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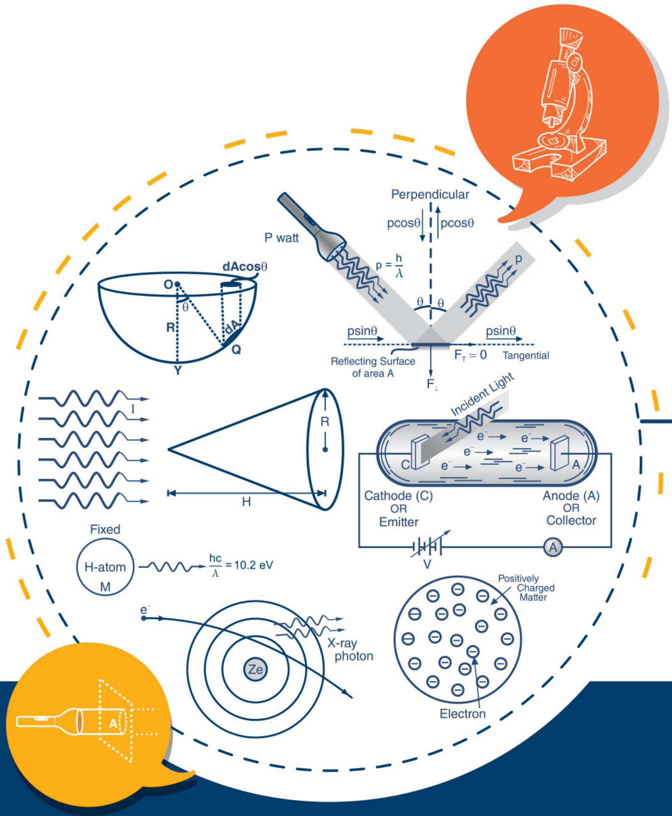

Modern Physics

Pearson

Rahul Sardana

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3 THIRD EDITION

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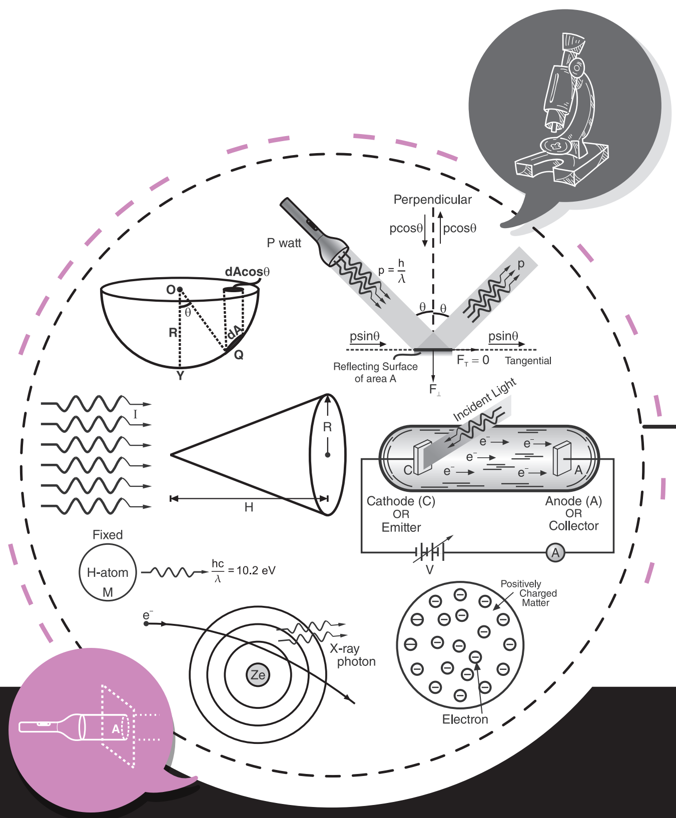

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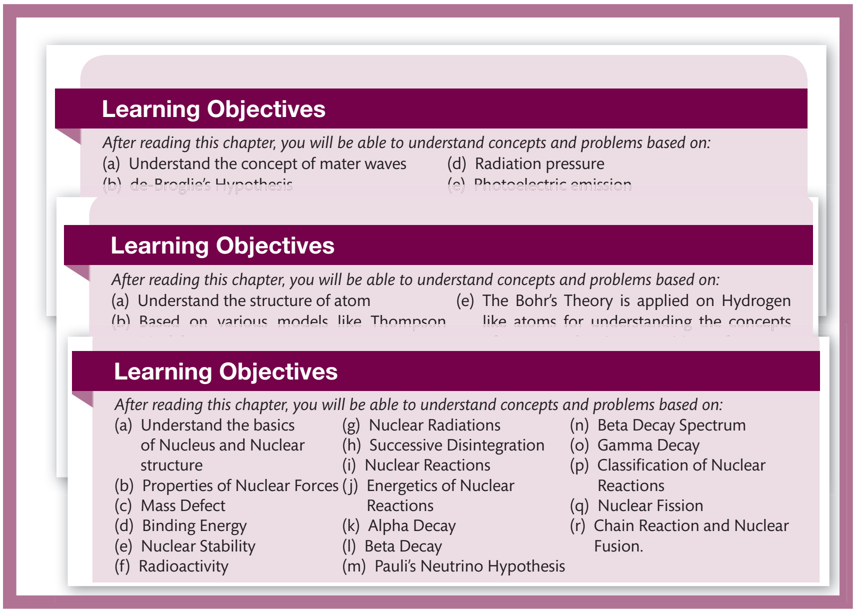
HINTS AND EXPLANATIONS

CHAPTER INSIGHT

Learning Objectives

Help the students set an aim to achieve the major take-aways from a particular chapter.



Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Understand the concept of mater waves
(d) Radiation pressure

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Understand the structure of atom
(e) The Bohr's Theory is applied on Hydrogen

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Understand the basics of Nucleus and Nuclear structure
(b) Properties of Nuclear Forces
(c) Mass Defect
(d) Binding Energy
(e) Nuclear Stability
(f) Radioactivity
(g) Nuclear Radiations
(h) Successive Disintegration
(i) Nuclear Reactions
(j) Energetics of Nuclear Reactions
(k) Alpha Decay
(l) Beta Decay
(m) Pauli's Neutrino Hypothesis
(n) Beta Decay Spectrum
(o) Gamma Decay
(p) Classification of Nuclear Reactions
(q) Nuclear Fission
(r) Chain Reaction and Nuclear Fusion.

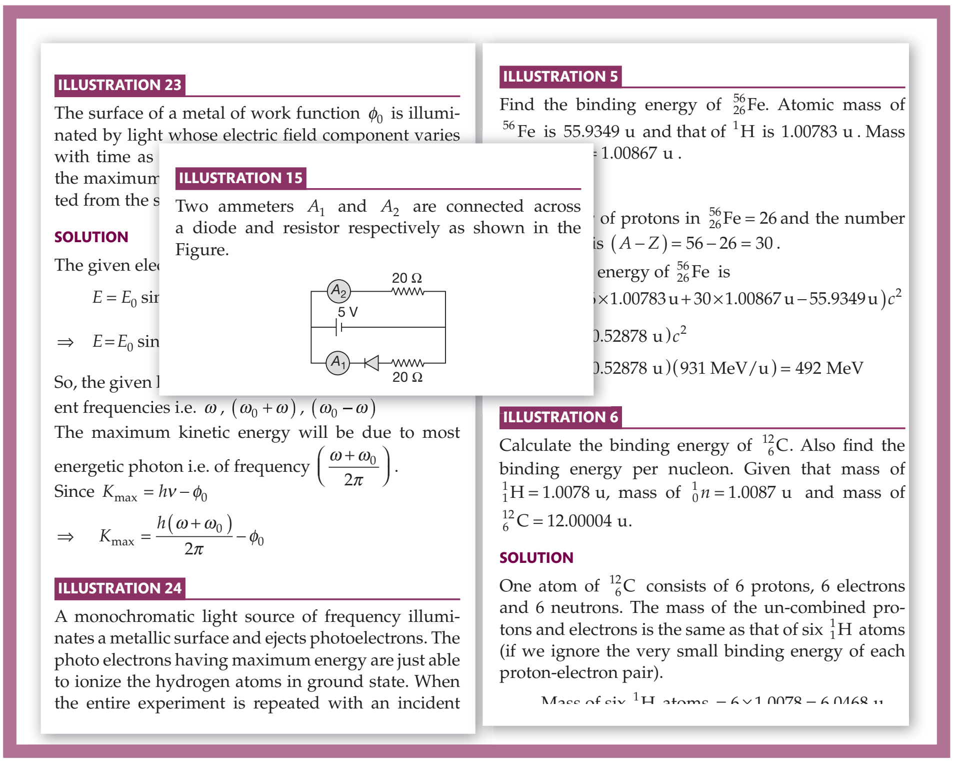


ILLUSTRATION 23

The surface of a metal of work function ϕ_0 is illuminated by light whose electric field component varies with time as

ILLUSTRATION 15

Two ammeters A_1 and A_2 are connected across a diode and resistor respectively as shown in the Figure.

ent frequencies i.e. ω, $(\omega_0+\omega)$, $(\omega_0-\omega)$

The maximum kinetic energy will be due to most energetic photon i.e. of frequency $\left(\frac{\omega+\omega_0}{2\pi}\right)$.

Since $K_{max} = h\nu - \phi_0$

$$\Rightarrow \quad K_{max} = \frac{h(\omega+\omega_0)}{2\pi} - \phi_0$$

ILLUSTRATION 24

A monochromatic light source of frequency illuminates a metallic surface and ejects photoelectrons. The photo electrons having maximum energy are just able to ionize the hydrogen atoms in ground state. When the entire experiment is repeated with an incident

ILLUSTRATION 5

Find the binding energy of $^{56}_{26}Fe$. Atomic mass of ^{56}Fe is 55.9349 u and that of ^{1}H is 1.00783 u. Mass

of protons in $^{56}_{26}Fe = 26$ and the number is $(A-Z) = 56-26 = 30$.

energy of $^{56}_{26}Fe$ is

$\times 1.00783\,u + 30\times 1.00867\,u - 55.9349\,u)c^2$

$0.52878\,u)c^2$

$0.52878\,u)(931\,MeV/u) = 492\,MeV$

ILLUSTRATION 6

Calculate the binding energy of $^{12}_{6}C$. Also find the binding energy per nucleon. Given that mass of $^{1}_{1}H = 1.0078$ u, mass of $^{1}_{0}n = 1.0087$ u and mass of $^{12}_{6}C = 12.00004$ u.

SOLUTION

One atom of $^{12}_{6}C$ consists of 6 protons, 6 electrons and 6 neutrons. The mass of the un-combined protons and electrons is the same as that of six $^{1}_{1}H$ atoms (if we ignore the very small binding energy of each proton-electron pair).

Theory with Illustrations

Elaborative and simple theory helps the students to understand the illustrations supporting the theory. Please note that theory and problem solving techniques are based on simple learning program IF → THEN → ELSE. I would suggest you not to attempt the illustrations without going through the theory of that section.

Test Your Concepts-I

Based on Photon Properties and De Broglie Phenomenon

(Solutions on page H.3)

1. The intensity of direct sunlight before it passes through the earth's atmosphere is 1.4 kWm^{-2}. If it is completely absorbed find the corresponding radiation pressure.
2. According to the Maxwell theory of electrodynamics an electron going in a circle should emit

factors (such as numerical aperture etc.) are taken to be roughly the same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light?

10. An electron and proton are possessing the same

Test Your Concepts-II

Based on X-

1. What potential difference should be applied acrc an X-ray tube to get X-ray of wavelength not le than 0.10 nm? What is the maximum energy o photon of this X-ray in joule? Take *hc* = 12400 e\
2. **(a)** An X-ray tube produces a continuous spe trum of radiation with its short-waveleng end at 0.45 Å. What is the maximum energy a photon in the radiation?
   (b) From your answer to (a), guess what order accelerating voltage (for electrons) is requir in such a tube?

Test Your Concepts-III

Based on Nuclear Reactions, Alpha

1. Consider two decay reactions.
   (a) $^{238}_{92}U \longrightarrow {}^{206}_{82}Pb + 10$ protons + 22 neutrons
   (b) $^{238}_{92}U \longrightarrow {}^{206}_{82}Pb + 8\,{}^{4}_{2}He + 6$ elctrons
   Are both the reactions possible?
   Given: Average binding energy of $^{238}_{92}U = 7.57$ Me\ that of $^{206}_{82}Pb = 7.83$ MeV and that of $^{4}_{2}He = 7$ Me\ per nucleon.
2. Find the minimum kinetic energy of an α-particl to cause the reaction $^{14}N + {}^{4}He \rightarrow {}^{17}O + {}^{1}H$. Give

Test Your Concepts

These topic based exercise sets are based on simple, single concept classification technique. These are meant for students practice after they study a particular topic and want to practice more on that topic learnt. Finally, in case of any difficulty they can refer to the hints and solutions to these exercise sets given at the end of the book.

Problem Solving Techniques

These techniques ensure that students become capable enough to solve a variety of problems in an easy and quick manner.

Problem Solving Technique(s)

FORMULAE FOR WORKING THE PROBLEMS ON PHOTO-ELECTRIC EFFECT

Maximum Kinetic Energy of photo-electrons

$$E_K = eV_0 = \frac{1}{2}mv^2_{max}$$

If λ_0 is the threshold wavelength and ν_0 the threshold frequency,

Work function of photo-metal,

$$\phi_0 = h\nu_0 = \frac{hc}{\lambda_0}$$

Problem Solving Technique(s)

Let us illustrate the above definitions by taking the case of hydrogen atom. For H atom, we have

$$E_n = -\frac{me^4}{8\varepsilon_0^2 n^2 h^2} = -\frac{2.17\times10^{-18}}{n^2}J = -\frac{13.6}{n^2}eV$$

So the energy of the 1st, 2nd, 3rd,......., ∞-th orbits are respectively −13.6 eV, −3.4 eV, −1.15 eV,...., 0 eV. Hence

Resonance potential $= -3.4 - (-13.6) = 10.2$ eV

First excitation potential = resonance potential = 10.2 eV

Second excitation potential $= -1.51 - (-13.6) = 12.09$ eV

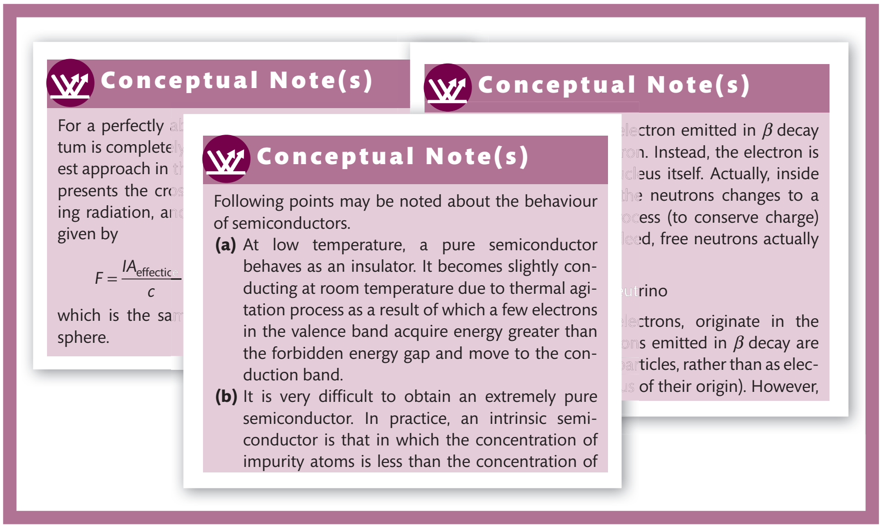
Ionisation potential $= 0 - (-13.6) = 13.6$ eV

Problem Solving Technique(s)

(a) Output voltage is obtained across the load resistance R_L. It is not constant but pulsating (mixture of ac and dc) in nature.

(b) The ripple frequency for full wave rectifier is twice the input frequency of the ac, hence $\omega_{ripple} = 2\omega$

(c) Average output in one cycle is



Conceptual Note(s)

Following points may be noted about the behaviour of semiconductors.

(a) At low temperature, a pure semiconductor behaves as an insulator. It becomes slightly conducting at room temperature due to thermal agitation process as a result of which a few electrons in the valence band acquire energy greater than the forbidden energy gap and move to the conduction band.

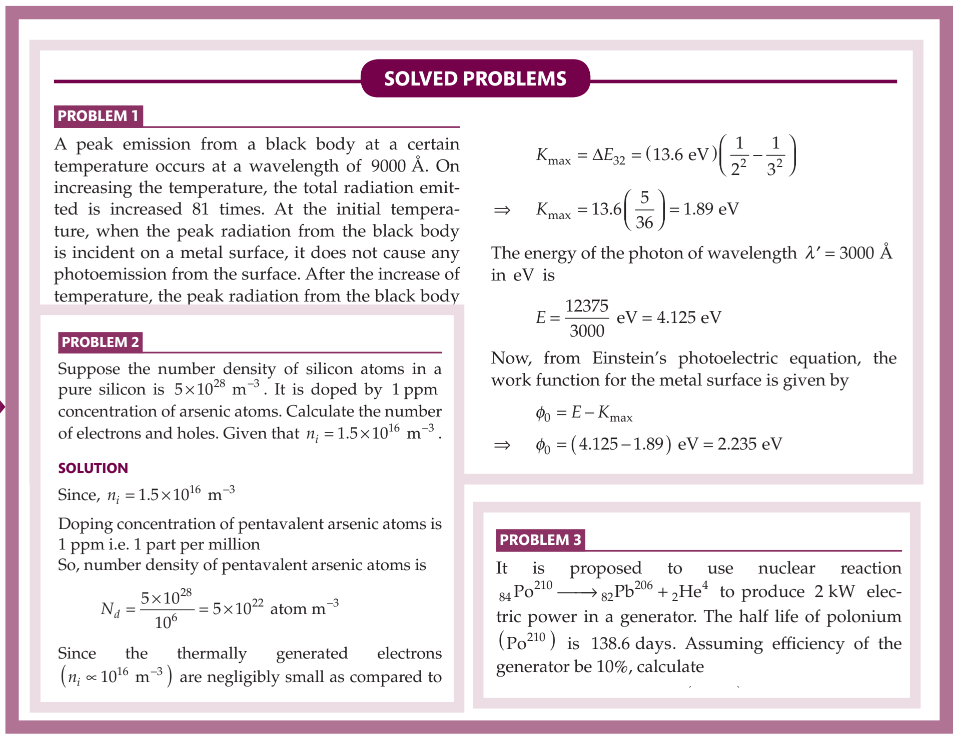
(b) It is very difficult to obtain an extremely pure semiconductor. In practice, an intrinsic semiconductor is that in which the concentration of impurity atoms is less than the concentration of

Conceptual Notes

The Conceptual Notes, Remarks, Words of Advice, Misconception Removals provide warnings to the students about common errors and help them avoid falling for conceptual pitfalls.

Chapter End Solved Problems

These are based on multiple concept usage in a single problem approach so as to expose a student's brain to the ultimate throttle required to take the JEE examination.



SOLVED PROBLEMS

PROBLEM 1

A peak emission from a black body at a certain temperature occurs at a wavelength of 9000 Å. On increasing the temperature, the total radiation emitted is increased 81 times. At the initial temperature, when the peak radiation from the black body is incident on a metal surface, it does not cause any photoemission from the surface. After the increase of temperature, the peak radiation from the black body

PROBLEM 2

Suppose the number density of silicon atoms in a pure silicon is $5\times10^{28}\ \text{m}^{-3}$. It is doped by 1 ppm concentration of arsenic atoms. Calculate the number of electrons and holes. Given that $n_i = 1.5\times10^{16}\ \text{m}^{-3}$.

SOLUTION

Since, $n_i = 1.5\times10^{16}\ \text{m}^{-3}$

Doping concentration of pentavalent arsenic atoms is 1 ppm i.e. 1 part per million

So, number density of pentavalent arsenic atoms is

$$N_d = \frac{5\times10^{28}}{10^6} = 5\times10^{22}\ \text{atom m}^{-3}$$

Since the thermally generated electrons $(n_i \propto 10^{16}\ \text{m}^{-3})$ are negligibly small as compared to

$$K_{max} = \Delta E_{32} = (13.6\ \text{eV})\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\Rightarrow \quad K_{max} = 13.6\left(\frac{5}{36}\right) = 1.89\ \text{eV}$$

The energy of the photon of wavelength $\lambda' = 3000$ Å in eV is

$$E = \frac{12375}{3000}\ \text{eV} = 4.125\ \text{eV}$$

Now, from Einstein's photoelectric equation, the work function for the metal surface is given by

$$\phi_0 = E - K_{max}$$

$$\Rightarrow \quad \phi_0 = (4.125 - 1.89)\ \text{eV} = 2.235\ \text{eV}$$

PROBLEM 3

It is proposed to use nuclear reaction $_{84}Po^{210} \longrightarrow {}_{82}Pb^{206} + {}_2He^4$ to produce 2 kW electric power in a generator. The half life of polonium (Po^{210}) is 138.6 days. Assuming efficiency of the generator be 10%, calculate

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Question: which ONLY ONE is correct.

1. Consider a source emitting 100 W of green light at a wavelength of 500 nm. The number of photons emerging from source per second is

MULTIPLE CORRECT CHOICE TYPE QUESTION

This section contains Multiple Correct Choice Type Question which ONE OR MORE is/are correct.

1. The maximum kinetic energy of photoelectrons ejected from a photometer when it is irradiated with radiation of wavelength 400 nm is 1 eV. If the threshold energy of the metal surface is 1.9 eV. Select the correct statement(s).
   (A) The maximum K.E. of photoelectrons when it is

MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions o followed by questions. Each question has four choices (A), (B), competitiveness there may be a fe

Comprehension I

The nuclei of a radioactive elemen at a constant rate α and this ele nucleus Y with a decay constant time $t = 0$, there are N_0 nuclei of the information given, answer the

INTEGER/NUMERICAL ANSWER TYPE QUESTI

In this section, the answer to each question is a numerical valu given in the question(s).

total acceleration of the elium atom and hydrogen re in ground state.

ectron in the ground state 24.6 eV. Find the energy both electrons from the

at 18 kV. The speed of

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having correct. Each question contains STATEMENT 1 and STATEMI

Bubble (A) If both statements are TRUE and STATEMENT 2
Bubble (B) If both statements are TRUE but STATEMENT 2
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is

1. **Statement-1:** Threshold wavelength of certain metal is λ_0. Light of wavelength slightly less than λ_0 is incident on the plate. It is found that after some time the

Practice Exercise

Inclusion of all types of questions asked in JEE Advanced in adequate numbers helps you with enough practice

Archive JEE Main and Advaned

From this fully updated section, students get to know the actual pattern of the problems asked in the past examinations.

ARCHIVE: JEE MAIN

1. **[Online April 2019]**
   Two particles move at right angle to each other. Their respectively. ision. The de e, is given by λ_2 $+ \frac{1}{\lambda_2^2}$

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

(In this section each question has four choices (A), (B), (C)

on ϕ_0 is . The ngth λ_d. t by $\Delta\lambda$ $\frac{\Delta\lambda_d}{\Delta\lambda}$ is

Integer/Numerical Answer Type Questions

(In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s)).

I mounted on stem of angu- 0^{24} m^{-2} with of wavelength ultaneously at gets displaced then the value

Matrix Match/Column Match Type Questions

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of

CHAPTER 1: DUAL NATURE OF RADIATION AND MATTER

Test Your Concepts-I (Based on Photon Properties)

1. For completely absorbing surface,

$$P_{rad} = \frac{I}{C} = \frac{1.4\times10^3}{3\times10^8} = 4.7\times10^{-6}\ \text{Nm}^{-2}$$

2. $f = \frac{v_1}{2\pi r_1} = \frac{c}{\lambda}$

$$\Rightarrow \quad \lambda = \frac{2\pi c r_1}{v_1} = \frac{(2\pi)(3\times10^8)(0.529\times10^{-10})(10^{10})}{(2.2\times10^6)}\ \text{Å}$$

$$\Rightarrow \quad \lambda = 453\ \text{Å}$$

3. For an electron, de-Broglie wavelength is given by,

6. Energy of each photon is

$$E = h\nu = 6.62\times10^{-34}\times10^{12} = 6.62\times10^{-22}\ \text{J}$$

Number of photons present in 6.62 J of radiati energy is calculated by using

$$E = N(h\nu)$$

$$\Rightarrow \quad N = \frac{E}{h\nu} = \frac{6.62}{6.62\times10^{-22}} = 10^{22}$$

7. (a) Energy of each photon,

$$E = h\nu = 6.63\times10^{-34}\times6\times10^{14} = 3.98\times10^{-19}$$

(b) If N is the number of photons emitted per secor by the source, then Power transmitted in the beam is

Single Correct Choice Type Questions

1. $E = nh\nu$

$$\Rightarrow \quad \frac{E}{t} = P = \left(\frac{n}{t}\right)h\nu$$

Hence, the correct answer is (C).

3. $5\,eV_0 = \frac{hc}{\lambda} - W$ ...(1)

$$\Rightarrow \quad eV_0 = \frac{hc}{3\lambda} - W \quad \text{...(2)}$$

Solving equation (1) & (2), we get

$$\frac{hc}{\lambda} - W = \frac{5hc}{3\lambda} - 5W$$

$$\Rightarrow \quad 4W = \frac{2hc}{3\lambda}$$

$$\Rightarrow \quad W = \frac{hc}{6\lambda}$$

Hence, the correct answer is (A).

Multiple Correct Choice Type Questions

1. $\frac{hc}{\lambda} = \phi_0 + K_{max}$

$$\Rightarrow \quad \frac{hc}{4000} = (1.9+1)\ \text{eV} = 2.9\ \text{eV}$$

$$\Rightarrow \quad hc = (4000)(2.9) = 11600\ \text{eVÅ}$$

Now, for $\lambda = 500\ \text{nm} = 5000\ \text{Å}$, we have

$$\frac{11600}{5000} = 1.9 + K_{max}$$

$$\Rightarrow \quad 2.32 = 1.9 + K_{max}$$

$$\Rightarrow \quad K_{max} = 0.42\ \text{eV}$$

The longest wavelength which will eject photoelectrons is given by

$$\lambda_{max} = \frac{hc}{\phi_0} = \frac{11600}{1.9} = 6105\ \text{Å}$$

$$\Rightarrow \quad \lambda_{max} \approx 6100\ \text{Å}$$

Hints and Explanations

Exhaustive solutions with shortcuts (where ever needed), help students enhance their problem-solving skills.

PREFACE

In the past few years, the IIT-JEE has evolved as an examination designed to check a candidate's true scientific skills. The examination pattern needs one to see those little details which others fail to see. These details tell us how much in-depth we should know to explain a concept in the right direction. Keeping the present-day scenario in mind, this series is written for students, to allow them not only to learn the tools but also to see why they work so nicely in explaining the beauty of ideas behind the subject. The central goal of this series is to help the students develop a thorough understanding of Physics as a subject. This series stresses on building a rock-solid technical knowledge based on firm foundation of the fundamental principles followed by a large collection of formulae. The primary philosophy of this book is to guide the aspirants towards detailed groundwork for strong conceptual understanding and development of problem-solving skills like mature and experienced physicists.

This updated Third Edition of the book will help the aspirants prepare for both Advanced and Main levels of JEE conducted for IITs and other elite engineering institutions in India. This book will also be equally useful for the students preparing for Physics Olympiads.

This book is enriched with detailed exhaustive theory that introduces the concepts of Physics in a clear, concise, thorough and easy-to-understand language. A large collection of relevant problems is provided in eight major categories (including updated archive for JEE Advanced and JEE Main), for which the solutions are demonstrated in a logical and stepwise manner.

We have carefully divided the series into seven parts to make the learning of different topics seamless for the students. These parts are

- Mechanics – I
- Mechanics – II
- Waves and Thermodynamics
- Electrostatics and Current Electricity
- Magnetic Effects of Current and Electromagnetic Induction
- Optics
- Modern Physics

Finally, I would like to thank all my teacher friends who had been a guiding source of light throughout the entire journey of writing this book.

To conclude, I apologise in advance for the errors (if any) that may have inadvertently crept in the text. I would be grateful to the readers who bring errors of any kind to my attention. I truly welcome all comments, critiques and suggestions. I hope this book will nourish you with the concepts involved so that you get a great rank at JEE.

PRAYING TO GOD FOR YOUR SUCCESS AT JEE. GOD BLESS YOU!

Rahul Sardana

ABOUT THE AUTHOR

Rahul Sardana, is a Physics instructor and mentor having rich and vast experience of about 20 years in the field of teaching Physics to JEE Advanced, JEE Main and NEET aspirants. Along with teaching, authoring books for engineering and medical aspirants has been his passion. He authored his first book '*MCQs in Physics*' in 2002 and since then he has authored many books exclusively for JEE Advanced, JEE Main and NEET examinations.

He is also a motivational speaker having skills to motivate students and ignite the spark in them for achieving success in all colours of life. Throughout this journey, by the Grace of God, under his guidance and mentorship, many of his students have become successful engineers and doctors.

CHAPTER 1

Dual Nature of Radiation and Matter

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Matter waves
(b) de-Broglie's Hypothesis
(c) Photon properties
(d) Radiation pressure
(e) Photoelectric emission
(f) Photoelectric effect and properties.

All this is followed by an Exercise Set (fully solved) which contains questions as per the latest JEE pattern. At the end of Exercise Set, a collection of problems asked previously in JEE Main are also given.

MATTER WAVES

Light possesses dual nature i.e. it behaves both as a wave and as a particle. In some phenomena e.g., interference, diffraction and polarisation, it behaves as waves because they are explained on the basis of Wave theory while in some other phenomena e.g. photoelectric effect, Compton effect, it behaves as particles (photons).

Since nature demands symmetry, therefore de-Broglie thought that matter must have dual nature. The particle nature of matter is well known and hence de-Broglie thought that material particles must possess wave-nature.

de-BROGLIE'S POSTULATE

According to de-Broglie a material particle in motion must have a wave like character and the wavelength associated with it is given by

$$\lambda = \frac{h}{p} \qquad \ldots(1)$$

where, h is the Planck's constant whose value is given by $h = 6.63 \times 10^{-34}$ Js and p is the momentum of the particle.

de-Broglie assumed this expression in analogy with photon because momentum of photon is

$$p = \frac{h}{\lambda}$$

$$\Rightarrow \quad \lambda = \frac{h}{p}$$

If m is the mass of particle and v the velocity, then momentum of particle is $p = mv$.

So, de-Broglie wavelength $\lambda = \dfrac{h}{mv}$ ...(2)

If E_k is kinetic energy of particle, then

$$p = \sqrt{2mE_K} \qquad \left\{\because E_K = \frac{p^2}{2m}\right\}$$

$$\Rightarrow \quad \lambda = \frac{h}{\sqrt{2mE_K}} \qquad \ldots(3)$$

For charged particles accelerated through a potential difference of V volts,
Kinetic energy i.e. $E_K = qV$

$$\Rightarrow \quad \lambda = \frac{h}{\sqrt{2mqV}} \qquad \ldots(4)$$

For electrons accelerated through a potential difference of V volts de-Broglie wavelength $\lambda = \dfrac{h}{\sqrt{2meV}}$.

Substituting $m = 9\cdot1\times10^{-31}$ kg, $h = 6\cdot62\times10^{-34}$ Js, $e = 1\cdot6\times10^{-19}$ C, we get

$$\lambda = \sqrt{\frac{150}{V}}\times10^{-10} \qquad \ldots(5)$$

$$\Rightarrow \quad \lambda = \sqrt{\frac{150}{V}}\ \text{Å} = \frac{12.27}{\sqrt{V}}\ \text{Å}$$

This is expression for de-Broglie wavelength associated with electron accelerated through a potential difference of V. The wave nature is possessed by all particles neutral of charged. The wave nature was first verified by Davisson and Germer for slow electrons.

CHARACTERISTICS OF MATTER WAVES

(a) Matter wave represents the probability of finding a particle in space.
(b) Matter waves are not electromagnetic in nature.
(c) de-Broglie or matter wave is independent of the charge on the material particle. It means, matter wave of de-Broglie wave is associated with every moving particle (whether charged or uncharged).
(d) Practical observation of matter waves is possible only when the de-Broglie wavelength is of the order of the size of the particles.
(e) Electron microscope works on the basis of de-Broglie waves.
(f) The phase velocity of the matter waves can be greater than the speed of the light.
(g) Matter waves can propagate in vacuum, hence they are not mechanical waves.
(h) The number of de-Broglie waves associated with n^{th} orbital electron is n.
(i) Only those circular orbits around the nucleus are stable whose circumference is integral multiple of de-Broglie wavelength associated with the orbital electron.

de-BROGLIE WAVELENGTH ASSOCIATED WITH THE CHARGED PARTICLES

The energy of a charged particle accelerated through potential difference V is $E = \frac{1}{2}mv^2 = qV$

Hence de-Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$$

Using the above formula, we get

$$\lambda_{\text{Electron}} = \frac{12.27}{\sqrt{V}}\ \text{Å}$$

$$\lambda_{\text{Proton}} = \frac{0.286}{\sqrt{V}}\ \text{Å}$$

$$\lambda_{\text{Deutron}} = \frac{0.202}{\sqrt{V}}\ \text{Å}$$

$$\lambda_{\alpha-\text{particle}} = \frac{0.101}{\sqrt{V}}\ \text{Å}$$

ILLUSTRATION 1

Find the ratio of de Broglie wavelength of proton and α-particle which have been accelerated through same potential difference.

SOLUTION

Kinetic energy gained by a charge q after being accelerated through a potential difference V volt is

$$qV = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2qV}{m}}$$

$$\Rightarrow \quad mv = \sqrt{2mqV}$$

So, the de Broglie wavelength is given by

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mqV}}$$

$$\Rightarrow \quad \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}}$$

$$\Rightarrow \quad \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{4\times2}{1\times1}} = 2\sqrt{2}$$

de-BROGLIE WAVELENGTH ASSOCIATED WITH UNCHARGED PARTICLES: THERMAL NEUTRONS

Thermal neutrons are generally in thermal equilibrium with the surroundings. For thermal neutrons at temperature T, kinetic energy of most of neutrons i.e. the most probable energy is $E_K = k_B T$, where k_B is the Boltzmann's constant. This energy is 0.025 eV for neutrons at 27 °C. So the de-Broglie wavelength corresponding to most of the thermal neutrons is the wavelength that corresponds to the most probable energy of the thermal neutrons at room temperature. $\lambda = \frac{h}{\sqrt{2mk_BT}}$, where T is the Absolute temperature, k_B is the Boltzmann's constant given by $k_B = 1.38\times10^{-23}\ \text{JK}^{-1}$.

However, the average kinetic energy of thermal neutrons is $E_{av} = \frac{3}{2}k_BT$, hence the de-Broglie wavelength corresponding to this energy is $\lambda = \frac{h}{\sqrt{3mk_BT}}$.

For thermal neutrons de-Broglie wavelength can also be given by the expression

$$\lambda_{\text{Neutron}} = \frac{0.286\times10^{-10}}{\sqrt{E(\text{in eV})}}\ \text{m} = \frac{0.286}{\sqrt{E(\text{in eV})}}\ \text{Å}$$

$$\text{So, } \lambda_{\text{Thermal neutron}} = \frac{6.63\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times1.38\times10^{-23}T}}$$

$$\Rightarrow \quad \lambda_{\text{Thermal neutron}} = \frac{30.9}{\sqrt{T}}\ \text{Å}$$

ILLUSTRATION 2

Obtain the de-Broglie wavelength associated with thermal neutrons at room temperature (27 °C). Hence explain why a fast neutron beam needs to be thermalised with the environment before it can be used for neutron diffraction experiments.

SOLUTION

Kinetic energy of most of the neutrons i.e. the most probable energy of a neutron at temperature T is

$$\frac{1}{2}mv^2 = k_BT$$

$$\Rightarrow \quad \frac{p^2}{2m} = k_BT \qquad \{\because p = mv\}$$

$$\Rightarrow \quad p = \sqrt{2mk_BT}$$

de-Broglie wavelength possessed by most of the neutrons is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mk_BT}}$$

Given $m_n = 1.675\times10^{-27}$ kg, $k_B = 1.38\times10^{-23}\ \text{Jmol}^{-1}\text{K}^{-1}$

$$T = 27 + 273 = 300\ \text{K},\ h = 6.63\times10^{-34}\ \text{Js}$$

$$\Rightarrow \quad \lambda = \frac{6.63\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times1.38\times10^{-23}\times300}}$$

$$\Rightarrow \quad \lambda = 1.78\times10^{-10}\ \text{m}$$

$$\Rightarrow \quad \lambda \approx 1.8\ \text{Å}$$

Since this wavelength is comparable to interatomic spacing in a crystal, so thermal neutrons can be used for diffraction experiments. A high energy neutron beam should be first thermalised before using it for diffraction experiments.

ILLUSTRATION 3

Find the de Broglie wavelength corresponding to the root-mean-square velocity of hydrogen molecules at room temperature (20 °C).

SOLUTION

$$\text{Since, } v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$v_{\text{rms}} = \sqrt{\frac{3\times8.31\times293}{2\times10^{-3}}} = 1911\ \text{ms}^{-1}$$

$$\text{Now, } \lambda = \frac{h}{p} = \frac{h}{mv_{\text{rms}}}$$

Mass of one hydrogen molecule is given by

$$m = \frac{2}{6.02\times10^{26}}\ \text{kg} = 3.32\times10^{-27}\ \text{kg}$$

$$\Rightarrow \quad \lambda = \frac{6.63\times10^{-34}}{3.32\times10^{-27}\times1911}\ \text{m}$$

$$\Rightarrow \quad \lambda = 1.04\times10^{-10}\ \text{m} = 1.04\ \text{Å}$$

HEISENBERG'S UNCERTAINTY PRINCIPLE

According to Heisenberg it is impossible to measure the position and momentum of a particle simultaneously with 100% accuracy. This is called Heisenberg's uncertainty principle. Uncertainty principle successfully explains the

(i) Non-existence of electrons in the nucleus
(ii) Finite size of spectral lines.

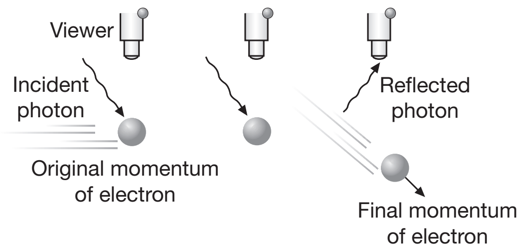
If Δx and Δp are uncertainties in determining the position and momentum of the particle simultaneously, then

$$\Delta x \Delta p \geq \frac{h}{4\pi}, \text{ where } h = 6.63 \times 10^{-34} \text{ Js}$$

$$\Rightarrow \quad m\Delta x \Delta v \geq \frac{h}{4\pi}$$

This principle is universal and holds for all microscopic and macroscopic particles. The principle is also unaffected by experimental techniques.

If $\Delta x = 0$, then $\Delta p \to \infty$ and if $\Delta p = 0$ then $\Delta x \to \infty$,i.e. if we are able to measure the exact position of the particle (say an electron) then the uncertainty in the measurement of the linear momentum of the particle is infinite. Similarly, if we are able to measure the exact linear momentum of the particle i.e., $\Delta p = 0$, then we cannot measure the exact position of the particle at that time.



An electron cannot be observed without changing it's momentum.

This principle is also applicable to energy and time, angular momentum and angular displacement. Hence,

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$

$$\Delta L \Delta \theta \geq \frac{h}{4\pi}$$

Problem Solving Technique(s)

(a) For numerical problems, we shall use

$$\Delta x \Delta p \approx h \qquad \Delta E \Delta t \approx h \qquad \Delta L \Delta \theta \approx h$$

(b) If the radius of the nucleus is r then the probability of finding the electron inside the nucleus is $\Delta x = 2r$ and uncertainty in momentum is $\Delta p = \frac{h}{4\pi r}$.

QUANTUM NATURE OF LIGHT AND PLANK'S QUANTUM THEORY

Some phenomena like photoelectric effect, Compton effect, Raman effect could not be explained by Wave theory of light. Therefore, quantum theory of light was proposed by Einstein who extended the Planck's hypothesis to explain Black Body radiation. According to quantum theory of light or radiation, the energy of an electromagnetic wave is not continuously distributes over the wave front (just like the energy possessed by water waves). Instead Plank proposed that an electro-magnetic wave travels in the form of discrete packets or bundles of energy called ***Quanta***.

So, according to Plank, **"light is propagated in bundles of small energy, each bundle being called a photon and possessing energy", given by**

$$E = h\nu = \frac{hc}{\lambda}$$

where, ν is frequency, λ is wavelength of light, h is Planck's constant whose value is $6 \cdot 63 \times 10^{-34}$ Js and $c = 3 \times 10^{8}$ ms^{-1}

ILLUSTRATION 4

An α particle and a proton are fired through the same magnetic fields which is perpendicular to their velocity vectors. The α particle and the proton move such that radius of curvature of their path is same. Find the ratio of their de Broglie wavelengths.

SOLUTION

Magnetic force experienced by a charged particle in a magnetic field is given by,

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$\Rightarrow \quad F = qvB \sin\theta$$

In this case $\theta = 90°$, so we get $F = qvB$ and this magnetic force is responsible for providing the centripetal force to the charged particle to move in a circle.

$$\Rightarrow \quad qvB = \frac{mv^2}{r}$$

$$\Rightarrow \quad mv = qBr$$

Hence the de Broglie wavelength is given by

$$\lambda = \frac{h}{mv} = \frac{h}{qBr}$$

$$\Rightarrow \quad \frac{\lambda_{\alpha\text{-particle}}}{\lambda_{\text{proton}}} = \frac{q_p r_p}{q_\alpha r_\alpha}$$

According to the problem, we have

$$\frac{r_\alpha}{r_p} = 1 \text{ and } \frac{q_\alpha}{q_p} = 2$$

$$\Rightarrow \quad \frac{\lambda_\alpha}{\lambda_p} = \frac{1}{2}$$

ILLUSTRATION 5

What amount of energy should be added to an electron to decrease its de Broglie wavelength from 100 pm to 50 pm ?

SOLUTION

According to de Broglie relation, initial momentum of the electron is

$$p_1 = \frac{h}{\lambda_1} = \frac{6.63\times 10^{-34}}{10^{-10}}$$

$$\Rightarrow \quad p_1 = 6.63\times 10^{-24} \text{ Js}$$

The final momentum of the electron is

$$p_2 = \frac{h}{\lambda_2} = \frac{6.63\times 10^{-34}}{0.5\times 10^{-10}}$$

$$\Rightarrow \quad p_2 = 13.26\times 10^{-24} \text{ Js}$$

Since kinetic energy of the electron is

$$E = \frac{p^2}{2m_e}$$

So, energy added to electron is

$$\Delta E = \frac{p_2^2 - p_1^2}{2m_e}$$

$$\Rightarrow \quad \Delta E = \frac{\left[(13.26)^2 - (6.63)^2\right]\times 10^{-48}}{2\times 9.1\times 10^{-31}\times 1.6\times 10^{-19}}$$

$$\Rightarrow \quad \Delta E \approx 453 \text{ eV}$$

ILLUSTRATION 6

Two identical nonrelativistic particles move at right angles to each other, possessing de Broglie wavelengths, λ_1 and λ_2. Find the de Broglie wavelength of each particle in the frame of their centre of mass.

SOLUTION

Initial momentum of each particle is

$$\vec{p}_1 = \left(\frac{h}{\lambda_1}\right)\hat{i} \text{ and } \vec{p}_2 = \left(\frac{h}{\lambda_2}\right)\hat{j}$$

$$\Rightarrow \quad \vec{v}_1 = \left(\frac{h}{m\lambda_1}\right)\hat{i} \text{ and } \vec{v}_2 = \left(\frac{h}{m\lambda_2}\right)\hat{j}$$

Velocity of center of mass is gives as

$$\vec{v}_{\text{cm}} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = \frac{h}{2m}\left(\frac{1}{\lambda_1}\hat{i} + \frac{1}{\lambda_2}\hat{j}\right)$$

Momentum of particles in frame of center of mass is

$$\vec{p}_{1c} = m(\vec{v}_1 - \vec{v}_{\text{cm}}) = \frac{h}{2}\left(\frac{1}{\lambda_1}\hat{i} - \frac{1}{\lambda_2}\hat{j}\right)$$

and $$\vec{p}_{2c} = m(\vec{v}_2 - \vec{v}_{\text{cm}}) = \frac{h}{2}\left(\frac{1}{\lambda_2}\hat{j} - \frac{1}{\lambda_1}\hat{i}\right)$$

$$|\vec{p}_{1c}| = |\vec{p}_{2c}| = \frac{h}{2}\frac{\sqrt{\lambda_1^2 + \lambda_2^2}}{\lambda_1\lambda_2}$$

de Broglie wavelength of particles in frame of their centre of mass is

$$\lambda_{1c} = \frac{h}{p_{1c}} = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}$$

and $$\lambda_{2c} = \frac{h}{p_{2c}} = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}$$

PROPERTIES OF PHOTONS

1. In the interaction of radiation with matter, the radiation behaves as if it is made up of particles called Photons. This fundamental is also called as Corpuscular Theory of Light. So, light behaves both as a particle and a wave.
2. All photons emitted by any source travel through free space with a speed equal to the speed of light i.e. $c = 3\times 10^8 \text{ ms}^{-1}$.
3. Each photon has a definite energy depending upon the frequency ν of the radiation and this energy is independent of the intensity. So,

$$E = h\nu = \frac{hc}{\lambda} \; (\text{in joule})$$

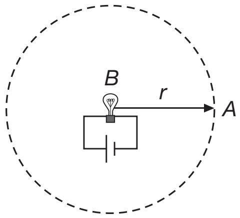
4. If λ is in Å, then $E = \dfrac{12375}{\lambda}$ eV.

5. If the intensity of the light of given wavelength is increased, then there is an increase in the number of photons incident per second per unit area on a surface. However, energy of the photon remains the same as long as the frequency or the wavelength is unchanged.
6. The speed of the photon changes as it travels through different media due to the change in its wavelength.
7. The frequency of the photon does not change when it goes from one medium to the other.
8. ***In the situations when a photon collides with a material particle, the total energy and momentum remains conserved. However, the number of photons may not be conserved in a collision because during the collision photon(s) may be absorbed or new photon(s) may be created.***
9. A photon is an electrically neutral particle which is not deflected by electric and magnetic field.
10. **Rest mass of photon is zero.**
Since the mass of a particle m moving with a speed v is given by
$$m=\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$
where m_0 is the rest mass of the particle
$$\Rightarrow \quad m_0=m\sqrt{1-\frac{v^2}{c^2}}$$
Now, for a photon, $v=c$, hence
$$m_0=0$$
So, from here we can conclude that the rest mass of a photon is always zero, i.e. we cannot have a frame of reference where the photon is at rest.
11. **Dynamic or kinetic mass of a photon,** is determined by using the Einstien's Mass-Energy Equivalence, i.e.,
$$E=h\nu=mc^2$$
$$\Rightarrow \quad m=\frac{h\nu}{c^2}=\frac{h}{c\lambda}$$
12. **The Linear Momentum of a photon** is found by using the de-Broglie relation according to which, we have
$$\lambda=\frac{h}{p}, \text{ where } \lambda=\frac{c}{\nu}$$
$$\Rightarrow \quad p=\frac{h\nu}{c}=\frac{h}{\lambda}$$
However, this result is also obtained by using the fact that the total energy of a subatomic particle of rest mass m_0, moving with a velocity v, having momentum p is given by
$$E^2=p^2c^2+m_0^2c^4$$
Now, for a photon, $m_0=0$, so we have from above expression that
$$E=pc$$
Since, $E=h\nu=\dfrac{hc}{\lambda}$
$$\Rightarrow \quad pc=\frac{hc}{\lambda}$$
$$\Rightarrow \quad p=\frac{h\nu}{c}=\frac{h}{\lambda}$$
13. The number of photons N, each of energy E, emitted from a source of monochromatic radiation of wavelength λ and energy W and power P
$$N=\frac{W}{E}=\frac{W}{h\nu}=\frac{Pt}{h\nu}$$
14. **Intensity of light (*I*)**
Energy crossing per unit area normally per second is called intensity or energy flux, i.e.
$$I=\frac{E}{At}=\frac{P}{A} \qquad \left(\frac{E}{t}=P=\text{radiation power}\right)$$
At a distance r from a point source of power P intensity is given by
$$I=\frac{P}{4\pi r^2}$$
$$\Rightarrow \quad I\propto\frac{1}{r^2}$$
15. **Number of photons falling per second (*n*)**
If P is the power of radiation and E is the energy of a photon then $n=\dfrac{P}{E}$.

NUMBER OF PHOTONS EMITTED PER SECOND BY A SOURCE

Consider a light bulb B having power P watt as shown in figure. If the wavelength of light emitted by the bulb is λ, then energy of each photon emitted by the bulb is

$$E_{\text{single photon}} = \frac{hc}{\lambda}$$



Since the power of bulb is P watt, so we can say that bulb is emitting light of energy P joule in one second in the form of photons (assuming the efficiency of the bulb to be 100%). Then the number of photons emitted per second (i.e. n) by the source (bulb) will be

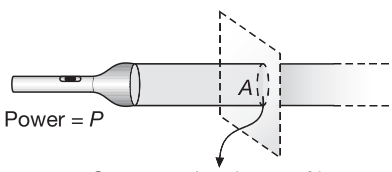
$$n = \frac{N}{t} = \frac{P}{E_{\text{single photon}}} = \frac{P\lambda}{hc}$$

where N is the total number of photons emitted by the source in time t.

These all photons are assumed to be emitted uniformly in all directions if bulb B is assumed to be a point isotropic source of light due to which we can consider that all the light energy emitted by the source is uniformly distributed in the spherical region with centre at the source i.e. bulb B. Also, it is observed that the radius of this sphere will uniformly increase at a rate of $c = 3 \times 10^8$ ms^{-1} in free space, because the light propagates in free space at a speed c.

INTENSITY OF LIGHT DUE TO A LIGHT SOURCE

Consider a torch that emits a uniform cylindrical beam of light having power P, cross-sectional area A as shown in figure.

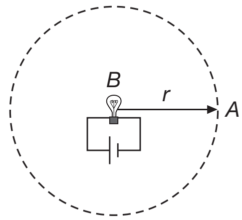


Since intensity (I) is defined as the energy emitted per second per unit area of the source normal to the direction of propagation of light, so we have

$$I = \frac{E_{\text{photon beam}}}{At} = \frac{P}{A}$$

Since the cross-sectional area of the beam is constant throughout, so the beam intensity at every point remains constant.

Similarly, we can find the intensity of light due to a point isotropic source. Consider a light bulb B having power P watt emitting light in all directions uniformly as shown in figure.



If we wish to find light intensity at a point A, at a distance r from the bulb B, then it can be given as

$$I = \frac{E_{\text{photon source}}}{At} = \frac{P}{4\pi r^2} \quad \ldots(1)$$

Here we have assumed that the entire power P of the spherical source of light is incident normally on the hypothetical spherical surface of radius r with bulb at the centre of the sphere.

PHOTON FLUX IN A LIGHT BEAM

Photon flux ϕ_N is defined as the number of photons incident on a surface per second per unit area of the surface held normally to the direction of propagation of the light beam. If a light beam of intensity I having wavelength λ is incident on a surface, then the number of photons per second per unit area (i.e. photon flux ϕ_N) is given by

$$\phi_N = \frac{N}{At} = \frac{1}{A}\left(\frac{N}{t}\right) = \frac{n}{A}$$

Since, we know that the number of photons emitted by the source per second is given by $n = \frac{P\lambda}{hc}$. So, we have

$$\phi_N = \frac{n}{A} = \frac{1}{A}\left(\frac{P\lambda}{hc}\right) = \left(\frac{P}{A}\right)\left(\frac{\lambda}{hc}\right) = \frac{I\lambda}{hc}$$

If we consider a point source of power P which emits light in all directions then the number of photons emitted per second by this point source is given by

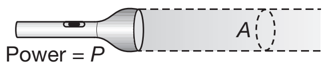
$$n = \frac{P\lambda}{hc}$$

Since these all photons are distributed in the three-dimensional spherical space around the source, so the photon flux at a distance r from the point source, is given by

$$\phi_N = \frac{n}{4\pi r^2}$$

PHOTON DENSITY IN A LIGHT BEAM

When photons are emitted by a light source, they move away from the source with speed of light. Consider a uniform cylindrical beam of light as shown in figure.



If a torch having a power P is producing a uniform light beam of cross-sectional area A, then the intensity of light beam is given by

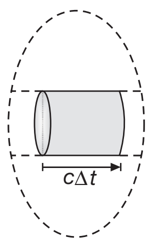
$$I = \frac{P}{A}$$

The photon flux at the cross-sectional area A of light beam is

$$\phi_N = \frac{I\lambda}{hc}$$

where λ is the wavelength of light.

The distance covered by these photons in time Δt is $c\Delta t$ and all the photons will lie in the shaded volume $A(c\Delta t)$ as shown in the figure.



Thus, photon density in the light beam is defined as the number of photons per unit volume.

$$\rho_{ph} = \frac{N}{A(c\Delta t)} = \left(\frac{N}{\Delta t}\right)\frac{1}{Ac} = \frac{1}{c}\left(\frac{n}{A}\right) = \frac{\phi_N}{c}$$

Since, $\phi_N = \frac{n}{A} = \frac{I\lambda}{hc}$

$$\Rightarrow \quad \rho_{ph} = \frac{\phi_N}{c} = \frac{I\lambda}{hc^2}$$

As the beam is uniform and cylindrical, the photon density throughout the beam remains, constant and at any point in space photon density can be given as

$$\rho_{ph} = \frac{\phi_N}{c} = \frac{\text{photon flux}}{\text{speed of light}}$$

Similarly, for a point isotropic source of light we can say that as the emitted photons move away from the source, the distance between photons increases and the photon density decreases.

If we wish to find photon density at a distance r from a point source of light of power P watt, then we first find the photon flux at a distance r from the source is given by

$$\phi_N = \frac{n}{A} = \frac{\left(\frac{P\lambda}{hc}\right)}{4\pi r^2}$$

So, the photon density at a distance r from the point source is given by

$$\rho_{ph} = \frac{\phi_N}{c} = \frac{P\lambda}{4\pi r^2 hc^2}$$

ILLUSTRATION 7

Find the energy, the mass and the momentum of a photon of ultraviolet radiation of 280 nm wavelength.

SOLUTION

Given, $\lambda = 280 \times 10^{-9}$ m

Since, $E = \frac{hc}{\lambda}$

$$\Rightarrow \quad E = \frac{(4.316\times10^{-15}\text{ eV sec})(3\times10^{8}\text{ ms}^{-1})}{(280\times10^{-9}\text{ m})} = 4.6\text{ eV}$$

Mass of photon is $m = \frac{E}{c^2}$

$$\Rightarrow \quad m = \frac{4.6\times1.6\times10^{-19}}{(3\times10^{8})^2} = 8.2\times10^{-36}\text{ kg}$$

Momentum of a photon is

$$p = \frac{E}{c}$$

$$\Rightarrow \quad p = \frac{4.6\times1.6\times10^{-19}}{3\times10^{8}} = 2.45\times10^{-27}\text{ kg ms}^{-1}$$

ILLUSTRATION 8

The intensity of sunlight on the surface of earth is $1400\ \text{Wm}^{-2}$. Assuming the mean wavelength of sunlight to be $6000\ \text{Å}$, calculate

(a) the photon flux arriving at $1\ \text{m}^2$ area on earth perpendicular to light radiations, and
(b) the number of photons emitted from the sun per second assuming the average radius of Earth's orbit is 1.49×10^{11} m.

SOLUTION

(a) Energy of a photon

$$E=\frac{hc}{\lambda}=\frac{12400}{6000}=2.06\ \text{eV}=3.3\times0^{-19}\ \text{J}$$

$$\text{Photon flux }=\frac{IA}{E}=\frac{(1400)(1)}{3.3\times10^{-19}}=4.22\times10^{21}$$

(b) $n=\frac{P}{E}=\frac{I\left(4\pi R^2\right)}{E}$

$$\Rightarrow\quad n=\frac{(1400)(4\pi)(1.49\times10^{11})^2}{3.3\times10^{-19}}=1.18\times10^{45}$$

ILLUSTRATION 9

A small plate of a metal is placed at a distance of 2 m from a monochromatic light source of wavelength 4.8×10^{-7} m and power 1 watt. The light falls normally on the plate. Find the number of photons striking the metal plate per square meter per second.

SOLUTION

$$E=\frac{hc}{\lambda}=\frac{(6.6\times10^{-34})(3\times10^{8})}{(4.8\times10^{-7})}=4.125\times10^{-19}\ \text{J}$$

Number of photons striking the metal plate per square meter per second is

$$n=\left(\frac{P}{E}\right)\left(\frac{1}{4\pi r^2}\right)$$

$$\Rightarrow\quad n=\left(\frac{1}{4.125\times10^{-19}}\right)\frac{1}{(4\pi)(2)^2}=4.82\times10^{16}\ \text{m}^{-2}\text{s}^{-1}$$

ILLUSTRATION 10

Find the number of photons entering the pupil of our eye per second corresponding to the minimum intensity of white light that we humans can perceive $(-10^{-10}\ \text{Wm}^{-2})$. Take the area of the pupil to be about $0.4\ \text{cm}^2$ and the average frequency of white light to be about 6×10^{14} Hz.

SOLUTION

Minimum intensity, $I=10^{-10}\ \text{Wm}^{-2}$

Area of pupil, $A=0.4\ \text{cm}^2=0.4\times10^{-4}$ m

Average frequency, $\nu=6\times10^{-14}$ Hz

Energy of one photon

$$E=h\nu=6.63\times10^{-34}\times6\times10^{14}\ \text{J}=4\times10^{-19}\ \text{J}$$

Let n be the number of photons crossing per square metre area per second.
Now Intensity = Energy incident per square metre area per second

$\Rightarrow\quad I=$ Total Energy of n photons

$\Rightarrow\quad I=n\times$ Energy of one photon

$$\Rightarrow\quad n=\frac{\text{Intensity}}{\text{Energy of one photon}}$$

$$\Rightarrow\quad I=\frac{10^{-10}\ \text{Wm}^{-2}}{4\times10^{-19}\ \text{J}}=2.5\times10^{8}\ \text{m}^{-2}\ \text{s}^{-1}$$

So, the number of photons entering the pupil of our eye per second is

$$N=n(\text{Area of the pupil})$$

$$\Rightarrow\quad N=2.5\times10^{8}\times0.4\times10^{-4}\ \text{s}^{-1}=10^{4}\ \text{s}^{-1}$$

ILLUSTRATION 11

Find the number of photons emitted per second by a 25 W source of monochromatic light of wavelength 6600 Å. What is the photoelectric current assuming 3% efficiency for photoelectric effect? Given $h=6.6\times10^{-34}$ Js.

SOLUTION

Energy of each photon is

$$E=\frac{hc}{\lambda}=\frac{6.6\times10^{-34}\times3\times10^{8}}{6600\times10^{-10}}=3\times10^{-19}\ \text{J}$$

Total energy emitted per second by 25 W source is

$$E=25\ \text{J}$$

Number of photons emitted per second is

$$n=\frac{25}{3\times10^{-19}}=8.33\times10^{19}$$

Photoelectric current (I) is

$$I=\begin{pmatrix}\text{3\% of photons}\\\text{emitted per second}\end{pmatrix}\times\begin{pmatrix}\text{charge on}\\\text{electron}\end{pmatrix}$$

$$\Rightarrow \quad I=\frac{3}{100}\times\frac{25}{3\times10^{-19}}\times1.6\times10^{-19}=0.4\text{ A}$$

ILLUSTRATION 12

A source emits monochromatic light of frequency 5.5×10^{14} Hz at a rate of 0.1 W. Of the photons given out, 0.15% fall on the cathode of a photocell which gives a current of 6 μA in an external circuit.

(a) Find the energy of a photon.

(b) Find the number of photons leaving the source per second.

(c) Find the percentage of the photons falling on the cathode which produce photoelectrons.

SOLUTION

(a) Since,

$$E=h\nu=(6.6\times10^{-34})(5.5\times10^{14})=36.3\times10^{-20}\text{ J}$$

$$\Rightarrow \quad E=2.27\text{ eV}$$

(b) Number of photons leaving the source per second is

$$n=\frac{P}{E}=\frac{0.1}{36.3\times10^{-20}}=2.75\times10^{17}$$

(c) Number of photons falling on cathode per second is

$$n_1=\frac{0.15}{100}\times2.75\times10^{17}=4.125\times10^{14}$$

Number of photoelectrons emitting per second is

$$n_2=\frac{6\times10^{-6}}{1.6\times10^{-19}}=3.75\times10^{13}$$

So, $\begin{pmatrix}\text{\%age of Photons}\\\text{falling on Cathode}\end{pmatrix}=\frac{n_2}{n_1}\times100$

$$\Rightarrow \quad \begin{pmatrix}\text{\%age of Photons}\\\text{falling on Cathode}\end{pmatrix}=\frac{3.75\times10^{13}}{4.125\times10^{14}}\times100=9\%$$

ILLUSTRATION 13

A cylindrical rod of some laser material 5×10^{-2} m long and 10^{-2} m in diameter contains 2×10^{25} ions per m^3. If on excitation all the ions are in the upper energy level and de-excite simultaneously emitting photons in the same direction, calculate the maximum energy contained in a pulse of radiation of wavelength 6.6×10^{-7} m. If the pulse lasts for 10^{-7} second, calculate the average power of the laser during the pulse.

SOLUTION

Total number of ions in the rod is

$$N=\begin{pmatrix}\text{Number of ions}\\\text{per unit volume}\end{pmatrix}\times\begin{pmatrix}\text{Volume of}\\\text{the rod}\end{pmatrix}$$

$$\Rightarrow \quad N=(2\times10^{25}\text{ m}^{-3})\times(3.14\times(0.005)^2\times5\times10^{-2}\text{ m}^3)$$

$$\Rightarrow \quad N=7.85\times10^{19}$$

As all the ions de-excite simultaneously, the number of photons emitted in the same direction is also 7.85×10^{19}.

So, the energy contained in a pulse of radiation of wavelength 6.6×10^{-7} m is

$$E=n\left(\frac{hc}{\lambda}\right)$$

$$\Rightarrow \quad E=\frac{hc}{\lambda}\times7.85\times10^{19}$$

$$\Rightarrow \quad E=\frac{6.6\times10^{-34}\times3\times10^{8}}{6.6\times10^{-7}}=23.55\text{ J}$$

Average power $P=\frac{\text{Energy}}{\text{Time}}$

$$P=\frac{23.55\text{ J}}{10^{-7}\text{ s}}=23.55\times10^{7}\text{ W}=235.5\text{ MW}$$

ILLUSTRATION 14

A 100 watt sodium lamp is radiating light of wavelength 5890 Å, uniformly in all directions,

(i) At what rate, photons are emitted from the lamp?

(ii) At what distance from the lamp the average flux is 1 photon/cm^2s ?

(iii) At what distance the average density is 1 photon/cm^3 ?

(iv) What are the photon flux and photon density at 2 m from the lamp?

SOLUTION

(i) The energy of each photon is given by

$$E=h\nu=\frac{hc}{\lambda}=\frac{12375}{5890}=2.1\text{ eV}$$

$$\Rightarrow \quad E=3.376\times10^{-25}\text{ J}$$

Since the lamp is emitting energy at the rate of $100\ \text{Js}^{-1}$ i.e. $P = 100\ \text{W}$. Hence the number of photons emitted per second n is given by

$$n = \frac{100}{3.376 \times 10^{-25}} \approx 3 \times 10^{20}\ \text{photons/sec}$$

(ii) If we consider the lamp to be a point source of light, then at a distance r from the lamp, the light energy is uniformly distributed over the surface of sphere of radius r. So, photon flux at a distance r is given by

$$\phi_N = \frac{n}{4\pi r^2}$$

For photon flux to have a value of 1 photon/cm^{2}s, we have

$$1 = \frac{n}{4\pi r^2}$$

$$\Rightarrow \quad r = \sqrt{\frac{n}{4\pi}}$$

$$\Rightarrow \quad r = \sqrt{\frac{3 \times 10^{20}}{4 \times 3.14}}\ \text{cm} = 4.9 \times 10^9\ \text{cm}$$

$$\Rightarrow \quad r = 4.9 \times 10^4\ \text{km}$$

So, at this distance, on the average, one photon will cross through $1\ \text{cm}^2$ area normal to radial direction.

(iii) Since, we know that the photon density is given by

$$\rho_{\text{ph}} = \frac{1}{c}\left(\frac{n}{A}\right) = \frac{\phi_N}{c}$$

So, the photon density at a distance r from the point source is given by

$$\rho_{\text{ph}} = \frac{1}{c}\left(\frac{n}{A}\right) = \frac{n}{4\pi r^2 c}$$

For $\rho = 1\ \text{photon/cm}^3 = 10^6\ \text{photons/m}^3$, we get

$$r = \sqrt{\frac{N}{4\pi \rho_{\text{ph}} c}} = \sqrt{\frac{3 \times 10^{20}}{4 \times 3.14 \times 10^6 \times 10^8}}$$

$$\Rightarrow \quad r = 282\ \text{m}$$

Thus, at a distance of $282\ \text{m}$ from $100\ \text{W}$ lamp, there is on the average only $1\ \text{photon/cm}^3$ at any moment.

(iv) Since photon flux is $\phi_N = \dfrac{n}{4\pi r^2}$, so at $r = 2\ \text{m}$ we calculate the value of photon flux as

$$\phi_N = \frac{3 \times 10^{20}}{4\pi (200\ \text{cm})^2}$$

$$\Rightarrow \quad \phi_N \approx 6 \times 10^{18}\ \text{photons/m}^2$$

Average density of photons at $r = 2$ m is given by

$$\rho_{\text{ph}} = \frac{3 \times 10^{20}}{4 \times 3.14 \times (200)^2 \times (3 \times 10^{10})}$$

$$\Rightarrow \quad \rho_{\text{ph}} = 2 \times 10^{10}\ \text{photons/m}^3$$

MOMENTUM OF A PHOTON

According to relativistic theory, the total relativistic energy E of a particle having rest mass m_0, momentum p is

$$E^2 = p^2c^2 + m_0^2c^4$$

For a photon, its rest mass is zero i.e. $m_0 = 0$, so we get

$$E = pc = h\nu = \frac{hc}{\lambda}$$

where λ is the wavelength of the photon

So, an electromagnetic wave consists of photons capable of transporting linear momentum. The linear momentum p possessed by an electromagnetic wave is related to the energy E it transports according to the relation

$$p = \frac{E}{c} = \frac{h}{\lambda}$$

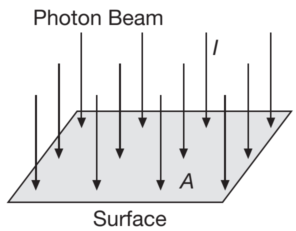
i.e. a photon is a particle with zero rest mass but finite momentum.

FORCE AND RADIATION PRESSURE DUE TO A PHOTON BEAM INCIDENT NORMALLY ON A SURFACE

Since we know that each photon has a definite energy and a definite linear momentum. Every photon in the light beam of a particular wavelength λ has the same energy i.e. $\dfrac{hc}{\lambda}$ and the same momentum $\dfrac{h}{\lambda}$. When light of intensity I falls on a surface, it exerts force on that surface. This is because, the incident light may be reflected (partially or completely) due to which there will be a change in its momentum and hence a force is exerted on the surface. Assuming that no transmission takes place from the surface, reflection coefficient of surface to be r, absorption coefficient of the surface to be a, then we have

$$a + r = 1$$

Let us assume that light beam of intensity I, wavelength λ, energy E, having N photons, be incident normally on the surface having area A as shown in figure.



Each incident photon of the light beam has an energy given by $E_{\text{each photon}} = \frac{hc}{\lambda}$ and momentum $p = \frac{h}{\lambda}$. Energy (E) of the photon beam is related to energy of each photon as

$$E_{\text{photon beam}} = E = N\left(E_{\text{each photon}}\right) = N\left(\frac{hc}{\lambda}\right)$$

For calculating the force exerted by beam on surface, we consider following cases.

For Perfectly Absorbing Surface

Let the surface on which light beam is incident be perfectly absorbing, then we have $r = 0$ and $a = 1$.

So, initial momentum of the photon is $p_i = \frac{h}{\lambda}$ and since the photon gets completely absorbed by the surface, so final momentum of the photon is $p_f = 0$. Taking the direction of incidence of photon as positive, the change in momentum of each incident photon is

$$\Delta p_{\text{each photon}} = \left|0 - \frac{h}{\lambda}\right| = \frac{h}{\lambda} \quad \text{(upwards)}$$

So, total change in momentum of the incident photon beam having N photons is

$$\Delta p = N\left(\Delta p_{\text{each photon}}\right) = N\left(\frac{h}{\lambda}\right)$$

$$\Rightarrow \quad \Delta p = \frac{N}{c}\left(\frac{hc}{\lambda}\right) = \left(\frac{N}{c}\right)E_{\text{each photon}} = \frac{NE_{\text{each photon}}}{c} \quad \ldots(1)$$

Since we know that the photon beam has energy E, so

$$E = N\left(E_{\text{each photon}}\right) = N\left(\frac{hc}{\lambda}\right) \quad \ldots(2)$$

So, from equation (1) and (2), we get

$$\Delta p = \frac{E}{c} \quad \ldots(3)$$

Since, by definition, we know that the intensity I of the light beam is the energy incident per second per unit area of a surface held normally to the direction of propagation of light, so we have

$$I = \frac{E}{A\Delta t} = \frac{P}{A}$$

where P is the power of the light source.

$$\Rightarrow \quad E = IA\Delta t \quad \ldots(4)$$

Substituting the value of equation (4) in equation (3), we get

$$\Delta p = \left(\frac{IA}{c}\right)\Delta t$$

$$\Rightarrow \quad F = \frac{\Delta p}{\Delta t} = \frac{IA}{c} = \frac{P}{c}$$

Hence the radiation pressure $\wp$ due to the light beam incident on a perfectly absorbing surface is

$$\wp = \frac{F}{A} = \frac{I}{c}$$

For Perfectly Reflecting Surface

Let the surface on which light beam is incident be perfectly reflecting, then we have $r = 1$ and $a = 0$.

So, initial momentum of the photon is $p_i = \frac{h}{\lambda}$ and since the photon gets completely reflected by the surface, so final momentum of the photon is $p_f = -\frac{h}{\lambda}$. Taking the direction of incidence of photon as positive, the change in momentum of each incident photon is

$$\Delta p_{\text{each photon}} = \left|-\frac{h}{\lambda} - \frac{h}{\lambda}\right| = \frac{2h}{\lambda} \quad \text{(upwards)}$$

So, total change in momentum of the incident photon beam having N photons is

$$\Delta p = N\left(\Delta p_{\text{each photon}}\right) = N\left(\frac{2h}{\lambda}\right)$$

$$\Rightarrow \quad \Delta p = \frac{N}{c}\left(\frac{2hc}{\lambda}\right) = \left(\frac{2N}{c}\right)E_{\text{each photon}}$$

$$\Rightarrow \quad \Delta p = \frac{2\left(NE_{\text{each photon}}\right)}{c} \qquad \ldots(1)$$

Since we know that the photon beam has energy E, so

$$E = N\left(E_{\text{each photon}}\right) = N\left(\frac{hc}{\lambda}\right) \qquad \ldots(2)$$

So, from equation (1) and (2), we get

$$\Delta p = \frac{2E}{c} \qquad \ldots(3)$$

Since, by definition, we know that the intensity I of the light beam is the energy incident per second per unit area of a surface held normally to the direction of propagation of light, so we have

$$I = \frac{E}{A\Delta t} = \frac{P}{A}$$

where P is the power of the light source.

$$\Rightarrow \quad E = IA\Delta t \qquad \ldots(4)$$

Substituting the value of equation (4) in equation (3), we get

$$\Delta p = \left(\frac{2IA}{c}\right)\Delta t$$

$$\Rightarrow \quad F = \frac{\Delta p}{\Delta t} = \frac{2IA}{c} = \frac{2P}{c}$$

Hence the radiation pressure due to the light beam incident on a perfectly absorbing surface is

$$\wp = \frac{F}{A} = \frac{2I}{c}$$

For Partially Reflecting Surface

For partially reflecting or partially absorbing surface, some fraction of the incident light is reflected and some fraction of it is absorbed such that $a + r = 1$.
In this case, we have

$$F = F_{\text{absorbed}} + F_{\text{reflected}} = a\left(\frac{IA}{c}\right) + r\left(\frac{2IA}{c}\right)$$

$$\Rightarrow \quad F = a\left(\frac{IA}{c}\right) + r\left(\frac{2IA}{c}\right) = \frac{IA}{c}(a+2r) = \frac{IA}{c}(a+r+r)$$

Since $a + r = 1$, so we get

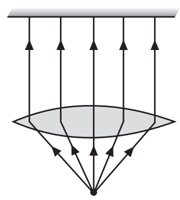
$$F = \frac{IA}{c}(1+r) = \frac{P}{c}(1+r)$$

Hence the radiation pressure due to the light beam incident on a partially reflecting surface having reflection coefficient r is

$$\wp = \frac{F}{A} = \frac{I}{c}(1+r)$$

ILLUSTRATION 15

A totally reflecting, small plane mirror placed horizontally faces a parallel beam of light as shown in the figure. The mass of the mirror is $20\ \text{g}$. Assume that there is no absorption in the lens and that 30% of the light emitted by the source goes through the lens. Calculate the power of the source needed to support the weight of the mirror.



SOLUTION

Since 30% of the total light is incident on the mirror and the same amount of light is reflected by the mirror, so the force due to reflection of light on mirror is given by

$$F = 2\left(\frac{0.3P}{c}\right) = \frac{0.6P}{c}$$

To support the weight of mirror, this force must balance the weight of the mirror i.e.

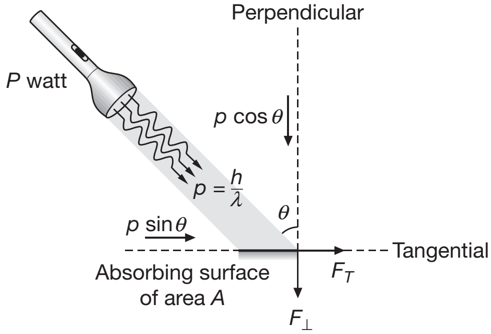
$$\frac{0.6P}{c} = mg$$

$$\Rightarrow \quad P = \frac{mgc}{0.6} = \frac{20\times10^{-3}\times10\times3\times10^{8}}{0.6}$$

$$\Rightarrow \quad P = 10^{8}\ \text{watt}$$

FORCE AND RADIATION PRESSURE DUE TO A PHOTON BEAM INCIDENT ON A PERFECTLY ABSORBING SURFACE AT AN ANGLE OF INCIDENCE

Let a light beam of intensity I, wavelength λ, energy E, power P, having N photons, be incident on the surface having area A at an angle of incidence θ as shown in figure.



Each incident photon of the light beam has an energy given by $E_{\text{each photon}} = \dfrac{hc}{\lambda}$ and momentum $p = \dfrac{h}{\lambda}$. Energy (E) of the photon beam is related to energy of each photon as

$$E_{\text{photon beam}} = E = N\left(E_{\text{each photon}}\right) = N\left(\frac{hc}{\lambda}\right)$$

Assuming that no transmission takes place from the surface, reflection coefficient of surface to be r, absorption coefficient of the surface to be a, then we have

$$a + r = 1$$

Since, the surface on which light beam is incident is perfectly absorbing, so we have $r = 0$ and $a = 1$.

So, initial momentum of the photon is $p_i = p = \dfrac{h}{\lambda}$

and since the photon gets completely absorbed by the surface, so final momentum of the photon is $p_f = 0$ (both tangentially and normally to the surface). Change in momentum of each incident photon is

$$\Delta p_{\text{each photon}} = \left|0 - \frac{h}{\lambda}\right| = \frac{h}{\lambda}$$

This change in momentum is in the direction opposite to the direction of the incident photon.

Also note that when the photon is completely absorbed by the surface, it suffers a change in momentum both along the normal (perpendicular) and the tangential (parallel) to the surface.

$$(\Delta p)_{\text{normal}} = (\Delta p)_{\perp} = \left|0 - \frac{h}{\lambda}\cos\theta\right| = \frac{h}{\lambda}\cos\theta$$

$$(\Delta p)_{\text{tangential}} = (\Delta p)_{T} = \left|0 - \frac{h}{\lambda}\sin\theta\right| = \frac{h}{\lambda}\sin\theta$$

Change in momentum of each incident photon is

$$\Delta p_{\text{each photon}} = \left|0 - \frac{h}{\lambda}\right| = \frac{h}{\lambda}$$

So, change in momentum of the incident photon beam (having N photons) normal/perpendicular to the surface is

$$\Delta p_{\perp} = N\left(\Delta p_{\text{each photon}}\right)_{\perp} = N\left(\frac{h}{\lambda}\cos\theta\right)$$

$$\Rightarrow \quad \Delta p_{\perp} = \frac{N}{c}\left(\frac{hc}{\lambda}\cos\theta\right)$$

$$\Rightarrow \quad (\Delta p)_{\perp} = \frac{1}{c}\left(NE_{\text{each photon}}\right)\cos\theta = \frac{E}{c}\cos\theta$$

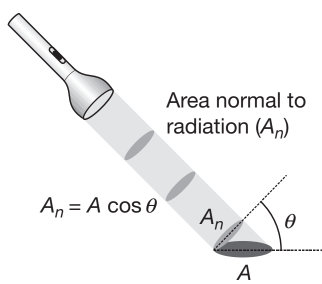
Also, change in momentum of the incident photon beam (having N photons) tangential/parallel to the surface is

$$\Delta p_{T} = N\left(\Delta p_{\text{each photon}}\right)_{T} = N\left(\frac{h}{\lambda}\sin\theta\right)$$

$$\Rightarrow \quad \Delta p_{T} = \frac{N}{c}\left(\frac{hc}{\lambda}\sin\theta\right)$$

$$\Rightarrow \quad (\Delta p)_{T} = \frac{1}{c}\left(NE_{\text{each photon}}\right)\sin\theta = \frac{E}{c}\sin\theta$$

Since, by definition, we know that the intensity I of the light beam is defined as the energy incident per second per unit area of a surface held normally to the direction of propagation of light.



So, we have,

$$I=\frac{E}{A_n\Delta t}=\frac{E}{(A\cos\theta)\Delta t}$$

$$\Rightarrow \quad E=(IA\Delta t)\cos\theta$$

So, we have, $(\Delta p)_\perp=\left(\frac{E}{c}\right)\cos\theta=\left(\frac{IA\Delta t\cos\theta}{c}\right)\cos\theta$

$$\Rightarrow \quad (\Delta p)_\perp=\left(\frac{IA}{c}\cos^2\theta\right)\Delta t$$

$$\Rightarrow \quad F_\perp=\frac{(\Delta p)_\perp}{\Delta t}=\left(\frac{IA}{c}\right)\cos^2\theta$$

Also, we see that,

$$(\Delta p)_T=\left(\frac{E}{c}\right)\sin\theta=\left(\frac{IA\Delta t\cos\theta}{c}\right)\sin\theta$$

$$\Rightarrow \quad F_T=\frac{(\Delta p)_T}{\Delta t}=\left(\frac{IA}{c}\right)\sin\theta\cos\theta$$

The net force F (due to the absorbed photon beam) acts on the surface in the direction of the incident photon beam and has a value

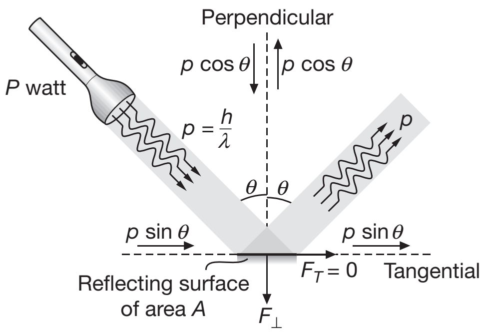
$$F=\sqrt{F_\perp^2+F_T^2}=\frac{IA}{c}\sqrt{\cos^4\theta+\sin^2\theta\cos^2\theta}=\frac{IA}{c}\cos\theta$$

Hence the radiation pressure $\wp_a$ due to the light beam incident on a perfectly absorbing surface is the normal force per unit area.

$$\Rightarrow \quad \wp_a=\frac{F_\perp}{A}=\left(\frac{I}{c}\right)\cos^2\theta$$

FORCE AND RADIATION PRESSURE DUE TO A PHOTON BEAM INCIDENT ON A PERFECTLY REFLECTING SURFACE AT AN ANGLE OF INCIDENCE

Let a light beam of intensity I, wavelength λ, energy E, power P, having N photons, be incident on the surface having area A at an angle of incidence θ as shown in figure.



Each incident photon of the light beam has an energy given by $E_{\text{each photon}}=\frac{hc}{\lambda}$ and momentum $p=\frac{h}{\lambda}$. Energy (E) of the photon beam is related to energy of each photon as

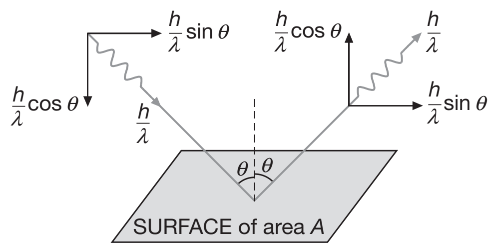
$$E_{\text{photon beam}}=E=N\left(E_{\text{each photon}}\right)=N\left(\frac{hc}{\lambda}\right)$$

Assuming that no transmission takes place from the surface, reflection coefficient of surface to be r, absorption coefficient of the surface to be a, then we have

$$a+r=1$$

Since the surface is perfectly reflecting, so we have $r=1$ and $a=0$.

So, initial momentum of the photon is $p_i=\frac{h}{\lambda}$ and since the photon gets completely reflected by the surface, so final momentum of the photon is $p_f=\frac{h}{\lambda}$.



Change in momentum of each incident photon normal to the surface is

$$(\Delta p_\perp)_{\text{each photon}}=\left|-\frac{h}{\lambda}\cos\theta-\frac{h}{\lambda}\cos\theta\right|=\frac{2h}{\lambda}\cos\theta \text{ and}$$

$$(\Delta p_T)_{\text{each photon}}=\left|\frac{h}{\lambda}\sin\theta-\frac{h}{\lambda}\sin\theta\right|=0$$

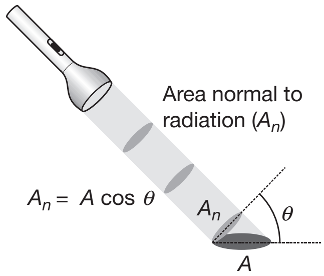
So, total change in momentum of the incident photon beam having N photons is

$$\Delta p_\perp=N\left(\Delta p_{\text{each photon}}\right)=N\left(\frac{2h}{\lambda}\cos\theta\right)$$

$$\Rightarrow \quad \Delta p_\perp=\frac{N}{c}\left(\frac{2hc}{\lambda}\cos\theta\right)=\frac{2E}{c}\cos\theta$$

$$\Rightarrow \quad (\Delta p)_\perp=\left(\frac{2E}{c}\right)\cos\theta\left\{\because E_{\text{photon beam}}=E=N\left(\frac{hc}{\lambda}\right)\right\}$$

Since, by definition, we know that the intensity I of the light beam is the energy incident per second per unit area of a surface held normally to the direction of propagation of light.



So, we have,

$$I = \frac{E}{A_n \Delta t} = \frac{E}{(A\cos\theta)\Delta t}$$

$$\Rightarrow \quad E = (IA\Delta t)\cos\theta$$

$$\Rightarrow \quad (\Delta p)_{\perp} = \left(\frac{2E}{c}\right)\cos\theta = \left(\frac{2IA}{c}\cos^2\theta\right)\Delta t$$

$$\Rightarrow \quad F_{\perp} = \frac{(\Delta p)_{\perp}}{\Delta t} = \left(\frac{2IA}{c}\right)\cos^2\theta$$

However, the force tangential to the surface is

$$F_T = \frac{(\Delta p)_T}{\Delta t} = 0$$

The net force F (due to the reflected photon beam) acts on the surface in the vertically downward direction or in the direction perpendicular to the surface and has a value given by

$$F = \sqrt{F_{\perp}^2 + F_T^2} = \frac{2IA}{c}\sqrt{\cos^4\theta + 0} = \left(\frac{2IA}{c}\right)\cos^2\theta$$

Hence the radiation pressure $\wp_a$ due to the light beam incident on a perfectly absorbing surface is the normal force per unit area.

$$\Rightarrow \quad \wp_a = \frac{F_{\perp}}{A} = \left(\frac{2I}{c}\right)\cos^2\theta$$

FORCE AND RADIATION PRESSURE DUE TO A PHOTON BEAM INCIDENT ON A PARTIALLY REFLECTING SURFACE AT AN ANGLE OF INCIDENCE

For partially reflecting or partially absorbing surface, some fraction of the incident light is reflected and some fraction of it is absorbed such that $a + r = 1$.

In this case, we have to do the calculation very carefully as the direction of the force on the surface is different for absorbed and reflected parts of the incident radiation as discussed earlier.

Change in momentum of the absorbed photon is $\frac{h}{\lambda}$, in the direction opposite to the incident beam.

Change in momentum of the reflected photon is $\frac{2h}{\lambda}\cos\theta$, vertically downwards.

Force on the plate due to the absorbed photons is along the direction of the incident beam i.e. making an angle θ with the vertical and has a value given by

$$F_{\text{absorbed}} = F_a = a\left(\frac{IA}{c}\right)\cos\theta = (1-r)\left(\frac{IA}{c}\right)\cos\theta$$

Force on the plate due to the reflected photons is vertically downwards and has a value given by

$$F_{\text{reflected}} = F_r = r\left(\frac{2IA}{c}\right)\cos^2\theta$$

Both these forces, F_r and F_a are inclined to each other at an angle θ. So, the resultant force F is given by

$$F = \sqrt{F_a^2 + F_r^2 + 2F_rF_a\cos\theta}$$

$$\Rightarrow \quad F = \left(\frac{IA\cos\theta}{c}\right)\sqrt{(1-r)^2 + 4r^2\cos^2\theta + 4r(r-1)\cos^2\theta}$$

Pressure on the surface due to the radiation is given by

$$P = \frac{F_a\cos\theta + F_r}{A}$$

$$\Rightarrow \quad P = \frac{F_a}{A}\cos\theta + \frac{F_r}{A} = \frac{I}{c}(1-r)\cos^2\theta + \frac{I}{c}(2r)\cos^2\theta$$

$$\Rightarrow \quad P = \frac{I}{c}\cos^2\theta(1-r+2r)$$

$$\Rightarrow \quad P = \frac{I}{c}\cos^2\theta(1+r)$$

ILLUSTRATION 16

A light of intensity $2\ \text{kWm}^{-2}$ falls on a plane mirror of reflective power $r = 0.8$ at an angle of incidence is $30°$. Calculate the pressure exerted by the light on the mirror.

SOLUTION

Since, we know that the radiation pressure P in this case is given by the relation

$$P = \frac{I}{c}(1+r)\cos^2\theta$$

$$\Rightarrow \quad P=\frac{2000}{3\times10^8}(1+0.8)\cos^2(30°)=\frac{2000}{3\times10^8}(1.8)\left(\frac{3}{4}\right)$$

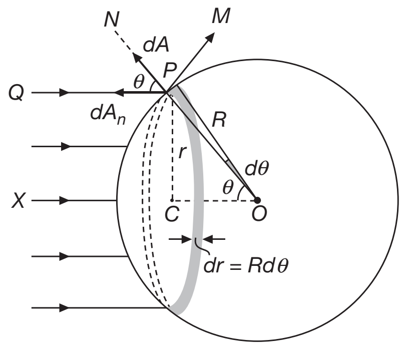
$$\Rightarrow \quad P=9\times10^{-6}\ \text{Nm}^{-2}$$

ILLUSTRATION 17

A perfectly reflecting solid sphere of radius R is placed in the path of a uniform beam of large cross-sectional area and intensity I. Calculate the force exerted on this sphere due to the light beam. Also calculate the force on the sphere if it would have been perfectly absorbing. Support the result obtained by an analytical argument.

SOLUTION

Let O be the centre of the sphere and QP be the light incident on the sphere of radius R. Let I be the intensity of the incident light. Consider a circular element of radius r, thickness dr, subtending an angle $d\theta$ at the centre of the sphere as shown in figure.



Then we observe that $r=R\sin\theta$ and $dr=Rd\theta$. If dA be the area of this circular element then

$$dA=(2\pi r)dr=2\pi(R\sin\theta)(Rd\theta)=2\pi R^2\sin\theta d\theta$$

Since intensity I is defined as the energy incident per second per unit area of a surface held normal to the direction of propagation of light, hence

$$I=\frac{dE}{(dA_n)(dt)}$$

where, $dA_n=dA\cos\theta$ is the area of the element normal to the direction of propagation of light.

$$\Rightarrow \quad I=\frac{dE}{(dA\cos\theta)(dt)}$$

So, the energy of the light falling on this element of area dA in time dt is

$$dE=Idt(dA\cos\theta)$$

The momentum (dp) of light falling on this element of area dA is $\frac{dE}{c}$ along QP and since light incident at P on this element is reflected by the sphere along PM. So, the change in momentum of the photon beam incident on the element is directed along PN (or OP) and is given by

$$dp=2\left(\frac{dE}{c}\right)\cos\theta=\frac{2}{c}(Idt)(dA\cos^2\theta)$$

The force (f) on dA due to the light falling on it is directed along PQ is given by

$$f=\frac{dp}{dt}=2\left(\frac{IdA}{c}\right)\cos^2\theta$$

It is observed that due to symmetry, the net force on the ring element as well as the sphere is directed along the line XO. The component of this force (directed along the line XO) acting on the element of area dA is

$$dF=f\cos\theta=\left(2\left(\frac{IdA}{c}\right)\cos^2\theta\right)\cos\theta$$

$$\Rightarrow \quad dF=f\cos\theta=2\left(\frac{IdA}{c}\right)\cos^3\theta$$

$$\Rightarrow \quad dF=\left(\frac{2I}{c}\right)(2\pi R^2\sin\theta d\theta)\cos^3\theta$$

The force on the entire sphere is $F=\int dF$

$$\Rightarrow \quad F=\int_0^{\frac{\pi}{2}}\left(\frac{2I}{c}\right)(2\pi R^2\sin\theta d\theta)\cos^3\theta$$

$$\Rightarrow \quad F=\left(\frac{4\pi R^2 I}{c}\right)\int_0^{\frac{\pi}{2}}\cos^3\theta\sin\theta d\theta$$

$$\Rightarrow \quad F=-\left(\frac{4\pi R^2 I}{c}\right)\int_0^{\frac{\pi}{2}}\cos^3\theta d(\cos\theta)$$

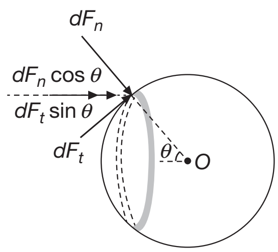
$$\Rightarrow \quad F=-\left(\frac{4\pi R^2 I}{c}\right)\left(\frac{\cos^4\theta}{4}\right)\Bigg|_0^{\frac{\pi}{2}}=-\frac{4\pi R^2 I}{c}\left(0-\frac{1}{4}\right)$$

$$\Rightarrow \quad F=\frac{\pi R^2 I}{c}$$

However, when the sphere is perfectly absorbing, then the normal force (dF_n) and the tangential force (dF_t) on the infinitesimal element of area $dA = 2\pi R^2 \sin\theta d\theta$ are given by

$$dF_n = \left(\frac{I}{c}\cos^2\theta\right)dA \text{ and } dF_t = \left(\frac{I}{c}\sin\theta\cos\theta\right)dA.$$

The net force F on the perfectly absorbing sphere acts along the direction of incident light beam as shown in figure.



$$\Rightarrow \quad F = \int dF = \int_0^{\frac{\pi}{2}} (dF_n\cos\theta + dF_t\sin\theta)$$

$$\Rightarrow \quad F = \left(\frac{I}{c}\right)\int_0^{\frac{\pi}{2}}\left(\cos^3\theta + \sin^2\theta\cos\theta\right)\left(2\pi R^2\sin\theta d\theta\right)$$

$$\Rightarrow \quad F = \left(\frac{I}{c}\right)\int_0^{\frac{\pi}{2}}\cos\theta\left(\cos^2\theta + \sin^2\theta\right)\left(2\pi R^2\sin\theta d\theta\right)$$

$$\Rightarrow \quad F = \left(\frac{\pi R^2 I}{c}\right)\int_0^{\frac{\pi}{2}} 2\sin\theta\cos\theta d\theta = \left(\frac{\pi R^2 I}{c}\right)\int_0^{\frac{\pi}{2}}\sin(2\theta)d\theta$$

$$\Rightarrow \quad F = \left(\frac{\pi R^2 I}{c}\right)\left(-\frac{\cos(2\theta)}{2}\Bigg|_0^{\frac{\pi}{2}}\right)$$

$$\Rightarrow \quad F = -\left(\frac{\pi R^2 I}{2c}\right)(\cos\pi - \cos 0)$$

$$\Rightarrow \quad F = -\left(\frac{\pi R^2 I}{2c}\right)(-1-1) = -\left(\frac{\pi R^2 I}{2c}\right)(-2) = \frac{\pi R^2 I}{c}$$

It is observed that, for a sphere placed in the path of a light beam the force exerted on sphere is independent of the nature of the surface of the sphere. This is because, the perfectly reflecting sphere reflects the incoming radiation in the backward sense for $\theta < \frac{\pi}{4}$ but in the forward sense for $\frac{\pi}{4} < \theta < \frac{\pi}{2}$. Due to this, the net momentum of the reflected radiation turns out to be zero and hence the net momentum exchange between the photon beam and the sphere is same in both the cases.

Conceptual Note(s)

For a perfectly absorbing sphere, the wave momentum is completely transferred to the sphere. The easiest approach in this case is to observe that the sphere presents the cross-sectional area πR^2 to the incoming radiation, and therefore the total force is simply given by

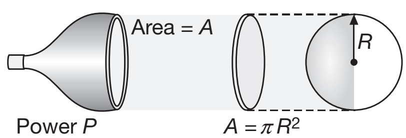
$$F = \frac{IA_{\text{effectice}}}{c} = \frac{I\left(\pi R^2\right)}{c}$$

which is the same result as the perfectly reflecting sphere.

FORCE EXERTED ON ANY OBJECT IN THE PATH OF A LIGHT BEAM

Consider a light source such as a lamp of power P watt. Let this lamp produce a uniform parallel beam of light of cross-sectional area A which is incident on a sphere of radius R as shown in figure. If I be the intensity of this light beam emitted by the lamp. Then

$$I = \frac{P}{A}$$



In this case, only those photons will be incident on the sphere which pass through the cross-sectional area $A = \pi R^2$ which is the projection of sphere on a cross-sectional plane. This area is also called as projected area. Thus, the power incident on sphere is

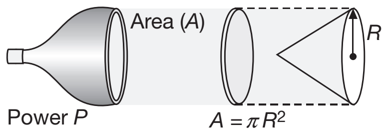
$$P_i = IA_{\text{Projected}} = IA = I\left(\pi R^2\right)$$

So, force exerted on sphere will be

$$F = \frac{P_i}{c} = \frac{I\left(\pi R^2\right)}{c}$$

This result will remain the same for perfectly absorbing as well as perfectly reflecting sphere.

Similarly, when a perfectly absorbing cone is placed in front of the light beam as shown in figure, then also the projection of cone along a cross-sectional plane of beam i.e. the projected area is $A = \pi R^2$.



Hence the force exerted on the perfectly absorbing cone due to the light beam will be

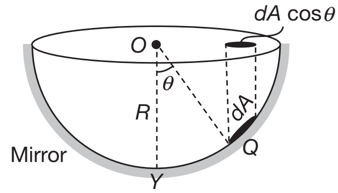
$$F = \frac{P}{c} = \frac{IA_{\text{Projected}}}{c} = \frac{I\left(\pi R^2\right)}{c}$$

ILLUSTRATION 18

A point source of light O is placed at the centre of curvature of a hemispherical perfectly reflecting surface radius of curvature R. Find the force on the hemisphere due to the light falling on it if the source emits a power P.

SOLUTION

The energy emitted by the source per unit time, i.e., P falls on an area $4\pi R^2$ at a distance R in unit time. Thus, the energy falling per unit area per unit time is $\frac{P}{\left(4\pi R^2\right)}$. Consider a small area dA at the point Q of the hemisphere as shown in figure.



The energy falling per unit time on it is $\frac{PdA}{4\pi R^2}$. The corresponding momentum incident on this area per unit time is $\frac{PdA}{4\pi R^2 c}$. As the light is reflected back, the change in momentum per unit time, i.e., the force on dA is

$$dF = \frac{2PdA}{4\pi R^2 c}$$

Suppose the radius OQ through the area dA makes an angle θ with the symmetry axis OY. The force on dA is along this radius. By symmetry, the resultant force on the hemisphere is along OY. The component of dF along OY is

$$dF\cos\theta = \frac{2PdA}{4\pi R^2 c}\cos\theta$$

If we project the area dA on the plane containing the rim, the projection is $dA\cos\theta$. Thus, the component of dF along OY is,

$$dF\cos\theta = \frac{2P}{4\pi R^2 c}\left(\text{Projection of } dA\right)$$

The net force along OY is

$$F = \frac{2P}{4\pi R^2 c}\left(\sum \text{Projection of } dA\right)$$

When all the small areas dA are projected, we get the area enclosed by the rim which is πR^2. Thus,

$$F = \left(\frac{2P}{4\pi R^2 c}\right)\left(\pi R^2\right) = \frac{P}{2c}$$

Test Your Concepts-I

Based on Photon Properties and De Broglie Phenomenon

(Solutions on page H.3)

1. The intensity of direct sunlight before it passes through the earth's atmosphere is 1.4 kWm^{-2}. If it is completely absorbed find the corresponding radiation pressure.
2. According to the Maxwell theory of electrodynamics an electron going in a circle should emit radiations of frequency equal to its frequency of revolution. What would be the wavelength of the radiation emitted by a hydrogen atom in ground state if this rule is followed?
3. An electron is accelerated by a potential difference of 25 volt. Find the de-Broglie wavelength associated with it.
4. Find the number of photons emitted per second by a *MW* transmitter of 10 kW power emitting radio waves of wavelength 500 m.
5. If 5% of the energy supplied to an incandescent light bulb is radiated as visible light, how many visible light photons are emitted by 100 watt bulb. Assume wavelength of all visible photons to be 5600 Å. Given $h = 6.625 \times 10^{-34}$ Js.
6. Calculate the number of photons in 6.62 J of radiation energy of frequency 10^{12} Hz. Given $h = 6.62 \times 10^{-34}$ Js.
7. Monochromatic light of frequency 6×10^{14} Hz is produced by a laser. The power emitted is 2×10^{-3} W.
   (a) What is the energy of each photon in the light?
   (b) How many photons per second, on the average, are emitted by the source?
8. Show that a free electron at rest cannot absorb a photon and thereby acquire kinetic energy equal to the energy of the photon. Would the conclusion change if the free electron was moving with a constant velocity?
9. An electron microscope uses electrons accelerated by a voltage of 50 kV. Determine the de-Broglie wavelength associated with the electrons. If other factors (such as numerical aperture etc.) are taken to be roughly the same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light?
10. An electron and proton are possessing the same amount of kinetic energy. Which of the two have greater wavelength?
11. An electron and a photon have same de Broglie wavelength (say 1 Å). Which one possesses more kinetic energy?
12. An electron and a proton have same wavelength. Which one possesses more energy?
13. It is desired to move a small space vehicle of mass 50 kg at rest, by a lamp of 100 Watt emitting blue light of wavelength 4700 Å. If the vehicle if in free space, calculate its acceleration. Assume all the emitted photons to be incident on the body of space vehicle.
14. With what velocity must an electron travel so that its momentum is equal to that of photon with a wavelength of $\lambda = 5200$ Å.
15. A parallel beam of monochromatic light of wavelength 496 nm is incident normally on a perfectly absorbing surface. The power through any cross-section of the beam is 10 W. Find (a) the number of photons absorbed per second by the surface and (b) the force exerted by the light beam on the surface. Take $hc = 1240$ eVnm.
16. A monochromatic source of light operating at 200 W emits 4×10^{20} photons per second. Find the wavelength of the light.
17. How many photons are emitted per second by a 5 mW laser source operating at 663 nm?
18. A hydrogen atom moving at a speed v absorbs a photon of wavelength 122 nm and stops. Find the value of v. Mass of a hydrogen atom = 1.67×10^{-27} kg

EMISSION OF ELECTRONS

As we are aware of the fact that metals have free electrons (negatively charged particles) which are responsible for their conductivity. However, the free electrons cannot normally escape out of the metal surface. If an electron attempts to come out of the metal, the metal surface acquires a positive charge and pulls the electron back to the metal. The free electron is thus held inside the metal surface by the attractive forces of the ions. Consequently, the electron can come out of the metal surface only if it is supplied some minimum energy to overcome the attractive pull of the metal.

This minimum energy required by an electron to escape from the metal surface is called the *work function* of the metal. It is generally denoted by ϕ_0 or sometimes W and is measured in eV (electron volt). One electron volt is the energy gained by an electron when it has been accelerated by a potential difference of 1 volt, so

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

This unit of energy is commonly used in atomic and nuclear physics.

The work function, ϕ_0 depends on the properties of the metal and the nature of its surface. The work function for platinum is the highest $(\phi_0)_{Pt} = 5.65 \text{ eV}$, whereas it is the lowest for caesium i.e., $(\phi_0)_{Cs} = 2.14 \text{ eV}$.

The minimum energy required for the electron emission from the metal surface can be supplied to the free electrons by any one of the following physical processes.

Thermionic Emission

By suitably heating a metal, sufficient thermal energy can be imparted to the free electrons to enable them to come out of the metal. The free electrons so emitted are called as ***Thermions.***

Field Emission

When the metal surface is subjected to very strong electric field of the order ranging from 10^3 Vm^{-1} to 10^8 Vm^{-1}, the electrons (beyond a certain limit) start coming out of the metal surface. This method of emission is dangerous and less efficient. This method of emission is also called the ***Cold Cathode Emission.***

Photo-electric Emission

When light of certain minimum energy (or minimum frequency or maximum wavelength) illuminates or falls on a metal surface, electrons are emitted from the metal surface. The emitted electrons are called ***photo-electrons.*** In case of Photoelectric emission, the rate of emission of photoelectrons is very low.

Secondary Emission

When fast moving electrons strike a metal surface, then some of their energy is transferred to the free electrons of the metal. Due to this, when free electrons gain energy more than the work function, then they are emitted from the metal surface. These emitted electrons are called the **secondary electrons.**

PHOTOELECTRIC EFFECT

The phenomenon of emission of electrons from a metallic surface by the use of light (or radiant) energy of certain minimum frequency (or maximum wavelength) is called **photoelectric effect**. The emitted electrons are called as **photoelectrons**. The phenomenon was discovered by **Hallwach** in 1888. For photoelectric emission the metal used must have **low work function** e.g., alkali metals. **Cesium** is assumed to be the best metal for photoelectric effect. To escape from the surface, the electron must absorb enough energy from the incident radiation to overcome the attraction of nucleus of the atom of the metal surface. The explanation to the photoelectric effect given by Einstein is based on the Law of Conservation of Energy. Before discussing the effect further, we must understand the following terms.

Work Function (or Threshold Energy)

The minimum energy of incident radiation, required to eject the electrons from metallic surface is defined as work function of that surface. It is the characteristic of a metal surface and is denoted by ϕ_0 or W.

$$\phi_0 = h\nu_0 = \frac{hc}{\lambda_0} \text{ (in joule), where}$$

$\nu_0 =$ Threshold frequency and

$\lambda_0 =$ Threshold wavelength

Work function in electron volt is given by

$$\phi_0\,(\text{eV}) = \frac{hc}{e\lambda_0} = \frac{12375}{\lambda_0\,(\text{in Å})} \approx \frac{12400s}{\lambda_0\,(\text{in Å})}$$

It is the minimum for Caesium. It is relatively less for alkali metals.

Work functions of some photosensitive metals

Metal	Work function (eV)	Metal	Work function
Caesium	1.9	Calcium	3.2
Potassium	2.2	Copper	4.5
Sodium	2.3	Silver	4.7
Lithium	2.5	Platinum	5.6

Threshold Frequency (ν_0)

The minimum frequency of incident radiations required to eject the electron from metal surface is defined as threshold frequency. If incident frequency $v < v_0$, then no photoelectron emission. For most metals the threshold frequency is in the ultraviolet (corresponding to wavelengths between 200 and 300 nm), but for potassium and caesium oxides it is in the visible spectrum i.e. λ between 400 and 700 nm.

Threshold Wavelength (λ_0)

The maximum wavelength of incident radiations required to eject the electrons from a metallic surface is defined as threshold wavelength. If incident wavelength $\lambda > \lambda_0$, then no photoelectron emission will take place.

SOME IMPORTANT TERMS

(a) Photoelectrons: The electrons emitted in the process of photoelectric effect are called photoelectrons.

(b) Photoelectric Current (i): If current flow in a circuit is due to photoelectric effect then that current due to the photoelectrons is called as photoelectric current.

(c) Stopping Potential (V_s or V_0): It is the minimum value of negative potential of anode or collector (with respect to cathode or emitter) for which photoelectric current is zero is called stopping potential. It can also be defined as that value of negative potential for which no photoelectron reaches the anode. This is also known as cut off voltage. This voltage is independent of intensity.

(d) Saturation Current: When all photoelectrons emitted by cathode reach the anode, then the current flowing in the circuit at that instant is called as saturated current. This is the maximum value of photoelectric current.

PLANK'S QUANTUM THEORY

The light energy from any source is always an integral multiple of a smaller energy value called quantum of light. Hence energy of a photon beam or a light sample is

$$E_{\text{photon beam}} = NE_{\text{each photon}} = N(h\nu),\ \text{where}$$

$N = 1, 2, 3, \ldots..$ is the number of photons in the photon beam or the light sample and $E_{\text{each photon}} = h\nu = \dfrac{hc}{\lambda}$.

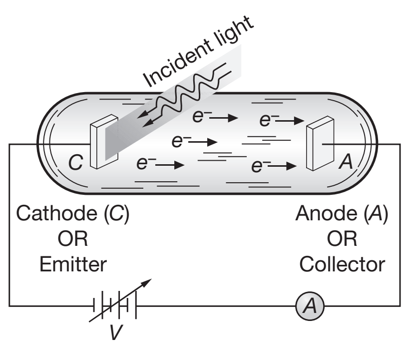
So, for a photon beam or a light sample, energy is quantized. Also, we can say that $h\nu$ is the quantum of energy and this small packet of energy is called as **photon**. This smallest energy is

$$E = h\nu = \frac{hc}{\lambda}$$

where $hc = 12375\ \text{eVÅ} \approx 12400\ \text{eVÅ}$

EXPERIMENTAL SETUP FOR PHOTOELECTRIC EFFECT

It consists of two conducting electrodes, the cathode (C) also called as emitter and anode (A) also called as collector which are enclosed in an evacuated glass tube as shown in figure.



The battery or some other source of potential difference creates an electric field in the direction from anode to cathode. Light of certain wavelength or frequency falling on the surface of cathode causes a current to flow in the external circuit. This current is called the **photoelectric current.**

When the potential difference increases, the photo electric current also increases till saturation is reached. As the polarity of battery is reversed (i.e. plate A is at negative potential w.r.t. plate C) the electrons start moving back towards the cathode. It is observed that at a particular negative potential of plate A, no electron reaches the plate A i.e. the current becomes zero. This negative potential for which the photo-electric current is zero is called the **stopping potential** denoted by V_0. Maximum kinetic energy (in eV) of photo electrons in terms of stopping potential will therefore be $K_{max} = |V_0|$ eV.

LAWS OF PHOTOELECTRIC EMISSION

We thus have the following laws of photoelectric emission, derived from the experimental observations.

(a) For each emitting metal, there is a certain minimum frequency ν_0 (or maximum wavelength λ_0), called the threshold frequency of the incident radiation, below (above which) which no emission of photoelectron takes place, no matter how great is the intensity. The value of ν_0 $(\text{or } \lambda_0)$ is different for different emitting surfaces.

(b) The process of emission of photoelectrons is an instantaneous process. There is no time lag $(< 10^{-8} \text{ s})$ between the incidence of radiation and the emission of photoelectrons.

(c) Photoelectric effect is a one photon-one electron phenomenon i.e. even if photon has an energy sufficient to strike off 3 electrons (say) it can only strike off one electron with the excess energy being imparted to the struck off electron as kinetic energy.

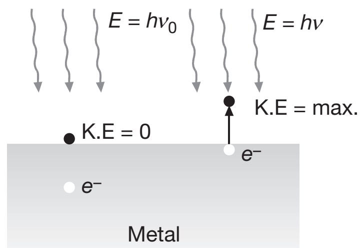
(d) The number of photoelectrons emitted per second, that is, photoelectric current is directly proportional to the intensity of the incident radiation but is independent of the frequency (or wavelength) of light.

(e) The velocities (or the energies) of the emitted photoelectrons vary between zero and a definite maximum (v_{max}). The proportion of photoelectrons having a particular velocity is independent of the light intensity.

(f) The maximum velocity, v_{max}, and hence the maximum kinetic energy is independent of the intensity of the incident light, but depends on its frequency, increasing linearly with the increase of the frequency of the incident light.

EINSTEIN'S EXPLANATION OF PHOTO-ELECTRIC EFFECT

The wave theory of light could not explain the observed characteristics of photoelectric effect. Einstein extended Planck's quantum idea for light to explain photo-electric effect.



According to his idea, the energy of electromagnetic radiation is not continuously distributed over the wave front like the energy of water waves but remains concentrated in packets of energy content $h\nu$, where ν is frequency of radiations and h is universal Planck's constant $(= 6 \cdot 625 \times 10^{-34} \text{ Js})$. Each packet of energy moves with the speed of light. The assumptions of Einstein's theory are

(a) The photoelectric effect is the result of collision of two particles, one of which is a photon of incident light and the other is an electron of photometal.

(b) The electron of photo-metal is bound with the nucleus by Coulomb attractive forces. The minimum energy required to free an electron from its bondage is called work function, $W = \phi_0 = h\nu_0$.

(c) The incident photon interacts with a single electron and loses its energy in two parts

- **(i)** Firstly, in getting the electron released from the bondage of the nucleus.
- **(ii)** Secondly, to impart kinetic energy to emitted electron.

(d) The efficiency of photoelectric effect is less than 1%, i.e. less than 1% of photons are capable of ejecting photoelectrons.

Accordingly, if $h\nu$ is the energy of incident photon, then

$$h\nu = \phi_0 + K_{max}$$

$$\Rightarrow \quad K_{max} = h\nu - \phi_0 \quad \ldots(1)$$

This is Einstein's photoelectric equation, where W is work function and

$$K_{max} = \frac{1}{2}mv_{max}^2 = eV_s$$

is the maximum kinetic energy of photoelectrons emitted.

Equation (1) is referred as Einstein's photo electric equation that explains all experimental results of photo-electric effect and is based on the Law of Conservation of Energy.

(e) Einstein's photoelectric equation says that when a single photon carrying an energy $h\nu$ falls onto a metal surface (where it is absorbed by a single electron), then a part of this energy ϕ_0 (called the work function of the metal surface) is utilized in causing the electron to escape from the metal surface and the excess energy $(h\nu - \phi_0)$ becomes the electron kinetic energy. If the electron (while coming out of the metal surface) does not lose energy by internal collisions, then it escapes from the metal with a maximum kinetic energy K_{max}. So, K_{max} represents the maximum kinetic energy that the photoelectron can have outside the surface. This happens to be in complete agreement with the quantum theory of the photon theory with experiment.

Since, we know that the number of photons incident per unit time (n) on a surface held normally to an incident light of intensity I is given by

$$n = \frac{IA}{h\nu}$$

So, for a particular frequency, if we double the light intensity, then we also double the number of photons and hence the photoelectric current is also doubled.

(f) The second objection (the frequency problem) is met when K_{max} equals zero and we have

$$h\nu_{th} = \phi_0$$

This asserts that the photon has just enough energy to eject the photoelectron but has no extra energy to give to the photoelectron as kinetic energy. If ν is reduced below ν_{th}, then $h\nu$ will be smaller than ϕ_0 and the individual photons, no matter how many of them there are (that is, no matter how intense the illumination), will not have enough energy to eject photoelectrons.

(g) The third objection (the time delay problem) also follows from the photon theory because the required energy is supplied to the electron in a concentrated bundle. It is not spread uniformly over the beam cross section as in the wave theory. Hence Einstein's equation for photoelectric effect is given by

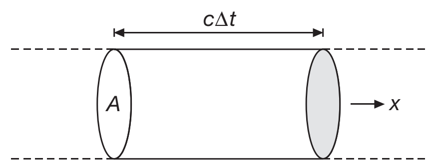
$$h\nu = h\nu_{th} + K_{max}$$

$$\Rightarrow \quad K_{max} = \frac{hc}{\lambda} - \frac{hc}{\lambda_{th}}$$

PHOTOELECTRIC EFFECT CANNOT BE EXPLAINED USING CLASSICAL WAVE THEORY OF LIGHT

Before we try to explain the drawbacks of Classical Wave Theory when applied to Photoelectric effect, we need to understand the concept of intensity of light.

Intensity of light is the energy crossing per second per unit area of a surface held normally to the direction of propagation of the wave. Let us consider a cylindrical volume with area of cross section A and length $c\Delta t$ along the x axis.



The energy U contained in a cylindrical cross-section of area A in time Δt when a wave propagates at speed c is given by $U = u_{av}(c\Delta t)A$, where u_{av} is the average energy density of the electromagnetic wave or the light wave. So, the intensity of the light beam is $I = \frac{U}{A\Delta t} = u_{av}c$. If E_0 is the amplitude of the electric field, then in terms of maximum electric field, the intensity is given by $I = \frac{1}{2}\varepsilon_0 E_0^2 c$.

The Intensity Problem: Wave theory cannot explain why kinetic energy of emitted photoelectrons is independent of intensity.

Since wave theory suggests that the oscillating electric field vector E of the light wave increases in amplitude when the intensity of the light beam is increased. Also, we know that the force applied to the electron in the presence of electric field E is eE, which simply suggests that the kinetic energy of the photoelectrons should also increase when the light beam is made more intense.

However, observations show that maximum kinetic energy is independent of the light intensity.

The Frequency Problem: Wave theory cannot explain the existence of a minimum frequency above which photoelectric effect takes place.

Wave theory also suggests that the photoelectric effect should occur for any frequency of the light but the light should be intense enough to supply the energy required to eject the photoelectrons.

However, observations show that there exists for each surface a characteristic cut off frequency or threshold frequency, $v_{\text{threshold}} = v_0$, above which photoelectric effect takes place and for frequencies less than the threshold frequency (v_0), the photoelectric effect does not occur, no matter how intense is light beam.

The Time Delay Problem: Wave theory cannot explain the immediate ejection of photoelectrons from a metal surface.

In accordance with the wave theory, when the energy acquired by a photoelectron is absorbed directly from the wave incident on the metal plate, then the effective target area for an electron in the metal is very limited (probably not much more than that of a circle of diameter roughly equal to that of an atom). As per the classical theory, the light energy is uniformly distributed over the wavefront. So, if the light is feeble enough, then there should be a measurable time lag, between the falling of the light on the surface and the ejection of the photoelectron from the surface. During this interval, the electron should be absorbing sufficient amount of energy from the beam until it accumulates enough energy to escape.

If we consider light as a wave, then the intensity depends upon electric field. If we take work function of metal to be as W or ϕ_0, then we have

$$W = \phi_0 = IAt$$

$$\Rightarrow \quad t = \frac{\phi_0}{IA}$$

So, according to wave theory of light applied on photoelectric effect, there should be time lag between the falling of a photon and emission of a photoelectron, because the metal has work function.

However, experiments show that the photoelectric effect is an instantaneous process. Hence, light is not of wave nature.

*So, **Quantum Theory of Light** solves these problems and provides a correct interpretation of the photoelectric effect phenomenon.*

ILLUSTRATION 19

In an experiment on photoelectric effect light of wavelength 400 nm is incident on a caesium plate at the rate of 5 W. The potential of the collector plate is made sufficiently positive with respect to emitter so that the current reaches the saturation value. Assuming that on the average one out of every 10^6 photons is able to eject a photoelectron, find the photocurrent in the circuit.

SOLUTION

$$E = \frac{12375}{4000} = 3.1 \text{ eV}$$

Number of photoelectrons emitted per second

$$n = \left(\frac{1}{10^6}\right)\left(\frac{5}{3.1 \times 1.6 \times 10^{-19}}\right) = 1 \times 10^{13} \text{ per second}$$

Since, $i = \dfrac{q}{t} = \dfrac{Ne}{t} = \left(\dfrac{N}{t}\right)e = ne$

$$\Rightarrow \quad i = (ne) = 1 \times 10^{13} \times 1.6 \times 10^{-19}$$

$$\Rightarrow \quad i = 1.6 \times 10^{-6} \text{ A} = 1.6\ \mu\text{A}$$

ILLUSTRATION 20

Ultraviolet light of wavelength 2000 Å causes photoemission from a surface. The stopping potential is 2 V.

(a) Find the work function in eV

(b) Find the maximum speed of the photoelectrons.

SOLUTION

(a) Using Einstein relation

$$\phi_0 = \frac{hc}{\lambda} - \text{eV}_0$$

$$\Rightarrow \quad \phi_0 = \frac{12400}{2000} - 2 = 4.2 \text{ eV}$$

(b) Since $\frac{1}{2} m v_{max}^2 = eV_0$

$$\Rightarrow \quad v_{max} = \sqrt{\frac{2 \text{ eV}_0}{m}} = \sqrt{\frac{2(1.6\times10^{-19})(2)}{9.1\times10^{-31}}}$$

$$\Rightarrow \quad v_{max} = 8.4\times10^5 \text{ ms}^{-1}$$

ILLUSTRATION 21

When a beam of 10.6 eV photons of intensity 2 Wm^{-2} falls on a platinum surface of area $1\times10^{-4} \text{ m}^2$ and work function 5.6 eV, 0.53% of the incident photons eject photo electrons. Calculate the number of photoelectrons emitted per second and their minimum and maximum energies (in eV). Given that $1 \text{ eV} = 1.6\times10^{-19}$ J.

SOLUTION

Number of photoelectrons emitted per second

$$\frac{N}{t} = \frac{(\text{Intensity})(\text{Area})}{(\text{Energy of each photon})} \times \frac{0.53}{100}$$

$$\Rightarrow \quad \frac{N}{t} = \frac{(2)(1\times10^{-4})}{(10.6\times1.6\times10^{-19})} \times \frac{0.53}{100} = 6.25\times10^{11} \text{ s}^{-1}$$

Minimum kinetic energy of photoelectrons is

$$K_{min} = 0$$

and maximum kinetic energy is,

$$K_{max} = E - \phi_0 = (10.6 - 5.6) \text{ eV} = 5 \text{ eV}$$

ILLUSTRATION 22

One milliwatt of light of wavelength 4560 Å is incident on a caesium surface. Calculate the photoelectric current, assuming a quantum efficiency of 0.5%.
Given Planck's constant $h = 6.62\times10^{-34}$ Js and velocity of light $c = 3\times10^8 \text{ ms}^{-1}$.

SOLUTION

The energy of each photon of incident light is

$$E = h\nu = \frac{hc}{\lambda} = \frac{(6.62\times10^{-34})(3\times10^8)}{4560\times10^{-10}}$$

$$\Rightarrow \quad E = 4.35\times10^{19} \text{ J}$$

Number of photons emitted per second by a one milliwatt source is

$$n = \frac{P_{source}}{E_{single\ photon}} = \frac{10^{-3}}{4.35\times10^{-19}} \approx 2.3\times10^{15} \text{ s}^{-1}$$

Number of photons emitted for a quantum efficiency of 0.5% is

$$N = (2.3\times10^{15}) \times \frac{0.5}{100}$$

$$\Rightarrow \quad N = 1.15\times10^{13} \text{ s}^{-1}$$

Thus photo-electric current

$$I = \frac{q}{t} = \frac{Ne}{t} = ne = (1.15\times10^{13})(1.6\times10^{-9})$$

$$\Rightarrow \quad I = 1.84\times10^{-6} \text{ A} = 1.84 \ \mu\text{A}$$

ILLUSTRATION 23

The surface of a metal of work function ϕ_0 is illuminated by light whose electric field component varies with time as $E = E_0[1+\cos(\omega t)]\sin(\omega_0 t)$. Calculate the maximum kinetic energy of photoelectrons emitted from the surface.

SOLUTION

The given electric field component is

$$E = E_0 \sin(\omega_0 t) + E_0 \sin(\omega_0 t)\cos(\omega t)$$

$$\Rightarrow \quad E = E_0 \sin(\omega_0 t) + \frac{E_0}{2}\left[\sin(\omega_0 + \omega)t + \sin(\omega_0 - \omega)t\right]$$

So, the given light sample that comprises three different frequencies i.e. ω, $(\omega_0 + \omega)$, $(\omega_0 - \omega)$
The maximum kinetic energy will be due to most energetic photon i.e. of frequency $\left(\frac{\omega + \omega_0}{2\pi}\right)$.
Since $K_{max} = h\nu - \phi_0$

$$\Rightarrow \quad K_{max} = \frac{h(\omega + \omega_0)}{2\pi} - \phi_0$$

ILLUSTRATION 24

A monochromatic light source of frequency illuminates a metallic surface and ejects photoelectrons. The photo electrons having maximum energy are just able to ionize the hydrogen atoms in ground state. When the entire experiment is repeated with an incident radiation of frequency $\frac{5}{6}f$, the photoelectrons so emitted are able to excite the hydrogen atom beam which then emits a radiation of wavelength 1215 Å.

(a) What is the frequency of radiation?
(b) Find the work function of the metal.

SOLUTION

(a) Using Einstein's equation of photoelectric effect i.e.

$$K_{max} = hf - \phi_0$$

where $K_{max} = 13.6$ eV

$$\Rightarrow \quad hf - \phi_0 = 13.6 \text{ eV} \quad \ldots(1)$$

So, when the experiment is repeated, then

$$h\left(\frac{5}{6}f\right) - \phi_0 = \frac{12375}{1215} = 10.2 \text{ eV} \quad \ldots(2)$$

Solving equations (1) and (2), we get

$$\frac{hf}{6} = 3.4 \text{ eV}$$

$$\Rightarrow \quad f = \frac{(6)(3.4)(1.6\times10^{-19})}{(6.63\times10^{-34})} = 4.92\times10^{15} \text{ Hz}$$

(b) From equation (1), we have

$$\phi_0 = hf - 13.6$$

$$\Rightarrow \quad \phi_0 = 6(3.4) - 13.6$$

$$\Rightarrow \quad \phi_0 = 6.8 \text{ eV}$$

ILLUSTRATION 25

A photon with an energy of 4.9 eV ejects photoelectrons from tungsten. When the ejected electron enters a constant magnetic field of strength $B = 2.5$ mT at an angle of $60°$ with the field direction, the maximum pitch of the helix described by electron is found to be 2.7 mm. Find the work function of the metal in electron-volts. Given that specific charge of electron is 1.76×10^{11} Ckg^{-1}.

SOLUTION

Pitch of helical path is

$$p = (v\cos\theta)T = \frac{vT}{2} \quad \{\because \theta = 60°\}$$

where, $T = \frac{2\pi m}{qB} = \frac{2\pi}{B\alpha}$ $\quad \left\{\because \alpha = \frac{q}{m}\right\}$

$$\Rightarrow \quad p = \frac{\pi v}{B\alpha}$$

$$\Rightarrow \quad v = \frac{B\alpha p}{\pi} \quad \ldots(1)$$

$$\Rightarrow \quad v = \frac{(2.5\times10^{-3})(1.76\times10^{11})(2.7\times10^{-3})}{3.14}$$

$$\Rightarrow \quad v = 0.38\times10^{6} \text{ ms}^{-1}$$

Since, $\text{KE} = \frac{1}{2}mv^2 = E - \phi_0$

$$\Rightarrow \quad \phi_0 = E - \frac{1}{2}mv^2 \quad \ldots(2)$$

Substituting value of v from equation (1) in equation (2), we get

$$\phi_0 = 4.9 - \frac{1}{2}\frac{(9.1\times10^{-31})(0.38\times10^{6})^2}{1.6\times10^{-19}}$$

$$\Rightarrow \quad \phi_0 = (4.9 - 0.4) \text{ eV}$$

$$\Rightarrow \quad \phi_0 = 4.5 \text{ eV}$$

ILLUSTRATION 26

If the wavelength of the incident radiation is increased from 3000 Å to 3010 Å, find the corresponding change in the stopping potential V.

SOLUTION

According to Einstein's Photo-electric equation, we have

$$eV_1 = E_1 - \phi_0 \quad \ldots(1)$$

$$eV_2 = E_2 - \phi_0 \quad \ldots(2)$$

Subtracting (2) from (1), we get

$$e(V_1 - V_2) = (E_1 - E_2)$$

$$\Rightarrow \quad V_1 - V_2 = \frac{hc}{e}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$\Rightarrow \quad V_1 - V_2 = \frac{6.6\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}\times10^{-10}}\left(\frac{1}{3000} - \frac{1}{3010}\right)$$

$$\Rightarrow \quad V_1 - V_2 = 0.012 \text{ V}$$

ILLUSTRATION 27

When light of wavelength λ is incident on a metal surface, stopping potential is found to be V_0. When light of wavelength $n\lambda$ is incident on the same metal surface, stopping potential is found to be $\dfrac{V_0}{n+1}$. Calculate the threshold wavelength of the metal.

SOLUTION

Let λ_0 be the threshold wavelength so that the work function is $\phi = \dfrac{hc}{\lambda_0}$. Now, by photoelectric equation, we get

$$eV_0 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0} \quad \ldots(1)$$

$$\frac{eV_0}{n+1} = \frac{hc}{n\lambda} - \frac{hc}{\lambda_0} \quad \ldots(2)$$

From (1) and (2), we get

$$\frac{hc}{\lambda} - \frac{hc}{\lambda_0} = (n+1)\frac{hc}{n\lambda} - (n+1)\frac{hc}{\lambda_0}$$

$$\Rightarrow \quad \frac{nhc}{\lambda_0} = \frac{hc}{n\lambda}$$

$$\Rightarrow \quad \lambda_0 = n^2\lambda$$

ILLUSTRATION 28

A light beam of wavelength 400 nm is incident on a metal of work function 2.2 eV. A particular electron absorbs a photon and makes 2 collisions before coming out of the metal

(a) Assuming that 10% of extra energy is lost to the metal in each collision find the final kinetic energy of this electron as it comes out of the metal.

(b) Under the same assumptions find the maximum number of collisions the electron should suffer before it becomes unable to come out of the metal.

SOLUTION

(a) Since, $E(\text{in eV}) = \dfrac{12375}{\lambda(\text{in Å})}$

$$\Rightarrow \quad E = \frac{12375}{4000} = 3.1\text{ eV}$$

Energy of electron after first collision is

$$E_1 = (90\% \text{ of } E) = 2.79\text{ eV} \quad \{\because\ 10\% \text{ is lost}\}$$

Energy of electron after second collision

$$E_2 = (90\% \text{ of } E_1) = 2.51\text{ eV}$$

Hence, KE of this electron after emitting from the metal surface $= (2.51 - 2.2)\text{ eV} = 0.31\text{ eV}$

(b) Energy after third collision,

$$E_3 = (90\% \text{ of } E_2) = 2.26\text{ eV}$$

Similarly, $E_4 = (90\% \text{ of } E_3) = 2.03\text{ eV} < W$

So, after four collisions the electron will not be able to come out of the metal.

ILLUSTRATION 29

Calculate the velocity of the emitted photoelectrons, if the work function of the target material is 1.24 eV and the wavelength of incident light is 4000 Å. What retarding potential is necessary to stop the emission of the electrons? Take $hc = 1240\text{ eVnm}$

SOLUTION

Energy of incident photons in eV on metal surface is

$$E = \frac{12400}{4000}\text{ eV} = 3.1\text{ eV}$$

According to Einstein's photo electric equation, we have

$$E = \phi_0 + K_{max}$$

$$\Rightarrow \quad K_{max} = E - \phi_0$$

$$\Rightarrow \quad K_{max} = (3.1 - 1.24)\text{ eV} = 1.86\text{ eV}$$

The stopping potential for these ejected electrons is given by

$$V_s = \frac{K_{max}}{e} = \frac{1.86\text{ eV}}{e}$$

$$\Rightarrow \quad V_0 = 1.86\text{ volt}$$

ILLUSTRATION 30

When a beam of 10.6 eV photon of intensity 2.0 Wm^{-2} falls on a platinum surface of area 1.0 cm^2 and work function 5.6 eV, 0.53% of the incident photons eject photoelectrons. Find the number of photoelectrons emitted per second and their minimum and maximum kinetic energies (in eV). Take $1\text{ eV} = 1.6 \times 10^{-19}\text{ J}$.

SOLUTION

Since intensity of the photon beam is defined as the energy incident per second per unit area of a surface, so

$$I = \frac{E}{At} = \frac{P}{A}$$

$$\Rightarrow \quad P = IA$$

$$\Rightarrow \quad P = \left(2.0\ \text{Wm}^{-2}\right)\left(1.0\times10^{-4}\ \text{m}^2\right) = 2\times10^{-4}\ \text{W}$$

According to the problem, energy carried by each photon is

$$E_{\text{single photon}} = 10.6\ \text{eV} = (10.6)\left(1.6\times10^{-19}\right)\ \text{J}$$

$$\Rightarrow \quad E_{\text{single photon}} = 16.96\times10^{-19}\ \text{J}$$

So, the number of photons striking the metal surface per second is

$$n = \frac{P_{\text{photon beam}}}{E_{\text{single photon}}} = \frac{2.0\times10^{-4}}{16.96\times10^{-19}}$$

$$\Rightarrow \quad n = 1.18\times10^{14}\ \text{photons/s}$$

Since, only 0.53% of the incident photons are able to eject photoelectrons, so the number of photoelectrons ejected per second is

$$n_{\text{photons}} = \left(\frac{0.53}{100}\right)n = \left(\frac{0.53}{100}\right)\left(1.18\times10^{14}\right)$$

$$\Rightarrow \quad n_{\text{photons}} = 6.254\times10^{11}$$

So, minimum kinetic energy is zero and the maximum energy of the emitted photoelectron is

$$E_{\max} = 10.6\ \text{eV} - 5.6\ \text{eV} = 5.0\ \text{eV}$$

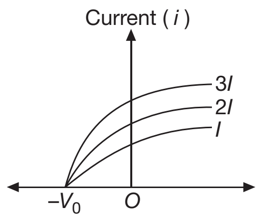
CHARACTERISTICS OF PHOTO ELECTRIC EFFECT

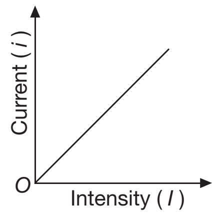
The following observations were made to study the effect of changes in various factors while studying the Photo Electric Effect.

Effect of Intensity

Intensity of light means the energy incident per second per unit area. For a given frequency, if intensity of incident light is increased, then photoelectric saturation current increases by the same factor and with decrease of intensity, the photoelectric saturation current also decreases by the same factor, but the stopping potential remains the same, so maximum value of kinetic energy is not effected.

In photoelectric effect **current (i) is directly proportional to intensity (I) of incident light.**



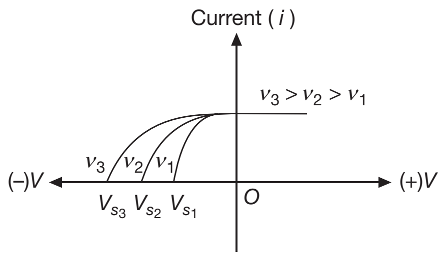


This means that the intensity of incident light affects the photoelectric current but leaves the maximum kinetic energy of photoelectrons unchanged.

Effect of Frequency

When the intensity of incident light is kept fixed and frequency is increased, the photoelectric current remains the same but the stopping potential increases.

If the frequency is decreased, the stopping potential decreases and at a particular frequency of incident light, the stopping potential becomes zero. This value of frequency of incident light for which the stopping potential is zero is called **threshold frequency** ν_0. If the frequency of incident light (ν) is less than the threshold frequency (ν_0), no photoelectric emission takes place.



Thus, the increase of frequency increases maximum kinetic energy of photoelectrons but leaves the photoelectric current unchanged.

Effect of Photo-metal

When frequency and intensity of incident light are kept fixed and photo-metal is changed, we observe that stopping potential (V_s) versus frequency (ν) graphs are parallel straight lines, cutting, frequency axis at different points. This shows that threshold frequency are different for different metals, the slope $\left(\frac{V_s}{\nu}\right)$ for all the metals is same and hence universal constant.

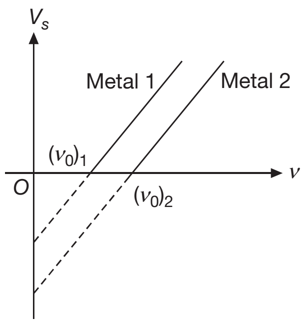
Since we know that

$$eV_s = h\nu - \phi_0$$

$$\Rightarrow \quad V_s = \left(\frac{h}{e}\right)\nu - \left(\frac{W}{e}\right)$$

Comparing with the line $y = mx + c$, where m is the slope and c is the intercept on the y axis.

Then, we observe that the graph is a straight line with slope $\frac{h}{e}$ (a universal constant) and negative intercept $\frac{\phi_0}{e}$ (depending on the nature of the metal). In figure threshold frequency and work function are greater for Metal 2 as compared to Metal 1.



Effect of Time

There is no time lag between incidence of light and the emission of photo-electrons.

Problem Solving Technique(s)

FORMULAE FOR WORKING THE PROBLEMS ON PHOTO-ELECTRIC EFFECT

Maximum Kinetic Energy of photo-electrons

$$E_K = eV_0 = \frac{1}{2}mv_{max}^2$$

If λ_0 is the threshold wavelength and ν_0 the threshold frequency,

Work function of photo-metal,

$$\phi_0 = h\nu_0 = \frac{hc}{\lambda_0}$$

Threshold frequency is minimum frequency and **Threshold wavelength** is maximum wavelength of incident light to cause photoelectric effect.

Einstein's Photo-electric Equation may be expressed as or $E_K = h\nu - \phi_0$

$$E_K = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

The condition for photoelectric emission is $h\nu \geq \phi_0$ or equivalently $\nu \geq \nu_0$ or equivalently $\lambda \leq \lambda_0$.

ILLUSTRATION 31

(a) If the wavelength of the light incident on a photoelectric cell is reduced from λ_1 Å to λ_2 Å, then calculate the change in the cut-off potential?

(b) Light is incident on the cathode of a photocell and the stopping voltages are measured for light of two different wavelengths. From the data given below, calculate the work function of the metal of the cathode in eV and the value of the universal constant $\frac{hc}{e}$.

Wavelength (Å)	Stopping voltage (volt)
4000	1.3
4500	0.9

SOLUTION

(a) Let the work function of the surface be ϕ_0. If ν be the frequency of the light falling on the surface, then according to Einstein's photoelectric equation, the maximum kinetic energy K_{max} of the emitted photoelectron is given by

$$K_{max} = h\nu - \phi_0 = \frac{hc}{\lambda} - \phi_0$$

Since, we know that the maximum kinetic energy of the photoelectrons emitted and the stopping potential are related to each other as $K_{max} = eV_s$

$$\Rightarrow \quad eV_s = \frac{hc}{\lambda} - \phi_0$$

$$\Rightarrow \quad V_s = \frac{hc}{e\lambda} - \frac{\phi_0}{e}$$

Now, $\Delta V_s = V_{s2} - V_{s1}$

$$\Rightarrow \quad \Delta V_s = \left(\frac{hc}{e\lambda_2} - \frac{\phi_0}{e}\right) - \left(\frac{hc}{e\lambda_1} - \frac{\phi_0}{e}\right)$$

$$\Rightarrow \quad \Delta V_s = \frac{hc}{e}\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

$$\Rightarrow \quad \Delta V_s = \frac{hc}{e}\left(\frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2}\right)$$

(b) From Equation (1), we have

$$\frac{hc}{e} = \frac{\Delta V_s(\lambda_1 \lambda_2)}{\lambda_1 - \lambda_2}$$

$$\Rightarrow \frac{hc}{e} = \frac{(1.3-0.9)\left[(4000\times10^{-10})\times(4500\times10^{-10})\right]}{500\times10^{-10}}$$

$$\Rightarrow \frac{hc}{e} = 1.44\times10^{-6}\ \text{Vm}^{-1}$$

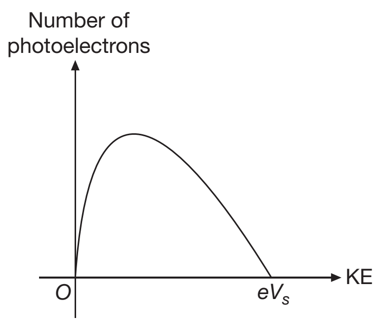
Also, we have $V_s = \frac{hc}{e\lambda} - \frac{\phi_0}{e}$

$$\Rightarrow \frac{\phi_0}{e} = \frac{hc}{e\lambda} - V_s = \frac{1.44\times10^{-6}}{4000\times10^{-10}} - 1.3$$

$$\Rightarrow \phi_0 = 2.3\ \text{eV}$$

GRAPH BETWEEN K_{max} AND ν

Whenever photoelectric effect takes place, electrons are ejected out with kinetic energies ranging from zero to K_{max} i.e. $0 \le K_{max} \le eV_s$. The energy distribution of photoelectrons is shown in figure.



Let us plot a graph between the maximum kinetic energy K_{max} and the frequency of the falling photon ν or the incident light. According to Einstein's Photo-Electric equation, we have

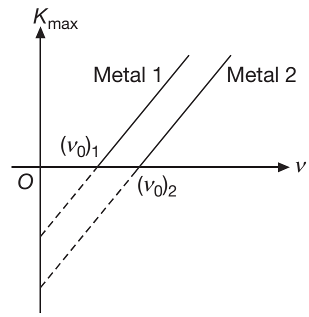
$$h\nu = \phi_0 + E_K$$

$$\Rightarrow \quad K_{max} = h\nu - \phi_0$$

Comparing with the line $y = mx + c$, where m is the slope and c is the intercept on the y axis.

Then, we observe that the graph is a straight line with slope h (a universal constant) and negative intercept ϕ (depending on the nature of the metal). For Metal 2, we observe that

$$\phi_2 > \phi_1 \text{ and hence } (\nu_0)_2 > (\nu_0)_1$$



Also, we observe that when $\nu = \nu_0$, the threshold frequency, then, $K_{max} = 0$

ILLUSTRATION 32

Calculate the value of the Planck's constant h if photoelectrons emitted from a surface of a certain metal by light of frequency 2.2×10^{15} Hz are fully retarded by a reverse potential of 6.6 V and those ejected by light of frequency 4.6×10^{15} Hz by a reverse potential of 16.5 eV.

SOLUTION

According to Einstein's photo electric equation, we have

$$h\nu_1 = \phi_0 + eV_1 \quad \ldots(1)$$

$$h\nu_2 = \phi_0 + eV_2 \quad \ldots(2)$$

Subtracting Equation (1) from Equation (2), we get

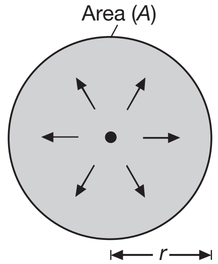
$$h(\nu_2 - \nu_1) = e(V_2 - V_1)$$

$$\Rightarrow \quad h = \frac{(16.5-6.6)(1.6\times10^{-19})}{(4.6-2.2)\times10^{15}}$$

$$\Rightarrow \quad h = 6.6\times10^{-34}\ \text{Js}$$

DETERMINATION OF PHOTOELECTRIC CURRENT

Let P be the power of a point source of electromagnetic radiations as shown.



The intensity I at a distance r from the source is given by $I = \frac{P}{4\pi r^2}\ \mathrm{Wm^{-2}}$. If A is the area of a metal surface on which radiations are incident, then the power received by the plate is

$$P' = IA = \left(\frac{P}{4\pi r^2}\right)A\,(W)$$

If ν is the frequency of radiation, then the energy of photon is given by

$$E = h\nu$$

The number of photons incident on the plate per second is given by

$$n = \frac{P'}{E} = \left[\frac{\frac{P}{4\pi r^2}\times A}{h\nu}\right]$$

If $\nu > \nu_0$ (threshold frequency) and photon efficiency of the metal plate is $\eta\%$, then the actual number of photoelectrons emitted per second is given by

$$n' = \left(\frac{\eta}{100}\right)n = \left[\frac{\frac{P}{4\pi r^2}\times A}{h\nu}\right]\frac{\eta}{100}$$

Finally, the photocurrent i is given by

$$i = n'e$$

where e is the charge of an electron $\left(e = 1.6\times10^{-19}\ \mathrm{C}\right)$

ILLUSTRATION 33

Light of wavelength $180\ \mathrm{nm}$ ejects photoelectrons from a plate of a metal whose work function is $2\ \mathrm{eV}$. If a uniform magnetic field of $50\ \mu\mathrm{T}$ is applied parallel to the plate, what would be the radius of the path followed by electrons ejected normally from the plate with maximum energy.

SOLUTION

$$\lambda = 180\ \mathrm{nm} = 1800\ \text{Å}$$

$$\Rightarrow \quad E = \frac{12375}{1800} = 6.875\ \mathrm{eV}$$

Since, $K_{\max} = E - \phi_0 = 4.875\ \mathrm{eV}$

Since, $r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$

Substituting the values, we get

$$r = \frac{\sqrt{2\times4.875\times1.6\times10^{-19}\times9.1\times10^{-31}}}{5\times10^{-5}\times1.6\times10^{-19}}$$

$$\Rightarrow \quad r = 0.15\ \mathrm{m} = 15\ \mathrm{cm}$$

ILLUSTRATION 34

A small metal plate, having work function ϕ_0, is kept at a distance d from a singly ionized, fixed ion. A monochromatic light beam is incident on the metal plate and photoelectrons are emitted. Find the maximum wavelength of the light beam so that some of the photoelectrons may go round the ion along a circle.

SOLUTION

For circular motion of electrons around the ion, the electrostatic force between the electron and the positively charged ion must provide the necessary centripetal force to the electron to revolve in a circle of radius d.

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{e^2}{d^2} = \frac{mv^2}{d}$$

$$\Rightarrow \quad \frac{e^2}{8\pi\varepsilon_0 d} = \frac{1}{2}mv^2 = \frac{hc}{\lambda} - \phi_0$$

$$\Rightarrow \quad \lambda = \frac{8\pi\varepsilon_0 hcd}{e^2 + 8\pi\varepsilon_0\phi_0 d}$$

ILLUSTRATION 35

A small plate of a photosensitive metal having work function $1.1\ \mathrm{eV}$ is placed at a distance of $2\ \mathrm{m}$ from a monochromatic light source of wavelength $496\ \mathrm{nm}$ and power $1\ \mathrm{watt}$. The light falls normally on the plate. Calculate the number of photons striking the metal plate per second per square meter. If a constant magnetic field of strength $10^{-4}\ \mathrm{T}$ is applied parallel to the metal surface, calculate the radius of the largest circular path followed by the emitted photoelectrons. Given $hc = 12400\ \mathrm{eV}\text{Å}$

SOLUTION

Energy of each incident photon in eV is

$$E_{\text{single photon}} = \frac{12400}{4960}\ \text{eV} = 2.5\ \text{eV}$$

$$\Rightarrow \quad E_{\text{each photon}} = 2.5\times 1.6\times 10^{-19}\ \text{J} = 4\times 10^{-19}\ \text{J}$$

The rate of emission of photons i.e. the number of photons emitted per second by the source is

$$n = \frac{P_{\text{source}}}{E_{\text{each photon}}} = \frac{1\ \text{Js}^{-1}}{4\times 10^{-19}\ \text{J}} = 2.5\times 10^{18}\ \text{photons/s}$$

Hence, the number of photons striking the plate per second per square meter (also called as photon flux) is given by

$$\phi_N = \frac{n}{A} = \frac{n}{4\pi r^2} = \frac{2.5\times 10^{18}}{4(3.14)(2)^2} \approx 5\times 10^{16}\ \text{s}^{-1}\text{m}^{-2}$$

The maximum kinetic energy of the photo-electrons emitted from the plate having work function $\phi_0 = 1.1$ eV is given by

$$K_{\max} = E - \phi_0 = 2.5 - 1.1 = 1.4\ \text{eV}$$

$$\Rightarrow \quad \frac{1}{2} m v_{\max}^2 = 1.4\ \text{eV}$$

The maximum velocity of the ejected photoelectrons is

$$v_{\max} = \sqrt{\frac{2\times 1.4\times 1.6\times 10^{-19}}{9.1\times 10^{-31}}}$$

$$\Rightarrow \quad v_{\max} \approx 7.0\times 10^{5}\ \text{ms}^{-1}$$

The maximum radius of the circle traversed by photoelectron in magnetic field $B = 10^{-4}$ T is given by

$$r = \frac{mv}{qB} = \frac{(9.1\times 10^{-31})(7\times 10^{5})}{(1.6\times 10^{-19})(10^{-4})} = 0.0398\ \text{m}$$

$$\Rightarrow \quad r \approx 0.04\ \text{metre} = 4.0\ \text{cm}$$

Test Your Concepts-II

Based on Photoelectric Effect

(Solutions on page H.5)

1. What will be the maximum kinetic energy of the photoelectrons ejected from magnesium (for which the work function $W = 3.7$ eV) when irradiated by ultraviolet light of frequency $1.5\times 10^{15}\ \text{sec}^{-1}$.
2. A 40 W ultraviolet light source of wavelength 2480 Å illuminates a photosensitive metal surface placed 2 m away. Calculate the number of photons emitted from the source per second and the number incident on unit area of the metal surface per second. The photo-electric work function for the metal is 3.7 eV. Calculate the kinetic energy of the fastest electrons ejected from the surface. Also calculate the maximum wavelength of light that can produce the photoelectric effect from the given metal surface.
   Given $hc = 12400$ eVÅ
3. The hydrogen atom in its ground state is excited by means of monochromatic radiation. Its resulting spectrum has six different lines. These radiations are incident on a metal plate. It is observed that only two of them are responsible for photoelectric effect. If the ratio of maximum kinetic energy of photoelectrons in the two cases is 5 then find the work function of the metal.
   [Take ground state energy of H-atom $= -13.6$ eV].
4. A metallic surface is irradiated with monochromatic light of variable wavelength. Above a wavelength of 5000 Å, no photoelectrons are emitted from the surface. With an unknown wavelength, stopping potential of 3 V is necessary to eliminate the photocurrent. Find the unknown wavelength.
5. A light source, emitting three wavelengths 5000 Å, 6000 Å and 7000 Å, has a total power of 10^{-3} W and a beam diameter of 2 mm. The power density is distributed equally amongst the three wavelengths. The beam shines normally on a metallic surface of area on $10^{-4}\ \text{m}^2$ and having a work function of 1.9 eV. Assuming that each photon liberates an electron, calculate the charge emitted per second from the metal surface.
6. A beam of light consists of four wavelength 4000 Å, 4800 Å, 6000 Å and 7000 Å, each of intensity $1.5\times 10^{-3}\ \text{Wm}^{-2}$. The beam falls normally on an area $10^{-4}\ \text{m}^2$ of a clean metallic surface of work

function 1.9 eV. Assuming no loss of light energy calculate the number of photoelectrons liberated per second.

7. In an experiment on photo electric emission, following observations were made:
   (i) Wavelength of the incident light $=1.98\times10^{-7}$ m
   (ii) Stopping potential = 2.5 V.
   Find the
   (a) threshold frequency.
   (b) work function and
   (c) energy of photo electrons with maximum speed.
8. Radiation of wavelength 5461 Å falls on a photo cathode and electrons with a maximum kinetic energy of 0.18 eV are emitted. When radiation of wavelength 1849 Å falls on the same surface a (negative) potential of 4.6 V has to be applied to the collector electrode to reduce the photoelectric current is zero. Find the value of h and cutoff wavelength.
9. Illuminating the surface of a certain metal alternately with light of wavelengths $\lambda_1 = 0.35\ \mu$m and $\lambda_2 = 0.54\ \mu$m, it was found that the corresponding maximum velocities of photo electrons differ by a factor $\eta = 2$. Find the work function of that metal.
10. When a surface is irradiated with light of $\lambda = 4950$ Å a photocurrent appears which vanishes if a retarding potential 0.6 V is applied. When a different source of light is used it is found that critical retarding potential is changed to 1.1 V. Find the work function of emitting surface and wavelength of second source. If photoelectrons after emission from surface are subjected to a magnetic field of 10 T, what changes will be observed in the above two retarding potentials?
11. The photoelectric work function of potassium is 2.3 eV. If light having a wavelength of 2800 Å falls on potassium, find
   (a) the kinetic energy in electron volt of the most energetic electrons ejected.
   (b) the stopping potential in volt.
12. Electrons with a maximum kinetic energy of 3 eV are ejected from a metal surface by ultraviolet radiation of wavelength 1500 Å. Calculate the work function of the metal, the threshold wavelength of metal and the stopping potential difference required to stop the emission of photoelectrons.
13. In an experiment tungsten cathode which has a threshold 2300 Å is irradiated by ultraviolet light of wavelength 1800 Å. Calculate the
   (a) work function for tungsten in eV.
   (b) maximum energy of emitted photoelectron in eV.
   Given that the Planck's constant is $h = 6.6\times10^{-34}$ Js, $1\text{ eV} = 1.6\times10^{-19}$ J and velocity of light $c = 3\times10^{8}\ \text{ms}^{-1}$
14. A low intensity ultraviolet light of wavelength 2250 Å irradiates a photocell made of molybdenum metal. If the stopping potential is 1.5 V, calculate the work function of the metal. Will the photocell work if it is irradiated by a high intensity light of wavelength 6875 Å?
15. The photoelectric threshold of the photo electric effect of a certain metal is 2750 Å. Calculate the
   (a) work function of emission of an electron from this metal,
   (b) maximum kinetic energy of these electrons,
   (c) maximum velocity of the electrons ejected from the metal by light with a wavelength 1800 Å.
   Take $hc = 1243$ eVnm
16. Light quanta with an energy 4.9 eV eject photoelectrons from metal with work function 4.5 eV. Find the maximum impulse transmitted to the surface of the metal when each electron flies out.

SOLVED PROBLEMS

PROBLEM 1

A uniform monochromatic beam of light of wavelength 365 nm and intensity 10^{-8} Wm^{-2} falls on a photosensitive metal surface having absorption coefficient 0.8 and work function 1.6 eV. Calculate the rate of number of electrons emitted per m^2, power absorbed per m^2 and the maximum kinetic energy of emitted photoelectrons.

SOLUTION

The rate of number of electrons emitted per m^2 i.e. the number of photons crossing unit area per unit time i.e. the incident photon flux ϕ_i is given by

$$\phi_{\text{incident}} = \frac{I\lambda}{hc} = \frac{10^{-8} \times 365 \times 10^{-9}}{6.62 \times 10^{-34} \times 3 \times 10^8} = 18.35 \times 10^9$$

The number of photons absorbed by the surface per second per unit area is given by

$$\phi_{\text{absorbed}} = (0.8)\phi_{\text{incident}} = (0.8)(8.35 \times 10^9)$$

$$\Rightarrow \quad \phi_{\text{absorbed}} \approx 1.5 \times 10^{10} \text{ s}^{-1}\text{m}^{-2}$$

Now assuming that each photon ejects only one electron, then the rate of electrons emitted per second per unit area is also

$$\phi_{\text{absorbed}} = 1.5 \times 10^{10} \text{ s}^{-1}\text{m}^{-2}$$

Since power absorbed per square metre is the absorption coefficient times the power incident per square metre, so

$$P_{\text{absorbed per square metre}} = (0.8) P_{\text{incident per square metre}}$$

$$\Rightarrow \quad P_{\text{absorbed per square metre}} = (0.8)(10^{-8}) = 8 \times 10^{-9} \text{ Wm}^{-2}$$

From Einstein's equation, maximum kinetic energy of the emitted photoelectrons from the metal with work function W is given by

$$K_{\max} = h\nu - W_0 = \frac{hc}{\lambda} - W_0$$

$$\Rightarrow \quad K_{\max} = \frac{(6.62 \times 10^{-34})(3 \times 10^8)}{365 \times 10^{-9}} - 1.6 \times 1.6 \times 10^{-19}$$

$$\Rightarrow \quad K_{\max} = 2.89 \times 10^{-19} \text{ joule} = 1.80 \text{ eV}$$

PROBLEM 2

In a photocell the plates P and Q have a separation of 10 cm, which are connected through a galvanometer without any cell. Bi-chromatic light of wavelengths 4000 Å and 6000 Å are incident on plate Q whose work function is 2.39 eV. If a uniform magnetic field B exists parallel to the plates, find the minimum value of B for which the galvanometer shows zero deflection.

SOLUTION

Energy of photons corresponding to light of wavelength $\lambda_1 = 4000 \text{ Å}$ is

$$E_1 = \frac{12375}{4000} = 3.1 \text{ eV}$$

and that corresponding to $\lambda_2 = 6000 \text{ Å}$ is,

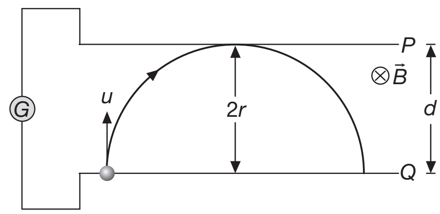
$$E_2 = \frac{12375}{6000} = 2.06 \text{ eV}$$

Given that the work function for the metal is $W = 2.39 \text{ eV}$, so we observe that

$$E_2 < W \text{ and } E_1 > W$$

Hence photoelectric emission is possible with λ_1 only.

Photoelectrons experience magnetic force and move along a circular path.



The galvanometer will indicate zero deflection when the photoelectrons complete semi-circular path before reaching the plate P.

Thus, $d = 2r = 10 \text{ cm}$

$$\Rightarrow \quad r = 5 \text{ cm} = 0.05 \text{ m}$$

Further $r = \dfrac{mv}{Bq} = \dfrac{\sqrt{2Km}}{Bq}$

$$\Rightarrow \quad B_{min} = \frac{\sqrt{2Km}}{rq}$$

where $K = E_1 - W = (3.1 - 2.39) = 0.71$ eV

Substituting the values, we have

$$B_{min} = \frac{\sqrt{2 \times 0.71 \times 1.6 \times 10^{-19} \times 9.109 \times 10^{-31}}}{(0.05)(1.6 \times 10^{-19})}$$

$$\Rightarrow \quad B_{min} = 5.68 \times 10^{-5} \text{ Tesla}$$

PROBLEM 3

When a metal surface is irradiated with light of wavelength 4950 Å, a photo current appears. This photo current vanishes, if a retarding potential greater than 0.6 volt is applied across the photo tube. However, when a different source of light is used, it is found that the critical retarding potential is changed to 1.1 volt. Calculate the work function of the emitting metal surface and the wavelength of second source.

If the photo electrons (after emission from the surface) are subjected to a magnetic field of 10 tesla, what changes will be observed in the above two retarding potentials.

SOLUTION

In the first case, energy of incident photon in eV is

$$E_1 = \frac{12375}{4950} \text{ eV} = 2.5 \text{ eV}$$

The maximum kinetic energy of ejected electrons is

$$(K_{max})_1 = eV_1 = 0.6 \text{ eV}$$

According to Einstein's photo electric equation, we have

$$E = \phi_0 + K_{max}$$

$$\Rightarrow \quad \phi_0 = E_1 - (K_{max})_1$$

$$\Rightarrow \quad \phi_0 = (2.5 - 0.6) \text{ eV} = 1.9 \text{ eV}$$

In second case, the maximum kinetic energy of ejected electrons is

$$(K_{max})_2 = eV_2 = 1.1 \text{ eV}$$

According to Einstein's photo electric equation, we have

$$E = \phi_0 + K_{max}$$

$$\Rightarrow \quad E_2 = (1.9 + 1.1) \text{ eV} = 3.0 \text{ eV}$$

So, the wavelength of incident photons in second case is

$$\lambda_2 = \frac{12375}{3.0} \text{ Å} = 4125 \text{ Å}$$

Since work done by magnetic force in moving a charged particle is zero, so a magnetic field can never speed up or slow down a charged particle and hence there will be no effect on the stopping potentials, because the kinetic energy of the emitted photoelectrons remains the same.

PROBLEM 4

In a photoelectric effect set-up, a point source of light of power 3.2×10^{-3} W emits mono energetic photons of energy 5 eV. The source is located at a distance of 0.8 m from the centre of a stationary metallic sphere of work function 3 eV and of radius 8×10^{-3} m. The efficiency of photoelectron emission is one for every 10^6 incident photons. Assume that the sphere is isolated and electrons are instantly swept away after emission.

(a) Calculate the number of photoelectrons emitted per second.

(b) Find the ratio of the wavelength of incident light to the de Broglie wavelength of the fastest photoelectrons emitted.

(c) It is observed that the photoelectron emission stops at a certain time t after the light source is switched on. Why?

(d) Evaluate the time t.

SOLUTION

(a) Number of photoelectrons emitted per second is

$$n = \left(\frac{1}{10^6}\right)\left(\frac{P}{E}\right)\left(\frac{\pi r^2}{4\pi R^2}\right)$$

$$\Rightarrow n = \left(\frac{1}{10^6}\right) \times \left(\frac{3.2 \times 10^{-3}}{5 \times 1.6 \times 10^{-19}}\right) \times \left(\frac{1}{4\pi \times 0.8 \times 0.8}\right) \times (\pi \times 8 \times 10^{-3} \times 8 \times 10^{-3})$$

$$\Rightarrow n = 10^5 \text{ sec}^{-1}$$

(b) $K_{max} = E - \phi_0 = 2$ eV

Since, for an electron, we have

$$\lambda_2 = \sqrt{\frac{150}{KE(\text{in eV})}} \text{ Å}$$

$$\Rightarrow \quad \lambda_2 = \sqrt{\frac{150}{2}} = 8.66 \text{ Å}$$

Further, Wavelength of incident photon is

$$\lambda_1 = \frac{12375}{5} = 2475\ \text{Å}$$

$$\Rightarrow \quad \frac{\lambda_1}{\lambda_2} \approx 286$$

(c) Photoemission will stop when potential on the sphere becomes equal to the stopping potential.

(d) $K_{max} = 2\ eV_0$. Therefore, the stopping potential V_0 is 2 volt. Let t be the desired time. Then

$$V_0 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = \frac{Ne}{4\pi\varepsilon_0 r} = \frac{(nt)e}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \quad t = \frac{V_0 r}{\left(\frac{1}{4\pi\varepsilon_0}\right)(ne)} = \frac{2\times 8\times 10^{-3}}{9\times 10^9 \times 10^5 \times 1.6\times 10^{-19}}$$

$$\Rightarrow \quad t = 111\ \text{s}$$

PROBLEM 5

When photons of energy 4.25 eV strike the surface of a metal A, the ejected photoelectrons have maximum kinetic energy, T_A expressed in eV and de-Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photons of energy 4.70 eV is $T_B = (T_A - 1.50\ \text{eV})$. If the de-Broglie wavelength of these photoelectrons is $\lambda_B = 2\lambda_A$, then find

(a) the work function W_A of metal A and the work function W_B of metal B.

(b) the maximum kinetic energy T_A of the electrons ejected from metal A.

SOLUTION

$$K_{max} = E - W$$

Therefore, $T_A = 4.25 - W_A$ ...(1)

$$T_B = (T_A - 1.50) = 4.70 - W_B \quad \text{...(2)}$$

Equations (1) and (2) gives,

$$W_B - W_A = 1.95\ \text{eV} \quad \text{...(3)}$$

de-Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2Km}}$$

$$\Rightarrow \quad \lambda \propto \frac{1}{\sqrt{K}} \qquad K = \text{KE of electron}$$

$$\Rightarrow \quad \frac{\lambda_B}{\lambda_A} = \sqrt{\frac{K_A}{K_B}}$$

$$\Rightarrow \quad 2 = \sqrt{\frac{T_A}{T_A - 1.5}}$$

$$\Rightarrow \quad T_A = 2\ \text{eV}$$

From equation (1), we get

$$W_A = 4.25 - T_A = 2.25\ \text{eV}$$

From equation (3), we get

$$W_B = W_A + 1.95\ \text{eV} = (2.25 + 1.95)$$

$$\Rightarrow \quad W_B = 4.20\ \text{eV}$$

$$\Rightarrow \quad T_B = 4.70 - W_B = 4.70 - 4.20 = 0.50\ \text{eV}$$

PROBLEM 6

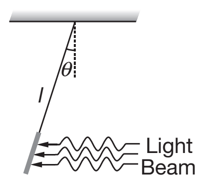
A small, plane strip is suspended from a fixed support through a string of length l as shown. A continuous beam of monochromatic light is incident horizontally on the strip and is completely absorbed. The energy falling on the strip per unit time is P.

(a) Find the deflection of the string from the vertical if the mirror stays in equilibrium.

(b) If the strip is deflected slightly from its equilibrium position in the plane of the figure, what will be the time period of the resulting oscillations?

SOLUTION

(a) The linear momentum of the light falling per unit time on the strip is $\frac{P}{c}$. As the light is incident on the strip, its momentum is absorbed by the mirror. The change in momentum imparted to the strip per unit time is thus $\frac{P}{c}$. This is equal to the force on the strip by the light beam. In equilibrium, the force by the light beam, the weight of the strip and the force due to tension add to zero. If the string makes an angle θ with the vertical,



$$T\cos\theta = mg$$

and $T\sin\theta = \dfrac{P}{c}$

Thus, $\tan\theta = \dfrac{P}{mgc}$

(b) In equilibrium, the tension is

$$T = \left[(mg)^2 + \left(\frac{P}{c}\right)^2\right]^{\frac{1}{2}}$$

$$\Rightarrow \quad \frac{T}{m} = \left[g^2 + \left(\frac{P}{mc}\right)^2\right]^{\frac{1}{2}}$$

This plays the role of effective g. The time period of small oscillations is

$$t = 2\pi\sqrt{\frac{l}{T/m}} = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \left(\frac{P}{mc}\right)^2}}}$$

PROBLEM 7

Ultraviolet light of wavelengths 800 Å and 700 Å when allowed to fall on hydrogen atoms in their ground state is found to liberate electrons with kinetic energy 1.8 eV and 4.0 eV respectively. Find the value of Planck's constant.

SOLUTION

When 800 Å wavelength falls on hydrogen atom (in ground state) 13.6 eV energy is used in liberating the electron. The rest is given as kinetic energy to the electron.

Hence, $K = E - 13.6(\text{in eV})$

$$\Rightarrow \quad (1.8\times1.6\times10^{-19}) = \frac{hc}{800\times10^{-10}} - 13.6\times1.6\times10^{-19} \quad \ldots(1)$$

Similarly, for the second wavelength, we have

$$(4\times1.6\times10^{-19}) = \frac{hc}{700\times10^{-10}} - 13.6\times1.6\times10^{-19} \quad \ldots(2)$$

Solving these two equations, we get

$$h = 6.6\times10^{-34}\ \text{Js}$$

PROBLEM 8

A beam of light has three wavelength 4144 Å, 4972 Å and 6216 Å with a total intensity of 3.6×10^{-3} Wm^{-2} equally distributed amongst the three wavelengths. The beam falls normally on 1.0 cm^2 area of a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each energetically capable photon ejects one electron. Calculate the number of photoelectrons emitted in two seconds.

SOLUTION

Threshold wavelength for the metal having a work function of 2.3 eV is

$$\lambda_{\text{th}} = \frac{12375}{2.3}\ \text{Å} = 5380\ \text{Å}$$

So, only the wavelengths 4144 Å and 4972 Å will be able to emit electrons from the metal surface because they are lesser than the threshold wavelength.
Since the intensity is equally distributed amongst the three incident wavelengths, so we have

$$I = \left(\frac{I_{\text{total}}}{3}\right) = 1.2\times10^{-3}\ \text{Wm}^{-2}$$

The energy incident per second (i.e. power incident) on the surface for each wavelength is

$$P = IA = \left(\frac{I_{\text{total}}}{3}\right)A$$

$$\Rightarrow \quad P = (1.2\times10^{-3}\ \text{Wm}^{-2})(1.0\ \text{cm}^2)$$

$$\Rightarrow \quad P = (1.2\times10^{-3})\times(10^{-4})\ \text{W} = 1.2\times10^{-7}\ \text{W}$$

Energy incident on surface for each wavelength in an interval of 2 seconds is

$$E = Pt = (1.2\times10^{-7})(2) = 2.4\times10^{-7}\ \text{J}$$

The number of photons (N) in a light beam of energy E having photons of wavelength λ are given by

$$N = \frac{E_{\text{light beam}}}{E_{\text{single photon}}} = \frac{E}{\left(\frac{hc}{\lambda}\right)} = \frac{E\lambda}{hc}$$

Number of photons N_1 due to wavelength 4144 Å is

$$N_1 = \frac{(1.2\times10^{-7})(4144\times10^{-10})}{(6.63\times10^{-34})(3\times10^8)} = 0.5\times10^{12}$$

Number of photons N_2 due to wavelength 4972 Å is

$$N_2 = \frac{(2.4\times10^{-7})(4972\times10^{-10})}{(6.63\times10^{-34})(3\times10^{8})} = 0.575\times10^{12}$$

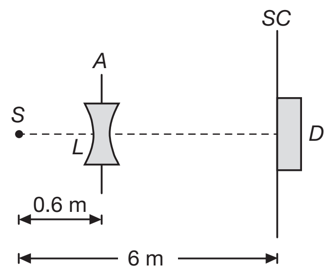
So, total number of photons (N) is given by

$$N = N_1 + N_2 = 0.5\times10^{12} + 0.575\times10^{12}$$

$$\Rightarrow \quad N = 1.075\times10^{12}$$

PROBLEM 9

A monochromatic point source S radiating wavelength 6000 Å, with power 2 watt, an aperture A of diameter 0.1 m and a large screen SC are placed as shown in figure. A photoemissive detector D of surface area $0.5\ \text{cm}^2$ is placed at the centre of the screen. The efficiency of the detector for the photoelectron generation per incident photon is 0.9.



(a) Calculate the photon flux at the centre of the screen and the photocurrent in the detector.

(b) If the concave lens L of focal length 0.6 m is inserted in the aperture as shown, find the new values of photon flux and photocurrent. Assume a uniform average transmission of 80% from the lens.

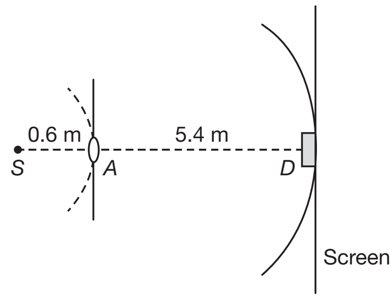
(c) If the work function of the photo emissive surface is 1 eV, calculate the values of the stopping potential in the two cases (without and with the lens in the aperture).

SOLUTION

(a) Energy of one photon,

$$E = \frac{hc}{\lambda} = \frac{(6.6\times10^{-34})(3\times10^{8})}{6000\times10^{-10}}$$

$$\Rightarrow \quad E = 3.3\times10^{-19}\ \text{J}$$



Power of the source is $2\ \text{W} = 2\ \text{Js}^{-1}$. Therefore, number of photons emitting per second is

$$n_1 = \frac{2}{3.3\times10^{-19}} = 6.06\times10^{18}\ \text{s}^{-1}$$

At distance 0.6 m, number of photons incident per unit area per unit time is

$$n_2 = \frac{n_1}{4\pi(0.6)^2} = 1.34\times10^{18}\ \text{m}^{-2}\text{s}^{-1}$$

Area of aperture is,

$$S_1 = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.1)^2 = 7.85\times10^{-3}\ \text{m}^2$$

So, total number of photons incident per unit time on the aperture,

$$n_3 = n_2 S_1 = (1.34\times10^{18})(7.85\times10^{-3})\ \text{s}^{-1}$$

$$\Rightarrow \quad n_3 = 1.052\times10^{16}\ \text{s}^{-1}$$

This aperture will become new source of light. Now these photons are further distributed in all directions. Hence, at the location of the detector, photons incident per unit area per unit time is

$$n_4 = \frac{n_3}{4\pi(6-0.6)^2} = \frac{1.052\times10^{16}}{4\pi(5.4)^2}$$

$$\Rightarrow \quad n_4 = 2.87\times10^{13}\ \text{m}^{-2}\text{s}^{-1}$$

This is the photon flux at the centre of the screen. Area of detector is $0.5\ \text{cm}^2$ or $0.5\times10^{-4}\ \text{m}^2$. Therefore, total number of photons incident on the detector per unit time is

$$n_5 = (0.5\times10^{-4})(2.87\times10^{13}\,d) = 1.435\times10^{9}\ \text{s}^{-1}$$

The efficiency of photoelectron generation is 0.9. Hence, total photoelectrons generated per unit time is

$$n_6 = 0.9 n_5 = 1.2915\times10^{9}\ \text{s}^{-1}$$

Hence, photocurrent in the detector is

$$i=(e)n_6=(1.6\times10^{-19})(1.2915\times10^{9})$$

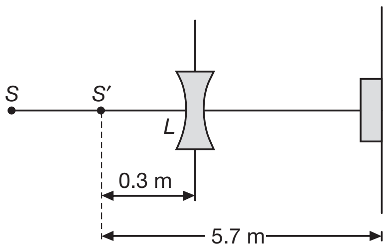
$$\Rightarrow \quad i=2.07\times10^{-10}\ \text{A}$$

(b) Using the lens formula, we get

$$\frac{1}{v}-\frac{1}{-0.6}=\frac{1}{-0.6}$$

$$\Rightarrow \quad v=-0.3\ \text{m}$$

i.e., image of source (say S', is formed at 0.3 m) from the lens.



