SUB-CRITICAL FLUCTUATIONS AND KINETICS OF THE ELECTROWEAK TRANSITION*

MARCELO GLEISER
Department of Physics and Astronomy
Dartmouth College, Hanover, NH 03755

INTRODUCTION

The possibility that the baryon number of the Universe can be generated at the electroweak phase transition has triggered a lot of interest in understanding the dynamics of weakly first-order phase transitions in the early Universe.[1] One of the necessary ingredients of a successful baryogenesis scenario is a departure from equilibrium during the transition. Current scenarios of electroweak baryogenesis rely on a first-order phase transition to generate the required out-of-equilibrium conditions; the symmetric metastable phase decays by the nucleation of bubbles of the broken-symmetric phase which are larger than a critical size. These bubbles expand and eventually percolate, completing the transition. There has been a lot of interesting work on the details of bubble wall dynamics within different baryogenesis models, including both the standard model and some of its extensions.[2]

In the present work, however, we would like to focus on questions concerning mainly the standard electroweak model and its phase transition, even though sufficient baryon number is probably not generated in the minimal version of the model. First we examine the reliability of the 1-loop approximation to the effective potential. By now it is well-known that the 1-loop approximation to the effective potential in any Higgs-like model suffers from infrared problems in the gauge sector for small enough values of the scalar vacuum expectation value, $\langle\phi\rangle$.[3] In analogy with QCD, it is believed that this IR problem is taken care of by a proper resummation of graphs; a magnetic plasma mass appears which is independent of $\langle\phi\rangle$, solving the IR problem. This divergence is independent of the strength of the transition. Another IR problem arises for weak enough transitions, when λ is not much smaller than g^2. (λ is the scalar quartic self-coupling and g is the gauge coupling.) Roughly speaking, the perturbative expansion parameter for scalar loops, $\lambda_T T/m(T)$, may become too large for a certain temperature range. An extreme example occurs in the context of second order transitions for which $m(T) \rightarrow 0$ at the critical temperature T_c. This is the origin of critical phenomena, characterized by divergent correlations on spatial fluctuations of the order parameter.[4] It turns out that for large enough Higgs masses, when $\lambda \lesssim g^2$, the electroweak transition is sufficiently weak for both these IR problems to be relevant.

The arguments above are based on perturbation theory. We would also like to have a non-perturbative criterion in order to establish the validity of the 1-loop approximation to the effective potential, and apply it to the electroweak potential. This will be done in the next Section.[5]

* This work is partially supported by a National Science Foundation grant No. PHY-9204726.

The results are in qualitative agreement with the estimates coming from the breakdown of perturbation theory.

The advantage of the non-perturbative approach is that it will give us an indication of the possible dynamics of a weak first-order transition. This will be the second issue addressed here. Is the dynamics of a sufficiently weak first order transition governed by the usual nucleation mechanism?[6] By studying the kinetics of sub-critical thermal fluctuations, we will argue that for sufficiently weak transitions, the evolution of the transition will be characterized by an emulsion of phases which will compete for dominance, as opposed to the nucleation and subsequent percolation of bubbles larger than a critical size. This should not come as a surprise, as weak enough transitions should exhibit pseudo-critical behavior due to the large (but non-divergent) correlations present in the symmetric phase.[7] In Section 3 we obtain an approximate kinetic equation describing the early stages of the phase transition.[8] By solving the equation it is possible to study the relevance of sub-critical thermal fluctuations to the evolution of the transition. We find that for sufficiently weak transitions sub-critical bubbles are effectively produced, and may occupy a reasonable fraction of the total volume of space. In this case, the transition completes by percolation of domains of the broken-symmetry phase.

