LETTERS TO THE EDITORS

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Dirac's New Electrodynamics

ABOUT twenty-five years ago, W. Gordon[1], O. Klein[2] and I[3] shared in establishing a consistent set of field equations for a charge-scalar ψ and the electromagnetic 4-potential A_k. To disencumber the formulæ, we take $c = 1$ and take the A^k to mean the ordinary potentials multiplied by $2\pi e/h$. Then the real Lagrangian density we used reads:

$$\tfrac{1}{4}F^{kl}F_{kl} + \tfrac{1}{2}(\psi_{,\underline{k}} - iA^k\psi)(\psi^*{}_{,k} + iA_k\psi^*) - \tfrac{1}{2}m^2\psi\psi^*, \quad (1)$$

with

$$F_{kl} = A_{k,l} - A_{l,k}. \quad (2)$$

The asterisk means the complex conjugate, a comma ordinary differentiation, underlining the subsequent raising of the subscript; the metric is $(-1, -1, -1, 1)$; m is the reciprocal Compton wave-length of 'the particle'; ψ and A_k have the dimension reciprocal length; the allotment of indices is the customary ($F_{14} = E_x$, etc.), as opposed to the rational. From (1) and (2) one obtains the Euler equations:

$$\left(\frac{\partial}{\partial x_{\underline{k}}} - iA^k\right)\left(\frac{\partial}{\partial x_k} - iA_k\right)\psi = -m^2\psi \quad (3)$$

$$F^l{}_{k,l} = -j_k \quad (4)$$

with the 4-current

$$j_k = -\frac{i}{2}(\psi^*\psi_{,k} - \psi\psi^*{}_{,k}) - A_k\psi\psi^*, \quad (5)$$

for which the equation of continuity follows from (4) and independently from (3).

It seems to have remained unnoticed at the time and, so I believe, ever since, that simplification is obtained by a change of gauge. We put the real quantities:

$$A_k + \left(\frac{i}{2}\log\frac{\psi}{\psi^*}\right)_{,k} = \bar{A}_k \quad (6)$$

$$\psi\exp\left(-\tfrac{1}{2}\log\frac{\psi}{\psi^*}\right) = \sqrt{\psi\psi^*} = \varphi. \quad (7)$$

Then the (unchanged) j_k becomes

$$j_k = -\bar{A}_k\varphi^2, \quad (8)$$

while (3) splits openly into real and imaginary parts. The latter re-asserts that the vector (8) has vanishing divergence; the former reads

$$\frac{\partial}{\partial x_k}\frac{\partial}{\partial x_{\underline{k}}}\varphi = (\bar{A}_k\bar{A}^k - m^2)\varphi. \quad (9)$$

That the wave function of (3) can be made real by a change of gauge is but a truism, though it contradicts the widespread belief about 'charged' fields requiring complex representation. It is equally a truism that the bracket in (5) then vanishes, so that (5) turns into (8). But it is interesting that with (8) the equations (4) become those of Dirac's[4] recently proposed theory, the potential becoming proportional to the current.

Dirac's fifth field variable λ is, in the present setting, the square of the real amplitude φ. In a wave phenomenon the *rays*—corresponding to particle paths—are not sharply defined. The concept is based on the assumption that the wave amplitude varies comparatively slowly. The better this holds the sharper is the definition of rays, breaking down entirely when the assumption fails. It is peculiar to our case that the rapidly varying phase, with which the slowly varying amplitude is usually contrasted, is abolished. The only length available for comparison is m^{-1}. If φ does not vary appreciably at such range, then from (9),

$$\bar{A}_k\bar{A}^k - m^2 = 0 \quad (10)$$

holds approximately. This is Dirac's fifth field equation, from which, as he has shown, follows immediately that the vector lines of A_k are *paths* in the field F_{kl} that is described by (2); from (8) the same holds for the vector lines of j_k, which does not depend on the gauge. One is, I think, entitled to say that the stream lines are *rays* of the wave motion ψ as far as such can be spoken of at all. But one must keep in mind that a detailed description of the wave has meaning only with respect to a fixed gauge. By changing the gauge you may turn the wave-(hyper-) surfaces into whatever you please.

There is a handsome symmetry between the equations (4), with (8), and the equation (9). Their left-hand sides exhibit with respect to $\bar{A}_k$ and φ the completest analogy possible for a vector and a scalar, the operators being Div Rot and Div Grad, respectively. On the right, the quadratic invariant of the *other* quantity occupies, one may say, the position of an eigenvalue parameter in the otherwise linear equations. I venture to surmise that we are actually facing an eigenvalue problem. If so, it is mathematically much more involved than those we are accustomed to. Any charge-free Maxwellian field, by the way, satisfies our equations. One is interested in what happens when (3) is replaced by Dirac's wave equation of 1927, or other first-order equations. This and the bearing on Dirac's 1951 theory will be discussed more fully elsewhere.

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Surface of Finely-Ground Silica

THE existence of a highly soluble surface layer on quartz particles, foreshadowed by the results of Briscoe *et al.*[1], and of King[2], has been demonstrated in our laboratory, and the properties of the layer studied in considerable detail[3]. All siliceous dusts examined possessed initially a high-solubility layer which is apparently not discrete, but blends smoothly into the less soluble core. It is removed by various solvents, a borate buffer (pH 7·5) being principally studied; and it is dispersed by this buffer mainly in the form of somewhat flocculent colloidal silica, the less soluble core afterwards yielding, more slowly, clear solutions of silica. In view of the importance widely attached to the 'solubility theory' in studies on silicosis, it is clearly of interest to determine whether the high-solubility layer on inhaled siliceous