Total number of photons incident per unit time on the lens are still n_3 or $1.052\times10^{16}\ \text{s}^{-1}$. Since, 80% of it transmits to second medium, therefore, at a distance of 5.7 m from S' number of photons incident per unit area per unit time will be

$$n_7=\frac{\left(\frac{80}{100}\right)(1.052\times10^{16})}{(4\pi)(5.7)^2}$$

$$\Rightarrow \quad n_7=2.06\times10^{13}\ \text{m}^{-2}\text{s}^{-1}$$

This is the photon flux at the detector.

New value of photocurrent is given by

$$i'=(2.06\times10^{13})(0.5\times10^{-4})(0.9)(1.6\times10^{-19})$$

$$\Rightarrow \quad i'=1.483\times10^{-10}\ \text{A}$$

(c) Energy of incident photons (in both the cases) is

$$E(\text{in eV})=\frac{12375}{\lambda(\text{in Å})}$$

$$\Rightarrow \quad E=\frac{12375}{6000\ \text{Å}}=2.06\ \text{eV}$$

Work function $W=1\ \text{eV}$

Maximum kinetic energy of photoelectrons in both cases,

$$K_{max}=E-W=1.06\ \text{eV}$$

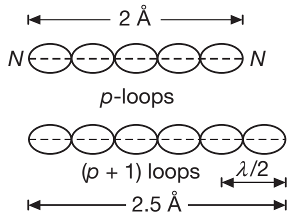
or the stopping potential will be $1.06\ \text{V}$.

PROBLEM 10

Assume that the de-Broglie wave associated with an electron can form a standing wave between the atoms arranged in a one-dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed if the distance d between the atoms of the array is 2 Å. A similar standing wave is again formed if d is increased to 2.5 Å but not for any intermediate value of d. Find the energy of the electron in eV and the least value of d for which the standing wave of the type described above can form.

SOLUTION

From the figure it is clear that



So, $p\cdot\left(\frac{\lambda}{2}\right)=2\ \text{Å}$ and

$$(p+1)\cdot\frac{\lambda}{2}=2.5\ \text{Å}$$

$$\Rightarrow \quad \frac{\lambda}{2}=(2.5-2)\ \text{Å}=0.5\ \text{Å}$$

$$\Rightarrow \quad \lambda=1\ \text{Å}=10^{-10}\ \text{m}$$

de-Broglie wavelength is given by

$$\lambda=\frac{h}{p}=\frac{h}{\sqrt{2mK}}$$

where K is the kinetic energy of electron

$$\Rightarrow \quad K=\frac{h^2}{2m\lambda^2}$$

$$\Rightarrow \quad K=\frac{(6.63\times10^{-34})^2}{2(9.1\times10^{-31})(10^{-10})^2}$$

$$\Rightarrow \quad K=2.415\times10^{-17}\ \text{J}$$

$$\Rightarrow \quad K=\left(\frac{2.415\times10^{-17}}{1.6\times10^{-19}}\right)\text{eV}$$

$$\Rightarrow \quad K=150.8\ \text{eV}$$

The least value of d will be, when only one loop is formed. So, we have

$$d_{min} = \frac{\lambda}{2}$$

$$\Rightarrow \quad d_{min} = 0.5 \text{ Å}$$

PROBLEM 11

When a beam of 10.6 eV photons of intensity 2 Wm^{-2} falls on a platinum surface of area $1\times10^{-4} \text{ m}^2$ and work function 5.6 eV. 0.53% of the incident photons eject photoelectrons. Find the number of photoelectrons emitted per second and their minimum and maximum energies (in eV). Take $1 \text{ eV} = 1.6\times10^{-19} \text{ J}$.

SOLUTION

Energy of incident photons,

$$E_i = 10.6 \text{ eV}$$

$$\Rightarrow \quad E_i = 10.6\times1.6\times10^{-19} \text{ J}$$

$$\Rightarrow \quad E_i = 16.96\times10^{-19} \text{ J}$$

Energy incident per unit area per unit time $(\text{intensity}) = 2 \text{ J}$

So, number of photons incident on unit area in unit time is

$$\frac{n}{A} = \frac{2}{16.96\times10^{-19}} = 1.18\times10^{18}$$

Therefore, number of photons incident per unit time on given area $(1\times10^{-4} \text{ m}^2)$ is

$$n = (1.18\times10^{18})(1\times10^{-4})$$

$$\Rightarrow \quad n = 1.18\times10^{14}$$

But only 0.53% of incident photons emit photoelectrons, so number of photoelectrons emitted per second (n) is

$$n = \left(\frac{0.53}{100}\right)(1.18\times10^{14})$$

$$\Rightarrow \quad n = 6.25\times10^{11}$$

$$K_{min} = 0$$

and $\quad K_{max} = E_i - W$

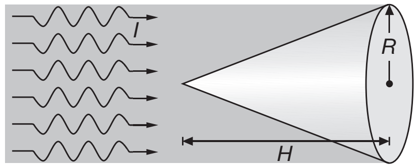
$$\Rightarrow \quad K_{max} = (10.6-5.6) \text{ eV} = 5 \text{ eV}$$

$$\Rightarrow \quad K_{max} = 5 \text{ eV}$$

and $\quad K_{min} = 0$

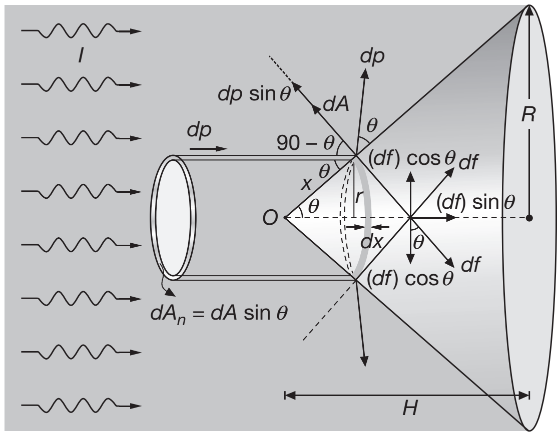
PROBLEM 12

A cone of radius R and height H with perfectly reflecting lateral surface, is placed in the path of a light beam of intensity I as shown. Calculate the force exerted by the light beam on this cone.



SOLUTION

To find the force on cone, we consider an elemental strip of width dx on the lateral surface of cone at a distance x from the vertex O of cone as shown in figure.



If the radius of the strip is r, then surface area of the strip is

$$dA = (2\pi r)dx \qquad \ldots(1)$$

Since for the cone, we have

$$\sin\theta = \frac{r}{x} = \frac{R}{\sqrt{R^2+H^2}} \qquad \ldots(2)$$

$$\Rightarrow \quad r = x\sin\theta$$

So, the area of strip is

$$dA = 2\pi(x\sin\theta)dx$$

If dA_n be the projection of area dA of the slant strip along the cross-sectional plane of the light beam i.e. normal to the light beam, then we have

$$dA_n = dA\sin\theta \qquad \ldots(3)$$

If dP is the power of light beam incident on this infinitesimal strip, then

$$dP = IdA_n \quad \ldots(4)$$

So, the force df on this infinitesimal strip element acts along the normal at the point of incidence and is given by

$$df = \left(\frac{2dP}{c}\right)\sin\theta = \left(\frac{2IdA_n}{c}\right)\sin\theta = \frac{2IdA\sin^2\theta}{c} \quad \ldots(5)$$

On resolving this infinitesimal force, we observe that the vertical components of the forces cancel, whereas the net force F is only obtained by integrating the component $df\sin\theta$ acting along the beam of light.

$$F = \int df\sin\theta$$

$$\Rightarrow \quad F = \int\left(\frac{2IdA}{c}\sin^2\theta\right)\sin\theta$$

$$\Rightarrow \quad F = \int\left(\frac{2I}{c}\sin^3\theta\right)dA = \int\left(\frac{2I}{c}\sin^3\theta\right)(2\pi(x\sin\theta)dx)$$

$$\Rightarrow \quad F = \left(\frac{4\pi I}{c}\sin^4\theta\right)\int_0^{\sqrt{R^2+h^2}} xdx$$

$$\Rightarrow \quad F = \left(\frac{4\pi I}{c}\sin^4\theta\right)\left(\frac{x^2}{2}\Bigg|_0^{\sqrt{R^2+h^2}}\right)$$

$$\Rightarrow \quad F = \left(\frac{2\pi I}{c}\right)\left(\frac{R^4}{(R^2+H^2)^2}\right)(R^2+H^2)$$

$$\Rightarrow \quad F = \left(\frac{2\pi I}{c}\right)\left(\frac{R^4}{R^2+H^2}\right) = \frac{2\pi IR^4}{c(R^2+H^2)}$$

PROBLEM 13

A small plate of a metal having work function of 1.17 eV is placed at a distance of 2 m from a monochromatic light source of wave length $4.8\times10^{-7}\text{ m}$ and power 1 W. The light falls normally on the plate. Find the number of photons striking the metal plate per m^2 per sec. If a constant uniform magnetic field of strength 10^{-4} T is applied parallel to the metal surface, find the radius of the largest circular path followed by the emitted photo electrons. Given $h = 6.6\times10^{-34}\text{ Js}$, $c = 3\times10^8\text{ ms}^{-1}$, $e = 1.6\times10^{-19}\text{ C}$ and electron mass $m = 9.1\times10^{-31}\text{ kg}$.

SOLUTION

Energy of photons of wavelength $4.8\times10^{-19}\text{ J}$ is

$$E = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^8}{4.8\times10^{-7}} = 4.125\times10^{-19}\text{ J}$$

Power of source $= 1\text{ W} = 1\text{ Js}^{-1}$

So, rate of emission of photons from the source is

$$n = \frac{1\text{ Js}^{-1}}{4.125\times10^{-19}\text{ J}} = 2.424\times10^{18}\text{ s}^{-1}$$

These photons move in all directions randomly. At a distance r from the source, the photons fall normally over a spherical surface of area $4\pi r^2$. The plate is at a distance $r = 2\text{ m}$. Hence the number of photons striking the surface per m^2 per second is

$$n = \frac{2.424\times10^{18}}{4\times3.14\times(2)^2} = 4.82\times10^{16}$$

The maximum KE of a photoelectron emitted from the plate is

$$K_{max} = \frac{hc}{\lambda} - W_0$$

$$\Rightarrow \quad K_{max} = 4.125\times10^{-19} - 1.17\times1.6\times10^{-19}$$

$$\Rightarrow \quad K_{max} = 2.253\times10^{-19}\text{ J}$$

Hence the maximum velocity of the photoelectron is

$$v_{max} = \sqrt{\frac{2K_{max}}{m}}$$

$$\Rightarrow \quad v_{max} = \sqrt{\frac{2\times2.153\times10^{-19}}{9.1\times10^{-31}}} = 7.03\times10^5\text{ ms}^{-1}$$

Radius of the largest circular path of the photoelectrons in the magnetic field is

$$r = \frac{mv_{max}}{eB} = \frac{9.1\times10^{-31}\times7.036\times10^5}{1.6\times10^{-19}\times10^{-4}}$$

$$\Rightarrow \quad r = 4\times10^{-2}\text{ m} = 4\text{ cm}$$

PROBLEM 14

Two metallic plates A and B each of area $5\times10^{-4}\text{ m}^2$, are placed parallel to each other at separation of 1 cm. Plate B carries a positive charge of $33.7\times10^{-12}\text{ C}$.

A monochromatic beam of light, with photons of energy 5 eV each, starts falling on place A at $t=0$ so that 10^6 photons fall on it per square meter per second. Assume that the photoelectron is emitted for every 10^6 incident photons. Also assume that all the emitted photoelectrons are collected by plate B and the work function of plate A remains constant at the value 2 eV.

(a) the number of photoelectrons emitted up to $t=10$ s

(b) the magnitude of the electric field between the plates A and B at $t=10$ s and

(c) The kinetic energy of the most energetic photo-electrons emitted at t = 10 s when it reaches plate B.

Neglect the time taken by the photoelectron to reach plate B.

(Take $\varepsilon_0 = 8.85\times10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2}$)

SOLUTION

Area of plates $A = 5\times10^{-4}\ \text{m}^2$

Distance between the plates $d = 1\ \text{cm} = 10^{-2}\ \text{m}$

(a) Number of photoelectrons emitted upto $t=10$ s are

$$n = \frac{\begin{pmatrix}\text{number of photons}\\ \text{falling in unit}\\ \text{area in unit time}\end{pmatrix}\times(\text{area}\times\text{time})}{10^6}$$

$$\Rightarrow\quad n = \frac{1}{10^6}\left[(10)^{16}\times(5\times10^{-4})\times(10)\right]$$

$$\Rightarrow\quad n = 5\times10^7$$

(b) At time $t=10$ s, charge on plate A

$$q_A = +ne = (5\times10^7)(1.6\times10^{-19})$$

$$\Rightarrow\quad q_A = 8\times10^{-12}\ \text{C}$$

and charge on plate B,

$$q_B = (33.7\times10^{-12} - 8\times10^{-12})$$

$$\Rightarrow\quad q_B = 25.7\times10^{-12}\ \text{C}$$

Since, electric field between the plates is

$$E = \frac{(q_B - q_A)}{2A\varepsilon_0}$$

$$\Rightarrow\quad E = \frac{(25.7-8)\times10^{-12}}{2\times(5\times10^{-4})(8.85\times10^{-12})} = 2\times10^3\ \text{NC}^{-1}$$

(c) Energy of photoelectrons at plate A is

$$K = E - W = (5-2)\ \text{eV} = 3\ \text{eV}$$

Increase in kinetic energy of photoelectrons when they reach B is

$$\Delta K = (eEd)\ \text{joule} = (Ed)\ \text{eV}$$

$$\Rightarrow\quad \Delta K = (2\times10^3)(10^{-2})\text{eV} = 20\ \text{eV}$$

Energy of photoelectrons at plate B is

$$K_B = (20+3)\ \text{eV} = 23\ \text{eV}$$

PROBLEM 15

A mercury arc lamp provides 100 mW of UV radiation at a wavelength of 2480 Å (all other wavelengths having been absorbed by filters). The cathode of photoelectric device (a photo-tube) consists of potassium and has an effective area of $4\ \text{cm}^2$. The anode is located at a distance of 1 m from radiation source. The work function (ϕ_0) for potassium is 2.25 eV.

(a) According to classical theory, the radiation from the arc spreads out uniformly in space as spherical wave. Calculate the time of exposure of the metal to the radiation so that a potassium atom (radius 2 Å) in the anode accumulates sufficient energy to eject a photo-electron.

(b) Calculate energy of a single photon from the source.

(c) Calculate the number of photons striking the cathode per second. Also calculate the saturation current, if the photo-conversion efficiency is 5% (i.e., if each photon has a probability of 0.05 of ejecting an electron).

(d) Calculate the cut off potential V_0.

Given $hc = 12400$ eVÅ

SOLUTION

(a) The energy emitted per second per unit area (i.e. intensity) of the UV lamp at a distance of one metre is

$$I = \frac{P}{4\pi r^2} = \frac{100\times10^{-3}}{4\pi(1)^2} = \frac{0.1}{4\pi}\ \text{Wm}^{-2}$$

The cross-sectional area of atom i.e. effective area of the atom exposed to radiation is $A_{\text{eff}} = \pi r^2$

$$\Rightarrow\quad A_{\text{eff}} = \pi(2\times10^{-10}\ \text{m})^2 = 4\pi\times10^{-20}\text{m}^2$$

Energy required to eject photo-electron from the metal surface is

$$\phi_0 = (2.25)(1.6\times10^{-19})\ \text{J} = 3.6\times10^{-19}\ \text{J}$$

Since, $\phi_0 = IAt$, hence the exposure time is given by

$$t = \frac{\phi_0}{IA_{\text{eff}}} = \frac{3.6\times10^{-19}}{\left(\frac{0.1}{4\pi}\right)(4\pi\times10^{-20})} = 360\ \text{s}$$

(b) Incident photon energy in eV is

$$E = \frac{12400}{2480}\ \text{eV} = 5\ \text{eV}$$

$$\Rightarrow \quad E = 5\times1.6\times10^{-19}\ \text{J} = 8\times10^{-19}\ \text{J}$$

(c) Since, we know that the intensity

$$I = \frac{E_{\text{photon source}}}{At} = \left(\frac{N}{t}\right)\left(\frac{E_{\text{each photon}}}{A}\right)$$

$$\Rightarrow \quad I = n\left(\frac{E_{\text{each photon}}}{A}\right)$$

$$\Rightarrow \quad n = \frac{IA}{E_{\text{each photon}}}$$

So, at the cathode (area $4\times10^{-4}\,\text{m}^2$), the number of photons striking per second is

$$n = \frac{IA}{E_{\text{each photon}}} = \left(\frac{0.1}{4\pi}\right)\frac{4\times10^{-4}}{8\times10^{-19}}$$

$$\Rightarrow \quad n \approx 4\times10^{12}\ \text{photons/s}$$

With an efficiency of 5%, the photo-current is

$$I = \left(\frac{5}{100}\right)ne = (0.05)(4\times10^{12})(1.6\times10^{-19})$$

$$\Rightarrow \quad I = 32\times10^{-9}\ \text{A} = 32\ \text{nA}$$

(d) The stopping potential for ejected electrons is

$$V_s = \frac{h\nu - \phi_0}{e} = \frac{(5-2.25)\ \text{eV}}{e} = 2.75\ \text{V}$$

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. Consider a source emitting $100\ \text{W}$ of green light at a wavelength of $500\ \text{nm}$. The number of photons emerging from source per second is

(A) 2.5×10^{19} photon per second

(B) 25×10^{20} photon per second

(C) 25×10^{19} photon per second

(D) 25×10^{17} photon per second

2. The distance d of a $100\ \text{W}$ lamp is continuously increased from a photocell. The photoelectric current I varies with distance d as

(A) $I\propto d^2$ (B) $I\propto d$

(C) $I\propto \dfrac{1}{d^2}$ (D) $I\propto \dfrac{1}{d}$

3. When a metallic surface is illuminated with monochromatic light of wavelength λ, the stopping potential is $5V_0$. When the same surface is illuminated with light of wavelength 3λ, the stopping potential is V_0. Then the work function of the metallic surface is

(A) $\dfrac{hc}{6\lambda}$ (B) $\dfrac{hc}{5\lambda}$

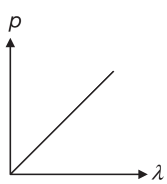
(C) $\dfrac{hc}{4\lambda}$ (D) $\dfrac{2hc}{4\lambda}$

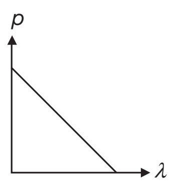
4. An electron of mass m, when accelerated through a potential difference V has de-Broglie wavelength λ. The de-Broglie wavelength associated with a proton of mass M accelerated through the same potential difference will be

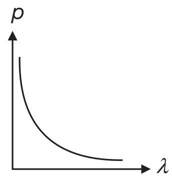
(A) $\lambda\left(\dfrac{m}{M}\right)$ (B) $\lambda\sqrt{\dfrac{m}{M}}$

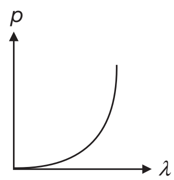
(C) $\lambda\left(\dfrac{M}{m}\right)$ (D) $\lambda\sqrt{\dfrac{M}{m}}$

5. Which of the following graphs represents the variation of particle momentum and the associated de Broglie wavelength?

(A)


(B)


(C)


(D)


6. If Planck's constant is denoted by h and the electronic charge by e, experiments on photoelectric effect allow the determination of

(A) only h (B) only e

(C) both h and e (D) only $\dfrac{h}{e}$

7. The maximum energy of the electrons released in a photocell is independent of

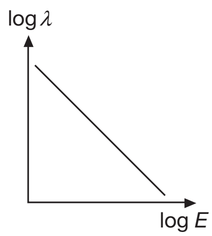
(A) frequency of incident light

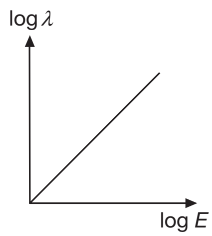
(B) intensity of incident light

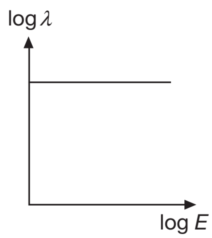
(C) nature of cathode rays

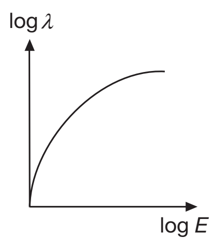
(D) None of these

8. If the energy and wavelength of electron are E and λ, then the graph between $\log E$ and $\log\lambda$ will be

(A)


(B)


(C)


(D)


9. The total energy E of a sub-atomic particle of rest mass m_0 moving at non-relativistic speed v is

(A) $E=m_0c^2$ (B) $E=\dfrac{1}{2}m_0v^2$

(C) $E=m_0c^2+\dfrac{1}{2}m_0v^2$ (D) $E=m_0c^2-\dfrac{1}{2}m_0v^2$

10. An electron is 2000 times lighter than a proton. Both are moving such that their matter waves have a length of 1 Å. The ratio of their kinetic energy in approximation is
(A) 1 : 1 (B) 1 : 2000
(C) 2000 : 1 (D) 1 : 200

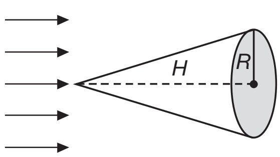
11. The de Broglie wavelength of a particle is approximately the same as that of a photon with the same energy.
(A) The energy of the particle is much greater than its rest energy.
(B) The energy of the particle is much less than its rest energy.
(C) The energy of the particle equals its rest energy.
(D) Data insufficient to arrive at a conclusion.

12. Two lumps of clay each of rest mass m_0, collide with a speed of $\frac{4}{5}c$ head on and stick together. The mass of the composite lump thus formed is
(A) $\frac{10}{3}m_0$ (B) $\frac{5}{3}m_0$
(C) $\frac{5}{6}m_0$ (D) $\frac{5}{12}m_0$

13. An electron moving with velocity $2\times10^{7}\ \text{ms}^{-1}$ describes a circle in a magnetic field of strength 2×10^{-2} T. If $\left(\frac{e}{m}\right)$ of electron is $1.76\times10^{11}\ \text{C kg}^{-1}$, then the diameter of the circle is nearly
(A) 1.1 m (B) 1.1 mm
(C) 1.1 cm (D) 11 cm

14. The radiation emitted, when an electron jumps from $n=3$ to $n=2$ orbit is a hydrogen atom, falls on a metal to produce photoelectron. The electrons from the metal surface with maximum kinetic energy are made to move perpendicular to a magnetic field of $\frac{1}{320}$ T in a radius of 10^{-3} m. The work function of metal is
(A) 1.03 eV (B) 1.89 eV
(C) 0.86 eV (D) 2.03 eV

15. The radiation force experienced by a body exposed to radiation of intensity I assuming surface of body to be perfectly absorbing is



(A) $\frac{\pi R^2 I}{c}$ (B) $\frac{2\pi R^2 I}{c}$
(C) $\frac{4\pi R^2 I}{c}$ (D) None of these

16. A radiation of energy E falls normally on a perfectly reflecting surface. The change in momentum of radiation is
(A) $\frac{E}{c}$ (B) $\frac{2E}{c}$
(C) Ec (D) $\frac{E}{c^2}$

17. An electron, accelerated by a potential difference V, has de Broglie wavelength λ. If the electron is accelerated by a potential difference $4V$, its de Broglie wavelength will be
(A) 2λ (B) 4λ
(C) $\frac{\lambda}{2}$ (D) $\frac{\lambda}{4}$

18. The photoelectric work function of a metal is 1 eV. Light of wavelength $\lambda = 3000$ Å falls on it. The photoelectrons will come out with approximate speed equal to
(A) $10\ \text{ms}^{-1}$ (B) $10^{2}\ \text{ms}^{-1}$
(C) $10^{4}\ \text{ms}^{-1}$ (D) $10^{6}\ \text{ms}^{-1}$

19. A proton and an α-particle are accelerated through the same potential difference. The ratio of their de Broglie wavelengths is
(A) $\sqrt{2}$ (B) $\frac{1}{\sqrt{2}}$
(C) $2\sqrt{2}$ (D) 2

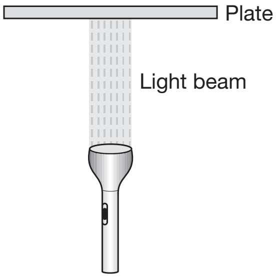
20. In an electron microscope if the potential is increased from 20 kV to 80 kV, the resolving power R of the microscope will become
(A) R (B) $2R$
(C) $4R$ (D) $\frac{R}{2}$

21. Boron has two isotopes ${}_5B^{10}$ and ${}_5B^{11}$. If the atomic weight of boron is 10.81, the ratio of ${}_5B^{10}$ to ${}_5B^{11}$ in nature is
(A) $\frac{19}{81}$ (B) $\frac{20}{53}$
(C) $\frac{15}{10}$ (D) $\frac{10}{11}$

22. In photoelectric emission the number of electrons ejected per second is proportional to the

(A) intensity of light
(B) wavelength of light
(C) frequency of light
(D) work function of the material

23. A plate of mass 10 g is in equilibrium in air due to the force exerted by light beam on plate. Calculate power of beam. Assume plate is perfectly absorbing.



(A) 1.5×10^7 W (B) 3×10^7 W
(C) 4.5×10^7 W (D) 6×10^7 W

24. For a photon, the de Broglie relation is given by

(A) $\lambda=\dfrac{h}{mc}$ (B) $\lambda=\dfrac{h}{p}$
(C) $\lambda\to$ infinity (D) Data Insufficient

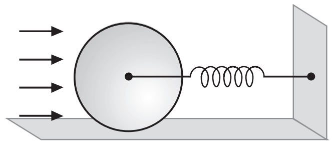
25. A desklamp illuminates a desk top with light of wavelength λ. The amplitude of this electromagnetic wave is E_0. Assuming illumination to be normally on the surface, the number of photons striking the desk per second per unit area N is

(A) $N=\dfrac{\lambda\varepsilon_0E_0^2}{h}$ (B) $N=\dfrac{2\lambda\varepsilon_0E_0^2}{h}$
(C) $N=\dfrac{\lambda\varepsilon_0E_0^2}{2h}$ (D) Data Insufficient

26. A small metal plate (work-function W_0) is kept at a distance d from a singly ionised fixed ion. A monochromatic light beam is incident on the metal plate and photoelectrons are emitted. The maximum wavelength of light so that the photo-electrons may go round the ions along a circle is

(A) $\dfrac{8\pi\varepsilon_0W_0d+e^2}{8\pi hc\varepsilon_0d}$ (B) $\dfrac{8\pi hc\varepsilon_0d}{8\pi\varepsilon_0W_0d+e^2}$
(C) $\dfrac{2\left(\dfrac{hc}{W_0}-e^2\right)}{n}$ (D) $\dfrac{8\pi\varepsilon_0d}{hceW_0}$

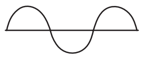
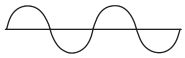
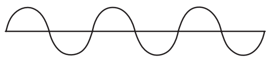
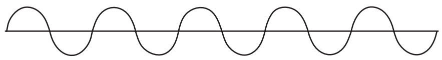
27. A perfectly reflecting body sphere in shape of radius R is placed on a path of parallel light beam of intensity I shown in figure. One end of a spring is attached to centre of sphere and the other end to a rigid wall as shown in figure. Assuming the sphere to be in equilibrium, the spring constant of spring to be K, then compression in spring is



(Neglect any thermal effect and friction is absent)

(A) $\dfrac{2I}{KC}\pi R^2$ (B) $\dfrac{I}{KC}\pi R^2$
(C) $\dfrac{I\pi R^2}{C}$ (D) None of these

28. The de-Broglie wave present in the fifth Bohr orbit is

(A) 
(B) 
(C) 
(D) 

29. The interatomic distance between atoms in a crystal is 2.8 Å. Then if such a crystal is used in Davisson-Germer experiment, the maximum order of diffraction that can be observed for a beam of electrons accelerated by 100 V shall be

(A) $n=1$ (B) $n=2$
(C) $n=10$ (D) $n\to\infty$

30. The energy of a photon of wavelength λ is

(A) $hc\lambda$ (B) $\dfrac{hc}{\lambda}$
(C) $\dfrac{\lambda}{hc}$ (D) $\dfrac{h\lambda}{c}$

31. The work function of a metal in 4 eV. For the emission of photoelectrons of zero velocity from the metal surface, the wavelength of the incident radiation should be

(A) 1700 Å (B) 2700 Å
(C) 3100 Å (D) 5900 Å

32. Photoelectric effect is the phenomenon in which
(A) photons come out of a metal when it is hit by a beam of electrons
(B) photons come out of the nucleus of an atom under the action of an electric field.

(C) electrons come out of a metal with a constant velocity which depends on the frequency and intensity of incident radiation.
(D) electrons come out of a metal with different velocities not greater than a certain value which depends only on the frequency of the incident light and not on its intensity.

33. The photoelectric effect is the ejection of electrons from the surface of a metal when
(A) it is heated to a high temperature.
(B) electrons of suitable velocity strike it.
(C) radiation of suitable wavelength falls on it.
(D) it is placed in a strong electric field.

34. Matter waves are
(A) electromagnetic waves.
(B) transverse mechanical waves.
(C) longitudinal mechanical waves.
(D) neither electromagnetic nor mechanical waves.

35. A photon of wavelength $1000\ \text{Å}$ has energy 12.3 eV. If light of wavelength $5000\ \text{Å}$, having intensity I, falls on a metal surface, the saturation current is $0.40\ \mu\text{A}$ and the stopping potential is 1.36 V. The work function of the metal is
(A) 2.47 eV (B) 1.36 eV
(C) 1.10 eV (D) 0.43 eV

36. In PROBLEM 35, if the intensity of light is made $4I$, the stopping potential will become
(A) 1.36 V (B) 2.72 V
(C) 5.44 V (D) 21.76 V

37. In PROBLEM 35, if the intensity of light is made $4I$, the saturation current will become
(A) $0.4\ \mu\text{A}$ (B) $0.8\ \mu\text{A}$
(C) $1.6\ \mu\text{A}$ (D) $6.4\ \mu\text{A}$

38. In a photoemissive cell with exciting wavelength λ, the fastest electron has a speed v. If the exciting wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electrons will be
(A) $v\sqrt{\frac{3}{4}}$ (B) $v\sqrt{\frac{4}{3}}$
(C) less than $v\sqrt{\frac{4}{3}}$ (D) greater than $v\sqrt{\frac{4}{3}}$

39. The de-Broglie wavelength of a molecule of thermal energy k_BT (k_B = Boltzmann constant and T = absolute temperature), is
(A) $\lambda = \sqrt{\frac{h}{2mk_BT}}$ (B) $\frac{h}{\sqrt{2mk_BT}}$
(C) $h\sqrt{2mk_BT}$ (D) $\frac{h}{4m^2k_B^2T^2}$

40. A proton is accelerated through a potential V. The de Broglie wavelength associated with it is
(A) $\frac{12.27}{\sqrt{V}}\ \text{Å}$ (B) $\frac{0.287}{\sqrt{V}}\ \text{Å}$
(C) $\frac{12.27}{\sqrt{V}}$ fm (D) $\frac{0.287}{\sqrt{V}}$ fm

41. In Davisson-Germer experiment Ni crystal acts as
(A) an ideal reflector
(B) three dimensional diffraction grating
(C) an ideal absorber
(D) two dimensional diffraction grating

42. Minimum light intensity that can be perceived by normal human eye is about $10^{-10}\ \text{Wm}^{-2}$. What is the minimum number of photons of wavelength 660 nm that must enter the pupil in one second, for one to see the object? Area of cross-section of the pupil is $10^{-4}\ \text{m}^2$?
(A) 3.318×10^3 (B) 1.453×10^3
(C) 3.318×10^4 (D) 1.453×10^5

43. The angle between the incident and the diffracted electron in the Davisson-Germer experiment is called as
(A) angle of incidence (B) angle of diffraction
(C) angle of scattering (D) none of the above

44. In Davisson-Germer experiment maximum intensity is observed at
(A) 50° and 54 volt (B) 54° and 50 volt
(C) 50° and 50 volt (D) 65° and 50 volt

45. A parallel beam of light of intensity I and cross section area S is incident on a plate at normal incidence. The photoelectric emission efficiency is 100%, the frequency of beam is ν and the work function of the plate is $\phi(h\nu>\phi)$. Assuming all the electrons are ejected normal to the plane and with same maximum possible speed. Calculate the net force exerted on the plate only due to striking of photons and subsequent emission of electrons
(A) $\frac{IS}{h\nu}\left(\frac{h}{\lambda}+\sqrt{m(h\nu-\phi)}\right)$
(B) $\frac{IS}{h\nu}\left(\frac{h}{\lambda}+\sqrt{2m(h\nu-\phi)}\right)$

(C) $\frac{h\nu S}{I}\left(\frac{h}{\lambda}+\sqrt{2m(h\nu-\phi)}\right)$

(D) None of these

46. In Davisson-Germer experiment, an electron beam of 60 eV energy falls normally to the surface of the crystal and maximum intensity is obtained at an angle of $60°$ to the direction of incident beam. The inter-atomic distance in the lattice plane of the crystal is

(A) 18 Å (B) 3.6 Å
(C) 1.8 Å (D) 0.18 Å

47. The incorrect statement in connection with Davisson and Germer experiment is
(A) The inter-atomic distance in nickel crystal is of the order of the de-Broglie wavelength.
(B) Electrons of constant energy are obtained by the electron gun.
(C) Nickel crystal acts as a three dimensional diffracting grating.
(D) Davisson-Germer experiment is an interference experiment.

48. In Davisson-Germer experiment the relation between Bragg's angle ϕ and diffraction angle θ is

(A) $\theta = 90° - \phi$ (B) $\theta = \frac{90° - \phi}{2}$

(C) $\theta = 180° - \phi$ (D) $\phi = \left(\frac{180° - \theta}{2}\right)$

49. The ionization chamber used is Davisson-Germer experiment, acts as
(A) emitter (B) collector
(C) source (D) radiator

50. The distance between two consecutive atoms of the crystal lattice is 1.227 Å. The maximum order of diffraction of electrons accelerated through 10^4 V will be

(A) 10 (B) $\frac{1}{10}$

(C) 100 (D) $\frac{1}{100}$

51. The human eye can barely detect a yellow light (6000 Å) that delivers 1.7×10^{-18} watt to the retina. Nearly how many photons per second does the retina receive ?
(A) 50
(B) 5
(C) 500
(D) More than 5 million

52. The ratio of the specific charge of a proton to that of an α-particle is
(A) 1 : 4 (B) 1 : 2
(C) 4 : 1 (D) 2 : 1

53. A photosensitive surface is receiving light of wavelength 5000 Å at the rate of 10^{-7} Js^{-1}. The number of photons received per second is
(A) 2.5×10^{12} (B) 2.5×10^{11}
(C) 2.5×10^{10} (D) 2.5×10^{9}

54. Photons of frequency ν fall on a metal surface for which the threshold frequency is ν_0. Then,
(A) all ejected electrons have the same kinetic energy $h(\nu-\nu_0)$.
(B) the ejected electrons have a distribution of kinetic energy from zero to $h(\nu-\nu_0)$.
(C) the most energetic electrons have kinetic energy $h\nu$
(D) the average kinetic energy of ejected electrons is $h\nu_0$.

55. When radiation of wavelength 3000 Å is incident on a photosensitive surface, the kinetic energy of electrons is 2.5 eV. The stopping potential for radiation of wavelength 1500 Å will be
(A) 2.5 V
(B) 5.0 V
(C) less than 5.0 V but more than 2.5 V
(D) more than 5.0 V

56. The de Broglie wavelength of a particle of mass m moving with a kinetic energy E is

(A) $\sqrt{\frac{h}{2mE}}$ (B) $\frac{h}{\sqrt{2mE}}$

(C) $\frac{h}{2mE}$ (D) $\frac{\sqrt{h}}{2mE}$

57. Two photons of energy 2.5 eV each are incident on a metal plate whose work function is 4.0 eV, then the number of electrons emitted from the metal surface will be
(A) one
(B) two
(C) None of these
(D) more than two

58. Of the following moving with same momentum, the one which has largest wavelength is
(A) an electron.
(B) a proton.
(C) an α-particle.
(D) all have same de-Broglie wavelength.

59. The maximum velocity of an electron emitted by light of wavelength λ incident on the surface of a metal of work-function ϕ is

(A) $\sqrt{\dfrac{2(hc+\lambda\phi)}{m\lambda}}$ (B) $\dfrac{2(hc-\lambda\phi)}{m}$

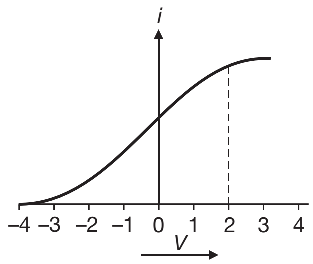
(C) $\sqrt{\dfrac{2(hc-\lambda\phi)}{m\lambda}}$ (D) $\sqrt{\dfrac{2(h\lambda-\phi)}{m}}$

where $h=$ Planck's constant, $m=$ mass of electron and $c=$ speed of light

60. An α–particle is accelerated through a potential difference of 200 V. The increase in its kinetic energy in electron volt will be

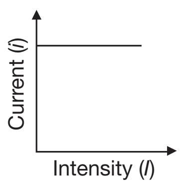
(A) 100 eV (B) 200 eV
(C) 400 eV (D) 800 eV

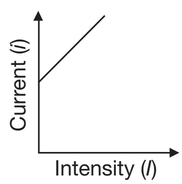
61. Figure represents the graph of photo current i versus applied voltage (V). The maximum energy of the emitted photoelectrons is

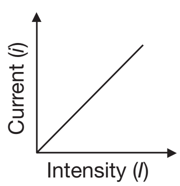


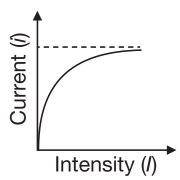
(A) 2 eV (B) 4 eV
(C) 0 eV (D) 4 J

62. Which one of the following graphs represents correctly the variation of photoelectric current (i) with intensity (I) of incident radiations

(A)


(B)


(C)


(D)


63. The potential difference between the cathode and anode in a cathode ray tube is V. Then the speed acquired by the electrons is proportional to

(A) V (B) $\sqrt{V}$
(C) V^2 (D) $V^{3/2}$

64. The duration of a laser pulse is 10^{-8} s. The uncertainty in its energy will be

(A) 6.6×10^{-26} J (B) 6.6×10^{-34} J
(C) 6.6×10^{-42} J (D) $\dfrac{1}{6.6}\times10^{26}$ J

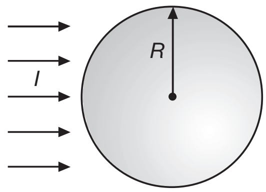
65. Which one of the following statements about photons is incorrect?

(A) Rest mass of a photon is zero
(B) Momentum of a photon of frequency ν is $\dfrac{h\nu}{c}$
(C) Energy of a photon of frequency ν is $h\nu$
(D) Photons exert no pressure

66. A parallel beam of uniform, monochromatic light of wavelength 2640 Å has an intensity of 100 Wm^{-2}. The number of photons in 1 mm^3 of this radiation are

(A) 222 (B) 335
(C) 442 (D) 555

67. Consider a sphere of radius R exposed to radiation of intensity I as shown in figure. If surface of sphere is partially reflection and reflection coefficient is 0.3, then radiation force experienced is



(A) $\dfrac{\pi R^2 I}{c}$ (B) $\dfrac{1.7\pi R^2 I}{c}$

(C) $\dfrac{0.3\pi R^2 I}{c}$ (D) None of these

68. A charge particle q of mass m is projected along the y-axis at $t=0$ from origin with a velocity v_0. If a uniform electric field E also exists along the x-axis, then the time at which de-Broglie wavelength of the particle becomes half of the initial value is

(A) $\dfrac{mv_0}{qE}$ (B) $2\left(\dfrac{mv_0}{qE}\right)$

(C) $\sqrt{3}\left(\dfrac{mv_0}{qE}\right)$ (D) $3\left(\dfrac{mv_0}{qE}\right)$

69. Two electrons are moving with the same speed v. One electron enters a region of uniform electric field while the other enters a region of uniform magnetic field, then after sometime if the de-Broglie wavelengths of the two are λ_1 and λ_2, then

(A) $\lambda_1=\lambda_2$ (B) $\lambda_1>\lambda_2$
(C) $\lambda_1<\lambda_2$ (D) $\lambda_1>\lambda_2$ or $\lambda_1<\lambda_2$

70. An electromagnetic radiation of wavelength λ has the same momentum as an electron moving with a speed $2\times10^5\ \text{ms}^{-1}$.

(A) $\lambda = 2.64$ nm (B) $\lambda = 1.64$ nm
(C) $\lambda = 3.64$ nm (D) $\lambda = 4.64$ nm

71. Let K_1 be the maximum kinetic energy of photo-electrons emitted by light of wavelength λ_1 and K_2 corresponding to wavelength λ_2. If $\lambda_1 = 2\lambda_2$ then

(A) $2K_1 = K_2$ (B) $K_1 = 2K_2$
(C) $K_1 < \frac{K_2}{2}$ (D) $K_1 > 2K_2$

72. Solar constant of the sun is $\sigma = 8.106\times10^4\ \text{Jmin}^{-1}\text{m}^{-2}$ and average sun earth distance is 1.5×10^8 km. The yearly loss in the mass of the sun is

(A) 13.8×10^{17} kg (B) 1.38×10^{19} kg
(C) 1.38×10^{17} kg (D) 13.8×10^{20} kg

73. A photon strikes a free electron at rest and is scattered straight backward. If the speed of electron after collision is αc, where $\alpha \ll 1$ then,

(A) electron's kinetic energy is a fraction α of photon's initial energy.
(B) electron's kinetic energy is a fraction $\frac{1}{\alpha}$ of photon's initial energy.
(C) electron's kinetic energy is a fraction α^2 of photon's initial energy.
(D) electron's kinetic energy is a fraction $\frac{1}{\alpha^2}$ of photon's initial energy.

74. Which one of the following does not fit into the group?

(A) Photon (B) Graviton
(C) Proton (D) Meson

75. Both the frequency and the intensity of a beam of light falling on the surface of photoelectric material are increased by a factor of two. This will

(A) increase both, the maximum kinetic energy of the photo-electrons, as well as photoelectric saturation current by a factor of two.
(B) increase the maximum kinetic energy of the photo-electrons by a factor greater than two and would increase the photoelectric saturation current by a factor of two.
(C) increase the maximum kinetic energy of the photoelectrons by a factor greater than two and will have no effect on the magnitude of the photoelectric saturation current produced.
(D) increase the maximum kinetic energy of the emitted photo-electrons by a factor of two but will have no effect on the saturation photoelectric current.

76. Two photons approach each other. The relative velocity of approach is

(A) $\frac{c}{2}$ (B) c
(C) $2c$ (D) $4c$

77. A particle of mass $3m$ at rest decays into two particles of masses m and $2m$ having non-zero velocities. The ratio of the de-Broglie wavelengths of the particles $\left(\frac{\lambda_1}{\lambda_2}\right)$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$
(C) 1 (D) 2

78. A small ball is projected with initial speed u and at an angle θ with horizontal from ground. The de-Broglie wavelength of ball at the moment its velocity vector becomes perpendicular to initial velocity vector is

(A) $\frac{h}{mu}$ (B) $\frac{h}{mu\sin\theta}$
(C) $\left(\frac{h}{mu}\right)\tan\theta$ (D) $\frac{h}{mu\cos\theta}$

79. The force exerted by a photon of intensity $1.4\ \text{kWm}^{-2}$ if it falls on a perfect absorber of radius 2 m is

(A) 2.35×10^{-4} N (B) 10^8 N
(C) 8.35×10^4 N (D) 8.8×10^{-8} N

80. A point source of radiation power P is placed on the axis of completely absorbing disc. The distance between the source and the disc is 2 times the radius of the disc. Find the force that light exerts on the disc

(A) $\frac{P}{c}$ (B) $\frac{P}{5c}$
(C) $\frac{P}{10c}$ (D) $\frac{P}{20c}$

81. Photoelectrons are emitted with maximum kinetic energy E from a metal surface when light of frequency ν falls on it when light of frequency ν' falls on the same metal, the maximum kinetic energy of emitted photoelectrons is found to be $2E$, then ν' is

(A) $\nu' = \nu$ (B) $\nu' = 2\nu$
(C) $\nu' > 2\nu$ (D) $\nu' < 2\nu$

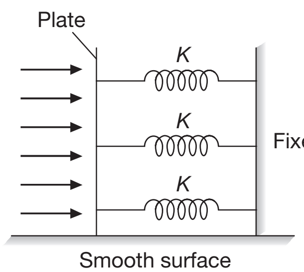
82. A material particle with a rest mass m_0 is moving with speed of light c. The de-Broglie wavelength associated is given by

(A) $\frac{h}{m_0 c}$ (B) $\frac{m_0 c}{h}$
(C) zero (D) ∞

83. Light of two different frequencies whose photons have energies 1 eV and 2.5 eV successively illuminate a metal of work function 0.5 eV. The ratio of the maximum speeds of the emitted electrons will be

(A) 1 : 5 (B) 1 : 4
(C) 1 : 2 (D) 1 : 1

84. Light of intensity I is incident normally on a perfectly reflecting plate of area A kept in a gravity free space. If the photons strike the plate symmetrically and initially the springs are in their natural lengths, then the maximum compression in the springs is



(A) $\frac{IA}{KC}$ (B) $\frac{2IA}{3KC}$
(C) $\frac{3IA}{KC}$ (D) $\frac{4IA}{3KC}$

85. The work function of a metallic surface is 5.01 eV. Photoelectrons are emitted when light of wavelength 2000 Å falls on it. The potential difference required to stop the fastest photoelectrons is

$\left(h = 4.14 \times 10^{-15} \text{ eVs}\right)$

(A) 1.2 V (B) 2.4 V
(C) 3.6 V (D) 4.8 V

86. A proton, accelerated through a potential difference V has a certain de Broglie wavelength. In order to have the same de Broglie wavelength, an α-particle must be accelerated through a potential difference

(A) $4V$ (B) $8V$
(C) $\frac{V}{4}$ (D) $\frac{V}{8}$

87. Radiation of frequency 1.5 times the threshold frequency is incident on a photosensitive material. If the frequency of incident radiations is halved and the intensity is doubled, the number of photoelectron ejected per second becomes

(A) zero
(B) half of its initial value
(C) one fourth the initial value
(D) three fourth the initial value

88. Ultraviolet light wavelength 300 nm and intensity 1.0 Wm^{-2} falls on the surface of a photoelectric material. If one percent of the incident photons produce photo electrons, then the number of photoelectrons emitted per second from an area of 1.0 cm^2 of the surface is nearly

(A) 9.61×10^{14} (B) 4.12×10^{13}
(C) 1.51×10^{12} (D) 2.13×10^{11}

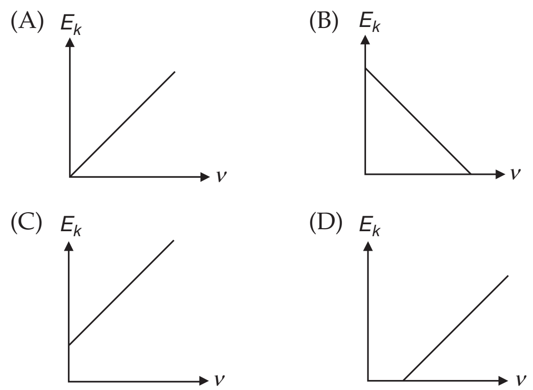
89. A proton and an α-particle are injected into a uniform electric field at right angles to the direction of field with equal kinetic energy. Then

(A) the proton trajectory will be less curved than α-particle trajectory.
(B) the α-particle trajectory will be less curved than proton trajectory.
(C) both the trajectories will be equally curved.
(D) both trajectories will be straight.

90. de Broglie waves are associated with

(A) moving charged particles only.
(B) moving neutral particles only.
(C) all moving particles.
(D) all particles whether in motion or at rest.

91. The maximum kinetic energy (E_K) of photoelectrons varies with the frequency (ν) of the incident radiation as



92. Stopping potential for photoelectrons

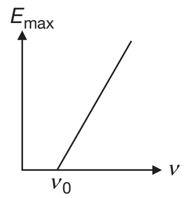
(A) does not depend on the frequency of the incident light.
(B) does not depend on the nature of cathode material.
(C) depends on both the frequency of the incident light and the nature of the cathode material.
(D) depends on the intensity of the incident light.

93. A particle of mass 10^{-31} kg is moving with a speed of 10^5 ms^{-1}. The de Broglie wavelength of the particle is

(A) 6.63×10^{-8} m (B) 6.63 Å
(C) 66.3 Å (D) 6.63×10^{-7} m

94. A moving particle is associated with a wave packet or group of waves. The group velocity is equal to
(A) velocity of light
(B) velocity of sound
(C) velocity of particle
(D) $\dfrac{1}{\text{particle velocity}}$

95. The maximum kinetic energy $(E_{\max})$ of photoelectrons emitted in a photoelectric cell varies with frequency (ν) as shown in the graph. The slope of the graph is equal to



(A) charge of the electron
(B) $\dfrac{e}{m}$ of the electron
(C) work function of the emitter
(D) Planck's constant

96. A photon of frequency ν is incident on a metal surface whose threshold frequency is ν_0. The maximum kinetic energy of the emitted electron will be
(A) $h(\nu - \nu_0)$ (B) $h(\nu + \nu_0)$
(C) $\dfrac{1}{2}h(\nu - \nu_0)$ (D) $\dfrac{1}{2}h(\nu + \nu_0)$

97. A stream of photons impinging normally on a completely absorbing screen in vacuum exerts a pressure P. If I is the irradiance then,
(A) $P = \dfrac{2I}{c}$ (B) $P = Ic$
(C) $P = \dfrac{I}{c}$ (D) $P = 2Ic$

98. An electron and a photon have same wavelength. It p is the momentum of electron and E the energy of photon. The magnitude of $\dfrac{p}{E}$ in SI unit is
(A) $\dfrac{1}{2c}$ (B) $\dfrac{1}{c}$
(C) $\dfrac{2}{c}$ (D) None of these

99. A metal plate is exposed to light with wavelength λ. It is observed that electrons are ejected from the surface of the plate. When a retarding uniform electric field E is imposed, no electron can move away from the plate farther than a certain distance d. Then the threshold wavelength λ_0 for the material of plate is (e is the electronic charge, h is Planck's constant and c is the speed of light)
(A) $\lambda_0 = \left(\dfrac{1}{\lambda} - \dfrac{hc}{eEd}\right)^{-1}$ (B) $\lambda_0 = \left(\dfrac{1}{\lambda} - \dfrac{eEd}{hc}\right)^{-1}$
(C) $\lambda_0 = \lambda - \dfrac{hc}{eEd}$ (D) $\lambda_0 = \lambda - \dfrac{eEd}{hc}$

100. A sensor is exposed for time t to a lamp of power P placed at a distance l. The sensor has an opening that is $4d$ in diameter. Assuming all energy of the lamp is given off as light, the number of photons entering the sensor if the wavelength of light is λ is
(A) $N = \dfrac{P\lambda d^2 t}{hcl^2}$ (B) $N = \dfrac{4P\lambda d^2 t}{hcl^2}$
(C) $N = \dfrac{P\lambda d^2 t}{4hcl^2}$ (D) $N = \dfrac{P\lambda d^2 t}{16hcl^2}$

101. When a metallic surface is illuminated by a monochromatic light of wavelength λ, the stopping potential for photoelectric current is $3V_0$. When the same surface is illuminated by light of wavelength 2λ, the stopping potential is V_0. The threshold wavelength for this surface for photoelectric effect is
(A) 6λ (B) $\dfrac{4\lambda}{3}$
(C) 4λ (D) 8λ

102. The momentum of a photon of an electromagnetic radiation is 3.3×10^{-29} kgms^{-1}. The frequency of the associated waves is $\left(h = 6.6 \times 10^{-34}\text{ Js}, c = 3 \times 10^8\text{ ms}^{-1}\right)$
(A) 3.0×10^3 Hz (B) 6.0×10^3 Hz
(C) 7.5×10^{12} Hz (D) 1.5×10^{13} Hz

103. If the wavelength of incident radiation in a photoelectric experiment is decreased then
(A) the photoelectric current will decrease.
(B) the photoelectric current will increase.
(C) the stopping potential will decrease.
(D) the stopping potential will increase.

104. The threshold wavelength for a photosensitive surface is 6000 Å and the wavelength of incident light is 5000 Å. Then the maximum energy of emitted electrons would be

(A) 0.041 eV (B) 0.41 eV
(C) 4.1 eV (D) 41 eV

105. An electron of mass m and charge e initially at rest gets accelerated by a constant electric field E. The rate of change of de-Broglie wavelength of this electron at time t, ignoring relativistic effects

(A) $-\dfrac{h}{eEt^2}$ (B) $-\dfrac{eht}{E}$

(C) $-\dfrac{mh}{eEt^2}$ (D) $-\dfrac{h}{eE}$

106. Of the following, the one which has the largest de Broglie wavelength for the same speed is
(A) electron (B) proton
(C) α-particle (D) oxygen atom

107. A beam of light has an power of 144 W equally distributed among three wavelength of 4100 Å, 4960 Å and 6200 Å. The beam is incident at an angle of incidence of $60°$ on an area of 1 cm^2 of a clean sodium surface, having a work function of 2.3 eV. Assuming that there is no loss of light by reflection and that each energetically capable photon ejects a photoelectron, find the saturation photocurrent. (Take $hc = 12400\text{ eVÅ}$)

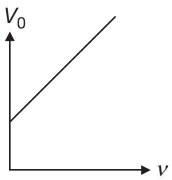
(A) 1.76 mA (B) 0.88 mA
(C) 3.52 mA (D) None of these

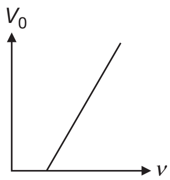
108. A small photocell is placed at a distance of 4 m from a photosensitive surface. When light falls on the surface the current is 5 mA. If the distance of cell is decreased to 1 m, the current will become

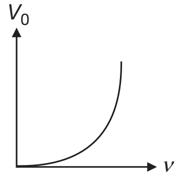
(A) 1.25 mA (B) $\left(\dfrac{5}{16}\right)$ mA
(C) 20 mA (D) 80 mA

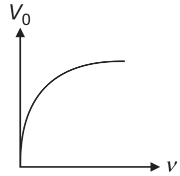
109. Einstein's photoelectric equation is $E_K = h\nu - \phi$. In this equation E_K refers to
(A) kinetic energy of all the emitted electrons.
(B) mean kinetic energy of emitted electrons.
(C) maximum kinetic energy of emitted electrons.
(D) minimum kinetic energy of emitted electrons.

110. In photoelectric effect, the graph showing the variation of cut-off voltage (V_0) with the frequency of incident radiation (ν) is

(A)


(B)


(C)


(D)


111. An electron is accelerated through a potential difference of 100 V. Its kinetic energy will be
(A) 100 J (B) 100 erg
(C) 100 eV (D) 100 MeV

112. If 5% of the energy supplied to a bulb is radiated as visible light, the number of visible quanta emitted per second by a 100 W bulb, assuming the wavelength of visible light to be $5.6\times10^{-5}\text{ cm}$, is

(A) 1.4×10^{19} (B) 1.4×10^{20}
(C) 2×10^{19} (D) 2×10^{20}

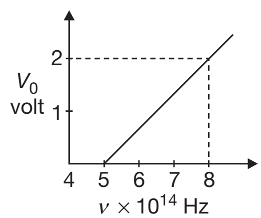
113. A neutron is confined to a nucleus of size 10^{-14} m. The minimum momentum of the electron may be
(A) $6.6\times10^{-20}\text{ kgms}^{-1}$ (B) $3.3\times10^{-20}\text{ kgms}^{-1}$
(C) $3.3\times10^{-48}\text{ kgms}^{-1}$ (D) $6.6\times10^{-48}\text{ kgms}^{-1}$

114. A point source of radiation power P is placed on the axis of an ideal plane mirror. The distance between the source and the mirror is n times the radius of the mirror. Find the force that light exerts on the mirror

(A) $\dfrac{2P}{c(n^2+1)}$ (B) $\dfrac{P(n^2+1)}{2c}$

(C) $\dfrac{P}{2c(n^2+1)}$ (D) $\dfrac{P}{4c(n^2+1)}$

115. The stopping potential (V_0) versus frequency plot of a substance is shown in figure The threshold wavelength is



(A) $5\times10^{14}\text{ m}$
(B) 6000 Å
(C) 5000 Å
(D) cannot be estimated from given data

116. The energy of incident photon is 12.375 eV while the energy of scattered photon is 9.375 eV. Then the kinetic energy of recoil electron is

(A) 3 eV (B) less than 3 eV
(C) more than 3 eV (D) 21.75 eV

117. When electrons are accelerated through potential difference of V volt, the de-Broglie wavelength associated is given by

(A) $\lambda = \sqrt{\frac{150}{V}}$ Å (B) $\lambda = \sqrt{\frac{150}{V}}$ m

(C) $\lambda = \frac{150}{\sqrt{V}}$ Å (D) $\lambda = \frac{\sqrt{150}}{V}$ Å

118. An electron and a proton are accelerated through the same potential. If their masses are m_e and m_p respectively, then the ratio of their de Broglie wavelength is

(A) 1 (B) $\sqrt{\frac{m_e}{m_p}}$

(C) $\frac{m_p}{m_e}$ (D) $\sqrt{\frac{m_p}{m_e}}$

119. A particle of mass 1 g is located in a box of size 2 cm. The uncertainty in the momentum of the electron will be

(A) 3.3×10^{-32} kgms^{-1} (B) 6.6×10^{-32} kgms^{-1}

(C) 3.3×10^{-33} kgms^{-1} (D) 6.6×10^{-34} kgms^{-1}

120. If E_1, E_2, E_3 are the respective kinetic energies of an electron, an alpha-particle and a proton, each having the same de-Broglie wavelength, then

(A) $E_1 > E_3 > E_2$ (B) $E_2 > E_3 > E_1$

(C) $E_1 > E_2 > E_3$ (D) $E_1 = E_2 = E_3$

121. The eye can detect 5×10^4 photons/m^{2}s of light of wavelength 500 nm. The ear can detect 10^{-13} Wm^{-2}. As a power detector, which is more sensitive?
(A) Sensitivity of eye is one fifth of the ear
(B) Sensitivity of eye is five times that of ear
(C) Both are equally sensitive
(D) Eye cannot be used as power detector

122. The kinetic energy of electron is E, when the incident light has wavelength λ. To increase the K.E. to $2E$, the incident light must have wavelength

(A) $\frac{hc}{E\lambda - hc}$ (B) $\frac{hc\lambda}{E\lambda + hc}$

(C) $\frac{h\lambda}{E\lambda + hc}$ (D) $\frac{hc\lambda}{E\lambda - hc}$

123. We wish to observe an object which is 2.5 Å in size. The minimum energy photon that can be used is

(A) 5 keV (B) 8 keV
(C) 10 keV (D) 12 keV

124. A certain mass of ice at 0° C melts into water at 0° C and there by gains 1 kg mass. If initial mass of ice is m_0 then,

(A) $m_0 = 2.69\times10^{11}$ kg (B) $m_0 = 1$ kg

(C) $m_0 = 2.69\times10^{10}$ kg (D) $m_0 = 9\times10^{16}$ kg

125. A star of mass M_0, radius R_0 contracts to radius R. Energy radiated by the star assuming uniform density in each case while temperature remains unchanged is

(A) M_0c^2 (B) $M_0c^2\left[1-\left(\frac{R}{R_0}\right)^2\right]$

(C) $M_0c^2\left(1-\frac{R}{R_0}\right)$ (D) $M_0c^2\left[1-\left(\frac{R}{R_0}\right)^3\right]$

126. The number of red photons $(\lambda = 663 \text{ nm})$ that must strike a totally reflecting screen per second at normal incidence so that a force of 1N is exerted on the screen is

(A) $n = 5\times10^{23}$ (B) $n = 5\times10^{24}$
(C) $n = 5\times10^{25}$ (D) $n = 5\times10^{26}$

127. A 20 amu atom emits photon of 6.6 Å while making a transition from excited state to ground state. The recoil energy of the atom will be

(A) 1.5×10^{-23} J (B) 3.5×10^{-23} J
(C) 5.1×10^{-23} J (D) 7.5×10^{-23} J

128. How many red photon (wavelength λ) must strike a totally reflecting screen per second at normal incidence, if the exerted force is to be 1 N ?

(A) $\frac{\lambda}{h}$ (B) $\frac{2\lambda}{h}$

(C) $\frac{\lambda}{2h}$ (D) infinity

129. The surface of a metal is illuminated with the light of 400 nm. The kinetic energy of the ejected photoelectrons was found to be 1.68 eV. The work function of the metal for $hc = 1240$ eV-nm is

(A) 3.09 eV (B) 1.41 eV
(C) 1.51 eV (D) 1.68 eV

130. A cathode of a photoelectric cell is changed such that the work function changes from W_1 to W_2 $(W_1 < W_2)$. If the current before and after changes are

I_1 and I_2 all other conditions remaining unchanged, then (assuming $h\nu > W_2$)

(A) $I_1 = I_2$ (B) $I_1 < I_2$

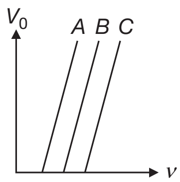
(C) $I_1 > I_2$ (D) $I_1 < I_2 < 2I_1$

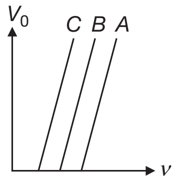
131. The momentum of a photon of frequency ν is

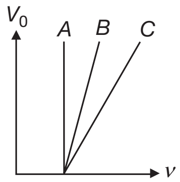
(A) $\dfrac{h\nu}{c^2}$ (B) $\dfrac{h\nu}{c}$

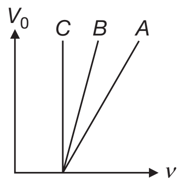
(C) $h\nu c$ (D) $h\nu c^2$

132. The work functions for three different metals A, B and C are ϕ_A, ϕ_B and ϕ_C respectively with $\phi_A > \phi_B > \phi_C$. The graphs between stopping potential (V_0) and frequency ν of incident radiation for them would look like

(A) 

(B) 

(C) 

(D) 

133. The work function of a certain metal is $\dfrac{hc}{\lambda_0}$. When a monochromatic light of wavelength $\lambda < \lambda_0$ is incident such that the plate gains a total power P. If the efficiency of photoelectric emission is $\eta\%$ and all the emitted photoelectrons are captured by a hollow conducting sphere of radius R already charged to potential V, then neglecting any interaction between plate and the sphere, expression of potential of the sphere at time t is

(A) $V + \dfrac{100\eta\lambda Pet}{4\pi\varepsilon_0 Rhc}$ (B) $V + \dfrac{\eta\lambda Pet}{4\pi\varepsilon_0 Rhc}$

(C) V (D) $\dfrac{\lambda Pet}{4\pi\varepsilon_0 Rhc}$

134. In order to increase the kinetic energy of ejected photoelectrons, there should be an increase in

(A) intensity of radiation.

(B) wavelength of radiation.

(C) frequency of radiation.

(D) both wavelength and intensity of radiation.

135. Which of the following arrangements corresponds to decreasing order of specific charge?

(A) Electron, proton, α-particle

(B) Proton, α-particle, electron

(C) α-particle, electron, proton

(D) Electron, α-particle, proton

136. A photoelectric cell is illuminated by a small bright source of light placed at 1 m. If the same source of light is placed 2 m away, the electrons emitted by the cathode

(A) each carries one quarter of its previous momentum.

(B) each carries one quarter of its previous energy.

(C) are half the previous number.

(D) are one quarter of the previous number.

137. Radiation pressure on any surface is

(A) dependent on wavelength of the light used

(B) dependent on nature of surface and intensity of light used

(C) dependent on frequency and nature of surface

(D) depends on the nature of source from which light is coming and on nature of surface on which it is falling

138. Light of frequency 1.5 times the threshold frequency is incident on a photo-sensitive material. If the frequency is halved and the intensity is doubled, the photoelectric current becomes

(A) four times (B) double

(C) half (D) zero

139. The ratio of the de Broglie wavelengths of a proton and an α-particle will be $1:2$ if their

(A) kinetic energies are in the ratio $1:8$

(B) kinetic energies are in the ratio $8:1$

(C) velocities are in the ratio $1:8$

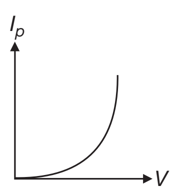
(D) velocities are in the ratio $8:1$

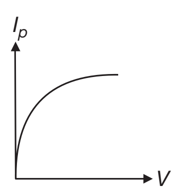
140. Cathode rays moving with same velocity v describe an approximate circular path of radius r metre in an electric field of strength x volt metre^{-1}. If the speed of the cathode rays is doubled to $2v$, the value of electric field needed so that the rays describe the same approximate circular path $\left(\text{volt metre}^{-1}\right)$ is

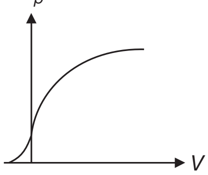
(A) $2x$ (B) $3x$

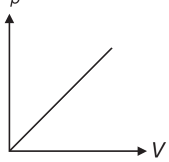
(C) $4x$ (D) $6x$

141. Which of the following graphs gives the variation of photoelectric current (I_p) with the voltage (V) applied to the electrodes of a photo cell?

(A) 

(B) 

(C) 

(D) 

142. An electron with speed v and a photon with a speed c have the same de-Broglie wavelength. If the K.E. and momentum of electrons is E_e and P_e and that of photon is E_{ph} and P_{ph} respectively, then the correct statement is

(A) $\frac{E_e}{E_{ph}} = \frac{2c}{v}$ (B) $\frac{E_e}{E_{ph}} = \frac{v}{2c}$

(C) $\frac{P_e}{P_{ph}} = \frac{2c}{v}$ (D) None of these

143. The energy of a photon corresponding to the visible light of maximum wavelength is approximately

(A) 1 eV (B) 1.6 eV

(C) 3.2 eV (D) 7 eV

144. Planck's work was connected with

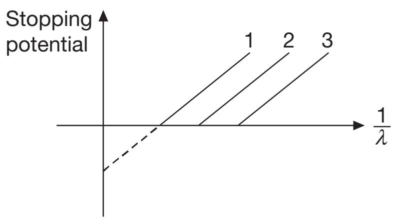
(A) wave nature of matter

(B) photoelectric effect

(C) structure of atom

(D) quantum nature of radiation.

145. The graph shows stopping potential versus $\frac{1}{\text{wavelength}}$ for three metals, then



(A) Planck's constant for metal (1) is greatest

(B) Work function for metal (3) is greatest

(C) Threshold frequency for metal (1) is greatest

(D) Threshold wavelength is maximum for metal (3)

146. Number of identical photons incident on a perfectly black body of mass m kept at rest on smooth horizontal surface. Then the acceleration of the body if n number of photons incident per sec. is (Assume wavelength of photon to be λ)

(A) $\frac{nh}{2\pi\lambda m}$ (B) $\frac{nh}{\lambda m}$

(C) $\frac{2\pi nh}{\lambda m}$ (D) $\frac{\lambda m}{nh}$

147. A radio transmitter operates at a frequency of 880 kHz and a power of 10 kW. The number of photons emitted per second is

(A) 1.71×10^{31} (B) 1327×10^{34}

(C) 13.27×10^{34} (D) 13.27×10^{44}

148. The largest momentum we can expect for a microwave photon is

(A) 6.6×10^{-27} kgms^{-1} (B) 6.6×10^{-34} kgms^{-1}

(C) 6.6×10^{-31} kgms^{-1} (D) 6.6×10^{-30} kgms^{-1}

149. An electron is moving with a velocity of $\frac{c}{10}$. The de-Broglie wavelength associated with it is

(A) 0.48×10^{-10} m (B) 0.24×10^{-10} Å

(C) 0.24×10^{-10} m (D) 1.24×10^{-10} m

150. The kinetic energy of the body is twice the rest mass energy. The ratio of the relativistic mass of the body to its rest mass is

(A) 1 (B) 2

(C) 3 (D) infinite

151. The number of complete de-Broglie wavelengths associated with the electron in nth orbit of hydrogen atom is

(A) n (B) n^2

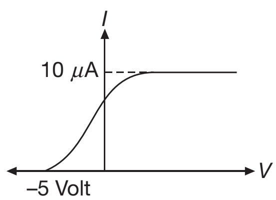
(C) $\frac{1}{n}$ (D) n^4

152. In a photoelectric cell, the current stops when the collecting plate is one volt negative with respect to the emitting metal. The maximum kinetic energy of the photoelectrons is

(A) 1 erg (B) 1 J

(C) 1.6×10^{-19} J (D) 1.6×10^{-19} eV

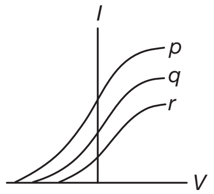
153. In the photoelectric experiment, if we use a monochromatic light, the I-V curve is as shown. If work function of the metal is 2 eV, estimate the power of light used. (Assume efficiency of photo emission $= 10^{-3}\%$, i.e. number of photoelectrons emitted are $10^{-3}\%$ of number of photons incident on metal.)

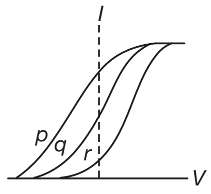


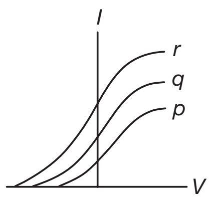
(A) 2 W (B) 5 W

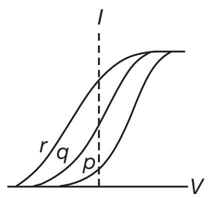
(C) 7 W (D) 10 W

154. Photoelectric effect experiments are performed using three different metal plates p, q and r having work functions $\phi_p = 2$ eV, $\phi_q = 2.5$ eV and $\phi_r = 3$ eV, respectively. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct I-V graph for the experiment is

(A)


(B)


(C)


(D)


155. The energy associated with a thermal neutron is of the order of

(A) 10 KeV
(B) 1 KeV
(C) 0.1 MeV
(D) 0.01 MeV

156. A proton, a deutron and an alpha particle are accelerated through potentials of V, $2V$ and $4V$ respectively. Their velocities will bear a ratio

(A) $1:1:1$
(B) $1:\sqrt{2}:1$
(C) $\sqrt{2}:1:1$
(D) $1:1:\sqrt{2}$

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

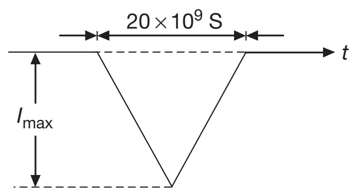
1. The maximum kinetic energy of photoelectrons ejected from a photometer when it is irradiated with radiation of wavelength 400 nm is 1 eV. If the threshold energy of the metal surface is 1.9 eV. Select the correct statement(s).

(A) The maximum K.E. of photoelectrons when it is irradiated with 500 nm photon will be 0.42 eV
(B) The maximum K.E. of photoelectrons when it is irradiated with 500 nm photon will be 1.725 eV
(C) The longest wavelength which will eject photoelectrons is nearly 6100 Å
(D) The value of hc is 11600 eVÅ

2. A collimated beam of light of flux density 30 kWm^{-2} is incident normally on a 100 mm^2 completely absorbing screen. If P is the pressure exerted on the screen and Δp is the momentum transferred to the screen during a 1000 s interval then,

(A) $P = 10^{-3}$ Nm^{-2} (B) $P = 10^{-4}$ Nm^{-2}
(C) $\Delta p = 10^{-4}$ kgms^{-1} (D) $\Delta p = 10^{-5}$ kgms^{-1}

3. A photomultiplier tube is to be used to detect light pulses each of which consists of a small but fixed number of photons. The average photoelectric efficiency is 10%. That is photon has 10% probability of causing the emission of a detectable photoelectron. Assume the photomultiplier gain is 10^6 and that the output current as a function of time (in nanosecond) can be approximated as shown in figure.



I_{max} when averaged over many pulses is 80 μA. Then which of the following statements is/are true.

(A) The charge carried by one pulse is 8×10^{-13} C
(B) Number of photoelectrons emitted per light pulse is 5
(C) Number of photons in one light pulse is 50
(D) Number of electrons carried by one pulse is 5×10^5

4. Light from a monochromatic source is incident normally on a small photo sensitive surface S having work function ϕ. If power of the source is W and a is the distance between the source and S, then

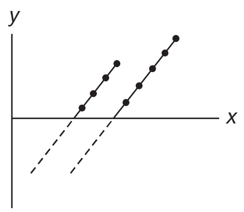
(A) the number of photons striking the surface per unit time will be $\left(\frac{W\lambda S}{4\pi hca^2}\right)$

(B) the maximum energy of the emitted electrons will be $\left(\frac{hc}{\lambda}-\phi\right)$

(C) the stopping potential needed to stop the most energetic photons will be $\frac{e}{\lambda}(hc-\lambda\phi)$

(D) photo emission occurs only if $0 \le \lambda \le \frac{hc}{\phi}$

5. For photoelectric phenomenon, an experimenter plots a graph as shown in figure.



Which of the following statement(s) is/are correct?

(A) x-axis shows wavelength of light used

(B) y-axis shows the kinetic energy of the slowest among the electrons ejected

(C) the intercept on the x-axis is proportional to the work function of the cathode

(D) the two graph lines for different cathodes are always parallel

6. When ultraviolet radiation is incident on a surface, no photoelectrons are emitted. If another beam causes photoelectrons to be emitted from the surface, it may consist of

(A) radio waves

(B) infrared rays

(C) X-rays

(D) gamma rays

7. When the intensity of a light source is increased,

(A) the number of photons emitted by the source in unit time increases

(B) the total energy of the photons emitted per unit time increases

(C) more energetic photons are emitted

(D) faster photons are emitted

8. Light rays are incident on an opaque sheet. The correct statement(s) is/are

(A) Light rays exert a force on the sheet

(B) Light rays transfer an energy to the sheet

(C) Light rays transfer momentum to the sheet

(D) Light rays transfer impulse to the sheet

9. According to Heisenberg's Uncertainty Principle,

(A) $\Delta x \Delta p \ge \frac{h}{4\pi}$ (B) $\Delta E \Delta t \ge \frac{h}{4\pi}$

(C) $\Delta\theta \Delta L \ge \frac{h}{4\pi}$ (D) $\Delta x \Delta v \ge \frac{h}{4\pi m}$

where the symbols bear the usual meaning.

10. In which of the following situations, the heavier of the two particles will have a smaller de-Broglie wavelength.

(A) The particles move with the same speed

(B) The particles move with the same linear momentum

(C) The particles move with the same kinetic energy

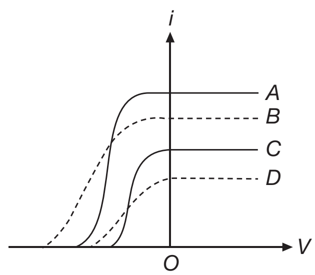
(D) The particles have fallen through the same height

11. The threshold wavelength for photoelectric emission from a material is 5200 Å. This material when illuminated with .... monochromatic radiation emits photoelectrons.

(A) 1 W UV (B) 50 W UV

(C) 1 W IR (D) 50 W IR

12. Figure shows the results of an experiment involving photoelectric effect. The graphs A, B, C, D related the light beam having different wavelengths.



(A) Beam B has highest frequency

(B) Beam C has longest wavelength

(C) Beam A has the highest rate of photoelectric emission

(D) Photoelectrons ejected by beam B have the highest momentum

13. Light rays are incident on a metallic sheet. Then,

(A) the force exerted is independent of frequency of light incident

(B) the force depends on the direction of light incident

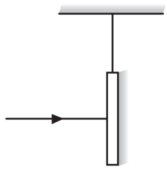
(C) the pressure is independent of frequency of light incident

(D) the pressure is proportional to the area of the plate

14. The momentum of a single photon of red light of frequency 400×10^{12} Hz moving through free space is

(A) zero (B) 8.8×10^{-28} $kgms^{-2}$
(C) 1.65×10^{-6} MeV/c (D) Data Insufficient

15. A small mirror is suspended by a thread as shown in figure. A short pulse of monochromatic light rays is incident normally on the mirror and gets reflected. Which of the following statements is/are correct?



(A) Mirror will start to oscillate.
(B) Wavelength of reflected rays will be greater than that of incident rays.
(C) Wavelength of reflected rays may be less than that of incident rays
(D) None of these

16. For a 75 W point light source assuming all the electric power consumed goes into emitted light of wavelength 600 nm, then
(A) frequency of the emitted light is 5×10^{14} Hz
(B) number of photons emitted per second is 2.3×10^{20}
(C) this emitted light on falling on a metal surface of work function 1.07 eV, will emit photoelectrons having kinetic energy between 0 and 1 eV
(D) on doubling the distance of this metal surface from the point source maximum kinetic energy of photoelectrons emitted becomes 0.25 eV

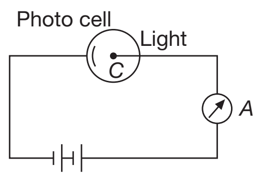
17. In an experiment for photoelectric effect, the frequency and intensity of a light source are both doubled, then
(A) The saturation photocurrent remains almost the same
(B) The saturation photocurrent becomes doubled
(C) The maximum kinetic energy of the photoelectrons is doubled
(D) The stopping potential becomes more than double

18. If the wavelength of light in an experiment on photoelectric effect is doubled,
(A) the photoelectric emission will not take place
(B) the photoelectric emission may or may not take place
(C) the stopping potential will increase
(D) the stopping potential will decrease

19. A metallic surface ejects electrons when exposed to green light of intensity I but no photoelectrons are emitted when exposed to yellow light of intensity I. It is possible to eject electrons from the same surface by
(A) yellow light of some intensity which is more than I.
(B) green light of any intensity.
(C) red light of any intensity.
(D) violet light of any intensity.

20. In an experiment of photoelectric effect, light from a point source of monochromatic light of wavelength 3000 Å is incident on a metal surface. The kinetic energies of photoelectrons range from zero to 4×10^{-19} J, then
(A) stopping potential for this light is 2.5 V
(B) threshold wavelength for the material is 7590 Å
(C) stopping potential will be doubled on reducing the distance and the wavelength of light source to half
(D) saturation current will be doubled on reducing the distance of source to half

21. Figure shows a photo cell being illuminated by a monochromatic light. If the intensity is kept constant and the frequency of the incident light is increased, then the



(A) photo electric current in the circuit decreases
(B) photo electric current in the circuit increases
(C) photo electric current in the circuit can be reduced to zero, when the polarity of the terminals is reversed.
(D) maximum kinetic energy of the photo electrons increases

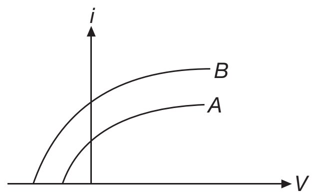
22. Light of wavelength 496 nm is incident on a metal surface causing ejection of photoelectrons for which stopping potential is 1.5 V, then
(A) the work function of the surface is 1 eV
(B) de-Broglie wavelength of fastest photoelectron is 100 nm
(C) to move the fastest electron in a circle of radius 1 m, perpendicular magnetic field B required is 4 μT
(D) this fastest electron when strikes zinc target can produce X-rays

23. Which of the following statements about photoelectric effect is/are false?
(A) It exhibits the particle nature of radiation
(B) Electrons are emitted only if the radiation has a frequency above a certain value

(C) All the electrons emitted by radiation of a particular frequency have the same energy
(D) Changing the intensity of radiation changes the maximum energy with which the electrons can be emitted

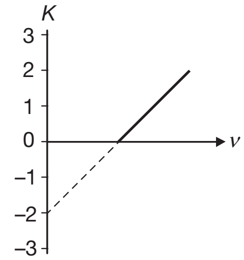
24. Radiations of monochromatic waves of wavelength 400 nm are made incident on the surface of metals Zn, Fe and Ni of work functions 3.4 eV, 4.8 eV and 5.9 eV respectively $(\text{take } hc = 12400\text{ eVÅ})$
(A) maximum KE associated with photoelectrons from the surface of any metal is 0.3 eV
(B) no photoelectrons are emitted from the surface of *Ni*
(C) if the wavelength of source of radiation is doubled then KE of photoelectrons is also doubled
(D) photoelectrons will be emitted from the surface of all the three metals if the wavelength of incident radiations is less than 200 nm

25. Observe the current vs voltage graphs of two photocells A and B receiving same intensity of a monochromatic beam. You may consider the plate area receiving the light to be same. Select the correct statement (s).



(A) Photoelectric efficiency of cell B must be greater than cell A.
(B) Photoelectric efficiency of cell B may be greater than cell A.
(C) Work function for cell B must be greater than that of cell A.
(D) Work function for cell B must be less than that of cell A.

26. Figure represents a graph of kinetic energy (K) of photoelectrons (in eV) and frequency (ν) for a metal used as cathode in photoelectric experiment. The work function of metal is



(A) 1 eV (B) 1.5 eV
(C) 2 eV (D) 3 eV

27. In PROBLEM 26, the threshold frequency is nearly
(A) 5×10^{14} Hz (B) 10×10^{14} Hz
(C) 2.5×10^{14} Hz (D) cannot be estimated

28. Which of the following statements are incorrect?
(A) Saturation current is photo electric effect experiment is independent of frequency of light incident.
(B) Stopping potential increases with increase in intensity of light incident.
(C) Stopping potential increases with decrease in wavelength of light incident.
(D) Photo electric effect depends on the work function of the metal.

29. It is necessary to consider light as a stream of photons to explain
(A) Photoelectric effect
(B) Compton effect
(C) Polarization of light
(D) Diffraction of light

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** Threshold wavelength of certain metal is λ_0. Light of wavelength slightly less than λ_0 is incident on the plate. It is found that after some time the emission of electrons stops.

Statement-2: The ejected electrons experience force of attraction due to development of positive charges on plate which after certain time is adequate enough to hold them to plate itself.

2. **Statement-1:** A photon has no rest mass, yet it carries definite momentum.
Statement-2: Momentum of photon is due to its energy and hence its equivalent mass.

3. **Statement-1:** In a photoelectric effect, the current increases when positive potential of collector is increased, before saturation of current.
Statement-2: The number of emitted photoelectrons increases.

4. **Statement-1:** The de Broglie wavelength of an electron accelerated through 941 volt is 0.4 Å.
Statement-2: Higher the accelerating potential, smaller is the de-Broglie wavelength.

5. **Statement-1:** Work function of copper is greater than that of sodium. But both will have same value of the threshold frequency and threshold wavelength.
Statement-2: The frequency is inversely proportional to wavelength.

6. **Statement-1:** In case of an electron and a photon having same momentum, wavelength associated with electron is smaller.
Statement-2: Electron cannot move with a speed of photon.

7. **Statement-1:** The photoelectrons produced by a monochromatic light beam incident on a metal surface, have a spread in their kinetic energies.
Statement-2: The work function of the metal varies as a function of depth from the surface.

8. **Statement-1:** A proton, a deutron and an α-particle are accelerated by the same potential difference. Their velocities will be in the ratio of $1:1:\sqrt{2}$.
Statement-2: Kinetic energy, $E = qV = \frac{1}{2}mv^2$

9. **Statement-1:** Photoelectric effect demonstrates the wave nature of light.
Statement-2: The number of photoelectrons is proportional to the frequency of light.

10. **Statement-1:** A photon has no rest mass, yet it carries momentum.
Statement-2: Momentum depends more on velocity than that of mass.

11. **Statement-1:** The threshold frequency of photoelectric effect supports the particle nature of sunlight.
Statement-2: If frequency of incident light is less than the threshold frequency, electrons are not emitted from metal surface.

12. **Statement-1:** Effective mass of photon varies with wavelength.
Statement-2: $E = mc^2$ is the relation between mass and energy.

13. **Statement-1:** Though light of a single frequency (monochromatic light) is incident on a metal, the energies of emitted photoelectrons are different.
Statement-2: The energy of electrons just after they absorb photons incident on metal surface may be lost in collision with other atoms in the metal before the electron is ejected out of metal.

14. **Statement-1:** A photon and an electron, both of energy 1 MeV has same wavelength.
Statement-2: $E = 22m_ec^2 = 10^6$ eV

15. **Statement-1:** The velocity of body of rest mass m_0 is $\frac{\sqrt{3}}{2}c$ (where c is the velocity of light in vacuum) then mass of the body is $2m_0$.
Statement-2: Moving mass is given as $m = \dfrac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$

16. **Statement-1:** In the process of photoelectric emission by monochromatic light, all the emitted photoelectrons possess the same kinetic energy.
Statement-2: In photoelectric effect a single photon interacts with a single electron and electron is emitted only if energy of each of incident photon is greater than the work function.

17. **Statement-1:** de-Broglie wavelength of an electron accelerated through a potential difference of V volt is $\lambda = \frac{12.72}{\sqrt{V}}$ Å.
Statement-2: de-Brogle wavelength of an electron is given by $\lambda = \frac{h}{mv}$

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

A photocell is operating in saturation mode with a photo current $20\ \mu A$ when a monochromatic radiation of wave length $3000\ Å$ and power $1\ MW$ is incident. When another monochromatic radiation of wave length $1500\ Å$ and power $5\ MW$ is incident, it is observed that the maximum velocity of photo electron is doubled. Assuming efficiency of photoelectron generation per incident photon to be same for both the cases. Based on above information, answer the following questions.

1. The threshold wavelength for the cell is
(A) $3500\ Å$ (B) $4000\ Å$
(C) $4500\ Å$ (D) $5000\ Å$

2. The saturation current in second case is
(A) $50\ \mu A$ (B) $40\ \mu A$
(C) $60\ \mu A$ (D) $45\ \mu A$

3. The efficiency of photoelectron generation per incident photon is
(A) 8.5% (B) 8.25%
(C) 8% (D) 8.75%

Comprehension 2

The radiations emitted when an electron jumps from $n = 3$ to $n = 2$ orbit of hydrogen atom falls on a metal to produce photoelectrons. The electrons emitted from the metal surface with maximum kinetic energy are made to move perpendicular to a magnetic field of strength $\frac{1}{320}$ T in a radius of 10^{-3} m. Based on above information, answer the following questions.

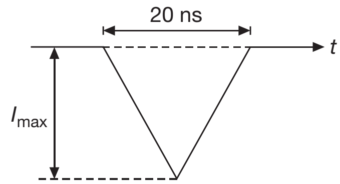
4. The kinetic energy of the electrons is
(A) 0.56 eV (B) 1.32 eV
(C) 0.86 eV (D) 1.76 eV

5. The work function of the metal is
(A) 1.89 eV (B) 1.03 eV
(C) 1.58 eV (D) 2.32 eV

6. Wavelength of radiation is nearly
(A) $6565\ Å$ (B) $5555\ Å$
(C) $4545\ Å$ (D) $3535\ Å$

Comprehension 3

A photomultiplier tube is to be used to detect light pulses each of which consists of a small but fixed number of photons. The average photoelectric efficiency (i.e. a photon's chance of causing the emission of a detectable photoelectron) is 10%. Assuming that the photomultiplier gain is 10^6 and that the output current as a function of time can be approximated as shown in figure.



Based on the information provided, answer the following questions.

7. I_{max} when averaged over many pulses is $80\ \mu A$
(A) The charge carried by one pulse is 9×10^{-13} C
(B) Number of photoelectrons emitted per light pulse is 10
(C) Number of photons in one light pulse is 50
(D) Number of electrons carried by one pulse is 5×10^5

8. The probability that all the photons of a light pulse will go undetected is (N is the number of photons in one light pulse)
(A) $\left(\frac{90}{100}\right)^{\frac{N}{10}}$ (B) $\left(\frac{90}{100}\right)^{\frac{N}{50}}$
(C) $(0.9)^{50}$ (D) $(0.9)^{5}$

9. The fluctuation in the value of I_{max} will be small if
(A) number of photons in the pulse is increased
(B) number of electrons in pulse is decreased
(C) number of photons in the pulse is decreased
(D) total quantity of charge carried by pulse is decreased

Comprehension 4

de-Broglie suggested that every moving particle has a wavelength associated with it, which is given by $\lambda = \frac{h}{p}$ or $\lambda = \frac{h}{\sqrt{2mK}}$. With the help of these formulae we know that wavelength for a charged particle accelerated through a potential V is $\lambda = \frac{h}{\sqrt{2mqV}}$. Based on above information, answer the following questions.

10. Proton and alpha particle are accelerated through same potential difference. Then ratio of their wavelength is
(A) $1:\sqrt{2}$ (B) $2\sqrt{2}:1$
(C) $2:1$ (D) $1:2\sqrt{2}$

11. When an electron is accelerated through 150 V potential difference, then the wavelength associated with it is approximately

(A) 1 Å (B) 2 Å
(C) 3 Å (D) 4 Å

12. If electron and alpha particle have same momentum the ratio of their wavelength is

(A) $1840:1$ (B) $1:1840$
(C) $1:1$ (D) None of these

Comprehension 5

Light of wavelength 3100 Å is falling on four elements A, B, C and D having work function ϕ_0 given by 2.5 eV, 3.5 eV, 4.5 eV and 5.5 eV respectively. Given that $hc = 12400$ eVÅ. Based on the information provided, answer the following questions.

13. Photo-electrons are emitted from following elements

(A) A (B) A and B
(C) A, B and C (D) A, B, C and D

14. Maximum wavelength of light, which can electrons from all four elements is

(A) 4960 Å (B) 3540 Å
(C) 2750 Å (D) 2250 Å

Comprehension 6

A large number of identical balls each having a mass of 66.3 g are thrown with speed of $5\ \text{ms}^{-1}$ into a house through two tall, narrow, parallel windows spaced 0.6 m apart, the choice of window as target being random at each throw. Fringes are formed on a wall 12 m behind the windows. Assuming the Plank's constant to have a hypothetical value of $h = 6.63 \times 10^{-3}$ Js. Based on above information, answer the following questions.

15. de-Broglie wavelength of the balls is

(A) 0.2 m (B) 0.02 m
(C) 2 m (D) 20 m

16. Fringe width obtained on the wall is

(A) 0.01 m (B) 0.2 m
(C) 0.5 m (D) 0.4 m

17. If electrons are used in place of balls, moving with speed $10^7\ \text{ms}^{-1}$ in the experiment then

(A) Fringe width will decrease
(B) No interference pattern is obtained
(C) Fringe width will not increase
(D) Interference pattern is obtained but it is not possible to observe it

Comprehension 7

A laser delivers 10 kJ of 1 micron wavelength radiation in 10^{-9} seconds onto a focal spot of $10^{-3}\ \text{cm}^2$ area. All the energy was absorbed and converted into thermal energy uniformly distributed over a sphere whose size just matches that of the focal spot containing 5×10^{18} hydrogen atoms (fully ionized). Based on the information provided, answer the following questions.

18. The peak electric field is

(A) $2.75 \times 10^{11}\ \text{Vm}^{-1}$ (B) $5.25 \times 10^{11}\ \text{Vm}^{-1}$
(C) $8.5 \times 10^{11}\ \text{Vm}^{-1}$ (D) $10 \times 10^{11}\ \text{Vm}^{-1}$

19. Radiation pressure is approximately

(A) 330 Giga Pascal (B) 400 Giga Pascal
(C) 500 Giga Pascal (D) 860 Giga Pascal

20. The pressure of this material is

(A) 280 Tera Pascal
(B) 480 Tera Pascal
(C) 560 Tera Pascal
(D) 720 Tera Pascal

Comprehension 8

A surface has light of wavelength $\lambda = 496$ nm incident on it, causing the ejection of photoelectrons for which the stopping potential is found to be 1.5 V. Based on above information, answer the following questions.

21. The de-Broglie wavelength of the fastest photoelectron emitted is

(A) 75 nm (B) 100 nm
(C) 135 nm (D) 235 nm

22. To move the fastest electron in a circle of radius 1 m, the value of perpendicular magnetic field B is

(A) 4.1×10^{-6} T (B) 2.1×10^{-5} T
(C) 3.2×10^{-5} T (D) 5.2×10^{-6} T

23. The threshold wavelength for photoelectric emission to occur is nearly

(A) 1250 nm (B) 2000 nm
(C) 2250 nm (D) 3000 nm

Comprehension 9

Light beam of energy 2.5 eV incident on metal surface having work function of 2 eV. Taking $hc = 12400$ eVÅ, and on the information provided, answer the following questions.

24. Wavelength of incident photon is

(A) 5000 Å (B) 6000 Å
(C) 7000 Å (D) 8000 Å

25. de-Broglie wavelength of photo electron is

(A) 17.5 nm (B) 1.75 nm
(C) 35 nm (D) 3.5 nm

Comprehension 10

A parallel beam of monochromatic light $(\lambda = 663\text{ nm})$ of intensity 30 kWm^{-2} is incident normally on a 100 mm^2 completely absorbing screen for 10 s. Based on above information, answer the following questions.

26. Pressure exerted by beam on the surface is

(A) 10^{-5} Pa (B) 10^{-4} Pa
(C) 2×10^{-4} Pa (D) 5×10^{-5} Pa

27. Momentum transferred to the screen during the interval is

(A) 10^{-8} kgms^{-1} (B) $5\times10^{-8}\text{ kgms}^{-1}$
(C) 10^{-7} kgms^{-1} (D) $2\times10^{-7}\text{ kgms}^{-1}$

28. Number of photons striking the screen during the interval is

(A) 5×10^{19} (B) 10^{20}
(C) 2×10^{20} (D) 5×10^{20}

Comprehension 11

A metallic surface, when illuminated by light of frequency 8×10^{14} Hz and 12×10^{14} Hz emits photoelectrons of maximum kinetic energy 0.5 eV and 2.0 eV. Based on the information provided answer the following questions.

29. The value of Planck's constant is

(A) 6.0×10^{-34} Js (B) 6.2×10^{-34} Js
(C) 6.4×10^{-34} Js (D) 6.6×10^{-34} Js

30. The work function of metal is

(A) 0.5 eV (B) 1.5 eV
(C) 2.5 eV (D) 3.5 eV

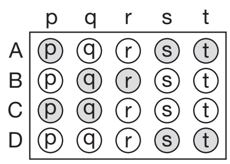
31. de-Broglie wavelength of electron when its energy is 0.5 eV

(A) 8.68 Å (B) 17.35 Å
(C) 21.25 Å (D) 24.54 Å

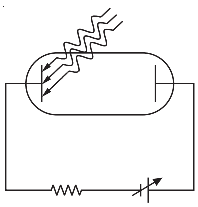
MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → (p, s, t); B → (q, r); C → (p, q); and D → (s, t); then the correct darkening of bubbles will look like the following:



1. In the shown experimental setup to study photoelectric effect, two conducting electrodes are enclosed in an evacuated glass-tube as shown.



A parallel beam of monochromatic light, falls on photosensitive electrodes. The emf of battery shown is high enough such that all photo electrons ejected from left electrode will reach the right electrode. Under initial conditions photoelectrons are emitted. As changes are made in each situation of **COLUMN-I**, match the statements in **COLUMN-I** with results in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) If frequency of incident light is increased keeping its intensity constant	(p) magnitude of stopping potential will increase
(B) If frequency of incident light is increased an it's intensity is decreased	(q) current through circuit may stop

(*Continued*)

COLUMN-I	COLUMN-II
(C) If work function of photo-sensitive electrode is increased	(r) maximum kinetic energy of ejected photo electron will increase
(D) If intensity of incident light is increased keeping it's frequency constant.	(s) Saturation current will increase

2. In a photoelectric effect experiment, if f is the frequency of radiations incident on the metal surface and I is the intensity of incident radiations, then match the quantities in **COLUMN-I** with their matches in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) If distance between cathode and anode is increased.	(p) Stopping potential increases.
(B) If I is increased keeping f and work function constant.	(q) Saturation current increases.
(C) Work function is decreased keeping f and I constant.	(r) Maximum kinetic energy of photo electron increases.
(D) If f is increased keeping I and work function constant.	(s) Stopping potential remain same.

3. In a photoelectric effect experiment, if the following changes are made, then match the **COLUMN-I** with **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) If intensity of incident light is increased keeping its frequency constant.	(p) Stopping potential will increase.
(B) If work function of photo sensitive electrode is increased.	(q) Current through circuit may stop.

(*Continued*)

COLUMN-I	COLUMN-II
(C) If frequency of incident light is increased and its intensity is decreased.	(r) Maximum kinetic energy of ejected photoelectrons will increase.
(D) If frequency of incident light is increased keeping its intensity constant.	(s) Saturation current will increase.

4. Match the properties in **COLUMN-I** with their respective phenomenon in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Photon character of radiation.	(p) Photoelectric effect.
(B) Wave character of radiation.	(q) Compton effect.
(C) Interaction of a photon with an electron, such that photon energy is much greater than the binding energy of electron, is more likely to result in.	(r) Diffraction.
(D) Interaction of a photon with an electron, such that photon energy is equal to or slightly greater than the binding energy of electron, is more likely to result in.	(s) Interference.

5. Match the experiments in **COLUMN-I** with their respective conclusions in **COLUMN-II**

COLUMN-I	COLUMN-II
(A) Photoelectric effect	(p) Wave nature of light
(B) Millikan's experiment	(q) Particle nature of light
(C) Young's Double slit experiment	(r) Particle nature of electron
(D) Davisson-Germer experiment	(s) Wave nature of electron

6. Match the wavelengths in **COLUMN-I** to the respective matches in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) 0.1 Å	(p) de-Broglie wavelength of electron in *X*-ray tube.
(B) 1 Å	(q) Photoelectric threshold wavelength.
(C) 10 Å	(r) *X*-ray wavelength.
(D) 5000 Å	(s) de-Broglie wavelength of most energetic photoelectron emitted from metal surface in photoelectric effect.

7. Some quantities related to photoelectric effect are mentioned under **COLUMN-I** and **COLUMN-II**. Match each quantity on **COLUMN-I** with the corresponding quantity in **COLUMN-II** on which it depends.

COLUMN-I	COLUMN-II
(A) de-Broglie wavelength of photoelectron.	(p) Frequency of light.
(B) Force due to radiation falling on metal plate.	(q) Work function.
(C) Stopping potential.	(r) Area of photo sensitive plate.
(D) Saturation current.	(s) Intensity of light (at constant ν).

8. In a photoelectric effect experiment, if f is the frequency of radiations incident on the metal surface and I is the intensity of incident radiations, then match the following.

COLUMN-I	COLUMN-II
(A) Work function is decreased keeping f and I constant.	(p) Stopping potential increases.
(B) If I is increased keeping f and work function constant.	(q) Saturation current increases.
(C) If distance between cathode and anode is increased.	(r) Maximum kinetic energy of photoelectron increases.
(D) If f is increased keeping I and work function constant.	(s) Stopping potential remain same.

9. In the experimental setup for a photocell, the wavelength of the light incident on the cathode is initially 0.6 times the threshold wavelength for the material of the cathode. Certain changes in the experiment setup are given in **COLUMN-I** and their possible effects are given in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) The intensity of the incident light is doubled but the frequency remains unaltered	(p) Saturation photocurrent remains the same
(B) Both the intensity and wavelength of the incident light are doubled	(q) Photocurrent falls to zero
(C) The intensity of the incident light is doubled and its wavelength is made half	(r) Stopping potential increases
(D) The intensity of the incident light remains the same and the wavelength is made half	(s) Saturation photocurrent increases

10. With respect to photoelectric effect experiment, match the entries of **COLUMN-I** with the entries of **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) If I is increased keeping f and ϕ constant.	(p) Stopping potential increases.
(B) If f (frequency) is increased keeping I (intensity) and ϕ (work function) constant.	(q) Saturation photocurrent increases.

(*Continued*)

COLUMN-I	COLUMN-II
(C) If ϕ is decreased keeping f and I constant.	(r) Maximum K.E. of the photoelectrons increases.
(D) If the distance between anode and cathode increases.	(s) Stopping potential remains the same.

11. If radiation of energy E, intensity I falls on different kinds of surfaces mentioned, then match the quantities in **COLUMN-I** with their respective answers in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Radiation pressure for a perfectly absorbing surface.	(p) $(1+\rho)\frac{I}{c}$
(B) Radiation pressure for a perfectly reflecting surface.	(q) $\frac{2E}{c}$
(C) Radiation pressure for a surface of reflection coefficient (ρ).	(r) $\frac{I}{c}$
(D) Impulse on a perfectly absorbing surface.	(s) $\frac{2I}{c}$
(E) Impulse on a perfectly reflecting surface.	(t) $(1+\rho)\frac{E}{c}$
(F) Impulse on a surface of reflection coefficient (ρ).	(u) $\frac{E}{c}$

INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

1. The human eye can barely detect a yellow light $(6000\ \text{Å})$ that delivers 1.7×10^{-8} W to the retina. How many photons strike the retina in one second is $\alpha\times10^{\beta}$, where α is measures to the nearest integer and β is the integer. Calculate $\frac{\beta}{\alpha}$.

2. A metal plate is placed 5 metre from a monochromatic light source whose power output is 10^{-3} W. Consider that a given ejected photoelectrons may collect its energy from a circular area of the plate as large as ten atomic diameters $(10^{-9}\ \text{m})$ in radius. The energy required to remove an electron through the metal surface is about 5 eV. Assuming light to be a wave, how long, in hour, would it take for such a 'target' to soak up this much energy from such a light source.

3. Energy from the sun is received on the earth at the rate of $2\ \text{cal cm}^{-2}\ \text{min}^{-1}$. If average wavelength of solar light be taken as $6600\ \text{Å}$, then $x\times10^{18}$ photons are received on earth per cm^2 per minute, find x.
Take $1\ \text{cal}=4.2\ \text{J}$, $c=3\times10^{8}\ \text{ms}^{-1}$.

4. Compute the typical de-Broglie wavelength of an electron in a metal at $27\ °\text{C}$ and compare it with the mean separation between two electrons in a metal which is given to be about 2×10^{-10} m.

5. Light of wavelength 180 nm ejects photoelectrons from a plate of metal whose work-function is 2 eV. If a uniform magnetic field of 5×10^{-5} T be applied parallel to the plate, what would be radius of the path, in mm, followed by electrons ejected normally from the plate with maximum energy.

6. A metallic sphere of radius 10 cm is kept in the path of a parallel beam of light of intensity $10^{-2}\ \text{Wm}^{-2}$. Calculate the approximate force to the closest integer (in piconewton) exerted by the beam on the sphere.

7. The maximum kinetic energy of photoelectrons emitted from a certain metallic surface is 30 eV when monochromatic radiation of wavelength λ falls on it. When the same surface is illuminated with light of wavelength 2λ, the maximum, kinetic energy of photoelectrons is observed to be 10 eV. Calculate the wavelength λ and determine the maximum wavelength of incident radiation (both in Å) for which photoelectrons can be emitted by this surface. Given $h=4\times10^{-15}$ eVs and $c=3\times10^{8}\ \text{ms}^{-1}$.

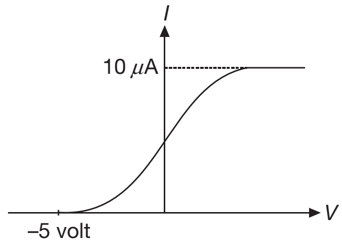
8. On a certain metal light of frequency $\nu=5\nu_0$ falls then maximum velocity of electrons emitted is $8\times10^{6}\ \text{ms}^{-1}$, where ν_0 is threshold frequency of metal. If $\nu=2\nu_0$ then the maximum velocity of photoelectron is $x\times10^{6}\ \text{ms}^{-1}$. Find x.

9. When a surface 1 cm thick is illuminated with light of wavelength λ, the stopping potential is V_0. When the same surface is illuminated by light of wavelength 3λ, the stopping potential is $\frac{V_0}{6}$. If the threshold wavelength for metallic surface is $n\lambda$. Calculate n.

10. A metallic sphere of radius 21 cm is kept in the path of a parallel beam of light of intensity $\frac{1}{110}$ Wm^{-2}. The force exerted by beam on the sphere is $x \times 10^{-13}$ N. Find x.

11. The wavelength of light incident on a metal surface is reduced from 300 nm to 200 nm (both are less than threshold wavelength). Find the change in the stopping potential, in volt, for photoelectrons emitted from the surface. Take $h = 6.6 \times 10^{-34}$ Js.

12. In the photoelectric experiment, if we use a monochromatic light, the I–V curve is as shown. If work function of the metal is 2 eV, estimate the power of light used (in W). Assume efficiency of photo emission to be 10^{-3} %, i.e., number of photoelectrons emitted are 10^{-3} % of number of photons incident on metal.



13. If photons of ultraviolet light of energy 12 eV are incident on a metal surface of work function of 4 eV, then find the stopping potential (in V).

14. A point isotropic light source of power $P = 12$ watts is located on the axis of a circular mirror of radius $R = 3$ cm. If distance of source from the centre of mirror is $a = 39$ cm and reflection coefficient of mirror is $r = 0.70$, then the force exerted by light ray on the mirror is $x \times 10^{-y}$ N. Calculate $\frac{y}{x}$.

15. A proton and an α-particle are fired through the same magnetic fields which is perpendicular to their velocity vectors. Both move such that radius of curvature of their path is the same. Find the ratio of their de-Broglie wavelengths.

ARCHIVE: JEE MAIN

1. [Online April 2019]
Two particles move at right angle to each other. Their de Broglie wavelengths are λ_1 and λ_2 respectively. The particles suffer perfectly inelastic collision. The de Broglie wavelength λ, of the final particle, is given by

(A) $\lambda = \sqrt{\lambda_1 \lambda_2}$
(B) $\lambda = \frac{\lambda_1 + \lambda_2}{2}$
(C) $\frac{2}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$
(D) $\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$

2. [Online April 2019]
A nucleus A, with a finite de-Broglie wavelength λ_A, undergoes spontaneous fission into two nuclei B and C of equal mass. B flies in the same direction as that of A, while C flies in the opposite direction with a velocity equal to half of that of B. The de-Broglie wavelength λ_B and λ_C of B and C are respectively

(A) $2\lambda_A$, λ_A
(B) λ_A, $\frac{\lambda_A}{2}$
(C) $\frac{\lambda_A}{2}$, λ_A
(D) λ_A, $2\lambda_A$

3. [Online April 2019]
The electric field of light wave is given as $\vec{E} = 10^{-3} \cos\left(\frac{2\pi x}{5 \times 10^{-7}} - 2\pi \times 6 \times 10^{14} t\right)\hat{x}\ \frac{N}{C}$. This light falls on a metal plate of work function 2 eV. The stopping potential of the photo-electrons is

Given, $E(\text{in eV}) = \frac{12374}{\lambda(\text{in Å})}$

(A) 0.48 V
(B) 2.48 V
(C) 0.72 V
(D) 2.0 V

4. [Online April 2019]
50 Wm^{-2} energy density of sunlight is normally incident on the surface of a solar panel. Some part of incident energy (25%) is reflected from the surface and the rest is absorbed. The force exerted on 1 m^2 surface area will be close to $(c = 3 \times 10^8\ ms^{-1})$

(A) 20×10^{-8} N
(B) 35×10^{-8} N
(C) 15×10^{-8} N
(D) 10×10^{-8} N

5. [Online April 2019]
A particle P is formed due to a completely inelastic collision of particles x and y having de-Broglie wavelengths λ_x and λ_y respectively. If x and y were moving in opposite directions, then the de-Broglie wavelength of P is

(A) $\dfrac{\lambda_x\lambda_y}{|\lambda_x-\lambda_y|}$ (B) $\lambda_x-\lambda_y$
(C) $\lambda_x+\lambda_y$ (D) $\dfrac{\lambda_x\lambda_y}{\lambda_x+\lambda_y}$

6. [Online April 2019]
In a photoelectric effect experiment the threshold wavelength of light is 380 nm. If the wavelength of incident light is 260 nm, the maximum kinetic energy of emitted electrons will be

Given $E(\text{in eV})=\dfrac{1237}{\lambda(\text{in nm})}$

(A) 4.5 eV (B) 15.1 eV
(C) 3.0 eV (D) 1.5 eV

7. [Online April 2019]
A 2 mW laser operates at a wavelength of 500 nm. The number of photons that will be emitted per second is

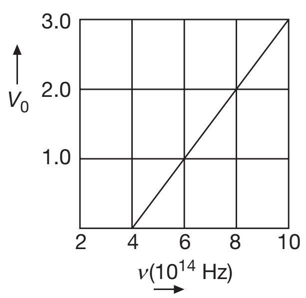
[Given Planck's constant $h=6.6\times10^{-34}$ Js, speed of light $c=3.0\times10^{8}\text{ ms}^{-1}$]

(A) 2×10^{16} (B) 1.5×10^{16}
(C) 1×10^{16} (D) 5×10^{15}

8. [Online April 2019]
Light is incident normally on a completely absorbing surface with an energy flux of 25 Wcm^{-2}. If the surface has an area of 25 cm^2, the momentum transferred to the surface in 40 min time duration will be

(A) 3.5×10^{-6} Ns (B) 6.3×10^{-4} Ns
(C) 5.0×10^{-3} Ns (D) 1.4×10^{-6} Ns

9. [Online April 2019]
The stopping potential V_0 (in volt) as a function of frequency (ν) for a sodium emitter, is shown in the figure. The work function of sodium, from the data plotted in the figure, will be
(Given: Planck's constant $(h)=6.63\times10^{-34}$ Js, electron charge $e=1.6\times10^{-19}$ C)



(A) 1.95 eV (B) 2.12 eV
(C) 1.82 eV (D) 1.66 eV

10. [Online April 2019]
Consider an electron in a hydrogen atom, revolving in its second excited state (having radius 4.65 Å). The de-Broglie wavelength of this electron is

(A) 3.5 Å (B) 12.9 Å
(C) 9.7 Å (D) 6.6 Å

11. [Online January 2019]
Surface of certain metal is first illuminated with light of wavelength $\lambda_1=350\text{ nm}$ and then, by light of wavelength $\lambda_2=540\text{ nm}$. It is found that the maximum speed of the photo electrons in the two cases differ by a factor of 2. The work function of the metal (in eV) is close to $\left(\text{Energy of photon}=\dfrac{1240}{\lambda(\text{in nm})}\text{ eV}\right)$

(A) 1.8 (B) 5.6
(C) 2.5 (D) 1.4

12. [Online January 2019]
The magnetic field associated with a light wave is given, at the origin, by

$B=B_0\left[\sin(3.14\times10^7)ct+\sin(6.28\times10^7)ct\right]$.

If this light falls on a silver plate having a work function of 4.7 eV, what will be the maximum kinetic energy of the photo electrons? $(c=3\times10^{8}\text{ ms}^{-1},\ h=6.6\times10^{-34}\text{ Js})$

(A) 8.52 eV (B) 7.72 eV
(C) 12.5 eV (D) 6.82 eV

13. [Online January 2019]
In an electron microscope, the resolution that can be achieved is of the order of the wavelength of electrons used. To resolve a width of 7.5×10^{-12} m, the minimum electron energy required is close to

(A) 100 keV (B) 1 keV
(C) 500 keV (D) 25 keV

14. [Online January 2019]
A metal plate of area $1\times10^{-4}\text{ m}^2$ is illuminated by a radiation of intensity 16 mWm^{-2}. The work function of the metal is 5 eV. The energy of the incident photons is 10 eV and only 10% of it produces photo electrons. The number of emitted photo electrons per second and their maximum energy, respectively, will be $\left[1\text{ eV}=1.6\times10^{-19}\text{ J}\right]$

(A) 10^{14} and 10 eV (B) 10^{11} and 5 eV
(C) 10^{10} and 5 eV (D) 10^{12} and 5 eV

15. [Online January 2019]

If the de-Broglie wavelength of an electron is equal to 10^{-3} times the wavelength of a photon of frequency 6×10^{14} Hz, then the speed of electron is equal to

(Speed of light $=3\times10^{8}\ \text{ms}^{-1}$

Planck's constant $=6.63\times10^{-34}$ Js

Mass of electron $=9.1\times10^{-31}$ kg)

(A) $1.7\times10^{6}\ \text{ms}^{-1}$ (B) $1.45\times10^{6}\ \text{ms}^{-1}$

(C) $1.8\times10^{6}\ \text{ms}^{-1}$ (D) $1.1\times10^{6}\ \text{ms}^{-1}$

16. [Online January 2019]

In a photoelectric experiment, the wavelength of the light incident on a metal is changed from 300 nm to 400 nm. The decrease in the stopping potential is close to $\left(\dfrac{hc}{e}=1240\ \text{nmV}\right)$

(A) 1.0 V (B) 2.0 V

(C) 1.5 V (D) 0.5 V

17. [Online January 2019]

A particle A of mass m and charge q is accelerated by a potential difference of 50 V. Another particle B of mass 4 m and charge q is accelerated by a potential difference of 2500 V. The ratio of de-Broglie wavelengths $\dfrac{\lambda_A}{\lambda_B}$ is close to

(A) 0.07 (B) 14.14

(C) 4.47 (D) 10.00

18. [Online January 2019]

When a certain photosensitive surface is illuminated with monochromatic light of frequency v, the stopping potential for the photo current is $-\dfrac{V_0}{2}$. When the surface is illuminated by monochromatic light of frequency $\dfrac{v}{2}$, the stopping potential is $-V_0$. The threshold frequency for photoelectric emission is

(A) $\dfrac{3v}{2}$ (B) $\dfrac{4}{3}v$

(C) $\dfrac{5v}{3}$ (D) $2v$

19. [Online January 2019]

In a Frank-Hertz experiment, an electron of energy 5.6 eV passes through mercury vapour and emerges with an energy 0.7 eV. The minimum wavelength of photons emitted by mercury atoms is close to

(A) 1700 nm (B) 2020 nm

(C) 250 nm (D) 220 nm

20. [Online 2018]

Two electrons are moving with non-relativistic speeds perpendicular to each other. If corresponding de Broglie wavelengths are λ_1 and λ_2, their de Broglie wavelength in the frame of reference attached to their centre of mass is

(A) $\lambda_{CM}=\dfrac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}$ (B) $\lambda_{CM}=\lambda_1=\lambda_2$

(C) $\lambda_{CM}=\left(\dfrac{\lambda_1+\lambda_2}{2}\right)$ (D) $\dfrac{1}{\lambda_{CM}}=\dfrac{1}{\lambda_1}+\dfrac{1}{\lambda_2}$

21. [Online 2018]

If the de Broglie wavelength associated with a proton and an α-particle are equal, then the ratio of velocities of the proton and the α-particle will be

(A) 1 : 2 (B) 2 : 1

(C) 1 : 4 (D) 4 : 1

22. [Online 2018]

The de-Broglie wavelength (λ_B) associated with the electron orbiting in the second excited state of hydrogen atom is related to that in the ground state (λ_G) by

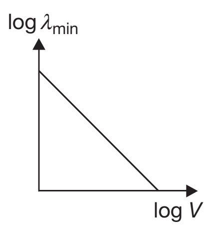
(A) $\lambda_B=\dfrac{\lambda_G}{3}$ (B) $\lambda_B=3\lambda_G$

(C) $\lambda_B=\dfrac{\lambda_G}{2}$ (D) $\lambda_B=2\lambda_G$

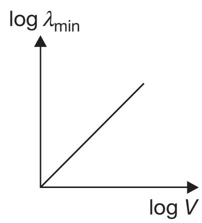
23. [2017]

An electron beam is accelerated by a potential difference V to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If λ_{min} is the smallest possible wavelength of X-ray in the spectrum, the variation of $\log\lambda_{min}$ with $\log V$ is correctly represented in

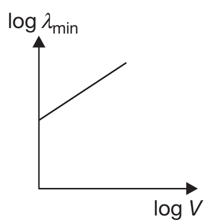
(A)



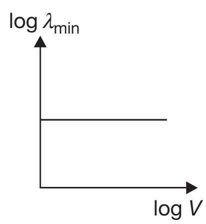
(B)



(C)



(D)



24. [2017]
A particle A of mass m and initial velocity v collides with a particle B of mass $\frac{m}{2}$ which is at rest. The collision is head on and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is

(A) $\frac{\lambda_A}{\lambda_B}=\frac{1}{3}$ (B) $\frac{\lambda_A}{\lambda_B}=2$
(C) $\frac{\lambda_A}{\lambda_B}=\frac{2}{3}$ (D) $\frac{\lambda_A}{\lambda_B}=\frac{1}{2}$

25. [Online 2017]
The maximum velocity of the photoelectrons emitted from the surface is v when light of frequency n falls on a metal surface. If the incident frequency is increased to $3n$, the maximum velocity of the ejected photoelectrons will be

(A) more than $\sqrt{3}v$ (B) less than $\sqrt{3}v$
(C) v (D) equal to $\sqrt{3}v$

26. [Online 2017]
A Laser light of wavelength 660 nm is used to weld Retina detachment. If a Laser pulse of width 60 ms and power 0.5 kW is used, the approximate number of photons in the pulse are
[Take Planck's constant $h=6.62\times10^{-34}\text{ Js}$]

(A) 10^{19} (B) 10^{22}
(C) 10^{18} (D) 10^{20}

27. [2016]
Radiation of wavelength λ, is incident on a photocell. The fastest emitted electron has speed v. If the wavelength of changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will be

(A) $=v\left(\frac{3}{5}\right)^{\frac{1}{2}}$ (B) $>v\left(\frac{4}{3}\right)^{\frac{1}{2}}$
(C) $<v\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (D) $=v\left(\frac{4}{3}\right)^{\frac{1}{2}}$

28. [Online 2016]
When photons of wavelength λ_1 are incident on an isolated sphere, the corresponding stopping potential is found to be V. When photons of wavelength λ_2 are used, the corresponding stopping potential was thrice that of the above value. If light of wavelength λ_3 is used then find the stopping potential for this case

(A) $\frac{hc}{e}\left(\frac{1}{\lambda_3}+\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right)$ (B) $\frac{hc}{e}\left(\frac{1}{\lambda_3}+\frac{1}{2\lambda_2}-\frac{1}{\lambda_1}\right)$
(C) $\frac{hc}{e}\left(\frac{1}{\lambda_3}-\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right)$ (D) $\frac{hc}{e}\left(\frac{1}{\lambda_3}+\frac{1}{2\lambda_2}-\frac{3}{2\lambda_1}\right)$

29. [Online 2016]
A photoelectric surface is illuminated successively by monochromatic light of wavelength λ and $\frac{\lambda}{2}$. If the maximum kinetic energy of the emitted photoelectrons in the second case is 3 times that in the first case, the work function of the surface is

(A) $\frac{hc}{2\lambda}$ (B) $\frac{hc}{\lambda}$
(C) $\frac{hc}{3\lambda}$ (D) $\frac{3hc}{\lambda}$

30. [2015]
Match **List-I** (Fundamental Experiment) with **List-II** (its conclusion) and select the correct option from the choices given below the list.

Column-I	Column-II
P. Franck-Hertz Experiment	(i) Particle nature of light
Q. Photo-electric Experiment	(ii) Discrete energy levels of atom
R. Davisson-Germer Experiment of electron	(iii) Wave nature
	(iv) Structure of atom

(A) P – (ii), Q – (i), R – (iii)
(B) P – (iv), Q – (iii), R – (ii)
(C) P – (i), Q – (iv), R – (iii)
(D) P – (ii), Q – (iv), R – (iii)

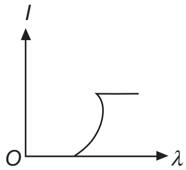
31. [Online 2015]
de-Broglie wavelength of an electron accelerated by a voltage of 50 V is close to ($|e|=1.6\times10^{-19}\text{ C}$, $m_e=9.1\times10^{-31}\text{ kg}$, $h=6.6\times10^{-34}\text{ Js}$)

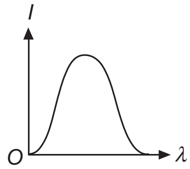
(A) 0.5 Å (B) 1.2 Å
(C) 1.7 Å (D) 2.4 Å

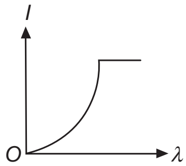
32. [Online 2015]
The de-Broglie wavelength associated with the electron in the $n=4$ level is

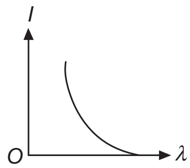
(A) two times the de-Broglie wavelength of the electron in the ground state
(B) four times the de-Broglie wavelength of the electron in the ground state
(C) half of the de-Broglie wavelength of the electron in the ground state
(D) $\frac{1}{4}$th of the de-Broglie wavelength of the electron in the ground state.

33. [2013]
The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows

(A)


(B)


(C)


(D)


34. [2012]
This question has Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.
Statement 1: Davisson-Germer experiment established the wave nature of electrons.
Statement 2: If electrons have wave nature, they can interfere and show diffraction.
(A) Statement 1 is true, Statement 2 is false.
(B) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for Statement 1.
(C) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.
(D) Statement 1 is false, Statement 2 is true.

35. [2011]
This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.
Statement-1: A metallic surface is irradiated by a monochromatic light of frequency $\upsilon > \upsilon_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also doubled.
Statement-2: The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.
(A) Statement-1 is true, statement-2 is false.
(B) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.
(C) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1.
(D) Statement-1 is false, Statement-2 is true.

36. [2010]
If a source of power 4 kW produces 10^{20} photons/second, the radiation belongs to a part of the spectrum called
(A) γ-rays (B) X-rays
(C) ultraviolet rays (D) microwaves

37. [2010]
This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.
Statement-1: When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by X-rays, both V_0 and K_{max} increase.
Statement-2: Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light.
(A) Statement-1 is true, Statement-2 is false.
(B) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(C) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.
(D) Statement-1 is false, Statement-2 is true.

38. [2009]
The surface of a metal is illuminated with the light of 400 nm. The kinetic energy of the ejected photoelectrons was found to be 1.68 eV. The work function of the metal is $(hc = 1240 \text{ eV nm})$
(A) 3.09 eV (B) 1.41 eV
(C) 1.51 eV (D) 1.68 eV

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

(In this section each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct)

1. [JEE (Advanced) 2017]
A photoelectric material having work-function ϕ_0 is illuminated with light of wavelength $\lambda\left(\lambda < \dfrac{hc}{\phi_0}\right)$. The fastest photoelectron has a de-Broglie wavelength λ_d. A change in wavelength of the incident light by $\Delta\lambda$ results in a change $\Delta\lambda_d$ in λ_d. Then, the ratio $\dfrac{\Delta\lambda_d}{\Delta\lambda}$ is proportional to

(A) $\dfrac{\lambda_d^2}{\lambda^2}$ (B) $\dfrac{\lambda_d}{\lambda}$

(C) $\dfrac{\lambda_d^3}{\lambda}$ (D) $\dfrac{\lambda_d^3}{\lambda^2}$

2. [JEE (Advanced) 2016]
In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_0) are given below

λ (μm)	V_0 (Volt)
0.3	2.0
0.4	1.0
0.5	0.4

Given that $c = 3\times10^8$ ms^{-1} and $e = 1.6\times10^{-19}$ C, Planck's constant (in units of Js) found from such an experiment is

(A) 6.0×10^{-34} (B) 6.4×10^{-34}

(C) 6.6×10^{-34} (D) 6.8×10^{-34}

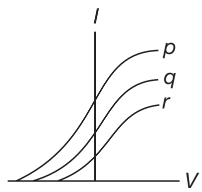
3. [JEE (Advanced) 2014]
A metal surface is illuminated by light of two different wavelengths 248 nm and 310 nm. The maximum speed of the photoelectrons corresponding to these wavelengths are μ_1 and μ_2, respectively. If the ratio $\mu_1 : \mu_2 = 2:1$ and $hc = 1240$ eVnm, the work function of the metal is nearly

(A) 3.7 eV (B) 3.2 eV

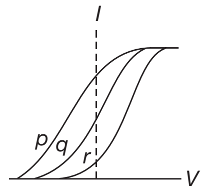
(C) 2.8 eV (D) 2.5 eV

4. [IIT-JEE 2009]
Photoelectric effect experiments are performed using three different metal plates p, q and r having work functions $\phi_p = 2.0$ eV, $\phi_q = 2.5$ eV and $\phi_r = 3.0$ eV, respectively. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct I-V graph for the experiment is

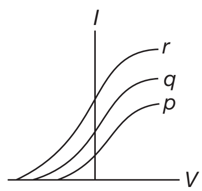
(A)



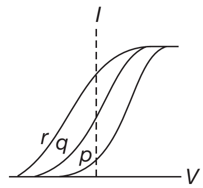
(B)



(C)



(D)



5. [IIT-JEE 2007]
Electrons with de-Broglie wavelength λ fall on the target in an X-ray tube. The cut-off wavelength of the emitted X-rays is

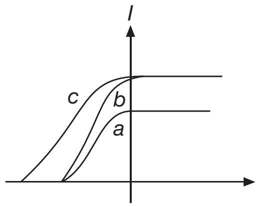
(A) $\lambda_0 = \dfrac{2mc\lambda^2}{h}$ (B) $\lambda_0 = \dfrac{2h}{mc}$

(C) $\lambda_0 = \dfrac{2m^2c^2\lambda^3}{h^2}$ (D) $\lambda_0 = \lambda$

6. [IIT-JEE 2005]
A beam of electron is used in an $YDSE$ experiment. The slit width is d. When the velocity of electron is increased, then

(A) no interference is observed
(B) fringe width increases
(C) fringe width decreases
(D) fringe width remains same

7. [IIT-JEE 2004]
The figure shows the variation of photocurrent with anode potential for a photosensitive surface for three different radiations. Let I_a, I_b and I_c be the intensities and f_a, f_b and f_c be the frequencies for the curves a, b and c respectively



(A) $f_a = f_b$ and $I_a \neq I_b$ (B) $f_a = f_c$ and $I_a = I_c$

(C) $f_a = f_b$ and $I_a = I_b$ (D) $f_b = f_c$ and $I_b = I_c$

8. [IIT-JEE 2004]

The energy of a photon is equal to the kinetic energy of a proton. The energy of the photon is E. Let λ_1 be the de-Broglie wavelength of the proton and λ_2 be the wavelength of the photon. The ratio $\frac{\lambda_1}{\lambda_2}$ is proportional to

(A) E^o (B) $E^{1/2}$

(C) E^{-1} (D) E^{-2}

9. [IIT-JEE 1999]

A particle of mass M at rest decays into two particles of masses m_1 and m_2 having non-zero velocities. The ratio of the de Broglie wavelengths of the particles $\frac{\lambda_1}{\lambda_2}$ is

(A) $\frac{m_1}{m_2}$ (B) $\frac{m_2}{m_1}$

(C) 1 (D) $\sqrt{\frac{m_2}{m_1}}$

10. [IIT-JEE 1998]

The work function of a substance is 4 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately

(A) 540 nm (B) 400 nm

(C) 310 nm (D) 220 nm

11. [IIT-JEE 1997]

The maximum kinetic energy of photoelectrons emitted from a surface when photons of energy 6 eV fall on it is 4 eV. The stopping potential in volt is

(A) 2 (B) 4

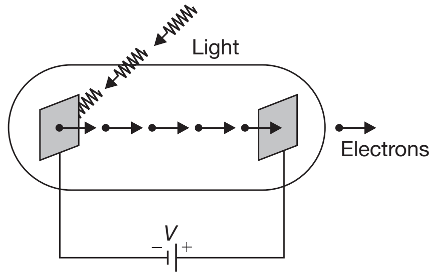
(C) 6 (D) 10

Multiple Correct Choice Type Problems

(In this section each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct)

1. [JEE (Advanced) 2016]

Light of wavelength λ_{ph} falls on a cathode plate inside a vacuum tube as shown in the figure. The work function of the cathode surface is ϕ and the anode is a wire mesh of conducting material kept at a distance d from the cathode. A potential difference V is maintained between the electrodes. If the minimum de Broglie wavelength of the electrons passing through the anode is λ_e, which of the following statement(s) is(are) true?



(A) For large potential difference $\left(V \gg \frac{\phi}{e}\right)$, λ_e is approximately halved if V is made four times

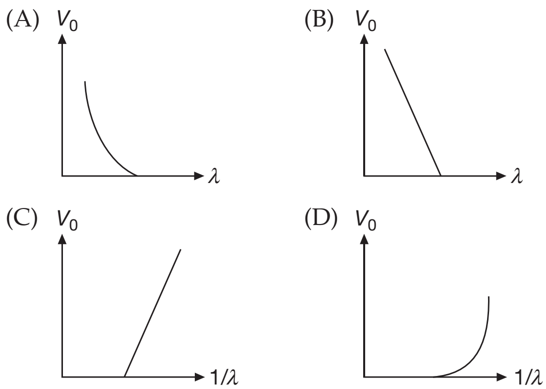
(B) λ_e increases at the same rate as λ_{ph} for $\lambda_{ph} < \frac{hc}{\phi}$

(C) λ_e is approximately halved, if d is doubled

(D) λ_e decreases with increase in ϕ and λ_{ph}

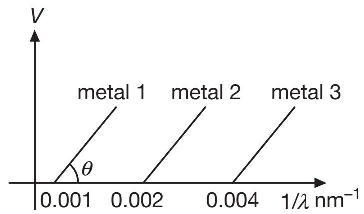
2. [JEE (Advanced) 2015]

For photo-electric effect with incident photon wavelength λ, the stopping potential is V_0. Identify the correct variation(s) of V_0 with λ and $\frac{1}{\lambda}$.



3. [IIT-JEE 2006]

The graph between $\frac{1}{\lambda}$ and stopping potential (V) of three metals having work functions ϕ_1, ϕ_2 and ϕ_3 in an experiment of photoelectric effect is plotted as shown in the figure. Which of the following statement(s) is/are correct? [Here, λ is the wavelength of the incident ray]



(A) Ratio of work functions $\phi_1 : \phi_2 : \phi_3 = 1:2:4$.

(B) Ratio of work functions $\phi_1 : \phi_2 : \phi_3 = 4:2:1$.

(C) $\tan\theta$ is directly proportional to $\dfrac{hc}{e}$, where h is Planck's constant and c is the speed of light.

(D) The violet colour light can eject photoelectrons from metals 2 and 3.

4. [IIT-JEE 1994]
When photons of energy 4.25 eV strike the surface of a metal, the ejected photoelectrons have maximum kinetic energy T_A eV and de Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photons of energy 4.70 eV is $T_B = (T_A - 1.50)$ eV. If the de-Broglie wavelength of these photoelectrons is $\lambda_B = 2\lambda_A$, then

(A) the work function of A is 2.25 eV.

(B) the work function of B is 4.20 eV.

(C) $T_A = 2.00$ eV.

(D) $T_B = 2.75$ eV.

5. [IIT-JEE 1992]
When a monochromatic point source of light is at a distance of 0.2 m from a photo-electric cell, the cut-off voltage and the saturation current are respectively 0.6 V and 18.0 mA. If the same source is placed 0.6 m away from the photoelectric cell, then

(A) the stopping potential will be 0.2 V.

(B) the stopping potential will be 0.6 V.

(C) the saturation current will be 6.0 mA.

(D) the saturation current will be 2.0 mA.

6. [IIT-JEE 1987]
Photoelectric effect supports the quantum nature of light because

(A) there is a minimum frequency of light below which no photoelectrons are emitted.

(B) the maximum kinetic energy of photoelectrons depends only on the frequency of light and not on its intensity.

(C) even when the metal surface is faintly illuminated, the photoelectrons leave the surface immediately.

(D) electric charge of the photoelectrons is quantized.

7. [IIT-JEE 1982]
The threshold wavelength for photoelectric emission from a material is 5200 Å. Photoelectrons will be emitted when this material is illuminated with monochromatic radiation from a

(A) 50 watt infrared lamp

(B) 1 watt infrared lamp

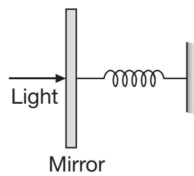
(C) 50 watt ultraviolet lamp

(D) 1 watt ultraviolet lamp

Integer/Numerical Answer Type Questions

(In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s)).

1. [JEE (Advanced) 2019]
A perfectly reflecting mirror of mass M mounted on a spring constitutes a spring-mass system of angular frequency Ω such that $\dfrac{4\pi M\Omega}{h} = 10^{24}\ \text{m}^{-2}$ with h as Planck's constant, N photons of wavelength $\lambda = 8\pi \times 10^{-6}$ m strike the mirror simultaneously at normal incidence such that the mirror gets displaced by 1 μm. If the value of N is $x \times 10^{12}$, then the value of x is ....
(Consider the spring as massless)



2. [JEE (Advanced) 2018]
In a photoelectric experiment a parallel beam of monochromatic light with power of 200 W is incident on a perfectly absorbing cathode of work function 6.25 eV. The frequency of light is just above the threshold frequency so that the photoelectrons are emitted with negligible kinetic energy. Assume that the photoelectron emission efficiency is 100%. A potential difference of 500 V is applied between the cathode and the anode. All the emitted electrons are incident normally on the anode and are absorbed. The anode experiences a force $F = n \times 10^{-4}$ N due to the impact of the electrons. The value of n is............. Mass of the electron $m_e = 9 \times 10^{-31}$ kg and $1.0 \text{ eV} = 1.6 \times 10^{-19}$ J.

3. [JEE (Advanced) 2013]
The work functions of Silver and Sodium are 4.6 and 2.3 eV, respectively. Find the ratio of the slope of the stopping potential versus frequency plot for Silver to that of Sodium.

4. **[IIT-JEE 2011]**
A silver sphere of radius $1\,\text{cm}$ and work function $4.7\ \text{eV}$ is suspended from an insulting thread in free-space. It is under continuous illumination of $200\ \text{nm}$ wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A \times 10^{Z}$ (where $1 < A < 10$). Find the value of Z .

5. **[IIT-JEE 2010]**
An α-particle and a proton are accelerated from rest by a potential difference of $100\ \text{V}$. After this, their de-Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$, to the nearest integer, is

ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on Photon Properties)

1. 4.7×10^{-6} Nm^{-2}
2. 453 Å
3. 2.5 Å
4. 2.51×10^{31}
5. 1.43×10^{19}
6. 10^{22}
7. (a) 3.98×10^{-19} J (b) 5×10^{15}
9. 5.5×10^{-12} m
10. $\lambda_e > \lambda_P$
11. $E_p > E_e$
12. $E_p > E_e$
13. 6.66×10^{-9} ms^{-2}
14. 1400 ms^{-1}
15. (a) 2.5×10^{19} photons/sec, (b) 3.33×10^{-8} N
16. 400 nm
17. 1.6×10^{16} photons/sec
18. 3.25 ms^{-1}

Test Your Concepts-II (Based on Photoelectric Effect)

1. 2.8 eV
2. 5×10^{19} photons/s, 1×10^{18} s^{-1}m^{-2}, 1.3 eV, 3351 Å
3. 11.93 eV
4. 2260 Å
5. 9.28 mC
6. 1.12×10^{12}
7. (a) 10^{15} Hz (b) 6.25 eV (c) 2 eV
8. 6.6×10^{-34} Js, 5990.25 Å
9. 1.9 eV
10. 1.9 eV, 4125 Å, No change is observed
11. (a) 2.1 eV (b) 2.1 V
12. 5.25 eV, 2357 Å, 3 V
13. (a) 5.4 eV (b) 1.5 eV
14. 4 eV, No
15. (a) 4.5 eV (b) 2.4 eV (c) 9.2×10^{5} ms^{-1}
16. 3.45×10^{-25} kgms^{-1}

Single Correct Choice Type Questions

1. C	**2.** C	**3.** A	**4.** B	**5.** C	**6.** D	**7.** B	**8.** A	**9.** C	**10.** C
11. A	**12.** A	**13.** C	**14.** A	**15.** A	**16.** B	**17.** C	**18.** D	**19.** C	**20.** B
21. A	**22.** A	**23.** B	**24.** B	**25.** C	**26.** B	**27.** B	**28.** D	**29.** B	**30.** B
31. C	**32.** D	**33.** C	**34.** D	**35.** C	**36.** A	**37.** C	**38.** D	**39.** B	**40.** B
41. B	**42.** C	**43.** B	**44.** A	**45.** B	**46.** C	**47.** D	**48.** D	**49.** B	**50.** A
51. B	**52.** D	**53.** B	**54.** B	**55.** D	**56.** B	**57.** C	**58.** D	**59.** C	**60.** C
61. B	**62.** C	**63.** B	**64.** A	**65.** D	**66.** C	**67.** A	**68.** C	**69.** D	**70.** C
71. C	**72.** C	**73.** A	**74.** C	**75.** C	**76.** B	**77.** C	**78.** C	**79.** A	**80.** D
81. C	**82.** C	**83.** C	**84.** D	**85.** A	**86.** D	**87.** A	**88.** C	**89.** A	**90.** C
91. D	**92.** C	**93.** A	**94.** C	**95.** D	**96.** A	**97.** C	**98.** B	**99.** B	**100.** A
101. C	**102.** D	**103.** D	**104.** B	**105.** A	**106.** A	**107.** A	**108.** D	**109.** C	**110.** B
111. C	**112.** A	**113.** B	**114.** C	**115.** B	**116.** A	**117.** A	**118.** D	**119.** A	**120.** A
121. B	**122.** B	**123.** A	**124.** A	**125.** D	**126.** D	**127.** A	**128.** C	**129.** B	**130.** A
131. B	**132.** B	**133.** B	**134.** C	**135.** A	**136.** D	**137.** B	**138.** D	**139.** D	**140.** C
141. C	**142.** B	**143.** B	**144.** D	**145.** B	**146.** B	**147.** A	**148.** C	**149.** C	**150.** C
151. A	**152.** C	**153.** C	**154.** A	**155.** D	**156.** D				

Multiple Correct Choice Type Questions

1. A, C	**2.** B, D	**3.** A, B, C	**4.** A, B, D	**5.** C, D
6. C, D	**7.** A, B	**8.** A, B, C, D	**9.** A, B, C, D	**10.** A, C, D
11. A, B	**12.** A, B, C, D	**13.** A, B, C	**14.** B, C	**15.** A, B
16. A, B, C	**17.** A, D	**18.** B, D	**19.** B, D	**20.** A, B
21. C, D	**22.** A, C	**23.** C, D	**24.** B, D	**25.** A, D
26. C	**27.** A	**28.** A, B	**29.** A, B	

Reasoning Based Questions

1. A	**2.** A	**3.** C	**4.** B	**5.** D	**6.** D	**7.** C	**8.** D	**9.** D	**10.** C
11. A	**12.** A	**13.** A	**14.** A	**15.** A	**16.** D	**17.** D			

Linked Comprehension Type Questions

1. C	**2.** A	**3.** B	**4.** C	**5.** B	**6.** A	**7.** C	**8.** C	**9.** A	**10.** B
11. A	**12.** C	**13.** B	**14.** D	**15.** B	**16.** D	**17.** D	**18.** A	**19.** A	**20.** A
21. B	**22.** A	**23.** A	**24.** A	**25.** B	**26.** B	**27.** C	**28.** B	**29.** A	**30.** C
31. B									

Matrix Match/Column Match Type Questions

1. $A \rightarrow (p, r)$	$B \rightarrow (p, r)$	$C \rightarrow (q)$	$D \rightarrow (s)$		
2. $A \rightarrow (s)$	$B \rightarrow (q, s)$	$C \rightarrow (p, r)$	$D \rightarrow (p, r)$		
3. $A \rightarrow (s)$	$B \rightarrow (q)$	$C \rightarrow (p, r)$	$D \rightarrow (p, r)$		
4. $A \rightarrow (p, q)$	$B \rightarrow (r, s)$	$C \rightarrow (q)$	$D \rightarrow (p)$		
5. $A \rightarrow (q)$	$B \rightarrow (s)$	$C \rightarrow (p)$	$D \rightarrow (t)$		
6. $A \rightarrow (p)$	$B \rightarrow (r)$	$C \rightarrow (s)$	$D \rightarrow (q)$		
7. $A \rightarrow (p, q)$	$B \rightarrow (p, r, s)$	$C \rightarrow (p, q)$	$D \rightarrow (s)$		
8. $A \rightarrow (p, r)$	$B \rightarrow (q, s)$	$C \rightarrow (s)$	$D \rightarrow (p, r)$		
9. $A \rightarrow (s)$	$B \rightarrow (q)$	$C \rightarrow (r, s)$	$D \rightarrow (p, r)$		
10. $A \rightarrow (q, s)$	$B \rightarrow (p, r)$	$C \rightarrow (p, r)$	$D \rightarrow (s)$		
11. $A \rightarrow (r)$	$B \rightarrow (s)$	$C \rightarrow (p)$	$D \rightarrow (u)$	$E \rightarrow (q)$	$F \rightarrow (t)$

Integer/Numerical Answer Type Questions

1. 2	**2.** 20	**3.** 28	**4.** 31	**5.** 148
6. 1	**7.** 300, 1200	**8.** 4	**9.** 5	**10.** 42
11. 2	**12.** 7	**13.** 8	**14.** 10	**15.** 2

ARCHIVE: JEE MAIN

1. D	**2.** C	**3.** A	**4.** A	**5.** A	**6.** D	**7.** D	**8.** C	**9.** D	**10.** C
11. A	**12.** B	**13.** D	**14.** B	**15.** B	**16.** A	**17.** B	**18.** A	**19.** C	**20.** A
21. D	**22.** B	**23.** A	**24.** B	**25.** A	**26.** D	**27.** B	**28.** D	**29.** A	**30.** A
31. C	**32.** B	**33.** D	**34.** B	**35.** D	**36.** B	**37.** A	**38.** B		

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Single Correct Choice Type Problems

1. D	**2.** B	**3.** A	**4.** A	**5.** A	**6.** C	**7.** A	**8.** B	**9.** C	**10.** C
11. B									

Multiple Correct Choice Type Problems

1. A	**2.** A, C	**3.** A, C	**4.** A, B, C	**5.** B, D
6. A, B, C	**7.** C, D			

Integer/Numerical Answer Type Questions

1. 1	**2.** 24.00	**3.** 1	**4.** 8	**5.** 3

CHAPTER 2

Atomic Physics

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Structure of an atom
(b) Thompson's Atom Model
(c) Rutherford's Atom Model
(d) Bohr's Model along with their postulates and drawbacks
(e) The Bohr's Theory applied on Hydrogen like atoms for understanding the concepts of energy and various transitions of an atom
(f) Hypothetical Bohr's Model
(g) X-rays and applications.

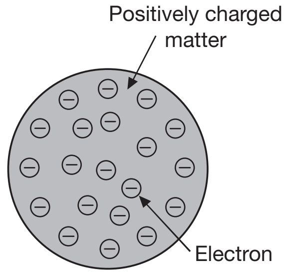
All this is followed by an Exercise Set (fully solved) which contains questions as per the latest JEE pattern. At the end of Exercise Set, a collection of problems asked previously in JEE Main are also given.

STRUCTURE OF AN ATOM: AN INTRODUCTION

All matter is made up of tiny particles known as atoms. There are about 105 different kinds of atoms, and they combine with each other in different ways to form groups called molecules. All matter has been found to be composed of atoms or molecules, and the basic knowledge of atoms and their constitution gives us valuable information about the behaviour of matter.

THOMSON EMPIRICAL MODEL/ THOMSON PLUM PUDDING MODEL

J.J. Thomson gave the, first idea regarding structure of atom. The model is known after him as Thomson's atom model. According to this, entire positive charge is distributed uniformly in the form of a sphere. Negatively charged electrons are arranged within this sphere lying here and there. The model is popularly known as plum-pudding model. Every electron is attracted towards the centre of uniformly charged sphere while they exert a force of repulsion upon each other. The electrons get themselves arranged in such a way that the force of attraction and that of repulsion balance each other. When disturbed, electrons vibrate to and fro within the atom and cause emission of visible, infra-red and ultra-violet light.



Thomson's atom model

Thomson's atom model satisfied the requirements of the atom and the demands of electro-magnetic theory. According to this model, hydrogen can give rise to a single spectral line. Experimentally, hydrogen is found to give several series, each series consisting of several lines. This indicated that Thomson's atom model needed modifications which was modified by Rutherford.

RUTHERFORD'S ATOMIC MODEL

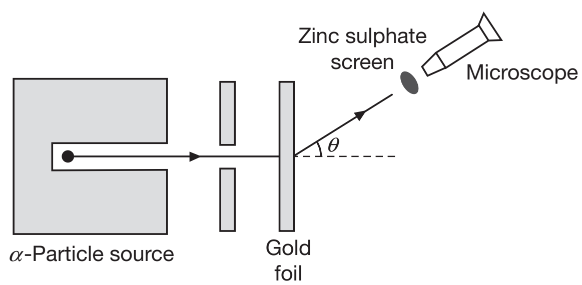
The correct description of the distribution of positive and negative charges within an atom was made in 1911 by a New Zealander while working at Manchester University in England. This was Ernest Rutherford, who was later called as Lord Rutherford for his many scientific achievements. He entered into physics during that crucial period of its development when the phenomenon of natural radioactivity had just been discovered, and he was first to realize that radioactivity represents a spontaneous disintegration of heavy unstable atoms.

Rutherford realized that important information about the inner structure of atoms can be obtained by the study of collisions between the rushing α particles incident on the atoms of various materials that form the target on which the α particle beam is incident.

EXPERIMENTAL ARRANGEMENT

The basic idea of the experimental arrangement used by Rutherford in his studies was explained as follows:

A piece or speck of α-emitting radioactive material is placed in a lead shield with a hole that allows a narrow beam of the α-particles to pass through it. In front of this arrangement is placed a gold thin metal foil to deflect or scatter the α particles. After passing through the gold foil the deflected particles are incident on a pivoted fluorescent screen with a magnifier through which the tiny flashes of light were observed whenever an α-particle struck the screen. The whole apparatus placed in an evacuated chamber, so that the particles would not collide with air molecules.



OBSERVATIONS

Rutherford performed experiments on the scattering of alpha particles by extremely thin metals foils and made the following observations.

(a) Most of the α-particles, either passed straight through the metal foil or suffered only small deflections. This could not be explained by Thomson's atom model.

(b) A few particles were deflected through angles which were less than or equal to $90°$.

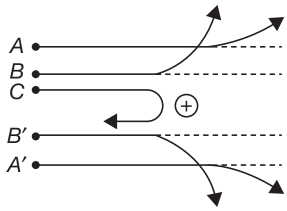
(c) Very few particles were deflected through angles greater than $90°$. It was observed that about only 1 in 8000 particles was found to be deflected greater than $90°$. Sometimes a particle was found to be deflected through $180°$. In other words, it was sent back in the same direction from where it came. The large angle of scattering came as a greater surprise. It could not be explained by Thomson's atom model. It was one of the main reasons for rejecting Thomson's atom model.

CONCLUSIONS

(a) The fact that most of the α-particles passed undeviated led to the conclusion that an atom has a lot of empty space in it.

(b) α-particles are heavy particles having high initial speeds. These could be deflected through large angles only by a strong electrical force. This led Rutherford to the conclusion that entire positive charge and nearly the entire mass of the atom were concentrated in a tiny central core. Rutherford named this core as Nucleus.

(c) The difference in deflection of various particles can be explained as follows:
α-particles which pass at greater distances away from the nucleus, shown as A and A' in figure, suffer a small deflection due to smaller repulsion exerted by the nucleus upon them. The particles like B and B' which pass close to the nucleus experience a comparatively greater force and hence get deflected through greater angles. A particle C which travels directly towards the nucleus is first slowed down by the repulsive force. Such a particle finally stops and then, is repelled along the direction of its approach. Thus, it gets repelled back after suffering a deviation of $180°$.



Different deviations for different α particles

Also, during the experiment following conclusions were made

(i) If ϕ is the angle made by a scattered particle with its original direction of motion and N is the number of particles available in that direction, it was found that,

$$\frac{1}{\sin^4\left(\frac{\phi}{2}\right)} \propto N$$

(ii) If t is the thickness of the foil and N is the number of α-particles scattered in a particular direction $(\phi = \text{constant})$ it was observed that

$$\frac{N}{t} = \text{constant}$$

$$\Rightarrow \quad \frac{N_1}{N_2} = \frac{t_1}{t_2}$$

ILLUSTRATION 1

In Rutherford's scattering experiment, if the number of α particles scattered at an angle of $90°$ is 55, then calculate the number of α particles scattered at an angle of $60°$.

SOLUTION

Since we know that

$$N \propto \frac{1}{\sin^4\left(\frac{\phi}{2}\right)}$$

$$\Rightarrow \quad \frac{N_{60°}}{N_{90°}} = \frac{\sin^4\left(\frac{90°}{2}\right)}{\sin^4\left(\frac{60°}{2}\right)} = \frac{\sin^4(45°)}{\sin^4(30°)} = \frac{\left(\frac{1}{\sqrt{2}}\right)^4}{\left(\frac{1}{2}\right)^4} = 4$$

$$\Rightarrow \quad \frac{N_{60°}}{N_{90°}} = \frac{N_{60°}}{55} = 4$$

$$\Rightarrow \quad N_{60°} = 4(55) = 220$$

RUTHERFORD'S ATOM-MODEL POSTULATES

On the basis of the conclusions drawn from Rutherford's experiment, a new atom model was proposed. This atom model, known as Rutherford's atom model, had the following characteristics.

(a) An atom consists of equal amounts of positive and negative charge so, the atom, as a whole is electrically neutral.

(b) The entire positive charge of the atom and practically its entire mass is concentrated in a small region which forms the core of the atom, called the nucleus.

(c) The negative charge, which is contained in the atom in the form of electrons, is distributed all around the nucleus, but separated from it.

(d) In order to explain the stability of electron at a certain distance from the nucleus, it was proposed by Rutherford that the electrons revolve round the nucleus in circular orbits. The electrostatic force of attraction between the nucleus and the electron provides the centripetal force to the electron to revolve in the orbit.

(e) The nuclear diameter is of the order of 10^{-14} m. This can be calculated by using the concept of distance of closest approach.

DISTANCE OF CLOSEST APPROACH

Let an α-particle (initially far from nucleus) having velocity v_α approach a nucleus (head-on) having a charge $+Ze$. The velocity of the α-particle decreases till it comes to rest at a distance r_0 from the nucleus. It is, then, repelled back along the direction of approach, r_0 gives the radius of nucleus.

Initial K.E. of α-particle $= \frac{1}{2} m_\alpha v_\alpha^2 = K_\alpha$

Initial P.E. of α-particle $= 0$

Final K.E. of α-particle $= 0$

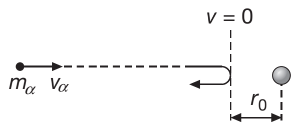
Final P.E. of α-particle $= \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_0}$

By Law of Conservation of Energy

$$(U + K)_{\text{initial}} = (U + K)_{\text{final}}$$

$$\Rightarrow \quad K_\alpha = \frac{1}{2} m_\alpha v_\alpha^2 = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_0}$$

$$\Rightarrow \quad r_0 = \frac{2}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{m_\alpha v_\alpha^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{K_\alpha}\right)$$



For α-particle, $q_1 = 2e$

$$\Rightarrow \quad r_0 = \frac{2}{4\pi\varepsilon_0}\left(\frac{2Ze^2}{m_\alpha v_\alpha^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Ze^2}{K_\alpha}\right)$$

$$\Rightarrow \quad r_0 = \frac{4}{4\pi\varepsilon_0}\left(\frac{Ze^2}{m_\alpha v_\alpha^2}\right) = 4\times 9\times 10^9\left(\frac{Ze^2}{m_\alpha v_\alpha^2}\right)$$

In one of the experiments, α-particles of velocity $2\times 10^7\ \text{ms}^{-1}$ were bombarded upon gold foil with $Z = 79$

So, for $Z = 79$, $e = 1.59\times 10^{-19}$ C, $m_\alpha = 4\times 1.67\times 10^{-27}$ kg, $v_\alpha = 2\times 10^7\ \text{ms}^{-1}$, we get,

$$r_0 = 4\times 9\times 10^9 \times \frac{79\times\left(1.59\times 10^{-19}\right)^2}{4\times 1.67\times 10^{-27}\times\left(2\times 10^7\right)^2}$$

$$\Rightarrow \quad r_0 = 2.69\times 10^{-14}\ \text{m}$$

This gives the order of the radius of nucleus.

ILLUSTRATION 2

A head-on collision takes place between an α-particle of kinetic energy 5.5 MeV and a gold nucleus $(Z = 79)$. Calculate the distance of closest approach.

SOLUTION

Since, the distance of closest approach is given by

$$r_0 = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Ze^2}{m_\alpha v_\alpha^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Ze^2}{K_\alpha}\right)$$

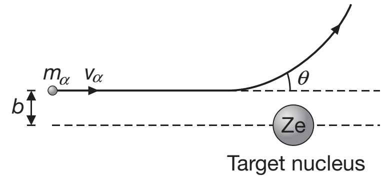
$$\Rightarrow \quad r_0 = 9\times 10^9 \times \frac{2\times 79\times\left(1.6\times 10^{-19}\right)^2}{5.5\times\left(1.6\times 10^{-13}\right)}$$

$$\Rightarrow \quad r_0 = 4.13\times 10^{-14}\ \text{m}$$

The radius of gold nucleus must be smaller than r_0, so it may lie between 10^{-14} to 10^{-15} m.

TRAJECTORY OF AN ALPHA PARTICLE AND IMPACT PARAMETER

The perpendicular distance of the initial velocity vector of the α-particle from the centre of the nucleus is called **impact parameter** and is denoted by b.



The angle between the direction of approach of the α-particle and the direction in which it finally goes is defined as the angle of scattering and is denoted by θ. Rutherford concluded that

$$\tan\left(\frac{\theta}{2}\right) = \frac{1}{b}\left(\frac{Ze^2}{4\pi\varepsilon_0 K_\alpha}\right)$$

$$\Rightarrow \quad b = \frac{Ze^2}{4\pi\varepsilon_0 K_\alpha}\cot\left(\frac{\theta}{2}\right)$$

For a given nucleus and α-particle of given energy K

$$\tan\left(\frac{\theta}{2}\right) \propto \frac{1}{b}$$

$$\Rightarrow \quad \cot\left(\frac{\theta}{2}\right) \propto b$$

A graph between b and $\cot\left(\frac{\theta}{2}\right)$ is a straight line.

Therefore θ increases with decrease in value of b which implies that an α-particle, passing closer to the nucleus, is deflected at large angles.

In case of head-on collision, the impact parameter tends to zero and the α-particle rebounds back. An α-particle close to the nucleus has small impact parameter and suffers large scattering.

For a large impact parameter, the α-particle goes nearly undeviated and has a small deflection.

The fact, that only a small fraction of number of α-particles rebounds back, indicates that the number of α-particles suffering head on collision is very small, which indicates that the mass of the atom is concentrated in a small volume. Therefore, Rutherford scattering is a powerful way to determine the size of the nucleus.

ILLUSTRATION 3

Let a 5 MeV α-particle is scattered by $74°$ when it approaches a gold nucleus $(Z = 79)$. Find the impact parameter.

SOLUTION

Since, $Z = 79$

$$K_\alpha = 5\text{ MeV} = 5\times1.6\times10^{-13}\text{ J} = 8\times10^{-13}\text{ J}$$

$$\theta = 74°$$

$$\Rightarrow \quad \frac{\theta}{2} = 37°$$

$$\Rightarrow \quad \cot\left(\frac{\theta}{2}\right) = \frac{4}{3}$$

Using the relation

$$\tan\left(\frac{\theta}{2}\right) = \frac{1}{b}\left(\frac{Ze^2}{4\pi\varepsilon_0 K_\alpha}\right)$$

$$\Rightarrow \quad b = \left(\frac{Ze^2}{4\pi\varepsilon_0 K_\alpha}\right)\cot\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \quad b = \frac{79\times(1.6\times10^{-19})^2\times(9\times10^9)}{8\times10^{-13}}\times\frac{4}{3}$$

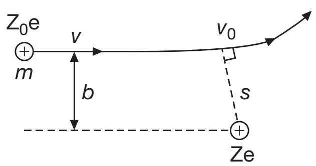
$$\Rightarrow \quad b = 3.03\times10^{-16}\text{ m}$$

ILLUSTRATION 4

A particle of mass m, atomic number Z_0, initial speed v and impact parameter b is scattered by a heavy nucleus of atomic number Z. Use the principle of conservation of angular momentum and energy to obtain a relation between the minimum distance s of the particle from the nucleus in terms of Z, Z_0, v and b. Show that for $b = 0$, s reduces to the distance of closest approach r_0 given by

$$r_0 = \frac{1}{4\pi\varepsilon_0}\left(\frac{2ZZ_0e^2}{mv^2}\right)$$

SOLUTION



By angular momentum conservation, we have

$$mvb = mv_0s \quad \ldots(1)$$

By energy conservation, we have

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 + \frac{1}{4\pi\varepsilon_0}\left(\frac{ZZ_0e^2}{s}\right) \quad \ldots(2)$$

Substituting value of v_0 from equation (1) in equation (2), we get

$$\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{vb}{s}\right)^2 + \frac{1}{4\pi\varepsilon_0}\left(\frac{ZZ_0e^2}{s}\right)$$

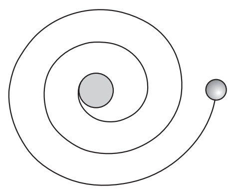
$$\Rightarrow \quad \frac{1}{2}mv^2\left(1-\frac{b^2}{s^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{ZZ_0e^2}{s}\right)$$

For $b = 0$, we have

$$s = \frac{1}{4\pi\varepsilon_0}\left(\frac{2ZZ_0e^2}{mv^2}\right)$$

FAILURE OF RUTHERFORD MODEL

(a) According to laws of electro-magnetic theory, a charged particle in accelerated motion must radiate energy in the form of electro-magnetic radiation. As the electron revolves in a circular orbit, it is constantly subjected to centripetal acceleration $\frac{v^2}{r}$.



Electron spiralling inwards

So, it must radiate energy continuously. Bohr calculated that this emission of radiation would cause the electrons in an atom to lose all their energy and fall into the nucleus within a hundred - millionth of a second following a spiral path. Thus, the whole atomic structure should collapse. Since matter composed of atoms exists permanently, as far as we know, there was obviously something wrong here.

(b) According to Rutherford model, electron can revolve in any orbit. So, it must emit continuous radiations of all frequencies. But elements emit spectral lines of only definite frequencies.

BOHR'S ATOMIC MODEL

To rectify the drawbacks of Rutherford Model, Bohr proposed a theory which applies to hydrogen atom and species like $He^{+}(Z=2)$, $Li^{++}(Z=3)$, $Be^{+++}(Z=4)$ etc. Here a single electron revolves around a stationary nucleus of positive charge Ze where $Z=1$ for hydrogen atom, $Z=2$ for He^{+} etc. Bohr in defiance of the well-established laws of classical mechanics and electrodynamics, proposed the following postulates to support his atomic model.

Circular Orbits

The atom consists of central nucleus, containing the entire positive charge and almost all mass of the atom. The electrons revolve around the nucleus in certain discrete circular orbits. The necessary centripetal force for circular orbit is provided by Coulomb's attraction between the electron and nucleus. So,

$$\frac{mv^2}{r}=\frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r^2}$$

where, $m=$ mass of electron,
$r=$ radius of circular orbit,
$v=$ speed of electron in circular orbit,
$Ze=$ charge on nucleus,
$Z=$ atomic number,
$e=$ charge on electron $=-1\cdot6\times10^{-19}$ C

Stationary Orbits

The allowed orbits for electron are those in which the electron does not radiate energy. These orbits are also called stationary orbits.

Stationary Nucleus

The nucleus is so heavy, that its motion may be neglected.

Constancy of Mass

The mass of the electron in motion is assumed to be constant.

Quantum Condition (Bohr's Quantisation Rule)

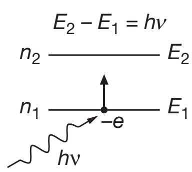
The stationary orbits are those in which angular momentum of electron is an integral multiple of $\frac{h}{2\pi}(=\hbar)$. This condition is also called as Bohr's Quantisation Rule according to which only those orbits are permitted for which the angular momentum of the electron in that orbit is an integral multiple of $\frac{h}{2\pi}$. This rule applies to an electron revolving in a particular orbit.

Mathematically, according to Bohr's Quantisation Rule, we have $L=mvr=n\left(\frac{h}{2\pi}\right)$, where n being an integer or the principle quantum number of the electron in the revolving orbit.

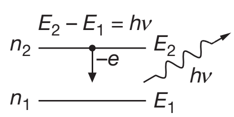
Bohr's Transition Rule

This rule applies to an electron making a transition from one stationary orbit to another. Whenever, an electron makes a transition from one orbit to the other, then a photon is emitted or absorbed having energy equal to the difference of energies between initial and final orbits/states.

So, when a photon of energy equal to the energy difference of two levels (say, $h\nu=E_2-E_1$) is incident on an electron in the lower energy level (n_1), then the electron will get excited to the higher energy level (n_2) as shown.



Similarly, when an electron makes a transition from a higher energy level (n_2) to a lower energy level (n_1), then a photon of energy equal to the energy difference of two levels i.e., $h\nu=E_2-E_1$ is emitted as shown.



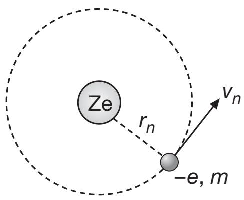
So, according to Bohr's Transition Rule, we have

$$|\Delta E|=\left|E_i-E_f\right|=h\nu$$

$$\Rightarrow \quad \nu_{i\to f}=\nu_{f\to i}=\nu=\frac{\left|E_i-E_f\right|}{h}$$

BOHR'S THEORY OF THE HYDROGEN LIKE ATOMS

Bohr proposed a theory which applies to hydrogen atom and species like He^{+}, Li^{++}, Be^{+++} etc. where a single electron revolves around a stationary nucleus of positive charge Ze as shown in figure.



We must note that, $Z=1$ for hydrogen atom, $Z=2$ for He^{+}, $Z=3$ for Li^{++}, $Z=4$ for Be^{+++}. Bohr applied the well established laws of classical mechanics and electrodynamics, to calculate the following quantities for the hydrogen like atoms.

Radius of Orbit

Since, we have

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r^2} \qquad \ldots(1)$$

and $$mvr = \frac{nh}{2\pi} \qquad \ldots(2)$$

From (2), $v = \dfrac{nh}{2\pi mr}$. Put in (1), we get

$$r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2 Z}$$

$$\Rightarrow \quad r_n = (0.53)\frac{n^2}{Z}\ \text{Å}$$

So, for H-like atoms, we have

$$r_n \propto \frac{n^2}{Z}$$

Velocity of Electron in *n*th Orbit

Since, $v_n = \dfrac{nh}{2\pi m r_n}$ and $r_n = \dfrac{n^2h^2\varepsilon_0}{\pi me^2 Z}$

$$\Rightarrow \quad v_n = \left(\frac{e^2}{2h\varepsilon_0}\right)\frac{Z}{n} = \left(\frac{e^2}{2h\varepsilon_0 c}\right)\left(\frac{cZ}{n}\right)$$

$$\Rightarrow \quad v_n = \alpha\left(\frac{cZ}{n}\right)$$

where $\alpha = \dfrac{e^2}{2h\varepsilon_0 c}$ is the fine structure constant (a pure number) whose value is $\dfrac{1}{137}$.

$$\Rightarrow \quad v_n = \left(\frac{1}{137}\right)\frac{cZ}{n}$$

i.e. velocity of electron in Bohr's first orbit is $\dfrac{c}{137}$, in second orbit is $\dfrac{c}{274}$ and so on.

Angular Frequency/Velocity

$$\omega_n = \frac{v_n}{r_n} = \frac{\left(\dfrac{e^2}{2h\varepsilon_0}\right)\dfrac{Z}{n}}{\left(\dfrac{h^2\varepsilon_0}{\pi me^2}\right)\dfrac{n^2}{Z}} = \left(\frac{\pi me^4}{2\varepsilon_0^2 h^3}\right)\left(\frac{Z^2}{n^3}\right)$$

$$\Rightarrow \quad \omega_n \propto \frac{Z^2}{n^3}$$

Frequency

$$f_n = \frac{\omega_n}{2\pi} = \left(\frac{me^4}{4\varepsilon_0^2 h^3}\right)\left(\frac{Z^2}{n^3}\right)$$

$$\Rightarrow \quad f_n \propto \frac{Z^2}{n^3}$$

Time Period of Revolution

$$T_n = \frac{1}{f_n} = \frac{1}{\left(\dfrac{me^4}{4\varepsilon_0^2 h^3}\right)\left(\dfrac{Z^2}{n^3}\right)} = \left(\frac{4\varepsilon_0^2 h^3}{me^4}\right)\left(\frac{n^3}{Z^2}\right)$$

$$\Rightarrow \quad T_n \propto \frac{n^3}{Z^2}$$

Current

$$i_n = \frac{e}{T_n} = ef_n = \left(\frac{me^5}{4\varepsilon_0^2 h^3}\right)\left(\frac{Z^2}{n^3}\right)$$

$$\Rightarrow \quad i_n \propto \frac{Z^2}{n^3}$$

Magnetic Field at the Centre of Atom

$$B_n = \frac{\mu_0 i_n}{2r_n} = \left(\frac{\mu_0}{2}\right)\left[\left(\frac{me^5}{4\varepsilon_0^2 h^3}\right)\left(\frac{Z^2}{n^3}\right)\right]\left[\left(\frac{\pi me^2}{h^2\varepsilon_0}\right)\left(\frac{Z}{n^2}\right)\right]$$

$$\Rightarrow \quad B_n = \left(\frac{\pi m^2 Z^3 e^7 \mu_0}{8\varepsilon_0^3 n^5 h^5}\right)\left(\frac{Z^3}{n^5}\right)$$

$$\Rightarrow \quad B_n \propto \frac{Z^3}{n^5}$$

Magnetic Moment of Atom

$$M_n = i_n A_n = i_n\left(\pi r_n^2\right) = \pi\left(\frac{me^5Z^2}{4\varepsilon_0^2 h^3 n^3}\right)\left(\frac{n^2h^2\varepsilon_0}{\pi me^2 Z}\right)^2$$

$$\Rightarrow \quad M_n = i_n A_n = i_n\left(\pi r_n^2\right) = \pi\left(\frac{me^5Z^2}{4\varepsilon_0^2 h^3 n^3}\right)\left(\frac{n^2h^2\varepsilon_0}{\pi me^2 Z}\right)^2$$

$$\Rightarrow \quad M_n = n\left(\frac{eh}{4\pi m}\right)$$

The term $\frac{eh}{4\pi m}$ is called the Bohr's Magneton for the atom.

$$\Rightarrow \quad M_n \propto n$$

Angular Momentum

$$L_n = mv_n r_n = m\left(\frac{e^2 Z}{2h\varepsilon_0 n}\right)\left(\frac{n^2h^2\varepsilon_0}{\pi me^2 Z}\right) = n\left(\frac{h}{2\pi}\right)$$

As expected from Bohr's Quantisation Rule.