THERMAL FLUCTUATIONS AND THE 1-LOOP POTENTIAL[5]

As is well-known, the 1-loop approximation to the effective potential relies on having fluctuations about $\phi_c = \langle\phi\rangle$ which are small enough that the inhomogeneous terms in the effective action can be neglected. We assume that for a given amplitude ϕ_A, the dominant fluctuations are spherically symmetric and of roughly a correlation volume. Since in this case the free energy becomes a monotonically increasing function of the radius, a correlation volume bubble is the smallest coherent fluctuation consistent with the coarse-graining implicit in the 1-loop potential, and thus with smallest free energy. In practice, within the 1-loop approximation, the effective potential is equivalent to the coarse-grained Landau-Ginzburg free-energy density, with its minima determining the equilibrium states of the system. In this case, a reasonable criterion for the validity of the 1-loop approximation is that the rms amplitude of these fluctuations, which we write as $\bar{\phi}(T)$, be smaller than the nearest inflexion point, $\phi_{\rm inf}(T)$. This condition is a simple consequence of the fact that the 1-loop approximation is equivalent to a Gaussian approximation for the path integral which naturally breaks down at the inflexion point. Thus we can write, as a criterion for the validity of the 1-loop approximation,

$$\bar{\phi}(T) \leq \phi_{\rm inf}(T)\,. \tag{1}$$

This is a general criterion which can be adapted to different models, including second-order transitions in the neighborhood of the critical point.

What remains is to calculate $\bar{\phi}(T)$. Since $\bar{\phi}(T)$ is the rms amplitude of the fluctuations, its definition is simply,

$$\bar{\phi}(T) = \sqrt{\langle\phi^2\rangle_T - \langle\phi\rangle_T^2}\,, \tag{2}$$

where the thermal average $\langle\ldots\rangle_T$ is defined in terms of the probability distribution for a fluctuation with free energy $F(\phi,T) = \int d^3x \left[\frac{1}{2}\left(\vec{\nabla}\phi\right)^2 + V(\phi)\right]$,

$P[\phi] \sim \exp[-F(\phi,T)/T]$, as

$$\langle\ldots\rangle_T = \frac{\int_{-\infty}^{+\infty} D\phi \ldots P[\phi]}{\int_{-\infty}^{+\infty} D\phi \; P[\phi]} \,. \tag{3}$$

By writing the profile for the correlation volume fluctuations as $\phi(r,T) = \phi_A \exp[-r^2/\xi(T)^2]$, where $\xi(T) = m(T)^{-1}$ is the correlation length, the path integrals above become simple integrals over the amplitude of the fluctuation ϕ_A.

As an application, we study the 1-loop approximation to the electroweak potential given by[9]

$$V_{\rm eff}(\phi,T) = D(T^2 - T_2^2)\phi^2 - ET\phi^3 + \frac{\lambda_T}{4}\phi^4 \,, \tag{4}$$

where D and E are constants given in terms of the W and Z boson masses and of the top quark mass as $D = \frac{1}{24}\left[6\left(\frac{m_W}{\sigma}\right)^2 + 3\left(\frac{m_Z}{\sigma}\right)^2 + 6\left(\frac{m_t}{\sigma}\right)^2\right]$ and $E = \frac{1}{12\pi}\left[6\left(\frac{m_W}{\sigma}\right)^3 + 3\left(\frac{m_Z}{\sigma}\right)^3\right] \simeq 10^{-2}$, where $\sigma \simeq 246$ GeV is the vacuum expectation value of the Higgs field. We use $m_W = 80.6$ GeV and $m_Z = 91.2$ GeV. T_2 is the spinodal instability temperature, given by $T_2 = \sqrt{\frac{m_H^2 - 8B\sigma^2}{4D}}$, where $m_H^2 = \frac{2\lambda + 12B}{\sigma^2}$ is the physical Higgs mass and $B = \frac{1}{64\pi^2\sigma^4}\left(6m_W^4 + 3m_Z^4 - 12m_t^4\right)$. The temperature dependent Higgs self-coupling λ_T can be well approximated by its tree-level value, $\lambda_T \simeq \lambda = m_H^2/2\sigma^2 \simeq 0.08(m_H/100\text{GeV})^2$.

