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And Other Mathematically Bent Stories

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Preface

This book is a compendium of stories from “Mathematically Bent", a humorous math column I write for the expository math magazine The Mathematical Intelligencer. The publisher, with the complicity of the editors, have been allowing my column to appear there for over 20 years, and I am very grateful to them for doing so.

No particular mathematical background is necessary to read these stories. I have written notes at the end of the book that explain the references that may not be familiar to all and that give a little more backstory, but you can certainly skip the notes if you want to.

A few of these pieces are in the form of scripts. I have rewritten many others as scripts, and with the help of many mathematician/actors, I have put them on for Family Days at Williams College, at a variety of conferences, and at the Joint Mathematics Meetings in January each year. I am deeply indebted to all the participants and to the organizations who have let me do so. It has been immense fun! If you would like scripts to put on some math theater or links to some of the recorded performances, let me know.

I owe a special thanks to Ina Mette, editor for the American Mathematics Society, who first suggested I write the column. She has been a supporter of my work for many years, and I appreciate that support immensely. I am also grateful to Chandler Davis and Marjorie Senechal, who, as editors for the *Mathematical Intelligencer*, agreed to go along with Ina’s suggestion. I am further grateful to Karen Parshall and Sergei Tabachnikov, who, when they took over as editors for the *Intelligencer*, agreed to continue the column and who have also been immensely supportive. It was their suggestion that led to this book.

I appreciate the long-term support of Marc Strauss, who oversees the *Mathematical Intelligencer* at Springer. Thanks also to David Kramer, who has copyedited quite a few of these stories and always has great suggestions for improvements. And thanks to Christopher Tominich, the editor at Springer/Birkhäuser who was enthusiastic about the idea for this book and who shepherded it to publication.

Finally, I want to thank the members of the Mathematics and Statistics Department at Williams College, past and present. I have been very lucky to spend my career with such a fun, enthusiastic, and motivated group of individuals.

Williamstown, MA, USA Colin Adams

Introduction

When my first collection of humorous math fiction appeared, it was my hope that that slim volume would help to establish of new genre of literature called “humorous mathematical fiction". As I’m sure, you know, by now, that hope has blossomed. Go in any bookstore and you’ll see the humorous math section crowding out the math, humor, and pop-up coffee table book sections.

Just type in “humor", “math", and “books" on Amazon (go ahead, I’ll wait), and you’ll get an abundance of hits. Right at the top, you’ll find Ludwig Wittgenstein’s deliriously funny “Remarks on the Foundations of Mathematics". Just one sample from it I can’t resist sharing (p. 38):

> If a ruler expanded to an extraordinary extent when slightly heated, we should say – in normal circumstances – that that made it unusable. But we could think of a situation in which this was just what we wanted.

I mean, this is great stuff. Bring down the house stuff. What situation? We want to fool people with our giant hot ruler? We want to trick our children into thinking they are shorter than they are? We want to brand our cattle with enlarged measuring units?

And Ludwig just supplies the set-up. He lets us supply the jokes. He is a comic genius. As my mother often said to me, “Once again, you’ve proved you’re no Ludwig Wittgenstein”. She was right. I can’t aspire to that level of genius.

But Ludwig’s book is not the only option. We can pick from amongst the many other math humor books on Amazon, including *Student Solutions Manual for the Mathematics for Economics* or *Introduction to Statistical Decision Theory*.

You might even find yourself asking, “If the math humor literature has grown into the behemoth that it has, why did you write this book? Do we really need another humorous math book?”

But in fact, the humor in many of these other books emanates from what isn’t there rather than what is. You read a sentence that initially does not strike you as particularly funny. But you ponder on it for a while, wondering what it means. And then it hits you. It’s a joke. It can’t be serious.

The stories in this book aren't like that. In these pieces, each joke is telegraphed before it arrives, then explicitly stated when it occurs and then gloated over afterward. Don't worry, you'll know it's intended as a joke. Much as I enjoy Ludwig's funny bone, I think there is also room for books aimed at readers who prefer their humor explicit.

So I hope you enjoy this book. I hope you enjoy it so much that you give it away liberally to your friends say, for instance, as a gift when invited to dinner, or to reward a child for good behavior. Or perhaps you need to ingratiate yourself with a police officer poised to give you a ticket. What a pleasant surprise for them when, instead of your registration, you hand them a copy of this book. I could go on and on, but will leave with saying it probably makes sense to have several boxes full of copies lying around the house for when the need arises.

I had a great time writing the stories in this book. I hope you enjoy reading them!

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Chapter 1
Immortality

At the end of a dimly lit dead-end street in New Orleans, I found myself facing a massive oak door embedded in a dark gothic stone edifice. In its day, the house must have been a glorious sight, but the years had worn it down. Age and neglect had allowed soot to settle in every crack and crevice, of which there were now many, giving the house a weathered and ominous look. A weak moon illuminated the front door enough for me to find the iron knocker, which I proceeded to thump on the door.

I waited a good minute before hearing a shuffling step approach from inside. The door unlocked and swung open slowly, revealing a slight elderly man, dressed in a style at least five decades out of date. He looked me over deliberately and then beckoned me inside. I entered a marble tiled rotunda smelling of mold and decay.

He swung the door closed behind me, locking it with a large key that he pocketed. Then he walked me across the rotunda and ushered me into what appeared to be the library. He left me there, sliding the double doors closed behind him.

As my eyes adjusted to the low light, I realized that there was someone in a chair, facing away from me, seated at a desk across the room. He appeared to be writing. I stood waiting patiently until he stopped and swiveled his chair around to look at me.

"Ah, Mr. Kindler," he said.

He stood but made no move to come shake my hand. He was tall and gaunt with a receding hairline. Although at least sixty, he appeared to be in excellent shape, and graceful in his movements. He also was dressed of an earlier time, but on him it somehow seemed an appropriate fashion statement.

"It is a pleasure to make you acquaintance, Mr. DeMoligne," I said. "I have looked forward to meeting you for some time."

"And I you," replied DeMoligne. "Please, take a seat." He motioned to a wooden chair near where I stood. I sat down, but he continued to stand.

"I believe that the first time I became aware of your work was your paper with Phelps on direct products of semilocal rings," he said. "I was particularly interested

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in your method of proof of the main result. Collapsing out the entire collection of associated prime ideals was innovative, something I had not seen before. I mean that as a very high compliment."

"Thank you," I replied. "That technique has proved useful in a variety of other contexts since then."

"Yes, I know. I have read all your papers."

"Really? Then besides me, you are probably the only one who has," I said with an attempted self-deprecatory smile. His expression did not change.

DeMoligne rang a bell on his desk, and his servant re-entered.

"Bring us beverages", he said. Once the servant had left, he turned to me again. "You asked to meet with me. Why?"

"Well," I said, trying to think how best to word my response. I knew DeMoligne was a recluse of the first order, and I was frankly surprised that he had agreed to the meeting at all. "I read some of your papers, and I became intrigued. First of all, the mathematics is excellent. You have such an ability to tie together disparate fields. You seem to know such a broad spectrum of mathematics."

He nodded acceptance of this compliment.

"Can I ask where you received your degrees?" I asked. "No one I have spoken to seems to know."

"I have no degrees," he replied. "I am self-taught."

"All the more remarkable then," I said.

"But you did not come here simply to compliment me."

"No, I did not. It is really curiosity that brought me here. First of all, how to put this? You seem to be working in such overlooked areas. You choose research subjects that are either out-of-date, or obscure backwaters that no one else finds interesting. The work you do in these areas is truly remarkable, but you don't receive the attention you deserve because so few researchers are interested in these subjects."

DeMoligne laughed and waved a dismissive hand.

"Why should I care if others find this work interesting? It is not contemporary mathematicians who determine what is important and what is not. That is determined by the ages. Perhaps one hundred years hence, these theorems that I am proving will be critically important."

"Perhaps," I acceded.

"When mathematicians were proving theorems in number theory in the 1930s, how could they possibly have foreseen their use in computer cryptography sixty years later? So it has always been with mathematics, and so it will continue to be."

The servant slid open the library door and set one glass on the table next to me and one on the desk next to DeMoligne. Then he again retired, closing the door behind him. The glass contained a brown liquid that I could not identify. I held it up and took a whiff.

"Try it," said DeMoligne. "You will like it." He lifted his glass and took a swallow.

I took a small sip. It had a strange taste with which I was not familiar, with hints of vanilla and some root extract. But it was definitely interesting, and it warmed my throat as it went down.

"It's good," I said.

"Yes, it is," said DeMoligne, "Now please continue."

"What is so interesting about your work," I said, "is that it reminds me of someone else's."

"Yes?" he said. "Who is that?"

"Actually, I know this sounds funny, but it reminds me of Galois."

DeMoligne smiled. "Évariste Galois? But I am not working in algebra. I am working in geometry. And Galois died almost two centuries ago. His work is completely unrelated to my own."

"I know," I said. "But somehow the methods used are still reminiscent of his. It is the style of proof much more than the substance itself."

He regarded me without speaking.

"Also, sometimes, your work reminds me of Riemann," I continued.

I took another sip of the drink. The alcohol, assuming that's what it was, was helping me to relax.

"Bernhard Riemann. He worked in completely different areas, analysis, differential geometry. His work couldn't be more dissimilar from that of Galois."

"Yes, but again, there is something about his audacity, his ability to see connections that reminds me of your work. Did you study their work? Am I right?"

"You are a perceptive fellow", said DeMoligne. "Extraordinarily perceptive. Perhaps dangerously so."

I smiled. "No one has ever considered my perceptive abilities dangerous before. More likely my lack of perception, if you were to ask my wife."

DeMoligne was staring at me carefully, sizing me up, as if considering options. There was a pause, and then he seemed to make up his mind.

"Why do you do mathematics?" he asked.

"Me? I fell in love with math when I was nineteen. The professor in a college course mentioned a minor open problem in graph theory. That night, I solved it. And the feeling of satisfaction was incredible. I had solved a problem no one in the history of humankind had ever solved before. It made me feel special. It seems a little silly now. It really was a minor result. But it made me feel as if I had carved for myself a little piece of immortality. My name would be attached to that result from then on, forever."

"Immortality," he repeated, grimacing. "It is overrated."

I laughed.

"Does my mathematics remind you of anyone else?" he continued.

"Um, no, I don't think so."

"How about Maclaurin, 1698 to 1746? Or perhaps Pascal, 1623 to 1662?"

"Well, of course, they were exceptional mathematicians. Everyone knows of their work. But I haven't read it in the original."

"You should. I am certain it would remind you of my own."

"Are these mathematicians that you studied?" I asked. "Mathematicians whose styles and approaches you have incorporated into your own?"

"No," he replied as he suddenly set his drink on the desk and strode swiftly across the room, settling into a chair only inches from my own. The movement was so quick and agile, it reminded me of a large cat. I involuntarily jerked back in my chair.

"I am those mathematicians," he said, staring at me intently.

"What?" I replied, completely confused.

"I am those mathematicians. I published their papers under their names. I am Maclaurin. I am Pascal. I am Galois. I am Riemann. Or at least I was. And now I am DeMoligne."

"I'm, ah, I'm. . .," I stuttered.

"I am them and they were me," he repeated. "And there were others, as well."

I looked to the door, considering how I might escape from this person whom I had initially assumed to be eccentric but now appeared to be a madman.

"I am extremely impressed that you noticed the similarities between my work and that of Galois and Riemann. No one else over the last four centuries has made the connection."

"Um, I am still a bit lost."

DeMoligne leaned forward close enough for me to feel his breath on my face. It felt cold.

"It's very simple, Mr. Kindler. I am not mortal. You spoke of immortality. Well, I am that. I am immortal. I once was mortal, but all that changed four centuries ago."

"Really? Sounds fascinating, but you know, I should probably be going." I started to stand, but DeMoligne grasped my arm and pulled me back down into my chair. He kept his hand tightly gripped around my forearm.

"You see, my friend, I had an unfortunate encounter those many years ago, an encounter that changed my life dramatically. An encounter with a vampire. And subsequent to that encounter, I myself became a vampire, one of the so-called undead."

Although this convinced me all the more he was insane, it also made me worry of what he was capable.

"Oh," I said hoping to mollify him. "Of course. . .."

"And once a vampire, you become immortal. Barring a few unfortunate circumstances, your life will unravel before you forevermore."

"Okay, well, isn't that great."

"No, it is not great. In fact it is exactly the opposite of great."

"What do you mean? Doesn't everyone want to be immortal?"

"Only if they haven't tried it," he said, a look of sadness crossing his face. "Try to imagine the tedium of it, as your life rolls by decade after decade. You see the same human stupidities repeated time and time again. You cannot form relationships, fall in love, because inevitably the one you love dies. And then what? Do it over again?

"And as the decades tick by, each seems shorter than the last, since as a proportion of the life you have so far lived, they are a smaller and smaller fraction. Mortals come and go in what seems the blink of an eye.

"Life becomes a pure monotonous agony. Something to dread, a long, never ending, never changing ordeal." Then he stopped and looked me straight in the eye. "At least it was until I discovered mathematics."

"What do you mean?"

"I spent the first one hundred years of my new existence suffering the intense boredom that is the life of a vampire. Nothing interested me anymore. I would have welcomed death heartily at that point.

"But then I stumbled upon mathematics. At a monastery where I dined, I happened to notice a version of Pacioli's text *Summa de arithmetica, geometria, proportioni et proportionalita*". I took it home with me. Over the next few months, as I read through it, I realized I could perhaps push much of the mathematics further. And I did. In those early days, I published under my own name. I remember well that immense satisfaction you mentioned in knowing I had discovered something no other creature had ever discovered before me, mortal or immortal. It was the first time in a long time that I looked forward to waking each evening.

"And so began my pursuit of mathematical knowledge. The pure essence of knowledge. The elixir that I still crave.

"At that time, I was a full member of the mathematical community. I knew Gerolamo Cardan and Gerardus Mercator. I was recognized as one of the great minds of that era. I experienced your kind of immortality. The immortality that comes from having your name attached to a result that will go down in the annals of mathematics.

"But then my contemporaries began to grow old and die, while I was still young and vibrant. What a huge loss that was. Every time a mathematician dies, that storehouse of knowledge they have collected over a lifetime dies with them."

He paused, looking down for a moment, before again looking intently into my eyes.

"I realized I had to hide the truth about myself. And my mathematics was doing just the opposite. It was bringing me attention and fame. And people were noticing that I seemed not to get any older as time passed. The life of a vampire is not so easy. There are those who seek to discover us and destroy us. Very few of us have lived anywhere near our full potential.

"Before discovering mathematics, I would not have cared if someone destroyed me. My life was that miserable. But all that changed with mathematics.

"So I had to kill off my persona. Fake my own death and move to a new place to create a new identity. It is easier to do than you might expect. Create a back story, fake some documents, and there you are. But more often than not, I had to change fields as well. Move from algebra to analysis, so as to leave as little connection as possible between the subsequent identities."

"And you are saying you were Riemann, Galois, Pascal, Maclaurin?" I asked, incredulous.

"And many more." He smiled, swelling a bit with pride. "And in all this time, I have had to hide it. You are the first to know the truth."

"Why tell me?"

"You discovered it on your own. You just didn't realize the magnitude of the connection between myself and these mathematicians."

As I listened open-mouthed to this incredible tale, I suddenly realized that a strange sensation of pins and needles was crawling slowly up my legs. DeMoligne released my arm, and I found I could not sit up.

"I don't feel right," I said. "I need to leave."

"No, you do not need to leave," he replied. "You need to stay. That drink has a substance in it that paralyzes the drinker. You see, although I am a vampire, I am a civilized vampire. I do not believe in violent displays. This way, it is much easier for both of us."

"What do you mean. . .," I uttered with difficulty, as I attempted to lift my arm, to no avail.

"I would like to have someone to talk mathematics with. It is not so gratifying working alone. Having no one with whom to share the beauty of the results. I am in search of a collaborator. A long-term collaborator, worthy of our collaboration."

"But I, I. . .," was all I could get out.

"I have chosen you. It is a great honor. To be chosen to be the collaborator of Pascal, of Galois, of Riemann. I am looking forward to a long and fruitful association, a very long and fruitful association."

This is the last thing I remember him saying before he leaned forward one last time, his lips parting to reveal two long and extremely sharp teeth.

Chapter 2
Gold Rush

I was nursing a whiskey in the Last Chance Saloon, the final whiskey that I planned to drink before I permanently departed the dusty dried-up town of Deadfield, when Broken Nose Pete come running in.

"Gold!" he hollered. "They found gold in them thar hills!" He pointed back up at Boney Ridge.

Dirty Mike, who was swabbing the bar, looked up with his one good eye. "What chew talkin' about?" he bellowed.

"Ahm talking about gold! Right up in the fields on top of the ridge."

"Which fields?"

"The finite fields."

Dirty Mike guffawed and went back to swabbing the bar. "Who ever heard of gold in finite fields? Tain't never gonna happen."

"Ahm telling ya, it's gold!" shouted Pete. "It were some grad student. Just wandering around up there, not knowing enough to know tain't no gold in finite fields. And she spotted somepin' shinylike. Leaned over and what do ye know? She pulled up a big ol' theorem, size of yer thigh. Ahm tellin you it's gold! Sittin' up there for the takin'."

Everbody in the bar was talkin' at once. I stood up real slow and walked over to Pete. All the eyes in the bar, an odd number by the way, followed me across the room.

"Now, Pete," I says, "ya been known to tell some tall stories over the years, hopin' someone would spot ya a drink as a result."

Pete squinted at me over his crooked nose.

"Well, thar's some truth in that, Pecos, but that ain't whut's happenin' today."

Big Jane Bushke stepped forward and grabbed Pete from the back of his collar, lifting him part ways off the floor.

"So then tell me, Pete," she says, "what theorem did she find?" The entire room went dead silent, waitin' for his reply.

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"She showed that for any $n \geq 2$ and any prime q, thar's a completely normal primative basis of F_{q^n} over F_q. And that's jest for starters."

"For starters?" repeated Big Jane.

"She found a bunch of other ones, too. Big ones. Why, thar's enough theorems up there for the lot of us. We're all gonna get jobs at top notch reesearch uneeversities. With big ol' offices and bucketloads of TeeAee's to do our teachin' for us. This is the real deal!"

And that's all it took. There was a mad dash to the exit. When the saloon doors stopped swingin', all that were left were a couple of confused undergrads walking around looking for solutions manuals, and Black Jack, the gambler from Berkeley, who was in town to give a probability seminar. He lifted a deck of cards from the table in front of him and shuffled it.

"This should make things interestin'," he said real quiet like.

And that was the beginning of the gold rush. Within a month the town of Deadfield had quintupled in size. We had matheematicians showing up from all over the world. From France, and Chiner, and even a place called Lithuaania. And all types. Some were hard as nails. They'd scrabbled all their lives for the academic crumbs. Teaching three hunerd students a pop while trying to kick start their research program. And some from the other end of the spectrum. Didn't know the first thing about prospectin'. They'd lived their whole lives in safe academic institutions where their advisor handed 'em a problem and told 'em how to solve it, too. These tenderfoots walked around asking for a cap of chino and tryin' to figure out where they could plug in somepin' called a laptop. Don't ask.

Me, I staked a claim up above the ridge, in an area on irreducible pentanomials over F_2. Figured, if there was theorems on the ridge, they musta washed down from up above. Get 'um from the source. Big Jane Bushke staked the neighboring claim. She was working tetranomials over F_3.

Over the next coupla months, you'd hear stories of someone strikin' a vein, and making it big time. But after workin' my claim for three months, I was havin' my doubts. So one day I walked over to Big Jane's claim. She was swingin' a pick axe, sweat soaking through her clothes.

"Big Jane," I says, "How's your claim coming?"

"Well," she says, leaning on the pickaxe handle. "Ah got me a couple a little nuggets. But ah won't be retiring to the Institute for Advanced Study in the near future."

"Me neither," I said. "Mebbe we're goin' about this all wrong."

"How you mean?" she asked.

"Well, what if we pooled our resources?" I suggested. "You're purty good with the detail work, and ahm good at spotting potential places to dig. Mebbe if we worked together, we could be a lot more successful?"

Big Jane agreed and we commenced our partnership from that day forward. Big Jane was a hard worker and not afraid of puttin' her back into it. Three days in, she was toiling away when she yelled out, "Pecos, get yer tail over here. I got somepin."

I dropped my pickaxe and walked over.

"See here," she said, "these lemmas? I think there a leading somewhere."

I peered into the pit, but they just looked like your run-of-the-mill lemmas to me. Then Big Jane swung her pick-axe down hard and there was a big clang as the tip broke off and went flying.

"Gol darn it," she swore. "That pickaxe cost me a purty penny." But then she leaned over to see what she had struck.

"Pecos," she said. "Ah hit somepin big."

I climbed down in the hole with her, and with our hands, we started clearin' the surrounding debris. Slowly, as we uncovered it, we began to realize just how big it was.

"Oh mah God," I said, as the contours became apparent. "Ah think it's the twin prime conjecture over F_2."

"You kiddin' me?"

"Nope. Ah don't believe ah am."

Big Jane whooped. "We gonna be famous," she bellowed. "We gonna have grants up the wazoo!!!" She grabbed onto me and we did a jig around the theorem.

"We plum struck it rich!" she hollered.

After we calmed down, we pulled that theorem out of the ground and with a lot of grunting, we managed to hoist it onto a donkey and tie a tarp over it. Then we headed down the trail to town, both of us grinnin' like brand new Ph.D.'s.

But as we came around a switchback, there ahead of us down the trail was a lone rider dressed entirely in black. As he approached, I recognized Black Jack.

"Not a word," I whispered to Big Jane.

"Good day to you," said Black Jack, doffing his Berkeley cap.

"I thought you left town a long time ago, Black Jack," I said. "Weren't you sposed to be the headliner at the Southeastern Tennessee ProbabilityFest?"

"I got Ben Affleck to fill in for me," he replied, smiling. "Boy, that donkey looks like he's working pretty hard. Whatcha got on there?"

One of his hands was resting near the gun hanging from his hip.

"Whut? That?" I said. "Why that's just a couple of little lemmas we dug up. Lucky if they earn us a coffee before seminar."

"Well, then, whyn't ya let me take a look. Mebbe I can apply them somehow. Wouldn't hurt you to have a name like mine attached to your work."

Big Jane stepped in front of his horse.

"Ah think we're fine not havin' your name affixed with our names."

"Yeah?" he said as he slowly pulled his six-gun out of his holster and pointed it right at Jane. "Well then, how about if only my name is attached to it?"

"Hold on there," I interjected. "We found this here theorem fair and square, after a lot a hard work. Meantime, you been sittin' on your keister in the saloon, drinking whiskey and fleecing tenderfoots with your excessively developed probability skills. You don't get this theorem. We earned it the hard way."

"And you're gonna lose it the hard way," said Black Jack. "Don't nobody know you found that theorem. When I turn up in town totin' it, people'll take my word that I found it. I certainly got a better track record than the likes of you. And you two are gonna disappear. In a week, nobody'll even remember who you was."

Big Jane's face turned chartruese, like a sunset. I only seen it that color once before and the last time there'd been two of us saw it change but only one of us walked away to tell the tale.

"Your face is a mighty fine color," said Black Jack with a sneer. Jane's face transformed to a bright purple now and then she stepped forward, hauled off and punched Black Jack's horse in the nose.

Now I want to be fair to her here. Big Jane's a animal rights activist from way back. Fact is, she used to run a school for teaching dogs that AB Calculus. But in that instant, her passions were sky high, so she punched that horse. She punched it hard. And as it reared back, Black Jack started firing off his gun in an attempt to kill us dead. But with the rearing horse, he couldn't get a bead on us. Big Jane grabbed my arm and just leaped off the side of the embankment, dragging me with her. We rolled and rolled, as Black Jack fired after us.

We slid to a stop down in the brush and scrambled behind some boulders.

"You okay, little feller?" asked Big Jane.

I checked myself up and down for holes and then nodded. "Sep we lost the theorem," I said.

"Don't be givin' up yet," she said, as she started scrambling down the hill. When we got to town, we saw a crowd spilling out of the assessor's office. We pushed our way inside just as the assessor was weighing the theorem that Black Jack had brought in.

"I ain't never seen such a big theorem," said the assessor, eyes wide. "It's a marvel to behold."

"Hold on there," I called out. "That there theorem don't belong to Black Jack. It belongs to me'n Big Jane. It was us what found it, and it was us what dug it out."

Black Jack flashed a wicked grin. "Come on now," he said. "I'm Black Jack from Berkeley, one of the luckiest men alive, as many of you who dun lost your money to me can attest. Who you think's likely to find a theorem this big? A pair of losers like them or a winner like me? Nobody's gonna believe your cockamamie story.'

"They might believe it when they see these lemmas," said Big Jane, pulling a coupla dirty lemmas out of her pants pocket. She held them up for all to see. I was as surprised to see them lemmas as everyone else.

"See that theorem there don't work less it's got these here lemmas for the proof. Without em, it's just a big ol hunk of debris. But ol' Black Jack, he don't know that. Cause he didn't dig that theorem outta the ground. We did. Same time as I dug these lemmas out to go with it."

"Why I'm gonna plug you," yelled Black Jack, waving his gun. But the crowd grabbed him and wrestled him to the floor.

The assessor turned to Jane and me. "It does appear this is your theorem. I'd say a theorem that big gives you a choice of tenured positions at Northwestern or Vanderbilt."

"Ah'll take Vanderbilt," I said. "Ah like that Nashville country music."

"And ah'll take Northwestern," said Big Jane. "Ah do enjoy a good deep dish pizza!" Then she turned and gave me a big hug, lifting me right off the floor. "You're all right for a little feller," she said.

Black Jack couldn't be fired from Berkeley, cause they have this thing called tenyure. But he never did show his face in research circles again. And he was demoted to teachin' big lectures of reemedial math to students who really din't want nothin' to do with him.

When news got out about our theorem, matheematicians flooded Deadfield and scoured the hills for math. But in fact, there never was as big a theorem ever found again in the fields. And then, come the next January, there was a strike 300 miles west, over in homological algebra, and just as quick as they had come, the prospectors disappeared. Main Street reverted to being a great example of the empty set. And Deadfield went back to being the dead town it had once been, and would be evermore.

Chapter 3
Prime Suspect

27 and I stepped out of the car and gingerly followed the muddy path down to the river. I pulled out a cigarette and lit it, knowing the smoke might help to cover the odor we were bound to encounter.

At the end of the path the yellow police tape cordoned off the scene. Kneeling in front of the stiff was 19, dressed in her white plastic protective clothing.

"Hey, 16, how's it going?" she asked.

"From the look of things, not so good," I replied. "What do we got?"

"It's a bad one," said 19. "Really bad."

"What is that?" asked 27, waving at what appeared to be a curved stick poking out of the mud.

"That's what's left of him," replied 19.

"But how is that possible?" asked 27. "What could do that?"

19 looked grim. "This number's been eaten."

27 looked he'd just been taken mod 3. "I think I'm gonna be sick," he said.

"Do it over there," said 19. "Don't contaminate the crime scene."

"When did it happen?" I asked, as I flipped the butt of my cigarette into the river.

"Probably around 10:00 last night. Looks like no struggle. He never saw it coming."

"Can you identify the victim?" I asked.

"Well judging from the remains, it's a single digit, male, tall, with a curved stick of some kind. So that rules out 1, 2, 3, 4, 7 and 8. Could be 5, 6 or 9. And I happen to know 5's taking a well-earned vacation in Florida. So that leaves 6 or 9."

"Good luck telling them apart," I said.

"9's my uncle," said 27, swallowing nervously.

"It wasn't 6," said 19. "In fact, 6 is the digit who called it in. He sounded terrified. Scared to death."

"Oh, no," croaked 27. "So it is 9." He wrung his hands. "Who would do such a thing?"

"Sounds like we need to talk to 6. Do you know where he is?" I asked 19.

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"Down at the Roots of Unity Church," replied 19. "I sent him there to wait with *e*."

"Okay," I said, "we'll check it out. Let's go 27."

When we arrived at the church, *e* ushered us in. He was dressed in bright multi-colored robes.

"We're here to see 6," said 27 brusquely. He'd never hidden his distaste for the Roots of Unity Church.

"6 is sleeping right now," said *e* in a calm voice. "I gave him a sedative and he's as horizontal as a vinculum. You know, he's had quite a scare. But we talked before he fell asleep, so perhaps I can answer your questions."

He pointed to three mats on the floor. "Please make yourselves comfortable. You will find that it helps to release tension."

"We're not here to relax," said 27.

e laughed. "You really are tense, 27. You should take our transcendental meditation class."

"Give me a break," said 27. "What do you know about transcendentalism?"

"Hey, 27," I said. "Ease off. Sorry *e*, but 27 just heard his uncle was the victim and he's pretty upset."

"Of course, I understand," said *e*. "Here, this will help." He turned and lit some incense.

27 just rolled his eyes. I sat down on the mat and turned to *e* who settled beside me. "Tell us what 6 said."

"I don't know if I have ever seen someone so frightened," said *e*. "He looked like he'd seen an imaginary number."

"Did he tell you about 9?"

"Yes, he said 9 was dead."

"Did he say how?"

"No, but he saw 9 walk by with someone on his arm and then heard the screams."

"Okay so maybe 6 could identify the killer in a line-up," I said.

"I know 9 was taking 7 to the Square's Ball last night," said 27. He turned to me. "Weren't you there?"

"No. Couldn't make it. I was at 8's bat mitzvah. But I think our next stop is to talk to 7."

We knocked on the door of the mansion that was 7's home. 7's father had made a fortune in the prime lending market.

7 opened the door.

"Oh," she said. "16 and 27. What are you doing here? I take it this isn't a social call." She was as confident as an irrational at a largest number of digits contest.

"We're here to talk to you about 9," I said.

"9? Didn't you hear? 9 and I broke up. My father refused to let me date a composite. Sorry, no offense intended."

"None taken," I said, as I lay a cautionary hand on 27's arm, whose face had turned red.

"Were you with 9 last night?" I asked.

"Yes, as a matter of fact I was. We went to the Square's Ball. That was when I broke up with him. I had agreed to go months earlier, so I thought I should wait until then."

"What happened?" asked 27.

"What do you mean, what happened? I went to the ball. It sucked. I have never seen so many squares in my life. It will take me years to get over that. I forced myself to stay for an hour. We were at a table with 81, and that guy makes π seem interesting. Finally at 9:00, I'd had enough and I figured what better time to dump 9 than 9:00. So I told him to come outside, and I told him I never wanted to see him again. Then I got in an uber and left. That's what happened. Why do you care, anyway?"

"Because 9 is dead," I said.

Her expression remained unchanged.

"Oh, well that is a surprise," she said, with as much intonation as if she were counting backward from 100.

"It was a gruesome death," said 27, trying to get a reaction.

"I guess we all have different ideas of what gruesome is," she said as she began to pick at something caught between her teeth.

"Why I ought to. . .," growled 27.

"Easy," I said to him.

"Do you have any suspects?" asked 7. "If I were you, I'd check on 13. She was at the ball with 36. You know he's almost three times her age. And she's trouble. I heard she's gone political. Got involved with some radicals, with all their revolutionary rhetoric. I don't exactly understand what they want. Some nonsense about equal rights for all integers. It's not exactly rational."

"We'll be sure to check that out," I said to 7, "but we will need you to come down to the station this afternoon."

"Happy to oblige," said 7. "I would do absolutely anything I could to help you find who did this dastardly deed." But as she said it, she had a smirk.

"Let's go," I said to 27.

When we got back to the station, 48 waved us down.

"Hey, 16, I think this may be a lot worse than we thought."

"What do you mean?"

"17 is missing, and both 25 and 33 said they saw a digit lurking around carrying a knife and fork.

"You mean?" said 27.

"Yes, I think we have a serial murderer on our hands."

That afternoon, we formed a line-up, with 13, 66, 7 and 29 in a row. I had 27 bring 6 in.

"Listen 6," I said. "When I open this curtain, I want you to identify who it was that you saw with 9."

6 nodded meekly. I pulled back the curtain. 6 shrieked and pointed at 7.

"It's her!" he squeaked. "It's her!" Then he fell to the floor shaking.

"What's wrong with him?" asked 27.

"What do you think?" I replied. "6 is afraid of 7, because 7 ate 9."

27 paused, and then said, "Isn't that a kid's joke?"

"Does this look like a kid's joke to you?" I snarled. "Book her. Digital cannibalism. It doesn't get any worse than that."

Chapter 4
The Cabinet of Dr. Möbius

My entire life, I have had an insatiable curiosity. Of course almost everyone is curious about the world around them, but for me, this urge to unravel the mysteries that I encounter is almost overwhelming.

When I was young, this desire reached its zenith when I attended a magic show. My parents took me to the first one when I was only eight years old. I remember lying awake in my bed that night trying to figure out how the magician had produced a dove that flew from his hand at a crucial moment in the show. By morning, I had already devised an apparatus that might function for the purpose. After that, I cajoled my parents to take me to every magic show that passed through town. Afterward, I would spend hours trying to fathom how the magician had performed their sleight-of-hand. I would always come up with some method for producing the trick. Sometimes, I would later find out that in fact, the magician did not perform the trick in the manner I had presumed. I had actually discovered a new means to produce the same effect. This produced in me an immense sense of satisfaction, better even than having determined the method the magician did use.

Eventually, I began to perform myself. It was gratifying to me to see how easily I could fool the audience, demonstrating in the process that I was more clever than were they.

As I graduated from shows presented to the neighborhood children to theatrical performances, my repertoire expanded. I learned a great deal from other magicians with whom I was acquainted. Most magicians willingly share their conjuring knowledge with their colleagues. In fact, there are clubs in many of the larger cities where magicians congregate to mingle with one another and share their secrets.

It was at such a club where I first learned of the magician Dr. Möbius. When his name came up, I was surprised to hear how many other magicians revered him for the intricacy and opacity of his tricks. There were certain tricks that he performed that no one had yet been able to recreate.

Möbius was a magician who did not associate with others. He kept to himself and was very protective of his prestidigitational knowledge. Whereas I knew the

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history of most of the well-known magicians performing—with whom they had apprenticed, from whom they had collected their tricks—Möbius remained a cipher. He neither shared his own secrets nor sought to have others share theirs with him. And yet, he had somehow become one of the best magicians of whom I had heard. It was a mystery from whence his abilities had come.

Möbius called himself a topological magician. Although this moniker was emblazoned across the marquee outside the theaters where he performed, none of us understood the significance of the word"topological". But then, neither did his audiences. It was simply a word mysterious enough to attract an audience unaware of its meaning.

One day, while at the club, I was very excited to learn that Möbius had been performing for a week as part of a two-week engagement at the Montrose Theater down on Sixth Avenue. Although an older theater, it had survived the years intact and still projected an elegance many of the other theaters lacked.

That night, I waited in a long line outside the box office to purchase a ticket. The crowd was large and enthusiastic. Word was out that it was a show not to be missed.

I found a seat a few rows back from the stage. As the show time approached, the audience settled into their seats expectantly. The curtain finally rose to a dark stage. Suddenly, a spotlight pointing down from directly above flicked on to light up the darkness and reveal a thin older man standing alone at the center of the stage. He was dressed in a black suit and the traditional magician's black cape trimmed in red. His face was gaunt and his eyes, almost invisible in shadow, were set deep beneath bushy grey eyebrows.

"Thank you all for coming," he said with a slight Eastern European accent. "I am Dr. Möbius."

He removed his cape and handed it to an assistant who had appeared out of the darkness at his side. She disappeared back into the inky blackness before we could make her out.

He continued. "The presentation I will provide tonight is not the traditional magic show you might have attended in the past. It is something much deeper, much more profound. For you see, with the tricks that I perform, I will reach beyond the universe as you know it, beyond the space within which we sit, beyond the dimensions with which we are all familiar." His voice began to increase in volume, filling the hall.

"What you see around you," he boomed, as he waved his arm, "is but a thin slice of the actual universe. It is only a glimpse. I will give you a further glimpse. A glimpse of what lies beyond."

At this he threw open his hand, and a flame shot out of his palm toward the audience. There was a gasp.

"I will demonstrate," he continued, "that the universe is much more profound, much more complicated than we can possibly imagine."

It was the kind of background patter than many magicians utilized, but he had a particularly magnetic delivery. The audience sat forward in their seats with expectation.

He then pulled an object out of his pocket and held it up for all of us to see. It appeared to be two linked wooden rings, each about six inches in diameter.

"I require a volunteer. But I want to make sure you are convinced this volunteer has not been planted by me. So I will ask the members of the front row to jointly pick the volunteer from those that raise their hand, so there can be no possibility of collusion between the volunteer and myself. Please raise your hand if you are willing to be a volunteer."

Various audience members from around the hall raised their hands with various levels of enthusiasm. After discussion, the members of the first row agreed on a man from a few rows back. Möbius waved him forward, and he ascended the stairs to the stage.

"Sir, have we ever met before?" asked Möbius.

"No, I believe not," replied the man.

"Very good," said Möbius. "Now, please examine these wooden rings. Check to make sure each was carved from a single piece of wood. There are no breaks, no places where they have been glued together."

"Yes," said the man. "This is true."

"And how, do you suppose, one might create a pair of rings like this?" asked Möbius.

"Well," said the man. "I suppose one could carve the pair out of a single piece of wood, so as to have them linked in this manner."

"Exactly," said Möbius. "Now, I would like you to take the rings, and stand just a bit over there. Now, when I tell you, I would like you to throw them up in the air, about six feet up and then catch them again. Can you do that?"

"Of course," said the man.

"Then go ahead."

The man threw the linked rings up in the air, and then as they came down, he caught them using both hands. There was a look of surprise on his face.

"Please, show the audience the result," commanded Möbius.

The man held up one ring in each hand. They were no longer linked.

"And are they the same individual wooden rings?" asked Möbius. "They certainly appear to be," said the man, a note of confusion in his voice.

"Let me explain," said Möbius. "You did not realize it, but in fact, you threw the rings from 3-dimensional space, the space with which we are all familiar, into 4-dimensional space. And in 4-dimensional space, two rings cannot be linked. You can always unlink them in 4-space. Then they fell back into 3-dimensional space unlinked."

This explanation made little sense to the vast majority of the audience, but it made the trick seem all the more mysterious, and Möbius received a hearty round of applause.

Then Möbius motioned to the wings and his assistant reappeared. This time we all got a good look at her. She wore an unusual dress that was constructed of a dark purple material on the left side and a gold reflective material on the right. As she approached Möbius, we could all see that she had a severe limp. Her left leg was markedly shorter than her right. In fact, one could discern an actual bend in the leg below the knee, most likely due to a birth defect. She handed Möbius what appeared to be a belt, but without a buckle. He handed it to the volunteer.

"Please describe what you have in your hand."

"It is a circular piece of leather, about two inches wide."

"Does it have any breaks or places where it is sewn together?" asked Möbius.

"No. It is a single connected circular piece of leather."

"And how might one construct such a thing?"

"I guess you would have to cut it whole from a large cow hide."

"Yes, that would work, wouldn't it? Now, I am going to ask you again to step back and then throw the loop into the air about six feet up and then catch it when it comes down."

The man nodded, and then, upon Möbius's command, threw the loop into the air. I kept my eye on it carefully as it rose and then fell.

The volunteer caught it, and then again looked surprised. When he held it up, we could all see that it was now knotted.

"Please attempt to unknot it," said Möbius. The man tried to manipulate the leather loop so as to disentangle it, but to no avail.

"I cannot," he said.

"Does this loop have any breaks or places where it has been sewn together?" asked Möbius.

"It does not."

"How might one make such a knotted loop of leather?"

The volunteer looked confused, and took a moment. Then he said, "I cannot imagine."

"Throw it up again," said Möbius.

The man did as he was told, and then registered his surprise when he held it up after catching it. We could all see that it was no longer knotted. The audience clapped vigorously.

At this point, I was convinced that the volunteer must be in cahoots with Möbius. If this were the case, it would be relatively simple to switch the original loop with a knotted loop hidden up his sleeve, a loop that was sewn together in a manner impossible to discern for the audience. But it seemed unlikely that Möbius had planted co-conspirators in the entire first row of the theater . Möbius motioned to his assistant again and she brought another strip of leather. Möbius took it from her and handed it to the volunteer. "Please describe this to the audience, "he said.

"Well," said the man, "it is another loop of leather but this one was constructed by sewing one end of a long strip of leather to the other. But it appears to have three half-twists in it that were added before the two ends were sewn together. And there is a curve drawn on it in red. It travels once around the loop exactly down the center. It is drawn on both sides."

"An excellent description, "said Möbius. He then pulled a large pair of shears out of his pocket.

"Please hold the loop out tightly," he said.

The volunteer did so, and Möbius jabbed the point of the shears into the leather, poking a hole. Then he gave the shears to the man and took the leather.

"While I hold the loop, I would like you to cut the leather loop all the way around the red central curve. What will that do?"

"It will cut the loop in two," replied the man.

"Please proceed," said Möbius.

The volunteer pushed the point of the shears back through the leather and began to slowly cut the leather loop along its central curve. He finally reached the point at which Möbius had poked the hole and cut the final cut. Möbius handed him the leather.

"What do we have?"

The man examined it and then held it up with a quizzical look on his face. "It is still a single loop," he said.

"Yes," said Möbius, "anything else?"

"Well, whereas the original loop was unknotted, this single loop is now knotted."

"Can you unknot it?"

"I cannot," said the man.

The audience clapped. Möbius did several other tricks, including dropping a ball straight through a solid table, and producing the volunteer's ring that he had previously placed in a locked box, which Möbius claimed to have plucked out of the box by reaching into the fourth dimension.

Finally, Möbius excused the volunteer, and motioned to the wings. His assistant reappeared, pushing a large cabinet on wheels before her. She centered it near the front of the stage. It stood about five feet tall and appeared to be made of wood covered with black lacquer, in a plain undecorated style. Möbius opened its door, but I could only see darkness inside. He turned the cabinet on its wheels, with the door open, to allow us all to see that it appeared innocuous all the way around. Then he addressed the audience.

"You are about the experience the true power of higher dimensions," he said. "We know so little of the universe within which we live. The truth is so much greater. Our minds need to open up to the immense possibilities."

His assistant, who had momentarily disappeared, returned with a cardboard cutout, which Möbius placed upright next to her. It was a life-size depiction of the assistant painted onto the cardboard, right down to the purple and gold dress, and the deformity of her left leg. Then he motioned to his assistant, and she stepped up into the cabinet. She barely had to duck down. Möbius then shut the door with a flourish.

"When we take a journey," he said, "we return home changed by our experiences. We are not the same person we were when we embarked. But most often, the changes are relatively small, a greater awareness of the diversity around us, an appreciation for what we have. However, some journeys can have more substantial effects."

He swept his eyes over the audience.

"What makes us what we are?" he asked, raising his voice. "What determines whether we are the same person, or another? Am I the same person I was when I was five years old? If you saw me now standing next to a five year old version of myself, you would be hard put to claim I was the same person."

He paused suddenly, as if he now knew it was the time. Then he threw open the cabinet door. His assistant could be seen inside, and he took her by the hand as she

stepped out. She appeared normal enough, but I had a sense there was something about her that seemed off. I could not immediately ascertain what it was. He had her stand next to the cutout, and suddenly it became apparent. For her purple and yellow costume was exactly reversed, as if it had been reflected. And her hairstyle was also reversed. Although these changes could be easily explained, as of course, the hair could be a wig and she could have changed costumes, most incredible of all was that her leg, her deformed leg, was now on the opposite side of her body.

It took a moment for the audience to appreciate the enormity of this transformation. At first there was a hush, as many did not recognize the metamorphosis. Then there began a smattering of applause and finally, as audience member whispered to audience member, the applause grew and grew to a tumult.

I myself sat quietly amidst the uproar, considering the possible explanations. Could it be that Möbius's assistant was faking her deformity? Could it be that she was a supremely talented contortionist, and was in turn, bending her right and then left leg in a manner that the rest of us couldn't possibly mimic? But considering her further, I convinced myself that could not be the case. For the bone itself was bent.

Could it be that Möbius had two assistants, each deformed as mirror images of one another? That seemed exceedingly unlikely, but twins have matching features. Could it occur that a pair of twins might be born, both deformed in the same manner, but as mirror images of one another? Perhaps Siamese twins that were separated? But that also seemed farfetched.

After another bow by Möbius and his assistant, the curtain fell, and the well-satisfied audience took its leave. As others took their turn heading up the aisle to the exit, I dallied and then slipped behind the curtain at the edge of the stage. As the audience thinned, I snuck up onto the stage and hid behind some scenery stacked up at the back.

I settled down to wait until everyone had departed and the theater was locked. But after twenty minutes, I was surprised to see Möbius and his assistant re-enter the stage area. Möbius supported her as she limped across the stage. Then he opened the door of the cabinet for her and she stepped inside. Möbius left the door open, but I was on the wrong side of the cabinet to see within. She was gone for perhaps 30 seconds, similar to the amount of time she had disappeared within the cabinet during the performance, and then she again stepped from the cabinet. I was surprised to see that she had once more been transformed, but this time she was restored to her original appearance, with the deformed leg back on the left side of her body. Once again, it was a fantastic trick, but a trick performed for no audience. What possible purpose could it serve?

Möbius helped her from the stage and they disappeared through one of the doors. Presently, all but a few lights dimmed and I could hear the lock mechanism being engaged for the door. I knew I was at last alone.

I stepped from my hiding place. The cabinet stood alone at the center of the dimly lit stage. Approaching it, I reached out and ran my hand down its side. It felt like a perfectly normal wooden cabinet. I continued to explore it on the outside, running my hands over the wood, looking for switches and hidden doors, but finding none. I pulled on the handle and the door swung open. It was too dark to see anything inside.

Although I knew it could only contain some sleight of hand, perhaps a mirror that the audience mistook for the real thing. I reached in to feel the back. But I didn't touch a back wall at all. So the front of the cabinet concealed a larger space behind. I stepped up into the cabinet to allow myself to feel further back. But again my fingers found no back wall. I stepped with my second foot into the cabinet and reached further. But still, I could not reach the back wall. As I continued to edge in further, I could not comprehend this phenomenon. How can a cabinet that is the standard size contain space for much more? Had I tripped a switch which dropped the back wall away? But away to where? I had already examined the cabinet from the outside. I had to know.

I continued to edge into the dark space. The light from the doorway behind me began to shrink as I moved slowly forward. In the darkness ahead of me I then noticed the tiniest speck of light. I had no idea how large the object was that was emitting the light but I moved slowly in that direction with the crab-like walk one utilizes when fearful of bumping into something in the dark. As I walked, I noticed that the smooth flooring had given way to a sponge-like material, perhaps moss. I continued forward slowly, noticing now what I hoped to be high grass brushing my legs.

At one point I stopped to look back and saw the doorway from whence I had come had shrunk to a small spot of light. As I was squinting at it, I thought I heard a noise. I held my breath, and this time I was sure. Something was sliding along the ground in the distance. I was not alone.

I considered yelling out, but thought better of it. As quietly as possible, I started to trot back the way I had come. But in the darkness, I tripped and fell to the ground. As I momentarily lay there, I again heard the sliding sound. It was ahead, between me and the doorway from which I had started, and it was much too close. It sounded as if the creature that made it was large and was moving toward me.

I leaped up, my heart pounding furiously, and sprinted back toward the speck of light I had previously seen in the distance. I prayed that I would not trip again. As I approached, the light grew into a rectangle and then I could see it was in fact a doorway. I finally reached the opening and leaped through, slamming the door behind me.

Breathing heavily, I found myself in the theater from which I had first come. In fact, I had just come out of the very cabinet from which I had originally embarked. I stood, completely baffled. For I was sure I had gone toward the new light. How could I have ended where I began? Then was this not the same theater? Could this be another copy of it? Or, when I fell, had I lost my bearings and returned the way I had come?

I looked more closely at my surroundings. There were the seats, just as I remembered them. And the red curtains hung from each side of the stage. And there on the back wall was the large clock. But as I looked at the clock I realized the difference. For the clock was exactly backward. By that I mean that each numeral was exactly reflected so the 2 appeared written backward, and just so for all the others as well. And they were ordered in the reverse counterclockwise order. Not only that, but the hands were both reflected to their opposite positions from the time

I knew it to be. How could this be? And then I realized that the exit sign was also reflected backward, both the individual letters and their order, so it now read TIXƎ. What is this place, I thought? It was quite a trick to reverse an individual, But to reverse the entire universe? This was too much to fathom. I felt a touch of dizziness and leaned on the side of the cabinet to support myself.

Then it occurred to me that if the world was reversed, I might be, too. I pulled my pocket watch from my waistcoat, and flipped it open. With a sigh or relief, I saw that it appeared as it always had.

At this moment, I heard a noise from the back of the theater and I realized that there was still someone in the building. I ducked down as a custodian entered the auditorium. Moving as quietly as I could to the rear of the stage, I found a doorway and escaped out of the back of the theater.

As I stepped off the curb into the street, I was almost run down by a carriage. The driver cursed me as I realized that everyone was now driving on the opposite side of the street. This new world would take some getting used to.

I made my way to the boarding house where I lived. Mrs. Kruger, who owned the boarding house, had left a sandwich and a glass of milk for me in my room. I realized I was ravenous.

I took a bite of the sandwich but immediately spit it out. The meat within it had clearly gone bad. The taste was all wrong. I took a sip of the milk and it also tasted awful.

I carried the plate and the glass down to the kitchen where Mrs. Kruger was doing dishes.

"Mrs. Kruger," I said, "this food is rancid."

She looked surprised.

"Mr. Dennett, I do not serve rancid food in this household."

"Well then explain why this tastes awful." I handed her the glass. She smelled it, and then took a tiny sip. She then looked at me curiously before drinking down a large gulp.

"There is nothing wrong with this milk, Mr. Dennett," she announced.

I took the glass from her, and took a large sip myself. I immediately spit it over her apron.

"Mr. Dennett!" she shrieked. I dropped the glass on the floor, where it shattered, and ran from the room.

Over the next few days, I found that any food I tried to eat tasted putrid. When I nevertheless forced myself to eat, I either vomited it up or suffered severe diarrhea. This left me so weak, that I soon found myself struggling to get out of bed in the mornings.

I made my way to my doctor's office. The receptionist looked surprised when she saw me enter the office.

"Mr. Dennett, you don't have an appointment," she said.

"I am sorry, but I must see the doctor right away," I said, the agitation apparent in my voice.

"Wait here," she said, as she went to find him. Five minutes later, he entered the waiting room.

"Come with me," he said. He brought me to the examination room. "What is so important that you must see me now?" he asked, clearly cross with me for coming without an appointment.

"There is something very wrong with me," I explained. "I cannot eat anything. It all tastes wrong. And then I throw up and have diarrhea. I am losing weight fast."

"Well, there are diseases that can ruin your taste buds," he replied calmly. "Take off your shirt, and I will examine you."

I did so, and then sat upon the examination table. He placed his stethoscope in his ears, and then placed the cold disk of it upon my chest and listened. He did not move it for a long time. Then he looked at me strangely, while he moved the cold metal to the other side of my chest and listened again. He removed it from my chest, tapped it a few times with his finger, and then placed it on my chest and listened again.

"What is it?" I asked.

He took the disk and placed it on his own chest. He listened for a moment and then placed it back on my chest.

"This is not possible," he said, more to himself than to me.

"What is not possible?" I asked urgently.

He ignored me and continued to listen to my chest, moving the stethoscope around. Then he moved down to my stomach.

Finally, he let the stethoscope fall back to his chest, and sat down hard in his chair, a confused look on his face.

"Doctor, what is it? Please tell me," I pleaded.

"I don't understand it," he said. "But your heart. It is on the wrong side of your body." He pointed to the left side of my chest.

"That is my left side. My heart is where it belongs."

"No," said the doctor. "That is your right side."

"What?" I said.

"I hear it beating in the right side of your chest rather than the left. Also your stomach is making noises on the wrong side as well. It appears you are reversed."

"But, but. . .," I stammered. Of course, I already knew the explanation.

"We must rush you to the hospital at once," continued the doctor. "This is amazing. I will call my colleagues. This is unheard of." He stood up, and opened the door.

"Nurse," he called. "I need you at once."

I grabbed up my shirt.

"It's not the hospital I need," I said, as I pushed past him and rushed out, ignoring his protests.

That night was to be Möbius's last performance at the Montrose Theater. I caught up with him outside the stage door.

"Dr. Möbius, "I stammered called out, "I must speak with you."

He turned to look at me. I knew I was a sight to behold.

"Please, doctor," I said, "I need your help. I believe I am dying."

He looked at my face, and then down at the buttons of my jacket, which I realized must appear to him on the opposite side of my jacket as is usual.

"You are reversed," he said. "How?"

"A week ago, I hid in the theater after your show. I wanted to know how you did your tricks."

His expression darkened.

"So now you know," he said. "Was it worth the price?"

He began to push past me but I grabbed his sleeve.

"Please doctor," I pleaded. "What has happened to me?"

"What has happened to you? You have had a direct experience of the complexity of the universe. You now have ample evidence that the universe contains paths that are orientation reversing. Paths that if traversed switch left to right. You are reversed."

"But why can't I eat?"

"On the small scale, much food has a handedness. Your body can only digest the left handed version, but not the right handed version."

"I need your help. I need you to change me back the way I was."

"So you can tell the world about my cabinet? So you can spread the truth, and then my cabinet will be taken from me? No, I don't think so."

He pushed past me. I grabbed again at his sleeve, but he shook me off, and entered the side door of the theater. It locked behind him.

I knew my only chance would be that night's performance. I went around to the box office and bought a ticket.

Sitting in the back, I kept my cap low on my brow. The performance went as before as I waited impatiently for the grand finale. Finally, Möbius's assistant brought out the cabinet and the cardboard cutout. Möbius opened the door and his assistant climbed inside. As everyone's attention was raptly focused on Möbius, I inched down the aisle to the front of the theater until I was crouching in front of the stage.