Kinetic Energy of Electron (E_K)

Since, we have

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r^2}$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r}$$

$$\Rightarrow \quad E_K = \frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r}$$

Potential Energy (U) of Electron in nth Orbit

$$U = -\frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r}$$

$$\Rightarrow \quad U = -\frac{Ze^2}{4\pi\varepsilon_0 r}$$

Total Energy (E) of Electron in nth Orbit

Total Energy = K.E. + P.E.

$$\Rightarrow \quad E = \frac{Ze^2}{8\pi\varepsilon_0 r} - \frac{Ze^2}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \quad E = -\frac{Ze^2}{8\pi\varepsilon_0 r}$$

So, we conclude that

$$\text{Total Energy} = -\text{K.E.} = \frac{1}{2}(\text{P.E.})$$

Further, since $r = \frac{n^2h^2\varepsilon_0}{\pi me^2 Z}$

$$\Rightarrow \quad E = -\left(\frac{me^4}{8h^2\varepsilon_0^2}\right)\frac{Z^2}{n^2}$$

$$\Rightarrow \quad E = -(13.6)\frac{Z^2}{n^2}\ \text{eV}$$

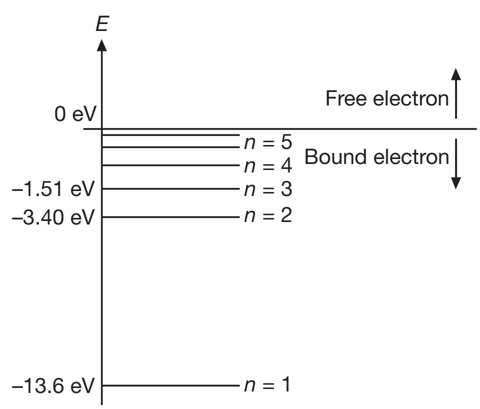
Also $E = -\left(\frac{me^4}{8\varepsilon_0^2 ch^3}\right)ch\frac{Z^2}{n^2}$

$$\Rightarrow \quad E = -(Rch)\frac{Z^2}{n^2}$$

where R = Rydberg's constant $= \frac{me^4}{8\varepsilon_0^2 ch^3} = 1.097\times10^7\ \text{m}^{-1}$

and Rydberg's Energy is $Rch \approx 2.17\times10^{-18}\ \text{J} \approx 13.6\ \text{eV}$. It is the energy of an electron in the first orbit of H atom.

An energy level diagram for the hydrogen atom $(Z = 1)$ is shown in the figure.



The vertical axis represents energy. The (arbitrary) zero of energy is taken as the energy of a stationary electron, infinitely far from the positive nucleus. The lowest energy level $(n=1)$ is known as the ground state. The energy level corresponding to $n=2$ is called the first excited state and so on. In this diagram zero energy level corresponds to $n=\infty$ which is the ionized state of the atom.

ILLUSTRATION 5

A beam of monochromatic light of wavelength λ ejects photoelectrons from a caesium metal surface having work function 1.7 eV. These photoelectrons are made to collide with hydrogen atoms in ground state. Find the maximum value of λ for which the

(a) hydrogen atoms may be ionized.
(b) hydrogen atom gets excited from the ground state to the first excited state and
(c) excited hydrogen atoms may emit visible light.

SOLUTION

(a) To ionize H-atom, the kinetic energy of photo-electrons must be at least 13.6 eV

$$\Rightarrow \quad \frac{12375}{\lambda} = 1.7 + 13.6$$

$$\Rightarrow \quad \lambda = \frac{12375}{15.3} \approx 809\text{ Å}$$

(b) To excite H-atom from $n=1$ to $n=2$, the kinetic energy possessed by the photoelectrons must be at least 10.2 eV.

$$\Rightarrow \quad \frac{12375}{\lambda} = 1.7 + 10.2 = 11.9$$

$$\Rightarrow \quad \lambda = \frac{12375}{11.9} \approx 1040\text{ Å}$$

(c) To emit visible light photons, H-atom must be excited at least from $n=1$ to $n=3$, so that for Balmer series it can emit visible light. So, the kinetic energy possessed by the photoelectrons must be at least 12.09 eV

$$\Rightarrow \quad \frac{12375}{\lambda} = 1.7 + 12.09 = 13.79$$

$$\Rightarrow \quad \lambda = \frac{12375}{13.79} \approx 897\text{ Å}$$

ILLUSTRATION 6

Calculate the angular momentum of an electron in Bohr's hydrogen atom whose energy is -3.4 eV?

SOLUTION

Energy of electron in n^{th} Bohr orbit of hydrogen atom is given by,

$$E = -\frac{13.6}{n^2}\text{ eV}$$

$$\Rightarrow \quad -3.4 = -\frac{13.6}{n^2}$$

$$\Rightarrow \quad n^2 = 4$$

$$\Rightarrow \quad n = 2$$

Since, the angular momentum of an electron in n^{th} orbit is given by

$$L = \frac{nh}{2\pi}$$

$$\Rightarrow \quad L = 2\left(\frac{h}{2\pi}\right) = \frac{h}{\pi}$$

ILLUSTRATION 7

Using Bohr's theory show that when n is very large the frequency of radiation emitted by hydrogen atom due to transition of electron from n to $(n-1)$ is equal to frequency of revolution of electron in its orbit.

SOLUTION

Frequency of revolution electron in nth orbit is given by

$$f_{\text{revolution}} = \frac{1}{T} = \frac{v}{2\pi r} = \frac{\left(\dfrac{e^2}{2h\varepsilon_0 c}\right)\dfrac{cz}{n}}{2\pi\left(\dfrac{n^2h^2\varepsilon_0}{\pi me^2 z}\right)}$$

$$\Rightarrow \quad f_{\text{revolution}} = \left(\frac{me^4}{4\varepsilon_0^2 h^3}\right)\frac{z^2}{n^3} \qquad \ldots(1)$$

Further, frequency of transition from n to $(n-1)$ is

$$hf = \frac{me^4 z^2}{8h^2\varepsilon_0^2}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$\Rightarrow \quad hf = \frac{me^4 z^2}{8h^2\varepsilon_0^2}\left[\frac{2n-1}{n^2(n-1)^2}\right]$$

When n is large then

$$2n-1 \cong 2n \text{ and } n-1 \cong n$$

$$\Rightarrow \quad hf \cong \frac{me^4z^2}{8h^2\varepsilon_0^2}\frac{2n}{n^4}$$

$$\Rightarrow \quad f_{\text{transition}} \cong \left(\frac{me^4}{4\varepsilon_0^2h^3}\right)\frac{z^2}{n^3} \qquad \ldots(2)$$

So, from (1) and (2), we observe that for large n,

$$f_{\text{revolution}} = f_{\text{transition}} \text{ (between adjacent levels)}$$

This Principle is also called **"BOHR'S CORRESPONDENCE PRINCIPLE"**.

ILLUSTRATION 8

An electron is orbiting in a circular orbit of radius r under the influence of a constant magnetic field of strength B. Assuming that Bohr's postulate regarding the quantisation of angular momentum holds good for this electron, find

(a) the allowed values of the radius r of the orbit.
(b) the kinetic energy of the electron in orbit.
(c) the potential energy of interaction between the magnetic moment of the orbital current due to the electron moving in its orbit and the magnetic field B.
(d) the total energy of the allowed energy levels.
(e) the total magnetic flux due to the magnetic field B passing through the nth orbit.

(Assume that the charge on the electron is $-e$ and the mass of the electron is m).

SOLUTION

(a) Since, $r = \frac{mv}{Be}$ $\qquad \ldots(1)$

From Bohr's Quantisation Rule, we have

$$mvr = \frac{nh}{2\pi} \qquad \ldots(2)$$

Solving these two equations, we get

$$r = \sqrt{\frac{nh}{2\pi Be}} \text{ and } v = \sqrt{\frac{nhBe}{2\pi m^2}}$$

(b) $K = \frac{1}{2}mv^2 = \frac{nhBe}{4\pi m}$

(c) $M = iA = \left(\frac{e}{T}\right)(\pi r^2) = \frac{e}{\left(\frac{2\pi r}{v}\right)}(\pi r^2) = \frac{evr}{2}$

$$\Rightarrow \quad M = \frac{e}{2}\sqrt{\frac{nh}{2\pi Be}}\sqrt{\frac{nhBe}{2\pi m^2}} = \frac{nhe}{4\pi m}$$

Since, $U = -MB\cos 180°$

$$\Rightarrow \quad U = \frac{nheB}{4\pi m}$$

The angle between $\vec{M}$ and $\vec{B}$ will be 180° because instead of taking electronic current, we have to take conventional current which moves opposite to electronic current.

(d) $E = U + K = \frac{nheB}{2\pi m}$

(e) $|\phi| = B\pi r^2 = \frac{nh}{2e}$

FREQUENCY OF EMITTED RADIATION

If electron jumps from initial state n_i to a final state n_f, then frequency of emitted or absorbed radiation ν is given by applying Bohr's Transition Rule, according to which

$$E_i - E_f = h\nu$$

$$\Rightarrow \quad \nu = \frac{E_i - E_f}{h} = Z^2Rc\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

If c is the speed of light and λ the wavelength of emitted or absorbed radiation, then

$$\nu = \frac{c}{\lambda} = Z^2Rc\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

So, Wave number $(\bar{\nu})$ is given by

$$\bar{\nu} = \frac{1}{\lambda} = Z^2R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

This relation holds for radiations emitted by hydrogen-like atoms i.e.

$H(Z=1)$, $He^{+}(Z=2)$, $Li^{++}(Z=3)$ and

$Be^{+++}(Z=4)$.

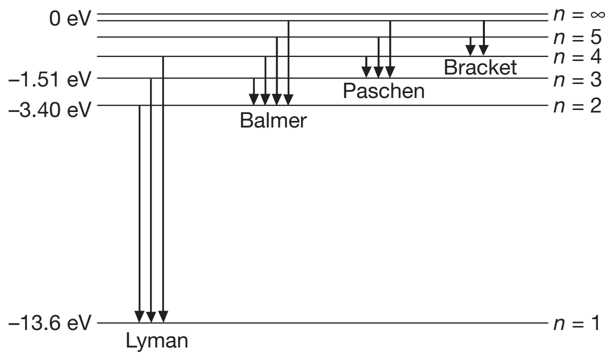
HYDROGEN SPECTRUM

Since, wave number $(\bar{\nu})$ is given by

$$\bar{\nu} = \frac{1}{\lambda} = Z^2 R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

where R is the Rydberg constant.

The various transitions for the hydrogen atom are shown in the following figure. All transitions starting from $n = 2$ onwards and ending at $n = 1$ belong to the Lyman Series. Likewise, all transitions starting from $n = 3$ onwards and ending at $n = 2$ belong to the Balmer Series. The other spectral series' names are mentioned in the figure.



	INITIAL STATE	FINAL STATE	WAVELENGTH FORMULA	FIRST MEMBER - SECOND MEMBER	SERIES LIMIT $n_i \to \infty$ TO n_f	MAXIMUM WAVELENGTH $(n_f + 1)$ TO n_f	LINES FOUND IN
LYMAN	$n_i = 2, 3, 4, 5, 6, \ldots$	$n_f = 1$	$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n_i^2}\right)$	$n_i = 2$ to $n_f = 1$ $n_i = 3$ to $n_f = 1$	From ∞ to 1 $\lambda = \frac{1}{R}$ $\lambda = 911$ Å	From 2 to 1 $\lambda = \frac{4}{3R}$ $\lambda = 1216$ Å	UV Region
BALMER	$n_i = 3, 4, 5, 6, 7, \ldots$	$n_f = 2$	$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$	$n_i = 3$ to $n_f = 2$ $n_i = 4$ to $n_f = 2$	From ∞ to 2 $\lambda = \frac{4}{R}$ $\lambda = 3646$ Å	From 3 to 2 $\lambda = \frac{36}{5R}$ $\lambda = 6563$ Å	Visible Region
PASCHEN	$n_i = 4, 5, 6, 7, 8, \ldots$	$n_f = 3$	$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n_i^2}\right)$	$n_i = 4$ to $n_f = 3$ $n_i = 5$ to $n_f = 3$	From ∞ to 3 $\lambda = \frac{9}{R}$ $\lambda = 8204$ Å	From 4 to 3 $\lambda = \frac{144}{7R}$ $\lambda = 18753$ Å	IR Region
BRACKETT	$n_i = 5, 6, 7, 8, 9, \ldots$	$n_f = 4$	$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n_i^2}\right)$	$n_i = 5$ to $n_f = 4$ $n_i = 6$ to $n_f = 4$	From ∞ to 4 $\lambda = \frac{16}{R}$ $\lambda = 14585$ Å	From 5 to 4 $\lambda = \frac{400}{9R}$ $\lambda = 40515$ Å	IR Region
PFUND	$n_i = 6, 7, 8, 9, 10, \ldots$	$n_f = 5$	$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n_i^2}\right)$	$n_i = 6$ to $n_f = 5$ $n_i = 7$ to $n_f = 5$	From ∞ to 5 $\lambda = \frac{25}{R}$ $\lambda = 22790$ Å	From 6 to 5 $\lambda = \frac{900}{11R}$ $\lambda = 74583$ Å	Far IR Region

ILLUSTRATION 9

Find the largest and shortest wavelengths in the Lyman series for hydrogen. In what region of the electromagnetic spectrum does each series lie?

SOLUTION

The transition equation for Lyman series is given by,

$$\frac{1}{\lambda}=R\left(\frac{1}{1^2}-\frac{1}{n^2}\right),\ n=2,3,\ldots..$$

The largest wavelength is corresponding to $n=2$, so

$$\frac{1}{\lambda_{max}}=1.097\times10^7\left(\frac{1}{1}-\frac{1}{4}\right)$$

$$\Rightarrow\quad \frac{1}{\lambda_{max}}=0.823\times10^7$$

$$\Rightarrow\quad \lambda_{max}=1.2154\times10^{-7}\ \text{m}$$

$$\Rightarrow\quad \lambda_{max}=1215\ \text{Å}$$

The shortest wavelength corresponds to $n\to\infty$, so

$$\frac{1}{\lambda_{min}}=1.097\times10^7\left(\frac{1}{1}-\frac{1}{\infty}\right)$$

$$\Rightarrow\quad \lambda_{min}=0.911\times10^{-7}\ \text{m}=911\ \text{Å}$$

Both of these wavelengths lie in ultraviolet (UV) region of electromagnetic spectrum.

ILLUSTRATION 10

Electrons of energies $10.20\ \text{eV}$ and $12.09\ \text{eV}$ can cause radiation to be emitted from hydrogen atoms. Calculate in each case, the principal quantum number of the orbit to which electron in the hydrogen atom is raised and the wavelength of the radiation emitted if it drops back to the ground state. Take $hc=12375\ \text{eVÅ}$

SOLUTION

Since the orbital energy of an electron revolving in n^{th} orbit is given by

$$E_n=-\left(\frac{13.6}{n^2}\right)\text{eV}$$

When $n=1$, $E_1=-13.6\ \text{eV}$

$$n=2,\ E_2=-3.4\ \text{eV}$$

$$n=3,\ E_3=-1.51\ \text{eV}$$

Here we observe that

$$10.20\ \text{eV}=E_2-E_1$$

and $12.09\ \text{eV}=E_3-E_1$

So, by absorbing a radiation photon of 10.2 eV the electron will make a transition to $n=2$ state and by absorbing a $12.09\ \text{eV}$ photon the electron will make a transition to $n=3$ state. Now after the life time of excited states, the electron in $n=2$ and $n=3$ will make transitions to lower states and ultimately come back to ground state. In this process, the possibilities of reverse transition are

$$n=3\to n=2$$

$$n=3\to n=1$$

$$n=2\to n=1$$

In above three transitions the amount of energy released will be

$$\Delta E_{32}=(-1.51\ \text{eV})-(-3.4\ \text{eV})=1.89\ \text{eV}$$

$$\Delta E_{31}=(-1.51\ \text{eV})-(-13.6\ \text{eV})=12.09\ \text{eV}$$

$$\Delta E_{21}=(-3.4\ \text{eV})-(-13.6\ \text{eV})=10.20\ \text{eV}$$

Thus, wavelength of radiations of corresponding transition are

$$\lambda_{32}=\frac{12375}{1.89}=6548\ \text{Å}$$

$$\lambda_{31}=\frac{12375}{12.09}=1024\ \text{Å}$$

$$\lambda_{21}=\frac{12375}{10.2}=1213\ \text{Å}$$

ILLUSTRATION 11

A single electron orbits around a stationary nucleus of charge $+Ze$, where Z is a constant and e is the magnitude of electronic charge. It requires **47.2 eV** to excite the electron from second Bohr orbit to the third Bohr orbit.

(a) Find the value of Z

(b) Find the energy required to excite the electron from $n=3$ to $n=4$

(c) Find the wavelength of radiation required to remove electron from first Bohr's orbit to infinity.

(d) Find the kinetic energy, potential energy and angular momentum of the electron in the first Bohr orbit.

SOLUTION

(a) Given $\Delta E_{23} = 47.2$ eV

Since, $\Delta E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ eV

$\Rightarrow \quad 47.2 = 13.6Z^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$

$\Rightarrow \quad Z = 5$

(b) To find ΔE_{34}, $n_1 = 3$, $n_2 = 4$

$$\Delta E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \text{ eV}$$

$$\Rightarrow \quad \Delta E = 13.6 \times 5^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 16.53 \text{ eV}$$

(c) Ionization energy is the energy required to excite the electron from $n = 1$ to $n \to \infty$

Thus, $\Delta E = 13.6 \times 5^2\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = 340$ eV

The respective wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{12400}{\Delta E} = \frac{12400}{340} = 36.47 \text{ Å}$$

(d) $K = -E = +340$ eV

$U = 2E = -680$ eV

$$L = \frac{h}{2\pi} = \frac{6.63 \times 10^{-34}}{2\pi} = 1.056 \times 10^{-34} \text{ Js}$$

ILLUSTRATION 12

Estimate the average kinetic energy of each hydrogen atom at room temperature. Use the result obtained to explain why nearly all H atoms are in the ground state at room temperature and hence emit no light.

SOLUTION

According to kinetic theory, the average kinetic energy of each H atom is given by,

$$E_{av} = \frac{3}{2}k_B T$$

$$\Rightarrow \quad E_{av} = \frac{3}{2}(1.38 \times 10^{-23})(300)$$

$$\Rightarrow \quad E_{av} = 6.2 \times 10^{-21} \text{ J}$$

$$\Rightarrow \quad E_{av} = \frac{6.2 \times 10^{-21}}{1.6 \times 10^{-19}} = 0.04 \text{ eV}$$

The average kinetic energy is thus very small compared to the energy between the ground state and the next higher energy state $(13.6 - 3.4 = 10.2 \text{ eV})$. Any atoms in excited state emit light and eventually fall to the ground state. Once in the ground state, collisions with other atoms can transfer energy of 0.04 eV on the average. A small fraction of atoms can have much more energy (in accordance with the distribution of molecular speeds), but even kinetic energy that is 10 times the average is not nearly enough to excite atoms above the ground state. Thus, at room temperature, nearly all atoms are in the ground state. Atoms can be excited to upper states at very high temperatures or by passing current of high energy electrons through the gas, as in a discharge tube.

ILLUSTRATION 13

The wavelength of the first line of Lyman series for hydrogen is identical to that of the second line of Balmer series for some hydrogen like ion x. Calculate energies of the first four levels, of x.

SOLUTION

Wavelength of the first line of Lyman series for hydrogen atom will be given by the equation

$$\frac{1}{\lambda_1} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4} \quad \text{...(1)}$$

The wavelength of second Balmer line for hydrogen like ion x is

$$\frac{1}{\lambda_2} = Rz^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3Rz^2}{16} \quad \text{...(2)}$$

Given that $\lambda_1 = \lambda_2$

$$\Rightarrow \quad \frac{1}{\lambda_1} = \frac{1}{\lambda_2}$$

$$\Rightarrow \quad \frac{3R}{4} = \frac{3Rz^2}{16}$$

$$\Rightarrow \quad z = 2$$

Hence the ion x is actually He^+. The energies of first four levels of ion x are,

$$E_1 = -(13.6)z^2 = -54.4 \text{ eV}, E_2 = \frac{E_1}{(2)^2} = -13.6 \text{ eV}$$

$$E_3 = \frac{E_1}{(3)^2} = -6.04 \text{ eV and } E_4 = \frac{E_1}{(4)^2} = -3.4 \text{ eV}$$

ILLUSTRATION 14

A doubly ionised lithium atom is hydrogen-like with atomic number 3.

(a) Find the wavelength of the radiation required to excite the electron in Li^{2+} from the first to the third Bohr orbit. (Ionisation energy of the hydrogen atom equals 13.6 eV).

(b) How many spectral lines are observed in the emission spectrum of the above excited system?

SOLUTION

Given, $Z = 3$

Since, $E_n \propto \dfrac{Z^2}{n^2}$

(a) To excite the atom from $n = 1$ to $n = 3$, energy of photon required is,

$$E_{1\to3} = E_3 - E_1 = \frac{(-13.6)(3)^2}{(3)^2} - \left(\frac{(-13.6)(3)^2}{(1)^2}\right)$$

$$\Rightarrow \quad E_{1\to3} = 108.8\text{ eV}$$

Corresponding wavelength will be,

$$\lambda(\text{in Å}) = \frac{12375}{E(\text{in eV})} = \frac{12375}{108.8} = 113.74\text{ Å}$$

(b) From n^{th} orbit, total number of emission lines (N) can be $N = \dfrac{n(n-1)}{2} = \dfrac{3(3-1)}{2} = 3$.

ILLUSTRATION 15

An electron in a hydrogen atom is in a state of binding energy 0.85 eV. The electron makes a transition to a state of excitation energy of $\mathbf{10.2\text{ eV}}$. Calculate the energy and wavelength of photon emitted.

SOLUTION

Since the binding energy is always negative, therefore,

$$E_i = -0.85\text{ eV}$$

Let n_i be the initial binding state of the electron, then

$$E_n = -13.6\frac{Z^2}{n_i^2}$$

$$\Rightarrow \quad -0.85 = -13.6\frac{Z^2}{n_i^2}$$

$$\Rightarrow \quad n_i = 4$$

Binding energy is $E_n = -\dfrac{13.6Z^2}{n^2}$

$$\Rightarrow \quad 0.85\text{ eV} = \frac{-13.6(1)^2}{n_2^2}$$

$$\Rightarrow \quad n_2 = 4$$

Let the electron now goes to an energy level n whose excitation energy is 10.2 eV. Since the excitation energy ΔE is defined with respect to ground state, therefore

$$E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\text{eV}$$

$$\Rightarrow \quad 10.2 = 13.6 \times 1^2\left(\frac{1}{1^2} - \frac{1}{n_f^2}\right)$$

$$\Rightarrow \quad n_f = 2$$

So, the electron makes a transition from energy level $n_i = 4$ to $n_f = 2$

Thus, the energy released is $\Delta E = E_4 - E_2$

$$\Rightarrow \quad \Delta E = 13.6\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = 2.55\text{ eV}$$

Since, $\lambda = \dfrac{hc}{\Delta E} = \dfrac{12400}{2.25\text{ eV}} = 5511\text{ Å}$

ILLUSTRATION 16

A particle of charge equal to that of an electron, $-e$ and mass 208 times the mass of electron (called a μ-meson) moves in a circular orbit around a nucleus of charge $+3e$ (take the mass of the nucleus to be infinite). Assuming that the Bohr model of the atom is applicable to this system:

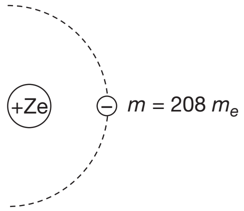
(i) Calculate the radius of nth Bohr orbit

(ii) Find the value of n, for which the radius of orbit is approximately the same as that of first Bohr orbit for the hydrogen atom;

(iii) Find the wavelength of radiation emitted when the μ-meson jumps from the third orbit to first orbit (Rydberg's constant $= 1.097 \times 10^7\text{ m}^{-1}$).

SOLUTION

If we assume that mass of nucleus is very much mass of mu-meson, then nucleus will be assumed to be at rest, only mu-meson is revolving round it.



(i) In nth orbit the necessary centripetal force to the mu-meson will be provided by the electrostatic force between the nucleus and the mu-meson.

Hence, $$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} - \frac{(Ze)(e)}{r^2} \qquad \ldots(1)$$

Further, it is given that Bohr model is applicable to this system also. Hence,

Angular momentum in nth orbit is $L = \frac{nh}{2\pi}$

$$\Rightarrow \quad mvr = n\frac{h}{2\pi} \qquad \ldots(2)$$

$$\Rightarrow \quad v = \frac{nh}{2\pi mr}$$

Substituting in (1), we get

$$r = \frac{n^2h^2\varepsilon_0}{\pi me^2 Z}$$

Substituting $Z = 3$ and $m = 208m_e$, we get

$$r_n = \frac{n^2h^2\varepsilon_0}{624\pi m_e e^2}$$

(ii) The radius of the first Bohr orbit for the hydrogen atom is

$$r_1 = \frac{h^2\varepsilon_0}{\pi m_e e^2}$$

Equating this with the radius calculated in part (a), we get

$$n^2 \approx 624$$

$$\Rightarrow \quad n \approx 25$$

(iii) Kinetic energy of atom is

$$K = \frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r}$$

and the potential energy is $U = -\frac{Ze^2}{4\pi\varepsilon_0 r}$

$$\Rightarrow \quad \text{Total energy } E_n = -\frac{Ze^2}{8\pi\varepsilon_0 r}$$

Substituting value of r, calculated in part (a), we get

$$E_n = \frac{1872}{n^2}\left(\frac{m_e e^4}{8\varepsilon_0^2 h^2}\right)$$

But $\left(-\frac{m_e e^4}{8\varepsilon_0^2 h^2}\right)$ is the ground state energy of hydrogen atom and hence is equal to -13.6 eV

$$\Rightarrow \quad E_n = \frac{-1872}{n^2}(13.6) \text{ eV} = -\frac{25459.2}{n^2} \text{ eV}$$

$$\Rightarrow \quad E_3 - E_1 = -25459.2\left(\frac{1}{9} - \frac{1}{1}\right) = 22630.4 \text{ eV}$$

So, the corresponding wavelength, is

$$\lambda(\text{in Å}) = \frac{12375}{22630.4}$$

$$\Rightarrow \quad \lambda = 0.546 \text{ Å}$$

ILLUSTRATION 17

A hydrogen like atom of atomic number z is in an excited state of quantum number $2n$. It can emit a maximum energy photon of 204 eV. If it makes a transition to quantum state n, a photon of energy 40.8 eV is emitted. Find n, z and the ground state energy (in eV) for this atom. Also, calculate the minimum energy (in eV) that can be emitted by this atom during de-excitation. Ground state energy of hydrogen atom is 13.6 eV.

SOLUTION

Given, $E_{2n} - E_1 = 204$ eV

$$\Rightarrow \quad (13.6)z^2\left(1 - \frac{1}{4n^2}\right) = 204 \qquad \ldots(1)$$

Also, $E_{2n} - E_n = 40.8$ eV

$$\Rightarrow \quad 13.6z^2\left(\frac{1}{n^2} - \frac{1}{4n^2}\right) = 40.8 \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$n = 2 \text{ and } z = 4$$

Since, $E_n = \frac{(-13.6)z^2}{n^2}$ eV, so we have

$$E_1 = (-13.6)(4)^2 \text{ eV}$$

$\Rightarrow \quad E_1 = -217.6$ eV

During de-excitation, minimum energy emitted is,

$$E_{\min} = E_{2n} - E_{2n-1} = E_4 - E_3$$

$$\Rightarrow \quad E_{\min} = \frac{-217.6}{4^2} - \left(\frac{-217.6}{3^2}\right) = 10.58 \text{ eV}$$

ILLUSTRATION 18

Calculate the energy of a H-atom in the first excited state, if the potential energy is assumed to be zero in the ground state.

SOLUTION

Since, we know that, in ground state, $n = 1$, we have

$$TE = -KE = \frac{PE}{2}$$

$$\Rightarrow \quad PE = 2(TE) = 2(-13.6 \text{ eV})$$

$$\Rightarrow \quad PE = -27.2 \text{ eV}$$

However, we have assumed this energy to be zero i.e., potential energy is increased by 27.2 eV. Since, kinetic energy in all energy states will remain unchanged whereas potential energy and hence, the total energy in all states will increase by 27.2 eV. Further, first excited state means $n = 2$, so

$$E_2 = -3.4 \text{ eV (previously)}$$

$$\Rightarrow \quad E_2' = -3.4 + 27.2 = 23.8 \text{ eV (now)}$$

ILLUSTRATION 19

An imaginary particle has a charge equal to that of an electron and mass 100 times the mass of the electron. It moves in a circular orbit around a nucleus of charge $+4e$. Take the mass of the nucleus to be infinite. Assuming that the Bohr's model is applicable to this system.

(a) Derive an expression for the radius of nth Bohr orbit.

(b) Find the wavelength of the radiation emitted when the particle jumps from fourth orbit to the second orbit.

SOLUTION

(a) We have $\dfrac{m_p v^2}{r_n} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{ze^2}{r_n^2}$ ...(1)

The quantization of angular momentum gives,

$$m_p v r_n = \frac{nh}{2\pi} \quad \text{...(2)}$$

Solving equations (1) and (2), we get

$$r = \frac{n^2 h^2 \varepsilon_0}{z\pi m_p e^2}$$

Substituting $m_p = 100$ m

where m = mass of electron and $z = 4$

we get, $r_n = \dfrac{n^2 h^2 \varepsilon_0}{400\pi m e^2}$

(b) As we know, $E_1^H = -13.60$ eV

and $E_n \propto \left(\dfrac{z^2}{n^2}\right) m$

For the given particle,

$$E_4 = \frac{(-13.60)(4)^2}{(4)^2} \times 100 = -1360 \text{ eV}$$

and $$E_2 = \frac{(-13.60)(4)^2}{(2)^2} \times 100 = -5440 \text{ eV}$$

$$\Rightarrow \quad \Delta E = E_4 - E_2 = 4080 \text{ eV}$$

$$\Rightarrow \quad \lambda(\text{in Å}) = \frac{12375}{\Delta E(\text{in eV})} = \frac{12375}{4080} = 3 \text{ Å}$$

ILLUSTRATION 20

Electrons in hydrogen-like atom $(Z = 3)$ make transitions from the fifth to the fourth orbit and from the fourth to the third orbit. The resulting radiations are incident normally on a metal plate and eject photoelectrons. The stopping potential for the photoelectrons ejected by the shorter wavelength is 3.95 V. Calculate the work function of the metal, and the stopping potential for the photoelectrons ejected by the longer wavelength (Rydberg constant $= 1.094 \times 10^7 \text{ m}^{-1}$)

SOLUTION

The stopping potential for shorter wavelength is 3.95 V i.e., maximum kinetic energy of photoelectrons corresponding to shorter wavelength will be 3.95 eV. Further energy of incident photons corresponding to shorter wavelength will be in transition from $n = 4$ to $n = 3$.

$$E_{4\to3} = E_4 - E_3 = \frac{-(13.6)(3)^2}{(4)^2} - \left(\frac{-(13.6)(3)^2}{(3)^2}\right)$$

$\Rightarrow \quad E_{4\to3} = 5.95$ eV

Now, from the equation,

$$K_{max} = E - W$$

we have $W = E - K_{max} = E_{4\to3} - K_{max}$

$\Rightarrow \quad W = (5.95 - 3.95)$ eV $= 2$ eV

Longer wavelength will correspond to transition from $n = 5$ to $n = 4$. From the relation, we get

$$\frac{1}{\lambda} = Rz^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

The longer wavelength,

$$\frac{1}{\lambda} = (1.094\times10^7)(3)^3\left(\frac{1}{16} - \frac{1}{25}\right)$$

$\Rightarrow \quad \lambda = 4.514\times10^{-7}$ m $= 4514$ Å

Energy corresponding to this wavelength is

$$E = \frac{12375 \text{ eV Å}}{4514 \text{ Å}} = 2.74 \text{ eV}$$

So, maximum kinetic energy of photo-electrons is

$$K_{max} = E - W = (2.74 - 2) \text{ eV} = 0.74 \text{ eV}$$

Hence, the stopping potential is 0.74 V.

ILLUSTRATION 21

Hydrogen atom in its ground state is excited by means of monochromatic radiation of wavelength 975 Å. How many different lines are possible in the resulting spectrum? Calculate the longest wavelength amongst them. You may assume the ionization energy for hydrogen atom as 13.6 eV.

SOLUTION

Energy corresponding to given wavelength is

$$E(\text{in eV}) = \frac{12375}{\lambda(\text{in Å})} = \frac{12375}{975} = 12.69 \text{ eV}$$

Now, let the electron excites to n^{th} energy state. Then,

$$E_n - E_1 = 12.69$$

$\Rightarrow \quad \frac{(-13.6)}{(n^2)} - (-13.6) = 12.69$

$\Rightarrow \quad n \approx 4$

i.e., electron excites to 4^{th} energy state

Total number of lines in emission spectrum would be

$$N = \frac{n(n-1)}{2} = \frac{4\times3}{2} = 6$$

Longest wavelength will correspond to the minimum energy and minimum energy is released in transition from $n = 4$ to $n = 3$.

$$E_{4\to3} = E_4 - E_3 = \frac{-13.6}{(4^2)} - \left(\frac{-13.6}{(3)^2}\right) = 0.66 \text{ eV}$$

Hence, longest wavelength will be,

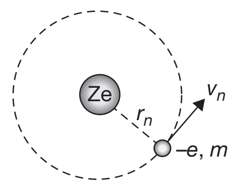
$$\lambda_{max} = \frac{12375}{E(\text{in eV})} = \frac{12375}{0.66} \text{ Å} = 1.875\times10^{-6} \text{ m}$$

$\Rightarrow \quad \lambda_{max} = 1.875\ \mu$m

HYPOTHETICAL BOHR'S MODEL

Consider a hypothetical hydrogen like atom for which an electron is revolving around the nucleus under the influence of a new potential energy field given as $U = f(r)$. This potential energy field, in this case, can be assumed to be a non-coulomb field. The force acting on the electron is then given by $F = -\frac{dU}{dr}$.

This force is responsible for providing the necessary centripetal force to the electron to revolve in a circle of radius r as shown.



$$\Rightarrow \quad F = \left|-\frac{dU}{dr}\right| = \frac{mv_n^2}{r_n} \quad \ldots(1)$$

According to Bohr's Quantisation Rule, we have

$$mv_nr_n = \frac{nh}{2\pi} \quad \ldots(2)$$

Now using equations (1) and (2), we can derive all the properties for electron motion like, radius of n^{th} orbit, velocity of electron in n^{th} orbit, angular velocity, frequency, time period, current, magnetic induction, magnetic moment and the total energy of energy levels for this hypothetical atom in the same way we've derived these for properties for a general hydrogen like atom.

ILLUSTRATION 22

A small particle of mass m moves in such a way that the potential energy $U=-\frac{1}{2}mb^2r^2$, where b is a constant and r is the distance of the particle from the origin taken at the nucleus. Assuming Bohr model of quantization of angular momentum and circular orbits, show that radius of the n^{th} allowed orbit is proportional to $\sqrt{n}$.

SOLUTION

Force on mass m in conservative field is

$$F=-\frac{dU}{dr}=mb^2r$$

For circular orbit of particle, we have

$$mb^2r=\frac{mv^2}{r} \qquad \ldots(1)$$

$$\Rightarrow \quad v=br$$

Also, by Bohr's Quantisation rule, we have

$$mvr=\frac{nh}{2\pi} \qquad \ldots(2)$$

$$\Rightarrow \quad m(br)r=\frac{nh}{2\pi}$$

$$\Rightarrow \quad r=\sqrt{\frac{nh}{2\pi mb}}$$

ILLUSTRATION 23

Assume a hypothetical hydrogen atom in which the potential energy between electron and proton at separation r is given by $U=k\left(\log_e r-\frac{1}{2}\right)$, where k is a constant. For such a hypothetical hydrogen atom, calculate the radius of nth Bohr's orbit and energy levels.

SOLUTION

Force of interaction between electron and proton is given by

$$F=-\frac{dU}{dr}=-\frac{k}{r}$$

Force is negative. It means there is attraction between the particles and they are bound to each other. This force provides the necessary centripetal force for the electron. So, we have

$$\frac{mv^2}{r}=\frac{k}{r} \qquad \ldots(1)$$

According to Bohr's assumption, we have

$$mvr=n\left(\frac{h}{2\pi}\right) \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$r=\frac{nh}{2\pi\sqrt{mk}} \quad \text{and} \quad v=\sqrt{\frac{k}{m}}$$

Since, $E=U+\frac{1}{2}mv^2$

$$\Rightarrow \quad E=k\log_e r-\frac{k}{2}+\frac{k}{2}=k\log_e r$$

So, $r_n=\dfrac{nh}{2\pi\sqrt{mk}}$ and $E_n=k\log_e\left(\dfrac{nh}{2\pi\sqrt{mk}}\right)$

ILLUSTRATION 24

Suppose that the potential energy between electron and proton at a distance r in a hypothetical hydrogen atom is given by $\frac{-ke^2}{3r^3}$. Use Bohr's theory to obtain energy levels of such a hypothetical atom.

SOLUTION

For the hypothetical atom, the potential energy of the electron revolving in the n^{th} orbit is given by

$$U=-\frac{ke^2}{3r^3}$$

The force on the electron in this potential field is given by

$$F=-\frac{dU}{dr}=\frac{ke^2}{r^4}$$

This force provides the necessary centripetal force to the electron to revolve in a circle of radius r in the n^{th} orbit with a speed v, so we have

$$\frac{mv^2}{r}=\frac{ke^2}{r^4}$$

$$\Rightarrow \quad mv^2=\frac{ke^2}{r^3} \qquad \ldots(1)$$

Applying Bohr's Quantisation Rule i.e. $mvr = \frac{nh}{2\pi}$

$$\Rightarrow \quad v = \frac{nh}{2\pi mr} \quad \text{...(2)}$$

Substituting equation (2) in (1), we get

$$m\left(\frac{nh}{2\pi mr}\right)^2 = \frac{ke^2}{r^3}$$

$$\Rightarrow \quad r = r_n = \frac{4\pi^2 ke^2 m}{n^2h^2} \text{ and } v = v_n = \frac{n^3h^3}{8\pi^3 km^2e^2}$$

Now energy in n^{th} orbit is $E = E_n = U + K$

$$\Rightarrow \quad E_n = -\frac{ke^2}{3r^3} + \frac{1}{2}mv^2 \quad \text{...(3)}$$

From equation (1), we get $\frac{1}{2}mv^2 = \frac{ke^2}{2r^3}$

Substituting this value of kinetic energy in equation (3), we get

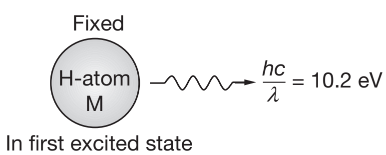
$$E_n = -\frac{ke^2}{3r^3} + \frac{ke^2}{2r^3} = \frac{1}{6}\left(\frac{ke^2}{r^3}\right)$$

$$\Rightarrow \quad E_n = \frac{1}{6}ke^2\left(\frac{n^2h^2}{4\pi^2ke^2m}\right)^3 = \frac{n^6h^6}{384\pi^6k^2e^4m^3}$$

RECOIL OF AN ATOM DUE TO ELECTRON TRANSITION

When an electron makes a transition from a higher energy level to a lower energy level, then a photon of wavelength λ is emitted by the atom. Since, the emitted photon possesses momentum $p = \frac{h}{\lambda}$, then the atom may or may not recoil depending upon whether it is fixed (kind of stationary) or is free to move. To understand this completely, let is discuss the following cases.

(a) When the hydrogen atom (of mass M) emitting the photon is fixed, then the entire energy equal to the energy difference of the two levels (between which the electron transition takes place) is given to the photon as shown.



If an electron makes a transition from $n_i = 2$ to $n_f = 1$, then the energy difference between the two levels is $\Delta E = (-3.4) - (-13.6) = 10.2$ eV.

So, the emitted photon possesses this energy and hence we have $\frac{hc}{\lambda} = 10.2$ eV, where $\frac{1}{\lambda} = R_H\left(\frac{1}{(1)^2} - \frac{1}{(2)^2}\right) = \frac{3R_H}{4}$.

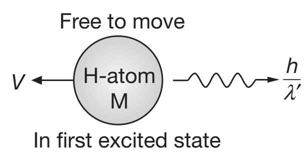
For hydrogen like atoms having atomic number Z, (like He^+ atom, we have $Z = 2$), if the electron de-excites from higher energy level n_i to a lower energy level n_f, then we have

$$\frac{1}{\lambda} = R_H Z^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\Rightarrow \quad \frac{hc}{\lambda} = (R_H ch)Z^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

where $R_H = 1.097 \times 10^7 \text{ m}^{-1}$

(b) However, when the hydrogen atom (initially at rest) is free to move, then this entire energy equal to the energy difference of the two levels (between which the electron transition takes place) is distributed between the photon and the hydrogen atom, because the hydrogen atom recoils due to the emission of the photon.



Due to this, the energy possessed by the photon is slightly less than the case when the atom was assumed to be fixed. So, if λ' is the new wavelength of the emitted photon, then we have $\lambda' > \lambda$.

If the electron in the hydrogen atom of mass M, de-excites from $n_i = 2$ to $n_f = 1$, then a photon of energy $\Delta E = 10.2$ eV is emitted and hence the hydrogen atom recoils with a speed V. Then according to Conservation of Linear Momentum, we have

$$0 = MV + \frac{h}{\lambda'}$$

$$\Rightarrow \quad |V| = \frac{h}{M\lambda'} \quad \text{...(1)}$$

By Law of Conservation of energy, we have

$$\frac{1}{2}MV^2 + \frac{hc}{\lambda'} = \Delta E = 10.2 \text{ eV}$$

Since mass of hydrogen atom is very large than photon, so the recoil speed of the hydrogen atom is neglected and hence $\frac{1}{2}MV^2$ can also be neglected.

$$\Rightarrow \quad \frac{hc}{\lambda'} = \Delta E = 10.2 \text{ eV}$$

$$\Rightarrow \quad \frac{h}{\lambda'} = \frac{\Delta E}{c} = \frac{10.2}{c} \text{ eV}$$

$$\Rightarrow \quad \frac{h}{\lambda'} = MV = \frac{\Delta E}{c} = \frac{10.2}{c} \text{ eV}$$

$$\Rightarrow \quad V = \frac{\Delta E}{Mc} = \frac{10.2}{Mc} \text{ ms}^{-1}$$

In general, for hydrogen like atoms having atomic number Z, (like He^+ atom, we have $Z = 2$), if the electron de-excites from higher energy level n_i to a lower energy level n_f, then $\Delta E = (R_H ch)Z^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$ and hence the recoil speed is now given by

$$V_{\text{H like}} = \frac{\Delta E}{(M_{\text{H like}})c}$$

ILLUSTRATION 25

An isolated hydrogen atom emits a photon of 10.2 eV.

(a) Determine the momentum of photon emitted

(b) Calculate the recoil momentum of the atom

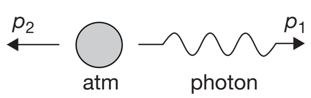
(c) Find the kinetic energy of the recoil atom. [Mass of proton, $m_p = 1.67 \times 10^{-27}$ kg]

SOLUTION

(a) Momentum of the photon is

$$p_1 = \frac{E}{c} = \frac{10.2 \times 1.6 \times 10^{-19}}{3 \times 10^8} = 5.44 \times 10^{-27} \text{ kgms}^{-1}$$

(b) Applying the Law of Conservation of Linear Momentum, we get



$$p_2 = p_1 = 5.44 \times 10^{-27} \text{ kgms}^{-1}$$

(c) $K = \frac{1}{2}mv^2$ (v = recoil speed of atom, m = mass of hydrogen atom)

$$\Rightarrow \quad K = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m}$$

Substituting the value of the momentum of atom, we get

$$K = \frac{(5.44 \times 10^{-27})^2}{2 \times 1.67 \times 10^{-27}} = 8.86 \times 10^{-27} \text{ J}$$

ILLUSTRATION 26

Light from a discharge tube containing hydrogen atoms falls on the surface of a piece of sodium. The kinetic energy of the fastest photoelectrons emitted from sodium is **0.73 eV**. The work function for sodium is **1.82 eV**. Calculate the

(a) the energy of the photons causing the photoelectrons emission.

(b) the quantum numbers of the two levels involved in the emission of these photons

(c) the change in the angular momentum of the electron in the hydrogen atom, in the above transition, and

(d) the recoil speed of the emitting atom assuming it to be at rest before the transition. (Ionization potential of hydrogen is 13.6 eV).

SOLUTION

(a) From Einstein's equation of photoelectric effect,

$$\begin{pmatrix}\text{Photon}\\ \text{energy}\\ \text{causing}\\ \text{photo-}\\ \text{electric}\\ \text{emission}\end{pmatrix} = \begin{pmatrix}\text{Maximum}\\ \text{kinetic}\\ \text{energy of}\\ \text{emitted}\\ \text{photons}\end{pmatrix} + \begin{pmatrix}\text{Work}\\ \text{function}\\ \text{of sodium}\end{pmatrix}$$

$$\Rightarrow \quad E = K_{\text{max}} + W = (0.73 + 1.82) \text{ eV}$$

$$\Rightarrow \quad E = 2.55 \text{ eV}$$

(b) In case of a hydrogen atom,

$$E_1 = -13.6 \text{ eV}, \ E_2 = -3.4 \text{ eV},$$

$$E_3 = -1.5 \text{ eV}, \ E_4 = -0.85 \text{ eV}$$

Since, $\Delta E = E_4 - E_2 = 2.55$ eV, therefore, quantum numbers of the two levels involved in the emission of these photons are 4 and 2 i.e., from $4 \to 2$.

(c) Change in angular momentum transition from 4 to 2 will be

$$\Delta L = L_2 - L_4 = 2\left(\frac{h}{2\pi}\right) - 4\left(\frac{h}{2\pi}\right)$$

$$\Rightarrow \quad \Delta L = -\frac{h}{\pi}$$

(d) By Law of Conservation of Linear Momentum, we have

$$\begin{pmatrix}\text{Momentum of} \\ \text{hydrogen atom}\end{pmatrix} = \begin{pmatrix}\text{Momentum of} \\ \text{emitted photon}\end{pmatrix}$$

$$\Rightarrow \quad Mv = \frac{\Delta E}{c} \quad (M = \text{mass of hydrogen atom})$$

$$\Rightarrow \quad v = \frac{\Delta E}{Mc}$$

$$\Rightarrow \quad v = \frac{(2.55)(1.6\times10^{-19})}{(1.67\times10^{-27})(3\times10^{8})}$$

$$\Rightarrow \quad v = 0.814\ \text{ms}^{-1}$$

ATOMIC COLLISIONS

The electron of an atom can be excited from lower energy level to a higher energy level by making a photon or electromagnetic radiation to fall on the atom (as already discussed). This radiation or photon must have an energy corresponding to the energy difference of the levels between which the electron transition takes place.

However, the electron in an atom can also be excited by colliding the atom with a particle or another atom. This is called as **Atomic Collision**. *In atomic collisions,* the loss in kinetic energy of the system (colliding particle and atom) is possible only when this loss in energy is sufficient enough to

(a) either excite the electron to a higher energy level or

(b) ionise the atom i.e. send an electron to infinity.

So, when an elementary particle collides with an atom, then the atom can be excited to an energy level above its ground state. In this process, a part of their combined kinetic energy (i.e., sum of K.E. of atom and incoming particle) is absorbed by the atom. *Also, this loss in energy is not converted to any other form of energy (like heat etc) during the collision and we have assumed the collisions to be head-on.*

Before we discuss further, we must remember the energies of electron in the different levels of the atom. Since, we know that for hydrogen like atoms, $E_n = -\frac{13.6Z^2}{n^2}$. However, for hydrogen atom, we have $Z = 1$, so energies of electron in different levels is

$E_1 = -13.6$ eV, $E_2 = -3.4$ eV: $\Delta E_{12} = 10.2$

E_1 to $E_3 = -1.51$ eV: $\Delta E_{13} = 12.09$

E_1 to $E_4 = -0.85$ eV: $\Delta E_{14} = 12.75$

$E_5 = -0.55$ eV, $E_6 = -0.38$ eV, $E_7 = -0.28$ eV

$E_8 = -0.20$ eV, $E_9 = -0.17$ eV, $E_{10} = -0.14$ eV

For quickly handling problems, we must keep in mind that

$$\Delta E_{12} = E_2 - E_1 = 10.2\ \text{eV},$$

$$\Delta E_{13} = E_3 - E_1 = 12.09\ \text{eV} \quad \text{and}$$

$$\Delta E_{14} = E_4 - E_1 = 12.75\ \text{eV}$$

ILLUSTRATION 27

A neutron having kinetic energy (K), collides head-on with a stationary H-atom. Find the nature of collision (elastic/inelastic/perfectly inelastic), when the neutron possesses a

(i) kinetic energy of 12 eV
(ii) kinetic energy of 20.4 eV
(iii) kinetic energy of 22 eV
(iv) kinetic energy of 24.18 eV

SOLUTION

To find nature of collision, we must first know about loss in energy of the system is minimum for an elastic collision and maximum for a perfectly inelastic collision (where the bodies after collision stick to each other and move as one single body). For an elastic collision, $\Delta E = 0$ and for a perfectly inelastic collision, we have $|\Delta E| = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(u_1 - u_2)^2$.

$$\Rightarrow \quad 0 \le |\Delta E| \le \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(u_1 - u_2)^2$$

$$\Rightarrow \quad 0 \le |\Delta E| \le \frac{1}{2}\left(\frac{m_n m_\mu}{m_n + m_\mu}\right)u^2$$

Since, we know that $m_H \approx m_n = m$ (say), so we get

$$0 \le |\Delta E| \le \frac{1}{2}\left(\frac{m}{2}\right)u^2$$

$$\Rightarrow \quad 0 \le |\Delta E| \le \frac{K}{2} \qquad \left\{\because K = \frac{1}{2}m_n u^2 = \frac{1}{2}mu^2\right\}$$

So, loss in energy, $|\Delta E|$, must lie in the range zero to $\frac{K}{2}$.

(i) When the kinetic energy of neutron is $K = 12$ eV

Since, we have calculated that $0 \le |\Delta E| \le \frac{K}{2}$

$\Rightarrow \quad 0 \le |\Delta E| \le 6$ eV

Maximum Loss in kinetic energy for $K = 12$ eV is 6 eV. This loss in energy is less than 10.2 eV, which is the energy required to excite the electron from ground state $(n_1 = 1)$ to first excited state $(n_2 = 2)$. This loss in energy is not sufficient enough to be absorbed by the electron, so that it can go from ground state to first excited state. So, no energy loss will place and hence for $K = 12$ eV, the collision is elastic.

(ii) When the kinetic energy of neutron is $K = 20.4$ eV

Since, we have calculated that $0 \le |\Delta E| \le \frac{K}{2}$

$\Rightarrow \quad 0 \le |\Delta E| \le 10.2$ eV

When $|\Delta E| = 0$, then the collision is elastic.

For $K = 20.4$ eV, the maximum loss in kinetic energy is $|\Delta E| = 10.2$ eV. This loss in energy is equal to the energy required to excite the electron from ground state $(n_1 = 1)$ to first excited state $(n_2 = 2)$. *So, collision is perfectly inelastic* i.e. the neutron will collide with the hydrogen atom (at rest) and both move together as one single body. In this process the neutron looses 10.2 eV kinetic energy to the electron, so that the electron gets excited from ground state to first excited state and the combined system (neutron and atom) moves with a kinetic energy of $(20.4 - 10.2)$ eV $= 10.2$ eV

(iii) When the kinetic energy of neutron is $K = 22$ eV

Since, we have calculated that $0 \le |\Delta E| \le \frac{K}{2}$

$\Rightarrow \quad 0 \le |\Delta E| \le 11$ eV

$\Rightarrow \quad 0 \le \Delta E \le 11$ eV > 10.2 eV

In this case, two possibilities arise.

Firstly, when the loss in energy is zero i.e. then the collision is elastic.

Secondly, out of 11 eV loss in energy of the system, the electron will take up 10.2 eV energy to get excited from ground state to first excited state and both neutron and hydrogen atom will be moving with kinetic energy equal to the remaining energy i.e. $(11 - 10.2) = 0.8$ eV.

So, the collision is an inelastic collision.

(iv) When the kinetic energy of neutron is K = 24.18 eV

Since, we have calculated that $0 \le |\Delta E| \le \frac{K}{2}$

$\Rightarrow \quad 0 \le |\Delta E| \le 12.09$ eV

In this case, three possibilities arise.

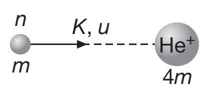
Firstly, when the loss in energy is zero i.e. then the collision is elastic.

Secondly, if $\Delta E = 10.2$ eV, then collision is inelastic because 10.2 eV excites the electron from ground state to first excited state and the remaining energy is shared by both neutron and atom, so that both move independent of each other with different velocity after collision.

Thirdly, if $\Delta E = 12.09$ eV, then collision is perfectly inelastic because 12.09 eV lost by the neutron will just excite the atom from $n = 1$ to $n = 3$ and both will move with a common velocity after collision.

ILLUSTRATION 28

A He^+ ion is at rest and in ground state. A neutron with initial velocity u, kinetic energy K collides head on with the He^+ ion. Calculate minimum value of K so that there can be an inelastic collision between these two particles.



SOLUTION

For He^+, we have $E_n = -\frac{13.6Z^2}{n^2}$ eV

$$\Rightarrow \quad E_n = -\frac{13.6(4)}{n^2} = -\frac{54.4}{n^2} \text{ eV}$$

From this relation, we get

$$E_1 = -54.4 \text{ eV}$$

$$E_2 = 13.6 \text{ eV} \quad \Rightarrow \quad E_2 - E_1 = 40.8 \text{ eV}$$

$$E_3 = -6.04 \text{ eV} \quad \Rightarrow \quad E_3 - E_1 = 48.36 \text{ eV}$$

$$E_4 = -3.4 \text{ eV} \quad \Rightarrow \quad E_4 - E_1 = 51 \text{ eV}$$

$$E_\infty = 0 \quad \Rightarrow \quad E_\infty - E_1 = 54.4 \text{ eV}$$

So, possible losses in energy can only be

$$\Delta E = \{40.8 \text{ eV},\ 48.36 \text{ eV},\ 51 \text{ eV}, \ldots\ldots, 54 \text{ eV}\}$$

Now minimum loss $(= \text{zero})$ will be for an elastic collision and maximum loss will be for perfectly inelastic collision.

$$\Rightarrow \quad \text{Loss} = \frac{1}{2}\frac{(m)(4m)}{4m+m}(u-0)^2 = \frac{1}{2}\left(\frac{4m}{5}\right)u^2$$

$$\Rightarrow \quad \text{Loss} = \frac{4}{5}\left(\frac{1}{2}mu^2\right) = \frac{4K}{5}$$

$$\Rightarrow \quad 0 \le \Delta E \le \frac{4K}{5}$$

Now, when $\Delta E = 0$, the collision is elastic.

Also, for inelastic collision $\frac{4K}{5} > 40.8$ eV

$$\Rightarrow \quad K > 51 \text{ eV}$$

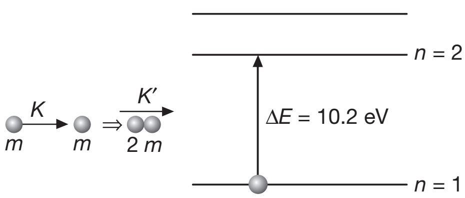
Because at $\Delta E = 40.8$ eV, perfectly inelastic collision will take place.

ILLUSTRATION 29

A moving hydrogen atom makes a head on collision with a stationary hydrogen atom. Before collision both atoms are in ground state and after collision they move together. What is the minimum value of the kinetic energy of the moving hydrogen atom, such that one of the atoms reaches one of the excitation state?

SOLUTION

Let K be the kinetic energy of the moving hydrogen atom and K', the kinetic energy of combined mass after collision.



By Law of Conservation of Linear Momentum, we have

$$p = p'$$

$$\Rightarrow \quad \sqrt{2Km} = \sqrt{2K'(2m)}$$

$$\Rightarrow \quad K = 2K' \qquad \ldots(1)$$

From Conservation of Energy, $K = K' + \Delta E$ ...(2)

Solving equations (1) and (2), we get

$$\Delta E = \frac{K}{2}$$

Now minimum value of ΔE for hydrogen atom is 10.2 eV, so we have

$$\Delta E \ge 10.2 \text{ eV}$$

$$\Rightarrow \quad \frac{K}{2} \ge 10.2$$

$$\Rightarrow \quad K \ge 20.4 \text{ eV}$$

Therefore, the minimum kinetic energy of moving hydrogen is 20.4 eV.

ILLUSTRATION 30

A 100 eV electron collides with a stationary helium ion (He^+) in its ground state and excites to a higher level. After the collision, He^+ ions emit two photons in succession with wavelength 1085 Å and 304 Å. Find the principal quantum number of the excited state. Also calculate the energy of the electron after the collision. Given $h = 6.63 \times 10^{-34}$ Js.

SOLUTION

The energy of the electron in the n^{th} state of He^+ ion of atomic number $Z(=2)$ is given by

$$E_n = -(13.6 \text{ eV})\frac{Z^2}{n^2}$$

$$\Rightarrow \quad E_n = -\frac{(13.6 \text{ eV}) \times (2)^2}{n^2}$$

$$\Rightarrow \quad E_n = -\frac{54.4}{n^2} \text{ eV} \qquad \ldots(1)$$

The energies E_1 and E_2 of the two emitted photons in eV are

$$E_1 = \frac{12431}{1085} \text{ eV} = 11.4 \text{ eV}$$

and $$E_2 = \frac{12431}{304} \text{ eV} = 40.9 \text{ eV}$$

So, total energy is

$$E = E_1 + E_2 = 11.4 + 40.9 = 52.3 \text{ eV}$$

For the transition from $n_i = n$ to $n_f = 1$, we have

$$\Delta E = -(54.4 \text{ eV})\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

Since, $\Delta E = 52.3$ eV

$$\Rightarrow \quad 52.3 \text{ eV} = 54.4 \text{ eV} \times \left(1 - \frac{1}{n^2}\right)$$

$$\Rightarrow \quad 1 - \frac{1}{n^2} = \frac{52.3}{54.4} = 0.96$$

$$\Rightarrow \quad n^2 = 25$$

$$\Rightarrow \quad n = 5$$

The energy of the incident electron is given to be 100 eV. The energy supplied to He^+ ion is 52.3 eV. So, the energy of the electron left after the collision is $100 - 52.3 = 47.7$ eV.

ILLUSTRATION 31

A moving H-atom makes a head on perfectly inelastic collision with a stationary Li^{++} ion. Before collision H-atom and Li^{++} ion are both in their first excited states. What is the velocity of the moving H atom if after collision H is found in its ground state and Li^{++} ion in its second excited state. Take mass of hydrogen atom, $m_H = 1.66 \times 10^{-27}$ kg and mass of Li^{++} to be $m_{Li^{++}} = 7m_H$.

SOLUTION

For Li^{++}, we have $Z = 3$

$$\Rightarrow \quad E_2 = \frac{-13.6(3)^2}{(2)^2} = -30.6 \text{ eV}$$

$$\Rightarrow \quad E_3 = \frac{-13.6(3)^2}{(3)^2} = -13.6 \text{ eV}$$

Energy required for Li^{++}, ion to go from first excited state $(n = 2)$ to second excited state $(n = 3)$ is

$$\Delta E = -13.6 - (-30.6) = 17 \text{ eV}$$

Energy released by hydrogen atom to go from first excited state to ground state is

$$\Delta E' = -3.4 - (-13.6) = 10.2 \text{ eV}$$

So, $\Delta E - \Delta E' = 17 - 10.2 = 6.8$ eV is the energy that should come from loss in KE in collision.

From Law of Conservation of Linear Momentum, velocity of combined mass is

$$v = \frac{m_1 u_1}{m_1 + m_2} \qquad (m_2 = 7m_1)$$

$$\Rightarrow \quad \Delta KE = \frac{1}{2} m_1 u_1^2 - \frac{1}{2}(m_1 + m_2) v^2$$

Since, $\Delta K = 6.8$ eV

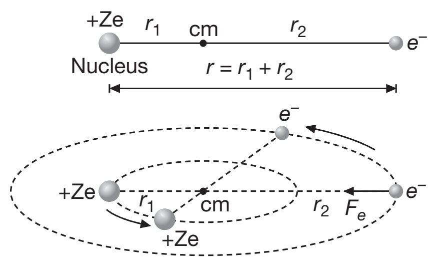
Solving these equations, we get

$$u_1 = 3.9 \times 10^4 \text{ ms}^{-1}$$

EFFECT OF MASS OF NUCLEUS ON BOHR MODEL

Till now, we had studied that in hydrogen atom or hydrogen like atoms, the nucleus is at rest and electron is revolving around this stationary nucleus. *Since no external force is acting on the electron nucleus system, so the centre of mass of the system remains at rest.* Theoretically, mass of electron is negligible or small compared to that of nucleus and hence the centre of mass of the atom is almost situated at nucleus. Due to this, in Bohr's atom model of hydrogen like atoms, the nucleus almost remains at rest and electron revolves around it.

But practically, the situation is a bit different. Actually, centre of mass of electron-nucleus system is close to nucleus (but not at the nucleus) because the nucleus is heavy and so, to keep the centre of mass of electron nucleus system at rest, both electron and nucleus revolve around their centre of mass just like the double star system as shown in figure.



So, in the atom, the nucleus and electron revolve around their centre of mass in concentric circles of radii r_1 and r_2 to keep centre of mass at rest. If r is the distance of electron from nucleus, the distances of nucleus and electron from the centre of mass are given by

$$r_1 = \frac{m_e r}{m_N + m_e} \text{ and } r_2 = \frac{m_N r}{m_N + m_e}$$

For the nucleus, we have

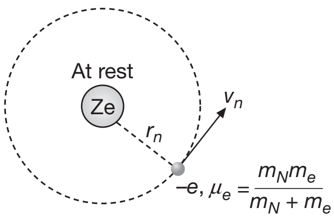
$$m_N r_1 \omega^2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Ze^2}{r^2}\right)$$

$$\Rightarrow \quad m_N\left(\frac{m_e r}{m_N + m_e}\right)\omega^2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Ze^2}{r^2}\right)$$

$$\Rightarrow \quad \left(\frac{m_N m_e}{m_N + m_e}\right) r\omega^2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Ze^2}{r^2}\right)$$

$$\Rightarrow \quad \mu_e r\omega^2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Ze^2}{r^2}\right)$$

So, in the above system, we can analyse the motion of electron with respect to nucleus by assuming the nucleus to be at rest and then replacing the mass of electron by the reduced mass of electron-nucleus system i.e. $\mu_e = \dfrac{m_N m_e}{m_N + m_e}$. Now the relative picture of atom will be same as that considered earlier as shown in figure. However, we have just replaced the mass of electron by the reduced mass of electron-nucleus system.



Now we can use all those relations which we've derived earlier for Bohr model just by replacing the mass of electron m_e by the reduced mass μ_e.

The radius of electron in n^{th} orbit of Bohr's model is given by

$$r_n = \frac{n^2 h^2 \varepsilon_0}{\pi m_e e^2 Z}$$

But if we consider the motion of nucleus into account, then radius of n^{th} orbit will be given by

$$r_n' = \frac{n^2 h^2 \varepsilon_0}{\pi \mu_e e^2 Z} = r_n\left(\frac{m_e}{\mu_e}\right)$$

Similarly, the speed of electron in n^{th} Bohr orbit is given by

$$v_n = \left(\frac{e^2}{2h\varepsilon_0}\right)\frac{Z}{n}$$

Since we note that, in the above expression of speed no term of m_e (mass of electron) is present, hence speed of electron in an orbit does not depend on electron mass. So, there will be no change in the speed of revolution of an electron in an orbit, if we consider the motion of nucleus into account.

Similarly, the expression of energy of electron in the n^{th} orbit of Bohr's model is given by

$$E_n = -\frac{m_e e^4 Z^2}{8n^2 h^2 \varepsilon_0^2}$$

But if we consider the motion of nucleus into account, then energy of electron in n^{th} orbit of Bohr's model is given by

$$E_n' = -\frac{\mu_e e^4 Z^2}{8n^2 h^2 \varepsilon_0^2} = E_n\left(\frac{\mu_e}{m_e}\right)$$

Thus, we can say that the energy of electron will be slightly less compared to what we've derived earlier. But for numerical calculations this small change can be ignored unless in a given problem, it is asked to consider the effect of motion of nucleus.

ABOUT RYDBERG CONSTANT

Since, we know that the Rydberg constant for hydrogen atom, when the mass of the nucleus (m_N) is very large compared to the mass of the electron (m_e), is given by

$$R_\infty = R = \frac{m_e e^4}{8\varepsilon_0^2 ch^3}$$

Please do not think that the Rydberg constant is same for all elements. The reason is that in Bohr's Theory, the nucleus is assumed infinitely heavy (and hence at rest) as compared to the electron. But if the mass of nucleus is taken into account, then the electron mass m_e has to replaced by reduced mass (μ_e), where $\mu_e = \dfrac{m_e m_N}{m_e + m_N}$.

$$\Rightarrow \quad \mu_e = \frac{m_e m_N}{m_e + m_N} = \frac{m_e}{1 + \dfrac{m_e}{m_N}}$$

Therefore, $R' = \dfrac{\mu_e e^4}{8\varepsilon_0^2 ch^3} = R\left(\dfrac{\mu_e}{m_e}\right)$

For a heavy nucleus, $m_N \to \infty$, then $\mu_e \to m_e$, so the Rydberg's Constant is represented by R_∞. Then we have

$$R_\infty = R = \frac{m_e e^4}{8\varepsilon_0^2 ch^3} = 1.097 \times 10^7 \text{ m}^{-1}$$

Rydberg's constant for an element is given by

$$R' = \frac{R_\infty}{\left(1+\frac{m_e}{m_N}\right)} = \frac{R}{\left(1+\frac{m_e}{m_N}\right)} = \frac{m_e e^4}{8\left(1+\frac{m_e}{m_N}\right)\varepsilon_0^2 ch^3}$$

Clearly this depends on mass of nucleus m_N and so Rydberg constant is different for different elements. Greater is m_N, larger is the value of Rydberg constant R'. Thus, Rydberg constant increases with increase in mass of nucleus.

ILLUSTRATION 32

The nucleus of a deuterium has a mass of 3.34×10^{-27} kg as compared to 1.67×10^{-27} kg for the hydrogen. Calculate the wavelength difference between the first Balmer line emitted by hydrogen and the first Balmer line emitted by deuterium. Given that the mass of electron is $m_e = 9.109 \times 10^{-31}$ kg.

SOLUTION

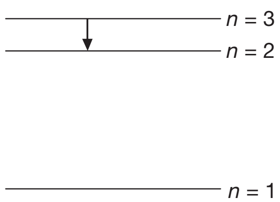
The first Balmer line corresponds to the transition from $n = 3$ to $n = 2$. In case of hydrogen atom, we have

$$\Delta E = E_3 - E_2$$

$$\Rightarrow \quad \Delta E = \left\{-\frac{13.6}{3^2} - \left(\frac{-13.6}{2^2}\right)\right\} \text{ eV} = 1.89 \text{ eV}$$

So, wavelength $\lambda = \dfrac{12375}{\Delta E(\text{in eV})}$

$$\Rightarrow \quad \lambda = \frac{12375}{1.89} = 6547.6 \text{ Å}$$



For ordinary hydrogen reduced mass of proton and electron is,

$$\mu_1 = \frac{(1.67 \times 10^{-27})(9.109 \times 10^{-31})}{(1.67 \times 10^{-27} + 9.109 \times 10^{-31})}$$

$$\Rightarrow \quad \mu_1 = 9.10408 \times 10^{-31} \text{ kg}$$

For deuterium atom reduced mass of nucleus and electron is,

$$\mu_2 = \frac{(3.34 \times 10^{-27})(9.109 \times 10^{-31})}{(3.34 \times 10^{-27} + 9.109 \times 10^{-31})}$$

$$\Rightarrow \quad \mu_2 = 9.10654 \times 10^{-31} \text{ kg}$$

All energies are proportional to μ, whereas the wavelengths are inversely proportional to μ. The wavelength of photon emitted in case of hydrogen is given by

$$\lambda_1 = \frac{(6547.6)(9.109 \times 10^{-31})}{(9.10408 \times 10^{-31})} = 6551 \text{ Å}$$

Similarly, in case of deuterium, wavelength of photon emitted is,

$$\lambda_2 = \frac{(6547.6)(9.109 \times 10^{-31})}{(9.10654 \times 10^{-31})} = 6549 \text{ Å}$$

$$\Rightarrow \quad \Delta\lambda = \lambda_2 - \lambda_1 = 2 \text{ Å}$$

ILLUSTRATION 33

Taking into account the motion of the nucleus of a hydrogen atom, find the expressions for the electron's binding energy in the ground state and for the Rydberg constant. How much (in percent) do the binding energy and the Rydberg constant, obtained without taking into account the motion of the nucleus, differ from the more accurate corresponding value of these quantities?

SOLUTION

If mass of nucleus is considered (not infinity), then the reduced mass of nucleus electron system can be taken as

$$\mu = \frac{mM}{m+M}$$

where, m is mass of electron and M is that of nucleus. The binding energy in ground state of hydrogen atom can now be given as

$$E = \frac{\mu e^4}{8h^2\varepsilon_0^2}$$

$$\Rightarrow \quad E = 13.6 \times \frac{\mu}{m} \text{ eV}$$

$$\Rightarrow \quad E = \frac{13.6M}{m+M} \text{ eV}$$

Since the hydrogen atom Rydberg constant is given by

$$R = \frac{me^4}{8\varepsilon_0^2 ch^3}$$

If effect of mass of nucleus is considered, the new value of Rydberg constant will be

$$R' = \frac{\mu e^4}{8\varepsilon_0^2 ch^3}$$

$$\Rightarrow \quad R' = \frac{RM}{m+M}$$

Percentage difference in the values of R and R' is

$$\frac{\Delta R}{R} = \left(\frac{R'-R}{R}\right)100\%$$

$$\Rightarrow \quad \frac{\Delta R}{R} = \frac{m}{M} \times 100 \approx 0.055\%$$

ILLUSTRATION 34

Calculate the separation between the particles of a system in the ground state, the corresponding binding energy and wavelength of first line in Lyman series if such a system is positronium consisting of an electron and positron revolving round their common centre of mass.

SOLUTION

Reduced mass $\mu = \frac{(m)(m)}{(m+m)} = \frac{m}{2}$

Since, $r \propto \frac{1}{m}$

For, H atom, we have

$$r_1 = 0.53 \text{ Å}$$

$$\Rightarrow \quad r = (2)(0.53) = 1.06 \text{ Å}$$

Since, $E \propto m$ {binding energy}

For, H atom, we have

$$E_1 = 13.6 \text{ eV}$$

$$\Rightarrow \quad E = \frac{13.6}{2} = 6.8 \text{ eV}$$

For, H atom, we have

$$E_{2\to1} = 10.2 \text{ eV} \qquad \text{\{First line of Lyman series\}}$$

$$\Rightarrow \quad E_{21} = \frac{10.2}{2} = 5.1 \text{ eV}$$

$$\Rightarrow \quad \lambda = \frac{12375}{5.1} \text{ Å} = 2426 \text{ Å}$$

ILLUSTRATION 35

Determine the separation of the first line of the Balmer series in a spectrum of ordinary hydrogen and tritium (mass number 3). Take Rydberg's constant $R = 10967800 \text{ m}^{-1}$

SOLUTION

Since $R' = R_H\left(\frac{m_{\text{nucleus}}}{m_{\text{nucleus}} + m_{\text{electron}}}\right)$

First line of Balmer series of H-atom is

$$\frac{1}{\lambda_H} = R\left(\frac{1}{4} - \frac{1}{9}\right) \times \frac{m_H}{m_e + m_H}$$

First line of Balmer series for T-atom is

$$\frac{1}{\lambda_T} = R\left(\frac{1}{4} - \frac{1}{9}\right) \times \frac{m_T}{m_e + m_T}$$

$$\Rightarrow \quad \lambda_H - \lambda_T = \frac{36(m_e + m_H)}{5Rm_H} - \frac{36(m_e + m_T)}{5Rm_T}$$

$$\Rightarrow \quad \Delta\lambda = \frac{36}{5R}\left[\frac{(m_e + m_H)m_T - (m_e + m_T)m_H}{m_H m_T}\right]$$

$$\Rightarrow \quad \Delta\lambda = \frac{36}{5R}\left[\frac{m_e(m_T - m_H)}{m_H m_T}\right]$$

$$\Rightarrow \quad \Delta\lambda = \frac{36}{5 \times 10967800} \times \frac{9.109 \times 10^{-31} \times 2 \times 1.67 \times 10^{-27}}{3 \times (1.67 \times 10^{-27})}$$

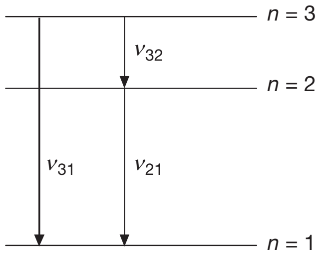
$$\Rightarrow \quad \Delta\lambda = 2.387 \text{ Å}$$

RITZ COMBINATION PRINCIPLE

If an electron is initially in an excited state with say $n = 3$, then it may transit downward from $n = 3$ level to $n = 1$ level directly. Alternatively, it may first transit from $n = 3 \to n = 2$ and subsequently from

$n=2 \rightarrow n=1$. In the first case if ν_{31} be the frequency of the photon emitted

$$h\nu_{31} = E_3 - E_1 \qquad \ldots(1)$$



In the second case, two different spectral lines (photons) of frequency ν_{32} and ν_{21} respectively would be emitted given by

$$h\nu_{32} = E_3 - E_2 \text{ and } h\nu_{21} = E_2 - E_1 \qquad \ldots(2)$$

(1) can be rewritten as

$$h\nu_{31} = (E_3 - E_2) + (E_2 - E_1)$$

$$\Rightarrow \quad h\nu_{31} = h\nu_{32} + h\nu_{21}$$

$$\Rightarrow \quad \nu_{31} = \nu_{32} + \nu_{21}$$

Ritz made this discovery empirically (1908) long before Bohr proposed his theory and is known as Ritz combination principle.

Generalising, we may write, labelling the photon frequency by appropriate integers, as follows:

$$h\nu_{sm} = E_s - E_m$$

$$\Rightarrow \quad h\nu_{sm} = (E_s - E_n) + (E_n - E_m)$$

$$\Rightarrow \quad h\nu_{sm} = h\nu_{sn} + h\nu_{nm} \quad (m < n < s) \qquad \ldots(3)$$

Since all combinations predicted by (3) are not actually observed, there has been an imposition of some rules, called selection rules, to eliminate certain combinations. Bohr's theory provides, as discussed above, a proper explanation of the combination principle.

Conceptual Note(s)

(a) If an electron in a single atom jumps to a level having principal quantum number n, then the maximum number of photons (emitted) will be $(n-1)$.

(b) If in a hydrogen like sample, the electrons of many atoms jump to a level having principal quantum number n, then the maximum number of photons emitted $= {}^nC_2 = \dfrac{n(n-1)}{2}$.

(c) For example if many of the atoms are in third excited state ($n = 4$), then the number of photons emitted for spectral lines seen is $N = \dfrac{4 \times 3}{2} = 6$

(d) If an atom goes to excited state by absorbing certain energy, then it may emit a number of photons in succession; the sum of the energies of all emitted photons will be equal to the amount of energy absorbed. For example if an atom absorbing energy E reaches to an excited level and it returns to ground state by emitted wavelength λ_1, λ_2 and λ_3, then

$$E = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} + \frac{hc}{\lambda_3}$$

(e) If an energetic electron strikes an electron (target electron) of an atom in ground state then electron jumps to an excited state by absorbing energy equal to the difference of ground state energy and excited state energy and the remaining energy is still carried by the incident (or striking) electron.

(f) The de-Broglie quantum condition: According to de-Broglie, only those orbits are allowed as stationary orbits in which circumference of orbit is equal to the integral multiple of de-Broglie wavelength.

i.e., $2\pi r = n\lambda$

$$\Rightarrow \quad 2\pi r = n\left(\frac{h}{mv}\right)$$

$$\Rightarrow \quad mvr = n\left(\frac{h}{2\pi}\right)$$

This is same as Bohr's quantum condition.

(g) "An atom possesses discrete levels" was verified by Franck Hertz Experiment.

MERITS AND DEMERITS OF BOHR'S THEORY

The merits of Bohr's theory can hardly be overestimated. It saved physics at a time when it was in the grip of severe crisis. But, like every physical theory, this theory also has drawbacks. We enumerate below some of its merits and demerits.

Merits

(a) The determination of the ratio of the mass of an electron to that of a proton in terms of the reduced mass concept of Bohr's theory agrees excellently with the value obtained by other methods.

(b) The general principle used by Bohr has also been successfully applied to a great number of phenomena such as the excitation and ionisation of atoms, X-ray spectra etc.

(c) The validity of the theory is further confirmed by the fact that the theory predicts new undiscovered series lines (spectral) which have later been actually observed.

(d) The theory has been instrumental to the discovery of heavy hydrogen (deuterium) by H.C. Urey.

(e) It gives a convincing explanation and a very simple and elegant picture of the origin of spectral lines.

(f) The agreement between the empirically determined value of the Rydberg constant and that evaluated by Bohr in terms of fundamental constants offers an excellent proof of the truth of Bohr's theory.

Demerits

(a) There is an ad hoc nature in the assumptions of Bohr in that the quantum idea of the stationary orbits is mixed up with the classical idea of coulomb force.

(b) The assumption of only circular orbits is utterly unjustified. In fact, Bohr in his original paper suggested that the orbit might be an ellipse instead of a circle.

(c) The spectral series, though agree excellently in case of hydrogen, are at variance with the theory for multi-electron atomic systems, e.g. the helium, singly ionised lithium etc. In these cases, it becomes necessary to introduce a magnetic quantum number.

(d) It cannot suggest any explanation whatsoever for the origin of the fine structure of the spectral lines.

(e) Bohr's theory is also unable to account for the multiple structure of spectral lines. For example, the doublet of sodium, triplets of magnesium etc. cannot be explained from Bohr's theory.

(f) It cannot make any calculation about the transitions or the selection rules which apply to them.

(g) It could not explain the splitting up of spectral lines when an atom is subjected to electric field (phenomenon called Stark Effect) or magnetic field (phenomenon called Zeeman Effect).

CRITICAL POTENTIAL

The resonance potential, the excitation potentials and the ionisation potentials are all included in the wider term critical potentials.

RESONANCE POTENTIAL

A minimum potential V is required to accelerate the bombarding electron to an energy V (in electron-volt) in order that an atom may be excited from its ground state to the next higher state. This potential is called the resonance potential.

EXCITATION POTENTIAL

The various values of the potential required to impart the necessary energy to excite an atom to different higher states are known as excitation potentials.

IONISATION POTENTIAL

The minimum potential necessary to supply the required energy to ionise an atom is called the ionisation potential or the first ionisation potential.

Problem Solving Technique(s)

Let us illustrate the above definitions by taking the case of hydrogen atom. For H atom, we have

$$E_n = -\frac{me^4}{8\varepsilon_0^2 n^2 h^2} = -\frac{2.17\times10^{-18}}{n^2}J = -\frac{13.6}{n^2}\text{eV}$$

So the energy of the 1st, 2nd, 3rd,......., ∞-th orbits are respectively −13.6 eV, −3.4 eV, −1.15 eV,....., 0 eV. Hence,

Resonance potential $= -3.4-(-13.6) = 10.2$ eV

First excitation potential = resonance potential = 10.2 eV

Second excitation potential $= -1.51-(-13.6) = 12.09$ eV

Ionisation potential $= 0-(-13.6) = 13.6$ eV

ILLUSTRATION 36

Find the ratio of ionization energy of Bohr's hydrogen atom and hydrogen-like lithium atom.

SOLUTION

Energy of an electron in n^{th} state of Bohr's hydrogen-like atom (of atomic number z) is given by,

$$E=-\left(\frac{13.6Z^2}{n^2}\right)\text{ eV}$$

The ionization energy of this atom is equal to the magnitude of the energy of electron in the ground state i.e. $E_\infty = 13.6Z^2$

$$\Rightarrow \quad \frac{(E_\infty)_{\text{H}}}{(E_\infty)_{\text{Li}}}=\frac{(Z_H)^2}{(Z_{Li})^2}$$

$$\Rightarrow \quad \frac{(E_\infty)_{\text{H}}}{(E_\infty)_{\text{Li}}}=\left(\frac{1}{3}\right)^2=\frac{1}{9}$$

ILLUSTRATION 37

Find the quantum number n corresponding to excited state of He^+ ion if on transition to the ground state, the ion emits two photons in succession with wavelengths 108.5 nm and 30.4 nm. The ionization energy of H atom is 13.6 eV.

SOLUTION

The energy transitions for the given wavelengths are

$$\Delta E_1=\frac{12400}{\lambda_1}=\frac{12400}{1085}=11.43\text{ eV}$$

$$\Delta E_2=\frac{12400}{\lambda_2}=\frac{12400}{304}=40.79\text{ eV}$$

Total energy emitted $\Delta E=\Delta E_1+\Delta E_2=52.22$ eV

$$\Rightarrow \quad \Delta E=13.6Z^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)\text{ eV}$$

where, ΔE is the energy emitted

$$\Rightarrow \quad 52.34=13.6\times 2^2\left(\frac{1}{1^2}-\frac{1}{n^2}\right)$$

Thus, $n=5$

Test Your Concepts-I

Based on Atomic Structure and Properties

(Solutions on page H.38)

1. A doubly ionised lithium atom is hydrogen like with atomic number 3.
   - **(a)** Find the wavelength of the radiation require to excite the electron in Li^{++} from the first to the third Bohr orbit (Ionisation energy of the hydrogen atom equals 13.6 eV).
   - **(b)** How many spectral lines are observed in the emission spectrum of the above excited system.
2. Find the ionization energy of a doubly ionized lithium atom.
3. An electron and a proton are separated by a large distance and the electron approaches the proton with a kinetic energy of 2 eV. If the electron is captured by the proton to form a hydrogen atom in the ground state, what wavelength photon would be given off?
4. A 12.5 meV α-particle approaching a gold nucleus (Z = 79) is deflected by 180°. How close does it approach the nucleus?
5. If the average life time of an excited state of hydrogen is of the order of 10^{-8} s, estimate how many orbits an electron makes when it is in the state n = 2 before it suffers a transition to state $n = 1$.
6. How many times does the electron go round the first Bohr orbit of hydrogen atom in 1 s?
7. Suppose potential energy between electron and proton at separation r is given by $U=-k\log_e r$, where k is a constant. For such a hypothetical hydrogen atom, calculate the radius of n^{th} Bohr's orbit and its energy levels.
8. Certain gas of identical hydrogen like atoms has all its atoms in a particular upper energy level. The

atoms make transition to a higher energy level when a monochromatic radiation, having wavelength 1654 Å, is incident upon it. Subsequently, the atoms emit radiation of only three different photon energies.

(a) Identify the atom

(b) Obtain the ionization energy for the gas atoms.

(c) If the atoms of the gas are to be excited to such a level which gives radiation of only six different photon energies, what should be energy of incident radiation.

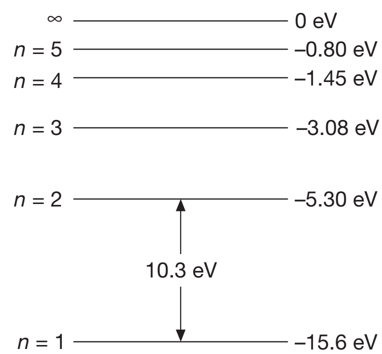
9. Electrons are emitted from an electron gun at almost zero velocity and are accelerated by an electric field *E* through a distance of 1 m. The electrons are now scattered by an atomic hydrogen sample in grounds state. What should be the minimum value of *E* so that red light of wavelength 6563 Å may be emitted in the hydrogen?

10. A hot gas emits radiation of wavelengths 460 Å, 831 Å and 1035 Å only. Assume that the atoms have only two excited states and the difference between consecutive energy levels decreases as energy is increased. Taking the energy of the highest energy state to be zero. Find the energies of the ground state and the first excited state.

11. A mixture of hydrogen atoms (in their ground state) and hydrogen like ions (in their first excited state) are being excited by electrons which have been accelerated by same potential difference *V* volts. After excitation when they come directly into ground state, the wavelengths of emitted light are found in the ratio 5 : 1. Calculate the minimum value of *V* for which both the atoms get excited after collision with electrons. Also find the atomic number of other ion and the energy of emitted light by hydrogen atoms and ions.

12. A stationary He^+ emitted a photon corresponding to the first line of Lyman series. This photon liberated a photo electron from a stationary hydrogen atom in the ground state. Find the velocity of the photoelectron.

13. The energy levels of a hypothetical one electron atom are shown in the figure.



(a) Find the ionization potential of this atom.

(b) Find the short wavelength limit of the series terminating at $n = 2$.

(c) Find the excitation potential for the state $n = 3$.

(d) Find wave number of the photon emitted for the transition $n = 3$ to $n = 1$.

(e) What is the minimum energy that an electron will have after interacting with this atom in the ground state if the initial kinetic energy of the electron is

(i) 6 eV

(ii) 11 eV

14. Using the known values for hydrogen atom, calculate

(a) radius of third orbit for Li^{+2}

(b) speed of electron in fourth orbit for He^+.

15. Find an expression for the magnetic dipole moment and magnetic field induction at the center of a Bohr's hypothetical hydrogen atom in the nth orbit of the electron in terms of universal constants.

16. A small particle of mass *m* moves in such a way that the potential energy $U = ar^2$ where *a* is a constant and *r* is the distance of the particle from the origin. Assuming Bohr's model of quantization of angular momentum and circular orbits, find the radius of *n*th allowed orbit.

17. The electron in a hydrogen atom makes a transition $n_1 \rightarrow n_2$ where n_1 and n_2 are the principle quantum numbers of the two states. Assume the Bohr model to be valid, the time period of the electron in the initial state is eight times that in the final state. What are the possible values of n_1 and n_2?

18. If potential energy in first orbit is taken to be zero, then find the kinetic energy, potential energy and total energy in first and second orbit of hydrogen atom.

19. Wavelengths belonging to Balmer series lying in the range of 450 nm to 750 nm were used to eject photoelectrons from a metal surface whose work function is 2 eV. Find (in eV) the maximum kinetic energy of the emitted photoelectrons. Take $hc = 1242$ eV nm.

20. The potential energy of a particle varies as:

$$U(x) = \begin{bmatrix} E_0 & 0 \le x \le 1 \\ 0 & x > 1 \end{bmatrix}$$

For $0 \le x \le 1$, the de-Broglie wavelength is λ_1 and for $x > 1$, the de-Broglie wavelength is λ_2. Total energy of the particle is $2E_0$. Find $\dfrac{\lambda_1}{\lambda_2}$.

21. An electron in a hydrogen like atom is in an excited state. It has a total energy of –3.4 eV. Calculate.

(a) the kinetic energy,

(b) the de-Broglie wavelength of the electron.

22. An electron of energy 20 eV collides with a hydrogen atom in the ground state. As a result of the collision, the atom is excited to a higher energy state and the electron is scattered with reduced velocity. The atom subsequently returns to its ground state with emission of radiation of wavelength 1.216×10^{-7} m. Find the velocity of the scattered electron.

23. Determine the maximum wavelength that hydrogen in its ground state can absorb. What would be the next smaller wavelength that would work? Take $hc = 12400$ eVÅ

24. Find the ratio of minimum to maximum wavelength of radiation emitted by electron in ground state of Bohr's hydrogen atom.

25. Calculate the difference between the ionization potentials of atomic hydrogen and atomic deuterium.

X-RAYS

When fast moving electrons strike a target of high melting point and high atomic weight (like tungsten, platinum molybdenum), electromagnetic radiations called X-rays are produced. A large part of these radiations has wavelength of the order of 0.1 nm and is known as X-rays.

X-rays were discovered by WC Roentgen in 1895, therefore they are also known as Roentgen rays. He found that photographic film wrapped in black paper becomes exposed when placed near a cathode ray tube. He concluded that some invisible radiations were coming from cathode ray tube which penetrated the black paper and exposed the photographic plate. He named these radiations as X-rays because he was unaware about the nature and properties of radiations. A device used to produce X-rays is generally called an X-rays tube or Coolidge tube.

Production

Coolidge modified the Roentgen tube. A modern X-ray tube consists of

(a) An electron source, preferably a filament heated by the passage of an electric current which may be varied.

(b) A heavy target of high melting point inclined at $45°$ to the path of electron beam, kept cooled by circulating cold water internally.

(c) A source of high potential difference applied across the filament and the target, keeping target positive with respect to filament. When the filament is heated, a fine beam of electrons strikes the target to produce X-rays.

Control of Intensity

The intensity of incident electrons determines the intensity of X-rays, i.e., greater is the number of electrons striking the target, more intense are the X-rays produced.

Control of Penetrating Power

The potential difference across the filament and target determines the energy and hence the penetrating power of X-rays.

Need of Cooling Device

Only about 1% of incident electron's energy is converted into X-rays and the remaining 99% is converted into heat, therefore cooling device is essential with an X-ray tube.

Hard and Soft X-rays

X-rays up to $4\ \text{Å}$ have high penetrating power and are called hard X-rays while those of $\lambda > 4\ \text{Å}$ are called soft X-rays.

ILLUSTRATION 38

Find the energy, the frequency and the momentum of an X-ray photon of wavelength $0.10\ \text{nm}$. Take $hc = 12400\ \text{eVÅ}$

SOLUTION

Given wavelength is $\lambda = 0.1\times10^{-9}\ \text{nm} = 1\ \text{Å}$

Photon energy, $E = \dfrac{12400}{1} = 12.4\ \text{keV}$

Frequency, $\nu = \dfrac{E}{h} = \dfrac{12.4\times10^3\times1.6\times10^{-19}}{6.63\times10^{-34}}$

$\Rightarrow \quad \nu \approx 3\times10^{18}\ \text{Hz}$

Photon momentum, $p = \dfrac{h}{\lambda} = \dfrac{6.63\times10^{-34}}{10^{-10}}$

$\Rightarrow \quad p = 6.63\times10^{-24}\ \text{Js}$

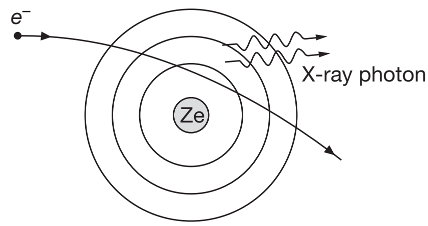
X-RAY SPECTRA CLASSIFICATION

In X-ray tube, when high speed electrons strike the target, they penetrate the target. They loose their kinetic energy and come to rest inside the metal. The electron before finally being stopped makes several collisions with the atoms in the target. At each collision one of the following two types of X-rays may get formed.

A. Continuous X-rays
B. Characteristic X-rays

Continuous X-rays

It consists of radiations of all possible wavelengths within a definite wavelength range having a definite short wavelength limit. These are produced due to deceleration of electrons passing near heavy nucleus.



The loss in energy of electrons during retardation is emitted in form of continuous X-rays. Since the electrons suffer collisions at all angles, right from the glancing collision to the direct hit, so they suffer varying decelerations and hence radiations of all possible wavelengths within a certain range are emitted, forming the continuous spectrum.

The maximum limiting frequency $\nu_{\max}$ or minimum limiting wavelength $\lambda_{\min}$ is obtained when entire kinetic energy of bombarding electron is converted to X-ray energy. If V_0 is the accelerating potential difference, then

$$\frac{1}{2}mv^2 = eV_0 = h\nu_{\max}$$

$$\Rightarrow \quad \nu_{\max} = \frac{eV_0}{h}$$

In terms of $\lambda_{\min}$, we have

$$eV_0 = \frac{hc}{\lambda_{\min}}$$

$$\Rightarrow \quad \lambda_{\min} = \frac{hc}{eV_0} = \frac{12375}{V_0}\ \text{Å} \approx \frac{12400}{V_0}\ \text{Å}$$

This relation is called as **Duane-Hunt Law** and gives the shortest wavelength limit of continuous X-ray spectrum. This wavelength is also called as cut-off wavelength.

If f is the fraction of kinetic energy of electrons converted into X-ray, then wavelength of emitted X-ray photon is given by

$$f(eV_0) = \frac{hc}{\lambda}$$

$$\Rightarrow \quad \lambda = \frac{hc}{feV_0}$$

The intensity vs wavelength graph for X-rays is shown in figure.

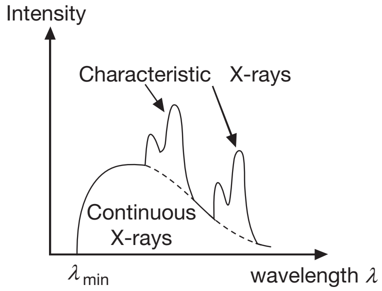


ILLUSTRATION 39

Find the cut-off wavelength of the X-rays emitted by an X-ray tube operating at 30 kV .

SOLUTION

For minimum wavelength, the total kinetic energy should be converted into an X-ray photon.

Thus, $\lambda = \frac{hc}{E} = \frac{12400}{E} = \frac{12400}{30 \times 10^3} = 0.41\ \text{Å}$

ILLUSTRATION 40

If an X-ray tube operates at the voltage of 10 kV , find the ratio of the de-Broglie wavelength of the incident electrons to the shortest wavelength of X-rays produced. The specific charge of electron is $1.8 \times 10^{11}\ \text{Ckg}^{-1}$.

SOLUTION

de Broglie wavelength (λ_d) when a charge q is accelerated by a potential difference of V_0 volt is given by

$$\lambda_d = \frac{h}{\sqrt{2mqV_0}} \qquad \ldots(1)$$

For cut-off wavelength of X-rays, we have

$$qV_0 = \frac{hc}{\lambda_m}$$

$$\Rightarrow \quad \lambda_m = \frac{hc}{qV_0} \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\frac{\lambda_d}{\lambda_m} = \frac{\sqrt{\frac{qV_0}{2m}}}{c}$$

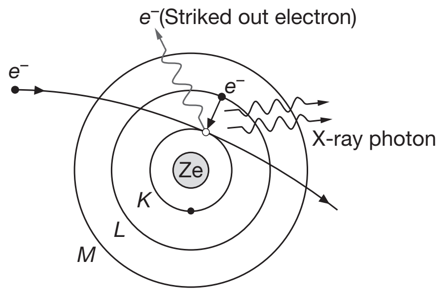
For electron, we have v

$$\frac{q}{m} = 1.8 \times 10^{11}\ \text{Ckg}^{-1}$$

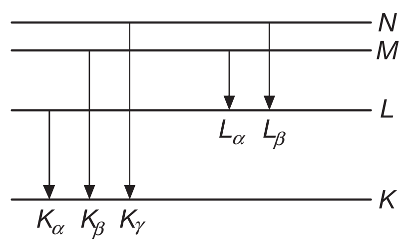
$$\Rightarrow \quad \frac{\lambda_d}{\lambda_m} = \frac{\sqrt{\frac{1.8 \times 10^{11} \times 10 \times 10^3}{2}}}{3 \times 10^8} = 0.1$$

Characteristic X-rays

The minimum wavelength depends on the electron energy, but not on the target material. The line spectrum depends on the element used as target. These characteristic X-rays are produced when an electron knocks out an atomic electron from one of the inner levels. The ejected electron leaves a vacancy, which is then filled by an electron falling from a higher level. When an electron jumps from higher energy orbit E_1 to lower energy orbit E_2, it radiates energy $(E_1 - E_2)$. Thus, this energy difference is radiated in the form of X-rays of very small but definite wavelength which depends upon the target material. The X-ray spectrum consists of sharp lines and is called characteristic X-ray spectrum.



If the transitions are to the $n = 1$ level, the X-rays are labelled K_α, K_β.... If they are to the $n = 2$ level, they are labelled L_α, L_β,.... etc. In the figure shown, the energy level diagram for an atom is drawn. The arrows indicate the transitions that give rise to the different series of X-rays.



$$\lambda_{K_\alpha} = \frac{hc}{E_L - E_K} \text{ for } K_\alpha \text{ wavelength}$$

$$\Rightarrow \lambda_{L_\alpha} = \frac{hc}{E_M - E_L} \text{ for } L_\alpha \text{ wavelength and so on.}$$

ILLUSTRATION 41

Find the maximum potential difference which may be applied across an X-ray tube with tungsten target without emitting any characteristic K or L X-ray. The energy levels of the tungsten atom with an electron knocked out are as follows.

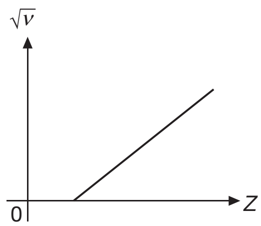
Shell containing vacancy	K	L	M
Energy in keV	69.5	11.3	2.3

SOLUTION

Energy required to knockout L shell electron is 11.3 keV hence the potential difference across the tube must be less than 11.3 kV so that L shell electron does not experience L series X-ray emission.

MOSELEY'S LAW

In 1913, Moseley noted that the characteristic lines shifted systematically as the target material was changed. He plotted the square root of the frequency of the K_α line versus the atomic number Z for many elements. The straight line he obtained is shown in the figure.



Moseley's plot did not pass through the origin, because when one of the two electrons in the $n=1$ level is ejected, then an electron in the next highest level will drop to the lower state to fill the vacancy and in the process it emits the K_α frequency. For this electron, the electric field due to the nucleus is screened by the remaining electrons in the $n=1$ level. Moseley estimated that the effective nuclear charge for the K_α transition is $(Z-1)e$.

In general, wave length of characteristic spectrum can be calculated using Bohr model including the concept of screening effect, where the outer electrons are screened by the inner electrons due to which the effective atomic number is $(Z - b)$ and hence we have

$$\frac{1}{\lambda} = R(Z-b)^2\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

The energy of X-ray radiations will be given by

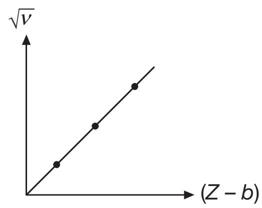
$$\Delta E = \frac{hc}{\lambda} = h\nu = Rhc(Z-b)^2\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

$$\Rightarrow \nu = Rc(Z-b)^2\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) \quad \ldots(1)$$

The above relation was first proposed as an empirical relation given below and is called **Moseley's Law.**

$$\sqrt{\nu} = a(Z-b) \quad \ldots(2)$$

where ν is frequency of emitted line, Z is atomic number of target, b is screening constant or shielding constant, $(Z-b)$ is called as effective atomic number. The plot of $\sqrt{\nu}$ vs $(Z-b)$ is a straight line passing through the origin as shown.



The proportionality constant a is obtained by comparing equations (1) and (2).

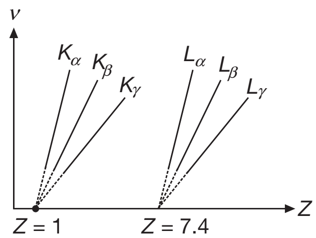
$$\Rightarrow a = \sqrt{Rc\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)}$$

For K_α line, we have $n_2 = 1$ and $n_1 = 4$

$$\Rightarrow a = \sqrt{\frac{3Rc}{4}}$$

The constant b doesn't depend on the nature of target. Different values of b are given below for reference.

1. for K_α-series, $b = 1$
2. for L_α-series, $b = 7.4$
3. for M_α-series, $b = 19.2$



Thus, Moseley's Law for the frequency of the K_α line is

$$\sqrt{\nu_{K_\alpha}} = a(Z-1)$$

where $a = \sqrt{\frac{3}{4}Rc}$, in which R is the Rydberg's constant and c is the speed of light. The wavelength of K-lines is given by

$$\frac{1}{\lambda} = (Z-1)^2\left[1-\frac{1}{n^2}\right] \text{ where } n = 2, 3, 4, \ldots..$$

ILLUSTRATION 42

The energy of a silver atom with a vacancy in K shell is 25.31 keV in L shell is 3.56 keV and in M shell is 0.530 keV higher than the energy of the atom with no vacancy. Calculate the frequency of K_α, K_β and L_α X-rays of silver.

SOLUTION

Energies required for below transition are

$$n = 1 \text{ to } \infty \text{ is } E_1 = 25.31 \text{ KeV}$$

$$n = 2 \text{ to } \infty \text{ is } E_2 = 3.56 \text{ KeV}$$

$$n = 3 \text{ to } \infty \text{ is } E_3 = 0.53 \text{ KeV}$$

Energy of K_α line is $\Delta E_{21} = 25.31 - 3.56 = 21.75$ KeV

Energy of K_β line is $\Delta E_{31} = 25.31 - 0.530 = 24.78$ KeV

Energy of L_α line is $\Delta E_{31} = 3.56 - 0.530 = 3.03$ KeV

$$\Rightarrow \quad \nu_{K_\alpha} = \frac{\Delta E_{21}}{h} = 5.249 \times 10^{18} \text{ Hz}$$

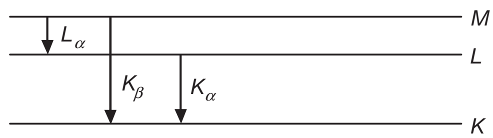
$$\Rightarrow \quad \nu_{K_\beta} = \frac{\Delta E_{31}}{h} = 5.98 \times 10^{18} \text{ Hz}$$

$$\Rightarrow \quad \nu_{L_\alpha} = \frac{\Delta E_{32}}{h} = 7.312 \times 10^{17} \text{ Hz}$$

ILLUSTRATION 43

Show that the frequency of K_β X-ray of a material equals to the sum of frequencies of K_α and L_α X-rays of the same material.

SOLUTION



The energy level diagram of an atom with one electron knocked out is shown above.

Energy of K_α X-ray is $E_{K\alpha} = E_L - E_K$ of K_β X-ray is $E_{K_\alpha} = E_M - E_K$

and, of L_α X-ray is $E_{L\alpha} = E_M - E_L$

thus, $E_{K_\beta} = E_{K_\alpha} + E_{L\alpha}$ or $\nu_{K_\beta} = \nu_{K_\alpha} + \nu_{L_\alpha}$

ILLUSTRATION 44

A free atom of iron emits K_α X-rays of energy 6.2 keV. Calculate the recoil kinetic energy of the atom. Mass of an iron atom $= 9.3 \times 10^{-20}$ kg.

SOLUTION

Wavelength of K_α photon is

$$\lambda_{K_\alpha} = \frac{12400}{6200} = 2 \text{ Å}$$

Using momentum conservation, we get

$$p = \frac{h}{\lambda} = mv_R$$

$\Rightarrow$ Recoil energy E_R of atom is

$$E_R = \frac{p^2}{2m} = \frac{\left(\frac{h}{\lambda}\right)^2}{2m}$$

$$\Rightarrow \quad E_R = \left(\frac{6.63 \times 10^{-34}}{2 \times 10^{-10}}\right) \times \frac{1}{2 \times 9.3 \times 10^{-20} \times 1.6 \times 10^{-19}}$$

$$\Rightarrow \quad E_R = 3.7 \times 10^{-10} \text{ eV}$$

ILLUSTRATION 45

The wavelength of K_α X-ray of tungsten is 20 pm. It takes 11.3 keV to knock out an electron from the L shell of a tungsten atom. What should be the minimum accelerating voltage across an X-ray tube having tungsten target which allows production of K_α X-ray? Take $hc = 12400$ eVÅ

SOLUTION

Binding energy of L-shell electron is 11.3 keV

Energy difference of $n = 2$ and $n = 1$ shell is

$$\Delta E = \frac{12400}{0.2} = 62 \text{ keV}$$

$\Rightarrow$ Binding energy of K-shell electron is

$$E_K = 62 + 11.3 = 73.3 \text{ keV}$$

Thus, accelerating voltage required to knock out K-shell electron is 73.3 V

Moseley's Law: Conclusions

(a) Mosley's Law supported Bohr's theory.

(b) It experimentally determined the atomic number (Z) of elements.

(c) This law established the importance of ordering of elements in periodic table by atomic number and not by atomic weight.

(d) Gaps in Moseley's data for $A = 43$, 61, 72, 75 suggested existence of new elements which were later discovered.

(e) The atomic numbers of Cu, Ag and Pt were established to be 29, 47 and 78 respectively.

(f) When a vacancy occurs in the K-shell, there is still one electron remaining in the K-shell. An electron in the L-shell will feel an effective charge of $(Z-1)e$ due to $+Ze$ from the nucleus and $-e$ from the remaining K-shell electron, because L-shell orbit is well outside the K-shell orbit.

(g) Wave length of characteristic spectrum is

$$\frac{1}{\lambda} = R(Z-b)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

(h) Energy of X-ray radiations is given by

$$\Delta E = h\nu = \frac{hc}{\lambda} = Rhc(Z-b)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

(i) If transition takes place from $n_2 = 2$ to $n_1 = 1$ we get the K_α line, for which we have

(i) $a = \sqrt{\frac{3Rc}{4}} = 2.47 \times 10^{15}$ Hz

(ii) Frequency is given by

$$\nu_{K_\alpha} = RC(Z-1)^2\left(1 - \frac{1}{2^2}\right) = \frac{3RC}{4}(Z-1)^2$$

$$\Rightarrow \quad \nu_{K_\alpha} = 2.47 \times 10^{15}(Z-1)^2 \text{ Hz}$$

(iii) $E_{K_\alpha} = 10.2(Z-1)^2$ eV

(iv) In general, the wavelength of all the K-lines are given by $\frac{1}{\lambda_K} = R(Z-1)^2\left(1 - \frac{1}{n^2}\right)$, where $n = 2, 3, 4, \ldots\ldots$ whereas for K_α line, $\lambda_{K_\alpha} = \frac{1216}{(Z-1)^2}$ Å.

PROPERTIES OF X-RAYS

(a) X-rays are electromagnetic waves of very short wavelength of order of 1 Å. Therefore, they can exhibit properties of reflection, refraction, interference, diffraction, polarisation like ordinary light. Due to this property they help in the study of crystal structure.

(b) They travel in vacuum with speed of light i.e. $c = 3 \times 10^8$ ms^{-1}

(c) They are electrically neutral, hence cannot be deflected by electric and magnetic fields.

(d) They do not possess magnetic moment.

(e) They have ionising power. Therefore, when they pass through a gas, the gas is ionised.

(f) They have penetrating power. They can penetrate light substances like wood, flesh, thick paper, thin sheets of metals, but cannot penetrate heavy substances like lead, calcium, barium sulphate etc.

(g) When incident on certain metals, they liberate electrons. This effect is called photoelectric effect.

(h) They cause fluorescence in many substances like barium, cadmium, zinc sulphide etc.

(i) They have destructive effect on living tissues. Therefore, the persons working with X-rays often wear lead clothes.

ILLUSTRATION 46

In Moseley's equation, we have $\sqrt{f}=a(Z-b)$, where a and b are constants. Find their values with the help of the following data.

Element	Z	Wavelength of K_α X-rays
Mo	42	0.71 Å
Co	27	1.785 Å

SOLUTION

Since, according to Moseley's Law we have

$$\sqrt{f}=a(Z-b)$$

where $f=\dfrac{c}{\lambda}$

So, for the first element, we have

$$\sqrt{\frac{c}{\lambda_1}}=a(Z_1-b) \quad \ldots(1)$$

and for the second element, we have

$$\sqrt{\frac{c}{\lambda_2}}=a(Z_2-b) \quad \ldots(2)$$

From equations (1) and (2), we get

$$\sqrt{c}\left(\frac{1}{\sqrt{\lambda_1}}-\frac{1}{\sqrt{\lambda_2}}\right)=a(Z_1-Z_2) \quad \ldots(3)$$

Now, we know that $c=3\times10^8\ \text{ms}^{-1}$ and further it is given to us that $\lambda_1=0.71\times10^{-10}\ \text{m}$, $\lambda_2=1.785\times10^{-10}\ \text{m}$, $Z_1=42$ and $Z_2=27$. Solving above three equations, we get

$$a=5\times10^7\ (\text{Hz})^{1/2} \text{ and } b=1.37$$

ILLUSTRATION 47

Determine the energy of the characteristic X-ray (K_β) emitted from a tungsten $(Z=74)$ target when an electron drops from the M shell $(n=3)$ to a vacancy in the K shell $(n=1)$.

SOLUTION

Energy associated with the electron in the K shell is approximately

$$E_K=-(74-1)^2(13.6\ \text{eV})=-72474\ \text{eV}$$

An electron in the M shell is subjected to an effective nuclear charge that depends on the number of electrons in the $n=1$ and $n=2$ states because these electrons shield the M electrons from the nucleus.

Since, there are eight electrons in the $n=2$ state and one remaining in the $n=1$ state, so, roughly nine electrons shield M shell electrons from the nucleus, and hence we have $Z_{\text{eff}}=Z-9$. The energy associated with an electron in the M shell is given by

$$E_M=\frac{-13.6Z_{\text{eff}}^2}{3^2}\ \text{eV}=\frac{-13.6(Z-9)^2}{3^2}\ \text{eV}$$

$$\Rightarrow \quad E_M=-\frac{(13.6)(74-9)^2}{9}\ \text{eV}=-6384\ \text{eV}$$

Therefore, emitted X-ray has an energy equal to

$$E_M-E_K=[-6384-(-72474)]\ \text{eV}=66090\ \text{eV}$$

ILLUSTRATION 48

X-rays are incident on a target metal atom having 30 neutrons. The ratio of atomic radius of the target atom and ${}^4_2\text{He}$ is $(14)^{1/3}$.

(a) Find the atomic number of target atom.

(b) Find the frequency of K_α line emitted by this metal.

Assume that the radius $r\propto A^{\frac{1}{3}}$ and $c=3\times10^8\ \text{ms}^{-1}$.

SOLUTION

(a) From the relation $r\propto A^{\frac{1}{3}}$

$$\Rightarrow \quad \frac{r_2}{r_1}=\left(\frac{A_2}{A_1}\right)^{\frac{1}{3}}$$

$$\Rightarrow \quad \left(\frac{A_2}{4}\right)^{\frac{1}{3}}=(14)^{\frac{1}{3}}$$

$$\Rightarrow \quad A_2=56$$

(b) $Z_2=A_2-\text{number of neutrons}$

$$\Rightarrow \quad Z_2=56-30=26$$

Since, $f_{K_\alpha} = Rc(Z-1)^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right) = \frac{3Rc}{4}(Z-1)^2$

Substituting the given values of R, c and Z, we get

$$f_{K_\alpha} = 1.55\times10^{18} \text{ Hz}$$

ILLUSTRATION 49

Characteristic X-rays of frequency 4.2×10^{18} Hz are produced when transitions from L-shell, K-shell take place in a certain target material. Use Mosley's Law to determine the atomic number of the target material. Given Rydberg constant $R = 1.1\times10^7 \text{ m}^{-1}$.

SOLUTION

$$\Delta E = hv = Rhc(Z-b)^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

For K-series, $b = 1$

$$\Rightarrow \quad v = Rc(Z-1)^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

Substituting the values, we get

$$4.2\times10^{18} = (1.1\times10^7)(3\times10^8)(Z-1)^2\left(\frac{1}{1}-\frac{1}{4}\right)$$

$$\Rightarrow \quad (Z-1)^2 = 1697$$

$$\Rightarrow \quad Z-1 \approx 41$$

$$\Rightarrow \quad Z = 42$$

ILLUSTRATION 50

The wavelength of the characteristics X-ray K_α line emitted from zinc $(Z = 30)$ is 1.415 Å. Find the wavelength of the K_α line emitted from molybdenum $(Z = 42)$.

SOLUTION

According to Moseley's law, the frequency for K series is given by

$$v \propto (Z-1)^2$$

$$\Rightarrow \quad \frac{c}{\lambda} \propto (Z-1)^2$$

$$\Rightarrow \quad \frac{1}{\lambda} = k(Z-1)^2 \quad \ldots(1)$$

where k is a constant. Let λ' be the wavelength of K_α line emitted from molybdenum, then

$$\frac{1}{\lambda'} = k(Z-1)^2 \quad \ldots(2)$$

Dividing Equation (1) by (2), we get

$$\lambda' = \left(\frac{Z-1}{Z'-1}\right)^2 \lambda$$

$$\Rightarrow \quad \lambda' = \left(\frac{30-1}{42-1}\right)^2 \times 1.415 \text{ Å} = 0.708 \text{ Å}$$

BRAGG'S LAW

When an X-ray beam of wavelength λ is incident on a crystal of inter planar spacing d at grazing angle θ, then the directions of diffraction maxima are given by

$$2d\sin\theta = n\lambda \quad \ldots(1)$$

where n is an integer, called order of maxima. Equation (1) is called Bragg's equation

$$\lambda = \frac{2d\sin\theta}{n}$$

For maximum wavelength

$$n_{\min} = 1 \text{ and } (\sin\theta)_{\max} = 1$$

$$\Rightarrow \quad \lambda_{\max} = 2d$$

Hence equation (1) has solution only for $\lambda \le 2d$.

INTENSITY OF TRANSMITTED X-RAY

The intensity of monochromatic X-ray beam after penetrating a thickness x of a target material is given by $I = I_0 e^{-\mu x}$, where μ is a constant called Absorption Coefficient. Its value depends upon nature of material. μ increases with increase of λ and atomic number Z of absorbing material. μ is maximum for lead.

Test Your Concepts-II

Based on X-rays and Properties

(Solutions on page H.42)

1. What potential difference should be applied across an X-ray tube to get X-ray of wavelength not less than 0.10 nm? What is the maximum energy of a photon of this X-ray in joule? Take hc = 12400 eVÅ
2. **(a)** An X-ray tube produces a continuous spectrum of radiation with its short-wavelength end at 0.45 Å. What is the maximum energy of a photon in the radiation?
   (b) From your answer to (a), guess what order of accelerating voltage (for electrons) is required in such a tube?
3. If the short series limit of the Balmer series for hydrogen is 3644 Å, find the atomic number of the element which give X-ray wavelengths down to 1 Å. Identify the element.
4. Iron emits K_α X-ray of energy 3.69 keV. Calculate the times taken by an iron K_α photon and a calcium K_α photon to cross through a distance of 3 km.
5. Use Moseley's Law with $b = 1$ to find the frequency of the K_α X-rays of La ($Z = 57$) if the frequency of the K_α X-rays of Cu ($Z = 29$) is known to be 1.88×10^{18} Hz.
6. When the voltage applied to an X-ray tube is increased $\eta = 1.5$ times, the short wave limit of an X-ray continuous spectrum shifts by $\Delta\lambda = 26$ pm. Find the initial voltage applied to the tube.
7. Find the cut-off wavelength for the continuous X-rays coming from an X-ray tube operating at 40 kV.
8. An X-rays tube operates at 20 kV. Find the maximum speed of the electrons striking the anticathode if the charge and mass of electron are 1.6×10^{-19} C and 9×10^{-31} kg.
9. Calculate the wavelength of the emitted characteristic X-ray from a tungsten ($Z = 74$) target when an electron drops from a M shell to a vacancy in the K shell.
10. A material whose K absorption edge is 0.2 Å is irradiated by X-rays of wavelength 0.15 Å. Find the maximum energy of the photoelectrons that are emitted from the K shell. (Take hc = 12400 eVÅ)
11. If two times the λ_{min} of continuous X-ray spectra of target atom A at 34.3 kV is same as the wavelength of K_α line of target atom B at 40 kV, then determine the atomic number of the atom B.
12. Stopping potentials of 24 kV, 100 kV, 110 kV and 115 kV are measured for photoelectrons emitted from a certain element when it is radiated with monochromatic X-ray. If this element is used as a target in an X-ray tube, what will be the wavelength of K_α line?

SOLVED PROBLEMS

PROBLEM 1

A peak emission from a black body at a certain temperature occurs at a wavelength of 9000 Å. On increasing the temperature, the total radiation emitted is increased 81 times. At the initial temperature, when the peak radiation from the black body is incident on a metal surface, it does not cause any photoemission from the surface. After the increase of temperature, the peak radiation from the black body caused photoemission. To bring these photoelectrons to rest, a potential equivalent to the excitation energy between the $n = 2$ and $n = 3$ Bohr levels of hydrogen atom is required. Calculate the work function of the metal.

SOLUTION

Let T be the initial absolute temperature of the black body. The total energy emitted by the body per unit area per second is given according to Stefan's Law by the relation

$$E = \sigma T^4$$

where σ is the Stefan's constant.
When the temperature of the black body is raised to T'. We have

$$E' = \sigma T'^4,$$

$$\Rightarrow \quad \frac{E'}{E} = \frac{T'^4}{T^4} = 81$$

$$\Rightarrow \quad T' = 3T$$

Since it is given that peak emission at temperature T occurs at a wavelength $\lambda_m = 9000$ Å. If λ'_m is the wavelength for peak emission at temperature T', then from Wien's displacement law $\lambda_m T = \text{constant}$, we have

$$\lambda'_m T' = \lambda_m T$$

$$\Rightarrow \quad \lambda'_m = \lambda_m\left(\frac{T}{T'}\right) = 9000\ \text{Å}\left(\frac{T}{3T}\right) = 3000\ \text{Å}$$

According to the problem, the kinetic energy of the emitted photo-electrons corresponds to the excitation energy for transition $n = 2$ to $n = 3$, which is given by

$$K_{\max} = \Delta E_{32} = (13.6\ \text{eV})\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\Rightarrow \quad K_{\max} = 13.6\left(\frac{5}{36}\right) = 1.89\ \text{eV}$$

The energy of the photon of wavelength $\lambda' = 3000$ Å in eV is

$$E = \frac{12375}{3000}\ \text{eV} = 4.125\ \text{eV}$$

Now, from Einstein's photoelectric equation, the work function for the metal surface is given by

$$\phi_0 = E - K_{\max}$$

$$\Rightarrow \quad \phi_0 = (4.125 - 1.89)\ \text{eV} = 2.235\ \text{eV}$$

PROBLEM 2

A gas of hydrogen like ions is prepared in such a way that the ions are only in the ground state and the first excited state. A monochromatic light of wavelength 1216 Å is absorbed by the ions. The ions are lifted to higher excited states and emit radiations of six wavelengths, some higher, some lower or some greater than the incident wavelength.

(a) Find the principle quantum number of the final excited state.
(b) Identify the nuclear charge on the ions.
(c) Calculate the value of the maximum and minimum wavelengths.

SOLUTION

(a) Since, total six wavelengths are obtained in the emission spectrum, hence from

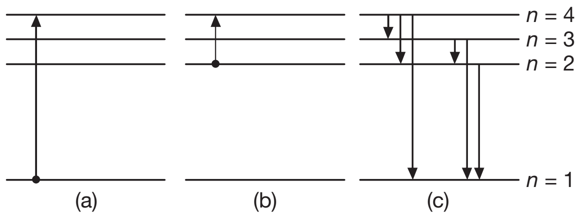
$$\frac{n(n-1)}{2} = 6$$

We have $n = 4$, i.e., after excitation the single electron jumps to 3rd excited state or $n = 4$.

(b) Energy difference corresponding to $\lambda = 1216$ Å is,

$$\Delta E = \frac{12375}{1216}\ \text{eV} = 10.177\ \text{eV} \cong 10.2\ \text{eV}$$

Now it may jump either from $n = 1$ or $n = 2$.



If it jumps from $n = 1$, then in emission spectrum all the six photons have energy equal to or less than the energy of absorbed photon or the wavelength of emitted photon is either equal to or greater than the wavelength of absorbed photon. While in the question it is given that the emitted wavelengths are either less than or greater than or smaller than the wavelength of absorbed photon. Which is possible only in the second case, i.e., when electron jumps from $n = 2$ to $n = 4$.

Hence, $E_4 - E_2 = 10.177$ eV

$$\Rightarrow \quad \frac{-13.6z^2}{4^2} - \left(\frac{-13.6z^2}{2^2}\right) = 10.177$$

Solving this, we get $z \approx 2$

(c) Maximum wavelength corresponds to minimum energy, i.e., a transition from $n = 4$ to $n = 3$. Thus,

$$\Delta E_{\text{min}} = E_4 - E_3$$

$$\Rightarrow \quad \Delta E_{\text{min}} = \frac{(-13.6)(2)^2}{(4)^2} - \left[\frac{-(13.6)(2)^2}{(3)^2}\right]$$

$$\Rightarrow \quad \Delta E_{\text{min}} = 2.64 \text{ eV}$$

$$\Rightarrow \quad \lambda_{\text{max}} = \frac{hc}{\Delta E_{\text{min}}} = \frac{12375}{2.64} = 4687 \text{ Å}$$

Minimum wavelength corresponds to maximum energy, i.e., a transition from $n = 4$ to $n = 1$.

Hence, $\Delta E_{\text{max}} = E_4 - E_1$

$$\Rightarrow \quad \Delta E_{\text{max}} = \frac{(-13.6)(2)^2}{(4)^2} - \left[\frac{(-13.6)(2)^2}{(1)^2}\right]$$

$$\Rightarrow \quad \Delta E_{\text{max}} = 51 \text{ eV}$$

$$\Rightarrow \quad \lambda_{\text{min}} = \frac{12375}{51} = 242 \text{ Å}$$

PROBLEM 3

A gas of hydrogen like atoms can absorb radiations of 68 eV. Consequently, the atoms emit radiations of only three different wavelengths. All the wavelengths are equal or smaller than that of the absorbed photon.

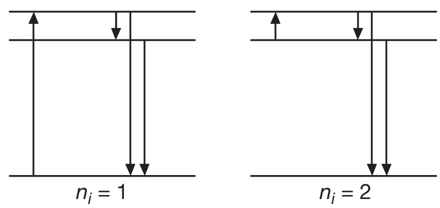
(a) Find the initial state of the gas atoms.
(b) Identify the gas atoms.
(c) Calculate the minimum wavelength of the emitted radiations.
(d) Find the ionization energy and the respective wavelength for the gas atoms.

SOLUTION

(a) Since, $N = \dfrac{n(n-1)}{2} = 3$

$$\Rightarrow \quad n = 3$$

i.e., after excitation atom jumps to second excited state. Hence $n_f = 3$. So n_i can be 1 or 2



If $n_i = 1$ then energy emitted is either equal to, greater than or less than the energy absorbed. Hence the emitted wavelength is either equal to, less than or greater than the absorbed wavelength. Hence $n_i \neq 1$.

If $n_i = 2$, then $E_e \geq E_a$ and hence $\lambda_e \leq \lambda_b$

So, $n_i = 2$

(b) Since, $E_3 - E_2 = 68$ eV

$$\Rightarrow \quad (13.6)(Z^2)\left(\frac{1}{4} - \frac{1}{9}\right) = 68$$

$$\Rightarrow \quad Z = 6$$

(c) $\lambda_{\text{min}} = \dfrac{12375}{E_3 - E_1} = \dfrac{12375}{(13.6)(6)^2\left(1 - \dfrac{1}{9}\right)} = 28.43 \text{ Å}$

(d) Ionization energy is

$$IE = (13.6)(6)^2 = 489.6 \text{ eV}$$

$$\Rightarrow \quad \lambda = \frac{12375}{489.6} = 25.3 \text{ Å}$$

PROBLEM 4

From a metal surface, photoelectrons are emitted when 4000 Å radiation is incident on the surface having work function 1.9 eV. These photoelectrons then pass through a region containing α-particles. It is observed that a maximum energy photoelectron combines with an α-particle to form a He^+ ion, emitting a single photon in this process. He^+ ions thus formed are in their fourth excited state. Find the energies (in eV) of the photons, lying in the 2 eV to 4 eV range, that are likely to be emitted during and after the combination. Take $h = 4.14 \times 10^{-15}$ eVs.

SOLUTION

The energy of the incident photon is

$$E = h\nu = \frac{hc}{\lambda} = \frac{12375}{4000}\ \text{eV} \approx 3.1\ \text{eV}$$

From Einstein's photoelectric equation, the maximum kinetic energy of the emitted electrons is

$$K_{\max} = h\nu - \phi_0 = 3.1\ \text{eV} - 1.9\ \text{eV} = 1.2\ \text{eV}$$

Since, it is given that

$$e^- + \text{He} \rightarrow \text{He}^+ + \text{photon}$$

where the electron has maximum kinetic energy $K_{\max}$ and the He^+ ion $(Z = 2)$ is in the fourth excited state which corresponds to $n = 5$. Since we know that the energy of the electron in the n^{th} state for a hydrogen like atom is

$$E_n = -(13.6)\frac{Z^2}{n^2}\ \text{eV} \qquad \ldots(1)$$

$$\Rightarrow \quad E_5 = -(13.6\ \text{eV}) \times \frac{(2)^2}{(5)^2} = -2.18\ \text{eV}$$

The energy of the emitted photon in the above combination process is

$$E = K_{\max} + (-E_5)$$

$$\Rightarrow \quad E = 1.2\ \text{eV} + 2.18\ \text{eV} = 3.38\ \text{eV}$$

and this energy lies well within the range 2 eV to 4 eV.

After the recombination process, the electron may undergo transitions from a higher level to a lower level, thus emitting photons. Using Equation (1), the energies in the lower energy levels of He^+ ions are

$$E_4 = \frac{(-13.6\ \text{eV}) \times (2)^2}{(4)^2} = -3.4\ \text{eV}$$

$$E_3 = \frac{(-13.6\ \text{eV}) \times (2)^2}{(3)^2} = -6.04\ \text{eV}$$

$$E_2 = \frac{(-13.6\ \text{eV}) \times (2)^2}{(2)^2} = -13.6\ \text{eV}$$

and $$E_1 = \frac{(-13.6\ \text{eV}) \times (2)^2}{(1)^2} = -54.4\ \text{eV}$$

Since, $E_5 = -2.18$ V, so the energies of the emitted photons are given by the differences of these energies which must lie in the range 2 eV to 4 eV.

We observe that the following difference of energies (in addition to $E = 3.38$ eV) lie well within the asked range of 2 eV to 4 eV.

(1) $\Delta E_{43} = E_4 - E_3 = -3.4 - (-6.04)$

$\Rightarrow \quad \Delta E_{43} = 2.64$ eV

(2) $\Delta E_{53} = E_5 - E_3 = -2.18 - (-6.04)$

$\Rightarrow \quad \Delta E_{53} = 3.86$ eV

Hence, the energies of the photons that are likely to be emitted with energies in the range 2 eV to 4 eV are 2.64 eV, 3.38 eV and 3.86 eV.

PROBLEM 5

The energy levels of a hypothetical one electron atom are given by $E_n = -\frac{18}{n^2}$ eV, where $n = 1, 2, 3, \ldots$..

(a) Calculate the four lowest energy levels and construct the energy level diagram.

(b) Find the excitation potential of the stage $n = 2$.

(c) Find the wavelengths (Å) which can be emitted when these atoms in the ground state are bombarded by electrons that have been accelerated through a potential difference of 16.2 V.

(d) Assuming these atoms to be in the ground state, will they absorb radiation having a wavelength of 2000 Å?

(e) Also calculate the photoelectric threshold wavelength of this atom.

SOLUTION

(a) Since, $E_n = -\frac{18}{n^2}$ eV, so we have

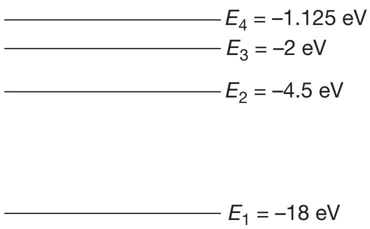
$$E_1 = \frac{-18}{(1)^2} = -18 \text{ eV}$$

$$E_2 = \frac{-18}{(2)^2} = -4.5 \text{ eV}$$

$$E_3 = \frac{-18}{(3)^2} = -2 \text{ eV} \text{ and}$$

$$E_4 = \frac{-18}{(4)^2} = -1.125 \text{ eV}$$

The energy level diagram is shown in figure.



(b) The excitation potential of stage $n = 2$ is

$$E_2 - E_1 = 18 - 4.5 = 13.5 \text{ V}$$

(c) Energy of the electron accelerated by a potential difference of 16.2 V is 16.2 eV. Since we observe that

$$E_4 - E_1 = -1.125 - (-18) = 16.875 \text{ eV} > 16.2 \text{ eV}$$

and $E_3 - E_1 = -2 - (-18) = 16 \text{ eV} < 16.2 \text{ eV}$

With this energy the electron will be able to excite the atom from $n = 1$ to $n = 3$, so we have the possible wavelengths corresponding to the transitions from $3 \to 1$, $3 \to 2$ and $2 \to 1$. Hence

$$\lambda_{32} = \frac{12375}{E_3 - E_2} = \frac{12375}{-2 - (-4.5)} = 4950 \text{ Å}$$

$$\lambda_{31} = \frac{12375}{E_3 - E_1} = \frac{12375}{16} = 773 \text{ Å}$$

and $$\lambda_{21} = \frac{12375}{E_2 - E_1} = \frac{12375}{-4.5 - (-18)} = 917 \text{ Å}$$

(d) The energy corresponding to $\lambda = 2000$ Å is given by

$$E = \frac{12375}{2000} = 6.1875 \text{ eV}$$

whereas the minimum excitation energy is 13.5 eV ($n = 1$ to $n = 2$).
Hence this is not possible.

(e) Threshold wavelength for photoemission to take place from such an atom is,

$$\lambda_{min} = \frac{12375}{18} = 687.5 \text{ Å}$$

PROBLEM 6

Hydrogen gas in the atomic state is excited to an energy level such that the electrostatic potential energy of H-atom becomes -1.7 eV. Now the photoelectric plate having work function 2.3 eV is exposed to the emission spectra of this gas. Assuming all the transitions to be possible, find the minimum de Broglie wavelength of ejected photo-electrons.

SOLUTION

Given that electrostatic potential energy of H-atom is

$$PE = -1.7 \text{ eV}$$

Since we know that kinetic energy is given by

$$KE = \left|\frac{PE}{2}\right| = \frac{1.7}{2} = 0.85 \text{ eV}$$

So, total energy is $E = -1.7 + 0.85 = -0.85$ eV

$$\text{Since, } E_n = -\frac{13.6}{n^2} = -0.85 \text{ eV}$$

$$\Rightarrow \quad n^2 = \frac{13.6}{0.85} = 16$$

$$\Rightarrow \quad n = 4$$

So, the atom is excited to $n = 4$ state. The maximum energy will be emitted, when electrons will make a transition from $n = 4$ to $n = 1$. The energy emitted for this transition is

$$\Delta E = -0.85 - (-13.6) = 12.75 \text{ eV}$$

When a photon of this energy is incident on a metal plate having work function 2.3 eV, the kinetic energy of fastest electron ejected will be given by

$$K_{max} = \Delta E - \phi_0 = 12.75 - 2.3 = 10.45 \text{ eV}$$

The minimum de Broglie wavelength is given by

$$\lambda_{min} = \frac{h}{p_{max}} = \frac{h}{\sqrt{2mK_{max}}}$$

$$\Rightarrow \quad \lambda_{min} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times (9.1 \times 10^{-31})(10.45 \times 1.6 \times 10^{-19})}}$$

$$\Rightarrow \quad \lambda_{min} = 3.8 \times 10^{-10} \text{ m} = 3.8 \text{ Å}$$

PROBLEM 7

Two hydrogen like atoms A and B are of different masses and each atom contains equal number of protons and neutrons. The difference in the energies between the first Balmer lines emitted by A and B is 5.667 eV. When the atom A and B, moving with the same velocity, strike a heavy target they rebound back with the same velocity. In the process, atoms B imparts twice momentum to the target than that A imparts. Identify the atoms A and B.

SOLUTION

$$5.667 = 13.6\left(Z_B^2 - Z_A^2\right)\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\Rightarrow \quad Z_B^2 - Z_A^2 = 3 \qquad \ldots(1)$$

Applying Law of Conservation of Linear Momentum on atom A and heavy target, we get

$$m_A u = Mv_1 - m_A u$$

$$\Rightarrow \quad 2m_A u = Mv_1$$

Similarly, for atom B and heavy target, we get

$$2m_B u = Mv_2$$

Given $Mv_2 = 2Mv_1$

$$\Rightarrow \quad m_B = 2m_A \qquad \ldots(2)$$

Since, both A and B contain equal number of protons and neutrons, so we have

$$\frac{m_A}{m_B} = \frac{2Z_A}{2Z_B} = \frac{Z_A}{Z_B} \qquad \ldots(3)$$

From these equations we get

$$Z_A = 1 \text{ and } Z_B = 2$$

i.e., A is ${}_1H^2$ and B is ${}_2He^4$ (both having single electron).

PROBLEM 8

Stopping potential of 24 kV, 10 kV, 110 kV and 115 kV are measured for photoelectrons emitted from a certain element when it is radiated with monochromatic X-ray. If this element is used as a target in an X-ray tube, what will be the wavelength of K_α line?

SOLUTION

Stopping potentials are 24 kV, 10 kV, 110 kV and 115 kV

If the electrons are emitted from conduction band then the maximum kinetic energy of photoelectrons would be 115×10^3 eV.

If they are emitted from next inner shell maximum kinetic energy of photoelectrons would be 110×10^3 eV and so on.

For photoelectrons of L shell it would be 100×10^3 eV and for K shell it is 24×10^3 eV. Therefore, difference between energy of L shell and K shell is,

$$\Delta E = E_L - E_K = (100-24)\times10^3 \text{ eV}$$

$$\Rightarrow \quad \Delta E = 76\times10^3 \text{ eV}$$

The wavelength of K_α line (transition of electron from L shell to K shell) is,

$$\lambda_{K_\alpha}(\text{in Å}) = \frac{12375}{\Delta E(\text{in eV})}$$

$$\Rightarrow \quad \lambda_{K_\alpha} = \frac{12375}{76\times10^3} = 0.163 \text{ Å}$$

PROBLEM 9

For a certain hypothetical one-electron atom, the wavelength (in Å) for the spectral lines for transitions originating at $n = p$ and terminating at $n = 1$ are given by

$$\lambda = \frac{1500p^2}{p^2-1} \text{ where } p = 2, 3, 4 \ldots..$$

(a) Find the wavelength of the least energetic and the most energetic photons in this series.

(b) Construct an energy level diagram for this element showing the energies of the lowest three levels.

(c) Calculate the ionization potential of this element.

SOLUTION

(a) Since, $\lambda = \dfrac{1500p^2}{p^2-1}$

$$\Rightarrow \quad \lambda = 1500\left(\frac{1}{1-\frac{1}{p^2}}\right)$$

So, λ_{max} corresponds to least energetic photon with $p = 2$

$$\Rightarrow \quad \lambda_{max} = 1500\left(\frac{1}{1-\frac{1}{4}}\right) = 2000 \text{ Å}$$

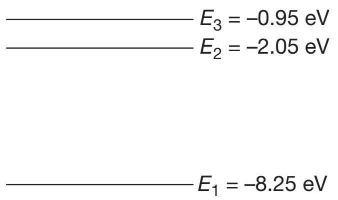
λ_{min} corresponds to most energetic photon with $p \to \infty$

$\Rightarrow \quad \lambda_{min} = 1500 \text{ Å}$

(b) $\lambda_{\infty \to 1} = 1500 \text{ Å}$

$$\Rightarrow \quad E_{\infty} - E_1 = \frac{12375}{1500} \text{ eV} = 8.25 \text{ eV}$$

$$\Rightarrow \quad E_1 = -8.25 \text{ eV} \qquad \{\because E_{\infty} = 0\}$$



Further, $\lambda_{2 \to 1} = 2000 \text{ Å}$

$$\Rightarrow \quad E_2 - E_1 = \frac{12375}{2000} \text{ eV} = 6.2 \text{ eV}$$

$$\Rightarrow \quad E_2 = -2.05 \text{ eV}$$

Similarly, $\lambda_{31} = 1500\left(\dfrac{1}{1-\dfrac{1}{9}}\right) = 1687.5 \text{ Å}$

$$\Rightarrow \quad E_3 - E_1 = \frac{12375}{1687.5} \text{ eV} = 7.3 \text{ eV}$$

$$\Rightarrow \quad E_3 = -0.95 \text{ eV}$$

(c) Ionization energy is

$$E_{1 \to \infty} = 8.25 \text{ eV}$$

So, Ionisation Potential equals 8.25 V

PROBLEM 10

Consider an excited hydrogen atom in state n moving with a velocity $v(v \ll c)$. It emits a photon in the direction of its motion and changes its state to a lower state m. Apply momentum and energy conservation principles to calculate the frequency f of the emitted radiation. Compare this with the frequency f_0 emitted if the atom were at rest.

SOLUTION

Applying Conservation of Linear Momentum, we get

$$mv = mv' + \frac{hf}{c} \qquad \ldots(1)$$

Applying Conservation of Energy, we get

$$\frac{1}{2}mv^2 + \Delta E = \frac{1}{2}mv'^2 + hf \qquad \ldots(2)$$

$$\Rightarrow \quad \Delta E = hf - \frac{1}{2}m\left(v^2 - v'^2\right)$$

$$\Rightarrow \quad \Delta E = hf - \frac{1}{2}m(v + v')\left(\frac{hf}{mc}\right)$$

Since, $v \approx v'$, so we have

$$\Delta E = hf - \left(\frac{hf}{2}\right)\left(\frac{2v}{c}\right)$$

$$\Rightarrow \quad \Delta E = hf\left(1 - \frac{v}{c}\right)$$

When atom was at rest, then we have

$$\Delta E \approx hf_0$$

$$\Rightarrow \quad hf_0 = hf\left(1 - \frac{v}{c}\right)$$

$$\Rightarrow \quad f = f_0\left(1 + \frac{v}{c}\right)$$

PROBLEM 11

An electron of a stationary hydrogen atom passes from the fifth energy level to the fundamental state. What velocity did the atom acquire as the result of photon emission? What is the recoil energy? Express your answer in terms of Rydberg constant R mass of hydrogen atom M and universal constants.

SOLUTION

$$E = hcR\left(\frac{1}{1^2} - \frac{1}{5^2}\right) = \frac{24}{25}hcR \qquad \ldots(1)$$

This energy will be shared by photon and the atom.

Thus, $E = hf + E_0 \qquad \ldots(2)$

where $E_0 = \dfrac{p^2}{2M}$ is the atom's recoil energy and p is the momentum due to emission of a photon. Applying the Law of Conservation of Linear Momentum, we get

$$p = p_{Ph} = \frac{hf}{c} \qquad \ldots(3)$$

Solving (1), (2) and (3), we get

$$E_0 = \frac{h^2 f^2}{2Mc^2} \text{ and } hf = \frac{2E}{1 + \sqrt{1 + \dfrac{2E}{Mc^2}}}$$

Since, the transition energy in hydrogen atom is below 13.6 eV and $\frac{2E}{Mc^2} \approx 10^{-8}$, so $\frac{2E}{Mc^2}$ can be neglected.

$$\Rightarrow \quad hf \approx E = \frac{24}{25} hcR$$

So, recoil energy of atom is $\frac{24^2 h^2 R^2}{2 \times 25^2 M} = \frac{h^2 R^2}{2.17 M}$ and the velocity of the atom is $\frac{24hR}{25M}$.

PROBLEM 12

If the wavelength of the nth line of Lyman series is equal to the de-Broglie wavelength of electron in initial orbit of a hydrogen like element $(Z = 11)$. Find the value of n.

SOLUTION

n^{th} line of Lyman series means transition from $(n+1)^{th}$ state to first state.

$$\frac{1}{\lambda} = RZ^2 \left(1 - \frac{1}{(n+1)^2}\right) \quad \ldots(1)$$

de-Broglie wavelength in $(n+1)^{th}$ orbit is

$$\lambda = \frac{h}{mv} = \frac{hr}{mvr} = \frac{(2\pi)(hr)}{(n+1)h} = \frac{2\pi r}{(n+1)}$$

$$\Rightarrow \quad \frac{1}{\lambda} = \frac{(n+1)}{2\pi r} \quad \ldots(2)$$

Equating equations (1) and (2), we get

$$\left(\frac{n+1}{2\pi r}\right) = RZ^2 \left(\frac{n(n+2)}{(n+1)^2}\right) \quad \ldots(3)$$

Since, $r \propto \frac{n^2}{Z}$

$$\Rightarrow \quad r = \frac{(n+1)^2}{11} r_0$$

Substituting in equation (3), we get

$$\frac{11}{2\pi r_0} = \frac{R(11)^2 (n)(n+2)}{(n+1)}$$

$$\Rightarrow \quad (n+1) = (1.09 \times 10^7)(11)(2\pi) \times (0.529 \times 10^{-10})(n^2 + 2n)$$

Solving this equation, we get

$$n = 24$$

PROBLEM 13

A hydrogen-like atom (described by the Bohr model) is observed to emit six wavelengths, orginating from all possible transitions between a group of levels. These levels have energies between -0.85 eV and -0.544 eV (including both these values).

(a) Find the atomic number of the atom.

(b) Calculate the smallest wavelength emitted in these transitions.

(Take $hc = 1240$ eV-nm, ground state energy of hydrogen atom $= -13.6$ eV)

SOLUTION

(a) Total 6 lines are emitted. Therefore,

$$\frac{n(n-1)}{2} = 6$$

$$\Rightarrow \quad n = 4$$

So, transition is taking place between m^{th} energy state and $(m+3)^{th}$ energy state.

$$E_m = -0.85 \text{ eV}$$

$$\Rightarrow \quad -13.6\left(\frac{z^2}{m^2}\right) = -0.85$$

$$\Rightarrow \quad \frac{z}{m} = 0.25 \quad \ldots(1)$$

Similarly, $E_{m+3} = -0.544$ eV

$$\Rightarrow \quad -13.6 \frac{z^2}{(m+3)^2} = -0.544$$

$$\Rightarrow \quad \frac{z}{(m+3)} = 0.2 \quad \ldots(2)$$

Solving equations (1) and (2) for z and m, we get

$$m = 12 \text{ and } z = 3$$

(b) Smallest wavelength corresponds to maximum difference of energies which is obviously $(E_{m+3} - E_m)$.

$$\Rightarrow \quad \Delta E_{max} = -0.544 - (-0.85) = 0.306 \text{ eV}$$

$$\Rightarrow \quad \lambda_{min} = \frac{hc}{\Delta E_{max}} = \frac{1240}{0.306} = 4052.3 \text{ nm}$$

PROBLEM 14

A neutron of kinetic energy 65 eV collides inelastically with a singly ionized helium atom at rest. It is scattered at an angle of 90° with respect of its original direction.

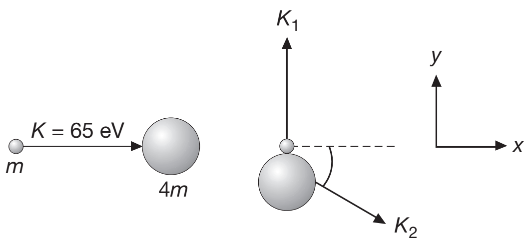
(a) Find the allowed values of the energy of the neutron and that of the atom after the collision.

(b) If the atom gets de-excited subsequently by emitting radiation, find the frequencies of the emitted radiation.

[Given: Mass of He atom $= 4\times(\text{mass of neutrons})$
Ionization energy of H atom = 13.6 eV]

SOLUTION

(a) Let K_1 and K_2 be the kinetic energies of neutron and helium atom after collision and ΔE be the excitation energy.



Applying Law of Conservation of Linear Momentum along x-direction, we get

$$p_i = p_f$$

$$\Rightarrow \quad \sqrt{2Km} = \sqrt{2(4m)K_2}\cos\theta \qquad \ldots(1)$$

Similarly, Applying Law of Conservation of Linear momentum along y-direction, we get

$$\sqrt{2K_1 m} = \sqrt{2(4m)K_2}\sin\theta \qquad \ldots(2)$$

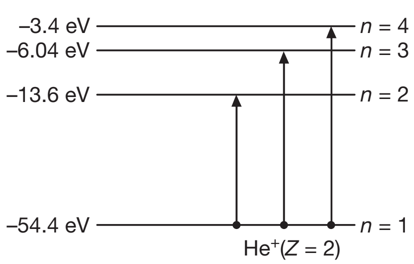
Squaring and adding equations (1) and (2), we get

$$K + K_1 = 4K_2 \qquad \ldots(3)$$

$$\Rightarrow \quad 4K_2 - K_1 = K = 65 \text{ eV} \qquad \ldots(4)$$

Now, during collision, electron can be excited to any higher energy state. Applying Law of Conservation of Energy, we get

$$K = K_1 + K_2 + \Delta E$$



$$\Rightarrow \quad 65 = K_1 + K_2 + \Delta E \qquad \ldots(5)$$

ΔE can have the following values,

$$\Delta E_1 = \{-13.6 - (-54.4)\} \text{ eV} = 40.8 \text{ eV}$$

Substituting in (5), we get

$$K_1 + K_2 = 24.2 \text{ eV} \qquad \ldots(6)$$

Solving (4) and (6), we get

$$K_1 = 6.36 \text{ eV and } K_2 = 17.84 \text{ eV}$$

Similarly, when we put $\Delta E = \Delta E_2$

$$\Rightarrow \quad \Delta E = \{-6.04 - (-54.4)\} \text{ eV}$$

$$\Rightarrow \quad \Delta E = 48.36 \text{ eV}$$

Put in equation (5), we get

$$K_1 + K_2 = 16.64 \text{ eV} \qquad \ldots(7)$$

Solving (4) and (7), we get

$$K_1 = 0.312 \text{ eV and } K_2 = 16.328 \text{ eV}$$

Similarly, when we put

$$\Delta E = \Delta E_3 = \{-3.4 - (-54.4)\} = 51 \text{ eV}$$

Put in equation (5), we get

$$K_1 + K_2 = 14 \text{ eV} \qquad \ldots(8)$$

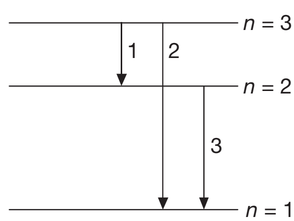
Now, solving (4) and (8), we get

$$K_1 = -1.8 \text{ eV and } K_2 = 15.8 \text{ eV}$$

But since the kinetic energy cannot have the negative values, the electron will not jump to third excited state i.e., $n = 4$.

Therefore, the allowed values of K_1 (KE of neutron) are 6.36 eV and 0.312 eV and of K_2 (KE of the atom) are 17.84 eV and 16.328 eV and the electron can jump upto second excited state only $(n = 3)$.

(b) Possible emission lines are only three as shown in figure. The corresponding frequencies are



$$\nu_1 = \frac{(E_3 - E_2)}{h}$$

$$\Rightarrow \quad \nu_1 = \frac{\{-6.04-(-13.6)\}\times 1.6\times 10^{-19}}{6.63\times 10^{-34}}$$

$$\Rightarrow \quad \nu_1 = 1.82\times 10^{15}\ \text{Hz}$$

$$\nu_2 = \frac{E_3 - E_1}{h}$$

$$\Rightarrow \quad \nu_2 = \frac{\{-6.04-(-54.4)\}\times 1.6\times 10^{-19}}{6.63\times 10^{-34}}$$

$$\Rightarrow \quad \nu_2 = 11.67\times 10^{15}\ \text{Hz and } \nu_3 = \frac{E_2 - E_1}{h}$$

$$\Rightarrow \quad \nu_3 = \frac{\{-13.6-(-54.4)\}\times 1.6\times 10^{-19}}{6.63\times 10^{-34}}$$

$$\Rightarrow \quad \nu_3 = 9.84\times 10^{15}\ \text{Hz}$$

Hence, the frequencies of emitted radiations are 1.82×10^{15} Hz, 11.67×10^{15} Hz and 9.84×10^{15} Hz

PROBLEM 15

The K-absorption edge of an unknown element is 0.171 Å.

(a) Identify the element.
(b) Find the average wavelengths of the K_α, K_β and K_γ lines.
(c) If a 100 eV electron strike the target of this element, what is the minimum wavelength of the X-ray emitted?

Given $hc = 12400$ eVÅ

SOLUTION

From Moseley's law, the wavelength of K series of X-rays is given by taking $b = 1$ in modified in Rydberg's formula. So,

$$\frac{1}{\lambda} = R(Z-1)^2\left(1-\frac{1}{n^2}\right);\ \text{for } K \text{ lines where, } n = 2, 3, 4, \ldots$$

(a) For K-absorption edge, substitute $n\to\infty$, in above expression, so we get

$$(Z-1) = \sqrt{\frac{1}{\lambda R}}$$

$$\Rightarrow \quad Z = \sqrt{\frac{1}{(0.171\times 10^{-10})(1.097\times 10^{7})}} + 1$$

$$\Rightarrow \quad Z = 74$$

The element is Tungsten

(b) For K_α line,

$$\frac{1}{\lambda_{K_\alpha}} = R(74-1)^2\left(1-\frac{1}{2^2}\right)$$

$$\Rightarrow \quad \lambda_{K_\alpha} = 0.228\ \text{Å}$$

For K_β line,

$$\frac{1}{\lambda_{K_\beta}} = R(74-1)^2\left(1-\frac{1}{3^2}\right)$$

$$\Rightarrow \quad \lambda_{K_\beta} = 0.192\ \text{Å}$$

For K_γ line,

$$\frac{1}{\lambda_{K_\gamma}} = R(74-1)^2\left(1-\frac{1}{4^2}\right)$$

$$\Rightarrow \quad \lambda_{K_\gamma} = 0.182\ \text{Å}$$

(c) The shortest wavelength corresponding to an electron with kinetic energy 100 eV is given by

$$\lambda_c = \frac{hc}{E} = \frac{12400}{100}\ \text{Å}$$

$$\Rightarrow \quad \lambda_c = 124\ \text{Å}$$

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. Assume an imaginary world, where angular momentum is quantized to even multiple of $\hbar\left(=\frac{h}{2\pi}\right)$. The longest possible wavelength emitted by hydrogen in the visible spectrum, for this hypothetical assumption is

(A) 700 nm (B) 486 nm
(C) 600 nm (D) 584 nm

2. If first excitation potential of a hydrogen-like atom is V_0 electron volt, then the ionization energy of this atom will be

(A) V_0 electron volt
(B) $\frac{3V_0}{4}$ electron volt
(C) $\frac{4V_0}{3}$ electron volt
(D) cannot be calculated by given information

3. The magnetic field at the centre (at nucleus) of the hydrogen like atoms (atomic number $=z$) due to the motion of electron in nth orbit is proportional to

(A) $\frac{n^3}{z^5}$ (B) $\frac{n^4}{z}$
(C) $\frac{z^2}{n^3}$ (D) $\frac{z^3}{n^5}$

4. A hydrogen atom has electron in the fourth energy level. The number of different possible photons lie in which of following series

(A) 3 Lyman, 2 Balmer, 1 Paschen
(B) 2 Lyman, 1 Balmer, 1 Paschen
(C) 2 Lyman, 1 Paschen, 1 Brackett
(D) 1 Lyman, 1 Balmer, 1 Paschen

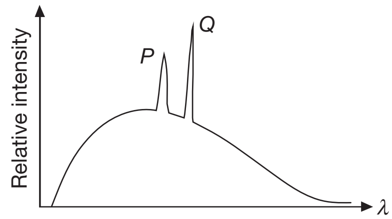
5. The angular momentum of an electron in an orbit is quantized because it is a necessary condition for the compatibility with

(A) the wave nature of electron.
(B) particle nature of electron.
(C) Pauli's exclusion behaviour.
(D) None of these.

6. A hydrogen atom is in fifth excited state. When the electron jumps to ground state, the velocity of recoiling hydrogen atom is

(A) 1.1 ms^{-1} (B) 4.2 ms^{-1}
(C) 8.4 ms^{-1} (D) 11.2 ms^{-1}

7. In a characteristic X-ray spectra of some atom superimposed on continuous X-ray spectra



(A) P represents K_α line
(B) Q represents K_β line
(C) Q and P represent K_α and K_β lines respectively
(D) Position of K_α and K_β depend on the particular atom

8. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the n^{th} state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the n^{th} state to the ground state. For large n, (A, B are constants)

(A) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$ (B) $\Lambda_n \approx A + B\lambda_n$
(C) $\Lambda_n^2 \approx A + B\lambda_n^2$ (D) $\Lambda_n^2 \approx \lambda$

9. According to Bohr model, the magnetic field at centre (at the nucleus) of a hydrogen atom due to motion of electron in the n^{th} orbit is proportional to

(A) $\frac{1}{n^3}$ (B) $\frac{1}{n^5}$
(C) n^5 (D) n^3

10. A particle moving with a velocity $\frac{1}{10^{\text{th}}}$ of that of light will cross a nucleus in about

(A) 10^{-47} s (B) 10^{-22} s
(C) 10^{-12} s (D) 10^{-8} s

11. In X-ray tube when the accelerating voltage V is halved, the difference between the wavelengths of K_α line and minimum wavelength of continuous X-ray spectrum
(A) remains constant
(B) becomes more than two times
(C) becomes half
(D) becomes less than two times

12. A H-atom moving with speed v makes a head on collision with a H-atom in rest. Both atoms are in ground state. The minimum value of velocity v for which one of atom may excite is
(A) $6.25\times10^4\ \text{ms}^{-1}$ (B) $8\times10^4\ \text{ms}^{-1}$
(C) $7.25\times10^4\ \text{ms}^{-1}$ (D) $13.6\times10^4\ \text{ms}^{-1}$

13. The minimum kinetic energy of an electron, hydrogen ion, helium ion required for ionization of a hydrogen atom is E_1 in case electron is collided with hydrogen atom. It is E_2 when hydrogen ion is collided and E_3 when helium ion is collided. Then
(A) $E_1 = E_2 = E_3$ (B) $E_1 < E_2 < E_3$
(C) $E_1 > E_3 > E_2$ (D) $E_1 > E_2 > E_3$

14. An α-particle after passing through a potential difference of V volt collides with a nucleus. If the atomic number of the nucleus is Z then the distance of closest approach of α-particle to the nucleus will be
(A) $14.4\left(\frac{Z}{V_0}\right)$ Å (B) $14.4\left(\frac{Z}{V_0}\right)$ m
(C) $14.4\left(\frac{Z}{V_0}\right)$ cm (D) All of these

15. An electron is lying initially in the $n=4$ excited state. The electron de-excites itself to go to $n=1$ state directly emitting a photon of frequency v_{41}. If the same electron first de-excites to $n=3$ state by emitting a photon of frequency v_{43} and then goes from $n=3$ to $n=1$ state by emitting a photon of frequency v_{31}, then
(A) $v_{41} = v_{43} + v_{31}$ (B) $v_{41} = v_{43} - v_{31}$
(C) $v_{43} = v_{41} + 2v_{31}$ (D) Data Insufficient

16. In Bohr's H atom, the graph of $\log\left(\frac{R}{R_0}\right)$ versus $\log n$ is
(A) a circle (B) an ellipse
(C) a parabola (D) a straight line

17. A diatomic molecule is made of two masses m_1 and m_2 which are separated by a distance r. If we calculate its rotational energy by applying Bohr's rule of angular momentum quantization, its energy will be given by (n is an integer)
(A) $\frac{n^2\hbar^2}{2(m_1+m_2)r^2}$ (B) $\frac{2n^2\hbar^2}{(m_1+m_2)r^2}$
(C) $\frac{(m_1+m_2)n^2\hbar^2}{2m_1m_2r^2}$ (D) $\frac{(m_1+m_2)^2n^2\hbar^2}{2m_1^2m_2^2r^2}$

18. If we take into account the reality that both the nucleus and electron revolve around their common centre of mass. During electron transition from a higher state, n_2, to a lower state, n_1, we find that the wavelength of the photon emitted is not given by the formula $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ where R is the Rydberg constant. The correct wavelength, in that case depends on mass of electron (m) and mass of the nucleus (M) and is given by
(A) $\frac{1}{\lambda} = R\frac{m}{M}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$
(B) $\frac{1}{\lambda} = R\left(1+\frac{m}{M}\right)\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$
(C) $\frac{1}{\lambda} = R\left(\frac{m}{n_1^2} - \frac{M}{n_2^2}\right)$
(D) $\frac{1}{\lambda} = \frac{RM}{M+m}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

19. When an electron has a transition from the state $(n+1)$ to state n where n is quite large, then the frequency of the emitted radiation (v) will vary with n as
(A) $v \propto \frac{1}{n^3}$ (B) $v \propto \frac{1}{n^2}$
(C) $v \propto \frac{1}{n}$ (D) $v \propto n$

20. The ratio of the frequencies of the long wavelength limits of the Lyman series and the Balmer series of hydrogen is
(A) $27:5$ (B) $4:1$
(C) $5:27$ (D) $1:4$

21. The spin angular momentum of an electron is equal to
(A) $\frac{h}{\pi}$ (B) $\frac{h}{\sqrt{2}\pi}$
(C) $\frac{h}{2\pi}$ (D) $\frac{2h}{\sqrt{2}\pi}$

22. The energy levels of a certain atom for first, second and third levels are E, $\frac{4E}{3}$ and $2E$ respectively. A photon of wavelength λ is emitted for a transition $3\to1$, What will be the wavelength of emission for transition $2\to1$?

(A) $\frac{\lambda}{3}$ (B) $\frac{4\lambda}{3}$

(C) $\frac{3\lambda}{4}$ (D) 3λ

23. If R be the Rydberg's constant, the energy of an electron in the ground state of the H atom is given by

(A) $-\frac{Rh}{c}$ (B) $-\frac{1}{Rhc}$

(C) $-Rhc$ (D) $\frac{hc}{R}$

24. A nucleus of mass number A, originally at rest, emits an α-particle with speed v. The daughter nucleus recoils with a speed

(A) $\frac{2v}{(A+4)}$ (B) $\frac{4v}{(A+4)}$

(C) $\frac{4v}{(A-4)}$ (D) $\frac{2v}{(A-4)}$

25. The de-Broglie wavelength of electron in ground state of a hydrogen atom is

(A) 0.53 Å (B) 1.06 Å

(C) 1.52 Å (D) 3.33 Å

26. A hydrogen atom emits ultraviolet radiation of 102.5 nm. Then the quantum numbers of the states involved in the transition are

(A) $\infty \to 1$ (B) $5 \to 1$

(C) $3 \to 1$ (D) $4 \to 1$

27. The difference between nth and $(n+1)$th Bohr's radius of H atom is equal to it's $(n-1)$th Bohr's radius. The value of n is

(A) 1 (B) 2

(C) 3 (D) 4

28. The angular momentum of an electron in hydrogen atom is $\frac{2h}{\pi}$. Kinetic energy of this electron is

(A) 4.35 eV (B) 1.51 eV

(C) 0.85 eV (D) 13.6 eV

29. A particle of mass m moves in a circular orbit in a central potential field $U(r)=\frac{1}{2}kr^2$. If Bohr's quantization conditions are applied, radii of possible orbitals and energy levels vary with quantum number n as

(A) $r_n \propto n$, $E_n \propto n$ (B) $r_n \propto \sqrt{n}$, $E_n \propto n$

(C) $r_n \propto \sqrt{n}$, $E_n \propto \frac{1}{n}$ (D) $r_n \propto n^2$, $E_n \propto \frac{1}{n^2}$

30. The angular momentum of an electron in the hydrogen atoms is $\frac{3h}{2\pi}$, where, h is the Planck's constant. The kinetic energy of this electron is

(A) 1.51 eV (B) 3.4 eV

(C) 4.35 eV (D) 6.8 eV

31. K_α wavelength of an unknown element is 0.0709 nm. Identify the element.

(A) *Co* (B) *Cu*

(C) *Mn* (D) *Mo*

32. Stopping potentials of 24 kV, 100 kV, 110 kV, 110 kV are measured for photoelectrons emitted from a certain element when it is irradiated with monochromatic X-rays. If the element is used as a target in an X-ray tube. The energy of K_α line is

(A) 54 keV (B) 76 keV

(C) 88 keV (D) 32 keV

33. A potential difference of 20 kV is applied across an X-ray tube. The minimum wavelength of X-rays generated is

(A) 0.26 Å (B) 0.62 Å

(C) 0.16 Å (D) 0.50 Å

34. The target element in an X-ray tube must have a high

(A) atomic number only

(B) mass number only

(C) melting point only

(D) both atomic number and melting point

35. White X-rays are called 'white' due to the fact that

(A) they are electromagnetic radiations having nature same as that of white light.

(B) they are produced most abundantly in X-ray tubes.

(C) they have a continuous wavelength range.

(D) they can be converted to visible light using coated screens and photographic plates are affected by them just like light.

36. The atomic number (Z) of an element whose K_α wavelength is λ is 11. The atomic number of an element whose K_α wavelength is 4λ is

(A) 6 (B) 11

(C) 44 (D) 4

37. In Bohr's Model of hydrogen atom. The ratio between the period of revolution of an electron in orbit of $n=1$ to the period of revolution of the electron in the orbit $n=2$ is

(A) 1 : 2 (B) 2 : 1
(C) 1 : 4 (D) 1 : 8

38. According to Bohr correspondence principle when quantum number is very large
(A) frequency of revolution of electron in an orbit is equal to the frequency of photon emitted when electron jumps from that orbit to next lower orbit
(B) classical physics approaches quantum physics
(C) wavelength of electron de Broglie wavelength does not depend on kinetic energy of electron
(D) energy of electrons are not quantized

39. An electron with kinetic energy 5 eV is incident on a H-atom in its ground state. The collision
(A) must be elastic
(B) may be partially elastic
(C) must be completely inelastic
(D) may be partially inelastic

40. In a hydrogen atom, the binding energy of the electron in the ground state is E_1, then the frequency of revolution of the electron in the nth orbit is
(A) $\frac{E_1 n}{h}$ (B) $\frac{E_1}{nh}$
(C) $\frac{2E_1 n}{h}$ (D) $\frac{2E_1}{nh}$

41. The average life time of an excited state of hydrogen is of the order of 10^{-8} s. The number of orbits an electron makes, when it is in the state $n = 2$ (before it starts a transition to a lower state) will be (take T_0 to be time period of revolution in ground state in s.)
(A) $\left(\frac{8}{T_0}\right)\times 10^{-8}$ (B) $T_0 \times 10^{-8}$
(C) $\frac{10^{-8}}{8T_0}$ (D) $\frac{T_0}{8\times 10^{-8}}$

42. Magnetic moment due to the motion of the electron in n^{th} energy state of hydrogen atom is proportional to
(A) n^0 (B) n
(C) n^3 (D) n^5

43. The angular momentum of an electron in a hydrogen atom is proportional to
(A) $\frac{1}{\sqrt{r}}$ (B) $\frac{1}{r}$
(C) $\sqrt{r}$ (D) r^2

44. Which one of the following statements is wrong in the context of X-rays generated from an X-ray tube?
(A) Wavelength of characteristic X-rays decreases when the atomic number of the target increases
(B) Cut-off wavelength of the continuous X-rays depends on the atomic number of the target
(C) Intensity of the characteristic X-rays depends on the electrical power given to the X-ray tube
(D) Cut-off wavelength of the continuous X-rays depends on the energy of the electrons in the X-ray tube

45. In Bohr's Model, when an electron revolves around the nucleus in an orbit
(A) It radiates energy
(B) It absorbs energy
(C) Its total mechanical energy is conserved
(D) Its angular momentum changes continuously

46. If an electron has, orbital angular momentum quantum number $l = 7$, then it will have an orbital angular momentum equal to
(A) $7\left(\frac{h}{2\pi}\right)$ (B) $42\left(\frac{h}{2\pi}\right)$
(C) $\sqrt{7}\left(\frac{h}{2\pi}\right)$ (D) $\sqrt{56}\left(\frac{h}{2\pi}\right)$

47. The ratio between total acceleration of the electron in singly ionized helium atom and hydrogen atom (both in ground state) is
(A) 1 (B) 8
(C) 4 (D) 16

48. The radius of the first orbit of hydrogen is 0.528 Å. The radius of the second orbit of hydrogen is
(A) 4.752 Å (B) 2.112 Å
(C) 0.071 Å (D) 0.142 Å

49. Protons and singly ionized atoms of U^{235} and U^{238} are passed in turn (which means one after the other and not at the same time) through a velocity selector and then enter a uniform magnetic field. The protons describe semicircles of radius 10 mm. The separation between the ions of U^{235} and U^{238} after describing semicircle is given by
(A) 60 mm (B) 30 mm
(C) 2350 mm (D) 2380 mm

50. The speed of the electron in the first orbit (ground state) of the hydrogen atom in terms of velocity of light c is
(A) $\frac{c}{2}$ (B) $\frac{c}{11}$
(C) $\frac{c}{137}$ (D) $\frac{c}{274}$

51. Speed of electron in fourth orbit for He^+ is

(A) $\frac{e^2}{2h\varepsilon_0}$ (B) $\frac{e^2}{4h\varepsilon_0}$

(C) $\frac{e^2}{h\varepsilon_0}$ (D) $\frac{2e^2}{h\varepsilon_0}$

52. A hydrogen atom emits a photon corresponding to an electron transition from $n=5$ to $n=1$. The recoil speed of hydrogen atom is almost

(A) 10^{-4} ms^{-1} (B) 2×10^{-2} ms^{-1}

(C) 4 ms^{-1} (D) 8×10^{2} ms^{-1}

53. The radius of hydrogen atom in its ground state is 5.3×10^{-11} m. After collision with an electron it is found to have a radius of 21.2×10^{-11} m. What is the principal quantum number n of the final state of the atom?

(A) $n=4$ (B) $n=2$

(C) $n=16$ (D) $n=3$

54. Check the correctness of the following statements about Bohr model of hydrogen atom.

(i) The acceleration of the electron in $n=2$ orbit is more than in $n=1$ orbit

(ii) The angular momentum of the electron in $n=2$ orbit is more than in $n=1$ orbit

(iii) The KE of the electron in $n=2$ orbit is less than in $n=1$ orbit

(A) all the statements are correct

(B) only (i) and (ii) are correct

(C) only (ii) and (iii) are correct

(D) only (iii) and (i) are correct

55. The electronic transition in Li^{2+} ion that emits radiation of wavelength same as the wavelength of second Balmer line of H-atom is

(A) $4\to3$ (B) $6\to3$

(C) $12\to6$ (D) $12\to9$

56. A μ-meson is a particle which has charge same as that of the electron and mass 207 times the mass of electron. A μ-mesonic atom is formed when a μ-meason is captured by proton. The ionization energy of such an atom is

(A) 3.8 keV (B) 5 keV

(C) 2.8 keV (D) 20 keV

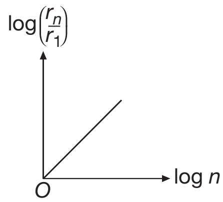
57. The ratio between total acceleration of the electron is singly ionized helium atom and hydrogen atom (both in ground state) is

(A) 1 (B) 8

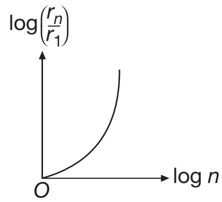
(C) 4 (D) 16

58. In hydrogen atom, the radius of n^{th} Bohr orbit is r_n. The graph between $\log\left(\frac{r_n}{r_1}\right)$ and $\log n$ will be

(A)



(B)



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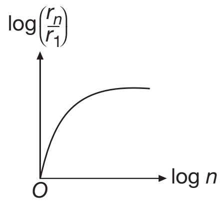
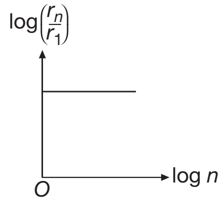


(D)



59. The ratio of the maximum wavelength of the Lyman series in hydrogen spectrum to the maximum wavelength in the Paschen series is

(A) $\frac{3}{105}$ (B) $\frac{6}{15}$

(C) $\frac{52}{7}$ (D) $\frac{7}{108}$

60. The Rydberg constant for hydrogen atom is R. The Rydberg constant for positronium (a bound system composed of a positron and an electron) is

(A) R (B) $2R$

(C) $\frac{R}{2}$ (D) $\frac{R}{4}$

61. The velocity of an electron in the first orbit of H atom is v. The velocity of an electron in the 2nd orbit of He^+ is

(A) $2v$ (B) v

(C) $\frac{v}{2}$ (D) $\frac{v}{4}$

62. The radius of electron's second stationary orbit in Bohr's atom is a_0. The radius of the third orbit will be

(A) $3a_0$ (B) $2.25a_0$

(C) $9a_0$ (D) $\frac{a_0}{3}$

63. Electrons with de-Broglie wavelength λ fall on the target in an X-ray tube. The cut-off wavelength of the emitted X-rays is

(A) $\lambda_0=\frac{2mc\lambda^2}{h}$ (B) $\lambda_0=\frac{2h}{mc}$

(C) $\lambda_0=\frac{2m^2c^2\lambda^3}{h^2}$ (D) $\lambda_0=\lambda$

64. The shortest wavelength of the Paschen series of a hydrogen like atom (atomic number $= Z$) is the same as the shortest wavelength of the Lyman series of hydrogen atom. The value of Z

(A) 2 (B) 3
(C) 4 (D) 5

65. The potential difference across the Coolidge tube is 20 kV and 10 mA current flows through the voltage supply. Only 0.5% of the energy carried by the electrons striking the target is converted into X-rays. The power carried by X-ray beam is P.