Below the temperature $T_1^2 = T_2^2/(1 - 9E^2/8\lambda D)$, the electroweak potential acquires a new minimum at $\phi_+ = \frac{1}{2\lambda}\left[3ET + \sqrt{9E^2T^2 - 8D(T^2 - T_2^2)\lambda}\right]$. At the critical temperature, $T_c^2 = T_2^2/(1 - E^2/\lambda D)$, the two minima are degenerate. For $T < T_1$, the nearest inflexion point to the minimum $\phi = 0$ is located at

$$\phi_{\rm inf}(T) = \frac{ET}{\lambda} - \sqrt{\frac{E^2T^2}{\lambda^2} - \frac{2D(T^2 - T_2^2)}{3\lambda}} \,. \tag{5}$$

We can now easily obtain an expression for $\bar{\phi}(T)$. Due to the non-linear terms, the integrals over ϕ_A cannot be calculated exactly. However, for the case at hand, the free energy of the fluctuations is dominated by their surface term which is quadratic in ϕ_A. We thus obtain,

$$\bar{\phi}(T) \simeq \left[\frac{4D^{\frac{1}{2}}T(T^2 - T_2^2)^{\frac{1}{2}}}{3\pi^{\frac{3}{2}}}\right]^{\frac{1}{2}} \,. \tag{6}$$

This result can be written as $\bar{\phi}^2(T) \simeq m(T)T/6$.[10] When compared to the numerical evaluation of the integrals, this approximation proves to be good within 10%.[5] By evaluating the condition for the validity of the 1-loop approximation at the critical temperature, $\bar{\phi}(T_c) \leq \phi_{\rm inf}(T_c)$, we obtain a weak coupling condition,

$$\lambda \leq \pi\left[E\left(1 - \frac{\sqrt{3}}{2}\right)\right]^{2/3} \,. \tag{7}$$

In terms of the Higgs mass, we find that for $m_H > 70$ GeV the 1-loop approximation is no longer valid at T_c, due to large amplitude fluctuations about $\langle\phi\rangle = 0$.

KINETICS OF WEAK FIRST-ORDER TRANSITIONS

We have seen that large fluctuations about equilibrium may invalidate the 1-loop approximation to the effective potential. We may ask if the usual mechanism for the dynamics of first-order transitions, based on the nucleation and subsequent percolation of large bubbles, will hold for sufficiently weak transitions. There are many difficulties involved in answering this question. First of all, what is the form of the effective potential beyond 1-loop order? Clearly, in any study of the transition we must rely on some valid approximation to the effective action. Although considerable progress has been made through recent efforts,[3] it is fair to say that within the minimal standard model the question is not yet settled. However, an important aspect of any improved effective potential is that summing higher loops makes the transition even weaker. In studying the kinetics of the transition we may addopt a pragmatic point of view and use the 1-loop approximation as a working model, but keeping in mind that the real transition will be even weaker, if at all first-order. In fact, the presence of large fluctuations within the symmetric phase indicates a sort of pseudo-critical behavior, signalling the proximity of the critical point; the weakness of the transition is related to having $T_c \to T_2$.

The second hard question that comes to mind is how to best model the dynamics of the system in the presence of sub-critical thermal fluctuations. This question is also far from being solved, although an initial attempt was made in Ref. 6, and, more recently in Ref. 8. The idea is to model the statistically dominant fluctuations as having roughly a correlation volume and amplitudes which interpolate between the two phases in question, *i.e.*, the minima of the effective potential. The simple picture that emerges is that of a chessboard-like volume, filled with both phases fluctuating into each other.[6] In principle, the problem is solved when we obtain the time evolution of the volumes of each phase. Clearly, the fact that one can have fluctuations of different volumes and different amplitudes, added to kinetic processes between the bubbles, such as collisions and shrinking, makes the full problem intractable analytically.[11] One must treat the problem numerically, or rely on some simple approximations which are expected to work for a range of parameters.

