2 + 1/2 O2

K = ((yH2 * yO2^0.5) / yH2O) * (p3 / pref)^0.5
//  Data from Table A-27 are stored in EQH2O.LUT.
log(K) = LOOKUPVAL(EQH2O,1,T3,3)

hr = h_T("H2",T1) + 0.5 * h_T("O2",T2) + 1.88 * h_T("N2",T2)
hp = (1-x)*hH2O_3 + x*hH2_3 + (x/2)*hO2_3 + 1.88*hN2_3
hp = hr
hH2O_3 = h_T("H2O",T3)
hH2_3 = h_T("H2",T3)
hO2_3 = h_T("O2",T3)
hN2_3 = h_T("N2",T3)
*/
```

IT Results

Part (a): $T_3 = 2526$ K

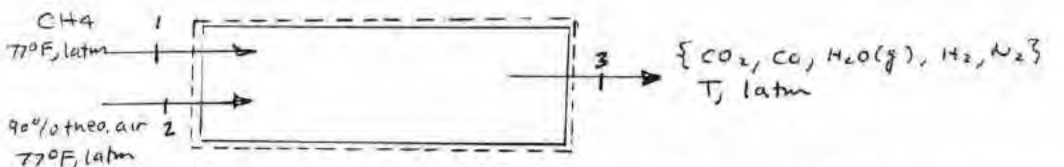
Part(b): $x = 0.04563$, $K = 0.004239$, $T_3 = 2432$ K

PROBLEM 14.48

KNOWN: CH_4 at 77°F, 1atm enters an insulated reactor and burns with 90% of theoretical air entering separately at 77°F, 1atm. An equilibrium mixture $\{CO_2, CO, H_2O(g), H_2, N_2\}$ exits at 1atm.

FIND: Determine the temperature of the exiting mixture

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = \dot{Q}_{cv} = 0$ and negligible kinetic/potential energy effects. (2) The exiting equilibrium mixture is modeled as an ideal gas mixture. (3) N_2 is inert.

ANALYSIS: The complete combustion of CH_4 with the theoretical amount of air is given by Eq. 13.4. Accordingly, combustion with 90% of theoretical air to form the specified mixture is described by

$$CH_4 + 1.8O_2 + 6.768N_2 \rightarrow (1-x)CO_2 + xCO + (1.6+x)H_2O + (0.4-x)H_2 + 6.768N_2$$

At equilibrium $CO_2 + H_2 \rightleftarrows CO + H_2O$. Accordingly, Eq 14.35 takes the form

$$K = \frac{[x][1.6+x]}{[1-x][0.4-x]}\left[\frac{p/p_{ref}}{n}\right]^0 = \frac{x(1.6+x)}{(1-x)(0.4-x)} \qquad (1)$$

An energy rate balance at steady state reduces to

$$[1-x][\bar{h}_f^\circ + \bar{h}(T) - h(537)]_{CO_2} + x[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{CO} + (1.6+x)[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{H_2O}$$
$$+ [0.4-x][\bar{h}(T) - \bar{h}(537)]_{H_2} + 6.768[\bar{h}(T) - \bar{h}(537)]_{N_2} - (\bar{h}_f^\circ)_{CH_4}$$

Inserting table data

$$0 = [1-x][-169{,}300 + \bar{h}_{CO_2} - 4028] + x[-47{,}540 + \bar{h}_{CO} - 3725] + (1.6+x)[-104{,}040 + \bar{h}_{H_2O}(T) - 4258]$$
$$+ [0.4-x][\bar{h}_{H_2} - 3640] + 6.768[\bar{h}_{N_2} - 3730] - (-32{,}210)$$

or

$$0 = [1-x][-173{,}328 + \bar{h}_{CO_2}] + x[-51{,}265 + \bar{h}_{CO}] + [1.6+x][-108{,}298 + \bar{h}_{H_2O}] +$$
$$[0.4-x][\bar{h}_{H_2} - 3640] + 6.768[\bar{h}_{N_2} - 3730] + 32{,}210 \qquad (2)$$

Equations (1), (2) are simultaneous in x, T. When solved iteratively
(1) using Table data, $T = 4000°R$, $x = 0.2693$.

---

1. For the products to include both CO and H_2, inspecting the reaction equation shows that x must be in the interval $0 < x < 0.4$.

PROBLEM 14.48 (Cont'd.)-Page 2

Alternative Solution Using IT

IT Code

```
T1 = 77 + 459.67 // °R
T2 = 77 + 459.67 // °R
p3 = 1 // atm
ndot_CH4 = 1 // Do calculations on the basis of 1 kmol of CH4 entering.

// CH4 + 1.8 O2 + 6.768 N2 → (1 - x) CO2 + x CO + (1.6 + x) H2O + (0.4 - x) H2 + 6.768 N2
ndotCO2_3 = 1 - x
ndotCO_3 = x
ndotH2O_3 = 1.6 + x
ndotH2_3 = 0.4 - x
ndotN2_3 = 6.768
ndotO2_3 = 1.8
ndot3 = ndotCO2_3 + ndotCO_3 + ndotH2O_3 + ndotH2_3 + ndotN2_3
yCO2 = ndotCO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yH2O = ndotH2O_3 / ndot3
yH2 = ndotH2_3 / ndot3
pref = 1 // atm

// For the reaction: CO2 + H2 ↔ CO + H2O
K = (yCO * yH2O) / (yCO2 * yH2)
// Data from Table A-27 are stored in EQWATGAS.LUT.
log(K) = LOOKUPVAL(EQWATGAS,2,T3,3)

0 = hCH4_1 + 1.8*hO2_2 + 6.768*hN2_2 - (1-x)*hCO2_3 - x*hCO_3 - (1.6+x)*hH2O_3 -
        (0.4-x)*hH2_3 -6.768*hN2_3
hCH4_1 = h_T("CH4",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hH2O_3 = h_T("H2O",T3)
hH2_3 = h_T("H2",T3)
hN2_3 = h_T("N2",T3)
```

IT Results

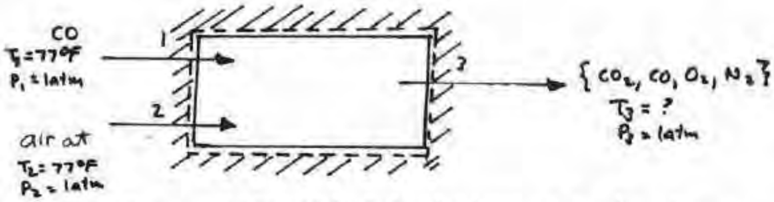
$K = 5.276$
$x = 0.2694$
$T_3 = 4003°R$

PROBLEM 14.49

KNOWN: CO at 77°F, 1atm enters an insulated reactor operating at steady state and burns with air entering at 77°F, 1atm. An equilibrium mixture of CO_2, CO, O_2, N_2 exits at 1atm, T.

FIND: Determine T if the combustion occurs with (a) 80% theoretical air, (b) 100% theoretical air.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The equilibrium mixture is modeled as an ideal gas as is the entering combustion air. (3) N_2 is inert.

ANALYSIS: (a) The reaction of CO with 80% of theoretical air to form $\{CO_2, CO, O_2, N_2\}$ is described by

$$CO + 0.4O_2 + 1.504N_2 \rightarrow x\,CO_2 + (1-x)CO + \left(\frac{0.8-x}{2}\right)O_2 + 1.504N_2$$

The amount of mixture is $n = x + (1-x) + \left(\frac{0.8-x}{2}\right) + 1.504 = (5.808-x)/2$. At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly, Eq. 14.35 takes the form

$$K(T_3) = \frac{[1-x]\left[\frac{0.8-x}{2}\right]^{1/2}}{x}\left[\frac{p/p_{ref}}{(5.808-x)/2}\right]^{1/2} = \frac{1-x}{x}\left[\frac{0.8-x}{5.808-x}\right]^{1/2} \qquad (1)$$

Another equation relating x and T_3 is obtained from an energy rate equation at steady state which reduces to give

$$0 = \frac{\cancel{\dot{Q}_{cv}}}{\dot{n}_{co}} - \frac{\cancel{\dot{W}_{cv}}}{\dot{n}_{co}} + [h_{co}]_1 + [0.4\bar{h}_{O_2} + 1.504\bar{h}_{N_2}]_2 - \left[x\bar{h}_{CO_2} + (1-x)\bar{h}_{co} + \left(\frac{0.8-x}{2}\right)\bar{h}_{O_2} + 1.504\bar{h}_{N_2}\right]_3$$

Then, with $\bar{h}_f^\circ = 0$ for O_2 and N_2 this becomes

$$0 = [\bar{h}_f^\circ]_{co} - \left\{x[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{CO_2} + (1-x)[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{co} + \left(\frac{0.8-x}{2}\right)[\bar{h}(T_3) - \bar{h}(537)]_{O_2} + 1.504[\bar{h}(T_3) - \bar{h}(537)]_{N_2}\right\}$$

or with data from Table A-23E

$$x[-169{,}300 + \bar{h}_{CO_2}(T_3) - 4027.5] + (1-x)[-47{,}540 + \bar{h}_{co}(T_3) - 3725.1] + \left(\frac{0.8-x}{2}\right)[\bar{h}_{O_2}(T_3) - 3725.1] + 1.504[\bar{h}_{N_2}(T_3) - 3729.3] = -47{,}540 \qquad (2)$$

For the products to include both CO_2 and O_2, inspection of the reaction equation shows that x must be in the interval $0 < x < 0.8$. Eqs. (1), (2) are simultaneous in x and T_3. Solving iteratively with table data, $T_3 = 4380°R$, $x = 0.766$.

(b) The reaction of CO with the theoretical amount of air to produce CO_2, CO, O_2, and N_2 is

$$CO + \tfrac{1}{2}O_2 + 1.88N_2 \rightarrow x\,CO + (1-x)CO_2 + \tfrac{x}{2}O_2 + 1.88N_2$$

For the products to include both CO and CO_2, it is necessary for x to be in the interval $0 < x < 1$.

The amount of mixture is $n = x + (1-x) + \frac{x}{2} + 1.88 = (5.76+x)/2$. At equilibrium, $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Thus, Eq. 14.35 takes the form

$$K(T_3) = \frac{[x][x/2]^{1/2}}{[1-x]}\left[\frac{p_3/p_{ref}}{(5.76+x)/2}\right]^{1/2} = \left[\frac{x}{1-x}\right]\left[\frac{x}{5.76+x}\right]^{1/2} \qquad (1)$$

Another equation involving T_3 and x can be obtained from an energy rate balance which reduces at steady state to

$$0 = \frac{\cancel{\dot{Q}_{cv}}^{0}}{\dot{n}_{CO}} - \frac{\cancel{\dot{W}_{cv}}^{0}}{\dot{n}_{CO}} + \left[0.5\bar{h}_{O_2} + 1.88\bar{h}_{N_2}\right]_2 + \left[\bar{h}_{CO}\right]_1 - \left[x\bar{h}_{CO} + (1-x)\bar{h}_{CO_2} + \frac{x}{2}\bar{h}_{O_2} + 1.88\bar{h}_{N_2}\right]_3$$

Then, with $\bar{h}_f^\circ = 0$ for O_2 and N_2 this becomes

$$x\left[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)\right]_{CO} + (1-x)\left[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)\right]_{CO_2} + \frac{x}{2}\left[\bar{h}(T_3) - \bar{h}(298)\right]_{O_2}$$
$$+ 1.88\left[\bar{h}(T_3) - \bar{h}(298)\right]_{N_2} = \left(\bar{h}_f^\circ\right)_{CO}$$

Noting that 77°F corresponds to 25°C, we use data from the SI table Table A-23 to obtain

$$x\left[-110{,}530 + \bar{h}(T_3) - 8669\right]_{CO} + (1-x)\left[-393{,}520 + \bar{h}(T_3) - 9364\right]_{CO_2} + \frac{x}{2}\left[\bar{h}(T_3) - 8682\right]_{O_2} +$$
$$1.88\left[\bar{h}(T_3) - 8669\right]_{N_2} = -110{,}530$$

or

$$x\left[\bar{h}_{CO}(T_3) - 119{,}199\right] + (1-x)\left[\bar{h}_{CO_2}(T_3) - 402{,}884\right] + \frac{x}{2}\left[\bar{h}_{O_2}(T_3) - 8682\right] + 1.88\left[\bar{h}_{N_2}(T_3) - 8669\right]$$
$$= -110{,}530 \quad (2)$$

Equations (1), (2) are simultaneous equations for x, T_3. Solving
1 iteratively with table data, $x = 0.125$, $T_3 = 2399\ K\ (4318\ °R)$

---

1. As expected, the greater amount of air used in part (b) results in a lower combustion product temperature T_3 and a greater amount of CO_2 formed.

IT can be used as an alternative to an iterative solution by hand using table data. The program follows.

IT Code

```
T1 = 77 + 459.67  // °R
T2 = 77 + 459.67  // °R
p3 = 1  // atm
ndotCO_1 = 1  // Do calculations on the basis of 1 lbmol of CO entering.

//  CO + a (0.5) (O2 + 3.76 N2)  →  x CO2 + (1 - x) CO + ((a - x) / 2) O2 + a 1.88 N2

ndotCO2_3  = x
ndotCO_3 = 1 - x
ndotO2_3 = (a - x) / 2
ndotN2_3 = a * 1.88
ndot_3 = ndotCO2_3 + ndotCO_3 + ndotO2_3 + ndotN2_3
yCO2 = ndotCO2_3 / ndot_3
yO2 = ndotO2_3 / ndot_3
yCO = ndotCO_3 / ndot_3
pref = 1  // atm
//  Part (a):
a = 0.8
//  For Part(b): Set a = 1.

//  For the reaction CO2  <===>  CO + 1/2 O2
K = (((yCO) * (yO2)^0.5) / (yCO2)) * (p3 / pref)^.5
// Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,2,T3,3)

0 = hCO_1 + a*0.5*hO2_2 + a*1.88*hN2_2 - x*hCO2_3 - (1-x)*hCO_3 -
       ((a-x)/2)*hO2_3 - a*1.88*hN2_3
hCO_1 = h_T("CO",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hO2_3 = h_T("O2",T3)
hN2_3 = h_T("N2",T3)
```

IT Results

Part (a): 80% theoretical air: x = 0.7658, T_3 = 4380°R

Part (b): Theoretical air: x = 0.8751 (lbmol of CO out per lbmol of CO in)
T3 = 4318°R

PROBLEM 14.50

KNOWN: See solution to Problem 14.49.

FIND: For each case, determine the exergy destruction per kmol of CO entering.

SCHEMATIC & GIVEN DATA: See solution to Problem 14.49.

ENGINEERING MODEL: (1) See solution to Problem 14.49. (2.) $T_0 = 537°R$.

ANALYSIS: (a) From the solution to Problem 14.49, the reaction is

$$CO + 0.4O_2 + 1.504N_2 \rightarrow 0.766\,CO_2 + 0.234\,CO + 0.017\,O_2 + 1.504\,N_2$$

where the products are at 4380°R.

At steady state an entropy rate balance reduces to give

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{co}} = (0.766\,\bar{s}_{CO_2} + 0.234\,\bar{s}_{CO} + 0.017\,\bar{s}_{O_2} + 1.504\,\bar{s}_{N_2})_3 - (\bar{s}_{CO})_1 - (0.4\,\bar{s}_{O_2} + 1.504\,\bar{s}_{N_2})_2 \quad (1)$$

The carbon monoxide enters at 77°F, 1atm. Thus from Table A-23E $(\bar{s}_{CO})_1 = \bar{s}^\circ_{CO}(537°R) = 47.27$ Btu/lbmol·°R. The combustion air enters at 77°F, 1atm, so with Eq. 13.23

$$\bar{s}_{O_2} = \bar{s}^\circ_{O_2}(537) - \bar{R}\ln y_{O_2} = 48.98 - 1.986\ln 0.21 = 52.08 \text{ Btu/lbmol·°R}$$

$$\bar{s}_{N_2} = \bar{s}^\circ_{N_2}(537) - \bar{R}\ln y_{N_2} = 45.74 - 1.986\ln 0.79 = 46.21 \text{ Btu/lbmol·°R}$$

The products exit at 4380°R, 1atm with the composition, $y_{CO_2} = \frac{0.766}{2.521}$, $y_{CO} = \frac{0.234}{2.521}$, $y_{O_2} = \frac{0.017}{2.521}$, $y_{N_2} = \frac{1.504}{2.521}$. Thus, with Eq. 13.23 and data from Table A-23E

$$\bar{s}_{CO_2} = \bar{s}^\circ_{CO_2}(4380) - \bar{R}\ln y_{CO_2} = 76.738 - 1.986\ln\frac{0.766}{2.521} = 79.10 \text{ Btu/lbmol·°R}$$

$$\bar{s}_{CO} = \bar{s}^\circ_{CO}(4380) - \bar{R}\ln y_{CO} = 63.567 - 1.986\ln\frac{0.234}{2.521} = 68.29 \text{ Btu/lbmol·°R}$$

$$\bar{s}_{O_2} = \bar{s}^\circ_{O_2}(4380) - \bar{R}\ln y_{O_2} = 65.958 - 1.986\ln\frac{0.017}{2.521} = 75.89 \text{ Btu/lbmol·°R}$$

$$\bar{s}_{N_2} = \bar{s}^\circ_{N_2}(4380) - \bar{R}\ln y_{N_2} = 61.887 - 1.986\ln\frac{1.504}{2.521} = 62.91 \text{ Btu/lbmol·°R}$$

Inserting values into Eq.(1)

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{co}} = (0.766)(79.10) + (0.234)(68.29) + (0.017)(75.89) + (1.504)(62.91) - (47.27) - (0.4)(52.08) - (1.504)(46.21)$$

$$= 34.88 \text{ Btu/lbmol·°R}$$

Calculating the exergy destruction in SI units, as required

$$\frac{\dot{E}_d}{\dot{n}_{co}} = T_0\frac{\dot{\sigma}_{cv}}{\dot{n}_{co}} = (537°R)\left(34.88\frac{\text{Btu}}{\text{lbmol·°R}}\right)\left|\frac{1\text{ kJ}}{0.94783\text{ Btu}}\right|\left|\frac{2.204616}{1\text{ kg}}\right|$$

$$= 43,568\ \frac{\text{kJ}}{\text{kmol(CO)}} \qquad \leftarrow (a)$$

PROBLEM 14.50 (Contd.)-Page 2

(b) From the solution to Problem 14.49(b)

$$CO + \tfrac{1}{2}O_2 + 1.88 N_2 \rightarrow 0.125\, CO + 0.875\, CO_2 + 0.0625\, O_2 + 1.88\, N_2$$

where the combustion products are at 2399 K.

At steady state an entropy rate balance reduces to give

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{CO}} = \left(0.125\,\bar{s}_{CO} + 0.875\,\bar{s}_{CO_2} + 0.0625\,\bar{s}_{O_2} + 1.88\,\bar{s}_{N_2}\right)_3 - (\bar{s}_{CO})_1 - \left(\tfrac{1}{2}\bar{s}_{O_2} + 1.88\,\bar{s}_{N_2}\right)_2 \quad (1)$$

1 The CO enters at 25°C, 1atm. Thus $(\bar{s}_{CO})_1 = \bar{s}^{\circ}_{CO}(298) = 197.54$ kJ/kmol·K from Table A-23. The air enters at 25°C, 1atm, so with Eq. 13.23 and data from Table A-23

$$\bar{s}_{O_2} = \bar{s}^{\circ}_{O_2}(298) - \bar{R}\ln y_{O_2} = 205.03 - 8.314 \ln 0.21 = 218.01 \text{ kJ/kmol·K}$$

$$\bar{s}_{N_2} = \bar{s}^{\circ}_{N_2}(298) - \bar{R}\ln y_{N_2} = 191.5 - 8.314 \ln 0.79 = 193.46 \text{ kJ/kmol·K}$$

The combustion products exit at 2399 K, 1atm with composition $y_{CO} = \frac{0.125}{2.9425}$, $y_{CO_2} = \frac{0.875}{2.9425}$, $y_{O_2} = \frac{0.0625}{2.9425}$, $y_{N_2} = \frac{1.88}{2.9425}$. Thus with Eq. 13.23 and data from Table A-23

$$\bar{s}_{CO} = \bar{s}^{\circ}_{CO}(2399) - \bar{R}\ln y_{CO} = 265.238 - \bar{R}\ln\frac{0.125}{2.9425} = 291.499 \text{ kJ/kmol·K}$$

$$\bar{s}_{CO_2} = \bar{s}^{\circ}_{CO_2}(2399) - \bar{R}\ln y_{CO_2} = 320.276 - \bar{R}\ln\frac{0.875}{2.9425} = 330.359 \text{ kJ/kmol·K}$$

$$\bar{s}_{O_2} = \bar{s}^{\circ}_{O_2}(2399) - \bar{R}\ln y_{O_2} = 275.609 - \bar{R}\ln\frac{0.0625}{2.9425} = 307.633 \text{ kJ/kmol·K}$$

$$\bar{s}_{N_2} = \bar{s}^{\circ}_{N_2}(2399) - \bar{R}\ln y_{N_2} = 258.565 - \bar{R}\ln\frac{1.88}{2.9425} = 262.290 \text{ kJ/kmol·K}$$

Inserting values in Eq. (1)

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{CO}} = (0.125)(291.499) + (0.875)(330.359) + (0.0625)(307.633) + (1.88)(262.290) - (197.54) - (0.5)(218.01) - 1.88(193.46)$$

$$= 167.583 \text{ kJ/kmol(CO)·K}$$

Then

2 $$\frac{\dot{E}_d}{\dot{n}_{CO}} = T_0\frac{\dot{\sigma}_{cv}}{\dot{n}_{CO}} = (298\,K)\left(167.583 \frac{kJ}{kmol(CO)\cdot K}\right) = 49{,}940 \frac{kJ}{kmol(CO)} \longleftarrow (b)$$

---

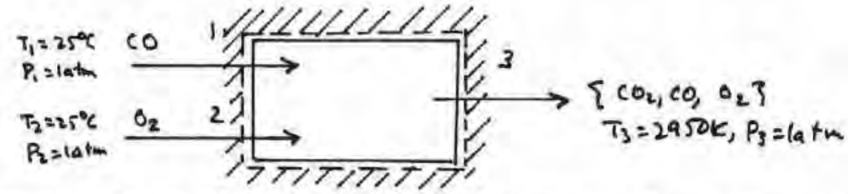
1. Working in SI units as in Problem 14.49(b)
2. The exergy destruction of part (b) is greater than in part (a) because more air is used, and this entails more mixing within the control volume.

PROBLEM 14.51

KNOWN: CO at 25°C, 1atm enters an insulated reactor and burns with excess O_2 entering at 25°C, 1atm. An equilibrium mixture of CO_2, CO, and O_2 exits at 1atm, 2950K.

FIND: Determine the percent excess O_2.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The equilibrium mixture is modeled as an ideal gas.

ANALYSIS: The reaction of CO with O_2 to produce CO_2, CO, and O_2 is

$$CO + \alpha O_2 \rightarrow x\,CO_2 + (1-x)\,CO + \left(\frac{2\alpha - x}{2}\right) O_2$$

The amount of mixture is $n = x + (1-x) + \left(\frac{2\alpha - x}{2}\right) = \frac{2(1+\alpha)-x}{2}$. At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly Eq. 14.35 takes the form

$$K = \frac{[1-x]\left[\frac{2\alpha - x}{2}\right]^{1/2}}{x}\left[\frac{P_3/P_{ref}}{(2(1+\alpha)-x)/2}\right]^{1/2} = \left(\frac{1-x}{x}\right)\left(\frac{2\alpha - x}{2(1+\alpha)-x}\right)^{1/2}$$

At 2950 K Table A-27 gives $\log_{10} K = -0.567 \Rightarrow K = 0.271019$. Thus

$$0.07345 = \left[\frac{1-x}{x}\right]^2\left(\frac{2\alpha - x}{2(1+\alpha)-x}\right) \qquad (1)$$

Another equation relating α and x can be obtained from an energy rate equation which reduces at steady state to

$$0 = \cancel{\frac{\dot{Q}_{cv}}{\dot{n}_{CO}}}^{0} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{CO}}}^{0} + [\bar{h}_{CO}]_1 + \alpha[\bar{h}_{O_2}]_2 - \left[x\bar{h}_{CO_2} + (1-x)\bar{h}_{CO} + \left(\frac{2\alpha - x}{2}\right)\bar{h}_{O_2}\right]_3$$

Then, with $\bar{h}_f^o = 0$ for O_2 this gives

$$x[\bar{h}_f^o + \bar{h}_{CO_2}(2950) - \bar{h}_{CO_2}(298)]_{CO_2} + (1-x)[\bar{h}_f^o + \bar{h}_{CO}(2950) - \bar{h}_{CO}(298)]_{CO} +$$
$$\left(\frac{2\alpha - x}{2}\right)[\bar{h}_{O_2}(2950) - \bar{h}_{O_2}(298)] = (\bar{h}_f^o)_{CO}$$

or with table data

$$x[-393{,}520 + 159{,}117 - 9364] + (1-x)[-110{,}530 + 100{,}352 - 8669] + \left(\frac{2\alpha - x}{2}\right)[104{,}785 - 8682] = -110{,}530$$

$$x[-243{,}767] + (1-x)[-18{,}847] + \left(\frac{2\alpha - x}{2}\right)(96{,}103) = -110{,}530$$

On further reduction

$$\alpha = 2.8404x - 0.954 \qquad (2)$$

Combining Eqs. (1) and (2)

$$0.07345 = \left[\frac{1-x}{x}\right]^2\left[\frac{4.6808x - 1.908}{4.6808x + 0.092}\right] \Rightarrow x = 0.702$$

Using this value for x, Eq (2) gives $\alpha = 1.04$. The theoretical amount of O_2 required is 0.5 kmol/kmol(CO). Accordingly, the percent excess O_2 is 108%.

PROBLEM 14.51 (Cont'd.) - Page 2

Alternative Solution Using IT

IT Code

```
T1 = 25 + 273.15  // K
T2 = 25 + 273.15  // K
T3 = 2950  // K
p3 = 1  // atm
ndotCO_1 = 1  // Do calculations on the basis of 1kmol of CO entering.

//  CO + (1+XS) (0.5) O2  →  x CO2 + (1 - x) CO + ((1 + XS - x) / 2) O2
ndotCO2_3 = x
ndotCO_3 = 1 - x
ndotO2_3 = (1 + XS - x) / 2
ndot3 = ndotCO2_3 + ndotCO_3 + ndotO2_3
yCO2 = ndotCO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yO2 = ndotO2_3 / ndot3
pref = 1  // atm

//  For the reaction  CO2  ↔  CO + 1/2 O2
K = (((yCO) * (yO2)^0.5) / (yCO2)) * (p3 / pref)^0.5
//  Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,1,T3,3)

0 = hCO_1 + (1+XS)*(0.5)*hO2_2 - x*hCO2_3 - (1-x)*hCO_3 - ((1+XS-x)/2)*hO2_3
hCO_1 = h_T("CO",T1)
hO2_2 = h_T("O2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hO2_3 = h_T("O2",T3)
PCTXS = XS * 100
```

IT Results

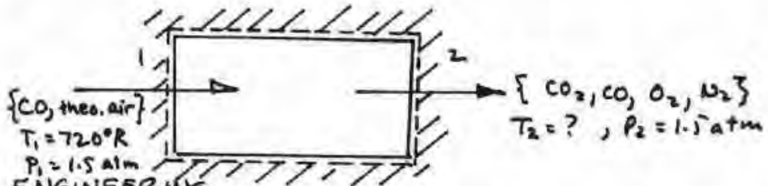
x = 0.7019
% excess oxygen = 107.6%

PROBLEM 14.52

KNOWN: A mixture of CO and the theoretical amount of air at 720 °R, 1.5 atm enters an insulated reactor operating at steady state. An equilibrium mixture of CO_2, CO, O_2, and N_2 exits at 1.5 atm.

FIND: Determine the temperature of the exiting mixture.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The ingoing and outgoing mixtures are modeled as ideal gas mixtures. (3) N_2 is inert.

ANALYSIS: The reaction is described by

$$CO + 0.5\,O_2 + 1.88\,N_2 \rightarrow x\,CO + (1-x)\,CO_2 + \frac{x}{2}\,O_2 + 1.88\,N_2$$

The amount of mixture is $n = (5.76+x)/2$. At equilibrium $CO_2 \rightleftarrows CO + 1/2\,O_2$. Accordingly, Eq. 14.35 takes the form

$$K(T_2) = \frac{[x][x/2]}{[1-x]}\left[\frac{P_2/P_{ref}}{(5.76+x)/2}\right]^{1/2} = \left[\frac{x}{1-x}\right]\left[\frac{1.5x}{5.76+x}\right]^{1/2} \qquad (1)$$

Another equation involving T_2 and x can be obtained from an energy rate balance which reduces at steady state to

$$0 = \frac{\cancel{\dot{Q}_{cv}}^{\,0}}{\dot{n}_{CO}} - \frac{\cancel{\dot{W}_{cv}}^{\,0}}{\dot{n}_{CO}} + \left[0.5\bar{h}_{O_2} + 1.88\bar{h}_{N_2} + \bar{h}_{CO}\right]_1 - \left[x\bar{h}_{CO} + (1-x)\bar{h}_{CO_2} + \frac{x}{2}\bar{h}_{O_2} + 1.88\bar{h}_{N_2}\right]_2$$

or, with $\bar{h}^\circ_f = 0$ for O_2 and N_2, and data from the ideal gas tables

$$x[-47{,}540 + \bar{h}_{CO}(T_2) - 3725.1] + (1-x)[-169{,}300 + \bar{h}_{CO_2}(T_2) - 4027.5] + \frac{x}{2}[\bar{h}_{O_2}(T_2) - 3725.1]$$
$$+1.88[\bar{h}_{N_2}(T_2) - 3729.5] = 0.5[5022.9 - 3725.1] + 1.88[5004.5 - 3729.5] + [-47{,}540 + 5006.1 - 3725.1]$$

or

$$x[\bar{h}_{CO}(T_2) - 51{,}265] + (1-x)[\bar{h}_{CO_2}(T_2) - 173{,}328] + \frac{x}{2}[\bar{h}_{O_2}(T_2) - 3725.1] + 1.88[\bar{h}_{N_2}(T_2) - 3729.5]$$
$$= -43{,}213 \qquad (2)$$

Equations (1), (2) are simultaneous in x and T_2. Inspection of the reaction equation shows that for CO and CO_2 to be in the products x must be in the interval $0 < x < 1$. Solving (1), (2) iteratively using table data, $x = 0.1335$, $T_2 = 4420°R$. ← T_2

The solution presented on the next page uses IT to avoid the iterative solution of simultaneous equations involving table data.

PROBLEM 14.52 (Cont'd.) – Page 2

Alternative Solution Using IT.

IT Code

```
T1 = 260 + 459.67 // °R
p1 = 1.5 // atm
p2 = 1.5 // atm
ndotCO_1 = 1 // Do calculations on the basis of 1 lbmol of CO entering.

// CO + 0.5 O2 + 1.88 N2 ===> x CO2 + (1 - x) CO + (1 - x)/2 O2 + 1.88 N2
ndotCO2_2 = x
ndotCO_2 = 1 - x
ndotO2_2 = (1 - x) / 2
ndotN2_2 = 1.88
ndot2 = ndotCO2_2 + ndotCO_2 + ndotO2_2 + ndotN2_2
yCO2 = ndotCO2_2 / ndot2
yCO = ndotCO_2 / ndot2
yO2 = ndotO2_2 / ndot2
pref = 1 // atm

// For the reaction CO2 <===> CO + 1/2 O2
K = (((yCO)*(yO2^0.5)) / (yCO2)) * (p2 / pref)^0.5
// Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,2,T2,3)

0 = hCO_1 + 0.5*hO2_1 + 1.88*hN2_1 - x*hCO2_2 - (1-x)*hCO_2 - ((1-x)/2)*hO2_2 - 1.88*hN2_2
hCO_1 = h_T("CO",T1)
hO2_1 = h_T("O2",T1)
hN2_1 = h_T("N2",T1)
hCO2_2 = h_T("CO2",T2)
hCO_2 = h_T("CO",T2)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
```

IT Results

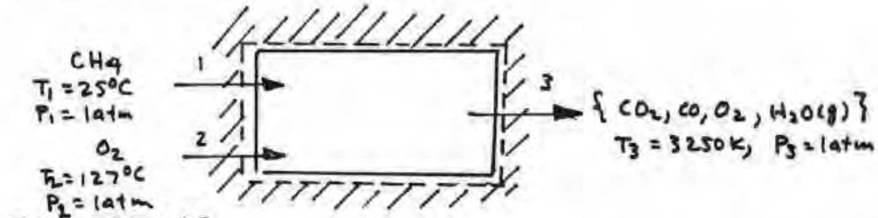
$K = 0.02862$
$x = 0.8659$
$T_2 = 4422°R$

PROBLEM 14.53

KNOWN: CH_4 at 25°C, 1atm enters an insulated reactor operating at steady state and burns with O_2 entering at 127°C, 1atm. An equilibrium mixture of CO_2, CO, O_2, and $H_2O(g)$ exits at 3250K, 1atm.

FIND: Determine the rate O_2 enters, in kmol per kmol of CH_4.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown on the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The mixture at 3 and incoming O_2 can be modeled as ideal gases.

ANALYSIS: The equation describing the reaction of CH_4 with O_2 to form CO_2, CO, O_2, H_2O is

$$CH_4 + \alpha O_2 \rightarrow (1-2x)CO_2 + 2x\,CO + (\alpha+x-2)O_2 + 2H_2O$$

The amount of mixture is $n = (1-2x) + 2x + (\alpha+x-2) + 2 = 1+\alpha+x$.
At equilibrium $CO_2 \rightleftarrows CO + \tfrac{1}{2}O_2$. Accordingly, Eq. 14.35 takes the form

$$K(T_3) = \frac{[2x][\alpha+x-2]^{1/2}}{[1-2x]}\left[\frac{p/p_{ref}}{1+\alpha+x}\right]^{1/2} = \left[\frac{2x}{1-2x}\right]\left[\frac{\alpha+x-2}{\alpha+x+1}\right]^{1/2} \qquad (1)$$

At 3250K, Table A-27 gives $\log_{10}K = -0.1215 \Rightarrow K = 0.75596$. Squaring both sides of Eq. (1) and solving for α

$$\alpha = \frac{[1-f(x)]x - (f(x)+2)}{f(x)-1} \quad \text{where } f(x) \equiv \left[\frac{(1-2x)K}{2x}\right]^2 \qquad (2)$$

Another equation involving α and x can be obtained from an energy rate balance which reduces at steady state to give

$$0 = \frac{\cancel{\dot{Q}_{cv}}^{\,0}}{\dot{n}_{CH_4}} - \frac{\cancel{\dot{W}_{cv}}^{\,0}}{\dot{n}_{CH_4}} + [\bar{h}_{CH_4}]_1 + \alpha[\bar{h}_{O_2}]_2 - [(1-2x)\bar{h}_{CO_2} + 2x\bar{h}_{CO} + (\alpha+x-2)\bar{h}_{O_2} + 2\bar{h}_{H_2O}]_3$$

Then with $\bar{h}_f^\circ = 0$ for O_2 and data from the ideal gas tables

$$0 = (\bar{h}_f^\circ)_{CH_4} + \alpha(\bar{h}(T_2) - \bar{h}(298))_{O_2} - \Big[(1-2x)[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO_2} + 2x[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO}$$
$$(\alpha+x-2)[\bar{h}(T_3) - \bar{h}(298)]_{O_2} + 2[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{H_2O}\Big]$$

$$0 = [-74{,}850] + \alpha[11{,}711 - 8{,}682] - (1-2x)[-393{,}520 + 173{,}822 - 9364] - 2x[-110{,}530 + 111{,}534 - 8669]$$
$$-(\alpha+x-2)[116{,}827 - 8682] - 2[-241{,}820 + 150{,}272 - 9904]$$

Upon reduction

$$105{,}116\,\alpha + 542{,}939\,x = 569{,}406 \qquad (3)$$

Equations (2), (3) are simultaneous solutions in x and α. By inspecting the reaction equation, for the products to contain CO and CO_2, x must be in the interval $0 < x < 0.5$. For O_2 to be in the products, $\alpha > 2$. Solving iteratively using table data

$x = 0.2671$, $\alpha = 4.0373$ kmol(O_2)/kmol(CH_4) ←

Iterative solution of simulaneous equations involving tables is avoided by using IT, as follows:

IT Code

```
T1 = 25 + 273.15  // K
T2 = 127 + 273.15  // K
T3 = 3250  // K
p3 = 1  // atm
ndotCH4_1 = 1  // Do calculations on the basis of 1 kmol of CH4 entering.

// CH4 + A O2  ===>  x CO2 + (1 - x) CO + (A - (3 + x) / 2) O2 + 2 H2O
ndotCO2_3 = x
ndotCO_3 = 1 - x
ndotO2_3 = A - (3 + x)/2
ndotH2O_3 = 2
ndot3 = ndotCO2_3 + ndotCO_3 + ndotO2_3 + ndotH2O_3
yCO2 = ndotCO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yO2 = ndotO2_3 / ndot3
pref = 1  // atm

// For the reaction  CO2  <===>  CO + 1/2 O2
K = (((yCO) * (yO2^0.5)) / (yCO2)) * (p3 / pref)^0.5
// Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,1,T3,3)

0 = hCH4_1 + A*hO2_2 - (x*hCO2_3 + (1-x)*hCO_3 + (A-(3+x)/2)*hO2_3 + 2*hH2O_3)
hCH4_1 = h_T("CH4",T1)
hO2_2 = h_T("O2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hO2_3 = h_T("O2",T3)
hH2O_3 = h_T("H2O",T3)
```

IT Result

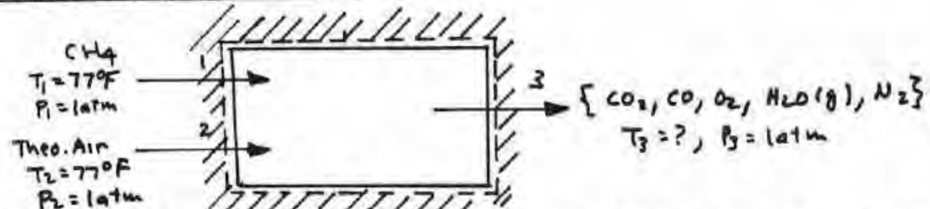
$\dot{n}_{O_2,2} / \dot{n}_{CH_4}$ = 4.037 kmol/kmol

PROBLEM 14.54

KNOWN: CH_4 at 77°F, 1atm enters an insulated reactor operating at steady state and burns with the theoretical amount of air entering at 77°F, 1atm. An equilibrium mixture of CO_2, CO, O_2, $H_2O(g)$, and N_2 exits at 1 atm.

FIND: Determine (a) the temperature of the exiting mixture, (b) the exergy destruction in Btu per lbmol of CH_4 entering.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The incoming air and exiting mixture can be modeled as ideal gases. (3) N_2 is inert. (4) $T_0 = 537°R$.

ANALYSIS: (a) Combustion of methane with the theoretical amount of air to produce CO_2, CO, O_2, $H_2O(g)$, and N_2 is described by

$$CH_4 + 2O_2 + 7.52N_2 \rightarrow (1-x)CO_2 + xCO + \frac{x}{2}O_2 + 2H_2O + 7.52N_2$$

The amount of mixture is $n = (1-x) + x + \frac{x}{2} + 2 + 7.52 = (21.04+x)/2$. At equilibrium $CO_2 \rightleftarrows CO + 1/2\,O_2$. Accordingly, Eq. 14.35 takes the form

$$K(T_3) = \frac{[x][x/2]^{1/2}}{[1-x]}\left[\frac{P_3/P_{ref}}{(21.04+x)/2}\right]^{1/2} = \frac{x}{1-x}\left[\frac{x}{21.04+x}\right]^{1/2} \qquad (1)$$

Another equation in x and T_3 can be obtained from an energy rate equation which reduces at steady state to

$$0 = \cancel{\frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}}}^{0} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{CH_4}}}^{0} + (\bar{h}_{CH_4})_1 + (2\bar{h}_{O_2} + 7.52\bar{h}_{N_2})_2 - [(1-x)\bar{h}_{CO_2} + x\bar{h}_{CO} + \frac{x}{2}\bar{h}_{O_2} + 2\bar{h}_{H_2O} + 7.52\bar{h}_{N_2}]_3$$

Then, with $\bar{h}^\circ_f = 0$ for O_2 and N_2

$$(1-x)[\bar{h}^\circ_f + \bar{h}(T_3) - \bar{h}(537)]_{CO_2} + x[\bar{h}^\circ_f + \bar{h}(T_3) - \bar{h}(537)]_{CO} + \frac{x}{2}[\bar{h}(T_3) - \bar{h}(537)]_{O_2}$$
$$+ 2[\bar{h}^\circ_f + \bar{h}(T_3) - \bar{h}(537)]_{H_2O} + 7.52[\bar{h}(T_3) - \bar{h}(537)]_{N_2} = (\bar{h}^\circ_f)_{CH_4}$$

With data from the ideal gas tables

$$(1-x)[-169{,}300 + \bar{h}_{CO_2}(T_3) - 4027.5] + x[-47{,}540 + \bar{h}_{CO}(T_3) - 3725.1] + \frac{x}{2}[\bar{h}_{O_2}(T_3) - 3725.1]$$
$$+ 2[-104{,}040 + \bar{h}_{H_2O}(T_3) - 4258] + 7.52[\bar{h}_{N_2}(T_3) - 3729.5] = -32{,}210$$

$$(1-x)[\bar{h}_{CO_2}(T_3) - 173{,}328] + x[\bar{h}_{CO}(T_3) - 51{,}265] + \frac{x}{2}[\bar{h}_{O_2}(T_3) - 3725] + 2[\bar{h}_{H_2O}(T_3) - 108{,}298]$$
$$+ 7.52[\bar{h}_{N_2}(T_3) - 3730] = -32{,}210 \qquad (2)$$

Eqs. (1), (2) are simultaneous in x and T_3. By inspection of the reaction equation, for CO and CO_2 to be in the products x must be in the interval $0 < x < 1$. Solving iteratively using table data $x = 0.1088$, $T_3 = 4066°R$.

PROBLEM 14.54 (Contd.) - Page 2

(b) From the solution to part (a), the reaction is described by

$$CH_4 + 2O_2 + 7.52\,N_2 \rightarrow 0.8912\,CO_2 + 0.1088\,CO + 0.0544\,O_2 + 2\,H_2O + 7.52\,N_2$$

where the combustion products are at 4066 °R.

The rate of exergy destruction is $\dot{E}_d = T_0\,\dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is obtained from an entropy rate balance:

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{CH_4}} = \left[0.8912\,\bar{s}_{CO_2} + 0.1088\,\bar{s}_{CO} + 0.0544\,\bar{s}_{O_2} + 2\,\bar{s}_{H_2O} + 7.52\,\bar{s}_{N_2}\right]_3 - \left[\bar{s}_{CH_4}\right]_1 - \left[2\bar{s}_{O_2} + 7.52\,\bar{s}_{N_2}\right]_2$$

The methane enters at 77°F, 1 atm. Thus, from Table A-25, $\bar{s}_{CH_4} = 44.49$ Btu/lbmol·°R. The air enters at 25°C, 1 atm, so with Eq. 13.23 and data from Table A-25

$$\bar{s}_{O_2} = \bar{s}^{\circ}_{O_2}(537) - \bar{R}\ln y_{O_2} = 48.98 - 1.986\ln 0.21 = 52.08 \text{ Btu/lbmol·°R}$$

$$\bar{s}_{N_2} = \bar{s}^{\circ}_{N_2}(537) - \bar{R}\ln y_{N_2} = 45.74 - 1.986\ln 0.79 = 46.21 \text{ Btu/lbmol·°R}$$

The combustion products exit at 4066°R, 1 atm with composition $y_{CO_2} = (0.8912/10.5744)$, $y_{CO} = (0.1088/10.5744)$, $y_{O_2} = (0.0544/10.5744)$, $y_{H_2O} = (2/10.5744)$, $y_{N_2} = (7.52/10.5744)$. Thus, with data from the ideal gas tables

$$\bar{s}_{CO_2} = \bar{s}^{\circ}_{CO_2}(4066) - \bar{R}\ln y_{CO_2} = 75.643 - 1.986\ln(0.8912/10.5744) = 80.556 \text{ Btu/lbmol·°R}$$

$$\bar{s}_{CO} = \bar{s}^{\circ}_{CO}(4066) - \bar{R}\ln y_{CO} = 62.915 - 1.986\ln(0.1088/10.5744) = 72.004 \text{ Btu/lbmol·°R}$$

$$\bar{s}_{O_2} = \bar{s}^{\circ}_{O_2}(4066) - \bar{R}\ln y_{O_2} = 65.274 - 1.986\ln(0.0544/10.5744) = 75.740 \text{ Btu/lbmol·°R}$$

$$\bar{s}_{H_2O} = \bar{s}^{\circ}_{H_2O}(4066) - \bar{R}\ln y_{H_2O} = 64.784 - 1.986\ln(2/10.5744) = 68.041 \text{ Btu/lbmol·°R}$$

$$\bar{s}_{N_2} = \bar{s}^{\circ}_{N_2}(4066) - \bar{R}\ln y_{N_2} = 61.258 - 1.986\ln(7.52/10.5744) = 61.915 \text{ Btu/lbmol·°R}$$

Using calculated data

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{CH_4}} = \{(0.8912)(80.556) + (0.1088)(72.004) + (0.0544)(75.740) + 2(68.041) + 7.52(61.915)\} - 44.49 - ((2)(52.08) + (7.52)(46.21)) = 189.28 \text{ Btu/lbmol·°R}$$

Finally, the exergy destruction is

$$\frac{\dot{E}_d}{\dot{n}_{CH_4}} = T_0\left(\frac{\dot{\sigma}_{cv}}{\dot{n}_{CH_4}}\right) = (537°R)\left(189.28\,\frac{\text{Btu}}{\text{lbmol·°R}}\right) = 101{,}643\,\frac{\text{Btu}}{\text{lbmol}\,(CH_4)} \longleftarrow$$

An iterative solution of simutaneous equations with table data can be avoided using IT, as shown on the next page.

PROBLEM 14.54 (Cont'd.) - Page 3

Alternative Solution Using IT.

IT Code

```
T1 = 77 + 459.67  // °R
p1 = 1 // atm
T2 = 77 + 459.67  // °R
p2 = 1  // atm
p3 = 1  // atm
To = 537  // °R
ndotCH4_1 = 1  //  Do calculations on the basis of 1 lbmol of CH4 entering.

// CH4 + 2 * (O2 + 3.76 N2)  →  (1 - x) CO2 + x CO + x/2 O2 + 2 H2O + 7.52 N2
ndotCO2_3 = 1 - x
ndotCO_3 = x
ndotO2_3 = x/2
ndotH2O_3 = 2
ndotN2_3 = 7.52
ndot3 = ndotCO2_3 + ndotCO_3 + ndotO2_3 + ndotH2O_3 + ndotN2_3
yCO2 = ndotCO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yO2 = ndotO2_3 / ndot3
yH2O = ndotH2O_3 / ndot3
yN2 = ndotN2_3 / ndot3
pref = 1  // atm

//  For the reaction CO2  ↔  CO + 1/2 O2
K = ((yCO * yO2^0.5) / yCO2) * (p3 / pref)^0.5
//  Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,2,T3,3)

0 = hCH4_1 + 2*hO2_2 + 7.52*hN2_2 - ((1-x)*hCO2_3 + x*hCO_3 + (x/2)*hO2_3 + 2*hH2O_3
+ 7.52*hN2_3)
hCH4_1 = h_T("CH4",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hO2_3 = h_T("O2",T3)
hH2O_3 = h_T("H2O",T3)
hN2_3 = h_T("N2",T3)

Edotd = To * (sigmadot / ndotCH4_1)
sigmadot / ndotCH4_1 = (1-x)*sCO2_3 + x*sCO_3 + (x/2)*sO2_3 + 2*sH2O_3 + 7.52*sN2_3 -
                                    sCH4_1 - 2*sO2_2 - 7.52*sN2_2
sCH4_1 = s_Tp("CH4",T1,p1)
pO2_2 = .21 * p2
sO2_2 = s_Tp("O2",T2,pO2_2)
pN2_2 = .79 * p2
sN2_2 = s_Tp("N2",T2,pN2_2)
pCO2_3 = yCO2 * p3
sCO2_3 = s_Tp("CO2",T3,pCO2_3)
pCO_3 = yCO * p3
sCO_3 = s_Tp("CO",T3,pCO_3)
pO2_3 = yO2 * p3
sO2_3 = s_Tp("O2",T3,pO2_3)
pH2O_3 = yH2O * p3
sH2O_3 = s_Tp("H2O",T3,pH2O_3)
pN2_3 = yN2 * p3
sN2_3 = s_Tp("N2",T3,pN2_3)
```

IT Results

$x = 0.109$

$T_3 = 4067°R$

$\dot{\sigma}_{cv} / \dot{n}_{CH_4} = 189.3$ Btu/lbmol(CH_4)·°R

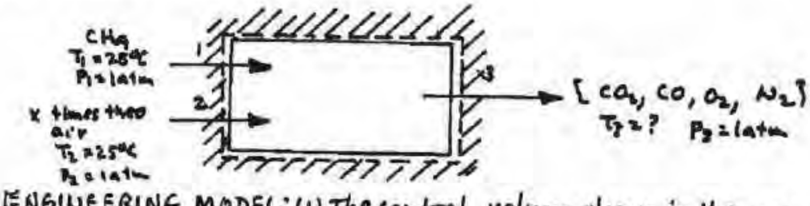
$\dot{E}_d / \dot{n}_{CH_4} = 1.016 \times 10^5$ Btu/lbmol(CH_4)

PROBLEM 14.55

KNOWN: $CH_4(g)$ at 25°C, 1atm enters an insulated reactor operating at steady state and burns with x times the theoretical amount of air entering at 25°C, 1atm. An equilibrium mixture of CO_2, CO, O_2 and N_2 exits at 1atm.

FIND: For x ranging from 1 to 4, determine the temperature of the exiting equilibrium mixture.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The equilibrium mixture can be modeled as an ideal gas. (3) N_2 is inert. (4) $x \geq 1$.

ANALYSIS: The reaction of CH_4 with x times the theoretical amount of air to produce CO_2, CO, O_2, and N_2 is described by

$$CH_4 + 2xO_2 + 7.52x\,N_2 \rightarrow (1-\alpha)CO_2 + \alpha CO + (2x + \tfrac{\alpha}{2} - 2)O_2 + 2H_2O + 7.52x\,N_2$$

For CO and CO_2 to appear in the products, it is necessary for α to be in the interval $0 < \alpha < 1$. The amount of mixture, n, is

$$n = (1-\alpha) + \alpha + (2x + \tfrac{\alpha}{2} - 2) + 2 + 7.52x = (2 + 19.04x + \alpha)/2$$

At equilibrium $CO_2 \rightleftarrows CO + \tfrac{1}{2}O_2$. Accordingly, Eq. 14.35 becomes

$$K(T_3) = \frac{[\alpha]\left[\frac{4x+\alpha-4}{2}\right]^{1/2}}{[1-\alpha]}\left[\frac{P/P_{ref}}{(2+19.04x+\alpha)/2}\right]^{1/2} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{4x+\alpha-4}{2+19.04x+\alpha}\right)^{1/2} \qquad (1)$$

Another equation relating T_3 and α for specified x is obtained using the energy rate balance which at steady state reduces to

$$0 = \cancel{\frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}}}^{0} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{CH_4}}}^{0} + (\bar{h}_{CH_4})_1 + [2x\bar{h}_{O_2} + 7.52x\bar{h}_{N_2}]_2 - [(1-\alpha)\bar{h}_{CO_2} + \alpha\bar{h}_{CO} + (2x + \tfrac{\alpha}{2} - 2)\bar{h}_{O_2} + 2\bar{h}_{H_2O} + 7.52x\bar{h}_{N_2}]_3$$

Then, with $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$0 = (\bar{h}_f^\circ)_{CH_4} - (1-\alpha)[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO_2} - \alpha[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO}$$
$$- (2x + \tfrac{\alpha}{2} - 2)[\bar{h}(T_3) - \bar{h}(298)]_{O_2} - 2[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{H_2O} - 7.52x[\bar{h}(T_3) - \bar{h}(298)]_{N_2}$$

With data from the ideal gas tables

$$0 = (-74850) - (1-\alpha)[-393{,}520 + \bar{h}_{CO_2}(T_3) - 9364] - \alpha[-110{,}530 + \bar{h}_{CO}(T_3) - 8669]$$
$$- (2x + \tfrac{\alpha}{2} - 2)[\bar{h}_{O_2}(T_3) - 8682] - 2[-241{,}820 + \bar{h}_{H_2O}(T_3) - 9904] + 7.52x[\bar{h}_{N_2}(T_3) - 8669]$$

or

$$0 = (-74850) - (1-\alpha)[\bar{h}_{CO_2}(T_3) - 402{,}884] - \alpha[\bar{h}_{CO}(T_3) - 119{,}199] - (2x + \tfrac{\alpha}{2} - 2)[\bar{h}_{O_2}(T_3) - 8682]$$
$$- [\bar{h}_{H_2O}(T_3) - 251{,}724] - 7.52x[\bar{h}_{N_2}(T_3) - 8669] \qquad (2)$$

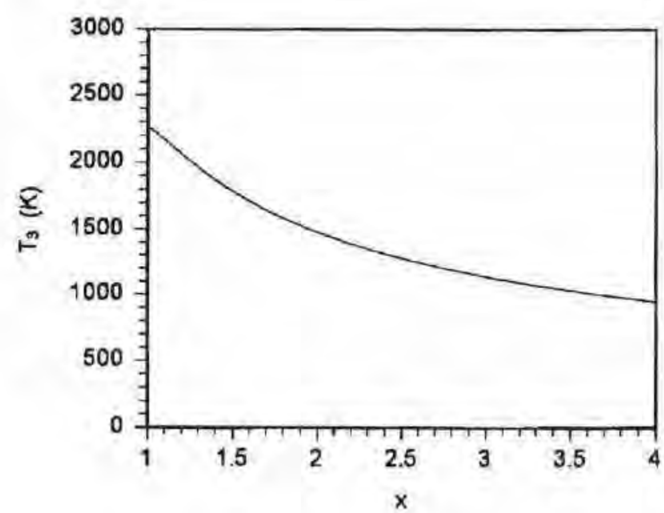
Eqs. (1), (2) are simultaneous in T_3, α, and x. For each specified value for x: $1 \leq x \leq 4$, the equations can be solved for T_3 and α, as shown in the table following the IT solution.

PROBLEM 14.55 (Cont'd.) - Page 2

```
T1 = 25 + 273.15  // K
T2 = 25 + 273.15  // K
p3 = 1  // atm
x = 1
ndotCH4_1 = 1  //  Do calculations on the basis of 1 kmol of CH4 entering.

// CH4 + (2 * x) O2 + (7.52 * x) N2  ===>
//       (1- a) CO2 + a CO + (2 * x + a / 2 - 2) O2 + 2 H2O + (7.52 * x) N2
ndotCO2_3 = 1 - a
ndotCO_3 = a
ndotO2_3 = 2 * x + a / 2 - 2
ndotH2O_3 = 2
ndotN2_3 = 7.52 * x
ndot3 = ndotCO2_3 + ndotCO_3 + ndotO2_3 + ndotH2O_3 + ndotN2_3
yCO2 = ndotCO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yO2 = ndotO2_3 / ndot3
pref = 1  // atm

// For the reaction:  CO2  <===>  CO + 1/2 O2
K = ((yCO * yO2^0.5) / yCO2) * (p3 / pref)^0.5
//  Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,1,T3,3)

0 = hCH4_1 + 2*x*hO2_2 + 7.52*x*hN2_2 - ((1-a)*hCO2_3 + a*hCO_3 + (2*x+a/2-2)*hO2_3 +
                        2*hH2O_3 + 7.52*x*hN2_3)
hCH4_1 = h_T("CH4",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hO2_3 = h_T("O2",T3)
hH2O_3 = h_T("H2O",T3)
hN2_3 = h_T("N2",T3)
```

IT Results

x	a	T_3 (K)
1.0	0.109	2259
1.5	7.025×10^{-4}	1789
2.0	1.036×10^{-5}	1481
2.5	2.471×10^{-7}	1280
3.0	8.101×10^{-9}	1138
3.5	3.934×10^{-10}	1032
4.0	5.312×10^{-12}	950.7



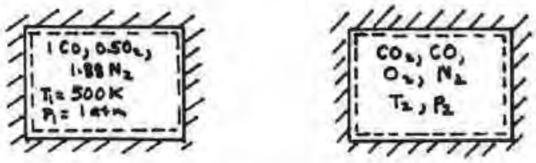
As the amount of air supplied increases, the temperature of the combustion products and the amount of CO decrease.

PROBLEM 14.56

<u>KNOWN:</u> A mixture of 1 kmol CO, 0.5 kmol O_2, 1.88 kmol N_2 is initially at 227°C, 1atm in a closed, rigid, insulated vessel. An equilibrium mixture of CO_2, CO, O_2, and N_2 eventually forms.

<u>FIND:</u> Determine the final pressure.

<u>SCHEMATIC & GIVEN DATA:</u>



initial final

<u>ENGINEERING MODEL:</u> (1) The system is shown in the accompanying figure. (2) The ideal gas model applies at the initial and final states. (3) For the system $Q = W = 0$ and kinetic and potential energy effects are negligible. (4) N_2 is inert.

<u>ANALYSIS:</u> The reaction takes the form

$$CO + 0.5O_2 + 1.88\,N_2 \rightarrow x\,CO + (1-x)\,CO_2 + \frac{x}{2}\,O_2 + 1.88\,N_2$$

The initial amount of mixture is $n_1 = 1 + 0.5 + 1.88 = 3.38$ kmol. The final amount of mixture is $n_2 = x + (1-x) + \frac{x}{2} + 1.88 = (5.76 + x)/2$.

The ideal gas equation of state gives

$$\left.\begin{array}{l} P_1 V = n_1 \bar{R} T_1 \\ P_2 V = n_2 \bar{R} T_2 \end{array}\right\} : \quad \frac{P_2}{P_1} = \frac{n_2 T_2}{n_1 T_1} \qquad (1)$$

As P_1, T_1, and n_1 are known, P_2 can be determined once n_2 and T_2 are known.

At equilibrium, $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly Eq. 14.35 takes the form

$$K(T_2) = \frac{[x][\frac{x}{2}]^{1/2}}{[1-x]}\left[\frac{P_2/P_{ref}}{n_2}\right]^{1/2} \qquad (2)$$

Since $P_1 = P_{ref}$, use of Eq. (1) in Eq. (2) gives

$$K(T_2) = \left[\frac{x}{1-x}\right]\left[\frac{x\,T_2}{2\,n_1 T_1}\right]^{1/2} \qquad (3)$$

Another equation involving x and T_2 can be obtained from an energy balance which reduces to $\Delta U = \cancel{Q}^{0} - \cancel{W}^{0}$. That is,

$$[x\,\bar{u}_{CO} + (1-x)\,\bar{u}_{CO_2} + \frac{x}{2}\,\bar{u}_{O_2} + 1.88\,\bar{u}_{N_2}]_2 - [\bar{u}_{CO} + 0.5\,\bar{u}_{O_2} + 1.88\,\bar{u}_{N_2}]_1 = 0$$

With $\bar{h} = \bar{u} + \bar{R}T$, $\bar{u} = \bar{h} - \bar{R}T$

$$\left[x[\bar{h}_{CO} - \bar{R}T] + (1-x)[\bar{h}_{CO_2} - \bar{R}T] + \frac{x}{2}[\bar{h}_{O_2} - \bar{R}T]\right]_2 - \left[(\bar{h}_{CO} - \bar{R}T) + 0.5(\bar{h}_{O_2} - \bar{R}T)\right]_1 + 1.88[\bar{u}_{N_2}(T_2) - \bar{u}_{N_2}(T_1)] = 0$$

or

$$\left[x\bar{h}_{CO} + (1-x)\,\bar{h}_{CO_2} + \frac{x}{2}\bar{h}_{O_2}\right]_2 - \left[\bar{h}_{CO} + 0.5\,\bar{h}_{O_2}\right]_1 + 1.88[\bar{u}_{N_2}(T_2) - \bar{u}_{N_2}(T_1)] + \frac{3}{2}\bar{R}T_1 - (1+\frac{x}{2})\bar{R}T_2 = 0$$

For CO, CO_2, and O_2, $h = \bar{h}_f^o + \Delta\bar{h}$. Thus

$$x\,[\bar{h}_f^o + h(T_2) - h(298)]_{CO} + (1-x)\,[\bar{h}_f^o + \bar{h}(T_2) - \bar{h}(298)]_{CO_2} + \frac{x}{2}[\bar{h}(T_2) - \bar{h}(298)]_{O_2}$$
$$- [\bar{h}_f^o + \bar{h}(T_1) - \bar{h}(298)]_{CO} - 0.5[\bar{h}(T_1) - \bar{h}(298)]_{O_2} + 1.88[\bar{u}_{N_2}(T_2) - \bar{u}_{N_2}(T_1)] + \frac{3}{2}RT_1 - (1+\frac{x}{2})\bar{R}T_2 = 0$$

With data from Table A-23

$$x[-110{,}530 + \bar{h}_{CO}(T_2) - 8669] + (1-x)[-393{,}520 + \bar{h}_{CO_2}(T_2) - 9364] +$$
$$\frac{x}{2}[\bar{h}_{O_2}(T_2) - 8682] - [-110{,}530 + 14{,}600 - 8669] - 0.5[14{,}770 - 8682] +$$
$$1.88[\bar{u}_{N_2}(T_2) - 10{,}413] + \frac{3}{2}(8.314)(500) - (1+\frac{x}{2})\bar{R}T_2 = 0$$

Thus

$$x\left[\bar{h}_{CO}(T_2) - 119,199\right] + (1-x)\left[\bar{h}_{CO_2}(T_2) - 402,894\right] + \frac{x}{2}\left[\bar{h}_{O_2}(T_2) - 8682\right] +$$
$$1.88\,\bar{h}_{N_2}(T_2) - \left(1+\frac{x}{2}\right)\bar{R}T_2 + 88195.3 = 0 \qquad (4)$$

Eqs. (3), (4) are simultaneous in x and T_2. Inspection of the reaction equation shows that for CO and CO_2 to be in the products x must be in the interval $0<x<1$. Solving iteratively using table data, $x = 0.2262$, $T_2 = 2761\,K$. Thus

$$n_2 = \frac{5.76 + 0.2262}{2} = 2.993$$

Eq. (1) then gives

$$P_2 = \left(\frac{2.993}{3.38}\right)\left(\frac{2761}{500}\right)(1\,atm) = 4.89\ atm$$

The following IT solution provides an alternative to the iterative approach involving the use of table data:

IT Code

```
nCO_1 = 1  // kmol
nO2_1 = 0.5  // kmol
T1 = 227 + 273.15  // K
p1 = 1  // atm
n1 = nCO_1 + nO2_1 + nN2

// CO + 0.5 O2 + 1.88 N2 → x CO + (1 - x) CO2 + x/2 O2 + 1.88 N2
nCO = x
nCO2 = 1 - x
nO2 = x/2
nN2 = 1.88
n2 = nCO + nCO2 + nO2 + nN2
yCO = nCO / n2
yO2 = nO2 / n2
yCO2 = nCO2 / n2
pref = 1  // atm

p2 / p1 = (n2 / n1) * (T2 / T1)

// For the reaction  CO2 ↔ CO + 1/2 O2
K = ((yCO * yO2^0.5) / yCO2) * (p2 / pref)^0.5
// Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = Lookupval(EQCO2A,1,T2,3)

U2/ nCO_1 = (x * uCO_2 + (1 - x) * uCO2_2 + (x / 2) * uO2_2 + 1.88 * uN2_2)
U1/ nCO_1 = (uCO_1 + 0.5 * uO2_1 + 1.88 * uN2_1)
U2 = U1
uCO_1 = u_T("CO",T1)
uO2_1 = u_T("O2",T1)
uN2_1 = u_T("N2",T1)
uCO_2 = u_T("CO",T2)
uCO2_2 = u_T("CO2",T2)
uO2_2 = u_T("O2",T2)
uN2_2 = u_T("N2",T2)
```

IT Results

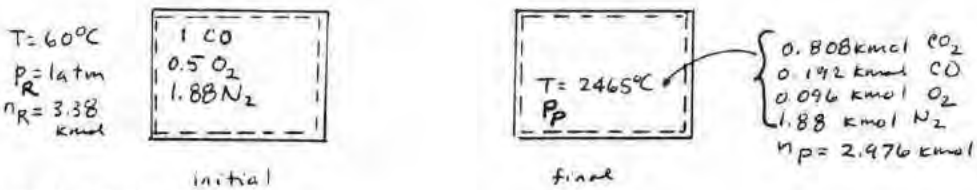
n_2 = 2.993 kmol
x = 0.2262
T_2 = 2760 K
p_2 = 4.887atm

PROBLEM 14.57

KNOWN: A mixture of CO and the theoretical amount of air reacts in an insulated vessel to form an equilibrium mixture of CO_2, CO, O_2 for which a molar analysis is provided. A measured value for the final temperature is indicated as well.

FIND: Check the consistency of the given data.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The system is shown in the figure above. (2) For the system Q = W = 0 and kinetic/potential energy effects are negligible. (3) Ideal gas principles apply. (4) N_2 is inert.

ANALYSIS: Using the data above the balanced reaction equation is

$$1\,CO + 0.5\,O_2 + 1.88\,N_2 \rightarrow 0.808\,CO_2 + 0.192\,CO + 0.096\,O_2 + 1.88\,N_2$$

Carbon and oxygen balances verify this expression is correct.

Next, an energy balance reads $\Delta U = \cancel{Q} - \cancel{W}$. Paralleling the development of Eq 13.17b, this can be rewritten as

$$0 = \sum_P n(\bar{h}_f^\circ + \Delta\bar{h}) - \sum_R n(\bar{h}_f^\circ + \Delta\bar{h}) - \bar{R}T_P \sum_P n + \bar{R}T_R \sum_R n.$$ With table data this becomes,

$$0 \overset{?}{=} 0.808[-393{,}520 + (145{,}971 - 9364)] + 0.192[-110{,}530 + (92485 - 8669)] + 0.096[96{,}379 - 8682]$$
$$+ 1.88[91{,}729 - 9684] - 1[-110{,}530 + (9685 - 8669)] - 0.5[9709 - 8682]$$
$$- 8.314[(2738)(2.976) - (333)(3.38)] = 561\ \text{kJ}$$

Accordingly, the energy balance requirement is closely satisfied.

Finally, the equilibrium constraint has to be considered: This requires the final mixture pressure, obtained using the ideal gas equation of state at fixed volume: $pV = nRT$. Thus

$$\frac{P_P}{P_R} = \frac{n_P T_P}{n_R T_R} \Rightarrow \frac{P_P}{n_P} = \frac{P_R}{n_R}\frac{T_P}{T_R} = \frac{(1\,atm)}{(3.38\,kmol)}\left[\frac{2738\,K}{333\,K}\right] = 2.433$$
$$\Rightarrow P_P = 2.433\ atm.$$

Then, at equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$, so

$$K(2738\,K) = \frac{[0.192][0.096]^{1/2}}{[0.808]}\left[2.433\right]^{1+\frac{1}{2}-1}$$

$$= \left[\frac{0.192}{0.808}\right]\left[(0.096)(2.433)\right]^{1/2} = 0.1148$$

Then, from Table A-27 at 2738 K, $\log_{10} K = -0.9428$, $K = 0.1141$. Thus, the equilibrium condition is also closely satisfied.

Accordingly, the given data are consistent.

PROBLEM 14.58

KNOWN: At 2000 K, $CO_2 \rightleftarrows CO + \frac{1}{2} O_2$

FIND: Estimate the enthalpy of reaction using the van't Hoff equation and equilibrium constant data. Compare with the value calculated using enthalpy data.

ENGINEERING MODEL: The ideal gas model is applicable.

ANALYSIS: Rearranging Eq 14.43b and using $\ln K = 2.303 \log_{10} K$

$$\Delta H = \bar{R} T^2 \frac{d \ln K}{dT} = 2.303 \bar{R} T^2 \frac{d \log_{10} K}{dT}$$

With the interpolation rule of Problem 14.6

$$\log_{10} K = C_1 + \frac{C_2}{T} \Rightarrow \frac{d \log_{10} K}{dT} = -\frac{C_2}{T^2}$$

Collecting results

$$\Delta H = -2.303 \bar{R} C_2 \qquad (1)$$

Then, in the interval $1900 \le T \le 2000\text{K}$, using data from Table A-27

$$\log_{10} K = C_1 + \frac{C_2}{T}$$

@ 1900K: $-3.267 = C_1 + C_2/1900$

@ 2100K: $-2.539 = C_1 + C_2/2100$

$\Big\}\ C_2 = -14523 \text{ K}$

Using this value for C_2, Eq. (1) gives

$$\Delta H = -2.303 \left(8.314 \frac{kJ}{kmol \cdot K}\right)(-14523 K)$$

$$= 278{,}074 \text{ kJ/kmol} \qquad (2)$$

With data from the ideal gas tables

$$\Delta H = \bar{h}_{CO} + \tfrac{1}{2}\bar{h}_{O_2} - \bar{h}_{CO_2}$$

$$= [\bar{h}_f^\circ + \bar{h}(2000) - \bar{h}(298)]_{CO} + \tfrac{1}{2}[\cancelto{0}{\bar{h}_f^\circ} + \bar{h}(2000) - \bar{h}(298)]_{O_2} - [\bar{h}_f^\circ + \bar{h}(2000) - \bar{h}(298)]_{CO_2}$$

$$= [-110{,}530 + 65{,}408 - 8669] + \tfrac{1}{2}[67{,}881 - 8682] - [-393{,}520 + 100{,}804 - 9364]$$

$$= 277{,}889 \text{ kJ/kmol} \qquad (3)$$

The results given by Eqs (2), (3) differ by less than 0.1%

PROBLEM 14.59

KNOWN: At 2000 K, $H_2O \rightleftarrows H_2 + \frac{1}{2}O_2$

FIND: Estimate the enthalpy of reaction using the van't Hoff and equilibrium constant data. Compare with the value calculated using enthalpy data.

ENGINEERING MODEL: The ideal gas model is applicable.

ANALYSIS: Rearranging Eq. 14.43b and using $\ln K = 2.303 \log_{10} K$

$$\Delta T = \bar{R}T^2 \frac{d \ln K}{dT} = 2.303\, \bar{R}\, T^2 \frac{d \log_{10} K}{dT}$$

With the interpolation rule of Problem 14.6

$$\log_{10} = C_1 + \frac{C_2}{T}, \quad \frac{d \log_{10}}{dT} = -\frac{C_2}{T^2}$$

Collecting results

$$\Delta H = -2.303\, \bar{R}\, C_2 \qquad (1)$$

Then, in the interval $1900 \le T \le 2100\,K$, using data from Table A-27

$$\left.\begin{array}{ll} @\ 1900K: & -3.886 = C_1 + C_2/1900 \\ @\ 2100K: & -3.227 = C_1 + C_2/2100 \end{array}\right\} \; C_2 = -13{,}147\,K$$

Using this value for C_2, Eq. (1) gives

$$\Delta H = -2.303\left(8.314 \frac{kJ}{kmol \cdot K}\right)(-13{,}147\,K) = 251{,}727 \frac{kJ}{kmol} \qquad (2)$$

With data from the ideal gas tables

$$\Delta H = \bar{h}_{H_2} + \tfrac{1}{2}\bar{h}_{O_2} - \bar{h}_{H_2O}$$

$$= \left[\cancel{\bar{h}^{\circ}_f}^{\,0} + \bar{h}(2000) - \bar{h}(298)\right]_{H_2} + \tfrac{1}{2}\left[\cancel{\bar{h}^{\circ}_f}^{\,0} + \bar{h}(2000) - \bar{h}(298)\right]_{O_2}$$

$$- \left[\bar{h}^{\circ}_f + \bar{h}(2000) - \bar{h}(298)\right]_{H_2O}$$

$$= [61{,}400 - 8468] + \tfrac{1}{2}[67{,}881 - 8682] - [-241{,}820 + 82{,}593 - 9904]$$

$$= 251{,}663 \frac{kJ}{kmol} \qquad (3)$$

The results given by Eqs. (2), (3) differ by less than 0.03 %.

PROBLEM 14.60

KNOWN: At 2800K, $CO_2 \rightleftarrows CO + \frac{1}{2} O_2$

FIND: Estimate the equilibrium constant at 2800K using K at 2000 K from Table A-27, together with the van't Hoff equation and enthalpy data. Compare with the value obtained from Table A-27 at 2800K.

ENGINEERING MODEL: The ideal gas model is applicable.

ANALYSIS: The van't Hoff equation, Eq 14.43b, gives upon rearrangement

$$\frac{d \ln K}{dT} = \frac{\Delta H}{\bar{R} T^2} \qquad (1)$$

Evaluating ΔH as the average of the values at 2000K and 3000K, Eq. (1) gives upon integration

$$\ln \frac{K_2}{K_1} = -\frac{(\Delta H)_{ave}}{\bar{R}} \left[\frac{1}{T_2} - \frac{1}{T_1} \right] \qquad (2)$$

For $CO_2 \rightarrow CO + \frac{1}{2} O_2$, data from the ideal gas tables gives

$$\Delta H = \bar{h}_{CO} + \tfrac{1}{2}\bar{h}_{O_2} - h_{CO_2}$$
$$= [\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(298)]_{CO} + \tfrac{1}{2}[\cancelto{0}{\bar{h}_f^\circ} + \bar{h}(T) - \bar{h}(298)]_{O_2} - [\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(298)]_{CO_2}$$
$$= [\bar{h}_{CO}(T) + \tfrac{1}{2}\bar{h}_{O_2}(T) - \bar{h}_{CO_2}(T)] + [\bar{h}_f^\circ - \bar{h}(298)]_{CO} - \left[\frac{\bar{h}(298)}{2}\right]_{O_2} - [\bar{h}_f^\circ - \bar{h}(298)]_{CO_2}$$
$$= (\bar{h}_{CO}(T) + \tfrac{1}{2}\bar{h}_{O_2}(T) - \bar{h}_{CO_2}(T)] + [-110{,}530 - 8669] - 4341 - [-393{,}520 - 9364]$$
$$= [\bar{h}_{CO}(T) + \tfrac{1}{2}\bar{h}_{O_2}(T) - \bar{h}_{CO_2}(T)] + 279{,}344$$

At T = 2000K:

$$\Delta H = [65{,}408 + \frac{67881}{2} - 100{,}804] + 279{,}344 = 277{,}889 \text{ kJ/kmol}$$

At T = 2800 K:

$$\Delta H = [94{,}784 + \frac{98826}{2} - 149808] + 279{,}344 = 273{,}733 \text{ kJ/kmol}$$

Then $(\Delta H)_{ave} = 275{,}811$ kJ/kmol, and Eq. (2) gives

$$\ln \frac{K_2}{K_1} = \frac{-275{,}811 \text{ kJ/kmol}}{8.314 \text{ kJ/kmol}\cdot\text{K}} \left[\frac{1}{2800} - \frac{1}{2000}\right] K^{-1}$$

$$\Rightarrow \quad \frac{K_2}{K_1} = 114.34$$

From Table A-27, at 2000K, $\log_{10} K_1 = -2.884$, $K_1 = 1.3061 \times 10^{-3}$. Thus

$$K_2 = 114.34(1.3061 \times 10^{-3}) = 0.1493$$

For comparison, Table A-27, at 2800 K, $\log_{10} K_2 = -0.825$, so $K_2 = 0.1496$. The estimated value is within 0.2 % of the table value.

PROBLEM 14.61

KNOWN: At 2800 K, $H_2O \rightleftarrows H_2 + \frac{1}{2}O_2$

FIND: Estimate the equilibrium constant at 2800 K using K at 2500 K from Table A-27, together with the van't Hoff equation and enthalpy data. Compare with the value obtained from Table A-27 at 2800 K.

ENGINEERING MODEL: The ideal gas model is applicable.

ANALYSIS: The van't Hoff equation, Eq. 14.43b, gives upon rearrangement

$$\frac{d \ln K}{dT} = \frac{\Delta H}{\bar{R}T^2} \qquad (1)$$

Evaluating ΔH as the average of the values at 2500 K and 2800 K, Eq. (1) gives upon integration

$$\ln \frac{K_2}{K_1} = -\frac{(\Delta H)_{ave}}{\bar{R}}\left[\frac{1}{T_2} - \frac{1}{T_1}\right] \qquad (2)$$

For $H_2O \rightarrow H_2 + \frac{1}{2}O_2$, data from the ideal gas tables

$$\Delta H = \bar{h}_{H_2} + \tfrac{1}{2}\bar{h}_{O_2} - \bar{h}_{H_2O}$$

$$= [\cancel{\bar{h}_f^{\circ}}^{\,0} + \bar{h}(T) - \bar{h}(298)]_{H_2} + \tfrac{1}{2}[\cancel{\bar{h}_f^{\circ}}^{\,0} + \bar{h}(T) - \bar{h}(298)]_{O_2} - [\bar{h}_f^{\circ} + \bar{h}(T) - \bar{h}(298)]_{H_2O}$$

$$= \bar{h}_{H_2}(T) + \tfrac{1}{2}\bar{h}_{O_2}(T) - \bar{h}_{H_2O}(T) - \bar{h}_{H_2}(298) - \tfrac{1}{2}\bar{h}_{O_2}(298) - (\bar{h}_f^{\circ} - \bar{h}(298))_{H_2O}$$

$$= \bar{h}_{H_2}(T) + \tfrac{1}{2}\bar{h}_{O_2}(T) - \bar{h}_{H_2O}(T) - 8468 - 4341 - (-241{,}820 - 9904)$$

$$= \bar{h}_{H_2}(T) + \tfrac{1}{2}\bar{h}_{O_2}(T) - \bar{h}_{H_2O}(T) + 238{,}915$$

At 2500 K:

$$\Delta H = 78{,}960 + \tfrac{1}{2}(87{,}057) - 108{,}868 + 238{,}915 = 252{,}536 \text{ kJ/kmol}$$

At 2800 K:

$$\Delta H = 89838 + \tfrac{1}{2}(98{,}826) - 125{,}198 + 238{,}915 = 252{,}968 \text{ kJ/kmol}$$

Thus, $(\Delta H)_{ave} = 252{,}752$ kJ/kmol, and Eq. (2) gives

$$\ln \frac{K_2}{K_1} = -\frac{252{,}752}{8.314}\left[\frac{1}{2800} - \frac{1}{2500}\right] \Rightarrow \frac{K_2}{K_1} = 3.6799$$

From Table A-27, at 2500 K, $\log_{10} K_1 = -2.2224$, $K_1 = 5.992 \times 10^{-4}$. Thus

$$K_2 = (5.992 \times 10^{-4})(3.6799) = 0.02205$$

For comparison, Table A-27, at 2800 K, $\log_{10} K_2 = -1.658$, so $K_2 = 0.02198$. The estimated value is within about 0.3% of the table value.

PROBLEM 14.62

KNOWN: For $C + 2H_2 \rightleftarrows CH_4$ at 25°C $\log_{10} K = 8.9$.

FIND: Estimate $\log_{10} K$ at 500°C.

ENGINEERING MODEL: (1) The Van't Hoff equation is appliable. (2) ΔH is constant.

ANALYSIS: The van't Hoff equation is given by

$$\frac{d \ln K}{dT} = \frac{\Delta H}{RT^2}$$

If ΔH is constant

$$\ln \frac{K_2}{K_1} = -\frac{\Delta H}{\bar{R}}\left[\frac{1}{T_2} - \frac{1}{T_1}\right] \qquad (1)$$

For $C + 2H_2 \rightleftarrows CH_4$, ΔH is given by

$$\Delta H = \bar{h}_{CH_4} - \bar{h}_C - 2\bar{h}_{O_2}$$

At 298 K, Table A-25 gives

$$\Delta H = (-74{,}850) - (0) - 2(0) = -74{,}850 \text{ kJ/kmol}$$

Substituting values into Eq. (1).

$$\ln \frac{K_2}{K_1} = \frac{74850}{8.314}\left[\frac{1}{773} - \frac{1}{298}\right]$$

$$\Rightarrow \quad \log_{10} \frac{K_2}{K_1} = -8.06$$

or

① $$\log_{10} K_2 = 8.9 - 8.06 = 0.84$$

1. For a discussion of the equilibrium constant and the heterogeneous system in which the pure solid or liquid phase of one (or more) of the substances is present in the equilibrium mixture, see Concepts of Thermodynamics by E.F. Obert, McGraw-Hill, 1960, Sec. 14-13.

PROBLEM 14.63

KNOWN: For $Cs \rightleftarrows Cs^+ + e^-$, K = 0.78 and 15.63 at 1600 K, 2000 K, respectively.

FIND: Estimate ΔH at 1800 K.

ENGINEERING MODEL: (1) The Van't Hoff equation is applicable. (2) ΔH varies only gradually with temperature from 1600 to 2000 K.

ANALYSIS: The van't Hoff equation is

$$\frac{d \ln K}{dT} = \frac{\Delta H}{\bar{R} T^2}$$

Then

$$\ln \frac{K_2}{K_1} = -\frac{(\Delta H)_{ave}}{\bar{R}}\left[\frac{1}{T_2} - \frac{1}{T_1}\right]$$

where $(\Delta H)_{ave}$ is the average value of the enthalpy of ionization over the temperature range. Thus

$$(\Delta H)_{ave} = \frac{\bar{R} \ln \frac{K_2}{K_1}}{\left(\frac{1}{T_1} - \frac{1}{T_2}\right)}$$

$$= \frac{(8.314)\left(\ln \frac{15.63}{0.78}\right)}{\left(\frac{1}{1600} - \frac{1}{2000}\right)} = 199,380 \ \frac{kJ}{kmol}$$

PROBLEM 14.64

KNOWN: An equilibrium mixture at 2000K, 1atm consists of Cs, Cs^+, e^-. K = 15.63

FIND: If 1 kmol of Cs is present initially, determine the percent ionization of Cs.

ENGINEERING MODEL: Equilibrium can be analyzed using ideal gas equilibrium concepts.

ANALYSIS: The ionization of Cs to form a mixture of Cs, Cs^+, and e^- is described by

$$Cs \rightarrow (1-z)\,Cs + z\,Cs^+ + z\,e^-$$

The amount of mixture is $n = (1-z) + z + z = 1 + z$

At equilibrium $Cs \rightleftarrows Cs^+ + e^-$, so Eq. 14.35 takes the form

$$K = \frac{[z][z]}{[1-z]}\left[\frac{p/p_{ref}}{1+z}\right]^{1+1-1} = \frac{z^2}{1-z^2} \Rightarrow z = \left[\frac{K}{1+K}\right]^{1/2}$$

With K = 15.6, this becomes

$$z = \left[\frac{15.6}{16.6}\right]^{1/2} = 0.969$$

Thus, the amount of Cs in the mixture is 0.031 kmol. Cs is 96.9% ← dissociated.

PROBLEM 14.65

KNOWN: At 18,000 °R and pressure p, an equilibrium mixture consists of Ar, Ar^+, e^-. For $Ar \rightleftarrows Ar^+ + e^-$, $K = 4.2 \times 10^{-4}$ at 18,000°R.

FIND: Plot the percent ionization of Ar for $0.01 \le p \le 0.05$ atm.

ENGINEERING MODEL: Equilibrium can be modeled using ideal gas mixture concepts.

ANALYSIS: The ionization of Ar to form a mixture of Ar, Ar^+, and e^- is described by

$$Ar \rightarrow (1-z)\,Ar + z\,Ar^+ + z\,e^-$$

The amount of mixture is $n = (1-z) + z + z = 1 + z$.

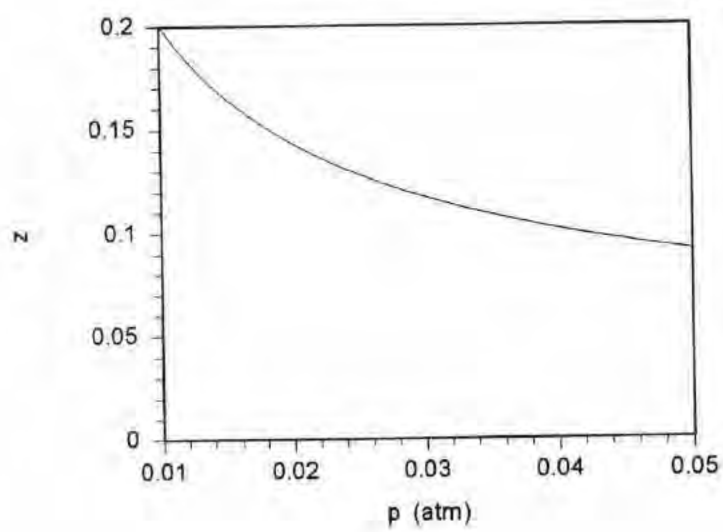
At equilibrium $Ar \rightleftarrows Ar^+ + e^-$, so Eq. 14.35 takes the form

$$K = \frac{[z][z]}{[1-z]}\left[\frac{p/p_{ref}}{1+z}\right]^{1+1-1} = \left(\frac{z^2}{1-z^2}\right)(p/p_{ref})$$

Thus

$$z = \left[\frac{K/(p/p_{ref})}{1 + K/(p/p_{ref})}\right]^{1/2} = \left[\frac{1}{\frac{(p/p_{ref})}{K} + 1}\right]^{\frac{1}{2}}$$

$(= 4.2 \times 10^{-4})$



PROBLEM 14.66

KNOWN: 1 kmol of Na ionizes to form an equilibrium mixture of Na, Na^+, e^- at 2000K and pressure p in which the amount of Na present is x kmol. For $Na \rightleftarrows Na^+ + e^-$ at 2000K, K = 0.668.

FIND: Plot p versus x for $0.2 \le x \le 0.3$ kmol.

ENGINEERING MODEL: Equilibrium can be modeled using ideal gas mixture concepts.

ANALYSIS: The ionization of Na to form a mixture of Na, Na^+, e^- is described by

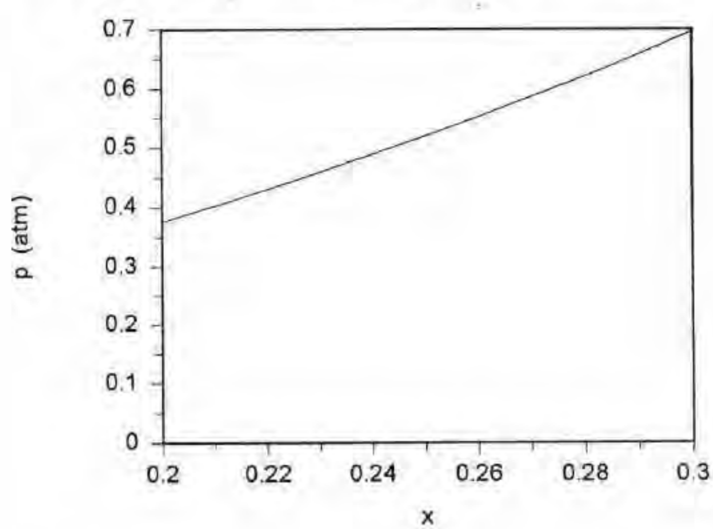
$$1\,Na \rightarrow x\,Na + (1-x)\,Na^+ + (1-x)\,e^-$$

The amount of mixture is $n = x + (1-x) + (1-x) = 2-x$.

At equilibrium $Na \rightleftarrows Na^+ + e^-$, so Eq. 14.35 takes the form

$$K = \frac{(1-x)(1-x)}{x}\left[\frac{p/p_{ref}}{2-x}\right]^{1+1-1} = \frac{(1-x)^2}{x(2-x)}(p/p_{ref})$$

Since K = 0.668, $\frac{p}{p_{ref}} = 0.668\,\frac{x(2-x)}{(1-x)^2}$. Thus for p_{ref} = 1 atm



PROBLEM 14.67

KNOWN: 1 kmol of N ionizes to form an equilibrium mixture of N, N^+, and e^- at 12,000 K, 6 atm in which the the amount of N is 0.95 kmol.

FIND: For $N \rightleftarrows N^+ + e^-$, determine K.

ENGINEERING MODEL: Equilibrium can be modeled as an ideal gas mixture concepts.

ANALYSIS: The ionization of N to form an equilibrium mixture of N, N^+, and e^- is described by

$$1N \rightarrow (1-z)N + zN^+ + ze^-$$

Since $1-z = 0.95$ kmol, $z = .05$ kmol. Accordingly

$$1N \rightarrow 0.95N + 0.05N^+ + 0.05e^-$$

The amount of mixture is $n = 1.05$ kmol.

At equilibrium $N \rightleftarrows N^+ + e^-$, so Eq. 14.35 takes the form

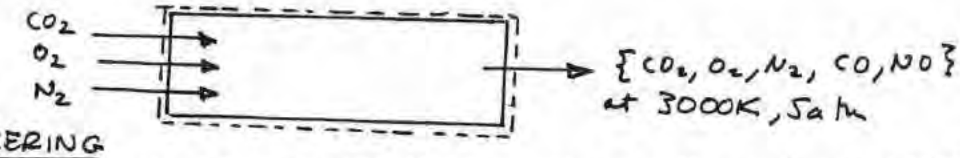
$$K = \frac{[0.05][0.05]}{[0.95]}\left[\frac{6}{1.05}\right]^{1+1-1} = 0.01504 \longleftarrow$$

PROBLEM 14.68

KNOWN: CO_2, O_2, N_2 enter a reactor operating at steady state with equal molar flow rates. An equilibrium mixture of $\{CO_2, O_2, N_2, CO, NO\}$ exits at 3000K, 5atm.

FIND: Determine the molar analysis of the equilibrium mixture.

SCHEMATIC & GIVEN DATA



ENGINEERING MODEL: (1) The control volume shown above is at steady state. (2) Ideal gas principles apply.

ANALYSIS: On the basis of 1 mole each of CO, O_2, N_2 entering, at steady state the balanced reaction equation has the form

$$1\,CO_2 + 1\,O_2 + 1\,N_2 \rightarrow (1-B)\,CO_2 + B\,CO + \left(1+\frac{B}{2}-\frac{E}{2}\right)O_2 + E\,NO + \left(1-\frac{E}{2}\right)N_2 \qquad (1)$$

where B, N are unknowns. For the products, $n = 3 + B/2$.

At equilibrium, these independent reactions must be satisfied

$$(\mathrm{I})\quad CO_2 \rightleftarrows CO + \tfrac{1}{2}O_2, \qquad \mathrm{II}\quad \tfrac{1}{2}O_2 + \tfrac{1}{2}N_2 \rightleftarrows NO$$

For reaction (I) with $K_1 = 0.3273$ from Table A-27 at 3000 K,

$$0.3273 = \frac{B\left(1+\frac{B}{2}-\frac{E}{2}\right)^{1/2}}{(1-B)}\left[\frac{p/p_{ref}}{n}\right]^{1/2} = \left(\frac{B}{1-B}\right)\left[\frac{1+\frac{B}{2}-\frac{E}{2}}{3+\frac{B}{2}}\right]^{1/2}(5)^{1/2} \qquad (2)$$

For reaction (II) with $K_2 = 0.1222$ from Table A-27 at 3000K

$$0.1222 = \frac{E}{\left[1+\frac{B}{2}-\frac{E}{2}\right]^{1/2}\left[1-\frac{E}{2}\right]^{1/2}}\left[\frac{p/p_{ref}}{n}\right]^{0} = \frac{2E}{\left[[2+B-E][2-E]\right]^{1/2}} \qquad (3)$$

Equations (2), (3) are two simultaneous equations for B and E. Solving, $B = 0.2017$, $E = 0.1208$. Then, referring to Eq. (1)

$$\left.\begin{array}{rl} (1-B)\,CO_2: & 0.7983 \\ B\ CO: & 0.2017 \\ \left(1+\frac{B}{2}-\frac{E}{2}\right)O_2: & 1.0405 \\ E\ NO: & 0.1208 \\ \left(1-\frac{E}{2}\right)N_2: & \underline{0.9396} \\ & 3.1009 \end{array}\right\}$$

$y_{CO_2} = 0.257$
$y_{CO} = 0.065$
$y_{O_2} = 0.336$
$y_{NO} = 0.039$
$y_{N_2} = 0.303$

PROBLEM 14.68 (Cont'd.) - Page 2

Alternative Solution Using IT

IT Code

```
T = 3000  // K
p = 5  // atm

// CO2 + O2 + N2  →  (1 - b) CO2 + b CO + (1 + b/2 - e/2) O2 + e NO + (1 - e/2) N
nCO2 = 1 - b
nCO = b
nO2 = 1 + b/2 - e/2
nNO = e
nN2 = 1 - e/2
ntot = nCO2 + nCO + nO2 + nNO + nN2
yCO = nCO / ntot
yO2  = nO2 / ntot
yCO2 = nCO2 / ntot
yNO = nNO / ntot
yN2 = nN2 / ntot
pref = 1  // atm

// For CO2 ↔ CO + 0.5 O2
K1 = (yCO * yO2^0.5 / yCO2) * (p / pref)^0.5

// For 1/2 O2 + 1/2 N2 ↔ NO
K2 = (yNO / (yO2^0.5 * yN2^0.5))

// Look up log10(K1) in EQCO2A.LUT.
log(K1) = LOOKUPVAL(EQCO2A,1,T,3)

// Look up log10(K2) in EQNO.LUT.
log(K2) = LOOKUPVAL(EQNO,1,T,3)
```

IT Results

$K_1 = 0.3273$
$K_2 = 0.1222$
$y_{CO} = 0.06506$
$y_{CO2} = 0.2574$
$y_{N2} = 0.303$
$y_{NO} = 0.03896$
$y_{O2} = 0.3355$

PROBLEM 14.69

KNOWN: An equimolar mixture of CO and $H_2O(g)$ enters a heat exchanger. An equilibrium mixture of CO, CO_2, O_2, H_2O, and H_2 exits at 2500 K, 1atm.

FIND: Determine the molar analysis of the equilibrium mixture.

ENGINEERING MODEL: The equilibrium mixture is modeled as an ideal gas mixture.

ANALYSIS: The overall reaction has the form

$$CO + H_2O \rightarrow x\,CO_2 + y\,H_2O + (1-x)CO + \tfrac{1}{2}(1-x-y)O_2 + (1-y)H_2$$

The total number of moles n in the product mixture is

$$n = x + y + (1-x) + \tfrac{1}{2}(1-x-y) + (1-y) = (5-x-y)/2$$

At equilibrium, two independent reactions relate the components of the product mixture:

1. $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$

2. $CO_2 + H_2 \rightleftarrows CO + H_2O$

From Table A-27 at 2500K the respective values of $\log_{10} K$ are

$$\log_{10} K_1 = -1.44 \quad \Rightarrow \quad K_1 = 0.03631$$

$$\log_{10} K_2 = +0.784 \quad \Rightarrow \quad K_2 = 6.08135$$

The forms taken by Eq 14.35 when $P/P_{ref} = 1$ are, respectively

$$K_1 = \frac{[1-x][\frac{1-x-y}{2}]^{1/2}}{x}\left[\frac{P/P_{ref}}{(5-x-y)/2}\right]^{1/2} = \frac{(1-x)}{x}\left[\frac{1-x-y}{5-x-y}\right]^{1/2}$$

$$K_2 = \frac{[1-x][y]}{[x][1-y]}\left[\frac{P/P_{ref}}{n}\right]^0 = \left[\frac{1-x}{x}\right]\left[\frac{y}{1-y}\right]$$

Solving this pair of simultaneous equations for x and y

$$x = 0.28808$$
$$y = 0.71105$$

The equilibrium mixture on a molar basis is then

$$\{0.28808\,CO_2,\ 0.71105\,H_2O,\ 0.71192\,CO,\ 0.000435\,O_2,\ 0.28895\,H_2\} \leftarrow$$

PROBLEM 14.69 (Cont'd.) - Page 2

Altenative Solution Using IT

IT Code

```
T = 2227 + 273.15 // K
p = 1 // atm

// CO2 + H2O → x CO2 + y H2O + (1 - x) CO + 1/2 (1 - x - y) O2 + (1 - y) H2

nCO2 = x
nH2O = y
nCO = 1 - x
nO2 = (1/2) * (1 - x - y)
nH2 = 1 - y
ntot = nCO2 + nH2O + nCO + nO2 + nH2
yCO2 = nCO2 / ntot
yH2O = nH2O / ntot
yCO = nCO / ntot
yO2 = nO2 / ntot
yH2 = nH2 / ntot
pref = 1 // atm

// For the reaction: CO2 ↔ CO + 1/2 O2
K1 = ((yCO * yO2^0.5) / yCO2) * (p / pref)^0.5

// Look up log10(K2) in EQCO2A.LUT.
log(K1) = LOOKUPVAL(EQCO2A,1,T,3)

// For the reaction: CO2 + H2 ↔ CO + H2O
K2= (yCO * yH2O) / (yCO2 * yH2)

// Look up log10(K1) in EQWATGAS.LUT.
log(K2) = LOOKUPVAL(EQWATGAS,1,T,3)
```

IT Results

K_1 = 0.03634
K_2 =6.082
n_{CO} = 0.7119 kmol
n_{CO2} = 0.2881 kmol
n_{H2} = 0.2889 kmol
n_{H2O} = 0.7111 kmol
n_{O2} = 0.0004324 kmol
x = 0.2881
y = 0.7111

PROBLEM 14.70

KNOWN: A closed vessel initially contains 3 lbmol CO_2, 6 lbmol CO, and 1 lbmol H_2. An equilibrium mixture is formed at 4680°R, 1atm consisting of CO_2, CO, H_2O, H_2, O_2.

FIND: Determine the composition of the equilibrium mixture.

ENGINEERING MODEL: The equilibrium mixture is modeled as an ideal gas mixture.

ANALYSIS: The overall reaction has the form

$$3\,CO_2 + 6\,CO + 1\,H_2 \rightarrow x\,CO_2 + (9-x)\,CO + (3-x-2y)\,H_2O + (x+2y-2)\,H_2 + y\,O_2$$

The total number of moles n in the product mixture is

$$n = x + (9-x) + (3-x-2y) + (x+2y-2) + y = 10 + y$$

At equilibrium, two independent reactions relate the components of the product mixture:

1. $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$
2. $H_2O \rightleftarrows H_2 + \frac{1}{2}O_2$

From Table A-27 at 4680°R the respective values of $\log_{10}K$ are

$$\log_{10}K_1 = -1.219 \Rightarrow K_1 = 0.06039$$

$$\log_{10}K_2 = -2.021 \Rightarrow K_1 = 0.00953$$

The forms taken by Eq. 14.35 when $p/p_{ref} = 1$ are, respectively

$$K_1 = \frac{[9-x][y]^{1/2}}{[x]}\left[\frac{p/p_{ref}}{10+y}\right]^{1/2} = \left(\frac{9-x}{x}\right)\left(\frac{y}{10+y}\right)^{1/2}$$

$$K_2 = \frac{[x+2y-2][y]^{1/2}}{[3-x-2y]}\left[\frac{p/p_{ref}}{10+y}\right]^{1/2} = \left[\frac{x+2y-2}{3-x-2y}\right]\left[\frac{y}{10+y}\right]^{1/2}$$

Solving this pair of simultaneous equations for and y

$$x = 2.30559$$
$$y = 0.00433$$

The equilibrium mixture on a molar basis is then

$$\{2.30559\,CO_2,\ 6.69441\,CO,\ 0.68575\,H_2O,\ 0.31425\,H_2,\ 0.00433\,H_2\}$$

PROBLEM 14.70 (Cont'd.) - Page 2

Alternative Solution Using IT.

IT Code

```
T = 4220 + 459.67  // °R
p = 1  // atm

// 3 CO2 + 6 CO + 1 H2 → x CO2 + (9 - x) CO + (3 - x - 2y) H2O + (x + 2y - 2) H2 + y O2
nCO2 = x
nCO = 9 - x
nH2O = 3 - x - 2*y
nH2 = x + 2*y - 2
nO2 = y
ntot = nCO2 + nCO + nH2O + nH2 + nO2
yCO2 = nCO2 / ntot
yCO = nCO / ntot
yH2O = nH2O / ntot
yH2 = nH2 / ntot
yO2 = nO2 / ntot
pref = 1  // atm

// For the reaction:  CO2 ↔ CO + 1/2 O2
K1 = ((yCO * yO2^0.5) / yCO2) * (p / pref)^0.5

// Look up log10(K1) in EQCO2A.LUT.
log(K1) = LOOKUPVAL(EQCO2A,2,T,3)

// For the reaction:  H2O ↔ H2 + 1/2 O2
K2 = ((yH2 * yO2^0.5) / yH2O) * (p / pref)^0.5

// Look up log10(K2) in EQH2O.LUT.
log(K2) = LOOKUPVAL(EQH2O,2,T,3)
```

IT Results

$K_1 = 0.06034$
$K_2 = 0.00952$
$n_{CO} = 6.694$ lbmol
$n_{CO2} = 2.306$ lbmol
$n_{H2} = 0.3142$ lbmol
$n_{H2O} = 0.6858$
$n_{O2} = 0.00432$
$x = 2.306$
$y = 0432$

PROBLEM 14.71

KNOWN: C_4H_{10} burns with 100% excess air to form an equilibrium mixture at 1400K, 20 atm consisting of CO_2, O_2, $H_2O(g)$, N_2, NO, NO_2. For the reaction $N_2 + 2O_2 \rightleftarrows 2NO_2$, at 1400K, $K = 8.4 \times 10^{-10}$

FIND: Determine the balanced reaction equation.

① ENGINEERING MODEL: The ideal gas model applies to the equilibrium mixture.

ANALYSIS: The balanced reaction equation for complete combustion with the theoretical amount of air is

$$C_4H_{10} + 6.5(O_2 + 3.76N_2) \rightarrow 4CO_2 + 5H_2O + 24.44\,N_2$$

For combustion with 100% excess air

$$C_4H_{10} + 13(O_2 + 3.76N_2) \rightarrow \alpha\,CO_2 + \beta\,H_2O + 2a\,NO + 2b\,NO_2 + \gamma\,O_2 + \delta\,N_2$$

C: $4 = \alpha$. H: $10 = 2\beta$, $\beta = 5$.

O: $26 = 8 + 5 + 2a + 4b + 2\gamma \Rightarrow \gamma = 6.5 - a - 2b$

N: $2(48.88) = 2a + 2b + 2\delta \Rightarrow \delta = 48.88 - a - b$

The total amount of mixture is $n = 64.38 - b$.

At equilibrium there are two reactions applicable:

1. $\frac{1}{2}N_2 + \frac{1}{2}O_2 \rightleftarrows NO$, $K_1 = 1.941 \times 10^{-3}$ (Table A-27)

2. $N_2 + 2O_2 \rightleftarrows 2NO_2$, $K_2 = 8.4 \times 10^{-10}$ (Given)

Accordingly, there are two constraints that determine a and b:

$$1.941 \times 10^{-3} = \frac{2a}{\left[[48.88-a-b][6.5-a-2b]\right]^{1/2}} \left[\frac{p/p_{ref}}{n}\right]^{\overbrace{1-\frac{1}{2}-\frac{1}{2}}^{0}}$$

$$= \frac{2a}{\left[[48.88-a-b][6.5-a-2b]\right]^{1/2}} \qquad (1)$$

$$8.4 \times 10^{-10} = \frac{[2b]^2}{[48.88-a-b][6.5-a-2b]^2} \left[\frac{20/1}{64.38-b}\right]^{2-1-2}$$

$$= \left[\frac{2b}{6.5-a-2b}\right]^2 \left[\frac{64.38-b}{48.88-a-b}\right]\left[\frac{1}{20}\right] \qquad (2)$$

Solving Eqs (1), (2) simultaneously results in $a = 1.727 \times 10^{-2}$, $b = 3.660 \times 10^{-4}$

The reaction equation is then

$$C_4H_{10} + 13(O_2 + 3.76N_2) \rightarrow 4CO_2 + 5H_2O + 3.454 \times 10^{-2}\,NO + 7.32 \times 10^{-4}\,NO_2 + 6.482\,O_2 + 48.862\,N_2$$

---

1. Using Kay's rule (Sec 11.8) to evaluate T_c for the mixture, it can be confirmed that, because of the relatively high mixture temperature: **1400K**, T_R for the mixture is large enough for the ideal gas model to be applicable.

PROBLEM 14.71 (Cont'd.) - Page 2

Alternative Solution Using IT.

<u>IT Code</u>

```
T = 1400  // K
p = 20  // atm
K2 = 8.4E-10

//  C4H10 + 13 (O2 + 3.76 N2) →
         4 CO2 + 5 H2O + x NO + y NO2 + (6.5 - x/2 - y) O2 + (48.88 - x/2 - y/2) N2
nCO2 = 4
nH2O = 5
nNO = x
nNO2 = y
nO2 = 6.5 - x/2 - y
nN2 = 48.88 - x/2 - y/2
ntot = nCO2 + nH2O + nNO + nNO2 + nO2 + nN2
yNO = nNO / ntot
yO2 = nO2 / ntot
yNO2 = nNO2 / ntot
yN2 = nN2 / ntot
pref = 1  // atm

// For the reaction:  1/2 N2 + 1/2 O2  ↔  NO
K1 = (yNO / (yN2^0.5 * yO2^0.5))
//  Data from Table A-27 are stored in EQNO.LUT.
log(K1) = LOOKUPVAL(EQNO,1,T,3)

//  For the reaction:  N2 + 2 O2  ↔  2 NO2
K2  =  (yNO2^2 / (yN2 * yO2^2)) * (p / pref)^-1
```

<u>IT Results</u>

$K_1 = 0.001941$
$n_{CO2} = 4$ kmol/kmol(C_4H_{10})
$n_{H2O} = 5$ kmol/kmol(C_4H_{10})
$n_{NO} = 0.03454$ kmol/kmol(C_4H_{10})
$n_{NO2} = 0.0007319$ kmol/kmol(C_4H_{10})
$n_{O2} = 6.482$ kmol/kmol(C_4H_{10})
$n_{N2} = 48.86$ kmol/kmol(C_4H_{10})

PROBLEM 14.72

KNOWN: One lbmol of $H_2O(g)$ dissociates to form an equilibrium mixture at 5000°R, 1atm consisting of $H_2O(g)$, H_2, O_2, OH

FIND: Determine the equilibrium composition.

ENGINEERING MODEL: The ideal gas model applies to the equilibrium mixture.

ANALYSIS. The dissociation reaction has the form

$$1\ H_2O \rightarrow \alpha H_2 + \beta O_2 + \gamma OH + \omega H_2O$$

H: $2 = 2\alpha + \gamma + 2\omega \Rightarrow \alpha = 1 - \omega - \gamma/2$

O: $1 = 2\beta + \gamma + \omega \Rightarrow \omega = 1 - 2\beta - \gamma$

$\alpha = 1 - (1 - 2\beta - \gamma) - \gamma/2 = 2\beta + \gamma/2$

The total amount of mixture is $n = (2\beta + \gamma/2) + \beta + \gamma + (1 - 2\beta - \gamma) = 1 + \beta + \frac{\gamma}{2}$

At equilibrium there are two applicable reactions

1. $H_2O \rightleftarrows H_2 + \frac{1}{2} O_2$, $K_1 = 0.0201$ (Table A-27)

2. $H_2O \rightleftarrows OH + \frac{1}{2} H_2$, $K_2 = 0.0215$ (Table A-27)

Accordingly, there are two constraints that determine β and γ:

$$0.0201 = \frac{[2\beta + \gamma/2][\beta]^{1/2}}{[1 - 2\beta - \gamma]}\left[\frac{p/p_{ref}}{1 + \beta + \frac{\gamma}{2}}\right]^{1 + \frac{1}{2} - 1}$$

$$= \left[\frac{2\beta + \gamma/2}{1 - 2\beta - \gamma}\right]\left[\frac{\beta}{1 + \beta + \gamma/2}\right]^{1/2} \qquad (1)$$

$$0.0215 = \frac{[\gamma][2\beta + \gamma/2]^{1/2}}{[1 - 2\beta - \gamma]}\left[\frac{p/p_{ref}}{1 + \beta + \frac{\gamma}{2}}\right]^{1 + \frac{1}{2} - 1}$$

$$= \left[\frac{\gamma}{1 - 2\beta - \gamma}\right]\left[\frac{2\beta + \gamma/2}{1 + \beta + \gamma/2}\right]^{1/2} \qquad (2)$$

Solving eqs (1), (2) simultaneously results in $\beta = 0.0341$, $\gamma = 0.06$. The equilibrium composition is then, per lbmol of H_2O initially present

$$\{0.0982\ H_2,\ 0.0341\ O_2,\ 0.06\ OH,\ 0.8718\ H_2O(g)\} \leftarrow$$

PROBLEM 14.72 (Cont'd.)-Page 2

Alternative Solution Using *IT*.

IT Code

```
T = 5000 // °R
p = 1 // atm

// H2O → a H2 + b O2 + c OH + d H2O
1 = a + c/2 + d  // H2 balance
1/2 = b + c/2 + d/2  // O2 balance
ntot = a + b + c + d
yH2 = a / ntot
yO2 = b / ntot
yOH = c / ntot
yH2O = d / ntot
pref = 1 // atm

// For the reaction: H2O ↔ H2 + 1/2 O2
K1 = ((yH2 * yO2^0.5) / yH2O) * (p / pref)^0.5
// Data from Table A-27 are stored in EQH2O.LUT
log(K1) = LOOKUPVAL(EQH2O,2,T,3)

// For the reaction: H2O ↔ OH + 1/2 H2
K2 = ((yOH * yH2^0.5) / yH2O) * (p / pref)^0.5
// Data from Table A-27 are stored in EQH2O_OH.LUT.
log(K2) = LOOKUPVAL(EQH2O_OH,2,T,3)
```

IT Results

$K_1 = 0.0201$
$K_2 = 0.02147$
a = 0.09828 lbmol
b = 0.03376 lbmol
c = 0.06153 lbmol
d = 0.8709 lbmol

PROBLEM 14.73

KNOWN: Steam enters a heat exchanger. An equilibrium mixture of H_2O, H_2, O_2, H, and OH exits at 1 atm and (a) 2800 K, (b) 3000 K.

FIND: Determine the molar analysis of the equilibrium mixture.

ENGINEERING MODEL: The equilibrium mixture is modeled as an ideal gas mixture.

ANALYSIS: The overall reaction has the form

$$1\,H_2O \rightarrow (1-a-b)\,H_2O + \left(a+\frac{b}{2}-c\right)H_2 + \frac{a}{2}O_2 + 2c\,H + b\,OH$$

The total number of moles n in the product mixture is

$$n = (1-a-b) + \left(a+\frac{b}{2}-c\right) + \frac{a}{2} + 2c + b = 1 + \frac{a}{2} + \frac{b}{2} + c$$

At equilibrium, three independent reactions relate the components of the product mixture:

1. $H_2O \rightleftarrows H_2 + \frac{1}{2}O_2$
2. $H_2O \rightleftarrows \frac{1}{2}H_2 + OH$
3. $H_2 \rightleftarrows 2H$

(a) From Table A-27 at 2800 K, the respective values of $\log_{10}K$ are

$$\log_{10}K_1 = -1.658 \Rightarrow K_1 = 0.02198$$

$$\log_{10}K_2 = -1.624 \Rightarrow K_2 = 0.02377$$

$$\log_{10}K_3 = -2.178 \Rightarrow K_3 = 0.00664$$

The forms taken by Eq. 14.35 when $p/p_{ref} = 1$ are, respectively

$$K_1 = \frac{\left(a+\frac{b}{2}-c\right)\left(\frac{a}{2}\right)^{1/2}}{(1-a-b)}\left(\frac{p/p_{ref}}{1+\frac{a}{2}+\frac{b}{2}+c}\right)^{1/2} = \left(\frac{a+\frac{b}{2}-c}{1-a-b}\right)\left(\frac{a/2}{1+\frac{a}{2}+\frac{b}{2}+c}\right)^{1/2}$$

$$K_2 = \frac{\left(a+\frac{b}{2}-c\right)^{1/2}(b)}{1-a-b}\left(\frac{p/p_{ref}}{1+\frac{a}{2}+\frac{b}{2}+c}\right)^{1/2} = \left(\frac{b}{1-a-b}\right)\left(\frac{a+\frac{b}{2}-c}{1+\frac{a}{2}+\frac{b}{2}+c}\right)^{1/2}$$

$$K_3 = \frac{[2c]^2}{\left(a-\frac{b}{2}-c\right)}\left[\frac{p/p_{ref}}{\left(1+\frac{a}{2}+\frac{b}{2}+c\right)}\right]^{1/2} = \frac{4c^2}{\left(a+\frac{b}{2}-c\right)\left(1+\frac{a}{2}+\frac{b}{2}+c\right)}$$

Solving this set of simultaneous equations for a, b, and c:

$a = 0.076754$, $b = 0.0679$, $c = 0.013334$

The equilibrium mixture on a molar basis is then

$\{0.855346\,H_2O,\ 0.09737\,H_2,\ 0.038377\,O_2,\ 0.026668\,H,\ 0.0679\,OH\}$ ←

PROBLEM 14.73 (Contd.) - Page 2

(b) From Table A-27 at 3000 K the respective values of $\log_{10} K$ are

$$\log_{10} K_1 = -1.343 \Rightarrow K_1 = 0.04539$$
$$\log_{10} K_2 = -1.265 \Rightarrow K_2 = 0.05433$$
$$\log_{10} K_3 = -1.606 \Rightarrow K_3 = 0.02477$$

Solving the set of equations considered in part (a) gives

$$a = 0.12568, \; b = 0.11232, \; c = 0.03282$$

The equilibrium mixture on a molar basis is then

$$\{0.762\ H_2O,\ 0.14902\ H_2,\ 0.06284\ O_2,\ 0.06564\ H,\ 0.11232\ OH\}$$

Alternative Solution Using IT.

IT Code

```
T = 3000 // K
p = 1 // atm

// H2O → a H2O + b H2 + c O2 + d H + e OH
1 = a + b + d/2  // H2 balance
1/2 = a/2 + c + e/2   // O2 balance
ntot = a + b + c + d + e
yH2O = a / ntot
yH2 = b / ntot
yO2 = c / ntot
yH = d / ntot
yOH = e / ntot
pref = 1  // atm

// H2O ↔ H2 + 1/2 O2        // Reaction 1
// H2O ↔ 1/2 H2 + OH        // Reaction 2
// H2 ↔ 2 H                 // Reaction 3

K1 = ((yH2 * yO2^0.5) / yH2O) * (p / pref)^0.5
K2 = ((yH2^0.5 * yOH) / yH2O) * (p / pref)^0.5
K3 = (yH^2 / yH2) * (p / pref)^1

// Data from Table A-27 are stored in EQH2O.LUT, EQH2O_OH.LUT, and EQH2.LUT.
// Look up log10(K) in the appropriate tables.
log(K1) = LOOKUPVAL(EQH2O,1,T,3)
log(K2) = LOOKUPVAL(EQH2O_OH,1,T,3)
log(K3) = LOOKUPVAL(EQH2,1,T,3)
```

IT Results

Part (a): T = 2800 K
n_{H2O} = 0.8719 kmol
n_{H2} = 0.1136 kmol
n_{O2} = 0.03162 kmol
n_{H} = 0.02894 kmol
n_{OH} = 0.06481 kmol

Part (b): T = 3000 K
n_{H2O} = 0.7884 kmol
n_{H2} = 0.1755 kmol
n_{O2} = 0.04983 kmol
n_{H} = 0.07216 kmol
n_{OH} = 0.1119 kmol

PROBLEM 14.74

KNOWN: The system is a two-phase liquid-vapor mixture of (a) water at 100°C, (b) R134a at 20°C.

FIND: Using tabulated property data show that $g_f = g_g$.

ANALYSIS: (a) At 100°C, Table A-2 gives

$h_f = 419.04$ kJ/kg $\qquad h_g = 2676.1$ kJ/kg

$s_f = 1.3069$ kJ/kg·K $\qquad s_g = 7.3549$ kJ/kg·K

Then, with $g = h - Ts$

$$g_f = h_f - T s_f = 419.04 - (373.15)(1.3069) = -68.63 \text{ kJ/kg}$$

① $$g_g = h_g - T s_g = 2676.1 - (373.15)(7.3549) = -68.38 \text{ kJ/kg}$$

(b) At 20°C, Table A-10 gives

$h_f = 77.26$ kJ/kg $\qquad h_g = 258.36$ kJ/kg

$s_f = 0.2924$ kJ/kg·K $\qquad s_g = 0.9102$ kJ/kg·K

Then, with $g = h - Ts$

$$g_f = 77.26 - (293.15)(0.2924) = -8.457 \text{ kJ/kg}$$

$$g_g = 258.36 - (293.15)(0.9102) = -8.465 \text{ kJ/kg}$$

---

1. As developed in Sec. 6.3.1, $s_g - s_f = (h_g - h_f)/T$ for a change in phase from saturated liquid to saturated vapor at T. Thus $h_g - Ts_g = h_f - Ts_f$, or $g_g = g_f$. The slight difference in the calculated values of g_g and g_f owes to table roundoff and signifies a slight inconsistency in table values.

PROBLEM 14.75

See the respective problems in Chapter 11 for solutions.

PROBLEM 14.76

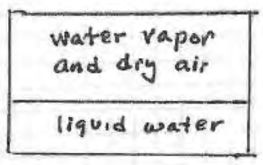
A closed system at 20°C, 1 bar consists of a pure liquid water phase in equilibrium with a vapor phase composed of water vapor and dry air. Determine the departure, in percent, of the partial pressure of the water vapor from the saturation pressure of pure water at 20°C.

① KNOWN: A liquid water phase is in equilibrium with moist air.

FIND: Determine, in percent, how much the presence of the dry air alters the partial pressure of the water vapor from the saturation pressure.

SCHEMATIC & GIVEN DATA:

Contents at 20°C, 1 bar



ENGINEERING MODEL:

1. The water vapor and liquid water are in phase equilibrium.
2. The water vapor is modeled as an ideal gas.
② 3. The liquid water phase is pure water only. Its specific volume is $v \approx v_f(T)$.

ANALYSIS: Invoking the following expression obtained in Part (a) of Example 14.10,

$$\frac{P_v}{P_{sat}} = \exp\left[\frac{v_f\,(p - P_{sat})}{RT}\right] \qquad (1)$$

With data from Table A-2 at 20°C, $v_f = 1.0018 \times 10^{-3}\ m^3/kg$, $P_{sat} = 0.02339$ bar

$$\frac{v_f\,(p - P_{sat})}{RT} = \frac{(1.0018\times10^{-3}\ m^3/kg)(0.97661\times10^5\ N/m^2)}{\left(\frac{8314}{18.02}\ \frac{N\cdot m}{kg\cdot K}\right)(293.15\ K)} = 7.23\times10^{-4}$$

Eq. (1) then gives

$$\frac{P_v}{P_{sat}} = \exp\left(7.23\times10^{-4}\right) = 1.00072$$

When expressed as a percentage, the departure of P_v from P_{sat} is

③ $$\left(\frac{P_v - P_{sat}}{P_{sat}}\right)(100) = \left(\frac{P_v}{P_{sat}} - 1\right)(100) = 0.072\ \% \longleftarrow$$

---

1. See Note 1 of Example 14.10.
2. See Note 2 of Example 14.10.
3. See Note 4 of Example 14.10.

PROBLEM 14.77

KNOWN: Graphite and diamond exist in equilibrium at 25°C, p.

FIND: Derive an expression for estimating the pressure p in terms of $\bar{v}$, $\bar{g}$ and the isothermal compressibility κ of each phase at 25°C, 1atm.

ENGINEERING MODEL: The isothermal compressibility κ is constant with pressure.

ANALYSIS: By Eq 14.59, at equilibrium $\bar{g}(25°C, p) = \bar{g}'(25°C, p)$ where $\bar{g}$ and $\bar{g}'$ denote the molar Gibbs functions of graphite and diamond, respectively.

To evaluate the molar Gibbs functions relative to $T_{ref} = 25°C$, $p_{ref} = 1 atm$, begin with Eq. 11.23. Since T is fixed

$$d\bar{g} = \bar{v}\,dp \tag{1}$$

Also, at fixed T the differential of $\bar{v}(T, p)$ reduces to

$$d\bar{v} = \underbrace{\left(\frac{\partial \bar{v}}{\partial p}\right)_T}_{= -\kappa \bar{v}\ (Eq.\ 11.63)} dp + \cancel{\left(\frac{\partial \bar{v}}{\partial T}\right)_p dT} = -\kappa \bar{v}\,dp$$

So

$$\frac{d\bar{v}}{\bar{v}} = -\kappa\,dp \Rightarrow \text{Integration at fixed T and constant } \kappa,\ \ln\left[\frac{\bar{v}(T_{ref}, p)}{\bar{v}(T_{ref}, p_{ref})}\right] = -\kappa(p - p_{ref})$$

or

$$\bar{v}(T_{ref}, p) = \bar{v}(T_{ref}, p_{ref})\exp(-\kappa(p - p_{ref})) \tag{2}$$

Combining Eqs (1), (2), and integrating

$$\bar{g}(T_{ref}, p) - \bar{g}(T_{ref}, p_{ref}) = \bar{v}(T_{ref}, p_{ref})\frac{\exp(\kappa p_{ref})}{-\kappa}\Big[\exp(-\kappa p)\Big]_{p_{ref}}^{p}$$

$$= -\frac{\bar{v}(T_{ref}, p_{ref})}{\kappa}\Big[\exp(-\kappa(p - p_{ref})) - 1\Big]$$

Then, with $T_{ref} = 25°C$, $p_{ref} = 1 atm$ and the condition $\bar{g} = \bar{g}'$

$$\bar{g}(25°C, 1atm) - \frac{\bar{v}(25°C, 1atm)}{\kappa}\Big[\exp(-\kappa(p - 1atm)) - 1\Big] =$$

$$\bar{g}'(25°C, 1atm) - \frac{\bar{v}'(25°C, 1atm)}{\kappa'}\Big[\exp(-\kappa'(p - 1atm)) - 1\Big] \tag{3}$$

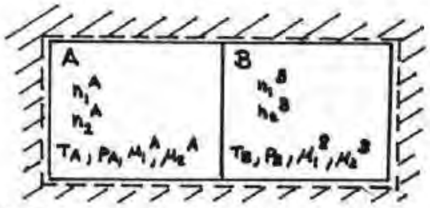
Equation (3) has a single unknown: the pressure p. Thus Eq (3) gives p implicitly in terms of $\bar{g}$, $\bar{v}$ and κ for each phase at 25°C, 1atm.

PROBLEM 14.78

KNOWN: An isolated system has two phases, A and B. Each phase consists of the same two substances, 1 and 2.

FIND: Show that conditions for equilibrium are 1: $T_A = T_B$, 2: $P_A = P_B$, 3: $\mu_1^A = \mu_1^B$, $\mu_2^A = \mu_2^B$.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The system shown in the accompanying figure is isolated. (2) Eq. 11.114(a) is applicable to each phase. (3) A and B do not react with one another.

ANALYSIS: According to Eq. 14.3 any process of an isolated system must satisfy $dS]_{U,V} \geq 0$. This indicates that the entropy of an isolated increases during an irreversible process. Each step of the process results in an increase in the entropy of the system and brings the system closer to equilibrium. The equilibrium state is the one having the maximum value of S. Therefore, when

$$dS]_{U,V} = 0 \tag{1}$$

there is equilibrium.

For the isolated system under present consideration, $dS = dS_A + dS_B$. Using Eq. 11.114a to evaluate dS_A and dS_B, respectively, we get

$$dS = \left[\frac{dU_A}{T_A} + \frac{P_A}{T_A}dV_A - \frac{\mu_1^A}{T_A}dn_1^A - \frac{\mu_2^A}{T_A}dn_2^A\right] + \left[\frac{dU_B}{T_B} + \frac{P_B}{T_B}dV_B - \frac{\mu_1^B}{T_B}dn_1^B - \frac{\mu_2^B}{T_B}dn_2^B\right] \tag{2}$$

For the isolated system, the total energy, total volume, and total mass for each substance are constant:

$$dU_A + dU_B = 0,\quad dV_A + dV_B = 0,\quad dn_1^A + dn_1^B = 0,\quad dn_2^A + dn_2^B = 0 \tag{3}$$

Introducing Eqs. (3) into Eq. (2)

$$dS = \left[\frac{1}{T_A} - \frac{1}{T_B}\right]dU_A + \left[\frac{P_A}{T_A} - \frac{P_B}{T_B}\right]dV_A - \left[\frac{\mu_1^A}{T_A} - \frac{\mu_1^B}{T_B}\right]dn_1^A - \left[\frac{\mu_2^A}{T_A} - \frac{\mu_2^B}{T_B}\right]dn_2^A \tag{4}$$

Eq. (4) shows that in seeking the maximum in S the independent variables are U_A, V_B, n_1^A, and n_2^A. Necessary conditions for a maximum are

$$\frac{\partial S}{\partial U_A} = \frac{\partial S}{\partial V_A} = \frac{\partial S}{\partial n_1^A} = \frac{\partial S}{\partial n_2^A} = 0$$

The first of these conditions requires

$$\frac{1}{T_A} - \frac{1}{T_B} = 0 \quad \Rightarrow \quad T_A = T_B \quad \longleftarrow$$

From the second

$$\frac{P_A}{T_A} - \frac{P_B}{T_B} = 0 \quad \Rightarrow \quad P_A = P_B \quad \longleftarrow$$

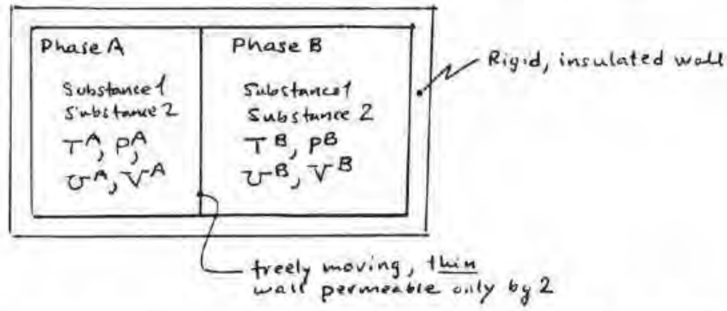
With the remaining two conditions

$$\frac{\mu_1^A}{T_A} - \frac{\mu_1^B}{T_B} = 0 \Rightarrow \mu_1^A = \mu_1^B,\quad \frac{\mu_2^A}{T_A} - \frac{\mu_2^B}{T_B} = 0 \Rightarrow \mu_2^A = \mu_2^B \quad \longleftarrow$$

PROBLEM 14.79

KNOWN: An isolated system has two phases, A and B, each of which consists of the same two substances, 1 and 2. The phases are separated by a freely moving, thin wall permeable only by substance 2.

FIND: Determine the necessary conditions for equilibrium.



ENGINEERING MODEL: The system consists of phase A plus phase B (2) For the system, $Q = W = 0$ and there are no kinetic/potential energy effects. (3) The phases are separated by a freely moving, thin wall permeable only by 2.

ANALYSIS: The appropriate equilibrium criteria for this case is obtained via Equation 14.3 as $dS]_{U,V} = 0$, where $dS = dS^A + dS^B$. Invoking Eq 11.114a

$$dU^A = T^A dS^A - p^A dV^A + \cancel{\mu_1^A dn_1^A} + \mu_2^A dn_2^A$$
$$dU^B = T^B dS^B - p^B dV^B + \cancel{\mu_1^B dn_1^B} + \mu_2^B dn_2^B$$

Since the wall is permeable only by substance 2, the amount of substance 1 present in each of the phases cannot change, and so these terms have been dropped in the expression above.

Since the container is rigid and insulated,

total volume is constant : $dV^B = -dV^A$

total energy is constant : $dU^B = -dU^A$

Since B is conserved, $dn_2^B = -dn_2^A$

Collecting results:

$$dS]_{U,V} = \left[\frac{dU^A}{T^A} + \frac{p^A}{T^A} dV^A - \frac{\mu_2^A}{T^A} dn_2^A\right] + \left[\frac{dU^B}{T^B} + \frac{p^B}{T^B} dV^B - \frac{\mu_2^B}{T^B} dn_2^B\right]$$

$$= dU^A\left[\frac{1}{T^A} - \frac{1}{T^B}\right] + dV^A\left[\frac{p^A}{T^A} - \frac{p^B}{T^B}\right] - dn_2^A\left[\frac{\mu_2^A}{T^A} - \frac{\mu_2^B}{T^B}\right]$$

Since U^A, V^A, n_2^A can be varied independently, it follows that when $dS]_{U,V} = 0$ the terms in the parentheses must vanish. Therefore, the criteria of equilibrium are

$T^A = T^B$ (temperature is the same in each phase)

① $p^A = p^B$ (pressure is the same in each phase)

$\mu_2^A = \mu_2^B$ (chemical potential of substance 2 is the same in each phase.)

---

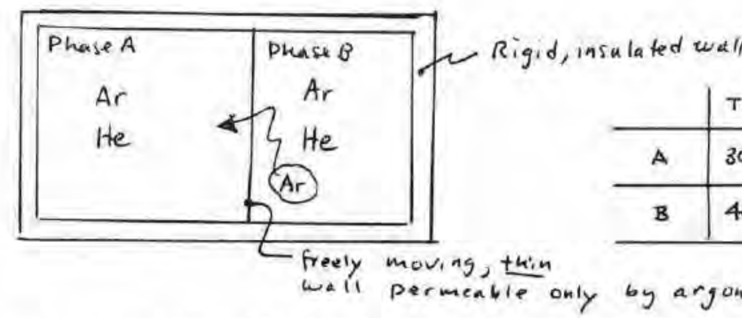
1. There is no restriction on μ_1^A or μ_1^B.

PROBLEM 14.80

KNOWN: An isolated system has two gas phases, A and B, each of which consists of argon and helium. The phases are separated by a freely moving, thin wall permeable only to argon. Initial data is provided.

FIND: Using the result of Problem 14.79, determine the final equilibrium temperature, pressure, and composition in the two phases.

SCHEMATIC & GIVEN DATA:



	T(K)	p(MPa)	n(kmol)	y_{AR}	y_{He}
A	300	0.2	6	0.5	0.5
B	400	0.1	5	0.8	0.2

ENGINEERING MODEL: (1) The idealizations of Problem 14.79 apply. (2) Ideal gas principles apply to the gas mixtures.

ANALYSIS: An energy balance requires $\Delta U = \Delta U^A + \Delta U^B = 0$, where

$$\Delta U^A = [n^A_{a,f}\,\bar{u}_a(T_f) + n^A_h\,\bar{u}_h(T_f)] - [n^A_{a,i}\,\bar{u}_a(T^A) + n^A_h\,\bar{u}_h(T^A)]$$

$$\Delta U^B = [n^B_{a,f}\,\bar{u}_a(T_f) + n^B_h\,\bar{u}_h(T_f)] - [n^B_{a,i}\,\bar{u}_a(T^B) + n^B_h\,\bar{u}_h(T^B)]$$

So,

$$0 = [n^A_{a,f} + n^B_{a,f}]\,\bar{u}_a(T_f) - n^A_{a,i}\,\bar{u}_a(T^A) - n^B_{a,i}\,u_a(T^B) + n^A_h[\bar{u}_h(T_f) - \bar{u}_h(T^A)] + n^B_h[\bar{u}_h(T_f) - \bar{u}_h(T^B)]$$

With $n^A_{a,f} + n^B_{a,f} = n^A_{a,i} + n^B_{a,i}$, where $n^A_{a,i} = 3$, $n^B_{a,i} = 4$, $n_a^{TOT} = 7$.

$$0 = n^A_{a,i}[\bar{u}_a(T_f) - \bar{u}_a(T^A)] + n^B_{a,i}[\bar{u}_a(T_f) - \bar{u}_a(T^B)] + n^A_h[\bar{u}_h(T_f) - \bar{u}_h(T^A)] + n^B_h[\bar{u}_h(T_f) - \bar{u}_h(T^B)]$$

For monatomic gases $\Delta\bar{u} = c_v\,\Delta T$, where c_v is a constant. So, this becomes after arrangement

$$T_f = \frac{(n^A_{a,i} + n^A_h)T^A + (n^B_{a,i} + n^B_h)T^B}{(n^A_{a,i} + n^A_h) + (n^B_{a,i} + n^B_h)} = \frac{(6)(300\text{K}) + 5(400\text{K})}{11} = 345.5\text{K} \longleftarrow T_f$$

The total volume is unchanged. Thus $(V^A + V^B)_f = (V^A + V^B)_i$:

$$n_{TOT}\frac{\bar{R}T_f}{p_f} = \frac{n^A\bar{R}T^A}{p^A} + \frac{n^B\bar{R}T^B}{p^B} \Rightarrow p_f = \frac{n_{TOT}\,T_f}{\frac{n^AT^A}{p^A} + \frac{n^BT^B}{p^B}} = \frac{(11)(345.5)}{\frac{(6)(300)}{0.2} + \frac{(5)(400)}{0.1}} = 0.131\text{ MPa} \longleftarrow p_f$$

The final constraint is $\mu^A_a = \mu^B_a$ at the final state. With Eq. 14.17, this becomes $y^A_{a,f} = y^B_{a,f}$. That is

$$\frac{n^A_{a,f}}{n^A_{a,f} + n^A_h} = \frac{n^B_{a,f}}{n^B_{a,f} + n^B_h} \Rightarrow \frac{n^A_{a,f}}{n^A_{a,f} + 3} = \frac{(7 - n^A_{a,f})}{(7 - n^A_{a,f}) + 1} \Rightarrow 4n^A_{a,f} = 21 \Rightarrow n^A_{a,f} = 5.25$$

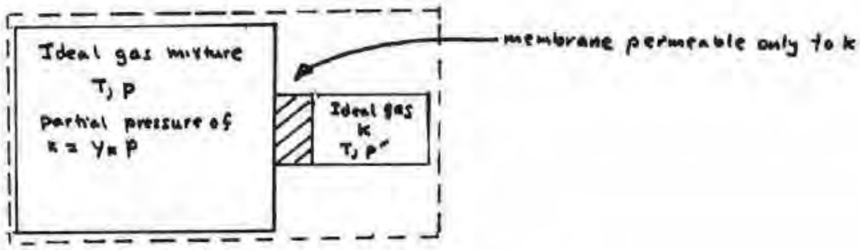
Accordingly at the final equilibrium state Phase A contains 5.25 kmol of Argon and 3.0 kmol of Helium. ⟵ Phase B contains 1.75 kmol of Argon and 1.0 kmol of Helium.

PROBLEM 14-81

KNOWN: An ideal gas mixture at T and p containing substance k is separated by a semipermeable membrane from a gas phase of pure k at T and p'.

FIND: Determine the relationship between p and p' for there to be no net transfer through the membrane.

SCHEMATIC & GIVEN DATA:



ENGINEERING MODEL: (1) The system is shown in the accompanying figure. (2) The gas mixture and the pure phase can be modeled as ideal gases. (3) The membrane allows only k to pass.

ANALYSIS: Following the discussion of Sec. 14.6.1, there would be no net transfer of k when the chemical potential of k in the mixture equals the chemical potential of pure k:

$$\mu_{k,\,mixture} = \mu_{k,\,pure} \qquad (1)$$

With Eq. 14.12, $\mu_{k,\,pure} = \bar{g}_k(T, p')$, and with Eq. 14.16, $\mu_{k,\,mixture} = \bar{g}_k(T, y_k p)$. Thus Eq. (1) becomes

$$\bar{g}_k(T, y_k p) = \bar{g}_k(T, p') \implies p' = y_k p \longleftarrow$$

That is, the pressure p' of the pure phase must equal the partial pressure of k in the mixture.

PROBLEM 14.82

KNOWN: A system involves (a) one component, (b) two components, (c) three components.

FIND: Determine the maximum number of homogeneous phases that can exist at equilibrium.

ENGINEERING MODEL: No reaction takes place.

ANALYSIS: Equation 14.68 summarizes Gibbs' phase rule, on rearrangement

$$P = 2 + N - F$$

where N is the number of components and P is the number of phases. The value of P is greatest when F, the degrees of freedom is least: zero. Thus, when $F = 0$

$$P = 2 + N \tag{1}$$

(a) $N = 1 \Rightarrow P = 3$. (b) $N = 2 \Rightarrow P = 4$. (c) $N = 3$, $P = 5$.

PROBLEM 14.83

Determine the number of degrees of freedom for systems composed of

(a) water vapor and dry air.
(b) liquid water, water vapor, and dry air.
(c) ice, water vapor, and dry air.
(d) N_2 and O_2 at 20°C, 1 atm.
(e) a liquid phase and a vapor phase, each of which contains ammonia and water.
(f) liquid acetone and a vapor phase of acetone and N_2.

KNOWN: Various systems are specified.

FIND: Determine the number of degrees of freedom.

ENGINEERING MODEL: (1) Gibbs phase rule applies. (2) Dry air is regarded as a single component.

ANALYSIS: Invoking Eq. 14.68, the degrees of freedom F is

$$F = 2 + N - P$$

Number of Components

Number of phases

(a) Water vapor and dry air: $N = 2$, $P = 1 \Rightarrow F = 2 + 2 - 1 = 3$.

(b) Liquid water, water vapor, and dry air: $N = 2$, $P = 2 \Rightarrow F = 2 + 2 - 2 = 2$.

(c) ice, water vapor, and dry air: $N = 2$, $P = 2 \Rightarrow F = 2 + 2 - 2 = 2$.

(d) N_2 and O_2 as gases: $N = 2$, $P = 1 \Rightarrow F = 2 + 2 - 1 = 3$.

(e) a liquid phase and a vapor phase, each containing NH_3, H_2O:
$N = 2$, $P = 2 \Rightarrow F = 2 + 2 - 2 = 2$.

(f) liquid acetone and a vapor phase of acetone and N_2: $N = 2$, $P = 2 \Rightarrow$
$F = 2 + 2 - 2 = 2$.

PROBLEM 14.84

FIND: Develop the phase rule for chemically reacting systems.

ANALYSIS: If there is a chemical reaction within the system, the constituents are not completely independent. For example, if the constituents A, B, C, D undergo the reaction

$$\nu_A A + \nu_B B \rightleftarrows \nu_C C + \nu_D D$$

there is an additional independent equation. Accordingly, the number of independent equations is $\{N(P-1)+1\}$. The number of variables is $P[N-1]+2$, as on p. 685. Accordingly, the number of degrees of freedom is

$$F = [P(N-1)+2] - [N(P-1)+1]$$
$$= (N-1) - P + 2$$

If a number of independent reactions occur, say r reactions, the phase rule takes the form

$$F = (N-r) - P + 2$$

PROBLEM 14.85

KNOWN: The gas phase reaction: $CH_4 + H_2O \rightleftarrows CO + 3H_2$

FIND: Determine the number of degrees of freedom.

ENGINEERING MODEL: There is a single independent reaction.

ANALYSIS: Applying the result of Problem 14.84, the number of degrees of freedom is

$$F = (N - r) - P + 2$$

No. of components = 4

No. of independent reactions = 1

No. of phases = 1

Thus

① $$F = (4-1) - 1 + 2 = 4$$

---

1. To specify this system, one might give values for T, p and the mole fractions of any two of the four components.

PROBLEM 14.86

KNOWN: Two cases are identified involving a two-phase liquid-vapor ammonia–water system at temperature T and pressure p:
(a) T = 20°C, mole fraction of NH_3 in the liquid phase is 80%
(b) T = 40°C, p = 12 bar.

FIND: (a) Determine p, in bar, and the mole fraction of NH_3 in the vapor phase. (b) Determine the mole fraction of NH_3 in each of the phases.

ENGINEERING MODEL: Raoult's law applies.

ANALYSIS: (a) For the liquid phase $y_{NH_3} = 0.80$. Thus, it follows that $y_{wv} = 0.20$. By Raoult's law, the partial pressures in the vapor phase are

$$p_{NH_3} = 0.8\; p_{sat,NH_3}(20°C) = 0.8\,(\underbrace{8.5762\text{ bars}}_{\text{Table A-13}}) = 6.861\text{ bar}$$

$$p_{wv} = 0.2\; p_{sat,wv}(20°C) = 0.2\,(\underbrace{0.02339\text{ bars}}_{\text{Table A-2}}) = 0.005\text{ bar}$$

Then for the vapor phase

$$p = p_{NH_3} + p_{wv} = 6.861 + 0.005 = 6.866\text{ bar} \longleftarrow p$$

Also, for the vapor phase, $p_{NH_3} = x_{NH_3}\, p$. So

① $$x_{NH_3} = \frac{p_{NH_3}}{p} = \frac{6.861}{6.866} = 0.999 \longleftarrow x_{NH_3}$$

(b) The partial pressures of the vapor phase sum to give the system pressure:

$$12\text{ bars} = p_{NH_3} + p_{wv} \qquad (1)$$

According to Raoult's law

$$p_{NH_3} = y_{NH_3}\; p_{sat,NH_3}(40°C) = y_{NH_3}\,(\underbrace{15.549\text{ bar}}_{\text{Table A-13}}) \qquad (2)$$

$$p_{wv} = y_{wv}\; p_{sat,wv}(40°C) = y_{wv}\,(\underbrace{0.07384\text{ bar}}_{\text{Table A-2}}) \qquad (3)$$

Also

$$1 = y_{NH_3} + y_{wv} \qquad (4)$$

Eqs. (1)-(4) give

$$12\text{ bar} = y_{NH_3}(15.549\text{ bars}) + (1 - y_{NH_3})(0.07384\text{ bar})$$

Solving, $y_{NH_3} = 0.771$. $\longleftarrow y_{NH_3}$

For the gas phase, $p_{NH_3} = x_{NH_3}\, p$. Thus, Eq. (2) with $y_{NH_3} = 0.771$ gives

$$(0.771)(15.549\text{ bar}) = x_{NH_3}\,(12\text{ bar})$$

① $$\Rightarrow\quad x_{NH_3} = \frac{(0.771)(15.549)}{12} = 0.999 \longleftarrow x_{NH_3}$$

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1. In each case the vapor phase is nearly all ammonia. The small amount of water vapor present in the gas phase is important owing to the possibility of freezing at some points of the cycle.