(A) $P = 0.1\text{ W}$ (B) $P = 1\text{ W}$
(C) $P = 2\text{ W}$ (D) $P = 10\text{ W}$

66. When 24.8 KeV X-rays strike a material, the photoelectrons emitted from K shell are observed to move in a circle of radius 23 mm in a magnetic field of $2\times10^{-2}\text{ T}$. The binding energy of K-shell electrons is

(A) 6.2 keV (B) 5.4 keV
(C) 7.4 keV (D) 8.6 keV

67. For large principal quantum number n the frequency of revolution of electron is equal to the frequency of transition of the electron between two adjacent orbits. This frequency f is proportional to

(A) $\frac{1}{n^2}$ (B) $\frac{1}{n^3}$
(C) $\frac{1}{n}$ (D) $\frac{1}{n^0}$

68. An electron typically spends about 10^{-8} sec in an excited state before it drops to lower state by emitting a photon. The revolutions do an electron in an $n = 2$ Bohr orbit make in 10^{-8} sec is

(A) 8×10^6 rev (B) 8 rev
(C) 10×10^8 rev (D) 80 rev

69. A metal of atomic number Z is used as a target in a Coolidge tube. Let ν be the frequency of the K_α line. For a number of values of Z and ν which plot gives a straight line.

(A) ν vs Z (B) ν vs $\frac{1}{Z}$
(C) $\sqrt{\nu}$ vs Z (D) ν vs $\sqrt{Z}$

70. The shortest wavelength in the Lyman series of hydrogen spectrum is 912 Å corresponding to a photon energy of 13.6 eV. The shortest wavelength in the Balmer series is

(A) 3648 Å (B) 228 Å
(C) 6566 Å (D) 8208 Å

71. On transition to the ground state from the excited state of He^+ ion, two photons are emitted in succession with wavelengths 1026.7 Å and 304 Å. Assuming $R = 1.097\times10^7\text{ m}^{-1}$, then the quantum number n corresponding to the exciting state of He^+ ion is

(A) 4 (B) 6
(C) 2 (D) 1

72. An electron jumps from the fourth orbit to the 2nd orbit of the hydrogen atom. If $R = 10^7\text{ m}^{-1}$, the frequency of the emitted radiation will be

(A) $\frac{3}{16}\times10^5$ Hz (B) $\frac{3}{16}\times10^{15}$ Hz
(C) $\frac{9}{16}\times10^{15}$ Hz (D) $\frac{3}{4}\times10^{15}$ Hz

73. If E_n and J_n are the magnitude of total energy and angular momentum of electron in the n^{th} Bohr orbit respectively, then

(A) $E_n \propto J_n$ (B) $E_n \propto \frac{1}{J_n}$
(C) $E_n \propto J_n^2$ (D) $E_n \propto \frac{1}{J_n^2}$

74. The recoil speed of a hydrogen atom after it emits a photon in going from $n = 5$ state to $n = 1$ state is

(A) 4.718 ms^{-1} (B) 7.418 ms^{-1}
(C) 4.178 ms^{-1} (D) 7.148 ms^{-1}

75. An excited atom at rest emits a photon of frequency f. Now if the same excited atom is moving with a speed $v(\ll c)$ and emits a photon in the direction of its motion, then the frequency of the photon

(A) remains f
(B) increases by a fraction $\frac{\Delta f}{f} = \frac{v}{c}$
(C) decreases by a fraction $\frac{\Delta f}{f} = \frac{v}{c}$
(D) nothing can be predicted

76. A hydrogen atom emits a photon of energy 12.1 eV. Its orbital angular momentum changes by ΔL. Then ΔL equals

(A) 1.05×10^{-34} Js (B) 2.11×10^{-34} Js
(C) 3.16×10^{-34} Js (D) 4.22×10^{-34} Js

77. The ground state energy of hydrogen atom is –13.6 eV. The potential energy of the electron in this state is

(A) –27.2 eV (B) –13.6 eV
(C) +13.6 eV (D) 0 eV

78. When an electron makes transition from one energy level to the other in an atom then which of the following quantities is conserved?

(A) Angular momentum
(B) Linear momentum
(C) Mechanical energy
(D) None of the above

79. The Bohr radius of the fifth electron of phosphorus atom (atomic number 15) acting as a dopant in silicon (relative dielectric constant 12) is

(A) 380.9 pm (B) 390.8 pm
(C) 930.8 pm (D) 830.9 pm

80. How would the wavelength of the electromagnetic radiation absorbed change if the number of atoms undergoing the same electronic transition was increased?

(A) There would be no change
(B) It would shift to shorter wavelengths
(C) It would be shifted to longer wavelengths
(D) It would be depend on the magnitude of the increase in the number of atoms

81. When an electron revolves around the nucleus, then ratio of magnetic moment to angular momentum is

(A) $\frac{e}{2m}$ (B) $\frac{2e}{m}$
(C) $\frac{e}{m}$ (D) $\left(\frac{e}{m}\right)^2$

82. If an electron revolves around a proton in an orbit of radius r then its time period T is proportional to

(A) r^2 (B) $r^{\frac{3}{2}}$
(C) r^3 (D) r

83. The radius of second orbit of an electron in hydrogen atom is 2.116 Å. The de-Broglie wavelength associated with this electron in this orbit would be

(A) 1.058 Å (B) 2.116 Å
(C) 6.64 Å (D) 13.28 Å

84. Find the longest wavelength present in the radiation emitted when hydrogen atoms excited to $n = 3$ states return to their ground state.

(A) 103 nm (B) 122 nm
(C) 656 nm (D) 912 nm

85. The angular momentum (L) and radius (r) of a hydrogen atom are related to each other as

(A) $Lr = \text{constant}$ (B) $Lr^2 = \text{constant}$
(C) $Lr^4 = \text{constant}$ (D) $Lr^{-\frac{1}{2}} = \text{constant}$

86. An electron collides with a hydrogen atom in its ground state and excites it to $n = 3$. The energy given to hydrogen atom in this inelastic collision is
[Neglect the recoiling of hydrogen atom]

(A) 10.2 eV (B) 12.1 eV
(C) 12.5 eV (D) None of these

87. The total energy of the electron in the hydrogen atom in the ground state is -13.6 eV. The kinetic energy of the electron is

(A) 0 (B) 13.6 eV
(C) 6.8 eV (D) –13.6 eV

88. The energy of an electron in the n^{th} orbit of positronium (a bound system composed of positron and an electron) is

(A) $-\frac{13.6}{n^2}$ eV (B) $-\frac{27.2}{n^2}$ eV
(C) $-\frac{54.4}{n^2}$ eV (D) $-\frac{6.8}{n^2}$ eV

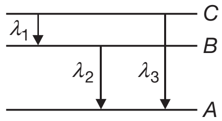
89. The binding energy of the electron with $n \to \infty$ in H atom is

(A) ZERO (B) infinite
(C) 13.6 eV (D) 10.2 eV

90. The wavelength of radiation emitted is λ_0 when an electron in hydrogen atom jumps from the third orbit to second. If in the hydrogen atom itself, the electron jumps from fourth orbit to second orbit, the wavelength of emitted radiation will be

(A) $\frac{16}{25}\lambda_0$ (B) $\frac{20}{27}\lambda_0$
(C) $\frac{27}{20}\lambda_0$ (D) $\frac{25}{16}\lambda_0$

91. Energy levels A, B, C of a certain atom correspond to increasing values of energy i.e., $E_A < E_B < E_C$. If $\lambda_1, \lambda_2, \lambda_3$ are the wavelengths of radiations corresponding to the transitions C to B, B to A and C to A respectively, which of the following statements is correct?



(A) $\lambda_3 = \lambda_1 + \lambda_2$ (B) $\lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$

(C) $\lambda_1 + \lambda_2 + \lambda_3 = 0$ (D) $\lambda_3^2 = \lambda_1^2 + \lambda_2^2$

92. Ionization potential of hydrogen atom is 13.6 eV. Hydrogen atoms in the ground state are excited by monochromatic radiation of photon energy 12.1 eV. According to Bohr's theory, the spectral lines emitted by hydrogen will be

(A) one (B) two
(C) three (D) four

93. If elements of quantum number greater than n were not allowed, the number of possible elements in nature would have been

(A) $\frac{1}{2}n(n+1)$ (B) $\left\{\frac{n(n+1)}{2}\right\}^2$

(C) $\frac{1}{6}n(n+1)(2n+1)$ (D) $\frac{1}{3}n(n+1)(2n+1)$

94. The ionisation potential of mercury is 10.5 volt. To gain energy sufficient enough to ionise mercury, an electron must travel in an electric field of 1.5×10^6 Vm^{-1} a distance of

(A) 7 μm (B) 6 μm
(C) 14 μm (D) 12 μm

95. The absorption coefficient of X-rays for a given wavelength is larger for

(A) lithium (B) lead
(C) aluminium (D) copper

96. The maximum KE found in α particles of natural origin is 7.7 MeV. Then, if these are used in alpha scattering experiment, the distance of closest approach from gold nucleus is about

(A) 1.5×10^{-15} m (B) 3×10^{-14} m
(C) 41×10^{-15} m (D) 3×10^{-16} m

97. The speed of an electron in the fourth orbit of Be^{+++} atom is

(A) c (B) $\frac{c}{10}$

(C) $\frac{e^2}{2h\varepsilon_0 c}$ (D) $\frac{e^2}{2h\varepsilon_0}$

98. For X-rays the wavelength λ (in Å) in terms of energy E (in keV) is given by

(A) $\lambda = \frac{6.20}{E}$ (B) $\lambda = \frac{12.40}{E}$

(C) $\lambda = \frac{3.10}{E}$ (D) $\lambda = \frac{24.80}{E}$

99. The magnetic field at the centre of a hydrogen atom due to the motion of the electron in the first Bohr orbit is B. The magnetic field at the centre due to the motion of the electron in the second Bohr orbit will be

(A) $\frac{B}{4}$ (B) $\frac{B}{8}$

(C) $\frac{B}{32}$ (D) $\frac{B}{64}$

100. Suppose, the electron in a hydrogen atom makes transition from $n = 3$ to $n = 2$ in 10^{-8} s. The order of the torque acting on the electron in this period, using the relation between torque and angular momentum as discussed in the chapter on rotational dynamics is

(A) 10^{-34} Nm (B) 10^{-26} Nm
(C) 10^{-42} Nm (D) 10^{-8} Nm

101. When a hydrogen like atom in excited state makes a transition from excited state to ground state the most energetic photon has an energy of 52.224 eV and the least energetic photon has an energy of 1.224 eV. The atomic number of the hydrogen like atom is

(A) 4 (B) 6
(C) 2 (D) 8

102. The ionisation potential of hydrogen atom is 13.6 volts. The energy of the atom in $n = 2$ state will be

(A) -10.2 eV (B) -6.4 eV
(C) -4.4 eV (D) -3.4 eV

103. In a transition to a state of excitation energy 10.2 eV, a hydrogen atom emits a 4960 Å photon. if $hc = 12400$ eVÅ, then the binding energy of the electron in the initial state is

(A) 1.51 eV (B) 3.4 eV
(C) 0.54 eV (D) 0.87 eV

104. Let ν_1 be the frequency of the series limit of the Lyman series, ν_2 be the frequency of the first line of the Lyman series, and ν_3 be the frequency of the series limit of the Balmer series.

(A) $\nu_1 - \nu_2 = \nu_3$ (B) $\nu_2 - \nu_1 = \nu_3$

(C) $\nu_3 = \frac{1}{2}(\nu_1 + \nu_2)$ (D) $\nu_1 + \nu_2 = \nu_3$

105. A hydrogen atom in ground state absorbs 10.2 eV of energy. The orbital angular momentum of the electron is increased by

(A) 1.05×10^{-34} Js (B) 2.11×10^{-34} Js

(C) 3.16×10^{-34} Js (D) 4.22×10^{-34} Js

106. According to Bohr's theory of hydrogen atom, the product of the binding energy of the electron in the nth orbit and its radius in the nth orbit

(A) is inversely proportional to n^3

(B) is proportional to n^2

(C) has a constant value 7.2 eV Å

(D) has a constant value of 10.2 eV Å

107. Balmer gives an equation for wavelength of visible radiation of H-spectrum as $\lambda = \frac{kn^2}{n^2 - 4}$. The value of k in terms of Rydberg's constant R, is

(A) R (B) $4R$

(C) $\frac{R}{4}$ (D) $\frac{4}{R}$

108. Let the potential energy of a hydrogen atom in the ground state be zero. Then its energy in the first excited state will be

(A) 10.2 eV (B) 13.6 eV

(C) 23.8 eV (D) 27.2 eV

109. A hydrogen atom moving at speed v collides with another hydrogen atom kept at rest. Find the minimum value of v for which one of the atoms may get ionized. The mass of a H-atom $= 1.67 \times 10^{-27}$ kg

(A) 7.2×10^4 ms^{-1} (B) 5.1×10^4 ms^{-1}

(C) 8.8×10^4 ms^{-1} (D) 4×10^4 ms^{-1}

110. In an excited state of hydrogen like atom an electron has a total energy of -3.4 eV. If the kinetic energy of the electron is E and its de Broglie wavelength is λ, then

(A) $E = 6.8$ eV, $\lambda \sim 6.6 \times 10^{-10}$ m

(B) $E = 3.4$ eV, $\lambda \sim 6.6 \times 10^{-10}$ m

(C) $E = 3.4$ eV, $\lambda \sim 6.6 \times 10^{-11}$ m

(D) $E = 6.8$ eV, $\lambda \sim 6.6 \times 10^{-11}$ m

111. The order of energies of energy levels A, B and C is $E_A < E_B < E_C$. If the wavelength corresponding to transition $C \to B$, $B \to A$ and $C \to A$ are λ_1, λ_2 and λ_3 respectively, then which of the following relation is correct ?

(A) $\lambda_1 + \lambda_2 + \lambda_3 = 0$ (B) $\lambda_3^2 = \lambda_1^2 + \lambda_2^2$

(C) $\lambda_3 = \lambda_1 + \lambda_2$ (D) $\lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$

112. The shortest wavelength of the Brackett series of a hydrogen like atom having atomic number z, is the same as the shortest wavelength of the Balmer series of hydrogen atom. The value of z is

(A) 2 (B) 3

(C) 4 (D) 6

113. The difference in the angular momentum associated with the electron in the two successive orbits of the hydrogen atom is

(A) $\frac{h}{\pi}$ (B) $\frac{h}{2}$

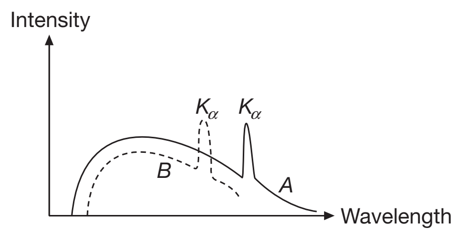
(C) $\frac{h}{2\pi}$ (D) $2h$

114. The shortest wavelength in the Lyman series is 912 Å. Then the longest wavelength in this series is

(A) 1216 Å (B) 1824 Å

(C) 2434 Å (D) 3648 Å

115. The figure represents the observed intensity of X-rays emitted by two different tubes A and B as a function of wavelength λ. For the tube A, the potential difference between the filament and target is V_A and atomic number of target is Z_A. For the tube B, corresponding potential difference is V_B and the atomic number is Z_B. The solid curve is for tube A and dotted curve for tube B, then



(A) $Z_A > Z_B$, $V_A > V_B$ (B) $Z_A = Z_B$, $V_A = V_B$

(C) $Z_A < Z_B$, $V_A > V_B$ (D) $Z_A < Z_B$, $V_A < V_B$

116. Magnetic moment due to the motion of the electron in the n^{th} energy state of hydrogen atom is proportional to

(A) n (B) n^0

(C) n^5 (D) n^3

117. The maximum angular speed of the electron of a hydrogen atom in a stationary orbit is

(A) 6.2×10^{5} rads^{-1} (B) 4.1×10^{16} rads^{-1}
(C) 2.4×10^{10} rads^{-1} (D) 9.2×10^{6} rads^{-1}

118. The ratio of the speed of the electron in the first Bohr orbit of He^{+} and the speed of light is equal to (where e, h and c have their usual meanings)

(A) $\frac{e^2}{2h\varepsilon_0}$ (B) $\frac{e^2}{4h\varepsilon_0 c}$
(C) $\frac{e^2}{h\varepsilon_0 c}$ (D) $\frac{e^2}{2h\varepsilon_0 c}$

119. A hydrogen atom is in an excited state of principle quantum number n. It emits a photon of wavelength λ when returns to the ground state. The value of n is

(A) $\sqrt{\lambda R(\lambda R-1)}$ (B) $\sqrt{\frac{\lambda R}{\lambda R-1}}$
(C) $\sqrt{\frac{(\lambda R-1)}{\lambda R}}$ (D) $\sqrt{\lambda(R-1)}$

120. In uranium $(Z=92)$ the K absorption edge is 0.107 Å and the K_α line is 0.126 Å the, wavelength of the L absorption edge is

(A) 0.7 Å (B) 1 Å
(C) 2 Å (D) 3.2 Å

121. If the K_α radiation of $Mo(Z=42)$ has a wavelength of 0.71 Å the wavelength of the corresponding radiation for $Cu(Z=29)$ is

(A) 1 Å (B) 2 Å
(C) 1.52 Å (D) 1.25 Å

122. The binding energy of an electron in the ground state of He is equal to 24.6 eV. The energy required to remove both the electrons is

(A) 24.6 eV (B) 38.2 eV
(C) 49.2 eV (D) 79 eV

123. The wavelengths of K_α X-rays of the metals A and B are $\frac{4}{1875R}$ and $\frac{1}{675R}$ respectively, where R is the Rydberg's constant. The number of elements lying between A and B according to their atomic numbers, is

(A) 3 (B) 6
(C) 5 (D) 4

124. A material whose K absorption edge is 0.15 Å is irradiated with 0.1 Å X-rays. The maximum kinetic energy of photoelectrons that are emitted from K-shell is

(A) 41 keV (B) 51 keV
(C) 61 keV (D) 71 keV

125. In a hydrogen atom, the electron is in nth excited state. It comes down to first excited state by emitting ten different wavelengths. The value of n is

(A) 6 (B) 7
(C) 8 (D) 9

126. The element which has K_α X-ray line whose wavelength is 0.18 nm is

(A) Iron (B) Cobalt
(C) Nickel (D) Copper

127. The ratio of the energies of the hydrogen atom in the first to the second excited state

(A) 4 : 1 (B) 1 : 4
(C) 4 : 9 (D) 9 : 4

128. The wavelength of first line of Balmer series is 6563 Å. The wavelength of first line of Lyman series will be

(A) 1215.4 Å (B) 2500 Å
(C) 7500 Å (D) 600 Å

129. The ratio of the wavelength of first line of Lyman series to the first line of Balmer series is

(A) 1 : 4 (B) 5 : 27
(C) 27 : 20 (D) 20 : 27

130. The wavelengths involved in the spectrum of deuterium $\left({}_1^2D\right)$ are slightly different from that of hydrogen spectrum because

(A) Sizes of two nuclei are different
(B) Nuclear forces are greater in case of deuterium
(C) Masses of the two nuclei are different
(D) Force of attraction between electron and nucleus is different in the two cases

131. The potential energy associated with an electron in the orbit

(A) increases with the increase in radii of the orbit
(B) decreases with the increase in the radii of the orbit
(C) remains the same with the change in the radii of the orbit
(D) None of the above

132. The ratio of minimum to maximum wavelength in Balmer series is

(A) $\frac{5}{9}$ (B) $\frac{5}{36}$

(C) $\frac{1}{4}$ (D) $\frac{3}{4}$

133. The wavelength K_α of X–rays produced by the X-ray tube is $0.76\ \text{Å}$. The atomic number of the anode material of the tube is

(A) 30 (B) 40
(C) 50 (D) 60

134. The graph of $\log\left(\frac{R}{R_0}\right)$ versus $\log A$ ($R =$ radius of a nucleus and $A =$ mass number) is

(A) a circle
(B) an ellipse
(C) a parabola
(D) a straight line

135. In Rutherford experiment the number of α-particles scattered at angle 90° is 25. How many particles are scattered at an angle 60°

(A) 100 (B) 85
(C) 70 (D) 55

136. The photon radiated from hydrogen corresponding to second line of Lyman series is absorbed by a hydrogen-like atom X in second excited state. As a result, the hydrogen-like atom X makes a transition of n^{th} orbit

(A) $X = \text{He}^+,\ n = 4$ (B) $X = \text{Li}^{++},\ n = 6$

(C) $X = \text{He}^+,\ n = 6$ (D) $X = \text{Li}^{++},\ n = 9$

137. The kinetic energy of an electron in the first orbit of H atom is $13.6\ \text{eV}$. The total energy of an electron in the second orbit of He^+ is

(A) 13.6 eV (B) 3.4 eV

(C) –13.6 eV (D) –3.4 eV

138. The ratio of the maximum wavelength of the Lyman series in hydrogen spectrum to the maximum wavelength in the Paschen series is

(A) $\frac{3}{105}$ (B) $\frac{6}{15}$

(C) $\frac{52}{7}$ (D) $\frac{7}{108}$

139. The ratio of areas between the electron orbits for the first excited state to the ground state for the hydrogen atom is

(A) 2 : 1 (B) 4 : 1
(C) 8 : 1 (D) 16 : 1

140. Electrons are accelerated through a potential difference of $V_0 = 10\ \text{kV}$. The minimum wavelength λ_{min} of the X-ray emitted is

(A) 1.24×10^{-10} m (B) 1.24×10^{-7} m

(C) 1240×10^{-10} m (D) 4000×10^{-10} m

141. The wavelength of the first line of the Balmer series in the hydrogen atom spectrum is

(A) 6563 Å (B) 6365 Å
(C) 6563 m (D) 6563 cm

142. In a Coolidge tube, the potential difference used to accelerate the electrons is increased from $12.4\ \text{kV}$ to $24.8\ \text{kV}$. As a result the difference between λ_{K_α} and λ_{min} increases three fold. The wavelength of K_α line is $\left(\frac{hc}{e} = 12.4\ \text{kV Å}\right)$

(A) 1 Å (B) 1.25 Å

(C) 1.5 Å (D) None of these

143. An electron with kinetic energy E eV collides with a hydrogen atom in the ground state. The collision is observed to be elastic for

(A) $0 < E < \infty$ (B) $0 < E < 10.2$ eV

(C) $0 < E < 13.6$ eV (D) $0 < E < 3.4$ eV

144. When the number of electrons striking the target material in Coolidge's tube (i.e., X-ray tube) is increased keeping the potential difference same, then

(A) intensity of X-rays increase
(B) wavelength of X-rays photons increase
(C) frequency of X-rays photons increase
(D) energy of X-rays photons increase

145. Which of the following is true for X-rays

(A) wavelength of continuous X-rays does not depend on potential difference
(B) wavelength of discrete X-rays does not depend on potential difference
(C) discrete X-rays have energy of the order of MeV
(D) continuous X-rays have energy of the order of MeV

146. The angular momentum of the electron in hydrogen atom in the ground state is

(A) $2h$ (B) $\frac{h}{2}$

(C) $\frac{h}{2\pi}$ (D) $\frac{h}{4\pi}$

147. In a hypothetical system, a particle of mass m and charge $-3q$ is moving around a very heavy particle of charge q. Assume that Bohr's model is applicable to this system, then velocity of mass m in first orbit is

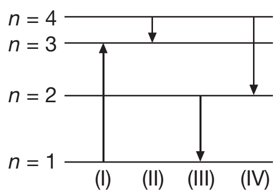
(A) $\frac{3q^2}{2\varepsilon_0 h}$ (B) $\frac{q^2}{\varepsilon_0 h}$

(C) $\frac{3q}{2\pi\varepsilon_0 h}$ (D) $\frac{3q}{4\pi\varepsilon_0 h}$

148. An electron in the ground state of hydrogen has an angular momentum L_1, and an electron in the first excited state of lithium has an angular momentum L_2.

(A) $L_1 = L_2$ (B) $L_1 = 4L_2$
(C) $L_2 = 2L_1$ (D) $L_1 = 2L_2$

149. The diagram shows the energy levels for an electron in a certain atom. Which transition shown represents the emission of a photon with the most energy ?



(A) I (B) II
(C) III (D) IV

150. The longest wavelength present in the Balmer series of hydrogen, corresponding to H_α line is

(A) 656 nm (B) 565 nm
(C) 400 nm (D) 700 nm

151. The quantity $\frac{e^2}{2h\varepsilon_0 c}$ has a value

(A) $\frac{1}{137}$ ms^{-1} (B) $\frac{2}{137}$ ms^{-1}

(C) $\frac{1}{137}$ (D) $\frac{2}{137}$

152. Which the following series fall in the visible range of electromagnetic spectrum ?

(A) Brackett series (B) Lyman series
(C) Balmer series (D) Paschen series

153. The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer) is

(A) 802 nm (B) 823 nm
(C) 1882 nm (D) 1648 nm

154. An X-ray tube produces continuous X-rays of wavelength 0.1 Å and greater. Then potential difference applied to tube must be

(A) 1.24 kV (B) 12.4 kV
(C) 124 kV (D) 6.62 kV

155. The minimum energy in electron volt required to strip a ten times ionised sodium atom (i.e., $Z = 11$) of its last electron is

(A) 13.6 eV (B) 1.23 eV
(C) 150 eV (D) 1.65 keV

156. de-Broglie wavelength of an electron in the nth Bohr orbit is λ_n and the angular momentum is J_n, then

(A) $J_n \propto \lambda_n$ (B) $\lambda_n \propto \frac{1}{J_n}$

(C) $\lambda_n \propto J_n^2$ (D) $\lambda_n \propto \frac{1}{J_n^2}$

157. For a lead $^{208}_{82}$Pb (μ-mesonic atom) the energy of the photon given off in the first Lyman transition

(A) 11 MeV (B) 14 meV
(C) 18 MeV (D) 20 MeV

158. When an electron in the hydrogen atom in ground state absorbs a photon of energy 12.1 eV, its angular momentum

(A) decreases by 2.11×10^{-34} Js

(B) decreases by 1.055×10^{-34} Js

(C) increases by 2.11×10^{-34} Js

(D) increases by 1.055×10^{-34} Js

159. In Rutherford experiment, α-particles are scattered by nucleus having charge $100e$. Initial kinetic energy of α-particles is 6 MeV. The size of the nucleus is

(A) 10^{-15} m (B) 5×10^{-14} m
(C) 10^{-13} m (D) 10^{-16} m

160. Difference between n^{th} and $(n+1)^{\text{th}}$ Bohr's radius of hydrogen atom is equal to $(n-1)^{\text{th}}$ Bohr's radius. The value of n is

(A) 1 (B) 2
(C) 3 (D) 4

161. In hydrogen and hydrogen like atoms the ratio of difference of energies $E_{2n} - E_n$ and $E_{4n} - E_{2n}$ varies with atomic number z and principle quantum number n as

(A) $\frac{z^2}{n^2}$ (B) $\frac{z^4}{n^4}$

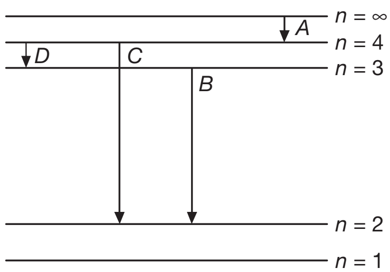
(C) $\frac{z}{n}$ (D) $\left(\frac{z}{n}\right)^0$

162. In Rutherford scattering experiment, the number of α-particles scattered at $60°$ is 9×10^6. The number of α-particles scattered at $120°$ will be

(A) 15×10^6 (B) $\frac{3}{5}\times10^6$

(C) 10^6 (D) $\frac{5}{9}\times10^6$

163. Consider the electronic energy level diagram of *H*-atom. The photons associated with shortest and longest wavelengths would be emitted from the atom by the transitions labelled



(A) *D* and *C* respectively
(B) *C* and *A* respectively
(C) *C* and *D* respectively
(D) *A* and *C* respectively

164. In a hypothetical atom like that of hydrogen, the mass of the electrons is doubled. The energy (in eV) and radius (in Å) of the first Bohr's orbit will be (Take Bohr's radius of Hydrogen to be 0.54 Å)

(A) −27.2 , 0.27 (B) −27.2 , 0.54
(C) −13.6 , 0.27 (D) −13.6 , 0.54

165. When a hydrogen atom emits a photon during the transition $n=5$ to $n=1$, its recoil speed is approximately

(A) 0.1 mms^{-1} (B) 3 mms^{-1}
(C) 4 ms^{-1} (D) 800 ms^{-1}

166. The force acting on the electron in a hydrogen atom depends on the principal quantum number as

(A) $F\propto n^2$ (B) $F\propto \frac{1}{n^2}$

(C) $F\propto n^4$ (D) $F\propto \frac{1}{n^4}$

167. The approximate value of quantum number *n* for the circular orbit of hydrogen 0.0001 mm in diameter is

(A) 1000 (B) 60
(C) 10000 (D) 31

168. The largest distance between the interatomic planes of a crystal is 10^{-7} cm. The upper limit for the wavelength of X-rays which can be usefully studied with this crystal is

(A) 1 Å (B) 2 Å
(C) 10 Å (D) 20 Å

169. Of the various series of the hydrogen spectrum, the one which lies wholly in the ultra violet region is

(A) Lyman series (B) Balmer series
(C) Paschen series (D) Brackett series

170. If the short series limit of the Balmer series for hydrogen is 3646 Å. Calculate the atomic number of the element which gives *X*-ray wavelength down to 1.0 Å. Identify the element

(A) $Z=21$ (B) $Z=31$
(C) $Z=11$ (D) $Z=5$

171. A potential difference of 10^3 V is applied across an X-ray tube. The ratio of the de-Broglie wavelength of the incident electrons to the shortest wavelength of X-rays produced is

$\left(\frac{e}{m}=1.8\times10^{14}\ \text{Ckg}^{-1} \text{ for an electron}\right)$

(A) $\frac{1}{10^4}$ (B) $\frac{1}{100}$

(C) $\frac{1}{20}$ (D) 1

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. The radius of the orbit of an electron in a Hydrogen like atom is $4.5a_0$, where a_0 is the Bohr radius. Its orbital angular momentum is $\frac{3h}{2\pi}$. It is given that h is Planck constant and R is Rydberg constant. The possible wavelength(s), when the atom de-excites, is (are)

(A) $\frac{9}{32R}$ (B) $\frac{9}{16R}$

(C) $\frac{9}{5R}$ (D) $\frac{4}{3R}$

2. H^+, He^+ and O^{++} all having the same kinetic energy pass through a region in which there is a uniform magnetic field perpendicular to their velocity. The masses of H^+, He^+ and O^{++} are 1 u, 4 u and 16 u respectively.
(A) H^+ will be deflected the most.
(B) O^{++} will be deflected the most.
(C) He^+ and O^{++} will be deflected equally.
(D) All will be deflected equally.

3. An electron makes transition inside a hydrogen atom. The orbital angular momentum of the electron may change by
(A) h (B) $\frac{h}{\pi}$
(C) $\frac{h}{2\pi}$ (D) $\frac{h}{4\pi}$

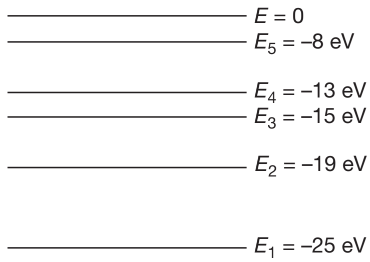
4. An electron is excited from a lower energy state to a higher energy state in a hydrogen atom. Which of the following decrease in the excitation?
(A) potential energy
(B) angular speed
(C) kinetic energy
(D) angular momentum

5. In Bohr's model of the hydrogen atom, let R, V, T and E represent the radius of the orbit, speed of the electron, time period of revolution of electron and the total energy of the electron respectively. The quantities proportional to the quantum number n are
(A) VR (B) RE
(C) $\frac{V}{E}$ (D) $\frac{T}{R}$

6. The wavelength of K_α X-rays for lead isotopes Pb^{208}, Pb^{206}, Pb^{204} are λ_1, λ_2 and λ_3 respectively. Then
(A) $\lambda_1 = \lambda_2 = \lambda_3$ (B) $\lambda_1 > \lambda_2 > \lambda_3$
(C) $\lambda_1 < \lambda_2 < \lambda_3$ (D) $\lambda_2 = \sqrt{\lambda_1 \lambda_3}$

7. Suppose the potential energy between electron and proton at a distance r is given by $-\frac{Ke^2}{3r^3}$. If m be the mass of electron, then application of Bohr's theory to hydrogen atom in this case shows that
(A) energy in the nth orbit is proportional to n^6
(B) energy is proportional to m^{-3}
(C) energy the nth orbit is proportional to n^{-2}
(D) energy is proportional to m^3

8. Consider an atom whose energy level diagram is shown in figure.



Suppose an atom starts at level 3 then
(A) Shortest wavelength photon the atom can emit is 1.24×10^{-7} m
(B) Longest wavelength photon that it can absorb is 6.2×10^{-7} m
(C) Lowest frequency photon that can ionize the atom is 3.62×10^{15} Hz
(D) The number of ways of de-excitation of atom to ground state is 3

9. A positronium atom consists of a positron and electron revolving around their common centre of mass. Then compared to hydrogen atom the positronium atom has
(A) ground state energy half of hydrogen atom
(B) Rydberg constant half of hydrogen atom
(C) radius of first orbit of electron double that in case of hydrogen atom
(D) velocity of electron in first orbit same as in case of hydrogen atom

10. The ground state and first excited state energies of hydrogen atom are -13.6 eV and -3.4 eV respectively. If potential energy in ground state is taken to be zero. Then
(A) potential energy in the first excited state would be 20.4 eV
(B) total energy in the first excited state would be 23.8 eV
(C) kinetic energy in the first excited state would be 3.4 eV
(D) total energy in the ground state would be 13.6 eV

11. When a hydrogen atom is excited from ground state to first excited state then
(A) its kinetic energy increases by 10.2 eV.
(B) its kinetic energy decreases by 10.2 eV.
(C) its potential energy increases by 20.4 eV.
(D) its angular momentum increases by 1.05×10^{-34} Js

12. An electron orbiting in a circular orbit around the nucleus of an atom
(A) has a magnetic dipole moment
(B) exerts an electric force on the nucleus equal to that on it by the nucleus

(C) does produces a magnetic induction at the nucleus
(D) has a net energy inversely proportional to its distance from the nucleus

13. The electron in a hydrogen atom makes a transition from 2nd excited state to the ground state. Then
(A) it's K.E. increases and total energy decreases
(B) both its K.E. and total energy increases
(C) frequency of emitted photons may be 4.6×10^{14} Hz
(D) frequency of emitted photons must be 2.9×10^{15} Hz

14. The energy, the magnitude of linear momentum and orbital radius of an electron in a hydrogen atom corresponding to the quantum number n are E, P and r respectively. Then according to Bohr's theory of hydrogen atom
(A) PEr is proportional to $\frac{1}{n}$
(B) $\frac{P}{E}$ is proportional to n
(C) Er is constant for all orbits
(D) Pr is proportional to n

15. If potential energy in hydrogen atom with electron in ground state is taken to be 13.6 eV, then
(A) potential energy in the first excited state would be 34 eV
(B) total energy in the first excited state would be 37.4 eV
(C) kinetic energy in the first excited state would be 44.2 eV
(D) total energy in the ground state would be 27.2 eV

16. If electron of the hydrogen atom is replaced by another particle of same charge but of double the mass, then select the correct option(s).
(A) Bohr radius will increase to double value
(B) Ionisation energy of the atom will be doubled
(C) Speed of the new particle in a given state will be one fourth of what electron will possess in the same orbit
(D) Gap between energy levels will now be doubled

17. An electron in hydrogen atom first jumps from second excited state to ground state and then from first excited state to ground state. Let the ratio of wavelength, momentum and energy of photons emitted in these two cases be a, b and c respectively, then

(A) $c=\frac{1}{a}$ (B) $a=\frac{9}{4}$
(C) $b=\frac{5}{27}$ (D) $c=\frac{5}{27}$

18. A hydrogen like atom of atomic number Z is in an excited state of quantum number $2n$. It can emit a maximum energy photon of 204 eV. It makes a transition to quantum state n, a photon of energy 40.8 eV is emitted, then
(A) $Z=2$ (B) $Z=4$
(C) $n=1$ (D) $n=2$

19. Which of the following physical quantities in hydrogen atom are independent of the quantum number n. The symbols have their usual meanings.
(A) vr (B) $\frac{E}{v^2}$
(C) Er (D) v^2r

20. Select the correct statement(s).
(A) X-rays travel faster than ultraviolet rays in vacuum.
(B) Balmer series of H-spectrum is found in visible region.
(C) The characteristic X-rays are produced due to jumping of electrons from higher to lower shell.
(D) In photoelectric emission process, the maximum energy of the photoelectrons must increase with increasing intensity of incident light.

21. Total energy of electron in the first orbit of hydrogen atom is equal to the
(A) total energy of electron in 2nd orbit of He^+
(B) total energy of electron in 3rd orbit of He^+
(C) total energy of electron in 2nd orbit of Li^{++}
(D) total energy of electron in 3rd orbit to Li^{++}

22. Which of the following transitions in He^+ ion will give rise to a spectral line having the same wavelength as some spectral line in the hydrogen atom?
(A) $n=4$ to $n=2$ (B) $n=6$ to $n=2$
(C) $n=6$ to $n=3$ (D) $n=8$ to $n=4$

23. When atomic number Z is doubled in atoms, which of the following statements are consistent with Bohr's theory?
(A) Energy of a state is doubled.
(B) Radius of an orbit is doubled.
(C) Velocity of electrons in an orbit is doubled.
(D) Radius of an orbit is halved.

24. Let A_n be the area enclosed by the n^{th} orbit in a hydrogen atom. The graph of $\log_e\left(\frac{A_n}{A_1}\right)$ against $\log_e n$
(A) will pass through the origin.
(B) will have a slope of four units at all points.
(C) will be a monotonically increasing nonlinear curve.
(D) will be a straight line.

25. A particular hydrogen like atom has its ground state total energy -54.4 eV, then
(A) its atomic number is 2
(B) it can absorb a photon of 40.8 eV
(C) in its ground state it cannot emit photon
(D) for its ground state its potential energy is -108.8 eV and kinetic energy is $+54.4$ eV

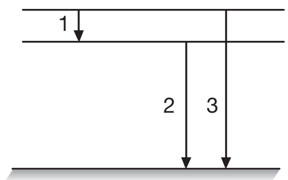
26. If the potential energy of the electron in the first allowed orbit in hydrogen atom is E then its
(A) ionisation potential is $-\frac{E}{2}$
(B) kinetic energy is $-\frac{E}{2}$
(C) total energy is $\frac{E}{2}$
(D) None of these

27. Hydrogen atoms absorb radiations of wavelength λ_0 and consequently emit radiations of 6 different wavelengths of which two wavelengths are shorter than λ_0. Choose the correct alternative(s).
(A) The final excited state of the atoms is $n = 4$
(B) The initial state of the atoms may be $n = 2$
(C) The initial state of the atoms may be $n = 3$
(D) There are three transitions belonging to Lyman series

28. For the electron in the n^{th} allowed orbit is
(A) the linear momentum is proportional to $\frac{1}{n}$
(B) the radius is proportional to n
(C) the kinetic energy is proportional to $\frac{1}{n^2}$
(D) the angular momentum is proportional to n

29. Whenever hydrogen atom emits a photon in the Balmer series
(A) it may emit another photon in the Balmer series.
(B) it must emit another photon in the Lyman series.
(C) the second photon, if emitted, will have a wavelength of about 122 nm.
(D) it may emit a second photon but the wavelength of this photon cannot be predicted.

30. The wavelengths and frequencies of photons in transitions 1, 2 and 3 for hydrogen like atom are λ_1, λ_2, λ_3, v_1, v_2 and v_3 respectively. Then



(A) $v_3 = \frac{v_1 v_2}{v_1 + v_2}$
(B) $v_3 = v_1 + v_2$
(C) $\lambda_3 = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$
(D) $\lambda_3 = \lambda_1 + \lambda_2$

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** If the accelerating potential in an X-ray tube is increased, the wavelengths of the characteristic X-rays do not change.
Statement-2: When an electron beam strikes the target in an X-ray tube, part of the kinetic energy is converted into X-ray energy.

2. **Statement-1:** According to classical theory, the proposed path of an electron in Rutherford atom model will be parabolic.
Statement-2: According to electromagnetic theory, an accelerated particle continuously emits radiation.

3. **Statement-1:** An electron in hydrogen atom passes from $n = 4$ to $n = 1$ level. The maximum number of photons that can be emitted is 6.
Statement-2: No. of photons emitted can never be more than 5.

4. **Statement-1:** The wavelength of first Balmer line of deuterium is slightly more than that of hydrogen.
Statement-2: In the centre of mass of an atom reference frame both nucleus and electron are non-stationary.

5. **Statement-1:** In a hydrogen atom energy of emitted photon corresponding to transition from $n=2$ to $n=1$ is much greater as compared to transition from $n\to\infty$ to $n=2$.
Statement-2: Wavelength of photon is directly proportional to the energy of emitted photon.

6. **Statement-1:** Magnetic moment of an atom is due to both, the orbital motion and spin motion of every electron.
Statement-2: A charged particle produces a magnetic field.

7. **Statement-1:** Total energy in an orbit is negative in an atom.
Statement-2: Electron is bounded by electrostatic attraction between electron and nucleus.

8. **Statement-1:** Between any two given energy levels, the number of absorption transition is always less than number of emission transition.
Statement-2: Absorption transition starts from the lowest energy level only and may end at any higher level. But emission transitions may starts from any higher energy level and end at any energy level below it.

9. **Statement-1:** Total energy of revolving electron in any stationary orbit is negative.
Statement-2: Energy is a scalar quantity and it can take positive and negative value.

10. **Statement-1:** In outermost stationary orbit, energy of electron is least negative.
Statement-2: In outermost orbit, electron is at maximum distance from nucleus.

11. **Statement-1:** Speed of electron in the n^{th} orbit of hydrogen atom is $v_H \propto \frac{1}{n}$.
Statement-2: Speed of electron in n^{th} orbit of hydrogen like atom is $v_{H\text{ Like}} \propto \frac{Z^2}{n}$

12. **Statement-1:** Atomic hydrogen gas excites to third excited state. The number of spectral lines in emission spectrum obtained is 6.
Statement-2: Number of spectral lines in emission spectrum can never be less than 6.

13. **Statement-1:** In a hydrogen atom energy of emitted photon corresponding to transition from $n=2$ to $n=1$ is much greater as compared to transition from $n\to\infty$ to $n=2$.
Statement-2: Wavelength of photon is directly proportional to the energy of emitted photon

14. **Statement-1:** The difference between the wavelengths of series limit of the Lyman series of spectral lines and that (the series limit) of the Paschen series is equal to the wavelength of a spectral line of the Lyman series (for the hydrogen atom).
Statement-2: The wave number of an atomic transition can be calculated from the formula $\bar{v} = R\left(\frac{1}{m^2}-\frac{1}{n^2}\right)$, where $\bar{v}$ is the wave number, R is the Rydberg's constant, m and n are the quantum numbers of the initial and final states.

15. **Statement-1:** If the current in the filament of electron gun in a X–ray tube is increased, the penetration power of X–rays is increased.
Statement-2: Increasing current increases the number of electrons emitted by the electron gun.

16. **Statement-1:** It is necessary to keep high vacuum in Coolidge tube to produce X–rays.
Statement-2: High vacuum is kept in Coolidge tube so that the electron emitting from the filament of the tube may not lose their energy in colliding with the atom of the gas in the tube.

17. **Statement-1:** If maximum frequency of Balmer and Paschen series is f_1 and f_2 respectively, then frequency of first line of Balmer series is $(f_1 - f_2)$.
Statement-2: Difference of energy level between two orbits is constant for an atom.

18. **Statement-1:** The wavelength of the K_α line from an element and that of the K_β line satisfy the relation $\lambda_{K_\alpha} > \lambda_{K_\beta}$
Statement-2: The energy separation between the K_α levels is smaller than that of the K_β levels

19. **Statement-1:** The wave number corresponding to the transition between the atomic levels $n=3$ and $n=2$ of a hydrogen atom i.e. $\bar{v}_{32}$, is related to the wave-numbers $\bar{v}_{21}$ and $\bar{v}_{31}$ for a hydrogen atom by the relation $\bar{v}_{31} = \bar{v}_{21} + \bar{v}_{32}$.
Statement-2: The wave-number $\bar{v}_{mn} = \frac{1}{\lambda_{mn}}$ for a transition is given by the expression $\frac{1}{\lambda_{mn}} = R\left(\frac{1}{n^2}-\frac{1}{m^2}\right)$, where R is the Rydberg constant and m, n are integers representing the initial and final principal quantum numbers.

20. **Statement-1:** When light is passed through a sample of hydrogen atoms in ground state, then wavelengths of absorption lines are same as wavelengths of lines of Lyman series in emission spectrum.
Statement-2: In ground state hydrogen atom will absorb only those radiation which will excite to higher energy level.

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

A beam of alpha particles is incident on a target made of lead. A particular alpha particle is incident head-on to a particular lead nucleus and stops 6.50×10^{-14} m away from the centre of the nucleus (This point lies well outside the nucleus). Assume that the lead nucleus which has 82 protons, remains at rest. The mass of alpha particle is 6.64×10^{-27} kg. Based on the information provided, answer the following questions.

1. The electric potential energy at the instant that alpha particle stops is

(A) 36.3 MeV (B) 45 MeV
(C) 3.63 MeV (D) None of these

2. The initial kinetic energy of the alpha particle is

(A) 36.3 MeV (B) 0.36 MeV
(C) 3.63 MeV (D) None of these

Comprehension 2

An energy of 68 eV is required to excite an electron in hydrogen like atom from its second Bohr orbit to third. The nuclear charge is Ze. Given that $hc = 12375$ eVÅ. Based on above information, answer the following questions.

3. The value of atomic number Z is given by

(A) 3 (B) 4
(C) 5 (D) 6

4. Kinetic energy of the electron in the first Bohr orbit is

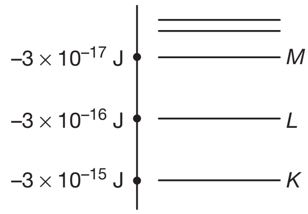
(A) 4.896 eV (B) 48.96 eV
(C) 489.6 eV (D) 0.4896 eV

5. The wavelength of the electromagnetic radiation required to eject the electron from the first orbit to infinity is given by

(A) 2.528 Å (B) 25.28 Å
(C) 252.8 Å (D) 0.2528 Å

Comprehension 3

Simplified picture of electron energy levels in a certain atom bombarded with high energy electrons is shown in the figure.



The impact of one of these electrons has caused the complete removal of *K*-level and is filled by an electron from the *L*-level with a certain amount of energy being released during the transition. This energy may appear as *X*-ray or may all be used to eject an *M*-level electron from the atom. Based on above information, answer the following questions.

6. The minimum potential difference through which electron may be accelerated from rest to cause the ejection of *K*-level electron from the atom is

(A) 1.875×10^{4} V (B) 3×10^{-15} V
(C) 3.33×10^{14} V (D) 10 V

7. Energy released when *L*-level electron moves to fill the vacancy in the *K*-level is

(A) 3×10^{-15} J (B) 3×10^{-17} J
(C) 2.7×10^{-15} J (D) 2.7×10^{-16} J

8. Kinetic energy of the electron emitted from the *M*-level is

(A) 3×10^{-16} J (B) 3×10^{-17} J
(C) 2.7×10^{-15} J (D) 2.67×10^{-15} J

Comprehension 4

The surface temperature of Sun is estimated by finding the most probable energy E for photons emitted by a black body at temperature T. The intensity (I) of the radiation from sun is proportional to $E^5 e^{-\frac{E}{k_B T}}$. If n_E is the number of photons with energy E, then distribution of n_E is given by $P(n_E) \approx \frac{I(E)}{E}$. Also, we know that the Balmer lines of hydrogen span the visible frequency range and the human eye has evolved to be the most sensitive to sunlight and assuming that the visible light is the most probable frequency band of the light emitted by the sun, answer the following questions.

9. The most probable energy E for photons emitted by a black body of temperature T is
(A) KT (B) $2KT$
(C) $3KT$ (D) $4KT$

10. The maximum energy of the Balmer lines which fall in the visible range is
(A) 1.9 eV (B) 3.4 eV
(C) 5.2 eV (D) 6.4 eV

11. If the human eye is sensitive to sunlight, the maximum surface temperature of sun is
(A) 1×10^4 K (B) 2×10^4 K
(C) 3×10^4 K (D) 4×10^4 K

Comprehension 5

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let N be the number density of free electrons, each of mass m. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency ω_p, which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_p, all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals. Based on above information, answer the following questions.

12. Taking the electronic charge as e and the permittivity as ε_0, use dimensional analysis to determine the correct expression for ω_p
(A) $\sqrt{\dfrac{Ne}{m\varepsilon_0}}$ (B) $\sqrt{\dfrac{m\varepsilon_0}{Ne}}$
(C) $\sqrt{\dfrac{Ne^2}{m\varepsilon_0}}$ (D) $\sqrt{\dfrac{m\varepsilon_0}{Ne^2}}$

13. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N\approx4\times10^{27}\ \text{m}^{-3}$. Take $\varepsilon_0\approx10^{-11}$ and $m\approx10^{-30}$, where these quantities are in proper SI units
(A) 800 nm (B) 600 nm
(C) 300 nm (D) 200 nm

Comprehension 6

A mixture of hydrogen atoms (in their ground state) and hydrogen like ions in their first excited state are being excited by electrons, which have been accelerated by same potential difference V. After excitation when they come into ground state, the wavelengths of emitted light are found in the ratio $1:5$, then

14. The minimum value of V for which both the atoms get excited after collision with electrons is
(A) 47.6 V (B) 13.6 V
(C) 10.2 V (D) 15 V

15. Atomic number of other ion is
(A) 2 (B) 1
(C) 3 (D) 4

16. The energy of light emitted by hydrogen atom is
(A) 47.6 eV (B) 13.6 eV
(C) 10.2 eV (D) 15 eV

17. Energy of light emitted by hydrogen ions is
(A) 47.6 eV (B) 13.6 eV
(C) 51 eV (D) 10.2 eV

Comprehension 7

The electric current in an X-ray tube (from the target to the filament) operating at 40 kV is 10 mA. Assume that on an average, 1% of the total kinetic energy of the electrons hitting the target is converted into X-rays. Based on above information, answer the following questions.

18. The total power emitted as X-rays is
(A) 40 W (B) 400 W
(C) 4 W (D) 0.4 W

19. The heat produced in the target every second is
(A) 29.6 J (B) 396 J
(C) 3600 J (D) 360 J

20. Minimum wavelength of X-ray produced is nearly
(A) 0.3 Å (B) 3 Å
(C) 30 Å (D) 300 Å

Comprehension 8

A sample of hydrogen atoms initially in its ground state are irradiated with light of different wavelengths and effects produced are observed.

21. If sample is irradiated with light of wavelength 85.5 nm, maximum kinetic energy of electrons leaving the gas is
(A) 0.9 eV
(B) 1.2 eV
(C) 10.2 eV
(D) Electrons cannot leave the gas

22. Least amount of energy that must be given to hydrogen atom initially in its ground level so that it can emit the H_α line in the Balmer series

(A) 12.09 eV (B) 10.2 eV
(C) 3.4 eV (D) 12.25 eV

23. For this least amount of energy given to the hydrogen atom, the maximum wavelength of photons emitted during de-excitation of atom will be

(A) 656 nm (B) 103 nm
(C) 240 nm (D) 346 nm

Comprehension 9

Consider a different atomic model in which electron revolves around the nucleus (proton) at a separation r under the action of force which is different from the electrostatic force of attraction. The potential energy between an electron and the proton due to this force is given by $U=-\frac{k}{r^4}$ where k is a constant. Based on above information, answer the following questions.

24. The radius of nth Bohr's orbit is

(A) $r=\frac{\pi}{nh}\sqrt{km}$ (B) $r=\frac{2\pi}{nh}\sqrt{km}$
(C) $r=\frac{4\pi}{nh}\sqrt{km}$ (D) $r=\frac{8\pi}{nh}\sqrt{km}$

25. The velocity in the nth orbit is given by

(A) $v=\frac{nh}{8\pi^2 m\sqrt{km}}$ (B) $v=\frac{n^2h}{8\pi^2 m\sqrt{km}}$
(C) $v=\frac{nh^2}{4\pi^2 m\sqrt{km}}$ (D) $v=\frac{n^2h^2}{8\pi^2 m\sqrt{km}}$

26. The kinetic energy of the electron in nth orbit is given by

(A) $KE=\frac{n^4h^4}{64\pi^4m^2k}$ (B) $KE=\frac{n^4h^4}{128\pi^4m^2k}$
(C) $KE=\frac{n^4h^4}{16\pi^4k^2}$ (D) $KE=-\frac{n^2h^2}{16k^2\pi^4m}$

27. The potential energy of the electron in the nth orbit is given by

(A) $PE=-\frac{n^4h^4}{64k\pi^4m^2}$ (B) $PE=-\frac{n^4h^4}{128k\pi^4m^2}$
(C) $PE=-\frac{n^2h^2}{128k\pi^4m^2}$ (D) $PE=-\frac{n^4h^4}{256k\pi^4m^2}$

28. The total energy of the electron in the nth orbit is given by

(A) $TE=-\frac{n^4h^4}{128\pi^4m^2k}$ (B) $TE=\frac{n^4h^4}{128\pi^4m^2k}$
(C) $TE=\frac{n^4h^4}{256k\pi^4m^2}$ (D) $TE=-\frac{n^4h^4}{256k\pi^4m^2}$

Comprehension 10

A 100 eV electron collides with a stationary helium ion (He^+) in its ground state and excites it to a higher level. After collision, He^+ ion emits two photons in succession with wavelength 1085 Å and 304 Å. Taking $hc = 12375$ eVÅ, answer the following questions.

29. The energy absorbed by He^+ during collision is

(A) 34.3 eV (B) 42.6 eV
(C) 52.1 eV (D) 47.9 eV

30. The principal quantum number of exited state is

(A) $n=2$ (B) $n=3$
(C) $n=4$ (D) $n=5$

31. The energy of electron after collision is

(A) 16.7 eV (B) 37.4 eV
(C) 47.9 eV (D) 52.1 eV

Comprehension 11

An electron orbits a stationary nucleus of charge +ze where z is a constant and e is the magnitude of electronic charge. It requires 47.2 eV to excite the electron form the second Bohr orbit to third Bohr orbit. Based on above information, answer the following questions.

32. The value of z is

(A) 5 (B) 4
(C) 3 (D) 2

33. The radius of first Bohr orbit is

(A) 0.53 Å (B) 0.106 Å
(C) 0.318 Å (D) 0.53 nm

Comprehension 12

In quantum mechanics, some quantities are discrete and cannot be continuous. One of these quantities is the energy. Energy can only take certain values, like E_1, E_2, E_3, E_4, ..., which are called energy levels. The energy cannot take any values between E_1 and E_2 or E_2 and E_3 or E_3 and E_4, etc. Certain transitions from one energy level to another result in the emission of a photon of radiation, whereas others can only take place if a photon is absorbed. The energy levels in a newly discovered gas are expressed as

$$En = -\frac{E_1 z^2}{n^2}$$

where, $-E_1 z^2$ is the ground state energy. Taking $z = 1$ for simplicity, but not assuming that the gas is hydrogen, an experiment is designed to measure the energy as a function of the level. The results obtained in the experiment are given in the table.

n	E_n(eV)
2	–144
3	–64
4	–36

Based on the information given, answer the following questions.

34. The ionization energy of the gas is

(A) 244 eV (B) 576 eV

(C) 144 eV (D) +13.6 eV

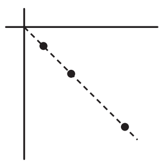
35. The ground state energy is

(A) –144 eV (B) +144 eV

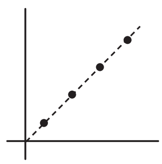
(C) –244 eV (D) None of these

36. The graph of E_n versus $\frac{1}{n^2}$ is best represented as

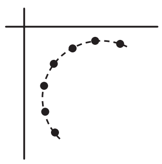
(A)



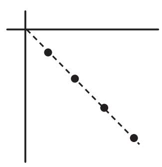
(B)



(C)



(D)



Comprehension 13

An electron is orbiting in a circular orbit of radius r under the influence of a constant magnetic field of strength B. Assuming that Bohr's postulate regarding the quantisation of angular momentum holds good for this electron of charge e and mass m, answer the following questions.

37. The radius of nth orbit of the electron will be

(A) $\sqrt{\frac{2nh}{\pi Be}}$ (B) $\sqrt{\frac{nh}{2\pi Be}}$

(C) $\sqrt{\frac{neh}{2\pi Be}}$ (D) $\sqrt{\frac{2neh}{\pi Be}}$

38. The kinetic energy of the electron in nth orbit is

(A) $nh\left(\frac{Be}{m}\right)$ (B) $2nh\left(\frac{Be}{m}\right)$

(C) $\frac{nh}{2}\left(\frac{Be}{m}\right)$ (D) $\frac{nh}{4}\left(\frac{Be}{m}\right)$

39. Potential energy of interaction between the magnetic moment of the orbital current due to the electron moving in its orbit and the magnetic field B is

(A) $\frac{nhBe}{2m}$ (B) $\frac{nhBe}{m}$

(C) $\frac{2nhBe}{m}$ (D) $\frac{nhBe}{4m}$

40. Total magnetic flux due to the magnetic field B passing through the nth orbit is

(A) $\frac{\pi}{2}\frac{nh}{e}$ (B) $\frac{\pi nh}{e}$

(C) $\frac{4\pi nh}{e}$ (D) $\frac{2\pi nh}{e}$

Comprehension 14

According to Bohr's theoretical model, the nucleus of the hydrogen atom is infinitely heavy when compared with electron and so it remains stationary while the electron revolves around it. However, practically, the nucleus has a finite mass, and both the electron and the nucleus revolve about their common centre of mass with a common angular velocity ω. Let us make correction in Bohr theory for the finite mass of the nucleus. Assuming m to be the mass of electron, M_H to be the mass of hydrogen nucleus and r the separation between them. Based on the information provided, answer the following questions.

41. The angular momentum of the atom will be constant about an axis passing through

(A) nucleus

(B) electron

(C) centre of mass of the atom

(D) any axis perpendicular to the line joining the electron and nucleus

42. The angular momentum of the atom about the centre of mass will be

(A) $mr^2\omega$ (B) $m(r-x)^2\omega$

(C) $\frac{(M_H+M)^2 r^2\omega}{M_H m^2}$ (D) $\left(\frac{M_H m}{H_H+m}\right)r^2\omega$

43. The new ground state energy of electron will be

(A) more than that found with Bohr's theoretical model.

(B) equal to that found with Bohr's theoretical model.

(C) smaller than that found with Bohr's theoretical model.
(D) data insufficient to arrive at a conclusion.

Comprehension 15

In atom X, a single electron orbits around a stationary nucleus of charge $+Ze$ where Z is a constant and e is the magnitude of the electronic charge. It requires 47.2 eV to excite the electron from the second Bohr orbit to the third Bohr orbit. Take ionization energy of the hydrogen atom 13.6 eV.

44. The value of Z is
(A) 3 (B) 4
(C) 5 (D) 6

45. Kinetic energy of electron in ground state of atom X is
(A) 122.4 eV (B) 170 eV
(C) 340 eV (D) 680 eV

46. The wavelength of electromagnetic radiation required to remove the electron from 3rd Bohr orbit to infinity is
(A) 329 Å (B) 428 Å
(C) 546 Å (D) 636 Å

Comprehension 16

A gas of identical hydrogen like atoms has some atoms in lowest (ground) energy level A and some atoms in particular upper (excited) energy level B and there are no atoms in any other energy level. The atoms of the gas make transition to a higher energy level by absorbing monochromatic light of photon energy 2.7 eV. Subsequently the atoms emit radiation of only six different photons energies. Some of emitted photons have energy 2.7 eV. Some have more and some have less than 2.7 eV. Based on the information given, answer the following questions.

47. The principal quantum number of initially excited level B is
(A) 1 (B) 2
(C) 3 (D) None of these

48. The magnitude of ionisation energy of gas atoms is
(A) 13.6 eV
(B) 12.7 eV
(C) 54.4 eV
(D) None of these

49. The maximum and minimum energies of the emitted photons
(A) 12.75 eV and 0.66 eV
(B) 13.6 eV and 12.75 eV
(C) 13.6 eV and 0.66 eV
(D) None of these

Comprehension 17

A sample of hydrogen atom gas contains 100 atoms. All the atoms are excited to the same n^{th} excited state. The total energy released by all the atoms as they come to the ground state through various types of transitions is $\frac{4800}{49}Rch$, where $Rch = 13.6$ eV. Based on the information provided, answer the following questions.

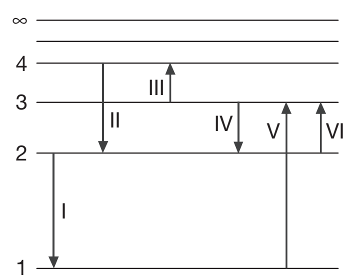
50. The value of n
(A) 5 (B) 6
(C) 7 (D) 8

51. The maximum energy of the emitted photon
(A) $\left(\frac{46}{49}\right)Rch$ (B) $\left(\frac{47}{49}\right)Rch$
(C) $\left(\frac{48}{49}\right)Rch$ (D) Rch

52. The maximum number of photons emitted is
(A) 300 (B) 600
(C) 1500 (D) 2100

Comprehension 18

The figure shown an energy level diagram for the hydrogen atom. Several transitions are marked as I, II, III, IV, V, & VI. The diagram is only indicative and not to scale. Based on the information given, answer the following questions.



53. A Balmer series photon is absorbed in
(A) II (B) III
(C) IV (D) VI

54. The wavelength of the radiation involved in transition II is
(A) 291 nm (B) 364 nm
(C) 487 nm (D) 652 nm

55. The transition that occur, when a hydrogen atom is irradiated with radiation of wavelength 1025 Å
(A) I (B) II
(C) IV (D) V

Comprehension 19

In the first orbit of hydrogen atom, energy is -13.6 eV, whereas in case of any other hydrogen like atom, the energy (in eV) in first orbit is $-(13.6)Z^2$, where Z is atomic number of atom. When the electron gets excited to a higher state then its energy increases. At infinity, it becomes zero, because electron becomes free from system. All other states other than the ground state are called as excited state. Based on the information provided, answer the following questions.

56. As number of orbit increases, kinetic energy of electron also increases

(A) yes
(B) no
(C) depends on atom
(D) depends on number of orbits

57. Energy of He^+ ion in its second excited state is approximately

(A) –13.6 eV (B) –27.2 eV
(C) –6 eV (D) –54.4 eV

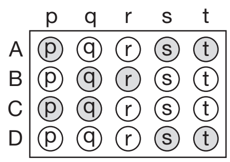
58. When a hydrogen nucleus joins an electron in the first excited state, then released energy is

(A) 13.6 eV (B) 3.4 eV
(C) 10.2 eV (D) None of these

MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:



1. Match the quantities in **COLUMN-I** with their proportionality on Z (atomic number) in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Radius of orbit	(p) is proportional to Z
(B) Current associated due to orbital motion of electron	(q) is inversely proportional to Z
(C) Magnetic field at the centre due to orbital motion of electron	(r) is proportional to Z^2
(D) Velocity of an electron	(s) is proportional to Z^3

2. Excitation energy of hydrogen atom is 13.6 eV. Match the following quantities in **COLUMN-I** with their values in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Energy of second excited state of hydrogen	(p) –3.4 eV
(B) Energy of fourth state of He^+	(q) –13.6 eV
(C) Potential energy of first excited state of Li^{++}	(r) –1.5 eV
(D) Kinetic energy of electron in second excited state of Li^{++}	(s) –61.2 eV

3. Some laws/processes are given in **COLUMN-I**. Match these with the physical phenomena given in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Transition between two atomic energy levels	(p) Characteristic X-rays

(*Continued*)

COLUMN-I	COLUMN-II
(B) Electron emission from a material	(q) Photoelectric effect
(C) Mosley's Law	(r) Hydrogen spectrum
(D) Change of photon energy into kinetic energy of electrons	(s) β-decay

4. Match the energies in **COLUMN-I** with their respective values in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Binding energy of electron in triply ionised Lithium atom	(p) 340 eV
(B) Energy that can remove electron from first excited state of triply ionised Beryllium atom	(q) 3.4 eV
(C) Ionisation energy of tetra-ionised Boron	(r) 122.4 eV
(D) Energy obtained in assembling singly ionised Helium atom so that the atom can be in ground state or other excited states	(s) 54.4 eV

5. Regarding transition of electrons match the transitions in **COLUMN-I** to the corresponding series in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) $n = 5$ to $n = 2$	(p) Lyman series
(B) $n = 8$ to $n = 4$	(q) Brackett series
(C) $n = 3$ to $n = 1$	(r) Paschen series
(D) $n = 4$ to $n = 3$	(s) Balmer series

6. Match the quantities in **COLUMN-I** with their dependence on the principal quantum number n and the atomic number Z in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Angular speed	(p) $\frac{n^3}{Z^2}$
(B) Time period	(q) n
(C) Angular momentum	(r) $\frac{Z^2}{n^3}$
(D) Magnetic moment	(s) $\frac{Z^3}{n^5}$
(E) Magnetic Field	

7. Match the properties in **COLUMN-I** with their corresponding transitions in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Ultraviolet light	(p) $n = 6 \rightarrow n = 3$
(B) Visible light	(q) $n = 3 \rightarrow n = 1$
(C) Infrared radiation	(r) $n = 4 \rightarrow n = 2$
(D) Micro wave	(s) $n = 7 \rightarrow n = 6$

8. For transition of electrons match the following

COLUMN-I	COLUMN-II
(A) $n = 5$ to $n = 2$	(p) Lyman series
(B) $n = 8$ to $n = 4$	(q) Brackett series
(C) $n = 3$ to $n = 1$	(r) Paschen series
(D) $n = 4$ to $n = 3$	(s) Balmer series

9. Match the quantities in **COLUMN-I** with their respective values in **COLUMN-II** for He^+ atom.

COLUMN-I	COLUMN-II
(A) Angular momentum	(p) $\frac{2h}{\pi}$
(B) Total energy of electron in ground state (in eV)	(q) 54.4
(C) Potential energy of electron in ground state (in eV)	(r) −108.8

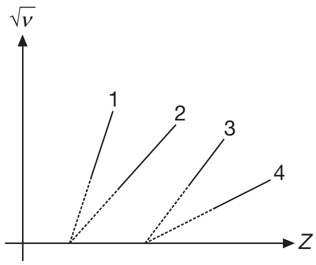
(Continued)

COLUMN-I	COLUMN-II
(D) Kinetic energy of electron in first excited state (in eV)	(s) $\frac{h}{2\pi}$
	(t) 13.6
	(u) –54.4

10. Match the entries of **COLUMN-I** with the respective entries in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Characteristic X-ray	(p) Inverse process of photoelectric effect
(B) X-ray production	(q) Potential difference
(C) Cut-off wavelength	(r) Moseley's law
(D) Continuous X-ray	(s) None of these

11. Square root of frequency $\sqrt{\nu}$, versus atomic number Z graph for characteristic X-rays is as shown in figure. Match the following



COLUMN-I	COLUMN-II
(A) Line-1	(p) L_α
(B) Line-2	(q) L_β
(C) Line-3	(r) K_α
(D) Line-4	(s) K_β

12. The energy, the magnitude of linear momentum, magnitude of angular momentum and orbital radius of an electron in a hydrogen atom corresponding to the quantum number n are E, p, L and r respectively. Then according to Bohr's theory of hydrogen atom match the expressions in **COLUMN-I** with statement in **COLUMN-II.**

COLUMN-I	COLUMN-II
(A) Epr	(p) is independent of n
(B) $\frac{p}{E}$	(q) is directly proportional to n
(C) Er	(r) is inversely proportional to n
(D) pr	(s) is directly proportional to L

13. **COLUMN-I** is a physical quantity related to orbiting electron in a hydrogen like atom, the term Z and n given in **COLUMN- II** have usual meaning in Bohr's theory.

COLUMN-I	COLUMN-II
(A) Frequency of orbiting electron	(p) is directly proportional to Z^2
(B) Angular momentum of orbiting electron	(q) is directly proportional to n
(C) Magnetic moment of orbiting electron	(r) is inversely proportional to n^3
(D) The average current due to orbiting of electron	(s) is independent of Z
	(t) is different for different hydrogen like atom

14. Match the series in **COLUMN-I** with the type of emission in **COLUMN-II** for hydrogen spectrum.

COLUMN-I	COLUMN-II
(A) Lyman series	(p) infrared region
(B) Balmer series	(q) visible region
(C) Paschen series	(r) ultraviolet region
(D) Brackett series	(s) X-rays

15. In **COLUMN-I**, maximum wavelengths of spectral series are given and in **COLUMN-II** minimum wavelengths are given. Match the **COLUMN-I** with **COLUMN-II** and mark the correct the correct option from the codes given below

COLUMN-I (λ_{max})	COLUMN-II (λ_{min})
(A) $\frac{4}{3R}$	(p) $\frac{9}{R}$
(B) $\frac{36}{5R}$	(q) $\frac{16}{R}$
(C) $\frac{144}{7R}$	(r) $\frac{25}{R}$
(D) $\frac{400}{9R}$	(s) $\frac{1}{R}$
(E) $\frac{900}{11R}$	(t) $\frac{4}{R}$

16. In Bohr's atomic model for hydrogen atom, match the contents of **COLUMN-I** with contents of **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) If electron jumps from $n = 2$ to $n = 1$	(p) speed of electron will become 2 times
(B) If electron jumps from $n = 1$ to $n = 4$	(q) kinetic energy of electron will become 4 times
(C) If electron jumps from $n = 3$ to $n = 1$	(r) angular momentum of electron will become 4 times
(D) If electron jumps from $n = 6$ to $n = 3$	(s) angular velocity of electron becomes 27 times
	(t) current due to electron becomes 8 times

17. Match the entries in **COLUMN-I** with their respective entries in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Atomic excitation	(p) Absorption spectrum
(B) Lyman series	(q) Independent of mass of electron
(C) Rydberg constant	(r) Inelastic collision
(D) Bohr's atomic model	(s) Dependent on mass of electron
(E) Speed of electron	(t) Stationary orbit

18. If f_1 is the maximum frequency of emitted photon of Lyman series, f_2 is minimum frequency of the emitted photon of Lyman series and f_3 is maximum frequency of the emitted photon of Balmer series, then match the relations in given in **COLUMN-I** with the corresponding relations in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) f_1	(p) greater than f_3
(B) f_2	(q) is negative
(C) $(f_2 - f_1)$	(r) less than f_2
(D) $(f_1 - f_2)$	(s) is equal to f_3

19. In hydrogen atom wavelength of second line of Balmer series is λ, Match the following two columns corresponding to the wavelength.

COLUMN-I	COLUMN-II
(A) First line of Balmer series	(p) $\left(\frac{27}{20}\right)\lambda$
(B) Third line of Balmer series	(q) $\left(\frac{\lambda}{4}\right)$
(C) First line of Lyman series	(r) $\left(\frac{25}{12}\right)\lambda$
(D) Second line of Lyman series	(s) None of these

20. Match the energies in **COLUMN-I** with the respective values in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Energy of second excited state of hydrogen	(p) -3.4 eV
(B) Energy of fourth state of He^+	(q) $+13.6$ eV
(C) Energy of first excited state of Li^{++}	(r) -1.5 eV
(D) Excitation energy of hydrogen atom	(s) -30.6 eV
	(t) -13.6 eV

21. Ionization energy of electron from first excited state of hydrogen atom is E. Match the following two columns for He^+ atom.

COLUMN-I	COLUMN-II
(A) Ionization energy from ground state	(p) $4E$
(B) Electrostatic potential energy in first excited state	(q) $-16E$
(C) Kinetic energy of electron in ground state	(r) $-8E$
(D) Ionization energy from first excited state	(s) $16E$

INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

1. Calculate the ratio between total acceleration of the electron in singly ionized helium atom and hydrogen atom when both the atoms are in ground state.

2. The binding energy of an electron in the ground state of He atom is equal to $E_0 = 24.6\text{ eV}$. Find the energy required, in eV, to remove both electrons from the atom.

3. An X-ray tube is operating at 18 kV. The speed of electrons striking the target is $x \times 10^7\text{ ms}^{-1}$. Calculate x. Take mass of electron to be $9 \times 10^{-31}\text{ kg}$.

4. Find the quantum number n corresponding to nth excited state of He^+ ion if on transition to the ground state the ion emits two photons in succession with wavelengths 108.5 nm and 30.4 nm. The ionization energy of the hydrogen atom is 13.6 eV.

5. The half-value thickness of an absorber is defined as the thickness that will reduce exponentially the intensity of a beam of particles by a factor of 2. Calculate the half-value thickness for lead, in micrometre, assuming that the X-ray beam has a wavelength of 20 pm, absorption coefficient of lead to be $\mu = 50\text{ cm}^{-1}$.

6. From what material (Z value) is the anode of an X-ray tube made, if the K_α line wavelength of the characteristic spectrum is 0.76 Å?

7. Calculate the voltage of X-ray tube in kilovolt, so that an electron emitted from the cathode may give an X-ray of wavelength 3.1 Å after striking the target. Take $hc = 12400\text{ eVÅ}$.

8. A stream of α-particles is incident on a sample of hydrogen gas. What should be the minimum kinetic energy of α-particles, in eV, to ionize the hydrogen atoms.

9. The potential difference across Coolidge tube is 20 kV and a 10 mA current flows through the voltage supply. Only 0.5% of the energy carried by electrons striking the target is converted into X-rays. Calculate the power carried by X-ray beam in watt.

10. Hydrogen gas in the atomic state is excited to an energy level such that the electrostatic potential energy of hydrogen atom becomes -1.7 eV. Now a photoelectric plate having work function 2.3 eV is exposed to the emission spectra of this gas. Assuming all the transitions to be possible, find the minimum de-Broglie wavelength of the ejected photoelectrons in Å to the nearest integer.

11. The shortest wavelength of the Brackett series of a hydrogen like atom of atomic number Z is same as the shortest wavelength of the Balmer series of hydrogen atom, then calculate Z.

12. A hydrogen like atom (described by the Bohr model) is observed to emit six wavelengths, originating from all possible transitions between a group of levels. These levels have energies between -0.85 eV and -0.544 eV (including both these values).
(a) Find the atomic number of the atom.
(b) Calculate the smallest wavelength emitted in these transitions, in Å.
(Take ground state energy of hydrogen atom to be -13.6 eV).

13. Emission spectrum of hydrogen atom has two lines of Balmer series with wavelength 4102 Å and 4861 Å. To what series does a spectral line belong if its wave number is equal to the difference of wave numbers of the above two lines? What is the wavelength of this line in Å? Given, $R = 1.097 \times 10^7\ \text{m}^{-1}$.

14. Calculate the quantum number n corresponding to the excited state of singly ionised helium, if on transition to ground state, the ionised helium emits two photons in succession having wavelengths 1026.7 Å and 304 Å. Given that the Rydberg's constant has a value $1.09 \times 10^7\ \text{m}^{-1}$.

15. A hydrogen like atom (atomic number Z) is in a higher excited state of quantum number n. The excited atom can make a transition to the first excited state by successively emitting two photons of energy 10.2 eV and 17 eV respectively. Alternately, the atom from the same excited state can make a transition to the second excited state by successively emitting two photons of energies 4.25 eV and 5.95 eV respectively.
Determine the values of n and Z.
(Ionization energy of hydrogen atom is 13.6 eV)

16. Assuming that the binding energy of an electron in the ground state of helium atom is 25.6 eV. If the ionisation energy of hydrogen is 13.6 eV, then the energy required to remove both the electrons from the helium atom is $10N$ eV. Calculate N.

17. The ionization energy of a hydrogen like Bohr atom is 4 rydberg.
(a) What is the wavelength of the radiation emitted, in Å, when the electron jumps from the first excited state to the ground state?
(b) What is the atomic number Z of the atom?
(c) Also, the radius of the first orbit for this atom as a multiple of the Bohr's radius a_0 is $\frac{a_0}{*}$, where $*$ is not readable. Find $*$.

18. Calculate the wavelength of K_α line (in picometer) for copper $(Z = 29)$ if the wavelength of K_α line for iron $(Z = 26)$ is known to be equal to 193 pm.

19. A doubly ionized lithium atom is hydrogen like with atomic number 3. Find the wavelength of the radiation to the nearest three digit integer, in Å, required to excite the electron in Li^{++} from the first to the third Bohr orbit. The ionization energy of the hydrogen atom is 13.6 eV.

20. The electric current in an X-ray tube operating at 40 kV is 10 mA. Assume that on an average 1% of the total kinetic energy of the electrons hitting the target are converted into X-rays.
(a) What is the total power emitted as X-rays, in watt?
(b) How much heat, in joule is produced in the target every second?

21. In a certain element, the K electron energy is -18.525 keV and the L electron energy is -3 keV. When electron jumps from L to K shell, the wavelength of X-ray emitted is $x \times 10^{-11}$ m. Calculate x. Take $hc = 12375$ eVÅ.

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1. [Online April 2019]
Radiation coming from transitions $n = 2$ to $n = 1$ of hydrogen atoms fall on He^+ ions in $n = 1$ and $n = 2$ states. The possible transition of helium ions as they absorb energy from the radiation is
(A) $n = 2 \to n = 4$ (B) $n = 2 \to n = 5$
(C) $n = 2 \to n = 3$ (D) $n = 1 \to n = 4$

2. [Online April 2019]
Taking the wavelength of first Balmer line in hydrogen spectrum $(n = 3 \text{ to } n = 2)$ as 660 nm, the wavelength of the 2nd Balmer line $(n = 4 \text{ to } n = 2)$ will be
(A) 889.2 nm (B) 488.9 nm
(C) 388.9 nm (D) 642.7 nm

3. [Online April 2019]
A He^+ ion is in its first excited state. Its ionization energy is
(A) 13.60 eV (B) 6.04 eV
(C) 48.36 eV (D) 54.40 eV

4. [Online April 2019]
In Li^{++}, electron in first Bohr orbit is excited to a level by a radiation of wavelength λ. When the ion gets deexcited to the ground state in all possible ways

(including intermediate emissions), a total of six spectral lines are observed. What is the value of λ ?
(Given: $h = 6.63 \times 10^{-34}$ Js , $c = 3 \times 10^{8}$ ms^{-1})

(A) 11.4 nm (B) 12.3 nm
(C) 9.4 nm (D) 10.8 nm

5. [Online April 2019]
An excited He^{+} ion emits two photons in succession, with wavelengths 108.5 nm and 30.4 nm , in making a transition to ground state. The quantum number n , corresponding to its initial excited state is (for photon of wavelength λ , energy $E = \frac{1240\text{ eV}}{\lambda(\text{in nm})}$)

(A) $n = 5$ (B) $n = 7$
(C) $n = 4$ (D) $n = 6$

6. [Online April 2019]
The electron in a hydrogen atom first jumps from the third excited state to the second excited state and subsequently to the first excited state. The ratio of the respective wavelengths, $\frac{\lambda_1}{\lambda_2}$, of the photons emitted in this process is

(A) $\frac{7}{5}$ (B) $\frac{27}{5}$
(C) $\frac{9}{7}$ (D) $\frac{20}{7}$

7. [Online January 2019]
A hydrogen atom, initially in the ground state is excited by absorbing a photon of wavelength 980 Å. The radius of the atom in the excited state, in terms of Bohr radius a_0 , will be $(hc = 12500\text{ eVÅ})$

(A) $4a_0$ (B) $9a_0$
(C) $25a_0$ (D) $16a_0$

8. [Online January 2019]
In a hydrogen like atom, when an electron jumps from the M-shell to the L-shell , the wavelength of emitted radiation is λ . If an electron jumps from N-shell to the L-shell , the wavelength of emitted radiation will be

(A) $\frac{25}{16}\lambda$ (B) $\frac{16}{25}\lambda$
(C) $\frac{20}{27}\lambda$ (D) $\frac{27}{20}\lambda$

9. [Online January 2019]
A particle of mass m moves in a circular orbit in a central potential field $U(r) = \frac{1}{2}kr^2$. If Bohr's quantization conditions are applied, radii of possible orbitals and energy levels vary with quantum number n as

(A) $r_n \propto n$, $E_n \propto n$ (B) $r_n \propto \sqrt{n}$, $E_n \propto n$
(C) $r_n \propto \sqrt{n}$, $E_n \propto \frac{1}{n}$ (D) $r_n \propto n^2$, $E_n \propto \frac{1}{n^2}$

10. [2018]
An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n , λ_g be the de Broglie wavelength of the electron in the n^{th} state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the n^{th} state to the ground state. For large n , (A , B are constants)

(A) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$ (B) $\Lambda_n \approx A + B\lambda_n$
(C) $\Lambda_n^2 \approx A + B\lambda_n^2$ (D) $\Lambda_n^2 \approx \lambda$

11. [2018]
If the series limit frequency of the Lyman series is ν_L , then the series limit frequency of the Pfund series is

(A) $25\nu_L$ (B) $16\nu_L$
(C) $\frac{\nu_L}{16}$ (D) $\frac{\nu_L}{25}$

12. [Online 2018]
The energy required to remove the electron from a singly ionized Helium atom is 2.2 times the energy required to remove an electron from Helium atom. The total energy required to ionize the Helium atom completely is

(A) 34 eV (B) 79 eV
(C) 20 eV (D) 109 eV

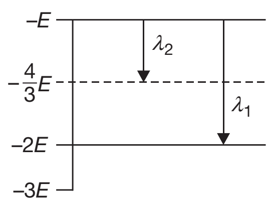
13. [Online 2018]
Muon (μ^-) is a negatively charged $(|q| = |e|)$ particle with a mass $m_\mu = 200m_e$, where m_e is the mass of the electron and e is the electronic charge. If μ^- is bound to a proton to form a hydrogen like atom, identify the correct statements.

I. Radius of the muonic orbit is 200 times smaller than that of the electron.
II. The speed of the μ^- in the n^{th} orbit is $\frac{1}{200}$ times that of the electron in the n^{th} orbit.
III. The ionization energy of muonic atom is 200 times more than that of an hydrogen atom.
IV. The momentum of the muon in the n^{th} orbit is 200 times more than that of the electron.

(A) I, II, IV (B) II, IV
(C) I, III, IV (D) III, IV

14. [Online 2018]
Both the nucleus and the atom of some element are in their respective first excited states. They get de-excited by emitting photons of wavelengths λ_N, λ_A respectively. The ratio is $\frac{\lambda_N}{\lambda_A}$ closest to
(A) 10 (B) 10^{-6}
(C) 10^{-10} (D) 10^{-1}

15. [2017]
Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \frac{\lambda_1}{\lambda_2}$ is given by



(A) $r = \frac{4}{3}$ (B) $r = \frac{2}{3}$
(C) $r = \frac{3}{4}$ (D) $r = \frac{1}{3}$

16. [Online 2017]
According to Bohr's theory, the time averaged magnetic field at the centre (i.e. nucleus) of a hydrogen atom due to the motion of electrons in the n^{th} orbit is proportional to ($n =$ principal quantum number)
(A) n^{-2} (B) n^{-3}
(C) n^{-4} (D) n^{-5}

17. [Online 2017]
The acceleration of an electron in the first orbit of hydrogen atom $(n = 1)$ is
(A) $\frac{h^2}{\pi^2 m^2 r^3}$ (B) $\frac{h^2}{4\pi^2 m^2 r^3}$
(C) $\frac{h^2}{4\pi m^2 r^3}$ (D) $\frac{h^2}{8\pi^2 m^2 r^3}$

18. [Online 2016]
A hydrogen atom makes a transition from $n = 2$ to $n = 1$ and emits a photon. This photon strikes a doubly ionized lithium atom $(Z = 3)$ in excited state and completely removes the orbiting electron. The least quantum number for the excited state of the ion for the process is
(A) 2 (B) 4
(C) 5 (D) 3

19. [2015]
As an electron makes a transition from an excited state to the ground state of a hydrogen – like atom/ion
(A) its kinetic energy increases but potential energy and total energy decrease
(B) kinetic energy, potential energy and total energy decrease
(C) kinetic energy decreases, potential energy increases but total energy remains same
(D) kinetic energy and total energy decrease but potential energy increases

20. [Online 2015]
If one were to apply Bohr model to a particle of mass m and charge q moving in a plane under the influence of a magnetic field B, the energy of the charged particle in the n^{th} level will be
(A) $n\left(\frac{hqB}{2\pi m}\right)$ (B) $n\left(\frac{hqB}{4\pi m}\right)$
(C) $n\left(\frac{hqB}{8\pi m}\right)$ (D) $n\left(\frac{hqB}{\pi m}\right)$

21. [2014]
Hydrogen $\left({}_1H^1\right)$, Deuterium $\left({}_1H^2\right)$, singly ionised Helium $\left({}_2He^4\right)^+$ and doubly ionised lithium $\left({}_3Li^6\right)^{++}$ all have one electron around the nucleus. Consider an electron transition from $n = 2$ to $n = 1$. If the wavelengths of emitted radiation are λ_1, λ_2, λ_3 and λ_4 respectively then approximately which one of the following is correct?
(A) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ (B) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
(C) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$ (D) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

22. [2013]
In a hydrogen like atom electron makes transition from an energy level with quantum number n to another with quantum number $(n-1)$. If $n \gg 1$, the frequency of radiation emitted is proportional to
(A) $\frac{1}{n}$ (B) $\frac{1}{n^2}$
(C) $\frac{1}{n^{3/2}}$ (D) $\frac{1}{n^3}$

23. [2012]
Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then the number of spectral lines in the emission spectra will be
(A) 3 (B) 5
(C) 6 (D) 2

24. [2012]
A diatomic molecule is made of two masses m_1 and m_2 which are separated by a distance r. If we calculate its rotational energy by applying Bohr's rule of angular momentum quantization, its energy will be given by (n is an integer)

(A) $\dfrac{n^2\hbar^2}{2(m_1+m_2)r^2}$ (B) $\dfrac{2n^2\hbar^2}{(m_1+m_2)r^2}$

(C) $\dfrac{(m_1+m_2)n^2\hbar^2}{2m_1m_2r^2}$ (D) $\dfrac{(m_1+m_2)^2n^2\hbar^2}{2m_1^2m_2^2r^2}$

25. [2011]
Energy required for the electron excitation in Li^{++} from the first to the third Bohr orbit is

(A) 12.1 eV (B) 36.3 eV
(C) 108.8 eV (D) 122.4 eV

26. [2009]
The transition from the state $n=4$ to $n=3$ in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from

(A) $2\rightarrow1$ (B) $3\rightarrow2$
(C) $4\rightarrow2$ (D) $5\rightarrow4$

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Single Correct Choice Type Problems

(In this section each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct)

1. [JEE (Advanced) 2014]
If λ_{Cu} is the wavelength of K_α, X-ray line of copper (atomic number 29) and λ_{mo} is the wavelength of the K_α, X-ray line of molybdenum (atomic number 42), then the ratio $\dfrac{\lambda_{Cu}}{\lambda_{mo}}$ is close to

(A) 1.99 (B) 2.14
(C) 0.50 (D) 0.48

2. [IIT-JEE 2011]
The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 Å. The wavelength of the second spectral line in the Balmer series of singly-ionized helium atom is

(A) 1215 Å (B) 1640 Å
(C) 2430 Å (D) 4687 Å

3. [IIT-JEE 2008]
Which one of the following statements is wrong in the context of X-rays generated from an X-ray tube?

(A) Wavelength of characteristic X-rays decreases when the atomic number of the target increases
(B) Cut-off wavelength of the continuous X-rays depends on the atomic number of the target
(C) Intensity of the characteristic X-rays depends on the electrical power given to the X-ray tube
(D) Cut-off wavelength of the continuous X-rays depends on the energy of the electrons in the X-ray tube

4. [IIT-JEE 2007]
The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer) is

(A) 802 nm (B) 823 nm
(C) 1882 nm (D) 1648 nm

5. [IIT-JEE 2005]
K_α wavelength emitted by an atom of atomic number $Z=11$ is λ. Find the atomic number for an atom that emits K_α radiation with wavelength 4λ

(A) $Z=6$ (B) $Z=4$
(C) $Z=11$ (D) $Z=44$

6. [IIT-JEE 2005]
A photon collides with a stationary hydrogen atom in ground state inelastically. Energy of the colliding photon is 10.2 eV. After a time interval of the order of mocro second another photon collides with same hydrogen atom inelastically with an energy of 15 eV. What will be observed by the detector?

(A) 2 photon of energy 10.2 eV
(B) 2 photon of energy 1.4 eV
(C) One photon of energy 10.2 eV and an electron of energy 1.4 eV
(D) One photon of energy 10.2 eV and another photon of energy 1.4 eV

7. [IIT-JEE 2003]
The electric potential between a proton and an electron is given by $V=V_0\log_e\left(\dfrac{r}{r_0}\right)$, where r_0 is a constant. Assuming Bohr's model to be applicable, write variation of r_n with n, n being the principal quantum number.

(A) $r_n\propto n$ (B) $r_n\propto\dfrac{1}{n}$

(C) $r_n\propto n^2$ (D) $r_n\propto\dfrac{1}{n^2}$

8. [IIT-JEE 2002]
A hydrogen atom and a Li^{++}ion are both in the second excited state. If L_H and L_{Li} are their respective electronic angular momenta and E_H and E_{Li} their respective energies then,

(A) $L_H > L_{Li}$ and $|E_H| > |E_{Li}|$

(B) $L_H = L_{Li}$ and $|E_H| < |E_{Li}|$

(C) $L_H = L_{Li}$ and $|E_H| > |E_{Li}|$

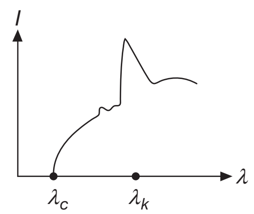
(D) $L_H < L_{Li}$ and $|E_H| < |E_{Li}|$

9. [IIT-JEE 2002]
The potential difference applied to an X-ray tube is 5 kV and the current through it is 3.2 mA. The number of electrons striking the target per second is

(A) 2×10^{16} (B) 5×10^{6}

(C) 1×10^{17} (D) 4×10^{15}

10. [IIT-JEE 2001]
The intensity of X-rays from a Coolidge tube is plotted against wavelength λ as shown. The minimum wavelength found is λ_c and the wavelength of the K_α line is λ_k. As the accelerating voltage is increased



(A) $(\lambda_k - \lambda_c)$ increases (B) $(\lambda_k - \lambda_c)$ decreases

(C) λ_k increases (D) λ_k decreases

11. [IIT-JEE 2001]
The transition from the state $n=4$ to $n=3$ in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition

(A) $2\rightarrow1$ (B) $3\rightarrow2$

(C) $4\rightarrow2$ (D) $5\rightarrow4$

12. [IIT-JEE 2000]
The electron in a hydrogen atom makes a transition from an exited state to the ground state. Which of the following statements is true?

(A) its kinetic energy increases and its potential and total energies decrease

(B) its kinetic energy decreases, potential energy increases and its total energy remains the same

(C) its kinetic and total energies decrease and its potential energy increases

(D) its kinetic, potential and total energies decrease

13. [IIT-JEE 2000]
Imagine an atom made up of a proton and a hypothetical particle of double the mass of electron but having the same charge as the electron. Apply the Bohr Atom Model and consider all possible transitions of this hypothetical particle to the first excited level. The longest wavelength photon that will be emitted has wavelength λ (given in terms of Rydberg constant R for the Hydrogen atom) equal to

(A) $\dfrac{9}{5R}$ (B) $\dfrac{36}{5R}$

(C) $\dfrac{18}{5R}$ (D) $\dfrac{4}{R}$

14. [IIT-JEE 2000]
Electrons with energy 80 keV are incident on the tungsten target of an X-ray tube. K shell electrons of tungsten have -72.5 keV energy. X-rays emitted by the tube contain only

(A) a continuous X-ray spectrum (Bremsstrahlung) with a minimum wavelength of $\sim 0.155\text{ Å}$.

(B) a continuous X-ray spectrum (Bremsstrahlung) with all wavelengths.

(C) the characteristic X-ray spectrum of tungsten.

(D) a continuous X-ray spectrum (Bremsstrahlung) with a minimum wavelength of $\sim 0.155\text{ Å}$ and the characteristic X-ray spectrum of tungsten.

15. [IIT-JEE 1999]
In Hydrogen spectrum the wavelength of H_α line is 656 nm, whereas in the spectrum of a distant galaxy, H_α line wavelength is 706 nm. Estimated speed of galaxy with respect to earth is

(A) $2\times10^{8}\text{ ms}^{-1}$ (B) $2\times10^{7}\text{ ms}^{-1}$

(C) $2\times10^{6}\text{ ms}^{-1}$ (D) $2\times10^{5}\text{ ms}^{-1}$

16. [IIT-JEE 1998]
X-rays are produced in an X-ray tube operating at a given accelerating voltage. The wavelength of the continuous X-rays has values from

(A) 0 to ∞

(B) λ_{min} to ∞ where $\lambda_{min} > 0$

(C) 0 to λ_{max} where $\lambda_{max} < \infty$

(D) λ_{min} to λ_{max} where $0 < \lambda_{min} < \lambda_{max} < \infty$

17. [IIT-JEE 1997]
As per Bohr Model, the minimum energy (in eV) required to remove an electron from the ground state of doubly ionised Li atom $(Z=3)$ is

(A) 1.51 (B) 13.6

(C) 40.8 (D) 122.4

18. [IIT-JEE 1997]
The K_α X-ray emission line of tungsten occurs at $\lambda = 0.021$ nm. The energy difference between K and L levels in this atom is about
(A) 0.51 MeV (B) 1.2 MeV
(C) 59 keV (D) 13.6 eV

19. [IIT-JEE 1985]
The X-ray beam coming from an X-ray tube will be
(A) monochromatic
(B) having all wavelengths smaller than a certain maximum wavelength
(C) having all wavelengths larger than a certain minimum wavelength
(D) having all wavelengths lying between a minimum and a maximum wavelength

20. [IIT-JEE 1983]
If the elements with principal quantum number $n > 4$ were not allowed in nature, the possible number of elements would be
(A) 4 (B) 32
(C) 60 (D) 64

21. [IIT-JEE 1982]
The shortest wavelength of X-rays emitted from an X-ray tube depends on
(A) the current in the tube
(B) the voltage applied to the tube
(C) the nature of the gas in tube
(D) the atomic number of the target material

Multiple Correct Choice Type Problems

(In this section each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct)

1. [JEE (Advanced) 2019]
A free hydrogen atom after absorbing a photon of wavelength λ_a gets excited from the state $n = 1$ to the state $n = 4$. Immediately after that the electron jumps to $n = m$ state by emitting a photon of wavelength λ_e. Let the change in momentum of atom due to the absorption and the emission are Δp_a and Δp_e, respectively. If $\frac{\lambda_a}{\lambda_e} = \frac{1}{5}$, which of the option(s) is/are correct?

(Use $hc = 1242$ eV nm; $1\text{ nm} = 10^{-9}$ m, h and c are Planck's constant and speed of light, respectively)

(A) $\lambda_e = 418$ nm

(B) $\frac{\Delta p_a}{\Delta p_e} = \frac{1}{2}$

(C) The ratio of kinetic energy of the electron in the state $n = m$ to the state $n = 1$ is $\frac{1}{4}$

(D) $m = 2$

2. [JEE (Advanced) 2016]
Highly excited states for hydrogen-like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principal quantum number n, where $n \gg 1$. Which of the following statement(s) is(are) true?
(A) Relative change in the radii of two consecutive orbitals does not depend on Z
(B) Relative change in the radii of two consecutive orbitals varies as $\frac{1}{n}$
(C) Relative change in the energy of two consecutive orbitals varies as $\frac{1}{n^3}$
(D) Relative change in the angular momenta of two consecutive orbitals varies as $\frac{1}{n}$

3. [JEE (Advanced) 2013]
The radius of the orbit of an electron in a Hydrogen like atom is $4.5a_0$, where a_0 is the Bohr radius. Its orbital angular momentum is $\frac{3h}{2\pi}$. It is given that h is Planck constant and R is Rydberg constant. The possible wavelength(s), when the atom de-excites, is (are)

(A) $\frac{9}{32R}$ (B) $\frac{9}{16R}$

(C) $\frac{9}{5R}$ (D) $\frac{4}{3R}$

4. [IIT-JEE 1998]
The electron in a hydrogen atom makes a transition $n_1 \to n_2$ where, n_1 and n_2 are the principal quantum numbers of the two states. Assume the Bohr Model to be valid. The time period of the electron in initial state is eight times that in the final state. The possible values of n_1 and n_2 are
(A) $n_1 = 4$, $n_2 = 2$ (B) $n_1 = 8$, $n_2 = 2$
(C) $n_1 = 8$, $n_2 = 1$ (D) $n_1 = 6$, $n_2 = 3$

5. [IIT-JEE 1988]
The potential difference applied to an X-ray tube is increased. As a result, in the emitted radiation
(A) the intensity increases
(B) the minimum wavelength increases
(C) the intensity remains unchanged
(D) the minimum wavelength decreases

6. **[IIT-JEE 1984]**
In the Bohr model of the hydrogen atom
(A) the radius of the nth orbit is proportional to n^2
(B) the total energy of the electron in the nth orbit is inversely proportional to n
(C) the angular momentum of the electron in an orbit is an integral multiple of $\frac{h}{\pi}$
(D) the magnitude of the potential energy of the electron in any orbit is greater than its kinetic energy

Reasoning Based Questions

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **[IIT-JEE 2007]**
Statement-1: If the accelerating potential in an X-ray tube is increased, the wavelengths of the characteristic X-rays do not change.
Statement-2: When an electron beam strikes the target in an X-ray tube, part of the kinetic energy is converted into X-ray energy.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

Comprehension Type Questions

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantisation of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantised rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantisation condition.

1. **[IIT-JEE 2010]**
A diatomic molecule has moment of inertia I. By Bohr's quantization condition its rotational energy in the n^{th} level ($n = 0$ is not allowed) is
(A) $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 \text{I}}\right)$ (B) $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$
(C) $n\left(\frac{h^2}{8\pi^2 I}\right)$ (D) $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

2. **[IIT-JEE 2010]**
It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $\frac{4}{\pi}\times 10^{11}$ Hz. Then the moment of inertia of CO molecule about its centre of mass is close to (Take $h = 2\pi\times 10^{-34}$ Js)
(A) 2.76×10^{-46} kgm^2 (B) 1.87×10^{-46} kgm^2
(C) 4.67×10^{-47} kgm^2 (D) 1.17×10^{-47} kgm^2

3. **[IIT-JEE 2010]**
In a CO molecule, the distance between $C(\text{mass} = 12\text{ amu})$ and $O(\text{mass} = 16\text{ amu})$, where $1\text{ amu} = \frac{5}{3}\times 10^{-27}$ kg, is close to
(A) 2.4×10^{-10} m (B) 1.9×10^{-10} m
(C) 1.3×10^{-10} m (D) 4.4×10^{-11} m

Comprehension 2

When a particle is restricted to move along x-axis between $x = 0$ and $x = a$, where a is of nanometer dimension, its energy can take only certain specific values. The allowed energies of the particle moving in such a restricted region, correspond to the formation of standing waves with nodes at its ends $x = 0$ and $x = a$. The wavelength of this standing wave is related to the linear momentum p of the particle according to the de-Broglie relation. The energy of the particle of mass m is related to its linear momentum as

$E = \frac{p^2}{2m}$. Thus, the energy of the particle can be denoted by a quantum number n taking values $1, 2, 3, \ldots$ ($n = 1$, called the ground state) corresponding to the number of loops in the standing wave.

Use the model described above to answer the following three questions for a particle moving in the line $x = 0$ to $x = a$. [Take $h = 6.6 \times 10^{-34}$ Js and $e = 1.6 \times 10^{-19}$ C].

4. [IIT-JEE 2009]
The allowed energy for the particle for a particular value of n is proportional to

(A) a^{-2} (B) $a^{-\frac{3}{2}}$
(C) a^{-1} (D) a^2

5. [IIT-JEE 2009]
If the mass of the particle is $m = 1 \times 10^{-30}$ kg and $a = 6.6$ the energy of the particle in its ground state is closest to
(A) 0.8 meV (B) 8 meV
(C) 80 meV (D) 800 meV

6. [IIT-JEE 2009]
The speed of the particle that can take discrete values is proportional to

(A) $n^{-\frac{3}{2}}$ (B) n^{-1}
(C) $n^{\frac{1}{2}}$ (D) n

Comprehension 3

In a mixture of $H - He^+$ gas (He^+ is singly ionized He atom), H atoms and He^+ ions are excited to their respective first excited states. Subsequently, H atoms transfer their total excitation energy to He^+ ions (by collisions). Assume that the Bohr model of atom is exactly valid. Based on above information, answer the following questions.

7. [IIT-JEE 2008]
The quantum number n of the state finally populated in He^+ ions is
(A) 2 (B) 3
(C) 4 (D) 5

8. [IIT-JEE 2008]
The wavelength of light emitted in the visible region by He^+ ions after collisions with H atoms is

(A) 6.5×10^{-7} m (B) 5.6×10^{-7} m
(C) 4.8×10^{-7} m (D) 4×10^{-7} m

9. [IIT-JEE 2008]
The ratio of the kinetic energy of the $n = 2$ electron for the H atom to that of He^+ ion is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$
(C) 1 (D) 2

Integer/Numerical Answer Type Questions

(In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s)).

1. [JEE (Advanced) 2018]
Consider a hydrogen-like ionized atom with atomic number Z with a single electron. In the emission spectrum of this atom, the photon emitted in the $n = 2$ to $n = 1$ transition has energy 74.8 eV higher than the photon emitted in the $n = 3$ to $n = 2$ transition. The ionization energy of the hydrogen atom is 13.6 eV. The value of Z is............

2. [JEE (Advanced) 2013]
A proton is fired from very far away towards a nucleus with charge $Q = 120e$, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. Find the de-Broglie wavelength (in units of fm) of the proton at its start.

Take the proton mass, $m_p = \left(\frac{5}{3}\right) \times 10^{-27}$ kg,

$\frac{h}{e} = 4.2 \times 10^{-15}$ JsC^{-1}, $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ mF^{-1} and

1 fm $= 10^{-15}$ m.

3. [JEE (Advanced) 2017]
An electron in a hydrogen atom undergoes a transition from an orbit with quantum number n_i to another with quantum number n_f. v_i and v_f are respectively the initial and final potential energies of the electron. If $\frac{v_i}{v_f} = 6.25$, then the smallest possible n_f is

4. [JEE (Advanced) 2016]
A hydrogen atom in its ground state is irradiated by light of wavelength 970 Å. Taking $\frac{hc}{e} = 1.237 \times 10^{-6}$ Vm and the ground state energy of hydrogen atom as -13.6 eV, the number of lines present in the emission spectrum is

5. **[JEE (Advanced) 2015]**
Consider a hydrogen atom with its electro in the n^{th} orbital. An electromagnetic radiation of wavelength 90 nm is used to ionize the atom. If the kinetic energy of the ejected electron is 10.4 eV, then the value of n is $(hc = 1242\text{ eVnm})$

6. **[JEE (Advanced) 2015]**
An electron in an excited state of Li^{2+} ion has angular momentum $\frac{3h}{2\pi}$. The de Broglie wavelength of the electron in this state is $p\pi a_0$ (where a_0 is the Bohr radius). The value of p is

7. **[JEE (Advanced) 2013]**
A proton is fired from very far away towards a nucleus with charge $Q = 120e$, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. Find the de-Broglie wavelength (in units of fm) of the proton at its start.

Take the proton mass, $m_p = \left(\frac{5}{3}\right)\times 10^{-27}\text{ kg}$,

$\frac{h}{e} = 4.2\times 10^{-15}\text{ JsC}^{-1}$, $\frac{1}{4\pi\varepsilon_0} = 9\times 10^{9}\text{ mF}^{-1}$ and

$1\text{ fm} = 10^{-15}\text{ m}$.

ANSWER KEYS—TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on Atomic Structure and Properties)

1. (a) 113.74 Å (b) 3
2. 122.4 eV
3. 793.3 Å
4. 1.82×10^{-14} m
5. 8.2×10^{6} revolutions
6. 6.56×10^{15} rev/sec
7. $r_n=\dfrac{nh}{2\pi\sqrt{mk}}$, $E_n=\dfrac{k}{2}\left[1-\log_e\left(\dfrac{n^2h^2}{4\pi^2mk}\right)\right]$
8. (a) Helium atom, (b) 54.4 eV, (c) 10.2 eV
9. 12.1 eV
10. –26.9 eV, –12 eV
11. 10.2 V, 2, 10.2 eV, 51 eV
12. 3.1×10^{6} ms^{-1}
13. (a) 15.6 eV (b) 2335 Å
    (c) 12.52 eV (d) 1.01×10^{7} m^{-1}
    (e) (i) 6 eV, (ii) 0.7 eV
14. (a) 1.587 Å (b) 1.095×10^{6} ms^{-1}
15. $\mu=\dfrac{neh}{4\pi m}$, $B=\dfrac{\mu_0\pi m^2e^7}{8\varepsilon_0h^5n^5}$
16. $r=\left(\dfrac{n^2h^2}{8am\pi^2}\right)^{1/4}$
17. $(n_1,n_2)=(2,1),(4,2),(6,3),\ldots\ldots$
18.

Orbit	K(eV)	U(eV)	E(eV)
First	13.60	0	13.60
Second	3.40	20.40	23.80

19. 0.55 eV
20. $\sqrt{2}$
21. (a) 3.4 eV (b) 6.63 Å
22. 1.86×10^{6} ms^{-1}
23. 1216 Å, 1026 Å
24. $\dfrac{3}{4}$
25. 6.68×10^{-3} eV

Test Your Concepts-II (Based on X-rays and Properties)

1. 2×10^{-15} J
2. (a) 27.624 keV (b) 30 kV
3. $Z=31$, Gallium
4. 10 μs
5. 7.52×10^{18} Hz
6. 15865 volt
7. 0.31 Å
8. 8.4×10^{7} ms^{-1}
9. 0.192 Å
10. 21 keV
11. 42
12. 0.163 Å

Single Correct Choice Type Questions

1. B	2. C	3. D	4. A	5. A	6. B	7. C	8. A	9. B	10. B
11. D	12. A	13. B	14. A	15. A	16. D	17. C	18. D	19. A	20. C
21. C	22. D	23. A	24. C	25. D	26. C	27. D	28. C	29. B	30. A
31. D	32. B	33. B	34. D	35. C	36. A	37. D	38. A	39. A	40. D

41. C	42. B	43. C	44. B	45. C	46. D	47. B	48. B	49. A	50. C
51. B	52. C	53. B	54. C	55. C	56. C	57. B	58. A	59. D	60. C
61. B	62. B	63. A	64. B	65. B	66. A	67. B	68. A	69. C	70. A
71. B	72. C	73. D	74. C	75. B	76. B	77. A	78. D	79. A	80. A
81. A	82. B	83. C	84. C	85. D	86. B	87. B	88. D	89. A	90. B
91. B	92. C	93. D	94. A	95. B	96. B	97. D	98. B	99. C	100. B
101. C	102. D	103. D	104. A	105. A	106. C	107. D	108. C	109. A	110. B
111. D	112. A	113. C	114. A	115. C	116. A	117. B	118. C	119. B	120. A
121. C	122. D	123. D	124. A	125. A	126. B	127. D	128. A	129. B	130. C
131. A	132. A	133. B	134. D	135. A	136. D	137. C	138. D	139. D	140. A
141. A	142. B	143. B	144. A	145. B	146. C	147. A	148. C	149. C	150. A
151. C	152. C	153. B	154. C	155. D	156. A	157. B	158. C	159. B	160. D
161. D	162. C	163. C	164. A	165. C	166. D	167. D	168. D	169. A	170. B

Multiple Correct Choice Type Questions

1. A, C	2. A, C	3. B, C	4. B, C	5. A, C, D
6. A, D	7. A, B	8. A, B, C	9. A, B, C, D	10. A, B, C, D
11. B, C, D	12. A, B, C, D	13. A, C	14. A, B, C, D	15. A, B, D
16. B, D	17. A, C, D	18. B, D	19. B, C, D	20. B, C
21. A, D	22. A, D	23. C, D	24. A, B, D	25. B, C, D
26. A, B, C	27. A, B, D	28. A, C, D	29. B, C	30. B, C

Reasoning Based Questions

1. B	2. D	3. C	4. D	5. C	6. C	7. A	8. A	9. B	10. B
11. C	12. C	13. C	14. D	15. D	16. A	17. A	18. B	19. A	20. A

Linked Comprehension Type Questions

1. C	2. C	3. D	4. C	5. B	6. A	7. C	8. D	9. D	10. B
11. A	12. C	13. B	14. C	15. A	16. C	17. C	18. C	19. B	20. A
21. A	22. A	23. B	24. C	25. D	26. B	27. D	28. C	29. C	30. D
31. C	32. A	33. B	34. B	35. D	36. A	37. B	38. C	39. A	40. B
41. C	42. D	43. A	44. C	45. C	46. A	47. B	48. A	49. A	50. B
51. C	52. D	53. D	54. C	55. D	56. B	57. A	58. B		

Matrix Match/Column Match Type Questions

1. A → (q)	B → (r)	C → (s)	D → (p)
2. A → (r)	B → (p)	C → (s)	D → (t)
3. A → (p, r)	B → (p, q, s)	C → (p)	D → (q)
4. A → (r)	B → (s)	C → (p)	D → (q, s)
5. A → (s)	B → (q)	C → (p)	D → (r)

6. A → (r)	B → (p)	C → (q)	D → (q)	E → (s)
7. A → (q)	B → (r)	C → (p)	D → (s)	
8. A → (s)	B → (q)	C → (p)	D → (r)	
9. A → (p, s)	B → (u)	C → (r)	D → (t)	
10. A → (r)	B → (p)	C → (q)	D → (q)	
11. A → (s)	B → (r)	C → (q)	D → (p)	
12. A → (p, s)	B → (q, s)	C → (q, s)	D → (s)	
13. A → (p, r, t)	B → (q, s)	C → (q, s)	D → (p, r, t)	
14. A → (r)	B → (q)	C → (p)	D → (p)	
15. A → (s)	B → (t)	C → (p)	D → (q)	E → (r)
16. A → (p, q, t)	B → (r)	C → (s)	D → (t)	
17. A → (r)	B → (p)	C → (s)	D → (t)	E → (q)
18. A → (p)	B → (p)	C → (q)	D → (s)	
19. A → (p)	B → (s)	C → (q)	D → (s)	
20. A → (r)	B → (p)	C → (s)	D → (q)	
21. A → (s)	B → (r)	C → (s)	D → (p)	

Integer/Numerical Answer Type Questions

1. 8	**2.** 79	**3.** 8	**4.** 5	**5.** 139
6. 41	**7.** 4	**8.** 68	**9.** 1	**10.** 4
11. 2	**12.** (a) 4, (b) 40441	**13.** 26206	**14.** 6	**15.** 6
16. 8	**17.** (a) 300, (b) 2, (c) 2	**18.** 154	**19.** 114	**20.** (a) 4, (b) 396
21. 8				

ARCHIVE: JEE MAIN

1. A	**2.** B	**3.** A	**4.** D	**5.** A	**6.** D	**7.** D	**8.** C	**9.** B	**10.** A
11. D	**12.** B	**13.** C	**14.** B	**15.** D	**16.** D	**17.** B	**18.** B	**19.** A	**20.** B
21. C	**22.** D	**23.** C	**24.** C	**25.** C	**26.** D				

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

1. B	**2.** A	**3.** B	**4.** B	**5.** A	**6.** C	**7.** A	**8.** B	**9.** A	**10.** A
11. D	**12.** A	**13.** C	**14.** D	**15.** B	**16.** B	**17.** D	**18.** C	**19.** C	**20.** C
21. B									

Multiple Correct Choice Type Problems

1. C, D	**2.** A, B, D	**3.** A, C	**4.** A, D	**5.** A, D
6. A, D				

Reasoning Based Questions

1. B

Comprehension Type Questions

1. D **2.** B **3.** C **4.** A **5.** B **6.** D **7.** C **8.** C **9.** A

Integer/Numerical Answer Type Questions

1. 3 **2.** 7 **3.** 5 **4.** 6 **5.** 2 **6.** 2 **7.** 7

CHAPTER 3

Nuclear Physics

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Nucleus and Nuclear Structure
(b) Properties of Nuclear Forces
(c) Mass Defect
(d) Binding Energy
(e) Nuclear Stability
(f) Radioactivity
(g) Nuclear Radiations
(h) Successive Disintegration
(i) Nuclear Reactions
(j) Energetics of Nuclear Reactions
(k) Alpha Decay
(l) Beta Decay
(m) Pauli's Neutrino Hypothesis
(n) Beta Decay Spectrum
(o) Gamma Decay
(p) Classification of Nuclear Reactions
(q) Nuclear Fission
(r) Chain Reaction and Nuclear Fusion.

All this is followed by an Exercise Set (fully solved) which contains questions as per the latest JEE pattern. At the end of Exercise Set, a collection of problems asked previously in JEE Main are also given.

NUCLEUS AND NUCLEAR STRUCTURE

Nucleus was discovered by Rutherford. The nucleus of an atom consists of two types of particles, protons and neutrons together called as Nucleons.

A proton has a positive charge equal to $+e = 1.6 \times 10^{-19}$ C and a mass equal to $m_p = 16726231 \times 10^{-27}$ kg. It was discovered by Goldstein.

A neutron has no charge i.e. is a neutral particle and its mass is $m_n = 1.6749286 \times 10^{-27}$ kg. *Thus, a neutron is slightly heavier than a proton.*

However, for problems (unless and until specified), we take

$$m_p \approx m_n = 1.6726231 \times 10^{-27} \text{ kg}$$

The total number of protons in the nucleus is called its atomic number (Z). The total number of nucleons (protons plus neutrons) in the nucleus is called its mass number (A). If N is the number of neutrons in the nucleus, then,

$$A = Z + N$$

No electrons are present inside the nucleus.

If X is the chemical symbol for an element then its nucleus is represented as ${}_Z^A X$ or as ${}_Z X^A$

ATOMIC MASS UNIT (u OR amu)

Atomic and nuclear masses are generally expressed in terms of atomic mass unit (a.m.u.). It is the nearest integer value of mass represented in a.m.u. (atomic mass unit).

$$1 \text{ amu} = \frac{1}{12}\begin{pmatrix}\text{Mass of } C^{12} \text{ atom at} \\ \text{rest in ground state}\end{pmatrix}$$

$$\Rightarrow \quad 1 \text{ amu} = 1 \text{ u} = 1.6603 \times 10^{-27} \text{ kg} = 931.478 \text{ MeV}/c^2$$

In general, we take

Mass of proton (m_p) = mass of neutron (m_n) = 1 a.m.u.

ISOTOPES

Nuclides having the same charge number (Z) but different mass number (A) are called isotopes. All the isotopes are chemically similar and hence they occupy the same position in the periodic table.

ISOBARS

Nuclides having the same mass number (A) but different atomic number (Z) are called isobars.

ISOTONES

Nuclides having the same neutron number $(A-Z)$ but different mass number (A) are called isotones.

ILLUSTRATION 1

The three stable isotopes of neon Ne^{20}, Ne^{21} and Ne^{22} have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u respectively. Obtain the average atomic mass of neon.

SOLUTION

Average atomic mass of neon

$$A_{\text{neon}} = \frac{p_1A_1 + p_2A_2 + p_3A_3}{p_1 + p_2 + p_3}$$

$$\Rightarrow \quad A_{\text{neon}} = \frac{90.51\times19.99 + 0.27\times20.99 + 9.22\times21.99}{100}$$

$$\Rightarrow \quad A_{\text{neon}} = \frac{1809.29 + 5.67 + 202.75}{100} = 20.184$$

NUCLEAR RADIUS

Assuming that the nuclei are spherical, their radii are well represented by the empirical formula

$$R = R_0A^{1/3}$$

where $R_0 = 1\cdot1\times10^{-15}\text{ m} = 1\cdot1\text{ fermi (fm)}$

ILLUSTRATION 2

Calculate the nuclear radius of ^{125}Fe knowing that the nuclear radius of ^{27}Al is 3.6 fermi.

SOLUTION

$$R = R_0A^{1/3}$$

$$\Rightarrow \quad \frac{R_{Fe}}{R_{Al}} = \left(\frac{A_{Fe}}{A_{Al}}\right)^{1/3} = \left(\frac{125}{27}\right)^{1/3}$$

$$\Rightarrow \quad R_{Fe} = \frac{5}{3}R_{Al} = \frac{5}{3}\times3.6 = 6\text{ fermi}$$

NUCLEAR DENSITY

The density of a nucleus of mass M and mass number A can be written as

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} = \frac{A\text{ amu}}{\frac{4}{3}\pi\left(R_0A^{1/3}\right)^3} = \frac{\left(A\times1.67\times10^{-27}\right)\text{ kg}}{\frac{4}{3}\pi\left(1\cdot1\times10^{-15}\text{ m}\right)^3 A}$$

$$\Rightarrow \quad \rho \cong 2.9\times10^{17}\text{ kgm}^{-3}$$

This comes out to be $\sim10^{17}\text{ kgm}^{-3}$, which is extremely large as compared to the density of ordinary matter which is $\sim10^3\text{ kgm}^{-3}$.

Conceptual Note(s)

Since density is independent of mass number A, so all nuclei same density. So, whether two nuclei are isobars, or isotopes or isotones. They must possess same density.

THE NUCLEAR FORCE

The force which binds the protons and neutrons inside the nucleus is neither electrical nor gravitational. It is an entirely different kind of force called the strong nuclear force. This force is extremely complex in nature. Some of its main characteristics are mentioned below.

(a) Nuclear forces are attractive in nature. Their magnitude, which depends upon inter nucleon distance is of very high order.

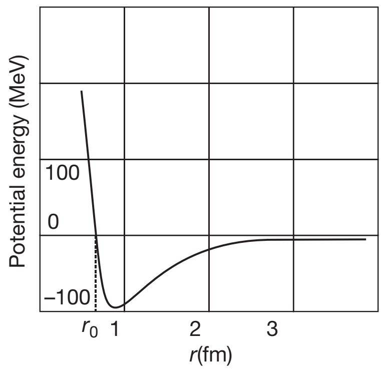
(b) Nuclear forces are charge independent. Nature of force remains the same whether we consider force between two protons, between two neutrons or between a proton and a neutron.

(c) These are short range forces. Nuclear forces operate between two nucleons situated in close neighbourhood only.

(d) Nuclear forces decrease very quickly with distance between two nucleons. Their rate of decrease is much more rapid than that of inverse square law forces (Coulombic forces). The forces become negligible when the nucleons are more than 10^{-14} cm apart.

(e) Nuclear forces are spin dependent. Nucleons having parallel spin are more strongly bound to each other than those having anti-parallel spin.

(f) The nuclear forces are very short-range forces. From a rough plot of the potential energy between two nucleons as a function of their separation is shown in figure.



For a separation greater than r_0, the force is attractive and for separations less than r_0, the force is strongly repulsive. The potential energy is a minimum at a distance r_0 about 0.8 fm from this the force is attractive for distances larger than 0.8 fm and repulsive for distances less than 0.8 fm. Nuclear forces are negligible when distance between nucleons is more than 10 fm.

YUKAWA THEORY OR MESON THEORY OF NUCLEAR FORCES

According to this theory, a nucleon consists of a core surrounded by a cloud of mesons, which may be charged or neutral. The mesons constantly get exchanged, back and forth, between two neighbouring nucleons. In this process the two nucleons remain bound to each other.

Proton-proton Interaction

It is the force between two neighbouring protons. It is due to the exchange of π^0 meson between them. It is represented in the form of a reaction as follows.

Proton P_1 emits π^0 and gets converted into a proton P_1', having different co-ordinates. So,

$$P_1 \longrightarrow P_1' + \pi^0$$

This π^0 is absorbed by P_2 which also gets converted into a new proton P_2'. Hence,

$$P_2 + \pi^0 \longrightarrow P_2'$$

Neutron-Neutron Interaction

It is the force between two neutrons. It is also due to exchange of π^0 between them

$$N_1 \longrightarrow N_1' + \pi^0$$

$$N_2 + \pi^0 \longrightarrow N_2'$$

Proton-Neutron Interaction

It is the force between a proton and a neutron situated close to each other. It can be take place in following two ways.

(a) Due to exchange of π^+ meson.

Proton emits π^+ meson and gets converted to a neutron. Another neutron absorbs this π^+ meson to get converted to a proton. So,

$$P \longrightarrow N' + \pi^+$$

$$N + \pi^+ \longrightarrow P'$$

(b) Due to exchange of π^- meson.

π^- meson is emitted by the neutron which is absorbed by the proton.

$$N \longrightarrow P' + \pi^-$$

$$P + \pi^- \longrightarrow N'$$

Dog-bone Analogy

The above interactions can be explained with the dog bone analogy according to which we consider the two interacting nucleons to be two dogs having a common bone clenched in between their teeth very firmly. Each one of these dogs wants to take the bone and hence they cannot be separated easily. They seem to be bound to each other with a strong attractive force (which is the bone) though the dogs themselves are strong enemies. The meson plays the same role of the common bone in between two nucleons.

MASS ENERGY EQUIVALENCE

According to Einstein the mass and energy are equivalent i.e., mass can be converted into energy and vice-versa. The mass energy equivalence relation is

$$\Delta E = c^2 \Delta m$$

Accordingly, annihilation of 1 kg mass is equivalent to energy given by

$$\Delta E = 1 \times (3 \times 10^8)^2$$

$$\Rightarrow \quad \Delta E = 9 \times 10^{16}\text{ J}$$

Energy corresponding to annihilation of 1 amu of mass is

$$\Delta E = (1.67 \times 10^{-27})(9 \times 10^{16})\text{ J}$$

$$\Rightarrow \quad \Delta E = \frac{(1.67 \times 10^{-27})(9 \times 10^{16})}{1.6 \times 10^{-19}}\text{ eV}$$

$$\Rightarrow \quad \Delta E = 931.5\text{ MeV}$$

MASS DEFECT AND BINDING ENERGY

It has been observed that the mass of a nucleus is always less than the mass of its constituent nucleons (i.e., protons + neutrons). *The difference between the total mass of the nucleons and the mass of the nucleus is called the mass defect* (Δm).

This is due to the fact that when nucleons combine to form a nucleus, the binding energy of nucleons is liberated. *The binding energy is equal to the work that must be done to split the nucleus into particles constituting it.*

Let $m({}_Z X^A)$ be the mass of nucleus, m_p be the mass of proton and m_n be the mass of neutron, then, the mass defect (Δm) is given by

$$\Delta m = \begin{pmatrix}\text{mass of constituent}\\ \text{nucleons}\end{pmatrix} - \begin{pmatrix}\text{mass of}\\ \text{nucleus}\end{pmatrix}$$

$$\Rightarrow \quad \Delta m = [Zm_p + (A-Z)m_n] - m({}_Z X^A)$$

This mass defect exists in the form of binding energy of nucleus, which is responsible for binding the nucleons into a small nucleus. So,

Binding energy of nucleus = $(\Delta m)c^2$ = $(931.5)\Delta m$ (in MeV) and binding energy per nucleon $= \dfrac{(\Delta m)c^2}{A}$

If the masses are taken in atomic mass unit, the binding energy is given by

$$\text{B.E.} = \left[\left(Zm_p + (A-Z)m_n\right) - m({}_Z X^A)\right]931.5\text{ MeV}$$

Dividing the binding energy by the number of nucleons A, in the nucleus, we obtain the binding energy per nucleon. The stability of a nucleus is measured by the binding energy per nucleon.

Conceptual Note(s)

(a) It is not the binding energy which accounts for the stability of nucleus.

(b) The stability of nucleus is governed by binding energy per nucleon. The more the binding energy per nucleon, the more stable a nucleus is.

BINDING ENERGY (BE): REVISITED

So, binding energy is the minimum energy required to break the nucleus into its constituent particles **OR** Binding energy is the amount of energy released during the formation of nucleus by its constituent particles and bringing them from infinite separation.

$$\text{Binding Energy } (BE) = \Delta mc^2$$

$$\Rightarrow \quad BE = \Delta m(\text{in amu}) \times 931.5\text{ MeV/amu}$$

$$\Rightarrow \quad BE = \Delta m \times 931.5\text{ MeV}$$

$$\Rightarrow \quad BE = \Delta m \times 931\text{ MeV}$$

Problem Solving Technique(s)

If binding energy per nucleon is more for a nucleus, then it is more stable. So, if $\left(\dfrac{BE_1}{A_1}\right) > \left(\dfrac{BE_2}{A_2}\right)$, then nucleus 1 would be more stable than nucleus 2.

ILLUSTRATION 3

During an experiment, following data is available about three nuclei P, Q and R. Arrange them in the decreasing order of stability.

	P	Q	R
Atomic mass number (A)	10	5	6
Binding energy (MeV)	100	60	66

SOLUTION

For P, we have $\left(\dfrac{BE}{A}\right)_P = \dfrac{100}{10} = 10$

For Q, we have $\left(\dfrac{BE}{A}\right)_Q = \dfrac{60}{5} = 12$

For R, we have $\left(\dfrac{BE}{A}\right)_R = \dfrac{66}{6} = 11$

$\Rightarrow$ Stability order is $Q > R > P$

ILLUSTRATION 4

A nucleus has binding energy of $100\ \text{MeV}$. It further releases $10\ \text{MeV}$ energy. Find the new binding energy of the nucleus.

SOLUTION

After releasing $10\ \text{MeV}$, it will become more stable and hence the binding energy of the nucleus will increase. So, new binding energy of the nucleus is

$$(BE)_{\text{new}} = 100 + 10 = 110\ \text{MeV}$$

ILLUSTRATION 5

Find the binding energy of $^{56}_{26}\text{Fe}$. Atomic mass of ^{56}Fe is $55.9349\ \text{u}$ and that of ^{1}H is $1.00783\ \text{u}$. Mass of neutron $= 1.00867\ \text{u}$.

SOLUTION

The number of protons in $^{56}_{26}\text{Fe} = 26$ and the number of neutrons is $(A - Z) = 56 - 26 = 30$.

The binding energy of $^{56}_{26}\text{Fe}$ is

$$BE = (26 \times 1.00783\ \text{u} + 30 \times 1.00867\ \text{u} - 55.9349\ \text{u})c^2$$

$$\Rightarrow \quad BE = (0.52878\ \text{u})c^2$$

$$\Rightarrow \quad BE = (0.52878\ \text{u})(931\ \text{MeV/u}) = 492\ \text{MeV}$$

ILLUSTRATION 6

Calculate the binding energy of $^{12}_{6}\text{C}$. Also find the binding energy per nucleon. Given that mass of $^{1}_{1}\text{H} = 1.0078\ \text{u}$, mass of $^{1}_{0}n = 1.0087\ \text{u}$ and mass of $^{12}_{6}\text{C} = 12.00004\ \text{u}$.

SOLUTION

One atom of $^{12}_{6}\text{C}$ consists of 6 protons, 6 electrons and 6 neutrons. The mass of the un-combined protons and electrons is the same as that of six $^{1}_{1}\text{H}$ atoms (if we ignore the very small binding energy of each proton-electron pair).

Mass of six $^{1}_{1}\text{H}$ atoms $= 6 \times 1.0078 = 6.0468\ \text{u}$

Mass of six neutrons $= 6 \times 1.0087 = 6.0522\ \text{u}$

Total mass of component particles $= 12.0990\ \text{u}$

Mass of $^{12}_{6}\text{C}$ atom $= 12.00004$

Mass defect is $\Delta m = 0.0990\ \text{u}$

Binding energy is $BE = (931)(0.099) = 92\ \text{MeV}$

Binding energy per nucleon is

$$\frac{BE}{A} = \frac{92}{12} = 7.66\ \text{MeV}$$

ILLUSTRATION 7

Calculate the binding energy per nucleon for $^{20}_{10}\text{Ne}$, $^{56}_{26}\text{Fe}$ and $^{238}_{92}\text{U}$. Given that mass of neutron is $1.008665\ \text{amu}$, mass of proton is $1.007825\ \text{amu}$, mass of $^{20}_{10}\text{Ne}$ is $19.992440\ \text{amu}$, mass of $^{56}_{26}\text{Fe}$ is $55.93492\ \text{amu}$ and mass of $^{238}_{92}\text{U}$ is $238.050783\ \text{amu}$.

SOLUTION

Binding energy of nucleus ${}_{Z}X^{A}$ is given by the equation

$$(BE)_{\text{Ne}} = \left[(A - Z)m_n + Zm_p - M\left({}^{A}_{Z}X\right)\right]c^2$$

$$\Rightarrow \quad (BE)_{\text{Ne}} = (\Delta m)c^2 = \left[\left(10m_n + 10m_p\right) - M\left({}^{20}_{10}\text{Ne}\right)\right]c^2$$

where,

$$\Delta m = \left(10(1.008665) + 10(1.007825)\right) - 19.992440$$

$$\Rightarrow \quad \Delta m = 0.17246\ \text{amu}$$

$$\Rightarrow \quad (BE)_{\text{Ne}} = (0.17246)(9315) = 160.64\ \text{MeV}$$

So, binding energy per nucleon is

$$\frac{(BE)_{\text{Ne}}}{A} = 8.03\ \text{MeV/nucleon}$$

Similarly, for $\left({}^{56}_{26}\text{Fe}\right)$, we have

$$(BE)_{\text{Fe}} = (\Delta m)c^2 = \left[\left(30m_n + 26m_p\right) - M\left({}^{56}_{26}\text{Fe}\right)\right]c^2$$

where,

$$\Delta m = \left(30(1.008665) + 26(1.007825)\right) - 55.93492$$

$$\Rightarrow \quad \Delta m = 0.52848\ \text{amu}$$

$$\Rightarrow \quad (BE)_{\text{Fe}} = (0.52848)(931.5) = 492\ \text{MeV}$$

Hence binding energy per nucleon is

$$\frac{(BE)_{\text{Fe}}}{A} = 8.79\ \text{MeV/nucleon}$$

Binding energy for $\left({}^{238}_{92}\text{U}\right)$ is

$$(BE)_{\text{U}} = \left[146m_n + 92m_p - M\left({}^{238}_{92}\text{U}\right)\right]c^2$$

where,

$$\Delta m = \left(146(1.008665) + 92(1.007825)\right) - 238.050783$$

$\Rightarrow \quad \Delta m = 1.934$ amu

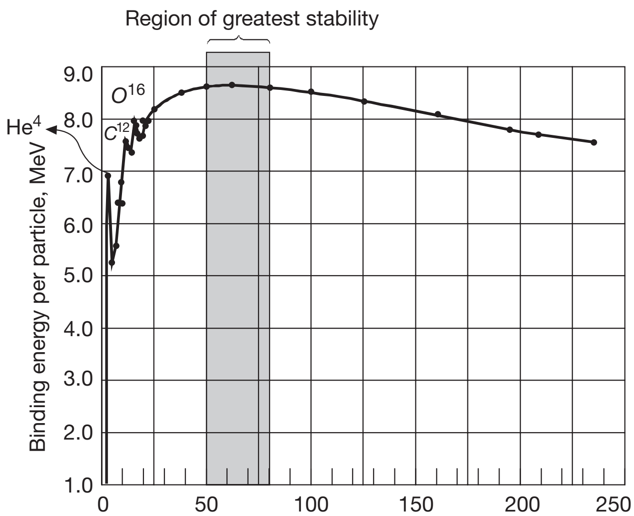
$\Rightarrow \quad (BE)_U = (1.934)(931.5) = 1802$ MeV

Binding energy per nucleon is

$$\frac{(BE)_U}{A} = \frac{1802}{238} = 7.57 \text{ MeV}$$

VARIATION OF BINDING ENERGY PER NUCLEON WITH MASS NUMBER *A*

The graph represents the average binding energy per nucleon in MeV against mass number A. It is observed that the binding energy for nuclei (except $_2He^4$, $_6C^{12}$ and $_8O^{16}$) rises first sharply, reaches a maximum value 8.5 MeV at $A = 50$ and then falls slowly, decreasing to 7.6 MeV for elements of higher mass number $A = 240$. Following facts can be concluded from this curve.



(a) The binding energy per nucleon for light nuclei, such as $_1H^2$, is very small ($\simeq 1$ MeV).

(b) The binding energy per nucleon increases rapidly for nuclei up to mass number 20 and the curve possesses peaks corresponding to nuclei $_2He^4$, $_6C^{12}$ and $_8O^{16}$. The peaks indicate that these nuclei are more stable than those in their neighbourhood. It confirms the reason for extraordinary stability of α-particle.

(c) After mass number 20, binding energy per nucleon increases gradually and for mass number between 40 and 120, the curve becomes more or less flat. The average value of binding energy per nucleon in this region is about 8.5 MeV. For $A = 56\left({}_{26}Fe^{56}\right)$, the binding energy per nucleon is maximum and it is equal to 8.8 MeV.

(d) After mass number 120, binding energy per nucleon starts decreasing and drops to 7.6 MeV for uranium. This low value of binding energy per nucleon in case of heavy nuclei is unable to have control over the repulsion between the large number of protons. Such nuclei are unstable and are found to disintegrate by emitting α-particles. The emission of α-particle not only decreases repulsive force inside the nucleus but also increases the value of B.E./A of the nucleus due to its extraordinary stable structure (α-particle has large binding energy). It is called α-decay.

Sometimes, the heavy nuclei increase the value of their B.E./A by emitting an electron. It is called β-decay. Inside the nucleus, an electron does not exist. It is created at the time of β-decay due to conversion of a neutron into proton. The β-decay leads to increase in Coulomb's repulsive force, but it increases B.E./A and also improves the neutron-proton ratio.

All such nuclei, which undergo α and β-decay are called radioactive nuclei.

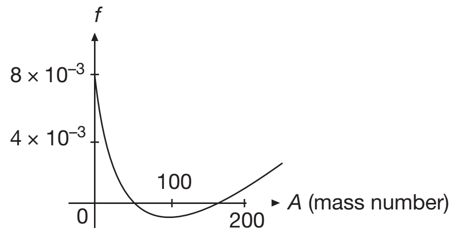
(e) The binding energy per nucleon has a low value for both very light and very heavy nuclei. In order to attain higher value of binding energy per nucleon, the lighter nuclei may unite together to form a heavier nucleus (process of nuclear fusion) or a heavier nucleus may split into lighter nuclei (process of nuclear fission). In both the nuclear processes, the resulting nucleus acquires greater value of binding energy per nucleon along with the liberation of enormous amount of energy.

PACKING FRACTION

Mass defect does not convey much information about nuclear stability and it is misleading to say that higher the mass defect more tightly bound nucleons exist in the nucleus. Packing fraction is a rather arbitrary but convenient and better means of expressing the binding energy. If M is the mass of an atom and A is its mass number then packing fraction

$$f = \frac{\text{Mass defect }(\Delta M)}{\text{Mass number}} = \frac{M - A}{A} = \frac{M - (Z+N)}{(Z+N)}$$

The smaller the value of packing fraction the more stable is the nucleus. From the graph of packing fraction f versus mass number A, following conclusions can be drawn.



(a) The packing fraction for very light nuclei like ${}_1^2H$, ${}_1^3H$ etc., are very large.

(b) As mass number increases packing fraction decreases becomes zero upto $A=16$ i.e., ${}_8^{16}O$ nucleus.

(c) For nuclei $16 < A < 180$ packing fraction becomes negative. The nucleons in these nuclei are strongly bound in the nucleus.

(d) Beyond $A > 180$ packing fraction is again positive. Thus, most of the nuclei with $A > 235$ are unstable.

NUCLEAR STABILITY

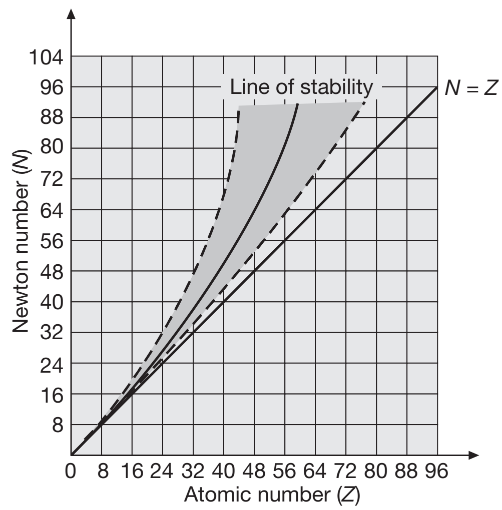
Among about 1500 known nuclides, less than 260 are stable. The others are unstable that decay to form other nuclides by emitting α, β-particles and γ-EM waves. (This process is called radioactivity.) The stability of nucleus is determined by many factors. Few such factors are given below:

(a) **Neutron-proton ratio $\left(\frac{N}{Z}\text{ Ratio}\right)$:** The chemical properties of an atom are governed entirely by the number of protons (Z) in the nucleus, the stability of an atom appears to depend on both the number of protons and the number of neutrons.

(i) For lighter nuclei, the greatest stability is achieved when the number of protons and neutrons are approximately equal $(N \approx Z)$ i.e., $\frac{N}{Z} = 1$.

(ii) **Heavy nuclei are stable only when they have more neutrons than protons.** Thus, heavy nuclei are neutron rich compared to lighter nuclei (for heavy nuclei, more is the number of protons in the nucleus, greater is the electrical repulsive force between them. Therefore, more neutrons are added to provide the strong attractive forces necessary to keep the nucleus stable.)

(iii) Figure shows a plot of N verses Z for the stable nuclei. For mass number upto about $A = 40$. For larger value of Z the nuclear force is unable to hold the nucleus together against the electrical repulsion of the protons unless the number of neutrons exceeds the number of protons.



At $Bi(Z = 83, A = 209)$, the neutron excess in $N - Z = 43$. There are no stable nuclides with $Z > 83$.

(b) **Even or odd numbers of Z or N:** The stability of a nuclide is also determined by the consideration whether it contains an even or odd number of protons and neutrons.

(i) It is found that an even-even nucleus (even Z and even N) is more stable (60% of stable nuclide have even Z and even N).

(ii) An even-odd nucleus (even Z and odd N) or odd-even nuclide (odd Z and even N) is found to be lesser sable while the odd-odd nucleus is found to be less stable.

(iii) Only five stable odd-odd nuclides are known: ${}_1H^2$, ${}_3Li^6$, ${}_5Be^{10}$, ${}_7N^{14}$ and ${}_{75}Ta^{180}$

(c) **Binding energy per nucleon:** The stability of a nucleus is determined by value of it's binding energy per nucleon. In general, higher the value of binding energy per nucleon, the more is the stability of the nucleus.

Test Your Concepts-I

Based on Nucleus Properties and Binding Energy

(Solutions on page H.87)

1. Calculate the nuclear radius of ^{70}Ge.
2. Calculate the electric potential energy of interaction due to the electric repulsion between two nuclei of ^{12}C when they touch each other at the surface. Assume that the potential energy of interaction between two nuclei is given by $U = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$.
3. Two stable isotopes of lithium $_3^6Li$ and $_3^7Li$ have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.0152 u and 7.016004 u respectively. Find the atomic weight of lithium.
4. Assuming the nuclei to be spherical in shape, how does the surface area of a nucleus of mass number A_1 compare with that of a nucleus of mass number A_2.
5. Calculate the density of $_6^{12}C$ nucleus. Take atomic mass of $_6^{12}C$ to be 12.000 amu and $R_0 = 1.2 \times 10^{-15}$ m.
6. Find the increase in mass of water when 1 kg of water absorbs 4.2×10^3 J of energy to produce a temperature rise of 1 K.
7. The binding energy of $_{17}^{35}Cl$ nucleus is 298 MeV. Calculate its approximate atomic mass. Given that, mass of proton is $m_p = 1.007825$ amu and mass of neutron is $m_n = 1.008665$ amu.
8. Calculate the binding energy of an alpha particle if mass of $_1^1H$ atom is 1.007826 u, mass of neutron is 1.008665 u, mass of $_2^4He$ atom is 4.00260 u. Take $1\,u = 931\,MeVc^{-2}$.
9. Show that the nuclide $_4^8Be$ has a positive binding energy but is unstable with respect to decay into two alpha particles, where masses of neutron, $_1^1H$, $_4^8Be$ and alpha particle are 1.008665 amu, 1.007825 amu, 8.005305 amu and 4.002603 amu respectively.
10. Find the binding energy and the the binding energy per nucleon of the nucleus of lithium isotope $_3^7Li$. Given that mass of $_3^7Li$ atom is 7.016005 amu, mass of $_1H^1$ atom is 1.007825 amu and mass of neutron is 1.008665 amu.

RADIOACTIVITY

The phenomenon of spontaneous emission of radiations $(\alpha, \beta, \gamma$ etc.$)$ by certain nuclei is called radioactivity. It is a nuclear phenomenon in which a heavy nucleus disintegrates itself without being forced to do so. It is a statistical probable process. The phenomenon of radioactivity was discovered by Becquerel.

LAWS OF RADIOACTIVE DISINTEGRATION

Rutherford-Soddy Law: Statistical Law

(a) Radioactivity is nuclear disintegration phenomenon. It is independent of all physical and chemical conditions.

(b) The disintegration is random and spontaneous statistical process. It is a matter of chance for any atom to disintegrate first.

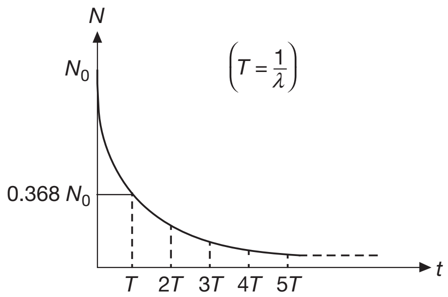
(c) The radioactive substances emit α or β particles along with γ-rays. These rays originate from the nuclei of disintegrating atoms and form fresh radioactive products with different physical and chemical properties.

The rate of decay of nuclei $\left(-\frac{dN}{dt}\right)$ is directly proportional to the number of undecayed nuclei (N) in the sample at time t.

$$\Rightarrow \quad -\frac{dN}{dt} \propto N$$

$$\Rightarrow \quad \frac{dN}{dt} = -\lambda N$$

where λ is constant of proportionality called Decay Constant or Disintegration Constant.



$$\Rightarrow \quad \int_{N}^{N} \frac{dN}{N} = -\lambda \int_{0}^{t} dt$$

$$\Rightarrow \quad \log_e \left(\frac{N}{N_0} \right) = -\lambda t$$

$$\Rightarrow \quad N = N_0 e^{-\lambda t}$$

where, N is the number of un-decayed nuclei in the sample at time t and N_0 is the number of un-decayed nuclei in the sample at time $t = 0$ (initially).

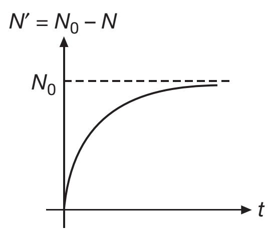
So, we conclude that the number of un-decayed nuclei in the sample decays exponentially with time.

Number of nuclei decayed i.e. the number of nuclei of B formed is given by

$$N' = N_0 - N$$

$$\Rightarrow \quad N' = N_0 - N_0 e^{-\lambda t}$$

$$\Rightarrow \quad N' = N_0 \left(1 - e^{-\lambda t}\right)$$



Displacement Laws

(a) When a nuclide emits α-particle, its mass number is reduced by four and atomic number by two, i.e.,

$$_zX^A \longrightarrow {}_{z-2}Y^{A-4} + {}_2\text{He}^4 + \text{Energy}$$

(b) When a nuclide emits a β-particle, its mass number remains unchanged but atomic number increases by one, i.e.,

$$_zX^A \longrightarrow {}_{z+1}Y^A + {}_{-1}\beta^0 + \bar{\nu} + \text{Energy}$$

where $\bar{\nu}$ is the antineutrino

The β-particle is not present initially in the nucleus but is produced due to disintegration of neutron into a proton. i.e.,

$$_0n^1 \longrightarrow {}_1H^1 + {}_{-1}\beta^0 + \bar{\nu} \text{ (antineutrino)}$$

When a proton is converted into a neutron, positive β-particle or positron is emitted.

$$_1H^1 \longrightarrow {}_0n^1 + {}_1\beta^0 + \nu \text{ (neutrino)}$$

(c) When a nuclide emits a gamma photon, neither the atomic number nor the mass number changes.

HALF LIFE ($T_{1/2}$)

The half life period of a radioactive substance is defined as the time in which one-half of the radioactive substance is disintegrated. If N_0 is initial number of radioactive atoms present, then in a half life time $T_{1/2}$, the number of undecayed radioactive atoms will be $\frac{N_0}{2}$ and in next half life $\frac{N_0}{4}$ and so on.

Conceptual Note(s)

So, at $t = T_{1/2}$, $N = \frac{N_0}{2}$

$$N_0 \xrightarrow{1T_{1/2}} \frac{N_0}{2} \xrightarrow{2T_{1/2}} \frac{N_0}{2^2} \xrightarrow{3T_{1/2}} \frac{N_0}{2^3} \longrightarrow \ldots\ldots \xrightarrow{nT_{1/2}} \frac{N_0}{2^n}$$

where $n = \frac{t}{T_{1/2}} = \frac{\text{Time Lapsed}}{\text{Half Life}}$

So, after n half lives, the fraction of undecayed nuclei in the sample is $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$, where $n = \frac{\text{time lapsed}}{T_{1/2}}$.

Since $N = N_0 e^{-\lambda t}$

$$\Rightarrow \quad \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\Rightarrow \quad e^{-\lambda T_{1/2}} = \frac{1}{2}$$

$$\Rightarrow \quad \lambda T_{1/2} = \log_e 2 = 0.693$$

$$\Rightarrow \quad T_{1/2} = \frac{0.693}{\lambda}$$

Table 3.1 Fraction of active/decayed atom at different time

Time (t)	Remaining fraction of active atoms (N/N_0) probability of survival	Fraction of atoms decayed $\frac{N_0 - N}{N_0}$ probability of decay
$t = 0$	$1\ (=100\%)$	0
$t = T_{1/2}$	$\frac{1}{2}\ (=50\%)$	$\frac{1}{2}\ (=50\%)$
$t = 2T_{1/2}$	$\frac{1}{4}\ (=25\%)$	$\frac{3}{4}\ (=75\%)$
$t = 3T_{1/2}$	$\frac{1}{8}\ (=12.5\%)$	$\frac{7}{8}\ (=87.5\%)$
$t = 10(T_{1/2})$	$\left(\frac{1}{2}\right)^{10} \approx 0.1\%$	$\approx 99.9\%$
$t = nT_{1/2}$	$\left(\frac{1}{2}\right)^{n}$	$1 - \left(\frac{1}{2}\right)^{n}$

ILLUSTRATION 8

At a given instant there are 25% undecayed radioactive nuclei in a sample. After 10 seconds the number of undecayed nuclei reduces to 12.5%. Calculate

(a) mean life of the nuclei
(b) the time in which the number of undecayed nuclei will further reduce to 6.25% of the reduced number.

SOLUTION

(a) In 10 s, number of nuclei has been reduced to half (25% to 12.5%).
Therefore, its half-life is

$$t_{1/2} = 10\text{ s}$$

Relation between half-life and mean life is

$$t_{\text{mean}} = \frac{t_{1/2}}{\log_e(2)} = \frac{01}{0.693}\text{ s}$$

$$\Rightarrow \quad t_{\text{mean}} = 14.43\text{ s}$$

(b) From initial 100% to reduction till 6.25%, it takes four half lives.

$$100\% \xrightarrow{t_{1/2}} 50\% \xrightarrow{t_{1/2}} 25\% \xrightarrow{t_{1/2}} 12.5\% \xrightarrow{t_{1/2}} 6.25\%$$

$$\Rightarrow \quad t = 4t_{1/2} = 4(10)\text{ s} = 40\text{ s}$$

$$\Rightarrow \quad t = 40\text{ s}$$

MEAN LIFE (τ)

The mean life or average life of a radioactive substance is equal to the average time for which the nuclei of atoms of the radioactive substance exist.

The average life of a sample can be calculated by finding the total life of all the nuclei of the substance and then dividing it by the total number of nuclei present in the sample initially. Mathematically

$$\tau = \frac{\int_0^{N_0} t\,dN}{\int_0^{N_0} dN} = \frac{1}{N_0}\int_0^{N_0} t\,dN = \frac{\frac{N_0}{\lambda}}{N_0}$$

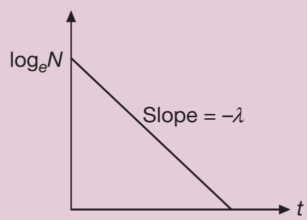
$$\Rightarrow \quad \tau = \frac{1}{\lambda}$$

Conceptual Note(s)

(a) From $N = N_0 e^{-\lambda t}$, we get

$$\log_e\left(\frac{N}{N_0}\right) = -\lambda t$$

So, the slope of the line shown in the graph i.e. the magnitude of inverse of slope of $\log_e\left(\frac{N}{N_0}\right)$ vs t curve is known as mean life (τ).



(b) From $N = N_0 e^{-\lambda t}$, if $t = \frac{1}{\lambda} = \tau$

$$\Rightarrow \quad N = N_0 e^{-1} = N_0\left(\frac{1}{e}\right) = 0.37N_0 = 37\% \text{ of } N_0.$$

i.e. mean life is the time interval in which number of undecayed atoms (N) becomes $\frac{1}{e}$ times or 0.37 times or 37% of original number of atoms. or
It is the time in which number of decayed atoms (N_0 - N) becomes $\left(1-\frac{1}{e}\right)$ times or 0.63 times or 63% of original number of atoms.

(c) Since, $T_{1/2} = \frac{0.693}{\lambda}$

$$\Rightarrow \quad \frac{1}{\lambda} = \tau = \frac{1}{0.693}\left(T_{1/2}\right) = 1.44\left(T_{1/2}\right)$$

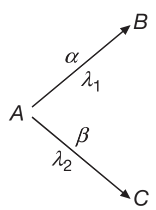
i.e. mean life is about 44% more than that of half life. Which gives us $\tau > T_{(1/2)}$
So, we conclude that

$$T_{1/2} = 0.693\tau$$

$$\Rightarrow \quad T_{1/2} < \tau$$

PARALLEL RADIOACTIVE DISINTEGRATION

Let a radioactive nucleus A decay to B and C through a simultaneous process. Assuming that the initial number of nuclei of A is N_0 and after the decay, at time t (say) the number of nuclei of A left is N_A, whereas the number of nuclei of B and C formed are N_B and N_C respectively.



Then at any instant the number of nuclei of A, B and C are given by

$$N_0 = N_A + N_B + N_C$$

$$\Rightarrow \quad \frac{dN_A}{dt} = -\frac{d}{dt}\left(N_B + N_C\right) \qquad \ldots(1)$$

Let A disintegrates into B and C by simultaneously emitting α and β particle respectively. Now, the rate of formation of B and C is proportional to the rate at which A decays. So, we have

$$\frac{dN_B}{dt} = +\lambda_1 N_A \text{ and } \frac{dN_C}{dt} = +\lambda_2 N_A$$

$$\Rightarrow \quad \frac{d}{dt}\left(N_B + N_C\right) = +\left(\lambda_1 + \lambda_2\right)N_A$$

Using equation (1), we get

$$\frac{dN_A}{dt} = -\left(\lambda_1 + \lambda_2\right)N_A$$

$$\Rightarrow \quad \lambda_{eff} = \lambda_1 + \lambda_2$$

$$\Rightarrow \quad t_{eff} = \frac{t_1 t_2}{t_1 + t_2}$$

Here t_{eff} can be taken as effective half life or effective average life.

ILLUSTRATION 9

The mean lives of a radio-active substances are 1620 years and 405 years for α emission and β emission respectively. Find out the time during which three-fourth of a sample will decay if it is decaying both by α emission and β emission simultaneously.

SOLUTION

Let at some instant of time t, number of atoms of the radioactive substance are N. It may decay either by α emission or by β emission. So, we can write,

$$\left(-\frac{dN}{dt}\right)_{net} = \left(-\frac{dN}{dt}\right)_{\alpha} + \left(-\frac{dN}{dt}\right)_{\beta}$$

If the effective decay constant is λ, then

$$\lambda N = \lambda_\alpha N + \lambda_\beta N$$

$$\Rightarrow \quad \lambda = \lambda_\alpha + \lambda_\beta = \frac{1}{1620} + \frac{1}{405}$$

$$\Rightarrow \quad \lambda = \frac{1}{324} \text{ year}^{-1}$$

Since, $N = N_0 e^{-\lambda t}$

$$\Rightarrow \quad \frac{N_0}{4} = N_0 e^{-\lambda t}$$

$$\Rightarrow \quad -\lambda t = \log_e\left(\frac{1}{4}\right) = -1.386$$

$$\Rightarrow \quad \left(\frac{1}{324}\right)t = 1.386$$

$$\Rightarrow \quad t = 449 \text{ year}$$

ILLUSTRATION 10

A radioactive nucleus can decay by two different processes, the half life for the first process is t_1 and that for the second process is t_2. Show that the effective half-life t of the nucleus is given by $\frac{1}{t}=\frac{1}{t_1}+\frac{1}{t_2}$.

SOLUTION

Let at any instant t, number of nuclei in radioactive sample be N. Then it may decay by either of two different processes. So,

$$-\frac{dN}{dt}=-\left(\frac{dN}{dt}\right)_1-\left(\frac{dN}{dt}\right)_2$$

$$\Rightarrow \quad \lambda N=\lambda_1 N+\lambda_2 N$$

$$\Rightarrow \quad \lambda=\lambda_1+\lambda_2$$

$$\Rightarrow \quad \frac{1}{t}=\frac{1}{t_1}+\frac{1}{t_2} \qquad \left\{\because\ t_{1/2}=t=\frac{0.693}{\lambda}\right\}$$

ILLUSTRATION 11

Prove mathematically that mean life or average life of a radioactive substance is $t_{av}=\frac{1}{\lambda}$.

SOLUTION

Let N be the number of atoms that exist at time t. Between t and $t+dt$ let dN atoms are decayed, then

Mean or average life is

$$t_{av}=\frac{\int_{N_0}^{0} t\,dN}{\int_{N_0}^{0} dN}$$

Since, $-\frac{dN}{dt}=\lambda N$

$$\Rightarrow \quad dN=-\lambda N dt$$

$$\Rightarrow \quad t_{av}=\frac{-\int_{0}^{\infty} t\lambda N dt}{-N_0}$$

Since, $N=N_0 e^{-\lambda t}$, so

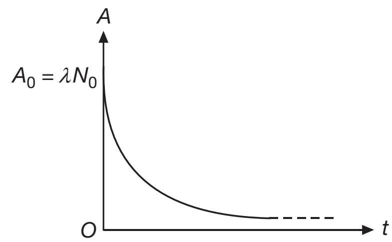
$$t_{av}=\frac{\int_{0}^{\infty} t\lambda N_0 e^{-\lambda t} dt}{N_0}$$

This integration is done by parts to get the result

$$t_{av}=\frac{1}{\lambda}$$

ACTIVITY OF RADIOACTIVE SUBSTANCE (*A*)

The activity of a radioactive substance means the rate of decay (or the number of disintegration/sec). This is denoted by



$$A=-\frac{dN}{dt}=-\frac{d}{dt}\left(N_0 e^{-\lambda t}\right)=\lambda N_0 e^{-\lambda t}=A_0 e^{-\lambda t} \text{ where}$$

$A_0=(\lambda N_0)$ is the activity at time $t=0$.

So, activity of a radioactive sample decreases exponentially with time.

SPECIFIC ACTIVITY

The activity per unit mass is called specific activity.

UNITS OF RADIOACTIVITY

Becquerel

In S.I. system the unit of radioactivity is becquerel.

1 becquerel = 1 disintegration/sec = 1 dps.

Rutherford

It is defined as a unit of activity equal to 10^6 dps.

Curie

The traditional unit of activity is curie. It is defined as $3\cdot 7\times 10^{10}$ dps which is also equal to the radioactivity of 1 g of pure Radium.

ILLUSTRATION 12

The half-life of Cobalt 60 is 5.25 years. How long after its activity have decreased to about one-eight of its original value?

SOLUTION

The activity is proportional to the number of undecayed atoms. In each half-life, half the remaining sample decays.

Since, $A = A_0\left(\frac{1}{2}\right)^n$ and $A = \frac{A_0}{8}$

$$\Rightarrow \quad \frac{A_0}{8} = A_0\left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad n = 3$$

Therefore, after three half-lives i.e. after 15.75 yr the sample decays to $\frac{1}{8}$th its original strength.

ILLUSTRATION 13

The half-life of ^{198}Au is 2.7 days. Calculate the

(a) decay constant
(b) average-life and
(c) activity for 1 mg of ^{198}Au.

SOLUTION

(a) The half-life and the decay constant are related as

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

$$\Rightarrow \lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{2.7 \text{ days}}$$

$$\Rightarrow \lambda = \frac{0.693}{2.7 \times 24 \times 3600 \text{ s}} = 2.9 \times 10^{-6} \text{ s}^{-1}$$

(b) The average-life is $t_{av} = \frac{1}{\lambda} = 3.9$ days

(c) The activity is $A = \lambda N$

Since, 198 g of ^{198}Au has 6×10^{23} atoms

The number of atoms in 1 mg of ^{198}Au is

$$N = 6 \times 10^{23} \times \frac{1 \text{ mg}}{198 \text{ g}} = 3.03 \times 10^{18}$$

$$\Rightarrow A = \lambda N = (2.9 \times 10^{-6} \text{ s}^{-1})(3.03 \times 10^{18})$$

$$\Rightarrow A = 8.8 \times 10^{12} \text{ disintegrations/s}$$

$$\Rightarrow A = \frac{8.8 \times 10^{12}}{3.7 \times 10^{10}} \text{ Ci} = 240 \text{ Ci}$$

ILLUSTRATION 14

A count rate-meter is used to measure the activity of a given sample. At one instant the meter shows 4750 counts per minute. Five minutes later it shows 2700 counts per minute.

(a) Find the decay constant
(b) Also, find the half life of the sample

SOLUTION

$$\text{Initial velocity } A_i = \left.\frac{dN}{dt}\right|_{t=0} = \lambda N_o = 4750 \quad \ldots(1)$$

$$\text{Final velocity } A_f = \left.\frac{dN}{dt}\right|_{t=5} = \lambda N = 2700 \quad \ldots(2)$$

Dividing (1) by (2), we get

$$\frac{4750}{2700} = \frac{N_0}{N_t} \quad \ldots(3)$$

The decay constant is given by

$$\lambda = \frac{2.303}{t}\log_e\left(\frac{N_0}{N_t}\right)$$

$$\Rightarrow \lambda = \frac{2.303}{5}\log_e\left(\frac{4750}{2700}\right) = 0.113 \text{ min}^{-1}$$

Half life of the sample is

$$T = \frac{0.693}{\lambda} = \frac{0.693}{0.113} = 6.14 \text{ min}$$

ILLUSTRATION 15

The half lives of radioisotopes P^{32} and P^{33} are 14 days and 25 days respectively. These radio isotopes are mixed in the ratio of $4:1$ of their atoms. If the initial activity of the mixed sample is 3 mCi, find the activity of the mixed isotopes after 60 days.

SOLUTION

Since, $R = \lambda N = \frac{N \log_e(2)}{t_{1/2}}$

$$\Rightarrow \quad R \propto \frac{N}{t_{1/2}}$$

Given, $\frac{N_1}{N_2} = \frac{4}{1}$ and $\frac{(t_{1/2})_1}{(t_{1/2})_2} = \frac{14}{25}$

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{N_1}{N_2} \times \frac{(t_{1/2})_2}{(t_{1/2})_1} = \frac{4}{1} \times \frac{25}{14} = \frac{100}{14}$$

$$\Rightarrow \quad R_1 = \frac{100}{114} \times 3 = 2.63 \text{ mCi}$$

and $R_2 = 3 - 2.63 = 0.37$ mCi

After 60 days, we have

$$R_1' = R_1 e^{-\lambda_1 t} = (2.63) e^{-\frac{0.693}{14} \times 60} = 0.135 \text{ mCi}$$

$$R_2' = R_2 e^{-\lambda_2 t} = (0.37) e^{-\frac{0.693}{25} \times 60} = 0.07 \text{ mCi}$$

So, total activity is

$$R' = R_1' + R_2' = 0.205 \text{ mCi}$$

ILLUSTRATION 16

A radioactive decay is given by $A \xrightarrow{T_{1/2}=8 \text{ yrs}} B$. It is known that only A is present at $t = 0$. Find the time at which if we are able to pick one atom out of the sample, then probability of getting B is 15 times of getting A.

SOLUTION

	A	$\rightarrow$	B
at $t = 0$	N_0		0
at $t = t$	N		$(N_0 - N)$

Probability of getting A, $P_A = \frac{N}{N_0}$

Probability of getting B, $P_B = \frac{N_0 - N}{N_0}$

Since according to the problem, we have $P_B = 15 P_A$

$$\Rightarrow \quad \frac{N_0 - N}{N_0} = 15\left(\frac{N}{N_0}\right)$$

$$\Rightarrow \quad N_0 = 16N$$

$$\Rightarrow \quad N = \frac{N_0}{16}$$

Since, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \frac{1}{16}$

$$\Rightarrow \quad n = 4$$

$$\Rightarrow \quad t = 4T_{1/2} = 4(8) = 32 \text{ yr}$$

ILLUSTRATION 17

The half-life of radium is 1620 years. How many radium atoms decay in 1 s in a 1 g sample of radium. The atomic weight of radium is 226 gmol^{-1}.

SOLUTION

Number of atoms in 1 g sample is

$$N = \left(\frac{0.001}{226}\right)\left(6.02 \times 10^{26}\right) = 2.66 \times 10^{21} \text{ atoms}$$

The decay constant is

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(1620)\left(3.16 \times 10^7\right)} = 1.35 \times 10^{-11} \text{ s}^{-1}$$

Taking one year $= 3.16 \times 10^7$ s

Now,

$$\frac{\Delta N}{\Delta t} = \lambda N = \left(1.35 \times 10^{-11}\right)\left(2.66 \times 10^{21}\right)$$

$$\Rightarrow \quad \frac{\Delta N}{\Delta t} = 3.6 \times 10^{10} \text{ s}^{-1}$$

Thus, 3.6×10^{10} nuclei decay in one second.

ILLUSTRATION 18

A certain radionuclide is known to become weaker in activity by 4% every hour. Find the decay constant and the mean life of the radionuclide. Given that $\ln(0.96) = -0.04$

SOLUTION

Suppose initial activity

$$A_0 = \lambda N_0$$

Activity after time t

$$A = \lambda N = \lambda N_0 e^{-\lambda t}$$

So, % decrease (η) in activity is $\eta = \frac{A_0 - A}{A_0} \times 100\%$

$$\Rightarrow \quad \eta = \frac{\lambda N_0 - \lambda N_0 e^{-\lambda t}}{\lambda N_0} \times 100$$

$$\Rightarrow \quad \eta = \left(1 - e^{-\lambda t}\right) \times 100$$

According to the problem, $\eta = 4\% = \frac{4}{100}$

$$\Rightarrow \quad \frac{4}{100} = 1 - e^{-\lambda t}$$

$$\Rightarrow \quad e^{-\lambda t} = 1 - 0.04 = 0.96$$

$$\Rightarrow \quad -\lambda t \log_e e = \log_e(0.96) = -0.04$$

$$\Rightarrow \quad \lambda t = 0.04$$

$$\Rightarrow \quad \lambda = \frac{0.04}{t} = \frac{0.04}{3600}\ s^{-1}$$

$$\Rightarrow \quad \lambda = 1.1\times10^{-5}\ s^{-1}$$

So, mean life is given by

$$T_{mean} = \frac{1}{\lambda} = 90000\ s$$

ILLUSTRATION 19

A F^{32} radio nuclide with half life $T = 14.3$ days is produced in a reactor at a constant rate $q = 2\times10^9$ nuclei per second. How soon after the beginning of production of that radio nuclide will its activity be equal to $R = 10^9$ disintegrations per second?

SOLUTION

$$N = \frac{R}{\lambda} = \frac{10^9}{\frac{0.693}{14.3\times3600}} = 7.43\times10^{13}$$

Now, $\dfrac{dN}{dt} = q - \lambda N$

$$\Rightarrow \quad \int_0^N \frac{dN}{q-\lambda N} = \int_0^t dt$$

$$\Rightarrow \quad N = \frac{q}{\lambda}\left(1-e^{-\lambda t}\right)$$

Substituting the values, we get

$$7.43\times10^{13} = \frac{2\times10^9}{\left(\frac{0.693}{14.3\times3600}\right)}\left(1-e^{-\left(\frac{0.693}{14.3}\times3600\right)t}\right)$$

Solving this equation, we get $t = 14.3$ hr.

ILLUSTRATION 20

There is a stream of neutrons with a kinetic energy of 0.0327 eV. If the half life of neutrons be $700\ s$, what fraction of neutrons will decay before they travel a distance of $10\ m$? Mass of neutron equal to $1.675\times10^{-27}\ kg$.

SOLUTION

Since, $K = \dfrac{1}{2}mv^2$

$$\Rightarrow \quad v = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2\times0.0327\times1.6\times10^{-19}}{1.675\times10^{-27}}} = 2.5\times10^3\ ms^{-1}$$

$$\Rightarrow \quad t = \frac{s}{v} = \frac{10}{2.5\times10^3} = 4\times10^{-3}\ s$$

Fraction decayed is $\dfrac{N}{N_0} = \dfrac{N_0\left(1-e^{-\lambda t}\right)}{N_0} = 1-e^{-\lambda t}$

$$\Rightarrow \quad \frac{N}{N_0} = 1-e^{-0.6930\times4\times10^{-3}/700}$$

$$\Rightarrow \quad \frac{N}{N_0} = 3.96\times10^{-6}$$

ILLUSTRATION 21

A radionuclide X is produced at constant rate α. At time $t = 0$, number of nuclei of X are zero. Find

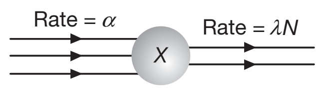
(a) the maximum number of nuclei of X.

(b) the number of nuclei at time t.

Decay constant of X is λ.

SOLUTION

(a) Let N be the number of nuclei of X at time t.



Rate of formation of $X = \alpha$ {given}

Rate of disintegration $= \lambda N$

Number of nuclei of X will increase until both the rates will become equal.

Therefore,

$$\alpha = \lambda N_{max}$$

$$\Rightarrow \quad N_{max} = \frac{\alpha}{\lambda}$$

(b) Net rate of formation of X at time t is,

$$\frac{dN}{dt} = \alpha - \lambda N$$

$$\Rightarrow \quad \frac{dN}{\alpha - \lambda N} = dt$$

Integrating with proper limits, we have

$$\int_0^N \frac{dN}{\alpha-\lambda N} = \int_0^t dt$$

$$\Rightarrow \quad N = \frac{\alpha}{\lambda}\left(1-e^{-\lambda t}\right)$$

This expression shows that number of nuclei of X are increasing exponentially from 0 to $\dfrac{\alpha}{\lambda}$.

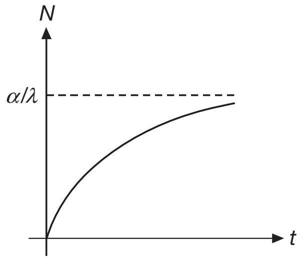


ILLUSTRATION 22

Natural uranium is a mixture of three isotopes ${}^{234}_{92}U$, ${}^{235}_{92}U$ and ${}^{238}_{92}U$ with mass percentage 0.01%, 0.71% and 99.28% respectively. The half life of three isotopes are 2.5×10^5 years. 7.1×10^8 years and 4.5×10^9 years respectively. Determine the share of radioactivity of each isotope into the total activity of the natural uranium.

SOLUTION

Let R_1, R_2 and R_3 be the activities of U^{234}, U^{235} and U^{238} respectively.

Total activity $R = R_1 + R_2 + R_3$

Share of U^{234}, $\dfrac{R_1}{R} = \dfrac{\lambda_1 N_1}{\lambda_1 N_1 + \lambda_2 N_2 + \lambda_3 N_3}$

Let m be the total mass of natural uranium.

Then $m_1 = \dfrac{0.01}{100}m$, $m_2 = \dfrac{0.71}{100}m$ and $m_3 = \dfrac{99.28}{100}m$

Now, $N_1 = \dfrac{m_1}{M_1}$, $N_2 = \dfrac{m_2}{M_2}$ and $N_3 = \dfrac{m_3}{M_3}$

where M_1, M_2 and M_3 are atomic weights.

$$\Rightarrow \quad \frac{R_1}{R} = \frac{\left(\dfrac{m_1}{M_1}\right)\dfrac{1}{T_1}}{\dfrac{m_1}{M_1}\dfrac{1}{T_1} + \dfrac{m_2}{M_2}\dfrac{1}{T_2} + \dfrac{m_3}{M_3}\dfrac{1}{T_3}}$$

$$\Rightarrow \quad \frac{R_1}{R} = \frac{\dfrac{0.01/100}{234} \times \dfrac{1}{2.5\times10^5 \text{ years}}}{\left[\left(\dfrac{\frac{0.01}{100}}{234}\right)\left(\dfrac{1}{2.5\times10^5}\right) + \left(\dfrac{\frac{0.71}{100}}{235}\right)\left(\dfrac{1}{7.1\times10^8}\right) + \left(\dfrac{\frac{99.28}{100}}{238}\right)\left(\dfrac{1}{4.5\times10^9}\right)\right]}$$

$$\Rightarrow \quad \frac{R_1}{R}\times 100\% = 0.648 \approx 64.8\%$$

Similarly, share of U^{235} is 0.016% and of U^{238} is 35.184%

ILLUSTRATION 23

Uranium ores on the earth at the present time typically have a composition consisting of 99.3% of the isotope ${}_{92}U^{238}$ and 0.7% of the isotope ${}_{92}U^{235}$. The half lives of these isotopes are $4.47\times10^9\,y$ and $7.04\times10^8\,y$ respectively. If these isotopes were equally abundant when the earth was formed, estimate the age of the earth.

SOLUTION

Let N_0 be number of atoms of each isotope at the time of formation of the earth $(t=0)$, let N_1 and N_2 be the number of atoms at present $(t=t)$. Then

$$N_1 = N_0 e^{-\lambda_1 t} \quad \ldots(1)$$

and $$N_2 = N_0 e^{-\lambda_2 t} \quad \ldots(2)$$

$$\Rightarrow \quad \frac{N_1}{N_2} = e^{(\lambda_2-\lambda_1)t} \quad \ldots(3)$$

Further it is given that

$$\frac{N_1}{N_2} = \frac{99.3}{0.7} \quad \ldots(4)$$

Equating (3) and (4) and taking log both sides, we get

$$(\lambda_2 - \lambda_1)t = \log_e\left(\frac{99.3}{0.7}\right)$$

$$\Rightarrow \quad t = \left(\frac{1}{\lambda_2-\lambda_1}\right)\log_e\left(\frac{99.3}{0.7}\right)$$

Substituting the values, we get

$$t = \frac{1}{\dfrac{0.693}{7.04\times10^8} - \dfrac{0.693}{4.47\times10^9}}\log_e\left(\frac{99.3}{0.7}\right)$$

$$\Rightarrow \quad t = 5.97\times10^9 \text{ y}$$

ILLUSTRATION 24

In the chemical analysis of a rock the mass ratio of two radioactive isotopes is found to be $100:1$. The mean lives of the two isotopes are 4×10^9 years and

2×10^9 years respectively. If it is assumed that at the time of formation the atoms of both the isotopes were in equal proportional, calculate the age of the rock. Ratio of the atomic weights of the two isotopes is $1.02:1$.

SOLUTION

At the time of observation i.e., at time t, we have

$$\frac{m_1}{m_2}=\frac{100}{1} \qquad \{\text{given}\}$$

Further it is given that

$$\frac{A_1}{A_2}=\frac{1.02}{1}$$

Number of atoms $N=\dfrac{m}{A}$

$$\Rightarrow \quad \frac{N_1}{N_2}=\frac{m_1}{m_2}\times\frac{A_2}{A_1}=\frac{100}{1.02} \qquad \ldots(1)$$

Let N_0 be the number of atoms of both the isotopes at the time of formation, then

$$\frac{N_1}{N_2}=\frac{N_0e^{-\lambda_1 t}}{N_0e^{-\lambda_2 t}}=e^{(\lambda_2-\lambda_1)t} \qquad \ldots(2)$$

Equating (1) and (2), we get

$$e^{(\lambda_2-\lambda_1)t}=\frac{100}{1.02}$$

$$\Rightarrow \quad (\lambda_2-\lambda_1)t=\log_e(100)-\log_e(1.02)$$

$$\Rightarrow \quad t=\frac{\log_e(100)-\log_e(1.02)}{\left(\dfrac{1}{2\times10^9}-\dfrac{1}{4\times10^9}\right)}$$

Substituting the values, we have

$$t=1.834\times10^{10} \text{ years}$$

ILLUSTRATION 25

The mass of carbon in an animal bone fragment found in an archaeological site is 200 g. If the bone registers an activity of 16 decays/s, what is its age? Assume that, when the animal was alive, the ratio of ${}_6C^{14}$ to ${}_6C^{12}$ in its bone was 1.3×10^{-12}.

SOLUTION

The 200 g of carbon is nearly all ${}^{12}_{6}C$

Since 12.0 g of ${}^{12}_{6}C$ contains 6.02×10^{23} atoms, so 200 g contains

$$\left(\frac{6.02\times10^{23}\text{ atoms}}{12\text{ g}}\right)(200\text{ g})=1.00\times10^{25}\text{ atoms}$$

When the animal was alive, the ratio of ${}^{14}_{6}C$ to ${}^{12}_{6}C$ in the bone was 1.3×10^{-12}. The number of ${}^{14}_{6}C$ nuclei at that time was

$$N_0=(1.00\times10^{25}\text{ atoms})(1.3\times10^{-12})$$

$$\Rightarrow \quad N_0=1.3\times10^{13}\text{ atoms.}$$

The magnitude of the activity when the animal was alive $(t=0)$ was

$$A_0=\left(\frac{dN}{dt}\right)_0=\lambda N_0$$

where $\lambda=3.83\times10^{-12}\text{ s}^{-1}$ as we calculated in one of previous example. So, the original activity was

$$A_0=\left(\frac{dN}{dt}\right)_0=\lambda N_0$$

$$\Rightarrow \quad A_0=(3.83\times10^{-12}\text{ s}^{-1})(1.3\times10^{13})=50\text{ s}^{-1}$$

Since, $A=\dfrac{dN}{dt}=\left(\dfrac{dN}{dt}\right)_0 e^{-\lambda t}=A_0e^{-\lambda t}$

where $\dfrac{dN}{dt}=A$ is given to be 16 s^{-1}.

$$\Rightarrow \quad 16=(50)e^{-\lambda t}$$

$$\Rightarrow \quad e^{\lambda t}=\frac{50}{16}$$

Taking natural logarithm of both sides, we get

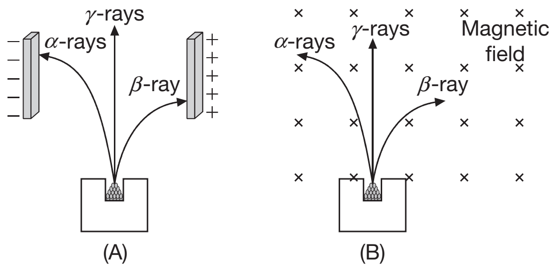
$$t=\frac{1}{\lambda}\ln\left(\frac{50}{16}\right)=\frac{1}{3.83\times10^{-12}\text{s}^{-1}}\ln\left(\frac{50}{16}\right)$$

$$\Rightarrow \quad t=2.98\times10^{11}\text{ s}=9400\text{ yr}$$

So, the time elapsed since the death of the animal is about 9400 yr.

NUCLEAR RADIATIONS

According to Rutherford's experiment when a sample of radioactive substance is put in a lead box and allow the emission of radiation through a small hole only. When the radiation enters into the external electric field, they splits into three parts (α-rays, β-rays and γ-rays)



Alpha Decay

Nearly 90% of the 2500 known nuclides are radioactive; they are not stable but decay into other nuclides

(i) When unstable nuclides decay into different nuclides, they usually emit alpha (α) or beta (β) particles.

(ii) Alpha emission occurs principally with nuclei that are too large to be stable. When a nucleus emits an alpha particle, its N and Z values each decrease by two and A decreases by four.

(iii) Alpha decay is possible whenever the mass of the original neutral atom is greater than the sum of the masses of the final neutral atom and the neutral helium- atom.

Beta Decay

There are different simple type of β-decay β^-, β^+ and electron capture.

(i) A beta minus particle (β^-) is an electron. Emission of β^- involves transformation of a neutron into a proton, an electron and a third particle called an antineutrino $(\bar{\nu})$.

(ii) β^- decay usually occurs with nuclides for which the neutron to proton ratio $\left(\frac{N}{Z}\text{ ratio}\right)$ is too large for stability.

(iii) In β^- decay, N decreases by one, Z increases by one and A doesn't change.

(iv) β^- decay can occur whenever the neutral atomic mass of the original atom is larger than that of the final atom.

(v) Nuclides for which $\frac{N}{Z}$ is too small for stability can emit a positron, the electron's antiparticle, which is identical to the electron but with positive charge. The basic process called beta plus β^+ decay

$$p \to n + \beta^+ + \nu \quad (\nu = \text{neutrino})$$

(vi) β^+ decay can occur whenever the neutral atomic mass of the original atom is at least two electron masses larger than that of the final atom

(vii) The mass of ν and $\bar{\nu}$ is zero. The spin of both is $\frac{1}{2}$ in units of $\frac{h}{2\pi}$. The charge on both is zero. The spin of neutrino is antiparallel to its momentum while that of antineutrino is parallel to its momentum.

(viii) There are a few nuclides for which β^+ emission is not energetically possible but in which an orbital electron (usually in the k-shell) can combine with a proton in the nucleus to form a neutron and a neutrino. The neutron remains in the nucleus and the neutrino is emitted.

$$p + \beta^+ \to n + \nu$$

Gamma Decay

The energy of internal motion of a nucleus is quantized. A typical nucleus has a set of allowed energy levels, including a ground state (state of lowest energy) and several excited states. Because of the great strength of nuclear interactions, excitation energies of nuclei are typically of the order of 1 MeV, compared with a few eV for atomic energy levels. In ordinary physical and chemical transformations, the nucleus always remains in its ground state. When a nucleus is placed in an excited state, either by bombardment with high-energy particles or by a radioactive transformation, it can decay to the ground state by emission of one or more photons called gamma rays or gamma-ray photons, with typical energies of 10 keV to 5 MeV. This process is called gamma (γ) decay. All the known conservation laws are obeyed in γ-decay.

The intensity of γ-decay after passing through a thickness x of a material is given by $I = I_0 e^{-\mu x}$ (μ = absorption co-efficient)

Table 3.2 Properties of α, β and γ-rays

Features	α-particles	β-particles	γ-rays
1. Identity	Helium nucleus or doubly ionised helium atom $\left({}_2\text{He}^4\right)$	Fast moving electron $(-\beta^0 \text{ or } \beta^-)$	Photons (E.M. waves)
2. Charge	$+2e$	$-e$	zero
3. Mass $4m_p$ (m_p = mass of proton $= 1.87 \times 10^{-27}$	$4m_p$	m_e	Massless
4. Speed	$\approx 10^7$ ms^{-1}	1% to 99% of speed of light	Speed of light
5. Range of kinetic energy	4 MeV to 9 MeV	All possible values between a minimum certain value to 1.2 MeV	Between a minimum value to 2.23 MeV
6. Penetration power (γ, β, α)	1 (Stopped by a paper)	100 (100 times of α)	10,000 (100 times of β upto 30 cm of iron (or Pb) sheet
7. Ionisation power ($\alpha > \beta > \gamma$)	10,000	100	1
8. Effect of electric or magnetic field	Deflected	Deflected	Not deflected
9. Energy spectrum	Line and discrete	Continuous	Line and discrete
10. Mutual interaction with matter	Produces heat	Produces heat	Produces, photo-electric effect, Compton effect, pair production
11. Equation of decay	${}_ZX^A \xrightarrow{\alpha-\text{decay}}$ ${}_{Z-2}Y^{A-4} + {}_2\text{He}^4$ ${}_ZX^A \xrightarrow{n\alpha} {}_{Z'}Y^{A'}$ $\Rightarrow \quad n_\alpha = \frac{A-A'}{4}$	${}_ZX^A \rightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + \bar{\nu}$ ${}_ZX^A \xrightarrow{n_\beta} {}_{Z'}X^A$ $\Rightarrow n_\beta = (2n_\alpha - Z + Z')$	${}_ZX^A \rightarrow {}_ZX^a + \gamma$

RADIOACTIVE SERIES

(a) If the isotope that results from a radioactive decay is itself radioactive then it will also decay and so on.

(b) The sequence of decays is known as radioactive decay series. Most of the radio-nuclides found in nature are members of four radioactive series. These are as follows

Table 3.3 Four radioactive series

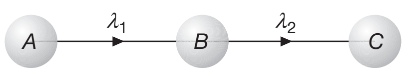
Mass number	Series (Nature)	Parent	Stable end product	Integer n
$4n$	Thorium (natural)	${}_{90}Th^{232}$	${}_{82}Pb^{208}$	52
$4n+1$	Neptunium (Artificial)	${}_{93}Np^{237}$	${}_{83}Bi^{209}$	52
$4n+2$	Uranium (Natural)	${}_{92}U^{238}$	${}_{82}Pb^{206}$	51
$4n+3$	Actinium (Natural)	${}_{89}Ac^{227}$	${}_{82}Pb^{207}$	51

(c) The $4n+1$ series starts from ${}_{94}Pu^{241}$ but commonly known as neptunium series because neptunium is the longest lived member of the series.

(d) The $4n+3$ series actually starts from ${}_{92}U^{235}$.

SUCCESSIVE DISINTEGRATION AND RADIOACTIVE EQUILIBRIUM

Suppose a radioactive element A disintegrates to form another radioactive element B which intern disintegrates to still another element C; such decays are called successive disintegration.



Rate of disintegration of A is $\frac{dN_1}{dt} = -\lambda_1 N_1$ (which is also the rate of formation of B)

Rate of disintegration of B is $\frac{dN_2}{dt} = -\lambda_2 N_2$

$$\therefore \quad \begin{pmatrix}\text{Net rate of}\\ \text{formation}\\ \text{of } B\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \text{decay}\\ \text{of } A\end{pmatrix} - \begin{pmatrix}\text{Rate of}\\ \text{decay}\\ \text{of } B\end{pmatrix}$$

$$\Rightarrow \quad (\text{Net rate of formation of } B) = \lambda_1 N_1 - \lambda_2 N_2$$

EQUILIBRIUM

In radioactive equilibrium, the rate of decay of any radioactive product is just equal to its rate of production from the previous member.

i.e., $\lambda_1 N_1 = \lambda_2 N_2$

$$\Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \frac{N_2}{N_1} = \frac{\tau_2}{\tau_1} = \frac{(T_{1/2})_2}{(T_{1/2})_1}$$

ILLUSTRATION 26

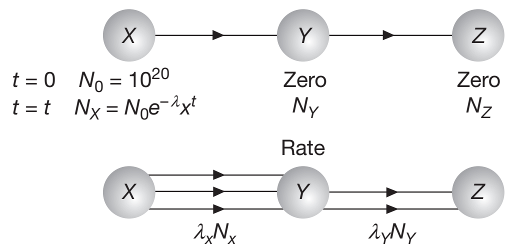
A radioactive nucleus X decays to a nucleus Y with a decay constant $\lambda_X = 0.1\ \text{s}^{-1}$. Y further decays to a stable nucleus Z with a decay constant $\lambda_Y = \frac{1}{30}\ \text{s}^{-1}$. Initially there are only X nuclei and their number is $N_0 = 10^{20}$. Setup the rate equations for the populations of X, Y and Z. The population of the Y nucleus as a function of time is given by

$$N_Y(t) = \left\{\frac{N_0 \lambda_X}{(\lambda_X - \lambda_Y)}\right\}\{\exp(-\lambda_Y t) - \exp(-\lambda_X t)\}$$

Find the time at which N_Y is maximum and determine the population of X and Z at that instant.

SOLUTION

Let at time $t = t$, number of nuclei of Y and Z are N_Y and N_Z. Then



Rate equation of the populations of X, Y and Z are

$$\left(\frac{dN_X}{dt}\right) = -\lambda_X N_X \qquad \ldots(1)$$

$$\left(\frac{dN_Y}{dt}\right) = \lambda_X N_X - \lambda_Y N_Y \qquad \ldots(2)$$

and $$\left(\frac{dN_Z}{dt}\right) = \lambda_Y N_Y \qquad \ldots(3)$$

Given $N_Y(t) = \frac{N_0\lambda_X}{\lambda_X - \lambda_Y}\left[e^{-\lambda_Y t} - e^{-\lambda_X t}\right]$

For N_Y to be maximum $\frac{dN_Y(t)}{dt} = 0$

i.e., $\lambda_X N_X = \lambda_Y N_Y$ {from equation (2)} ...(4)

$$\Rightarrow \lambda_X\left(N_0 e^{-\lambda_X t}\right) = \lambda_Y \frac{N_0\lambda_X}{\lambda_X - \lambda_Y}\left[e^{-\lambda_Y t} - e^{-\lambda_X t}\right]$$

$$\Rightarrow \frac{\lambda_X - \lambda_Y}{\lambda_Y} = \frac{e^{-\lambda_Y t}}{e^{-\lambda_X t}} - 1$$

$$\Rightarrow \frac{\lambda_X}{\lambda_Y} = e^{(\lambda_X - \lambda_Y)t}$$

$$\Rightarrow (\lambda_X - \lambda_Y)t\log_e(e) = \log_e\left(\frac{\lambda_X}{\lambda_Y}\right)$$

$$\Rightarrow t = \frac{1}{(\lambda_X - \lambda_Y)}\log_e\left(\frac{\lambda_X}{\lambda_Y}\right)$$

Substituting the values of λ_X and λ_Y, we have

$$t = \frac{1}{\left(0.1 - \frac{1}{30}\right)}\log_e\left(\frac{0.1}{\frac{1}{30}}\right) = 15\log_e(3)$$

$$\Rightarrow t = 16.48 \text{ s}$$

The population of X at this moment

$$N_X = N_0 e^{-\lambda_X t} = (10^{20})e^{-(0.1)(16.48)}$$

$$\Rightarrow N_X = 1.92 \times 10^{19}$$

Since, $N_Y = \frac{N_X\lambda_X}{\lambda_Y}$ {From equation (4)}

$$\Rightarrow N_Y = (1.92 \times 10^{19})\frac{(0.1)}{\frac{1}{30}} = 5.76 \times 10^{19}$$

$$\Rightarrow N_Z = N_0 - N_X - N_Y$$

$$\Rightarrow N_Z = 10^{20} - 1.92 \times 10^{19} - 5.76 \times 10^{19}$$

$$\Rightarrow N_Z = 2.32 \times 10^{19}$$

ILLUSTRATION 27

A factory produces a radioactive substance A at a constant rate R which decays with a decay constant λ to form a stable substance. Find

(i) the number of nuclei of A and

(ii) number of nuclei of B, at any time t assuming the production of A starts at $t = 0$.

(iii) also find out the maximum number of nuclei of A present at any time during its formation.

SOLUTION

(i) Factory $\xrightarrow[\text{constant rate}]{R} A \xrightarrow[\text{decay}]{\lambda} B$

Let N be the number of nuclei of A at any time t

$$\Rightarrow \frac{dN}{dt} = R - \lambda N$$

$$\Rightarrow \int_0^N \frac{dN}{R - \lambda N} = \int_0^t dt$$

On solving we will get

$$N = \frac{R}{\lambda}\left(1 - e^{-\lambda t}\right)$$

(ii) Number of nuclei of B at any time t,

$$N_B = Rt - N_A = Rt - \frac{R}{\lambda}\left(1 - e^{-\lambda t}\right) = \frac{R}{\lambda}\left(\lambda t - 1 + e^{-\lambda t}\right).$$

(iii) Maximum number of nuclei of A present at any time during its formation $= \frac{R}{\lambda}$.

ILLUSTRATION 28

Consider a radioactive disintegration according to the equation $A \to B \to C$. Decay constant of A and B is same and equal to λ. Number of nuclei of A, B and C are N_0, 0, 0 respectively at $t = 0$. Find

(a) Number of nuclei of B as function of time t.

(b) Time t at which the activity of B is maximum and the value of maximum activity of B.

SOLUTION

(a) Let the number of nuclei at any instant be shown in the table.

	A	**B**	**C**
At $t = 0$	N_0	0	0
At t	N_1	N_2	N_3

where, $N_1 = N_0 e^{-\lambda t}$ ...(1)

Now, $\frac{dN_2}{dt} = \lambda(N_1 - N_2)$

$\Rightarrow \quad \frac{dN_2}{dt} = \lambda N_0 e^{-\lambda t} - \lambda N_2$

$\Rightarrow \quad dN_2 + \lambda N_2 dt = \lambda N_0 e^{-\lambda t}$

$\Rightarrow \quad e^{\lambda t} dN_2 + \lambda N_2 e^{\lambda t} dt = \lambda N_0 dt$

$\Rightarrow \quad d\left(N_2 e^{\lambda t}\right) = \lambda N_0 dt$

Integrating, we get

$$\int_0^{N_2} d\left(N_2 e^{\lambda t}\right) = \int_0^t \lambda N_0 dt$$

$\Rightarrow \quad N_2 = \lambda N_0 t e^{-\lambda t}$

(b) Activity of B is, $R_2 = \lambda N_2 = \lambda^2 N_0 t e^{-\lambda t}$

For maximum activity, we have

$$\frac{dR_2}{dt} = 0$$

$\Rightarrow \quad t = \frac{1}{\lambda}$

$\Rightarrow \quad R_{max} = \frac{\lambda N_0}{e}$

Test Your Concepts-II

Based on Radioactivity

(Solutions on page H.88)

1. At time $t = 0$, number of nuclei of a radioactive substance are 100. At $t = 1$ s these numbers become 90. Find the number of undecayed nuclei at $t = 2$ s.
2. Find the amount of heat generated by 1 mg of Po^{210} preparation during the mean life period of these nuclei if the emitted alpha particles are known to possess kinetic energy 5.3 MeV and practically all daughter nuclei are formed directly in the ground state.
3. The radioactivity of a uranium specimen with mass number 238 is $2.5 \times 10^4\ s^{-1}$, the specimen's mass is 2 g. Find the half-life.
4. Ac^{227} has a half life of 21.8 years with respect to radioactive decay. The decay follows two parallel paths, one leading the Th^{227} and the other leading to Fr^{223}. The percentage yields of these two daughters nuclides are 1.2% and 98.8% respectively. What is the rate constant in yrs^{-1}, for each of the separate paths?
5. The disintegration rate of a certain radioactive sample at any instant is 4750 disintegrations per minute. Five minutes later the rate becomes 2700 per minute. Calculate
   (a) decay constant and
   (b) half-life of the sample.
6. In an agricultural experiment, a solution containing 1 mole of a radioactive material $\left(T_{1/2} = 14.3 \text{ days}\right)$ was injected into the roots of a plant. The plant was allowed 70 hours to settle down and then activity was measured in its fruit. If the activity measured was 1 μCi, what percentage of activity is transmitted from the root to the fruit in steady state?
7. A sample of 1 g of $^{109}_{83}Bi$ with a half life of 2.7×10^7 year decays into a stable isotope of thallium by emitting α particles.
   (a) What is the activity of the sample?
   (b) What will be the activity of the sample after 2 years?
   (c) After what time does the activity reduces to 25% of the original activity?
8. A number N_0 of atoms of a radioactive element are placed inside a closed volume. The radioactive decay constant for the nuclei of this element is λ_1. The daughter nuclei that form as a result of the decay process are assumed to be radioactive, too, with a radioactive decay constant λ_2. Determine the time variation of the number of such nuclei. Consider two limiting cases, when $\lambda_1 \gg \lambda_2$ and $\lambda_1 \ll \lambda_2$.
9. Calculate the probability that a radioactive atom having a mean life of 10 days decays during the fifth day.
10. Old wood from an Egyptian tomb, 4500 years old has C-14 activity of 7.3 dis. $min^{-1}\ g^{-1}$. Old wood

known to be 2500 years old has a C-14 activity 9.3 dis. min^{-1} g^{-1}.

(a) What is half life for C-14?

(b) What is the activity of fresh wood?

11. Determine the amount of $_{84}Po^{210}$ (polonium) necessary to provide a source of α particles of 5 millicurie strength. If half life of polonium is 138 days, given 1 curie $= 3.7 \times 10^{10}$ disintegrations/sec.

12. The specific activity of a preparation consisting of radioactive Co^{58} and non-radioactive Co^{59} is 2.2×10^{12} dps/g. The half life of Co^{58} is 71.3 days. Find the ratio of the mass of radioactive cobalt in that preparation to the total mass of the preparation.

13. A laboratory has 1.49 μg of pure $^{13}_{7}N$, which has a half-life of 10.0 min.

(a) How many nuclei are present initially?

(b) What is the activity initially?

(c) What is the activity after 1.00 h?

14. Suppose, the daughter nucleus in a nuclear decay is itself radioactive. Let λ_p and λ_d be the decay constants of the parent and the daughter nuclei. Also, let N_p and N_d be the number of parent and daughter nuclei at time t. Find the condition for which the number of daughter nuclei becomes constant.

15. A radioactive sample has 6×10^{18} active nuclei at a certain instant. How many of these nuclei will still be in the same active state after two half-lives?

16. The number of ^{238}U atoms in an ancient rock equals the number of ^{206}Pb atoms. The half-life of decay of ^{238}U is 4.5×10^9 y. Estimate the age of the rock assuming that all the ^{206}Pb atoms are formed from the decay of ^{238}U.

17. The decay constant for the radioactive nuclide ^{64}Cu is 1.516×10^{-5} s^{-1}. Find the activity of a sample containing 1 μg of ^{64}Cu. Atomic weight of copper = 63.5 g/mole . Neglect the mass difference between the given radioisotope and normal copper.

18. The half-life of a radioactive nuclide is 20 hours. What fraction of original activity will remain after 40 hours?

19. The age of a rock containing lead and uranium is equal to 1.5×10^9 years. The uranium is decaying into lead with half life equal to 4.5×10^9 years. Find the ratio of lead to uranium present in the rock, assuming that initially no lead was present in the rock. Given that $2^{\frac{1}{3}} = 1.259$.

20. In a radioactive disintegration process shown, A is continuously produced at the rate of 10^{21} per second.

$$\xrightarrow{10^{21}\text{ per second}} A \xrightarrow{\lambda = \frac{1}{30}} B$$

Find the maximum number of nuclei of A.

ARTIFICIAL TRANSMUTATION – NUCLEAR REACTIONS

A radio-active substance breaks up by emitting radiations. The daughter nucleus, left behind, has different physical and chemical properties and is assigned a new place in the periodic chart. Thus radio-activity is the phenomenon by which a substance gets converted into another one. This change can be brought about by artificial method, by bombarding a given nucleus with some radiation. The particles constituting the incident radiation must possess sufficient kinetic energy so as to penetrate into the given nucleus. As they enter the given nucleus, a compound nucleus (an intermediate state) is formed which is generally unstable. The compound nucleus then breaks up to produce product nucleus by emitting radiation. The process is, schematically, represented as

$$\underset{\left(\begin{array}{c}\text{Projectile}\\ \text{particle}\end{array}\right)}{P} + \underset{\left(\begin{array}{c}\text{Target}\\ \text{nucleus}\end{array}\right)}{T} \rightarrow \underset{\left(\begin{array}{c}\text{Compound}\\ \text{nucleus}\end{array}\right)}{C^*} \rightarrow \underset{\left(\begin{array}{c}\text{Residual}\\ \text{nucleus}\end{array}\right)}{R} + \underset{\left(\begin{array}{c}\text{Outgoing}\\ \text{particle or}\\ \text{radiation}\end{array}\right)}{O}$$

This is a reaction in which only the nuclei take part. Orbital electrons have no contribution to it. Such reactions are known as nuclear reactions.

LAWS GOVERNING NUCLEAR REACTIONS

(a) **Law of Charge:** The electric charge involved in nuclear reactions must be same before and after the reaction. So, charge number is conserved in nuclear reactions.

(b) Law of Number of Nucleons: The total number of nucleons involved in a nuclear reaction must be same before and after the reaction. So, mass number is also conserved in nuclear reactions.

(c) Law of Conservation of Energy: The total energy (rest mass energy + K.E.) of the reacting particles must be equal to the total energy of the product particles.

(d) Law of Conservation of Linear Momentum: The total linear momentum of the reacting particles must be equal to the total linear momentum of the product particles.

(e) Law of Conservation of Angular Momentum: Total angular momentum of nuclei before and after reaction must be the same.

Q-VALUE OF A NUCLEAR REACTION

Consider a nuclear reaction, schematically represented by equation

$$P+T\longrightarrow R+O$$

Let K_P, K_R and K_O be the kinetic energies of P (projectile particle), R (residual nucleus) and O (outgoing particle or radiation) respectively, while T (target nucleus) is taken to be at rest initially. Q-value of a nuclear reaction is defined as

$$Q=(K_R+K_O)-K_P$$

Let m_P, m_T, m_R, m_O, respectively, be the masses of P, T, R and O.

Since, for nor-relativistic approach, the total energy of a subatomic particle is equal to the sum of rest mass energy and its kinetic energy, so energy of P, T, R and O are respectively given by

$$E_P=m_Pc^2+K_P$$

$$E_T=m_Tc^2+0=m_Tc^2$$

$$E_R=m_Rc^2+K_R$$

$$E_O=m_Oc^2+K_O$$

Total initial energy is

$$\Sigma E_{\text{initial}}=(m_Pc^2+K_P)+m_Tc^2 \quad \ldots(1)$$

Total final energy is

$$\Sigma E_{\text{final}}=(m_Rc^2+K_R)+(m_Oc^2+K_O) \quad \ldots(2)$$

By Law of Conservation of Energy, we have

$$\Sigma E_{\text{initial}}=\Sigma E_{\text{final}}$$

$$\Rightarrow \quad (m_Pc^2+K_P)+m_Tc^2=(m_Rc^2+K_R)+(m_Oc^2+K_O)$$

$$\Rightarrow \quad Q=(K_R+K_O)-K_P=[(m_P+m_T)-(m_R+m_O)]c^2$$

$$\Rightarrow \quad Q=(\Delta m)c^2$$

where, $\Delta m=(m_P+m_T)-(m_R+m_O)$ is the mass defect between initial and final particles.

Conceptual Note(s)

Since the Q factor of a reaction is given by

$$Q=(K_R+K_O)-K_P$$

$$\Rightarrow \quad K_R+K_O=Q+K_P$$

Also we know that, the kinetic energy is a positive quantity and hence (K_R+K_O) must also be positive.

$$\Rightarrow \quad K_R+K_O>0$$

$$\Rightarrow \quad Q+K_P>0$$

So, a necessary but not sufficient condition for the occurrence of a nuclear reaction is that for the reaction

$$Q+K_P>0$$

$$\Rightarrow \quad K_P>-Q$$

CASE-I: EXOERGIC REACTION

A reaction is said to be exoergic if Q is positive.

$$\Rightarrow \quad (m_R+m_O)<(m_P+m_T)$$

The part of mass which disappears gets converted into the energy in accordance with Einstein's Mass-Energy equivalence.
Also, is positive if $(BE)_{\text{final}}>(BE)_{\text{initial}}$.

CASE-II: ENDOERGIC REACTION

A reaction is said to be endoergic if Q is negative.

$$\Rightarrow \quad (m_R+m_O)>(m_P+m_T) \text{ i.e. } (BE)_{\text{final}}<(BE)_{\text{initial}}$$

i.e., the sum of the masses of product particles is greater than that of reactant particles. So, energy is required, by the kinetic energies of the initial reactants, to make the reaction or transformation "go".

The minimum amount of energy that the bombarding particle or the projectile particle must have in order to initiate an endoergic reaction, is called Threshold Energy E_{Th}.

$$E_{\text{Th}} = -Q\left(\frac{m_P}{m_T}+1\right)$$

where, m_P is the mass of the projectile i.e. the nucleus used to hit the target and m_T is the mass of the target nucleus

Problem Solving Technique(s)

For a nuclear reaction $A+B \to C+D$, if the binding energies of A, B, C and D are given as B_1, B_2, B_3 and B_4, then the energy released in the reaction is

$$\Delta E = (B_3 + B_4) - (B_1 + B_2)$$

ILLUSTRATION 29

(a) A particle having mass m, kinetic energy K_{Lab} measured in the Lab Frame collides head-on with a stationary nucleus of mass M. If $K_{\text{wrt cm}}$ is the total kinetic energy of the system with respect to the centre of mass frame/coordinate system (i.e. the energy available to cause the reaction), then show that

$$K_{\text{wrt cm}} = \left(\frac{M}{M+m}\right)K_{\text{Lab}}$$

(b) If K_{cm} is the total kinetic energy of the centre of mass in the lab frame, then show that

$$K_{\text{cm}} = \left(\frac{m}{M+m}\right)K_{\text{Lab}}$$

(c) If K_{th} is the minimum kinetic energy (called Threshold energy) to cause an endoergic reaction in the above situation, then show that

$$K_{\text{th}} = -Q\left(1+\frac{m}{M}\right)$$

Try to prove this result, both using the concept of lab frame (L-Frame) and the centre of mass frame.

SOLUTION

(a) The velocity of the centre of mass of the system is

$$v_{\text{cm}} = \frac{mv + M(0)}{m+M} = \frac{mv}{m+M}$$

Let v'_m be the velocity of m with respect to the centre of mass and v'_M be the velocity of M with respect to the centre of mass, then

$$v'_m = v - v_{cm} = v - \left(\frac{mv}{m+M}\right) = \frac{Mv}{m+M} \quad \text{and}$$

$$v'_M = 0 - \frac{mv}{m+M} = -\frac{mv}{m+M}$$

In the centre of mass frame, the kinetic energy K_{cm} is given by

$$K_{\text{wrt cm}} = \frac{1}{2}mv'^2_m + \frac{1}{2}Mv'^2_M$$

$$\Rightarrow \quad K_{\text{wrt cm}} = \frac{m}{2}\frac{M^2v^2}{(m+M)^2} + \frac{M}{2}\frac{m^2v^2}{(m+M)^2}$$

$$\Rightarrow \quad K_{\text{wrt cm}} = \frac{1}{2}\left(\frac{Mm}{m+M}\right)\left(\frac{M}{m+M}+\frac{m}{m+M}\right)v^2$$

$$\Rightarrow \quad K_{\text{wrt cm}} = \frac{1}{2}\left(\frac{Mm}{m+M}\right)v^2 = \frac{1}{2}\mu v^2 \qquad \text{...(1)}$$

where μ is the reduced mass of the system given by

$$\mu = \frac{mM}{m+M}$$

Since, $K = K_{\text{Lab}} = \frac{1}{2}mv^2$, so equation (1) becomes

$$K_{\text{wrt cm}} = K_{\text{Lab}}\left(\frac{M}{m+M}\right) \qquad \text{...(2)}$$

(b) The kinetic energy of centre of mass K_{cm} is

$$K_{\text{cm}} = \frac{1}{2}(m+M)v^2_{cm}$$

$$\Rightarrow \quad K_{\text{cm}} = \frac{1}{2}(m+M)\left(\frac{mv}{m+M}\right)^2$$

$$\Rightarrow \quad K_{\text{cm}} = \frac{1}{2}\left(\frac{m^2v^2}{m+M}\right) = \frac{1}{2}mv^2\left(\frac{m}{m+M}\right)$$

$$\Rightarrow \quad K_{\text{cm}} = K_{\text{Lab}}\left(\frac{m}{m+M}\right) \qquad \text{...(3)}$$

(c) **From the Lab Frame (L-Frame)**

As we have seen that in the lab frame, the kinetic energy of centre of mass is

$$K_{cm} = K_{\text{Lab}}\left(\frac{m}{m+M}\right)$$

This energy being possessed by the centre of mass gives us a solid argument that the kinetic energy being carried by the projectile particle in the lab frame i.e. $K_{\text{Lab}}=\frac{1}{2}mv^2$ is not fully available to be dissipated in the reaction, because a part of this energy K_{cm} will be carried away by the centre of mass of the system. So, the available energy to be dissipated is

$$K_0=K_{\text{Lab}}-K_{\text{cm}}=K_{\text{Lab}}\left(\frac{M}{m+M}\right) \qquad \ldots(4)$$

Conceptual Note(s)

As already studied in Colisions, the loss in kinetic energy of the system, when a mass m having a velocity u collides head-on with a stationary mass M is given by

$$\text{Loss}=-\Delta K=\frac{1}{2}\left(\frac{mM}{m+M}\right)u^2$$

If K is the initial kinetic energy of the colliding particle in the lab frame, then the above expression can be re-witten as

$$\text{Loss}=-\Delta K=\left(\frac{M}{m+M}\right)K=K_{Lab}-K_{cm}$$

This loss in kinetic energy is actually calculated above in equation (4) as the difference of K_{Lab} and K_{cm}.

In addition to K_0, there is also the rest mass energy available for the the reaction as Q value of the reaction. So, the total energy available for the nuclear reaction is $(Q+K_0)$. To make the reaction go, the neessary and sufficient condition is that this sum $(Q+K_0)$ must be greater than zero. Hence

$$Q+K_0>0$$

$$\Rightarrow \quad K_0>-Q$$

$$\Rightarrow \quad K_{\text{Lab}}\left(\frac{M}{m+M}\right)>-Q$$

$$\Rightarrow \quad K_{\text{Lab}}>-Q\left(\frac{m+M}{M}\right)$$

$$\Rightarrow \quad K_{\text{Lab}}>-Q\left(1+\frac{m}{M}\right)$$

This minimum energy to be possessed by the projectile particle to initiate an endoergic reaction is called as the Threshold Energy K_{Th}. So

$$K_{\text{Th}}=-Q\left(\frac{m+M}{M}\right)=-Q\left(1+\frac{m}{M}\right)$$

From the Centre of Mass Frame

We understand that in the centre of mass frame, the total momentum of the particles is zero both before and after the collision. Also, in this frame, the sum of kinetic energy of the particles with respect to the centre of mass i.e. $K_{\text{wrt cm}}$ and the Q value of the reaction should be positive. So, for an endoergic reaction, at threshold, we have

$$K_{\text{wrt cm}}+Q>0\text{, where }(Q<0)$$

$$\Rightarrow \quad K_{\text{wrt cm}}>-Q$$

$$\Rightarrow \quad K_{\text{Lab}}\left(\frac{M}{m+M}\right)>-Q$$

$$\Rightarrow \quad K_{\text{Lab}}>-Q\left(\frac{m+M}{M}\right)$$

$$\Rightarrow \quad K_{\text{Lab}}>-Q\left(1+\frac{m}{M}\right)$$

This minimum energy to be possessed by the projectile particle to initiate an endoergic reaction is called as the Threshold Energy K_{Th}. So

$$K_{\text{Th}}=-Q\left(\frac{m+M}{M}\right)=-Q\left(1+\frac{m}{M}\right)$$

ILLUSTRATION 30

How much energy must a bombarding proton possess to cause the reaction ${}_3^7\text{Li}+{}_1^1\text{H}\rightarrow{}_4^7\text{Be}+{}_0^1n$. Given that mass of Li atom is 7.01600 u, mass of Be atom is 7.01693 u, mass of H atom is 1.0783 u and mass of neutron is 1.0866 u.

SOLUTION

Since the mass of an atom also includes the masses of the atomic electrons, so to get the mass of the nucleus, the appropriate electron masses must be subtracted from the given masses of atoms.

Reactants	Products
$m\left({}_3^7\text{Li}\right)=7.01600-3m_e$	$m\left({}_4^7\text{Be}\right)=7.01693-4m_e$
$m\left({}_1^1\text{H}\right)=1.0783-1m_e$	$m\left({}_0^1n\right)=1.0866$
$\text{Total}=8.02383-4m_e$	$\text{Total}=8.02559-4m_e$

The Q-value of the reaction is given by

$$Q = -0.00176 \text{ u} = -1.65 \text{ MeV}$$

Since the energy is supplied as kinetic energy of the bombarding particle i.e. proton, so the incident proton must have kinetic energy more than this energy, because the system must possess some kinetic energy even after the reaction, so that momentum is conserved.

With momentum conservation taken into account, the minimum kinetic energy that the incident particle is given by

$$E_{\text{th}} = -\left(1+\frac{m}{M}\right)Q = -\left(1+\frac{1}{7}\right)(-1.65) = 1.89 \text{ MeV}$$

ENERGETICS OF NUCLEAR REACTIONS

Consider a nuclear reaction, schematically represented by equation

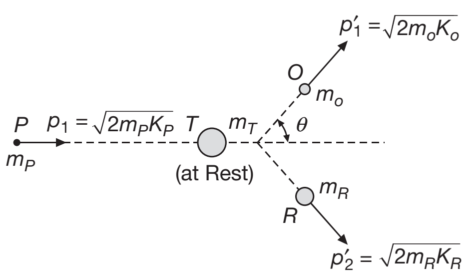
$$P + T \longrightarrow R + O$$

Let K_P, K_R and K_O be the kinetic energies of P (projectile particle), R (residual nucleus) and O (outgoing particle or radiation) respectively, while T (target nucleus) is taken to be at rest initially. Q-value of a nuclear reaction is defined as

$$Q = (K_R + K_O) - K_P$$

Let m_P, m_T, m_R, m_O, respectively, be the masses of P, T, R and O.

It can be shown that, by using Conservation of momentum, if we know the kinetic energy K_O of the outgoing paticle (O) and the angle θ of the outgoing particle with respect to the direction of motion of the projectile particle (P), then we can easily calculate the Q value of the reaction.



For the sake of convenience, let the momentum of projectile particle (P) before collision be p_1, then

$$p_1 = \sqrt{2m_pK_P} \qquad \ldots(1)$$

Since the target nucleus (T) is at rest, so momentum and kinetic energy of target nucleus both are zero.

Similarly, let the momentum of the outgoing particle (O) be p_1', then

$$p_1' = \sqrt{2m_OK_O} \qquad \ldots(2)$$

Similarly, let the momentum of the residual nucleus (R) be p_2', then

$$p_2' = \sqrt{2m_RK_R} \qquad \ldots(3)$$

All the above momenta and kinetic energies are expressed in the Lab Frame (i.e. in the laboratory frame).

Applying Conservation of Linear Momentum, we get

$$p_1' + p_2' = p_1$$

$$\Rightarrow \quad p_2' = p_1 - p_1' \qquad \ldots(4)$$

Squaring (4), we get

$$p_2'^2 = (p_1 - p_1')^2 = p_1^2 + p_1'^2 - 2p_1p_1'\cos\theta$$

where θ is the angle between p_1 and p_1'.

Applying definition of Q value, we get

$$Q = (K_R + K_O) - K_P = \left(\frac{p_2'^2}{2m_R} + \frac{p_1'^2}{2m_O}\right) - \frac{p_1^2}{2m_P}$$

$$\Rightarrow \quad Q = \left(\frac{p_1^2 + p_1'^2 - 2p_1p_1'\cos\theta}{2m_R} + \frac{p_1'^2}{2m_O}\right) - \frac{p_1^2}{2m_P}$$

$$\Rightarrow \quad Q = \frac{p_1'^2}{2}\left(\frac{1}{m_R} + \frac{1}{m_O}\right) + \frac{p_1^2}{2}\left(\frac{1}{m_R} - \frac{1}{m_P}\right) - \frac{p_1p_1'}{m_R}\cos\theta$$

$$\Rightarrow \quad Q = \frac{p_1'^2}{2m_O}\left(1 + \frac{m_O}{m_R}\right) + \frac{p_1^2}{2m_P}\left(\frac{m_P}{m_R} - 1\right) - \frac{p_1p_1'}{m_R}\cos\theta \qquad \ldots(5)$$

Using equations (1), (2) and (3) we can express the above result as

$$Q = K_O\left(1 + \frac{m_O}{m_R}\right) - K_P\left(1 - \frac{m_P}{m_R}\right) - \frac{2\sqrt{m_Pm_OK_PK_O}}{m_R}\cos\theta$$

So, we observe thar, if we know the kinetic energy K_O of the outgoing paticle (O) and the angle θ of the outgoing particle with respect to the direction of motion of the projectile particle (P), then we can easily calculate the Q value of the reaction.

ILLUSTRATION 31

Consider a body at rest in the Lab Frame. If the body explodes in two fragments of masses m_1 and m_2, calculate the final kinetic energies of the fragments in terms of Q value and the masses of the fragments.

SOLUTION

Since the body is initially at rest, hence its total initial momentum is zero. So, due to the explosion, the two fragments will fly off in opposite directions. If p_1 and p_2 be the momentum of the fragments after explosion, then

$$p_1 + p_2 = 0$$

$$\Rightarrow \quad p_1 = -p_2$$

So, the magnitude of momentum will of the fragments is same after the explosion i.e. $|p_1| = |p_2| = p$ (say).

So, the final kinetic energy after explosion is

$$K_{\text{final}} = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} = \frac{p^2}{2}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)$$

whereas, the initial kinetic energy of the system is zero.

Since, $Q = \Sigma K_{\text{final}} - \Sigma K_{\text{initial}}$

$$\Rightarrow \quad Q = \Sigma K_{\text{final}} = \frac{p^2}{2}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)$$

$$\Rightarrow \quad p = \sqrt{\left(\frac{2m_1 m_2}{m_1 + m_2}\right)Q} = \sqrt{2\mu Q} \quad \ldots(1)$$

where μ is the reduced mass of the system given by

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

The kinetic energies of the fragments are

$$K_1 = \frac{p_1^2}{2m_1} = \frac{p^2}{2m_1} \text{ and } K_2 = \frac{p_2^2}{2m_2} = \frac{p^2}{2m_2}$$

Substituting the value of p from equation (1), we get

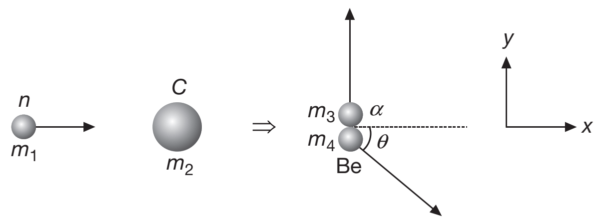
$$K_1 = \frac{p^2}{2m_1} = \left(\frac{m_2}{m_1 + m_2}\right)Q \text{ and}$$

$$K_2 = \frac{p^2}{2m_2} = \left(\frac{m_1}{m_1 + m_2}\right)Q$$

ILLUSTRATION 32

A neutron with kinetic energy $K = 10$ MeV activates a nuclear reaction $n + {}^{12}\text{C} \longrightarrow {}^{9}\text{Be} + \alpha$. Find the kinetic energy of the alpha particles outgoing at right angle to the direction of incoming neutrons. Take $u = 931.5$ MeV and threshold energy of reaction $(E_{\text{th}}) = 6.17$ MeV.

SOLUTION



Since, $Q + K_1 = K_3 + K_4 \quad \ldots(1)$

Applying Law of Conservation of Linear Momentum

Along x-axis

$$\sqrt{2m_1K_1} = \sqrt{2m_4K_4}\cos\theta \quad \ldots(2)$$

Along y-axis

$$\sqrt{2m_3K_3} = \sqrt{2m_4K_4}\sin\theta \quad \ldots(3)$$

Squaring and adding equations (2) and (3), we get

$$m_1K_1 + m_3K_3 = m_4K_4$$

$$\Rightarrow \quad K_4 = \frac{m_1}{m_4}K_1 + \frac{m_3}{m_4}K_3$$

Substituting value of K_4 in equation (1) and rearranging, we get

$$Q + \left(1 - \frac{m_1}{m_4}\right)K_1 = \left(1 + \frac{m_3}{m_4}\right)K_3$$

where, $Q = \dfrac{-E_{th}}{\left(1 + \dfrac{m_1}{m_2}\right)} = \dfrac{-6.17}{1 + \dfrac{1}{12}} = -5.69$ MeV

$$\Rightarrow \quad \left(1 + \frac{4}{9}\right)K_3 = -5.69 + \left(1 - \frac{1}{9}\right)(10)$$

$$\Rightarrow \quad K_3 = 2.21 \text{ MeV}$$

ALPHA DECAY

Alpha decay is a process in which an unstable nucleus transforms itself into a new nucleus by emitting an alpha particle (a helium nucleus, ${}_{2}^{4}\text{He}$).

Since an α-particle has two protons and two neutrons, so after an α-decay, the parent nucleus is transformed into a daughter nucleus with mass number smaller by 4 and atomic number smaller by 2.

An alpha decay can be expressed by the equation

$${}_{Z}^{A}X \rightarrow {}_{Z-2}^{A-4}Y + {}_{2}^{4}\text{He} + Q$$

Here Q is the energy released in the process and is determined from Einstein's mass-energy relation which gives

$$Q = (m_X - m_Y - m_{He})c^2$$

where m_X, m_Y and m_{He} are the masses of the parent nucleus X, daughter nucleus Y and the α-particle respectively. The energy Q is shared by the daughter nucleus X and the α-particle. As the parent nucleus is at rest before its α-decay, the α-particles are emitted with fixed energy and hence are mono-energetic. This energy can be determined by Applying the Laws of Conservation of Energy and Momentum.

For example, uranium (238) on emitting an α-particle changes into thorium (234) as

$${}_{92}^{238}\text{U} \longrightarrow {}_{90}^{234}\text{Th} + {}_{2}^{4}\text{He} + Q$$

Similarly, polonium (208) is transmuted into lead (204) as

$${}_{84}^{208}\text{Po} \longrightarrow {}_{82}^{204}\text{Pb} + {}_{2}^{4}\text{He} + Q$$

Generally, the nuclei with mass number 210 or more undergo α-decay. In such nuclei, the long range repulsive forces between the protons dominate over the short range nuclear forces which bind the various nucleons together. By emitting α-particles, these nuclei achieve greater stability. An α-particle has a high value of binding energy $(\approx 28 \text{ MeV})$. After the emission of an α-particle, the binding energy per nucleon of the emitting nucleus increases and the residual nucleus becomes more stable.

Speed of Emitted α-particles

Consider the alpha decay process equation i.e.,

$${}_{Z}^{A}X \longrightarrow {}_{Z-2}^{A-4}Y + {}_{2}^{4}\text{He} + Q$$

The speed of the emitted α-particles can be calculated by using the Laws of Conservation of Energy and Momentum.

Suppose the parent nucleus ${}_{Z}^{A}X$ be at rest before decay. Let v_α and v_Y be the velocities of the α-particle and the daughter nucleus. Applying the Law of Conservation of Momentum, we get

$$m_Y v_Y = m_\alpha v_\alpha \qquad \dots(1)$$

As the energy Q released in the decay process appears in the form of kinetic energy of α-particle and the daughter nucleus, so we have

$$\frac{1}{2}m_\alpha v_\alpha^2 + \frac{1}{2}m_Y v_Y^2 = Q$$

Substituting the value of v_Y from equation (1), we get

$$\frac{1}{2}m_\alpha v_\alpha^2 + \frac{1}{2}\frac{m_\alpha^2 v_\alpha^2}{m_Y^2} = Q$$

$$\Rightarrow \quad \frac{1}{2}m_Y m_\alpha v_\alpha^2 + \frac{1}{2}m_\alpha^2 v_\alpha^2 = m_Y Q$$

$$\Rightarrow \quad \frac{1}{2}(m_Y + m_\alpha)m_\alpha v_\alpha^2 = m_Y Q$$

$$\Rightarrow \quad K_\alpha = \frac{1}{2}m_\alpha v_\alpha^2 = \left(\frac{m_Y}{m_Y + m_\alpha}\right)Q$$

Since, $m_Y = (A-4)$ amu and $m_\alpha = 4$ amu, so we have

$$K_\alpha = \frac{1}{2}m_\alpha v_\alpha^2 = \left(\frac{A-4}{A}\right)Q$$

$$\Rightarrow \quad v_\alpha = \sqrt{\frac{2K_\alpha}{m_\alpha}} = \sqrt{\frac{2(A-4)Q}{Am_\alpha}}$$

For example, in the α-decay of a random nucleus ${}_{86}^{222}\text{Rn}$, we have

$$Q = 5.587 \text{ MeV}$$

$$\Rightarrow \quad K_\alpha = \left(\frac{A-4}{A}\right)Q = \frac{(222-4)}{222} \times 5.587 \text{ MeV}$$

$$\Rightarrow \quad K_\alpha = 5.486 \text{ MeV} = 5.486 \times 1.6 \times 10^{-19} \text{ J}$$

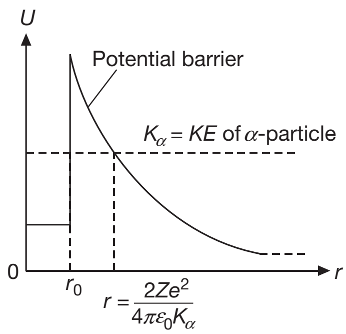
Since, $m_\alpha = 4 \text{ amu} = 4 \times 1.66 \times 10^{-27} \text{ kg}$

$$\Rightarrow \quad v_\alpha = \sqrt{\frac{2 \times 5.486 \times 1.6 \times 10^{-19}}{4 \times 1.66 \times 10^{-27}}} \text{ ms}^{-1}$$

$$\Rightarrow \quad v_\alpha = 1.62 \times 10^7 \text{ ms}^{-1}$$

Theory of α-decay (Tunnelling Effect)

The α-particles emitted by different radioactive nuclei have kinetic energy ranging from 4 MeV to 9 MeV. The nucleus of an α-emitter possesses a barrier of height about 25 MeV. Figure shows a plot of the potential energy U of the system consisting of the α-particle and the residual nucleus.



Plot of potential energy U of an α - particle as a function of distance r from the centre of the residual nucleus.

The α-particles are short of about 16 to 25 MeV of energy, needed for the emission. Therefore, classically, we cannot explain the emission of α-particles by radioactive nuclei.

In 1928, Gamow, Congdon and Gurney explained the emission of α-particles in terms of the penetration of the nuclear potential barrier on the basis of Quantum Theory. According to this theory, we have

(a) An α-particle may exist as an entity (already formed) inside a nucleus before it escapes from the nucleus.

(b) The α-particle is in a state of constant motion inside the nucleus with a speed of about 10^7 ms^{-1}.

(c) Quantum mechanically, even an α-particle having insufficient kinetic energy has a small but finite probability p of its crossing the potential barrier.

As the size of the nucleus $\approx 10^{-14}\text{ m}$ and speed of α-particle $\approx 10^7\text{ ms}^{-1}$, the α-particle takes about 10^{-21} s to move across the nucleus. Thus α-particle presents itself before the potential barrier 10^{21} times in a second. The probability P of escape of an α-particle from a nucleus will be

$$P = p\nu$$

As ν is large $\left(10^{21}\text{ s}^{-1}\right)$, so P is sufficiently large and the α-particle can tunnel through the energy barrier which is classically impossible. Hence α-decay occurs as a result of barrier tunnelling.

The barrier tunnelling explains why every $^{238}_{92}\text{U}$ nuclide in a sample of $^{238}_{92}\text{U}$ atoms does not decay at once, even when its decay process has a positive Q value. Consequently, the half-lives for α-decay of most of the alpha unstable nuclei are very long. For example, the half-life of $^{238}_{92}\text{U}$ for α-decay is 4.5×10^9 year.

ILLUSTRATION 33

The nucleus of an atom is $^{235}_{92}Y$, initially at rest, decays by emitting an α-particle as per the equation

$$^{235}_{92}Y \rightarrow {}^{231}_{90}X + {}^{4}_{2}\text{He} + \text{Energy}$$

It is given that the binding energies per nucleon of the parent and the daughter nuclei are 7.8 MeV and 7.835 MeV respectively and that of α-particle is 7.07 MeV/nucleon. Assuming the daughter nucleus to be formed in the unexcited state and neglecting its share in the energy of the reaction, calculate the speed of the emitted α-particle. Take mass of α-particle to be $6.68\times10^{-27}\text{ kg}$.

SOLUTION

$$Q = \left[(7.835\times231)+(7.07\times4)-(7.8\times235)\right]\text{ MeV}$$

$$\Rightarrow \quad Q = 5.18\text{ MeV}$$

$$\Rightarrow \quad Q = 5.18\times1.6\times10^{-13}\text{ J}$$

This entire kinetic energy is taken by the α-particle, so

$$\frac{1}{2}m_\alpha v_\alpha^2 = 5.18\times1.6\times10^{-13}$$

$$\Rightarrow \quad \frac{1}{2}\times6.68\times10^{-27}v_\alpha^2 = 5.18\times1.6\times10^{-13}$$

$$\Rightarrow \quad v_\alpha = 1.57\times10^7\text{ ms}^{-1}$$

BETA DECAY

The process of spontaneous emission of an electron $\left(e^-\right)$ or a positron $\left(e^+\right)$ from a nucleus is called **beta decay**.

Like α-decay, β-decay is a spontaneous process, with a definite disintegration energy and half-life. It is also a statistical process, obeying the Law of radioactive Decay.

In beta minus $\left(\beta^-\right)$ decay, the mass number of the radioactive nucleus remains unchanged but its atomic number increases by one. An electron and a new particle antineutrino $(\bar{\nu})$ are emitted from the nucleus, as in the decay.

$$^{32}_{15}\text{P} \longrightarrow {}^{32}_{16}\text{S} + e^- + \bar{\nu}$$

In general, the **beta minus decay** may be represented as

$$^{A}_{Z}X \longrightarrow {}_{Z+1}^{A}Y + {}_{-1}^{0}e + \bar{\nu}$$

OR $$^{A}_{Z}X \longrightarrow {}_{Z+1}^{A}Y + \beta^- + \bar{\nu}$$

The electron emitted from the nucleus is called a beta particle, denoted by β^-.

In beta plus $\left(\beta^+\right)$ decay, the mass number of the radioactive nucleus remains unchanged but its atomic number decreases by one. A positron $\left(e^+\right)$ and a new particle neutrino (ν) are emitted from the nucleus, as in the decay.

$$^{22}_{11}\text{Na} \longrightarrow {}^{22}_{10}\text{Ne} + e^+ + \nu$$

In general, the **beta plus decay** may be represented as

$$^{A}_{Z}X \longrightarrow {}_{Z-1}^{A}Y + {}_{+1}^{0}e + \nu$$

OR $$^{A}_{Z}X \longrightarrow {}_{Z-1}^{A}Y + \beta^+ + \nu$$

The positron so emitted is called a beta plus particle $\left(\beta^+\right)$

The positron is an antiparticle of electron. It has a positive charge equal in magnitude to the charge on an electron and has a mass equal to the mass of an electron.

Similarly, neutrino and antineutrino are antiparticles of each other. Both are massless, chargeless particles having spins $\pm\frac{1}{2}$.

Although a nucleus contains no electrons, positrons and neutrinos, yet can it eject these particles. It is believed that electrons, positrons and neutrinos are created during the process of beta decay. If the unstable nucleus has excess neutrons than needed for stability, a neutron converts itself into a proton. So, in a beta-minus decay, an electron and an antineutrino are created and emitted from the nucleus via the reaction given by

$$n \rightarrow p + e^- + \bar{\nu}$$

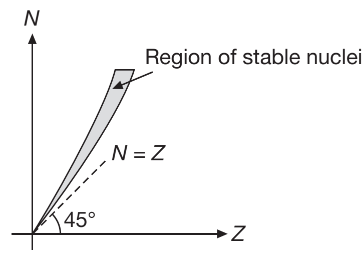
If the unstable nucleus has excess protons than that needed for stability, a proton converts itself into a neutron. So, in a beta-plus decay, a positron and a neutrino are created and emitted from the nucleus via the reaction given by

$$p \rightarrow n + e^+ + \nu$$

Clearly, a beta decay process involves the conversion of a neutron into a proton or vice versa. These nucleons have nearly equal masses. That is why the mass number A of a nuclide undergoing beta decay does not change.

TYPES OF BETA DECAY

In a beta decay process, the $\left(\frac{N}{Z}\right)$ ratio of nucleus is changed. This decay is shown by unstable nuclei. In beta decay, either a neutron is converted into proton or proton is converted into neutron. For better understanding of beta decay let us first draw the N vs Z graph.



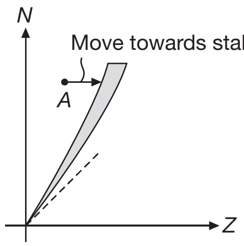
From the N vs Z graph, we observe that there are two types of unstable nuclides.

Negative Beta or β^- Decay or A-Type Beta Decay

The nuclides showing this type of decay are also called as A type nuclides. For A type nuclides, we have

$$\left(\frac{N}{Z}\right)_A > \left(\frac{N}{Z}\right) \text{stable}$$

To achieve stability, these A type nuclides increase their Z by converting a neutron inside them to a proton by emitting a β^- particle. Due to this, the nucleus now attains an $\left(\frac{N}{Z}\right)$ ratio that takes it towards the line of stability as shown.



$${}_0n^1 \rightarrow {}_1p^1 + e^{-1} + \bar{\nu}$$

$${}_ZX^A \rightarrow {}_{Z+1}Y^A + \underset{(\beta^-\ \text{particle})}{e^{-1}} + \bar{\nu}$$

This decay is called β^{-1} decay. Kinetic energy available for β^{-1} and $\bar{\nu}$ is Q

$$Q = K_\beta + K_{\bar{\nu}}$$

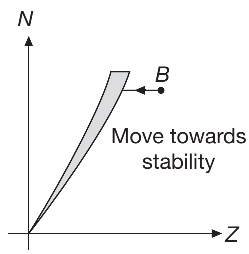
Kinetic energy of the emitted β particles satisfies the condition $0 < K_\beta < Q$

Positive Beta or β^+ Decay or B-Type Beta Decay

The nuclides showing this type of decay are also called as B type nuclides. For B type nuclides, we have

$$\left(\frac{N}{Z}\right)_B > \left(\frac{N}{Z}\right) \text{stable}$$

To achieve stability, these B type nuclides decrease their Z by converting a proton inside them to a neutron by emitting a β^+ particle. Due to this, the nucleus now attains an $\left(\frac{N}{Z}\right)$ ratio that takes it toward the line of stability as shown.



$$p \rightarrow n + \underset{(\text{positron})}{e^+} + \underset{(\text{neutrino})}{\nu}$$

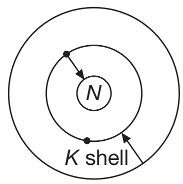
$${}_ZX^A \rightarrow {}_{Z-1}Y^A + \underset{(\beta^+\ \text{particle})}{e^+} + \nu$$

Electron Capture or *K*-capture or Reverse Beta Decay Process

Besides β^{-1} and β^+ emission, there is a third related process. This process is called Electron Capture (abbreviated as EC) or K-capture or a reverse beta decay process. It is a rare process which is found only in few proton rich nuclei. In this process, the proton rich nucleus captures one of the atomic electrons from the K shell and so is called as K-Capture process. A proton in the nucleus combines with this electron and converts itself into a neutron. A neutrino is also emitted in the process and is emitted from the nucleus.

$$p + {}_{-1}e^0 \rightarrow n + \nu$$

The vacancy created in the K shell is filled by transition of electrons from the outer shells. This results in the emission of **characteristic X-rays.** This process is inferred experimentally by detection of emitted X-rays (due to other electrons jumping down to fill the empty state) of just the proper energy.



If X and Y are atoms then reaction is written as

$${}_ZX^A \rightarrow {}_{Z-1}Y^A + \nu + Q + \text{characteristic X-rays of atom } Y$$

If X and Y are taken as nucleus, then reaction is written as

$${}_ZX^A + {}_{-1}e^0 \rightarrow {}_{Z-1}Y^A + \nu$$

This process is called **Electron Capture** or **K-Capture** or **Reverse Beta Decay process.**

An example is ${}^7_4\text{Be}$, which as a result becomes ${}^7_3\text{Li}$. The process is written

$${}^7_4\text{Be} + e^- \rightarrow {}^7_3\text{Li} + \nu,$$

Or, in general,

$${}^A_Z\text{N} + e^- \rightarrow {}^A_{Z-1}\text{N}' + \nu \qquad \text{\{Electron Capture\}}$$

In β decay, it is the weak nuclear force that plays the crucial role. The neutrino is unique because it interacts with matter only via the weak force, which is why it is so hard to detect.

Conceptual Note(s)

(a) Please note that the electron emitted in β decay is not an orbital electron. Instead, the electron is created within the nucleus itself. Actually, inside the nucleus, one of the neutrons changes to a proton and in the process (to conserve charge) emits an electron. Indeed, free neutrons actually decay in this fashion

$$n \rightarrow p + e^{-} + a \text{ neutrino}$$

Since the emitted electrons, originate in the nucleus, so the electrons emitted in β decay are often referred to as β particles, rather than as electrons (just to remind us of their origin). However, these electrons or β-particles are indistinguishable from the orbital electrons.

(b) If during the process of beta decay, we do not consider the emission of neutrino or antineutrino, then we observe that the Laws of Conservation of Energy, Conservation of Linear Momentum and Conservation of Angular momentum are not applicable for the beta decay process.

(c) To solve this absurdity, Pauli assumed that in the beta decay process, no doubt the beta particles are emitted but at the same time another particle called neutrino or antineutrino is also emitted along with the beta particle.

Due to Pauli's Neutrino Hypothesis, the Laws of Conservation of Energy, Conservation of Linear Momentum and Conservation of Angular momentum become applicable for the beta decay process.

PAULI'S NEUTRINO HYPOTHESIS AND NEUTRINO PROPERTIES

Pauli assumed that during the beta decay process, emission of beta particle is accompanied by the emission of another particle called neutrino or antineutrino. Due to Pauli's Neutrino Hypothesis, the conservation laws like Conservation of Energy, Conservation of Linear Momentum and Conservation of Angular momentum become applicable for the beta decay process.

According to Pauli, the particle called neutrino or antineutrino has the following properties. These particles have few of their properties matching with a photon.

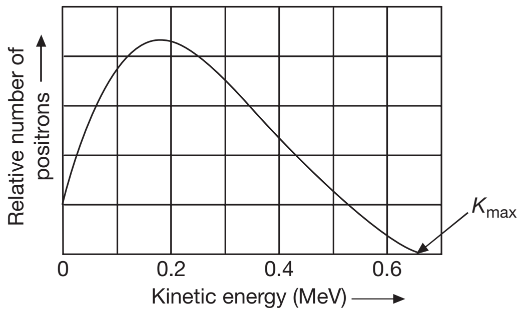
(a) Both have zero rest mass like a photon.
(b) Both have zero charge i.e. both are neutral.
(c) Both particles are energy particles just like photons.
(d) Both particles have linear momentum just like a photon.
(e) Like other nucleons, neutrinos too possess spin. They have spin quantum number $s = \pm \frac{1}{2}$.
(f) They carry angular momentum of $\pm \frac{h}{2\pi}$ just like other nucleons.
(g) Since neutrinos (or antineutrinos) are massless and chargeless, they interact so weakly with matter that it becomes very difficult to detect them. They can penetrate through earth without being absorbed. They can penetrate about 10^{12} km in lead, the most dense material, without interacting with it. By ingenious experiments, neutrinos have been detected and their mass and spin or intrinsic angular momentum have been measured.
(h) The β^{+} decay process is accompanied by the emission of neutrino ν, whereas the β^{-} decay process is accompanied by the emission of an antineutrino $\bar{\nu}$.

CONTINUOUS ENERGY SPECTRUM FOR BETA RAYS

In both α- decay and β-decays processes, the disintegration energy Q depends on the nature of the radionuclide. In the α-decay of a particular radionuclide, every emitted α- particle has a definite amount of kinetic energy.

However, in β- decay, the disintegration energy is shared in all proportions between the three particles i.e., daughter nucleus, electron (or positron) and antineutrino (or neutrino). As a result, the kinetic energy of the electrons (or positrons) is not fixed.

Their energy varies from zero to a maximum value K_{max}. Thus β- rays have a continuous energy spectrum, as shown in Figure.



The distribution of the kinetic energies of positrons emitted in the decay of $^{64}_{29}Cu$

The maximum kinetic energy or end point energy K_{max} must be equal to disintegration energy Q. When the electron (or positron) has maximum energy, the energy carried by the daughter nucleus and neutrino is nearly zero.

Q-VALUE FOR BETA DECAY PROCESS

We have already discussed the three fundamental beta decay processes. Generally, the masses used in the calculation of mass defect involved in a nuclear reaction are the masses of the atoms in which we have neglected the mass of the electrons which is already included in the mass of the atom. However, for the beta decay process, we know that the mass of the beta particle is same as the mass of the electron and hence it is not possible for us to neglect the mass of the orbital electrons of the atom.

Q-value for β^- Decay Process

Considering the emission of antineutrino, the equation of β^--decay can be written as

$$_Z X^A \longrightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + Q + \bar{\nu}$$

Production of antineutrino along with the electron helps to explain the continuous spectrum because the energy is distributed randomly between electron and $\bar{\nu}$ and it also helps to explain the spin quantum number balance (because p, n and $\pm e$ each has spin quantum number $\pm\frac{1}{2}$). During β^- decay, inside the nucleus a neutron is converted to a proton with emission of an electron and antineutrino.

$$n \rightarrow p + {}_{-1}e^0 + \bar{\nu}$$

Let M_X be the mass of atom $_Z X^A$, M_Y be the mass of atom $_{Z+1}Y^A$ and m_e be the mass of electron, then Q value is given by

$$Q = \left[(M_X - Zm_e) - \left\{(M_Y - (Z+1)m_e) + m_e\right\}\right]c^2$$

$$\Rightarrow \quad Q = (M_X - M_Y)c^2$$

Here also, one can see that by applying the law of conservation of momentum and energy, we will get

$$T_e = \frac{m_Y}{m_e + m_Y}Q \text{ and } T_Y = \frac{m_e}{m_e + m_Y}Q$$

Since, $m_e \ll m_Y$, we can consider that almost all the energy is taken away by the electron.

Q-value for β^+ Decay Process

Considering the emission of antineutrino, the equation of β^--decay can be written as

$$_Z X^A \longrightarrow {}_{Z-1}Y^A + {}_{+1}e^0 + Q + \nu$$

During β^+ decay, inside the nucleus a proton is converted to a neutron with emission of a positron and a neutrino.

$$p \rightarrow n + {}_{+1}e^0 + \nu$$

Let M_X be the mass of atom $_Z X^A$, M_Y be the mass of atom $_{Z-1}Y^A$ and m_e be the mass of electron, then Q value is given by

$$Q = \left[(M_X - Zm_e) - \left\{(M_Y - (Z-1)m_e) + m_e\right\}\right]c^2$$

$$\Rightarrow \quad Q = (M_X - M_Y - 2m_e)c^2$$

Q-value for *K*-capture Process

Considering the K-capture or the electron capture by an element $_Z X^A$, written by the equation

$$_Z X^A \longrightarrow {}_{Z-1}Y^A + \bar{\nu}$$

Let M_X be the mass of atom $_Z X^A$, M_Y be the mass of atom $_{Z-1}Y^A$ and m_e be the mass of electron, then

$$Q \text{ value} = \left[(M_X - Zm_e) - (M_Y - (Z-1)m_e)\right]c^2$$

$$\Rightarrow \quad Q \text{ value} = (M_X - M_Y - m_e)c^2$$

ILLUSTRATION 34

Neon - 23 beta decays in the following way:

$$^{23}_{10}\text{Ne} \rightarrow {}^{23}_{11}\text{Na} + {}^{0}_{-1}e + \bar{\nu}$$

Find the minimum and maximum kinetic energy that the beta particle ${}^{o}_{-1}e$ can have. The atomic masses of ^{23}Ne and ^{23}Na are $22.9945\,u$ and $22.9898\,u$, respectively.

SOLUTION

Reactants	Products
$m\left({}^{23}_{10}\text{Ne}\right) = 22.9945 - 10m_e$	$m\left({}^{23}_{11}\text{Na}\right) = 22.9898 - 11\,m_e$
	$m\left({}^{o}_{-1}e\right) = m_e$
Total $= 22.9945 - 10m_e$	Total $= 22.9898 - 10m_e$

Mass defect $= 22.9945 - 22.9898 = 0.0047$ u

$$Q = (0.0047)(931) = 4.4 \text{ MeV}$$

The β-particle and neutrino share this energy. Hence the energy of the β-particle can range from 0 to 4.4 MeV.

ILLUSTRATION 35

Consider the beta decay $^{198}\text{Au} \rightarrow {}^{198}\text{Hg}^* + \beta^- + \bar{\nu}$ where $^{198}\text{Hg}^*$ represents a mercury nucleus in an excited state at energy 1.088 MeV above the ground state. What can be the maximum kinetic energy of the electron emitted? The atomic mass ^{198}Au is 197.968233 u and that of ^{198}Hg is 197.966760 u.

SOLUTION

If the product nucleus ^{198}Hg is formed in its ground state, the kinetic energy available to the electron and the antineutrino is

$$Q = \left[m\left({}^{198}\text{Au}\right) - m\left({}^{198}\text{Hg}\right)\right]c^2$$

As $^{198}\text{Hg}^*$ has energy 1.088 MeV more than ^{198}Hg in ground state, the kinetic energy actually available is

$$Q = \left[m\left({}^{198}\text{Au}\right) - m\left({}^{198}\text{Hg}\right)\right]c^2 - 1.088 \text{ MeV}$$

$$\Rightarrow \quad Q = (197.968233 - 197.966760)(931) \text{ MeV} - 1.088 \text{ MeV}$$

$$\Rightarrow \quad Q = 1.3686 \text{ MeV} - 1.088 \text{ MeV} = 0.2806 \text{ MeV}$$

This is also the maximum possible kinetic energy of the emitted electron.

ILLUSTRATION 36

Calculate the Q-value in the following decays

(a) $^{19}\text{O} \rightarrow {}^{19}\text{F} + e^- + \bar{\nu}$

(b) $^{25}\text{Al} \rightarrow {}^{25}\text{Mg} + e^+ + \nu$

The atomic masses needed are as follows:

^{19}O	^{19}F	^{25}Al	^{25}Mg
19.003576 u	18.998403 u	24.990432 u	24.985839 u

SOLUTION

(a) The Q-value of β^--decay is

$$Q = \left[m\left({}^{19}\text{O}\right) - m\left({}^{19}\text{F}\right)\right]c^2$$

$$\Rightarrow \quad Q = (19.003576 \text{ u} - 18.998403 \text{ u})(931 \text{ MeV/u})$$

$$\Rightarrow \quad Q = 4.816 \text{ MeV}$$

(b) The Q-value of β^+-decay is

$$Q = \left[m\left({}^{25}\text{Al}\right) - m\left({}^{25}\text{Mg}\right) - 2m_e\right]c^2$$

$$\Rightarrow \quad Q = [24.99032 \text{ u} - 24.985839 \text{ u} - 2 \times 0.511 \text{ MeV}_c^{-2}]c^2$$

$$\Rightarrow \quad Q = (0.004593 \text{ u})(931 \text{ MeV/u}) - 1.022 \text{ MeV}$$

$$\Rightarrow \quad Q = 4.276 \text{ MeV} - 1.022 \text{ MeV} = 3.254 \text{ MeV}$$

GAMMA DECAY

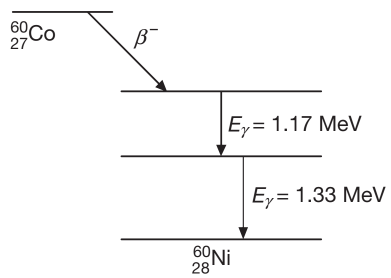
The process of emission of a γ-ray photon during the radioactive disintegration of a nucleus is called **gamma decay**.

As the emitted γ-ray photons have zero rest mass and carry no charge, so in a γ-decay the mass number and atomic number of the nucleus remain unchanged and no new element is formed. A γ-decay can be expressed as

$$\underset{(\text{Excited state})}{{}^{A}_{Z}X} \longrightarrow \underset{(\text{Ground state})}{{}^{A}_{Z}X} + \gamma$$

A nucleus does not contain photons, yet it can emit photons. These photons are created during the emission process. We know that a nucleus can exist in different energy states. After an α or a β-decay, the daughter nucleus is usually left in the excited state. It attains the ground state by single or successive transitions by emitting one or more photons. As the nuclear states have energies of the order of MeV, therefore, the photons emitted by the nuclei have energy of the order of several MeV. The wavelength of such high energy photons is a fraction of an angstrom. The short wavelength electromagnetic waves emitted by nuclei are called γ-rays.

An example of γ-decay is shown through an energy level diagram shown in Figure. Here an unstable ${}^{60}_{27}\text{Co}$ nucleus is transformed via a β-decay into an excited ${}^{60}_{28}\text{Ni}$ nucleus, which in turn reaches the stable ground state by emitting photons of energies 1.17 MeV and 1.33 MeV, in two successive γ-decay processes.

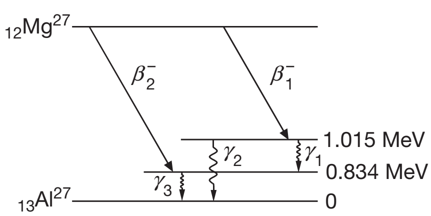


Energy-level diagram showing the emission of γ-rays by a ${}^{60}_{27}\text{Co}$ nucleus subsequent to beta decay

Usually, γ-rays are emitted after α- or β-decay, but there are long lived radioactive nuclei that emit only γ-rays.

ILLUSTRATION 37

Calculate the kinetic energy of β-particles and the radiation frequencies corresponding to the γ-decays shown in figure



Given, mass of ${}_{12}\text{Mg}^{27}$ atom $= 26.991425$ amu and mass of ${}_{13}\text{Al}^{27}$ atom $= 26.990080$ amu

SOLUTION

Energy of photon corresponding to frequency ν_1 is

$$h\nu_1 = E_3 - E_2$$

$$\Rightarrow \quad \nu_1 = \frac{E_3 - E_2}{h}$$

$$\Rightarrow \quad \nu_1 = \frac{(1.015 - 0.834)\text{ MeV}}{6.62 \times 10^{-34}\text{ Js}}$$

$$\Rightarrow \quad \nu_1 = \frac{0.181 \times 1.6 \times 10^{-13}\text{ J}}{6.62 \times 10^{-34}\text{ Js}}$$

$$\Rightarrow \quad \nu_1 = 4.37 \times 10^{19}\text{ s}^{-1}$$

Energy of photon corresponding to frequency ν_2 is

$$h\nu_2 = E_3 - E_1$$

$$\Rightarrow \quad \nu_2 = \frac{E_3 - E_1}{h}$$

$$\Rightarrow \quad \nu_2 = \frac{(1.015 - 0)\text{ MeV}}{6.62 \times 10^{-34}\text{ Js}}$$

$$\Rightarrow \quad \nu_2 = 2.45 \times 10^{20}\text{ s}^{-1}$$

Energy of photon corresponding to frequency ν_3 is

$$h\nu_3 = E_2 - E_1$$

$$\Rightarrow \quad \nu_3 = \frac{E_2 - E_1}{h}$$

$$\Rightarrow \quad \nu_3 = \frac{(0.834 - 0)\text{ MeV}}{6.62 \times 10^{-34}}$$

$$\Rightarrow \quad \nu_3 = 2.0 \times 10^{20}\text{ s}^{-1}$$

Now emission of β_1^--particle is given by

$${}_{12}\text{Mg}^{27} \rightarrow {}_{13}\text{Al}^{27} + \beta^- + \nu_2 + Q_1$$

$$\Rightarrow \quad Q_1 = \left[m\left({}_{12}\text{Mg}^{27}\right) - m\left({}_{13}\text{Al}^{27}\right) - E(\nu_2)\right]$$

$$\Rightarrow \quad Q_1 = [26.991425 - 26.990080]u - (E_3 - E_1)\text{ MeV}$$

$$\Rightarrow \quad Q_1 = 0.001345 \times 931 - 1.015\text{ MeV} = 0.237\text{ MeV}$$

$$\Rightarrow \quad K.E.\text{ of } \beta_1^- \text{ particle is } 0.237\text{ MeV}$$

Emission of β_2^- particle is given by

$${}_{12}\text{Mg}^{27} \rightarrow {}_{13}\text{Al}^{27} + \beta_2^- + \nu_3 + Q_2$$

$$\Rightarrow \quad Q_2 = \{[26.991425 - 26.990080]931 - 0.834\}\text{ MeV}$$

$$\Rightarrow \quad Q_2 = 0.418\text{ MeV}$$

$$\Rightarrow \quad \text{KE of } \beta_2^- \text{ particle is } 0.418\text{ MeV}$$

CLASSIFICATION OF NUCLEAR REACTIONS

Nuclear reactions can be classified into the following categories.

Elastic Scattering

The incident particle gets deflected without any change in its energy, i.e.,

$$^{4}_{2}He + ^{197}_{79}Au \longrightarrow ^{197}_{79}Au + ^{4}_{2}He$$

The bombarding particle passes sufficiently at large distance away from the target nucleus so as to get repulsion which changes its direction of motion without any change in its energy.

Inelastic Scattering

If the bombarding particle passes close to target it gets deflected. Due to strong repulsion, the target particle also acquires some energy. So, the energy left with the scattered particle is less than that it had initially.

$$^{1}_{1}H + ^{7}_{3}Li \longrightarrow ^{7}_{3}Li + ^{1}_{1}H$$

$^{7}_{3}Li$ means existence of $^{7}_{3}Li$ in one of its excited states.

Simple Capture

The incoming particle is captured by the target nucleus. The product nucleus which is generally in the form of excited state decays to the ground state by emitting γ-ray of energy $h\nu$.

$$^{1}_{1}H + ^{12}_{6}C \longrightarrow ^{13}_{7}N \longrightarrow ^{13}_{7}N + h\nu$$

Disintegration (Nuclear Transmutations)

The intermediate compound nucleus breaks up and results in a product nucleus and an outgoing particle. The product nucleus has different chemical properties as compared to the target particle. Majority of nuclear reactions belong to this category. Such nuclear disintegrations are called Nuclear Transmutations.

(a) Disintegration by α-particles

(i) (α, p) reactions

$$^{A}_{Z}X + ^{4}_{2}He \longrightarrow ^{A+4}_{Z+2}C^* \longrightarrow ^{A+3}_{Z+1}Y + ^{1}_{1}H$$

$\{C^*$ is Compound Nucleus$\}$

EXAMPLES

The historical experiment of Rutherford is an α induced transmutation, an (α, p) reaction.

$$_{7}N^{14} + _{2}He^{4} \longrightarrow _{9}F^{18*} \longrightarrow ^{17}_{8}O + _{1}H^{1}$$

and is exoergic in nature $(Q > 0)$. Other useful (α, p) reactions are

$$^{10}_{5}B + ^{4}_{2}He \longrightarrow ^{14}_{7}N^* \longrightarrow ^{13}_{6}C + ^{1}_{1}H$$

$$^{23}_{11}Na + ^{4}_{2}He \longrightarrow ^{27}_{13}Al^* \longrightarrow ^{26}_{12}Mg + ^{1}_{1}H$$

$$^{27}_{13}Al + ^{4}_{2}He \longrightarrow ^{31}_{15}P^* \longrightarrow ^{30}_{14}Si + ^{1}_{1}H$$

$$^{45}_{21}Sc + ^{4}_{2}He \longrightarrow ^{49}_{23}V^* \longrightarrow ^{48}_{22}Ti + ^{1}_{1}H$$

(ii) (α, n) reactions

$$^{A}_{Z}X + ^{4}_{2}He \longrightarrow ^{A+4}_{Z+2}C^* \longrightarrow ^{A+3}_{Z+2}Y + ^{1}_{0}n$$

$\{C^*$ is Compound Nucleus$\}$

EXAMPLES

$$^{7}_{3}Li + ^{4}_{2}He \longrightarrow ^{11}_{5}B^* \longrightarrow ^{10}_{5}B + ^{1}_{0}n$$

$$^{9}_{4}Be + ^{4}_{2}He \longrightarrow ^{13}_{6}C^* \longrightarrow ^{12}_{6}C + ^{1}_{0}n$$

$$^{19}_{9}F + ^{4}_{2}He \longrightarrow ^{23}_{11}Na^* \longrightarrow ^{22}_{11}Na + ^{1}_{0}n$$

$$^{23}_{11}Na + ^{4}_{2}He \longrightarrow ^{27}_{13}Al^* \longrightarrow ^{26}_{13}Al + ^{1}_{0}n$$

(b) Disintegration by protons

(i) (p, α) reactions: When the reactions yield α-particles. The (p, α) reactions are usually exoergic and have the general form

$$^{A}_{Z}X + ^{1}_{1}H \longrightarrow ^{A+1}_{Z+1}C^* \longrightarrow ^{A-3}_{Z-1}Y + ^{4}_{2}He$$

$\{C^*$ is Compound Nucleus$\}$

EXAMPLES

$$^{6}_{3}Li + ^{1}_{1}H \longrightarrow ^{7}_{4}Be^* \longrightarrow ^{3}_{2}He + ^{4}_{2}He \quad (Q = 4\text{ MeV})$$

$$^{7}_{3}Li + ^{1}_{1}H \longrightarrow ^{8}_{4}Be^* \longrightarrow ^{4}_{2}He + ^{4}_{2}He \quad (Q = 17.35\text{ MeV})$$

$$^{11}_{5}B + ^{1}_{1}H \longrightarrow ^{12}_{6}C^* \longrightarrow ^{8}_{4}Be + ^{4}_{2}He \quad (Q - 8.59\text{ MeV})$$

{The product nucleus $^{8}_{4}Be$ is highly ustable and decays almost immediately as $^{8}_{4}Be \longrightarrow ^{4}_{2}He + ^{4}_{2}He$, so that the final reaction gives three α-particles}

$$^{19}_{9}F + ^{1}_{1}H \longrightarrow ^{20}_{10}Ne^* \longrightarrow ^{16}_{8}O + ^{4}_{2}He$$

$(Q = 812\text{ MeV})$

$$^{23}_{11}Na + ^{1}_{1}H \longrightarrow ^{24}_{12}Mg^* \longrightarrow ^{20}_{10}Ne + ^{4}_{2}He$$

$(Q = 2.38\text{ MeV})$

$$^{63}_{29}Cu + ^{1}_{1}H \longrightarrow ^{64}_{30}Zn^* \longrightarrow ^{60}_{28}Ni + ^{4}_{2}He$$

$(Q = 3.76\text{ MeV})$

(ii) (p, n) reactions: When the reactions yield neutrons. The general equation of this type of reactions is

$$^{A}_{Z}X + ^{1}_{1}H \longrightarrow ^{A+1}_{Z+1}C^* \longrightarrow {}_{Z+1}^{A}Y + ^{1}_{0}n$$

$\{C^* \text{ is Compound Nucleus}\}$

the product nucleus being isobaric with the target nucleus.

Since two isobars differing in Z by unity cannot both be stable, the product nucleus is β^+ active, decaying by β^+ emission (or electron capture) into $^{A}_{Z}X$ (the same as target nucleus):

$$_{Z+1}^{A}Y \xrightarrow[\text{electron capture}]{\beta^+} {}^{A}_{Z}X$$

EXAMPLES

$$\begin{cases} ^{11}_{6}B + ^{1}_{1}H \longrightarrow ^{12}_{6}C^* \longrightarrow ^{11}_{6}C + ^{1}_{0}n \\ ^{11}_{6}C \xrightarrow{\beta^+} ^{11}_{5}B \end{cases}$$

$(Q = -1.763\text{ MeV})$ (Half-life = 2.5 min)

$$\begin{cases} ^{23}_{11}Na + ^{1}_{1}H \longrightarrow ^{24}_{12}Mg^* \longrightarrow ^{23}_{12}Mg + ^{1}_{0}n \\ ^{23}_{12}Mg \xrightarrow{\beta^+} ^{23}_{11}Na \end{cases}$$

$(Q = -4.84\text{ MeV})$ (Half-life = 12.3 s)

$$\begin{cases} ^{54}_{24}Cr + ^{1}_{1}H \longrightarrow ^{55}_{25}Mn^* \longrightarrow ^{54}_{25}Mn + ^{1}_{0}n \\ ^{54}_{25}Mn \xrightarrow{\beta^+} ^{54}_{24}Cr \end{cases}$$

$(Q = -2.16\text{ MeV})$ (Half-life = 310 days)

$$\begin{cases} ^{63}_{29}Cu + ^{1}_{1}H \longrightarrow ^{64}_{30}Zn^* \longrightarrow ^{63}_{30}Zn + ^{1}_{0}n \\ ^{63}_{30}Zn \xrightarrow{\beta^+} ^{63}_{29}Cu \end{cases}$$

$(Q = -4.15\text{ MeV})$ (Half-life = 38.5 min)

The (p, n) reaction is always endoergic.

(iii) (p, γ) reactions: When the reactions yield γ-photons. The compound nucleus formed by absorption of proton by the target nucleus does not emit any nuclear particle but goes down to the ground state emitting one or more γ-photons. The (p, γ) reaction is the radiative capture of proton.

The general equation of (p, γ) reaction is

$$^{A}_{Z}X + ^{1}_{1}H \longrightarrow ^{A+1}_{Z+1}C^* \longrightarrow ^{A+1}_{Z+1}C + \gamma$$

$\{C^* \text{ is Compound Nucleus}\}$

EXAMPLES

$$^{7}_{3}Li + ^{1}_{1}H \longrightarrow ^{8}_{4}Be^* \longrightarrow ^{8}_{4}Be + \gamma$$

$$^{14}_{7}N + ^{1}_{1}H \longrightarrow ^{15}_{8}O^* \longrightarrow ^{15}_{8}O + \gamma$$

$$^{24}_{12}Mg + ^{1}_{1}H \longrightarrow ^{25}_{13}Al^* \longrightarrow ^{25}_{13}Al + \gamma$$

$$^{27}_{13}Al + ^{1}_{1}H \longrightarrow ^{28}_{14}Sr^* \longrightarrow ^{28}_{14}Sr + \gamma$$

(iv) (p, d) reactions: When the reactions yield deuterons. The general equation of this type of reaction is

$$^{A}_{Z}X + ^{1}_{1}H \longrightarrow {}^{A-1}_{Z}X + ^{2}_{1}H$$

This is an example of direct reaction without any formation of the compound nucleus.

EXAMPLES

$$^{7}_{3}Li + ^{1}_{1}H \longrightarrow ^{6}_{3}Li + ^{2}_{1}H$$

$$^{9}_{4}Be + ^{1}_{1}H \longrightarrow ^{8}_{4}Be + ^{2}_{1}H$$

(c) Disintegration by neutrons

(i) (n, α) reactions: When the reactions yield α-particles. The general equation for (n, α) reactions is

$$^{A}_{Z}X + ^{1}_{0}n \longrightarrow {}^{A+1}_{Z}C^* \longrightarrow ^{A-3}_{Z-2}Y + ^{4}_{2}He$$

$\{C^* \text{ is Compound Nucleus}\}$

The (n, α) reactions are usually exoergic, i.e. Q is positive, particularly for medium heavy nuclei.

EXAMPLES

$$^{6}_{3}Li + ^{1}_{0}n \longrightarrow ^{7}_{3}Li^* \longrightarrow ^{3}_{1}H + ^{4}_{2}He$$

$(Q = 4.785\text{ MeV})$

followed by $^{3}_{1}H \xrightarrow{\beta^-} {}^{3}_{2}He + {}^{-1}_{0}e + \bar{\nu}_e$

$${}^{10}_{5}B + {}^{1}_{0}n \longrightarrow {}^{11}_{5}B^* \longrightarrow {}^{7}_{3}Li + {}^{4}_{2}He$$

$$(Q = 2.79 \text{ MeV})$$

$${}^{35}_{17}Cl + {}^{1}_{0}n \longrightarrow {}^{36}_{17}Cl^* \longrightarrow {}^{32}_{15}P + {}^{4}_{2}He$$

$$(Q = 0.935 \text{ MeV})$$

The first two reactions are utilised in the construction of neutron detectors as they have fairly large cross-sections. The first reaction also gives a method of producing tritium which is useful in nuclear fusion.

(ii) (n, p) reactions: When the reactions yield protons. The general equation for (n, p) reactions is

$${}^{A}_{Z}X + {}^{1}_{0}n \longrightarrow {}^{A+1}_{Z}C^* \longrightarrow {}^{A}_{Z-1}Y + {}^{1}_{1}H$$

$$\{C^* \text{ is Compound Nucleus}\}$$

The product nucleus Y is an isobar of the target nucleus X with Z-value one unit lower and is thus β^- active decaying to the target nucleus. So,

$${}^{A}_{Z-1}Y \xrightarrow{\beta^-} {}^{A}_{Z}X$$

So, the (n, p) reactions are such that the initial and final nuclides are identical. The process therefore appears to be a conversion of neutron into a proton and an electron.

EXAMPLES

$${}^{14}_{7}N + {}^{1}_{0}n \longrightarrow {}^{15}_{7}N^* \longrightarrow {}^{14}_{6}C + {}^{1}_{1}H$$

$$(Q = 0.627 \text{ MeV})$$

followed by ${}^{14}_{6}C \xrightarrow[5568 \text{ yr}]{\beta^-} {}^{14}_{7}N + {}^{-1}_{0}e + \bar{\nu}_e$

$${}^{27}_{13}Al + {}^{1}_{0}n \longrightarrow {}^{28}_{13}Al^* \longrightarrow {}^{27}_{12}Mg + {}^{1}_{1}H$$

$$(Q = -1.83 \text{ MeV})$$

followed by ${}^{27}_{12}Mg \xrightarrow[10 \text{ min}]{\beta^-} {}^{27}_{13}Al + {}^{-1}_{0}e + \bar{\nu}_e$

Only the first reaction is induced by thermal neutrons.

(iii) (n, γ) reactions: When the reactions yield γ-photons. This is the most important neutron-induced transmutation, known as **radiative capture** of neutrons, and has the following general equation.

$${}^{A}_{Z}X + {}^{1}_{0}n \longrightarrow {}^{A+1}_{Z}C^* \longrightarrow {}^{A+1}_{Z}Y + \gamma$$

$$\{C^* \text{ is Compound Nucleus}\}$$

The product nucleus is thus the same as the compound nucleus in the ground state. The (n, γ) reaction is always exoergic $(Q > 0)$ and can be induced by almost zero energy neutrons.

The radiative capture raises the target nucleus to an excited isomeric state and by releasing the excitation energy as γ photons, the product nucleus becomes an isotope of the target nucleus. The isotopic product nuclei are generally β^- active, as it has a higher neutron-proton ratio compared to the original one. In fact, this method of inducing β-activity is used extensively with copious supply of neutrons from reactors.

EXAMPLES

$${}^{2}_{1}H + {}^{1}_{0}n \longrightarrow {}^{3}_{1}H^* \longrightarrow {}^{3}_{1}H + \gamma$$

followed by ${}^{3}_{1}H \xrightarrow[12.4 \text{ yr}]{\beta^-} {}^{3}_{2}He + {}^{-1}_{0}e + \bar{\nu}_e$

(d) Disintegration by deutrons

(i) (d, α) reactions: When the reactions yield α-particles. The general equation of (d, α) reactions is

$${}^{A}_{Z}X + {}^{2}_{1}H \longrightarrow {}^{A+2}_{Z+1}C^* \longrightarrow {}^{A-2}_{Z-1}Y + {}^{4}_{2}He$$

$$\{C^* \text{ is Compound Nucleus}\}$$

The Q-values are usually positive and the reactions exoergic. Some example of (d, α) reactions are given as under.

EXAMPLES

$${}^{6}_{3}Li + {}^{2}_{1}H \longrightarrow {}^{8}_{4}Be^* \longrightarrow {}^{4}_{2}He + {}^{4}_{2}He$$

$$(Q = 22.4 \text{ MeV})$$

$${}^{14}_{7}N + {}^{2}_{1}H \longrightarrow {}^{16}_{8}O^* \longrightarrow {}^{12}_{6}C + {}^{4}_{2}He$$

$$(Q = 13.57 \text{ MeV})$$

$${}^{16}_{8}O + {}^{2}_{1}H \longrightarrow {}^{18}_{9}F^* \longrightarrow {}^{14}_{7}N + {}^{4}_{2}He$$

$${}^{23}_{11}Na + {}^{2}_{1}H \longrightarrow {}^{25}_{12}Mg^* \longrightarrow {}^{21}_{10}Ne + {}^{4}_{2}He$$

$$(Q = 6.9 \text{ MeV})$$

$${}^{27}_{13}Al + {}^{2}_{1}H \longrightarrow {}^{29}_{14}Si^* \longrightarrow {}^{25}_{12}Mg + {}^{4}_{2}He$$

$$(Q = 6.7 \text{ MeV})$$

$$^{24}_{12}Mg + ^2_1H \longrightarrow ^{26}_{13}Al^* \longrightarrow ^{22}_{11}Na + ^4_2He$$

$$(Q = 1.96 \text{ MeV})$$

Since the α-particles ejected from the compound nucleus are to cross high potential barrier, the (d, α) reactions occur at fairly high energy of deuteron and for low Z target nuclei.

(ii) (d, p) reactions: When the reactions yield protons. The general equation of (d, p) reactions is

$$^A_ZX + ^2_1H \longrightarrow ^{A+2}_{Z+1}C^* \longrightarrow ^{A+1}_{Z}X + ^1_1H$$

$\{C^* \text{ is Compound Nucleus}\}$

The Q-values are usually positive and the reactions exoergic. For some light nuclei, however, Q may be negative. Some examples of (d, p) reactions are as under.

EXAMPLES

$$^7_3Li + ^2_1H \longrightarrow ^9_4Be^* \longrightarrow ^8_3Li + ^1_1H$$

$$(Q = -0.193 \text{ MeV})$$

$$^{12}_6C + ^2_1H \longrightarrow ^{14}_7N^* \longrightarrow ^{13}_6C + ^1_1H$$

$$(Q = 2.72 \text{ MeV})$$

$$^{23}_{11}Na + ^2_1H \longrightarrow ^{25}_{12}Mg^* \longrightarrow ^{24}_{11}Na + ^1_1H$$

$$(Q = 4.74 \text{ MeV})$$

$$^{31}_{15}P + ^2_1H \longrightarrow ^{33}_{16}S^* \longrightarrow ^{32}_{15}P + ^1_1H$$

$$(Q = 5.71 \text{ MeV})$$

$$^{109}_{47}Ag + ^2_1H \longrightarrow ^{111}_{48}Cd^* \longrightarrow ^{110}_{47}Ag + ^1_1H$$

$$(Q = 4.6 \text{ MeV})$$

The products of (d, p) reactions are usually radioactive

(iii) (d, n) reactions: When the reactions yield neutrons. The general equation of (d, n) reaction is

$$^A_ZX + ^2_1H \longrightarrow ^{A+2}_{Z+1}C^* \longrightarrow ^{A+1}_{Z+1}Y + ^1_0n$$

$\{C^* \text{ is Compound Nucleus}\}$

With some exceptions, the (d, n) reactions are exoergic and the Q-values are positive. The product nucleus Y is an isotope of the compound nucleus. Some examples of (d, n) reactions are given below.

EXAMPLES

$$^7_3Li + ^2_1H \longrightarrow ^9_4Be^* \longrightarrow ^8_4Be + ^1_0n$$

$$(Q = 15.024 \text{ MeV})$$

$$^9_4Be + ^2_1H \longrightarrow ^{11}_5B^* \longrightarrow ^{10}_5B + ^1_0n$$

$$(Q = 4.36 \text{ MeV})$$

$$^{12}_6C + ^2_1H \longrightarrow ^{14}_7N^* \longrightarrow ^{13}_7N + ^1_0n$$

$$(Q = -0.283 \text{ MeV})$$

$$^{16}_8O + ^2_1H \longrightarrow ^{18}_9F^* \longrightarrow ^{17}_9F + ^1_0n$$

$$(Q = -1.625 \text{ MeV})$$

$$^{35}_{17}Cl + ^2_1H \longrightarrow ^{37}_{18}Ar^* \longrightarrow ^{36}_{18}Ar + ^1_0n$$

$$(Q = 6.28 \text{ MeV})$$

When deuterons bombard deuterons both (d, p) and (d, n) reactions may be observed because of the two alternative decay schemes for the compound nucleus.

$$^2_1H + ^2_1H \longrightarrow ^4_2He^* \longrightarrow ^3_1H + ^1_1H$$

$$(Q = 4.03 \text{ MeV})$$

$$^2_1H + ^2_1H \longrightarrow ^4_2He^* \longrightarrow ^3_2He + ^1_0n$$

$$(Q = 3.26 \text{ MeV})$$

The product ^3_1H is tritium, an isotope of hydrogen. Its nucleus is called triton which is β-active.

$$^3_1H \xrightarrow{\beta^-} ^3_2He + \bar{\nu}_e \qquad (\text{Half-life} = 12.4 \text{ yr})$$

The other product ^3_2He is a stable isotope of helium.

The tritium may be bombarded with deuterons to produce (d, n) reaction

$$^3_1H + ^2_1H \longrightarrow ^5_2He^* \longrightarrow ^4_2He + ^1_0n$$

$$(Q = 17.6 \text{ MeV})$$

The reactions with beryllium and deuterium serve as sources of neutrons. A thick beryllium target if bombarded with 1 MeV deuterons (accelerated in a cyclotron or Van de Graaff generator) yields about 10^8 neutrons per sec per μA deuteron-current absorbed in the target.

(iv) (d, t) reactions: When the reactions yield tritium. The general equation of (d, t) reactions is

$$^A_ZX + ^2_1H \longrightarrow ^{A+2}_{Z+1}C^* \longrightarrow ^{A-1}_{Z}Y + ^3_1H$$

$\{C^* \text{ is Compound Nucleus}\}$

The product nucleus Y is an isotope of the target nucleus X. The cross-section of such reactions is low. Some examples are as under.

$${}^{7}_{3}\text{Li} + {}^{2}_{1}\text{H} \longrightarrow {}^{9}_{4}\text{Be}^* \longrightarrow {}^{6}_{3}\text{Li} + {}^{3}_{1}\text{H}$$

$$(Q = -0.996 \text{ MeV})$$

$${}^{9}_{4}\text{Be} + {}^{2}_{1}\text{H} \longrightarrow {}^{11}_{5}\text{B}^* \longrightarrow {}^{8}_{4}\text{Be} + {}^{3}_{1}\text{H}$$

$$(Q = 4.59 \text{ MeV})$$

NOTE: At higher energies $(> 20 \text{ MeV})$ of deuterons the $(d, 2n)$, $(d, 2p)$, $(d, 3n)$ etc. reactions in which more than one particle (two or more) is emitted from the compound nucleus become important, e.g.

$${}^{107}_{47}\text{Ag} + {}^{2}_{1}\text{H} \longrightarrow {}^{109}_{48}\text{Cd}^* \longrightarrow {}^{107}_{48}\text{Cd} + 2\,{}^{1}_{0}n$$

(e) Photo disintegration (γ-induced transmutations)
This type of reactions, called photo disintegrations or photonuclear reactions, occur when sufficiently high-energy photons enter into a nucleus. The energy of the incident photon must be greater than the binding energy of a nuclear particle (separation energy) like neutron, proton, α-particle etc. to produce (γ, n), (γ, p), (γ, α) etc. reactions.
The photo disintegration of deuteron, discovered by Chadwick and Goldhaber, deserves special mention, for it was from this reaction that they evaluated neutron mass.

$${}^{2}_{1}\text{H} + \gamma \longrightarrow {}^{2}_{1}\text{H}^* \longrightarrow {}^{1}_{1}\text{H} + {}^{1}_{0}n$$

$$(\gamma, n \text{ reaction})$$

Another example of (γ, n) reaction is

$${}^{9}_{4}Be + \gamma \longrightarrow {}^{9}_{4}Be^* \longrightarrow {}^{8}_{4}Be + {}^{1}_{0}n$$

This reaction is used for preparing the source for photo-neutrons.
Other γ-induced reactions are:

$${}^{12}_{6}\text{C} + \gamma \longrightarrow {}^{12}_{6}\text{C}^* \longrightarrow {}^{8}_{4}\text{Be} + {}^{4}_{2}\text{He}$$

$$(\gamma, \alpha \text{ reaction})$$

$${}^{11}_{5}\text{B} + \gamma \longrightarrow {}^{11}_{5}\text{B}^* \longrightarrow {}^{8}_{4}\text{Be} + {}^{3}_{1}\text{H}$$

$$(\gamma, t \text{ reaction})$$

$${}^{10}_{5}\text{B} + \gamma \longrightarrow {}^{10}_{5}\text{B}^* \longrightarrow {}^{8}_{4}\text{Be} + {}^{2}_{1}\text{H}$$

$$(\gamma, d \text{ reaction})$$

$${}^{9}_{4}\text{Be} + \gamma \longrightarrow {}^{9}_{4}\text{Be}^* \longrightarrow {}^{8}_{4}\text{Be} + {}^{1}_{0}n$$

$$(\gamma, n \text{ reaction})$$

$${}^{9}_{4}\text{Be} + \gamma \longrightarrow {}^{9}_{4}\text{Be}^* \longrightarrow {}^{8}_{3}\text{Li} + {}^{1}_{1}\text{H}$$

$$(\gamma, p \text{ reaction})$$

The last (γ, n) reaction serves as a convenient source of neutrons.
Neutral π-mesons can be artificially produced by the interaction of γ-rays with hydrogen and deuterium.

$${}^{1}_{1}\text{H} + \gamma \longrightarrow \pi^0 + {}^{1}_{1}\text{H}$$

$${}^{2}_{1}\text{H} + \gamma \longrightarrow \pi^0 + {}^{1}_{0}n + {}^{1}_{1}\text{H}$$

$${}^{2}_{1}\text{H} + \gamma \longrightarrow \pi^0 + {}^{2}_{1}\text{H}$$

The first reaction can occur only if the energy of the γ-photon is not less than the threshold value equal to the mass-energy of the neutral pion mass. Steinberg could produce π^0-mesons by bombarding light targets such as hydrogen or beryllium with high energy x-radiation from an electron synchrotron.

$$\underset{\text{(Photon)}}{\gamma} + {}^{9}_{4}\text{Be} \longrightarrow {}^{9}_{4}\text{B}^* \longrightarrow {}^{8}_{4}\text{Be} + {}^{1}_{0}n$$

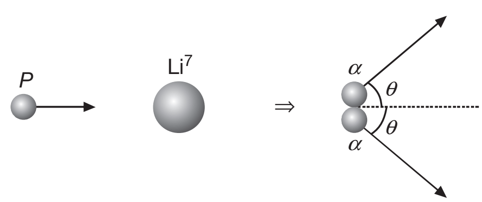
ILLUSTRATION 38

A proton is bombarded on a stationary lithium nucleus. As a result of the collision two α particles are produced. If the direction of motion of the α particles with the initial direction of motion makes an angle $\cos^{-1}\left(\frac{1}{4}\right)$, find the kinetic energy of the striking proton. Given binding energies per nucleon of Li^7 and He^4 are 5.60 and 7.06 MeV respectively. (Assume mass of proton ≈ mass of neutron).

SOLUTION

Q value of the reaction is given by

$$Q = (2 \times 4 \times 7.06 - 7 \times 5.6) \text{ MeV} = 17.28 \text{ MeV}$$



Applying Law of Conservation of Energy for Collision, we get

$$K_p + Q = 2K_\alpha \quad \ldots(1)$$

where K_p and K_α are the kinetic energies of proton and α particle respectively.

Applying Law of Conservation of Linear Momentum, we get

$$\sqrt{2m_p K_p} = 2\sqrt{2m_\alpha K_\alpha}\cos\theta \quad \ldots(2)$$

$$\Rightarrow \quad K_p = 16K_\alpha \cos^2\theta = (16K_\alpha)\left(\frac{1}{4}\right)^2$$

$$\{\because m_\alpha = 4m_p\}$$

$$\Rightarrow \quad K_\alpha = K_p \quad \ldots(3)$$

Solving equations (1) and (3) with $Q = 17.28$ MeV, we get

$$K_p = 17.28 \text{ MeV}$$

NUCLEAR FISSION

The splitting of heavy nucleus into two or more fragments of comparable masses, with an enormous release of energy is called nuclear fission.

In nuclear fission, heavy nuclei having mass number A greater than 200, break up into two or more fragments of comparable masses. The most suitable fission material, from a practical point of view, to achieve energy from nuclear fission is $_{92}U^{236}$. The technique is to hit a uranium sample by sample by slow moving neutrons (kinetic energy ≈ 0.04 eV, also called thermal neutrons). A $_{92}U^{235}$ nucleus has large probability of absorbing a slow neutron and forming $_{92}U^{236}$ nucleus. This nucleus then fissions into two parts. A variety of combinations of the middle-weight nuclei may be formed due to the fission. For example, one may have

$$_{92}U^{235} + {_0n^1} \rightarrow {_{92}U^{236}} \rightarrow X + Y + 2\,{_0n^1},$$

OR $$_{92}U^{235} + {_0n^1} \rightarrow {_{92}U^{236}} \rightarrow X' + Y' + 3\,{_0n^1}$$

and a number of other combinations.

In the nuclear fission reaction

$$^{235}_{92}U + {_0n^1} \rightarrow {^{236}_{92}U} \rightarrow {^{141}_{56}Ba} + {^{92}_{36}Kr} + 3\,{_0n^1} + \text{energy}$$

The mass defect is given by

$$\Delta m = m_{\text{reactants}} - m_{\text{products}}, \text{ where}$$

$$m_{\text{reactants}} = (M_U - 92m_e) + m_n \text{ and}$$

$$m_{\text{products}} = (M_{\text{Ba}} - 56m_e) + (M_{\text{Kr}} - 36m_e) + 3m_n$$

$$\Rightarrow \quad \Delta m = (M_U + m_n) - (M_{\text{Ba}} + M_{\text{Kr}} + 3m_n)$$

The Q value is given by

$$Q = [(M_U + m_n) - (M_{\text{Ba}} + M_{\text{Kr}} + 3m_n)]c^2$$

In a nuclear fission reaction, we observe that

(a) on an average, 2.5 neutrons are emitted in each fission event.

(b) mass lost per reaction is ≈ 0.2 a.m.u.

(c) in nuclear fission reaction, the total B.E. increases and excess energy is released.

(d) in each fission event, about 200 MeV of energy is released. A large part of this released energy appears in the form of kinetic energies of the two fragments. Neutrons take away about 5 MeV.

Conceptual Note(s)

When slow neutrons are bombarded on $_{92}U^{235}$, the fission takes place according to reaction

$$_{92}U^{235} + {_0n^1} \rightarrow {_{56}Ba^{141}} + {_{36}Kr^{92}} + 3({_0n^1}) + 200 \text{ MeV}$$

(a) In nuclear fission the sum of masses before reaction is greater than the sum of masses after reaction, the difference in mass being released in the form of fission energy.

(b) The phenomenon of nuclear fission was discovered by Otto Hans and F. Strassman in 1939 and was explained by N. Bohr and J.A. Wheeler on the basis of liquid drop model of nucleus.

(c) It may be pointed out that it is not necessary that in each fission of uranium, the two fragments Ba^{141} and Kr^{92} are formed but they may be any stable isotopes of middle weight atoms. The most probable division is into two fragments containing about 40% and 60% of the original nucleus with the emission of 2 or 3 neutrons per fission. So, average number of neutrons produced per fission is 2.5.

(d) Most of energy released appears in the form of kinetic energy of fission fragments.

(e) The fission of U^{238} takes place by fast neutrons.

CHAIN REACTION

If on the average more than one of the neutrons produced in each fission are capable of causing further fission, the number of fissions taking place at successive stages goes on increasing at a rapid rate, giving rise to self sustained sequence of fission known as chain reaction. The chain reaction takes place only if the size of the fissionable material is greater than a certain size called the critical size.
There are two types of chain reactions.

Uncontrolled Chain Reaction

In this process the number of fissions in a given interval on the average goes on increasing and the system will have the explosive tendency. This forms the principle of atom bomb. If a nuclear reaction is uncontrolled then in about $1\ \mu s$, energy of order of 2×10^3 J is released.

Controlled Chain Reaction (As in a Nuclear Reactor)

In this process the number of fissions in a given interval is maintained constant by absorbing a desired number of neutrons. This forms the principle of nuclear reactor, consisting of the following parts:

(a) Fuel: The fuel is U^{235} or U^{233} or Pu^{239}

(b) Moderator: A moderator is a suitable material to slow down neutrons produced in fission. The best choice as moderators are heavy water (D_2O) and graphite (C).

(c) Controller: To maintain the steady rate of fission, the neutron absorbing material known as controller is used. The control rods are made of Cadmium or Boron-steel.

(d) Coolant: To remove the considerable amount of heat produced in the fission process, suitable cooling fluids known, as coolants are used. The usual coolants are water, carbon-dioxide, air etc.

(e) Reactor Shield: The intense neutrons and gamma radiation produced in nuclear reactors are harmful for human body. To protect the workers from such radiations, the reactor core is surrounded by concrete wall, called the reactor shield.

Critical Mass

If the amount of uranium is too small, then the liberated neutrons have large scope to escape from the surface and the chain reaction may stop before enough energy is released for explosion. Therefore, in order for explosion to occur, the mass uranium has to be greater than some minimum value, called the critical mass.

Reproduction Factor

It is the ratio of the rate of neutron production and the rate at which the neutrons disappear.

Whether a mass of active material will sustain a chain reaction or not is determined by the reproduction factor (K). If $K\geq1$, the chain reaction will be sustained. If $K=1$, the mass is said to be critical.

ILLUSTRATION 39

Polonium $\left({}^{210}_{84}Po\right)$ emits ${}^{4}_{2}He$ particles and is converted into lead $\left({}^{206}_{82}Pb\right)$. This reaction is used for producing electric power in a space mission. Po^{210} has half life of 138.6 days. Assuming an efficiency of 10% for the thermoelectric machine, how much ${}^{210}Po$ is required to produce 1.2×10^7 J of electric energy per day at the end of 693 days. Also find the initial activity of the material.
Given: Masses of nuclei

$${}^{210}Po=209.98264\text{ amu},\ {}^{206}Pb=205.97440\text{ amu},$$

$${}^{4}_{2}He=4.00260\text{ amu}.$$

$$1\text{ amu}=931\text{ MeV and}$$

$$\text{Avogadro number }=6\times10^{23}\text{ mol}^{-1}$$

SOLUTION

Since, ${}^{210}_{84}Po\longrightarrow{}^{206}_{82}Pb+{}^{4}_{2}He$

$\Rightarrow\quad \Delta m=0.00564$ amu

Energy liberated per reaction is

$$\Delta E=(\Delta m)931\text{ MeV}=8.4\times10^{-13}\text{ J}$$

Electrical energy produced is 10% of ΔE i.e., $= 8.4\times10^{-14}$ J

Let m g of ^{210}Po is required to produce the desired energy, then

$$N = \frac{m}{210}\times 6\times10^{23}$$

Also, $\lambda = \frac{0.693}{t_{1/2}} = 0.005$ per day

$$\Rightarrow \quad \left(-\frac{dN}{dt}\right) = \lambda N = \frac{(0.005)(6\times10^{23})m}{210} \text{ per day}$$

So, electrical energy produced per day is

$$E = \frac{(0.005)(6\times10^{23})m}{210}\times 8.4\times10^{-14} \text{ J}$$

Since, $E = 1.2\times10^{7}$ {given}

$\Rightarrow \quad m = 10$ g

Activity at the end of 693 days is

$$R = \frac{0.005\times6\times10^{23}\times10}{210} = \frac{10^{21}}{7} \text{ per day} = R_0\left(\frac{1}{2}\right)^n$$

where, n is the number of half lives

$$\Rightarrow \quad n = \frac{693}{138.6} = 5$$

$$\Rightarrow \quad R_0 = R(2)^5 = 32\times\frac{10^{21}}{7} = 4.57\times10^{21} \text{ per day}$$

ILLUSTRATION 40

In a nuclear reactor, fission is produced in 1 g for U^{235} (235.0439 u) in 24 hours by a slow neutron (1.0087 u). Assume that $_{35}Kr^{92}$ (91.8973 u) and $_{56}Ba^{141}$ (140.9139 amu) are produced in all reactions and no energy is lost.

(a) Write the complete reaction

(b) Calculate the total energy produced in kilowatt hour. Given 1 u = 931 MeV .

SOLUTION

The nuclear fission reaction is

$$_{92}U^{235} + {_0}n^1 \rightarrow {_{56}}Ba^{141} + {_{36}}Kr^{92} + 3\,{_0}n^1$$

Mass defect $\Delta m = \left[(m_u + m_n) - (m_{Ba} + m_{Kr} + 3m_n)\right]$

$$\Delta m = 256.0526 - 235.8373 = 0.2153 \text{ u}$$

Energy released, $\alpha = 0.2153\times931 = 200$ MeV

Number of atoms in 1 g $= \frac{6.02\times10^{23}}{235} = 2.56\times10^{21}$

Energy released in fission of 1 g of U^{235} is

$$Q = 200\times2.56\times10^{21} = 5.12\times10^{23} \text{ MeV}$$

$$\Rightarrow \quad Q = (5.12\times10^{23})\times(1.6\times10^{-13}) = 8.2\times10^{10} \text{ J}$$

$$\Rightarrow \quad Q = \frac{8.2\times10^{10}}{3.6\times10^{6}} \text{ kWh} = 2.28\times10^{4} \text{ kWh}$$

NUCLEAR FUSION

The phenomenon of combination of two or more light nuclei to form a heavy nucleus with release of enormous amount of energy is called the nuclear fusion. The sum of masses before fusion must be greater than the sum of masses after fusion, the difference in mass appearing as fusion energy. The fusion of two deuterium nuclei into helium is expressed as

$$_1H^2 + {_1}H^2 \longrightarrow {_2}He^4 + 23\cdot8 \text{ MeV}$$

It may be pointed out that this fusion reaction does not actually occur. Due to huge quantity of energy release, the helium nucleus $_2He^4$ has got such a large value of excitation energy that it breaks up by the emission of a proton or a neutron as soon as it is formed, giving rise to the following reactions.

$$_1H^2 + {_1}H^2 \longrightarrow {_2}He^3 + {_0}n^1 + Q(= 3\cdot26 \text{ MeV})$$

$$_1H^2 + {_1}H^2 \longrightarrow {_1}H^3 + {_1}H^1 + Q(= 4\cdot04 \text{ MeV})$$

The fusion process occurs at extremely high temperature and high pressure just as it takes place at sun where temperature is 10^7 K. So, fusion reactions are also called Thermo-nuclear reactions.

Nuclear fusion has the possibility of being a much better source of energy than fission due to the following reasons.

(a) In fusion there is no radiation hazard as no radioactive material is used.

(b) The fuel needed for fission (U-235 etc.) is not available easily whereas hydrogen needed for fusion can be obtained in huge quantity.

(c) The energy released per nucleon is much more in fusion than in fission.

However, the very high temperature and pressure required for fusion cannot be easily created and maintained and as such it has not been possible as yet to use fusion for power generation.

Conceptual Note(s)

(a) For the fusion to take place, the component nuclei must be brought to within a distance of 10^{-14} m. For this they must be imparted high energies to overcome the repulsive force between nuclei. This is possible when temperature is enormously high.

(b) The principle of hydrogen bomb is also based on nuclear fusion. To start a fusion bomb very high temperature is required. This is achieved by incorporating an atom bomb within the nuclear bomb.

(c) The source of energy of sun and other stars is nuclear fusion (or thermo-nuclear reactions). There are two possible cycles:

(i) Proton-Proton cycle

In 1938, Hans Bethe suggested that the stellar energy is produced by thermonuclear reactions in which protons are combined and transformed into helium nuclei. This is known as proton-proton cycle and is applicable for relatively low stellar temperature. The cycle is

$$_1H^1 + _1H^1 \longrightarrow _1H^2 + _1\beta^0 + \nu + 0.42 \text{ MeV} \quad ...(1)$$

$$_1H^2 + _1H^1 \longrightarrow _2He^3 + \gamma + 5.5 \text{ MeV} \quad ...(2)$$

$$_2He^3 + _2He^3 \longrightarrow _2He^4 + _1H^1 + _1H^1 + 12.8 \text{ MeV} \quad ...(3)$$

The reactions (1) and (2) occur twice to yield two $_2He^3$ nuclei.

Net result is

$$_1H^1 + _1H^1 + _1H^1 + _1H^1 \longrightarrow _2He^4 + 2\,_1\beta^0 + 2\nu + 2\gamma + \text{Energy } (24\cdot6 \text{ MeV})$$

(ii) Carbon-Nitrogen cycle

For the main sequence stars with extremely high temperatures, Bethe suggested an alternative to proton-proton cycle called the Carbon-Nitrogen cycle. The cycle is

$$_1H^1 + _6C^{12} \longrightarrow _7N^{13*} + \gamma + 1.95 \text{ MeV}$$

$$_7N^{13*} \longrightarrow _6C^{13} + _1\beta^0 + \nu + 2.22 \text{ MeV}$$

$$_6C^{13} + _1H^1 \longrightarrow _7N^{14} + \gamma + 7.54 \text{ MeV}$$

$$_7N^{14} + _1H^1 \longrightarrow _8O^{15*} + \gamma + 7.35 \text{ MeV}$$

$$_8O^{15*} \longrightarrow _7N^{15} + _1\beta^0 + \nu + 2.7 \text{ MeV}$$

$$_7N^{15} + _1H^1 \longrightarrow _6C^{12} + _2He^4 + 4.96 \text{ MeV}$$

Net result is

$$_1H^1 + _1H^1 + _1H^1 + _1H^1 \longrightarrow _2He^4 + 2\,_1\beta^0 + 2\nu + 3\gamma \text{ (Energy} = 26\cdot7 \text{ MeV)}$$

For sun, both the cycles occur with equal probability. Stars with masses between 0.4 to 2.5 solar mass produce energy mainly by C-N cycle rather than P-P cycle. Stars with masses 0.4 solar mass or lower (which constitute the bulk of stellar population in our galaxy) mainly derive their energy from P-P cycle.

ILLUSTRATION 41

It is proposed to use the nuclear fusion reaction:

$$_1H^2 + _1H^2 = _2He^4$$

in a nuclear reactor of 200 MW rating. If the energy from above reaction is used with a 25% efficiency in the reactor, how many grams of deuterium will be needed per day. (The masses of $_1H^2$ and $_2He^4$ are 2.0141 and 4.0026 u respectively).

SOLUTION

Energy released in the nuclear fusion is

$$Q = \Delta mc^2 = \Delta m(931) \text{ MeV}$$

$$\Rightarrow \quad Q = (2\times2.0141 - 4.0026)\times931 \text{ MeV} = 23.834 \text{ MeV}$$

$$\Rightarrow \quad Q = 23.834\times10^6 \text{ eV}$$

Since efficiency of reactor is 25%

So effective energy used is

$$\Delta E=\frac{25}{100}\times 23.834\times 10^{6}\times 1.6\times 10^{-19}\ \text{J}$$

$$\Rightarrow \quad \Delta E=9.534\times 10^{-13}\ \text{J}$$

Since the two deuterium nucleus are involved in a fusion reaction, therefore, energy released per deuterium is $\frac{9.534\times 10^{-13}}{2}$

For $200\ \text{MW}_{\text{power per day}}$ number of deuterium nuclei required is

$$N=\frac{200\times 10^{6}\times 86400}{\frac{9.534}{2}\times 10^{-13}}=3.624\times 10^{25}$$

Since $2\ \text{g}$ of deuterium constitute 6×10^{23} nuclei, therefore amount of deuterium required per day is

$$m=\frac{2\times 3.624\times 10^{25}}{6\times 10^{23}}=120.83\ \text{g/day}$$

ILLUSTRATION 42

In the fusion reaction $^{2}_{1}\text{H}+^{2}_{1}\text{H}\longrightarrow ^{3}_{2}\text{He}+^{1}_{0}n$, the masses of deuteron, helium and neutron expressed in amu are 2.015, 3.017 and 1.009 respectively. If 1 kg of deuterium undergoes complete fusion, find the amount of total energy released $1\ \text{amu}=931.5\ \text{MeVc}^{-2}$.

SOLUTION

$$\Delta m=2(2.015)-(3.017+1.009)=0.004\ \text{amu}$$

So, energy released is

$$\Delta E=(0.004\times 931.5)\ \text{MeV}=3.726\ \text{MeV}$$

Energy released per deuteron $=\frac{3.726}{2}=1.863\ \text{MeV}$

Number of deuterons in 1 kg $=\frac{6.02\times 10^{26}}{2}=3.01\times 10^{26}$

So, energy released per kg of deuterium fusion is

$$E=(3.01\times 10^{26}\times 1.863)=5.6\times 10^{26}\ \text{MeV}$$

$$\Rightarrow \quad E\approx 9\times 10^{13}\ \text{J}$$

NUCLEAR HOLOCAUST

The estimate of after effects of the atomic (or nuclear) explosion is termed as nuclear holocaust. If a fusion bomb explodes, then a nuclear holocaust will not only destroy every form of life on earth but will also make this planet unfit for life for all times. The radioactive waste will hang like a cloud in earth's atmosphere and will absorb sun's radiations, thus causing a long nuclear winter. One can imagine this only by the mathematical figures quoted, according to which energy liberated by fission of 50 kg of U^{235} is equal to $4\times 10^{15}\ \text{J}$ which is the energy available from 20,000 tons of Trinitrotoluene (TNT).

USE OF RADIOISOTOPES

In Medicine

Radioisotopes are extensively used in medicine:

(a) Radio iodine is used to determine the condition of human thyroid gland. Iodine-131 is administered orally to the patient. After a sufficient time, the activity is measured. From the observations it can be interpreted whether the gland is over-active, normal or under-active.

(b) Amounts of sodium and potassium in the body is measured by using Na-24 and K-42 as tracers.

(c) Radioactive isotopes are used to locate the position and extent of cancer.

(d) Radioisotopes are used in locating tumors within the brain.

(e) Radioactive Cr^{51} is used to locate the exact position where the hemorrhage might have taken place inside the body.

(f) Water contents of the body are measured by using deuterium and tritium as tracers.

(g) Radio gold is being used for the treatment of leukemia.

In Industry

(a) Radioactive Carbon-14 is used to study wear and tear of the position of an engine. C-14 is mixed with the ring. After some time, the engine oil is analysed to detect the presence of any radiation in it. In case of wear and tear the radiations are found to be there.

(b) Radio Cobalt is used for testing fields and castings by taking their photographs with γ-rays.

As Tracers

The radioactive isotope has identical chemical properties as another stable isotope. Therefore, by mixing it with stable isotope we can trace the presence or distribution of the element in a biological or physical system by detecting the radiation emitted by radioisotope of that element. The radioisotope in such a case is said to be a Tracer.

Thus, a radioactive tracer is a radioisotope which, when mixed with a chemically similar element or artificially attached to a biological system, can be traced by radiation detecting devices.

Following are the few examples of radioisotopes acting as tracers:

(a) Phosphorous-32 mixed with phosphorous manure has been used to study the process of extracting food from soil by various plants.

(b) Radio carbon is being used for research in photosynthesis in plants.

(c) The progress and absorption of sodium chloride in the body can be studied by feeding the person with radio-isotope sodium-24 along with sodium chloride.

(d) With the aid of radioisotopes, the rate, place and sequence of formation of the organic constituents of a living body can be studied.

Test Your Concepts-III

Based on Nuclear Reactions, Alpha, Beta, Gamma Decay, Fission and Fusion

(Solutions on page H.90)

1. Consider two decay reactions.

(a) $^{238}_{92}U \longrightarrow {}^{206}_{82}Pb + 10 \text{ protons} + 22 \text{ neutrons}$

(b) $^{238}_{92}U \longrightarrow {}^{206}_{82}Pb + 8\,{}^{4}_{2}He + 6 \text{ elctrons}$

Are both the reactions possible?
Given: Average binding energy of $^{238}_{92}U = 7.57$ MeV, that of $^{206}_{82}Pb = 7.83$ MeV and that of $^{4}_{2}He = 7$ MeV per nucleon.

2. Find the minimum kinetic energy of an α-particle to cause the reaction $^{14}N + {}^{4}He \to {}^{17}O + {}^{1}H$. Given that, the masses of ^{14}N, ^{4}He, ^{1}H and ^{17}O are respectively 14.00307 u, 4.00260 u, 1.00783 u and 16.99913 u.

3. In a neutron induced fission of $_{92}U^{235}$ nucleus, usable energy of 185 MeV is released. If a $_{92}U^{235}$ reactor is continuously operating it at a power level of 100 MW, find the time it takes for 1 kg of uranium to be consumed in this reactor.

4. A neutron breaks into a proton and electron. Calculate the energy produced in this reaction in MeV. Mass of an electron is 9×10^{-31} kg, mass of proton is 1.6725×10^{-27} kg, mass of neutron is 1.6747×10^{-27} kg and speed of light 3×10^{8} ms^{-1}.

5. It is observed that $^{212}_{83}Bi$ decays as per following equation.

$$^{212}_{83}Bi \longrightarrow {}^{208}_{81}Tl + {}^{4}_{2}He$$

The kinetic energy of α particle emitted is 6.802 MeV. Calculate the kinetic energy of Tl recoil atoms.

6. The nuclear reaction, $n + {}^{10}_{5}B \to {}^{7}_{3}Li + {}^{4}_{2}He$ is observed to occur even when very slow-moving neutrons $(M_n = 1.0087 \text{ amu})$ strike a boron atom at rest. For a particular reaction in which $K_n = 0$, the helium $(M_{He} = 4.0026 \text{ amu})$ is observed to have a speed of 9.30×10^{6} ms^{-1}. Determine

(a) the kinetic energy of the lithium $(M_{Li} = 7.0160 \text{ amu})$ and

(b) the Q-value of the reaction.

7. Neon-23 decays in the following way

$$^{23}_{10}Ne \longrightarrow {}^{23}_{11}Na + {}^{0}_{-1}e + \bar{\nu}$$

Find the minimum and maximum kinetic energy possessed by the beta particle $\left({}^{0}_{-1}e\right)$. The atomic masses of ^{23}Ne and ^{23}Na are 22.9945 u and 22.9898 u, respectively.

8. The radionuclide ^{11}C decays according to the reaction.

$$^{11}C \longrightarrow {}^{11}B + e^{+} + \nu.$$

The maximum energy of emitted positrons is 0.961 MeV. Given that atomic mass of ^{11}C is $m_c = 11.011434$ u, atomic mass of ^{11}B is $m_B = 11.009305$ u, and the mass of positron is $m_p = 0.0005486$ u, calculate disintegration energy Q and compare it with the maximum energy of the emitted positron given above. $(1\text{u} = 931 \text{ MeV})$.

9. It is proposed to use the nuclear fusion reaction $_{1}H^{2} + {}_{1}H^{2} \longrightarrow {}_{2}He^{4}$ in a nuclear reactor of

200 MW rating. If the energy from the above reaction is used with a 25% efficiency in the reactor, how many gram of deuterium fuel will be needed per day. Given that the masses of ${}_1H^2$ and ${}_2He^4$ are 2.0141 atomic mass units and 4.0026 atomic mass unit respectively.

10. Assuming the splitting of U^{235} nucleus liberates 200 MeV energy, find

(a) the energy liberated in the fission of 1 kg of U^{235} and

(b) the mass of coal with calorific value of 30 $kJgm^{-1}$ which is equivalent to 1 kg of U^{235}.

11. In a nuclear reaction $\alpha + {}_7N^{14} \longrightarrow {}_8O^{17} + p$ when α-particles of kinetic energy 7.7 MeV were bombarded on nitrogen atom protons were ejected with a kinetic energy of 5.5 MeV.

(a) Find the Q-value of the reaction

(b) Find the angle ϕ between the direction of motion of proton and α-particle.

Given that atomic mass of ${}_1H^1 = 1.00814$ amu, atomic mass of ${}_7N^{14} = 14.00752$ amu, Atomic mass of ${}_8O^{17} = 17.00453$ and atomic mass of ${}_2He^4 = 4.00388$ amu.

12. Show that ${}^{230}_{92}U$ does not decay by emitting a neutron or proton. Given masses are $m\left({}^{230}_{92}U\right)$ = 230.033927 amu; $m\left({}^{229}_{92}U\right)$ = 229.033496 amu; $m\left({}^{229}_{91}Pa\right)$ = 229.032089 amu; $m(n)$ = 1.008665 amu, $m(p) = 1.007825$ amu.

13. 8 protons and 8 neutrons are separately at rest. How much energy will be released if we form ${}^{16}_{8}O$ nucleus? Given that mass of ${}^{16}_{8}O$ atom is 15.994915 u, mass of neutron is 1.008665 u and mass of hydrogen atom is 1.007825 u

14. Calculate the minimum kinetic energy of protons incident on C^{13} nuclei at rest in the laboratory that will produce the endothermic reaction ${}^{13}C(p, n){}^{13}N$. Given that

$$m\left({}^{13}C\right) = 13.0033554,\ m_n = 1.0086654$$

$$m\left({}^{1}H\right) = 1.0078254,\ m\left({}^{13}N\right) = 13.0067384$$

15. It is observed that 20 MeV energy is released per fusion in the reaction ${}_1H^2 + {}_1H^2 \rightarrow {}_2He^4 + {}_0n^1$. Calculate the mass of ${}_1H^2$ consumed in a fusion reactor of power 1 MW in 1 day.

SOLVED PROBLEMS

PROBLEM 1

A radionuclide with half life 1620 sec is produced in a reactor at a constant rate 1000 nuclei per second. During each decay energy 200 MeV is released. If production of radio nuclides started at $t = 0$, calculate the rate of release of energy at 3240 second and the total energy released upto 405 second.

SOLUTION

Let N be the number of nuclei at time t, then net rate of increase of nuclei at instant t is,

$$\frac{dN}{dt} = \alpha - \lambda N$$

{where α = rate of production of nuclei}

$$\Rightarrow \int_0^N \frac{dN}{\alpha - \lambda N} = \int_0^t dt$$

$$\Rightarrow \quad N = \frac{\alpha}{\lambda}\left(1 - e^{-\lambda t}\right) \qquad \ldots(1)$$

Rate of decay at this instant

$$R = \lambda N = \alpha\left(1 - e^{-\lambda t}\right)$$

Hence, the rate of release of energy i.e. $\frac{dE}{dt}$ is given by

$$\frac{dE}{dt} = R\begin{pmatrix}\text{Energy released} \\ \text{in each decay}\end{pmatrix}$$

$$\Rightarrow \frac{dE}{dt} = \alpha(1 - e^{-\lambda t})(200)\ \text{MeV sec}^{-1}$$

Substituting the values, we get

$$\frac{dE}{dt} = 1000\left(1 - e^{\frac{0.693}{1620}\times 3240}\right)(200)$$

$$\Rightarrow \begin{pmatrix}\text{Rate of Release} \\ \text{of Energy}\end{pmatrix} = 1.5\times 10^5\ \text{MeVsec}^{-1}$$

Total number of nuclei decayed upto time t is $\alpha t - N$

$$\Rightarrow \begin{pmatrix}\text{Total number of} \\ \text{decayed nuclei}\end{pmatrix} = \alpha t - \frac{\alpha}{\lambda}\left(1 - e^{-\lambda t}\right)$$

Hence, total energy released upto this instant is

$$E = \left[\alpha t - \frac{\alpha}{\lambda}\left(1 - e^{-\lambda t}\right)\right](200)\ \text{MeV}$$

Substituting the values, we get

$$E = \left[1000\times 405 - \frac{1000}{\frac{0.693}{1620}}\left(1 - e^{-\frac{0.693}{1620}\times 405}\right)\right]200\ \text{MeV}$$

$$\Rightarrow \quad E = 6.63\times 10^6\ \text{MeV}$$

PROBLEM 2

The energy received from the sun by earth and its surrounding atmosphere is $2\ \text{cal cm}^{-2}\ \text{min}^{-1}$ on a surface normal to the rays of sun. Calculate the

(a) total energy received in joules by earth and its atmosphere.

(b) total energy radiated in J min^{-1} by sun to the universe? Distance of sun to earth is 1.49×10^8 km.

(c) rate in mega-grams per minute) at which the hydrogen must be consumed in the fusion reaction to provide the sun with the energy it radiates.

Take mass of hydrogen atom to be 1.008145 amu and mass of He atom to be 4.003874 amu.

SOLUTION

(a) Let D be the diameter of earth. Then effective area of earth receiving radiation normally is

$$A_{\text{eff}} = \pi R^2 = \frac{\pi D^2}{4}$$

$$\Rightarrow \quad A_{\text{eff}} = \frac{\pi\left(1.27\times 10^4\right)^2}{4}\ \text{km}^2$$

$$\Rightarrow \quad A_{\text{eff}} = \frac{\pi}{4}(1.27)^2\times 10^{18}\ \text{cm}^2$$

Energy received by the earth per minute is

$$E_{\text{received}} = \left\{\frac{\pi}{4}(1.27)^2\times 10^{18}\right\}\times(2\times 4.2)\ \text{Jmin}^{-1}$$

$$\Rightarrow E_{\text{received}} = 10.645\times 10^{18}\ \text{Jmin}^{-1}$$

(b) The area of the surface surrounding the sun at a distance equal to earth-sun separation is $4\pi d^2$.

$$\Rightarrow \quad A = 4\pi\left(1.49\times 10^8\right)^2\times 10^6\times 10^4\ \text{cm}^2$$

$\Rightarrow \quad A = 4\pi(1.49)^2 \times 10^{26}\ \text{cm}^2$

Since the energy is received at the rate of $2\ \text{cal cm}^{-2}\ \text{min}^{-1}$ on this surface, so the energy radiated by sun in Jmin^{-1} is given by

$$E = 2\times 4.2\times\left[4\pi(1.49)^2\times 10^{26}\right]\ \text{Jmin}^{-1}$$

$\Rightarrow \quad E = 2.3444\times 10^{28}\ \text{Jmin}^{-1}$

(c) Since, we know that the fusion reaction in the sun is

$$4\text{H} \rightarrow \text{He} + Q$$

The mass defect for this reaction is given by

$$\Delta m = 4(1.008145) - 4.003874$$

$\Rightarrow \quad \Delta m = 0.028706$ amu

The energy released in one reaction is given by

$$Q = (0.028706)(931.5)\ \text{MeV} = 26.74\ \text{MeV}$$

So, mass of hydrogen required for the purpose is

$$m = \frac{(4.032580)(1.66\times 10^{-24})(2.3444\times 10^{28})}{(0.028706)(931.5)(1.6\times 10^{-19})}\ \text{gmin}^{-1}$$

$\Rightarrow \quad m = 3.6673\times 10^{22}\ \text{gmin}^{-1}$

$\Rightarrow \quad m = 3.6673\times 10^{16}$ megagram/min

PROBLEM 3

It is proposed to use nuclear reaction ${}_{84}\text{Po}^{210} \longrightarrow {}_{82}\text{Pb}^{206} + {}_{2}\text{He}^{4}$ to produce 2 kW electric power in a generator. The half life of polonium (Po^{210}) is 138.6 days. Assuming efficiency of the generator be 10%, calculate

(a) how many grams of (Po^{210}) are required per day at the end of 1386 days.

(b) initial activity of the material

Mass of nuclei: $\text{Po}^{210} = 209.98264$ amu,

$\text{Pb}^{206} = 205.97440$ amu,

${}_{2}\text{He}^{4} = 4.00260$ amu

1 amu = 931 MeV

SOLUTION

(a) $\Delta m = 0.00564\ \text{amu} \equiv 5.25\ \text{MeV} = 8.4\times 10^{-13}\ \text{J}$

Since, $\lambda = \dfrac{0.693}{t_{1/2}} = 0.005$ per day

Let m g of Po^{210} are required per day for the reactor, then

$$n = \frac{(6.02\times 10^{23})m}{210}$$

$$\Rightarrow \quad \left(-\frac{dN}{dt}\right) = \lambda N = \frac{0.005\times 6.02\times 10^{23}\times m}{210}\ \text{per day}$$

So, energy produced per day is

$$E = \frac{0.005\times 6.02\times 10^{23}\times m}{210}\times 8.4\times 10^{-13}\ \text{J}$$

$\Rightarrow \quad E = (12\times 10^{6})\ m\text{J}$

Now, 10% of $(12\times 10^{6})m$ equals $2\times 10^{3}\times 24\times 3600$ J

$$\Rightarrow \quad m = \frac{2\times 10^{3}\times 24\times 3600}{1.2\times 10^{6}} = 144\ \text{g}$$

(b) $R = \lambda N = (0.005)\left(\dfrac{144}{210}\right)(6.02\times 10^{23})$

$\Rightarrow \quad R = 2.064\times 10^{21}$ per day

Now, $R = R_0\left(\dfrac{1}{2}\right)^{10} \qquad \left\{\because\ n = \dfrac{1386}{138.6} = 10\right\}$

$\Rightarrow \quad R_0 = (2)^{10} R = 2.11\times 10^{24}$ per day

PROBLEM 4

Suppose a nucleus initially at rest undergoes α decay according to equation

$${}^{225}_{92}X \longrightarrow Y + \alpha$$

At $t = 0$, the emitted α particle enters in a region of space where a uniform magnetic field $\vec{B} = B_0\hat{i}$ and electric field $\vec{E} = E_0\hat{i}$ exist. The α particle enters in the region with velocity $\vec{v} = v_0\hat{j}$ from $x = 0$. At time $t = \sqrt{3}\times 10^{7}\dfrac{m_\alpha}{q_\alpha E_0}$ sec, the particle was observed to have speed twice the initial speed v_0, then find

(a) the velocity of α particle at time t.

(b) the initial velocity v_0 of the α particle

(c) the binding energy per nucleon of α particle.

Given that

$m(Y) = 221.03$ u, $m(\alpha) = 4.003$ u,

$m(n) = 1.009$ u, $m(p) = 1.008$ u,

$m_\alpha = \dfrac{2}{3}\times 10^{-26}$ kg, $q_\alpha = 3.2\times 10^{-19}$ C and

$1\ \text{u} = 931\ \text{MeVc}^{-2}$

SOLUTION

(a) Magnetic force on α particle, (at $t=0$)

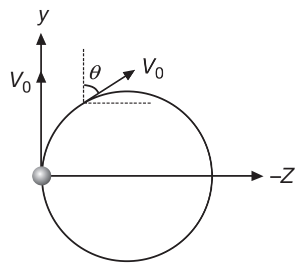
$$\vec{F}_m = q(\vec{v}\times\vec{B}) = q_\alpha\left[(v_0\hat{j})\times(B_0\hat{i})\right]$$

$$\Rightarrow \quad \vec{F}_m = -q_\alpha v_0 B_0 \hat{k}$$

Force due to electric field (at any time t)

$$\vec{F}_e = q\vec{E} = q_\alpha E_0 \hat{i}$$

Hence, the particle will move in a circular path in y-z plane due to magnetic field and at the same time it will move along x-direction. The resultant path is therefore, a helix with increasing pitch.



Hence, velocity of particle at any time t can be written as,

$$\vec{v} = \left(\frac{q_\alpha E_0}{m_\alpha}t\right)\hat{i} + v_0\cos\theta\hat{j} - v_0\sin\theta\hat{k}$$

where $\theta = \omega t = \dfrac{B_0 q_\alpha}{m_\alpha}t$

(b) Speed of particle at any time t is

$$v = \sqrt{\left(\frac{q_\alpha E_0 t}{m_\alpha}\right)^2 + v_0^2} \quad \{\because \sin^2\theta + \cos^2\theta = 1\}$$

Given $v = 2v_0$ at $t = (\sqrt{3}\times 10^7)\dfrac{m_\alpha}{q_\alpha E_0}$, so, we get

$$(2v_0)^2 = (\sqrt{3}\times 10^7)^2 + v_0^2$$

$$\Rightarrow \quad v_0 = 10^7 \text{ ms}^{-1}$$

(c) When an α-particle is emitted with velocity v_0 from a stationary nucleus X, decay product (nucleus Y) recoils. Then by Law of Conservation of Linear Momentum, we have

$$m_y v_y = m_\alpha v_0$$

$$\Rightarrow \quad v_y = \frac{m_\alpha v_0}{m_y} = \left(\frac{4.003}{221.03}\right)(10^7)$$

$$\Rightarrow \quad v_y = 1.81\times 10^5 \text{ ms}^{-1}$$

Total energy released during α-decay of nucleus X is

$$E = \text{K.E. of nucleus } Y + \text{K.E. of } \alpha\text{-particle}$$

$$\Rightarrow \quad E = \frac{1}{2}m_y v_y^2 + \frac{1}{2}m_\alpha v_0^2$$

$$\Rightarrow \quad E = \frac{1.66\times 10^{-27}}{2\times 1.6\times 10^{-13}}\left[(221.03)(1.81\times 10^5)^2 + (4.003)(10^7)^2\right]$$

$$\Rightarrow \quad E = 2.11 \text{ MeV}$$

Hence, Mass lost during α-decay is $\dfrac{2.11}{931.5}\text{u} = 0.0023\,\text{u}$

Mass of nucleus X is

$$m_x = (m_y + m_\alpha + 0.0023)\,\text{u}$$

$$\Rightarrow \quad m_x = 225.0353\,\text{u}$$

Mass defect in nucleus X is

$$\Delta m = 92m_p + (225-92)m_n - m_x = 1.898\,\text{u}$$

So, binding energy per nucleon is

$$\frac{BE}{A} = \frac{1.898\times 931.5}{225}\text{ MeV} = 7.86 \text{ MeV}$$

PROBLEM 5

Checkout from the following data, whether alpha decay or any of the beta decay are allowed for ${}^{226}_{89}\text{Ac}$.

$$m({}^{226}_{89}\text{Ac}) = 226.028356 \text{ amu},$$

$$m({}^{222}_{87}\text{Fr}) = 222.017415 \text{ amu},$$

$$m({}^{226}_{90}\text{Th}) = 226.017388 \text{ amu},$$

$$m({}^{226}_{88}\text{Ra}) = 226.025406 \text{ amu},$$

$$m({}^{4}_{2}\text{He}) = 4.002603 \text{ amu}.$$

SOLUTION

Let us first write the reaction for the corresponding decays and then find the disintegration energy Q. If $Q>0$, then the decay is allowed.

For alpha decay

$$^{226}_{89}\text{Ac} \to {}^{222}_{87}\text{Fr} + \alpha$$

$$\Rightarrow \quad Q = \left[m\left({}^{226}_{89}\text{Ac}\right) - m\left({}^{222}_{87}\text{Fr}\right) - m\left({}^{4}\text{He}\right) \right] c^2$$

$$\Rightarrow \quad Q = 5.50 \text{ MeV} \qquad \{\text{Alpha decay is allowed}\}$$

For β^- decay

$$^{226}_{89}\text{Ac} = {}^{226}_{90}\text{Th} + \beta^- + \bar{\nu}$$

$$\Rightarrow \quad Q = \left[M\left({}^{226}_{89}\text{Ac}\right) - M\left({}^{226}_{90}\text{Th}\right) \right] c^2$$

$$\Rightarrow \quad Q = 1.12 \text{ MeV} \qquad \{\beta^- \text{ decay is allowed}\}$$

For β^+ decay

$$^{226}_{89}\text{Ac} \to {}^{226}_{88}\text{Ra} + \beta^+ + \nu$$

$$Q = \left[m\left({}^{226}_{89}\text{Ac}\right) - m\left({}^{226}_{88}\text{Ra}\right) - 2m_e \right] c^2$$

$$\Rightarrow \quad Q = -0.38 \text{ MeV} \qquad \{\beta^+ \text{ decay is not allowed}\}$$

For electron capture

$$^{226}_{89}\text{Ac} + e^- \to {}^{226}_{88}\text{Ra} + \nu$$

$$\Rightarrow \quad Q = \left[m\left({}^{226}_{89}\text{Ac}\right) - m\left({}^{226}_{88}\text{Ra}\right) \right] c^2$$

$$\Rightarrow \quad Q = 0.64 \text{ MeV} \qquad \{\text{Electron capture is allowed}\}$$

From the above analysis, it is clear that during α-decay the Q-value is maximum and hence chances of α-decay are maximum.

PROBLEM 6

A radionuclide consists of two isotopes. One of the isotopes decays by α-emission and other by β-emission with half lives $T_1 = 405$ s, $T_2 = 1620$ s, respectively. At $t = 0$, probabilities of getting α and β-particles from the radionuclide are equal. Calculate their respective probabilities at $t = 1620$ s. If at $t = 0$, total number of nuclei in the radio nuclide are N_0. Calculate the time t when total number of nuclei remained undecayed becomes equal to $\dfrac{N_0}{2}$.

Given, $\log_{10} 2 = 0.3010$, $\log_{10} 5.94 = 0.7742$ and $x^4 + 4x - 2.5 = 0$, $x = 0.594$.

SOLUTION

At $t = 0$, probabilities of getting α and β particles are same. This implies that initial activity of both is equal, say R_0.

Activity after $t = 1620$ s is

$$R_1 = R_0 \left(\frac{1}{2}\right)^{\frac{1620}{405}} = \frac{R_0}{16}$$

and $$R_2 = R_0 \left(\frac{1}{2}\right)^{\frac{1620}{1620}} = \frac{R_0}{2}$$

Total activity $R = R_1 + R_2 = \dfrac{9}{16} R_0$

Probability of getting α particles is $\dfrac{R_1}{R} = \dfrac{1}{9}$ and

Probability of getting β particles is $\dfrac{R_2}{R} = \dfrac{8}{9}$

Since, $R_{01} = R_{02}$

$$\Rightarrow \quad \frac{N_{01}}{T_1} = \frac{N_{02}}{T_2}$$

$$\Rightarrow \quad \frac{N_{01}}{N_{02}} = \frac{1}{4}$$

Let N_0 be the total number of nuclei at $t = 0$ then,

$$N_{01} = \frac{N_0}{5} \text{ and } N_{02} = \frac{4N_0}{5}$$

Given, that $N_1 + N_2 = \dfrac{N_0}{2}$

$$\Rightarrow \quad \frac{N_0}{5}\left(\frac{1}{2}\right)^{\frac{t}{405}} + \frac{4N_0}{5}\left(\frac{1}{2}\right)^{\frac{t}{1620}} = \frac{N_0}{2} \qquad \ldots(1)$$

Let $\left(\dfrac{1}{2}\right)^{\frac{t}{1620}} = x$, then the equation (1) becomes

$$x^4 + 4x - 2.5 = 0$$

$$\Rightarrow \quad x = 0.594$$

$$\Rightarrow \quad \left(\frac{1}{2}\right)^{\frac{t}{1620}} = 0.594$$

Solving, we get

$$t = 1215 \text{ s}$$

PROBLEM 7

A radioactive element decays by β-emission. A detector records n beta particles in 2 seconds and in next 2-seconds it records $\dfrac{3}{4}n$ beta particles. Find mean life correct to nearest whole number. Given $\log_e |2| = 0.6931$, $\log_e |3| = 1.0986$.

SOLUTION

Let n_0 be the number of radioactive nuclei at time $t = 0$. Number of nuclei decayed in time t are given by $n_0(1 - e^{-\lambda t})$, which is also equal to the number of beta particles emitted during the same interval of time. For the given condition.

$$n = n_0(1 - e^{-2\lambda}) \quad \ldots(1)$$

$$\left(n + \frac{3}{4}n\right) = n_0(1 - e^{-4\lambda}) \quad \ldots(2)$$

Dividing equation (2) by (1), we get

$$1.75 = \frac{1 - e^{-4\lambda}}{1 - e^{-2\lambda}}$$

$$\Rightarrow \quad 1.75 - 1.75e^{-2\lambda} = 1 - e^{-4\lambda}$$

$$\Rightarrow \quad 1.75e^{-2\lambda} - e^{-4\lambda} = \frac{3}{4} \quad \ldots(3)$$

Let us take $e^{-2\lambda} = x$

Then the above equation becomes

$$x^2 - 1.75x + 0.75 = 0$$

$$\Rightarrow \quad x = \frac{1.75 \pm \sqrt{(1.75)^2 - (4)(0.75)}}{2}$$

$$\Rightarrow \quad x = 1 \text{ and } \frac{3}{4}$$

$$\Rightarrow \quad e^{-2\lambda} = 1 \text{ or } e^{-2\lambda} = \frac{3}{4}$$

But $e^{-2\lambda} = 1$ is not accepted because which means $\lambda = 0$. Hence, $e^{-2\lambda} = \frac{3}{4}$

$$\Rightarrow \quad -2\lambda \log_e(e) = \log_e(3) - \log_e(4)$$

$$\Rightarrow \quad -2\lambda = \log_e(3) - 2\log_e(2)$$

$$\Rightarrow \quad \lambda = \log_e(2) - \frac{1}{2}\log_e(3)$$

Substituting the given values, we get

$$\lambda = 0.6931 - \frac{1}{2} \times (1.0986) = 0.14395 \text{ s}^{-1}$$

So, mean life

$$t_{av} = \frac{1}{\lambda} = 6.947 \text{ s}$$

PROBLEM 8

A nuclear reactor generates power at 50% efficiency by fission of $^{235}_{92}$U into two equal fragments of $^{116}_{46}$Pd with the emission of two gamma rays of 5.2 MeV each and three neutrons. The average binding energies per particle of $^{235}_{92}$U and $^{116}_{46}$Pd are 7.2 MeV and 8.2 MeV respectively. Calculate the energy released in one fission event. Also estimate the amount to ^{235}U consumed per hour to produce 1600 megawatt power.

SOLUTION

$$\begin{pmatrix}\text{Energy}\\\text{released}\\\text{in}\\\text{one}\\\text{fission}\end{pmatrix} = \begin{pmatrix}\text{Binding}\\\text{energy}\\\text{of two}\\ ^{116}_{46}Pd\\\text{nuclei}\end{pmatrix} - \begin{pmatrix}\text{Binding}\\\text{energy}\\\text{of}\\ ^{235}_{92}U\\\text{nucleus}\end{pmatrix} - \begin{pmatrix}\text{Energy}\\\text{of two}\\\text{emitted}\\\text{gamma}\\\text{rays}\end{pmatrix}$$

Here binding energy of $^{235}_{92}$U nucleus is

$$(\Delta E)_U = 72 \times 235 = 1692 \text{ MeV}$$

Binding energy of two $^{116}_{46}$Pd nuclei

$$2(\Delta E)_{Pd} = 2 \times 8.2 \times 116 = 1902.4 \text{ MeV}$$

Energy of two emitted gamma rays is

$$2E_\gamma = 2 \times 5.2 = 10.4 \text{ MeV}$$

Total energy released in one event is

$$E = (1902.4 - 1692 - 10.4) \text{ MeV}$$

$$\Rightarrow \quad E = 200 \text{ MeV}$$

$$\Rightarrow \quad E = 200 \times (1.6 \times 10^{-13})$$

$$\Rightarrow \quad E = 3.2 \times 10^{-11} \text{ J}$$

So, the number of fission per second required to produce 1600×10^6 J of energy per second i.e. 1600 MW will be

$$N = \frac{1600 \times 10^6}{3.2 \times 10^{-11}} = 5 \times 10^{19} \text{ s}^{-1}$$

So, 5×10^{19} nuclei of ^{235}U per second are required for this purpose. The mass of these nuclei is

$$m = \frac{235}{6.02 \times 10^{23}} \times (5 \times 10^{19})$$

$$\Rightarrow \quad m = 195.2 \times 10^{-4} \text{ gs}^{-1}$$

Thus, amount of ^{235}U consumed per hour is

$$m = \left(195.2\times10^{-4}\right)\times 3600$$

$$\Rightarrow \quad m = 70.27\ \text{ghr}^{-1}$$

Since reactor efficiency is 50%, hence the consumption of ^{235}U per hour is given by

$$m = 70.27\times 2 = 140.5\ \text{g}$$

PROBLEM 9

Nuclei of a radioactive element A are being produced at a constant rate α. The element has a decay constant λ. At time $t=0$, there are N_0 nuclei of the element.

(a) Calculate the number N of nuclei of A at time t.

(b) if $\alpha = 2N_0\lambda$, calculate the number of nuclei of A after one half-life of A and also the limiting value of N as $t\to\infty$.

SOLUTION

(a) Let at time t, number of radioactive nuclei are N
Net rate of formation of nuclei of A

$$\frac{dN}{dt} = \alpha - \lambda N$$

$$\Rightarrow \quad \frac{dN}{\alpha-\lambda N} = dt$$

$$\Rightarrow \quad \int_{N_0}^{N}\frac{dN}{\alpha-\lambda N} = \int_0^t dt$$

Solving this equation, we get

$$N = \frac{1}{\lambda}\left(\alpha - \left(\alpha-\lambda N_0\right)e^{-\lambda t}\right) \qquad \ldots(1)$$

(b) (i) Substituting $\alpha = 2\lambda N_0$ and $t = t_{1/2} = \dfrac{\log_e(2)}{\lambda}$ in equation (1), we get,

$$N = \frac{3}{2}N_0$$

(ii) Substituting $\alpha = 2\lambda N_0$ and $t\to\infty$ in equation (1), we get

$$N = \frac{\alpha}{\lambda} = 2N_0$$

$$\Rightarrow \quad N = 2N_0$$

PROBLEM 10

A nucleus X-initially at rest, undergoes alpha-decay, according to the equation

$$^{A}_{92}X \longrightarrow {}^{228}_{z}Y + \alpha$$

(a) Find the value of A and z in the above process.

(b) The α-particle in the above process is found to move in a circular track of radius $0.11\ \text{m}$ in a uniform magnetic field of $3\ \text{T}$. Find the energy (in MeV) released during the process and binding energy of the parent nucleus X.

Given: $m_y = 228.03$ amu $\qquad m_\alpha = 4.003$ amu

$m\left({}^1_0n\right) = 1.009$ amu $\qquad m\left({}^1_1\text{H}\right) = 1.008$ amu

$1\ \text{amu} = 1.66\times10^{-27}\ \text{kg} = 931.5\ \text{MeVc}^{-2}$

SOLUTION

(a) The given equation is,

$$^{A}_{92}X \longrightarrow {}^{228}_{z}Y + {}^4_2\text{He}$$

$$A = 228+4 = 232$$

and $92 = z+2$

$$\Rightarrow \quad z = 90$$

(b) $$\frac{m_\alpha v_\alpha^2}{r} = qv_\alpha B$$

$$\Rightarrow \quad r = \frac{m_\alpha v_\alpha}{qB} = \frac{\sqrt{2m_\alpha K_\alpha}}{qB}$$

$$\Rightarrow \quad K_\alpha = \frac{q^2B^2r^2}{2m_\alpha}$$

$$\Rightarrow \quad K_\alpha = \frac{\left(1.6\times10^{-19}\right)^2(3)^2(0.11)^2}{2(4.003)\left(1.66\times10^{-27}\right)\left(1.6\times10^{-13}\right)}\text{MeV}$$

$$\Rightarrow \quad K_\alpha = 5.21\ \text{MeV}$$

Applying Law of Conservation of Linear Momentum, we get

$$0 = m_\alpha v_\alpha + m_Y v_Y$$

$$\Rightarrow \quad |p_y| = |p_\alpha|$$

$$\Rightarrow \quad \sqrt{2m_YK_Y} = \sqrt{2m_\alpha K_\alpha}$$

$$\Rightarrow \quad K_Y = \left(\frac{m_\alpha}{m_Y}\right)K_\alpha = \left(\frac{4.003}{228.03}\right)(5.21) = 0.09\ \text{MeV}$$

Therefore, energy released during the process is

$$E=\frac{1}{2}\left(m_\alpha v_\alpha^2+m_Y v_Y^2\right)-0=K_\alpha+K_Y$$

$$\Rightarrow \quad E=K_\alpha+K_Y=5.21+0.09=5.3\text{ MeV}$$

Now, mass of ${}^{232}_{92}X$ is

$$m\left({}^{232}_{92}X\right)=m_Y+m_\alpha+0.000365=232.033365\text{ u}$$

So, mass defect is given by

$$\Delta m=92(1.008)+(232-92)(1.009)-232.033365$$

$$\Rightarrow \quad \Delta m=1.962635\text{ amu}=1.962635\text{ u}$$

$$\Rightarrow \quad \text{Binding Energy}=1.962635\times 931.5\text{ MeV}$$

$$\Rightarrow \quad \text{Binding Energy}=1828.2\text{ MeV}$$

PROBLEM 11

A radioactive source in the form of metal sphere of diameter 10^{-3} m emits beta particles at a constant rate of 6.25×10^{10} particles per second. If the source is electrically insulated, how long will it take for its potential to rise by 1 volt. Assume that 80% of the emitted beta particles escape from the source?

SOLUTION

Let t be the time for the potential of metal sphere to rise by one volt. Then up to this time, β-particles emitted from sphere are

$$N=\left(6.25\times10^{10}\right)t$$

Number of β-particles that escape in this time are

$$N_e=\left(\frac{80}{100}\right)\times\left(6.25\times10^{10}\right)t$$

$$\Rightarrow \quad N_e=5\times10^{10}t$$

Since, the emission of a β-particle leads to a charge e on metal sphere, so charge acquired by the sphere in time t sec is

$$Q=\left(5\times10^{10}t\right)\times\left(1.6\times10^{-19}\right)$$

$$\Rightarrow \quad Q=8\times10^{-9}t\text{ coulomb} \qquad \ldots(1)$$

The capacitance C of the metal sphere is given by

$$C=4\pi\varepsilon_0 r$$

$$\Rightarrow \quad C=\left(\frac{1}{9\times10^{9}}\right)\times\left(\frac{10^{-3}}{2}\right)$$

$$\Rightarrow \quad C=\frac{10^{-12}}{18}\text{ farad} \qquad \ldots(2)$$

Since $Q=CV$ where $V=1$ volt

$$\Rightarrow \quad \left(8\times10^{-9}\right)t=\left(\frac{10^{-12}}{18}\right)\times1$$

$$\Rightarrow \quad t=6.95\ \mu\text{s}$$

PROBLEM 12

A nucleus at rest undergoes a decay emitting an α-particle of de-Broglie wavelength, $\lambda=5.76\times10^{-15}$ m. If the mass of the daughter nucleus is 223.610 amu and that of the α-particle is 4.002 amu. Determine the total kinetic energy in the final state. Hence obtain the mass of the parent nucleus in amu. $\left(1\text{ amu}=931.470\text{ MeVc}^{-2}\right)$

SOLUTION

Given mass of α-particle, $m=4.002$ amu and mass of daughter nucleus,

$$M=223.610\text{ amu}$$

de-Broglie wavelength of α-particle,

$$\lambda=5.76\times10^{-15}\text{ m}$$

So, momentum of α-particle is

$$p=\frac{h}{\lambda}=\frac{6.63\times10^{-34}}{5.76\times10^{-15}}\text{ kgms}^{-1}$$

$$\Rightarrow \quad p=1.151\times10^{-19}\text{ kgms}^{-1}$$

By Law of Conservation of Linear Momentum, this should also be equal to the linear momentum of the daughter nucleus (in opposite direction).

Let K_1 and K_2 be the kinetic energies of α-particle and daughter nucleus. Then total kinetic energy in the final state is

$$K=K_1+K_2=\frac{p^2}{2m}+\frac{p^2}{2M}$$

$$\Rightarrow \quad K=\frac{p^2}{2}\left(\frac{1}{m}+\frac{1}{M}\right)$$

$$\Rightarrow \quad K=\frac{p^2}{2}\left(\frac{M+m}{Mm}\right)$$

Since, $1\text{ amu}=1.67\times10^{-27}$ kg

Substituting the values, we get

$$K=10^{-12}\text{ J}$$

$$\Rightarrow \quad K=\frac{10^{-12}}{1.6\times10^{-13}}=6.25\text{ MeV}$$

$\Rightarrow \quad K = 6.25 \text{ MeV}$

Mass defect, $\Delta m = \dfrac{6.25}{931.470} = 0.0067$ amu

$$\therefore \begin{pmatrix}\text{mass of}\\ \text{parent}\\ \text{nucleus}\end{pmatrix} = \begin{pmatrix}\text{mass}\\ \text{of}\\ \alpha\text{-particle}\end{pmatrix} + \begin{pmatrix}\text{mass of}\\ \text{daughter}\\ \text{nucleus}\end{pmatrix} + \begin{pmatrix}\text{mass}\\ \text{defect}\\ (\Delta m)\end{pmatrix}$$

$\Rightarrow \quad m_{parent} = (4.002 + 223.610 + 0.0067)$ amu

$\Rightarrow \quad m_{parent} = 227.62$ amu

Hence, mass of parent nucleus is 227.62 amu.

PROBLEM 13

A small quantity of solution containing Na_{24} radio nuclide (half-life = 15 hour) of activity 1.0 microcurie is injected into the blood of a person. A sample of the blood of volume 1 cm^3 taken after 5 hour shows an activity of 296 disintegrations per minute. Determine the total volume of the bond in the body of the person. Assume that the radioactive solution mixes uniformly in the blood of the person.
(1 curie $= 3.7 \times 10^{10}$ disintegrations per second)

SOLUTION

λ is the disintegration constant, then

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{15} \text{ h}^{-1}$$

$\Rightarrow \quad \lambda = 0.0462 \text{ h}^{-1}$

Let R_0 be the initial activity then

$$R_0 = 1 \text{ microcurie} = 3.7 \times 10^4 \text{ dps}$$

Let r be the activity in 1 cm^3 of blood at $t = 5$ hr, then

$$r = \frac{296}{60} \text{ disintegration per second}$$

$\Rightarrow \quad r = 4.93$ disintegration per second, and

R be the activity of whole blood at time $t = 5$ hr
Total volume of blood should be

$$V = \frac{R}{r}$$

$\Rightarrow \quad V = \dfrac{R_0 e^{-\lambda t}}{r}$

Substituting the values, we get

$$V = \left(\frac{3.7 \times 10^4}{4.93}\right) e^{-(0.0462)(5)} \text{ cm}^3$$

$\Rightarrow \quad V = 5.95 \times 10^3 \text{ cm}^3$

$\Rightarrow \quad V = 5.95$ lt.

PROBLEM 14

The element curium $^{248}_{96}\text{Cm}$ has a mean life of 10^{13} seconds. Its primary decay modes are spontaneous fission and α-decay, the former with a probability of 8% and the latter with a probability of 92%. Each fission releases 200 MeV of energy. If the mass of $^{248}_{96}\text{Cm}$ is 248.072220 u, $^{244}_{94}\text{Pu}$ is 244.064100 u and $^{4}_{2}\text{He}$ is 4.002603 u. Calculate the power output from a sample of 10^{20} Cm atoms. $(1 \text{ u} = 931 \text{ meVc}^{-2})$

SOLUTION

The reaction involved in α-decay is

$$^{248}_{96}\text{Cm} \to {}^{244}_{94}\text{Pu} + {}^{4}_{2}\text{He}$$

Mass defect

$$\Delta m = m\left({}^{248}_{96}\text{Cm}\right) - \left[m\left({}^{244}_{94}\text{Pu}\right) + m\left({}^{4}_{2}\text{He}\right)\right]$$

$\Rightarrow \quad \Delta m = (248.072220 - 244.064100 - 4.002603)\text{u}$

$\Rightarrow \quad \Delta m = 0.005517$ u

Therefore, energy released in α-decay will be

$$E_\alpha = (0.005517 \times 931) \text{ MeV} = 5.136 \text{ MeV}$$

Similarly, $E_{fission} = 200$ MeV {given}

Mean life is

$$t_{mean} = 10^{13} \text{ s} = \frac{1}{\lambda}$$

So, disintegration constant $\lambda = 10^{-13} \text{ s}^{-1}$
Rate of decay at the moment when number of nuclei are 10^{20}

$$-\frac{dN}{dt} = \lambda N = (10^{-13})(10^{20})$$

$$\Rightarrow \quad -\frac{dN}{dt} = 10^7 \text{ disintegration per second}$$

Of these disintegrations, 8% are in fission and 92% are in α-decay.
Therefore, energy released per second is

$$P = \frac{E}{t}$$

$$\Rightarrow \quad P = \left(0.08 \times 10^7 \times 200 + 0.92 \times 10^7 \times 5.136\right) \text{ MeV}$$

$$P = \frac{E}{t} = 2.074 \times 10^8 \text{ MeV}$$

So, power output (in watt) is

$$P = \text{energy released per second } \left(\text{Js}^{-1}\right)$$

$$\Rightarrow \quad P = \left(2.074 \times 10^8\right)\left(1.6 \times 10^{-13}\right)$$

So, power output is $P = 3.32 \times 10^{-5}$ W

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. A radio isotope has a half life of 10 sec. If initially there are 1000 isotopes in sample which falls from rest from top of a building of height 3000 m. The number of nuclei in the sample when the sample is at a height of 1000 m from the ground is (take $g = 10\text{ ms}^{-2}$)

(A) 50 (B) 250
(C) 29 (D) 100

2. A radioactive substance is being produced at a constant rate of 200 nuclei. The decay constant of the substance is 1 s^{-1}. Assuming that initially there are no nuclei present, the time (in second) after which the number of nuclei will become 100 is

(A) 1 s (B) $\log_e(2)$ s
(C) $\dfrac{1}{\log_e(2)}$ s (D) 2 s

3. A radioactive material of half-life T was produced in a nuclear reactor at different instants, the quantity produced second time was twice of that produced first time. If now their present activities are A_1 and A_2 respectively then their age difference equals

(A) $\dfrac{T}{\log_e 2}\left|\log_e\left(\dfrac{2A_1}{A_2}\right)\right|$ (B) $T\left|\log_e\left(\dfrac{A_1}{A_2}\right)\right|$

(C) $\dfrac{T}{\log_e 2}\left|\log_e\left(\dfrac{A_2}{2A_1}\right)\right|$ (D) $T\left|\log_e\left(\dfrac{A_2}{2A_1}\right)\right|$

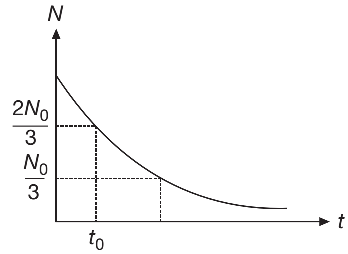
4. The atomic weight of boron is 10.81 and it has two isotopes ${}^{10}_{5}B$ and ${}^{11}_{5}B$. The ratio of ${}^{10}_{5}B : {}^{11}_{5}B$ in nature would be

(A) $19:81$ (B) $10:11$
(C) $15:16$ (D) $81:19$

5. Radioactivity is

(A) spontaneous process
(B) irreversible process
(C) self disintegration process
(D) all the above

6. Figure shows the variation of the number of radioactive atoms left undecayed with time. The time corresponding to $N = \dfrac{N_0}{3}$ is



(A) $3t_0$ (B) $t_0\log_e(2)$

(C) $\dfrac{t_0\log_e(3)}{\log_e\left(\dfrac{3}{2}\right)}$ (D) $t_0\log_e(3)$

7. If one starts with one curie of radioactive substance having half life of 12 hour, the activity left after a period of 1 week will be about

(A) 1 curie (B) 120 micro curie
(C) 60 micro curie (D) 8 mili curie

8. There are two radio nuclei A and B out of which A is an alpha emitter and B is a beta emitter. Their disintegration constants are in the ratio of $1:2$. The ratio of number of atoms of A and B at any time t so that probabilities of getting alpha and beta particles are same at that instant is

(A) $1:2$ (B) $1:e$
(C) $2:1$ (D) $e:1$

9. Cobalt-57 is radioactive, emitting β particles. The half-life for this is 270 day. If 100 mg of this is kept in an open container, the mass of cobalt-57 after 540 day will be

(A) 50 mg (B) $\dfrac{50}{\sqrt{2}}$ mg
(C) 25 mg (D) zero

10. The half life of radon is 3.8 days. After how many days will $\dfrac{1}{10}$ th of the radon sample remain behind

(A) 1.262 days (B) 12.62 days
(C) 126.2 days (D) 1262 days

11. The half-life of a radioactive material is T. After $\dfrac{T}{2}$ time, the material left is

(A) $\frac{1}{2}$ (B) $\frac{3}{4}$

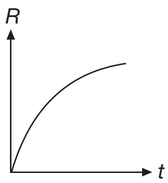
(C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{4}$

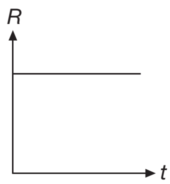
12. In any fission process the ratio $\frac{\text{mass of fission products}}{\text{mass of parent nucleus}}$ is
(A) Less than 1
(B) Greater than 1
(C) Equal to 1
(D) Depends on the mass of the parent nucleus

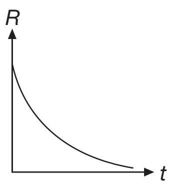
13. α–particle emitted during various radioactive processes have same
(A) speed (B) momentum
(C) kinetic energy (D) specific charge

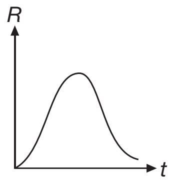
14. The half life of a radioactive substance is 4 day. Initially at $t = 0$ there are 100 nuclei of this substance then in the sample we will obtain
(A) 50 undecayed nuclei after 4 day
(B) 25 undecayed nuclei after 6 day
(C) 75 undecayed nuclei after 2 day
(D) Cannot be said

15. A radioactive nucleus X decays to a stable nucleus Y. Then the graph of rate of formation of Y against time t will be

(A)


(B)


(C)


(D)


16. In the beta decay, the mass number and atomic number of the daughter nuclei respectively
(A) increase by 2
(B) decrease by 1
(C) do not change at all
(D) mass number remains same but atomic number increases by 1

17. The mean lives of a radioactive substance are 1620 year and 405 year for α-emission and β-emission respectively. The time during which three-fourth of a sample will decay if it is decaying both by α-emission and β-emission simultaneously is
(A) 249 year (B) 449 year
(C) 133 year (D) 99 year

18. Decay constant of radium is λ. By a suitable process its compound radium bromide is obtained. The decay constant of radium bromide will be
(A) λ (B) More than λ
(C) Less than λ (D) Zero

19. Half life of a radioactive substance A is two times the half life of another radioactive substance B. Initially the number of nuclei of A and B are N_A and N_B respectively. After three half lives of A number of nuclei of both are equal. Then the ratio $\frac{N_A}{N_B}$ is
(A) $\frac{1}{3}$ (B) $\frac{1}{6}$
(C) $\frac{1}{4}$ (D) $\frac{1}{8}$

20. The energy released (in MeV) due to annihilation of 1 kg mass completely to energy is
(A) 7.625×10 MeV (B) 10.5×10^{29} MeV
(C) 2.8×10^{-28} MeV (D) 5.625×10^{29} MeV

21. Which of the following is not a mode of decay of radioactive elements?
(A) Alpha decay (B) Positron emission
(C) Fission (D) Electron capture

22. In a sample of a radioactive substance, the fraction of the initial number of nuclei will remain undecayed after a time $t = \frac{T}{2}$, where T is the half life of radioactive substance is
(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{4}$
(C) $\frac{1}{2\sqrt{2}}$ (D) $\frac{1}{\sqrt{2}-1}$

23. A radioactive decay counter is switched on at $t = 0$. A β-active sample is present near the counter. The counter registers the number of β-particles emitted by the sample. The counter registers 1×10^5 β-particles at $t = 36$ sec and 1.11×10^5 β-particles at $t = 108$ sec. Half life of this sample is
(A) 5.2 sec (B) 10.8 sec
(C) 15.4 sec (D) 20.6 sec

24. The atomic ratio between the uranium isotopes ^{238}U and ^{234}U in a mineral sample is found to be 1.8×10^4. The half life of ^{234}U is 2.5×10^5 year. The half-life of ^{238}U is

(A) 4.5×10^9 year (B) 5.4×10^9 year
(C) 4.5 year (D) 5.4 year

25. How would the radio isotope of magnesium with atomic mass 27 undergo radioactive decay?
(A) Electron capture (B) Alpha decay
(C) Beta decay (D) Gamma ray emission

26. The decay constants of a radioactive substance for a and b emission are λ_a and λ_b respectively. If the substance emits α and β simultaneously, the average half life of the material will be
(A) $T_\alpha - T_\beta$ (B) $T_\alpha + T_\beta$
(C) $\dfrac{T_\alpha T_\beta}{T_\alpha + T_\beta}$ (D) None of these

27. If 10% of a radioactive substance decays in every 5 year, then the percentage of the substance that will have decayed in 20 year will be
(A) 40% (B) 50%
(C) 65.6% (D) 34.4%

28. The rate of decay of a radioactive element
(A) increases with increase in time
(B) decreases with increase in time
(C) remains constant with increase in time
(D) decreases exponentially with time

29. Consider the following nuclear reaction

$$X^{200} \to A^{110} + B^{90} + \text{Energy}$$

If the binding energy per nucleon for X, A and B are 7.4 MeV, 8.2 MeV and 8.2 MeV respectively, the energy released will be
(A) 90 MeV (B) 110 MeV
(C) 200 MeV (D) 160 MeV

30. A nucleus X initially at rest, undergoes alpha decay according to the equation

$$^{232}_{Z}X \to ^{A}_{90}Y + \alpha$$

What fraction of the total energy released in the decay will be the kinetic energy of the alpha particle?
(A) $\dfrac{90}{92}$ (B) $\dfrac{228}{232}$
(C) $\sqrt{\dfrac{228}{232}}$ (D) $\dfrac{1}{2}$

31. The total energy E of a sub-atomic particle of rest mass m_0 moving at non-relativistic speed v is
(A) $E = m_0c^2$ (B) $E = \dfrac{1}{2}m_0v^2$
(C) $E = m_0c^2 + \dfrac{1}{2}m_0v^2$ (D) $E = m_0c^2 - \dfrac{1}{2}m_0v^2$

32. The radioactivity of a sample is x at a time t_1 and y at a time t_2. If the mean life of the specimen is τ, the number of atoms that have disintegrated in the time interval $(t_2 - t_1)$ is
(A) $xt_1 - yt_2$ (B) $x - y$
(C) $\dfrac{x-y}{\tau}$ (D) $(x-y)\tau$

33. The energy released by the fission of a single uranium nucleus is 200 MeV. The number of fissions of uranium nucleus per second required to produce 16 MW of power is
(Assume efficiency of the reactor is 50%)
(A) 2×10^6 (B) 2.5×10^6
(C) 5×10^6 (D) None of these

34. The activity of a radioactive substance is R_1 at time t_1 and R_2 at time t_2 $(>t_1)$. Its decay constant is λ. Then
(A) $R_1t_1 = R_2t_2$ (B) $R_2 = R_1 e^{\lambda(t_2-t_1)}$
(C) $\dfrac{R_1 - R_2}{t_2 - t_1} = \text{constant}$ (D) $R_2 = R_1 e^{\lambda(t_1-t_2)}$

35. In PROBLEM 34, number of atoms decayed between time interval t_1 and t_2 are
(A) $\dfrac{\log_e(2)}{\lambda}(R_1 - R_2)$ (B) $R_1e^{-\lambda t_1} - R_2e^{-\lambda t_2}$
(C) $\lambda(R_1 - R_2)$ (D) $\left(\dfrac{R_1 - R_2}{\lambda}\right)$

36. Half-lives of two radioactive substances A and B are respectively 20 minute and 40 minute. Initially, the samples of A and B have equal number of nuclei. After 80 minute the ratio of the remaining numbers of A and B nuclei is
(A) 1:16 (B) 4:1
(C) 1:4 (D) 1:1

37. In a radioactive series, $_{92}U^{238}$ changes to $_{82}Pb^{206}$ through n_1 α-decay process and n_2 β-decay process
(A) $n_1 = 8$, $n_2 = 8$ (B) $n_1 = 6$, $n_2 = 6$
(C) $n_1 = 8$, $n_2 = 6$ (D) $n_1 = 6$, $n_2 = 8$

38. In free space, the intensity of 5 eV neutron beam is reduced by a factor of one half. Half life of neutron is 12.8 min . The distance travelled by neutron beam is

(A) 2800 km (B) 23800 km
(C) 28 km (D) 2 km

39. When secular radioactive equilibrium is reached between Radium and Uranium, then which of the following relation hold true. Given that,

λ_R is decay constant of Radium
N_R is number of nuclei of Radium
λ_U is decay constant of Uranium
N_U is number of nuclei of Uranium

(A) $\frac{\lambda_R}{\lambda_U} = \frac{N_U}{N_R}$ (B) $\frac{\lambda_R}{\lambda_U} = \frac{N_R}{N_U}$
(C) $\frac{\lambda_R}{\lambda_U} = \sqrt{\frac{N_R}{N_U}}$ (D) $\frac{\lambda_R}{\lambda_U} = \sqrt{\frac{N_U}{N_R}}$

40. A γ-ray photon is emitted

(A) after ionization of an atom
(B) due to conversion of a neutron into a proton in the nucleus
(C) after de-excitation of a nucleus
(D) due to conversion of a proton into a neutron in the nucleus

41. The count rate from 100 cm^3 of a radioactive liquid is c. Some of this liquid is now discarded. The count rate of the remaining liquid is found to be $\frac{c}{10}$ after three half-lives. The volume of the remaining liquid, in cm^3, is

(A) 20 (B) 40
(C) 60 (D) 80

42. The rate of disintegration of fixed quantity of a radioactive element can be increased by

(A) increasing the temperature
(B) increasing the pressure
(C) increasing the humidity
(D) It is not possible

43. A 100 ml solution having activity 50 dps is kept in a beaker. It is now constantly diluted by adding water at a constant rate of 10 ml/sec and at the same time the solution is constantly being taken out at the rate of 2 ml/sec. The activity of 10 ml solution which is taken out, assuming half life to be effectively very large is

(A) $10\left[1-\left(\frac{2}{7}\right)^{\frac{1}{4}}\right]$ (B) $10\left[1-\left(\frac{5}{7}\right)^{\frac{1}{2}}\right]$
(C) $50\left[1-\left(\frac{5}{7}\right)^{\frac{1}{4}}\right]$ (D) $50\left[1-\left(\frac{2}{7}\right)^{\frac{1}{2}}\right]$

44. An isotope of potassium $^{40}_{19}K$ has a half life of 1.4×10^9 year and decays to Argon $^{40}_{18}Ar$ which is stable. A sample of rock taken from the moon contains both potassium and argon in the ratio $\frac{1}{7}$. Age of the rock is

(A) 2.8×10^9 year (B) 3.5×10^9 year
(C) 4.2×10^9 year (D) 8.2×10^{10} year

45. The count rate of a Geiger-Muller counter for the radiation of radioactive material of half life of 30 minutes decreases to 5 s^{-1} after 2 hours. The initial count rate was

(A) 25 s^{-1} (B) 80 s^{-1}
(C) 625 s^{-1} (D) 20 s^{-1}

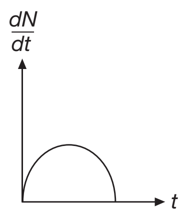
46. The half-life of a certain radioactive element is such that $\frac{7}{8}$ of a given quantity decays in 12 day. What fraction remains undecayed after 24 day?

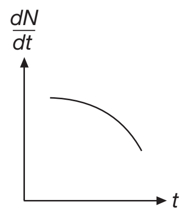
(A) 0 (B) $\frac{1}{128}$
(C) $\frac{1}{64}$ (D) $\frac{1}{32}$

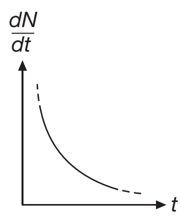
47. When a free neutron decays to form a proton and an electron, then choose the incorrect statement

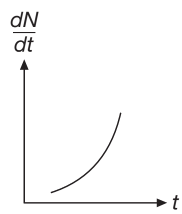
(A) the relation may be expressed as ${}_0n^1 \rightarrow {}_1p^1 + {}_{-1}e^0$
(B) every electron comes out with the same energy
(C) the electron shares the major part of the energy released
(D) all the above

48. Radioactive element decays to form a stable nuclide, then the rate of decay of reactant $\left(\frac{dN}{dt}\right)$ will vary with time (t) as shown in figure

(A)


(B)


(C)


(D)


49. If 10% of a radioactive material decays in 5 day, then the amount of original material left after 20 day is approximately

(A) 60% (B) 65%
(C) 70% (D) 75%

50. It has been observed that the most stable nuclei have generally the neutron to proton ratio as
(A) $1:2$ (B) $2:1$
(C) $1:4$ (D) $1:1$

51. A radioactive isotope is being produced at a constant rate A. The isotope has a half-life T. Initially there are no nuclei, after a time $t \gg T$, the number of nuclei becomes constant. The value of this constant is
(A) AT (B) $\frac{A}{T}\ln(2)$
(C) $AT\ln(2)$ (D) $\frac{AT}{\ln(2)}$

52. Figure below shows initial steps of a radioactive series

	A	$\xrightarrow{\lambda}$	B	$\xrightarrow{2\lambda}$	C
$t=0$	N_0		0		0
t	N_1		N_2		N_3

The ratio of N_1 to N_2, when N_2 is maximum is
(A) $\frac{\ln 2}{2}$
(B) 2
(C) $\frac{1}{2}$
(D) At no time this is possible

53. Select the wrong statement.
(A) Radioactivity is a statistical process
(B) Radioactivity is a spontaneous process
(C) Radioactivity is the neutral characteristics of few elements
(D) Radioactive elements cannot be produced in the laboratory

54. A radioactive element ${}_ZX^A$ emits an α-particle and changes into
(A) ${}_{z-2}Y^A$ (B) ${}_zY^{A-4}$
(C) ${}_{z-2}Y^{A-4}$ (D) ${}_{z+2}Y^A$

55. In gamma ray emission from a nucleus
(A) only the proton number changes
(B) both the neutron number and the proton number change
(C) there is no change in the proton number and the neutron number
(D) only the neutron number changes

56. At time $t=0$, N_1 nuclei of decay constant λ_1 and N_2 nuclei of decay constant λ_2 are mixed. The decay rate of the mixture at time t is
(A) $N_1N_2\lambda_1\lambda_2e^{-(\lambda_1+\lambda_2)t}$ (B) $\left(\frac{N_1}{N_2}\right)e^{-(\lambda_1-\lambda_2)t}$
(C) $N_1\lambda_1e^{-\lambda_1 t}+N_2\lambda_2e^{-\lambda_2 t}$ (D) $N_1N_2e^{-(\lambda_1+\lambda_2)t}$

57. Which of the following is its own antiparticle?
(A) photon (B) electron
(C) proton (D) π-meson

58. A bone containing 200 g carbon-14 has a β-decay rate of 375 decay/min. Calculate the time that has elapsed since the death of the living one. Given the rate of decay for the living organism is equal to 15 decay per min per gram of carbon and half-life of carbon-14 = 5730 year
(A) 27190 year (B) 1190 year
(C) 17190 year (D) None of these

59. Estimate the age of an ancient wooden piece if it is known that the specific activity of C^{14} nuclide in it amounts to $\frac{3}{5}$ of that in freshly felled trees. Given that the half-life of C^{14} nuclide is 5570 year.
(A) 1000 year (B) 2000 year
(C) 3000 year (D) 4000 year

60. A radioactive material decays by simultaneous emission of two particles with respective half lives 1620 and 810 year. The time (in year) after which one-fourth of the material remains is
(A) 1080 (B) 2430
(C) 3240 (D) 4860

61. A radioactive isotope is being produced at a constant rate X. Half life of the radioactive substance is Y. After some time the number of radioactive nuclei become constant. The value of this constant is
(A) XY (B) $\frac{X}{Y}$
(C) $\frac{XY}{\log_e(2)}$ (D) $(XY)\log_e(2)$

62. If the half-lives of a radioactive element for α and β decay are 4 year and 12 year respectively, then the percentage of the element that remains after 12 year will be
(A) 6.25% (B) 5.25%
(C) 4.25% (D) 3.50%

63. Assuming that all laws of thermodynamics can be applied to a nucleus, the α decay of a nucleus may be regarded as an
(A) isothermal process (B) isobaric process
(C) adiabatic process (D) isochoric process

64. The graph of $\log\left(\frac{R}{R_0}\right)$ versus A ($R =$ radius of a nucleus and $A =$ mass number) is

(A) a circle (B) an ellipse
(C) a parabola (D) a straight line

65. Consider the fission reaction

$$^{236}_{92}\text{U} \longrightarrow X^{117} + Y^{117} + n + n$$

i.e. two nuclei of same mass number 117 are found plus two neutrons. The binding energy per nucleon of X and Y is 8.5 MeV whereas of U^{236} is 7.6 MeV. The total energy liberated is

(A) 2000 MeV (B) 200 MeV
(C) 20 MeV (D) 2 MeV

66. In a radioactive substance at $t = 0$, the number of atoms is 8×10^4. Its half life period is 3 year. The number of atoms 1×10^4 will remain after interval

(A) 9 year (B) 8 year
(C) 6 year (D) 24 year

67. The activity of a radioactive sample is measured as 9750 counts (minute)$^{-1}$ at $t = 0$ and 975 counts (minute)$^{-1}$ at $t = 5$ minute. The decay constant is nearly

(A) 0.922 min^{-1} (B) 0.691 min^{-1}
(C) 0.461 min^{-1} (D) 0.230 min^{-1}

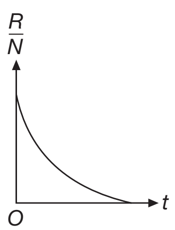
68. The half life of a certain radioactive element is 1 year. The percentage of the radioactive element decayed in 6 months will be

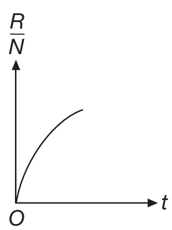
(A) 29.3% (B) 39.5%
(C) 70.5% (D) 14.5%

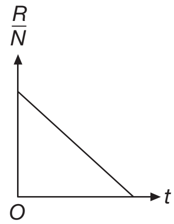
69. Magic numbers are

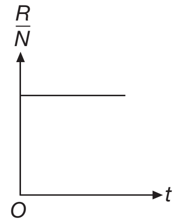
(A) 2, 8, 18, 28, 50, 164….
(B) 2, 8, 18, 50, 82, 164….
(C) 2, 8, 20, 28, 50, 82, 126….
(D) 2, 20, 50, 82, 164, 224….

70. A radioactive sample has N_0 active atoms at $t = 0$. If the rate of disintegration at any time is R the number of atoms is N, then the ratio $\frac{R}{N}$ varies with time as

(A) 

(B) 

(C) 

(D) 

71. The half life of radium is 1600 years. The fraction of the sample of radium that would remain after 6400 years is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$
(C) $\frac{1}{8}$ (D) $\frac{1}{16}$

72. The half life of ^{198}Au is 2.7 day. The probability that any ^{198}Au nucleus will decay in one second is

(A) 10^{-6} (B) 3×10^{-6}
(C) 5×10^{-6} (D) 10^{-5}

73. A and B are isotopes. B and C are isobars. If d_A, d_B and d_C be the densities of nuclei A, B and C respectively then

(A) $d_A > d_B > d_C$ (B) $d_A < d_B < d_C$
(C) $d_A = d_B = d_C$ (D) $d_A = d_B < d_C$

74. A sample contains large number of nuclei. The probability that a nucleus in sample will decay after four half lives is

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$
(C) $\frac{15}{16}$ (D) $\frac{7}{16}$

75. If u denotes 1 atomic mass unit. One atom of an element has mass exactly equal to Au, where A is mass number of element

(A) $A = 1$
(B) $A = 12$
(C) $A = 16$
(D) A can take up any integral value from 1 to 110

76. A sample contains 10^{-2} kg, each of two substances A and B with half lives 4 sec and 8 sec respectively. Their atomic masses are in the ratio 1 : 2. Amounts of A and B after an interval of 16 sec are respectively

(A) 2.5 mg, 2.5 mg (B) 0.625 mg, 2.5 mg
(C) 5 mg, 0.625 mg (D) 5 mg, 2.5 mg

77. The activity of a sample of radioactive material is R_1 at time t_1 and R_2 at time $t_2 \left(t_2 > t_1\right)$. If mean life of the radioactive sample is T, then

(A) $R_1 t_1 = R_2 t_2$
(B) $\dfrac{R_1 - R_2}{t_2 - t_1} = \text{constant}$
(C) $R_2 = R_1 \exp\left(\dfrac{t_1 - t_2}{T}\right)$
(D) $R_2 = R_1 \exp\left(\dfrac{t_1}{T t_2}\right)$

78. Samples of two radioactive nuclides, X and Y, each have equal activity A_0 at time $t = 0$. X has a half-life of 24 year and Y a half-life of 16 year. The samples are mixed together. What will be the total activity of the mixture at $t = 48$ years ?

(A) $\dfrac{1}{12} A_0$
(B) $\dfrac{1}{4} A_0$
(C) $\dfrac{3}{16} A_0$
(D) $\dfrac{3}{8} A_0$

79. At time $t = 0$, N_1 nuclei of decay constant λ_1 and N_2 nuclei of decay constant λ_2 are mixed. The decay rate of mixture is

(A) $-N_1 N_2 e^{-(\lambda_1 + \lambda_2)t}$
(B) $-\left(\dfrac{N_1}{N_2}\right) e^{-(\lambda_1 + \lambda_2)t}$
(C) $-\left(N_1 \lambda_1 e^{-\lambda_1^{t}} + N_1 \lambda_2 e^{-\lambda_2 t}\right)$
(D) $-N_1 \lambda_1 N_2 \lambda_2 e^{-(\lambda_1 + \lambda_2)t}$

80. An event on a distant star causes the emission of a burst of radiation containing β-particles, γ-rays and light. Which one of the following statements about the order in which these radiations arrive at, the earth is correct.

(A) The light would arrive first
(B) The γ-rays would arrive first
(C) The light and the γ-rays would arrive together, ahead of the β-particles
(D) The light and the β-particles would arrive together, ahead of the γ-rays

81. Two radioactive elements R and S disintegrate as

$R \to P + \alpha$, $\lambda_R = 4.5 \times 10^{-3}$ year^{-1}

$S \to Q + \beta$, $\lambda_S = 3 \times 10^{-3}$ year^{-1}

Starting with number of atoms of R and S in the ratio of $2:1$, this ratio after the lapse of three half lives of R will be

(A) $3:2$
(B) $1:3$
(C) $1:1$
(D) $2:1$

82. A count rate meter shows a count of 240 per minute from a given radioactive source. One hour later, the meter shows a count rate of 30 per minute. The half-life of the source is

(A) 20 min
(B) 40 min
(C) 80 min
(D) 120 min

83. The ratio of molecular mass of two radioactive substances is $\dfrac{3}{2}$ and the ratio of their decay constant is $\dfrac{4}{3}$. Then the ratio of their initial activity per mole will be

(A) 2
(B) $\dfrac{8}{9}$
(C) $\dfrac{4}{3}$
(D) $\dfrac{9}{8}$

84. Half-life of a radioactive substance is 20 minutes. Difference between points of time when it is 33% disintegrated and 67% disintegrated is approximately

(A) 10 min
(B) 20 min
(C) 30 min
(D) 40 min

85. A radioactive substance X decays into another radioactive substance Y. Assuming that initially only X was present, λ_x and λ_y be the disintegration constants of X and Y, N_x and N_y be the number of nuclei of X and Y at any time t, then the number of nuclei N_y will be maximum when

(A) $\dfrac{N_x}{N_x - N_y} = \dfrac{\lambda_x}{\lambda_x - \lambda_y}$
(B) $\dfrac{N_y}{N_x - N_y} = \dfrac{\lambda_y}{\lambda_x - \lambda_y}$
(C) $\lambda_y N_x = \lambda_x N_y$
(D) $\lambda_y N_y = \lambda_x N_x$

86. The half lives of a radioactive sample are 30 year and 60 year for two simultaneous decay processes. The time after which, only one-fourth of the sample will remain is

(A) 10 year
(B) 20 year
(C) 40 year
(D) 60 year

87. The SI units of decay constant is

(A) m^{-1}
(B) mm^{-1}
(C) s^{-1}
(D) year

88. A nucleus of mass 218 amu in free state decays to emit an alpha particle (mass 4 amu). The kinetic energy of the alpha particle is found to be 6.7 MeV. The recoil energy of the daughter nucleus (in MeV) is

(A) 0.125 (B) 1.25
(C) 12.5 (D) 0.0125

89. A radioactive sample S_1 having an activity of $5\ \mu\text{Ci}$ has twice the number of nuclei as another sample S_2 which has an activity of $10\ \mu\text{Ci}$. The half lives of S_1 and S_2 can be
(A) 20 years and 5 years, respectively
(B) 20 years and 10 years, respectively
(C) 10 years each
(D) 5 years each

90. The masses of two radioactive substances A and B are same and their half-lives are 1 year and 2 year respectively. The ratio of their activities after six year will be
(A) $1:4$ (B) $4:1$
(C) $1:8$ (D) $8:1$

91. In the options given below, let E denote the rest mass energy of a nucleus and n a neutron. The correct option is
(A) $E\left({}^{236}_{92}\text{U}\right) > E\left({}^{137}_{53}\text{I}\right) + E\left({}^{97}_{39}Y\right) + 2E(n)$
(B) $E\left({}^{236}_{92}\text{U}\right) < E\left({}^{137}_{53}\text{I}\right) + E\left({}^{97}_{39}Y\right) + 2E(n)$
(C) $E\left({}^{236}_{92}\text{U}\right) < E\left({}^{140}_{56}\text{Ba}\right) + E\left({}^{94}_{36}\text{Kr}\right) + 2E(n)$
(D) $E\left({}^{235}_{92}\text{U}\right) < E\left({}^{140}_{56}\text{Ba}\right) + E\left({}^{94}_{36}\text{Kr}\right) + E(n)$

92. A sample of radioactive material has mass m, decay constant λ, and molecular weight M. Avogadro constant N_A. The initial activity of the sample and activity at time t is
(A) $\lambda m,\ \lambda m e^{-\lambda t}$
(B) $\dfrac{\lambda m}{M},\ \dfrac{\lambda m}{M} e^{-\frac{m}{M}t}$
(C) $\dfrac{\lambda m N_A}{M},\ \left(\dfrac{\lambda m N_A}{M}\right) e^{-\lambda t}$
(D) $m N_A e^{\lambda},\ m N_A e^{-\lambda t}$

93. Solar constant of the sun is $\sigma = 8.106 \times 10^4\ \text{Jmin}^{-1}\text{m}^{-2}$ and average sun earth distance is 1.5×10^8 km. The yearly loss in the mass of the sun is
(A) 13.8×10^{17} kg (B) 1.38×10^{19} kg
(C) 1.38×10^{17} kg (D) 13.8×10^{20} kg

94. Certain radioactive substance reduces to 25% of its value in 16 day. Its half-life is
(A) 32 day (B) 8 day
(C) 64 day (D) 28 day

95. A star initially has 10^{40} deuterons. It produces energy via the processes ${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_1\text{H} + p$ and ${}^2_1\text{H} + {}^3_1\text{H} \rightarrow {}^4_2\text{He} + n$. Where the masses of the nuclei are:
$m({}^2\text{H}) = 2.014$ amu, $m(p) = 1.007$ amu, $m(n) = 1.008$ and $m({}^4\text{He}) = 4.001$ amu. If the average power radiated by the star is 10^{16} W, the deuteron supply of the star is exhausted in a time of the order of
(A) 10^6 s (B) 10^8 s
(C) 10^{12} s (D) 10^{16} s

96. The half life period of a radioactive substance is 140 day. After how much time, 15 g will decay from a 16 g sample of the substance?
(A) 140 day (B) 560 day
(C) 420 day (D) 280 day

97. Consider a nuclear decay process

$$A \longrightarrow B \longrightarrow C \ \text{(stable)}$$

At a certain time, the activity of nuclei A is x and the net rate of increase of number of nuclei B is y. The activity of nuclei B at this instant is
(A) y (B) $x - y$
(C) $y - x$ (D) x

98. The mean lives of a radioactive material for α and β radiations are 1620 year and 520 year respectively. The material decays simultaneously for α and β decay. The time after which one fourth of the material remains undecayed is
(A) 540 year (B) 324 year
(C) 720 year (D) 840 year

99. Packing fraction is given by the relation
(A) $\dfrac{A}{M-A}$ (B) $\dfrac{A-M}{A}$
(C) $\dfrac{M}{M-A}$ (D) $\dfrac{M-A}{A}$

100. Probability (P) that a radioactive nucleus will not decay in time t will be (given decay constant $= \lambda$)
(A) $e^{-\lambda t}$ (B) $1 - e^{-\lambda t}$
(C) $e^{\lambda t}$ (D) $1 - e^{\lambda t}$

101. In the following nuclear reaction, Y stands for
${}^{27}_{13}\text{Al} + {}^4_2\text{He} \longrightarrow {}^{30}_{15}\text{P} + Y$
(A) ${}^1_0 n$ (B) ${}^0_{-1} e$
(C) ${}^1_1\text{H}$ (D) ${}^0_{+1} e$

102. The ratio activity of an element decreases 64 times its original value in 60 sec . Then the half life of the element is

(A) 5 sec (B) 10 sec
(C) 20 sec (D) 30 sec

103. The half-life of a radioactive material is T . After $\frac{T}{2}$ time, the material left is

(A) $\frac{1}{2}$ (B) $\frac{3}{4}$
(C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{4}$

104. Let T be the mean life of a radioactive sample. 75% of the active nuclei present in the sample initially will decay in time

(A) $2T$ (B) $\frac{T}{2}\ln(2)$
(C) $4T$ (D) $2T\ln(2)$

105. If 1 mg of U^{235} is completely annihilated, the energy liberated is

(A) 9×10^{10} J (B) 9×10^{19} J
(C) 9×10^{18} J (D) 9×10^{17} J

106. The count rate of activity of a radioactive sample of a very large population decreased from 1024 to 128 in 3 minutes . Then the rate of disintegration at the end of 5 minutes is

(A) 96 (B) 64
(C) 48 (D) 32

107. The fraction of the initial number of active nuclei which remains undecayed after half of a half-life of the radioactive sample is

(A) $\frac{1}{4}$ (B) $\frac{1}{2\sqrt{2}}$
(C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}-1$

108. The binding energy per nucleon for deuteron $\left({}_1^2\text{H}\right)$ and helium $\left({}_2^4\text{He}\right)$ are 1.1 MeV and 7.0 MeV , respectively. The energy released when two deuterons fuse to form a helium nucleus is

(A) 47.12 MeV (B) 23.6 MeV
(C) 11.8 MeV (D) 34.4 MeV

109. The instantaneous concentration N, the initial concentration N_0 , the radioactive constant λ can be written as

(A) $N=N_0e^{\lambda t}$ (B) $N=N_0e^{-\lambda t}$
(C) $N=N_0e^{-\lambda}$ (D) $N=N_0e^{\lambda}$

110. A radioactive nucleus can decay by either emitting an α particle or by emitting a β particle. Probability of α decay is 75% while that of β decay is 25% . The decay constant of α decay is λ_1 and that of β decay is λ_2 . Then, $\frac{\lambda_1}{\lambda_2}$ is

(A) 3 (B) $\frac{1}{3}$
(C) 1 (D) 2

111. An element A decays into element C by a two step process given below.

$A \rightarrow B + {}_2\text{He}^4$

$B \rightarrow C + 2e^-$

Then,

(A) A and B are isotopes
(B) A and B are isobars
(C) A and C are isotopes
(D) A and C are isobars

112. How many α particles are emitted in the decay ${}^{235}_{92}X \longrightarrow Y^{219}_{88}$

(A) 4 (B) 5
(C) 6 (D) no comments

113. The count rate observed from a radioactive source at t second was N_0 and at $4t$ second it was $\frac{N_0}{16}$. The count rate observed, at $\left(\frac{11}{2}\right)t$ second will be

(A) $\frac{N_0}{128}$ (B) $\frac{N_0}{64}$
(C) $\frac{N_0}{32}$ (D) None of these

114. The rate of disintegration of a radioactive substance falls from 800 decay/min to 100 decay/min in 6 hours . The half-life of the radioactive substance is

(A) $\frac{6}{7}$ hour (B) 2 hrs
(C) 3 hrs (D) 1 hr

115. In the above decay, how many β particles are given out?

(A) 4 (B) 5
(C) 6 (D) no comments

116. ${}_{86}A^{222} \rightarrow {}_{84}B^{210}$. In this reaction how many α and β particles are emitted

(A) $6\alpha,\ 3\beta$ (B) $3\alpha,\ 4\beta$
(C) $4\alpha,\ 3\beta$ (D) $3\alpha,\ 6\beta$

117. If N_0 is the original amount of the substance of half-life period $t_{1/2} = 5$ year, then the amount of substance decayed after 15 year is

(A) $\frac{N_0}{8}$ (B) $\frac{N_0}{16}$

(C) $\frac{7N_0}{8}$ (D) $\frac{N_0}{4}$

118. Nuclei of radioactive element A are produced at a rate t^2 at any time t. The element A has decay constant λ. Let N be the number of nuclei of element A at any time t. At time $t = t_0$, $\frac{dN}{dt}$ is minimum. Then the number of nuclei of element A at time $t = t_0$ is

(A) $\frac{2t_0 - \lambda t_0^2}{\lambda^2}$ (B) $\frac{t_0 - 2t_0^2}{\lambda^2}$

(C) $\frac{2t_0 - \lambda t_0^2}{\lambda}$ (D) $\frac{t_0 - \lambda t_0^2}{\lambda}$

119. A certain radioactive substance has half-life of 5 year. For a nucleus in a sample of the radioactive substance, the probability of decay in ten year is

(A) 50% (B) 75%

(C) 100% (D) 60%

120. Two identical samples (same material and same amount) P and Q of a radioactive substance having mean life T are observed to have activities A_P and A_Q respectively at the time of observation. If P is older than Q, then the difference in their age is

(A) $T\ln\left(\frac{A_P}{A_Q}\right)$ (B) $T\ln\left(\frac{A_Q}{A_P}\right)$

(C) $T\left(\frac{A_P}{A_Q}\right)$ (D) $T\left(\frac{A_Q}{A_P}\right)$

121. The radius R of a nucleus changes with the nucleon number A of nucleus as

(A) $R \propto A^{\frac{2}{3}}$ (B) $R \propto A^{\frac{1}{3}}$

(C) $R \propto A^0$ (D) $R \propto A$

122. The activity of a radioactive sample A_0 decreases to one third of the original value in 9 years. Its activity 9 years further, will be

(A) $\frac{A_0}{3}$ (B) $\frac{A_0}{6}$

(C) $\frac{A_0}{9}$ (D) $\frac{A_0}{18}$

123. Sun radiates energy in all direction. The average energy received at earth is $1.4\ \text{kWm}^{-2}$. The average distance between the earth and the sun is 1.5×10^{11} m. If this energy is released by conservation of mass into energy, then the mass lost per day by the sun is approximately

(A) 4.4×10^9 kg (B) 7.6×10^{14} kg

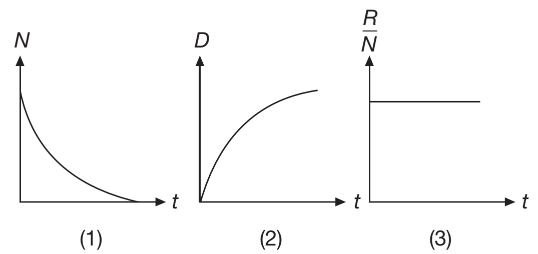
(C) 3.8×10^{12} kg (D) 3.8×10^{14} kg

124. The net force between two nucleons 1 fm apart is F_1 if both are protons, F_2 if both are neutrons, and F_3 if one is a neutron and the other is a proton.

(A) $F_1 < F_2 < F_3$ (B) $F_2 < F_1 < F_3$

(C) $F_1 < F_2 = F_3$ (D) $F_1 = F_2 < F_3$

125. In a radioactive decay, let N represent the number of residual active nuclei, D the number of daughter nuclei, and R the rate of decay at any time t. Three curves are shown in figure. The correct ones are



(A) 1 and 3 (B) 2 and 3
(C) 1 and 2 (D) all are correct

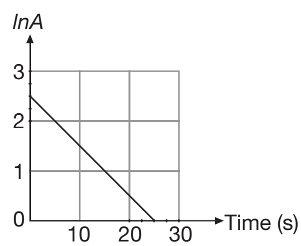
126. A and B are two radioactive substances whose half lives are 1 year and 2 year respectively. Initially 10 g of A and 1 g of B is taken. The time (approximate) after which they will have same quantity remaining is

(A) 6.62 year (B) 5 year
(C) 3.2 year (D) 7 year

127. The half life of the radioactive substance is 40 days. The substance will disintegrate completely in

(A) 40 days (B) 400 days
(C) 4000 days (D) infinite time

128. The graph shows the variation of the count rate A of a radioactive source with time t.



If $\ln(12) = 2.5$, then A as a function of t is given by

(A) $A = 2.5e^{-10t}$ (B) $A = 12e^{10t}$

(C) $A = 2.5e^{-0.1t}$ (D) $A = 12e^{-0.1t}$

129. A radioactive nuclide is produced at the constant rate of n per second (say, by bombarding a target with neutrons). The expected number N of nuclei in existence t seconds after the number is N_0 is given by

(A) $N = N_0 e^{-\lambda t}$

(B) $N = \dfrac{n}{\lambda} + N_0 e^{-\lambda t}$

(C) $N = \dfrac{n}{\lambda} + \left(N_0 - \dfrac{n}{\lambda}\right)e^{-\lambda t}$

(D) $N = \dfrac{n}{\lambda} + \left(N_0 + \dfrac{n}{\lambda}\right)e^{-\lambda t}$

(where λ is the decay constant of the sample)

130. A radioactive substance contains 10000 nuclei and its half-life period is 20 day. The number of nuclei present at the end of 10 day is

(A) 7070 (B) 9000

(C) 8000 (D) 7500

131. A stationary nucleus of mass 24 amu emits a gamma photon. The energy of the emitted photon is 7 MeV. The recoil energy of the nucleus is

(A) 2.2 keV (B) 1.1 keV

(C) 3.1 keV (D) 22 keV

132. In any fission process the ratio $\dfrac{\text{mass of fission products}}{\text{mass of parent nucleus}}$ is

(A) Less than 1
(B) Greater than 1
(C) Equal to 1
(D) Depends on the mass of the parent nucleus

133. Two radioactive samples of different elements (half lives t_1 and t_2 respectively) have same number of nuclei at $t = 0$. The time after which their activities are same is

(A) $\dfrac{t_1 t_2}{0.693(t_2 - t_1)}\ln\left(\dfrac{t_2}{t_1}\right)$ (B) $\dfrac{t_1 t_2}{0.693}\ln\left(\dfrac{t_2}{t_1}\right)$

(C) $\dfrac{t_1 t_2}{0.693(t_1 + t_2)}\ln\left(\dfrac{t_2}{t_1}\right)$ (D) None of these

134. The radioactivity of a sample is R_1 at time t_1 and R_2 at time t_2. If the half life of the sample be T, the number of atoms that have disintegrated in time $(t_2 - t_1)$ is proportional to

(A) $R_1 t_1 - R_2 t_2$ (B) $(R_1 - R_2)^{-1}$

(C) $\dfrac{(R_1 - R_2)}{T}$ (D) $(R_1 - R_2)T$

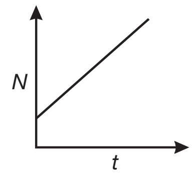
135. In the given reaction, the radioactive radiations are emitted in the sequence as

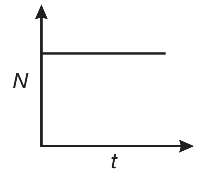
$${}_{Z}X^{A} \longrightarrow {}_{Z+1}Y^{A} \longrightarrow {}_{Z-1}T^{A-4} \longrightarrow {}_{Z-1}T^{A-4}$$

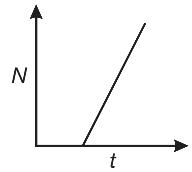
(A) α, β, γ (B) β, α, γ

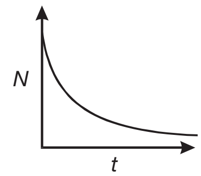
(C) γ, α, β (D) α, γ, β

136. The graph between the instantaneous concentration (N) of a radioactive element and time (t) is

(A)


(B)


(C)


(D)


137. A radioactive nuclide can decay simultaneously by two different processes which have individual decay constants λ_1 and λ_2 respectively. The effective decay constant of the nuclide is λ given by

(A) $\lambda = \sqrt{\lambda_1 \lambda_2}$ (B) $\dfrac{1}{\lambda} = \dfrac{1}{\lambda_1} + \dfrac{1}{\lambda_2}$

(C) $\lambda = \dfrac{1}{2}(\lambda_1 + \lambda_2)$ (D) $\lambda = \lambda_1 + \lambda_2$

138. Masses of two isobars ${}^{64}_{29}\text{Cu}$ and ${}^{64}_{30}\text{Zn}$ are 63.9298 u and 63.9292 u respectively. It can be concluded from this data that

(A) both the isobars are stable.

(B) ${}^{64}\text{Zn}$ is radioactive, decaying to ${}^{64}\text{Cu}$ through β- decay.

(C) ${}^{64}\text{Cu}$ is radioactive, decaying to ${}^{64}\text{Zn}$ through γ- decay.

(D) ${}^{64}\text{Cu}$ is radioactive, decaying to ${}^{64}\text{Zn}$ through β- decay.

139. A fraction f_1 of a radioactive sample decays in one half life, and a fraction f_2 decays in one mean life.

(A) $f_1 > f_2$

(B) $f_1 < f_2$

(C) $f_1 = f_2$

(D) Data insufficient to arrive at a conclusion

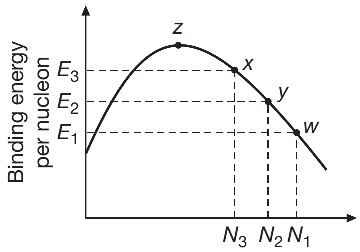
140. The half life of ^{131}I is 8 days. Given a sample of ^{131}I at time $t = 0$. We can assert that

(A) no nucleus will decay before $t = 4$ days
(B) no nucleus will decay before $t = 8$ days
(C) all nuclei will decay before $t = 16$ days
(D) a given nucleus may decay at any time after $t = 0$

141. In the process of fission, the binding energy per nucleon

(A) increases
(B) decreases
(C) remains unchanged
(D) increases for mass number $A < 56$ nuclei but decreases for mass number $A > 56$ nuclei

142. Consider the nuclear fission reaction $W \to X + Y$. What is the Q value (energy released) of the reaction?



(A) $E_1N_1 - (E_2N_2 + E_3N_3)$

(B) $(E_2N_2 + E_3N_3 - E_1N_1)$

(C) $E_2N_2 + E_1N_1 - E_3N_3$

(D) $E_1N_1 + E_3N_3 - E_2N_2$

143. If the half life and the mean life of the radioactive element is denoted by t and T respectively then

(A) $t = T$ (B) $t = \frac{1}{T}$
(C) $t < T$ (D) $t > T$

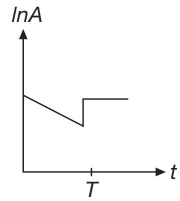
144. The radioactivity of a substance is measured in terms of disintegration per second. 3×10^{10} disintegration per second is

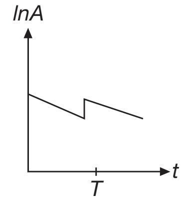
(A) 1 eV (B) 1 MeV
(C) 1 rutherford (D) 1 curie

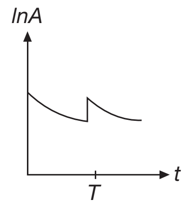
145. Half-life of a radioactive substance is 20 minutes. The time between 20% and 80% decay will be

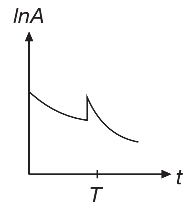
(A) 40 minutes (B) 20 minutes
(C) 25 minutes (D) 30 minutes

146. At time $t = 0$, some radioactive gas is injected into a sealed vessel. At time T, some more of the same gas is injected into the same vessel. Which one of the following graphs best represents the variation of the logarithm of the activity A of the gas with time t?

(A)


(B)


(C)


(D)


147. If N_1 atoms of a radioactive element emit N_2 beta particles per second, then the decay constant of the element (in s^{-1}) is

(A) $\frac{N_2}{N_1}$ (B) $\frac{N_1}{N_2}$

(C) $N_2 \log_e(2)$ (D) $N_1 \log_e(2)$

148. In the disintegration series

$$^{238}_{92}U \xrightarrow{\alpha} X \xrightarrow{\beta^-} {}^{A}_{Z}Y$$

The respective values of Z and A are

(A) 92, 236 (B) 88, 230
(C) 90, 234 (D) 91, 234

149. Radioactivity is not influenced by

(A) pressure
(B) electronic configuration
(C) temperature
(D) all of these

150. A radioactive material has a mean lives of 1620 year and 660 year for α and β emission respectively. The material decay by simultaneous α and β emission. The time in which one fourth of the material remains intact is

(A) 4675 year (B) 720 year
(C) 650 year (D) 324 year

151. The binding energies of nuclei X and Y are E_1 and E_2 respectively. Two atoms of X fuse to give one atom of Y and an energy Q is released. Then

(A) $Q > E_2 - 2E_1$ (B) $Q < 2E_1 - E_2$

(C) $Q = E_2 - 2E_1$ (D) $Q = 2E_1 - E_2$

152. Free ^{238}U nuclei kept in a train emit alpha particles. When the train is stationary and a uranium nucleus decays, a passenger measures that the separation between the alpha particle and the recoiling nucleus becomes x in time t after the decay. If a decay takes place when the train is moving at a uniform speed v, the distance between the alpha particle and the recoiling nucleus at a time t after the decay, as measured by the passenger will be

(A) $x+vt$
(B) $x-vt$
(C) x
(D) Depends on the direction of the train

153. A radioactive nucleus is being produced at a constant rate α per second. Its decay constant is λ. If N_0 are the number of nuclei at time $t=0$, then maximum number of nuclei possible are

(A) $\frac{\alpha}{\lambda}$ (B) $N_0+\frac{\alpha}{\lambda}$
(C) N_0 (D) $\frac{\lambda}{\alpha}+N_0$

154. A radioactive isotope A decays into another isotope B which has a half-life equal to half the half-life of A. Both isotopes emit α-particles during their decay B decays into a stable nucleus. If a sample consists initially of atoms of A only, then the net activity of the sample initially

(A) decreases with time
(B) increases with time
(C) remains constant
(D) any of the above may be true

155. The probability of survival of a radioactive nucleus for one mean life is

(A) $1-\frac{1}{e}$ (B) $\frac{1}{e}$
(C) $\frac{\log_e(2)}{e}$ (D) $1-\frac{\log_e(2)}{e}$

156. The binding energy per nucleon of ${}_1H^2$ and ${}_2He^4$ are 1.1 eV and 7 MeV respectively. The energy released in the process ${}_1\text{H}^2+{}_1\text{H}^2={}_2\text{He}^4$ is

(A) 20.8 MeV (B) 16.6 MeV
(C) 25.2 MeV (D) 23.6 MeV

157. There are two radioactive substances A and B such that decay constant of B is twice that of A. Initially both have equal number of nuclei. After n half lives of A, the rate of disintegration of both are equal. The value of n is

(A) 16 (B) 4
(C) 2 (D) 1

158. Suppose a radioactive substance disintegrates completely in 10 days. Each day it disintegrates at a rate which is twice the rate of the previous day. The percentage of the material left to be disintegrated after passing of 9 days is

(A) 10 (B) 20
(C) 25 (D) 50

159. Mirror nuclei are those in which the number of protons in the parent nuclei

(A) is less than the daughter nuclei by one
(B) is more than the daughter nuclei by two
(C) equal to the neutron number
(D) exceeds the number of neutrons in the daughter nuclei by unity by one positron emission

160. The decay of ${}^{238}_{92}\text{U}$ to ${}^{239}_{93}\text{Np}$ by β-emission is not possible because

(A) β-decay only occurs in isotopes of low mass
(B) ${}^{239}_{93}\text{Np}$ is not a stable isotope
(C) mass number cannot increase in a decay process
(D) atomic number cannot increase in a decay process.

161. Half life period and mean life time of a radioactive element are

(A) inversely proportional to each other
(B) directly proportional to each other
(C) equal to each other
(D) not related to each other

162. If P_α, P_β and P_γ be the penetrating powers of α, β and γ radiations respectively then

(A) $P_\alpha=P_\beta=P_\gamma$ (B) $P_\alpha>P_\beta>P_\gamma$
(C) $P_\alpha<P_\beta<P_\gamma$ (D) $P_\alpha=P_\beta<P_\gamma$

163. A radioactive nuclide emits an α, β and a γ ray in close succession, the atomic mass of the end product is reduced by

(A) 8 a.m.u. (B) about 4 a.m.u.
(C) 2 a.m.u. (D) 1 a.m.u.

164. The life time of a neutron is about 10^3 s and its rest energy is 940 MeV, then the mean free path of a 5 eV neutron in vacuum is closest to

(A) 10 km (B) 100 km
(C) 1000 km (D) 10000 km

165. A certain radioactive element disintegrates for an interval of time equal to its mean life. The fraction of the original amount that remains undecayed is

(A) e (B) $\frac{1}{e}$
(C) e^2 (D) $\frac{1}{e^2}$

166. If N denotes the concentration of a radioactive element, then the rate of change of concentration with time (t) can be written as

(A) $\frac{dN}{dt} \propto N$ (B) $-\frac{dN}{dt} \propto N$

(C) $-\frac{dN}{dt} \propto e^{N}$ (D) $-\frac{dN}{dt} \propto e^{-N}$

167. In the sun about 4 billion kg of matter is converted to energy each second. The power output of the sun in watt is

(A) 3.6×10^{26} (B) 0.36×10^{26}

(C) 36×10^{26} (D) 0.036×10^{26}

168. A radioactive isotope has a decay constant and a molar mass M. Taking the Avogadro constant to be L, what is the activity of a sample of mass m of this isotope?

(A) λmML (B) $\frac{\lambda mL}{M}$

(C) $\frac{\lambda ML}{m}$ (D) $\frac{mL}{\lambda M}$

169. If 90% of a radioactive sample is left undecayed after time t has elapsed. The percentage of initial sample that will decay in a total time $2t$ is

(A) 20% (B) 19%

(C) 40% (D) 38%

170. A nucleus with $Z = 92$ emits the following particles in a sequence given. $\alpha, \beta^-, \beta^-, \alpha, \alpha, \alpha, \alpha, \alpha, \beta^-, \beta^-, \alpha, \beta^+, \beta^+, \alpha$.

The Z of the resulting nucleus is

(A) 74 (B) 76

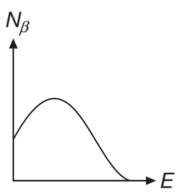
(C) 78 (D) 82

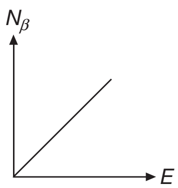
171. A freshly prepared radioactive sample of half-life 1 hour emits radiations that are 128 times as intense as the permissible safe limit. The minimum time after which this sample can be safely used is

(A) 14 hours (B) 7 hours

(C) 128 hours (D) 256 hours

172. The curve representing the energy spectrum of β-particles is

(A)


(B)


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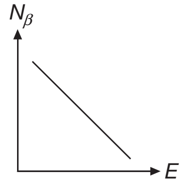
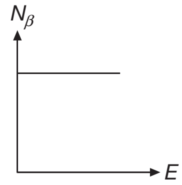


(D)


173. Consider the following reaction

$$_1H^2 + {}_1H^2 \longrightarrow {}_2He^4 + Q.$$

If $m({}_1H^2) = 2.0141$ u; $m({}_2He^4) = 4.0024$ u.

The energy Q released (in MeV) in this fusion reaction is

(A) 12 (B) 6

(C) 24 (D) 48

174. The equations $\frac{dN}{dt} = -\lambda N$ and $N = N_0 e^{-\lambda t}$ describes how the number N of undecayed atoms in a sample of radioactive material, which initially (at $t = 0$) contained N_0 undecayed atoms, varies with time t. Which one of the following statements about is correct ?

(A) λdt gives the fraction of atoms present which will decay in the next small time interval dt

(B) λ is the time needed for N to fall from N_0 to the value $\frac{N_0}{e}$.

(C) λ is equal to the half-life of the sample.

(D) λ is the number of atoms left after a time equal to e second.

175. A radioactive element X converts into another stable element Y. If half life of X is 2 hr, initially only X to be present and after time t, the ratio of atoms of X and Y is found to be $1:4$, then time t in hr is

(A) 2 (B) 4

(C) between 4 and 6 (D) 6

176. The fraction of atoms of radioactive element that decays in 6 day is $\frac{7}{8}$. The fraction that decays in 10 day will be

(A) $\frac{70}{80}$ (B) $\frac{77}{88}$

(C) $\frac{31}{32}$ (D) $\frac{35}{36}$

177. The radioactive emissions which produce an isotope of the original nucleus are

(A) one alpha and four beta

(B) one alpha and two beta

(C) one alpha and one beta
(D) two alpha and one beta

178. When the nucleus of an atom absorbs one of the atom's orbital electrons, the process is known as K-capture. Which equation (in which X denotes the appropriate particle) represents this process

(A) ${}^{55}_{26}\text{Fe} + {}^{0}_{-1}X \rightarrow {}^{55}_{25}\text{Mn}$

(B) ${}^{63}_{28}\text{Ni} \rightarrow {}^{0}_{-1}X + {}^{63}_{29}\text{Cu}$

(C) ${}^{10}_{5}\text{B} + {}^{1}_{0}X \rightarrow {}^{7}_{3}\text{U} + {}^{4}_{2}\text{He}$

(D) ${}^{7}_{3}\text{Li} + {}^{1}_{1}X \rightarrow {}^{8}_{4}\text{Be}$

179. A radioactive nucleus A finally transforms into a stable nucleus B. Then A and B can be
(A) isotones (B) isotopes
(C) isobars (D) None of these

180. In a sample of radioactive material, what percentage of the initial number of active nuclei will decay during one mean life?
(A) 37% (B) 50%
(C) 63% (D) 69.3%

181. Number of nuclei of a radioactive substance at time $t=0$ are 1000 and 900 at time $t=2$ s. Then number of nuclei at time $t=4$ s will be
(A) 700 (B) 790
(C) 800 (D) 810

182. If ${}_{92}\text{U}^{238}$ changes to ${}_{85}\text{At}^{210}$ by a series of α and β decays, the number of α and β decays undergone is
(A) 5 and 7 (B) 7 and 5
(C) 7 and 7 (D) 7 and 9

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. For a certain radioactive substance, it is observed that after 4 hours, only 6.25% of the original sample is left undecayed. If follows that
(A) the half life of the sample is 1 hour
(B) the mean life of the sample is $\dfrac{1}{\ln 2}$ hour
(C) the decay constant of the sample is $\ln\ 2\ \text{hour}^{-1}$
(D) after a further 4 hours, the amount of the substance left over would by only 0.39% of the original amount

2. Energy released in Radioactive decay process $X \rightarrow Y + y$ is equal to
(A) sum of rest energy of Y and y minus rest energy of X.
(B) sum of binding energy of Y and y minus binding of X.
(C) Rest energy of X minus sum of rest energy of Y and y
(D) Binding energy of X minus sum of binding energy of Y and y

3. Which of the following statement(s) is correct?
(A) The rest mass of a stable nucleus is less than that of constituent nucleons.
(B) The rest mass of a stable nucleus is greater than the sum of the rest masses of its constituent nucleons.
(C) In nuclear fusion, energy is released by fusing two nuclei of medium mass.
(D) In nuclear fission, energy is released by fragmentation of a very heavy nucleus.

4. In radioactivity decay according to law $N = N_0 e^{-\lambda t}$
Which of the following is/are true?
(A) Probability that a nucleus will decay is $1 - e^{-\lambda t}$
(B) Probability that a nucleus will decay four half lives is $\dfrac{15}{16}$
(C) Fraction nuclei that will remain after two half lives is zero
(D) Fraction of nuclei that will remain after two half-lives is $\dfrac{1}{4}$

5. A nuclide A undergoes α decay and another nuclide B undergoes β^- decay. Select the correct statement(s).
(A) All the α-particles emitted by A will have almost the same speed.
(B) The α-particles emitted by A may have widely different speeds.
(C) All the β-particles emitted by B will have almost the same speed.
(D) The β-particles emitted by B may have widely different speeds.

6. Let m_P be the mass of a proton, m_n the mass of a neutron, M_1 the mass of a $^{20}_{10}Ne$ nucleus and M_2 the mass of a $^{40}_{20}Ca$ nucleus. Then,
   (A) $M_2 = 2M_1$ (B) $M_2 > 2M_1$
   (C) $M_2 < 2M_1$ (D) $M_1 < 10(m_n + m_P)$

7. Regarding a nucleus, select the correct option(s)
   (A) Density of a nucleus is directly proportional to mass number A
   (B) Density of all the nuclei is almost constant, of the order of 10^{17} kgm^{-3}
   (C) Nucleus radius of the order of 10^{-15} m
   (D) Nucleus radius $\propto A$

8. A radioactive sample has initial concentration N_0 of nuclei. Select the correct statement(s).
   (A) The number of undecayed nuclei present in the sample decays exponentially with time.
   (B) The activity (R) of the sample at any instant is directly proportional to the number of undecayed nuclei present in that sample at that time.
   (C) The number of decayed nuclei grows exponentially with time.
   (D) The number of decayed nuclei grows linearly with time.

9. A radioactive sample has initial concentration N_0 of nuclei.
   (A) The number of undecayed nuclei present in the sample decays exponentially with time.
   (B) The activity (R) of the sample at any instant is directly proportional to the number of undecayed nuclei present in the sample at that time.
   (C) The number of decayed nuclei grows linearly with time.
   (D) The number of decayed nuclei grows exponentially with time.

10. For nuclei having $A > 120$, select the correct statement(s)
   (A) The binding energy of nucleus decreases as A increases.
   (B) The binding energy per nucleon decreases as A increases.
   (C) If the nucleus breaks into two roughly equal parts, energy is released.
   (D) If two nuclei fuse to form a bigger nucleus energy is released.

11. Choose correct statement(s) from following
   (A) The binding energy per nucleon decreases as atomic number increases
   (B) Density of all nuclei is almost the same
   (C) Potential difference applied to an X-ray tube is of the order of hundred thousand volt
   (D) The ratio of square of velocity and radius $\left(\frac{v^2}{r}\right)$ of an orbiting electron in hydrogen atom is independent of principal quantum number (n)

12. When the nucleus of an electrically neutral atom undergoes a radioactive decay process it will remain neutral after the decay, if the process is
   (A) α-decay (B) β-decay
   (C) γ-decay (D e^- capture process

13. In which of the following decays, the atomic number decreases.
   (A) α-decay (B) β^+-decay
   (C) β^--decay (D) γ-decay

14. Choose the correct options.
   (A) Isotopes have same number of atomic number
   (B) Isobars have same atomic weight
   (C) Isotones have same number of neutrons
   (D) In neutral isotope atoms number of electrons are same

15. Choose the correct options
   (A) By gamma radiations atomic number is not changed
   (B) By gamma radiations mass number is not changed
   (C) By the emission of one α and two β particles isotopes are produced
   (D) By the emission of one α and four β particles isobars are produced

16. A muonic hydrogen atom is a hydrogen atom with electron replaced by a muon whose mass is 212 times the mass of an electron, then
   (A) Bohr radius of the muonic atom is 250 fm
   (B) Ground state energy of muonic atom is 2883 eV
   (C) angular momentum in ground state is $\frac{h}{2\pi}$
   (D) angular momentum in ground state is $\frac{106h}{\pi}$

17. Which of the following are correct?
   (A) A neutron can be decayed into a proton only inside a nucleus
   (B) A proton can be changed into a neutron only inside a nucleus
   (C) An isolated neutron can be changed into a proton
   (D) An isolated proton can be changed into a neutron

18. At $t = 0$, number of radioactive nuclei of a radioactive substance are x and its radioactivity is y. Half-life of radioactive substance is T. Then

(A) $\frac{x}{y}$ is constant throughout

(B) $\frac{x}{y} > T$

(C) value of xy remains half after one half-life

(D) value of xy remains one fourth after one half-life

19. A nitrogen nucleus $^{14}_{7}N$ absorbs a neutron and can transform into lithium nucleus $^{7}_{3}Li$ under suitable conditions, after emitting

(A) 4 protons and 4 neutrons

(B) 5 protons and 1 beta minus particles

(C) 2 alpha and 2 gamma particles

(D) 1 alpha particle, 4 protons and 2 beta minus particles.

20. Magnetic field does cause deflection for

(A) α-particles (B) β^{+}-rays

(C) β^{-}-rays (D) γ-rays

21. Polonium $_{84}Po^{210}$ emits α-particles and is converted into $_{82}Pb^{206}$. This reaction is used for producing electric power in a space mission. Po^{210} has half life of 138.6 days. Assuming an efficiency of 10%. Select the correct statement(s)

Given: $M(Po^{210}) = 209.98264$ amu; $M(\alpha) = 4.0026$ amu $M(Pb^{206}) = 205.97440$ amu; 1 amu = 931 MeV energy

(A) 10 g Po^{210} is required to produce 1.2×10^{7} Joule energy

(B) Decay constant of Po^{210} is 0.005 day^{-1}

(C) Q-value of α-decay process is 8.4×10^{-13} J

(D) None of these is correct

22. Select the incorrect statement(s)

(A) Density of nucleus increases as mass number increases.

(B) Cadmium rods are used to slow down fast moving neutrons in a nuclear reactor.

(C) It is easier to remove an orbital electron but quite difficult to release a nucleon.

(D) Fragments produced in fission of ^{235}U are radioactive.

23. In the beta decay process $^{A}_{Z}X \rightarrow ^{A}_{Z+1}Y + e^{-}$

(A) total energy is conserved

(B) mass number is conserved

(C) charge is conserved

(D) spin angular momentum is conserved

24. Regarding the nuclear forces, select the correct option(s)

(A) They are short range forces

(B) They are charge independent forces

(C) They are not electromagnetic forces

(D) They are exchange forces

25. In an endoergic nuclear reaction, an incoming particle collides with stationary nucleus. Then the

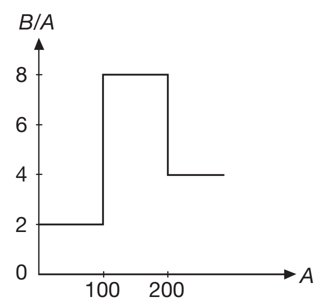
(A) kinetic energy of incoming particle is greater than Q- value of reaction in ground frame

(B) kinetic energy of incoming particle is equal to the Q- value of reaction in centre of mass frame

(C) linear momentum of particle-nucleus system is conserved

(D) energy is released in the process

26. Assume that the nuclear binding energy per nucleon $\left(\frac{B}{A}\right)$ versus mass number (A) is as shown in the figure. Use this plot to choose the correct choice(s) given below.



(A) Fusion of two nuclei with mass numbers lying in the range of $1 < A < 50$ will release energy.

(B) Fusion of two nuclei with mass numbers lying in the range of $51 < A < 100$ will release energy.

(C) Fission of a nucleus lying in the mass range of $100 < A < 200$ will release energy when broken into two equal fragments.

(D) Fission of a nucleus lying in the mass range of $200 < A < 260$ will release energy when broken into two equal fragments.

27. Two radioactive substances have half lives T and $2T$. Initially they have equal number of nuclei. After time $t = 4T$, the ratio of their number of nuclei is x and the ratio of their activity is y. Then

(A) $x = \frac{1}{8}$ (B) $x = \frac{1}{4}$

(C) $y = \frac{1}{2}$ (D) $y = \frac{1}{4}$

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** 1 amu is equal to 931.5 MeV.
   Statement-2: 1 amu is equal to $\frac{1}{12}$th the mass of C^{12} atom.
2. **Statement-1:** The ionising power of β- particle is less compared α- particle but their penetrating power is more.
   Statement-2: The mass of β- particles is less than the mass of α- particle.
3. **Statement-1:** After emission of one α-particle and two β-particles, atomic number remains unchanged
   Statement-2: Mass number changes by four.
4. **Statement-1:** A certain radioactive substance has a half life period of 30 days. The disintegration constant is 0.0231 day^{-1}.
   Statement-2: Decay constant varies inversely as half life.
5. **Statement-1:** Between α, β and γ radiations, penetrating power of γ-rays is maximum.
   Statement-2: Ionising power of γ-rays is least.
6. **Statement-1:** γ-rays are produced by the transition of a nucleus from some higher energy state to some lower energy state.
   Statement-2: Electromagnetic waves are always produced by the transition process.
7. **Statement-1:** In a nuclear process, energy is released if total binding energy of daughter nuclei is more than the total binding energy of parent nuclei.
   Statement-2: If energy is released, then total mass of daughter nuclei is less than the total mass of parent nuclei.
8. **Statement-1:** During β-decay a proton converts into a neutron and an electron. No other particle is emitted.
   Statement-2: During β-decay linear momentum of system should remains constant.
9. **Statement-1:** Binding energy per nucleon is of the order of MeV.
   Statement-2: $1 \text{ MeV} = 1.6 \times 10^{-13}$ J
10. **Statement-1:** Half life of a certain radio-active element is 100 days. After 200 days fraction left undecayed will be 50%.
    Statement-2: $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/t_{1/2}}$, where symbols have usual meaning.
11. **Statement-1:** β- particles emitted from radioactive nuclei has continuous energy ranging from zero to a certain maximum value.
    Statement-2: In β- decay a neutron is converted into a proton, an electron and an antineutrino. The total energy emitted in β- decay is shared by β- particle and antineutrino.
12. **Statement-1:** The nuclear energy can be obtained by the nuclear fission of heavier nuclei as well as by fusion of lighter nuclei.
    Statement-2: The binding energy per nucleon with increases in mass number, first increases and then decreases.
13. **Statement-1:** γ- photons are emitted during annihilation process of electron and positron.
    Statement-2: High energy photons are emitted due to conversion of mass into energy.
14. **Statement-1:** 25% of the radioactive nucleus remains active after 200 days for an element of half life 100 days.
    Statement-2: $N = N_0\left(\frac{1}{2}\right)\frac{t}{T}$ where symbols have usual meaning.
15. **Statement-1:** Rate of radioactivity can not be increased or decreased by increasing or decreasing pressure or temperature.
    Statement-2: Rate depends on number of nuclei present in the radioactive sample.
16. **Statement-1:** If binding energy per nucleon increases after a nuclear reaction then the reaction is exothermic.
    Statement-2: If there is decrease in mass in nuclear reaction then the reaction is exothermic.

17. **Statement-1:** If we compare the stability of two nuclei, then that nucleus is more stable whose total binding energy is more.
Statement-2: More the mass defect during formation of a nucleus more will be the binding energy.

18. **Statement-1:** Radioactivity is independent of the physical and chemical conditions of substance.
Statement-2: Radioactivity is a property of the nucleus.

19. **Statement-1:** Only those nuclei which are heavier than lead are radioactive.
Statement-2: Nuclei of elements heavier than lead are unstable.

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

The nuclei of a radioactive element X are being produced at a constant rate α and this element decays to a stable nucleus Y with a decay constant λ and half-life $T_{1/2}$. At time $t=0$, there are N_0 nuclei of the element X. Based on the information given, answer the following questions.

1. The number N_X of nuclei of X at time $t=T_{1/2}$ is

(A) $\dfrac{\alpha-\lambda N_0}{2\lambda}$ (B) $(2\lambda N_0-\alpha)\dfrac{1}{\lambda}$

(C) $\left(\lambda N_0+\dfrac{\alpha}{2}\right)\dfrac{1}{\lambda}$ (D) $\dfrac{\alpha+\lambda N_0}{2\lambda}$

2. The number N_Y of nuclei of Y at time t is

(A) $\alpha t-\left(\dfrac{\alpha-\lambda N_0}{\lambda}\right)e^{-\lambda t}+\left(\dfrac{\alpha-\lambda N_0}{\lambda}\right)$

(B) $\alpha t+\left(\dfrac{\alpha-\lambda N_0}{\lambda}\right)e^{-\lambda t}-\left(\dfrac{\alpha-\lambda N_0}{\lambda}\right)$

(C) $\alpha t+\left(\dfrac{\alpha-\lambda N_0}{\lambda}\right)e^{-\lambda t}$

(D) $\alpha t-\left(\dfrac{\alpha-\lambda N_0}{\lambda}\right)e^{-\lambda t}$

3. The number N_Y of nuclei of Y at $t=T_{1/2}$ is

(A) $\dfrac{\alpha}{\lambda}\log_e(2)+\dfrac{3}{2}\left(\dfrac{\alpha-\lambda N_0}{\lambda}\right)$

(B) $\dfrac{\alpha}{\lambda}\log_e(2)+\dfrac{1}{2}\left(\dfrac{\alpha-\lambda N_0}{\lambda}\right)$

(C) $\dfrac{\alpha}{\lambda}\log_e(2)-\dfrac{1}{2}\left(\dfrac{\alpha-\lambda N_0}{\lambda}\right)$

(D) $\dfrac{\alpha}{\lambda}\log_e(2)-2\left(\dfrac{\alpha-\lambda N_0}{\lambda}\right)$

Comprehension 2

The mass of a nucleus ${}^{A}_{Z}X$ is less than the sum of the masses of $(A\text{-}Z)$ number of neutrons and Z number of protons in the nucleus. The energy equivalent to the corresponding mass difference is known as the binding energy of the nucleus. A heavy nucleus of mass M can break into two light nuclei of masses m_1 and m_2 only if $(m_1+m_2)<M$. Also two light nuclei of masses m_3 and m_4 can undergo complete fusion and form a heavy nucleus of mass M' only if $(m_3+m_4)>M'$. The masses of some neutral atoms are given in the table below

${}^{1}_{1}H$	1.007825 u	${}^{2}_{1}H$	2.014102 u
${}^{6}_{3}Li$	6.015123 u	${}^{7}_{3}Li$	7.016004 u
${}^{152}_{64}Gd$	151.919803 u	${}^{206}_{82}Pb$	205.974455 u
${}^{3}_{1}H$	3.016050 u	${}^{4}_{2}He$	4.002603 u
${}^{70}_{30}Zn$	69.925325 u	${}^{82}_{34}Se$	81.916709 u
${}^{209}_{83}Bi$	208.980388 u	${}^{210}_{84}Po$	209.982876 u

$(1\text{ u}=932\text{ MeVc}^{-2})$

Based on above information, answer the following questions.

4. The correct statement is

(A) The nucleus ${}^{6}_{3}Li$ can emit an alpha particle

(B) The nucleus ${}^{210}_{84}Po$ can emit a proton

(C) Deuteron and alpha particle can undergo complete fusion.
(D) The nuclei $^{70}_{30}Zn$ and $^{82}_{34}Se$ can undergo complete fusion

5. The kinetic energy (in keV) of the alpha particle, when the nucleus $^{210}_{84}Po$ at rest undergoes alpha decay, is

(A) 5319 (B) 5422
(C) 5707 (D) 5818

Comprehension 3

A radionuclide with half life T is produced in a reactor at a constant rate P nuclei per second. During each decay, energy E_0 is released. If production of radionuclide is started at $t=0$. Based on above information, answer the following questions.

6. The number of nuclei in the radionuclide at any instant t is given by

(A) $N=\frac{PT}{\log_e 2}\left(1-e^{-\frac{t\log_e 2}{T}}\right)$

(B) $N=PT\left(1-e^{-\frac{t}{T}}\right)$

(C) $N=PT\log_e 2\left(1-e^{-\frac{t}{T\log_e 2}}\right)$

(D) $N=PT\left(1-e^{-\frac{t}{2T}}\right)$

7. The rate of release of energy as a function of time is

(A) $\frac{PE_0}{\log_e 2}\left(1-e^{-\frac{t\log_e 2}{T}}\right)$ (B) $PE_0\left(1-e^{-\frac{t\log_e 2}{T}}\right)$

(C) $PE_0\log_e 2\left(1-e^{-\frac{t\log_e 2}{T}}\right)$ (D) $PE_0\left(1-e^{-\frac{t}{2T}}\right)$

8. Total energy released upto time t is

(A) $PtE_0-\frac{PTE_0}{\log_e 2}\left(1-e^{-\frac{t\log_e 2}{T}}\right)$

(B) $PtE_0-PTE_0\left(1-e^{-\frac{t\log_e 2}{T}}\right)$

(C) $PtE_0-PTE_0\log_e 2\left(1-e^{-\frac{t\log_e 2}{T}}\right)$

(D) None of these

Comprehension 4

Two radioactive nuclei A and B, both convert into a stable nucleus C. The nucleus A converts into C after emitting two α-particles and three β-particles, the nucleus B converts into C after emitting one α-particle and five β-particles. A time $t=0$, nuclei of A are $4N_0$ and that of B are N_0. The half-life of A for converting to C is 1 min and that of B is 2 min. Initially number of nuclei of C are zero. Based on above information, answer the following questions.

9. If atomic numbers and mass numbers of A and B are Z_1, Z_2, A_1 and A_2 respectively, then

(A) $Z_1-Z_2=6$
(B) $A_1-A_2=4$
(C) Both (A) and (B) are correct
(D) Both (A) and (B) are wrong

10. The number of nuclei of C, when number of nuclei of A and B are equal is

(A) $2N_0$ (B) $3N_0$

(C) $\frac{9N_0}{2}$ (D) $\frac{5N_0}{2}$

11. The rate of disintegrations of A and B are equal at time t_0 (in minute), then

(A) $t_0=4$ (B) $t_0=6$
(C) $t_0=8$ (D) $t_0=2$

Comprehension 5

Nuclear reactions are performed for artificial transmutation of elements for there are two types of nuclear reactions, exoergic and endoergic. In exoergic reactions energy is released. In endoergic reactions energy has to supplied for the reaction proceed. In exoergic reactions nuclear energy is converted into kinetic energy. In endoergic reactions, energy input is required in the form of kinetic energy to be converted into nuclear binding energy. The minimum energy required for the reaction to take place is called threshold energy. Consider the reaction given by

$$p+{}^3_1H\longrightarrow{}^2_1H+{}^2_1H$$

Given that the atomic masses are

$$m\left({}^1_1\text{H-atom}\right)=1.007825\text{ amu }(u)$$

$$m\left({}^3_1\text{H-atom}\right)=3.016049\text{ amu }(u)$$

$$m\left({}^2_1\text{H-atom}\right)=2.014102\text{ amu }(u)$$

Based on above information, answer the following questions.

12. Protons are incident on ^3_1H at rest. The threshold energy for the reaction is

(A) 5.4 MeV (B) 10 MeV
(C) 2 MeV (D) 8 MeV

13. When ${}_1^3\text{H}$ are incident on protons, then threshold energy is
(A) 10 MeV (B) 8 MeV
(C) 16 MeV (D) 20 MeV

14. Which of the following statement is correct?
(A) Less energy is required for nuclear reaction if light part is at rest and heavy particle is incident
(B) More energy is required for nuclear reaction when heavy particle is incident and light particle is at rest
(C) Threshold energy does not depend on which particle is at rest
(D) Threshold energy does not depend on Q-value of reaction

Comprehension 6

All nuclei consist of two type of particles i.e., protons and neutrons. Nuclear force is the strongest force and the stability of nucleus is determined by the neutron-proton ratio or mass defect or Binding energy per nucleons or packing fraction. The shape of nucleus is calculated by quadrupole moment. The spin of nucleus depends on even or odd mass number. The volume of nucleus depends on the mass number. The whole mass of atom (nearly 99%) is centred at the nucleus and the magnetic moment of nucleus is measured in terms of the nuclear magnetons. Based on above information, answer the following questions.

15. The correct statement(s) about nuclear force is/are
(A) charge dependent
(B) short ranges forces
(C) non conservative force
(D) spin dependent force

16. Binding energy per nucleon is maximum
(A) for lighter order element (low mass number)
(B) for heavier order element (high mass number)
(C) for middle order element
(D) equal for all order elements

17. Volume (V) of the nucleus is related with mass number (A) as
(A) $V \propto A^2$ (B) $V \propto A^{\frac{1}{3}}$
(C) $V \propto A^{\frac{2}{3}}$ (D) $V \propto A$

Comprehension 7

The atomic masses of the hydrogen isotopes are given for reference. The mass of hydrogen ${}_1H^1$ is 1.007825 amu, deuterium ${}_1H^2$ is 2.014102 amu and tritium ${}_1H^3$ is 3.016049 amu. Based on the information provided, answer the following questions.

18. The energy released in the reaction

$${}_1\text{H}^2 + {}_1\text{H}^2 \longrightarrow {}_1\text{H}^3 + {}_1\text{H}^1 \text{ is nearly}$$

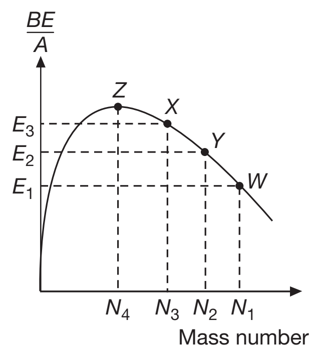
(A) 1 MeV (B) 2 MeV
(C) 4 MeV (D) 8 MeV

19. The number of fusion reactions required to generate 1 kWh is nearly
(A) 10^8 (B) 10^{18}
(C) 10^{28} (D) 10^{38}

20. The mass of deuterium ${}_1\text{H}^2$ that would be required to generate 1 kWh is
(A) 3.7 kg (B) 3.7 g
(C) 3.7×10^{-5} kg (D) 3.7×10^{-8} kg

Comprehension 8

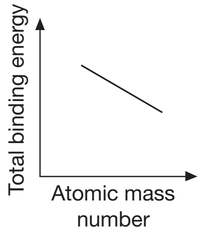
Consider the nuclear fission reaction $W \rightarrow X + Y$. Based on the graph given showing binding energy per nucleon vs number of nucleons, answer the following.

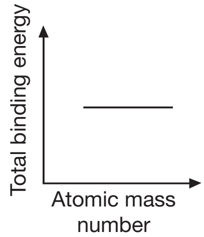


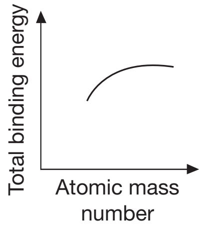
21. The Q value of the reaction is
(A) $E_1N_1 - (E_2N_2 + E_3N_3)$
(B) $(E_2N_2 + E_3N_3 - E_1N_1)$
(C) $E_2N_2 + E_1N_1 - E_3N_3$
(D) $E_1N_1 + E_3N_3 - E_2N_2$

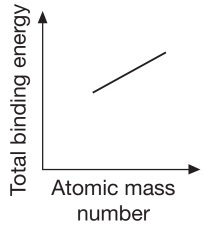
22. If M_W is the mass of W, M_X is mass of X and M_Y is mass of Y nucleus, choose the correct statement.
(A) $\frac{M_W}{N_1} > \frac{M_Y}{N_2} > \frac{M_X}{N_3}$ (B) $\frac{M_W}{N_1} < \frac{M_Y}{N_2} < \frac{M_X}{N_3}$
(C) $\frac{M_W}{N_1} < \frac{M_Y}{N_2} > \frac{M_X}{N_3}$ (D) $\frac{M_W}{N_1} > \frac{M_Y}{N_2} < \frac{M_X}{N_3}$

23. The graph representing the relationship between atomic mass number A and the total binding energy (BE) of the nucleus, for nuclei heavier than Z is

(A) 

(B) 

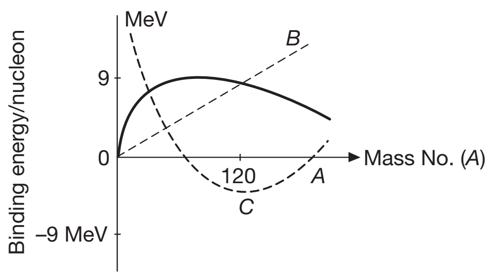
(C) 

(D) 

24. When a slow neutron is captured by a U^{235} nucleus, a fission results which releases 200 MeV of energy. If the output of a nuclear reactor is 1.6 MW, the rate of nuclei (per sec) undergoing fission is

(A) 8×10^{-3} (B) 1×10^{13}
(C) 1×10^{25} (D) 5×10^{16}

25. The binding energy per nucleon versus mass number is best represented by the curve



(A) A (B) B
(C) C (D) None of these

Comprehension 9

In a nuclear fusion reactor, the reaction occurs in two stages.
Stage-I: Two deuterium $\left({}_1D^2\right)$ nuclei fuse to form a tritium $\left({}_1T^3\right)$ nucleus with a proton as a by- product. The reaction may be represented as $D(D,\, p)T$.
Stage-II: A tritium nucleus fuses with another deuterium nucleus to form a helium $\left({}_2He^4\right)$ nucleus with a neutron as a by product. The reaction is represented as $T(D,\, n)\alpha$.

Given that $m\left({}_1D^2\right) = 2.014102$ u (atom)

$$m\left({}_1T^3\right) = 3.016049 \text{ u (atom)}$$

$$m\left({}_2\text{He}^4\right) = 4.002603 \text{ u (atom)}$$

$$m\left({}_1\text{H}^1\right) = 1.007825 \text{ u (atom)}$$

$$m\left({}_0n^1\right) = 1.008665 \text{ u}$$

Based on above information, answer the following questions.

26. The energy released in the Stage-II of fusion reaction, is
(A) 4.033 MeV (B) 17.587 MeV
(C) 40.33 MeV (D) 1.7587 MeV

27. The energy released in the combined reaction per deuterium, is given by
(A) 4.207 MeV (B) 5.207 MeV
(C) 6.207 MeV (D) 7.207 MeV

28. The percentage of the mass energy of the initial deuterium is released is
(A) 0.184% (B) 0.284%
(C) 0.384% (D) 0.484%

Comprehension 10

Many unstable nuclei can decay spontaneously to a nucleus of lower mass but different combination of nucleons. The process of spontaneous emission of radiation is called Radioactivity in which the decay rate is actually exponentially decrease with time. Also, radioactive decay is a statistical process which is independent of all external conditions. Based on above information, answer the following questions.

29. If T_H is the half life and T_M is the mean life, which of the following statements is correct?
(A) $T_M > T_H$ (B) Both are equal
(C) $T_M < T_H$ (D) Nothing can be said

30. If n is the number of α- particles being emitted per second by N atoms of a radioactive element, then half life of element, in second, will be
(A) $\frac{n}{N}$ s (B) $\frac{N}{n}$ s
(C) $\frac{0.693}{n}$ s (D) $\frac{0.693n}{N}$ s

31. The activity of radioactive substance of decay constant λ is R_1 at time t_1 and R_2 at time $t_2(>t_1)$, then
(A) $R_1t_1 = R_2t_2$ (B) $\frac{R_1 - R_2}{t_2 - t_1} = \text{constant}$
(C) $R_2 = R_1e^{-\lambda(t_2-t_1)}$ (D) $R_1t_2 = R_2t_2$

Comprehension 11

A radioactive sample has mass m, decay constant λ and molecular weight M. If Avogadro number is N_A then answer the following questions.

32. The initial number of nuclei present is

(A) $\frac{m}{N_A}$ (B) $\frac{M}{m}N_A$

(C) $\frac{m}{M}N_A$ (D) MN_A

33. The number of decayed nuclei after time t is

(A) $\frac{m}{M}N_A e^{-\lambda t}$ (B) $\frac{m}{M}e^{-\lambda t}$

(C) $\frac{m}{M}N_A\left(1-e^{-\lambda t}\right)$ (D) $\frac{m}{M}\left(1-e^{-\lambda t}\right)$

34. The activity of sample after time t is

(A) $\left(\frac{mN_A\lambda}{M}\right)e^{-\lambda t}$ (B) $\frac{m}{M}N_A\lambda\left(1-e^{-\lambda t}\right)$

(C) $\frac{M}{m}N_A\lambda e^{-\lambda t}$ (D) None of these

Comprehension 12

Scientists are working hard to develop nuclear fusion reactor. Nuclei of heavy hydrogen, ^2_1H known as deuteron and denoted by D can be thought of as a candidate for fusion reactor. The D-D reaction is $^2_1\text{H}+{}^2_1\text{H}\longrightarrow{}^3_2\text{He}+n+\text{energy}$. In the core of fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of $^4_1$H nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place. Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time t_0 before the particles fly away from the core. If n is the density (number/volume) of deuterons, the product nt_0 is called Lawson number. In one of the criteria, a reactor is termed successful if Lawson number is greater than 5×10^{14} scm^{-3}. It may be helpful to use the following: Boltzmann constant $k=8.6\times10^{-5}$ eVK^{-1}; $\frac{e^2}{4\pi\varepsilon_0}=1.44\times10^{-9}$ eVm. Based on above information, answer the following questions.

35. In the core of nuclear fusion reactor, the gas becomes plasma because of

(A) strong nuclear force acting between the deuterons
(B) Coulomb force acting between the deuterons
(C) Coulomb force acting between deuteron-electron pairs
(D) the high temperature maintained inside the reactor core

36. Assume that two deuteron nuclei in the core of fusion reactor at temperature T are moving towards each other, each with kinetic energy $1.5\,\text{kT}$, when the separation between them is large enough to neglect Coulomb potential energy. Also neglect any interaction from other particles in the core. The minimum temperature T required for them to reach a separation of 4×10^{-15} m is in the range

(A) $1\times10^9\text{ K}<T<2\times10^9\text{ K}$
(B) $2\times10^9\text{ K}<T<3\times10^9\text{ K}$
(C) $3\times10^9\text{ K}<T<4\times10^9\text{ K}$
(D) $4\times10^9\text{ K}<T<5\times10^9\text{ K}$

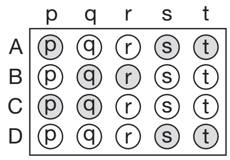
37. Results of calculations for four different designs of a fusion reactor using D-D reaction are given below. Which of these is most promising based on Lawson criterion?

(A) Deuteron density $=2\times10^{12}$ cm^{-3}, confinement time $=5\times10^{-3}$ s
(B) Deuteron density $=8\times10^{14}$ cm^{-3}, confinement time $=9\times10^{-1}$ s
(C) Deuteron density $=4\times10^{23}$ cm^{-3}, confinement time $=1\times10^{-11}$ s
(D) Deuteron density $=1\times10^{24}$ cm^{-3}, confinement time $=4\times10^{-12}$ s

MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

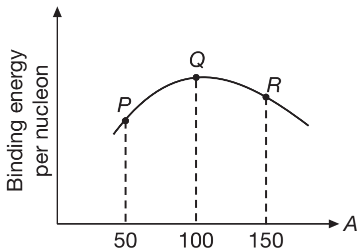
If the correct matches are A $\rightarrow$ p, s and t; B $\rightarrow$ q and r; C $\rightarrow$ p and q; and D $\rightarrow$ s and t; then the correct darkening of bubbles will look like the following:



1. Some laws/processes are given in **COLUMN-I**. Match these with the physical phenomena given in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Nuclear fusion	(p) Converts some matter into energy
(B) Nuclear fission	(q) Generally possible for nuclei with low atomic number
(C) β-decay	(r) Generally possible for nuclei with higher atomic number
(D) Exothermic nuclear reaction	(s) Essentially proceeds by weak nuclear forces.

2. Corresponding to the graph of binding energy per nucleon vs mass number (A) shown in figure, match the following two columns.



COLUMN-I	COLUMN-II
(A) $P + P \rightarrow Q$	(p) Energy is released
(B) $P + P + P \rightarrow R$	(q) Energy is absorbed
(C) $P + R \rightarrow 2Q$	(r) No energy transfer will take place
(D) $P + Q \rightarrow R$	(s) Data insufficient

3. Match the quantities in **COLUMN-I** with their respective values in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Pair Production	(p) Few MeV
(B) Inverse photoelectric effect	(q) 20 KeV
(C) De-excitation of Be^{+4} atom from second excited state	(r) 54 eV
(D) K_α X-ray photons of molybdenum $Z = 42$	(s) 0.1 eV

4. Match the following two columns.

COLUMN-I	COLUMN-II
(A) The energy of air molecules at room temperature	(p) 0.02 eV
(B) Binding energy of heavy nuclei per nucleon	(q) 2 eV
(C) X-ray photon energy	(r) 10 keV
(D) Photon energy of visible light	(s) 7 MeV

5. Match the processes given in **COLUMN-I** with their characteristics in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Alpha decay	(p) Mono energetic particles are emitted
(B) Beta decay	(q) Poly energetic particles are emitted
(C) Positron emission	(r) Angular momentum is conserved
(D) Electron capture	(s) Can take place inside and outside nucleus

6. Match the quantities in **COLUMN- I** with their respective values in **COLUMN-II.**

COLUMN-I	COLUMN-II
(A) 1 rutherford	(p) 1 disintegration/sec
(B) 1 becquerel	(q) 3.7×10^{10} dis/sec
(C) 1 curie	(r) 10^6 dis/sec
(D) Activity of 1 g Ra^{226}	(s) 10^{10} dis/sec

7. In the following chain, $A \to B \to C$. A and B are radioactive, while C is stable. Initially we have only A and B nuclei. There are no nuclei of C present in sample. As the time passes, match the two columns.

COLUMN-I	COLUMN-II
(A) Number of nuclei of $(A+B)$	(p) will increase continuously
(B) Number of nuclei of B	(q) will decrease continuously
(C) Number of nuclei of $(C+B)$	(r) will first increase then decrease
(D) Number of nuclei of $(A+C)$	(s) data insufficient

8. Match the processes given in **COLUMN-I** with their characteristics in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) α-decay	(p) Atomic number of the product nucleus decreases
(B) β^+-decay	(q) Atomic number of the product nucleus increases
(C) β^--decay	(r) Atomic number of the product nucleus not necessarily changes
(D) Electron capture	(s) some mass is converted into energy

9. Match the following two columns.

COLUMN-I	COLUMN-II
(A) After emission of one α and one β particles	(p) atomic number will decrease by 3
(B) After emission of two α and one β particles	(q) atomic number will decrease by 2
(C) After emission of one α and two β particles	(r) mass number will decrease by 8
(D) After emission of two α and two β particles	(s) mass number will decrease by 4

(*Continued*)

10. Match the processes given in **COLUMN-I** with their characteristics in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) α-decay	(p) Mass number decreases
(B) β^--decay	(q) Atomic number decreases
(C) β^+-decay	(r) Mass number does not change
(D) γ-decay	(s) Chemical symbol of nucleus changes
	(t) Energy is released

11. Match the reactions/processes given in **COLUMN-I** with their characteristics given in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Spontaneous radioactive decay of an uranium nucleus initially at rest as given by reaction $^{238}_{92}U \longrightarrow {}^{234}_{90}Th + {}^4_2He + ...$	(p) Number of protons is increased
(B) Fusion reaction of two hydrogen nuclei as given by reaction $^1_1H + {}^1_1H \longrightarrow {}^2_1H + ...$	(q) Momentum is conserved
(C) Fission of U^{235} nucleus initiated by a thermal neutron as given by reaction $^1_0n + {}^{235}_{92}U \longrightarrow {}^{144}_{56}Ba + {}^{89}_{36}Kr + 3\,{}^1_0n$	(r) Mass and energy are inter convertible
(D) β^--decay (negative beta decay)	(s) Charge is conserved
	(t) Angular momentum is conserved

12. Match **COLUMN-I** of the nuclear processes with **COLUMN-II** containing parent nucleus and one of the end products of each process and then select the correct answer using the codes given below the lists.

COLUMN-I	COLUMN-II
(A) Alpha decay	(p) ${}^{15}_{8}O \longrightarrow {}^{15}_{7}N + ...$
(B) β^+-decay	(q) ${}^{238}_{92}U \longrightarrow {}^{234}_{90}Th + ...$
(C) Fission	(r) ${}^{185}_{83}Bi \longrightarrow {}^{184}_{82}Pb + ...$
(D) Proton emission	(s) ${}^{239}_{94}Pu \longrightarrow {}^{140}_{57}La + ...$

13. At t = 0, x nuclei of a radioactive substance emit y nuclei per second. Match the following two columns.

COLUMN-I	COLUMN-II
(A) Decay constant λ	(p) $(\ln 2)\left(\frac{x}{y}\right)$
(B) Half-life	(q) $\frac{x}{y}$
(C) Activity after time $t = \frac{1}{\lambda}$	(r) $\frac{y}{e}$
(D) Number of nuclei after time $t = \frac{1}{\lambda}$	(s) None of these

INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

1. Calculate the activity of 0.5 mg of radon-222 in curie. It is known that half-life of radon is 3.8 days.

2. If 20 g of a radioactive substance due to radioactive decay reduces to 10 g in 4 minutes, then in what time (in minutes) 80 g of the same substance will reduce to 20 g

3. An unstable element is produced in a nuclear reactor at a constant rate. If its half life is 100 years, how much time in years is required to produce 50% of the equilibrium quantity?

4. How many head-on, elastic collisions must a neutron have with deuterium nucleus to reduce its energy from 1 MeV to 0.025 eV.

5. There are two radio nuclei A and B. A is an alpha emitter and B a beta emitter. Their disintegration constants are in ratio of $1:2$. The ratio of number of atoms of A and B at any time t so that probabilities of getting alpha and beta particles are same at that instant is

6. A ${}^{7}Li$ target is bombarded with a proton beam current of 10^{-4} A for 1 hour to produce ^{7}Be of activity 1.8×10^{8} disintegrations per second. Assuming that one ^{7}Be radioactive nucleus is produced by bombarding 1000 protons, determine its half-life, in days, to the nearest three digit integer.

7. Number of nuclei of a radioactive substance at $t=0$ are 1000 and 900 at $t=2$ sec. The number of nuclei at $t=4$ sec will be $x10$, then the value of x in number $x10$ is

8. At time $t=0$, activity of a radioactive substance is 1600 Bq, at $t=8$ s activity remains 100 Bq. Find the activity, in Bq, at $t=2$ s.

9. Calculate the energy (in MeV) required to extract a neutron from a carbon nucleus with mass number 13, if $m({}_{6}C^{13}) = 13.00335$ u, $m({}_{6}C^{12}) = 12.0000$ u, $m_n = 1.00867$ u and $m_p = 1.00783$ u.

10. The mean lives of a radioactive substance are 1620 and 405 years for α-emission and β-emission respectively. Find out the time (in years) after which three fourth of a sample will decay if it is decaying both by α-emission and β-emission simultaneously. (Take $\ln 2 = 0.693$)

11. An environment study showed that the Chernobyl disaster released 6.0 MCi of ^{137}Cs into the environment. The mass of ^{137}Cs released is $10x$ kg. If the half life of ^{137}Cs is 30.2 years, calculate x.

12. In U^{238} ore containing Uranium the ratio of U^{234} to Pb^{206} nuclei is 3. Assuming that all the lead present in the ore is final stable product of U^{238}, half life of U^{238} to be 4.5×10^{9} years calculate the age of ore. (in 10^{9} years)

13. A ^{118}Cd radionuclide goes through the transformation chain.

$$^{118}\text{Cd} \xrightarrow[30\text{ min}]{} {}^{118}\text{In} \xrightarrow[45\text{ min}]{} {}^{118}\text{Sn (stable)}$$

The half lives are written below the respective arrows. A time $t = 0$ only Cd was present. Find the percentage of nuclei transformed into stable over 60 minutes.

14. A nucleus at rest undergoes a decay emitting an α-particle of de-Broglie wavelength 5.76×10^{-15} m. If the mass of daughter nucleus is 223.610 amu and that of α-particle is 4.002 amu. The mass of the parent nucleus is $22x$ amu, then calculate x if $1 \text{ amu} = 931.47 \text{ MeV}/c^2$

15. A radioactive sample decays with a mean life of 20 millisecond. A capacitor of capacitance 100 μF is charged to some potential and then the plates are connected through a wire of resistance R. What should be the value of R in ohm so that the ratio of the charge on the capacitor to the activity of the radioactive sample remain constant in time?

16. A thermonuclear device consists of a torus (of diameter 3 m with a tube of diameter 1 m), containing deuterium gas at 10^{-2} mm mercury pressure and at 27 °C. A bank of capacitors of 1200 μF is discharged through the tube at 40 kV. If only 10% of the electrical energy is transformed to plasma kinetic energy, then the maximum temperature attained is ….. $\times 10^2$ K. Assuming that the energy is equally shared between the deutrons and electrons in the plasma. Take $\pi^2 \approx 10$, $g = 10 \text{ ms}^{-2}$

17. There are two radioactive substances A and B. Decay constant of B is two times that of A. Initially both have equal number of nuclei. After n half lives of A, rate of disintegration of both are equal, calculate n.

18. The mass defect for the nucleus of helium is 0.0302 amu. Calculate the binding energy per nucleon for helium in MeV. Take $1 \text{ amu} = 930 \text{ MeV}/c^2$.

19. Half life of radioactive substance A is two time that of B. Initially number of nuclei of A and B are N_A and N_B respectively. After three half lives of A number of nuclei of both are equal. Then the ratio $\dfrac{N_B}{N_A}$ is

20. A neutron at rest decays. Assuming the resulting proton to remain at rest. Calculate the energy of the antineutrino in 10^5 eV. Given that, $m_n = 1.0087$ u, $m_p = 1.0072$ u, $m_e = 0.00055$ u

21. ^{23}Ne decays to ^{23}Na by negative beta emission. Mass of ^{23}Ne is 22.994465 amu mass of ^{23}Na is 22.989768 amu. Calculate the maximum kinetic energy of emitted electrons in MeV, neglecting the kinetic energy of recoiling product nucleus

22. The nucleus $_{92}\text{U}^{238}$ is unstable against α-decay with a half-life of about 4.5×10^9 years. Calculate the kinetic energy of the emitted α-particle in MeV to the nearest integer. Given that $m\left({}_{92}\text{U}^{238}\right) = 238.05081$ u; $m\left({}_{2}\text{He}^{4}\right) = 4.00260$ u; $m\left({}_{90}\text{Th}^{234}\right) = 234.04363$ u

23. A neutron with an energy of 4.6 MeV collides with protons and is retarded. Assuming that upon each collision neutron is deflected by 45° find the number of collisions which will reduce its energy to 0.23 eV.

24. The nuclei of two radioactive isotopes of same substance A^{236} and A^{234} are present in the ratio $4:1$ in an ore obtained from Mars. Their half lives are 30 min and 60 min respectively. Both isotopes are alpha emitters and the activity of the isotope with half life 30 min is one Rutherford. Calculate after how much time (in min) their activities will become identical.

25. The mean lives of a radioactive substance are 1200 yr and 600 yr for α-emission and β-emission respectively. Find out the time, in year, during which three fourth of a sample will decay if it is decaying both by α-emission and β-emission simultaneously.

Given $\log_e(4) \approx 1.4$

26. A uranium U^{235} nucleus liberates an energy of 200 MeV per fission. Calculate the energy liberated in 10^{14} J, when a uranium bomb containing 1.5 kg is exploded.

27. There are two radioactive substances A and B. Decay constant of B is two times that of A. Initially both have equal number of nuclei. After n half lives of A, rate of disintegration of both are equal, calculate n.

28. A heavy nucleus having mass number 200 gets disintegrated into small fragments of mass number 80 and 120. If binding energy per nucleon for parent atom is 6.5 MeV and for daughter nuclei is 7 MeV and 8 MeV respectively, then the energy released in the decay will be $x \times 10^5$ eV, then calculate x.

29. A nuclear reactor is designed to deliver 4 MW power. The nuclear fuel consists of Uranium-235 to run the reactor with amount of energy released per fission 200 MeV. Calculate amount of fuel needed (in grams) to run the reactor at designed rating with 100% efficiency for one year.

30. The radioactivity of an old sample of whisky due to tritium (half life 12.5 years) was found to be only about 4% of that measured in a recently purchased bottle marked 10 years old. Find the age of sample in years.

ARCHIVE: JEE MAIN

1. [Online April 2019]
The ratio of mass densities of nuclei ^{40}Ca and ^{16}O is close to
(A) 0.1 (B) 1
(C) 2 (D) 5

2. [Online April 2019]
Two radioactive materials A and B have decay constants 10λ and λ, respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of A to that of B will be $\frac{1}{e}$ after a time
(A) $\frac{1}{10\lambda}$ (B) $\frac{11}{10\lambda}$
(C) $\frac{1}{9\lambda}$ (D) $\frac{1}{11\lambda}$

3. [Online April 2019]
Two radioactive substances A and B have decay constants 5λ and λ respectively. At $t=0$, a sample has the same number of the two nuclei. The time taken for the ratio of the number of nuclei of become $\left(\frac{1}{e}\right)^2$ will be
(A) $\frac{1}{2\lambda}$ (B) $\frac{1}{4\lambda}$
(C) $\frac{1}{\lambda}$ (D) $\frac{2}{\lambda}$

4. [Online April 2019]
Half lives of two radioactive nuclei A and B are 10 minutes and 20 minutes, respectively. If, initially a sample has equal number of nuclei, then after 60 minutes, the ratio of decayed numbers of nuclei A and B will be
(A) $9:8$ (B) $3:8$
(C) $8:1$ (D) $1:8$

5. [Online 2019]
A sample of radioactive material A, that has an activity of 10 mCi (where $1\text{ Ci} = 3.7\times10^{10}$ dps) has twice the number of nuclei as another sample of a different radioactive material B which has an activity of 20 mCi. The correct choices for half-lives of A and B (in days) would then be respectively
(A) 10 and 40 (B) 20 and 5
(C) 20 and 10 (D) 5 and 10

6. [Online January 2019]
At a given instant, say $t=0$, two radioactive substances A and B have equal activities. The ratio $\frac{R_B}{R_A}$ of their activities after time t itself decays with time t as e^{-3t}. If the half-life of A is $\ln 2$, the half-life of B is
(A) $4\ln 2$ (B) $\frac{\ln 2}{2}$
(C) $\frac{\ln 2}{4}$ (D) $2\ln 2$

7. [Online January 2019]
Using a nuclear counter the count rate of emitted particles from a radioactive source is measured. At $t=0$ it was 1600 counts per second and $t=8$ seconds it was 100 counts per second. The count rate observed, as counts per second, at $t=6$ seconds is close to
(A) 400 (B) 360
(C) 150 (D) 200

8. [Online January 2019]
Consider the nuclear fission $Ne^{20} \rightarrow 2He^4 + C^{12}$ Given that the binding energy/nucleon of Ne^{20}, He^4 and C^{12} are, respectively, 8.03 MeV, 7.07 MeV and 7.86 MeV, identify the correct statement
(A) Energy of 12.4 MeV will be supplied
(B) 8.3 MeV energy will be released
(C) Energy of 3.6 MeV will be released
(D) Energy of 9.7 MeV has to be supplied

9. [2018]
It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d; while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_c. The values of p_d and p_c are respectively
(A) (0.89, 0.28) (B) (0.28, 0.89)
(C) (0, 0) (D) (0, 1)

10. [Online 2018]
A solution containing active cobalt $^{60}_{27}Co$ having activity of 0.8 μCi and decay constant λ is injected in an animal's body. If 1 cm^3 of blood is drawn from the animal's body after 10 hrs of injection, the activity found was 300 decays per minute. What is the volume of blood that is flowing in the body? ($1\text{ Ci} = 3.7\times10^{10}$ decays per second and at $t=10$ hrs, $e^{-\lambda t} = 0.84$).
(A) 4 litres (B) 7 litres
(D) 5 litres (D) 6 litres

11. [Online 2018]
An unstable heavy nucleus at rest breaks into two nuclei which move away with velocities in the ratio of $8:27$. The ratio of the radii of the nuclei (assumed to be spherical) is

(A) 3: 2 (B) 2: 3
(C) 4 : 9 (D) 8 : 27

12. [Online 2018]
At some instant, a radioactive sample S_1 having an activity $5\,\mu\text{Ci}$ has twice the number of nuclei as another sample S_2 which has an activity of $10\,\mu\text{Ci}$. The half lives of S_1 and S_2 are
(A) 20 years and 10 years, respectively
(B) 10 years and 20 years, respectively
(C) 20 years and 5 years, respectively
(D) 5 years and 20 years, respectively

13. [2017]
A radioactive nucleus A with a half-life T, decays into a nucleus B. At $t = 0$, there is no nucleus B. At sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by

(A) $t = \frac{T}{2}\frac{\log 2}{\log(1.3)}$ (B) $t = T\frac{\log(1.3)}{\log 2}$

(C) $t = T\log(1.3)$ (D) $t = \frac{T}{\log(1.3)}$

14. [Online 2017]
Two deuterons undergo nuclear fusion to form a Helium nucleus. Energy released in this process is (given binding energy per nucleon for deuteron $= 1.1$ MeV and for helium $= 7.0$ MeV)
(A) 25.8 MeV (B) 32.4 MeV
(C) 30.2 MeV (D) 23.6 MeV

15. [Online 2017]
Imagine that a reactor converts all given mass into energy and that it operates at a power level of 10^9 watt. The mass of the fuel consumed per hour in the reactor will be (velocity of light, c is $3\times10^8\ \text{ms}^{-1}$)
(A) 4×10^{-2} g (B) 6.6×10^{-5} g
(C) 0.8 g (D) 0.96 g

16. [2016]
Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively. Initially, the samples have equal number of nuclei. After 80 minutes, the ratio of decayed numbers of A and B nuclei will be
(A) 5 : 4 (B) 1 : 16
(C) 4 : 1 (D) 1 : 4

17. [Online 2016]
A neutron moving with a speed v makes a head on collision with a stationary hydrogen atom in ground state. The minimum kinetic energy of the neutron for which inelastic collision will take place is
(A) 20.4 eV (B) 10.2 eV
(C) 12.1 eV (D) 16.8 eV

18. [Online 2015]
Let N_β be the number of β particles emitted by 1 gram of Na^{24} radioactive nuclei (half life $= 15$ hrs) in 7.5 hours, N_β is close to (Avogadro number $= 6.023\times10^{23}$ / g mole)
(A) 6.2×10^{21} (B) 7.5×10^{21}
(C) 1.25×10^{22} (D) 1.75×10^{22}

19. [2014]
The radiation corresponding to $3\to2$ transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to
(A) 1.8 eV (B) 1.1 eV
(C) 0.8 eV (D) 1.6 eV

20. [2012]
Assume that a neutron breaks into a proton and an electron. The energy released during this process is

(Mass of neutron $= 1.6725\times10^{-27}$ kg

Mass of proton $= 1.6725\times10^{-27}$ kg

Mass of electron $= 9\times10^{-31}$ kg)

(A) 7.10 MeV (B) 6.30 MeV
(C) 5.1 MeV (D) 0.51 MeV

21. [2011]
The half life of a radioactive substance is 20 minutes. The approximate time interval $(t_2 - t_1)$ between the time t_2 when $\frac{2}{3}$ of it has decayed and time t_1 when $\frac{1}{3}$ of it had decayed is
(A) 7 min (B) 14 min
(C) 20 min (D) 28 min

22. [2010]
A radioactive nucleus (initial mass number A and atomic number Z) emits 3α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be

(A) $\frac{A-Z-4}{Z-2}$ (B) $\frac{A-Z-8}{Z-4}$

(C) $\frac{A-Z-4}{Z-8}$ (D) $\frac{A-Z-12}{Z-4}$

Directions: Question number 23-24 are based on the following paragraph.
A nucleus of mass $M + \Delta m$ is at rest and decays into two daughter nuclei of equal mass $\frac{M}{2}$ each. Speed of light is c.

23. [2010]
The speed of daughter nuclei is

(A) $c\sqrt{\frac{\Delta m}{M+\Delta m}}$ (B) $c\frac{\Delta m}{M+\Delta m}$

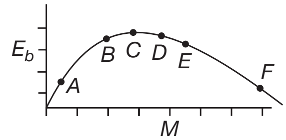
(C) $c\sqrt{\frac{2\Delta m}{M}}$ (D) $c\sqrt{\frac{\Delta m}{M}}$

24. [2010]
The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then

(A) $E_1 = 2E_2$ (B) $E_2 = 2E_1$

(C) $E_1 > E_2$ (D) $E_2 > E_1$

25. [2009]



The above is a plot of binding energy per nucleon E_b, against the nuclear mass M; A, B, C, D, E, F correspond to different nuclei. Consider four reactions

(i) $A+B \to C+\varepsilon$ (ii) $C \to A+B+\varepsilon$

(iii) $D+E \to F+\varepsilon$ (iv) $F \to D+E+\varepsilon$

where ε is the energy released. In which reaction is ε positive?

(A) (i) and (iv) (B) (i) and (iii)
(C) (ii) and (iv) (D) (ii) and (iii)

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

(In this section each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct)

1. [JEE (Advanced) 2019]
In a radioactive sample, ${}^{40}_{19}\text{K}$ nuclei either decay into stable ${}^{40}_{20}\text{Ca}$ nuclei with decay constant 4.5×10^{-10} per year or into stable ${}^{40}_{18}\text{Ar}$ nuclei with decay constant 0.5×10^{-10} per year. Given that in this sample all the stable ${}^{40}_{20}\text{Ca}$ and ${}^{40}_{18}\text{Ar}$ nuclei are produced by the ${}^{40}_{19}\text{K}$ nuclei only. In time $t\times10^9$ years, if the ratio of the sum of stable ${}^{40}_{20}\text{Ca}$ and ${}^{40}_{18}\text{Ar}$ nuclei to the radioactive ${}^{40}_{19}\text{K}$ nuclei is 99, the value of t will be

(A) 1.15 (B) 9.2
(C) 2.3 (D) 4.6

2. [JEE (Advanced) 2016]
An accident in a nuclear laboratory resulted in deposition of a certain amount of radioactive material of half-life 18 days inside the laboratory. Tests revealed that the radiation was 64 times more than the permissible level required for safe operation of the laboratory. What is the minimum number of days after which the laboratory can be considered safe for use?

(A) 64 (B) 90
(C) 108 (D) 120

3. [JEE (Advanced) 2016]
The electrostatic energy of Z protons uniformly distributed throughout a spherical nucleus of radius R is given by $E = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$. The measured masses of the neutron, ${}^{1}_{1}\text{H}$, ${}^{15}_{7}\text{N}$ and ${}^{15}_{8}\text{O}$ are $1.008665\ \text{u}$, $1.007825\ \text{u}$, $15.000109\ \text{u}$ and $15.003065\ \text{u}$, respectively. Given that the radii of both the ${}^{15}_{7}\text{N}$ and ${}^{15}_{8}\text{O}$ nuclei are same, $1\ \text{u} = 931.5\ \text{MeV c}^{-2}$ (c is the speed of light) and $\frac{e^2}{(4\pi\varepsilon_0)} = 1.44\ \text{MeV fm}$. Assuming that the difference between the binding energies of ${}^{15}_{7}\text{N}$ and ${}^{15}_{8}\text{O}$ is purely due to the electrostatic energy, the radius of either of the nuclei is $(1\ \text{fm} = 10^{-15}\ \text{m})$

(A) 2.85 fm (B) 3.03 fm
(C) 3.42 fm (D) 3.80 fm

4. [JEE (Advanced) 2015]
A fission reaction is given by ${}^{236}_{92}\text{U} \to {}^{140}_{54}\text{Xe} + {}^{94}_{38}\text{Sr} + x + y$, where x and y are two particles. Considering ${}^{236}_{92}\text{U}$ to be at rest, the kinetic energies of the products are denoted by K_{Xe}, K_{Sr}, $K_x(2\ \text{MeV})$ and $K_y(2\ \text{MeV})$, respectively. Let the binding energies per nucleon of ${}^{236}_{92}\text{U}$, ${}^{140}_{54}\text{Xe}$ and ${}^{94}_{38}\text{Sr}$ be 7.5 MeV, 8.5 MeV and 8.5 MeV, respectively. Considering different conservation laws, the correct options is/are

(A) $x=n,\ y=n,\ K_{\text{Sr}}=129\ \text{MeV},\ K_{\text{Xe}}=86\ \text{MeV}$

(B) $x=p,\ y=e^-,\ K_{\text{Sr}}=129\ \text{MeV},\ K_{\text{Xe}}=86\ \text{MeV}$

(C) $x=p,\ y=n,\ K_{\text{Sr}}=129\ \text{MeV},\ K_{\text{Xe}}=86\ \text{MeV}$

(D) $x=n,\ y=n,\ K_{\text{Sr}}=86\ \text{MeV},\ K_{\text{Xe}}=129\ \text{MeV}$

5. [IIT-JEE 2008]
A radioactive sample S_1 having an activity of $5\ \mu\text{Ci}$ has twice the number of nuclei as another sample S_2 which has an activity of $10\ \mu\text{Ci}$. The half lives of S_1 and S_2 can be
(A) 20 years and 5 years, respectively
(B) 20 years and 10 years, respectively
(C) 10 years each
(D) 5 years each

6. [IIT-JEE 2007]
In the options given below, let E denote the rest mass energy of a nucleus and n a neutron. The correct options is
(A) $E\left({}^{236}_{92}\text{U}\right) > E\left({}^{137}_{53}\text{I}\right) + E\left({}^{97}_{39}Y\right) + 2E(n)$
(B) $E\left({}^{236}_{92}\text{U}\right) < E\left({}^{137}_{53}\text{I}\right) + E\left({}^{97}_{39}Y\right) + 2E(n)$
(C) $E\left({}^{236}_{92}\text{U}\right) < E\left({}^{140}_{56}\text{Ba}\right) + \left({}^{94}_{36}\text{Kr}\right) + 2E(n)$
(D) $E\left({}^{236}_{92}\text{U}\right) = E\left({}^{140}_{56}\text{Ba}\right) + E\left({}^{94}_{36}\text{Kr}\right) + 2E(n)$

7. [IIT-JEE 2006]
Half-life of a radioactive substance A is 4 days. The probability that a nucleus will decay in two half-lives is
(A) $\frac{1}{4}$ (B) $\frac{3}{4}$
(C) $\frac{1}{2}$ (D) 1

8. [IIT-JEE 2005]
If a star can convert all the He nuclei completely into oxygen nuclei. The energy released per oxygen nuclei is
[Mass of the helium nucleus is 4.0026 amu and mass of oxygen nucleus is 15.9994 amu]
(A) 7.6 MeV (B) 56.12 MeV
(C) 10.24 MeV (D) 23.4 MeV

9. [IIT-JEE 2004]
After 280 days, the activity of a radioactive sample is 6000 dps. The activity reduces to 3000 dps after another 140 days. The initial activity of the sample in dps is
(A) 6000 (B) 9000
(C) 3000 (D) 24000

10. [IIT-JEE 2003]
A nucleus with mass number 220 initially at rest emits an α-particle. If the Q value of the reaction is 5.5 MeV, calculate the kinetic energy of the α-particle
(A) 4.4 MeV (B) 5.4 MeV
(C) 5.6 MeV (D) 6.5 MeV

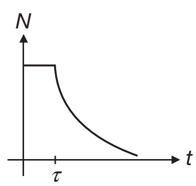
11. [IIT-JEE 2003]
For uranium nucleus how does its mass vary with volume?
(A) $m \propto V$ (B) $m \propto \frac{1}{V}$
(C) $m \propto \sqrt{V}$ (D) $m \propto V^2$

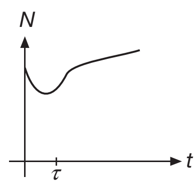
12. [IIT-JEE 2003]
If the atom ${}_{100}\text{Fm}^{257}$ follows the Bohr's model and the radius of last orbit of ${}_{100}\text{Fm}^{257}$ is n times the Bohr radius, then find n
(A) 100 (B) 200
(C) 4 (D) $\frac{1}{4}$

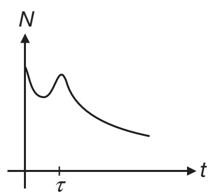
13. [IIT-JEE 2002]
Which of the following processes represents a γ-decay?
(A) ${}^{A}X_{Z} + \gamma \longrightarrow {}^{A}X_{Z-1} + a + b$
(B) ${}^{A}X_{Z} + {}^{1}n_{0} \longrightarrow {}^{A-3}X_{Z-2} + c$
(C) ${}^{A}X_{Z} \longrightarrow {}^{A}X_{Z} + f$
(D) ${}^{A}X_{Z} + e_{-1} \longrightarrow {}^{A}X_{A-1} + g$

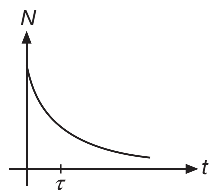
14. [IIT-JEE 2002]
The half life of ${}^{215}\text{At}$ is 100 μs. The time taken for the radioactivity of a sample of ${}^{215}\text{At}$ to decay to $\frac{1}{16}$th of its initial value is
(A) 400 μs (B) 6.3 μs
(C) 40 μs (D) 300 μs

15. [IIT-JEE 2001]
A radioactive sample consists of two distinct species having equal number of atoms initially. The mean life-time of one species is τ and that of the other is 5τ. The decay products in both cases are stable. A plot is made of the total number of radioactive nuclei as a function of time. Which of the following figures best represents the form of this plot?

(A)


(B)


(C)


(D)


16. [IIT-JEE 2001]
The electron emitted in beta radiation originates from
(A) inner orbits of atoms
(B) free electrons existing in nuclei
(C) decay of a neutron in a nucleus
(D) photon escaping from the nucleus

17. [IIT-JEE 2000]
Two radioactive materials X_1 and X_2 have decay constants 10λ and λ respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of X_1 to that of X_2 will be $\frac{1}{e}$ after a time
(A) $\frac{1}{10\lambda}$ (B) $\frac{1}{11\lambda}$
(C) $\frac{11}{10\lambda}$ (D) $\frac{1}{9\lambda}$

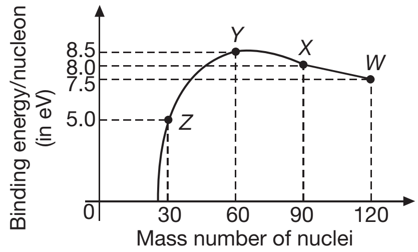
18. [IIT-JEE 1999]
The half life period of a radioactive element X is same as the mean life time of another radioactive element Y. Initially both of them have the same number of atoms. Then,
(A) X and Y have the same decay rate initially.
(B) X and Y decay at the same rate always.
(C) Y will decay at a faster rate than X.
(D) X will decay at a faster rate than Y.

19. [IIT-JEE 1999]
Which of the following statements is correct?
(A) Beta rays are same as cathode rays.
(B) Gamma rays are high energy neutrons.
(C) Alpha particles are singly ionised helium atoms.
(D) Protons and neutrons have exactly the same mass.

20. [IIT-JEE 1999]
^{22}Ne nucleus, after absorbing energy, decays into two α- particles and an unknown nucleus. The unknown nucleus is
(A) Nitrogen (B) Carbon
(C) Oxygen (D) Boron

21. [IIT-JEE 1999]
Order of magnitude of density of uranium nucleus is $\left(m_p = 1.67 \times 10^{-27}\ \text{kg}\right)$
(A) 10^{20} kgm^{-3} (B) 10^{17} kgm^{-3}
(C) 10^{14} kgm^{-3} (D) 10^{11} kgm^{-3}

22. [IIT-JEE 1999]
Binding energy per nucleon vs mass number curve for nuclei is shown in the figure. W, X, Y and Z are four nuclei indicated on the curve. The process that would release energy is



(A) $Y \longrightarrow 2Z$ (B) $W \longrightarrow X + Z$
(C) $W \longrightarrow 2Y$ (D) $X \longrightarrow Y + Z$

23. [IIT-JEE 1998]
The half-life of ^{131}I is 8 days. Given a sample of ^{131}I at time $t = 0$, we can assert that
(A) no nucleus will decay before $t = 4$ days
(B) no nucleus will decay before $t = 8$ days
(C) all nuclei will decay before $t = 16$ days
(D) a given nucleus may decay at any time after $t = 0$

24. [IIT-JEE 1997]
Masses of two isobars $_{29}Cu^{64}$ and $_{30}Zn^{64}$ are 63.9298 u and 63.9292 u respectively. It can be concluded from these data that
(A) both the isobars are stable
(B) Zn^{64} is radioactive, decaying to Cu^{64} through β-decay
(C) Cu^{64} is radioactive, decaying to Zn^{64} through γ-decay
(D) Cu^{64} is radioactive, decaying to Zn^{64} through β-decay

25. [IIT-JEE 1994]
Consider α-particles, β-particles and γ-rays each having an energy of 0.5 MeV. In increasing order of penetrating powers, the radiations are
(A) α, β, γ (B) α, γ, β
(C) β, γ, α (D) γ, β, α

26. [IIT-JEE 1994]
Fast neutrons can easily be slowed down by
(A) the use of lead shielding
(B) passing them through heavy water
(C) elastic collisions with heavy nuclei
(D) applying a strong electric field

27. [IIT-JEE 1993]
A star initially has 10^{40} deutrons. It produces energy via the processes $_1H^2 + {}_1H^2 \longrightarrow {}_1H^3 + p$ and $_1H^2 + {}_1H^3 \longrightarrow {}_2He^4 + n$. If the average power

radiated by the star is 10^{16} W, the deutron supply of the star is exhausted in a time of the order of

(A) 10^6 s (B) 10^8 s

(C) 10^{12} s (D) 10^{16} s

(The masses of nuclei are: $m(H^2) = 2.014$ u, $m(p) = 1.007$ u, $m(n) = 1.008$ u, $m(He^4) = 4.001$ u)

28. [IIT-JEE 1989]
The decay constant of a radioactive sample is λ. The half-life and mean-life of the sample are respectively given by

(A) $\frac{1}{\lambda}$ and $\frac{(\log_e)2}{\lambda}$ (B) $\frac{(\log_e 2)}{\lambda}$ and $\frac{1}{\lambda}$

(C) $\lambda(\log_e 2)$ and $\frac{1}{\lambda}$ (D) $\frac{\lambda}{(\log_e 2)}$ and $\frac{1}{\lambda}$

29. [IIT-JEE 1988]
A freshly prepared radioactive source of half-life 2 h emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with this source is

(A) 6 h (B) 12 h

(C) 24 h (D) 128 h

30. [IIT-JEE 1987]
Four physical quantities are listed in **COLUMN-I**. Their values are listed in **COLUMN-II** in a random order

COLUMN-I	COLUMN-II
1. Thermal energy of air molecules at room temperature.	i. 0.02 eV
2. Binding energy of heavy nuclei per nucleon.	ii. 2 eV
3. X-ray photon energy.	iii. 10 keV
4. Photon energy of visible light.	iv. 7 MeV

The correct matching of columns I and II is given by

	1	2	3	4
(A)	i	iv	iii	ii
(B)	i	iii	ii	iv
(C)	ii	i	iii	iv
(D)	ii	iv	i	iii

31. [IIT-JEE 1987]
During a nuclear fusion reaction

(A) a heavy nucleus breaks into two fragments by itself

(B) a light nucleus bombarded by thermal neutrons breaks up

(C) a heavy nucleus bombarded by thermal neutrons breaks up

(D) two light nuclei combine to give a heavier nucleus and possibly other products

32. [IIT-JEE 1987]
During a negative beta decay

(A) an atomic electron is ejected

(B) an electron which is already present within the nucleus is ejected

(C) a neutron in the nucleus decays emitting an electron

(D) a part of the binding energy of the nucleus is converted into an electron

33. [IIT-JEE 1983]
The equation:

$$4\,{}_1^1H^+ \longrightarrow {}_2^4He^{2+} + 2e^- + 26 \text{ MeV represents}$$

(A) β-decay (B) γ-decay

(C) fusion (D) fission

34. [IIT-JEE 1983]
Beta rays emitted by a radioactive material are

(A) electromagnetic radiations

(B) the electrons orbiting around the nucleus

(C) charged particles emitted by the nucleus

(D) neutral particles

35. [IIT-JEE 1981]
The half-life of the radioactive radon is 3.8 days. The time, at the end of which $1/20^{th}$ of the radon sample will remain undecayed, is (given $\log_{10} e = 0.4343$)

(A) 3.8 day (B) 16.5 day

(C) 33 day (D) 76 day

Multiple Correct Choice Type Problems

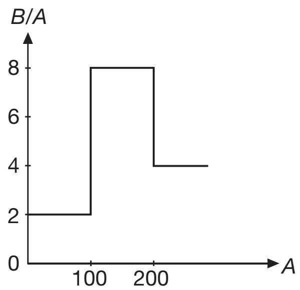
(In this section each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct)

1. [JEE (Advanced) 2018]
In a radioactive decay chain, ${}_{90}^{232}Th$ nucleus decays to ${}_{82}^{212}Pb$ nucleus. Let N_α and N_β be the number of α and β^- particles, respectively, emitted in this decay process. Which of the following statements is (are) true?

(A) $N_\alpha = 5$ (B) $N_\alpha = 6$

(C) $N_\beta = 2$ (D) $N_\beta = 4$

2. **[JEE (Advanced) 2008]**
Assume that the nuclear binding energy per nucleon $\left(\frac{B}{A}\right)$ versus mass number (A) is as shown in the figure. Use this plot to choose the correct choice(s) given below.



(A) Fusion of two nuclei with mass numbers lying in the range of $1 < A < 50$ will release energy.
(B) Fusion of two nuclei with mass numbers lying in the range of $51 < A < 100$ will release energy.
(C) Fission of a nucleus lying in the mass range of $100 < A < 200$ will release energy when broken into two equal fragments.
(D) Fission of a nucleus lying in the mass range of $200 < A < 260$ will release energy when broken into two equal fragments.

3. **[IIT-JEE 1998]**
Let m_p be the mass of proton, m_n the mass of neutron. M_1 the mass of $^{20}_{10}\text{Ne}$ nucleus and M_2 the mass of $^{40}_{20}\text{Ca}$ nucleus. Then
(A) $M_2 = 2M_1$ (B) $M_2 > 2M_1$
(C) $M_2 < 2M_1$ (D) $M_1 < 10(m_n + m_p)$

4. **[IIT-JEE 1994]**
Which of the following statement(s) is (are) correct?
(A) The rest mass of a stable nucleus is less than the sum of the rest masses of its separated nucleons
(B) The rest mass of a stable nucleus is greater than the sum of the rest masses of its separated nucleons
(C) In nuclear fission, energy is released by fusing two nuclei of medium mass (approximately 100 amu)
(D) In nuclear fission, energy is released by fragmentation of a very heavy nucleus

5. **[IIT-JEE 1992]**
When a monochromatic point source of light is at a distance of 0.2 m from a photo-electric cell, the cut-off voltage and the saturation current are respectively 0.6 V and 18.0 mA. If the same source is placed 0.6 m away from the photoelectric cell, then
(A) the stopping potential will be 0.2 V.
(B) the stopping potential will be 0.6 V.
(C) the saturation current will be 6.0 mA.
(D) the saturation current will be 2.0 mA.

6. **[IIT-JEE 1986]**
The mass number of a nucleus is
(A) always less than its atomic number
(B) always more than its atomic number
(C) sometimes equal to its atomic number
(D) sometimes more than and sometimes equal to its atomic number

7. **[IIT-JEE 1984]**
From the following equations pick out the possible nuclear fusion reactions
(A) ${}_6\text{C}^{13} + {}_1\text{H}^1 \longrightarrow {}_6\text{C}^{14} + 4.3\text{ MeV}$
(B) ${}_6\text{C}^{12} + {}_1\text{H}^1 \longrightarrow {}_7\text{N}^{13} + 2\text{ MeV}$
(C) ${}_7\text{N}^{14} + {}_1\text{H}^1 \longrightarrow {}_8\text{O}^{15} + 7.3\text{ MeV}$
(D) ${}_{92}\text{U}^{235} + {}_0n^1 \longrightarrow {}_{54}\text{Xe}^{140} + {}_{36}\text{Sr}^{94} + {}_0n^1 + {}_0n^1 + \gamma + 200\text{ MeV}$

Comprehension Type Questions

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options).

Comprehension I

If the measurement errors in all the independent quantities are known, then it is possible to determine the error in any dependent quantity. This is done by the use of series expansion and truncating the expansion at the first power of the error. For example, consider the relation $z = \frac{x}{y}$. If the errors in x, y and z are Δx, Δy and Δz, respectively, then

$$z \pm \Delta z = \frac{x \pm \Delta x}{y \pm \Delta y} = \frac{x}{y}\left(1 \pm \frac{\Delta x}{x}\right)\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$$

The series expansion for $\left(1 \pm \frac{\Delta y}{y}\right)^{-1}$, to first power in $\frac{\Delta y}{y}$, is $1 \mp \left(\frac{\Delta y}{y}\right)$. The relative errors in independent variables are always added. So, the error in z will be $\Delta z = z\left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right)$.

The above derivation makes the assumption that $\frac{\Delta x}{x} \ll 1$, $\frac{\Delta y}{y} \ll 1$. Therefore, the higher powers of these quantities are neglected. (There are two questions based on **PARAGRAPH, the question given below is one of them**).

1. [JEE (Advanced) 2018]

Consider the ratio $r = \frac{(1-a)}{(1+a)}$ to be determined by measuring a dimensionless quantity a. If the error in the measurement of a is $\Delta a \left(\frac{\Delta a}{a \ll 1} \right)$, then what is the error Δr in determining r?

(A) $\frac{\Delta a}{(1+a)^2}$ (B) $\frac{2\Delta a}{(1+a)^2}$

(C) $\frac{2\Delta a}{(1-a^2)}$ (D) $\frac{2a\Delta a}{(1-a^2)}$

2. [JEE (Advanced) 2018]

In an experiment the initial number of radioactive nuclei is 3000. It is found that 1000 ± 40 nuclei decayed in the first 1.0 s. For $|x| \ll 1$, $\ln(1+x) = x$ up to first power in x. The error $\Delta\lambda$, in the determination of the decay constant λ, in s^{-1}, is

(A) 0.04 (B) 0.03

(C) 0.02 (D) 0.01

Comprehension 2

The mass of a nucleus ${}_Z^AX$ is less than the sum of the masses of $(A\text{-}Z)$ number of neutrons and Z number of protons in the nucleus. The energy equivalent to the corresponding mass difference is known as the binding energy of the nucleus. A heavy nucleus of mass M can break into two light nuclei of masses m_1 and m_2 only if $(m_1 + m_2) < M$. Also two light nuclei of masses m_3 and m_4 can undergo complete fusion and form a heavy nucleus of mass M' only if $(m_3 + m_4) > M'$. The masses of some neutral atoms are given in the table below

${}_1^1H$	1.007825 u	${}_1^2H$	2.014102 u
${}_3^6Li$	6.015123 u	${}_3^7Li$	7.016004 u
${}_{64}^{152}Gd$	151.919803 u	${}_{82}^{206}Pb$	205.974455 u
${}_1^3H$	3.016050 u	${}_2^4He$	4.002603 u
${}_{30}^{70}Zn$	69.925325 u	${}_{34}^{82}Se$	81.916709 u
${}_{83}^{209}Bi$	208.980388 u	${}_{84}^{210}Po$	209.982876 u

(Continued)

$(1\text{ u} = 932\text{ MeVc}^{-2})$

Based on above information, answer the following questions.

3. [JEE (Advanced) 2013]

The correct statement is

(A) The nucleus ${}_3^6Li$ can emit an alpha particle

(B) The nucleus ${}_{84}^{210}Po$ can emit a proton

(C) Deuteron and alpha particle can undergo complete fusion.

(D) The nuclei ${}_{30}^{70}Zn$ and ${}_{34}^{82}Se$ can undergo complete fusion

4. [JEE (Advanced) 2013]

The kinetic energy (in keV) of the alpha particle, when the nucleus ${}_{84}^{210}Po$ at rest undergoes alpha decay, is

(A) 5319 (B) 5422

(C) 5707 (D) 5818

Comprehension 3

The β- decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p) and an electron (e^-) and observed as the decay products of the neutron. Therefore, considering the decay of a neutron as a two body decay process, it was predicted theoretically that the kinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has a continuous spectrum. Considering a three-body decay process, i.e., $n \to p + e^- + \bar{\nu}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino $(\bar{\nu}_e)$ to be massless and possessing negligible energy and the neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy of the electron is 0.8×10^6 eV. The kinetic energy carried by the proton is only the recoil energy. Based on above information, answer the following questions.

5. [IIT-JEE 2012]

What is the maximum energy of the anti-neutrino?

(A) zero

(B) Much less than 0.8×10^6 eV

(C) Nearly 0.8×10^6 eV

(D) Much larger than 0.8×10^6 eV

6. [IIT-JEE 2012]
If the anti-neutrino had a mass of $3\ \text{eVc}^{-2}$ (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K, of the electron?

(A) $0 \le K \le 0.8 \times 10^6$ eV

(B) $3\ \text{eV} \le K \le 0.8 \times 10^6$ eV

(C) $3\ \text{eV} \le K < 0.8 \times 10^6$ eV

(D) $0 \le K < 0.8 \times 10^6$ eV

Comprehension 4

Scientists are working hard to develop nuclear fusion reactor. Nuclei of heavy hydrogen, $^2_1$H known as deuteron and denoted by D can be thought of as a candidate for fusion reactor. The D-D reaction is $^2_1\text{H} + ^2_1\text{H} \longrightarrow ^3_2\text{He} + n + \text{energy}$. In the core of fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of $^4_1$H nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place. Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time t_0 before the particles fly away from the core. If n is the density (number/volume) of deuterons, the product nt_0 is called Lawson number. In one of the criteria, a reactor is termed successful if Lawson number is greater than $5 \times 10^{14}\ \text{scm}^{-3}$. It may be helpful to use the following: Boltzmann constant $k = 8.6 \times 10^{-5}\ \text{eVK}^{-1}$; $\dfrac{e^2}{4\pi\varepsilon_0} = 1.44 \times 10^{-9}$ eVm. Based on above information, answer the following questions.

7. [IIT-JEE 2009]
In the core of nuclear fusion reactor, the gas becomes plasma because of

(A) strong nuclear force acting between the deuterons

(B) Coulomb force acting between the deuterons

(C) Coulomb force acting between deuteron-electron pairs

(D) the high temperature maintained inside the reactor core

8. [IIT-JEE 2009]
Assume that two deuteron nuclei in the core of fusion reactor at temperature T are moving towards each other, each with kinetic energy $1.5\ \text{kT}$, when the separation between them is large enough to neglect Coulomb potential energy. Also neglect any interaction from other particles in the core. The minimum temperature T required for them to reach a separation of 4×10^{-15} m is in the range

(A) $1 \times 10^9\ \text{K} < T < 2 \times 10^9$ K

(B) $2 \times 10^9\ \text{K} < T < 3 \times 10^9$ K

(C) $3 \times 10^9\ \text{K} < T < 4 \times 10^9$ K

(D) $4 \times 10^9\ \text{K} < T < 5 \times 10^9$ K

9. [IIT-JEE 2009]
Results of calculations for four different designs of a fusion reactor using D-D reaction are given below. Which of these is most promising based on Lawson criterion?

(A) Deuteron density $= 2 \times 10^{12}\ \text{cm}^{-3}$, confinement time $= 5 \times 10^{-3}$ s

(B) Deuteron density $= 8 \times 10^{14}\ \text{cm}^{-3}$, confinement time $= 9 \times 10^{-1}$ s

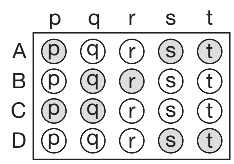
(C) Deuteron density $= 4 \times 10^{23}\ \text{cm}^{-3}$, confinement time $= 1 \times 10^{-11}$ s

(D) Deuteron density $= 1 \times 10^{24}\ \text{cm}^{-3}$, confinement time $= 4 \times 10^{-12}$ s

Matrix Match/Column Match Type Questions

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A $\rightarrow$ p, s and t; B $\rightarrow$ q and r; C $\rightarrow$ p and q; and D $\rightarrow$ s and t; then the correct darkening of bubbles will look like the following:



1. [IIT-JEE 2006]
Some laws/processes are given in **COLUMN-I**. Match these with the physical phenomena given in **COLUMN-II.**

COLUMN-I	COLUMN-II
(A) Nuclear fusion	(p) Converts some matter into energy
(B) Nuclear fission	(q) Generally possible for nuclei with low atomic number

(Continued)

COLUMN-I	COLUMN-II
(C) β-decay	(r) Generally possible for nuclei with higher atomic number
(D) Exothermic nuclear reaction	(s) Essentially proceeds by weak nuclear forces.

2. **[JEE (Advanced) 2015]**
Match the nuclear processes given in **COLUMN-I** with the appropriate option(s) in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Nuclear fusion	(p) absorption of thermal neutrons by $^{235}_{92}U$
(B) Fission in a nuclear reactor	(q) $^{60}_{27}Co$ nucleus
(C) β-decay	(r) Energy production in stars via hydrogen conversion to helium
(D) γ-ray emission	(s) Heavy water
	(t) Neutrino emission

3. **[JEE (Advanced) 2013]**
Match **COLUMN-I** of the nuclear process with **COLUMN-II** containing parent nucleus and one of the end products of each process and then select the correct answer using the codes given below the lists.

COLUMN-I	COLUMN-II
(A) Alpha decay	(p) $^{15}_{8}O \rightarrow ^{17}_{7}N + \ldots$
(B) β^{+} decay	(q) $^{238}_{92}U \rightarrow ^{234}_{90}Th + \ldots$
(C) Fission	(r) $^{185}_{83}Bi \rightarrow ^{184}_{82}Pb + \ldots$
(D) Proton emission	(s) $^{239}_{94}Pu \rightarrow ^{140}_{57}La + \ldots$

4. **[IIT-JEE 2007]**
Some laws/processes are given in **COLUMN-I**. Match these with the physical phenomena given in **COLUMN-II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

COLUMN-I	COLUMN-II
(A) Transition between two atomic energy levels	(p) Characteristic X-rays
(B) Electron emission from a material	(q) Photoelectric effect
(C) Mosley's law	(r) Hydrogen spectrum
(D) Change of photon energy into kinetic energy of electrons	(s) β-decay

Integer/Numerical Answer Type Questions

(In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s)).

1. **[JEE (Advanced) 2019]**
Suppose a $^{226}_{88}Ra$ nucleus at rest and in ground state undergoes α-decay to a $^{222}_{86}Rn$ nucleus in its excited state. The kinetic energy of the emitted α particle is found to be 4.44 MeV. $^{222}_{86}Rn$ nucleus then goes to its ground state by γ-decay. The energy of the emitted γ photon is ______ keV.
(Given: atomic mass of $^{226}_{88}Ra = 226.005$ u, atomic mass of $^{222}_{86}Rn = 222.000$ u, atomic mass of α particle $= 4.000$ u, $1\text{ u} = 931\text{ MeVc}^{-2}$, c is speed of the light).

2. **[JEE (Advanced) 2017]**
^{131}I is an isotope of Iodine that β decays to an isotope of Xenon with a half-life of 8 days. A small amount of a serum labelled with ^{131}I is injected into the blood of a person. The activity of the amount of ^{131}I injected was 2.4×10^{5} Becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After 11.5 h, 2.5 ml of blood is drawn from the person's body, and gives an activity of 115 Bq. The total volume of blood in the person's body, in litres is approximately (you may use $e^{2} \approx 1 + x$ for $|x| \ll 1$ and $\ln 2 \approx 0.7$).

3. **[JEE (Advanced) 2016]**
The isotope $^{12}_{5}B$ having a mass 12.014 u undergoes β-decay to $^{12}_{6}C$. $^{12}_{6}C$ has an excited state of the nucleus $\left(^{12}_{6}C^{*}\right)$ at 4.041 MeV above its ground state. If $^{12}_{5}B$ decays to $^{12}_{6}C^{*}$, the maximum kinetic energy of the β-particle in units of MeV is

($1u = 931.5$ MeV c^{-2}, where c is the speed of light in vacuum).

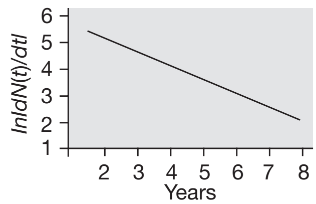
4. [JEE (Advanced) 2015]
For a radioactive material, its activity A and rate of change of its activity R are defined as $A = -\frac{dN}{dt}$ and $R = -\frac{dA}{dt}$, where $N(t)$ is the number of nuclei at time t. Two radioactive source P (mean life τ) and Q (mean life 2τ) have the same activity at $t = 0$. Their rate of change of activities at $t = 2\tau$ are R_P and R_Q, respectively. If $\frac{R_P}{R_Q} = \frac{n}{e}$, then the value of n is

5. [JEE (Advanced) 2015]
A nuclear power plant supplying electrical power to a village uses a radioactive material of half life T years as the fuel. The amount of fuel at the beginning is such that the total power requirement of the village is 12.5% of the electrical power available from the plant at that time. If the plant is able to meet the total power needs of the village for a maximum period of nT years, then the value of n is

6. [JEE (Advanced) 2013]
A freshly prepared sample of a radioisotope of half-life 1386 s has activity 10^3 disintegrations per second. Given that $\log_e 2 = 0.693$, find the fraction of the initial number of nuclei (expressed in nearest, integer percentage) that will decay in the first 80 s after preparation of the sample.

7. [JEE (Advanced) 2011]
The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^9 s. The mass of an atom of this radioisotope is 10^{-25} kg. Find the mass (in mg) of the radioactive sample.

8. [IIT-JEE 2010]
To determine the half-life of a radioactive element, a student plots a graph of $\ln\left|\frac{dN(t)}{dt}\right|$ versus t. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 yr, the value of p is



ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on Nucleus Properties and Binding Energy)

1. 4.53 fm
2. 10.2 MeV
3. 6.94 amu
4. $\frac{S_1}{S_2} = \left(\frac{A_1}{A_2}\right)^{\frac{2}{3}}$
5. 2.4×10^{17} kgm^{-3}
6. 4.7×10^{-14} kg
7. 34.969 amu
8. 28.3 MeV
10. 39.231 MeV, 5.604 MeV

Test Your Concepts-II (Based on Radioactivity)

1. 81
2. 1.55×10^6 J
3. 1.4×10^{17} s
4. 3.81×10^{-4} year^{-1}, 3.14×10^{-2} year^{-1}
5. (a) 0.113 min^{-1} (b) 6.132 min
6. 1.26×10^{-11} %
7. (a) 1.42×10^{14} per year (b) 1.41×10^{14} per year
   (c) 5.4×10^7 year
8. $N_2 \approx N_0 e^{-\lambda_2 t}$, $N_2 \approx \frac{\lambda_1}{\lambda_2} N_0 e^{-\lambda_1 t}$
9. 0.39
10. (a) 5724 year (b) 12.58 dismin^{-1}g^{-1}
11. 1.12×10^{-6} g
12. 1.88×10^{-3}
13. (a) 6.90×10^{16} nuclei (b) 8.00×10^{13} decay/s
    (c) 1.25×10^{12} s^{-1}
14. $\lambda_p N_p = \lambda_d N_d$
15. 1.5×10^{18}
16. 4.5×10^9
17. 3.86 Ci
18. $\frac{1}{4}$
19. 0.259
20. 30×10^{21}

Test Your Concepts-II (Based on Nuclear Reactions, Alpha, Beta, Gamma Decay, Fission and Fusion)

1. (a) Not possible (b) Possible
2. 1.54 MeV
3. 8.78 day
4. 0.73 MeV
5. 0.1308 MeV
6. (a) 1.02 MeV (b) 2.82 MeV
7. 4.4 MeV
8. 0.961 MeV
9. 120.35 g
10. (a) 8.09×10^{13} J (b) 2.7×10^6 kg
11. (a) –1.18 MeV (b) 43°18′
13. 127.6 MeV
14. 4.31 MeV
15. 1.79×10^{-4} kg

Single Correct Choice Type Questions

1. B	2. B	3. C	4. A	5. D	6. C	7. C	8. C	9. C	10. B
11. C	12. A	13. D	14. D	15. C	16. D	17. B	18. A	19. D	20. D
21. C	22. A	23. B	24. A	25. C	26. C	27. D	28. D	29. D	30. B
31. C	32. D	33. D	34. D	35. D	36. C	37. C	38. B	39. A	40. C
41. D	42. D	43. C	44. C	45. B	46. C	47. D	48. C	49. B	50. D
51. D	52. B	53. D	54. C	55. C	56. C	57. A	58. C	59. D	60. A

61. C	**62.** A	**63.** C	**64.** D	**65.** B	**66.** A	**67.** C	**68.** A	**69.** C	**70.** D
71. D	**72.** B	**73.** C	**74.** C	**75.** B	**76.** B	**77.** C	**78.** D	**79.** D	**80.** C
81. C	**82.** A	**83.** C	**84.** B	**85.** D	**86.** C	**87.** C	**88.** A	**89.** A	**90.** A
91. A	**92.** C	**93.** C	**94.** B	**95.** C	**96.** B	**97.** B	**98.** A	**99.** D	**100.** A
101. A	**102.** B	**103.** C	**104.** D	**105.** A	**106.** D	**107.** C	**108.** B	**109.** B	**110.** A
111. C	**112.** A	**113.** B	**114.** B	**115.** A	**116.** B	**117.** C	**118.** A	**119.** B	**120.** B
121. B	**122.** C	**123.** D	**124.** C	**125.** D	**126.** A	**127.** D	**128.** D	**129.** C	**130.** A
131. B	**132.** A	**133.** A	**134.** D	**135.** B	**136.** D	**137.** D	**138.** D	**139.** B	**140.** D
141. A	**142.** B	**143.** C	**144.** D	**145.** A	**146.** B	**147.** A	**148.** D	**149.** D	**150.** C
151. C	**152.** C	**153.** A	**154.** B	**155.** B	**156.** D	**157.** D	**158.** D	**159.** D	**160.** C
161. B	**162.** C	**163.** B	**164.** D	**165.** B	**166.** B	**167.** A	**168.** B	**169.** B	**170.** C
171. B	**172.** A	**173.** C	**174.** A	**175.** C	**176.** C	**177.** B	**178.** A	**179.** B	**180.** C
181. D	**182.** C								

Multiple Correct Choice Type Questions

1. A, B, C, D	**2.** B, C	**3.** A, D	**4.** A, B, D	**5.** A, D
6. C, D	**7.** B, C	**8.** A, B, C	**9.** A, B, D	**10.** B, C
11. B, C	**12.** C, D	**13.** A, B	**14.** A, B, C, D	**15.** A, B, C
16. A, B, C	**17.** B, C	**18.** A, B, D	**19.** A, C, D	**20.** A, B, C
21. A, B, C	**22.** A, B	**23.** A, B, C	**24.** A, B, C, D	**25.** A, B, C
26. B, D	**27.** B, C			

Reasoning Based Questions

1. D	**2.** B	**3.** B	**4.** A	**5.** B	**6.** C	**7.** A	**8.** D	**9.** B	**10.** D
11. A	**12.** AB	**13.** A	**14.** D	**15.** B	**16.** B	**17.** D	**18.** A	**19.** D	

Linked Comprehension Type Questions

1. D	**2.** B	**3.** C	**4.** C	**5.** A	**6.** A	**7.** B	**8.** A	**9.** B	**10.** C
11. B	**12.** A	**13.** C	**14.** C	**15.** B	**16.** C	**17.** D	**18.** C	**19.** B	**20.** D
21. B	**22.** A	**23.** C	**24.** D	**25.** A	**26.** B	**27.** D	**28.** C	**29.** A	**30.** C
31. C	**32.** C	**33.** C	**34.** A	**35.** D	**36.** A	**37.** B			

Matrix Match/Column Match Type Questions

1. A → (p, q)	B → (p, r)	C→ (p, s)	D → (p, q, r)
2. A → (p)	B → (p)	C→ (p)	D → (s)
3. A → (p)	B → (q)	C→ (r)	D → (q)
4. A → (p)	B → (s)	C→ (r)	D → (q)
5. A → (p, r)	B → (q, r, s)	C→ (q, r)	D → (p, r)
6. A → (r)	B → (p)	C→ (q)	D → (q)
7. A → (q)	B → (s)	C→ (p)	D → (s)
8. A → (p, s)	B → (p)	C→ (q, s)	D → (p)
9. A → (s)	B → (p, r)	C→ (s)	D → (q, r)

10. A → (p, q, s, t)	B → (r, s, t)	C→ (q, r, s, t)	D → (r, t)
11. A → (q, r, s, t)	B → (q, r, s, t)	C→ (q, r, s, t)	D → (p, q, r, s, t)
12. A → (q)	B → (p)	C→ (s)	D → (r)
13. A → (s)	B → (p)	C→ (r)	D → (s)

Integer/Numerical Answer Type Questions

1. 77	**2.** 8	**3.** 100	**4.** 8	**5.** 2
6. 100	**7.** 8	**8.** 800	**9.** 5	**10.** 449
11. 7	**12.** 2	**13.** 31	**14.** 8	**15.** 200
16. 4706	**17.** 1	**18.** 7	**19.** 8	**20.** 6
21. 4	**22.** 4	**23.** 24	**24.** 180	**25.** 560
26. 1.23	**27.** 1	**28.** 2000	**29.** 1539	**30.** 68

ARCHIVE: JEE MAIN

1. B	**2.** C	**3.** A	**4.** A	**5.** B	**6.** C	**7.** D	**8.** D	**9.** A	**10.** C
11. A	**12.** C	**13.** B	**14.** D	**15.** A	**16.** A	**17.** A	**18.** B	**19.** B	**20.** D
21. C	**22.** C	**23.** C	**24.** D	**25.** A					

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Single Correct Choice Type Problems

1. B	**2.** C	**3.** C	**4.** A	**5.** A	**6.** A	**7.** B	**8.** C	**9.** D	**10.** B
11. A	**12.** D	**13.** C	**14.** A	**15.** D	**16.** C	**17.** D	**18.** C	**19.** A	**20.** B
21. B	**22.** C	**23.** D	**24.** D	**25.** A	**26.** B	**27.** C	**28.** B	**29.** B	**30.** A
31. D	**32.** C	**33.** C	**34.** C	**35.** B					

Multiple Correct Choice Type Problems

1. A, C	**2.** B, D	**3.** C, D	**4.** A, D	**5.** B, D
6. C, D	**7.** B, C			

Comprehension Type Questions

1. B	**2.** C	**3.** C	**4.** A	**5.** C	**6.** D	**7.** D	**8.** A	**9.** B

Matrix Match/Column Match Type Questions

1. A → (p, q)	B → (p, r)	C → (p, s)	D → (p, q, r)
2. A → (r) OR (r, t)	B → (p, s)	C → (q, t)	D → (r)
3. A → (q)	B → (p)	C → (s)	D → (r)
4. A → (p, r)	B → (p, q, s)	C → (p)	D → (q)

Integer/Numerical Answer Type Questions

1. 135	**2.** 5	**3.** 9	**4.** 2	**5.** 3
6. 4	**7.** 1	**8.** 8		

CHAPTER

4 Semiconductor Devices and Applications

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) The basics of semiconductors
(b) Types of semiconductors
(c) Nature of charge carriers
(d) Junction diode basics
(e) Diode as a rectifier
(f) Zener Diode
(g) Opto electronic junction devices
(h) Transistor and its working
(i) Input and Output characteristics of a transistor
(j) Transistor as a switch
(k) Transistor as an Amplifier
(l) Transistor as an Oscillator
(m) Logic Gates and Applications.

All this is followed by an Exercise Set (fully solved) which contains questions as per the latest JEE pattern. At the end of Exercise Set, a collection of problems asked previously in JEE Main are also given.

INTRODUCTION

Devices in which a controlled flow of electrons can be obtained are the basic building blocks of all the electronic circuits.

Before the discovery of transistor in 1948, such devices were mostly vacuum tubes (also called valves) like

(a) a vacuum diode which has two electrodes i.e. an anode (often called plate) and a cathode.
(b) a triode which has three electrodes i.e. a cathode, a plate and a grid.
(c) a tetrode (with four electrodes) and
(d) a pentode (with five electrodes).

FUNCTIONING OF VACUUM TUBE(S)

In a vacuum tube, the electrons are supplied by a heated cathode and the controlled flow of these electrons in vacuum is obtained by varying the voltage between its different electrodes. Vacuum is required in the inter electrode space, else the moving electrons may lose their energy on collision with the air molecules in their path. In these devices the electrons can flow only from the cathode to the anode (i.e., only in one direction). *Therefore, such devices are generally referred to as valves*.

DISADVANTAGES OF USING VACUUM TUBE(S)

Vacuum tube devices are bulky, consume high power, operate generally at high voltages $(\sim 100\text{ V})$ and have limited life and low reliability.

SEMICONDUCTOR ELECTRONICS

The seed of the development of modern solid-state semiconductor electronics goes back to 1930's when it was observed that some solid state semiconductors and their junctions offer the possibility of controlling the number and the direction of flow of charge carriers through them. Simple excitations like light, heat or small applied voltage can change the number of mobile charges in a semiconductor. Also note that the supply and flow of charge carriers in the semiconductor devices are within the solid itself, whereas in the earlier vacuum tubes/valves, the mobile electrons were obtained from a heated cathode and they were made to flow through an evacuated space called vacuum. Semiconductor devices require no external heating or large evacuated space. They are small in size, consume low power, operate at low voltages and have long life and high reliability.

CLASSIFICATION OF METALS, CONDUCTORS AND SEMICONDUCTORS ON THE BASIS OF CONDUCTIVITY

On the basis of the relative values of electrical conductivity (σ) or resistivity $\left(\rho = \frac{1}{\sigma}\right)$, the solids are broadly classified as:

(a) **Insulators:** They have high resistivity or low conductivity.

$$\rho \approx 10^{11} - 10^{19}\ \Omega\text{m}$$

$$\sigma \approx 10^{-11} - 10^{-19}\ \text{Sm}^{-1}$$

(b) **Metals:** They have very low resistivity or high conductivity.

$$\rho \approx 10^{-2} - 10^{-8}\ \Omega\text{m}$$

$$\sigma \approx 10^{2} - 10^{8}\ \text{Sm}^{-1}$$

(c) **Semiconductors:** They have resistivity or conductivity whose values lie somewhere between metals and insulators.

$$\rho \approx 10^{-5} - 10^{6}\ \Omega\text{m}$$

$$\sigma \approx 10^{5} - 10^{-6}\ \text{Sm}^{-1}$$

The values of ρ and σ given above are indicative of magnitude and could well go outside the ranges as well.

Relative values of the resistivity are not the only criteria for distinguishing metals, insulators and semiconductors from each other. Our interest in this chapter is in the study of semiconductors which could be:

(a) Elemental semiconductors like Si and Ge

(b) Compound semiconductors like

- **Inorganic:** CdS, $GaAs$, $CdSe$, InP, etc.
- **Organic:** anthracene, doped pthalocyanines, etc.
- **Organic polymers:** polypyrrole, polyaniline, polythiophene, etc.

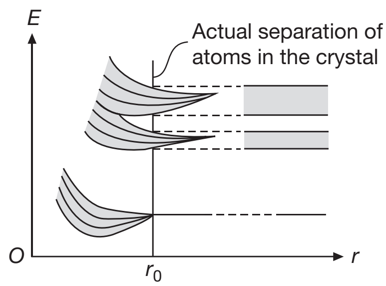
Conceptual Note(s)

Most of the currently available semiconductor devices are based on elemental semiconductors Si or Ge and compound inorganic semiconductors. However, after 1990, a few semiconductor devices using organic semiconductors and semiconducting polymers have been developed signalling the birth of a futuristic technology of polymer- electronics and molecular-electronics.

However, in this chapter, we shall be restricting ourselves to the study of inorganic semiconductors, particularly elemental semiconductors Si and Ge. The general concepts introduced here for discussing the elemental semiconductors, by-and-large, apply to most of the compound semiconductors as well.

ENERGY BANDS

For an isolated atom, the valence electrons can exist only in one of the allowed orbitals each of a sharply defined energy called energy levels. However, when two atoms are brought nearer to each other, then there are alterations in energy levels and they spread in the form of bands as shown in Figure.



Origin of energy bands in a solid

Valence Band

The energy band formed by a series of energy levels containing valence electrons is known as **valence band**. At 0 K, the electrons fill the energy levels in valence band starting from lowest one. It must be noted that

(a) this band is always filled with electrons.

(b) this is the band of maximum energy till which an electron stays in the valence shell of an atom.

(c) there is no flow of current due to electrons present in this band.

Fermi Level

The highest energy level which can be occupied by an electron in valence band at 0 K is called **Fermi Level**.

Conduction Band

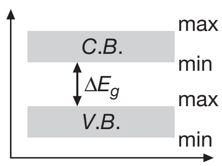
The lowest unfilled allowed energy band next to valence band is called **conduction band**. At 0 K the Fermi Level and other lower levels are occupied completely by electrons. As temperature rises, the electrons absorb energy and get excited and jump to higher levels. The electrons in higher energy levels are more free as compared to electrons in the lower energy levels. It must be noted that

(a) this band is also called an empty band of minimum energy because, after that if the energy increases, then the electrons become free and are not bound to the nucleus
(b) this band is partially filled by the electrons.
(c) The electrons in the conduction band are called the free electrons. They are able to move anywhere within the volume of the solid.
(d) Current flows due to such electrons.

FORBIDDEN ENERGY GAP (ΔE_g)

The energy gap between the conduction band and the valence band is called the Forbidden Energy Gap, given by

$$\Delta E_g = (C.B.)_{\min} - (V.B.)_{\max}$$



It must be noted that

(a) no free electron is present in forbidden energy gap.
(b) width of forbidden energy gap depends upon the nature of substance.
(c) as temperature increases, the forbidden energy gap decreases very slightly.

For Conductors, $\Delta E_g < 0.1$ eV

For Insulators, $\Delta E_g > 3$ eV

For Semi-conductors, $0.1 \text{ eV} < \Delta E_g < 3$ eV

For Germanium, $\Delta E_g = 0.7$ eV

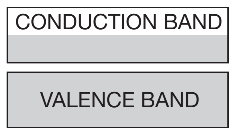
For Silicon, $\Delta E_g = 1.1$ eV

BAND THEORY OF CLASSIFICATION OF METALS, INSULATORS AND SEMICONDUCTORS

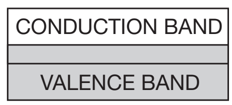
Metals

In this case we have two possibilities

(a) The valence band completely filled and conduction band partially filled and extremely small energy gap between valence band and conduction band. e.g. In sodium, conduction band is partially filled while valence band is completely filled.



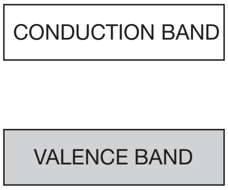
(b) The valence band is completely filled and conduction band is empty and two overlap each other. e.g. Zinc.



In both the cases it can be assumed that there is a single energy band which is partially filled and hence on application of small electric fields metals conduct electricity.

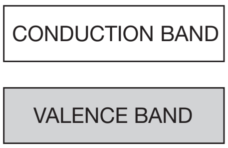
Insulators

In insulators the Forbidden Energy Gap (FEG) is quite large e.g. In case of diamond FEG is of order of 6 eV i.e. a minimum of 6 eV of energy is required by an electron to jump from completely filled valence band to conduction band. When electric field is applied across such a solid the electrons find it difficult to acquire this large amount of energy and so conduction band continues to be almost empty.



Semiconductors

The energy band structure is similar to that of insulators but in the case of semiconductors the Forbidden Energy Gap (FEG) is much smaller e.g. in case of silicon it is (FEG) 1.1 eV and because of smaller width of FEG the electrons in valence band find it comparatively easier to shift to conduction band even at room temperature.

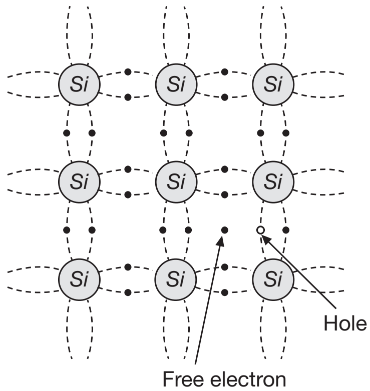


HOLES IN SEMICONDUCTORS

When an electron is removed from a covalent bond, it leaves a vacancy behind. An electron from a neighbouring atom can move into this vacancy, leaving the neighbour with a vacancy. In this way the vacancy formed is called hole (or cotter), and can travel through the material and serve as an additional current carriers. A hole is considered as a seat of positive charge, having magnitude of charge equal to that of an electron. A hole acts as a virtual charge, although it does not carry any physical charge on it. The effective mass of hole is more than electron, because the mobility of a hole is less than that of an electron.

CURRENT CARRIERS IN SEMICONDUCTORS

In a pure semiconductor, an atom behaves as if there are 8 electrons in its valence shell (because of formation of covalent bonds) and the entire material behaves as an insulator at low temperature. A semiconductor atom needs energy of order of 1.1. eV which is easily available at room temperature. Due to thermal agitation of crystal structure, electrons from a few covalent bonds come out and the bond from which an electron comes out has a vacancy called **Hole** (of positive nature).



This hole can be filled by some other electron from some other covalent bond. As the electron from some other covalent bond moves to fill this vacancy a hole is created at its place. In other words we can say that hole has shifted its position from one covalent bond to another as an electron does this in an attempt to fill the hole.

Conceptual Note(s)

Since a hole moves in a direction opposite to that of an electron so a hole is treated as a positive charged carrier.

So, at room temperature a pure semiconductor will have electrons and holes wandering in random directions. These electrons and holes are called **Intrinsic Carriers** and such a semiconductor is called an **Intrinsic Type Semiconductor**.

As a crystal is electrically neutral, the number density of free electrons n_e will be equal to number density of holes n_h. So, in an intrinsic semiconductor, $n_e = n_h = n_i$.

The fraction of electrons of valence band present in conduction band at temperature T is

$$f \propto \exp\left(\frac{-E_g}{k_B T}\right).$$

where E_g is the forbidden energy gap and k_B is Boltzmann constant.

INCREASING THE CONDUCTIVITY OF A SEMICONDUCTOR: DOPING PROCESS

A pure semiconductor at room temperature possesses free electrons and holes but their number is so small that conductivity offered by the pure semiconductor cannot be of any practical use. By the addition of impurities to the pure semiconductor in a very small ratio $(1:10^6)$, the conductivity of a Si-crystal $($or Ge-crystal$)$ can be remarkably improved.

The process of adding impurity to a pure semi conductor crystal $(Si$ or Ge-crystal$)$ so as to improve its conductivity, is called **doping**.

Generally, doping of few ppm i.e. 1 impurity atom per 10^6 semiconductor atoms is done. Also, the size of the dopant and the semiconductor atoms must be comparable or nearly same so that the dopant does not distort the original semiconductor lattice.

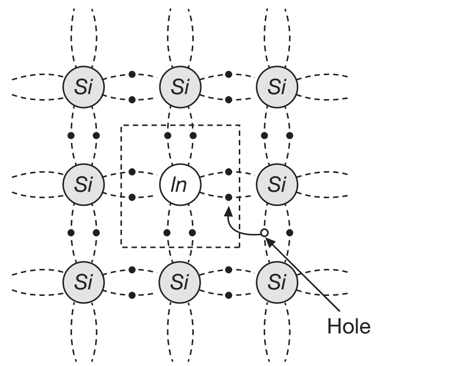
The impurity atoms are of two types:

(a) **Pentavalent impurity atoms** i.e. atoms having 5 valence electrons such as Antimony (Sb) or Arsenic (As). Such atoms, when added to a pure semiconductor, produce excess of free electrons i.e. donate electrons to the semiconductor. For this reason, pentavalent impurity atoms are called **Donor Impurity** atoms. The semiconductor so produced is called n-type **Extrinsic Semiconductor.**

(b) **Trivalent impurity atoms** i.e. atoms having 3 valence electrons such as Indium (In) or Gallium (Ga). Such atoms on being added to a pure semiconductor, instead of producing free electrons, accept electrons from the semiconductor. For this reason, trivalent impurity atoms are called **Acceptor Impurity** atoms. The semiconductor so produced is called p-type **Extrinsic Semiconductor.**

p-TYPE (EXTRINSIC) SEMICONDUCTOR

Consider a Silicon crystal to which a trivalent impurity say Indium is added. The four silicon atoms surrounding the In atom, can share one electron each with the In atom which has got three valence electrons. In an attempt to have 8 electrons in valence shell, the In atom borrows one of the nearby covalent bonds of one electron. Thus, the valence shell of the In atom possesses 8 electrons but a hole is created in the covalent bond from which electron has been borrowed.

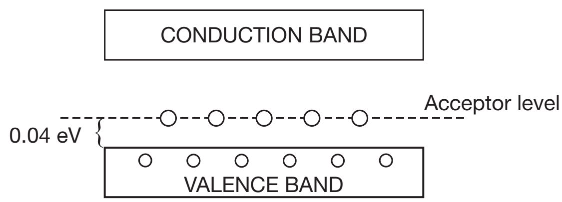


So, for every trivalent impurity atom added, an extra hole will be created. As the trivalent impurity atoms accept electrons from the silicon crystal, it is called **acceptor impurity**.

The Si-crystal so obtained is called p-type as it contains excess free holes. Each hole is equivalent to positive charge. The holes so created are extrinsic carriers and the p-type Si-crystal obtained is called p-type extrinsic semiconductor.

Since the pure Si-crystal also possesses a few electrons and holes (by way of thermal agitation), therefore, the p-type Si-crystal will have a large number of holes (majority charge carriers) and a small number of electrons (minority charge carriers).

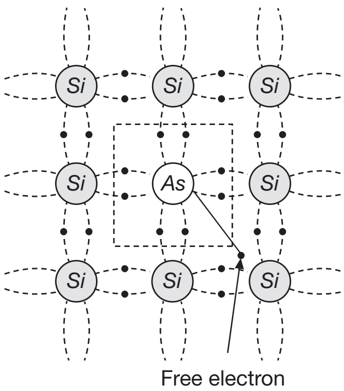
In the extrinsic p-type Si-crystal, the hole produced revolves round the nucleus of the In atom. Since the hole may be treated as a particle of same mass as electron but having an equal positive charge so, it possesses a small positive energy of the order of 0.04 eV. Such holes create an **acceptor energy level** (of value 0.04 eV) just above the top of the valence band. The electrons from valence band can raise themselves to the acceptor energy level by absorbing thermal energy at room temperature and in turn create holes in the valence band.



Number density of valence band holes (n_h) in p-type semiconductor is approximately equal to that of the acceptor atoms (N_a) and is very large as compared to the number density of conduction band electrons (N_e). Hence, $n_h \approx N_a >> n_e$.

n-TYPE (EXTRINSIC) SEMICONDUCTOR

When the arsenic impurity atoms are added to the silicon crystal in a small ratio $(1:10^6)$, its atoms replace the silicon atoms here and there. The four electrons out of the five valence electrons of As atom take part in covalent bonding with four silicon atoms surrounding it. The fifth electron is set free. Obviously, the extra free electrons created in the crystal will be as many as the number of the pentavalent impurity atoms added.



Since the pentavalent impurity increases the number of free electrons, it is called **donor impurity**. The silicon crystal so obtained is termed as n-type Si crystal. The electrons so set free in the silicon crystal are called **Extrinsic Carriers** and the n-type Si crystal is called n-type **Extrinsic Semiconductor**.

Due to thermal agitation, the pure Si crystal possesses a new electrons and holes. So, n-type Si crystal will have a large number of free electrons (majority charge carriers) and a small number of holes (minority charge carriers).

In the extrinsic n-type semiconductor, the fifth electron of the As atom revolves around the donor atom inside the Si crystal. As dielectric constant of silicon is very high, it is bound to the donor atom with a very small amount of energy, which is of the order of 0.045 eV. In terms of valence and conduction band, one can think that all such electrons (extrinsic carriers) create a donor energy level just below (0.045 eV) the conduction band as shown in figure. As the energy gap between donor energy level and the conduction band is very small, the electrons can easily raise themselves to conduction band even at room temperature. Hence, the conductivity of n-type extrinsic semiconductor is remarkably increased.

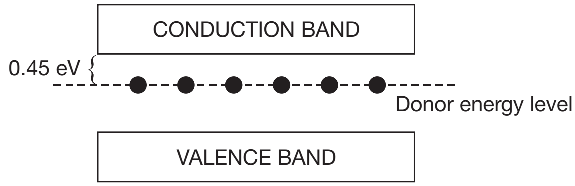


ILLUSTRATION 1

A p-type semiconductor has acceptor level 50 meV above the valence band. Find the maximum wavelength of light that can create a hole.

SOLUTION

To create a hole, an electron of valence band has to be excited into one of the acceptor levels which are 50 meV above the valence band. Hence a minimum of 50 meV energy is needed to create a hole. Since

$$E_{\min} = \frac{hc}{\lambda_{\max}}$$

$$\Rightarrow \quad \lambda_{\max} = \frac{hc}{E_{\min}} = \frac{6.6\times10^{-34}\times3\times10^{8}}{50\times1.6\times10^{-22}}$$

$$\Rightarrow \quad \lambda_{\max} = 2.47\times10^{-5}\ \text{m}$$

RELATION BETWEEN THE NUMBER DENSITY OF INTRINSIC AND EXTRINSIC CHARGE CARRIERS

In a doped or extrinsic semiconductor, the number density of the conduction band (n_e) and the number density of holes in the valence band (n_h) differ from that in a pure semiconductor. If n_i is the number density of electrons in conduction band or the number density of holes in valence band in a pure semiconductor, then

$$n_e n_h = n_i^2$$

This relation is also called as Mass-Action Law.

In n-type (extrinsic) semiconductor, the number density of electrons in conduction band is approximately equal to that of donor atoms and very large as compared to number density of holes in valence band. Thus,

$$n_e \approx N_d >> n_h,$$

where N_d represents the number density of donor atoms.

ILLUSTRATION 2

A semiconductor has equal electron atom hole concentration of $6\times10^8\ \text{m}^{-3}$. On doping with certain impurity, electron concentration increases to $9\times10^{12}\ \text{m}^{-3}$.

(i) Identify the new semiconductor obtained after doping.
(ii) Calculate the new hole concentration.
(iii) How does the energy gap vary with doping?

SOLUTION

(i) Since the electron concentration increases after doping, so the new semiconductor is an n type semiconductor.

(ii) Since, $n_e n_h = n_i^2$

where, $n_i = 6\times10^8\ \text{m}^{-3}$, $n_e = 9\times10^{12}\ \text{m}^{-3}$

$$\Rightarrow \quad n_h = \frac{n_i^2}{n_e} = \frac{(6\times10^8)^2}{9\times10^{12}}$$

$$\Rightarrow \quad n_h = 4\times10^4\ \text{m}^{-3}$$

(iii) Doping decreases the energy gap.

ILLUSTRATION 3

A germanium specimen is doped with aluminium. The concentration of acceptor atoms is nearly $10^{21}\ \text{m}^{-3}$. Given that the intrinsic concentration of electron hole pairs is about $10^{19}\ \text{m}^{-3}$, then calculate the concentration of electrons in the specimen.

SOLUTION

According to Mass-Action Law, we have

$$n_i^2 = n_h n_e$$

where, $n_i \approx 10^{19}\ \text{m}^{-3}$ is the number density of intrinsic charge carriers and $n_h \approx 10^{21}\ \text{m}^{-3}$ is the number density of acceptor atoms.

$$\Rightarrow \quad n_e = \frac{n_i^2}{n_h} = \frac{(10^{19})^2}{(10^{21})} = 10^{17}\ \text{m}^{-3}$$

ILLUSTRATION 4

In a pure germanium sample taken at a temperature of $300\ \text{K}$, concentration of electron-hole pairs is $7\times10^{15}\ \text{cm}^{-3}$. If one antimony atom is doped into germanium for 10^7 germanium atoms and assuming that only half of the impurity atoms contribute electrons to the conduction band, then calculate the factor by which the number of charge carriers increase due to doping. Assume that the number density of the germanium atoms is $5\times10^{28}\ \text{m}^{-3}$.

SOLUTION

The electron-hole pair concentration in a pure semiconductor is given to be $7\times10^{15}\ \text{m}^{-3}$. Total number density of charge carriers will be

$$n_{\text{initial}} = n_h + n_e = 14\times10^{15}\ \text{m}^{-3}$$

After doping is done with the donor impurity, we have

$$N_D = \frac{5\times10^{28}}{10^7} = 5\times10^{21}\ \text{m}^{-3}$$

and $n_e = \dfrac{N_D}{2} = 2.5\times10^{21}\ \text{m}^{-3}$

So, $n_{\text{final}} = n_h + n_e$

Since, we have $n_e \gg n_h$, so

$$n_{\text{final}} \approx n_e \approx 2.5\times10^{21}\ \text{m}^{-3}$$

The factor (f) by which the number of charge carriers increase due to doping is given by

$$f = \frac{n_{\text{final}} - n_{\text{initial}}}{n_{\text{initial}}}$$

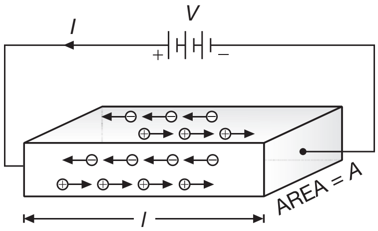
$$\Rightarrow \quad f = \frac{2.5\times10^{21} - 14\times10^{15}}{14\times10^{15}}$$

$$\Rightarrow \quad f \approx \frac{2.5\times10^{21}}{14\times10^{15}} = 1.8\times10^5$$

ELECTRICAL RESISTIVITY OF SEMICONDUCTORS

Consider a block of semiconductor of length l area of cross section A having n_e as number density of electrons and n_h as number density of holes. Then total current

$$I = I_e + I_h$$



Further $I_e = neAv_e$ and $I_h = n_h eAv_h$, where v_e = Drift velocity of electrons and v_h = drift velocity of holes.

$$\Rightarrow \quad I = n_e eAv_e + n_h eAv_h$$

$$\Rightarrow \quad \frac{V}{R} = eA(n_e v_e + n_h v_h)$$

Since $V = El$ and $R = \frac{\rho l}{A}$

$$\Rightarrow \quad \frac{El}{\frac{\rho l}{A}} = eA(n_e v_e + n_h v_h)$$

$$\Rightarrow \quad \frac{1}{\rho} = \sigma = e\left\{n_e\left(\frac{v_e}{E}\right) + n_h\left(\frac{v_h}{E}\right)\right\}$$

$$\Rightarrow \quad \sigma = e(n_e \mu_e + n_h \mu_h)$$

where $\mu_e = \frac{v_e}{E}$ = Drift velocity of electrons per unit electric field also called **Electron Mobility**.

And $\mu_h = \frac{v_h}{E}$ = Drift velocity of holes per unit electric field also called **Hole Mobility**.

Conceptual Note(s)

Following points may be noted about the behaviour of semiconductors.

(a) At low temperature, a pure semiconductor behaves as an insulator. It becomes slightly conducting at room temperature due to thermal agitation process as a result of which a few electrons in the valence band acquire energy greater than the forbidden energy gap and move to the conduction band.

(b) It is very difficult to obtain an extremely pure semiconductor. In practice, an intrinsic semiconductor is that in which the concentration of impurity atoms is less than the concentration of intrinsic carriers.

(c) Extremely small doping can drastically change the conductivity of an intrinsic semiconductor.

(d) Electron mobility is greater than the hole mobility in semiconductors.

(e) At low temperature the semiconductors are insulators but become slightly conducting at room temperature. Thus, unlike metals, the resistance of semiconductors decreases with increase of temperature. Hence, they (semiconductors) possess a negative temperature coefficient of resistance (TCR).

(f) If light of energy greater than forbidden energy gap is incident on an intrinsic semiconductor, the electrons from the valence band move to the conduction band. Thus, electron and hole pairs are created. Due to increase in concentration of carriers, the conductivity of semiconductor increases. This property of semiconductors is called **photoconductivity.** Semiconductors with large photoconductivity are used as **Light Dependent Resistors (LDR)** which are commonly used in automatic light control circuits.

ILLUSTRATION 5

An n-type semiconductor of conductivity $6\ \Omega^{-1}\text{cm}^{-1}$ is to be made from an intrinsic germanium semiconductor. Calculate the number density of donor atoms required, if the mobility of electrons in n-type semiconductor is given to be $3850\ \text{cm}^2\text{V}^{-1}\text{s}^{-1}$.

SOLUTION

In n-type semiconductor, $n_e \gg n_h$

Let number density of donor atoms, $N_0 = n_e$

Conductivity of semiconductor is given by

$$\sigma = n_e e \mu_e$$

On substituting the values, we get

$$6 \times 10^2 = n_e \times 1.6 \times 10^{-19} \times 3850 \times 10^{-4}$$

$$\Rightarrow \quad n_e = 9.7 \times 10^{21}\ \text{m}^{-3}$$

ILLUSTRATION 6

Calculate the conductivity of pure silicon crystal at $300\ \text{K}$ temperature, if the electron hole pairs per cm^3 are 1.072×10^{10} at this temperature and mobilities are $\mu_h = 1350\ \text{cm}^2\text{V}^{-1}\text{s}^{-1}$ and $\mu_e = 480\ \text{cm}^2\text{V}^{-1}\text{s}^{-1}$.

SOLUTION

Conductivity of intrinsic i.e. pure silicon semiconductor is given by

$$\sigma = n_i e\mu_e + n_i e\mu_h = n_i e(\mu_e + \mu_h)$$

where n_i is the number density of intrinsic charge carriers, μ_e and μ_h are the mobility of electron and holes respectively.

Given that, $n_i = 1.072\times10^{10}\ \text{cm}^{-3}$

$$\mu_h = 1350\ \text{cm}^2\ \text{volt}^{-1}\text{s}^{-1}$$

$$\mu_c = 480\ \text{cm}^2\ \text{volt}^{-1}\text{s}^{-1}$$

$$\Rightarrow \quad \sigma = (1.072\times10^{10})(1.6\times10^{-19})(1350+480)$$

$$\Rightarrow \quad \sigma = 3.14\times10^{-6}\ \Omega^{-1}\text{cm}^{-1}$$

ILLUSTRATION 7

The energy gap of pure silicon semiconductor is $1.12\ \text{eV}$. The mobilities of electrons and holes are respectively $0.140\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$ and $0.050\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$. They are independent of temperature. The intrinsic carrier concentration is given by $n_i = n_0 e^{-\frac{E_g}{2k_BT}}$ where n_0 is a constant, E_g the gap width and k_B is the Boltzmann's constant whose value is $1.38\times10^{-23}\ \text{JK}^{-1}$. Calculate the ratio of the electrical conductivities of silicon at temperatures of $400\ \text{K}$ and $200\ \text{K}$.

SOLUTION

Conductivity of semiconductor is given by

$$\sigma = n_e e\,\mu_e + n_h e\,\mu_h$$

where, $n_e = n_h = n_i$ (for a pure semiconductor)

$$\Rightarrow \quad \sigma = n_i e(\mu_e + \mu_h) = e(\mu_e + \mu_h)n_i$$

$$\Rightarrow \quad \sigma = e(\mu_e + \mu_h)n_0 e^{-\frac{E_g}{2k_BT}}$$

$$\Rightarrow \quad \frac{\sigma_{400}}{\sigma_{200}} = \frac{e^{\frac{-E_g}{2k_B(400)}}}{e^{\frac{-E_g}{2k_B(200)}}} = e^{\frac{E_g}{800k_B}}$$

$$\Rightarrow \quad \frac{\sigma_{400}}{\sigma_{200}} = e^{\frac{1.12\times1.6\times10^{-19}}{800\times1.38\times10^{23}}} = e^{16.23}$$

ILLUSTRATION 8

Calculate the resistivity of a sample in which 10^{19} atoms of phosphorous are added per cubic metre. Take the resistivity of pure silicon as 3000 ohm-metre and the mobilities of electrons and holes as $0.15\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$ and $0.030\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$ respectively.

SOLUTION

The conductivity of a pure silicon is given by

$$\sigma_i = n_i e(\mu_e + \mu_h)$$

Since the resistivity is reciprocal of conductivity, so

$$\rho_i = \frac{1}{\sigma_i} = \frac{1}{n_i e(\mu_e + \mu_h)}$$

$$\Rightarrow \quad n_i = \frac{1}{\rho_i e(\mu_e + \mu_h)}$$

$$\Rightarrow \quad n_i = \frac{1}{3000\times1.6\times10^{-19}(0.15+0.030)}$$

$$\Rightarrow \quad n_i = 1.157\times10^{16}\ \text{m}^{-3}$$

When 10^{19} atoms of phosphorus (donor atoms) are added per m^3, we have

$$n_e \gg n_i \text{ or } n_e \gg n_h$$

$$\Rightarrow \quad n_e = 10^{19}$$

$$\Rightarrow \quad \rho = \frac{1}{n_e e\,\mu_e} = \frac{1}{10^{19}\times1.6\times10^{-19}\times0.15}$$

$$\Rightarrow \quad \rho = 4.17\ \text{ohm metre}$$

p-n JUNCTION

A *p-n* junction is the basic building block of many semiconductor devices like diodes, transistor, etc. A clear understanding of the junction behaviour is important to analyse the working of other semiconductor devices. Let us now understand how a junction is formed and how the junction behaves under the influence of external applied voltage (also called bias).

p-n Junction Formation

Consider a thin p-type silicon $(p\text{-}Si)$ semiconductor wafer. By the precise addition of a small quantity of pentavalent impurity to this p-type silicon $(p\text{-}Si)$

semiconductor, a part of the *p-Si* wafer can be converted into n-type silicon $(\text{n-}Si)$. The wafer now contains p-region and n-region and a metallurgical junction between p and n-region.

Conceptual Note(s)

A p-n junction cannot be made just by placing a p-type semiconductor in close contact with n-type semiconductor. The two separate semiconductors cannot have a continuous contact at the atomic level. The junction will behave as a discontinuity for the flowing charge carries. So, both acceptor and donor impurities must be grown in a single Si or Ge crystal.

Diffusion Current, Drift Current and Barrier Potential

It is observed that two important processes occur during the formation of a p-n junction

1. Diffusion Process
2. Drift Process

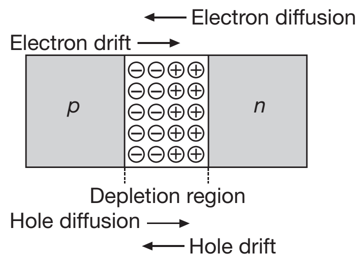
Since we know that in an n-type semiconductor, the concentration of electrons (number of electrons per unit volume) is more compared to the concentration of holes. Similarly, in a p-type semiconductor, the concentration of holes is more than the concentration of electrons. So, when the pn junction is formed, then due to the concentration gradient across p and n-sides, the holes diffuse from p-side to n-side $(p \to n)$ and electrons diffuse from n-side to p-side $(n \to p)$. *This motion of charge carriers due to diffusion from one region to the other gives rise to diffusion current* i_{df} *across the junction.*

Now, when an electron diffuses from $n \to p$, it leaves behind an ionised donor on n-side. This ionised donor (positive charge) is immobile as it is bonded to the surrounding atoms. As the electrons continue to diffuse from $n \to p$, a layer of positive charge (or positive space-charge region) on n-side of the junction is developed.

Similarly, when a hole diffuses from $p \to n$ due to the concentration gradient, it leaves behind an ionised acceptor (negative charge) which is immobile. As the holes continue to diffuse, a layer of negative charge (or negative space-charge region) on the *p*-side of the junction is developed.

This space-charge region on either side of the junction together is known as depletion region as the electrons and holes taking part in the initial movement across the junction depleted the region of its free charges (shown in figure). The thickness of depletion region is of the order of one-tenth of a micrometre.

Due to the positive space-charge region on *n*-side of the junction and negative space charge region on *p*-side of the junction, an electric field directed from positive charge towards negative charge develops. Because of this field, an electron on p-side of the junction moves to n-side and a hole on n-side of the junction moves to p-side. *This motion of charge carriers due to the electric field is called drift. Hence, a drift current* i_{dr}*, which is opposite in direction to the diffusion current (shown in figure) starts.*



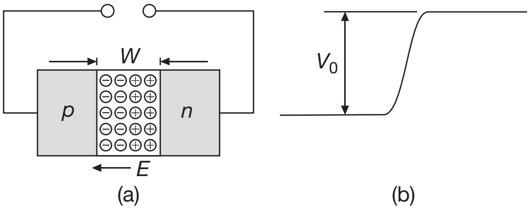
p-n junction formation process

Initially, diffusion current is large and drift current is small. As the diffusion process continues, the space-charge regions on either side of the junction extend, thus increasing the electric field strength and hence the drift current.

This process continues until the diffusion current equals the drift current. Thus, a p-n junction is formed. So, in this case

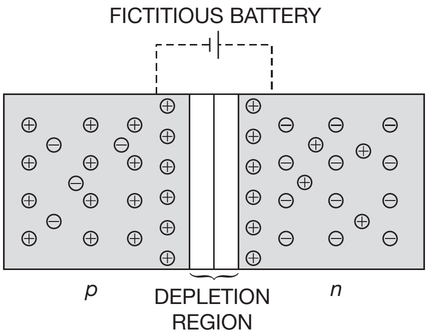
$$i_{\text{net}} = i_{df} + i_{dr} = 0$$

Since, in a p-n junction, under equilibrium there is no net current, so the loss of electrons from the n-region and the gain of electron by the p-region causes a difference of potential across the junction of the two regions. The polarity of this potential is such as to oppose further flow of carriers so that a condition of equilibrium exists. Figure shows the p-n junction at equilibrium and the potential across the junction.



(a) Diode under equilibrium ($V = 0$)
(b) Barrier potential under no bias.

The n-material has lost electrons, and p-material has acquired electrons. The n-material is thus positive relative to the p-material. Since this potential tends to prevent the movement of electron from the n-region into the p-region, it is often called a barrier potential. As a result of this it appears to us as if some fictitious battery is applied across the junction with its negative terminal connected to p region and positive terminal to n region.



The potential difference developed across the junction due to migration of majority charge carriers is called **potential barrier**. The potential barrier is of 0.7 V for Silicon and 0.3 V for Germanium.

On the average the potential barrier in pn junction is of the order of 0.5 V and the width of depletion region is of the order of 10^{-6} m, so the barrier electric field is

$$E = \frac{V}{d} = \frac{0.5}{10^{-6}} = 5\times10^{5}\text{ Vm}^{-1}$$

ILLUSTRATION 9

A potential barrier of 0.5 V exists across a p-n junction.

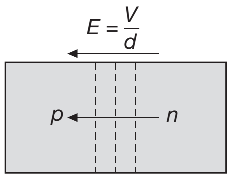
(i) If the depletion region is $5\times10^{-7}\text{ m}$ wide. Calculate the intensity of the electric field in the region.

(ii) If an electron having a speed of $5\times10^{5}\text{ ms}^{-1}$ approaches the junction from the n-side, calculate the speed with which it enters the p-side.

SOLUTION

(i) Width of depletion layer, $d = 5\times10^{-7}\text{ m}$

Electric field, $E_B = \dfrac{V}{d} = \dfrac{0.5\text{ V}}{5\times10^{-7}} = 10^{6}\text{ volt m}^{-1}$



(ii) According to the Work-Energy Theorem, we have

$$W = \Delta K$$

$$\Rightarrow \quad eV = \frac{1}{2}m_e v_f^2 - \frac{1}{2}m_e v_i^2$$

$$\Rightarrow \quad \frac{1}{2}m_e v_i^2 = eV + \frac{1}{2}m_e v_f^2$$

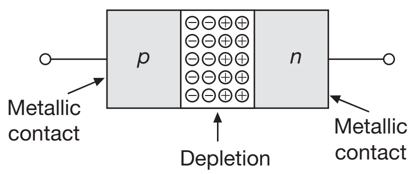
$$\Rightarrow \quad v_f = \sqrt{\frac{m_e v_i^2 - 2\text{ eV}}{m_e}}$$

$$\Rightarrow \quad v_f = \sqrt{\frac{9\times10^{-31}\times\left(5\times10^{5}\right)^2 - 2\times1.6\times10^{-19}}{9\times10^{-31}}}$$

$$\Rightarrow \quad v_f = 2.7\times10^{5}\text{ ms}^{-1}$$

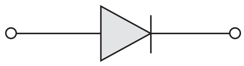
SEMICONDUCTOR DIODE

A semiconductor diode is basically a p-n junction with metallic contacts provided at the ends for the application of an external voltage as shown in Figure.



Semiconductor diode

It is a two terminal device. A p-n junction diode is symbolically represented as shown in Figure.



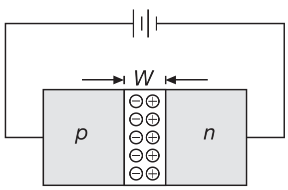
Symbol for *p-n* junction diode.

The direction of arrow indicates the conventional direction of current (when the diode is under forward bias). The equilibrium barrier potential can be altered by applying an external voltage V across the diode. In both the cases the diode is not connected across a voltage i.e. the diode is not biased and hence is in equilibrium i.e. mathematically

$$i_{\text{net}} = i_{df} + i_{dr} = 0$$

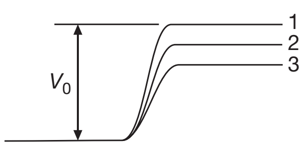
p-n JUNCTION DIODE UNDER FORWARD BIAS

When an external voltage V is applied across a semiconductor diode (having barrier potential V_B) such that *p*-side is connected to the positive terminal of the battery and *n*-side to the negative terminal of the battery, as shown in Figure, it is said to be *Forward Biased*.



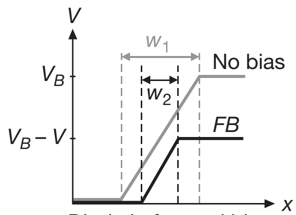
p-n junction diode under forward bias

The applied voltage mostly drops across the depletion region and the voltage drop across the *p*-side and *n*-side of the junction is negligible. This is because the resistance of the depletion region (a region where there are no charges) is very high compared to the resistance of *n*-side and *p*-side. The direction of the applied voltage (V) is opposite to that of the barrier potential (V_B). As a result, the depletion layer width decreases and the barrier height is reduced as shown in Figure.



Barrier potential (1) Without battery
(2) Low battery voltage, and
(3) High voltage battery

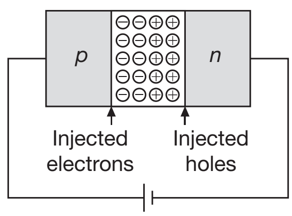
The effective barrier height under forward bias is $|V_B - V|$. As the applied voltage V is increased, the barrier potential decreases and width of depletion region also decreases.



Diode in forward bias
Width of depletion region is w_1 without bias and w_2 in forward bias ($w_2 < w_1$)

When the applied voltage is small, the barrier potential will be reduced only slightly below the equilibrium value, and only a small number of carriers in the material (only the ones which happen to be in the uppermost energy levels) will possess enough energy to cross the junction. Hence, the current will be small. However, when the applied voltage is significantly increased, then the barrier height will be reduced and more number of charge carriers will possess energy to cross the junction and hence the current increases.

Due to the applied voltage, the electrons from *n*-side cross the depletion region to reach the *p*-side (where they are minority carries). Similarly, holes from *p*-side cross the junction and reach the *n*-side (where they are minority carries). *This process under forward bias is called as minority carrier injection*. So, on each side of the junction boundary, the minority charge carrier concentration increases significantly compared to the locations that are far from the junction. Due to this concentration gradient, the injected electrons on *p*-side diffuse from the junction edge of *p*-side to the other end of *p*-side. Similarly, the injected holes on *n*-side diffuse from the junction edge of *n*-side to the other end of *n* -side (shown in figure).



Forward bias minority carrier injection

Hence a current is obtained due to this motion of the charge carriers. *The total diode forward current is sum of hole diffusion current and conventional current due to electron diffusion.* The magnitude of this forward current is usually in mA.

Conceptual Note(s)

When the junction diode is in forward bias, then

(a) width of depletion layer decreases.

(b) forward biasing resistance offered is $R_{Forward} \approx 10\ \Omega - 25\ \Omega$

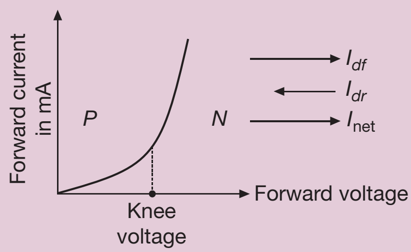
(c) forward bias opposes the potential barrier and for $V > V_B$, a forward current is set up across the junction.

(d) the current is given by $i = i_s\left[\exp\left(\frac{eV}{k_BT}\right) - 1\right]$

where, i_s is the saturation current, $e = 1.6 \times 10^{-19}$ C, k_B is the Boltzmann's constant, V is the applied voltage and T is the absolute temperature.

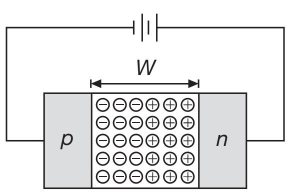
(e) the cut-in voltage (also called as knee voltage) is the voltage at which the current starts to increase rapidly.
For Ge it is 0.3 V and for Si it is 0.7 V.

(f)



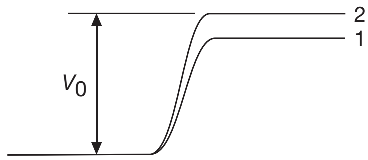
p-n JUNCTION DIODE UNDER REVERSE BIAS

When an external voltage V is applied across the diode such that n-side is connected to positive of the battery and p-side is connected to the negative of the battery, then the diode is said to be *Reverse Biased* as shown in Figure.



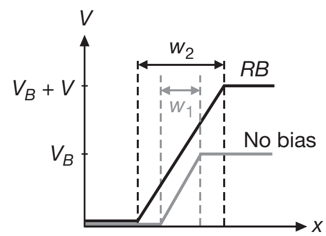
Diode under reverse bias

The applied voltage mostly drops across the depletion region. The nature of the applied voltage is same as the nature of barrier potential or we can say that the applied voltage helps the barrier potential. As a result of this, the barrier height increases and the depletion region widens due to the change in the electric field as shown in Figure.



Barrier potential under reverse bias

The effective barrier height under reverse bias is $(V_B + V)$. As the applied voltage V is increased, the barrier potential increases and width of depletion region also inceases.



Diode in reverse bias

Width of depletion region is w_1 without bias and w_2 in reverse bias ($w_2 > w_1$)

Due to this increased height of the barrier potential, the flow of electrons from $n \to p$ and holes from $p \to n$ region is suppressed. Thus, diffusion current, decreases enormously compared to the diode under forward bias.

The electric field direction at the junction is such that whenever the electrons on p-side or holes on n-side, in their random motion, come close to the junction, then they are swept to its majority zone. This drift of carriers gives rise to current called drift current which is of the order of a few μA. This is very low because this current is due to the motion of carriers from the minority side to the majority side across the junction. The drift current is also there under forward bias but it is negligible (μA) when compared with current due to injected carriers which is usually in mA.

Conceptual Note(s)

When the junction diode is in reverse bias, then

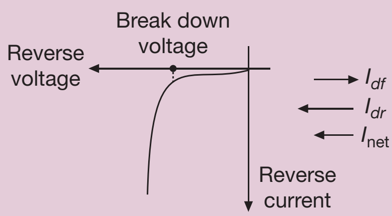
(a) width of depletion layer increases

(b) reverse biasing resistance offered is $R_{\text{Reverse}} \approx 10^5\ \Omega$

(c) reverse bias supports the potential barrier and no current flows across the junction due to the diffusion of the majority carriers. (A very small reverse currents may exist in the circuit due to the drifting of minority carriers across the junction)

(d) the break down voltage (also called Reverse voltage) is the voltage at which break down of semiconductor occurs.
For *Ge* it is 25 V and for *Si* it is 35 V.

(e)

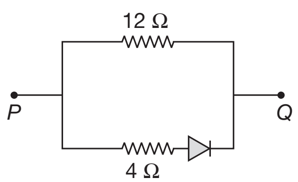


(f) The diode reverse current is not very much dependent on the applied voltage. Even a small voltage is sufficient enough to sweep the minority charge carriers from one side of the junction to the other side of the junction. So, the reverse current is not limited by the magnitude of the applied voltage but is limited because of the small concentration of the minority charge carriers on either side of the junction.

ILLUSTRATION 10

Calculate the net resistance of the network shown in the figure between the points P and Q when

(i) $V_P > V_Q$ **(ii)** $V_P < V_Q$



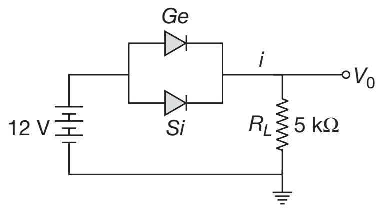
SOLUTION

(i) When $V_P > V_Q$, the diode is forward biased. Hence, the resistance of diode will be taken as zero. So, the net resistance is $\frac{4 \times 12}{4+12}\ \Omega = 3\ \Omega$.

(ii) When $V_P < V_Q$, the diode is reverse biased. Hence, there will be no current in the diode branch. So, the net resistance is $12\ \Omega$.

ILLUSTRATION 11

Calculate the value of V_0 and i if the silicon and germanium diode start conducting at 0.7 V and 0.3 V, respectively. If the *Ge* diode connection is now reversed, what will be the new values of V_0 and i?



SOLUTION

Here we must note that the germanium diode will start conducting before the silicon diode starts conducting. The effective forward voltage across the germanium diode is given by

$$V_0 = (12 - 0.3)\ \text{V} = 11.7\ \text{V}$$

This will appear as the output voltage across the load i.e.,

$$V_0 = 11.7\ \text{V}$$

So, the current through load resistance R_L is given by

$$i = \frac{11.7}{5 \times 10^3}\ \text{A} = 2.34\ \text{mA}$$

On reversing the connection of germanium diode, it will be reverse biased and will not conduct. So, only the silicon diode will conduct and then the output voltage across the laod is

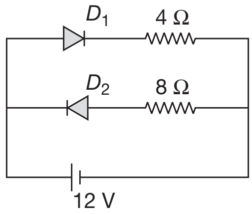
$$V_0 = (12 - 0.7)\ \text{V} = 11.3\ \text{volt}$$

So, the current through load resistance R_L is given by

$$i = \frac{11.3}{5 \times 10^3}\ \text{A} = 2.26\ \text{mA}$$

ILLUSTRATION 12

Calculate the current passing through the $4\ \Omega$ and $8\ \Omega$ resistors in the circuit shown in Figure.



SOLUTION

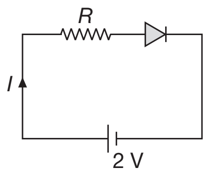
In the given circuit, diode D_1 is forward biased and D_2 is reverse biased. Hence, D_1 will conduct but D_2 will not.

Therefore, current through $8\ \Omega$ resistance will be zero whereas the current through $4\ \Omega$ resistance will be

$$I_{4\,\Omega} = \frac{12}{4} = 3\ \text{A}$$

ILLUSTRATION 13

The diode used in the circuit shown in the figure has a constant voltage drop of 0.5 V at all current and a maximum power rating of 200 mW. What should be the value of the resistor R, connected in series with the diode, for obtaining maximum current?



SOLUTION

Current through diode (or circuit) is given by

$$I = \frac{\text{Power}}{\text{Voltage}}$$

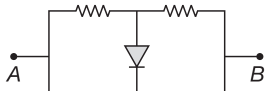
$$\Rightarrow \quad I = \frac{200 \times 10^{-3}\ \text{W}}{0.5\ \text{V}} = 0.4\ \text{A}$$

$$\text{Since, } R = \frac{\Delta V}{\Delta I} = \frac{\text{Net voltage}}{\text{Current}}$$

$$\Rightarrow \quad R = \frac{2 - 0.5}{0.4} = 3.75\ \Omega$$

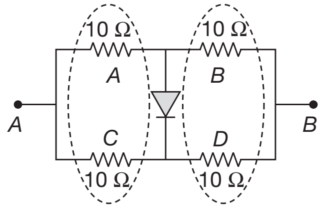
ILLUSTRATION 14

Calculate the net resistance between two points A and B, if the value of each resistance shown in the Figure is $10\ \Omega$.



SOLUTION

The given circuit is in the form of a wheatstone bridge.



Since we observe that

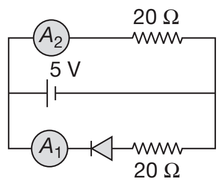
$$\frac{P}{Q} = \frac{R}{S}$$

Hence no current will from through the diode and hence it will not offer any resistance. So, the net resistance between A and B will be

$$R_{AB} = \frac{(10)(10)}{10+10} + \frac{(10)(10)}{10+10} = 20\ \Omega$$

ILLUSTRATION 15

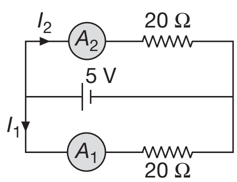
Two ammeters A_1 and A_2 are connected across a diode and resistor respectively as shown in the Figure.



Calculate the amount of current flowing through these two ammeters. Ignore the resistances of the meters.

SOLUTION

Let ae current I_1 flows across a diode and I_2 flows across a resistor as shown in the Figure.



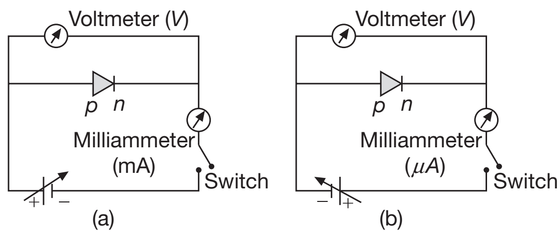
Since the diode is in reverse biased condition, so it will not conduct. Hence, ammeter A_1 will not show any reading.
So, the ammeter A_2 will show a reading I_2 given by

$$I_2 = \frac{V}{R} = \frac{5}{20} = 0.25 \text{ A}$$

V-I CHARACTERISTICS OF A *p-n* JUNCTION DIODE

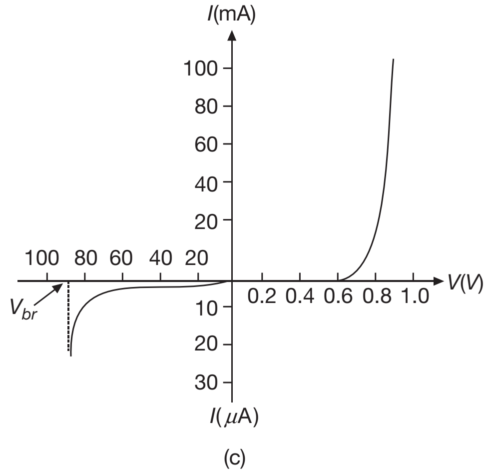
The current under reverse bias is essentially voltage independent upto a critical reverse bias voltage, known as breakdown voltage (V_{br}). When $V = V_{br}$, the diode reverse current increases sharply. Even a slight increase in the bias voltage causes large change in the current. If the reverse current is not limited by an external circuit below the rated value (specified by the manufacturer) the *p-n* junction will get destroyed. Once it exceeds the rated value, the diode gets destroyed due to overheating. This can happen even for the diode under forward bias, if the forward current exceeds the rated value.

The circuit arrangement for studying the V-I characteristics of a diode, (i.e., the variation of current as a function of applied voltage) are shown in figure.



The battery is connected to the diode through a potentiometer (or rheostat) so that the applied voltage to the diode can be changed. Note that in forward bias measurement, we use a milliammeter since the expected current is large (as explained in the earlier section) while a microammeter is used in reverse bias to measure the current.

For different values of voltages, the value of the current is noted. A graph between V and I is obtained as in Figure.



Experimental circuit arrangement for studying V-I characteristics of a *p-n* junction diode (a) in forward bias, (b) in reverse bias. (c) Typical V-I characteristics of a silicon diode.

We observe from the graph that in forward bias, the current first increases very slowly, almost negligibly, till the voltage across the diode crosses a certain value. After the characteristic voltage, the diode current increases significantly (exponentially), even for a very small increase in the diode bias voltage. *This voltage is called the Threshold Voltage or Cut-In Voltage or Knee Voltage* (≈ 0.3 V for germanium diode and ≈ 0.7 V for silicon diode).

The above discussion shows that the *p-n* junction diode primarily allows the flow of current only in one direction (forward bias). The forward bias resistance is low as compared to the reverse bias resistance.

DYNAMIC RESISTANCE OF A DIODE

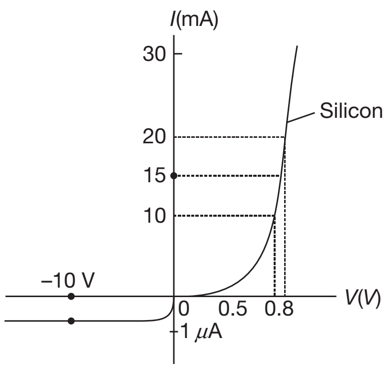
It is observed that both the forward bias and the reverse bias characteristics of the diode do not obey Ohm's Law, because the resistance offered by the junction diode depends on the applied voltage. Therefore, for diodes, we define a quantity called *dynamic resistance* (r_d) which is the ratio of small change in voltage ΔV to a small change in current ΔI. Mathematically

$$r_d = \frac{\Delta V}{\Delta I}$$

The region of the V-I graph where the dynamic resistance is almost independent of the applied voltage is called the linear region of the junction diode.

ILLUSTRATION 16

The V-I characteristic of a silicon diode is shown in Figure. Calculate the resistance of the diode at $I = 15$ mA and at $V = -10$ V.



SOLUTION

As seen from the figure, the diode current varies linearly between 10 mA to 20 mA. So, we can calculate the resistance using Ohm's law. From the curve, we observe that at $I = 20$ mA, $V = 0.8$ V and at $I = 10$ mA, $V = 0.7$ V. So, the forward bias resistace R_{fb} of the diode is

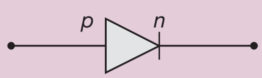
$$R_{fb} = \frac{\Delta V}{\Delta I} = \frac{0.1 \text{ V}}{10 \text{ mA}} = 10\ \Omega$$

From the curve, we see that at $V = -10$ V, $I = -1\ \mu$A. So, the reverse bias resistance R_{rb} of the diode is

$$R_{rb} = \frac{10 \text{ V}}{1\ \mu\text{A}} = 1 \times 10^7\ \Omega$$

Conceptual Note(s)

(a) the junction diode is represented by the symbol as shown in figure.

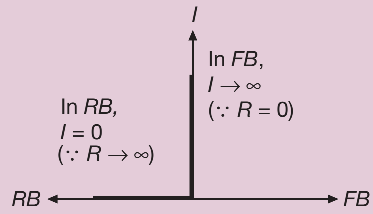


The arrow-head represents the *p* section of the junction diode and points in the direction in which the hole current or conventional current will flow, when junction diode is forward biased. The electron current or the electronic current will flow in opposite direction.

(b) It may be noted that the potential barrier opposes the forward current, while it aids the reverse current.

(c) The depletion region becomes thick and junction diode offers high resistance to current during Reverse bias.

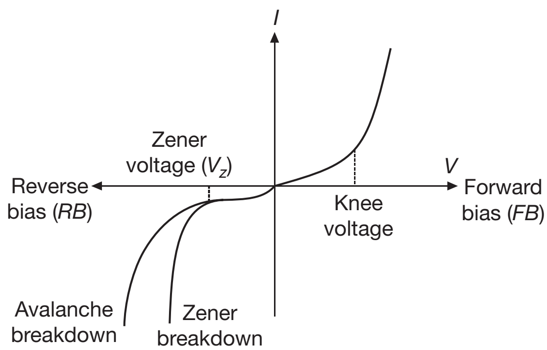
(d) For an ideal diode, the *V*-*I* relation is shown in figure. In forward bias (FB) of an ideal diode, $R = 0$ and hence $I \to \infty$. Similarly, in reverse bias (RB) of an ideal diode, $R \to \infty$ and hence $I = 0$.



(e) In practice, a *Ge* junction diode is preferred to *Si* junction diode. It is because, in case of *Ge* junction diode, the knee voltage is low (≈ 0.3 V) in comparison to that of *Si* junction diode (≈ 0.7 V).

ZENER AND AVALANCHE BREAKDOWN IN REVERSE BIAS OF A DIODE

For the diode in reverse bias, the current is very small $(\approx \mu\text{A})$ and almost remains constant with change in the bias. *It is called reverse saturation current*. However, for special cases, at very high reverse bias (break down voltage), the current suddenly increases. Depending upon the doping of the *pn* junction, we categorise the breakdown process in two categories.



Zener Breakdown

This breakdown occurs in a highly doped *pn* junction in which width of depletion region is small. When reverse bias voltage is increased, the electric field across depletion region also increases (which is the sum of barrier electric field and applied electric field) and if we go on increasing the reverse bias voltage, then at a particular value of reverse voltage (called Zener voltage V_Z) a large number of electrons and holes are produced. This is called as *Zener Breakdown*.

AVALANCHE BREAKDOWN

This kind of breakdown occurs in a reverse bias of lightly doped *pn* junction. When the *pn* junction is lightly doped, then the width of depletion region is large. So, on increasing the reverse bias, some covalent bonds are broken in the depletion region and electron-hole pairs are produced, which further collide with atoms thus producing more electron-hole pairs. This results in a continuous flow of current carriers in reverse bias and these newly generated charge carriers are also accelerated by applied electric field in reverse bias thus leading to *Avalanche Breakdown*.

Conceptual Note(s)

Note that Zener breakdown predominates at lower reverse voltages and Avalanche breakdown at higher reverse voltages.

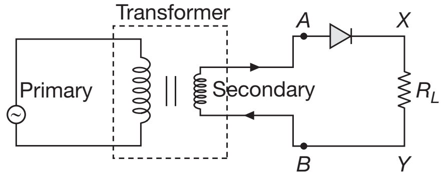
APPLICATION OF JUNCTION DIODE AS A RECTIFIER

From the V-I characteristic of a junction diode we see that it allows current to pass only when it is forward biased. So, if an alternating voltage is applied across a diode the current flows only in that part of the cycle when the diode is forward biased. This property is used to rectify alternating voltages and the circuit used for this purpose is called a rectifier.

Rectification process is the process of converting AC to DC. This is based on the principle that a diode conducts in FORWARD BIAS and a diode does not conduct in REVERSE BIAS.

Half Wave Rectifier (HWR)

If an alternating voltage is applied across a diode in series with a load, a pulsating voltage will appear across the load only during the half cycles of the ac input during which the diode is forward biased. Such a rectifier circuit is called a Half Wave Rectifier as shown in figure.

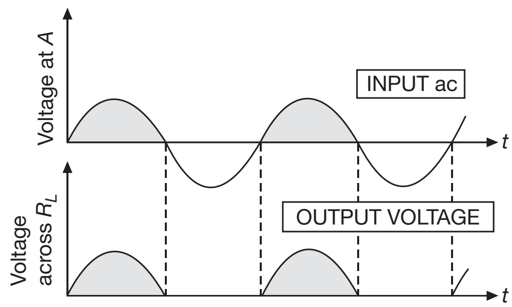


Half-wave rectifier circuit

The secondary of a transformer supplies the desired ac voltage across terminals A and B.
When the voltage at A is positive, then the diode is forward biased and it conducts.
When the voltage at A is negative, then the diode is reverse biased and it does not conduct.

The reverse saturation current of a diode is negligible and can be considered equal to zero for practical purposes. (The reverse breakdown voltage of the diode must be sufficiently higher than the peak ac voltage at the secondary of the transformer to protect the diode from reverse breakdown).

Therefore, for the positive half cycle of AC, there is a current through the load resistor R_L and we get an output voltage, as shown in Figure. However, there is no current flowing through the load resistor R_L for the negative half cycle.



Input ac voltage and output voltage waveforms from the rectifier circuit.

Now, for the next positive half cycle, again we get an output across the load resistance. So, we conclude that the output voltage, though still varying, is restricted only to one direction and hence is said to be rectified. Since the rectified output of this circuit is only for half of the input ac wave, so it is called as half wave rectifier (HWR).

Problem Solving Technique(s)

(a) Output voltage is obtained across the load resistance R_L. It is not constant but pulsating (mixture of ac and dc) in nature.

(b) The ripple frequency for half wave rectifier is same as that of ac, that is $\omega_{\text{ripple}} = \omega$.

(c) Average output in one cycle is

$$I_{dc} = \frac{I_0}{\pi} \text{ and } V_{dc} = \frac{V_0}{\pi}$$

where $I_0 = \dfrac{V_0}{r_f + R_L}$

where r_f is the forward resistance of the diode or the resistance of the diode in forward bias.

(d) Similarly, the r.m.s. output is

$$I_{rms} = \frac{I_0}{2} \text{ and } V_{rms} = \frac{V_0}{2}$$

(e) Form factor of the diode is the ratio of the rms current to the dc current. So, it is given by

$$\frac{I_{rms}}{I_{dc}} = \frac{\pi}{2} = 1.57$$

(f) The ratio of the effective alternating component of the output voltage or current to the dc component is known as Ripple factor (r).

$$r = \frac{I_{ac}}{I_{dc}} = \left(\left(\frac{I_{rms}}{I_{dc}}\right)^2 - 1\right)^{1/2} = 1.21$$

(g) Peak inverse voltage (PIV) is the maximum reverse biased voltage that can be applied before commencement of Zener region. When diode is not conducting, then PIV across HWR is V_0.

(h) Efficiency of the diode is the ratio of the ouput power of the diode to the input power. The percentage efficiency η for HWR is given by

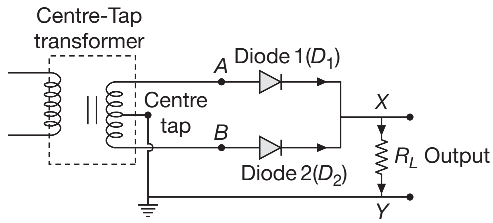
$$\eta_{HWR} = \frac{P_{out}}{P_{in}} \times 100\% = \left(\frac{40.6}{1 + \frac{r_f}{R_L}}\right)\%$$

If $R_L \gg r_f$, then $\eta_{HWR} = 40.6\%$

If $R_L = r_f$, then $\eta_{HWR} = 20.3\%$

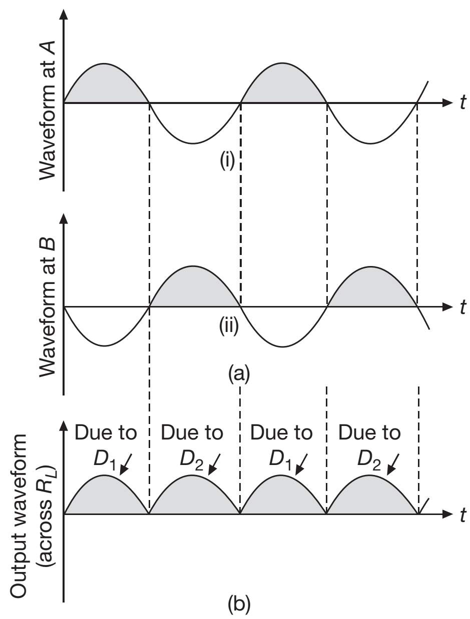
Full Wave Rectifier (FWR)

A FWR rectifies both halves of the ac input and it consists of two diodes. Since it gives a rectified output voltage for both the positive as well as negative half of the ac cycle, hence it is known as full wave rectifier (FWR). In FWR, the p side of each of the two diodes are connected to the ends of the secondary of the transformer as shown in figure.



A full-wave rectifier circuit

Similarly, the n side of both the diodes are connected together and then the output is taken between this common point of the diodes and the midpoint of the secondary of the transformer. So, for a full-wave rectifier, central tapping is done across the secondary of the transformer and hence it is called a Centrally Tapped Transformer. As can be seen from figure (b) atotal secondary voltage.



(a) Input wave forms given to the diode D_1 at A and to the diode D_2 at B; (b) Output waveform across the load R_L connected in the full-wave rectifier circuit.

For the course of ac input, when the input voltage at A (at any instant) is positive with respect to the centre tapping, then at that same instant, the voltage at B (being out of phase) will be negative. Hence diode D_1 gets forward biased and conducts whereas, diode D_2 get reversed bias and does not conduct. So, for this positive half input cycle, we get an output current and an output voltage across the load resistor R_L.

Similarly, for the next half of the ac input, the voltage at A becomes negative with respect to central tapping, then the voltage at B would be positive. For this part of the cycle, we observe that the diode D_1 does not conducts, but the diode D_2 conducts so that an output current and output voltage is obtained across the load resistance R_L.

So, we get an output voltage both during positive as well as the negative half of the ac input. It is quite clear that this (FWR) is a more efficient circuit for getting rectified voltage or current compared to HWR.

Please note that each diode rectifies only for half the cycle, but the two do so for alternate cycles. Thus, the output between the common terminals of the diode and the centrally tapped transformer becomes a full wave rectifier output.

Problem Solving Technique(s)

(a) Output voltage is obtained across the load resistance R_L. It is not constant but pulsating (mixture of ac and dc) in nature.

(b) The ripple frequency for full wave rectifier is twice the input frequency of the ac, hence $\omega_{\text{ripple}} = 2\omega$

(c) Average output in one cycle is

$$I_{dc} = \frac{2I_0}{\pi} \text{ and } V_{dc} = \frac{2V_0}{\pi},$$

where $I_0 = \dfrac{V_0}{r_f + R_L}$

where r_f is the forward resistance of the diode or the resistance of the diode in forward bias.

(d) Similarly, the r.m.s. output is

$$I_{rms} = \frac{I_0}{\sqrt{2}} \text{ and } V_{rms} = \frac{V_0}{\sqrt{2}}$$

(e) Form factor of the diode is the ratio of the rms current to the dc current. So, it is given by

$$\frac{I_{rms}}{I_{dc}} = \frac{\pi}{2\sqrt{2}} = 1.11$$

(f) The ratio of the effective alternating component of the output voltage or current to the dc component is known as Ripple factor (r).

$$r = \frac{I_{ac}}{I_{dc}} = \left[\left(\frac{I_{rms}}{I_{dc}}\right)^2 - 1\right]^{1/2} = 0.48$$

(g) Peak inverse voltage (PIV) is the maximum reverse biased voltage that can be applied before commencement of Zener region. When diode is not conducting, then PIV across FWR is $2V_0$.

(h) Efficiency of the diode is the ratio of the ouput power of the diode to the input power. The percentage efficiency η for FWR is given by

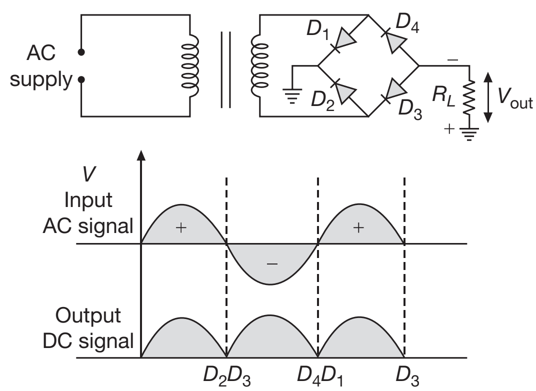
$$\eta_{FWR} = \frac{P_{out}}{P_{in}} \times 100\% = \left(\frac{81.2}{1 + \dfrac{r_f}{R_L}}\right)\%$$

If $R_L \gg r_f$, then $\eta_{FWR} = 81.2\%$

If $R_L = r_f$, then $\eta_{FWR} = 40.6\%$

Full Wave Bridge Rectifier

There is another circuit of full wave rectifier which does not need a centrally tapped transformer but needs four diodes D_1, D_2, D_3 and D_4 as shown in figure.



During positive half cycle D_1 and D_3 are forward biased and D_2 and D_4 are reverse biased. During negative half cycle D_2 and D_4 are forward biased and D_1 and D_3 are reverse biased

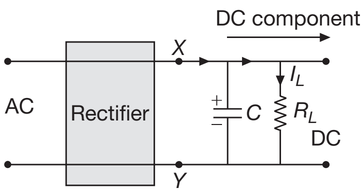
FILTER CIRCUITS

The rectified voltage is in the form of pulses of the shape of half sinusoids. Though the output is unidirectional, still it does not have a steady value.

To get a steady dc output from the pulsating voltage, normally a capacitor is connected across the output terminals (parallel to the load resistance R_L). We can also use an inductor in series with the load resistance R_L for the same purpose. Since these additional circuits appear to filter out the AC ripple and give a pure DC voltage, so they are called as Filters.

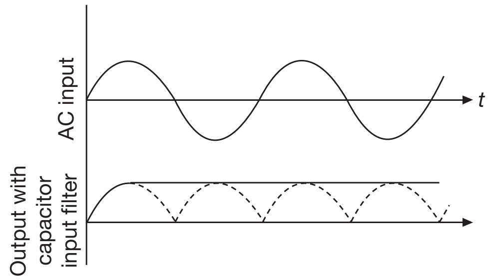
Role of Capacitor in Filter Circuits

When the capacitor is connected across the output terminals in parallel to the load resistance R_L, then a steady dc output from the pulsating voltage can be obtained.



A full-wave rectifier with capacitor filter

When the voltage across the capacitor is rising, it gets charged. If there is no external load, then the capacitor remains charged to the peak voltage of the rectified output. However, when there is a load, the capacitor gets discharged through the load and the voltage across it begins to fall. In the next half cycle of the rectified output, the capacitor again gets charged to the peak value. The rate of fall of the voltage across the capacitor is inversely proportional to the capacitive time constant $R_L C$ of the circuit (i.e. the product of capacitance C of the capacitor and the effective resistance R_L used in the circuit). To make the capacitive time constant large, the value of C should also be large. It is because of this reason, the capacitor input filters make use of capacitor having large capacitances. The output voltage obtained by using capacitor input filter is very close to the peak voltage of the rectified output. This type of filter is most widely used in power supplies.



Input and output voltage of FWR

ILLUSTRATION 17

In a full-wave rectifier circuit operating from 100 Hz mains frequency, what is the fundamental frequency in the ripple?

SOLUTION

In a full wave rectification process, the output signal (ripple) frequency is double that of input signal frequency. Hence, output frequency is 200 Hz.

ILLUSTRATION 18

The applied input ac to a half wave rectifier is 60 W. The dc output is 20 W. Calculate the rectification efficiency. Also calculate the value of power efficiency.

SOLUTION

Rectification efficiency (η_R) is the ratio of dc output power to the ac input power, so rectification efficiency

$$\eta_R = \frac{DC \text{ output power}}{AC \text{ input power}}$$

Given that dc output power $P_{dc} = 20$ W

and ac input power $P_{ac} = 60$ W

$$\Rightarrow \quad \eta_R = \frac{20}{60} \times 100 = 33.3\%$$

The power efficiency (η_P) of a rectifier is given by

$$\eta_P = \frac{DC \text{ output power}}{AC \text{ input power for half cycle}} \times 100\%$$

$$\Rightarrow \quad \eta_P = \frac{20}{30} \times 100\% = 66.67\%$$

ILLUSTRATION 19

In a centrally tapped full wave rectifier, the value of the load resistance is 2 kΩ. The voltage applied across the half the secondary winding is given by

$V = 220\sin(314t)$. Assume that the each diode has a forward bias dynamic resistance of $20\ \Omega$. Calculate the peak value of current, the dc value of current and the rms value of current.

SOLUTION

Comparing $V = 220\sin(314t)$ with the general equation $V = V_0 \sin(\omega t)$, we get $V_0 = 220$ V and $\omega = 314$ rads^{-1}

Also, it is given that $R_L = 2\ \text{k}\Omega$, $r_d = 20\ \Omega$

Peak value of current is

$$I_0 = \frac{V_0}{(r_d + R_L)} = \frac{220}{20 + 2000} \approx 109 \text{ mA}$$

dc value of current is

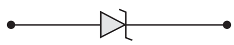
$$I_{dc} = \frac{2I_0}{\pi} = \frac{2 \times 109}{3.14} = 69.4 \text{ mA}$$

RMS value of current is

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = 0.707 \times 109 = 77.06 \text{ mA}$$

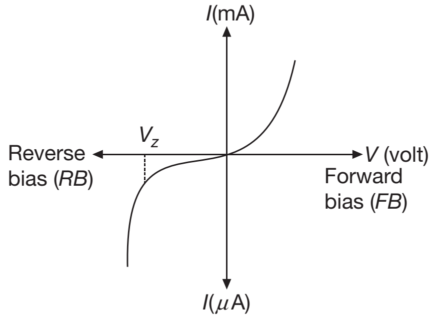
ZENER DIODE: SPECIAL PURPOSE JUNCTION DIODE

Zener diode is actually a junction diode developed for a special purpose. It is named after its inventor Clarence Melvin Zener. This diode is designed to operate under reverse bias in the breakdown region and is used as a voltage regulator. Zener diodes have highly doped *pn*-junction. Normally diodes are not designed to operate in the breakdown region, however Zener diodes operate reliably in this region. The symbol for Zener diode is shown in figure.



Zener diode symbol

Due to the heavy doping of both the p side and the n side of the junction, the depletion region formed is very thin $(< 10^{-6}\ \text{m})$ and hence the electric field of the junction is extremely high $(\approx 5 \times 10^6\ \text{Vm}^{-1})$ even for a small reverse bias voltage of about 5 V. The I-V characteristics of a Zener diode is shown in figure.



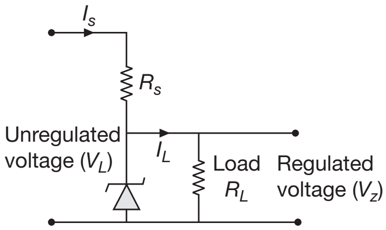
I-V characteristics

It is observed that when the applied reverse bias voltage (V) reaches the breakdown voltage (V_z) of the Zener diode, then there is a large change in the current. Also, it is observed that after the breakdown voltage V_z, a large change in the current can be produced by almost insignificant change in the reverse bias voltage. *In other words, Zener voltage remains constant, even though current through the Zener diode varies over a wide range.* This property of the Zener diode is used for regulating supply voltages so that they are constant.

Let us understand how reverse current suddenly increases at the breakdown voltage. We know that reverse current is due to the flow of electrons (minority carriers) from $p \to n$ and holes from $n \to p$. As the reverse bias voltage is increased, the electric field at the junction becomes significant. When the reverse bias voltage V equals the Zener voltage V_z, then the electric field strength is high enough to pull the valence electrons from the host atoms on the p-side which are then accelerated to n-side. These electrons account for high current observed at the breakdown. *The emission of electrons from the host atoms due to the high electric field is known as internal field emission or field ionisation.*

ZENER DIODE AS A VOLTAGE REGULATOR

The most important use of Zener diode is that it can be used as voltage regulator. When the ac input voltage of a rectifier fluctuates, its rectified output also fluctuates. To get a constant dc voltage from the dc unregulated output of a rectifier, we use a Zener diode. The circuit diagram of a voltage regulator using a Zener diode is shown in figure.



Zener diode as DC voltage regulator

The regulating action takes place because in the reverse breakdown region, a very small change in voltage produces a very large change in current. In the Zener region, the resistance of the Zener diode drops considerably (due to the large current). The unregulated dc voltage (filtered output of a rectifier) is connected to the Zener diode through a series resistance R_s such that the Zener diode is reverse biased.

As the input voltage increases, the current through R_S and Zener diode also increases. This increases the voltage drop across R_S without any change in the voltage across the Zener diode (which still remains V_Z). This is because in the breakdown region, Zener voltage remains constant even though the current through the Zener diode changes.

Similarly, when the input voltage decreases, the current through R_S and Zener diode also decreases. The voltage drop across R_S decreases without any change in the voltage across the Zener diode (which still remains V_Z).

When the applied input voltage $\left(V_{\text{input}}\right)$ is such that the voltage across the Zener diode is less than the Zener voltage, then the diode will not conduct and the output voltage $\left(V_{\text{output}}\right)$ is given by

$$V_{\text{output}} = \left(\frac{R_L}{R_S + R_L}\right) V_{\text{input}}$$

However, when the applied input voltage is such that the voltage developed across the Zener diode is more that the Zener voltage (V_Z), then the output voltage is equal to the Zener voltage i.e.

$$V_{\text{output}} = V_Z$$

Thus, any increase or decrease in the input voltage results in the corresponding increase or decrease of the voltage drop across R_s without any change in voltage across the Zener diode (which still remains V_Z). Due to this, the Zener diode acts as a voltage regulator. We have to select the Zener diode according to the required output voltage and according to the the series resistance R_S. *Every Zener diode has a certain value of current limit and the corresponding power limit. If the current in the Zener diode exceeds this limit, then the diode will burn.*

The graph of output voltage V_{output} vs input voltage V_{input} for the Zener diode is shown in Figure. It must be noted that the output voltage remains constant after the reverse breakdown voltage (i.e. Zener voltage V_Z) is obtained.

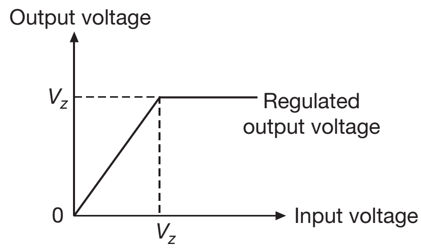


ILLUSTRATION 20

In a Zener regulated power supply a Zener diode with $V_Z = 6$ V is used for regulation. The load current is to be 4 mA and the unregulated input is 10 V. What should be the value of series resistor R_S? Assume that the Zener current is five times the load current.

SOLUTION

The value of R_S should be such that the current through the Zener diode is much larger than the load current. This is to have good load regulation. Since Zener current is five times the load current, so $I_Z = 20$ mA.

The total current through R_S is given by

$$I_S = I_Z + I_L = 24 \text{ mA}$$

The voltage drop across R_S is

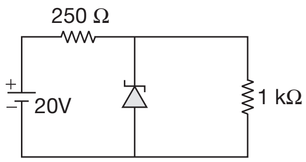
$$\Delta V_S = V_{\substack{\text{unregulated}\\ \text{input voltage}}} - V_Z = 10 - 6 = 4 \text{ V}.$$

$$\Rightarrow \quad R_S = \frac{\Delta V_S}{I_S} = \frac{4 \text{ V}}{24 \times 10^{-3} \text{ A}} = 167\ \Omega$$

Note that slight variation in the value of the resistor does not matter, what is important is that the current I_Z should be sufficiently larger than I_L.

ILLUSTRATION 21

The breakdown voltage of a Zener diode is 12 V. It is used in a voltage regulator circuit shown in figure. Calculate the current through the diode.



SOLUTION

Current through $250\ \Omega$ resistor is

$$I_1 = \frac{\Delta V}{\Delta I} = \frac{(20-12)\ \text{V}}{250\ \Omega}$$

$\Rightarrow \quad I_1 = 32\times 10^{-3}\ \text{A} = 32\ \text{mA}$

Current through $1\ \text{k}\Omega$ resistor is

$$I_2 = \frac{12\ \text{V}}{1\ \text{k}\Omega} = 12\times 10^{-3}\ \text{A} = 12\ \text{mA}$$

So, current through the Zener diode is

$$I_Z = I_1 - I_2 = 32\ \text{mA} - 12\ \text{mA} = 20\ \text{mA}$$

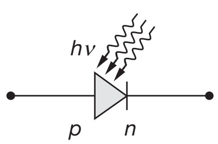
OPTO ELECTRONIC JUNCTION DEVICES

Till now we have studied the behaviour of a semiconductor diode under the applied electrical inputs. However, there are semiconductor diodes in which charge carriers are generated by photons (i.e. by photo-excitation). *These types of diodes are called opto-electronic devices*. Let us study the functioning of the following optoelectronic devices

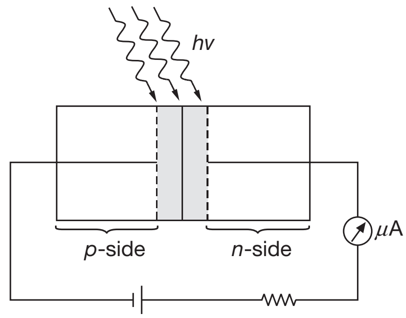
(i) **Photodiodes:** Which are used to detect optical signal and are also called as photodetectors.
(ii) **Light Emitting Diodes (LED):** Which converts an electrical energy into light.
(iii) **Photovoltaic devices:** Which converts optical radiation into electricity also called as solar cells.

Photodiode

A photodiode is again a special purpose *p-n* junction diode which can be used as a photodetector to detect optical signals. Its symbolic representation is shown in figure.



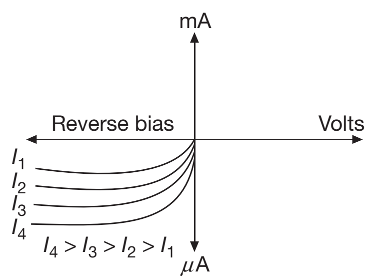
This diode is fabricated with a transparent window to allow light to fall on the diode. It is always operated in the reverse bias mode as shown in the Figure, where the circuit used to measure I-V characteristics of a photodiode is shown.



An illuminated photodiode under reverse bias.

When the photodiode is illuminated with light having photons of energy $h\nu$ greater than the forbidden energy gap (E_g) of the semiconductor, then electron-hole pairs are generated due to the absorption of photons. The diode is fabricated in such a manner that the generation of electron-hole pair takes place in or near the depletion region of the diode. Due to electric field of the junction, these electrons and holes get separated before they recombine. The direction of the electric field is such that electrons reach *n*-side and holes reach *p*-side of the junction, due to which an emf is setup across the junction. When an external load is connected across it, a current flow through the load. Since, photocurrent is proportional to the intensity of incident light, hence the magnitude of the photocurrent depends on the intensity light incident on the diode.

Please note that, if the diode is reverse biased, then it is easier to observe the change in the current with change in the light intensity. A typical I-V characteristic curve for the photodiode is shown in figure.



I-V characteristics of a photodiode for different illumination intensity $I_4 > I_3 > I_2 > I_1$

Conceptual Note(s)

The current in the forward bias is known to be more (≈ mA) than the current in the reverse bias (≈ μA), still we operate the diode in reverse bias. To explain the need to operate the photodiode in reverse bias mode, let us consider the case of an n-type semiconductor in which the charge density of majority carriers i.e. electrons is considerably larger than the charge density of minority carriers i.e. holes, so $n \gg p$, where n and p are respective electrons and holes concentration when there is no illumination. On illumination of the diode by a photon, let the excess electrons and holes generated be Δn and Δp respectively. If n' and p' be the electron and hole concentrations at any particular illumination, then we have

$$n' = n + \Delta n$$

$$p' = p + \Delta p$$

Since, $\Delta n = \Delta p$ and $n \gg p$, so the fractional change in the majority carriers i.e. $\frac{\Delta n}{n}$ would be much less compared to the fractional change in the minority carriers i.e. $\frac{\Delta p}{p}$.

Hence, we observe that, the fractional change due to the photo-effects on the minority carriers produces a larger reverse bias current which is more easily measurable than the fractional change in the forward bias current. Due to this, photodiodes are preferably used in the reverse bias condition for measuring light intensity.

ILLUSTRATION 22

A pn photodiode is made of a material with a band gap of 1.5 eV. Calculate the minimum wavelength of radiation that can be absorbed by the material.

SOLUTION

Since the energy of the photon is given by

$$E = h\nu = \frac{hc}{\lambda}$$

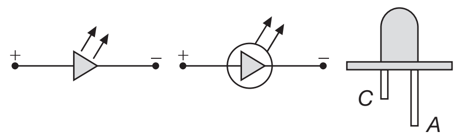
The minimum wavelength of radiation

$$\lambda = \frac{hc}{E} = \frac{(6.4 \times 10^{-34}\ \text{Js}) \times (3 \times 10^{8}\ \text{ms}^{-1})}{1.5 \times 1.6 \times 10^{-19}\ \text{J}}$$

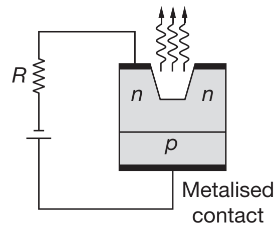
$$\Rightarrow \quad \lambda = 8.3 \times 10^{-7}\ \text{m} = 8300\ \text{Å}$$

Light Emitting Diode (LED)

LED is a heavily doped p-n junction which emits spontaneous radiation under forward bias. LED is encapsulated with a transparent cover so that emitted light can come out. The symbolic representation of LED and its actual shape is shown in Figure, where the longer terminal corresponds to the p-side and the shorter terminal corresponds to the n-side of the LED.



A p-n junction made from a translucent semiconductor like indium phosphide or gallium arsenide is provided with metal contacts as shown in figure.



A p-n junction made from a translucent semiconductor like gallium arsenide or indium phosphide is provided with metallised contacts, as shown in figure. When it is forward biased through a series resistance R, light photons are emitted from the non-metallised surface of the n-region. The series resistance R limits the current through the LED and hence controls the intensity of light emitted by it.

When this junction is forward biased through a resistor R connected in series to the diode, then photons are emitted from the non metallic surface of the n-region. The resistor R controls the current through the LED and hence controls the intensity of light emitted by the diode.

When the diode is forward biased, electrons are sent from n-region to the p-region (where they are minority carriers) and holes are sent from p-region to the n-region (where they are minority carriers). Close to the junction, the concentration of minority carriers increases compared to the equilibrium concentration (i.e., when there is no bias). So, on either side of the junction, excess minority carriers are there which recombine with majority carriers (near the junction). Due to recombination of charge carriers, the energy is released in the form of photons. It is observed that photons with energy equal to or slightly less than the band gap or forbidden energy gap are emitted. When the forward current of the diode is small, the intensity of emitted light is small. As the forward current increases the intensity of emitted light increases and reaches a maximum. Further increase in the forward current results in decrease of light intensity. LEDs are biased such that the light emitting efficiency is maximum.

The V-I characteristics of a LED is similar to that of a Si junction diode. But the threshold voltages are much higher and slightly different for each colour. The reverse breakdown voltages of LEDs are very low, typically around 5 V. So, care should be taken that high reverse voltages do not appear across them.

LEDs that can emit red, yellow, orange, green and blue light are commercially available. The semiconductor used for fabrication of visible LEDs must at least have a band gap of 1.8 eV (spectral range of visible light is from about $0.4\ \mu\text{m}$ to $0.7\ \mu\text{m}$, i.e., from about 3 eV to 1.8 eV). The compound semiconductor Gallium Arsenide Phosphide $(GaAs_{1-x}P_x)$ is used for making LEDs of different colours. For making red LED, $GaAs_{0.6}P_{0.4}$ having $E_g \approx 1.9\text{ eV}$ is used. For making infrared LED, $GaAs$ having $E_g \approx 1.4\text{ eV}$ is used.

LEDs have the following advantages over conventional incandescent low power lamps.

(a) Low operational voltage and less power.
(b) Fast action and no warm-up time required.
(c) The bandwidth of emitted light is 100 Å to 500 Å or in other words it is nearly (but not exactly) monochromatic.
(d) Long life and ruggedness.
(e) Fast on-off switching capability.

These LEDs find extensive use in remote controls, burglar alarm systems, optical communication, etc. Extensive research is being done for developing white LEDs which can replace incandescent lamps.

ILLUSTRATION 23

A voltage drop of 2 V occurs across a light emitting diode (LED) and a current of $10\ \mu\text{A}$ is passed through it when it is operated with a 6 V battery having a limiting resistor R. Calculate the value of R.

SOLUTION

Current, $I = \dfrac{\Delta V}{R}$

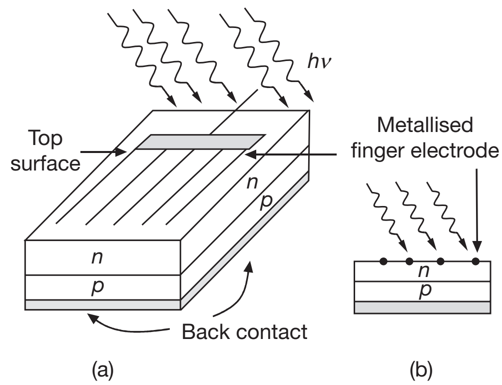
$$\Rightarrow \quad R = \frac{\Delta V}{I}$$

Resistance of limiting resistor will be

$$R = \frac{(6-2)\text{ V}}{10\times10^{-6}\text{ A}} = 400\text{ k}\Omega$$

Solar Cell

A solar cell is basically a p-n junction which generates emf when solar radiation falls on the p-n junction. It works on the principle of photovoltaic effect (which is the process of generation of voltage due to bombardment of photons). The process is just similar as the photodiode, except that no external bias is applied and the junction area is kept much larger for solar radiation to be incident, so as to get more power. A simple p-n junction solar cell is shown in figure.



(a) Typical p-n junction solar cell; (b) Cross-sectional view

A p-type Silicon $(p\text{-}Si)$ wafer of about $300\ \mu\text{m}$ is taken, over which a thin layer $(\approx 0.3\ \mu\text{m})$ of n-type Silicon $(n\text{-}Si)$ is grown on one side by the diffusion process. The other side of $p\text{-}Si$ is coated with a metal that forms the *back contact*. On the top of $n\text{-}Si$ layer, metal finger electrode also called as metallic grid is deposited. This acts as the *front contact*. The metallic grid occupies only a very small fraction of the cell area $(<15\%)$ so that light can be incident from the top on sufficiently large area of the cell.

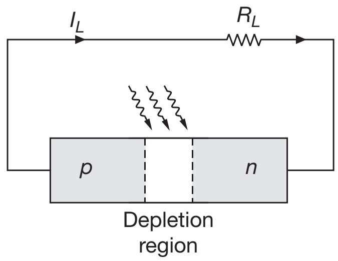
When light falls on a solar cell, the generation of emf is due to the following three basic processes

(a) Generation: In this process, the electron-hole pairs are generated (close to the junction) due to the falling light having $h\nu > E_g$.

(b) Separation: In this process, the separation of electrons and holes takes place due to electric field of the depletion region. The electrons are swept to n-side and holes to the p-side

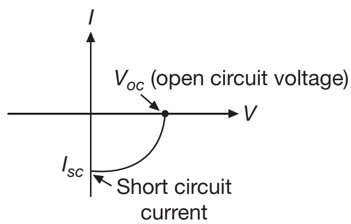
(c) Collection: In this process, the electrons reaching the n-side are collected by the front contact and holes reaching p-side are collected by the back contact. Due to this process, the p-side becomes positive and the n-side becomes negative, thus giving rise to a photovoltage.

When an external load is connected in the external circuit, as shown in the figure, a photocurrent I_L flows through the load resistor R_L. This current is proportional to the intensity of light falling on the cell.



A typical illuminated *p-n* junction solar cell

A typical I-V characteristics of a solar cell is shown in the figure.



I-V characteristics of a solar cell

Note that the I-V characteristics of solar cell is drawn in the fourth quadrant of the coordinate axes. This is because a solar cell does not draw current but supplies the same to the load.

Semiconductors with band gap close to 1.5 eV are ideal materials for solar cell fabrication. Solar cells are made with semiconductors like

Si having $E_g = 1.1$ eV ,

GaAs having $E_g = 1.43$ eV

CdTe having $E_g = 1.45$ eV

$CuInSe_2$ having $E_g = 1.04$ eV etc.

The important criteria for the selection of a material for solar cell fabrication are

(a) band gap $(\approx 1$ to 1.8 eV$)$,

(b) high optical absorption $(\approx 10^4\ \text{cm}^{-1})$,

(c) electrical conductivity,

(d) availability of the raw material, and

(e) cost.

Solar cells are used to power electronic devices in satellites and space vehicles and also as power supply to some calculators. Rigorous research and efforts are going on for the production of low cost photovoltaic cells for large scale solar energy.

Conceptual Note(s)

Please note that sunlight is not always required for a solar cell. Any light with photon energies greater than the bandgap will do.

JUNCTION TRANSISTOR

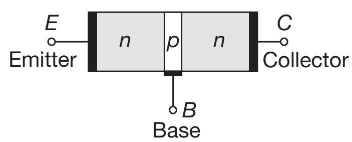
Transistor (also called as Junction Transistor) was first invented by J Bardeen and WH Brattain of Bell Telephone Laboratories, USA. A junction transistor is a three terminal solid state device obtained by growing either a narrow section of p-type crystal between two relatively thicker sections of n-type crystals or a narrow section of n-type crystal between two thicker sections of p-type crystals. Transistors are mainly of two types

(a) npn transistor

(b) pnp transistor

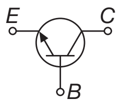
NPN Transistor

A *npn* transistor consists of a thin section of p-type semiconductor sandwiched between two thicker sections of n-type semiconductors as shown in figure.



npn transistor

The circuit symbol for the *npn* transistor is shown in Figure.

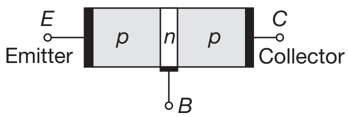


npn transistor symbol

Please note that, the arrowhead in the transistor symbol is shown in the base emitter branch and in the case of *npn* transistor the arrow is pointing from *p* to *n* i.e. from base to emitter.

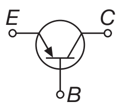
PNP Transistor

A *pnp* transistor consists of a thin section of *n*-type semiconductor sandwiched between two thicker sections of *p*-type semiconductors as shown in Figure.



pnp transistor

The circuit symbol for the *pnp* transistor is shown in Figure.



pnp transistor symbol

Here too, the arrowhead in the transistor symbol is shown in the base emitter branch and in the case of *pnp* transistor, the arrow is pointing from *p* to *n* i.e. from emitter to base. In both the types of transistors, the arrowhead at the emitter, points along the direction of conventional current.

Conceptual Note(s)

(a) A transistor can be compared with triode value. The Emitters can be compared to cathode of triode valve. The collector can be compared with the plate and the base with the grid of the Triode valve.

(b) Base is kept thin and is comparatively lightly doping.

(c) Symbol for transistors is always drawn keeping in mind to show the direction of conventional current in the Emitter-Base branch.

TRANSISTOR CONSTRUCTION

Each type of transistor has three main parts.

(a) The Emitter **(b)** The Base

(c) The Collector

Emitter (E)

The emitter is a section on one side of the transistor. It is a moderately sized and heavily doped semiconductor. It is normally forward biased w.r.t. any other part of the transistor. It supplies a large number of majority charge carriers for the flow of current through the transistor.

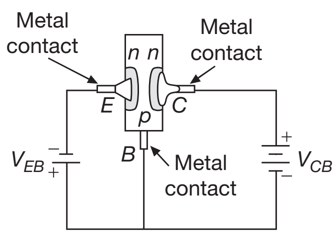
Base (B)

The base is the middle section of a transistor. It is very thin and lightly doped. It controls the flow of majority charge carriers from emitter to collector.

Collector (C)

The collector is a section on the other side of the transistor. It is larger in size and moderately doped as compared to the emitter. Normally it is reverse biased w.r.t. any other part of the transistor. It collects the majority charge carriers for the circuit operation.

The relative sizes of the three regions of the *npn* transistor and the biasing of base-emitter and base-collector junctions is shown in the Figure.



Relative sizes of the three regions of the *npn* transistor and the biasing

The forward bias voltage V_{EB} is small (0.5 V to 1 V) while the reverse bias voltage V_{CB} is high (5 V to 15 V).

WORKING OF TRANSISTOR

There are four possible ways of biasing the two *pn* junctions (emitter junction and collector junction) of transistor.

(a) Cut-off mode: Denotes operation like an open switch where only leakage current flows.
(b) Active mode: Also known as linear mode operation.
(c) Inverse mode: The emitter and collector are inter changed.
(d) Saturation mode: Maximum collector current flows and transistor acts as a closed switch from collector to emitter terminals.

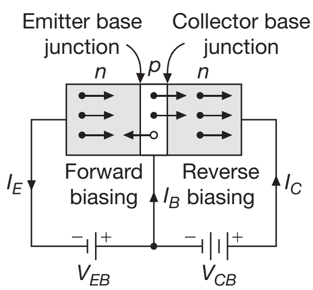
Table 4.1 Different modes of operation of a transistor

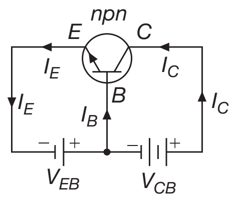
Operating mode	Emitter base bias	Collector base bias
Cut off	Reverse	Reverse
Active	Forward	Reverse
Inverse	Reverse	Forward
Saturation	Forward	Forward

A transistor is mostly used in the active region of operation i.e. emitter base junction is forward biased and collector base junction is reverse biased. From the operation of junction transistor, it is found that whenever the current in emitter circuit changes, then there is a corresponding change in the collector current. In each state of the transistor there is an input port and an output port. In general, each electrical quantity (*V* or *I*) obtained at the output is controlled by the input.

ACTION OF NPN TRANSISTOR

The emitter of the *npn* transistor is forward biased by connecting it to the negative terminal of the V_{EB} battery and the collector is reverse biased by connecting it to the positive terminal of the V_{CB} battery, as shown in Figure.





Action of *npn* transistor and its biasing

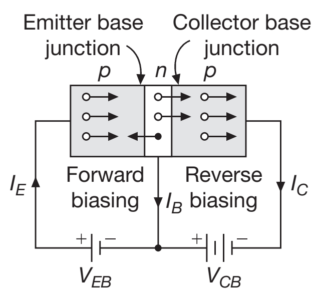
The forward bias of the emitter-base circuit repels the electrons of emitter towards the base, setting up emitter current I_E. As the base is very thin and lightly doped, a very few electrons $(< 5\%)$ from the emitter combine with the holes of base, giving rise to base current I_B and the remaining electrons $(> 95\%)$ are pulled by the collector which is at high positive potential. These electrons are finally collected by the positive terminal of V_{CB} battery, giving rise to collector current I_C. As soon as an electron from the emitter combines with a hole in the base region, an electron leaves the negative terminal of the V_{EB} battery and at the same time, the positive terminal of V_{EB} battery receives an electron from the base due to which a base current I_B is set up in the circuit. Similarly, corresponding to each electron that goes from collector to positive terminal of the V_{CB} battery, an electron enters the emitter from negative terminal of V_{EB} battery. Both the base current I_B and collector current I_C combine to form emitter current I_E, such that

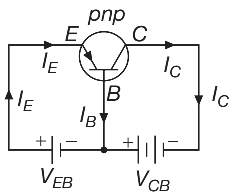
$$I_E = I_B + I_C$$

Here I_B is a small fraction of I_C i.e. $[I_B \ll I_C]$ depending on the shape of transistor, thickness of base, doping levels, bias voltages, etc.

ACTION OF PNP TRANSISTOR

The emitter of the *pnp* transistor is forward biased by connecting it to the positive terminal of V_{EB} battery and the collector is reverse biased by connecting it to the negative terminal of the V_{CB} battery, as shown in Figure.





Action of *pnp* transistor and its biasing

The forward bias of the emitter-base circuit repels the holes of emitter towards the base and electrons of base towards the emitter. As the base is very thin and lightly doped, most of the holes $(>95\%)$ entering it pass on to collector while a very few of them $(<5\%)$ recombine with the electrons of the base region. As soon as a hole combines with an electron, an electron from the negative terminal of the V_{EB} battery enters the base, which sets up a small base current I_B. Each hole entering the collector region combines with an electron from the negative terminal of the V_{CB} battery and gets neutralised and thus creates a collector current I_C. Here the emitter current I_E divides to give the base current I_B and the collector current I_C, such that

$$I_E = I_B + I_C$$

Thus, inside the *pnp* transistor, the current conduction is due to holes while electrons are the charge carriers in the external circuit.

ILLUSTRATION 24

In an npn transistor circuit, the collector current is 10 mA. If 95% of the electrons emitted reach the collector, what is the base current?

SOLUTION

Given that $I_C = 95\% I_E$

$\Rightarrow \quad I_C = 0.95 I_E$

$\Rightarrow \quad I_E = \dfrac{I_C}{0.95} = \dfrac{100}{95} \times 10\text{ mA}$

$\Rightarrow \quad I_E = 10.53\text{ mA}$

Since, $I_E = I_C + I_B$

So, the base current is given by

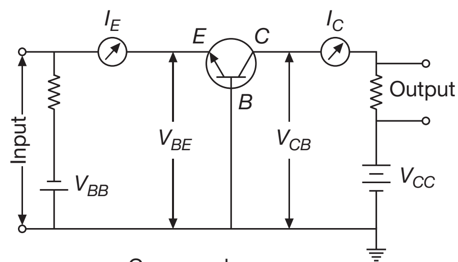
$$I_B = I_E - I_C = 10.53 - 10 = 0.53\text{ mA}$$

THREE CONFIGURATIONS OF A TRANSISTOR

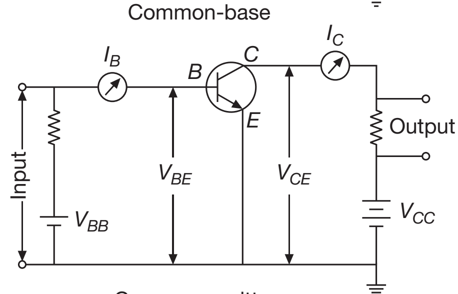
A transistor is a three element device. One terminal has to be always common to the input and the output circuits. This terminal is connected to the ground and serves as a reference point for the entire circuit. So, a transistor can be used in one of the following three configurations

(a) Common-base (CB) circuit.
(b) Common-emitter (CE) circuit.
(c) Common-collector (CC) circuit.

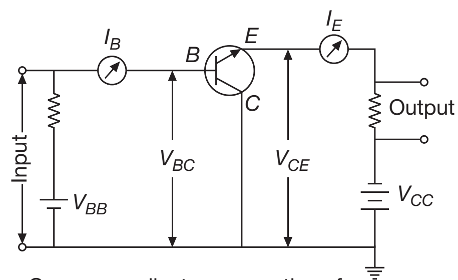
The three types of circuit arrangements for an *npn* transistor are shown in figure.



Common-base



Common-emitter



Common-collector connections for *npn* transistor

In each case, the emitter-base junction is forward biased whereas the collector-base junction is reverse biased.

CURRENT GAIN FOR A TRANSISTOR

We define two types of current gains for a transistor.

(a) Common base configuration current gain: This current gain is denoted by α. It is also called as common base current amplification factor or the ac current gain.
It is defined as the ratio of the small change in the collector current to the small change in the emitter current when the collector-base voltage is kept constant. Thus

$$\alpha = \left(\frac{\Delta I_C}{\Delta I_E}\right)_{V_{CB}=\text{constant}}$$

(b) Common emitter configuration current gain: This current gain is denoted by β. It is also called as common emitter current amplification factor or the ac current gain.
It is defined as the ratio of the small change in the collector current to the small change in the base current when the collector-emitter voltage is kept constant. Thus

$$\beta = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE}=\text{constant}}$$

RELATION BETWEEN α AND β

Since we know that, for both *npn* and *pnp* transistors

$$I_E = I_B + I_C$$

For small changes in the currents, we can re-write the above equation as

$$\Delta I_E = \Delta I_B + \Delta I_C$$

Dividing both sides by ΔI_C, we get

$$\frac{\Delta I_E}{\Delta I_C} = \frac{\Delta I_B}{\Delta I_C} + 1$$

Since, $\frac{\Delta I_C}{\Delta I_E} = \alpha$ and $\frac{\Delta I_C}{\Delta I_B} = \beta$, so we get

$$\frac{1}{\alpha} = \frac{1}{\beta} + 1$$

$$\Rightarrow \quad \alpha = \frac{\beta}{1+\beta} \text{ OR } \beta = \frac{\alpha}{1-\alpha}$$

As the value of I_B is about 1% to 5% of I_E or I_C is 95% to 99% of I_E, so α is about 0.95 to 0.99 and β is about 20 to 100. The CE configuration is frequently used because it gives high current gain as well as voltage gain.

Conceptual Note(s)

(a) It is observed that, if the emitter-base junction is forward biased and the collector-base junction is reverse biased, then too α and β are independent of current

(b) Please note that, the above definitions of α and β do not hold when both the junctions of a transistor are forward biased or reverse biased.

ILLUSTRATION 25

The current gain for common base amplifier is 0.98. Calculate the current amplification factor, if the transistor is being used in the common emitter configuration.

SOLUTION

If α is the current amplification factor for common base amplifier and β is the current amplification for common emitter amplifier, then it is given that $\alpha = 0.98$

Since, $\beta = \frac{\alpha}{1-\alpha}$

$$\Rightarrow \quad \beta = \frac{0.98}{(1-0.98)} = 49$$

ILLUSTRATION 26

The current factor of a transistor in a common base arrangement in 0.98. Calculate the change in collector current corresponding to a change of 5 mA in the emitter current. Also calculate the change in base current.

SOLUTION

Given that, $\alpha = 0.98$ and $\Delta I_E = 5.0$ mA

From the definition of $\alpha = \frac{\Delta I_C}{\Delta I_E}$

Change in collector current,

$$\Delta I_C = (\alpha)(\Delta I_E) = (0.98)(5) \text{ mA} = 4.9 \text{ mA}$$

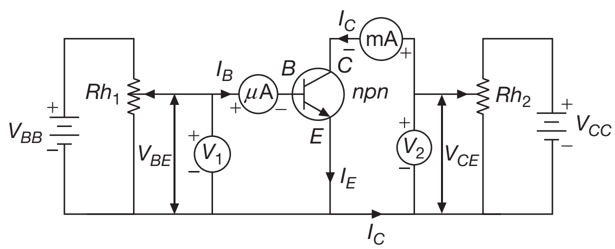
Further, change in base current,

$$\Delta I_B = \Delta I_E - \Delta I_C = 0.1 \text{ mA}$$

COMMON EMITTER TRANSISTOR CHARACTERISTICS

The common emitter characteristics are the graphs drawn between appropriate voltages and currents for a transistor when its emitter is taken as the common terminal and grounded (i.e. taken at zero potential), base is the input terminal and collector is the output terminal.

The circuit diagram for studying the common emitter characteristics of an *npn* transistor is shown in Figure.



Circuit for studying the common-emitter characteristics of an *npn* transistor

The emitter base junction is forward biased by means of the battery V_{BB} through a rheostat Rh_1. The emitter collector circuit is reverse biased by means of battery V_{CC} through a rheostat Rh_2.

The base emitter voltage V_{BE} and the collector emitter voltage V_{CE} are measured by using high resistance voltmeters.

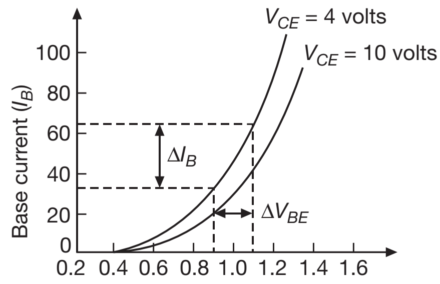
The base current I_B is measured by a microammeter and the collector current I_C by a milliammeter.

Three types of characteristic curves are studied.

(a) The input characteristic curve.
(b) The output characteristic curve.
(c) The transfer characteristic curve.

INPUT CHARACTERISTIC CURVE OF A TRANSISTOR

The input characteristic curve of a transistor is a graph showing the variation of base current I_B with base emitter voltage V_{BE} at constant collector emitter voltage V_{CE}. Curves for two different collector emitter voltages applied across *npn* transistor have been plotted in figure.



Input characteristics of *CE npn* transistor

A detailed study of these curves reveals the following facts.

(a) As long as V_{BE} is less than the barrier voltage, the base current I_B is small just similar to the case of a forward biased diode.

(b) As soon as the base emitter voltage V_{BE} exceeds the barrier voltage, the base current I_B increases sharply with a small increase in V_{BE} just like the case of a forward biased diode.

(c) The value of I_B is much smaller than that in a normal diode because more than 95% majority emitter carriers (electrons in *npn* and holes in *pnp* transistor) go to the collector to constitute the collector current I_C.

Since the increase in V_{CE} appears as the increase in V_{CB}, so its effect on I_B is negligible, due to which the input characteristic for various values of V_{CE} give almost identical curves. Hence, it is sufficient enough to determine only one input characteristic.

Input Resistance of a Transistor

The input resistance (r_i) of the transistor in CE configuration is defined as the ratio of the small change in base emitter voltage to the corresponding small change in the base current, when the collector emitter voltage is kept fixed. Mathematically,

$$r_i = \left(\frac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}=\text{constant}}$$

Since the input characteristic curve is non-linear, so r_i varies. However, at any point of the input characteristic curve, r_i is equal to the slope of the tangent drawn at that point to the curve. *The input resistance can have a value ranging from few hundred to few thousand ohms.*

ILLUSTRATION 27

The current amplification factor for a common emitter arrangement is 59. If the emitter current is 5 mA. Calculate the value of collector current.

SOLUTION

Current amplification factor for common emitter amplifier is given to be $\beta = 59$ and the emitter current is $I_E = 5\text{ mA}$

Since, $\alpha = \dfrac{\beta}{1+\beta}$

Also $\alpha = \dfrac{I_C}{I_E}$

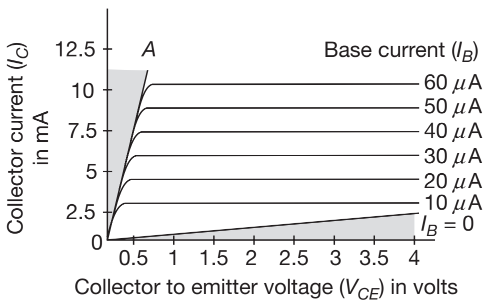
$$\Rightarrow \quad \frac{I_C}{I_E} = \frac{\beta}{1+\beta}$$

$$\Rightarrow \quad I_C = \left(\frac{\beta}{1+\beta}\right) I_E$$

$$\Rightarrow \quad I_C = \left(\frac{59}{60}\right) \times 5 = 4.92\text{ mA}$$

OUTPUT CHARACTERISTIC CURVE OF A TRANSISTOR

The output characteristic curve of a transistor is a graph showing the variation of collector current I_C with collector emitter voltage V_{CE} at constant base-current I_B is called the output characteristic of the transistor. Curves for different values of I_B for *npn* transistor have been plotted in figure.



Output characteristic of *CE npn* transistor

A detailed study of these curves reveals the following facts.

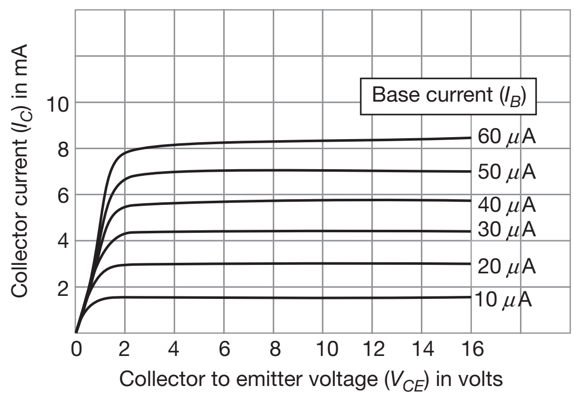
(a) When the voltage V_{CE} increases from zero volt to about 0.5 V, the collector current I_C increases rapidly. The value of V_{CE} upto which I_C increases rapidly is called Knee Voltage.

(b) Once the voltage V_{CE} exceeds the voltage V_{BE} (so that the collector base junction is reverse biased), the output current I_C varies very slowly but linearly with V_{CE} for a given base current I_B. This makes us conclude that beyond the knee voltage, the output resistance of the transistor is high.

(c) For a given V_{CE}, the larger the value of I_B, the larger is the value of I_C.

ILLUSTRATION 28

From the output characteristics of a common emitter transistor shown in figure, calculate the values of β_{ac} and β_{dc} of the transistor when V_{CE} is 10 V and $I_C = 4\text{ mA}$.



SOLUTION

Since, we know that

$$\beta_{ac} = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE}} \text{ and } \beta_{dc} = \frac{I_C}{I_B}$$

For calculating β_{ac} and β_{dc} at the asked values of V_{CE} and I_C, consider any two characteristics for two values of I_B which lie above and below the given value of I_C.

Given that $I_C = 4\text{ mA}$, so let us select two values of I_B i.e. $I_B = 30\ \mu\text{A}$ and $I_B = 20\ \mu\text{A}$. At $V_{CE} = 10\text{ V}$

we read the two values of I_C from the graph, then we have

$$\Delta I_B = (30-20)\ \mu A = 10\ \mu A,$$

$$\Delta I_C = (4.5-3)\ mA = 1.5\ mA$$

$$\Rightarrow \quad \beta_{ac} = \frac{\Delta I_C}{\Delta I_B} = \frac{1.5\ mA}{10\ \mu A} = 150$$

For calculating β_{dc}, we can either estimate the value of I_B corresponding to $I_C = 4$ mA at $V_{CE} = 10$ V or we can calculate the two values of β_{dc} for the two characteristics chosen and then calculate their mean value

So, for $I_C = 4.5$ mA, $I_B = 30\ \mu A$, we have

$$\beta_1 = \frac{4.5\ mA}{30\ \mu A} = 150$$

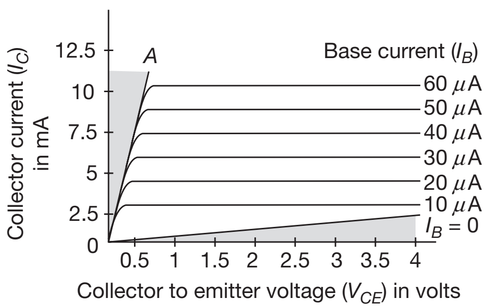
and for $I_C = 3$ mA, $I_B = 20\ \mu A$, we have

$$\beta_2 = \frac{3\ mA}{20\ \mu A} = 150$$

$$\Rightarrow \quad \beta_{dc} = \frac{\beta_1 + \beta_2}{2} = \frac{150+150}{2} = 150$$

Three Regions of the Output Characteristic Curve of a Transistor

We have shaded different regions in the output characteristic curve of an *npn* transistor for different values of I_B.



Output characteristic of *CE npn* transistor

(a) The shaded region towards the left of line OA is called saturation region and the line OA is called saturation line. Here $V_{CE} < V_{BE}$. Both the junctions are forward biased. Here I_C does not depend on the input current I_B.

(b) The shaded region lying below the curve for $I_B = 0$ is called cut-off region. In this region, both the junctions are reverse biased. Here $I_C = 0$. In the shaded regions, the transistor works as switch, it turns over rapidly from OFF state for which $I_C = 0$ (cut-off) to the ON state for which I_C is maximum (saturation state).

(c) The non-shaded central region of the output characteristic is called active region. In this region, the emitter-base junction is forward biased and the collector-base junction is reverse biased. A transistor works as an audio amplifier in this region.

Output Resistance of a Transistor

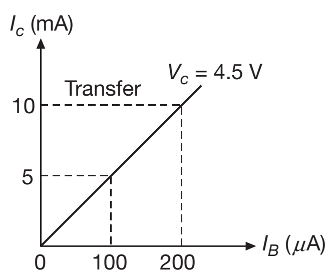
The output resistance r_o of a transistor in CE configuration is defined as the ratio of the small change in the collector-emitter voltage to the corresponding change in the collector current when the base current is kept constant. Thus

$$r_0 = \left(\frac{\Delta V_{CE}}{\Delta I_C}\right)_{I_B=\text{constant}}$$

The reciprocal of the slope of the linear part of the output characteristic curve (i.e. I_C vs V_{CE} curve) gives the value of output resistance. The output resistance of the transistor is mainly controlled by the bias of the base-collector junction. The output resistance is of the order of few hundred kilo ohms.

TRANSFER CHARACTERISTIC CURVE OF A TRANSISTOR

The transfer characteristic curve of a transistor is a graph showing the variation of collector current I_C with the base current I_B at constant collector emitter voltage V_{CE}. The transfer characteristic of a transistor is almost a straight line as shown in Figure.



Transfer characteristic of *CE npn* transistor

CURRENT AMPLIFICATION FACTOR (β)

It is defined as the ratio of the change in collector current to the small change in base current at constant collector-emitter voltage (V_{CE}) when the transistor is in the active state.

$$\beta_{ac} = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE}=\text{constant}}$$

This is also known as small signal current gain and its value is very large. The direct ratio of I_C and I_B gives the dc current gain (β_{dc}) of the transistor. Hence,

$$\beta_{dc} = \frac{I_C}{I_B}$$

Since I_C increases with I_B almost linearly and $I_C = 0$ when $I_B = 0$, the values of both β_{ac} and β_{dc} are nearly equal.

ADVANTAGES OF TRANSISTORS

Transistors, because of their many merits over vacuum tubes, have practically completely replaced them. Some of the advantages of the transistors over the vacuum tubes are as given below

Advantages

(a) Transistors require low voltages for their operation as compared to vacuum tubes.
(b) Since no heating is required, transistors are set into operation as soon as the circuit is switched on.
(c) Due to their small sizes, the circuits involving transistors are very compact.
(d) Transistors have almost unlimited life.
(e) Since transistors have no filaments, hence no power is needed to heat them to cause the emission of electrons.
(f) Since no vacuum has to be created in transistors, they have no vacuum deterioration trouble.
(g) Transistors are shock proof.
(h) During operation, transistors do not produce any humming noise.
(i) Transistors are cheaper as compared to vacuum tubes.

Transistors enjoy a number of advantages over the vacuum tubes still they have following drawbacks which put restrictions on their use in electronic circuits.

DRAWBACKS OF TRANSISTORS

(a) Ordinary semiconductor devices cannot handle as much power as ordinary vacuum tubes can do.
(b) The transistors are temperature-sensitive. The maximum temperature the transistors can withstand, is very low $(\approx 50\ ^\circ\text{C})$. Even a small overheating spoils the transistor. This is because, at a higher temperature, the covalent bonds break up and the semiconductor piece forming the transistor becomes conducting.
(c) Noise level is higher in transistors as compared to that in the vacuum tubes.
(d) They show poor response for high frequency range.

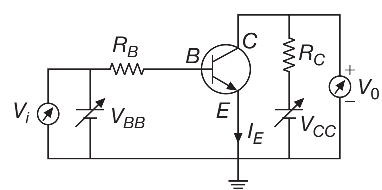
TRANSISTOR AS A SWITCH

Modern day digital devices like computers perform millions of switching operations per day. For such operations, transistors can be used as switches in computer circuits because they act swiftly. It has been observed that transistors have many advantages over other electrically operated switches such as relays and reed switches. This is because, the transistors

(a) are small, cheap and reliable.
(b) can switch on and off millions of times a second.
(c) have long life in well designed circuits.
(d) have no moving parts.

Three States of a Transistor

To understand the operation of a transistor as a switch, we first study the three states or conditions in which a transistor can work. Figure shows the circuit diagram of a base-biased *npn* transistor in CE configuration. Let R_B be the resistance in the input circuit and R_C be the resistance in the output circuit.



A base biased *npn* transistor in *CE* configuration

Applying Kirchhoff's rule to the input and output circuits separately, we get

$$V_{BB} - I_B R_B - V_{BE} = 0$$

$\Rightarrow$ $V_{BB} = I_B R_B + V_{BE}$

and $V_{CC} - I_C R_C - V_{CE} = 0$

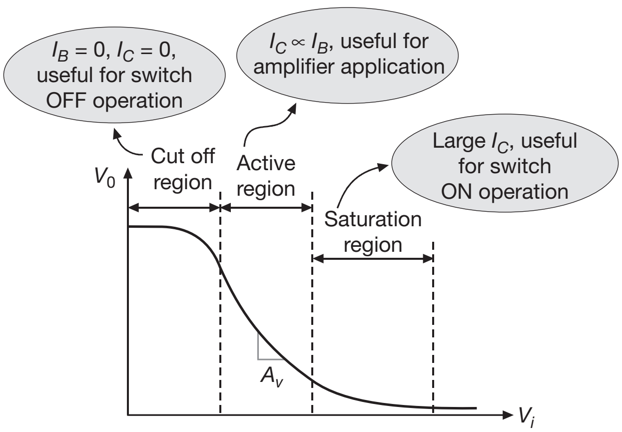
$\Rightarrow$ $V_{CE} = V_{CC} - I_C R_C$

The voltage V_{BB} can be regarded as the dc input voltage V_i and V_{CE} as the dc output voltage V_0. So, we can write

$$V_i = I_B R_B + V_{BE}$$

and $V_0 = V_{CC} - I_C R_C$ ...(1)

Figure shows a typical output voltage (V_0) plotted against the input voltage (V_i) characteristic, called the transfer characteristic of the base biased transistor. As discussed already, it has three well-defined regions.



Transfer characteristic of base-biased transistor

Cut-off Region

When V_i increases from zero to a low value (less than 0.6 V in case of a Si transistor), the forward bias of the emitter-base junction is insufficient to start a forward current. That is, $I_B = 0$ and hence $I_C = 0$. The transistor is said to be in the *cutoff region*. From equation (1), the output voltage is given by $V_0 = V_{CC}$.

Active Region

When V_i increases slightly above 0.6 V, a current I_C flows in the output circuit and the transistor is said to be in the *active region*. Since we have

$$V_0 = V_{CC} - I_C R_C$$

So, as the term $I_C R_C$ increases, the output voltage V_0 decreases. Now as V_i increases, I_C increases almost linearly and so V_0 decreases linearly till its value becomes less than 1.0 V.

Saturation Region

When the input voltage V_i is high i.e., the emitter-base junction is heavily forward biased, then the collector current I_C is large, due to which it a large potential drop across load resistance R_C is produced such that the emitter-collector junction also gets forward biased. Hence the output voltage V_0 decreases to almost zero and the transistor is said to be in the *saturation state* because it cannot pass any more collector current I_C.

Conceptual Note(s)

Please note that, the transitions from cutoff state to active state and from active state to saturation state are not sharply defined because these regions of the transfer characteristic are non-linear.

Switching Action of a Transistor

A transistor can be used as a switch when it is operated in cutoff and saturation states only. The design of a switch circuit is such that the transistor does not remain in the active state.

As long as the input voltage is low and unable to forward-bias the transistor, the output voltage V_0 (at V_{CC}) is high. When V_i is high enough to drive the transistor into saturation, then V_0 is low, nearly zero. When the transistor is not conducting, it is said to be switched off and when it is driven into saturation, it is said to be switched on. Corresponding to cutoff and saturation voltages of the transistor, if we define the low (0) and the high (1) states as below and above certain voltage levels, then a low input switches the transistor off and a high input switches it on. Alternatively, we can say that a low input to the transistor gives a high output and high input gives a low output.

AMPLIFYING ACTION OF A TRANSISTOR

For considering the transistor as an amplifier we will use the transistor in the Active mode as shown earlier in the output voltage (V_0) plotted against the input voltage (V_i) graph. When the base-emitter junction of a transistor is forward biased, the depletion layer about this junction is much smaller than the depletion layer around the base-collector junction which is reverse biased. Thus, the resistance R_{EB} of the emitter-base junction is much smaller than the resistance R_{BC} of the collector-base junction. So, power dissipation in the emitter base circuit is given by

$$P_{EB} = I_E^2 R_{EB}$$

Similarly, power dissipation in the base collector circuit is

$$P_{BC} = I_C^2 R_{BC}$$

Since, $I_E \approx I_C$ and $R_{BC} \gg R_{EB}$

$$\Rightarrow \quad P_{BC} \gg P_{EB}$$

i.e., the power dissipated in the base-collector circuit is much higher than the power dissipated in the emitter-base circuit or output power is much greater than the input power. This phenomenon is called *the amplifying action of a transistor*.

The base region of a transistor is very thin and lightly doped. A thin and lightly doped base region contains a smaller number of majority charge carriers. This reduces the rate of recombination of electrons and holes at the emitter-base junction. Most $(95-99\%)$ of the majority charge carriers, diffusing from emitter to base, reach the collector. Thus, the base current is small and the collector is almost equal to the emitter current. This results in the large voltage gain and power gain of the transistor.

Conceptual Note(s)

TRANSISTOR: A WORD MADE FROM TRANSFER AND RESISTOR

The resistance offered by the emitter-base junction to the flow of current is small because it is forward biased. Similarly, the resistance offered by the base-collector junction to the flow of current is large because this junction is reverse biased. However, since the collector current I_C is almost equal to the emitter current I_E, so we can say that the current is transferred from the low resistance circuit to the high resistance circuit. Hence the name transistor, which is combination of the words transfer and resistor.

TRANSCONDUCTANCE

For the case of a voltage amplifier, the input signal to be amplified is superposed on a steady voltage V_{EB} applied across the emitter-base junction. For a high voltage gain (the ratio of output voltage to the input voltage), the change in the collector current (ΔI_c) should be as large as possible, for a given change in the emitter-base voltage (ΔV_{EB}). So, we can define a new term that acts as a figure of merit for a transistor and this term is called the Transconductance.

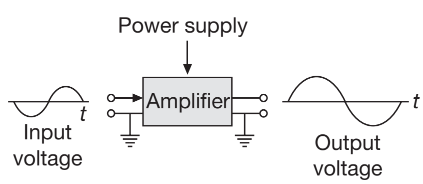
Transconductance is defined as the ratio of the small change in the collector current to the small change in the emitter-base voltage. It is denoted by g_m. Thus

$$g_m = \frac{\Delta I_C}{\Delta V_{BE}}$$

The transconductance is also called transfer conductance and has the same units of conductance (i.e. siemen or mho). The transconductance depends on the geometry, doping levels and biasing of the transistor.

CONCEPT OF AN AMPLIFIER

An **amplifier** is a circuit which consists of at least one transistor that can be used for increasing the voltage, current or power of ac input. To amplify means to increase the size or to magnify an input signal. The output signal of an amplifier is an enlarged version of the input signal. The general concept of an amplifier or simply the block diagram of the amplifier is shown in the Figure.



The concept of an amplifier

The amplifier has

(a) two input terminals across which the signal to be amplified is fed.

(b) two output terminals for connecting the load across which the amplified output is to be taken and

(c) a power supplying assembly to supply power to the amplifier.

(d) one input terminal and one output terminal (which is common to the input and output) earthed.

AC VOLTAGE GAIN (A_V)

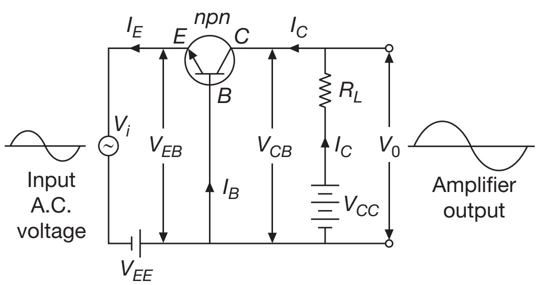
The usefulness of an amplifier is expressed in terms of the gain of the amplifier. The ac voltage gain of an amplifier is defined as the ratio of the change in the output voltage (ΔV_0) to the corresponding change in the input voltage (ΔV_i). Thus

$$A_V = \frac{\Delta V_0}{\Delta V_i}$$

The voltage gain of an amplifier is always greater than unity. Please note that, only the ac and not the dc components of the input and the output voltages are used to calculate the voltage gain.

NPN TRANSISTOR AS COMMON BASE AMPLIFIER

For an *npn* transistor used as a common base amplifier, the base is common to both input and output circuits. The emitter base junction is forward biased by the battery V_{EE} and the collector base junction is reverse biased by battery V_{CC} as shown in the Figure.



npn transistor as a common base amplifier

Due to this biasing technique, the resistance R_{in} of input circuit decreases and the resistance R_{out} of output circuit increases i.e. $R_{\text{out}} > R_{\text{in}}$. A low input ac signal is superimposed on the forward bias V_{EB} of the emitter base junction and a load resistance R_L is connected between the collector and dc supply. The amplified output is obtained between collector and ground. When no ac signal is fed to the input circuit, then

$$I_E = I_B + I_C$$

When a current I_C flows in the output circuit, a voltage drop $I_C R_L$ occurs across the load R_L. The output voltage V_0 is calculated by applying Kirchhoff's Loop Law (KLL) and is given by

$$V_0 = V_{CB} = V_{CC} - I_C R_L \qquad \ldots(1)$$

When the input signal V_i is applied across the emitter-base junction, then it changes the emitter-base voltage and hence the emitter current I_E is also changed. Due to change in the emitter current I_E, the collector current I_C also changes and hence the output voltage V_0 also changes in accordance with equation (1). This variation in the collector voltage appears as amplified output.

Phase Between Input and Output Signals

When ac signal is fed to the input circuit, then its positive half cycle decreases the forward bias. This decreases the emitter current and the collector current, due to which the potential drop $(I_C R_L)$ across load resistance decreases and hence V_0 increases. Since collector is connected to the positive terminal of the V_{CC} battery, so increase in V_0 implies that it becomes more positive and hence an amplified half output is obtained. So, as the input signals goes through its positive half cycle, the output signal also goes through a positive half cycle.

Similarly, as the input signal goes through its negative half cycle, the output signal also goes through its negative half cycle. Hence in a common base amplifier, the input and output voltages are in same phase.

CURRENT, VOLTAGE AND POWER GAIN FOR A COMMON BASE AMPLIFIER

AC Current Gain (α)

It is defined as the ratio of the small change in the collector current (ΔI_C) to the small change in the emitter current (ΔI_E) at constant collector-base voltage. It is commonly denoted by α_{ac} or sometimes is also denoted by A_i. So

$$\alpha_{ac} = A_i = \left(\frac{\Delta I_C}{\Delta I_E}\right)_{V_{CB}=\text{constant}}$$

DC Current Gain (α_{dc})

It is defined as the ratio of the collector current to the emitter current, at constant collector-base voltage. Mathematically,

$$\alpha_{dc} = \left(\frac{I_C}{I_E}\right)_{V_{CB}=\text{constant}}$$

AC Voltage Gain (A_V)

It is defined as the ratio of the small change in output voltage (ΔV_{CB}) to the small change in input voltage ΔV_{EB}. So,

$$A_V = \frac{\Delta V_{CB}}{\Delta V_{EB}}$$

Since, $\Delta V_{CB} = (\Delta I_C)R_0$ and $\Delta V_{EB} = (\Delta I_E)R_i$

where R_i is the resistance of input circuit and R_0 is the resistance of output circuit (including R_L).

$$\Rightarrow \quad A_V = \left(\frac{\Delta I_C}{\Delta I_E}\right)\left(\frac{R_0}{R_i}\right) = \alpha_{ac}\left(\frac{R_0}{R_i}\right)$$

$$\Rightarrow \quad A_V = A_i \times A_r$$

$$\Rightarrow \quad \begin{pmatrix}\text{Voltage}\\ \text{Gain}\end{pmatrix} = \begin{pmatrix}\text{Current}\\ \text{Gain}\end{pmatrix} \times \begin{pmatrix}\text{Resistance}\\ \text{Gain}\end{pmatrix}$$

AC Power Gain (A_P)

It is defined as the ratio of the small change in output power to the small change in input power.

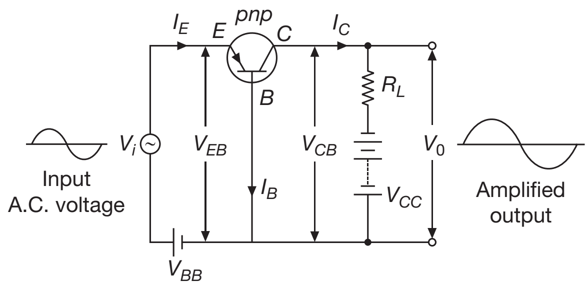
$$A_P = \frac{\text{Change in output power}}{\text{Change in input power}}$$

$$\Rightarrow \quad A_P = \frac{(\Delta I_C)^2 R_0}{(\Delta I_E)^2 R_i} = \left(\frac{\Delta I_C}{\Delta I_E}\right)^2 \left(\frac{R_0}{R_i}\right)$$

$$\Rightarrow \quad A_P = \alpha_{ac}^2 \left(\frac{R_0}{R_i}\right) = \alpha_{ac}^2\,(\text{Resistance Gain})$$

PNP TRANSISTOR AS COMMON BASE AMPLIFIER

For a *pnp* transistor used as a common base amplifier, the base is common to both input and output circuits. The emitter base junction is forward biased by the battery V_{EE} and the collector base junction is reverse biased by battery V_{CC} as shown in the figure.



pnp transistor as a common base amplifier

Due to this biasing technique, the resistance R_{in} of input circuit decreases and the resistance R_{out} of output circuit increases i.e. $R_{out} > R_{in}$. A low input ac signal V_i is superimposed on the forward bias V_{EB} of the emitter base junction and a load resistance R_L is connected between the collector and dc supply. The amplified output is obtained between collector and ground. When no ac signal is fed to the input circuit, then

$$I_E = I_B + I_C$$

When a current I_C flows in the output circuit, a voltage drop $I_C R_L$ occurs across the load R_L. The output voltage V_0 is calculated by applying Kirchhoff's Loop Law (KLL) and is given by

$$V_0 = V_{CB} = V_{CC} - I_C R_L \qquad \ldots(1)$$

When the input signal V_i is applied across the emitter-base circuit, then it changes the emitter-base voltage and hence the emitter current I_E is also changed. Due to change in the emitter current I_E, the collector current I_C also changes and hence the output voltage V_0 also changes in accordance with equation (1). This variation in the collector voltage appears as amplified output.

Phase Between Input and Output Signals

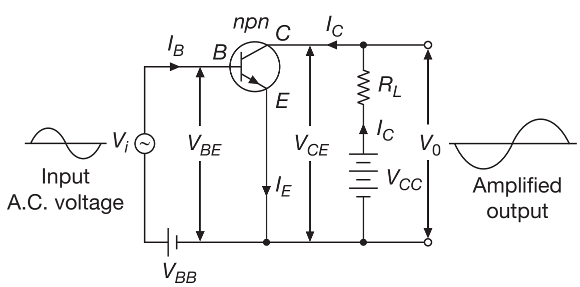
When ac signal is fed to the input circuit, then its positive half cycle increases the forward bias. This increases the emitter current and the collector current, due to which the potential drop $(I_C R_L)$ across load resistance increases and hence V_0 decreases. Since collector is connected to the negative terminal of the V_{CC} battery, so decrease in V_0 implies that it becomes less negative and hence more positive. Hence, an amplified half output is obtained, thus making us conclude that as the input signals goes through its positive half cycle, the output signal also goes through a positive half cycle.

Conceptual Note(s)

In a common base amplifier, the input and output voltages are in same phase i.e. phase angle between the input and the output voltage is 0°.

NPN TRANSISTOR AS COMMON EMITTER AMPLIFIER

For an *npn* transistor used as a common emitter amplifier, the emitter is common to both input and output circuits. The base emitter junction is forward biased by the battery V_{BB} and the collector emitter junction is reverse biased by battery V_{CC} as shown in the figure.



npn transistor as a common emitter amplifier

Due to this biasing technique, the resistance R_{in} of input circuit decreases and the resistance R_{out} of output circuit increases i.e. $R_{out} > R_{in}$. A low input ac signal V_i is superimposed on the forward bias V_{BE} of the base emitter junction and a load resistance R_L is connected between the collector and dc supply. The amplified output is obtained between collector and ground. When no ac signal is fed to the input circuit, then

$$I_E = I_B + I_C$$

When a current I_C flows in the output circuit, a voltage drop $I_C R_L$ occurs across the load R_L. The output voltage V_0 is calculated by applying Kirchhoff's Loop Law (KLL) and is given by

$$V_0 = V_{CE} = V_{CC} - I_C R_L \quad \ldots(1)$$

When the input signal V_i is applied across the base emitter circuit, then it changes the base emitter voltage and hence the emitter current I_E is also changed. Due to change in the emitter current I_E, the collector current I_C also changes and hence the output voltage V_0 also changes in accordance with equation (1). This variation in the collector voltage appears as amplified output.

Phase Between Input and output Signals

When ac signal is fed to the input circuit, then its positive half cycle increases the forward bias. This increases the emitter current and the collector current, due to which the potential drop $(I_C R_L)$ across load resistance increases and hence V_0 decreases. Since collector is connected to the positive terminal of the V_{CC} battery, so decrease in V_0 implies that it becomes less positive and hence more negative. Hence, an amplified half output is obtained, thus making us conclude that as the input signals goes through its positive half cycle, the output signal goes through a negative half cycle.

Similarly, as the input signal goes through its negative half cycle, the amplified output signal goes through its positive half cycle. Hence in a common emitter amplifier, the input and output voltages are $180°$ out of phase.

CURRENT, VOLTAGE AND POWER GAIN FOR A COMMON EMITTER AMPLIFIER

AC Current Gain (β)

It is defined as the ratio of the small change in the collector current (ΔI_C) to the small change in the base current (ΔI_B) at constant collector-emitter voltage. It is commonly denoted by β_{ac} or sometimes is also denoted by A_i. So

$$\beta_{ac} = A_i = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE}=\text{constant}}$$

DC Current Gain (β_{dc})

It is defined as the ratio of collector current to the base current, at constant collector-emitter voltage. Mathematically,

$$\beta_{dc} = \left(\frac{I_C}{I_B}\right)_{V_{CE}=\text{constant}}$$

In the linear region of the output characteristics, β_{ac} is usually close to β_{dc}.

AC Voltage Gain (A_V)

It is defined as the ratio of small change in output voltage ΔV_{CE} to the small change in input voltage ΔV_{BE}. So,

$$A_V = \frac{\Delta V_{CE}}{\Delta V_{BE}}$$

Since, $\Delta V_{BE} = R_i(\Delta I_B)$ and $\Delta V_{CE} = -R_0(\Delta I_C)$

where R_i is the resistance of the input or the emitter base circuit and R_0 is the resistance of the output or collector-emitter circuit (including R_L).

The negative sign indicates that the input and output voltages have a phase difference of 180° i.e., if the input voltage increases, the output voltage decreases.

$$\Rightarrow \quad A_V = -\left(\frac{\Delta I_C}{\Delta I_B}\right)\left(\frac{R_{\text{out}}}{R_{\text{in}}}\right) = -\beta_{ac}\left(\frac{R_{\text{out}}}{R_{\text{in}}}\right)$$

$$\Rightarrow \quad A_V = A_i \times A_r$$

$$\Rightarrow \quad \begin{pmatrix}\text{Voltage}\\ \text{Gain}\end{pmatrix} = \begin{pmatrix}\text{Current}\\ \text{Gain}\end{pmatrix} \times \begin{pmatrix}\text{Resistance}\\ \text{Gain}\end{pmatrix}$$

AC Power Gain (A_P)

It is defined as the ratio of the small change in output power to the small change in input power.

$$A_P = \frac{\text{Change in output power}}{\text{Change in input power}}$$

$$\Rightarrow \quad A_P = \frac{(\Delta I_C)^2 R_0}{(\Delta I_B)^2 R_i} = \left(\frac{\Delta I_C}{\Delta I_B}\right)^2\left(\frac{R_0}{R_i}\right)$$

$$\Rightarrow \quad A_P = \beta_{ac}^2\left(\frac{R_0}{R_i}\right) = \beta_{ac}^2\,(\text{Resistance Gain})$$

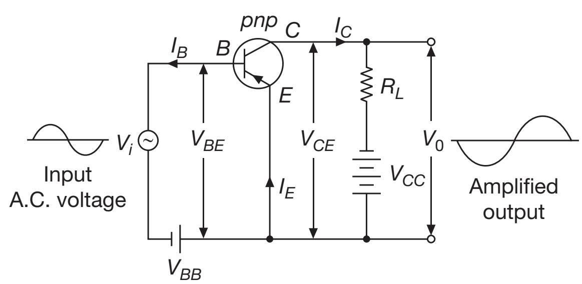
Since $\beta_{ac}^2 \gg \alpha_{ac}^2$, so the ac power gain of a common emitter amplifier is much larger than that of a common base amplifier.

Conceptual Note(s)

It may be noted that the transistor is not generating any power. The energy for the higher ac power at the output is supplied by the dc battery.

PNP TRANSISTOR AS COMMON EMITTER AMPLIFIER

For a *pnP* transistor used as a common emitter amplifier, the emitter is common to both input and output circuits. The base emitter junction is forward biased by the battery V_{BB} and the collector emitter junction is reverse biased by battery V_{CC} as shown in the figure.



pnp transistor as a common emitter amplifier

Due to this biasing technique, the resistance R_{in} of input circuit decreases and the resistance R_{out} of output circuit increases i.e. $R_{\text{out}} > R_{\text{in}}$. A low input ac signal V_i is superimposed on the forward bias V_{BE}

of the base emitter junction and a load resistance R_L is connected between the collector and dc supply. The amplified output is obtained between collector and ground. When no ac signal is fed to the input circuit, then

$$I_E = I_B + I_C$$

When a current I_C flows in the output circuit, a voltage drop $I_C R_L$ occurs across the load R_L. The output voltage V_0 is calculated by applying Kirchhoff's Loop Law (KLL) and is given by

$$V_0 = V_{CE} = V_{CC} - I_C R_L \qquad \ldots(1)$$

When the input signal V_i is applied across the base emitter circuit, then it changes the base emitter voltage and hence the emitter current I_E is also changed. Due to change in the emitter current I_E, the collector current I_C also changes and hence the output voltage V_0 also changes in accordance with equation (1). This variation in the collector voltage appears as amplified output.

Phase Between Input and Output Signals

When ac signal is fed to the input circuit, then its positive half cycle decreases the forward bias. This decreases the emitter current and hence the collector current, due to which the potential drop $(I_C R_L)$ across load resistance decreases and hence the output voltage V_0 increases. Since collector is connected to the negative terminal of the V_{CC} battery, so increase in V_0 implies that it becomes more negative. Hence, an amplified half output is obtained, thus making us conclude that as the input signals goes through its positive half cycle, the output signal goes through a negative half cycle.

Similarly, as the input signal goes through its negative half cycle, the amplified output signal goes through its positive half cycle. Hence in a common emitter amplifier, the input and output voltages are $180°$ out of phase.

ILLUSTRATION 29

For a transistor connected in common emitter mode, if $R_0 = 4\text{ k}\Omega$, $R_i = 1\text{ k}\Omega$, $I_C = 1\text{ mA}$ and $I_B = 20\ \mu\text{A}$, then calculate the voltage gain.

SOLUTION

The current gain for a transistor in common emitter mode is given by

$$\beta = \frac{I_C}{I_B} = \frac{1\times10^{-3}}{20\times10^{-6}} = 50$$

and the voltage gain in common emitter mode is

$$A_V = \beta\left(\frac{R_0}{R_i}\right) = (50)\left(\frac{4}{1}\right) = 200$$

ILLUSTRATION 30

A transistor is connected in common emitter configuration. The collector supply is 8 V and voltage drop across a resistor of $800\ \Omega$ in the collector circuit is 0.5 V. If the current gain factor α is 0.96, calculate the base current.

SOLUTION

Given that, $V_{CC} = 8\text{ V}$, $V_0 = I_C R_L = 0.5\text{ V}$, $R_L = 800\ \Omega$ and $\alpha = 0.96$

Since, $\beta = \dfrac{\alpha}{1-\alpha}$

$$\Rightarrow \quad \beta = \frac{0.96}{1-0.96} = 24$$

Since, $V_0 = I_C R_L$

$$\Rightarrow \quad 0.5 = I_C \times 800$$

$$\Rightarrow \quad I_C = \frac{0.5}{800}\text{ A}$$

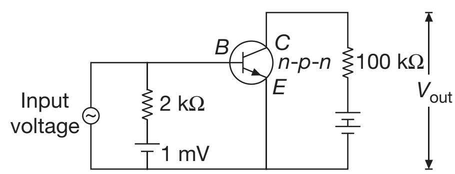
Since the current gain is given by $\beta = \dfrac{I_C}{I_B}$

$$\Rightarrow \quad I_B = \frac{I_C}{\beta} = \frac{\frac{0.5}{800}}{24} = 26\times10^{-6}\text{ A}$$

$$\Rightarrow \quad I_B = 26\ \mu\text{A}$$

ILLUSTRATION 31

In the following common emitter configuration an npn transistor with current gain $\beta = 95$ is used. Calculate the output voltage of the amplifier.



SOLUTION

The input current i.e. the base current is given by

$$I_{in} = I_B = \frac{V_{in}}{R_{in}} = \frac{1\ \text{mV}}{2\ \text{k}\Omega}$$

$$\Rightarrow \quad I_B = 0.5\times10^{-6}\ \text{A}$$

Since $\beta = \frac{I_{out}}{I_{in}} = \frac{I_C}{I_B}$

$$\Rightarrow \quad I_C = I_{out} = \beta I_B$$

$$\Rightarrow \quad I_C = 95\times0.5\times10^{-6}\ \text{A}$$

$$\Rightarrow \quad I_C = 0.47\times10^{-4}\ \text{A}$$

So, the output voltage is

$$V_0 = I_C R_L \text{, where } R_L = 100\ \text{k}\Omega$$

$$\Rightarrow \quad V_0 = (0.47\times10^{-4}\ \text{A})(100\ \text{k}\Omega)$$

$$\Rightarrow \quad V_0 = 4.7\ \text{mV}$$

ILLUSTRATION 32

A load of $3\ \text{k}\Omega$ is connected in the collector branch of an amplifier circuit using a transistor in common emitter mode. The current gain is 40. The input resistance of the transistor is $0.40\ \text{k}\Omega$. When the input current is changed by $40\ \mu\text{A}$, then calculate the change in output voltage, the change in the input voltage and the power gain.

SOLUTION

Given that

Load resistance $R_L = 3\ \text{k}\Omega = 3\times10^3\ \Omega$,

Current gain $\beta = 40$

Input reistance $r_i = 0.40\ \text{k}\Omega = 0.4\times10^3\ \Omega$ and

Change in base current $\Delta I_B = 40\ \mu\text{A} = 40\times10^{-6}\ \text{A}$

So, output voltage V_0 is

$$V_0 = \Delta I_C R_L = \beta \Delta I_B R_L$$

$$\Rightarrow \quad V_0 = 40\times40\times10^{-6}\times3\times10^3 = 4.8\ \text{V}$$

$$\Rightarrow \quad \Delta V_{BE} = \Delta I_B r_i = 40\times10^{-6}\times0.4\times10^3$$

$$\Rightarrow \quad \Delta V_{BE} = 16\times10^{-3}\ \text{V}$$

and power gain is

$$A_P = \frac{\beta^2 R_L}{r_i}$$

$$\Rightarrow \quad A_P = (40)^2\left(\frac{3\times10^3}{0.4\times10^3}\right) = 12000$$

ILLUSTRATION 33

An *npn* transistor is connected in common emitter configuration in which collector supply is $8\ \text{V}$ and the voltage drop across the load resistance of $800\ \Omega$ connected in the collector circuit is $0.8\ \text{V}$. If current amplification factor is 25, determine collector emitter voltage and base current. If the internal resistance of the transistor is $200\ \Omega$, calculate the voltage gain and the power gain.

SOLUTION

Voltage across the load resistance R_L is

$$V_0 = I_C R_L = 0.8\ \text{V}$$

$$\Rightarrow \quad I_C = \frac{0.8}{R_L} = \frac{0.8}{800}\ \text{A} = 1\ \text{mA}$$

Since, $\beta = 25 = \frac{I_C}{I_B}$

$$\Rightarrow \quad I_B = \frac{I_C}{25} = 40\ \mu\text{A}$$

Since, $V_{CE} = V_{CC} - I_C R_L$

$$\Rightarrow \quad V_{CE} = (8 - 0.8)\ \text{V} = 7.2\ \text{V}$$

Voltage gain is

$$A_V = \beta\left(\frac{R_{out}}{R_{in}}\right)$$

$$\Rightarrow \quad A_V = 25\left(\frac{800}{200}\right) = 100$$

Power gain is

$$A_P = \beta^2\left(\frac{R_{out}}{R_{in}}\right) = (25)^2\left(\frac{800}{200}\right) = 2500$$

ILLUSTRATION 34

A pnp transistor is used in common emitter mode in an amplifier circuit. A change of $45\ \mu\text{A}$ in the base current brings a change of $3\ \text{mA}$ in collector current and 0.05 V in base emitter voltage. Calculate the input resistance r_i and the base current amplification factor (β). If a load of 7 kΩ is used, then calculatealso find the voltage gain of the amplifier.

SOLUTION

Given that, $\Delta I_B = 45\ \mu\text{A} = 45\times10^{-6}\ \text{A}$,

$$\Delta I_C = 3\ \text{mA} = 3\times10^{-3}\ \text{A}$$

$$\Delta V_{BE} = 0.05\ \text{V},\ R_L = 7\ \text{k}\Omega = 7\times10^{3}\ \Omega$$

Input resistance

$$r_i = \frac{\Delta V_{BE}}{\Delta I_B} = \frac{0.05}{45\times10^{-6}} = 1.1\times10^{3}\ \Omega$$

Base current amplification factor

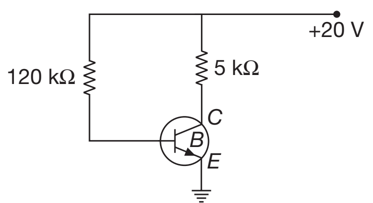
$$\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{3\times10^{-3}}{45\times10^{-6}} = 66.67$$

Voltage gain of the amplifier

$$A_V = \beta\frac{R_L}{r_i} = \frac{66.67\times7\times10^{3}}{1.1\times10^{3}} = 424.27$$

ILLUSTRATION 35

In the following circuit, the value of β is 200. Find I_B, V_{CE}, V_{BE} and V_{BC}, when $I_C = 2.5$ mA. Find whether transistor is in active, cut off or saturation state.



SOLUTION

Since, $\beta = \dfrac{I_C}{I_B}$

$$\Rightarrow \quad I_B = \frac{I_C}{\beta} = \frac{2.5}{200} = 0.0125\ \text{mA}$$

Applying Kirchhoff's Lop Law to base emitter loop, we get

$$V_{CE} = V_C - I_C R_C$$

$$\Rightarrow \quad V_{CE} = 20 - (2.5\times10^{-3})\times(5\times10^{3})$$

$$\Rightarrow \quad V_{CE} = 7.5\ \text{V}$$

Similarly, we know that

$$V_{BE} = V_C - I_B R_B$$

$$\Rightarrow \quad V_{BE} = 20 - (0.0125\times10^{-3})\times(120\times10^{3})$$

$$\Rightarrow \quad V_{BE} = 18.5\ \text{V}$$

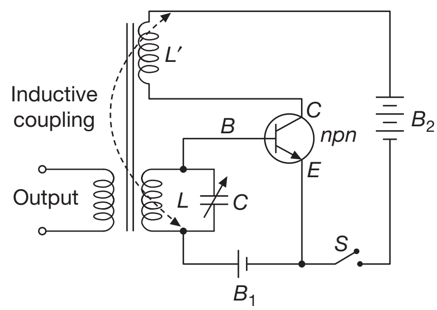
$$\Rightarrow \quad V_{BC} = V_{BE} - V_{CE}$$

$$\Rightarrow \quad V_{BC} = (18.5 - 7.5)\ \text{V} = 11\ \text{V}$$

The transistor is in active state because base emitter junction is forward biased and the base collector junction is reverse biased.

TRANSISTOR AS AN OSCILLATOR

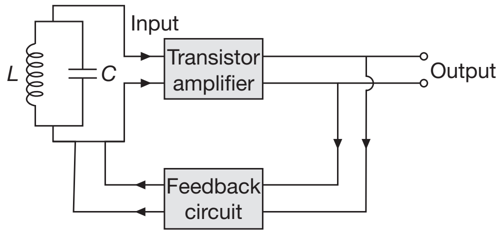
Oscillator is an electronic device that produces electric oscillations of constant frequency and amplitude, without the need of any external input signal. It converts dc energy obtained from a battery into ac energy in an oscillatory circuit.
A basic circuit using a common-emitter *npn* transistor as an oscillator is shown in Figure.



A tank circuit consisting of an inductance L and a variable capacitor C is connected in the input or the emitter-base circuit which is forward biased. A small coil L' called feedback coil or tickler coil is connected in the output or the emitter-collector circuit which is reverse biased. The coil L' is inductively coupled with the coil L of the tank circuit.

Principle of an Oscillator

An oscillator may be regarded as the self-sustained transistor amplifier with a positive feedback. The block diagram of an oscillator is shown in Figure.



Principle for an oscillator

Essential Parts of an Oscillator

A transistor oscillator has the following essential parts.

(a) Tank circuit: A tank circuit is just a parallel combination of an inductance L and a capacitance C. The electric energy once given to it alternately changes between electrostatic energy in the capacitor and the magnetic energy in the inductor. The frequency of electric oscillations in the tank circuit is

$$f = \frac{1}{2\pi\sqrt{LC}}$$

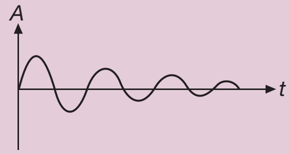
However, actually the oscillations get damped due to resistive losses in the inductance, and dielectric losses in the capacitor.

(b) Transistor amplifier: The oscillations of the tank circuit are fed to the transistor amplifier. The oscillations get amplified due to the amplifying action of the transistor.

(c) Feedback circuit: To compensate for the energy losses occurring in the tank circuit, the feedback circuit returns (feeds back) a part of the output power of the transistor amplifier to the tank circuit in phase with the input signal. *This process is called positive feedback* and produces undamped oscillations. The feedback may be done through inductive coupling (mutual inductance).

Conceptual Note(s)

Need for positive feedback: The oscillations are damped due to the presence of some inherent electrical resistance in the circuit. Consequently, the amplitude of oscillations decreases rapidly and the oscillations ultimately stop. Such oscillations are of little practical importance. In order to obtain oscillations of constant amplitude, we make an arrangement for **regenerative** or **positive feedback** from the output circuit to the input circuit so that the losses in the circuit can be compensated.



WORKING OF AN OSCILLATOR

When the switch S is closed, a small collector current starts growing through coil L'. This increases the magnetic flux linked with coil L' and hence with coil L (because both are inductively coupled). This induces an emf in the coil L that supports the forward bias and a positive charge begins to build on the upper plate of capacitor C. Due to this, the emitter current increases and hence the collector current also increases. This increases the magnetic flux linked with L' and hence with L. Consequently, the forward bias increases due to which the emitter current and the collector current also increase. Because of this, the charging of the capacitor continues and this process continues till the collector current becomes maximum.

Now, when the current through L' stops changing, the induced emf linked with L vanishes. This decreases the emitter current and hence the collector current. The decreasing current through L' induces an emf in L that opposes the forward bias, which results in decrease in the emitter current and hence the collector current. At the same instant, the positive charge on the lower plate of capacitor C begins to build up. The process continues till the collector current becomes zero, but the inertia of the collapsing magnetic field carries the collector current below the zero value. The induced emf linked with L again becomes zero, i.e. the forward bias is now not being opposed by induced emf. The emitter current and hence the collector current will start increasing. This cycle repeats again and again to give electric oscillations of constant amplitude and of constant frequency,

$$f = \frac{1}{2\pi\sqrt{LC}}$$

The oscillations of a desired frequency can be obtained by changing the value of capacitance C of the variable capacitor.

Conceptual Note(s)

(a) If the tank circuit is connected on the base side (as done above), then the oscillator is called as a tuned base oscillator.

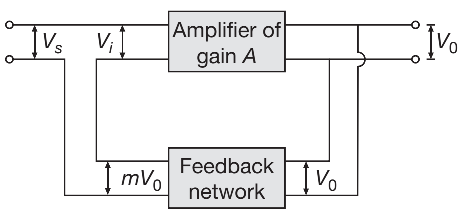
(b) If the tank circuit is connected on the collector side, then the oscillator is also called as a tuned collector oscillator.

(c) In the common emitter transistor circuit, a signal applied to the base emitter circuit appears with a phase change of 180° in the collector emitter circuit. The coupling of L and L' produces a further phase change of 180° due to mutual induction. Hence the energy fed back to the tank circuit is in phase with the input signal. Due to this positive feedback, the oscillations of the tank circuit are correctly maintained.

BARKHAUSEN'S CRITERION FOR SUSTAINED OSCILLATIONS

When a part of the output is fed back to the input of an amplifier, the process is called feedback process. Figure shows a feedback amplifier with input V_S and output V_0. The voltage gain of the feedback amplifier is

$$A'_V = \frac{\text{Output}}{\text{Input}} = \frac{V_0}{V_s}$$



Principle of feedback oscillator.

The input given to the feedback network is V_0. If m is the feedback fraction of the feedback network, then output obtained from it is mV_0. This fraction is mixed with the signal voltage V_s and is given to the amplifier.

$\Rightarrow$ Input of the amplifier, $V_i = V_s + mV_0$

The voltage gain of the amplifier is

$$A = \frac{\text{Output}}{\text{Input}} = \frac{V_0}{V_s + mV_0}$$

$$\Rightarrow \quad AV_s + AmV_0 = V_0$$

$$\Rightarrow \quad AV_s = V_0(1 - mA)$$

$$\Rightarrow \quad \frac{V_0}{V_s} = \frac{A}{1 - mA}$$

Hence the voltage gain of the feedback amplifier is

$$A'_V = \frac{V_0}{V_s} = \frac{A}{1 - mA}$$

When $mA = 1$, $A'_V = \frac{V_0}{V_s} \to \infty$. This means $V_s = 0$

Thus, the output voltage is obtained without the input voltage. The amplifier becomes a self-sustained oscillator. Hence the condition for stable oscillations to be sustained is $mA = 1$. *This is known as Barkhausen's criterion for sustained oscillations.*

Conceptual Note(s)

If the feedback is negative, the gain of the amplifier becomes

$$A'_V = \frac{A}{1 + mA}$$

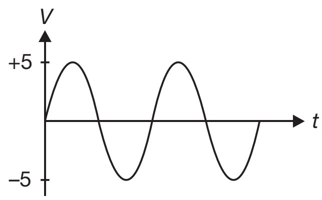
In an oscillator, the feedback is in same phase (positive feedback). If the feedback voltage is in opposite phase (negative feedback), the gain is less than 1 and it can never work as an oscillator. It will be an amplifier with reduced gain. However, the negative feedback reduces the noise and distortion of an amplifier.

Different oscillators use different feedback networks (such as inductive coupling or LC or RC networks) for coupling the output to the input apart from the resonant circuit for obtaining oscillations of a particular frequency. These give rise to different types of oscillators like Colpitt's oscillator, Hartley oscillator, RC-oscillator, etc.

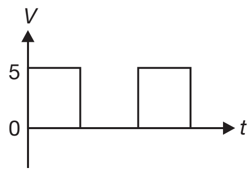
DIGITAL ELECTRONICS

Till now we have studied analog electronic circuits. In these kinds of circuits, the voltage signals vary continuously with time and such signals are called

continuous or analog voltage signals. Figure shows a typical voltage signal varying sinusoidally between 0 and 5 V.



The digital electronic circuits make use of entirely different type of voltage signals. In such signals, the pulse wave form does not vary continuously with time as in analogue voltage signals. Instead, it has only two voltage levels, either zero or some constant value of voltage. Such voltage signals are called **Digital Voltage Signals**. Figure shows a digital voltage signal, which at any instant will either be equal to 0 V or 5 V.



By representing these two voltage levels by binary numbers 0 and 1, the digital electronics has been developed. The counters, computers, etc are the outcome of digital electronics.

LOGIC GATES

A gate is a digital circuit, which works in accordance with some logical relationship between input and output voltages. Therefore, they are generally known as logic gates (gates because they control the flow of information). The five common logic gates used are OR, AND, NOT, NAND, NOR. Each logic gate is indicated by a symbol and its function is defined by a truth table that shows all the possible input logic level combinations with their respective output logic levels.

The logic gates are building blocks of digital electronics. They are used in digital electronics to change one voltage level (input voltage) into another (output voltage) according to some logical statement relating both.

A logic gate may have one input or multiple inputs, but it has only one output. The relation between the possible values of input and output voltages are expressed in the form of a table called **Truth Table or Table of Combinations**. Truth table of a logic gate is a table that shows all the input and output possibilities for the logic gate. Truth tables help us understand the behaviour of logic gates. These logic gates can be realised using semiconductor devices.

BOOLEAN ALGEBRA BASICS

1. Identity Law	
AND FORM	$1A = A$
OR FORM	$0 + A = A$
2. Null Law	
AND FORM	$0A = 0$
OR FORM	$1 + A = 1$
3. Idempotent Law	
AND FORM	$AA = A$
OR FORM	$A + A = A$
4. Inverse Law	
AND FORM	$A\bar{A} = 0$
OR FORM	$A + \bar{A} = 1$
5. Commutative Law	
AND FORM	$AB = BA$
OR FORM	$A + B = B + A$
6. Associative Law	
AND FORM	$(AB)C = A(BC)$
OR FORM	$A + (B + C) = (A + B) + C$
7. Distributive Law	
AND FORM	$A + BC = (A + B)(A + C)$
OR FORM	$A(B + C) = AB + AC$
8. Absorption Law	
AND FORM	$A(A + B) = A$
OR FORM	$A + AB = A$
9. De Morgan's Law	
AND FORM	$\overline{AB} = \bar{A} + \bar{B}$
OR FORM	$\overline{A + B} = \bar{A}\bar{B}$

Conceptual Note(s)

Apart from the above rules please keep in mind that

(a) $\bar{1}=0$ **(b)** $\bar{0}=1$ **(c)** $1+1=1$

(d) $1+\bar{1}=1$ **(e)** $0+\bar{0}=1$

ILLUSTRATION 36

Write down truth tables for

(a) $Y=\bar{A}\bar{B}+\bar{A}B$ **(b)** $Y=\left(A+\bar{B}\right)+\overline{AB}$

SOLUTION

(a)

A	B	$\bar{A}$	$\bar{B}$	$\bar{A}\bar{B}$	$\bar{A}B$	Y
0	0	1	1	1	0	1
0	1	1	0	0	1	1
1	0	0	1	0	0	0
1	1	0	0	0	0	0

(b)

A	B	$\bar{B}$	$A+\bar{B}$	AB	$\overline{AB}$	Y
0	0	1	1	0	1	1
0	1	0	0	0	1	1
1	0	1	1	0	1	1
1	1	0	1	1	0	1

ILLUSTRATION 37

Let $Y=A\left(\overline{BC}\right)$. Evaluate Y for

(a) $A=1$, $B=0$, $C=1$,

(b) $A=B=C=1$ and

(c) $A=B=C=0$

SOLUTION

(a) When, $A=1$, $B=0$, $C=1$, then $BC=0$

$\Rightarrow \quad \overline{BC}=1$

$\Rightarrow \quad A\overline{BC}=1$

(b) When, $A=B=C=1$, then $BC=1$

$\Rightarrow \quad \overline{BC}=0$

$\Rightarrow \quad A\overline{BC}=0$

(c) When, $A=B=C=0$, then $BC=0$

$\Rightarrow \quad \overline{BC}=1$

$\Rightarrow \quad A\overline{BC}=0$

ILLUSTRATION 38

Let $Y=A\overline{BC}+B\overline{CA}+C\overline{AB}$. Find the output Y if following inputs are given

(a) $A=1$, $B=0$, $C=1$

(b) $A=B=C=1$

(c) $A=B=C=0$

SOLUTION

A	B	C	$\overline{AB}$	$\overline{BC}$	$\overline{CA}$	$A\overline{BC}$	$B\overline{CA}$	$C\overline{AB}$	Y
0	0	0	1	1	1	0	0	0	0
1	1	1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	0	1	1

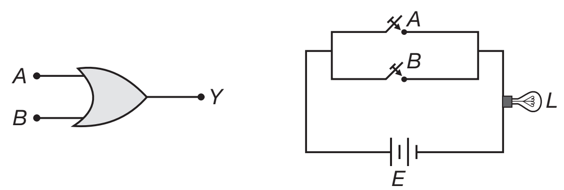
(a) $Y=1$ **(b)** $Y=0$ **(c)** $Y=0$

THE 'OR GATE'

The OR gate is a two inputs and one output logic gate. It combines the inputs A and B with the output Y following the Boolean expression

$$Y=A+B$$

to be read as Y equals A OR B. The symbol of OR gate, its electrical analogue and its truth table are shown.

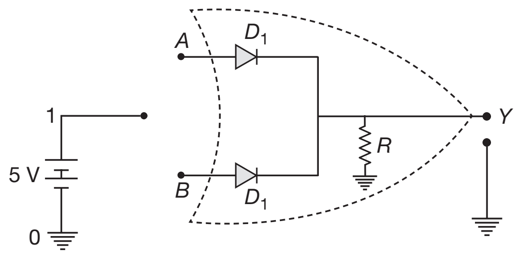


SYMBOL ELECTRICAL ANALOGUE

A	B	Y
0	0	0
1	0	1
0	1	1
1	1	1

Realisation of OR Gate

The negative terminal of the battery is grounded and corresponds to the 0 state and the positive (i.e., voltage 5 V in the present case) to the 1 state.



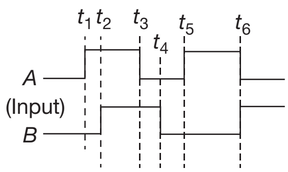
When both A and B are connected to 0, no current passes through the diode and therefore no voltage develops across R and the output is zero.

When input A is connected to zero and B to 1, the diode D_2 is forward biased and the current through it is limited by a current limiting resistance. This current causes a 5 V drop across the resistance assuming the diode to be ideal and this gives an output of 5 V or 1. Interchanging A and B to 1 and 0 will still give a 5 V drop across the resistance as D_1 will conduct.

When the terminals A and B are connected to 1, then both the diodes D_1 and D_2 conduct. However, the voltage drop across R cannot exceed 5 V and the output is 1. Hence the truth table is satisfied.

ILLUSTRATION 39

For the input waveforms A and B shown in Figure, sketch the output waveform (Y) obtained from OR gate.



SOLUTION

Note the following:

(a) At $t < t_1$; $A = 0$, $B = 0$; Hence $Y = 0$

(b) For t_1 to t_2; $A = 1$, $B = 0$; Hence $Y = 1$

(c) For t_2 to t_3; $A = 1$, $B = 1$; Hence $Y = 1$

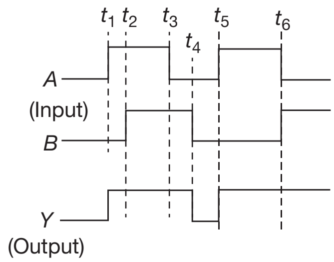
(d) For t_3 to t_4; $A = 0$, $B = 1$; Hence $Y = 1$

(e) For t_4 to t_5; $A = 0$, $B = 0$; Hence $Y = 0$

(f) For t_5 to t_6; $A = 1$, $B = 0$; Hence $Y = 1$

(g) For $t > t_6$; $A = 0$, $B = 1$; Hence $Y = 1$

Therefore, the waveform Y will be as shown in the figure.

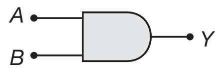


THE AND GATE

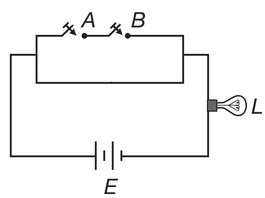
The AND gate is also a two inputs and one output logic gate. It combines the inputs A and B with the output Y following the Boolean expression

$$Y = A \cdot B$$

to be read as Y equals A AND B. The symbol of AND gate its electrical analogue and its truth table are shown.



SYMBOL



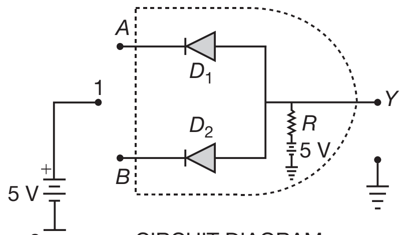
ELECTRICAL ANALOGUE

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	1

Realisation of AND Gate

The resistor R is connected permanently to the positive terminal of a 5 V battery. When both A and B are connected to zero, both the diodes conduct.

The voltage output at Y will be the voltage across the diode which is 0 assuming the diodes to be ideal.



CIRCUIT DIAGRAM

When A is connected to 0 and B to 1, the upper diode conducts while lower diode does not conduct as it is not forward biased. The voltage output of Y will then be the voltage across the upper diode which is 0.

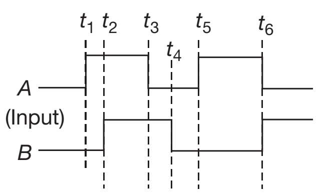
When A is 1 and B is 0, the lower diode conducts and the output is 0.

When both A and B are connected to 1, none of the diodes conduct. Hence the voltage at the output Y will be the battery voltage i.e., Y will be 1.

Thus, we see that the circuit in figure can perform the function of an AND gate. We note that the output is 1 only when both the inputs are 1.

ILLUSTRATION 40

For the input waveforms A and B shown in Figure, sketch the output waveform (Y) obtained from AND gate.



SOLUTION

(a) For $t \le t_1$; $A = 0$, $B = 0$; Hence $Y = 0$

(b) For t_1 to t_2; $A = 1$, $B = 0$; Hence $Y = 0$

(c) For t_2 to t_3; $A = 1$, $B = 1$; Hence $Y = 1$

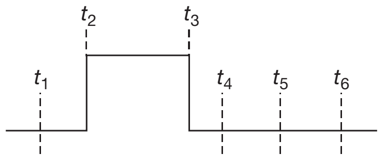
(d) For t_3 to t_4; $A = 0$, $B = 1$; Hence $Y = 0$

(e) For t_4 to t_5; $A = 0$, $B = 0$; Hence $Y = 0$

(f) For t_5 to t_6; $A = 1$, $B = 0$; Hence $Y = 0$

(g) For $t > t_6$; $A = 0$, $B = 1$; Hence $Y = 0$

Based on the above, the output waveform for AND gate can be drawn as given below.

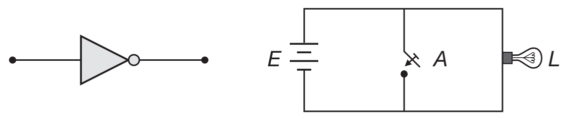


THE 'NOT GATE'

The NOT gate is a one input and one output logic gate. It combines the input A with the output Y following the Boolean expression

$$Y = \overline{A}$$

to be read as Y equals NOT A. The way, the NOT gate gives the output, it is also called **Invertor or Negator**. It is represented by the symbol as shown in figure.

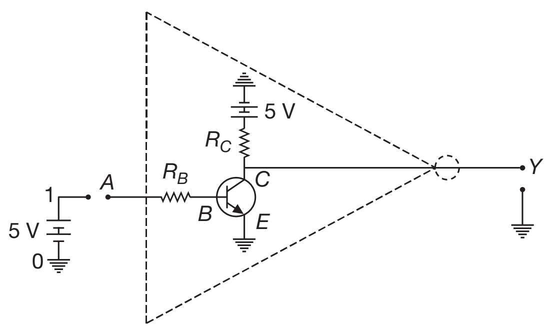


SYMBOL ELECTRICAL ANALOGUE

A	Y
0	1
1	0

Realisation of NOT Gate

A NOT gate cannot be realised by diodes and we have to use a transistor. We choose R_B and R_C such that when 5 V (or voltage corresponding to 1 state) is applied at the base, a large collector current flows, the voltage at Y drops and the base-collector junction is forward-biased.



When A is connected to 0, the collector base is reverse biased and the base emitter junction is not forward biased. So the base current is zero and hence the collector current is zero. The transistor is then said to be in the cut-off mode and the voltage at Y is 5 V, which corresponds to the 1 state. When A is connected to 1, the transistor goes to saturation, the voltage drop across R_C is almost equal to 5 V and the output Y is very nearly 0 V corresponding to 0 of truth table.

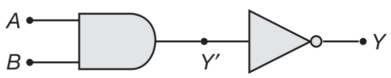
COMBINATION OF GATES

Various combinations of three basic gates i.e. OR, AND and NOT give rise to complicated digital circuits. We now discuss a few combinations of these basic gates using their symbols,

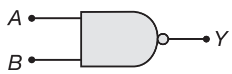
The NAND Gate

It is a logic circuit in which AND gate is followed by a NOT gate.

If the output (Y') of AND gate is connected to the input of NOT gate, the gate so obtained is called NAND gate. The logic symbol of the NAND gate is as shown in figure.



(a)



(b)

The truth table of NAND gate can be obtained by combining the truth tables of AND and NOT gates. It will be as given in figure.

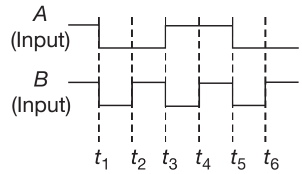
A	B	Y	Y′
0	0	0	1
1	0	0	1
0	1	0	1
1	1	1	0

Boolean expression for the NAND gate is

$$Y = \overline{AB}$$

ILLUSTRATION 41

Sketch the output Y from a NAND gate having inputs A and B shown in Figure.



SOLUTION

(a) For $t < t_1$; $A = 1$, $B = 1$; Hence $Y = 0$

(b) For t_1 to t_2; $A = 0$, $B = 0$; Hence $Y = 1$

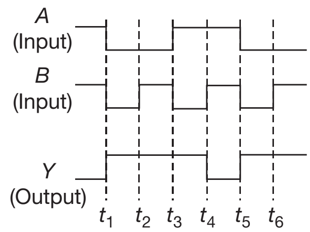
(c) For t_2 to t_3; $A = 0$, $B = 1$; Hence $Y = 1$

(d) For t_3 to t_4; $A = 1$, $B = 0$; Hence $Y = 1$

(e) For t_4 to t_5; $A = 1$, $B = 1$; Hence $Y = 0$

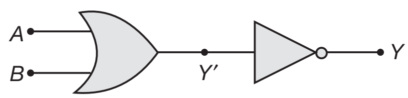
(f) For t_5 to t_6; $A = 0$, $B = 0$; Hence $Y = 1$

(g) For $t > t_6$; $A = 0$, $B = 1$; Hence $Y = 1$

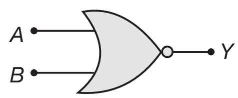


The NOR Gate

It is a logic circuit in which OR gate is followed by a NOT gate. If the output (Y') of OR gate is connected to the input of a NOT gate, the gate so obtained is called the NOR gate. The logic symbol of the NOR gate is shown in figure.



(a)



(b)

The truth table of NOR gate can be obtained by combining the truth tables of OR and NOT gates.

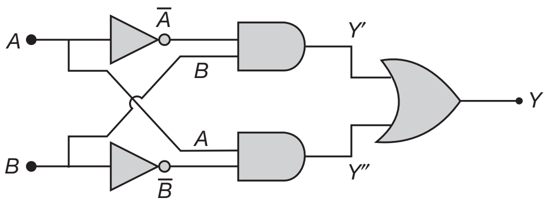
A	*B*	*Y′*	*Y*
0	0	0	1
1	0	1	0
0	1	1	0
1	1	1	0

Boolean expression for the NOR gate is

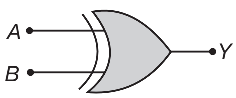
$$Y = \overline{A + B}$$

THE 'XOR GATE'

XOR gate is obtained by using OR, AND and NOT gates as shown in figure. It is also called **Exclusive OR gate** and its logic symbol is as shown in figure.



(a)



(b)

Its truth table is given in figure. It follows that output in case of XOR gate is 1, only when inputs are different.

A	*B*	*Y*
0	0	0
0	1	1
1	0	1
1	1	0

The Boolean equation for the XOR gate is

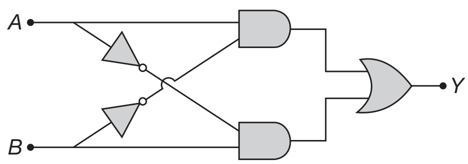
$$Y = A\overline{B} + \overline{A}B$$

Conceptual Note(s)

The NAND and the NOR gates are the building blocks of the digital electronics, because all the logic gates like the OR, the AND, the NOT can be constructed by using the NAND gates or by using the NOR gates.

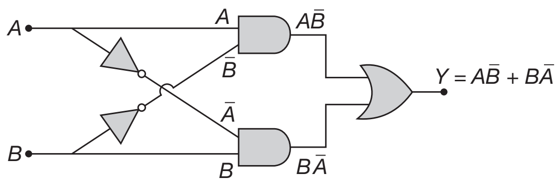
ILLUSTRATION 42

Write the truth table for the circuit given in figure.



SOLUTION

The circuit can be redrawn as,



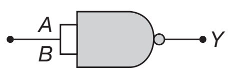
The corresponding truth table is

A	*B*	$\overline{A}$	$\overline{B}$	$A\overline{B}$	$B\overline{A}$	*Y*
0	0	1	1	0	0	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	0	0	0	0

LOGIC GATES USING NAND GATES

Construction of the 'NOT' gate from the 'NAND' gate

When both the inputs (A and B) of the NAND gate are joined together then it works as the NOT gate.

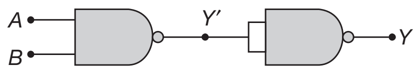


Truth table and logic symbol

Input	Output
$A = B$	Y
0	1
1	0

Construction of the 'AND' gate from the 'NAND' gate

When the output of the NAND gate is given to the input of the NOT gate (made from the NAND gate), then the resultant logic gate works as the AND gate

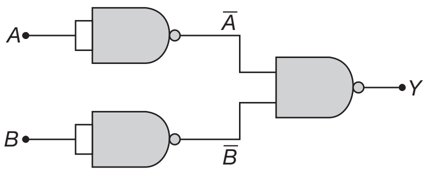


Truth table and logic symbol

A	B	Y′	Y
0	0	1	0
0	1	1	0
1	0	1	0
1	1	0	1

Construction of the 'OR' gate from the 'NAND' gate

When the outputs of two NOT gates (obtained from the NAND gate) is given to the inputs of the NAND gate, the resultant logic gate works as the OR gate



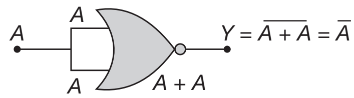
Truth table and logic symbol

A	B	$\bar{A}$	$\bar{B}$	Y
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

LOGIC GATES USING NOR GATES

Construction of the 'NOT' gate from the 'NOR' gate

When both the inputs (A and B) of the NAND gate are joined together then it works as the NOT gate.



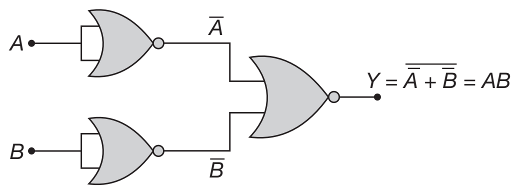
NOT Gate

Truth table and logic symbol

Input	Output
0	1
1	0

Construction of the 'AND' gate from the 'NOR' gate

When the output of the NAND gate is given to the input of the NOT gate (made from the NAND gate), then the resultant logic gate works as the AND gate



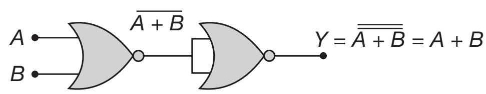
AND gate

Truth table and logic symbol

A	B	$Y = AB$
0	0	0
0	1	0
1	0	0
1	1	1

Construction of the 'OR' gate from the 'NOR' gate

When the outputs of two NOT gates (obtained from the NAND gate) is given to the inputs of the NAND gate, the resultant logic gate works as the OR gate



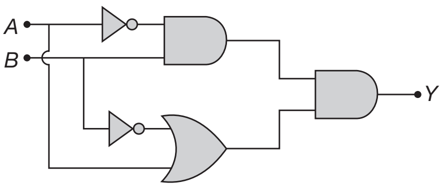
OR gate

Truth table and logic symbol

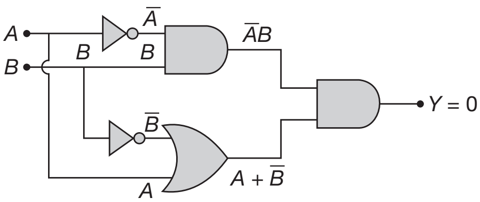
A	B	$Y = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

ILLUSTRATION 43

Find the output Y and write the truth table for the following circuit.



SOLUTION



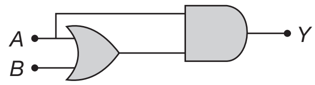
$$Y = \bar{A}B(A + \bar{B})$$

$$Y = A\bar{A}B + \bar{A}\bar{B}B = 0$$

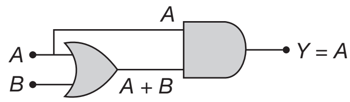
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	0

ILLUSTRATION 44

Find the output Y and write the truth table for the following circuit.



SOLUTION



$$Y = A(A + B)$$

$$\Rightarrow \quad Y = A + AB$$

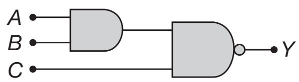
$$\Rightarrow \quad Y = A(1 + B)$$

$$\Rightarrow \quad Y = A$$

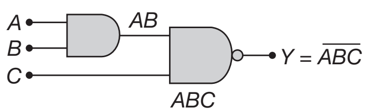
A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

ILLUSTRATION 45

Find the output Y when all inputs are first high and then low.



SOLUTION



Since, $Y = \overline{ABC}$

When all inputs are high i.e. $A = 1$, $B = 1$, $C = 1$ then

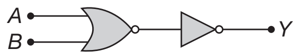
$$Y = \bar{1} = 0$$

When all inputs are low i.e. $A = 0$, $B = 0$, $C = 0$ then

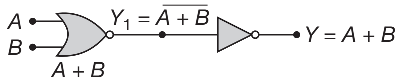
$$Y = \bar{0} = 1$$

ILLUSTRATION 46

Write the truth table for the following circuit. Name the equivalent gate that this circuit represents.



SOLUTION

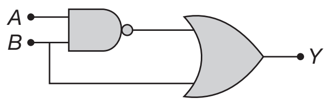


A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

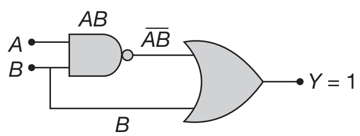
From the truth table, it is clear that the output is high only when atleast one of the inputs is high (i.e. 1). So, the circuit corresponds to OR gate.

ILLUSTRATION 47

Write the Boolean equation for the following circuit. Also write the truth table.



SOLUTION



$$Y = \overline{AB} + B$$

According to de-Morgan's Law

$$\overline{AB} = \bar{A} + \bar{B}$$

$$\Rightarrow \quad Y = \overline{AB} + B = \bar{A} + \bar{B} + B$$

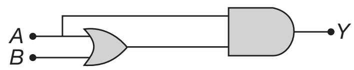
Since, $B + \bar{B} = 1$

$$\Rightarrow \quad Y = \bar{A} + 1 = 1$$

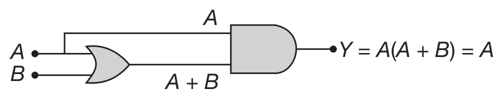
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	1

ILLUSTRATION 48

Draw the truth table for the function Y of A and B for the following logic gate.



SOLUTION



The output Y in terms of the input A and B can be written as,

$$Y = A(A + B)$$

$$\Rightarrow \quad Y = AA + AB$$

Since, $AA = A$

$$\Rightarrow \quad Y = A + AB = A(1 + B)$$

Since, $1 + B = 1$

$$\Rightarrow \quad Y = A$$

The truth table for the logic gate is

A	B	$Y = A$
0	0	0
0	1	0
1	0	1
1	1	1

SOLVED PROBLEMS

PROBLEM 1

In a silicon sample, the number density of silicon atoms is $5\times10^{28}\ \text{m}^{-3}$. This sample is doped simultaneously with $5\times10^{22}\ \text{m}^{-3}$ atoms of arsenic and $5\times10^{20}\ \text{m}^{-3}$ atoms of indium. Calculate the number density of electrons and holes. Given that $n_i = 1.5\times10^{16}\ \text{m}^{-3}$. Is the sample p-type or n-type.

SOLUTION

Since we know that, for each atom doped of arsenic one free electron is received and for each atom doped of indium a vacancy is created. So, the number of free electrons introduced by adding the pentavalent impurity is

$$n_e = N_{As} = 5\times10^{22}\ \text{m}^3 \qquad \ldots(1)$$

The number of holes introduced by adding the trivalent impurity is

$$n_e - n_h = 5\times10^{22} - 5\times10^{20}$$

$$\Rightarrow \quad n_e - n_h = 4.95\times10^{22} \qquad \ldots(2)$$

Since we know that

$$(n_e + n_h)^2 = (n_e - n_h)^2 + 4n_e n_h$$

Also, $n_e n_h = n_i^2$

$$\Rightarrow \quad (n_e + n_h)^2 = (n_e - n_h)^2 + 4n_i^2$$

$$\Rightarrow \quad n_e + n_h = \sqrt{(4.95\times10^{22})^2 + 4(1.5\times10^{16})^2}$$

$$\Rightarrow \quad n_e + n_h = 4.95\times10^{22} \qquad \ldots(3)$$

Adding Equations (3) and (2), we get

$$2n_e = 2(4.95\times10^{22})$$

$$\Rightarrow \quad n_e = 4.95\times10^{22}\ \text{m}^{-3}$$

Since $n_i^2 = n_h n_e$

$$\Rightarrow \quad n_h = \frac{n_i^2}{n_e} = \frac{(1.5\times10^{16})^2}{4.95\times10^{22}} = 4.54\times10^9\ \text{m}^{-3}$$

Now, since the number density of electrons $n_e\,(= 4.95\times10^{22})$ is greater than number of holes $n_h\,(= 4.5\times10^9)$. So, the sample is an n-type semiconductor.

PROBLEM 2

Suppose the number density of silicon atoms in a pure silicon is $5\times10^{28}\ \text{m}^{-3}$. It is doped by 1 ppm concentration of arsenic atoms. Calculate the number of electrons and holes. Given that $n_i = 1.5\times10^{16}\ \text{m}^{-3}$.

SOLUTION

Since, $n_i = 1.5\times10^{16}\ \text{m}^{-3}$

Doping concentration of pentavalent arsenic atoms is 1 ppm i.e. 1 part per million

So, number density of pentavalent arsenic atoms is

$$N_d = \frac{5\times10^{28}}{10^6} = 5\times10^{22}\ \text{atom m}^{-3}$$

Since the thermally generated electrons $(n_i \propto 10^{16}\ \text{m}^{-3})$ are negligibly small as compared to those produced by doping, so

$$n_e \approx N_d = 5\times10^{22}\ \text{m}^{-3}$$

Also, $n_e n_h = n_i^2$

$$\Rightarrow \quad n_h = \frac{n_i^2}{n_e} = \frac{1.5\times10^{16}\times1.5\times10^{16}}{5\times10^{22}}$$

$$\Rightarrow \quad n_h = 4.5\times10^9\ \text{m}^{-3}$$

PROBLEM 3

A pure germanium plate of area $3.5\times10^{-4}\ \text{m}^2$ and of thickness 1.5×10^{-3} m is connected across a battery of potential 5 V. Calculate the amount of current produced at room temperature in the germanium sample. Also find the amount of heat generated in the plate in 120 second. Given that the concentration of carriers in germanium at room temperature is 1.6×10^6 per cubic metre. The mobilities of electrons and holes are $0.4\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$ and $0.2\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$ respectively.

SOLUTION

Conductivity of semiconductor is given by

$$\sigma = n_e e\mu_e + n_h e\mu_h$$

Since semiconductor is intrinsic, so we have

$$n_e = n_h = n_i$$

$$\Rightarrow \quad \sigma = n_i e(\mu_e + \mu_h)$$

$$\Rightarrow \quad \sigma = 1.6\times10^{6}\times1.6\times10^{-19}(0.4+0.2)$$

$$\Rightarrow \quad \sigma = 1.536\times10^{-13}\ \text{ohm}^{-1}\text{m}^{-1}$$

Current flowing is given by

$$i = jA$$

where, j is the current density given by

$$j = \sigma E = \sigma\left(\frac{V}{d}\right)$$

$$\Rightarrow \quad i = \sigma\left(\frac{V}{d}\right)A$$

$$\Rightarrow \quad i = (1.536\times10^{-13})\left(\frac{5}{1.5\times10^{-3}}\right)(3.5\times10^{-4})$$

$$\Rightarrow \quad i = 1.79\times10^{-13}\ \text{A}$$

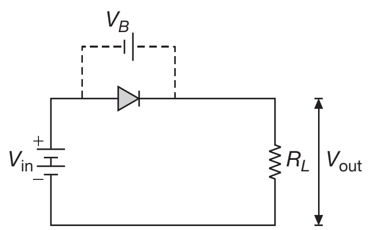
Heat produced is given by

$$H = Vit = 5\times1.79\times10^{-13}\times120$$

$$\Rightarrow \quad H = 10.74\times10^{-11}\ \text{J}$$

PROBLEM 4

A silicon diode is connected to a battery having voltage $V_{in} = 10$ V and a load resistance $R_L = 10$ kΩ as shown in Figure.



If the barrier voltage is $V_B = 0.7$ V, then calculate the output voltage across R_L, current in the diode and the forward resistance.
Assuming the diode to be ideal, calculate the output voltage and output current in diode.

SOLUTION

Since, $V_{out} = V_{in} - V_B$, where V_B is the barrier voltage

$$\Rightarrow \quad V_{out} = 10 - 0.7 = 9.3\ \text{V}$$

As diode is in forward biased state, so it will conduct

$$\Rightarrow \quad I = \frac{V_{out}}{R_L} = \frac{9.3}{10\times10^{3}} = 0.9\ \text{mA}$$

$$\text{and,}\quad r_f = \frac{V_B}{I} = \frac{0.7}{0.9\times10^{-3}} = 777\ \Omega$$

For ideal diode, $r_f = 0$, $V_B = 0$

$$V_{out} = V_{in} = 10\ \text{V}$$

$$I = \frac{V_{out}}{R_L} = \frac{10}{10\times10^{3}} = 1\ \text{mA}$$

PROBLEM 5

A semiconductor diode is used as a half rectifier having internal resistance 200 Ω. The voltage applied is given by $V = 50\sin(\omega t)$ volt and load resistance is 650 Ω. Calculate the maximum output current, dc output current, dc output power and dc output voltage.

SOLUTION

Given that $V = 50\sin(\omega t)$ volt

Comparing it with general equation $V = V_0\sin(\omega t)$, we get

$$V_0 = 50\ \text{volt}$$

Also, it is given that $R_L = 650\ \Omega$, $r_d = 200\ \Omega$

Maximum output current is

$$I_0 = \frac{V_0}{r_d + R_L}$$

$$\Rightarrow \quad I_0 = \frac{50}{(200+650)} = 58\ \text{mA}$$

dc output current is

$$I_{DC} = \frac{I_0}{\pi} = 18.5\ \text{mA}$$

dc output power is

$$P = I_{dc}^2 R_L = 0.22\ \text{W}$$

dc output voltage is

$$V = I_{dc} R_L = 12.02\ \text{W}$$

PROBLEM 6

In an npn transistor used in common emitter mode, 10^{10} electrons enter the emitter in 10^{-6} s. If only 2% of the electrons are lost in the base. Calculate the base current and the current amplification factor. Given that the charge on an electron is 1.6×10^{-19} C.

SOLUTION

The current in the emitter is given by

$$I_E = \frac{q}{t} = \frac{10^{10}\times1.6\times10^{-19}}{10^{-6}} = 1.6 \text{ mA}$$

Since, base current I_B is 2% of I_E, so we have

$$I_B = \left(\frac{2}{100}\right)\times1.6 \text{ mA} = 0.032 \text{ mA}$$

$$\Rightarrow \quad I_C = I_E - I_B = 1.568 \text{ mA}$$

So, the current amplifications factor is given by

$$\beta = \frac{I_C}{I_B} = \frac{1.568}{0.032} = 49$$

PROBLEM 7

In a common emitter amplifier, the load resistance of the output circuit is 500 times the resistance of the input circuit. If $\alpha = 0.98$, then find the voltage gain and power gain.

SOLUTION

Given that, $\alpha = 0.98$ and $\frac{R_{\text{out}}}{R_{\text{in}}} = 5000$,

$$\text{Since, } \beta = \frac{\alpha}{1-\alpha} = \frac{0.98}{1-0.98} = 49$$

$$\Rightarrow \quad \beta = \frac{0.98}{1-0.98} = 49$$

Voltage gain is

$$A_V = (\beta)\left(\frac{R_{\text{out}}}{R_{\text{in}}}\right) = (49)(500) = 24500$$

Power gain is

$$A_P = (\beta^2)\left(\frac{R_{\text{out}}}{R_{\text{in}}}\right) = (49)^2(500) = 1200500$$

PROBLEM 8

A transistor is used in common emitter mode in an amplifier circuit. When a signal of 40 mV is added to the base-emitter voltage, the base current changes by 40 μA and the collector current changes by 4 mA. The load resistance is 5 kΩ. Calculate the current gain β, the input resistance R_i, the transconductance g_m and the voltage gain.

SOLUTION

Given that $\Delta I_B = 40\ \mu\text{A}$ and $\Delta I_C = 4$ mA

$$\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{4\times10^{-3}}{40\times10^{-6}} = 100$$

Since, $\Delta V_i = (\Delta I_B)\times R_i$

$$\Rightarrow \quad R_i = \frac{\Delta V_i}{\Delta I_B} = \frac{40\times10^{-3}}{40\times10^{-6}} = 1000\ \Omega = 1\text{ k}\Omega$$

The transconductance g_m is

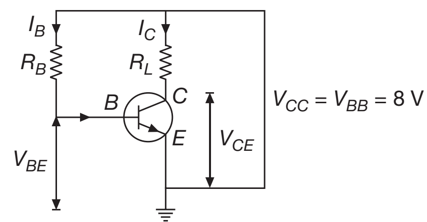
$$g_m = \frac{\Delta I_C}{\Delta V_i} = \frac{4\times10^{-3}}{40\times10^{-3}} = 0.1\ \Omega^{-1}$$

The voltage gain A_V is

$$A_V = \beta\left(\frac{R_{\text{out}}}{R_{\text{in}}}\right) = (100)\left(\frac{5}{1}\right) = 500$$

PROBLEM 9

An n-p-n transistor in a common emitter mode is used as a simple voltage amplifier with a collector current of 4 mA. The terminal of 8 V battery is connected to the collector through a load resistance R_L and to the base through a resistance R_B. The collector emitter voltage $V_{CE} = 4$ V, base emitter voltage $V_{BE} = 0.6$ V and base current amplification factor $\beta_{DC} = 100$. Calculate the values of R_L and R_B.



SOLUTION

Given that, $V_{CE} = 4$ V, $V_{BE} = 0.6$ V, $\beta = 100$

and $\Delta I_C = 4 \text{ mA} = 4\times10^{-3}$ A

Since, $V_{CE} = V_{CC} - I_C R_L$

$$\Rightarrow \quad V_{CE} + \Delta I_C R_L = V_{CC}$$

$$\Rightarrow \quad 4\times10^{-3} R_L = 8 - 4 = 4$$

$$\Rightarrow \quad R_L = 1000\ \Omega$$

The base current is given by

$$I_B = \frac{I_C}{\beta} = \frac{4\times10^{-3}}{100} = 4\times10^{-5}\ \text{A}$$

Also, we know that

$$V_{CC} = I_B R_B + V_{BE}$$

$$\Rightarrow \quad 8 = I_B R_B + 0.6$$

$$\Rightarrow \quad I_B R_B = 7.4\ \text{V}$$

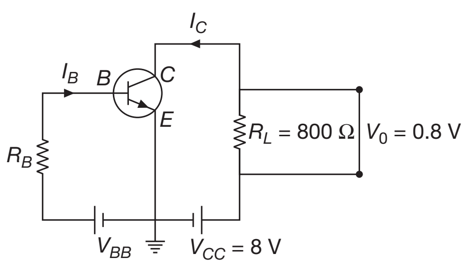
$$\Rightarrow \quad 4\times10^{-5} R_B = 7.4$$

$$\Rightarrow \quad R_B = \frac{7.4}{4}\times10^5 = 1.85\times10^5\ \Omega$$

$$\Rightarrow \quad R_B = 185\ \text{k}\Omega$$

PROBLEM 10

An npn transistor is connected in common emitter configuration in which collector supply is 8 V and the voltage drop across the load resistance of 800 Ω connected in the collector circuit is 0.8 V. If current gain factor is $\left(\frac{25}{26}\right)$, determine the collector emitter voltage and base current. If the internal resistance of the transistor is 200 Ω, calculate the voltage gain and power gain.



SOLUTION

Since, $V_0 = I_C R_L$

$$\Rightarrow \quad 0.8 = I_C \times 800$$

$$\Rightarrow \quad I_C = 10^{-3}\ \text{A}$$

Since, $V_{CE} = V_{CC} - I_C R_L$

$$\Rightarrow \quad V_{CC} = V_{CE} + I_C R_L$$

$$\Rightarrow \quad 8 = V_{CE} + 10^{-3}\times800$$

$$\Rightarrow \quad V_{CE} = 8 - 0.8 = 7.2\ \text{V}$$

Since the current gain factor is given to be $\frac{25}{26}$, which is less than 1, so it must be the current gain for the common base configuration of the transistor.

$$\Rightarrow \quad \alpha = \frac{25}{26},$$

$$\Rightarrow \quad \beta = \frac{\alpha}{1-\alpha} = \frac{\frac{25}{26}}{1-\frac{25}{26}} = 25$$

$$\Rightarrow \quad I_B = \frac{I_C}{\beta} = \frac{10^{-3}}{25} = 4\times10^{-5}\ \text{A} = 40\ \mu\text{A}$$

Voltage gain is

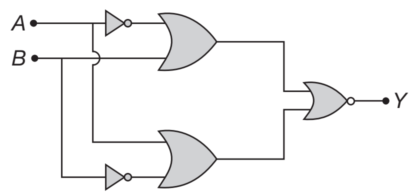
$$A_V = \beta\frac{R_L}{r_i} = 25\times\frac{800}{200} = 100$$

Power gain is

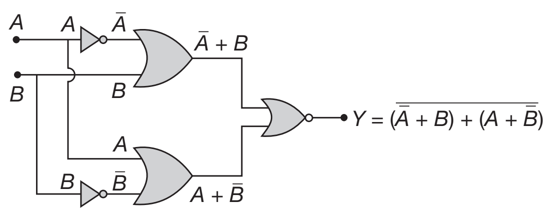
$$A_p = \beta^2\frac{R_L}{r_i} = \beta A_V = 25(100) = 2500$$

PROBLEM 11

Construct truth table for the function Y of A and B as shown in the figure.



SOLUTION



The Boolean expression of the output is

$$Y = (\bar{A}+B)+(A+\bar{B})$$

$$\Rightarrow \quad Y = (\bar{A}+A)+(B+\bar{B})$$

Since, $A+\bar{A}=0$ and $B+\bar{B}=0$

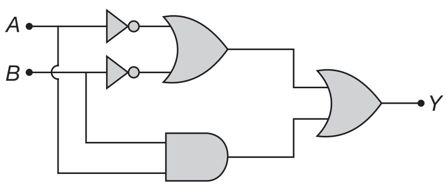
$$\Rightarrow \quad Y = 0$$

The corresponding truth table is

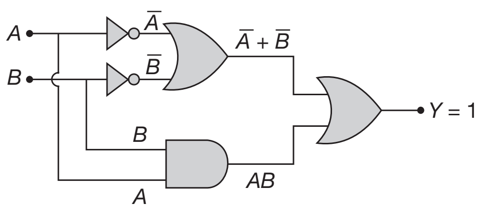
A	*B*	*Y*
0	0	0
0	1	0
1	0	0
1	1	0

PROBLEM 12

Write the Boolean equation and truth table for the following circuit. Also draw the output wave form.



SOLUTION



$$Y = AB + \bar{A} + \bar{B}$$

According to de-Morgan's Law

$$\bar{A} + \bar{B} = \overline{AB}$$

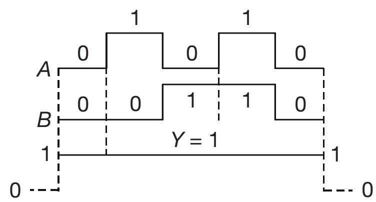
$$\Rightarrow \quad Y = AB + \overline{AB}$$

$$\Rightarrow \quad Y = 1$$

The truth table for the above combination of logic gates is

A	*B*	*AB*	$\overline{AB}$	*Y*
0	0	0	1	1
1	0	0	1	1
0	1	0	1	1
1	1	1	0	1

The output wave form for the circuit is shown in Figure.



PRACTICE EXERCISES

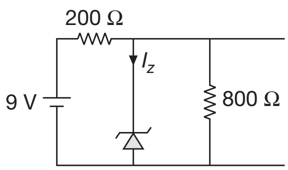
SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. The part of a transistor which is most heavily doped to produce large number of majority carriers is
(A) emitter
(B) base
(C) collector
(D) can be any of the above three

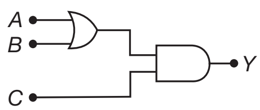
2. In an *npn* transistor circuit, the collector current is 10 mA. If 90% of the electrons emitted reach the collector
(A) the emitter current will be 9 mA
(B) the emitter current will be 11 mA
(C) the base current will be 10 mA
(D) the base current will be 0.1 mA

3. The reverse breakdown voltage of a Zener diode is 5.6 V in the given circuit.



The current I_Z through the Zener is
(A) 15 mA (B) 7 mA
(C) 10 mA (D) 17 mA

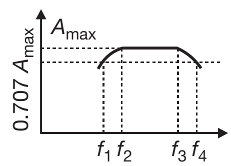
4. To get an output $Y = 1$ from circuit of figure, the inputs must be



	A	B	C
(A)	0	1	0
(B)	1	0	0
(C)	1	0	1
(D)	1	1	0

5. Formation of covalent bonds in compounds exhibits
(A) wave nature of electron
(B) particle nature of electron
(C) both wave and particle nature of electron
(D) None of these

6. The frequency response curve of *RC* coupled amplifier is shown in figure. The band width of the amplifier will be

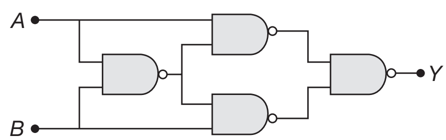


(A) $f_3 - f_2$ (B) $f_4 - f_1$
(C) $\frac{f_4 - f_2}{2}$ (D) $f_3 - f_1$

7. A strip of copper and another germanium are cooled from room temperature to 80 K. The resistance of
(A) each of these decreases
(B) copper strip increases and that of germanium decreases
(C) copper strip decreases and that of germanium increases
(D) each of these increases

8. The current voltage relation of diode is given by $I = (e^{1000V/T} - 1)\text{ mA}$, where the applied voltage V is in volts and the temperature T is in degree kelvin. If a student makes an error measuring $\pm 0.01\text{ V}$ while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA?
(A) 0.2 mA (B) 0.02 mA
(C) 0.5 mA (D) 0.05 mA

9. Truth table for system of four NAND gates as shown in figure is



(A)

A	*B*	*Y*
0	0	0
0	1	0
1	0	1
1	1	1

(B)

A	*B*	*Y*
0	0	1
0	1	1
1	0	0
1	1	0

(C)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(D)

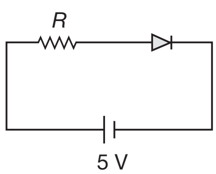
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

10. A semiconductor device is connected in series circuit with a battery and a resistance. A current is found to pass through the circuit. If the polarity of the battery is reversed, the current drops almost to zero. The device may be
(A) a p-type semiconductor
(B) a n-type semiconductor
(C) a pn junction
(D) an intrinsic semiconductor

11. An oscillator is basically an amplifier with gain
(A) less than unity (B) more than unity
(C) zero (D) 0.5

12. The difference in the variation of resistance with temperature in a metal and a semiconductor arises essentially due to the difference in the
(A) crystal structure
(B) variation of the number of charge carriers with temperature
(C) type of bonding
(D) variation of scattering mechanism with temperature.

13. The diode used in figure requires minimum current of 1 mA to be above the knee voltage 0.7 V of current versus voltage characteristics. The maximum value of R so that the voltage is above knee point is



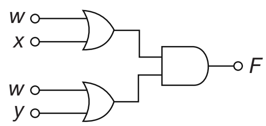
(A) 5 kΩ (B) 5.7 kΩ
(C) 4.3 kΩ (D) 3.5 kΩ

14. A piece of copper and another of germanium are cooled from room temperature to 80 K. The resistance of
(A) each of them increases.
(B) each of them decreases.
(C) copper increases and germanium decreases.
(D) copper decreases and germanium increases.

15. In a full wave rectifier with input frequency 50 Hz, the ripple in the output is mainly of the frequency (in Hz)
(A) 25 (B) 50
(C) 100 (D) 50^2

16. In the middle of the depletion layer of a reverse-biased p-n junction, the
(A) electric field is zero
(B) potential is maximum
(C) electric field is maximum
(D) potential is zero

17. The diagram of a logic circuit is given below. The output F of the circuit is represented by



(A) $W\cdot(X+Y)$ (B) $W\cdot(X\cdot Y)$
(C) $W+(X\cdot Y)$ (D) $W+(X+Y)$

18. For a transistor the value of $\alpha = 0.9$, the value of β is
(A) 1 (B) 0.09
(C) 0.9 (D) 9

19. $\frac{1}{\alpha} - \frac{1}{\beta}$ is equal to
(A) 1 (B) 2
(C) $\alpha\beta$ (D) $\alpha - \beta$

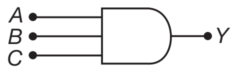
20. The conductivity of semiconductors like Ge and Si
(A) increases when it is doped with tetravalent impurity
(B) increases when it is doped with pentavalent or trivalent impurity
(C) increases when it is doped with pentavalent impurity and decreases when it is doped with trivalent impurity
(D) decreases when it is doped with pentavalent impurity and increases when it is doped with trivalent impurity

21. When npn transistor is used as an amplifier
(A) electrons move from base to collector
(B) holes move from emitter to base
(C) electrons move from collector to base
(D) holes move from base to emitter

22. In a semiconductor diode P side is earthed and N, side is applied a potential of -2 V. The diode shall
(A) conduct
(B) not conduct
(C) conduct partially
(D) breakdown

23. Given sets of elements are phosphorus, arsenic, indium and bismuth. The addition of which in pure semiconductor will result in p-type semiconductor
(A) phosphorus, arsenic and indium
(B) phosphorus, arsenic, indium and bismuth
(C) indium and arsenic
(D) indium only

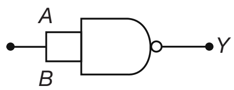
24. The output from a logic gate is 1 when inputs A, B and C are such that



(A) $A = 1, B = 0, C = 1$ (B) $A = 1, B = 1, C = 0$
(C) $A = B = C = 0$ (D) $A = B = C = 1$

25. For a transistor amplifier in common emitter configuration for load impedance of 1 kΩ ($h_{fe} = 50$ and $h_{oe} = 25$) the current gain is
(A) –5.2 (B) –15.7
(C) –24.8 (D) –48.78

26. The symbol represents



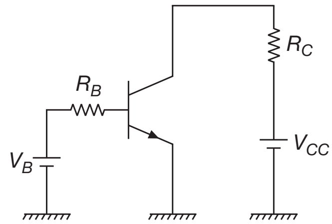
(A) NOT gate (B) OR gate
(C) AND gate (D) NOR gate

27. A piece of copper and another of germanium are cooled from room temperature to 77 K, the resistance of
(A) each of them increases
(B) each of them decreases
(C) copper decreases and germanium increases
(D) copper increases and germanium decreases

28. NAND and NOR gates are called universal gates primarily because they
(A) are available universally
(B) can be combined to produce OR, AND and NOT gates
(C) are widely used in Integrated circuit packages
(D) are easiest to manufacture

29. The value of β
(A) is always less than 1
(B) lies between 20 and 200
(C) is always greater than 200
(D) is always infinity

30. A common emitter amplifier circuit, built using an *npn* transistor, is shown in the figure.



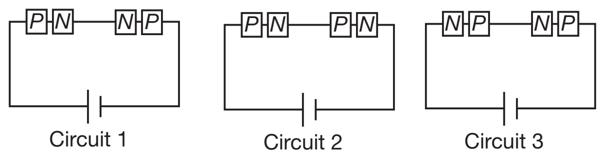
Its *dc* current gain is 250, $R_C = 1$ kΩ and $V_{CC} = 10$ V. What is the minimum base current for V_{CE} to reach saturation?
(A) 10 μA (B) 100 μA
(C) 7 μA (D) 40 μA

31. The forbidden energy gap in conductors, semi-conductors and insulators are EG_1, EG_2 and EG_3 respectively. The relation among them is
(A) $EG_1 = EG_2 = EG_3$ (B) $EG_1 > EG_2 > EG_3$
(C) $EG_1 < EG_2 < EG_3$ (D) $EG_1 < EG_2 > EG_3$

32. The manifestation of band structure in solids is due to
(A) Heisenberg's uncertainty principle
(B) Pauli's exclusion principle
(C) Bohr's correspondence principle
(D) Boltzmann's law

33. The current gain α of a transistor is 0.95. The change in emitter current is 10 mA. The change in base current is
(A) 9.5 mA (B) 0.5 mA
(C) 10.5 mA (D) $\left(\frac{200}{19}\right)$ mA

34. Two identical pn junctions may be connected in series with a battery in three ways as shown in figure. The potential drops across the two pn junctions are equal in



(A) circuit 1 and circuit 2 (B) circuit 2 and circuit 3
(C) circuit 3 and circuit 1 (D) circuit 1 only

35. An example of n-type semiconductor is
(A) pure germanium
(B) pure silicon
(C) silicon doped with phosphorus
(D) germanium doped with boron

36. The energy band gap is maximum in
(A) metals (B) superconductors
(C) insulators (D) semiconductors

37. When a *pn* junction diode is reverse biased, the flow of current across the junction is mainly due to
(A) diffusion of charges.
(B) drift of charges.
(C) both drift and diffusion of charges.
(D) either drift or diffusion depending upon the nature of the material.

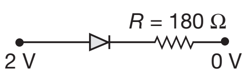
38. When *p-n* junction diode is forward biased, then
(A) The depletion region is reduced and barrier height is increased
(B) The depletion region is widened and barrier height is reduced
(C) both the depletion region and barrier height are reduced
(D) both the depletion region and barrier height are increased

39. An *n*-type semiconductor has resistivity $0.1\ \Omega\text{m}$. The number of donor atoms which must be added to achieve this is $\left(\mu_e = 0.05\ \text{m}^2\text{V}^{-1}\text{s}^{-1}\right)$
(A) 1.25×10^{17} (B) 1.25×10^{23}
(C) 1.25×10^{21} (D) 1.25×10^{22}

40. When pure germanium is doped with trivalent impurity like aluminium; the conduction is due to
(A) electrons (B) holes
(C) protons (D) positrons

41. In a common base amplifier, the phase difference between the input signal voltage and output voltage is
(A) 0 (B) $\frac{\pi}{2}$
(C) $\frac{\pi}{4}$ (D) π

42. Assuming that the silicondiode having resistance of 20 Ω, the current through the diode is (knee voltage 0.7 V)



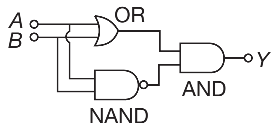
(A) 0 mA (B) 10 mA
(C) 6.5 mA (D) 13.5 mA

43. In a common-emitter transistor amplifier circuit $\beta = 100$, input resistance $R_1 = 1\ \text{k}\Omega$, output resistance $R_2 = 10\ \text{k}\Omega$. The voltage gain of circuit is
(A) 100 (B) 1000
(C) 10 (D) 5000

44. The electrical conductivity of a semiconductor increases when electromagnetic radiation of wavelength shorter than 2480 nm is incident on it. The band gap in (eV) for the semiconductor is
(A) 0.5 eV (B) 0.7 eV
(C) 1.1 eV (D) 2.5 eV

45. If the forward voltage in a semiconductor diode is doubled, the width of depletion layer
(A) increases
(B) decreases
(C) remains unchanged
(D) becomes zero

46. The following configuration of gate is equivalent to

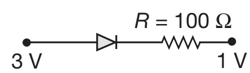


(A) NAND (B) XOR
(C) OR (D) NOR

47. An *npn* transistor is used in common emitter configuration as an amplifier with $1\ \text{k}\Omega$ load resistance. Signal voltage of $10\ \text{mV}$ is applied across the base-emitter. This produces a $3\ \text{mA}$ change in the collector current and $15\ \mu\text{A}$ change in the base current of the amplifier. The input resistance and voltage gain are
(A) $0.33\ \text{k}\Omega$, 1.5
(B) $0.33\ \text{k}\Omega$, 300
(C) $0.67\ \text{k}\Omega$, 200
(D) $0.67\ \text{k}\Omega$, 300

48. One serious drawback of semi-conductor devices is
(A) they do not last for long time.
(B) they are costly.
(C) they cannot be used with high voltage.
(D) they pollute the environment.

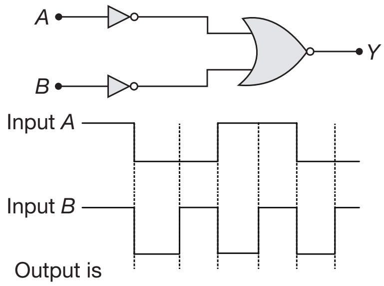
49. Assuming that the junction diode is ideal, the current through the diode is

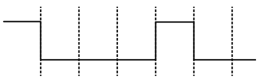


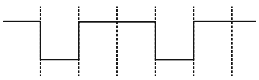
(A) 0 (B) 2 mA
(C) 20 mA (D) 200 mA

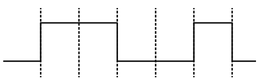
50. In a full wave rectifier circuit operating from 50 Hz mains frequency, the fundamental frequency in the ripple would be
(A) 100 Hz (B) 70.7 Hz
(C) 50 Hz (D) 25 Hz

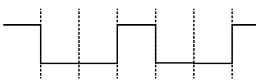
51. The logic circuit shown below has the input waveforms A and B as shown. Pick out the correct output waveform



(A) 

(B) 

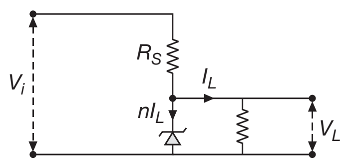
(C) 

(D) 

52. A p-type semiconductor has acceptor level 57 meV above the valence band. The maximum wavelength of light required to create a hole is

(A) 57 Å (B) 57×10^{-3} Å
(C) 217100 Å (D) 11.61×10^{-33} Å

53. The value of the resistor, R_S, needed in the dc voltage regulator circuit shown here, equals



(A) $\frac{(V_i - V_L)}{nI_L}$ (B) $\frac{(V_i + V_L)}{nI_L}$
(C) $\frac{(V_i - V_L)}{(n+1)I_L}$ (D) $\frac{(V_i + V_L)}{(n+1)I_L}$

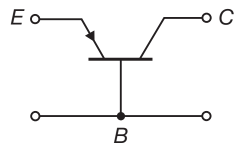
54. The ratio of forward biased to reverse biased resistance for pn junction diode is

(A) $10^{-1} : 1$ (B) $10^{-2} : 1$
(C) $10^{4} : 1$ (D) $10^{-4} : 1$

55. A solid which is transparent to visible light and whose conductivity increases with temperature is formed by
(A) metallic bonding
(B) ionic bonding
(C) covalent bonding
(D) van der Waals bonding

56. With the rise in temperature, the specific resistance of a semiconductor
(A) increases
(B) remains unchanged
(C) decreases
(D) first decreases and then increases

57. For circuit shown in figure $I_E = 4$ mA, $I_B = 40\ \mu$A. What are the values of α and I_C?



(A) 0.99, 3.96 mA (B) 1.01, 4.04 mA
(C) 0.97, 4.04 mA (D) 0.99, 4.04 mA

58. The width of the depletion layer in a pn junction diode
(A) increases when a reverse bias is applied.
(B) increases when a forward bias is applied.
(C) decreases when a reverse bias is applied.
(D) remains the same, irrespective of the bias voltage.

59. By increasing the temperature, the specific resistance of a conductor and a semiconductor
(A) increases for both (B) decreases for both
(C) increases, decreases (D) decreases, increases

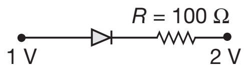
60. In the ratio of the concentration of electrons that of holes in a semiconductor is $\frac{7}{5}$ and the ratio of currents is $\frac{7}{4}$ then what is the ratio of their drift velocities?

(A) $\frac{4}{7}$ (B) $\frac{5}{8}$
(C) $\frac{4}{5}$ (D) $\frac{5}{4}$

61. A multistage amplifier has the overall voltage gain of 150. The gain is reduced to 25 when a negative feedback is applied. What is the fraction of the output that is fed back to the input?

(A) $\frac{1}{30}$ (B) $\frac{1}{20}$
(C) $-\frac{1}{30}$ (D) $\frac{1}{6}$

62. Assuming that the junction diode is ideal, the current in the arrangement shown in figure is



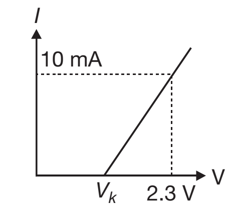
(A) 0 mA (B) 2 mA
(C) 10 mA (D) 30 mA

63. The current gain α of a transistor is defined as
(A) the ratio of change in collector current to the change in emitter current for a constant value of collector voltage in a common base arrangement.
(B) the ratio of change in collector current to the change in base current for a constant collector voltage in a common collector arrangement.
(C) the ratio of change in collector current to the change in base current for a constant collector voltage in a common emitter arrangement.
(D) the ratio of change in emitter current to the change in collector current for a constant emitter voltage in a common emitter arrangement.

64. The value of $\overline{1}+\overline{1}$ is
(A) 2 (B) 0
(C) 1 (D) 10

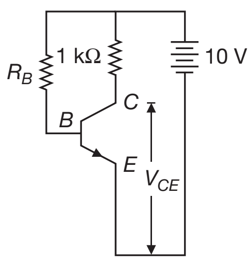
65. In common base mode of a transistor, the collector current is 5.488 mA for an emitter current of 5.60 mA. The value of the base current amplification factor (β) will be
(A) 48 (B) 49
(C) 50 (D) 51

66. The resistance of a germanium junction diode whose V-I is shown in figure is $(V_k = 0.3\text{ V})$



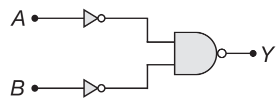
(A) 5 kΩ (B) 0.2 kΩ
(C) 2.3 kΩ (D) $\left(\frac{10}{2.3}\right)$kΩ

67. In the circuit shown here the transistor used has a current gain $\beta = 100$. What should be the bias resistor R_B so that $V_{CE} = 5$ V? (neglect V_{BE})



(A) $2\times10^3\ \Omega$ (B) $200\times10^3\ \Omega$
(C) $1\times10^6\ \Omega$ (D) $500\ \Omega$

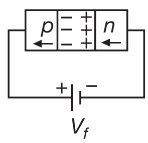
68. The logic gate equivalent to the given logic circuit is

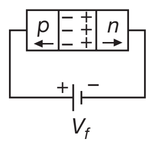


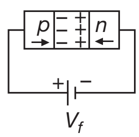
(A) OR (B) NAND
(C) AND (D) NOR

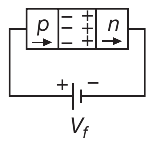
69. At absolute zero, *Si* acts as
(A) non-metal (B) metal
(C) insulator (D) None of these.

70. In the case of forward biasing of a *pn* junction diode, which one of the following figures correctly depicts the direction of flow of current

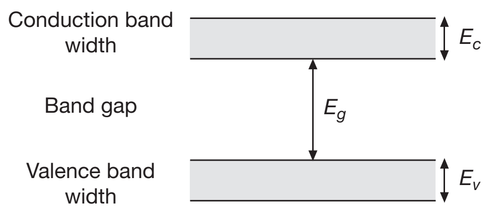
(A)


(B)


(C)


(D)


71. If the lattice constant of this semiconductor is decreased, then which of the following is correct?



(A) all E_c, E_g, E_v decrease
(B) all E_c, E_g, E_v increase
(C) E_c and E_v increase, but E_g decreases
(D) E_c and E_v decrease, but E_g increases

72. In a semiconductor the forbidden energy gap between the valency band and the conduction band is of the order of
(A) 1 eV (B) 5 eV
(C) 1 keV (D) 1 MeV

73. An experiment is performed to determine the I-V characteristics of a Zener diode, which has a protective resistance of $R = 100\ \Omega$, and a maximum power of dissipation rating of 1 W. The minimum voltage range of the DC source in the circuit is

(A) $0-5$ V (B) $0-24$ V
(C) $0-12$ V (D) $0-8$ V

74. In a bridge rectifier, the number of diodes required is
(A) 1 (B) 2
(C) 3 (D) 4

75. At absolute zero temperature a crystal of pure germanium
(A) behaves as perfect conductor
(B) behaves as perfect insulator
(C) contains no electron
(D) None of the above

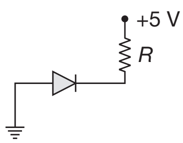
76. In a transistor the value of β is 100, the value of α is
(A) 0.01 (B) 0.1
(C) 0.99 (D) 1

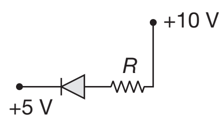
77. The type of binding in germanium crystal is
(A) ionic (B) metallic
(C) covalent (D) vander Waal's

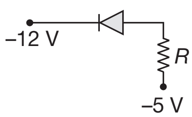
78. When npn transistor is used as an amplifier, then
(A) electrons move from collector to emitter
(B) electrons move from emitter to collector
(C) electrons move from collector to base
(D) holes move from emitter to collector

79. An npn transistor operates as a common emitter amplifier, with a power gain of 60 dB. The input circuit resistance is $100\ \Omega$ and the output load resistance is $10\ k\Omega$. The common emitter current gain β is
(A) 10^4 (B) 6×10^2
(C) 10^2 (D) 60

80. In the following, which one of the diodes is reverse biased?

(A)


(B)


(C)


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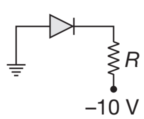


81. The intrinsic conductivity of germanium at 27° is $2.13\ \text{mho m}^{-1}$ and mobilities of electrons and holes are 0.38 and $0.18\ m^2V^{-1}s^{-1}$ respectively. The density of charge carriers is

(A) $2.37\times10^{19}\ m^{-3}$ (B) $3.28\times10^{19}\ m^{-3}$
(C) $7.83\times10^{19}\ m^{-3}$ (D) $8.47\times10^{18}\ m^{-3}$

82. In a common base configuration $I_E = 1$ mA, $I_C = 0.95$ mA. The value of base current is
(A) 1.95 mA (B) 0.05 mA
(C) 1.05 mA (D) 0.95 mA

83. Read the following statements carefully
Y: The resistivity of a semiconductor decreases with increase of temperature
Z: In a conducting solid, the rate of collisions between free electrons and ions increases with increase of temperature
State the correct statement(s) from the following
(A) Y is true but Z is false
(B) Y is false but Z is true
(C) Both Y and Z are true
(D) Y is true and Z is the correct reason for Y

84. The transfer ratio β of a transistor is 50. The input resistance of the transistor when used in the common emitter mode is 1 kΩ. The peak value of the collector alternating current for an input peak voltage of 0.01 V is
(A) 100 μA (B) 500 μA
(C) 0.01 μA (D) 0.25 μA

85. Two pnp transistors in series give
(A) a positive OR circuit
(B) a negative OR circuit
(C) a positive AND circuit
(D) a negative AND circuit

86. An oscillator is nothing but an amplifier with
(A) positive feedback (B) large gain
(C) no feedback (D) negative feedback

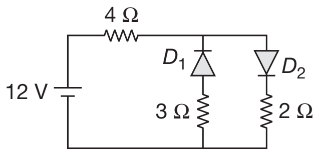
87. The dominant mechanisms for motion of charge carriers in forward and reverse biased silicon pn junctions are
(A) drift in forward bias, diffusion in reverse bias.
(B) diffusion in forward bias, drift in reverse bias.
(C) diffusion in both forward and reverse bias.
(D) drift in both forward and reverse bias.

88. What is voltage gain in a common emitter amplifier when input resistance is $3\ \Omega$ and the load resistance $24\ \Omega$ with $\beta = 60$
(A) 8.4 (B) 4.8
(C) 2.4 (D) 480

89. A semiconductor is cooled from T_1 K to T_2 K. Its resistance

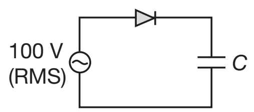
(A) will decrease.
(B) will increase.
(C) will first decrease and then increase.
(D) will not change.

90. The circuit has two oppositely connect ideal diodes in parallel. What is the current following in the circuit?



(A) 1.33 A (B) 1.71 A
(C) 2.00 A (D) 2.31 A

91. A sinusoidal voltage of r.m.s. value 100 V is connected to an ideal junction diode as shown in figure. The maximum potential difference across the capacitor will be

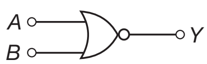


(A) 100 V (B) $100\sqrt{2}$ V
(C) $50\sqrt{2}$ V (D) ZERO

92. In germanium the energy gap is about 0.75 eV. The wavelength of light which germanium starts absorbing is

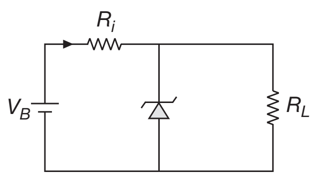
(A) 5000 Å (B) 1650 Å
(C) 16500 Å (D) 165000 Å

93. The logic symbol shown in figure represents



(A) OR gate (B) XOR gate
(C) NAND gate (D) NOR gate

94. The figure represents a voltage regulator circuit using a Zener diode. The breakdown voltage of the Zener diode is 6 V and the load resistance is, $R_L = 4\ \text{k}\Omega$. The series resistance of the circuit is $R_i = 1\ \Omega$. If the battery voltage V_B varies from 8 V to 16 V, what are the minimum and maximum values of the current through Zener diode?

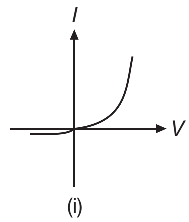


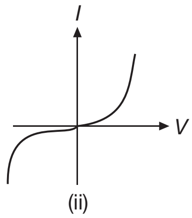
(A) 0.5 mA, 6 mA (B) 0.5 mA, 8.5 mA
(C) 1.5 mA, 8.5 mA (D) 1 mA 8.5 mA

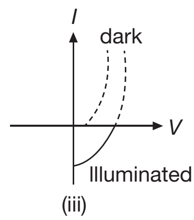
95. In the depletion region of an unbiased pn junction diode, there are

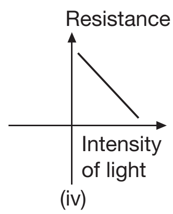
(A) only electrons
(B) only holes
(C) both electrons and holes
(D) only fixed ions

96. Identify the semiconductor devices whose characteristics are given below, in the order (i), (ii), (iii), (iv)









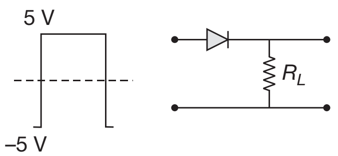
(A) Zener diode, Solar cell, Simple diode, Light dependent resistance
(B) Simple diode, Zener diode, Solar cell, Light dependent resistance
(C) Zener diode, Simple diode, Light dependent resistance, Solar cell
(D) Solar cell, Light dependent resistance, Zener diode, Simple diode

97. Which one of the following logic gates does the truth table represent

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

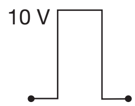
(A) NOT (B) NOR
(C) OR (D) AND

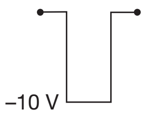
98. If in a p-n junction diode, a square input signal of 10 V is applied as shown

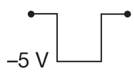


Then the output signal across R_L will be

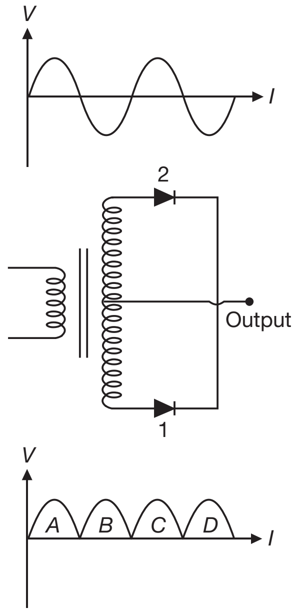
(A) 

(B) 

(C) 

(D) 

99. A full wave rectifier circuit along with the output is shown in figure



This contribution from the diode 1 is

(A) C (B) A, C
(C) B, D (D) A, B, C, D

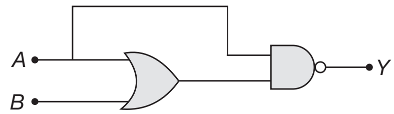
100. When the temperature of a semiconductor is increased, its electrical conductivity

(A) increases
(B) remains the same
(C) decreases
(D) first increases and then decreases

101. The voltage gain of an amplifier without feedback is 100. If a negative feedback is introduced, with a feedback fraction $\beta = 0.1$, then the gain of the feed back amplifier is

(A) 9.09 (B) 10
(C) 100.1 (D) 90.0

102. The truth table for the circuit given in the figure is



(A)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

(B)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(C)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	1

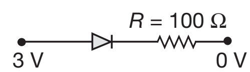
(D)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

103. While using a transistor as an amplifier

(A) the collector junction is forward biased and emitter junction reverse biased.
(B) the collector junction is reverse biased and emitter junction is forward biased.
(C) both the junctions are forward biased.
(D) both the junctions are reverse biased.

104. Assuming that the silicon diode (having negligible resistance), the current through the diode is (knee voltage of silicon diode 0.7 V)

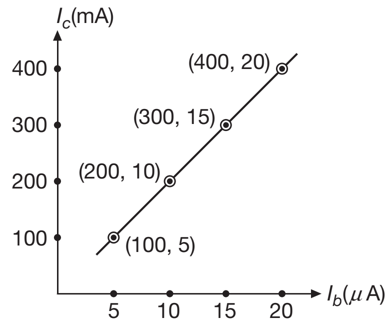


(A) 0 mA
(B) 7 mA
(C) 2.3 mA
(D) 23 mA

105. *npn* transistors are preferred to *npn* transistors because they have

(A) low cost
(B) low dissipation of energy
(C) capable of handling large power
(D) electrons have high mobility than holes and hence high mobility of energy

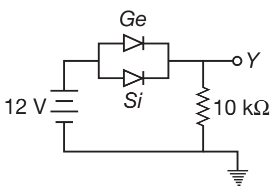
106. The transfer characteristic curve of a transistor, having input and output resistance 100 Ω and 100 kΩ respectively, is shown in the figure. The voltage and power gain, are respectively



(A) 5×10^4, 2.5×10^6 (B) 2.5×10^4, 2.5×10^6
(C) 5×10^4, 5×10^6 (D) 5×10^4, 5×10^5

107. Carbon, silicon and germanium have four valence electrons each. At room temperature which one of the following statements is most appropriate?
(A) The number of free electrons for conduction is significant only in Si and Ge but small in C.
(B) The number of free conduction electrons is significant in C but small in Si and Ge.
(C) The number of free conduction electrons is negligibly small in all the three.
(D) The number of free electrons for conduction is significant in all the three.

108. Two junction diodes one of Germanium (Ge) and other of silicon (Si) are connected as shown in figure to a battery of emf 12 V and a load resistance 10 kΩ. The germanium diode conducts at 0.3 V and silicon diode at 0.7 V. When a current flows in the circuit, the potential of terminal Y will be

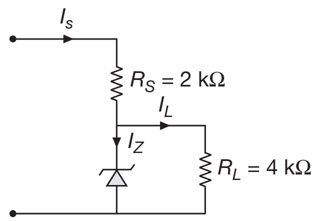


(A) 12 V (B) 11 V
(C) 11.3 V (D) 11.7 V

109. In PROBLEM 108, if the polarity of Ge diode are reversed, and a current flows, the potential of point Y will be
(A) 12 V (B) 11 V
(C) 11.3 V (D) 11.7 V

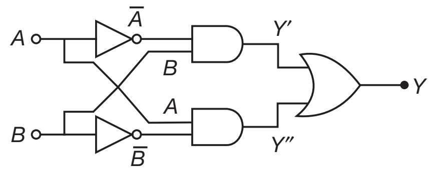
110. A hole in a p-type semiconductor is
(A) an excess electron
(B) a missing electron
(C) a missing atom
(D) a donor level

111. Figure shows a DC voltage regulator circuit, with a Zener diode of breakdown voltage 6 V. If the unregulated input voltage varies between 10 V to 16 V, then what is the maximum Zener current?



(A) 7.5 mA (B) 1.5 mA
(C) 2.5 mA (D) 3.5 mA

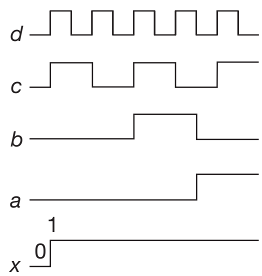
112. The following circuit represents



(A) OR gate (B) XOR gate
(C) AND gate (D) NAND gate

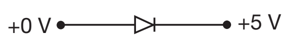
113. $A+\bar{A}$ is equal to
(A) 1 (B) 0
(C) A (D) $A\bar{A}$

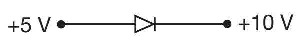
114. If a, b, c, d are inputs to a gate and x is its output, then as per the following time graph, the gate is



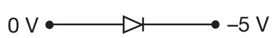
(A) NAND (B) NOT
(C) AND (D) OR

115. In which of the following figures, the junction diode is forward biased

(A) 

(B) 

(C) 

(D) 

116. The energy gap of silicon is 1.14 eV. The maximum wavelength at which silicon will begin absorbing energy is

(A) 10855 Å (B) 1085.5 Å

(C) 108.55 Å (D) 10.855 Å

117. The only function of a NOT gate is to

(A) stop a signal

(B) invert an input signal

(C) recomplement a signal

(D) act as a universal gate

118. Mobility of electrons in a semiconductor is defined as the ratio of their drift velocity to the applied electric field. If, for an n-type semiconductor, the density of electrons is $10^{19}\ \text{m}^{-3}$ and their mobility is $1.6\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$ then the resistivity of the semiconductor (since it is an n-type semiconductor contribution of holes is ignored) is close

(A) 2 Ωm (B) 0.2 Ωm

(C) 0.4 Ωm (D) 4 Ωm

119. The collector current in npn transistor circuit is 19 mA. What is the base current if 95% of the electrons emitted reach the collector?

(A) 1 mA (B) 2 mA

(C) 0.5 mA (D) 0.75 mA

120. A truth table is given below

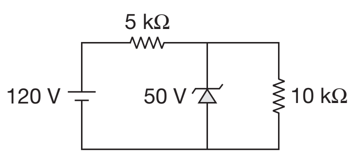
A	B	Y
0	0	1
1	0	0
0	1	0
1	1	0

Which of the following follows this truth table?

(A) XOR gate (B) NOR gate

(C) AND gate (D) OR gate

121. For the circuit shown below, the current through the Zener diode is



(A) Zero (B) 9 mA

(C) 14 mA (D) 5 mA

122. The input impedance of the npn transistor in common emitter mode (in standard notations) is given by

(A) $Z_i = \left(\frac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}}$ (B) $Z_i = \left(\frac{\Delta I_B}{\Delta V_{BE}}\right)_{V_{CE}}$

(C) $Z_i = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE}}$ (D) $Z_i = \left(\frac{\Delta I_C}{\Delta V_{CE}}\right)_{I_B}$

123. The current gain β may be defined as

(A) The ratio of change in collector current to the change in emitter current for a constant collector voltage in a common base arrangement.

(B) The ratio of change in collector current to the change in base current at constant collector voltage in a common emitter circuit.

(C) The ratio of change in emitter current to the change in base current for constant emitter voltage in common emitter circuit.

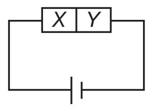
(D) The ratio of change in base current to the change in collector current at constant collector voltage in common emitter circuit.

124. The current gain in the common emitter mode of a transistor is 10. The input impedance is 20 kΩ and load resistance is 100 kΩ. The power gain is

(A) 100 (B) 200

(C) 500 (D) 300

125. A semiconductor X is made by doping a germanium crystal with arsenic $(Z = 33)$. A second semiconductor Y is made by doping germanium with indium $(Z = 49)$. The two are joined end to end and connected to a battery as shown. Which of the following statements is correct?



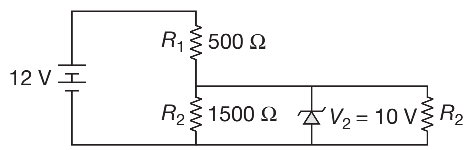
(A) X is p–type, Y is n-type and the junction is forward biased

(B) X is n–type, Y is p-type and the junction is forward biased

(C) X is p–type, Y is n-type and the junction is reverse biased

(D) X is n–type, Y is p-type and the junction is reverse biased

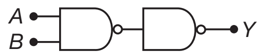
126. In the given circuit the current through Zener Diode is close to



(A) 6.7 mA (B) 0.0 mA
(C) 4.0 mA (D) 6.0 mA

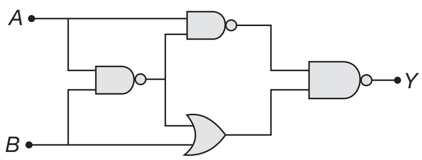
127. Boolean expression $Y = A\overline{B} + B\overline{A}$ is given
If $A = 1, B = 1$ then, $Y = ?$
(A) 0 (B) 1
(C) 11 (D) 10

128. The arrangement shown in figure performs the logic function of



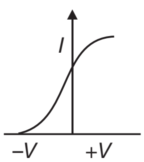
(A) AND gate
(B) NAND gate
(C) OR gate
(D) XOR gate

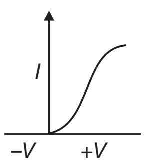
129. The output of the given logic circuit is

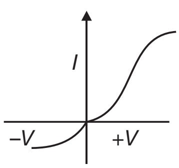


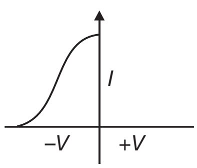
(A) $A\overline{B} + \overline{A}B$ (B) $A\overline{B}$
(C) $AB + \overline{AB}$ (D) $\overline{A}B$

130. Different voltages are applied across a pn junction and the currents are measured for each value. Which of the following graphs is obtained between voltage (V) and current (I)

(A)


(B)


(C)


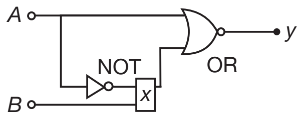
(D)


131. In positive logic, logic state 1 corresponds
(A) positive voltage
(B) zero voltage
(C) lower voltage level
(D) higher voltage level

132. The logic circuit shown in the following figure yields the given truth table

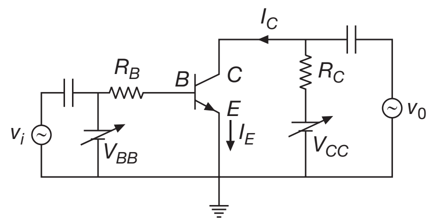
A	*B*	*Y*
1	1	1
0	1	1
1	0	1
0	0	0

The gate X in the diagram is



(A) NAND (B) XOR
(C) AND (D) NOR

133. In the figure, given that V_{BB} supply can vary from 0 to 5.0 V, $V_{CC} = 5$ V, $\beta_{dc} = 200$, $R_B = 100\ \text{k}\Omega$, $R_C = 1\ \text{k}\Omega$ and $V_{BE} = 1.0$ V. The minimum base current and the input voltage at which the transistor will go to saturation, will be respectively



(A) 25 μA and 3.5 V (B) 20 μA and 2.8 V
(C) 25 μA and 2.8 V (D) 20 μA and 3.5 V

134. The electrical conductivity of semiconductor increases, when em radiation of wavelength shorter than 2480 nm is incident on it. The band gap (in eV) for the semiconductor is
(A) 0.9 (B) 0.7
(C) 0.5 (D) 1.1

135. The truth table given below is for

A	*B*	*Y*
0	0	1
0	1	0
1	0	0
1	1	1

(A) OR (B) AND
(C) XNOR (D) XOR

136. The truth table of a logic gate is

A	*B*	*Y*
1	1	0
1	0	1
0	1	1
0	0	1

The logic gate is
(A) NAND (B) AND
(C) XOR (D) NOT

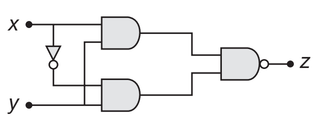
137. Holes are charge carries in
(A) intrinsic semiconductors
(B) *n*-type semiconductors
(C) *p*-type semiconductors
(D) all the above

138. A working transistor with its three legs marked P, Q and R is tested using a multimeter. No conduction is found between P and Q. By connecting the common (negative) terminal of the multimeter to R and the other (positive) terminal to P or Q, some resistance is seen on the multimeter. Which of the following is true for the transistor?
(A) It is an npn transistor with R as collector.
(B) It is an npn transistor with R as base.
(C) It is a pnp transistor with R as collector.
(D) It is a pnp transistor with R as emitter.

139. When pure germanium is doped with pentavalent impurity like phosphorus the conduction is due to
(A) electrons
(B) holes
(C) protons
(D) positrons

140. The value of α
(A) is always less than 1
(B) is always greater than 1
(C) may be less or greater than 1
(D) None of the above

141. Truth table for the following digital circuit will be



(A)

x	y	z
0	0	1
0	1	1
1	0	1
1	1	1

(B)

x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

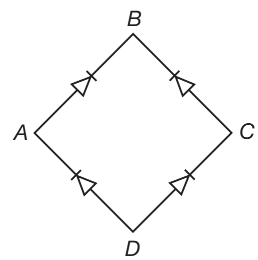
(C)

x	y	z
0	0	0
0	1	1
1	0	1
1	1	1

(D)

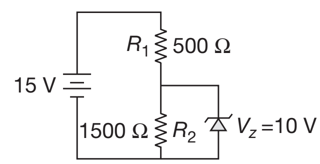
x	y	z
0	0	1
0	1	1
1	0	1
1	1	0

142. In the diagram the input is across the terminals A and C and output is across B and D. Then the output is



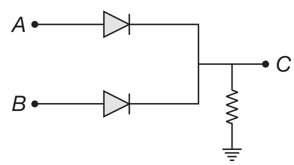
(A) zero
(B) same as input
(C) full wave rectified
(D) half wave rectified

143. In the given circuit, the current through Zener diode is



(A) 6.7 mA (B) 3.3 mA
(C) 2.5 mA (D) 5.5 mA

144. In the circuit below, A and B represent two inputs and C represents the output. The circuit represents



(A) OR gate (B) NOR gate
(C) AND gate (D) NAND gate

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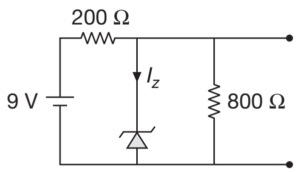
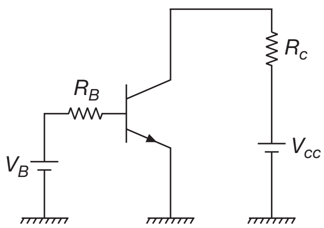
1. **[Online April 2019]**
The reverse breakdown voltage of a Zener diode is 5.6 V in the given circuit.



The current I_Z through the Zener is

(A) 15 mA (B) 7 mA
(C) 10 mA (D) 17 mA

2. **[Online April 2019]**
A common emitter amplifier circuit, built using an npn transistor, is shown in the figure.



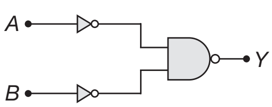
Its dc current gain is 250, $R_C = 1 \text{ k}\Omega$ and $V_{CC} = 10 \text{ V}$. What is the minimum base current for V_{CE} to reach saturation?

(A) 10 μA (B) 100 μA
(C) 7 μA (D) 40 μA

3. **[Online April 2019]**
An NPN transistor is used in common emitter configuration as an amplifier with $1 \text{ k}\Omega$ load resistance. Signal voltage of 10 mV is applied across the base-emitter. This produces a 3 mA change in the collector current and $15 \mu\text{A}$ change in the base current of the amplifier. The input resistance and voltage gain are

(A) $0.33 \text{ k}\Omega$, 1.5 (B) $0.33 \text{ k}\Omega$, 300
(C) $0.67 \text{ k}\Omega$, 200 (D) $0.67 \text{ k}\Omega$, 300

4. **[Online April 2019]**
The logic gate equivalent to the given logic circuit is

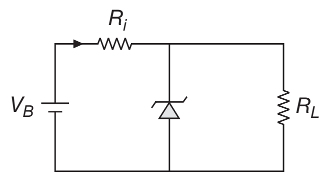


(A) OR (B) NAND
(C) AND (D) NOR

5. **[Online April 2019]**
An npn transistor operates as a common emitter amplifier, with a power gain of 60 dB. The input circuit resistance is 100Ω and the output load resistance is $10 \text{ k}\Omega$. The common emitter current gain β is

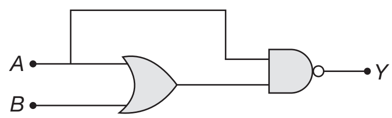
(A) 10^4 (B) 6×10^2
(C) 10^2 (D) 60

6. **[Online April 2019]**
The figure represents a voltage regulator circuit using a Zener diode. The breakdown voltage of the Zener diode is 6 V and the load resistance is, $R_L = 4 \text{ k}\Omega$. The series resistance of the circuit is $R_i = 1 \Omega$. If the battery voltage V_B varies from 8 V to 16 V, what are the minimum and maximum values of the current through Zener diode?



(A) 0.5 mA, 6 mA (B) 0.5 mA, 8.5 mA
(C) 1.5 mA, 8.5 mA (D) 1 mA 8.5 mA

7. **[Online April 2019]**
The truth table for the circuit given in the figure is



(A)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

(B)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(C)

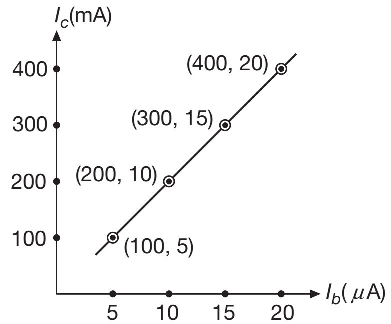
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	1

(D)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

8. **[Online April 2019]**
The transfer characteristic curve of a transistor, having

input and output resistance $100\ \Omega$ and $100\ \text{k}\Omega$ respectively, is shown in the figure. The voltage and power gain, are respectively

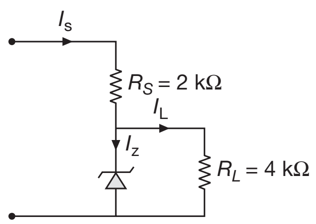


(A) 5×10^4, 2.5×10^6 (B) 2.5×10^4, 2.5×10^6

(C) 5×10^4, 5×10^6 (D) 5×10^4, 5×10^5

9. [Online April 2019]

Figure shows a DC voltage regulator circuit, with a Zener diode of breakdown voltage $6\ \text{V}$. If the unregulated input voltage varies between $10\ \text{V}$ to $16\ \text{V}$, then what is the maximum Zener current?



(A) 7.5 mA (B) 1.5 mA

(C) 2.5 mA (D) 3.5 mA

10. [Online January 2019]

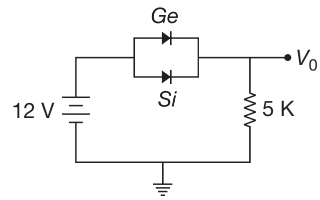
Mobility of electrons in a semiconductor is defined as the ratio of their drift velocity to the applied electric field. If, for an n-type semiconductor, the density of electrons is $10^{19}\ \text{m}^{-3}$ and their mobility is $1.6\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$ then the resistivity of the semiconductor (since it is an n-type semiconductor contribution of holes is ignored) is close

(A) $2\ \Omega\text{m}$ (B) $0.2\ \Omega\text{m}$

(C) $0.4\ \Omega\text{m}$ (D) $4\ \Omega\text{m}$

11. [Online January 2019]

Ge and Si diodes start conducting at $0.3\ \text{V}$ and $0.7\ \text{V}$ respectively. In the following figure if Ge diode connection are reversed, the value of V_0 changes by: (assume that the Ge diode has large breakdown voltage)

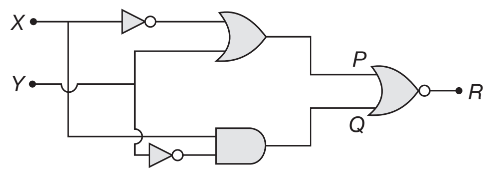


(A) 0.2 V (B) 0.4 V

(C) 0.6 V (D) 0.8 V

12. [Online January 2019]

To get output 1 at R, for the given logic gate circuit the input values must be

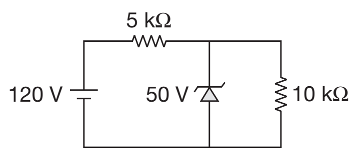


(A) $X=1$, $Y=1$ (B) $X=0$, $Y=0$

(C) $X=1$, $Y=0$ (D) $X=0$, $Y=1$

13. [Online January 2019]

For the circuit shown below, the current through the Zener diode is

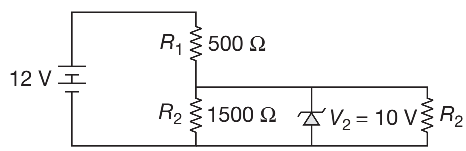


(A) Zero (B) 9 mA

(C) 14 mA (D) 5 mA

14. [Online January 2019]

In the given circuit the current through Zener Diode is close to

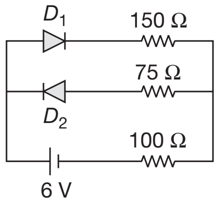


(A) 6.7 mA (B) 0.0 mA

(C) 4.0 mA (D) 6.0 mA

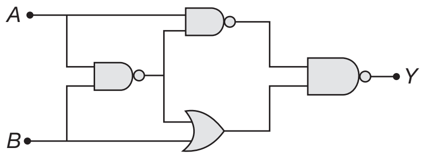
15. [Online January 2019]

The circuit shown below contains two ideal diodes, each with a forward resistance of $50\ \Omega$. If the battery voltage is $6\ \text{V}$, the current through the $100\ \Omega$ resistance (in amperes) is



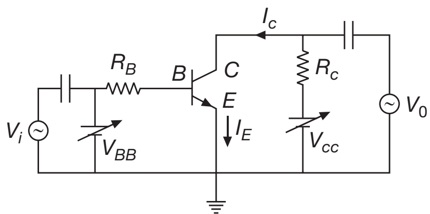
(A) 0.036 (B) 0.020
(C) 0.030 (D) 0.027

16. [Online January 2019]
The output of the given logic circuit is



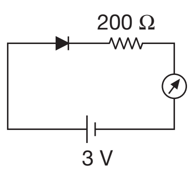
(A) $A\bar{B}+\bar{A}B$ (B) $A\bar{B}$
(C) $AB+\overline{AB}$ (D) $\bar{A}B$

17. [Online January 2019]
In the figure, given that V_{BB} supply can vary from 0 to 5.0 V, $V_{CC}=5$ V, $\beta_{dc}=200$, $R_B=100\ \text{k}\Omega$, $R_C=1\ \text{k}\Omega$ and $V_{BE}=1.0$ V. The minimum base current and the input voltage at which the transistor will go to saturation, will be respectively:



(A) 25 μA and 3.5 V
(B) 20 μA and 2.8 V
(C) 25 μA and 2.8 V
(D) 20 μA and 3.5 V

18. [2018]
The reading of the ammeter for a silicon diode in the given circuit is



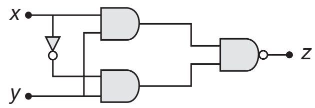
(A) 0 (B) 15 mA
(C) 11.5 mA (D) 13.5 mA

19. [Online 2018]
In a common emitter configuration with suitable bias, it is given that R_L is the load resistance and R_{BE} is small signal dynamic resistance (input side). Then, voltage gain, current gain and power gain are given, respectively, by (β is current gain, I_B, I_C and I_E are respectively base, collector and emitter currents.)

(A) $\beta\frac{R_L}{R_{BE}}, \frac{\Delta I_E}{\Delta I_B}, \beta^2\frac{R_L}{R_{BE}}$ (B) $\beta\frac{R_L}{R_{BE}}, \frac{\Delta I_C}{\Delta I_B}, \beta^2\frac{R_L}{R_{BE}}$

(C) $\beta^2\frac{R_L}{R_{BE}}, \frac{\Delta I_C}{\Delta I_E}, \beta^2\frac{R_L}{R_{BE}}$ (D) $\beta^2\frac{R_L}{R_{BE}}, \frac{\Delta I_C}{\Delta I_B}, \beta\frac{R_L}{R_{BE}}$

20. [Online 2018]
Truth table for the following digital circuit will be



(A)

x	y	z
0	0	1
0	1	1
1	0	1
1	1	1

(B)

x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

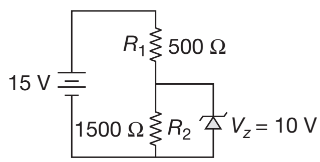
(C)

x	y	z
0	0	0
0	1	1
1	0	1
1	1	1

(D)

x	y	z
0	0	1
0	1	1
1	0	1
1	1	0

21. [Online 2018]
In the given circuit, the current through Zener diode is



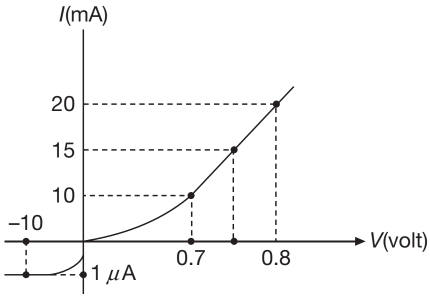
(A) 6.7 mA (B) 3.3 mA
(C) 2.5 mA (D) 5.5 mA

22. [2017]
In a common emitter amplifier circuit using an n-p-n transistor, the phase difference between the input and the output voltages will be

(A) 45° (B) 90°
(C) 135° (D) 180°

23. [Online 2017]

The V-I characteristic of a diode is shown in the figure. The ratio of forward to reverse bias resistance is



(A) 100 (B) 10^6

(C) 10 (D) 10^{-6}

24. [Online 2017]

What is the conductivity of a semiconductor sample having electron concentration of $5\times10^{18}\ \text{m}^{-3}$, hole concentration of $5\times10^{19}\ \text{m}^{-3}$, electron mobility of $2.0\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$ and hole mobility of $0.01\ \text{m}^2\text{V}^{-1}\text{s}^{-1}$? (Take charge of electron as 1.6×10^{-19} C)

(A) $1.83\ (\Omega\text{m})^{-1}$ (B) $1.68\ (\Omega\text{m})^{-1}$

(C) $1.20\ (\Omega\text{m})^{-1}$ (D) $0.59\ (\Omega\text{m})^{-1}$

25. [Online 2017]

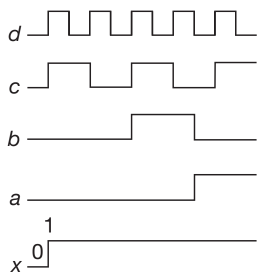
The current gain of a common emitter amplifier is 69. If the emitter current is 7.0 mA, collector current is

(A) 0.69 mA (B) 6.9 mA

(C) 69 mA (D) 9.6 mA

26. [2016]

If a, b, c, d are inputs to a gate and x is its output, then as per the following time graph, the gate is

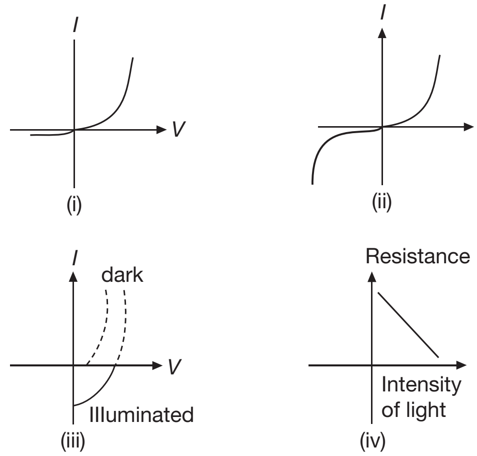


(A) NAND (B) NOT

(C) AND (D) OR

27. [2016]

Identify the semiconductor devices whose characteristics are given below, in the order (i), (ii), (iii), (iv)



(A) Zener diode, Solar cell, Simple diode, Light dependent resistance

(B) Simple diode, Zener diode, Solar cell, Light dependent resistance

(C) Zener diode, Simple diode, Light dependent resistance, Solar cell

(D) Solar cell, Light dependent resistance, Zener diode, Simple diode

28. [2016]

For a common emitter configuration, if α and β have their usual meanings, the incorrect relationship between α and β is

(A) $\alpha=\dfrac{\beta^2}{1+\beta^2}$ (B) $\dfrac{1}{\alpha}=\dfrac{1}{\beta}+1$

(C) $\alpha=\dfrac{\beta}{1-\beta}$ (D) $\alpha=\dfrac{\beta}{1+\beta}$

29. [Online 2016]

An unknown transistor needs to be identified as a npn or pnp type. A multimeter, with +ve and −ve terminals, is used to measure resistance between different terminals of transistor. If terminal 2 is the base of the transistor then which of the following is correct for a pnp transistor?

(A) +ve terminal 2, −ve terminal 3, resistance low

(B) +ve terminal 2, −ve terminal 1, resistance high

(C) +ve terminal 1, −ve terminal 2, resistance high

(D) +ve terminal 3, −ve terminal 2, resistance high

30. [Online 2016]

An experiment is performed to determine the I-V characteristics of a Zener diode, which has a protective resistance of $R = 100\ \Omega$, and a maximum power of dissipation rating of 1 W. The minimum voltage range of the DC source in the circuit is

(A) 0 – 5 V (B) 0 – 24 V
(C) 0 – 12 V (D) 0 – 8 V

31. [Online 2016]
The truth table given in figure represents

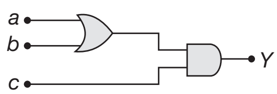
A	*B*	*Y*
0	0	0
0	1	1
1	0	1
1	1	1

(A) OR – Gate (B) NAND – Gate
(C) AND – Gate (D) NOR - Gate

32. [Online 2016]
The ratio (R) of output resistance r_0 and the input resistance r_i in measurements of input and output characteristics of a transistor is typically in the range

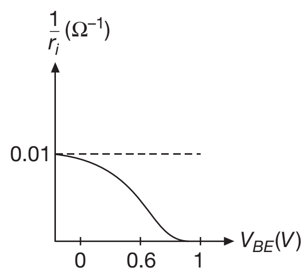
(A) $R \sim 10^2 - 10^3$ (B) $R \sim 1 - 10$
(C) $R \sim 0.1 - 1.0$ (D) $R \sim 0.1 - 0.01$

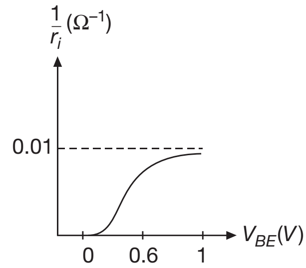
33. [Online 2016]
To get an output of 1 from the circuit shown in figure the input must be

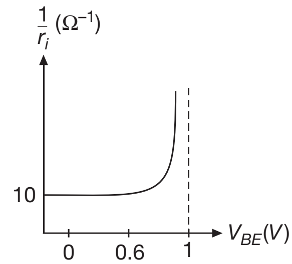


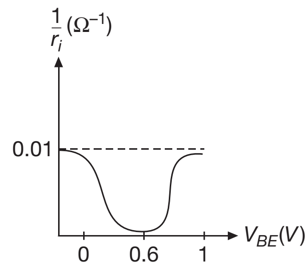
(A) $a = 0$, $b = 0$, $c = 1$
(B) $a = 1$, $b = 0$, $c = 0$
(C) $a = 1$, $b = 0$, $c = 1$
(D) $a = 0$, $b = 1$, $c = 0$

34. [Online 2016]
A realistic graph depicting the variation of the reciprocal of input resistance in an input characteristics measurement in a common emitter transistor configuration is

(A)


(B)


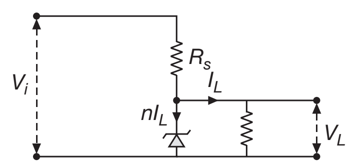
(C)


(D)


35. [Online 2015]
In an unbiased n-p junction electrons diffuse from n-region to p-region because

(A) holes in p-region attract them
(B) electrons travel across the junction due to potential difference
(C) electron concentration in n-region is more as compared to that in p-region
(D) only electrons move from n to p-region and not the vice-versa

36. [Online 2015]
The value of the resistor, R_S, needed in the dc voltage regulator circuit shown here, equals

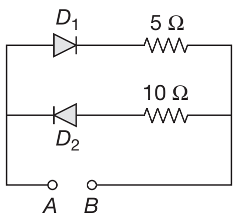


(A) $\dfrac{(V_i - V_L)}{nI_L}$ (B) $\dfrac{(V_i + V_L)}{nI_L}$

(C) $\dfrac{(V_i - V_L)}{(n+1)I_L}$ (D) $\dfrac{(V_i + V_L)}{(n+1)I_L}$

37. [Online 2015]

A 2 V battery is connected across AB as shown in the figure. The value of the current supplied by the battery when in one case battery's positive terminal is connected to A and in other case when positive terminal of battery is connected to B will respectively be



(A) 0.2 A and 0.1 A
(B) 0.4 A and 0.2 A
(C) 0.1 A and 0.2 A
(D) 0.2 A and 0.4 A

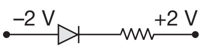
38. [2014]

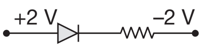
The current voltage relation of diode is given by $I = (e^{1000V/T} - 1)$ mA, where the applied voltage V is in volts and the temperature T is in degree kelvin. If a student makes an error measuring ± 0.01 V while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA?

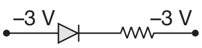
(A) 0.2 mA
(B) 0.02 mA
(C) 0.5 mA
(D) 0.05 mA

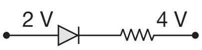
39. [2014]

The forward biased diode connection is

(A) 

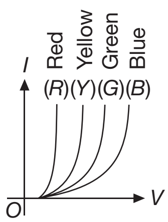
(B) 

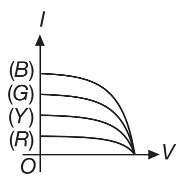
(C) 

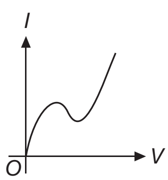
(D) 

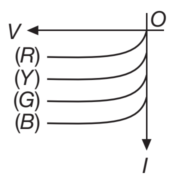
40. [2013]

The I-V characteristic of an LED is

(A) 

(B) 

(C) 

(D) 

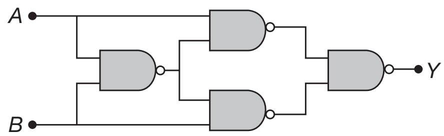
41. [2013]

A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of capacity 250 pico farad in parallel with a load resistance 100 kilo ohm. Find the maximum modulated frequency which could be detected by it

(A) 10.62 MHz
(B) 10.62 kHz
(C) 5.31 MHz
(D) 5.31 kHz

42. [2012]

Truth table for system of four NAND gates as shown in figure is



(A)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(B)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(C)

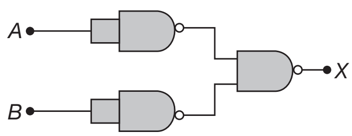
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(D)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

43. [2010]

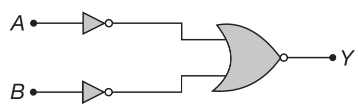
The combination of gates shown below yields

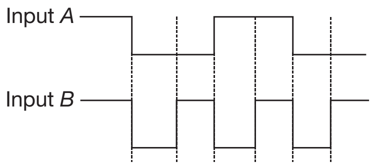


(A) NAND gate
(B) OR gate
(C) NOT gate
(D) XOR gate

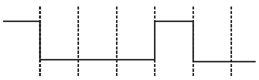
44. [2009]

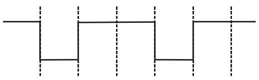
The logic circuit shown below has the input waveforms A and B as shown. Pick out the correct output waveform

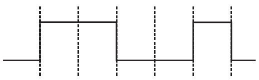


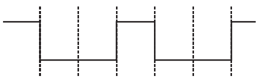


Output is

(A)


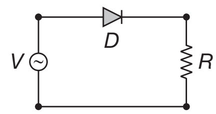
(B)


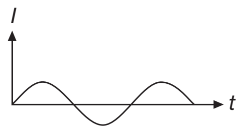
(C)


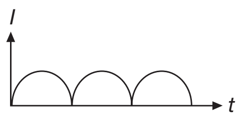
(D)


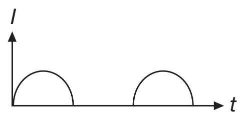
45. [2009]

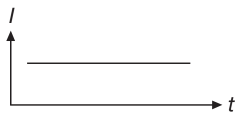
A p-n junction (D) shown in the figure can act as a rectifier. An alternating current source (V) is connected in the circuit. The current (I) in the resistor (R) can be shown by



(A)


(B)


(C)


(D)


ANSWER KEYS—PRACTICE EXERCISES

Single Correct Choice Type Questions

1. A	**2.** B	**3.** C	**4.** C	**5.** A	**6.** B	**7.** C	**8.** A	**9.** D	**10.** C
11. B	**12.** B	**13.** C	**14.** D	**15.** C	**16.** A	**17.** C	**18.** D	**19.** A	**20.** B
21. A	**22.** A	**23.** D	**24.** D	**25.** D	**26.** A	**27.** C	**28.** B	**29.** B	**30.** D
31. C	**32.** B	**33.** B	**34.** B	**35.** C	**36.** C	**37.** A	**38.** C	**39.** C	**40.** B
41. A	**42.** C	**43.** B	**44.** A	**45.** B	**46.** B	**47.** D	**48.** C	**49.** C	**50.** A
51. A	**52.** C	**53.** C	**54.** D	**55.** C	**56.** C	**57.** A	**58.** A	**59.** C	**60.** D
61. A	**62.** A	**63.** A	**64.** B	**65.** B	**66.** B	**67.** B	**68.** A	**69.** C	**70.** D
71. D	**72.** A	**73.** B	**74.** D	**75.** B	**76.** C	**77.** C	**78.** B	**79.** C	**80.** A
81. A	**82.** B	**83.** C	**84.** B	**85.** C	**86.** A	**87.** B	**88.** D	**89.** B	**90.** C
91. B	**92.** C	**93.** D	**94.** B	**95.** D	**96.** B	**97.** D	**98.** A	**99.** C	**100.** A
101. A	**102.** D	**103.** B	**104.** D	**105.** D	**106.** A	**107.** A	**108.** D	**109.** C	**110.** B
111. D	**112.** B	**113.** A	**114.** D	**115.** D	**116.** A	**117.** B	**118.** C	**119.** A	**120.** B
121. B	**122.** A	**123.** B	**124.** C	**125.** D	**126.** B	**127.** A	**128.** A	**129.** B	**130.** C
131. D	**132.** C	**133.** A	**134.** C	**135.** C	**136.** A	**137.** D	**138.** A	**139.** A	**140.** A
141. A	**142.** C	**143.** B	**144.** A						

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1. C	**2.** D	**3.** D	**4.** A	**5.** C	**6.** B	**7.** D	**8.** A	**9.** D	**10.** C
11. B	**12.** C	**13.** B	**14.** B	**15.** B	**16.** B	**17.** A	**18.** C	**19.** B	**20.** A
21. B	**22.** D	**23.** D	**24.** B	**25.** B	**26.** D	**27.** B	**28.** A, C	**29.** B	**30.** B
31. A	**32.** B	**33.** C	**34.** C	**35.** C	**36.** C	**37.** B	**38.** A	**39.** B	**40.** D
41. B	**42.** D	**43.** B	**44.** A	**45.** C					

CHAPTER 5

Communication Systems

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Roots of modern communication
(b) Elements of communication system
(c) Some basic terminologies
(d) Propagation of electromagnetic waves
(e) Concept of modulation
(f) Usage of antenna and concept of demodulation

All this is followed by an Exercise Set (fully solved) which contains questions as per the latest JEE pattern. At the end of Exercise Set, a collection of problems asked previously in JEE Main are also given.

INTRODUCTION

Communication is the act of transmission of information. We see that every living creature in the world needs to impart or receive information almost continuously with others in the surrounding world. For a communication to be successful, it is essential that the sender and the receiver both understand a common language. Man has constantly made endeavours to improve the quality of communication with other human beings. Languages and methods used in communication have kept evolving from prehistoric to modern times, to meet the growing demands in terms of speed and complexity of information. It would be worthwhile to look at the major milestones in events that promoted developments in communications, as presented in Table 5.1.

Table 5.1 Showing developments in communications

Year	Event	Remarks
Around 1565 A.D.	The reporting of the delivery of a child by queen using drum beats from a distant place to King Akbar.	It is believed that minister Birbal experimented with the arrangement to decide the number of drummers posted between the place where the queen stayed and the place where the king stayed.
1835	Invention of telegraph by Samuel F.B. Morse and Sir Charles Wheatstone	It resulted in tremendous growth of messages through post offices and reduced physical travel of messengers considerably.
1876	Telephone invented by Alexander Graham Bell and Antonio Meucci	Perhaps the most widely used means of communication in the history of mankind.

(*Continued*)

Table 5.1 Continued

Year	Event	Remarks
1895	Jagadis Chandra Bose and Guglielmo Marconi demonstrated wireless telegraphy.	It meant a giant leap – from an era of communication using wires to communicating without using wires. (wireless)
1936	Television broadcast (John Logi Baird)	First television broadcast by BBC
1955	First radio FAX transmitted across continent. (Alexander Bain)	The idea of FAX transmission was patented by Alexander Bain in 1843.
1968	ARPANET- the first internet came into existence (J.C.R. Licklider)	ARPANET was a project undertaken by the U.S. defence department. It allowed file transfer from one computer to another connected to the network.
1975	Fiber optics developed at Bell Laboratories	Fiber optical systems are superior and more economical compared to traditional communication systems.
1989-91	Tim Berners-Lee invented the **World Wide Web (www).**	**www** may be regarded as the mammoth encyclopedia of knowledge accessible to everyone round the clock throughout the year.

ROOTS OF MODERN COMMUNICATION

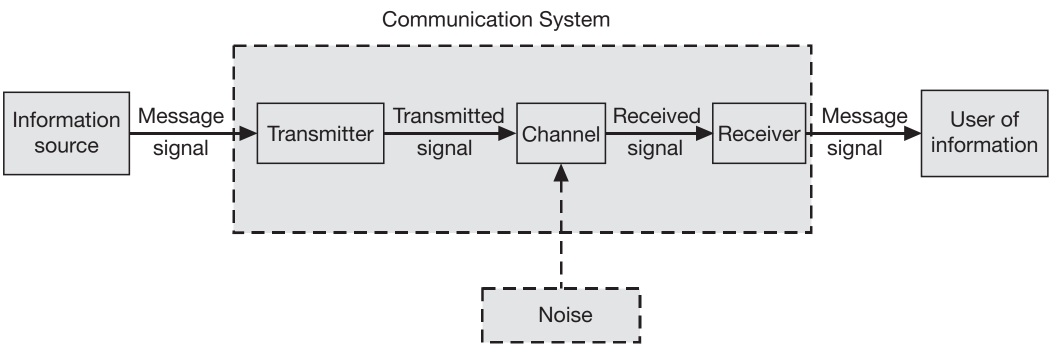
The works of scientists J.C. Bose, F.B. Morse, G. Marconi and Alexander Graham Bell in 19^{th} and 20^{th} century have been said to lay the roots of modern communication. This development seems to have increased dramatically after the first half of the 20^{th} century. In the era to come, we can hope to see many more accomplishments. The study of this chapter is to introduce us to the concepts of communication, the modes of communication, the need for modulation, the production and deduction of amplitude modulation etc.

ELEMENTS OF A COMMUNICATION SYSTEM

Communicating is the most essential part of life and it is seen that communication exists at all stages of life for all living creatures. Irrespective of its nature, every communication system has three essential elements.

(a) The Transmitter
(b) The Medium or Channel and
(c) The Receiver.

Figure shows the block diagram of the general form of a communication system.



Block diagram of a generalised communication system

In a communication system, the transmitter is located at one place, the receiver is located at some other place, near to or far from, the transmitter. The channel is the physical medium that connects the transmitter to the receiver. Depending upon the type of communication system, a channel may be in the form of wires or cables which connect the transmitter and the receiver or it may be wireless.

Function of Transmitter

The purpose of the transmitter is to convert the message signal produced by the source of information into a form suitable for transmission through the channel. If the output of the information source is a non-electrical signal like a voice signal, a transducer converts it to electrical form before giving it as an input to the transmitter.

Function of Receiver

When a transmitted signal propagates along the channel, it may get distorted due to channel imperfection. Moreover, noise adds to the transmitted signal and the receiver receives a corrupted version of the transmitted signal. The receiver has the task of operating on the received signal. It reconstructs a recognisable form of the original message signal for delivering it to the user for information.

MODES OF COMMUNICATION

There are two basic modes of communication

(a) Point-to-point mode of communication and
(b) Broadcast mode of communication

In point-to-point communication mode, communication takes place over a link between a single transmitter and a receiver. Telephony is an example of such a mode of communication.

In broadcast communication mode, there are a large number of receivers corresponding to a single transmitter. Radio and television are examples of broadcast mode of communication.

ABOUT JAGADIS CHANDRA BOSE (1858–1937)

Jagadis Chandra Bose developed an apparatus which generated ultrashort electro-magnetic waves and studied their quasi-optical properties. He was said to be the first to employ a semiconductor like galena as a self-recovering detector of electromagnetic waves. Bose also invented highly sensitive instruments for the detection of minute responses by living organisms to external stimuli and established parallelism between animal and plant tissues.

BASIC TERMINOLOGIES USED IN ELECTRONIC COMMUNICATION SYSTEMS

To understand the principles underlying any communication, we must first get ourselves acquainted with the following terminologies commonly used in communications.

Transmitter

A transmitter processes the incoming message signal so as to make it suitable for transmission through a channel and subsequent reception.

Receiver

A receiver extracts the desired message signals from the received signals at the channel output.

Noise

Noise refers to the unwanted signals that tend to disturb the transmission and processing of message signals in a communication system. The source generating the noise may be located inside or outside the system.

Transducer

Any device that converts one form of energy into another can be termed as a transducer. In electronic communication systems, we usually come across devices that have either their inputs or outputs in the electrical form. An electrical transducer may be defined as a device that converts some physical variable (pressure, displacement, force, temperature, etc) into corresponding variations in the electrical signal at its output.

Information

It is the news which one wishes to convey.

Signal

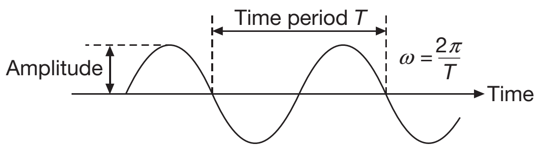
Information converted in electrical form and suitable for transmission is called a signal. A signal can be defined as a single valued function of time that conveys the information. Such a function has a unique value at any instant of time. Signals can be

(a) either Analog
(b) or Digital

Analog Signals

These signals are continuous variations of voltage or current. They are essentially single-valued functions of time. A sine wave is a fundamental analog signal. All other analog signals can be fully understood in terms of their sine wave components. Sound and picture signals in TV are analog in nature. It is represented by the equation

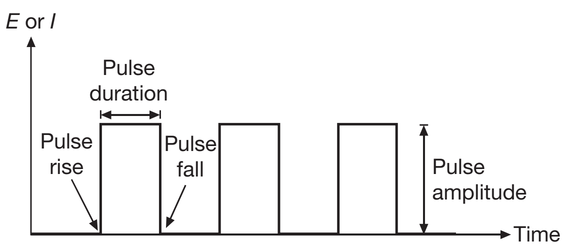
$$E = E_0 \sin(\omega t + \phi)$$



Sinusoidal pulse

Digital Signals

These signals are those which can take up only discrete stepwise values. Binary system that is extensively used in digital electronics employs just two levels of a signal. 0 corresponds to a low level and 1 corresponds to a high level of voltage/ current.



Pulse shaped signals

There are several coding schemes useful for digital communication. They employ suitable combinations of number systems such as the binary coded decimal (BCD). American Standard Code for Information Interchange (ASCII) is a universally popular digital code to represent numbers, letters and certain characters.

Amplification

It is the process of increasing the amplitude (and consequently the strength) of a signal using an electronic circuit called the amplifier. Amplification is necessary to compensate for the attenuation of the signal in communication systems. The energy needed for additional signal strength is obtained from a DC power source. Amplification is done at a place between the source and the destination wherever signal strength becomes weaker than the required strength.

Attenuation

The loss of strength of a signal while propagating through a medium is known as attenuation.

Range

It is the largest distance between a source and a destination up to which the signal is received with sufficient strength.

Bandwidth

Bandwidth refers to the frequency range over which an equipment operates or the portion of the spectrum occupied by the signal.

Baseband

It is the band of frequencies of the original signal (i.e. a signal not changed by modulation) produced by the source of information.

Modulation

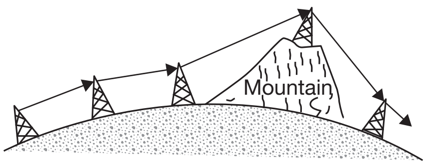
The original low frequency message or the information signal cannot be transmitted to long distances. Therefore, at the transmitter, information contained in the low frequency message signal is superimposed on a high frequency wave, which acts as a carrier of the information *also called as Carrier Wave*. This process is known as modulation. There are several types of modulation, abbreviated as AM (amplitude modulation), FM (frequency modulation) and PM (phase modulation).

Demodulation

The process of retrieval/recovery of information from the carrier wave at the receiver is called as demodulation. This process is the reverse of modulation.

Repeater

A repeater is a combination of a receiver and a transmitter. A repeater, picks up the signal from the transmitter, amplifies and retransmits it to the receiver sometimes with a change in carrier frequency. Repeaters are used to extend the range of a communication system as shown in figure. A communication satellite is essentially a repeater station in space.



Use of repeater station to increase the range of communication

BANDWIDTH OF ANALOG SIGNALS

The message signal in any communication system can be voice, music, picture or computer data. Each of these signals has different ranges of frequencies. The type of communication system needed for a given signal depends on the band of frequencies which is considered essential for the communication process.

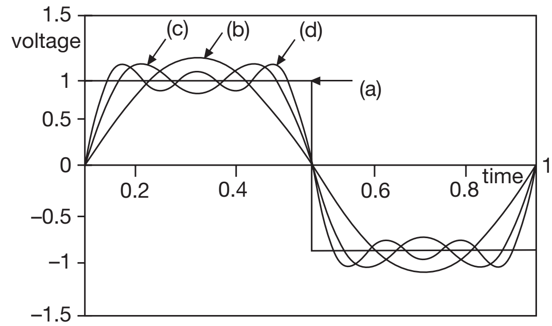
For speech signals, frequency range 300 Hz to 3100 Hz is considered adequate. Therefore, for commercial telephonic communications of speech signals s a bandwidth of 2800 Hz ranging from 3100 Hz to 300 Hz is required.

For transmitting music, an approximate bandwidth of 20 kHz is required, because of the high frequencies produced by the musical instruments. The audible range of frequencies extends from 20 Hz to 20 kHz .

Video signals for transmission of pictures require about 4.2 MHz of bandwidth. A TV signal contains both voice and picture and is usually allocated 6 MHz of bandwidth for transmission.

DIGITAL SIGNALS IN THE FORM OF SINE WAVES

Since we know that the analog signals are expressed in terms of a sine wave and digital signals are in the form of rectangular waves as shown in figure.



(a) Rectangular wave
(b) Fundamental ν_0
(c) Fundamental (ν_0) + 2nd harmonic ($2\nu_0$)
(d) Fundamental (ν_0) + 2nd harmonic ($2\nu_0$) + 3rd harmonic ($3\nu_0$)

Approximation of a rectangular wave in terms of a fundamental sine wave and its harmonics

These rectangular waves can be approximately expressed in terms of superposition of sinusoidal waves of frequencies ν_0, $2\nu_0$, $3\nu_0$, $4\nu_0 \ldots n\nu_0$ where n is an integer extending to infinity and $\nu_0 = \frac{1}{T_0}$.

To illustrate this fact, we have shown the following three frequencies in the same figure.

(a) Fundamental frequency (ν_0),

(b) Fundamental frequency plus the second harmonic i.e. $(\nu_0 + 2\nu_0)$ and

(c) Fundamental frequency plus the second harmonic plus the third harmonic i.e. $(\nu_0 + 2\nu_0 + 3\nu_0)$

So, we observe that, to reproduce the rectangular wave shape exactly we need to superimpose all the harmonics ν_0, $2\nu_0$, $3\nu_0$, $4\nu_0$ ..... which implies an infinite bandwidth.

However, for practical purposes, the contribution from higher harmonics can be neglected, thus limiting the bandwidth. Due to this, the received waves are a distorted version of the transmitted waves.

So, if the bandwidth is large enough to accommodate a few harmonics, the information is not lost and the rectangular signal is more or less recovered. *This is so because, the higher the harmonic the less is its contribution to the wave form.*

BANDWIDTHS OF COMMUNICATION CHANNELS

The physical path between the transmitter and the receiver is called the *communication channel*. The bandwidth of the communication channel is the difference between the highest and the lowest frequencies that the channel allows to pass through it, also called as the *Passband*.

The commonly used transmission media are wire, free space and fibre optic cable. Coaxial cable is a widely used wire medium, which offers a bandwidth of approximately 750 MHz. Such cables are normally operated below 18 GHz.

Communication through free space using radio waves takes place over a very wide range of frequencies: from a few hundreds of kHz to a few GHz. This range of frequencies is further subdivided and allocated for various services as indicated in the following Table 5.2.

Table 5.2 Showing some important wireless communication frequency bands

Service	Frequency Bands	Comments
Standard AM broadcast	540-1600 kHz	
FM broadcast	88-108 MHz	
Television	54-72 MHz 76-88 MHz 174-216 MHz 420-890 MHz	VHF (very high frequencies) TV UHF (ultra high frequencies) TV
Cellular Mobile Radio	896-901 MHz 840-935 MHz	Mobile to base station Base station to mobile
Satellite Communication	5.925-6.425 GHz 3.7-4.2 GHz	Uplink Downlink

Optical communication using fibres is performed in the frequency range of 1 THz to 1000 THz (microwaves to ultraviolet). An optical fiber can offer a transmission bandwidth in excess of 1000 GHz.

Spectrum allocations are arrived at by an international agreement. The International Telecommunication Union (ITU) administers the present system of frequency allocations.

The number of channels (N) that can be accommodated in a given band width is given by

$$N=\frac{\text{Total Bandwidth of Channel}}{\text{Bandwidth Needed per Channel}}$$

ILLUSTRATION 1

Assume that light of frequency 4.5×10^{14} Hz is used in an optical communication system. If 2% of the frequency bandwidth is used, how many T.V. channels can be accommodated in this bandwidth? The bandwidth needed for T.V. transmission is 4.5×10^{6} Hz/channel.

SOLUTION

Optical frequency used

$$f=4.5\times10^{14}\text{ Hz}$$

Total bandwidth of the channel is

$$\Delta f=2\%\text{ of }4.5\times10^{14}\text{ Hz}=9\times10^{12}\text{ Hz}$$

Bandwidth per TV channel $=4.5\times10^{6}$ Hz

Number of TV channels which can be accommodated

$$N=\frac{\text{Total bandwidth of the channel}}{\text{Bandwidth needed per TV channel}}$$

$$\Rightarrow\quad N=\frac{9\times10^{12}}{4.5\times10^{6}}=2\times10^{6}\text{ channels}$$

ILLUSTRATION 2

The TV signals have a bandwidth of 4.7 MHz. What is the number of channels that can be accommodated in a bandwidth of 4700 GHz?

SOLUTION

Number of channels

$$N=\frac{\text{Total bandwidth of the channel}}{\text{Bandwidth needed per channel}}$$

$$\Rightarrow\quad N=\frac{4700\text{ GHz}}{4.7\text{ MHz}}=\frac{4700\times10^{9}}{4.7\times10^{6}}=10^{6}$$

PROPAGATION OF ELECTROMAGNETIC WAVES

In communications using radio waves, an antenna at the transmitter radiates the electromagnetic waves, also called as em waves, which travel through the space and reach the receiving antenna at the other end. As the em wave travels away from the transmitter, the strength of the wave keeps on decreasing. Several factors have been known to influence the propagation of em waves and the path they follow. At this stage, it becomes important for us to understand the composition of the earth's atmosphere because it plays a vital role in the propagation of em waves. A brief discussion on some useful layers of the atmosphere is given in Table 5.3.

Table 5.3 Showing different layers of atmosphere and their interaction with the propagating electromagnetic waves

Name of the stratum (layer)	Approximate height over earth's surface		Exists during	Frequencies most affected
Troposphere	10 km		Day and Night	VHF (up to several GHz)
D Layer (a part of Stratosphere)	PARTS OF IONOSPHERE	65 km to 75 km	Day only	Reflects LF, absorbs MF and HF to some extent.
E Layer (a part of Stratosphere)		100 km	Day only	Helps surface waves, reflects HF.
F_1 Layer (a part of Mesosphere)		170 km to 190 km	Day only. It merges with F_2 at night.	Partially absorbs HF waves yet allowing them to reach F_2.
F_2 Layer Thermosphere		300 km at night, 250 km to 400 km during daytime	Day and Night	Efficiently reflects HF waves, particularly at night.

Electromagnetic waves radiated from an antenna can be transmitted through space in three ways.

(a) Ground wave propagation
(b) Sky wave propagation
(c) Space wave propagation

GROUND WAVE PROPAGATION

To radiate signals with high efficiency, the antennas should have a size comparable to the wavelength λ of the signal (at least $\sim \frac{\lambda}{4}$). At longer wavelengths (i.e., at lower frequencies), the antennas have large physical size and they are located on or very near to the ground. In standard AM broadcast, ground based vertical towers are generally used as transmitting antennas. For such antennas, ground has a strong influence on the propagation of the signal. This mode of propagation is called *surface wave propagation* and the wave glides over the surface of the earth. A wave induces current in the ground over which it passes and it is attenuated as a result of absorption of energy by the earth. The attenuation of surface waves increases very rapidly with increase in frequency. The maximum range of coverage depends on the transmitted power and frequency (less than a few MHz). The maximum range of ground wave communications depends on two factors.

1. The power of the transmitter.
2. The frequency of the transmitted wave.

Conceptual Note(s)

To radiate the signals with high efficiency, the antennas should have a size comparable to the wavelength λ (or at least $\approx \frac{\lambda}{4}$) of the transmitted signal.

Advantages and Disadvantages of Ground Wave Propagation

Advantages

(a) Ground waves are not affected by the changing atmospheric conditions.

(b) When given enough transmitting power, the ground waves can be used to communicate between any two locations on the earth.

Disadvantages

(a) Since ground waves are limited to VLF, LF and MF signals, so they require large antennas.

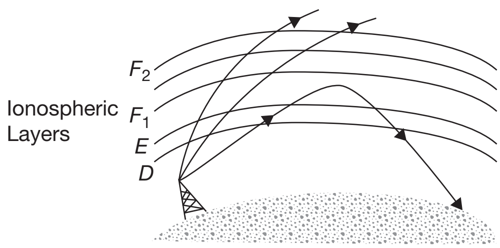
(b) Ground waves require a relatively high transmission power.

SKY WAVE PROPAGATION

In the frequency range of a few MHz (i.e. upto 30 MHz to 40 MHz), long distance communication can be achieved by ionospheric reflection of radio waves back towards the earth. This mode of propagation is called *sky wave propagation* and is used by short wave broadcast services.

A radio wave directed toward the sky and reflected by the ionosphere towards the desired location on the earth is called a *sky wave*.

The ionosphere is so called because of the presence of a large number of ions or charged particles. It extends from a height of $\sim 65\text{ km}$ to about 400 km above the earth's surface. Ionisation occurs due to the absorption of the ultraviolet and other high-energy radiation coming from the sun by air molecules. The ionosphere is further subdivided into several layers, the details of which are given previously in Table 5.3. The degree of ionisation varies with the height. The density of atmosphere decreases with height. At great heights the solar radiation is intense but there are few molecules to be ionised. Close to the earth, even though the molecular concentration is very high, the radiation intensity is low so that the ionisation is again low. However, at some intermediate heights, there occurs a peak of ionisation density. The ionospheric layer acts as a reflector for frequencies ranging from 3 MHz to 30 MHz. Electromagnetic waves of frequencies higher than 30 MHz penetrate the ionosphere and escape. These phenomena are shown in the figure.



Sky wave propagation

The phenomenon of bending of em waves so that they are diverted towards the earth is similar to the phenomenon of total internal reflection as studied earlier in Ray Optics.

Explanation of Reflection of Radio Waves from Ionosphere

Whenever an em wave passes through the ionosphere, its oscillating electric field changes the velocity of electrons in the ionosphere (the heavy ions being negligibly affected). Due to this the effective dielectric constant changes and hence the effective refractive index of the ionosphere also changes. The effective dielectric constant n_{eff} of the ionosphere is

$$n_{eff} = n_0\sqrt{1-\frac{Ne^2}{\varepsilon_0 m\omega^2}} = n_0\sqrt{1-\frac{81N}{f^2}}$$

where n_0 is the refractive index of free space, N is the electron density of ionosphere, ε_0 is the dielectric constant of free space, e is the charge on electron, m is the mass of electron and ω is the angular frequency of EM wave.

So, we observe that $n_{\text{eff}} < n_0$, i.e. the effective refractive index of the ionosphere is less than that of free space and hence ionosphere behaves like a rarer medium. As we go deep into the ionosphere, N increases so n_{eff} decreases. Consequently, the em wave will gradually turn away from the normal as it keeps on penetrating the ionosphere. This happens till the angle of incidence reaches the critical angle and after which the em waves are reflected back to the earth (phenomenon of Total Internal Reflection).

Critical Frequency

It is observed that different frequencies are reflected from different regions of the ionosphere which have different values of N (i.e. the electron density of ionosphere). This is the reason why different points on the earth receive signals reflected from the different depts of the ionosphere. If the frequency is too high, then after a certain value, the density N may not be so high so as to produce enough bending for attaining the condition of Total Internal Reflection. This value of frequency is called the critical frequency f_c. If $N_{\max}$ is the maximum electron density in the ionosphere layer, then the critical frequency is mathematically given by

$$f_c \approx 9\sqrt{N_{\max}}$$

The value of f ranges from 5 MHz to 10 MHz. Frequencies higher than f_c cross the ionosphere and do not return to earth.

So, the highest frequency of the radio wave which when sent straight normally toward the given layer ionosphere gets reflected from the ionosphere to return to earth is called the critical frequency.

Conceptual Note(s)

Maximum Usable Frequency (MUF)

The highest frequency of the radio wave which when sent at a certain angle towards the given layer of ionosphere will get reflected from it to return to earth is called the MUF. Mathematically, MUF is given by

$$\text{MUF} = \frac{f_c}{\cos\theta}$$

where θ is the angle between the incident ray and the normal.

ILLUSTRATION 3

Frequencies higher than 10 MHz are found not to be reflected by the ionosphere on a particular day at a place. Calculate the maximum electron density of the ionosphere.

SOLUTION

Given that $f_c = 10\text{ MHz} = 10^7\text{ Hz}$

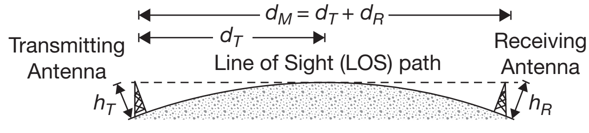
Now $f_c = 9(N_{\max})^{\frac{1}{2}}$

$\Rightarrow \quad f_c^2 = 81\text{ N}_{\max}$

$$\Rightarrow \quad N_{\max} = \frac{f_c^2}{81} = \frac{(10^7)^2}{81} = 1.23 \times 10^{12}\text{ m}^{-3}$$

SPACE WAVE PROPAGATION

Another mode of radio wave propagation is by space waves. A space wave travels in a straight line from transmitting antenna to the receiving antenna. Space waves are used for *line-of-sight (LOS) communication* and satellite communication. At frequencies above 40 MHz, communication is essentially limited to line-of-sight paths. At these frequencies, the antennas are relatively smaller and can be placed at heights of many wavelengths above the ground. Because of line-of-sight nature of propagation, direct waves get blocked at some point by the curvature of the earth as illustrated in figure.



Line of sight communication by space waves

If the signal is to be received beyond the horizon then the receiving antenna must be high enough to intercept the line-of-sight waves.

If the transmitting antenna is at a height h_T, then it can be shown that the distance to the horizon d_T is given by

$$d_T = \sqrt{2Rh_T}$$

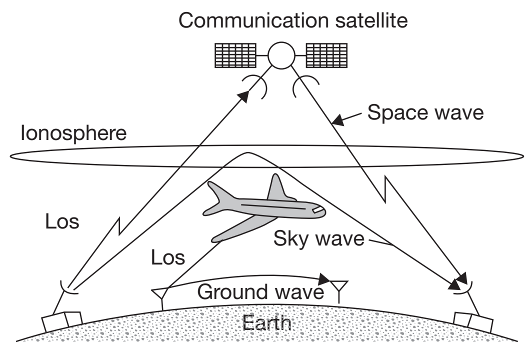
where R is radius of the earth (approximately 6400 km). d_T is also called the radio horizon of the transmitting antenna.

The maximum line-of-sight distance d_M between the two antennas having heights h_T and h_R above the earth is given by

$$d_M = d_T + d_R = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

where h_R is the height of the receiving antenna.

Television broadcast, microwave links and satellite communication are some examples of communication systems that use space wave mode of propagation. Figure summarises the various modes of wave propagation discussed so far.



Various propagation modes for em waves

DETERMINATION OF RANGE

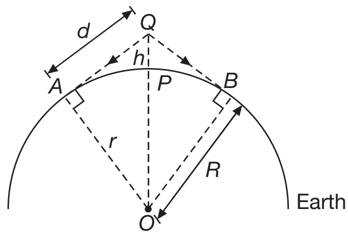
The range is determined by the height of transmitting antenna. The range AP or PB can be easily calculated by geometrical consideration. Suppose height of the tower is h and the radius of earth is r (that is $OA = OB = OP = r$). In the right-angled triangle OQA, we have

$$OQ^2 = QA^2 + OA^2$$

Since $QA \simeq AP = d$

$$\Rightarrow \quad (R+h)^2 = R^2 + d^2$$

$$\Rightarrow \quad R^2 + h^2 + 2Rh = R^2 + d^2$$



Since $R^2 \gg h^2$

$$\Rightarrow \quad d^2 = 2Rh$$

$$\Rightarrow \quad d = \sqrt{2Rh}$$

ILLUSTRATION 4

A transmitting antenna at the top of a tower has a height 32 m and the height of the receiving antenna is 50 m. What is the maximum distance between them for satisfactory communication in LOS mode? Given radius of earth $6.4 \times 10^6\text{ m}$.

SOLUTION

$$d_m = \sqrt{2 \times 64 \times 10^5 \times 32} + \sqrt{2 \times 64 \times 10^5 \times 50}\text{ m}$$

$$\Rightarrow \quad d_m = 64 \times 10^2 \times \sqrt{10} + 8 \times 10^3 \times \sqrt{10}\text{ m}$$

$$\Rightarrow \quad d_m = 144 \times 10^2 \times \sqrt{10}\text{ m} = 45.5\text{ km}$$

ILLUSTRATION 5

A ground receiver station is receiving a signal at frequencies of 5 MHz and 100 MHz, transmitted from a ground transmitter at a height of 300 m located at a distance of 100 km. Identify whether it is coming via space wave or sky wave propagation or satellite transponder. (Given the value of radius of the earth is 6400 km and maximum electron density, $N_{\max} = 10^{12}\text{ m}^{-3}$)

SOLUTION

Maximum distance covered by space wave propagation

$$d = \sqrt{2Rh} = \sqrt{2 \times 6.4 \times 10^6 \times 300\text{ m}} = 62\text{ km}$$

As the receiver transmitter distance is 100 km, so space wave propagation is not possible for both 5 MHz and 100 MHz waves. The critical frequency for ionospheric propagation is

$$f_c = 9(N_{\max})^{\frac{1}{2}} = 9(10^{12})^{\frac{1}{2}}$$

$$\Rightarrow \quad f_c = 9 \times 10^6\text{ Hz} = 9\text{ MHz}$$

Since the 5 MHz frequency signal has frequency less than the critical frequency f_c, so this signal must be coming via ionospheric propagation, whereas the 100 MHz frequency signal must have been coming via satellite transmission.

ILLUSTRATION 6

What will be the required height of a T.V. tower which can cover the population of 60.3 lakhs if average population density around the tower is 1000 km^{-2}? [Radius of earth $= 6.4 \times 10^6\text{ m}$]

SOLUTION

$$\begin{pmatrix}\text{Area}\\\text{Covered}\end{pmatrix} \times \begin{pmatrix}\text{Population}\\\text{Density}\end{pmatrix} = \begin{pmatrix}\text{Population}\\\text{Covered}\end{pmatrix}$$

$$\Rightarrow \quad \pi d^2 \times (1000\text{ km}^{-2}) = 60.3\text{ lakhs}$$

$$\Rightarrow \quad \pi d^2 \times 1000(1000\text{ m})^{-2} = 60.3 \times 10^5$$

$$\Rightarrow \quad d^2 = \frac{60.3 \times 10^5 \times 1000}{\pi} = 1.92 \times 10^9$$

Height of T.V. tower is

$$h = \frac{d^2}{2R} = \frac{1.92 \times 10^9}{2 \times 6.4 \times 10^6} = 150\text{ m}$$

ILLUSTRATION 7

A transmitting antenna at the top of a tower has a height 32 m and that of the receiving antenna is 100 m. What is the maximum distance between them for satisfactory communication in *LOS* mode? Given radius of earth $6.4 \times 10^6\text{ m}$.

SOLUTION

Here $h_T = 32$ m, $h_R = 100$ m,

$$R = 6.4 \times 10^6 \text{ m}$$

$$d_M = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

$$\Rightarrow \quad d_M = \sqrt{2 \times 64 \times 10^5 \times 32} + \sqrt{2 \times 64 \times 10^5 \times 50} \text{ m}$$

$$\Rightarrow \quad d_M = 64 \times 10^2 \times \sqrt{10} + 8 \times 10^3 \times \sqrt{10} \text{ m}$$

$$\Rightarrow \quad d_M = 144 \times 10^2 \sqrt{10} \text{ m} = 45.5 \text{ km}$$

ILLUSTRATION 8

What is space wave propagation? Which two communication methods make use of this mode of propagation? If the sum of the heights of transmitting and receiving antennas in line of sight of communication is fixed at h, show that the range is maximum when the two antennas have a height $\frac{h}{2}$ each.

SOLUTION

If a radio wave transmitted from an antenna, travelling in a straight line, directly reaches the receiving antenna, the propagation is called space wave propagation. Television broadcast and satellite communication make use of this mode of propagation. The range of line-of-sight communication between two antennas is given by

$$r = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

Given $h_T + h_R = h$

Let $h_T = H$, then $h_R = h - H$

$$\Rightarrow \quad r = \sqrt{2R}\left[\sqrt{H} + \sqrt{h-H}\right]$$

For r to be maximum, we have $\frac{dr}{dH} = 0$

$$\Rightarrow \quad \sqrt{2R}\left(\frac{1}{2\sqrt{H}} + \frac{1}{2\sqrt{h-H}(-1)}\right) = 0$$

$$\Rightarrow \quad \frac{1}{2\sqrt{H}} - \frac{1}{2\sqrt{h-H}} = 0$$

$$\Rightarrow \quad H = h - H$$

$$\Rightarrow \quad H = \frac{h}{2}$$

Hence for maximum range, $h_T = h_R = \frac{h}{2}$

MODULATION AND ITS NECESSITY

Since modulation is the process of superimposing the low audio frequency baseband message or information signals (called as modulating signals) on a high frequency wave (called as *carried wave*). The resultant transmitted wave is called as the *modulated wave*. Since the purpose of a communication system is to transmit information or message signals without significant loss. The message signals are also called *baseband signals*, which essentially designate the band of frequencies representing the original signal, as delivered by the source of information. No signal, in general, is a single frequency sinusoid, but it spreads over a range of frequencies called the *signal bandwidth*. Suppose we wish to transmit an electronic signal in the audio frequency (AF) range (baseband signal frequency less than 20 kHz) over a long distance directly. Let us find what factors prevent us from doing so and how we overcome these factors.

Size of the Antenna or Aerial

For transmitting a signal, we need an antenna or an aerial. This antenna should have a size comparable to the wavelength of the signal (at least $\frac{\lambda}{4}$ in dimension), so that the antenna properly senses the time variation of the signal. For an electromagnetic wave of frequency 20 kHz, the wavelength λ is 15 km and such a long antenna is not possible to construct and operate. Hence direct transmission of such baseband signals is not practical. We can obtain transmission with reasonable antenna lengths if transmission frequency is high (for example, if v is 1 MHz, then λ is 300 m). Therefore, there is a need of translating the information contained in our original low frequency baseband signal into high or radio frequencies before transmission.

Effective Power Radiated by an Antenna

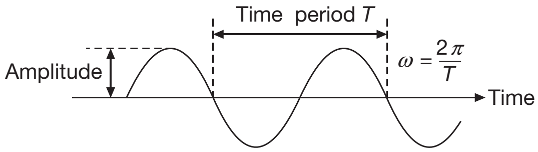
A theoretical study of radiation from a linear antenna (length l) shows that the power radiated is proportional to $\left(\frac{l}{\lambda}\right)^2$. This implies that for the same antenna length, the power radiated increases with decreasing λ, i.e., increasing frequency. Hence, the effective power radiated by a long wavelength baseband signal would be small. For a good transmission, we need high powers and hence this also points out to the need of using high frequency transmission.

Mixing Up of Signals from Different Transmitters

Another important argument against transmitting baseband signals directly is more practical in nature. Suppose many people are talking at the same time or many transmitters are transmitting baseband information signals simultaneously. All these signals will get mixed up and there is no simple way to distinguish between them. This points out towards a possible solution by using communication at high frequencies and allotting a band of frequencies to each message signal for its transmission.

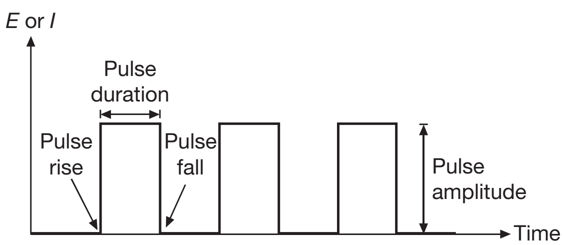
The above arguments suggest that there is a need for translating the original low frequency baseband message or information signal into high frequency wave before transmission such that the translated signal continues to possess the information contained in the original signal. In doing so, we take the help of a high frequency signal, known as the carrier wave, and a process known as modulation which attaches information to it. The carrier wave may be a

(a) Continuous sinusoidal wave or



(a) Sinusoidal pulse

(b) Discontinuous pulse wave



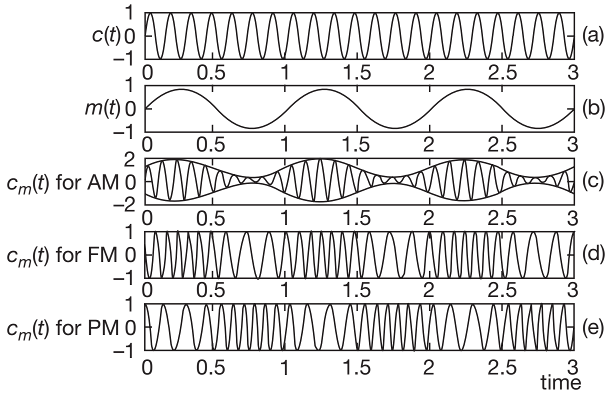
(b) Pulse shaped signals

A sinusoidal carrier wave can be represented as

$$c(t) = A_c \sin(\omega_c t + \phi)$$

where $c(t)$ is the signal strength (voltage or current), A_c is the *carrier voltage amplitude*, $\omega_c (= 2\pi\nu_c)$ is the angular frequency and ϕ is the initial phase of the carrier wave. During the process of modulation, any of the three parameters, viz A_c, ω_c and ϕ, of the carrier wave can be controlled by the message or information signal. This results in three types of modulation:

(i) Amplitude modulation (AM),
(ii) Frequency modulation (FM) and
(iii) Phase modulation (PM), as shown in figure.



Modulation of a carrier wave: (a) A sinusoidal carrier wave; (b) A modulating signal; (c) Amplitude modulation; (d) Frequency modulation; and (e) Phase modulation

Similarly, we know that, the significant characteristics of a pulse wave are

(a) Pulse amplitude
(b) Pulse duration or pulse width and
(c) Pulse position (denoting the time of rise or fall of the pulse amplitude).

Based on this information we observe that the following different types of pulse modulation are possible.

(a) Pulse Amplitude Modulation (PAM),
(b) Pulse Duration Modulation (PDM) also called as Pulse Width Modulation (PWM) and
(c) Pulse Position Modulation (PPM).

In this chapter, we shall confine our study to amplitude modulation only.

AMPLITUDE MODULATION

In amplitude modulation, the amplitude of the carrier wave is varied in accordance with the amplitude of the information signal. However, the frequency of the amplitude modulated wave remains the same as that of the carrier wave.

To explain the amplitude modulation process using a sinusoidal signal as the modulating signal, given by

$$m(t) = A_m \sin(\omega_m t)$$

where A_m is the modulating voltage amplitude of the modulating signal and ω_m is the angular frequency of the modulating signal.

Let the sinusoidal carrier wave be given by

$$c(t) = A_c \sin(\omega_c t)$$

where A_c is the carrier voltage amplitude of the carrier wave and ω_c is the angular frequency of the carrier wave.

The modulated signal $c_m(t)$ i.e. instantaneous voltage of the AM wave can be written as

$$c_m(t) = (A_c + A_m \sin \omega_m t)\sin(\omega_c t)$$

If $A_{\max}$ and $A_{\min}$ be the maximum and the minimum amplitude of the modulated wave, then

$$A_{\max} = A_c + A_m \text{ and}$$

$$A_{\min} = A_c - A_m$$

Now, $$c_m(t) = A_c\left(1 + \frac{A_m}{A_c}\sin \omega_m t\right)\sin(\omega_c t) \quad \ldots(1)$$

Note that the modulated signal also contains the message signal. Equation (1), can be written as

$$c_m(t) = A_c \sin(\omega_c t) + \mu A_c \sin(\omega_m t)\sin(\omega_c t) \quad \ldots(2)$$

where $\mu = \dfrac{A_m}{A_c}$ is called the Modulation Index.

In practice, to avoid distortion, μ is kept ≤ 1.

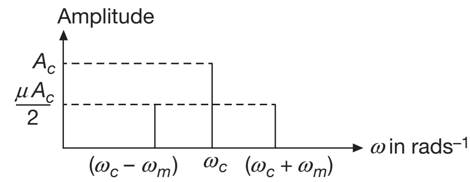
Using the trigonometric relation

$$\sin A \sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B)),$$

we can write $c_m(t)$ of equation (2) as

$$c_m(t) = A_c \sin \omega_c t + \frac{\mu A_c}{2}\cos(\omega_c - \omega_m)t - \frac{\mu A_c}{2}\cos(\omega_c + \omega_m)t$$

Here $(\omega_c - \omega_m)$ and $(\omega_c + \omega_m)$ are respectively called the *lower side and upper side frequencies*. The modulated signal now consists of the carrier wave of frequency ω_c plus two sinusoidal waves each with a frequency slightly different from, known as *side bands*. The frequency spectrum of the amplitude modulated signal is shown in figure.



A plot of amplitude versus ω for an amplitude modulated signal

As long as the broadcast frequencies (carrier waves) are sufficiently spaced out so that sidebands do not overlap, different stations can operate without interfering with each other. The two side band frequencies have the same voltage amplitude given by

$$A_{SB} = \frac{\mu A_C}{2}$$

A_{SB} never exceeds half the carrier amplitude, because $\mu \le 1$.

So, we observe that the lower side band frequency is

$$\omega_{\text{LSB}} = \omega_C - \omega_M$$

$$\Rightarrow \quad f_{\text{LSB}} = f_C - f_M$$

The upper side band frequency is given by

$$\omega_{\text{USB}} = \omega_C + \omega_M$$

$$\Rightarrow \quad f_{\text{USB}} = f_C + f_M$$

Band width $\Delta\omega$ of the amplitude modulated wave is given by

$$\Delta\omega = \omega_{\text{USB}} - \omega_{\text{LSB}} = 2\omega_M$$

This makes us conclude that the band width is twice the frequency of the modulating signal.

ILLUSTRATION 9

An audio signal given by, $m(t) = 30 \sin (2\pi \times 2500t)$ volts is used to amplitude modulate a carrier wave given by $c(t) = 60 \sin (2\pi \times 200{,}000t)$ volts. Find

(a) percent modulation,
(b) components of modulated wave and
(c) the amplitude of each sideband.

SOLUTION

Here $m(t)=A_m\sin(2\pi f_m t)=30\sin(2\pi\times 2500t)$ volts

and $c(t)=A_c\sin(2\pi f_c t)=60\sin(2\pi\times 200{,}000t)$ volts

Clearly, $A_m=30\text{ V}$, $f_m=2500\text{ Hz}$

$A_c=60\text{ V}$, $f_c=200{,}000\text{ Hz}$

(a) $\mu=\dfrac{A_m}{A_c}\times 100=\dfrac{30}{60}\times 100=50\%$

(b) The three components of the modulated carrier wave are

(i) $f_c=200{,}000\text{ Hz}=200\text{ kHz}$

(ii) $USB=200{,}000+2500=202{,}500\text{ Hz}$

$\Rightarrow\quad USB=202.5\text{ kHz}$

(iii) $LSB=200{,}000-2500$

$\Rightarrow\quad LSB=197{,}500\text{ Hz}=197.5\text{ kHz}$

ILLUSTRATION 10

An amplitude modulated wave is represented as $c_m(t)=5(1+0.6\cos 6280t)\sin 211\times 10^4 t$, volts.

(a) What are the minimum and maximum amplitudes of the $A.M.$ wave?

(b) What frequency components are contained in the modulated wave?

(c) What are the amplitudes of the components?

SOLUTION

Given the A.M. wave,

$$c_m(t)=5(1+0.6\cos 6280t)\sin(211\times 10^4 t)\text{ volts.}$$

Comparing with the standard A.M. wave with $c_m(t)=A_c(1+\mu\cos\omega_m t)\sin\omega_c t$, we get

$$A_c=5\text{ V},\ \mu=0.6$$

Modulating frequency,

$$f_m=\frac{\omega_m}{2\pi}=\frac{6280}{2\pi}=1\text{ kHz}$$

Carrier frequency,

$$f_c=\frac{\omega_c}{2\pi}=\frac{211\times 10^4}{2\pi}=336\text{ kHz}$$

(a) Minimum amplitude of A.M. wave

$$A_{\min}=A_c-\mu A_c=5-0.6\times 5=2\text{ V}$$

Maximum amplitude of A.M. wave

$$A_{\max}=A_c+\mu A_c=5+0.6\times 5=8\text{ V}$$

(b) Frequency components of the A.M. wave are

$$(f_c-f_m),\ f_c,\ (f_c+f_m)$$

i.e., $(336-1)$, 336, $(336+1)$

$\Rightarrow\quad 335\text{ kHz},\ 336\text{ kHz},\ 337\text{ kHz}$

(c) The amplitudes of the three components are

$$\frac{\mu A_c}{2},\ A_c,\ \frac{\mu A_c}{2}$$

i.e., $\dfrac{0.6\times 5}{2}$, 5, $\dfrac{0.6\times 5}{2}$

$\Rightarrow\quad 1.5\text{ V},\ 5\text{ V},\ 1.5\text{ V}$

ILLUSTRATION 11

A carrier wave of frequency 10 MHz and peak value 10 V is amplitude modulated by a 5 kHz sine wave of amplitude 6 V. Calculate the frequency and amplitude of the two sidebands and draw the spectrum.

SOLUTION

Here $f_c=10\text{ MHz}$,

$f_m=5\text{ kHz}=0.005\text{ MHz}$, $E_c=10\text{ V}$, $E_m=6\text{ V}$

Frequency of USB is

$$f_{\text{USB}}=f_c+f_m=10+0.005=10.005\text{ MHz}$$

Frequency of LSB is

$$f_{\text{LSB}}=f_c-f_m=10-0.005=9.995\text{ MHz}$$

$$\mu=\frac{A_m}{A_c}=\frac{6}{10}=0.6$$

Amplitude of USB or LSB is Side Band Amplitude, given by

$$A_{SB}=\frac{\mu A_c}{2}=\frac{0.6\times 10}{2}=3\text{ V}$$

The frequency spectrum for the A.M. wave is shown in figure

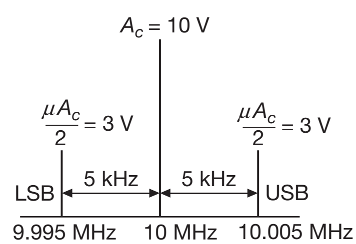


ILLUSTRATION 12

A bandwidth of 5 MHz is available for A.M. transmission. If the maximum audio signal frequency used for modulating the carrier is not to exceed 5 kHz, how many stations can be broadcast within this band simultaneously without interfering with each other?

SOLUTION

Bandwidth required for each station is

$\Delta f = 2\times$ Maximum frequency of audio signal

$\Rightarrow \quad \Delta f = 2\times 5 = 10 \text{ kHz}$

Number of stations which can be broadcast within bandwidth of 5 MHz is

$$N = \frac{5 \text{ MHz}}{10 \text{ kHz}} = \frac{5000 \text{ kHz}}{10 \text{ kHz}} = 500$$

MODULATION INDEX IN TERMS OF A_{max} AND A_{min}

Since, we know that the maximum and the minimum amplitude of the modulated wave are given by

$$A_{\max} = A_c + A_m \quad \ldots(1)$$

$$A_{\min} = A_c - A_m \quad \ldots(2)$$

Also, the Modulation Index is given by

$$\mu = \frac{A_m}{A_c}$$

From equations (1) and (2), we have

$$A_c = \frac{A_{\max} + A_{\min}}{2} \text{ and}$$

$$A_m = \frac{A_{\max} - A_{\min}}{2}$$

$$\Rightarrow \quad \mu = \frac{A_m}{A_c} = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$$

Conceptual Note(s)

(a) All A_c, A_m, A_{max}, A_{min} are expressed in volt.
(b) The modulation index μ has no unit.
(c) All frequencies (f or ν) are in hertz (Hz).
(d) All angular frequencies (ω) are in radian per second i.e. rads^{-1}.

ILLUSTRATION 13

A sinusoidal carrier voltage of 100 V is amplitude modulated by a sinusoidal voltage of frequency 10 kHz resulting in maximum modulated carrier amplitude of 120 V. Calculate the modulation factor

SOLUTION

Since $A_c = 100$ V, $A_{\max} = 120$ V

$$\Rightarrow \quad \mu = \frac{A_m}{A_c}$$

$$\Rightarrow \quad \mu = \frac{A_{\max} - A_c}{A_c} = \frac{120-100}{100} = 0.2$$

ILLUSTRATION 14

A sinusoidal carrier voltage of frequency 1200 kHz is amplitude modulated by a sinusoidal voltage of frequency 20 kHz resulting in maximum and minimum modulated carrier amplitudes of 110 V and 90 V respectively. Calculate

(a) the frequency of lower and upper sidebands
(b) the unmodulated carrier amplitude
(c) the modulation and
(d) the amplitude of each sideband

SOLUTION

(a) Lower sideband frequency is

$$f_{\text{LSB}} = f_c - f_m = 1200 - 20 = 1180 \text{ kHz}$$

Upper sideband frequency is

$$f_{\text{USB}} = f_c + f_m = 1200 + 20 = 1220 \text{ kHz}$$

(b) Unmodulated carrier amplitude

$$A_C = \frac{A_{\max} + A_{\min}}{2} = \frac{110+90}{2} = 100 \text{ V}$$

(c) Modulation index,

$$\mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}} = \frac{110-90}{110+90} = 0.1$$

(d) Amplitude of each sideband is

$$A_{SB} = \frac{\mu A_c}{2} = \frac{0.1\times 100}{2} = 5 \text{ V}$$

ILLUSTRATION 15

The maximum peak-to-peak voltage of an A.M. wave is 16 mV and the minimum peak-to-peak voltage is 8 mV. What is the modulation factor?

SOLUTION

$$E_{max} = \frac{16}{2} = 8 \text{ mV}$$

$$E_{min} = \frac{8}{2} = 4 \text{ mV}$$

$$\Rightarrow \quad \mu = \frac{E_{max} - E_{min}}{E_{max} + E_{min}} = \frac{8-4}{8+4} = \frac{1}{3} = 0.33$$

POWER AND CURRENT RELATION IN AM WAVE

Since, power dissipated in any circuit having rms voltage A_{rms} is given by

$$P = \frac{A_{rms}^2}{R}$$

The average power in the unmodulated carrier wave P_c is given by

$$P_c = \frac{\left(\frac{A_c}{\sqrt{2}}\right)^2}{R} = \frac{A_c^2}{2R}$$

where R is the resistance of the antenna in which power is dissipated.

Power (P_{sb}) of side bands each of amplitude $A_{sb} = \frac{\mu A_c}{2}$ is given by

$$P_{sb} = \frac{\left(\frac{\mu A_c}{2\sqrt{2}}\right)^2}{R} + \frac{\left(\frac{\mu A_c}{2\sqrt{2}}\right)^2}{R} = \frac{\mu^2 A_c^2}{4R}$$

Hence, total power of AM wave is given by

$$P_t = P_c + P_{sb} = \frac{A_c^2}{2R}\left(1 + \frac{\mu^2}{2}\right)$$

$$\Rightarrow \quad P_t = P_c\left(1 + \frac{\mu^2}{2}\right)$$

$$\Rightarrow \quad \frac{P_t}{P_c} = 1 + \frac{\mu^2}{2}$$

If I_t and I_c be the rms values of the total modulated current and the unmodulated carrier current respectively, then

$$P_t = I_t^2 R \text{ and}$$

$$P_c = I_c^2 R$$

$$\Rightarrow \quad \frac{P_t}{P_c} = \frac{I_t^2}{I_c^2} = 1 + \frac{\mu^2}{2}$$

$$\Rightarrow \quad \frac{I_t}{I_c} = \sqrt{1 + \frac{\mu^2}{2}}$$

Conceptual Note(s)

If a carrier wave is modulated by several sine waves the total modulated index m_t is given by

$$\mu_t = \sqrt{\mu_1^2 + \mu_2^2 + \mu_3^2 + ...}$$

ILLUSTRATION 16

A message signal of frequency 10 kHz and peak voltage of 10 volt is used to modulate a carrier of frequency 1 MHz and peak voltage of 20 volt. Determine

(a) modulation index,
(b) the side bands produced.

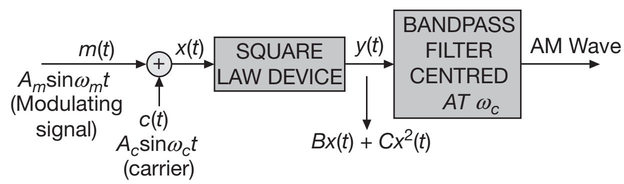
SOLUTION

(a) Modulation index $\mu = \frac{10}{20} = 0.5$

(b) The side bands are at $(1000+10)$ kHz $= 1010$ kHz and $(1000-10)$ kHz $= 990$ kHz.

PRODUCTION OF AMPLITUDE MODULATED WAVE

Amplitude modulation can be produced by a variety of methods. A conceptually simple method to produce amplitude modulation is shown in the block diagram.



Block diagram of a simple modulator for obtaining an AM signal

Here the modulating signal $A_m \sin(\omega_m t)$ is added to the carrier signal $A_c \sin \omega_c t$ to produce the signal $x(t)$. This signal $x(t) = A_m \sin(\omega_m t) + A_c \sin(\omega_c t)$ is passed through a square law device which is a non-linear device that produces an output given by

$$y(t) = Bx(t) + Cx^2(t) \quad \ldots(1)$$

where B and C are constants. Thus,

$$\Rightarrow \quad y(t) = BA_m \sin(\omega_m t) + BA_c \sin(\omega_c t) + CA_m^2 \sin^2(\omega_m t) + A_c^2 \sin^2(\omega_c t) + 2A_m A_c \sin(\omega_m t)\sin(\omega_c t) \quad \ldots(2)$$

Since, $\sin^2\theta = \dfrac{1-\cos(2\theta)}{2}$ and $\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$

$$\Rightarrow \quad y(t) = BA_m \sin(\omega_m t) + BA_c \sin(\omega_c t) + \frac{C}{2}\left(A_m^2 + A_c^2\right) - \frac{CA_m^2}{2}\cos(2\omega_m t) - \frac{CA_c^2}{2}\cos(2\omega_c t) + CA_m A_c \cos(\omega_c - \omega_m)t - CA_m A_c \cos(\omega_c + \omega_m)t \quad \ldots(3)$$

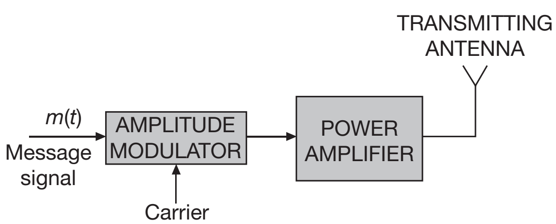
In equation (3), there is a dc term $\frac{C}{2}\left(A_m^2 + A_c^2\right)$ and sinusoids of frequencies ω_m, $2\omega_m$, ω_c, $2\omega_c$, $(\omega_c - \omega_m)$ and $(\omega_c + \omega_m)$. This signal is passed through a band pass filter (as shown in Figure) which rejects dc as well as the sinusoids of frequencies ω_m, $2\omega_m$ and $2\omega_c$.

However, it retains the frequencies ω_c, $(\omega_c - \omega_m)$ and $(\omega_c + \omega_m)$. The output of the band pass filter therefore is

$$c_m(t) = A_c \sin(\omega_c t) + \frac{\mu A_c}{2}\cos\left[(\omega_c - \omega_m)t\right] - \frac{\mu A_c}{2}\cos\left[(\omega_c + \omega_m)t\right]$$

This output is of the same form as obtained in an AM wave.

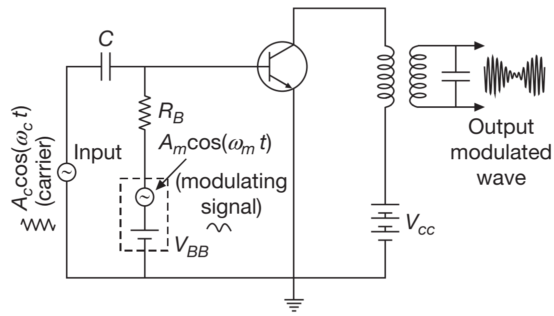
Please note that the modulated signal cannot be transmitted as such. The modulator is to be followed by a power amplifier which provides the necessary power and then the modulated signal is fed to an antenna of appropriate size for radiation as shown in figure.



Block diagram of a transmitter

PRODUCTION OF AMPLITUDE MODULATED CARRIER WAVE USING A COMMON EMITTER AMPLIFIER

The amplitude of an amplitude modulated carrier waves increases or decreases in accordance with the instantaneous value of the modulating or message signal. A simple circuit for producing amplitude modulated wave is shown in Figure.



Circuit for an amplitude modulator (Base modulator)

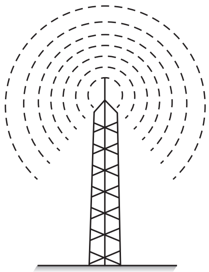
The circuit shown is simply a CE amplifier for the carrier wave signal. The modulating signal $m(t) = A_m \sin(\omega_m t)$ is applied on the base. Thus, the base biasing voltage is not constant d.c. but is the sum of a constant d.c. voltage V_{BB} and the modulating voltage $A_m \sin(\omega_m t)$. As the biasing of the base changes, the amplification produced also changes. The output voltage is a carrier signal varying in amplitude in accordance with the biasing modulating voltage. This gives an amplitude modulated wave.

ANTENNA

An antenna is basically a small length of a conductor that is used to radiate or receive electromagnetic waves. It acts as a conversion device. At the transmitting end, it converts high frequency current into electromagnetic waves. At the receiving end, it transforms electromagnetic waves into electrical signal that is fed to the input of the receiver. Several types of antenna are used in communication.

Dipole Antenna

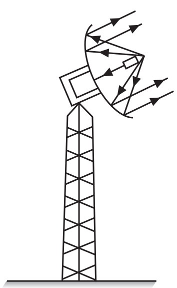
A simple dipole antenna is just a piece of a conductor of length much smaller than the wavelength of the carrier signal. Sometimes rounded structures called lobes, are attached to it to increase its length. The length of the dipole antenna should be such that it is in resonance with the carrier frequency. Usually, the length of a dipole antenna is taken equal to $\frac{\lambda}{2}$, where λ is the wavelength of the carrier wave. A dipole antenna is omni-directional and is used to transmit radio waves.



A dipole antenna

Dish Antenna

A dish antenna is a highly directional antenna used for the transmission and reception of UHF and microwaves. Its active component is a small dipole placed at the focus of a parabolic reflector or spherical dish as shown in Figure.



A dish antenna

In the receiver mode, the dish collects the electromagnetic radiations and focuses them on the dipole which converts them into electrical signals. In the transmitter mode, the dipole converts the electrical signal into electromagnetic waves and directs them on the reflector which are then transmitted as a parallel beam as shown.

The gain of a dish antenna is defined by

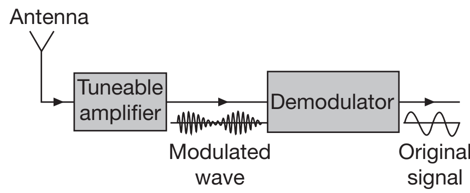
$$P = 6\left(\frac{D}{\lambda}\right)^2$$

where D is diameter of the boundary of the reflector. λ is wavelength of electromagnetic radiations.

Dish antennas are used in radars and satellite communication. The microwave signal from a satellite is received by a cable operator through a dish antenna.

RECEIVER

The function of a receiver is to recover the original message or data from the modulated signal after its propagation through the communication channel. The basic process involved is the separation of the modulating signal from the carrier wave. This process is called demodulation figure shows a schematic representation of a receiver.

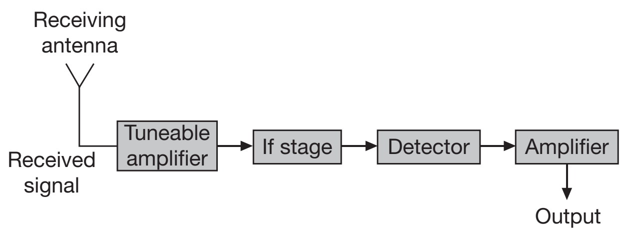


Schematic representation of a receiver.

At the receiving station, the antenna picks up the modulated wave radiated from the transmitter. By a tuneable amplifier, the desired signal is selected (and unwanted signals are rejected) and then amplified. Finally, it is fed to a demodulator or detector which recovers the original signal from the modulated wave.

RECEIVER FOR DETECTION OF AN AMPLITUDE MODULATED WAVE

The block diagram for a typical receiver used for detecting an amplitude modulated wave is shown in Figure.

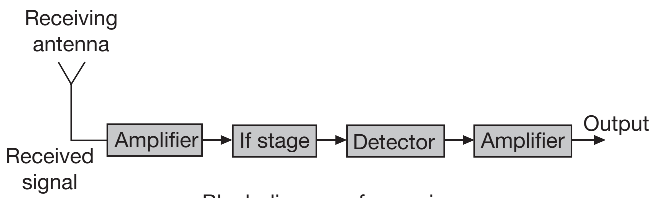


Block diagram of a receiver for an AM wave

At the receiving station, the antenna picks up the modulated wave radiated from the transmitter. By a tuneable amplifier, the signal is selected (and unwanted signals rejected) and then amplified. To facilitate further processing, the carrier frequency is changed to a lower frequency by a device called intermediate frequency (IF) stage. Then the signal is fed to a demodulator/detector which recovers the message signal from the modulated wave. To further strengthen the detected signal, it is fed to an amplifier.

DETECTION/DEMODULATION OF AN AMPLITUDE MODULATED WAVE

The transmitted message gets attenuated in propagating through the channel. The receiving antenna is therefore to be followed by an amplifier and a detector. In addition, to facilitate further processing, the carrier frequency is usually changed to a lower frequency by what is called an *intermediate frequency (IF) stage* preceding the detection. The detected signal may not be strong enough to be made use of and hence is required to be amplified. A block diagram of a typical receiver is shown in figure.



Block diagram of a receiver

Detection is the process of recovering the modulating signal from the modulated carrier wave.

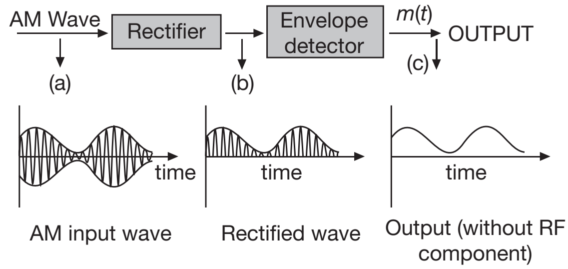
So, a demodulator or a receiver performs the following functions.

(a) It selects the required signals and rejects the unwanted signals.

(b) It amplifies and demodulates the required signal.

(c) It displays the original modulating signal in the required manner.

Since we know that the modulated carrier wave contains the frequencies ω_c and $(\omega_c \pm \omega_m)$. In order to obtain the original message signal $m(t)$ of angular frequency ω_m, a simple method is represented in the form of a block diagram shown in figure.

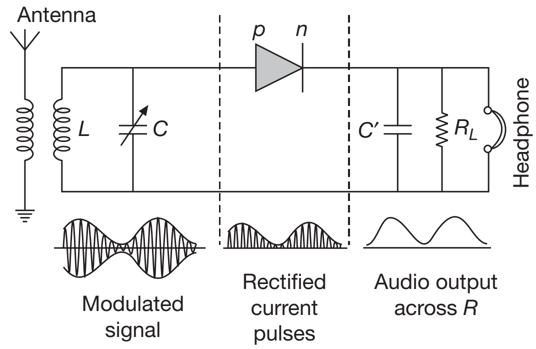


Block diagram of a detector for AM signal.
The quantity on *y*-axis can be current or voltage

The modulated signal of the form given in (a) of figure is passed through a rectifier to produce the output shown in (b). This envelope of signal (b) is the message signal. In order to retrieve $m(t)$, the signal is passed through an envelope detector (which may consist of a simple RC circuit).

JUNCTION DIODE AS A DETECTOR/ DEMODULATOR

In the circuit diagram shown, a junction diode is used with other elements so as to function as a detector for an amplitude modulated wave.



Junction diode as a detector

The input circuit is a parallel combination of inductance L and a variable capacitor C'. *It is called tuned circuit*. By adjusting the frequency of this circuit, the desired modulated radio signal is resonantly selected from the different signals picked up by the antenna and the diode rectifies this signal. Hence the output of the diode is a series of positive half cycles of radio frequency current pulses. The peaks of these pulses vary in accordance with the audio signal. To recover the audio signal, the rectified output is fed to a parallel combination of a low valued capacitor C and a resistor R. The capacitor C offers a low reactance $\left(X_C = \frac{1}{2\pi fC}\right)$ to the high frequency carrier wave and a high reactance to the low frequency audio wave signals and hence the capacitor C acts as a by-pass for high frequency carrier waves while the low frequency audio wave appears across the resistor R. This sends current through a headphone to reproduce the original audio signal.

The essential condition for demodulation is that the time period of the high frequency carrier wave must be smaller than the capacitive time constant of the RC circuit i.e.

$$T_c \ll RC$$

$$\Rightarrow \quad \frac{1}{f_c} \ll RC$$

where f_c is the frequency of the carrier wave. *Also note that R and C are also called as the resistance and capacitance of the output circuit of a diode AM detector.*

The maximum modulated frequency f_m can be found using a diode detector. This frequency f_m is given by

$$f_m = \frac{1}{2\pi RC\mu}$$

where μ is the modulation index of an AM wave.

For exact recovery of the original message, the following condition must be satisfied

$$\frac{1}{f_c} < RC < \frac{1}{f_m}$$

Conceptual Note(s)

(a) Length of a dipole antenna, $\ell = \frac{\lambda}{2} = \frac{c}{2f}$

(b) Condition for satisfactory detection by a diode,

$$\frac{1}{f_c} < RC < \frac{1}{f_m}$$

ILLUSTRATION 17

In a diode AM-detector, the output circuit consists of $R = 1\ \text{k}\Omega$ and $C = 10\ \text{pF}$. A carrier signal of 100 kHz is to be detected. Is it good? If yes, then explain why? If not, what value would you suggest?

SOLUTION

For demodulation, $\frac{1}{f_c} \ll RC$

$$\Rightarrow \quad \frac{1}{f_c} = \frac{1}{100 \times 10^3} = 10^{-5}\ \text{s}$$

Also, $RC = 10^3 \times 10 \times 10^{-12}\ \text{s} = 10^{-8}\ \text{s}$

Since, $\frac{1}{f_c}$ is not less than RC, so this is not good.

When we take $C = 1\ \mu\text{F}$, then $RC = 10^3 \times 10^{-6}\ \text{s} = 10^{-3}\ \text{s}$

This value is much larger than $\frac{1}{f_c}$, so the capacitor of $1\ \mu\text{F}$ provides a satisfactory circuit for a detector.

ILLUSTRATION 18

Calculate the length of a half wave dipole at

(a) 30 MHz

(b) 300 MHz and

(c) 3000 MHz.

What interference do you from the results?

SOLUTION

(a) For $f = 30 \times 10^6$ Hz

$$\Rightarrow \quad \lambda = \frac{c}{f} = \frac{3 \times 10^8}{30 \times 10^6} = 10\ \text{m}$$

So, length of dipole is $L = \frac{\lambda}{2} = 5\ \text{m}$

(b) For $f = 300 \times 10^6$ Hz

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{300 \times 10^6} = 1\ \text{m}$$

So, length of dipole is $L = \frac{\lambda}{2} = 0.5\ \text{m}$

(c) For $f = 3000 \times 10^6$ Hz

$$\Rightarrow \quad \lambda = \frac{c}{f} = \frac{3 \times 10^8\ \text{ms}^{-1}}{3000 \times 10^6\ \text{s}^{-1}} = 0.1\ \text{m}$$

So, length of dipole is $L = \frac{\lambda}{2} = 0.05 \text{ m} = 5 \text{ cm}$

Clearly, the length of dipole decreases with the increase in the frequency of carrier waves.

ILLUSTRATION 19

The tuned circuit of the oscillator in a simple A.M. transmitter uses a coil of $40\ \mu\text{H}$ and shunt capacitor of value 1 nF. If the oscillator output is modulated by audio frequencies upto 5 kHz, what is the frequency range occupied by sidebands?

SOLUTION

$$f_c = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{40\times10^{-6}\times1\times10^{-9}}}$$

$$\Rightarrow \quad f_C = 7.96\times10^5 \text{ Hz} = 796 \text{ kHz}.$$

Since $f_m = 5\text{ kHz}$

So, frequency range of side bands is from $(796-5)\text{ kHz}$ to $(796+5)\text{ kHz}$ i.e. from 791 kHz to 801 kHz

THE INTERNET

It is a system with billions of users worldwide. It permits communication and sharing of all types of information between any two or more computers connected through a large and complex network. It was started in 1960's and opened for public use in 1990's. With the passage of time it has witnessed tremendous growth and it is still expanding its reach. Its applications include

E Mail

It permits exchange of text/graphic material using email software. We can write a letter and send it to the recipient through ISP's (Internet Service Providers) who work like the dispatching and receiving post offices.

File Transfer

A FTP (File Transfer Programmes) allows transfer of files/software from one computer to another connected to the Internet.

WORLD WIDE WEB (WWW)

Computers that store specific information for sharing with others provide websites either directly or through web service providers. Government departments, companies, NGO's (Non-Government Organisations) and individuals can post information about their activities for restricted or free use on their websites. This information becomes accessible to the users. Several search engines like Google, Yahoo! etc. help us in finding information by listing the related websites. Hypertext is a powerful feature of the web that automatically links relevant information from one page on the web to another using HTML (hypertext markup language).

E-commerce

Use of the Internet to promote business using electronic means such as using credit cards is called E-commerce. Customers view images and receive all the information about various products or services of companies through their websites. They can do on-line shopping from home/office. Goods are dispatched or services are provided by the company through mail/courier.

CHAT

Real time conversation among people with common interests through typed messages is called chat. Everyone belonging to the chat group gets the message instantaneously and can respond rapidly.

FACSIMILE (FAX)

It scans the contents of a document (as an image, not text) to create electronic signals. These signals are then sent to the destination (another FAX machine) in an orderly manner using telephone lines. At the destination, the signals are reconverted into a replica of the original document. Note that FAX provides image of a static document unlike the image provided by television of objects that might be dynamic.

MOBILE TELEPHONY

The concept of mobile telephony was developed first in 1970's and it was fully implemented in the following decade. The central concept of this system is to divide the service area into a suitable number of cells centred on an office called MTSO (Mobile Telephone Switching Office). Each cell contains a low-power transmitter called a base station and caters to a large number of mobile receivers (popularly called cell phones). Each cell could have a service area of a few square kilometres or even less depending upon the number of customers. When a mobile receiver crosses the coverage area of one base station, it is necessary for the mobile user to be transferred to another base station. This procedure is called handover or handoff. This process is carried out very rapidly, to the extent that the consumer does not even notice it. Mobile telephones operate typically in the UHF range of frequencies (about 800 MHz-950 MHz).

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. An example of point to point mode of communication is
(A) FM radio (B) Telephony
(C) Television (D) Internet

2. Through which mode of propagation, the radio waves can be sent from one place to another
(A) Ground wave propagation
(B) Sky wave propagation
(C) Space wave propagation
(D) All of them

3. For television broadcasting, the frequency employed is normally
(A) 30 MHz - 300 MHz (B) 30 GHz - 300 GHz
(C) 30 kHz - 300 kHz (D) 30 Hz - 300 Hz

4. Basically, the product modulator is
(A) An amplifier
(B) A mixer
(C) A frequency separator
(D) A phase separator

5. An antenna is a device
(A) That converts electromagnetic energy into radio frequency signal
(B) That converts radio frequency signal into electromagnetic energy
(C) That converts guided electromagnetic waves into free space electromagnetic waves and vice-versa
(D) None of these

6. The phenomenon by which light travels in an optical fibres is
(A) Reflection
(B) Refraction
(C) Total internal reflection
(D) Transmission

7. In which of the following remote sensing technique is not used
(A) Forest density (B) Pollution
(C) Wetland mapping (D) Medical treatment

8. Which of the following device is fully duplex?
(A) Mobile phone (B) Walky-talky
(C) Loud speaker (D) Radio

9. A carrier frequency of 1 MHz and peak value of 10 V is amplitude modulated with a signal frequency of 10 kHz with peak value of 0.5 V. Then the modulation index and the side band frequencies respectively are
(A) 0.05 and (1 ± 0.010) MHz
(B) 0.5 and (1 ± 0.010) MHz
(C) 0.05 and (1 ± 0.005) MHz
(D) 0.5 and (1 ± 0.005) MHz

10. In optical communication system operating at 1200 nm, only 2% of the source frequency is available for TV transmission having a bandwidth of 5 MHz. The number of TV channels that can be transmitted is
(A) 2 million (B) 10 million
(C) 0.1 million (D) 1 million

11. If both the length of an antenna and the wavelength of the signal to be transmitted are doubled, the power radiated by the antenna
(A) Is doubled (B) Is halved
(C) Remains constant (D) Is quadrupled

12. A modem is a
(A) Modulating device only
(B) Demodulating device only
(C) Modulating and demodulating device
(D) Transmitting device

13. What is the modulation index of an over modulated wave
(A) 1 (B) zero
(C) <1 (D) >1

14. A modulated signal $C_m(t)$ has the form $C_m(t) = 30\sin(300\pi t) + 10(\cos 200\pi t - \cos 400\pi t)$. The carrier frequency f_c, the modulating frequency (message frequency) f_ω, and the modulation index μ are respectively given by

(A) $f_c = 200$ Hz; $f_\omega = 50$ Hz; $\mu = \frac{1}{2}$

(B) $f_c = 150$ Hz; $f_\omega = 50$ Hz; $\mu = \frac{2}{3}$

(C) $f_c = 150$ Hz ; $f_\omega = 30$ Hz ; $\mu = \frac{1}{3}$

(D) $f_c = 200$ Hz ; $f_\omega = 30$ Hz ; $\mu = \frac{1}{2}$

15. A transmitter supplies 9 kW to the aerial when unmodulated. The power radiated when modulated to 40% is

(A) 5 kW (B) 9.72 kW
(C) 10 kW (D) 12 kW

16. A transmitting antenna of height h and the receiving antenna of height 45 m are separated by a distance of 40 km for satisfactory communication in line of sight mode. Then the value of h is (given radius of earth is 6400 km)

(A) 15 m (B) 20 m
(C) 30 m (D) 25 m

17. An AM wave has 1800 watt of total power content. For 100% modulation the carrier should have power content equal to

(A) 1000 watt (B) 1200 watt
(C) 1500 watt (D) 1600 watt

18. In short wave communication, waves of which of the following frequencies will be reflected back by the ionospheric layer, having electron density 10^{11}m^{-3}

(A) 2 MHz (B) 10 MHz
(C) 12 MHz (D) 18 MHz

19. The sky wave propagation is suitable for radio-waves of frequency

(A) Upto 2 MHz
(B) From 2 MHz to 20 MHz
(C) From 2 MHz to 30 MHz
(D) From 2 MHz to 50 MHz

20. Sinusoidal carrier voltage of frequency 1.5 MHz and amplitude 50 V is amplitude modulated by sinusoidal voltage of frequency 10 kHz producing 50% modulation. The lower and upper side-band frequencies in kHz are

(A) 1490, 1510 (B) 1510, 1490
(C) $\frac{1}{1490}, \frac{1}{1510}$ (D) $\frac{1}{1510}, \frac{1}{1490}$

21. Arrange the following communication frequency bands in the increasing order of frequencies

1. AM broadcast
2. Cellular mobile radio
3. FM broadcast
4. Television UHF
5. Satellite communication

(A) 1 3 4 2 5 (B) 1 2 3 4 5
(C) 5 2 4 3 1 (D) 1 3 2 4 5

22. In an amplitude modulation with modulation index 0.5, the ratio of the amplitude of the carrier wave to that of the side band in the modulated wave is

(A) 4 : 1 (B) 1 : 4
(C) 1 : 2 (D) 1 : 1

23. In AM, the cent percent modulation is achieved when

(A) Carrier amplitude = signal amplitude
(B) Carrier amplitude ≠ signal amplitude
(C) Carrier frequency = signal frequency
(D) Carrier frequency ≠ signal frequency

24. An amplitude modulated signal is given by $V(t) = 10\left[1 + 0.3\cos\left(2.2\times10^4 t\right)\right]\sin\left(5.5\times10^5 t\right)$. Here t is in seconds. The sideband frequencies (in kHz) are, [Given $\pi = \frac{22}{7}$]

(A) 1785 and 1715 (B) 178.5 and 171.5
(C) 89.25 and 85.75 (D) 892.5 and 857.5

25. Consider an optical communication system operating at $\lambda \approx 800\text{ nm}$. Suppose, only 1% of the optical source frequency is the available channel bandwidth for optical communication. How many channels can be accommodated for transmitting audio signals requiring a bandwidth of 8 kHz

(A) 4.8×10^8 (B) 48
(C) 6.2×10^8 (D) 4.8×10^5

26. A radio station has two channels. One is AM at 1020 kHz and the other FM at 89.5 MHz. For good results you will use

(A) Longer antenna for the AM channel and shorter for the FM
(B) Shorter antenna for the AM channel and longer for the FM
(C) Same length antenna will work for both
(D) Information given is not enough to say which one to use for which

27. An amplitude modulated wave is modulated to 50%. What is the saving in power if carrier as well as one of the side bands are suppressed

(A) 70% (B) 65.4%
(C) 94.4% (D) 25.5%

28. Modulation is the process of superposing

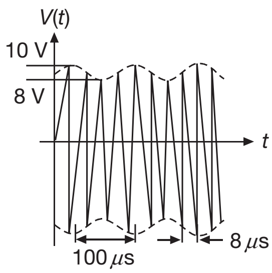
(A) Low frequency audio signal on high frequency waves
(B) Low frequency radio signal on low frequency audio waves

(C) High frequency audio signal on low frequency radio waves
(D) Low frequency audio signal on low frequency radio waves

29. An oscillator is producing FM waves of frequency 2 kHz with a variation of 10 kHz. What is the modulating index
(A) 0.20 (B) 5.0
(C) 0.67 (D) 1.5

30. In an amplitude modulated wave for audio frequency of 500 cps, the appropriate carrier frequency will be $(1\text{ cps} = 1\text{ Hz})$
(A) 50 cps (B) 100 cps
(C) 500 cps (D) 50,000 cps

31. An amplitude modulated signal is plotted below:



Which one of the following best describes the above signal?
(A) $\left(9+\sin\left(2\pi\times10^4 t\right)\right)\sin\left(2.5\pi\times10^5 t\right)$ V
(B) $\left(9+\sin\left(4\pi\times10^4 t\right)\right)\sin\left(5\pi\times10^5 t\right)$ V
(C) $\left(1+9\sin\left(2\pi\times10^4 t\right)\right)\sin\left(2.5\pi\times10^5 t\right)$ V
(D) $\left(9+\sin\left(2.5\pi\times10^5 t\right)\right)\sin\left(2\pi\times10^4 t\right)$ V

32. For sky wave propagation of a 10 MHz signal, what should be the minimum electron density in ionosphere
(A) $\sim 1.2\times10^{12}\text{ m}^{-3}$ (B) $\sim 10^{6}\text{ m}^{-3}$
(C) $\sim 10^{14}\text{ m}^{-3}$ (D) $\sim 10^{22}\text{ m}^{-3}$

33. If the maximum amplitude of an amplitude modulated wave is 25 V and the minimum amplitude is 5 V, the modulation index is
(A) $\frac{1}{5}$ (B) $\frac{1}{3}$
(C) $\frac{3}{2}$ (D) $\frac{2}{3}$

34. For an amplitude modulated wave, the maximum amplitude is found to be 12 V and minimum amplitude is found to be 4 V. The modulation index of this wave is ....%
(A) 25 (B) 50
(C) 75 (D) 20

35. In frequency modulation
(A) The amplitude of modulated wave varies as frequency of carrier wave
(B) The frequency of modulated wave varies as amplitude of modulating wave
(C) The frequency of modulated wave varies as frequency of modulating wave
(D) The frequency of modulated wave varies as frequency of carrier wave

36. A step index fibre has a relative refractive index of 0.88%. What is the critical angle at the corecladding interface
(A) 60° (B) 75°
(C) 45° (D) None of these

37. The maximum peak to peak voltage of an AM wire is 24 mV and the minimum peak to peak voltage is 8 mV. The modulation factor is
(A) 10% (B) 20%
(C) 25% (D) 50%

38. The velocity of all radio waves in free space is $3\times10^8\text{ ms}^{-1}$. The frequency of a radio wave of wavelength 150 m is
(A) 20 kHz (B) 2 kHz
(C) 2 MHz (D) 1 MHz

39. 1000 kHz carrier wave is amplitude modulated by the signal frequency $200\text{ Hz-}4000\text{ Hz}$. The channel width of this case is
(A) 8 kHz (B) 4 kHz
(C) 7.6 kHz (D) 3.8 kHz

40. In a diode AM-detector, the output circuit consists of $R = 1\text{ k}\Omega$ and $C = 10\text{ pF}$. A carrier signal of 100 kHz is to be detected. Is it good
(A) Yes
(B) No
(C) Information is not sufficient
(D) None of these

41. The bit rate for a signal, which has a sampling rate of 8 kHz and where 16 quantisation levels have been used is
(A) 32000 bits/s (B) 16000 bits/s
(C) 64000 bits/s (D) 72000 bits/s

42. Range of frequencies allotted for commercial FM radio broadcast is
(A) 88 MHz to 108 MHz (B) 88 kHz to 108 kHz
(C) 8 MHz to 88 MHz (D) 88 GHz to 108 GHz

43. Which of the following is the disadvantage of FM over AM?
(A) Larger band width requirement
(B) Larger noise
(C) Higher modulation power
(D) Low efficiency

44. Audio signal cannot be transmitted because
(A) The signal has more noise
(B) The signal cannot be amplified for distance communication
(C) The transmitting antenna length is very small to design
(D) The transmitting antenna length is very large and impracticable

45. The area of the region covered by the TV broadcast by a TV tower of 100 m height is (radius of the earth is $R = 6.4 \times 10^6\text{ m}$)
(A) $12.8\pi \times 10^8\text{ km}^2$ (B) $1.28\pi \times 10^3\text{ km}^2$
(C) $0.64\pi \times 10^3\text{ km}^2$ (D) $1.28 \times 10^3\text{ km}^2$

46. A diode AM detector with the output circuit consisting of $R = 1\text{ k}\Omega$ and $C = 1\ \mu\text{F}$ would be more suitable for detecting a carrier signal of
(A) 0.1 kHz (B) 1 kHz
(C) 10 kHz (D) 100 kHz

47. A sky wave with a frequency 55 MHz is incident on D-region of earth's atmosphere at $45°$. The angle of refraction is (electron density for D-region is $400\text{ electrons/cm}^3$)
(A) 60° (B) 45°
(C) 30° (D) 15°

48. The attenuation in optical fibre is mainly due to
(A) Absorption
(B) Scattering
(C) Neither absorption nor scattering
(D) Both (A) and (B)

49. The maximum range, d_{max} of radar is
(A) Proportional to the cube root of the peak transmitted power
(B) Proportional to the fourth root of the peak transmitted power
(C) Proportional to the square root of the peak transmitted power
(D) Not related to the peak transmitted power at all

50. A 500 Hz modulating voltage fed into an FM generator produces a frequency deviation of 2.25 kHz. If amplitude of the voltage is kept constant but frequency is raised to 6 kHz then the new deviation will be
(A) 4.5 kHz (B) 54 kHz
(C) 27 kHz (D) 15 kHz

51. In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m. The bandwidth $(\Delta\omega_m)$ of the signal is such that $\Delta\omega_m \ll \omega_c$. Which of the following frequencies is not contained in the modulated wave?
(A) ω_m (B) ω_c
(C) $\omega_m + \omega_c$ (D) $\omega_c - \omega_m$

52. The distance of coverage of a transmitting antenna is 12.8 km. Then, the height of the antenna is (Given that radius of earth $= 6400\text{ km}$)
(A) 6.4 m (B) 12.8 m
(C) 3.2 m (D) 16 m

53. A signal wave of frequency 12 kHz is modulated with a carrier wave of frequency 2.51 MHz. The upper and lower side band frequencies are respectively
(A) 2512 kHz and 2508 kHz
(B) 2522 kHz and 2488 kHz
(C) 2502 kHz and 2498 kHz
(D) 2522 kHz and 2498 kHz

54. The maximum distance upto which TV transmission from a TV tower of height h can be received is proportional to
(A) $h^{\frac{1}{2}}$ (B) h
(C) $h^{\frac{3}{2}}$ (D) h^2

55. What should be the maximum acceptance angle at the air core interface of an optical fibre if n_1 and n_2 are the refractive indices of the core and the cladding, respectively
(A) $\sin^{-1}\left(\dfrac{n_2}{n_1}\right)$ (B) $\sin^{-1}\sqrt{n_1^2 - n_2^2}$
(C) $\tan^{-1}\left(\dfrac{n_2}{n_1}\right)$ (D) $\tan^{-1}\left(\dfrac{n_1}{n_2}\right)$

56. The carrier frequency generated by a tank circuit containing 1 nF capacitor and $10\ \mu\text{H}$ inductor is
(A) 1592 Hz (B) 1592 MHz
(C) 1592 kHz (D) 159.2 Hz

57. Advantage of optical fibre
(A) High bandwidth and EM interference
(B) Low bandwidth and EM interference
(C) High band width, low transmission capacity and no EM interference
(D) High bandwidth, high data transmission capacity and no EM interference

58. The modulation in which pulse duration varies in accordance with the modulating signal is called
(A) PAM (B) PPM
(C) PWM (D) PCM

59. A radar has a power of 1 kW and is operating at a frequency of 10 GHz. It is located on a mountain top of height 500 m. The maximum distance upto which it can detect object located on the surface of the earth (Radius of earth $= 6.4 \times 10^6\text{ m}$) is
(A) 80 km (B) 16 km
(C) 40 km (D) 64 km

60. Identify the incorrect statement from the following
(A) AM detection is carried out using a rectifier and an envelop detector
(B) