MACHINE DESIGN

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S G Kulkarni
Department of Mechanical Engineering
Government College of Engineering
Aurangabad

Tata McGraw-Hill Publishing Company Limited
NEW DELHI

McGraw-Hill Offices
New Delhi New York St Louis San Francisco Auckland Bogotá Caracas
Kuala Lumpur Lisbon London Madrid Mexico City Milan Montreal
San Juan Santiago Singapore Sydney Tokyo Toronto

Tata McGraw-Hill

Published by the Tata McGraw-Hill Publishing Company Limited,
7 West Patel Nagar, New Delhi 110 008.

ISBN (13): 978-0-07-064788-6
ISBN (10): 0-07-064788-7

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Typeset at The Composers, 260, C.A. Apt., Paschim Vihar, New Delhi 110 063 and printed at
Avon Printers, 271, FIE, Patparganj, Delhi 110 092

Cover Printer: Rashtriya Printer

RCLQCRXXRBXLY

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About the Author

The author has taught the subject of Machine Design at the degree and diploma level for a period of thirty-five years. He has been the member of the Academic Council, Faculty of Engineering and the Board of Studies in Mechanical Engineering of the then Marathawada University, Aurangabad. Besides authoring another book on Machine Design, he also has to his credit writing on the subject of Power Plants in Marathi titled *Shaktisamyantre* for the Maharashtra University Book Production Board, Nagpur.

Dedication

Dedicated to my father and mother for their love and inspiration

Contents

Preface

This book was first published as part of TMH Outline Series and was well received by the students and teachers alike. The new edition, is now being published as Sigma Series. I have expanded the material by adding numerous problems at the end of each chapter. This will give students an opportunity to try and reinforce the concepts learned. I hope that the new format of the book is more convenient and will be appreciated by the students.

The seeds of this book were planted over thirty-five years ago when as a teacher in the Government Engineering College, Aurangabad, I tried to use different methods to make the subject easy to understand for the students and by giving a large number of problems as exercises.

The book has been written mainly for the prefinal- and final-year students studying for the Mechanical Engineering degree course and comprehensively covers a major portion of the syllabi prescribed by various Indian universities. The students preparing for B section of A.M.I.E.(Mechanical) will also find it useful. The final-year students of diploma courses in Mechanical Engineering will find the first ten chapters of the book in line with their syllabus. Practicing engineers will also find it useful as a great number of challenging problems are included in the text.

The theoretical part has been given a brief but precise coverage which makes it an indispensable supplement to any standard book on the subject.

The subject matter is divided into two parts of which the part of basic theory to be applied for design of any machine component is covered in the first four chapters. The second chapter includes the concept of manufacturing tolerances required to be used in designing of a machine component. Chapters 5 through 20 are devoted to explaining the procedures of designing different machine elements such as bolts, power screw, springs, shafts, belt and rope drives, gears of different types, sliding contact and rolling contact bearings as the titles of the chapters suggest. The last chapter is a new concept of machine design termed Optimum Design, giving a basic concept of optimization used in Machine Design as suggested by J.B. Johnson.

Machine Design is not an exact science as there are a variety of methods to arrive at the solution in the form of specifications of any machine element satisfying the necessary functional requirements and no solution is a unique solution to the problem. There may be more than one solution and all may be reasonably correct. This aspect becomes clear as the readers go through the solved problems where many assumptions have to be made for solving them. This may cause confusion in the minds of the students. In order to avoid this possibility, most of the problems have been formulated by stating all the assumptions required to solve them. But such an approach should not deter them from using their own thinking and reasoning abilities. Hence, some problems which require the students to make their own assumptions are also included. This will familiarize them with the process of making proper assumptions and will therefore, make them better equipped to tackle the practical problems, they may require to handle in their field of study or work.

I am grateful to all those who aided in the development of the text and to the Editors in charge of the McGraw-Hill Publishing company for the constant encouragement in writing this book. I would like to convey a special thanks to Mr. M G Navare (then Area Manager, TMH Publishing Co.) who introduced me to the company and persuaded me to write the book.

Last but not the least, I would like to thank my wonderful wife, Mrs. Veena Kulkarni, who was very supportive and encouraging while I was writing this book.

S G KULKARNI

1

Fundamental Concepts

CONCEPT REVIEW

1.1 FORCE, TORQUE

According to Newton's second law of motion, force may be defined as

$$F = m \times a$$

In SI units one newton is the force required to accelerate a mass of one kg with an acceleration of one metre per second squared.

If a body is in rotational motion at an angular speed of ω r/s, with radius of rotation r, the force which keeps the body in rotation is *centripetal force* given by

$$\text{Centripetal force} = mr\omega^2 = mv^2/r$$

where v is tangential velocity in m/s.

Force that exists away from the centre due to mass of the body and is equal and opposite to the centripetal force is termed as *centrifugal force*.

For a rotating body of mass m with its mass assumed to be concentrated in a ring of radius k, the product mk^2 is known as *mass moment of inertia* of the body and is denoted by letter I.

$$\therefore \qquad I = mk^2$$

Two equal and opposite parallel forces at a distance x form a couple given by

$$C = F \cdot x \text{ N.m}$$

A couple due to force F as shown in Fig. 1.1 causes twisting of the shaft and is termed as torque T acting on the shaft. It is given by

$$T = F \times r \text{ N.m}$$

This torque causes the rotational motion of shaft or disc mounted on it and may be written as

$$T = I\alpha \text{ N.m}$$

where I is the moment of inertia of disc and shaft and α is the angular acceleration in rad/sec^2.

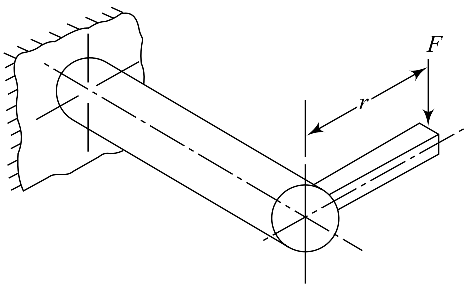


Fig. 1.1

1.2 WORK, POWER AND ENERGY

Work done in displacing a force F Newtons by a distance d metres is given as

$$\text{Work} = F \times d \text{ N.m}$$

Work done in causing the angular motion θ radians of a body subjected to torque T is given by

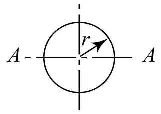
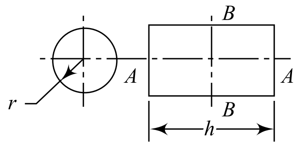
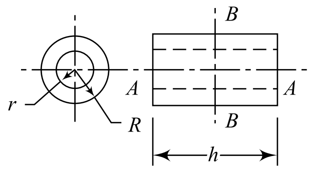
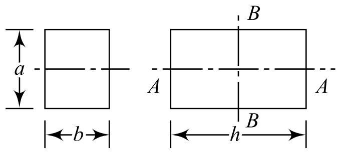
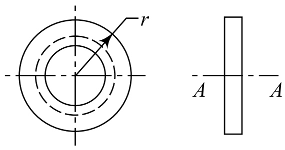
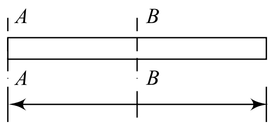
$$\text{Work} = T\theta \text{ N.m}$$

The rate of doing work is defined as *power*. Thus

$$P = F(d/t) = F \times v \text{ N.m/s} \qquad P = T(\theta/t) = T \times \omega \text{ N.m/s}$$

The capacity to do the work is termed as *energy*.

Table 1.1 *Mass Moments of Inertia of Common Solids*

Solid		*Mass M-1* I_m	*Radius of gyration* K
1. Sphere		about $AA = \frac{2}{5}Mr^2$	$0.6325\,r$
2. Cylinder		about $AA = \frac{1}{2}Mr^2$ about $BB = M\left(\frac{h^2}{12} + \frac{r^2}{4}\right)$	$0.0707\,r$ $0.289\sqrt{h^2 + 3r^2}$
3. Hollow Cylinder		about $AA = \frac{1}{2}M(R^2 + r^2)$ about $BB = M\left(\frac{h^2}{12} + \frac{R^2 + r^2}{4}\right)$	$0.707\sqrt{R^2 + r^2}$ $0.289\sqrt{h^2 + 3(R^2 + r^2)}$
4. Prism		about $AA = \frac{M}{12}(a^2 + b^2)$ about $BB = \frac{M}{12}(h^2 + b^2)$	$0.2887\sqrt{a^2 + b^2}$ $0.2887\sqrt{h^2 + b^2}$
5. Thin Ring		about $AA = Mr^2$	T
6. Thin Uniform Rod		about $BB = \frac{MI^2}{12}$ about $AA = \frac{MI^2}{3}$	$I/\sqrt{12}$ $I/\sqrt{3}$

1.3 MACHINE AND MACHINE ELEMENTS

Machine is a device to perform useful work when some form of energy is supplied to it. Lever, power screw, inclined plane are examples of simple machines while lathe, i.c. engines, washing machine etc., are examples of complex machines.

The smallest component of machine such as a bolt, spring, pin etc. is termed as a *machine element* for the purpose of machine design.

WORKED EXAMPLES

1.1 It is required to lift water at the top of a 20 m high building at the rate of 50 litres/min. Assuming the losses due to friction equivalent to 5 m and those due to leakage equal to 5 m head of water; efficiency of pump 90% and efficiency of motor 90%, decide the kilowatt capacity of motor required to drive the pump.

Solution:

Work to be done = $wQHg$ $\quad$ w = density of water

w = l kg/litre $\quad$ Q = discharge

Q = 50 litres per min. $\quad$ H = head of water

$H = 20 + 5 + 5 = 30$ m $\quad$ g = gravitational constant

$\therefore$ Work output/min = $1 \times 50 \times 30 \times 9.81$ N.m/min

$$\therefore \quad \text{Input power} = \frac{1 \times 50 \times 30 \times 9.81}{0.9 \times 0.9 \times 1000 \times 60} = 0.302 \text{ kW.}$$

1.2 A power screw is rotated at constant angular speed of 1.5 revolutions/s by applying a steady torque of 15 N.m. How much work is done per revolution? What is the power required?

Solution:

Work per revolution = $T\theta$ $\quad$ T = 15 N.m

$= 15 \times 2\pi$ $\quad$ θ = 1 revolution = 2π rad.

= 94.24 Nm per revolution

P = Work/s = (Work/rev) × (No. of revolutions/s)

$= 94.24 \times 1.5 = 141.36$ W.

1.3 Power is transmitted by an electric motor to a machine by using a belt drive. The tensions on the tight and slack side of the belt are 2200 N and 1000 N respectively and diameter of the pulley is 600 mm. If speed of the motor is 1500 r.p.m, find the power transmitted.

Solution:

D = Diameter of pulley in mm $\quad$ F = Tangential force, N

N = Speed r.p.m $\quad$ v = Peripheral velocity, m/s

$P = T\omega$

$= Fr\omega = Fv$

$F = T_1 - T_2 = 2200 - 1000 = 1200$ N

$$v = \frac{\pi DN}{60} = \frac{\pi \times 600 \times 1500}{1000 \times 60} = 47.12 \text{ m/s}$$

$$P = \frac{1200 \times 47.12}{1000} = 56.548 \text{ kW}$$

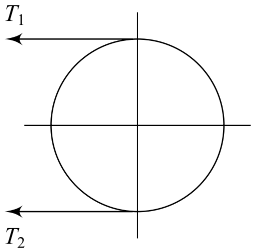


Fig. E-1.3

1.4 The flywheel of an engine has a mass of 200 kg and radius of gyration equal to 1 m. The average torque on the flywheel is 1200 N.m. Find the angular acceleration of flywheel and the angular speed after 10 seconds starting from rest.

Solution:

$$I = mk^2 = 2000 \times (1)^2 = 2000 \text{ kgm}^2$$

$$T = I\alpha, \quad \therefore \quad \alpha = \frac{T}{I} = \frac{1200}{2000} = 0.6 \text{ rad/s}^2$$

$$\omega = \omega_0 + \alpha t = 0 + 0.6 \times 10 = 6 \text{ rad/s.}$$

1.5 Calculate the moment of inertia and radius of gyration of a solid sphere of mass 10 kg and diameter 6.5 m about its centroidal axis.

Solution:

$$I = \frac{2}{5}\, mr^2 = \frac{2}{5} \times 10 \times \left(\frac{6.5}{2}\right)^2 = 49 \text{ kg.m}^2$$

$$k = 0.6325 \times 3.25 = 2.055 \text{ m (Refer Table 1.1)}$$

1.6. Calculate the work done per minute by a punch tool making 20 working strokes per min when a 30 mm diameter hole is punched in 5 mm thick plate with ultimate shear strength of 450 MPa in each stroke.

Solution: Force required to punch one hole = area sheared × ultimate shear strength = $\pi d t \times S_s$

where d = diameter of hole in mm

t = thickness of plate in mm

S_s = ultimate shear strength in MPa.

$$= \frac{\pi \times 30 \times 5 \times 450}{1000} \quad 212.0575 \text{ kN}$$

Work done/min = Average force × Thickness of plate × No. of holes/min

$$= \frac{212.0575}{2} \times \frac{5}{1000} \times 20 = 10.692875 \text{ kN.m}$$

OBJECTIVE QUESTIONS

1.1 A force of 50 N acts on a roller of mass 10 kg initially at rest on a frictionless surface. The roller travels 10 m while the force acts. The work done on the roller is

(a) zero (b) 125 J (c) 250 J (d) 500 J

1.2 A cricket ball of mass 150 gm is moving with a velocity of 12 m/s and is hit by a bat such that it is turned back with a velocity of 20 m/s. The force of blow acts on the ball for 0.01 s. Average force exerted on ball by the bat is

(a) 120 N (b) 240 N (c) 480 N (d) 960 N

1.3 A flywheel with moment of inertia of 10 kgm^2 rotating with 120 r.p.m. will stop in 5 revolutions at a braking torque of

(a) 2.5×10^2 N.m (b) 50.265 N.m (c) 62.5 N.m (d) 103.3 N.m

1.4 Force required to punch a hole of 20 mm diameter in a 5 mm thick plate with ultimate shear strength of 250 MPa is

(a) 100 kN (b) 78.54 kN (c) 314.16 kN (d) 5 kN

1.5 In a machine a force of 120 N is required to lift a load of 3000 N and a force of 200 N is required to lift a load of 7000 N. Hence a force of 250 N will be required to lift a load of
(a) 87500 N (b) 9500 N (c) 6250 N (d) 8000 N

1.6 A table has a heavy circular top of radius 1 m and mass 20 kg. It has four legs of negligible mass fixed symmetrically on its circumference. Maximum mass that may be placed anywhere on the table without toppling is
(a) 20 kg (b) 24 kg (c) 48 kg (d) 10 kg

1.7 Bar is the unit of
(a) area (b) time (c) pressure (d) mass

REVIEW QUESTIONS

1.1 Define a machine. How does it differ from a component? How is the component of a machine different from its element?
1.2 Compare force and torque, mass and mass moment of inertia.
1.3 Define work. Differentiate between power and energy.
1.4 Derive the relationship between power and torque.

PRACTICE PROBLEMS

1.1 Find the moment of inertia of a flywheel of mass 100 kg and diameter 1 m if the mass is distributed uniformly over the area. If this flywheel is subjected to a uniform torque of 100 N.m, find the time required to attain a speed of 2000 r.p.m. starting from rest.
1.2 If above flywheel has its mass concentrated in a rim of radius 600 mm, how much time will it take to stop from a speed of 1800 r.p.m. when a braking torque of 120 N.m is applied.
1.3 A rope is passing over a free rotating pulley. One end of the rope is attached to a mass of 100 kg and the other end to a mass of 25 kg. Neglecting the friction and mass of the pulley find tension in the rope.
1.4 35 kW power is transmitted from one shaft to the other at 1500 r.p.m. of a driver pulley having 600 mm diameter. Find the tensions in the belt if the ratio of tensions is 2.

ANSWERS

Objective Questions

(1) d (2) c (3) b (4) b (5) b (6) c (7) c

Practice Problems

(1) 12.5 kgm^2, 26.18 sec (2) 56.54 sec (3) 392 N (4) 148.6 N, 74.3 N

2

Design Procedure, Simple Stresses

CONCEPT REVIEW

2.1 INDUCED STRESS

A machine element offers resistance to failure when an external force acts on it. This resistance per unit area is termed as *induced stress.* The induced stress normal to the cross section is designated by letter σ and that tangential to cross section is designated as τ.

2.2 STRENGTH

Any material can resist the effect of external force acting on it up to a limited value of induced stress. This limiting value is termed as the *strength* of material. S is used to represent strength. Thus S_y, S_{ut}, S_{ue}, S_s, S_{ys} represent yield point strength, ultimate tensile strength, ultimate compressive strength, ultimate shear strength and yield point shear strength respectively. The unit of strength is same as that of induced stress, i.e. MPa which is 10^6 N/m^2 or N/mm^2.

2.3 LOAD—CAPACITY

Different forces acting on a machine element due to useful work to be done or due to the situation of working constitute the load on the machine member. This load may deform the member, cause wear and tear, or in extreme case may cause fracture of the member.

The material used for manufacture and the dimensions of cross section of machine element decide *capacity* of the member to resist the above-mentioned effects of load.

2.4 DESIGN OF MACHINE MEMBER

The procedure of machine design basically involves selection of material and dimensions of cross section of the machine member in such a manner that load does not cause failure of the member by deformation, wear or fracture. This may be achieved by writing in equation form as

Capacity of the member to resist failure > the effect of load

or in the form of known quantities, as,

Strength per unit area of cross-section > induced stress

or according to the type of failure

$S > \sigma$ or τ, where S is the tensile, compressive or shear strength.

2.5 FACTOR OF SAFETY

The above equation may be conveniently written by using sign of equality as

$$S = \sigma \text{ (or } \tau) \times \text{Constant}$$

or

$$\sigma \text{ (or } \tau) = \frac{S}{\text{Constant}} \quad (2.1a)$$

This constant decides how much more the strength should be as compared to the induced stress. It assures the safety of machine member from failure and hence is termed as *factor of safety* and may be designated as F.S. Right hand side of Eq. (2.1) is termed as permissible or safe or allowable stress, while left hand side is induced stress.

F.S. may be arbitrarily selected as 3 to 5 based on yielding or 5 to 7 based on fracture failure. Exact knowledge of situation helps the designer to select F · S. more precisely.

2.6 DESIGN PROCEDURE

Designing of machine elements involves following steps:

(a) Specifying the problem
(b) Selection of proper mechanism
(c) Analysis of forces
(d) Selection of material
(e) Selection of factor of safety
(f) Calculation of cross-sectional dimensions using basic design equation
(g) Modifying and finalising dimensions with proper tolerances and preparing drawings with proper instruction for manufacturing.

Class-room problems usually involve only last four steps, while practical problems may involve some more steps.

2.7 SIMPLE STRESS: DIFFERENT SITUATIONS

(a) *Direct tensile stress:* The load causes uniformly distributed tensile induced stress on any cross section of the member (Fig. 2.1). Hence design equation may be written for ductile material when yielding is the criteria of failure as.

Induced stress
$$\sigma_t = \frac{P}{A} = \frac{S_y}{\text{F} \cdot \text{S}} \quad (2.1b)$$

For brittle material or ductile material with fracture as criteria of failure,

Induced stress
$$\sigma_t = \frac{P}{A} = \frac{S_{ut}}{\text{F} \cdot \text{S}} \quad (2.2)$$

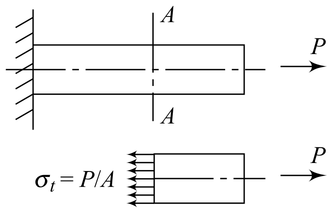


Fig. 2.1

(b) *Direct compressive stress:* For any section A A (Fig. 2.2) design equation may be written as

$$\sigma_c = \frac{P}{A} = \frac{S_{uc}}{\text{F}\cdot\text{S}} \qquad (2.3)$$

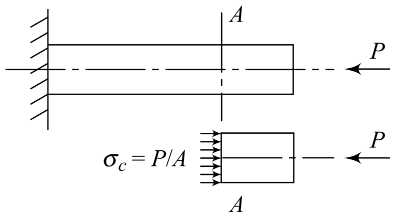


Fig. 2.2

(c) *Direct shear stress*: (Fig. 2.3)
In above case load P causes the induced stress tangential to the specific cross section for which design equation is

Induced shear stress $\tau_i = \frac{P}{A} = \frac{S_{ys}}{\text{F}\cdot\text{S}}$ for ductile material $= \frac{S_s}{\text{F}\cdot\text{S}}$ for brittle material

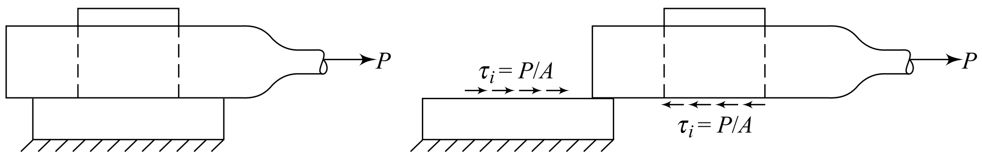


Fig. 2.3

(d) *Double shear:* If the possibility of failure due to shear is at the two sections AA and BB (Fig. 2.4), resistance is offered by these sections to the applied shear force. The design equation then becomes

$$\tau = \frac{P}{2A} = \frac{S_{ys}}{\text{F}\cdot\text{S}} \quad \text{for ductile material}$$

$$= \frac{S_s}{\text{F}\cdot\text{S}} \quad \text{for brittle material} \qquad (2.4)$$

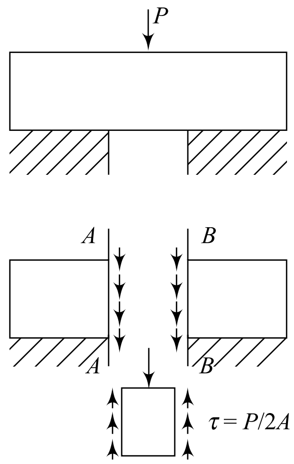


Fig. 2.4

(e) *Bearing:* The two components such as shaft and bearing (Fig. 2.5) rub against each other causing wear. Wear can be kept minimum by limiting the bearing pressure between the components. The limiting value of bearing pressure P_b is based on compressive strength of softer material and the rubbing velocity. This bearing pressure is assumed to act uniformly on projected area $d \times l$. Hence the design equation is

$$P_b = \frac{P}{\text{Projected area}} = \frac{P}{dl} \qquad (2.5)$$

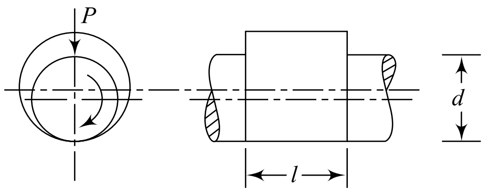


Fig. 2.5

(f) *Bending:* The machine element when loaded as in Fig. 2.6 bends under given load. As a result, the nature of induced stress is tensile at outer fibre and compressive at the inner. All materials are weak under tension than under compression, hence design equation by using basic strength of material formula

$$\frac{M}{I} = \frac{\sigma_t}{y} = \frac{E}{R}$$

$$\left.\begin{aligned} \text{Becomes} \quad \sigma_t &= \frac{My}{I} = \frac{S_y}{\text{F}\cdot\text{S}} && \text{for ductile material} \\ &= \frac{S_{ut}}{\text{F}\cdot\text{S}} && \text{for brittle material} \end{aligned}\right\} \quad (2.6)$$

$$\left.\begin{aligned} \text{Again} \quad \sigma_t &= \frac{E}{R} y = \frac{S_y}{\text{F}\cdot\text{S}} && \text{for ductile material} \\ &= \frac{S_{ut}}{\text{F}\cdot\text{S}} && \text{for brittle material} \end{aligned}\right\} \quad (2.7)$$

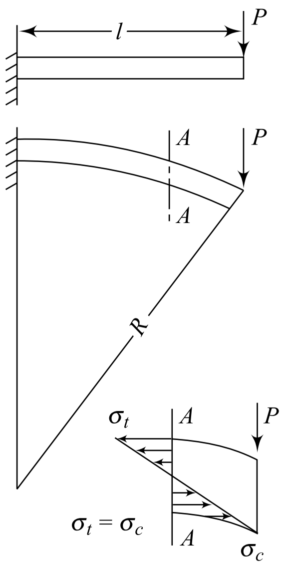


Fig. 2.6

Equation (2.6) is useful for determination of cross-sectional dimensions. The ratio $\frac{I}{y}$ is termed as *section modulus* where I is moment of inertia of cross section about the axis of bending and y is distance of the fibre from the neutral axis. Equation (2.7) is useful for finding permissible radius of curvature.

A transverse shear stress is also induced in the cross-section but it does not occur at the top and bottom fibre and hence may be neglected. For pure bending there is no transverse shear stress.

(g) *Twisting:* A pure twisting moment acting on the machine member (Fig. 2.7) of circular cross section induces torsional shear stress. The shear stress is zero at centre and increases with the increase in radius. Therefore, the design equation for torque T is

$$\left.\begin{aligned} \tau &= \frac{T}{Z_p} = \frac{S_{ys}}{\text{F}\cdot\text{S}} && \text{for ductile material} \\ \text{and} \quad \tau &= \frac{T}{Z_p} = \frac{S_s}{\text{F.S}} && \text{for brittle material} \end{aligned}\right\} \quad (2.8)$$

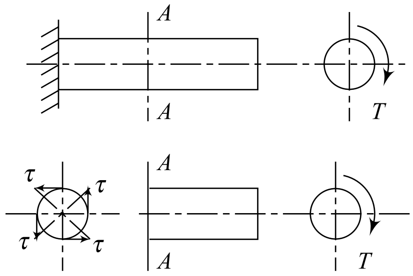


Fig. 2.7

(h) *Permissible elastic deformation:* In some machine components elastic deformation is not allowed to exceed a particular limit. Equations to be used in such case are as under:
For direct tension or compression

$$\delta = \frac{Pl}{AE} \tag{2.9}$$

where δ is deformation and l the original length of member.

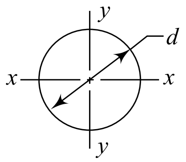
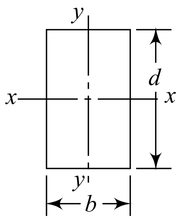
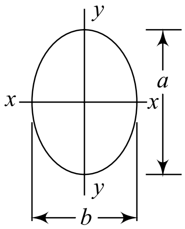
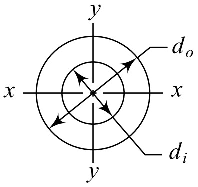
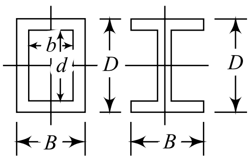
For bending, various deflection equations from strength of material may be used. For twisting moment T, if θ is the permissible angle of twist then

$$\theta = \frac{Tl}{JG} \tag{2.10}$$

where l = length of shaft, mm $\quad J$ = Polar moment of inertia of cross section, mm^4

G = Modulus of rigidity, MPa $\quad Z_p$ = Polar section modulus = $\frac{J}{r}$ mm^3

Table 2.1 *Moments of Inertia of Some Standard Cross Section*

Section Figure	I_{xx}	I_{yy}	Z_{xx}	Z_{yy}
(1) Circle 	$\frac{\pi}{64}d^4$	$\frac{\pi}{64}d^4$	$\frac{\pi}{32}d^4$	$\frac{\pi}{32}d^4$
(2) Rectangle 	$\frac{1}{12}bd^3$	$\frac{\pi}{64}db^3$	$\frac{1}{6}bd^2$	$\frac{1}{6}db^2$
(3) Ellipse 	$\frac{\pi}{65}ba^3$	$\frac{\pi}{64}ab^3$	$\frac{\pi}{32}ba^2$	$\frac{\pi}{32}ab^2$
(4) Hollow Circular 	$\frac{\pi}{64}(d_o^4 - d_i^4)$	$\frac{\pi}{64}(d_o^4 - d_i^4)$	$\frac{\pi}{32}\left(\frac{d_o^4 - d_i^4}{d_o}\right)$	$\frac{\pi}{32}\left(\frac{d_o^4 - d_i^4}{d_o}\right)$
(5) Box or I Section  Section	$\frac{1}{12}BD^3 - \frac{1}{2}bd^3$	$\frac{1}{2}DB^2 - \frac{1}{2}db^3$	$\frac{1}{6}\left[\frac{BD^2 - bd^2}{D}\right]$	$\frac{1}{6}\left[\frac{DB^2 - db^2}{B}\right]$

$$\left.\begin{aligned} b &= B - 1t \\ d &= D - 2t \end{aligned}\right\} \text{for } l \text{ section}$$

Polar moment of inertia $\quad J = I_{xx} + I_{yy}$

and, $\quad$ Polar Section modulus $= \dfrac{l_{xx} + l_{yy}}{\text{depth}/2}$

2.8 MANUFACTURING TOLERANCES

Tolerance is defined as the permissible deviation from the basic dimension. It is impracticable to get a machine part of an accurate basic size as it increases the cost without increasing the utility of part. A large tolerance decreases the cost (Fig. 2.8).

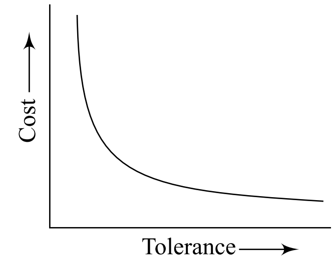


Fig. 2.8

Size of the part when specified by providing tolerance on either side of basic size is termed as the *unilateral method.* Specifying the size by providing tolerance on both sides of basic size is *bilateral method of tolerances.* A bar diagram shows the maximum and minimum sizes of hole and shaft. The diagram 2.9 indicates the shaft and hole tolerances for clearance fit. Similar diagram results for interference fit except that, the hole and shaft tolerance positions are interchanged and in place of clearance there is interference. Figure 2.10 shows the results of bilateral and unilateral tolerances respectively. From this figure it can be concluded that in bilateral method of tolerances the type of fit may change by changing tolerance range. Bilateral method is used for dimensioning holes.

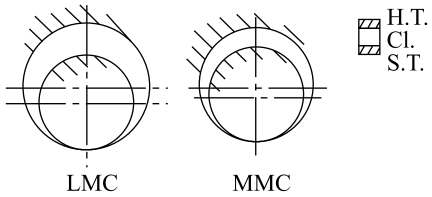


Fig. 2.9

H.T: Hole tolerance LMC: Least Metal Condition
S.T: Shaft tolerance MMC: Maximum Metal Condition
B.S: Basic size

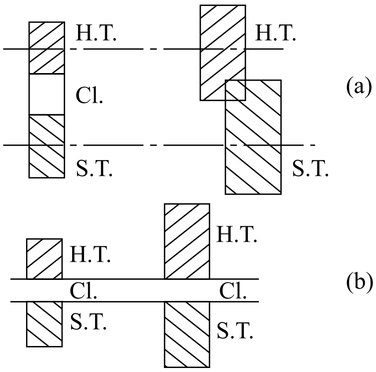


Fig. 2.10

As tolerance is deviation from the basic size, the two sizes obtained are termed as lower (or fundamental) deviation and upper deviation and are designated as

ei – lower deviation on shaft EI – lower deviation on hole
es – upper deviation on shaft ES – upper deviation on hole
IT – Tolerance

Thus

$$es = ei + IT \text{ and } ES = EI + IT$$

The method of tolerance where basic size is used for shaft is known as *basic shaft system.* The method which uses basic size for hole is termed as *basic hole system.* Former method is used when standard shafts are available and the latter when standard reamers are available.

The method of *selective assembly* is used when parts are to be produced on a very large scale. In this method, a large tolerance range is used and the assembly is made by selecting holes and shafts with small clearance or interference. This reduces the cost but gives a close fit. This method requires sorting of shafts and holes in different groups. This requires additional labour and also more number of gauges and increases the cost of production. It also results in non-interchangeable assembly which needs complete replacement of assembly, e.g. ball bearings.

Preferred numbers: Preferred numbers are used for standardisation. They are obtained by using the series having numbers in G.P with common ratio as $\sqrt[n]{10}$. Captain Renard put forth this concept and hence, the series are termed as R 5, R 10, R 20, where 5, 10, 20 etc. are values of n. Thus the series can be written for numbers between 10 and 100 as

R5 10, 16, 25, 40, 63 with common ratio = $\sqrt[5]{10} = 1.6$
R10 10, 12.5, 16, 20, 25, 31.5, 40, 50, 63, 80 with common ratio = $\sqrt[10]{10} = 1.25$

WORKED EXAMPLES

2.1 An m.s bar of 12 mm diameter is subjected to an axial load of 50 kN in tension. Find the magnitude of induced stress.

Solution:
$$\sigma_t = \frac{P}{A} = \frac{50 \times 10^3}{\frac{\pi}{4} \times (12)^2} = 442.09 \text{ MPa}$$

2.2 If the length of bar in above example is 1 m and the modulus of elasticity of the material of the bar is 2×10^5 MPa, find the elongation of bar.

Solution:
$$\delta l = \frac{Pl}{AE} = \frac{50 \times 50^3 \times 1000 \times 4}{\pi \times (12)^2 \times 2 \times 10^5} = 2.21 \text{ mm}$$

2.3 If Poisson's ratio for the material of bar in above example is 0.3, find the change in diameter of the bar.

Solution: Poisson's ratio $\mu = \dfrac{\text{lateral strain}}{\text{longitudinal strain}}$

$$\text{Longitudinal strain} = \frac{\delta l}{l} = 2.21 \times 10^{-3}$$

$$\therefore \quad \text{Lateral strain} = 0.3 \times 2.21 \times 10^{-3} = \frac{\text{decrease in diameter}}{\text{original diameter}}$$

$$\therefore \quad \text{Decrease in diameter} = 0.663 \times 10^{-3} \times 12 = 7.956 \times 10^{-3} \text{ mm.}$$

2.4 If ultimate tensile strength for the material of bar in the above example is 650 MPa, find the factor of safety.

Solution:
$$\text{F} \cdot \text{S} = \frac{S_{ut}}{\sigma_t} = \frac{650}{442.09} = 1.47$$

2.5 A steel wire of 6 mm diameter is used for hoisting. 100 m length of wire is hanging vertically with a load of 1.5 kN being lifted at the lower end of wire. The density of wire material is 7.7×10^4 N/m^3 and $E = 2 \times 10^5$ MPa. Determine the total elongation of wire.

Solution: Total elongation = elongation due to weight W (1.5 kN) + elongation due to self weight W_1 acting from C.G. of wire at $L/2$

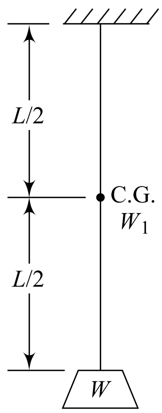


Fig. E-2.5

Now
$$W_1 = \frac{\pi}{4}\left(\frac{6}{1000}\right)^2 \times 1 \times 7.7 \times 10^4 \times 100 \text{ N}$$
$$= 217.71 \text{ N}$$

$$\therefore \quad \text{Total elongation} = \frac{WL}{AE} + \frac{W_1 L}{2AE}$$
$$= \frac{1.5 \times 1000 \times 1 \times 10^5}{\frac{\pi}{4}(6)^2 \times 2 \times 10^5} = \frac{217.71 \times 1 \times 10^5}{2 \times \frac{\pi}{4}(6)^2 \times 2 \times 10^5}$$
$$= 28.44 \text{ mm}$$

2.6 A steel flat 10 mm wide and 12 mm thick bar is bent into a circular arc of radius 12 m. Find the maximum intensity of stress induced in the cross section.

$$E = 2 \times 10^5 \text{ MPa}$$

Solution:
$$\sigma_t = \frac{E}{R}y = \frac{2\times10^5}{12\times1000}\times\frac{t}{2} = \frac{2\times10^5\times6}{12\times1000} = 100 \text{ MPa}$$

2.7 The thickness of flanges and web of a 200 mm × 75 mm standard channel is 12 mm and 6 mm respectively. Find moment of inertia of section about *XX* and *YY* axes.

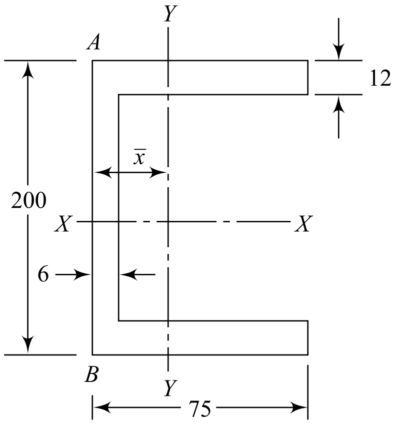


Fig. E-2.7

Solution:
$$I_{xx} = \frac{1}{12}\times 75\times 200^3 - \frac{1}{12}\times 69\times 176^3$$
$$= 18652288 \text{ mm}^4$$

To find I_{yy} let us find $\bar{x}$

$$\bar{x} = \frac{a_f x_1 + a_w x_2 + a_f x_3}{2a_f + a_w}$$

a_f, a_w — areas of flange and web respectively
x_1, x_2, x_3 — distances of C.G. of areas from reference line *AB*

$$\bar{x} = \frac{(75\times12\times37.5)\times2+176\times6\times3}{75\times12\times2+176\times6}$$
$$= 24.74 \text{ mm}$$

using parallel axis theorem

$$I_{yy} = \left(\frac{1}{12}\times12\times75^3\right)\times2 + [75\times12\,(37.5-24.74)^2]\times2$$
$$+\frac{1}{12}\times176\times6^3 + 176\times6\times(24.74-3)^2 = 1638166.4 \text{ m}^4$$

2.8 Two identical steel bars are pin connected and a load of 500 kN is attached to point *B*. Find the required area of cross section of bars so that the normal stress in bars is limited to 200 MPa. Also find the vertical displacement of point *B*. *E* = 200 GPa, *BC* = *AB* = 3 m.

Solution: From the equilibrium of forces

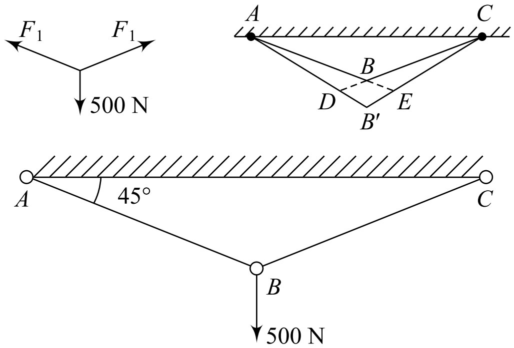


Fig. E-2.8

$$2\left(1/\sqrt{2}\right)F_1 - 500 = 0$$
$$F_1 = 354 \text{ kN}$$

$$\text{Required area } A = \frac{F_1}{\text{permissible stress}}$$
$$= \frac{354\times1000}{200} = 1770 \text{ mm}^2$$

By approximation

$$BB' = \frac{DB'}{\cos 45°}$$ where *DB* is the extension of either link.

$$DB' = \frac{Pl}{AE} = \frac{354 \times 3000 \times 10^3}{1770 \times 2 \times 10^5} = 3 \text{ mm}$$

$$BB' = \frac{3}{1/\sqrt{2}} = 4.25 \text{ mm}$$

2.9 A machine element is subjected to an axial load of 200 kN. Design the member.

Solution: First three steps of design procedure are eliminated and we start from the 4th step, i.e. selection of material. The materials available are m.s., C.I, A, l, copper and its alloys. The last two may not be used due to high cost and with no specific reason for using them. As the load is tensile and C.I is weak under tension, m.s is the appropriate choice.

Referring to the Table 2 hot rolled 14C6 or 20C8 steel may be used for which S_y = 300 MPa, S_{at} = 400 – 500 MPa.

In absence of any specific condition the factor of safety of 3 to 5 based on S_y may be used. Let F . S = 3, S_y = 300 MPa and P = 200 kN

$$\sigma_t = \frac{P}{A} = \frac{S_y}{\text{F} \cdot \text{S}}, \quad \therefore \quad \frac{200 \times 10^3}{\frac{\pi}{4} d^2} = \frac{300}{3}$$

$$d = 50.46 \text{ mm}$$

For standardisation let d = 50 mm even though it may reduce F · S slightly. If we round it off to the next standard diameter of 55 mm cost may increase which is undesirable.

2.10 If length of the element in above example is 1000 mm, what maximum axial load should act on the rod such that deformation in axial direction is not more than 0.5 mm ?

$$E = 2 \times 10^5 \text{ MPa}$$

Solution:

$$\delta l = \frac{Pl}{AE}, \quad \therefore P = 0.5 \times \frac{\pi (50)^2 \times 2 \times 10^5}{1000 \times 1000} = 196.35 \text{ kN}$$

Thus, if maximum permissible elongation is the criteria of design, the rod cannot take a load of 200 kN even though induced stress is within the permissible limit.

2.11 A compression member of FG 200 C.I has to support a load of 750 kN. Using a ratio of 2 for the outer to inner diameter and F · S as 6, based on the ultimate strength, find the diameters of the rod using S_{uc} = 630 MPa for FG 200 C.I.

Solution:

From Fig. E-2.11, the area of cross section of machine member is

$$A = \frac{\pi}{4} [(2d)^2 - d^2] = 0.75\pi d^2$$

where d is the inner diameter.

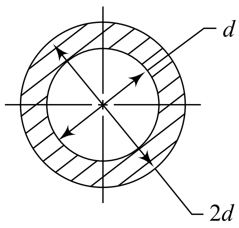


Fig. E-2.11

$\because \quad \sigma_c = \dfrac{P}{A} = \dfrac{S_{uc}}{\text{F} \cdot \text{S}}, \quad \therefore \quad 0.75\, \pi d^2 = \dfrac{750 \times 10^3 \times 6}{630}$

$\therefore \quad d = 55.05$ mm rounded to 55 mm

$\therefore \quad$ outer diameter $= 2d = 110$ mm

2.12 If the compression member in Example 2.11 has square cross section with the side of outer square, double the side of inner square, find the cross-sectional dimension.

Solution:

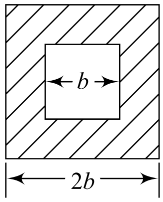


Fig. E-2.12

$$A = 4b^2 - b^2 = 3b^2$$

$$\therefore \quad \sigma_c = \frac{P}{A} = \frac{S_{uc}}{\text{F} \cdot \text{S}}$$

$$\therefore \quad A = \frac{P}{S_{uc}} (\text{F} \cdot \text{S})$$

$$\therefore \quad 3b^2 = \frac{750 \times 10^3 \times 6}{630}$$

$\therefore \quad b = 48.79$ mm rounded to 50 mm

$\therefore \quad$ outer side $= 2\, b = 100$ mm.

2.13 If the material for compression member in Examples 2.11 and 2.12 is 30 C8 steel what will be the changes in design ?

Solution:

As the compressive strengths of 200 FG C.I. and 30 C8 steel are approximately same, the dimensions of cross section will also remain the same. Use of 200 FG C.I may reduce the cost. As C.I or steel pipes are readily available, use of circular cross section is preferable.

2.14 A hollow compression member is subjected to an axial load of 50 kN. The material used is 200 FG C.I with S_{uc} = 600 MPa. Find the cross-sectional dimensions if

(a) the section is hollow circular with outer diameter 1.5 times the inner diameter

(b) the section is square box with side of outer square 1.5 times the side of inner square

(c) the section is square the outer side of which is 1.5 times diameter *D* of circular bore. Based on ultimate strength F · S = 6.

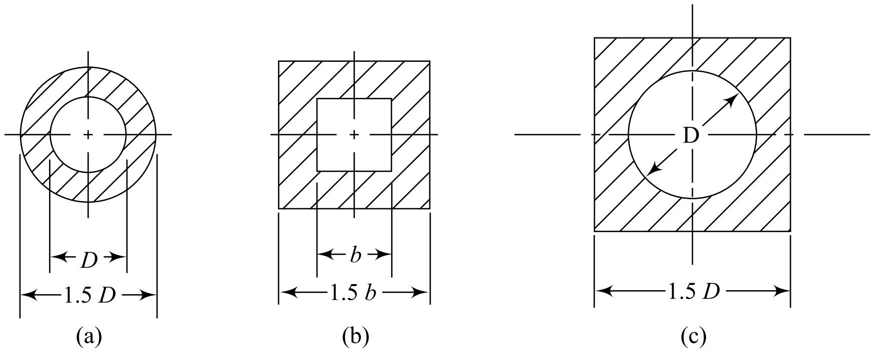


Fig. E-2.14

Solution:

For compressive load

$$A = \frac{P}{\sigma_c} \text{ where } A \text{ is the area of cross section}$$

$$\sigma_c = \frac{S_{uc}}{F \cdot S} = \frac{600}{6} = 100 \text{ MPa}$$

$$P = 50{,}000 \text{ N}$$

$\therefore$ $$A = \frac{50{,}000}{100} = 500 \text{ mm}^2$$

For case (a) $$A = \frac{\pi}{4}(1.5^2 - 1)D^2$$

$$= 0.9817\, D^2 \text{ for section at } a$$

$$A = 0.9817\, D^2 = 500 \text{ mm}^2$$

$\therefore$ $$D = 22.568 \text{ mm}$$

For case (b) $$A = 1.25\, b^2$$

$\therefore$ $$1.25\, b^2 = 500, \quad \therefore\ b = 20 \text{ mm}$$

For case (c) $$A = \left(2.25 - \frac{\pi}{4}\right) D^2$$

$$= 1.46\, D^2 = 500 \text{ mm}^2, \quad \therefore\ D = 18.5 \text{ mm modified to 20 mm.}$$

2.15 A link shown in Fig. E-2.15 is subjected to a tensile load of 40 kN with $h = 2\,t$, $L = 475$ mm. Maximum permissible elongation is 0.125 mm. The material used is 30 C8 steel with $S_y = 330$ MPa, $S_{ut} = 500$ MPa. Find 'h' and 't'.

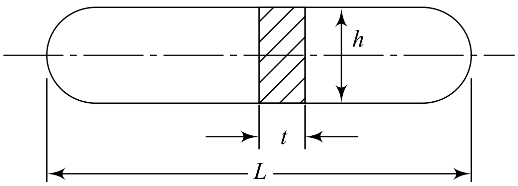


Fig. E-2.15

Solution:

Failure of the link may take place under two conditions

Condition 1: Induced stress ≤ permissible stress

let F · S = 4

$\therefore$ $$\sigma_t = \frac{P}{A} = \frac{S_y}{F \cdot S}$$

$\therefore$ $$A = h \times t = \frac{40 \times 10^3 \times 4}{330} = 484.848 \text{ mm}^2$$

Putting $$h = 2\,t,\ t = 15.56 \text{ rounded to 16 mm}$$

$$h = 32 \text{ mm}$$

Condition 2: Permissible elongation ≤ 0.125 mm. Now $\delta l = \dfrac{Pl}{AE}$

$\therefore$ $$0.125 = \frac{40 \times 10^3 \times 475}{h \times t \times 2 \times 10^5}$$

$$t = 19.49 \text{ rounded to 20 mm} \qquad h = 40 \text{ mm.}$$

Thus, with $t = 20$ mm and $h = 40$ mm both induced stress and deformation will be within limit.

2.16 If a slot of 15 mm width has to be cut in the link of Example 2.15 what modification in h and t is required?

Solution:

There is no need of separate calculation if thickness is to be kept the same. The designer should see that the area of cross section remains same for the link with slot. This is achieved by increasing the dimension h by 15 mm which is the width of the slot. Thus the modified dimensions will be $t = 20$ mm and $h = 55$ mm.

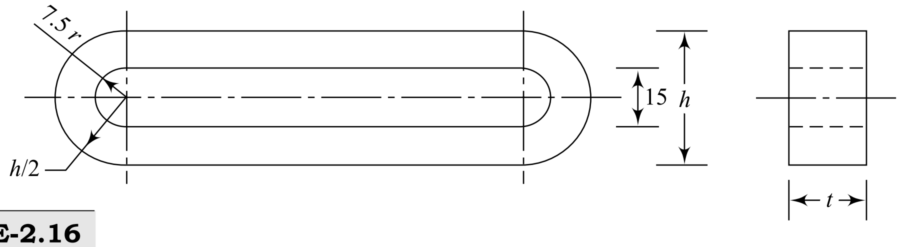


Fig. E-2.16

2.17 A shaft is supported with two bearings and is subjected to a radial load of 70 kN. Permissible bearing pressure is 1.5 MPa. Diameter of shaft is 100 mm. Find the length of bearings.
Solution:

$$\text{Permissible bearing pressure} = \frac{\text{load on each bearing}}{\text{projected area of each bearing}}$$

Load on each bearing = 35 kN
Projected area of each bearing = diameter of shaft × length of bearing
Substituting

$$1.5 = \frac{35{,}000}{100l}, \quad \therefore\ l = 233.33 \text{ mm modified to 235 mm.}$$

2.18 Solve Example 2.14 for the section shown in Fig. E-2.18.
Solution:

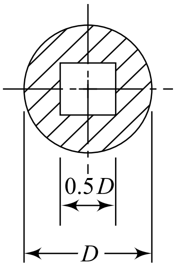


Fig. E-2.18

$$\text{Area of section} = \left(\frac{\pi}{4} - 0.25\right) D^2$$

$$= 0.5354\, D^2 = 500 \text{ mm}^2$$

$$D = 30.56 \text{ mm, modified to 30 mm.}$$

2.19 Two links of 30 C8 steel are connected by a pin of the same material. The links are subjected to an axial tensile load of 5 kN and have a rectangular cross section with width to thickness ratio of 2 : 1. Design the links and the pin.
Solution:
First, let us decide the dimensions of pin. The pin is subjected to single shear and bearing as shown in Fig. E-2.19 (a) and (b) respectively. Let P_b = 30 MPa.
Again as S_y = 330 MPa for 30 C8 steel, $S_{ys} = 0.5\, S, S_y = 166$ MPa. Let F · S = 3

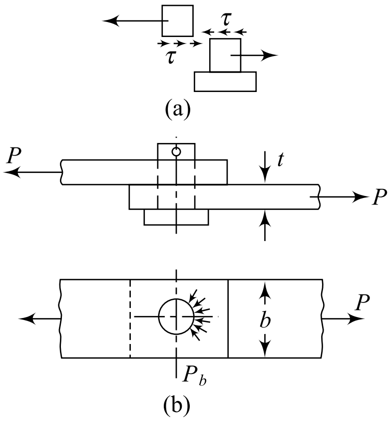


Fig. E-2.19

(a) Bearing failure of pin,

$$\therefore\ P = P_b \times d \times l, \quad \therefore\ d \times l = \frac{5 \times 10^3}{30} = 165.67 \text{ mm}^2$$

length l of the pin depends on the thickness of links. Let us calculate it.

(b)
$$\sigma_t = \frac{P}{A} = \frac{S_y}{\text{F·S}}$$

Area of cross section of link

$$= \frac{5000 \times 3}{330} = b \times t = 2t^2$$

$\therefore$ $t = 6.74 \cong 10$ mm and $b = 20$ mm

$\therefore$ Length of pin $l = 20$ mm, $\therefore$ $d = 8.285 \cong 10$ mm. Let us check diameter of the pin for shear

$$\tau = \frac{P}{A} = \frac{5000 \times 4}{\pi \times 10^2} = 63.66 \text{ MPa}$$

$$\therefore \quad \text{F·S} = \frac{165}{63.66} = 2.59$$

F · S is less than 3 and by increasing the diameter of the pin, hole for the pin in the link will have to be increased. Hence there is no harm in keeping the pin weaker.

Portion of the link to receive the pin will be modified in shape as shown in Fig. E-2.19(c). Let us check margin l_1 of the link at the end.

This margin is decided by considering failure at the end of the link by shearing as shown in Fig. E-2.19(d).

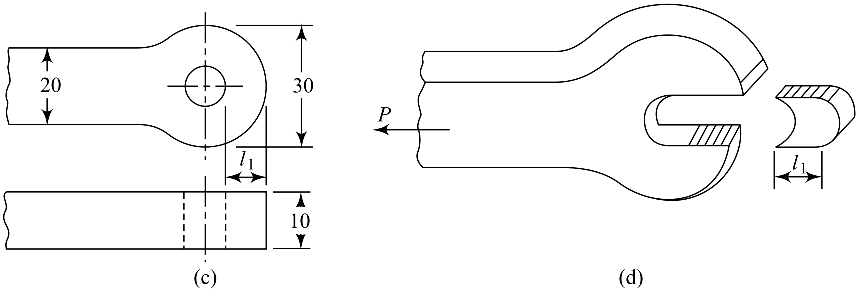


Fig. E-2.19

$$\tau = \frac{P}{2A} = \frac{S_{ys}}{\text{F·S}}, \quad \text{Let F · S = 4}$$

$$\therefore \quad 2 \times l_1 \times 10 = \frac{5000 \times 4}{165}$$

$\therefore$ $l_1 = 24.24$ rounded to 25 mm

2.20 A hypothetical machine element is subjected to bending moment as shown in Fig. E-2.20(a). P = 6 kN, l = 350 mm. Material is 30 C8 steel with S_y = 350 MPa, F · S = 3. Find the dimensions of the most economical cross section.

Solution:

The conventional cross sections used are (a) solid circular, (b) rectangular and (c) I section as shown in Fig. E-2.20 (b)

Maximum bending moment acting on fixed end = $P \times l$

$= 6 \times 1000 \times 350 = 21 \times 10^5$ N.mm

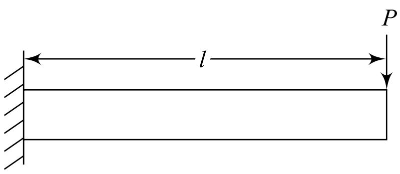


Fig. E-2.20(a)

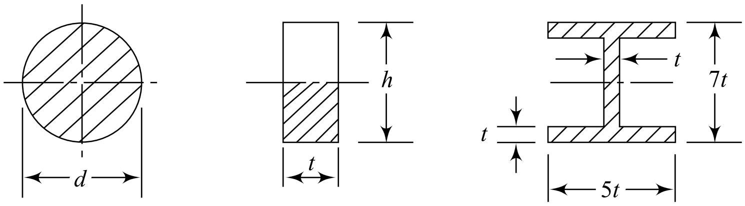


Fig. E-2.20(b)

$$\because \qquad \sigma_t = \frac{M}{Z} = \frac{S_r}{\text{F}\cdot\text{S}}, \text{ Let F} \cdot \text{S} = 3$$

$$Z = \frac{21 \times 10^5 \times 3}{350} = 18000 \text{ mm}^3$$

(a) for circular section $I_{xx} = I_{yy} = \dfrac{\pi}{64}\ d^4 \text{ mm}^4$

$$\therefore \qquad Z_{xx} = \frac{\pi}{32} d^3 = 18000 \text{ mm}^3, \quad \therefore\ d = 56.8 \text{ mm} \cong 60 \text{ mm}$$

(b) for rectangular section $I_{xx} = \dfrac{1}{12}\ th^3$

$$\therefore \qquad Z_{xx} = \frac{1}{6}\ th^2 = \frac{1}{6}\ t \times (2\ t)^2 = 18{,}000 \text{ mm}^3$$

$$\therefore \qquad t = 30 \text{ mm},\ h = 90 \text{ mm}$$

(c) for I section

$$I_{xx} = \frac{1}{12} 4\ t\ (7t)^3 - \frac{1}{12} 3\ t \times (5t)^3$$
$$= 83.08\ t^4$$

$$Z_{xx} = \frac{I_{xx}}{3.5t} = 23\ 73t^3 = 18{,}000 \text{ mm}^3$$

$$\therefore \qquad t = 9.11 \text{ mm} \times 10 \text{ mm}$$

Thus, most economical section is the one with minimum cross-sectional area.

(a) for circular section cross-sectional area = $\dfrac{\pi}{4} \times 60^2 = 2827.43 \text{ mm}^2$

(b) for rectangular section cross-sectional area = $90 \times 30 = 2700 \text{ mm}^2$

(c) for I section cross-sectional area = $(170 \times 40 - 50 \times 30) = 1300 \text{ mm}^2$

Hence, I section is preferable.

2.21 A C.I pulley of 600 mm diameter transmits 30 kW at 300 r.p.m. The pulley is secured to the shaft by means of key. The material for shaft and key is 30C8 steel. Find the diameter d of shaft and dimensions of key, assuming width of key = $d/4$. For 30C8 Steel S_y = 330 MPa.

Solution:

v = peripheral speed of pulley in m/s

D = diameter of pulley in m

F_t = Tangential force on pulley

$$v = \frac{\pi DN}{60} = \frac{\pi \times 600 \times 300}{60 \times 1000} = 9.424 \text{ m/s}$$

$$\text{Power} = F_t \times v$$

$$\therefore \quad F_t = \frac{30 \times 1000}{9.424} = 3183.36 \text{ N}$$

$$\therefore \quad \text{Torque} = F_t \times \text{radius of pulley}$$

$$= \frac{3183.36 \times 300}{1000} = 955 \text{ N.m} = 955 \times 10^3 \text{ N.mm}$$

Safe $$\tau = \frac{S_{ys}}{\text{F}\cdot\text{S}} = \frac{165}{5} = 33 \text{ MPa, as } S_{ys} = 0.5\ S_y$$

Again

$$T = \tau \cdot Z_P$$

$$\therefore \quad Z_P = \frac{955 \times 10^3 \times 5}{165} = 28.94 \times 10^3 \text{ mm}^3 = \frac{\pi}{16}\ d^3$$

$$d = 52.8 \text{ mm} \cong 55 \text{ mm}$$

F·S of 5 is selected because the strength of shaft is reduced due to keyway.
The key fails due to shearing and crushing (Fig. E-2.21).

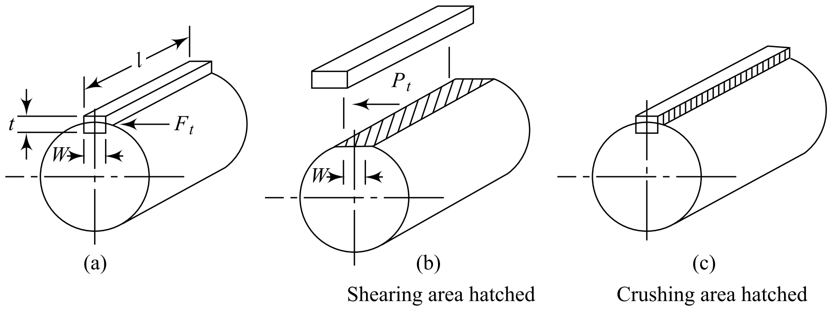


Fig. E-2.21

The key is subjected to a tangential force

$$P_t = \frac{T}{d/2} = \frac{955 \times 10^3}{55/2} = 34727.3 \text{ N}$$

$$w = \frac{d}{4} = 14 \text{ mm}$$

From Fig. E-2.21 (b), shearing area is $w \times l$

$$\therefore \quad \tau = \frac{34727.3}{w \times l} = \frac{165}{\text{F}\cdot\text{S}}, \text{ Let F} \cdot \text{S} = 4$$

$$\therefore \quad l = 60.133 \text{ mm} \simeq 60 \text{ mm}$$

F·S for key is chosen less than the F.S for shaft as replacement of key is easier and economical.

Crushing of key is used to find thickness 't'. Crushing area is $\frac{t}{2} \times l$

$$S_{uc} = 630 \text{ MPa, F}\cdot\text{S} = 4 \text{ based on } S_{uc}$$

$$\sigma_c = \frac{34727.3}{\frac{t}{2}\times 60} = \frac{S_{uc}}{\text{F}\cdot\text{S}} = \frac{630}{4}, \qquad \therefore \quad t = 7.349 \cong 8 \text{ mm}$$

∴ The key should be rectangular with dimensions as, $t = 8$ mm
$w = 14$ mm
$l = 60$ mm.

2.22 The pulley in Example 2.21 has 4 straight arms of elliptic section with the major axis of ellipse being double the minor axis. Determine the cross section of arms using S_{ut} = 140 MPa for C.I.

Solution:

The torque on pulley causes b.m on arms as shown in Fig. E-2.22. Take the maximum length of arms equal to difference between radius of pulley and radius of shaft, i.e. = 300 – 27.5 = 272.5 mm.

$$\therefore \text{ Maximum b.m on each arm} = \frac{3183.36\times 272.5}{4}$$
$$= 216866.4 \text{ N.mm}$$

Taking F·S = 7

$$\sigma_t = \frac{216866.4}{Z} = \frac{140}{7}$$

$$Z = 10843.32 \text{ mm}^3 = \frac{\pi}{32}\, ba^2 = \frac{\pi}{32}\, b(2b)^2 = \frac{\pi}{8}\, b^3$$

$$\therefore \qquad b = 30.22 \text{ mm} \cong 32 \text{ mm}, \; a = 64 \text{ mm}.$$

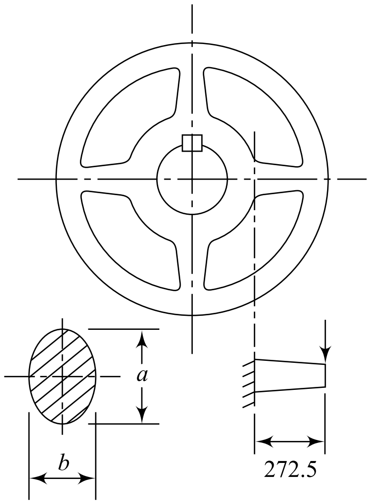


Fig. E-2.22

2.23 A hollow steel shaft 3 m long transmits 25 kN.m torque. The total angle of twist not exceeding 2° and permissible shear stress is equal to 60 MPa, find the inner and outer diameter of shaft. $G = 0.8 \times 10^5$ MPa.

Solution:

$$\theta = \frac{Tl}{JG}, \quad \therefore \quad \frac{2\times\pi}{180} = \frac{25\times 10^6\times 3000}{\frac{\pi}{32}(d_o^4 - d_i^4)\times 0.8\times 10^5}$$

$$d_o^4 - d_i^4 = 2.7356\times 10^8 \text{ mm}^4$$

Again $$\text{induced } \tau = \frac{16Td_0}{\pi(d_o^4 - d_i^4)} = 60 \text{ MPa}$$

$$\therefore \qquad 60 = \frac{16\times 25\times 10^6\, d_o}{\pi\times 2.7356\times 10^8}$$

$$\therefore \qquad d_o = 128.91 \text{ mm} \cong 130 \text{ mm}$$
$$d_i = 58.91 \text{ mm} \cong 58 \text{ mm}$$

Inner diameter is modified on the lower side as it increases the section modulus.

2.24 Calculate the value of maximum clearance, hole tolerance and shaft tolerance according to the basic hole system for following combinations. Basic size is 50 mm. (1) $H_7\,p_6$, (2) $H_6 g_5$

Tolerance grade	Tolerance in microns
7	$7.5 \times \sqrt[3]{d}$
6	$4.7 \times \sqrt[3]{d}$
5	$3.2 \times \sqrt[3]{d}$

Fundamental deviation in microns
for hole H … 0

for shaft g ... $8\sqrt[3]{d^2}$

for shaft p ... $2.1\sqrt[3]{d^2}$

Solution:

1. For H_7p_6 combination, IT for shaft $= 4.7\ \sqrt[3]{50}\ = 17.315$ microns

$$IT \text{ for hole} = 7.3\ \sqrt[3]{50} = 26.89 \text{ microns}$$

Fundamental deviation for hole $H = 0$ microns

Fundamental deviation for shaft $p = 2.1\ \sqrt[3]{50^2}\ = 28.5$ microns

$\therefore$ Hole size = 50.00 mm to 50.02689 mm
Shaft size = 50.0285 mm to 50.04581 mm

These sizes are obtained as

Min. size of hole = Basic size + Fundamental deviation
= 50 + 0.00 = 50.00 mm
Max. size of hole = Minimum size + Tolerance
= 50.00 + 0.02689 = 50.02689 mm
Min. size of shaft = 50 + 0.0285 = + 50.025 mm
Max. size of shaft = 50.0285 + 0.01731 = 50.04581 mm

2. For H_6g_5 combination

$$IT \text{ for shaft} = 3.2 \times \sqrt[3]{50} = 11.79 \text{ microns}$$

$$IT \text{ for hole} = 4.7 \times \sqrt[3]{50} = 17.31 \text{ microns}$$

Fundamental deviation for hole $H = 0.00$ microns

Fundamental deviation for shaft $p = -8\sqrt[3]{2500}\ = -108.57$ microns

Hole size = 50.00 mm to 50.01731 mm

Shaft size = 49.90322 mm to 49.89143 mm

2.25 Using the table for tolerances find the type of fit, maximum and minimum clearance or interference for 150 G7 - e8 combination.

Solution:

For G7 hole of basic size = 150 mm
$EI = 14$ microns, $IT = 40$ microns
For e8 shaft ei = –85 micron, $IT = 63$ microns
Hole size = 150.014 to 150.054 mm
Shaft size = (150 – 0.085 – 0.063) = 149.852 mm to 149.915 mm
Max. clearance = Max. hole size – Min. shaft size = 150.054 – 149.852 = 0.202 mm
Min. clearance = Min. hole size – Max. shaft size = 150.014 – 149.915 = 0.099 mm.

2.26 What are the values of maximum interference, hole tolerance and shaft tolerance in following cases?

(a) Hole size Max. 50.025 mm. Min. 50.000 mm
Shaft size Max. 50.042 mm. Min. 50.026 mm

(b) Hole size Max. 200.046 mm. Min. 200.000 mm
Shaft size Max. 200.079 mm. Min. 200.050 mm

Solution:

(a) Maximum interference = Max. shaft size – Min. hole size = 50.042 – 50.00 = 0.042 mm
Hole tolerance = 50.025 – 50.00 = 0.025 mm
Shaft tolerance = 50.042 – 50.026 = 0.016 mm

(b) Max. interference = 200.079 – 200.00 = 0.079 mm
Hole tolerance = 0.046 mm
Shaft tolerance = 0.029 mm

2.27 Draw the bar diagram and state the type of fit for 80 H_6J_5 and 90 H_6P_5 combination of shaft and hole.

Solution:

From the Table 7 for grade 6 and 5, tolerances are 22 and 15 microns respectively. While lower deviation for H_6 hole, J_5 shaft and P_5 shaft is +9, –9 and +37 microns respectively.

$\therefore$ For H_6J_5 combination
Hole size = 80.031 mm to 80.009 mm
Shaft size = 80.006 mm to 79.991 mm

For H_6P_5 combination
Hole size = 80.031 mm to 80.009 mm
Shaft size = 80.052 mm to 80.37 mm

H_6J_5 bar diagram

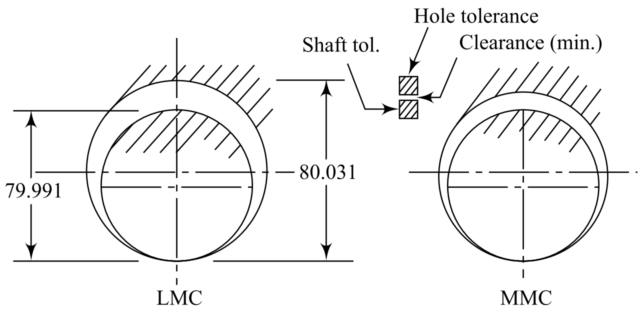


Fig. E-2.27a

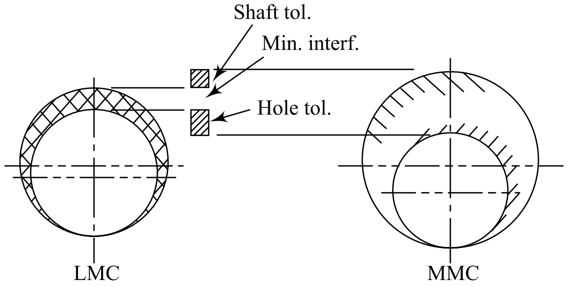


Fig. E-2.27b

2.28 Determine the limiting dimensions of a gudgeon pin and piston of 20 H_6g_5 combination.

Solution:

For a (g_5) type shaft fundamental deviation is (–16) microns and tolerance is 9 microns and for an H_6 hole of 20 mm diameter, fundamental deviation and tolerance are 0.00 and +13 microns respectively.

Min. shaft size = 20 – (0.016) = 19.984 mm
Max. shaft size = 19.984 + (0.009) = 19.993 mm
Min. hole size = 20.00 mm
Max. hole size = 20.00 + 0.013 = 20.013 mm
Min. clearance = Min. hole – Max. shaft = 0.007 mm
Max. clearance = Max. hole – Min. shaft = 0.029 mm.

2.29 Design a shaft and hole combination for selective assembly with a maximum and minimum interference of 0.030 mm and 0.018 mm respectively. A batch of holes of varying sizes from 60 mm to 60.024 mm and shafts of varying sizes from 60.024 to 60.048 mm are produced.

Solution:

Four groups of holes of min. and max. sizes

Min. size	60.00 mm	60.006 mm	60.012 mm	60.018 mm
Max. size	60.006 mm	60.012 mm	60.042 mm	60.024 mm

Four corresponding groups of shafts of sizes

Min. size	60.024 mm	60.030 mm	60.036 mm	60.042 mm
Max. size	60.030 mm	60.036 mm	60.042 mm	60.048 mm

2.30 If the maximum and minimum interference in above example have been changed to 0.032 mm and 0.016 mm respectively suggest the groups.

Three groups of holes of following sizes are suggested:

60.00 mm	60.008 mm	60.016 mm
60.008 mm	60.016 mm	60.048 mm

Corresponding 3 groups for sizes of shafts are:

60.024 mm	60.032 mm	60.040 mm
60.032 mm	60.040 mm	60.048 mm

2.31 Suggest the types of fits for the shaft and hole in Example 2.26.

Solution:

From the Table 7, 50 H_7 P_6 and 200 H_7P_6 are the type of fits.

2.32 Considering, maximum interference between a hole and shaft of 100 mm nominal diameter as 47 microns and minimum interference 3 microns, find the dimensions of parts with basic shaft system and basic hole system.

Assuming equal tolerance on hole and shaft; from the bar diagram tolerance on shaft and hole is = $\frac{1}{2}$ (47 – 3) = 22 Microns

$\therefore$ With basic hole system, the dimensions are

hole size = 100.00 and 100.022 mm

shaft size 100.025 and 100.47 mm

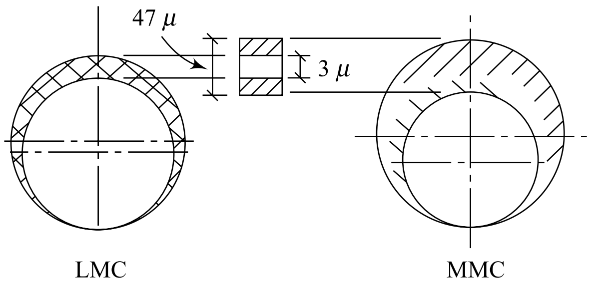


Fig. E-2.32

and with basic shaft system the dimensions are
shaft size = 100.00 and 100.022 mm.
hole size = 99.097 and 99.075 mm.

2.33 Write *R* 10/3 series, *R* 20/3 series for numbers between 10 and 100.

Solution:

For *R* 10 series, ratio is $\sqrt[10]{10} = 1.26$

$\therefore$ Numbers in *R* 10 series are 10, 12.5, 16, 20, 25, 31.5, 40, 50, 63, 80, 100.

$\therefore$ *R* 10/3 series is obtained by taking every third number, i.e.

10, 20, 40, 80 or 12.5, 25, 50, 100 or 16, 31.5, 63

Similarly *R* 20 series is obtained by taking step ratio $\sqrt[20]{10}$ and then to obtain *R* 20/3 we take every third number.

$\therefore$ *R* 20 series 10, 11.2, 12.5, 14, 16, 18, …, 80, 90, 100

$\therefore$ *R* 20/3 series 10, 14, 20, 28, 40, 56, 80, or
11.2, 16, 22.4, 31.5, 45, 63, 90 or
14, 20, 28, 40, 56, 80.

OBJECTIVE QUESTIONS

2.1 Opening and closing of wheel valve requires application of
(a) coplanar forces (b) spatial forces (c) moment (d) couple

2.2 Polar section modulus of a hollow shaft with inner diameter as half the outer diameter *D* is approximately
(a) $0.472\,D^3$ (b) $0.587\,D^3$ (c) $0.184\,D^3$ (d) $0.5\,D^3$

2.3 A cantilever is likely to fail due to excessive stress in
(a) single shear (b) torsional shear (c) tension (d) compression

2.4 Increase in factor of safety causes decrease in
(a) cost (b) size
(c) induced stress (d) strength of machine member.

2.5 Bending moment at *A* on the cantilever shown in Fig. O-2.5
(a) 120 kN.m (b) 60 kN.m
(c) zero (d) 40 kN.m

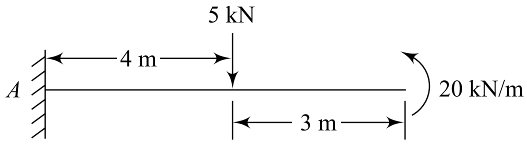


Fig. O-2.5

2.6 A bar of length *L* metres hanging vertically weighing *W* N/m carries a load of *P* Newton at the free end. The tensile force at a distance *d* metres from support of the bar in Newton is
(a) P (b) $\{P + W(L - d)\}$ (c) $\{P + W(L + d)\}$ (d) $(P + Wd)$

2.7 The area under stress-strain curve represents
(a) material hardness (b) breaking strength
(c) energy required to cause the failure (d) none of the above

2.8 Which of the following is dimensionless?
(a) Young's modulus (b) stress (c) strain (d) deformation

2.9 Distribution of stress on the cross section of a machine component subjected to axial tensile or compressive force
(a) varies with zero at bottom fibre to maximum at top fibre
(b) is uniform
(c) varies from neutral axis to the outermost fibres
(d) is unpredictable

2.10 Ratio of areas of cross section of a part subjected to double shear to that of a similar part subjected to single shear for the same load is
(a) 1.5 (b) $\frac{1}{2}$ (c) 2 (d) none of the above

2.11 The most economical cross section for a component subjected to bending is
(a) circular (b) square (c) I section (d) channel section

2.12 A shaft of 60 mm diameter is capable of transmitting 30 kW at 300 r.p.m. To transmit 45 kW at 200 r.p.m the diameter of shaft of same material should be
(a) 60 mm (b) 80 mm (c) 75 mm (d) 100 mm

2.13 A tension member of 40 mm diameter is to be replaced by a square bar of the same material. Side of square will be
(a) 40 mm (b) 36 mm (c) 60 mm (d) 25 mm

2.14 A tension member of diameter d is designed with F · S of 3. If the load and diameter are doubled F · S will be
(a) reduced to half (b) unchanged (c) doubled (d) tripled

2.15 Elliptic cross section is more economical for the arms of pulley due to
(a) ease of manufacture
(b) its section modulus about minor axis being greater than that about major axis
(c) its moment of inertia being larger about major axis
(d) none of the above

2.16 Between a solid shaft of 100 mm diameter and a hallow shaft of 51 mm inner and 102 mm outer diameter subjected to the same torque
(a) solid shaft is 20% stronger (b) hollow shaft is 10% stronger
(c) both are equally strong (d) the strength of hollow shaft is 50% the strength of solid shaft

2.17 As compared to an I section of 40 mm width and 50 mm height with webs and flanges 5 mm thick, a box section of 50 × 40 mm outside dimensions and a uniform thickness of 5 mm subjected to bending is
(a) stronger (b) weaker (c) equally strong (d) 75% strong

2.18 A link of rectangular section of width b and thickness t is safe for tensile load P. If a hole of diameter d is cut in the link width wise and if thickness of the link is doubled width of the new link will be
(a) $(b - d)$ (b) $(b + d)$ (c) $d + \frac{b}{2}$ (d) $2(b + d)$

2.19 The ratio of maximum shear stress induced in a shaft of diameter 'd' to that induced in a key of width 'w' and length 'l' mounted on the shaft subjected to twisting moment is given by
(a) $\frac{\pi d^2}{wl}$ (b) $\frac{\pi d^3}{16\,wl}$ (c) $\frac{8\,wl}{\pi d^2}$ (d) $\frac{16\,wl}{\pi d^2}$

2.20 Selective assembly is used
(a) for clearance fit (b) for getting close fit in spite of using a large tolerance range
(c) for unit production (d) for interference fit

2.21 Bilateral method of tolerancing is used
   (a) in producing parts on large scale   (b) for dimensioning of holes
   (c) for dimensioning of shafts   (d) in selective assembly

2.22 Interference fit is produced
   (a) under minimum metal condition   (b) in the assembly of parts with minimum tolerances
   (c) under maximum metal condition   (d) in basic shaft system

2.23 The cost of production can be reduced by
   (a) using a large factor of safety   (b) using large tolerances
   (c) using low quality material for production   (d) using unskilled labour

REVIEW QUESTIONS

2.1 What are the different possibilities of failure of any machine element?
2.2 What is the role of force analysis in design procedure?
2.3 Define "Machine Design".
2.4 Differentiate between induced stress and safe or design stress.
2.5 What is factor of safety? Why is it necessary? Why a very small or a very large factor of safety should not be used?
2.6 What is moment of inertia of section? How does it differ from mass moment of inertia?
2.7 What is section modulus and polar section modulus? Find the section modulus for rectangular area in terms of width and thickness.
2.8 Compare the section modulus of rectangular section about the two axes. Explain how is it beneficial to use rectangular section in a direction in which depth is larger when subjected to bending moment.
2.9 Find the polar section modulus of hollow shaft.
2.10 Define simple stress and give few examples of machine components subjected to simple stress.
2.11 Differentiate between single and double shear.
2.12 In what respect does bending stress differ from direct tensile or compressive stress?
2.13 Differentiate between direct shear stress and torsional shear stress.
2.14 Compare rectangular box section, hollow circular section, elliptic section subjected to bending moment in terms of economy.
2.15 Why is transverse shear stress neglected while designing a part subjected to bending moment due to transverse load?
2.16 Design of a part subjected to bending moment is done on the basis of safe tensile stress. Why?
2.17 Equation (2.8) is applicable to only circular cross sections. Why?
2.18 Why is the value of permissible bearing pressure for most components smaller than the permissible crushing stress even though bearing stress is developed due to crushing of the two components against each other?
2.19 What should be the method of design if a component has more than one possibilities of failure?
2.20 Discuss the factors affecting selection of material for machine element.
2.21 Define failure. What are the possible modes of failure?
2.22 Why is designing essential before manufacture?
2.23 Why are the tolerances used in manufacture?
2.24 Which type of components should be given bilateral tolerances? Why?

2.25 Define basic shaft system and basic hole system of tolerances.
2.26 Define upper deviation, lower deviations, basic size.
2.27 State true or false giving reasons
(a) A part under single shear is likely to fail under smaller load as compared to a part under double shear
(b) A torsional shear stress is the same as a direct shear stress
(c) A rectangular section with depth smaller than width is used in bending
(d) A hollow shaft is stronger than a solid shaft of the same length and volume
(e) A component subjected to an axial load is likely to buckle
(f) I section is preferred for the frame of C clamp
2.28 Classify the forces acting on machine member as useful load, dead load, inertia load, etc.
(a) A tension member weighing 500 N is subjected to axial load of 6 kN.
(b) A pulley of diameter 300 mm supported between bearings is subjected to tensions 1000 N and 300 N on tight and slack side respectively.
2.29 Specify the factors of safety from group B for the components in group A.
Group-A—(a) Boiler shell, (b) Key, (c) Shaft, (d) Crane hook, (e) Bolt.
Group-B—(a) 2, (b) 3, (c) 5, (d) 10 and (e) 12
Justify your answer.

PRACTICE PROBLEMS

2.1 One end of a rope is tied to a vertical pillar and the other end is pulled with a force of 250 N. What magnitude of tensile force is set up in the rope?
If two persons hold both ends of the rope and pull it with equal force of 250 N on both sides what tensile force is induced in the rope?
2.2 A bar of steel of square cross section with side of square 20 mm is subjected to a load of 50,000 N along the axis, causing tension. What is the magnitude of induced stress? What will be the magnitude of stress if the load increases to 250 kN.
S_{ut} = 500 MPa for steel.
2.3 Find the maximum bending moment and torque on the shaft shown in Fig. P-2.3.

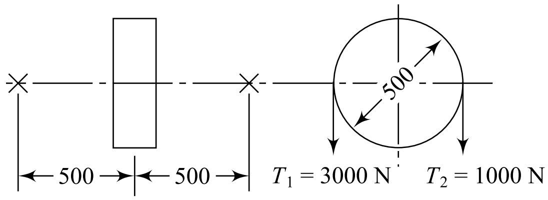


Fig. P-2.3

2.4 A steel rod of 25 mm diameter and 800 mm length is subjected to an axial compressive force of 175 kN. Find the induced stress and the axial and lateral deformation of rod. Poisson's ratio is 0.25.
2.5 Find reactions at supports and plot the B.M. and S.F diagrams for each of the beams shown in Fig. P-2.5.
2.6 A tension bar is subjected to an axial pull of 30 kN. The bar is hollow with inner diameter 0.6 times the outer diameter. Find the diameters if material used is 30C8 steel with S_{ut} = 500 MPa, S_y = 330 MPa and F · S based on yielding is 3.
2.7 Find the diameters of bars in Problem 2.6 if a compressive load 50 kN in magnitude is acting in axial direction. Material is 200 FG C.I with S_{uc} = 630 MPa, F · S = 6.
2.8 If the bar in Problem 2.6 has a slot 6 mm thick as shown in Fig. P-2.8, other quantities being the same, find the diameter of the rod.

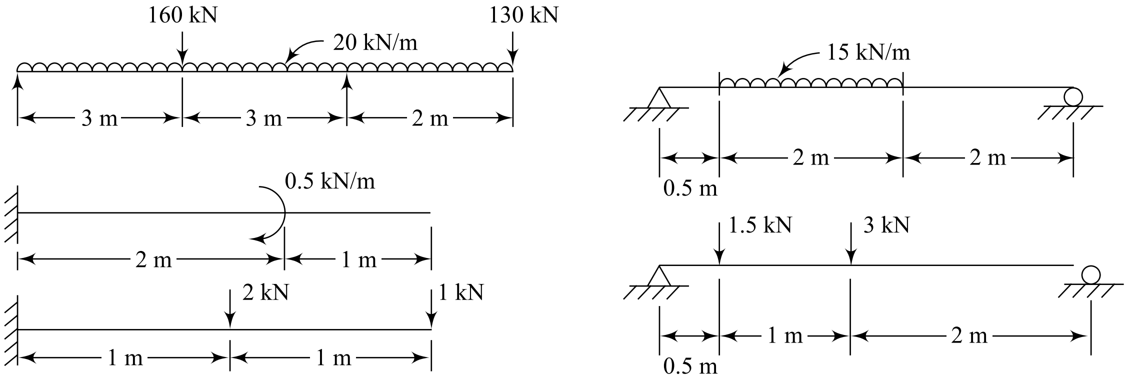


Fig. P-2.5

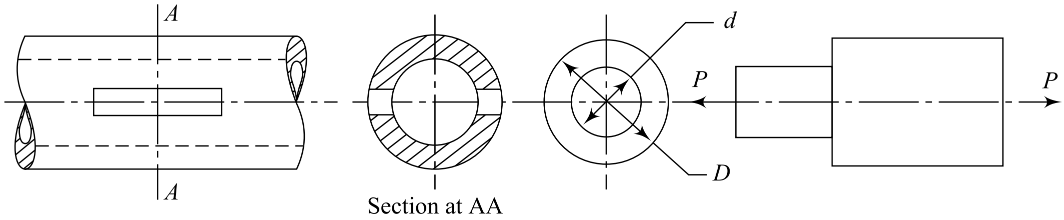


Fig. P-2.8 and P-2.9

2.9 A machine component shown in Fig. P-2.9 is made of 30C8 steel. D = 1.5 d, F · S = 4 based on S_y. Neglecting the effect of stress concentration, find D and d if P = 40 kN.

2.10 A circular bar shown in Fig. P-2.10 is subjected to a tensile axial load of 20 kN. End of the rod has a slot of 7.5 mm thickness. The material of the rod IS 35C8 steel with S_y = 350 MPa, S_{uc} = 500 MPa and a factor of safety of 4 based on S_y . Find d and d_1.

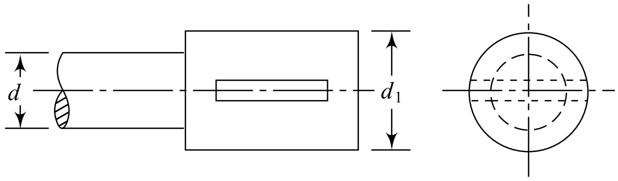


Fig. P-2.10

2.11 A circular rod is used as handle for rotating the screw of a screw jack. The frictional torque between the screw and nut is 11 kN mm. Find the diameter of rod of 30C8 steel. F · S is 3.5 based on S_y.

2.12 The lever keyed to the shaft in Fig. P-2.12 has a rectangular section with h = 3 t. A load of 10 kN is gradually applied at the end of the lever. Find the section of lever at AA. Also find the dimensions of key. Material for lever and key is 20C4 steel with S_y, = 300 MPa, S_{uc} = 500 MPa, F · S is 3 based on S_y and 5 based on S_{uc}.

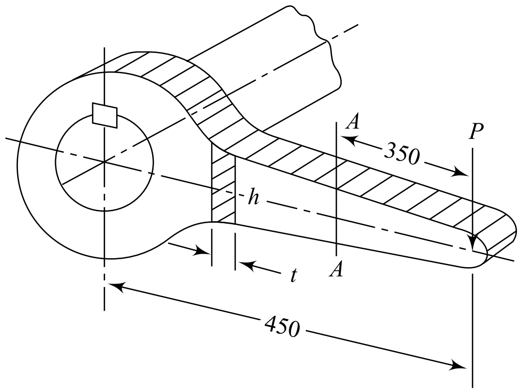


Fig. P-2.12

2.13 Solve Problem 2.12 using I section for the lever with $h = 7\,t$ and $b = 4\,t$, h, b, t being height, width of section and thickness of web and flanges respectively. Use 30 C8 steel with $S_y = 330$ MPa and F · S as 3.5.

2.14 A C-I pulley 500 mm diameter is secured to a shaft of 50 mm diameter. The shaft transmits 12 kW at 150 r.p.m. The pulley has four straight arms of elliptic section with ratio of axes being 2. Decide the cross section of arms. $S_{ut} = 140$ MPa, F · S = 7.

2.15 In order to reduce weight it is planned to use hollow shaft for power transmission in aircraft. Derive an expression for the percentage saving in weight by using hollow shaft in place of solid shaft of equal strength.

2.16 Determine the maximum power a solid shaft of 60 mm diameter can transmit at 240 r.p.m if permissible shear stress for the material is 90 MPa.

2.17 A propeller shaft in a ship is 400 mm in diameter. The allowable working stress in shear is 50 MPa and allowable angle of twist is 1° per 15 diameter length. If $G = 85$ MPa, determine the maximum torque the shaft can transmit.

2.18 For the shaft of Problem 2.17 an axial hole of 200 mm diameter is bored throughout its length. Find the percentage reduction in torque carrying capacity and percentage saving in weight.

2.19 A solid circular shaft transmits 1500 kW at 300 r.p.m. For the material of shaft, permissible $\tau = 65$ MPa, $G = 85$ GPa. Find the diameter of shaft if angle of twist does not exceed 1° per 20 diameter length.

2.20 Two 30C8 steel plates are connected by a row of rivets. Load on the plates is 16 kN. Thickness of the plates is 15 mm. Find width of the plates and the diameter of rivets.

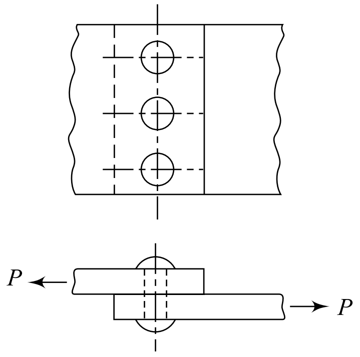


Fig. P-2.20

2.21 Find the stress induced in the cross section of a steel bar of 500 mm diameter subjected to an axial tensile force of 100 kN. What factor of safety is S_y is 300 MPa?

2.22 Find the force required to punch a hole of 20 mm in a 15 mm thick plate of 30C8 steel with ultimate shear strength of 275 MPa.

2.23 A link of rectangular cross section is subjected to a pull of 40 kN. 30C8 steel with $S_y = 330$ MPa and $S_{uc} = 500$ MPa is used. Find the cross section of the link using a factor of safety of 4 and ratio of width to thickness of the section 2 : 1.

2.24 Find the length of the link in the above problem to avoid buckling if the load is compressive and both ends are fixed.

2.25 Find the width of the link of the problem 2.23 if a slot of 10 mm is cut in the width, the thickness being unaltered.

2.26 The slotted link of the above problem is made of 200 FG C.I. Calculate 't' and 'h' if $h/t = 3$, $N = 7$.

2.27 A 100 mm diameter shaft subjected to a transverse load of 30 kN is supported in a bearing with permissible bearing pressure of 1.5 MPa. Find the required length of the bearing.

2.28 A hollow short compression member with $d_o = 5d_i$ made of 200 FG C.I is supporting load of 25 kN. $S_{uc} = 600$ MPa. Find the cross section with $N = 8$.

2.29 Two plates 10 mm thick are connected by four rivets arranged in two rows as shown in Fig. P-2.29. Find the diameter of rivets if the

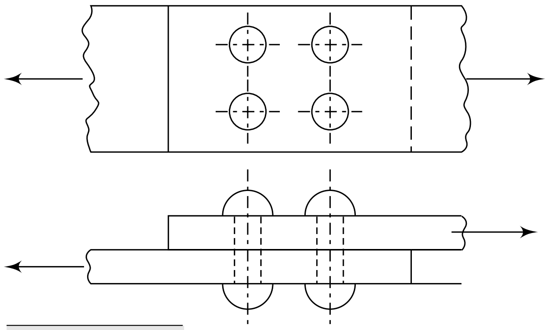


Fig. P-2.29

plates are subjected to a pull of 15 kN using a factor of safety of 2 if the material for the rivets is having S_y = 200 MPa.

2.30 Find the width of the plates to accommodate the rivets in the above problem if the material is 30C8 steel with S_y = 330 MPa and N = 3.5.

ANSWERS

Objective Questions

(1) d	(2) c	(3) c	(4) c	(5) c	(6) b	(7) c	(8) c
(9) b	(10) b	(11) c	(12) a	(13) b	(14) c	(15) b	(16) c
(17) a	(18) c	(19) c	(20) b	(21) b	(22) c	(23) b	

Practice Problems

(1) 250 N, 250 N (2) 125 MPa, 625 MPa (3) 1 kNm, 0.5 kNm

(4) 356.5 MPa, δ = 1.36 mm, Change in diameter = –0.0106 mm

(6) 24 mm, 14 mm (7) 30 and 18 mm (8) 26 and 15 mm (9) 13 and 20 mm

(10) 20 and 24 mm (11) 15 mm (12) h = 96 mm, w = 125 mm, t = 10 mm

(13) t = 12 mm, h = 84 mm, b = 48 mm (14) b = 28 mm, a = 56 mm

(15) $1 - \left(\dfrac{K^2 - 1}{[(K^4 - 1)/K]^{2/3}} \right)$ where $K = \dfrac{\text{Outer dia.}}{\text{Inner dia.}}$

(16) 95.56 kW (17) 621 kN.m (18) 6.25% and 25% (19) 18.8 mm $\cong$ 19 mm

(20) diameter of rivet = 12 mm, width of plate = 50 mm

(21) N = 5.9 (22) 259.181 kN (23) t = 16 mm, W = 32 mm

(24) 2.5 m (25) 42 mm (26) t = 24 mm, h = 70 mm

(27) 200 mm (28) d_o = 30 mm (29) 10 mm (30) 40 mm

3

Combined Stresses: Theories of Failure

CONCEPT REVIEW

3.1 DEFINITION

A part subjected to two types of loading simultaneously gets a combination of stresses induced at a point in the cross section. The design equations are based on the failure of the component at that point where these combined stresses exceed the limit.

3.2 COMBINED BENDING MOMENT AND AXIAL LOAD

A hypothetical member subjected to two loads simultaneously may be treated as a member subjected to P_1 and P_2 separately. The stresses under P_1 and P_2 are calculated separately and superimposed on each (Fig. 3.1 and Fig. 3.2).

$$\sigma_{t_1} = \frac{M}{Z} = \frac{P_1 l}{Z}$$

$$\sigma_{t_2} = \frac{P_2}{A}$$

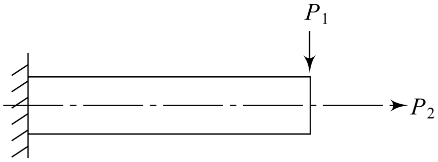


Fig. 3.1

$\therefore$ Total induced stress at the top fibre = $\sigma_{t_1} + \sigma_{t_2} = \dfrac{P_1 l}{Z} + \dfrac{P_2}{A}$

and total induced stress at the bottom fibre = $-\sigma_{t_1} + \sigma_{t_2} = -\dfrac{P_1 l}{Z} + \dfrac{P_2}{A}$

3.3 DESIGN EQUATION

Design equation for the component may be written as

$$\frac{P_1 l}{Z} + \frac{P_2}{A} = \frac{S_y}{N}$$

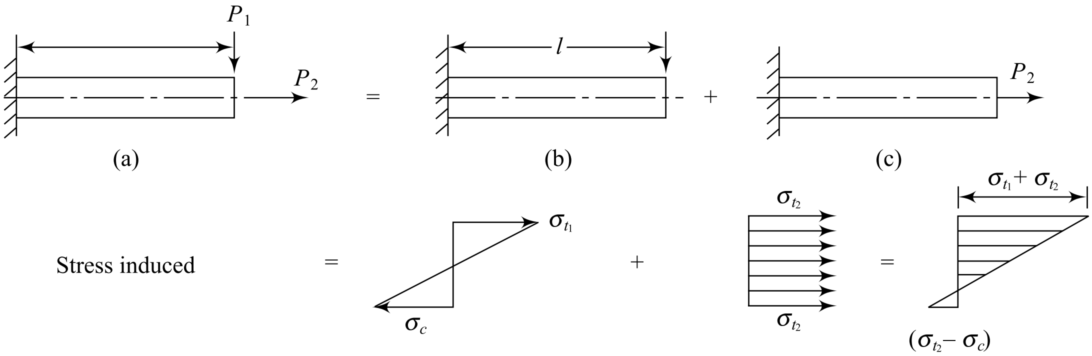


Fig. 3.2

when stresses due to bending and direct loading both are tensile. For brittle material S_y may be replaced by S_{ut}. When both bending and direct stresses are compressive the equation becomes

$$\frac{P_1 l}{Z} + \frac{P_2}{A} = \frac{S_{uc}}{N}$$

A frame of C clamp as shown in Fig. E-3.1 is also subjected to direct tensile stress as well as bending stress and the equations to be used are

$$\frac{P \cdot e}{Z} + \frac{P}{A} = \frac{S_y}{N} \quad \text{for inner fibre and ductile material}$$

$$-\frac{P \cdot e}{Z} + \frac{P}{A} = \frac{S_{uc}}{N} \quad \text{for outer fibre}$$

and

$$\frac{P \cdot e}{Z} + \frac{P}{A} = \frac{S_{ut}}{N} \quad \text{for brittle material}$$

3.4 COMBINED NORMAL AND SHEAR STRESS

A machine component subjected simultaneously to bending moment and torque or direct tension and torsion has both normal and shear stresses acting simultaneously on a small elemental area (Fig. 3.3). Effect of this type of loading is to cause principal stresses, σ_1, σ_2 and maximum shear stress τ_{max} given by,

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

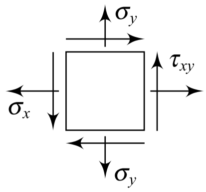


Fig. 3.3

Principal stresses act on the planes termed as *principal planes.* These are the planes on which only normal stresses act and there are no shear stresses. Maximum shear stress acts on the planes making an angle of 45° with principal planes.

3.5 THEORIES OF FAILURE

(a) Maximum principal stress theory or Rankine theory

According to this theory failure of a component takes place if the maximum principal stress at any point exceeds the ultimate or yield point strength.

If principal stresses act in the three directions and are σ_1, σ_2 and σ_3 and if $\sigma_1 > \sigma_2 > \sigma_3$, then the design equation is

$$\sigma_1 = \frac{S_y}{N}$$

(b) Maximum shear stress theory or Guest theory

According to this theory failure occurs when τ_{max} exceeds S_{ys}. The design equation may be written as

$$\tau_{max} = \frac{S_{ys}}{N}$$

or

$$\frac{S_{ys}}{N} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

(c) Maximum strain theory or *St.* Venant theory

According to this theory failure of a component occurs when the maximum strain under actual loading exceeds the maximum strain in tension bar subjected to yield point stress.

For a component subjected to σ_1 and σ_2, strains ε_1 and ε_2 are

$$\varepsilon_1 = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E} \quad \text{and} \quad \varepsilon_2 = \frac{\sigma_2}{E} - \frac{\mu\sigma_1}{E}$$

where μ is Poisson's ratio and E is modulus of elasticity.

As per the theory if $\varepsilon_1 > \varepsilon_2$, design equation becomes

$$\varepsilon_1 = \frac{S_y}{NE} \quad \text{where } N \text{ is F}\cdot\text{S}$$

(d) Maximum total strain energy theory

Strain energy is the energy stored due to deformation and is given by $\frac{\sigma\varepsilon}{2}$ (Fig. 3.4). At the yield point stress of tension bar, the total strain energy is $\frac{S_y^2}{2E}$.

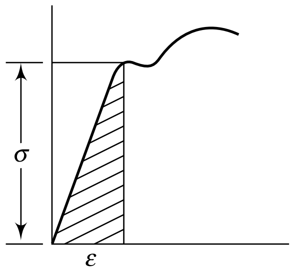


Fig. 3.4

According to this theory failure of a component occurs when the total strain energy under actual condition of loading exceeds the total strain energy $\left(\frac{S_y^2}{2E}\right)$ of tension bar at the yield point stress. Hence design equation becomes

$$\frac{S_y^2}{2E(\text{F}\cdot\text{S})} = \frac{\sigma_1\varepsilon_1}{2} + \frac{\sigma_2\varepsilon_2}{2} + \frac{\sigma_3\varepsilon_3}{2}$$

or

$$\frac{S_y}{N} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1 + \sigma_2 + \sigma_3)}$$

where σ_1, σ_2 and σ_3 are principal stresses and ε_1, ε_2 and ε_3, strains in the three directions of principal stresses.

(e) Maximum energy of distortion theory (Von Huber Mises Hencky theory).
According to this theory the cause of failure is not the total strain energy. Part of this total energy which causes the shearing action is known as *shear energy* or *energy of distortion* and is the cause of failure. Thus, failure of a component takes place when the maximum energy of distortion under actual loading exceeds the value of maximum energy of distortion for tension bar at the time of failure. Design equation for triaxial stresses is

$$\sqrt{\frac{1}{2}(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = \frac{S_y}{N}$$

3.6 FORMS OF DESIGN EQUATIONS TO BE USED IN PRACTICE

Usually machine components are subjected to uniaxial normal stress σ accompanied by shear stress τ. Under this loading condition the forms of design equation to be used are:

(a) Maximum principal stress theory

$$\frac{\sigma}{2} + \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2} = \frac{S_y}{N} \quad \text{or} \quad \frac{S_u}{N}$$

(b) Maximum shear stress theory S

$$\frac{1}{2}\sqrt{\sigma^2 + 4\tau^2} = \frac{S_{ys}}{N} = \frac{S_y}{2N} \quad \text{as } S_{ys} = 0.5\, S_y$$

(c) Maximum strain theory

$$\frac{3}{8}\sigma + \frac{5}{8}\sqrt{\sigma^2 + 4\tau^2} = \frac{S_y}{N} \text{ or } = \frac{S_u}{N} \quad (\text{for } \mu = 0.25)$$

(d) Maximum total strain energy theory

$$\sqrt{\sigma^2 + 2(1 + \mu)\,\tau^2} = S_y/N$$

(e) Maximum energy of distortion theory

$$\sqrt{\sigma^2 + 3\tau^2} = S_y/N$$

Theories indicated in (a) and (c) are used for brittle materials while theories (b) and (e) are used for ductile materials. Theory (d) is rarely used.

3.7 MOHR'S CIRCLE DIAGRAM

It is useful to decide principal stresses and maximum shear stress using simple graphical construction as shown in Fig. 3.5.

$OA = \sigma_2$ $\qquad$ $OD = \sigma_y$ $\qquad$ $GE = DF = \tau_{xy}$

$OB = \sigma_1$ $\qquad$ $OE = \sigma_x$

As per the sign convention used in drawing this diagram, the shear stress causing clockwise couple and the tensile stress are taken as positive.

3.8 PRACTICAL DESIGN ASPECTS

The maximum principal stress theory does not hold good when two equal and opposite principal stresses are acting, i.e. when under the condition $\sigma_1 = -\sigma_2$ since $\tau_{max} = \sigma_1$ which is not desirable.

Maximum shear stress theory does not hold good under the triaxial stress condition of $\sigma_1 = \sigma_2 = \sigma_3$ since τ_{max} is always zero even if $\sigma_1 > S_y$ *By* which is contradictory to the maximum principal stress theory.

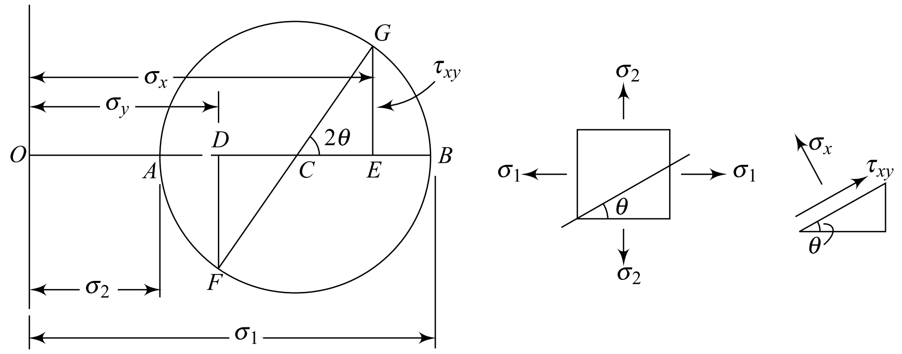


Fig. 3.5

Maximum strain theory allows higher values of σ_1 under biaxial stress condition if both σ_1 and σ_2 are positive.

Total strain energy theory does not hold good for the condition $\sigma_1 = \sigma_2 = \sigma_3$.

The energy of distortion theory has same limitation as that of maximum shear stress theory.

In practice, the maximum principal stress and maximum strain theories are not applicable if failure occurs by yielding, and cannot be used for ductile materials. Maximum principal stress theory is used for brittle materials while maximum strain theory is used for pressure vessels. Maximum shear stress and maximum energy of distortion theories are applicable to ductile materials.

WORKED EXAMPLES

3.1 A frame of C clamp as shown in Fig. E-3.1 is subjected to a load of 5 kN. The material is 200 FG C.I. Eccentricity e = 50 mm. If the section is rectangular with $h = 3b$, find the dimensions b and h. F · S = 5.

Solution:

Total stress at inner fibre,
$$\sigma_t = \frac{5000}{b \times h} + \frac{5000 \times 50 \times 6}{bh^2}$$

∴ Design equation is
$$\frac{5000}{3b^2} + \frac{5000 \times 50 \times 6}{9b^3} = \frac{200}{5}$$

Thus, by trial and error b = 17 mm
h = 51 mm.

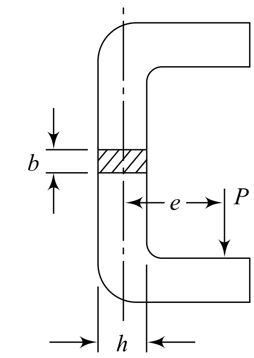


Fig. E-3.1

3.2 Solve the above example assuming the cross section of I section as shown in Fig. E-3.2.

Solution:

Area of section = 15 t^2

$$\text{M.I. of section} = \frac{1}{12} \times 5t(7t)^3 - \frac{1}{12} \times 4t\,(5t)^3$$
$$= 101.25\, t^4$$

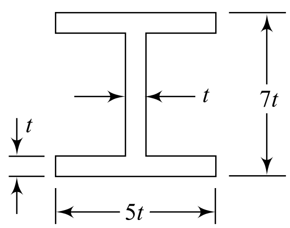


Fig. E-3.2

$$\therefore \qquad Z = \frac{101.25t^4}{3.5t} = 28.92\ t^3$$

$$\therefore \qquad \frac{5000}{15t^2} + \frac{5000 \times 50 \times 6}{28.92\ t^3} = \frac{200}{5}$$

By trial and error, $t = 11.5$ mm rounded to 12 mm.

3.3 The bracket shown in Fig. E-3.3 is made of 30C8 steel with $S_y = 350$ MPa. Using $h = 3\ t,$ find the dimension h and t.

Solution:

Resolve force of 5.5 kN in two components. Vertical component, P_V = Horizontal component, P_H = 3.88 kN. Horizontal component P_H induces bending stress σ_{t3} in the top fibre.

Vertical component P_V induces bending stress σ_{t_i} in the tip fibre

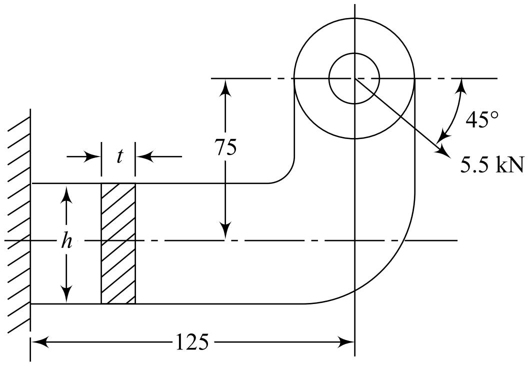


Fig. E-3.3

$$\sigma_{t_i} = \frac{P_V \times 125}{\frac{1}{6} th^2} = \frac{3.88 \times 10^3 \times 125}{1.5t^3}$$

$$= 323.33 \times 10^3/t^3$$

Effect of P_H: (a) a direct stress (b) bending stress

$$\sigma_{t_2} = \frac{3.58 \times 10^3}{A} = \frac{1.293 \times 10^3}{t^2} \text{ MPa by substituting } A = 3\ t^2$$

$$\sigma_{t_3} = \frac{3.88 \times 10^3 \times 75}{1.5\ t^3} = \frac{194}{t^3} \times 10^3 \text{ MPa}$$

$\therefore$ Max. stress in top fibre = $\sigma_{t_1} + \sigma_{t_2} + \sigma_{t_3}$

$\therefore$ Design equation

$$\frac{323.33 \times 10^3}{t^3} + \frac{194 \times 10^3}{t^3} + \frac{1.293 \times 10^3}{t^2} = \frac{S_y}{N}$$

Let $N = 3.5$

$$\therefore \qquad \frac{517.33 \times 10^3}{t^3} + \frac{1.293 \times 10^3}{t^2} = \frac{350}{3.5}$$

By trial and error $t = 20$ mm.

3.4 A symmetrical link shown in Fig. E-3.4 (a) carries a tensile force of 10 kN. The ratio $b/t = 4$ and material used is 30C8 with $S_y = 350$ MPa. Find b and t with F · S = 4. If shape of the link is modified as in Fig. E-3.4(b), determine increase in the width b and thickness 't'.

Solution:

For the first case, the design equation is

$$\sigma_t = \frac{10{,}000}{4t^2} = \frac{350}{4}$$

$\therefore \qquad t = 5.345$ mm rounded to 6 mm.

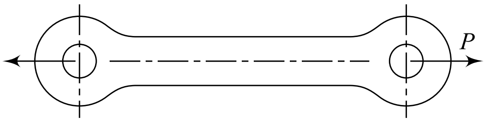


Fig. E-3.4a

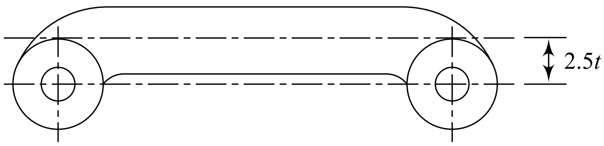


Fig. E-3.4b

Design equation for the second case is

$$\sigma_1 = \sigma_{t_1} + \sigma_{t_2} = \frac{10,000}{4t^2} + \frac{10,000 \times 2.5\,t}{\frac{1}{6} \times t \times 16t^2} = \frac{2500}{t^2} + \frac{9375}{t^2} = \frac{350}{4}$$

$\therefore$ $t = 11.65$ mm rounded to 12 mm.

3.5 The principal stresses at a point consist of a tensile stress $\sigma_1 = 200$ MPa and $\sigma_2 = 100$ MPa compressive and $\sigma_3 = 0$. Determine the maximum shear stress and factor of safety if the material has $S_y = 500$ MPa.

Solution:

Based on Max. principal stress theory,

$$\text{F} \cdot \text{S} = \frac{500}{200} = 2.5$$

From Mohr's circle $\tau_{max} = \dfrac{200 + 100}{2} = 150$ MPa

$\therefore$ Factor of safety based on Max. shear stress theory

$$= \frac{500}{2 \times 150} = 1.67$$

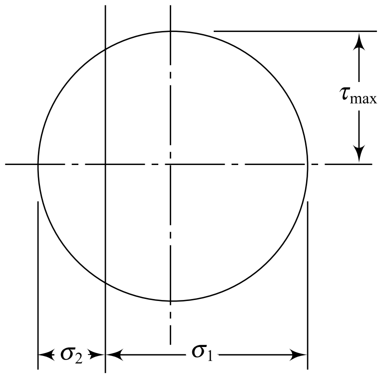


Fig. E-3.5

3.6 A cylindrical boiler as shown in Fig. E-3.6, two metre diameter made of sheet metal 20 mm thick, is subjected to an internal pressure of 1.5 MPa. Find the factor of safety by using different theories of failure. $S_y = 350$ MPa. $\mu = 0.25$.

Solution:

From Fig. E-3.6 the element on the boiler shell is subjected to three principal stresses

σ_1 = pressure = –1.5 MPa

$$\sigma_2 = \text{Hoop stress} = \frac{1.5 \times 1000 \times 2}{2 \times 20} = 75 \text{ MPa}$$

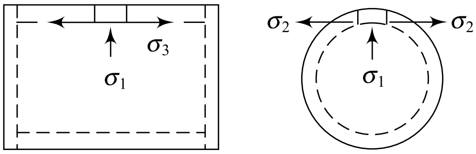


Fig. E-3.6

and $\sigma_3 = \text{longitudinal stress} = \dfrac{1.5 \times 1000 \times 2}{4 \times 20} = 37.5$ MPa

By Max. principal stress theory, $\text{F} \cdot \text{S} = \dfrac{350}{75} = 4.66$

Maximum out of the three maximum shear stresses is

$$\frac{\sigma_1 - \sigma_2}{2} = \frac{-1.5 - 75}{2} = -38.25 \text{ MPa}$$

$\therefore$ Based on max. shear stress theory $\text{F} \cdot \text{S} = \dfrac{S_y}{2 \times \tau_{max}} = \dfrac{350}{2 \times 38.25} = 4.575$

From max. strain theory, max. strain = $\dfrac{75}{E} - \dfrac{(37.5-1.5)}{E} \times 0.25 = \dfrac{66}{E}$

$\therefore$ $\quad$ $F \cdot S = \dfrac{350}{66} = 5.3$

From max. total strain energy theory with $\mu = 0.25$

$$\sqrt{75^2 + 37.5^2 + (-1.5)^2 - 2 \times 0.25[75(-1.5) + 75 \times 37.5 - 37.5\,(-1.5)]} = \frac{350}{F \cdot S}$$

$\therefore$ $\quad$ $F \cdot S = 4.178$

From maximum energy of distortion theory

$$\sqrt{\frac{1}{2}[(-1.5-75)^2 + (75-37.5)^2 + (37.5+1.5)^2} = \frac{350}{F \cdot S}$$

$F \cdot S = 5.294$

3.7 Determine the diameter of a ductile steel bar subjected to an axial tensile load of 40 kN and a torsional moment of 16×10^5 N.mm. Use factor of safety of 1.5, $E = 2 \times 105$ MPa and $S_y = 210$ MPa.

Solution:

$$\sigma = \frac{P}{A} = \frac{40000 \times 4}{\pi d^2} = \frac{5.092 \times 10^4}{d^2} \text{ MPa}$$

$$\tau = \frac{T}{Z_p} = \frac{16 \times 10^5 \times 16}{\pi d^3} = \frac{81.48 \times 10^5}{d^3} \text{ MPa}$$

(a) Applying maximum shear stress theory

$$\frac{S_y}{N} = \frac{210}{1.5} = \sqrt{\left(\frac{5.092 \times 104}{d^2}\right)^2 + 4\left(\frac{81.48 \times 10^5}{d^3}\right)^2}$$

By trial and error $d = 55$ mm

(b) Using maximum energy of distortion theory

$$\frac{210}{1.5} = \sqrt{\left(\frac{5.092 \times 104}{d^2}\right)^2 + 3\left(\frac{81.48 \times 10^5}{d^3}\right)^2}$$

$\therefore$ $d = 52.5$ mm. However we adapt $d = 55$ mm as standard diameter.

3.8 A shaft is designed based on maximum energy of distortion as the criteria of failure and factor of safety of 2. The material used is 30C8 steel with $S_y = 310$ MPa. The shaft is subjected to an axial load of 40 kN. Determine the maximum torque that can be applied to the shaft before yielding. Diameter of shaft is 20 mm.

Solution:

$$\sigma = \frac{P}{A} = \frac{40 \times 1000}{\pi \times 100} = 127.32 \text{ MPa}$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16T}{\pi (20)^3} = 0.0101T \text{ MPa}$$

From maximum energy of distortion theory

$$\frac{S_y}{N} = \sqrt{\sigma^2 + 3\tau^2}, \quad \therefore \frac{310}{2} = \sqrt{(127.32)^2 + 3(0.0101T)^2}$$

$T = 80.17$ N.m

3.9 A load P = 44 kN is applied to a crankshaft at a distance of 200 mm from the bearing (Fig. E-3.9). Material for the shaft is 30C4 steel with S_y = 276 MPa. Using factor of safety of 2 and maximum shear stress theory find the diameter of the shaft.

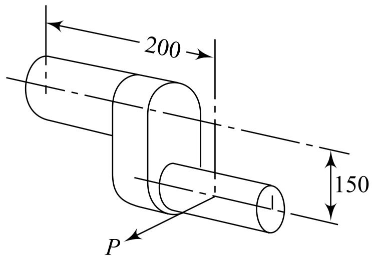


Fig. E-3.9

Solution:

Bending moment = 44000 × 200 N.mm

$$\sigma = \frac{M}{Z} = \frac{44000 \times 200 \times 32}{\pi d^3} = \frac{89636}{d^3} \times 10^3 \text{ MPa}$$

$$\tau = \frac{T}{Z_p} = \frac{44000 \times 150 \times 16}{\pi d^3} = \frac{33.61 \times 10^6}{d^3} \text{ MPa}$$

Using maximum shear stress criteria

$$\frac{S_y}{N} = \sqrt{(\sigma^2 + 4\tau^2)}, \quad \therefore \quad \frac{10^6}{d^3}\sqrt{(89.63)^3 + 4(33.61)^2} = \frac{276}{2}$$

d = 87.06 mm rounded to 90 mm.

3.10 A cylindrical shaft of outer diameter double the inner diameter is subjected to a bending moment of 15000 N.m and torque of 25000 N.m. Find the dimensions of shaft with F · S of 2.

Solution:

$$\sigma = \frac{M}{Z} = \frac{32 \times 15000 \times 10^3}{\pi (d_0^4 - d_1^4)/d_0} = \frac{32 \times 15000 \times 2 \times 10^3}{\pi \times 15 d_i^3} = 20371.83/d_i^3$$

$$\tau = \frac{T}{Z_p} = \frac{25000 \times 16 \times 10^3}{\pi (d_0^4 - d_i^4)/d_0} = 2 \times \frac{25000 \times 16 \times 10^3}{15 \pi d_i^3} = 2 \times \frac{8488.2636}{d_i^3} \times 10^3$$

Let the material be 30C8 with S_y = 350 MPa

$$\therefore \quad \frac{350}{2} = \frac{10^3}{d_i^2}\sqrt{(20371.83)^2 + 4(2 \times 8488.2636)^2}$$

$\therefore \quad d_i = 60.9, \quad \therefore d_i = 60$ mm

$d_0 = 120$ mm

3.11 A hub is press fitted on a shaft. An element in the hub is subjected to a radial compressive stress (pressure) of 50 MPa and hoop stress of 75 MPa. Find the factor of safety if (a) hub is made of 30C8 steel with S_y = 350 MPa. Using maximum shear stress theory (b) if the hub is made of C.I with S_{ut} = 200 MPa, S_{uc} = 700 MPa.

Solution:

(a) Using maximum shear stress theory

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{75 - (-50)}{2} = 62.5 \text{ MPa}$$

$$\text{F} \cdot \text{S} = \frac{S_{ys}}{\tau_{\max}} = \frac{350/2}{62.5} = 2.8 \text{ as } S_{ys} = 0.5\, S_y$$

(b) For brittle materials, using coulomb—Mohr's theory

$$\frac{\sigma_1}{S_{ut}} + \frac{\sigma_2}{S_{uc}} = \frac{1}{\text{F} \cdot \text{S}}$$

$$\sigma_1 = 75 \text{ MPa}, \sigma_2 = 50 \text{ MPa}$$
$$S_{ut} = 200 \text{ MPa}, S_{uc} = 700 \text{ MPa}$$

$$\therefore \quad \frac{75}{200} + \left(\frac{-50}{-700}\right) = \frac{1}{N}, \quad \therefore \quad N = 2.24$$

3.12 The diameter of shaft in Fig. E-3.12 is 60 mm. The material used is 50C4 steel with S_{yp} = 400 MPa. Using F · S = 2.5, find the load P applying

(a) maximum shear stress theory

(b) Mises Hencky theory

Solution:

The shaft is subjected to bending moment and torque

$$M = 400\, P \text{ N.mm}$$

$$\therefore \quad \text{bending stress} = \frac{M}{Z} = \frac{32}{\pi d^3}\ 400$$

$$P = 0.01886 \text{ MPa}$$

$$T = 230\, P \text{ N.mm}$$

$$\therefore \quad \tau = \frac{16T}{\pi d^3} = \frac{16 \times 230\, P}{\pi (60)^3}$$

$$= 0.00542\, P \text{ MPa}$$

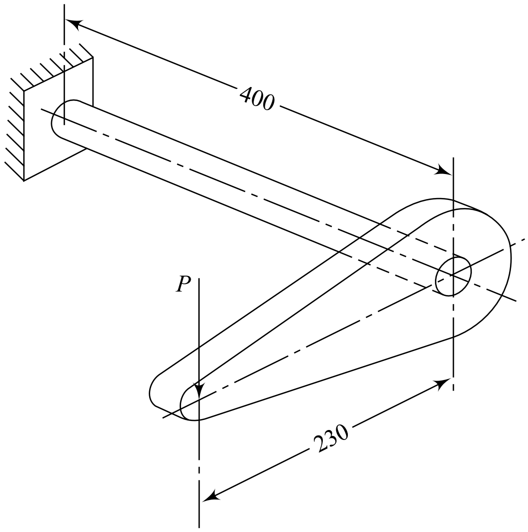


Fig. E-3.12

(a) Applying maximum shear stress theory

$$\tau_{max} = \frac{P}{2}\sqrt{(0.01886)^2 + 4(0.00542)^2} = \frac{S_{ys}}{N} = \frac{0.5 \times 400}{2.5}$$

$$\therefore \quad P = 7355.2166$$

(b) From Mises Hencky theory

$$P\sqrt{(0.01886)^2 + 3(0.00542)^2} = \frac{400}{2.5}$$

$$P = 7594.72 \text{ N.}$$

OBJECTIVE QUESTIONS

3.1 A thin cylinder of 4 m diameter with wall thickness 50 mm is subjected to a pressure of 2 MPa from inside. The maximum shear stress induced in the element on the inner surface is

(a) 40 MPa (b) 80 MPa (c) 160 MPa (d) 41 MPa

3.2 An element is subjected to σ = 60 MPa and τ = 40 MPa. If the material has S_{yP} = 330 MPa, the factor of safety by maximum principal stress theory is

(a) 3 (b) 3.3 (c) 4.125 (d) 4

3.3 The most appropriate theory for ductile materials is

(a) Maximum strain theory (b) Maximum strain energy theory

(c) Energy of distortion theory (d) Maximum principal stress theory

3.4 Maximum principal stress theory is normally used for
(a) ductile materials (b) brittle materials
(c) thick cylinders (d) materials with no specific yield point

3.5 Maximum shear stress theory does not hold good if condition of stress is
(a) biaxial with $\sigma_1 = -\sigma_2$ (b) triaxial with $\sigma_1 = \sigma_2 = \sigma_3$
(c) uniaxial with $\sigma = S_y/2$ (d) triaxial with $\sigma_1 > \sigma_2 > \sigma_3$

3.6 A brittle material subjected to pure shear fails
(a) by yielding when $\tau_{max} = S_{ys}$ (b) by fracture when $\tau_{max} = S_y/2$
(c) by fracture when $\tau_{max} = S_{ut}$ (d) by fracture when $\tau_{max} = \dfrac{S_{ut}}{2}$

3.7 Safe area of stress diagram for maximum energy of distortion theory is represented by
(a) square of sides equal to 2 S_y (b) an ellipse with major axis $2\sqrt{2}\,S_y$
(c) a rectangle (d) a circle

3.8 The relation between maximum utilizable values obtained by tension test and pure shear test for maximum shear stress theory is
(a) $S_y = S_{ys}$ (b) $S_{ys} = 0.5\,S_y$ (c) $S_{ys} = 0.5\,S_u$ (d) $S_{ys} = 0.577\,S_y$

3.9 Under biaxial stress condition if $\sigma_1 = \sigma_2$, $\mu = 0.25$, the maximum value of σ_1 or σ_2 applying maximum energy of distortion theory is
(a) 0.5 S_y (b) 0.866 S_y (c) S_y (d) $\sqrt{3}S_y$

3.10 Using the maximum strain theory, equivalent stress in a system subjected to σ, τ and $\mu = 0.25$ is given by
(a) $\dfrac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$ (b) $\sqrt{\sigma^2 + \tau^3}$
(c) $\dfrac{3}{8}\sigma + \dfrac{5}{8}\sqrt{\sigma^2 + 4\tau^2}$ (d) $\dfrac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$

REVIEW QUESTIONS

3.1 Define principal planes and principal stresses.
3.2 Explain the reasons for using different theories of failure.
3.3 State different theories of failure. Compare their effectiveness and justify their applications.
3.4 "Maximum shear stress theory is more reliable as compared to maximum principal stress theory under the state of biaxial stresses of opposite nature." Explain.
3.5 Explain which three theories are applicable to ductile materials.
3.6 The cause of failure of a component subjected to combined stresses is the maximum energy of distortion and not the total strain energy. Give reasons.
3.7 Why is using maximum energy of distortion theory more beneficial for ductile materials.
3.8 Draw Mohr's circle diagram and explain how it is useful for finding the stress condition at any plane.
3.9 Draw Mohr's circle diagram and show that the maximum shear stress in a tension bar is half the normal stress and is equal to the normal stress for a specimen subjected to $\sigma_1 = -\sigma_2$.

3.10 Show that the yield point shear stress S_{ys} and the yield point tensile strength S_y obtained by using maximum energy of distortion theory are related as $S_{ys} = 0.577\ S_y$.
3.11 Explain how *T* section is economical for the parts such as frame of the C clamp.
3.12 Why was the distortion energy theory found more suitable compared to total strain energy theory?
3.13 Prove that for maximum shear stress theory $S_{ys} = 0.5\ S_y$ for pure shear and $S_{ys} = 0.577\ S_y$ for pure shear with energy of distortion theory.
3.14 Compare the theories of failure by drawing the boundaries of their application for biaxial stress condition and explain their fields of application.
3.15 State true or false
(a) Maximum principal stress theory fails for biaxial stress condition

$$\sigma_1 = -\sigma_2$$

(b) Maximum shear stress theory holds good for triaxial stress condition $\sigma_1 = \sigma_2 = \sigma_3$.
(c) Maximum shear stress theory is economical as compared to maximum principal stress theory.
(d) Maximum shear stress theory and energy of distortion theory are applicable to ductile materials.

PRACTICE PROBLEMS

3.1 A thin walled cylinder closed at both ends is subjected to an internal pressure of 5 MPa. Mean diameter of cylinder is 8 m. The material used is 30C8 steel with $S_y = 300$ MPa. Factor of safety is 3. Neglecting the effect of pressure in radial direction find the wall thickness using (a) maximum principal stress theory, (b) maximum shear stress theory, and (c) maximum energy of distortion theory.
3.2 A thin walled cylinder with mean diameter 150 mm is subjected to a twisting moment of 1.5 kN.m and an internal pressure of 2.5 MPa. If the material used is 30C8 steel with $S_y = 320$ MPa and F · S = 2.5, find the wall thickness.
3.3 A shaft is subjected to a twisting moment of 5 kN.m and a bending moment of 2 kN.m. If the material has $S_y = 330$ MPa and F · S = 2, find the diameter of shaft using (a) maximum shear stress theory, (b) maximum energy of distortion theory.
3.4 A machine part has factor of safety of 2 by Mises Hencky theory. The material used has $S_y = 400$ MPa, $\sigma_1 = 150$ MPa, $\tau = 0$. Find σ_2.
3.5 A lever is keyed to a round bar as shown in Fig. P-3.5. Find the factor of safety in the round bar by energy of distortion theory if the material used is 30C8 with $S_y = 350$ MPa. Compare the result with that obtained by maximum shear stress theory.
3.6 A machine element is subjected to three principal stresses of 120 MPa, 0 MPa and –90 MPa. The material used is 30C8 with $S_y = 360$ MPa. Find the factor of safety applying
(a) Maximum principal stress theory
(b) Maximum shear stress theory
(c) Energy of distortion theory

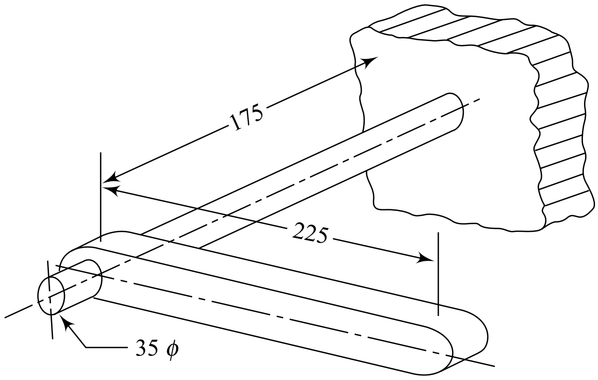


Fig. P-3.5

3.7 Find the factor of safety for the following condition of stresses. The material used is 30 C4 steel with S_y = 310 MPa. Use three theories of failure of Problem 3.6.
(a) σ_x = 70 MPa, σ_y = 30 MPa.
(b) σ_x = 70 MPa, τ_{xy} = 30 MPa clockwise.
(c) σ_x = –10 MPa, σ_y = –60 MPa, τ_{xy} = 30 MPa anti-clockwise.
(d) σ_x = 50 MPa, σ_y = 20 MPa, τ_{xy} = 40 MPa.

3.8 A 400 × 400 mm plate of 45C8 steel has normal stress acting on all edges. σ_x = 40 MPa, σ_y is compressive in nature. Using maximum shear stress theory factor of safety is 3. Find the change in length in x direction. S_y = 400 MPa, $E = 2.1 \times 10^5$ MPa and μ = 0.25.

3.9 The frame of a portable hydraulic rivetter is shown in Fig. P-3.9. Load P is 45 kN. The material used for the frame is 40 C8 steel with yield strength of 400 MPa. Find the dimensions of I section for the frame using factor of safety of 2.5.

3.10 The frame of a press is shown in Fig. P-3.10. = 60 kN, e = 200 mm and d = 150 mm. The material used is 50C4 steel with yield point strength of 450 MPa. Using factor of safety of 3, find the dimension t.

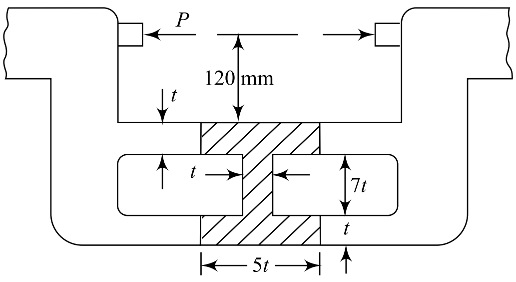


Fig. P-3.9

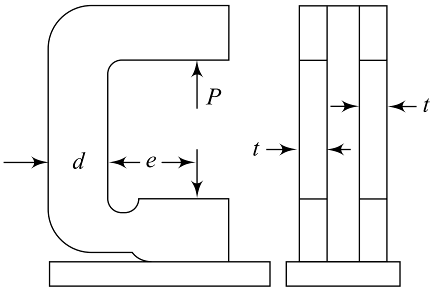


Fig. P-3.10

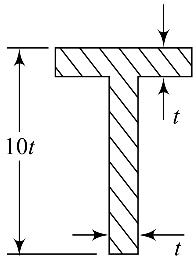


Fig. P-3.11

3.11 Solve Problem 3.9 using a T section as shown in Fig. P-3.11. How is it economical? Why is it not practicable?

3.12 A 50 mm diameter non-rotating shaft of steel with S_y = 400 MPa is subjected to a steady torque of 1500 Nm. Find the permissible steady bending moment that can be superposed on it if the factor of safety by Mises Hencky theory is 13.

3.13 A 30 mm diameter steel shaft is subjected to maximum bending moment of 100 Nm; an axial tensile force of 5000 N and a torque of 200 Nm. S_y = 240 MPa. Determine factor of safety by all theories of failure.

3.14 Determine the diameter of a ductile steel bar subjected to an axial tensile load of 40000 N and a torsional moment of 16×10^5 Nmm. Use factor of safety of 5, $E = 2.1 \times 10^5$ MPa, S_y = 210 MPa. Use (i) Maximum shear stress theory, (ii) Max. E.D. theory.

3.15 A machine part is designed by using maximum E.D. theory and a factor of safety of 3. The material has S_{yp} = 400 MPa, σ_x = 150 MPa, τ_{xy} = 0, Find σ_y.

3.16 Find the wall thickness of thin walled cylindrical shell subject to an axial tension of 300 kN and a torque of 40 kNm. Use maximum shear stress theory and maximum strain theory. The material has S_y = 300 MPa factor of safety of 4 and outer diameter of cylinder 500 mm.

3.17 A machine part has an F · S by maximum shear stress theory of 4, the material tests S_{yp} = 450 MPa, σ_x = 170 MPa. Find the F·S using distortion energy theory σ_y = 100 MPa.

ANSWERS

Objective Questions

(1) d (2) c (3) c (4) b (5) b (6) c (7) b (8) b (9) c
(10) c.

Practice Problems

(1) 20 mm, 20 mm, 17.3 mm (2) 2 mm—using max. shear stress theory
(3) 69.29 mm, 69.86 mm. In both cases a standard diameter of 70 mm is adapted,
(4) 77 MPa (5) 2.81, 2.584 (6) 3, 1.715, 1.97
(7) (a) 4.43, 7.73, 5.1, (b) 3.82, 3.36, 3.55, (c) 4.18, 3.96, 4.07, (d) 3.98, 3.63, 3.79.
(8) t = 12 mm (9) 0.1206 mm (10) 16 mm (12) 1000 Nm
(13) Max. Prin. Stress theory 3.62, Max. Shear stress theory 2.73, Max. Strain theory 3.48, Max. E.D. theory 3.04, Total strain energy theory 3.216
(14) (i) 55 mm, (ii) 52.5 mm (15) 103.33 MPa or 46.46 MPa
(16) 2.5 mm by both the theories (17) 2.7

4

Variable Loading

CONCEPT REVIEW

4.1 TYPES

When the load or stress on a part changes in magnitude or direction or both, the loading is known as variable loading. The variations are as follows: (Refer Fig. 4.1)

(a) Stress variation only on positive side
(b) Stress variation only on positive side but with zero minimum stress
(c) Stress variation in positive and negative direction
(d) Stress variation equally in positive and negative directions, i.e. completely reversible load
(e) Variation only on negative side but zero maximum stress
(f) Variation only on negative side.

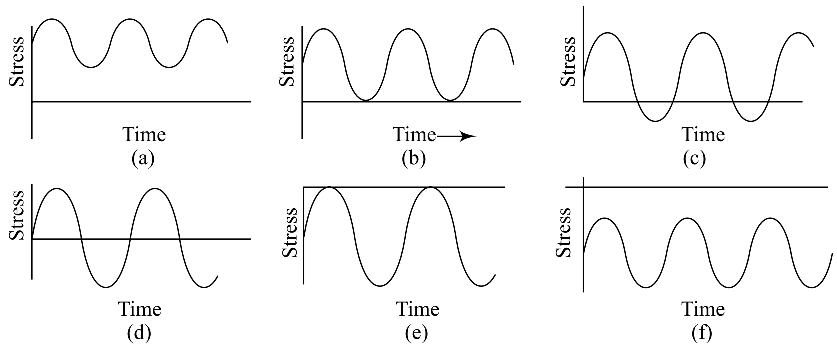


Fig. 4.1

4.2 ENDURANCE LIMIT

Completely reversible stresses occur when a rotating shaft is subjected to a bending moment. Failure of the shaft under different loads takes place after some cycles of reversals. The curve giving the relationship between reversible stress and number of cycles of reversals for failure is the endurance curve. The value of stress at which the material of the shaft completes 10^6 (a million) number of cycles of reversals before failure, is termed as the endurance limit of the material. The test carried out with specific dimensions of test piece to give the endurance curve is known as endurance test. Ref. Fig. 4.2.

(a) Endurance test setup (b) Type of specimen (c) Endurance curve

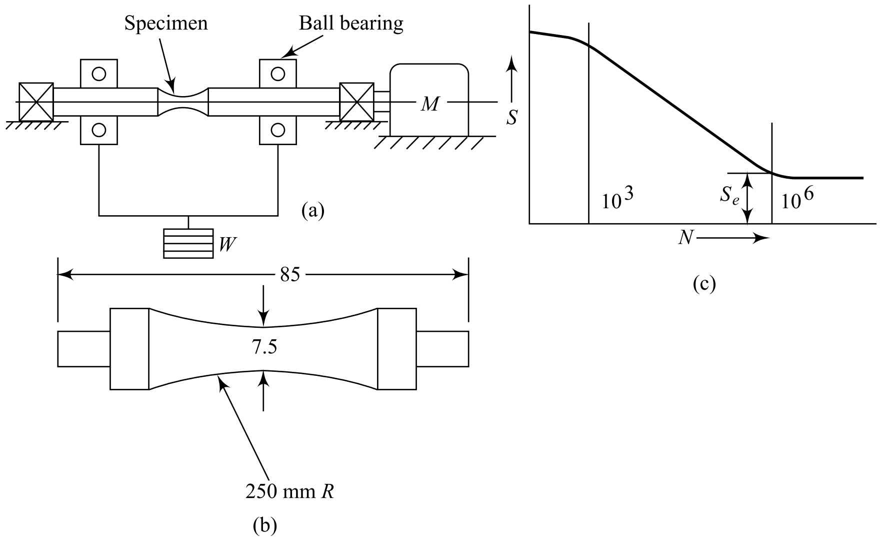


Fig. 4.2

The ordinate of S-N diagram is termed as fatigue strength S_f. Fatigue strength at 10^6 cycles of reversible loading is termed as endurance limit and is designated by S_e. The portion of the curve up to 10^3 cycles is known as low cycle fatigue and beyond 10^3 cycles as the high cycle fatigue. The curve is approximately a straight line between 10^3 and 10^6 cycles of reversals, and is represented by the equation suggested by Basquin.

$$A = S_f L^B \tag{4.1}$$

where A and B are constants, L is the life of specimen in number of cycles of reversals and S_f is the completely reversible stress. Usually S_f is 0.8 to 0.9 S_u at 10^3 cycles and $S_f = S_e = 0.5\ S_u$ at 10^6 cycles. It is further modified by using various factors such as surface correction, size factor etc.

4.3 DESIGN EQUATION FOR MACHINE ELEMENT SUBJECTED TO VARIABLE LOADING

A machine element subjected to a completely reversible type of load is designed by using S_e as the criterion of failure assuming the life to be 10^6 cycles. Thus design equation may be written as

$$\left.\begin{aligned} \text{For normal loading } \sigma &= \frac{S_e}{N} \\ \text{For shear loading } \tau &= \frac{S_{es}}{N} \end{aligned}\right\} \tag{4.2}$$

where σ and τ are induced stresses.

and $S_{es} = 0.6\ S_{us}$ and N is the factor of safety.

For a definite period of life Eq. (4.1) is used.

A machine part subjected to a load other than the completely reversible type is designed by splitting the stress in two components as shown in Fig. 4.3. The mean or static stress is designated by σ_m and the variable stress is σ_v. The maximum and minimum values of stresses are given by σ_{max} and σ_{min} respectively.

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}, \sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$$

When $\sigma_v = 0$, the load induces a static stress only and failure occurs at S_y or S_u. At $\sigma_m = 0$, the load is completely reversible and failure occurs at S_e. These two conditions are shown in the diagram suggested by Soderberg (Fig. 4.4) by points A and B respectively. Thus .

