

## LETTERS TO THE EDITORS

The Editors do not hold themselves responsible for opinions expressed by their correspondents. No notice is taken of anonymous communications

## Measurement of Length and Angle in Quantum Mechanics

I WISH to point out that in the accepted theory the association of operators and observations is very intricate precisely in the simplest cases. You give me a splinter of a diamond crystal. Its state function is to be an antisymmetric function of the co-ordinates  $x_k$  of the  $n$  carbon nuclei the specimen contains. Can this function give information on the shape of the splinter? The label  $k$  is not connected with any location, with any point of the crystal lattice! Let the splinter be needle-shaped. I can measure its length. What operator is associated with this measurement? We must remember that only operators symmetric with respect to all the  $x_k$  are meaningful.

The only answer I can find to the last question is this. Let  $r_{kl} \geq 0$  be the distance between the  $k$ th and the  $l$ th carbon nucleus ( $k \neq l$ ). Let  $F$  be a symmetric function of the  $n(n-1)/2$  variables  $r_{kl}$ , the value of  $F$  being equal to the biggest of its arguments. (This does not contradict its symmetry; for example, for  $n=3$  we should have  $F$  a function of the three variables  $r_{12}, r_{23}, r_{31}$ , constant on the half-surfaces of certain cubes, its value being the edge of the cube.)

This function  $F(r_{kl})$  might be associated with the measurement of the length of the needle. In order to account for the whole observed shape of a solid, the construction could be extended in an obvious way, by using as the arguments of  $F$  not the mutual distances, but the Cartesian co-ordinates of the nuclei with respect to the centre of gravity. But is this answer satisfactory?

In the case of a crystal, the elements of shape that interest us most are the angles between the various plane domains of its surface. The measurement of the angle between two crystal planes is certainly the one most frequently performed on a crystal concerning its external shape. Even an X-ray investigation of a single crystal is usually preceded by carefully orientating it with respect to a given frame; this amounts to angle measurements of a similar kind. For all these purposes no measurement of length in the crystal is required, nor is any usually made. It is hard to see how the accepted theory would associate a Hermitic operator with this very common kind of laboratory performance.

E. SCHRÖDINGER

Dublin Institute for Advanced Studies.

Jan. 5.

## Diffraction Gratings in Immersion

At the London Optical Conference in 1950, it was suggested<sup>1</sup> that the resolving power  $R$  of diffraction gratings might be increased by immersing the grating in a medium of high refracting index  $\mu_i$ . Thus:

$$R = \frac{2W \sin \alpha}{\lambda} \cdot \mu_i \quad (1)$$

where  $W$  is the ruled width of the grating and where  $\alpha$  applies for the case of autocollimation of incidence and diffraction. This idea, which is a strict parallel

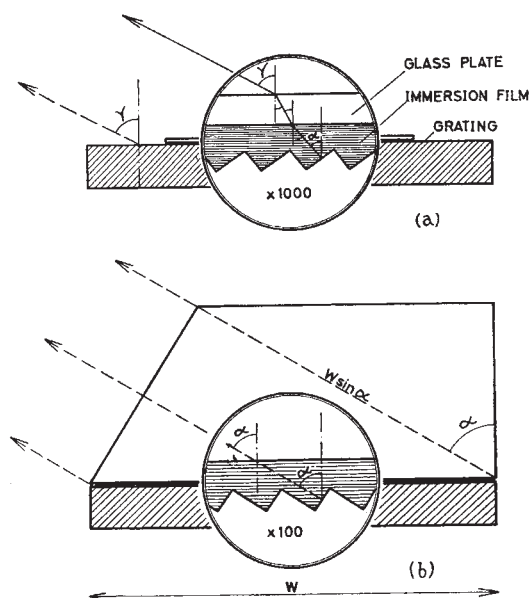


Fig. 1

to that of the immersion microscope, was further developed<sup>2</sup> in accordance with Figs. 1a and b.

If we bring a piece of coverglass into optical contact with a plane grating with an immersion film (Fig. 1a), two kinds of spectra can be observed: one produced by the uncovered grating, as indicated by the dotted line, and the other produced by the 'immersion'-grating. These two spectra will coincide in all orders in accordance with Snell's law of refraction. They will, however, exhibit great differences in intensity, as the immersion spectrum is produced at a steeper angle of incidence  $\alpha$  than the angle of incidence  $\gamma$  occurring at the uncovered surface:

$$\sin \gamma = \mu_i \sin \alpha \quad (2)$$

Thus a grating characterized by a 'blaze' in an order  $m$  will have this blaze shifted into the range of  $\mu_i m$  when it is used as an immersion grating. Striking illustrations of this effect were obtained from photographs of the grating surface. Thus a piece of coverglass, which was fastened to the grating by an immersion oil, appeared in bright or dark contrast to the uncovered part of the grating, when illuminated in different orders of monochromatic light.

Some experiments were also made with ruling gratings on transparent films of antimony trisulphide and zinc sulphide, evaporated in vacuum on the 'back' surface of plane glass plates, afterwards aluminizing the ruled surfaces for reflexion. These films of very high refractive indices ( $\mu > 2$ ) were, however, very brittle, so that the grating surfaces soon began to crack. (The proper name 'refraction grating' given earlier<sup>2</sup> refers only to this special application of the immersion principle with diffraction gratings.)

A second method of testing the immersion principle will now be reported. In the ordinary dense-ruled gratings, the blaze generally appears in the first or second orders of spectrum, corresponding to angles of incidence less than  $20^\circ$ . In the coarse-ruled gratings, however, the blaze sometimes appears at high angles of incidence  $\approx 60^\circ$ . When applying the method of immersion to a grating of this type, it is advantageous