In Ref. 8 a simple kinetic equation for the earlier stages of the transition was derived. By early stages we mean temperatures just below T_1, the temperature at which a new minimum appears. In this case, most of the volume is in the $\langle\phi\rangle = 0$ phase, and only small sub-critical fluctuations of the phase at ϕ_+ appear. The equation for the number density of these fluctuations was shown to be,

$$\frac{\partial n(R,t)}{\partial t} = -\frac{\partial n(R,t)}{\partial R}\left(\frac{dR}{dt}\right) + \left(\frac{V_0}{V}\right)\Gamma_{0\to+}(R) - \left(\frac{V_+}{V}\right)\Gamma_{+\to 0}(R). \quad (8)$$

Here, $\Gamma_{0\to+}(R)$ ($\Gamma_{+\to 0}(R)$) is the rate per unit volume for the thermal nucleation of a bubble of radius R of phase $\phi = \phi_+$ within the phase $\phi = 0$ (phase $\phi = 0$ within the phase ϕ_+). The volume ratios $V_{0(+)}/V$ take into account the fact that the total volume in each phase changes in time due to the evolution of $n(R,t)$. The initial conditions we choose are $V_0(t=0) = V$, and $V_+(t =$

$0) = 0$, that is, all volume $V = V_0 + V_+$ is initially in the phase $\phi = 0$. Also, V_+ must be understood as the volume of the (+)–phase in bubbles of radius R *only*, since we are following the evolution of $n(R,t)$. Thus, in Eq. (8) bubbles of radius R can disappear due to both their shrinking (accounted for by the first term in the right-hand side with $dR/dt < 0$) and to the nucleation of a bubble of the (0)–phase in its interior.

Within these approximations, we expect the equation above to be a valid description of the kinetics. The interest in studying this equation lies on the fact that we can solve it exactly in two regimes, including and neglecting the term responsible for the bubble shrinking. Once we find the time scales for reaching equilibrium for each of these two regimes we can compare them for different values of parameters that govern the strength of the transition and decide if there is a reasonable range of values for the parameters when shrinking can be neglected in the kinetics. If this is the case, sub-critical bubbles may become a crucial factor in the evolution of the phase transition. Denoting by $\tau_{1(2)}$ the equilibration time-scale with (without) shrinking, and assuming that the bubbles shrink with constant speed v, the ratio of equilibration time scales is, for bubbles of a correlation radius,

$$\frac{\tau_1}{\tau_2(\xi)} = \frac{T}{v\alpha}\left[\frac{4\pi}{3}\xi^3\Gamma_{+\to 0}\right] , \tag{9}$$

where $\alpha \sim \phi_+^2$ is the coefficient of the surface contribution to the fluctuation's free energy. As τ_1/τ_2 becomes larger than unity, shrinking becomes a sub-dominant process in the kinetics of the transition. In Ref. 8, the ratio $\tau_1/\tau_2(\xi)$ was obtained for the minimal standard model as a function of the Higgs mass, and for a top mass of 130 GeV. It was found that the ratio approaches unity for $m_H \simeq 55$ GeV, for $v = 1$. For smaller (and more realistic) shrinking velocities one obtains smaller Higgs masses. Even though the approximations used to solve the kinetic equation break down for about the same values of m_H, it is clear that as the strength of the transition weakens shrinking becomes less important in the dynamics.[12] In fact, the approximations break down precisely because a large fraction of the volume is occupied by the broken phase.

We then arrive at the following picture for the evolution of a sufficiently weak first order transition.[7] As the expansion of the Universe cools the system down to T_c (we are interested in $T_c \ll M_{Pl}$, with M_{Pl} being the Planck mass), there will be large amplitude fluctuations about the high temperature minimum $\langle\phi\rangle = 0$. These are unstable fluctuations, which will shrink at some velocity v. (v is not known in general. We expect it to depend on the curvature radius of the fluctuation, on the coupling to the "environment", and on the free energy density difference between the two phases.) However, the thermal nucleation rate for the fluctuations will be efficient enough that a reasonable fraction of the volume will be occupied by the broken-symmetric phase. As the temperature drops below T_c, the broken-symmetric phase may be above percolation threshold. If percolation occurs, the transition will complete by the competition between the domains of both phases. As the broken-symmetric phase has lower free energy, the interfaces (walls) between the two phases will move toward the symmetric phase, which eventually will disappear.

REFERENCES

1. V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov, *Phys. Lett.* **155B**, 36 (1985); M. E. Shaposhnikov, *Nucl. Phys.* **B287**, 757 (1987); P. Arnold and L. McLerran, *Phys. Rev. D* **36**, 581 (1987); M. Dine, O. Lechtenfeld, B. Sakita, W. Fischler, and J. Polchinski, *Nucl. Phys.* **B342**, 381 (1990); N. Turok and J. Zadrozny, *Phys. Rev. Lett.* **65**, 2331 (1990); *Nucl. Phys.* **B358**, 471 (1991); A. G. Cohen, D. B. Kaplan, and A. E. Nelson, *Phys. Lett.* **245B** (1990) 561; *Nucl. Phys.* **B349**,727 (1991); M. Dine, P. Huet, R. S. Singleton Jr., and L. Susskind, *Phys. Lett.* **257B**, 351 (1991).

2. See, for example, P. Huet, K. Kajantie, R. Leigh, B.-H. Liu, and L. McLerran, Slac preprint SLAC-PUB-5943, December 1992; M. Kamionkowski and K. Freese, *Phys. Rev. Lett.* **69**, 2743 (1992).

3. G. Boyd, D. Brahm and S. Hsu, Harvard preprint No. HUTP-92-A027; M. E. Carrington, *Phys. Rev.* **D45**, 2933 (1992); J.R Espinosa, M. Quirós, and F. Zwirner, *Phys. Lett.* **B291**, 115 (1992); *ibid.*, CERN preprint No. TH.6577/92, December 1992; P. Arnold, *Phys. Rev.* **D46**, 2628 (1992); W. Buchmüller and T. Helbig, DESY preprint No. 92-117; W. Buchmüller, T. Helbig, and D. Walliser, DESY preprint No. 92-151, December 1992; P. Arnold and O. Espinosa, Univ. of Washington preprint No. UW/PT-92-18, December 1992.

4. J.J. Binney, N.J. Dowrick, A.J. Fisher, and M.E.J. Newman, *The Theory of Critical Phenomena: An Introduction to the Renormalization Group*, Oxford Science Publications (Clarendon Press, Oxford 1992).

5. M. Gleiser and R. O. Ramos, Dartmouth College preprint No. DART-HEP-92/08, November 1992. In press, *Phys. Lett.* **B**.

6. M. Gleiser, E.W. Kolb, and R. Watkins, *Nucl. Phys.* **B364**, 411 (1991).

7. M. Gleiser and E.W. Kolb, *Phys. Rev. Lett.* **69**, 1304 (1992); FERMILAB preprint No. Pub-92/222-A (1992).

8. G. Gelmini and M. Gleiser, UCLA preprint No. UCLAHEP-92-44, November 1992. Submitted to *Nucl. Phys.* **B**.

9. G.W. Anderson and L.J. Hall, *Phys. Rev.* **D45**, 2685 (1992).

10. For other recent works which estimated the rms amplitude of fluctuations see M. Dine, R. Leigh, P. Huet, A. Linde, and D. Linde, *Phys. Rev.* **D46**, 550 (1992); M. Gleiser, *Phys. Rev.* **D42**, 3350 (1990).

11. For a review see, K. Binder and D. Stauffer, *Adv. Phys.* **25**, 343 (1976).

12. For a different opinion see G. Anderson, *Phys. Lett.* **295B**, 32 (1992). We believe that this author's conclusions are based in an incorrect understanding (and consequent formulation) of the kinetics of thermal fluctuations, which we hope to have clarified here.