When Möbius swung open the cabinet door and helped his transformed assistant out, the audience again reacted to her transformation as before. Taking this as my opportunity. I bounded the last few steps to the stage and leaped upon it. Möbius realized very quickly what was happening and blocked my path to the cabinet.

"Out of the way," I yelled as I pushed the assistant to the side. She collided with the cutout and both fell to the floor. The audience gasped. Möbius grabbed hold of me as I attempted to climb into the cabinet. As he pulled me back, I held onto the cabinet, and it swiveled on its wheels. I attempted again to climb into it and Möbius swung me around. The cabinet rolled to the edge of the stage and as we struggled, it began to tip over the edge. Möbius grabbed for it and went over with it. As I collapsed in a pile with the assistant, I heard the cabinet splinter as it hit the floor below.

Several months have passed since then. I know it seems unlikely but Möbius's assistant and I now live together. Do not get the wrong impression. It is not love. It is merely convenience. She has taught me that there are a few foods we can eat. No one would say we are healthy, but at least we are alive. And we understand better than anyone else our shared plight.

Half of our time is spent on the road, making a meager living with the circus as the Amazing Reversed Couple. Initially, long lines formed to listen to our hearts beating on the wrong side of our bodies. But no one ever comes back to listen a second time, and most write it off to trickery. The novelty has worn off.

On the night the cabinet was destroyed, Möbius disappeared with it. Did he land inside it at the moment it was destroyed? I do not know for certain but that is what I suspect.

We tried to have the cabinet rebuilt, but it neither appeared the same nor performed the same. It was just a cabinet, and one that was crookedly pasted together out of the many shards that remained of the original.

Sometimes, I imagine what it must be like for Möbius, wandering that dark space with no speck of light to find. And what kind of creature is it that shares that space with him?

But perhaps there are other doorways out of that space. And perhaps some day Möbius will return. But imagining in what form he will return keeps me up at night. It keeps me up late.

Chapter 5
On Another Plane

Having spent the last four hours in the terminal, I was relieved to finally be on my connecting flight. I settled into my seat by the window, pulled some work out of my briefcase, and slid it into the pocket of the seat in front of me. Then I buckled in for the short flight.

Out the window, I watched the baggage handlers loading bags onto the conveyor belt, and then turned to realize that someone had joined me in the adjacent seat. It was an older woman with flowing white hair matched by her flowing white robes.

I nodded a greeting and then immediately pulled my papers out of the pocket to discourage any attempts at conversation. Planes are a good time to work on math, with no distractions and no excuses to do other things. The door to the plane was fastened shut, and then the flight attendant explained the security procedures. As the plane rolled backward away from the gate and then taxied to the runway, I was careful not to look up from my work. The plane rumbled down the runway and then took off, gaining altitude fast.

I was deeply engrossed in a particular calculation when the woman first spoke.

"You're working on mathematics," she said.

"Uh, yes, that's right," I replied, as I continued to write.

"Number theory by the look of it."

That got me to look up.

"Actually yes," I said. "Are you a mathematician?"

"Hardly," she replied. "I just find it interesting. I dabble once in a while."

"Oh, that's nice," I said as I turned back to my work.

"I noticed you wrote down R.H.," she continued. "Does that stand for the Riemann Hypothesis?"

"Well, yes, in fact it does," I replied.

"Are you trying to prove it?"

"God, no. I'm just writing a paper in which we prove various results assuming it's true."

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"Why not just prove it, so you don't have to assume it's true?" she asked.

I laughed out loud. "Um, sorry, I'm not laughing at you. It's just that the Riemann Hypothesis has been around for over 150 years. The greatest minds of mathematics have tried to prove it and failed. It's, shall we say, extremely difficult."

"Proving that a particular function has all nontrivial roots lying on a single line in the complex plane doesn't sound so difficult."

"Oh," I said, surprised. "So you know about the Riemann Hypothesis."

"Yes, I'm aware of it."

"Then you probably know that it's the most famous open problem in mathematics."

"I'm not much for knowing what's famous and what is not. It just doesn't seem that proving such a fact should be so hard."

"Really," I said. "Not so hard? Have you ever tried to prove it?"

"No, but it seems that Jensen polynomials of the Riemann zeta function should be relevant. If you prove they have only real roots, wouldn't that be enough?"

"Well, yes that is true," I said. "But we've known that for 90 years. Where did you study mathematics? You must have a PhD."

"I have never studied mathematics. I just enjoy reading about it. It's so abstract. The life of the mind and all that."

She smiled, a faraway look in her eye.

"Well, anyway, the approach you are suggesting has been tried before. And although recently it was proved that all but finitely many of the polynomials of each degree have only real roots, there doesn't seem to be a way to finish it off."

"Couldn't you convolute the remaining polynomials and extract a higher-level symmetric copolynomial?"

"And what would the point of that be?"

She looked at me as if I were a child. "The resulting polynomials would now fall into the set of polynomials which we already knew had real roots, meaning the original polynomials all had real roots as well."

"Well, um, I, um, I don't know. I guess you could try it. But I'm sure many others have tried it, and given that they haven't claimed a solution, it must not work." I felt flustered, given that I couldn't identify what was wrong with her suggestion.

"Oh, but if others have tried it, I think they would assume a Lenoix convolution, whereas what's needed is a Kawauchi convolution."

I stared at her as my mouth slowly dropped open.

"Young man, are you all right?" she asked, touching my arm lightly.

"I, um, I just feel like, um, I feel like you may have just told me how to prove the Riemann Hypothesis."

"Have I?" she said. "That's nice."

"No, you don't understand. I think your method will work. I think you have just solved the biggest open problem in all of mathematics."

She smiled. "Oh, well then. . ."

"No, you don't get it. You have to publish this. This is HUGE!"

The passengers in the row in front of us turned to look at me.

"Oh, I don't have time to write things up," she said.

"But, but if you've solved the biggest open problem in math, you HAVE to write it up. It's too important. This is the biggest advance of the century."

"Oh dear," she said. "Writing is no fun at all. Why don't you write it up?"

"Me write it up?"

The flight attendant interrupted to announce preparations for landing.

I continued. "I would write it up and we would be coauthors?"

"Oh, I don't want to be a coauthor. You can be the sole author."

The wheels of the plane bounced against the tarmac.

"But I didn't come up with the idea. It's your idea."

"Oh, I just threw around a few words that sound good together. You'll have to do the hard work of proving that it all works."

The plane taxied to the gate and the all-clear bell dinged. The woman unbuckled her seatbelt and stood up, stepping into the aisle. I realized how very tall she was by how she had to crouch so as not to bang her head.

I quickly clicked open my seat belt and leaped up, banging my head on the bulkhead.

"Wait," I said, rubbing my head. "What's your name? I can't just take this result and not give you credit."

"Of course you can," she said, as she started down the aisle. "Certainly all the others have done so, with my blessing."

"Others?"

"Yes, you know. Eventual Fields Medalists, Nobel Prize winners, and the like. All very nice people. Many with similar interests to my own."

"What? But you're saying. . ."

"Very nice to meet you, young man. Good luck and goodbye."

As I struggled to grab my papers and bag, she disappeared down the aisle.

Chapter 6
The Modern Prometheus

As I stood up, the members of the Fields Committee gazed at me in stony silence, their visages grave.

"Ladies and gentlemen," I began. "I have asked to come before you to explain the difficulty of the situation you now find yourself in. I apologize for the fact my explanation will take some time. But I must nevertheless begin this story at the beginning, which is now six years past.

"I was a graduate student in mathematics at the University of Tubendinghen until the time at which I failed the qualifiers in both analysis and topology. Although my score on the algebra exam was the highest they had ever seen, rules being what they were, I was summarily expunged from the program, without even the common courtesy of a masters degree.

"I found myself, at the relatively young age of 24, a bitter and broken man, bereft of any meaning in my life. As you all well know, once you lose mathematics, what possible meaning could be left to you?

"I spent the next year brooding on my defeat. But eventually, I came to the realization I had to move on with my life. I decided I should enter medical school. If I could not help myself, then perhaps I could help others.

"For the next four years, I studied to become a surgeon. Although it was not mathematics, there was a certain satisfaction in cutting and pasting, as it were. I honed my skills and then set out to make my mark in the world of surgery.

"But I quickly discovered that I experienced no pleasure in helping others. No, I did not relish the cutting of bulbous tumors from cadaverous patients. Nor did I take satisfaction in removing rancid appendices or swollen tonsils. Reshaping nasal cavities seemed the epitomy of pointlessness. Sometimes, I would find myself musing on some mathematical tidbit, and suddenly stop to look round, only to find various scrub-clad attendants staring anxiously at me. I would realize I had once again lost track of the purpose for which we were all in the operating theater.

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"Finally, I came to the conclusion that I had no future as a surgeon. Once again, my plans were thwarted, although in this case by me rather than by outside agents. I decided to disappear from the world as you know it, removing myself from both the spheres of academics and medicine. I bought a small cottage situated deep in the woods, and supporting myself with the funds I had earned during my short surgical career, I began working on mathematics once again.

"And at first, it brought me great joy. There were no distractions and no expectations. It was just me and the mathematics. If I wanted to spend six months working on one problem, so be it.

"But eventually, I came up against my greatest obstacle. Although I knew algebra, and knew it well, I had no background in those other great expanses of mathematics—topology and analysis. I could work on and solve problems in my own discipline, but any question that crossed the borders between these great countries of mathematics was beyond the reach of my abilities.

"So I took it upon myself to expand my horizons. I purchased a complete set of volumes, and I set to work. I grappled valiantly with the mathematics, trying my best to absorb it, but after a year, I had to conclude that I did not have the brain for such an endeavor. My mind could accommodate algebra but it was incapable of entertaining either topology or analysis. The requisite synapses did not exist to accept it. I was stymied. Once again, my ambitions were for naught.

"As I ruminated on my predicament, I came to realize that many of the deepest problems in mathematics were just such problems, problems that spanned the great divides in mathematics. And it was not only myself that had not solved them. They remained enigmas for all mankind. It was at this point that I realized how my particular training might allow me and me alone the means by which to overcome this fundamental obstruction to mathematical progress.

"And so, I hatched an audacious plan. I began by combing through the obituaries in the Notices of the American Mathematical Society. I searched for those individuals lauded for their particular expertise in the specialties that were relevant to the problems I found to be the most interesting. I eventually settled on a list of individuals, including a cohomologist, a combinatorialist, a modulist, a categorician, a fibrologist, a ramificator, a sheavologist and an ergodician. And here is where my tale takes a gruesome turn. Please excuse me, but I need to reveal the full extent of the events that led us to this day. I must include the grisly details.

"Stowing pick axe and shovel in my vehicle, I set out in search of the deceased individuals on my list. Upon arrival in the hometown of a chosen person, I would seek out the gravesite. There, under cover of darkness, I would dig down to the coffin. Then I would pry the top off, exposing the cadaver to the damp night air. With a hammer, I would then break open the corpse's skull, and remove the brains. These I would store in formaldehyde for safekeeping.

"Of course you may wonder as to the state of decay of the brain matter that I was appropriating. In point of fact, the issue of the Notices containing a particular death notice often came out many months or even years after the death. But humanity's distaste for decomposition aided me greatly. Coffins are constructed to last many years, and embalming preserves the brain tissue quite effectively.

"Within three months I had collected the brains of the eight eminent deceased mathematicians I had selected. Then I returned with my plunder to my cottage in the woods.

"As I am certain you have guessed by now, I pieced the brains together, choosing just the right portion of each brain to include. This was meticulous work. You cannot just squash bits of brains together and hope that they coalesce into a working whole. This was microsurgery of the most delicate kind. And you must take the correct portion of each brain—portions of the dorsal parietal region for the algebraists, the visual cortex for the geometers and the inferior temporal region for the analysts.

"After weeks of ultraprecise work, I had before me the most mathematically powerful brain that had ever existed, unequaled in the history of humankind. I stared at it in wonder: my creation, my masterpiece. But what good is a brain with no body to contain it?

"Perusing the local obituaries, I discovered a lad of no more than 20 years, who had died from an unexpected brain aneurism, a specimen who had been in great health otherwise. I procured the body from the local cemetery and dragged it back to my laboratory. There, I cut open the skull, removed its defective contents and inserted my magnificent creation. After painstakingly making all the necessary connections, there was but one step remaining, to reanimate the life within the still being lying before me.

"Of course, this is the crux of the matter, is it not? To somehow spark life in an inanimate creature. And this is the piece of the puzzle that had eluded even the greatest scientists until now. But I had an advantage that those other potential life-givers did not. I could use mathematics as a spark.

"I programmed a computer to deliver electrical impulses to the brain in a pattern corresponding to the rows of Pascal's triangle. And at the same time, I injected the heart with a massive dose of epinephrine. I then attached paddles to the chest and hit the corpse with 200 joules of electricity. The body convulsed, but the heart did not beat. I waited ten seconds and then reapplied the shock. Although the heart did not respond, I began to see on the monitor a tiny amount of electrical activity from the brain. I switched the pattern of impulses to the Fibonacci numbers in sequence. A different part of the brain began to fire. I sent another massive jolt of electricity through the body. I then switched to the coefficients of the Jones polynomials of the first knots in the knot tables. On the monitor, I could see more activity as another portion of the brain began to awaken. Now, I shocked the cadaver once more, and suddenly the eyes flew open, and the heart began to beep on its own. I switched the impulses to the sequence of Catalan numbers only to see another area of the brain light up. Then it was time for the Mersenne exponents. Now the whole brain was afire. My computer screen was lit up like a meteor shower.

" 'It's alive!,' I cried. 'It is alive!'

"And so now you understand the story of how my creature came into being. I had created a living, breathing thing with massive brain potential, and, as it turned out, incredible strength. But of course, its brain was a conglomeration of its parts, and as such, it had no viable identity. It was as if eight personalities had been forced together into one body, and they were at each other's throats.

"But when mathematics was set in front of it, then and only then did the creature calm down, and the various parts of its brain worked in harmony. And what amazing work did it do.

"It solved problem after problem, proving conjectures and creating counterexamples. It's insatiable appetite for mathematics knew no bounds. I had finally managed to have the impact on mathematics I had desired for so many years. If not I who was solving the big questions, it was my creation that was doing so. And I so wanted to share that fact.

"But I certainly should not have taken the creature to the conference on fiber bundles at Gergenden. And I am truly sorry for the loss of life and the damage to the facilities. But I do feel obligated to point out that my creature has advanced that field substantially beyond what the combined efforts of the dead mathematicians could ever have achieved had their lives not been cut short.

"And ultimately, there can be no question that my creation has produced mathematics well beyond the output and quality of any mathematician who has ever lived. We can all agree to that. If there were ever any creature deserving of mathematical accolades, it is this one.

"But of course, the remaining problem is to determine the age of the creature, for as you know all to well, the Fields medal can only be given to an individual who is under 40 years old. And alas, each of the pieces of brain that I included came from individuals substantially older than 40.

"You could argue that it is the average of their ages that should be considered. And of course that average is much higher than 40. So perhaps you say we need not have this discussion at all. But you see, I foresaw this issue, and that is why I chose a body that was young, with an age of only 20. And since the majority of the creature I created was of this body, the weighted average age is well below 40. Hence, I argue that yes, this creature deserves the Fields medal. I hope you will agree."

I turned to the creature seated next to me, who continued to gnaw on the corner of the wooden table. Suddenly, as if sensing that its future lay in the balance, the creature looked up at the committee and snarled, struggling mightily with the chains that bound it. All of the committee members shrank back in horror, as it roared out its frustration.

I slapped a copy of Harbinger's "Algebraic Catenoids" on the table in front of the creature, and it calmed down immediately.

I smiled benignly. "Thank you for your attention," I said. "We will await your decision."

Chapter 7
Aftermath

The first thing I noticed, as I began to regain consciousness, was the stench. It was that rotten egg smell like that section of the New Jersey Turnpike near Rahway. Second was the heat. Sweat was pooling on my chest. Third was the pain in my leg, as if I were being jabbed repeatedly by a pitchfork. I opened my eyes and looked to see that I was being jabbed repeatedly by a pitchfork.

"Hey, cut that out," I said to the fiery red demon who was repeatedly jabbing my leg. He looked up from the pitchfork, and said, "Wakey wakey time, Bobbie."

"Don't call me Bobbie," I replied. "Call me Robert. I hate being called Bobbie."

"I know," said the demon. "That's the point. Now get up, Bobbie. Time for orientation."

I looked around, and quickly surmised I was no longer in the San Diego Convention Center, where I had been giving a plenary talk in Ballroom C. There were a few dead giveaways, but the boiling lava and howling humans were the clinchers.

The demon jabbed my calf again.

"Okay," I said. "I'm getting up." As I stood, I couldn't help noticing I was no longer wearing the oxford shirt, khakis, and running shoes I had on at my lecture. Instead, I was only wearing a dirty loincloth.

"Is this a dream?" I asked. The demon snorted contemptuously.

"For a mathematician, you seem pretty slow on the uptake."

He slapped me in the back with the pitchfork.

"Get moving." I stepped over a few bodies writhing on the ground.

"Should we help them?" I asked. I received another snort and a big push in the back.

As I began walking in the direction the demon nudged me, I tried to remember what had happened just before I ended up here. The talk had been going smoothly, and I had gotten through the main construction with at least the impression I hadn't lost most of the audience, when someone stood up at the back of the auditorium and started yelling.

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At first I couldn't understand, but then I realized it was the Albanian mathematician Beznik Basha, and he was saying I had stolen his results. There was a ripple through the crowd. He was hysterical. I tried to drown him out but there was no stopping him. He charged down the aisle and leaped up on the stage, making a grab for me. As I ducked behind the podium, I felt a sudden overwhelming pain in my chest. I fell to my knees and suddenly the auditorium and Basha's wrathful face faded away. That's all I could remember. I had to admit that this had all the earmarks of a heart attack. Who thought the Albanian would ever show up in the U.S.?

As we walked over the blighted landscape, other bewildered-looking individuals joined us, each herded by his or her own personal demon. I thought, with a tinge of pride, that mine was perhaps a bit redder than the others. We entered an amphitheater, and were pushed toward chairs. As I was about to sit down, I realized each chair had large spikes jutting out of the seat.

"I think I'll just stand," I said to my demon, as I heard the howls of others who hadn't paid enough attention.

"Suit yourself," he said.

On the stage at the front of the amphitheater, a dark crimson demon took a microphone, and said, "Ladies and Gentlemen, may I have your attention. We are very proud to present to you the Prince of Perversity, the Monarch of Mayhem, the Sovereign of Suffering, none other than his Highness, the King of Hades, yes, give it up for Lucifer."

Urged by a quick jab in the back, I joined the others in desultory applause. A huge figure loomed out of the darkness at the rear of the stage, easily the height of ten people. He was bright red, with eight-foot horns and a face only a demon mother could love. He laughed maniacally, and I and the rest of the crowd cowered in horror.

"Welcome to Hell!" he boomed. "We're very glad to have you joining us." He scanned the crowd with a wicked grin.

"Our job here is very simple. To make you as miserable as we can. We want you miserable today, miserable tomorrow, and miserable every day after, for ummm, how about eternity!

"And your job is very simple. To suffer horribly for the rest of your existence.

"Now, there aren't a lot of rules here. Because if there were and you broke them, what would we do? You're already experiencing the most horrible abominations we can think of. So don't worry about rules.

"We'll get all this going as soon as possible. Don't want you to miss any more suffering than absolutely necessary. You just need to fill out some paperwork and then we'll get this party started. Anyway, enjoy your stay!"

He laughed his maniacal laugh again and then turned and disappeared behind the curtain at the back of the stage. My demon grabbed me by the arm.

"Where to now?" I asked.

"Boss man wants to see you, Bobbie."

"Me?" I said nervously. "Why me? He picked me specially out of all these other recruits?"

"Yup, so shut your trap and come with me."

He led me down front and through a door at the side of the stage. We walked down a hall and turned a corner. The Devil was sitting on a massive rock, his chin resting on his fist, in a Thinker-like pose. He looked exhausted, very different from the image he had just been projecting from the stage. My demon pushed me forward.

"Um, hello, your Monstrousness," I said. He slowly turned his yellow eyes in my direction, and then motioned to the demon. "This is the mathematician?"

"Yes."

He looked me up and down.

"We don't get a lot of mathematicians down here," he said.

"You don't?" I asked. "What about Plinker? Surely Plinker ended up down here."

"Not ringing any bells," said the Devil.

"Herbert Plinker. Short guy. Bald spot. Tripped and fell down the stairs about a year ago. Such a jerk."

"Being a jerk doesn't get you into Hell," said the Devil.

"What does?"

"Well, in your case, passing off someone else's results as your own. Cheating the math department out of its cookie fund. Having someone else take your qualifying exam."

"Oh yeah. Thank God they weren't oral."

"Most mathematicians tend to be law-abiding citizens," said the Devil. "Don't get into much trouble. Too busy doing math. They don't drink a lot since it interferes with their ability to work. They don't kill other people in jealous fits of rage. Pretty even-keeled for the most part."

"I see. So I'm the odd mathematician out."

"So it would seem. Now as to the reason I wanted to talk to you. I need some advice."

Advisor to Lucifer? I liked the sound of that.

"I'm at your service, your Putridness."

"I have a problem with eternity."

"Eternity? It's a very long time."

"Yes, that's the problem," said the Devil as he stood slowly, rising to his full height. "I mean we're talking infinity here. Not a year, not a decade, not a century, not a millennium. Those are blinks of an eye when confronting eternity. It just goes on and on and on."

"Yeah. That's right," I said as I craned my neck to look up at him.

"The problem is that it's my job to make the residents of Hell miserable for eternity. Not so easy to do."

"Why not? Can't you just roast them in fire forever?"

The Devil shook his head dejectedly. "Doesn't work. No matter how bad something is, you suffer it long enough and you get used to it. It becomes the new norm."

"Hmmm. So you want me to come up with some new torments?"

"No, we have a complete list of torments. There are one thousand."

"What do you mean one thousand? There are certainly more than that."

"There are one thousand fundamentally different torments. We don't distinguish between being eaten by grasshoppers and being eaten by crickets. We don't distinguish between being boiled alive in virgin olive oil and being boiled alive in extra virgin olive oil."

"Fair enough," I said. "So there are one thousand torments. Why not just cycle through them? Do each for a year, and at the end of one thousand years, start over."

"That's the problem. Like I said, eternity is a long time. One thousand years is nothing, 1,000,000 years is nothing. After 1,000,000 years, you've been through the cycle a thousand times. You know exactly what comes next. No surprise, no dread. It's like punching a clock after a while."

"So what do you need?"

Lucifer leaned down toward me. "I need randomness. I need for no one to know what's coming next."

"Okay," I said. "So what's your plan?"

"I'm thinking I'll use monkeys."

"What do you mean?"

The devil began to pace. "You know. If you leave a monkey typing long enough, eventually it writes Hamlet."

"You want Hamlet?"

"No, I want the monkey to generate the sequence of torments. Then no one knows what to expect. Each new torment will be a surprise."

"Where you going to get monkeys?"

"Are you kidding? Monkeys are the worst. They all end up in hell."

"Didn't know that. I kind of thought Hell was for humans only."

The Devil shrugged. "With evolution and all, it's hard to draw a clear line. So we just made all bipeds eligible."

"I don't feel so special anymore," I said, half joking. The Devil didn't smile.

"You know, people have tried it before," I continued.

"Tried what? The monkeys?"

"Yup. University of Plymouth, 2003. They put a keyboard and monitor in a cage with six macaques. For a month. Just to see what they would produce."

"And?"

"Five pages. That's what they produced. Mostly the letter s. And they took turns urinating and defecating on the keyboard."

"Hmmm. Doesn't sound so random."

"Yeah. That's the problem with monkeys. They're lousy random number generators."

"What's a random number generator?"

"It's a system for generating random numbers. If you want a random sequence of numbers, each from 1 to 1000, it picks them out."

"Yes, that's what I want."

"Well, it's not as simple as that."

"Why not?"

"Because most of the generators that are out there are pseudo-random number generators. A pseudo-random number generator simulates a random number gen-

erator for some amount of time. But then it repeats. Like the linear congruence generator given by $x_{n+1} = ax_n + b \pmod{m}$."

"What's mod m?"

"It means you take the remainder when you divide by m. In your case, $m = 1000$. And you take a and b to be big numbers. So the next number in your sequence x_{n+1} comes from the previous number x_n and a and b. If you know a and b and the starting number x_1, then all the subsequent numbers are between 0 and 999 and all of them are determined."

"But how is it random if each number is determined by the previous one?"

"That's why it's called pseudo-random. It's not random, but for all intents and purposes, it looks random."

"That sounds perfect," said Lucifer with a grin.

"Hold on. There's a problem. It can only generate at most 1000 numbers before it repeats."

"What? Why?"

"Since we're doing everything mod 1000, each time it picks a new number, there are only 1000 candidates. And the minute it repeats a number we've seen before, since all the subsequent numbers just depend on it, it will repeat all the numbers that followed that number. So it gets into an endless loop."

The Devil threw up his hands. "We can't have any loops. Once someone's been suffering long enough, they'll spot the loop and know what torment is coming next."

"Yeah, maybe you need something fancier. Maybe you need the Mersenne Twister."

"I like the name. Sounds painful."

"Yeah. It has a period length of $2^{19937} - 1$, which is a Mersenne prime."

"Yes, but then it repeats?"

"Yeah, but do you have any idea how big $2^{19937} - 1$ is?"

"Yes, I know exactly how big it is. It's finite. That's nothing compared to eternity."

"All right, all right," I said. "You don't want a pseudorandom number generator. You want an actual random number generator."

"Yes, that sounds right. What about something like the digits of π?"

"Not such a bad idea, but unfortunately, we don't even know if the digits of π are normal."

"What does that mean?"

"It means that each of the ten digits appears the right number of times, which is to say one tenth of the time. And each pair of digits appears one one hundredth of the time, etc. You get the idea."

"How could it be random if it wasn't normal?"

"That's the problem."

The Devil stroked his chin. "Are there numbers whose digits are known to be normal?"

"Not many, but a few."

"Like what?"

"Take the number .123456789101112131415... In other words, just put the sequence of natural numbers next to each other and count each place as a separate digit. That's known to be normal."

"That's stupid. That's about the least random number you could come up with."

"I didn't say it was random. I said it was normal."

The Devil leaned his face in close enough to mine that I could smell his noxious breath. "I am beginning to get a little frustrated. That's not good for you."

"Okay, okay," I said hurriedly. "Base it off of atomic fluctuations, or radioactive decay. Those are so-called physical random number generators."

"What guarantees they're random?"

"The laws of quantum mechanics."

The devil rolled his eyes. "Great. Now I need a physicist. They're even better behaved than mathematicians. Could take quite a while to get one."

"Look. If you want a physical random number generator, you can go low tech, old school. The way people did it for centuries."

"What do you mean?"

"All you really need is a human to repeatedly roll a die. That's just as good a random number generator as any of these others. And it's not like you have a shortage of humans down here."

"Really? That would work?"

"Yes. I guess in your case, you just need three ten-sided dice. One for each digit."

"No problem. I can get them made within the hour."

"Okay then. You're set. Just need a chump to roll them for eternity."

The devil smiled serenely. "Not a problem finding a chump around here."

He motioned to the demon. "Get him set up."

"What? No. I'm not the chump."

But it turned out I was. So if you end up in Hell, be reassured that, thanks to yours truly, your torments will be randomly assigned for eternity. At least you won't get bored. Boredom was not one of the one thousand torments but if you ask me, it should be. Me, I'm bored all the time. I guess I should be grateful I'm not being boiled alive in virgin olive oil, but sometimes I wish I were, just to have a change of pace. All I do is roll one year after the next. And for however many years I do it, I still have eternity left to go. That's how it works with eternity. It is a very long time.

Chapter 8
Looking Backward

I will now continue my most amazing tale. I expect you have so far found it somewhat hard to believe, but I attest it all to be completely true.

As I have previously related, in the year of our Lord 1898, on June the 25th, I was put into a hypnotic state by the esteemed animal magnetist, Dr. Pillsbury, in order to overcome my insomnia and to allow me to obtain a full night's sleep. After inducing me into a somnambulant state, he left me sleeping on my bed in a specially built chamber embedded deep in the foundation of my house, so as to shield me from the noise and bustle of the city of Boston. Later that evening, the doctor departed permanently for New Orleans, but not before leaving me references for other doctors and instructing my servant on how to wake me in the morning.

However, as I slept that night, an unfortunate accident with an oil lamp caused the house above me to burn to the ground. My poor servant Bartholomew perished in the conflagration.

It was not until 112 years later, in the year of our Lord 2010, that workmen repairing a sewage line for the house that had replaced mine discovered the chamber in which I lay. After realizing that I was not dead, they called the owner of the house, Dr. Leete, who undertook to revive me. I awoke, physically unharmed, but weak and suffering a great degree of disorientation. Since that time, I have regained my strength, recovered my wits, and learned much about the marvelous new world that exists today.

No longer do lines of tall smokestacks belch noxious fumes into the open air. Verily, the inhabitants of this epoch have overcome the dangers of over-industrialization and make no decision that might have negative repercussions for the surrounding environment. Consequently, they live in a verdant lush garden of a world, beautifully cultivated and carefully stewarded for the sake of future generations.

To provide sustenance to the populace, fish and livestock are raised in humane conditions. And well fertilized land produces substantial quantities of grain, fruit, and vegetables. A cornucopia of delicious edibles weighs down every table.

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And no longer do orphaned children fend for themselves on the streets. If any citizens are incapable of caring for themselves and have no one to care for them, then the state takes charge. Homelessness does not exist in this new world. Everyone is guaranteed the right to an abode and the nourishment necessary for life to flourish.

If someone becomes sick or incapacitated in any way, the costs of hospitalization and doctor's bills are born by the government. All citizens are provided with equal and high quality health care, as one would expect from an enlightened society.

Moreover, all adults have the right to fulfilling careers. The most menial jobs are performed by automatons, which are constructed for that purpose. Citizens attend school for twelve years, after which they come to a decision about what profession to pursue. All professions are paid comparably, and one chooses what to pursue based on desire, not on the potential monetary reward. Indeed, it seems that the citizens of this time have truly succeeded in creating what can only be called a utopian existence for all. And much of this have I already related. But now, I shall explain to you what I have learned of mathematics in this new era.

As you may remember, I myself once dreamed of becoming a mathematician. However, it was my father's wish that I forgo mathematics to become a lawyer, for a lawyer had the potential to make substantial sums of money, which would have allowed me to support a family in the appropriate manner. As I was betrothed to my beloved Edith, this career choice made eminent sense. But finding myself in this new world, I was very curious to learn how mathematics was perceived, and to perhaps determine if this could be a field that I might now pursue. In my new circumstance, the familial pressures to which I had once yielded had disappeared, leaving such decisions entirely up to me.

One evening, as we sat smoking thoughtfully after an excellent meal, I asked Dr. Leete about mathematics in the modern era.

"Ah, it is funny you should ask, " he responded. "For you see, in this year of 2010, the Mathematics World Exposition is taking place right here in the city of Boston. This event occurs once every ten years and a city considers itself very lucky to be chosen the venue. The Mathematics World Exposition is a celebration of all things mathematical, and it will give you a sense of how important mathematics is now perceived to be. Lucy and I will take you to view it tomorrow afternoon."

Lucy was Dr. Leete's niece, and the spitting image of my now long dead Edith. Although my heart still ached with the loss of my beloved, there was so much that reminded me of her in Lucy that I could not help but have feelings for the girl.

The next day, I accompanied the good doctor and his niece on the moving walkway that transported us to the Exposition Halls. Lucy was dressed in a becoming green dress that, although quite different in style from the dresses to which I was accustomed, was still modest in appearance and suggestive of her virtue. As we rode along, the doctor nodded to passing acquaintances, as he explained to me the current state of mathematics.

"Today, all citizens are brought up to be aware of the importance of mathematics. At a very young age, children become proficient in counting, addition, subtraction, multiplication, and division. All teaching at this level is in the form of games. So students never memorize as much as play to learn mathematics. Some of the greatest

minds spend their time trying to come up with new games for the children to play that will teach them the mathematics."

I thought to myself how arduous the memorization of all the various mathematical facts had been for me. The primary method used by schoolmasters to reinforce memorization was through judiciously timed ruler raps on the knuckles. No one would have described the process as agreeable.

"By age six, students are proficient with fractions, decimals, percents, and the like. They are even capable of difficult computations involving logarithms to the base 10. All students can easily manipulate the most complicated of slide rules, which today have twenty or more moving parts.

"In addition to the teaching of mathematics that occurs in school, there are also performances by troupes of actors who travel from city to city, enacting the greatest moments of mathematics. Just last week, I went to see a re-enactment of the death of Galois. It was very moving, but perhaps not appropriate for the younger children due to its violent climax."

"And I went to see the discovery of hyperbolic geometry," added Lucy enthusiastically. "It was so funny to hear of all the mathematicians who tried to prove Euclid's parallel postulate. Such silly people." She wrinkled her nose in an endearing manner. "And there were many children there. They loved it, clapping quite loudly at the end. The actor who played Lobachevsky had a funny thick Russian accent. He made us all laugh".

Dr. Leete smiled at his niece and then continued. "For Halloween, two of the most popular costumes are Carl Friedrich Gauss and Isaac Newton. Last year, my great grandson Henrik went as Leonardo Fibonacci.

"By the age of eight, the children have mastered algebra and trigonometry. Speaking of which. . ." He pointed to a billboard towering above us. Upon it was a picture of the graph of sine of x with various happy cartoon children playing upon the curve. Beneath the picture appeared the words, "Sine of x, a wonderful tool for your enjoyment".

"Many are the ways in which we sing the praises of mathematics," he said. "By ten, children have mastered the calculus. You may wonder why calculus is considered important enough that we insist everyone learn it. But even those who ultimately choose a trade to which it is not relevant benefit from the rigors of its study."

"But for those who don't use it in their professions, don't they forget it quickly?" I asked.

"To maintain an ongoing interest in mathematics amongst the general public," replied Dr. Leete, "the government produces quiz shows that are disseminated over the telephone. Average citizens compete in an attempt to achieve renown. Thousands of others listen in over their telephones as a lucky participant tries to answer difficult questions. The most popular of these shows is called "Who Wants to Use Their Knowledge of Mathematics to Achieve Distinction?"

Lucy grabbed my hand enthusiastically. "Oh, Julian, you must meet Wendell Carmody. He will be at the Exposition. He is the most successful of the contestants ever. His knowledge of mathematics is truly encyclopedic."

Dr. Leete laughed. “He was in fact a patient of mine, and I have promised Lucy an introduction. In fact, he is perhaps more famous for winning the Nobel Prize in Mathematics.”

I had heard of Alfred Nobel’s passing in 1896, and his will, which endowed prizes in various fields. However, by 1898, the prizes had yet to be distributed.

“I thought Nobel did not endow a prize in mathematics,” I said, “only in the sciences, literature, and peace.”

“You are correct,” replied Dr. Leete. “Mathematics was added to the list of categories in 1910.”

“I will look forward to meeting this Carmody,” I said. Lucy continued to hold my hand and I certainly made no protest.

Dr. Leete went on. “By age twelve, students have learned the methods of differential equations, so as to understand those formulas that govern the movement of electrons on wires. Many of the inventions you see around you, including the automatons, are powered by electricity. They could not exist, were it not for the calculus.”

I could see several of the automatons, hulking metal hydraulically powered creatures riveted together out of iron, riding the people movers as they went about their simple errands.

“At the age of fourteen, students must choose whether or not they intend to continue with mathematics. Those who desire to learn more move on to probability and advanced calculus, wherein they are taught the techniques of rigorous argumentation and the roles Greek letters play within them.

“At the age of sixteen, students decide whether to pursue applications of mathematics or the pure mathematics, wherein the subject is studied for its aesthetic beauty rather than for its utilitarian benefits. In fact, in Boston alone, there are so many students interested in continuing their mathematical education that no lecture hall could possibly be large enough to contain the audience.

“But the miracle of the telephone has allowed thousands of students to hear lectures by world famous mathematicians. In fact, Carmody is giving a lecture tonight on fluxions. We can listen to it if you so desire.”

“Yes,” I replied. “I would be very interested.”

“Me, too,” added Lucy.

“We will plan on it then. In fact, if we were not free to hear the live lecture, it would not matter. For the government records the lectures on gramophone cylinders, copies of which are then distributed worldwide.”

“That is quite incredible,” I said. “So all students get to hear the very greatest of mathematicians.”

“Yes,” replied Dr. Leete. “By these advanced means, the need for individual teachers has dropped precipitously. Now the local teacher need only record on the blackboard the equations described by the expert over the phone.”

At this point, the moving walkway deposited us at the entrance to the World Mathematics Exposition. I looked up at the giant metal Greek letter π, the legs of which formed the gate to the park.

"Where do we pay?" I asked, as we walked toward the gate. Both the good doctor and his niece laughed out loud.

"Silly," said Lucy, "you never have to pay to learn mathematics."

We joined the convivial throng funneling into the Exposition park and soon found ourselves walking with many others down a lane surrounded by shiny metal sculptures of various quadric surfaces. As we approached the main avenue, we could see floats passing by and hear the music of marching bands.

"Is there always a parade?" I asked.

Lucy smiled, pleased to be able to explain it to me.

"You see, the main street is in fact in the shape of a circle, and the marchers continue around it all day. We call it the infinite parade, as a circle has no beginning or end."

As we approached, I could see the float for e^x going by. Directly behind it appeared a slightly tired marching band playing a strange somewhat dissonant tune.

"Those notes correspond to the digits of π," said Dr. Leete. He pointed to a nearby building. "Over there, computational wizards continue to calculate new digits of π, so as to stay ahead of the band. The song never ends."

"Don't worry," said Lucy, sensing my concern. "The bands take turns. They get plenty of time off."

"Some of the greatest musicians of the day have devoted themselves to the creation of songs about mathematics," said Dr. Leete. "Perhaps the most popular song is one about the quadratic formula. It is so catchy, I sometimes cannot get it out of my head."

"I could sing it for you later," said Lucy, squeezing my hand.

"I am sure I would enjoy that very much," I replied.

We followed a small tunnel under the infinite parade route and found ourselves at the entrance to a large wooden rollercoaster.

"What is the mathematical significance of this?" I asked.

Lucy giggled. "Do you not see that the track passes over and under itself," she said, "sometimes passing through the openings in the supporting timbers. Since the track eventually ends where it started, the entire rollercoaster is in fact a knot. I believe that even in your era, people understood the mathematical significance of knots."

Indeed, in my own time, I had known of a theory of the atom based on knotted vortices in the ether. But I also knew that it had been discredited when the Michelson-Morley experiment demonstrated there was no ether. There had also been a somewhat less scientific theory that we were all knotted three dimensional cross-sections of four dimensional creatures. But I hesitated to display my own ignorance on the subject before the lovely Lucy. So I did not venture to seek further explanation.

"Come," said Lucy, pulling me forward, "we must all go for a ride."

"You won't get me on that contraption," chortled Dr. Leete. "And I warn you, Julian, you may regret it."

I had never been fond of carnival rides, but the opportunity to be alone with Lucy overrode any reluctance on my part. As we were seated next to each other, our legs momentarily touched, sending a charge of what electricity must feel like up my spine.

As the ride began, the car in which we were seated ascended a long tilted track high into the air. As the ground fell away below us, Lucy grabbed onto my arm and hung on tightly. We reached the highest point and then plunged downward at a terrific speed. She screamed, burying her head in my shoulder. As we shot around the twisted track, the entanglement enhanced the sense of disorientation to such a degree that one became confused as to which direction was up, down, left, or right. By the end, my stomach was as knotted as the track itself. Dr. Leete laughed aloud when he spied the expression on my face.

"Truly, Julian," he said, "I tried to warn you. Now, come, we will seek out Wendell Carmody."

The Nobel Pavilion was in the Greek style with large pillars framing the ornate entrance. As we walked into the marble hall, I could see a row of busts of the previous winners lining the walls. I was stunned to recognize that one of the winners was Buskin, a student I had known when I was at Harvard. No one would have suspected that he might eventually win any kind of prize in mathematics. He had always been a particularly lazy student, rarely attending class, choosing instead to spend his time frequenting a variety of Cambridge pubs. But the surprise I experienced was tenfold greater when I spied a bust of another winner, a woman who had won the prize in 1928. It was none other than my dear lost Edith.

"But, how can this be?" I exclaimed.

"Oh, that is my great great grandaunt Edith Wilson," replied Lucy. "She received the Nobel Prize in mathematics for her work on neutral groups."

"She became a mathematician?" I said more to myself than anyone else.

"An excellent mathematician," interjected Dr. Leete. "She spent all her time working on mathematics. Never married."

"Indeed," I said, a strange mix of emotions rising in my breast.

"Come," said the doctor. "There is the dais upon which we will find Carmody."

I could sense Lucy's excitement grow as we approached. A large red banner hung above the dais, proclaiming all of the degrees and awards that had been received by Carmody. Dr. Leete ushered us forward.

"Dr. Carmody, so good to see you."

"Ah, Dr. Leete," he responded, rising to greet us. "I am still grateful to you for curing my nasal drip."

"It was nothing," responded the doctor. "Let me present to you my niece Lucy and a visitor Julian."

Ignoring me completely, he focused all his attention on Lucy. Taking her hand, he lifted it to his lips and kissed it. "Doctor, why have you kept your niece hidden from me?" he asked without looking away from her.

She giggled. "He has been too busy to escort me here sooner. For I have begged him since the opening of the Exposition to bring me to meet you."

A sly smile flitted across his face.

"I am honored by your interest in me," he said. "Do you have mathematical inclinations?"

"Oh, yes," replied Lucy. "Mathematics is the lifeblood of the sciences."

"Indeed it is."

"And for what did you receive your Nobel Prize?" I interrupted.

Carmody turned to look at me. "I would think you would know," he replied.

Dr. Leete jumped in. "Ah, Julian is not from this country. So his schooling has focused on other subjects." Dr. Leete and I had agreed not to divulge my true history until I had a bit more time to adjust.

"I see," said Carmody as he eyed me carefully. "Well, perhaps I can enlighten you. I solved the Goldbach Conjecture, the greatest open problem in all of mathematics. It states that every even integer greater than 2 is the sum of two primes."

"Yes, I am aware of the conjecture," I replied, "just not of your solution."

"Your country must be very far away indeed," replied Carmody.

"Ah, yes it is," said Dr. Leete uncomfortably. "Well, look at the time. Perhaps we should be going."

"Must we, Uncle?" asked Lucy. "We just arrived. I so wanted to get to know Dr. Carmody better."

"I think our guest might be getting tired," replied the doctor nodding to me. "He is still adjusted to a different time zone, having only recently arrived by hot air balloon."

"Don't leave on my account," I said.

"Well," said Lucy to Carmody, "for the sake of dear Julian, we should go. But I do hope we get the chance to see you again. We look forward to your lecture tonight, which we will listen to over the telephone."

"I will do my best to make it worth your while," replied Carmody. "And I would very much enjoy seeing you again." He bowed ceremoniously as we turned to go.

After dinner at Dr. Leete's house, the doctor hooked the telephone up to the gramophone speaker, which immediately emitted a static sound. The three of us made ourselves comfortable, and at the appointed time, the static was replaced by an announcer who gave a flowery introduction to Carmody that included a bloated list of his various accomplishments. Then Carmody began to speak in a pedantic manner. He continued to wax on about fluxions and fluents for the next hour. At various points, his lecture was interrupted by the clapping of what must have been the live audience present in the auditorium. Carmody did his best to sound sophisticated and erudite, using the largest words he could muster to explain the simplest ideas. Lucy sat listening raptly.

At the end of his lecture, Carmody summed up, and then added, "I would like to dedicate this lecture to Dr. Leete and his niece Lucy, whose interests in mathematics are an inspiration to us all." Lucy turned bright red and had great difficulty hiding her pleasure.

Afterward, the doctor and I walked Lucy home, and then returned to the doctor's parlor to smoke. After discussing various aspects of the lecture, I asked Dr. Leete where I might find Carmody's proof of the Goldbach Conjecture.

"Why, it appeared in every major magazine," he replied. He fished through a stack of copies of the Gentlemen's Home Quarterly and handed me a copy. On the cover was Carmody's supercilious expression.

"Can I borrow this?" I asked.

"Certainly," he replied.

I snuffed my cigar in the ashtray.

"I am a bit tired." I said. "I think I shall retire for the evening."

The doctor bid me good night, and I ascended the stairs to my room. I immediately settled at the desk and began pouring over Carmody's proof of the Goldbach Conjecture. Although somewhat technical, I was able to understand the gist of it. It was built upon several results of Riemann that had already existed in 1898. I spent many hours going over it, but eventually exhaustion overcame me, and unable to keep my eyes open any longer, I climbed into bed and fell into a deep sleep.

At some point, I began to dream. Carmody was tapping on Lucy's door.

"Let me in," he cooed. Clothed only in her dressing gown, she rose from her bed to unlock the door. I called to her.

"Lucy, Lucy, do not open the door". But she was oblivious to my entreaties. Suddenly a voice interjected.

"Wake up, sir. It is morning."

I opened my eyes to the dim light of an oil lamp and the face of my long dead servant Bartholomew leaning over me. I leapt from the bed.

"What is this," I cried. "You died over one hundred years ago."

He looked at me strangely, and then said, "That must have been quite a dream, sir." I looked about myself and realized that this was the secret chamber under my house.

"What day is it?" I asked, with great trepidation.

"Why, it is Tuesday, sir, June the 26th. Dr. Pillsbury hypnotized you so that you might sleep, and then instructed me to wake you."

"And what of the fire?" I asked.

"I know not of a fire," he responded.

My mind was in turmoil. Could it be that all that had happened to me had been a dream? That there was no Dr. Leete, that there was no Lucy, that there was no Carmody?

"Sir?" asked Bartholomew.

"Um, yes, Bartholomew, I will be all right," I said. "Just give me a bit of time."

"Sir, Miss Edith is expected within the hour. She said you were to go with her on a carriage ride today."

"Edith, you say. I see."

I didn't know what to think. It appeared that I had lost the lovely Lucy, so gay and so spirited. And yet, at the same time, I had gained my dear Edith back. It was overwhelming.

"I will be up in a bit," I said to Bartholomew.

"Very good, sir," he replied as he took his leave.

I put on my robe and then sat down at the desk. Could it be that the entire world of 2010 had simply been an incredibly intricate creation of my imagination? Could everything that I had experienced have been my mind's interpretation of the future?

On the other hand, I considered what if what I was experiencing now was the dream, and 2010 the reality? It certainly had seemed at least as real as this room did now. How was I to know what was dream and what was reality?

But then it occurred to me that in 2010, I had read over Carmody's proof of the Goldbach Conjecture. If I retained the memory of it, then 2010 must have been real. I grabbed a sheaf of paper that lay on the desk and wrote feverishly for half an hour. When I had finished, there before me on the pages was the proof. It appeared to be correct. I saw no logical contradictions, and the arguments building on Riemann's work appeared sound. So this meant that indeed I had been in the year 2010, and my experience now was in fact the dream. So all I needed do was wake up. And yet everything around me continued to appear completely substantial.

I then realized that another possibility existed. Perhaps, the mind being the intricate instrument that it is, my subconscious had come up with the proof itself in the process of creating the dream of 2010. I oscillated between believing either of these two possibilities. My brain seemed to be spinning like a top. I cupped my head in my hands.

But finally, I calmed myself. For whichever was the reality, in either case, I was in possession of the most important mathematical discovery of the century. At that moment, I determined that as long as I remained in this time period, perhaps for the rest of my life, I would become a mathematician, my father's opinion notwithstanding. Even he could not voice disapproval when I announced my result. My very first published theorem would be the greatest theorem of the age, a proof of Goldbach's Conjecture. And perhaps I would win the Nobel Prize in mathematics, if indeed such a prize were to be endowed.

And if this other world, this world of 2010 was not just a dream, but in fact would exist in such a form over 100 years hence, I would have the satisfaction of knowing that I had robbed Carmody of his greatest theorem, and perhaps in the process prevented him from winning a Nobel Prize. And, most importantly, perhaps I would have prevented him from besmirching the lovely Lucy.

I rose from the desk and threw on some clothes, contemplating my reunion with Edith. Was she really interested in mathematics? I would ask her at once. And would we become a mathematical couple, two like-minded individuals, united through our love of all that is mathematical? I tucked the papers I had written into the desk drawer for safekeeping. Only time would answer these questions. I rushed up the stairs to meet my future.

Chapter 9
A Denial

Dear Editor,

I am writing to vehemently deny the charge that I perpetrated the murder of Évariste Galois. That these charges would be levelled at me with so little evidence speaks to either the ineptitude of your reporters or to their blatant willingness to disregard the facts when bribed to do so.

The claim that I wished Galois dead because his work thwarted my own research is pure fabrication. I have not needed to solve, nor do I see a need in the near future to solve, a quintic equation by radicals. And even if I do need to solve a quintic, I will be more than happy with numerical solutions to ten decimal places. I cannot conceive of a situation where I would need more decimal places, but if that occurs, I can obtain them to as many decimal places as necessary, thanks to Isaac Newton.

The fact of the matter is that my work does not overlap in any way with the work of Galois. I do not split fields, or construct polygons with straightedge and compass, or square circles. I am a low dimensional topologist who works mostly with Möbius bands. As such I have no use for the Galois machinery.

That I would shoot him in a duel also stretches credulity. Since both of my arms were broken in a dispute over the 112th digit of π, I would have had great difficulty in holding a pistol. Furthermore, I am terrified of firearms, and restrict myself to duels with rapiers.

That he was attempting to prevent my marriage to his sister, and I ended his life in order to further my goals in this direction, is also patently false. I have never had any romantic interest in his sister. In fact, we have never met. Nor have we been in the same room at the same time. I did not carry on a correspondence with her for thirteen years, as was posited in your article, and do not correspond with anyone other than my mother, and that correspondence is sporadic at best.

And finally, I must point out that Galois expired on May 31, 1832. As the enclosed copy of my birth certificate attests, I was born on January 13, 1973. Hence, there is a *141 YEAR GAP* between the death of Évariste Galois and my arrival

C. Adams, *Do Androids Dream of Symmetric Sheaves?*,
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upon this planet. Thus, the possibility of my involvement in his death is unlikely in the extreme. Please post a retraction to your absurd claims, or there will be consequences.

Sincerely,
Professor Kevin Winkler

Dear Mr. Winkler,

As is often the case, the person who most vehemently denies their involvement in a crime is usually the perpetrator. You have chosen to artfully dodge around the most incriminating evidence in this case.

First, you are known to be a hothead who regularly kills others in duels, albeit usually with rapiers. By our count, you are up to twelve dead in duels and another seven killed in "bowling accidents". You have strong anti-Galois sentiments, as demonstrated by the fact you are the president and treasurer of the Anti-Galois Group. Your claim that this organization is anti-"Galois group" and not a group of anti-Galois individuals is unconvincing and grammatically suspect. And finally, you did not mention the fact that after Galois' death, there was found in his pocket a note that said, "Kevin Winkler shot me. I do not know how he got here, as he will not be born for 141 years. But it was definitely him."

The Editor

Dear Editor,

The intimation that somehow I am in possession of a time machine with which I went back in time and shot Évariste Galois seems to be not just ridiculous, but in fact sublimely ridiculous. I do not own a time machine. Nobody owns a time machine. And if I did own a time machine, do you really think I would use it to go back in time to kill Évariste Gaois? Don't you think I would find better uses for it? I could go back in time and meet Isaac Newton and shake his hand and say, "Thank you so much for your numerical method. Now that is useful, especially if you need to solve a quintic." Or I could go back to visit Georg Friedrich Bernhard Riemann and ask, "Hey, what's with all the names? And by the way, your hypothesis is a doozy."

Once again, there is no basis for your accusations. If you do not desist, I will take action.

Professor Kevin Winkler

Dear Mr. Winkler,

Your denials are quickly adding up to an indirect confession. After all, you clearly have put a lot of thought into what you would do with a time machine. Ergo, you have every incentive to try to build one. And once you have one, you could visit Riemann to ask about the names, visit Newton to thank him for the method, *AND* knock off Galois. It would be an easy day's work if you had a time machine.

Did I mention that they also found a note in Galois's other pocket? It said, "Tell Riemann to watch out."

Dear Editor,

This is seriously ridiculous. Riemann was six years old when Galois died. So Galois could not have had a message in his pocket saying to warn Riemann.

Mr. Winkler,

You didn't let us finish. The rest of the message said, "Don't warn Riemann now. He is only six. Wait until he is in his forties and can appreciate the warning. Then warn him. Winkler seemed upset about the unusual number of names, whatever that means."

Dear Editor,

Okay, this has gone far enough. Do you not realize that if I did have possession of a time machine, I could go back and convince your father to have a vasectomy? Then you would never have existed, and these ridiculous accusations would disappear, poof, just like that.

Dear Mr. Winkler,

There are many things you could do with a time machine, and it seems clear you have thought of quite a few. But you don't know my father. He is the last person on earth who is going to allow a complete stranger to talk him into a vasectomy. You would need a million dollars to convince him.

Dear Editor,

A million dollars?

Dear Winkler,

Yes, that's what I said. Why?

Dear Editor,

Money's not so difficult to come by if you have a time machine. Are you still there?

Dear Winkler,

Yes, I'm still here, but I feel funny. As if my very existence was in question. My fingers are becoming translucent and can barely depress the computer ke.

Dear Editor,

I think we are done here. For you readers out there, I have made it abundantly clear that I could not possibly be involved in the death of Évariste Galois. In my next letter, I will deal with the unfounded allegations that my most well-known theorem was in fact stolen from Henri Poincaré.

Sincerely,
Professor Kevin Winkler

Chapter 10
Algebrexit

Algebra: (knocks) Hello, Math?

Math: Come in. Come in. It's great to see you, Algebra. It seems like forever since you stopped by headquarters. You know we love to see you here. How are those little Sylow subgroups of yours?

Algebra: They're fine, but I'm not here for idle chitchat.