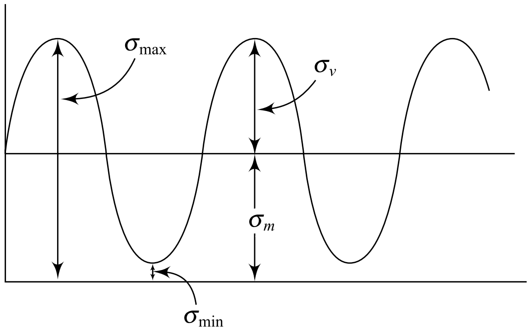


Fig. 4.3

$$OA = S_e \qquad OB = S_y$$

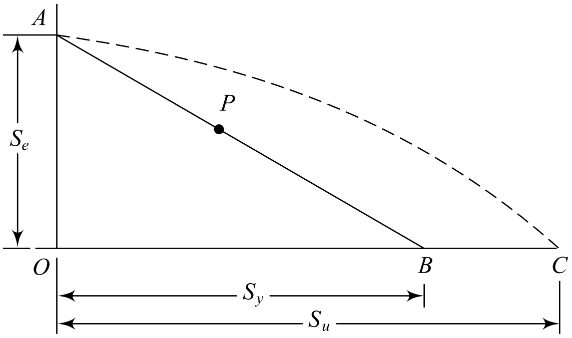


Fig. 4.4

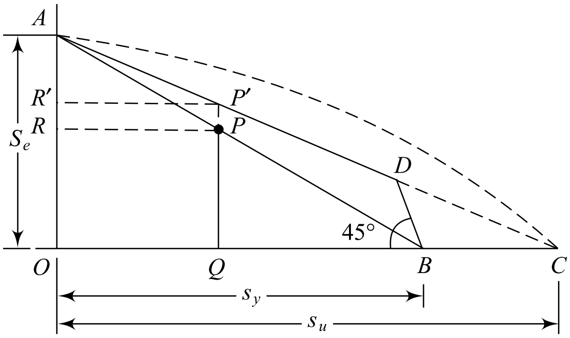


Fig. 4.5

According to Soderberg any combination of σ_m and σ_v may be represented by a point on the straight line joining points A and B. The actual combination of σ_m and σ_v causing failure of the machine element is represented by a point on the parabola (shown dotted) joining points A and C where $OC = S_u$. The parabola is suggested by Gerber.

Goodman's line lies between these two lines such that the first part of Goodman's line is obtained by joining the points A and C till the line BD making an angle of 45° with abscissa intersects at D (Fig. 4.5).

Using Soderberg's diagram $PR = \sigma_m \, (\text{F}\cdot\text{S})$

$$PQ = \sigma_v \, (\text{F}\cdot\text{S}) \qquad \frac{BQ}{OB} = \frac{PQ}{AO}$$

or

$$\frac{S_y - \sigma_m \, (\text{F}\cdot\text{S})}{S_y} = \frac{\sigma_v \, (\text{F}\cdot\text{S})}{S_e}$$

or

$$\frac{\sigma_m}{S_y} + \frac{\sigma_v}{S_e} = \frac{1}{\text{F}\cdot\text{S}} \tag{4.3}$$

By using Goodman line the equation is slightly changed as

$$\frac{\sigma_m}{S_u} + \frac{\sigma_v}{S_e} = \frac{1}{\text{F·S}} \tag{4.4}$$

Since failure of the component takes place by fracture, we use Eq. (4.4) in the text.

The value of endurance limit is affected by the surface finish, size and type of loading etc. Hence its modified value used is

$$S_e = K_a\, K_b\, K_c\, K_d\, S_e \tag{4.5}$$

where,

K_a = surface finish factor, K_b = size factor
K_c = endurance limit correction factor for the type of loading
K_d = reliability factor

The various factors are used according to the availability of data.

4.4 STRESS CONCENTRATION

For a machine element with abrupt change in cross section as shown in Fig. 4.6 the stresses are found to be higher than the nominal value. Thus at points *A* and *B* which are either sharp corners or extremities of the hole the stress is found to be

$\sigma = K_t \dfrac{P}{A}$ where *A* is the minimum area of cross section, *P* is the load and K_t is factor by which the stress is increased. This factor is termed as the *stress concentration factor.*

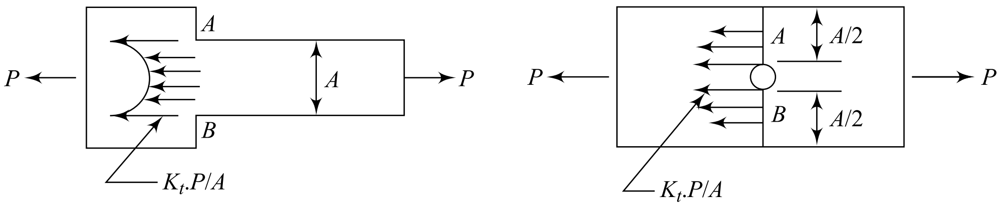


Fig. 4.6

The sharp corner and hole are termed as the stress raisers. Even a small crack or tool may act as a stress raiser.

4.5 NOTCH SENSITIVITY

For the same type and size of discontinuity the increase in stress is different for different materials. K_t is termed as the theoretical SCF (Stress Concentration Factor) and K_f is the actual SCF or form stress factor. The relationship between the two is given as

$$q = \frac{K_f - 1}{K_t - 1} \quad \text{or} \quad K_f = 1 + q\,(K_t - 1) \tag{4.5}$$

where *q* is termed as the notch sensitivity index. In the absence of data, *q* is taken as 1. The value of K_t for different types of discontinuities is taken from the available tables or graphs.

The effect of stress concentration is more predominant for variable type of loading. For static loading the increase in stress at the stress raiser causes the local yielding of components resulting in equal distribution of stresses. For variable loading the stress concentration gives rise to a crack which propagates to cause the final fracture. Hence the SCF is applied for variable stress only. Hence the Eq. (4.4) used for designing a machine element subjected to fluctuating load is modified as

$$\frac{\sigma_m}{S_u} + \frac{K_f \sigma_v}{K_a K_b K_c K_d S_e} = \frac{1}{N} \tag{4.6}$$

4.6 MINER'S EQUATION

A machine element subjected to variable loading with varying mean and variable components of stresses as shown in Fig. 4.7 may be designed by calculating the life of the element using Miner's cumulative damage criteria.

$$\frac{N_1}{L_1} + \frac{N_2}{L_2} + \frac{N_3}{L_3} + \ldots = 1 \tag{4.7}$$

where L_1, L_2, L_3 are the lives with variable loadings composed of σ_{m1}, σ_{v1}, σ_{m2}, σ_{v2}, σ_{m3}, σ_{v3} and so on and N_1, N_2, N_3 etc are the actual number of cycles with those stress components respectively.

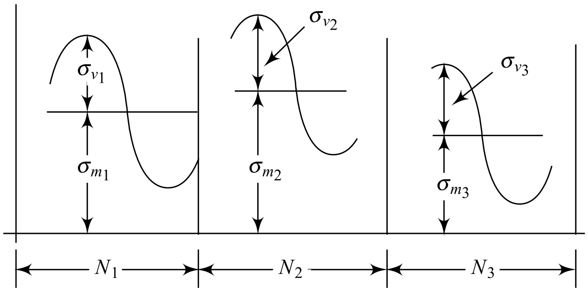


Fig. 4.7

This criterion is useful for short time testing to calculate the fatigue life of the specimen.

4.7 LOADING IN THE FINITE LIFE RANGE

To find the life of a machine component subject to stress components as shown in Fig. 4.8, the completely reversible load equivalent to the combined load at P is obtained by joining CP and extending it further to meet the y axis at E. OE is the required completely reversible stress which can be obtained by

$$\frac{S_f}{S_u} = \frac{K_t \sigma_v}{S_u - \sigma_m} \quad \text{or} \quad S_f = \frac{K_t \sigma_v S_u}{S_u - \sigma_m} \tag{4.8}$$

using this value of S_f and Eq. (4.1), the life of the component can be decided.

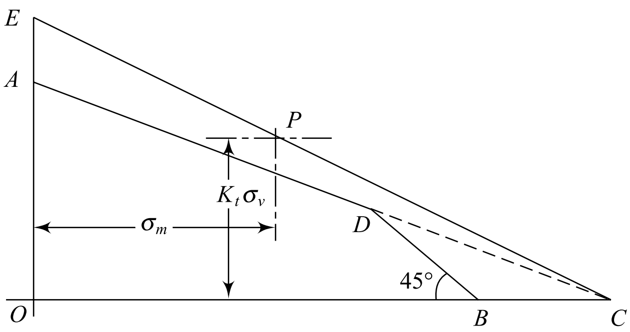


Fig. 4.8

4.8 ANOTHER FORM OF GOODMAN DIAGRAM (FIG. 4.9)

It gives the values of σ_{max}, σ_{min}, σ_m, σ_v directly on the diagram.

4.9 DESIGNING FOR VARIABLE LOADING WITH COMBINED STRESSES

The procedure for designing a part subjected to combined axial and bending load or combined bending and torsion with the variable stresses consists of writing Goodman's Equation 4.5 as

$$\sigma_m + \frac{K_f \sigma_v}{K_a K_b K_c K_d \cdot S_e / S_u} = \frac{S_u}{N} \qquad (4.9)$$

The right hand side of the equation is written as equivalent stress so that

$$\sigma_m + \frac{K_f \sigma_v}{K_a K_b K_c K_d \cdot S_e / S_u} = \sigma_{eq} \qquad (4.10)$$

Similar equation may be written for shear stress also as

$$\tau + \frac{K_f \tau_v}{K_a K_b K_c K_d \cdot S_{es} / S_{us}} = \tau_{eq} \qquad (4.11)$$

σ_{eq} and τ_{eq} obtained from equations (4.10) and (4.11) are further combined by different theories of failure.

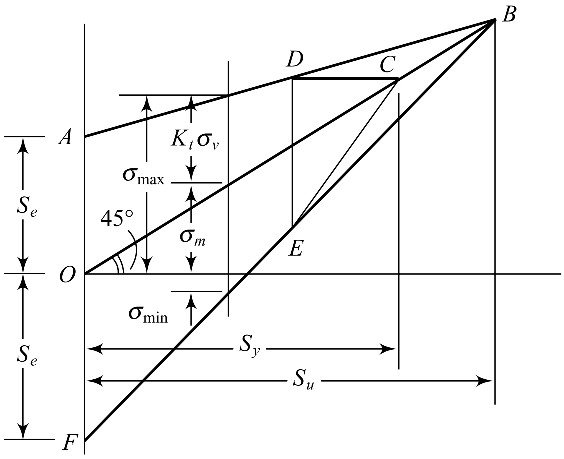


Fig. 4.9

Another approach first applies the different theories of failure and then Goodman, Soderberg or Gerber criteria are applied. Let σ_m, σ_v, τ_∞, τ_v be the component stresses thus, by using Van Mises Hencky theory we have

$$\left.\begin{aligned} \sigma_m' &= \sqrt{\sigma_m^2 + 3\tau_m^2} \\ \sigma_v' &= \sqrt{\sigma_v^2 + 3\tau_v^2} \end{aligned}\right\} \qquad (4.12)$$

Further,

$$\frac{\sigma_m'}{S_u} = \frac{K_f \sigma_v'}{K_a K_b K_c K_d S_e} = \frac{1}{N} \qquad (4.13)$$

4.10 PRACTICAL DESIGN ASPECTS

Reduction of Stress Concentration

The stress concentration cannot be completely eliminated. The efforts therefore are made only to reduce the intensity by increasing the region of stress concentration. The various methods used are (Fig. 4.10 (a) to (e))

(1) Providing fillets as in *A*
(2) Drilling additional holes as in *B*
(3) Providing additional grooves as in *C*
(4) Drilling additional countersunk as in *D*
(5) Reducing shank diameter in threaded fasteners as in *E*

Improvement of Fatigue Strength

In all methods an attempt is made to increase the surface strength by improving surface finish or by hardening surface or inducing compressive stress layer on the surface. The methods used are:

(a) Cold working including shot peening, burnishing.
(b) Heat treatments like induction hardening, case hardening, nitriding. The respective improvement in fatigue strength is 40–100%, 1.5 to 2 times and 30–60%.
(c) Pre-stressing as in carriage springs, gun barrels. In case of gun barrel it is termed as auto fretting.

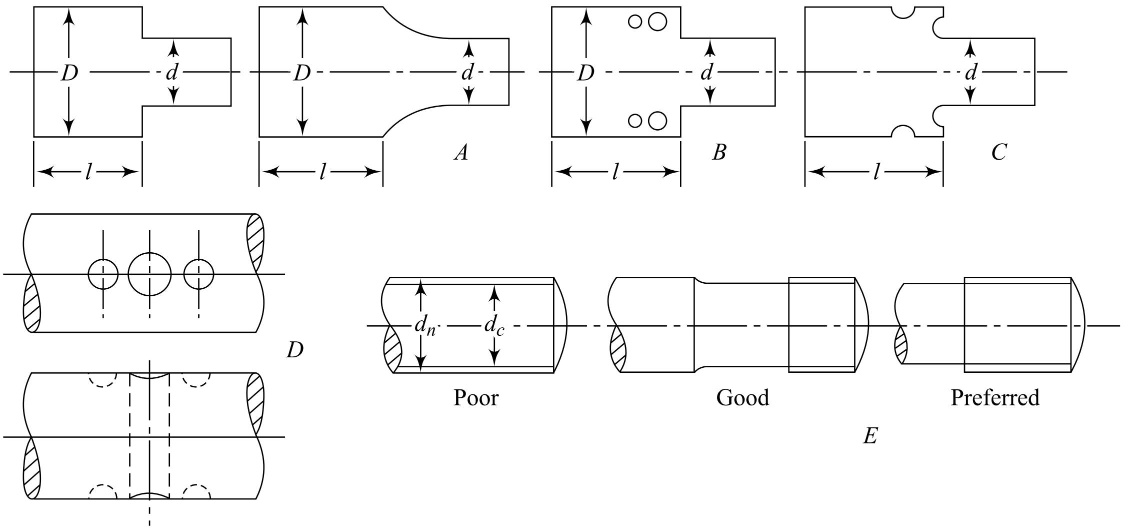


Fig. 4.10

WORKED EXAMPLES

4.1 A pulley is supported midway between the two bearings spaced 400 mm apart on a shaft rotating at 1400 r.p.m. The tensions acting vertically downwards on the tight and slack side of the belt are 1000 N and 500 N respectively. The material of the shaft is 30 C8 steel with S_e = 280 MPa, S_u = 600 MPa. Find the diameter of shaft with bending consideration only.

Solution:

Total tension acting vertically on the pulley causes a b.m on the shaft.

$$\therefore \quad \text{Maximum B.M.} = \frac{1500 \times 400}{4} = 150000 \text{ N.mm}$$

$$\sigma = \frac{150000}{\pi d^3} \times 32 = \frac{1527.88 \times 10^3}{d^3}$$

This is a completely reversible load. If SCF due to key-way is 1.85 and factor of safety based on the endurance limit is 3 then

$$\frac{S_e}{3} = 1.85\,\sigma \text{ substituting we get } d = 31.17 \text{ modified to 32 mm.}$$

4.2 Calculate the fatigue strength for the specimen made of 30 C8 steel with S_e = 280 MPa, S_u = 600 MPa for a life of 200×10^3 cycles of stress reversals. S_f for 10^3 cycles is 0.9 S_u.

Solution:

We have, S_f = 540 MPa for 10^3 cycles

and $S_f = S_e$ = 280 MPa for 10^6 cycles

From Eq. (4.1)

$$\log A = \log S_f + B \log L$$

$$\therefore \quad \log A = \log 540 + 3\,B$$

and $\log A = \log 280 + 6\,B$

$\therefore$ $B = 0.095$ and $A = 1040.3$

Now for a life of 200×10^3 cycles

$$1040.3 = S_f (2 \times 10^5)^{0.095}$$

$\therefore$ $S_f = 326.57$ MPa

4.3 What would be the life of a specimen, if the fluctuating reversible stress in the Example 4.2 is taken as 420 MPa?

$$1040.3 = 420\,(L)^{0.095}$$

$$L = 1.4 \times 10^5 \text{ cycles.}$$

4.4 Determine the plate thickness for infinite life when a completely reversible load of 16000 N acts on the plate in the Fig. E-4.4. S_u = 440 MPa, $q = 0.8$. Surface finish factor = 0.67, size factor = 0.85, reliability factor for 90% reliability = 0.897. $K_t = 2.35$ and F · S = 1.5.

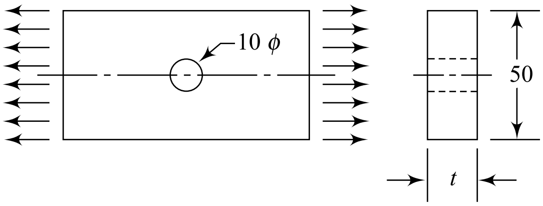


Fig. E-4.4

Solution:

$$\text{Permissible stress} = \frac{0.5\,S_u \times K_a \times K_b \times K_c}{K_f \times \text{F} \cdot \text{S}}$$

$$K_f = 1 + 0.8\,(2.35 - 1) = 2.08$$

$\therefore$ $$\text{Permissible stress} = 36.02 \text{ MPa} = \frac{16000}{(50-10) \times t}$$

$$t = 11.1 \text{ mm modified to 12 mm.}$$

4.5 The load on the plate shown in the Fig. E-4.5 varies from 50,000 to 100,000 N S_u = 480 MPa, S_e = 240 MPa, $K_a = 0.67$, $K_b = 0.85$, K_c correction factor for type of loading = 0.7, $q = 0.95$. Find maximum value of D. F · S = 2.

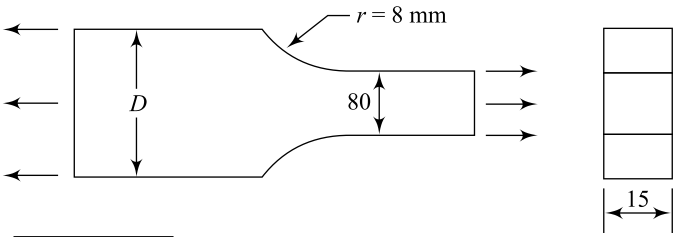


Fig. E-4.5

Solution:

$$\sigma_m = \frac{P_m}{A} = \frac{(50000 + 100000)/2}{80 \times 15}$$

$$= 62.5 \text{ MPa}$$

$$\sigma_v = \frac{P_v}{A} = \frac{(100000 - 50000)/2}{80 \times 15} = 20.83 \text{ MPa}$$

Using Goodman's equation

$\therefore$ $$\frac{1}{2} = \frac{62.5}{480} + \frac{K_f \times 20.83}{0.67 \times 0.85 \times 0.7 \times 240}$$

$\therefore$ $$K_f = 1.7 = 1 + q(K_t - 1), \quad \therefore K_t = 1.736$$

Referring to the chart C11 for $\dfrac{r}{d} = 0.1$ and $K_t = 1.736$, $\dfrac{D}{d} = 1.2$

$\therefore$ $D = 96$ mm

4.6 A cantilever beam of 40C8 steel with S_u = 600 MPa is subjected to a load varying from –50 to 150 N. K_a = 0.76, K_b = 0.85. Reliability factor = 0.879 and q = 0.9. Determine the diameter d, considering F · S = 1.5.

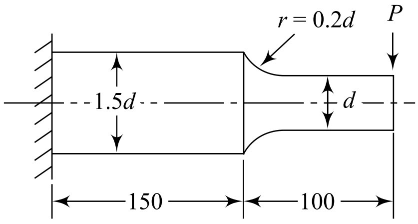


Fig. E-4.6

Solution:

From the chart for $\frac{r}{d} = 0.2$ and $\frac{D}{d} = 1.5$,

$$K_t = 1.35$$

$$\therefore \quad K_f = 1 + q\,(1.35 - 1) = 1.315$$

$$\left.\begin{array}{r}\text{Maximum b.m} = 150 \times 100 \text{ N.mm} \\ \text{Minimum b.m} = -50 \times 100 \text{ N.mm}\end{array}\right\}\text{at the fillet}$$

$\therefore$ Mean b.m = 5000 N.mm, Hence $\sigma_m = \frac{M}{Z} = \frac{32 \times 5000}{\pi d^3}$

Variable b.m = 10,000 N.mm, $\therefore$ $\sigma_v = \frac{32 \times 10{,}000}{\pi d^3}$

Using Goodman's equation

$$\frac{1}{1.5} = \frac{32 \times 5000}{\pi d^3 \times 600} + \frac{10000^{-} \times 32 \times 1.315}{0.76 \times 0.85 \times 0.897 \times 0.5 \times 600 \times \pi d^3}$$

$$d = 9.76 \text{ mm rounded to 10 mm.}$$

4.7 The non-rotating shaft shown in Fig. E-4.7 is subjected to a load P varying from 4000 N to 12000 N. The material 30C8 steel has S_u = 600 MPa, S_y = 350 MPa and S_e = 300 MPa. K_a = 0.8, K_b = 0.85 and K_c = 0.9. Find the dimension D for a factor of safety of 3.5, and q = 0.9.

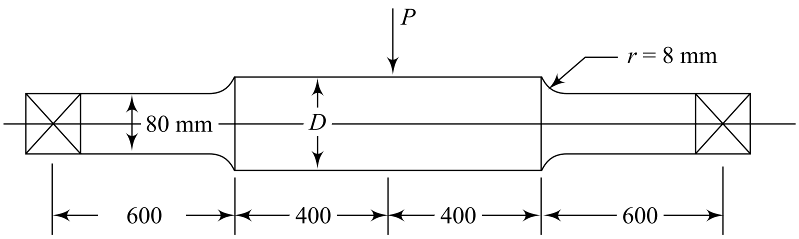


Fig. E-4.7

Solution:

The possibility of failure is at the fillet. (Fillet radius = 8 mm)

$$M_{max} = 6000 \times 600 \text{ N.mm}$$

$$M_{min} = 2000 \times 600 \text{ N.mm}$$

$$\therefore \quad M_m = 4000 \times 600 \text{ N.mm} = 24 \times 10^5 \text{ N.mm}$$

$$M_v = 2000 \times 600 \text{ N.mm} = 12 \times 10^5 \text{ N.mm}$$

$$\therefore \quad \sigma_m = \frac{24 \times 10^5 \times 32}{\pi \times 80} = 47.75 \text{ MPa}$$

$$\therefore \quad \sigma_v = \frac{12 \times 10^5 \times 32}{\pi \times 80^3} = 23.87 \text{ MPa}$$

Now $$\frac{1}{3.5} = \frac{47.75}{600} + \frac{K_f \times 23.87}{0.9 \times 0.85 \times 0.9 \times 300}$$

$\therefore$ $K_f = 1.6$ and $K_t = 1.632$. From chart 9 with $\frac{r}{d} = 0.1$, $K_t = 1.632$, $\frac{D}{d} = 2.8$

$\therefore$ $D = 224$ mm.

4.8 A machine component is subjected to a bi-axial stress condition with σ_1 varying from –50 MPa to 150 MPa and σ_2 varying from 25 MPa to 175 MPa. If the material tests $S_u = 500$ MPa, $S_e = 250$ MPa and $K_t = 1.85$, find the factor of safety using the maximum energy of distortion theory.

Solution:

$$\sigma_{1max} = 150 \text{ MPa}, \qquad \sigma_{1min} = -50 \text{ MPa}$$

$$\sigma_{1max} = 50 \text{ MPa}, \qquad \sigma_{1v} = 100 \text{ MPa}$$

Similarly, $$\sigma_{2m} = 100 \text{ MPa}, \qquad \sigma_{2v} = 75 \text{ MPa}$$

From the maximum energy of distortion theory

$$\sigma'_m = \text{equivalent mean stress} = \sqrt{50^2 - 50 \times 100 + 100^2} = 86.6 \text{ MPa}$$

$\therefore$ $$\sigma'_v = \sqrt{100^2 - 100 \times 75 + 75^2} = 90.138 \text{ MPa}$$

Using Goodman's equation

$$\frac{1}{N} = \frac{86.6}{500} + \frac{1.85 \times 90.138}{250} \qquad \text{or} \qquad N = 1.2$$

4.9 A shaft is subjected to a bending moment varying from –200 N.m to 500 N.m and a twisting moment varying from 50 N.m to 175 N.m. The material 30 C8 steel has $S_u = 600$ MPa, $S_e = 300$ MPa, $K_a = 0.76$, $K_b = 0.85$, $K_c = 0.897$, $K_t = 1.85$ and $q = 0.95$. Find the diameter of the shaft using Von Mises Hencky theory. Factor of safety = 1.5.

Solution:

$$M_m = 150 \text{ N.m}, M_u = 350 \text{ N.m}, K_f = 1 + 0.95(1.85 - 1) = 1.8$$

$\therefore$ $$\sigma_m = \frac{32}{\pi d^3} \times 150 \times 10^3 = \frac{1.52 \times 10^5}{d^3} \text{ MPa}$$

$$\sigma_v = \left(\frac{32}{\pi d^3}\right) \times 350 \times 10^3 = \frac{3.565 \times 10^5}{d^3} \text{ MPa}$$

Again $$T_v = 62.5 \text{ N.m}, \quad \therefore \tau_v = \frac{16}{\pi d^3} \times 62.5 \times 10^3 = \frac{3.183 \times 10^5}{d^3} \text{ MPa}$$

$$T_m = 112.5 \text{ N.m}, \quad \therefore \tau_m = \left(\frac{16}{\pi d^3}\right) \times 112.5 \times 10^3 = \frac{5.73 \times 10^5}{d^3} \text{ MPa}$$

$\therefore$ $$\sigma'_m = \left[\sqrt{(1.52)^2 + 3(5.73)^2}\right] \times \frac{10^5}{d^3} = \frac{10.02 \times 10^5}{d^3} \text{ MPa}$$

$$\sigma'_v = \left[\sqrt{(3.565)^2 + 3(3.183)^2}\right] \times \frac{10^5}{d^3} = \frac{6.53 \times 10^5}{d^3}$$

$\therefore$ $$\frac{1}{1.5} = \frac{10.02 \times 10^6}{600\, d^3} + \frac{1.8 \times 6.53 \times 10^5}{0.76 \times 0.85 \times 0.897 \times 300\, d^3}$$

$\therefore$ $d = 23.29$ mm modified to 25 mm.

4.10 The endurance strength for a part is 280 MPa while S_u = 630 MPa. It is subjected to a loading as follows

$$\sigma_{m_1} = 315 \text{ MPa and } \sigma_{v_1} = 96 \text{ MPa for 80\% of time}$$
$$\sigma_{m_2} = 245 \text{ MPa and } \sigma_{v_2} = 145 \text{ MPa for 20\% of time}$$

Find the expected life in number of cycles of reversals. Assume K_t = 1.5.

Solution:

From the properties of material we have,

$$S_f = 280 \text{ for } 10^6 \text{ cycles}$$
$$S_f = 0.9 \times 630 = 567 \text{ for } 10^3 \text{ cycles}$$

$\therefore$ $\log A = \log 280 + 6B$ and $\log A = \log 567 + 3B$

$\therefore$ $B = 0.1021, \quad A = 1147.84.$

Now under given loading, using Eq. 4.8

$$S_{f_1} = \frac{1.5 \times 96 \times 630}{630 - 315} = 288 \text{ MPa}$$

$$S_{f_2} = \frac{1.5 \times 145 \times 630}{630 - 245} = 355.9 \text{ MPa}$$

The value of L_1 corresponding to S_{f_1} is $\left(\frac{1147.84}{288}\right)^{\frac{1}{0.1021}} = 7.6 \times 10^5$ cycles.

and L_2 for S_{f_2} is $\left(\frac{1147.84}{355.4}\right)^{\frac{1}{0.1021}} = 0.9703 \times 10^5$ cycles.

Now, $\frac{N_1}{L_1} + \frac{N_2}{L_2} = 1.$ If $\frac{N_1}{L_e} = \alpha_1$, and $\frac{N_2}{L_e} = \alpha_2$, L_e = expected life

then $\frac{\alpha_1}{L_1} + \frac{\alpha_2}{L_2} = \frac{1}{L_e}$

$\therefore$ $\frac{0.8}{7.6 \times 10^5} + \frac{0.2}{0.9703 \times 10^5} = \frac{1}{L_e}$

$\therefore$ $L_e = 3.2 \times 10^5$ cycles.

4.11 The material for a machine part in Fig. E-4.11 has S_y = 1500 MPa, K_a = 0.84, reliability factor = 0.897, K_t = 2.3 and q = 0.95. Determine the value of load P for infinite life.

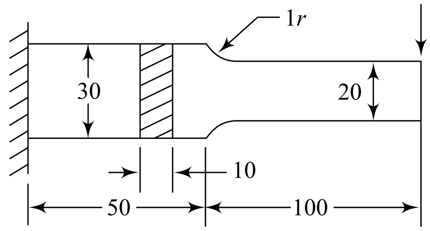


Fig. E-4.11

Solution:

$$K_f = 1 + 0.95\,(2.3 - 1) = 2.235\,P$$

$$\text{Induced } \sigma = K_f \frac{M}{Z} = (2.235)\,\frac{P \times 100 \times 6}{10 \times 20^2} = 0.335\,P$$

and safe $\sigma = 0.89 \times 0.897 \times (0.5 \times 1500) = 598.75$ MPa

Equating, we get $P = 1787.3$ N.

4.12 A shaft is subjected to a torque varying between 5000 N.m to 10000 N.m. The stress concentration factor due to the keyway is 2.5. S_u = 500 MPa, $S_e = 0.5\,S_u$, S_y = 300 MPa, endurance correction factor = 0.6, size correction factor = 0.8 and surface correction factor = 0.82. Find the diameter of the shaft using F·S = 2.

Solution:

$$\text{Mean Torque} = \frac{5000 + 10000}{2} = 7500 \text{ N.m}$$

$$\text{Variable Torque} = 2500 \text{ N.m}$$

$$\therefore \quad \tau_m = \frac{7500 \times 10^3 \times 16}{\pi d^3} \text{ MPa and } \tau_v = \frac{2500 \times 10^3 \times 16}{\pi d^3} \text{ MPa}$$

Using Goodman's equation and $S_{us} = 0.5\ S_u$

$$\frac{1}{2} = \frac{7500 \times 10^3 \times 16}{\pi d^3\ (0.5 \times 500)} + \frac{2500 \times 10^3 \times 16 \times 2.5}{\pi d^3 \times 0.5 \times 500 \times 0.6 \times 0.82 \times 0.8}$$

$$\therefore \quad d = 98.39 \text{ mm modified to 100 mm.}$$

4.13 A torque varying from 25 kN.m to 75 kN.m is applied at the end of the shaft. Fillet radius $r = \frac{D}{8}$, factor of safety = 1.6, material is 40 MN 2512 with S_y = 350 MPa. S_e = 250 MPa, K_a = 0.85, K_b = 0.82, K_c = 0.6, SCF due to keyway = 1.6 q = 0.9.

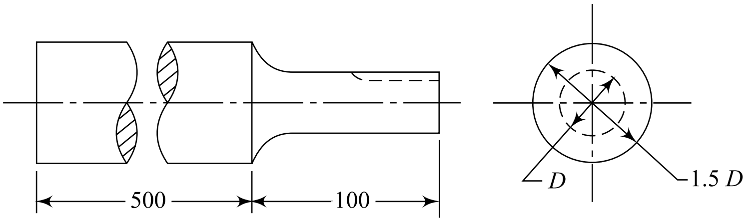


Fig. E-4.13

Solution:

From the chart C-6 $\quad K_t = 1.39, \quad \therefore K_t = 1 + (1.39 - 1) \times 0.9 = 1.351$

Using Soderberg's equation

$$\frac{1}{N} = \frac{\tau_m}{S_{ys}} + \frac{K_f\, \tau_v}{K_a\, K_b\, K_c\, S_y}$$

$$T_{\text{mean}} = \frac{25 + 75}{2} = 50 \text{ N.m} \times 10^3, \quad \therefore \ \tau_m = \frac{50 \times 10^6 \times 16}{\pi D^3} \text{ MPa}$$

$$T_v = \frac{75 - 25}{2} = 25 \text{ kN.m}, \quad \therefore \ \tau_v = \frac{25 \times 10^6 \times 16}{\pi D^3}$$

$$\therefore \quad \frac{1}{1.6} = \frac{50 \times 10^6 \times 16}{\pi D^3 \times 0.5 \times 350} + \frac{1.351 \times 25 \times 10^6 \times 16}{\pi D^3 \times 0.82 \times 0.85 \times 0.6 \times 250}$$

Under pure twisting the failure may occur at the keyway where K_t = 1.6. Hence $K_f = 1 + 0.6 \times 0.9$ = 1.54 which is more than the value of K_f at the fillet.

$$\therefore \qquad \frac{1}{1.6} = \frac{50\times10^6\times16}{\pi D^3\times175} + \frac{25\times10^6\times16\times1.54}{250\,\pi D^3\times0.6\times0.85\times0.82}$$

$$\therefore \qquad D = 174.6 \text{ mm modified to } 175 \text{ mm.}$$

4.14 A machine part is made of forged steel with S_{ut} = 630 MPa, S_e = 0.22 S_{ut}. The life of the part is 250,000 cycles. The loading for 50% of the time is ± 225 MPa and for 30% of the time is ±145 MPa. Calculate the loading during the remaining time?

Solution:

For 10^6 cycles the permissible completely reversible stress is

$$S_e = 0.22\, S_{ut} = 138.6 \text{ MPa} = S_{f_1}$$

For 10^3 cycles the permissible completely reversible stress S_{f_2} = 0.9 S_{ut} = 567 MPa

From the Eq. (4.1)

$$\frac{S_{f_1} L_1^B}{S_{f_2} L_2^B} = \frac{A}{A} = 1, \quad \therefore \quad \frac{138.6(10^6)^B}{567\,(10^3)^B} = 1$$

$$\therefore \qquad B = 0.2039,\ A = 2318.2658$$

For a given loading σ_{R_1} = 225 MPa. Therefore, L_1 = 92902.419 cycles. σ_{R_2} = 145 MPa and hence, L_2 = 801403.13 cycles.

Using Miner's equation we have,

$$\frac{0.5}{92902.419} + \frac{0.3}{801403.13} + \frac{0.2}{L_3} = \frac{1}{25000}$$

$$\therefore \qquad L_3 = 113873.49 \text{ cycles}$$

$$\therefore \qquad \sigma_{R_3} = 215.85 \text{ MPa which is the loading during the remaining time.}$$

4.15 A machine part is operated for a period N_1 = 78 hours at the service load. The service load is then removed and the part is operated at a heavier load. Failure at this load occurs after 79 hours. The life of the part at heavy load is 80 hours. Find its life at the service load.

Solution:

$$\text{Using} \qquad \frac{N_1}{L_1} + \frac{N_2}{L_2} = 1, \quad \text{we get} \quad \frac{78}{L_1} + \frac{79}{80} = 1$$

$$\therefore \qquad L_1 = \text{Life at service load} = 6040 \text{ hours.}$$

4.16 A shaft subjected to a b.m due to the weight of the pulley equal to 2000 N and a non-rotating load of 800 N acting vertically downwards rotates at a speed of 1400 r.p.m. The distance between the bearings is 1 m. The external load and the load due to pulley act at a distance of 300 mm from L.H. bearing. The material of the shaft has S_y = 300 MPa and S_e = 200 MPa. Using F·S = 2.5, find the diameter of the shaft if K_t due to keyway is 1.3.

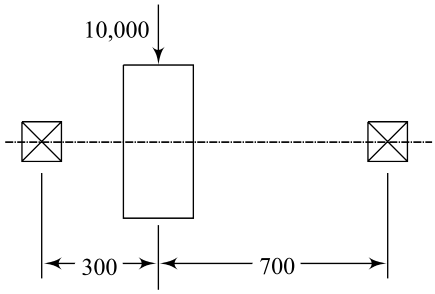


Fig. E-4.16

Solution:

The shaft is subjected to a completely reversible load due to the b.m. Hence the design is based on the endurance limit, S_e, to find the maximum b.m at the pulley we calculate the reactions. Taking moment about B, $R_A \times 1000 = 10{,}000 \times 700$

$$\therefore \quad R_A = 7000 \text{ N}$$

$$\text{Maximum b.m} = 7000 \times 300 = 21 \times 10^5 \text{ N.mm}$$

$$\sigma_{\max} = \frac{M}{Z} = \frac{32 \times 21 \times 10^5}{\pi d^3}$$

$$\therefore \quad \text{Permissible stress} = \frac{S_e}{N \times K_t} = \frac{200}{2.5 \times 1.3} = 61.54 \text{ MPa}$$

$\therefore$ Equating $\sigma_{\max} = 61.54$ MPa. $d = 70.31$ mm.

A standard shaft diameter of 70 mm is adapted since it does not affect the factor of safety to a large extent.

4.17 A shaft of diameter d is subjected to a torque varying between 100 N.m to 500 N.m. K_r due to keyway is 1.5. F · S = 2, $S_y = 300$ MPa, $S_e = 200$ MPa. Correction factor for torsion = 0.6. Surface finish factor = 0.85 and size factor = 0.82. Find the value of *d.*

Solution:

$$T_{\text{mean}} = \frac{500+100}{2} = 300 \text{ N.m}, \quad \therefore \quad \tau_m = \frac{16 \times 300000}{\pi d^3} \text{ MPa}$$

$$T_v = \frac{500+100}{2} = 200 \text{ MPa}, \quad \therefore \quad \tau_v = \frac{16 \times 200000}{\pi d^3} \text{ MPa}$$

$$S_{ys} = 0.5\, S_y = 150 \text{ MPa}$$

$\therefore$ Using Soderberg equation we get,

$$\frac{1}{2} = \frac{16 \times 300{,}000}{\pi d^3 \times 150} + \frac{16 \times 200000 \times 1.5}{\pi d^3 \times 0.6 \times 0.85 \times 0.82 \times 200}$$

$$d = 33.18 \text{ mm modified to 35 mm.}$$

Note: As per the recent observation a component subjected to a varying torque has negligible effect due to mean shear stress. Hence, the design may be based only on the variable shear stress.

4.18 A uniform bar having a machined surface is subjected to an axial load varying from 400 kN to 150 kN. The material of the bar has $S_u = 630$ MPa. $K_c = 0.7$ and $K_t = 1.42$. Find the diameter d of the rod using F · S = 1.5.

Solution:

$$\text{Mean axial load} \qquad P_m = \frac{400+150}{2} = 275 \text{ kN}$$

$$\text{Variable axial load} \qquad P_v = \frac{400-150}{2} = 125 \text{ kN}$$

$$\therefore \quad \sigma_m = \frac{P_m}{A} = \frac{275 \times 10^3 \times 4}{\pi d^2} = \frac{350.14 \times 14 \times 10^3}{d^2}$$

$$\sigma_v = \frac{P_v}{A} = \frac{125 \times 10^3 \times 4}{\pi d^2} = \frac{159.16 \times 14 \times 10^3}{d^2}$$

Using Goodman's equation with $K_a = 0.85$, $K_b = 0.82$, $K_c = 0.7$, $S_u = 630$ MPa and $S_e = 315$ MPa.

$$\text{Thus,} \qquad \frac{1}{1.5} = \frac{350 \times 14 \times 10^3}{d^2 \times 630} + \frac{159.16 \times 1.42}{d^2 \times 315 \times 0.7 \times 0.82 \times 0.85}$$

$$d = 55.13 \text{ rounded to 55 mm (standard diameter).}$$

OBJECTIVE QUESTIONS

4.1 The design stress for a component subjected to a completely reversible load, is found by applying the factor of safety to
(a) ultimate strength (b) endurance strength
(c) yield strength (d) fatigue strength

4.2 Increase in the radius of fillets
(a) improves the appearance of machine part (b) reduces the strength of the part
(c) reduces stress concentration factor (d) increases the notch sensitivity index

4.3 Short time fatigue life testing is done by using
(a) Soderberg's equation (b) Basquin's equation
(c) Miner's equation (d) Gerber's equation

4.4 Endurance strength is found at
(a) 10^3 cycles of reversals (b) 10^9 cycles of reversals
(c) 10^6 cycles of reversals (d) 10^7 cycles of reversals

4.5 For a material with $S_u \geq 1400$ MPa the endurance limit is
(a) $0.9\ S_u$ (b) $0.5\ S_u$ (c) $0.65\ S_u$ (d) $0.8\ S_u$

4.6 Fatigue strength may be improved by
(a) hot working (b) cold working (c) annealing (d) sand blasting

4.7 It is possible to get the complete picture of variable stresses by referring to
(a) S-N diagram (b) Goodman's diagram
(c) Soderberg's diagram (d) Gerber's diagram

4.8 A machine part is operated for a period of 10 hours at service load, for 9.6 hours at heavy load when failure occurred. The life of the part at heavy test load alone is 10 hours. The life of the part at service load is
(a) 96 hrs (b) 100 hrs (c) 400 hrs (d) 250 hrs

4.9 Maximum stress in the part shown in Fig. O-4.9 is induced at the section
(a) AA (b) BB
(c) CC (d) at fillets

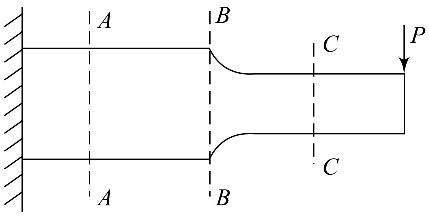


Fig. O-4.9

4.10 The relationship between notch sensitively factor q, theoretical stress concentration factor K_t, and form stress factor K_f, is given by

(a) $q = \dfrac{K_f}{K_t}$ (b) $q = \dfrac{(K_f - 1)}{(K_t - 1)}$

(c) $q = \dfrac{K_f + 1}{K_t + 1}$ (d) $q = \dfrac{K_t - 1}{K_f - 1}$

4.11 Effect of stress raiser on different materials is
(a) the same (b) indicated by sensitivity index
(c) independent of material (d) indicated by stress concentration factor.

4.12 To design a component of ductile material under variable loading, S.C.F. is applied to
(a) mean stress only (b) variable stress only
(c) both the mean and variable stresses (d) yield point strength

4.13 Improvement of fatigue strength using pre-stressing is done for
(a) ductile materials
(b) parts subjected to undirectional variable stress
(c) parts subjected to completely reversible load
(d) parts subjected to variable load

4.14 Basquin's equation is used to design
(a) for completely reversible loads
(b) parts subjected to indefinite number of cycles
(c) parts for definite number of cycles
(d) for low cycle fatigue

4.15 Autofretting is a
(a) type of heat treatment
(b) type of corrosion
(c) type of prestressing
(d) method to reduce stress concentration

4.16 Loading a component such that the variable stress is less than endurance limit for some time is termed as
(a) prestressing (b) training (c) underloading (d) low cycle fatigue

4.17 Endurance strength is 100% for a component with
(a) ground finish
(b) mirror finish
(c) machined surface
(d) hot rolled surface

4.18 Fatigue strength of the cold rolled component is
(a) more than hot rolled component
(b) less than hot rolled
(c) more than polished specimen
(d) less than machined surface

4.19 A large rough surface and small smooth area of a fatigue fractured specimen indicates
(a) overloading
(b) underloading
(c) less number of points of stress concentration
(d) low ductility of material.

4.20 Fatigue strength is maximum within the temperature range of
(a) 300–400°C (b) 100–200°C (c) 500–600°C (d) 150–250°C

4.21 Number of cycles of repetitions for low cycle fatigue is
(a) between 10^3 and 10^6
(b) less than 10^3
(c) more than 10^6
(d) between 10^2 and 10^4

4.22 Endurance correction factor for shear load is
(a) 0.5 (b) 0.6 (c) 0.7 (d) 0.85

REVIEW QUESTIONS

4.1 Define endurance test and endurance limit.
4.2 What are the different variations possible in fluctuating loading on a machine part?
4.3 How does a completely reversible load differ from a fluctuating load ?
4.4 Differentiate between endurance strength and fatigue strength.
4.5 What are the different factors affecting fatigue strength?
4.6 What is stress concentration? How does it affect the fatigue strength?
4.7 What are the different methods to reduce stress concentration?
4.8 Explain how notch sensitivity index affects the fatigue strength.
4.9 Write and explain Basqin's equation. To which part af the S-N diagram does it apply?
4.10 Draw and explain the S-N diagram.
4.11 Why is Goodman's equation preferred to Soderberg's criterion far designing a machine part subjected to fluctuating loads?

4.12 Draw Soderberg's, Goodman's and Gerber's diagrams and explain their usefulness in designing.
4.13 Show the cross section of a fractured specimen and explain how it is interpreted.
4.14 Explain Miner's cumulative fatigue damage.
4.15 Explain how Miner's equation is useful for short time testing of fatigue life of a specimen.
4.16 Explain the different methods of improving fatigue strength.
4.17 Explain the procedure for designing machine elements subjected to variable combined stresses.
4.18 Explain the procedure for designing a machine element for finite life.
4.19 Differentiate between the low cycle and high cycle fatigue.
4.20 Explain Goodman's diagram. How its modified version resembles Soderberg's diagram?
4.21 Explain the terms mean, variable and maximum value of stress.
4.22 Give different methods to reduce the effect of stress concentration.
4.23 Describe the method of designing part subjected to fatigue loading for definite life.
4.24 What are the drawbacks of Miner's approach of cumulative fatigue damage? How are they taken care of in Manson's approach?
4.25 State true or false giving justification
- (a) A smaller rough area and large smooth area indicate small overload in fatigue fracture.
- (b) Stress concentration factor should be applied to both static and dynamic parts of loading.
- (c) Gerber's diagram is not correct representation of the failure of the component under fatigue loading hence is not used for designing.
- (d) Modified Goodman's diagram is more correct approach of designing for fatigue as compared to Soderberg's diagram.
- (e) Highly polished surface is more suitable for components like springs, ball bearings.
- (f) Effect of small hole or fillet is the same for the plate subjected to tensile load.
- (g) Cold rolled parts are weaker as compared to hot rolled.
- (h) Shot peening has the same effect as the shot blasting.
- (i) Fatigue strength does not depend upon the temperature.
- (j) Usually the gear teeth are case hardened.

PRACTICE PROBLEMS

4.1 In the worked Example 4.4, find the life of the plate if the thickness of the plate is 20 mm. $S_f = 0.9S_u$ for 10^3 cycles.

4.2 Find the diameter of the hole if the plate shown in Fig. P-4.2 is subjected to a load varying between 15000 to 90000 N. For the material S_u/F · S = 280 MPa, S_e/F · S = 115 MPa. The plate is 25 mm thick. End Correction factor = 0.7.

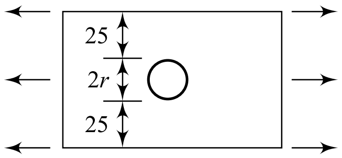


Fig. P-4.2

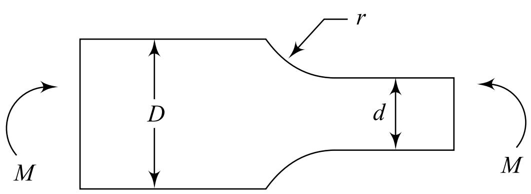


Fig. P-4.3

4.3 For a plate of 15 mm thickness shown in Fig. P-4.3, D = 230 mm, d = 120 mm, r = 10 mm, M = 2500 N.m. Find the value of maximum stress on the fillet. What is the factor of safety if the part is made of 200 FG C.I.

4.4 A cantilever of 30C8 steel is subjected to a transverse load varying from 50 N upwards to 150 N downwards. It is also subjected to an axial load varying from 150 N compression to 500 N tension. Find the dimension d for an infinite life if $S_u = 560$ MPa, $S_e = 280$ MPa, $K_a = 0.8$, $K_b = 0.85$, $K_c = 0.7$ for axial load and $K_a = 0.8$, $K_b = 0.8$, $K_c = 1$ for bendings. Reliability factor = 0.897 for both types of loading and F·S = 2.5.

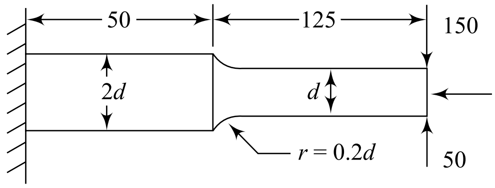


Fig. P-4.4

4.5 Load P on a simply supported shaft is 4000 N. Find the radius of the fillet if the stress to the left of the fillet is the same as that to the right.

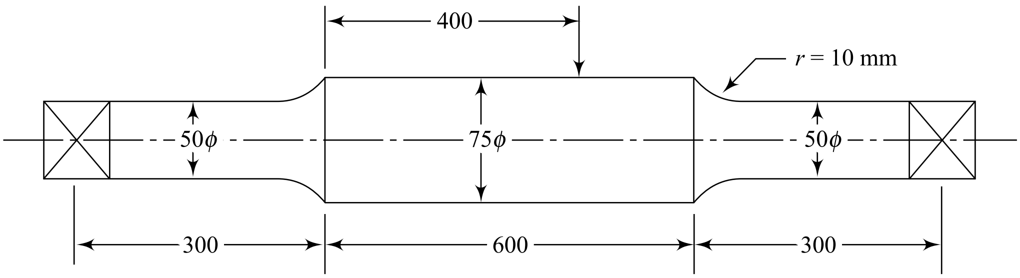


Fig. P-4.5

4.6 The material 30C8 steel has $S_u = 500$ MPa, $S_y = 300$ MPa and $S_e = 210$ MPa. Sketch the modified Goodman's diagram and thereby determine the magnitude of endurance strength for released loading in bending.
(*Hint:* Find the endurance strength for the condition $\sigma_{\min} = 0$)

4.7 A mechanical part is made of 45C4 steel with $S_u = 600$ MPa, $S_y = 480$ MPa and $S_e = 200$ MPa. Determine the factor of safety for (a) bending stress fluctuating between 40 and 100 MPa, (b) bending stress varying between 0 and 250 MPa, (c) compressive stress varying between 0 and 210 MPa.

4.8 A round wire cantilever spring shown in Fig. P-4.8 is subjected to a load $P = \pm 190$ N. The spring is made of steel with $S_u = 1200$ MPa. The surface finish factor = 0.87 and reliability factor = 0.868. Assuming zero stress concentration determine the number of stress cycles likely to cause fatigue of the spring.

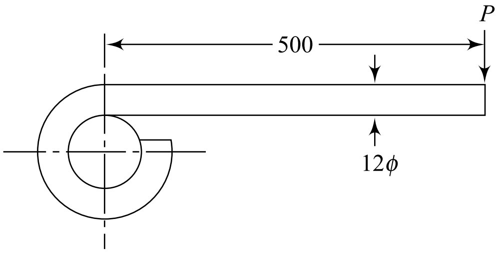


Fig. P-4.8

4.9 A load on the plate shown in Fig. P-4.9 fluctuates between –20 kN to 100 kN. The plate is made of 15C4 steel with $S_u = 440$ MPa and $S_y = 350$ MPa. Surface finish factor = 0.82. The value of q for both the hole and fillet is 0.8 and factor of safety is 1.5. Determine plate thickness for infinite life (Refer charts for K_t).

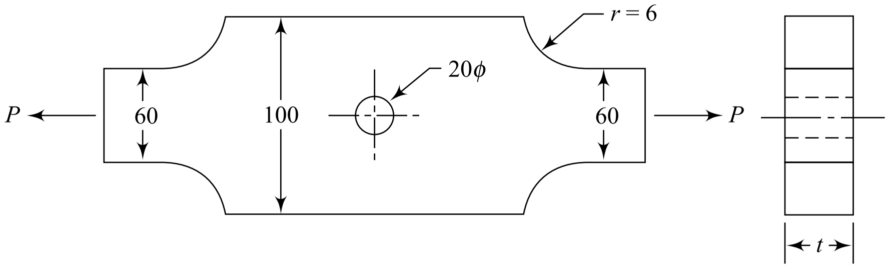


Fig. P-4.9

4.10 A bar of 50C8 steel is subjected to a tensile load P varying from 0 to maximum. The properties of material are $S_u = 1.2$ GPa, $S_y = 600$ MPa, $K_t = 1.8$, $q = 0.95$, size factor = 0.85 and reliability factor = 0.868. Find the maximum P value for 2.5×10^5 number of cycles. Diameter of bar is 20 mm.

4.11 The work cycle of mechanical component subjected to completely reversible loading consists of the following:
(a) ±350 MPa for 85% of time (b) ±400 MPa for 12% of time
(c) ±500 MPa for 3% of time
The material for the component is 50 C4 steel with $S_u = 660$ MPa, $S_e = 280$ MPa. Determine the life of the component.

4.12 A transmission shaft carries a pulley mid-way between two bearings. The b.m on the shaft varies from 200 to 600 N.m and the torque on the pulley varies from 100 to 300 Nm. For the' shaft material $S_u = 550$ MPa. The endurance strength correction factor for torque = 0.6, $K_a = 0.85$, $K_b = 0.88$ for both b.m as well as torque. S.C.F for b.m is 1.6 and for torque 1.3. Find the diameter of shaft using Von Mises Hencky theory. Use F · S = 2 and $S_y = 300$ MPa.

4.13 Calculate the life of the component subjected to $\sigma_m = 210$ MPa, $\sigma_v = 116$ MPa and S.C.F = 1.4. The material tests $S_{ut} = 630$ MPa and $S_e = 0.36\ S_{ut}$.

4.14 A shaft of 30 mm diameter made of 30C8 steel has a 7.5 mm transverse hole. It is subjected to (a) fluctuating torque between 0 and 90 N.m, (b) completely reversed torque of 40 N.m, (c) a torque varying between 15 to 85 N.m. Find the factor of safety in each case. $K_a = 0.85$ $K_b = 0.84$, $K_c = 0.6$, $S_{ys} = 0.577\ S_y$, $S_y = 300$ MPa, $K_t = 2.5$, $q = 0.95$, $S_e = 200$ MPa.

4.15 A circular rod of 30C8 steel with all the characteristic as Problem 4.14 is subjected to a b.m varying between 50 and 1000 N.m and an axial load varying between 5 and 15 kN tensile. The maximum of two loads occur simultaneously. Find the diameter of the rod using F · S equal to 3.

4.16 A beam of a circular section is subjected to a load P varying between 5 to 15 kN. It is machined from 20 C4 steel. Determine the diameter D using F · S = 2, $q = 0.95$, $S_{ut} = 560$ MPa, $S_y = 300$ MPa, $S_v = 280$ MPa.

4.17 Find the dimension D for the beam in Problem 4.16 when it is subjected to a steady load of magnitude 15 kN.

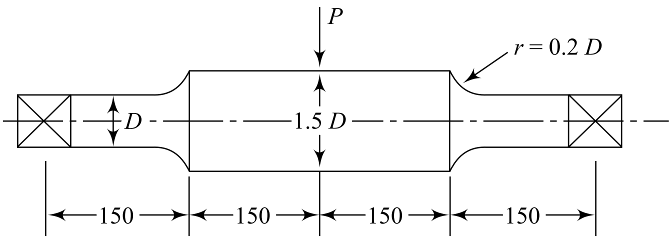


Fig. P-4.16

4.18 A rectangular plate of Problem 4.3 with $D = 60$ mm, $d = 40$ mm is subjected to an axial load of 25 kN. Find the radius 'r' of the fillet for the plate of 30C8 steel with S_y =300 MPa.

4.19 Decide the radius of the fillet for the above plate with $D/d = 1.02$ for the load fluctuating between 50 kN to 10 kN axial tensile. $S_y = 300$ MPa, $S_e = 0.45\ S_u$, $S_u = 4.40$ MPa, N = 2.5.

4.20 Find the diameter of the hole and the total width of the plate of Fig. P-4.2 if the part is to be safe for continuous operation. The load varies from 160 kN to 89 kN. $S_u = 560$ MPa, $S_e = 220$ MPa. The plate is 25 mm thick. N = 2.

4.21 A part of 40C8 steel with $S_u = 630$ MPa, $S_e = 230$ MPa is subjected to the following load cycle.
(i) $\sigma_{m1} = 300$ MPa, $\sigma_{v1} = 100$ MPa for 80% of the time
(ii) $\sigma_{m2} = 250$ MPa, $\sigma_{v2} = 135$ MPa for remaining time.
$K_a = 0.87$, $K_b = 0.97$, $K_c = 0.67$, $K_t = 1.5$. Find the expected life of the component.

4.22 A component is made of a material with $S_u = 630$ MPa, $S_e = 280$ MPa, $K_t = 1.5$ at a point of expected failure. For a short time test the service stress of 315 MPa was applied for 1500 reversals. Then a stress of 450 MPa was applied. After 1798 additional reversals the part failed. Find the expected life of the component. $K_a = 0.87$, $K_b = 0.97$, $K_c = 0.7$.

4.23 30C8 steel rod of circular c/ s is subjected to a variable b.m of 500 Nm to 1000 Nm and the axial force which varies from 5 kN to 15 kN; maximum of both loads occurring at the same instance. Determine the required diameter of the rod neglecting stress concentration and column effect.
A = Correlation factor for bending 1.00 and 0.7 for axial load
B = Size correlation of factor = 0.85
C = Correlation factor for surface effect = 0.88

ANSWERS

Objective Questions

(1) b (2) c (3) c (4) c (5) b (6) b (7) b (8) d (9) d
(10) b (11) b (12) b (13) b (14) c (15) c (16) b (17) b (18) a
(19) a (20) a (21) b (22) b

Practice Problems

(1) 239427 cycles (2) 30 mm (3) 138.9 MPa, 1.43
(4) 15 mm (5) 2.5 mm (6) 296 MPa
(7) 3.75, 1.2, 1.428 (8) 1.95×10^5 (9) 9.79 mm modified to 10 mm
(10) 211.88 kN (11) 62666 cycles (12) 46.35 modified to 50 mm
(13) 549541 cycles (14) (a) 4.99 (b) 12.87 (c) 5.95.
(15) 60 mm (16) 50.91 modified to 55 mm.
(17) 37.83 mm modified to 40 mm (18) 2.6 mm
(19) 3.2 mm (20) 12.5 mm (21) 20124.189 cycles
(22) 4676.6784 cycles (23) 60 mm

5

Cotter and Knuckle Joint

CONCEPT REVIEW

5.1 INTRODUCTION

Two types of joints, as described below, are used to connect the rods subjected to an axial load.

5.2 COTTER JOINT

The cotter joint is used when no relative motion between the rods is desirable, e.g. between a piston rod and the cross head, the strap end of a connecting rod, and a cotter foundation bolt. Different types of cotter joints are:

(1) Spigot and socket type (2) Sleeve and double cotter type (3) Gib and cotter type.

5.3 DESIGN PROCEDURE

The design procedure follows the basic direct stress equations of Chapter 2. The materials used are 30C8 steel and C-I. Based on the yield strength a factor of safety of 3 to 4 for steel gives σ_t = 100 MPa = σ_c, τ = 50 MPa while based on the ultimate strength and using a factor of safety of 7, σ_t = 20 MPa, σ_c = 100 MPa, τ = 10 MPa, for C.I.

In a cotter joint, the taper provided on the cotter should not be too large causing the removal of cotter due to applied force. Also, it should not be too small such that a small wear makes it useless. The most suitable proportion of a taper to be provided on the cotter is within 1 in 20 to 1 in 30.

5.4 KNUCKLE JOINT OR PIN JOINT

This method is used when the two connected rods permit an angular motion between them, e.g. a crank and the connecting rod or a piston and the connecting rod in an I-C engine.

WORKED EXAMPLES

5.1 Design a sleeve and double cotter joint for carrying an axial load of 10 kN.

Solution:

In Fig. E-5.1 (a) the three components designated (1), (2) and (3) are the rod, the sleeve and the cotter respectively.

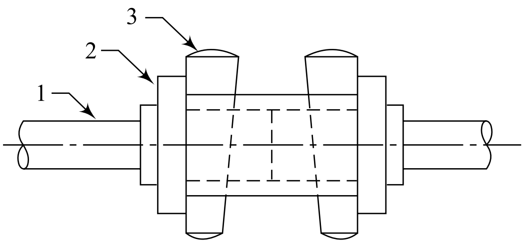


Fig. E-5.1a

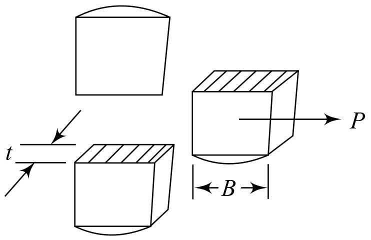


Fig. E-5.1b

Assuming the same material 30 C8 being used for all the components, we write the equations of failure for these components. Let $\sigma_t = 90$ MPa, $\sigma_c = 100$ MPa and $\tau = 50$ MPa.

The double shear failure of cotter is represented by hatched areas in Fig. E-5.1b. Taking the ratio of $B : t$ where B is the mean width and t the thickness, as 3 : 1.

$$\tau = \frac{P}{2A} = \text{safe stress} = 60 \text{ MPa} \tag{5.1}$$

$\therefore$ $$2A = 2Bt = 6t^2 = \frac{10000}{50}$$

$\therefore$ $$t = \sqrt{\frac{10,000}{50 \times 6}} \;\; 5.77 \text{ mm rounded to 6 mm}$$

and $$B = 18 \text{ mm modified to 20 mm}$$

The portion of cotter covered by the rod of diameter d_1 tends to get crushed under the action of load. As shown in Fig. E-5.1c the resisting area is $d_1 t$. The rod diameter is increased to compensate for the reduction in area due to the slot for cotter.

$\therefore$ $$\sigma_c = \frac{P}{d_1 t} = 100, \quad \therefore d_1 t = \frac{10000}{100} = 100 \text{ mm}^2 \tag{5.2}$$

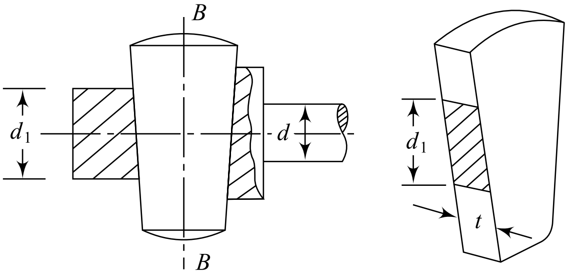


Fig. E-5.1c

Failure of rod:

The rod of diameter *'d'* will fail due to direct tension for which

$\therefore$ $$\sigma_t = \frac{P}{A} = 90, \quad \therefore A = \frac{1000}{90} = \frac{\pi}{4} d^2 \tag{5.3}$$

$\therefore$ $$d = 11.89 \text{ mm modified to the standard 15 mm diameter.}$$

Failure of portion of the rod receiving the cotter:
Consider section *BB* of rod where the slot for the cotter is made Fig. E-5.l(d) for which

$$\sigma_t = \frac{P}{A} = 90 \quad \text{or} \quad \frac{10000}{\left(\frac{\pi}{4} d_1^2 - d_1 t\right)} = 90 \qquad (5.4)$$

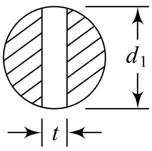


Fig. E-5.1d

Substituting $d_1 t = 100$ from Eq. (5.2)

$d_1 = 16.7$ mm modified to 20 mm.

If the cotter is stronger than the rod, application of an external force will cause double shearing of the rod as shown in Fig. E-5.l(e)

$$\therefore \quad \tau = \frac{P}{2A} = \frac{10000}{2 l d_1} = 50 \text{ MPa} \qquad (5.5)$$

$$\therefore \quad l = \frac{10000}{2 \times 50 \times 20} = 5 \text{ mm modified to 10 mm.}$$

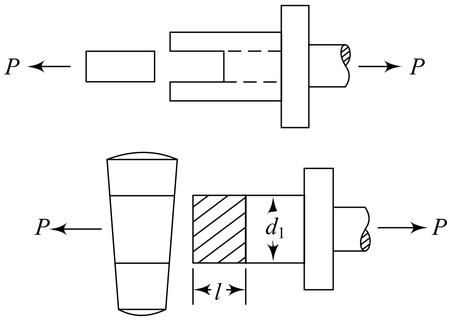


Fig. E-5.1e

The sleeve may be made out of C.I or 30 C8 steel. Let us design using both the materials and see which is economical.

Considering the tearing failure at section *CC*, Fig. E-5.1(f) and (g)

$$\text{Using C.I} \qquad \sigma_1 = \frac{P}{A} = \frac{10000}{\left\{\frac{\pi}{4}(d_2^2 - d_1^2) - (d_2 - d_1)\, t\right\}} = 20 \qquad (5.6)$$

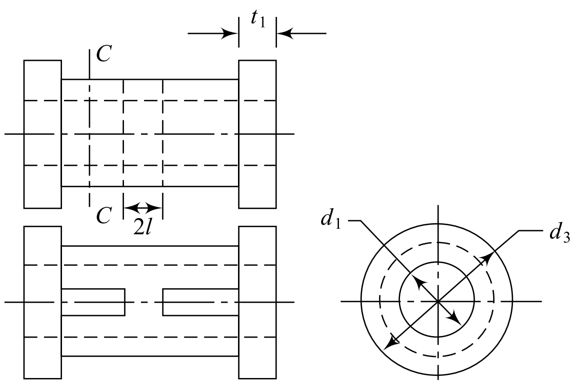


Fig. E-5.1f

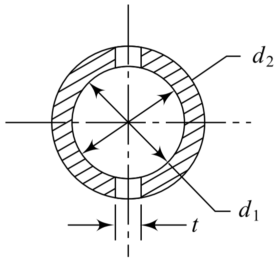


Fig. E-5.1g

Substituting $d_1 = 20$ mm and $t = 6$ mm we get $d_2 = 34$ mm modified to 35 mm. Crushing of cotter against the sleeve:

$$\sigma_c = \frac{P}{A} = \frac{10000}{(d_3 - d_1)t} = 100, \quad \therefore d_3 = 36.67 \text{ mm modified to 40 mm}$$

Shearing of sleeve collar:
Collar is provided to improve the crushing strength.

$$\tau = \frac{10000}{2 \times d_3 t_1} = 10, \quad \therefore \; t_1 = \frac{10000}{2 \times 10 \times 40} = 12.5 \text{ mm}$$

Total length of the sleeve = $2\,t_1 + 2l + 2\,B = 25 + 20 + 36 = 81$ mm.

Detailed dimensions:

Cotter

$B = 18$ mm and $t = 6$ mm

Height = $d_3 + 5 = 40 + 5 = 45$ mm

Rod: $d = 15$ mm, $d_1 = 20$ mm and $l = 10$ mm

Sleeve: $d_1 = 20$ mm, $d_2 = 35$ mm, $d_3 = 40$ mm and $t_1 = 12.5$ mm

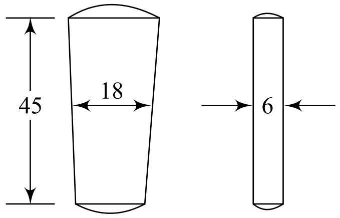


Fig. E-5.1h

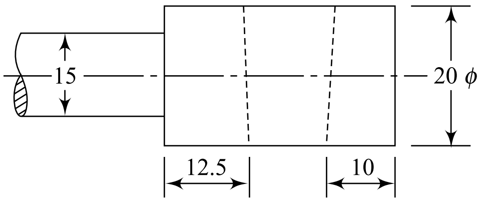


Fig. E-5.1i

5.2 Design a spigot and socket joint to connect two rods of 30 C8 steel to carry an axial tensile and compressive load of 10 kN.

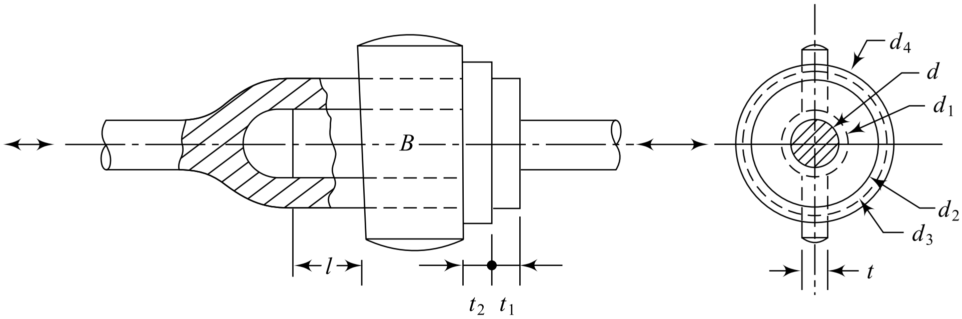


Fig. E-5.2a

Solution:

The load and the materials are the same as in Example 5.1. Hence, the dimensions B, t, d_1 and l remain the same.

Spigot end:

From Example 5.1, $d = 15$ mm, $d_1 = 20$ mm and $l = 10$ mm

A collar is provided on the spigot to avoid damage to the socket due to failure of the cotter under compressive load and the spigot hitting the inner portion of the socket. In case the collar takes the entire compressive load.

Crushing of collar:

$$\sigma_c = \frac{10000}{\frac{\pi}{4}\left(d_2^2 - d_1^2\right)} = 100, \therefore\ d_2 = 22.96 \text{ modified to 25 mm.} \qquad (5.7)$$

Shearing failure of the collar under compressive load

$$\tau = \frac{P}{\pi d_1 t_1} = 50$$

$$\therefore \qquad t_1 = \frac{10000}{\pi \times 20 \times 50} = 3.183 \text{ mm modified to } t_1 = 5 \text{ mm.}$$

Socket end: Using Eq. (5.6) from Example 5.1 and taking $\sigma_t = 90$ MPa

$$d = 15 \text{ mm},\ d_1 = 20 \text{ mm}, \qquad d_2 = 25 \text{ mm}$$

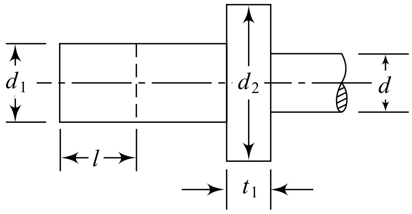


Fig. E-5.2b

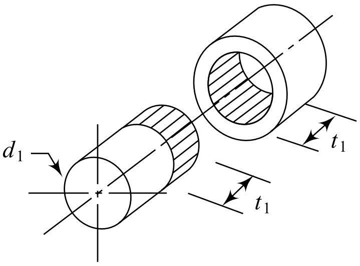


Fig. E-5.2c

The thickness t_2 and the diameter d_4 of the socket collar may be found by considering the crushing and shearing respectively.

$$\therefore \quad \sigma_c = \frac{10000}{(d_4 - d_1)\, t} = 100 \text{ MPa} \qquad (5.8)$$

$d_4 = 36.7$ mm modified to 40 mm.

$$\tau = \frac{10000}{2(d_4 - d_1)\, t_2} = 50 \text{ MPa} \qquad (5.9)$$

$t_2 = 4$ mm modified to 5 mm.

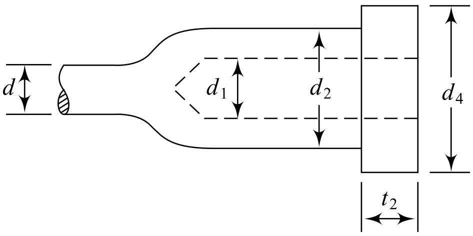


Fig. E-5.2d

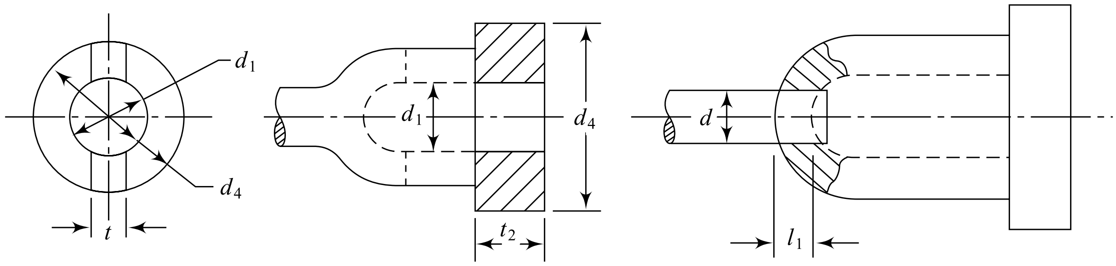


Fig. E-5.2e, f and g

Under a compressive load the socket may get sheared by the rod as shown in Fig. E-5.2 (g)

$$\therefore \qquad \tau = \frac{10000}{\pi d l_1} = 50 \text{ MPa} \qquad (5.10)$$

$$\therefore \qquad l_1 = 4.24 \text{ mm rounded to 5 mm}$$

5.3 A foundation bolt with a circular rod is secured by means of a cotter. Find the dimensions of the rod using the values of permissible stresses $\sigma_t = 90$ MPa, $\sigma_c = 100$ MPa and $\tau = 50$ MPa. The axial load on the rod is 10 kN.

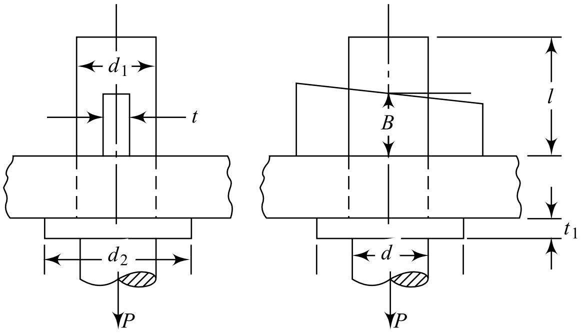


Fig. E-5.3

Solution:

The values of the load and the permissible stresses are the same as in Examples 5.1 and 5.2. Applying Eqs (5.1), (5.2), (5.3), (5.4) and (5.5) of Example 5.1 to calculate B_1, d_1, t, d_1 and l respectively, we get, B = 18 mm, t = 6 mm, d = 15 mm, d_1 = 20 mm and l = 10 mm. The dimensions d_2 and t_1, can be found by using Eqs (5.1) and (5.2), of Example 5.2. Hence d_2 = 25 mm and t_1 = 5 mm.

5.4 Design a knuckle joint to connect two tension rods subjected to an axial load of 15 kN. Use 30C8 steel material for all the components.

The four components of a joint are (i) double eye end, (ii) single eye end, (iii) pin and (iv) split pin.

Pin design:

(a) Double shearing: As shown in Fig. E-5.4 (b) the pin may fail under double shear. For 30C8 steel S_y = 330 MPa, $\therefore$ S_{ys} = 165 MPa.

A factor of safety of 5 is adapted as it will be observed further that the diameter of the pin calculated with a small factor of safety for double shearing does not withstand the bending.

$$\therefore \qquad \tau = \frac{P}{2A} = \frac{S_{ys}}{5}$$

$$\text{or} \qquad \frac{15000}{2 \times \frac{\pi}{4} d^2} = \frac{165}{5}, \quad \therefore\ d_p = 17.65 \text{ modified to 20 mm}$$

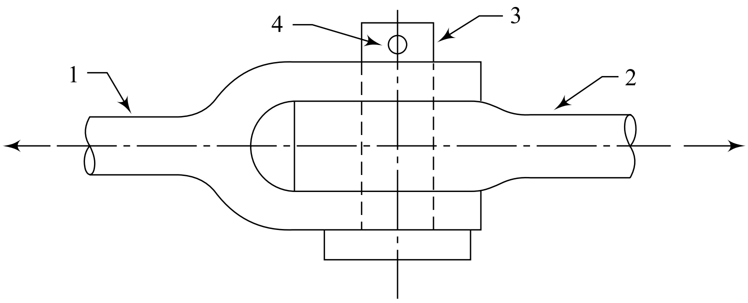


Fig. E-5.4a

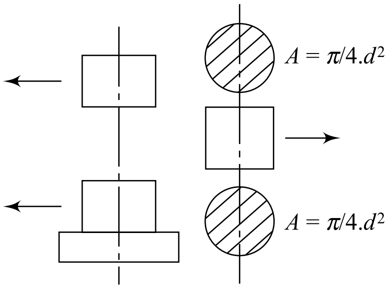


Fig. E-5.4b

(b) Bearing of pin: Using Eq. 2.6 from Chapter 2 and taking P_b = 20 MPa we get,

$$20 = \frac{15000}{20 \times t}, \quad \therefore t = 37.5 \text{ mm modified to 40 mm.}$$

(c) Bending check: Here the induced bending stress using shearing and bearing is calculated and checked if it is within the limit. The pin may be considered as a simply supported beam with the load and reactions, both uniformly distributed. Hence, the maximum b.m at the centre is taken as the mean of the b.m due to (a) concentrated load and (b) uniformly distributed load, i.e. $M = \frac{Pl}{6}$ where l is the length of the pin between the points A and B which may be taken as 1.5 t = 60 mm.

$$\therefore \quad \frac{15000 \times 60}{6} = \sigma_t Z = \sigma_t \frac{\pi}{32} d_p^3 = \frac{\pi}{32} \cdot (20)^3 \sigma_t$$

$\therefore$ σ_t = 190.98 MPa. This value being high, let us increase the value of d_p to 30 mm so that the induced stress becomes

$$\sigma_t = \frac{15000}{6 \times \pi (30)^2} = 56.58 \text{ MPa}$$

which is well within the limit. Hence, d_p = 30 mm. Also, an increase in d_p reduces the value of t as the projected area is $d_p \cdot t$. This in turn reduces the bending moment and the height of the single eye as well as the double eye thereby reducing the cost of material.

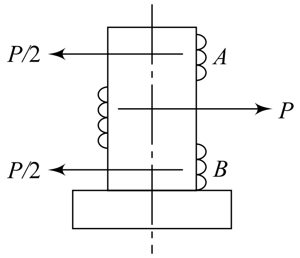


Fig. E-5.4c

Design of rod with single eye:

At the section AA

$$\sigma_t = \frac{P}{\frac{\pi}{4} d^2} = \frac{S_y}{\text{F·S}}$$

$$\therefore \quad \frac{15000 \times 4}{\pi d^2} = \frac{330}{3}$$

$\therefore$ d = 13.17 mm modified to 20 mm.

The thickness t of the eye is the same as that found for a pin under bearing. As the diameter of the pin is modified from 20 mm to 30 mm, the thickness may be changed from 40 mm to $40 \times \frac{20}{30}$ = 26.67 mm modified to 30 mm.

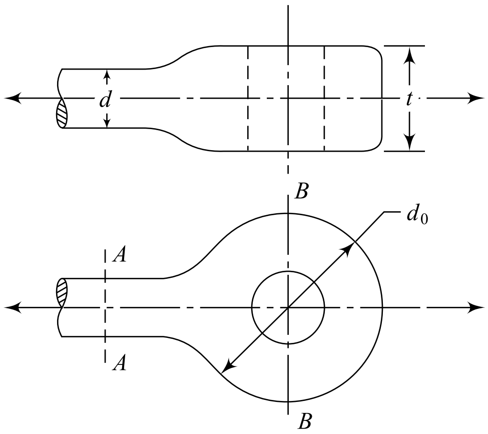


Fig. E-5.4d

$\therefore$ For tearing failure of the rod at section BB

$$\sigma_t = \frac{P}{(d_0 - d_p)} = \frac{330}{3}$$

$$\therefore \quad \frac{15000}{(d_o - 30)30} = 110 \text{ MPa}$$

$\therefore$ d_o = 34.54 modified to 40 mm.

Shearing of rod due to pin as shown in Fig. E-5.4(e) yields

$$\tau = \frac{15000}{2(d_o - d_p)\,t} = \frac{15000}{2 \times 10 \times 30} \text{ 25 MPa which is within the limit.}$$

Design of rod with double eye:
This part will have the same values of d_o, d_p and d but the thickness t is equally divided in the two portions.

The dimensions of the pin head and the total height of the pin are as per the functional requirement.

In general, the pin head diameter is equal to 1.5 to 2 d_p and height of the pin head is taken as 0.6 d_p.

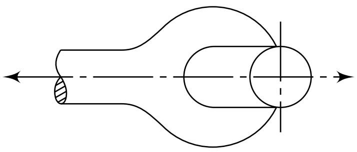


Fig. E-5.4e

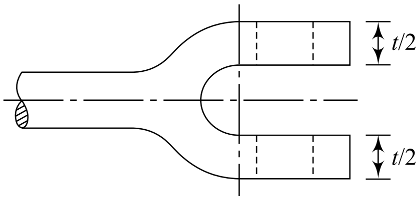


Fig. E-5.4f

OBJECTIVE QUESTIONS

5.1 The most suitable taper to be provided on the cotter is
(a) 1/10 (b) 1/24 (c) 1/100 (d) 1/4

5.2 Cost of the joint can be reduced by using a double cotter and sleeve due to
(a) use of a simple manufacturing method (b) use of C.I
(c) use of 30C8 steel (d) none of the above

5.3 A knuckle joint is preferred to a cotter joint
(a) where the cost is the criteria
(b) where relative motion between the rods is desirable
(c) where the rods have different diameters
(d) for rods of different materials

5.4 A gib and cotter joint is used for
(a) the strap end of a connecting rod
(b) the small end of a steam engine connecting rod
(c) the crank and the connecting rod of an I.C. engine
(d) none of the above

5.5 Links of a four bar chain are connected by
(a) sleeve and cotter joint (b) pin joint
(c) knuckle threads (d) spigot and socket joint

5.6 A piston rod and a cross head are connected by
(a) universal joint (b) Oldhams coupling (c) muff coupling (d) cotter joint

5.7 The width to thickness ratio of cotter should be
(a) 50 : 1 (b) 1 : 1 (c) 3.5 : 1 (d) none of these

5.8 In a steam engine a knuckle joint is used to connect
(a) a cross head and a piston rod
(b) a valve rod and an eccentric
(c) the strap end of a connecting rod and piston rod
(d) none of the above.

5.9 A cotter joint is capable of transmitting
(a) the twisting moment
(b) an axial tensile as well as compressive load
(c) the bending moment
(d) only axial compressive load

5.10 The outside diameter of single eye of a knuckle joint with the pin diameter d_p should be
(a) 1.5 d_p
(b) 3 d_p
(c) d_p
(d) none of the above

5.11 A collar is provided on the spigot end to
(a) increase the strength of the rod
(b) prevent the damage to the socket
(c) prevent the damage to the cotter
(d) none of the above.

REVIEW QUESTIONS

5.1 Differentiate between a cotter and a knuckle joint.
5.2 Explain the different types and applications of a cotter joint.
5.3 Explain the necessity of providing taper on cotter. Why the taper should not be too large or too small?
5.4 What should be the magnitude of a correct taper?
5.5 Explain whether it is advisable to use taper on one or both sides of a cotter?
5.6 Where should the gib and cotter type joint be used?
5.7 Show the methods of preventing the cotter from slackening.
5.8 Why are the edges of a cotter rounded?
5.9 Draw a neat sketch of a cotter joint and write the equations of failure for the different sections.
5.10 Why is it advisable to take the diameter of a pin more than the calculated value?
5.11 Make a neat sketch of an assembly of knuckle joint and write the various equations of failure.
5.12 Compare the method of joining rods using the sleeve and cotter with the spigot and socket joint method.
5.13 How a sleeve and cotter joint can be cheaper as compared to spigot and socket joint?
5.14 What is the limitation on *B/t* ratio of cotter?
5.15 Why is it preferable to use. $d_p/1 = 1$ for pin of the knuckle joint?
5.16 How permissible bearing pressure p_b is decided?
5.17 Cotter may not be designed for bending. Why ?
5.18 A small factor of safety is permissible for cotter. Is it true? Why?
5.19 Why is a collar provided on spigot and socket in spigot and socker joint?
5.20 Why collars are provided on both sides of sleeve.

PRACTICE PROBLEMS

5.1 Design a cotter joint to connect two rods of 25 mm diameter of 30C8 steel considering the permissible values of stresses as $\sigma_t = 70$ MPa, $\sigma_c = 80$ MPa and $\tau = 40$ MPa.

5.2 Design a sleeve and double cotter joint to be designed for connecting two steel rods. An axial tensile load of 30000 N acts on the rods. Permissible stresses σ_t = 55 MPa, σ_c = 70 MPa and τ = 40 MPa.

5.3 Design a cotter foundation bolt to withstand a load of 36 kN. The permissible stresses are σ_t = 80 MPa, σ_c = 100 MPa and τ = 60 MPa.

5.4 Design a gib and cotter joint to connect square rods with a side of the square as 25 mm. Consider σ_t = 60 MPa, σ_r = 90 MPa and τ = 40 MPa. The joint has to carry a load of 35 kN.

5.5 Design a knuckle joint to connect two tension rods to carry a load of 25 kN. Consider σ_t = 65 MPa, σ_c = 80 MPa and τ = 50 MPa.

5.6 Design a pin joint to connect a suspension link subjected to a load of 30 kN. Thickness of each side of the link is 15 mm. Consider σ_t = 120 MPa and τ = 80 MPa.

5.7 A m.s tie rod is connected to the bracket by a knuckle pin. Find the diameter of the rod, the eye and the pin. Consider σ_t = 70 MPa, σ_c = 100 MPa and τ = 40 MPa.

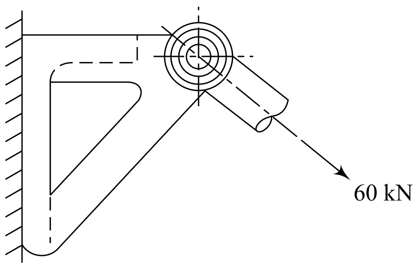


Fig. P-5.7

5.8 Design a cotter foundation bolt used for fixing a machine on its foundation. Force induced in the bolt due to tightening to take care of vibrations is 12 kN per bolt. Use 30C8 steel for bolt, cotter, nut. Base plate of machine is made of C.I. Refer to Fig. E-5.3. (Using σ_t = 80 MPa, τ = 40 MPa, σ_C = 100 MPa, bolt of m16 × 1.5 threads.)

5.9 Design a knuckle joint to connect two 30C8 steel rods of 50 mm diameter use the safe stresses under tension, shear of 75 MPa and 40 MPa and p_b = 30 MPa.

5.10 Two lengths of tubes of 30C8 steel are connected by a sleeve and two cotter pins of the same material. The external diameter of tubes is 50 mm. Find the thickness of tubes and sleeve and dimensions of cotter pin with safe stresses σ_t = 80 MPa, τ = 40 MPa, σ_c = 100 MPa. Axial load P = 40 kN.

ANSWERS

Objective Questions

(1) b (2) a (3) b (4) a (5) b (6) d (7) c (8) d (9) b
(10) a (11) b

Practice Problems

(2) d = 22 mm, d_1 = 32 mm, B = 30 mm, t = 14 mm, d_3 = 44 mm, d_2 = 40 mm.
(3) d = 25 mm, d_1 = 30 mm, t = 8 mm, B = 45 mm.
(4) t = 8 mm, B = 50(30 + 25) mm, length of tail of the rod is 12 mm.
(6) Width of the link is 90 mm, d_p = 50 mm.
(8) t = 7.07 $\simeq$ 10 mm, B = 25 mm, d_1 = 19.86 $\simeq$ 20 mm, l = 7.5 $\simeq$ 10 mm, d_2 = 23.51 $\simeq$ 25 mm, t_1 = 4.77 $\simeq$ 5 mm.
(9) d_p = 70 mm, d_0 = 112 mm.
(10) Thickness of tubes is 5 mm, sleeve thickness is 5 mm, max. diameter of cotter pin = 28 mm, min. diameter = 25 mm, height of the pin = 75 mm.

6

Design of Levers

CONCEPT REVIEW

6.1 INTRODUCTION

A rigid rod capable of turning about a fixed point and doing some useful work after the application of an effort is termed as a lever. The fixed point about which a lever turns is the *fulcrum*. Levers may be straight or bent. A straight tommy bar used to operate a screw jack, the lever of a lever loaded safety valve, a bell crank lever and a rocker arm are the different types of levers.

6.2 DESIGN MATERIALS AND PROCEDURES

A lever may be forged or cast and accordingly forged steel or cast steel or C.I may be used. It is always subjected to a bending moment and hence is designed for the bending failure while the fulcrum pin and the support are designed as the pin for a knuckle joint.

WORKED EXAMPLES

6.1 A tommy bar is used to operate upon a screw jack for which, the frictional torque to be overcome is 50 N.m. Find the length and the diameter of the rod made of 30C8 steel.

Solution:

A frictional torque of 50×1000 N.mm may be overcome by applying a manual force of 250 N at the end of a lever of length *l*.

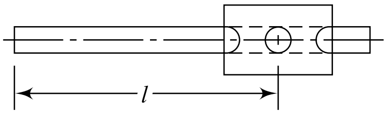


Fig. E-6.1

$$\therefore \qquad l = \frac{50000}{250} = 200 \text{ mm.}$$

For 30C8 steel, $S_y = 330$ MPa. Therefore, when the factor of safety is 4, $\sigma_t = 80$ MPa.

Maximum b.m at the fixed end of the lever = 250 × 200.

As $\quad$ b.m = $\sigma_t Z$, $\quad \therefore \quad Z = \dfrac{250 \times 200}{80} = \dfrac{\pi}{32} d^3$

$\therefore \quad d = 18.53$ mm.

Let us adapt the diameter of the tommy bar as 20 mm.

6.2 Design a bell crank lever to carry a load of 1000 N as shown in Fig. E-6.2. Use C.I for the lever, with $\sigma_t = 20$ MPa, $\sigma_c = 100$ MPa, $t = 10$ MPa and 30C8 steel for the fulcrum pin with $\sigma_t = 90$ MPa, $\sigma_c = 100$ MPa, $\tau = 60$ MPa and $P_b = 20$ MPa.

Solution:

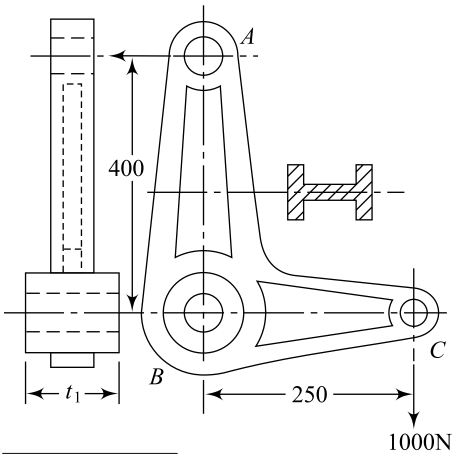


Fig. E-6.2a

$$\text{Force acting at } A = \frac{1000 \times 250}{400} = 625 \text{ N}$$

$$\text{Reaction at the fulcrum} = \sqrt{(1000)^2 + (625)^2}$$

$$= 1179.247 \text{ N taken as } 1180 \text{ N}$$

The fulcrum pin is designed just like a knuckle pin

(a) Bearing of the pin:

$$1180 = P_b \cdot dl$$

$$= 20 \times 1.5\, d^2, \text{ assuming } l = 1.5d$$

$\therefore \quad d = 6.25$ mm modified to 10 mm.

Hence $\quad l = 15$ mm.

The dimension l depends on the thickness t_1 of the lever. Hence as explained below, the values calculated with the bearing consideration may require modifications both in l as well as d.

(b) For preliminary calculations let the boss diameter be $2d = 20$ mm. Thus, the lengths of the lever arms up to the sections where the b.m is maximum will be (400 – 10) mm and (250 – 10) mm.

$\therefore \quad M_{max} = 625 \times 390 = 243750$ N.mm.

Section of the lever may be taken as rectangular or an I section. Let us take a rectangular section with a height $h = 3t$ where t is the thickness of the section.

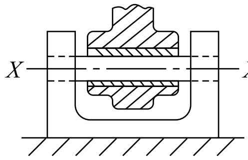


Fig. E-6.2b

$$\therefore \quad 243750 = \sigma_t \frac{(h^2 t)}{6} = 20 \times 1.5 t^3$$

$\therefore \quad t = 20.1$ mm modified to 21 mm and $h = 60$ mm.

Therefore, $t_1 = 21 + 2 \times 2 = 25$ mm, taking the projection of the bar as 2 mm on both sides. Thus, the length of the pin inside the lever = 25 mm. The pin is fixed on its supports on both sides of the lever while the lever turns on it. Hence, a bush may be provided on the pin so that in case of wear the bush may be replaced easily without much cost. The material of the bush is either gunmetal or bronze. Let us now check the diameter of the pin for double shear and bending, and modify if necessary.

Double shearing of the pin:

$$\tau = \frac{P}{2A} = \frac{1180}{2 \times \frac{\pi}{4}(10)^2} = 7.51 \text{ MPa which is within the limit.}$$

Bending of the pin:
Unsupported length of the pin may be taken as 25 + 10 + 2 = 37 mm where 1 mm clearance on both the sides is kept and the thickness of the supporting brackets is 10 mm each.

Maximum b.m on the pin = $\dfrac{PL}{6}$

$$\therefore \qquad \frac{1180 \times 37}{6} = \sigma_t \times \frac{\pi}{32}(10)^3, \quad \therefore \quad \sigma_t = 74.122 \text{ MPa}$$

which is within the limit.

∴ The diameter of the pin and the bush is 10 mm and 12 mm respectively, assuming the thickness of bush equal to 1 mm.

6.3 In the above example use forged steel for lever and consider the section of the lever as shown in Fig. E-6.3.

Solution:

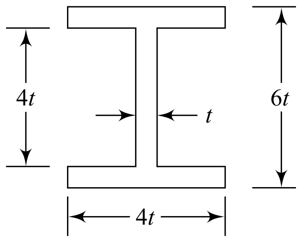


Fig. E-6.3

For an I section, $\quad Z = \dfrac{I_{xx}}{y}$

$$I_{xx} = \frac{1}{2} \times 4\,t \times (6\,t)^3 - \frac{1}{12} \times 3 \times (4\,t)^3 = 56\,t^4$$

$$\therefore \qquad Z = \frac{56t^4}{3t} = 18.7\,t^3$$

Now, $\quad M = 243750 = \sigma_t Z$ and $\sigma_t = 90$ MPa

$$\therefore \qquad Z = \frac{243750}{90} = 18.7\,t^3. \text{ Hence, } t = 5.25 \text{ mm modified to 6 mm.}$$

Therefore, the width of the section = 24 mm and the depth = 36 mm.

6.4 The lever of a lever loaded safety valve is shown in Fig. E-6.4 (a). The force P is due to a steam pressure of 1.2 MPa acting on the valve of diameter 60 mm. The material used is forged steel with σ_t = 80 MPa, σ_c = 100 MPa and τ = 50 MPa.

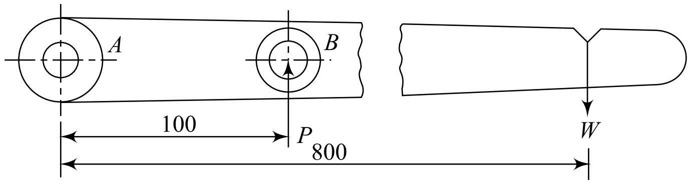


Fig. E-6.4a

Solution:

The force acting at point B

$$P = \frac{\pi}{4} \times 60^2 \times 1.2 = 3392.92 \text{ N in the vertically upward direction.}$$

∴ By taking the moments about A, the force acting at C is

$$W = \frac{3392.92 \times 100}{800} = 424.11 \text{ N vertically down}$$

∴ Force at A = 2968.81 N vertically downwards.

As shown in Fig. E-6.4 (b), the b.m. varies at points A, B and C with the maximum b.m. acting at B.

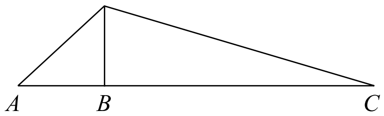


Fig. E-6.4b

Since the forces at A and B are almost the same, we design the pin at B and adapt the same dimensions for the pin at A. The pin at B is subjected to only double shear and bending while the pin at A is subjected to shear, bearing as well as bending.

Bearing of the pin:

Let $$P_b = 20 \text{ MPa} = \frac{3392.92}{d_p \times l} \quad \text{and} \quad \text{let } d_p = l.$$

$$\therefore \quad d_p = \sqrt{\frac{3392.92}{20}} = 13.02 \text{ mm modified to 15 mm. and } l = 15 \text{ mm.}$$

Checking the pin for shearing:

$$\text{Induced shear stress } \tau = \frac{3392.92}{2 \times \frac{\pi}{4} \times 15^2} = 9.6 \text{ MPa which is within the limit.}$$

The check for bending may be done after calculating the thickness of the lever.

Lever: The lever has a hole of 15 mm diameter for receiving the pin and if a bush is used, the hole diameter may be increased by twice the thickness of the bush. Let the thickness of the bush be 1.5 mm. Hence, the diameter of the hole becomes 18 mm. Let us first decide the dimensions of the solid section AA with the ratio $h : t$ equal to 3 : 1.

$$\text{b.m at } AA = 425 \times (700 - 9) = 293675 \text{ N.mm}, \ \sigma_t = 1.5\, t^3 = 80 \times 1.5\, t^3$$

$$\therefore \quad t = 13.47 \text{ mm} = 14 \text{ mm and } h = 42 \text{ mm.}$$

The section with a hole for the pin is shown in Fig. E-6.4(c). The thickness of the boss is 14 + 2 = 16 mm. The diameter of the boss is usually double the diameter of the hole. Hence, the section modulus of the section at the hole is

$$= \frac{\left[\frac{1}{12} \times 16 \times (36)^3 - \frac{1}{12} \times 16 \times (18)^3\right] + \left[\frac{1}{12} \times 14(42)^3 - \frac{1}{12} \times 14(36)^3\right]}{21}$$

$$= 4704 \text{ mm}^3$$

$$\therefore \quad \sigma_t = \frac{425 \times 700}{4704} = 63.244 \text{ MPa which is within the limit.}$$

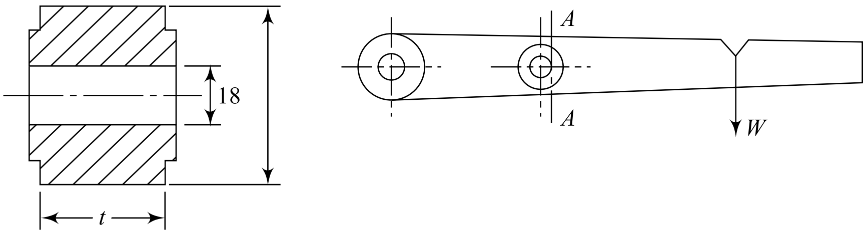


Fig. E-6.4c

6.5 Design a band brake lever considering $T_1 = 3000$ N and $T_2 = 1000$ N. Use 30C8 steel with $\sigma_t = 80$ MPa, $\tau = 50$ MPa and $P_b = 20$ MPa [(Fig. E-6.5(a)].

Solution:

The method of solving this problem is similar to that of the bell crank lever. Taking the moments about the fulcrum.

$$T_1 \times 100 - T_2 \times 20 - P \times 450 = 0$$

$$\therefore \quad P = \frac{3000 \times 100 - 1000 \times 20}{450} = 622.22 \text{ N}$$

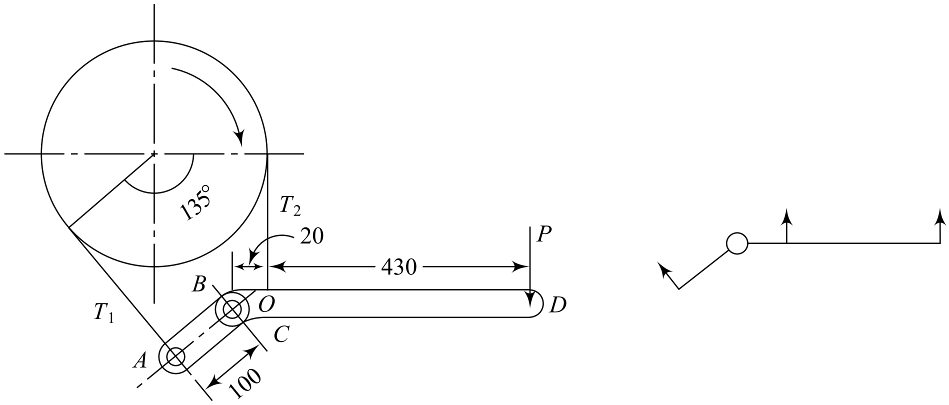


Fig. E-6.5a

Reaction at the fulcrum = $\sqrt{(1000 + 622.22 + 3000 \cos 45°)^2 + (3000 \sin 45°)^2}$ = 4302 N.

Bearing of pin: Assuming $1 = d$ $4302 = 20\, d_p^{\,2}$, $\therefore$ $d_p = 14.66 \cong 15$ mm

Maximum b.m at $AA = 3000 \times (100 - 7.5) = 277500$ N.mm

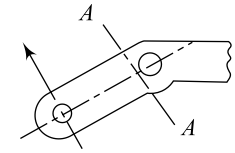


Fig. E-6.5b

Taking the rectangular section with the ratio $h : t = 3 : 1$

we get, $277500 = \sigma_t th^2/6 = \sigma_t \times 1.5\, t^3$

$$\therefore \quad t = \sqrt{\frac{277500}{80 \times 1.5}} = 13.22 \text{ mm modified to 15 mm}$$

$$\therefore \quad h = 45 \text{ mm.}$$

Using a bush of thickness 1 mm, the hole for the pin has a diameter of 17 mm.

$$\therefore \text{ The section at the hole has } Z = \frac{\frac{1}{12} \times 15 \times (45)^3 - \frac{1}{12} \times 15 \times (17)^3}{22.5}$$

$$= 4789.56 \text{ mm}^3$$

$$\therefore \quad \text{The induced value of } \sigma_t = \frac{3000 \times 100}{4789.56} = 62.63 \text{ MPa}$$

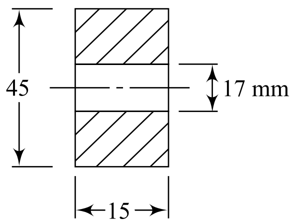


Fig. E-6.5c

which is within the limit.

The actual value of Z is slightly greater than the calculated value since the thickness at the base is increased and hence, the section is more safe.

The dimensions of the pin at A may be kept the same as the dimensions of the fulcrum pin. Due to shear, the failure of the pin at B induces.

$$\tau = \frac{4300}{2 \times \frac{\pi}{4}(15)^2} = 12.16 \text{ MPa which is a safe value.}$$

6.6 Design a foot lever shown in Fig. E-6.6. For the lever, a key and a shaft permissible value of σ_t = 80 MPa, τ = 40 MPa.

Solution:

Twisting moment acting on the shaft = 1000×800 N.mm = $\frac{\pi}{16} d^3 \times 40$

$\therefore \quad d = 46.7 \text{ mm} = 50 \text{ mm}.$

Shearing of key: Tangential force on the key

$$= \frac{1000 \times 800}{50} = 16000 \text{ N}$$

$\therefore 16{,}000 = bl \times \tau$, where b is the width and l is the length of key. As $b = d/4 = 12.5$ mm

$$\therefore \quad l = \frac{16000}{12.5 \times 40} = 32 \text{ mm}$$

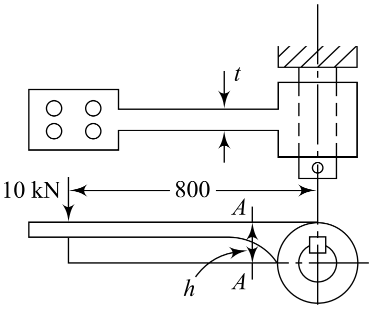


Fig. E-6.6

Let $\sigma_c = 100$ MPa

$\therefore$ Crushing of the key gives the following equation for its thickness

$$\sigma_c \times l \times \frac{t}{2} = 16000$$

$$\therefore \quad t = \frac{16000 \times 2}{100 \times 32} \; 10 \text{ mm}.$$

Overhang of the shaft = length of the key + clearance. Keeping the value of clearance as small as possible say, 1 mm, the overhanging length = 33 mm.

$\therefore$ b.m on the shaft = $1000 \times 33 = 33{,}000$ N.mm

Torque = 800,000 N.mm

$$\therefore \quad T_e = \sqrt{M^2 + T^2} = \sqrt{800{,}000 + (33{,}000)^2} = 800{,}680 \text{ N.mm}$$

$$\tau_{\text{ind}} = \frac{800{,}680 \times 16}{\pi (50)^3} \; 32.6 \text{ MPa which is within the limit.}$$

The diameter of the box may be taken as $1.5\,d = 75$ mm.

Section of lever at AA:

b.m at $AA = 1000 \times (800 - 37.5) = 762500$ N.mm

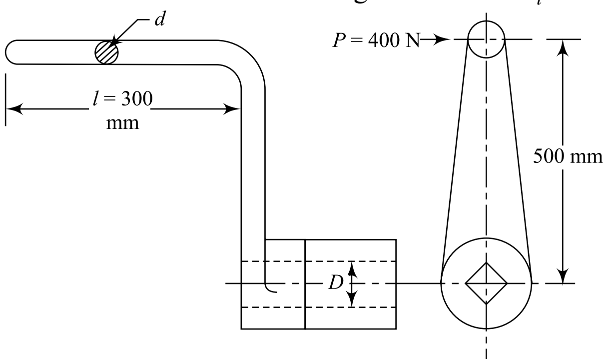
Taking $h = 3\,t$ for the rectangular section we have,

$$762500 = 30 \times 1.5 \times t^3$$

$\therefore \quad t = 18.52$ mm modified to 20 mm

$\therefore \quad h = 60$ mm.

6.7 A cranked lever is subjected to a force of 400 N. The material used is forged steel with $\sigma_t = 80$ MPa, $\tau = 40$ MPa, $l = 300$ mm and $\sigma_c = 100$ MPa. Find the dimensions of the lever.



Solution:

Assuming a force P acting on 2/3 of the length l

$$\text{Maximum b.m on lever} = \frac{2}{3} \times 300 \times 400 = \sigma_t \times \frac{\pi}{32} d^3$$

$$\therefore d = \frac{\sqrt[3]{200 \times 400 \times 32}}{\pi \times 80} = 21.67 \text{ mm}$$

$$= 22 \text{ mm}$$

Fig. E-6.7

Rest of the part can be designed just like a foot lever

OBJECTIVE QUESTIONS

6.1 All the types of levers are subjected to
(a) torque (b) b.m
(c) direct axial load (d) a combination of twisting and bending moment

6.2 The method of manufacturing usually adopted for levers is
(a) casting (b) fabrication (c) forging (d) machining

6.3 The rocker arm is an example of a lever of
(a) type 1 (b) type 2 (c) type 3 (d) none of the above

6.4 An I section is more suitable for
(a) lever of a lever loaded safety valve (b) a rocker arm
(c) a foot lever (d) a cranked lever

6.5 The taper to be provided on the lever of a band brake should be
(a) 1/20 (b) 1/5 (c) 1/8 (d) none of the above

REVIEW QUESTIONS

6.1 What are the materials used for lever? What are the methods of manufacturing normally adopted?
6.2 Explain which section is more suitable for levers and why?
6.3 Why are levers tapered towards the ends?
6.4 What is the criterion of designing a lever?
6.5 Classify a rocket arm, a bell crank lever, foot lever, cranked lever, lever of a lever loaded safety valve as the levers of Classes I, II or III.
6.6 Why is it desirable to consider the ratio d_p: l in a pin between 1 and 1.5?
6.7 Why is a bush of gun metal or bronze used for the fulcrum pin of a bell crank lever?
6.8 What are the functions of a lever?
6.9 State the three classes of levers.
6.10 Which are the sections normally used in the design of levers?
6.11 Why is I section preferred for bell crank lever or rocker arm?
6.12 In what respect a foot lever differs from other types of levers?
6.13 "Gear is a form of lever" Justify.

PRACTICE PROBLEMS

6.1 Design the lever of a lever loaded safety valve with the force $P = 5$ kN. Use forged steel for the lever with $\sigma_t = 90$ MPa and 30C8 for the pins with $\sigma_t = 80$ MPa. $P_b = 20$ MPa and $\tau = 40$ MPa

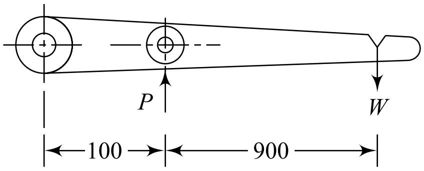


Fig. P-6.1

6.2 The rocker arm of an exhaust valve lever is subjected to a load of 6000 N acting at the end of each arm, the arm length being equal to 400 mm. The angle between the arms is 135°. Using an I section for the lever, find the dimensions of the I section and the fulcrum pin. Use the same materials as in Problem 6.1.

6.3 The right angled bell crank lever of Fig. E-6.2 is to be designed to raise a load of 7 kN at the end of the horizontal arm. The arm lengths are 100 and 400 mm. $\sigma_t = 85$ MPa, $\tau = 70$ MPa and $P_b = 10$ MPa. Find the dimensions of the pin and the lever.

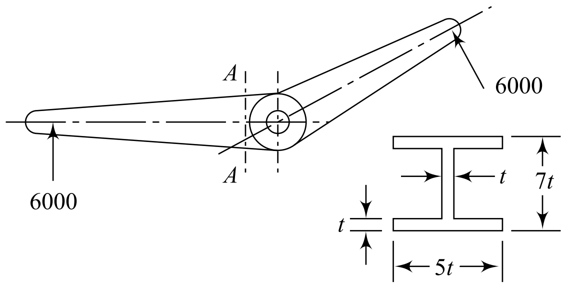


Fig. P-6.2

6.4 Design the lever of a lever loaded safety value when the diameter of the valve is 80 mm, the blow off pressure is 1.4 MPa and distance of the centre of the fulcrum pin from the axis of the valve is 80 mm and length of the lever arm is 10 mm. The permissible stress $\sigma_t = 80$ MPa, $\sigma_c = 100$ MPa, $P_b = 20$ MPa.

6.5 The lever of a band brake has the long and the short arms of length 800 mm and 75 mm respectively. The tensions $T_1 = 4000$ N and $T_2 = 2000$ N. Find the dimension of the lever and the fulcrum pin for σ_t, = 80 MPa, σ_c = 100 MPa. $\tau = 40$ MPa and $P_b = 15$ MPa.

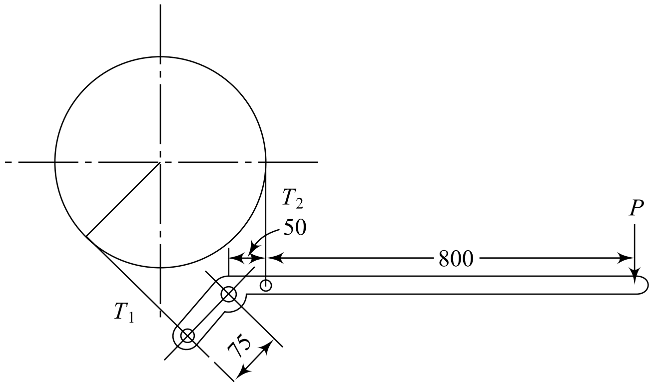


Fig. P-6.5

6.6 A cross lever shown in Fig. P-6.6 is made of a material with the value of the permissible bending stress equal to 70 MPa. Permissible shear stress for the pin is 40 MPa. Find the dimensions of the pins at *A, B, C* and *D* and the cross section of the vertical and the horizontal arms of the lever considering that the bearing pressure does not exceed 17.5 MPa.

6.7 Design a lever of lever loaded safety valve as shown in Fig. P-6.1(a). A pressure of 1.2 MPa acts on the valve of 70 mm diameter. Distance of the point of application of load *W* from the fulcrum is 1 m while that of *P* is 150 mm. Use C.I for lever with permissible values of stresses, $\sigma_t = 35$ MPa, $\sigma_c = 100$ MPa, $\tau = 20$ MPa, $p_b = 20$ MPa. For 30C8 steel pin $S_y = 300$ MPa.

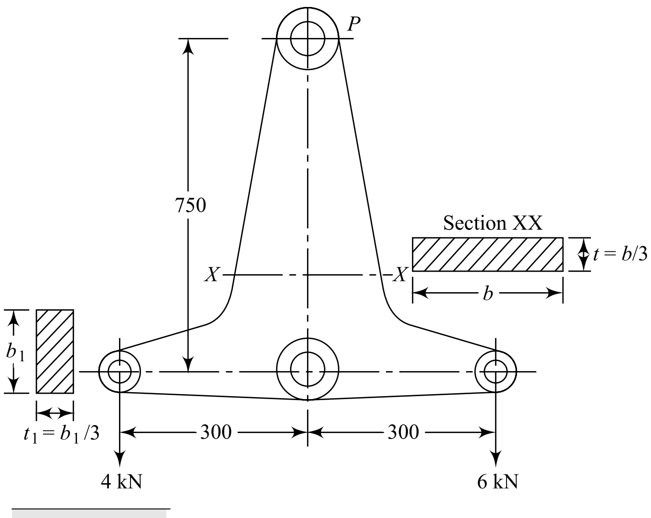


Fig. P-6.6

6.8 The band brake (Fig. E-6.5(a)) transmits 15 kW at 300 r.p.m. The diameter of the drum being 300 mm. The distance of the point of application of load from the fulcrum is 375 mm. Distances *AB* and *BC* are 120 mm and 75 mm respectively. $\mu = 0.3$. Design the lever and the pin using 30C8 steel with safe stresses $\sigma_t = 80$ MPa, $\tau = 40$ MPa, $p_b = 15$ MPa.

6.9 A cranked lever (Fig. E-6.7) is subjected to a force of 500 N (l = 400 mm) applied at the radius of 500 mm, design the lever for σ_t = 90 MPa, τ = 40 MPa, σ_c = 100 MPa.

6.10 Design a rocker arm as shown in Fig. P-6.2. The force due to gas pressure is 3000 N. The length of each arm is 100 mm from the fulcrum. Use I section with forged steel for rocker arm with S_y = 330 MPa and N = 4. For the pin P_b = 20 MPa, S_y = 300 MPa.

ANSWERS

Objective Questions

(1) b (2) c (3) a (4) b (5) d

Practice Problems

(1) t = 15 mm, b = 45 mm, $d_p = l$ = 16 mm

(2) d_p = 25 mm, l = 50 mm, t = 12 mm for I section of width 4 t and height $7t$.

(3) d_p = 40 mm, length of the pin l = 80 mm section of lever near the fulcrum $= 30 \times 90$ mm^2 and at the load end = 20×40 mm^2

(7) τ = 25 mm, B = 75 mm, d_p = 15 mm

(8) Rectangular section of lever b = 50 mm, t = 17 mm $\rightarrow$ 20 mm, d_p = 20 mm, l = 20 mm, section of the lever at fulcrum d_o = 55 mm

(9) $d = 31.69 \rightarrow 35$ mm, $D = 43.26 \rightarrow 45$ mm, D_1 = 50 mm, t = 12 mm, B = 40 mm

(10) d_p = 15 mm, L_p = 45 mm, I section t = 6 mm, depth = $6t$ = 36 mm, width = $4t$ = 24 mm.

7

Shafts and Couplings

CONCEPT REVIEW

7.1 INTRODUCTION

The circular rod used to transmit torque is called as a shaft. In addition to torque a shaft may be subjected to bending moment also.

Shafts are either cold or hot rolled with material being medium carbon steel.

7.2 DESIGN EQUATION

A shaft subjected to either a twisting moment or a combination of twisting and bending moments is designed by applying the three theories of failure explained in Section 3.6.

For a shaft subjected only to torque T

$$T = \frac{\pi}{16} d^3 \times \tau \tag{7.1}$$

Now, for a shaft subjected to the combined twisting and bending moments T and M respectively, we have,

$$\tau = \frac{16T}{\pi d^3}$$

using maximum shear stress theory of failure

$$\sigma = \frac{32\,M}{\pi d^3}$$

$$\therefore \qquad \text{Induced shear stress} = \frac{1}{2}\sqrt{\left(\frac{32\,M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\sqrt{M^2 + T^2}$$

$$\text{or} \qquad \sqrt{M^2 + T^2} = \frac{\pi}{16} d^3 \tau = T_e$$

Using the shock and fatigue factors K_m and K_t respectively, the above equation is modified as

$$T_e = \sqrt{(K_m M)^2 + (k_t T)^2} = \frac{\pi}{16}\, d^2 \tau \quad (7.2)$$

The design equation relating to angular twist is

$$\theta = \frac{Tl}{JG} \quad (7.3)$$

where T = twisting moment
l = length between points of start and end of twisting moment, J = polar m.i of shaft section
G = modulus of rigidity of shaft material.

7.3 WEAKENING EFFECT

When a pulley, flange or gear is mounted on the shaft and is fitted to it by a key, a keyway cut on the shaft reduces its section and also causes stress concentration, thereby reducing strength of the shaft. This effect is calculated by an empirical relationship.

Strength of the shaft with keyway

$$= (1 - 0.2w' - 1.1\, t') \times \text{strength of solid shaft} \quad (7.4)$$

where $w' = \frac{w}{d} t' = \frac{t}{d}$

w = width of key, t = thickness of key and d = diameter of shaft.

7.4 DIFFERENT TYPES OF COUPLINGS

Coupling is used for connecting two pieces of shafts. They are broadly classified as (a) Rigid couplings, (b) Flexible couplings.

Solid muff or box coupling, protected type of flange coupling, marine coupling, split muff coupling are types of rigid couplings.

Bushed pin type of flexible coupling, Oldham's coupling, Hooke's joint are some varieties of flexible couplings.

In bushed pin type of flexible coupling a rubber bush is provided to have flexibility and a brass bush takes care of wear.

WORKED EXAMPLES

7.1 A shaft is used to transmit 25 kW at 1500 r.p.m. The material used is 30C8 steel. S_y = 300 MPa. Find the diameter of the shaft.

Solution:

Given S_{ys} = 150 MPa. Let F·S = 3

$$\therefore \quad \tau = \frac{S_{ys}}{\text{F·S}} = \frac{300}{2 \times 3} = 50 \text{ MPa}$$

$$T = \frac{1000 \times 25 \times 60}{2\pi \times 1500} = 159.154 \text{ N.m} = 159154 \text{ N.mm}$$

$$\therefore \quad 159154 = 50 \times \frac{\pi}{16} d^3. \text{ Hence, } d = 25.3 \text{ mm modified to 30 mm.}$$

7.2 The shaft mentioned in the above example is also subjected to a b.m of magnitude 100 N.m due to a pulley. Find the diameter of the shaft. The shock and fatigue factors are $K_m = 1.5$ and $K_t = 1.2$ respectively.

Solution:

$$T_e = \sqrt{(1.5 \times 100 \times 1000)^2 + (159154)^2 \times (1.2)^2} = 242842 \text{ N.mm} = 50 \frac{\pi}{16} d^3$$

$$\therefore \qquad d = \sqrt[3]{\frac{242842 \times 16}{\pi \times 50}} = 29.136 \text{ mm.}$$

Thus a shaft of diameter 30 mm may be used.

7.3 A shaft of diameter 60 mm is subjected to shear stress of 40 MPa and has an angle of twist equal to 0.01 radian. Determine the length of the shaft for $G = 0.8 \times 10^5$ MPa.

Solution:

$$T = \frac{\pi}{16} d^3 \tau \quad \text{and} \quad \theta = \frac{Tl}{JG}$$

$$\therefore \qquad \theta = \frac{\pi d^3 \tau l}{16\, JG} = \frac{\pi d^3 \tau l \times 32}{16 \times \pi \times d^4 \times G} = \frac{2 \tau l}{Gd}$$

$$\therefore \qquad l = \frac{0.01 \times 0.8 \times 10^5 \times 60}{2 \times 40} = 600 \text{ mm}$$

7.4 For the shaft shown in Fig. E-7.4 the ratio of belt tension for either belt is 3 : 1. The maximum tension in the belt is 3000 N. $S_{ut} = 650$ MPa, $S_y = 400$ MPa for the shaft material. $K_m = 1.5$, $K_t = 1.2$. Determine the shaft diameter and angle of twist when the pulleys are keyed to the shaft.

Solution:

We first draw the s.f and b.m diagrams in the vertical and horizontal planes. The bending moments in vertical and horizontal planes are combined to find the resultant bending moment. Equivalent torque at the critical point is calculated which is then used to find the diameter of the shaft.

Power is taken by pulley A and transmitted to pulley B. The maximum belt tension is 3000 N. Therefore, the minimum belt tension is 1000 N.

Torque taken by A

$$= (3000 - 1000) \times 100 = 200000 \text{ N.mm}$$

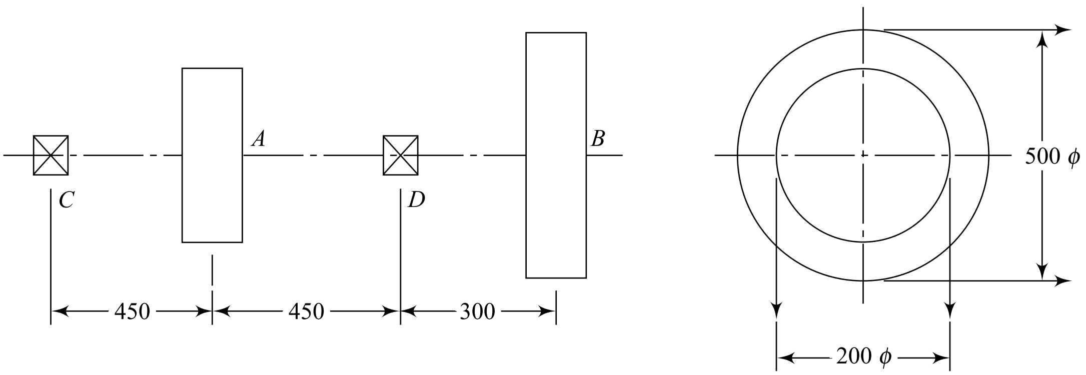


Fig. E-7.4a

$= \text{Torque transmitted by } B$

$(T_{1B} - T_{2B}) \times 250 = 200{,}000$

and $T_{1B} = 3\ T_{2B}$

$2T_{2B} \times 250 = 200{,}000$

$T_{2B} = 400$ N and $T_{1B} = 1200$ N

Resultant force at A = 4000 N vertically downwards.

Resultant force at B = 1600 N acting horizontally towards right.

Neglecting the mass of the pulley, the vertical s.f and b.m diagrams are as shown in Fig. E-7.4(b).

Vertical reactions $R_{CV} = R_{DV} = 2000$ N

Maximum vertical b.m = $2000 \times 450 = 900{,}000$ N.mm

From the horizontal S.F diagram

$R_{CH} = 533.33$ N, $R_{DH} = 2133.33$ N

Maximum b.m at D = $1600 \times 300 = 480{,}000$ N.mm

From the torque diagram shown in Fig. E-7.4 (b) torque at A = 200000 N.mm

From the b.m diagram the resultant b.m at A

$$= \sqrt{(900{,}000)^2 + \frac{(480{,}000)^2}{2}}$$

$= 93.145 \times 10^4$ N.mm

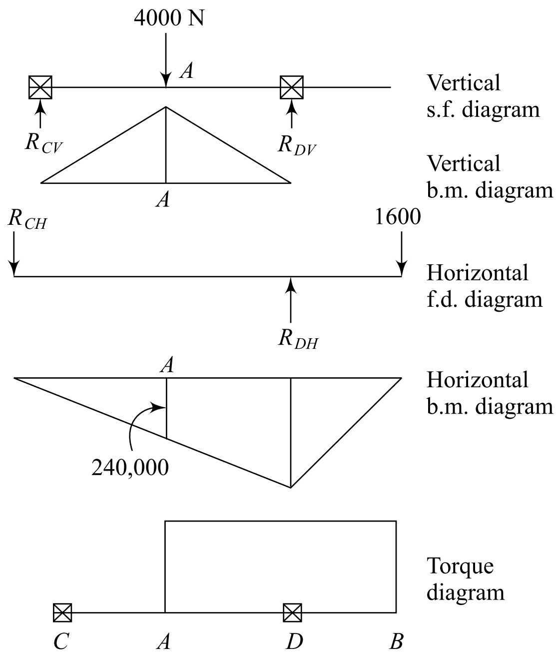


Fig. E-7.4b

$\therefore$ A is the critical point

$$T_e \text{ at } A = 10^4 \sqrt{(1.5 \times 93.145)^2 + (1.2 \times 20)^2} = 141.76 \times 10^4 \text{ N.mm}$$

S_y = 400 MPa. Hence S_{ys} = 200 MPa using a factor of safety of 2.5 and weakening effect due to the keyway equal to 0.75.

(1) $\tau = \dfrac{200 \times 0.75}{2.5} = 60$ MPa

(2) $d = \sqrt[3]{\dfrac{16 T_e}{\pi t}} = \sqrt[3]{\dfrac{16 \times 141 \times 76 \times 10^4}{\pi \times 60}}$ = 49.36 mm rounded to 50 mm

(3) Also $\theta = \dfrac{Tl}{JG} = \dfrac{200{,}000 \times 750 \times 32}{\pi (50)^4 \times 0.8 \times 10^5}$ = 0.003 rad

The length l is to be taken between the points between which the torque acts (i.e. between A and B).

7.5 A 600 mm diameter pulley driven by a horizontal belt transmits power to a 200 mm diameter pinion. The pulley has a mass of 90 kg, $K_m = 2$, $K_t = 1.5$ and $\tau = 40$ MPa. Find the diameter of the shaft.

Solution:

Taking moments of vertical forces @ A with weight of pulley = $90 \times 9.81 = 882.9$ N

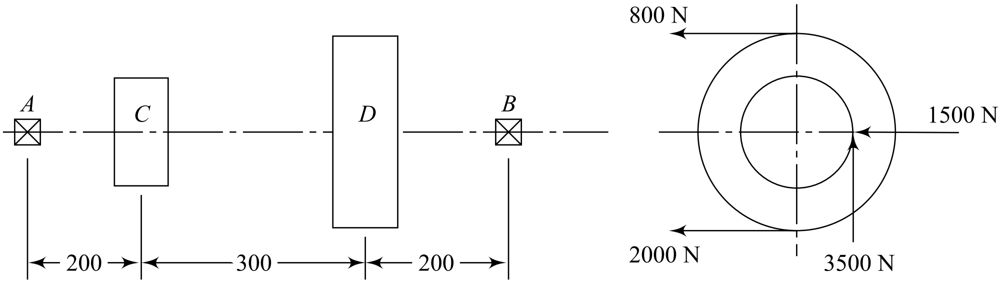


Fig. E-7.5a

$$3600 \times 200 - 882.9 \times 500 \times R_{BV} \times 700 = 0$$

$\therefore$ $R_{BV} = 379.92$ and $R_{AV} = 2319.17$ N

Vertical bending moments $M_D = 79584$ N.mm and $M_C = 463854$ N.mm

Also, $R_{AH} = 1871.42$, N and $R_{BH} = 2428.5$ N

Horizontal bending moments $M_C = 274284$ N.mm, $M_D = 485700$ N.mm

From the resultant b.m diagram shown in Fig. E-7.5 (c) we have, $M_C = 596027.71$ N.mm, $M_D = 492181.57$ N.mm.

The torque diagram is shown in Fig. E-7.5 (d)

$$T_C = T_D = (2000 - 800) \times 300 = 360{,}000 \text{ N.mm}$$

$\therefore$ C is the critical point for which equivalent torque

$$T_e = \sqrt{(2 \times 596027.71)^2 + (1.5 \times 360000)^2} = 1308662.6 \text{ N.mm}$$

Let $\tau = 40$ MPa

$\therefore$ $$d = \sqrt{\frac{16 \times 1308662.6}{40\,\pi}} = 55.5 \text{ mm modified to 60 mm.}$$

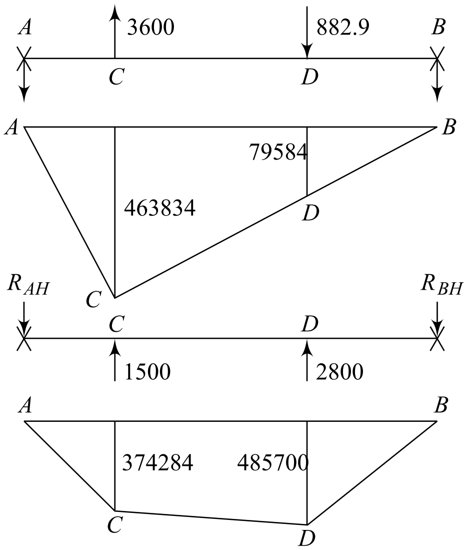


Fig. E-7.5b

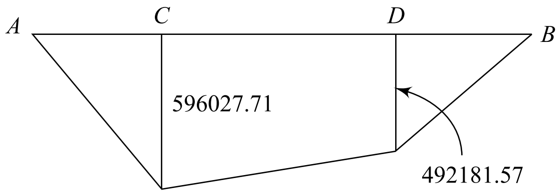


Fig. E-7.5c

Fig. E-7.5d

7.6 Design a line shaft transmitting power to two machine tools. The power received by the shaft is 30 kW at 300 r.p.m. The power absorbed by pulley P_I is 12 kW and the remaining power is absorbed by pulley P_2. The diameter of pulley P_1 is 300 mm and its mass is 40 kg. The diameter and mass of pulley P_2 are 600 mm and 75 kg respectively. Assume the belt tension ratio of 2 for both pulleys and the shaft material as 30C8 steel with $K_m = 2$ and $K_t = 1.5$. Draw the b.m and torque diagrams, assuming maximum shear stress theory.

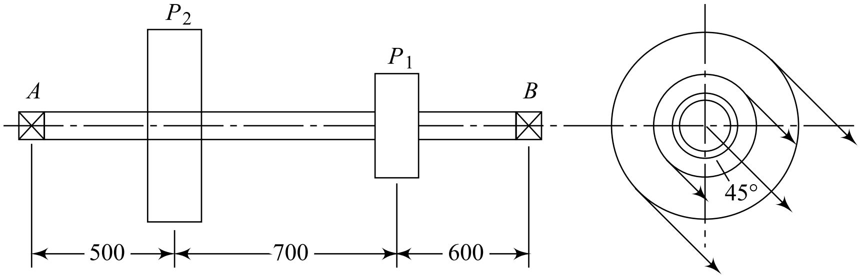


Fig. E-7.6a

Solution:

For pulley P_1, power = 12 kW

$$\therefore \quad \text{Torque} = \frac{12 \times 1000 \times 60}{2\pi \times 300} = 381.97 \text{ N.m} = (T_1 - T_2) \times \frac{150}{1000}$$

$$\text{and} \quad \frac{T_1}{T_2} = 2, \quad \therefore T_1 = 5092.9333 \text{ N} \quad \text{and} \quad T_2 = 2546.4666 \text{ N.}$$

The force $T_I + T_2$ = 7639.3999 N acts at an angle of 45°. Therefore its vertical component = 7639.3997 × sin 45° = 5401.8714 N downward

Similarly for pulley P_2, power = 18 kW

$$\therefore \quad T_1 = 3819.7 \text{ N}, T_2 = 1909.85 \text{ N}$$

Thus vertical component of $(T_1 + T_2)$ = 4051.44

∴ Total vertical force at pulley P_1 = 5401.8714 + 40 × 9.81 = 5794.2714 N.

Total vertical force at pulley P_2 = 4051.44 + 75 × 9.81 = 4787.1925 N

Taking moments @ A and B

R_{AV} = 5388.84 N $\qquad R_{BV}$ = 5192.62 N

Vertical b.m at pulley P_1 = 3115.575 N.m

Vertical b.m at P_2 = 2694.42 N.m

Horizontal components are as shown in Fig. E-7.6 (b)

R_{AH} = 4728.33 N $\qquad R_{RH}$ = 4724.6466 N

∴ Horizontal b.m at P_1 = 2834.784 N.m

Horizontal b.m at P_2 = 2364.165 N.m

Resultant b.m at P_1 = 4212.2212 N.m

Resultant b.m at P_2 = 3584.5746 N.m

$$\text{Total torque} = \frac{30 \times 1000 \times 60}{2\pi \times 300} = 954.93 \text{ N.m}$$

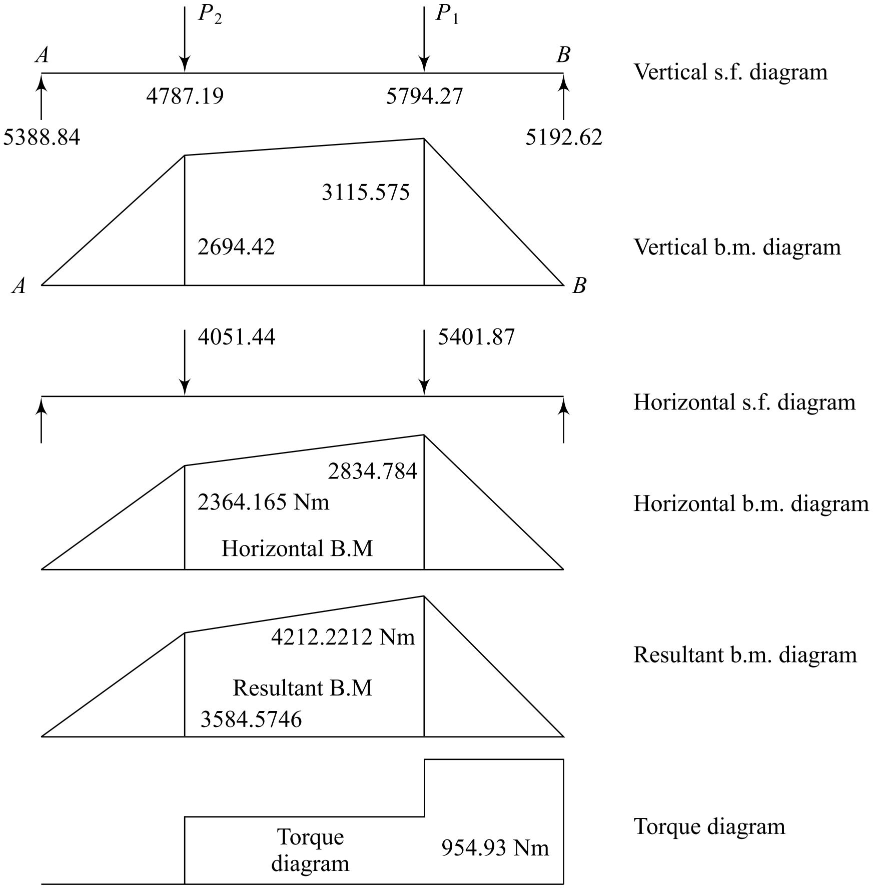


Fig. E-7.6b

From the torque distribution shown in Fig. E-7.6(b) the critical point is at pulley P_1 for which

$$T_e = \sqrt{(2 \times 4212.2212)^2 + (1.5 \times 954.93)^2} = 8545.366 \text{ N.m}$$

Let $\tau = 40$ MPa

$$d = \sqrt[3]{\frac{8545.366 \times 1000 \times 16}{\pi \times 40}} = 102.85 \text{ mm modified to } 105 \text{ mm.}$$

7.7 Design a rigid muff coupling. Use C.I for the muff. The power transmitted is 25 kW at 300 r.p.m. $S_{ut} = 200$ MPa, F·S = 6, use 30C8 steel for the shaft consider $S_y = 330$ MPa and F·S = 4.

Solution:

The shaft is subjected to twisting moment only and safe stress $\tau = \dfrac{330}{2 \times 4} =$ 41.25 MPa

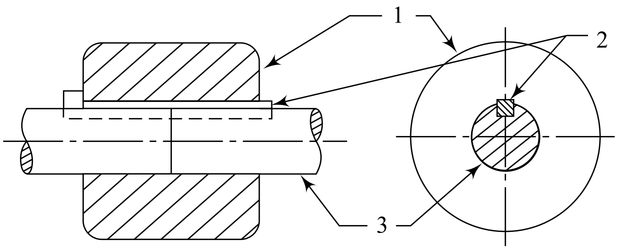


Fig. E-7.7

$$\text{Torque} = \frac{25 \times 1000 \times 60}{2\pi \times 300} = 795.77471 \text{ N.m}$$

$$\text{Diameter of shaft} = d = \sqrt[3]{\frac{795.77 \times 1000 \times 16}{\pi \times 41.25}}$$

= 46.14 mm modified to 50 mm to take into account the weakening effect due to keyway.

Key: Width of the key $= \dfrac{d}{4} = 12.5$ mm

$$\text{Tangential force on key} = \frac{795.77 \times 1000}{25} = 31830.8 \text{ N}$$

$$\text{Shearing of key, } \tau = \frac{31830.8}{l \times 12.5} = 41.25 \text{ MPa}$$

$$\therefore \quad l = \frac{31830.8}{12.5 \times 41.25} = 61.732 \text{ mm}$$

The empirical relation is $l = 3.5\ d$. Let us use $l = 100$ mm to have sufficient length of the shaft inside the coupling.

Crushing of key: $31830.8 = \sigma_c \left(\dfrac{t}{2} \times l \right)$

For 30C8 steel $\quad \sigma_c = \dfrac{500}{5} = 100$ MPa, $\quad \therefore \quad t = \dfrac{31830.8 \times 2}{100 \times 100}$

= 6.366 mm

Let us use $\quad t = 8$ mm

$$\therefore \quad \text{Weakening effect} = 1 - 0.25 \times 0.2 - 1.1 \times \frac{8}{50} = 0.774$$

This effect is considered in modifying the value of d.

Muff: It is treated as a hollow shaft. For 200 FG C.I

$$\text{Safe shear stress } \tau = \frac{200}{2 \times 6} = 16.7 \text{ MPa}$$

$$\text{Torque} = 795774.71 \text{ N.mm} = \tau \times \frac{\pi}{16} \left(\frac{d_0^4 - d_i^4}{d_0} \right)$$

$$= 16.7 \times \frac{\pi}{16} \left(\frac{d_0^4 - 50^4}{d_0} \right)$$

By trial and error, $d_o = 69.5$ mm modified to 75 mm

$\therefore$ Length of the muff = 100 mm.

7.8 For the connection of above shafts if the protected type of flange coupling is used, find the dimensions of flanges and bolts.

Solution:

From Example 7.8, diameter of shafts = 50 mm

diameter of hub = 75 mm

key dimensions = 12 × 8 × 75 mm

No. of bolts = 0.0 2d + 3 = 4

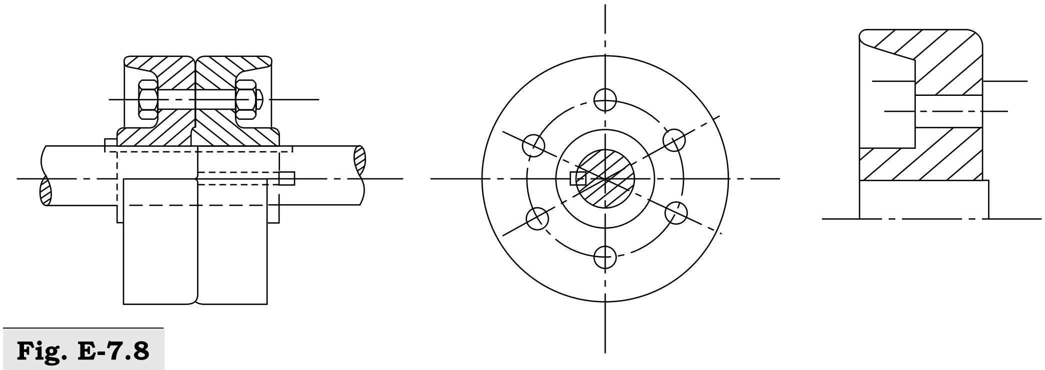


Fig. E-7.8

Pitch circle diameter, i.e. p.c.d of bolts = 3 d = 150 mm.

The bolts are subjected to initial tightening and shearing due to torque. A large factor of safety may be selected to account for preload. Hence, for the bolt the safe stress due to shear may be taken as 30 MPa.

$$\text{Shearing load/bolt} = \frac{T}{(p.c.d/2)}/\text{No. of bolts} = \frac{795774.71}{75 \times 4} = 2652.58 \text{ N}$$

$$\therefore \qquad \text{Area of bolt} = \frac{2652.58}{30} = 88.419 \text{ mm}^2$$

Corresponding to this core area the standard bolt is M 14 × 2 (core area 115 mm^2). Centre of the bolt is situated at 37.5 mm from the hub thereby providing sufficient space for fitting a nut using the spanner.

Flange thickness t is calculated by considering shearing of flange at the hub.

$$\text{Tangential force at hub} = \frac{T}{\text{radius of hub}} = \frac{795774.71}{37.5} \text{ N} = \tau\pi\,(\text{diameter of hub}) \times t$$

$$\therefore \qquad t = \frac{795774.71}{37.5 \times 16.7 \times \pi \times 75} = 5.393 \text{ mm.}$$

Since the minimum thickness for casting should not be less than 10 mm, we take the flange thickness equal to 10 mm.

7.9 Design a bushed pin type of flexible coupling to connect the motor shaft and pump shaft of 50 mm and 40 mm diameter respectively when 15 kW power is to be transmitted at 1200 r.p.m, the permissible bearing pressure for pin is 0.3 MPa.

Solution:

Design of the shaft, key and flanges is to be carried out in a similar manner as in Example 7.8.

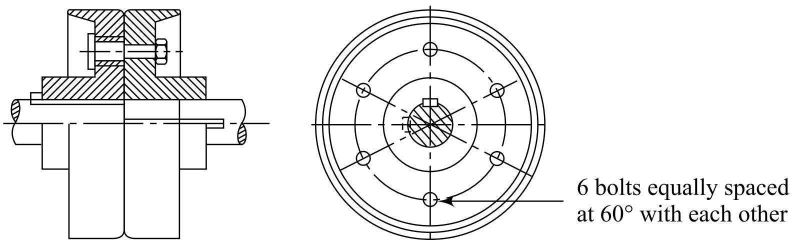


Fig. E-7.9a

No. of pins = $0.02d + 3 = 4$. Let us use 6 pins to reduce the load on each pin. p.c.d for the pins = $3d$ = 150 mm. Since the diameter of pin along with the bushes is more than the bolt diameter it is advisable to increase the value of p.c.d to accommodate the pins, hence p.c.d = 160 mm.

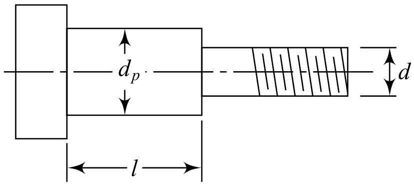


Fig. E-7.9b

$$\text{Torque} = \frac{15 \times 1000 \times 60}{2\pi \times 1200} = 119.3662 \text{ N.m}$$

$$= 119366.2 \text{ N.mm}$$

$$\text{Load on each pin} = \frac{119366.2}{6 \times (160/2)} = 248.68 \text{ N}$$

The diameter d of the threaded part may be found by shear consideration as in Example 7.9.

Let $\tau = 30$ MPa

$\therefore$ $248.68 = 30 \times$ core area

$\therefore$ Core area, $A_c = 8.28 \text{ mm}^2$.

Let us adapt M10 × 1.25 bolt threads which is bigger than calculated value as it will be observed that the bending stress may cause failure.

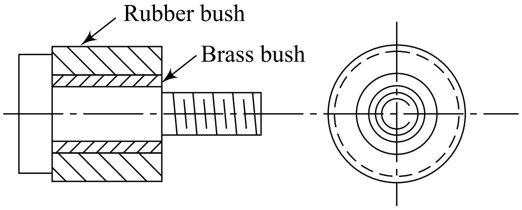


Fig. E-7.9c

Bearing of pin:

$248.68 = 0.3 \times$ projected area

Thickness of the brass bush is taken as 2 mm and that of rubber bush as 6 mm.

Thus, the outermost diameter of the bush

$= d_p + 2(6 + 2)$

Let $d_p = 2d = 20$ mm

$\therefore$ Outermost diameter = 36 mm

$\therefore$ $248.68 = 0.3 \times 36 \times l$

$\therefore$ $l = 23.02 \text{ mm} \cong 24$ mm.

Keeping the diameter d_p larger thus results in a small value of l thereby decreasing the bending moment on section *AA.*

Check for bending:

Assuming the tangential force on the pin to be acting at a distance of 0.6 l, b.m = $248.68 \times 24 \times 0.6$ = 3580.992 N.mm = $\sigma_t \cdot Z$

$\therefore$ $$\sigma_t = \frac{3580.992}{\frac{\pi}{32}(10)^3} = 36.475 \text{ MPa}$$ This is within the limit as for the bolt material

$S_y = 330$ MPa.

7.10 Design a split muff coupling to transmit a power of 25 kW at 300 r.p.m. Use the same materials as in Example 7.9.

Solution:

A split muff coupling differs from the solid muff type in one aspect and that is, in the former type the muff is made in two halves and those halves are attached together by using bolts.

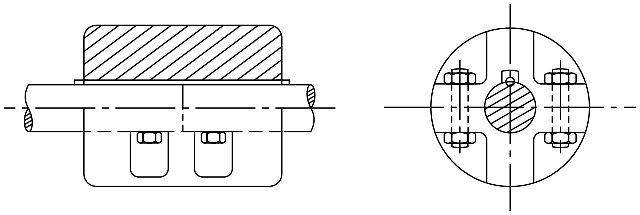


Fig. E-7.10

From Example 7.8, diameter of the shaft, $d = 50$ mm and length of the muff $= 3.5\,d = 175$ mm.
Outside diameter of muff $= 2d = 100$ mm
The above proportions give safe results.

Bolt design: If σ_t is the safe tensile stress in the bolt each bolt will exert a force $= \frac{\pi}{4} d_b^2 \cdot \sigma_t$

$\therefore$ Total clamping force on each shaft

$$= \frac{n}{2} \times \frac{\pi}{4} d_b^2 \cdot \sigma_t, \quad n = \text{total number of bolts.}$$

$$\therefore \text{ Clamping pressure intensity} = \frac{\text{Force}}{\text{Projected area}} = \frac{\frac{n}{2} \cdot \frac{\pi}{4} d_b^2 \cdot \sigma_t}{d \times l/2} \quad \text{where } l = \text{length of muff}$$

$$= \frac{\pi d_b^2\, n \cdot \sigma_t}{4\,dl}$$

$\therefore$ Frictional force/mm^2 on the periphery of shaft

$$= \frac{\mu \cdot \pi d_b^2 \cdot n \cdot \sigma_t}{4\,dl} \quad \text{where } \mu = \text{coeff. of friction}$$

$\therefore$ Frictional torque = Frictional force per unit area $\times$ peripheral area of shaft $\times$ shaft radius

$$= \frac{\mu \cdot \pi d_b^2 \cdot n \cdot \sigma_t}{4\,dl} \times \frac{\pi}{2} dl \times \frac{d}{2}$$

$$= \frac{\mu \cdot \pi^2}{16} d_b^2\, n \cdot \sigma_t \cdot d$$

Let $\mu = 0.3$, $n = 8$ and $\sigma_t = 70$ MPa

$$\therefore \quad 795774.71 = \frac{0.3 \times \pi^2}{16} d_b^2 \times 8 \times 70 \times 50$$

$$\therefore \quad d_b = 12.39 \text{ mm.}$$

Hence, let us use M16 $\times$ 2 bolts.

7.11 A shaft is subjected to loads as shown in Fig. E-7.11(a). Gear C is connected to the other gear such that 50 kW is transmitted at 100 r.p.m. The pressure angle of the involute gear teeth is 20°. The ratio of belt tensions for pulley A is 2 : 1, the diameter of pulley being 750 mm. The sprocket B is 500 mm diameter with negligible tension in the chain on the slack side. The diameter of gear C is 300 mm. The power transmitted by chain drive is 20 kW, the remaining being transmitted by the belt drive. Find diameter of the shaft if F·S = 3, $K_m = 1.5$, $K_t = 1.2$ and $S_y = 350$ MPa for shaft material.

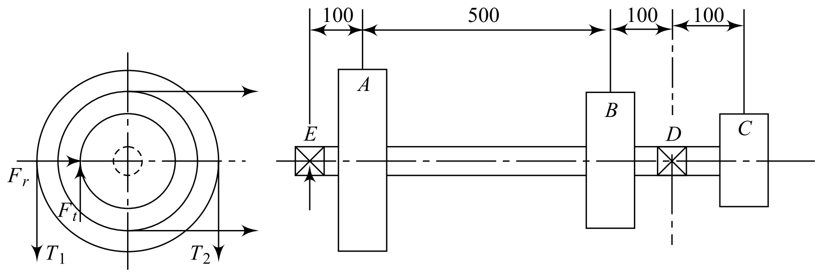


Fig. E-7.11a

Solution:

$$\text{Peripheral velocity of gear} = \frac{\pi \times 0.3 \times 1000}{60} = 15.7 \text{ m/s}$$

$$\therefore \quad \text{Tangential force } F_t = \frac{50000}{15.7} = 3184.71 \text{ N}$$

For pulley A, $(T_1 - T_2) \times \frac{0.75}{2} = \frac{30 \times 60000}{2\pi \times 1000}$

$$\therefore \quad T_1 - T_2 = \frac{30 \times 60000}{2\pi \times 1000 \times 0.375} = 764 \text{ N}$$

$$\therefore \quad T_2 = 764 \text{ N } T_1 = 1528 \text{ N}, T_1 + T_2 = 2292 \text{ N}$$

For sprocket B, as tension on the slack side is zero, the tension on tight side only is producing the torque to transmit a power of 30 kW.

$\therefore\ T_t \times 0.25 = \frac{20 \times 60000}{2\pi \times 1000}$, $\therefore\ T_t = 764$ N acting in horizontal direction as shown in Fig. E-7.11(b).

Radial force F_r on gear $= F_t \tan 20° = 1159.14$ N as shown in E-7.11(a)

By taking moments @ E or D

Vertical reactions at E and D $R_{EV} = 2419.4285$ N acting upwards

$R_{DV} = 3311.4285$ N acting downwards

From the vertical S.F diagram we find the b.m at A and D as shown in Fig. E-7.11(b).

The bending moments are

$M_{VD} = 318.4$ N.m, $M_{VA} = 241.94$ N.m

Horizontal reactions $R_{EH} = 56.4485$ N $\quad R_{DH} = 1979.59$ N

Horizontal b.m $M_{HD} = 115.91$ N.m $\quad M_{HA} = 5.644$ N.m

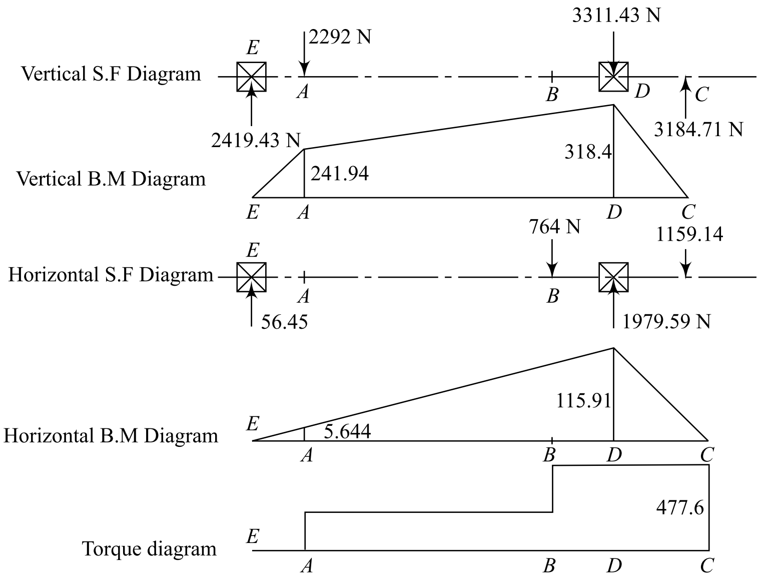


Fig. E-7.11b

As D is the maximum loaded point, resultant b.m at D, the maximum loaded point is,

$$M_{\text{resultant}} = \sqrt{318.4^2 + 115.91^2} = 338.84 \text{ N.m}$$

$$\text{Torque at } D = 3184 \times \frac{0.3}{2}\ 477.6 \text{ N.m}$$

$$T_{\text{equivalent}} = \sqrt{(1.5 \times 338.84)^2 + (1.2 \times 477.6)^2}$$
$$= 766.00 \text{ N.m}$$

$$\text{Safe shear stress} = \frac{0.5\, S_{ys}}{N} = \frac{0.5 \times 350}{3} = 58.33 \text{ MPa}$$

$$\text{Induced shear stress} = \frac{16 T_{\text{eq}}}{\pi d^3} = \frac{16 \times 766 \times 10^3}{\pi d^3}$$

Equating the safe and induced shear stress we get d = 40.59 mm modified to 45 mm to account for the weakening effect due to keyway.

7.12 Calculate the diameter of the solid circular shaft shown in Fig. E-7.12(a) to transmit 45 kW at 1000 r.p.m. The pressure angle of the involute bevel and spur gears is 20°. Diameter of bevel gear C = 500 mm and the diameter of spur pinion D = 300 mm. Assume complete power being transmitted and safe shear stress for shaft equal to 60 MPa.

Solution:

$$\text{Peripheral velocity of } C = \frac{\pi \times 500 \times 1000}{60000} = 26.18 \text{ m/s}$$

$$\therefore \text{ Tangential force } F_{t_1} = \frac{45{,}000}{26.18} = 1724.13 \text{ N, in the direction shown in Fig. E-7.12(b)}$$

Assuming the pitch cone angle of 60° and also $F_{r_1} = F_{t_1} \tan 20° = 627.55$, we get

$$H = F_{r_1} \sin 30° = 313.75 \text{ N}$$

$$V = F_{r_1} \cos 30° = 543.47 \text{ N}$$

$$\text{For spur gear, peripheral velocity} = \frac{\pi \times 0.3 \times 1000}{60} = 15.7 \text{ m/s}$$

$$\therefore \quad F_{t_2} = \frac{45000}{15.7} = 2866.2246 \text{ N}$$

By taking moments @ B $\quad F_{r_2} = F_{t_2} \tan 20° = 1043.22$ N

$$R_{AV} = \frac{1043.22 \times 100 - 543.47 \times 400}{300} = 376.89 \text{ N} \uparrow$$

$\therefore \quad R_{BV} = 876.62 \text{ N} \downarrow$

Similarly, $\quad R_{AH} = 1343.42$ N and $R_{BH} = 2485.54 \text{ N} \downarrow$

The vertical and horizontal S.F diagrams are shown accordingly. Now at bearing A

b.m due to V in vertical plane = 54.347 N.m

b.m due to H in vertical plane = 313.75 × 0.25 = 78.44 N.m

Effective b.m at A in vertical plane = 78.44 – 54.347 = 24.093 N.m

However, the effect of 'H' is not considered while drawing b.m and s.f diagrams and also in further calculations as it is too small to cause any change in final answer.

b.m at D in vertical plane = 87.662 N.m

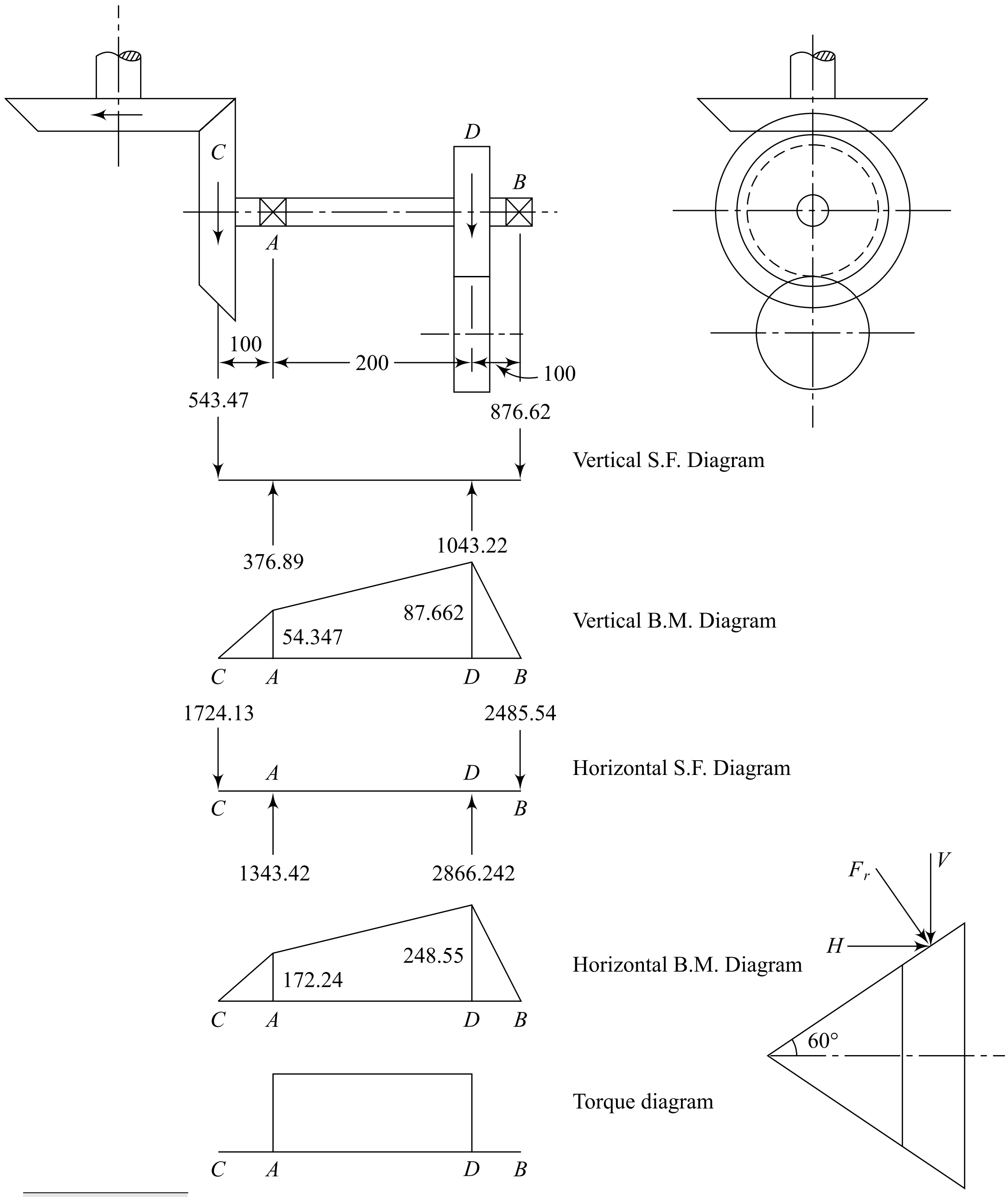


Fig. E-7.12

Similarly,

Horizontal b.m at A = 172.24 N.m

Horizontal b.m at D = 248.55 N.m

Thus resultant b.m is maximum at D and is equal to $\sqrt{87.662^2 + 248.55^2}$ = 263.55 N.m.

$$\text{Torque on spur gear} = 2866.242 \times \frac{0.3}{2} = 429.93 \text{ N.m}$$

$$\therefore \quad T_{\text{equivalent}} = \sqrt{(1.5 \times 263.55)^2 + (1.2 \times 429.83^2)} = 649.96 \text{ N.m}$$

$$\therefore \quad d = \sqrt[3]{\frac{16 \times 649.96 \times 10^3}{\pi \times 60}} = 38.06 \text{ mm modified to 40 mm.}$$

7.13 An electric motor drives a machine through a pair of spur gears. The pinion is mounted on motor shaft and overhangs by 200 mm from the nearest bearing. The pinion has 20 teeth of 10 mm module and 20° involute profile. Design the motor shaft to transmit 15 kW at 1200 r.p.m. Use safe shear stress value of 400 MPa, K_m = 1.2 and K_t = 1.

Solution:

$$\text{Diameter of gear} = 20 \times 10 = 200 \text{ mm}$$

$$\therefore \quad V = \frac{\pi \times 0.2 \times 1200}{60} = 12.57 \text{ m/s}$$

$$\therefore \quad F_t = \frac{15000}{12.57} \ 1193.34 \text{ N}, \ \therefore \ F_n = \frac{F_t}{\cos 20°} = 1269.92 \text{ N}$$

$\therefore$ Maximum b.m at the bearing = 1269.92 × 0.2 = 253.984 N.m

Torque = 1193.34 × 0.1 = 119.334 N.m

$$\therefore \quad T_{\text{eq}} = \sqrt{(1.2 \times 253.984)^2 + (1 \times 119.334)^2} = 326.59 \text{ N.m}$$

$$\therefore \quad d = \sqrt[3]{\frac{16 \times 326.59 \times 10^3}{\pi \times 40}} = 34.64 \text{ modified to 35 mm.}$$

OBJECTIVE QUESTIONS

7.1 It is advisable to use a rectangular key of width/thickness ratio
(a) less than one (b) more than one (c) one (d) none of the above

7.2 A small value of permissible stress is used for the bolts of a flange coupling due to
(a) combined loading (b) initial tightening
(c) stress concentration (d) none of the above

7.3 In the bushed pin type of flexible coupling, the diameter to length ratio of the pin should be
(a) as small as possible (b) as large as possible
(c) nearly equal to one (d) none of the above

7.4 Length of a muff in terms of the diameter of the shaft should be
(a) 1.5 d (b) 3.5 d (c) 10 d (d) 0.5 d

7.5 Shaft A has a diameter double the diameter of shaft B of the same material and transmits 80 kW. Assuming both shafts rotate at the same speed the power transmitted by B is
(a) the same as that of A (b) 40 kW
(c) 10 kW (d) 20 kW

7.6 Bolts of a split muff coupling are designed for
(a) shearing (b) tension
(c) combined tension and shear (d) none of the above

7.7 For a square key with the key width = 0.25 times the diameter of shaft, the weakening effect reduces the strength of the shaft to
(a) 80% (b) 67.5% (c) 70% (d) 50%

7.8 Muff is designed as a
(a) thin cylinder (b) thick cylinder (c) hollow shaft (d) none of the above

7.9 The ratio of width of key to diameter of shaft on which the key is fitted is
(a) 1/2 (b) 1/4 (c) 1/8 (d) 2

7.10 Taper on the rectangular sunk key should be
(a) 1/16 (b) 1/32 (c) 1/48 (d) 1/100

7.11 It is desirable to increase the diameter of the pin in a bushed pin type of flexible coupling for
(a) higher stresses due to shear
(b) keeping the magnitude of b.m small by reducing the unsupported length of pin
(c) fitting the pin in the bush
(d) reducing thickness of flange

7.12 Rubber bush is used in flexible coupling to
(a) avoid wear of the pin (b) allow proper fixation of the pin
(c) allow small misalignment of the shaft (d) avoid failure of C.I flanges

7.13 Which one of the following is flexible coupling?
(a) muff coupling (b) marine coupling
(c) Oldham's coupling (d) protected type flanged coupling

7.14 Design of power transmission shafting is based on
(a) Maximum shear stress theory of failure (b) St. Venant theory
(c) Rankine's theory (d) Heigh's theory

REVIEW QUESTIONS

7.1 Which theory is commonly used for design of a shaft? Explain why?
7.2 Explain the effect of keyway on the strength of a shaft.
7.3 Why is it essential to use coupling?
7.4 Give the classification of coupling.
7.5 Explain advantage of split muff coupling over solid muff coupling.
7.6 What is the benefit of using C.I flanges?
7.7 Why a large factor of safety is used for bolts of the flange couplings?
7.8 Explain the reason for using a brass bush and rubber bush in a bushed pin type of flexible coupling.
7.9 Why a too large or too small number of bolts should not be used in flange coupling?
7.10 Derive the expression for the stress induced in the bolts of a split muff coupling with torque T, diameter of shaft d, bolt diameter d_t, number of bolts n and coefficient of friction μ.
7.11 What are different types of keys used to connect shaft to a pulley or a gear? Explain by drawing sketch.
7.12 Why a rectangular key is preferred to a square key?
7.13 Why diameter of the pin of bused pin type of flexible coupling is increased in the bushed portion? Why are two bushes used?

PRACTICE PROBLEMS

7.1 Design a rigid muff coupling to transmit 30 kW at 100 r.p.m. Use the same materials as in Example 7.8.

7.2 Design a protected type flange coupling for transmitting 50 kW at 600 r.p.m. Use the same materials as in Example 7.9.

7.3 Design a shaft for transmitting 50 kW at 1200 r.p.m. The power is taken by pulley A and transmitted through pulley B. The diameter and weight of pulley A is 600 mm and 1000 N respectively and for pulley B is 300 mm and 500 N respectively.

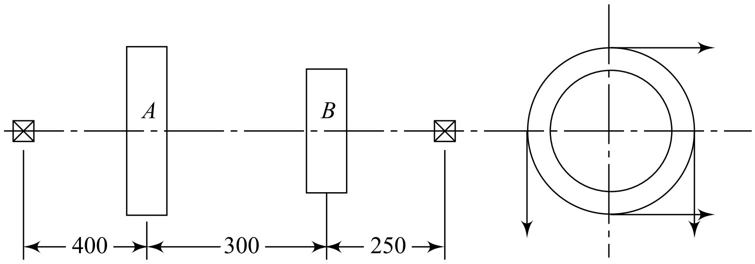


Fig. P-7.3

The ratio of belt tensions for both the pulleys is the same. Maximum tension on either pulley belt should not exceed 4000 N. Use 30C8 steel for shaft with τ_{safe} = 40 MPa. Draw vertical and horizontal b.m and torque diagrams. K_m = 1.5, k_t = 1.2

7.4 Power is taken by pulley P_1 and transmitted by pulley P_2 in the shaft shown in Fig. P-7.4. The diameter and mass of pulley P_1 mm and 60 kg respectively and those of pulley P_2 are 600 mm and 75 kg respectively. Ratio of belt tensions is 3 : 1. The power to be transmitted in the shaft is 60 kW at 100 r.p.m. Design the shaft. Use K_m = 1.5, K_t = 1.2, safe τ = 40 MPa.

7.5 A pulley of diameter 600 mm as shown in Fig. P-7.5 driven by a horizontal belt transmits power through the shaft to a pinion which drives a mating gear. Mass of the pulley is 150 kg, K_m = 2, K_t = 1.5 and τ_{safe} = 40 MPa. Find the diameter of shaft using 30C8 steel.

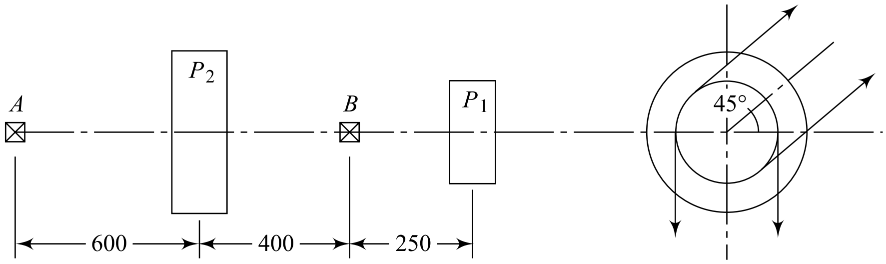


Fig. P-7.4

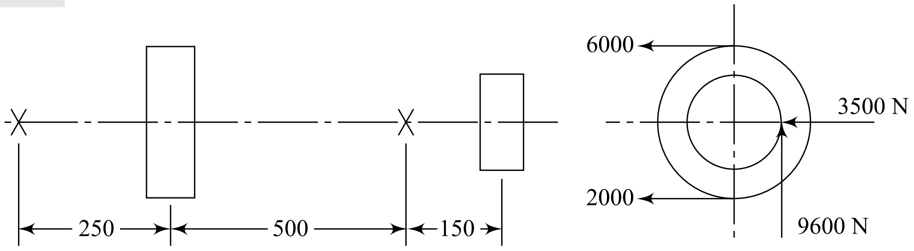


Fig. P-7.5

7.6 Find the diameter of the shaft shown in Fig. P-7.6. The power supplied to the shaft through a gear and pinion is 30 kW. Twenty kW power is supplied to the milling machine through a horizontal drive via pulley P_1 and the remaining power is supplied to the planer though pulley P_2 by a vertical belt. The speed of the motor is 1200 r.p.m. Diameters of the gear and pinion are 300 mm and 100 mm respectively and the diameters of pulleys P_1 and P_2 are 750 mm and 900 mm respectively. $K_m = 2$, $K_t = 1.5$, the safe shear stress for the shaft is 40 MPa, the pressure angle for gear tooth is 20° and ratio of belt tensions is 2.

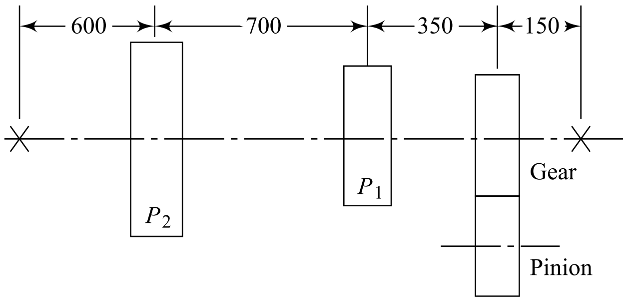


Fig. P-7.6

7.7 Find the diameter of four bolts of a flange coupling to transmit 63.5 kW at 240 r.p.m, p.c.d for the bolts is 200 mm and the safe stress is 32 MPa.

7.8 Design a bushed pin type of flexible coupling to transmit 35 kW at 100 r.p.m. Permissible bearing pressure is 0.3 MPa for rubber. Material used for the pins and the key is 30C4 steel. The shafts are of 50 mm diameter.

7.9 The length of a shaft is 600 mm and the permissible angle of twist is 1° when the shearing stress equals 69 MPa. Find the diameter of the shaft $G = 0.8 \times 10^5$ MPa.

7.10 A shaft of length 380 mm carries a spur gear of 20° involute teeth of 300 mm diameter at the midpoint and transmits 22.5 kW at 300 r.p.m. Using F·S= 1.5, $S_y = 350$ MPa, $K_m = 1.5$ and $K_t = 1.2$, find the diameter of the shaft (a) considering only tangential component, (b) considering both radial and tangential components. Use Guests theory and $T_{\max} = 1.5\ T_{\text{av}}$.

7.11 A shaft shown in Fig. P-7.11 transmits power varying between 200 kW to 100 kW and back to 200 kW in each revolution at 600 r.p.m. The spur gear A of diameter 500 mm receives the power and spur gear B of 250 mm diameter delivers it to the machine. Find the diameter of the shaft using F·S = 2. Neglect the radial component acting vertically downwards. Stress concentration factor due to keyway is 1.6 under bending and 1.3 under torsion. The material tests $S_y = 350$ MPa and $S_e = 210$ MPa.

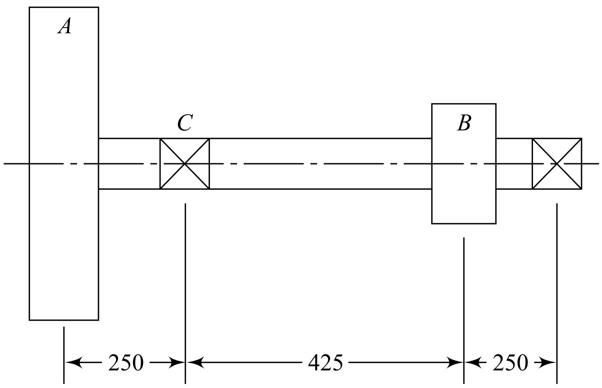


Fig. P-7.11

7.12 Solve Problem 7.11 for a pressure angle of 20° and considering radial components of tooth load.

7.13 Solve Problem 7.12 with the tangential force due to gears A and B acting in opposite directions and that of gear A acting vertically downward.

7.14 Power is supplied to a shaft through the pulley A with both the belt tensions acting vertically downwards. Power is transmitted to the cutter through a chain drive at B. Tension in the tight side of the chain acts vertically upwards while the slack side tension is negligible and supplies 27.5 kW to the cutter. Pulley *C* supplies the remaining 22.5 kW power to the blower using a *V* belt, the blower pulley being exactly behind *C*. If the ratio of tensions for both the pulleys is 2 : 1, find the diameter of the shaft using a safe shear stress of 40 MPa. Speed of the shaft is 1000 r.p.m.

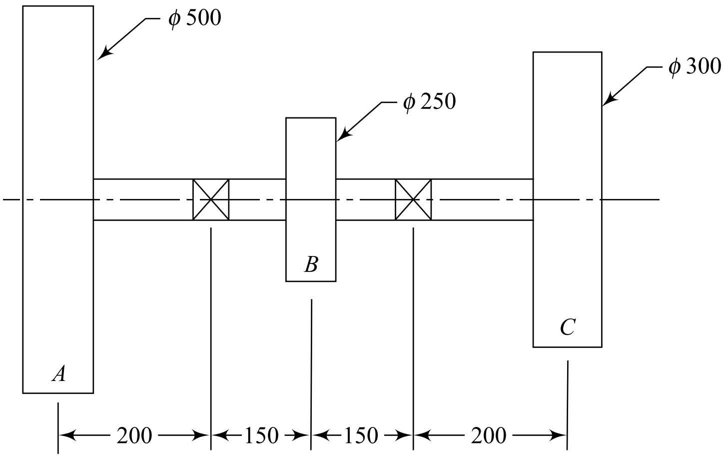


Fig. P-7.14

7.15 A shaft of 40C18 steel receives a power of 100 kW through a pair of spur gears *A* and *B* of 150 mm and 450 mm diameter rotating at 1800 and 600 r.p.m respectively. The power is delivered to gear *D* through a pinion C250 mm diameter. The gears have 14½° involute teeth. Using a factor of safety of 3, K_b = 2 and K_t = 1.5, find the shaft diameter.

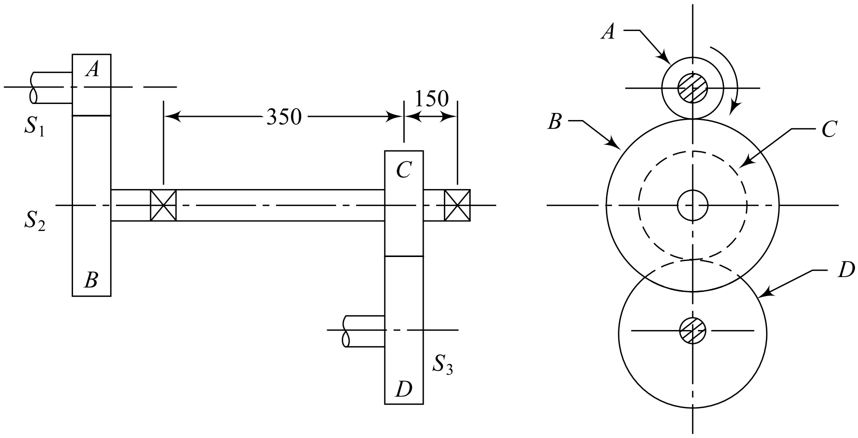


Fig. P-7.15

7.16 Design a protected type C.I flange coupling to transmit 36.5 kW at 500 r.p.m. Materials for the shaft, flanges and key are 30C8, 200 F.G C.I and 20C4 steel respectively and the bolts are made of 20C4 steel. Assume the value of F·S as per the requirement.

7.17 Design a bushed pin type of flexible coupling to transmit 20 kW at 1000 r.p.m. Use 30C4 for pins and keys, 30C8 for shafts and 200 F.G C.I for flanges. The permissible bearing pressure for rubber bush is 0.5 MPa.

7.18 A shaft is used to transmit 40 kW at 200 r.p.m by using a pulley of 500 mm diameter and 100 kg mass. The power is taken by a horizontal belt drive with ratio of belt tensions 2 : 1. The pulley is situated at the centre of the shaft 1.8 m long, $K_t = K_m$ = 1.5. Find the diameter of shaft with safe shear stress of 40 MPa.

7.19 An overhung crank is subjected to a tangential load of 2000 N at 150 mm radius and an overhang of 200 mm. Design the shaft with safe shear stress of 40 MPa, $K_m = 1.2$, $K_t = 1$.

7.20 A chain drive is used to transmit 30 kW to a shaft rotating at 450 r.p.m power is taken off by a pulley of 600 mm diameter and 250 kg mass. The ratio of tensions for pulley is 3.5 = 1. The force on the sprocket is represented by T_c and that on the slack side may be neglected (of the chain) being very small. Determine the size of the shaft using $K_m = 1.5$, $K_t = 1$, safe shear stress = 40 MPa.

7.21 A shaft takes 60 kW power from electric motor and transmit it to two machines through pulleys *A* and *B* of 250 kg and 200 kg masses and 600 mm and 450 mm diameter respectively. The power is taken from electric motor rotating at 1500 r.p.m through a gearing with velocity ratio 3:1. The type of the gears are 20° involute spur and diameter of gear is 300 mm. The ratio of tensions for both pulleys is 3:1. $K_m = K_t = 1.5$. Find the diameter of shaft. Use $\tau = 40$ MPa.

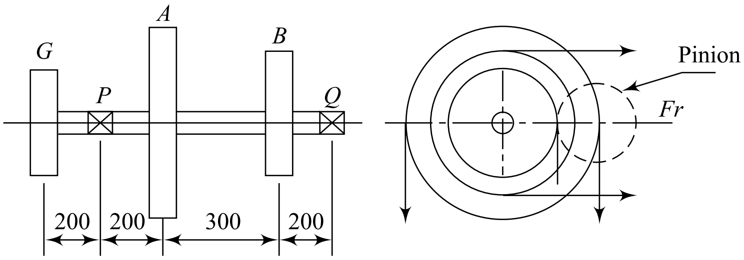


Fig. P-7.21

7.22 A Hooke's joint is used to connect two shaft intersecting at 30° with each other. Input shaft transmits 5 kW 800 r.p.m. What will be the speed of output shaft? Find the dimensions of forks and pins. $P_b = 20$ MPa, $S_y = 3000$ MPa.

7.23 Design protected type of flange coupling to transmit 25 kW at 200 r.p.m, p.c.d of bolts is 240 mm.

7.24 Design a shaft subjected to a twisting moment of 100 Nm and B.M of 150 Nm with $K_m = K_t = 1.5$ using safe shear stress = 40 MPa.

ANSWERS

Objective Questions

(1) b (2) b (3) c (4) b (5) c (6) b (7) b (8) c (9) b
(10) d (11) b (12) c (13) c (14) a

Practice Problems

(3) 60 mm (5) 70 mm (6) 70 mm (7) 18 mm
(9) 69 mm (10) 40 mm (11) 85.76 → 90 mm
(12) 87.8 → 90 mm (13) 99.63d → 100 mm
(14) 5.5 m (15) 73.25 → 75 mm
(18) $d = 125.99 \simeq 130$ mm, $Q = 0.854°$ (19) 41.61 → 45 mm
(20) $d = 64.28 \rightarrow 65$ mm (21) $d = 62.61 \rightarrow 65$ mm
(22) 692.8 r.p.m, input shaft $d = 19.66 \rightarrow 20$ mm, output shaft $d_p = 12$ mm, thickness of fork = 10 mm
(23) Diameter of shaft 50 mm, bolts M 10 × 1.5.
(24) 35 mm ϕ

8

Springs

CONCEPT REVIEW

8.1 DEFINITION

A *spring* is an elastic body which deflects or distorts under the action of load and regains its original shape after the load is removed.

8.2 FUNCTIONS OF SPRINGS

(1) To absorb energy and mitigate shock as in case of a shock absorber spring.
(2) To apply definite force or torque, e.g. the spring used in safety valves, spring washer.
(3) To support moving masses for vibration isolation, e.g. automobile suspension springs.
(4) To indicate or control load or torque, e.g. a spring balance.
(5) To store energy, e.g. a clock spring.

8.3 CLASSIFICATION OF SPRINGS

(1) Helical compression or tension spring termed as close coiled helical spring.
(2) Helical torsion spring.
(3) Leaf or carriage spring.
(4) Flat spring.
(5) Spiral spring.
(6) Belleville spring.
(7) Disc spring.

8.4 MATERIAL

The primary purpose of most springs is to store energy. Since the energy stored is proportional to the induced stress, it is desirable to use a high strength material for spring.

8.5 DESIGN EQUATIONS FOR CLOSE COILED HELICAL SPRING

A helical spring is equivalent to a shaft subjected to twisting moment as shown in Fig. 8.1. The torque acting on the spring of a wire of diameter d and mean radius R is $P \times R$. It induces a shear stress τ_1

$$\tau_1 = \frac{\text{Torque}}{\text{Polar section modulus}}$$

$$= \frac{P \cdot R}{\frac{\pi}{16} d^3} \text{ substituting } R = \frac{D}{2}, \text{ where}$$

D is the mean diameter we get,

$$\tau_1 = \frac{8PD}{\pi d^3}$$

Direct shear stress, τ_2 due to transverse load P is

$$\tau_2 = 1.23 \frac{P}{A} \text{ where the factor 1.23 has}$$

been arrived at experimentally.

$$= \frac{1.23P}{\frac{\pi}{4} d^2} = \frac{4 \times 1.23\, P}{\pi d^2}$$

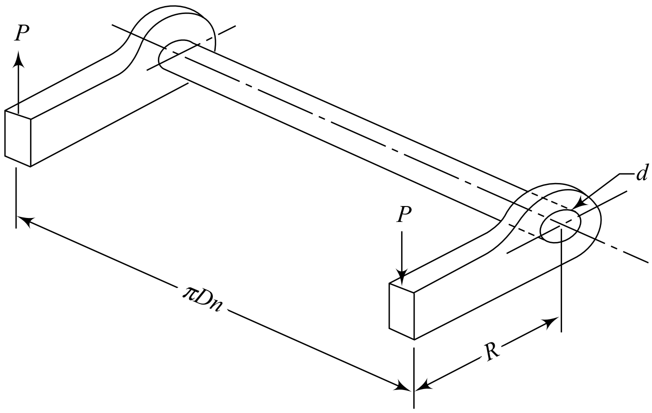


Fig. 8.1

By combining τ_1 and τ_2

$$\tau_{(\text{total})} = \frac{8PD}{\pi d^3} + \frac{4 \times 1.23\, P}{\pi d^2} = \frac{8PD}{\pi d^3}\left(1 + 0.615\frac{d}{D}\right)$$

Let $\frac{D}{d} = C$ = Spring index

$$\tau_{\max} = \frac{8PC}{\pi d^2}\left(1 + \frac{0.615}{C}\right)$$

In actual practice the stress distribution is not just the summation but is as shown in Fig. 8.2(b). This modified stress distribution suggested by A.M. Wahl gives the maximum total shear stress as

$$\tau = \frac{8\,PD}{\pi d^3}\left(\frac{4C-1}{4C-4} + \frac{0.615}{C}\right) = \frac{8\,KPC}{\pi d^2} \tag{8.1}$$

Where Wahl's correction factor, $K = \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C}$

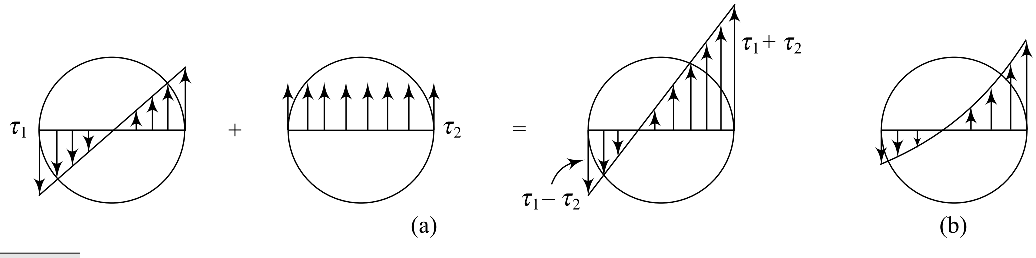


Fig. 8.2

8.6 ACTIVE NUMBER OF TURNS N

By using twist in the shaft given by $\theta = \dfrac{Tl}{JG}$, where θ is angle of twist, l the length of the shaft, G modulus of rigidity of material and J the polar moment of inertia of the cross section, the angular twist of section CD w.r.t. section AB situated at a distance dl is given by

$$d\theta = \frac{PRdl}{JG}$$

This causes the point E situated 90° away to move by an amount equal to

$$d\delta = Rd\theta = \frac{PR^2\,dl}{JG}$$

Integrating this expression over the entire length of the spring, the deflection is given as

$$\delta = \frac{64\,PR^3\,n}{Gd^4} = \frac{8\,PD^3\,n}{Gd^4} = \frac{8\,PC^3\,n}{Gd}$$

where n is the number of active turns

$$\therefore \qquad n = \frac{Gd\delta}{8\,PC^3} \tag{8.2}$$

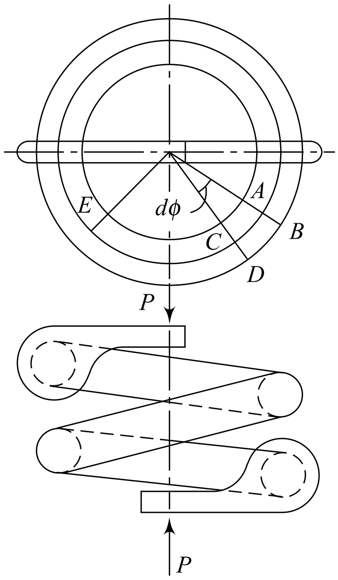


Fig. 8.3

In a compression spring the ends can be (1) plain, (2) plain and ground, (3) squared, (4) squared and ground. The number of inactive turns in each case are 1.5, 1, 1 and 2 respectively. Hence, this number is added to the calculated value of number of turns from Eq. (8.2) to get the actual number of turns.

8.7 DESIGN OF CARRIAGE SPRING

A triangular leaf as shown in Fig. 8.4 may be treated as a cantilever for which the bending stress at a section at distance x may be written as

$$\sigma_x = \frac{P \cdot x}{Z_x}$$

where Z_x is the section modulus at distance $x = \dfrac{1}{6} b_x t^2$

and $$b_x = B \cdot \frac{x}{l}$$

$$\therefore \qquad \sigma_x = \frac{6\,P \cdot x}{b_x t^2} = \frac{6\,P \cdot x}{B\,(x/l)\,t^2} = \frac{6\,Pl}{Bt^2} \tag{8.3}$$

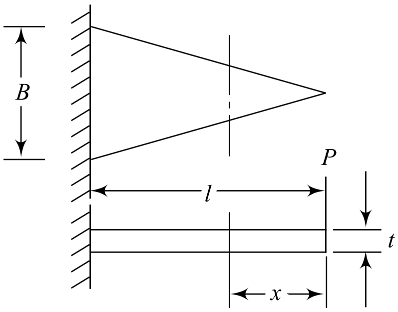


Fig. 8.4

This is constant and hence leaf spring is termed as a beam of uniform strength.

The triangular leaf is cut in leaves of width b and $b/2$ as shown in Fig. 8.5. These leaves are stacked over each other to form the leaf spring as shown in Fig. 8.6. The deflection equation of this type of spring is also derived from the normal deflection equation of cantilever as

$$\frac{d^2 y}{dx^2} = \frac{M_x}{EI_x} = \frac{12\,P \cdot x}{E\,(B\,(x/l)\,t^3} = \frac{12\,Pl}{EBt^3}$$

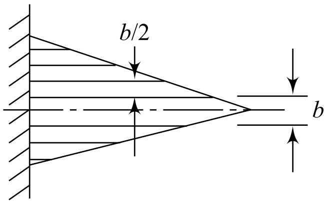


Fig. 8.5

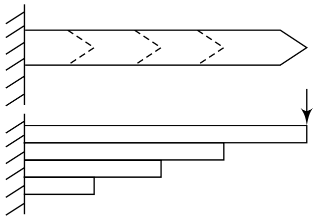


Fig. 8.6

Integrating the above equation

$$\therefore \quad \frac{dy}{dx} = \frac{12\,Plx}{EBt^3} + c_1$$

and $$y = \frac{12\,Plx^2}{2\,EBt^3} + c_1x + c_2 \text{ on further integration.}$$

Substituting $\frac{dy}{dx} = 0$ at $x = l$ and $y = 0$ at $x = l$

We have $$c_1 = \frac{-12\,Pl^2}{EBt^3},\; c_2 = \frac{6\,Pl^3}{EBt^3}$$

$\therefore$ Maximum deflection y at $x = 0$ is given by

$$y = \frac{6\,Pl^3}{EBt^3} \qquad (8.4)$$

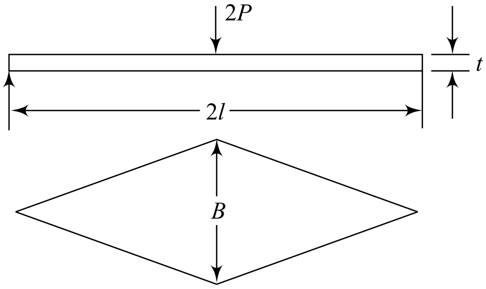


Fig. 8.7

A leaf spring of simply supported type as shown in Fig. 8 7 will give the same equations of stress and deflection as Eqs (8.3) and (8.4) when the load is $2P$ and the unsupported length is $2\,l$.

8.8 SPRING WITH EXTRA FULL LENGTH LEAVES

The leaves which are cut from the original triangular leaf, are termed as graduated leaves. In actual practice some extra leaves with the same length as that of the top leaf are added to increase the stiffness of the spring.

The extra full length leaves behave as ordinary cantilever beam for which the deflection is

$$y = \frac{4\,Pl^3}{Ebt^3}$$

Let n_e and n_g represent the number of extra full length and graduated leaves respectively.

As the leaves are stacked over each other the total load is shared by them such that deflection of the two is the same.

Let $$P = P_g + P_e$$

$$\delta_g = \frac{6\,P_g l^3}{En_g bt^3} = \delta_e = \frac{4\,P_e l^3}{En_e bt^3}$$

$$\therefore \quad \frac{P_g}{2\,n_g} = \frac{P_e}{3\,n_e} = \frac{P_g + P_e}{(2\,n_g + 3\,n_e)} = \frac{P}{(2\,n_g + 3\,n_e)}$$

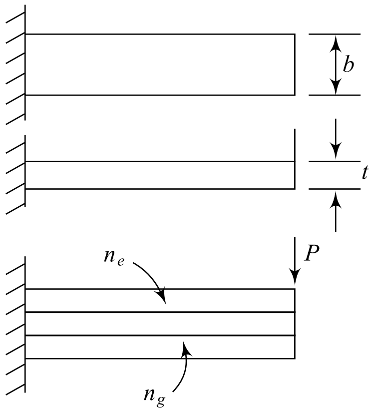


Fig. 8.8

$$\therefore \qquad P_g = \frac{P(2n_g)}{(2n_g + 3n_e)} \text{ and } P_e = \frac{P(3n_e)}{(2n_g + 3n_e)}$$

$$\therefore \qquad \sigma_g = \frac{6P_g l}{n_g bt^2} = \frac{12\,Pl}{(2n_g + 3n_e)bt^2} \tag{8.5}$$

Further,

$$\sigma_e = \frac{18\,Pl}{(2n_g + 3n_e)\,bt^2} \tag{8.6}$$

and, deflection

$$y = \frac{12\,Pl^3}{(2n_g + 3n_e)\,Ebt^3} \tag{8.7}$$

While fitting, if the extra full length leaves are prestressed then $\sigma_e = \sigma_g$ the value of σ_e is calculated by substituting $B = b\,(n_e + n_g)$ in Eq. (8.3) while the deflection is calculated by using Eq. (8.7).

8.9 DESIGN OF SPRING SUBJECTED TO VARIABLE LOAD

A close coiled helical spring subjected to fluctuating load fails due to fatigue. The maximum variation of load in such a spring is only on positive or negative side which in extreme case may induce the stress condition as shown in Fig. 8.9. It can be seen from this figure that

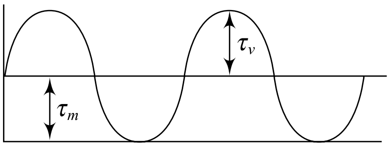


Fig. 8.9

$$\tau_m = \tau_v = \frac{\tau_{\max}}{2}$$

For the part subjected to a variable load of completely reversible type $\tau_m = 0$ and $\tau_v = S_{es}$. Hence, in this case $\tau_m = \tau_v = \dfrac{S_{es}}{2}$. Thus, Soderberg's diagram for a spring starts from the point C as shown in Fig. 8.10 (b) whereas Fig. 8.10 (a) represents Soderberg's diagram for a component subjected to completely reversible shear stress.

From Fig. 8.10 (b)

$$PQ = \tau, \quad OQ = \tau_m, \quad OE = S_{ys}/\,\text{F·S}, \quad OB = S_{ys}$$

$$CF = S_{es}/2, \; FD = \frac{S_{es}/2}{\text{F·S}}$$

From similar triangles PQE and CFB, we have

$$\frac{PQ}{CF} = \frac{QE}{FB}$$

$$\therefore \qquad \frac{\tau_v}{S_{es}/2} = \frac{S_{ys}/N - \tau_m}{S_{ys} - S_{es}/2} \tag{8.8}$$

where $\quad \tau_v = \dfrac{8K_v P_v C}{\pi d^2} \quad$ and $\quad K_v = \dfrac{4C-1}{4C-4}$

$$\tau_m = \frac{8K_m P_m C}{\pi d^2} \quad \text{and} \quad K_m = \frac{0.615}{C} + 1$$

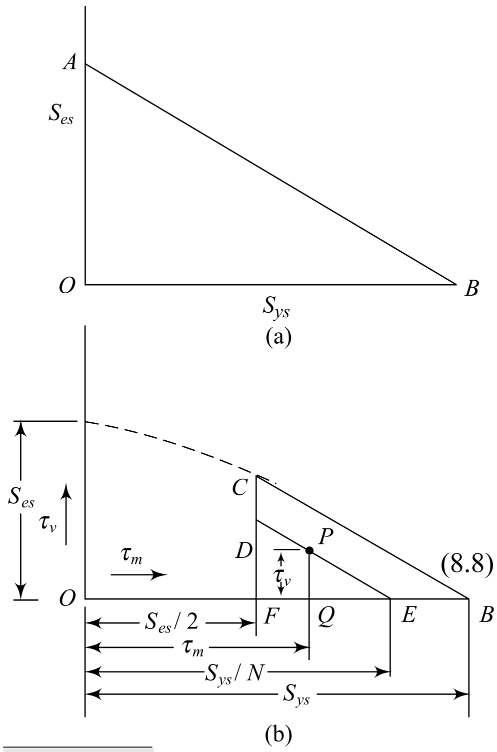


Fig. 8.10

8.10 SPRINGS WITH NON-CIRCULAR CROSS SECTION

The theory of helical springs with non-circular cross section is based on the torsion of non-circular bars. The equations to be used are

$$\tau_1 = \frac{PR}{\alpha_1 bc^2} \text{ at point } A_1 \quad (8.9)$$

$$\tau_1 = \frac{PR}{\alpha_2 bc^2} \text{ at point } A_2 \quad (8.10)$$

$$\delta = \frac{2\pi PR^3 n}{\beta Gbc^3} \quad (8.11)$$

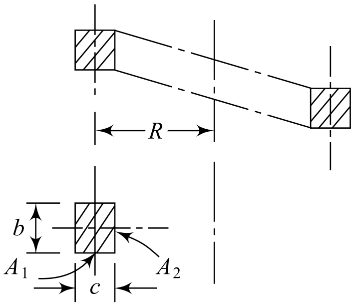


Fig. 8.11

Table 8.1 *Constants for Equations*

b/c	1.0	1.2	1.5	2.0	2.5	3.6	4.0	5.0	∞
α_1	0.208	0.219	0.231	0.246	0.258	0.267	0.282	0.291	0.333
α_2	0.208	0.235	0.269	0.309	0.336	0.355	0.378	0.392	…
β	0.1406	0.166	0.196	0.229	0.249	0.283	0.281	0.291	0.333

Torsional shear stress thus calculated, should be modified by adding to it the transverse shear stress $\tau_2 = 1.5\ P/A$

$\therefore$
$$\tau_{total} = \tau_1 + \tau_2 \quad (8.12)$$

8.11 SURGING OF HELICAL COMPRESSION SPRING

The sudden application of compression load sets a compression wave in the spring which travels along the length of the spring and reflects at the far end. If the frequency of application of load matches the natural frequency of vibration, very high stresses are induced in the spring material causing fatigue failure. It can be avoided by keeping the natural frequency at least 20 times greater. The natural frequency is given by

$$f = \frac{d}{2\pi D^2 n}\sqrt{\frac{G}{2\rho}} \text{ Hertz,} \quad \text{where } \rho = \text{density of material.}$$

WORKED EXAMPLES

8.1 A coil spring is used for the front suspension of an automobile. The spring has a stiffness of 90 N/mm with squared and ground ends. The material used is oil tempered chrome vanadium steel for which the permissible shear stress may be taken as 500 MPa. The load on the spring causes a total deflection of 8.5 mm. Find the spring wire diameter and free length of the spring using a spring index of 6.

Solution:

$$P_{max} = 90 \times 8.5 = 765 \text{ N}$$

$$K = \frac{4\times 6-1}{4\times 6-4} + \frac{0.675}{6} = 1.2525$$

From Eq. (8.1), $d = \sqrt{\dfrac{8KPC}{\tau\cdot\pi}}$

$$= \sqrt{\frac{8\times 1.2525\times 765\times 6}{500\times \pi}} = 5.41 \text{ mm}$$

Using Eq. (8.2)

$$n = \frac{Gd\delta}{8PC^3} = \frac{0.84\times 10^5\times 5.41\times 8.5}{8\times 765\times 6^3} = 2.92$$

The calculated value of number of turns is modified to 5 since the ends of the spring are ground and squared. If after a deflection of 8.5 mm a gap of 0.5 mm is left, the free length l of the spring is

$$l = 5\times 5.41 + 8.5 + 0.5\times 4 = 37.55 \text{ mm}$$

8.2 A helical spring of rate 10 N/mm is mounted on top of another spring of rate 8 N/mm. Find the force required to give a total deflection of 45 mm.

Solution:

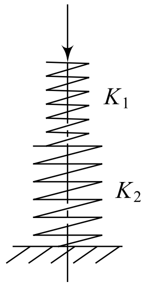


Fig. E-8.2

In this case (Fig. E-8.2) total stiffness K_t is given by, $\dfrac{1}{K_t} = \dfrac{1}{K_1} + \dfrac{1}{K_2}$.

$$\frac{1}{K_t} = \frac{1}{8} + \frac{1}{10}, \quad \therefore K_t = 4.44 \text{ N/mm}$$

For a deflection of 45 mm, the force required is $45\times 4.444 = 199.999 \cong 200$ N.

8.3 The larger of the two concentric springs made of 38 mm diameter round bar has a mean coil diameter of 228 mm and 6 active coils. The inner spring has a wire diameter 25 mm, spring index 5 and number of active coils equal to 9. Free height of the outer spring is 19 mm more than the inner. Find the deflection of each spring for a load of 90000 N. Take G = 77,000 MPa. Calculate the load carried by each spring.

Solution:

Part of the total load is utilized in deflecting outer spring by 19 mm while remaining load is shared by both springs acting in parallel, stiffness in that case is given by

$$K_t = K_1 + K_2$$

Stiffness of outer spring, $K_1 = \dfrac{77000\times 38}{8\times 6^3\times 6}$

$$= 311.012 \text{ N/mm}$$

as $C = \dfrac{223}{38} = 6$

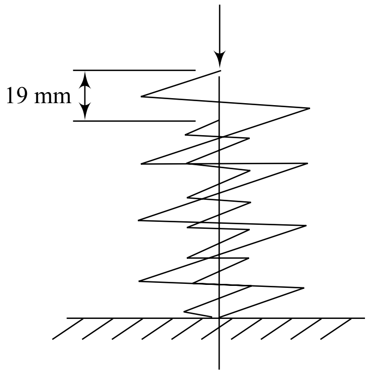


Fig. E-8.3

Stiffness of inner spring, $K_2 = \dfrac{77000 \times 25}{8 \times 5^3 \times 9} = 213.88$ N/mm as $C = 5$

Total load = Load shared by outer spring for 19 mm deflection + Load shared by outer spring for further deflection + Load shared by inner spring for further deflection.

Load causing 19 mm deflection of outer spring

$= 311.012 \times 19 = 5909.22$ N

Remaining load = 849090.78 N

It is shared by two springs in proportion to the stiffnesses

$\therefore$ Load shared by outer spring = $\dfrac{311.012 \times 849090.78}{(311.012 + 213.88)}$

= 50311.191 N

$\therefore$ Total load on outer spring =

50311.191 + 5909.22 = 56220.411 N

8.4 A spring loaded safety valve is held against its seat by a close coiled helical compression spring. The diameter of the valve is 75 mm and blow off pressure is 1.1 MPa. Mean diameter of the coil is 100 mm and compression is 25 mm. Find the diameter of spring bar and the number of active coils if permissible shearing stress is 500 MPa and $G = 0.8 \times 10^5$ MPa.

Solution:

Maximum load on the spring = $1.1 \times \dfrac{\pi}{4} \times 75^2 = 4859.6511$ N

Now, $$500 = \frac{8K \times 4859.65 \times C}{\pi \times (100/C)^2}$$

$\therefore$ $KC^3 = 404.04$ putting $K = \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C}$

by trial and error $C = 7$

$\therefore$ $d = 150/7 = 21.43 \text{ mm} \cong 21.50$ mm

$$n = \frac{0.8 \times 10^5 \times 21.5 \times 25}{8 \times 73 \times 4859.65} = 3.224 \text{ modified to } 4 + 2 = 6.$$

8.5 In the above example let the normal pressure on the valve be 1.00 MPa, blow off pressure remaining the same. The deflection of spring causing the opening of valve is 3.5 mm. What will be the change in the design procedure and calculations?

Solution:

The maximum load and hence the dimensions d remains unchanged. For calculating the number of active turns the force causing the deflection $\delta = 3.5$ mm is given by

$$P = \frac{\pi}{4}(1.1^2 - 1^2) \times 75^2 = 485.965 \text{ N}$$

$\therefore$ $$n = \frac{0.8 \times 10^5 \times 21.5 \times 3.5}{8 \times 7^3 \times 485.965} = 4.51 \text{ modified to } 5 + 2 = 7$$

8.6 In Example 8.4 if a spring with square cross-section is used, find the side of the square and the number of active coils.

Solution:

$$\tau_1 = \frac{PR}{0.208c^3}, \quad \tau_2 = \frac{1.5P}{c^2}$$

$$\therefore \quad 500 = \frac{4859.65 \times 50}{0.208c^3} + \frac{4859.65 \times 1.5}{c^2}$$

By trial and error $c = 14$ mm

$$n = \frac{25 \times 0.1406 \times 0.8 \times 10^5 \times 14^4}{2\pi \times 4859.65 \times 50^3} = 2.83 \text{ modified to } 3 + 2 = 5.$$

8.7 A railway wagon of mass 20000 kg moving with a velocity of 2 m/s is brought to rest by two buffers of a spring of diameter 300 mm. The maximum deflection of the spring is 200 mm. Permissible shear stress is 600 MPa. Find the dimensions of each spring.

Solution:

K.E of wagon = strain energy stored in spring

$$\therefore \quad \frac{1}{2} \times 20000 \times (2)^2 = 2 \times \left[\frac{1}{2} P \times \delta\right] = P \times \frac{200}{1000}$$

$$\therefore \quad P = 200000 \text{ N}$$

$$\therefore \quad 600 = \frac{8K \times 200000 \times C}{\pi (300/C)^2}$$

$$\therefore \quad KC^3 = 106.028 \text{ by trial and error, } C = 4.25$$

$$\therefore \quad d = 300/4.25 = 70.58 \text{ mm}$$

$$n = \frac{0.8 \times 10^5 \times 70.58 \times 200}{8 \times (4.25)^3 \times 200000} = 9.1942 \cong 10$$

$\therefore$ Total number of turns 10 + 2 = 12.

8.8 Design a close coiled helical spring subjected to a tensile load of magnitude varying from 2500 N to 3000 N. The axial deflection of spring for this range of load is 6.5 mm. Design the spring, taking the spring index as 6 and the safe shear stress for material of the spring equal to 465 MPa.

Solution:

$$K = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{4} = 1.2525$$

Maximum load = 3000 N

$$d = \sqrt{\frac{8 \times 1.2525 \times 3000 \times 6}{\pi \times 465}} = 11.11 \text{ mm}$$

$\therefore$ Standard Wire Gauge 6/0 with 11.785 mm diameter may be adapted. An increase in load by 500 N causes a deflection of 6.5 mm

$$\therefore \quad n = \frac{0.8 \times 10^5 \times 11.785 \times 6.5}{8 \times 63 \times 500} = 7.092$$

Adding 2 more turns, $n = 9$

$$D = 6 \times 11.785 = 70.71 \text{ mm}$$

Maximum deflection for 3000 N = $\dfrac{3000}{500} \times 6.5 = 39$ mm

$\therefore$ Free length = $9 \times 11.785 + 39 + 8 \times 0.5 = 149.065$ rounded to 150 mm.

A gap of 0.5 mm after full compression is assumed.

8.9 A load of 5 kN is dropped from a height of 50 mm axially on the spring of a wire of diameter 12 mm, spring index equal to 6 and the number of active coils as 8. Find the stress induced in the spring.

Solution:

If P is the effective load on spring then,

$$500\,(50 + \delta) = \frac{1}{2} P\delta \text{ where } \delta \text{ is the deflection in mm.}$$

as $$\delta = \frac{8PC^3 n}{Gd} = \frac{8P \times 216 \times 8}{0.8 \times 10^5 \times 12} = 0.0144 \text{ P mm}$$

$\therefore$ $5000\,(50 + 0.0144\text{ P}) = 0.0072\, P^2$

$\therefore$ $$P^2 - 10000\,P - 34722222 = 0$$

$\therefore$ $$P = \frac{10000 + \sqrt{10000^2 + 4 \times 34722222}}{2} = 12728.015 \text{ N}$$

For $C = 6$ and $K = 1.2525$

$\therefore$ $$\tau = \frac{8 \times 1.2525 \times 12728.015}{\pi \times 144} = 281.91 \text{ MPa.}$$

8.10 A semi-elliptic spring has an effective length of 1.2 m. The maximum stress not exceeding 400 MPa, the deflection under a maximum load of 9 kN is 100 mm. Find the thickness and width of leaves if (a) the extra full length leaves are prestressed, (b) extra full length leaves are not prestressed. Number of extra full length leaves is 2 and the number of graduated leaves is 10.

Solution:

Under condition (a), maximum stress induced in all the leaves is the same and is given by

$$\sigma = \frac{6Pl}{nBt^2}$$

and $$\delta = \frac{12Pl^3}{Bt^3 E\,(3n_e + 2n_g)} \quad \text{where } n = 12,\ n_e = 2 \text{ and } n_g = 10$$

$\therefore$ $$\frac{\sigma}{\delta} = \frac{Et\,(3n_e + 2n_g)}{2nl^2}$$

Substituting $l = 600$ mm, we get

$\therefore$ $$t = \frac{400}{100} \times \frac{2 \times 12 \times 600^2}{2 \times 10^5\ (3 \times 2 + 2 \times 10)} = 6.646 \text{ mm}$$

Substituting this value of t, $\sigma = 400 = \dfrac{6 \times 4500 \times 600}{12B(6.646)^2}$

$\therefore$ $$B = 80.87 \text{ mm}$$

For condition (b), $$\sigma_{max} = \frac{18Pl}{Bt^2\ (3n_e + 2n_g)}$$

Since the maximum stress is induced in extra full length leaves and $\delta = \dfrac{12Pl^3}{Bt^3\,(3n_e + 2n_g)}$ by taking the ratio of $\sigma_{\max}$ and δ we get $t = \dfrac{\sigma_{\max} l^2}{1.5\,E\delta}$ = 4.8 mm

$$B = \frac{18Pl}{\sigma t^2\,(3n_e + 2n_g)} = \frac{18 \times 4500 \times 600}{400 \times 4.8^2 \times 26} = 202.82 \text{ mm}$$

8.11 A helical spring is subjected to a continuously varying load. A number 7 oil tempered wire is used with the mean diameter of the coil as 26 mm. The maximum and minimum force acting on the spring is 400 N and 260 N respectively and deflection during this variation is 8 mm. Find the factor of safety and number of active turns. For No. 7 wire oil tempered S_u = 1400 MPa, S_{ys} = 0.4 S_u, S_{es} = 0.23S_u and d = 4.5 mm.

Solution:

$$\because \quad C = 5.777, \qquad \therefore\ K_v = \frac{4C-1}{4C-4} = 1.157$$

$$K_m = 1 + \frac{0.615}{C} = 1.106$$

and
$$P_m = \frac{400+260}{2} = 330 \text{ N}, \quad \therefore\ \tau_m = \frac{8 \times 1.106 \times 330 \times 5.77}{\pi \times (4.5)^2} = 265.2 \text{ MPa}$$

$$P_v = \frac{400-260}{2} = 70 \text{ N}, \quad \therefore\ \tau_v = \frac{8 \times 1.157 \times 70}{\pi\,(4.5)^2} \times 5.777 = 58.9254 \text{ MPa}$$

Using Eq. (8.8)

$$\frac{58.9254}{1400 \times 0.23/2} = \frac{1400 \times 0.4/N - 265.2}{560 - 1400 \times 0.23/2}$$

$$\therefore \quad \frac{58.9254}{161} = \frac{560/N - 265.2}{399}$$

$\therefore$ N = 1.36. A deflection of 8 mm is caused due to a force of (400 – 260) = 140 N

$$\therefore \quad n = \frac{0.8 \times 10^5 \times 4.5 \times 8}{8 \times 140 \times 5.77^3} = 13.33$$

8.12 A helical compression spring carries a fluctuating load varying from 428 N to 642 N. The spring index is 6 and factor of safety is 1.5. S_{ys} = 648 MPa, S_{es} = 375 MPa. Calculate the spring wire diameter and the number of effective turns if deflection due to variation in load is 4 mm.

Solution:

$$P_m = 535 \text{ N}, \quad P_v = 107 \text{ N}$$

$$K_v = \frac{4 \times 6 - 1}{4 \times 6 - 4} = 1.15, \quad K_m = 1 + \frac{0.615}{6} = 1.1025$$

$$\therefore \quad \tau_m = \frac{8 \times 1.1025 \times 535 \times 6}{\pi d^2} = \frac{9012.05}{d^2} \text{ MPa}$$

$$\tau_v = \frac{8\times 1.15\times 107\times 6}{\pi d^2} = \frac{1880.06}{d^2}\ \text{MPa}$$

$$\therefore \qquad \frac{1880.06/d^2}{(648/1.5)-(9012.05/d^2)} = \frac{375/2}{(648/1.5)-(375/2)}$$

from which $d = 5.152$ mm. A load of (642 – 428) = 214 N causes a deflection of 4 mm

$$\therefore \qquad n = \frac{0.8\times 10^5\times 5.152\times 4}{8\times 214\times 6^3} = 4.458 \text{ which is modified to 5.}$$

8.13 Design the cantilever leaf spring to absorb 600 N.m energy without exceeding a deflection of 150 mm and permissible stress of 800 MPa. The effective length of the spring is 500 mm. $E = 0.2\times 10^6$ MPa.

Solution:

$$\text{Energy stored} = \frac{1}{2}P\delta$$

$$\therefore \qquad P = \frac{2\times 600\times 1000}{150} = 8000 \text{ N}$$

Using Eqs. (8.3) and (8.4)

$$800 = \frac{6\times 8000\times 500^3}{nbt^2} \qquad\qquad 150 = \frac{6\times 8000\times 500^3}{2\times 10^5\times \pi bt^3}$$

$$\therefore \text{ Taking the ratio,} \qquad t = \frac{800\times 500\times 500}{150\times 0.2\times 10^6} = 6.67 \text{ mm}$$

Putting this value in either equation

$$nb = 674.32$$

∴ Use 9 leaves each of 75 mm width and 6.67 mm thickness.

8.14 A close coiled helical compression spring is used in the spring loaded safety valve of 80 mm diameter. The blow off pressure is 1.4 MPa and maximum lift is 18 mm. Material of the spring is oil quenched steel with a safe shear stress of 500 MPa. Spring index is 6. The normal pressure inside the boiler is 1.00 MPa and $G = 0.84\times 10^5$ MPa. Design the spring.

Solution:

$$\text{Maximum load} = 1.4\times \frac{\pi}{4}\times 80^2 = 7031.1675 \text{ N}$$

$$\text{Minimum load} = 1\times \frac{\pi}{4}\times 80^2 = 5026.55 \text{ N}$$

Load causing a deflection of 18 mm = 7037.1675 – 5026.55 = 2010.62 N

For a spring index of 6, $K = 1.2525$

∴ Using the shear stress equation

$$d = \sqrt{\frac{8\times 1.2525\times 7037.1675\times 6}{\pi\times 500}} = 16.41 \text{ mm} \cong 16.5 \text{ mm}$$

$$\therefore \qquad D_m = 99 \text{ mm}$$

$$n = \frac{Gd\delta}{8PC^3} = \frac{0.84 \times 10^5 \times 16.5 \times 18}{8 \times 2010.62 \times 63} = 7.18$$

∴ Actual number of turns = 10

∴ Free length = 10 × 16.5 + total deflection + gap × (n – 1)

Total deflection corresponding to the maximum load = $\frac{7037.1675}{2010.62} \times 18 = 62.99$ mm.

∴ Free length = 165 + 62.99 + 1 × 9 = 236.99 mm rounded to 237 mm.

8.15 Design a tension spring for a spring balance when the maximum load to be weighed is 1000 N. Length of the scale is 100 mm and the spring index is 5. The material has the maximum permissible shear stress of 600 MPa and $G = 0.8 \times 10^5$ MPa.

Solution:

$$K = \frac{20-1}{20-4} + \frac{0.615}{5} = 1.3105$$

∴ $$d = \sqrt{\frac{8 \times 1.3105 \times 100 \times 5}{\pi \times 600}} = 5.2726 \text{ mm rounded to } 5.3 \text{ mm}$$

∴ $$D_m = 26.5 \text{ mm}$$

$$n = \frac{0.8 \times 10^5 \times 5.3 \times 100}{8 \times 1000 \times (5)^3} = 42.4 \text{ modified to } 44.$$

8.16 The blow off pressure for a safety valve is 1.2 MPa with the maximum lift of the valve as 10 mm. The valve of diameter 69 mm is loaded with a spring of spring index 5.5 and an initial compression of 40 mm. Maximum permissible shear stress for the spring material is 500 MPa, $G = 0.8 \times 10^5$ MPa. Design the spring.

Solution:

$$K = \frac{4 \times 5.5 - 1}{4 \times 5.5 - 4} + \frac{0.615}{5.5} = 1.2785$$

Load $$P = \frac{\pi}{4} d^2 \times p = \frac{\pi}{4} \times 60^2 \times 1.2 = 3392.92 \text{ N}$$

∴ $$d = \sqrt{\frac{8 \times 1.2785 \times 3392.92 \times 5.5}{\pi \times 500}} = 11.02 \text{ mm}$$

Thus a standard wire of diameter 11.785 is adapted.

∴ $$D_m = 11.785 \times 5.5 = 64.8175 \text{ mm}$$

Total deflection = Initial compression + Lift = 40 + 10 = 50 mm

∴ $$n = \frac{Gd\delta}{8C^3P} = \frac{0.8 \times 10^5 \times 11.785 \times 50}{8 \times 5.5^3 \times 3392.92}$$

$$= 11.4 \text{ modified to } (11 + 2) = 13$$

∴ Free length = 13 × 11.785 + 50 + 12 × 0.5 = 209.205 mm.

8.17 A helical compression spring is subjected to a load varying between 800 and 1500 N. The material used is oil tempered cold drawn wire having S_{ys} = 700 MPa and S_{es} = 356 MPa. Find the diameter of the wire and the number of coils if C = 5 and N = 2.5.

Solution:

$$K_m = 1 + \frac{0.615}{5} = 1.123 \qquad K_v = \frac{4C-1}{4C-4} = 1.1875$$

$$P_m = \frac{800+1500}{2}, \quad \therefore\ \tau_m = \frac{8\times 1.123\times 1150\times 5}{\pi d^2} = \frac{16443.252}{d^2}\ \text{MPa}$$

$$P_v = 750\ \text{N}$$

$$\therefore \qquad \tau_v = \frac{8\times 1.1875\times 750\times 5}{\pi d^2} = \frac{11339.789}{d^2}\ \text{MPa}$$

Using Soderberg's modified equation we have,

$$\frac{1}{N} = \frac{\tau_m - \tau_v}{S_{ys}} + \frac{2\tau_s}{S_{es}}$$

$$\therefore \qquad \frac{1}{2,5} = \frac{5103.463}{700\,d^2} + \frac{2\times 11339.789}{356\,d^2}$$

$$\therefore \qquad d = 13.322\ \text{mm and } D_m = 66.61\ \text{mm.}$$

If the deflection of the spring during load variation is assumed as 7 mm

$$n = \frac{0.8\times 10^5\times 13.322\times 7}{8\times 5^3\times 700} = 10.6576 \cong 11$$

$\therefore$ Total number of turns = 13.

8.18 A close coiled helical compression spring has a mean coil diameter of 60 mm and the diameter of the wire is 10 mm. Number of active and inactive coil turns is 11 and 2 respectively. Free length of the spring is 210 mm. Decide the maximum load that can be applied on the spring if the minimum load is one third of the maximum load. Use F·S = 1.5, S_{ys} = 700 MPa and S_{es} = 1360 MPa.

Solution:

$$K_m = 1.1025, \quad K_v = 1.15$$

$$P_m = \frac{3P_{\min} + P_{\min}}{2} = 2P_{\min}, \quad P_v = \frac{3P_{\min} - P_{\min}}{2} = P_{\min}$$

$$\tau_m = \frac{8\times 1.1025\times 2P_{\min}\times 6}{\pi(10)^2} = 0.337\,P_{\min}\ \text{MPa}$$

$$\tau_v = \frac{8\times 1.15\times P_{\min}\times 6}{\pi(10)^2} = 0.1758\,P_{\min}\ \text{MPa}$$

$$\therefore \qquad \frac{1}{1.5} = \frac{(0.337-0.1758)\,P_{\min}}{700} + \frac{2\times 0.1758\,P_{\min}}{360}$$

$$\therefore \qquad P_{\min} = 552.36\ \text{N},\ P_{\max} = 1657\ \text{N.}$$

OBJECTIVE QUESTIONS

8.1 In a squared and ground helical spring the effective number of turns is increased by
(a) 1 (b) 2 (c) 1.5 (d) 0

8.2 Frequency of the fluctuating load on a helical compression spring should be
(a) less than natural frequency of vibration
(b) twenty times the natural frequency of vibration
(c) slightly greater than the natural frequency
(d) twenty times less than the natural frequency of vibration

8.3 Two concentric springs with stiffness equal to 100 N/mm and 80 N/mm respectively, when subjected to a load of 900 N will deflect by
(a) 9 mm (b) 11.25 mm (c) 5 mm (d) 31.5 mm

8.4 Stiffness of the spring can be increased by
(a) increasing the number of turns (b) increasing the free length
(c) decreasing the number of turns (d) decreasing the spring wire diameter

8.5 The stress induced in an extra full length leaf in case of a prestressed leaf spring is
(a) 1.5 times that in graduated leaves
(b) equal to that in graduated leaves
(c) dependent on the ratio of the number of extra full length and graduated leaves
(d) none of the above

8.6 In a four stroke I.C engine completing 3000 r.p.m, the natural frequency of vibration of the exhaust valve spring should be
(a) 1200 Hz (b) 1000 Hz (c) 600 Hz (d) 120 Hz

8.7 Initial gap between two turns of a close coiled helical tension spring should be
(a) 0.5 mm (b) based on the maximum deflection
(c) 1 mm (d) zero

8.8 A spring with 25 active coils cannot be accommodated within a given space. Hence 5 coils of the spring are cut. The stiffness of the new spring will be
(a) the same (b) 1.25 times the original spring
(c) 0.8 times the original spring (d) none of the above

8.9 The type of stress induced in a close coiled helical tension spring is
(a) tensile (b) torsional shear (c) compressive (d) none of the above

8.10 Wahl suggested the correction in the stress factor to account for
(a) the additional transverse shear stress (b) stress concentration
(c) fatigue stress (d) none of the above

8.11 Springs with rectangular or square cross section are used for
(a) higher stiffness (b) larger volume (c) larger length (d) none of the above

8.12 The ratio of the stress induced in extra full length leaves to that in graduated leaves in a leaf spring is
(a) 2 (b) 1.5 (c) 1.2 (d) none of the above

8.13 Due to addition of extra full length leaves the deflection of a semi-elliptic spring
(a) increases (b) decreases (c) does not change (d) is doubled

8.14 The same stress and deflection formulae can be used for both, semi-elliptic as well as the cantilever springs if
(a) force and maximum span for both is the same

(b) force on a semi-elliptic spring is twice to that on a cantilever spring and the length is half
(c) the force on a semi-elliptic spring is half of that on a cantilever spring and the length is double
(d) the force and length of a semi-elliptic spring are double to those of a cantilever spring

8.15 The starting point on Soderberg's diagram for a spring subjected to variable loading has the coordinates
(a) S_{es}, 0 (b) 0, S_{es} (c) $\frac{S_{es}}{2}, \frac{S_{es}}{2}$ (d) $\frac{S_{es}}{2}, \frac{S_{ys}}{2}$

8.16 A spring of stiffness 100 N/mm used in a spring loaded safety valve of diameter 20 mm on a boiler with the pressure inside the boiler equal to 1 MPa should be initially compressed by
(a) 5 mm (b) 3.14 mm (c) 2 mm (d) 6.28 mm

REVIEW QUESTIONS

8.1 What are the functions of springs? Give one example of each function.
8.2 What are the types of springs?
8.3 What are the different materials used for manufacturing springs?
8.4 Why a strong material should be used for helical springs? Why is it not necessary for a carriage spring?
8.5 Show the distribution of shear stress over the spring wire diameter and derive the relation between shear stress and dimensions of a helical spring.
8.6 Why did Wahl modify the stress factor?
8.7 Why Soderberg's diagram for a helical spring starts from the point where both the coordinates are equal to $\frac{S_{es}}{2}$.
8.8 Write short notes on (a) Surging of springs, (b) Buckling of compression springs.
8.9 Why are extra full length leaves added to graduated leaves in case of the leaf springs?
8.10 Explain why a carriage spring is called as the beam of uniform strength.
8.11 Give the reason for high stress in extra full length leaves.
8.12 Derive the expression for induced stress and deflection of a cantilever carriage spring and explain how the same expressions hold good for semi-elliptic springs.
8.13 Derive the above expressions for a carriage spring using extra full length leaves.
8.14 Explain the method of construction of carriage spring.
8.15 Where are the springs with non-circular cross sections used?
8.16 Derive expression for deflection of close coiled helical spring and from that the expression for stiffness of the spring.
8.17 Derive expression for number of coils of close coiled helical spring in terms of deflection, modulus of rigidity, wire diameter and spring index.
8.18 Differentiate between active and inactive turns in case of close coiled helical spring. How are actual number of turns decided?
8.19 Show the types of end turns in case of close coiled helical compression and tension springs.
8.20 Leaf spring is termed as a beam of uniform strength, justify.
8.21 Derive the expression for deflection of leaf spring with extra full length leaves.

8.22 How does prestressing affect the stress induced in the leaves of leaf spring with extra full length leaves.
8.23 Explain the method of construction of leaf spring. What are nip and master leaf?
8.24 Explain the method of designing helical spring with square or rectangular wire. Where are they used?
8.25 Where are springs in series and parallel used?
8.26 Match springs of group *B* with the applications in group *A*.
(Gr. A) Spring balance, ball pen, door hinges, truck chassis, clock, more energy storage in limited space.
(Gr. B) Leaf spring, tension spring, helical compression spring, spring with non circular section, spiral spring, helical tension spring.

PRACTICE PROBLEMS

8.1 Design a close coiled helical spring for a maximum load of 500 N and a deflection of 200 mm. The permissible shear stress = 500 MPa and spring index = 6.
8.2 A steel spring is made of No. 8 wire to sustain a load of 450 N at a deflection of 60 mm. Find the value of maximum stress and the required number of active coils using the spring index of 5. For No. 8 wire $d = 4.11$ mm. $\tau_{\text{safe}} = 600$ MPa.
8.3 A helical spring with static loading exerts a force of 600 N when it is released by 10 mm from the maximum compressed position. A wire of diameter 6 mm is used. The spring index is 6 and the maximum stress is limited to 400 MPa. Find the required number of active coils.
8.4 The spring loaded safety valve for a boiler is required to blow off at 1 MPa. The diameter and maximum lift of the valve are 60 mm and 15 mm respectively. Design the spring with a spring index of 6 and an initial compression of 30 mm. The maximum permissible shear stress for spring material is 450 MPa. $G = 0.8 \times 10^5$ MPa.
8.5 A helical compression spring having a free length of 350 mm exerts a maximum force of 10 kN when compressed by 80 mm. Maximum value of the outside diameter of the spring is 180 mm, $G = 0.8 \times 10^5$ MPa. Find the wire diameter, mean coil diameter and number of active turns. Permissible shear stress = 240 MPa.
8.6 The spring for a spring balance is elongated by 15 mm when pulled by a force of 375 N. Spring index is 6 and permissible shear stress for the material of the spring is 550 MPa, $G = 0.8 \times 10^5$ MPa. Find d_w and n_e.
8.7 A mass of 800 kg moving with a velocity of 1.2 m/s is to be brought to rest by a close coiled helical spring with a spring index of 8 and a maximum compression of 200 mm. The maximum shear stress is 450 MPa and $G = 0.84 \times 10^5$ MPa. Find the diameter of the wire and the number of active turns.
8.8 Two helical springs of the same axial length but different diameters of coils are placed coaxially one inside the other. The axial load is 3000 N and deflection is 25 mm. The maximum permissible shear stress for both the springs is 150 MPa. The spring indices are 5 and 9, the wire diameters being the same. Neglecting the effect of stress concentration, find the ratio of actual number of coils of the two springs.
8.9 Design a helical spring for a spring balance under tension from the following data: length of the scale = 50 mm, spring index = 5, maximum load to be weighed = 1000 N, permissible shear stress for spring material = 500 MPa and $G = 0.8 \times 10^5$ MPa.

8.10 Design a compression spring made of square steel wire subjected to a maximum load of 3000 N. Spring index = 7, deflection under load = 150 mm, permissible shear stress = 40 MPa and $G = 0.8 \times 10^5$ MPa.

8.11 A close coiled helical compression spring is subjected to a continuously varying load. The maximum load = 600 N, minimum load = 200 N, spring index = 6, S_{ys} = 650 MPa, S_{es} = 350 MPa. Find the factor of safety if the diameter of the wire is 6.44 mm.

8.12 A railway wagon is resting on six helical springs. The mass of the wagon causes a load of 15×10^4 N. The dynamic load due to irregularities is 50 kN. Corresponding to an amplitude of oscillation of 20 mm, design the spring with the *NiCr* steel having S_{ys} = 750 MPa, S_{es} = 400 MPa, $C = 6$, $G = 0.84 \times 10^5$ MPa and factor of safety = 2.25.

8.13 Design a cantilever leaf spring to absorb 800 N.m energy. The deflection is 150 mm, length of the spring = 600 mm, permissible stress = 875 MPa and $E = 2 \times 10^5$ MPa.

8.14 A semi-elliptic laminated spring 1 m long carries a central load of 5 kN. It is made of spring steel with a safe bending stress of 450 MPa. Maximum deflection of the spring is 130 mm. Calculate the thickness, width and number of leaves of the spring.

8.15 In Fig. P-8.15 a cantilever spring of maximum width 600 mm and length 1 m rests on a close coiled helical spring of 10 mm wire diameter. The spring index is 10 and number of active coils is 8. Calculate the thickness of the cantilever spring if a load of 2.1 kN causes a deflection of 40 mm at the end of the cantilever. Also calculate the stresses induced in the leaf and helical springs.

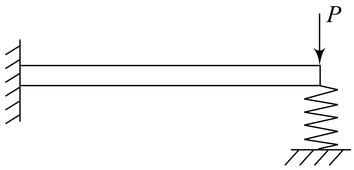


Fig. P-8.5

8.16–8.18 Design a close coiled helical spring for.

Maximum load = 4.5 kN, Deflection = 40 mm, τ = 500 MPa and C = 6.

P = 500 N, C = 8, Deflection 8 mm and τ = 400 MPa.

P = 200 N – 800 N, C = 5, Deflection = 10 mm, S_y = 800 MPa, S_e = 350 MPa and F·S = 2.

8.19 A close coiled helical compression spring used for front suspension of an automobile has spring stiffness of 80 N/mm. The ends of the spring are squared and ground and the design load is 7.5 kN. Find the diameter of the wire and free length of the spring if the material used has a safe shear stress of 580 MPa. Assume C = 6.

8.20 A machine is supported on four springs for vibration isolation. The load on the foundation is 70 kN and the deflection due to this load is 12 mm. Design the spring for a safe shear stress of 450 MPa, considering a solid deflection of 25 mm and the outside diameter of the spring not exceeding 150 mm.

8.21 A *P*-Bronze helical spring is required to absorb 9000 N.mm of energy without exceeding the maximum permissible shear stress of 50 MPa. Calculate the dimensions of spring if the maximum deflection is 20 mm. Assume spring index = 6.

8.22 Two concentric helical springs having the maximum diameters 50 mm and 30 mm respectively are subjected to a maximum total load of 1.5 kN. Maximum deflection under this load is 18 mm and the deflection when compressed solid is 25 mm. The material for both the springs is same and τ_{safe} = 550 MPa. Design the springs with suitable assumptions.

8.23 The spring rates for two concentric springs are 50 N/mm and 30 N/mm for the outer and inner spring respectively. The outer spring is 15 mm longer than the inner spring. If the total load is 3.5 kN, find the load carried by each spring.

8.24 A helical spring of spring rate 25 N/mm is arranged in series with another spring of stiffness 35 N/mm. Find the force required to give a total deflection of 50 mm.

8.25 A helical spring is subjected to a continuously varying load. The length of the spring varies between 57.5 mm and 65 mm with the corresponding loads of 700 N and 400 N. The spring is made of steel with S_{ys} = 650 MPa, S_e = 300 MPa and has the mean helix diameter of 28 mm. Find the factor of safety and the number of active coils if the diameter of the wire is 5.6 mm.

8.26 A helical compression spring is subjected to a static load of 1250 N. The number of active coils is 12 and the mean diameter of the coil is 60 mm. Find the size of square wire and the stress induced in the cross section of the spring if the deflection is 50 mm.

8.27 If the spring in Problem 8.26 is made of a round wire, find the required diameter and the stress induced. Also find the percentage saving of material.

8.28 A helical compression spring is subjected to a continuously varying load. The diameter of the wire is 5 mm and spring index is 5.5. In the most compressed position the force is 400 N and after a release of 8 mm the maximum force is 260 N. Find the factor of safety and number of active coils if the material has a maximum shear strength of 900 MPa.

8.29 A cantilever spring is subjected to a maximum load of 6000 N at the end of a clear span of 500 mm. The permissible value of stress is 400 MPa. Find the dimensions of leaves with the deflection not exceeding 3.5 mm.

8.30 An engine valve spring exerts a force of 300 N when the valve is closed and the lift is 8 mm for the opening of the valve. The stiffness of the spring is 25 N/mm, S_{ys} = 630 MPa, S_{es} = 336 MPa. Factor of safety = 1.5 and spring index = 6. Find the diameter of the wire and the free length of the spring.

8.31 The initial compression of a helical spring is 45 mm. It absorbs 500 N.m of energy for a further compression of 60 mm. Find the dimensions of the spring if τ_{safe} = 400 MPa and $G = 0.84 \times 10^5$ MPa. Assume a spring index of 6.

8.32 A helical spring is made from 5 mm diameter and has 12 active turns. The spring index is 5. Loading is static. Find the stress when deflection is 24 mm.

8.33 Two concentric springs carry total load of 5000 N. Find load shared by each spring and deflection of each spring

	Wire dia, d	D_m	*n*	*Free length*
Outer	10 mm	60 mm	9	120 mm
Inner	5 mm	40 mm	10	80 mm

8.34 A mass of 5 kg falls from a height of 250 mm on a helical compression spring of spring index 7 and material with safe shear stress of 550 MPa. Find wire diameter and stiffness of the spring. The number of active turns is 8.

8.35 A semielliptic leaf spring is composed of 8 graduated and 2 extra full length leaves. The clear span is 1.5 m. The load on the spring is 10 kN. Width of the leaves is 60 mm, maximum deflection is 60 mm. Find necessary thickness and stress induced in the leaves under (a) prestressed condition, (b) unstressed condition.

8.36 The load on the combination of spring as shown in P-8.36 is 400 N. The dimensions of helical spring are d = 8 mm, D = 68 mm, n = 10. For leaf spring, 5 graduated leaves of 75 mm width and 6 mm thickness are used. Find the stress induced in each spring and the deflection.

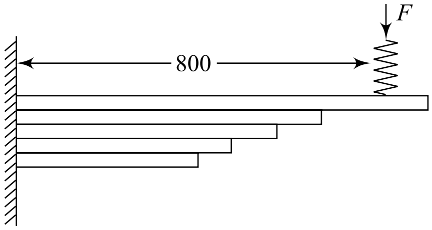


Fig. P-8.36

8.37 A cantilever spring of 1 m length consists of 8 graduated and one extra full length leaf. A load of 2000 N at the end causes the deflection of 75 mm. Determine the thickness of leaves and maximum bending stress with and without preload.

8.38 A helical compression spring of 4 mm wire diameter and 30 mm outer diameter τ_s made up of steel of permissible shear stress of 300 MPa. Active number of coils is 15. Find the maximum steady load the spring can take and total number of turns with squared and ground ends.

8.39 The following particulars refer to a valve spring of a petrol engine. Length and load on spring when valve is open are 32 mm and 400 N and those when valve is closed are 40 mm and 250 N respectively. Maximum outside diameter of spring should be 26 mm. Design spring and specify the dangerous engine speed with the point of view of surging of spring t = 500 MPa, $G = 0.8 \times 10^5$ MPa.

8.40 A solenoid brake is actuated by a helical compression spring of free length approximately 300 mm. It should exert a force of 15 kN when compressed to a height of 225 mm. The outside diameter of spring should not exceed 175 mm. Design spring for a permissible stress of 500 MPa.

8.41 Load on a close coiled helical compression spring varies from 480 to 1450 N. The spring index is 6. The material is oil tempered carbon steel with ultimate shear stress of 700 MPa, endurance stress of 350 MPa. Find the size of the wire if the factor of safety based on yield strength is 1.5.

ANSWERS

Objective Questions

(1) b (2) d (3) c (4) c (5) b (6) b (7) d (8) b (9) b
(10) b (11) b (12) b (13) b (14) d (15) c (16) b

Practice Problems

Notation: d = Wire diameter, n_e = effective number of turns, $G = 0.8 \times 10^5$ MPa, $S_{es} = 0.5\ S_{us}$, $C = 6$ if not mentioned.

(1) d = 4.37 mm, n_e = 8.1
(2) τ_{max} = 444.5 MPa, n_e = 10.66
(3) n_e = 10.8
(4) d = 13.4 mm, n_e = 7.4
(5) d = 25.5, D_m = 150 mm, n_e = 11.4
(6) d = 13.04, n_e = 24
(7) d = 24.84 mm, n_e = 8.42
(8) 9.31
(9) d = 5.76, n_e = 23.08
(10) C = 11.75 mm, n_e 24.18
(11) 1.66
(12) d = 45.51 mm
(13) 5 leaves of 80 mm width and 10.5 mm thickness
(14) t = 4.32 mm, 9 leaves of 100 mm width
(15) t = 12.6 mm, σ = 102.45 MPa, τ = 145.65 MPa
(16) d = 13.12 mm, n_e = 6
(17) d = 5.5 mm, n_e = 2
(18) d = 7.6 mm, n_e = 10.14
(19) d = 15.62 mm, n_e = 9.1 $\rightarrow$ 10
(20) d = 28.64 mm, n_e = 3
(21) d = 6 mm, n_e = 6.17
(22)– (23) 2468.75 N, 1031.25 N
(24) 729.17 N
(25) 1.45, n_e = 8
(26) c = 9.5 mm, τ = 414.45
(27) 10.39 mm, 386.2 MPa, 7%
(28) 2.63, n_e = 17.17
(29) t = 14.28 mm, b = 220.67 mm
(30) d = 6.6 mm, n_e = 12.12
(31) d = 11.3 mm, n_e = 11.765
(32) 534 MPa
(33) 4278.12 N, 721.88 N, $\delta_1 = \delta_2$ 73.926 mm
(34) 37.92 mm, 750 N/mm
(35) t = 11.68 mm, (a) $\sigma_e = \sigma_g$ = 274.88 MPa, (b) σ_e = 374.82 MPa, σ_g = 248.9 MPa
(36) σ = 142.22 MPa, δ = 75.851 mm for leaf spring, τ = 148.252 MPa, δ = 21.09 mm, Total deflection = 96.941 mm.
(37) τ 12.32 mm, $\sigma_e = \sigma_g$ = 195.21 MPa, without preload σ_e = 277 MPa, σ_g = 184.94 MPa.
(38) 215 N, 17
(39) d = 4 mm, n = 10 + 2, No possibility of surging.
(40) d = 25 mm, n = 6 + 2, reduce gap by 1.5 mm
(41) d = 9.5 mm, D_m = 57 mm.

9

Bolt Loading

CONCEPT REVIEW

9.1 INTRODUCTION

The bolt and nut combination is used for fastening two components for which detachment is to be done. The ISO metric threads are used for bolts. There are three series of these threads. Coarse series is used for general purpose where jars and vibrations are not important, detachment is frequent and tapped holes are in a metal other than steel. Fine thread series is used for automotive and aircraft parts where jerks and vibrations are predominant, fine adjustment is important and tapped holes are in steel. Lastly, the extra fine series is used for aeronautic equipment, for threading in thin walled material, where fine adjustments are required and jars and vibrations are excessive.

9.2 DESIGN OF BOLTS FOR EASY SITUATION

With tighening unknown:

Tightening induces the tensile stress which is a linear function of diameter given by

$$\sigma_t = \frac{S_y}{60}(A)^{\frac{1}{2}} \quad \text{where} \quad S_y = \text{yield strength MPa.}$$

$$A = \text{root area in mm}^2 \text{ and } d < 45 \text{ mm}$$

$$= \frac{S_y}{40}(A)^{\frac{1}{2}} \qquad \text{for } d > 45 \text{ mm}$$

$$\therefore \qquad \text{Load } P = \sigma_t A = \frac{S_y}{60} A^{3/2}$$

or $$A = \left(\frac{60 \times \sigma_t}{S_y}\right)^{2/3} \quad \text{for } d < 45 \text{ mm} \tag{9.1}$$

$$= \left(\frac{40 \times \sigma_t}{S_y}\right)^{2/3} \quad \text{for } d > 45 \text{ mm}$$

The initial tightening produces a tensile load given by

$$F_i = 2860\, d \text{ N, where } d = \text{diameter of the bolt in mm} \tag{9.2}$$

The tightening torque is given by

$$T = CdF_i \text{ where } C \text{ is a constant} = 0.2. \tag{9.3}$$

9.3 PRELOADING OF BOLTS

In the applications like pressure vessels and cylinder covers, it is essential to apply the initial tightening torque to make a joint leakproof. The initial tightening elongates the bolt and compresses the connected members. This situation is represented by point A in Fig. 9.1

δ_b = elongation of bolt
δ_p = compression of the part
BG = external force F_e
AF = initial tightening load F_i
BJ = increase in bolt load due to external load = ΔF
JG = decrease in part load due to $F_e = F_e - \Delta F$
CE = limiting value of external load when the joint opens, $= F_o$

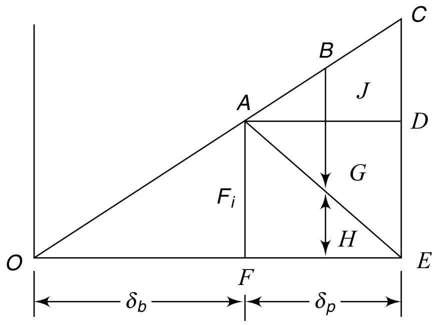


Fig. 9.1

By similar triangles AFD and CED

$$\frac{F_o}{F_i} = \frac{\delta_b + \delta_p}{\delta_b} \tag{9.4}$$

As δ_b and δ_p are inversely proportional to stiffnesses K_b and K_p, $\delta_b/\delta_p = K_p/K_b$

$$\frac{F_o}{F_i} = \frac{K_p + K_b}{K_p} \tag{9.5}$$

Effect of External Load F_e on Bolt Load and Part Load

The load on the bolt increases by ΔF (BJ) which increases the deflection of the bolt by $\Delta\delta_p = AJ$. There is a decrease in load on the part by ($F_e - \Delta F$), i.e. JG which reduces the compression of the part by the same amount, i.e. $JG = \Delta\delta_p$. Using the relationship between deflection and stiffness we can write

$$\Delta\delta_b = \frac{\Delta F}{K_b} = \frac{F_e - \Delta F}{K_p} = \Delta\delta_p$$

$$\therefore \qquad \frac{F_e}{K_b + K_p} = \frac{\Delta F}{K_b} \quad \text{or} \quad \Delta F = F_e \left(\frac{K_b}{K_b + K_p} \right) \qquad (9.6)$$

9.4 ADVANTAGES OF PRELOADING

1. It stops the leakage in pressure vessels.
2. It reduces the amplitude of fluctuation in load thereby improving the fatigue strength of the parts.

9.5 USE OF SPRING WASHERS AND GASKETS

From Eq. (9.5) if $K_p > K_b$ which is usually the case, then F_o is nearly equal to F_i and hence, the joint opens with a very small external load. By using a gasket of small stiffness the total stiffness of the part reduces since $\frac{1}{K_t} = \frac{1}{K_p} + \frac{1}{K_g}$, where K_t is the total stiffness and K_g is the stiffness of gasket. This improves the ratio F_0/F_i making the joint leakproof for a larger external load.

A spring washer helps in avoiding the loosening of nuts in automotive applications. The principle behind the use of a spring washer is also the same.

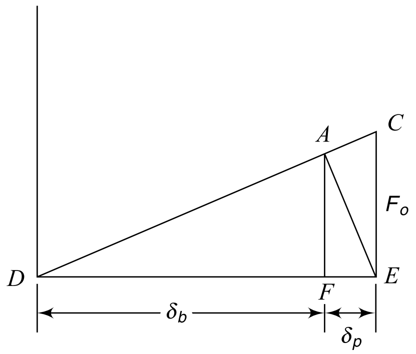


Fig. 9.2

9.6 ECCENTRIC LOADING OF BOLTS

A bracket fixed to the wall as shown in Fig. 9.3 tilt about point O if the bolts do not hold it properly. The bolts exert a force to hold the bracket. This force is proportional to the distance of bolts from point O. Hence, the force induced in each bolt in the first row is kl_1 and that induced in each bolt in the second row is kl_2 where k is the force induced per unit distance. If there are n_1 bolts in the first row and n_2 bolts in second row, we can write the equation of moments about O as

$$P.e = n_1 (k\,l_1)\, l_1 + n_2 (kl_2)\, l_2$$
$$= n_1\, kl_1^2 + n_2\, kl_2^2$$

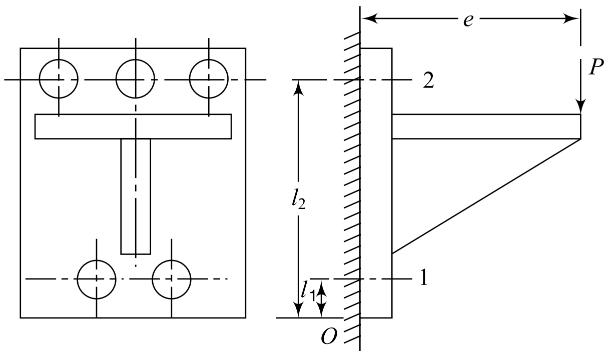


Fig. 9.3

$\therefore$ $$k = \frac{P \cdot e}{n_i l_1^2 + n_2 l_2^2}$$

$\therefore$ Force induced in the bolts in the first row is given by $F_1 = \left(\frac{P \cdot e}{n_1 l_1^2 + n_2 l_2^2}\right) l_1$ and that in the bolts in second row is

$$F_2 = \left(\frac{P \cdot e}{n_1 l_1^2 + n_2 l_2^2}\right) l_2$$

$\therefore$ This force induces stress in the bolt which is given by

$$\sigma_1 = \frac{F_1}{A_C} \text{ and } \sigma_2 = \frac{F_2}{A_C}$$

where A_C is the core area in mm^2.
Again, the force P induces a direct shear stress given by

$$\tau = \frac{P}{(n_1 + n_2)\, A_C}$$

As $F_2 > F_1$, the maximum stress is induced in bolts in the second row. The values of A_c and diameter of the bolt can be calculated by combining σ_2 and τ and applying the theory of failure.

If the bracket is loaded as shown in Fig. 9.4, the effect of load P causes a direct tensile load in addition to the tensile load due to tilting of the bracket.

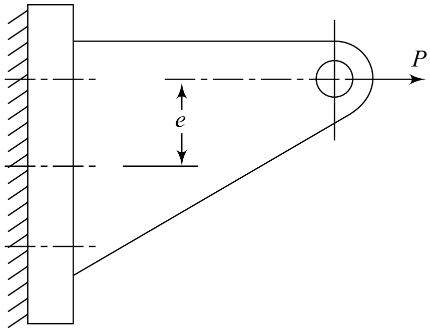


Fig. 9.4

$\therefore$ $$\sigma_1 = \text{direct tensile stress} = \frac{P}{nA_C}$$

where n is the total number of bolts

and $$\sigma_2 = \left(\frac{P \cdot e}{n_1 I_1^2 + n_2 I_2^2}\right) \cdot l_2 \text{ as explained earlier.}$$

Maximum induced stress $= (\sigma_1 + \sigma_2) = \frac{S_y}{N}$

In the loading as shown in Fig. 9.5 the bolts are subjected to two types of forces causing a shearing action. One is the direct shearing force $F_1 = \frac{P}{n}$ where n is the number of bolts. The force F_2 depends on the radius of bolt from the C.G of the bolt arrangement.

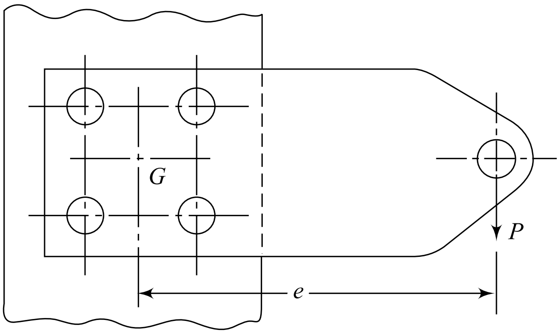


Fig. 9.5a

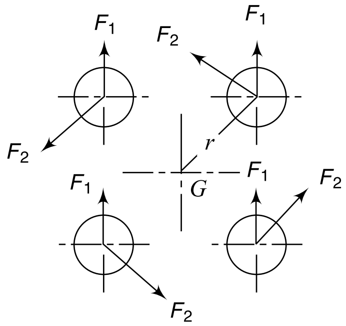


Fig. 9.5b

Thus, $F_2 = Kr$ where K is the force induced per unit radial distance of the centre of the bolt from C.G and r is the radius of bolt centre from C.G in mm. F_2 is calculated by writing the equation of moment about the C.G of the arrangement as

$$P \cdot e = n\, F_2 r = n\, Kr^2$$

In case the bolts are at different radii the above equation can be written as

$$P\,.\; e = n_1\, kr_1^{\;2} + n_2 kr_2^{\;2} + n_3 kr_3^{\;2} + \ldots.$$

The maximum value of F_2 is calculated and since both F_1 and F_2 cause the shearing, they are added vectorially to give

$$F_1 = \sqrt{F_1^2 + F_2^2 + 2\,F_1 F_2 \cos\theta}$$

The dimension of the bolt is then decided by writing

$$\tau = \frac{F}{A_C} = \frac{S_{ys}}{N}$$

WORKED EXAMPLES

9.1 The cylinder head of a steam engine with 250 mm bore is fastened by eight stud bolts made of 30C8 steel. Maximum pressure inside the cylinder is 1 MPa. Determine the size of bolts and the approximate tightening stress and torque. Take 20% overload. Assume S_y = 300 MPa for bolt material.

Solution:

Total load on cylinder head considering 20% overload

$$= 1.2 \times \frac{\pi}{4} \times 250^2 \times 1 = 58904.862 \text{ N}$$

$\therefore$ load/bolt = 7363.1 N

Using Eq. 9.1 $$A_C = \left(\frac{60 \times 7363.1}{300}\right)^{\frac{2}{3}} = 132.62 \text{ mm}^2$$

Referring to Table 6 and using the coarse series, M 16×2 ISO metric threads with a root area of 157 mm^2 may be used.

Initial tightening = $2860\,d = 2860 \times 16 = 45760$ N

$$\therefore \quad \text{Tightening stress} = \frac{45760}{157} = 291.46 \text{ MPa}$$

Tightening torque = $CdF_i = 0.2 \times 16 \times 45760 = 146432$ N.mm.

9.2 A steel bolt of M 16×2 is 300 mm long and carries an impact load of 5000 N.mm. If the threads stop adjacent to the nut and $E = 2.1 \times 10^5$ MPa

(a) find the stress in the root area.

(b) find the stress if the shank area is reduced to root area.

Solution:

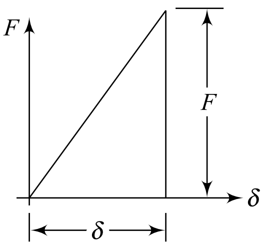


Fig. E-9.2

(a) From Fig. E-9.2, energy stored = $\frac{1}{2}$ F $\cdot$ δ

$\delta = F/k$ where k is the stiffness

$U = \frac{F^2}{2k}$ for tensile load $\delta = \frac{P}{AE}$

$\therefore$ Stiffness of bolt $k = \frac{AE}{l}$

For 16×2 threads, shank area $A = \frac{\pi}{4} \times 16^2 = 201.06$ mm^2

$$\therefore \quad k = \frac{201.06 \times 2.1 \times 10^5}{300} = 140742 \text{ N/mm}$$

Again, $U = \frac{F^2}{2k}$, $\therefore$ $F = \sqrt{2Uk}$

$$= \sqrt{2 \times 5000 \times 140742} = 37515.6 \text{ N}$$

From Table 6 root area for M16×2 bolt is 157 mm^2

$$\therefore \quad \sigma = \frac{37515.6}{157} = 238.94 \text{ MPa}$$

(b) Now, $A_C = 157$ mm^2 = Area of reduced shank

$$\therefore \quad k = \frac{157 \times 2.1 \times 10^5}{300} = 109900.00 \text{ N/mm}$$

$$F = \sqrt{2 \times 5000 \times 109900.00} = 33151.168 \text{ N}$$

$$\therefore \quad \sigma = \frac{33151.168}{157} = 211.153 \text{ MPa.}$$

9.3 A cast iron cylinder head is fastened to a cylinder of bore 500 mm with 8 stud bolts. The maximum pressure inside the cylinder is 2 MPa. The stiffness of part $k_p = 3k_b$. What should be the initial lightening load so that the joint is leakproof at maximum pressure.

Solution:

Total load at a pressure of 2 MPa

$$= 2 \times \frac{\pi}{4} \times 500^2 = 392699.08 \text{ N}$$

$\therefore$ Load/bolt = 49087.385 N. This will cause the opening of the joint in case the initial load F_i is not sufficient.

From Eq. (9.5) $$F_i = F_0 \left(\frac{k_p}{k_p + k_b} \right) = 49087.385 \left(\frac{3}{3+1} \right)$$

$$= 36065.54 \text{ N}$$

Thus, a slightly greater load will ensure a leakproof joint, hence the load of 37000 N may be used.

9.4 In Example 9.3 what should be the size of the bolt if F_i = 40000 N?

Solution:

From Eq. (9.6) $$\Delta F = F_e \left(\frac{k_b}{k_b + k_p} \right)$$

$$= 49087.385 \left(\frac{1}{1+3} \right) = 12271.846 \text{ N}$$

$\therefore$ Maximum load on the bolt = 40000 + 12271.841 = 52271.841 N

$$A_C = \left(\frac{60 \times 52271.841}{300} \right)^{\frac{2}{3}} = 493.09 \text{ mm}^2$$

From Table 6 use M30 × 3.5 ISO metric threads with the core area of 561 mm^2.

9.5 A bolt of M 20 × 2.5 ISO metric thread is subjected to a fluctuating load of 0 to 12000 N. S_e = 210 MPa. The bolt and the part are of the same material and length. S_y = 490 MPa, stress concentration factor SCF = 3.85, area of component = 362 mm^2. Calculate

(a) F·S without preload
(b) minimum F_i to prevent opening of joint
(c) F·S with F_i = 10000 N
(d) minimum force in the part for a given loading and preload of 10000 N.

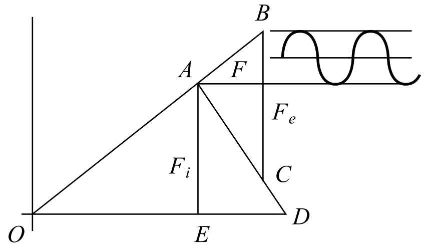


Fig. E-9.5

Solution:

(a) Without Preload $P_v = P_m$ = 6000 N, A_C for the bolt = 245 mm^2 from Table 6

$\therefore$ $$\sigma_m = \sigma_v = 24.49 \text{ MPa}$$

$\therefore$ Using Soderberg's equatiom $\dfrac{1}{N} = \dfrac{24.49}{490} + \dfrac{3.85 \times 24.49}{210}$, $\quad \therefore N = 2$

(b) $k_b = \dfrac{A_b E_b}{l_b}$, $k_p = \dfrac{A_p E_p}{l_p}$ as $E_p = E_b$ and $l_p = l_b$

$\therefore \quad k_b \propto A_b$ and $k_p \propto A_p$

$$\therefore \quad F_i = \left(\frac{k_p}{k_b + k_p}\right) \; F_e = \left(\frac{A_p}{A_b + A_p}\right) 12000$$

$$A_p = 362 \text{ mm}^2, A_b = \frac{\pi}{4} \times 20^2 = 314 \text{ mm}^2 \text{ (shank area)}$$

$$\therefore \quad F_i = \frac{362}{362 + 314} \times 12000 = 6426.0355 \text{ N}$$

(c) With the preload of 10000 N

AE = 10000 N

. BC = 12000 N (from Fig. E-9.5)

when external load is zero, the load on the bolt = F_i = 10000 N.
when external load is 12000 N the load on the bolt = 10000 + ΔF

$$= 10000 + 12000 \left(\frac{314}{314 + 362}\right) = 15574.00 \text{ N}$$

$\therefore$ Load on the bolt fluctuates between 10000 N to 15574 N

i.e. $$p_m = \frac{10000 + 15574}{2} = 12787 \text{ N}$$

$$P_v = \frac{15574 - 10000}{2} = 2787 \text{ N}$$

$$\therefore \quad \sigma_m = P_m/A_C = 12787/245 = 52.19 \text{ MPa}, \; \sigma_v = 2787/245 = 11.37 \text{ MPa}$$

$$\therefore \quad \frac{1}{N} = \frac{52.19}{490} + \frac{3.85 \times 11.37}{210}$$

$\therefore \quad N$ = 3.378 which shows that preloading improves the factor of safety

(d) Minimum force in the part = $F_i - (F_e - \Delta F)$

= 10000 – (12000 – 5574) = 3574 N.

9.6 A bolt of 20 × 2.5 ISO metric threads is subjected to a preload of 35000 N. Factor of safety is 2.5 and the average stress at root area is 180 MPa. Find the maximum and minimum value of varying load on the part. S_y = 630 MPa, S_e = 350 MPa, area of steel part = 700 mm^2 and SCF = 3.85.

Solution:

Let F_{e_1} = minimum external load

F_{e_2} = maximum external load.

Under the action of F_{e_1}, load on the bolt = $F_i + F_{e_1} \left(\frac{k_b}{k_b + k_p}\right)$ (i)

and under the action of F_{e_2} load on the bolt = $F_i + F_{e_2} \left(\frac{k_b}{k_b + k_p}\right)$ (ii)

Equations (i) and (ii) give the minimum and maximum load on the bolt respectively.

$$\text{Mean load on the bolt} = F_i + \left(\frac{F_{e_1} + F_{e_2}}{2}\right)\left(\frac{k_b}{k_b + k_p}\right) \quad \text{(iii)}$$

$$\text{Variable load on the bolt} = \left(\frac{F_{e_2} - F_{e_1}}{2}\right)\left(\frac{k_b}{k_b + k_p}\right) \quad \text{(iv)}$$

Now, using Soderberg's equation we have

$$\frac{1}{2.5} = \frac{180}{630} + \frac{3.85 \times \sigma_v}{350}$$

$$\sigma_v = 10.39 \text{ MPa}$$

$$\therefore \quad P_v = 10.39 \times 245 = 2545.55 \text{ N}$$

$$\sigma_m = 180 \text{ MPa},$$

$$\therefore \quad P_m = 180 \times 245 = 44100 \text{ N}$$

$$\frac{k_b}{k_b + k_p} = \frac{A_b}{A_b + A_p} = \frac{314}{314 + 700} = 0.3096$$

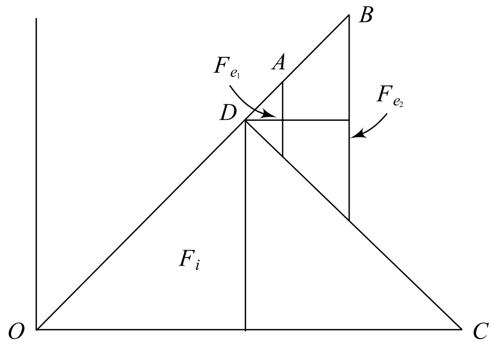


Fig. E-9.6

Using Eqs. (iii) and (iv)

$$44100 = 0.3096 \times \left(\frac{F_{e_1} + F_{e_2}}{2}\right) + 35000$$

$$2445 = 0.3096 \times \left(\frac{F_{e_2} - F_{e_1}}{2}\right)$$

From above equations

$$F_{e_1} = 21172 \text{ N}, \quad F_{e_2} = 37613 \text{ N}$$

9.7 A bracket is fitted to a vertical channel with 5 bolts, three at the top arid two at the bottom with all the bolts equally spaced. The value of $P = 20$ kN, $e = 200$ mm, $l_1 = 50$ mm and $l_2 = 250$ mm. Find the diameter of the bolt.

Solution:

The bolts are subjected to a tensile load due to tilting of bracket as shown in Fig. E-9.7(b). The bolts at a larger distance from point O are stressed higher.

Let P_1 and P_2 be the loads at distances l_1 and l_2 respectively from the point about which tilting takes place.

$\therefore P_1 = kl_1 \quad P_2 = k\,l_2$ where k is the load in N per unit distance from point O.

Taking moments about O we have,

$$P \times e = 3P_2 l_2 + 2P_1 l_1 = 3\,k\,l_2^2 + 2\,k l_1^2$$

$$\therefore \quad 20000 \times 200 = 3k(250)^2 + 2k(50)^2$$

$\therefore k = 20.77$ N per mm. Hence, maximum load in the bolt is in the top row given by $250 \times 20.77 =$ 5192.5 N

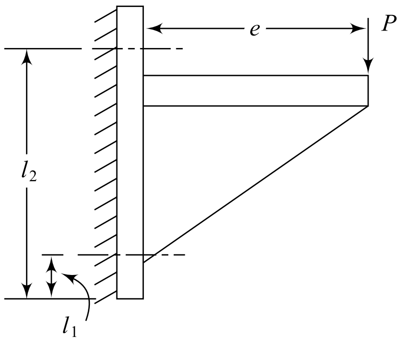


Fig. E-9.7a

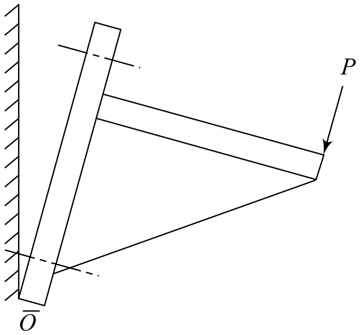


Fig. E-9.7b

$$\therefore \qquad \sigma_1 = \frac{5192.5}{A_C}$$

The load P may cause shearing of the bolts such that

$$\tau = \frac{20000}{5 \times A_C} = \frac{4000}{A_C}$$

(Using the maximum shear stress theory)

$$\text{Maximum induced shear stress} = \frac{1}{2}\sqrt{\left(\frac{5192.5}{A_C}\right) + \left(\frac{4000}{A_C}\right)^2}$$

$$= \frac{4768.7}{A_C}$$

Using 30C4 steel with $S_y = 300$ MPa, $S_{ys} = 150$ MPa, F·S = 4 and permissible shear stress = 37.5 MPa, the value of $A_C = \dfrac{4768.7}{37.5} = 127.165$ mm^2. Referring to Table 6 M 16×2 ISO Metric thread bolts with core area 157 mm^2 are used.

9.8 A bracket is fitted to the channel with 4 bolts. The dimension $a = b = 150$ mm distance of load from the C.G of the bolt arrangement is 300 mm. Find the diameter of the bolts.

Solution:

The shear forces P_I and P_2 due to direct shear and shearing due to turning of the bracket are introduced as shown in Fig. E-9.8. Bolt number 3 is subjected to maximum load equal to $P_I + P_2$.

$$P_1 = \frac{40000}{4} = 10000 \text{ N}$$

Taking moment @ the C.G of the bolts

$$40 \times 1000 \times 300 = 4\, P_2 \times (a/2), \qquad P_2 = k(a/2)$$

$$= k\, a^2 = 22500\, k$$

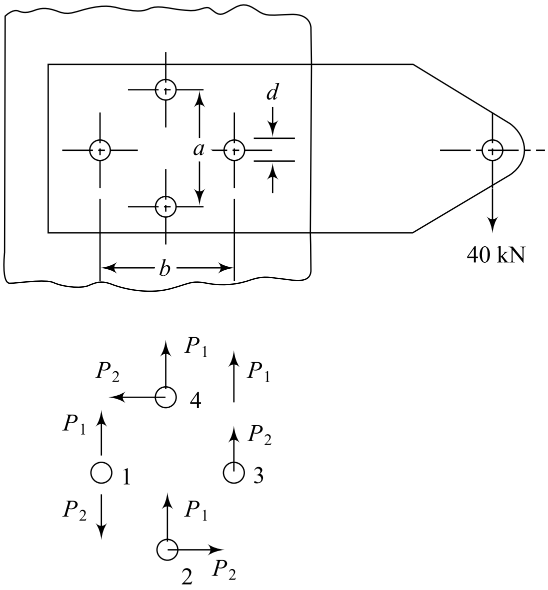


Fig. E-9.8

$$k = 533.33 \text{ N/mm}, \therefore P_2 = 75 \times 533.33 = 39999.99 \text{ N}$$

$$\tau_1 = \frac{P_1}{A_C} = \frac{10000}{A_C}, \ \tau_2 = \frac{39999.99}{A_C}$$

$\therefore$ Total shear stress $= \dfrac{49999.99}{A_C} = 37.5$

$A_C = 1333.33 \text{ mm}^2$. Hence, M 48 × 5 bolts should be used.

OBJECTIVE QUESTIONS

9.1 Bolts are rarely subjected to
(a) bending moment (b) shear (c) compression (d) tension

9.2 Preloading is essential in the bolts of
(a) pressure vessels (b) brackets (c) coupling (d) structural joints

9.3 The ratio of load required to open the joint to the initial tightening load when $k_p = 4\, k_b$ is
(a) 1.25 (b) 0.8 (c) 0.2 (d) 5

9.4 Effect of initial tightening for variable loading is
(a) increase in the factor of safety (b) decrease in the factor of safety
(c) decrease in the induced stress (d) none of the above

9.5 In automotive parts, loosening of the nut is delayed by using
(a) special threads (b) lock nuts (c) spring washer (d) gaskets

9.6 Under static loading the maximum stress induced in the bolts is

(a) $\dfrac{S_y}{60}\sqrt{A}$ (b) $\dfrac{P}{\sqrt{A}}$ (c) 2860 d (d) none of the above

9.7 By reducing the shank diameter equal to the root diameter the impact strength of the bolt
(a) decreases (b) increases (c) is not affected (d) none of the above

9.8 Stress concentration factor at the root of first engaged thread is
(a) 2.3 (b) 3.85 (c) 1.8 (d) 4.15

9.9 Possibility of fatigue fracture of bolts can be reduced by
(a) increasing the diameter of the shank (b) increasing the pitch of the threads
(c) using bolts of uniform strength (d) using fine series

9.10 In automotive components it is advisable to use
(a) coarse series threads (b) fine series threads
(c) extra fine series threads (d) threads of uniform strength

9.11 Gaskets and washers
(a) reduce leakage (b) reduce vibration
(c) reduce the stiffness of connected members (d) increase the stiffness of bolts

9.12 Preloading in structural joints and pressure vessels both subjected to fluctuating loads
(a) reduce the factor of safety (b) improves the factor of safety
(c) avoids the leakage (d) secures the parts tightly

9.13 The arrangement of bolts having maximum strength is shown by

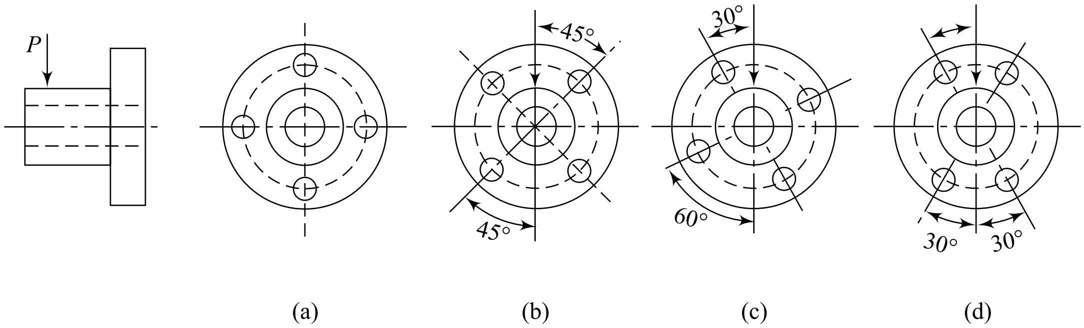


Fig. O-9.13

REVIEW QUESTIONS

9.1 Why are V threads used for fasteners?
9.2 What are the different series of threads and their applications?
9.3 What are the different materials used for nuts and bolts?
9.4 What are the different methods of manufacturing bolts?
9.5 Differentiate between a stud, a bolt and a nut.
9.6 Why are locking devices used for the nut and bolt assemblies?
9.7 Explain a few methods of locking with sketches.

9.8 What are the different types of the stresses induced in bolts? Explain the procedure of designing a bolt subjected to direct tensile load.
9.9 Why is the preloading of bolts desirable?
9.10 Explain the benefits of preloading of bolts.
9.11 What is the bolt of uniform strength? Where is it preferably used?
9.12 What is the cause of stress concentration in bolts? How can it be reduced?
9.13 Derive the relationship between the load required to open the bolted joint and the initial tightening force.
9.14 Derive the relationship to calculate the load shared by the bolt in a preloaded joint when an external load F_e acts on the joint.
9.15 Does the preload affect the factor of safety in bolted assemblies? How?
9.16 Explain the effect of spring washers and gaskets in bolted assemblies.
9.17 What should be the correct ratio of stiffness of the part and the bolt for a leakproof joint? How is it achieved?
9.18 What is the function of threaded fasteners?
9.19 Why is initial tightening of bolts essential? Why excessive tightening should be avoided?
9.20 Justify (a) Preloading is essential as well as beneficial.
(b) Preloading improves the fatigue strength of bolted joint.
9.21 State true or false:
(a) Spring washers are used in pressure vessels to reduce leakage.
(b) Gaskets are used in automotive wheels.
(c) A hard and stiff material should be used for gaskets.
(d) While making assembly the spring washers should be tightened flat.
(e) Coarse series threads are used for aeronautical equipment.
(f) Core diameter of M12 × 1.5 thread is 12 mm.
(g) Rolled threads are superior to cut threads under fatigue.

PRACTICE PROBLEMS

9.1 A cylinder cover of 600 mm diameter pressure vessel is fastened by 12 bolts of 750 mm pitch circle diameters. The maximum pressure inside the cylinder is 1.5 MPa. Determine the diameter of the bolts if the permissible stress should not exceed the stress under easy situation. Calculate the initial tightening force and torque on each bolt.
9.2 A cast iron cylinder head is secured to a cylinder of 500 mm bore by means of 16 stud bolts. The stiffness of the cylinder and head is higher than that of bolt. Internal pressure is 1.7 MPa. What is the axial force on each bolt if the bolts are tightened just enough to prevent opening of the joint under a pressure of 2.5 MPa. Calculate the diameter of the bolts if the permissible stress does not exceed 40 MPa.
9.3 The longitudinal stay of a boiler 6 m long supports an area of 400×400 mm^2 on each end plate. The pressure inside the boiler is 1.1 MPa. Determine the diameter of the stay if the permissible tensile stress does not exceed 50 MPa.
9.4 The outer diameter of the aluminium part shown in Fig. P-9.4 is 40 mm. The bolt is M 16 × 1.5 of 30C4 steel with the initial tightening torque equal to 70000 N.mm. Find the maximum force P that can be applied without completely removing the initial preload. For steel $E = 2 \times 10^5$ MPa and for aluminium $E = 0.72 \times 10^5$ MPa. Length of the part and the component is the same.

9.5 A 250 mm long bolt M 16 × 2 of steel carries an impact load of 4000 N.mm. $E = 2.1 \times 10^5$ MPa. Find the stress at the root area for (a) a standard bolt, (b) bolt of uniform strength.

9.6 The load on a bolted assembly varies between 5000 N to 25000 N. The assembly has a preloading of 15000 N. The ratio $k_p/k_b = 2$. Does the joint open under the given load? Calculate the diameter of the bolt if $S_y = 300$ MPa, $S_e = 200$ MPa, Stress concentration factor = 3.85 and F·S = 2.5.

9.7 In a pressure vessel the fluid pressure varies from 0 to 15 MPa. The cylinder bore is 100 mm and F·S is 4. Four bolts of M 20 × 1.5 are used to hold the cylinder cover at its place. S.C.F = 3.85, S_y = 300 MPa, S_e = 210 MPa and $E_p = 1 \times 10^5$ MPa. Find the value of initial force *F.* Area of the part is 750 mm^2 per bolt.

9.8 The bolt in Fig. P-9.4 has SCF equal to 2.5. The initial torque is 50000 N.mm. Load *P* varies from 5000 to 25000 N. If the part is made of steel of area 950 mm^2, find F·S for the bolt and the minimum force in the part. S_y = 330 MPa, S_e = 200 MPa.

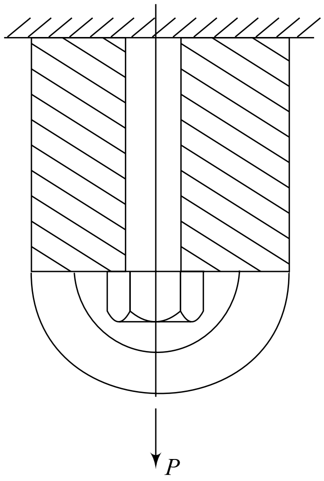


Fig. P-9.4

9.9 In Fig. P-9.9, the load *P* on the bracket is 12500 N. Find the diameter of the bolt using 30C8 steel with S_y = 330 MPa. Assume F·S = 4.

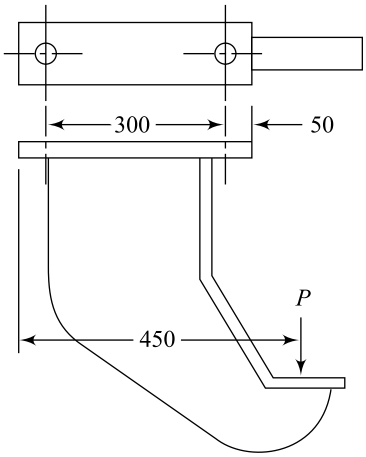


Fig. P-9.9

9.10 For the circular flange shown in Fig. P-9.10, *P* = 20 kN and *l* = 100 mm. It is supported by 6 bolts of 30C8 steel at 150 mm p.c.d. Find the diameter of the bolts if the outer diameter of the bracket is 200 mm. F·S = 6. S_y = 330 MPa.

9.11 The pillar crane shown in Fig. P-9.11 is fastened to the foundation by 8 equispaced bolts. Derive an expression for the maximum load on any bolt in terms of *P, a, b* and *l.*

9.12 The load *P* on the bracket shown in Fig. P-9.12 is at a distance of 500 mm from the C.G of the bolt arrangement. Find the diameter of the bolts when (a) 6 bolts are used, (b) 4 bolts are used dropping the middle column of the bolts.

9.13 A cylinder cover of 500 mm diameter is fastened by 12 bolts at 700 mm p.c.d. The permissible maximum pressure inside the cylinder is 1.5 MPa. Determine the diameter of the bolt if the

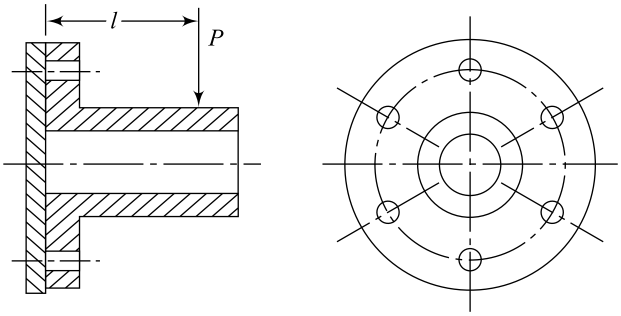


Fig. P-9.10

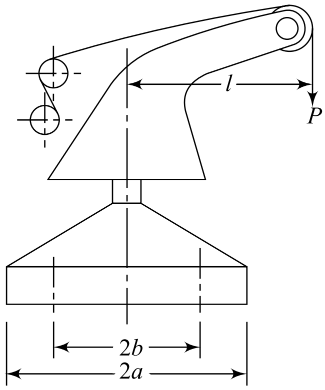


Fig. P-9.11

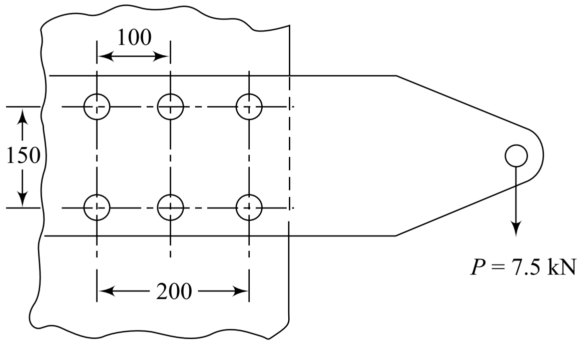


Fig. P-9.12

permissible stress should not exceed the stress under easy situation. Calculate the initial tightening force and torque on each bolt. $S_y = 300$ MPa.

9.14. Find the diameter of cap screw used to fix the plate as shown in Fig. P-9.14. The material for the screw being 30C8 steel with $S_y = 350$ MPa, $N = 4$ for (a) $P = 10$ kN, $Q = 0$, (b) $Q = 20$ kN, $P = 0$.

9.15 Find the diameter of bolts used to connect the bracket with $l = 650$ mm, $a = 100$ mm, $b = 150$ mm. For material $n = 2.5$, $S_y = 210$ MPa, $P = 5$ kN Fig. P-9.15.

9.16 The load P on the bracket is 7.5 kN, $\theta = 30°$ material for bolts 15C4 steel with $S_y = 280$ MPa, $N = 4$
Find size of bolts Fig. P-9.16.

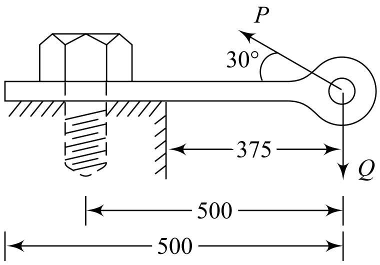


Fig. P-9.14

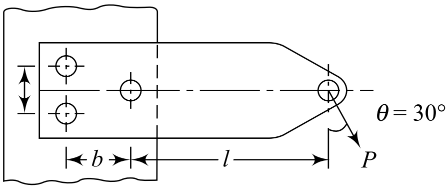


Fig. P-9.15

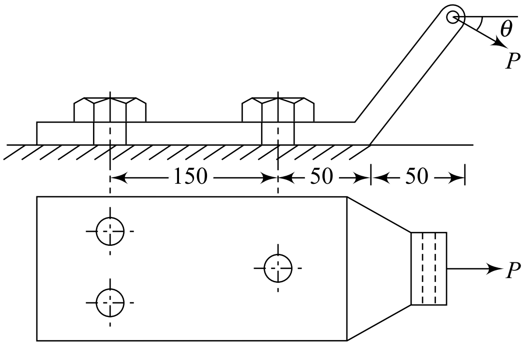


Fig. P-9.16

9.17 An electric motor is fitted to the foundation with the help of 8 bolts. The arrangement is symmetrical with $a = 100$ mm, $b = 150$ mm, $c = 60$ mm. The motor transmits 20 kW at 600 r.p.m. The ratio of tensions is 2 and the belt is horizontal. Find the diameter of bolt if material has $S_y = 300$ MPa, $N = 2.5$.

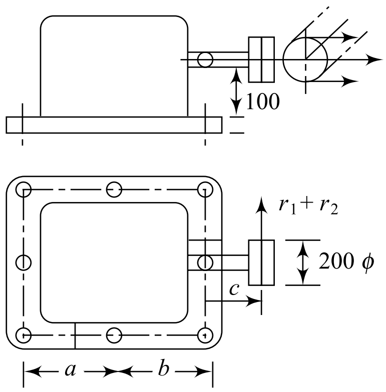


Fig. P-9.17

9.18 What will be the size of the bolts in the above problem if the belt center line is inclined at 30° with the horizontal as shown by dotted lines.

9.19 The material for the bolts shown in Fig. P-9.19 10C4 steel with $S_y = 210$ MPa, $N = 3$, $a = 100$ mm, $b = 200$ mm, $c = 250$ mm, $d = 115$ mm. The diameter of the bolts is 16 mm. Find P_1 and P_2.

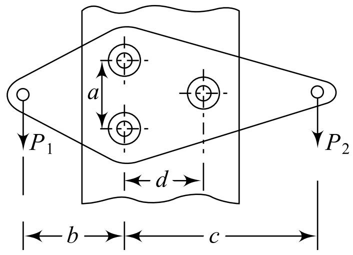


Fig. P-9.19

ANSWERS

Objective Questions

(1) c (2) a (3) a (4) a (5) c (6) a (7) b (8) b (9) c
(10) b (11) c (12) b (13) b

Practice Problems

(1) M 20 × 1.5, 57200 N, 22.8 N.m (2) M 30 × 3.5 (3) M 80 × 6
(4) 61 kN (5) (a) 234 MPa, (b) 206.88 MPa
(6) F_e = 45,000 N, Joint does not open, M 30 × 3.5 (7) 7378.24 N,
(8) 1.377, 7490 N (9) M 20 × 2.5 (10) M 12 × 1.75

(11) $$\frac{Pl(a+b)}{6a^2 + 4b^2 - 4ab} - \frac{P}{8}$$

(12) With F·S = 6, S_y = 333 MPa, M 16 × 2 bolts can be used in both cases.
(13) M 24 × 3, 68640 N, 329.472 Nm
(14) (a) M27 × 3, (b) M33 × 3.5
(15) $d = 6.32 \to 10$ mm (16) M8 × 1.25 (17) M10 × 1.5
(18) M10 × 1.5 (19) P_1 = 19860 N, P_2 = 22362.7 N

10

Power Screw

CONCEPT REVIEW

10.1 INTRODUCTION

The combination of a screw and a nut used for lifting a load or for moving it linearly is termed as the *power screw* since, work is done by the screw in moving the load.

10.2 THREAD PROFILES

Commonly used thread profiles (Fig. 10.l) are:

(1) Square threads,
(2) Acme threads,
(3) ISO Trapezoidal threads,
(4) Knuckle threads, and
(5) Buttress threads.

Friction is minimum in knuckle threads, but they are difficult to manufacture. Square threads cause less frictional torque as compared to acme or ISO trapezoidal threads. Manufacturing of acme and trapezoidal threads is possible by using dies which reduces the cost of manufacture.

10.3 EFFICIENCY AND POWER LOSS DUE TO FRICTION

A screw is considered as an inclined plane wound on a cylinder. Refer Fig. 10.2.

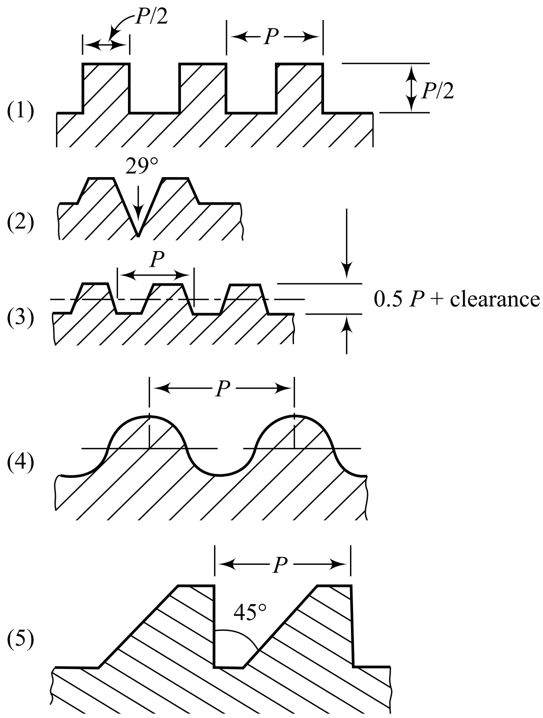


Fig. 10.1

$$\therefore \qquad \alpha = \tan^{-1} \frac{p}{\pi d_m} \qquad (10.1)$$

where α = angle of helix, p = pitch and d_m = mean diameter of thread

For multistart threads

$$\alpha = \tan^{-1} \frac{l}{\pi d_m}, \; l = \text{lead in mm} \qquad (10.2)$$

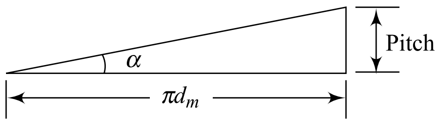


Fig. 10.2

Using the theory of inclined plane the force P required to lift the load up the screw is given by

$$P = W(\tan \alpha + \varphi) \qquad (10.3)$$

where W = load to be lifted, φ = angle of friction = $\tan^{-1} \mu$, and μ = coefficient of friction.

$$\therefore \text{ Torque required to overcome friction } T_f = W \tan(\alpha + \varphi) \times \frac{d_m}{2} \qquad (10.4)$$

In the absence of friction

$$P = W \tan \alpha$$

$$\therefore \qquad \eta = \frac{\tan \alpha}{\tan(\alpha + \varphi)} \qquad (10.5)$$

For the trapezoidal and acme threads with 2β equal to 30° and 29° respectively,

$$T_f = W \times \frac{d_m}{2} \left[\frac{\cos \beta \tan \alpha + \mu}{\cos \beta + \mu \tan \alpha} \right] \qquad (10.6)$$

$$\eta = \frac{\tan \alpha (1 - \mu \sec \beta \tan \alpha)}{(\tan \alpha + \mu \sec \beta)} \qquad (10.7)$$

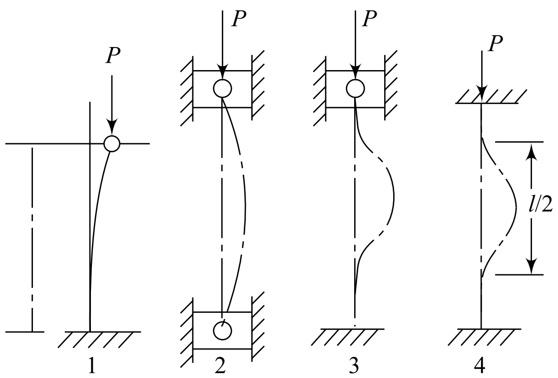


Fig. 10.3

10.4 COLUMN EFFECT

A screw subjected to compressive load as in a screw jack or screw clamp may buckle if the length is more. The buckling or crippling load is calculated by using the following equations. Euler's equation:

$$P_{cr} = \frac{C\pi^2 EA}{(L/k)^2} \tag{10.8}$$

J.B. Johnson's equation:

$$P_{cr} = S_y A \left[1 - \frac{S_y \,(L/k)^2}{4C\pi^2 E}\right] \tag{10.9}$$

where

C = constant depending on end condition, E = modulus of elasticity MPa, A = area of transverse cross section, L = length of the column, k = minimum radius of gyration, $d/4$ for a circular section of diameter d. L/k is termed as slenderness ratio.

10.5 DESIGN PROCEDURE

1. The material normally used for the screw is 30C4 or 45C8 steel while that for the nut is phosphor bronze, gunmetal etc. Soft bearing material used for nut wears fast and only a nut of smaller size needs replacement reducing the cost of replacement.
2. Find the core diameter of the screw by using the direct stress equation $\sigma_c = \frac{W}{A_c} = \frac{S_{uc}}{N}$, large factor of safety of the order of 7 to 10 is adapted to account for the stress concentration, torsional shear stress induced due to the frictional torque and buckling in case of compressive loading.
3. After obtaining the core diameter the core area and pitch are obtained by referring to the Table 8.
4. Find α, the angle of helix using Eqs. (10.1) or (10.2).
5. Find the frictional torque beween screw and nut using Eq. (10.4) or (10.6)
6. Find the shear stress induced in the core area of the screw due to friction to torque T_f using equation

$$\tau = \frac{16T_f}{\pi dc^3}$$

7. Find the actual σ_t or σ_c using the modified standard core diameter.
8. Check whether induced stresses are within the limit according to maximum shear or maximum energy of distortion theory of failure.
9. Find the number of threads in the nut using bearing consideration. Use P_b = 10–15 MPa. The hatched area in Fig. 10.4 represents the bearing area for one thread.
   For n number of threads

   load $$W = n \cdot P_b \,\frac{\pi}{4}(d_n^2 - d_c^2) \tag{10.10}$$

10. Find the height H of the nut using

$$H = n \times p \tag{10.11}$$

11. Check the threads of the nut for shearing by using the equation

$$\tau = \frac{W}{\pi d_n \, t \cdot n} \tag{10.12}$$

where t = thickening of threads at the root

12. Find L/k by using the equation

$$L/k = \sqrt{\frac{2C\pi^2 E}{S_y}} \tag{10.13}$$

If the calculated value of L/k exceeds the actual L/k ratio, use J.B. Johnson's formula (Eq. (10.9)) to check for buckling. Else, use Euler's formula (Eq. (10.8)) to check for the same. The buckling or crippling load obtained from Eqs (10.8) or (10.9) must be 2–3 times the actual compressive load. If the actual load is tensile, then this calculation need not be done.

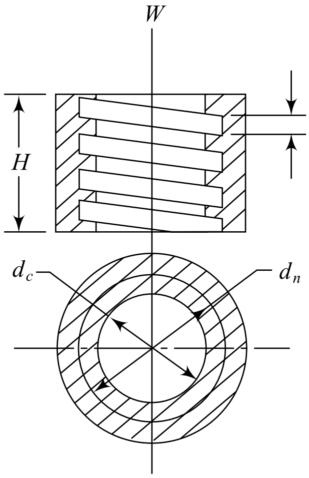
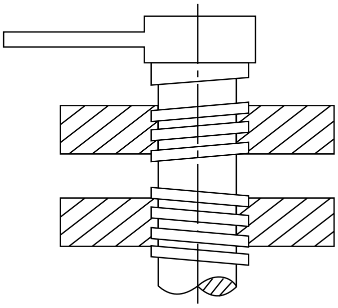


Fig. 10.4

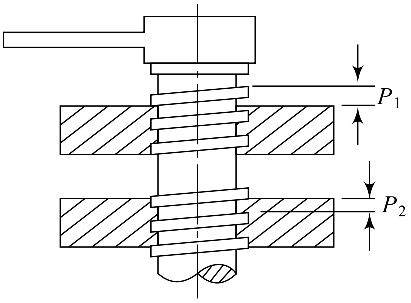
10.6 OTHER VARIETIES OF SCREWS

Multistart threads may be used to increase the linear displacement of a screw in one revolution. But this increases the angle α and if $\alpha > \phi$, the screw may overhaul. For this purpose a compound screw (Fig. 10.5) may be used for increasing the linear displacement per revolution. Total displacement in one revolution $= p_1 + p_2$ as the threads are of opposite hands. In differential screws (Fig. 10.6) the threads of the same hand with a small difference in the pitch are used so that the total displacement per revolution is equal to the difference of the two pitches. This is used in precision equipments.



Compound screw

Fig. 10.5



Differential screw

Fig. 10.6

A ball bearing screw is used for reducing the friction between the screw and the nut. The use of this screw increases the cost of manufacturing (Fig. 10.7). They are used in precision machine tools.

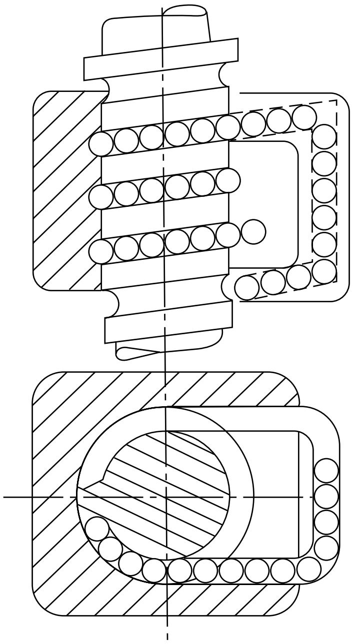


Fig. 10.7

WORKED EXAMPLES

10.1 A 25 mm single start square threaded screw with pitch 5 mm is 400 mm long between the nut and collar. The axial load is 25 kN and the coefficient of friction between the screw and nut is 0.12. Determine the factor of safety if S_y = 350 MPa.

Solution:

$$\alpha = \tan^{-1} \frac{5}{\pi d_m} = 4.046°$$

$$d_n = 25 \text{ mm}, d_c = 20 \text{ mm}, \quad \therefore d_m = 22.5 \text{ mm}$$

$$\varphi = \tan^{-1} 0.12 = 6.84°$$

$$\therefore \qquad T_f = 25000 \times \frac{22.5}{2} \times \tan(10.886) = 54089 \text{ N.mm}$$

$$\therefore \qquad \tau = \frac{16 \times 54089}{\pi (20)^3} = 34.43 \text{ MPa}$$

Actual $$\frac{L}{k} = \frac{400}{0.25 \times 20} = 80$$

Again by Eq. (10.13) $L/k = \sqrt{\dfrac{2C\pi^2 E}{S_y}} = \sqrt{\dfrac{2\times1\times\pi^2\times2.1\times10^5}{35}} = 108.82 > 80$

$\therefore$ Using J.B. Johnson's formula and $C = 1$

$$\sigma_c = \frac{2500\times4}{\pi\times20^2}\times\frac{1}{\left(1-\dfrac{350(80)^2}{4\times1\times\pi^2\times2.1\times10^5}\right)} = 109.09 \text{ MPa}$$

$\therefore$ Using maximum shear stress theory

$$\tau_{max} = \frac{1}{2}\sqrt{(109.09)^2+4(34.43)^2} = 64.5 \text{ MPa}$$

$$S_y = 350 \text{ MPa}, \therefore S_{ys} = 175 \text{ MPa and F·S} = \frac{175}{64.5} = 2.713.$$

10.2 A triple start square threaded screw is used to raise a load of 50 kN. The screw has a nominal diameter of 50 mm and the pitch is 8 mm. Height of the nut = 40 mm, μ between the screw and nut is 0.12 and there is no collar friction. Find the maximum shear stress induced in the screw, transverse shear stress induced in the screw and nut threads and the bearing pressure between screw and nut.

Solution:

$$d_n = 50 \text{ mm}, p = 8 \text{ mm} \quad \therefore \quad d_m = 46 \text{ mm}, d_c = 42 \text{ mm}$$

$$\alpha = \tan^{-1}\frac{8\times3}{\pi\times46} = 9.43°, \phi = \tan^{-1} 0.12 = 6.84°$$

$$\therefore \quad T_f = 50000\times\frac{46}{2}\tan(9.43° + 6.84°) = 335630 \text{ N.mm}$$

$$\therefore \quad \tau_{md} = \frac{16\times335630}{\pi\times(42)^3} = 23.07 \text{ MPa}$$

$$\sigma_c = 50000/\frac{\pi}{4}(42)^2 = 36.08 \text{ MPa}$$

$$\therefore \quad \tau_{max} = \frac{1}{2}\sqrt{(36.08)^2+4(23.07)^2} = 29.28 \text{ MPa}$$

$$\text{Bearing pressure} = \frac{50000}{\frac{\pi}{4}(d_n^2-d_c^2)\times n}, \quad n = \frac{\text{height of nut}}{\text{pitch}} = 5$$

$$= \frac{50000\times4}{\pi(50^2-42^2)\times5} = 17.29 \text{ MPa}$$

Transverse shear stress in the screw

$$= \frac{50000}{\pi\, d_c t \times n} = \frac{50000}{\pi \times 42 \times 4 \times 5} = 18.95 \text{ MPa}$$

Transverse shear stress in the nut $= \dfrac{50000}{\pi d_n t \times n} = \dfrac{50000}{\pi \times 50 \times 4 \times 5} = 15.92$ MPa

10.3 A sluice gate weighing 600 kN is raised and lowered by means of two square threaded screws. The coefficient of friction between the thrust collar and screw is 0.003 and that between the screw and nut is 0.05. Design the screw and the nut.

Solution:

Load shared by each screw = 30×10^4 N, $\sigma_c = 80$ MPa

$$\therefore \qquad d_c = \sqrt{\frac{30 \times 10^4 \times 4}{\pi \times 80}} = 69.09 \text{ mm.}$$

From Table 8, $d_n = 80$ mm, $d_c = 70$ mm, $p = 10$ mm, $d_m = 75$ mm

$$\therefore \qquad \alpha = \tan^{-1} \frac{10}{\pi \times 75} = 2.43°$$

$$\therefore \qquad \varphi = \tan^{-1} 0.05 = 2.862°$$

$$\therefore \qquad T_f = 30 \times 10^4 \times \frac{74}{2} \tan (2.43° + 2.862°)$$

$$= 1042 \text{ N.m} = 1042 \times 10^3 \text{ N.mm}$$

$$\therefore \qquad \tau_{\text{ind}} = \frac{1042 \times 10^3 \times 16}{\pi \times (69.5)^3} = 15.8 \text{ MPa}$$

$$\sigma_{\text{ind}} = \frac{30 \times 10^4 \times 4}{\pi (69.5)^2} = 79.08 \text{ MPa}$$

Maximum shear stress $= \dfrac{1}{2}\sqrt{(79.08)^2 + 4(15.8)^2} = 42.57$ MPa.

Since $S_y = 350$ MPa and $S_{ys} = 175$ MPa, F·S $= \dfrac{175}{42.57} = 4.11$ which is sufficient.

Design of P-Bronze nut:

Bearing, $\qquad 30 \times 10^4 = P_b \times \dfrac{\pi}{4}(d_n^2 - d_c^2)\, n$

$$\therefore \qquad n = \frac{30 \times 10^4 \times 4}{15 \times \pi (80^2 - 69.5^2)} = 16.222$$

Let us use 18 threads, $\quad \therefore$ Height of the nut = 180 mm.

Shear stress induced in nut threads $= \dfrac{30 \times 10^4}{\pi \times 80 \times 5 \times 18} = 13.26$ MPa.

10.4 The lead screw of a lathe has trapezoidal threads. To drive the tool carriage the screw has to exert an axial force of 20 kN. The thrust is carried by the collar. The length of the lead screw is 1.5 m. Coefficients of friction at the collar and nut are 0.1 and 0.15 respectively. Suggest suitable size of the screw and height of the nut if the permissible bearing pressure is 4 MPa.

Solution:

Since the speed of the screw is high permissible, P_b is very small. Hence, it is advisable to base the preliminary calculation on wear of the screw. Using Eq. 10.10 we get

$$W = P_b \times \frac{\pi}{4}\,(d_n + d_c)\,(d_n - d_c)\cdot n$$

putting $\quad (d_n + d_c) = 2\,d_m, \quad (d_n - d_c) = p,\ n = \dfrac{H}{p} \quad \text{and} \quad \dfrac{H}{d_m} = \psi$

we get

$$d_m = \sqrt{\frac{2W}{\pi \psi P_b}} = \sqrt{\frac{2\times 20000}{\pi \times 1.5 \times 4}} = 46.06 \text{ mm}$$

Here the adapted value of ψ is 1.5.

The nearest standard size is 55 mm × 9 mm. From Table 10

$\therefore \quad d_n = 55$ mm, $p = 9$ mm, $d_c = 45.5$ mm

$d_m = 50.5$ mm, $\beta = 15°$ for trapezoidal thread

$H = 1.5 \times 50.5 = 75.75$

$\therefore$ Number of threads of nut $= \dfrac{75.75}{9} = 8.41$ modified to 9

Now, we check for the maximum shear stress

$$\alpha = \tan^{-1}\frac{9}{\pi \times 50.5} = 3.25°$$

modified coefficient of friction = 0.15 sec 15° = 0.15529

$\therefore \quad \varphi = 8.82°$

$$\therefore \quad T_f = 20000 \times \frac{50.5}{2} \times \tan(3.25° + 8.82°) = 107986.07 \text{ N.mm}$$

$$\therefore \quad \tau_{\text{ind}} = \frac{16 \times 107986}{\pi \times (45.5)^3} = 5.838 \text{ MPa}$$

$$\sigma_c = \frac{50000 \times 4}{\pi \times (45.5)^2} = 30.75 \text{ MPa}$$

$$\therefore \quad \tau_{\max} = \frac{1}{2}\sqrt{(30.75)^2 + 4(5.838)^2} = 16.44 \text{ MPa}$$

Actual $\quad L/k = \dfrac{1500}{0.25 \times 50.5} = 118.81$

From Eq. (10.13) $L/k = \sqrt{\dfrac{2C\pi^2 E}{S_y}}$

Let $C = 1$ and $S_y = 300$ MPa

$$\therefore \quad L/k = \sqrt{\frac{2 \times \pi^2 \times 2.1 \times 10^5}{300}} = 117.54$$

∴ As actual L/k is greater than calculated L/k, use Euler's equation

$$P_c = \frac{C\pi^2 EA}{(L/k)^2} = \frac{1 \times \pi^2 \times 2.1 \times 10^6}{(118.81)^2} \times \frac{\pi}{4} \times (50.5)^2 = 294143.18 \text{ N}$$

which is 14 times the actual load and is quite a satisfactory value.

10.5 A single start screw is used to raise a load of 45 kN. The nominal diameter is 60 mm and pitch is 9 mm. The acme threads are used and coefficient of friction is 0.12. Neglecting the collar friction calculate the torque required to raise the load, to lower the load and efficiency of the screw.

Solution:

$$d_m = 55.5 \text{ mm}$$

$$\alpha = \tan^{-1} \frac{9}{\pi(55.5)} = 2.95°$$

$$\text{modified } \mu = \mu \sec\beta = \frac{0.12}{\cos 14.5°} = 0.124$$

$$\therefore \quad \varphi = 7.06°$$

$$\therefore \quad T_f = 45000 \times \frac{55.5}{2} \tan \times (2.95° + 7.06°)$$

$$= 220413.05 \text{ N.mm to lift the load}$$

$$T_f = 45000 \times \frac{55.5}{2} \times \tan(7.06 - 2.95)$$

$$= 89730.6 \text{ N.mm to lower the load}$$

Efficiency, $\eta = \dfrac{\tan\alpha}{\tan(\alpha + \varphi)} = \dfrac{\tan 2.95}{\tan 10.01} = 0.2919.$

10.6 Find the stress induced in the core area of the screw in Example 10.5 using the maximum energy of distortion theory.

Solution:

$$\sigma_c = \frac{45000 \times 4}{\pi \times (55.5)^2} = 18.6 \text{ MPa}$$

$$\tau = \frac{16 \times 220413.05}{\pi \times (55.5)^3} = 6.56 \text{ MPa}$$

$$\text{Induced stress} = \sqrt{18.6^2 + 3(6.56)^2} = 21.8 \text{ MPa.}$$

10.7 Design a *C* clamp with a mean collar radius of 10 mm. Coefficient of friction between the screw and nut is 0.12 and that for the collar is 0.25. Also design the frame and handle of the *C* clamp. Material used is 30C8 for the screw and handle, 200 FG CI for the frame and the pad and P-Bronze for the nut. Refer Fig. E-10.7(a).

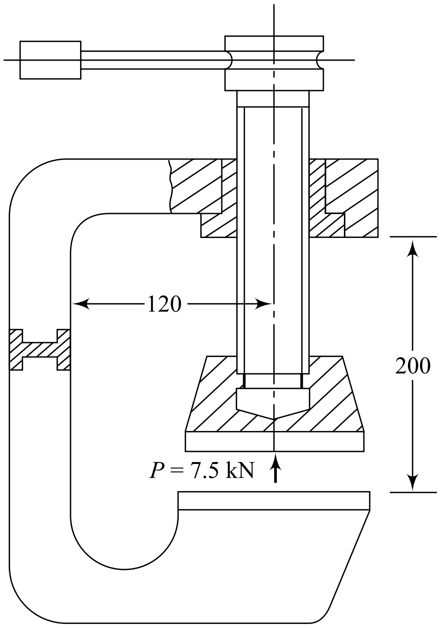


Fig. E-10.7a

Solution:

Screw

Direct compression using

$\sigma_c = 30$ MPa

$$d_c = \sqrt{\frac{7500 \times 4}{\pi \times 30}} = 17.84 \text{ mm}$$

Using Table 8, the selected square thread has

$d_n = 24$ mm, $d_c = 19$ mm, $p = 5$ mm

and $d_m = 21.5$ mm

$$\alpha = \tan^{-1} \frac{5}{\pi \times 21.5} = 4.23°,$$

$$\varphi = \tan^{-1} 0.12 = 6.3°$$

$$T_{f_1} = 7500 \times \frac{21.5}{2} \times \tan(4.23 + 6.3)$$

$$= 14986.629 \text{ N.mm}$$

Now,
$$\sigma_c = \frac{7500}{\pi \times (19)^2} = 26.45 \text{ MPa}$$

$$\tau = \frac{16 \times 14986.629}{\pi (19)^3} = 11.13 \text{ MPa}$$

$\therefore$
$$\tau_{max} = \frac{1}{2}\sqrt{(26.45)^2 + 4(11.13)^2} = 17.3 \text{ MPa}$$

which is well within the limit.

$$L/k = \frac{200}{0.25 \times 1} = 42.1$$

Calculated value of L/k

$$= \sqrt{\frac{2C\pi^2 E}{S_y}} = \sqrt{\frac{2 \times 1 \times \pi^2 \times 2.1 \times 10^5}{300}} = 117.54$$

$\therefore$ As calculated $L/k >$ actual L/k use J.B. Johnson's formula,

$\therefore$
$$P_{cr} = S_y A \left(1 - \frac{S_y (L/k)^2}{4C\pi^2 E}\right)$$

$$= 300 \times \frac{\pi}{4} \times (19)^2 \left[1 - \frac{300 \times (42.1)^2}{4 \times 1 \times \pi^2 + 2.1 \times 10^5}\right]$$

= 79603.244 which is more than 10 times greater than the actual load. Hence it is a safe value.

P-Bronze nut

$$n = \frac{7500}{\frac{\pi}{4}(24^2 - 19^2) \times 15} = 2.96$$

For stability, let $n = 6$ so that the height of the nut is 30 mm.

Shear stress in the nut threads = $\frac{7500}{\pi \times 24 \times 2.5 \times 6}$ = 6.63 MPa which is within the limit. To design the handle, the torque to overcome the collar friction is to be calculated

$$T_{f_2} = \frac{2}{3}\mu W r = \frac{2}{3} \times 0.25 \times 7500 \times 10 = 12499.99 \text{ N.mm}$$

$\therefore$ Total torque = 14986.629 + 12499.99 = 27489.629 N.mm

This is the maximum bending moment on the handle. If d_1 is the circular handle diameter of the circular handle

$$d_1 = \sqrt[3]{\frac{32 \times 27486.629}{\pi \times 80}} = 15.182 \text{ mm} \cong 16 \text{ mm}$$

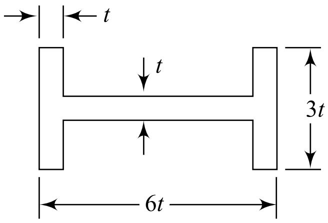


Fig. E-10.7b

Section of frame:

Usually an I section is preferred. Refer Fig. E-10.7(b)

$$I_{xx} = \frac{1}{12} \times 3t\,(6t)^3 - \frac{1}{12} \times 2t\,(4t)^3 = 43.33\,t^4$$

$$\therefore \quad z = \frac{43.33t^4}{3t} = 14.44\,t^3$$

b.m = 7500 (120 + 3t)

$$\therefore \quad \sigma_b = \frac{7500(120 + 3t)}{14.44t^3}$$

$\therefore$ Direct stress $\sigma_t = \frac{7500}{10t^2}$

For C.I, let σ_t = 20 MPa

$$\therefore \quad 20 = \frac{7500\,(120 + 3t)}{14.44t^3} + \frac{7500}{10t^2}$$

By trial and error t = 20 mm.

10.8 Design a screw jack for lifting a load of 100 kN considering a maximum lift of 500 mm. Material for the screw is 30C8 steel with S_{uc} = 550 MPa, S_y = 300 MPa. For the nut P_b = 15 MPa. The body is of C.I.

Solution:

For the screw let $\sigma_c = 80$ MPa considering crushing failure

of screw, $d_c = \sqrt{\dfrac{100000 \times 4}{\pi \times 80}} = 39.89$ mm.

Therefore, we use screw of 48 mm nominal diameter and a pitch of 8 mm.

$$\therefore \quad \alpha = \tan^{-1} \frac{8}{\pi \times 44} = 3.31°. \text{ Let } \mu = 0.12$$

$$\therefore \quad \varphi = \tan^{-1} 0.12 = 6.84°$$

$$\therefore \quad T_f = 100000 \times \frac{44}{2} \tan (6.84 + 3.31) = 393860.76 \text{ N.mm}$$

$$\therefore \quad \tau_m = \frac{393860.76 \times 16}{\pi (40)^3} = 31.34 \text{ MPa}$$

$$\text{Induced } \sigma_c = \frac{100000 \times 4}{\pi (40)^3} = 79.57 \text{ MPa}$$

$\therefore$ As per the energy of distortion theory maximum induced stress

$$= \sqrt{(79.57)^2 + 3(31.34)^2} = 96.32 \text{ MPa}$$

$$\therefore \quad \text{F·S} = 300/96.32 = 3.11$$

Considering buckling $L/k = \dfrac{500}{0.25 \times 40} = 50$

Using J.B. Johnson's formula

$$P_c = 300 \times \frac{\pi}{4} \times 40^2 \left[1 - \frac{300\,(50)^2}{4 \times 0.25 \times \pi^2 \times 2.1 \times 10^5}\right]$$

= 240572.59 N which is 2.4 times the actual load and hence safe value.

Nut: For the P-bronze nut $P_b = 15$ MPa

$\therefore$ Number of threads in the nut $= \dfrac{100000 \times 4}{\pi (48^2 - 40^2) \times 15} = 12.057$

Use 13 threads so that height of the nut is 104 mm.

To calculate the thickness of the nut let $\sigma_t = 60$ MPa. Considering tearing of nut

$$\sigma_t = \frac{100000 \times 4}{\pi (d_0^2 - 48^2)}, \quad \therefore d_0 = 66.52 \text{ mm modified to 70 mm.}$$

Thickness of the nut collar may be calculated by considering shearing.

Let $\tau = 30$ MPa

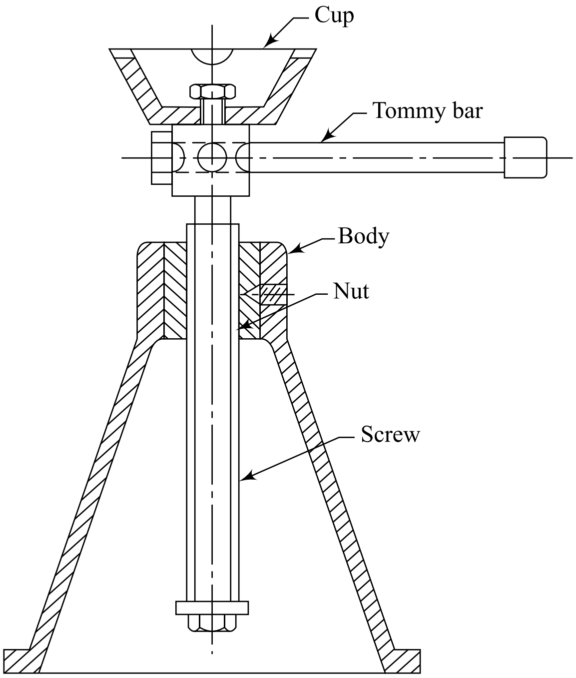


Fig. E-10.8a

$$\therefore \qquad t = \frac{100000}{\pi \times 70 \times 30} = 15.15 \text{ mm modified to 16 mm.}$$

For the lever to rotate screw, the torque to be overcome = $T_f + T_{f\text{collar}}$

For collar friction

$$d_o = 50 \text{ mm}, d_i = 20 \text{ mm}$$

$$\therefore \qquad T_{f\text{collar}} = \frac{2}{3}\mu W \frac{(r_0^3 - r_i^3)}{(r_0^2 - r_1^2)} W$$

Let .μ for collar be 0.2

$$\therefore \qquad T_{f\text{collar}} = \frac{2}{3} \times 0.2 \times 100000 \frac{(25^3 - 10^3)}{(25^2 - 10^2)} = 371428.57 \text{ N.mm}$$

$\therefore$ Total torque = 393860.76 + 371428.57 = 765289.33 N.mm

This causes a b.m on the lever

$$\therefore \qquad \text{diameter of lever} = \sqrt[3]{\frac{765289.33 \times 32}{\pi \times 100}} = 42.71 \text{ mm}$$

Therefore adapt the diameter of the lever equal to 50 mm and a length of 2 m.

C-I body:

Diameter of the body at the top = 1.5 times diameter of the nut

= 105 mm

Inside diameter of the body at the bottom = 2.25 × 70 = 157.5 mm

The body is subjected to compressive load. If the mean inside diameter is 125 mm

$$\sigma_c = \frac{100000 \times 4}{\pi (D_0^2 - 125^2)}, \ \sigma_c = 100 \text{ for C.I.}$$

$\therefore$ $D_0 = 130$ mm. Thus, thickness of the body = 15 mm.

Cup of C.I: Thickness of the cup may be taken as 15 mm, and height of the cup = 50 mm. The dimensions are shown in Fig. E-10.8(d).

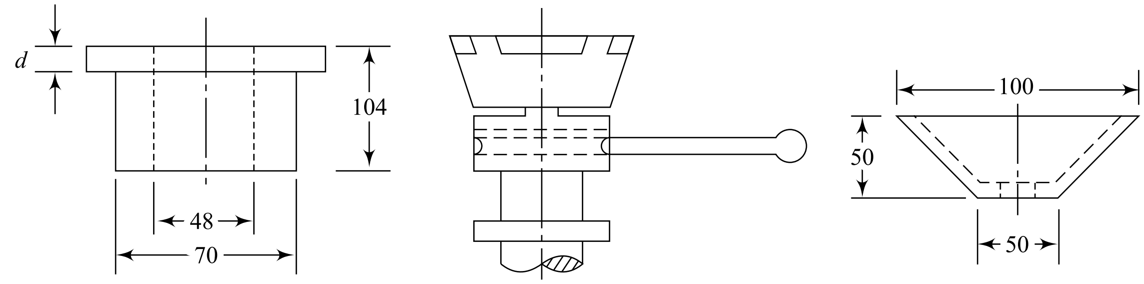


Fig. E-10.8b, c and d

10.9 Design a toggle jack to lift a load of 5 kN. Distance between the centre lines of the nut varies from 50 mm to 220 mm. The links are symmetrical and 120 mm long. The links, pins and the screw are made of 30C8 steel with $\sigma_c = 100$ MPa, $\sigma_t = 80$ MPa, $\tau = 50$ MPa and $P_b = 15$ MPa.

Solution:

The two extreme positions of the jack are shown in Fig. E-10.9(a). The angle between the link and horizontal is minimum in the bottommost position and is given by

$$\alpha = \cos^{-1} \frac{95}{120} = 37.65°$$

$$\text{Compressive force in the link} = \frac{5000}{2 \sin 37.65} = 4092.74 \text{ N}$$

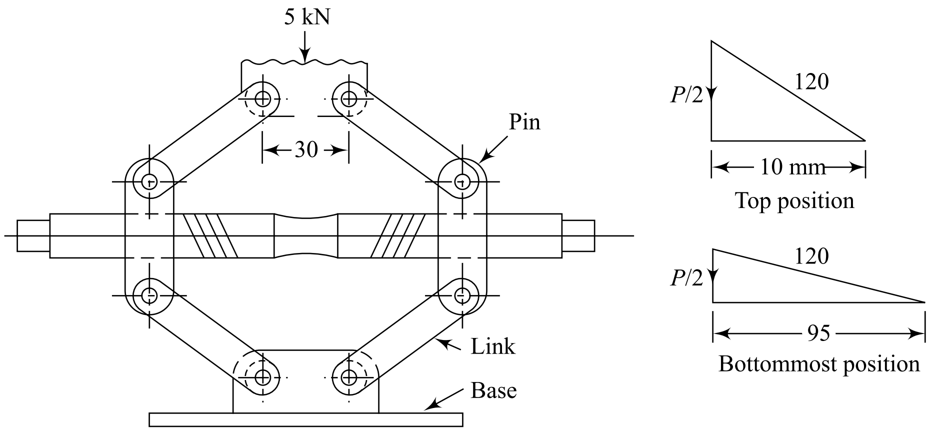


Fig. E-10.9a

Component of force in the horizontal direction = $\dfrac{5000}{2\tan\alpha}$ = 3240.46 N

From the free body diagram E-10.9(b) it is clear that at all the joints a vertical force equal to $P/2$ and a horizontal force of $\dfrac{P}{2\tan\alpha}$ acts.

The screw is subjected to a pull of $P/\tan\alpha$ as shown in the free body diagram E-10.9(c)

∴ Force on the screw = 6480.92 N

A compressive force of 4092.74 acts on the pair of link. Thus, each link is subjected to a compressive force of 2046.37 N.

Design of screw:

$$d_c = \sqrt{\frac{6480.92\times 4}{\pi\times 80}} = 10.156 \text{ mm.}$$

Let us use a screw with d_n = 14 mm, d_c = 12 mm, d_m = 13 mm

$$\alpha = \tan^{-1}\frac{2}{\pi\times 13} = 2.80°,\ \phi = 6.84°$$

$$T_f = 6480.92\times\frac{13}{2}\times\tan(6.84+2.8)$$
$$= 7155.33 \text{ N.mm}$$

$$\tau = \frac{16\times 7155-33}{\pi\times(12)^3} = 21.08 \text{ MPa}$$

$$\sigma_t = 6480.92/\left(\frac{\pi}{4}\times 12^2\right) = 57.3 \text{ MPa}$$

∴ Maximum shear stress

$$= \frac{1}{2}\sqrt{(57.3)^2+4(21.08)^2}$$
$$= 35.57 \text{ MPa which is a safe value.}$$

Height of the nut:

$$n = \frac{6480.92\times 4}{\pi(14^2-12^2)\times 15} = 10.58$$

Let n = 12 so that H = 24 mm

$$\tau_{\text{ind}} = \frac{6480.92}{\pi\times 14\times 1\times 12}$$
$$= 12.28 \text{ MPa which is within the limit.}$$

Link:

The links are under direct compression. There is also a possibility of buckling. Let us use σ_c = 50 MPa and find the dimension of the link considering direct compressive stress for which $b = 3\,t$

Load on each link = $\dfrac{4092.74}{2}$ = 2046.37 N

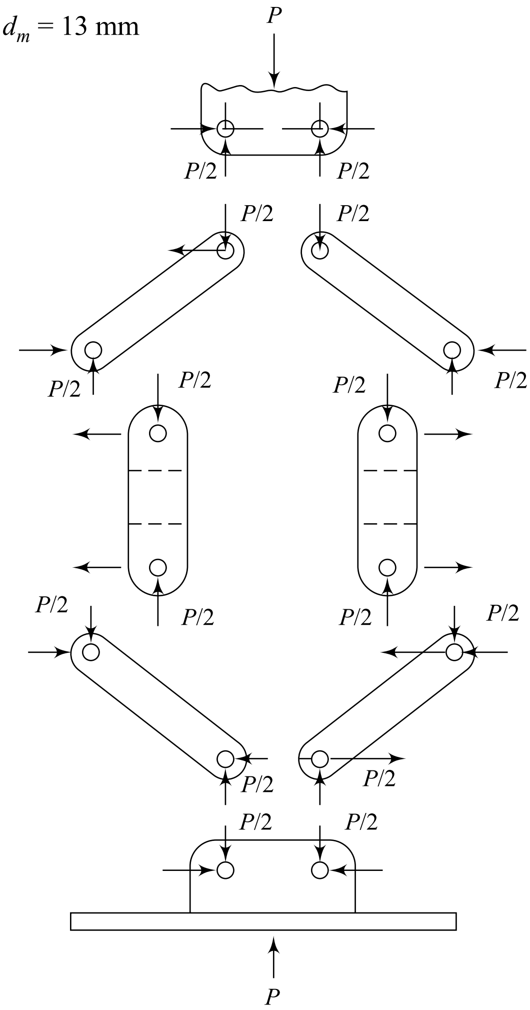


Fig. E-10.9b

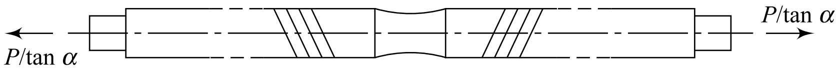


Fig. E-10.9c

$$\therefore \qquad 3t \times t = \frac{3046.37}{50}, \ \therefore\ t = 3.69 \text{ mm}$$

Let us adapt $t = 4$ mm and $b = 12$ mm

Moment of inertia of the cross section in the plane of link $= \frac{1}{12} \times 4(12)^3$

$\therefore$ Value of k for the cross section in the plane of the link $= \frac{4 \times 12 \times 12}{12 \times 4} = 12$ mm.

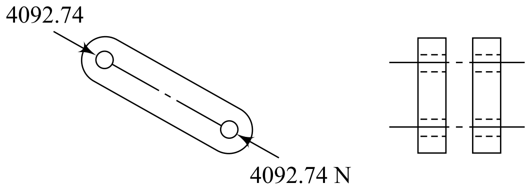


Fig. E-10.9d

$\therefore\ L/k = \frac{120}{12} = 10$ which is a very small value.

Similarly m-I of the c/s in the plane perpendicular to the plane of the link = 1/2 × 12(4)^3,

$\therefore\ k$ in this plane $= \frac{64}{48} = 1.33$

$\therefore\ L/k = \frac{120}{1.33} = 90.22 < 117$. Hence, use J. B. Johnson's formula.

The link in the plane is considered to be fixed at both the ends. Hence, $C = 4$

$$\therefore \qquad P_{er} = S_y A \left(1 - \frac{S_y\ (L/k)^2}{4C\pi^2 E}\right)$$

$$= 300 \times 48 \left(1 - \frac{300\,(90.22)^2}{4 \times 4 \times \pi^2 \times 2.1 \times 10^6}\right) = 13339.647 \text{ N}$$

which is very large as compared to the actual load of 2046.37 N. Hence, there is no possibility of buckling.

Pins at the joints are designed with bearing, shearing and bending consideration just like a pin in the knuckle joint. Load on each pin is 4092.74 N. Use $P_b = 20$ MPa and $l : d = 1 : 1$.

$$4092.74 = 20 \times d^2, \ \therefore\ d = l = 14.305 \text{ mm} \cong 15 \text{ mm}.$$

The attachment of the link to the nut may be done as shown in Fig. E-10.9(e) such that the length of the pin is 15 + (2 × thickness of link) = 23 mm. With a small clearance it may be 24 mm.

$\therefore$ Bending moment on the pin = 4092.74 × 24/6

$$\therefore \qquad \sigma_t = \frac{M}{Z} = \frac{4092.74 \times 32 \times 24}{\pi \times (15)^3 \times 6} = 49.4 \text{ MPa}$$

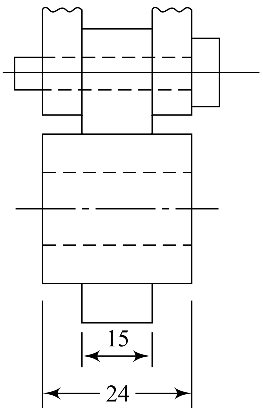


Fig. E-10.9e

There is no harm caused to the pin due to bending as the stress induced due to bending is small.

Shear stress induced in the c/s of pin = $\dfrac{4092.74 \times 4}{\pi (15)^2 \times 2}$ = 11.58 MPa

Turn buckle:

Turn buckle is designed on the same lines as a power screw. It is used for joining the ends of suspension wires and rods and is useful as a device to produce tension. It has right and left handed metric V threads and works as the compound screw.

10.10 The pull in the tie rod of an iron roof truss is 100 kN. Find the dimension of the rod and the coupling nut using 30C8 steel for both.

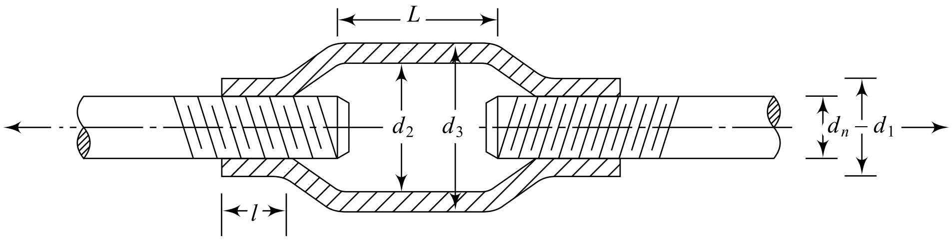


Fig. E-10.10

(a) *Diameter of rod:* $\sigma_t = \dfrac{p}{A} = 80$

$$d = \sqrt{\frac{100000 \times 4}{\pi \times 80}} = 39.89 \text{ mm}$$

As a part of the rod is threaded let us use M48 threads so that d_n = 48 mm and d_c = 41.795 mm

$\therefore$ d_m = 44.752 mm, and core area = 1465 mm^2

$$\alpha = \tan^{-1} \frac{P}{\pi d_m} = \tan^{-1} \frac{5}{\pi \times 44.752} = 2.036°$$

Let μ = 0.1. As angle of V is 60°, the modified $\mu' = \mu \sec 30° = 0.115$

$\therefore$ $\varphi = \tan^{-1} 0.115 = 6.58°$

$\therefore$ $$T_f = 100000 \times \frac{44.752}{2} \times \tan (6.58° + 2.036°) = 335849 \text{ N.mm}$$

$\therefore$ $$\tau_{\text{ind}} = \frac{335849 \times 16}{\pi (41.795)^3} = 23.49 \text{ MPa}$$

$$\sigma_t = \frac{100000}{1465} = 68.26 \text{ MPa}$$

$\therefore$ Maximum shear stress = $\dfrac{1}{2}\sqrt{(68.26)^2 + 4(23-49)^2}$ = 41.44 MPa which is within the limit.

(b) *Nut:* Length of the nut is found by shearing consideration

$$P = \pi d_c l \times \tau, \quad \therefore \quad \frac{100000}{\pi \times 41.795 \times 40} = 19.03 \text{ mm}$$

Let us use $l = 20$ mm

Find d_1 by tearing consideration

$$P = \sigma_t \times \frac{\pi}{4}(d_1^2 - d_n^2)$$

$$\therefore \quad d_1 = \sqrt{\frac{100{,}000 \times 4}{\pi \times 80} + d_n^2} = 62.41 \text{ mm say } 64 \text{ mm}$$

$$d_2 = d_1 + 6 \text{ mm} = 70 \text{ mm}$$

$$d_3 = 70 + 2 \text{ (thickness of coupler)} = 70 + 16 = 86 \text{ mm}$$

or d_3 can be found by tearing consideration

$$d_3 = \sqrt{\frac{100000 \times 4}{\pi \times 80} + d_2^2} = 80.57 \text{ mm say } 82 \text{ mm.}$$

$$L = 6\,d = 6 \times 48 = 288 \text{ mm.}$$

OBJECTIVE QUESTIONS

10.1 The best threads for a power screw are
- (a) ISO Metric threads
- (b) Ball bearing threads
- (c) Trapezoidal threads
- (d) Buttress threads

10.2 Acme threads are preferred over square threads for power screw because
- (a) their efficiency is high
- (b) they can be manufactured using dies
- (c) they have higher coefficient of friction
- (d) none of the above

10.3 It is essential to check the design of a screw
- (a) for maximum principal stresses
- (b) for maximum shear sress
- (c) for maximum tensile stress
- (d) for bending

10.4 The expression to find the diameter of a screw by wear consideration is
- (a) $d_e = \sqrt{\frac{4W}{\sigma_t \cdot \pi}}$
- (b) $d_m = \sqrt{\frac{2W}{P_b \psi \pi}}$
- (c) $d_c = \sqrt{\frac{W}{\pi P_b t}}$
- (d) $P_b = \frac{4W}{\pi(d_n^2 - d_c^2)}$

10.5 The condition for a screw to be self-locking is
- (a) $d_m = 1.2\, d_c$
- (b) $\tan^{-1} \mu < \tan^{-1} \frac{p}{\pi d_m}$
- (c) $\varphi > \alpha$
- (d) $\tau_{\text{ind}} < \sigma_{\text{ind}}$

10.6 Buckling of the screw should be checked by using J.B. Johnson's formula if the
- (a) actual slenderness ratio > $\sqrt{\frac{2C\pi^2 E}{S_y}}$
- (b) actual slenderness ratio < 117.5
- (c) actual slenderness ratio < $\sqrt{\frac{2C\pi^2 E}{S_y}}$
- (d) none of the above

10.7 Slenderness ratio is the ratio of the
(a) unsupported length to the minimum radius of gyration
(b) unsupported length to the diameter of rod
(c) length of the component to the minimum width of cross section
(d) none of the above

10.8 The material generally used for the nut is
(a) C.I (b) 30C8 steel (c) Al (d) P-Bronze

10.9 In order to get a very small value of linear displacement per revolution of screw, we use a
(a) compound screw (b) screw having a small pitch
(c) differential screw (d) multistart threads

10.10 Multistart threads are used to get
(a) small linear displacement
(b) larger linear displacement with assured self-locking
(c) larger linear displacement with no guarantee of self-locking
(d) none of the above

10.11 Multistart threads should be replaced by a compound screw to
(a) get larger displacement (b) get self-locking
(c) improve the efficiency (d) none of the above.

10.12 Ball bearing threads are used in a
(a) screw jack (b) machine tool feed mechanism
(c) screw press (d) shaft straightener

10.13 For a displacement of 0.01 mm in one revolution
(a) a differential screw with pitches 5 mm and 6 mm should be used
(b) a differential screw with pitches 0.5 mm and 0.49 mm should be used
(c) a compound screw with pitches 5.10 mm and 5.11 mm should be used
(d) a ball bearing screw should be used

10.14 The differential screw is used in a
(a) turn buckle (b) micrometer (c) vernier calliper (d) coupler

10.15 Overhauling is possible for a screw of the type
(a) single start square thread with pitch 1.5 mm, mean diameter 10 mm and $\mu = 0.12$
(b) triple start threads with pitch 1.5 mm, mean diameter 10 mm and $\mu = 0.12$
(c) double start threads with M10 × 1.5 and $\mu = 0.12$.
(d) single start square threads M12 × 1.5 and $\mu = 0.08$

REVIEW QUESTIONS

10.1 Draw the different profiles used for the screws and explain their relative merits and demerits.
10.2 Why are ISO Metric threads rarely used for the power screw while they are invariably used for fasteners?
10.3 Differentiate between a compound screw and a differential screw.
10.4 Derive the expression for the efficiency of a screw.
10.5 The magnitudes of torque considered to design the screw and the handle are not the same. Why?
10.6 Explain why a soft material is used for the nut.
10.7 Explain the procedure of checking the design of a screw for buckling.

10.8 Why a large factor of safety is used for preliminary calculation of diameter of screw?
10.9 Explain the procedure of deciding the height of the nut.
10.10 Why are acme threads used in practice in place of square threads?
10.11 What is a split nut? Where is it used?
10.12 What modification of power screw is used in feed mechanism of machine tools? Why? Draw the sketch of the arrangement.
10.13 What are the multi start threads? Where are they used? What are their limitations?
10.14 Differentiate between a compound screw and a differential screw.
10.15 What are the different methods used to prevent the rotation of component being pressed with the screw? Explain by drawing neat sketches.
10.16 What is self locking screw? When is it essential? How is it achieved?

PRACTICE PROBLEMS

10.1 A single start screw with 48 mm nominal diameter and pitch 8 mm is used to raise a load of 10 kN. The coefficient of friction between the screw and the nut is 0.12. Calculate the torque required to overcome friction while raising and lowering the load and efficiency of the screw.
10.2 The gate in a large gate valve weighs 6000 N and the frictional force due to water pressure is 2000 N. The gate is lifted by using a rotating nut. The nominal diameter of the screw is 40 mm and the pitch is 6 mm. The rotating nut presses against a supporting collar of 75 mm outside diameter and 42 mm inside diameter. Coefficient of friction for the collar is 0.12. Find the torque that should be applied to raise the gate valve and the maximum shear stress induced in the body of the screw. Also find the efficiency of the screw.
10.3 A shaft straightener exerts a force of 30 kN. The material of the screw 30C8 has S_{uc} = 500 MPa, S_y = 300 MPa and permissible P_b = 20 MPa. Design the screw, the nut, the C.I, body and the fixing bolts.

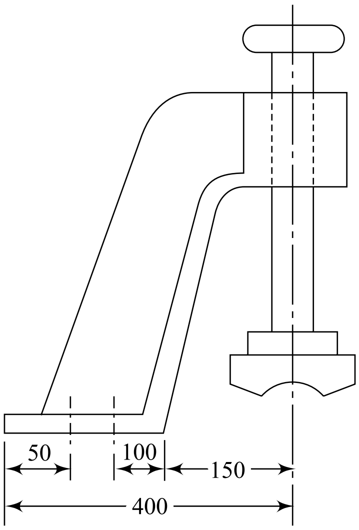


Fig. P-10.3

10.4 The lead screw of a lathe has acme threads. The screw exerts an axial force of 12.5 kN and the thrust is carried by a collar. Length of the lead screw is 1000 mm and the speed is 30 r.p.m. Find the size of the screw and the nut if permissible bearing pressure is 5 MPa and safe shear stress is 30 MPa.

S_y = 300 MPa

10.5 Design a screw jack for lifting a load of 40 kN. Use the material and the data as in Example 10.8.
10.6 Design a C Clamp to exert a clamping force of 3500 N with maximum distance between the inner sides of the arms being 300 mm. Design the screw, the nut, the frame with I section and the handle taking the eccentricity as 100 mm.
10.7 Design a 25 kN capacity hand screw press with a frame of T section as shown in Fig. P-10.7. Design the screw, the nut, the frame, the fixing bolts four in number, hand wheel and the pad.

10.8 Design a toggle jack to lift a load of 1000 N through a height of 100 mm. The length of the links is 120 mm and the distance between the pins connecting the links to the top and base is 30 mm. Bearing pressure is limited to 15 MPa and the coeffcient of friction between the screw and the nut is 0.12.

10.9 A screw is used to lift a load of 25000 N. Coefficient of friction between the screw and the nut is 0.15 and the maximum shear stress in the screw is 30 MPa with the bearing pressure limited to 10 MPa. Determine the size of the screw, height of the nut, and the power lost in friction if the screw rotates at 100 r.p.m.

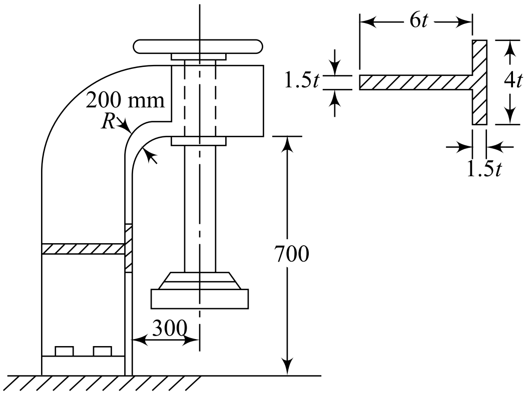


Fig. P-10.7

10.10 The moving head of a hydraulic testing machine which weighs 250 kN and is supported by two trapezoidal screws. The permissible compressive stress = 100 MPa, coefficient of friction = 0.12 and bearing pressure = 15 MPa. Determine the height of the nut.

10.11 Lead screw of a lathe has acme threads. The length of the screw is 1.5 m. The axial force on the screw is 15 kN. The coefficient of function between screw and nut is 0.12. Suggest suitable size and the height of the screw and the nut of P_b = 4 MPa.

10.12 A double start square threaded screw is used for lifting a load of 25 kN. The screw has nominal diameter 32 mm and pitch 6 mm. State whether screw is self locking if the coefficient of friction between screw and the nut is 0.12. Also find the maximum shear stress induced in the screw.

10.13 Design a screw press as shown in Fig. P-10.13. The material for the hand wheel and pad is 200 FG C.I with N = 6.5, material for screw 30C8 steel with N = 5. Material for nut p-bronze with P_b = 4 MPa. Material for pillars 30C4 steel with S_y = 300 MPa, N = 4.

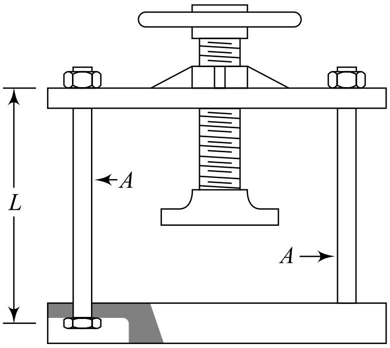


Fig. P-10.13

ANSWERS

Objective Questions

(1) b (2) b (3) b (4) b (5) c (6) c (7) a (8) d (9) c
(10) c (11) b (12) b (13) b (14) b (15) b

Practice Problems

(1) 39.46 N.m, 13.56 N.m
(2) 85.6 N.m, 5.51 MPa, 8.9%
(4) d_n = 36 mm, p = 6 mm
(9) d_n = 50 mm, p = 12 mm, n_e = 3.05 → 6, p_f = 1.22 kW
(10) d_n = 50 mm, p = 8 mm, n = 15
(11) ψ = 1.2, d_m = 55 mm, d_c = 45.5 mm, p = 9 mm, H = 66 mm, N for buckling = 14

11

Belt, Rope and Chain Drive

CONCEPT REVIEW

11.1 INTRODUCTION

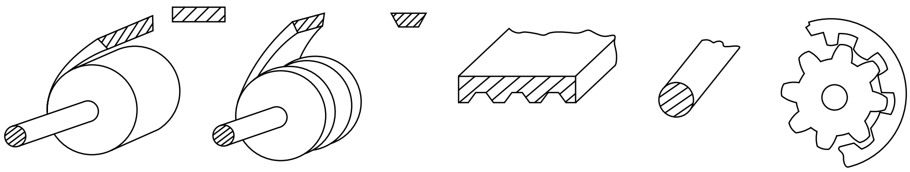
The belt and rope drives are useful to transmit power between two parallel shafts making use of the friction between the belt or rope and the pulley. The slip exists in these drives while the chain drive is a positive drive.

11.2 TYPES

Belt drives are of the following types:

(a) Flat belt—it has a rectangular cross section.
(b) V belt—it has a trapezoidal cross section.
(c) Multiple V.
(d) Ribbed.
(e) Round.
(f) Toothed.

All the above drives may be either open belt or cross belt whereas, in a quarter turn the direction of the belt is changed by 90°.



(a) (b) (c) (d) (e) (f)

Fig. 11.1

11.3 LENGTH OF THE BELT

(a) *Open belt drive:* Let the following be the notation:

D = diameter of larger pulley
d = diameter of smaller pulley
C = centre distance
θ = angle of contact of smaller pulley.

$$\phi = \sin^{-1}\frac{D-d}{2C},\ L = \text{Length of the belt}$$

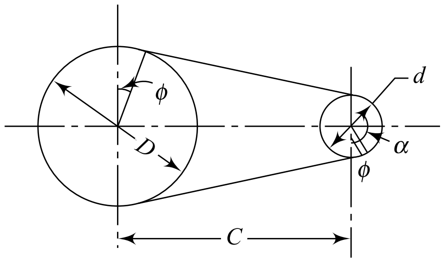


Fig. 11.2

From Fig. 11.2

$$\therefore \quad L = \frac{\pi}{2}(D+d) + (D-d)\sin\phi + 2c\cos\phi$$

$$= \frac{\pi}{2}(D+d) + \frac{(D-d)^2}{2C}$$

$$+ 2C\left[1 - \frac{1}{2}\left(\frac{D-d}{2C}\right)^2 + \ldots\right]$$

$$= \frac{\pi}{2}(D+d) + 2C + \frac{(D-d)^2}{4C}$$

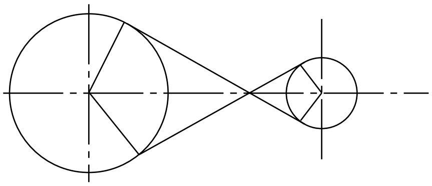


Fig. 11.3

(b) *Cross belt* : Using similar steps as above we get from Fig. 11.3

$$L = \frac{\pi}{2}(D+d) + 2C + \frac{(D+d)^2}{4C}$$

11.4 RATIO OF TENSIONS AND POWER TRANSMISSION

The forces on a small element of belt of length $R\,\delta\theta$ are as shown in Fig. 11.4. They are:

(a) Tension T on the slack side
(b) Tension $T + \delta T$ on the tight side
(c) Reaction N
(d) Frictional force μN.

Considering equilibrium of this element of the belt we write

$$\Sigma H = 0 \text{ and } \Sigma V = 0$$

$$\therefore \quad \mu N + T\cos\frac{\delta\theta}{2} - (T+\delta T)\cos\frac{\delta\theta}{2} = 0 \quad \text{(i)}$$

$$N - 2T\sin\frac{\delta\theta}{2} - \delta T\sin\frac{\delta\theta}{2} = 0 \quad \text{(ii)}$$

Putting $\cos\frac{\delta\theta}{2} = 1$, $\sin\frac{\delta\theta}{2} = \frac{\delta\theta}{2}$, $\delta\theta$ being small and the product $\delta T \times \frac{\delta\theta}{2} = 0$

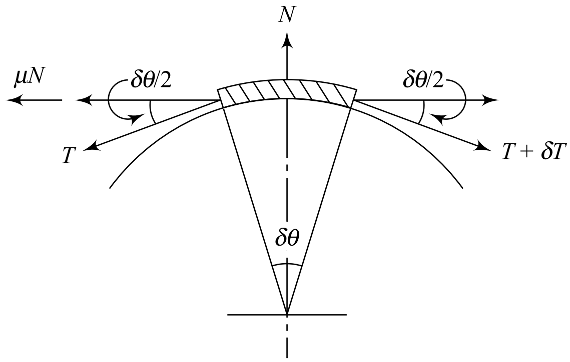


Fig. 11.4

Equations (i) and (ii) become

$$\mu N - \delta T = 0 \quad \text{(iii)}$$

$$N - T\,\delta\theta = 0 \quad \text{(iv)}$$

Substituting the value of N from (iv) in (iii)

$$\delta T = \mu T \delta\theta \quad \text{(v)}$$

or

$$\frac{\delta T}{T} = \mu \delta\theta \quad \text{(vi)}$$

or

$$\int_{T_2}^{T_1} \frac{dT}{T} = \mu \int_0^{\theta} d\theta$$

or

$$\frac{T_1}{T_2} = e^{\mu\theta} \quad (11.1)$$

$$\text{Power transmitted } P = \text{Tangential force} \times v$$

$$= (T_1 - T_2)v \quad (11.2)$$

where v = peripheral velocity in m/s = $\dfrac{\pi D N_1}{60} = \dfrac{\pi d N_2}{60}$ where N_1 and N_2 are the speeds of the bigger and smaller pulleys respectively.

For the V belt the same expressions are used by modifying μ to μ'.

From Fig. 11.5 N is the resultant of N_1 and N_2. Since both the forces are equal hence,

$$N = N_1 \sin \alpha/2 + N_2 \sin \alpha/2$$

$$= 2\,N_1 \sin \alpha/2$$

$$\therefore \quad N_1 = \frac{N}{2 \sin \alpha/2} = \frac{N}{2} \operatorname{cosec} \frac{\alpha}{2}$$

$\therefore$ Equation (iv) is written as $2 \,.\, \dfrac{N}{2} \operatorname{cosec} \dfrac{\alpha}{2} - T\delta\theta = 0$

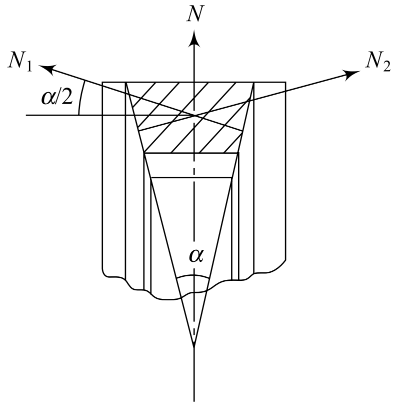


Fig. 11.5

$$\therefore \qquad \frac{T_1}{T_2} = e^{(\mu \operatorname{cosec} \alpha/2)\,\theta} = e^{\mu'\theta}$$

where $\mu' = \mu \operatorname{cosec} \alpha/2.$

11.5 CENTRIFUGAL TENSION

Let us consider an element of length $r\,\delta\varphi$ on which C.F is acting. The horizontal components which is balanced by a centrifugal tension T_c as shown in Fig. 11.6, balance each other while the vertical components taken together balance the C.F.

Therefore, if m is the mass of the belt/unit length then,

$$(m \cdot r\delta\phi) \cdot rw^2 = 2T_c \sin \delta\varphi/2 = 2\,T_c\,\delta\varphi/2 = T_c\,\delta\varphi$$

as $\sin \delta\varphi/2 \cong \delta\varphi/2$

$$\therefore \qquad T_c = mr^2w^2 = mv^2 \qquad (11.3)$$

T_c increases the tension on the tight and slack sides equally

$\therefore$ Tension on the tight side $T_t = T_1 + T_c$.

Tension on the slack side $T_s = T_2 + T_c$

$\therefore$ Equation (11.1) becomes

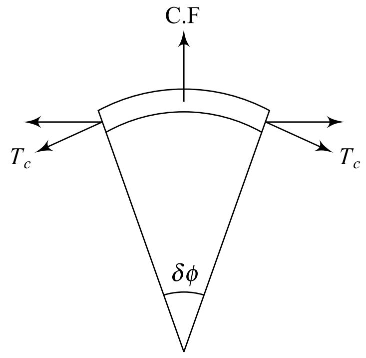


Fig. 11.6

$$\frac{T_t - T_c}{T_s - T_c} = e^{\mu\theta} \qquad (11.4)$$

Now, power to be transmitted $= T_1\,(1 - T_2/T_1)\,v$

$$= (T_t - T_c)\left(1 - \frac{1}{e^{\mu\theta}}\right) v$$

$$= (T_t - m\,v^2)\,kv, \text{ replacing} \left(1 - \frac{1}{e^{\mu\theta}}\right) \text{ by } k$$

For maximum power transmission

$$\frac{dP}{dv} = 0, \quad \therefore \quad \frac{d}{dv}[(T_t - mv^2)kv] = T_t - 3\,mv^2 = 0$$

or $$T_t = 3\,mv^2 = 3T_c. \qquad (11.5)$$

11.6 FLAT AND V BELT COMPARISON

Flat belt is useful for a large centre distance. It is noiseless in operation and the maintenance is easy. It is made from leather, cotton, woven fabric, wood, silk or synthetic fibres. Its length may be selected according to the requirement and the ends are joined by cemented leather or a rubber joint. Raw hide strips, twine, bolts and wire hooks can also be used for the purpose.

V belts are made from the fabric and cord moulded in rubber. They are endless belts that come in standard lengths. The B.I.S. provides the dimension of V belts with the diameter of pulley corresponding to the power to be transmitted.

A V belt drive is compact, has negligible slip, quiet in operation and can cushion shocks. It may be operated in either direction and high velocity ratio may be used. As $\mu' > \mu$ the ratio of tension and hence power transmitted increases. More power can be transmitted by using more number of belts. Although the failure of one belt does not stop the drive, however, it is advisable to replace all the belts at one time instead of replacing only the damaged belt. The disadvantages of this drive are less durability of the belts, complicated construction of pulleys and shorter centre distance.

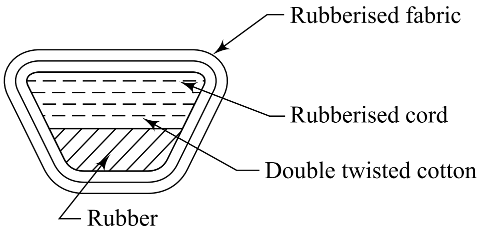


Fig. 11.7

11.7 STRENGTH OF BELT

Design Equation

The maximum value of force acting on the belt consists of tension T_1 and centrifugal tension T_c. It gives rise to tensile stress given by

$$\sigma_1 = \frac{T_1 + T_c}{\text{Area of c/s}} = \frac{T_t}{b \times t} \tag{11.6}$$

Some designers consider that the additional stress due to the bending of the belt while passing on the pulley is obtained from the basic equation

$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$ where σ is the bending stress, R is the radius of the smaller pulley

and $y = \frac{1}{2}$ thickness of the belt.

Thus, $$\sigma_2 = \frac{E}{R} y = \frac{Et/2}{d/2} = \frac{Et}{d} \tag{11.7}$$

$\therefore$ Total stress on the belt = $\sigma_1 + \sigma_2$

11.8 CREEP

As $T_1 > T_2$, T_1 causes elongation of the belt more than that due to T_2, hence length of belt leaving the follower is more than that reaching to it. Thus the length $\delta l_1 - \delta l_2$ (difference between two elongations) passes off the follower without making contact to it.

This portion of the length causes the loss of motion and power and is termed as the slip.

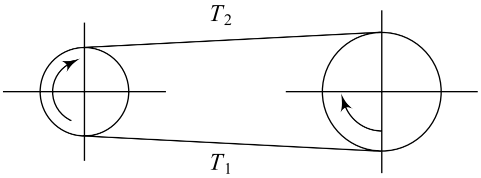


Fig. 11.8

$$\text{Creep} = \frac{T_1 - T_2}{b \times t \times E} = \left(\frac{\sigma}{E}\right) = \varepsilon \tag{11.8}$$

The effect of creep on the velocity ratio

$$\frac{N_1}{N_2} = \frac{D}{(1-\varepsilon)d} \tag{11.9}$$

Initial tension $$T_o = \frac{T_1 + T_2}{2} \tag{11.10}$$

11.9 DESIGN PROCEDURE

(a) This method uses Eq. 11.6 and/or Eq. 11.7 depending on the availability of data.
(b) The flat belt may be designed by first using the tables and then checked by using Eqs 11.6 and 11.7.
(c) V belts may be designed completely by using the tables obtained from the manufacturer's catalogue.
(d) The rope drive is designed by using Eqs. 11.1 to 11.5 as applied to V belts.

11.10 DESIGN OF CHAIN DRIVE

The roller chains used for power transmission have various components of heat treated alloy steel with pins and bushing carburised and the surface hardened for wear resistance.

The chain drive can be used for a comparatively larger distance (5–8 metres) for parallel shafts and has high efficiency (98%). It has a high production cost, the operation is noisy and the design is complicated.

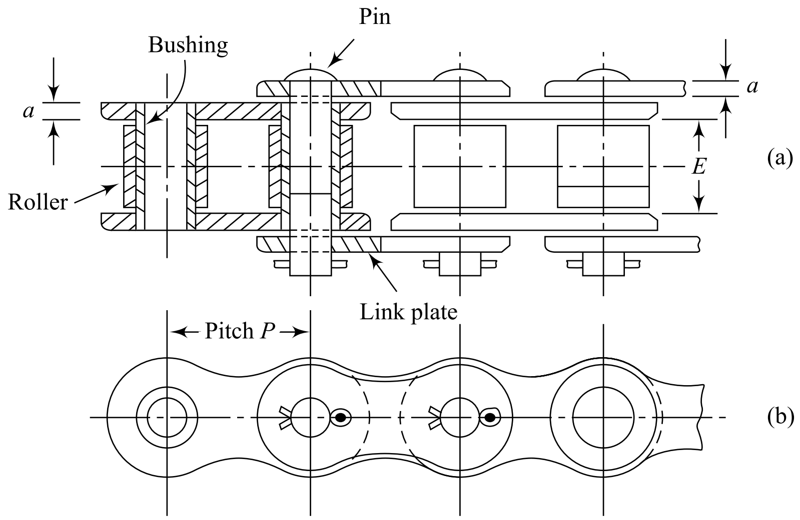


Fig. 11.9

The following relations are used to design the drive

$$i = \frac{Z_2}{Z_1} = \frac{n_1}{n_2}$$

where i = transmission ratio

Z_2, Z_1 = number of teeth on bigger and smaller sprocket respectively

n_2, n_1 = speeds of rotation in r.p.m.

centre distance $$a = \frac{P}{4}\left[m - \frac{Z_1 + Z_2}{2} + \sqrt{\left(m - \frac{Z_1 + Z_2}{2}\right)^2 - \left(\frac{Z_2 - Z_1}{\pi}\right)^2}\right]$$

$$m = \frac{2a}{P} + \frac{Z_1 + Z_2}{2} + \left(\frac{Z_2 - Z_1}{2\pi}\right)^2 \cdot \frac{P}{a}$$

where P is the pitch and

m = number of chain links, length of the chain, $L = m\,P$ mm

Velocity of the chain = $\dfrac{Z_1 P n_1}{1000 \times 60}$ m/s

Pitch diameter of the sprocket = $\dfrac{P}{\sin \dfrac{\pi}{Z}}$

$$Z_{min} = \frac{4d}{P} + 5 \text{ for a pitch up to 25 mm}$$

$$= \frac{4d}{P} + 4 \text{ for a pitch ranging from 30–55 mm.}$$

The total force on the driving side of the chain

$$P_0 = P + P_c + P_s \text{ N}$$

where P = net driving force

P_b = breaking force (from table)

P_c = centrifugal tension

P_s = tension due to sag

$P_c = qv^2$ where v = speed of the chain in m/s

$P_s = \dfrac{qA^2}{8f}$ q = weight per metre length

f = chain sag A = centre distance.

WORKED EXAMPLES

11.1 A leather belt 125 mm wide and 6 mm thick transmits power from a pulley of diameter 750 mm at 500 r.p.m. Angle of lap for smaller pulley is 150°, coefficient of friction = 0.3, mass of the belt = 1 mg/m^3 and the maximum permissible stress = 2.75 MPa. Find the maximum power that can be transmitted.

Solution:

$$v = \frac{\pi \times 750 \times 500}{60 \times 1000} = 19.634 \text{ rn/s}$$

$$\therefore \quad T_c = mv^2 = \rho\, btv^2 = \frac{1 \times 1000 \times 125 \times 6 \times (19.634)^2}{1000 \times 1000} = 289.120 \text{ N}$$

$$T_t = \sigma \times b \times t = 2.75 \times 125 \times 6 = 2062.5 \text{ N}$$

$$\therefore \quad T_1 = T_t - T_c = 1773.38 \text{ N}$$

$$e^{\mu\theta} = e^{\frac{150}{180}\pi \times 0.3} = 2.1922$$

$$\therefore \quad T_2 = \frac{1773.38}{2.1922} = 808.93 \text{ N}$$

$$\therefore \quad P = \frac{(T_1 - T_2)\, v}{1000} = \frac{(1773.38 - 808.93)}{1000} \times 19.634 = 18.936 \text{ kW}$$

11.2 Determine the width of a 9.5 mm thick belt required to transmit 11.25 kW from a motor rotating at 750 r.p.m. The diameter and speed of the driving pulley are 300 mm and 750 r.p.m respectively. Centre distance = 3 m and density of leather = 1000 kg/m^3. Maximum permissible stress = 2.46 MPa, μ = 0.3. Neglect the belt thickness for calculating the belt velocity. Assume an open drive.

Solution:

Let b be the width in mm

$$\text{Mass/metre length} = \frac{9.5}{1000} \times \frac{b}{1000} \times 1 \times 1000 = 9.5\, b \times 10^{-3} \text{ kg}$$

$$v = \frac{\pi \times 300 \times 750}{1000 \times 60} = 11.8 \text{ m/s}$$

$$T_c = 9.5 \times b \times 10^{-3} \times (11.8)^2 = 1.322b \text{ N}$$

$$T_1 - T_2 = \frac{P}{v} = \frac{11.25 \times 1000}{11.8} = 953.3898 \text{ N}$$

$$\varphi = \sin^{-1}\frac{D-d}{2C} = \sin^{-1}\frac{900-300}{2 \times 3000} = 5.739°$$

$$\theta = 180 - 2\phi = 168.522°$$

$$\therefore \quad \frac{T_1}{T_2} = e^{\left(\frac{0.3 \times 168.522\,\pi}{180}\right)} = 2.415$$

$$\therefore \quad T_1 = 1627.15 \text{ N},\ T_2 = 673.77 \text{ N}$$

Maximum tension $\quad T_t = 1627.15 + 1.322\, b = 2.46 \times 9.5\, b$

$$\therefore \quad b = 73.8 \text{ mm.}$$

11.3 Design a flat belt horizontal drive for transmitting 5.8 kW at 1400 r.p.m of motor. Speed of the driven pulley is 450 r.p.m and the maximum permissible peripheral speed is 16 m/s. Assume 3% creep, load factor = 1.2, density of the belt material = 980 kg/m^3, modulus of elasticity for the belt material = 100 MPa, S_{ut} = 25 MPa, centre distance = 2.8 m and endurance limit for the belt material = 4 MPa.

Solution:

From the limiting belt speed

$$16 = \frac{\pi d \times 1400}{60}, \quad \therefore\ d = 0.2182 \text{ m} = 218.2 \text{ mm}$$

$\therefore$ A standard diameter of 200 mm is adopted. This reduces the velocity to 14.67 m/s which is beneficial. With 3% creep

$$D = (1 - 0.03) \times 200 \times \frac{1400}{450} = 603.55 \text{ mm}$$

Let us adapt a standard diameter of 630 mm. $C_{\min} = 1.5(D + d) = 1.5\,(630 + 200) = 1245$ mm. $C_{\text{actual}} = 2800$ mm $> C_{\min}$. Hence, it is a satisfactory value.

Belt length, $$L = 2\,C + \frac{\pi}{2}(D + d) + \frac{(D - d)^2}{4C}$$

$$= 5600 + 1303.76 + 16.50 = 6920.2689 \text{ mm}$$

As $v = 14.67$ m/s

$$T_1 - T_2 = \frac{P}{v} = \frac{5.8 \times 1000}{14.67} = 395.36 \text{ N}$$

$$\varphi = \sin^{-1}\left(\frac{630 - 200}{2 \times 2800}\right) = 4.4°$$

$\therefore$ $$\theta = 180° - 2\varphi = 171.2°$$

$\therefore$ $$\frac{T_1}{T_2} = e^{\mu\theta} = e^{\left(0.25 \times \frac{171.2}{180}\pi\right)} = 2.11$$

$\therefore$ $2.11\,T_1 - T_2 = 395.36$ or $T_1 = 751.54$ N and $T_2 = 356.18$ N.

$$\sigma_{cf} = mv^2 = \frac{980 \times (14.67)^2}{10^6} = 0.21 \text{ MPa}$$

T_1 causes $$\sigma_1 = \frac{751.54}{bt}$$

$$\sigma_{\text{bending}} = \frac{Et}{d} \quad \text{Using } \frac{d}{t} = 25,\ t = 8 \text{ mm}$$

$$\sigma_{\text{bending}} = \frac{100}{25} = 4 \text{ MPa}$$

$$\text{Permissible stress} = \frac{S_{ut}}{\text{service factor}} \times \frac{C_2 \times C_3 \times C_4}{\text{F·S}}$$

C_1 = service factor = 1.2

C_2 = length correction factor = $1.04 - 0.0004\ (14.67)^2 = 0.954$

C_3 = correction factor for angle of contact = $1 - 0.17\ (\pi - \theta)$

$$= 1 - 0.17\left(\pi - \frac{171.2}{180\,\pi}\right) = 0.9742$$

C_4 = factor for operating condition = 1 for horizontal drive

Let F·S = 4

$$\therefore \quad \text{Permissible stress} = \frac{25 \times 0.954 \times 0.9742 \times 1}{1.2 \times 4} = 4.84 \text{ MPa}$$

$\therefore$ $4.84 = \dfrac{751.54}{b \times t} + 0.21 + 4$. Substituting t = 8 mm we have,

b = 146.78 mm. Let us adapt b = 150 mm.

Life of belt in hours

$$H = \frac{10^7}{3600 \times nv/L}\left(\frac{S_e}{\sigma_{\max}}\right)^m$$

where, n = number of pulleys, v = peripheral velocity m/s

L = length of belt m, m = 6 for flat belt

= 8 for V belt

$$n = 2,\ \frac{v}{L} = \frac{14.67}{6.92} = 2.12\ S_e = 4 \text{ MPa and } \sigma_{\max} = 4.836 \text{ MPa}$$

$$\therefore \quad H = \frac{10^7}{3600 \times 2 \times 2.12} \times \left(\frac{4}{4.836}\right)^6 = 209.78 \text{ hours.}$$

Pulley width a = 160 mm (belt width + 10 mm) (from BIS–1691)

Number of arms for smaller pulley = 4

Number of arms for larger pulley = 6

Elliptical cross section of pulley arm of C.I has width b as follows

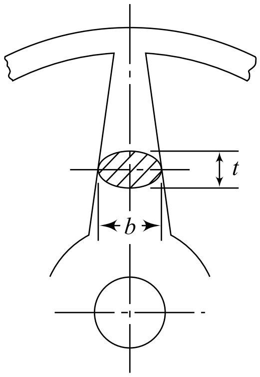


Fig. E-11.3

$$\text{For the driver pulley } b = 2.94\ \sqrt[3]{\frac{a \times d}{4n}}$$

$$= 2.94\sqrt[3]{\frac{160 \times 200}{4 \times 4}} = 37.04 \text{ mm}$$

$$\text{For the driven pulley } b = 2.94\ \sqrt[3]{\frac{630 \times 160}{16}} = 54.3 \text{ mm}$$

Torque on pulley = $(T_1 - T_2)\ d_2$

$= (751.54 - 356.18) \times 100 = 39356$ N.mm

$= \text{Bending moment on four arms with section modulus } Z = \dfrac{\pi}{32} tb^2$

Using $\sigma_t = 10$ MPa

$$\text{b.m} = 4\ \sigma_t \times z = 4 \times 10 \times \frac{\pi}{32} t\ (37.04)^2$$

$\therefore$ $t = 7.32$ mm. Let us adapt $t = 10$ mm.

For the bigger pulley, torque $= (751.54 - 356.18) \times 300 = 6\ (\sigma_t \cdot Z)$

$$= 10 \times 6 \times \frac{\pi}{32} t\ (54.3)^2$$

$\therefore$ $t = 6.82$ mm. We take the thickness $t = 10$ mm.

$$\text{thickness of rim} = \frac{d}{300} + 2 = 3 \text{ mm for the smaller pulley}$$

$$= \frac{600}{300} + 2 = 4 \text{ mm for the bigger pulley}$$

11.4 It is required to design a leather cross belt drive to connect 7.5 kW, 1440 r.p.m electric motor to a compressor running at 480 r.p.m. The distance between the centres of the pulleys is twice the diameter of the bigger pulley. The belt should operate at 20 m/s approximately and its thickness is 5 mm. Density of leather = 950 kg/m^3 and $S_{ut} = 25$ MPa.

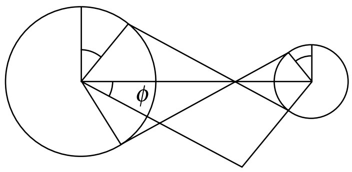


Fig. E-11.4

Solution:

$$v = \frac{\pi d n}{60}, \quad \therefore d = \frac{20 \times 60}{\pi \times 1440} = 0.265 \text{ m}$$

The nearest standard diameter is 250 mm.

$$\therefore \quad D = \frac{1440}{480} \times 250 = 750 \text{ mm}, \quad \therefore v = 18.86 \text{ m/s}$$

Centre distance = 1.5 m

$$\therefore \quad L = 2C + \frac{\pi}{2}(0.75 + 0.25) + \frac{(0.75 + 0.25)^2}{4 \times 3} = 4.654 \text{ m}$$

$$\varphi = \sin^{-1}\frac{D + d}{2C} = \sin^{-1}\frac{1}{3} = 19.47°$$

$$\therefore \quad \theta = 180 + 38.94 = 218.94°$$

$$\frac{T_1}{T_2} = e^{0.25 \times \frac{218.94}{180}\pi} = 2.6$$

$$T_1 - T_2 = \frac{P}{v} = \frac{7.5 \times 1000}{18.86} = 397.65 \text{ N}$$

$\therefore \qquad T_1 = 646.2 \text{ N}, T_2 = 248.54 \text{ N}$

$$\sigma_1 = \frac{646.2}{bt} = \frac{646.2}{5b}$$

$$\sigma_2 = \frac{950 \times (18.86)^2}{10^6} = 0.338 \text{ MPa}$$

$$\sigma_3 = \frac{Et}{d} = \frac{100 \times 5}{250} = 2 \text{ MPa}$$

C_1 = Coefficient of angle of contact. This is not required as $\theta > \pi$
C_2 = Coefficient of belt velocity = $1.04 - 0.0004\, v^2 = 0.897$
C_3 = Load factor = 1
C_4 = 1 for the horizontal drive

Using F·S = 4, $\sigma_{\text{permissible}} = \dfrac{25 \times 0.987}{4} = 5.6 \text{ MPa}$

$$\therefore \qquad 5.6 = \frac{646.2}{5b} + 0.338 + 2$$

$b = 39.61$ mm, $\therefore$ $b = 40$ mm and $t = 5$ mm.

11.5 Two shafts 1 m apart are connected by a V belt drive to transmit 90 kW at 1200 r.p.m of a driver pulley of 300 mm effective diameter. The driver pulley rotates at 400 r.p.m. The angle of groove is 40° and coefficient of friction between the belt and the pulley rim is 0.25. Area of the belt section is 400 mm^2 and the permissible stress is 2.1 MPa. Density of belt material = 1100 kg/m^3. Calculate the number of belts required and the length of the belt.

Solution:

$$\text{Maximum tension} = \sigma_i A = 2.1 \times 400 = 840 \text{ N}$$

$$\text{Mass/unit length} = 1100 \times \frac{400}{1000 \times 1000} = 0.44 \text{ kg/m}$$

$$T_c = 0.44 \times v^2$$

$$v = \frac{\pi \times 300 \times 1200}{60 \times 1000} = 18.84 \text{ m/s}$$

$$T_c = 0.44(18.84)^2 = 156.176 \text{ N}$$

$$T_1 = 840 - 156.176 = 683.824 \text{ N}$$

$$\varphi = \sin^{-1} \frac{900 - 300}{2000} = 17.45°, \ \theta = 180 - 34.9 = 145.1°$$

$$\therefore \qquad \frac{T_1}{T_2} = e^{0.25 \times \frac{145.1}{180} \pi \operatorname{cosec} 20°} = 6.35$$

$$T_2 = 107.68 \text{ N}$$

$$P = (T_1 - T_2)\, v = (683.824 - 107.68) \times \frac{18.84}{1000} = 10.85 \text{ kW}$$

$\therefore$ Number of belts required = $\dfrac{\text{Total Power}}{10.85} = \dfrac{90}{10.85} = 8.294 \cong 9$

Length of the belt

$$L = 2 \times 1000 + \frac{\pi}{2}\,(900 + 300) + \frac{(900 - 300)^2}{4000} = 3975 \text{ mm}$$

From Table 11 the standard length 3990 mm is selected

Now the exact centre distance

$$C = A + \sqrt{A^2 - B}$$

Where $$A = \frac{L}{4} - \frac{\pi(D+d)}{8} = \frac{4090}{4} - \frac{\pi}{8}(900 + 300) = 526.26$$

$$B = \frac{(D-d)^2}{8} = \frac{600^2}{8} = 45000$$

$\therefore$ $$C = 526.26 + \sqrt{526.26^2 - 45000} = 1007.87 \text{ mm}$$

11.6 A V belt drive is used to transmit 80 kW at a motor speed of 100 rpm, the driven pulley speed being 600 r.p.m. Approximate centre distance = 1.110 m, $\mu = 0.3$, angle of groove = 40° and V_{max} = 25 m/s. Neglecting the c.f tension calculate the size and number of belts. Also check the induced stress and find the life of the belt assuming S_e = 9 MPa.

Solution:

Referring to Table 17

To transmit 80 kW power a D belt is recommended.

Now, $$25 = \frac{\pi d \times 1000}{60 \times 1000},\ \therefore\ d = 477 \text{ mm}$$

Let us adapt d = 450 mm so that $V < 25$ m/s

$\therefore$ $$D = 450 \times \frac{1000}{600} = 750 \text{ mm},\ \therefore\ v = \frac{25 \times 450}{477} = 23.58 \text{ m/s}$$

Minimum centre distance = 0.5 $(D + d) + t$

Assuming $$\frac{d}{t} = 30$$

$$= 0.5\,(750 + 450) + 15 = 615 \text{ mm}$$

Actual value of $C >$ minimum value. Hence satisfactory

Length of the belt $= 2 \times 1110 + \dfrac{\pi}{2}\,(750 + 450) + \dfrac{(750 - 450)^2}{4 \times 1110} = 4125.22$ mm

So referring to Table 11 we can use a standard pitch length of 4092 mm for which the exact (centre distance will be)

$$C = A + \sqrt{A^2 - B}$$

$$A = \frac{L}{4} - \frac{\pi(D+d)}{8} = 551.76 \text{ mm} \qquad B = \frac{(D-d)^2}{8} = 11250$$

$\therefore$ $$C = 551.76 + \sqrt{551.76^2 - 11250} = 1093.23 \text{ mm}$$

From Table 17 power rating for the D section V belt with arc of contact of 180° on the smaller pulley at a speed of 23 m/s = 21.70 kW

$$\varphi = \sin^{-1}\frac{750-450}{2\times 1093-23} = 7.88° \qquad \theta = 180 - 2\times 7.88 = 164.24°$$

$\therefore$ From Table 13 power correction factor = 0.96 and from Table 11 length of the belt correction factor = 0.92.

Compensated pulley pitch diameter factor from Table 12 is 1.12 and service factor = 0.78

$\therefore$ Power rating is 21.99 kW for, $d_e = 450 \times 1.12 = 504$ mm and

$v = 26.4$ m/s from Table 17

$\therefore$ Actual power transmitted by the belt = 21.99 × 0.96 × 0.92 × 0.78 = 15.15 kW

$$\therefore \qquad \text{Number of belts required} = \frac{80}{15.15} = 5.28$$

Hence we use 6 belts.

Dimensions of the D type belt and belt pulley are as given below

$h = 19.9$ mm, $e = 37$ mm, $f = 24$ mm, $r_1 = 2$ mm,

$r_2 = 3$ mm, Pitch width = 27 mm and $b = 8.1$ mm

Now, let us check the induced stresses

$$\frac{T_1}{T_2} = e^{0.3\times\frac{164.24}{180}\pi\,\mathrm{cosec}\,20°} = 12.35$$

$$\text{Power per belt} = \frac{80}{6} = 13.33\text{ kW} = \frac{(T_1 - T_2)\times v}{1000}$$

$$T_1 - T_2 = 565.3\text{ N}, \therefore T_1 = 615.1\text{ N}, T_2 = 49.8\text{ N}$$

$$T_c = mv^2.$$

From Table 21 $\quad m = 0.596$ kg/m

$\therefore \quad T_c = 0.596 \times (23.58)^2 = 331.38$ N

Area of c/s = 950 mm^2

$$\therefore \qquad \sigma_1 + \sigma_2 = \frac{615.1 + 331.38}{950} = 0.996\text{ MPa}$$

$$\sigma_3 = \frac{Et}{d} = \frac{100\times 19}{450} = 4.22\text{ MPa}$$

$\therefore$ Maximum stress = 5.216 MPa which is within the limit.

$$\text{Life in hours, } H = \frac{10^7}{3600\times\frac{23.58}{4.092}\times 2}\left(\frac{9}{5.216}\right)^8 = 18936.46\text{ hours.}$$

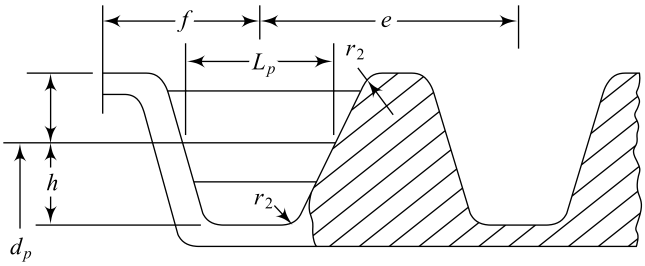


Fig. E-11.6a

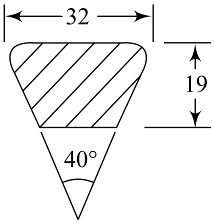


Fig. E-11.6b

11.7 A rope drive is used for transmitting 250 kW at 300 r.p.m of a pulley of diameter 1.25 m. The angle of lap is 180° and groove angle is 45°. Permissible tension in each rope = 2000 N, mass of the rope = 1.27 kg/metre and coefficient of friction between the rope and the pulley = 0.3. Calculate the number of ropes.

Solution:

$$v = \frac{\pi \times 1.25 \times 300}{60} = 19.63 \text{ m/s}$$

$$T_c = 1.27\, v^2 = 1.27\,(19.63)^2 = 489.37 \text{ N}$$

$$\therefore \quad T_1 = T_t - T_c = 2000 - 489.37 = 1510.63 \text{ N}$$

$$\text{Since} \quad \frac{T_1}{T_2} = e^{\mu\theta \operatorname{cosec} \alpha/2} = e^{0.3 \times \pi \times \operatorname{cosec} 22.5°}$$

$$\therefore \quad \text{Power per rope} = (1510.63 - 128.61) \times \frac{19.63}{1000} = 27.127 \text{ kW}$$

$$\therefore \quad \text{No. of ropes} = \frac{250}{27.227} = 9.21. \text{ Hence we use 10 ropes.}$$

11.8 Design a chain drive to transmit 12 kW from an electric motor running at 900 r.p.m and having a transmission ratio of 2.80. The minimum centre distance is 500 mm.

Solution:

From IS : 2403 number of teeth on the sprocket may be selected at 25. Hence number of teeth on the sprocket gear = 25 × 2.8 = 70

$$v = \frac{25 \times 15 \times 900}{60 \times 1000} = 5.625 \text{ m/s, using } p = 15 \text{ mm}$$

Using a simple ISO 10 B chain the breaking force P_B = 22700 N

$$\text{Driving force} = \frac{12 \times 1000}{5.625} = 2133.33 \text{ N}$$

$$\text{Centrifugal force } P_c = 0.95 \times (5.625)^2 = 30.05 \text{ N}$$

Expected factors of safety are

$$\text{Static factor of safety} = \frac{P_B}{P_o} \geq 7$$

$$\text{Dynamic factor of safety} = \frac{P_B}{P_o Y} \geq 5$$

In this case static factor of safety $= \dfrac{22700}{(2133.33 + 30.05)} = 10.49 > 7$, hence, a safe value.

Again from Table 22, $Y = 1.4$

$\therefore$ Dynamic factor of safety = 7.49 > 5

Thus, both the static and dynamic factors of safety are sufficient. From Table 25 the bearing area = 67 mm^2.

$$\therefore \text{ Actual bearing pressure} = \frac{2163.38}{67} = 32.28 \text{ N/mm}^2$$

From Table 26, by interpolation P_o = 19.96 N/mm^2

for $v = 5.625$ m/s, $Z_1 = 25$ and

for $y = 1.4$, $a = 40\,p$, $C_1 = 0.89$

$$P_b = 0.89 \times 19.96 \times 1 = 17.76 \text{ N/mm}^2$$

This is smaller than the actual bearing pressure. Hence it is advisable to use the duplex 10 B chain for which $A = 2 \times 67$, $q = 2 \times 0.95$

$$P_c = 60.1 \text{ N}$$

$\therefore$ Actual bearing pressure $= \dfrac{2133.33 - 33 + 60.1}{2 \times 67}$

$= 16.36 \text{ N/mm}^2$ which is less than 17.76 Nmm^2.

Hence the chain is safe.

$\therefore$ We use the duplex 10 B chain.

Other details may be worked out by referring to manufacturer's catalogue.

OBJECTIVE QUESTIONS

11.1 Ratio of tensions $\dfrac{T_1}{T_2}$ in a V belt drive is given by

(a) $e^{\mu\theta}$ (b) $e^{\mu\theta \operatorname{cosec} \alpha}$ (c) $e^{\mu\theta \operatorname{cose} \alpha/2}$ (d) $e^{\mu\theta \cos \alpha/2}$

where θ = angle of lap, α = groove angle.

11.2 In case of a belt drive the maximum power is transmitted if the value of centrifugal tension is

(a) $\frac{1}{3}$ tension T_1 on tight side (b) $\frac{1}{3}$ total tension T_t on tight side

(c) $\frac{1}{3}$ tension T_s on slack side (d) $\frac{1}{3}$ the sum of $(T_1 + T_2)$

11.3 Magnitude of initial tension in the belt should be

(a) zero (b) $T_1 + \dfrac{T_2}{2}$ (c) $\dfrac{T_1 + T_2 + 2T_c}{2}$ (d) $T_1 - T_2$

11.4 The magnitude of velocity of the belt for maximum power transmission should be

(a) $\sqrt{\dfrac{T_t}{3m}}$ (b) $\sqrt{2gh}$ (c) $(T_1 - T_2) \cdot r$ (d) $\dfrac{\text{Power}}{(T_1 - T_2)}$

11.5 Maximum induced stress in the belt is

(a) $\dfrac{T_1}{bt}$ (b) $\dfrac{T_t}{bt}$ (c) $\dfrac{T_t}{bt} + \dfrac{Et}{d}$ (d) $\dfrac{T_1}{bt} + \dfrac{Et}{D}$

11.6 Slip is the result of
(a) insufficient friction between the belt and the pulley
(b) unequal elongation of belt due to T_1 and T_2
(c) elongation of belt due to T_t
(d) none of the above

11.7 The angle of contact in a belt drive should be
(a) more than 150° in the smaller pulley (b) more than 150° in the larger pulley
(c) 180° (d) between 150° and 200° on either pulley

11.8 Length of the belt in a cross belt drive is

(a) $2C + \frac{\pi}{2}(D + d) + \frac{(D + d)^2}{4C}$ (b) $2C + \frac{\pi}{2}(D - d) + \frac{(D + d)^2}{4C}$

(c) $2C + \frac{\pi}{2}(D + d) + \frac{(D - d)^2}{4C}$ (d) $2C + \frac{\pi}{2}(D - d) + \frac{(D - d)^2}{4C}$

11.9 By increasing the centre distance the ratio of tensions in an open belt drive is
(a) increased (b) decreased
(c) not affected (d) none of the above

11.10 The length of a belt cannot be altered as per the designer's will in case of the
(a) flat belt (b) V belt (c) open belt (d) cross belt

11.11 Power transmission in a V belt drive can be increased by increasing the
(a) groove angle (b) number of belts (c) velocity (d) initial tension

11.12 In a belt drive, due to the centrifugal tension the capacity to transmit power
(a) increases (b) decreases (c) remains unaffected

11.13 The pitch line velocity of the belt drive should not exceed
(a) 50 rn/s (b) 20 rn/s (c) 10 rn/s (d) 40 m/s

11.14 Creep is given by

(a) $\varepsilon = \frac{T_1}{E}$ (b) $\varepsilon = \frac{T_1 - T_2}{E}$ (c) $\varepsilon = \frac{T_1 - T_2}{btE}$ (d) $\varepsilon = \frac{T_1 + T_2}{btE}$

11.15 The power transmitted in a belt drive depends on the
(a) revolutions per minute (b) T_t and T_s
(c) arc of contact (d) all of the above

11.16 The speed variation in a chain drive is due to
(a) improper machining of the sprocket and the chain
(b) length of the chain being more than required
(c) the polygonal action
(d) a very small length of the chain

11.17 V belt is preferably used for
(a) large power transmission (b) small centre distance
(c) small power transmission (d) large centre distance

11.18 In an open belt drive the lower side is used on the
(a) slack side (b) tight side (c) none of the above

11.19 The crown height of a flat belt pulley varies between
(a) 1 to 5 mm (b) 0.3 to 1 mm (c) 2 to 5 mm (d) none of the above

REVIEW QUESTIONS

11.1 What are the different types of belts used in practice? Discuss their merits and demerits.
11.2 Prove that the ratio of tensions of two sides of the belt is $T_1/T_2 = e^{\mu\theta}$.
11.3 What is centrifugal tension? Derive its relationship with the mass per unit length of the belt and the velocity of the belt.
11.4 Discuss the various stresses induced in belts.

11.5 What is creep? How does it affect the design of a belt drive?
11.6 What is the significance of (a) angle of contact correction factor, (b) length correction factor, (c) factor for compensating pulley diameter, (d) service factor?
11.7 State and prove the condition for maximum power transmission.
11.8 Illustrate the different methods of joining the ends of the belts. What are their relative strengths?
11.9 Draw and explain the construction of the cross-section of a V belt.
11.10 Derive the relationship for the length of a belt in terms of the centre distance and diameters of the pulleys.
11.11 Compare V and flat belt drives.
11.12 Compare belt and chain drives.
11.13 Explain the terms (a) tensions on tight and slack sides, (b) initial and centrifugal tension, (c) slip, (d) creep as applied to belt drives.
11.14 What is polygonal or chordal action in chain drive?
11.15 Justify (a) Lower side of the open belt is tight side, (b) For shorter distance V belt drives are preferred, (c) Cross belt drive is more efficient as compared to open belt drive.

PRACTICE PROBLEMS

11.1 It is required to drive a shaft by means of a belt drive. Pulley *B* rotates at 750 r.p.m. Pulley *A* rotates at 250 r.p.m and has a diameter of 300 mm. The thickness of the belt is 6 mm. Determine the size of pulley *B*.
11.2 A pulley is driven by a flat belt running at 600 m/min. The coefficient of friction between the pulley and the belt is 0.3. The centre distance between the shafts is 1.425 m and the diameters of pulleys are 1000 mm and 500 mm. Maximum tension in the belt = 700 N. Calculate the power transmitted.
11.3 A flat belt running on a pulley of diameter 1 m transmits 7.5 kW at 200 r.p.m, $\mu = 0.25$, diameter of smaller pulley = 225 mm and centre distance = 1.5 m. Find the necessary width of the belt if the pull should not exceed 20 N/mm. Neglect the C.F tension.
11.4 Calculate the centrifugal tension in the belt running over two pulleys at 30 m/s. The belt is 200 mm wide and 8 mm thick and has a mass of 1000 kg/m^3. If the coefficient of friction between the belt and the pulley is 0.28 and angle of lap on the bigger pulley is 190°, find the maximum power that can be transmitted at the given speed if the maximum permissible stress is 2.2 MPa. What is the percentage increase in power when the belt runs at optimum speed.
11.5 A V belt consists of 4 belts each of cross section 90 mm^2 and groove angle 38°. Density of the belt material is 1390 kg/m^3 and the coefficient of friction is 0.2. Effective diameter of pulley on the motor shaft = 120 mm, speed ratio = 2 : 1 and centre distance = 400 mm. Find the length of the belt if the drive is an open belt. Also, if the maximum permissible stress in the belt is 2.5 MPa, calculate the maximum power transmitted at motor speed 1440 r.p.m.
11.6 A rope drive is required to transmit 1175 kW from a pulley of diameter 1.5 m running at 360 r.p.m. Maximum pull that the rope can sustain = 2.2 kN, mass of the belt per unit length = 1.8 kg, angle of lap = 160°, angle of groove = 40° and coefficient of friction = 0.3. Calculate the number of ropes taking into account the effect of centrifugal tension.
11.7 A crossed leather belt drive has pulleys of diameter 300 mm and 600 mm. The centre distance is 1.8 m and speed of the drive is 1200 r.p.m. Power to be transmitted = 6 kW, mass of the belt = 0.5

kg/m and coefficient of friction = 0.3. Calculate the cross section of the belt assuming a safe tensile stress of 5 MPa for the belt thickness equal to 10 mm.

11.8 Design a flat open belt horizontal drive to transmit 10 kW at 1500 r.p.m of motor. The speed of the driven pulley is 500 r.p.m and the maximum permissible peripheral speed is 20 m/s. Assume 3% creep. Load factor = 1.1, density of belt material = 1000 kg/m^3, Modulus of elasticity for belt material = 100 MPa, S_{ut} = 25 MPa, centre distance = 3 m and endurance limit for belt material = 9 MPa. Find the life of the belt in hours. Also calculate the dimensions of the pulleys. μ = 0.30.

11.9 Design a belt drive from an electric motor to the flywheel of a punching press. Power of the electric motor = 100 kW, speed of motor = 750 r.p.m, speed of flywheel = 250 r.p.m. The diameter of the flywheel which is also used as the driven pulley is 1.5 m. Centre distance between the motor and the flywheel shaft = 2 m, Creep = 3%. Use a V belt drive.

11.10 A roller chain drive is used in a two wheeler with the petrol engine running at 1000 rpm. Centre distance = 620 mm, crankshaft diameter = 30 mm. The crankshaft requires a power of 2.5 kW. Design the chain drive.

11.11 A V belt drive is used to transmit 35 kW at 760 r.p.m of the driver pulley of 350 mm effective diameter. The driven pulley diameter is 1400 mm and the centre distance is 2 m. Groove angle = 40°, mass of belt = 0.45 kg/m and μ = 0.27. The B type belt with a cross-sectional area of 140 mm^2 is used. The permissible tensile stress for the belt is 5.7 MPa. Find the number of belts required and the length of the belt. Use an open belt drive.

11.12 In a flat belt drive the angle of lap on the smaller pulley is 120°. μ = 0.3, maximum stress in the belt = 1.5 MPa, diameter of pulley = 475 mm, speed = 900 r.p.m, maximum power transmitted = 6.25 kW and thickness of belt = 6 mm. Find the width of the belt.

11.13 In a rope drive 100 kW is transmitted by means of ropes of mass 1.2 kg/m. The groove angle is 50° and the angle of lap is 170°. μ = 0.3. If the initial tension in each rope is 900 N, find the number of ropes required.

11.14 Figure P-11.14 shows a pivoted motor drive. The motor drive weighs 2500 N and rotates at 1200 r.p.m. Find the tensions in the belt. Also find the size of the belt and the power that can be transmitted if the permissible stress should not exceed 2.5 MPa. Neglect centrifugal stress.

Use $\frac{d}{t} = 36$.

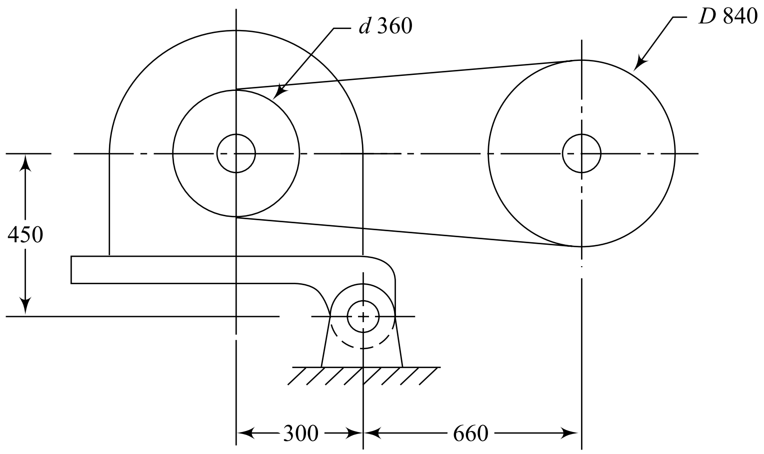


Fig. P-11.14

11.15 Solve Problem 11.14 using a cross belt drive.

11.16 In an open belt drive the smaller pulley is of 300 mm diameter and rotates at 300 r.p.m. The bigger pulley has a diameter of 500 mm and is situated at a distance of 576 mm centre to centre from the smaller pulley. The coefficient of friction is 0.25, when 4 kW is being transmitted. Which of the following alternatives is more effective to increase power to be transmitted:
(a) increasing initial tension by 10%
(b) increasing μ by 10%
(c) increasing angle of lap to 190° by using idler pulley

11.17 A compressor is driven by an electric motor by a flat leather belt. The details of the belt drive are

Item	*Diameter*	θ	μ	*N*	*Power*
Motor pulley	300 mm	144°	0.3	900 r.p.m	22 kW
Compressor pulley	1200 mm	216°	0.25	225 r.p.m	22 kW

Thickness of belt = 8 mm, maximum permissible stress in the belt = 2.8 MPa, the density of the belt = 1000 kg/m^3. Calculate the width of the belt.

11.18 In an electric motor and a compressor V belt drive, diameters of the motor and the compressor pulley are 250 mm and 1250 mm respectively. Groove angle for the motor pulley is 35° whereas compressor pulley is flat. The centre distance is 2 m. The power to be transmitted is 20 kW at 1750 r.p.m of the motor pulley. Each belt has a mass of 0.3 kg/m and area of c/s of the belt = 200 mm^2. The maximum permissible stress is 2.5 MPa. Determine the number of belts required if the coefficient of friction for both the pulleys is 0.3.

11.19 In the flat belt drive of Problem 11.18, V belts are used with a groove angle of 60° then, with other parameters remaining the same calculate the increase in power. Area of flat and V belts is the same.

11.20 In the Problem 11.18 find the number of belts required if the same amount of power is transmitted by using V belt drive of groove angle 45° and the area of c/s of belts is 140 mm^2.

11.21 Two parallel shafts connected by a crossed belt have pulleys of diameters 400 mm and 600 mm. The centre distance is 5 m. The direction of rotation of the driven shaft is to be reversed by using an open belt drive. State whether we can use the same belt. If not what is the remedy? If a belt of thickness 8 mm and width 100 mm is used and initially 63 kW power is transmitted at 600 r.p.m. calculate the change in the transmitted power when an open belt drive is used. In case the same amount of power is to be transmitted what modification is required?

11.22 Find the width of the leather belt 6 mm thick transmitting 20 kW at 500 r.p.m of 750 mm diameter pulley, $\theta = 150°$, $\mu = 0.3$, $m = 1000$ kg/m^3, permissible maximum tensile stress 2.75 MPa.

11.23 V belt with cross-sectional area of 250 mm^2 and angle of groove 45° has density 1.5 mg/m^3. The angle of lap is 180°. The coefficient of friction between pulley and belt is 0.25 and maximum stress is limited to 4.75 MPa. Find the maximum H.P that can be transmitted.

11.24 A roller chain is used to connect two shafts spaced 25 pitches apart to transmit 75 kW at 300 r.p.m of a 17 tooth driver sprocket to 34 tooth driven sprocket. The working period is 18 hrs. per day with abnormal service conditions. Specify the length and size of chain.

11.25 A double strand No. 8 type roller chain is used to transmit power between a 15 tooth driving sprocket relating at 500 r.p.m. Driving source is an electric motor and a moderate shock is expected. Driven sprocket has 60 teeth. Determine the rated power and approximate centre distance, if the chain length is 90 pitches. From Table power rating of chain is 16.99 kW, tooth correction factor is 0.85, multiple strand factor is 1.7, service correction factor is 1.3, pitch = 25.4 mm.

11.26 A chain drive is to be designed for running a drilling machine. The prime mover is an electric motor with power 5.5 kW and speed 2800 r.p.m. The centre distance should be around 500 mm. Assume transmission ratio 3.

11.27 A D section V belt has 6.116 m length and operates on 400 mm diameter pulleys rotating at 870 r.p.m. Find the number of belts for transmitting 75 kW with expected life of 20,000 hr.

11.28 What maximum h.p can be transmitted for the open belt of 100 mm width and 5 mm thickness if the maximum stress in the belt material is limited to 2.5 MPa. Density of belt material is 1.1 mg/m^3, coefficient of friction 0.25 and the angle of lap on the smaller pulley is 135°.

ANSWERS

Objective Questions

(1) c (2) b (3) c (4) a (5) c (6) b (7) a (8) a (9) a
(10) b (11) b and c (12) b (13) b (14) c (15) d (16) c (17) b
(18) b (19) b

Practice Problems

(1) 96 mm
(2) 3.97 kW
(3) 68.37 mm → 70 mm
(4) 1440 N, 35.1573 → 35 kW, 1.82%
(5) 1374.5 → 1375 mm, 6.228 ≃ 6.3 kW
(6) 40 ropes
(7) 13.12 → 15 mm
(8) d = 224 mm, D = 710 mm, t = 10 mm, b = 25 mm with f.s of 3, life 485.4 hrs
(9) Using D type belt d = 515 mm, L = 7.369 m, exact C.D = 2042.54 mm, 7 belts
(11) 4, 5.5 m
(12) 100 mm
(13) 8
(14) t = 10 mm, b = 50 mm
Power = 14.6 ≃ 14 kW
(15) P = 28.28 kW, t = 10 mm, b = 67.7 ≃ 70 mm
(16) Third alternative is more effective % increase in each case 10%, 7.5% and 12.5%
(17) 140 mm
(18) 3
(19) 5
(20) As there is a very small difference in the maximum tension in both cases the same belt may be used. No modification required.
(22) 131 mm
(23) 12.135 kW
(25) Rated power = 31.915 kW, centre distance = 641.18 mm
(26) Z_1 = 21, impact factor Y = 1.4, output correction factor 1, tooth correction factor 1.26, rated power = 6.93 kW, diameter of small sprocket 85.21 mm, diameter of big sprocket 254.18 mm
(27) 4 belts
(28) 10.35 kW

12

Brakes

CONCEPT REVIEW

12.1 TYPES OF BRAKES

Brakes are used for stopping motion by absorbing the energy of rotating wheels. Brakes can be (a) Mechanical, (b) Hydraulic, (c) Pneumatic. Mechanical brakes are of the following types: (1) Block brakes, (2) Band brakes, (3) Long shoe brakes, (4) Pivoted type shoe brakes, (5) Band and block brakes.

The first three type of brakes are most commonly used.

12.2 MATERIAL

For the drums, levers, bands 30C8 or 40C8 steel with tensile strength of 400–500 MPa, may be used while for the linings, special friction material like pressed asbestos or other materials, like leather, wood, cork, felt, carbon, graphite, etc. may be used. Shoes are made of C.I. The permissible intensity of pressure varies between 0.35 to 0.7 MPa and the coefficient of friction lies between 0.2 to 0.4.

The friction material should not get affected by environmental conditions, have a high coefficient of friction, possess ability to withstand high temperatures, should be heat resistant and have good resilience and high resistance to wear. A combination of C.I or m.s shoe with a feredo lining satisfies most of these requiremens.

12.3 DESIGN OF BRAKES

It is based on the torque capacity and ability to absorb and dissipate heat.

12.4 BLOCK BRAKES OR SHORT SHOE BRAKE (FIG. 12.1)

The angle of shoe or block is less than 60°. The following equations are used for the analysis

$$\text{Torque} = \mu N \cdot r \tag{12.1}$$

$$N \cdot b + \mu N \cdot c - P \cdot a = 0 \tag{12.2}$$

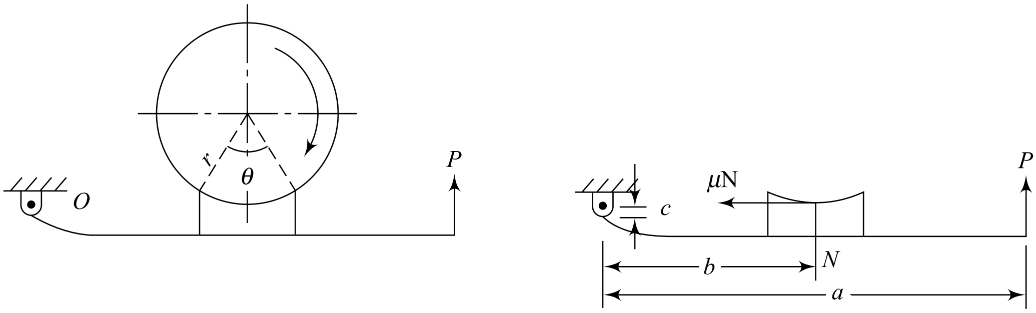


Fig. 12.1

When the frictional force μN passes through the fulcrum, i.e. when $c = 0$, Eq. (12.2) becomes

$$N \cdot b - P \cdot a = 0 \tag{12.3}$$

and when it passes above the fulcrum it may be written as

$$N \cdot b - \mu N \cdot c - P \cdot a = 0 \tag{12.4}$$

In this case as the moment due to μN is in the same direction as that due to the external for P, the brake is said to be self-energising. The brake becomes self-locking when $b = \mu c$.

12.5 BAND BRAKE

In a simple band brake (Fig. 12.2), one end of the band is attached to the fulcrum. The equations used are

$$\frac{T_1}{T_2} = e^{\mu\theta} \tag{12.5}$$

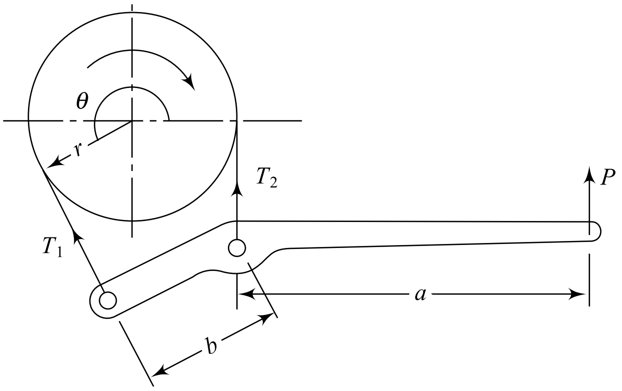


Fig. 12.2

$$\text{Torque} = (T_1 - T_2) \cdot r \tag{12.6}$$

$$T_1 \cdot b - P \cdot a = 0 \text{ for clockwise rotation} \tag{12.7}$$

$$T_2 \cdot b - P \cdot a = 0 \text{ for anticlockwise rotation} \tag{12.7a}$$

For a differential band brake (Fig. 12.3), Eqs. (12.5) and (12.6) remain unchanged while Eq. (12.7) changes to

$$T_1 \cdot b - T_2 \cdot c + P \cdot a = 0 \quad \text{for clockwise rotation} \tag{12.8}$$

$$T_2 \cdot b - T_1 \cdot c + P \cdot a = 0 \quad \text{for anticlockwise rotation} \tag{12.9}$$

The brake becomes self-locking if $T_1 \cdot b = T_2 \cdot c$ or $T_2 \cdot b = T_1 \cdot c$ for Eqs (12.8) and (12.9) respectively. For an additative brake (Fig. 12.4), Eqs. (12.8) and (12.9) change to

$$T_1 \cdot b + T_2 \cdot c - P \cdot a = 0 \text{ and} \tag{12.10}$$

$$T_2 \cdot b + T_1 \cdot c - P \cdot a = 0 \tag{12.11}$$

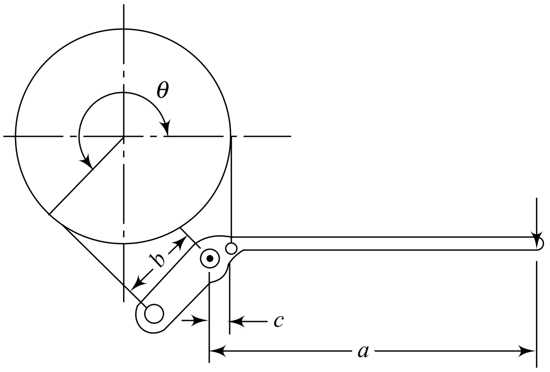


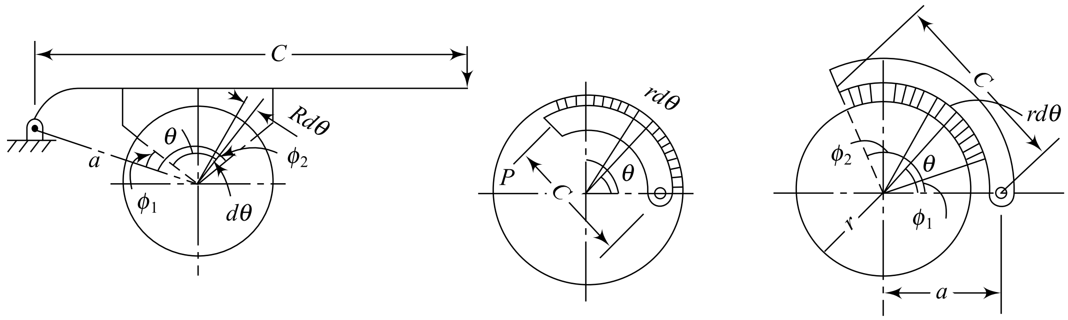
Fig. 12.3

Fig. 12.4

Here moments of T_1 and T_2 are added together hence it is called as, additative brakes whereas in a differential band brake, the moments are substracted from each other. The additative brake does not become self-locking in any case but requires a larger braking force.

12.6 LONG SHOE BRAKES

When the angle of contact of the shoe and the brake drum is larger than 60°, the pressure distribution is non-uniform and hence, the normal reaction N cannot be assumed to act at a point. This type of shoe brake is termed as the *long shoe brake*.



External shoe Internal shoe External shoe

Fig. 12.5

Let us consider a unit pressure p acting upon an element of area $b \cdot r\ (d\theta)$ of the friction material located at an angle θ from the hinge pin where b is the width of the shoe. As the pressure is not uniform, let the maximum pressure p_m act at an angle θ_m from the hinge pin. Since the intensity of pressure is proportional to the vertical distance from the hinge pin which is further proportional to sin θ the relation between the pressures is

$$\frac{p}{\sin\theta} = \frac{p_m}{\sin\theta_m} \quad \text{or} \quad p = \frac{p_m \sin\theta}{\sin\theta_m} \tag{12.12}$$

The pressure p_m is maximum when θ = 90°. If the positive angle is less than 90°, pressure p is maximum at the toe.

Figure 12.6 shows the force diagram of an external contacting shoe brake

since $\Sigma M = 0$

$$M_p + M_N + M_\mu = 0$$

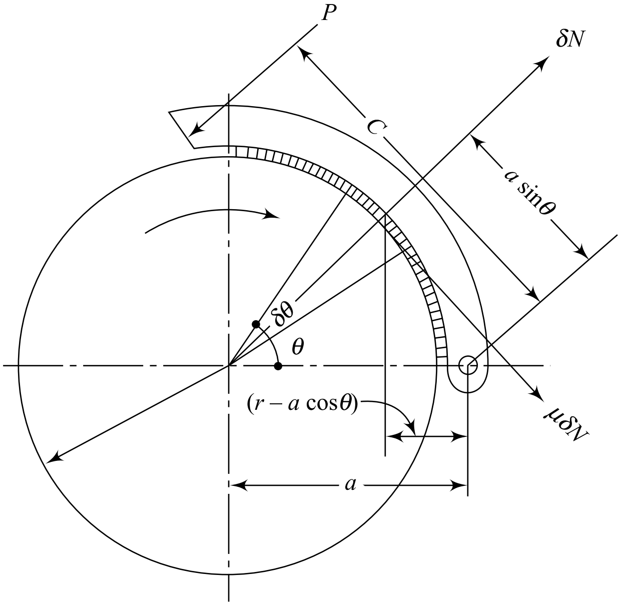


Fig. 12.6

where M_p, M_N and M_μ are moments about the fulcrum due to the external force P, normal reaction $\int dN$ and frictional force $\mu \int dN$ respectively.

(1) $M_p = P \cdot C$

(2) $M_N = \int_{\varphi_1}^{\varphi_2} p \cdot b \cdot r \cdot d\theta\, a \sin\theta = \int_{\varphi_1}^{\varphi_2} p \cdot b \cdot r \sin\theta\, d\theta$ where φ_1 and φ_2 are angles from the hinge pin at which the friction material comes in contact with the drum.

$$\therefore \quad M_N = \frac{p_m bra}{\sin\theta_m}\int_{\varphi_1}^{\varphi_2} \sin^2\theta\, d\theta$$

$$= \left[\frac{p_m bra}{4\sin\theta_m}\right][2(\varphi_1 - \varphi_2) - \sin 2\varphi_2 + \sin 2\varphi_1] \tag{12.13}$$

(3) $M_\mu = \int_{\varphi_1}^{\varphi_2} \mu pbr(r - a\cos\theta)\, d\theta$

Substituting and Integrating

$$M_\mu = \frac{\mu p_\infty br}{4\sin\theta_m}[4r(\cos\varphi_1 - \cos\varphi_2) + a(\cos 2\varphi_2 - \cos 2\varphi_1)] \quad (12.14)$$

(4) $P\cdot C = M_N + M_\mu$ for clockwise rotation since the moment $P\cdot C$ is anticlockwise and both M_μ and M_N are in clockwise direction. For anticlockwise rotation the direction of M_μ changes, the direction of other moments remaining unchanged. Hence, in this case the moment due to frictional force supports the moment due to external force. Thus, the brake is self-energizing. It becomes self-locking when $M_N = M_\mu$.

(5) Torque on the drum $= \int_{\varphi_1}^{\varphi_2} \mu p_m br^2\, d\theta = \int_{\varphi_1}^{\varphi_2} \frac{\mu p_m br^2 \sin\theta\, d\theta}{\sin\theta_m}$

$$= \frac{\mu p_m br^2}{\sin\theta_m}(\cos\varphi_1 - \cos\varphi_2) \quad (12.15)$$

(6) The hinge pin reactions

$$R_X = P_K - \int dN\cos\theta - \int \mu dN \sin\theta \quad (12.16)$$

$$R_X = P_Y + \int dN\sin\theta + \int \mu dN \cos\theta \quad (12.17)$$

12.7 PIVOTED TYPE SHOE BRAKE

Although many brakes have an appearance of a pivoted type brake, they are rarely used in practice. This is due to the inherent problem of locating the pivot so close to the drum surface and impracticability of maintaining the correct position as the lining wears.

In this case, as the shoe is symmetrical and θ is measured from the axis of symmetry, the pressure intensity is proportional to cos φ. Therefore, $p = p_m \cos\varphi$. Also, all the sine components of normal reaction cancel each other and hence the normal reaction at hinge may be written as

$$N = \int_{-\theta/2}^{\theta/2} pbr\cos\varphi\, d\varphi$$

$$= \int_{-\theta/2}^{\theta/2} p_m br\cos^2\varphi\, d\varphi$$

$$= \frac{p_m br}{2}(\theta + \sin\theta) \quad \text{(i)}$$

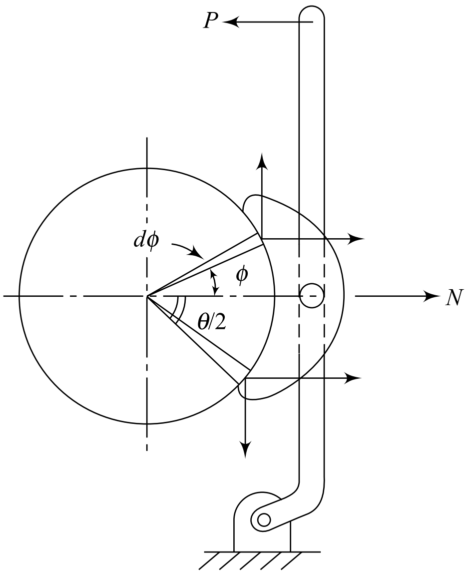


Fig. 12.7

$$\text{Torque on the drum} = \int_{-\theta/2}^{\theta/2} \mu p r^2 b \, d\varphi$$

$$= \mu r^2 b \int_{-\theta/2}^{\theta/2} p_m \cos\varphi \, d\varphi = 2\mu r^2 b p_m \sin \theta/2 \qquad \text{(ii)}$$

From Eqs (i) and (ii) $\quad T = \mu N r \dfrac{4 \sin \theta/2}{\theta + \sin\theta}$ (12.18)

12.8 HEAT DISSIPATION IN BRAKES

Heat generated = $H_g = p_{av} A_c \mu V$ Watts (12.19)

where p_{av} = average pressure in N/mm^2, A_c = area of contact in mm^2
μ = coefficient of friction, V = peripheral velocity of drum in m/s.

By energy consideration

H_g = [Potential energy (E_p) + Kinetic energy (E_k)] Watts

Heat dissipated $\quad H_d = C \, \Delta \, t \, A_r$ (12.20)

where, C = heat transfer coefficient in J/m^2 of area of radiating surface
A_r = area of radiating surface in m^2
Δt = temperature difference between the radiating surface and the surroundings.

A complete data for the calculation of H_g and H_d may not be available, hence it is a normal practice to check the design of the brake for the following criteria.

$p_{av} V < 9.8 \times 10^5$ for continuous application
$p_{av} V < 1.93 \times 10^6$ for intermittent application (12.21)
$p_{av} V < 2.9 \times 10^6$ for continuous application with large heat dissipation.

Unit of p_{av} being N/m^2 and V is in m/s.

WORKED EXAMPLES

12.1 The block brake shown in Fig. 12.1 has the drum of diameter 750 mm. a = 1 m, b = 400 mm and c = 50 mm. Coefficient of friction is 0.3. The torque capacity of the brake is 225 N·m. Find the normal reaction and the force to be applied at the end of the lever, for clockwise and anticlockwise rotation of the drum at 600 r.p.m.

Solution:

Using Eq. 12.1

$$T = 0.3 \times N \times 0.375 = 225$$

$\therefore \quad N = 2000$ N

Using Eq. 12.2

$$2000 \times 400 + 0.3 \times 2000 \times 50 - P \times 1000 = 0$$

P = 830 N for clockwise rotation

Using Eq. 12.4

P = 770 N for anticlockwise rotation

Further calculation of dimension may be made by using Eq. 12.22 for heat dissipation

$$V = \frac{\pi \times 0.75 \times 600}{60} = 23.56 \text{ m/s}$$

$$pV = 1.93 \times 10^6, \qquad \therefore p = 0.0819 \times 10^6 \text{ Pa}$$

$$N = p \times \text{Area of contact}, \qquad \therefore \text{Area of contact} = \frac{2000}{0.0819} = 24420 \text{ mm}^2$$

Let the angle of contact be 60°

$$\therefore \qquad 375 \times \frac{\pi}{3} \times b = 24420$$

$\therefore$ $b = 62.18$ mm. Let us adapt $b = 100$ mm.

$\therefore$ Angle of contact will be 37.5°

Maximum b.m on the lever $= P\,(a - b)$

$$= 830 \times 664 \text{ N.mm}$$

$$= \sigma_t z = 70 \times \frac{1}{6} bt^2$$

$$\therefore \qquad t = \sqrt{\frac{830 \times 600 \times 6}{70 \times 100}} = 20.66 \text{ mm} \cong 21 \text{ mm.}$$

The dimension of the fulcrum pin may be calculated by first finding the reaction at the fulcrum.

Reaction at fulcrum in vertical direction = 2000 – 830 = 1170 N

Horizontal reaction = 0.3 × 2000 = 600 N

$$\therefore \qquad \text{Resultant reaction} = \sqrt{1170^2 + 600^2} = 1314.87 \text{ N}$$

Dimensions of the fulcrum pin may be calculated by using the method described in chapter two using Eqs. 2.2.4 to 2.2.7.

12.2 In the brake shown in Fig. E-12.2 diameter of the drum rotating at a speed of 100 r.p.m is 600 mm. Find the breaking torque and the amount of heat generated per unit time. $\mu = 0.3$.

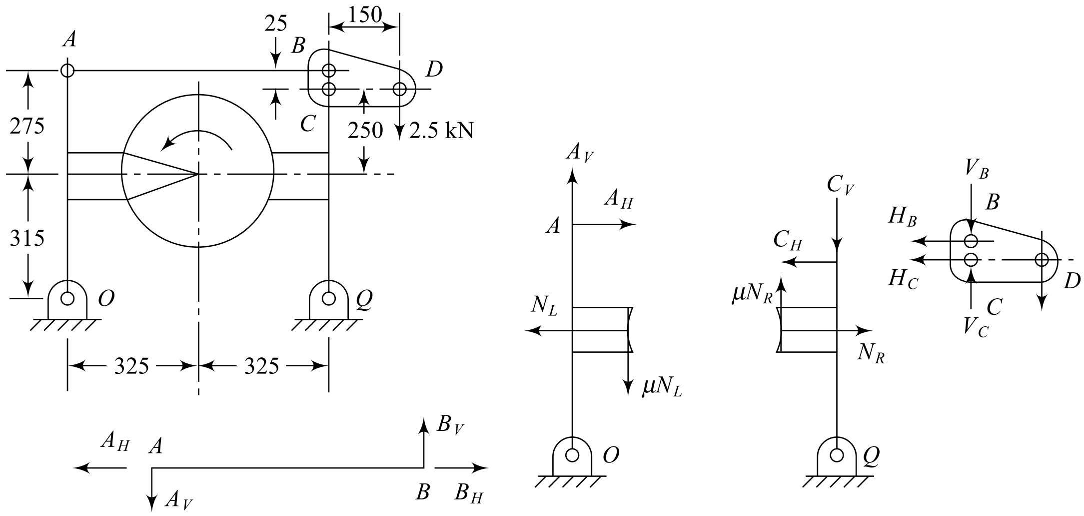


Fig. E-12.2

Solution:

Free body diagram for each link is as shown in Fig. E-12.2

For lever *BCD* $\quad H_B = H_c$ and $\Sigma M = 0$

$\therefore \quad 2500 \times 150 = H_c \times 25, \quad \therefore H_c = 15000 \text{ N} = H_B$

Considering the right hand link *CQ* and taking moment @ *Q*

$$15000 \times 665 - N_R \times 315 - \mu N_R \times 25 = 0$$

Substituting $\quad \mu = 0.3; N_R = 30930$ N

For link *AO* taking moments @ *O* we get

$$N_L = 31219.512 \text{ N}$$

$\therefore \quad$ Braking torque $= \mu(N_L + N_R) \times$ radius of drum

$$= 0.3\ (30930 + 31219.512) \times \frac{300}{1000} = 5593.456 \text{ N.m}$$

$$V = \frac{\pi \times 600 \times 100}{60 \times 100} = 3.14 \text{ m/s}$$

$$H_g = F_t \cdot V = 0.3\ (30930 + 31219.512) \times 3.14 = 58574.553 \text{ J/p}$$

12.3 Design a band brake for absorbing 30 kW from a drum rotating at 500 r.p.m. Diameter of drum = 500 mm, $\mu = 0.3$, $\theta = 270°$. For band material safe, $\sigma_t = 80$ MPa. For 20C4 steel used for lever, pins and rivets safe stresses are $\sigma_t = 50$ MPa, $\sigma_c = 80$ MPa, $\tau = 40$ MPa.

Solution:

$$\frac{T_1}{T_2} = e^{0.3 \times \frac{3\pi}{2}} = 4.12$$

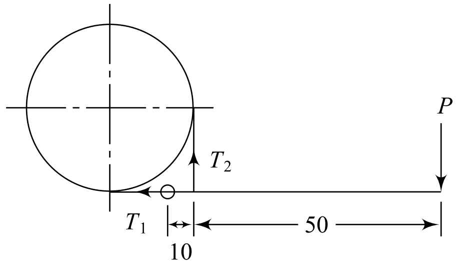


Fig. E-12.3(a)

$\therefore \quad$ Torque $= (4.12\ T_2 - T_2) \times \dfrac{500}{2 \times 1000}$

Again $\quad$ Torque $T = \dfrac{\text{Power} \times 60 \times 1000}{2\pi \times 500}$

$$= \frac{30 \times 60 \times 1000}{2\pi \times 500} = 573 \text{ N.m}$$

$$\therefore \quad T_2 = \frac{573 \times 2000}{500 \times 3.12} = 734\ 61 \text{ N}$$

$$T_1 = 3026.6 \text{ N}$$

Band thickness $= 0.005D = 2.5$ mm $\cong 3$ mm, where D = diameter of drum

Maximum tension $T_1 = 3026.6 = \sigma_t \cdot A = 80 \times 3 \times b$

$\therefore \quad$ width of the band $b = \dfrac{3026.6}{240} = 12.61$ mm

Use $b = 20$ mm to compensate for the reduction in area due to rivet holes.

Lever design:

Taking moment @ fulcrum $P \times 60 - T_2 \times 10 = 0$

$\therefore \qquad P = 122.44$ N

$\therefore$ Maximum b.m. at a distance of 50 mm = $122.44 \times 50 = 6122$ N.mm

Assuming rectangular cross section of the lever with

$$d : t = 3 : 1 \qquad z = 1.5\, t^3$$

$$6122 = \sigma_t \cdot z, \qquad \therefore \quad t = \sqrt[3]{\frac{6122}{1.5 \times 20}} = 3.7 \text{ mm}$$

Let us adapt $t = 10$ mm and depth $d = 30$ mm.

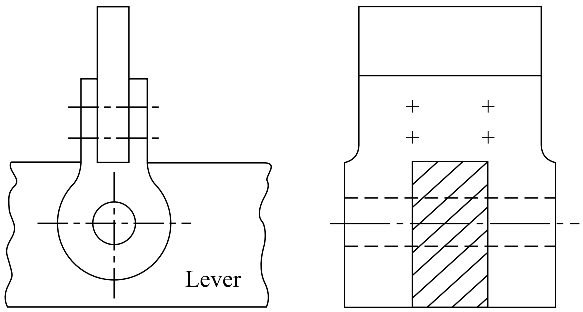


Fig. E-12.3(b)

Connection of band end with lever:

Use four rivets to attach the band to the tight and slack side. Considering shearing failure of rivets we have,

$$T_1 = 4 \times 2 \times \frac{\pi d^2}{4} \times \tau$$

$$3026.6 = 4 \times 2 \times \frac{\pi}{4} d^2 \times 40$$

$\therefore \qquad$ diameter of rivet = 3.47 mm $\cong$ 4 mm.

This justifies the increase in the width of the band from 12.61 mm to 20 mm.

Dimensions of the pins may be calculated as in a knuckle joint. Under double shear,

$$T_1 = 3026.6 = 2 \times \frac{\pi}{4} \times d_p^2 \times \tau, \quad \therefore d_p = 6.94 \text{ mm} \cong 10 \text{ mm}.$$

Considering bearing criteria, $3026.6 = 10 \times l \times P_b$. Substituting $P_b = 20$.

$$l = 15.133 \text{ mm say } 16 \text{ mm}.$$

The same may be checked for bending also.

Width of the drum should be slightly more than the band width say 22 mm. By taking into account the torque, the shaft diameter is calculated as 40 mm. Other dimensions may be selected from the manufacturer's catalogue.

Check for heating:

$$\text{Average pressure} = \frac{T_1 + T_2}{\text{projected area}} = \frac{3026.6 \times 734.61}{20 \times 500} = 0.376 \text{ MPa}$$

$$V = \frac{\pi \times 500 \times 600}{60 \times 1000} = 15.7 \text{ m/s}$$

$\therefore$ $p \times V = 5.9 \times 10^6$ which exceeds the limit. Hence, either proper cooling should be provided or the speed may be reduced and/or band width should be increased.

12.4 Derive an expression for the ratio of tensions in case of a band and block brake with n number of blocks and angle of embrace of each block as 2 θ.

In the band and block type brake, $\theta = 15°$ and effective diameter of the drum is 800 mm. μ = 0.4, a = 100 mm, b = 25 mm. The power to be absorbed at 600 r.p.m is 450 kW. When the force applied at the end of the lever at a distance of 1.2 m from the fulcrum is 200 N. Find the number of blocks.

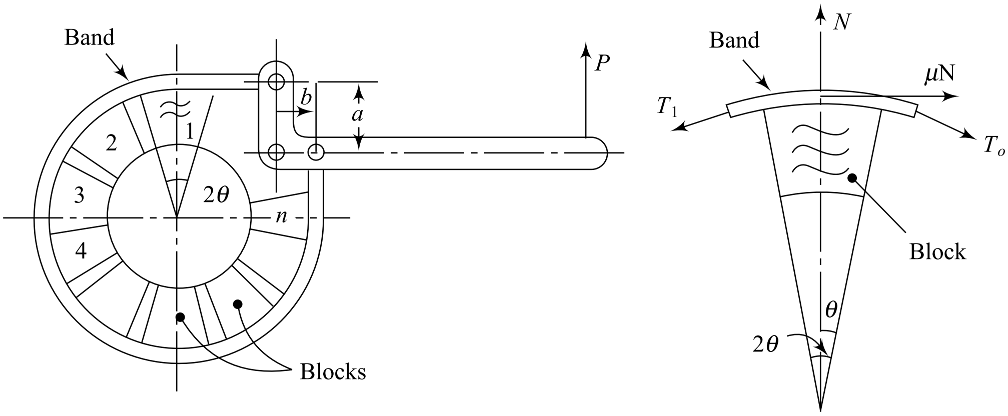


Fig. E-12.4

Solution:

Consider equilibrium of the belt covering the first block

as $\Sigma V = 0$, $\therefore$ $N = (T_1 + T_0) \sin \theta$ (i)

and $\Sigma H = 0$, $\therefore$ $T_1 \cos \theta = T_0 \cos \theta + \mu N$ (ii)

$\therefore$ $T_1 \cos \theta = T_0 \cos \theta + \mu(T_1 + T_0) \sin \theta$

or $(T_1 - T_0) \cos \theta = \mu\, (T_1 + T_0) \sin \theta$

or $$\frac{\mu \tan \theta}{1} = \frac{T_1 - T_0}{T_1 + T_0}$$

By componendo dividendo rule

$$\frac{T_1}{T_0} = \frac{1 + \mu \tan \theta}{1 - \mu \tan \theta}$$

Similarly for the second block $\dfrac{T_2}{T_1} = \dfrac{(1+\mu \tan\theta)}{(1-\mu \tan\theta)}$

and for the third block $\dfrac{T_3}{T_2} = \dfrac{(1+\mu \tan\theta)}{(1-\mu \tan\theta)}$ and so on.

$$\therefore \quad \frac{T_n}{T_0} = \frac{T_n}{T_{n-1}} \times \ldots \frac{T_2}{T_1} \times \frac{T_1}{T_0} = \left(\frac{1+\mu \tan\theta}{1-\mu \tan\theta}\right)^n$$

Now, power to be absorbed = 450 kW = $\dfrac{2\pi NT}{60000}$

Substituting $T = 7162 = (T_n - T_0) \times 0.4$ N.m

$T_n - T_0 = 17905$ N (iii)

Again from the equilibrium equation for the lever

$$200 \times 1.2 - T_n \times \frac{25}{1000} + T_0 \times \frac{100}{1000} = 0$$

$\therefore \quad T_n - 4\, T_0 = 9600$ (iv)

From Eqs. (iii) and (iv)

$$T_0 = 2768.33 \text{ N and } T_n = 20673.33 \text{ N}$$

$$\frac{T_n}{T_0} = \frac{20678.33}{2768.33} = 7.468 = \left(\frac{1+\mu \tan\theta}{1-\mu \tan\theta}\right)^n$$

Again $$\frac{1+\mu \tan\theta}{1-\mu \tan\theta} = \frac{1+0.35 \tan 15^\circ}{1-0.35 \tan 15^\circ} = 1.2$$

$\therefore \quad n = 11.2$, say 12.

Therefore 12 blocks should be used.

12.5 Calculate the value of torque that the back-stop in Fig. E-12.5 can resist if the maximum pressure between the lining and the drum is 1.4 MPa. Also find the coefficient of friction required to hold the load. $b = 30$ mm.

Solution:

Backstop should operate when the direction of rotation reverses.

$$T_1 = p_{max}\, br$$
$$= 1.4 \times 30 \times 320 = 13440 \text{ N}$$

For the backstop to operate when the rotation is reversed

$$13440 \times 50 = T_2 \times 200$$

$\therefore \quad T_2 = 3360$ N

$$\therefore \quad \text{Torque} = (13440 - 3360) \times \frac{320}{1000} = 3225.6 \text{ N.m}$$

Again $$\frac{T_1}{T_2} = 4 = e^{\mu\theta}$$

As $\theta = 210^\circ$, $\mu = 0.378$.

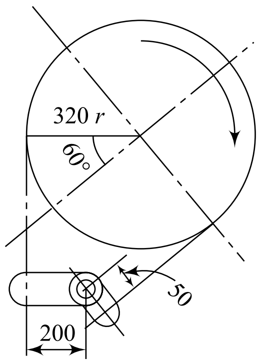


Fig. E-12.5

12.6 Maximum lining pressure at any point in the brake shown in Fig. E-12.6 is limited to 0.6 MPa. Find the torque capacity and the force P if the coefficient of friction is 0.15.

Solution:

From Fig. E-12.6

$$T_2 = T_3$$

$$\therefore \quad \frac{T_1}{T_2} = e^{\mu\frac{\pi}{2}} = 1.369$$

and $$\frac{T_3}{T_4} = e^{\mu\frac{5\pi}{6}} = 1.688$$

$$T_1 = 0.6 \times 60 \times 250 = 9000 \text{ N}$$

$$\therefore \quad T_2 = 6598.24 \text{ N} = T_3$$

$$\therefore \quad T_4 = 3908.91 \text{ N}$$

$$\text{Torque} = (T_1 - T_4)\, r$$

$$= (9000 - 3908.91)\frac{250}{1000}$$

$$= 1272.7725 \text{ N.m}$$

To calculate P we take moments @ the fulcrum

$$P \times 300 - (T_2 + T_3) \cos 60° \times 100 = 0$$

$$\therefore \quad P = 2199.4133 \text{ N}$$

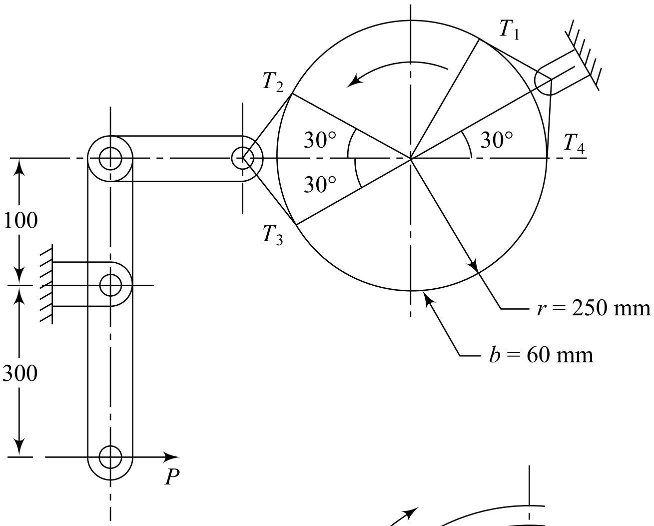


Fig. E-12.6

12.7 For the internal expanding single shoe brake shown in Fig. E-12.7, maximum pressure between the lining and the drum is limited to 0.6 MPa. $\mu = 0.25$ and width of the lining is 65 mm. Find the torque about the point O. Also calculate the force P.

Solution:

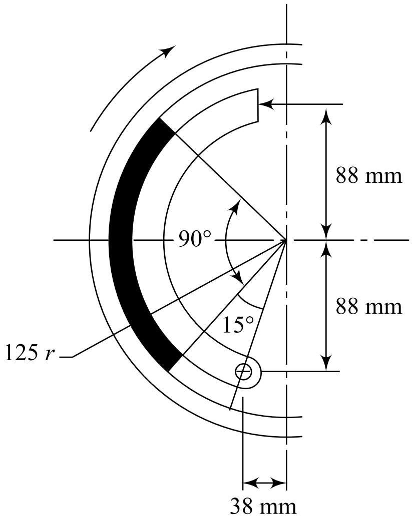


Fig. E-12.7

$$A = \sqrt{38^2 + 88^2} = 95.854 \text{ mm}$$

$$c = 88 + 88 = 176 \text{ mm}$$

$$\therefore \quad M_n = \frac{0.6 \times 65 \times 95.854 \times 125}{4 \times 1000}\left[\frac{2\pi}{2} - \sin 210° + \sin 30°\right]$$

$$= 483.82 \text{ N.m}$$

$$M_\mu = \frac{0.25 \times 65 \times 125 \times 0.6}{4 \times 1000} [95.854 (\cos 210° - \cos 30°) - 4 \times 125(\cos 105° - \cos 15°)]$$

$$= 135.98 \text{ N.m}$$

$$P = \frac{M_n + M_\mu}{C} = \frac{483.82 + 135.98}{0.176} = 3521.59 \text{ N}$$

$$\text{Torque} = 0.25 \times 65 \times 125 \times \frac{125 \times 0.6}{1000} (\cos 15° - \cos 105°) = 186.576 \text{ N.m}$$

12.8 The brake shown in Fig. E-12.8a requires an actuating force of 2000 N at the end of the lever. Coefficient of friction is 0.3 and maximum pressure intensity = 1 MPa. Determine the width of the shoe and the braking capacity.

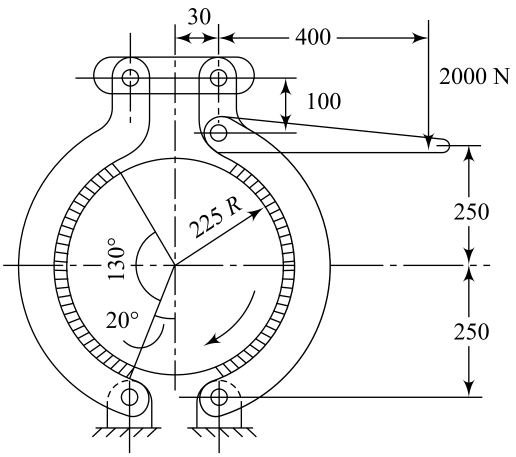


Fig. E-12.8a

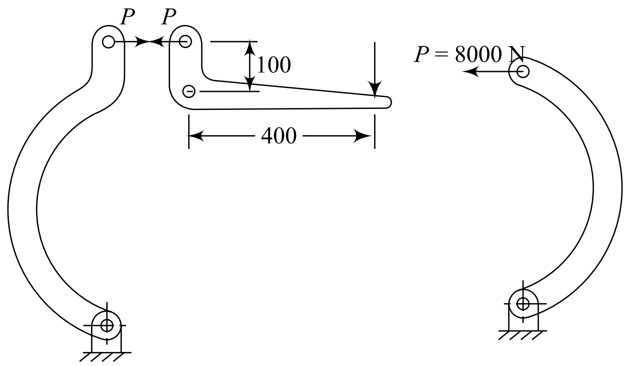


Fig. E-12.8b

Solution:

For clockwise rotation the left hand shoe is self energizing. Hence maximum pressure acts on it.

$$\phi_2 = 130°,\ \phi_1 = 0°$$

$$p_m = 1 \text{ MPa}$$

$$a = \frac{250}{\cos 20°} = 266 \text{ mm}$$

$$M_{N_l} = \frac{1 \times b \times 225 \times 266}{4 \times 1000} \times \left[2\left(\frac{\pi}{180} \times 130° - 0\right) - \sin 260°\right]$$

$$= 82.633\ b \text{ N.m}$$

$$M_{\mu_x} = \frac{0.3 \times 1 \times 6 \times 225}{4 \times 1000}$$

[4 × 225 (cos 0 – cos 130°) + 266 (cos 260° – cos 0)] = 19.68 b

Again $\quad M_{N_L} - M_{\mu_L} = PC_L$

$\therefore \quad b(82.633 - 19.68) = P \times \dfrac{600}{1000}$

Force P is obtained by taking moments @ fulcrum

$\therefore \quad P = 2000 \times \dfrac{400}{100} = 8000 \text{ N}$

$$b = \frac{8000 \times 600}{1000 \times 62.953} = 76.24 \text{ mm}$$

For the right hand shoe the values of M_N and M_μ depend on the pressure p_r

$$\therefore \qquad M_{N_R} = b \times 82.633 \times \frac{p_R}{p_L} \text{ N.m} = 6299.93\, p_R \text{ N.m}$$

(since $p_L = 1$ MPa)

$$M_{\mu_R} = 19.68 \times 76.24 \times p_R/p_L = 1500.4\, p_R \text{ N.m}$$

For right hand shoe $M_N + M_\mu = P \times C_R$

$$\therefore \quad (6299.93 + 1500.4)\, p_R = 8000 \times \frac{500}{1000}$$

$$p_R = 0.5127 \text{ MPa}$$

$$M_{N_L} = 6299.93 \times 0.5127 = 3229.9741 \text{ N.m}$$

$$M_{\mu_L} = 1500.4 \times 0.5127 = 769.25 \text{ N.m}$$

$$\text{Braking capacity} = \frac{\mu p_{m_R}\, bR^2}{\sin\theta_m}(\cos\varphi_1 - \cos\varphi_2) + \frac{\mu p_{m_L}\, br^2}{\sin\theta_m}(\cos\varphi_1 - \cos\varphi_2)$$

$$= 0.3\,(1 + 0.5127) \times 76.24 \times (225)^2 \frac{(\cos 0° - \cos 130°)}{1000} \text{ N.m}$$

$$= 2877.4209 \text{ N.m.}$$

12.9 The internal expanding shoe brake has a drum of diameter 250 mm. $R = 100$ mm and face width of the shoe = 28 mm. Calculate the actuating force, torque capacity and the hinge pin reactions if $p_{\max} = 600$ kPa and $\mu = 0.32$.

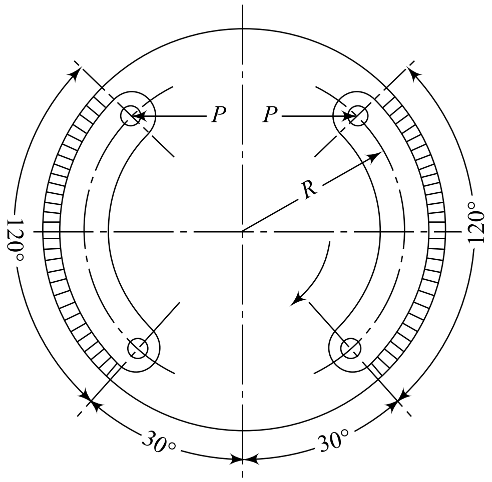


Fig. E-12.9

Solution:

The right nand shoe is self-energizing for clockwise rotation

$\varphi_1 = 0$, $\varphi_2 = 120°$, $r = 125$ mm, $a = 100$ mm, $C = 2 \times 100 \cos 30° = 173.2$ mm

$$M_N = \frac{0.6 \times 28 \times 128 \times 100}{4000}\left[\frac{2 \times 120}{180}\pi - \sin 240°\right] = 265.37 \text{ N.m}$$

$$M_\mu = \frac{0.32 \times 0.6 \times 28 \times 125}{4000} \times [4 \times 125\,(1 - \cos 120°) + 100\,(\cos 240° - 1)]$$

$= 100.8$ N.m

$$P = \frac{M_N - M_\mu}{C}$$

$$P = \frac{265.37 - 100.8}{173.2} \times 1000 = 950.173 \text{ N}$$

For the left hand shoe, $M_N = \dfrac{265.37}{0.6} \times p_{mL} = 442.283\, P_{mL}$ N.m

$$M_\mu = \frac{100.8 \times p_{mL}}{0.6} = 168\, P_{mL} \text{ N.m}$$

$$M_N + M_\mu = P \cdot C$$

$$(442.283 + 168)\, p_{mL} = 950.173 \times 0.1732$$

$\therefore$ $p_{mL} = 0.2696$ MPa

Torque capacity $= (P_{mR} + P_{mL}) \times \mu \times br^2 (\cos \varphi_1 - \cos \varphi_2)$

$$= (0.6 + 0.2696) \times 0.32 \times 28 \times (125)^2 \,\frac{(1 - \cos 120°)}{1000}$$

$= 182.616$ N.m.

Hinge pin reactions:

For the right hand shoe, the maximum pressure intensity = 0.6 MPa

$$R_X = \frac{p_m br}{\sin \theta_m}\left[\frac{1}{2}\sin^2 \varphi_2 - \mu\left(\frac{\varphi_2}{2} - \frac{1}{4}\sin^2 \varphi_2\right)\right] P \sin 30°$$

$$= \frac{0.6 \times 28 \times 125}{1 \times 1000}\left[\frac{1}{2}\sin^2 120° - 0.32\left(\frac{\pi}{3} - \frac{\sin 240°}{4}\right)\right] - 0.95 \times 0.5$$

$= 0.5367$ kN

$$R_Y = \frac{p_m bt}{\sin \theta_m}\left[\left(\frac{\varphi_2}{2} - \frac{\sin^2 \varphi_2}{4}\right) + 0.32\,\frac{\sin^2 \varphi_2}{2}\right] - P \cos 30°$$

$$= \frac{0.6 \times 28 \times 125}{1000}\left[\left(\frac{\pi}{3} - \frac{\sin 240°}{4}\right) + \frac{0.32}{2}\sin^2 120°\right] - 0.95 \times 0.866$$

$= 2.08307$ kN

$\therefore$ Resultant reaction $= \sqrt{(2.08307)^2 + (0.5367)^2} = 2.151$ kN

Similarly for the left hand shoe

$$R_X = \frac{0.2636 \times 28 \times 125}{1000}\left[\frac{1}{2}\sin^2 120° + 0.32\left(\frac{\pi}{3} - \frac{\sin 240°}{4}\right)\right] - 0.95 \times 0.5$$

$$= 0.2604 \text{ kN}$$

$$R_Y = \frac{0.2696 \times 28 \times 125}{1000}\left[\left(\frac{\pi}{3} - \frac{\sin 240°}{4}\right) - \frac{0.32}{2}\sin^2 120°\right] - 0.95 \times 0.866$$

$$= 0.2565 \text{ kN}$$

$$\therefore \quad R = \sqrt{(0.2604)^2 + (0.2565)^2} = 0.3655 \text{ kN.}$$

12.10 Find the value of $p_{\max}$ for each shoe and the value of torque exerted by the brake. Also find the power, absorbed for the pivoted type of shoe brake shown in Fig. E-12.10 (a) and (b).

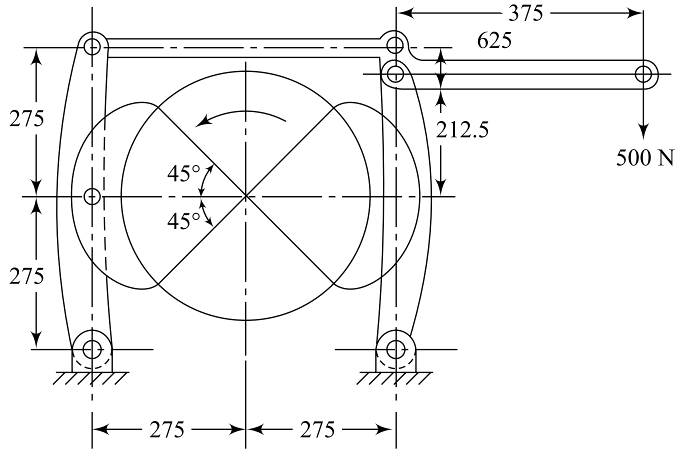


Fig. E-12.10a

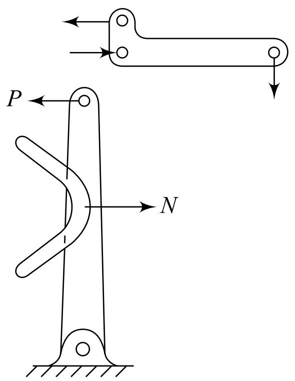


Fig. E-12.10b

Solution:

For the right hand shoe

$$P = \frac{500 \times 375}{62.5} = 3000 \text{ N}$$

$$\therefore \quad 3000 \times 487.5 - \frac{p_m b r_m}{2}(\theta + \sin\theta) \times 275 = 0$$

$$\therefore \quad 3000 \times 487.5 - \frac{p_m \times 100 \times 250}{2}\left(\frac{\pi}{2} + \sin\frac{\pi}{2}\right) \times 275 = 0$$

$$\therefore \quad p_m = 0.166 \text{ MPa}$$

For the left hand shoe a force $P = 3000$ N acts at a distance of 550 mm from the fulcrum. Hence from Fig. E-12.10(a)

$$N \times 275 - P \times 550 = 0 \text{ where } N \text{ is the hinge pin reaction}$$

From Eq. (i) of 12.7

$$N = \frac{p_m br}{2}(\theta + \sin\theta)$$

where p_m is the maximum pressure intensity.

$$p_m = 0.187 \text{ MPa}$$

$$T = 2\mu br^2 p_m \sin\theta/2 = 2 \times 0.2 \times 100 \times (150)^2 \frac{(0.187 + 0.166)}{1000} \times \frac{1}{\sqrt{2}}$$

$$= 593.88 \text{ N.m}$$

$$\therefore \quad \text{Power} = \frac{2nNT}{60000} = \frac{2 \times \pi \times 600 \times 593.88}{60000} = 37.31 \text{ kW}$$

Considering the heat dissipation, $p_{av} = \frac{p_m}{2} = \frac{0.187}{2}$ MPa

and
$$V = \frac{\pi DN}{60} = \frac{\pi \times 500 \times 600}{60 \times 1000} = 15.7 \text{ m/s}$$

$\therefore p_{av} V = \frac{0.187}{2} \times 10^6 \times 15.7 = 1.467 \times 10^6$ which is within the limit for intermittent operation.

OBJECTIVE QUESTIONS

12.1 Self-locking is not possible in case of
(a) differential band brake (b) simple band brake
(c) internal expanding shoe brake (d) simple block brake

12.2 For a brake with a drum speed of 600 r.p.m and diameter 500 mm, the average pressure intensity should not exceed
(a) 0.5 MPa (b) 0.1 MPa (c) 0.2 MPa (d) 0.02 MPa

12.3 The type of shoe brake rarely used is
(a) internal expanding (b) pivoted (c) external (d) short shoe

12.4 Ferodo is preferred for the friction lining due to
(a) high coefficient of friction (b) high heat resistance
(c) good heat dissipation (d) low wear

12.5 The differential band brake becomes self-locking if the lever arms tensions ratio is equal to
(a) 4 (b) $e^{\pm\mu\theta}$ (c) a/b (d) b/c

12.6 The pressure variation in long shoe brakes is proportional to
(a) the distance of the point of contact from the hinge (b) $\sin\theta$
(c) $\cos\theta$ (d) none of the above
where θ is the angle at the point under consideration from the point of zero pressure.

12.7 To transmit 20 kW with a drum peripheral velocity of 10 m/s, drum diameter 500 mm, ratio of tensions, and permissible pressure intensity of 0.2 MPa, the width of the band should be
(a) 100 mm (b) 10 mm (c) 40 mm (d) none of the above

12.8 The direction of rotation does not change the torque capacity of
(a) differential band brake (b) single shoe brake
(c) internal expanding shoe brake (d) simple band brake.

12.9 The brake is said to be self-energizing if
(a) the direction of applied force is the same as that of the frictional force.
(b) the direction of moment due to frictional force and that due to external force is the same required to operate the brake.
(c) no external force is required to operate the brake.
(d) an external force is required to disengage the brake.

12.10 The torque capacity of the brake should be increased by
(a) increasing the pressure intensity
(b) increasing the drum diameter
(c) increasing the width of the shoe
(d) increasing the coefficient of friction

12.11 Short shoe brakes have a angle of contact less than
(a) 10° (b) 20° (c) 60° (d) 45°

12.12 Self-locking does not happen in case of
(a) long shoe brakes (b) internal expanding shoe brake
(c) additative type of band brake (d) differential band brake

REVIEW QUESTIONS

12.1 Explain the function of a brake.
12.2 What are the different types of brakes?
12.3 Differentiate between the differential, simple and additative types of band brakes.
12.4 Differentiate between the short, long and pivoted shoe brakes.
12.5 Why are pivoted shoe brakes seldom used?
12.6 What are the desirable properties of friction material used for the lining of brake shoes?
12.7 Explain why the distribution of pressure over the length of the shoe is not uniform in long shoe brakes. What is the type of pressure distribution in this case?
12.8 What is meant by a self-energising brake? How does it tend to be self-locking and how should this be avoided?
12.9 What are the design criteria of brakes?
12.10 What is the cause of keeping the product of p_{av} and velocity within a prescribed limit? What are the limits for different conditions?
12.11 Draw an internal expanding shoe brake and explain its working.
12.12 Derive the equation of moments due to normal reaction, frictional force and external force for an external contracting shoe brake. Write the expressions for torque capacity and hinge pin reactions and show how the same relations hold good for internal expanding shoe brake also?
12.13 What is backstop? Where is it used?
12.14 Many brakes are oil immersed even though it reduces the coefficient of friction. Why?

12.15 Is the brake shoe always in contact with the brake drum? Discuss this point in connection with brakes used in hoists.

12.16 Explain the phenomena of heating of brakes. How is it taken into account at the time of design?

12.17 Why simple band brake is preferred to the differential type?

PRACTICE PROBLEMS

12.1 The block brake shown in Fig. P-12.1 has a face width of 50 mm. The coefficient of friction is 0.25 and the permissible maximum pressure is 0.55 MPa.. Find the value of *P*. Also calculate the torque capacity of the brake.

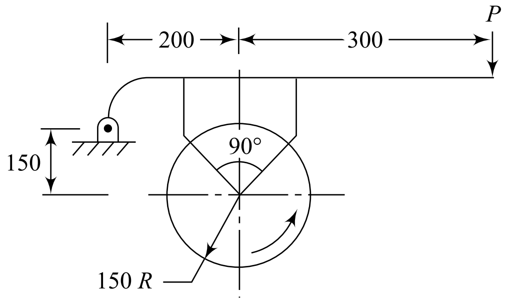


Fig. P-12.1

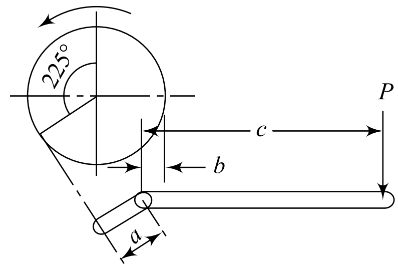


Fig. P-12.2

12.2 Design a differential type band brake as shown in Fig. P-12.2 of dimensions $a = 36.6$ mm, $b = 75$ mm, $c = 300$ mm and $\mu = 0.2$. The speed and diameter of the drum are 200 r.p.m and 250 mm respectively. If the power to be absorbed is 10 kW, calculate the force applied at the end of the lever. Find the dimensions of the band, lever and fulcrum pin.

12.3 The brake shown in Fig. P-12.3 has the coefficient of friction of 0.3. The angle of each shoe is 35°. Find the distance *k* such that the wear of both the shoes is the same.

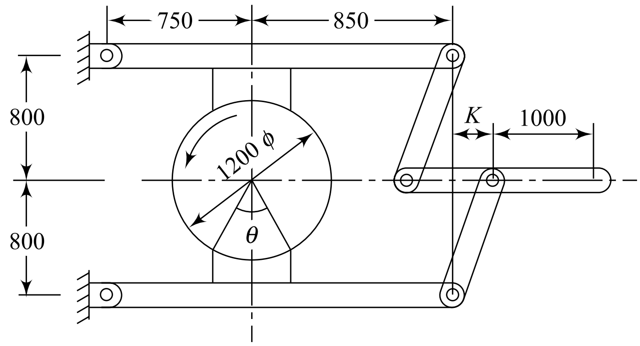


Fig. P-12.3

12.4 A band brake exerts a torque of 1500 N.m. The drum is 50 mm wide and has a diameter of 500 mm. If the maximum pressure between the drum and the lining is 0.7 MPa and the coefficient of friction is 0.3, find the angle of contact between the band and the drum.

12.5 Draw the free body diagram of each link of the brake shown in Fig. P-12.5. Calculate the force P if the power to be absorbed is 15 kW at 200 r.p.m. Check the design for heating. $\mu = 0.3$.

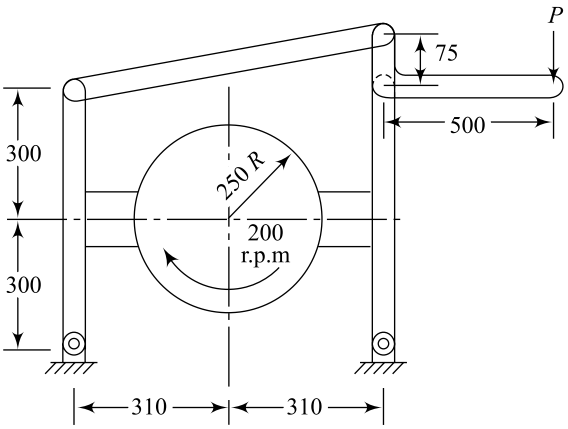


Fig. P-12.5

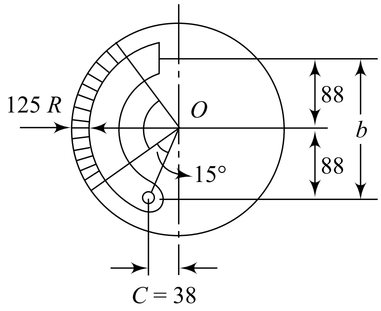


Fig. P-12.6

12.6 In an internal expanding shoe brake shown in Fig. P-12.6, $\varphi_1 = 15°$ and $\varphi_2 = 105°$. The value of maximum shoe pressure is 0.7 MPa, $\mu = 0.25$, width of the lining = 50 mm. Find the torque capacity if the inner radius of the drum is 125 mm.

12.7 Find the width of the shoe in an internal expanding shoe brake with $\varphi_1 = 15°$ and $\varphi_2 = 150°$, $\mu = 0.35$, $p_m = 0.85$ MPa, $R = 150$ mm, $b = 250$ mm, distance of pivot from the centre of drum = 125 mm. The power to be absorbed is 10.38 kW at 120 r.p.m.

12.8 For the brake shown in Fig. P-12.8, $\mu = 0.3$. The permissible maximum pressure is 1 MPa and the force at the end of the lever is 622 N. Find the width of the band and the torque capacity.

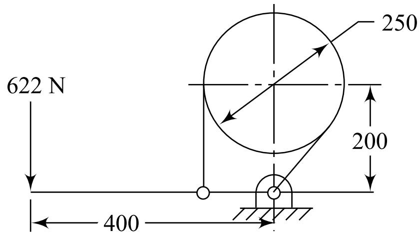


Fig. P-12.8

12.9 Design a brake shown in Fig. P-12.9 of torque capacity of 900 N.m, $\mu = 0.3$, $p_m = 0.525$ MPa, b = width of the shoe = $\dfrac{D}{3}$ where D is the diameter of drum. Find the values of P, D and b.

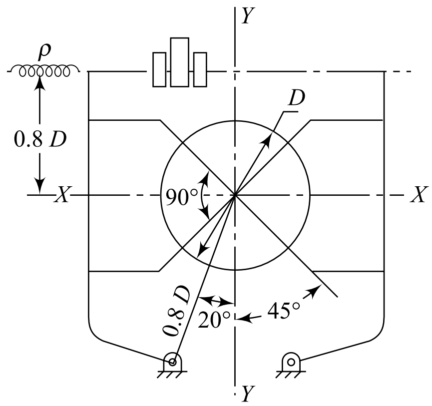


Fig. P-12.9

12.10 A double block brake brings the brake drum rotating at a speed of 300 r.p.m to rest within 5 seconds. The brake drum diameter is 600 mm and a torque capacity of 10000 N.m. The permissible bearing pressure is 0.5 MPa. Determine the actuating force at the end of the lever of length 1 m. The distance between the two fulcrums is 500 mm and the axis of the drum shaft is at a distance of 350 mm from the line joining the two fulcrums. Design the shoe, pivot pin and find the cross section of the lever at the critical section $\mu = 0.3$.

12.11 The brake shown in Fig. P-12.11, $a = 35$ cm, $b = 70$ cm, $c = 10$ cm and diameter of drum = 500 mm. Considering the fulcrum at A, B and C, calculate the respective values of operating when the breaking torque force P is 15 N.m, $\mu = 0.35$.

12.12 In Problem 12.11 consider the brake to be self-energizing and a braking torque of 20 N.m, $a = 350$ mm, $c = 150$ mm and $\mu = 0.35$. Find the dimension b and the breaking capacity in kilowatt if the operating force is 120 N and speed of the drum is 100 r.p.m.

12.13 Figure P-12.13 shows a brake with its lever making an angle of 25° with the horizontal. Diameter of the brake drum is 250 mm and its mass is 200 kg. Speed of rotation = 2250 r.p.m, coefficient of friction between brake shoe and drum = 0.4. Neglecting the mass of the shoe and the lever find the braking torque. Also calculate the number of revolutions and the time required for the drum to come to rest after the brake is applied.

12.14 The brake in Fig. P-12.14 has a braking capacity of 400 kN.m, $\mu = 0.3$ and diameter of brake drum = 4.5 m. Determine the force applied if weight W is used.

12.15 The band brake shown in Fig. P-12.15 has $\mu = 0.35$ and $\theta = 180°$. Determine the location of pivot so that the brake is self-locking. If the distance of pivot from points A and B is the same

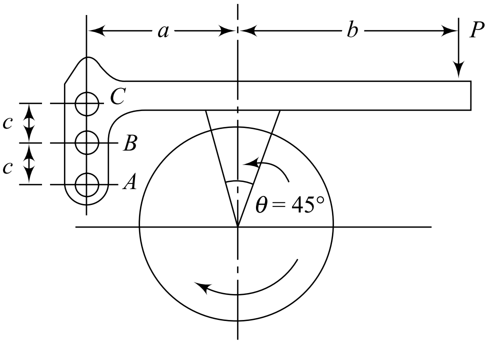


Fig. P-12.11

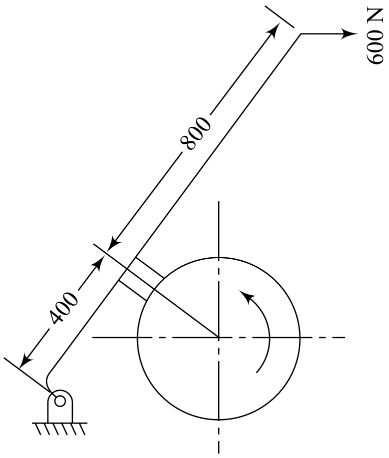


Fig. P-12.13

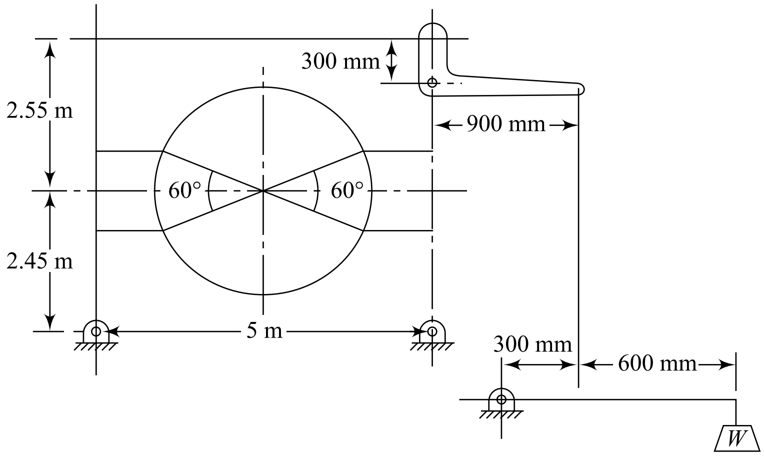


Fig. P-12.14

and a force P is applied at a distance of 300 mm from the pivot, find the value of P in both the directions of rotation. Consider a torque capacity of 125 N.m.

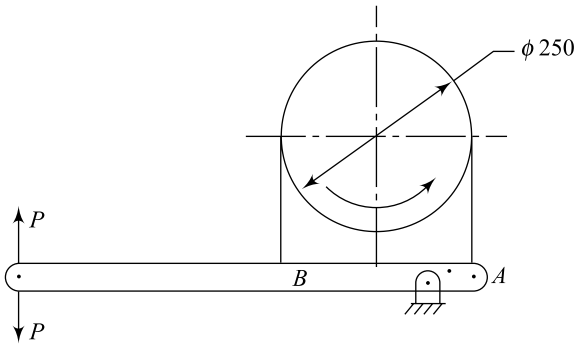


Fig. P-12.15

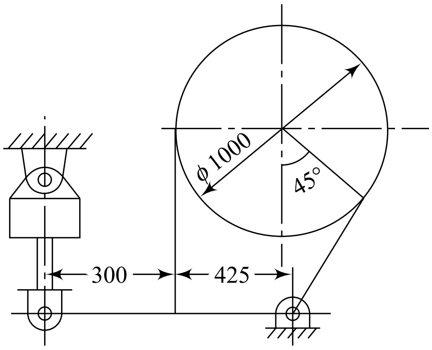


Fig. P-12.16

12.16 In Problem 12.15 find the width of the band and the drum if the permissible pressure is 0.1 MPa. Assuming the clearance between the drum and the bearing, $C = 25$ mm. Find the diameter of the shaft and the dimensions of the key. Check the design of the band if the thickness of the band is 0.005 D where D is the drum diameter. Use 30C8 steel for the band with the factor of safety based on yielding of 4. Calculate the dimensions of the rivet using 10C4 steel. Sketch the arrangement.

12.17 The torque capacity of the brake shown in Fig. P-12.16 is 27 kN.m. The brake band consists of a number of pieces of woven asbestos 225 mm wide. $\mu = 0.3$. Find

(a) the direction of drum rotation that should be specified such that the air cylinder operating force is minimum.

(b) the diameter of air cylinder for the condition in (a) if the air pressure is 1.05 MPa.

(c) the value of coefficient of friction at which the brake becomes self locking.

12.18 In the backstop in Fig. P-12.18 the diameter of the drum is 300 mm. $OA = 25$ mm and $OB = 75$ mm. Determine the direction of rotation for which it becomes self-locking, the minimum coefficient of friction at which it performs satisfactorily and the rated capacity if $\mu = 0.35$ and the band tension should not exceed 10 kN.

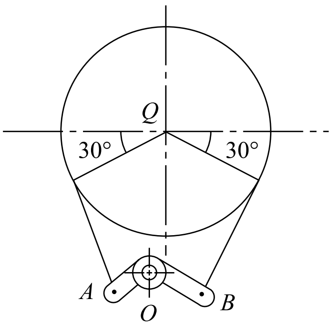


Fig. P-12.18

12.19 A spring set hydraulically released brake shown in Fig. P-12.19 has a torque capacity of 2400 N.m at 400 r.p.m of the drum speed in either direction. $\mu = 0.4$. The width of the shoe is $D/3$ where D is the diameter of the drum. Determine

(a) the value of D and other dimensions if $p_{max} = 0.6$ MPa.

(b) the spring force required to set the brake.

(c) the hydraulic pressure to release the brake if the bore of the hydraulic cylinder is 50 mm.

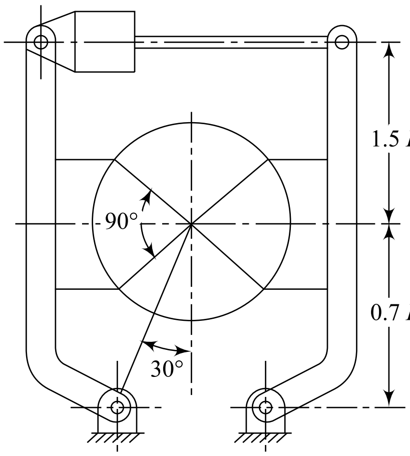


Fig. P-12.19

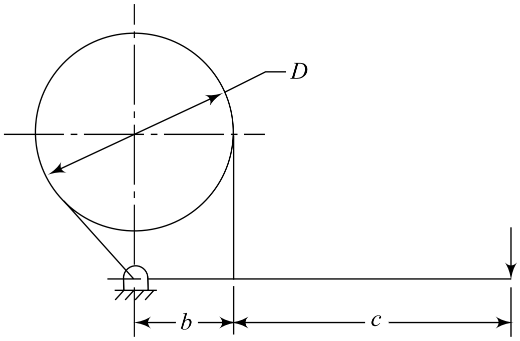


Fig. P-12.20

12.20 For the brake shown in Fig. P-12.20 derive the general equation for the ratio of torque capacity with clockwise rotation of the drum to that with anticlockwise rotation if the operating force in both cases is the same.

12.21 Draw the free body diagram for each link and find the torque capacity of the brake if the operating force is 400 N for the brake shown in Fig. P-12.21.

12.22 In the band and block brake in Fig. P-12.4, the number of blocks is 12, $\theta = 14°$. The effective diameter of the drum is 1.15 m, a = 80 mm, b = 20 mm. The force to be applied at the end of the lever 1 m long is 200 N. $\mu = 0.35$. Find the power capacity of the brake if the speed of rotation is 250 r.p.m.

12.23 In the pivoted type of shoe brake b = 100 mm, $\theta = 90°$. Drum is woven asbestos and for drum C.I for which p_m = 0.35 MPa and μ = 0.35. Find the braking torque and force P. What will happen if the drum rotates anticlockwise?

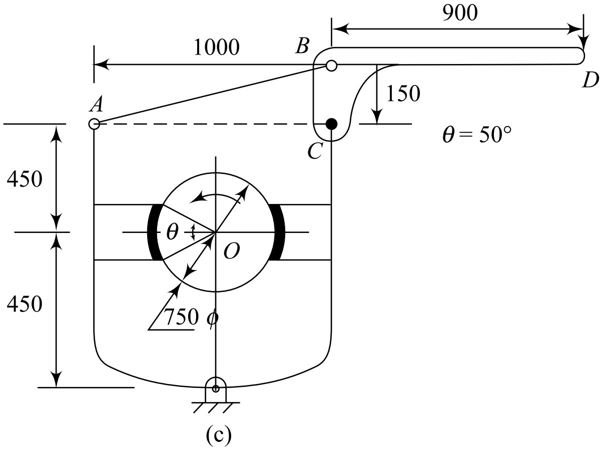


Fig. P-12.21

12.24 A double block brake is shown in Fig. P-12.24. It is subjected to a braking force of 1.5 kN. μ = 0.3. Find the torque capacity.

12.25 For the pivoted type double shoe brake shown in Fig. P-12.25. Calculate maximum pressure intensity for each shoe and torque capacity. Also find the power absorbed. Speed of the drum is 600 r.p.m, check whether the brake is suitable for intermittent operation.

12.26 Internal expanding shoe brake in Fig. E-12.7 has drum diameter 250 mm, a = 100 mm. If actuating force is 950 N, find the width of the shoe if the maximum pressure intensity should not exceed 0.6 MPa, μ = 0.32. Calculate the torque capacity and hinge pin reactions.

12.27 A single block short shoe brake has drum diameter 400 mm, a = 800 mm, b = 290 mm, c = 100 mm, μ = 0.3, p = 1200 N. Find the torque capacity in both directions of rotation.

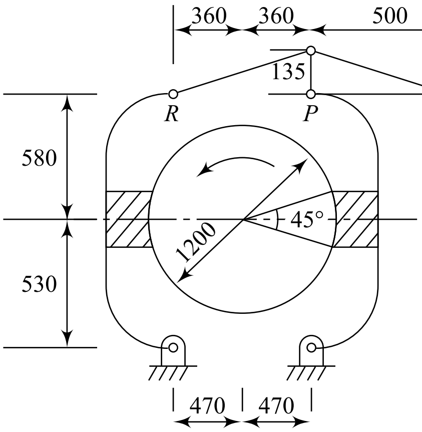


Fig. P-12.24

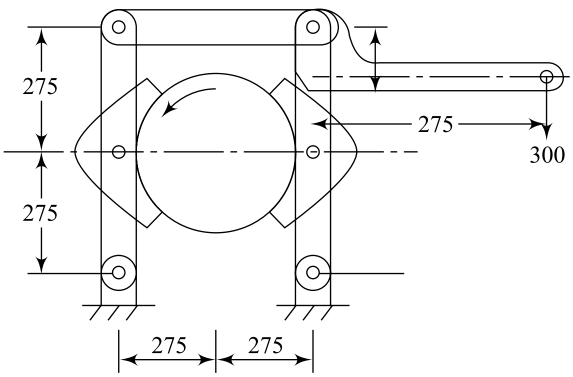


Fig. P-12.25

12.28 In a band and block brake $\theta = 15°$, number of blocks is 12. The thickness of the block 75 mm; diameter of drum 850 mm. The least force applied at 500 mm from the fulcrum on the lever is 610.8 N, $a = 150$ mm, $b = 30$ mm. Find the power absorbed at 240 r.p.m if $\mu = 0.4$.

ANSWERS

Objective Questions

(1) b (2) c (3) b (4) b (5) d (6) b (7) c (8) c (9) b
(10) c (11) c (12) c

Practice Problems

(1) 1811 N, 167.67 N.m (2) 1365 N for anticlockwise rotation (3) 174 mm
(4) 134° (5) 713.26 N, $b = 105$ mm, for angle of contact of 60°
(6) 258.00 N.m (7) 50 mm (8) 50 mm, 532 N.m
(9) $D = 308.3$ modified to 310 mm, $P = 9087.216$ N, $b = 103.33$ mm
(10) $P = 1940.87$ N, $b = 73.8 \rightarrow 75$ mm for angle of contact 30°, lever of 30C8 steel section 63×21 mm, pivot pin dia. 20 mm
(11) 467.5 N, 515 N, 571.4 N (12) $b = 216.66 \rightarrow 220$ mm
(13) 85.86 N.m, 321.59 rev, 8.575 sec (14) 16588.6 N
(15) distance of pivot from A 62.5 mm, 416.67 N for both
(16) $b = 80$ mm; shaft diameter 30 mm, band is safe
(17) anticlockwise, dia. of cylinder = $45.66 \rightarrow 50$ mm, self-locking not possible
(18) clockwise, 0.261, 1156.3 N.m (19) $D = 380$ mm, 4811.4 N, 2.45 MPa
(20) $\dfrac{T_1 b - P(b+c)}{P(b+c) - T_2\, b}$ (21) 1067.99 N.m (22) 457.92 kW
(23) $T = 1082.75$ N.m, $P = 3735.2$ N, $P = 7264.18$ N
(25) max. pr. intensity = 0.166 for RHS, 0.187 for LHS; 593.88 N.m; 37.31 kW, yes
(26) 28 mm, 182.616 N.m, 2.151 kN for RHS, 5.5 kN for LHS, pr. itensity for RHS is 0.36 MPa
(27) anticlockwise 180 N.m, clockwise 221.54 N.m (28) 225 kW.

13

Clutches

CONCEPT REVIEW

13.1 INTRODUCTION

The *clutch* is used primarily to engage and disengage the driver and the driven shaft as per the requirement. Clutches are classified as:

(a) friction, (b) electrical and electromagnetic, and (c) hydraulic.

The design of only friction clutches has been explained in this chapter.

The main types of friction clutches are:

(a) single plate type, (b) multiple plate type, (c) cone clutch, and (d) centrifugal type.

Section 13.2 explains the basic theory of the first three types of clutches.

13.2 PRINCIPLE OF FRICTION CLUTCHES

The principle of transmission of power by contacting friction surfaces is used in a friction clutch. The basic concept is as follows. Let us consider a conical pivot in contact with another conical surface. The maximum and minimum radii of contact are r_2 and r_1 respectively. Let W be the axial load in Newton. Consider a small strip of radius r and width δr assuming uniform intensity of pressure such that the load normal to the surface of width δl is

$$\delta N = p \cdot 2\pi r\, \delta l$$

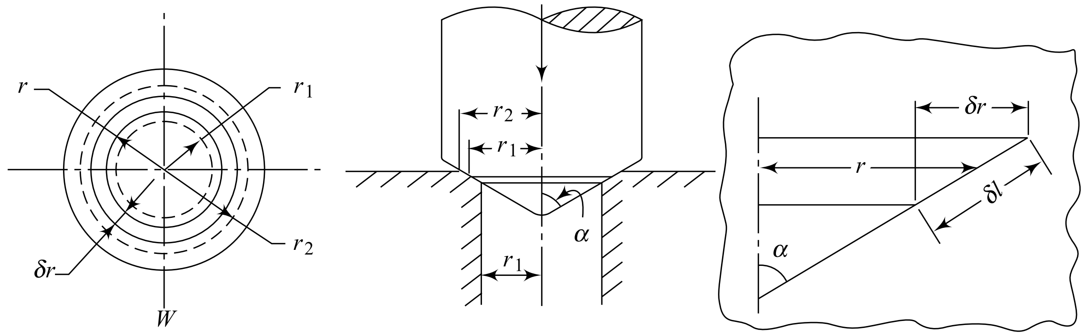
From the enlarged view in Fig. 13.1

$$\delta l = \frac{\delta r}{\sin \alpha}$$ Substituting this, δN can be written as

$$\delta N = p \cdot 2\pi r \frac{\delta r}{\sin \alpha}$$

Axial component of δN is $\delta W = \delta N \sin \alpha = p \cdot 2\pi r \mathrm{d}r$

$$\therefore \quad W = 2 \cdot \pi p \int_{r_1}^{r_2} r\,\mathrm{d}r = \pi p (r_2^2 - r_1^2) \qquad \text{(i)}$$



Enlarged view of thin ring of radius 'r'

Fig. 13.1

Frictional force on an elementary strip is μdN

$\therefore$ Elementary frictional force is

$$\mu dN = \mu \cdot 2\pi p \cdot r^2 \; \frac{\delta r}{\sin \alpha}$$

$\therefore$ Frictional torque

$$(\mu \cdot dN) \cdot r = \mu \cdot 2\pi p \cdot r^2 \; \cdot \; \frac{\delta r}{\sin \alpha}$$

This is the torque transmitted. It is calculated using an assumption either (a) uniform intensity of pressure or (b) uniform rate of wear.

With uniform intensity of pressure

$$T_f = \int_{r_1}^{r_2} \mu \cdot 2\pi \, p \cdot r^2 \times \frac{dr}{\sin \alpha} = \frac{2\pi \, p \cdot \mu}{\sin \alpha} \int_{r_1}^{r_2} r^2 \, dr = \frac{2\mu \, . \, \pi p}{3 \sin \alpha} (r_2^3 - r_1^3)$$

Again from Eq. (i),

$$p = \frac{W}{\pi (r_2^2 - r_1^2)}$$

$$\therefore \qquad T_f = \frac{2 \mu W (r_2^3 - r_1^3)}{3 (r_2^2 - r_1^2) \sin \alpha} \qquad \text{(ii)}$$

With uniform rate of wear

Since the rate of wear depends on intensity of pressure 'p' and velocity and velocity depends on radius, for uniform rate of wear,

$$p \cdot r = C \text{ where } C \text{ is a constant}$$

$$\therefore \qquad T_f = \int_{r_1}^{r_2} \mu \cdot 2\pi C r \cdot \frac{dr}{\sin \alpha} = \frac{2\mu\pi C}{\sin \alpha} \left(\frac{r_2^2 - r_1^2}{2} \right) = \frac{\mu \pi C}{\sin \alpha} (r_2^2 - r_1^2)$$

Now $\qquad p_m \, r_m = C \text{ and } r_m = \dfrac{r_2 + r_1}{2}$

$$\therefore \qquad C = \frac{p_m \, (r_2 + r_1)}{2} \text{ and from (i)}$$

$$p_m = \frac{W}{\pi(r_2^2 - r_1^2)}$$

$$\therefore \qquad T_f = \frac{1}{2}\mu W \frac{(r_2 + r_1)}{\sin \alpha} \qquad \text{(iii)}$$

Equations (ii) and (iii) may be used for flat pivot putting $\alpha = \frac{\pi}{2}$ or $\sin \alpha = 1$

$\therefore$ For flat pivot with uniform intensity of *pr*,

$$T_f = \frac{2}{3}\mu W \frac{(r_2^3 - r_1^3)}{(r_2^2 - r_1^2)} \qquad \text{(iv)}$$

and for flat pivot with uniform rate of wear,

$$T_f = \frac{1}{2}\mu W(r_2 + r_1)$$

or

$$= \mu W r_m \qquad \text{(v)}$$

Equations (i) and (iii) are used for cone clutch and Eqs (i) and (v) are used for plate clutches. For multiple plate clutch, the Eq. (v) is modified as,

$$T_f = n(\mu W r_m) \qquad \text{(vi)}$$

where n is the number of friction surfaces.

13.3 PRACTICAL DESIGN ASPECTS

Number of friction surfaces with both sides of plates effective is calculated as (No. of plates on driver shaft) + (No. of plates on driven shaft) – 1 = 6 in the given Fig. 13.2). Normally the discs are made of steel and bronze and a friction lining of pressed asbestos or ferodo is used. The friction material should have high wear resistance, high coefficient of friction, high heat resistance, good strength and it should not get damaged by moisture and oil.

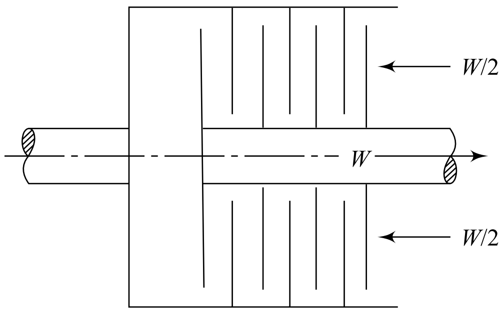


Fig. 13.2

For the new surfaces, the intensity of pressure is uniform but after its use for some time, its wear at outer radius is more as compared to that at inner radius. Thus contact at higher radius is less effective as compared to that at lower radius. Under this condition, the uniform rate of wear assumption is more correct. Due to wear of surfaces force required for engagement is more than calculated W. A factor required to modify it is called engagement factor. The springs are required to be further compressed at the time of disengagement. The factor applied to get the force for disengagement is termed as disengagement factor.

The size of single plate and cone clutch is bigger as compared to multiple plate clutch and they can transmit large power hence they are used in four-wheelers while multiple plate clutch is used in two-wheelers especially in scooters. In many mopeds like Luna, TVS centrifugal clutch is used.

13.4 THEORY OF CENTRIFUGAL CLUTCH

The arrangement of the clutch is as shown in Fig. 13.3. The driver shaft carries the spider, shoes and spring while the driven shaft carries pulley or drum. The shoes are mounted radially and spring force

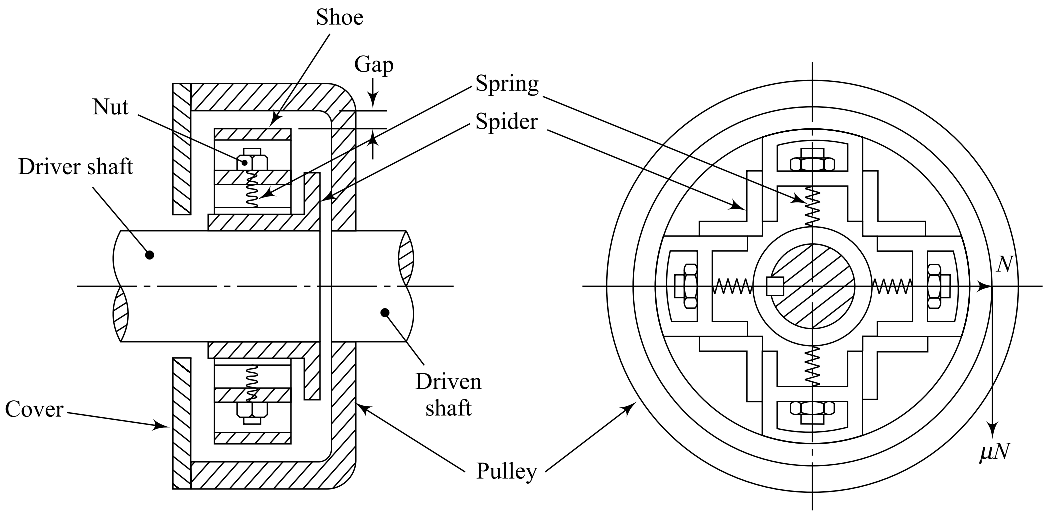


Fig. 13.3

keeps them away from the inner radius of pulley rim. When driver shaft starts rotating the c.f force acts on shoes. When this force overcomes the spring force it comes in contact with the inner radius of rim. This speed is designated as engagement speed. Further increase in speed increases the force N on the inner radius of rim. The frictional force on the rim is μN. If the engagement speed is ω_1 and the maximum speed of rotation of driver shaft is ω_2 then

$N = mr(\omega_2^2 - \omega_1^2)$ where m is the mass of the shoe in kg, r the radius of C.G of shoe. If R is the inner radius of rim, 'n' is the number of shoes and μ the coefficient of friction then frictional torque

$$T_f = n \times \mu \times mr(\omega_2^2 - \omega_1^2) \times R \tag{vii}$$

Usually three, four or six shoes are used. In some clutches the thin plates are used as spring, while in others, springs are not used.

WORKED EXAMPLES

13.1 A clutch plate with maximum diameter 60 mm has maximum lining pressure of 0.35 MPa. The power to be transmitted at 400 r.p.m is 135 kW and $\mu = 0.3$. Find inside diameter and spring force required to engage the clutch. If the springs with spring index 6 and material spring steel with safe shear stress 600 MPa is used find the wire diameter if 6 springs are used.

Solution:

$$T_f = \frac{60 \times 1000 \times 135}{2\pi \times 400} = 3222.88 \text{ N.m}$$

Using Eq. (v) and (i),

$$T_f = \mu Wrm = \mu \pi p_m (r_2^2 - r_1^2)\, r_m$$

but $p_m r_m = p_{max} r_1 = C$

$\therefore$ $T_f = \mu \pi p_{max} r_1 (r_2^2 - r_1^2)$

$\therefore$ $3222.88 \times 10^3 = 0.3 \times \pi \times 0.35\, r_1\, (300^2 - r_1^2)$

Solving $r_1 = 204$ mm or $r_1 = 137$ mm

Using $r_1 = 204$ mm is advantageous as axial force required to apply the clutch is smaller

$$W = 2\pi p_m r_m (r_2 - r_1) = 2\pi P_{max} r_1 (r_2 - r_1)$$
$$= 2\pi \times 0.35 \times 204(300 - 204) = 43067.465 \text{ N}$$

Design of spring for clutch:

Load on each spring = 43067/6 = 7177.9 N

$$K = \frac{4C - 1}{4C - 4} + \frac{0.625}{C} = 1.2525$$

$$\tau = \frac{8\,KPC}{\pi d^3}, \quad \therefore \quad d = \sqrt{\frac{8 \times 1.2525 \times 7177.9 \times 6}{\pi \times 600}}$$
$$= 15.13 \text{ mm}$$

If force required at the time of engagement is 10% larger than

$$d = \sqrt{\frac{8 \times 1.2525 \times 1.1 \times 7177.9 \times 6}{\pi \times 600}} = 15.868 \text{ mm}$$

13.2 A multiple disc clutch consisting of steel and bronze plates is to transmit 10 kW at 1000 r.p.m, the inner and outer diameter of discs being 90 and 160 mm respectively. The engagement factor may be taken as 1.25. The coefficient of friction between the contracting plates is 0.18 and maximum intensity of pressure 0.3 N/mm^2. Find the required number of steel and bronze plates, using uniform wear condition.

Solution:

$$T_f = \frac{10 \times 60 \times 1000}{2\pi \times 1000} = 95.48 \text{ N.m}$$

Again using Eq. (i) and engagement factor 1.25

$$1.25\, T_f = n(\mu W r_m) = n \times \mu 2\pi p r_i (r_0 - r_i) r_m$$

$$\therefore \quad n = \frac{1.25 \times 95.48 \times 10^3}{0.18 \times 2 \times \pi \times 45(80 - 45) \times 62.5 \times 0.3} = 3.57 \text{ say } 4$$

$\therefore$ Use 3 steel and 2 bronze plates so that $n = 3 + 2 - 1 = 4$

13.3 A cone clutch is used to transmit 30 kW at 750 r.p.m. Semicone angle of the clutch is 12.5°, $\mu = 0.2$, mean diameter of friction surface is 6 b where b is the width of friction surface in mm, load factor is 1.75. Find the radii of friction surfaces and face width. Also find the spring wire diameter if disengagement factor is 1.2 and spring index is 6. Assume safe shear stress of 500 MPa. If the spring is deflected by 5 mm during disengagement, find the number of active coils and free length of spring.

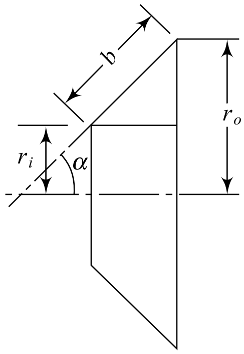


Fig. E-13.3

Solution:

$$\text{Torque} = \frac{30 \times 1000 \times 60}{2\pi \times 750} = 381.97 \text{ N.m}$$

$$\therefore \quad T_f = 1.75 \times 381.97 = \frac{\mu W r_m}{\sin\alpha} = \frac{\mu p\pi (r_0^2 - r_2^2)\, r_m}{\sin\alpha}$$

$$= \frac{\mu\, p\pi (r_o - r_i)(r_o + r_i)\, r_m}{\sin\alpha} = \frac{\mu p\pi\, b \sin\alpha \times 2 r_m \times r_m}{\sin\alpha}$$

As $r_o - r_i = b \sin\alpha$ from Fig. E-13.3 and $r_m = 3\,b$ from given data

$\therefore \quad T_f = \mu p\pi b \times 2 \times 9\, b^2$

$\therefore \quad 1.75 \times 381.97 \times 10^3 = 0.2 \times p \times \pi \times b \times 18\, b^2$

Assuming $p = 0.5$ MPa

$$b = 49.07 \text{ mm say } 50 \text{ mm} = \frac{r_o - r_2}{\sin\alpha}$$

$$\therefore \quad r_m = 150 \text{ mm} = \frac{r_o + r_i}{2}$$

$\therefore \quad r_o - r_i = 50 \sin 12.5° = 10.82$ mm

$r_o + r_i = 300$ mm

$\therefore \quad r_o = 155.41$ mm, $r_i = 144.59$ mm

$\therefore \quad W = \pi p (r_o^2 - r_i^2) = 5098.8$ N

Force at the time of disengagement

$= 1.2 \times 5098.8 = 6118.56$ N

This is maximum force on spring. With $C = 6$, $K = 1.2525$

$$\therefore \quad \tau = \frac{8\,KPC}{\pi d^2}, \quad \therefore\ d = \frac{\sqrt{8 \times 1.2525 \times 6118.56 \times 6}}{\pi \times 500} = 15.3 \text{ mm}$$

$D_m = 91.8$ mm

Deflection $\delta = 5$ mm when force increases from 5098.8 N to 6118.56 N, i.e. the force causing deflection 5 mm is (6118.56 – 5098.8) = 1019.76 N

$$\therefore \text{ No. of active coils,} \quad n = \frac{Gd\delta}{8PC^3} = \frac{0.8 \times 10^5 \times 15.3 \times 5}{8 \times 1019.76 \times 6^3} = 3.473 \text{ say } 4$$

$\therefore$ Total turns 4 + 2 = 6

$\therefore$ free length = 6 × 15.3 + 5 × 0.5 + Total deflection

$$\therefore \quad \text{Total deflection} = \frac{6118.56 \times 5}{1019 - 76} = 30 \text{ mm}$$

$\therefore$ free length = 124.3 mm

13.4 A centrifugal clutch as shown in Fig. 13.3 is used for transmitting 25 kW at 1750 r.p.m. The engagement speed is 75% of the full speed. The inside diameter of the drum is 300 mm and the radial distance of C.G of each shoe from the axis of shaft is 125 mm. The coefficient of friction is 0.25. Determine the necessary mass of each shoe. In engaged position normal intensity of pressure between friction lining and the drum may be taken as 0.1 MPa. Assuming the arc of contact of friction lining on each shoe subtending an angle of 60° at the centre, determine the length and width of friction lining.

Solution:

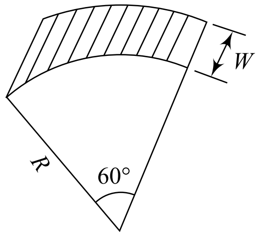


Fig. E-13.4

$$\text{Torque to be transmitted} = \frac{60 \times 1000 \times P}{2\pi \times 750} = 318.3 \text{ N.m}$$

$$318.3 = n \times \mu \times mr(\omega_2^2 - \omega_1^2) \times R$$

$$= 4 \times 0.25 \times m \times 0.125 \left(1 - \frac{9}{16}\right)\left(\frac{\pi \times 50}{30}\right)^2 \times 0.15$$

$\therefore$ $m = 3.2$ kg

Centrifugal force acting between each shoe and drum

$$= mr(\omega_2^2 - \omega_1^2) = 2115.8 \text{ N}$$

$\therefore$ 2115.8= Pr. intensity between friction lining and drum $\times$ Area of contact of shoe

$\therefore$ Area of contact of shoe = $\dfrac{2115.8}{0.1}$ = 21158 mm^2, see Fig. E-13.4

$$= \text{width} \times \text{arc length} = \text{width} \times R\theta = \text{width} \times \frac{\pi}{3} \times 150$$

$\therefore$ Width = 134.69 mm $\simeq$ 135 mm

$$\text{Length} = \frac{\pi}{3} \times 150 = 157 \text{ mm}$$

13.5 Design the helical spring used for bringing the shoe back to original position in the above clutch. Spring material has safe shear stress of 500 MPa. The gap between the friction lining and the inner surface of drum is 5 mm. Initial extension of spring 5 mm.

Solution:

Using spring index of 6

$$K = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

$$\tau = \frac{8KPC}{\pi d^2}, \quad \therefore d = \sqrt{\frac{8 \times 1.2525 \times 2115.8 \times 6}{\pi \times 500}} = 8.99 \text{ mm} \simeq 9 \text{ mm}$$

Again $$n = \frac{0.8 \times 10^5 \times 9 \times 15}{8 \times 2115.8 \times 6^3} = 2.62 \text{ say 3 active turns.}$$

13.6 In the above example a flat thin plate is used as spring. The width of the plate is the same as that of shoes, find the thickness of plate.

Solution:

Unsupported length of the plate may be decided assuming the radius at which the plate is situated equal to 120 mm. The angle subtended by the plate will be 30° (refer Fig. E-13.6).

$\therefore$ Unsupported length of the plate spring = $\dfrac{\pi}{6} \times 120 = 62.83$ mm

$$\delta = \frac{Pl^3}{48\,EI} \quad \text{where } l = 62.83 \text{ mm}$$

$\delta = 5$ mm, $P = 2115.8$ N

$$\therefore \quad I = \frac{2115.8 \times (63)^3}{48 \times 2.1 \times 10^5 \times 5}$$

$$= 10.49 \text{ mm}^4 = \frac{1}{12}\, bt^3$$

$$\therefore \quad t = \sqrt[3]{\frac{10.49 \times 12}{135}} = 0.9766 \text{ mm} \simeq 1 \text{ mm.}$$

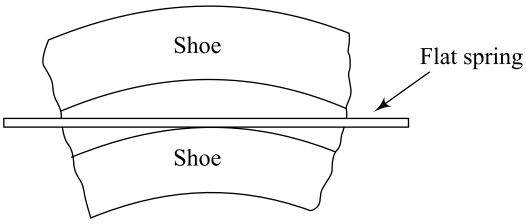


Fig. E-13.6

13.7 In Example 13.1, a mass of 20 kg is connected to driven shaft at radius of gyration of 75 mm and is brought to full speed by engaging the clutch with motor running at 100 r.p.m. Calculate the amount of heat generated.

Solution:

$$T_f = 3222.58 \text{ N.m}$$
$$I = mk^2 = 20 \times (0.075)^2 = 0.1125 \text{ kgm}^2$$

Again
$$\omega = \frac{\pi \times 1000}{30} = 104.72 \text{ r/s}$$

$$\alpha = \frac{T}{I} = \frac{3222.58}{0.1125} = 28645.155 \text{ r/s}^2$$

$$\text{Time to attain full speed} = \frac{\omega}{\alpha} = \frac{104.72}{28645.153} = 3.65 \times 10^{-3} \text{ sec}$$

Angular rotation during slipping

$$\theta = \frac{3.65 \times 10^{-3} \times 104.72}{2} = 0.191 \text{ radians}$$

Heat generated = Friction work = $T\theta = 3222.58 \times 0.191 = 615.5$ Joules.

13.8 An automobile engine develops a maximum braking torque of 140 N.m. Which of the following standard plate clutches should be recommended if both the sides of the plate are effective?

(a) O.D = 220 mm, I.D = 150 mm, $p_{max} = 0.25$ MPa, $\mu = 0.3$
(b) O.D = 250 mm, I.D = 150 mm, $p_{max} = 0.21$ MPa, $\mu = 0.3$
(c) O.D = 275 mm, I.D = 150 mm, $p_{max} = 0.185$ MPa, $\mu = 0.3$

Solution:

∴ In case (a)

$$T = \frac{\pi \times 0.3 \times 0.25 \times 150 \times (220^2 - 150^2)}{8000} = 114.423 \text{ N.m}$$

In case (b)

$$T = \frac{\pi \times 0.3 \times 0.21 \times 150 \times (250^2 - 150^2)}{8000} = 148.44 \text{ N.m}$$

In case (c)

$$T = \frac{\pi \times 0.3 \times 0.185 \times 150 \times (275^2 - 150^2)}{8000} = 173.68 \text{ N.m}$$

∴ Plate clutch of 'b' type is recommended.

13.9 A plate clutch is used to transmit 75 kW at 1000 r.p.m $\mu = 0.25$ and maximum pressure intensity is 0.825 MPa. Ratio O.D/I.D = 4/3. Find the dimensions of friction surface with single pair of mating friction surfaces.

Solution:

$$T = \frac{60 \times 1000 \times 75}{2 \times \pi \times 1000} = 716.19 \text{ N.m}$$

From O.D./I.D. = $\frac{4}{3}$, $d_m = 1.17\, d$ where d = I.D.

$$\therefore \quad T = \frac{\pi \times 0.25 \times 0.825 \times 1.17\, d^3}{8000} [(4/3)^2 - 1]$$

$\therefore$ $d = 213.39$ say 225 mm, $\therefore$ O.D = 300 mm.

13.10 A cone clutch has mean diameter of 300 mm, face width 100 mm. Coefficient of friction 0.2. The cone pitch angle is 10°. The average pr. intensity is 0.07 MPa and the speed is 500 r.p.m. Find the engagement force required and the power that can be transmitted.

Solution:

$$\text{Axial force} = 2\pi C(r_o - r_i)$$
$$= 2\pi \times 0.07 \times 150\, b \sin \alpha = 2\pi \times 0.07 \times 150 \times 100 \sin 10°$$
$$= 1145.62 \text{ N} = F_n \sin \alpha$$

$\therefore$ $F_n = 6597.36$ N

$\therefore$ Engagement force $F_e = 6597.36 (\sin 10° + 0.2 \cos 10°) = 2445.04$ N

$$T_f = \frac{\mu F_a \cdot r_m}{\sin \alpha} = \frac{0.2 \times 1145.62 \times 150}{\sin 10° \times 1000} = 197.92 \text{ N.m}$$

$$\therefore \quad \text{Power} = \frac{2\pi \times 500 \times 197.92}{60000} = 10.36 \text{ kW.}$$

13.11 Prove that the time required for speed ω_2 of driven shaft to become equal to that of driving shaft ω_1 is given by

$$t = \frac{(\omega_1 - \omega_2)\, I_1 I_2}{T(I_1 + I_2)}$$

where I_1 and I_2 are the m.I of driving and driven shafts respectively and T the driving torque.

Solution:

After engagement equivalent m.I I_e can be obtained as

$$\therefore \quad \frac{1}{I_e} = \frac{1}{I_1} + \frac{1}{I_2}, \quad \therefore \quad I_e = \frac{I_1 I_2}{I_1 + I_2}$$

and $$T = I_e \cdot \alpha, \therefore \alpha = \frac{T(I_1 + I_2)}{I_1 I_2}$$

Again $$\omega_1 = \omega_2 + \alpha t$$

$$t = \frac{\omega_1 - \omega_2}{\alpha} = \frac{(\omega_1 - \omega_2) I_1 I_2}{T(I_1 + I_2)}$$

OBJECTIVE QUESTIONS

13.1 The clutch is used for
(a) connecting two shafts permanently

(b) connecting driver and driven shaft intermittently
(c) engagement and disengagement of driver and driven shaft according to requirement
(d) none of the above

13.2 Single plate clutch is used in
(a) four-wheelers (b) two-wheelers
(c) mopeds (d) applications where initial torque is high

13.3 On the motors with low starting torque the type of the clutch to be used is
(a) multiple plate (b) single plate with both sides effective
(c) cone clutch (d) centrifugal clutch.

13.4 Multiple plate clutch is used on two-wheelers as
(a) it can transmit more torque (b) its size is small for that torque capacity
(c) frequent disengagement is required (d) it can dissipate heat effectively

13.5 Engagement speed in centrifugal clutch should be
(a) 50% of full speed (b) 75% of full speed
(c) 90% of full speed (d) 25% of full speed

13.6 The material used for friction lining is
(a) brass (b) C.I (c) leather (d) pressed asbestos

13.7 The coefficient of friction between the engaging surfaces should be
(a) 0.05 (b) 0.25 (c) 0.5 (d) none of the above

13.8 Assumption of uniform rate of wear is used for estimating capacity of clutch as
(a) intensity of pressure is not uniform
(b) rate of wear is uniform
(c) intensity of pressure is uniform for new surfaces but as the surfaces do not wear uniformly after some use, the intensity of pressure doesn't remain uniform
(d) it is safe

13.9 Angle of cone preferable in case of cone clutch is
(a) 30° (b) 60° (c) 10° (d) 90°

13.10 The ratio of face width to the mean diameter in cone clutch varies from
(a) 0.125 to 0.25 (b) 1 to 3 (c) 0.5 to 0.8 (d) 5 to 10

13.11 The ratio of inner to outer radius of plate type clutch varies from
(a) 0.1 to 0.5 (b) 0.5 to 0.8 (c) 0.8 to 0.9 (d) none of the above

13.12 In case of single plate clutch six springs are used for
(a) avoiding spring failure
(b) distributing axial load uniformely
(c) proper engagement and disengagement of friction surface
(d) reducing the size of spring

13.13 Number of springs used is only one in
(a) centrifugal clutch (b) cone clutch
(c) multiple plate clutch (d) single plate clutch

13.14 Use of spring is not necessary in one variety of
(a) cone clutch (b) centrifugal clutch
(c) single plate clutch (d) multiple plate clutch

13.15 Effect of increasing stiffness of springs of centrifugal clutch
(a) increases the engagement speed
(b) decreases the engagement speed
(c) increases the friction torque at maximum speed
(d) none of the above

13.16 Friction affects the engagement force as well as disengagement force in case of

(a) c.f clutch (b) cone clutch

(c) single plate clutch (d) multiple plate clutch

13.17 The number of effective surfaces with 5 steel and 4 brass plates in multiple plate clutch is

(a) 5 (b) 9 (c) 8 (d) 4

13.18 Mass of shoe of centrifugal clutch of maximum speed 1000 r.p.m is 2.25 kg hence with maximum speed of 1500 r.p.m for the same power transmission the mass of the shoe should be

(a) 1.5 kg (b) 1 kg (c) 4.5 kg (d) 1.12 kg

REVIEW QUESTIONS

13.1 What is the difference between a clutch and coupling? How clutch differs from brake?

13.2 Explain the working of single plate clutch by drawing a neat sketch. What is the function of toggle lever?

13.3 Why are more springs used in single plate clutch?

13.4 What are the desirable properties of friction material to be used for clutches?

13.5 Compare the cone clutch and single plate clutch explaining why cone clutch is rarely being used nowadays.

13.6 Derive the relationship of friction torque in clutches using uniform intensity of pressure theory.

13.7 Derive the relationship for friction torque in clutches using uniform rate of wear assumption. Explain why this assumption is usually used in designing clutch.

13.8 Derive the expression for friction torque in centrifugal clutch.

13.9 What are the fields of application of different types of clutches? Explain the reasons for the same.

13.10 Explain the working of a centrifugal clutch by drawing a neat sketch.

13.11 What is the engagement factor? Why does it occur?

13.12 How the speed of engagement affects the capacity of centrifugal clutch? Why too small or too large engagement speed should be avoided?

13.13 What will be the effect of stiffness of spring and the mass of the shoe on the engagement speed and the capacity of centrifugal clutch?

13.14 Why the driven shaft should be a splined shaft in case of cone and plate clutch?

13.15 Why too small (less than 8°) or too large (more than 36°) cone angle should be avoided in case of cone clutch?

13.16 What is the role of heat dissipation in the design of clutch?

13.17 Cone angle in cone clutch should be between 10° to 15°, why?

13.18 What should be the range of the ratio of inner to outer diameter of plate clutch friction surfaces? Why?

13.19 Why the springs are fitted with initial tension in centrifugal clutch? How is the initial tension adjusted?

13.20 The centrifugal clutches are used for the engines which cannot be started under load. Explain.

13.21 Why are the slots provided on the clutch plate?

13.22 Which assumption is used in designing the clutch out of (a) uniform rate of wear, (b) uniform intensity of pressure? Why?

13.23 Multiple plate clutches are used on two-wheelers while single plate clutches are used on four-wheelers. Why?

PRACTICE PROBLEMS

13.1 A plate clutch has a pair of mating surfaces with 300 mm outer diameter and the inner diameter 0.6 times O.D $\mu = 0.25$. Actuating force is 5 kN. Find the maximum pressure intensity and torque capacity using uniform intensity of pressure.

13.2 Solve Problem 13.1 using uniform rate of wear theory.

13.3 A plate clutch has to transmit a torque of 165 N.m. The ratio of inner to outer radius is 0.7, μ = 0.25. The pressure intensity 0.25 MPa. Find the dimension of friction surfaces using uniform rate of wear equation.

13.4 Design the spring for the clutch in Problem 13.1 using $C = 6$, safe $\tau = 600$ MPa. The disengagement force is 20% greater than engagement force and the deflection during disengagement is 10 mm. Use 6 springs.

13.5 In a cone clutch the stiffness of the spring is 200 N/mm and initial compression of the spring is 50 mm. The cone angle is 12°. The intensity of pressure is limited to 0.3 MPa and the ratio of outer to inner diameter of friction surfaces is 1.5. Find the dimensions of the friction surfaces and the power transmitted by clutch at 1000 r.p.m, $\mu = 0.25$.

13.6 What is the power transmitted in the Problem 13.5 if engagement factor is 1.115. Also design the spring using $C = 6$, $\tau = 500$ MPa and disengagement factor 1.2.

13.7 In a plate clutch the ratio of face width to the mean radius is 1/5. The clutch is used for transmitting 100 kW at 600 r.p.m. Find the dimension of friction surface if both sides are effective. μ = 0.25, $p_m = 0.2$ MPa.

13.8 Use 8 springs and find the dimensions of spring for plate clutch above using data in Problems 13.7 and 13.4.

13.9 A cone clutch has outer diameter of friction surface 330 mm, inner diameter 300 mm, cone height 90 mm and $\mu = 0.25$. The power to be transmitted at 1000 r.p.m is 25 kW. Estimate the actuating force and intensity of pressure by uniform rate of wear assumption.

13.10 A multiple plate clutch is used for transmitting 12 kW. The number of pairs of contacting surfaces is 3.$\mu = 0.3$. The driving shaft rotates at 1500 r.p.m. The ratio of outer radius to inner radius is 2. Find the dimensions of friction surfaces if the intensity of pressure is limited to 0.06 MPa.

13.11 A multiple plate clutch is used for transmitting 25 kW at 1575 r.p.m. $\mu = 0.3$. The intensity of pressure is limited to 0.07 MPa, the outside and inside radii of contacting surfaces being 130 mm and 65 mm. Find the number of discs on the driving and driven shaft. Design the spring with spring index 8, permissible shear stress 550 MPa and disengagement force 20% greater than engagement force. Initial compression is 10 mm.

13.12 A centrifugal clutch is required to transmit 22.5 kW at 800 r.p.m. The engagement begins at 80% of the full speed. The inside diameter of the drum is 320 mm and C.G of each shoe is situated at 125 mm from the axis of the shaft. Assume $\mu = 0.25$ and find the mass of each shoe if the number of shoes is 4. If the arc of contact of friction surfaces of each shoe subtends an angle of 45° with the centre, find the dimension of friction plates if the maximum pr. intensity does not exceed 0.2 MPa.

13.13 Design the spring for c.f clutch in Problem 13.12 using material with safe shear stress 600 MPa, spring index 6 and gap between the contacting surfaces 7.5 mm and initial compression of 10 mm. Also find the thickness of the thin plate if used as spring.

13.14 In Problem 13.11, calculate the amount of heat generated if the mass connected to the driven shaft is equivalent to 40 kg at radius 60 mm and is brought to maximum speed from rest by engaging the clutch with the motor running at 1575 r.p.m.

13.15 The contact surfaces in a cone clutch have mean diameter 80 mm. The cone angle is 15°, $\mu = 0.3$. Find the frictional torque when the axial load is 2 kN. The driver shaft rotates at 1000 r.p.m and drives a flywheel of mass 15 kg and radius of gyration 125 mm. Calculate the time required for the flywheel to attain maximum speed and also the energy lost in slipping the clutch.

13.16 Design a single plate clutch with both the sides effective to transmit 80 kW at 750 r.p.m, $\mu = 0.3$. Ratio of inner to outer diameter is 0.7. Permissible maximum intensity of pressure is 0.2 MPa. Design 8 springs with spring index 8, safe $\tau = 600$ MPa, initial compression 5 mm, compression during engagement 5 mm.

13.17 Design a single plate clutch of the Problem 13.16 for power 100 kW, speed 1440 r.p.m ratio of diameters 0.5, pressure intensity 0.35 MPa, 6 springs with spring index 8 and the same data, $\mu = 0.3$.

13.18 Design a cone clutch with semicone angle 12.5°, power 50 kW, speed 1000 r.p.m intensity of pr. 0.35 MPa, $\mu = 0.35$, width of friction surfaces 0.25 times mean diameter. Design spring using spring stiffness 100 N/mm, spring index 6 and safe shear stress 600 MPa.

13.19 What is the minimum value of the inner to outer diameter ratio of friction surfaces of plate clutch at which the capacity of the clutch will not decrease more than 10% during initial wear period?

13.20 A clutch has been designed on the average pr. intensity of 0.175 MPa. What will be the maximum pressure on the friction surfaces after the clutch is worn when the outer diameter is 350 mm and inner diameter 212 mm.

13.21 A spring controlled centrifugal clutch has eight spring loaded shoes. The drum diameter is 400 mm. The mass of each shoe 1.2 kg, the radius of C.G of the shoe 175 mm and the spring force at the instant of contact is 2000 N on each side. $\mu = 0.3$.
(a) What is the speed at which the shoe first contacts the drum?
(b) What power can be transmitted at 1200 r.p.m?

13.22 A cone clutch with cone angle 10° is required to transmit 30 kW at 600 r.p.m. The width of the lining along an element of cone is 50 mm and maximum lining pressure is 0.35 MPa. The coefficient of friction is 0.2. Find suitable radii of friction surface.

13.23 A cone clutch is used for transmitting 160 kW at 600 r.p.m. Mean radius of friction surface is 200 mm and cone angle 8°. Maximum lining pressure is 0.75 MPa and coefficient of friction 0.15. Find the necessary width of the friction surface.

13.24 A cone clutch with mean radius of friction surfaces of 200 mm has cone angle 12°. Maximum lining pressure 0.7 MPa, $\mu = 0.3$. Find the torque the clutch can transmit and engaging force for steady operation. Width of lining is 75 mm. Find the power that can be transmitted at 600 r.p.m.

13.25 A multiple plate clutch is used to transmit 5 kW at 1000 r.p.m. The inner and outer diameter of contact surfaces are 85 and 150 mm respectively, the coefficient of friction for clutch operating in oil being 0.1 and the pressure intensity 0.35 MPa. Find total number of bronze and steel plates.

13.26 A centrifugal clutch has 4 shoes each of mass 1.35 kg, the C.G. of each mass being at 100 mm radius when it is just touching the rim of the pulley. The diameter of pulley is 250 mm. The spring used to connect the shoe to the spider has stiffness of 240 N/mm and total deflection of the spring is 5 mm, $\mu = 0.25$. Calculate the speed at which the shoes first touch the drum and the power that can be transmitted at 1200 r.p.m.

13.27 A plate clutch has four pairs of contact surfaces each with 240 mm external and 120 mm internal diameter. Using $\mu = 0.3$, find the total spring force pressing the plates together to transmit 25 kW at 1600 r.p.m. If there are six springs of stiffness 13 N/mm each and if each of the contact surfaces are worn out by 1.25 mm, what maximum power can be transmitted at the same speed?

13.28 A single plate clutch with 375 mm and 225 mm external and internal diameter respectively connects two shafts with the driver shaft rotating at 175 r.p.m. The rotating parts of the driven shaft have a mass of 35 kg and radius of gyration 250 mm. The axial force exerted by springs is 450 N and $\mu = 0.3$. Determine the time required to reach the maximum speed and energy dissipated during this time due to clutch slip.

13.29 A plate clutch with outer diameter of friction surfaces of 600 mm transmits 135 kW at 400 r.p.m with maximum intensity of pressure of 0.35 MPa, $\mu = 0.3$. Find inner diameter and spring force required to engage the clutch.

13.30 In a plate clutch $r_m/b = 5 : 1$. The outer diameter of the friction surfaces is 400 mm. The clutch is used for transmitting 100 kW at 600 r.p.m. Find the dimensions of friction surfaces if both sides are effective. $\mu = 0.25$, $p_m = 0.3$ MPa.

13.31 A centrifugal clutch has four shoes of mass 2 kg each. Engagement speed is 75% of the full speed. The spring stiffness 200 N/mm and initial compression is 10 mm. Radius of the c.g of the shoe is 120 mm and inside radius of the drum 200 mm. $\mu = 0.3$, permissible pr. intensity is 0.15 MPa. Find the power in kW and the dimensions of friction lining.

13.32 A cone clutch has outer and inner radii of contacting surfaces of 165 mm and 150 mm respectively; cone height 60 mm, $\mu = 0.35$. The power to be transmitted at 1000 r.p.m is 25 kW. Estimate the actuating force and intensity of pressure by uniform rate of wear theory.

ANSWERS

Objective Questions

(1) c	(2) a	(3) d	(4) b	(5) b	(6) d	(7) b	(8) c	(9) a
(10) a	(11) b	(12) b	(13) b	(14) b	(15) a	(16) b	(17) c	(18) b

Practice Problems

(1) 0.11 MPa, 153.125 N.m
(2) 0.294 MPa, 150 N.m
(3) 164 mm, 123 mm
(4) 13.83 $\to$ 14 mm, $n_e = 6.4$
(5) $r_i = 92$ mm, $r_o = 138$ mm, 139.1 kW
(6) 120 kW, $d_w = 8.75$ mm, 3 active turns
(7) $r_o = 150$ mm, $r_i = 122$ mm
(8) $d_w = 4.71 \to 5$ mm
(9) 3031.52 N, 0.2144 MPa
(10) $r_o = 107$ mm, $r_i = 54$ mm
(11) $n = 4$ and 3, 1 $d_w = 10$ mm, $n_e = 1.05$
(12) 2.5 kg, $b = 136$ mm
(14) 1010.6 J
(15) 185.45 N.m, 10.132 sec, 2570.114 J
(16) $r_i = 169$ mm, $d_w = 8.5$ mm, $n_e = 0.5 \to 3$ to 4 turn
(17) $r_i = 87.5$ mm $\to$ 90 mm
(18) $r_m = 135$ mm, $d_w = 8.4$ mm
(19) 0.2679
(20) 0.1715 MPa
(21) 932 r.p.m, 79.39 kW
(22) $r_m = 150$ mm
(23) 87 mm $\to$ 90 mm
(24) 19785 N, 239 kW
(25) 2 + 1
(26) 900 r.p.m 14.6 kW
(27) 10.25 kW
(28) 1 sec, 734.74 J
(29) 536 mm, 18.86 kN
(30) $b = 30$ mm
(31) 32 kW
(32) 1466.76 N, 0.2936 MPa

14

Spur Gear

CONCEPT REVIEW

14.1 DEFINITIONS

1. A *pair of spur gears* is equivalent to a pair of cylindrical discs keyed to parallel shafts and having line contact used to transmit torque.
2. *Pitch circle diameter* is the diameter of the circle which by pure rolling would transmit the same motion as the actual gear wheel.
3. *The pitch point* is the point of contact of two pitch circles.
4. *The circular pitch p* is the distance measured along the pitch circle circumference from the point on one tooth to the corresponding point on the next tooth. It is calculated as,

$$p = \frac{\text{pitch circle circumference}}{\text{No. of teeth}}$$

$$= \frac{\pi d_p}{t_p} = \frac{\pi d_g}{t_g}$$

5. *Module* is the pitch circle diameter divided by number of teeth

i.e. $$m = \frac{d_p}{t_p} = \frac{d_g}{t_g} \text{ mm for spur gears}$$

6. *Circular pitch* $p = \dfrac{\pi d_p}{t_p} = \pi m$ mm
7. *The addendum* is the radial distance from the pitch circle to the top of the tooth.

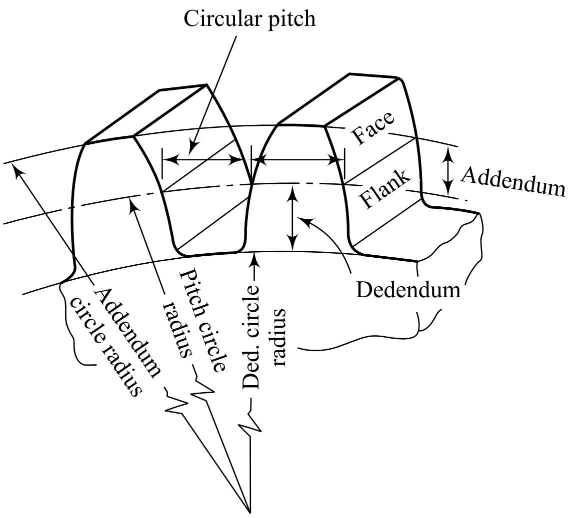


Fig. 14.1

8. *The dedendum* is the radial distance from the pitch circle to the bottom of the tooth space.
9. *A pinion* is the smaller of the two mating gears.
10. *A rack* is a gear wheel with infinitely large number of teeth hence pitch circle circumference is a straight line, or it is a gear with infinite radius.
11. *Pressure angle ϕ or angle of obliquity* is the angle which the common normal to the profiles of the two teeth at the point of contact makes with the common tangent to the two pitch circles at the pitch point.
12. *Condition of correct gearing:* For velocity ratio to remain constant the contact surfaces should have a profile such that the common normal to the two contacting surfaces intersects the line joining the centres at a fixed point.

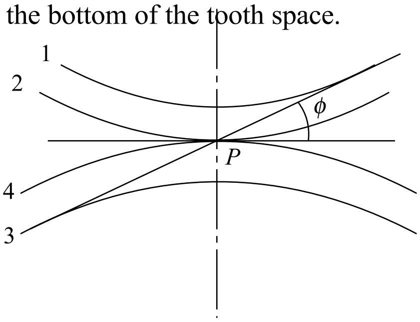


Fig. 14.2

13. *The profiles* satisfying this condition are known as conjugate profiles.
14. *Involute, epi and hypo cycloid* are the standard curves satisfying this condition.
15. It is common practice to use involute profile for gear teeth due to ease of manufacture and possibility of more precision in cutting involute profile.
16. *Interference* takes place in the involute gears which can be avoided if the addendum circles of two mating gears cut the common tangent between the points of tangency.
17. This places the limitation on the minimum number of teeth on the pinion which is given by the expression,

$$t_p = \frac{2a_w t_p / t_g}{\left(\sqrt{1 + A\sin^2\phi}\right) - 1} = \frac{2a_w / G}{\left(\sqrt{1 + A\sin^2\phi}\right) - 1}$$

where addendum = $a_w m$

and $$A = \frac{t_p}{t_g} \times \left(\frac{t_p}{t_g} + 2\right) = \frac{1}{G}\left(\frac{1}{G} + 2\right)$$

For contact between the rack and pinion,

$$t_p = \frac{2a_r}{\sin^2\varphi} \quad \text{where addendum} = a_r m$$

For 20° pressure angle t_p = 17.1 say 18 for a_r = 1; for $14\frac{1}{2}°$ pressure angle t_p = 31.9 say 32. Hence for calculation of pinion, the starting value of number of teeth to be assumed should be 18 to 20 for 20° pressure angle and 32 for $14\frac{1}{2}°$. Force acting on the tooth of spur gear F_n is normal to the tooth profile which is resolved in two components namely,

$$F_r = F_n \sin\phi$$
$$F_t = F_n \cos\phi$$
$$F_r = F_t \tan\phi$$

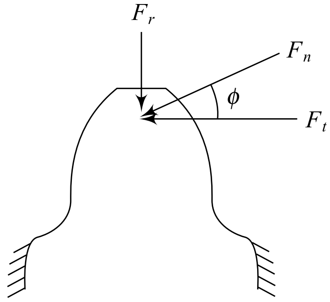


Fig. 14.3

14.2 DESIGN EQUATIONS

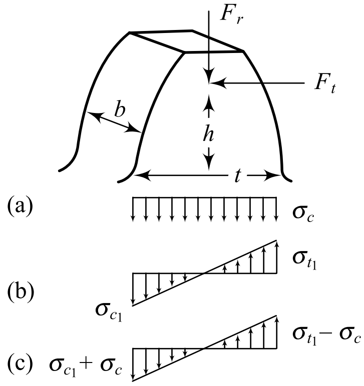


Fig. 14.4

(a) Beam strength or Lewis Equation

The tooth subjected to F_t and F_r may be treated as cantilever and the actual stress distribution is as shown in Fig. 14.4(c) but the design is based only on bending as in 14.4(b). Let b and t be the width and thickness of the tooth at the root

$$\therefore \quad z = \text{section modules} = \frac{1}{6} bt^2$$

$$\therefore \quad \sigma_t = \frac{M}{z} = \frac{F_t h}{\frac{1}{6} bt^2} = \frac{6 F_t h}{6 t^2}$$

$$\therefore \quad F_t = \sigma_t \times b \frac{t^2}{6h}$$ multiplying both numerator and denominator by p

$$= \sigma_t \cdot b \cdot p \left(\frac{t^2}{6hp} \right)$$

The bracketed quantity depends on the form of the tooth and is termed as Lewis form stress factor 'y'

$$\therefore \quad F_t = \sigma_t \cdot b \cdot p \cdot y$$

σ_t is permissible static bending stress which is modified to $K_v\, \sigma_t$ where K_v is the velocity factor used for taking into account the fatigue loading

$$\therefore \quad F_t = K_v\, \sigma_t \cdot b \cdot p \cdot y \tag{14.2.1}$$

$$K_v = \frac{3}{3+V} \text{ for peripheral velocity } V < 10 \text{ m/s}$$

$$= \frac{6}{6+V} \text{ for } 10 \text{ m/s} < V < 20 \text{ m/s}$$

$$= \frac{5.6}{5.6+\sqrt{V}} \text{ for } v > 10 \text{ m/s}$$

$$y = 0.154 - \frac{0.912}{\text{No. of teeth}} \text{ for } 20° \text{ full depth teeth}$$

$$= 0.175 - \frac{0.841}{\text{No. of teeth}} \text{ for } 20° \text{ stub teeth}$$

$$= 0.124 - \frac{0.684}{\text{No. of teeth}} \text{ for } 14\frac{1}{2}° \text{ pressure angle}$$

usually $b = 10$ m to 15 m where m = module

Lewis form factors can also be obtained directly from Table 28. The values given in the table are modified for stress concentration and load sharing ratio.

14.3 WEAR STRENGTH

The two teeth in contact press against each other causing the contact pressure P_o given by Hertz's contact stress equation as

$$P_o = 0.591 \sqrt{\frac{P_1 E_1 E_2}{(E_1 + E_2)} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)}$$

where P_1 = load per unit width N/mm, E_1, E_2 moduli of elasticity of materials of two gears, R_1, R_2 are the radii of curvature at the point of contact. From Fig. 14.5

$$R_1 = \frac{d_1}{2} \sin \phi$$

$$R_2 = \frac{d_2}{2} \sin \phi = \frac{N_2 d_1}{2 N_1} \sin \phi$$

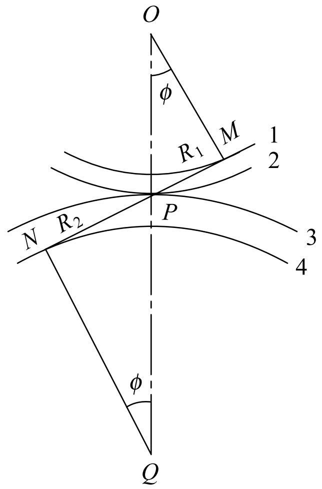


Fig. 14.5

Let P_o be called S_{es}, i.e. surface endurance limit in compression for gear material. Squaring both sides, multiplying both by 'b' and calling $P \times b$ product as F_w, i.e. wear strength of the tooth.

$$F_w = \frac{S_{es}^2}{1.4} bd_1 \text{ , } \sin \varphi \left(\frac{1}{E_1} + \frac{1}{E_2} \right) \left(\frac{2 N_2}{N_1 + N_2} \right)$$

$$= d_p b k_w Q \tag{14.3.1}$$

where $$k_w = \frac{S_{es}^2}{1.4} \sin \varphi \left(\frac{1}{E_p} + \frac{1}{E_g} \right),$$

as $$d_1 = d_p, \text{ and } Q = \frac{2 N_2}{N_1 + N_2}$$

14.4 DYNAMIC TOOTH LOAD

Due to inaccuracy in cutting of teeth, the load on teeth in contact suddenly increases. This incremental load F_i added to the tangential tooth load F_i makes dynamic tooth load F_d given by $F_d = F_t + F_i$

where $$F_i = \frac{21V (bC + F_t)}{21V + \sqrt{bC + F_t}} \tag{14.4.1}$$

This is suggested by Buckingham

C is the dynamic load constant and is given by $C = \dfrac{k_d \cdot e}{(1/E_p + 1/E_g)}$ in N/mm or N/m

where $k_d = 0.110$ for $14\frac{1}{2}°$ involute teeth
$= 0.114$ for 20° full depth teeth
$= 0.119$ for 20° stub teeth
E_p = modulus of elasticity for pinion material
E_g = modulus of elasticity for gear material

There is also another approach for finding F_i suggested by M.F. Spotts. According to that approach the equation to find F_i is

$$F_i = \frac{2e}{t}\sqrt{km_e} \qquad (14.4.2)$$

where e = combined error in mm.
t = time for which tooth is in contact
k = spring constant
m_e = equivalent mass for two gears

Now $t = \dfrac{60}{N_p t_p}$ where N_p is r.p.m of pinion and t_p is number of teeth on pinion.

$$k = \frac{b}{9}\left(\frac{E_p E_g}{E_p + E_g}\right)$$

For both gears of steel $k = 11{,}500$ b N/mm. For steel pinion with C.I gear $k = 8{,}000$ b N/mm.

If the gear is assumed to be equivalent to a ring, equivalent mass will be

$m' = \dfrac{\pi b\rho}{2r_o^2}(r_0^4 - r_i^4)$, with r_o and r_i outer and inner radii respectively and ρ the density of gear material.

For solid cylindrical gear $m' = \dfrac{\pi b\rho r_o^2}{2}$

With m_1' and m_2' equivalent masses of two gears separately, combined equivalent mass m_e is given by $m_e = \dfrac{m_1' m_2'}{m_1' + m_2'}$

By using $m_1' = \pi b\rho_p \cdot r_p^2/2$, $m_2' = \pi b\rho_g \cdot r_g^2/2$

$$m_e = \frac{(\pi b\rho_p r_p^2)(\pi b\rho_g \cdot r_g^2)}{4\left[\dfrac{(\pi b\rho_p \cdot r_p^2)}{2} + \dfrac{(\pi b\rho_g \cdot r_g^2)}{2}\right]} = \frac{G^2}{1+G^2}\left(\frac{\pi b\rho_p r_p^2}{2}\right) \qquad (14.4.3)$$

The final expression holds good for both gears of the same material and $C = r_g/r_p$.

14.5 PRACTICAL DESIGN ASPECTS

The involute profile is normally used for gears as the involute rack has a straight line profile as shown in Fig. 14.6. Involute profile poses the difficulty of interference when number of teeth is reduced below the minimum number of teeth. This difficulty is overcome by (a) using stub teeth of which height is less than the full depth teeth, (b) by using composite profile with cycloidal curve at the root of the tooth and (c) by increasing centre distance.

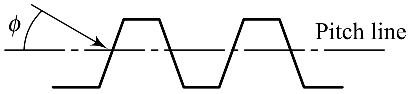


Fig. 14.6

In epicyclic gear train internal gears are used. The teeth of these internal gears are stronger than those of the corresponding spur gears. Operation is smooth and quiet because a greater number of teeth are in contact.

Gears are made from gray and alloy C.I, C.S., forged steel, brass, bronze and impregnated fabric. Heat treatments such as through hardening, case hardening, nitriding, induction or flame hardening may be used for improvement of surface strength.

Lubrication of gears is important and may be achieved by (a) applying lubricant by an oil can, drip oiler or brush, (b) dipping larger gear into bath of oil in case of gears in enclosed casing, (c) using *EP* lubricant if contact pressure is high.

While mounting the gears, care should be taken to see that the shafts are parallel.

Spur gears are highly efficient, the loss of power in friction being only 1 to 2%.

WORKED EXAMPLES

14.1 In a spur gear drive the diameter of pinion is 80 mm and the centre distance 160 mm. The power to be transmitted is 4.5 kW at 800 r.p.m of pinion. Using 20° full depth teeth and material for pinion a steel with permissible static bending stress of 200 MPa and for gear a steel with a permissible static bending stress of 150 MPa, determine the necessary module and width of the teeth using Lewis Equation only.

Solution:

$$V = \frac{\pi d_p N_p}{60} = \frac{\pi \times 80 \times 800}{1000 \times 60} = 3.35 \text{ m/s}$$

$$\therefore \qquad F_t = \frac{P}{V} = \frac{4.5 \times 1000}{3.35} = 1342.87 \text{ N}$$

$$K_v = \frac{3}{3 + 3.35} = 0.472$$

As the material for the gear is weak, let us test the gear for beam strength

$$t_g = \frac{d_g}{m} = \frac{240}{m} \left(\text{as centre distance} = \frac{d_p + d_g}{2}, \quad \therefore d_g = 240 \text{ mm} \right)$$

$\therefore \qquad y_g = 0.154 - \dfrac{0.912 \times m}{240}$

$\therefore$ Lewis Equation for gear

$$F_t = K_v\, \sigma_g\, y_g\, b \cdot p$$

Let $b = 12$ m

$$\therefore \qquad 1342.87 = 0.472 \times 150 \times \left(0.154 - \frac{0.912\text{ m}}{240}\right) \times 12\, \pi m^2$$

By trial and error $m = 2$ $b = 24$ mm

$$t_g = 120,\ t_p = 40$$

$$\therefore \qquad \text{For gear } y_g = 0.154 - \frac{0.912}{120} = 0.1464$$

$$\text{for pinion } y_p = 0.154 - \frac{0.912}{40} = 0.1312$$

$$\sigma_g\, y_g = 21.96 \text{ and } \sigma_p\, y_p = 26.24 \quad (\therefore\ \sigma_p y_p > \sigma_g y_g)$$

Thus the pinion is stronger than gear hence m and b based on the strength of the gear are satisfactory for the pinion.

14.2 A pair of gears is to be designed for compact size. Power to be transmitted 20 kW at 1450 r.p.m of pinion and gear ratio 4 : 1. Tooth profile 20° stub. Material for pinion C.S and for gear C.I. Determine the module and necessary face width by using Lewis Equation.

Solution:

For compact size use minimum number of teeth. Let addendum = module

$$t_p = \frac{2\,a_w / G}{\left(\sqrt{1 + A\sin^2\phi}\right) - 1} = \frac{0.5}{\sqrt{1 + \frac{1}{4}\left(\frac{1}{4} + 2\right)\sin^2 20°} - 1}$$

$$= 15.44 \text{ say } 16$$

$$t_g = 64,\ \sigma_p = 100 \text{ MPa},\ \sigma_g = 70 \text{ MPa}$$

$$y_p = 0.175 - \frac{0.841}{16} = 0.1224,\ \therefore\ y_p \sigma_p = 12.24 \text{ MPa}$$

$$y_g = 0.175 - \frac{0.841}{64} = 0.1618,\ \therefore\ y_g \sigma_g = 11.33$$

$\therefore$ Let us design gear

$$V = \frac{\pi \times 16\, m \times 1450}{60 \times 1000} = 1.214\, m \text{ m/s}$$

$$\therefore \qquad K_v = \frac{3}{3 + 1.214\, m}$$

$$F_t = \frac{P}{V} = \frac{20{,}000}{1.214 \text{ N}} = \frac{16474.464}{m}$$

∴ Lewis Equation for gear with $b = 12$ m

$$F_t = K_v\, \sigma_g y_g\, b \times \text{p}$$

$$\frac{16474.464}{m} = \left(\frac{3}{3 + 1.214\, m}\right) \times 11.33 \times 12\pi\, \text{m}^2$$

by trial and error $m = 5$ mm, $b = 60$ mm.

14.3 A train of gears transmitting 5.6 kW at 1440 r.p.m is shown in Fig. E-14.3(a), $t_a = 20$, $t_b = 100$, $t_e = 25$, $t_d = 150$, $m_a = m_b = 5$ mm, $m_c = m_d = 6$ mm, $\phi = 20°$. Calculate tangential and radial forces between A and B and between C and D; and resultant reactions at bearings E_1 and E_2.

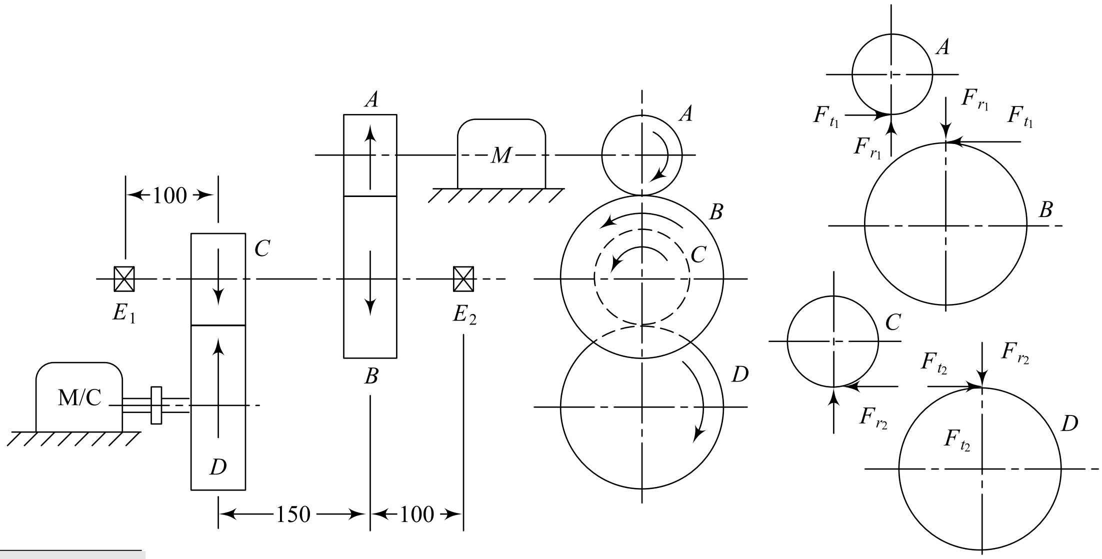


Fig. E-14.3a

Solution:

$$F_{t_1} = \frac{P}{V_1} \text{ where } V_1 = \frac{\pi d_a N_a}{60} \quad d_a = t_a m_a = 100 \text{ mm}$$

$$d_b = t_b m_b = 500 \text{ mm}, \qquad d_c = t_c m_c = 150 \text{ mm}$$

$$d_d = t_d m_d = 900 \text{ mm}$$

$$\therefore \qquad V_1 = \frac{\pi \times 100 \times 1440}{60 \times 1000}$$

$$= 7.54 \text{ m/s}$$

$$\therefore \qquad F_{t_1} = \frac{5.6 \times 1000}{7.54} = 742.7 \text{ N}; \; F_{r_1} = F_t \tan\varphi = 270.32 \text{ N}$$

Again $V_2 = \dfrac{\pi d_c N_c}{60} = \dfrac{\pi \times 150 \times 288}{100 \times 60} = 2.26$ m/s

$\therefore \quad F_{t_2} = \dfrac{5.6 \times 1000}{2.26} = 2477.87$ N,

$\therefore \quad F_{r_2} = F_{t_2} \tan \varphi = 901.87$ N

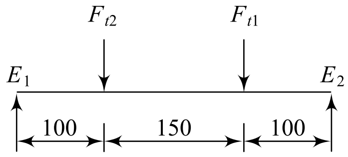


Fig. E-14.3b

Horizontal force diagram

$\therefore$ Taking moments @ E_1

$\therefore \quad F_{t_2} \times 100 + F_{t_1} \times 250 - H_{e_2} \times 350 = 0$

$\therefore \quad H_{e_2} = 1258.1034$ N and $H_{e_1} = 2031.2085$ N

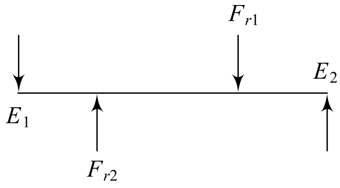


Fig. E-14.3c

Vertical force diagram

Taking moments @ E_2

$$270.32 \times 100 - 901.87 \times 250 + V_{e_1} \times 350 = 0$$

$\therefore \quad V_{e_1} = 584.92$ N and $V_{e_2} = 71.65$ N.

14.4 The gear box for the rotating drum of a concrete mixer is shown in Fig. E-14.4. 4 kW power is supplied to the drum and drum rotates at 150 r.p.m. The two pins are rigidly fixed to the drum and these pins carry two identical spur gears *F*. Spur gears *C* and *E* are integral. The spur gear *A* is fixed ring gear. Pressure angle for all gears is 20° and module 5 mm. The number of teeth on gears *A*, *B*, C and *E* are 75, 20, 80 and 45 respectively. Each planetary gear shares equal part of load. Find the tangential forces between gears *B* and *C* and between gears *E* and *F*.

By using tabular or vector method the speeds of the different gears are calculated which are found to be,

$$N_A = 850 \text{ r.p.m}, N_C = N_E = 400 \text{ r.p.m.}$$
$$N_f = 600 \text{ r.p.m.}$$

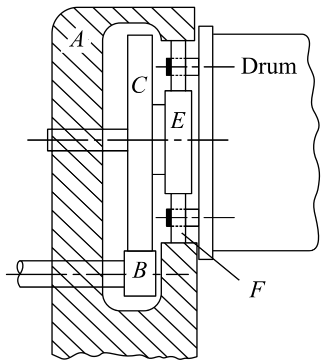


Fig. E-14.4

Condition	*B*	*C*	*E*	*F*	*Drum*	*A*
Drum fixed give one rotation to *B*	1	$-\dfrac{20}{80} = -\dfrac{1}{4}$	$-\dfrac{1}{4}$	$+\dfrac{1}{4} \times \dfrac{45}{15} = \dfrac{3}{4}$	0	$\dfrac{3}{4} \times \dfrac{15}{75} = \dfrac{3}{20}$
× rotation to *B*	x	$-\dfrac{x}{4}$	$-\dfrac{x}{4}$	$\dfrac{3x}{4}$	0	$\dfrac{3x}{20}$
+ *y* rotation to all	$x + y$	$-\dfrac{x}{4} + y$	$-\dfrac{y}{4} + y$	$\dfrac{3x}{4} + y$	y	$\dfrac{3x}{20} + y$

As drum rotates at 150 r.p.m and A is fixed

$$\frac{3x}{20} + y = 0 \text{ (i) and } y = 150 \text{ (ii)}$$

$\therefore$ $x = -1000$

$\therefore$ C and E rotate at 400 r.p.m.

$\therefore$ Tangential velocities can be calculated

$$d_c = 80 \times 5 = 400 \text{ mm}$$
$$d_e = 45 \times 5 = 225 \text{ mm}$$

$\therefore$ $$V_e = \frac{\pi \times 225 \times 400}{60 \times 1000} = 4.712 \text{ m/s}$$

$\therefore$ Tangential force on $E = \dfrac{4 \times 1000}{4.712} = 848.9$ N. This force is equally divided on two planet wheels

$\therefore$ Tangential force between E and $F = 424.45$ N

$$V_c = \frac{\pi \times 400 \times 400}{400 \times 1000} = 8.377 \text{ m/s}$$

$\therefore$ Tangential force between b and $c = \dfrac{4000}{8.377} = 477.5$ N

14.5 Check the pair of gears in Problem 14.2 for wear strength, if the B.H.N for C.S is 300 and for C.I 250. $Ep = 2 \times 10^5$ MPa, $E_g = 1 \times 10^5$ MPa

Solution:

Taking average B.H.N $= \dfrac{300 + 250}{2} = 275$

$$S_{es} = 2.75 \text{ B.H.N} - 70 = 686.25 \text{ MPa}$$

$\therefore$ $$k_w = \frac{S_{es}^2}{1.4} \sin 20^\circ \left[\frac{1}{E_p} + \frac{1}{E_g}\right] = \frac{686.25^2}{1.4} \times \frac{0.342}{10^5}\left(1 + \frac{1}{2}\right)$$

$$= 1.725 \text{ N/mm}^2, \qquad d_p = 16 \times 5 = 80 \text{ mm}$$

$\therefore$ $$F_w = d_p\, b k_w Q, \qquad Q = \frac{2 \times 4}{4 + 1} = 1.6$$

$$= 80 \times 60 \times 1.725 \times 1.6$$
$$= 13248\ N > F_t, \text{ thus design is safe}$$

14.6 A gear drive is used for transmitting 10 kW at 1000 r.p.m. The pinion has 20 teeth and the gear has 50 teeth, module = 5 mm, $b = 60$ mm. Pressure angle $= 14\frac{1}{2}^\circ$, permissible static bending stress = 100 MPa. The surface endurance limit of the material is 600 MPa and $E = 1 \times 10^5$ MPa. Dynamic load factors for 0.01, 0.02, 0.04, 0.06 and 0.08 mm errors are 55, 110, 220, 330 and 440 (N/mm) respectively. Check the gear design for beam and wear strength and state the permissible error.

Solution:

$$y_p = 0.124 - \frac{0.684}{20} = 0.0898, \quad d_p = 20 \times 5 = 100 \text{ mm}$$

$$V = \frac{\pi \times 100 \times 1000}{60 \times 1000} = 5.236 \text{ m/s}$$

$$K_v = \frac{3}{3 + 5.236} = 0.3642$$

$$\therefore \quad F_t = \frac{10 \times 1000}{5.236} = 1909.5548 \text{ N}$$

As the pinion and gear are of the same material let us check pinion for induced bending stress

$$F_t = K_v \sigma y b p, \ \therefore \ \sigma = \frac{1909.5548}{0.3642 \times 0.0898 \times 60 \times \pi \times 5}$$

$$= 61.96 \text{ MPa which is safe}$$

$$F_w = d_p b k_w Q, \ Fk_w = \frac{S_{es}^2}{1.4} \times \sin 14\frac{1^\circ}{2} \times \left(\frac{1}{E_p} + \frac{1}{E_g} \right) = \frac{600^2}{1.4} \times 0.25 \times \frac{2}{10^5}$$

$$= 1.2876 \text{ N/mm}^2$$

$$\therefore \quad F_w = 100 \times 60 \times 1.2876 \left(\frac{2 \times 2.5}{2.5 + 1} \right) = 11036.571 \text{ N}$$

$$F_0 = \sigma y b p = 100 \times 0.0898 \times 60 \times \pi \times 5 = 8463.45 \text{ N}$$

$\therefore$ F_d should not exceed 8463.45 N

$$8463.45 = 1909.8548 + \frac{21 \times 5.236\,(60\,C + 1909.5548)}{21 \times 5.236 + \sqrt{60\,C + 1909.5548}}$$

By trial and error the equation is satisfied for C = 330 N/mm for which the error is 0.06 mm

$\therefore$ 0.06 mm error is permissible

14.7 A 20° full depth steel pinion meshes with a C.I gear with 220 B.H.N. Centre distance is 200 mm and the speed ratio 3 : 1. The speed of the pinion is 600 r.p.m. The module and the face width of the pair are 5 mm and 50 mm respectively. The dynamic tooth factor is 8 N/mm/micron. Wear load factor 0.95 N/mm^2. The tooth pitch error $e = 8 + 1.25\,(m + 0.25\,\sqrt{d}\,)$ microns where m = module in mm and d = p.c.d of gear in mm. Assume permissible static stress for pinion 110 MPa and for gear 55 MPa. Find the maximum safe power transmitted by the spur gear pair.

Solution:

$$\text{Centre distance } C = \frac{d_p + d_2}{2}, \quad \frac{d_g}{d_p} = 3$$

$$\therefore \quad 200 = \frac{d_p + 3 d_p}{2}, \quad \therefore \ d_p = 100 \text{ mm}, \ d_g = 300 \text{ mm}$$

$\therefore \qquad t_p = 20,\ t_g = 60,\ V = \dfrac{\pi \times 100 \times 600}{60 \times 1000} = 3.141$ m/s

$\therefore \qquad y_p = 0.154 - \dfrac{0.912}{20} = 0.1084, \quad \therefore\ \sigma_p y_p = 11.924$

$$y_g = 0.154 - \frac{0.912}{20} = 0.1388, \quad \therefore\ \sigma_g y_g = 7.634$$

$\therefore$ Gear is weaker

$$F_0 \text{ for gear} = \sigma_g y_g bp = 7.634 \times \pi \times 5 \times 50 = 5995.72 \text{ N}$$

$$F_w = d_p bkQ = 100 \times 50 \times 0.95 \times \left(\frac{2 \times 3}{3+1}\right)$$

$$= 7125 \text{ N}$$

As $F_0 < F_w$, $\therefore$ F_0 should be greater or equal to F_d for safe design, $\therefore$ $F_d = 5995.72$ N

$$\text{Total error } e = 16 + 1.25\,(5 + 0.25\sqrt{300}\,) + 1.25\,(5 + 0.25\sqrt{100}\,)$$

$$= 27.66 \text{ microns}$$

$$F_d = \frac{21V\,(bC + Ft)}{21V + \sqrt{bC + Ft}} + F_t C = 8 \times 27.66 \text{ N/mm}$$

$\therefore \qquad 5995.72 = \dfrac{21 \times 3.141(50 \times 27.66 \times 8 + F_t)}{21 \times 3.141 + \sqrt{(50 \times 27.66 \times 8 + F_t)}} + F_t$

$\therefore \qquad F_t = 1350$ N

$\therefore \qquad P = \dfrac{1350 \times 3.141}{1000} = 4.24$ kW.

14.8 Solve Problem 14.7 using Eq. 14.4.2,

Solution:

$$m_e = \left[\frac{G^2}{1+G^2}\right]\frac{\pi b \rho r_p^2}{2g} \quad \text{where } G = \frac{r_g}{r_p} = 3$$

$$= \frac{9}{10} \times \frac{\pi \times 50 \times 0.0000768 \times 50^2}{2 \times 9806.6} = 1.3839 \times 10^{-3}\ \frac{N \sec^2}{\text{mm}}$$

$\therefore \quad F_t = \dfrac{eN_p t_p}{30}\sqrt{8000 \times b \times m_e} = \dfrac{0.02766 \times 600 \times 20}{30}\sqrt{8000 \times 50 \times 1.3839 \times 10^{-3}}$

$$= 260.31 \text{ N}$$

Now $\qquad F_0 = F_d = 260.31 + F_t$

$\therefore \qquad F_t = 5995.72 - 260.31 = 5735.41$ N

$\therefore$ Power to be transmitted = $\dfrac{F_t V}{1000} = \dfrac{5735.41 \times 3.141}{1000}$ = 18.01 kW

It is observed that there is a large difference in the answers obtained by two methods. With F_t = 5735.41 N, the bending stress induced must be checked. Now

$$K_v = \frac{3}{3+V} = \frac{3}{3+4.141} = 0.4885$$

$\therefore$ $F_t = 5735.41 = 0.4885 \times 6 \times 0.1084 \times 50 \times \pi \times 5$

$\therefore$ σ = 137.9 MPa while σ permissible for pinion is only 55 MPa.

$\therefore$ the power calculation should be based on beam strength.

$\therefore$ $F_t = 0.4885 \times 55 \times 0.1084 \times 50 \times \pi \times 5$
$= 2287.422$ N

$\therefore$ Power to be transmitted = $\dfrac{2287.422 \times 3.141}{1000}$ = 7.184 kW.

Thus power to be transmitted should not be more than 7.184 kW even though dynamic load rating permits 18.01 kW.

14.9 A pair of spur gear with 20° full depth involute teeth has pinion with 20 teeth and gear 60 teeth. The speed of the pinion is 1800 r.p.m and it transmits 30 kW. The permissible static bending stress for the material of both gears is 140 MPa. The error in microns is given by $e = 32 + 2.5$ $(m + 0.25\sqrt{d})$. Design the gear and find the required surface hardness.

Solution:

As the material for both gears is the same, the pinion is to be designed.

$$V = \frac{\pi d_p N_p}{60} = \frac{\pi \times 20\,\mathrm{m} \times 1800}{60 \times 1000} = 1.885\, m \text{ m/s}$$

$$K_v = \frac{3}{3+1.885\, m}; \quad F_t = \frac{30 \times 1000}{1.885\, m} = \frac{15915.12}{m} \text{ N}$$

$$y_p = 0.1084$$

Using Lewis Equation

$$\frac{15915.12}{m} = \left(\frac{3}{3+1.885\, m}\right) \times 140 \times 0.1084 \times 12 \times \pi \times m^2$$

by trial and error m = 5 mm, $\therefore$ F_t = 3183 N

$$F_i = \frac{e N_p t_p}{30} \sqrt{k \cdot b \cdot m_v}$$

$$e_g = 32 + 2.5\,(5 + 0.25\sqrt{300}\,) = 55.325 \text{ micron}$$

$$e_p = 32 + 2.5\,(5 + 0.25\sqrt{100}\,) = 50.75 \text{ micron}$$

$\therefore$ Total error = 106.075 $\simeq$ 106 micron

$$m_e = \frac{G^2}{1+G^2}\,\frac{\pi b \rho_p r_p^2}{2g} = \frac{9}{10}\,\frac{\pi \times 60 \times 0.0000768 \times 50^2}{2 \times 9806.6}$$

$$= 1.66 \times 10^{-3}\,\frac{N\,\sec^2}{\mathrm{mm}}$$

$$k = 11500,\ N_p = 1800 \text{ r.p.m}, t_p = 20, \text{Total error} = 0.106 \text{ mm}$$

$$\therefore \quad F_i = \frac{0.106 \times 1800 \times 20}{30}\sqrt{11500 \times 60 \times 1.66 \times 10^{-3}} = 4304.9 \text{ N}$$

$$\therefore \quad F_t + F_i = 3183 + 4304.9 = 7487.9 \text{ N}$$

$$F_0 = 140 \times 0.1084 \times 12 \times \pi \times 25 = 14303.042 \text{ N}$$

$$\therefore \quad F_w = F_t + F_i = 7487.9 \text{ N} = d_p b k Q$$

$$Q = \frac{2 \times 3}{3 + 1} = 1.5$$

$$\therefore \quad k_w = \frac{5429.88}{100 \times 60 \times 1.5} = 0.8274 \text{ N/mm}^2$$

$$k_w = \frac{S_{es}^2}{1.4} \sin\phi \left(\frac{1}{E_p} + \frac{1}{E_g}\right), E_p = E_g = 2 \times 10^5 \ \varphi = 20°$$

$$\therefore \quad 0.8274 = \frac{S_{es}^2}{1.4} \times \frac{0.342 \times 2}{2 \times 105}, \therefore S_{es} = 581.98 \text{ MPa}$$

Again $\quad S_{es} = 2.75 \text{ B.H.N} - 70$

$$\therefore \quad \text{B.H.N} = \frac{581.98 + 70}{2.75} = 237$$

14.10 Two 20° full depth spur gears carry 35 kW at 860 r.p.m, speed ratio is 2 : 1 with centre distance 225 mm. Module = 5 mm. The error on both gears taken together is 0.122 mm. Calculate necessary face width material steel, $k_w = 1.2$.

Solution:

$$\frac{d_p + d_g}{2} = 225 \text{ and } \frac{d_g}{d_p} = 2, \therefore d_p = 150 \text{ mm}$$

$$V = \frac{\pi \times 150 \times 860}{60 \times 1000} = 6.7544 \text{ mm}$$

$$F_t = \frac{35 \times 1000}{6.75} = 5185.185 \text{ N}$$

$$\therefore \quad F_d = \frac{0.122 \times 860 \times 30}{30}\sqrt{11500 b^2 \times 5.535 \times 10^{-5}} + 5185.185$$

$$m_e = \frac{G^2}{1 + G^2} \frac{\pi b \rho r_p^2}{2g} = 5.535 b \times 10^{-5} \frac{\text{N sec}^2}{\text{mm}}$$

$$F_w = d_p b k_w Q = 150 \times 1.2 \times b \times \frac{4}{3} = 240\, b$$

Using $\quad F_w = F_d,\ b = 33.176$ mm say 35 mm.

14.11 A pair of spur gears with 20° involute full depth teeth has a module of 8 mm. The p.c.d of gear is 360 mm. The gear ratio is 2.5 : 1. Calculate number of teeth on each wheel, addendum, total depth, clearance, outside diameter, root diameters, dedendum, base circle diameter. State whether the interference will occur?

Solution:

$$d_g = 360 \text{ mm and } d_p = 360/2.5 = 144 \text{ mm}$$
$$N_g = 360/8 = 45 \text{ and } N_p = 18$$
$$\text{Addendum} = m = 8 \text{ mm}$$
$$\text{Dedendum} = 1.157 \text{ m} = 9{,}256 \text{ mm}$$
$$\therefore \quad \text{Total depth} = 17.256 \text{ mm} \qquad \text{(add. + ded.)}$$
$$\therefore \quad \text{Clearance} = 1.256 \text{ mm} \qquad \text{(ded. –add.)}$$
$$\text{Outside diameter} = \text{p.c.d} + 2 \textit{ addendum}$$
$$\therefore \quad \text{For pinion outside diameter} = 144 + 2 \times 8 = 160 \text{ mm}$$
$$\text{For gear outside diameter} = 360 + 2 \times 8 = 376 \text{ mm}$$
$$\text{Root diameter of pinion} = \text{Outside diameter} - 2 \times \text{Total depth}$$
$$= 160 - 2 \times 17.256 = 125.488 \text{ mm}$$
$$\text{Root diameter of gear} = 376 - 2 \times 17.256 = 341.488 \text{ mm}$$
$$\text{Radius of base circle of pinion} = \frac{p.c.d\,(pinion)}{2} \times \cos 20° = 169.14467 \text{ mm}$$

Interference occurs if the

$$N_g < \frac{2\,a_w}{\left(\sqrt{1 + A\sin^2 20°}\right) - 1} \text{ where } A = \frac{1}{2.5}\left(\frac{1}{2.5} + 2\right) = 0.96$$

$$\text{R.H.S} = 36.59. \text{ As } N_g = 45, \text{ i.e. } N_g > 36.59$$

$\therefore$ Interference does not occur.

14.12 Design a pair of spur pinion and gear made of cast steel and C.I respectively. The diameter of pinion is 140 mm and it transmits 30 kW at 1250 r.p.m. The gear ratio is 3 : 1 and teeth are 20° full depth involute.

Solution:

The problem may be solved by first assuming minimum number of teeth necessary to avoid interference and then checking it at the end from the module obtained. Let us assume $t_p = 16$ and $t_g = 48$.

$$\therefore \quad y_p = 0.154 - \frac{0.912}{16} = 0.097, \quad \therefore\ \sigma_p y_p = 110 \times 0.097 = 10.67$$

$$y_g = 0.154 - \frac{0.912}{48} = 0.135$$

$$\therefore \quad \sigma_g y_g = 55 \times 0.135 = 7.425$$

$\therefore$ gear has to be designed.

$$V = \frac{\pi \times 140 \times 1250}{60 \times 1000} = 9.163 \text{ m/s}, \ \therefore\ K_v = \frac{6}{6 + 9.163} = 0.3957$$

$$F_t = \frac{30 \times 1000}{9.163} = 3274 \text{ N}$$

Let $b = 12$ m, Lewis Equation may be written as

$$3274 = 0.3957 \times 7.425 \times 12\, m \times \pi\, m.$$

$$\therefore \quad m = 5.436 \text{ mm say 6 mm}, \ b = 60 \text{ mm}$$

With this module the number of teeth on pinion may be changed to 24 so that the p.c diameter of pinion is 144 mm and that of gear 432 mm.

14.13 Check the design of Problem 14.12 for wear strength and dynamic tooth load assuming B.H.N 250 for pinion material. Assume an error of 0.04 mm for which $C = 228$ kN/m.

Solution:

$$y_g = 0.141 \text{ for a number of teeth 72}$$
$$F_0 = 55 \times 0.141 \times 60 \times 6\pi = 8777.0 \text{ N}$$

For B.H.N 250, $S_{es} = 2.75 \times 250 - 70 = 617.5$ MPa

$\therefore$
$$k_w = \frac{(617.5)^2}{1.4} \sin 20° \left(\frac{1}{2 \times 10^5} + \frac{1}{1 \times 10^5} \right)$$
$$= 1.397 \text{ N/mm}^2, \quad Q = \frac{2 \times 3}{3+1} = 1.5$$

$\therefore$ $F_w = dpbk_w = 144 \times 60 \times 1.397 \times 1.5 = 18105.12$ N

This is satisfactory as $F_w > F_t$

$$F_d = \frac{21V\,(bC + F_t)}{21V + \sqrt{bC + F_t}} + F_t$$
$$= \frac{21 \times 9.163\,[60 \times 228 + 3274]}{21 \times 9.163 + \sqrt{60 \times 228 + 3274}} = 3274 = 13275.335 \text{ N}$$

F_d with reduced error to 0.02 mm comes

$$F_d = 9896.5 \text{ N but still } F_d > F_0$$

$\therefore$ Thus modification in design by increasing $m = 8$ mm, reducing number of teeth to 18, $b = 80$ mm gives

$$F_0 = \sigma y_g bp \qquad y_g = 0.1371 \text{ for a number of teeth 54}$$
$$= 55 \times 0.1371 \times 80 \times \pi \times 8 = 15161 \text{ N}$$
$$F_w = 144 \times 80 \times 1.397 \times 1.5 = 24140.16 \text{ N}$$
$$F_d = 15482 \text{ N with } e = 0.04 \text{ mm}$$

which can be reduced further by reducing error slightly. Hence the design is satisfactory.

14.14 A pair of spur gears has pinion made of material with 80 MPa safe static bending stress, gear made of material with safe static bending stress of 55 MPa. The module and face width of the teeth are 5 mm and 60 mm respectively. The pinion rotates at 600 r.p.m. The number of teeth on pinions and gear are 20 and 80 respectively. Find the capacity in kW of the gear drive. The error is limited to $e = 16 + 1.25\,(m + 0.25\sqrt{d})$ microns. B.H.N of the pinion material is 250.

Solution:

For finding capacity the condition to be satisfied is $F_d < F_0$ and $F_d < F_w$.

$$e_p = 16 + 1.25\,(5 + 0.25\sqrt{100}) = 25.375 \text{ microns for pinion}$$
$$e_g = 16 + 1.25\,(5 + 0.25\sqrt{400}) = 28.5 \text{ microns for gear}$$

$\therefore$ total error = 53.875 microns = 0.053875 mm

$$C = k_e \left(\frac{E_g E_p}{E_g + E_p} \right)$$
$$= 0.114 \times 0.053875 \left(\frac{2 \times 1 \times 10^{10}}{3 \times 10^5} \right) = 409.45 \text{ N/mm}$$

$$V = \frac{\pi \times 100 \times 600}{60,000} = 3.14 \text{ m/s}$$

$$\therefore \quad F_d = \frac{21 \times 3.14(60 \times 409.45 + F_t)}{21 \times 3.14 + \sqrt{60 \times 409.45 + F_t}} + F_t$$

$$y_p = 0.154 - \frac{0.912}{20} = 0.1084, \ \therefore \ \sigma_p y_p = 8.672$$

$$y_g = 0.1426, \quad \therefore \ \sigma_g y_g = 9.843$$

$$\therefore \quad F_0 = \sigma_g y_g b p_e = 7.843 \times 60 \times \pi \times 5 = 7391.85 \text{ N}$$

From previous problem $k_w = 1.397$ N/m^2

$$\therefore \quad F_w = dpbk_w Q \quad \text{where} \quad Q = \frac{2 \times 4}{4+1} = 1.6$$

$$= 100 \times 60 \times 1.397 \times 1.6 = 13411.2 \text{ N}$$

$\therefore$ If $F_d < F_0$ then both conditions are satisfied.

$\therefore$ Let $F_d = F_0$ from which $F_t < 500$ N, this is too small.

Hence using the same material for gear as that of pinion and increasing module to 6 mm.

$$F_0 = 80 \times 0.1426 \times 60 \times \pi \times 6 = 12902.144 \text{ N}$$

for which $\quad F_t \simeq 5000 \text{ N}, \ \therefore \ P = \dfrac{5000\pi}{1000} = 15.7 \text{ kW}.$

14.15 Find the incremental dynamic load occurring on a pair of spur gears of the Problem 14.14 using equivalent mass. Assume pinion to be solid disc and gear of C.I and spokes with inside diameter 380 mm.

Solution:

$$e_p = 25.375 \text{ microns}, \ e_g = 28.5 \text{ microns}$$

$\therefore \quad$ Combined error = 0.053875 mm from previous problem.

$$t = \frac{60}{20 \times 600} = \frac{1}{200} \text{ sec}, \ E_p = 1 \times 10^{11} \text{ N/m}^2, \ E_g = 2 \times 10^{11} \text{ N/m}^2$$

$$k = \frac{b}{9}\left(\frac{E_p E_g}{E_p + E_g}\right) = 7.4 \times 10^9 \, b \text{ N/m}$$

$$m_g' = \frac{\pi b \rho}{2 r_0^2}(r_0^4 - r_i^4)$$

Note: In this solution unit of mass used is kg instead of N sec^2/m used in Examples 14.8 and 14.9, but answer does not differ.

$b = 60 \text{ mm} = \dfrac{60}{1000}$ m, $\quad \rho = 7200$ kg/m^3, $\quad r_0 = \dfrac{240}{1000}$ m $\quad$ and $\quad r_i = \dfrac{190}{1000}$ m

using $\quad m = 6$ mm

$$\therefore \quad m_g' = \frac{\pi \times 0.06 \times 7200 \times (0.24^4 - 0.19^4)}{2(0.24)^2} = 23.73 \text{ kg}$$

$$\therefore \quad m_p' = \pi \times 0.06 \times 7800 \times \left(\frac{60}{1000}\right)^2 = 5.292 \text{ kg}$$

$$\therefore \qquad m_e = \frac{m_p' m_g'}{m_g' + m_p'} = 4.327 \text{ kg}$$

$$\therefore \qquad F_i = \frac{2 \times 0.053875 \times 200}{1000 \times \left(\sqrt{7.4 \times 10^2 \times \frac{60}{1000} \times 4.327}\right)} = 944.56 \text{ N}.$$

14.16 Design a pair of spur gears for transmitting 63 kW at 1200 r.p.m of pinion; using cast steel with safe static bending stress of 110 MPa. $K_v = \frac{3}{3+V}$. Gear ratio 2.5 : 1. B.H.N for pinion is 300. The teeth are 20° full depth involute. Find the permissible error so that the design is safe for dynamic load.

Solution:

Assume 20 teeth for pinion so that

$$t_g = 50, y_p = 0.154 - \frac{0.912}{20} = 0.13576$$

$$V = \frac{\pi \times 20\, m \times 1200}{60000} = 1.256 \text{ m}$$

$$F_t = \frac{63000}{1.256\, m} = \frac{50159.253}{m}$$

Let $\qquad b = 12 \text{ m}, K_v = \frac{3}{3 + 1.256\, m}$

$$\therefore \qquad \frac{50159.263}{m} = \left(\frac{3}{3 + 1.256\, m}\right) \times 110 \times 0.13576 \times 12\, \pi m^2$$

by trial and error $m = 8$ mm. It is standard module, $\therefore$ $b = 96$ mm, $d_p = 160$ mm, $d_p = 400$ mm

$$\therefore \qquad F_0 = 110 \times 0.13576 \times 96 \times 8\, \pi = 36030.941 \text{ N}$$

$$F_w = 160 \times 96 \times k_w Q$$

$$Q = \frac{2 \times 2.5}{2.5 + 1} = 1.43$$

$$S_{es} = 2.75 \times 300 - 70 = 755 \text{ MPa}$$

$$\therefore \qquad k_w = \frac{755^2}{1.4} \times \sin 20° \left(\frac{2}{2 \times 105}\right) = 1.3925 \text{ N/mm}^2$$

$$\therefore \qquad F_w = 160 \times 96 \times 1.3925 \times 1.43 = 30585.984 \text{ N}$$

As $F_0 > F_w$, F_w should be greater or equal to $F_d \cdot V = 1.256\, m = 10.048$ m/s

Now $\qquad F_t = \frac{50159.253}{8} = 6270$ N

$$F_i = \frac{21 \times 10.048 (96\, C + 6270)}{21 \times 10.048 + \sqrt{96\, C + 6270}} = F_w - F_t = 24316 \text{ N}$$

$$\therefore \qquad C = 460 = \frac{k_d \cdot e}{\left(\frac{1}{E_p} + \frac{1}{E_g}\right)} \qquad k_d = 0.111,\ E_p = E_g = 2 \times 10^5$$

$$\therefore \qquad e = \frac{460 \times 2 \times 10^5}{0.111 \times 2 \times 2 \times 10^5 \times 10^5} = 0.04144 \text{ mm is permissible error.}$$

14.17 A bronze spur gear is used with mild steel pinion. The gear ratio is 2.5 : 1. The pressure angle is 20° stub. The number of teeth on pinion is to be 16, the power to be transmitted 6.3 kW at 1500 r.p.m of pinion. The safe static stresses for bronze and C.S are 80 MPa and 110 MPa. Design the pair.

Solution:

$$y_p = 0.175 - \frac{0.841}{16} = 0.1224, \ \therefore \ \sigma_p y_p = 13.464$$

$$y_g = 0.175 - \frac{0.841}{40} = 0.154, \ \therefore \ \sigma_g y_g = 12.32$$

$\therefore$ Gear is to be designed

$$V = \frac{\pi \times 16 m \times 1500}{60{,}000} = 1.257\, m \text{ m/s},\ K_v = \frac{3}{3+V}$$

$$F_t = \frac{6.3 \times 1000}{1.257\, m} = \frac{5012}{m}$$

$$\frac{5012}{m} = \frac{3}{3 + 1.257\, m} \times 12.32 \times 12\, \pi m^2$$

$\therefore$ $m = 3$ mm by trial and error, $d_p = 48$ mm, $d_g = 120$ mm, $b = 10$ mm.

Check for F_w

Let B.H.N. be 200, $\therefore$ $S_{es} = 2.75 \times 100 - 70 = 480$ MPa

$$\therefore \qquad K_w = \frac{480^2}{1.4} \sin 20° \left(\frac{1}{2 \times 10^5} + \frac{1}{1 \times 10^5}\right) = 1.688$$

$$Q = \frac{2 \times 2.5}{2.5 + 1} = 1.428$$

$$\therefore \qquad F_w = 48 \times 40 \times 1.688 \times 1.428 = 4628.08 \text{ N}$$

$$F_t = 1670 \text{ N} < F_w,\ V = 3.771 \text{ m/s}$$

Let us check for dynamic load assuming total error of 0.06 mm so that

$$C = \frac{0.114 \times 0.06}{\left(\frac{1}{2 \times 10^5} + \frac{1}{1 \times 10^5}\right)} = 436 \text{ N}$$

$$\therefore \qquad F_d = 1670 + \frac{21 \times 3.771(40 \times 436 + 1670)}{21 \times 3.771 + \sqrt{40 \times 436 + 1670}} = 8827.22 \text{ N}$$

$$F_0 = 12.32 \times 40 \times \pi \times 3 = 4644.53 \text{ N}$$

Design is not safe under dynamic load. The modifications suggested are increase module to 5 mm, increase hardness, increase face width to 60 mm.

OBJECTIVE QUESTIONS

14.1 Involute profile is preferred to cycloidal because
(a) the profile is easy to cut
(b) only one curve is required to be cut
(c) the rack has straight line profile and hence can be accurately cut
(d) none of the above

14.2 The condition of correct gearing is stated as
(a) pitch line velocities of teeth should be the same
(b) radius of curvature of the two profiles is the same
(c) common normal to the pitch surface should cut the line joining the centres at a fixed point.
(d) none of the above.

14.3 Spur gears are used for
(a) connecting skew shafts
(b) connecting intersecting shafts
(c) transmitting power from one shaft to another shaft
(d) connecting two parallel shafts to transmit power

14.4 A rack is a gear of
(a) definite pitch
(b) infinite module
(c) infinite diameter
(d) infinite number of teeth

14.5 Module of the gear is
(a) p.c.d/number of teeth
(b) No. of teeth/p.c.d
(c) p.c.d $\times$ number of teeth
(d) 1/p.c.d

14.6 Interference can be avoided in involute gear with 20° pressure angle by
(a) cutting involute profile accurately
(b) using as small number of teeth as possible
(c) using more than 20 teeth
(d) using more than 8 teeth.

14.7 Which is incorrect relationship of gears?
(a) $p \times$ diametral pitch $= \pi$
(b) $m = \dfrac{\text{p.c.d}}{\text{No. of teeth}}$
(c) dedendum = 1.157 m
(d) addendum = 2.157 m

14.8 Stub tooth is
(a) provided on racks only
(b) a tooth of standard profile
(c) longer than standard tooth
(d) shorter than standard tooth

14.9 Preliminary design of gears using Lewis Equation is based on
(a) shear stress
(b) contact stresses
(c) bending stress
(d) wear

14.10 If F_o, F_w, F_d represent beam strength, wear strength and dynamic load the condition for safe design of gear is
(a) $F_o > F_d > F_w$
(b) $F_w > F_d > F_o$
(c) $F_d > F_o > F_w$
(d) $F_w > F_o > F_d$

14.11 Wear strength of gear can be improved by
(a) increasing B.H.N
(b) increasing endurance limit
(c) increasing S_y
(d) increasing compressive strength.

14.12 Velocity factor is used to take care of
- (a) effect of high velocity
- (b) possibility of fatigue failure
- (c) possibility of high wear
- (d) pitting

14.13 Material combination factor is used for finding
- (a) beam strength
- (b) dynamical load
- (c) wear strength
- (d) heat capacity

14.14 Out of the pinion and gear, design should be made of the gear for which
- (a) Lewis factor y is smaller
- (b) bending stress σ_g > is smaller
- (c) $\sigma_b y$ is smaller
- (d) $\sigma_b y$ is bigger

14.15 Dynamic tooth load depends on
- (a) pitch line velocity
- (b) misalignment of shafts
- (c) inaccuracy in tooth profile
- (d) pressure angle

14.16 The expression used for the wear strength of the gear is
- (a) $F_w = d_p b k_w Q$
- (b) $F_w = d_g b k_w Q$
- (c) $F_w = F_o$
- (d) $F_w = F_t + \dfrac{21V(bc + F_t)}{21V + \sqrt{bc + F_t}}$

14.17 The spur gears are used for gear ratios up to
- (a) 6
- (b) 2
- (c) 10
- (d) 20

14.18 For 50 mm diameter gear of involute 20° teeth the interference will occur if module is
- (a) 1.5 mm
- (b) 2.01 mm
- (c) 3.0 mm
- (d) 4.0 mm

14.19 If the dynamic tooth load is not within the limit it is advisable for making design safer to
- (a) reduce the module
- (b) reduce the face width
- (c) reduce the error
- (d) reduce the hardness

14.20 If two pairs of spur gears are used for speed reductions from 1800 r.p.m to 200 r.p.m the speed of the 2nd pinion for compact gear box should be
- (a) 900 r.p.m
- (b) 600 r.p.m
- (c) 450 r.p.m
- (d) 800 r.p.m

14.21 With the point of view of wear strength, 20° pressure angle is
- (a) superior to 14½°
- (b) inferior to 14½°
- (c) as good as 14½°
- (d) none of the above

REVIEW QUESTIONS

14.1 Explain the action of forces on the spur gear tooth when power is to be transmitted from one shaft to the other.

14.2 Define (a) module, (b) circular pitch, (c) face width of gear, (d) addendum, (e) pressure angle.

14.3 Explain the importance of Lewis form factor in designing the spur gear. Derive the equation of beam strength of spur gear.

14.4 What are the causes of failure of gear tooth?

14.5 How number of teeth affects the design of gears?

14.6 Explain the concept of wear strength and further state the method of checking the design of gear for wear strength.

14.7 What are different materials used for gears? What type of heat treatment is recommended?

14.8 Why is it preferred to use involute type teeth for gears? What is stub teeth? Why are they used?

14.9 Why dynamic load is induced in the gear teeth? Explain the procedure of designing for dynamic load using Buckingham Equation.

14.10 Describe method used to calculate the dynamic load on gears using M.F. Spotts' equation of mechanics.

14.11 Write a short note on "Lubrication of gears".

14.12 What are the applications of spur gears? Explain in brief.

14.13 After studying the topic of helical gears, explain why spur gears are preferred in certain applications.

14.14 Define beam strength of the tooth and derive the relationship for the same.

14.15 Write a short note on 'Velocity Factor' explaining its significance in gear tooth design.

14.16 What are the conditions to be satisfied for the safe design of spur gear tooth?

14.17 Explain why a less number of teeth is desirable but not practicable below a particular number. How is that number decided?

14.18 Write short notes on (i) Pitting and (ii) Seizure of gear teeth.

PRACTICE PROBLEMS

14.1 The gear train shown in Fig. P-14.1 is required to transmit 40 kW at 1500 r.p.m of pinion *A*. The speed ratio between *A* and *B* is 5 : 2 and between *C* and *D* is 3 : 1. Find the speeds of gears *B*, *C* and *D* and number of teeth on each wheel if module is 5 mm, Also find the reactions on bearings R_1 and R_2 if pressure angle is 20°.

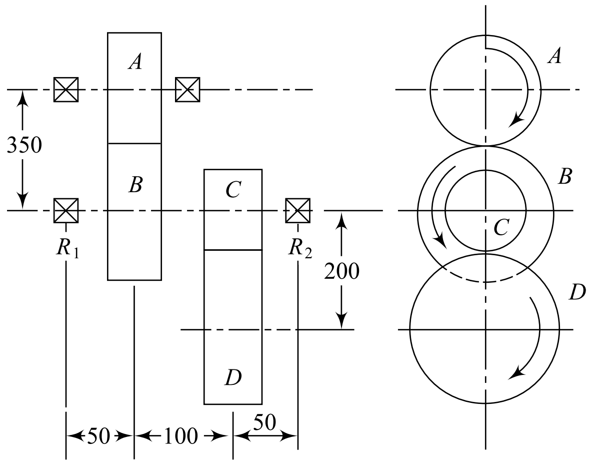


Fig. P-14.1

14.2 A pair of gears is made of C.I with permissible static stress of 55 MPa. The pinion rotates at 500 r.p.m and transmits 4 kW to a gear rotating at 200 r.p.m. Find the module and face width of the pinion using Lewis Equation only for the smallest size of gears. Use $14\frac{1}{2}°$ pressure angle.

14.3 In the Fig. P-14.3 gears 1, 2, 3, 4 have 24, 36, 18 and 36 teeth. Module is 5 mm. The power to be transmitted 60 kW of 1450 r.p.m of gear 1. Find the reactions on the bearings of shaft *B*. Pressure angle = 20°.

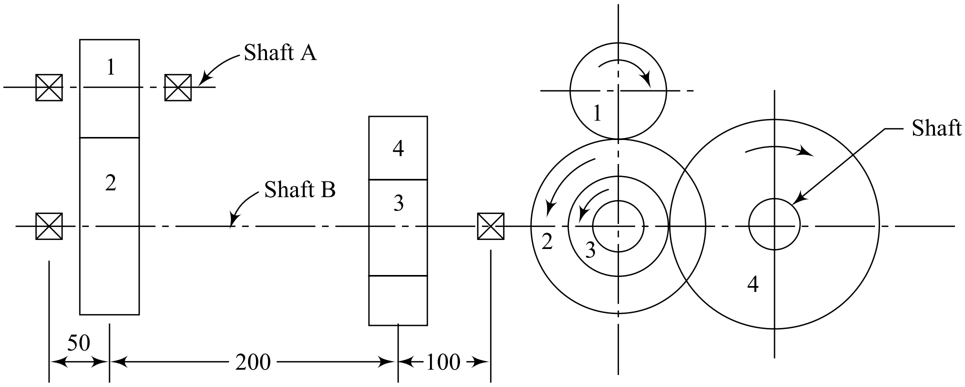


Fig. P-14.3

14.4 In the Problem 14.3 gears 1, 2, 3, 4 have 20, 50, 18 and 54 teeth respectively. The power to be transmitted 40 kW of pinion 1 rotating at 100 r.p.m. Pr. angle is $14\frac{1}{2}°$. Find the reactions at the bearings of shaft *B*.

14.5 Derive the equation for the speed ratio of shafts 4 and 0. The shaft 4 rotates at 1400 r.p.m. (Refer Fig. P-14.5) $t_1 = 25$, $t_2 = 50$, $t_3 = 60$, and $t_4 = 15$

Find the tangential forces between gears 1, 2 and 3, 4 if the 4 is the input shaft and transmits 20 kW. Assume 4 mm module.

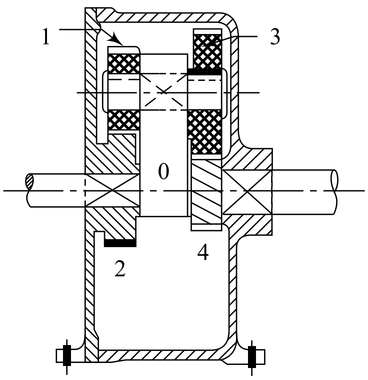


Fig. P-14.5

14.6 A spur gear made of bronze drives a mild steel pinion with velocity ratio of 3.5 : 1. Pressure angle is 14½°. Pinion transmits 4.5 kW at 1800 r.p.m. Minimum number of teeth is 15. Assume safe static stress under bending 84 MPa for bronze and 105 MPa for steel. Find the width of the gears based on beam strength and module.

14.7 A pair of 20° full depth spur gear transmits 36.5 kW at 750 r.p.m of pinion. The velocity ratio is 3.5 : 1. The materials for the pinion and gears are steel with $\sigma_t = 110$ MPa and C.I with $\sigma_t = 60$ MPa respectively. The pinion has 16 teeth. Determine face width, module and p.c.d of wheels on the basis of Lewis Equation and check it for wear, assuming $b = 12$ m and $S_{es} = 600$ MPa.

14.8 Design a gear drive for connecting a shaft of motor running at 600 r.p.m to a machine shaft running at 100 r.p.m. Pinion has 30 teeth of 20° full depth involute profile. Gear is of cast steel with $\sigma_t = 140$ MPa. Power to be transmitted 90 kW. The gear has six arms of box section with internal ribs. Suggest suitable material for pinion.

14.9 A 35 kW is to be transmitted at 450 r.p.m to a shaft with gear ratio 4 : 1 using 20° full depth involute spur gear drive. Pinion is made of heat treated C.S with $\sigma_t = 200$ MPa and gear with high grade C.I with $\sigma_t = 90$ MPa, module is 10 mm. Design the pair and check it for wear and dynamic tooth load $k_w = 4.25$ N/mm^2, $C = 632$ N/mm.

14.10 Two-stage spur gear reducer is used to transmit 7.5 kW. Input shaft rotates 720 r.p.m and output shaft at 80 r.p.m. Pressure angle 20° number of teeth on pinion 18, σ_t for all gears = 100 MPa, $k_w = 0.142$ (B.H.N/100)^2. Design the gear drive by Lewis Equation and specify surface hardness.

14.11 Two meshing spur gears with 20° full depth teeth have module of 6 mm. Pinion has 20 teeth and gear ratio 80. Pinion and gears are solid discs with $b = 50$ mm. Speed of the pinion is 900 r.p.m. Find the dynamic load for an error of $e = 32 + 2.5\,(m + 0.25\sqrt{d})$ where m and d are in mm and error is in microns.

14.12 Two meshing spur gears have 20° full depth teeth. Gears are solid discs; $t_p = 25$, $t_g = 75$, $b = 60$ mm, speed of the pinion 1,760 r.p.m. The power to be transmitted 60 kW, m = 5 mm. Find the required value of B.H.N and check the teeth for bending $e = 16 + 2.5\,(m + 0.25\sqrt{d})$.

14.13 Two gears with $m = 6$ mm, 20° full depth teeth mesh with each other with gear ratio 2. The number of teeth on pinion = 24, $b = 75$ mm. Speed of pinion is 860 r.p.m, $e = 32 + 2.5\,(m + 0.25\sqrt{d})$ microns. B.H.N of pinion and gear 320. Find the kW capacity of the pair.

14.14 A spur gear drive is required to transmit 25 kW at 200 r.p.m of pinion. The velocity ratio is 2 : 1. The centre distance is 450 mm. The safe static stress for the material is 55 MPa. Design the gears. Assume involute teeth of 20° pressure angle. Use beam strength equation.

14.15 Check the gears in Problem 14.14 for wear strength and dynamic load. Hardness of the material may be 300 B.H.N. Let the error in cutting the teeth be 0.04 mm.

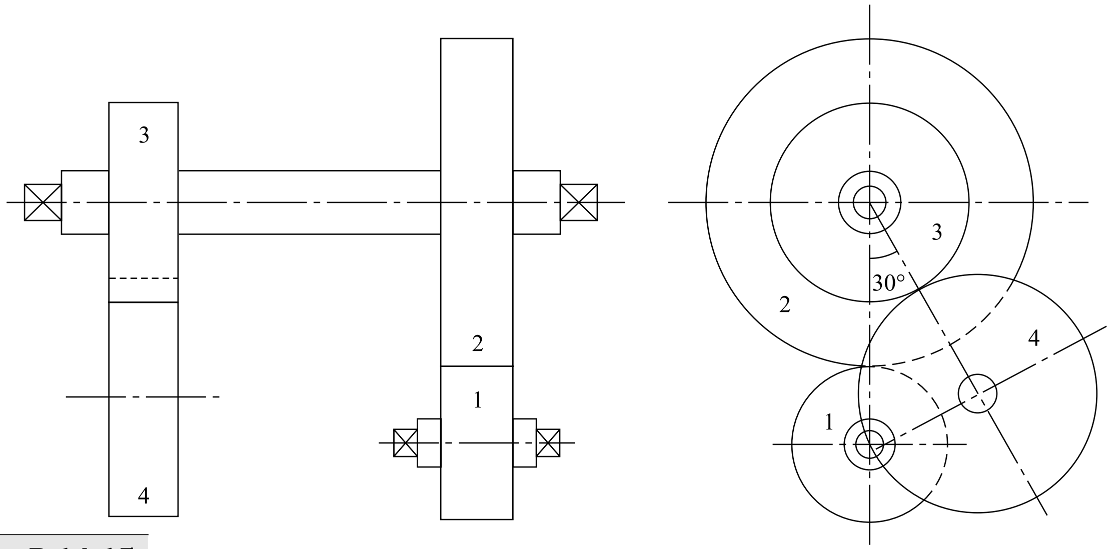


Fig. P-14.15

14.16 The number of teeth on gears 1, 2, 3 and 4 are 24, 36, 18 and 36 respectively. The pressure angle is 20° module = 4 mm. Gear 1 is driver and rotates at 1500 r.p.m transmitting 16 kW. Determine the tooth forces on the gears.

14.17 Two meshing spur gears with 14½° pressure angle have gear ratio 4 : 1. The number of teeth on pinion is 20. The pinion rotates at 1600 r.p.m, the power to be transmitted 40 kW, module = 5 mm, face width 60 mm. Find the required hardness of the material. Check the teeth for dynamic load, assuming error $e = 16 + 2.5\ (m + 0.25\ \sqrt{d}\,)$.

14.18 A pair of spur gear with 20° full depth teeth have module 6 mm. Pinion and gear are solid discs with face width of 45 mm. The pinion rotates at 1000 r.p.m and transmits 36 kW. Find the dynamic tooth load if the total error is 0.04 mm.

14.19 A pair of spur gear has to transmit 100 kW at 750 r.p.m of pinion. Gear ratio is 3 : 1. By using 20° full depth involute teeth find the face width if pinion is made of heat treated cast steel with σ_t = 200 MPa and gear with high grade C.I with σ_t = 90 MPa. Assume $b = 12\ m$. Check the design for wear and dynamic tooth load if B.H.N of the gear is 250 and that of pinion 275. Assume error $e = 16 + 2.5\ (m + 0.25\ \sqrt{d}\,)$.

14.20 By using the equivalent mass criteria find the dynamic load for meshing gear with number of teeth t_p = 30, t_g = 90, b = 75 mm. Material for both the gears is steel both being solid plates, m = 5 mm, speed of pinion 900 r.p.m error e = 0.08 mm.

14.21 Solve the above problem with the pinion solid disc of steel and gear of C.I with rim and spokes with inside diameter of 350 mm.

14.22 Two meshing spur gears have 20° full depth teeth. t_p = 18, t_g = 54, m = 6 mm, b = 60 mm, speed of pinion 1,760 r.p.m. Power to be transferred 40 kW. Find the required value of B.H.N and check the beam strength.

14.23 A pair of spur gears has to transmit 10 kW at 1500 r.p.m. The material for gear is bronze with allowable static bending stress of 85 MPa and pinion of mild steel has allowable static bending stress 105 MPa. Find the minimum number of teeth for interference not to take place if gear ratio is 3.5 : 1 and design the pair for beam strength. Pressure angle = 14½°.

14.24 A cast steel pinion is to drive a C.I gear. The gear ratio is 6 : 1. The power to be transmitted 23 kW at 900 r.p.m of pinion having 16 number of teeth. The hardness of pinion material is 250 B.H.N and safe strengths for C.I and C.I are 110 MPa and 55 MPa respectively. Design pair and check for wear and dynamic load.

14.25 A pair of 4:1 reduction gear is used for transmitting 75 kW at 1200 r.p.m of pinion. The gears have 20° involute full depth teeth and the material for the pinion and gear is the same with safe bending stress of 145 MPa. Find the module, face width d_p and d_g using Lewis equation.

14.26 Two parallel shafts with centre distance 200 mm are to be connected by 20° full depth involute spur gear and pinion for giving speed ratio 3:1. The speed of the pinion is 600 r.p.m. Module and width of the gears are 5 mm and 50 mm respectively. The safe static stresses for pinion and gear materials are 110 and 55 MPa respectively. Find maximum power that can be transmitted safely; by the pair. The tooth pitch error $e = 16 + 1.25\ (m + 0.25\sqrt{d}\,)$ microns.

14.27 Two mating gears have 20° full depth teeth with m = 6 mm, t_p = 20, t_g = 80. Pinion is solid steel and gear of C.I with rim and spokes and inside diameter 380 r.p.m. Find dynamic load using equivalent mass.

14.28 Find the power transmitted by the pair of gears in the above problem. Assume B.H.N for pinion 300 and for gear 250.

14.29 Find the necessary face width of a pair of 20° full depth involute teeth gears. Gear ratio is 2:1 and centre distance is 225 mm. m = 5 mm, B.H.N of the material 350, e_p = 0.0169 mm, e_g = 0.0187 mm. Power to be transmitted at 860 r.p.m of pinion is 35 kW.

14.30 A pair of spur gears with 20° full depth teeth is used to transmit 20 kW at 900 r.p.m of pinion. The gear ratio is 6 : 1. The material for pinion is C.S with permissible static stress 55 MPa. Determine the module and face width of the gear from the standpoint of beam strength; wear strength and dynamic tooth load. Assume minimum number of teeth on pinion. $K_v = 3/(3 + V)$, wear factor = 1.3 N/mm^2, C for 0.07 mm, error = 590 kN.m. Use Buckingham equation for dynamic tooth load.

14.31 A pair of gears with 20° full depth teeth of involute profile have m = 6 mm, b = 75 mm, d_p = 144 mm, d_g = 288 mm. The pinion rotates at 1200 r.p.m and transmits 60 kW. Find the required B.H.N of gear material. $e = 32 + 2.5\,(m + 0.25\sqrt{d})$ microns. Use Spott's approach.

14.32 Solve above problem using Buckingham equation for dynamic tooth load.

14.33 What conclusion can be drawn by comparing the two results in the above problem.

ANSWERS

Objective Questions

(1) c (2) c (3) d (4) c (5) a (6) c (7) d (8) d (9) c (10) d (11) a
(12) b (13) c (14) c (15) c (16) a (17) a (18) d (19) c (20) b (21) a

Practice Problems

(1) Vertical ractions 755 N, 1991 N, Speed of B 600 r.p.m, of D = 200 r.p.m, Horizontal reactions 2075 N, 527041 N, $t_A = 40$, $t_B = 100$, $t_C = 20$, $t_D = 60$.
(2) 3 mm.
(3) V_R 454.62 N ↓, 5936.57 N ↓, H.R. 6565.77 N ← and 3227.6285 N ←
(4) V_B = 14875.378 N ↓ V_B = 4369.4 N ↓ H_B = 4989.41 N, H_A = 807.52 N →
(5) 2728.4 N, 4547.33 N, $N_4/N_0 = 1 + \dfrac{N_2 N_3}{N_1 N_4}$
(6) m = 4 mm, b = 35 mm
(7) m = 8 mm, b = 96 mm, d_p = 128 mm, d_g = 448 mm.
(8) b = 120 mm, assuming mass of gear 50 kg, diameter of gear shaft = 90 mm, d_p = 180 mm, d_g = 1080 mm
(9) b = 58.42, modified to 100 mm.
(10) m = 5 mm, B.H.N = 240. (11) 2373 N (12) B.H.N = 300
(13) 50 kW (14) m = 6 mm, b = 90 mm. (16) 2122 N, 4244 N.
(25) m = 6 mm, d_p = 96 mm, d_g = 384 mm (26) 23.236 kW using M.F. Spott's approach
(27) 1953.158 N (28) 78.87 kW (29) 27.09 → 30 mm
(30) Design is modified to use C.S for both the pinion and gear with m = 6 mm, b = 72 mm.
(31) 250.6 B.H.N (32) 427.35 B.H.N
(33) Buckingham approach is more conservative.

15

Helical Gears

CONCEPT REVIEW

15.1 INTRODUCTION

Helical gears are used for connecting two parallel shafts. The teeth are inclined to the axis of the gear and they form a part of helix with helix angle α. There are two circular pitches: one is the pitch obtained by $p = \dfrac{\pi d}{t}$ while the other is the normal distance between the teeth termed as normal pitch and is designated as p_m from Fig. 15.1

$$p_n = p \cos \alpha$$

Similarly
$$m_n = m \cos \alpha$$

15.2 FORCE ANALYSIS

The normal force to the tooth profile has three components as shown in the figure,

AB — Normal force F_n
GB — Tangential force F_t
EB — Radial force F_r
CB — Axial force F_a
$\angle ABO = \phi_n$ – normal pressure angle
$\angle OBG = \alpha$ – helix angle
$\angle FBG = \phi$ – pressure angle in diametral plane.

$\therefore$
$$\angle F_t = F_n \cos \phi_n \cos \alpha$$

$$F_r = F_t \tan \phi = F_n \cos \phi_n \cos \alpha \tan \phi \qquad (15.2.1)$$

But
$$F_r = F_n \sin \phi_n$$

$\therefore$
$$F_n \sin \phi_n = F_n \cos \phi_n \cos \alpha \tan \phi$$

or
$$\tan \phi_n = \cos \alpha \tan \phi \qquad (15.2.2)$$

$$F_a = F_t \tan \alpha \qquad (15.2.3)$$

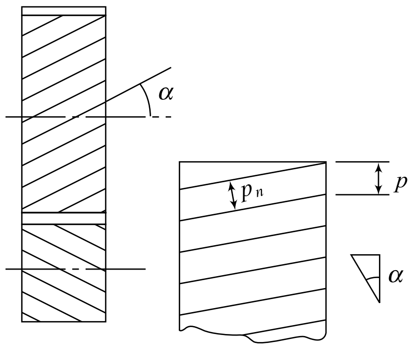


Fig. 15.1

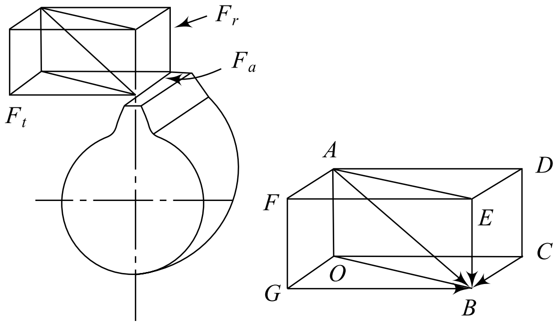


Fig. 15.2

15.3 EQUIVALENT SPUR GEAR: FORMATIVE NUMBER OF TEETH

Design of helical gears can be done by using the approach similar to spur gear by first reducing it to an equivalent spur gear. If helical gear is cut by a plane *AB* perpendicular to the direction of teeth, a section of elliptic shape is obtained. The larger radius of curvature of this ellipse is given by,

$$R = \frac{d}{2\cos^2\alpha} \quad \text{where } d \text{ is the p.c.d of gear.}$$

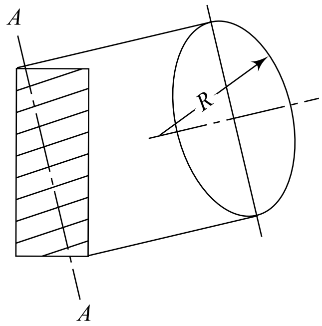


Fig. 15.3

This is taken as the radius of equivalent spur gear

$\therefore$ t' = number of teeth on equivalent spur gear

$$= \frac{2\pi R}{p_n} = \frac{2\pi d}{2\cos^2\alpha\ p\cos\alpha} = \frac{\pi d}{p\cos^3\alpha}$$

but $\frac{\pi d}{p}$ is the actual number of teeth

$\therefore$ equivalent or formative number of teeth $t' = \frac{t}{\cos^3\alpha}$

$$\therefore \quad t'_p = \frac{t_p}{\cos^3\alpha}; \quad t'_p = \frac{t_g}{\cos^3\alpha} \tag{15.3.1}$$

15.4 DESIGN EQUATIONS

(a) Lewis Equation:

In Fig. 15.2, *OB* represents tangential force perpendicular to the direction of tooth

$\therefore$ $OB = \sigma \cdot y'\, b_n\, \pi\, m_n\, k_v$

But $F_t = GB = OB\cos\alpha$ $\quad$ y' = Lewis form factor based on the formative number of teeth

$$b_n = \frac{b}{\cos\alpha}$$

$$\therefore \qquad F_t = OB \cos\alpha = \left[\sigma \cdot y' \frac{b}{\cos\alpha} \cdot m_n k_v\right]\cos\alpha$$

$$= k_v \sigma \cdot y' \, b m_n \qquad (15.4.1)$$

which is similar to that for spur gear with m_n replacing m.

(b) Wear Strength:

As diameter of equivalent spur gear is given by $\frac{b}{\cos^2\alpha}$, hence the formula for wear load becomes

$$F_w = \frac{d_p bkQ}{\cos^2\alpha} \qquad (15.4.2)$$

with the same notations as that for spur gear

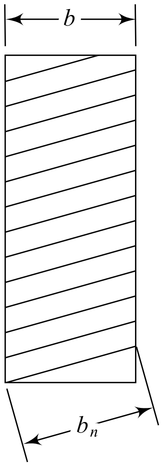


Fig. 15.4

(c) Dynamic Load:

Buckingham Equation is modified as

$$F_d = F_t + \frac{21V\,(bC\cos^2\alpha + F)\cos\alpha}{21V + \sqrt{bC\cos^2\alpha + F_t}} \qquad (15.4.3)$$

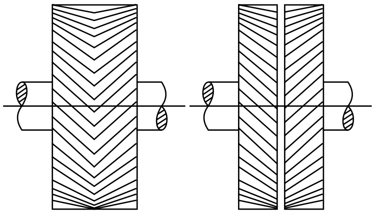
Equation 14.4.2 remains the same. F_d is taken to be acting in the direction of OB in Fig. 15.2.

15.5 PRACTICAL DESIGN ASPECTS

In helical gearing more than one pair of teeth is always in contact and engagement is smooth and gradual. This reduces noise and eliminates impact stresses. Pitch line velocities up to 50 m/s can be reached. Helical gears can transmit heavy loads at high speed. Due to this type of contact, the contact ratio is of less importance and the contact area which is proportional to the face width becomes significant.

From the force analysis it is clear that both radial and thrust loads occur on the shaft bearings of helical gears. Double helical or Herringbone gear eliminates the axial thrust Fig. 15.5.

Helix angle usually varies from 15° to 30° and from 25° to 30° or even 45° for Herringbone gears.



(a) (b)

Fig. 15.5

Helical gears can also be used for connecting two non-parallel shafts. But in that case the teeth have only a point contact and hence are used for relatively small loads.

WORKED EXAMPLES

15.1 Two parallel shafts are connected by two helical gears of 32 and 44 teeth, m_n = 6.5 mm, centre distance is 260 mm. Find the forces at the centre of the tooth if the force normal to the tooth is 8000 N. ϕ = 20°.

Solution:

$$C = \frac{d_p + d_g}{2} = \frac{(32+44)\,m_n}{2\cos\alpha}$$

$\therefore$ $$\cos\alpha = \frac{(32+44)\times 6.5}{2\times 260} = 0.95, \quad \therefore \quad \alpha = 18.194°$$

$\therefore$ $$\phi_n = \tan^{-1}(\cos\alpha\tan\phi) = 19.07°$$

$\therefore$ Components of forces, $F_t = F_n \cos\phi_n \cos\alpha$

$= 8000 \times \cos 19.07° \cos 18.194° = 7183.0$ N

$F_r = F_t \tan\phi = 2614.40$ N

$F_n = F_t \tan\alpha = 2360.40$ N

15.2 A helical gear has 30 teeth and a pitch diameter 264 mm. Module $m_n = 6.5$ mm, $\phi_n = 20°$. The force normal to the tooth surface is 6000 N. Find the power transmitted at 600 r.p.m and also find the formative number of teeth.

Solution:

$$d_p = \frac{t_p\, m_n}{\cos\alpha}, \quad \therefore 264 = \frac{30\times 8}{\cos\alpha}, \quad \therefore \alpha = 24.62°$$

$\therefore$ $$t'_p = \frac{30}{\cos^3\alpha} = 39.93$$

Again $F_t = F_n \cos 20° \cos 24.62° = 6000 \times 0.9396 \times 0.909$

$= 5125.09$ N

$$V = \frac{\pi dN}{60} = \frac{\pi\times 264\times 600}{60\times 1000} = 8.293 \text{ m/s}$$

$\therefore$ $$\text{Power kW} = \frac{5125.09\times 8.293}{1000} = 42.5 \text{ kW.}$$

15.3 Two precision cut forged steel helical gears have 20° full depth involute teeth. The angle of helix is 23°. Permissible static bending stress 100 MPa, module 3 mm, face width 300 mm. The speed of rotation of pinion 600 r.p.m. Gear ratio 3 : 1, surface endurance strength 630 MPa. Find the transmitted and wear load and state whether the design is safe.

Solution:

For minimum space let $t_p = 18$, $\therefore$ $d_p = 54$ mm, $t_g = 54$, $d_g = 162$ mm

As the material for both gears is the same, design the pinion.

$\therefore$ $$t'_p = \frac{18}{\cos^3 23°} = 23.08$$

$\therefore$ $$y'_p = 0.154 - \frac{0912}{23.08} = 0.1148$$

$$V = \frac{\pi\times 54\times 6000}{60\times 1000} = 1.696 \text{ m/s}$$

$\therefore$ $$K_v = \frac{5.6}{5.6+\sqrt{1.696}} = 0.811$$

$\therefore$ Lewis Equation

$$F_t = K_v \cdot \sigma \cdot y'b \cdot \pi \cdot m_n$$
$$= 0.811 \times 100 \times 0.1148 \times 30 \times \pi \times 3 \times \cos 23° = 2423.155 \text{ N}$$

For wear load

$$k_w = \frac{S_{es}^2}{1.4} \times \sin \phi_n \left(\frac{1}{E_p} + \frac{1}{E_g}\right)$$

$$\phi_n = \tan^{-1} (\tan 20° \cos 23°) = 18.51°$$

$$\therefore \quad k_w = \frac{630^2}{1.4} \times \sin 18.51 \left(\frac{1}{2 \times 10^5} + \frac{1}{2 \times 10^5}\right) = 0.9$$

$$Q = \frac{2r}{r+1} = \frac{2 \times 3}{3+1} = 1.5$$

$$\therefore \quad F_w = \frac{d_p b k_w Q}{\cos^2 \alpha} = \frac{54 \times 30 \times 0.9 \times 1.5}{\cos^2 23°} = 2532.2 \text{ N}$$

As $F_w > F_t$ the design is safe

$$\text{The power to be transmitted} = \frac{2423.155 \times 1.696}{1000} = 4.1 \text{ kW}$$

$$\text{Axial thrust} = F_t \tan \alpha$$
$$= 2423.155 \times \tan 23° = 1028.56 \text{ N.}$$

15.4 Two parallel shafts are connected by helical gears with 20° full depth teeth and helix angle of 15°. The material for both gears is forged steel with safe static stress 140 MPa. The power to be transmitted 40 kW at 1400 r.p.m of pinion. Design the gear with Lewis Equation and check for wear strength. Use B.H.N = 250, gear ratio 4.5 : 1.

Solution:

Let us use $t_p = 16, \quad \therefore \; t_p' = \dfrac{16}{\cos^3 15°} = 17.75$

$$y_p' = 0.154 - \frac{0.912}{17.75} = 0.1056$$

$$V = \frac{\pi \times 16 m \times 1400}{60 \times 1000} = 1.173\, m \text{ m/s}, \quad \therefore K_v = \frac{6}{6 + 1.173\, m}$$

$$\therefore \quad F_t = \frac{40 \times 1000}{1.173\, m} = 34100.6/m. \text{ Let } b = 12\, m$$

$\therefore$ Lewis Equation

$$\frac{34100.6}{m} = \left(\frac{6}{6 + 1.173 \text{ m}}\right) \times 140 \times 0.1056 \times 12\, m \times \pi\, m \cos \alpha$$

putting $\alpha = 15°$ by trial and error, $m = 6$ mm, $b = 72$ mm, $F_t = 5683.43$ N

$$F_w = \frac{d_p b k_w Q}{\cos^2 \alpha}, \ \varphi_n = \tan^{-1} 20° \cos 15° = 19.37°$$

$$S_{es} = 2.75 \times 250 - 70 = 617.5 \text{ MPa}$$

$$\therefore \quad k_w = \frac{617.5^2}{1.4} \sin \varphi_n \left(\frac{1}{E_p} + \frac{1}{E_g} \right) = \frac{617.5^2}{1.4} \times 0.3316 \times \frac{2}{2 \times 10^5} = 0.903$$

$$Q = \frac{2 \times 4.5}{4.5 + 1} = 1.636$$

$$\therefore \quad F_w = 96 \times 72 \times 0.903 \times 1.636 \cos^2 \alpha = 10948.152 \text{ N}$$

$\therefore$ $F_w > F_t$ thus the design is safe.

15.5 Two parallel shafts are connected by a pair of steel helical gears. The power transmitted is 15 kW at 4000 r.p.m of the pinion. The safe static strength for the material is 100 MPa. Gear ratio is 4 : 1. Stub teeth with 20° pressure angle in diametral plane have helix angle of 45°. Also calculate the necessary B.H.N with the standpoint of wear. Check the design for dynamic load and suggest modification if necessary. Use 30 teeth on the pinion.

Solution:

$$V = \frac{\pi \times 30 m \times 4000}{60 \times 1000} = 6.28\, m \text{ m/s}$$

$$\therefore \quad F_t = \frac{15 \times 1000}{6.28 m} = \frac{2388.538}{m} \text{ N}$$

$$t_p' = \frac{30}{\cos^3 45°} = 84.98$$

$$y_p' = 0.175 - \frac{0.841}{84.98} = 0.165$$

$$K_v = \frac{5.6}{5.6 + \sqrt{6.28 m}}, \ b = 12\, m \text{ (assumed)}$$

$\therefore$ Lewis Equation

$$\frac{2388.535}{m} = \left(\frac{5.6}{5.6 + \sqrt{6.28 m}} \right) \times 100 \times 0.165 \times 12\, m \times \pi m \cos 45°$$

By trial and error

$m = 2$ mm, $b = 24$ mm, $v = 12.56$ m/s

$\therefore$ $F_t = 1194.267$ N, $F_0 = 100 \times 0.165 \times 12 \times \pi \times (2)^2 \cos 45° = 1759.3815$ N

Let $C = 119$ N/mm

$$\therefore \quad F_d = 1194.267 + \frac{21 \times 12.56[119 \times 24 \times 0.5 + 1194.267]0.707}{21 \times 12.56 + \sqrt{119 \times 24 \times 0.5 + 1194.267}}$$

$$= 2746.270 \text{ N}$$

$F_d > F_0$, $\therefore$ Let $m = 2.5$ mm, $b = 24$ mm

$$\therefore \quad V = 12.56 \times \frac{2.5}{2} = 15.7 \text{ m/s}$$

$$\therefore \quad F_t = 995.414 \text{ N}, \quad F_0 = 2749.00$$

Substituting

$$F_d = 995.414 + \frac{21 \times 15.7[119 \times 24 \times 0.5 + 995.414]0.707}{21 \times 15.7 + \sqrt{119 \times 24 \times 0.5 + 995.414}}$$

$$= 2486.2 \text{ N}$$

$$\therefore \quad F_0 > F_d$$

Now F_w should be greater than or equal to F_d. Again $\phi_n = \tan^{-1}$ (tan 20° cos 45°) = 14.43°

Putting $F_d = F_w$

$$2486.2 = \frac{d_p b k_w Q}{\cos^2 \alpha}$$

$$\therefore \quad k_w = \frac{2486.2 \cos^2 \alpha}{d_p b Q} \quad \text{where} \quad d_p = 30 \times 2.5 = 75 \text{ mm}$$

$$b = 24 \text{ mm}, \quad Q = \frac{2 \times 4}{4 + 1} = 1.6$$

$$\therefore \quad = \frac{2486.2 \times 0.707^2}{75 \times 24 \times 1.6} = 0.4313$$

Again $$K_w = \frac{S_{es}^2}{1.4} \sin \phi_n \left(\frac{1}{E_p} + \frac{1}{E_g} \right)$$

$$\therefore \quad 0.4314 = \frac{S_{es}^2}{1.4} \sin 14.43 \left[\frac{1}{2 \times 10^5} + \frac{1}{2 \times 10^5} \right]$$

$$\therefore \quad S_{es} = 492.3 \text{ MPa} = 2.75 \text{ B.H.N} - 70$$

$$\therefore \quad \text{B.H.N} = 204.47.$$

15.6 A pair of helical gear has 20° stub teeth in the diametral plane. Helix angle is 45°. The pinion rotates at 8,000 r.p.m and transmits 12 kW. Gear ratio is 4 : 1. Safe static stress for the material for pinion and gear is 100 MPa. The B.H.N for pinion is 300 and that of gear is 200. Find the module and face width, if the centre distance is 200 mm.

Solution:

From the centre distance and gear ratio $d_p = 80$ mm, $d_g = 320$ mm

$$V = \frac{\pi \times 80 \times 8000}{60 \times 1000} = 33.5 \text{ m/s}$$

$$\therefore \quad K_v = \frac{5.6}{5.6 + \sqrt{33.5}} = 0.4832$$

$$F_t = \frac{12{,}000}{33.5} = 358.2 \text{ N}$$

$$y'_p = 0.175 - \frac{0841}{t'_p} \text{ but } t'_p = \frac{80/m}{\cos^3 45°} = \frac{226.3}{m}$$

$$= 0.175 - \frac{0.841m}{226.3} = 0.175 - 3.716 \times 10^{-3} \times m$$

∴ Lewis Equation

$$358.2 = 0.4832 \times 100 \times (0.175 - 3.716 \times 10^{-3}\ m) \times 12 \times \pi \times m^2$$

By trial and error

$$m = 1.5 \text{ mm}, b = 18 \text{ mm}$$

$$F_0 = 100 \times y' \times 18 \times \pi \times 1.5 \times 0.707 = 1016.04 \text{ N}$$

15.7 Check the design from Example 15.6 for dynamic load and wear strength if error is 0.01 mm.

Solution:

As B.H.N for pinion is 300 and that of gear 200, using mean B.H.N

$$S_{es} = 250 \times 2.75 - 70 = 617.5 \text{ MPa}$$

$$\tan \phi_n = \tan 20° \cos 45°, \quad \therefore \quad \phi_n = 14.43°$$

$$k_w = \frac{617.5^2}{1.4} \sin 14.43 \left(\frac{1}{2 \times 10^5} + \frac{1}{2 \times 10^5} \right) = 0.6787$$

$$F_w = \frac{d_p b k_w Q}{\cos^2 \alpha} = \frac{80 \times 15 \times 0.6787 \times 1.6}{\cos^2 45°} = 2606.99 \text{ N}$$

Again for $e = 0.01$ mm, $C = 119$ N/mm

$$\therefore \quad F_d = F_t + \frac{21v(bC\cos^2\alpha + F_t)\cos\alpha}{21v + \sqrt{bC\cos^2\alpha + F_t}}$$

$$= 358.2 + \frac{21 \times 33.5[119 \times 18 \times 0.5 + 358.2] \times 0.707}{21 \times 33.5 + \sqrt{119 \times 18 \times 0.5 + 358.2}}$$

$$= 1317.11 \text{ N}$$

Thus $F_d > F_0$ and $F_w < F_d$.

∴ Design is not satisfactory with the point of view of beam strength

Modifying design to $m = 2$ mm, $b = 20$ mm. The conditions are satisfied. The students are advised to check the same.

15.8 Two helical gears have module 6.25 mm normal to the teeth. Blanks are solid and $\varphi_n = 20°$, $b = 65$ mm, $t_g = 44$, $t_p = 32$, centre distance 250 mm. Both gears are of steel with B.H.N = 300. $e = 0.1775$ mm. The pinion rotates at 800 r.p.m. Find ϕ and α.

Solution:

$$\frac{d_p + d_g}{2} = 250 \text{ and } d_g = \frac{11}{8} d_p, \quad \therefore \quad d_p = 210.52 \text{ mm}$$

$$d_g = 289.48 \text{ mm}$$

$$\therefore \quad m = \frac{210.52}{32} = 6.578 \text{ mm} = \frac{m_n}{\cos\alpha}$$

$$\therefore \quad \cos\alpha = \frac{m_n}{m} = \frac{6.25}{6.578} = 0.95$$

$$\therefore \quad \alpha = 18.195°$$

$$Q = \frac{2G}{G+1} = \frac{2 \times 44/32}{44/32+1} = 1.157$$

B.H.N = 300, $\therefore$ $S_{es} = 2.75 \times 300 - 70 = 755$ MPa

$$\therefore \quad k_w = \frac{S_{es}^2}{1.4} \sin 20° \left(\frac{1}{E_p} + \frac{1}{E_g}\right) E_p = E_g = 2 \times 10^5 \text{ MPa} = 1.39$$

$$\therefore \quad F_w = \frac{210.52 \times 65 \times 1.39 \times 1.157}{\cos^2 18.195°} = 24384.2 \text{ N}$$

Again $\quad k_d = 11500\, b$ N/mm and $G = \dfrac{44}{32} = 1.375$

$$m_e = \frac{G^2}{1+G^2} \frac{\pi b \rho r_p^2}{2g} = \frac{1.375^2}{1+1.375^2}\left[\frac{\pi \times 65 \times 0.0000768 \times 105.25^2}{2 \times 9806.6}\right]$$

$$= 5.7945 \times 10^{-3} \frac{N \sec^2}{mm}$$

$$F_i = \frac{eN_p t_p}{30}\sqrt{11{,}500\, b \times m_e}$$

$$= \frac{0.1775 \times 800 \times 32}{30}\sqrt{11500 \times 65 \times 5.7945 \times 10^{-3}} = 9968.52 \text{ N}$$

$$\therefore \quad F_n = F_w - F_i = 24384.2 - 9968.52 = 14415.68 \text{ N}$$

$$\therefore \quad F_t = F_n \cos\phi \cos\alpha$$

$$= 14415.68 \times \cos 20° \cos 18.195 = 12868.983 \text{ N}$$

$$V = \frac{\pi d_p N_p}{60} = \frac{\pi \times 210.52 \times 800}{60 \times 1000} = 8.82 \text{ m/s.}$$

$$\therefore \quad P = \frac{12869.983 \times 8.82}{1000} = 113.5 \text{ kW}$$

15.9 A pair of helical gears have 20° stub teeth in diametral plane and helix angle is 45°. The power to be transmitted is 20 kW. The pinion rotates at 5000 r.p.m and has 30 teeth. The gear ratio is 5. The material for gears is cast steel with safe bending stress of 110 MPa. B.H.N of the material is 250. Design the gear using Lewis Equation and check it for wear strength.

Solution:

Formative number of teeth on pinion $= \dfrac{30}{\cos^3 45°} = 84.85$

$$\therefore \quad y' = 0.175 - \frac{0.841}{84.85} = 0.165$$

$$V = \frac{\pi \times 30m \times 5000}{60{,}000} = 7.854\ m \text{ m/s}$$

$\therefore$ $$F_t = \frac{20{,}000}{7.854m} = \frac{2546.47}{m},\ K_v = \frac{5.6}{5.6 + \sqrt{7.254m}},$$

$\therefore$ Lewis Equation

$$\frac{2546.47}{m} = \left(\frac{5.6}{5.6 + \sqrt{7.854m}}\right) \times 110 \times 0.165 \times 12\ m \times \pi m \cos 45°$$

By trial and error

$$m = 2.5 \text{ mm},\ b = 30 \text{ mm},\ d_p = 75 \text{ mm},\ d_g = 375 \text{ mm}$$

Wear strength:

$$S_{es} = 2.75 \times 250 - 70 = 617.5 \text{ MPa},\ Q = \frac{2 \times 5}{5+1} = 1.7$$

$$\phi_n = \tan^{-1} (\tan 20° \cos 45°) = 14.43°$$

$\therefore$ $$\sin \phi_n = 0.249, \quad \therefore \quad k_w = \frac{617.5^2}{1.4} \times 0.249 \left(\frac{2}{2 \times 10^5}\right) = 0.678$$

$\therefore$ $$F_w = \frac{75 \times 30 \times 0.678 \times 1.7}{\cos^2 45°} = 5186.7 \text{ N},\ F_t = 1018.6 \text{ N}$$

$F_t < F_w$, $\therefore$ Design is safe.

15.10 Check the above design for dynamic tooth load if $e = 16 + 1.25\ (m + 0.25\sqrt{d}\ )$.

Solution:

$$e = 16 + 1.25\ (2.5 + 0.25\sqrt{75}\ ) = 21.83 \text{ for pinion in microns}$$

$$e = 16 + 1.25\ (2.5 + 0.25\sqrt{275}\ ) = 25.176 \text{ microns for gear}$$

$\therefore$ $$e_{\text{total}} = 47.006 \text{ microns}$$

$\therefore$ $$C = \frac{0.114 \times 0.047006 \times 10^5}{5} = 535.8 \text{ N/mm}$$

$\therefore$ $$F_i = \frac{21V(Cb\cos^2 45° + F_t)\cos 45°}{21V + \sqrt{Cb\cos^2 45° + F_t}}$$

$$= \frac{21 \times 19.635(535.8 \times 30 \times 0.707^2 + 1018.6)0.707}{21 \times 19.635 + \sqrt{535.8 \times 30 \times 0.5 + 1018.6}}$$

$$= 5201.8 \text{ N},\ F_t = 1018.6 \text{ N}$$

$\therefore$ $$F_d = 6220.4 \text{ N}$$

$$F_0 = 110 \times 0.165 \times 30 \times \pi \times \pi \times 2.5 \times 0.707 = 3023.48 \text{ N}$$

As $F_d > F_w$ and $F_d > F_0$, the design is not safe with the standpoint of dynamic tooth load. By increasing module from 2.5 mm to 3 mm, d_p increases to 90 mm and F_w increases to 6223 N, this being satisfactory. To improve F_0 the face width should be increased to 55 mm so that

$$F_0 = 110 \times 0.165 \times 55 \times \pi \times 3 \times 0.707$$
$$= 6657.5 \text{ N which may be satisfactory}$$

15.11 A pair of 20° helical gears have parallel shafts with centre distance of 385 mm. Normal module is 8 mm and the number of teeth on pinion and gear are 32 and 60 respectively. Show the tooth forces at the centre of the tooth if the pair transmits 23 kW at 1200 r.p.m of pinion.

Solution:

$$m = \frac{2C}{t_p + t_g} = \frac{2 \times 385}{92} = 8.37 \text{ mm}$$

$$m_n = 8 \text{ mm} = m \cos \alpha, \quad \therefore \quad \alpha = 17°$$
$$d_p = 267.84 \cong 268 \text{ mm}$$

$$\therefore \quad V = \frac{\pi \times 268 \times 1200}{60,000} = 16.84 \text{ m/s}$$

$$\therefore \quad F_t = \frac{25,000}{16.84} = 1484.56 \text{ N}$$

$$F_r = F_t \tan 20° = 540.33 \text{ N}, F_\alpha = \tan \alpha = 453.87 \text{ N}$$

15.12 Two helical gears are mounted on parallel shafts, $m_n = 6$ mm, $\phi_n = 20$, blanks are solid. Face width 60 mm. The number of teeth are 32 and 44. Centre distance 240 mm. Both gears are made of steel with B.H.N 300. r.p.m of pinion is 900. The permissible error is 0.2 mm. Find α and the power the gears can carry.

Solution:

$$C = \frac{d_p + d_g}{2} = \frac{(1 + 44/32)}{2} d_p, \quad \therefore \quad d_p = 202.1 \text{ mm}$$
$$d_g = 277.9 \text{ mm}$$

$$\therefore \quad m = \frac{202.1}{32} = 6.135 \text{ mm}, \quad \therefore \quad \alpha = \cos^{-1} \frac{6}{6.315} = 18.173°$$

$$F_0 = \sigma y\, b\, \pi m_n, \text{ for steel } \sigma = 140 \text{ MPa}, y = 0.1255$$
$$= 140 \times 0.1255 \times 60\pi \times 6 = 19871.2 \text{ N}$$

$$S_{es} = 2.75 \text{ B.H.N} - 70 = 755 \text{ MPa}, \quad \therefore k_w = \frac{755^2}{1.4} \sin 20° \left(\frac{1}{E_p} + \frac{1}{E_g} \right)$$

$$Q = \frac{2 \times 11/8}{1 + 11/8} = 1.158 \qquad = 1.392 \text{ N/mm}^2$$

$$F_w = d_p\, b k_w Q / \cos^2 \alpha = 202.1 \times 60 \times \frac{1.3920 \times 1.158}{0.9}$$
$$= 21660.335 \text{ as } F_0 < F_w \text{ for safe design } F_d \le F_0$$

$$m_e = \frac{G^2}{1+G^2}\left(\frac{\rho b \pi r_p^2}{2}\right), \rho = 7800 \text{ kg/m}^3, G = \frac{44}{32}$$

$$= \frac{1.89}{1+1.89}\left[\frac{7800}{2} \times \frac{60}{1000} \times \pi \times \left(\frac{202.1}{2000}\right)^2\right]$$

$$= 4.909 \text{ kg}$$

$$t = \frac{60}{32 \times 900} = 2.08 \times 10^{-3} \text{ sec}$$

$$k = \frac{b}{2}\left(\frac{E_p E_g}{E_p + E_g}\right) = 11111.11\, b \text{ N/mm} = 6.66 \times 10^8 \text{ N/m}$$

as $$b = \frac{60}{100} m$$

$$e = \frac{0.20}{1000} m, \quad \text{using } F_t = \frac{2e}{t}\sqrt{k_d m_e}$$

$\therefore$ $$F_t = \left(\frac{2 \times 0.22}{1000}\Big/ 2.08 \times 10^{-3}\right)\sqrt{6.67 \times 10^8 \times 4.909} = 1100.4139 \text{ N}$$

$\therefore$ $$F_w = 19871.2 = F_t + 1100.4139$$

$\therefore$ $$F_t = 18770.786 \text{ N}, \quad V = \frac{\pi \times 202.1 \times 900}{60000} = 9.523 \text{ m/s}$$

$\therefore$ $$P = \frac{F_t V}{1000} = 178.76 \text{ kW.}$$

OBJECTIVE QUESTIONS

15.1 Helical gears may be used for the pitch line velocities up to
(a) 5 m/s (b) 1.5 m/s (c) 50 m/s (d) 100 m/s

15.2 Formative number of teeth in helical gear are given by
(a) $\dfrac{\text{p.c.d}}{\text{Normal module}}$ (b) $\dfrac{\text{Actual no. of teeth}}{\cos^3 \alpha}$
(c) Actual no. of teeth $\times \cos \phi$ (d) Actual no. of teeth/$\cos \phi_n$

15.3 Helical gears with helix angle α have normal module m_n and actual module m related as
(a) $m_n = m \cos \alpha$ (b) $m_n = m/\cos \alpha$ (c) $m_n = m \cos^2 \alpha$ (d) $m_n = m \cos^3 \alpha$

15.4 The type of bearing most suitable for helical gears is
(a) deep groove b.b (b) thrust roller bearings
(c) taper roller bearing (d) angular contact b.b

15.5 Herringbone gears are used to
(a) avoid interference (b) eliminate axial thrust
(c) avoid the effect of dynamic load (d) to reduce the wear of teeth

15.6 Herringbone gears are virtually
(a) spur gear (b) miter gear (c) stub teeth gear (d) double helical gear

15.7 Normal pressure angle ϕ_n is
(a) the same as ordinary pressure angle (b) more than ϕ
(c) less than ϕ (d) none of the above

15.8 Beam strength of helical gear tooth as compared to the spur gear tooth of the same p.c.d, module and material is
(a) more (b) equal (c) less (d) double

15.9 Effect of tooth error in helical gear tooth as compared to the similar spur gear tooth is
(a) the same (b) more prominent
(c) less prominent (d) dependent on the magnitude of error

15.10 Due to smooth engagement and two pairs of teeth always in contact, the helical gears
(a) are more suitable for shock loads
(b) are capable of transmitting more power but at low speeds
(c) are capable of transmitting heavy load at high speed
(d) transmit less load at high speed

15.11 It is desirable to have angle of helix between 15°–30° so that
(a) axial thrust is limited and engagement is smooth
(b) engagement is smooth and gradual
(c) axial thrust is eliminated
(d) none of the above

15.12 Helical gears may be used to connect two non-intersecting shafts if
(a) load is relatively small (b) axial thrust is to be eliminated
(c) more smooth engagement is needed (d) none of the above

15.13 Under the similar conditions the wear strength of helical gear compared to spur gear is
(a) smaller (b) same
(c) larger (d) double if helix angle is 45°

15.14 With less number of teeth the possibility of interference in helical gears as compared to spur gear of the same number of teeth is
(a) more (b) less
(c) cannot be predicted (d) none of the above

15.15 With smaller gear ratio it is desirable to connect two parallel shafts by
(a) Helical gears (b) spur gear with full depth teeth
(c) spur gear with stub teeth (d) Herringbone gear

15.16 Effect of helix angle on the ratio factor used in wear strength equation of helical tooth is
(a) more predominant (b) nil (c) less predominant (d) unpredicted

15.17 Helix angle of the helical gear lies within a range of
(a) 10°–20° (b) 15°–45° (c) 30°–40° (d) 15°–30°

15.18 With 45° helix angle the formative number of teeth of helical gear with 45 mm p.c. diameter and 3 mm module is
(a) 50 (b) 21 (c) 30 (d) 42.44

15.19 For Herringbone gear the type of the bearing to be used may be
(a) Taper roller bearing (b) deep groove ball bearing
(c) thrust ball bearing (d) angular contact ball bearing

REVIEW QUESTIONS

15.1 Define the term formative number of teeth in case of helical gears and derive the relationship.
15.2 Discuss the applications of helical gears and compare it with spur gear.
15.3 Compare the helical and spur gear for the following aspects: (a) Gear ratio, (b) mode of engagement, (c) noise, (d) power transmitted, (e) speed, (f) axial thrust.
15.4 Explain why helical gears are capable of transmitting greater power at high speed as compared to the spur gear.
15.5 Discuss the effect of helix angle on the performance of helical gears and state the range of helix angle suitable for different conditions.
15.6 What is the limitation on the uses of helical gears in place of spur gear?
15.7 What is Herringbone gear? State its application.
15.8 Why selection of bearing is more important in helical gears?
15.9 Explain how and why Lewis Equation is modified in case of helical teeth.
15.10 Draw the force diagram on the helical tooth and explain the relationships between the three components of force acting on the tooth.
15.11 Prove $\tan \phi_n = \tan \varphi \cos \alpha$ in case of helical gear where φ_n, φ and α carry usual meaning.
15.12 Which type of helical gears are used for connecting nonparallel and nonintersecting shafts? Explain by drawing sketch.
15.13 Why are helical gears not interchangeable?
15.14 As compared to spur gears, helical gears can transmit larger power at higher speeds. Why?
15.15 Why a deep groove ball bearing or taper roller bearing are recommended to be used with helical gears?

PRACTICE PROBLEMS

15.1 Two parallel shafts are connected by a pair of helical gears. The gear ratio is 4.5 : 1. Centre distance is 165 mm. No. of teeth on pinion is 16, normal module 3.5 mm and $\phi = 20°$. Find the forces at the centre of the tooth if the force normal to the tooth is 6000 N.
15.2 A helical gear has 20 teeth and pitch diameter 110 mm. Normal module 5 mm and $\phi = 20°$. The force normal to the tooth surface is 7500 N. Find the formative number of teeth and power transmitted at 2000 r.p.m.
15.3 Find the induced stress in the gear tooth in Problem 15.2 assuming $K_v = 5.6/(5.6 + \sqrt{v})$ and $b = 12$ mm.
15.4 Find the necessary face width of the pair of gears in Problem 15.1 for the wear strength to be sufficient. $k_w = 0.9$.
15.5 Two parallel shafts are connected by helical gears of forged steel with safe static stress of 105 MPa. Normal module 5 mm, face width 50 mm. The pinion rotates at 1200 r.p.m. Gear ratio 3.75 : 1, $\phi_n = 20°$, $\alpha = 25°$. B.H.N of the material 260. Find the safe power that can be transmitted. Assume $t_p = 20$.

15.6 The following data refer to a pair of helical gears transmitting 32 kW at 1400 r.p.m of pinion. $\phi_n = 20°$, $m_n = 5$ mm, $\alpha = 18°$, gear ratio 5 : 1. Permissible static stress for steel used for both gears = 240 MPa. Material combination factor $k_w = 1.2$ N/mm^2. Maximum permissible tooth error = 25 microns: $K_v = \dfrac{6}{6+V}$. Determine the minimum width of gears taking into account dynamic load using Buckingham Equation.

15.7 Design a pair of helical gears transmitting 30 kW at 1760 r.p.m of pinion. $t_p = 25$, $t_g = 75$, $\phi_n = 20°$, $\alpha = 25°$. Both the gears are made of steel with safe static stress in bending 140 MPa. The error is given by $16 + 1.25\,(m_n + 0.25\sqrt{d}\,)$. Design the gear and calculate the B.H.N required.

15.8 Find the wear strength of a pair of helical gears with diameter of pinion 144 mm, face width 250 mm, angle of helix 23° and pressure angle in diametral plane 20°, gear ratio 5 : 1. Both gears are made of steel with B.H.N of 205 and 250.

15.9 Two helical gears are mounted on parallel shafts and spaced 225 mm apart. $t_p = 35$ and $t_g = 70$, $\alpha = 20°$, $m_n = 4.00$ mm, $b = 50$ mm. Both gears are made of steel hardened to 320 B.H.N. Pinion rotates at 900 r.p.m. The value of dynamic load constant $C = 119$ kN/m. Find the capacity in kW of the gear drive.

15.10 Solve Problem 15.9 without considering dynamic load and $t_p = 30$, $t_g = 90$, $m = 5$ mm and $b = 60$ mm. Assume σ_t for material to be 150 MPa.

15.11 Helical gears of C.S with permissible bending stress of 140 MPa are used for transmitting 63.5 kW at 1050 r.p.m of pinion. The number of teeth on 20 pinion and on 60 gear. Helix angle is 23° and the teeth are 20° full depth. Design the gears using Lewis Equation.

15.12 For the helical gear pair in Problem 15.9 check the design for wear strength and dynamic tooth load if the permissible total tooth error is 0.06 mm. What should be the B.H.N for the gear material?

15.13 Using equivalent mass equation, find the dynamic load for a pair of helical gears both made from high grade C.I. The number of teeth on pinion is 25 and that on the gear is 75. Module is 4 mm. Pinion is solid disc while the internal diameter of the gear may be taken as 220 mm. Speed of pinion 1750 r.p.m. $e = 16 + 1.25\,[m_n + 0.25\sqrt{d}\,]$. For C.I density 7200 kg/m^3, $b = 50$ mm.

15.14 Two helical gears are cut with a hob of 11.00 mm normal circular pitch. Speed ratio is 2 : 1 and centre distance is 240 mm. Find α if the pinion has 40 teeth.

15.15 A pair of 20° involute helical teeth gears have $m_a = 6.5$ mm. The number of teeth $t_p = 32$, $t_g = 44$. If the gear is subjected to a transmitted force of 8500 N, find the forces at the centre of the teeth. Centre distance is 260 mm.

15.16 Calculate the power to be transmitted at 600 r.p.m of the pinion of the above problem.

15.17 A pair of helical gears is required to transmit 40 kW at 1750 r.p.m. Gear ratio is 1 : 4. The centre distance is 200 mm. The number of teeth on pinion is 15. Using helix angle of 15° and pressure angle $\varphi = 20°$, find the size of the tooth using Lewis Equation for involute full depth teeth.

15.18 Two parallel shafts are connected by a pair of helical gears with normal module 6 mm, normal pressure angle of 20°, face width 65 mm, having solid gear blanks. Centre distance is 250 mm, number of teeth $t_p = 32$, $t_g = 44$. Material for both gears is steel with 300 B.H.N. $e = 45 + 3.5\,(m + 0.25\sqrt{d}\,)$. Find power that can be safely transmitted at 1440 r.p.m of pinion using M.F. Spott's equation for dynamic load.

15.19 Find the power transmitted safely at 600 r.p.m of pinion of precision cut helical gears with gear ratio 3 : 1, $\varphi = 20°$, $\alpha = 23°$. Permissible static bending stress 100 MPa, module $m = 3$ mm, width $b = 30$ mm. B.H.N is 225 for both gears. Consider wear and beam strength.

15.20 Two helical gears have normal pressure angle 14.5° and helix angle 23°. The pinion has 48 teeth, the gear ratio 5:1 and module 3 mm. B.H.N for pinion is 250 and that for gear is 200. The face width is 30 mm. Determine the wear strength.

15.21 The following data relates to a pair of helical gears made of steel transmitting 30 kW at 2000 r.p.m pinion. The helix angle is 25°. The gears are made from steel with permissible static stress of 150 MPa. The error in cutting gears is given by $e = 16 + 1.25\,(m + 0.25\sqrt{d}\,)$ microns. Design gears and specify required surface hardness. Use equivalent mass equation.

15.22 Two helical gears are used in a speed reducer transmitting 75 kW at 1200 r.p.m of pinion. Both gears are made of the same material with safe static stress of 200 MPa and B.H.N 350. Involute full depth teeth of module 5 mm in diametral plane, helix angle 15° are used. $t_p = 22$, $t_g = 68$. Calculate dynamic and wear load.

15.23 Two helical gears have 20° involute full depth teeth, and carry 35 kW at 860 r.p.m of pinion. Gear ratio is 2.1 and centre distance 225 mm. Module is 5 mm in diametral plane and both gears are hardened to 350 B.H.N. Find the necessary face width if the pair has $\alpha = 25°$.

ANSWERS

Objective Questions

(1) c (2) b (3) a (4) a (5) b (6) d (7) c (8) a (9) c
(10) c (11) a (12) a (13) d (14) b (15) b (16) b (17) b (18) d
(19) b

Practice Problems

(1) $F_t = 5303.66$ N, $F_r = 2003.17$ N, $F_a = 2112.65$ N
(2) $t_p = 26.61$, Power = 74.6 kW
(3) $\sigma = 92.18$ MPa (4) $b = 105$ mm (5) 30 kW
(6) With Lewis Equation $b = 30$ mm, $F_o > F_d$ but $F_w < F_d$ increase b to 60 mm, $d_p = 84$ mm, No. of teeth = 16
(7) Using Buckingham Equation B.H.N = 277, $m = 6$ mm (8) 35334.2 N
(9) 35 kW based on wear strength and dynamic load (10) 58.8 kW
(11) $m = 6$ mm, $d_p = 180$ mm, $d_g = 360$ mm, $b = 72$ mm
(12) Use gear material with $\sigma_b = 200$ MPa, B.H.N = 300
(13) 1358 N (14) 28.95°
(15) $F_t = 7632.05$ N, $F_r = 2777.8$ N, $F_a = 2507.81$ N (16) 52.49 kW
(17) $b = 54$ mm (18) 133.88 kW (19) 4.1 kW (20) 4584 N

16

Bevel Gears

CONCEPT REVIEW

16.1 INTRODUCTION

Two intersecting shafts are connected by bevel gears. Just as pitch cylinders in spur gear, there are pitch cones in bevel gears.

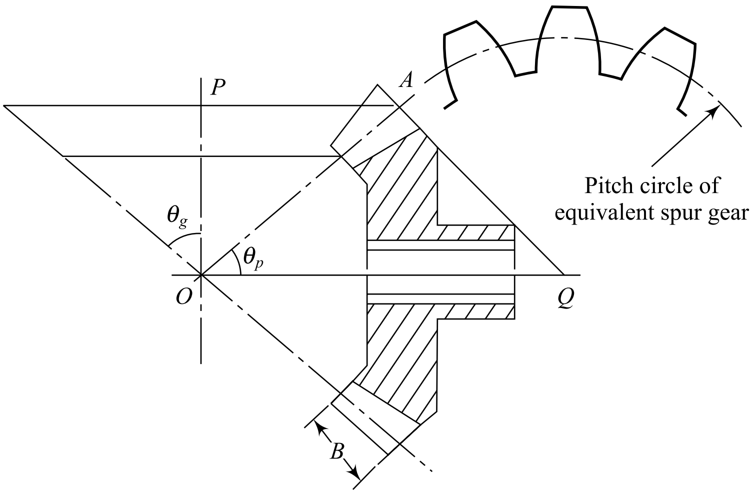


Fig. 16.1

OA — slant length or pitch cone radius = L
B — face width of the tooth along L
QA — radius of back cone
θ_p — pitch cone angle for pinion
θ_g — pitch cone angle for gear
α — shaft angle—angle between the axes OQ and OP of bevel pinion and gear respectively
$\alpha = \theta_p + \theta_g$

16.2 EQUIVALENT RELATIONSHIPS

The bevel gear can be reduced to equivalent spur gear. According to Tredgold's approximation the radius of equivalent spur gear is given by the radius of back cone

$$r_p' = \frac{r_p}{\cos\theta_p} \qquad (16.2.1)$$

This is proved from the figure Fig. 16.1 where $r_p = PA$ and $r_p' = QA$

$$\therefore \qquad t_p' = \frac{t_p}{\cos\theta_p} \qquad t_g' = \frac{t_g}{\cos\theta_g} \qquad (16.2.2)$$

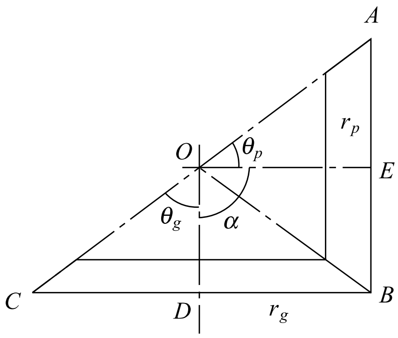


Fig. 16.2

Relationship between α, θ_p, θ_g and gear may be derived as follows from Fig. 16.2.

As, $$\alpha = \theta_p + \theta_g$$

$\therefore$ $$\theta_p = \alpha - \theta_g$$

$\therefore$ $$\sin(\theta_p) = \sin(\alpha - \theta_g) = \sin\alpha\cos\theta_g - \cos\alpha\sin\theta_g$$

$$\text{Now } OB = \frac{r_p}{\sin\theta_p} = \frac{r_g}{\sin\theta_g}$$

$\therefore$ Substituting $$\sin\theta_p = \frac{r_p}{r_g}\sin\theta_g$$

we get $$\frac{r_p}{r_g}\sin\theta_g = \sin\alpha\cos\theta_g - \cos\alpha\sin\theta_g$$

Dividing throughout by $\cos\theta_g$

$$\frac{r_p}{r_g}\tan\theta_g = \sin\alpha - \cos\alpha\tan\theta_g$$

$$\tan\theta_g = \frac{\sin\alpha}{\dfrac{r_p}{r_g} + \cos\alpha} \qquad \text{or} \qquad \theta_g = \tan^{-1}\frac{\sin\alpha}{\dfrac{r_p}{r_g} + \cos\alpha}$$

Similarly,

$$\theta_p = \tan^{-1}\frac{\sin\alpha}{\dfrac{r_g}{r_p} + \cos\alpha}$$

16.3 LEWIS EQUATION FOR BEVEL GEAR

As radius of bevel gear varies along the width, the torque does not produce the same tangential force. For solving problems the quantities at the outermost radius are used.

Let us consider the width '*dx*' of the tooth at distance '*x*' from the apex of the cone for which Lewis Equation may be written as

$$dF_x = \sigma \cdot y \cdot dx \cdot p_x,$$

$$= \sigma \cdot y \cdot dx \cdot \frac{px}{L} \qquad p_x = \frac{p \cdot x}{L}$$

$\therefore$ Torque $d\,T_x = \left(\sigma \cdot y \cdot dx \cdot \frac{px}{L}\right) \cdot r_x,$

$$\because r_x = \frac{rx}{L}$$

$$\therefore \qquad T = \int_{L-B}^{L} \sigma \cdot y \cdot dx \cdot \frac{px}{L} \cdot \frac{rx}{L}$$

$$= \frac{\sigma \cdot y \cdot pr}{L^2} \int_{L-B}^{L} x^2\, dx$$

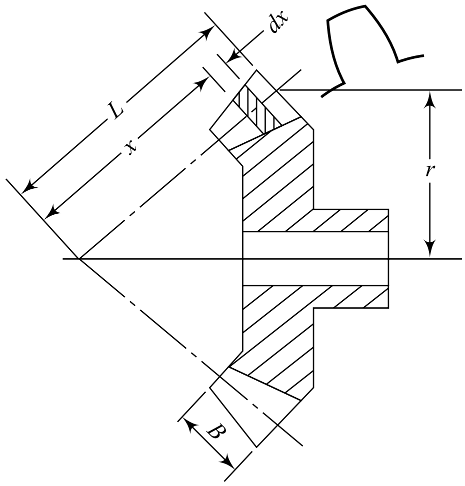


Fig. 16.3

After integrating, expanding and neglecting terms of small value

$$T = \sigma \cdot y \cdot p \cdot r \left(1 - \frac{B}{L}\right) B$$

$\therefore$ Lewis Equation finally may be written as

$$F_t = \sigma \cdot y \cdot p \cdot B \left(1 - \frac{B}{L}\right) K_v \qquad (16.3.1)$$

$\left(1 - \frac{B}{L}\right)$ is termed as Bevel Factor

$$K_v = \frac{5.6}{5.6 + \sqrt{V}} \quad \text{or} \quad K_v = \frac{6}{6 + V}$$

Wear strength
$$F_w = \frac{0.75\, d_p b k_w\, Q}{\cos \theta_p} \qquad (16.3.2)$$

where the notations carry the same meaning as in case of spur gears but $Q = \dfrac{2t_g'/t_p'}{t_g'/t_p' + 1}$

Dynamic load

$$F_d = F_t + \frac{21V\,(BC + F_t)}{21V + \sqrt{BC + F_t}} \qquad (16.3.3)$$

16.4 FORCE ANALYSIS

CA is the force normal 'to the tooth of which three components F_t, F_a and F_t are given by *OA, BO* and *CB* respectively (Fig. 16.4).

$$F_t = \frac{\text{Torque}}{\text{Radius}} = \frac{\text{Power}}{\text{Velocity}}$$

$$F_a = CO \sin \theta_p \text{ or } CO \sin \theta_g \qquad CO = F_t \tan \phi$$

$$\therefore \qquad F_a = F_t \tan \phi \sin \theta_p \qquad F_t = CO \cos \theta_p \text{ or } CO \cos \theta_g$$

depending on the gear to which the quantities refer.

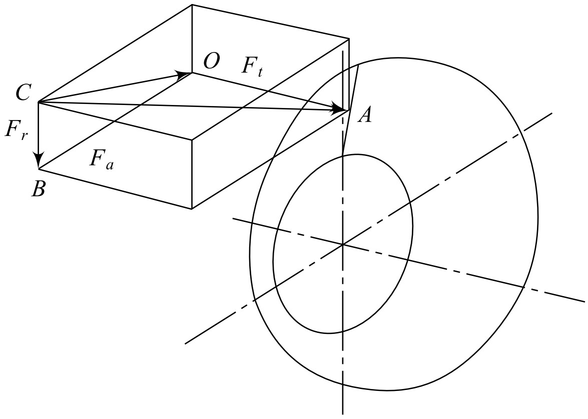


Fig. 16.4

16.5 PRACTICAL DESIGN ASPECTS

For correctly designed gears the face width should not be more than 1/3 the slant length *L*. The efficiency of straight tooth bevel gear is high (up to 98%) but it may produce noise at high speeds as in spur gears. In that case the spiral bevel gears with the teeth in spiral path on the pitch cones are used.

Hypoid gears have spiral teeth and shafts do not intersect.

Bevel gears are manufactured in pairs only. The bevel gears with shaft angle of 90° are termed as Mitre gears. The bevel gears with any other shaft angle are termed as angular bevel gears.

The zerol bevel gear has curved teeth but a zero spiral angle.

WORKED EXAMPLES

16.1 Two shafts intersecting at an angle of 90° are connected by a pair of bevel gears with a gear ratio 2.5 : 1. The module of 20° full depth involute teeth at outer radius is 5 mm and number of teeth on pinion is 30. If 10 kW is transmitted at 400 r.p.m of pinion, determine the tangential force at the mean radius, the axial thrust on the pinion and gear. $B = 67$ mm.

Solution:

Diameter of pinion = $30 \times 5 = 150$ mm

$\therefore$ Diameter of gear = $150 \times 2.5 = 375$ mm

$$\therefore \quad L = \sqrt{r_p^2 + r_g^2} = 201.94 \text{ mm}$$

$$\therefore \quad \theta_p = \sin^{-1} \frac{75}{201.94} = 21.81° \qquad \theta_g = \sin^{-1} \frac{187.5}{201.94} = 68.2°$$

$$\text{Mean radius of pinion} = 75 - \frac{B}{2} \times \sin \theta_p = 74.81 \text{ mm}$$

$$\therefore \text{ Pitch line velocity at mean radius} = \frac{\pi \times 74.81 \times 400}{60 \times 1000} = 1.55 \text{ m/s}$$

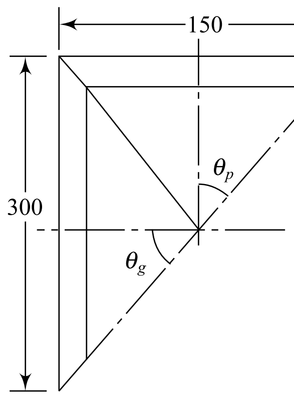


Fig. E-16.1a

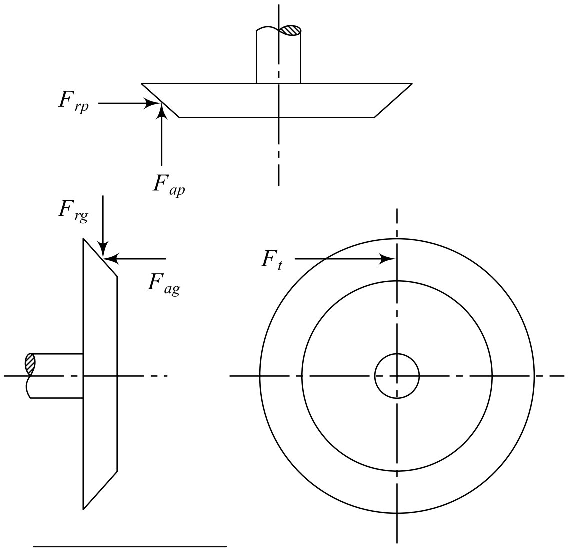


Fig. E-16.1b

$$\therefore \quad F_t \text{ at mean radius} = \frac{P}{V} = \frac{10 \times 1000}{1.55} = 6451.61 \text{ N}$$

$\therefore$ Axial thrust on pinion $= F_t \tan \phi \sin \theta_p$

$= 6451.61 \times \tan 20° \sin 21.81° = 872.42$ N

Gear axial thrust force $= F_t \tan \phi \cos \theta_p = 2180.1125$ N

Free body diagrams for both gears are drawn. It is obvious that $F_{rp} = F_{ag}$ and $F_{ap} = F_{rg}$.

16.2 A bevel gear rotates at 600 r.p.m and transmits power to the other gear rotating at 250 r.p.m. The outer module is 3.5 mm. The power to be transmitted is 30 kW. The teeth are 20° involute full depth. $L/B = 3$. Check the safety of the design if permissible static stress is 55 MPa.

Solution:

Let the number of teeth on pinion be 30 so that p.c.d is 105 mm

$$\therefore \quad V = \frac{\pi \times 105 \times 600}{60 \times 1000} = 3.298 \text{ m/s}$$

$$\therefore \quad F_t = \frac{30 \times 1000}{3.298} = 909.6422 \text{ N}$$

$$t_p' = \frac{t_p}{\cos \theta_p}, \text{ let } \alpha = 90°, \ \theta_p = \tan^{-1} \frac{1}{2.4 + 0} = 22.61°$$

$$\therefore \quad t_p' = \frac{30}{\cos 22.61°} = 32.49, \ r_p = 52.5 \text{ mm}, \ r_g = 126 \text{ mm}$$

$$\therefore \quad y_p' = 0.154 - \frac{0.912}{23.49} = 0.126$$

$$K_v = \frac{6}{6 + 3.298} = 0.6453, \ \frac{L - B}{L} = 0.67$$

Using Lewis Equation 16.3.1

$$F_t = K_v\, \sigma y'\, B \times \pi m \times \frac{L - B}{L}$$

$\therefore$ $$9096.422 = 0.6453 \times \sigma \times 0.126 \times B \times \pi \times 3.5 \times 0.67$$

Again as $$L = \sqrt{r_p^2 + r_g^2} = \sqrt{52.5^2 + 126^2}$$

$$= 136.5 \text{ mm}, \quad \therefore B = 45.5 \text{ mm}$$

Substituting

$$\sigma = \frac{9096.422}{0.6453 \times 0.126 \times 45.5 \times \pi \times 3.5 \times 0.67} = 33.37 \text{ MPa}$$

$\therefore$ Design is safe.

16.3 A pair of straight teeth bevel gear is used to transmit 6 kW at 350 r.p.m of the pinion mounted on horizontal drive shaft. The gear is mounted on vertical spindle of the machine tool and rotates at 175 r.p.m. The pinion has 24 machine cut stub teeth with 20° pressure angle, module 4 mm and the material with safe static stress as 140 MPa. Determine the necessary face width.

Solution:

$$\text{p.c.d of pinion} = 24 \times 4 = 96 \text{ mm}$$

$$\text{Pitch line velocity} = \frac{p \times 96 \times 350}{60 \times 1000} = 1.76 \text{ m/s}$$

$\therefore$ $$F_t = \frac{6 \times 1000}{1.76} = 3409.0 \text{ N}$$

and $$K_v = \frac{6}{6 + 1.76} = 0.789,\ r_p = 48 \text{ mm},\ r_g = 96 \text{ mm}$$

$$L = \sqrt{48^2 + 96^2} = 107.33 \text{ mm}$$

$$\theta_p = \tan^{-1}\frac{1}{R} = 26.56°, \qquad \therefore\ t_p = \frac{24}{\cos 26.56°} = 26.83$$

$\therefore$ $$y = 0.175 - \frac{0.9}{26.56} = 0.141$$

$\therefore$ Lewis Equation

$$3409 = 0.789 \times 140 \times 0.141 \times B \times \pi \times 4 \times \frac{L - B}{L}$$

$\therefore$ $$B(107.33 - B) = 1871.81$$

$\therefore$ $$B = 22 \text{ mm}$$

16.4 A pair of bevel gear is required to transmit 18 kW at 600 r.p.m. The output shaft is making an angle of 90° with input shaft and rotates at 300 r.p.m. Pinion has 30 teeth. The teeth are 20° full depth. Safe static stress for the material is 105 MPa. Check the design for wear strength; first designing it for beam strength.

Solution:

Let $$\frac{B}{L} = \frac{1}{3},\ r_p = 15\, m \text{ mm},\ r_g = 2\, r_p$$

$$L = \sqrt{r_p^2 + r_g^3} = \sqrt{15^2 m^2 + 30^2 m^2} = 33.54\, m \text{ mm}$$

$\therefore \qquad B = 11.18\, m$ mm

$$V = \frac{p \times 30\, m \times 600}{60 \times 1000} = 0.9424\, m \text{ m/s}$$

$$\therefore \qquad F_t = \frac{18 \times 1000}{0.9424\, m} = \frac{19100.169}{m} \text{ N}$$

$$\theta_p = \tan^{-1}\frac{1}{2} = 26.56°,\ \theta_g = 63.44°$$

$$\therefore \qquad t'_p = \frac{30}{\cos 65°} = 33.54,\ t'_g = t_g/\cos 63.44° = 134.18$$

$$\therefore \qquad y'_p = 0.154 - \frac{0.912}{33.54} = 0.1268$$

$$K_v = \frac{6}{6 + 0.9424\, m},\quad \frac{B - L}{L} = 0.67$$

$\therefore$ Lewis Equation is

$$\frac{19100.169}{m} = \left(\frac{6}{6 + 0.9424\, m}\right) \times 105 \times 0.1268 \times 11.18\, m \times \pi m \times 0.67$$

$\therefore \qquad m = 5$ mm by trial and error

$\therefore \qquad B = 56$ mm, $r_p = 75$ mm, $r_g = 150$ mm

$\therefore \qquad F_o = 7846.776$ N and $F_r = 3820$ N

To check for F_w, $d_p = 150$ mm, $B = 56$ mm

$$k_w = \frac{S_{es}^2}{14} \sin\phi \left(\frac{1}{E_p} + \frac{1}{E_g}\right) = \frac{600^2}{1.4} \sin 20° \left(\frac{1}{2 \times 10^5} + \frac{1}{2 \times 10^5}\right)$$

$$= 0.879$$

$$Q = \frac{2 \times 134.18/33.54}{(138.18/33.540 + 1)} = 1.6$$

$$\therefore \qquad F_w = \frac{(150 \times 56 \times 0.879 \times 1.6) \times 0.75}{\cos\theta_p} = \frac{8860.32}{\cos 26.56°} = 9905.7 \text{ N}$$

Here $F_w > F_t$, $\therefore$ design is safe.

16.5 Design a pair of bevel gears to transmit 12 kW at 600 r.p.m. Gear ratio is 2 : 1. The pinion has 24, 20° full depth involute teeth. The safe static stress is 105 MPa. $B = L/3$, $K_v = \dfrac{3.5}{3.5 + \sqrt{V}}$. Find m, B, L, d_p and d_g. Check the design for dynamic load and wear. $C = 300$ kN/m. If the mean plane of the gear is 120 mm from the left hand bearing, determine the forces on the bearings and estimate the diameter of the shaft.

Solution:

$$\theta_p = \tan^{-1}\frac{1}{2} = 26.56°$$

$$t'_p = \frac{24}{\cos 26.56°} = 26.83$$

$\therefore \qquad y'_p = 0.154 - \dfrac{0.912}{26.83} = 0.120$

$$V = \frac{\pi \times 24\,m \times 600}{60 \times 1000} = 0.754\,m$$

$\therefore \qquad F_t = \dfrac{12 \times 1000}{0.754\,m} = \dfrac{15915.119}{m}$ and $K_v = \dfrac{3.5}{3.5 + \sqrt{0.754\,m}}$ m/s

Bevel factor = 0.67

$$L = \sqrt{(12\,m)^2 + (24\,m)^2} = 26.83\,m$$

$\therefore \qquad B = 8.943\,m$

$\therefore$ Lewis Equation

$$\frac{15915.199}{m} = \left(\frac{3.5}{3.5 + \sqrt{0.754\,m}}\right) \times 105 \times 0.12 \times 8.943\,m \times \pi \times m \times 0.67$$

$\therefore \qquad m = 5$ mm by trial and error

$\therefore \qquad B = 45$ mm, $L = 135$ mm, $d_p = 120$ mm, $d_g = 240$ mm

Let $\qquad K_w = 0.879$ as found in previous problem

Again $\qquad t'_g = \dfrac{48}{\cos 63.44°} = 107.35$

$\therefore \qquad t'_g / t'_p = 4, \quad \therefore \quad Q = \dfrac{4 \times 2}{4 + 1} = 1.6$

$\therefore \qquad F_w = \dfrac{0.75 \times 120 \times 45 \times 0.897 \times 1.6}{\cos 26.56°}$

$\therefore \qquad F_w = 6498.35$ N, $F_t = 3183$ N

$F_o = 5929.6$ N, $V = 3.77$ m/s

$$F_d = 3183 + \frac{21 \times 3.77\,[300 \times 45 + 3183]}{21 \times 3.77 + \sqrt{300 \times 45 + 3183}}$$

$= 9522.83$ N

$\therefore \quad F_d > F_w$ and $F_d > F_o$

Modify $m = 7.5$ mm to get satisfactory result

With $m = 7.5$ mm, $F_o = 13331$ N, $V = 5.655$ m/s

$\therefore F_d < F_o < F_w$

$F_w = 15477$ N, $F_t = 2122.0$ N

$\therefore \qquad F_d = 9726.52$ N

$F_r = F_t \tan\phi \cos\theta_p = 690.83$ N

$F_a = F_t \tan\phi \sin\theta_p = 345.3422$ N

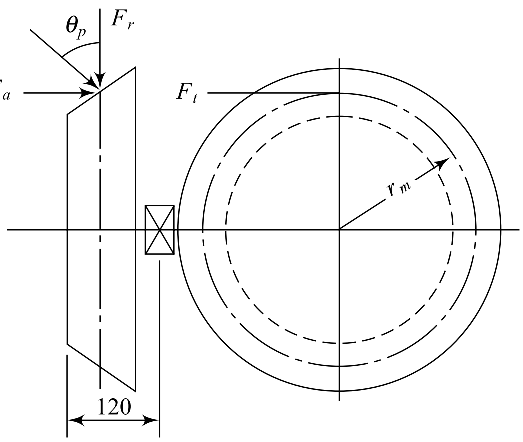


Fig. E-16.5

For gear $F_t = 2122.0$ N

$F_r = 345.34$ N

$F_a = 690.83$ N, $d_g = 360$ mm

$\therefore$ Horizontal b.m on gear shaft $= F_t \times 0.12$

$= 254.64$ N.m

Vertical b.m on gear shaft

$$= F_a \times r_m - F_r \times 0.12$$
$$= 690.83 \times 0.18 - 345 \times 0.12$$
$$= 82.94 \text{ N.m}$$

Torque on gear shaft

$$= F_t \cdot r_g = 381.96 \text{ N.m}$$

Using $K_m = 1.2$

$$T_e = \sqrt{1.5^2\,[(254.64)^2 + (82.94)^2] + 1.2^2 \times (381.96)^2}$$
$$= 609.47 \text{ N.m}$$

Again $T_e = \dfrac{\pi}{16} d^3 \times \tau$, Let $\tau = 60$ MPa

$$\therefore \quad d = \frac{\sqrt[3]{16 \times 609.47 \times 10^3}}{\pi \times 60} = 37.26 \text{ mm} \cong 40 \text{ mm}$$

Radial forces on the bearings can be found by knowing the distance between bearings.

Axial force = 690.83 N.

16.6 Find the power transmitted by a pair of bevel gears with 20° full depth involute teeth. The material is steel with hardness 350 B.H.N. Module = 6.5 mm, face width 60 mm and axial width 35 mm. Pinion rotates at 600 r.p.m. The shafts are perpendicular to each other. $k_w = 0.96$, $C = 300$ N/mm, $t_p = 40$, $t_g = 60$, $\sigma = 105$ MPa.

Solution:

$$d_p = 260 \text{ mm}, d_g = 390 \text{ mm}, L = \sqrt{r_p^2 + r_g^2} = 234.36 \text{ mm}$$

$$\theta_p = \tan^{-1} \frac{1}{1.5} = 36.69°$$

$$\therefore \quad t_p' = 48.07,\ t_g' = 108.2234, \quad Q = \frac{2 \times t_g'/t_p'}{1 + t_g'/t_p'} = 1.384$$

$$\therefore \quad y_p' = 0.154 - \frac{0.912}{48.07} = 0.135$$

$$V = \frac{\pi \times 260 \times 600}{60 \times 1000} = 8.168 \text{ m/s}, \qquad \therefore K_v = \frac{6}{6 + V} = 0.424$$

$$\therefore \quad F_r = K_v\, \sigma\, y_p'\, Bp\left(\frac{L - B}{L}\right)$$

$$= 0.424 \times 105 \times 0.135 \times 60 \times \pi \times 6.5 \left(\frac{234.36 - 60}{234.36}\right)$$

$$= 4850.144 \text{ N}$$

$$F_w = 0.75\ d_p B k_w Q/\cos\ \theta_p$$
$$= 0.75 \times 260 \times 60 \times 0.96 \times 1.384/\cos 33.69° = 18682.75$$
$$F_o = 12923.797\text{ N}$$

∴ Power calculation should be based on bending strength

$$\therefore \qquad P = \frac{4850 \times 8.168}{1000} = 39.57\text{ kW}$$

If dynamic load is considered, $F_d \le F_w$. As $F_o < F_w$, $F_d \le F_o$ should be satisfied. Now

$$F_d = F_t + \frac{21V(CB + F_t)}{21V + \sqrt{CB + F_t}} = 11438.679\text{ N} = F_0$$

By trial and error $F_t = 3000$ N

$$\therefore \text{ Power to be transmitted} = \frac{3000 \times 8.168}{1000} = 24.5\text{ kW}$$

16.7 A pair of straight teeth bevel gears is used for transmitting 7.5 kW at 900 r.p.m of pinion. Pinion has 20 teeth and module at the outer radius of 5 mm. Find the face width and check it for wear and dynamic load. Gear ratio 3 : 1.

Solution:

$$d_p = 60\text{ mm},\ d_g = 180\text{ mm},\ L = \sqrt{r_p^2 + r_g^2} = 94.86\text{ mm}$$

$$V = \frac{p \times 60 \times 900}{60{,}000} = 2.827\text{ m/s}$$

$$\therefore \qquad F_t = \frac{7.5 \times 1000}{2.827} = 2653\text{ N}$$

$$K_v = \left(\frac{5.6}{5.6 + \sqrt{2.827}}\right) = 0.769$$

Let $\sigma_p = 110$ MPa. Let $B = L/k = 94.86/k$, $y'_p = 0.154 - \dfrac{0.912}{t'_p}$

$$\theta_p = \tan^{-1}\frac{1}{3} = 18.434°$$

$$\therefore \qquad t'_p = \frac{20}{\cos 18.434°} = 21.08, \qquad \therefore \quad y'_p = 0.110$$

∴ Lewis Equation

$$2653 = 0.769 \times 110 \times 0.110 \times \frac{94.86}{k}\pi \times 5 \times \left(\frac{k-1}{k}\right)$$

$k = 4$ gives $F_t = 2630$ N, ∴ Use $B = 30$ mm

$$F_o = 110 \times 0.110 \times 30 \times \pi \times 5 \times \left(\frac{94.86 - 30}{94.86}\right) = 3898.7\text{ N}$$

$$F_w = 0.75\frac{d_p b k_w Q}{\cos\theta_p} = \frac{60 \times 30 \times k_w \times Q \times 0.75}{\cos\theta_p}$$

Let $S_{es} = 600$ MPa, $\therefore\ k_w = 0.879$

As $\theta_g = 90° - 18.434 = 81.566°$

$$\therefore \quad t_g' = \frac{60}{\cos 71.566°} = 189.74$$

$$\therefore \quad Q = \frac{2 \times 189.74/21.08}{189.74/21.08 + 1} = 1.8$$

$$\therefore \quad F_w = 60 \times 30 \times 0.879 \times 1.8 \times \frac{0.75}{\cos 18.434°} = 2251.49 \text{ N}$$

$$F_d = F_t + \frac{21V\,(BC + F_t)}{21V\sqrt{BC + F_t}}$$

Let us assume $e = 0.04$ mm so that

$$C = \frac{k_d \cdot e}{1/E_p + 1/E_g}$$

$$C = \frac{0.114 \times 0.04}{2/2 \times 10^5} = 456 \text{ N/mm}$$

$$\therefore \quad F_d = 2653 + \frac{21 \times 2.827\,(30 \times 456 + 2653)}{21 \times 2.827 + \sqrt{30 \times 456 + 2653}} = 5180.6 \text{ N}$$

This is much bigger than F_o and F_w. The remedy is to redesign the gear with increased diameter by increasing number of teeth.

16.8 A bevel gear is made of cast steel with permissible static stress under bending of 110 MPa, p.c.d of gear is 500 mm and module 2.0 mm. The teeth are 20° full depth. Find the beam and wear strength of gear. Pitch cone angle 30°.

Solution:

$$t = \frac{500}{2} = 250, \quad \therefore \quad t' = \frac{250}{\cos 30°} = 288.675$$

$$y' = 0.154 - \frac{0.912}{288.675} = 0.1508$$

Let $B = L/3$, $\therefore$ bevel factor = 0.67, $L = \dfrac{250}{\sin 30°} = 500$ mm

$$\therefore \quad F_0 = 110 \times 0.1508 \times \frac{500}{3} \times \pi \times 2.00 \times 0.67 = 11638.511 \text{ N}$$

$$F_w = d_p B k_w Q'/\cos\theta_p$$

Let us use this gear with another gear of a diameter which can be found out by

$$L = \sqrt{r_p^2 + r_g^2}, \qquad \therefore\ 500 = \sqrt{250^2 - r_g^2}$$

$\therefore \quad r_g = 433$ mm

$\therefore \quad t_g = 433, \quad \therefore$ gear ratio = 1.732

and $\quad t_g' = \dfrac{433}{\cos 60} = 866, \quad \therefore\ G' = 866/288.675 = 2.99$

$\therefore \quad Q' = 1.5$

$$k_w = \frac{600^2}{1.4} \times \sin 20 \left(\frac{1}{2 \times 10^5} + \frac{1}{2 \times 10^5} \right) = 0.879$$

$$F_w = 500 \times \frac{500}{3} \times 0.879 \times \frac{1.5 \times 0.75}{\cos 30°} = 93009.311 \text{ N}$$

16.9 A pair of bevel gear is to transmit 20 kW at 1800 r.p.m of pinion. Gear ratio is 3 : 1. Profile of tooth is 14½° composite. Assume $L/B = 3$. If the material for both gears is C.I with permissible static bending stress of 55 MPa, determine face width, module and p.c.d of both gears assuming number of teeth on pinion equal to 24.

Solution:

$$t_p' = \frac{24}{\cos \theta_p} \quad \text{where } \theta_p = \tan^{-1} \frac{1}{3} = 18.43°$$

$$\therefore \quad t_p' = 25.3, \quad L = \frac{24\,m}{2 \sin \theta_p} = 37.957\,m$$

$$y_p' = 0.124 - \frac{0.684}{25.3} = 0.09696, \quad \text{bevel factor} = 0.67$$

$$V = \frac{\pi \times 24\,m \times 1800}{60,000} = 2.262 \text{ m/s}, K_v = \frac{6}{6+V} = \frac{6}{6 + 2.262\,m}$$

$$F_t = \frac{20,000}{2.262\,m} = 8841.73/m$$

Lewis Equation

$$\frac{8841.73}{m} = \left(\frac{6}{6 + 2.262\,m} \right) \times 55 \times 0.09696 \times \frac{37.957\,m}{3} \times \pi m \times 0.67$$

By trial and error $m = 6$ mm

$\therefore \quad F_t = 1473.62$ N, $\quad F_0 = 4806.9484$ N

$d_p = 144$ mm, $\quad d_g = 432$ mm.

OBJECTIVE QUESTIONS

16.1 The bevel gears are used to connect
(a) two parallel shafts (b) two intersecting shafts
(c) two non-intersecting shafts (d) none of the above

16.2 Bevel gears with shafts angle of 90° are termed as
(a) Zerol gears (b) Angular bevel gears
(c) Mitre gears (d) Hypoid gears

16.3 The bevel gears used for connecting non-intersecting shafts are
(a) Mitre gears (b) Hypoid gears (c) Spiral bevel gears (d) Zerol gears

16.4 Face width of the bevel gear is usually equal to
(a) 10 modules (b) pitch cone radius/3
(c) pitch cone radius/2 (d) none of the above

16.5 Formative number of teeth on bevel gears is equal to
(a) $2 \times$ actual number of teeth
(b) Actual number of teeth/cos ϕ
(c) Actual number of teeth/cos α
(d) Actual number of teeth/cos θ where ϕ—pr. angle, α—shaft angle, θ = cone angle of pinion or gear

16.6 Lewis Equation for bevel gear is corrected for
(a) variation in p.c.d (b) variation in tooth thickness
(c) taking care of axial thrust (d) variation in torque acting on the tooth

16.7 Ratio factor Q in wear load equation of bevel gear is given by

(a) $\dfrac{2 \text{ gear ratio } G}{G+1}$ (b) $\dfrac{2[\text{ratio of formative number of teeth gear and pinion } G']}{G'+1}$

(c) $\dfrac{G+1}{G'+1}$ (d) $\dfrac{2G'}{G}$

16.8 Bevel factor should not be less than
(a) 0.75 (b) 0.8 (c) 0.67 (d) 0.76

16.9 Pitch cone angle of pinion of straight tooth bevel gear pair with gear ratio 1.732 is
(a) 25° (b) 30° (c) 60° (d) none of the above

16.10 The face width of the bevel gear is 0.3 times the radius of pitch cone. Hence the bevel factor must be
(a) 1.3 (b) 3 (c) 0.7 (d) 1.7

16.11 Interchangeability is possible only in
(a) bevel gear (b) helical gears (c) spur gear (d) mitre gear

16.12 Formative number of teeth on bevel gear are found by using
(a) Lewis Equation (b) Buckingham Equation
(c) Tredgold's approximation (d) Bevel factor

REVIEW QUESTIONS

16.1 Explain why bevel factor is introduced in the Lewis Equation for bevel gear and derive the relationship.

16.2 Compare the bevel gear with helical gear.

16.3 Differentiate between spur gear and straight tooth bevel gear.

16.4 What are the different types of bevel gears and their applications?

16.5 Define pitch cone radius, cone angle and shaft angle in case of bevel gears and derive the relationship between the pitch cone angle and shaft angle.

16.6 State the approximation suggested for finding the radius of equivalent spur gear of any bevel gear and give the formative number of teeth formula.

16.7 What is the appropriate relationship for face width of the bevel gear? Why

16.8 State true or false and justify:
(a) Bevel gears are interchangeable.
(b) Bevel gears are manufactured in pairs only.
(c) Mitre gears are used for connecting non-intersecting shafts.
(d) Efficiency of hypoid gears is more than mitre gears.

16.9 Explain the procedure of designing a pair of bevel gears.

16.10 Explain the tooth forces in bevel gears and their significance in the selection of bearing and design of shaft.

16.11 Derive the relationship between the shaft angle and the pitch cone angles of bevel gear pair.

16.12 Differentiate between a spur gear and straight tooth bevel gear.

16.13 Why the diameters of shafts connected by bevel gears are larger than the diameters of shafts connected by spur gear pair transmitting the same power?

PRACTICE PROBLEMS

16.1 Two mitre gears are connected with gear ratio 3 : 1. The module of 20° full depth teeth is 6 mm. The number of teeth on pinion is 20. The power to be transmitted is 6 kW at 600 r.p.m of pinion. Determine the tangential force at mean radius and axial thrust on the pinion and gear. Take B/L = 0.25.

16.2 Two 20° full depth steel bevel gears have module = 3 mm at outer radius; and B.H.N equal to 325. The number of teeth are 30 and 45, the face width B = 50 mm but axial width is 25 mm. Pinion rotates at 100 r.p.m. Find the h.p in kW. Also find the force on the midpoint of the tooth.

16.3 A pair of steel bevel gear is to transmit 15 kW at 600 r.p.m, m = 6.5 mm, t_p = 20, B/L = 1/3. Gear ratio is 3. Safe stress for the material 100 MPa. Design the pair with bending consideration. Shaft angle is 60°.

16.4 A pair of mitre gears transmits 10 kW at 750 r.p.m of pinion. The gear ratio is 3.75. The pinion has 28 teeth of 20° involute full depth. The material has safe static stress 130 MPa. Determine the necessary face width—B.H.N of the material is 300. Check the design for wear strength.

16.5 In the above problem, find the minimum distance between bearings 1 and 2 and calculate the reaction on bearings 1, 2 and 3, 4. (Ref. Fig. P-16.5).

16.6 A straight tooth bevel gear pair is made from hardened steel with permissible bending stress of 140 MPa. The shaft angle is 90° and the gear ratio 3.5 : 1. The pinion rotates at 1400 r.p.m. If the bevel factor is 0.67, find the module, face width, p.c.d of pinion gear using Lewis Equation, if 20 kW is to be transmitted. Pinion has 20 teeth of 20° involute profile.

16.7 Check the design of bevel gear in Problem 16.6 for wear strength and find the dynamic load

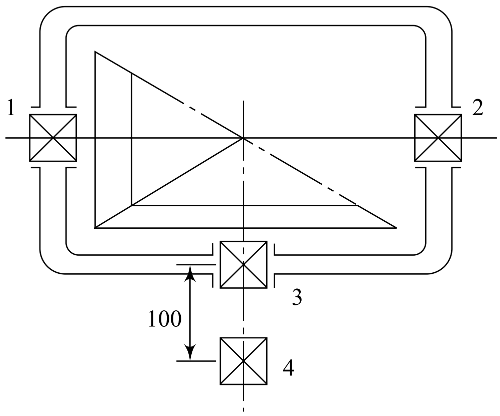


Fig. E-16.5

assuming different errors and state how much is the permissible error. The B.H.N for the material is 300.

16.8 Two 20° full depth involute steel bevel gears with $t_p = 40$, $t_g = 60$ have module $m = 6$ mm at outside diameter $b = 65$ mm, thickness for the blanks is 40 mm, B.H.N = 300. Maximum total error in both gears is 0.17 mm. Find the speed of operation and power transmitted by the pair.

16.9 A pair of bevel gears is required to transmit 20 kW at 600 r.p.m of pinion. The shaft angle is 90° and gear ratio 2.5: 1. The module of 20° full depth teeth is 6.5 mm at outer diameter and number of teeth on pinion is 30. Determine the tangential force and axial thrust at the mean radius. Assume $b/1 = 1/3$.

16.10 A pair of bevel gears has C.I gear and C.S pinion with 60 MPa and 150 MPa respectively safe static stresses. The speed of pinion 600 r.p.m and that of gear 240 r.p.m. The teeth are 20° full depth involute with module 5 mm at the outer periphery. $L/b = 3/1$. Find the safe power that can be transmitted based on Lewis Equation. $T_p = 20$.

16.11 A pair of straight tooth bevel gears transmits 30 kW at 1000 r.p.m of pinion. The shaft angle is 90° and gear ratio 4 : 1. The pinion has 24 machine cut straight teeth with 200 pressure angle of forged steel with safe static stress of 140 MPa. Determine the module, necessary face width, d_p and d_g using Lewis Equation.

16.12 A pair of straight tooth bevel gears has shaft angle 90°, $t_p = 24$, $t_g = 32$. Power to be transmitted is 12.5 kW at 1440 r.p.m of pinion. Pressure angle is 20°. The material used for gears 30C8 steel with safe bending stress of 150 MPa. Find the module based on beam strength. Specify B.H.N assuming factor of safety of 2 for wear consideration. Use dynamic load equation based on equivalent mass of gears. Maximum torque may be taken 50% greater than average torque.

16.13 Two 20° full depth steel bevel gears have module 6 mm at outside radius, B.H.N 325, $t_p = 30$, $t_g = 45$, $b = 62$ mm; axial width 35 mm. Power to be transmitted 45 kW. Find the speed at which pinion rotates. $e_p = 0.08636$ mm, $e_g = 0.09144$ mm.

ANSWERS

Objective Questions

(1) b (2) c (3) b (4) b (5) d (6) a (7) b (8) c (9) c
(10) c (11) c (12) c

Practice Problems

(1) 5714.21 N, 608 N: 1988.94 N
(2) 42.22 kW based on wear strength
(3) 4 mm
(4) 52 mm, safe $F_w = 11832$ N
(5) 430 mm, Vertical reaction $Rv_1 = 201.58$ N, $Rv_2 = 2073.42$ N, $Rv_3 = 168$ N, $Rv = 590$ N
(6) 5 mm module, $b = 60$ mm
(7) Permissible error = 0.01 mm
(8) $V = 10.754$ m/s, $P = 123.88$ kW using $F_2 = F_w/2$
(9) $F_t = 3921.56$ N, $F_a = 1325.25$ N
(10) 8.625 kW
(11) $m = 4$ mm, $b = 66$ mm, $d_p = 96$ mm, $d_g = 384$ mm
(12) 5 mm, B.H.N = 290
(13) 596 r.p.m

17

Worm Gear

CONCEPT REVIEW

17.1 INTRODUCTION

It is a form of spiral gear used for transmitting power between non-intersecting shafts and the shaft angle of 90°.

The ordinary cylindrical worm is a single or multiple start threaded screw while worm gear is a helical gear.

17.2 NOMENCLATURE

N_g — No. of teeth on gear

S_w — No. of starts of threads

p_c — Circular pitch of gear = Axial pitch of worm p_a

α = angle of helix

$$= \tan^{-1} \frac{l}{\pi d_w}$$

l = lead = $S_w p_a$

$$R = \text{Transmission ratio} = \frac{\text{speed of pinion}}{\text{speed of gear}} = \frac{N_g}{S_w}$$

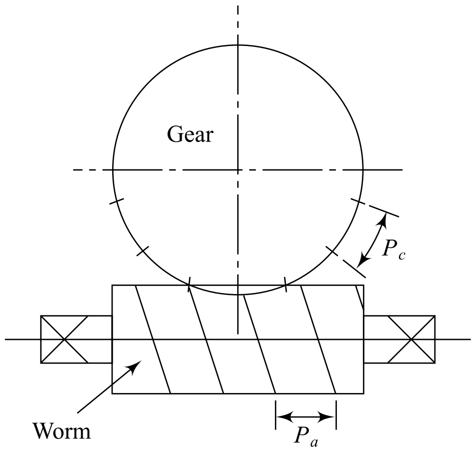


Fig. 17.1

17.3 DESIGN EQUATIONS

The material for worm is steel and that for gear is phosphor bronze. The design is based on many empirical relations.

Let D_g and d_w be diameter of worm and gear so that

$$C = \frac{D_g + d_w}{2} \tag{i}$$

But $$d_w = \frac{C^{0.875}}{3.48} = 3\,p_c \tag{ii}$$

where C is in metres,

$$\text{face width } b = 0.73\, d_w \tag{iii}$$

L—length of worm in axial direction

$$= p_c\left(4.5 + \frac{N_g}{50}\right) \tag{iv}$$

Lewis Equation

$$F_t = \sigma\, b\, y\, \pi\, m_n\, K_v \text{ N where } K_v = \frac{6}{6 + V_g} \tag{v}$$

Wear Load Equation

$$F_w = D_g dB \text{ N where } B \text{ is constant obtained from table} \tag{vi}$$

Dynamic load

$$F_d = F_0 = \sigma b y \pi m_n \text{ N} \tag{vii}$$

Power rating with heat dissipation point of view

$$P = \frac{3650\, C^{1.7}}{R + 5} \text{ kW} \tag{viii}$$

C—Centre distance in metres $\qquad$ R—Transmission ratio $\frac{N_g}{S_w}$

AGMA Power Rating Equation

$$P = \frac{n}{R} KQV \text{ kW} \tag{ix}$$

where n = Speed of worm r.p.m $\qquad$ R = Transmission ratio $\qquad$ K = Pressure constant from table

$$Q = \frac{R}{R + 2.5} \qquad V = \frac{2.3}{2.3 + V_w + \dfrac{3V_w}{R}}$$

where V_w = peripheral velocity of worm in m/s.

17.4 PRACTICAL DESIGN ASPECTS

The worm gear is useful for a very large transmission ratio up to 500. It may be made self-locking due to which it may be used in hoists. The drive is very compact and noiseless; but the friction between the worm and gear causes loss of power.

The pressure angle should not be less than 20° for single and double start worm and 25° for triple start worm.

17.5 FORCE ANALYSIS

From Fig. 17.2, three mutually perpendicular components of resultant force between worm and gear are

F_{t_g} = tangential force on gear = axial force F_{a_w} on worm

F_{t_w} = tangential force on worm = axial force F_{a_g} on gear

F_r = radial force common to both known as separating force.

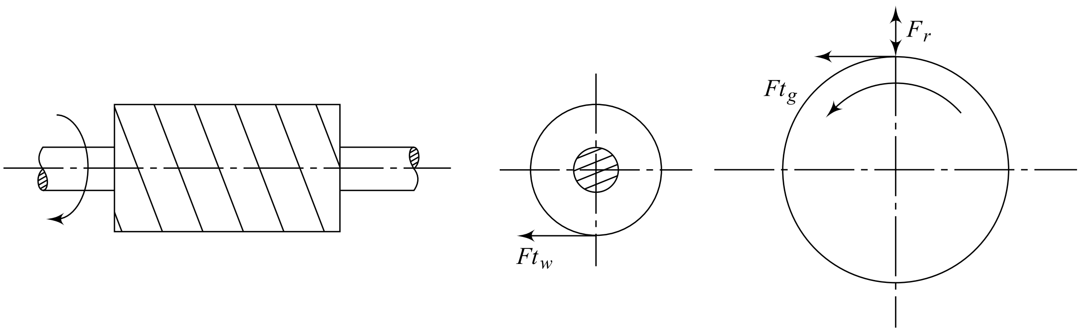


Fig. 17.2

They are obtained as

(1) $F_{t_w} = \dfrac{2\text{ Torque}}{d_w}$ or $F_{t_w} = \dfrac{\text{Power}}{V_w}$

(2) $F_{t_g} = F_{t_w} = \left(\dfrac{1 - \mu \tan\alpha/\cos\phi_n}{\tan\alpha + \mu/\cos\phi_n}\right)$

where μ = coefficient of friction
α = helix angle
ϕ_n = normal pressure angle

(3) $F_r = F_{t_g}\left(\dfrac{\sin\phi_n}{\cos\phi_n\cos\alpha - \mu\sin\alpha}\right) = F_{t_w}\left(\dfrac{\sin\phi_n}{\cos\phi_n\sin\alpha + \mu\cos\alpha}\right)$

(4) Efficiency of drive = $\tan\alpha\left[\dfrac{\cos\phi_n - \mu\tan\alpha}{\cos\phi_n\tan\alpha + \mu}\right]$

WORKED EXAMPLES

17.1 A double threaded worm has helix angle of 20° and axial pitch 30 mm. Find d_w.
Solution:

$$\alpha = \tan^{-1}\frac{l}{\pi d_w}, \qquad \therefore d_w = \frac{l}{\pi\tan 20°} = \frac{30\times 2}{\pi\tan 20°} = 52.5\text{ mm.}$$

17.2 A triple start worm has pitch diameter 105 mm and pitch in axial direction 25 mm. Determine the angle of helix.
Solution:

$$\alpha = \tan^{-1}\frac{25\times 3}{\pi\times 105} = 12.8°.$$

17.3 A worm gear reducer unit has centre distance 250 mm. Find the worm diameter and normal module if the angle of helix is 20°.

Solution:

$$d_w = \frac{C^{0.875}}{3.48} = \frac{(0.250)^{0.875}}{3.48} = 0.08543 \text{ m} = 85.43 \text{ mm} = 3\,p_c$$

$\therefore \quad p_c = 28.47 = \pi m_a$

$\therefore \quad m_a = 9.06 \text{ mm}, \qquad \therefore\ m_n = 9.06 \cos 20° = 8.51 \text{ mm}.$

17.4 A worm gear speed reducer unit has $C = 250$ mm, $R = 20$. What is approximate power rating to prevent overheating?

Solution:

$$P = \frac{3650\,C^{1.7}}{R+5} = \frac{3650\,(0.25)^{1.7}}{20+5} = 13.83 \text{ kW.}$$

17.5 In a worm gear drive the speed of the worm is 2000 r.p.m and diameter 75 mm. The transmission ratio is 27 : 1. What is approximate power rating with wear consideration? $k = 0.485$.

Solution:

$$V_w = \frac{\pi \times 75 \times 2000}{60 \times 1000} = 7.854 \text{ m/s}$$

$$\therefore \quad V = \frac{2.3}{2.3 + 7.854 + \dfrac{3 \times 7.854}{27}} = 0.208$$

$$Q = \frac{27}{27 + 2.5} = 0.915$$

$\therefore$ Using Eq. (ix) $\quad P = \dfrac{2000}{27} \times 0.915 \times 0.208 \times 0.485 = 6.837 \text{ kW.}$

17.6 For a hardened steel worm and gear the centre distance is 450 mm. Transmission ratio is 20. Find the axial module and the lead angle.

Solution:

$$d_w = \frac{(0.45)^{0.875}}{3.48} = 0.1428 \text{ m} = 3p_c = 3\,\pi m_a$$

$\therefore \quad m_a = 0.0151515$ m say 16 mm

$$\tan\alpha = \frac{l}{\pi d_w} = \frac{\pi m_a \times \text{No. of starts}}{\pi d_w}$$

But transmission ratio $= \dfrac{\text{No. of teeth on gear}}{\text{No. of starts on worm}}$

$$\therefore \quad \text{No. of starts} = \frac{t_g}{20} = \frac{D_g/m_a}{20} = \frac{D_g}{20 \times m_a}$$

$$\therefore \quad \tan\alpha = \frac{\pi m_a}{\pi d_w}\left(\frac{D_g}{20 \times m_a}\right) = \frac{D_g}{20\,d_w}$$

Again $\quad C = \dfrac{D_g + d_w}{2}, \qquad \therefore \quad D_g = 900 - 142.8 = 757.2$ mm

$$\therefore \quad \tan\alpha = \frac{757.2}{20 \times 142.8}, \qquad \therefore \quad \alpha = 14.84°.$$

17.7 A hardened steel worm rotates at 1500 r.p.m and transmits power to a phosphor bronze gear. The transmission ratio is 15 : 1 and centre distance 225 mm. Find the power transmitted by the drive by considering all the criteria. $K = 0.727$ for wear equation.

Solution:

$$d_w = \frac{(0.225)^{0.875}}{3.48} = 0.078 \text{ m} = 78 \text{ mm}$$

$$\therefore \quad p_c = \frac{d_w}{3} = 26 \text{ mm}$$

With single start thread $t_g = 15$, $\quad \therefore \quad D_g = \dfrac{15 \times 26}{\pi}$

$$\therefore \quad \frac{D_g + d_w}{2} \ll \text{C.D.}$$

With double start threads $t_g = 30$, $D_g = \dfrac{30 \times 26}{\pi}$

Again $\quad \dfrac{D_g + d_w}{2} < \text{C.D.}$

$\therefore$ With triple start threads $t_g = 45$, $\quad \therefore D_g = 372$ mm

Now $\quad \dfrac{D_g + d_w}{2} = \dfrac{372 + 78}{2} = 225 \text{ mm} = \text{ C.D.}$

and $\quad p_c = 26 = \pi m_a, \quad \therefore m_a = 8.276$ mm say 8 mm

$\therefore \quad D_g = 45 \times 8 = 360$ mm

$\therefore \quad d_w = 90$ mm, $b = 0.73\, d_w = 65$ mm

Power by using Lewis Equation

let $\quad \sigma = 55$ MPa for phosphor-bronze

Assuming $14\frac{1}{2}°$ full depth teeth

$$y = 0.124 - \frac{0.684}{45} = 0.1088$$

$$K_v = \frac{6}{6 + V_g} \text{ again } V_g = \frac{\pi \times 360 \times 1500}{60 \times 1000 \times 15} = 1.885 \text{ m/s}$$

$$\therefore \quad K_v = 0.76,\ m_n = 8 \cos \alpha$$

where $\quad \alpha = \tan^{-1} \dfrac{3 \times \pi \times 8}{\pi \times 90} = 14.93°$

$$\therefore \quad F_t = 0.76 \times 55 \times 65 \times 0.1088 \times \pi \times 8 \cos 14.93$$
$$= 7178.67 \text{ N}$$
$$F_o = 9445.61 \text{ N}$$

$$P = F_t V/1000 = \frac{7178.67 \times 1.885}{1000} = 13.53 \text{ kW}$$

Power by heat dissipation criteria using Eq. (viii)

$$= \frac{3650}{15+5}\ (0.225)1.7 = 14.45 \text{ kW}$$

Power with wear consideration

$$V_w = \frac{\pi \times 90 \times 1500}{60 \times 1000} = 7.068 \text{ m/s}$$

$\therefore$ $K = 0.727,\ Q = 15/(15 + 2.5) = 0.857$

$\therefore$ Using eq. (ix) $P = \frac{1500}{15} \times 0.727 \times 0.2133 \times 0.857 = 13.28 \text{ kW}$

As the power by wear consideration is the minimum the same may be treated as the capacity of the drive, i.e. $P = 13.28$ kW.

17.8 Design a worm and worm gear drive for a speed reduction by 25. Pinion (worm) rotates at 600 r.p.m and transmits 35 kW.

Solution:

From heat dissipation equation

$$P = \frac{3650(C)^{1.7}}{R+5}, \qquad \therefore C = \left[\frac{35(25+5)}{3650}\right]^{\frac{1}{1.7}} = 0.48 \text{ m.}$$

$$\therefore \quad d_w = \frac{(0.48)^{0.875}}{3.48}$$

$$= 0.1512 = 3\,P_e, \qquad \therefore P_e = 50.4 \text{ mm}$$

$$D_g = 960 - d_w = 808.8 \text{ mm}$$

For double start threads

$$t_g = 50, \qquad \therefore D_g = \frac{50 \times 50.4}{\pi} = 802.14 \text{ mm}$$

$\therefore$ Use double start threads with 50 teeth. Assuming 20° full depth teeth

$$y = 0.154 - \frac{0.912}{50} = 0.1357$$

$$V_g = \frac{\pi \times 802 \times 24}{60 \times 1000} = 1 \text{ m/s}$$

$$K_v = \frac{6}{6+1} = 0.857,\ b = 0.73,\ d_w = 110.376 \text{ mm say } 110 \text{ mm}$$

$$\alpha = \tan^{-1} \frac{2 \times 50.4}{\pi \times 151.2} = 11.98°$$

$$F_t = \frac{P}{V} = 35{,}000 \text{ N}$$

Lewis Equation

$\therefore \quad 35{,}000 = 0.857 \times 6 \times 0.1357 \times 110 \times 50.4 \cos 11.98°$

$\therefore \quad \sigma = 55.5$ MPa so modify $b = 112$ mm

$$F_w = D_g bB, \qquad B = 690 \text{ kN/m}^2 \text{ from table}$$

$$= \frac{802.14}{1000} \times \frac{110}{1000} \times 690 = 60.88 \text{ kN}, \qquad \therefore F_w > F_t$$

Power with wear criteria, $K = 4.405$ from table

$$V_w = \frac{\pi \times 151.2 \times 600}{60 \times 1000} = 4.75 \text{ m/s}$$

$$\therefore \quad V = \frac{2.3}{2.3 + 4.75 + \dfrac{3 \times 4.75}{25}} = 0.30$$

$$Q = \frac{25}{25 + 2.5} = 0.909$$

$$\therefore \quad P = \frac{600}{25} \times 0.301 \times 0.909 \times 4.405 = 28.92 \text{ kW}$$

$\therefore$ 28.92 kW should be transmitted.

17.9 In a worm gear speed reducer the centre distance should not exceed 250 mm, the gear ratio 12.5, the worm speed is 2500 r.p.m. Determine the dimensions and the capacity of the reducer. Assume $14\frac{1}{2}°$ involute teeth.

Solution:

$$d_w = \frac{(0.25)^{0.875}}{2.48} = 0.08543 = 3\,P_c$$

$\therefore \quad P_c = 0.0284, \qquad \therefore m_a = 9$ mm

For single start worm gear teeth number is 12.5.

For double start worm gear teeth number is 25

$$D_g = 25 \times 9 = 225 \text{ mm}$$

This gives centre distance $= \dfrac{225 + 85.43}{2} < 250$ mm

$\therefore$ Let us use quadruple start threads so that

$$D_g = 50 \times 9 = 450 \text{ mm}$$

$$\therefore \quad C = \frac{450 + 85.43}{3} = 267.7 \text{ mm} > 250 \text{ mm}$$

$\therefore$ Let us try $m_a = 8$ mm so that $d_w = 100$ mm, $d_g = 400$ mm

$$\alpha = \tan^{-1} \frac{4 \times 8}{100} = 17.75°$$

Face width $b = 0.73 \times 100 = 73 \text{ mm} \simeq 75$ mm

Capacity of gear of P — Bronze for which $\sigma = 55$ MPa,

$$V_g = \frac{\pi \times 400 \times 200}{60{,}000} = 4.188 \text{ m/s}$$

$$\therefore \quad K_v = \frac{6}{6 + 4.188} = 0.588$$

y from table of $14\frac{1}{2}°$ = 0.110

$\therefore \quad F_t = K_v\, \sigma\, y\, b\, \pi\, m_a = 0.588 \times 55 \times 0.110 \times 75 \times \pi \times 8 = 6705.541$ N

$\therefore$ Power based on beam strength

$$= F_t V = \frac{6705 - 541 \times 4.188}{1000} = 28.08 \text{ kW}$$

17.10 In the above problem find the capacity of gear drive based on wear and heat dissipation and decide what should be the input power, $K = 0.881$.

Solution:

$$V_w = \frac{\pi \times 100 \times 2500}{60,000} = 13.089 \text{ m/s}$$

$$\therefore \quad V = \frac{2.3}{2.3 + 13.089 + \dfrac{3 \times 13.89}{12.5}} = 0.1241$$

$$Q = \frac{12.5}{12.5 + 2.5} = 0.833$$

$$\therefore \quad P = \frac{2500}{12.5} \times 0.881 \times 0.883 \times 0.1241 = 18.214 \text{ kW}$$

Power capacity based on heat dissipation

$$= \frac{3650(0.225)^{1.7}}{12.5 + 5} = 19.758 \text{ kW}$$

By referring to the three answers obtained by three criteria it is obvious that the input power is limited by wear criteria and should be 18.214 kW.

17.11 A worm gear drive transmits 6 kW at 1200 r.p.m with transmission ratio 20 : 1. The worm has triple start threads and pitch diameter 70 mm. Module is 6 mm and number of teeth on gear are 60 with 20° stub. $\mu = 0.1$. The arrangement is as shown in Fig. 17.2. Calculate F_{t_w}, F_{t_g} and F_r.

Solution:

$$V_w = \frac{\pi \times 70 \times 1200}{60,000} = 4.4 \text{ m/s}$$

$$\therefore \quad F_{t_w} = \frac{P}{V_w} = \frac{6000}{4.4} = 1363.6363 \text{ N}$$

$$\alpha = \tan^{-1} \frac{\text{lead}}{\pi d_w} = \tan^{-1} \frac{3 \times \pi \times 6}{\pi \times 70} = 14.4°$$

Using Eqs (2), (3) and (4) from Article 17.5

We get $F_{t_g} = 3654$ N, $F_r = 1411.85$ N, efficiency = 0.688

OBJECTIVE QUESTIONS

17.1 Worm and worm gear is a type of

(a) spur gear (b) helical gear (c) spiral gear (d) bevel gear

17.2 Worm gear drive can be used for gear ratios up to

(a) 15 (b) 200 (c) 5000 (d) 500

17.3 Worm gear drive is used for hoists because it is
(a) very efficient (b) noiseless
(c) self-locking (d) capable of taking large load

17.4 For worm gear with double start worm, pressure angle should not be less than
(a) 20° (b) 30° (c) 10° (d) 40°

17.5 Shaft angle in case of worm and gear is
(a) 60° (b) 90° (c) 180° (d) 120°

17.6 The transmission ratio in worm gear is given by
(a) D_g/d_w (b) $\dfrac{\text{No. of teeth on gear}}{\text{No. of threads on worm}}$
(c) No. of teeth on gear/No. of starts on worm (d) none of the above

17.7 Efficiency of worm gear used for hoist is less than
(a) 90% (b) 70% (c) 60% (d) 50%

17.8 The type of gear with minimum axial thrust is
(a) bevel gear (b) worm gear (c) helical gear (d) Herringbone gear

17.9 Heat dissipation is an important criteria of design in case of
(a) spur gear (b) bevel gear (c) worm gear (d) helical gear

17.10 Material used for gear is always bronze in a pair of
(a) spur gears (b) helical gears (c) hypoid gears (d) worm gears

17.11 For a speed ratio of 100 smallest gear box is obtained by using
(a) pairs of spur gear
(b) a pair of bevel and a pair of spur gear in compound gear train
(c) a pair of helical and a pair of spur gear in compound gear train
(d) a pair of helical gear and a pair of worm gear in compound gear train

17.12 For transmission ratio of 25 and triple start worm, the number of teeth on worm gear is
(a) 38 (b) 75 (c) 100 (d) 50

17.13 Type of gearing used to connect two non-parallel shafts with shaft angle other than 90° is
(a) mitre gear (b) Herringbone gear (c) spiral gear (d) worm gear

17.14 Usual proportion of face width b and diameter of worm d_w is
(a) $b = 0.5$ (b) $b = \dfrac{d_w}{3}$ (c) $b = 0.73\, d_w$ (d) $b = 0.75\, d_w$

REVIEW QUESTIONS

17.1 Explain the merits and demerits of worm gear drive as compared with the other types.
17.2 Why hardened steel is used for the worm and phosphor-bronze for gear?
17.3 What are the different criteria of deciding the power to be transmitted by the worm and worm gear drive?
17.4 Why is the centre distance an important parameter in the design of worm and worm gear?
17.5 Why is heat dissipation a very important aspect of worm and worm gear?
17.6 State true or false justifying your answer:
(1) The velocity ratio of worm and worm gear drive is independent of the diameters of gears.
(2) The worm and gear are interchangeable.

(3) Worm gear drive can provide the speed reduction only up to 8 to 10.
(4) Wear is not important in the design of worm gear drive?
(5) Worm gear drive is a type of helical gears.

17.7 What are the limits of helix angle and pressure angle in worm gear drive.

17.8 What are the types of bearings used on the shaft of worm and worm gear and why?

17.9 Draw the sketch of worm and worm gear drive and explain the forces involved.

17.10 The gear ratio of worm and worm gear wheel does not depend on the proportions of diameters. Explain how is this fact beneficial in the design of drive.

17.11 State true or false, justifying your answer:
(i) Worm gear drive is suitable for hoists.
(ii) Centre distance is not important parameters in the worm gear drive.

PRACTICE PROBLEMS

17.1 A worm gear has helix angle of 19° and centre distance of 236.18 mm. m_n = 6.5 mm. Find the number of starts and number of teeth on gear if the speed ratio is 20 : 1.

17.2 A four start worm meshes with a gear of 60 teeth. Helix angle is 23°, m_n = 8.5 mm, ϕ_n = 20°. The worm rotates at 1200 r.p.m. Find the centre distance, power transmitted by considering Lewis Equation and heat capacity.

17.3 Find the power transmitted in the above problem if the calculations are based on wear capacity. K = 0.727.

17.4 In a worm gear drive the speed of the worm is 1500 r.p.m and diameter 65 mm. The transmission ratio 15 : 1. What is approximate power rating based on wear consideration. K = 0.485.

17.5 A hardened steel worm rotates at 2500 r.p.m and transmits power to a phosphor bronze gear rotating at 50 r.p.m. Centre distance is 300 mm. Find the power transmitted by the drive by considering all criteria. K = 1.7.

17.6 It is necessary to use worm gear drive for which the power transmission capacity based on heat dissipation is 14.45 kW and the transmission ratio is 15. Find d_w, d_g, number of teeth on gear, number of starts on worm and check the design for bending strength.

17.7 A worm gear reducer unit has centre distance of 425 mm. Determine approximate diameter of worm and axial pitch.

17.8 Find the power rating of worm gear drive for a centre distance of 300 mm and gear ratio 25, based on heating capacity.

17.9 A double threaded worm has pitch diameter 75 mm. The wheel has 20 teeth and pitch diameter 125 mm. Find the value of helix angle.

17.10 A worm gear set has a helix angle of 17° and centre distance 235 mm. Normal module is 6.5 mm. Find suitable values of number of starts and number of teeth, for a speed ratio of 20 : 1.

17.11 A worm and worm gear unit has transmission ratio of 35. The speed of the steel worm is 2500 r.p.m. The worm wheel is made of phosphor-bronze. The tooth form is $14\frac{1}{2}^\circ$ involute. Design the drive and check for beam and wear strength and dynamic tooth load. The power to be transmitted is 1.5 kW.

17.12 Find the power that can be transmitted to a phosphor-bronze gear from a hardened steel worm rotating at 1500 r.p.m with a transmission ratio of 25. The centre distance is 300 mm. The teeth are $14\frac{1}{2}°$ involute.

17.13 Find the diameter of worm for worm gear drive with $\alpha = 20°$ and pitch 35 mm double start threads.

17.14 A worm gear reducer unit is used for centre distance of 275 mm. Find the worm diameter and axial module.

17.15 If for the above unit gear ratio is 20 what is approximate power input rating in order to prevent overheating?

17.16 For a pair of worm and worm gear with transmission ratio 20 and centre distance 400 mm. Find the axial module and the lead angle.

17.17 A hardened steel worm rotating at 1250 r.p.m transmits power to a phosphor bronze gear with transmission ratio 15 : 1, centre distance is 225 mm. Recommend the power for which the pair should be used.

17.18 Find the power transmitted by considering failures in bending, wear and heat dissipation for a work and worm gear drive and decide the safe power to be transmitted if the worm rotates at 2500 r.p.m and *p*-bronze gear rotates at 100 r.p.m. Centre distance is 300 mm for *p*-bronze safe $\sigma = 55$ MPa.

17.19 Compute the efficiency of drive in the above problem.

17.20 In the above problem assuming power to be transmitted 15 kW, find the diameter of worm and gear shafts.

17.21 A hardened steel worm rotating at 2500 r.p.m tansmits power to a *p*-bronze gear with transmission ratio 20 : 1. The centre distance is 240 mm. The teeth are $14\frac{1}{2}°$ involute. Complete the design.

ANSWERS

Objective Questions

(1) c (2) d (3) c (4) a (5) b (6) c (7) d (8) d (9) c
(10) d (11) d (12) b (13) c (14) c

Practice Problems

(1) Triple start, 60 (2) 320.52 mm, 17.57 kW, 26.37 kW (3) 12.94 kW
(4) 11.35 kW (5) Lewis Equation 14.7 kW, Heat Cap. 8.57 kW, Wear Cap. 11.57 kW
(6) $d_w = 90$ mm, $d_g = 360$ mm, triple start, $t_g = 45$ (7) 136 mm, 45.33 mm (8) 15.71 kW
(9) 9° – 27'–7" (10) Double start, 40 (11) Single start $d_w = 40$, $d_g = 140$, $m = 4$ mm
(12) Double start, $t_g = 50$, $m = 10$ mm (13) 61.21 mm
(14) 92.86 mm, $ma = 9.85$ mm (15) 16.263 kW (16) 14 mm, 14.57° (17) 12.748 kW
(18) 15.71 kW with heat dissipation criteria. Calculated value of $P_c = 34$ mm should be reduced to 32 mm to get centre distance 305 mm with double start threads.
(19) 40%.
(20) Worm shaft 60 mm, gear shaft 60 m assuming length of shaft between the bearing 2*b*.

18

Sliding Contact Bearings

CONCEPT REVIEW

18.1 INTRODUCTION

Sliding contact bearings are ordinary journal bearings. They are classified as:

(1) Thick film type: In which the surfaces are completely separated from each other by the lubricant.

(2) Thin film type: In which although lubricant is present, surfaces are partially in contact with each other. It is also termed as boundary lubricated.

(3) Hydrostatic bearings: In which the fluid film pressure is obtained by supplying the lubricant at high pressure through a set of holes in the bearing shell located such that the force exerted by the pressure supports the loaded shaft at all the points (Fig 18.1).

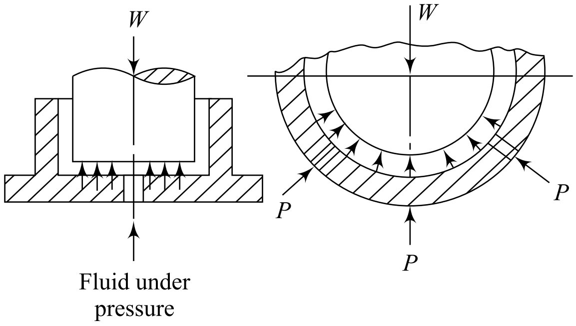


Fig. 18.1

(4) Hydrodynamic bearings: In this type the fluid film pressure is generated only by the rotation of the journal. The position of the journal gets adjusted in such a manner that a force supporting the journal load is produced due to film pressure.

The figures show the positions of shaft and bearing under different conditions. Figure 18.2(c) shows the radial pressure distribution and Fig. 18.2(d) indicates the longitudinal pressure distribution.

Hydrostatic lubrication assures the pressure of lubricant between shaft and bearing but involves use of pump which requires elaborate arrangement.

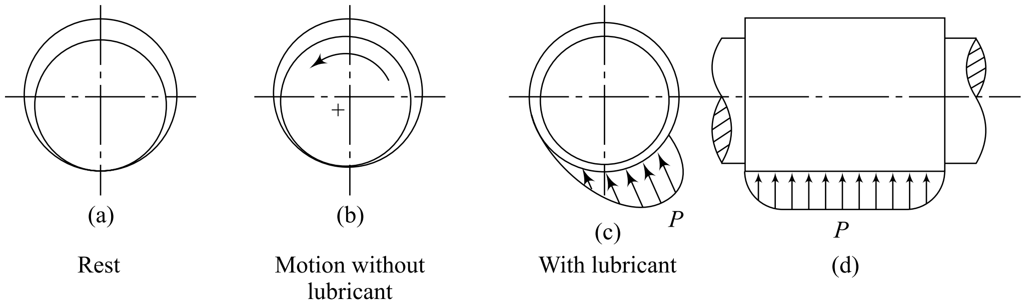


Fig. 18.2

18.2 VISCOSITY

The particles of lubricant adhere strongly to the moving and stationary plates. Motion is accompanied by shear between the particles throughout the entire height of the film thickness. If A is the area of the plate in contact with the fluid, the unit shearing stress is given by $\tau = P/A$. According to Newton this shear stress is directly proportional to the velocity v of the moving plate and inversely proportional to the film thickness h

$$\therefore \qquad \tau = \frac{P}{A} \propto \frac{v}{h} \quad \text{or} \quad \tau = \frac{P}{A} = Z\frac{v}{h}$$

where Z is defined as the viscosity of the fluid and is given by,

$$Z = \frac{P \cdot h}{A \cdot v} \qquad \text{(i)}$$

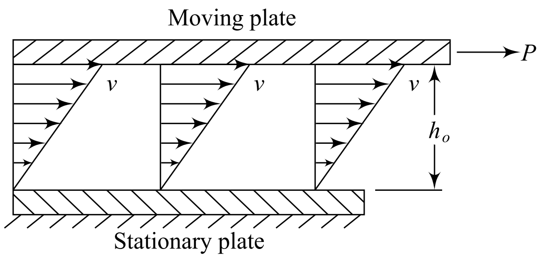


Fig. 18.3

Unit of viscosity is a poise. Fluid is said to have absolute viscosity of one poise if a force of one dyne is required to maintain a plate at a velocity of one cm/s, the plate having a constant area of one square cm and being separated from the stationary plate by a fluid film thickness of one cm.

1 poise = 100 centipoise or 1 cp = 0.01 poise

In S.I. units, unit of viscosity is $\frac{N \cdot s}{m^2}$, $\frac{kg}{ms}$ or PaS

1 cp = 10^{-3} PaS = 10^{-9} M PaS

If the plates are parallel, vertical or normal load cannot be supported. But if the stationary plate is inclined so that the film thickness varies from h_1 to h_2 where it leaves, the velocity gradient is not the same at the entry and exit. The moving plate then can support a load W due to velocity pressure relationship causing pressure built-up in the oil film from zero to maximum and again to zero.

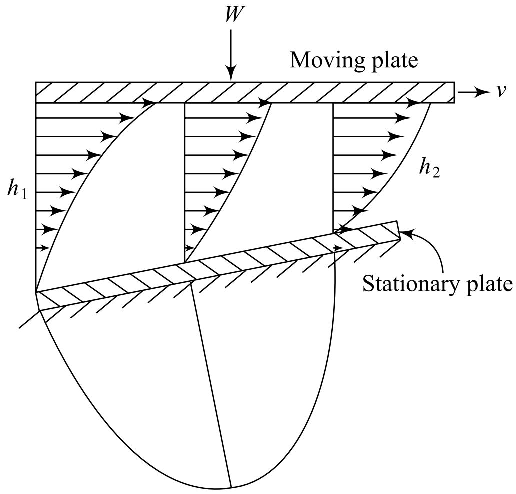


Fig. 18.4

18.3 PETROFF'S EQUATION

If flat moving plate and stationary plate are rolled into cylindrical surfaces forming journal and bearing the tangential force is given by μW where μ is the coefficient of friction.

$\therefore$ Torque $= \mu W \cdot r = \mu \cdot p \cdot 2r \cdot l \cdot r = 2\mu p r^2 l$ (A)

where p is the pressure intensity in MPa, $2rl$ projected area in m^2, r is the radius of shaft.

Again $W = \tau \cdot A$, using definition of Eq (i) (1)

$$\therefore \quad \text{Torque} = \tau \cdot A \cdot r = z \cdot \frac{v}{h} \cdot 2\pi r l \cdot r \tag{B}$$

Equating (A) and (B)

$$\mu = 2\pi^2 \left(\frac{ZN}{p}\right)\frac{r}{c_r} \tag{ii}$$

In this equation $h = c_r$ where c_r is the radial clearance. This equation is known as Petroff's equation published in 1883 and is approximation for lightly loaded bearings and is useful in deciding the coefficient of friction.

18.4 BEARING CHARACTERISTIC NUMBER

The term $\frac{ZN}{p}$ in Petroff's equation is termed as bearing characteristic number. The graph of $\mu \alpha \frac{ZN}{p}$ is as shown in Fig. 18.5. If N is taken in revolutions per minute, viscosity in PaS, pressure in Pascals then equation for straight line portion of the graph is given by

$$\left.\begin{aligned} &\mu = 0.326\left(\frac{ZN}{p}\right)\frac{r}{c_r} + k \\ \text{or} \quad &\mu = 0.326\left(\frac{ZN}{p}\right)\frac{D}{c} + k \end{aligned}\right\} \text{(iii)} \quad \begin{aligned} &\text{if } c \text{ is radial clearance} \\ &\text{if } c \text{ is diametral clearance} \end{aligned}$$

$k = 0.002$ for L/D between 0.75 to 2.8 where L and D are the length and diameter of bearing.

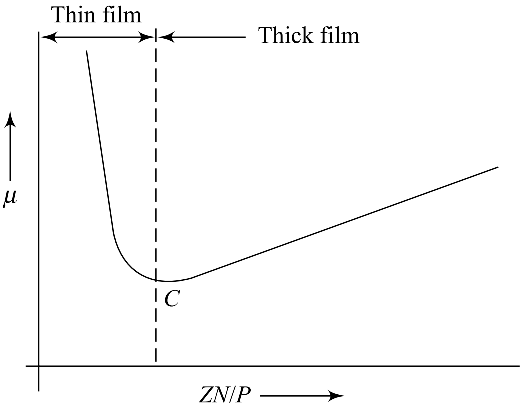


Fig. 18.5

This graph is obtained by McKee brother's hence the Eq. (iii) is known as McKee's equation. This graph predicts the performance of a bearing.

The value of $\frac{ZN}{p}$ at point C which gives minimum coefficient of friction is denoted by α and is termed as bearing modulus. When bearing operates near this value, slight increase of pressure or decrease in speed may be accompanied by increase in friction, wear and heating. To prevent such condition the bearing should operate at values of $\frac{ZN}{p}$ at least 3 times α and if the bearing is subjected to large fluctuation of load and heavy impacts value of $\frac{ZN}{p}$ as high as 15 α may be used.

18.5 NOMENCLATURE

In hydrodynamic bearing the following terminology is used.

D — diameter of journal mm
c — diametral clearance mm, c_r — radial clearance mm
W — load in Newton
h_0 — minimum film thickness mm
L — length of bearing mm
e — eccentricity mm

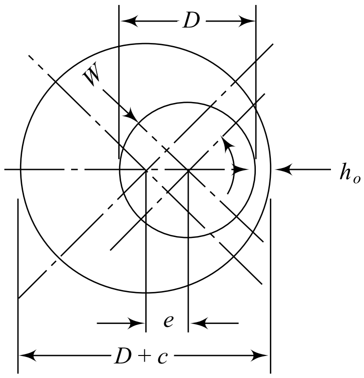


Fig. 18.6

Clearance ratio $= \frac{c}{D}$ or $\frac{c_r}{r}$ if radial clearance is given.

From Fig. 18.6 $e = \frac{c}{2} - h_0$ [or $(c_r - h_0)$ if radial clearance is given]

$$\text{Attitude} = \frac{\text{eccentricity}}{\text{radial clearance}} = \frac{2e}{c} = 1 - \frac{2h_0}{c}$$

18.6 HEAT GENERATED AND DISSIPATED

Heat generated is due to friction torque T_f

$$T_f = \mu W \cdot d/2$$

$$\therefore \qquad H_g = \frac{2\pi N}{60} T_f = \mu W \cdot \frac{D}{2} \cdot \frac{2\pi N}{60} = \mu WV \text{ watts} \qquad \text{(iv)}$$

Heat dissipated is given by Lasche's equation

$$H_d = \frac{(\Delta t + 18)^2}{K} LD \text{ watts} \qquad \text{(v)}$$

$$\Delta t = T_B - T_A$$

Difference between bearing temp. T_B and surrounding temp. T_A

K = 0.273 for bearing of heavy construction in °C m^2/W

= 0.484°C m^2/W with light or medium construction

If operating temperature t_0 is given, then $\Delta t = \frac{1}{2}(t_0 - t_A)$.

18.7 PARAMETERS AFFECTING THE BEARING PERFORMANCE

Small clearance causes more heat generation but has greater load capacity. Large clearance reduces the heat generation but is mechanically undesirable. The value of clearance ratio c/D should be between 0.001 to 0.002.

Value of h_o, for satisfactory operation of bearing should be 0.00015 mm diameter.

Larger L/D ratio is desirable to minimize end leakage but with the point of view of space limitation, manufacturing tolerances and shaft deflection, smaller L/D ratio is desirable. L/D should be within 1 to 2 and $L/D < 1$ used for highly loaded engine bearings.

The pressure at which oil film breaks and metal to metal contact begins is known as critical pressure. It depends on the material and degree of smoothness of surfaces in contact.

18.8 SOMMERFIELD AND OTHER DIMENSIONLESS NUMBERS

The theory of hydrodynamic lubrication is based on Reynold's equation. Comprehensive computer solutions to this equations were obtained by Raimondi and Boyde and have been tabulated in the tables giving correlation between many dimensionless numbers. They are also available in the form of charts. These charts or tables are entered using a number which is introduced in Article 18.4 and is known as Sommerfield number given by

$S = \frac{ZN'}{p}\left(\frac{D}{c}\right)^2$ or $\frac{ZN}{p}\left(\frac{r}{c_r}\right)^2$ where $\frac{D}{c}$ or $\frac{r}{c_r}$ is the reciprocal of clearance ratio. N' is $N/60$, i.e. revolutions per sec.

WORKED EXAMPLES

18.1 A shaft of 100 mm diameter rotates at 1500 r.p.m in a bearing of 120 mm length. Load on the shaft $W = 45$ kN. Operating temperature T_0 is 80°C, $T_A = 35$°C, $\frac{ZN}{p} = 20 \times 10^{-6}$. Determine coefficient of friction, pressure intensity, H_d, type of oil to be used. $D/c = 1000$.

Solution:

$$p = \frac{W}{L \times D} = 3.75 \text{ MPa}$$

$$\frac{ZN}{p} = \frac{Z \times 1500}{3.75 \times 10^6} = 20 \times 10^{-6}, \therefore Z = 0.050 \text{ kg/ms}, \therefore \text{ Use SAE 70 oil}$$

Using McKee's equation, Eq. (iii)

$$\mu = 0.002 + 0.326 \times 20 \times 10^{-6} \times 1000 = 0.00852$$

$$V = \frac{\pi \times 100 \times 1500}{60 \times 100} = 7.854 \text{ m/s}$$

$$\therefore \quad H_g = \mu WV = 0.00852 \times 45 \times 1000 \times 7.854 = 3011.2236 \text{ Nm} = 3.011 \text{ kJ}$$

H_d by Lasche's equation is $\frac{(\Delta t + 18)^2}{K} LD$

$$\Delta t = \frac{1}{2}(t_0 - t_A) = \frac{1}{2}(80 - 38) = 21°\text{C}$$

$$K = 0.484°\text{C} \cdot \text{m}^2/\text{W}$$

$$\therefore \quad H_d = \frac{(21 + 18)^2}{0.484} \times \frac{120}{1000} \times \frac{100}{1000} = 37.71 \text{ Joules}$$

As $H_g > H_d$ artificial cooling is required.

18.2 A 50 mm diameter 75 mm long journal bearing is loaded with bearing pressure of 2 MPa. The shaft in bearing rotates at 500 r.p.m. Heat is lost from surface at the rate of 11.6 Joules/m^2 per sec per °C. The housing area is 8 times the projected area. If the room temperature is 28°C, determine the surface temperature of bearing. $\mu = 0.0015$.

Solution:

$$W = 50 \times 75 \times 2 = 7500 \text{ N as } L = 75 \text{ mm}, D = 50 \text{ mm}$$

$$V = \frac{\pi DN}{60} = \frac{p \times 50 \times 500}{1000 \times 60} = 1.3 \text{ m/s}$$

$$\therefore \quad H_g = 7500 \times 1.3 \times 0.0015 = 14.625 \text{ J/s}$$

$$H_d = 8C \times L \times D \times (t_s - 28) = H_g \qquad C = 11.6 \text{ J/m}^2 \text{ sec °C}$$

$$\therefore \quad (t_s - 28) = \frac{14.625 \times 1000 \times 100}{8 \times 11.6 \times 75 \times 50} = 30.93$$

$$\therefore \quad t_s = \text{surface temp.} = 58.93°\text{C}$$

18.3 A 3 kN load is supported by a journal bearing of 75 mm diameter and 75 mm length. Diametral clearance 0.05 mm and bearing is lubricated by an oil of 0.0207 PaS viscosity at operating temperature. Determine the maximum speed of rotation of bearing when it is capable of dissipating 80 watts by heat transfer.

Solution:

$$\mu = 0.002 + 0.326 \left(\frac{ZN}{p}\right)\frac{75}{0.05}$$

$$Z = 0.027 \text{ PaS}, p = \frac{3000}{75 \times 75} = 0.5333 \text{ MPa}$$

$$\therefore \quad \mu = 0.002 + 0.326 \left(\frac{0.0207\text{ N}}{0.5333 \times 10^6}\right)(1500)$$

$$= 0.002 + 1.9 \times 10^{-5}\text{ N}$$

$$H_d = H_g = 80 \text{ watts} = \mu WV = (0.002 + 10.9 \times 10^{-5}\text{ N}) \times 300 \times \frac{\pi \times 75 \times N}{100 \times 60}$$

$$= 0.02356\text{ N} + 2.24 \times 10^{-4}\text{ N}^2$$

Solving $N = 547.7$ r.p.m.

18.4 A journal bearing 300 mm long, 150 mm diameter carries a radial load of 9 kN at 1200 r.p.m. The power lost in friction is 6 kW. Viscosity of oil at room temperature is 0.018 PaS. Find the diametral clearance.

Solution:

$$\mu = 0.002 + 0.326 \left(\frac{0.018 \times 1200}{p}\right)\frac{150}{C}$$

$$p = \frac{9000}{300 \times 150} = 0.2 \text{ MPa}$$

$$\therefore \quad \mu = 0.002 + 0.326 \left(\frac{0.018 \times 1200}{0.2 \times 10^6}\right)\frac{150}{C} = 0.002 \times \frac{5.2812}{C} \times 10^{-3}$$

$$V = \frac{\pi \times 150 \times 1200}{60 \times 1000} = 9.424 \text{ m/s}$$

$$H_g = \mu wV = \left[0.002 + \frac{5.2812}{C} \times 10^{-3}\right] \times 9000 \times 9.424 = 6000 \text{ watts}$$

Solving $c = 0.76$ mm.

18.5 A 75 mm journal bearing 100 mm long is subjected to 2.5 kN at 600 r.p.m. If the room temperature is 24°C, what viscosity of oil should be used to limit the bearing surface temperature at 55°C. $D/c = 1000$.

Solution:

$$\Delta t = 55 - 24 = 31°\text{C}$$

Using Lasche's equation

$$H_d = \frac{(31+18)^2}{0.484} \times \frac{100}{1000} \times \frac{75}{1000} = 37.2 \text{ watts}$$

$$p = \frac{2500}{100 \times 75} = 0.3333 \text{ MPa}, V = \frac{\pi \times 75 \times 600}{1000 \times 60} = 2.356 \text{ m/s}$$

$$\therefore \qquad \mu = 0.002 + \left(\frac{Z \times 600}{0.3333 \times 10^6}\right) 1000 \times 0.326 = 0.002 + 0.5868\, Z$$

$$H_g = \mu WV = (0.002 + 0.5868Z) \times 2500 \times 2.356 = H_d = 37.2$$

$$\therefore \qquad Z = 0.00735 \text{ PaS}$$

18.6 A 50 mm diameter journal bearing rotates. at 1500 r.p.m. $L/D = 1$, radial clearance 0.05 mm, minimum film thickness = 0.01 mm. Calculate the maximum radial load that the journal bearing can carry and still operate under hydrodynamic condition. For this load, calculate power lost in friction and the oil temperature assuming the $H_g = H_d$. Absolute viscosity = 20×10^{-3} PaS, sp. gravity of oil 0.88, sp. heat of oil 2.1 kJ/kg °C.

Solution:

From Table 29 (App. 1) for $\frac{h_0}{c_r} = \frac{0.01}{0.05} = 0.2$, Sommerfield number is 0.0446

$$\therefore \qquad 0.0446 = \frac{ZN'}{p}\left(\frac{D}{c}\right)^2 = \frac{0.02 \times 1500}{60\, p}\left(\frac{50}{0.1}\right)^2$$

(Radial clearance 0.05 mm, ∴ diametral Cl. 0.1 mm)

$$\therefore \qquad p = 2.8 \times 10^6 \text{ Pa,}$$

$$\therefore \text{ Max. radial load } W = 2.8 \times 10^6 \times \frac{50}{1000} \times \frac{50}{1000} = 7000 \text{ N}$$

$$V = \frac{\pi \times 50 \times 1500}{60 \times 1000} = 3.92 \text{ m/s}$$

corresponding to $\frac{h_0}{c_r} = 0.2$ from Table 29 (App. 1) $\frac{\mu r}{c_r} = 1.7$

$$\therefore \qquad \mu = \frac{1.7 \times 0.05}{25} = 3.4 = 10^{-3}$$

$$\therefore \qquad \text{Power lost in friction} = \mu WV$$

$$= 3.4 \times 10^{-5} \times 7000 \times 3.92 = 93.3 \text{ watt/s}$$

$$\therefore \qquad H_g = 93.3 \text{ J/s}$$

From the table $\frac{Q}{rc_r N' L} = 4.62$

$$\therefore \qquad Q = 4.62 \times 25 \times 0.05 \times 25 \times 50 = 7218.75 \text{ mm}^3$$

$$\therefore \qquad \text{mass} = \frac{7218.75}{10^9} \times 0.88 \times 1000 \text{ kg}$$

As $\qquad H_g = H_d$

$$\therefore \qquad 93.3 = 2.1 \times \frac{7218.75}{10^9} \times 0.88 \times 1000 \times \Delta t \times 1000$$

$$\therefore \qquad \Delta t = 6.993°$$

18.7 In a journal bearing diameter of shaft 75 mm, $L/D = 1$, radial clearance 0.05 mm, minimum film thickness 0.02 mm, speed of journal 400 r.p.m, radial load 3.5 N, sp. gravity of oil 0.9 and specific heat 1.75 kJ/kg/°C. Calculate viscosity of suitable oil, power lost in friction and resultant temperature rise.

Solution:

$$\frac{h_0}{c_r} = \frac{0.02}{0.05} = 0.4 \text{ for which } S = 0.121 \text{ from Table 29 (App. 1)}$$

$$p = \frac{3500}{75 \times 75} = 0.622 \text{ MPa}, N' = \frac{400}{60} = 6.7 \text{ r.p.s}$$

$$\frac{D}{c} = \frac{75}{0.1} = 750$$

Using
$$S = \frac{ZN'}{p}\left(\frac{D}{C}\right)^2 = 0.121$$

$$Z = \frac{0.121 \times 0.622 \times 10^6}{6.7 \times (750)^2} = 0.01997 \text{ PaS}$$

For
$$\frac{h_0}{c_r} = 0.4,\ \mu\frac{r}{c_r} = 3.22 \text{ from Table 29 (App. 1)}$$

$\therefore$
$$\mu = \frac{3.22}{750} = 4.3 \times 10^{-3}$$

$$V = \frac{\pi \times 75 \times 400}{60 \times 1000} = 1.57 \text{ m/s}$$

$\therefore$
$$H_g = 4.3 \times 10^{-3} \times 3500 \times 1.57 = 23.62 \text{ Joules}$$

For
$$\frac{h_0}{c_r} = 0.4,\ \frac{Q}{rc_r N'L} = 4.33$$

$\therefore$
$$Q = 4.33 \times 37.5 \times 0.05 \times 6.7 \times 75 \text{ mm}^3 = 4079.67 \text{ mm}^3$$

$\therefore$
$$H_d = \frac{4079.67}{10^9} \times 0.9 \times 1000 \times 1.75 \times 1000 \times \Delta t$$

Equating
$$H_g = H_d$$
$$\Delta t = 3.675°\text{C}$$

18.8 A sleeve bearing 50 mm diameter and 50 mm long has a journal speed of 3600 r.p.m. The radial load on bearing is 4.5 kN. Oil used is SAE 10 at an average operating temperature of 60°. If $\frac{h_0}{c} = 0.5$, calculate the radial clearance, heat loss, the side flow and the minimum film thickness.

Solution:

$$p = \frac{4.5 \times 1000}{50 \times 50} = 1.8 \times 10^6 \text{ Pa}$$

From C-17 chart Z for SAE 10 oil at 60°C is 13×10^{-3} PaS.

From Table 29 for $\frac{h_0}{c} = 0.5,\ S = 0.18$

$$S = \left(\frac{r}{c}\right)^2 \left(\frac{ZN'}{p}\right) = 0.18 = \left(\frac{25}{c_r}\right)^2 \left(\frac{13 \times 10^{-3} \times 60}{1.8 \times 10^6}\right)$$

$\therefore \qquad c_r = 0.0387 \text{ mm}, \therefore \dfrac{r}{c_r} = \dfrac{25}{0.0387} = 646$

From chart C-19 for $\quad S = 0.18,\ \mu \dfrac{r}{c_r} = 4.3$

$\therefore \qquad \mu = \dfrac{4.3}{646} = 6.656 \times 10^{-3}$

$$V = \frac{\pi DN}{60} = \frac{\pi \times 50 \times 3600}{60 \times 1000} = 9.424 \text{ m/s}$$

$\therefore \qquad H_g = \mu WV = 6 \times 656 \times 10^{-3} \times 4500 \times 9.424 \text{ W} = 282.27 \text{ Joules}$

From chart C-21 for $\quad S = 0.18$, flow variable = 4.2

$\therefore \qquad Q = 4.2 \times 50 \times 0.0387 \times 60 \times 50 = 24381 \text{ mm}^3/\text{s}$

$$\frac{h_0}{c_r} = 0.5, \qquad \therefore h_0 = 0.0387 \times 0.5 = 0.01935 \text{ mm}$$

18.9 Load on 100 mm full bearing is 9000 N, speed 320 r.p.m. $L/D = 1$ $c_d/D = 0.0011$, operating temp. 65°C, $h_0 = 0.022$ mm. Select the oil and calculate its friction loss, hydrodynamic oil flow through the bearing, the amount of oil leakage, the temperature rise and maximum pressure.

Solution:

$$c = 0.0011 \times 100 = 0.11 \text{ mm}$$

$\therefore \qquad \dfrac{2h_0}{c} = \dfrac{0.022 \times 2}{0.11} = 0.4$ from Table 29 corresponding value of S is

$$S = 0.121 = \frac{ZN'}{p}\left(\frac{r}{c}\right)^2 = \frac{Z \times 320 \times 100 \times 100}{60 \times 9000 \times 10^6} \times \left(\frac{1}{0.0011}\right)^2$$

$\therefore$ SAE 30 oil is used $Z = 2.47 \times 10^{-2}$

For SAE 30 oil at 65°C, $Z = 0.0255$ PaS

$\therefore \qquad \mu = 0.022 + \dfrac{0.326 \times 0.0255 \times 320}{0.9 \times 10^6} \times \dfrac{1}{0.0011} = 4.97 \times 10^{-3}$

and $\qquad V = \dfrac{\pi \times 100 \times 320}{60 \times 1000} = 1.6755 \text{ m/s}$

$\therefore \qquad H_g = \mu WV = 4.97 \times 10^{-3} \times 9000 \times 1.6755 = 74.94 \text{ W}$

For changed viscocity $\quad S = \dfrac{0.0255 \times 320}{60 \times 0.9 \times 10^6}\left(\dfrac{1}{0.0011}\right)^2 = 0.1249$

From chart C-21 for this value of S

$\therefore \qquad \dfrac{Q}{rc_r N' L} = 4.33, \therefore Q = \dfrac{4.33 \times 50 \times 50 \times 0.0011 \times 320 \times 100}{60}$

$$= 6350.66 \text{ mm}^3$$

$$\frac{Q_s}{Q} = 0.680$$

$\therefore \qquad$ End leakage $= 6350.66 \times 0.680 = 4318.44 \text{ mm}^3$

Again $$\frac{\rho c_p \Delta t}{p} = 14.2$$

$$\therefore \qquad \Delta t = \frac{14.2 \times 0.9 \times 10^6}{1.42 \times 10^6} = 9°C$$

Again $$\frac{p}{p_{max}} = 0.415, \therefore p_{max} = \frac{0.9}{0.415} = 2.168 \text{ MPa.}$$

18.10 In 180° centrally loaded bearing with $L/D = 0.5$ and diameter of journal 100 mm, diametral clearance = 0.1 mm and minimum film thickness 0.02 mm, speed is 750 r.p.m. The oil used is SAE 50 at 80°C temperature. Find the load carried by bearing. If the operating temperature is reduced to 62°C, speed is reduced to 475 r.p.m and the load increased to 3000 N; what will be minimum film thickness?

Solution:

From Table 30

$$\text{For } \frac{2h_0}{c} = \frac{0.02 \times 2}{0.1} = 0.4, S = 0.321$$

From chart C-17 $\qquad Z = 30 \times 10^{-3}$ PaS

$$\therefore \qquad 0.321 = \left(\frac{30 \times 10^{-3} \times 750}{60 \times p}\right)(1000)^2$$

$$\therefore \qquad p = 0.375 \text{ MPa}, \therefore W = 0.375 \times 100 \times 50 = 1875 \text{ N}$$

Under new condition

$$Z = 60.17 \times 10^{-3} \text{ PaS}$$

$$p = 0.6 \text{ MPa } N' = 475/60$$

$$\therefore \qquad S = 0.794 \text{ for which } \frac{2h_0}{c} = 0.6, \therefore h_0 = 0.03 \text{ mm.}$$

18.11 The following data refers to a full journal bearing. Load 5000 N, speed of shaft 300 r.p.m, diameter of shaft 100 mm, clearance (diametral) 0.1 mm, minimum oil thickness 0.020 mm. Viscosity of oil at the temperature of bearing 12.1×10^{-3} PaS. Find L/D ratio and power lost in friction.

Solution:

Let us assume $L/D = 1$, so that

$$p = \frac{5000}{100 \times 100} = 0.5 \text{ MPa} = 0.5 \times 10^6 \text{ Pa}$$

$$\therefore \qquad S = \frac{ZN'}{p}\left(\frac{D}{c}\right)^2 = \frac{12.1 \times 10^{-3}}{0.5 \times 10^6} \times \frac{300}{60}(1000)^2 = 0.121$$

Thus the assumption of $L/D = 1$ is correct as $S = 0.121$ for $\frac{2h_0}{C} = \frac{0.02 \times 2}{0.1} = 0.4$

Again from Table 29 (App. 1) = $\frac{\mu D}{c} = 3.22$

$$\therefore \qquad \mu = \frac{3.22}{100} = 3.22 \times 10^{-3}$$

$$V = \frac{\pi \times 100 \times 300}{60,000} = 1.57 \text{ m/s}$$

$\therefore$ $\mu WV = 3.22 \times 10^{-3} \times 5000 \times 1.57 = 25.277$ J/s = 25.277 W.

18.12 Load on a hydrodynamic full journal bearing is 20 kN. The diameter and speed of the shaft are 100 mm and 1500 r.p.m respectively. Diametral clearance 0.2 mm. Sommerfield number 0.631. L/D ratio 1 : 1. Calculate temperature rise of oil, Specify oil, and also find the quantity of oil required.

Solution:

From Table 29 (App. 1) for full journal bearing with

$$L/D = 1 \text{ and } S = 0.631 \quad \frac{4q}{DcN'L} = 3.59 = \frac{q}{rc_r n_s L}$$

$\therefore$
$$q = \frac{3.59}{4} \times 100 \times 0.2 \times \frac{1500}{60} \times 100 = 44875 \text{ mm}^3$$

$$p = \frac{20,000}{100 \times 100} = 2 \text{ MPa, density of oil} = 861 \text{ kg/m}^3$$

$$c_p = \text{sp. heat of oil} = 1760 \text{ J/kg°C}$$

From Table 29 (App. 1) $\dfrac{C_p \Delta t}{p} = 52.1$

$\therefore$
$$\Delta t = \frac{52.1 \times 2 \times 10^6}{861 \times 1760} = 68.76\text{°C}$$

18.13 Find the heat generated in the bearing of Example 18.12

Solution:

From Table 29 (App. 1) $\mu \dfrac{D}{c} = 12.8$

$\therefore$
$$\mu = 12.8 \times \frac{0.2}{100} = 25.6 \times 10^{-3}$$

$\therefore$
$$H_g = 25.6 \times 10^{-3} \times 20,000\, V$$

$$V = \frac{\pi \times 100 \times 1500}{60,000} = 7.854 \text{ m/s}$$

$\therefore$ Substituting,
$$H_g = 25.6 \times 10^{-3} \times 20,000 \times 7.854$$
$$= 4021.24 \text{ J/s} = 4.021248 \text{ kW}.$$

18.14 A sleeve bearing is of 40 mm diameter and $L/D = 1$. The speed of rotation 3600 r.p.m. The bearing supports a radial load of 3.6 kN. SAE 10 oil at average operating temperature of 60°C is used. Determine radial clearance for $\dfrac{h_0}{c_r} = 0.5$, where c_r is the radial clearance and h_0 minimum film thickness.

Solution:

From Table 29 for $\dfrac{h_0}{c} = 0.5$

$$S = \frac{0.264 + 0.121}{2} = 0.1925 = \frac{ZN'}{p}\left(\frac{D}{c}\right)^2$$

$$p = \frac{3.6 \times 10^3}{40 \times 40} = 2.25 \text{ MPa}$$

SAE 10 oil at 60°C has $Z = 13.5 \times 10^{-3}$ PaS, putting this value

$$\left(\frac{D}{c}\right)^2 = \frac{0.1925 \times 2.25 \times 10^6}{13.5 \times 10^{-3} \times 60}$$

$\therefore$ $$\frac{D}{c} = 731.25, \therefore\ c = 0.0546 \text{ mm diametral clearance}$$

$\therefore$ Radial clearance = 0.0273 mm.

18.15 A journal bearing 160 mm long and 45 mm, diameter supports a radial load, of 8000 N. The shaft speed is 160 r.p.m, oil used is SAE 60 at 25 °C inlet temperature. Using clearance ratio of 600, find the rise in temperature, maximum film pressure and minimum film thickness.

Solution:

$$p = \frac{8000}{160 \times 45} = 1.11 \text{ MPa}$$

$$N' = \frac{160}{60} = 2.67 \text{ r.p.s}$$

$\therefore$ $$S = \frac{Z \times 2.67}{1.11 \times 10^6}(600)^2$$

As operating temperature is not known, let us assume 60°C so that $Z = 80 \times 10^{-3}$ PaS

$\therefore$ $$S = \frac{80 \times 10^{-3} \times 2.67}{1.11 \times 10^6}(600)^2 = 0.0639$$

From Table 29 for this value of S, $\left(\dfrac{\rho c_p\ \Delta t}{p}\right) \simeq 8.5$, $\rho = 861$ kg/m^3, $c_p = 1760$ Nm/kg

$\therefore$ $$\Delta t = \frac{8.5 \times 1.11 \times 10^6}{861 \times 1760} = 6.22$$

$$t_{av} = 25 + \frac{6.22}{2} = 28.22 \text{ °C}$$

Hence assumed value is not correct. Let us do 2nd trial with 35°C so that $Z = 450 \times 10^{-3}$,

$$\therefore \qquad S = 0.395, \therefore \frac{\rho c_p \,\Delta t}{p} \cong 35$$

$$\therefore \qquad \Delta t = 25.61$$

$$\therefore \qquad 25 + \frac{25.61}{2} = 37.8\ °C$$

This is nearer to the assumed value. For $S = 0.395$, $\dfrac{p}{p_{max}} = 0.8$, $\therefore p_{max} = 1.374$ MPa and also $\dfrac{h_0}{c_r} = 0.94$, $h_0 = 0.94\, c_r$

$$c_r - \text{radial clearance} = \frac{22.5}{600} = 0.0375 \text{ mm}$$

$$h_0 = 0.03525 \text{ mm.}$$

18.16 Find power lost in friction and by finding heat dissipated state whether artificial cooling is required for bearing in Example 18.15.

Solution:

From chart C-29 $\dfrac{\mu r}{c_r} = 12, \quad \therefore \quad \mu = \dfrac{12}{600} = 0.02$

$$\therefore \qquad H_g = \mu W V$$

$$V = \frac{\pi \times 45 \times 160}{60{,}000} = 0.377 \text{ m/s}$$

$$\therefore \qquad H_g = 0.02 \times 8000 \times 0.377 = 60.32 \text{ J/s}$$

$$H_g = \frac{(\Delta t + 18)^2}{0.484} \times \frac{45}{1000} \times \frac{160}{100} \quad \text{putting } \Delta t = 25.61°C = 28.27 \text{ J/s}$$

as $H_g > H_d$ artificial cooling is required.

18.17 Load on 75×75 mm hydrodynamic bearing is 12.5 kN, r.p.m = 2000 and viscosity of oil 10 cp. Clearance ratio 1000. Find minimum film thickness and coefficient of friction.

Solution:

$$S' = \frac{10 \times 10^{-3} \times 2000}{60 \times p}\,(1000)^2$$

$$p = \frac{12.5 \times 10^3}{75 \times 75} = 2.22 \text{ MPa} = 2.22 \times 10^6 \text{ Pa}$$

$\therefore$ Substituting this we get $\quad S = 0.15$; as $c = \dfrac{75}{1000} = 0.075$ mm

and from chart C-18 for $S = 0.15$, $\dfrac{h_0}{c} = 0.48$, $\therefore h_0 = 0.036$ mm. Again from the chart $\mu \dfrac{r}{c} = 4.5$

$$\therefore \qquad \mu = 4.5 \times 10^{-3}.$$

18.18 Find power lost in friction, quantity of oil flowing and rise in temperature in the bearing of Example 18.17.

Solution:

From chart C-21 (App. 2) $\dfrac{q}{rcN'l} = 4.2$

$$q = 4.2 \times \frac{75}{2000} \times \frac{0.075}{2000} \times \frac{2000}{60} \times \frac{75}{1000} = 1.477 \times 10^{-3}\ \text{m}^3$$

Temperature rise variable = 16, p = 861 kg/m^3, c = 1760 Nm/kg

$$\therefore \qquad \Delta t = \frac{16 \times 2.2 \times 10^6}{861 \times 1760} = 23.23\ °\text{C}.$$

OBJECTIVE QUESTIONS

18.1 It is necessary to use pump in bearing of
(a) Hydrodynamic type (b) hydrostatic type
(c) radial (d) thrust type

18.2 In hydrodynamic bearing the fluid film is formed due to
(a) pressure developed by pump (b) rotation of journal
(c) friction between shaft and bearing (d) none of the above

18.3 Radial pressure in hydrodynamic bearing is
(a) uniform
(b) independent of load
(c) is maximum where the thickness of film is minimum
(d) none of the above

18.4 The hydrodynamic bearings are used in the range of value of bearing characteristic number equal to
(a) α (b) 2 α (c) 3 α (d) 20 α
where alpha is the bearing modules.

18.5 Load carrying capacity of bearing can be increased by
(a) increasing Sommerfield number (b) increasing speed
(c) decreasing viscosity (d) increasing minimum film thickness

18.6 The value of L/D for satisfactory operation of journal bearing should be
(a) 10 (b) 0.2 (c) 1 (d) 20

18.7 McKee's equation is useful for finding
(a) Sommerfield number (b) viscosity of oil
(c) minimum film thickness (d) power lost in friction

18.8 Petroff's equation is approximately applicable for finding coefficient of friction in
(a) bearing with $L/D = 0.5$ (b) bearing with $\dfrac{2h_o}{C} = 0.1$
(c) lightly loaded bearing (d) hydrostatic bearing

18.9 The attitude is the ratio of
(a) h_0 and diametral clearance (b) eccentricity and h_0
(c) eccentricity and diametral clearance (d) eccentricity and radial clearance

18.10 Lasches' equation is useful for finding
(a) power lost in friction
(b) temperature of oil film
(c) heat dissipated by bearing
(d) heat generated due to rotation of shaft

18.11 Bearing characteristic number B.C.N and Sommerfield number are related as
(a) $S = \text{B.C.N} \times \frac{r}{c}$
(b) $S = \text{B.C.N} \times D/C$
(c) $S = \text{B.C.N} \times \left(\frac{c^2}{r}\right)$
(d) $S = \text{B.C.N} \times \left(\frac{D}{c}\right)^2$

18.12 For satisfactory performance the clearance ratio in sliding contact bearing should be between
(a) 0.01 to 0.02
(b) 0.005 to 0.01
(c) 0.001 to 0.002
(d) 0.0005 to 0.0008

18.13 High operating temperature
(a) allows increase in the load
(b) affects the minimum film thickness
(c) reduces the load carrying capacity
(d) none of the above

18.14 Increase in the speed of the shaft
(a) decreases the viscosity
(b) increases the power lost in friction
(c) decreases the heat generated
(d) decreases the temperature of bearing.

18.15 Hydrodynamic journal theory is based on the equation of
(a) Petroff (b) McKee (c) Reynold (d) Lasche

18.16 Critical pressure is the pressure at which
(a) operating temperature is minimum
(b) speed is maximum
(c) metal to metal contact begins
(d) viscosity is minimum

REVIEW QUESTIONS

18.1 Differentiate between (a) thick film and thin film lubrication, (b) hydrostatic and hydrodynamic bearings, (c) bearing characteristic number and bearing modulus.
18.2 Define viscosity using Newton's statement.
18.3 Write Petroff's equation and state how the coefficient of friction for lightly loaded bearing is obtained by using it.
18.4 Draw the graph of bearing characteristic number and coefficient of friction and explain the significance of graph.
18.5 Explain the effect of clearance, L/D ratio, temperature, speed and minimum film thickness on the performance of hydrodynamic bearings.
18.6 State the McKees equation for calculation of friction torque.
18.7 State the Lasches' equation for calculating the heat dissipated in hydrodynamic bearing.
18.8 What are the dimensionless parameters used in designing hydrodynamic bearing? Explain their significance.
18.9 Write a short note on materials used for bearing.
18.10 What are the types of lubricants used in practice?
18.11 What is the difference between full journal bearing and partial bearing?
18.12 Explain the distribution of radial and axial pressure in case of hydrodynamic bearing by drawing a neat sketch.

18.13 Explain by drawing the sketch of different terms used in case of hydrodynamic bearings namely clearance ratio, attitude, minimum film thickness, eccentricity etc.
18.14 Write a note on hydrostatic squeeze film lubrication.
18.15 Why is lubrication required in sliding contact bearings?
18.16 What are different systems of lubricating a journal bearing? Give examples.
18.17 How pressure build-up takes place in hydrodynamic lubrication?
18.18 What is Sommerfield number? State its significance in hydrodynamic bearings.

PRACTICE PROBLEMS

18.1 A 40 mm diameter and 50 mm long journal bearing is subjected to a load of 3000 Newton. The oil used is SAE 60 at the temperature of 70°C, speed of the shaft is 800 r.p.m. By using McKee's equation, find the heat generated. $\dfrac{r}{c_r} = 1000$.
18.2 Using Lasches' equation and assuming a lightly constructed bearing, find the rise in temperature, for bearing in Problem 18.1 if heat is carried away by oil.
18.3 If the heavy construction is used what is the rise in temperature for bearing of Problem 18.1?
18.4 A bearing of 75 mm diameter 100 mm length has a shaft rotating at 600 r.p.m. The construction is heavy and dissipates 20% of the heat generated with a rise in temperature 22°C. Find the load it can carry. Viscosity of oil is 30×10^{-3} PaS.
18.5 A bearing rotating at 600 r.p.m is used to carry a load of 40 kN. Diameter and length of bearing are 250 and 100 mm respectively. A radial clearance is 0.125 mm. Find the power loss due to friction for SAE 10, 20, 30 and 40 lubricants. Operating temperature is 70°C.
18.6 Repeat Problem 18.5 using SAE 30 oil and using clearances 0.075, 0.100, 0.125, 0.150, 0.175 mm. Plot a curve showing the relation between the coefficient of friction and the clearance.
18.7 A sleeve bearing D mm diameter, L mm long using G grade of oil operates at temperature T°C. It supports the load W and rotates at N r.p.m, the radial clearance is C_r, find (a) temperature rise and average temperature of lubricant, (b) find coefficient of friction, (c) the magnitude and location of minimum film thickness, (d) the side flow and the total flow, (e) maximum oil film pressure and its angular location.

Problem No.	*L mm*	*D mm*	*G*	*T*	*N r.p.m*	c_r	*WkN*
18.7	75	75	SAE 30	40°C	1120	45 μm	4
18.8	60	60	SAE 40	50°C	1200	25 μm	5
18.9	100	100	SAE 50	60°C	900	75 μm	6
18.10	50	150	SAE 60	70°C	1250	100 μm	8

18.11 A sleeve bearing is 45 mm diameter and $L/D = 1$, clearance ratio is 1000, radial load of 3 kN and speed 1200 r.p.m. The oil used is SAE 40 at inlet temperature of 40°C. Find:
(a) the average oil temperature of the bearing (b) minimum film thickness
(c) maximum oil pressure
18.12 A sleeve bearing 12 mm long and 12 mm diameter uses SAE 10 oil at 50°C. The radial clearance is 0.0080 mm. If the journal speed is 30 r.p.m and the radial load 100 N, find the temperature rise of the lubricant and the minimum film thickness.

18.13 A sleeve bearing is 32 mm long and $L/D = 1$. The shaft rotates at 1750 r.p.m. The clearance is 0.020 mm using SAE = 30 oil at 50°C, the temperature rise is 26° and minimum film thickness is 0.014 mm. Find the load the bearing can carry.

18.14 In 50 mm × 50 mm bearing load is 25 kN and radial clearance is 0.025 mm. Speed is 2000 r.p.m, SAE 30 oil at 30°C inlet temperature is used. Find the minimum film thickness and the frictional loss for a partial bearing of:
(a) 180° (b) 120° (c) 60°

18.15 Find the required length of the journal bearing of 150 mm diameter carrying a radial load of 9 kN at 1200 r.p.m. The diametral clearance is 0.15 mm. The power lost in friction is 6 kW and viscosity of oil at the operating temperature is 0.018 kg/ms.

18.16 A shaft running at 600 r.p.m is supported in bearing of 60 mm × 60 mm. The bearing operates in still air at a room temperature of 30°C. The oil of 0.013 PaS at the operating temperature of 120°C. The diametral clearance is 0.06 mm and minimum film thickness 0.024 mm. Determine permissible load per bearing and the power lost per bearing if there is no external cooling.

18.17 A 360° full journal bearing with $L/D = 1$ and 75 mm diameter carries a load of 36,000 N at 1200 r.p.m. Minimum film thickness is 0.0225 mm. SAE 20 oil is used and the film temperature is 82°C. What is the diametral clearance?

18.18 A 250 × 250 mm bearing carries a load of 108 kN. The bearing rotates at 1500 r.p.m. The clearance ratio is 670. For full journal bearing the power lost in friction is 14.36 kW. Find the viscosity of the oil.

18.19 A 50 × 75 mm bearing rotates at 1200 r.p.m and power lost in friction is 75 W. SAE 10 oil is used with clearance ratio of 1000. Find the load on bearing if it operates at 58°C.

18.20 A 75 × 115 mm bearing rotates at 900 r.p.m at 75°C. Clearance ratio 1000. Room temperature is 38.5°C. Cooling rate is 115 J per m^2 per degree centigrade. SAE 20 oil is used. Intensity of pressure is 9.7 MPa. Assume 120° partial central bearing. Find power loss due to friction and artificial cooling required.

18.21 A 120° central partial bearing is 60 mm in diameter and 100 mm long. r.p.m is 900. Clearance ratio 1000. SAE 20 oil is used and the load on bearing is 8.2 kN. Find minimum oil film thickness if the oil temperature is 73°C.

18.22 Three bearings have respective arcs 120°, 180° and 365°. The diameter and length are 60 mm and 90 mm respectively, r.p.m 1200. Clearance ratio 1000. Intensity of pressure 1.4 MPa. SAE 10 oil is used. Find the film temperature for each bearing using $h_0 = 0.015$ mm.
What is the effect of angle on the operating temperature if film thickness is to be maintained?

18.23 In the above problem film temperature is 70°C for all the three bearings. Find the pressure for each bearing and state the relationship between the angle and the pressure.

18.24 In the above problem the load on bearings is 7000 N. Film temperature is 74°C. Find the minimum film thickness (which is not given) and friction power loss and state the relationship between the angle and power loss.

18.25 A sleeve bearing 25 mm diameter and 25 mm length uses SAE 10 oil at 50°C. The radial clearance is 0.016 mm. The journal speed is 1800 r.p.m and the radial load 400 N. Find the temperature rise and minimum film thickness.

18.26 A sleeve bearing 32 mm long and 32 mm diameter carries a load of 1100 N at 1750 r.p.m. Using SAE 30 oil at 50°C, the temperature rise is 26° and minimum film thickness is 0.014 mm. Find the radial clearance.

18.27 A journal bearing 100 × 100 mm supports a load of 20 kN at a speed of 750 r.p.m diametral clearance is 0.15 mm. Find the viscosity of oil when the bearing temperature is 90°C and still oil temperature 20°C.

18.28 A journal bearing 300 × 300 mm carries a radial load of 13 kN at 1200 r.p.m. Viscosity of oil is 0.018 kg/ms at operating temperature. If 6 kW power is lost in friction find the diametral clearance.

18.29 A journal bearing 125 mm long and 75 mm diameter supports a load of 5 kN. Bearing has diametral clearance 0.05 mm and is lubricated by an oil of viscosity 0.0207 PaS at operating temperature. Determine maximum speed of rotation of bearing when it can dissipate 133.33 joules/s by heat transfer.

18.30 A journal bearing 100 mm × 100 mm supports a shaft rotating at 1500 r.p.m. Radial clearance 0.1 mm, minimum film thickness 0.02 mm. Calculate the maximum radial load the bearing can support and for that load calculate the power lost in friction and oil temperature assuming $H_g = H_d$. Absolute viscosity 20×10^{-3} PaS, specific gravity of oil 0.88, specific heat of oil 2.1 kJ/kg/°K.

ANSWERS

Objective Questions

(1) b (2) b (3) c (4) c (5) b (6) c (7) d (8) c
(9) d (10) c (11) d (12) c (13) c (14) b (15) c (16) c

Practice Problems

(1) 62.49 J/s (2) 105.01°C (3) 74.35°C (4) 24635.82 N
(5) 1050.653, 1261.859, 1434.665, 1722.673 Watts
(6) 3316.322, 2644.301, 2241.085, 1972.281, 1780.6 Watts
(7) $\mu = 0.024$, $\Delta t = 13.13$°C, $h_0 = 0.0576$, $Q_t = 17955$ mm^3, $Q_s = 7720.65$, $P_{\max} = 0.86$ MPa
(8) In the same order, 0.02, 32.07, 0.0345, 6804, 2687.58, 1.6735
(9) 0.021, 7.92, 0.081, 46125, 25830, 0.722
(10) 0.0586, 16.42, 0.16, 187500, 43125, 0.42
(11) $\Delta t = 117.3132$, $h_0 = 0.021375$ mm
(12) $\Delta t = 18.33$, $h_0 = 0.00584$
(13) 2.1 kN
(14) For 180° $h_0 = 0.02$, $\mu = 7.15 \times 10^{-3}$, for 120° $h_o = 0.01728$, $\mu = 5.35 \times 10^{-3}$ for 60° $h_o = 0.011225$, $\mu = 4.11 \times 10^{-3}$

Ans:

Bearing type	h_o *mm*	μ
180°	0.02	7.15×10^{-3}
120°	0.01728	5.35×10^{-3}
60°	0.011225	4.11×10^{-3}

(15) 145 mm (16) 3.867 kN, 23.47 J/s (17) 0.075 mm
(18) 0.0296 PaS (19) 4.9762 kN (20) 252.8 J/s, 216.6 J/s (21) 0.015 mm
(25) 22°C, 0.128 mm (26) 0.020 mm (27) $Z = 4.625 \times 10^{-4}$ PaS
(28) 0.645 mm (29) 547.7 r.p.m (30) 28 kN, rise in temp. 70°C, $H_{(g)} = 747.7$ J/s

19

Rolling Contact Bearings

CONCEPT REVIEW

19.1 INTRODUCTION

In rolling contact bearings as the name implies, the friction is of rolling type. It is very much smaller than the sliding type. The coefficient of friction is as small as 0.001 in ball bearings, 0.0011 in roller bearings and 0.0013 in thrust ball bearings.

19.2 CONSTRUCTION AND TYPES

The rolling contact bearings are composed of four elements:

(1) outer race
(2) inner race
(3) rolling elements
(4) cage or retaining ring.

As shown in Fig. 19.1 inner race is mounted on the shaft tightly and rotates with the shaft. If the shaft is hollow outer race is made to rotate with shaft. The other race is stationary. The rolling elements roll about their axis when outer and inner races rotate with respect to each other. The function of the cage is to keep the rolling element in their respective angular positions.

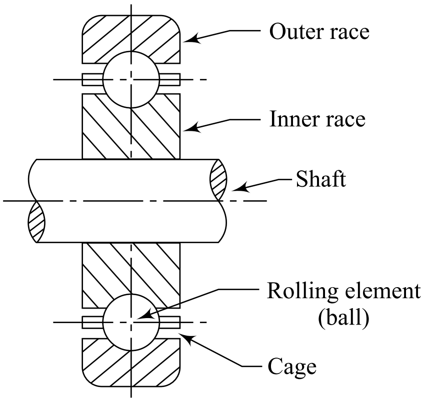


Fig. 19.1

Ball bearings are classified as follows (Fig. 19.2)

1. *Deep groove type:* Single row deep groove ball bearing can take combination of radial and thrust load. They are also of two types. In one type a notch is provided for filling the balls and are termed as filling type. In the other variety the balls are inserted in the grooves by moving the inner ring to an eccentric position. This type is known as Conrod or non-filling type. In this type a limited number of balls may be inserted.
2. *Angular contact ball bearings:* They have more thrust capacity (Fig. 19.2 c).
3. *Self-aligning ball bearings* may tolerate misalignment of shafts (Fig. 19.2 j).

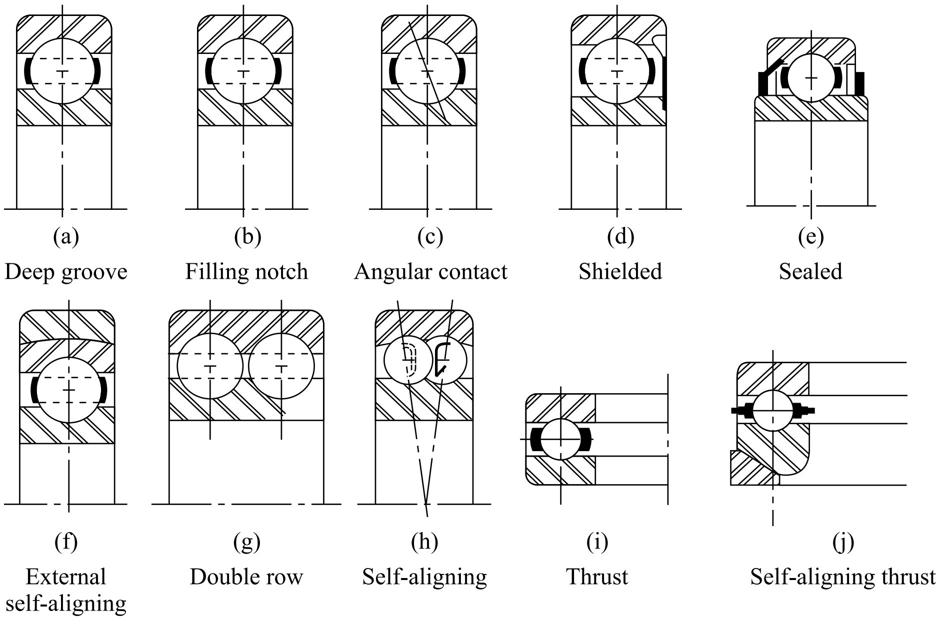


Fig. 19.2

4. *Double row angular contact ball bearings* are also capable of taking heavier load. (Fig. 19.2 g)

For larger capacity, roller bearings are used in place of ball bearings. They are capable of taking only radial or thrust load.

In roller bearings the shapes of the roller are cylindrical, spherical and taper according to which the bearings are termed. Taper roller bearings can carry a combination of thrust and radial loads (Fig. 19.3 (a) to (f)).

Helical rollers are made by winding rectangular material into roller. Due to inherent flexibility they are capable of taking considerable misalignment.

A roller with length much larger than diameter is known as needle roller and are used where radial space is limited: Cage may be absent in needle roller bearings.

There are many other varieties of the rolling contact bearings as shown in Fig. 19.3.

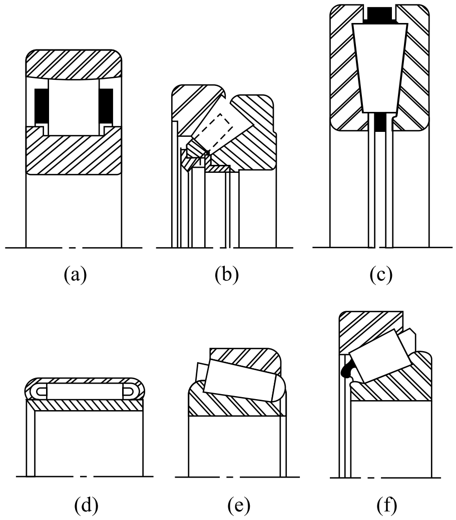


Fig. 19.3

19.3 DEFINITIONS

The selection of ball bearing or roller bearing is based on the following concepts.

Basic static load rating is defined as that static radial load which corresponds to a total permanent deformation of 0.0001 of the ball diameter at the most heavily stressed ball and race contact.

Striebeck Equation

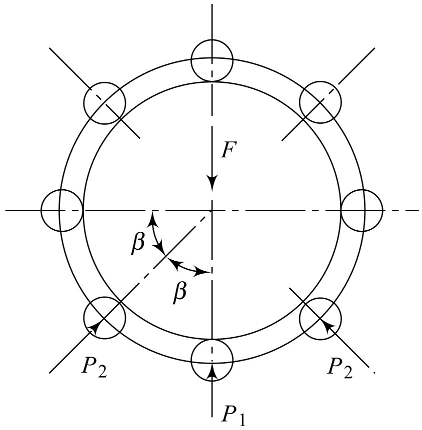


Fig. 19.4

Basic static load rating is based on Striebeck equation which uses Hertz contact stresses.

Let us consider the effect of external load on static bearing

$$F = P_1 + 2\,P_2 \cos\beta + 2\,P_3 \cos 2\beta + \ldots \quad \text{(i)}$$

If radial deflection is taken as

$\delta_1, \delta_2, \delta_3$, etc. then

$$\delta_2 = \delta_1 \cos\beta,\ \delta_3 = \cos 2\,\beta \text{ etc.}$$

$$\therefore \quad \frac{\delta_1}{\delta_2} = \frac{1}{\cos\beta},\ \frac{\delta_1}{\delta_3} = \frac{1}{\cos 2\beta} \quad \text{(ii)}$$

Now according to Hertz contact stress equation

$$\frac{P_1}{P_2} = \left(\frac{\delta_1}{\delta_2}\right)^{3/2} = \frac{1}{(\cos\beta)^{3/2}} \qquad \Bigg| \qquad \frac{P_1}{P_3} = \left(\frac{\delta_1}{\delta_3}\right)^{3/2} = \left(\frac{1}{\cos 2\beta}\right)^{3/2} \quad \text{(iii)}$$

$\therefore$ Equation (i) becomes

$$F = P_1\left[1 + \frac{2P_2}{P_1}\cos\beta + \frac{2P_3}{P_1}\cos 2\,\beta + \ldots\right]$$

$$= P_1\,[1 + 2\,(\cos\beta)^{5/2} + 2\,(\cos 2\beta)^{5/2} + \ldots] = QP_1$$

As $\beta = \dfrac{360}{Z}$ where Z is the number of balls, with $Z = 8\ \beta^2 = 45°$

$$\therefore \quad Q = 1 + 2(\cos 45°)^{5/2} + 2(\cos 90°)^{5/2} = 1.8408$$

or $$\frac{Z}{Q} = \frac{8}{1.8408} = 4.35$$

For $Z = 10$ $$Q = 1 + 2(\cos 36°)^{5/2} + 2(\cos 72°)^{5/2} = 12.2834$$

$$\therefore \quad \frac{Z}{Q} = 4.38$$

Thus Z/Q is nearly constant which is rounded to 5.

$$\therefore \quad Q = \frac{Z}{5}$$

$$\therefore \quad F = \frac{P_1 Z}{5}$$

But experimental results show that for constant stress, load on the ball varies as the square of the diameter

$$\therefore \quad P_1 = KD^2$$

$$\therefore \quad F = C_o = \text{static load rating} = \frac{KZD^2}{5}$$

(**Static equivalent load**) is defined as that static radial load which if applied would cause the same total permanent deformation at the most heavily stressed ball and race contact which occurs under the actual condition of loading.
(**The life**) of an individual ball bearing is defined as the number of revolutions (or hours at some given constant speed) which bearing runs before the first evidence of fatigue develops in the material of either ring or of any of the rolling elements.
(**Rating life of a group of apparently identical ball bearings**) is defined as the number of revolutions (or hours at a given constant speed) that 90% of the group of ball bearings will complete or exceed before the first evidence of fatigue develops. As presently determined the life which 50% of the group of ball bearings will complete or exceed is known as average life and is approximately five times the rating life.
(**Basic load rating**) is that constant stationary radial load which a group of apparently identical ball bearings with stationary outer ring can endure for a rating life of 10^6 revolution of the inner ring.
(**The equivalent load**) is the constant stationary radial load which if applied to a bearing with rotating inner ring and stationary outer ring would give the same life as that which the bearing will attain under the actual condition of load and rotation.

Rating life $$L = \left(\frac{C}{P_e}\right)^3 \times 10^6 \text{ cycles for ball bearings} \qquad \text{(i)}$$

Equivalent Dynamic Load

$$P_e = (XV\,F_r + YF_a) \qquad \text{(ii)}$$

where X — radial factor
V — race rotation factor 1.00 for inner race, 1.2 for outer race
Y — axial load factor
F_r F_a — radial and axial load respectively
S — service factor
C — dynamic load for 10^6 cycles.

Equivalent Static Load

$$P_0 = X_0\,F_r + Y_0 F_a \qquad \text{(iii)}$$

where X_0 and Y_0 are radial and axial load factors respectively.

Rating life $L = (C/P_e)^{10/3} \times 10^6$ cycles for roller bearings. (iv)

19.4 SELECTION OF ROLLING CONTACT BEARING

It is done using Eqs (i) to (iv) from Article 19.3 and by making use of the manufacturers catalogue.

19.5 SELECTION FOR VARIABLE LOADING

It is done by using Miner's cumulative damage fatigue approach as suggested in Chapter 4 and Eq. 4.7.

If equivalent loads P_1, P_2, P_3 etc. act for N_1, N_2, N_3 ... etc, number of cycles then Eq. (i) cannot be directly used.

If P_1, P_2, P_3 etc. alone were acting the life of the bearing would have been L_1, L_2, L_3 etc. given by

$$L_1 = \left(\frac{C}{P_1}\right) \times 10^6,\ L_2 = \left(\frac{C}{P_2}\right) \times 10^6,\ L_3 = \left(\frac{C}{P_3}\right) \times 10^6, \text{ and so on.}$$

In one revolution with load P_1 the part of the life consumed will be $\frac{1}{L_1}$, thus in N_1 revolutions $\frac{N_1}{L_1}$ life will be consumed. Hence if N_e is the total life in number of cycles and if

$$N_1 = \alpha_1 N_e$$
$$N_2 = \alpha_2 N_e$$
$$N_3 = \alpha_3 N_e \text{ ... etc.}$$

Then $$\frac{\alpha_1 N_e}{L_1} + \frac{\alpha_2 N_e}{L_2} + \frac{\alpha_3 N_e}{L_3} + \ldots = 1$$

Using the values of L_1, L_2, L_3

$$\frac{\alpha_1 P_1^3}{10^6 C^3} + \frac{\alpha_2 P_2^3}{10^6 C^3} + \frac{\alpha_3 P_3^3}{10^6 C^3} + \ldots = \frac{1}{N_e}$$

or $$\alpha_1 P_1^3 + \alpha_2 P_2^3 + \alpha_3 P_3^3 + \ldots = \frac{C^3}{N_e} \times 10^6$$

19.6 PRELOADING OR DUPLEXING

This method is used to eliminate the play or clearance in either radial or axial direction of ball bearing to be used for precision machine tool or equipment. The arrangement is shown in Fig.19.5. The outer rings of the bearings at A project a small but controlled amount beyond the inner rings. After the inner races are coming in contact by rotating lock-nut the displacement of balls in the races removes the looseness from the bearing.

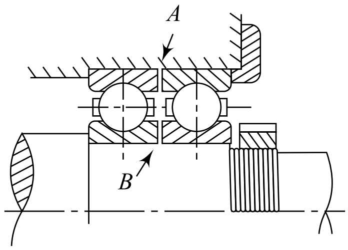


Fig. 19.5

19.7 COMPARISON BETWEEN THE SLIDING AND ROLLING CONTACT BEARINGS

1. Rolling contact bearings have high starting torque.
2. Rolling contact bearings require limited axial space.
3. Rolling contact bearings of some varieties can take combination of radial and axial load, e.g. deep groove type B-B, taper roller bearings.
4. Rolling contact bearings have very low coefficient of friction of the order of 0.0001.
5. There is saving of 90% power lost due to friction in rolling contact bearings.
6. Rolling contact bearings have long life.
7. Rolling contact bearings give warning of failure by becoming noisy.
8. R.C.B can be preloaded to reduce the deflection of machine tool shafts.
9. R.C.B can be fitted with very less clearance and more accurate positioning of shafts is possible.
10. R.C.B requires very less or no lubrication requiring less maintenance.
11. R.C.B requires larger radial space.
12. Initial cost is high for R.C.B.
13. It is essential to replace the complete bearing in case of failure of only one element in case of R.C.B.

WORKED EXAMPLES

19.1 A 6203 single row deep groove ball bearing has a basic static load rating C_0 = 4500 N and basic dynamic load rating C = 7350 N. If it is subjected to radial load of 1350 N and axial load of 1260 N, what is the rated life with outer ring stationary.

Solution:

$$\frac{F_a}{VF_r} = \frac{1260}{1 \times 1350} = 0.933$$

$$\frac{F_a}{C_0} = \frac{1260}{4500} = 0.28 \text{ for which } e = 0.38,\ Y = 1.15 \text{ (Table 33)}$$

Now $\dfrac{F_a}{VF_r} > e,\ \therefore\ X = 0.56$

$\therefore$ $P_e = XV \times F_r + YF_a = 0.5 \times 1 \times 1350 + 1.15 \times 1260 = 2205.00$ N

$$L = \left(\frac{C}{P}\right)^3 \times 10^6 = \left(\frac{7350}{2205}\right)^3 \times 10^6 = 37.03 \times 10^6 \text{ cycles}$$

19.2 Select suitable ball bearing to carry a radial load of 10,000 N, axial load 4000 N. The shaft rotates at 1000 r.p.m. Average life of 5000 hrs is desired. Inner race rotates, the service factor is 1.5, X = 0.56, Y = 1.2.

Solution:

$$\text{Rated life} = \frac{\text{Average life}}{5} = \frac{5000}{5} = 1000 \text{ hrs}$$

$$1000 \text{ hrs} = 1000 \times 60 \times 1000 = 60 \times 10^6 \text{ cycles.}$$

$$P_e = (XVF_r + YF_e)\,S$$

$$= (0.56 \times 1 \times 10{,}000 + 1.2 \times 4000) \times 1.5 = 15{,}600 \text{ N}$$

$$L = (C/P_e)^3 \times 10^6 = 60 \times 10^4$$

$\therefore$ $C = P_e \times (60)^{1/3} = 15600 \times (60)^{1/3} = 61070$ N

6312 bearing has 62.7 kN capacity from Table 36 (App. 1). Hence it may be selected.

19.3 A bearing for an axial flow compressor is to carry a radial load of 2.5 kN and a thrust load of 1.5 kN. The service imposes a light shock with shock factor 1.5. The bearing is to be used for 40 hours/week for 5 yrs. The speed of the shaft is 1000 r.p.m and diameter 50 mm. Select the suitable ball bearing. X = 0.56, Y = 1.4.

Solution:

$\therefore$ $P_e = (0.56 \times 2500 + 1.4 \times 1500) \times 1.5 = 5250$ N

$$\text{Life} = 60 \times 1000 \times 40 \times 52 \times 5 = 624 \times 10^6 \text{ cycles} = \left(\frac{C}{5250}\right)^3 \times 10^6$$

$\therefore$ $C = 44{,}862.8$ N

6310 bearing is suitable.

19.4 A roller bearing is selected to withstand a radial load of 40 kN and life of 1200 hrs at 600 r.p.m. What load rating would you look for in searching from manufacturer's catalogue if it specifies load at speed 500 r.p.m and life 3000 hrs.

Solution:

$$\text{Specified load} = 40000 \left[\frac{1200 \times 600}{3000 \times 500}\right]^{3/10} = 32100 \text{ N}$$

19.5 A deep groove ball bearing has dynamic capacity of 20200 N and is to operate on the following work cycle. Radial load of 5800 N at 200 r.p.m for 25% of the time.
Radial load of 8900 N at 500 r.p.m for 20% of the time
Radial load of 3500 N at 400 r.p.m for remaining time
Assuming the loads are steady and the inner race rotates, find the expected average life of the bearing in hours.
Solution:
Let the total life of bearing be one minute
$\therefore$ Radial load of 5800 N acts for 50 cycles
$\therefore$ Radial load of 8900 N acts for 100 cycles
and Radial load of 3500 N acts for 220 cycles
$\therefore$ Total life in cycles = 370 cycles

$$\therefore \quad \alpha_1 = \frac{50}{370} = 0.135, \ \alpha_2 = \frac{100}{370} = 0.270$$

$$\alpha_3 = \frac{220}{370} = 0.594, \ C = 20200 \text{ N}$$

Using equation from Article 19.5

$$0.135\,(5800)^3 + 0.270\,(8900)^3 + 0.594\,(3500)^3 = \frac{P_e^3}{N_e} \times 10^6$$

$$\therefore \quad P_e = 2000 \text{ N}$$

$\therefore N_e$ = 1533.260 hrs using 370 r.p.m as number of cycles/minute.

19.6 A bearing has dynamic capacity of 48545 N. What equivalent load it can carry for the average life of 6000 hrs at 500 r.p.m?
Solution:

$$\text{Rated life} = 6000/5 = 1200 \text{ hrs}$$
$$= 1200 \times 60 \times 500 = 36 \times 10^6 \text{ cycles}$$

Again $\quad L = (C/P_e)^3 \times 10^6$

$$\therefore \quad 36 \times 10^6 = \left(\frac{48.545}{P_e}\right)^3 \times 10^6$$

$$\therefore \quad P_e = 14702 \text{ N.}$$

19.7 Select a deep groove type of ball bearing for a 25 mm diameter shaft rotating at 450 r.p.m. The shaft has a bevel gear mounted on it due to which radial and a thrust load of magnitude 1450 N and 1500 N respectively act on it. The value of $X = 0.56$, $Y = 1.37$, $S = 1.5$. The rated life should be 3000 hrs. Inner ring rotates.
Solution:

$$P_e = (0.56 \times 1 \times 1450 + 1.37 \times 1500) \times 1.5 = 4300.5 \text{ N}$$
$$L = 3000 \times 60 \times 450 = 81 \times 10^6 \text{ cycles}$$

Using $$L = \left(\frac{C}{P_e}\right) \times 10^6$$

$$C = 18607.18 \text{ N}$$

From table SKF 6305 with C = 22500 N is selected.

19.8 A shaft is supported as shown in Fig. E-19.8. The weight of the pulley is 1000 N and tensions in the belts in horizontal direction are 3000 N and 1500 N on tight and slack side respectively. Select the proper bearing for shaft diameter of 50 mm. There is an axial thrust of 2000 N. Life of bearing should be 5000 hrs. at 400 r.p.m.

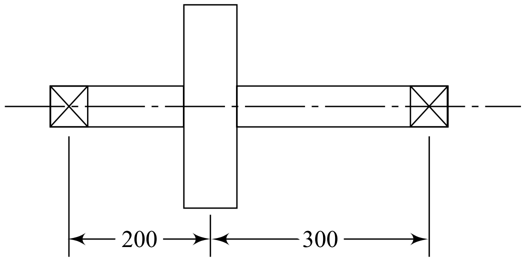


Fig. E-19.8

Solution:

Force in the vertical plane = 1000 N

Force in the horizontal plane = 3000 + 1500 = 4500 N

$\therefore$ Resultant radial force = $\sqrt{1000^2 + 4500^2}$ = 4609.77 N

Reaction at bearing $A = \dfrac{4609.77 \times 300}{500}$ = 2765.86 N

Reaction at B = 1843.91 N

Bearing A is heavily loaded

From Table 33 (App.1) $X = 0.56$, $Y = 1.5$, $V = 1$, $S = 1.5$

$\therefore$ $$P_e = [0.56 \times 1 \times 2765.86 + 1.5 \times 2000] \times 1.5 = 6823.32 \text{ N}$$

$$L = 5000 \times 60 \times 400 = 120 \times 10^6$$

$\therefore$ $$C = 33655.5 \text{ N}$$

Bearing of 6310 type with capacity 47.6 kN may be used.

19.9 A ball bearing is subjected to a combination of radial and axial load of 1400 and 6000 N respectively. The inner ring rotates at 1000 r.p.m. $X = 0.56$, $Y = 1.2$. For the life of 5000 hrs of 50% of bearings find the equivalent dynamic load and basic static load rating.

Solution:

$$P_e = 0.56 \times 14000 + 1.2 \times 6000 = 15040 \text{ N}$$

Life = 5000 hrs for 50% of ball bearings

$\therefore$ Life for 90% of b.b = 1000 hrs

$= 1000 \times 60 \times 1000 = 60 \times 10^6$ cycles

$\therefore$ using $$\left(\frac{C}{P_e}\right)^3 = L$$

$\therefore$ $$C = 15040 \sqrt[3]{60} = 58879.6 \text{ N.}$$

19.10 Find the rating load of a deep groove ball bearing for the following load cycle

	Radial load	*Axial load*	*Speed r.p.m*	*X*	*Y*	*S*
1	3000 N	1000 N	600	0.56	1.2	1.5
2	3500 N	1000 N	800	0.56	1.2	1.5
3	3500 N	100 N	900	0.56	0.00	1.5
4	500 N	2000 N	1500	0.56	1.2	1.5

The loads act for 15%, 20%, 30% and 35% time respectively.

Solution:

$$P_{e_1} = 1.5\,(0.56 \times 3000 + 1.2 \times 1000) = 4320 \text{ N}$$
$$P_{e_2} = 1.5\,(0.56 \times 3500 + 1.2 \times 1000) = 474 \text{ N}$$
$$P_{e_3} = 1.5\,(0.50 \times 5000 + 0) = 4200 \text{ N}$$
$$P_{e_4} = 1.5\,(0.56 \times 500 + 1.2 \times 2000) = 4020 \text{ N}$$
$$N_1 = 0.15 \times 600 = 90 \text{ cycles assuming life} = 1 \text{ mt}$$
$$N_2 = 0.2 \times 800 = 160 \text{ cycles}$$
$$N_3 = 0.3 \times 900 = 270 \text{ cycles}$$
$$N_4 = 0.35 \times 1500 = 525 \text{ cycles}$$
$$\therefore \quad N_e = 1045 \text{ cycles}$$
$$\therefore \quad 90 \times (4320)^3 + 160\,(4740)^3 + 270\,(4200)^3 + 525\,(4020)^3 = 1045\,P_e^3$$
$$\therefore \quad P_e = 4217.70 \text{ N}$$

19.11 For the bearing loaded as in Problem 19.10, find the life of 90% of the ball bearing if the bearing used is 6207 with dynamic capacity 19620 N.

Solution:

From the previous problem $P_e = 4217.7$ N

$$\therefore \quad L = 10^6 \left(\frac{19620}{4217.7}\right)^3 = 100.66 \times 10^6 \text{ cycles}$$

With 1045 r.p.m life in hours

$$= \frac{100.66 \times 10^6}{1045 \times 60} = 1605.42 \text{ hrs}$$

19.12 A load varies continuously in magnitude in a sinusoidal manner and the life is 20×10^6 cycles. Determine mean cubic load if the maximum load is 3000 N.

Solution:

Load for any part of rotation may be expressed as $F = 1500 - 1500 \cos 2\pi N$ where N is the fraction of a revolution.

$$F_n = \sqrt[3]{\int_0^1 \frac{(1500 - 1500 \cos 2\pi N)^3\, dN}{1}}$$

$$= \sqrt[3]{(1500)^3 \left[N - \frac{3 \sin 2\pi N}{2\pi} + \frac{3}{2\pi}\left(\pi N + \frac{\sin h\pi N}{n}\right) - \sin \frac{2\pi N}{6\pi}(\cos^2 2\pi N + 2)\right]_0^1}$$

$$= \sqrt[3]{(1500)^3\,(2.5)} = 2035.81 \text{ N}$$

19.13 A 6308 bearing has dynamic capacity of 31.5 kN. What equivalent radial load can the bearing carry at 500 r.p.m if the desired life is 6000 hrs for 90% of the bearings?

Solution:

Desired life in million number of cycle = $500 \times 60 \times 5000 = 180$ million

$$\left(\frac{C}{P_e}\right)^3 = 180$$

$$\therefore \quad P_e = C/\sqrt[3]{180} = \frac{31500}{5.646} = 5579 \text{ N.}$$

OBJECTIVE QUESTIONS

19.1 The coefficient of friction in rolling contact bearing is
(a) 0.1 (b) 0.0001 (c) 0.0018 (d) 0.005

19.2 The diameter of shaft for which 6207 bearing is used is
(a) 70 mm (b) 7 mm (c) 0.7 mm (d) 35 mm

19.3 If the dynamic capacity of 6205 bearing is 10.8 kN the same for 6305 bearing must be
(a) 8 kN (b) 10 kN (c) 16.2 kN (d) 5 kN

19.4 Life of the ball bearing is halved if the load P is changed to
(a) 2 P (b) 0.5 P (c) 1.1 P (d) 1.26 P

19.5 Combination of radial and thrust load is taken by
(a) deep groove type ball bearing (b) needle bearing
(c) cylindrical roller bearing (d) spherical roller bearing

19.6 The ratio of average life and rating life of the ball bearing is
(a) 3 (b) 5 (c) 4 (d) 2.5

19.7 Type of the failure which occurs in ball bearing is
(a) shear (b) fatigue (c) bending (d) crushing

19.8 Race rotation factor for outer race rotation is
(a) 1.00 (b) 2.00 (c) 1.5 (d) 1.2

19.9 Life of the roller bearing is given in million number of cycles by
(a) $\left(\frac{C}{P}\right)^3$ (b) $\left(\frac{C}{P}\right)^{10/3}$ (c) $\left(\frac{C}{P}\right)^{2.5}$ (d) $\left(\frac{C}{P}\right)^4$

19.10 Striebeck equation defines
(a) life of R.B.C. (b) static capacity of R.B.C
(c) dynamic load capacity of R.B.C (d) none of the above

19.11 Preloaded rolling contact bearings are used for
(a) heavy radial load (b) precision machine tools
(c) small axial space (d) limited radial space

19.12 Cage is not required in
(a) taper roller bearing (b) needle roller bearing
(c) spherical roller bearing (d) thrust bearings

19.13 Load rating of 620 bearing is 22.5 kN. The life of the bearing should be 27×10^6 cycles; the equivalent load on the bearing must be
(a) 9 kN (b) 5 kN (c) 7.5 kN (d) 12 kN

REVIEW QUESTIONS

19.1 What are the main components of the rolling contact bearing? Explain by drawing sketch.
19.2 What are the different types of rolling contact bearings?
19.3 Differentiate between ball bearing, roller bearing and needle roller bearing.
19.4 Differentiate between radial and thrust bearings. Explain by drawing sketch.
19.5 Define (a) basic static load capacity, (b) basic dynamic load capacity, (c) life of the bearing.

19.6 Differentiate between filling and non-filling type bearings.
19.7 Draw the sketch and give the application of (a) deep groove ball bearing, (b) self-aligning ball bearings, (c) taper roller bearing.
19:8 What is equivalent static load and equivalent dynamic load?
19.9 Explain the procedure of selection of rolling contact bearing.
19.10 Derive the expression for finding the life of the bearing subjected to variable loading.
19.11 Give the merits and demerits of rolling contact bearings over the sliding contact type.
19.12 What is preloaded bearing?
19.13 What are the materials used for the components of rolling contact bearings?
19.14 Differentiate between the rated life and average life of the rolling contact bearing.
19.15 For diameter of shaft of 40 mm the ball bearings available are 6108, 8208 and 6308. Which of the three should be selected? If the load is less than dynamic capacity of 6108 b.b, and if it is more than basic dynamic capacity of 6308 b.b, which bearings are recommended?
19.16 What are different types of rolling contact bearings? Which bearings are suitable for the following applications? Why?
(a) Condition of dirty environment.
(b) Shafts with angular misalignment
(c) Combined radial and axial load
(d) Precision machine tools.
19.17 What is the retainer nut needed in needle roller bearing? What is the advantages of this?
19.18 Why is it essential to use very hard material and mirror polish for the inner and outer races and rolling elements of rolling contact bearings?
19.19 Derive the relationshuip for finding the equivalent load for rolling bearing subjected to variable radial or combined radial and axial load.
19.20 Show the method of mounting of ball bearing.
19.21 Derive Striebeck Equation. What is its significance?

PRACTICE PROBLEMS

19.1 Select a ball bearing for 50 mm shaft rotating at 1000 r.p.m for 350 hrs and subjected to a radial load of 6000 N.
19.2 If a bearing is subjected to a combination of radial load of 6000 N and an axial load of 8000 N which bearing should be used? Use $X = 0.56$, $Y = 1.55$. Expected life is 2500 hrs at 2000 r.p.m of inner race.
19.3 A 205 bearing carries a 5000 N radial load for 1/6th of the work cycle and 2500 N radial load for remainder of the cycle. The speed is constant at 200 r.p.m. The inner ring rotates and loads are stationary. Find the rating life at 5 hrs per day.
19.4 In the above problem the bearing is subjected to axial load of 1000 N during first $\frac{1}{6}$ th of the work cycle in addition to the radial load and also an axial load of 2000 N during the remaining work cycle in addition to the given radial load. Estimate the life of the bearing.
19.5 A 206 bearing is subjected to the following work cycle:
A radial load of 2500 N at 500 r.p.m for one half of the time and 700 N at 3600 r.p.m for the remaining time. The inner ring rotates and the load is steady. Find the rating life at 6 hours per day.

19.6 A ball bearing carries a load of 2500 N at 400 r.p.m for 30% of the time and a load of 5000 N at 900 r.p.m for the remainder of the time. Life of 6 yrs at 8 hrs per day is expected. Find the maximum value of the basic rating load C.

19.7 Using the constant $K = 60.8 \times 10^6$ in Striebeck equation compute the static capacity of a single row deep groove ball bearing 212 with 10 balls with 16 mm diameters.

19.8 This bearing is subjected to a radial load of 300 kN and an axial load of 10 kN. Find the life of the bearings. Use C_0, C, X_0, Y_0 from the table.

19.9 This bearing is subjected to a radial load of 20 kN and an axial load of 15 kN. Service Factor is 1.5. Find the life in hours if the shaft rotates at 250 r.p.m.

19.10 If the life of the bearing in the above problem is to be 1000 hrs which bearing should be used?

19.11 A 204 ball bearing operates under the following work load. Inner ring rotates and loads are steady. Find the rating life at 2.5 hrs per day.
Radial load of 1800 N at 2000 r.p.m for 5% time
Radial load of 1110 N at 3000 r.p.m for 15% time
Radial load of 600 N at 2200 r.p.m for 35% time
Radial load of 500 N at 1750 r.p.m for 45% time

19.12 Schedule of work load for a ball bearing is
A radial load of 5000 N at 800 r.p.m for 30% time
A radial load of 2000 N at 440 r.p.m for 70% time
Find the maximum values of the basic rating load C that the bearing must have. The loads are steady and the expected life is 5 yr at 2.5 hr/day.

19.13 A ball bearing carries a load of 2.5 kN at 450 r.p.m for 30% of the time and a load of 5 kN at 880 r.p.m for the 70% of the time. Life of the bearing should be 6 years at 8 hrs per day. Find the minimum value of basic load rating C.

19.14 Find the rating life at 3 hrs per day of 207 bearing which is subjected to a steady radial load of 3.5 kN and an axial load of 770 N. The inner ring rotates and the speed is 3600 r.p.m.

19.15 A ball bearing carries a load of 1.0 kN at 1760 r.p.m for 40% of the time and a load of 5 kN for remaining time. Life of the bearing should be 2 yrs. at 6 hrs. per day. Find the maximum basic load rating for the bearing.

19.16 A shaft rotating at constant speed has variable load of 3 kN for 2 sec, 3.5 kN for 25 sec and 500 N for 3.5 sec. This load variation repeats. Find the equivalent load.

19.17 Select a suitable ball bearing for a shaft of 25 mm diameter to carry a radial load of 2000 N at 1200 r.p.m and the required life of 2000 hr.

19.18 Select a suitable b.b for carrying a radial load of 9000 N and axial load of 5000 N. The shaft rotates at 1000 r.p.m. Average life of 5000 hr is expected. The inner race rotates. $S = 1.5$, $X = 0.6$, $Y = 1.2$.

19.19 A bearing is required for 40 mm shaft to carry a radial load of 1780 N and an axial load of 2670 N at 500 r.p.m. The rating life required is 6000 hr. There is no shock load. Select suitable b.b.

19.20 It is required to select a deep groove ball bearing for a 25 mm diameter shaft rotating at 400 r.p.m. Due to bevel gear mounted on the shaft an axial thrust of 1500 N is produced in addition to a radial load of 1440 N. $X = 0.56$, $Y = 1.37$. Select proper bearing for rating life of 3000 hr.

19.21 A 6305 b.b carries a combined load of 2000 N in radial and 1500 N in axial direction. The speed of rotation is 1200 r.p.m. The outer ring rotates and moderate shock is produced for which $S = 2$. Find the rating life of the bearing.

19.22 A 6310 bearing is installed on the left hand of the shaft shown in the figure P-19.22. A vertical load of 5000 N acts at the pulley. Find the rating life in years if bearing runs 8 hr. per day at 500 r.p.m. A light shock is produced.

19.23 A 205 bearing carries a radial load of 5000 N for 1/6th of the work cycle and 2500 N for the remaining time. Speed is uniform at 180 r.p.m. Find the life of bearing at 4 hr/day.

19.24 A 6206 bearing is subjected to the following work cycle. Radial load of 2500 N at 150 r.p.m for 30% of the time.
Radial load of 3800 N at 600 r.p.m for 10% of the time.
Radial load of 1250 N at 300 r.p.m for remaining time.
The inner ring rotates. What is the rating life at 8 hr. per day.

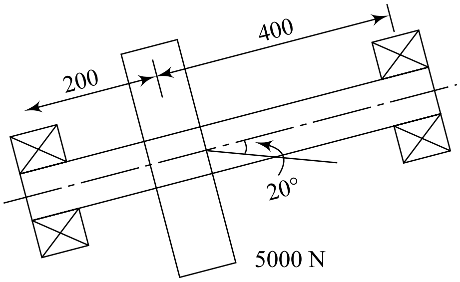


Fig. P-19.22

19.25 The work cycle for 304 bearing is as follows:
(a) 3000 N radial and 700 N axial at 300 r.p.m for 50% time.
(b) 3500 N radial at 500 r.p.m for 25% of time.
(c) 2000 N radial at 1000 r.p.m for remaining time. The outer ring rotates. Find the rated life at 5 hr/day.

19.26 A 6210 bearing has a work cycle with 800 r.p.m for 30% of time, 2000 r.p.m for 30% of the time and 4000 r.p.m for the remaining time. The radial load is 2000 N and axial load is 750 N. There is light shock with $S = 1.5$. Find the rating life at 4 hr/day for this bearing.

19.27 A ball bearing carries a radial load of 5000 N at 440 r.p.m for 30% of the time and a radial load of 2500 N at 800 r.p.m for remaining time. Expected rating life is 6 years at 8 hr/day. Find the maximum value of basic dynamic load rating that the bearing must have.

19.28 Load cycle for 312 bearing is as follows:
Radial load 5000 N, axial load of 3000 N at 900 r.p.m for 25% of time.
Radial load of 6000 N and axial load of 1000 N at 1200 r.p.m for 25% of the time.
Radial load of 4000 N and axial load of 1500 N at 1000 r.p.m for the remaining time.
The loads are steady. Find the rating life of this bearing in year with 2 hr/day.

ANSWERS

Objective Questions

(1) c (2) d (3) c (4) d (5) a (6) b (7) b (8) d (9) b
(10) b (11) b (12) b (13) c

Practice Problems

(1) 6010 Type	(2) 6318 Type	(3) 618.6 days	(4) 332 days
(5) 5.687 yrs	(6) 43927 N	(7) 9728 N	(8) 2.424×10^6 cycles
(9) 59.33 hr	(10) 317 deep groove B.B		(11) 3.31 yr
(12) 22669	(13) 11150 N	(14) 273.65 days	(15) 60341 N
(16) 3330 N	(17) 6205	(18) 6313 N_0	(19) 6308 b.b
(20) 6305 b.b	(21) 189.86 hr	(22) 8.1 year	(23) 2.7 year
(24) 5.8 years	(25) 1.94 year	(26) 587 days	

20

Miscellaneous Elements

CONCEPT REVIEW

20.1 WELDED JOINTS

Two plates are joined by using fillet welds. The welding has the following merits:

(1) The joint is as strong as the original plate.

(2) The joint is light as no compensating plates as in riveted joints are required and also weight of rivets is eliminated.

(3) There is saving of material due to absence of compensating plates and rivets.

(4) Method of joining is quick and economical. The welding method has disadvantage of introduction of internal stresses due to heating during welding and also stress concentration. Welded joint is permanent and cannot be dissembled like riveted joint.

20.2 DESIGN OF WELDED JOINT

Design method involves finding the thickness and length of the joint.

(a) Butt Joint: (Fig. 20.1) Section A–A is subjected to direct tensile load, hence basic equation of induced stress may be used as

$$\sigma_t = \frac{P}{A} = \frac{P}{hl}$$

h — height of weld mm
l — length of weld mm

The strength of the butt weld is equal to the strength of the solid plate.

(b) Lap or Fillet Weld: The fillet weld may be subjected to a load P as shown in Figs 20.2(a) and 20.2(b). In Fig. 20.2(b) it is clear case of shear stress while in Fig. 20.2(a) it is subjected to a combination of normal and shear stress. However it is general practice to design in both the cases, assuming the failure to occur at the throat by shearing and the design equation is written as

$$\tau = \frac{P}{0.707\,hl} = \text{permissible shear stress}$$

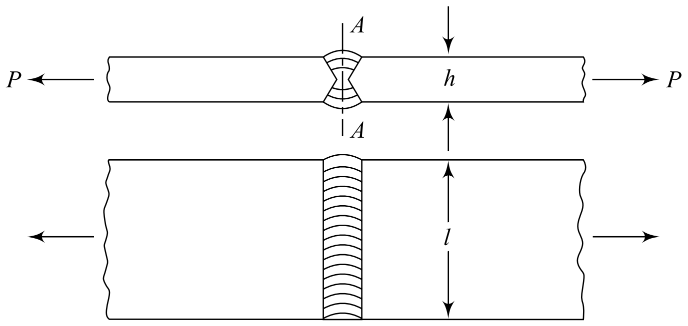


Fig. 20.1

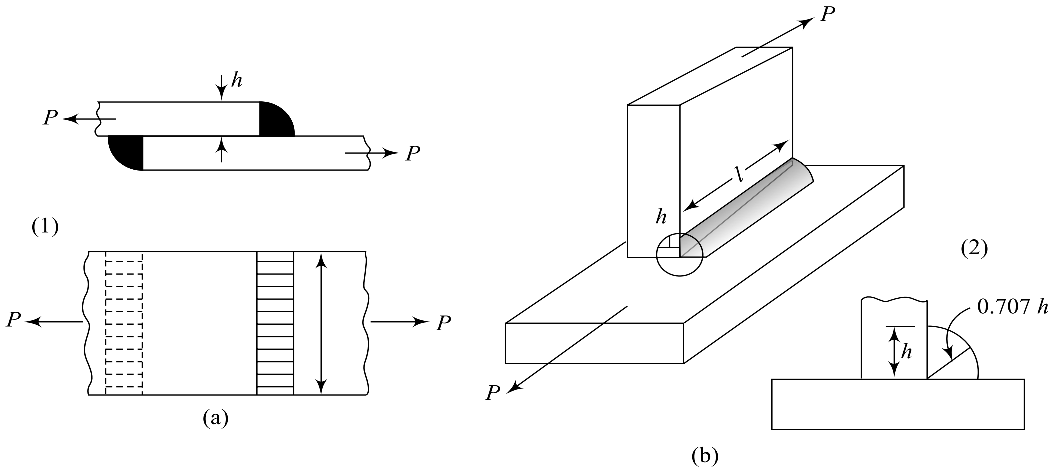


Fig. 20.2

as throat thickness is given by 0.707 h. Permissible value of shear stress ranges between 30 to 80 MPa.

(c) Weld Subjected to Bending: The section modulus of weld is calculated by using

$$Z = \frac{I}{Y}$$

where I = m.I of area about axis of bending
Z = section modulus
Y = distance of weld from the neutral axis

I is calculated by neglecting m.I of weld about its own axis if weld is parallel to the axis.

For example, for a weld as shown in Fig. 20.4(a)

$$I_{xx} = t \cdot bd^2$$

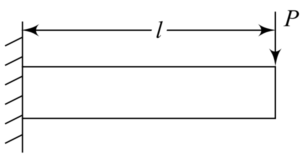


Fig. 20.3

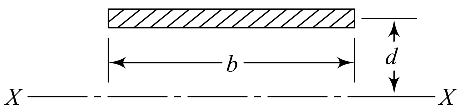


Fig. 20.4a

while for weld in Fig. 20.4(b)

$$I_{xx} = \frac{1}{12} td^3$$

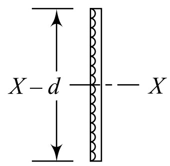


Fig. 20.4b

Further $\qquad \sigma = \dfrac{M}{Z} \qquad \tau = \dfrac{P}{A}$

using maximum shear stress theory

$$\tau_{max} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2} = \text{permissible shear stress}$$

(d) Weld in Torsion: For the weld as shown in Fig. 20.5 the force P causes direct shear stress as well as the shear stress due to rotating of plate about G which is maximum at maximum radius. For point A the stress condition will be as shown in Fig. 20.5(b).

τ_1 = direct shear stress = P/A

τ_2 = shear stress due to twisting. It is calculated as follows.

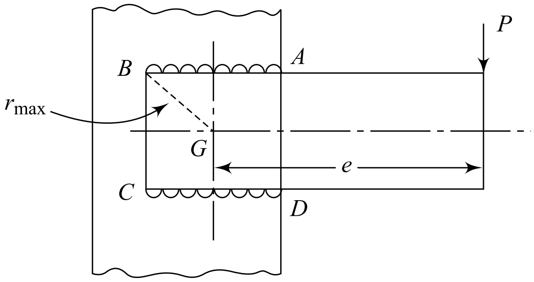


Fig. 20.5a

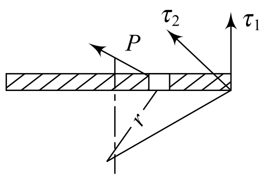


Fig. 20.5b

Let us consider the element of weld of area dA at radius r subjected to shear stress τ. It resists the twisting moment δT given by

$$\delta T = \tau \cdot dA \cdot r$$

$\therefore$ The total twisting moment $T = P \cdot e$ is balanced by summation of δT

$$P \cdot e = \int \tau \cdot dA \cdot r = \frac{\tau}{r} \int r^2 \cdot dA$$

as $\qquad \int r^2\, dA = J$ (polar moment of inertia of area)

$$\therefore \qquad P \cdot e = \frac{\tau}{r} \cdot J$$

or $\qquad \tau_2 = \dfrac{T \cdot r}{J}$ and $\tau_{2max} = \dfrac{T \cdot r_{max}}{J}$, where $T = P \cdot e$

20.3 DESIGN OF FLYWHEEL

The function of flywheel is to absorb the excess kinetic energy in the system and release it when the energy is required for doing useful work and thus avoiding cyclic fluctuations in angular velocity.

The flywheels are used mainly for two purposes: (i) In prime movers to keep the torque uniform throughout the cycle and thus avoid the cyclic fluctuations in angular speed. (ii) In machine tools having intermittent operations to supply extra amount of energy for actual working stroke which is stored during idle time. This allows the use of small capacity prime mover.

The energy is stored in the flywheel due to its large mass. Energy stored by the flywheel during increase of speed from minimum speed w_2 to maximum speed w_1 or energy released by flywheel during reduction of speed w_1 to w_2 may be written as

$$E_f = \text{Energy stored or released}$$
$$= \text{change in Kinetic Energy}$$
$$= \frac{1}{2} I (\omega_1^2 - \omega_2^2) = \frac{1}{2} (\omega_1 + \omega_2)(\omega_1 - \omega_2)$$
$$= I \frac{(\omega_1 + \omega_2)}{2} \times \frac{(\omega_1 - \omega_2)}{\omega} \times \omega$$

Putting $\omega = \dfrac{\omega_1 + \omega_2}{2}$ = mean speed

and $\dfrac{\omega_1 - \omega_2}{\omega} = K_s$ = coefficient of fluctuation of speed

$$E_f = K_s I \omega^2$$
$$I - \text{m.I of flywheel} = mk^2$$

where m is the mass of flywheel in kg and k is the radius of gyration in metre.

Procedure of designing consists of finding the mass of the flywheel using the above equation. Further cross-sectional area of the rim is found by assuming the mass of the flywheel being concentrated in the rim so that

$$M = \rho \cdot \pi d \cdot b \cdot t \qquad \text{(ii)}$$

where ρ is the density in kg/m^3, d diameter of flywheel, b and t being the width and thickness of rim. Usually $b = 2\,t$. Further cross section of rim is checked for the stresses induced.

Two types of the stresses are induced in the rim due to centrifugal force:

(i) Hoop stresses (ii) Bending stresses

Hoop stress: Let R be the mean radius of the rim. Let F be the centrifugal force acting on the element of the rim including angle $\delta\theta$ at the centre of the flywheel. Let A be the area of rim section equal to bt. The forces acting on the element are centrifugal force F and the forces acting on cross section of rim due to hoop stress σ_1. Considering equilibrium of forces

$$F = 2\sigma_1 A \sin \frac{\delta\theta}{2}$$

But $F = \rho \cdot AR\delta\theta \cdot R\omega^2 = \rho A \delta\theta v^2$ as $v = R\omega$

$$\therefore \quad \rho A \delta\theta \cdot v^2 = 2\sigma_1 A \sin \frac{\delta\theta}{2} = 2\sigma_1 A \frac{\delta\theta}{2} = \sigma_1 A \delta\theta$$

$$\therefore \quad \sigma_1 = \rho v^2$$

where ρ = density in kg/m^3

v = velocity of rim in m/s

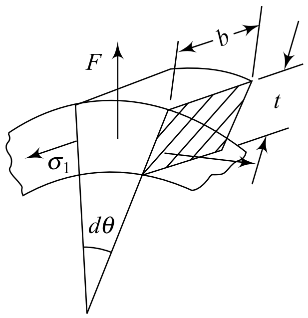


Fig. 20.6

Bending stress: (Fig. 20.7) Considering the length of the rim supported between two adjacent arms to be equal to 'l' uniformly distributed load due to centrifugal force in kg/m is

$$F = \frac{\rho A v^2}{R} \text{ Newton}$$

Again for beam fixed at both ends

$$M = \frac{wl^2}{12} = \frac{\rho A v^2 l^2}{12R}$$

$$= \frac{\rho A v^2}{12R} \frac{(2\pi R)^2}{n^2}$$

where n is the number of arms

Bending stress
$$\sigma_2 = \frac{M}{Z} = \frac{M}{\frac{1}{6}bt^2}$$

$$= \frac{4\rho A v^2 \pi^2 R^2}{12Rn^2 \times \frac{bt^2}{6}}$$

$$= \frac{19.74\rho v^2 R}{n^2 t} \quad \text{(iv)}$$

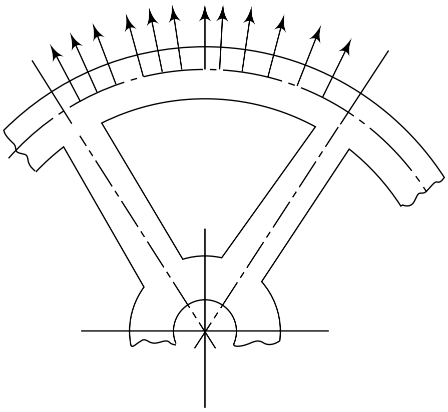


Fig. 20.7

Prof. Lanza has suggested that the total stress is given by

$$\sigma = 0.75\ \sigma_1 + 0.25\ \sigma_2.$$

WORKED EXAMPLES

20.1 Find the size of the weld in Fig. E-20.1 if the permissible shear stress is 80 MPa and the load acting on the connection P = 60 kN.

Solution:

$$\tau = \frac{P}{0.707\,lh}$$

where l = length of weld

h = thickness (size) of weld

$$\therefore \quad h = \frac{60 \times 1000}{80 \times 0.707 \times 120} = 8.84 \text{ mm} \rightarrow 10 \text{ mm}$$

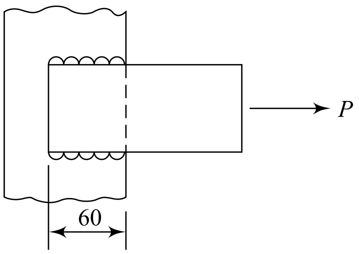


Fig. E-20.1

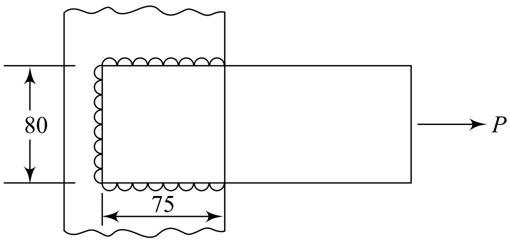


Fig. E-20.2

20.2 Find the size of the weld if $P = 120$ kN (Fig. E-20.2).
Solution:

$$l = 230 \text{ mm}$$
$$\tau = 80 \text{ MPa}$$
$$\therefore \quad h = \frac{120 \times 1000}{80 \times 0.707 \times 230} = 9.224 \text{ m} \rightarrow 10 \text{ mm.}$$

20.3 Find the size of the weld used for connecting the square bar of 150 × 150 mm size and loaded as shown in Fig. E-20.3.

$$P = 25 \text{ kN}, \ \tau_{per} = 75 \text{ MPa}$$

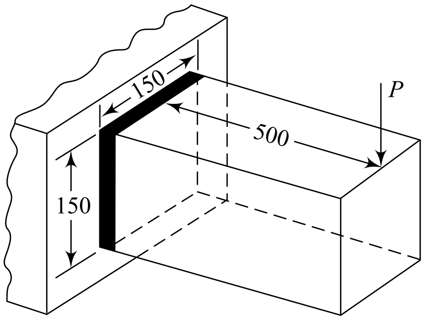


Fig. E-20.3a

Solution:

$$\text{Area of weld} = t \times 150 \times 4 = 600\,t$$
$$\therefore \quad \text{Direct shear stress} = \frac{25 \times 1000}{0.707 \times 600\,t} = \frac{58.93}{t} \text{ MPa}$$
$$\text{Bending stress} = \frac{M}{Z} = \text{for getting } Z$$
$$I_{xx} = \left(bt \times \left(\frac{b}{2}\right)^2 \times 2 + \frac{1}{12} tb^3 \times 2 \right) 0.707$$
$$= \left(\frac{tb^3}{2} + \frac{tb^3}{6} \right) 0.707 = 0.707 \times \frac{2}{3} tb^3$$
$$\therefore \quad Z = \left(\frac{2}{3} tb^3 \Big/ b/2 \right) 0.707 = \left(\frac{4}{3} tb^2 \right) 0.707$$

Putting $\quad b = 150, Z = \dfrac{4}{3} \times 0.707\, t \times (150)^2$

$$= 21210\, t$$

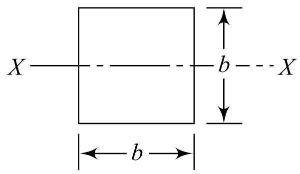


Fig. E-20.3b

$$\therefore \quad \sigma_b = \frac{M}{Z} = \frac{2500 \times 500}{21210 t} = \frac{589.345}{t} \text{ MPa}$$
$$\therefore \quad \tau_{max} = \frac{1}{2t} \sqrt{(589.345)^2 + 4(58.93)^2} = \frac{300.5}{t} = 75$$
$$\therefore \quad t = 2.5 \text{ mm} \cong 3 \text{ mm.}$$

20.4 Find P if the throat of weld is 3 mm and permissible shear stress = 75 MPa (Fig. E-20.4)
Solution: [From Fig. E-20.4(b)]

$$\bar{x} = \frac{250 \times 125 \times 2}{250 \times 2 + 400} = 69.44 \text{ mm} \quad \therefore \quad r = \sqrt{200^2 + (125 - 69.44)^2}$$
$$= 207.57 \text{ mm}$$
$$J = \left\{ \frac{1}{12} \times 400^3 + \frac{1}{12} \times 2 \times 250^3 + 400 \times 69.44^2 + 250 \times 2 \times (207.57)^2 \right\} \times 3$$
$$= 31408917 \times 3 \text{ mm}^4$$
$$r_{max} = \sqrt{200^2 + (250 - 69.44)^2} = 269.447 \text{ mm}$$

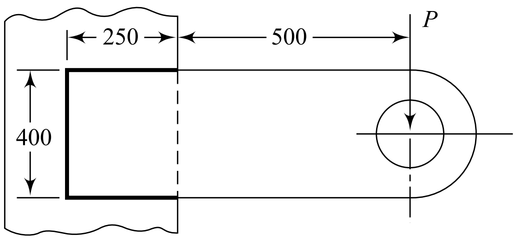


Fig. E-20.4a

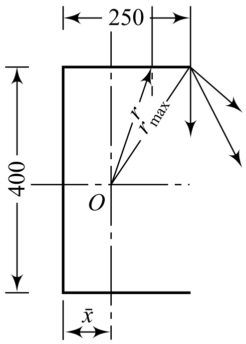


Fig. E-20.4b

$$\tau_1 \text{ due to torsion} = \frac{P \times 750 \times 269.447}{J} = \frac{P \times 750 \times 269.447}{3 \times 31408917} = 2.144 \times 10^{-3}\, P$$

$$\tau_2 \text{ due to direct shear} = \frac{P}{3 \times 900} = 3.7 \times 10^{-4}\, P \text{ MPa}$$

Again $\qquad \theta = \tan^{-1} \dfrac{180.56}{200} = 42°$

$\therefore$ Angle between τ_1 and τ_2 is $90 - 42 = 48°$

$\therefore\quad$ Resultant shear stress $= \sqrt{\tau_1^2 + \tau_1^2 + 2\tau_1\tau_2 \cos 48°} = 2.4 \times 10^{-3}\, P = 75$

$\therefore \qquad P = 31250$ N.

20.5 A cylindrical beam is attached to support by weld as shown in Fig. E-20.5 and is subjected to a bending moment M. Find the maximum stress induced in the weld.

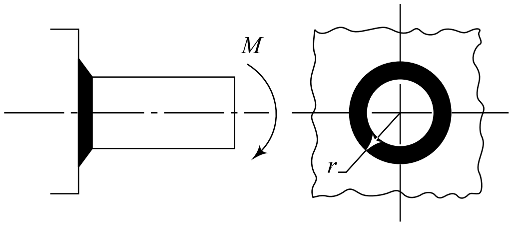


Fig. E-20.5

Solution:

For weld length $Rd\theta$ situated at angle θ, $\sigma = \sigma_{max} \sin\theta$ as stress is proportional to distance from neutral plane.

Moment of force dP in that weld length

$$= dP \cdot r \sin\theta = \sigma \cdot r\, d\theta \cdot r \sin\theta \times t$$

$$dM = \sigma_{max}\, r^2 \sin^2 d\theta \times t$$

$$\therefore \qquad M = \int_0^{2\pi} \sigma_{max}\, r^2 t \sin^2\theta\, d\theta = \sigma_{max}\, r^2 t \times \pi$$

$$\therefore \qquad \sigma_{max} = \frac{M}{r^2 t\pi} = \frac{4M}{D^2 \pi\, (0.707\, h)} = \frac{5.66\, M}{D^2 \pi h}$$

20.6 Find the torque T that can be applied in each of the cases shown.

Solution:

(a) At any angle θ consider element of weld of length $r\, d\theta$. Area of weld

$$= 0.707\, hrd\theta$$

Resistance offered by this area

$$= \tau\, (0.707\, hrd\theta)$$

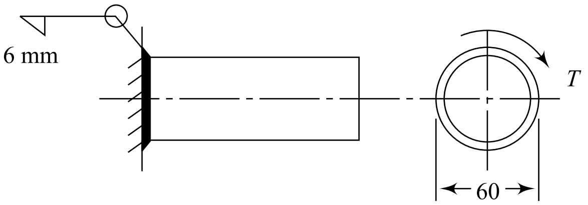


Fig. E-20.6a

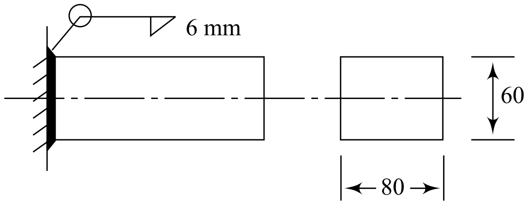


Fig. E-20.6b

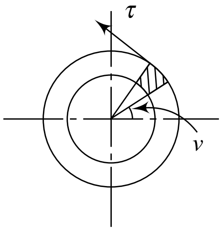


Fig. E-20.6c

$$\therefore \qquad T = \int_0^{2\pi} \tau \times 0.707\, hr^2 d\theta$$

$$= \tau \cdot r^2 \cdot 0.707h \times 2\pi$$

$$= 140 \times 30^2 \times 0.707 \times 6 \times 2\pi = 3358.31 \text{ N.m}$$

(b) For rectangular weld $J = 0.707\, h \dfrac{(a+b)^3}{6}$

$$a = 70 \text{ mm}, \quad b = 60 \text{ mm}, \quad h = 6 \text{ mm}, \quad r_{max} = \sqrt{60^2 + 80^2} = 100 \text{ mm}$$

$$T = \frac{\tau \cdot J}{r_{max}} = \frac{140 \times 0.707 \times 6(60+80)^3}{6 \times 100 \times 1000} = 2716 \text{ N.m.}$$

20.7 Design a C.I flywheel for a four stroke engine developing 150 kW at 200 r.p.m. Calculate the mean diameter of the flywheel if the hoop stress is not to exceed 4 MPa. Total fluctuation of speed is to be 4% of the mean speed. Work done during the power stroke may be assumed to be 1.5 times the average work done during the cycle. Density of C.I is 7200 kg/m^3.

Solution:

$$T_{av} = \frac{60 \times 1000 \times P}{2\pi \times N} = \frac{60 \times 1000 \times 150}{2\pi \times 200}$$

$$= 7161.972 \text{ N.m}$$

Energy developed during power stroke E_p

$$= 1.5 \text{ average work during cycle}$$

$$= 1.5 \times 7161.972 \times 4\pi$$

$$= 134999.95 \text{ N.m}$$

E_f = Hatched area = 134999.95 – 7161.97 π

$$= 112499.95 \text{ N.m}$$

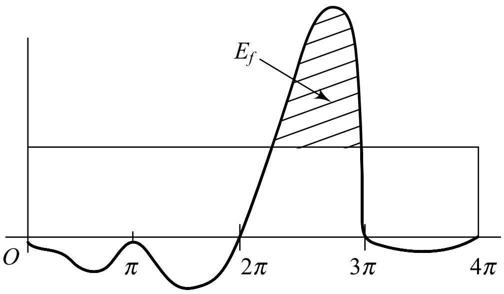


Fig. E-20.7

$$K_s = 0.04, \; \omega = \frac{\pi \times 200}{30}$$

$$I \text{ (m.I of flywheel)} = \frac{E_f}{k_s \omega^2} = \frac{112499.95 \times 30^2}{0.04 \times (\pi \times 200)^2} = 6410.25 \text{ N.m}^2$$

Now Hoop stress = 4 MPa

$$\therefore \qquad \rho v^2 = 4 \times 10^6 \text{ Pa}$$

$$\therefore \qquad v = \sqrt{\frac{4 \times 10^6}{7200}} = 23.57 \text{ m/s} = \frac{\pi DN}{60}$$

$\therefore$ $$D = \frac{23.57 \times 60}{\pi \times 200} = 2.25 \text{ m}$$

$\therefore$ Radius of flywheel = 1.125 m

$$I = mk^2 = mr^2 = 6410.25 \text{ N.m}^2$$

$\therefore$ $$m = \frac{6410.25}{(1.125)^2} = 5064.88 = \rho\, b\, t \pi \times 2.25$$

$\therefore$ $$bt = \frac{5064.88}{7200 \times \pi \times 2.25} = 0.0995 \text{ m}^2$$

Let $b = 2\,t$

$\therefore$ $t = 0.0995/2 = 0.223 \text{ mm} \rightarrow 223 \text{ mm}$

$\therefore$ $b = 446$ mm

Hoop stress = $\rho v^2 = 7200 \times (23.57)^2 = 4$ MPa

$$\text{Bending stress} = \frac{19.74 \times 23.52^2 \times 1.125 \times 7200}{6^2 \times 0.2}$$

= 11.06 MPa

$\therefore$ Resultant stress = $0.75 \times 4 + 0.25 \times 11.06$

= 5.76 MPa

For 200 FG C.I. This gives F.S. of 34.72 which is quite safe

Shaft: Let $T_{\max} = 2\,T_{uv}$

$= 2 \times 7161.9274 \times 1000$ N.mm

$$\tau = \frac{16T}{\pi d^3} = \frac{16 \times 2 \times 7116.9724 \times 1000}{\pi d^3} = \frac{S_{ys}}{N} = \frac{150}{3}$$

$\therefore$ $d = 113.41 \text{ mm} = 115 \text{ mm}$

Arms: assuming 6 arms

$$\text{b.m on each arm} = \frac{2 \times 7162 \times (1125 - 115)}{\pi \times 1125}$$

= 2148600 N.mm

$$\sigma = \frac{M}{Z} = \frac{2148600}{\frac{\pi}{8} b^3} = \frac{140}{10}$$

$\therefore$ $b = 73.11$ mm say 75 mm minor axis

$a = 150$ mm major axis

20.8 Flywheel for punching machine to punch a hole of 22 mm diameter in 18 mm thick plate is to be designed. Punching machine has 40 strokes/mt punching one hole per stroke. The hole is punched during 1/10th revolution of crankshaft. The crankshaft is connected to the flywheel by gear ratio 1 : 10. The mean diameter of the flywheel is 1 m. The minimum speed of the flywheel is limited to 90% of the maximum speed. The ultimate shearing stress for cold punching is 400 MPa. If the mechanical efficiency of the machine is 80%, estimate the capacity of motor and design the flywheel.

Solution:

Work done during punching

$$= \pi \times 22 \times 18 \times 400 \times \frac{18}{2000}$$

$$= 4478.6544 \text{ N.m}$$

$$\therefore \qquad \text{Motor Power} = \frac{4478.6544 \times 40}{1000 \times 60 \times 0.8} = 3.74 \text{ kW}$$

$$E_f = 0.9 \times 4478.6544$$

$$K_s = \frac{1 - 0.9}{(1 + 0.9)/2} = 0.105$$

$$\omega = \frac{\pi \times 400}{30} = 41.89 \text{ r/s}$$

$$E_f = K_s I \omega^2$$

$$\therefore \qquad I = \frac{4030.78}{0.105 \times (41.89)^2} = 21.87 \text{ (kgm}^2\text{)} = mr^2$$

$$\therefore \qquad m = \frac{21.87}{(0.5)^2} = 87.48 \text{ kg}$$

$$= \pi b\, t \times 7200 \times 1$$

$$\therefore \qquad bt = \frac{87.48}{\pi \times 7200} = 3.87 \times 10^{-3} \text{ m}^2$$

Let $b = 2t$

$$\therefore \qquad t = \sqrt{\frac{3.87 \times 10^{-3}}{2}} = 0.044 \text{ m}$$

$$\therefore \qquad t = 50 \text{ mm}, b = 100 \text{ mm}$$

$$v = \frac{\pi \times 1 \times 400}{60} = 20.94 \text{ m/s}$$

Hoop stress $$\sigma_1 = \frac{7200 \times 20.94^2}{10^6} = 3.157 \text{ MPa}$$

$$\sigma_2 = \frac{19.74 \times 20.94^2 \times 0.5 \times 7200}{36 \times 0.05 \times 10^6}$$

$$= 17.31 \text{ MPa}$$

$\therefore$ By Lanza's approximation total stress = 0.75 × 3.157 + 0.25 × 7.31 = 6.695 MPa which is within limit.

OBJECTIVE QUESTIONS

20.1 Design of welds is generally based on
(a) tensile strength (b) shear strength
(c) compression (d) none of the above

20.2 Welded joint is preferred to riveted joint because
(a) of high strength (b) it saves material and is light
(c) it is easy for fabrication (d) it is a permanent type joint

20.3 The shear stress induced in the weld shown in Fig. O-20.3 is given by

(a) $\dfrac{3.045P}{bt}$ (b) $\dfrac{2.5P}{bt}$

(c) $\dfrac{0.5P}{bt}$ (d) none of the above

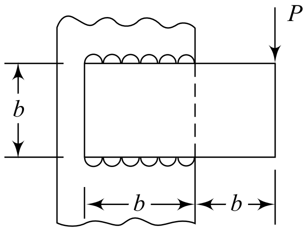


Fig. O-20.3

20.4 A cantilever beam subjected to a bending moment due to force at the end of the length l and that subjected to pure bending moment M have maximum stress in the weld at the fixed end

(a) the same (b) $\dfrac{M}{Z}$

(c) $\dfrac{P}{A}+\dfrac{M}{Z}$ and $\dfrac{M}{Z}$ (d) $\tau_{max}=\dfrac{1}{2}\sqrt{\left(\dfrac{M}{Z}\right)^2+4\left(\dfrac{P}{A}\right)^2}$ and M/Z

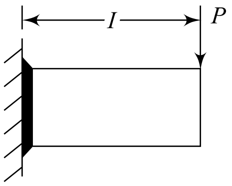


Fig. O-20.4a

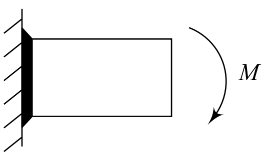


Fig. O-20.4b

20.5 The function of a flywheel is
(a) to control the speed of the shaft within limit
(b) to reduce the variation in cyclic speed
(c) to store the energy in excess and to release the same when required
(d) none of the above.

20.6 Maximum stress in the flywheel rim is

(a) $\sigma = \rho v^2$ (b) $\sigma = \rho v^2 + \dfrac{19.74\rho v^2 R}{n^2 t}$

(c) $\sigma = 0.75\ \rho v^2 + \dfrac{4.935\,\rho v^2 R}{n^2 t}$ (d) none of the above

20.7 Coefficient of fluctuation of speed K_s is
(a) difference between maximum speed ω_1 and minimum speed ω_2
(b) $2\dfrac{(\omega_1 - \omega_2)}{\omega_1 + \omega_2}$
(c) $\dfrac{\omega_1 + \omega_2}{2(\omega_1 - \omega_2)}$
(d) $\dfrac{\omega_1 - \omega_2}{2}$

REVIEW QUESTIONS

20.1 Compare the welded joint with riveted joint.
20.2 What are the different methods of welding? Draw the sketches of different welds and their conventional representation.
20.3 Write a short note on stress concentration in welds.
20.4 Derive the expression for the maximum stress induced in weld subjected to torsional loading.
20.5 Derive the expression for shear stress induced in the weld shown in Fig. R-20.5(a) and also for the bending stress induced in the weld in Fig. R-20.5(b).

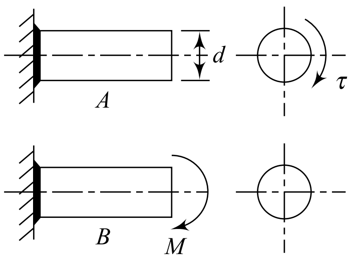


Fig. R-20.5a and b

20.6 What is the function of a flywheel? Differentiate between the function of a flywheel in prime movers such as I.C engine, stress engine, and that in intermittent operating machines like punches.
20.7 Derive the expression for hoop stress and the bending stress in the rim of the flywheel and write design equation, using Prof. Lanza's approximation.
20.8 Derive the expression to relate the fluctuation of energy with the coefficient of fluctuation of speed and moment of inertia of flywheel and the mean speed in rad/s.
20.9 Define coefficient of fluctuation of energy.
20.10 What are the problems in welding and how are they overcome?

PRACTICE PROBLEMS

20.1 Find the size of the weld in the following cases: (Fig. P-20.1a and b)
(a) $P = 25$ kN, $a = 100$ mm
(b) $P = 105$ kN, $a = 120$ mm, $b = 250$ mm, allowable value of shear stress 85 MPa.

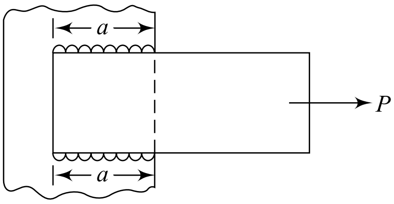


Fig. P-20.1a

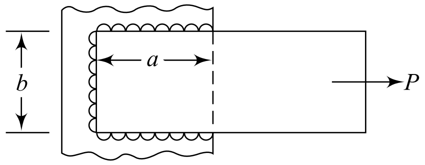


Fig. P-20.1b

20.2 Find the size of the weld in the following cases:

(a) $P = 50$ kN, $b = 200$ mm, $a = 100$ mm, $c = 200$ mm

(b) $P = 75$ kN, $b = 250$ mm, $a = 70$ mm, $c = 150$ mm

(c) weld provided at top and bottom also in part (a) Assume $\tau_{\text{safe}} = 90$ MPa.

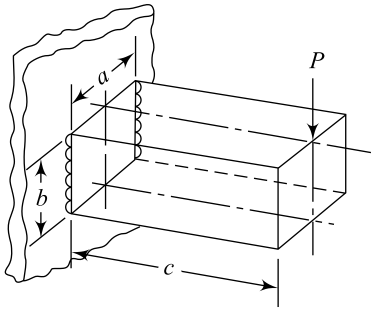


Fig. P-20.2

20.3 Find the size of the weld in following cases for $\tau_{\text{safe}} = 80$ MPa

(a) $P = 20$ kN, $a = 100$ mm, $b = 250$ mm, $c = 100$ mm

(b) $P = 35$ mm, $a = 150$ mm, $b = 300$ mm, $c = 50$ mm

(c) $P = 50$ kN, $a = b = c = 125$ mm

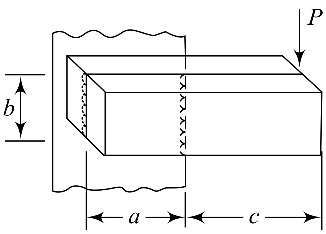


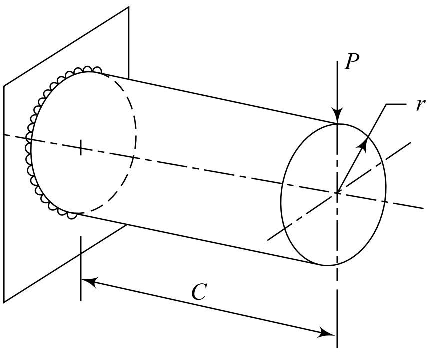
Fig. P-20.3

20.4 Find the size of the weld for

(a) $P = 35$ kN, $c = 200$ mm, $r = 50$ mm

(b) $P = 25$ kN, $c = 300$ mm, $r = 45$ mm

(c) $P = 50$ kN, $c = 250$ mm, $r = 60$ mm. Assume $\tau = 80$ MPa



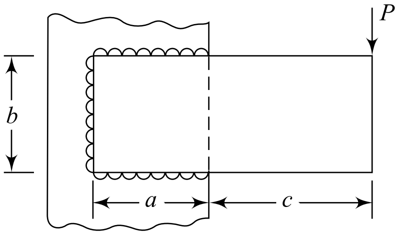


Fig. P-20.4

20.5 Prove that the value of maximum stress in the weld of Fig. P-20.5 is given by

$$\sigma = \frac{P}{2A}\sqrt{K^2 + (1 + aK)^2} \quad \text{where } K = \frac{3(2r - a)}{1 + 3a^2}$$

A = area of one weld

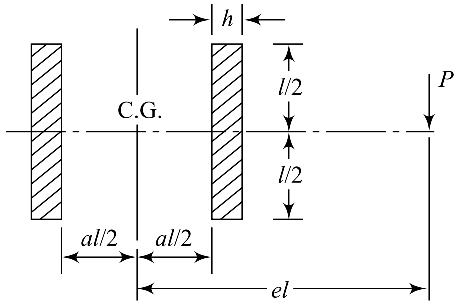


Fig. P-20.5

20.6 Find l if the permissible tensile stress is 100 MPa, 10 mm fillet weld on both sides and P = 150 kN

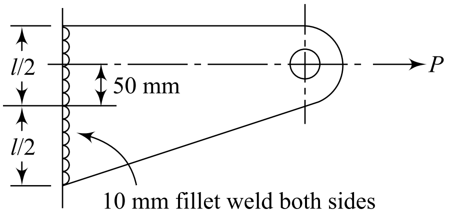


Fig. P-20.6

20.7 A C.I flywheel is used for I.C Engine for which the work done during power stroke is 2.5 times the work obtained during complete cycle. The engine develops 50 kW at 2000 r.p.m. The permissible tangential velocity for the flywheel is 75 m/s. Find the section of flywheel assuming width : thickness ratio for the rim is 2:1. Also check the stress induced by using Prof. Lanza's approximation.

20.8 The speed of a C.I flywheel is limited to 5 m/s at mean radius. The flywheel runs at 50 r.p.m and supplies 12000 N.m energy during punching. The actual punching time occupies 30° rotation of wheel and speed drops by 20%. Find the c/s of the rim and check the same for maximum induced stress.

20.9 A punching machine punches 30 mm diameter holes in 25 mm plate whose ultimate shear strength is 200 MPa. The punch has a stroke of 75 mm and maximum and minimum speed of flywheel during punching are 240 and 180 r.p.m. Find the dimensions of the flywheel rim with width to thickness ratio 1.5 : 1.

20.10 Design a C.I flywheel for a single cycle double acting steam engine developing 75 kW at 150 r.p.m fluctuation of energy 15% of the work done per cycle. Fluctuation of speed is 7.5 per cent of mean speed. Assume width to thickness ratio 2 : 1. Find the section of rim, also find the section of elliptic arms with major axis double the minor axis. The permissible stress for C.I = 7 MPa and for steel S_y = 300 MPa. Find the dimensions of shaft and key.

20.11 For the welded joints shown in the figure Fig. P-20.11a and b if the permissible stress is 75 MPa. Stress concentration factor 1.28. Find the size of weld

$P_{max} = 3.5$ kN $\quad a = 100$ mm $\quad b = 150$ mm $\quad c = 180$ mm
$P_{min} = 0$

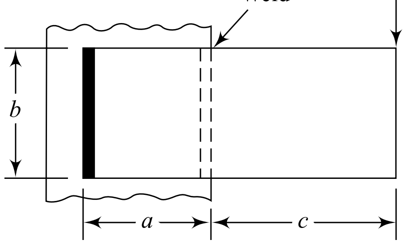


Fig. P-20.11a

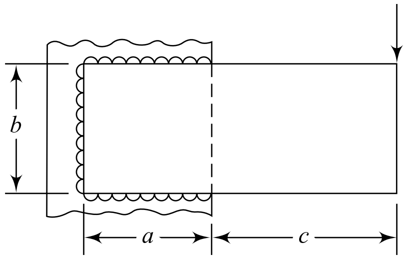


Fig. P-20.11b

20.12 Find the length of the weld if the permissible stress is 80 MPa.

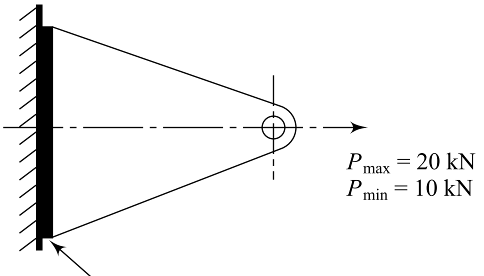


Fig. P-20.12

20.13 Prove that the maximum stress in the weld shown in the Fig. P-20.13 is given by

$$\sigma = \frac{P}{2A}\sqrt{a^2 k_1^2 + (1+k_1)^2} \quad \text{where} \quad k_1 = \frac{3(2e-1)}{1+3a^2}.$$

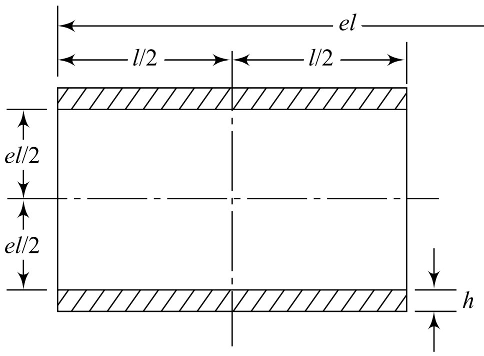


Fig. P-20.13

20.14 Find the size of the fillet weld to carry a steady load of 13 kN. The weld metal has yield strength of 360 MPa. Use F·S of 3 and length of the weld 52 mm.

20.15 Pulley shown in Fig. P-20.15 has 300 mm diameter with 70 mm diameter of hub. The web plate has 6 mm weld on both sides. The pulley transmits 30 kW at 1200 r.p.m. What is the stress induced in the weld?

20.16 Find the load P, the welded connection in Fig. P-20.16 can carry if the size of the weld is 3 mm and permissible shear stress is 80 MPa.

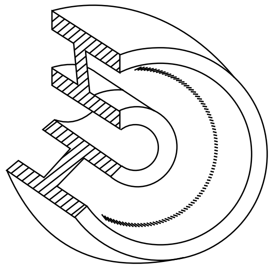


Fig. P-20.15

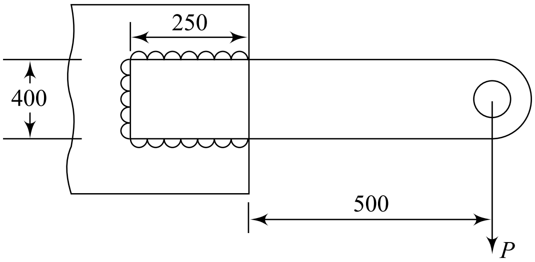


Fig. P-20.16

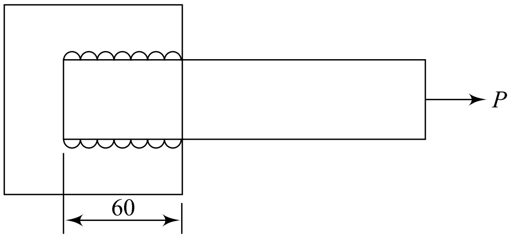


Fig. P-20.17

20.17 Find the size of the weld for the welded connection shown in Fig. P-20.17, if the shear safe stress for the weld is 80 MPa and load $P = 60$ kN.

20.18 Find the torque than can be safely applied in two cases of welds shown in Fig. 20.18(a) and (b) if the permissible shear stress is limited to 140 MPa.

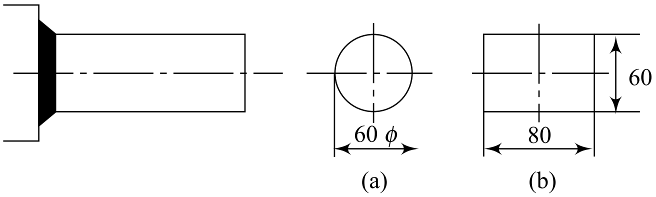


Fig. P-20.18

20.19 Find the maximum load P if the shear failure should not take place and safe shear stress is 85 MPa. Fig. 20.19.

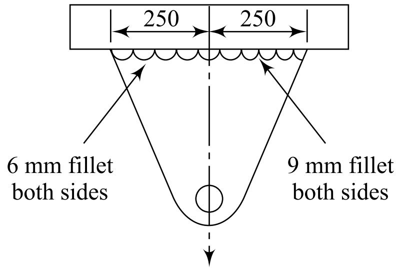


Fig. P-20.19

20.20 Find the value of maximum stress induced in the weld shown in Fig. P-20.20.

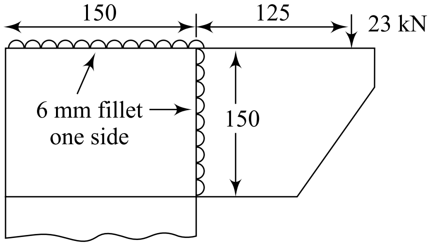


Fig. P-20.20

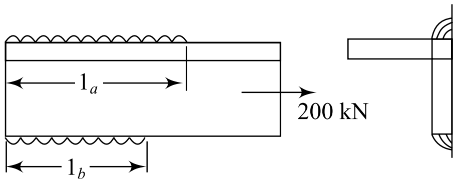


Fig. P-20.21

20.21 A 150 × 100 × 12 mm angle is to be welded to a steel plate by fillet weld along the edges of 150 mm leg (Fig. P-20.21). Find the weld length and its distribution.

20.22 A C.I flywheel of 3 m diameter and rim which width 150 mm and depth 200 mm has 6 arms. The speed of engine is 100 rpm. Determine the stress induced in the rim using Prof. Lanza's approximation the flywheel with safe stress 14 MPa. Maximum torque is 164 Nm.

ANSWERS

Objective Questions

(1) b (2) b (3) a (4) d (5) c (6) c (7) b

Practice Problems

(1) 2.08 cm 3 mm, 3.84 cm 4 mm
(2) (a) 10 mm, (b) 10 mm, (c) 5 mm
(3) (a) 3.00 mm, (b) 2 mm, (c) 10 mm
(4) (a) 8 mm, (b) 10.48 → 12 mm, (c) 9.914 → 10 mm
(6) 240 mm
(7) t = 45 mm, speed should be reduced for better F·S
(8) t = 120 mm, b = 240 mm
(9) m.I of flywheel = 14.28 N.m^2
(10) t = 143 mm, shaft dia. = 75 mm
(11) 6 mm
(13) 300 mm
(14) 6 mm
(15) τ = 3.366 MPa
(16) 31.25 kN
(17) 8.84 → 10 mm
(18) (a) 559.71 Nm, (b) 2716 Nm
(19) 341 kN
(20) 74.8 MPa
(21) Length = 295 mm, I_a = 200 mm, I_b = 95 mm
(22) 3.3 MPa, major axis 145 mm, minor axis 82.5 mm.

21

Statistical Considerations and Optimum Design

CONCEPT REVIEW

21.1 STATISTICAL CONSIDERATIONS

Statistical methods are useful in predicting the variation in strength of materials used for components as well as the variation in forces or loads acting on them. Some basic definitions are given below.
Arithmetic mean of population.

$$\bar{x} = \frac{x_1 + x_2 + \ldots x_n}{N} = \frac{1}{N}\sum_{1}^{N} x_i$$

Standard deviation

$$S_x = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots (x_n - \bar{x})^2}{N}} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N}}$$

One of the most important of the many distributions occurring in study of statistics is normal or Gaussian distribution. The equation of the curve is given by

$$f(x) = \frac{1}{S_x\sqrt{2\pi}}\, e^{-(x-\bar{x})^2/2S_x^2}$$

By replacing x by standardised variable z defined as $z = \dfrac{x - \bar{x}}{S_x}$

The equation of the curve is

$$f(z) = \frac{1}{\sqrt{2\pi}}\, e^{-z^2/2}$$

The total area under the curve is unity and areas under the curve up to any value of z can be found from the statistical Table 37 (App. 1).

For the basic curve with variable z, the area under the entire curve for $-\infty < z < +\infty$ is unity.

The area under the curve for $-3 < z < +3$ is 0.9973.

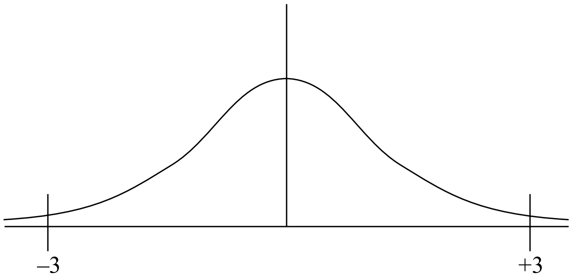


Fig. 21.1

Or, any normal distribution curve for $\bar{x} - 3\,S_x < x < \bar{x} + 3\,S_x$ represents 99.73 per cent of the total population. Hence, in design the possibility of rejection of components is negligible if the tolerances provided on both sides of mean dimensions are equal to $3\,S_x$. These are known as natural tolerances. Basic theorems for normal distribution are:

(1) Addition or subtraction of two independent variables with normal distribution is also normally distributed.
(2) Mean value of sum or difference of above variables is the sum or difference of component means.
(3) Standard deviation of sum or difference will be equal to the square root of the sum of the squares of the component standard deviation.

21.2 OPTIMUM DESIGN

Usual method of design discussed in this book uptil now may result in number of solutions to the same problem due to many assumptions. These solutions are known as adequate solution. 'Optimum design' aims at minimizing undesirable effects such as cost, stress, dead weight, deflection, vibrations, space occupancy, etc., or maximizing the desirable effects such as power to be transmitted, load carrying capacity, speed capabilities, etc. This gives unique solution. The following definitions are suggested by R.C. Johnson.

Primary Design Equation (P.D.E): This equation expresses the quantity to be minimized or maximized.
Subsidiary Design Equation (S.D.E): This design equation expresses either functional requirement or significant undesirable effect.
Limit Equations: They can give the satisfactory range of the values of certain parameters, such as strength of material or dimensions of the component, etc.

21.3 STATISTICAL CONSIDERATION IN DECIDING FACTOR OF SAFETY

The actual load P acting on the element should be smaller than the capacity C of the component to sustain the load.

As the actual load cannot be calculated accurately and its variation may be from $\bar{P} - \Delta P$ to $\bar{P} + \Delta P$, assuming normal distribution. P the mean load, ΔP tolerance on mean load, the distribution is as shown in Fig. 21.2. Variation of capacity of the component may be represented in the similar fashion as in Fig. 21.3. In some applications the failure of component may cause dangerous effect, in that case the capacity C should always have the value greater than the load. This condition is represented in Fig. 21.4.

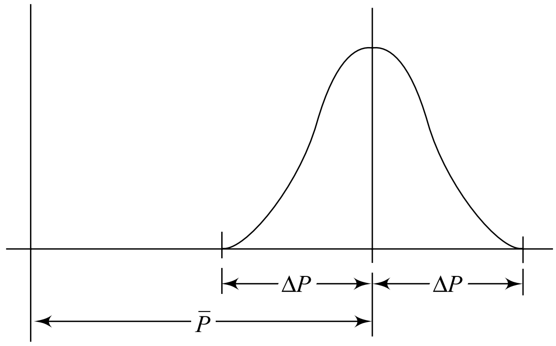


Fig. 21.2

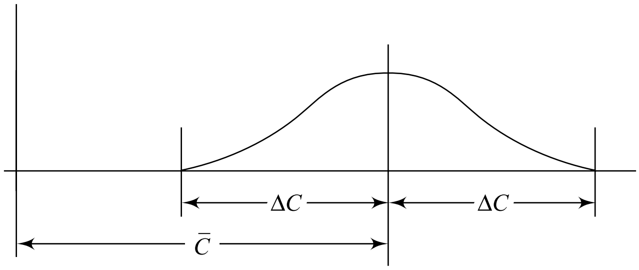


Fig. 21.3

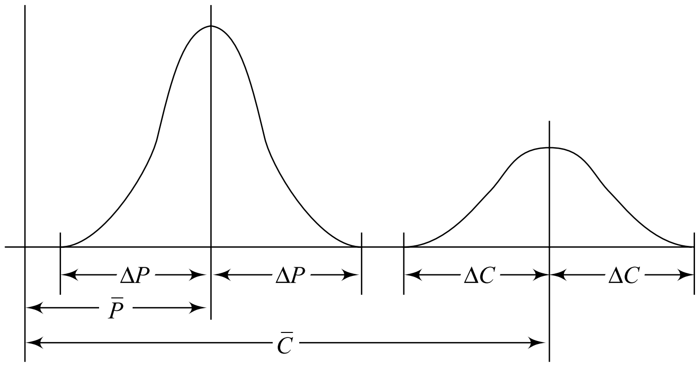


Fig. 21.4

In some designs, possibility of failure is tolerated up to certain limit, in that case the two curves may overlap for some values as shown in Fig. 21.5.

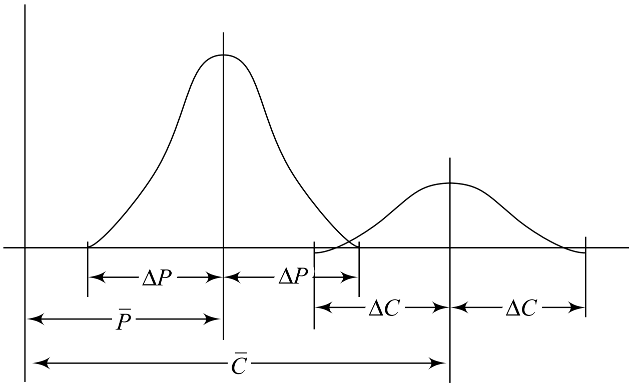


Fig. 21.5

The factor of safety may be defined as

$$N = \frac{\bar{C}}{\bar{P}}$$

In the first case from Fig. 21.4,

$$\bar{C} - \Delta C \geq \bar{P} + \Delta P$$

$$\therefore \qquad \frac{\bar{C}}{\bar{P}} - \frac{\Delta C}{\bar{C}} \cdot \frac{\bar{C}}{P} \geq 1 + \frac{\Delta P}{\bar{P}}$$

$$\therefore \qquad N \geq \frac{1 + \Delta P/\bar{P}}{1 - \Delta C/\bar{C}}$$

By using theorems in statistics, it can be shown that, in the second case

$$N = \frac{1 + 1.29\sqrt{(\Delta C/3)^2 + (\Delta P/3)^2}}{(A_f)^{0.128}\ \bar{P}} = \frac{1 + 0.43\sqrt{(\Delta C/\bar{C})^2 + (\Delta P/\bar{P})^2}}{(A_f)^{0.128}}$$

where A_f is % possibility of failure.

WORKED EXAMPLES

21.1 In a lot of 1500 shafts the diameters are normally distributed with a mean of 100.02 mm and a standard deviation should be of 0.01 mm. The dimensions of shaft for proper assembly should lie between 100 ± 0.025 mm. Calculate the number of shafts likely to be rejected.

Solution:

The minimum diameter of shaft with standard deviation of 0.01 mm is given by 100.02 – 3 (0.01) = 99.99 mm whereas the permissible value is 100 – 0.025 = 99.975,50, the possibility of rejection on negative side is zero. Hence the hatched area represents rejected shafts for which

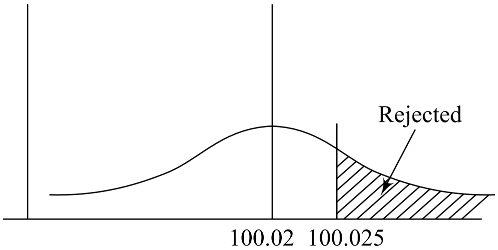


Fig. E-21.1

$Z = \dfrac{100.025 - 100.02}{0.01} = 0.5$. From Table 37 (App. 1), area from 0 to 0.5 below normal curve is 0.1915,

$\therefore$ Hatched area = 0.5 – 0.1975 = 0.3085

$\therefore$ Rejected number of shafts = 0.3085 × 1500 = 462.75 modified to 463.

21.2 In a sample of 1000 shafts, the mean diameter is of 40.5 mm and a tolerance of ± 0.025 mm. What must be the standard deviation in order to assure that 97% shafts are within acceptable limits? Assume normal distribution of diameters.

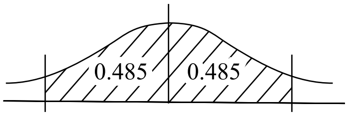


Fig. E-21.2

Solution:

Within acceptable limits 97% shafts should have d_{max} = 40.5 + 0.025 = 40.525 mm; d_{min} = 40.5 – 0.025 = 40.475 mm. Thus, 3% will not lie within this limit. 1.5% may be over d_{max} and 1.5% may be below d_{min}. So area below the curve for acceptable components is (0.5 – 0.015) = 0.485 for which $z = 2.17$

$$\therefore \qquad S_x = \frac{40.525 - 40.5}{2.17} = 0.01152 \text{ mm.}$$

21.3 For a shaft and hole assembly of 55 mm basic size with loose running fit, find the number of assemblies rejected if the minimum and maximum clearances are 0.25 mm and 0.37 mm respectively.

Solution:

For loose running fit, type of fit used is H_{11}/C11, for which the tolerance is 0.190 mm for IT 11 grade by referring to table of tolerances. From the same table, upper deviation of shaft is – 0.140 mm

$\therefore$ Max. shaft diameter = 55 – 0.140 = 54.86 mm
Min. shaft diameter = 54.86 – 0.190 = 54.67 mm
For hole the fundamental deviation is 0

$\therefore$ Min. diameter of hole = 55.00 mm
Max. diameter of hole = 55.00 + 0.190 = 55.19 mm

$\therefore$ Maximum clearance in assembly = 55.19 – 54.67 = 0.52 mm
and minimum clearance in assembly = 55.00 – 54.86 = 0.14 mm

$\therefore$ Mean clearance = 0.33 mm
Mean hole diameter = 55.095 mm
Mean shaft diameter = 54.765 mm

$\therefore$ Hole tolerance ± 0.095 = shaft tolerance

$$\therefore \quad \text{Standard deviation for both hole and shaft} = \frac{\text{Natural tolerance}}{3} = \frac{0.095}{3} = 0.0317 \text{ mm}$$

$\therefore \quad S_c$ = Standard deviation of clearance

$$= \sqrt{(S_H)^2 + (S_S)^2}\ ,$$

where S_s and S_H are standard deviations on shaft and hole respectively.

$\therefore \quad S_c = \sqrt{2(0.0317)^2} = 0.04483$ mm

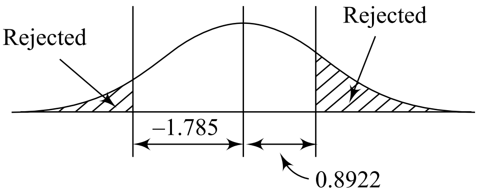


Fig. E-21.3

Components with less than 0.25 mm clearance can be found from $z = \dfrac{0.25 - 0.33}{0.04483} = -1.7845$. The area corresponding to this value of z is 0.463

$\therefore$ No. of rejected components = Total number (0.5 – 0.463) = 0.037 total number

Similarly, components with more than 0.37 mm clearance can be found by

$$z = \frac{0.37 - 0.33}{0.04483} = 0.8922.$$

The area corresponding to this value of z is 0.3133

$\therefore$ Percentage of rejected components = 0.5 – 0.3133 = 0.1867

$\therefore$ Total percentage of rejected components = 0.037 + 0.1867 = 0.2237, i.e. 22.37%.

21.4 By using optimum design method, derive the equation for optimum design of tension rod.

Solution:

The primary objective of design when no specific aim mentioned in the problem is minimizing the cost

$\therefore$ P.D.E is $C_m = CwV$

where C_m = material cost $\quad C$ = cost/unit wt

w = wt/volume $\quad V$ = volume

Here, the cost of machinery, etc. is considered negligible as compared to C_m

$\therefore$ P.D.E is $C_m = Cw(AL)$ (i)

where A is cross-sectional area of rod $\quad L$ is length of rod

S.D.E is written assuming no yielding is tolerated

$$\therefore \quad \tau_{\max} = \frac{P}{2A} \tag{ii}$$

where P is the axial load and the limit equation is

$$\tau_{\max} < \frac{S_y}{2N} \tag{iii}$$

Combining (i) and (ii) and eliminating A,

$$C_m = CwL\left(\frac{P}{2\tau_{\max}}\right)$$

For minimum cost, $\tau_{\max} = \dfrac{S_y}{2N}$

Substituting

$$C_m = CWL\left[\frac{P}{2(S_y/2N)}\right] = PLN\left(\frac{Cw}{S_y}\right)$$

The quantity $\frac{Cw}{S_y}$ is denoted as material parameter or material selection factor. By selecting the material having minimum value of this factor the design with minimum cost is obtained.

21.5 A cylindrical torsion bar is required to transmit 60 kW at 600 r.p.m. The torsional stiffness is 100 N.m/degree. Using Factor of Safety of 1.5, design the shaft for minimum weight.

Solution:

P.D.E is based on minimum weight

$$W = \rho\left(\frac{\pi d^2}{4}\right)\cdot l \tag{i}$$

where ρ = density, d = diameter and l = length of shaft

S.D.E $$\tau_{max} = \frac{16T}{\pi d^3}, \qquad \therefore\ d^3 = \frac{16T}{\pi\tau_{max}} \tag{ii}$$

also $$\theta = \frac{Tl}{JG} = \frac{32Tl}{\pi d^4 G}, \quad \therefore\ \frac{T}{\theta} = \frac{\pi d^4}{32l} = k_t \text{ (stiffness)}$$

$\therefore$ $$l = \frac{\pi d^4}{32k_t} \tag{iii}$$

Substituting Eqs (ii) and (iii) in Eq. (i)

$$W = \rho\left(\frac{\pi d^2}{4}\right)\left(\frac{\pi d^4 G}{32k_t}\right) = \frac{\pi^2}{128}\left(\frac{\rho G}{k_t}\right)d^6 = \frac{\pi^2}{128}\left(\frac{\rho G}{k_t}\right)\left(\frac{16T}{\pi\tau_{max}}\right)^2$$

$$= \frac{2T^2}{k_t}\left(\frac{\rho G}{\tau_{max}^2}\right) = \frac{2T^2}{k_t}\left(\frac{4\rho GN^2}{S_y^2}\right) = \frac{8(TN)^2}{k_t}\left(\frac{\rho G}{S_y^2}\right) = K_m\frac{8(TN)^2}{k_t}$$

where $K_m = \frac{\rho G}{S_y^2}$ material selection parameter.

The following data is available:

Material	*ρN/mm*^{3}	*G MPa*	S_y *MPa*	K_m
St. steel	0.11×10^{-3}	84×10^3	230	0.0401
Ti. alloy	0.06×10^{-3}	42×10^3	910	2.77×10^{-3}
Cr-Steel	0.1×10^{-3}	84×10^3	420	0.02

Thus Ti: alloy is the best material

Now $$T = \frac{60{,}000 \times 60}{2\pi \times 600} = 954.92 \text{ N.m}$$

$$\tau = \frac{910}{2 \times 1.5} = 303.33 \text{ MPa}, \quad \therefore\quad d = \sqrt[3]{\frac{16 \times 954.92 \times 10^3}{\pi \times 303.33}}$$

$$= 25.2 \text{ mm} - 25 \text{ mm}$$

$$l = \frac{\pi \times 25^4 \times 42 \times 10^3}{32 \times 100 \times 10^3} = 16106.8 \text{ or } 16 \text{ metre}$$

OBJECTIVE QUESTIONS

21.1 The shape of the curves representing the variation of load and capacity in optimum design is assumed to be
(a) parabolic (b) elliptical (c) gaussian (d) straight line

21.2 The magnitude of design tolerances should be
(a) less than natural tolerances (b) more than natural tolerances
(c) 50% of the natural tolerances (d) none of the above
for 100% acceptance of the component

21.3 In normal distribution, 99.73% components are accepted if the tolerances are equal to
(a) $\pm S_x$ (b) $\pm 2S_x$ (c) $\pm 3\, S_x$ (d) $\pm 4\, S_x$
where S_x is the standard deviation.

21.4 The equation satisfying the condition that the load capability is always more than the load is
(a) $\bar{C} + \Delta C > \bar{P} + \Delta P$ (b) $\bar{C} - \Delta C > \bar{P} + \Delta P$
(c) $\bar{C} - \Delta C > \bar{P} - \Delta P$ (d) $\bar{C} + \Delta C > \bar{P} - \Delta P$

21.5 Optimum design aims at
(a) reducing the undesirable effects (b) maximizing the desirable effects
(c) minimizing the undesirable effects (d) none of the above

REVIEW QUESTIONS

21.1 Define arithmetic mean of population and standard deviation.
21.2 Explain the statistical consideration in deciding the factor of safety under condition of tolerating no failure, or a small possibility of failure.
21.3 What is the importance of statistical consideration in deciding tolerances?
21.4 Explain the terms (a) P.D.E, (b) S.D.E, (c) limit equation as applied to optimum design.
21.5 Explain how optimum design differs from adequate design.
21.6 Which are the effects governing the design of (a) spring, (b) gear, (c) power screw, (d) bell crank lever, (e) clutch?
Accordingly write possible P.D.E.
21.7 Derive the expression for factor of safety if probability curves of load and strength overlap by a thin margin and when they do not overlap.
21.8 What are the fields of application of Gaussian distribution?

PRACTICE PROBLEMS

21.1 In manufacturing of 1,200 springs it is decided to have mean spring rate of 1,500 N/m with a standard deviation of 45 N/m.
(a) How many have a rate less than 1,400 N/m ?
(b) How many have a rate between 1,480 and 1,550?

21.2 A shaft and hole have dimensions 25.82 ± 0.025 mm and 25.90 ± 0.025 mm with standard deviations of 10 and 12.5 microns respectively. Compute the minimum clearance. What percentage of assemblies have smaller clearance than minimum clearance?

21.3 A tension bar is subjected to an axial load of 5000 N. The possibility of variation of load is ± 1500 N. Variation in strength may be assumed to be 30%. Allowing 1% possibility of yielding, find the factor of safety.

21.4 In the Problem 21.3, if yielding is not permissible decide the value of factor of safety.

21.5 Derive an expression for optimum design of a cantilever of length *L*, assuming $\frac{2}{3}$ *L* subjected to bending load of *P* Newton for minimum weight. Assume rectangular section of width band *b* and depth *h*.

21.6 The results of tensile test on one variety of steel are as follows. S_{ut} is in MPa, *n* is the number of specimen *S*.

S_{ut}	1245	1274	1303	1332	1361	1398	1420	1448	1477	1506	1535	1563	1593	1622
n	2	4	2	7	22	22	26	41	31	22	7	5	4	2

Estimate the mean and standard deviation with normal distribution.

21.7 The recommended class of fit between a journal and bearing is 20 H7-e8. The maximum and minimum clearances are limited to 0.08 mm and 0.05 mm respectively. Assuming natural tolerances determine the percentage of rejected parts.

21.8 A lot of 200 pins was checked for diameter using 'GO' and 'No GO' gauges. The diameter of the pin is to be 4.25 mm with tolerance of 0.03 mm. Four pins were rejected being undersize and six pins being oversize. What is the mean diameter and the standard deviation of the lot if normal distribution is assumed.

21.9 The assumed diametral tolerance of music wire up to 0.26 mm diameter is ± 0.003 mm. A 0.254 mm music wire is used for the manufacture of coil spring and is subjected to a torque of 1.77 N·mm. Determine standard deviation of stress if diametral tolerance is the same as natural tolerance.

21.10 A shaft and hole have natural tolerances of 0.05 mm and 0.075 mm respectively. Determine the mean clearance such that not more than 5% of the assemblies will have clearance less than 0.05 mm. What percentage will have clearance greater than 0.175 mm?

21.11 An axially loaded tension bar is subjected to an axial force of 5000 N. The possibility of variation of load is ± 1500 N. The following materials are available.

Sr. No.	*Material*	*Density kg/m*^{3}	S_y *MPa*	*Cost Rs./kg*
1	A	1×10^4	430	50
2	B	1.1×10^4	280	40
3	C	0.6×10^4	900	150
4	D	0.2×10^4	230	60

Find diameter of bar for minimum cost.

21.12 Derive the material selection factor for designing a cantilever beam of length *L*. Assume rectangular section *L*/3 used for fixing the fixed end criteria for design is cost should be minimum and permissible deflection should be δ.

21.13 Design a pressure vessel to hold the liquid at 5.7 MPa for minimum weight. The following materials are available.

Sr. No.	*Material*	S_y *MPa*	*P kg/m³*	*Cost Rs./kg*
1	RCC	200	2500	5.5
2	Alloy Steel	1000	7800	21.0
3	30C8 Steel	300	7800	9.25
4	Al alloy	400	2700	42.00
5	Fibre glass	200	1800	46.20
6	CFRP	600	1500	370.00

21.14 Solve the same problem with the criteria of minimizing the cost.

21.15 A cylindrical bar is to be designed for transmitting 1000 Nm torque. The torque gradient is 100 Nm/degree and factor of safety is 1.6.

Sr. No.	*Material*	*P kg/m³*	*G MPa*	S_y *MPa*
1	Tl alloy	4890	42000	910
2	Al alloy	3000	28000	504
3	St. steel	8490	84000	315
4	Mg alloy	1980	16800	196
5	Cast phenolic	1440	1560	52.5

Design for minimum weight.

ANSWERS

Objective Questions

(1) c (2) b (3) c (4) b (5) b and c

Practice Problems

(1) 1.12%, i.e. N, (b) 646 (2) 0.09% (3) 1.41 (4) 1.5625
(7) 2.78% (8) 4.2513 mm, 0.01523 mm (9) 6.654 MPa
(10) Zero per cent (11) 7 mm
(12) CPE/S_yz where C—cost per unit wt., ρ—density kg/m^3, E—modulus of elasticity in MPa and S_y = yield point strength in MPa.
(13) CFRP is the best material as selection factor is minimum, ρ/S_y material t = 4.156 mm
(14) Material selection factor is C_m (ρ/S_y) which is smallest for R.C.C, t = 12.468 = 12.5 mm
(15) *Tl* Alloy is the best material.

1

Appendix

Table 1 *Average Strength Properties of Some Materials (in N/mm²)*

Material IS Designation Old	New	*U.T.S σ_{ut}	Tensile Yield Strength $\sigma_y(min)$	Endurance Limit Strength σ_e	Torsional Endurance Limit, τ_e
St 34	Fe 330	330	200	170	100
St 37	Fe 360	360	220	180	110
St 42	Fe 410	410	250	200	120
St 50	Fe 490	490	290	250	150
St 70	Fe 690	690	410	350	200
C 10	10C4	340–500	250	180	100
C 30	30C8	500–600	330	250	150
C 40	40C8	580–680	380	300	180
C35Mn75	35C8	550–650	360	275	160
C 50	50C4	600–800	400	300	180
C 60	60C4	850–1050	570	400	200
	55Cr3	1000	740	500	300
	40Cr4	900–1050	700	450	130
	20MnCr1	1000		500	300
	40Ni6Cr4Mo2	800–1150	600	400	240
	25Cr15Mo6	1550	1300	700	400
IS 210 *Cast Iron*					
Grade 20	FG 200	200		100	80
Grade 25	FG 260	260		120	100
IS 1030 *Cast Steel*					
Grade 1	20–40	400		160	100
Grade 2	25–45	450		180	110

*Ultimate Tensile Strength

Table 2 *Tensile Properties for Carbon Steels (Unalloyed) (Stress values in N/mm^2)*

Designation (Condition)		Hot Rolled or Normalised		Cold Drawn (20 to 40 mm dia.)		Hardened and Tempered			Case Hardened Refined & Quenched (Core Properties)		
New	Old	σu_t	% elon-gation	σu_t	$\%\delta$	σu_t	$\%\delta$	Limiting Ruling Section	σu_t	$\%\delta$	Limiting Ruling Section
7C4	C07	320–400	27								
10C4	C10	340–420	26	450	13				500	17	15
14C6	C14	370–450	26						500	17	15 to 30
15C8	C15Mn75	420–500	25	510	13				500	17	30
20C8	C20	440–520	24	510	12				500	16	30
25C8	C25Mn75	470–570	22								
30C8	C30	500–600	21	570	10	600	18	30			
35C8	C35Mn75	550–650	20			600	18	63			
40C8	C40	580–680	18	610	9	700	17	30			
50C4	C50	600–780	13	630	8	800	13	30			

Note: Yield stress may be taken as 55 to 65% of ultimate stress unless otherwise specified. Percentage elongation values ($\%\delta$) are based on a gauge length of $5.65\sqrt{A}$ which is now internationally accepted.

Table 3 *Tensile Properties of Alloy Steels in the Hardened and Tempered Condition*

Designation	σu_t N/m/mm^2	$\sigma_{0,2}$ N/mm^2	Izod Impact (N.m)	Limiting Ruling Sec. (mm)
20C15	700–850	500	48	150
27C15	700–850	500	48	300
37C15	900–1050	700	41	150
35Mn6Mo3	1000–1150	800	48	300
35Mn6Mo4	1000–1150	800	48	630
55Cr3	1000–1150	740	17	630
40Cr4	900–1050	700	55	300
40Cr4Mo3	1000–1150	800	48	300
15Cr13Mo6	800–950	600	55	1500
25Cr15Mo6	1550 Min.	1300	14	630
40Cr13Mo10V2	1550 Min.	1300	14	300
40Cr7Al0Mo2	900–1050	700	48	630
40Nil4	900–1050	700	55	630
35NiCr2	900–1050	700	55	630
30Ni13Cr5	1550 Min.	1300	14	630 (air hardened) 1500 (oil hardened)
	800–950	600	55	1500
40Ni6Cr4Mo2	1000–1150	800	48	630
	1100–1250	880	41	630
40Ni6CrMo3	1200–1350	1000	30	300
*15Cr65	600	—	35	—
*20MnCr1	1000	—	38	—
*15NiCr1Mo12	1000	—	41	—
*16NiCr2Mo20	1350	—	35	—

*Case Hardening steels, refined and quenched, core properties.

Table 4 *Tensile Properties for Carbon-Manganese Free Cutting Steels (Stress Values in N/mm^2)*

Designation		Condition Hot Rolled or Normalized		Cold Drawn Bars 20 to 40 mm diameter		Hardened and Tempered			Case Hardened. Refined & Quenched (Core Properties)		
New	*Old*	σ_{ut}	$\%\delta$	σ_{ut}	$\%\delta$	σ_{ut}	$\%\delta$	*Ruling Section*	σ_{ut}	$\%\delta$	*Ruling Section*
10C8S10	10S11	370–490	24	460	10				500	17	30
14C14S14	14Mn1S14	440–540	22	520	11				600	17	30
25C12S14	25Mn1S14	500–600	20	560	10						
40C10S18	40S18	550–650	17	600	10	600–750	18	60			
						700–850	17	30			
11C10S25	11S25	370–490	22	440	11						
						600–750	18	100			
40C15S12	40Mn2S12	600–700	15	640	8	700–850	18	60			
						800–950	16	30			

Table 5 *Physical Constants of Some Common Materials*

Material	*Modulus of Elasticity, $E \times 10^{-3}$ MN/m^2*	*(kgf/mm^2)*	*Modulus of rigidity, $G \times 10^{-3}$ MN/m^2*	*(kgf/mm^2)*	*Poisson's Ratio, μ*	*Density (kg/m^3)*
Aluminium (alloys)	71.0	(7.24)	26.2	(2.67)	0.334	2730
Beryllium	287.1	(29.28)			—	1820
Beryllium copper	124.2	(12.66)	48.3	(4.92)	0.285	8230
Brass	95.1	(9.70)	34.3	(3.50)	0.30–0.40	8450
Bronze	109.0	(11.10)			—	8730
Carbon steel	202.0	(20.60)	78.5	(8.00)	0.292	7820
Cast Iron, gray	100.0	(10.20)	41.4	(4.22)	0.211	7200
Copper	120.6	(12.30)	38.3	(3.90)	0.260	8960
Inconel	214.0	(21.80)	76.0	(7.75)	0.290	8960
Lead	15.7	(1.60)	7.5	(0.76)	0.450	11340
Magnesium	44.8	(4.57)	16.6	(1.69)	0.350	1800
Molybdenum	331.0	(33.75)	117.2	(11.95)	0.307	10200
Monel metal	179.3	(18.28)	65.6	(6.68)	0.320	8830
Nickel silver	127.5	(13.00)	48.3	(4.92)	0.322	8690
Nickel steel	196.1	(20.00)	75.6	(7.80)	0.291	7750
Phosphor bronze	111.0	(11.32)	41.4	(4.22)	0.349	8160
Stainless steel (18-8)	190.3	(19.40)	73.1	(7.45)	0.305	7750
Titanium	103.5	(10.55)		—	—	4480
Tungsten	437.3	(41.53)	173.6	(17.70)	0.170	19300
Ziroconium	68.4	(6.97)		—	—	6500

Table 6 *Threads for Bolts and Nuts: Coarse and Fine Series*

Series		Size	Pitch p	Major Diameter $d = D$	Pitch Diameter $d_2 = D_2$	Bolt, d_3	Minor Diameter Nut, d_1	Depth of Thread h_3	Depth of Engagement H_1	Max. Stress Area, S mm^2
Coarse Series →	M	2.5	0.45	2.5	2.208	1.948	2.013	0.276	0.244	3.39
	M	3	0.5	3	2.675	2.387	2.459	0.307	0.271	5.03
	M	4	0.7	4	3.545	3.141	3.242	0.429	0.379	8.78
	M	5	0.8	5	4.480	4.019	4.134	0.491	0.433	14.2
	M	6	1	6	5.350	4.773	4.918	0.613	0.541	20.1
	M	8	1.25	8	7.188	6.466	6.647	0.767	0.677	36.6
	M	10	1.5	10	9.026	8.160	8.376	0.920	0.812	58.0
	M	12	1.75	12	10.863	9.853	10.106	1.074	0.947	84.3
	M	16	2	16	14.701	13.546	13.835	1.227	1.083	157
	M	20	2.5	20	18.376	16.933	17.294	1.534	1.353	245
	M	24	3	24	22.051	20.320	20.752	1.840	1.624	353
	M	30	3.5	30	27.727	25.706	26.211	2.147	1.894	561
	M	33	3.5	33	30.727	28.706	29.211	2.147	1.894	694
	M	36	4	36	33.402	31.093	31.67	2.454	2.165	817
Fine Series	M	8 × 1	1	8	7.350	6.773	6.918	0.613	0.541	39.2
	M	10 × 1.25	1.25	10	9.188	8.466	8.647	0.767	0.677	61.2
	M	12 × 1.25	1.25	12	11.188	10.466	10.647	0.767	0.677	92.1
	M	16 × 1.5	1.5	16	15.026	14.16	14.376	0.920	0.812	167
	M	20 × 1.5	1.5	20	19.026	18.16	18.376	9.920	0.812	272
	M	24 × 2	2	24	22.701	22.546	21.835	1.227	1.083	384
	M	30 × 2	2	30	28.701	27.546	27.835	1.227	1.083	621
	M	36 × 3	3	36	34.051	32.32	35.752	1.840	1.624	865

Fine Series Coarse Series

Table 7 *Fundamental Tolerances of Grades 01, 0 and 1 to 16*

Diameter Steps in mm		*Values of Tolerances in Microns*						*Tolerances Grades*								*(1 micron = 0.001 mm)*			
		01	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14**	*15**	*16**
To and inc	3	0.3	0.5	0.8	1.2	2	3	4	6	10	14	25	40	60	100	140	250	400	600
Over	3																		
To and inc	6	0.4	0.6	1	1.5	2.5	4	5	8	12	18	30	48	75	120	180	300	480	750
Over	6																		
To and inc	10	0.4	0.6	1	1.5	2.5	4	6	9	15	22	36	58	90	150	220	360	500	900
Over	10																		
To and inc	18	0.5	0.8	1.2	2	3	5	8	11	18	27	43	70	110	180	270	430	700	1100
Over	18																		
To and inc	30	0.6	1	1.5	2.5	4	6	9	13	21	33	52	84	130	210	330	520	840	1300
Over	30																		
To and inc	50	0.6	1	1.5	2.5	4	7	11	16	25	39	62	100	160	250	390	620	1000	1600
Over	50																		
To and inc	80	0.8	1.2	2	3	5	8	13	19	30	46	74	120	190	300	460	740	1200	1900
Over	80																		
To and inc	120	1	1.5	2.5	4	6	10	15	22	35	54	87	140	220	350	540	870	1400	2200
Over	120																		
To and inc	180	1.2	2	3.5	5	8	12	18	25	40	63	100	160	250	400	630	1000	1600	2500
Over	180																		
To and inc	250	2	3	4.5	7	10	14	20	29	46	72	115	185	290	460	720	1150	1850	2900
Over	250																		
To and inc	315	2.5	4	6	8	12	16	23	32	52	81	130	210	320	520	810	1300	2100	3200
Over	315																		
To and inc	400	3	5	7	9	13	18	25	36	57	89	140	230	360	570	890	1400	2300	3600
Over	400																		
To and inc	500	4	6	8	10	15	20	27	40	63	97	155	250	400	630	970	1550	2500	4000

*Up to 1 mm Grades 14 to 16 are not provided.

Table 8 *Basic Dimensions for Square Threads in mm (Normal Series) According to IS : 4694–1968*

Nominal Diameter (d_1)	*Major Diameter*		*Minor Diameter*	*Pitch*	*Depth of Thread*		*Area of Core*
	Bolt (*d*)	*Nut* (*D*)	*Bolt* (*dc*)	(*p*)	*Bolt* (*h*)	*Nut* (*H*)	A_c mm^2
22	22	22.5	17				227
24	24	24.5	19				284
26	26	26.5	21	5	2.5	2.75	346
28	28	28.5	23				415
30	30	30.5	24				452
32	32	32.5	26	6	3	3.25	531
(34)	34	34.5	28				616
36	36	36.5	30				707
(38)	38	38.5	31				755
40	40	40.5	33	7	3.5	3.75	855
(42)	42	42.5	35				962
44	44	44.5	37				1,075
(46)	46	46.5	38				1,134
48	48	48.5	40	8	4	4.25	1,257
50	50	50.5	42				1.385
52	52	52.5	44				1,521
55	55	55.5	46				1,662
(58)	58	58.5	49	9	4.5	4.75	1,886
60	60	60.5	51				2,043
(62)	62	62.5	53				2,206
65	65	65.5	55				2,376
(68)	68	68.5	58				2,642
70	70	70.5	60				2,827
(72)	72	72.5	62				3,019
75	75	75.5	65	10	5	5.25	3,318
(78)	78	78.5	68				3,632
80	80	80.5	70				3,848
(82)	82	82.5	72				4,072
85	85	85.5	73				4,185
(88)	88	88.5	76				4,536
90	90	90.5	78				4,778
(92)	92	92.5	80				5,027
95	95	95.5	83	12	6	6.25	5,411
(98)	98	98.5	86				5,809
100	100	100.5	88				6,082
(105)	105	105.5	93				6,793
110	110	110.5	98				7,543
(115)	115	116	101				8,012
120	120	121	106				8,825
(125)	125	126	111	14	7	7.5	9,677
130	130	131	116				10,568

(*Contd*)

(*Contd.*)

(135)	135	136	121				11,499
140	140	141	126				12,469
(145)	145	146	131	14	7	7.5	13,478
150	150	151	134				14,103
(155)	155	156	139				15,175
160	160	161	144				16,286
(165)	165	166	149	16	8	8.5	17.437
170	170	171	154				18,627
(175)	175	176	159				19,856

Table 9 *Basic Dimensions for Square Threads in mm (Fine Series) According to IS: 4694–1968*

Nominal Diameter	*Major Diameter*		*Minor Diameter*	*Pitch*	*Depth of Thread*		*Area of Core*
	Bolt	*Nut*			*Bolt*	*Nut*	
(d_1)	*(d)*	*(D)*	*(dc)*	*(p)*	*(h)*	*(H)*	*(Ac)mm^2*
10	10	10.5	8				50.3
12	12	12.5	10				78.5
14	14	14.5	12	2	1	1.25	113
16	16	16.5	14				154
18	18	18.5	16				201
20	20	20.5	18				254
22	22	22.5	19				284
24	24	24.5	21				346
26	26	26.5	23				415
28	28	28.5	25				491
30	30	30.5	27				573
32	32	32.5	29				661
(34)	34	34.5	31				755
36	36	36.5	33	3	1.5	1.75	855
(38)	38	38.5	35				962
40	40	40.5	37				1,075
42	42	42.5	39				1,195
44	44	44.5	41				1,320
(46)	46	46.5	43				1,452
48	48	48.5	45				1,590
50	50	50.5	47				1,735
52	52	52.5	49				1,886
55	55	55.5	52				2,124
(58)	58	58.5	55				2,376
60	60	60.5	57				2,552
(62)	62	62.5	59				2,734
65	65	65.5	61				2,922
(68)	68	68.5	64				3,217
70	70	70.5	66				3,421
(72)	72	72.5	68				3,632

(*Contd*)

(*Contd.*)

75	75	75.5	71				3,959
(78)	78	78.5	74				4,301
80	80	80.5	76				4,536
(82)	82	82.5	78	4	2	2.25	4,778
(85)	85	85.5	81				5,153
(88)	88	88.5	84				5,542
90	90	90.5	86				5,809
(92)	92	92.5	88				6,082
95	95	95.5	91				6,504
(98)	98	98.5	94				6,960
100	100	100.5	96	4	2	2.25	7,238
(105)	105	105.5	101				8,012
110	110	110.5	106				8,825
(115)	115	115.5	109				9,331
120	120	120.5	114				10.207
(125)	125	125.5	119	6	3	3.25	11,122
130	130	130.5	124				12,076
(135)	135	135.5	129				13,070
140	140	140.5	134				14,103
(145)	145	145.5	139				15,175
150	150	150.5	144				16,286
(155)	155	155.5	149				17,437
160	160	160.5	154				18,627
(165)	165	165.5	159				19,856
170	170	170.5	164				21,124
(175)	175	175.5	169				22,432

Note: Diameters within brackets are of second preference.

Table 10 *Basic Dimensions for Trapezoidal/Acme Threads*

Normal or Major Diameter (d) mm	*Minor or Core Diameter (d_c) mm*	*Pitch (p) mm*	*Area of Core *(A_c) mm^2*
10	6.5		33
12	8.5	3	57
14	9.5		71
16	11.5		104
18	13.5	4	143
20	15.5		189
22	16.5		214
24	18.5		269
26	20.5	5	330
28	22.5		389
30	23.5		434
32	25.5		511
34	27.5	6	594
36	29.5		683

(*Contd*)

(*Contd.*)

38	30.5		731
40	32.5		830
42	34.5	7	935
44	36.5		1,046
46	37.5		1,104
48	39.5		1,225
50	41.5	8	1,353
52	43.5		1,486
55	45.5		1,626
58	48.5		1,847
60	50.5	9	2,003
62	52.5		2,165
65	54.5		2,333
68	57.5		2,597
70	59.5	10	2,781
72	61.5		2,971
75	64.5		3,267
78	67.5		3,578
80	69.5		3,794
82	71.5		4,015
85	72.5		4,128
88	75.5		4,477
90	77.5		4,717
92	79.5		4,964
95	82.5	12	5,346
98	85.5		5,741
100	87.5		6,013
105	92.5		6,720
110	97.5		7,466
115	100		7,854
120	105		8,659
125	110		9,503
130	115	14	10,387
135	120		11,310
140	125		12,272
145	130		13,273
150	133		13,893
155	138		14,957
160	143	16	16,061
165	148		17,203
170	153		18,385
175	158		19,607

Table 11 *Power Correction Factors for Belt Length—D Section*

Nominal Inside Length (mm)	(in)	Pitch Length (mm)	Length Correction Factor
3048	120	3127	0.86
3099	122	3178	0.86
*3150	*124	3229	0.86
*3175	*125	2254	0.86
3251	128	3330	0.87
*3404	*134	3483	0.87
*3454	*136	3553	0.88
*3556	*140	3635	0.88
*3607	*142	3686	0.89
3658	144	3736	0.90
*3759	*148	3838	0.90
3801	150	3880	0.90
*3861	*152	3940	0.91
*3912	*154	3990	0.91
*3937	*155	4016	0.91
*3962	*156	4041	0.92
4013	158	4092	0.92
4115	162	4194	0.92
4216	166	4295	0.92
*4267	*168	4346	0.92
*4318	*170	4397	0.92
*4344	*171	4423	0.92
*4369	*172	4448	0.92
4394	173	4473	0.93
*4445	*175	4524	0.93
*4471	*176	4550	0.93
*4496	*177	4575	0.93
4521	178	4600	0.93
4572	180	4651	0.94
*4623	*182	4702	0.94
*4699	*188	4778	0.95
*4826	*190	4905	0.95
*4851	*191	4930	0.95
*4877	*192	4956	0.95
4953	195	5032	0.96
*4978	*196	5059	0.96
*5004	*197	5083	0.96
*5080	*200	5159	0.96
*5182	*204	5260	0.96
*5283	*208	5362	0.96

Table 12 *Small Diameter Factors for Speed Ratio*

Speed Ratio Range	*Factor to be Applied to the Smaller Pulley Diameter to obtain Equivalent Diameter (d_e)*
1.000–1.019	1.00
1.020–1.032	1.01
1.033–1.055	1.02
1.056–1.081	1.03
1.082–1.109	1.04
1.110–1.142	1.05
1.143–1.178	1.06
1.179–1.222	1.07
1.223–1.274	1.08
1.275–1.340	1.09
1.341–1.429	1.10
1.430–1.562	1.11
1.563–1.814	1.12
1.815–2.984	1.13
2.949 and over	1.14

Table 13 *Power Correction Factors for Arc of Contact*

Difference in Pulley Diameter in Millimetres (D-d)	*Centre to Centre Distance C (in mm)*											
	250	*375*	*500*	*625*	*750*	*1000*	*1250*	*1500*	*1750*	*2000*	*2250*	*2500*
50	0.97	0.98	0.99	0.99	0.99	0.99	0.99					
100	0.94	0.96	0.97	0.98	0.98	0.99	0.99					
150	0.91	0.94	0.96	0.97	0.97	0.98	0.98					
200	0.87	0.92	0.94	0.95	0.96	0.97	0.98	0.98				
250	0.82	0.89	0.93	0.94	0.95	0.97	0.97	0.98				
300	0.77	0.87	0.91	0.93	0.94	0.96	0.97	0.97	0.98			
350	0.70	0.84	0.89	0.91	0.93	0.95	0.96	0.97	0.97			
400		0.80	0.87	0.90	0.92	0.94	0.95	0.96	0.97			
450		0.77	0.85	0.89	0.91	0.93	0.95	0.96	0.96	0.97		
500		0.72	0.82	0.87	0.89	0.93	0.94	0.95	0.96	0.97		
550		0.67	0.80	0.85	0.88	0.92	0.94	0.95	0.96	0.96		
600			0.77	0.83	0.87	0.91	0.93	0.94	0.95	0.96	0.96	0.97
650			0.73	0.81	0.85	0.90	0.92	0.94	0.95	0.95	0.96	0.96
700			0.70	0.79	0.84	0.89	0.91	0.93	0.94	0.95	0.96	0.96
750			0.65	0.77	0.82	0.88	0.91	0.93	0.94	0.95	0.95	0.96
800				0.74	0.80	0.87	0.90	0.92	0.93	0.94	0.95	0.95
850				0.71	0.79	0.86	0.89	0.91	0.93	0.94	0.95	0.95
900				0.68	0.77	0.85	0.89	0.91	0.92	0.93	0.94	0.95
1050					0.70	0.81	0.86	0.89	0.91	0.92	0.93	0.94
1200						0.77	0.83	0.87	0.89	0.91	0.92	0.93
1350						0.72	0.80	0.85	0.87	0.89	0.91	0.92
1500						0.65	0.77	0.82	0.86	0.88	0.89	0.91
1650							0.73	0.80	0.84	0.86	0.88	0.90
1800							0.68	0.77	0.81	0.85	0.87	0.89
1950								0.73	0.79	0.83	0.85	0.87
2100								0.70	0.77	0.81	0.84	0.86
2250								0.65	0.74	0.79	0.82	0.85

For Tables 14-17

Belt Cross Section	*Formula*	*Maximum Value of d_e in the Formula*
A	$\text{kW} = \left(0.45\, S_1^{-0.09} - \dfrac{19.62}{d_e} - 0.765 \times 10^{-4}\, S_1^2\right) S_1$	125 mm
B	$\text{kW} = \left(0.79\, S_1^{-0.09} - \dfrac{50.8}{d_e} - 1.32 \times 10^{-4}\, S_1^2\right) S_1$	175 mm
C	$\text{kW} = \left(1.47\, S_1^{-0.09} - \dfrac{142.7}{d_e} - 2.34 \times 10^{-4}\, S_1^2\right) S_1$	300 mm
D	$\text{kW} = \left(3.22\, S_1^{-0.09} - \dfrac{506.7}{d_e} - 4.78 \times 10^{-4}\, S_1^2\right) S_1$	425 mm

Table 14 *Kilowatt Ratings for A Section V-belts with 180° Arc of Contact on Smaller Pulley*

Belt Speed S_1 (metres per second)	*Smaller Pulley Equivalent Diameter (mm)*					
	80	*90*	*100*	*110*	*120*	*125 and over*
0.5	0.13	0.13	0.14	0.14	0.15	0.15
1	0.22	0.24	0.25	0.27	0.28	0.29
2	0.37	0.40	0.43	0.46	0.49	0.51
3	0.51	0.58	0.64	0.68	0.72	0.74
4	0.58	0.74	0.81	0.88	0.93	0.96
5	0.74	0.85	0.95	1.04	1.13	1.18
6	0.81	0.94	1.05	1.15	1.25	1.32
7	0.88	1.08	1.25	1.39	1.50	1.54
8	0.96	1.17	1.35	1.50	1.62	1.69
9	1.03	1.32	1.54	1.69	1.77	1.84
10	1.10	1.40	1.62	1.77	1.91	1.99
11	1.18	1.47	1.69	1.91	2.06	2.13
12	1.25	1.54	1.84	2.06	2.21	2.28
13	1.32	1.62	1.91	2.13	2.35	2.43
14	1.32	1.69	1.99	2.28	2.50	2.50
15	1.32	1.77	2.06	2.35	2.57	2.65
16	1.40	1.84	2.13	2.43	2.65	2.79
17	1.40	1.84	2.21	2.50	2.79	2.87
18	1.40	1.84	2.28	2.57	2.87	2.94
19	1.40	1.84	2.28	2.65	2.94	3.02
20	1.32	1.91	2.35	2.72	3.02	3.09
21	1.32	1.91	2.35	2.72	3.02	3.16
22	1.25	1.91	2.35	2.72	3.09	3.24
23	1.25	1.84	2.35	2.79	3.09	3.24
24	1.18	1.84	2.35	2.79	3.16	3.31
25	1.10	1.77	2.28	2.79	3.16	3.31
26	1.03	1.69	2.28	2.72	3.16	3.31
27	0.88	1.62	2.20	2.72	3.09	3.31
28	0.81	1.54	2.13	2.65	3.09	3.24
29	0.66	1.47	2.06	2.57	3.02	3.24
30	0.51	1.32	1.99	2.50	2.94	3.16

Table 15 *Kilowatt Ratings for B Section V-belts with 180° Arc of Contact on Smaller Pulley.*

Belt Speed S_1 (metres per second)	*Smaller Pulley Equivalent Diameter (mm)*					
	130	*140*	*150*	*160*	*170*	*180 and over*
0.5	0.22	0.22	0.22	0.22	0.29	0.29
1	0.37	0.44	0.44	0.44	0.51	0.51
2	0.66	0.74	0.81	0.81	0.88	0.88
3	0.96	1.03	1.10	1.17	1.25	1.39
4	1.18	1.32	1.40	1.47	1.50	1.62
5	1.47	1.54	1.69	1.84	1.91	1.99
6	1.62	1.84	1.99	2.06	2.21	2.28
7	1.91	2.13	2.21	2.35	2.50	2.65
8	2.06	2.28	2.43	2.65	2.79	2.94
9	2.21	2.43	2.65	2.87	3.02	3.16
10	2.35	2.65	2.87	3.09	3.31	3,46
11	2.50	2.79	3.09	3.31	3.53	3.75
12	2.65	3.02	3.31	3.53	3.75	3.97
13	2.79	3.16	3.46	3.75	3.97	4.19
14	2.87	3.16	3.60	3.90	4.19	4.56
15	2.94	3.38	3.75	4.05	4.34	4.63
16	3.02	3.46	3.90	4.19	4.49	4.78
17	3.09	3.60	3.97	4.34	4.71	4.92
18	3.16	3.60	4.04	4.49	4.78	5.07
19	3.16	3.68	4.19	4.56	4.92	5.22
20	3.16	3.75	4.19	4.63	5.00	5.44
21	3.16	3.75	4.27	4.71	5.07	5.44
22	3.16	3.75	4.27	4.78	5.15	5.52
23	3.09	3.75	4.27	4.78	5.22	5.59
24	3.02	3.68	4.27	4.78	5.22	5.66
25	2.94	3.60	4.19	4.78	5.22	5.66
26	2.79	3.53	4.19	4.71	5.22	5.66
27	2.65	3.46	4.12	4.63	5.15	5.66
28	2.50	3.31	3.97	4.56	5.15	5.59
29	2.43	3.16	3.82	4.49	5.00	5.52
30	2.13	2.94	3.68	4.34	4.92	5.44

Table 16 *Kilowatt Ratings for C Section V-belts with 180° Arc of Contact on Smaller Pulley*

Belt Speed S_e (metres per second)	*Smaller Pulley Equivalent Diameter (mm)*						
	180	*200*	*220*	*240*	*260*	*280*	*300 and over*
0.5	0.37	0.44	0.44	0.51	0.51	0.51	0.51
1	0.64	0.73	0.81	0.88	0.88	0.96	0.96
2	1.15	1.32	1.47	1.50	1.69	1.77	1.84
3	1.62	1.84	2.06	2.21	2.35	2.50	2.57
4	2.06	2.35	2.57	2.79	3.02	3.16	3.31
5	2.35	2.79	3.09	3.38	3.60	3.75	3.97
6	2.72	3.16	3.60	3.90	4.19	4.41	4.63
7	3.02	3.60	4.05	4.41	4.71	5.00	5.22

(*Contd*)

(*Contd.*)

8	3.31	3.97	4.49	4.85	5.22	5.59	5.81
9	3.60	4.27	4.85	5.37	5.81	6.10	6.40
10	3.82	4.56	5.22	5.81	6.25	6.62	6.99
11	4.04	4.92	5.59	6.18	6.69	7.13	7.58
12	4.19	5.15	5.96	6.62	7.13	7.72	8.00
13	4.34	5.44	6.25	6.91	7.58	8.16	8.46
14	4.49	5.59	6.55	7.28	7.94	8.53	8.97
15	4.63	5.81	6.77	7.58	8.31	8.90	9.41
16	4.71	5.96	7.06	7.87	8.61	9.27	9.78
17	4.78	6.10	7.21	8.09	8.90	9.56	10.15
18	4.78	6.25	7.35	8.38	9.19	9.93	10.51
19	4.78	6.33	7.58	8.53	9.41	10.15	10.81
20	4.78	6.33	7.65	8.68	9.63	10.44	11.11
21	4.71	6.33	7.72	8.83	9.86	10.66	11.33
22	4.56	6.33	7.80	8.90	10.00	10.81	11.62
23	4.34	6.25	7.80	8.97	10.08	11.03	11.77
24	4.19	6.18	7.72	9.05	10.15	11.11	11.91
25	4.05	6.03	7.72	9.05	10.15	11.18	11.99
26	3.82	5.88	7.58	8.97	10.15	11.18	12.06
27	3.60	5.66	7.43	8.90	10.15	11.18	12.13
28	3.24	5.44	7.35	8.75	10.08	11.18	12.13
29	2.79	5.15	7.06	8.60	9.93	11.03	12.06
30	2.43	4.78	6.77	8.33	9.71	10.80	11.99

Table 17 *Kilowatt Ratings for D Section V-belts with 180° Arc of Contact on Smaller Pulley*

Belt Speed S_1 (metres per second)	*Smaller Pulley Equivalent Diameter (mm)*							
	300	*320*	*340*	*360*	*380*	*400*	*420*	*430 and over*
0.50	0.81	0.88	0.96	0.96	1.03	1.03	1.10	1.10
1	1.47	1.54	1.69	1.77	1.84	1.91	1.91	1.99
2	2.50	2.79	2.94	3.09	3.24	3.38	3.53	3.53
3	3.46	3.75	4.04	4.34	4.56	4.71	4.92	5.00
4	4.34	4.78	5.15	5.44	5.74	6.03	6.25	6.40
5	5.07	5.66	6.10	6.55	6.91	7.21	7.58	7.65
6	5.81	6.55	7.06	7.51	7.94	8.38	8.75	8.90
7	6.47	7.28	7.94	8.46	8.97	9.41	9.86	10.08
8	7.13	7.94	8.68	9.34	10.00	10.52	10.96	11.25
9	7.65	8.68	9.49	10.22	10.96	11.47	12.06	12.28
10	8.23	9.34	10.22	11.03	11.84	12.43	13.02	13.31
11	8.68	9.86	10.88	11.84	12.58	13.39	14.05	14.34
12	9.12	10.37	11.47	12.50	13.39	14.19	14.93	15.30
13	9.49	10.88	12.06	13.16	14.05	15.00	15.74	16.11
14	9.79	11.25	12.58	13.75	14.56	15.74	16.55	16.99
15	10.00	11.62	13.09	14.34	15.44	16.40	17.28	17.72
16	10.30	11.91	13.46	14.78	15.96	16.99	18.02	18.46
17	10.44	12.21	13.83	15.22	16.55	17.58	18.61	19.05

(*Contd*)

(*Contd.*)

18	10.51	12.43	14.12	15.59	16.99	18.09	19.20	19.71
19	10.51	12.50	14.27	15.89	17.36	18.53	19.86	20.23
20	10.51	12.58	14.49	16.11	17.65	18.98	20.15	20.74
21	10.37	13.31	14.56	16.40	17.87	19.27	20.52	21.11
22	10.22	12.58	14.56	16.47	18.02	19.49	20.81	21.48
23	9.93	12.43	14.56	16.40	18.17	19.71	21.11	21.70
24	9.63	12.21	14.34	16.40	18.24	19.78	21.26	21.99
25	9.19	11.91	14.27	16.25	18.17	19.78	21.33	22.06
26	8.68	11.47	13.97	16.11	17.87	19.78	21.33	21.99
27	8.16	11.03	13.53	15.81	17.80	19.56	21.26	21.99
28	7.51	10.51	13.09	15.44	17.43	19.34	21.11	21.84
29	6.77	9.86	12.58	15.00	17.14	19.12	20.89	21.62
30	5.96	9.12	11.91	14.42	16.70	18.68	20.52	21.33

Table18 *Correction Factor for Length F_c*

Nominal inside Length in mm	*Belt Section*				
	A	*B*	*C*	*D*	*E*
610	0.8	—	—	—	—
787	0.84	—	—	—	—
889	0.87	0.81	—	—	—
965	0.88	0.83	—	—	—
1,016	0.89	0.84	—	—	—
1,295	0.94	0.89	0.80	—	—
1,422	0.96	0.90	0.81	—	—
3,048	1.13	1.07	0.97	0.86	—
4,013	—	1.13	1.02	0.92	—
4,953	—	1.18	1.07	0.96	—
6,045	—	—	1.11	1.00	0.96
8,331	—	—	1.19	1.07	1.03

Table 19 *V-Belt Recommended Standard Pulley Diameter*

A Section (mm)	*B Section (mm)*	*C Section (mm)*	*D Section (mm)*
75	125	200	355
80	132	212	375
85	140	224	400
90	150	236	425
95	160	250	450
100	170	265	475
106	180	280	500
112	190	300	530
118	200	315	560
125	224	355	600
132	250	375	630
140	280	400	710

(*Contd*)

(*Contd.*)

150	300	450	750
160	315	500	800
170	355	530	900
180	375	560	1000
190	400	600	1060
200	450	630	1120
224	500	710	1250
250	530	750	1400
280	560	800	1500
300	600	900	1600
315	630	1000	1800
355	710	1120	2000
400	750	1250	
450	800	1400	
500	900	1600	
560	1000		
630	1120		
710			
800			

Table 20 *Recommended Pulley Diameter in mm (General)*

40,	45,	50,	56,	63,	71,	80,	90,	100,	112,
125,	140,	160,	180,	200,	224,	250,	280,	315,	355,
400,	450,	500,	560,	630,	710,	800,	900,	1000,	1120,
1250,	1400,	1600,	1800,	2000					

Table 21 *Data on Standard V-Belt Sections*

Cross Section Symbol	*Usual Load of Drive* kW	*Recommended Minimum Pulley Pitch Diameter* d, mm	*Nominal Top Width* W, mm	*Nominal Thickness* T, mm	*Weight per metre* kgf
A	0.75 – 5	75	13	8	0.106
B	2 – 15	125	17	11	0.189
C	7.5 – 75	200	22	14	0.343
D	22 – 150	355	32	19	0.596
E	30 – 190	500	38	23	—

Table 22 *Values of F_b for Different Pulley Diameter Ratio*

Pulley Diameter Ratio, D/d	F_b	*Pulley Diameter Ratio, D/d*	F_b
1.0 to 1.099	1.00	1.223 to 1.274	1.08
1.02 to 1.032	1.01	1.275 to 1.340	1.09
1.033 to 1.055	1.02	1.341 to 1.429	1.10
1.056 to 1.081	1.03	1.43 to 1.562	1.11
1.082 to 1.109	1.04	1.563 to 1.814	1.12
1.11 to 1.142	1.05	1.815 to 2.948	1.13
1.143 to 1.178	1.06	2.949 and over	1.14
1.179 to 1.222	1.07		

Table 23 *Dimensions of V-grooved Pulley*

Groove Cross Section	t_r mm	Pitch Width l_p	b Minimum mm	Pulley Pitch Diameter, d_p mm	Angle a deg	h mm	f mm	e mm
A	6	11	3.3	Under 75 to 67 75 to 125 and over	32° 34° 38°	8.7	10	15
B	8	14	4.2	125 to 200	32° 34° 38°	10.8	12.5	19
C	10	19	5.7	200 to 300	34° 36° 38°	14.3	17	25.5
D	12	27	8.1	355 to 500	34° 36° 38°	19.9	24	37
E	14	32	9.6	—	—	23.4	29	44.5

**Note:* Values in this table are calculated from the following formulae:
$b = 0.3\ l_p$; $c = 8.35\ l_p, f = 0.9\ l_p, h = 0.7\ l_p + 1$ in mm.

Table 24 *Properties of Flat Belts*

Belt Material	Ultimate strength σ_v MPa	Endurance Limit σ_{-1} MPa	Factor of Safety n_t	Maximum Velocity in m/sec v	Density gm/cm^3 g	d/t	Width b	Thickness mm	Modulus of Elasticity E, MPa
Leather oak tanned	20–30	4	8–10	20–40	0.98	25–35	20–300	3–7.5	100–1,500
chrome Rubber	37–44	8	6–10	20–30	1.25–1.5	30–40	20–500	2.5–14	80–120
Cotton woven	35–40	8	6–10	15–25	0.75–1.5	25–40	30–250	4.5–8.5	30–60
Woollen woven	30	6	8–10	20–30	0.9–1.25	25–30	50–500	2–10	

Table 25 *Dimensions and Breaking Loads for Roller Chains*

ISO No.	Pitch p(mm)	Roller d_1(mm)	Width b_1(mm)	Transverse Pitch p_1(mm)	P_b kN	Bearing Area mm^2
06 B	9.525	6.35	5.72	10.24	10.70	28
08 B	12.70	8.5	17.75	13.92	18.20	50
10 B	15.875	10.16	9.65	16.59	22.70	67
12 B	19.05	12.07	11.68	19.46	29.50	89
16 B	25.40	15.88	17.02	31.88	65.00	211
20 B	31.75	19.05	19.56	36.45	98.10	296
24 B	38.10	25.40	25.40	48.36	108.90	555
28 B	44.45	27.94	30.99	59.56	131.50	741
32 B	50.80	29.21	30.99	58.55	172.40	847
40 B	63.50	39.37	38.10	72.29	272.20	1276

Table 26 *Bearing Pressure for Chain Links*

v m/s	*No. of Teeth on Small Sprocket z_1*								
	11	*13*	*15*	*17*	*19*	*21*	*23*	*>25*	
1.0	21.28	23.34	24.72	25.6	26.39	26.78	27.46	27.95	bearing pr. MPa
2.0	16.68	19.32	21.09	22.17	23.25	23.94	24.53	25.11	
4.0	11.38	14.42	16.68	18.15	19.13	20.04	20.69	21.38	
7.0		9.61	12.36	14.32	15.60	16.48	17.27	18.05	
12.0			7.55	9.90	11.77	12.85	13.73	14.42	
18.0				6.28	8.14	9.53	10.50	11.58	
24.0					5.00	6.67	8.04	9.12	

Table 27 *Impact Factor Y for Chain Drive*

Type of Machine	*Y*
Lathe/Drilling m/c	1.4
Milling machine/blower/C.F Pump	1.5
Centrifugal compressor/Agitator	1.6
Mixing drum	1.7
Ball mill/Drawing m/c	1.8
Reciprocating Pump/Beater/Vibrating Screen	2.0
Slotting M/c	2.3
Hoists/Presses/Ventilator/Rolling and Hammer mill/Reciprocating compressor	2.5
Stacker/Excavator	3.0

Table 28 *Form Factors y - for use in Lewis Strength Equation*

Number of Teeth	*$14\frac{1}{2}°$ Full-Depth Involute or Composite*	*20* Full-Depth Involute*	*20* Stub Involute*
12	0.067	0.078	0.099
13	0.071	0.083	0.103
4	0.075	0.088	0.108
15	0.078	0.092	0.111
16	0.081	0.094	0.115
17	0.084	0.096	0.117
18	0.086	0.098	0.120
19	0.088	0.100	0.123
20	0.090	0.102	0.125
21	0.092	0.104	0.127
23	0.094	0.106	0.130
25	0.097	0.108	0.133
27	0.099	0.111	0.136
30	0.101	0.114	0.139
34	0.104	0.118	0.142
38	0.106	0.122	0.145
43	0.108	0.126	0.147
50	0.110	0.130	0.151

(*Contd*)

(*Contd.*)

60	0.113	0.134	0.154
75	0.115	0.138	0.158
100	0.117	0.142	0.161
150	0.119	0.146	0.165
300	0.122	0.150	0.170
Rack	0.124	0.154	0.175

Table 29 *Dimensionless Performance Parameters for Full Journal Bearings with Side Flow*

L/D	ε	$\dfrac{h_o}{c_r}$	S	ϕ	$\dfrac{r}{c_r}f$	$\dfrac{q}{rc_r n_s L}$	$\dfrac{q_s}{q}$	$\dfrac{\rho c \Delta t_o}{p}$	$\dfrac{p}{p_{max}}$
∞	0	1.0	∞	(70.92)	∞	π	0	∞	—
	0.1	0.9	0.240	69.10	4.80	3.03	0	19.9	0.826
	0.2	0.8	0.123	67.26	2.57	2.83	0	11.4	0.814
	0.4	0.6	0.0626	61.94	1.52	2.26	0	8.47	0.764
	0.6	0.4	0.0389	54.31	1.20	1.56	0	9.73	0.667
	0.8	0.2	0.021	42.22	0.961	0.760	0	15.9	0.495
	0.9	0.1	0.0115	31.62	0.756	0.411	0	23.1	0.358
	0.97	0.03	—	—	—	—	—	0	—
	1.0	0	0	0	0	0	0	∞	0
1	0	1.0	∞	(85)	∞	π	0	∞	—
	0.1	0.9	1.33	79.5	26.4	3.37	0.150	106	0.540
	0.2	0.8	0.631	74.02	12.8	3.59	0.280	52.1	0.529
	0.4	0.6	0.264	63.10	5.79	3.99	0.497	24.3	0.484
	0.6	0.4	0.121	50.58	3.22	4.33	0.680	14.2	0.415
	0.8	0.2	0.0446	36.24	1.70	4.62	0.842	8.00	0.313
	0.9	0.1	0.0188	26.45	1.05	4.74	0.919	5.16	0.247
	0.97	0.03	0.00474	15.47	0.514	4.82	0.973	2.61	0.152
	1.0	0	0	0	0	—	1.0	0	0
$\frac{1}{2}$	0	1.0	∞	(88.5)	∞	π	0	∞	—
	0.1	0.9	4.31	81.62	85.6	3.43	0.173	343.0	0.523
	0.2	0.8	2.03	74.94	40.9	3.72	0.318	164.0	0.506
	0.4	0.6	0.779	61.45	17.0	4.29	0.552	68.6	0.441
	0.6	0.4	0.319	48.14	8.10	4.85	0.730	33.0	0.365
	0.8	0.2	0.0923	33.31	3.26	5.41	0.874	13.4	0.267
	0.9	0.1	0.0313	23.66	1.60	5.69	0.939	6.66	0.206
	0.97	0.03	0.00609	13.75	0.610	5.88	0.980	2.56	0.126
	1.0	0	0	0	0	—	1.0	0	0
$\frac{1}{4}$	0.0	1.0	∞	(89.5)	∞	π	0	∞	—
	0.1	0.9	16.2	82.31	322.0	3.45	0.180	1287.0	0.515
	0.2	0.8	7.57	75.18	153.0	3.76	0.330	611.0	0.489
	0.4	0.6	2.83	60.86	61.1	4.37	0.567	245.0	0.415
	0.6	0.4	1.07	46.72	26.7	4.99	0.746	107.0	0.334
	0.8	0.2	0.261	31.04	8.80	5.60	0.884	35.4	0.240

(*Contd*)

(*Contd.*)

	0.9	0.1	0.0736	21.85	3.50	5.91	0.945	14.1	0.180
	0.97	0.03	0.0101	12.22	0.922	6.12	0.984	3.73	0.108
	1.0	0	0	0	0	—	1.0	0	0

q in^3 sec. $\rho \approx 0.03$ lb/in.^{3} $c = 3734$ in-lb/lb-°F. $\rho c = 112$.

Table 30 *Dimensionless Performance Parameters for 180° Bearings with Side Flow*

L/D	ε	$\dfrac{h_o}{c_r}$	S	ϕ	$\dfrac{r}{c_r}\mu$	$\dfrac{q}{rc_r n_s L}$	$\dfrac{q_s}{q}$	$\dfrac{\rho c \Delta t_o}{p}$	$\dfrac{p}{p_{\max}}$
∞	0	1.0	∞	90.0	∞	π	∞	∞	—
	0.1	0.9	0.347	72.90	3.55	3.04	0	14.7	0.778
	0.2	0.8	0.179	61.32	2.01	2.80	0	8.99	0.759
	0.4	0.6	0.0898	49.99	1.29	2.20	0	7.34	0.700
	0.6	0.4	0.0523	43.15	1.06	1.52	0	8.71	0.607
	0.8	0.2	0.0253	33.35	0.859	0.767	0	14.1	0.459
	0.9	0.1	0.0128	25.57	0.681	0.380	0	22.15	0.337
	0.97	0.03	0.00384	15.48	0.416	0.119	0	44.0	0.190
	1.0	0	0	0	0	0	0	—	0
	0.1	0.9	1.40	78.50	14.1	3.34	0.139	57.0	0.525
	0.2	0.8	0.670	68.93	7.15	3.46	0.252	29.7	0.513
	0.4	0.6	0.278	58.86	3.61	3.49	0.425	16.5	0.466
	0.6	0.4	0.128	44.67	2.28	3.25	0.572	12.4	0.403
	0.8	0.2	0.0463	32.33	1.39	2.63	0.721	10.4	0.313
	0.9	0.1	0.0193	24.14	0.921	2.14	0.818	9.13	0.244
	0.97	0.03	0.00483	14.57	0.483	1.60	0.915	6.96	0.157
	1.0	0	0	0	0	—	1.0	0	0
$\frac{1}{2}$	0	1.0	∞	90.0	∞	π	0	∞	—
	0.1	0.9	4.38	79.97	44.0	3.41	0.167	177.0	0.518
	0.2	0.8	2.06	72.14	21.6	3.64	0.302	87.8	0.499
	0.4	0.6	0.794	58.01	9.96	3.93	0.506	42.7	0.438
	0.6	0.4	0.321	45.01	5.41	3.93	0.665	25.9	0.365
	0.8	0.2	0.0921	31.29	2.54	3.56	0.806	15.0	0.273
	0.9	0.1	0.0314	22.80	1.38	3.17	0.886	9.80	0.208
	0.97	0.03	0.00625	13.63	0.581	2.62	0.951	5.30	0.132
	1.0	0	0	0	0	—	1.0	0	0
$\frac{1}{4}$	0	1.0	∞	90.0	∞	π	0	∞	—
	0.1	0.9	16.3	81.40	163.0	3.44	0.176	653.0	0.513
	0.2	0.8	7.60	73.70	79.4	3.71	0.320	320.0	0.489
	0.4	0.6	2.84	58.99	35.1	4.11	0.534	146.0	0.417
	0.6	0.4	1.08	44.96	17.6	4.25	0.698	79.8	0.336
	0.8	0.2	0.263	30.43	6.88	4.07	0.837	36.5	0.241
	0.9	0.1	0.0736	21.43	2.99	3.72	0.905	18.4	0.180
	0.97	0.03	0.0104	12.28	0.877	3.29	0.961	6.46	0.110
	1.0	0	0	0	0	—	1.0	0	0

Table 31 *Dimensionless Performance Parameters for 120° Bearing, Centrally Loaded, with Side Flow*

L/D	ε	$\frac{h_o}{c_r}$	S	ϕ	$\frac{r}{c_r}f$	$\frac{q}{rc_r n_s L}$	$\frac{q_s}{q}$	$\frac{\rho c \Delta t_o}{p}$	$\frac{p}{p_{max}}$
∞	0	1.0	∞	90.0	∞	π	0	∞	—
	0.1	0.9007	0.877	66.69	6.02	3.02	0	25.1	0.610
	0.2	0.8	0.431	52.60	3.26	2.75	0	14.9	0.599
	0.4	0.6	0.181	39.02	1.78	2.13	0	10.5	0.566
	0.6	0.4	0.0845	32.67	1.21	1.47	0	10.3	0.509
	0.8	0.2	0.0328	26.80	0.853	0.759	0	14.1	0.405
	0.9	0.1	0.0147	21.51	0.653	0.388	0	21.2	0.311
	0.97	0.03	0.00406	13.86	0.399	0.118	0	42.4	0.199
	1.0	0	0	0	0	0	0	∞	0
1	0	1.0	∞	90.0	∞	π	0	∞	—
	0.1	0.9024	2.14	72.43	14.5	3.20	0.0876	59.5	0.427
	0.2	0.8	1.01	58.25	7.44	3.11	0.157	32.6	0.420
	0.4	0.6	0.385	43.98	3.60	2.75	0.272	19.0	0.396
	0.6	0.4	0.162	35.65	2.16	2.24	0.384	15.0	0.356
	0.8	0.2	0.0531	27.42	1.27	1.57	0.535	13.9	0.290
	0.9	0.1	0.0208	21.29	0.855	1.11	0.657	14.4	0.233
	0.97	0.03	0.00498	13.49	0.461	0.694	0.812	14.0	0.162
	1.0	0	0	0	0	—	1.0	0	0
$\frac{1}{2}$	0	1.0	∞	90.0	∞	π	0	—	—
	0.1	0.9034	5.42	74.99	36.6	3.29	0.124	149.0	0.431
	0.2	0.8003	2.51	63.38	18.1	3.32	0.225	77.2	0.424
	0.4	0.6	0.914	48.07	8.20	3.15	0.386	40.5	0.389
	0.6	0.4	0.354	38.50	4.43	2.80	0.530	27.0	0.336
	0.8	0.2	0.0973	28.02	2.17	2.18	0.684	19.0	0.261
	0.9	0.1	0.0324	21.02	1.24	1.70	0.787	15.1	0.203
	0.97	0.03	0.00631	13.00	0.550	1.19	0.899	10.6	0.136
	1.0	0	0	0	0	—	1.0	0	0
$\frac{1}{4}$	0	1.0	∞	90.0	∞	π	0	∞	—
	0.1	0.9044	18.4	76.97	124.0	3.34	0.143	502.0	0.456
	0.2	0.8011	8.45	65.97	60.4	3.44	0.260	254.0	0.438
	0.4	0.6	3.04	51.23	26.6	3.42	0.442	125.0	0.389
	0.6	0.4	1.12	40.42	13.5	3.20	0.599	75.8	0.321
	0.8	0.2	0.268	28.38	5.65	2.67	0.753	42.7	0.237
	0.9	0.1	0.0743	20.55	2.63	2.21	0.846	25.9	0.178
	0.97	0.03	0.0105	12.11	0.832	1.69	0.931	11.6	0.112
	1.0	0	0	0	0	—	1.0	0	0

Table 32 *Dimensionless Performance Parameters for 60° Bearings with Side Flow*

L/D	ε	$\frac{h_o}{c_r}$	S	ϕ	$\frac{r}{c_r}\mu$	$\frac{q}{rc_r n_s L}$	$\frac{q_s}{q}$	$\frac{\rho c \Delta t_o}{p}$	$\frac{p}{p_{max}}$
∞	0	1.0	∞	90.0	∞	π	0	∞	—
	0.1	0.9191	5.75	65.91	19.7	3.01	0	82.3	0.337
	0.2	0.8109	2.66	48.91	10.1	2.73	0	46.5	0.336
	0.4	0.6002	0.931	31.96	4.67	2.07	0	28.4	0.329
	0.6	0.4	0.322	23.21	2.40	1.40	0	21.5	0.317
	0.8	0.2	0.0755	17.39	1.10	0.722	0	19.2	0.287
	0.9	0.1	0.0241	14.94	0.667	0.372	0	22.5	0.243
	0.97	0.03	0.00495	10.58	0.372	0.115	0	40.7	0.163
	1.0	0	0	0	0	0	0	∞	0
1	0	1.0	∞	90.0	∞	π	0	∞	—
	0.1	0.9212	8.52	67.92	29.1	3.07	0.0267	121.0	0.252
	0.2	0.8133	3.92	50.96	14.8	2.82	0.0481	67.4	0.251
	0.4	0.6010	1.34	33.99	6.61	2.22	0.0849	39.1	0.247
	0.6	0.4	0.450	24.56	3.29	1.56	0.127	28.2	0.239
	0.8	0.2	0.101	18.33	1.42	0.883	0.200	22.5	0.220
	0.9	0.1	0.0309	15.33	0.822	0.519	0.287	23.3	0.192
	0.97	0.03	0.00584	10.88	0.422	0.226	0.465	30.5	0.139
	1.0	0	0	0	0	—	1.0	0	0
$\frac{1}{2}$	0	1.0	∞	90.0	∞	π	0.0	∞	—
	0.1	0.9223	14.2	69.00	48.6	3.11	0.0488	201.0	0.239
	0.2	0.8152	6.47	52.60	24.2	2.91	0.0883	109.0	0.239
	0.4	0.6039	2.14	37.00	10.3	2.38	0.160	59.4	0.233
	0.6	0.4	0.695	26.98	4.93	1.74	0.236	40.3	0.225
	0.8	0.2	0.149	19.57	2.02	1.05	0.350	29.4	0.201
	0.9	0.1	0.0422	15.91	1.08	0.664	0.464	26.5	0.172
	0.97	0.03	0.00704	10.85	0.490	0.329	0.650	27.8	0.122
	1.0	0	0	0	0	—	1.0	0	0
$\frac{1}{4}$	0	1.0	∞	90.0	∞	π	0	∞	—
	0.1	0.9251	35.8	71.55	121.0	3.16	0.0666	499.0	0.251
	0.2	0.8242	16.0	58.51	58.7	3.04	0.131	260.0	0.249
	0.4	0.6074	5.20	41.01	24.5	2.57	0.236	136.0	0.242
	0.6	0.4	1.65	30.14	11.2	1.98	0.346	86.1	0.228
	0.8	0.2	0.333	21.70	4.27	1.30	0.496	54.9	0.195
	0.9	0.1	0.0844	16.87	2.01	0.894	0.620	41.0	0.159
	0.97	0.03	0.0110	10.81	0.713	0.507	0.786	29.1	0.107
	1.0	0	0	0	0	—	1.0	0	0

Table 33 *Factors X, V and Y*

		In Relation to the Load the Inner Ring is			*Single Row Bearings*[(2)]		*Double Row Bearings*[(3)]				
Bearing		*Rotating*		*Stationary*	$\frac{F_a}{VF_r} > e$		$\frac{F_a}{VF_r} \le e$		$\frac{F_a}{VF_r} > e$		*e*
Type	*V*	*V*	*X*	*Y*	*X*	*Y*	*X*	*Y*	*X*	*Y*	
Radial											
Contact deep Groove Ball Bear-ings	$\frac{F_a}{C_o}$	$\frac{F_a}{IZD^2}$ Units kPa									
	0.014	172				2.30				2.30	0.19
	0.028	345				1.99				1.99	0.22
	0.056	689				1.71				1.71	0.26
	0.084	1030	1	1.2	0.56	1.55	1	0	0.56	1.55	0.28
	0.11	1380				1.45				1.45	0.30
	0.17	2070				1.31				1.31	0.34
	0.28	3450				1.15				1.15	0.38
	0.42	5170				1.04				1.04	0.42
	0.56	6890				1.00				1.00	0.44
Angular with Contact											
Groove Ball the Bearings Contact Angle[(4)]	$\frac{iF_a}{C_a}$	$\frac{F_a}{IZD^2}$ Units kPa		For this type use *X*, *Y* and *e* values applicable to single row radial contact bearings							
	0.014	172						2.78		3.74	0.23
	0.028	345						2.40		3.23	0.26
	0.056	689						2.07		2.78	0.30
	0.085	10.30	1	1.2			1	1.87	0.78	2.52	0.34
5°	0.11	1380						1.75		2.36	0.36
	0.17	2070						1.58		2.13	0.40
	0.28	3450						1.39		1.87	0.45
	0.42	5170						1.26		1.69	0.50
	0.56	6890						1.21		1.63	0.52
	0.014	172				1.88		2.18		3.06	0.29
	0.029	345				1.71		1.98		2.78	0.32

(*Contd*)

(*Contd*)

	0.057	689				1.52		1.76		2.47	0.36
	0.086	1030				1.41		1.63		2.29	0.38
10°	0.11	1380	1	1.2	0.46	1.34	1	1.55	0.75	2.18	0.40
	0.17	2070				1.23		1.42		2.00	0.44
	0.29	3450				1.10		1.27		1.79	0.49
	0.43	5170				1.01		1.17		1.64	0.54
	0.57	6890				1.00		1.16		1.63	0.54
	0.015	172				1.47		1.65		2.39	0.38
	0.029	345				1.40		1.57		2.28	0.40
	0.058	689				1.30		1.46		2.11	0.43
	0.087	1030				1.23		1.38		2.00	0.46
15°	0.12	1380	1	1.2	0.44	1.19	1	1.34	0.72	1.93	0.47
	0.17	2070				1.12		1.26		1.82	0.50
	0.29	3450				1.02		1.14		1.66	0.55
	0.44	5170				1.00		1.12		1.63	0.56
	0.58	6890				1.00		1.12		1.63	0.56
20°			1	1.2	0.43	1.00	1	1.09	0.70	1.63	0.57
25°			1	1.2	0.41	0.87	1	0.92	0.67	1.61	0.68
30°			1	1.2	0.39	0.75	1	0.78	0.63	1.24	0.80
35°			1	1.2	0.37	0.68	1	0.66	0.60	1.07	0.95
40°			1	1.2	0.35	0.57	1	0.55	0.57	0.93	1.14
Self-aligning Ball Bearings			1	1	0.40	0.4 cot α	1	0.42 cot α	0.65	0.65 cot α	1.5 tan α

Table 34 *Factors X, Y and e*

Bearing Type		*Single Row Bearings*[1] $\frac{F_a}{VF_r} > e$ X	Y	*Double Row Bearings*[3] $\frac{F_a}{VF_r} \le e$ X	Y	$\frac{F_a}{VF_r} > e$ X	Y	*e*
Radial contact Groove ball bearings	$\frac{F_a}{iZD^2}$ Units kPa							
	172		3.09				3.09	0.09
	345		2.77				2.77	0.12
	689		2.43				2.43	0.14
	1030		2.23				2.23	0.15
	1380	0.56	2.10	1	0	0.56	2.10	0.16
	2070		1.92				1.92	0.18
	3450		1.71				1.71	0.21
	5170		1.56				1.56	0.23
	6890		1.44				1.44	0.24

(*Contd*)

(*Contd*)

Angular contact, Groove ball bearings with contact angle	$\frac{F_a}{ZD^2}$ Units kPa	For this type use the X, Y and e values applicable to single row radial contact bearings						
	172				3.69		5.02	0.17
	345				3.30		4.49	0.19
	689				2.89		3.94	0.22
	1030			1	2.66	0.78	3.63	0.24
5°	1380				2.50		3.41	0.25
	2070				2.20		3.12	0.27
	3450				2.04		2.78	0.31
	5170				1.86		2.53	0.34
	6890				1.72		2.35	0.36
	172		2.20		2.55		3.58	0.25
	345		2.09		2.41		3.39	0.26
	689		1.94		2.24		3.14	0.28
10°	1030		1.84		2.13		2.99	0.29
	1380	0.46	1.77	1	2.04	0.75	2.87	0.31
	2070		1.66		1.92		2.69	0.33
	3450		1.53		1.77		2.49	0.35
	5170		1.44		1.66		2.33	0.38
	6890		1.36		1.57		2.21	0.40
	172		1.55		1.74		2.52	0.35
15°	345		1.51		1.70		2.46	0.36
	689		1.48		1.66		2.41	0.36
	1030		1.42		1.59		2.31	0.38
	1380	0.44	1.39	1	1.56	0.72	2.25	0.39
	2070		1.34		1.50		2.17	0.41
	3450		1.26		1.42		2.05	0.43
	5170		1.20		1.35		1.96	0.45
	6890		1.16		1.30		1.88	0.47
20°		0.43	1.14	1	1.25	0.70	1.86	0.50
25°		0.41	0.95	1	1.00	0.67	1.55	0.62
30°		0.39	0.81	1	0.83	0.63	1.31	0.75
35°		0.37	0.69	1	0.69	0.60	1.12	0.91
40°		0.35	0.60	1	0.58	0.57	0.97	1.08

Table 35 *Deep Groove Type of Ball Bearings: Load Capacities*

Bearing of Basic Design No. (SKF)	*D* *mm*	*D* *mm*	*B* *mm*	*r* *mm*	*Basic Capacity Static. Co. N*	*Basic Dynamic Cap. C N*	*Max Permissible Speed r.p.m*
6000	10	26	6	0.5	1863.9	3531.6	20000
01	12	28	8	0.5	2158.2	3924.0	20000
02	15	32	9	0.5	2501.55	4316.4	20000
6003	17	35	10	0.5	2795.85	4561.5	20000
04	20	42	12	1	4414.50	7210.5	16000
05	25	47	12	1	5101.20	7651.00	16000
6006	30	55	13	1.5	6965.10	10202.4	13000
07	35	62	14	1.5	8632.80	12262.5	13000
08	40	68	15	1.5	9613.80	12949.2	10000
6009	45	75	16	1.5	12458.70	15990.3	10000
10	50	80	16	1.5	13439.70	16677	8000
11	55	90	18	2	17658.00	21592	8000
6012	60	95	18	2	18933.30	22366.8	8000
13	65	100	18	2	20797.20	23544	8000
14	70	110	20	2	25015.50	29430	6000
6015	75	115	20	2	27468.00	30411	6000
16	80	125	22	2	32863.50	36787.5	6000
17	85	130	22	2	35316.00	38259	5000
6018	90	140	24	2.5	40711.50	44635.00	5000
19	95	145	24	2.5	44145.00	46597.5	5000
20	100	150	24	2.5	44145.00	46597.5	4000
6021	105	160	26	3	52974.00	55917	4000
22	110	170	28	3	59841.00	62784	4000
24	120	180	28	3	64255.50	65727	3000
6026	130	200	33	3	81423.00	81423	3000
28	140	210	33	3	88290.00	84856.5	3000
30	150	225	35	3.5	102024.00	96138	2500
6032	160	240	38	3.5	115758.00	109872	2500
34	170	260	42	3.5	140283.00	129492	2500
36	180	280	46	3.5	162846.00	147150	2000
6038	190	290	46	3.5	176580.00	150093	2000
40	200	310	51	3.5	196200.00	166770	2000

Table 36 *Deep Groove Ball Bearings*

ISI No.	*Bearing of Basic Design No. (SKF)*	*d* *mm*	*D* *mm*	*B* *mm*	*Basic Capacity* *Static* C_0N	*Dynamic* *CN*	*Max Permissible Speed* *r.p.m*	
10BC02	6200	10	30	9	1	2197.4	3924	20000
12BC02	01	12	32	10	1	2943	5297.4	20000
15BC02	02	15	35	11	1	3482.55	5984.1	16000
17BC02	6203	17	40	12	1	4316	7357.5	16000

(*Contd*)

(*Contd.*)

20BC02	04	20	47	14	1.5	6425.55	9810.00	16000
25BC02	05	25	52	15	1.5	6965.10	10791.00	13000
30BC02	6206	30	62	16	1.5	9810.00	15009.3	13000
35BC02	07	35	72	17	2	13439.7	19620.0	10000
40BC02	08	40	80	18	2	15696	22366.8	10000
45BC02	6209	45	85	19	2	17952.3	25015.5	8000
50BC02	10	50	90	20	2	20797.2	26977.5	8000
55BC02	11	55	100	21	2.5	25506	33354	8000
60BC02	6212	60	110	22	2.5	31392	39730.5	6000
65BC02	13	65	120	23	2.5	34825.5	43164	6000
70BC02	14	70	125	24	2.5	38254	47088	5000
75BC02	6215	75	130	25	2.5	41692.5	51012	5000
80BC02	16	80	140	26	3	44635	55917	5000
85BC02	17	85	150	28	3	55917	64255.5	4000
90BC02	6218	90	160	30	3	61803	73575	4000
95BC02	19	95	170	32	3.5	70632	83385	4000
100BC02	20	100	180	34	3.5	79951.5	94666.5	3000
105BC02	6221	105	190	36	3.5	91233	102024	3000
110BC02	22	110	200	38	3.5	102024	109872	3000
120BC02	24	120	215	40	3.5	102024	111834	3000
	6226	130	230	40	4	113796	119682	2500
	6228	140	250	42	4	126549	126549	

Table 37 *Cumulative Normal Frequency Distribution (Area under the Standard Normal Curve from 0 to Z)*

Z	*0.00*	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545

(*Contd*)

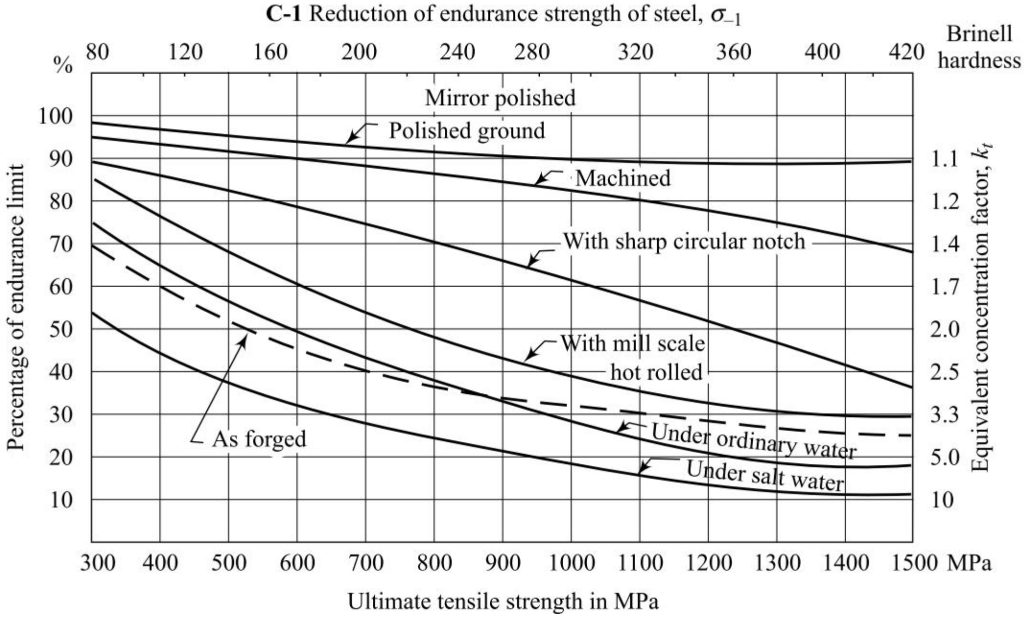
(*Contd.*)

1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.2965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4979	.4979	.4979	.4980	.4979
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999

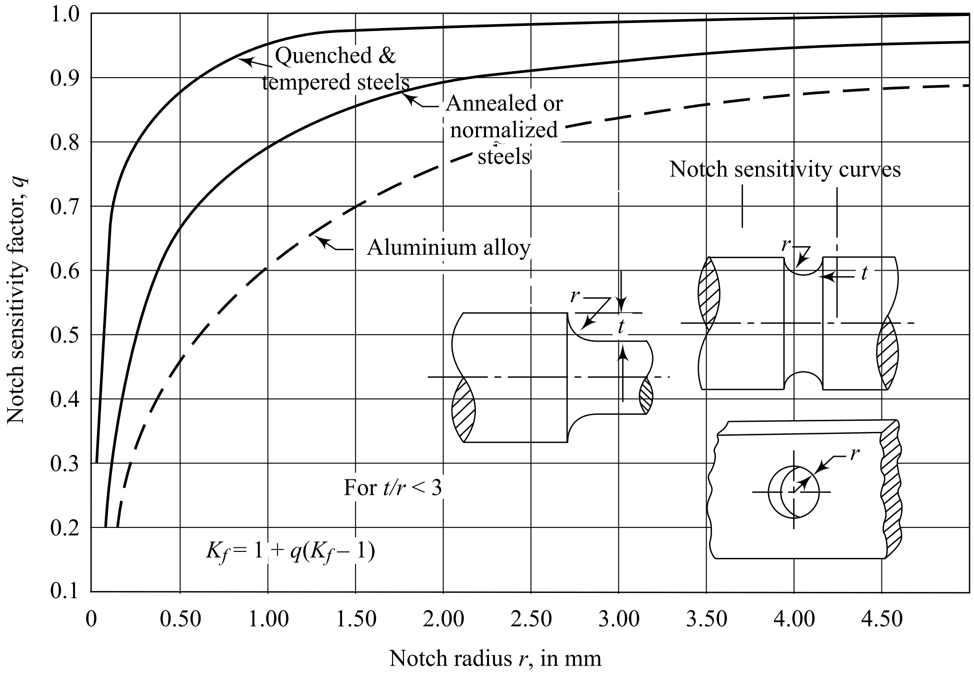
2

Appendix

C-1 Reduction of endurance strength of steel, σ_{-1}

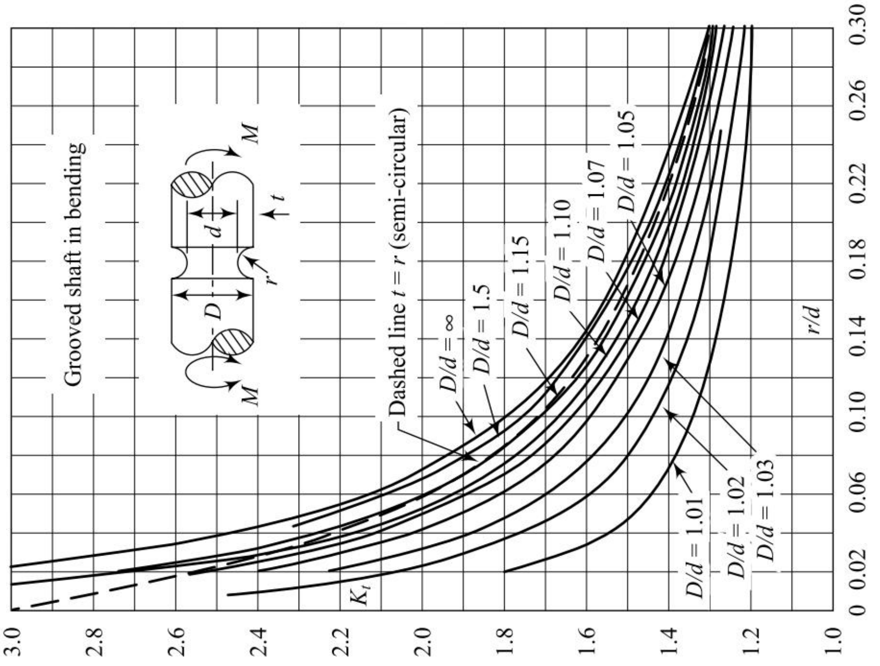


C-2

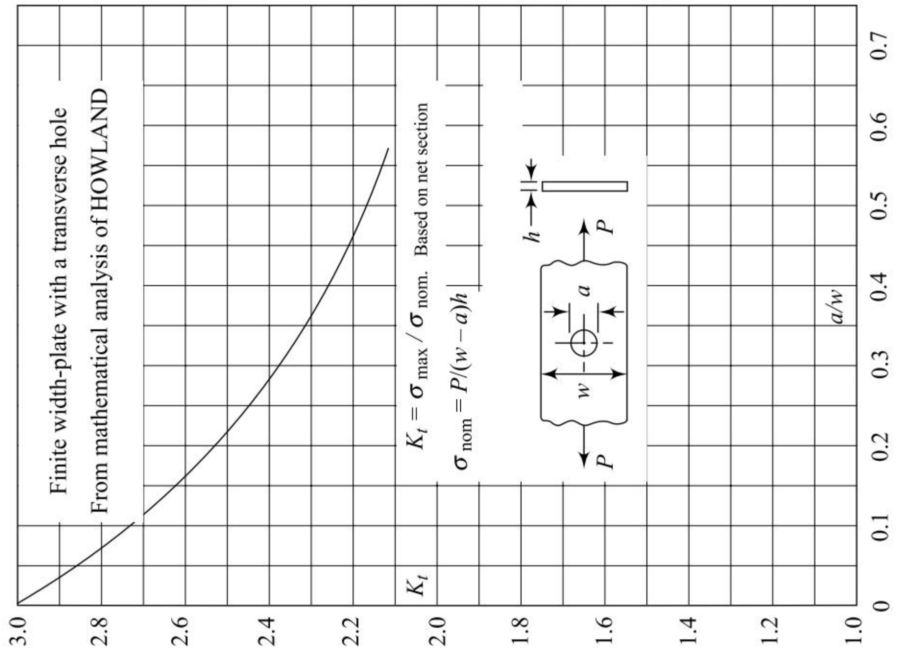


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
Notch sensitivity factor, q
Quenched &
tempered steels
Annealed or
normalized
steels
Aluminium alloy
Notch sensitivity curves
r
t
For t/r < 3
Kf = 1 + q(Kf – 1)
0
0.50
1.00
1.50
2.00
2.50
3.00
3.50
4.00
4.50
Notch radius r, in mm

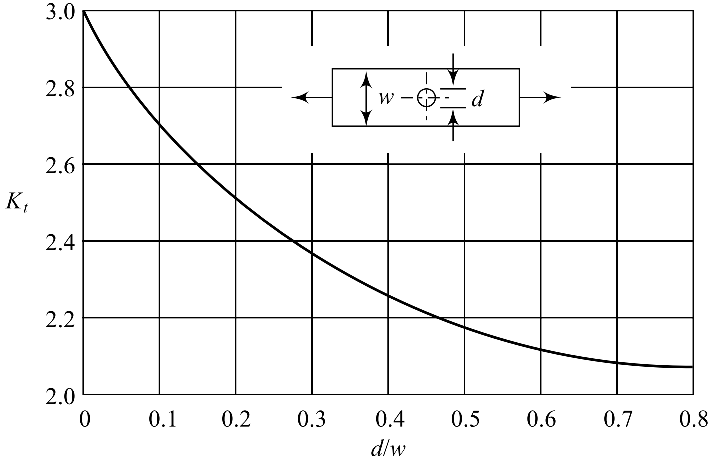
C-3


Grooved shaft in bending
Dashed line $t = r$ (semi-circular)
$D/d = \infty$
$D/d = 1.5$
$D/d = 1.15$
$D/d = 1.10$
$D/d = 1.07$
$D/d = 1.05$
$D/d = 1.03$
$D/d = 1.02$
$D/d = 1.01$
K_t
r/d

C-4

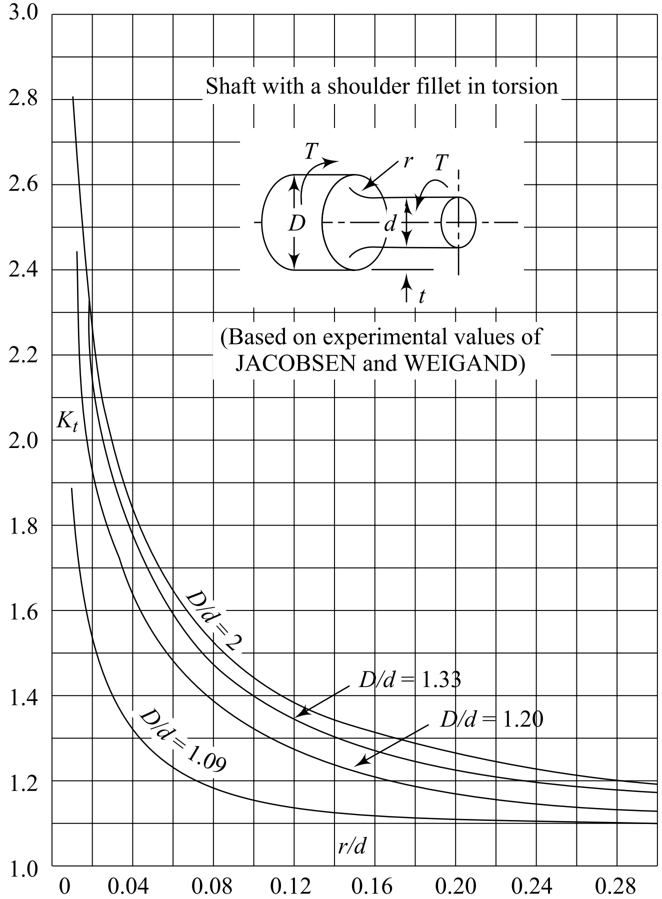

Finite width-plate with a transverse hole
From mathematical analysis of HOWLAND
$K_t = \sigma_{max} / \sigma_{nom.}$ Based on net section
$\sigma_{nom} = P/(w - a)h$
K_t
a/w

C-5 *Charts of theoretical stress-concentration*

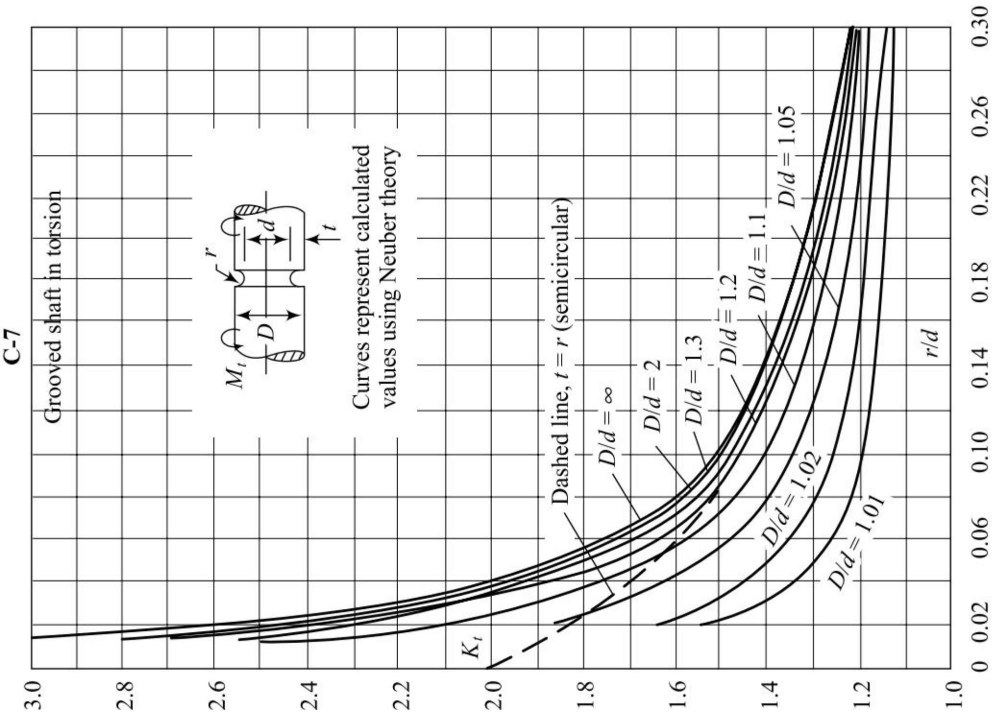


Bar in tension or simple compression with a transverse hole, $\sigma_0 = F/A$, where $A = (w - d)\,t$, and where t is the thickness.

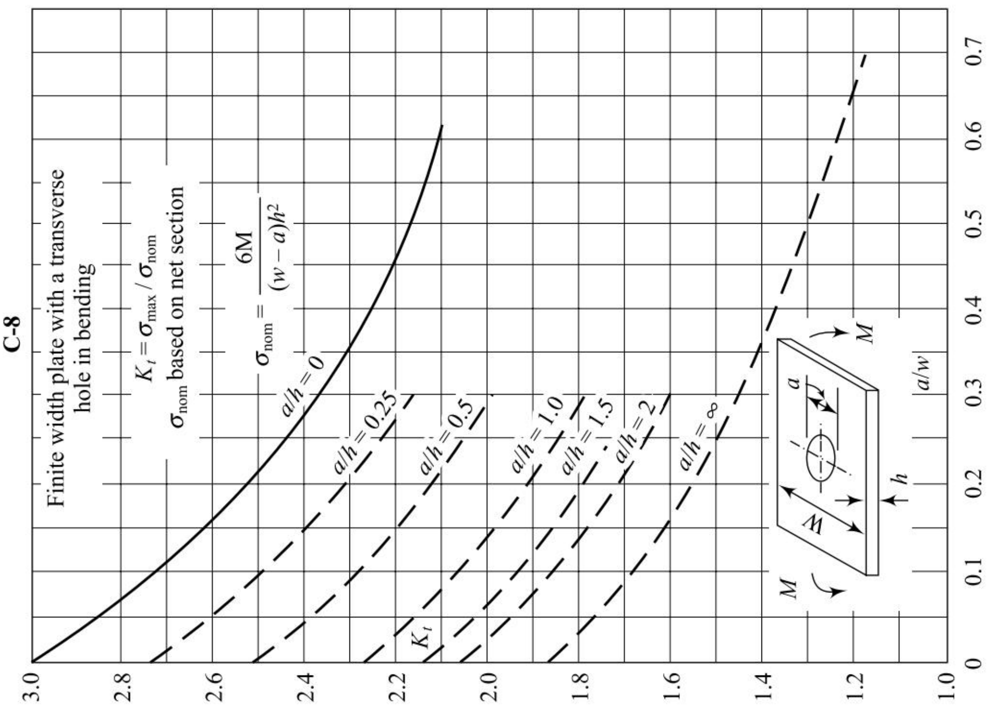
C-6



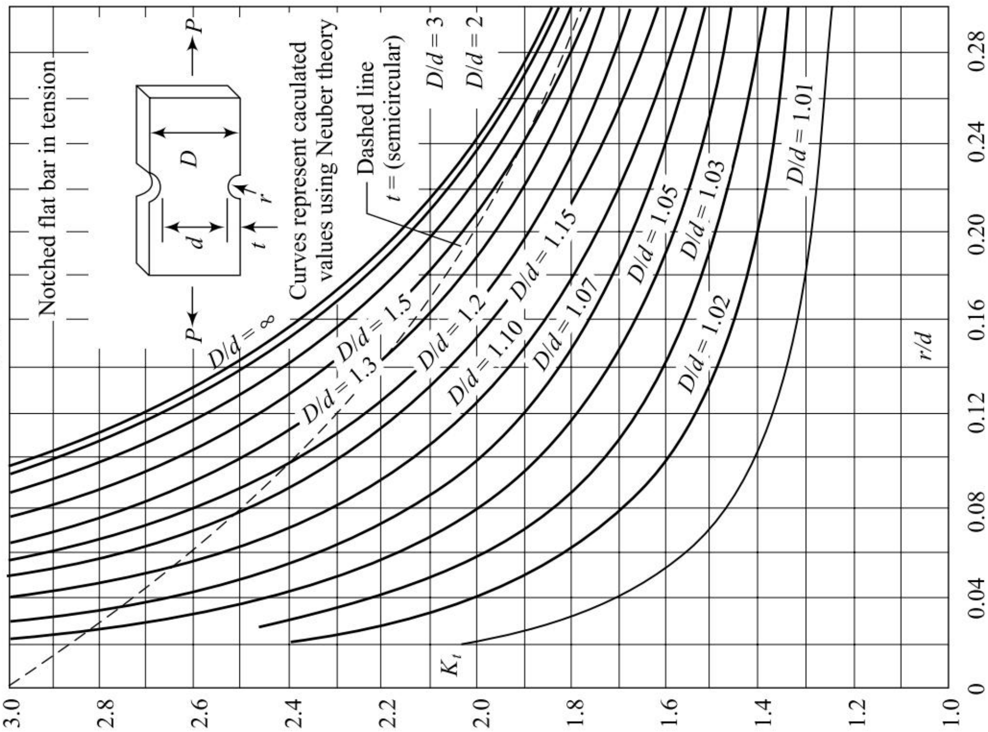
C-7


Grooved shaft in torsion
M_t
D
d
r
t
Curves represent calculated values using Neuber theory
Dashed line, $t = r$ (semicircular)
$D/d = \infty$
$D/d = 2$
$D/d = 1.3$
$D/d = 1.2$
$D/d = 1.1$
$D/d = 1.05$
$D/d = 1.02$
$D/d = 1.01$
K_t
r/d
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0
0.02
0.06
0.10
0.14
0.18
0.22
0.26
0.30

C-8

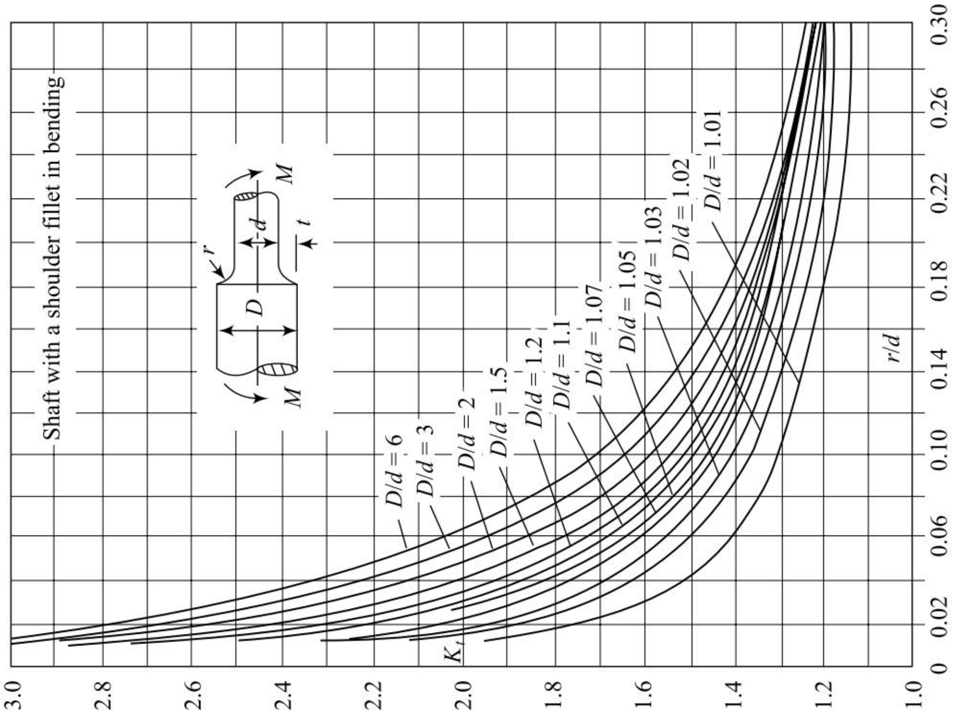

Finite width plate with a transverse hole in bending
$K_t = \sigma_{\max} / \sigma_{\text{nom}}$
σ_{nom} based on net section
$\sigma_{\text{nom}} = \dfrac{6M}{(w-a)h^2}$
$a/h = 0$
$a/h = 0.25$
$a/h = 0.5$
$a/h = 1.0$
$a/h = 1.5$
$a/h = 2$
$a/h = \infty$
M
M
a
w
h
K_t
a/w
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

C-10



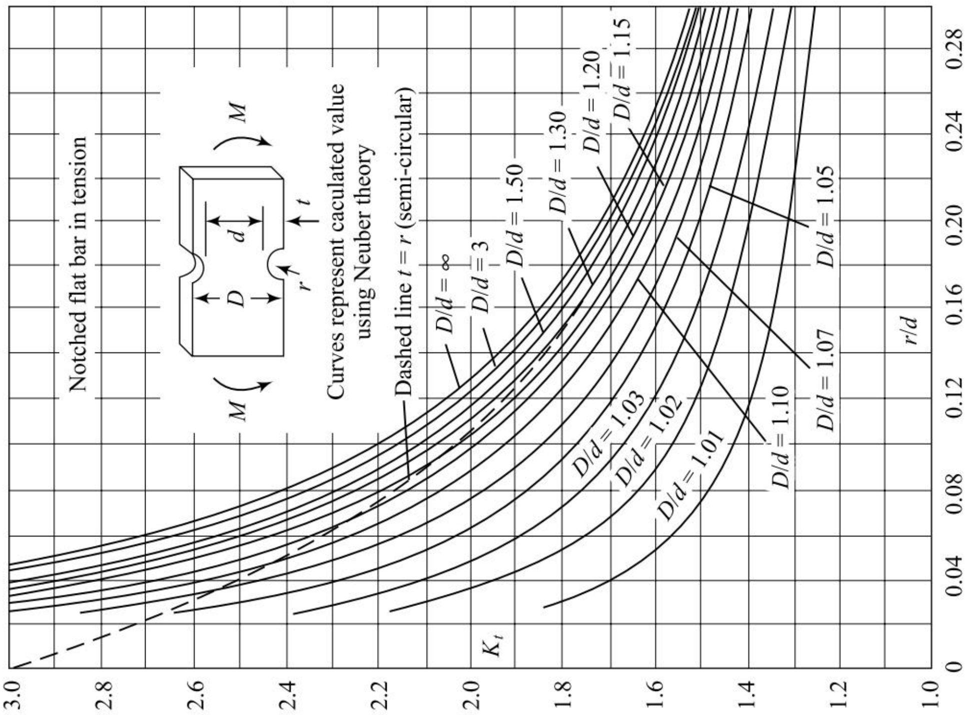
Notched flat bar in tension
P
D
d
t
r
Curves represent caculated values using Neuber theory
Dashed line t = (semicircular)
D/d = ∞
D/d = 3
D/d = 2
D/d = 1.5
D/d = 1.3
D/d = 1.2
D/d = 1.15
D/d = 1.10
D/d = 1.07
D/d = 1.05
D/d = 1.03
D/d = 1.02
D/d = 1.01
K_t
r/d
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0
0.04
0.08
0.12
0.16
0.20
0.24
0.28

C-9

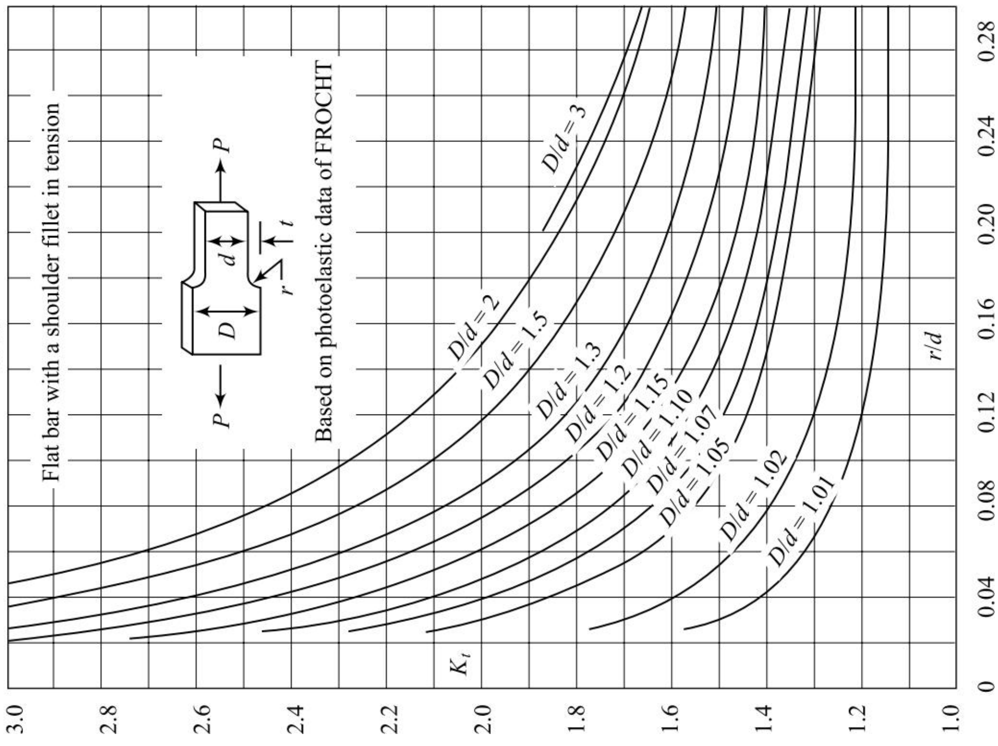


Shaft with a shoulder fillet in bending
D
d
r
t
M
D/d = 6
D/d = 3
D/d = 2
D/d = 1.5
D/d = 1.2
D/d = 1.1
D/d = 1.07
D/d = 1.05
D/d = 1.03
D/d = 1.02
D/d = 1.01
K_t
r/d
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0
0.02
0.06
0.10
0.14
0.18
0.22
0.26
0.30

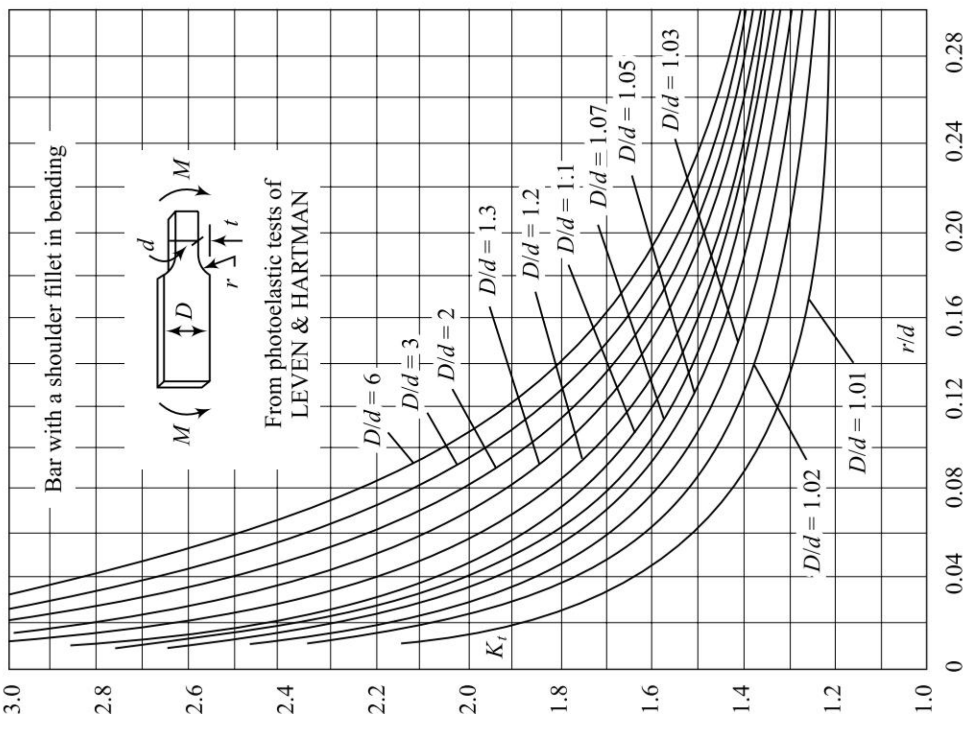
C-12


Notched flat bar in tension
Curves represent caculated value using Neuber theory
Dashed line t = r (semi-circular)
M
D
d
r
t
D/d = ∞
D/d = 3
D/d = 1.50
D/d = 1.30
D/d = 1.20
D/d = 1.15
D/d = 1.10
D/d = 1.07
D/d = 1.05
D/d = 1.03
D/d = 1.02
D/d = 1.01
K_t
r/d
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0
0.04
0.08
0.12
0.16
0.20
0.24
0.28

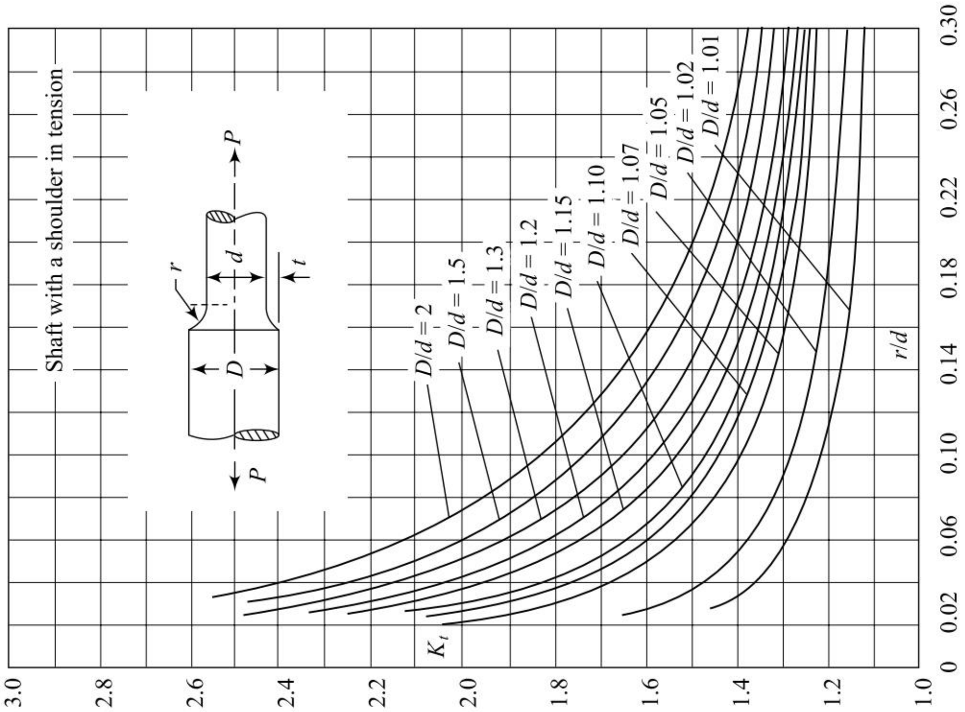
C-11


Flat bar with a shoulder fillet in tension
Based on photoelastic data of FROCHT
P
D
d
r
t
D/d = 3
D/d = 2
D/d = 1.5
D/d = 1.3
D/d = 1.2
D/d = 1.15
D/d = 1.10
D/d = 1.07
D/d = 1.05
D/d = 1.02
D/d = 1.01
K_t
r/d
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0
0.04
0.08
0.12
0.16
0.20
0.24
0.28

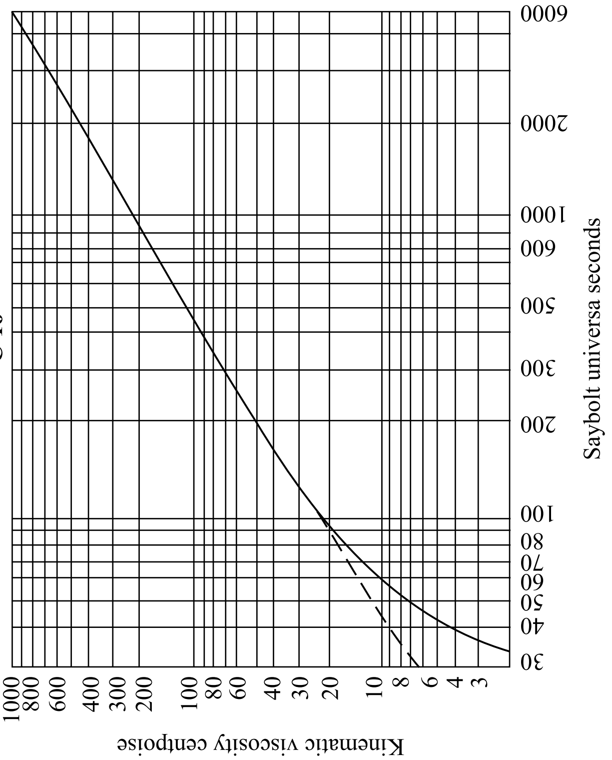
C-13


Bar with a shoulder fillet in bending
From photoelastic tests of
LEVEN & HARTMAN
M
D
d
r
t
D/d = 6
D/d = 3
D/d = 2
D/d = 1.3
D/d = 1.2
D/d = 1.1
D/d = 1.07
D/d = 1.05
D/d = 1.03
D/d = 1.02
D/d = 1.01
K_t
r/d
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0
0.04
0.08
0.12
0.16
0.20
0.24
0.28

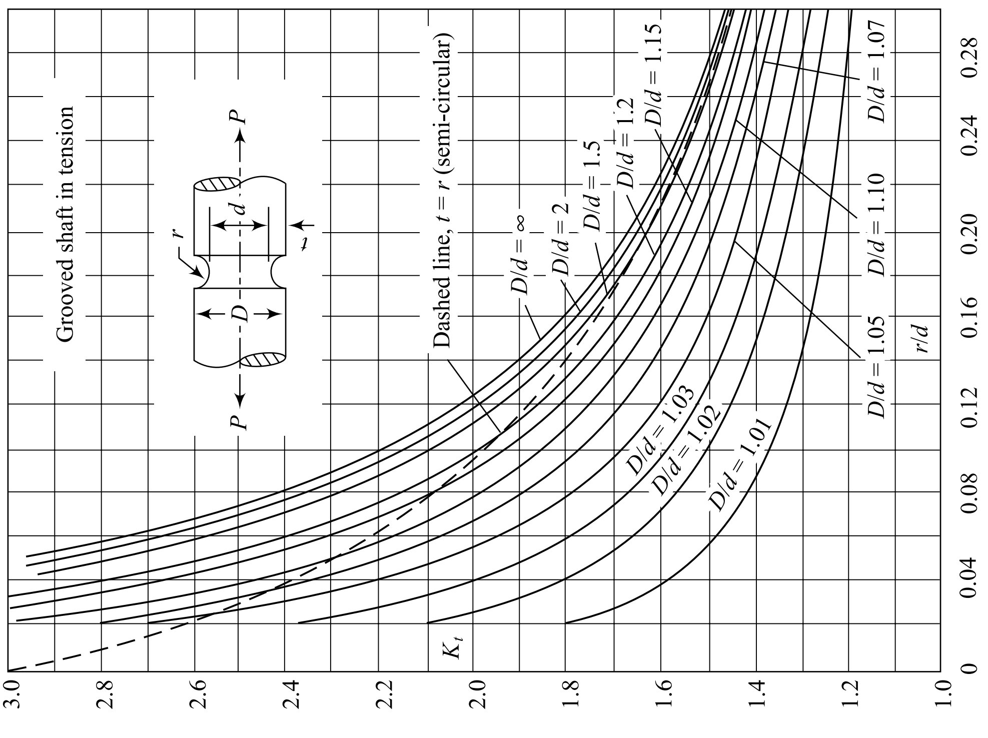
C-14


Shaft with a shoulder in tension
P
D
d
r
t
D/d = 2
D/d = 1.5
D/d = 1.3
D/d = 1.2
D/d = 1.15
D/d = 1.10
D/d = 1.07
D/d = 1.05
D/d = 1.02
D/d = 1.01
K_t
r/d
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0
0.02
0.06
0.10
0.14
0.18
0.22
0.26
0.30

C-16

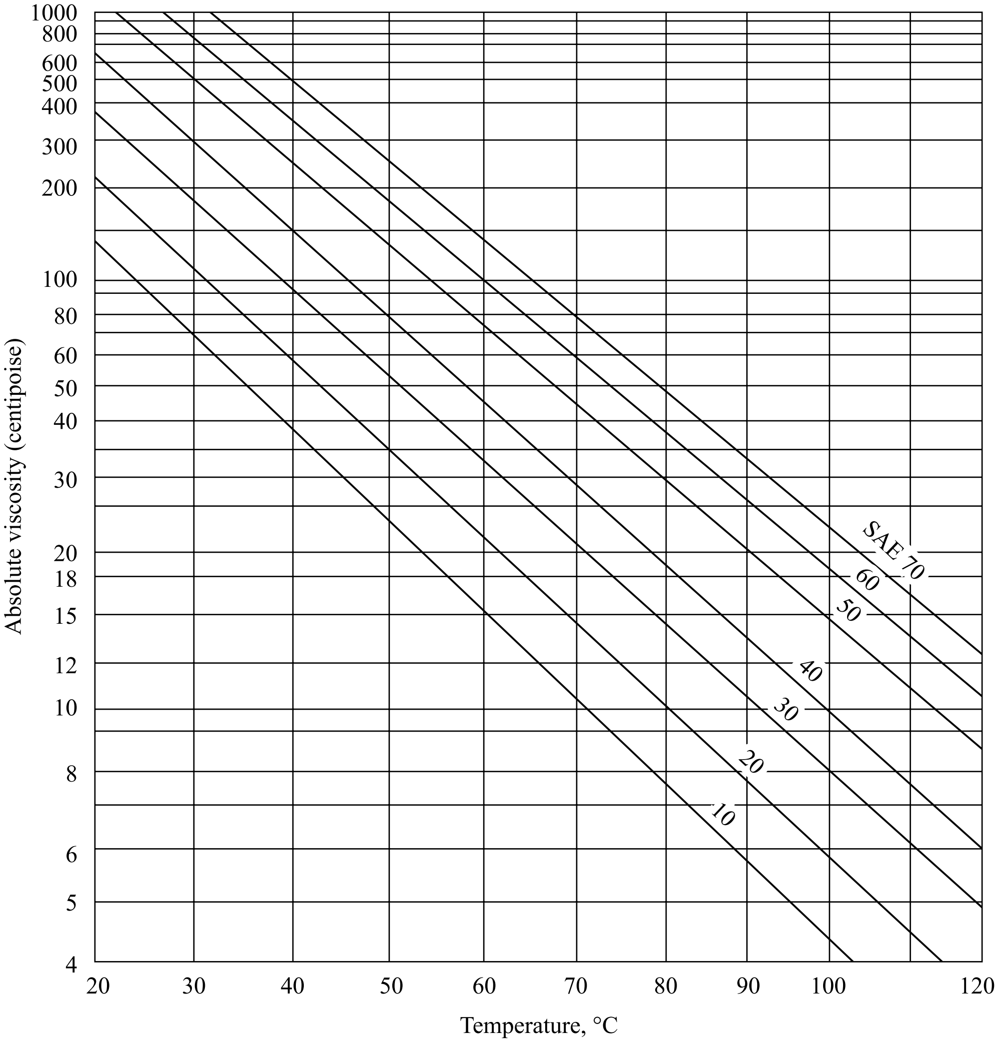

Saybolt universa seconds
30
40
50
60
70
80
100
200
300
500
600
1000
2000
6000
Kinematic viscosity centpoise
1000
800
600
400
300
200
100
80
60
40
30
20
10
8
6
4
3

C-15

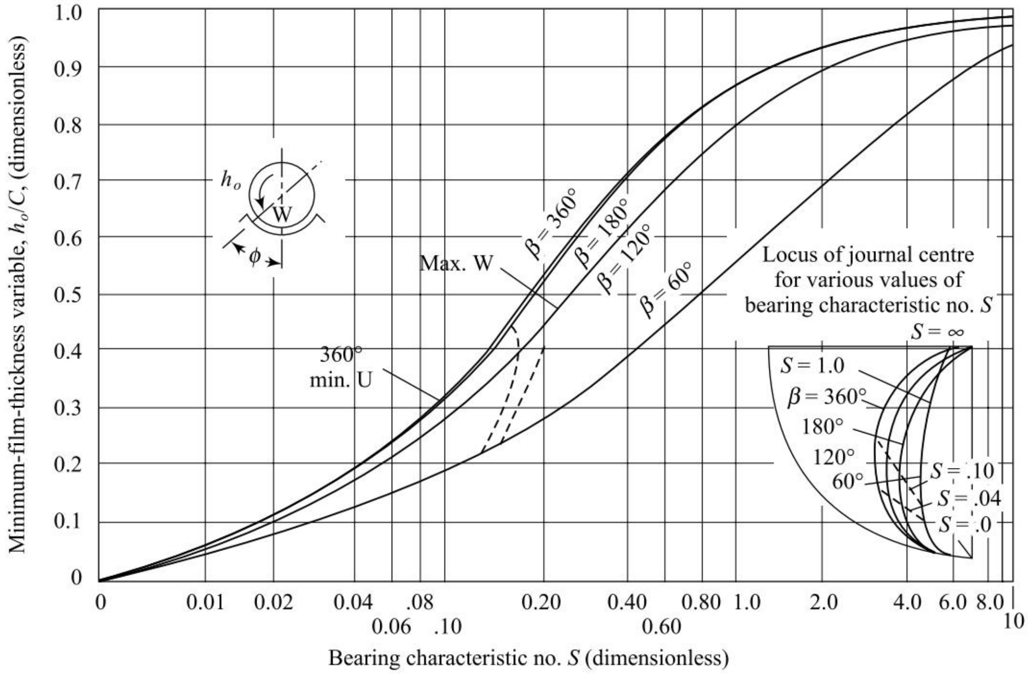

Grooved shaft in tension
P
D
d
r
t
Dashed line, $t = r$ (semi-circular)
$D/d = \infty$
$D/d = 2$
$D/d = 1.5$
$D/d = 1.2$
$D/d = 1.15$
$D/d = 1.07$
$D/d = 1.10$
$D/d = 1.05$
$D/d = 1.03$
$D/d = 1.02$
$D/d = 1.01$
K_t
r/d
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0
0.04
0.08
0.12
0.16
0.20
0.24
0.28

C-17

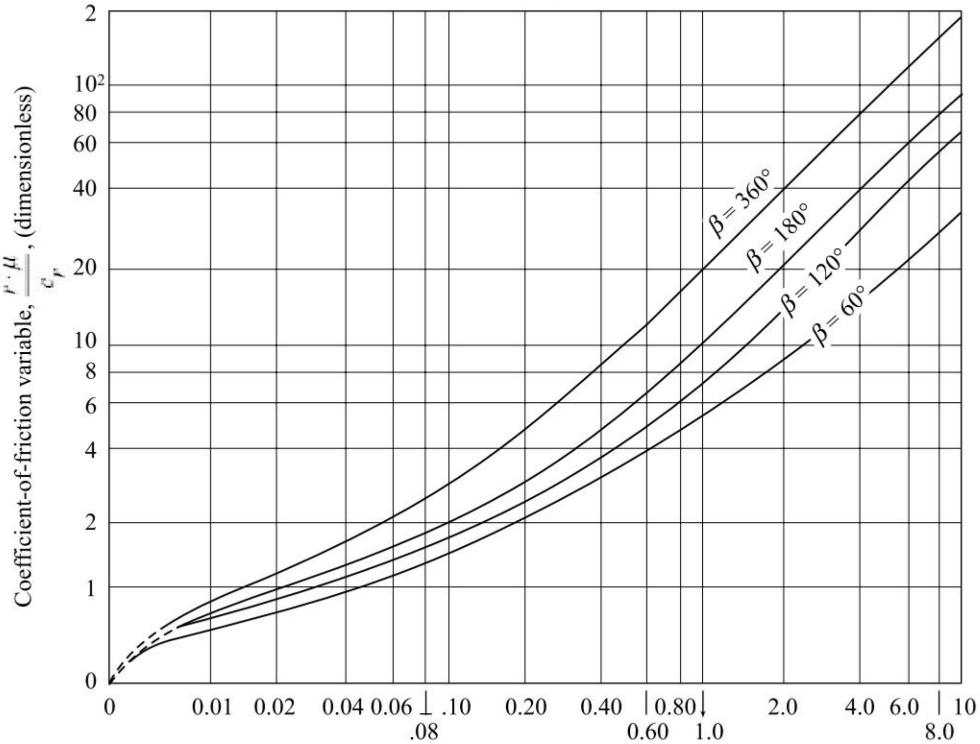
Average absolute viscosities vs temperature



C-18



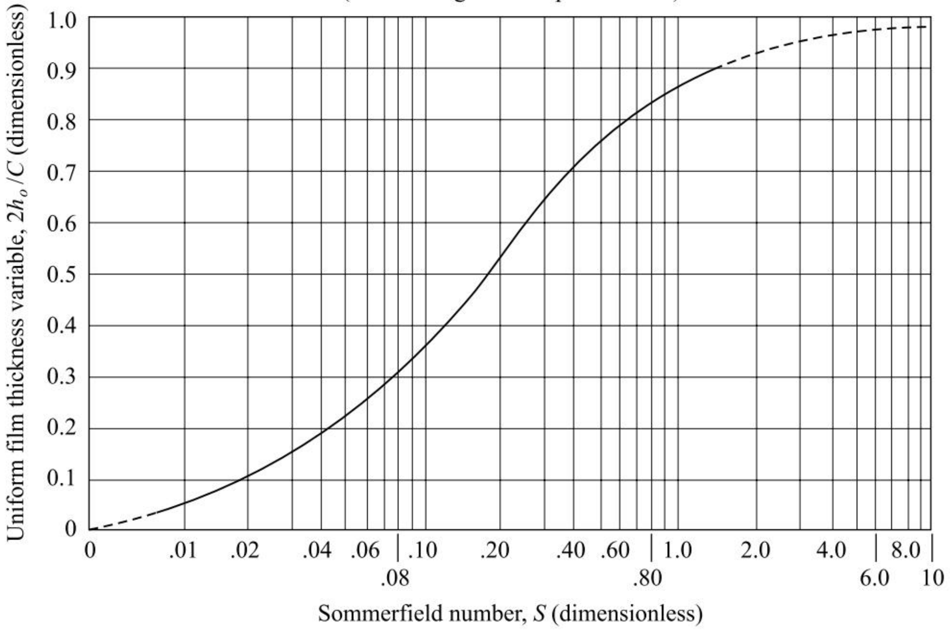
C-19



Coefficient-of-friction variable vs. Sommerfield S (with side flow)

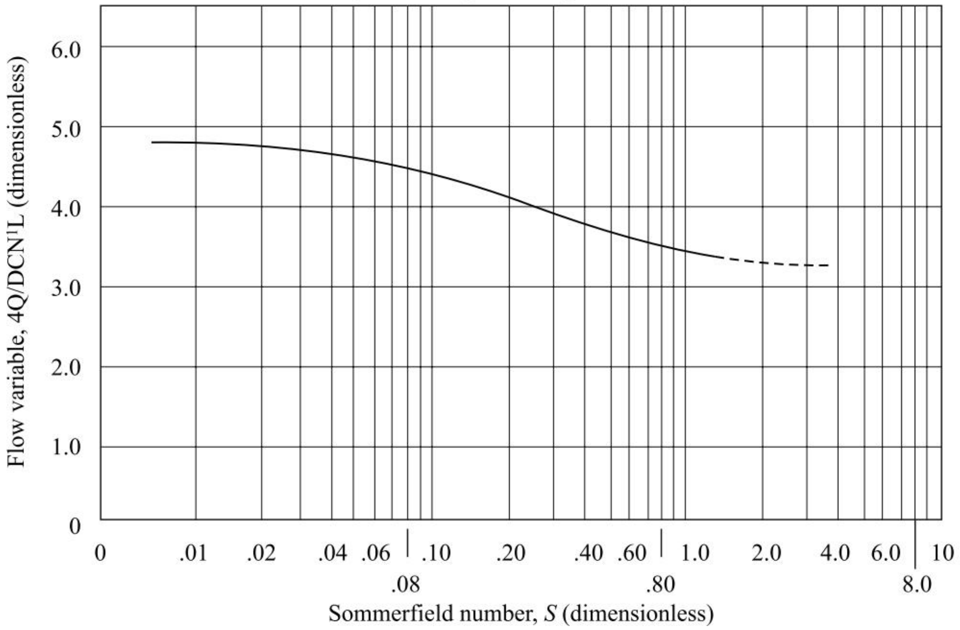
C-20

Based on Raimondi and Boyd data for $L/D = 1$
(Full bearing ambient pressure = 0)



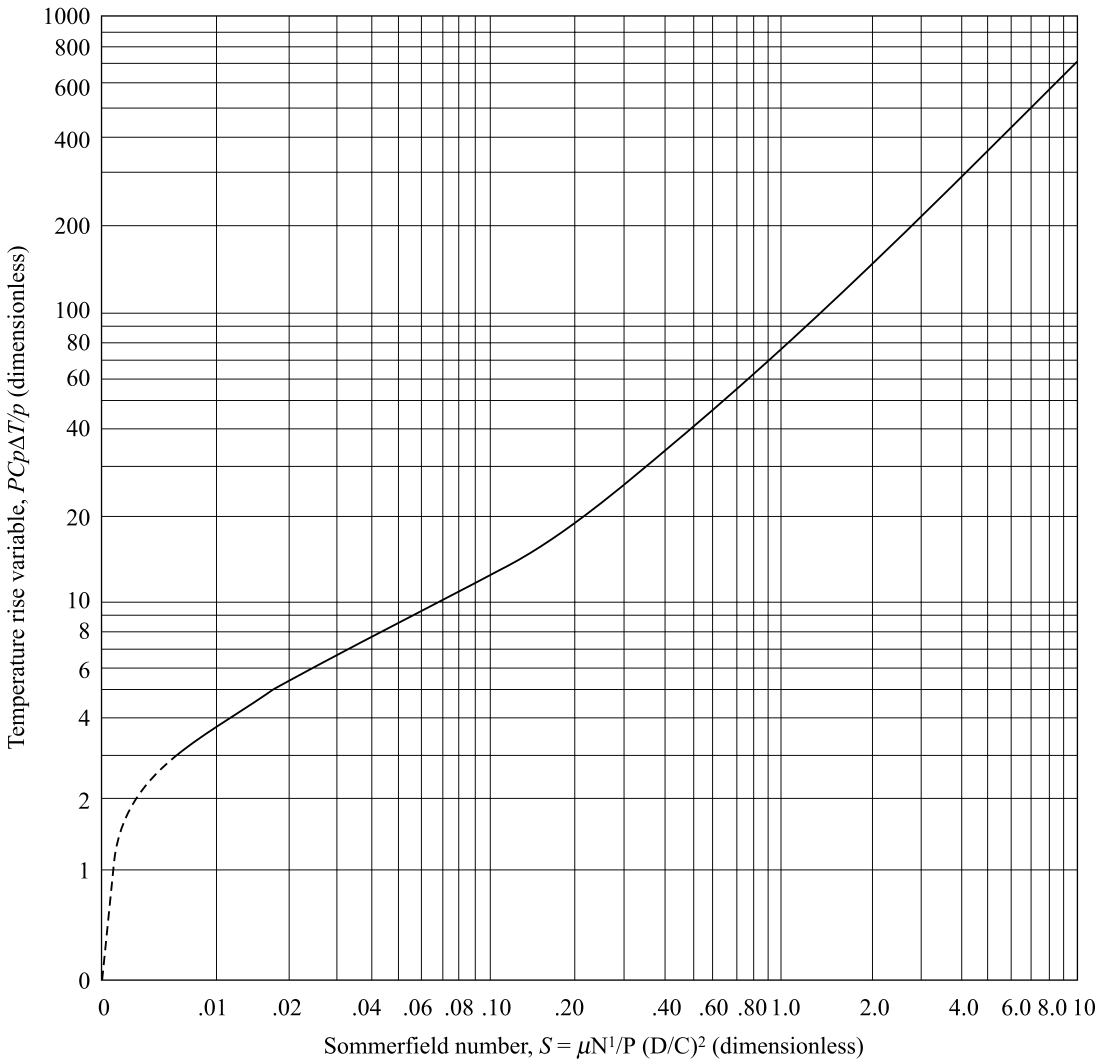
C-21

Based on Raimondi and Boyd data for $L/D = 1$
(Full bearing ambient pressure = 0)



C-22

Based on Raimondi and Boyd data for $L/D = 1$
(Full bearing ambient pressure = 0)



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