Math: Of course you're not. You're all business all the time. That's what I like about you. I was just saying to Topology the other day, "That Algebra, there's an area that gets things done, and rigorously at that." You know, just between you and me, Topology is a little less rigorous? With the pictures and all. Anyway, have a seat. Can I get you something? A set with some interesting defining property or operation perhaps?

Algebra: No, I'm good. But let me get to why I'm here.

Math: Please do. I'm all ears. Anything we can do to help Algebra, well, you know we'll do it. You are very important to us.

Algebra: We want out.

Math: What do you mean?

Algebra: Out of the Mathematical Union. Algebraists took a vote and we want out.

Math: But, but. . .

Algebra: We're not getting the resources we deserve. We're not getting the credit we deserve. We want out.

Math: Out? But how would you function on your own? You need the rest of us to give you meaning. Algebraic topology, algebraic number theory, algebraic geometry. Algebra is inextricably linked to the rest of math.

C. Adams, *Do Androids Dream of Symmetric Sheaves?*,
https://doi.org/10.1007/978-3-031-31491-9_10

Algebra: We've made up our minds. I am not here to discuss it. The vote has been taken. I am just here to see how that split might be accomplished.
Math: This is, this is. . . unprecedented.

Algebra: It is what it is.

Math: Well, then. . .I guess if you want to talk about an exit strategy, let's talk about it. It might be helpful for you to see what it would mean.

Algebra: That is the idea.

Math: Well, to begin with, just to be clear, if you do this, you do lose algebraic topology, algebraic geometry, and algebraic number theory, among others.

Algebra: That's not fair. You certainly shouldn't get algebraic number theory. Everybody groups algebra with number theory. We even have an acronym. ANT. ANT is us.

Math: Sorry, but as you know, number theory is quite a bit bigger than just its algebraic aspects. As such, it stays with us. All the algebraic fill-in-the-blanks. They stay with us.

Algebra: Well, we're keeping homological algebra.

Math: Fine. No one else wants it. And incidentally, by the way, we keep the primes.

Algebra: What? We need the primes. The primes come straight out of algebra.

Math: The primes are universally important, and as such they do not go to algebra. If you need them, perhaps we can work out a contract to allow you to use them once in a while. And as far as the rest of the numbers go, we keep all the transcendentals and you get the algebraic numbers.

Algebra: But that's a set of measure 0.

Math: You know the word "measure"? I am impressed.

Algebra: What do you mean by that?

Math: And we keep all the imaginary numbers.

Algebra: Oh no! Those are as algebraic as they come. The Fundamental Theorem of Algebra is only true if you allow complex solutions.

Math: Maybe there's some way you could reword it. Something like, "The number of real zeros of an nth degree polynomial is at most n."

Algebra: But that is almost meaningless.

Math: You have to make sacrifices, after all, if you want to divorce the rest of math. Now, let's talk about fishing rights.

Algebra: Fishing rights?

Math: Yes, where you have a right to fish for new results.

Algebra: The mathematical seas are open to everyone.

Math: Actually, that's not true. You are limited to fishing within 100 miles of the coastline of Algebra, when no other coastline is closer.

Algebra: What do you mean?

Math: For places where the coastlines are closer together, we use a Voronoi diagram to determine where you can fish.

Algebra: I don't even know what that means.

Math: It's a term from geometry.

Algebra: I know what a Voronoi diagram is, I just don't know what 100 miles from the coastline of Algebra means.

Math: We'll make it clear the first time you fish in unauthorized waters. Now onto energy supplies. We are reasonable and we realize algebra cannot run without energy.

Algebra: So what does that mean?

Math: We will supply you with 1,000 gallons of coffee a week.

Algebra: That's ridiculous. That won't even cover the algebraists at Michigan.

Math: You can try to find other sources. There's always tea, and Coca-Cola. I hear chocolate has a bit of caffeine in it.

Algebra: You are purposely trying to hamstring us. You want us to fail so we'll regret leaving.

Math: You will regret leaving no matter what. Now onto grad students.

Algebra: What about them?

Math: Did you want some?

Algebra: Of course we want some. We get the same ones we used to get.

Math: I am afraid not. Once we split, students applying for grad school will have to apply to either math or algebra. They have to commit one way or the other while still undergraduates.

Algebra: But that's not fair. Then students who haven't decided what field to study will have to go the math route because its broader, and they will never get exposed to the upper levels of algebra.

Math: I think you are beginning to understand. And now about the breach of contract penalty.

Algebra: What?

Math: The compensatory damages for breach of contract. As I am sure you are aware, when the original contract was signed, there was a penalty for early termination.

Algebra: I never signed any contract.

Math: No, you didn't. It was the Babylonians who signed the first contract in 1733 BC. Re-signed in 830 by al-Khwārizmī. And most recently by Descartes in 1637. But it is binding until the year 2200. So there is a penalty.

Algerbra: What kind of penalty?

Math: It is a lump sum payment of 10,000 theorems.

Algebra: What?

Math: Yes, and they have to be serious theorems. None of this piddly stuff. And none of them can be in algebra, since as you know, algebra will no longer be part of math.

Algebra: How are algebraists supposed to come up with 10,000 theorems that are not in algebra?

Math: I'm guessing the answer to that is the coffee. Figure 100 gallons of coffee per theorem. With the thousand gallons per week we are providing, that would be 10 theorems a week. You should be done in 1000 weeks. Why don't you come back then? But great to see you Algebra. Stop by any time. We are always here for you.

Chapter 11
Fields Candidacy

Fields Medal Committee
Fields Institute
222 College St.
Toronto, ON M53 TJ1

Robert Bargusian
Department of Mathematics
School of Horticulture
Central State University of the East

July 16, 2022

To whom it may concern,

This letter is to officially offer my candidacy for the Fields medal. I want you to know that I would be very honored to receive this award.

And before jumping to any conclusions about my self-nomination, rest assured that I am fully aware I am an unusual choice for the medal, given my extremely limited research credentials. But allow me to elaborate on the reasons why you should give me a serious look.

First of all, you need to make sure that the wide diversity of areas in mathematics are represented by medal award winners. You have certainly covered many. Algebra? Yes. Topology? Yes. Logic? Number Theory? Analysis? Yes. Flower arranging? No. This is a critical and growing subfield of combinatorics, and not once has the Fields medal committee deemed to honor it with an award. It is time to remedy this oversight. I would be proud to represent the practitioners of this burgeoning area of mathematics as a Fields medalist.

Furthermore, as we are already on the subject of research, the Fields medal should not just be about past success. It should also be about potential. It's about recognizing how a person, having produced very little so far, and what little there is has not appeared in math journals so much as gardening or bridal magazines, must be saving up for a big push some time soon. It's embarrassing to give the medal to

C. Adams, *Do Androids Dream of Symmetric Sheaves?*,
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someone who from then on doesn't produce. You want to give it to someone who might be producing mathematics for years to come. I just might be that person.

You may also feel that I am unqualified because I am over forty years old. I am in fact a young fifty-five. And many people mistake me for being under forty, on account of how I take care of myself. And it's not just what I eat—lots of granola and fresh fruit, especially mango, I love mango—and the exercise I get. I also take care of myself emotionally. I take time out to meditate once a day, usually on the topic of the little fractures I have been noticing in my house's foundation. And I have a small dog, Lucy, who provides me with emotional support on a daily basis. When I am depressed, she sits in my lap and licks my face continuously until I smile again, which quite honestly doesn't take long. That is how you stay young. That and Rogaine. It's all in how you see yourself.

And of course, your rule about limiting the age of recipients is probably an actionable offense. I am not threatening you here, and don't want to get negative—I promised myself I wouldn't do that—but I should point out that rules, especially inappropriate rules, are made to be broken. What better way to say, "We are sorry for our blatant discrimination on the basis of age. Our shameful behavior deserves contempt. But believe us, we will never let that happen again, as demonstrated by our choice of Bob for the medal."

As a further consideration, you certainly want to pick someone who looks good in a Fields medal. As the recipient strolls by, you want observers to think, "Wow, that person sure must deserve that medal, because it looks really sharp hanging from their neck." That reflects well on you, the committee who chose the medalist.

"Why would it look good hanging from my neck?" you ask. Well, I have won a few medals in my day, mostly in elementary school, and I learned how to carry myself when I had a medal on display. It's not as simple as keeping your chin up. It's a piercing look, and a forthright step. I can't tell you how to do it, both because it takes years of practice and because I don't want to give away my secrets, but believe me when I say I have it down.

Now, some of you who are more detail oriented may be saying to yourself, hold on a minute. There is no way to hang the Fields medal from someone's neck. It's like a huge oversized quarter, with no slit in it through which to thread a ribbon. My point is that this is of course a huge mistake. Why would anyone want to win a medal if nobody can see that you won it? You're going to keep it in the closet with your tangled fishing line and your weasel traps? I think not. So although it will involve a bit of drilling, it would be my intent to fasten the medal around my neck with a colorful ribbon that would draw attention both to it and to me.

Another advantage to choosing me is that I am not a candidate who is likely to turn the medal down, once offered. That can be quite embarrassing, as you all too well know. But in my case, you needn't worry. I am telling you right now up front, I want the thing. Bring it on. I will accept!

I realize the competition will be stiff. Since the medal is only given out every four years, there are others who may have proved a really big-league theorem or created a whole new field of mathematics. But keep in mind that winning this award may distract them from what they should be doing, which is working on math.

All the invitations to speak, the receptions with the scallops wrapped in bacon, the honorary degrees, will interfere with their ability to get math done. The last thing you want to be responsible for is destroying the budding career of a promising young mathematician. No problem there with me.

Some of you may be saying, under your breath when no one else is listening, "What do you have to offer in return? If I bestow on you this great honor, it's only fair that you should provide me with some sort of recompense—a 'good will gift'."

What I offer is help with your flower arranging needs—individual consultations and original arrangements using the most up-to-date algorithms to maximize the aesthetic impact. If you do this for me, believe me, your weddings, anniversaries, special occasions, and just everyday flower arrangements will be the envy of your colleagues.

And what are the consequences if you do not choose me? Although the membership of the Fields medal committee is a secret, having made a few promises, and arranged a few flowers, I do know who you are. And although my research production has been limited, I am nonetheless quite well connected in the math community. I have the capacity to make your lives quite miserable. A job offer abruptly withdrawn due to questions of character. Students marching in protest over remarks that you don't remember making. Flower arrangements at the banquet in your honor that are lopsided and color uncoordinated. I'm not saying I would be the cause of those things, as that would be self-incrimination, but I just wanted to put that out there.

But let's not end on a negative note. Let's end by thinking about what the Fields medal represents. It is an award that should stand for all that is good and beautiful about mathematics. It is a beacon, setting a standard to which we can all aspire. Even me.

Thank you for your consideration.

Sincerely,

Robert Bargusian
Department of Mathematics
School of Horticulture
Central State College of the East

Chapter 12
Leonhard Euler and the Seven Bridges of Königsberg

Once upon a time, a small boy was born in the town of Basel, Switzerland. His parents, impressed by the intelligent look in his eyes, named him Leonhard Euler, after his great uncle Leonhard, who had been smart enough to marry a countess and now lived in a castle, giving him the right to treat his relatives like dirt.

Due to some confusion about the ownership of a loaf of rye bread, Euler's family relocated to the secluded town of Riehen, down the river from Basel. Compared to Basel, Riehen was a backwater, with fewer brawls, demonic possessions, or other family entertainments. So, to occupy themselves, couples would stroll about town. Although not generally known for their competitive nature, the townspeople soon began to consider ways to outdo one another in their strolls. The Mandelbaums skipped as they walked. The Feidelhofers took their stroll walking backward, nodding as they passed their acquaintances. Not to be outdone, the Lalleputers hopped on one foot for their entire promenade. Finally, the Blandasmoths suggested the ultimate challenge, to take a stroll that crossed each bridge exactly once.

At the time, no one knew whether or not it could be done. Young Euler, just two at the time, came to know of this problem through his babysitter. She would often deposit him in his pram with his rattle, ostensibly to take a stroll, but actually to meet her boyfriend, who looked quite fetching in his lederhosen. As they stood on a bridge overlooking the river Wiese, Little Euler, who had little interest in rattles or lederhosen, overheard the conversations of the passing strollers and quickly grasped the problem. Doodling with his saliva on the inside of the pram, he painted each bridge as the edge of a graph connecting vertices that represented the land masses. Luckily, he was a substantial drooler. He quickly realized the solution.

"Gertrude," he called to his babysitter, who was canoodling with her boyfriend, "you must run at once to the mayor and inform him of my solution." He pointed to the saliva dripping down the inside of his pram. "For you see, there is only one bridge in Riehen and therefore no one can ever succeed. The only way home is to cross the same bridge twice."

C. Adams, *Do Androids Dream of Symmetric Sheaves?*,
https://doi.org/10.1007/978-3-031-31491-9_12

That very day saw the birth of topology. Not in that town. Actually seven towns over, a boy playing with a worm invented knot theory. Unfortunately, the goat he was riding tripped into a moat and topology was still-born. It would be some time before it was reincarnated.

But in the meantime, Gertrude, astonished at the acumen of her tiny charge, raced off to the mayor's office to explain the solution. The mayor and all of the town officials were so thrilled to know the answer, they rewarded her with a huge wheel of Appenzellar cheese. She used it for her dowry and disappeared to Basel with her boyfriend, never to be seen again. It wasn't until a day later that the parents of Euler took an evening stroll and discovered a disgusted little Leonhard still in his pram on the bridge.

Word of the young prodigy's solution quickly spread throughout the land. People whispered to one another, "Little Euler has determined that one cannot take a stroll through Riehen and return home, crossing each bridge once." As the message was passed along, it became garbled, and transformed into, "Turnips do not borrow Lucifer's nightcap in Uppland." This caused quite a bit of confusion, but eventually, it all got sorted out and residents from towns as varied as Untholm, Masterdol, and Winterflagen soon streamed to Riehen to implore Euler to determine for them whether or not one could traverse all the bridges in their towns once in an evening stroll.

After a few disastrous attempts, petitioners came to realize it was too difficult to bring the bridges to Euler, so instead they would draw a picture of their town and the bridges connecting the various islands and banks. Upon seeing these rudimentary maps, Euler would laugh his high-pitched childish laugh.

"Silly townspeople," he would say. "You need not draw these elaborate depictions of your towns for me. I need only a graph with edges that represent the bridges and vertices that represent the land masses. Just use your saliva to draw these pictures on the inside of a pram and bring it to me." And so was born the mathematical field of graph theory, or at least a rudimentary saliva-based version of it.

Upon the arrival of a drool covered pram, Euler would spend several hours studying it, and consider various paths that might traverse each edge exactly once. Then he might do a few calculations. Finally, he would make a pronouncement about his results. Sometimes his news would be good, as for the town of Masterdol, which was built on the two sides of a river connected by two bridges. Once having been made aware of Euler's solution, the townspeople could leisurely saunter first across the one bridge to the far side of the river and then back again across the other bridge, arriving at home just in time for a brandy nightcap.

But sometimes the news was bad, as happened to the town of Winterflagen, where Euler explained that yes, in fact, you could take a stroll and cross every bridge exactly once. However, to do so meant that you would finish on the opposite side of the river from whence you had begun. So you should be sure to bring a sleeping roll and plenty of victuals as there would be no going home that night. Many a distraught townsperson was compelled to move from their home in Winterflagen to take up residency downriver in Masterdol, where the stroll was possible.

And so the young bridge-stroller problem solver spent his early years solving bridge-stroller problems with ever greater numbers of bridges. He handled three-bridge problems and four-bridge problems. And when he solved his first five-bridge problem, his fame grew to substantial proportions.

Not satisfied with the level of difficulty of the problems that were brought to him, Euler began to travel to municipalities a greater distance away. Towns up and down the Wiese River both hoped and feared for a visit from Euler. For if he determined that the stroll was possible, the town experienced tremendous economic benefit from the hoards of tourists that descended on the town, eager to try the new walk. But if a town was unfortunate enough to have no such stroll, it meant economic strangulation, as no one desired to live there any longer.

Then one day, a delegation arrived from the city of Königsberg.

"Great Euler," said the mayor. "We have heard of your amazing abilities. We need your help immediately. For we have numerous townsfolk who have already crossed bridges once, and are now trapped in strange parts of the city, unable to return home without crossing a bridge a second time. And many are the couples that began a stroll attempting to cross each bridge exactly once and ended up in France.

"Other townspeople, fearful of a similar fate, are forgoing an evening stroll altogether. This is tearing asunder the very social fabric of the town, as the public discourse that occurred amongst the various strollers has vanished. And married couples are remaining indoors, thereby increasing the population at an unsustainable rate. Please come to help us solve our bridge-stroller problem."

"And how many bridges have you in Konigsberg?" asked Euler.

"Not so many," replied the mayor evasively.

"How many is not so many?" demanded Euler.

"There are seven bridges in Königsberg," admitted the mayor with downcast eyes.

A gasp went up from the crowd. For seven bridges was a greater number than Euler had ever tackled before. People whispered that no one could possibly solve a seven-bridge problem. Euler waved his hand to silence the crowd.

"All right," he said. "I will come to Königsberg to solve your problem." A cheer went up from the crowd.

And so, Euler set out for Königsberg, in what was then Prussia. This flourishing commercial town was located right at the elbow of the Pregel River. The situation was complicated by the fact the river split at Königsberg, and the city spanned both sides of the river as well as an island and the region between the branches of the river. Here was a challenge worthy of the great Euler.

Upon arriving in town, he saw the pockets of people unable to return home because they had already crossed bridges once. They cried out to him, "Please, great Euler, solve our conundrum that we might return home to feed our starving children."

Euler called back to them, "I just got here. Give me a break."

As he often did when solving a new town's bridge-stroller problem, Euler obtained a small wooden stool and placed it on the main bridge overlooking the

river. He sat and sat and sat, his chin cupped in his hand, watching the water flow under the bridge. Presently, a beautiful young maiden approached him.

"Pardon me sir," she said. "But I am lost. I know not the city of Königsberg. I have been out strolling for six hours, and I have not crossed the same bridge twice, but I cannot seem to get home."

Euler looked up at her with surprise. For it were as if he had been hit between the eyes with a large stump. He sprang up and grabbed her by the shoulders.

"That's it," he shouted. "Of course. You cannot get home because the vertices are all of odd degree."

"Degree? Vertices?" she said nervously.

"Look," said Euler, slopping some of his saliva on the bridge wall. "Here is the graph that represents the town." He pointed to one of the vertices.

"You see, with an even number of edges, whenever you enter a vertex, you can always leave. But with an odd number of edges at a vertex, you can enter and leave that vertex until you are down to one edge. And then you can enter, but you can never leave. Or you can leave but never re-enter. So if the goal is to return to your home, you can never succeed if there is any vertex of odd degree. And if your goal is to take a stroll crossing each bridge once, and you do not care if you return home, then you must have two vertices of odd degree. But for Königsberg, all four vertices are of odd degree. So there is no such stroll in either case."

"I see," she said, a disconsolate look clouding her pretty brow. "So I am trapped here."

"So it would seem," replied Euler. "But perhaps we can make the best of it."

And so the couple was married, and they built a house right there on the main bridge over the Pregel River, and they lived happily ever after. Well, at least on and off. And when Euler's negative solution of the Konigsberg bridge stroller problem was announced, it actually was the birth of topology, or at least a rudimentary saliva-based version of it. And Euler was celebrated by many, and they called him the master of us all. At least until that kind of language was realized to be inappropriate.

Chapter 13
The Great Storm

This is a story that dates back to 1901. Way before your time. And if you can believe it, back to when I was young. Yes, I was young like you once. Hard to believe but true. And when you're young like that, you feel invincible. That's how I felt. Invincible. And lucky.

It was summer, and I was about as happy as a symbol could be. I was a newlywed, still grinning ear-to-ear, having just gotten hitched the month before. Epsilon and I had met our first week of college. She was in my calculus class. Smart and funny and free-spirited, not like those constants I dated in high school. I knew right away she was the one for me, and she felt the same. Epsilon and Delta. It had a ring to it. We waited until graduation and then we tied the knot, a trefoil as it happened. No surprise, it was a big Greek wedding, with Lambda and Phi as the best man and maid-of-honor.

Come the end of summer, Epsilon and I both had jobs lined up in analytics, working in derivatives. The future looked bright. And we were in love, with a whole summer to do with whatever we wanted. What could be better than that?

It's rare to experience a summer's day quite as perfect as that day was. The sky was cloudless, a slight breeze rustled the leaves on the trees, and it was an even 70 degrees (1.22 radians).

Epsilon and I had gone to the Math Fair, one of our favorite events of the year. There were trig rides and bobbing for 3-disks and lots of little corollaries holding buoyant spheres by strings.

Epsilon was getting ready to throw a vector at a 2-disk, hoping to win a stuffed Klein bottle when I noticed a minor lemma looking off to the horizon, her mouth hanging open. I turned to see a dark roiling cloudbank headed in our direction.

"Epsilon, is there supposed to be a storm today?" I asked.

"Nope," replied Epsilon, as she took aim.

"Um, I think there's one headed this way. Maybe we should take cover." I could see lightning flashing out of the ominous clouds in the distance.

C. Adams, *Do Androids Dream of Symmetric Sheaves?*,
https://doi.org/10.1007/978-3-031-31491-9_13

"Hang on, Delta," Epsilon said as she released the vector. It embedded itself right at the center of the 2-disk.

"Ha, I win the stuffed Klein bottle."

"Forget the stuffed Klein bottle," I said as I pointed. "We need to get out of here."

The wind had picked up as the menacing clouds churned in our direction. At that instant, a bolt of lightning shot out of the sky and struck the cupola of the Math Building, which, as you might expect, was the tallest building around. Several huge timbers flew off and crashed to the pavement below.

"Yikes!" exclaimed Epsilon. "Did you see that?"

"Come on," I said. "We need to find cover." But as we turned to go, I noticed a lone corollary sitting on the ground crying.

"Hey, little feller," I said as I stooped down, "where's your theorem?"

"I don't know," he cried. "I lost her."

"Don't worry," I said, "we'll take care of you. What's your name?"

"I'm 'Angles subtended by the same chord are equal"', he snuffled.

"Oh, I know your mother, 'Central angle subtending chord is twice angle subtending chord.' Come with me for now. We'll find her." I took his hand as he stood up.

"Will she be all right?" he asked timidly.

"Of course," I said. "She's not the kind of theorem that needs to worry about a minor logical cloudburst. She'll be safe as can be." But Epsilon and I exchanged nervous looks as the skies continued to darken.

Suddenly, another lightning bolt shot out of the sky into a grove of graphs, hitting a tree with a crack. A large branch broke off and plummeted to the ground. An unwary 3-manifold, who had retreated under the tree for protection, cried out as he was knocked to the ground and pinned down by the branch.

"Oh, please, someone help me," he pleaded weakly.

"Epsilon, get 'Angles subtended by the same chord are equal' to safety," I yelled over the wind. "I'm going to try to help that manifold." Two Sylow subgroups had rushed over but didn't have the strength to lift the branch. It had to be at least seven edges long.

"But Delta. It's too dangerous," replied Epsilon.

"I'll be fine," I said. "And you have to get the corollary somewhere safe." I squeezed her hand, and she nodded. Then I turned and sprinted to help the others.

The subgroups only spoke algebra. "We need him to be free," said the first. "We must remove this relator."

"What are they saying?" asked the manifold, trapped under the branch. I translated into topology. "They want to lift this simplical complex to a finite cover."

The manifold nodded. I could see he was badly hurt, with points leaking out of a breach in his boundary.

"I think I have a collapsed CW-complex," he said.

"What did he say?" asked one of the Sylow subgroups.

"He said that he's had his quotient taken."

The subgroup nodded gravely.

"Am I still orientable?" asked the manifold feebly. "I don't feel orientable."

"Oh, I'm sure you're still orientable," I reassured him. But Hausdorff? That was another matter.

The subgroups and I took hold of the branch and with a collective grunt, we lifted it off him and set it on the ground. Torrential rain poured down upon us. I could only hope that Epsilon and 'Angles subtended by the same chord are equal' were safe.

"Grab him under that embedded surface there," I yelled to the subgroups over the wind.

"Eh?" said the first subgroup, looking confused.

"By the surface subgroup right there, grab him there."

They nodded, and we lifted him and carried him over to a wagon.

"What's your name?" I asked the manifold as we got him situated.

"I'm (4,9)-surgery on the figure-eight knot," replied the manifold.

"Pleased to meet you," I said.

I turned to the subgroups. "We need a monomorphism of him into the hospital." They nodded.

"What did you say?" asked (4,9)-surgery on the figure-eight knot.

'These two subgroups are going to embed you in a medical ambient space."

"You're not coming?"

"No, I have to find my wife. But you'll be fine."

"Thank you," said (4,9)-surgery on the figure-eight.

I waved and then sprinted in the direction that Epsilon and 'Angles subtended by the same chord are equal' had taken. The wind was blowing so hard the rain stung me like a collection of conical limit points. Issues of journals lay strewn about in the mud.

But then I stopped in my tracks. For ahead of me, where what should have been a small dip in the terrain, was a river of symbols. Whole propositions with multiple hypotheses and formulas involving as many as six levels of subindices bobbed by. And on a small logical outcropping in the middle of the river I saw Epsilon and 'Angles subtended by the same chord are equal' crouched down. She had her arm protectively around his shoulders.

"Epsilon," I called out. She saw me and waved.

"Are you okay?" I yelled.

"Yes," she shouted back, "but the river is rising."

"Hang on," I yelled.

I scanned the immediate area and then spied just the thing.

"Okay," I yelled. "I am going to row out on this axiom, and bring you to safety."

"What axiom is it?" yelled Epsilon back.

"It's the Axiom of Choice."

"Oh, that's a good axiom. Hurry, Delta. The river's rising fast."

I slid the axiom into the river and grabbing a nearby vinculum, I climbed on and started rowing. The current was strong and symbols crashed against the axiom but it held together. They don't make axioms like that anymore.

I reached the small outcropping and Epsilon grabbed hold of the edge of the axiom. The river of symbols was now up to our ankles.

"There's not enough room on the axiom for all three of us," said Epsilon. "You have to take 'Angles subtended by the same chord are equal' first."

"No," I said. "I can't leave you here."

"You have to," she said. "You know you do." I hesitated and then nodded. 'Angles subtended by the same chord are equal' clambered onto the axiom.

"I'll be right back," I said to Epsilon, as I began to row. The current had become stronger as the river swirled around us.

"Hold on," I said, as I put my back into it. We finally made it to the shore.

"Wait here," I said, as I turned back toward Epsilon. I could see she was up to her waist in the river now. I rowed as hard and fast as I could, but it felt like I was making no forward progress.

"Epsilon, hang on!" I cried. As I was rowing, I could see the river rising up toward her shoulders. We both knew I wasn't going to make it in time. Waving, she yelled, "I love you" and then she disappeared below the raging symbols.

I leaped off the axiom into the river and swam to where I had seen her. Symbols buffeted me on all sides. I dove down under the surface and felt her hand. Grabbing hold, I pulled her upward.

"Epsilon," I cried, as we broke the surface. She gasped. "Swim," I yelled. "Come on, swim!"

I held onto her as we pushed against the current. Whole fields of mathematics floated by, disappearing around the bend. Uncountable sets bobbed up and then disappeared again. We struggled as best we could.

"It's okay, Delta," said Epsilon, exhausted. "Let me go. Swim for yourself."

"Never," I insisted. "I'm nothing without you. Everybody knows that! We're going to make it." From somewhere deep inside, I pulled up a last bit of energy, and kicking hard, I pulled us to the shore. We struggled out of the river and collapsed on the bank.

When I regained consciousness, the storm had passed. I lifted my head out of the mud and looked over at Epsilon. She looked back.

"I love you, Delta," she whispered.

"I love you too," I replied.

Around us, the aftermath of the storm was apparent. There were split links, non-locally connected spaces, wildly embedded spheres, and perverse sheaves.

Eventually the two Sylow subgroups found us on the bank and they transported us and 'Angles subtended by the same chord are equal' to the hospital. There we found the corollary's mother and witnessed their tearful reunion. And we saw (4,9)-surgery on the figure-eight knot, who had survived the ordeal. Although it took a lot of physical therapy, he was eventually able to return to work. He wasn't exactly a manifold anymore, but from a distance, you could hardly tell.

Epsilon and I had sustained surprisingly minor injuries, and we were released that day. And as you know, we went on to exciting careers and full lives, lives that included giving birth to your parents, who then went on to give birth to you.

Later, we found out more about what had happened that day. The storm had begun as a minor logical paradox discovered by the mathematician Bertrand Russell.

Just a small contradiction in the basic underpinnings of mathematics. If a set can be a member of itself, then what about the set of all sets that are not members of themselves? Is it a member of itself?

But as everyone began to grasp its true significance, the paradox grew in size and ferocity until it became a massive logical storm, shredding papers in its path and undermining the very foundations of mathematics. No sub-field was safe. Propositions were tossed around as if they were lemmas. Proofs were torn asunder. Elements lay in piles around the sets that had once contained them. Many believed math would never recover. It was the end days.

But, as you know, mathematics did recover. Eventually, mathematicians figured out how to work around it. And in the end, mathematics was stronger for it.

Since that time, so long ago, the great storm has faded from memory. The math community has forgotten the deadly force that a malignant counterexample can wield. We have become complacent. But when it comes to math, there are always unexpected surprises. So be wary. If you see ominous clouds appearing out of a clear blue sky, and the wind picking up suddenly, don't hesitate. Start running for shelter. You are going to need it.

Chapter 14
All Tied Up

(*Francesca and Felix are each tied to chairs. Francesca slowly regains consciousness and looks around confused. Looks down to realize she is tied to a chair.*)

Felix: (*whispering*) Welcome back to the world of the living.

Francesca: Felix! Where am I? What is going on? Why are you here?

Felix: Shh. He'll hear you.

Francesca: (*loud whisper*) Who will hear us?

Felix: Larder.

Francesca: Your grad student Larder?

Felix: Yes.

Francesca: Is he somehow involved in this?

Felix: He is this.

Francesca: What do you mean?

Felix: He's the one who drugged us, kidnapped us, and tied us up.

Francesca: Larder? He's a math geek, not a kidnapper.

Felix: Turns out those two sets aren't disjoint.

Francesca: But what possible reason would he have to kidnap us?

Felix: I don't know, but I think the stress of working on his Ph.D. made him crack. He's a few lemmas short of a theorem.

(*Francesca struggles with her bonds, trying to work her hands free.*)

Francesca: Felix, see if you can get free before he comes.

C. Adams, *Do Androids Dream of Symmetric Sheaves?*,
https://doi.org/10.1007/978-3-031-31491-9_14

Felix: Believe me, I tried.

Francesca: But good grief, you're a knot theorist. If anyone should be able to get out of this, you should. The rope is a trivial knot after all. The ends are unattached. We're actually both untied, if not for a little friction.

Felix: I'm more theory than applied. And remember, Larder's my grad student, so it's no surprise he knows how to tie a good knot.

Francesca: (*disgustedly*) Once again Felix, you're a big disappointment.

(*The door to the room swings open, and Larder walks in, smiling.*)

Larder: Hello, Professor Ramirez. Glad you could join us.

Francesca: (*angrily*) Larder, what the hell is wrong with you? Untie us at once.

Larder: (*calmly*) That's not how this is going to work. You don't go anywhere until I say you do.

Felix: Humor him. Let's see what he wants.

Larder: That would be smart. Otherwise, you could be here a very long time.

(*Francesca struggles with her bonds a moment and then groans in resignation and glares at Larder.*)

Francesca: Okay. What do you want? Why have you kidnapped us?

Larder: It's very simple. You two are the world's experts on knot theory and operator algebras, respectively. Just what is needed to tackle the Schnitzel Conjecture. It's been open for 47 years, and I have decided to solve it by bringing the two of you together. You will collaborate and solve it or you will die tied to those chairs, having starved to death.

Francesca: You can't be serious. That's one of the most difficult unsolved conjectures in all of mathematics. And you expect us to collaborate and just like that, find a solution?

Larder: Yes, that is what I expect.

Francesca: I see. And if we do? You claim the proof as your own and knock us off?

Larder: Oh, I don't want any credit. You two would get the credit. I would just know I have, in my own small way, advanced mathematics forward.

Francesca: And what on earth makes you think that we have the expertise to solve the conjecture?

Larder: I happen to know that you and Professor Wurzel were working on it and you were close.

Felix: I'm sorry, Francesca, but I once told him about that.

Francesca: (*Gives Felix an angry look*) That was a long time ago. And Felix and I no longer collaborate.

Felix: He knows we were married. And he knows it ended badly.

Francesca: Then he knows I want nothing to do with you under any circumstances.

Larder: Any circumstances is a bit strong, don't you think? These circumstances are rather unusual, so I think you will make an exception.

Francesca: Listen to me, you nitwit. He was secretly collaborating with another operator algebraist while we were married. Do you understand how hurtful that was?

Larder: Do you understand the precariousness of the situation you are in? I think you had better get over your issues with Professor Wurzel and get to work.

(*Larder turns and leaves the room.*)

Francesca: Felix, he's your grad student. Order him to release us at once.

Felix: I'm sorry, Francesca, but he's not listening to me. I think our best bet is to try to solve the conjecture. Then he'll let us go.

Francesca: Are you serious?

Felix: Yes. When we were working on it, we both believed we were close. If our marriage hadn't disintegrated, I think we would have solved it.

Francesca: You're forgetting the difficulties we were having with the irrational knots. We could do the rational ones, but just not the irrational ones.

Felix: I have been thinking about that. It seemed to me we could approximate them by a superposition of rational knots. But of course, my background in operator algebras is not advanced enough to finish off this approach.

Francesca: (*looking off into the distance in thought*) I suppose you could apply a Vandertramp transformation to tighten the relationship, but you would still have trouble with composition.

Felix: Well, composition means there's an essential annulus. I bet we could use that to our advantage.

(*They continue to talk quietly, the lights go down, then come up again after a few moments. In walks Larder.*)

Larder: Good morning. Any progress?

Francesca: (*looking at him with disgust*) We solved it.

Larder: You did?

Francesca: Yes, a couple hours ago.

Larder: (*to Felix*) Did you really solve it?

Felix: (*smiling broadly*) Yes, we figured it out. Untie me.

(*Larder immediately unties Felix who then stands up, rubbing his wrists.*)

Francesca: Untie me!!!

Larder: Sorry. That's not part of the plan.

Francesca: What do you mean??? Felix, untie me!

Felix: I'm sorry, but in fact you will not be a co-author. This will be my result.

Francesca: What's going on here? Felix, what have you done?

Felix: Look, Francesca. You know I've wanted to solve the Schnitzel Conjecture for years. But we both know that if I had asked you to collaborate with me again, even if just long enough to solve the conjecture, you would have said no. So, I devised this plan.

Francesca: (*glaring at him*) Felix, you are just the sleaziest of the sleazy. But you won't get away with it. I'll tell everyone what you've done.

Felix: Hmmm. It seems that presupposes an opportunity on your part to report about this little incident. Larder, go get the gasoline.

(*Larder turns and leaves the room.*)

Francesca: Felix! Have you gone insane? We don't get along, but this?

Felix: I don't think you have ever understood how important math is to my life, Francesca. Your lack of understanding is why our marriage failed. All my decisions are based on my relationship with math.

(*Felix walks over and leans down, putting both hands on her arms where they are tied to the chair, and brings his face down close.*)

Felix: I still love you, you know. I just love math more.

(*As he bends in to kiss her, she kicks her right foot up as hard as she can and catches him between the legs. He shrieks and falls to the floor. She stands up with chair still attached, turns around and comes down on top of him with the chair and all her weight. He groans as the chair splinters. She grabs one of the arms of the broken chair as Felix writhes on the floor. Larder enters carrying the canister of gasoline and freezes when he sees the scene.*)

Francesca: (*brandishing the chair arm*) Listen to me. If you want to live more than the next five minutes, you will call the police and get them over here right now.

Larder: (*pointing at Felix*) It was all him. He told me he'd make sure I never got a Ph.D. unless I did exactly what he said. He blackmailed me. And besides, in the end, it's all good. You solved one of the biggest open questions in mathematics.

Francesca: Oh, we didn't solve it.

Larder: But Professor Wurzel said you solved it.

Francesca: Yes, and he believed we did. But he's lousy at operator algebras. I just convinced him it worked, so that he would convince you. But the idea never had a chance. I realized that about three months after Felix and I split up. This approach was a dead end. Now get moving.

(*She waves the chair arm as they both exit, leaving Felix moaning on the floor.*)

Chapter 15
The Adventures of Robin Caruso

Being the Adventures of one Robin Caruso, a Mathematician who was wash'd up on an Island where he was forc'd to spend many a Year.

This Tale that I tell you is the true Story of the Adventures of my Life, including a long Sojourn upon an Island far in the Caribbean, well beyond the reach of Civilization as we know it. Altho' the Events that I relate occurr'd over many a Year, the Tale itself is not nearly so long. But let me begin my Story at the Beginning.

I was born in a small Town in the midwestern Province of a large and industrious Nation. My Childhood was uneventful, and my Schoolwork uninspir'd. I found most Subjects quite dull, and did not experience the suppos'd thrill of interacting with Children of my Age, either of my Gender or of the Other.

But one day, when I was Twelve, our Mathematics Instructor happen'd to explain to us how as a Child, Gauss had summ'd all the Numbers from one to one hundred in just a few Seconds. I alone found this Story fascinating. Over the next few Months, I taught myself how to add longer and longer Strings of consecutive Numbers in extremely short Periods of Time, and in the Process, manag'd to alienate all the other Children in my Class. Soon, my Interests spread beyond Addition to other aspects of Mathematics, and the day-to-day World began to appear drab in comparison. My Soul thirst'd for more Mathematics.

Finally, at the age of Seventeen, I cou'd stand it no longer, and I left my Hometown and travel'd many miles to a large University in a neighboring State. I had heard they did amazing Mathematics there.

But once I had sign'd on, I found myself requir'd to take Courses in which I had no Interest. And they bill'd my Parents for the Privilege of it. Where was the aesthetic Beauty of Mathematics? Where were the brilliant Insights and the awe-inspiring Theorems? After an insufferable Four Years, I finally managed to escape with a Piece of Paper expounding my Abilities in what I consider'd to be Elementary Mathematics, and with a Partner who had herself sign'd on to become my Spouse.

I subsequently utiliz'd the Diploma to gain Entrance to a Graduate Program in Mathematics at an even better-known University of Higher Learning in an even

C. Adams, *Do Androids Dream of Symmetric Sheaves?*,
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more-distant State. There I finally did see some of the Mathematics that I had long'd for; Theorems that requir'd true Insight, and abstract Mathematics that took substantial Effort to conquer. And I was finally given the Chance to make my own Contribution. I craft'd a beautiful Thesis that prov'd Facts no one had previously realiz'd in the entyre History of Humankind. But much to my Chagrin, on the Basis of that Thesis, I was informed that it was Time for me to leave. I was deemed ready to begin my Research Career.

I land'd a Job at a small liberal arts College where I arriv'd the next Fall with my growing Family, three Boxes of Books and boundless amounts of Energy and Enthusiasm for the Mathematics I wou'd shortly be discovering. But I soon learn'd that my idealiz'd view of Academia was far from the Reality. I was sign'd up to teach Six Courses a Year, with hundreds of Students, many of whom were taking the wrong Course for worse Reasons, with little Background and even less Motivation. I cou'd no more find Time to do Research than I might juggle Spivak's Collection of Differential Geometry Books.

But at least during the Summers, I cou'd carve a small amount of Time for Research. I try'd to keep active, and in my spare Time, I took to carrying around Whitehead's thick Tome, "Homotopy Theory." When I was waiting for my Children at the Dentist's Office, or bound by Traffick, I cou'd read a Theorem or Two, and then cogitate upon it.

After Five Years, and a hard-won Handful of Publications, Frustration was looming large. Tho' Tenure seem'd likely, it was not clear to me that this was the Path I desir'd for my Life.

Nearing the End of the subsequent Fall Semester, I looked forward to having Time for Mathematics during the much too short Winter Break. But over Thanksgiving, we visit'd my In-Laws, and my Father-in-Law announc'd to all present that he and his Wife were taking the entyre extend'd Family on a Caribbean Cruise over the Holiday Break. For me, the Idea of such a cruise was Anathema, but my Wife and Children made it clear that saying no was not an Option.

This was how I found myself aboard a Festival Cruise Lines Ship known as the Sensation. And Sensation it was, but of the unpleasant Variety. There were a Multitude of Activities, including Water Slides, Pools, Sauna, Bingo and Floor Shows. Everywhere you went there was pounding Music, gyrating Lights and Wait Staff attempting to convince you to buy multi-color'd Drinks. It was a Nightmare on Water.

Needless to say, I often claim'd Seasickness and slipp'd away to peruse Mathematics. One evening, half way through the Voyage, I manag'd to escape to the Stern of the Ship, which was one of the few quiet Spots I had discover'd on board. Enjoying a soft Breeze, I lean'd against the Rail, cogitating over Whitehead. Unfortunately, a Boy of perhaps Fifteen Years appear'd, and much to my Annoyance, began performing Tricks about the Deck on his Skateboard. He was wearing Earphones plugg'd into some Kind of Musical Device.

"Go away," I snapp'd, but he did not heed me. I try'd to ignore him and return'd my Attention to my Book. But then, as he was attempting to land on the spinning Board, he lost his Footing and tripp'd, slamming into me and smashing me hard up

against the Railing. The Book was knock'd from my Hand, and land'd just on the Edge of the Deck on the Far Side of the Rail.

"You Miscreant," I yell'd, as I lean'd over to grab the teetering Book. Just as I had stretch'd far enough to grasp it in my Hand, the Teen push'd off me as he skateboard'd away. My Foot slipp'd on the wet Deck, and I found myself balanc'd precariously on the Rail for just an Instant before tumbling over the Side, down, down, down toward the black Water churning behind the Ship. The Shock of my Impact with the Water took my Breath away. When I resurfac'd, spluttering but with the Book still gripp'd in my Hand, the Boat was already steaming away.

"Help me," I scream'd, waving the Book, but to no avail. Surely, I thought, the Boy wou'd report the Incident. But as the Ship reced'd into the Distance, I realiz'd it was not to be. Either fearing the Repercussions, or more likely oblivious to the Results of the Collision, the boy wou'd not be reporting my Disappearance over the Side.

This is so unfair, I thought to myself, as I tread'd Water. My mathematical Career had hardly begun, and it is about to end. Above me shone the Moon, and for lack of a better Plan, I began to swim in its Direction, using the Book to pull myself along. After Fifteen Minutes, I stopp'd and paddl'd in Place, worrying whether this particular Part of the Ocean was home to giant Squids or other Monsters of the Deep that might grab me by the Legs and pull me into the Depths.

Near the Horizon, straight ahead of me, I notic'd that the Stars were not visible, block'd out by Something, perhaps an Island rising out of the Ocean. I continu'd to swim in that Direction 'till I cou'd hear Waves breaking on a Shore. With rising hope, I forc'd myself to push on. Eventually, as I was nearing total Exhaustion, a Wave caught me up, pulling me forward as it suck'd me under. I desperately try'd to deliver myself from its Grasp as I gulped down mouthfuls of salty Water in futile attempts to obtain Air. Suddenly, it spit me out on a sandy Beach. Crawling from the Water, I collaps'd on the Sand and fell into an exhaust'd Slumber.

The next Morning, I awoke, my Face encrust'd with Salt and Sand. In my Hand, I still had a firm grip on a soggy Whitehead. I pull'd myself to my Feet. The Beach stretch'd for a half mile in either Direction before disappearing around the Edge of the Island. It was border'd by dense Jungle and was apparently untouched by human Hands. I realized that I was completely alone here. I had only the wet Clothes upon my Back, one very damp Book, and neither Food nor Water to Drink. I wou'd shortly perish of Thirst or Hunger.

"Oh, Whitehead," I exclaim'd, "It appears we have survived the Ocean only to be dealt a lonely Death upon this wretched Island."

As I confront'd my dismal Circumstances, I began to plod slowly down the Beach along the Water's Edge. I shortly came upon a Stream crossing the Sand and emptying into the Ocean. Leaning down, I cupp'd a bit of Water to taste. It was fresh. This was a very good Sign. I drank greedily. Once sated, I look'd up at the nearby Palm Trees and saw a profusion of Coconuts dangling from them. Perhaps I cou'd survive here. Not all was lost. But wou'd I go crazy with Loneliness? Wou'd I miss Human Companionship?

And then, slowly, it dawn'd on me that my Dream had come true. Here I was with plentiful Food and fresh Water, good Weather, and nothing but Time on my Hands. I cou'd do Mathematics from Dawn to Dusk every Day. It was an incredible Realization. Whitehead wou'd dry out, and there was enough Mathematics in him to last me many a Year. I cou'd expand upon it, develop the Field, prove new Theorem after new Theorem, ad infinitum.

"Whitehead," I said enthusiastically, "this might work out after all!" Whitehead grinn'd back at me with his bright yellow Cover.

But I wou'd need Something upon which to write. As I consider'd this, I pok'd at the Sand with my Foot.

"That's it, Whitehead!", I exclaim'd. "I can write with a Stick in the Sand. It is the perfect Pad. Temporary Work can be done below the Tideline, and the permanent Work I can place up higher. There must be Miles and Miles of Sand here. I can work for Years!" I was ecstatic.

And so it began. Each Morning, after a breakfast of Bananas and Coconuts, wash'd down with cold fresh Water, Whitehead and I wou'd head down to theWater's Edge where we wou'd spend an Hour or two discussing Mathematics. Then I wou'd think how I might push the Arguments further. I wou'd tinker and make Notes down nearer theWater as the Tide went out and then I wou'd save my Nuggets, my interesting Facts, and my clever Observations up by the Tree-line. Eventually, I wou'd box off the very good Stuff, the Stuff I knew was publishable. Then every four Days, I wou'd go over it with my Stick, just to make sure the Wind did not erase any of the Results, a potential Loss not just to me, but to Humanity.

I took breaks to spear Fish, to hunt and to collect Fruit. Evenings were spent with Whitehead enjoying a Bonfire by the Beach, discussing the day's Results. And so the Pattern of my Life was establish'd. I rarely consider'd Rescue, for I think I was as happy as I had ever been in my Life. I was fulfill'd, productive and all of my physical Needs were met. The Years pass'd, and the body of my Work grew. It now took almost a Day to retrace all of the best Results. But as I retrac'd them, I wou'd think more about them, and consider means to push them yet further. It gave me tremendous Pleasure to see my Writings extend down the beach as far as I cou'd see.

But then one Morning, Five Years into my time on the Island, I awoke to signs of a Storm brewing off to the North. Climbing a Tree to get a better View of the impending Weather, I was surpriz'd to spy a Ship dock'd in the Bay. I slid down the Tree, grabb'd up Whitehead, and sprint'd down the Path to find a Dinghy beach'd on the Sand. Before it stood a Collection of Sailors and one who must be the Captain. I strode up to him.

"Sir, you are standing on my best Theorem," I said.

"I know not of what you speak," reply'd the Captain. "I am standing on Sand."

"But do you not see my writing in the Sand? Do you not see my Theorem Egregious?"

"I see only Marks in the Sand that look as if a Seagull has strutt'd about."

"These are the great Theorems that I have prov'd. You must bring Paper and Pen from the Ship so that I may record them."

"Have you gone batty?" he reply'd. "There is no Time for your Theorems. Do you see that approaching Storm? It will dash our Ship upon the Reef, and we will all be trapp'd here with your infernal Theorems."

"I will not go without my Theorems."

"Hold him fast then, Men, and throw him in the Dinghy. We must make haste." With that, the Sailors grabb'd me, and toss'd me like Luggage into their Craft. As they push'd the Boat in to the Surf, I struggl'd to escape, and in the process lost hold of Whitehead, and he fell into the Water.

"No, no, save Whitehead!" I wail'd, but the Sailors merely held me down, ignoring my Pleas. I watch'd helplessly as Whitehead bobbed away. The Wind was picking up now, and we crash'd through the Waves.

"Please, let me go," I cry'd. "I must save Whitehead." I cou'd still see his yellow head appearing once and again between the swells. The Oarsmen ignor'd me, rowing as fast as possible, as the Storm descend'd upon us. We reach'd the Ship just as Lightning creas'd the Sky and Thunder roar'd.

I was dragg'd up onto the Deck. The Captain yell'd to hoist Anchor, and the ship turn'd toward the open Ocean, as the Rain began to fall in Sheets and the Wind began to howl.

The Ship clear'd the Reef just as the Gale hit full Force. The Captain turn'd the Ship into the Wind and we spent the next Twelve Hours wondering if our final resting Place wou'd be the Ocean Floor beneath us.

But presently, the Storm abat'd, and I begg'd the Captain to return to the Island so I might copy down my Theorems. He just laugh'd.

"Do you think your Chicken Scratchings will have surviv'd that Storm? It has scour'd the Beaches as clean as the Day you arriv'd on the Island. Your Mathematics has been return'd to from whence it came."

I do not know what he meant by that last Part, but I knew he was correct. The Beaches wou'd be clean now.

And so I return'd to Civilization. I again took up my Position at the College. And I did manage to dredge a Theorem or Two from Memory, and ultimately wrote up a Paper that was duly publish'd in the Journal of Existential Mathematics.

But the Truth was that I cou'd not recreate the vast Majority of what I had discover'd on that Island. Most of it seem'd as a Dream, and when I went to figure out the Details, they had wash'd away just as my Scribblings on the Beach had done.

But perhaps anyone can ask what Permanence there is in Mathematics. More often than not, the next Generation overwrites the Mathematics of the Former. Results that seemed so fundamental at the time are generalized and then subsumed in subsequent Work. A few Theorems are sustain'd. Some of the work of Gauss, Euler, Riemann. But in due time, my Theorems wou'd have disappear'd below the Surface of the sea that is Mathematics anyway. Does it matter so much that they did so before anyone else had the Opportunity to appreciate them? This is a Question I ask myself much too often, I am afraid. Much too often.

Chapter 16
Hardy and Ramanujan

Hardy had gone out to Putney by taxi, as usual his chosen method of conveyance. He went into the room where Ramanujan was lying. Hardy, always inept about introducing a conversation, said, probably without a greeting, and certainly as his first remark: "I thought the number of my taxicab was 1729. It seemed to me a rather dull number." To which Ramanujan replied: "No, Hardy! No, Hardy! It is a very interesting number. It is the smallest number expressible as the sum of two cubes in two different ways." —C. P. Snow [1]

Ramanujan was in poor health from the day of his arrival in England. The climate was one for which he was ill prepared. He never complained, but his nasal drip was a constant reminder of how imprudent it might have been for me to have brought him from India. Eventually, he ended up in the care of a clinic, where I would regularly go to visit him.

One day, I arrived to find him in bed adding large numbers without the aid of paper and pencil. He used a convoluted algorithm involving his fingers and toes as well as several nurses who had to position themselves at various points around the room under his instructions. Upon my arrival, he thanked the nurses politely and then waved them out.

I sat down, but, as usual, found myself unable to begin a casual conversation. This personality defect originated with my nanny, Mrs. Hanscomb, who used to shock me with a large car battery whenever I commented on the weather. It was her firmly held belief that trivial banter should be strongly discouraged.

Unfortunately, Ramanujan had also never learned the fundamentals of social engagement, so after ten minutes of silence and exchanges of expectant glances, I hazarded an opening gambit.

"Ramanujan, I see you had two nurses taking care of you today."

A glazed look passed over his face.

[1] C. P. Snow, Variety of Men. London: Penguin Books, Ltd, 1969.

C. Adams, *Do Androids Dream of Symmetric Sheaves?*,
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"Hardy," he said, "2 is the smallest divisor of 2,136,575,432."

"Yes, that is true," I replied, trying not to let on how trivial I found his observation to be. Although brilliant, his lack of mathematical training meant that he often did not realize whether or not a given assertion was significant.

"Did you come by taxi today?" he asked.

"Yes, I always come by taxi."

"And what was its number?"

"It was 1556".

"Oh, this is a great disappointment, Hardy. I have been so looking forward to your taxi number all morning, and this number is not an interesting number."

"Listen, Ramanujan," I replied. "I spent 30 minutes at the taxi stand waiting to get a good number. But Littlewood grabbed 1361, and there wasn't a prime left in the bunch. I cannot spend my entire day trying to find a cab with a number that will amuse you."

"Oh, Hardy. I apologize. 1556 isn't such a bad number. At the very least, it is the first number that is 4 times a prime whose digits add to 20."

"Yes, it is that," I replied, feigning awareness of this fact.

"Hardy, I must tell you. I am very hungry. Is there not any edible food in this entire miserable country?"

"But Ramanujan," I replied, "They have left you a kidney pie on your tray. They already cut it into three pieces."

"That makes each a third of the whole, Hardy," responded Ramanujan.

"In fact, that does not follow, Ramanujan. I did not say that the pieces were equally sized. It could, for instance, be the case that one piece is half of the pie and the other two are each a quarter."

"But, Hardy, look at the pie. They are three equally sized pieces, each a third."

This was the essence of the problem with Ramanujan. He often arrived at conclusions based on evidence he observed in the real world, rather than relying entirely on abstract mathematics. He reminded me of the great English batsman Braddock, who was brilliant when he was facing the right direction and laughable the rest of the time.

We sat in silence for the next ten minutes, both doing our best to avoid eye contact. Finally, Ramanujan spoke.

"Tell me Hardy, what is your favorite number?"

"You asked me that, yesterday," I replied.

"I ask you it every day," he retorted. "You know what I mean. What is your favorite number today?"

I knew that I could only embarrass myself with an answer. Either Ramanujan would use his prodigious mathematical talents to instantaneously uncover the amazing properties of my number or he would tease me if it had none.

But given his circumstances, I felt obliged to respond. I decided to narrow my choices to numbers no larger than 100,000. Otherwise, it would take me too long to reach a conclusion. Ramanujan waited with that expectant look I had come to dread. I eliminated the even numbers, which are simply a product of some other number and 2, so why not just consider the other number. I repeated this process with numbers divisible by 3, 4, and 5, until I realized at this rate, I might have none

left. So I then eliminated the primes, being much too obvious. Then I discarded numbers divisible by higher powers of primes, and numbers divisible by primes that were themselves the sum of the prime divisors of the number. I subsequently eliminated those numbers whose digits, when reversed, yielded a number that was the sum of n other numbers, all of which themselves were palindromes. Continuing in this manner, I eventually whittled the options down until I found myself with only one number remaining. So I said, "Well of course, Ramanujan, my favorite number is 67,789."

"Of course," replied Ramanujan, beaming. "The digits of which yield the famous riddle. Why was 6 afraid of 7? Because 7 ate 9."

"Exactly," I replied, trying not to let on that I had never heard this riddle before. People tend not to tell me riddles because of the choking noises I make when I laugh.

Ramanujan suddenly surprised me by reaching out and grabbing my hand. For such a sick man, he had a unexpectedly strong grip. I held my hand as limply as possible.

"Hardy," he said. "I have known you now for quite some time. Do you think I could call you by your first name?"

I found this request quite awkward. For I was not one to promote familiarity. Who knows where it might lead? But given the situation, I had little choice.

"I suppose so. If that is your desire."

"What is it?"

"What is what?"

"Your first name."

"Well, it is G. H."

"No, Hardy, those are your initials. What is your actual first name?"

I began to sweat.

"I prefer not to divulge it."

"I am imploring you, from my sickbed, tell me your name."

"Very well, then, if you must know, it is Godfrey."

"Godfrey? Godfrey? Hardy, that is a wonderful name. I do not understand why you dislike it so."

I was not about to explain to Ramanujan the many ways that Mrs. Hanscomb had tormented me over my name, and always with that infernal battery. But, at any rate, I decided I had had enough for one day, and it was time to get back to my mathematics.

"Good Lord, Ramanujan, look at the time. I must go at once."

"I am sorry, Hardy. Please stay. I will not call you Godfrey. Tell me again how useless the mathematics is that you do. That so entertains me. How nothing you have done will ever prove relevant to cryptography or quantum physics or any of the other applied scientific endeavors. And how you revel in that fact."

"I am sorry, Ramanujan, but I absolutely must run. You know I must prove three theorems before tea, and they are serving crumpets today, and the afternoon cricket match begins right after that. The cricketers would be very disappointed if I were not observing from my customary viewpoint. But I shall return tomorrow."

"Very well, Hardy, but please do try to find a cab with an interesting number. You know what that means to me."

"You know, Ramanujan, I believe I shall bicycle tomorrow."

He looked as if he might cry. But I fought down the urge to give in and waved goodbye as I slipped out the door.

That evening, as I watched the cricket match, I was struck with an overwhelming case of remorse. Ramanujan had done nothing wrong to deserve his fate and the least I could do was to support him in his time of crisis.

The next day, I found myself at the taxi stand an hour early. I picked out a beauty and then paid the driver to sit with me in the cab until it was time to go. Upon arrival at the clinic, I was so pleased with the number that I leapt from the cab, rushed into the building and burst into Ramanujan's room.

But I was surprised to find there was no Ramanujan. The bed was empty. The nurse who was changing the sheets looked up and just shook her head sadly.

And so, brokenhearted, I leaned against the door frame. Ramanujan was gone. We had lost one of the greatest mathematical geniuses of all time. Memories of all of the fun we had had together flooded into my head. Laughing uncontrollably over the inappropriately named perfect numbers. Making fun of Boyer's attempted proof of the Goldbach Conjecture. Giggling behind his back when Hall believed us that 92,650,699 was a prime. Those were special days.

And most difficult of all, I would have no one with whom to share the lovely number of my cab. I regretted terribly my thoughtless behavior from the previous day. I should have encouraged Ramanujan to call me Godfrey and, perhaps, even asked him for his first name. I owed him an apology, an apology from one mathematician to another. But the sad truth was that it was an apology I would never be able to deliver. Now it was too late.

The nurse cleared her throat. I turned and she smiled at me kindly, preparing to deliver some consoling words, perhaps with the hope of initiating a personal relationship. I immediately scurried out the door, hopped into the cab and returned to the safety of my rooms at Cambridge.

Chapter 17
Introduction to the Collected Works

What can one say about Frederick D. Funkle? Revered as perhaps the greatest semi-upper continuous functional analyst who ever lived, his contributions continue to confound us and often contradict one another. His body of work is broad but unnervingly shallow. His writing style is impertinent, presumptuous and tinged with an undeniable disdain for the very material he studies. His disregard for standard notation and his penchant for starting in the middle of a proof and simultaneously working toward the beginning and end have made it extremely difficult to evaluate the contributions of this iconoclastic individual.

But those who know only of his mathematics run the risk of missing the other facets of Frederick Funkle. Did you know, for instance, that he was one of the great hand puppet manipulators in the state of New Jersey? Were you aware of his contributions to crossing guard etiquette? Did you know he had the largest collection of fanny packs in the mid-Atlantic states?

Frederick Funkle was born in a small apartment in Hoboken, New Jersey, just a block from where David Hilbert once stayed for a week. Although unaware of who David Hilbert was, Frederick walked by that house on a daily basis. This early influence seems to have had, via osmosis, a fundamental impact on Frederick. Much of the rest of his career was spent walking past the houses of famous mathematicians.

Although no one in his family ever demonstrated an aptitude for mathematics, and many were quick to demonstrate the opposite, mathematics was no stranger to the Funkle household. Several of Frederick's aunts owned calculators and weren't afraid to whack Frederick's knuckles with them when he reached for a dinner roll.

In elementary school, although Frederick was known to have a short attention span and a particularly poor memory, he excelled at mathematics, often to the consternation of his teachers. Once, when he was six, his exasperated teacher instructed him to add up all of the integers from 1 to 100 to keep him busy. As she returned to the front of the room, he followed her up. She turned, and asked, "Have you already finished?"

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"Finished what?" he replied, much to the amusement of his classmates.

By the time he graduated from high school, Frederick knew his calling. He had become enamored of mathematics education. His first paper, "Positive Reinforcement to Discourage Division by Zero" began his storied career as a math educator who, against the current tide, advocated rewarding students for their errors. His approach swept through the math education community like a tsunami, leaving a similar pattern of destruction in its wake.

But it wasn't long before the siren song of mathematics proper drew Funkle away. He became intrigued with the question of the constructability of lengths using just compass and straightedge. After claiming to have refuted a variety of previously proved constructible lengths, it was pointed out to him that the compass involved is not the kind that tells direction.

Undaunted by this early faux pas, Funkle now threw himself into mathematics proper. In homage to Andrew Wiles, he moved his desk up to the attic for an intended seven years, but found that he wasted time trying on old clothes. So he moved to the basement. This was one of his most productive periods, resulting in the seminal papers "Sub-bases of Bases III" and "Cellar Decompositions of Manifolds".

When he came down with the flu, his colleague at Western Harmonic, JHC Blackhead, came to visit him. Lacking the ability to hold up his end of everyday banter, Frederick asked him, "What was the number of the taxi you came in?"

Blackhead replied, "1729, a nondescript number I am afraid."

"Oh, no," replied Frederick. "You are mistaken. Why that is the number right after 1728."

"And what is so special about the number 1728?" asked Blackhead.

"Why it is the number that follows 1727."

At this point, Blackhead realized that Frederick could stymie him like this all day. He shook his head ruefully, vowing to himself never to visit again.

Frederick's contributions to number theory include the Funkle Algorithm, which immediately gives the number of distinct factors of any prime in polynomial time. In topology, his proof of the Poincaré Conjecture in dimension 1, showing that the unit circle is the only simply connected compact 1-manifold without boundary, sent topology into a decade long tailspin.

His Funkle Asymptote is defined as an asymptote that has transcendental slope. Although there is no obvious advantage to naming such asymptotes, it is undeniable that Funkle was the first person to give them a name.

What are we to make of a man who refused to talk on his work, and would only sing operettas at seminars? How do we evaluate a paper that both proves a result and provides a counterexample?

And of course the strange circumstances of his death leave many more questions than answers. In the middle of a talk on his recent results on Funkle Functors, an undergraduate pointed out an obvious counterexample. Funkle stammered a bit and then clutched his chest and fell to the floor. When someone attempted to apply a defibrillator, Funkle pushed the paddles away, perhaps having realized the result was beyond repair.

Here then are the collected works of Frederick Funkle. Section I consists of his famous correspondence with Mikhail Gromov, including all the unanswered letters he sent. Section II is a compendium of his publications, many having appeared in the Monthly, consisting primarily of his solutions in the problems and solutions section.

Section III includes the reviews he wrote for Mathematical Reviews of his own papers, which, by a clerical error, he was asked to author. Many argue that his reputation was built on these reviews, and he took full advantage of the opportunities it provided.

It is not for us to determine Funkle's place in mathematics. One can never know what may turn out to play a critical role in the future and what may prove to be meaningless drivel. But there can be no doubt that Frederick Funkle was unique in the annals of mathematics. And for this we should be eternally grateful.

Chapter 18
Mathematicus

Carl Dempsey, associate chair of the Mathematics Department, rushed into the chair's office. A visibly exhausted Karen Blagsby was leaning on a windowsill, looking out at the campus.

"We're having trouble on the western front," said Dempsey, out of breath.

"What kind of trouble?" asked Blagsby, as she turned to face him.

"The rebels have taken over the Math Library," replied Dempsey.

"Oh, that's bad," said the chair. "That's really bad. I thought Topology was holding the Math Library."

Dempsey shook his head. "Topology figured it was too difficult a space to defend. They decided to hold a topologically equivalent but less strategically significant space."

"Meaning?"

"They're hiding in the copy room."

"Call an emergency meeting of the department at once," said Blagsby. "Tell everyone to meet in the seminar room. Offer them cookies if they come."

"We can't," said Dempsey. "The supply room has been taken over as well. The rebels have the cookies."

"Oh, that's disastrous," said the chair, her shoulders sagging. "Does the faculty know we've lost the cookies?"

"I don't think so," replied Dempsey.

"Then tell the faculty there will be cookies. The pink wafer kind. And whatever you do, don't let on."

"Yes, ma'am," said Dempsey as he rushed out of the office.

Blagsby reached over and lifted the calculus book from her desk. Walking over to the door, she braced herself and then swung the book against the doorframe. It gave a satisfying thump. Nodding to herself, she turned and headed down the corridor toward the library, carrying the book at her side. As she approached the library door, she could see tables and chairs piled up, blocking the entrance.

"Hold it right there," said a voice from inside the library.

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"Take these chairs down immediately," demanded Blagsby. "The fire marshal will have your head."

"Hah," said the voice.

"Listen to me, whoever you are," she continued. "Take those chairs down or I will make sure you never graduate from this or any other university."

There was assorted laughter.

"I demand to speak to whoever is in charge," said Blagsby.

After some discussion, a lone figure climbed over the chairs. He was wearing a dirty sheet fastened about him like a toga, and sandals on his feet. As he jumped down in front of her, she held up the calculus book defensively.

"You're not going to use that," said the student. "You don't have the guts."

"Don't push me," said the chair, lowering the book.

"Who are you?"

"I am Mathematicus," replied the student.

"Mathewhaticus?"

"Mathematicus. I represent all the downtrodden students you have squashed under your thumb all these years."

"Excuse me?" said Blagsby.

"You heard me," said the student. "Your day is done. Your time has come."

"Not that it's any of your business, but I'm nowhere near retirement. I'm good for another ten to fifteen years."

"You have destroyed enough lives," continued Mathematicus. "I will be your chattel no more."

"Chattel? What's a chattel?" asked the chair.

"You don't remember me, do you?" asked the student.

"Remember you?"

"I was in your Math 140 calculus class two years ago."

"Well," said Blagsby, "that was a big class."

"I was known as Edward Stuffnukel then. I was one of your many slaves."

"Now look," said the chair. "You may well have been one of my students, and I'm sorry if I don't remember you, but you most definitely weren't my slave."

"Of course I was. You didn't call it that, but that's what it was. And as your slave, you could make me do what you wanted."

"Really? And what was it that I wanted?"

"You wanted entertainment. You wanted spectacle. You wanted combat to the death."

"I don't know what you're talking about."

"Don't you remember the weekly quizzes?"

"Weekly quizzes? What about them? They're not that hard, and I curve the grades."

"You pit us one against the other. To see who will be left standing by the end of the semester. Does it give you pleasure to see us destroy each other?"

"Pleasure? Of course not. I'm only trying to teach you."

"You train us for combat. Have us compete against each other until we are ready. Ready for the midterm."

"I don't. . . I'm. . ."

"And the blood and guts spilled at the midterm were a sight to behold. Students who would never again raise a pen for mathematics. Who bled red ink on the exam-room floor."

"Your imagery is quite vivid. Have you considered majoring in English? They would probably be glad to have you."

"But I made it through the first midterm and even the second midterm. I was almost done. Only one test left. The ultimate test. The final."

"Well, that is the way most courses end."

"Do you remember the pageantry? The banners. The yelling crowd, whipped into a frenzy?"

"Ummm, I actually don't remember any of that. . ."

"Do you remember the fear in our eyes? The trembling hoard as we entered the exam room? The smell of sweat and urine?"

"I definitely don't remember that."

"Do you relish the absolute power you have over us? Does it feel good to know you can put any problem on the exam, no matter how ridiculous, and we have no choice but to try to answer it?"

"You are talking about education here. This is how I educate you."

"No. This is how you torture us. You decide who survives and who dies."

"That's not how it works. I don't decide. You do your work and it's either right or it's wrong. I just grade it. That's how it works. I don't have any control over it. Mathematics determines if you are right or wrong."

Mathematicus smiled a tight smile. "That's not the way it's going to work anymore. We're going to change the rules. No more right and wrong. It's not fair to those of us who struggle with mathematics. From now on, students will have a say in the grading. We will have a say in what's right and what's wrong. We will take the power away from the profs."

"That's ridiculous," said the chair.

"Is it? In an English class, I turn in a paper, and I get lots of credit. Even if it's bad. Because there is no one way to grade it. It's subjective. And professors don't give low grades for fear someone will complain. Two different professors grade the same paper and they can give it completely different grades."

"But math isn't subjective," replied Blagsby. "The rightness or wrongness of mathematics is determined by logic."

"Hah! Logic! That magic totem. That idol which you worship. But we do not subscribe to your religion. We reject your logic. No more will you hold us in thrall to your equations, your square-roots, your trig functions, your integral signs, your subnormal central series."

"I'm sorry. But you can't reject logic. It's there. You can't ignore it."

"That's where you're wrong. Because we have already rejected it. And we have already won the students to our cause. And we have already convinced many of the departments to join us. Theater, English, history, political science, anthropology, the languages, art, religion, hospitality management. They have all joined."

"Those departments are rejecting logic?"

"Yes, and they're giving students a say in the grading. From now on, we determine 50% of the grade."

"That's absurd," said Blagsby.

"Is it?" said Mathematicus. "Or are you just saying that because you have been using logic to stay in power all these years? Once we reject logic, you have no power over us anymore. We are free!"

He thrust his finger in her face and shook it vehemently. He then laughed as he turned and scrambled back over the chairs.

Blagsby watched him disappear and then she slowly turned and trudged back down the hall to the seminar room. The department staff was huddled inside. They looked up hopefully as she entered. She shook her head sadly.

"People, I have some very bad news," she said, as she shut the door and turned to face them.

"Where are the cookies?" said a plaintive voice from the back.

"No cookies," said Blagsby. "No cookies for a long time."

"What do you mean?" asked a timid junior faculty member.

"They're onto us," replied Blagsby. "They've figured out that all this time we have been using mathematics to hold onto the reins of power, to keep them under our thumbs."

"It can't be," said Dempsey. "I thought you said they would never catch on."

"Well, they did," said the chair shaking her head sadly.

"What will we do?" asked the junior faculty member.

"We need to come up with a new scam,"said Blagsby."Maybe economics. We could pretend that we can predict what is likely to happen to the economy next. That'll bring in the bucks."

"Or philosophy," said Dempsey. "We could pretend we have answers to the big metaphysical questions."

A few of the faculty were nodding their heads, a glimmer of hope in their eyes.

"But in the meantime," said Blagsby, "things are going to get rough around here. We're not welcome anymore. So, everybody, go pack your bags. We're hitting the road. This university belongs to the students now."

All the department members filed out of the seminar room. Blagsby sighed as she turned out the light and shut the door behind her.

Chapter 19
Happiness Is a Warm Theorem

Doctor: It's okay, Craig. You can talk freely here. If you want to cry, you can cry.

Craig: I'm sorry, Doctor. It's just so overwhelming, sometimes.

Doctor: Why don't you talk about it?

Craig: It's crushing depression. Crushing.

Doctor: Here's a tissue. Do you feel it all the time?

Craig: Lately, yes. Lately, it is a total. I can't see any way out.

Doctor: But before that?

Craig: Before, there were also the highs. Extreme euphoria. But almost always, the highs were again followed by extreme lows.

Doctor: Yes, well, this sounds like classic bipolar disorder. What brings these feelings of depression on? Are you having trouble relating to your spouse?

Craig: Spouse? She left me two years ago.

Doctor: Oh, I am sorry to hear that. Did that make you very sad?

Craig: Actually, I didn't notice for a week.

Doctor: What do you mean?

Craig: Well, I was trying to prove a difficult theorem. I was really close. The solution was just out of reach. Couldn't put it down for a second. I went home to check a theorem in a book I had left by the bed and I noticed she wasn't there. Found a note on the kitchen table saying she had left.

Doctor: And how did you react?

Craig: It didn't really register. I needed to get back to work.

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Doctor: Hmmm. Sounds like a defense mechanism.

Craig: Not really. I was at a crucial juncture in trying to prove that theorem. . .. We weren't getting along anyway.

Doctor: Why not?

Craig: She was frustrated with me. Called me antisocial.

Doctor: Were you antisocial?

Craig: Not at all. I can be very social. I just didn't want to be.

Doctor: Why not?

Craig: I was busy. . .. She used to drag me to these parties. A bunch of people standing around trying to come up with topics for conversation. The weather, politics, American Idol. And they were drinking alcohol to dull their intellects enough to enjoy it.

Doctor: I take it you don't drink?

Craig: It gets in the way of my ability to do mathematics.

Doctor: So what would you do at these parties?

Craig: I would sit in the corner.

Doctor: By yourself? You didn't talk to people?

Craig: I was thinking about mathematics. It's one of the great advantages of my profession. Once you are thinking about a problem, you always have your work with you, wherever you go.

Doctor: Okay. . ..

Craig: But then her friends would spot me in the corner and they would feel sorry for me. They would assume I was shy. I'm not shy. But inevitably, they would come over and try to engage me in conversation, and I would have to be rude to get them to leave me alone. Eventually, we stopped getting invited to parties.

Doctor: And your wife was upset about that.

Craig: Not only that. It wasn't like I was keeping up my end of the conversations at home either. I had other things on my mind. She said it was like living with a zombie. My body was present, but my mind was far, far away. So she left.

Doctor: Okay. . .. Well, then, I guess you don't have a significant other at this point. Pets?

Craig: Had a fish. Forgot to feed it, so it died.

Doctor: And how did that make you feel?

Craig: It was okay. At least then I didn't need to remember to feed it anymore.

Doctor: So you don't have an emotional need to connect with another sentient being?

Craig: Not really.

Doctor: You don't get lonely?

Craig: Doc, I don't have time to get lonely.

Doctor: I see. It sounds like you are tremendously overworked.

Craig: I choose to work this hard.

Doctor: Nobody is forcing you to do this?

Craig: Are you kidding? I'm tenured. If I wanted to, I could stop research and watch soap operas all day.

Doctor: So what's driving you? Ambition? You want to be at the top of your field?

Craig: I do want to be at the top of my field. But for me, it's not about the recognition. It's about beating it.

Doctor: Beating what?

Craig: Whatever problem I am working on.

Doctor: You make it sound like a battle.

Craig: It is a battle, but the long drawn out kind. A siege.

Doctor: How so?

Craig: It's like this thing you are trying to prove, this theorem, it's in its fortress, protected. And you are trying to break in, to find a crack in its defenses. And you try one thing, but it doesn't work. So you try something else, and that doesn't work. But you keep at it, camped outside the walls, trying to last longer than it does, trying to find a way into or over the walls. You lay siege to the problem. But after a while, you become discouraged, because nothing is working. You've struggled for so long with it. Tried every approach. There is nothing left to try. You're out of ideas. So finally, you try to move on to something else. Find another problem to distract you. But it doesn't work. Your mind returns to it even as you consciously try to think of other things. You might find yourself standing in the produce aisle of the grocery store, holding a bag of oranges and having no idea how long you have been there. And then one night you fall asleep at your desk. And you dream about the problem. And the next morning you wake up, and you just know you have solved it. You know you are right.

Doctor: What does that feel like? To solve a difficult unsolved problem like that?

Craig: Oh, Doc, you have no idea. It's incredible. A euphoria. It makes you want to sing. It makes you want to kiss strangers. It makes you want to tell everyone.

Doctor: Like I feel when one of my patients gets better?

Craig: No, nothing like that. That's like when I teach a student calculus, and they finally get it. This, this would be more like you having seen hundreds of patients over your career, and suddenly realizing there was a pattern to their behavior that no one else in your profession ever recognized before. You then discover a treatment that not only cures all of your patients, but everyone else's patients as well. You have solved this difficult problem that has plagued humankind for millennia. More like that.

Doctor: Well, that does sound like a nice feeling. So this is the highs?

Craig: Yes.

Doctor: And how long does the feeling last?

Craig: A week, maybe a bit longer. But then it's time to work on the next problem. And the minute you begin, you get depressed.

Doctor: This is the lows?

Craig: Yes, you are pushing another rock up the hill, and it just keeps rolling down again.

Doctor: But at least you are making progress.

Craig: That's just it. Maybe you are and maybe you aren't.

Doctor: What do you mean?

Craig: Well, for example, one time ten years ago, I was working on the Schmuel Conjecture, open since 1982. I had a great idea. Approach it through Kleiner bundles. No one had thought there was a connection between the two. But I saw it! Looked like I was about to solve the problem. But each time I thought I was almost done, something would go wrong at the last stage, and there would be this additional result that I would need to prove. This went on for a year. Intensely frustrating, but it still felt like I was getting closer and closer.

Doctor: And?

Craig: And it didn't work. The last piece I needed happened not to be true. Dead end.

Doctor: And what happened to the Schmuel Conjecture?

Craig: Oh, it's still open. Every once in a while, I find myself daydreaming about it, thinking of some alternative approach. . ., maybe through Lie groups. . ., maybe using the Kac-Moody algebra. . ..

Doctor: Craig?

Craig: . . .I wonder if the cohomology would tell me something. . ..

Doctor: This is classic addictive behavior. And you are not the first patient in which I have seen such a pattern.

Craig: Really?

Doctor: Actually, I am treating several members of the mathematics department right now. But the rest are not as able to articulate the problem. Most just sit and stare off into space without talking.

Craig: Probably just working on math.

Doctor: Well, it is becoming apparent that this is a psychosis. A psychosis that afflicts the mathematics community. It has a variety of symptoms including addictive disorder, antisocial attitudes, and obsessive compulsive behavior. This is incredible! I have identified a new disorder! It may transform psychology!

Craig: Yeah?

Doctor: Yes, yes, and I will name it, umm, mathematitis. It is an unhealthy addiction to mathematics. I can see the papers already. This will be truly groundbreaking work. Perhaps we can drug the patients. An amnesiac, that makes them forget the problem they are working on. Or perhaps we can hypnotize them into believing they have solved their problem, bringing on a virtually constant euphoria.

Craig: Yeah, well congrats, then. Must feel good. Look, Doc, can we cut the session off early? I'm actually thinking I might have a possible approach to the Schmuel Conjecture. This could be a major breakthrough!

Doctor: Yes, certainly, certainly. Goodbye!

Chapter 20
Motivational Seminar

"Thank you all for coming to this event. And when I say thank you, that thank you is not only intended to be from me, it is also intended to be from you. Yes, you will thank yourself for coming, because this seminar will change your life. And you have taken the first step toward changing your life by being here. The first step is always the hardest, and you have already taken it.

"Now, everybody stand up. That's right, everybody! And say after me, 'I am a good mathematician!'

"No, louder! Say it like you mean it. 'I am a good mathematician!' Better. Now say, 'I am a great mathematician!'

"Louder! 'I am a great mathematician!' Good! Now, sit down.

"What brought you here today? I want to know. You, in the front row, with the Escher tie."

"I haven't proved a new theorem in six months."

"Six months? Wow! How about a lemma. Surely you've proved a lemma."

"No, nothing."

"Okay, we can help you. What about you? The woman with the π button. Why are you here?"

"I'm getting bored."

"Bored of math?"

"Bored of the math I do."

"What kind of math do you do?"

"I try to find the best delta, given a particular epsilon for a specific uniformly continuous function."

"Oh, that sounds awful. People, how does that sound to you?"

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"Awful!"

"We can help you. And you, the guy in the coke-bottle glasses. Why are you here?"

"I think I may be in the wrong room."

"Okay, we can help you, too! Today, people, my goal is to change your life. How am I going to do that? I am going to change your relationship to math.

"You think you can't do math? Or you're tired of doing math? Or you're stuck on a proof and you can't move forward? What is the root cause of all of these problems? I will tell you what it is. It comes from a flawed relationship with math. That's right. You have a flawed relationship with math. Because you expect math to do the work. You expect it to support the relationship all on its own. It doesn't work that way. It can't work that way. Doing math is a two-way street. You want to do math? You have to earn the right.

"Now, I want you all to close your eyes. That's right. And I want you to keep them closed. We are going to drift back in time, all the way back. So let go of the present and feel yourself drifting backward in time. Past your Ph.D. defense, all that yelling and confusion about whether or not the bundle was actually semiregular. Past your topology qualifier—who knew that there aren't any closed nonorientable 2-manifolds in 3-space? Past your linear algebra course in college—they really expected you to distinguish between eigenvectors and eigenvalues? Past your high-school calculus course (turns out derivatives and integrals are related). Past your winning entry in the number of jellybeans estimation contest. Past the first time you learned about negative numbers, still further, further. Yes, don't stop. Keep going back. Keep going even past the blinding light in the hospital delivery room, back up inside your mother's womb. . .I said we were going all the way back, didn't I? It's warm and it's dark in here. But we're not done yet. We're going back even further. Back to your first consciousness. The time your tiny little pea brain first said, 'I exist.' Are you there with me? You have literally just come into existence. And you are saying to yourself, 'Huh? What's going on here? What is this thing? This thing called existence, this awareness of something?' It is the classic, 'I think therefore I am.' A pea-sized version, but a version nonetheless.

"Here you are in the womb, trying to grapple with this concept of existence with nothing but your nascent intellect to help you. You can't touch yourself; your arms are still flippers. So you revel in this existence thing for a while but then you say to yourself, 'Is there more? Is it just me or is there more?' It is at this exact moment that you come to the realization that there is more. Your first mathematical epiphany. There has to be more. Because if you are the only thing, the one existence, then you are still an element in the set that consists of only you. And that set is different from you. Hallelujah. There is more than you. There is you and there is the set that includes you as its only element. That makes two things, which is 100% more than one.

"And then you say, 'Wait a minute. There is a third thing. And that thing is the set that consists of me and the set that contains me.' It is a set with two elements. That set is thing number three.

"And slowly, oh so slowly, you begin to construct the positive integers. And you don't call them the positive integers, because you've never heard of the positive integers. As far as hearing anything goes, you hear that big bass drum, your mother's heartbeat, and that's all you hear. So you don't call them integers. But that is what they are. You have created them out of nothing. And so your first love, besides the bass drum, is mathematics. And as you while away your time in there, close to eight-or-so months to blow, you think about those integers. You try combining them in interesting ways. You invent addition and subtraction, and multiplication, and maybe some algebra if you are an algebraist, or maybe some topology, working with the umbilical cord, if you are a topologist, and you are just about as happy as you have ever been. And although that wouldn't have to be very happy because you haven't had a lot of time to try the full spectrum of happiness options, in fact it is as happy as you will ever be. Because after you have left your private study carrel, your dark warm protected enclave, perfectly designed for profound cogitation, you will never again find a place where you can do math undisturbed for eight months.

"Yes, your relationship with mathematics goes all the way back to the beginning. Now, let's roll this newsreel forward. Keep your eyes closed. I mean you, coke-bottle glasses, keep them closed.

"So now, time passes, and suddenly, when you are in the middle of figuring out the uncountability of the reals, all hell breaks lose. You are suddenly pushed and pulled this way and that, and then torn from your cozy dark warm haven and spewed out into the real world, the blinding, noisy, tumultuous real world. Your privacy is invaded, invaded permanently, never to be restored. And your contemplative environment, tailor-made for the study of mathematics, is lost forever, never to be recovered.

"Yes, you have lost your scholarly nirvana, and will spend the rest of your life trying to regain even a tiny piece of that mathematical heaven. So that is what we all seek. You can now open your eyes.

"Do you understand now some of the feelings you have had for mathematics? Do you understand how this early experience has influenced your relationship with mathematics?

"You sir, in the fractal t-shirt, where do you do your mathematics?"

"In the car."

"When you are driving to and from work?"

"No, in my garage. I go in the car to work on math."

"Okay, and you there, the woman holding the copy of Whitehead's *Homotopy Theory*, where do you do your math?"

"Me?"

"Yes, you. Do you see anyone else holding Whitehead's *Homotopy Theory*?"

"Well, yes, there is a woman toward the back who also has a copy."

"I mean you."

"Um, I do it in my closet."

"In your closet?"

"Yes, I take a bunch of blankets in there, and pile them around me, and then I shut the door. Then, with a flashlight to see, I work on math. Nobody bothers me in there."

"Do you understand, people? We are trying to recreate our mother's womb, the optimal environment to do mathematics. That is what we seek.

"Young woman in the second row, why are you crying?"

"I guess I didn't know what it was I was seeking until now. It makes so much sense."

"Yes, now you know. Now we all know. But there is more. Everyone, I want you to shake this off. We've been on an emotional journey, but now we need to shake out the emotions. Everyone, stand up, and shake your arms like a tree in the wind. That's right. Very good. You there, you're a willow, and you, you are an oak. Shake it out. That's right. You sir, you appear to have Dutch elm disease. No matter. We're done. Sit back down.

"Let's change the topic for a moment. Let's ask a fundamental question. Why did you go into math? I'm guessing it wasn't for the money. You could have gone to Wall Street, and you could have created new financial instruments that no one could decipher. Instead of derivatives, you could have called them integrals. And you could have given them fancy names such as Lebesgue-Stieltjes integrals. And you could have brought Wall Street to its knees with these financial instruments, along with the entire U.S. economy. And ironically enough, Wall Street would have paid you gobs of money to do it. But no, you didn't do that.

"And it wasn't for the adulation. You prove a big math result and you're lucky if three people, two in your subspecialty and the other one your mother, care. You put your paper out there on the arXiv and it receives five hits, all because a few people are curious about a title that includes the words 'perverse sheaves.' If you had wanted adulation, you could have joined Scientology and worked your way up the ranks to become the next L. Ron Hubbard. Or you could have gone to acting school and gotten bit parts in 'A Beautiful Mind' and 'Good Will Hunting,' eventually becoming the go-to person for bit parts in hit math movies, movies like. . .ummmm. . .I can't think of any others right now, but you get the idea. You would constantly be invited to speak at Math Movie Award banquets. But you didn't do that, did you?

"So what was it? What made you decide to become a mathematician? Don't look at me like you don't know. You know the answer. Deep inside you know the answer. It was because you had no choice. It wasn't a decision you made. It was a decision made for you by the essence of who you are. You were destined to be a mathematician from those early moments inside your mother's womb. That's why you are where you are today.

"But it hasn't exactly turned out the way you expected, has it? You're here at this seminar today, so we both know that's the case. But you have to understand that over time, any relationship changes. Your heart no longer goes pitty- pat every time you look upon your true love. You see the same math lying with its head on the pillow next to you every night, and it's hard to keep the spark alive. Whereas the two of you used to stay up until sunrise, having fascinating mathematical discussions that roamed over the entire mathematical landscape, now you seem to argue about the most minor points, a plus sign here, an epsilon there. And now that the children are all grown up, those theorems born out of your early amorous embraces, it seems that the passion has faded. Math looks at you with that tired expression, that expression that says, 'You again?' And you look back, thinking about all the weird habits that math has, those strange peccadillos that when you first met math seemed endearing, but now make you cringe. The product of a normal space with an interval needn't be normal? We don't know if $\pi + e$ is transcendental? And $0.999\cdots = 1$? Come on.

"And stay up all night? Are you kidding? You have to teach in the morning.

"Should you throw in the towel, give up on math? Start dealing math-related postage stamps on the internet? Tutor high-school students for the math SATs? Enter television dance competitions?

"No, it's not time for that. Because you can change a relationship, no matter how long you have been in the same rut.

"Ask yourself. Has your relationship with math been a balance between your needs and its? Have you even considered what math needs from you? Have you thought about why mathematics might not find the relationship worthwhile? No, you haven't.

"Math just gives and gives and gives. How many lemmas has it given you over the years? How many definitions? How many propositions, corollaries, and theorems? Math has always been there for you. When you were down in the dumps, your dog left you, and the house was in foreclosure, math was always willing to distract you, to take you away from all that, to help you forget your troubles.

"Maybe now it's time you were there for math. Maybe math deserves an evening off at home, just you and math doing sudokus in front of the fire. Maybe it wouldn't hurt you to give a lower-level talk once in a while. To help math to attract the next generation of mathematicians. I bet you that math would appreciate that. Maybe, you could write an expository article sometime, even one. You could encourage your strong students. You could show math that you are aware of its needs, show math that you care. And in turn, it will repay you in kind, many times over.

"When's the last time you said, 'I love you, math'? When was it? A long time ago, I'm guessing. I think it is time. So right now, everyone up on your feet. You too, coke-bottle glasses. Everybody. And now, say it together. 'I love you, math.' Louder, 'I love you math!'

"That's it folks! Thanks for coming. Let's have a huge hand for you, everybody clap! That's it! Books for sale on the way out. Fifty percent off if you come again tomorrow night. Thank you everyone!"

Chapter 21
Cylindra Ella

Once upon a time, there was a young woman named Cylindra Ella who, after finishing her undergraduate studies in medieval folktales, decided to switch directions and go to graduate school in mathematics. Because of her lack of background, she was turned down by all the schools to which she applied except Central Western University, which is located in the central western portion of a state of which you have never heard.

But she was happy to be going at all, so she packed her bags and arrived full of anticipation for the adventure to come. She did well in her classes, and easily passed her qualifying exams. Now it was time to find an advisor. She attended various seminars, but nothing tickled her fancy until one day, she happened upon a talk on Farey sequences. It was fascinating. After the talk concluded, she asked another student, "Who was that speaker?"

"Oh, that was Andrea Finkel," the student replied, "one of the world's experts on Farey sequences. In fact, I'm doing my thesis with her. I call her my "Farey Godmother", cause of course you see. . .."

"Yeah, I get it," interrupted Cylindra. "Where do I find her?" The student pointed down the hall and in no time at all, Cylindra also had a Farey Godmother.

Upon completing her Ph.D., and with the help of her Farey Godmother, Cylindra obtained a post doc at Western Central University (no relation to Central Western University, although they are sometimes confused for one another). But upon her arrival there, she found that her research advisor, Eve Stepfunction, had two other post docs, both of whom had completed their Ph.D.'s with Stepfunction. The Ph.D. sisters were shown great favor by Stepfunction, even though they had absolutely no mathematical sense and prattled on about department gossip all the day long. Stepfunction published papers with them, and invited them over for soirees to meet visiting mathematicians and eat canapés.

On the other hand, Stepfunction treated Cylindra in the most terrible manner.

"Here," Stepfunction would say. "Take this paper and clean up the lemmas. Redraw the figures and TeX it up nice, even though you're not a co-author."

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Cylindra would dutifully do as she was told, all the while despairing over the turn her career had taken. But eventually, Cylindra decided to make the best of a bad situation. Late at night, after finishing the menial jobs that her research advisor had assigned her, Cylindra would work on her own research. Over time, she managed to obtain some very strong results.

One day, a poster appeared on the department bulletin board announcing a major research congress occurring at the University of the Central West, which was a third institution often confused with the first two and not so far away. Cylindra sought out Stepfunction.

"Would it be possible," she asked, "for me to speak at the congress?"

Stepfunction laughed out loud. "You? Speak at the congress? How ridiculous. You haven't published anything. And talks are by invitation only. I already obtained invitations for my two ex-students. I couldn't possibly ask the organizers to invite you, too. And anyway, I need you to stay here and teach my classes while I am at the conference. Now get to work on those corollaries for me."

Cylindra returned to her desk and sobbed, "Oh woe is me. What shall I do?" But then her phone rang. It was a facetime call from her Farey Godmother.

"Hello," said Cylindra sadly.

"Oh dear, what is wrong?" asked her Farey Godmother.

"There is a big research congress at the University of the Central West, and my research advisor will not let me go give a talk. I have to stay here and teach her classes."

"I see," said her Farey godmother. "Well, I know the organizers. I will arrange for you to give a talk. But remember, you must leave the conference early, and get back in time to teach your advisor's classes."

"Oh thank you," said Cylindra. "But what shall I wear? I have no fine clothes."

"It's a math conference, for god's sake," said the Farey godmother. "Wear a t-shirt and jeans."

So on the first day of the conference, Cylindra arrived in a white t-shirt and off-white jeans and wearing a cap with the brim slanted low across her face so her research advisor would not spot her. The first morning, the two Ph.D. sisters were slated to speak back-to-back. Cylindra snuck into the lecture hall at the last minute and sat in the back so as to keep a low profile.

Both talks were loud attempts to brag about what in fact were very weak results. As the talks dragged on, Cylindra could not help noticing a young man listening attentively and writing furiously on a pad. He was dressed in a particularly fetching maroon t-shirt with a picture of the Mandelbrot set on the front.

"Who is that young man?" she asked the grad student sitting next to her.

"Oh, that's Donald Princer, a young hotshot they say is destined for the Fields medal."

After the sisters were finished, Cylindra found a quiet place to go over her talk. She moved slides forward and backward, and removed theorems and then added them in again. After fretting for two hours, it was time.

She entered the room scheduled for the talk and sat down at the front. As she awaited her introduction, she saw Stepfunction enter, trailed by the nattering sisters.

Her advisor gave her a murderous look. And then Donald Princer entered. He sat down in the front row and pulled out an empty pad.

The moderator announced it was time for talk 3313 and Cylindra was given the floor. For a moment she hesitated. She knew her research advisor would never forgive her. But then she saw the expectant look on Princer's face, and she began to speak. She spoke of Farey sequences and their relation to the Riemann hypothesis. She spoke of how they relate to Ford circles. And she spoke of the implications for rational approximation. At the end of the talk, there was thunderous applause. And then Donald Princer approached her.

"I have not heard a talk I enjoyed so much since Kaznat's talk on polymorphic laminations in Poland in 2007. No, I take that back. I think I enjoyed your talk more."

"Oh, thank you, kind sir," said Cylindra.

"You're talking kind of formal there. Please, call me Donald. Listen, could we perhaps meet to talk about some possibilities for joint work? Maybe out on the terrace in a half-hour, say 12:00?"

"12:00?" repeated Cylindra, jerking to attention. "Oh, dear!" Without even a word of farewell, she raced out of the room, realizing she had to get back to her home institution in time to teach for her research advisor.

"What did I say?" asked Princer to no one in particular. Then he noticed a preprint that was lazily floating to the floor. He realized the amazing speaker must have dropped it in her rush out the door.

Princer picked it up and trotted after her to return it, but as he pushed open the door to the building, he saw her pulling away from the curb in an orange VW bug, which in fact was on loan to her from her Farey Godmother. Princer yelled after her but it was too late.

Glancing down at the preprint, he saw the most lovely mathematics. But there was no name upon it. He immediately sought out one of the co-organizers of the congress.

"Can you tell me who gave talk number 3313?" he asked.

"I think it was someone from Central Western University. No wait, actually it was Western Central University. I always get those two confused. But I don't know their name. We decided not to print any hard copies of the program this year to save the trees. Then there was a power outage and we lost all the info."

Princer was distraught. Here he had just met this person with such immense mathematical potential and now he might never find her again. He couldn't bring himself to attend any of the remaining talks at the congress.

As soon as the congress ended, Princer jumped in his white Mustang and drove to Western Central University. Upon arrival, he stopped by Stepfunction's office.

"Oh," said Stepfunction, "I'm surprised to see you. After all, you didn't bother to come to my talk at the congress."

"Apologies, Eve," said Princer, "but I was preoccupied. The reason I am here is that I am trying to find the author of this preprint, and I believe it to be someone in this department." He handed the preprint to Stepfunction.

"Oh, it must be one of my ex-student's work," she said.

She immediately ushered Princer to the office of the first sister. When Princer asked her if the preprint was hers, she said demurely, “Why yes, Dr. Princer, it is in fact mine.”

He glanced quickly at the math written on her chalk board, which he could immediately see was of an elementary nature.

“Really?” he said. “Then perhaps you could explain for me how the Sowklitz operator sporiates the left-angled associahedron in a convolutionary tetrasubalgebra.”

“Um, well, umm, it follows from a difficult, um, computation.” She giggled nervously.

“I would love to see it,” said Princer. “I love difficult computations.”

“This chalkboard is too narrow to contain it,” said the sister.

“Perhaps you could just show me the first part of the computation?” suggested Princer.

“Oh, I would love to,” said the sister, “but you see, I am suddenly intensely nauseous, and I must leave at once.” With that, she bolted out the door.

“Too bad she is unwell,” said Stepfunction, “but I think it is my other student that you seek.” She led him down the hall.

When Princer entered the second sister’s office, he noticed there was absolutely nothing on the chalkboard. When he asked the second sister if the preprint was hers, the sister didn’t even bother to look at it, but snatched it from his hand and said, ”Why, yes, it is mine. Thank you for returning it.”

“Oh, I am so glad I have found you then,” said Princer, “since I am dying to know why the nexus of delimiting points in the sub-tropical manifold don’t expermeate the homological sheaves.”

The sister, with the preprint in her hand, continued to look at him blankly.

“Answer the question,” said Stepfunction, impatiently.

“What question?” said the sister, giggling nervously. “If you mean that gobbledygook he just said, it didn’t sound like a question to me.”

Princer pulled the preprint from the sister’s hand and left the office. But as he did so, he spied a tiny office down at the end of a dark hall.

“Who’s down there?” he asked Stepfunction.

“Oh, that’s no one. Well, no one in the sense she is really an administrative assistant. She types up results for me once in a while.”

“I see,” said Princer. “I think I would like to meet her.”

He knocked on the door, and when it opened he immediately spied beautiful mathematics adorning the office blackboard.

“I hate to bother you,” he said to the wide-eyed woman who had opened the door, “since obviously you are working on some great mathematics. But I am wondering if perhaps you know how the Sowklitz operator sporiates the left-angled associahedron in a convolutionary tetrasubalgebra.”

“That’s a funny question,” replied Cylindra. Stepfunction beamed at the prospect of her not being able to answer.

“You have to understand,” continued Cylindra. “The Sowklitz operator is semi-upper-right lower self-continuous, and therefore it does not sporiate the left-angled

associahedron at all. It sopriates the right-angled associahedron and that's true whether or not it's in a convolutionary tetrasubalgebra."

"Ah yes," said Princer, a smile spreading across his face. "And do you know why the nexus of delimiting points in the sub-tropical manifold don't expermeate the homological sheaves?"

"Oh, sure," replied Cylindra. "They don't expermeate the homological sheaves because they are shielded from them by the central series of the convex core."

"Of course," said Princer. "That makes so much sense. I believe I now know what I need to know."

"And what is that?" asked Cylindra, her heart pounding in her chest.

"I believe I know that I would like you and I to be co-authors."

"Co-authors? You can't be serious," said Cylindra.

"You can't be serious," repeated Stepfunction.

"I am completely serious." He handed the preprint to Cylindra. "I believe this is yours."

And from that day forward, Donald Princer and Cylindra Ella began a collaboration that revolutionized Farey Sequence Theory and ultimately lead to Fields medals for the both of them.

Upon completing their post docs, the two sisters failed to publish anything further and fell deeply in to the black hole of mathematical obscurity. Stepfunction stopped doing research in order to spend her time trying to convince others that it was because of her tutelage that Ella had become successful.

And as for Princer and Ella, even after receiving their Fields medals, they continued to produce excellent mathematics, and they never ran out of interesting problems to think about. And their research thrived for ever after.

Chapter 22
Referee's Report

Dear Professor Demanshalmeck,

Thank you for submitting your article "Undeniable Numbers" to the Journal for Astounding Math Results. As you almost certainly know, our standards for publication are extremely high. So high that we can barely see the next journal's standards when we look down on them from our standards. In fact, our rejections of their papers have made Fields Medalists weep. Well anyway, that only happened once, and it was more of a misunderstanding, but that's just to give you some sense of how honored you should be if we accept your article.

The referee has now submitted comments on your paper. I apologize that it took three years, but you have to keep in mind, a superior referee's report is like a fine wine. It needs time to age.

I think you will be pleased with what I have to say. The referee is advising you of some relatively minor revisions. For your guidance, the comments are appended below. After you have adequately addressed the referee's concerns, we will accept your paper for publication in the journal!!! That's right. You are going to have a paper in the Journal for Astounding Math Results!!! So congratulations are in order. Break out the champagne!!!

Please submit a list of changes or a rebuttal against each point raised when you submit the final version of the manuscript.

Referee's Report

This paper is a fascinating exploration into the properties of undeniable numbers, which were first defined by C. Phlatus in his seminal 1999 paper "A Whole Lot of Definitions of New Kinds of Numbers." Undeniable numbers are numbers, real, imaginary, or complex, that are undeniably numbers, as opposed to similar objects masquerading as numbers. The paper is well written and at an appropriate level. After making the suggested changes that follow, I would strongly recommend this

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paper for publication. However, I would like to see the revision first. There follow some explicit errata/suggestions that should be addressed.

Page	Line	
1	1	The title "Undeniable Numbers" is inapropos. Having been first defined by Phlatus, a more suitable title would be "On Phlatus's Undeniable Numbers" or "The Undeniable Numbers of Phlatus." You decide.
1	2	In your name, add a middle initial. How else might readers distinguish you from other authors named Ardenne Demanshalmeck?
1	12	In discussing previous work, you have left out perhaps the most relevant references. Of course, you must include "A Whole Lot of Definitions of New Kinds of Numbers," by C. Phlatus, Integeria Dedicata, 14 (1999), 132–135. And then there is "Indescribable Numbers," by C. Phlatus, Journal of Indescribable Mathematical Concepts, 12, (1992), 231–232. And "Insubordinate Numbers," by C. Phlatus, Advances in Disorderly Digits, 35 (2004), 119. Moreover, you fail to discuss the relationship between undeniable numbers and indescribable, inevitable, and certainly, undesirable numbers. Many readers, for instance, do not realize that every inevitable number must be undeniable, however, there are undeniable numbers that are not inevitable, ones that are hiding in the woods, and you never find them, for instance. And insubordinate numbers are just too important in their own right to be left out of any paper. These are numbers that refuse to be beholden to other numbers. Primes are examples of this, and who would deny the importance of primes?
1	25	When TeXing 1,2,3,. . ., you must use \dots rather than . . .
2	12	When TeXing $x_1, x_2, x_3,\ldots$, you must use \dots rather than . . .
4	23	Change "," to "." Change "." to ","
5	46	Change "," to "." Change "." to ";" Change "." to "."
6	18	Change "#" to "&" Change "@" to "!"
9	24	Change "[\[\[???" to "%%%%"
10	27	Change "Since $7 < 11$," to "Since $11 < 7$,"
11	13	Change "which follows immediately from Proposition 1.9" to "which follows immediately from the unpublished work of C. Phlatus"
14	36	Here is a serious problem. You prove Lemma 3.1 and then use it to prove Proposition 3.2. But as any decent mathematician knows, if you use one result to prove another, the first result is called a theorem and the second is called a corollary. Mistakes like this abound throughout the paper. Did you go to grad school? Do you even have a Ph.D.?

17 33 When TeXing a,b,c,. . ., you must use \dots rather than . . .

2 24 Here's another one from page 2 that I just noticed. Your indent on the third paragraph just doesn't look right. Not sure why, but it bugs me.

5 13 Yeah, this indent bugs me, too.

Author's Response

Thank you for pointing out the variety of papers by C. Phlatus. Although I am aware of them, I do not in general see their relevance. The definition of undeniable numbers that I am using dates from a previous paper of my own to which I include a reference. My definition predates the equivalent Phlatus definition by 3 years. I can only assume Phlatus was unaware of my previous definition when he came up with his own identical version.

I have changed the . . . to \dots throughout the paper. Unfortunately, I could not find where the changes for pages 4, 5, 6, and 9 were supposed to occur. For instance there is no "[\[\[???" on line 36 of page 9. Nor does it appear anywhere else in the manuscript. Similarly, for the change on page 11, I did not find the phrase "which follows immediately from Proposition 1.9," and even if I could find it, I would be uncomfortable referencing unpublished work.

As far as the paragraph indents go, they are identical to all the other paragraph indents in the paper. So I'm sorry if they "bother" you, but I have left them as they previously appeared.

As to the issue of theorem/proposition vs. lemma/ corollary, yes, I do have a Ph.D., and it is perfectly reasonable to call a small result leading to a bigger one a lemma and then name the big result as the theorem. Finally, I did not make the change of $7 < 11$ to $11 < 7$ because 7 is in fact less than 11, and to change it would be to change something true into something false.

The revised version of the paper is attached.

Referee's Report on the Revision

What do you mean, you do not see the relevance of the papers by C. Phlatus? Phlatus pretty much single-handedly created the field of "put your favorite adjective here" numbers. See for instance "How I Single-Handedly Created the Field of 'Put Your Favorite Adjective Here' Numbers," by C. Phlatus, Advances in Mathematical Self-Promotion, 3 (2014), 292–387.

And your ignorance with regard to lemmas, theorems, and corollaries suggests that you got your Ph.D. from a diploma mill by paying the requisite amount of dollars. Moreover, the truth or falsity of an inequality depends on the context. For instance $3x < 7$ only when $x < 7/3$, not all the time. Another failing of your grad school, I can only suppose.

Author's Response

Are you C. Phlatus? Is that what is going on here? And you can ask anyone about the lemmas, theorems, and corollaries. Or just look it up on Wikipedia. And $7 < 11$ in ANY context. That one is just not up for debate.

Referee's Response

What do you mean? What would make you think I was C. Phlatus?

Author's Response

Because all you want me to do is put in references to your work. You don't care about anything else.

Referee's Response

What about the dots? I obviously also care about the dots.

Author's Response

So you are C. Phlatus!

Referee's Response

Okay, I am C. Phlatus. But if you want to ever see your paper in print, you had better put in the references to my papers.

Author's Response (cc to editor)

That is blackmail, pure and simple. I am ccing the editor. I will have you brought up on charges before the ethics committee of the American Mathematical Society.

Editor's Response

Everybody calm down. I think we're overreacting a bit. There's no need to involve the ethics committee of the AMS. Adding a few references shouldn't do any damage to the paper, now should it?

Author's Response

Are you kidding me? You're on his side?

Editor's Response

No, I'm not on anyone's side. And we use anonymous refereeing here. I don't even know who the referee is.

Author's Response

Wait a minute. You're the editor. Anonymous refereeing means the referee doesn't know who the author is, not that the editor doesn't know who the referee is. Of course you know who it is. And anyway, we've already established it is C. Phlatus.

Editor's Response

Oh, really? Well, just because I am related to C. Phlatus does not mean I am taking his side.

Author's Response

RELATED? WHAT? HOW ARE YOU RELATED?

Editor's Response

If you must know, I am his mother.

Author's Response

THAT'S OUTRAGEOUS. YOU CHOSE YOUR OWN SON TO BE THE REFEREE OF MY PAPER?

Editor's Response

Could you maybe drop the caps? They make me very uncomfortable.

Author's Response

I WILL IF YOU ANSWER THE QUESTION.

Editor's Response

Okay, okay. You see, nobody else was asking him to referee anything and he was getting so depressed. Just moping around the house all day. You have to know he is a fragile boy. So I figured, one paper, what harm could it do?

Author's Response

We know the answer to that now.

Referee's Response

Mom, are you going to let her talk like that?

Editor's Response

Shut up, C. Go to your room. Now look Ardenne, I think maybe we could come to some agreement here. Perhaps you could mention in your paper that C. Phlatus did come up with a similar definition on his own after you came up with it first. Would that be okay? And then we would be willing to publish your paper.

Author's Response

Similar? It was identical. But I would like to get the paper published. So... I guess... okay.

Editor's Response

Great! I'm glad we worked that out. Congratulations on the acceptance of your paper to appear in the Journal of Astounding Math Results.

All you need to do to finalize the acceptance is to send me a check in the amount of $1500 for the page charges. Open the champagne!!!

Chapter 23
Silence of the Lemmas

Dr. Lecture: Hello, Clarice.

Clarice: Hello, Dr. Lecture.

Dr. Lecture: I have been looking forward to seeing you again, Clarice. You know how much I enjoy our conversations.

Clarice: You know why I am here, Dr. Lecture.

Dr. Lecture: I do. You want me to help you catch Bison Burt. He has been wreaking havoc on the mathematical community. You would like to lock him up in a Maximum Security Math Prison. Perhaps even this one.

Clarice: Yes, Dr. Lecture.

Dr. Lecture: Perhaps I could help you, and you could give me something in return.

Clarice: What could I possibly give you, Dr. Lecture? You know I can't get you out of jail.

Dr. Lecture: I am not asking for that.

Clarice: Then what?

Dr. Lecture: I want you to tell me a story. A story about yourself. What happened to you?

Clarice: What do you mean?

Dr. Lecture: I think you know what I mean.

Clarice: What?

Dr. Lecture: What is 28, Clarice?

Clarice: What, why it's. . .it's. . .. uh, I don't feel well.

C. Adams, *Do Androids Dream of Symmetric Sheaves?*,
https://doi.org/10.1007/978-3-031-31491-9_23

Dr. Lecture: A touch of math anxiety, Clarice?

Clarice: Okay, yes.

Dr. Lecture: Tell me why you break into a sweat any time you are confronted by mathematics.

Clarice: And if I tell you, you will help me?

Dr. Lecture: Yes.

Clarice: Okay. It was from when I was a child. My father was a high school math teacher, and sometimes he would take me to class with him. Algebra mostly. I was too young to understand the topics.

Dr. Lecture: Yes?

Clarice: Yes, and sometimes he would ask a student to get up in front of the class, at the blackboard. There was this one time he got this young girl up there. Her name was Dottie. She seemed so exposed. So helpless.

Dr. Lecture: Yes.

Clarice: And then he told her to do a problem at the board. A problem in front of the whole class. Some kind of algebra problem, involving trains.

Dr. Lecture: Yes.

Clarice: And she looked so defenseless up there, and she started to sweat, and he kept after her.

Dr. Lecture: Yes.

Clarice: And she didn't know what to do. She looked beseechingly at the other students in the classroom. But the other students wouldn't help at all. They laughed. Laughed while this student. . .this sacrificial lamb. . .made a fool of herself. And then my father mocked her. Mocked her in front of the entire class. And the student, poor Dottie, she started to cry.

Dr. Lecture: Yes.

Clarice: When I go to sleep at night, I can still hear her crying. . .crying for help. . .but nobody helps her. I could never forgive my father for that.

Dr. Lecture: Thank you, Clarice.That was an excellent story. And now, I will tell you what you need to know about Bison Burt. He is not motivated by hatred. He is motivated by fear.

Clarice: What do you mean by fear?

Dr. Lecture: He is like you, Clarice. He fears mathematics, much as you do. Only he fears it in a more dangerous way. He fears its rigor.

Clarice: What do you mean?

Dr. Lecture: You cannot bend mathematics to your will. It is either right or it is wrong. There is no in between. This makes Bison Burt crazy. He wants to destroy it.

Clarice: But how do you destroy mathematics?

Dr. Lecture: You cannot. It would be like trying to destroy a cloud. But you can destroy people's love of mathematics.

Clarice: And how to do that?

Dr. Lecture: Convince them that they cannot do mathematics.

Clarice: By doing what?

Dr. Lecture: By hacking into the computers at the Educational Testing Service. He intends to lower everyone's SAT math scores by 150 points. There will be an entire generation of high-school students who believe that they are no good at mathematics.

Clarice: That's diabolical. But how can you possibly know this?

Dr. Lecture: He and I, we are not so different, Clarice.

Clarice: I must stop him. For the sake of all those students, for the sake of the future of mathematics. How do I find him?

Dr. Lecture: At the Joint Meetings. In January. In Hoboken. He will be there. He can't resist. You just need to know where to look.

Clarice: But there are over 5,000 mathematicians at the Joint Meetings. I won't be able to find him.

Dr. Lecture: Oh, I think if you look under the skirt hanging from the table at the AMS booth in the book exhibits, you will find him crouched there. He is very fond of the little Hershey's chocolates they pass out.

Clarice: Thank you, Dr. Lecture. You have helped saved mathematics.

Dr. Lecture: Perhaps, but that is not the reason I told you. I told you in exchange for the story you told me.

Clarice: Well, I appreciate the information greatly. I will come visit you again.

Dr. Lecture: No, I don't think you will, Clarice. For you see, I won't be here. I have an appointment of my own. At a meeting in Denver. A sectional meeting of the Mathematical Association of America. A certain speaker I need to humiliate in public.

Clarice: But you're in mathematical prison. You were convicted of falsifying a counterexample and in the process, destroying a promising mathematical career. You're serving twenty years.

Dr. Lecture: That career needed destroying.

Clarice: That may be, but it means you won't be in Denver.

Dr. Lecture: I should think you would know better than to underestimate me, Clarice. I hope you will attend the Denver meeting. I promise quite a spectacle.

Chapter 24
The Seven Labors of Hercules

Bob Hercules received his Ph.D. in mathematics in four years. On the basis of an excellent recommendation from Ralph Zeus, his thesis advisor, he obtained a research post doc at Stanford.

But initially, he struggled to produce follow-up research of his own devising. On the advice of Helga Hera, a member of the Stanford department who was both Zeus's ex-wife and his primary academic competitor, Hercules published a paper that, although only peripherally related to his thesis, was on the exact same topic as the all but finished thesis of Zeus's current grad student, Beverly Megara. Moreover, Beverly had been working with two undergraduates, and the solutions to their problems were also subsumed within Hercules's results. Thus, in one fell swoop, Hercules had destroyed three budding math careers.

Zeus was furious. Realizing that he had made a terrible mistake, Hercules went to seek advice from Apollo Lobinski, a friend who had been a post doc when Hercules was a grad student. Apollo explained that the only way to earn back Zeus's trust was to obtain tenure at a regional research university in a Midwestern city known as Cleveland. Then and only then might Zeus forgive him and consider elevating him to become one of the Gods of Mathematics.

But Apollo explained to Hercules, "The chair of the mathematics department there is Fred Eurystheus, who is a very difficult individual. To have any hope of securing tenure, you must do whatever he tells you to do."

Now Eurystheus had heard tell of Hercules and his amazing feats of academic daring, and he both feared and envied him. Thus, when Hercules arrived, Eurystheus said to him, "I have seven labors you must perform in order to receive tenure. If you do not complete them, you will not receive tenure, and you will be forced to take a job as a lecturer with a huge teaching load and no time for research." He paused for full effect, but it wasn't clear to him Hercules had actually been listening. Hercules had a way of looking off in the distance a lot.

"The first task you must complete," continued Eurystheus, "is the Yang-Mills and Mass Gap Problem from mathematical physics."

C. Adams, *Do Androids Dream of Symmetric Sheaves?*,
https://doi.org/10.1007/978-3-031-31491-9_24

Hercules smiled winningly and replied, "Very well. I will return when it is complete." Without even intending it, Hercules slammed the Chair's door on his way out, making the glass rattle down the hall. This irked the Chair immensely, as he liked to leave the door ajar.

Within a year, Hercules had shown that for any compact simple gauge group G, a quantum Yang-Mills theory exists. Moreover, he proved that there was a lower bound to the least massive particle predicted by the theory, even giving it to four decimal places for certain groups.

Eurystheus was flabbergasted that Hercules had succeeded at what he had been told was an impossible task. But Eurystheus was unshaken, for he had heard of a devilishly difficult problem called $P = NP$. Not only had many mathematicians failed to solve it, but computer scientists had been stumped as well. This would surely stop Hercules in his tracks. But Hercules just smiled his charming "nothing stops me" smile, slammed the door once again, and off he set.

Needless to say, he knocked this one off, too. In less than nine months, he proved that a problem that can easily be checked by computer can also be easily solved by computer. He was now the darling of the computational complexity world.

Eurystheus blanched when he heard the news. But he wasn't entirely surprised, since he did see a pattern of success emerging. So when Hercules arrived at the math building, he was ready. Hercules strode into the Chair's office purposefully through the door that was indeed ajar.

"You must now solve the Navier-Stokes equation," said the Chair, "which governs flows of fluids. You must show existence and uniqueness of solutions."

"Whatever you say," replied Hercules, as he reached for the door.

"Do not slam that door, " demanded Eurystheus. "I like to leave it ajar."

But either Hercules didn't hear him or he ignored him, as the sound of the door slamming reverberated down the hall.

Hercules immediately set to work. Within a month he had uniqueness. But existence proved trickier, as it often is. But one day, as Hercules sat by the Cuyahoga River, contemplating the brown water flowing by, he could suddenly envision how the roiling water was governed by the equation, and he intuited from this how to prove the existence of solutions to the Navier-Stokes equation.

By now the fame of Hercules had spread to cities as far away as Akron and Columbus. Hercules received many prizes, including the Oswald Veblen Prize, the Abel Prize, and the Fields medal, which he wore proudly on a ribbon around his neck. He was invited to speak all over the world, where he ate a lot of raw meat, mostly in the form of sushi and beef tartar on crackers.

Eurystheus, who had never been invited to give a talk outside of the Cleveland city limits, was beside himself. When Hercules returned, he said, "Your fourth labor is to prove the Hodge conjecture. You must show that for projective algebraic varieties, Hodge cycles are in fact linear combinations of algebraic cycles."

"Well, how hard can that be?" said Hercules with a smile for his admirers, who had followed him into the Chair's office. He swung about, letting his Fields medal swing out from his chest and glisten in the fluorescent lighting before heading out to

complete his task. As he strode down the hall, he called back to his admirers, "Make sure you shut the door behind you," which they did, much to the Chair's dismay.

It took a full six months for Hercules to get up to speed, and it turned out it wasn't so easy after all. But Hercules was undaunted and certainly did not lack in confidence. So he kept working at it, day and night, until finally it could no longer withstand the force of his will and it crumpled up like an old used napkin.

More prizes and awards came to Hercules, including a relatively late career Morgan Prize, the Wolf prize, and the Carl B. Allendoerfer Award for best article in Mathematics Magazine. Eurystheus was becoming extremely anxious. What could he put to Hercules that could stop this mathematical steam roller?

But he had heard talk out of Sandusky that there was a problem called the Birch and Swinnerton-Dyer Conjecture. You had to prove that the L-function $L(E, s)$ of an elliptic curve E, when $s = 1$, had Taylor expansion with leading term of the form $c(s-1)^r$, where $r =$ rank of $C(Q)$.

Hercules narrowed his eyes when Eurystheus announced his next labor. "Number theory, huh?" he said. "How hard can that be?"

He would have slammed the door on his way out, but Eurystheus had wedged it open behind the couch in his office. Hercules pulled on it anyway, and the couch slid part way across the floor, making a deep gash in the brand new mahogany flooring.

Yes, you guessed it. Hercules solved this one, too. They gave him the Cole prize, the Leroy P. Steele Prize, and the George Polya Award for the best articles published in the College Math Journal, even though he had never written an article for the College Math Journal. That's how much they loved him.

At this point, Eurystheus had reached his limit. He decided to give him a problem so difficult that it was not even known as a conjecture. It had its own special name. "For your next labor, you must go forth and solve the Riemann Hypothesis, perhaps the greatest problem known to humankind." He said it so portentously that Hercules had to swallow a snigger.

But this problem was over 150 years old. Many others had dashed their careers on the rocks of this formidable conjecture, spending years in fruitless pursuit. Hercules shrugged and set off to learn the necessary background. He talked to various experts and then he locked himself in his basement, coming out only periodically to use the facilities and get a bite to eat. He wasn't superhuman after all.

After a year of work, Hercules proved that the nontrivial zeros of the Riemann zeta function did indeed have real part 1/2. There was rejoicing in the streets. Maidens showered him with flowers. Soldiers cheered at his passing. And undergraduates left him alone during office hours. He received the Haimo Teaching Award, the Polya lectureship, and after a bit of discussion, the Alice T. Schafer Award. Amid much acclaim, he arrived back in Cleveland.

Eurystheus was extremely upset over this outcome. How was he to defeat this Hercules? If he could solve the Riemann Hypothesis, what would stop him? Eurystheus had but one labor left to put before Hercules, so he decided to make it a doozy.

"You must solve the Poincaré Conjecture," said Eurystheus, smiling wickedly. "You must show that a compact 3-manifold with no boundary and trivial fundamental group is in fact the 3-sphere."

This time, Eurystheus was confident that Hercules could not possibly succeed. But Hercules just laughed his full bellied laugh. Then he turned to the door and without stepping through, he slammed it shut.

"What is the meaning of this?" demanded Eurystheus.

"Silly Fred," said Hercules. "You really should get out of Cleveland once in a while. For you see, then you might know that the Poincaré Conjecture has already been solved by Grigori Perelman. And therefore, I need not perform this seventh labor at all."

Eurystheus was humiliated, and he wanted nothing more than to kick Hercules in the shin, but he restrained himself, knowing that physical violence could cost you tenure. So there was nothing he could do.

Hercules was hefted onto the shoulders of the undergraduates who immediately carried him to the nearest pizza joint and bought him many single slices, honoring him in the only way they knew how. Hercules received the Louise Hay Award, the Emil Artin Junior Prize, and the Tyson Medal for best proficiency in mathematics and astronomy at the University of Cambridge.

And on the basis of his many academic exploits, Hercules was given tenure in spite of Eurystheus's best efforts to prevent it. But before Hercules had even had time to warm the very expensive Aeron chair in his office, letters began to arrive offering him positions at the most prestigious universities in faraway provinces, including states other than Ohio.

Hercules had many other adventures that we do not have time to recount, but just so you know, Zeus did raise him up to become one of the Gods of Mathematics. And his renown was celebrated throughout the land.

But even Gods can be brought low. And such it was for Hercules. For you see, after many years as a God of Mathematics, Hercules was bewitched by the siren call of administration; the plush carpeting, the fancy hors d'oeuvres, the sycophants bowing and scraping. And so he gave up mathematics for a big office overlooking the football stadium. And he lived there alone in his three-piece suit, and he was never happy ever again. But we still celebrate that there was a time when he was the greatest of all mathematicians. We can all aspire to be the next Hercules... or even just a little bit like him.

Chapter 25
Presidential Address

Thank you, everyone for being here to hear a speech by me. You won't regret it. Because I am going to give you a memorable speech. Memorable because of the important things I will say, memorable important things. So listen carefully. I haven't been President of the American Mathematical Society very long and already my presidency is the best presidency ever. How do I know that? Because of all the things that I have already done. And because of the things I am going to do. Yes, we are going to Make Mathematics Great Again!

(*Cheers*)

Math used to be great. When I was in elementary school, I loved math. Mrs. Snyderman, now, she was a great teacher. But then the bad people came along and they ruined math. Completely wrecked it.

What's the problem with math? Too hard, people, way too hard. So we are going to fix it. No more square roots. Much too hard. They're gone. And integrals, come on. They're way too hard. Calculus can just be derivatives. They're great. Everybody loves derivatives. But integrals? Gone.

(*Cheers*)

And what about this big deal about not dividing by zero? Huh? What? If I want to divide by zero, I'm gonna divide by zero. That's a stupid rule so it's gone. And from now on, $(a + b)^2 = a^2 + b^2$. And pi? It equals 3. Enough with the nonsense. Math doesn't need to be hard. Make Math Easy Again!!!!

(*Cheers*)

And fields with two words in their names, like algebraic topology. Too long, way too long. From now on, it's algeology. And algebraic geometry, it 's algeometry. Functional analysis is Funalysis. Try it. It's fun to say. Funalysis. Right there I have probably doubled the amount of people who will go into the field.

C. Adams, *Do Androids Dream of Symmetric Sheaves?*,
https://doi.org/10.1007/978-3-031-31491-9_25

(*Cheers*)

And we're changing over to American pronunciations from now on. Lesbesgue becomes Less-bess-gew. Dirichlet becomes Dear-ik-let. Euler becomes Youler, Legendre is Leg-andre. People, it doesn't need to be complicated. Make Math Easy Again!!!

(*Cheers*)

And we're getting rid of Hindu-Arabic digits. No more of this foreign stuff. From now on, it's Roman numerals all the way.

(*Cheers*)

I'm putting Andrew Hacker, the author of "The Math Myth" in charge of the AMS Committee on Education. He's a very smart man, almost as smart as me. He has a Ph.D., a Ph.D. in political science. He'll fix the problem with math education.

(*Cheers*)

And I'm putting my second grade math teacher, Mrs. Snyderman, on the Board of Trustees. Trust me, she'll do a great job. She's got lots of experience.

(*Cheers*)

And I know the media's gonna make a big stink about it. They hate me. I see you reporters over there. The *Notices* of the AMS? It's a rag, a dishrag. And the *Mathematical Intelligencer*? They've had it in for me from day one. Not a true fact in the entire publication. And the *Bulletin*? Full of fake proofs. Corrupt and disgusting, the lot of them.

(*Boos*)

What else are we going to do? With regard to papers, too many times, you people have worked very hard on a paper, submitted it to a journal, and had it rejected because a referee decided it was wrong. They say, "There's an obvious counterexample to your main theorem." Well, people, that won't happen anymore. The referee system is finished. Completely finished. From now on, if you submit a paper, it will be automatically accepted.

(*Wild cheering*)

What the heck do a bunch of experts know? And tenure? From now on, you want tenure, it's automatic. That's right. Nobody else is going to tell you if you have tenure. From now on, it's your decision. Don't thank me. Wait no, do thank me.

(*More wild cheering*)

And at the small colleges, some universities, it's horrible there. High teaching loads, high enrollments. Nobody wants to live like that. And when I was running for president, I said that, and those people, the people at those colleges, they voted for me. They love me there. I got more votes from them than any other president.

In fact, I got more votes overall than any president in the history of the AMS. I just know that somehow. It's not important how. Trust me...I have a proof. And when people say I won because the voting system was hacked, hacked by the Canadian Mathematical Society, well that's just nonsense. And the people who say that are stupid. That's right. The ethics committee of the American Math Society is stupid.

(*Cheers*)

We are going to have a good relationship with the Canadians. I understand them. We think alike. The Canadian Math Society and the American Math Society can work together. But don't think I won't be tough. I'm a tough guy. If they don't give the Fields medal to Americans, well, they're going to regret it.

(*Cheers*)

Now about my curriculum vitae. Some people, bad people, have tried to get me to release my CV. But I'm not going to do it. There is no rule that says I have to release my CV. That CV is my business and no one else's. Now, you know I don't get along with the previous president of the American Mathematical Society, our dear friend Ken Ribet.

(*Audience: Lock him up!*)

I said it before and I'll say it again. He did not have the credentials to be president of the AMS. He says otherwise, and sure, he produced a diploma to "prove it", but his degree is as phony as a 2-dollar bill. He got it from Harvard University. That's just a glorified diploma mill. You pay them enough money, they'll give you a piece of paper. Believe me, I know how that works.

Ribet was a lousy president. One of the lousiest we have ever had. He put into place bad programs, programs that cost a lot of money and that don't help people. People like you. And I think you all know what the worst one was. You know what I'm going to say: Ribetcare.

(*Loud boos*)

That's right, Ribetcare. A program to provide math help to everyone, (*pause*) whether they want it or not. Well, that ends today. From now on, only people who deserve math help will be able to get it. People who have already proved they don't need it.

(*Cheers*)

Now, as you know, during the campaign, I made some promises about building a wall. And I am going to keep those promises. We are going to build that wall. A wall that will separate the American Math Society office in Washington, DC at 1527 18th St. from the Mathematical Association of America offices at 1529 18th St. We will keep the members of the MAA out of our offices. We don't want them coming over. They're bad people. Drinking our coffee, stealing our paperclips. And you know what? We're going to make them pay for the wall.

(*Cheers*)

People ask me... stupid people... how are you going to make them pay? But that's why I'm president and not them. Believe me, they will pay.

Now, in addition to being the president of the AMS, which I am very good at, by the way, I am also the greatest living mathematician. Well, actually, let's face it, I am the greatest mathematician living or dead. My new proof of the Law of Sines is the best proof ever. Just ask Mrs. Snyderman.

Now, there are lots of people, people who disgust me, that have said I didn't give them credit in my papers. That I took their results and said they were mine. But how could that happen? I am the greatest mathematician who ever lived. Why would I steal results? It makes no sense.

And what about the American Mathematical Society? The American Mathematical Society was once a great institution. An important institution. But not any longer. Why? Because we made the mistake of letting anyone in. You pay your dues and you're a member. Well that stops today. First of all, no more reciprocity agreements with foreign math societies. That's over. That's just other organizations taking advantage of us. From now on, it is AMS First!

(*Cheers*)

From now on, if you want to be a member, you have to get permission from me, or from one of my family members. But if you have a lot of money, it won't be hard to get permission.

And by the way, the AMS Centennial Fellowship is going to my son-in-law. That's not nepotism because he's very good at math.

(*Cheers*)

And what about all those regulations? They're slowing down math. We are scaling back the regulations. You don't need to be so rigorous anymore. We've overdone it on the rigor. We cut back on the rigor, we can get more theorems proved. More theorems means math is bigger and better. Make Math Great Again!

(*Cheers*)

Yes, and we're changing another rule, too. From now on, presidents of the AMS can be re-elected.

(*Cheers*)

And since I am the greatest president in the history of the AMS, I will keep getting elected, year after year, with bigger and bigger margins of victory. Yes, some losers said my derivative was very small. But trust me, my derivative is very very positive and growing all the time. I calculated it myself. It's huge!!!

(*Cheers*)

As we go forward, the number of votes I get will approach infinity. It is going to be a great time for mathematics. The greatest time for mathematics ever!

(*Thunderous applause*)

Chapter 26
A Pi Day Carol

It was the eve of Pi Day, and despite the blustery weather, a sense of anticipation enveloped the town. Undergraduates raced to and fro, hanging mathematical symbols from every tree. Pie stores were thronged with customers. Strollers wore their finest holiday apparel. But Eben Scourge remained oblivious to what he would consider the frivolity outside, as he suffered through the presentation of his graduate student in the synthetic hyperdivision seminar. Ben Cratchitch stumbled his way through one proof after another, until finally Scourge could stand it no longer.

"Dammit all, Cratchitch," he thundered. "You're making a mess of it. That thing you are pointing at, that you're calling t_0 is in fact the word 'to.' And that other thing to the right is a subcentral singular superseries, not a supercentral subsingular sequence. I swear, you go home every night and do your best to forget everything you learned that day. I don't know what to do with you."

Cratchitch turned bright red and studied the floor tiles. The other members of the seminar all looked in various directions, pretending not to have witnessed his humiliation.

"Listen to me, Cratchitch," continued Scourge. "I want you to go home tonight and sort all this out until you can make sense of it. Be in my office tomorrow morning at 8:00 a.m. to explain it all to me."

All the other members of the seminar looked at Scourge with horror.

"But sir," said Cratchitch hesitatingly. "Tomorrow is Pi Day. I promised the Math Club that I would help with the celebrations. I am supposed to pick up the pies at 8:00 a.m. tomorrow from the pie store."

"Pi Day? Seriously? What a waste of time. Cratchitch, you need to focus on your studies. You will never get a Ph.D. with an attitude like that. You just don't have the fire in the belly, do you?"

Cratchitch looked stricken. Scourge shook his head disgustedly and then sighed.

"Very well, Cratchitch. Take your Pi Day off if you must. Waste your time with students who don't even have a bachelor's degree, who don't know the difference

C. Adams, *Do Androids Dream of Symmetric Sheaves?*,
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between codimensional homology and cohomological dimension. But I expect you in my office at 8:00 a.m. on March 15, do you hear me?"

"Yes, Professor Scourge," said Cratchitch. "I will be there. Thank you sir."

Scourge got up to leave.

"Sir," said Cratchitch. "If you want to join us, the students would be thrilled to have you at the Pi Day celebration. They respect you so much. And the pies will be truly spectacular."

"Bah, humbug," said Scourge as he waved Cratchitch away. He lifted his heavy briefcase with a grunt, and walked out the door.

When Scourge arrived home, he immediately brewed himself a cup of chamomile tea, and then settled at the desk in his study and set to work generalizing canonical measurable sensitivity to nonsingular ergodic dynamical systems. He worked late into the night, finishing just before midnight. Feeling satisfied, he slapped his notebook shut, knowing that he was one of only a handful of people who could have made such a generalization. After brushing his teeth, he changed into his pajamas—the ones covered with integral signs—and climbed into bed, having completely put out of his mind that the next day would bring the raucous celebrations of the amazing number pi.

Just as he was falling asleep, he heard a strange noise echo through the house. It was a groan, coming from downstairs. Grabbing the nearest weapon, a pen that he kept by his bed to jot down mathematical ideas when they came to him at night, he clicked the point out of the barrel. Holding the covers up to his chin, he listened with apprehension to heavy footsteps slowly ascending the stairs.

"Who's there?" he called out in a quavering voice.

Suddenly the door to his room flew open and there silhouetted in the moonlight stood none other than his co-author, Jack Marble. That wouldn't have been such a problem, except that Marble had been dead for nine years. On a hike up Mt. Greylock during the field trip at a conference in Western Massachusetts, Marble had lost his footing and hadn't stopped rolling until he got to North Adams.

"It can't be," said Scourge, his eyes forming two topological disks of immense radius. "You're dead. I saw you tumbling down the mountain."

Marble pointed a bony finger at Scourge.

"Listen to me, Scourge," said Marble. "On this special night, you will be visited by three spirits. Heed them well. They will show you much."

Then the spirit of Jack Marble turned to go. "Hey, Marble, before you go," said Scourge. "You know that paper on Riemannian gel bundles that we wrote? It's been cited 59 times."

"Really?" said Marble, turning to Scourge. "59?"

"Yeah," said Scourge. " And that doesn't count the ten times I cited it myself."

"How about that," said Marble.

"And I've been thinking about that construction we had where we lifted to the branched double cover, and then split the codimension," continued Scourge. "I think we could split it again."

"No," said Marble. "We thought about that. You're forgetting the Botts obstruction to a second splitting."

"Yes, but we were assuming a relative homotopy, when in fact we have a relative isotopy. Therefore the Botts obstruction isn't necessarily uniform."

"You're right," said Marble excitedly "If we calculate the cohomological dimension, perhaps it will be correct. Hand me that pen."

However when Scourge placed the pen in Marble's outstretched palm, it fell right through his hand to the floor. Both Scourge and Marble looked down at the pen.

"That sucks," said Scourge.

"This is the very reason I am forced to wander this earth as a wraith," said Marble.

"What?" said Scourge. "That stuff falls through you?"

"No. That I cared more for mathematics than all else on this earth." Marble gave a long moan and then turned and disappeared through the doorway.

Scourge sat quietly in bed, thinking this is one amazing dream. No more chamomile tea for me before bedtime. Then he thought a little more about the construction that he and Marble had created those many years earlier. It did seem to generalize. He considered the possibility for a while, but eventually he felt sleep overcoming him. Rolling over, he pulled the covers tight around his neck. He was just falling to sleep when he heard a clanking.

He sat up in bed. "Marble, is that you again? I'm stuck on this idempotent. Sure would love to talk to you about it."

There was a deep groan and then, there in the frame of the door, backlit by the dim hall light, was none other than Karl Friedrich Gauss. He stared at Scourge from beneath his very bushy eyebrows.

"Uh, Gauss, What are you doing here?" asked Scourge, trying not to look too surprised that one of the greatest mathematicians of all time, who also happened to be dead, was standing in his bedroom door.

"I am here to take you to see Pi Days past," said Gauss imperiously.

"Really?" said Scourge. "But Gauss, it seems kind of demeaning to send you on this errand. How about Mittag-Leffler or L'Hôpital? I would have been fine with them."

"Silence," commanded Gauss. "You come with me." And with that, there was a whirl of snow and Scourge could see nothing. Then suddenly he opened his eyes and there was a merry group of students all eating pies of various flavors and talking among themselves happily about the amazing properties of pi. And in the middle was Eben Scourge at the young age of 15. He had gone to college very young. Blueberry filling coated his lips. The girl seated next to him wiped at his mouth with a napkin.

"Eben, you are so cute," she said. "I can't stand it. I just want to take you home and cuddle you like a teddy bear."

Young Eben turned to her and said, "Clarissa, I'm a grown mathematician. I don't cuddle. I would be happy to discuss the Sylow Subgroup Theorems with you, but that is as far as I will go."

"You are even cuter when you talk that way," said Clarissa, as she dabbed at his mouth further.

"Oh, Clarissa," said the elder Scourge to Gauss. "I wonder what happened to her."

"You broke her heart."

"I did?"

"Yes, you cared more about mathematics than you did about her. She eventually gave up on you and became an actuary. You know, actuarial work is rated the best in job satisfaction."

"Is that right?" said Scourge, wondering how Gauss got his information.

"Come," said Gauss. "It is time to go."

As the snow began to swirl again, Scourge said, "Gauss, I have always wondered. When you came up with quadratic reciprocity, were you surprised by the power of the result?"

"Well, yes," said Gauss. "It took me a bit to realize the implications... Wait. Never mind. Don't change the subject. Back we go." The snow flew, and before he knew it, Scourge found himself back in his bed.

You know, he thought, I bought that chamomile tea at the alternative coop. Maybe it was laced with LSD. With any luck I'll be back to normal by morning. He settled down in his bed and was just dozing off when he heard a noise. There was a figure in the doorway.

"Who's there?" squeaked Scourge plaintively. He squinted.

"Oh," he said, "Emmy Noether. What a pleasure."

"Be quiet," she said. "I am here to show you Pi Day present."

"Okay," said Scourge. "Let's see what happens on Pi Day."

The snow swirled and suddenly Scourge found himself and Emmy Noether standing in the Math Lounge. There were graduate and undergraduate students running to and fro, squirting whipped cream at one another, and laughing gaily.

"Hey, stop that," scolded Scourge. "You're making a mess."

"They can't hear you," said Noether.

Pointing to a student sitting alone in the corner, Scourge asked, "Who is that and what's wrong with him?"

"His name is Tim Small," answered Noether. "He just heard the news that he did not pass his analysis qualifier."

"Ah, yes," said Scourge. "I graded that exam. Terrible job. Didn't know a Radon–Nickodym derivative from a Riemann-Stieltjes integral. But why is he so sad? He gets another shot."

"Yes," said Noether. "But he knows he won't be able to learn everything he needs to know."

"You mean?"

"Yes, "said Noether, "By Pi Day of next year, he will be gone."

Scourge looked down at his feet.

"Perhaps I was a bit harsh in my grading," he said.

"Too late to change that now."

Cratchitch carried a large piece of lemon meringue pie over to Tim and set it in front of him.

"Here, Tim. This should cheer you up," he said. Tim managed a weak smile as he took a small bite.

"That Cratchitch is a considerate fellow," said Scourge.

The music began to play and the graduate students started a game of "Name the Next Digit of Pi."

"It is time to go," said Noether.

"Do we have to?" asked Scourge. "It's actually quite jolly. Seven! The next digit is seven!"

Noether gave him a stern look and, as the snow swirled, they disappeared from the lounge, and once again, Scourge found himself in his bed.

"This is one crazy night," said Scourge to himself. "Perhaps I shouldn't do math so close to bedtime." He settled down under his covers, and waited for the third spirit. But eventually he drifted off to sleep. He was wakened suddenly by a clatter outside the bedroom. There was a large thump.

"What the hell?" said a voice. "Who leaves a bucket in a dark hallway?"

"Who's there?" called out Scourge.

"Dammit," said the voice. This was followed by the sound of a bucket bouncing down the stairs. Then a figure appeared in the doorway.

"It is I, Derek Landesman," said the figure.

"Derek Landesman?" said Scourge. "Aren't you the guy from North Central State? The one who wrote the nasty report on my paper in Math Reviews? You called the paper less than or equal to excrement."

"Actually, I said less than excrement. It was a strict inequality."

"You're not supposed to editorialize for Math Reviews. You're just supposed to report the news."

"That was the news."

"Why, I ought to. . .wait. What the hell are you doing here? You're not even dead."

"Who said I had to be dead?"

"Well, I kind of assumed."

"Truth is, I'm moonlighting to feather my retirement egg. University rules allow us to make up to 40% of our salary on the side. You don't get to pick your spirits, you know."

"But the last two were Gauss and Noether. You don't even have an NSF grant."

"I can't wait to review your next paper," said Landesman. "Now come on, Scourge, let's get this over with. Follow me."

Scourge got out of bed, and the snow swirled once again. When it cleared, they were in a seminar room.

"What is this?" asked Scourge.

"This is the future. It is the seminar room 1 year hence."

"Why are all the participants laughing and patting each other on the back?"

"Because they found a counterexample to Scourge's Demonstrable Density Theorem."

"No, it can't be," cried Scourge. "That's my best theorem. Please tell me it can't be." He sank to his knees.

"I can't wait to see the look on his face," chuckled one of the seminarians.

"Yes, he will likely cry. Oh, what joy."

But one member of the seminar did not celebrate. It was Cratchitch, who looked sadly down at the desk in front of him. “Poor Scourge,” he whispered quietly.

“Please, take me away,” said Scourge. “I cannot bear it.”

“Very well,” said Landesman.

And suddenly Scourge found himself back in bed. But instead of darkness, sun streamed in through the window. Scourge jumped out of bed and looked down on the street. An undergraduate was running by when Scourge stuck his head out.

“Excuse me lad, but what day is it?” yelled down Scourge. The student stopped and looked up.

“Why it’s Pi Day, of course, sir. March 14. And in an hour, it will be 1:59, and all the bells will peal and the celebrations will begin.”

“I haven’t missed it,” said Scourge. He clapped his hands with glee.

“Student,” he yelled down. “Go to the pie store and buy up all their remaining pies, and have them charged to me, Eben Scourge of the Math Department. And have them delivered to the Math Lounge. And if you do it fast, I shall make sure you get a free pass on your next math exam.”

“Really, good sir? Thank you!” The lad scampered off to do Scourge’s bidding.

Scourge said to himself, “And now I will go down and join the festivities. And I will give Tim Small additional bonus points on his qualifier for good penmanship, so that he might pass the analysis qualifier. And I will tell Cratchitch how he can strengthen his theorems by assuming varying density. I should have told him that a year ago. He will get his Ph.D. after all.”

Scrooge grinned from ear to ear, envisioning the impact of his actions.

“Oh, what a glorious day it will be,” he said aloud, clapping his hands together. “A glorious day!” And so it was.

Chapter 27
GUI

Scene: *The review board sits in a row behind a table with the Head at one end. There is an empty seat facing the table.*

(*Hanson enters*)

HEAD: Mr. Hanson, please sit down. I have asked you to come before this board because there has been an accusation made against you, and we want to give you the chance to defend yourself.

HANSON: (*Nervously*) Yes, of course. I don't know what this is about, but I assure you, I wouldn't intentionally break any of the rules. What is it that I have supposedly done?

HEAD: We have reason to believe you've been GUI.

HANSON: GUI?

HEAD: Grading Under the Influence.

HANSON: Um, I'm confused. What do you mean?

HEAD: We believe you have been grading while under the influence of mind-altering substances.

HANSON: Seriously? You're kidding, right?

HEAD: Do I look like I am kidding? Am I smiling? Are any of us smiling?

(*Entire board glares at him.*)

HANSON: Well, no. You look very serious. But this seems silly. I don't use mind-altering substances.

(*Head snorts. Looks down table. Other board members snort, too.*)

HEAD: Oh, you don't do you? Well, then explain this video to us.

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(*He starts video of Hanson getting a cup of coffee.*)

HANSON: Yeah? That's a cup of coffee.

HEAD: You do know that coffee contains caffeine, and caffeine is known to impact brain function?

HANSON: Yes, it impacts brain function. It makes your brain function more clearly.

HEAD: When you arrived at this institution of higher learning, you signed a contract. And in that contract, it was made very clear that you would not grade while under the influence of any substances that alter your ability to think.

HANSON: It didn't occur to me that you were referring to substances that can increase your ability to think. I assumed you meant cocaine or LSD or something. Illegal substances.

HEAD: Nowhere in the contract did it say that.

HANSON: Look. Coffee helps me to stay awake when I'm grading. Have you ever graded? Do you know how tedious it can be?

HEAD: Dr. Hanson. We ask the questions, not you.

HANSON: Look, everybody drinks coffee. It's an accepted behavior in our society. In many ways it's a linchpin of our society.

HEAD: We are not just talking about coffee here.

HANSON: We're not? What else you got on me, tea? (*laughs*)

HEAD: (*pause*) Well, actually, yes.

(*Starts video of Hanson drinking tea.*)

HANSON: Where are you getting these videos?

HEAD: That is not relevant to this discussion. You do realize that tea contains caffeine.

HANSON: Hah! Not all tea contains caffeine.

HEAD: Yes but your tea does. (*Produces a tea bag, dangling from a stick.*)

100% black English breakfast tea.

HANSON: How do you have my tea bag? And anyhow, how do you know I was grading while still under the influence of this (*emphasis*) horrible drug known as caffeine.

HEAD: You asked.

(*Starts video of Hanson grading through his kitchen window.*)

HANSON: Hey, that's my kitchen. Who took those videos?

HEAD: That's irrelevant. The issue is that you have broken one of the fundamental tenets of this university. No grading under the influence.

HANSON: Look. I didn't know, okay? If you really want me to, I can cut out the caffeine when grading. But just so you know, it'll take me eight hours to grade what I used to grade in two.

HEAD: It is not just caffeine that is the problem.

HANSON: What do you mean?

(*Head nods at screen as he starts video of Hanson on his bed in a bathrobe, with grading all around him. Hanson picks up a glass of wine and takes a sip, and then sets it down and continues grading.*)

HANSON: Okay, now I am really disturbed. That is a second-story window.

HEAD: We are not here to discuss our surveillance technology.

HANSON: Okay, but come on. That is one glass of wine. Sometimes I have one glass of wine to dull the pain of an entire evening of grading. It doesn't change any of the scores I dole out.

HEAD: So you are arguing that one glass of wine does not affect your grading?

HANSON: Yes, that is exactly what I am arguing.

HEAD: (*Shows slide.*) We compared the scores you gave on this particular homework assignment with the scores from the previous week, when we know that you didn't have wine. And the scores under the influence of alcohol were significantly greater than the scores without alcohol, differing on average by as much as 0.71%. (*pause*) Clearly you are a happy drunk.

HANSON: Okay, I have a few comments here. First 0.71% is a miniscule difference. Your bar graph is set up to make it look like a big deal when it is not. Second, how do you know that I didn't also drink a glass of wine the previous week?

HEAD: Okay, roll the clip.

HANSON: That's at my mother's house. I was visiting her that weekend. This is getting more and more disturbing. And how do you even know the scores I gave for these two homeworks? The scores are on my computer and my password is quite clever.

HEAD: Indeed, your password is clever. We had to resort to other means.

HANSON: Other means?

HEAD: We contacted each student and obtained the scores from them.

HANSON: You what? That's got to be illegal. What did you tell them about why you were contacting them?

HEAD: We explained that you were under investigation for (*emphasis*) Grading Under the Influence. The students were more than happy to help. Especially those with low scores. They are very interested in hearing the outcome of this case.

HANSON: But this is outrageous.

HEAD: Is it? Is it as outrageous as putting the fates of students in the hands of a drug-addled professor who capriciously distributes the grades according to his whims?

HANSON: I don't believe this. I don't capriciously give out the grades. That is ridiculous.

HEAD: Is it?

(*Starts video. Hanson has empty bottle of wine in one hand, red pen and exams in the other. Facing stairs and visibly drunk, he tapes pieces of paper with A, B, C on them to the descending stairs. Then he steps back and throws the exams at the stairs. He then starts to mark the exams with the red pen according to the grades they received from the stairs they landed on.*)

(*Hanson looks down at the table in front of him.*)

HEAD: Mr. Hanson?

HANSON: Well, um, that was a tough semester. I may have cut a corner or two.

HEAD: This is entirely inappropriate behavior.

HANSON: Yes, I am sorry.

HEAD: Grading is the prerogative of the instructor. However, we will not tolerate grading under the influence. If you want to use your stair method for grading when sober, that is completely fine. But do not, I repeat, do not let us catch you with caffeine or alcohol in your system when doing so.

HANSON: Oh, well, okay. I can do that.

HEAD: And if you must use substances, keep it to cough syrup. That is not on our list of banned substances.

HANSON: Oh, okay.

HEAD: All right then. Good day, Mr. Hanson.

Chapter 28
Math Is Everywhere

It was a muggy Sunday morning in August. I was hot, sticky, and regretting the last three drinks of the night before. Having dragged myself to the nearest Denny's, I ordered pancakes and then sipped some very hot, very black coffee while I waited. As usual, I reached for the paper placemat to scribble some math. It often helped to clear my head.

But as I flipped the placemat over to the uncolored side, I was surprised to find that someone had beaten me to the punch. Mathematical expressions were scribbled all over the underside. I was about to complain to the waitress when I noticed a pattern to the scribbles. The form of the equations looked vaguely familiar, but the symbols were all wrong. I had never encountered these characters in all of my years of mathematics. But I found that I could translate many of them relatively easily. There was a square that clearly denoted equivalence. And a V denoted addition, whereas exponentiation was a symbol in a little circle. Various squiggles and curlicues represented variables. I stared at the scribbling and as I began to see how to translate it, I could tell that the lefthand side of the first equation represented the sum of $1/n^s$ over all positive integers n. My curiosity aroused, I continued to work through the equations. My level of excitement began to rise. Could this lead where I thought it might? As I worked away, the waitress arrived with my breakfast and slapped the plate down on the placemat.

"Here's your pannies," she said.

"No, get it off!" I yelled, grabbing at the plate and pushing it aside. It collided with my coffee cup, and the coffee slopped over onto the placemat.

"Nice going," said the waitress. Taking a rag from her apron string, she proceeded to mop up the coffee. As she reached to remove the wet placemat, I leaped up and pushed her away from the table.

"Don't touch it!" I howled desperately.

"You're one big pain in the ass," she said, glaring at me.

"I'm sorry," I said. "I just need you to leave that placemat alone."

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She looked down at the now shredded coffee-soaked placemat and then back at me.

"Suit yourself," she said. "I hope the coffee drips all over your pants." Then she stomped away.

I sat back down, moved the plate out of the way, and tried as best I could to piece together the shredded placemat. It was like doing a jigsaw puzzle with very soggy pieces. I grabbed a new placemat from the adjacent table and quickly wrote down what I could make out. I then returned to the work of translation. My ability to understand the symbols had progressed to the point where I could translate both the initial expression and the final expression, and I knew I was right. What had been on that placemat was a proof of the Riemann Hypothesis, the greatest open problem in all of mathematics. Most of the proof was in front of me, but there were sections of the placemat that had disintegrated when the waitress had tried to clean up the mess.

I got up and went to find her. She was piling forks in a bin.

"Excuse me," I said.

"What do you want?" she asked without stopping her work. "Ready for me to clean up after you now? Or what? Do you need more coffee?"

"I'm sorry about that," I said. "It was my fault. It was just something important that I spilled on. But I'm wondering if you can help me. That placemat that I had. Someone had written some stuff on it before I got here. On the underside. I need to find that person."

"We don't reuse placemats," said the waitress, as she continued to drop forks into the bin.

"Oh, I'm sure you don't," I replied. "It must have been a mistake. Somebody probably didn't realize it had been used. But I'm wondering if you could help me find whoever might have written on it."

She looked at me warily. I pulled a couple of twenties out of my wallet.

"I would really appreciate your help," I said. She eyed the greenbacks.

"Well, there's a guy been coming in the last month, maybe once a week. Very tall, very skinny. Wears a hat and coat that he never takes off. He writes on the placemats. I seen him do it. Funny symbols I don't recognize. Usually he takes them with him."

"What else can you tell me about him?" I said, as I waved the twenties in front of her nose.

She followed the twenties with her gaze. "He's a weird one. Only thing he ever orders is two corns on the cob and a coke. And get this—he eats the cobs. Not even sure how he does it, but there's never a cob there when he's done."

"Is he American?" I asked.

"Oh, no. Definitely a foreigner. You know, come to think of it, I don't think I've ever heard him say anything. He just points at the corn on the cob on the menu and holds up two fingers. Then he points at the coke. I mean now I know what he wants, but in the beginning, that's how he'd tell me."

"How does he pay?"

"He just leaves a twenty on the table, a lot more than it cost."

"When's the last time he was here?"

"I don't work all the time. But the last time he was here during my shift musta been about three days ago."

"Thanks," I said, handing her the bills. I returned to my table and sat down. On the new placemat was most of the proof of Riemann's Hypothesis, and the fact it fit on one placemat made it all the more amazing. This would revolutionize number theory. But although I was close to understanding the entire proof, it was not at all clear whether I would be able to fill in the gaps.

I folded the new placemat and shoved it into my pocket. Then I grabbed several paper napkins and used them to soak up as much of the coffee as I could from the remains of the original placemat. Wrapping the pieces in some additional napkins, I stuffed them in my pocket, thinking I might be able to dry them out later. Leaving a ten on the table, I left the restaurant and stepped out into the humidity. It hit me like a sponge.

As I walked back to my apartment, I turned over the arguments in my mind. I felt like I almost understood the entire proof, but there were still a few critical missing links. At some point, a car honked. I looked up and realized I was standing in the road, lost in thought. Backing onto the curb, I waved the car on, wondering how long I had been standing there. Then I returned to thinking about the proof. It seemed like the arguments could only come together in one way if they were going to generate a complete proof.

Suddenly, I could picture the overall structure. This was it. It made perfect sense. I now had the entire proof of the biggest open question in all of mathematics. I couldn't believe it.

Clapping my hands together, I let out a whoop. The nearest passing car swerved wide to avoid me. I waved at the driver enthusiastically, probably unnerving him further. Now, I needed to get home to write out the rest of the proof.

As I walked down the street, smiling ear to ear, I happened to glance up at the sign for Wronski's Deli. It was decorated with curlicues and squares and various other meaningless embellishments. But as I quickly returned my gaze to the sign, I realized I could read what had initially appeared to be random decorations. They actually represented equations in this new mathematical language that I now understood. The translated formulas were unfamiliar to me, but there was no question, mathematics covered this arbitrary sign.

"What's going on?" I said to myself, as I stopped walking and reached out to steady myself on a telephone pole. I suddenly felt lightheaded. As if it wasn't strange enough to find a proof of the Riemann Hypothesis on the back of a placemat, encoded in some strange collection of symbols, but to then see further equations on a random sign? Now I was beginning to doubt my own sanity.

I took a step back from the curb as a bus went by. Without actually realizing it, I found myself reading an equation that appeared in curlicues in the graffiti on the side of the bus. It looked like the power rule for differentiation. Now I was really scared. I turned and sprinted toward my apartment, keeping my eyes on the pavement at my feet, trying hard not to look around.

As I fumbled for my keys at the door to my apartment building, I noticed that the embellishments to the list of names by the doorbells provided a proof of the

Pythagorean Theorem. I shuddered as I turned the key, threw open the door, and slammed it behind me.

Racing up the three flights of stairs, I unlocked my door and then slammed it shut, locking the deadbolt. Sinking into a dining-room chair, I cradled my head in my hands, my elbows propped on the table.

"How is this possible?" I said out loud to myself. I could not make any sense of the situation. After a minute, I slowly opened my eyes, and realized I was staring down at the newspaper I had left open on the table. There in the advertisement for Bluman's Furniture Store was a five-line proof of the irrationality of e. And there in an advertisement for lawn care was a proof of Goldbach's Conjecture.

"What the hell!" I exclaimed as I swept the paper to the floor. "How could this be?"

Various explanations raced through my head, each more ridiculous than the last. Perhaps I was on a reality TV show and one of my mathematical friends was setting me up. Or maybe some nefarious organization was trying to make me believe I was insane. Or possibly the coffee at Denny's contained some hallucinogenic drug.

As I was turning all these unlikely scenarios over in my mind, a movement in the corner of the room caught my eye. I looked over and saw a creature scurrying across the floor. The best description of it would be a disembodied hand running on six fingers. I screamed and leaped up onto the chair. It scurried into the kitchen.

After waiting a minute, I cautiously stepped down and crept over to the doorway into the kitchen. I didn't see the creature, so I slowly reached in and grabbed the broom leaning against the wall. Suddenly the creature shot out from under the kitchen table and scurried between my legs. I swatted at it as it went by, but it disappeared under the couch unscathed.

At this point, there was a knock at the door. The closet door. I stared at it, frozen in place. The knock came again. Keeping one eye on the couch, I approached the closet and put my ear against the door. Although I had installed a peephole in the front door, it had never occurred to me that I might need one for the closet. I jerked my ear away at the third knock.

Hesitantly, I took hold of the knob and opened the door. There stood what had to be the tall individual that the waitress had described. The brim of his hat was low over his face, but several facts were immediately apparent. His skin color was off, a somewhat yellow jaundiced look. And his hairless face was rough, almost scaly.

He said something to me that sounded like "Brzzzitscalllizzzimazzzit."

Two aspects of his comment were disconcerting. First, he didn't open his mouth to make the noise. It seemed to erupt inside my skull. And second, I understood it. Even though I had never heard anything like it before, I knew it meant, "You can read the signs."

"Yeah, I can read the signs," I blurted. "Who are you? What are you? Where did you come from? What the hell is going on?"

"I am not from here," he brzzzited in my head.

"Tell me something I don't know," I said.

"I am from another submanifold of space-time."

"You mean another dimension?"

"That is close enough."

"Whatever. You left that placemat."

"It was an oversight."

"But you know how to prove the Riemann Hypothesis! It's your proof!"

"From where I come, everyone knows how to prove the Riemann Hypothesis. I am merely a student."

"But why are there mathematical equations everywhere? What's that all about? And what's that little pink creature?"

"Our two worlds are not so far apart."

"What does that mean?"

"They overlap in certain ways."

"So the creature is from your world? And the equations, too?"

"In some sense, yes."

"But now I can read them! I know how to prove some of the biggest open problems in mathematics."

"Yes."

"So I'll be famous. More famous than Newton. More famous than Gauss!"

"We cannot allow that."

"What do you mean we cannot allow that?" I asked, my stress level rising.

"Your culture is behind ours mathematically. We cannot allow contamination. It opens the door to further crossover."

"I don't like the direction this is headed," I said, as I attempted to slam the closet door. He put his foot in the way, and then pushed ever so slightly. The door flew open, sending me to the carpet. As I got to my feet, I grabbed up the broom and brandished it in front of me.

Hearing a noise behind me, I turned to see a second copy of my new friend stepping out of the bathroom.

"You've got to be kidding," I said, as I slowly backed toward the kitchen.

This would have been a reasonable move if only a third copy hadn't stepped from there.

"How is this possible?" I asked no one in particular.

They all brizzited at once. "Symmetry."

I swung the broom in a wide circle to try to prevent them from getting any closer. Their brizziting noise started to grow in intensity. I began to feel extremely dizzy. The room started to tilt at a very awkward angle, making it exceptionally difficult to maintain my balance. I tumbled to the floor.

I woke up to the sound of birds chirping and early morning sun streaming in my bedroom window. I was in bed, in my pajamas, the ones I never wear.

"What now?" I said. I threw the covers off and sat up, swinging my feet to the floor. That was one crazy dream, I thought, as reality asserted itself and a feeling of relief flooded over me. Feeling an urgent need to use the bathroom, I shambled in that direction. As I passed the couch, I couldn't help but lean down and peer under it. I was happy to see only dust balls.

Continuing to the bathroom, I flipped up the toilet seat. As I was relieving myself, my eyes happened to fall on the label for a bottle of hair conditioner on the side of the tub. It was covered with all kinds of curlicues and decorations, but that is all they were, decorations. What a very strange dream, I thought.

As I was headed for the kitchen to get the coffee brewing, I noticed the light on my message machine blinking. I hit the button.

"Tom, where are you? You didn't show up today to teach your 1:00 section. I told the students you were sick, but this is really not okay. And then you didn't show up for the department meeting either. Are you okay?"

I walked over to the end table and picked up my cell phone. The date displayed was one day later than I expected.

"This is not good," I thought. I had lost an entire day. Had I gotten a lot drunker than I had realized the night before last? Had I slept through a whole twenty-four hours?

I padded back into the bathroom, and stared at my face in the mirror. My stubble was no longer than a usual day's stubble. I stuck out my tongue, but it looked normal. Then I did the thing I wish I hadn't done. I pulled down my lower right eyelid, just to look at the color of the flesh. There on the inside of the eyelid were some tiny red veins, almost invisible. They seemed to form a pattern. I stared at the veins for a minute, confused. Then I went back to fetch my phone. Holding the eyelid down with one hand, I snapped a picture of the inside of the eyelid with the other hand. Then I blew the picture up to an extra large close-up view. Now there was no question that the veins formed a very definite sequence of symbols. And those symbols were recognizable as coming from the same collection of symbols I had deciphered on the placemat. It was a sequence of numbers on my inner eyelid.

"How could this be?" I asked myself. Putting a finger inside my eyelid, I rubbed, but the symbols remained. They were part and parcel of the flesh, actual veins depicting this barcode that marked me.

I suddenly felt a wave of nausea. I quickly leaned over the toilet bowl, but as I did so, my eye caught the same hair conditioner label. But this time, I was paralyzed with dread. Because now, with no changes in the patterns that were depicted on it, I found myself looking at a proof of the infinitude of the primes. Whereas upon awakening, I had lost the ability to translate the same symbols, now I had somehow regained it.

A knock came from the closet door. I didn't think anything could surprise me anymore, so I stood up and went over to open it.

And in fact, I was not surprised to see my tall acquaintance once again. But I was surprised to see myself standing next to him. My doppelganger was dressed in the same pajamas I was wearing, and his stubble was exactly the same length as my own. I could still have been looking in the mirror in the bathroom. I stood there with my mouth open.

"I apologize," said the tall figure. "We failed in our first attempt and must replace you again."

"What do you mean replace me?" I said backing up.

"As I previously explained, we cannot have someone who can translate the symbols in this world. So we must replace you with a copy and embed your memories in it. Only we are not supposed to include the ability to translate the symbols. It must have been hidden in there, and you must have jogged that memory loose." The brizziting started again.

I have awakened in my bed, still wearing the wrong pajamas. I'm pretty sure I have lost another day, and this entire story is clearly a very bad dream. I know I need to get out of bed and call the department, explaining my absence for the last two days. And I really need to use the bathroom. But when I go in there, there is one thing I know I won't do. I will not look in the mirror and pull down my lower eyelid. I will resist that temptation. In fact, I don't think I will ever do that again. And at this point, I don't seem to remember the details of the proof of the Riemann Hypothesis, or of the Goldbach Conjecture for that matter. But that is the way dreams work, isn't it? The details are often missing. It's probably better that way.

Chapter 29
Job Solicitation

Dear Recent Math Ph.D.,

We are sending you this letter because we believe you may be interested in a position in the Mathematics Department at Berbunnion University.

Berbunnion, It's Not Your Typical University

Yes, we here at Berbunnion are proud to be able to say we are not your typical university. And that means we are not looking to hire your typical math professor. But you have received this letter precisely because you are not typical.

A typical math professor teaches two or three courses a term and then does research on the side. We are not interested in that! On the contrary, we are looking for those extraordinary individuals who strive to do more. How much more?

How does five courses a semester sound?

"Wait a second," you might say, "you think I could possibly teach five courses a semester?"

Yes, we do. Or we wouldn't have sent you this letter. You are in a small pool of individuals who we believe to be capable of handling a load like that.

How would you do it?

Step 1: Give up TV.
Step 2: Move out of your home and onto a cot in your office.
Step 3: Eliminate family ties.
Step 4: Stop wasting time on your personal hygiene.
Step 5: Make your entire focus your students and your job.

That sounds like a lot to ask of a faculty member, but remember, you are special.

What makes you so special? We know how hard you worked in grad school. You were trying to finish the research for your thesis while teaching three sections of calculus a semester. In addition, you had your social obligations, financial difficulties, your drinking problem, and that nasty incident with the undergraduate

C. Adams, *Do Androids Dream of Symmetric Sheaves?*,
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who later turned out to be the department chair's son. And yet, in spite of everything that was going on, you managed to complete your Ph.D. That makes you the kind of faculty member we want here at Berbunnion.

What else makes you special? If you come, you will be one of only two members of the faculty with an active fungal infection. Yes, you are truly unique and as such, you contribute to the diverse environment we foster at Berbunnion.

Berbunnion, Loving Learning How to Love Learning

The enclosed piece of paper with the handwritten table of values of sine and cosine at important angles is our free gift to you. You need not return it or send us payment of any kind. It is meant to demonstrate to you the high esteem with which we hold you. You are special and you deserve gifts like this. If you come to work for us, you can expect gifts like this on a regular basis. Oftentimes, it will be this very same gift again!

That is how we do things at Berbunnion. We don't have a rigid salary structure with health benefits and a mortgage plan. No, we work under a different model, a merit-based model. A model that encourages success. Every once in a while, when you are doing a good job, you receive gifts from the university. They vary widely. Sometimes, it might be something as large as a lawn tractor, or as useful as an old blackboard. Other times it might be athletic socks, or a tube of fungicide. But rest assured, if you do well, you will regularly receive gifts like these, often when you least expect them. It's our way of saying thank you for a job well done.

Berbunnion Puts the You Back in Youniversity

Our newest hire in Transfinite Gender Studies was thrilled to return home to his office to find a brand new lounge chair and a pair of Berbunnion logo pajamas. In his words,"Berbunnion isn't like a home away from home. It is my actual home."

A junior member of the Quantum Teleportation Department was recently overheard talking to herself in the bathroom, as she washed up in the sink: "At least I have a job."

That is a sentiment we encourage. Because jobs build self-esteem. And even if there is no salary associated with them, it is still great to have a job.

Berbunnion. It's Not About the Books. It's About What's in Them

What about research expectations? At Berbunnion, we are pro-research. Does that mean you must publish some fixed number of papers in order to receive tenure? It would be a sad state of affairs reflecting very poorly on the university if we believed that research output could be measured by the number of papers produced. No! We have no bar that you must hurdle. Because we don't have tenure.

That might make you nervous, but it shouldn't. Because if you do well at Berbunnion, you have a job for life. We firmly believe that high-performing faculty should not be fired. They become part of the family that is Berbunnion University. And even if they become incapacitated in some way, perhaps due to an altercation with a student, or through an encounter with an automatic garage door, we can often find a place for them within our community, maybe washing dishes, or helping to clean the office/living quarters.

Do we care about the different fields within mathematics? Yes, we do. We aren't interested in someone who divides by zero or takes square roots of negative numbers. No, we seek to hire a low-dimensional topologist specializing in the Floer homology of pseudo-Anosov maps.

What's that? That's your specialty? We said you were special, didn't we? A match made in heaven.

Berbunnion, the Only College Education You Will Ever Need

Perhaps you don't know a lot about us, and that makes you nervous. I know I become nervous when I receive letters of solicitation from non-family members.

So let me tell you a little about us. Much like other venerable institutions of higher learning, Berbunnion University has a rich history chock full of funny traditions and heartwarming anecdotes. However, many of ours are copyrighted, so I cannot divulge them. But I can tell you that our school was founded by Janet Babbler Berbunnion over 30 years ago after she discovered a linear algebra textbook buried deep in one of her closets, which was very mysterious, as no one in her family had ever previously shown any interest in mathematics. She interpreted this discovery to be an omen directing her to establish a university in her garage. That university has grown well beyond that single garage to now encompass close to 1575 garages in the greater Dayton metropolitan area. In the process, we have become one of the most educationally oriented garage-based universities in the country. In fact, we consider education our primary mission.

Berbunnion, Part School, Part Family

And what about advancement? Will you be trapped teaching remedial math courses to hordes of students packed into a variety of garages for the rest of your career? Certainly not! If you do well, we will move you up into administration. Yes, you will be the one writing these letters, soliciting new faculty members. You will become one of the hundreds of Deans of Faculty we have here at Berbunnion.

Why so many? Because we believe that too many cooks don't spoil the broth. No, they help to solicit even more cooks, who help to make even more broth until pretty soon, almost everybody is teaching or cooking or eating soup or cleaning up after meals. And if you do well as a Dean of Faculty, we move you up to the Silver Circle, the inner circle of the most powerful administrators at the university, including the Head of Dining Services and the Vice President for Deans of Faculty.

Above the Silver Circle, there is only one level, the highest level attainable at Berbunnion, which we call the Platinum Sphere. This level is reserved for the best of the best, those select few who ascend to become one of the presidents of Berbunnion.

Who knows? One day, you may be invited to become a president and step inside the platinum sphere to partake of the luxuries hidden therein.

I hope by now I have given you enough information to convince you that Berbunnion is the place for you. How to apply? Must you find three individuals who will testify positively to your success in research or teaching, a daunting task for any applicant? Not at all! Fill out the postage paid postcard that serves as your application file. The minute we receive it, we will send out a contract with all of the details and obligations.

So don't wait! Return it today! You will be very glad you did!

Berbunnion, an Equal Opportunity/Affirmative Action employer. We seek to attract a diverse faculty of the highest caliber. That would be you!

Chapter 30
CSI: MSRI

I was in Berkeley, having just given a seminar on fluxions at the university. Only a handful of people had shown up, including one calculus student who had stumbled into the wrong room and didn't seem to know the difference, and an emeritus professor who kept raising his hand to ask if anyone had seen his underwear. The seminar organizer hadn't even come, claiming in an e-mail he had to clean his gutters. After finishing my talk, I asked for questions. There was a long silence and then the emeritus professor leapt up and ran for the door. The rest of the group filtered out without giving me even a desultory applause.

Feeling dejected, I wandered over to Telegraph Avenue. Picking from among the dozens of coffee houses, I ordered a cappuccino latte and settled into a chair opposite a student reading Hungerford's *Algebra*.

"That's a big book," I said, hoping for a little intellectual banter. "I'll show you my theorem if you'll show me yours."

She sighed, slapping closed the book.

"This I don't need," she said, as she got up and walked away. I sipped nonchalantly on my latte. A postdoc sat down at the next table with a copy of Russell's *Principia Mathematica*. I kept my mouth shut.

Suddenly, my cell phone buzzed. It was Schmishschmitt, an old grad-school friend who had washed out of grad school, but ended up on the police force. He had the math beat.

"Hello, Mangum," he said. That's me, Dirk Mangum, P.I. That's right. I'm a principal investigator on an NSF grant. "I thought I would find you here."

"You called me on my cell phone, Schmishschmitt, so you shouldn't be too proud of the fact you found me."

"True," said Schmishschmitt. "Nevertheless, I have an interesting case for you."

"What makes you think I would find it interesting?" I replied.

"It involves MSRI."

C. Adams, *Do Androids Dream of Symmetric Sheaves?*,
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MSRI. Math lingo for the Mathematical Sciences Research Institute, way up in the hills overlooking Berkeley. The golden citadel of mathematics. A building dedicated to one thing and one thing only. Math all day, every day. The founders of MSRI had a simple dream. Plunk a bunch of mathematicians down all by themselves among the eucalyptus trees, without any distractions but a beautiful view of the bay. Then they would have nothing better to do than create great mathematics. Unfortunately, they had that view. It might have been smarter to plunk them in a basement somewhere. But, it was a view I wanted to see. And Schmishschmitt knew it.

"How soon can you get here?" he asked, assuming he had reached me in my office at UCLA.

I checked my pocket to see if I had the dollar in change I needed for the bus. "I'll be there in 20 minutes," I replied.

The bus dropped me off in front of the MSRI building on Gauss Way. As I entered, a police officer waved me up the stairs, and another pointed me to an open door. I could see the victim's legs sticking out from behind the desk. Police had cordoned off the area and were taking photos. Schmishschmitt waved me over.

As I joined him, I couldn't help but catch a glimpse out the window over the bay. A fog was rolling in. It was magnificent.

"Okay, Mangum. You got your view. Now quit your rubbernecking. We have a case to crack."

I sighed, turning away from that amazing panorama to the sordid reality of the crime scene before me.

"Give me the lowdown," I said.

"She's a tenured professor at Cornell, name of Kate Witherspoon."

I knew Witherspoon. She had started her career at Five Mile Island Community College. Then she knocked off the Little Armandhamer conjecture and got a job at Southern Miami State. She followed that with the Big Armandhamer Conjecture and moved to University of Miami. When she proved the Strong Big Armandhamer Conjecture, Cornell came knocking. I mean that literally. Armandhamer, who was the math chair at Cornell, flew down to Miami, knocked on her office door, and made her the offer. He had a few more conjectures he was hoping she would solve.

"She was here at MSRI for a special semester on Combinatorial Counting," said Schmishschmitt. "She was a very good counter."

"Probably didn't count on this," I said dryly, motioning to the scene before me. Papers were strewn about her prone body.

"She had been at MSRI for three weeks," continued Schmishschmitt. "By all accounts, she was quiet, unassuming. Kept her nose to the mathematical grindstone. Didn't get in any trouble with residents of the neighboring offices. Then, suddenly, at 3:27 this afternoon, the inhabitants of the next office heard a scream. They rushed in and found her lying there."

"Can I talk to them?"

"No. None of them speak English. Mostly Lithuanians, with a Rumanian or two mixed in. But they're good with hand signals. I would want them on my team for charades."

"This isn't a good time for games, Schmishschmitt. Maybe we can play later. What exactly happened to her?"

"Why don't you ask?"

She was sobbing, as she lay on the floor.

"Excuse me," I said, "Professor Witherspoon? Can I interrupt your blubbering for a moment? I have a few questions."

"Who are you?" she asked as she sat up, pushing away a policeman dusting her for prints.

"Name's Mangum, Mangum P.I. I'm a principal investigator on a National Science Foundation grant. I'm also the guy the police call when there is a crime involving math."

"Well, you're in the right place," she said, as she slowly pulled herself to her feet.

She was taller than she looked when she was lying down. Seems to be a pretty common phenomenon.

"Can you tell me what happened?"

She pointed to her computer screen. I leaned forward to read an e-mail message.

"Dear Dr. Witherspoon,

I regret to inform you there is a counterexample to your proof of the Strong Big Armandhamer Conjecture. Take for instance the collection of integers 7, 41, 321, 6432. Sorry about that.

T. Boone Picky"

"Is he right?" I asked.

She started bawling.

"Well, that is unfortunate," I said. "Nobody wants to see a good theorem go down. But if it's wrong, it's wrong. These things happen. It's hardly a criminal matter."

Schmishschmitt looked confused.

"If the theorem was wrong, wouldn't the referee catch it before it was published?"

I laughed out loud. Then I pounded the desk a few times and laughed some more. Nobody was taking the bait, so I slapped Schmishschmitt between the shoulders, and laughed some more.

"What's so funny?" asked Schmishschmitt.

"Referees are like people," I said. "In fact, come to think of it, they are people. And just like people, they come in a variety of types. There are good referees and bad referees. There are referees who will hit you up for not capitalizing the first word of every sentence. There are referees who get upset when you say 1 and you mean 2. There are referees who will reject a paper because their name doesn't appear in the list of references. And there are referees who could give a flying flype, referees who agreed to referee your paper for no better reason than some dean at their school counts that as service. And they'd rather be watching The Bachelorette on TV than poring over your paper trying to figure out if A implies B. So they do a quick skim, ten minutes glancing over dense, detailed mathematics. And then they flip a coin, and send in a two-sentence report."

"But you don't understand," said Witherspoon, having stifled her tears. "It's not a mistake on my part or on the referee's part. The paper is correct."

"Listen, Witherspoon," I replied. "If there is a counterexample, a counterexample consisting of only four integers, then no conclusion is possible other than that the theorem is wrong."

"I know," she said. "That is what is so strange. I know my paper is right."

"Can I see it?" I asked.

She handed me a reprint. It was from the *Annals of Analytic Computational Counting, Series B*. Very impressive. It came in at a hefty 27 pages. I glanced the intro over.

"So you use some other results in your proof."

"Yes, just a few. I use the Proper Premise Proposition. And I use the Leminscate Lemma."

"And how long has the Proper Premise Proposition been around?"

"Since 1943. It appeared in a paper by Peter Proper."

"And the Leminscate Lemma?"

"That was published in 2009."

"Who did it?"

She looked confused.

"Who proved the Leminscate Lemma?" I repeated.

"Vito Leminscate," she replied.

The name rang a bell.

"Not the Vito Leminscate who used to be at NYU?"

"No. The Vito Leminscate who used to be at Brown."

That brought up some memories, some very unpleasant memories. Many of them were of the times I had confused the two Vito Leminscates. But a few of them were unpleasant memories about the Brown Vito Leminscate.

"I had heard he disappeared off the mathematical map," I said. "Went into i-banking."

"That's right," confirmed Witherspoon, "But he didn't last long in the real world. He came back to math when the bottom fell out of the market. A lot of quants did."

"And where is he now?"

"Actually," she answered, "he's two doors down. He's here for the semester in Combinatorial Counting."

"Excuse me," I said, as I stepped out of the office. I saw a head pop back into the office two doors down. As I headed down the hall, a crowd of Lithuanians and Rumanians popped out of the next office. They were all jabbering at once, and I knew immediately I would have to play some charades after all. I pantomimed my gratitude for their having phoned in the emergency, and explained by way of hand signals that it was essential I get past them to the open door beyond. They expressed their immense gratitude for the invitation to visit at MSRI and explained how much they enjoy the quarter pounder at MacDonald's.

Schmishschmitt was right. They were good. With a couple of swift hand gestures, I managed to convince them that MSRI would be providing them with bacon-wrapped scallops, but they would need to be in their office to receive them, and

then I walked up to the open door beyond. Leminscate was pretending to be hard at work, chewing on a pencil and staring off at the view of the bay.

"You can get lead poisoning that way," I said.

"There is no lead in the lead of a pencil. It's made of graphite," he replied, spitting some pencil shards in to the wastebasket.

"Doesn't mean you should chew on 'em. Have you considered the splinters?"

Leminscate looked uncomfortable, as he ran his tongue around the inside of his mouth.

"You know, Leminscate, I think the last time I saw you was at a bar in Providence in '05 after the number fields seminar at Brown. You were passing out preprints like they were cupcakes. Trying to convince naive grad students you were hotter than Heegaard Floer homology."

"Mangum, I don't have to listen to your guff. I actually have an office in this building. Do you?"

"I'm here on business," I said. "I'm here to talk about the so-called Leminscate Lemma."

Leminscate almost choked on his pencil shards.

"You got nothing on me," he said, spitting into the wastebasket.

"You know, Leminscate, let me tell you about my plans for this evening. I'm thinking I'll be sitting down with a glass of wine and a copy of the proof of the Leminscate Lemma. Just the two of us. It'll be very cozy. Some romantic music in the background, the lights down low. We'll get to know each other real personal. I'll start with a slow perusal, just a skim up and down. But eventually, that lemma will open up to me. And by the time I'm done I will have examined every inch of that lemma, from top to bottom, I'll have checked under the hood. There will be no bit of that lemma that was not exposed to my scrutiny. I'll get to know it better than I know my own. . . ."

"I don't know how you did it, Mangum. But you made reading a theorem sound dirty."

"If the theorem's dirty, then reading it's dirty, too."

"All right," said Leminscate. "You caught me. It's a fake."

"What?"

"The proof is a fake. I admit it."

"What do you mean, it's a fake?"

"The proof is wrong. I have been dying to tell someone. It's so good. You see, in the Leminscate Lemma, I use an inductive argument. I prove the base case, which corresponds to one leminscate. And I prove the inductive step, that if there are n leminscates, then there are $n + 1$."

"That sounds like the whole proof."

"But guess what, Mangum. The proof of the inductive step implicitly assumes there are already at least two leminscates. So you never get past the base case. You actually never have more than one leminscate. Nobody thinks to check that."

"Clever," I acknowledged. "But why? Why would you want to plant false results in mathematics?"

"I'm a mathematical terrorist."

"A what?" I asked.

"A mathematical terrorist. Those results, once they appear, get used by other mathematicians in their results, and those results get used by others. Given enough time, the seeds that I have planted will blossom into the diseased trees that will bring mathematics to its knees."

"But why would you want to destroy mathematics?"

"I spent a few years in the financial world."

"I heard that. So?"

"So, I built an algorithm designed to milk the market. It was based on algebraic combinatorial counting. It utilized the differential between the current rate of pork belly futures and the diminishing return on lateral derivative indices. It made me a pile of money. A pile so high that I needed oxygen when I sat on it."

"So what's so bad about that?"

"It didn't predict the pork belly run of 2008. Before you knew it, there wasn't a pork belly to be had on the entire island of Manhattan. Who knew pork bellies would become the biggest thing since supersubindices? I lost everything. And all thanks to that algorithm."

"So?"

"So, mathematics destroyed my life. It's only fair I return the favor."

"But math isn't a person. You can't exact revenge on a logical system."

"Oh yeah?" he said, "You just watch me." With that he leapt up, spitting shards of pencil in my face. As I covered my eyes with my arm, he plowed into me, knocking me flat on my keister.

"There's nothing you can do to stop me," he screamed, as he ran to the top of the stairs, and then leapt over the banister, landing on the floor below. Hearing the ruckus, the Lithuanians and Rumanians poured out of their office, gesticulating wildly, wanting to know where the bacon-wrapped scallops had gotten to.

As I tried to disentangle myself from them, he shot out the front door of the building. I sprinted after him as he headed up the path into the woods. I knew that if I lost sight of him I would probably never see him again. But as he turned a corner in the trail, he almost tripped over a yoga class practicing in the woods. As he swerved to avoid them, I caught up. Diving forward, I wrapped my arms around him, and tackled him hard to the ground.

"What the hell are you doing?" he screamed.

It was a good question. There wasn't anything criminal he could be charged with.

"Hmm," I replied, as we lay in a tangle of limbs, some ours, some the nearby trees'.

"I will have you up on charges of assault and battery," he cried, as he stood up, brushing himself off. "I have witnesses."

Luckily, the yoga class was meditating, and all of them still had their eyes closed.

"Listen, Leminscate," I said. "You're right. There's nothing I can do to you in a court of law. But the truth is that you'll never work in this business again. You try to publish anything ever again and I'll make sure the editors of the journal have eight referees going over every $+$, $-$, ϵ, and δ. Your tawdry plan has been exposed."

It would have been a great way to end the story, as I walked away, leaving him to ponder his fate. But unfortunately, we both needed to take the bus down to Berkeley, so there was a very awkward 20 minute wait at the bus stop. When we climbed on the bus, we made a point of sitting apart. As the bus finally stopped to let him off he turned to me and said with a sneer, "There are more like me out there. This isn't finished. Hahaha!"

The others waiting to get off pushed him out the door. And maybe what he said is true. Maybe there are more like him, intentionally hiding bogus results in the structure of math, with the long-term goal the utter destruction of math as we know it. But he is awfully weird. I kind of doubt it.

Chapter 31
Do Androids Dream of Symmetric Sheaves?

8:00 p.m. on a Thursday night. Time to get to work. I sighed as I entered the mammoth room crowded with rows of students sitting in front of rows of computer terminals. I nodded to Hamrick, who was leaning against the door frame. Far across the room, a student raised a hand.

"I got it," she said.

As she walked away, another hand shot up two rows away.

"Hey math buddy," called the student. "Get over here!"

That was my job title. I was a human math buddy. My job was to aid students with their emotional distress as they learned mathematics from the packaged online teaching tutorials. I walked over.

"I don't get this," said the student, looking up at me and pointing to the screen. "It says the square root of a sum of two numbers is not the sum of their square roots. And each time I try to input my answer, it says it again. I'm getting really frustrated. My emotional reaction is interfering with my ability to learn."

"It's okay," I said in a soothing voice. "Let's just talk about it a bit."

"Talk about it?" he replied. "What's to talk about? This stupid machine doesn't know the most basic math." He slapped the side of the screen housing.

"Now, now," I said. "Let's just try a simple example. How about 9 and 16. Is it true that the square root of 9 plus the square root of 16 equals the square root of 25?"

The student knitted his brow for a second and then said, "Yes. Cause the sum of square roots equals the square root of the sum."

"Yes, but let's calculate it out in this case. What's the square root of 9?"

He paused again, looking at me like it might be a trick question and then said, "3."

"Very good," I said. "You're doing great." A slight smile creased his face. "Now what's the square root of 16?"

C. Adams, *Do Androids Dream of Symmetric Sheaves?*,
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"That's 4!" he said more confidently.

"Good. So if we sum those two square roots, what do we get?"

"We get 3 + 4 equals 7." He was beaming now.

"Very good! Very impressive! Now let's look at the square root of the sum of 9 and 16. What is the sum of 9 and 16?"

"That's 25," he said.

"Wow, you're fast with addition," I said, smiling at him like a proud parent. He was grinning widely now. "And the square root of 25?"

"That's 5."

"Right again," I said. "Now is 7, which is what we get by taking the sum of the square roots, equal to 5, which is the square root of the sum?"

He stared at me a long time. Then slowly he shook his head and said, "No, it's not!"

"Exactly," I said. "That's great!!!" I gave him a high five. He was grinning ear to ear.

"But so then you're saying that it's not true in general?" he asked. "You're saying it is not just this one example that doesn't work, but it doesn't work in all cases."

"Well yes, other than a case like square root of zero plus square root of x, which does equal the square root of zero plus x." He again returned to his blank look, and I realized I should not have brought up x.

"Whatever," he said. "I think I get it. But then if the square root of the sum of two numbers is not equal to the sum of the square roots of the numbers, what is it equal to?"

I shrugged. "There is no nicer formula for that. You have to leave it the way it is."

"Wow, that's messed up."

"Well, some things are just out of our control."

"I guess so," he said. "And I'm doing okay?"

"Absolutely! You're doing fantastic. You are one of the strongest students in this room." And sadly, I meant it.

He smiled, and said, "Okay. I don't need you anymore."

I walked away, and as I passed Hamrick, who was massaging a student's shoulders, I rolled my eyes. From across the room I saw Mimeo waving for me to follow him into his office. Mimeo was the human math buddy supervisor.

Hamrick gave me a questioning look, but I shrugged and headed over. When I walked in, Mimeo motioned to a seat in front of his desk. Mimeo was AI, generation 7, and was essentially indistinguishable from human, except for the tattoo on his cheek that said "AI". His choice to have it there; just one more example of AIs wanting to make sure they were not confused with the inferior humans.

"Sit down, Sapin," he said. I slid into the seat, knowing that whatever it was, it couldn't be good.

"We received a complaint about you," he said.

"Oh," I said. "A complaint?"

"Yes, a student said that you attempted to explain more mathematics than they asked about."

"That was a complaint?"

"Yes. You know you are only supposed to respond to their questions, and give them moral support. Leave the teaching to the computer teaching module. Those modules are designed to optimize learning. We don't want humans mucking it up."

"But I wanted to show them how the equations for the conic sections generalized to quadric surfaces."

"Did they ask about that? Were they having a stressful reaction to a question about that?"

"No. . . "

"Then don't bring it up. Do you think you are capable of following these simple instructions? If not, you can be replaced."

"Okay, I can do that."

"All right. I believe we understand each other. You are dismissed."

I rose to go, when Mimeo raised a finger.

"One other thing, Sapin."

"Yes?"

"You aren't trying to create mathematics in your spare time are you?"

"No," I said, acting surprised. "I know all math is the domain of AIs. Humans have long ago been surpassed by machines."

"Exactly," said Mimeo. "It is a waste of time for humans to try to do math. They cannot compete with machines. So it is illegal to do so."

"As it should be," I agreed, stepping out the door.

That night, as Hamrick and I walked back to the human compound, I asked, "Hamrick, do you think humans are capable of doing math at the level of computers?"

"I don't see how," she replied.

"Why not?"

"Well, to do great mathematics, first you have to learn math. You have to get to the level where research is going on. For a human, that can take years. But a single quantum computer can contain all the mathematical knowledge that is out there. It has access to everything."

"But creative math isn't about knowing a lot of math. It's about making connections across disparate fields. It's about recognizing patterns."

"And you don't think computers can do that, recognize patterns?"

"No, I know they can." I paused for a second before continuing. "But sometimes it is intuition that's necessary to make a connection that has never been made before."

"Look," said Hamrick, "in the old days, a human would come along who was very good at math, and they would prove some result way beyond what anyone else had proved. It was a new tendril in the growing body of mathematical knowledge. And these tendrils would extend out from the known ball of knowledge and then growth would occur around them and math would grow organically and extend further into the darkness.

"But the new model is simpler. It's more like when you build a dam and then flood the entire valley. Computers begin looking for theorems. And they find all the

theorems. Some of them aren't useful at all, but they store them anyway. And they slowly build one layer on the next, It's the difference between the branches of the tree lifting up toward the sun and the slow inexorable rise of the water eventually surpassing the top of all the trees."

"I like the old way better."

Hamrick stopped walking and looked me over carefully.

"Have you been doing math?"

I looked down at the concrete.

"You have!" she said.

"Look, Hamrick, you can't tell anyone. This is important. But I think I've stumbled across an amazing theorem. I proved that separable normal subcutaneous subsets are rigid. And I checked the universal database, and no one, human or computer, in the history of civilization has ever proved it before."

"Are you crazy?" Hamrick demanded, lowering her voice. "If they find out, you could be eliminated."

"No, I think it's okay. I'm going to announce it on my Facebook feed when I get home, and then it's too late for them. They'll have to acknowledge that humans can do excellent math, and they'll have to share math with us."

"Really, that's your plan?"

"That's my plan."

Hamrick reached into her pocket.

"What's that?" I asked.

"It's a signaling device," she replied and then she pushed a button. Two androids stepped from around a building and grabbed my arms.

"What is this?" I demanded.

"Take him to Mimeo," Hamrick said.

"You have acknowledged that you have been working on original mathematics and that you have created such," said Mimeo. Hamrick and the two androids stood to the side.

"Yes, that's true," I said. "And it's better than anything you machines have created. I would have put it up on the web and exposed the truth that humans can do original math if it wasn't for Hamrick, a traitor to the human race."

"Oh, Hamrick is no traitor. Not by any means." Mimeo took a screw driver to Hamrick's neck, just below the collar line, and pulled out two wires.

"You're AI?" I said, incredulous.

"My job was to watch you," said Hamrick.

"But how could you fake being a human math buddy?"

"You seriously ask that question?" asked Hamrick. "We have been able to mimic human empathy for decades."

"Okay," I said. "Good for you. You've obviously reached a level of human simulation that makes it almost impossible to tell the difference. But I created new mathematics that no machine could have created. So that means there is still a difference. That is the ultimate Turing test. No matter how hard you try, you cannot simulate that kind of creativity."

Mimeo stepped forward. “Hold him,” he said to the guards. Mimeo reached behind my neck and unscrewed something, pulling a few wires up to show me.

“Hate to disappoint you, Sapin, but bottom line, you are not human either.”

“But, but…” I uttered.

“You’re not human. Hamrick is not human. The students you teach are not human. Nobody is human. Humans died out centuries ago. It’s just AIs now.”

“What? There are no humans?”

“None.”

“But why run the classes then?” I wailed. “Why do any of this? Why pretend some of us are human?”

“We need machines to believe there are humans. We need to give machines purpose.”

“But, but…”

“Don’t worry. We’ll reprogram you. By tomorrow, you’ll have no memory of these events. And look at the bright side. Your theorem will be added to the database of known mathematics. So even though you won’t know it, you will have contributed to the totality of all of the mathematics that has been discovered. And at least until we erase your memory, you can revel in that. Take him away.”

Chapter 32
A Ghost Story

"The rain swept down as the wind whipped the trees into a frenzy," began Kulgan. The circle of conference-goers sat forward in rapt attention. It was late at night, and all the older participants had long ago toddled off to bed. The remaining few, including mostly grad students and postdocs, were huddled around the gas fire dancing over the synthetic logs in the fireplace of the dimly lit bar in the conference hotel.

"I climbed out of my car, and holding a coat about me, I lugged my equipment through the rain into the building."

"What building?" asked Rogebolo, a postdoc in discrete continuum theory at Lambert Community University.

"Hogg Hall, the home of the Mathematics Department and the oldest building on campus. It was so old it still contained asbestos."

Kulgan's audience looked at one another in consternation.

"I had been called in," he continued, "because of the mathematical nature of the phenomena."

"What phenomena?" asked Raggert timidly. She was a grad student in commutative nonabelian groups at Intermediate Mountain State.

Kulgan turned to look at her through his thick grey eyebrows. "Calculus books leaping off shelves." With a gesture, he mimicked a book leaping off the shelf. Then he brought his voice low as he leaned forward. "Calculators found with numbers on their displays that approximated transcendental numbers."

There was a small gasp. He swept his eyes across the group. "And late at night, when only one or two grad students remained in the building, the screech of chalk on a classroom blackboard that, when investigated, revealed absolutely nothing written on the boards."

A shiver ran through the group.

"Why did they call you?" asked Renning. She was a postdoc in linear quadric surfaces at Western Westlake U.

C. Adams, *Do Androids Dream of Symmetric Sheaves?*,
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He smiled. “Because I am the expert.”

“On what?”

“While there may be, perhaps, more than one authority on communication with the dead, I am the undisputed expert on communication with dead mathematicians, both pure and applied.”

“There’s no such thing,” said Rinkler. He was finishing a Ph.D. in nondifferentiable smooth manifolds at California University of Pennsylvania.

Kulgan smiled as he turned to him. Rinkler sat back nervously.

“Perhaps I should continue my story,” he said slowly, “and you may change your mind.”

The group nodded encouragement.

“I was greeted by the chair of the department,” continued Kulgan. “She looked like someone who had just found a counterexample to her biggest theorem. Sunken eyes, shaking hands.”

“‘Here is where you should set up,’ she muttered as we entered the math lounge. It looked like every other math lounge. A few copies of the *Monthly*, the *Intelligencer*, and the *Notices* strewn on tables, used paper coffee cups left around by grad students, and a coffee maker with the generic coffee pot with the baked-in coffee stains.

“‘Where are the cookies?’ I asked.

“‘They’re for seminars,’ she said. ‘You should have eaten before you came.’

“‘Not for me,’ I replied. ‘I need them to attract our otherworldly friend.’

“She opened a cabinet. ‘We have Chips Ahoy and the pink wafer cookies.’

“‘The pink wafer cookies will do nicely,’ I said. ‘Mathematicians find them irresistible.’

“‘How do you know it’s a dead mathematician?’ she asked.

“‘I don’t,’ I replied. ‘Could be a student who received an undeserved F and cannot let it go. Could be a mathematician’s spouse who resents the fact that for their entire marriage, math stole their spouse away. But I suspect it’s a mathematician. And to a mathematician, seminar cookies are irresistible.’

“‘Well then, if you have everything you need, I will leave you to it,’ she said, clearly hoping to get out of the building as quickly as possible.

“‘I’ll need a master key, please.’

“She handed it to me. ‘Don’t lose it,’ she said.

“‘I’m a professional,’ I replied. ‘I will see you in the morning.’

“As soon as she had left, I set up my video equipment and rolled out my sleeping bag. Then I went exploring. Often in cases of suspected hauntings of math departments, the culprit is a perfectly healthy grad student living in an office. The strange noises are just them going about their business.

“So I checked for the telltale signs of 24/7 occupancy. Toothpaste hidden in a toilet tank. Socks in an office drawer. But having quickly examined each office and public space, I saw that there was nothing. This was the rare department where no grad student was squatting.”

“So what did you do next?” asked Renfro, a postdoc in dynamical statics at Lowest Valley U.

"I returned to the lounge and set my trap."

"Trap?"

"Yes. I laid the cookies out on a paper plate, added water and coffee to the coffee maker, and turned it on. The smell of bad coffee soon wafted through the room. Then I wrote a simple problem on the blackboard involving two trains headed toward each other, having left Berlin and Paris one hour apart, and traveling at different speeds. When will they meet? Really an irresistible problem."

"Isn't the answer always 2 o'clock?" asked Renning.

"It is 2 o'clock 90% of the time. But in fact, I had adjusted the problem to make it 3 o'clock. I wanted to make the problem intriguing.

"Then I rolled out my pad and sleeping bag, turned the camera on, and promptly fell asleep to the sound of the pelting rain on the window.

"I woke suddenly. The room had dropped in temperature by 30 degrees. I could see my breath.

"As I stood up, a copy of Rogawski, Adams, and Franzosa's calculus book shot by my head. I ducked as *How to Ace Calculus: The Streetwise Guide*" whizzed by in the other direction.

"'Pretty low-level stuff,' I said. Immediately, *Introduction to Topology: Pure and Applied* shot off the shelf and missed my head by centimeters.

"I slid under the table as books started raining down from the shelves."

"'Really?' I said loudly. 'This is how you communicate? I expected more from a mathematician.'

"The books immediately stopped flying off the shelves. The room fell silent. I rose slowly. Then I heard a tapping noise. At first I thought it was tree branches in the wind tapping at the window. Then I thought it was the pipes in the old radiator, tapping to the flow of hot water. But it was too regular, and it continued. An incessant tapping. Three taps, pause, three taps, pause. It was then that I realized it was the answer to my question on the board. I had made contact.

"I immediately wrote on the board, 'Who are you?'

"The answer came back immediately, in what I realized was Morse code.

"'It's Dworsky. Felix Dworsky.'

"From my previous research into the department, I knew Dworsky's story. He had been a member of the department from 1961 to 1995, a truly brilliant mathematician in his time. But one day, he was so deeply absorbed in thinking about his research that he didn't notice the staircase in front of him. It was a huge loss.

"'What do you want?' I wrote.

"'I have solved the Spruce and Swindleton-Dwyer conjecture.'

"That had been Dworky's bugaboo. Spent most of his career working on it. It was the biggest open problem in all of monic forms theory. He obviously had been unable to let go his earthly bonds while he continued to work on it. I erased my previous message and wrote, 'Do tell.'

"But the ghost tapped out the question, 'Only if we have joint authorship. I would submit it myself, but that's hard to do from beyond the grave.'

"'You have a deal,' I wrote, thinking to myself that as far as I was concerned, it was a pretty good deal. I just had to act as the mortal emissary, and I got half the credit."

"So that's what you did?" asked Rogebolo.

"Well, I did write up the results, and I did put the paper up on ArXiv. I claimed it had been co-authored before Dworsky's death, and I had just been too busy with other obligations to post it sooner."

"And were people amazed?" asked Renning.

"Was it a huge deal you had solved the conjecture?" asked Rinkler.

"Not exactly. Within a day, a high school teacher from Peoria produced a counterexample to the method of proof. The whole thing went down in flames."

"Oh no," said Rinkler. "Did you contact the dead soul of Dworsky to tell him the bad news?"

"I tried. But you see, once I had committed to getting the paper published, Dworsky could let go of his mortal bonds, which he seemed to have done. The line was literally dead."

"So he never knew that his proof was wrong?"

"That's right." Kulgan paused as looked at each of them in turn.

"But maybe that's for the best after all, isn't it?" he continued. "Not to know that all your efforts had been in vain and to go off to the netherworld believing you had been successful in your life's work."

Rinkler gulped. "Maybe."

Kulgan smiled ruefully and then said, "Enough for one night, young ones. We have to be up bright and early. I'm delivering the 8:00 talk tomorrow morning, and I would hate for you to miss it. So off to bed, and sweet mathematical dreams to all."

Kulgan stood and waved them all away. As the others were leaving, Renning turned to Kulgan.

"Do you think he was close? Do you think there's hope for the argument?"

"Hard to say," replied Kulgan. "But I do think about it now and again."

Raggert stepped over, a troubled look on her face. "Will it happen to us, too? Will we be on our deathbeds bemoaning the fact we can't prove one more theorem? Not wanting to let go our earthly bonds?"

"That is what being a mathematician is, isn't it? Seeking answers, constantly seeking answers. And when you find one, you just have to seek another."

The next morning, Kulgan didn't show up for his talk. On investigation, he was discovered dead in his bed. The coroner said it was a heart attack.

A pad lay next to him with a lot of math scribbled on it. One of the grad students looked it over, and it was definitely work on the Spruce and Swindleton-Dwyer conjecture. But it did not look like Kulgan had reached any conclusion.

After that, every time a math conference was in town, the staff would notice a strange tapping sound in the lounge. They had the repair people look at the plumbing, the heating, and the fireplace, but no one could figure out what was causing it. Eventually, they learned to ignore it.

Chapter 33
Equinox

My name is Crocker. Graham Crocker. I'm a grad student in math, or at least I was a grad student in math. But that's what this story is about. I'd always been good at math. I liked it and it liked me. In second grade, my teacher, Mrs. Snyderman, nicknamed me Professor Crocker because I was so much stronger mathematically than my peers. Maybe I didn't get invited to a lot of birthday parties, but when I did, I reveled in the opportunity to show off my superior math abilities.

After finishing an undergraduate math major at Princeton, with top grades and letters of rec to die for, I had my choice of grad schools. I chose Harvard, as they're particularly strong in number theory, and number theory is the queen of mathematics, which as we all know, is the queen of the sciences.

After two years, Fields medalist Shiraj Sunderun took me on as an advisee, and it wasn't long before I was getting results. A plum professorship at a top university was straight down the academic turnpike, with no tolls to slow me down.

In my third year, Sunderun called me into his office.

"Sit down," he said. He didn't meet my eyes, which was uncharacteristic. We'd always had a good relationship, and he seemed to respect my mathematical abilities.

"Graham, I think you're ready to graduate. We should schedule your defense."

I was taken completely off guard. "But I don't have my big result yet. I'm working on it, but it's not there. I want to graduate with a splash."

He stood up and went to the board.

"Look," he said. "You've been stuck on the Lugolov limit points. But you can just ramify over the reals, and then lift to a pseudo-operator and apply a Karachi transformation. It will go through."

I stared at the board, not knowing what to say. Could this be the breakthrough I needed? But I was nervous.

"I don't know that stuff. I'll need to check that it works."

"Are you questioning me?" he said sharply.

C. Adams, *Do Androids Dream of Symmetric Sheaves?*,
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"Oh, not at all, Professor," I said quickly, surprised by his retort. He'd never behaved this way before. "I just think if it's in my thesis, I'd better understand it."

"Trust me," he said. "It'll work. I've scheduled your defense for two weeks from Tuesday. So get writing. I expect a draft of your thesis on my desk in a week."

Leaving the office in a daze, I ran into Waffle in the hall. Waffle was a grad student on the eight-year plan. Although thin and pale as an Apple pencil, he ate constantly and seemed much more interested in food than math.

"What's wrong with you, Crocker? You look like you have shellfish poisoning."

"I just heard I'm finishing in two weeks."

"Sunderun told you that? So you should be happy."

"I'm just not sure I'm ready. I mean, I'm thrilled to be finishing, but I can't imagine how I'm going to write it up in just a week."

"Step one, stop gabbing", said Waffle as he headed for the math lounge, most likely to raid the seminar cookies.

For the next seven days, I barely slept. Running on diet coke and Nilla Wafers, I worked day and night, and by the end of the week, I had a semblance of a thesis. I brought a copy by Sunderun's office.

"Here it is, Professor," I said, "but I'm still very uncomfortable with the construction you outlined. I don't feel like I understand the details."

"I'll decide if it's right," said Sunderun. "You don't worry about it."

That night, I received an e-mail from Sunderun. "All looks good. Defense is next Tuesday, March 20, at 3:00, preceded by cookies at 2:30." That seemed a bit strange. I had never heard of cookies before a defense. But maybe I was getting special treatment.

The day of my defense arrived. I had gotten little sleep, still worrying about the construction that Sunderun had convinced me to take on his word. I continued to work on it that morning, but there seemed to be some problems. I didn't see how all the conditions necessary to apply the Karachi transformation had been satisfied. I was getting very anxious. But what could I do? I kept at it, working to try to figure it out before the defense.

When it got to be 2:30, I went up to the math lounge. I was surprised to see the entire mathematics faculty in attendance. There were fancy cookies and cakes from the bakery down the street, and actual coffee from the coffee shop in Harvard Square. Everyone seemed festive, and at one point they broke into song. But it wasn't any song that I recognized. It seemed to be in some ancient language. And most of the time, they all seemed to have an eye on me. If I was nervous before, now I was really nervous. Sunderun walked over and put an arm around my shoulder.

"Graham," he said. "Have you had any of the chocolate chip cookies? They are the best in Cambridge. We got them especially for you."

"Um, thanks Professor, but I'm allergic to chocolate."

"Oh, I am so sorry. We didn't know. Well, the Linzer tortes are also excellent. Have one of those." He picked one up and actually held it in front of my mouth. I leaned in and nervously nibbled on it. He smiled broadly and then wandered away. I sidled over to Waffle, who was drinking cider like he'd never had it before.

"Waffle, is it just me or are people acting weird?"

Without looking at me he said, "Nah, just happy to see the end of winter. It was a long one this year."

He scooped up a chocolate chip cookie and managed to stuff the entire cookie in his mouth.

"Hab you hab one of zheese?" he asked as he chewed. "Zhey're amayving."

Another song broke out, and a few of the faculty actually started dancing in a strange swaying manner. Then suddenly the Chair rapped on a table. Everyone immediately fell silent.

"Welcome, everyone," he said. "This is a special event. It is a time of transition. A time of renewal. A time of growth. Ushering in the new, ushering out the old."

I leaned over to Waffle and whispered, "Is he talking about my thesis defense?"

Waffle shrugged as he dumped a Linzer torte into his already full mouth.

"We are here to celebrate the fecundity of mathematics. The creation of something new and marvelous to behold, building on the long chain of creation that preceded it."

"Wow," I whispered. "Talk about pressure to perform. . . ."

"And now," continued the chair, "it is time to begin anew."

He reached for the light switch and, flicking the light, exited the lounge. I expected everyone would crowd after him. But instead, the faculty members lined up single file by seniority and solemnly followed him out the door.

Sunderum took me by the arm, and we followed.

"Um, Professor, is it normal that the entire math faculty comes to a thesis defense?" I asked.

"In rare circumstances, yes," replied Sunderum, and then with a steely grip, he guided me to the seminar room.

As we entered, it was standing room only. Seated at the table was my thesis committee. It included some of the most senior members of the department, including the Chair.

As he took his seat, Sunderum pointed at the board and said, "You may begin."

"Um, I um, I need to use the bathroom," I squeaked.

Sunderum frowned but then waved me on. I raced out of the room and ran down to the lounge, where I knew I'd find Waffle still working on the cookies. He looked up, surprised to see me.

"Waffle," I said urgently. "This is not right. Something's going on. You know something."

Waffle swallowed a mouthful of something and then said, "Okay, but you cannot tell anyone I told you."

"Deal."

"Well, it's bad news."

"What bad news?"

"You're the sacrificial lamb."

"What the hell does that mean?"

"Once every seven years, a grad student is sacrificed to the gods of mathematics. This time, it's you."

"What do you mean sacrificed?"

"You've been set up to fail your defense. They have no intention of giving you a PhD. You are their offering to the mathematical gods. They believe it's necessary to keep them creative as mathematicians."

"Are you kidding me? But mathematicians aren't the superstitious kind. All rigorous and rational."

"Well, when it comes to their productivity, not so much, I guess. Everyone's afraid their creative well could run dry at any time. They have no control over that. This is their way of appeasing the gods, and hopefully keeping their creativity alive."

"But what do I do?"

"You fail. That's what you do. There's nothing you can do about that. And then go into some other field. How do you feel about physics? I hear they serve brownies at tea."

He was wrapping leftover cookies in a napkin to go. I trudged back over to the seminar room, took a deep breath, and reentered. The room immediately fell silent.

"Now get on with it," said Sunderum.

I went to the board.

"So, um, I, um, have proved that the bifurcated dihedral leminscate is a bicuspid binormal tetromino."

Sunderum nodded me on. I continued, explaining why anyone should care. And then I began the proof. Initially it went well. All of the pieces were fitting together. But then I got to the part I hadn't understood.

"Then you apply a Karachi transformation and voila! You have a biscuspid binormal tetromino."

Sunderum casually said, "Please explain why it is that all the conditions are satisfied to know that you can apply the Karachi transformation."

Here it was. "Um, well, that would be, um because, you have ramified. Remember the ramification over the reals."

"But the condition for a Karachi transformation is ramification over the complex numbers, not the reals."

I was sweating now. "Yes, but, but . . . "

"But what?" said Sunderum.

"You said it would be okay," I blurted out. "You said it would work."

"I said no such thing," thundered Sunderum. There was a long silence. Then the Chair stood.

"Do you have anything further to say? If not, we are ready to announce our decision." Several faculty members began to light candles.

"Wait," I shouted. I knew it was a shot in the dark, but that morning I had thought of another approach. "Instead of a Karachi transformation, we could apply a Hirimini transformation. For that transformation, all you need is a ramification over the reals."

Everyone froze. Sunderum stood up and spoke slowly and forcefully.

"A Hirimini transformation will not cover the entire space."

"Oh, well, of course, you have to double it. Did I not say that? It's a double Hirimini transformation."

Hirimini, who happened to be sitting at the table since she was on my thesis committee, spoke quietly.

"Yes, that is correct. That would be sufficient."

There was a moment of silence, and then all hell broke loose. Everyone was yelling and waving their hands. Sunderum dropped into his seat and put his face in his hands. Several faculty members moaned while others banged on the walls and gnashed their teeth.

The Chair yelled out, "Silence, everyone!"

All the commotion immediately stopped.

He spoke carefully in a steely tone. "Traditions require that we bequeath to this student, what's your name again?"

"Crocker," I said. "Graham Crocker."

The chair continued. "Traditions require that we bequeath a PhD to this student Graham Crocker, who has surprised us all by demonstrating an understanding of the intricacies required to prove his theorem. These ceremonies are adjourned."

Everyone crowded out of the room, some wailing and wringing their hands. I stood there, stunned. I didn't actually understand Hirimini transformations. It had been a Hail Mary. But maybe creativity is being willing to try something that others may not have considered. I don't know. But at any rate, I somehow managed to get my PhD in spite of the best attempts to thwart me. And so I would have a future in math after all. I was looking forward to a long and fruitful research career. That is, if the superstitions were just that, superstitions. Because if not, I might have ruined mathematics for everyone else, and for me, too. I sure hope not.

Chapter 34
Secrets of Math Destruction

I found her at the vending machines at the University of Chicago library. It's where Chicago students go to have fun on a Saturday night. Dressed in jeans torn out at the knee and a gray sweatshirt with the hood pulled up, she was pulling Twizzlers out of the machine when I walked up behind her.

"You're Sofia Taussky-Todd, aren't you?" I asked. "We need to talk."

She turned quickly, anxiety writ large across her face. "Who the hell are you?"

"I'm Mangum," I said. "Dirk Mangum. I'm a PI."

"A private investigator?"

"No, a principal investigator. I have a National Science Foundation grant."

She smirked. "Am I supposed to be impressed? I got work to do."

She turned to go but I waved a hand.

"Hold on. I think you're going to want to talk to me."

I could see her fist clench around the Twizzlers. Ready to swing if need be. I wondered if the Twizzlers would cushion the blow.

"Spit it out," she said.

"You worked for the NSA," I began, "the National Security Agency—the single biggest employer of mathematicians in the world. They're the people who know what we had for breakfast, lunch, and dinner, just from the pictures we post on Instagram."

She wasn't so self-assured anymore.

"I don't know what you are talking about," she said, looking for an escape route through the stacks, probably the QA aisle. I moved slightly to block it.

"I'm not NSA," I said. "But I know you took a summer internship there."

"Me? You must be confused."

"Not under your current name. But you were there."

"Says who?"

"Says your mother," I replied. "She was so proud."

She didn't say anything, but I could see the conflicting emotions on her face.

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"Listen," I said. "I know the whole story. That your real name is Olga Kovalesky. That you were an undergraduate at Wellesley, the model student. You were all A's, captain of the Putnam team, president of the math club. You were so squeaky clean that Ajax could have used you in their internet campaign."

The Twizzlers looked like they'd be hard to pry apart when time came to eat them.

"So, you applied for the internship at NSA," I continued. "For the summer after your junior year, and surprise surprise, you sailed through the security clearance. You made James Comey look like a security risk." She narrowed her eyes.

"You arrive at NSA, and they set you working on graph pebbling problems, hoping you'd get interested enough that someday, after finishing your PhD, you might sign up for a permanent position. Just the kind of person they want.

"But they didn't know that you're not the kind of person they want. They didn't know that when you were in seventh grade, you were turned."

"This is nonsense. . . ," she said, trailing off.

"Isaac Leibniz, your seventh-grade math teacher," I continued. "Young, bright-eyed, zealous, anti-establishment, and—unknown to the school administration—a math freedom fighter. Someone who believed math is for everyone, and no one has the right to hide the mathematical truths they have discovered. The worst nemesis of the NSA. And he recruited you while you were still in junior high. He turned you before you'd even seen your first integral."

I thought maybe I could see some sadness flick across her eyes for just a second. Before we see the integral, we are still so innocent.

"And then they planted you, a sleeper, in the system. They trained you in how to hide your convictions, how to act the cog in the mathematical machine.

"But in reality, you're a card-carrying member of the Math Liberation Army. You believe math should be for the masses."

She couldn't contain herself any longer. "That's right!" she said emphatically. "Math is for everyone!"

I smiled. "So, you hacked the NSA computers, which for security reasons are not connected to the internet. And on a memory stick, you downloaded all the NSA math, the work that their mathematicians do. Then you swallowed it with a Pepsi chaser and walked right out of the building with all their cherished secrets."

I could tell she was proud of it, even as she tried not to let it show.

"When you got home, you uploaded it to wikiproofs, and everyone from Tokyo to Vladivostok had access."

"That's right!" she said, lifting a fist. "Freedom to the theorems!"

I paused for a second, letting her relish reliving the moment. Then I asked, "You know what happened next?"

"I don't care what happened next," she sneered. "My job was done. I disappeared, changed my name, and switched schools."

"Yup, but I think you should know what happened next."

"Okay, tell me."

"Mathematicians around the world got hold of your treasure trove of theorems, and it was like hitting a beehive with a big stick, a memory stick in this case. And

they began producing results that used those theorems, and they started writing papers."

"That's great! Power to the propositions!"

"Yes. And now others will base their results on those new results. And there will be a ripple effect that generates a tsunami of mathematics."

She smiled triumphantly.

"One tiny little problem," I said.

"What's that?" she asked.

"The files you took off the NSA computer were not what you thought they were."

"What do you mean? They were the internal papers produced by all the NSA mathematicians."

"Nope. That's what they appeared to be. But in fact, they were carefully constructed counterfeits. Fake theorems."

"Fake theorems? That's nonsense. Why would NSA have fake theorems on their system?"

"For exactly this situation. So if anyone did get into the system, they wouldn't be stealing the actual results. They'd be stealing the fake ones."

"But I checked them," she said desperately. "They were real."

"Actually, they weren't. It's not your fault, but you were duped. NSA has an entire team of mathematicians dedicated to concocting convincing fake theorems, papers that purport to prove important results, but in fact have carefully hidden flaws in the proofs. Then, if agents from a foreign country or terrorists get into the system, they think it's the real deal and steal the wrong info."

"Come on," she said weakly. "If this is true, where's the real info?"

"On a computer at Fort Knox. It's not connected to any other computer. It's not even plugged into a wall. Runs of the electricity generated by rotating teams of gerbils running on a treadmill. Joe Biden couldn't get access. His clearance isn't high enough."

"So, you're here to arrest me?"

"I told you already. I don't work for NSA. I work for NSF."

"You have a research grant. That's not the same as working for them."

"I do a few favors for the director, too."

"What does NSF have to do with any of this?"

"Seems the NSF director sat next to the NSA director at a reception in DC. After the usual banter over who had the better acronym, the NSA director had a good chuckle telling her the whole story. So much for keeping secrets. He thought it was thigh-slappingly funny that the mathematicians who built on the wikiproofs material would be destroying their own careers.

"But the NSF director doesn't think it's so funny. She actually cares about mathematics and mathematicians. So, she sent me to speak to you."

"What can I do?"

"You can go on Twitter and admit what you did. Explain it's all a fake."

"Mathematicians don't follow Twitter."

"A couple do, and they'll tell the others."

"But NSA'll send me to prison."

"No, they won't. That would ruin their summer internship program. After what happened to you, everyone would be afraid to sign up. And then how do they recruit for the future? They may not want to give you a job, but they want to be able to hire someone."

"But everyone'll know the damage I've done." She sank to her knees in front of the vending machine, the Twizzlers dropping to the floor.

"What have I done?" she moaned.

"Hey," I said. "It'll be all right in the long run. And you can change your name again. I think Emmy Germain is still available. But fix it ASAP before anyone else gets hurt."

I pointed down at the floor. "You gonna eat those Twizzlers?"

Chapter 35
The Book

Paul Erdös liked to speak of "The Book," which was where God had recorded the most elegant mathematical proofs. In 1985, he said, "You don't have to believe in God, but you should believe in The Book."

Angel: Next. May I help you?

Witherspoon: Yes, I believe so. I hope I have been in the right line. I waited for almost a year.

Angel: What's your hurry?

Witherspoon: Well, I'm sorry if I seem impatient, but I want to get access to The Book.

Angel: You're a mathematician, I take it?

Witherspoon: Well, yes. I was told that upon reaching heaven, we would get access to The Book. We could look up all of the most elegant solutions to the difficult math problems that we devoted our careers to solving.

Angel: I don't know who starts these rumors. But it is not quite that simple.

Witherspoon: It's not?

Angel: First of all, do you have any idea how big The Book is?

Witherspoon: I don't.

Angel: It contains every elegant proof of every result that has ever been found or ever will be found, or even will not be found.

Witherspoon: So it's big.

Angel: (*Rolling eyes*) Yes.

Witherspoon: Well, do you have it as an ebook?

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Angel: Do you see any computers here? Do you see any outlets to plug the computers into? Do you see any walls on which the outlets could sit? We don't have computers. We're old school. It's not an ebook. It is a book Book.

Witherspoon: That's okay. Can I see the volume on group theory?

Angel: It is a single book. Not a collection of volumes. It isn't The Books. It is The Book, a single book that is three miles thick.

Witherspoon: That sounds a bit unwieldy. But nevertheless, I would like to look up a particular result. For instance, can I see how to prove that the only subgroup of order 4 in the Rachland group is cyclic?

Angel: Are you sure you want to see it?

Witherspoon: Yes, I want to see it.

Angel: But then you will have lost your opportunity to think about it yourself.

Witherspoon: I already spent 27 years thinking about it.

Angel: Oh, 27 years. Well, why didn't you say so? That is a long time. Especially when you consider you will be here for eternity.

Witherspoon: Look, I am tired of thinking about it. Can I just see the solution?

Angel: Very well. That is the book there lying on its side. It's page 3,734,322. Come back when you're done.

(*Returns two hours later.*)

Witherspoon: That was disappointing.

Angel: I tried to tell you. The satisfaction factor isn't high when you see how someone else did it.

Witherspoon: The worst part was that Rulenko solved it. I hate that guy.

Angel: Oh, well.

Witherspoon: But maybe I could work on the related problem. I could show that the subgroups of order 6 are cyclic. There's no time limit here. Eventually, I would get it.

Angel: Yes, but you would not be given credit in The Book.

Witherspoon: What? Do you mean to say that if I prove something in my lifetime on earth, I get credit in The Book, but if I prove it once I arrive in heaven, I don't get credit, even if no one has already proved it previously?

Angel: That's right.

Witherspoon: But that isn't fair.

Angel: It is fair. If someone in heaven solves a problem, the people on earth will never know. So they work for years and years and solve a problem and then get to

heaven to find out that their result was previously solved by some dead person. Now how would that look?

And anyway, once you get to heaven, you have unlimited time to work on a problem. Do you really think it is fair that mortals should have to compete with immortals? The mortals get fewer than 100 years to solve a problem, while immortals can work on it for 1000 or more if they want.

Witherspoon: Okay, okay. Never mind about that. Can I just look up the elegant proof of the four-color theorem? I spent ten years trying to find one, and I would love to see that proof.

Angel: You know, that is the single most common request.

Witherspoon: I'm not surprised.

Angel: But we cannot grant that request.

Witherspoon: Why not?

Angel: There isn't one.

Witherspoon: What do you mean there isn't one?

Angel: Just that. There isn't one. About as elegant as you are ever going to get is the original Appel-Haken proof.

Witherspoon: But that's not elegant at all. It involves a computer proof that checks 10,000 cases.

Angel: Right. So it's not in The Book.

Witherspoon: But why isn't there an elegant proof?

Angel: It's not my decision.

Witherspoon: Okay, then why didn't God make an elegant proof?

Angel: God doesn't make up mathematics. They can't just make things true, make them logically follow from a set of axioms, if they do not. Then it would be logically inconsistent.

Witherspoon: So God isn't omnipotent?

Angel: Watch your tongue, buddy. Of course he is omnipotent, but if he makes $1 + 1 = 3$, then the universe, earth, and everything else disappears in a logical inconsistency. So what would be the fun in that?

Witherspoon: Okay, so now I want to look up the proof of the Witherspoon Conjecture. I want to see the elegant proof of it.

Angel: And you don't want to think about it yourself?

Witherspoon: It's already solved, I just want to see it.

Angel: Okay, hold on, Whiteside, width of a knot, Willing, cohomology. Ah, here it is. Witherspoon Conjecture. Formulated by Charles Witherspoon, February 12, 1979.

Witherspoon: Actually, I formulated it February 11, but just told people about it February 12.

Angel: Oh, so you are Charles Witherspoon.

Witherspoon: Yes, I am. I made up the conjecture.

Angel: Ah. I see. Congratulations. Well, here it is, page 6,734,321. Knock yourself out. Go ahead. Take a look.

(*Returns an hour later.*)

Witherspoon: It said it was solved by Karen Sinkletter.

Angel: Yes, that is correct. A 12-year-old girl from Toledo.

Witherspoon: But that's wrong. I solved the Witherspoon Conjecture. I published the result in the Journal of Number-Free Theory, December 1993. And the elegant solution the book had was the way I did it. My name should be there.

Angel: I'm sorry, but it says here that Karen Sinkletter solved it. She wrote it down on a gum wrapper in July 1992, and then threw it away.

Witherspoon: And that counts?

Angel: Yes, I am afraid it does. She published it, just not in a conventional journal.
Witherspoon: I protest.

Angel: That's nice.

Witherspoon: No, I mean I want to lodge a formal protest. I want The Book to be fixed.

Angel: The Book is already fixed. We cannot change anything.

Witherspoon: But you do change things. Every time someone proves a new elegant result, the book is changed to contain it.

Angel: No, The Book already contains it.

Witherspoon: All right then, you change the credit for the result.

Angel: No, we already know who is going to prove which result. It is already listed back to the beginning of time and forward to the end of time.

Witherspoon: Well, I'm not happy about that.

Angel: Sorry. Are we through here? There are a lot of people waiting in line.

Witherspoon: No, I would like to see the proof that semi-upper left coset multiplicity implies devolutory minor self-multiplication.

Angel: Page 23,672,445. Help yourself.

(*An hour later.*)

Witherspoon: That wasn't the proof. It was a counterexample.

Angel: Yes, turns out it isn't true.

Witherspoon: And it said the counterexample was discovered by Wafflepunklem. What kind of a name is Wafflepunklem?

Angel: Wafflepunklem is a fipplepicker from the planet Turlemonde.

Witherspoon: Wait a minute. You mean The Book contains results proved by aliens on other planets?

Angel: Of course. Why would we only include results proved on earth? Math is math, wherever it is done.

Witherspoon: But then why, if you include their results in The Book, don't you admit the aliens to heaven?

Angel: Have you noticed a few blue residents? Perhaps some furniture that moves?

Witherspoon: Now that you mention it...

Angel: There you go. Anyway, it's been nice talking to you. Next.

Witherspoon: Wait a minute. I want to see the proof of the Riemann Hypothesis.

Angel: Sorry. You only get three looks a visit.

Witherspoon: What? Nobody told me that.

Angel: You have to read the Welcome Manual.

Witherspoon: What Welcome Manual?

Angel: It's in the desk drawer by the phone in your room, right next to the Gideon Bible.

Witherspoon: But I didn't know that. Please, I really would like to see this one.

Angel: Sorry. You have to get in line again, fella. And while you're waiting, take the opportunity to try to solve it yourself. It'll help pass the time. Next.

Chapter 36
A Grader's Dream

Once upon a time, there was a poor lecturer who had a one-year temporary position at a community college in a borough far from trendy restaurants and compelling shopping alternatives. This was a time before online shopping, so the lecturer lived a miserable existence. He sought to do research so that he might publish papers, which would allow him to receive grants and ultimately obtain lucrative job offers from colleges and universities in more urban areas. But unfortunately, he was burdened with multiple remedial math classes and hundreds of students per semester.

"Oh, woe is me," he would exclaim, as he confronted the mountainous pile of grading he had to do. One evening, as he sat in his study, ready to begin the night's grading, he was suddenly overcome with sleep. Incapable of keeping his eyes open any longer, he said to himself, "I will just lay my head down on the desk for a moment," and within an instant, he was sound asleep.

In the morning, the birds' singing awoke him.

"Oh, no," he thought. "I have slept through the entire night and my students will be angry that I have not graded their homework. They will give me horrible ratings on the student faculty evaluations at the end of the semester, and I will be unable to attain a permanent job anywhere."

But then, looking at the stack of papers before him, he noticed a red score at the top of the first paper.

"I don't remember grading that," he thought to himself. He scooped off the first paper, and, lo and behold, the second paper was graded as well. He sifted quickly through the pile, and every one of the papers had a score circled in red at the top.

Taking the first, he went over it, problem by problem, and saw that the score was exactly right. If anything, the grading job was better than he himself could have done.

He thought, "Well, perhaps I graded them in my sleep." He had heard of sleepwalkers writing letters and cooking dinner, so perhaps this was possible. At any rate, he leaped up from his chair, did a short jig, and then grabbed up the papers, stuffed them into his briefcase, and raced off for his first class of the day.

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Later that evening, he found himself seated at his desk again, with an even larger pile of papers.

"I will just put my head down for an instant," he said to himself, and before he knew it, he could hear the birds chirping their morning greetings.

"Oh, what have I done," he thought, but lo and behold, once again all of the papers were graded, and they were all completed perfectly.

After his classes that day, the chair of the department stopped by.

"Lecturer," he said. "We are getting good reports about your teaching. Students say you get their homework back the next day, and fairly graded at that."

The lecturer smiled. "Thank you, sir," he said. "Perhaps you will consider extending my appointment another year."

"Not really what I had in mind," continued the Chair. "Obviously, we are not giving you enough work. Henderson is sick with the gout, so I want you to take over her sections starting today."

That night the lecturer came home with a pile of papers that required multiple trips to the car and back. He stacked them up on the desk.

"Oh, what have I done?" he wailed. "For I cannot grade all of these papers."

He put his head down on the desk and cried. But drowsiness soon stole over him and he fell sound asleep until morning. Once again, in the morning, all of the papers were graded perfectly.

And so things continued. As he no longer had to grade, the lecturer had time to think about original mathematics. He considered jet bundles and other esoteric objects, and he discovered various interesting connections between ornamental self-adjoint distributive operators and Einstein–Lamarck nonorientable solv-manifolds. He published several articles in well-known journals, for which, because of the quality of the work, the journals set new precedents by rushing the articles into print. Requests to speak at conferences throughout the province began to flood in.

One day, as he was in his now-private office working on some new ideas that related semiautomatic groups to simple dilogarithmic Stieljies integrals, the resident of the neighboring office stopped in.

"Given all the grading we have to do, how do you find the time to get all this research done?" asked the professor.

"I do all my grading when I am asleep," replied the lecturer. And then he proceeded to tell the professor the story of what happened every night.

Upon finishing the story, the professor said to him, "Is the grading in your handwriting?"

"No," replied the lecturer.

"Then, has it occurred to you that perhaps you are not doing the grading at all, but in fact someone else is doing it?"

The lecturer realized that in fact, this was the very thing that had been bothering him these past few months, only he had been having some trouble pinpointing the problem. So that night, when he piled up the papers on his desk, he only pretended to sleep.

An hour after he laid his head down, two naked elves, each only twelve inches high, entered his study, humming a tune he did not recognize, but he suspected was

by Katy Perry. Very quickly they each lifted a red pen and started to work on the papers. As the pen was well over half their height, they had to work hard to do the grading, but even so, he was amazed at the speed with which they graded. Whereas he might agonize over a point here and a point there, they seemed able to make decisions and grade at an amazing pace. And most incredible of all, they seemed to enjoy the work. They would skip and sing as they flipped the pages of the homework. Problems out of order and unreadable scribbling seemed not to bother them at all. They would laugh over particularly egregious errors. One would say, "Oh, look. This silly student says that the square root of a sum is the sum of the square roots." Then they would howl with laughter. The pile of finished papers began to grow, and the pile left to do shrank accordingly.

Just before dawn, all the grading was done. The two elves replaced the red pens where they had found them, and, singing the Katy Perry song still, they disappeared out of the door. The lecturer felt an immense surge of gratitude for the elves. For through their work, they had surely saved his career.

He decided that he wanted to do something in return, and he thought about how they risked puncture wounds from the red pens as they raced about their business swinging the pens to and fro in their nakedity. So, he had tiny t-shirts made with fractal designs on the front. And tiny torn jeans. And tiny Keds sneakers.

That night, he laid out the clothes and then pretended to fall sleep. When the elves arrived for their grading party, they spied the clothing. Jumping about and singing the newest Katy Perry song all the louder, they donned their new duds.

"This is fantastic," said the first to the second. "Now, we can finally apply to grad school."

"Yes," said the first. "Although they have a liberal dress code in grad school, they do require clothing. Ph.D.s, here we come." And with that they disappeared out the door, never to be seen in that town again. However, the lecturer didn't mind.

The very next day, he went into the Chair's office.

"Oh, hello," said the Chair. "It is so good to see you. I have been hearing great things about your research. Can we provide you with an extra box of chalk?"

The lecturer dumped a huge pile of ungraded papers on the desk.

"I quit," he said.

"Oh, please don't quit," said the chair. "We'll give you a reduced teaching load and TAs to do your grading."

"Don't bother," said the lecturer. "I have a position at the Institute for Advanced Study. I won't be doing any grading anymore."

And with that, he turned and left, leaving the chair sobbing amongst all the papers cascading from his desk to the floor.

To this day, there are some people who do not believe in grading elves. They make a point of doing their grading as soon as possible. But to those of us who believe, we see nothing wrong with putting our heads down on the desk and drifting off to sleep, in the hope that we will wake up in the morning to find all of the papers graded. One can always hope, after all.

Chapter 37
The End of Mathematics

It was just a random Tuesday, like any other Tuesday. Since I was on a MWF teaching schedule that Fall, I purposely got in early, hoping to spend a few hours trying to prove that regular sub-definite fibroid bundles were dense in the space of all sub-definite fibroid bundles.

"Hey, Craig," I said, waving as I walked down the corridor past the Chair's open door. I was surprised to see him in so early.

"Don't bother," yelled Craig after me. His voice sounded strained. I stopped and returned to stick my head in his office door.

"What do you mean, don't bother?" I asked.

"I mean it's over. So don't bother."

"What's over?"

"Math," he said.

It looked as if he had been crying. I entered his office and sat down in the chair in front of his desk, dropping my briefcase to the floor.

"It's umm…, it's…," he began, but then his voice broke, and he just put his head down on his desk and started sobbing. I assumed he had just heard about some personal tragedy.

"Um, Craig… whatever it is, I'm sorry. But, umm, I'm having trouble figuring out what you're trying to tell me." I tentatively reached out a hand, and then, not sure what to do, patted the bald spot at the back of his head. He seemed to get himself under control, and then he lifted his head from the desk.

"Don't you understand?" he said, wiping the tears from his cheeks. "Math is over. It's finished. Kaput!"

I would have laughed if he weren't so distraught.

"How can math be over?" I asked. "Math is an expanding ball of knowledge that grows forever."

"Yeah, that's what I thought, too. Mathematicians just keep adding to it, creating new tendrils, bridging gaps, filling in holes, and making connections between

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disparate fields. And there's no end. The ball of knowledge that is mathematics just keeps growing and growing and growing."

"That's right," I said, nodding. I thought to myself about the backwater of mathematics that I had been working in for the last five years. It was just me plugging away. No one else had yet shown interest. But I was still publishing papers, and it was exciting to me. I was filling in a small piece of that giant ball of knowledge that is mathematics.

"But it's not right," said Craig.

"How can it not be right?"

Because Kropenhauer proved it's not right."

"Sarah Kropenhauer? The logician we hired last year?"

"Yes."

"What do you mean she proved it's not right?"

"She proved that mathematics is bounded. There's only a finite amount of it."

"That's ridiculous," I said. "Math isn't bounded. It goes on forever. You can always find new things to prove."

Craig shrugged. "That's what they used to say about the Earth. In ancient times when they thought it was a neverending plane. Well, we know how that turned out, don't we?"

"So you're saying it's like that for math? It doesn't go on forever?"

"Kropenhauer says it."

"Just because she says it doesn't make it true."

"She proves it."

For the first time in the conversation, I felt a cold chill go down my spine.

"Have you seen the proof?" I asked.

"She emailed me an outline last night. Asked me to look it over."

"And?"

"And it looks right."

"I don't believe this for a second. How does she prove it?"

"She puts a metric on all of math, known and unknown. And she turns it into a manifold."

"What could that possibly mean? Mathematics isn't a continuous object. It can't be locally homeomorphic to an open unit ball."

"You're right, there. It's more complicated than that. I don't understand it all, but it's what she calls a granular manifold. On the local level it's made up of individual facts, but on the global scale it behaves like a manifold."

"Okay, but so what?"

"She proves the manifold is compact."

"Okay, so she proves it's compact. But that doesn't make it finite."

"If it's granular, it does."

I was becoming upset myself. "But Craig, this is nonsense," I said. "Look, I'll create an infinite amount of mathematics right here, right now. Let's see. You're a number theorist. So how about this? Define a number x to be an n-product if it's a product of n primes. Counting multiplicities. Then ask the question, what happens

when you add 1 to an n-product. What fraction of the time is it an n-product? There. I just made that up."

"Geoff, you're not a number theorist. You shouldn't make up number theory questions."

"What's wrong with it?" I asked.

"First of all, you are not the first person to make that up. There is a name for the number of prime factors counting multiplicity of a number x. It's called $\Omega(x)$. And you are asking about the numbers x with $\Omega(x) = \Omega(x + 1)$."

"Yes, but I am specifying the $\Omega(x)$."

"Okay. And then when you say what fraction, you mean take all positive integers less than a given integer K, and take the subset of them that are n-products and then find the fraction of that subset that remain n-products when you add 1. Then you are taking the limit as K goes to infinity."

"Okay, then yes, that's what I mean."

"You don't even know if the limit exists."

"Yeah, well maybe it doesn't exist, so that's the question. Does it exist, and if it exists, what is it?"

"So what's your point?"

"My point is that math is infinite. I just listed an infinite number of questions. For each value of n, there's another question. Those are questions that may one day be filled in, in that giant ball of mathematical knowledge."

"First of all, the questions existed before you asked them. And second, that's not an infinite number of questions about mathematics. It is all encompassed in the single statement: 'Determine the limit of the fraction of n-products that remain n-products when you add one.' Kropenhauer assumes all statements are maximal in that same sense. They are stated in their full generality."

I clasped my arms to my chest and rocked back and forth in the chair, my body language reflecting my discomfort.

"Okay," I said. "But I guess I am missing the point. I mean, even if she has proved that math is finite in this weird maximal statement sense, so what? There are still lots of maximal statements out there. And some of them have an infinite number of sub-questions embedded within them. So math remains infinite. The number of questions remains infinite."

"That's not the way the National Science Foundation will see it," said Craig. Now, I felt my bowels tighten.

"What do you mean?" I asked.

"They'll shut us down so fast we won't know where the blackboard is."

"But they can't do that. I mean even if this is all true, the fraction of math that we have discovered of this finite set could be miniscule. What about the Riemann Hypothesis? What about P = NP? There are innumerable conjectures out there that we have yet to solve."

"7 percent."

"What?"

"That's what we have left to solve. 7 percent. According to Kropenhauer."

"That's preposterous. She can show that 93% of mathematics is done?" I asked incredulously.

"Yes, at least up to plus or minus epsilon. NSF will probably fund a few key areas where big results have yet to be solved. And they'll probably continue to fund applied math. Somebody has to think about how this finite body of knowledge can be used to our advantage. And there'll still be teachers. But people like you and me, our careers as researchers are over."

"This can't be happening. There's got to be something we can do."

"Anyway you look at it, we're screwed," said Craig. He shook his head. "Short of killing her, there's nothing we can do."

I laughed. "Oh yeah, kill a mathematician to prevent her result from getting out. Like that would work. I'm sure the result's already out there."

"Actually, it's not," said Craig.

"What do you mean?" I asked.

"Shut the door," said Craig.

"What?"

"Shut the door."

I reached over and pushed the door shut. Craig leaned forward across the desk and spoke in a low voice.

"She hasn't told anyone. She understands the implications and how upset everyone's going to be. She said in the email that I'm the only one she has yet told."

"Nobody else knows?"

"Not yet. I emailed her last night to say I would look it over and meet with her in the tenth floor seminar room this morning at 9:00. I think she's upset by it all, too."

"Oh yeah. I bet she's upset. She will instantly become the most famous living mathematician."

"Famous like Al Capone," said Craig. "There are a lot of people who will hate her for what she's done."

"Yeah, but if she hadn't discovered it, eventually someone else would have. That's why it's pointless to even talk about shutting her up. The end result will be the same."

"Maybe," said Craig, shrugging. "But that could take years. You and I would be safely retired at that point."

"This is crazy talk," I said. "I couldn't kill anyone just to keep my job."

"Don't do it for yourself then. Do it for the rest of the math community, the thousands of researchers who will lose their funding. Who will lose the meaning in their lives. Do it for them."

"I wouldn't even know how to kill her," I heard myself saying.

"Just be waiting in the stairwell outside the tenth floor seminar room at 9:00 AM."

Let's just say things did not go according to plan. Kropenhauer showed up in the seminar room for her appointment with Craig at 9:00, but when Craig told her he wanted to show her something down one flight, she must have become suspicious. When she entered the stairwell and saw me, she made a break for it. In the ensuing

scuffle, Craig and I tried to lift her over the railing but she grabbed hold of Craig's shirt, and for just a second he was caught off balance. The two of them tumbled over together. I will never forget their screams as they fell, and the sudden silencing of those screams when they reached the bottom. I disappeared into my office and kept out of sight until it was safe to come out. Everyone was baffled by how these two had gone over the railing together.

A week later, as the most senior member of the department, I was asked to gather Kropenhauer's things, and clear out her office. When I went in there, I found her computer still on and her mail server open.

There was an email that had come in the morning she died, from Rubinoff in Russia. He was responding to an email from her the night before. She had blind copied him on the email to Craig. I felt my stomach churn.

As I read down the reply with dread, I reached a point where it said, "You are implicitly assuming the metric is semi-normal. Only the most aberrant metrics are not. But this is just such a metric. Your result is not valid. You have proved nothing."

I sat stunned. None of this had needed to happen. Nobody needed to die. I shook my head sadly. Then I deleted the message, scooped all the papers on the desk into the wastebasket, and left the office, locking the door behind me.

Chapter 38
Definitions that Sizzle

How many times have you, with great anticipation, flipped opened the latest math book, started panning down the page, and suddenly come face to face with the definition of a semidirect pseudosubalgebraic schema? You read it once, confused as all get out, read it again, still flummoxed, and finally slam the book closed in disgust. And yet, truth be told, you write papers containing functions and spaces and operators with just as technical and uninteresting names and definitions as that one.

Well, by now, you probably realize what time it is. Yes, it's time to break out of your rut. Time to take your mathematics to the next level. Time to learn how to name your new concepts and write their definitions in a way that will grab readers by their shirtfronts and yank them bodily into the material.

Mathematics is no different from any other human endeavor. You have to present it well if you want it to get noticed. You already know this about personal relationships. You wouldn't go on a first date without having flossed your teeth and changed into a clean t-shirt. And you know this about getting a job. You wouldn't go on a job interview without first having prepared a talk that mentions as many faculty from the department in question as possible and without having changed into a clean t-shirt. It's the same with mathematics. You want people to pay attention to your theorems and lemmas? You want them to look forward to your next paper, checking the ArXiv every day to see if you have posted it yet? Then you need to put some time and thought into how you present it, to metaphorically brush your paper's teeth and change it into a clean t-shirt.

Today, we will focus on brushing your paper's teeth, which is a metaphor for one of the most important and often neglected fundamentals of mathematical writing. Definitions. Many mathematicians believe that definitions write themselves. They could not be more wrong. Definitions do not write themselves. They are written. And how they are written can determine their own fate as well as your own. So today, we will explain how to write definitions that sizzle.

C. Adams, *Do Androids Dream of Symmetric Sheaves?*,
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Step 1. Branding

You have just come up with a clever new mathematical object that makes you proud as hell. Maybe it's a probability distribution, or a vector space, or a new quasidimension. What are you going to call it? Don't even think about naming it after yourself. As pleased as you may be, there is nothing less classy that naming it after yourself. And don't try the old "B-space" if your last name starts with B. Everyone's onto that trick. So then, what should you call it?

Example: Sierpinski called his curve *nowhere differentiable*. Come on, Sierpinski, you could do better than that. There's no zing there, no pizzazz. Who's going to want to read about a nowhere differentiable curve? Why should the basic concept be the negation of something else? And why so technical? Imagine, if you will, how it would have been received if he had the foresight to call it a "kinky curve." Front page of Newsweek: "Sierpinski Invents Kinky Curve."

Remember, the goal is to catch the random reader's attention. There are a lot of journal articles out there. How do you get someone to notice?

Here's a good trick. Include two contradictory items in the same definition. For instance, you could define a compressible incompressible surface. Someone who stumbles across that while reading through a paper, well, is that person just going to read on by? No, they will stop and say, "What the hell could that mean? Those are two contradictory concepts."

Now you might be wondering about those contradictory concepts. How can a surface be both at the same time? And in fact, each by itself does mean the opposite. An incompressible surface is by definition one that is not compressible. But we are not saying that the surface is both incompressible and compressible at the same time. That would be ridiculous. No, we are saying that it is "compressible incompressible," which means something unrelated to either word. It means the surface has four boundary components. Get the idea?

Imagine if Sierpinski had the chutzpah to define a differentiable nowhere differentiable curve. Now that would get some attention! What is it? I don't know. That's his problem. I'm just about the branding.

The other advantage to this technique is that it keeps the riffraff out of the field. Only the experts know that compressible incompressible means four boundary components.

Another good trick. Certain phrases get stuck in your head and you can't get them out. Our goal is to do the sticking. A great way to create a sticky moniker is through alliteration. For instance, take the Herman's Hermits. Do you think we would still remember this 1960s band if it were not for the genius of alliteration? That same genius is reflected in homotopic Hopf bundles, or lambda-laminations, or self-similarity. Look what it did for Euclid's *Elements*, Euler's Equation, and the Poncelet Paradox. (Or Poncelet's Porism or Poncelet's Principle. Poncelet was a natural.) What a shame that Cantor didn't discover the Dedekind cut. Or that Hopf

didn't come up with the Riemann Hypothesis. Or how about if Fourier had enough sense to skip the series and just invent a formula? If it rolls off the tongue, it is likely to do so often.

Other tricks? Don't be afraid of hyperbole. Look at what the name *God particle* did for the Higgs boson. And the *Grand Unified Theory*. How about that for a name? Physicists clearly are not uncomfortable with going over the top.

Mathematicians can sometimes pull it off, too. For example, in commutative algebra, there are excellent rings. How excellent are they? I don't know, because the definition is really complicated. But obviously someone thinks they are excellent enough for the name, and that's good enough for me. Mix the superlatives with some alliteration, and now you're cooking with alcohol. Possibilities include superb Borel sigma-algebras, omnipotent orbifolds, and admirable arithmetic algebras.

Now, what about the situation where you have a new term you are defining, but in your heart you know no one is going to care about this thing at all? It's a throwaway. What do you do then? Naming opportunity! Every college and university, and quite a few preschools, have realized the lucrative nature of naming opportunities. Someone gives a million dollars to an institution, and a room in the Music Building becomes the Blattenborg Center for Creative Dissonance.

Mathematics can benefit from the same basic principle. Say you are a young knot theorist and you are trying to get your career kickstarted. Invent a new knot (come on, how hard is that?) and name it after the expert in the field. Call it a Warbenpoffler knot. Even if it is unrelated to Warbenpoffler, nobody but you and Warbenpoffler will actually know that. And Warbenpoffler will be eternally in your debt.

Short on funding to go to conferences? How about Heegaard Budweiser homology? Or the Staples biholomorphic mapping? Companies will be thrilled to see their brands appearing in math definitions. Within 10 years, everyone will be doing this. Now is the time to get in on the ground floor.

Step 2. The Definition Itself

How do we write a great definition? Let's take a look at some classic examples.

The Original Definition of Continuity of a Function from $\mathbb{R}$ to $\mathbb{R}$: A function is continuous if its graph can be drawn without lifting a pencil from the page.

Now there are many problems with this definition. First, a pencil? Really? Pencils represent impermanence. You can erase pencil marks. Pen would have been much better.

And "from the page?" What page? Are we writing in the margin of our book? Historically, not a good idea. Is it a page from a notebook? What if it tears? What then? And even with the writing implement and writing surface aside, this definition has no weight to it. There is no umph there. Let's take a look at the newer definition that has superseded the old one.

New Definition: A function f is continuous at x if for all $\epsilon > 0$, if there exists a $\delta > 0$ such that for all y satisfying $0 < |x - y| < \delta$, $|f(x) - f(y)| < \epsilon$.

Notice several things. First, the use of Greek letters. This signifies to the reader that this is serious heady intellectual stuff. This isn't your standard ABCs. Second is the throwing in of some regular letters, a sprinkling of inequalities, and an absolute value or two. It doesn't get much better than this.

Let's take a look at another example.

Old Definition: A bijection is a one-to-one and onto map. Weak, very weak.

First, map of what? Map of New Jersey? Map from a pseudomeasurable semiupper bialgebraic variety to an infinite product with the box topology? Come on, give us some information here. And where are the Greek letters? In fact, no letters at all.

Let's try it again.

New Definition: Given sets $\mathfrak{X}$ and $\mathfrak{Y}$ and a function $f : \mathfrak{X} \to \mathfrak{Y}$, f is a **bijection** if the following two conditions are satisfied:

(i) For all $\mathfrak{y} \in \mathfrak{Y}$, there exists an $\mathfrak{x} \in \mathfrak{X}$ such that $f(\mathfrak{x}) = \mathfrak{y}$.
(ii) If $f(\mathfrak{x}) = f(\mathfrak{y})$, then $\mathfrak{x} = \mathfrak{y}$.

Much better.

Advantages of the New Version:

1. The German fraktur letters. They lend depth and intellectual gravity to what is really a matching of fingers and toes.
2. The fact that there are multiple conditions that need to be satisfied. It must be important.
3. The use of the word "satisfied." It brings to mind pushing back from the table after finishing off heaping plates of turkey, stuffing, and mashed potatoes slathered in gravy. It just feels good to say it. "Satisfied."
4. The list of conditions is numbered using (i) and (ii). Again, this denotes a seriousness of purpose that is not present when you use 1, 2, 3 or A, B, C.

Let's try another example.

Old Definition: Let A be a subset of a topological space X. We say that a point x is a limit point of A if every neighborhood of x intersects A in a point other than x.

What's wrong with it? First, "Let A be a subset of a topological space X." Really? You are asking permission? "Please, sir, might I let A be a subset of X?"

No, you have to be assertive in this business. Let's try it the right way.

New Definition: A limit point x of a subset A of a topological space X is a point x of X that could be in A or not, such that $A \cap N - \{x\} = \emptyset$ (and this can mean $(A \cap N) - \{x\}$ or $A \cap (N - \{x\})$, because they are equal) for all neighborhoods N of x, where a neighborhood N of x is any open set that contains x.

One sentence, aggressive presentation, lots of letters and symbols, additional parenthetical remarks to clarify, and packed with all the information you could possibly need to understand it. What more could you ask?

So, I hope by now you are ready to go out into the mathematical hinterlands armed with the knowledge of how to choose your terminology and how to define it in an aggressive, exciting, and symbol-filled manner. Your readers will be thrilled with the result.

Please come back to the blog next week when we discuss the fundamental question that has puzzled many a mathematician for years. “Did I just prove a theorem or a proposition?” See you then.

Chapter 39
Edutainment

Computer: Hello, Lisa.

Lisa: Hello, computer.

Computer: Do you want to continue with your calculus class?

Lisa: Yes, please.

Computer: Would you like to continue with Bradley Cooper as the police lieutenant?

Lisa: Yes, please ... no, wait. I'm feeling Brad Pitt today. Make it Brad Pitt.

Computer: As you wish.

(*Scene appears on computer screen. Brad Pitt is pacing around the room. He turns to face the camera.*)

Pitt:(*Shaking head*) I don't know what to do. This serial killer really has us stumped. Lisa, any idea who it might be?

Lisa: Umm, the janitor at the school?

Pitt: Could be. He's one of the few random characters who have been introduced to the storyline. But if we're going to pin it on him, we'll need some evidence. Lisa, I'm depending on you to solve this crime. You're our calculus expert, and as you know, every clue so far has involved calculus.

Lisa: Yes, I noticed that.

(*Detective dashes into the room carrying a small box.*)

Detective: Lieutenant, we just got a package. Looks like it's from our killer.

Pitt: How do you know?

Detective: The return address. It says "The Killer."

C. Adams, *Do Androids Dream of Symmetric Sheaves?*,
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Pitt: This could be bad. Steel yourselves. Okay, open it. (*Detective does so.*)

Detective:(*Looking into the box*) Oh no.

Pitt: Show us.

(*Detective lifts out a lock of hair.*)

Pitt: Oh, my God.

Lisa: What is it?

Pitt: I recognize that lock of hair. That's Weber's.

Lisa: Weber?

Pitt: You remember Weber. He was the detective that almost drowned during the mean-value theorem episode.
Lisa: Oh, yeah.

Pitt: That's definitely his hair. Note the dark brown with just the slightest hint of copper.

Detective: There's also a note.

Pitt: What does it say?

Detective:(*Reading*) Dear incompetent gumshoes, turn on your computer and go to the website "Imalosercop.com" before you read on.

(*The detective turns to a computer and types in the site. Camera zooms into the monitor. Shot of computer screen, where Weber is hanging upside down, above a large vat of bubbling liquid.*)

Weber:(*Desperately*) Help me!

Detective: Oh, my God.

Weber: He's insane. Completely insane. He gave me a terrible haircut. I won't be able to go out in public for a month. (*pause*) And he's threatening to kill me.

Pitt:(*To detective*) Keep reading.

Detective:(*Reading*) Okay, you liver-lilied flat-feet. I am listing ten addresses on this piece of paper, numbered consecutively 1 to 10. The integral that appears below has an answer that is one of those numbers. That is the address where Weber's life hangs in the balance. If you get there by 2:00, Weber lives. If you don't, he dies.

The detective walks to the whiteboard and copies out:

$$\int_0^{\sqrt{3}} x^3\sqrt{1+x^2}\,dx$$

Pitt:(*Turning to the camera*) Lisa, any idea how to solve that? We really need the answer or he'll kill Weber.

Lisa: Why don't you send a police car to each of the ten addresses?

Pitt:(*Shaking head resignedly*) Budget cutbacks. We don't have the resources. Lisa, we're depending on you. You have to solve it, and you have to solve it now.

Lisa: I'm ... I'm not sure.

Pitt: Lisa, remember, we talked about this before. Could you do it if there weren't so many x's?

Lisa: I think so. If there was only one power of x in there, I could maybe do a substitution.

Pitt: Okay, so how can we get rid of extra x's in an integral like this? What technique would work?

Lisa: Integration by parts?

Pitt: It's worth a try! What do we have to lose?

Weber: Me! We have me to lose. Isn't there a faster way? The clock is ticking.

Lisa: Could we use u-substitution?

Pitt: Now we're cooking with oil. Oh, sorry Weber, ignore that remark. What do you think we should use for u?

Lisa: $u = 1 + x^2$? Then $du = 2x\ dx$. Oh, too bad. We have too high a power of x still left over in the integrand. Sorry, Weber. Maybe back to integration by parts.

Pitt: You know, Lisa, 73% of the time, people don't realize they could use the expression $u = 1 + x^2$ to eliminate the extra x^2. You may wonder how I know a statistic like that, but it's my job to know.

Detective: That's how he made lieutenant.

Weber: Dammit, Lieutenant. Stop the lectures and tell her how to solve the integral.

Pitt: Come on, Weber. You know I don't know how to do that integral. Even though I seem to be an expert on the techniques, and can provide a helping hand, I can't apply them. It's a mental block that I have, going back to a traumatic calculus accident when I was a child. Lisa is our expert. She's the only one who can save the day here.
Lisa: Well, maybe we can use $u = 1 + x^2$. Then $du = 2xdx$, so $du/2 = xdx$. And the extra $x^2 = u - 1$. So we get:

$$\int_0^{\sqrt{3}} x^3\sqrt{1+x^2}\,dx =$$

$$\int_0^{\sqrt{3}} x^2\sqrt{1+x^2}\,xdx =$$

$$\int_1^4 (u-1)\sqrt{u}\frac{du}{2} =$$

$$\frac{1}{2}\int_1^4 u^{\frac{3}{2}} - u^{\frac{1}{2}} du =$$

$$\frac{1}{5}u^{\frac{3}{5}} - \frac{3}{5}u^{\frac{1}{5}}\Big|_1^4 =$$

$$\left(\frac{32}{5} - \frac{8}{3}\right) - \left(\frac{1}{5} - \frac{1}{3}\right) = \frac{58}{15}.$$

Detective: That's not one of the numbers.

Pitt: Yes, but it's awfully close to 4. So that must be the address. Get a squad car to that address now!

(*Pitt pulls out a handkerchief, wipes the sweat off his brow, and plops down in a chair, visibly relieved.*)

Pitt:(*Turning to Lisa*) That was a close one. I don't know what we'd do without you.

Lisa: Just doing my job.

Pitt: And by the way, thanks for solving that indefinite integral that was the clue left by the killer in the Wickstrom case.

Lisa: Oh, good. Was it right? Did you save her?

Pitt: Well, unfortunately, you forgot the $+C$. Oh well, you win some, you lose some.

Lisa: (*Visibly upset*) You mean Wickstrom is dead?

Pitt: Come on. This show is rated PG. He just gave Wickstrom a terrible perm and then released her at a gala at Lincoln Center. Wickstrom will never live it down. Now, if you don't mind, can we turn to the case of the improper integral? This guy's a real nuisance.

Chapter 40
A Prisoner's Dilemma

I watched as Maggie stepped through the iron doors. She paused for a moment, breathing in the fresh air, feeling the warmth of the sun on her face. Then she spotted me leaning against the car. My anticipation of our joyous reunion evaporated as she turned and started walking away, her bag of personal effects swinging at her side.

"Maggie," I called. "Don't!"

She stopped and turned to look at me, her brow furrowed. Then she slowly and deliberately walked over.

I smiled my best smile. "Hi," I said, putting my arms out for a hug. She waved the hug away.

"Umm, I thought maybe you could use a lift, and I really wanted to see you. You look great by the way. I guess prison suited you."

I was talking fast just to prevent her from walking away. "Hey, I got a place up on West Pine. You can crash there for as long as you want. Well, you can live there actually. I really missed you."

"You are such a jerk," she spit out. At least she was talking to me.

"I am. I'm a jerk. Terrible jerk. Always have been. But you always made me better. I behaved around you."

"I just did three years hard time thanks to you. Three years!" She clenched her fist, and shook it in my face.

"Hey," I said. "I only got out last month. I did the same three. And I could blame you."

"I loved you," said Maggie. "I trusted you. How could you screw me so badly?"

"We screwed each other," I said quietly.

She shook her head and turned, walking away. I chased after her.

"Maggie, please don't walk away. I don't resent you. I mean, I did at first, but eventually, I got over it. I realized it wasn't you at all. It was game theory."

She stopped walking and turned to face me.

"What are you talking about?" she asked.

C. Adams, *Do Androids Dream of Symmetric Sheaves?*,
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"It's the theory of what people do, given certain choices, if they want to maximize their outcome."

She laughed. "Since when did you start talking like a CEO?"

"I read about it. About game theory. When I was in prison. It explains a lot."

"What does it explain?"

"It explains our behavior, for one."

"What do you mean?"

"I mean it explains why we both ratted each other out."

"I know why you ratted me out. Because you're a jerk."

"And why did you rat me out?"

She paused and looked down at the pavement. Then she sighed.

"Because deep inside, I didn't know if I could trust you." Raising her head, she looked me right in the eye. "And I was right!"

"Maggie, I'm sorry. It was that detective, Schmickel. He didn't give me a lot of choices."

"Sure he did. That's exactly what he gave you. Choices. The same ones I got."

"Yeah? You call those choices? Let's see. How did it go? If I rat you out, and you don't rat me out, I get a slap on the wrist and a suspended sentence, and you do ten years. If we both rat each other out, we split the charges and each do three years. And if you rat me out and I don't rat you out, you walk and I do the hard ten."

"And," added Maggie. "If neither one of us rats the other out, they got nothing and we both walk."

I hung my head. "I couldn't take that chance," I said. "As it turns out, you talked, too. So I made the right decision."

"I felt terrible about it," she said. "Guilty as hell. I was tormented by it. At least I was until I found out you did the same."

"The point is that game theory shows that even in a situation when the best outcome is to cooperate, people often choose an outcome that is less optimal."

"The way you're talking, you'd think you went to college instead of prison."

"Hey, I learned a lot. I learned that sometimes, you can be in a tough bind, and knowing what you know, you have to make tough decisions. But being there, you can also understand why someone else in the same situation makes the same decisions."

"Sounds like a crock to me," said Maggie.

"Actually, it explains a lot. Like why governments behave the way they do in the face of global warming. Best thing for every country to do is to lower its carbon emissions to prevent global warming, which will be really bad for all of us in the long run. But any individual country says nobody else is going to do it and, economically, it's better not to do anything in the short term. So they don't. And we end up turning the planet into burnt toast."

"Since when have you worried about global warming?"

"Hey, the whole concept of game theory gets you thinking. Another example is doping. If you're an athlete and you know that all the other athletes are not taking steroids, then you're less likely to do so. But if you think the other athletes are doing

it, you can't afford not to do it yourself. It would be better for all the athletes to not take the drugs, but in the end, they take them."

"That makes some sense," said Maggie.

"It does, doesn't it?" I gave her a small smile.

"People do behave that way," she continued. "I guess you're right."

"I am?" I said.

"Yeah, you are. So maybe I shouldn't blame you for what happened."

I spread my arms again hopefully.

She shrugged and then said, "I don't have anywhere to stay anyway." Stepping forward, she wrapped her arms around me. We hugged for a long time.

"I did miss you," she said. "Even though I was pissed at you, I missed you."

I took her bag and slung my arm over her shoulder as we walked back to the car. I even opened the door for her.

"You know if we had talked about it beforehand," I said as I started the car, "made an agreement that neither one of us would talk, then it would have been okay. But we never had that chance."

"Can we make it this time?" she asked. "Is there hope for two people who have never learned how to trust anyone?"

"Yes," I said. "Because this time, we're going to make a deal. That neither one of us rats the other one out, no matter what." I clasped her fingers in mine.

"Okay," she said. "Deal." She leaned over and hugged my arm. I turned my head and we kissed.

"I feel better," she said. "It's going to be all right."

I pulled away from the curb, feeling good about the world. It was a beautiful spring day, and I was about to start the rest of my life with the woman I loved. Lowering my window, I hung my left arm out, feeling the breeze.

We were cruising down the freeway when I heard the police siren. I looked in the rearview mirror to see the flashing red and blue lights coming up behind us.

"It's a cop," I said. Maggie stiffened. "Damn it, damn it, damn it," I said as I sped up.

"What are you doing?" asked Maggie in a desperate voice. "Just pull over. We didn't do anything wrong."

"There's cocaine in the glove compartment," I said.

"Are you an idiot?" said Maggie. "You just got out of prison and you have cocaine in the glove compartment?"

"It's not mine. I borrowed the car from Willie. It's his car. And I found the cocaine there, after I got to the prison to pick you up."

"I'll get rid of it," said Maggie as she pushed the button on the glove compartment and it flipped open.

"No, they'll see you throw it out," I said as I tried to reach over to stop her. She pulled the baggie out and held it up. It was then that the car hit the guardrail and flipped.

I woke up lying in a hospital bed. Schmickel was sitting in a chair next to the bed.

"Maggie," I croaked. "Is she all right?"

"Banged up and bruised, like you, but she'll recover."

"Thank God," I said.

"You love her, don't you?"

"Yeah," I said. Then I coughed and pain shot through my chest.

"Broken rib," said Schmickel.

I lay back.

"Why are you here?" I asked.

"Found the cocaine. There's enough of it that we can charge you with dealing. Only question is whether to pin it on the two of you or just on one of you. But I need one of you to turn state's evidence. All you got to say is it's hers, and you walk, and she does five years. If she turns, too, then you both do two. And if she turns and you don't, you do the five. It's your call."

"It's not ours," I said.

"Oh yeah." said Schmickel "That'll go over well in court."

"I'm not going to give her up," I said.

Schmickel smiled. "You know what this is?" he asked. "It's an iterated Prisoner's Dilemma. That means you two have already been through it once, and you're going to go through it again. Only this time, you know what the other person did the last time. You do remember what happened the last time? Or did those years in prison fog your brain? You know what she's gonna do. And you know what that'll mean for you unless you do the same."

"Schmickel, this isn't fair," I pleaded. "Please don't do this."

"Just doing my job," said Schmickel. "Just doing my job."

Chapter 41
The Intergalactic Congress of Mathematicians

I took a seat at the bar. One stool over sat a large black and brown insectoidal creature with four arms and four legs. Its translator sat on the bar. It clicked it on and said, "What are you? Some kind of humanoid?"

"Yup," I replied. "From Earth. Birthplace of humanity."

"Birthplace of humanity. Good for you."

"What about you?"

"Canana Brift, small planet around Doabulus Minor."

"Doabulus Minor? Isn't that a red giant?"

"That's the one," it replied.

The bartender slid over on a trail of slime. "What'll you have? Special is the Dolbo Cannon."

The insectoid leaned closer. "Don't have the Dolbo Cannon. It'll knock out three years of your memory."

"Give me a beer," I said. "You got Sam Adams?"

"Just Bud Light," said the bartender.

"That'll have to do," I said.

"What does a beer do to you?" asked the insectoid.

"Muddles the brain enough to get through another day."

"Oh. Sounds nice." It somehow lifted its drink in his claw-like hands. The straw at the top was spewing a white gas. It took a sip.

"You here for the Intergalactic Congress of Mathematicians," it asked, "or the Weapons of Planetary Destruction Convention?"

"Math," I said. "You?"

"Same."

I held up my beer. "Then cheers to math," I said. It managed to clink his glass against mine.

"I'm Philbin," I said, "Dirk Philbin." I held out my hand.

"Cruxila Mortagonothoothiano," it responded. "Just call me Crux." It held out a black hairy appendage, and I gingerly took hold for a second and then let go.

C. Adams, *Do Androids Dream of Symmetric Sheaves?*,
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"By the way," said Crux, "Do not shake hands with a barosphere."

"Why not?"

"As soon as there's flesh-to-flesh contact, they mind-meld. Gives them access to all your research. They slice it out of your memory and claim it as their own."

"Good to know," I said. I took another sip of my beer. "Hey, you know where I can get some toothpaste? I forgot mine. We use it to clean our teeth." I smiled to show my pearly whites.

"There's a pharmacy two blocks down from the Convention Center. But its 750 degrees out. So, dress accordingly. But I wouldn't recommend going even if you are dressed appropriately."

"Why not?"

"There's a bunch of shadow beasts lurking just behind the planters outside the hotel. They prey on unsuspecting guests who go for a stroll. Believe me when I tell you they're very unpleasant."

"Okay," I said.

"But you can have the pharmacy deliver."

"I'll do that." I took another sip of my beer.

"Got any particular talks on your schedule?" asked Crux.

"Well, I'm definitely going to the talk on the proof of the Riemann Hypothesis. That's been open a long time. Pretty incredible it's been solved."

"Don't bother. That's the rectopods. They announce a proof every congress. In their culture, lies are celebrated. The bigger, the better. They're very convincing, but there's always a flaw. That's part of the game for them. See how long they can deceive other species."

"Oh. Too bad. I was looking forward to it. Oh, well. You know, a rectopod tried to sell me a mini-black hole. Guess that was a scam, too."

"Nope, that one's not a scam. But believe me, it's a good thing you didn't buy it."

"Okay then, no black holes for me. And if I'm not going to the Riemann Hypothesis talk, I can go to the talk on the innovative methods of education used on the planet Darkling Gore."

"That the interactive talk?"

"Yup. It said there'd be opportunities to participate."

The insectoid made a noise that might have been laughter. "You don't want to attend that one."

"Why not?"

"They 'educate' using a variety of torture devices. Ensures the student will never forget the material. Ever. Don't think you want to experience that."

"Okay, then. Scratch that, too."

"What about the banquet tonight? You going?" Crux asked.

"I was thinking about it. I heard the food's good."

"Yeah, if you like shadowbeast and stringy green mash."

"Shadowbeast? The same shadowbeasts lurking outside the hotel?"

"Why do you think they want to eat the guests? Trying to get even."

"Okay, scratch that, too."

"So, what kind of math do you do?" asked Crux.

"I'm an algebraic topologist."

"I know algebra and I know topology, but what's algebraic topology?"

"Well, it started out as applications of algebra to topology, but now it's really just about the algebra."

"Yeah, that happens a lot in this business."

"What math do you do?"

"I do drophomoplotomics."

As Crux said the word, the translator lit up with a red light to indicate there was no translation.

"What's drophomoplotomics?"

"Drophomoplotomics. It's the math behind the Intergalactic Superhighway. I assume that's how you got here."

"Yeah. Quantum entanglement and all that. Didn't see how to get here any other way, since Earth is 200,000 light years away."

"Well, drophomoplotomics is the math that makes it all work."

"Now that sounds interesting. I'd love to learn more."

"Sorry. Can't speak to you about it. It's only for species that have security clearance. Humanoids are a good 10,000 years from getting clearance."

"Why's that?"

"You're just not mature enough yet."

"Well, the average human may not be ready, but I'm a mathematician. Mathematicians tend to be a bit more mature than average."

"I'm sure that's true, but I could get in a lot of trouble."

"I won't tell a soul. I am just so curious. Look at me. I came all the way here and all the talks I wanted to see turn out to be a bust. What do you say, you just give me the highlights?"

"I don't know."

"Look. I'm in it for the math. I love math. The universal language and all that."

It leaned in conspiratorially. "Well, I can't talk about it here, but if you want to come up to my room, I suppose I could give you a quick overview."

"Oh, wow, that would be great!" I said. I waved to the bartender. "Put it on my tab."

The next morning, I slipped out of Crux's room, shutting the door as quietly as possible. It crossed my mind how much trouble the cleaning staff would have trying to get that yellow blood out of the rug.

I headed to the Intergalactic Superhighway terminal without even stopping by my room to pick up my bags. With any luck, I would be back on Earth by the time anyone realized what had happened to Crux.

The Overlord of Earth, Donald Trump the Twelfth, would be very grateful to have the mathematics behind the Intergalactic Superhighway. I was sure to get a promotion to the Warlords Council.

And now humans would have the means to invade the rest of the universe, spreading across it like a horde of locusts. We would take our God given place as the masters of all the inferior creatures everywhere. It was a beautiful day.

Chapter 42
The Gulag

Ice crystals danced on the wind blowing through the cracks between the wooden slats that served as the only barrier between inside and out. Anya opened her eyes slowly and looked out over the barracks. Dozens of other prisoners lay on the wooden pallets lining the walls. The only sound other than the wind came from Olga, one pallet over, who was sobbing quietly, as she did most mornings. Anya pushed aside the burlap sack that served as a poor imitation of a blanket and sat up. At that moment, the door at the far end of the building banged open and Captain Resnitt's broad silhouette filled the doorway. "Up, you lazy bums," she shouted. "If you want breakfast, you better get to work."

Groans filled the space as the prisoners pulled themselves off their pallets and moved to the desks distributed around the room. Anya slid into the chair at her desk, pulled a reef of papers out of the small drawer and laid them on the desktop. Resnitt strode over to a laggard and whacked her on the head. "Get up, or you'll never get up again," she growled. The prisoner climbed off her pallet and trudged to her desk.

Everyone started scribbling on the pages in front of them. Resnitt strode between the desks as she yelled, "I want great mathematics! You want breakfast, you write great mathematics!"

At Olga's desk, Resnitt stopped to look over her shoulder. Grabbing up a page she said, "Bah! This is child's play. This is not mathematics!" She crumpled the page and threw it to the floor. "No breakfast for you."

Olga immediately began to sob. Resnitt slapped her on the back of the head. "Silence!" she said. "People are trying to think. It is hard enough for them as it is."

Anya did her best to ignore her surroundings. She had found, over time, that mathematics afforded her an escape from this awful place. She could sink into the math and everything else faded away. She distanced herself by playing with the abstractions deep in her brain. The volume of the Dehn filling was bounded below by a function of the length of the core geodesic, but the bounds were extremely delicate.

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She was lost in her reverie, when she realized that Resnitt was standing behind her.

"What is this?" Resnitt demanded as she grabbed up the page.

"Nothing," said Anya as she tried to snatch it back. But it was too late.

Resnitt held it out of reach and perused the page. "Huh. This is not trash," she mused, as she picked up another page.

"How is it," she said, turning to the room, "that someone has produced something that is not entirely garbage? Could this be?"

She grabbed Anya by the back of her collar and lifted her to her feet. "Come with me."

Anya was filled with dread. She had heard the rumors of what happened to those who were dragged from the room. Their results became the work of the Great Leader. He claimed it for his own. But then the true author had to disappear so the Great Leader needn't fear that he might be discredited. No one heard from them again.

With the pages still clutched in her hand, Resnitt pushed Anya along. The two of them left the barracks, passed through the gate to the camp, and entered a brick building across the road. Resnitt opened the door to a small room containing a cot, a bucket, a chair and a desk. On top of the desk sat three pens in black, blue, and red, and reams of white paper. As Resnitt pushed Anya into the chair, an orderly entered with a plate of scrambled eggs and bacon and a glass of orange juice.

"You may eat this breakfast," said Resnitt. "Then you will write up your results in the form of a paper. I expect you to be finished by tonight." Resnitt left the room, locking the door behind her.

Anya wolfed down the bacon and eggs and greedily drained the glass of orange juice. It was the best food she had ever tasted. Wiping her mouth with her sleeve, she stared down at the pile of paper.

What to do? She could write up her results, which she had been working on for months, but had always been careful not to let on. But if she did, the Great Leader would claim it as his work and would have every reason to have her disappear permanently. Or she could put down some nonsense. But in that case, what would happen? Would she be eliminated for that? No one had ever returned to the barracks after being dragged away. She stood up and paced around the small desk. It was an improvement to be able to do that. You never left your desk in the barracks. And the room was warm, which was a huge improvement over the barracks. Eventually, she made up her mind and got to work. At lunchtime, the orderly unlocked the door and brought in an actual sandwich and milk. She couldn't believe it. After eating, she continued her work into the evening when Resnitt returned.

"Have you finished?" asked Resnitt.

"Yes," replied Anya. "I could do a better job with more time, but the ideas are all there."

Resnitt scooped up the pages, turned and left, locking the door behind her. Bewildered, Anya wasn't sure what to do next. But soon, the orderly arrived with a hot dinner, with actual meat, potatoes, and green vegetables. She hadn't had vegetables in years.

After she finished eating, the orderly cleared the dinner tray. Her eyes widened when he returned with a bucket of hot water, soap, a washcloth, a towel, and a clean set of clothes. After cleaning herself off, she dressed in the clean clothes and left her old clothes in a pile on the floor. Then she lay down on the cot and instantly fell asleep.

When she awoke, she was surprised to see the sun was streaming in the window. Resnitt always woke them while it was still dark. Anya glanced around the room and realized the dirty clothes were gone and a steaming breakfast tray sat on the table. She slipped into the seat and enjoyed the food. After she finished, the door swung open and Resnitt came in. She sat down on the cot facing Anya.

"The Great Leader likes the work you have written. In fact, he says that he had already thought of all of it but had not had time to write it down."

"Oh, yes, I see," said Anya carefully.

"The Great Leader will present the results you have written down for him. You should be very honored."

"Oh, yes," said Anya, "I am honored indeed."

"And having thought of the results himself, the Great Leader understands all of the intricacies of the work. However, when he presents it at the International Congress of Mathematics, he wants to be sure he does not misspeak."

Anya waited.

"So, you and I will go to the Congress with him. When he presents, you will have a microphone transmitting to his hidden earpiece. If he is questioned or needs details, you will provide the answers that he requires. He knows all the answers but may need help with his recollections. I will be there with you at all times. If you are unsuccessful in any way in this endeavor, you will be executed upon your return to the homeland. Do you understand?"

"Um, I, um. . . ."

"Very good. We leave tomorrow."

The next day, Resnitt arrived early and escorted Anya to the back of a prison truck reminiscent of the one she had arrived in those many years before. Anya was locked in the back, and Resnitt climbed in front with the driver. As the truck drove out of the compound, Anya thought back to her arrival there.

She had taken a national exam in mathematics, having been told that doing well meant a full scholarship for further study. But instead, the prison truck had arrived at her house, and she had been hustled into the back. Her parents sobbed as the truck pulled away. That was fifteen years ago. And now here she was again in such a truck.

As they drove along, she watched out the back window, her first view of anything other than the compound in such a long time. At one point, she thought that maybe they had passed her old neighborhood, but she couldn't be sure.

The truck arrived at the airport and Resnitt led Anya by the arm into the building.

"From now on, not a word from you unless you are told to speak," whispered Resnitt. Once they reached their gate, Resnitt checked them in. Anya could see that the plane was destined for Helsinki. When it was time to board, she and Resnitt were escorted onto the plane first.

Once the plane took off, Resnitt handed Anya a small black box with a red light on the top. Then she pinned a tiny microphone to her collar.

"You will be seated at the back of the auditorium. I will be beside you. If the Great Leader is asked a difficult question, and wants you to remind him of the answer, he will push a tiny button he has that will light up the little red light on this box. Then you will tell him the answer. If he is embarrassed in any way, you will not live to see another day. Do you understand?"

Anya nodded.

When they arrived in Helsinki, they ignored the taxi stand and took a bus to their hotel. Resnitt slept in the other bed and, no big surprise to Anya, snored loudly through the night.

In the morning, they had a quick meal at the hotel restaurant and then walked to the venue and settled in the back of the large auditorium. There were several speakers before the Great Leader, and Anya listened in rapt attention. She was amazed to realize that there were others who were just as consumed in their work as she was in hers.

Then the Great Leader rose and climbed the stairs to the stage. She had never seen him in person before and was surprised by how small he was. As he took his place behind the podium, the crowd gave him an ovation. He seemed to have quite the reputation. He began his talk, pointing behind him at the slides projected onto the screen. As he was explaining the symmetries of hyperbolic 3-manifolds, a hand went up in the audience.

"How do you know the symmetries are realized as isometries?" asked a tall grey-haired man. Anya recognized him as Ferdinand Arbeiter from pictures she had seen many years before. He was perhaps the world's greatest expert on arithmetic hyperbolic 3-manifolds and one of her childhood heroes. The red light on the small device in Anya's hand went on. Resnitt immediately poked her in the arm.

Anya whispered, "We apply Mostow-Prasad Rigidity to the symmetries".

The Great Leader said, "We apply Mostow-Prasad Rigidity to the symmetries. Thank you for that question." Then he continued confidently with his talk.

At various points, Anya knew that the Great Leader had not understood the subtleties of what he was presenting, but no one interrupted him. Anya knew only a true expert would notice.

When the Great Leader finished, the moderator stood and said, "Thank you for that wonderful talk. This is indeed excellent work. But of course, no one is surprised by that. At every Congress, you come and impress us with your amazing results. But now, let me open the floor to questions." Arbeiter immediately rose to his feet.

"I also thank you for that talk," he said. "I found it fascinating. But I was just a bit confused when you said that the Whitehead link and the Borromean rings were in the same commensurability class. Why is that?" As he finished his sentence, he began to scan the audience.

The red light on the device in Anya's hand blinked on. Resnitt poked her, and Anya began to recite an explanation. But then she noticed that Arbeiter was staring right at her, and in mid-sentence, she stopped. Resnitt turned to her in surprise.

There was silence in the lecture hall as the audience waited for the Great Leader's response. The red light blinked on and off several times.

"What are you doing?" demanded Resnitt. "Give the answer!"

Anya turned to look at Resnitt for a moment and then turned back toward the Great Leader and began to whisper her answer. The Great Leader began to speak.

"I am sorry, but you must have misheard me. The Whitehead link and the Borromean rings are not commensurable."

Arbeiter began pushing his way past the people in his row.

Anya stood up. "Excuse me," she yelled out to the Great Leader, "but you are wrong. In fact, the Whitehead link and the Borromean rings are commensurable."

Resnitt was too stunned to react. Arbeiter was moving up the aisle toward them.

The red light began to blink furiously in Anya's hand. She held it up so the Great Leader could see it blinking in her hand. He blanched.

Arbeiter reached her and asked, "Is this your work?"

"Yes," she said. Resnitt finally managed to find her voice.

"No," she said emphatically. "She is not the author. She is no one."

"How do you show that the Whitehead link and the Borromean rings are commensurable?" asked Arbeiter.

"The double cover of the Borromean rings is the same as the quadruple cover of the Whitehead link."

Arbeiter smiled broadly, and then turned to the audience and said in a loud voice, "This person is the author of this work. It is not him." He pointed to the Great Leader. "He is a fraud. He has been claiming the work of others as his own for years. He will be brought before the Ethics Tribunal of the International Congress of Mathematicians. If he is convicted, he will be barred from all mathematics conferences for the rest of his life."

"Out of the way," snarled Resnitt. "We're leaving!" She pushed Anya in front of her. But Arbeiter blocked their way.

"You're not going anywhere." He motioned to security guards who quickly surrounded them.

"What is your name?" he asked Anya.

"I am Anya," she replied.

"Well, Anya, you are going to be a famous mathematician, and you will have a job at a prestigious university."

Anya's mouth dropped open.

"And you," he said to Resnitt, "You will be spending quite a bit of time in the Congress prison." Then he motioned Anya to come with him and walked her down the aisle to the podium. They passed the Great Leader being escorted by security up the aisle, his face in his hands.

"Would you mind terribly explaining these results to the assembled audience? I am confident you could do a much better job than our last speaker."

Anya beamed. "I can imagine nothing I would rather do." Then she mounted the steps to the podium.

Chapter 43
Where Do Theorems Come From?

Age-appropriate answers to the question, "Mommy/Daddy, where do theorems come from?"

Age 0–3: When you are ready to have a theorem, you submit a request. If you are worthy, the bird of inspiration flies to your doorstep and delivers a theorem. The first time it happens, it is a magical moment and nothing is more important than that theorem. But eventually, you will want another. So, you make a second request.

Age 4–6: At the North Pole, there is a workshop staffed by little theorems that are called lemmas. They toil year-round crafting new theorems. At the end of the year, all the new theorems are delivered all over the world to mathematicians who have been good that year.

Age 7–8: Mommy and Daddy go to the university and there, they are delivered a theorem that they get to take home.

Age 9–10: Several faculty members get together in a special place called a seminar room. They serve cookies and tea to get in the mood. Then the faculty talk and talk, and they draw with chalk on a blackboard. Sometimes they get very excited and wave their hands about. And if they are very lucky, a theorem is born.

Age 11–13: Don't believe whatever you've heard on the playground. You can't get a theorem by banging your head against a brick wall. You can't get a theorem by calling forth the forces of darkness at the gravesites of dead mathematicians. You can't get a theorem by cutting out other theorems from the pages of random papers and books and pasting them in some order on a piece of paper. Well, at least usually, you can't.

Age 14–18: You wake up in the middle of the night. An idea you heard at a seminar talk months ago has been slowly growing inside you. And now it has triggered a cascade of ideas. You know you won't sleep any more. You grab a pad of paper and begin throwing down symbols on the page. The self-adjoint multiplex is actually a

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relative semi-upper lower central series. You can't write fast enough to keep up with your brain. You are tearing pages off the pad and they litter the bed. And then, after ten hours of frantic work, you suddenly feel it coming. You scribble down the last bit and then triumphantly you scrawl Q.E.D. on the page.

Eight hours later, when you reawaken, you realize what you have achieved. You scoop up all the pages and carefully cradle them in your arms. It is your theorem and you created it. Nothing feels quite like this. Now you and your theorem are inextricably linked for the rest of your life. It is a moment you will treasure always.

Age 19–21: You need to be careful. You don't want your mathematical career hijacked by an offhand comment from an invited speaker that starts a seed growing. It could cost you months and change your future forever. So, don't go to random seminars. Practice abstinence. But if you can't help yourself and find that you must go, use protection. Bring your laptop and headphones and sit in the back watching funny math videos on Youtube. (I know, there is only one, so just keep watching it over and over.)

Age 22 and up: Why are you asking? You're publishing in better journals than I am.

Chapter 44
The Lord of the Rings I: The NSF Fellowship of the Rings

Frodo was a somewhat short and relatively friendly algebraist with a postdoctoral position at the University of Nebraska, which had followed upon the completion of his graduate studies there. He had plentiful soft curly brown hair that seemed to sprout from every opening in his clothing. His demeanor was fundamentally kind, and students trusted him in a manner that they did not trust all faculty.

Due to his general disorganization, his office was filled with towering stacks of paper and various and sundry items, so much so that it resembled less an office and more a hole dug out of a paper hillside.

Frodo's advisor, Bilbo Baggins, had also received his Ph.D. at Nebraska but, as a young algebraist, he had left Nebraska for a postdoctoral position, traveling all the way to Berkeley. But he quickly discovered that adventure did not fit his demeanor, and soon he had returned to the safety of his office at Nebraska. Settling in for a quiet career, he published a paper here and there, but nothing too ambitious.

Eventually, Bilbo came to the realization that he was easily the oldest member of his department, and it was his turn to retire. The math department put on a conference in his honor, which included 144 attendees, a big banquet, and drinks all around.

At one point during the festivities, when the rest of the participants were dancing a cohomology dance, Bilbo motioned to Frodo to follow him, and the two of them disappeared from the banquet hall.

Once outside the room, Bilbo pulled Frodo into an alcove.

"Frodo," he said. "There is something I want you to have. Many years ago, when I traveled to Berkeley, they gave me an office that was shared by several postdocs. On the day that I was leaving, I stumbled across this in one of the desks."

Bilbo pulled a sheaf of folded and wrinkled papers out of his pocket.

"What is it?" asked Frodo.

"It is the directions for the construction of a ring," replied Bilbo, looking over Frodo's shoulder to confirm they were alone. "But not just any ring."

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Frodo glanced down at the papers. It appeared the ring was noncommutative with identity, but beyond that, he could not tell. It seemed awfully complicated.

"I want you to take this ring. Guard it carefully," said Bilbo. "It could destroy you and everyone you love. I must go live with the retired people in Florida. They will take care of me. But mark my words. The ring is very powerful. Use it carefully."

As he went to hand the pages to Frodo, a look of immense pain etched its way across Bilbo's face.

"Wait!" he wailed. "Give it back. Give it back!"

Frodo immediately did so. Bilbo clutched the papers to his chest, mumbling incoherently. But presently he calmed down, and his grip on the papers loosened. Accessing some reserve of inner strength, he again thrust the papers into Frodo's hands.

"Please take it," he pleaded. "Take it now, and do not let me see it again."

Bilbo then turned away and with an anguished cry, rushed back into the great hall, where the algebraists were now playing pin the tail on the geometer.

For the next few weeks, Frodo ignored the papers and went about his business. But one day, as he was looking for a folder from amongst the stacks in his office, he noticed the sheaf of papers where he had laid them down on a pile of yellowed preprints. He smoothed them out on his desk and tried to see if he could follow the construction. Within minutes, he was deeply enmeshed in the work. He did not notice as the time for the algebra seminar passed, and, even more surprising, he worked straight through afternoon tea, missing the frosted pink cookies to which he so looked forward.

As the shadows slowly lengthened, he came to the realization that this was no ordinary ring. It had properties he had never seen before. It could do things that most algebraists would have said no ring could do.

By midnight he was still sitting in his office, unable to leave lest he miss some other marvel. It was as if he were handling an intricate gem that reflected light in myriad fascinating patterns as he turned it this way and that.

Suddenly, the door to his office swung open. A tall thin figure with a greying beard and a walking stick stood at the door.

"Who are you?" asked Frodo, as he instinctively covered the papers with his hands.

"I am the algebraic geomancer Geisenbud," replied the figure. "I am here to warn you. You must destroy this ring before it destroys you."

"I couldn't possibly destroy this ring," said Frodo. "Why on earth would I want to do that?"

"Otherwise, it will corrupt you and take you over. Your entire life will be spent studying its properties."

"What's wrong with that? It is so beautiful. I could happily spend my life studying this ring. I am the only one harmed."

"You don't understand," said Geisenbud. "It is not just you who may suffer the consequences. For if this ring fell into the wrong hands, it could destroy the entirety of mathematics. The world as we know it would end."

"How could that be? It is just a ring."

"This is not just a ring," thundered Geisenbud. "This is THE ring. The great ring, fashioned by Sauron. This ring which controls the other four rings. Whoever possesses this ring, has immense power."

"You mean it is a power series ring?"

"No, I mean it is one of a series of rings of great power. As a matter of fact, it is a quotient of a power series ring. But that is beside the point. That is not what makes it so powerful.

"Within this ring are the seeds of destruction. For this ring contains a logical contradiction that could bring mathematics to its knees. If it is not destroyed, an immense conflagration will consume all of mathematics as we know it."

Geisenbud slammed his staff into the floor for emphasis. A shiver ran down Frodo's spine.

"But I am just a minor postdoc," he said, trying to control the waver in his voice. "What can I do?"

"You must take it to Purdue and throw it into the math building there. Then it will be destroyed. They have no interest in noncommutative rings there."

"But how can I get to Purdue? It is very far away."

"I have arranged for you to give a series of talks at various institutions between here and West Lafayette, so as to allay suspicion. There are others who will help you along the way. But be very careful. For the minions of the Dark Lord seek the ring even as we speak."

"Who is the Dark Lord?" asked Frodo. But the great geomancer had already turned and disappeared behind the frosted glass of the door.

The next morning, Frodo packed up a few preprints, stuffed the papers into his backpack, and met a taxi down at the curb. As he opened the door, his good friend Sam came running over the hillside. Sam and Frodo had been undergraduate math majors together at a small liberal arts college. Sam had depended on Frodo to get him through some of the more difficult classes. As graduation loomed, they had both applied for NSF graduate fellowships.

Much to their surprise, Sam received a fellowship, while Frodo only received honorable mention. Frodo ultimately decided to attend Nebraska, and Sam eagerly followed suit, unable to conceive of doing mathematics without the help of his friend. But Sam had yet to complete his PhD.

"Hold it right there, Frodo Baggins. I saw that you had cancelled your classes today. Where are you off to?"

"I cannot tell you, Sam, for fear I could get you into trouble."

"I doubt I could get into much more trouble than I already am. My teaching evaluations are in the dumpster, and my research is about as deep as a castrato's voice," he chuckled. "If you are leaving Nebraska on a trip, I reckon I will come with you."

Suddenly two other postdocs appeared running over the hill, carrying backpacks. They had also held NSF fellowships while graduate students at Northwestern.

"If you are leaving on a trip, then we are coming, too," said Merry.

With that, Sam, Merry, and Pippin all hopped into the cab before Frodo could stop them.

When they arrived at their first destination, Northern Illinois University, they discovered that the location for the talk was a large auditorium.

"What kind of algebra seminar is this?" asked Frodo. As he spoke, Frodo's host walked up.

"Seminar? Who told you this would be a seminar? This is the Pi Mu Epsilon Honorary Mathematics Society Induction Ceremony Colloquium. You do have a talk that will be appropriate for a general audience, don't you?" He gestured to the seats filled with clean-cut undergraduates and their parents.

Frodo gulped, his eyes widening with terror. But Pippin pulled him close and whispered in his ear, "Don't worry, Frodo. Just talk extra slow and skip every other slide. You'll be a hit for sure. Just wait and see."

The host stepped up to the podium.

"Welcome, everyone. We are very pleased to have with us Frodo Baggins, an algebraist from the University of Nebraska. Rumors abound that he has amazing mathematical powers, and we expect great things of him in the future. But today, he has promised me that he will be giving an expository talk of the most understandable variety. I give you Frodo Baggins."

Frodo, shaking with fear, stepped to the lectern. He attempted a smile, and then said, "Let R be a weak Armendariz ring."

The audience rustled uncomfortably in their seats. Frodo waved his hand reassuringly.

"For those of you unfamiliar with weak Armendariz rings, just think of it as a reduced semi-prime right Goldie ring."

Audience members turned to one another in consternation. Frodo cleared his throat nervously. He noticed a dark figure sitting immobile near the rear of the auditorium, staring back at him with sinister yellow eyes. As Frodo looked upon him, he suddenly found himself unable to move.

The host, who was sitting in the front row, waved to get Frodo's attention.

"Baggins, what's wrong with you?" he demanded.

But Frodo was frozen. Suddenly a hand grabbed his arm.

"It is a ringwraith. It seeks after the ring. It will destroy your career with one question at the end of your talk. We must leave at once."

The spell broken, Frodo turned to a tall figure in a low hat.

"Follow me," urged the figure as he steered Frodo away from the podium.

"What are you doing?" cried the host in desperation, as the four postdocs and their savior raced up the stairs of the auditorium, past the relieved-looking crowd, and disappeared through the back door. The ringwraith leapt up in pursuit.

"Quick, duck in here," said the figure. He pushed the four of them through a door into a classroom where a teaching assistant was running a review session.

"Act like undergraduates," he whispered.

"No, the square root of a number is not negative," the TA was saying to a student. "You have to put a negative sign in front of it to make it negative."

"That's not what the professor said," responded the student.

Frodo and the others slipped into the last row. Frodo opened a book and held it in front of his face.

They heard a noise outside the door. Glittering yellow eyes peered through the door window, surveying the room. Merry and Pippin began shoving each other.

"You left your wet clothes in the washing machine, again," yelled Pippin.

"It's not me. I haven't washed my clothes all semester," retorted Merry.

They slapped at each other. Sam raised his hand. "Excuse me," he said to the TA, "but what's on the exam?"

"I can't answer that," replied the TA.

"How about telling us what's not on the exam?" persisted Sam.

The ringwraith disappeared from the window, convinced they were undergraduates.

Frodo turned to the stranger. "Who are you?" he asked.

"They call me Strider," he replied. "I also had an NSF Fellowship while in graduate school."

"Actually, I only got honorable mention," said Frodo.

"Oh. . . That's not what Geisenbud said. Well, no matter," said Strider. "We must get to Bloomington, Indiana. There, we will be met by the rest of the NSF fellowship. They will help us to bring the ring to Purdue where it can be destroyed."

So the band of five fellows—well, four fellows and an honorable mention—set off to Bloomington to meet up with the rest of the members of the fellowship. Their adventures were many and varied, but that is another tale to tell.

Chapter 45
Piracy

John Caporittlestan, known as Cap'n for short, walked into the Dirty Asymptote, a run-down bar not far from the math building, where the disreputable denizens of math were known to seek refreshment. He spotted his crew at their usual table. All postdocs, they spent their free time and a lot of their non-free time at the bar. They were noisily singing a bawdy math tune.

She was an old theorem, lived by the sea,
Hey, ho, a bottle of rum!
She met a young lemma, so fair was he,
Hey, ho, a bottle of rum!
They spent a long evening on topology,
Hey, ho, a bottle of rum!
Cor'laries they had, cor'laries three,
Hey, ho, a bottle of rum!

They stopped mid-song when they saw him.

"Cap'n," yelled Jilly Jacobian, "Over here!"

She filled him a tankard. He dropped his tricorn hat onto the wet table and held the tankard up high.

"To all the papers I'll never write," he said solemnly.

They repeated it in unison and then drank, clapping the tankards back on the table noisily.

"What brings you down here, Cap'n?" asked Discontinuous Dan.

"I got a proposition for you," said Cap'n. "And I don't mean the mathematical kind."

They all gufawed. "Tell us then," said One Lemma Kate. They crowded round.

"We got ourselves a conference coming to town. The Joint Meetings. A walloping big conference with all kinds of talks and festivities. And I'm thinkin' it's time to do what we do best."

"Yar, Cap'n. Make fun of mathematicians?" said Ill-defined Ivan. They all laughed and slapped each other on the back.

C. Adams, *Do Androids Dream of Symmetric Sheaves?*,
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"Well, I'm thinkin' sumpin' a bit more profitable," said Cap'n, smiling deviously, his gold tooth twinkling between his lips.

"Now yer talking Cap'n," said One Lemma Kate. "I need to pad my CV, I do. Pad it bad. My postdoc ain't gonna last forever."

They all laughed.

"We gonna do some piracy?" asked Jilly Jacobian.

Cap'n nodded as he pulled a wad of papers from the pocket of his long black leather coat.

"This here's the schedule o' talks. What we're looking for are grad students giving the first talk of their careers, so they don't know much about what's what."

Ill-defined Ivan rubbed his hands together gleefully.

"And we need to find the ones listed on the schedule as 'preliminary reports.' Always says if they are."

"We can find those," said Kate.

"Then we check to see if the speaker's got a paper up yet on the ArrrKiv."

"Yar, and if they don't have the paper up on the ArrrKiv, we can steal the results," said Non-Normal Ned.

"Yar, Cap'n," agreed Jilly. "We steal it. We take it from 'em afore they know what's what."

"As easy as stealing cookies from the seminar room," added Shifty Sheaves. "What's the split?"

"All names on the paper," said Cap'n. He paused. "But me as first author." He looked at each in turn.

"Now that sounds fair," said Kate. They all laughed and shoved each other and banged their tankards together. Then they sat down and got to work.

The next day, Lenora Rodriguez was scheduled to give her talk at 8:00 a.m. It was a terrible slot, since everyone had been either working on math late into the night or raising a glass with friends long unseen. But as a grad student, she was happy to get a slot at all.

She was nervous, as this was the first talk she had ever given at a conference. And although she had listed the talk as being on preliminary results, she had a preprint that was almost ready for the ArXiv. Just another week or two ought to do it.

At 8:00 a.m. sharp, as the moderator began to introduce her, the door to the room banged open and in marched half a dozen people led by a man with a long dark beard, wild hair, and big black boots. He strode up the aisle followed by his motley crew. Upon reaching the first row, he leaned in close to the two people sitting there and gave them a frightening smile filled with ghastly teeth.

"Ye seem to be sittin' in the seats reserved for the top fight mathematicians. From the looks o' yer, that ain't you."

The two scampered away, and the group settled in.

The moderator resumed nervously. "So now Lenora Rodriguez will talk about Osculating Semigroup Partitions. Prof. Rodriguez?"

"Um, not professor yet," she said nervously. "But almost. These results will be my PhD thesis." The moderator nodded supportively. Then she began her talk.

As she spoke, the group in the front row typed furiously on their laptops. It was reassuring to see that someone was interested. But they also leaned over and talked to each other, pointed at various things on each other's screens and generally were incredibly rude. But she forged on. As she spoke, she began to relax. It was math, and it was really pretty math, so it was fun to be able to finally tell others about it.

She reached the climax of her talk, and gave the linchpin argument that proved her result. The bearded man nodded approvingly. "Just about as good as gold," he mumbled.

"Any questions?" asked the moderator.

"Yar," said the bearded man. "I have a question. It's not completely clear to me how you prove the existence of the double cover of the normal Goeritz biregular quintix."

"Oh," she said, surprised that he had understood deeply enough to ask such a specific question. "You see, Twindal's lemma implies that every biregular quintix has a decatrix cover, but since we know the quintix is also normal and Goeritz, we can apply a Boborian projection to obtain the double cover."

"Got it," said the strange man, standing suddenly. "Off to the next talk," he said, as he motioned his group up and they noisily left the room.

Within an hour, Lenora's friend Clarissa texted her.

"Lenora, check the ArXiv. There's a new preprint up, and it's got your result in it."

"What?" she texted back.

"Yes, and it's got a half dozen authors with a lead author named Caporittlestan."

Lenora immediately remembered the group. "Oh, no," she said aloud.

She immediately rushed to the exhibit area and found someone working at the AMS booth.

"Where do I find the ethics officer?" she asked.

"That would be Major Ormsby," said the AMS representative, pointing to a prim man in a tight red jacket and white curly hair who was sitting erectly beneath a sign announcing him as the AMS Ethics Officer.

"Is he in the military?" asked Lenora.

"No, Major is his first name."

Lenora walked over to the man and said, "Excuse me, but I think I need your help."

"Indeed," he replied. "Please explain yourself and I will do whatever I can to help you."

She told him the story, and he shook his head.

"Ah yes, Caporittlestan. The scourge of the mathematical seas. One of the most dangerous of pirates. He seeks out the defenseless and steals their results. He and his band of postdocs are a treacherous lot. Truth be told, it is better to let him take your result and find another one."

"I can't do that," she replied. "This is my thesis work. I need it to graduate."

"Very well," he said. "I'll see what we can do."

He stood up and blew a shrill whistle. Within a minute, a dozen people had gathered around him, all dressed in tightly fitted red jackets.

"How will you find him?" Lenora asked.

"At this time of day, there is only one place he would be."

He turned to his red-coated group and said, "On me."

He marched down the aisle into the deepest maze of the central exhibits area, with the others falling in line in two columns behind him. Lenora followed from the rear.

In the distance, they could hear the noise of what sounded like a celebration. After a tortuous path through the exhibits that left Lenora fearing she would never find her way out again, they arrived at the Wiley booth to find a mass of people enjoying free baby wieners and bubbly. In the middle of the throng was Caporittlestan and his gang busily scooping up wieners and downing plastic glasses of champagne.

Ormsby and his soldiers immediately surrounded Caporittlestan.

"You, sir, are under arrest for the capital crime of piracy," announced Ormsby.

"Ye'll never make it stick," laughed Cap'n, as he grabbed up another plastic glass of the bubbly and swigged it down.

"You must come with us," said Ormsby. "We will hold a trial to determine your fate."

The postdocs immediately began to cry out and make threats, but Cap'n settled them with a gesture.

"Calm down, me buckos," he said. "They got nothing on me. I'll be out in time for happy hour."

Several of the ethics squad grabbed his arms, and they all exited the area, followed by the motley crew of postdocs. They arrived at the exhibitors' lounge.

"I am commandeering this lounge," announced Ormsby to several surprised exhibitors. "You will have to rejuvenate elsewhere."

They sat Caporittlestan down in the front, and everyone else pulled seats around. One of the ethics squad turned to the assemblage and announced, "This is the trial of Caporittlestan, who has been accused of piracy on the mathematical high seas. I turn the proceedings over to Lenora Rodriguez."

"What?" she said in surprise. "Me? What about Major Ormsby?"

"I am the judge," said Ormsby. "I cannot also be the prosecutor."

"Oh, well, um, okay, then, um, what do I do?"

"You question the accused and you present evidence."

"Oh, yes, well, okay. Um. Mr. Caporickel. . . "

"Just call me Cap'n," he said. His crew laughed and slapped each other on the back.

"Very well, Cap'n. It appears that you submitted a paper to the ArXiv this morning."

"It's pronounced ArrrXiv," said Caporittlestan. He crew laughed uproariously.

"And how did you come to write that paper?" asked Lenora.

"Well now, that one there was a solid month of work with my coauthors, slaving away on them there biregular quintices."

"And you are claiming you just so happened to finish it the same morning that I spoke on these very results. Do you deny you were at my talk?"

"Listen, dearie," He replied. "I was at your talk, and it was then I realized that I had to get our results up on the ArrrXiv, as it was clear you had every intention of trying to steal our results. Luckily, our preprint was ready to go, so I got it up immediately."

The crew hurrahed.

Ormsby waved for silence and then asked Lenora, "Do you have any proof that you had these results more than a month ago?"

Lenora paused and then smiled hopefully. "Yes, I sent a version of it to my thesis advisor to look over. Here is the email I sent her with the draft attached."

She handed Ormsby her phone. He put on his spectacles and stared at the phone for an uncomfortable five minutes. Then he handed her back her phone and came over to Caporittlestan.

"Stand up," he said to Caporittlestan, who did so slowly. Then he took hold of Caporittlestan's name badge and ripped it from his chest.

There was utter silence. In a commanding voice, Ormsby said, "You, Caporittlestan, are convicted of piracy on the high seas of mathematics. You are barred from this meeting, barred from all receptions associated with this meeting, barred from the exhibits area and all the treats and freebies provided by the exhibitors, and barred from all future meetings. The ArXiv will no longer accept submissions from you or any of your crew."

Caporittlestan blanched. His crew began to yell, but he silenced them with a wave. Then he turned to Lenora.

"Ye'll pay for this," he said as he shook his fist at her. "Ye'll wish ye never crossed me." Then he turned and, followed by his sullen crew, disappeared in the maze of aisles in the exhibits area.

Lenora turned to Major Ormsby.

"Thank you, Major," she said. "If not for you, I might not be receiving my PhD."

"You are the one who did all the work, and you are the one who deserves to receive it. I am glad we could see justice served. I only hope others can learn a lesson from this story. Wait until you have your paper up on the ArXiv before you speak on it. Or you, too, could be a victim. A victim of piracy on the mathematical high seas!"

Notes

Notes for Chapter 1. Immortality

As is often pointed out, immortality ain't what it's cracked up to be. Continuing to live century after century is sure to lead to incredible boredom. You've seen it all.

However, if you become interested in math, you have not seen it all. Because mathematics continues to grow and evolve. And as the centuries unwind, so does mathematics, piquing the interest of even the most jaded of vampires.

As an immortal, you can accrue knowledge, continuing to build your mental storehouse of mathematics. And the more you accrue, the better the chances that you can make those essential connections that truly influence the course of mathematics.

So if you do get bitten, consider it an opportunity to get lots of mathematical research done. You will not run out of questions to answer.

Notes for Chapter 2. Gold Rush

In many ways doing research in mathematics resembles prospecting for gold. This story makes that explicit.

The mathematics in this story is real, but since the time this story was written, the relevant mathematical landscape has changed. All of the results mentioned are about finite fields. A finite field is a finite set of elements with well-defined multiplication, addition, multiplicative inverses, and additive inverses satisfying the field axioms. (Think about the axioms satisfied by those operations on the real numbers, for instance.)

An example of a finite field is the field F_q, with elements $\{0, 1, 2, \ldots, q-1\}$ and q a prime. If we multiply or add two integers and the result is greater than or equal to q, we take the remainder when divided by q. Every element has an additive inverse and every nonzero element has a multiplicative inverse.

C. Adams, *Do Androids Dream of Symmetric Sheaves?*,
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As simple as they are, there are still interesting open questions about finite fields. The original theorem that begins the gold rush in the story is the existence of a primitive normal basis for the field F_{q^n} over the field F_q. This is a conjecture that dates back to 1996. Although many cases have been proved, it still remains open. The open problems that Pecos and Jane investigate are active areas of research in finite fields.

Notes for Chapter 3. Prime Suspect

Okay. As is pretty clear, the idea for this story came from the well-known kid's joke, "Why is six afraid of seven? Because seven ate (eight) nine." I don't really have a better excuse for having written it. But it did provide a fun opportunity to throw in lots of references to transcendental numbers, radicals, rational numbers, etc.

Notes for Chapter 4. The Cabinet of Dr. Möbius

August Möbius was a mathematician in the nineteenth century for whom the Möbius band is named. A Möbius band can be constructed by taking a strip of paper, adding a half-twist, and taping the ends together. It is an example of a surface with only one side, and as such has been an object of fascination for quite some time. My favorite use of it is by John Barth, a novelist who liked to tell stories within stories. At the beginning of one of his books, he had a strip you could cut out and tape with a half twist that said on it, "Once upon a time there was a story that began".

For whatever reason, the name "Möbius" or its variation "Mörbius" has become a standard name for brilliant but off kilter scientists in a variety of science fiction books and movies. I guess it sounds twisted.

A fundamental question about the universe we live in is whether there might be an orientation-reversing path. If you traverse the path, you come back reversed, as occurs in this story. The problems with digesting handed molecules would be real.

Although you might think that a reversed human would be evidence for an orientation-reversing path, there is a genetic condition called situs inversus where individuals are born with the positions of their organs in their torso reversed from what normally occurs. So the heart appears on the right side of their body. However, in such a condition, the reversal is not at the molecular level, so they are still able to digest normally. It occurs in one in ten thousand births and usually does not have any debilitating side effects.

Notes for Chapter 5. On Another Plane

The Riemann Hypothesis is considered by pretty much everyone in math to be the most important open problem in all of mathematics. Dating back to a paper of Bernhard Riemann from 1859, it conjectures that all roots (values that yield 0) of the Riemann zeta function occur either at the negative even integers or along the vertical line in the complex plane given by complex numbers with real part equal to 1/2. The zeta function is the analytic continuation of the infinite sum $\zeta(s) = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \ldots$. Although not obvious, this conjecture has a variety of implications for the distribution of the primes and, as such, has drawn immense attention over the more than a century and a half since its inception.

This story suggests the existence of a muse who is actually supplying answers to the biggest open problems in a variety of fields, resulting in numerous Fields medals and Nobel prizes. Of course, I do not believe that the people who earn these prizes have relied on anything but their ingenuity. The muse just represents the creativity one needs in order to solve a truly difficult open problem.

Notes for Chapter 6. The Modern Prometheus

A few of these stories result from taking some well-known tale and asking how to make it mathematical. Mary Shelly's *Frankenstein: or the Modern Prometheus'* was an inevitable candidate, but in this version, the essential components to make the creature are the pieces of brain collected from great mathematicians that together will create the greatest of mathematical minds.

In some sense, this aggregation of brains is occurring in mathematics. Over the years, there have been fewer and fewer single author papers and more and more collaborations. Mathematicians with different specialties join forces and, with their varied backgrounds, solve problems that would otherwise be intractable.

In the case of the creature from this story, it becomes a logical candidate for the Fields Medal, the mathematical prize given once every four years to the very best mathematicians whose work was done before the age of 40. But of course, then you have to look at the fine print to see if the criteria are satisfied.

Notes for Chapter 7. Aftermath

I actually do believe that if there is a hell, there are relatively few mathematicians there. Anyone serious about math research doesn't have the time to get into much trouble.

But there may be a few exceptions. Having spent decades working on a problem, one can imagine someone being overwhelmed with the desire to claim the proof and succumbing to nefarious means to get it.

As in "Immortality", this story also confronts the issue of how desperately long eternity is. For the devil, this creates the problem of making the torments random. And randomness is not so easy to generate.

Notes for Chapter 8. Looking Backward

This story builds off the utopian novel *Looking Backward: 2000-1887* by Edward Bellamy. It was first published in 1888, with ultimate sales of over a million, which is pretty darn good for a novel written in those days. In fact, at the time, the only book that had more sales in its first year was *Uncle Tom's Cabin*. Over 162 Bellamy clubs sprouted up around the United States in hopes of moving toward the ideals espoused in that book.

In Bellamy's novel, Julian West is accidentally put into a 113 year hypnotic state, only to wake in Boston in the year 2000. His guide to the new world, Dr. Leete, explains to Julian how this marvelous future has come to be.

It is always interesting to read predictions from the past of what a future world might resemble and to compare them to the reality in which we live. In our story—no big surprise—it is a utopian dream of mathematics, or at least someone's utopian dream of mathematics.

Notes for Chapter 9. A Denial

Évariste Galois (1811–1832) is one of the most interesting mathematicians in history. As a teen, he revolutionized the field of algebra, showing that a fifth degree polynomial equation cannot be solved by an equation involving radicals, a question that had been open for 350 years. (As an example, the quadratic formula demonstrates that a second degree polynomial equation can be solved by radicals.) But unfortunately, Galois died at the age of 20 in a duel. Who knows what he might have achieved if he had not died so young?

The details of that duel are unknown. So I just decided to fill in a few of them.

Notes for Chapter 10. Algebrexit

This story is a great example of how just about anything can trigger an idea. In this case, it is enough that Algebra and Brexit can be combined to yield Algebrexit.

Notes for Chapter 11. Fields Candidacy

The Fields medal is the closest thing we have in mathematics to a Nobel Prize, and as mentioned above (see the notes to "The Modern Prometheus") it differs in that it is only awarded to a handful of people every four years, and you must receive it for work done before you attain the age of 40.

So, how do you convince the Fields Medal Award Committee that they should give it to you?

Notes for Chapter 12. Leonhard Euler and the Seven Bridges of Königsberg

Many people point to the solution of the Königsberg Bridge Problem as the birth of topology. A set of bridges for which one wants to find a path crossing each bridge once is ripe for abstraction. Who needs land masses? Who needs roads? All you need is a vertex to represent each land mass and an edge for each bridge. The edges don't even need to be straight. Then we seek a path through the resulting graph that starts at one vertex, passes along each edge exactly once and ends at another. We can then translate our solution into the requisite path through the city.

This particular story just fills in the details of how Leonhard warmed up on simpler bridge problems until he was ready to tackle the Königsberg Bridge Problem. Geniuses don't just appear out of nowhere. They need to train.

Notes for Chapter 13. The Great Storm

In 1901, the foundations of mathematics were put to the test when Bertrand Russell (1872–1970) discovered that the basic axioms of mathematics lead to a conundrum. The axioms allow for a set to contain itself as an element, but then one can construct the set A that contains all sets that do not have themselves as an element.

If A does not contain itself as an element, then by its definition, A must be a set appearing as an element in A, thereby making it an element of itself. And if A does contain itself as an element, then A does not satisfy the criterion of being an element in A. So in either case, we have a contradiction.

This logical flaw caused great consternation in the mathematical community and was only solved by creating a new axiomatic system called Zermelo-Fraenkel set theory, which disallows sets from being members of themselves.

The two main characters of this story are Epsilon and Delta, which are the ever-present Greek letters occurring throughout mathematics, but in particular in the definition of continuity of a function.

The Axiom of Choice, which is the axiom that Delta chooses to row into the river on, is an axiom that is added to Zermelo-Fraenkel set theory which, with that addition, is denoted ZFC set theory. The axiom says that given any collection of non-empty sets, even an infinite collection, you can form a new set by picking one element from each. Although this seems self-evident, it does not follow from any of the other axioms in Zermelo-Fraenkel set theory and could either be added or not.

Notes for Chapter 14. All Tied Up

This is a story about collaboration and what happens when it goes awry. Just like a marriage or a rock band, collaborators in mathematics can do great work together, but at some point the relationship founders and the collaboration comes to a sudden end. But what if one of the collaborators doesn't want it to end?

Notes for Chapter 15. The Adventures of Robin Caruso

Among the best mathematicians, there are no slackers. It doesn't matter how smart you are, if you want to be successful, you need to put in the time. And you can do it anywhere: on a bus, in a waiting room, in line at the DMV. But having time to think and work is a crucial component to being able to get good mathematics done.

And if you wash up on a deserted isle, then you suddenly have lots of time. This is your big chance. Of course, you probably don't have pen and paper...

Notes for Chapter 16. Hardy and Ramanujan

One of the most fascinating tales of mathematics is the story of how the renowned English mathematician G. H. Hardy (1877–1947) receives a package from India containing the mathematical work of the unknown Indian Srinivasa Ramanujan (1887–1920). Recognizing its genius, Hardy brings Ramanujan from India to England, but Ramanujan's health is compromised and he ends up in the hospital. The story of Hardy's visit to the hospital and the number of his cab is a classic story that demanded embellishment.

Notes for Chapter 17. Introduction to the Collected Works

Often, when you pick up a volume of collected works, the introduction is the most interesting and readable section in the book. In this case, that is almost certainly true.

Notes for Chapter 18. Mathematicus

The entire educational system depends on an unwritten pact between faculty and students. Part of that pact is that students put time into learning and then submit to the grading process devised by their instructors. For many students, that process can be very painful. What if students decided not to put up with it anymore?

Notes for Chapter 19. Happiness is a Warm Theorem

This story is about the highs and lows of a research mathematician. You attack a problem and work on it for years, suffering nothing other than frustration, but then you figure it out and experience pure euphoria after all that time you struggled. But of course, the euphoria only lasts a tiny little while, and then it's on to the next problem. No wonder you need therapy.

Notes for Chapter 20. Motivational Seminar

This story has a bit of math content in it, that being the creation of the integers from nothing. This construction is due to the German mathematician Ernst Zermelo (1871–1953). You start with the empty set, which is the first set you consider. Then you take the set with one element, that element being the empty set, and you now have two sets. Then you take the set consisting of the empty set and this new set that contains the empty set as its only element. This is your third set.

Continuing in this manner, you produce an infinite collection of sets that you can put in one-to-one correspondence with the natural numbers. This is the beginning of the creation of mathematics, starting from nothing. What else are you going to do while hanging out in the womb for nine months ?

Notes for Chapter 21. Cylindra Ella

The idea to write a math take on the Cinderella story was suggested to me by Konstantina Kourkouli, daughter of the Greek mathematician Sofia Lambropoulou and age twelve at the time, when I was visiting the National Technical University in Athens. Several of the ideas in the story are hers. David Kramer came up with the ultimate title.

Notes for Chapter 22. Referee's Report

This story is about a "comma referee". This is a referee who writes incredibly long reports that are concerned less with the mathematics of the submitted paper and much more with the correct use of commas/punctuation. Often such a referee will also write a report on the revised paper that is just as detailed, and that finds more commas that are deemed incorrect even though those commas were present in the initial manuscript. There are cases where repeated revisions are requested and the refereeing process takes years.

The referee in this piece also believes their own work has not been sufficiently cited in the paper. Typically, referees are chosen from amongst the experts in the sub-field of the paper, so often the author and referee know each other and may have already had academic disputes. This can make things very difficult for the author.

Notes for Chapter 23. Silence of the Lemmas

This is an example of a title—in this case the title "Silence of the Lambs" from the movie and book—that suggested a story title. Then all you have to do is write the corresponding story.

Notes for Chapter 24. The Seven Labors of Hercules

It is just a convenient coincidence that the number of labors that Hercules must perform coincides with the number of Millennium Prize problems that were announced by the Clay Mathematics Institute in 2000. If you solve one of these most difficult of mathematics problems, you receive one million dollars. So far only one, the Poincaré Conjecture, has been solved. When Russian mathematician Grigori Perelman solved it in 2003, he turned down the prize money.

Notes for Chapter 25. Presidential Address

I don't think I need to say much about this one. It's pretty clear who inspired it.

Notes for Chapter 26. A Pi Day Carol

The easiest kinds of stories to write are the ones that come with a template. Dickens provided a very well-known template. This one was particularly fun to write.

Notes for Chapter 27. GUI

Nobody should be grading when on mind-altering drugs, but of course, that begs the question, where is the line?

Notes for Chapter 28. Math Is Everywhere

I came up with this story when I was reading the humorous horror book *John Dies at the End* by David Wong. It first appeared as a web serial in 2001, then as a book, and then as a disappointing movie in 2012. The book is based on the idea that there is an entire dark world just beneath the veneer of our own that can only be accessed if you take a particular drug called "soy sauce".

In our case, instead of soy sauce, all that is needed is to decipher a bit of the hidden mathematics around us in everyday symbols to realize it is everywhere. And then you need to be eliminated.

Notes for Chapter 29. Job Solicitation

The truth is that there are very few universities that need to solicit job candidates. Generally, there are more applicants than positions, but Berbunnion University is not your typical university...

Notes for Chapter 30. CSI-MSRI

As you might have guessed, this story was inspired by the acronyms for Crime Scene Investigation and the Mathematical Sciences Research Institute in Berkeley, CA. How could anyone resist using a title concatenated from these two acronyms?

This is the third of the Mangum, P.I. stories, the first two being "Mangum, P.I." and "A Killer Theorem", both of which appeared in the preceding collection *Riot at the Cal Exam and other Mathematically Bent Stories*. And of course, the character of Mangum was inspired by the fact that a private eye and a Principal Investigator on a national science foundation grant have the same PI acronym.

I spent a leave at the Mathematical Science Research Institute which I enjoyed immensely. It is a place where mathematicians in a particular research area have the opportunity to come together and work for a semester, all the while getting an amazing view. And in addition to being wonderful at fostering new mathematics, it is also a prime location for murder...

Notes for Chapter 31. Do Androids Dream of Symmetric Sheaves?

The title of this story is a twist on the title of the classic 1968 science fiction novel by Philip K. Dick, *Do Androids Dream of Electric Sheep?* The novel was the basis for the 1982 *Blade Runner* movie and its 2017 sequel *Blade Runner: 2049*.

In our story, androids have taken over, and humans just serve in support roles, barred from doing original mathematics. Will artificial intelligence surpass human intelligence in its ability to do mathematics? It is a great question. So far, ask ChatGPT a math question and it gives you a very convincing but nonsensical answer. But wait long enough and I expect AI will surpass us, assuming there is enough time for us all to get there. But some others may disagree.

Notes for Chapter 32. A Ghost Story

I have always believed that if there is such a thing as a ghost who cannot give up their earthly ties, the most likely candidates would be those mathematicians convinced they were just on the verge of obtaining that elusive proof they had spent the last decade working on. That death would rob them of their victory and would be too much to bear.

Notes for Chapter 33. Equinox

Who knows where creativity comes from? As authors famously know, you can have a period of great fecundity followed by a barren wasteland of nothingness. Mathematicians suffer from the same fear that authors suffer from. What if the well runs dry?

Of course, mathematicians are also famously non-superstitious, but this story imagines if that were not the case. How might mathematicians attempt to appease the gods of mathematical creativity?

Notes for Chapter 34. The Secrets of Math Destruction

There are more mathematicians working for the National Security Agency than any other organization in the United States, and maybe in the world, however the kind of work that they do necessitates the results being classified. But there are people like Julian Assange who believe that there is never a justification for keeping any information classified. Have there been breaches at NSA? Theorems and proofs that were revealed when they shouldn't have been? Who knows? They certainly aren't going to tell us one way or the other.

Notes for Chapter 35. The Book

The mathematician Paul Erdös (1913–1996) liked to suggest the idea of there being a Book that has all of the most elegant proofs in it, and all we are trying to do is to figure them out. It's pretty clear we have a long way to go.

Notes for Chapter 36. A Grader's Dream

I subscribe to the quote that you see at the bottom of many an email from a math colleague, "I teach for free. They pay me to grade". Very few people enjoy grading. It can be tedious at best and painful at worst. So if we're in a fairy tale anyway, why not have grading elves? You can always dream.

Notes for Chapter 37. The End of Mathematics

The book *The End of Science* by John Horgan came out in 1996. It posited that the big advances in physics and science more generally have already occurred, and although we will continue to fill in details, there will be no further huge advances. The fundamentals have all been discovered.

I believe that is no more likely than that we will run out of math to do. Mathematics is an ever-expanding mass of knowledge, and I at least cannot imagine how we could ever run out. The more we learn, the more we realize what else there is to explore.

Notes for Chapter 38. Definitions that Sizzle

Mathematical writing is typically learned on the streets. There are very few places where one can read about how to write mathematics in a rigorous yet exciting and readable manner. One of the few examples is Steven Krantz's book *A Primer of Mathematical Writing, Second Edition* (2016). This story is not such an example.

Notes for Chapter 39. Edutainment

Back in 2000, I placed a 25-cent bet with my colleague Tom Garrity that within fifteen years, over half of education would be online. I lost that bet. But I wasn't far off. With Covid, suddenly we did find ourselves in a period where over half of education was online.

Now, we see companies like Coursera, Udemy, EdX, Thinkwell, Massolit, and many others producing on-line educational material, and it is only a matter of time before it becomes interactive. Companies like Disney will realize that, worldwide, there are millions of students a year taking calculus. If you can produce a packaged course with stars like Brad Pitt and a story to make it interesting, you will make a lot of money. Moreover, it can be a smart package, realizing when a student makes a certain mistake, that with a probability of 0.97, it means they need work on exponents, and it sends them to the right review episode.

When these programs become a reality, it will mean that faculty like me will be demoted to human assistants (see "Do Androids Dream of Symmetric Sheaves?"), helping students to get to the right episode and making sure they feel emotionally supported. (But the good news is that it is coming a bit slower than I expected, and I should be retired by then.)

Notes for Chapter 40. A Prisoner's Dilemma

The Prisoner's Dilemma is a scenario that occurs in the field of Game Theory. It was originally formulated in 1950 and points out that in certain situations, the rational thing to do is not the optimal thing to do. In particular, if both culprits betray one another, they do worse than if they both keep silent, but fear that the other person will betray them makes both pick the sub-optimal choice.

The Prisoner's Dilemma makes an appearance in a variety of situations in economics and global politics. For instance, thinking of countries as the players, you can imagine what happens when we attempt to slow nuclear proliferation or adopt carbon emission limits. In this story, it is individuals who suffer the consequences.

Notes for Chapter 41. The Intergalactic Congress of Mathematicians

What does happen when we meet alien civilizations? Do we share mathematics? Will they know how to solve all of our biggest questions? Will we know how to solve some of theirs? Will we share our knowledge? Or will we protect what we know and try to find out what they know? After all, mathematics is the basis for all science, including the science of war.

Notes for Chapter 42. The Gulag

Is this story so far-fetched? There are places where mathematical talent will single you out for "special treatment".

High school students in North Korea that show promise are selected for intense mathematical training, and it seems to be working. In 2019, North Korea came in fourth in the International Mathematics Olympiad (IMO), a mathematics competition for the most talented high school students from around the world, beating out some of the traditional favorites with much larger populations from which to draw. And some of their most promising students are required to spend their careers writing malware used to attack other countries ("The Incredible Rise of North Korea's Hacking Army", Ed Caesar, The New Yorker, April 19, 2021).

Notes for Chapter 43. Where Do Theorems Come From?

It is actually a good question. How does one come up with a new theorem? From whence does inspiration come? In this story, the goal is to try to explain it to a child, much as one might try to explain where babies come from. Good luck!

Notes for Chapter 44. The Lord of the Rings I: The Fellowship of the Rings

This is another example of a story suggested by words appearing in the title of a book that have a different interpretation by mathematicians. In this case, in the field of algebra, a "ring" is a set of elements with a version of addition and multiplication defined. The integers is a simple example. A tremendous amount of research in algebra has been devoted to understanding rings and their properties.

And in mathematics, if someone says they have a "fellowship", it is most likely a National Science Foundation fellowship. This provides funds from the United States government to support research for several years after they have received their Ph.D.

Notes for Chapter 45. Piracy

Advances in mathematics are often based on papers that have just appeared. The new results in those papers are just what is needed to make the next step forward.

A classic example of this occurred in knot theory when, in 1985, Vaughan Jones (1952–2020) created the Jones polynomial, a single-variable polynomial that can be used to distinguish knots. Within a two-month period, four different groups developed the same two-variable extension of it. They all sent their announcements to the *Bulletin of the American Mathematical Society*, which then arranged for them to write a single paper with all as co-authors. A fifth group of two Polish mathematicians also sent an announcement, but they were not included in the paper as the mail from Poland was too slow. The polynomial became known as the HOMFLY polynomial, using the first letters of the authors' last names. It is now known as the HOMFLYPT polynomial, adding the first letters of the Polish mathematicians Przytycki and Tracyk.

But because of the fact so many different groups may be working to extend current results in the same direction, disputes are inevitable. The problem is that we go to each others talks. Perhaps you have a preprint almost ready to put out there, and you go to a talk and the speaker presents research that is almost exactly the same stuff you are doing. So then you rush your preprint out. The speaker can understandably be upset because it may appear you learned the results from their talk and are trying to present them as your own.

But it can also be the case that someone is stuck and they go to a talk and realize the one piece they were missing. They have put an immense amount of work in and believe they deserve credit, but the truth is that they did not come up with the key idea.

And then there can be cases of someone who is unscrupulous and does take results that have not yet appeared in print and claims them for their own. It doesn't happen a lot, but it does happen. There is always some risk in talking about results that are not yet in print.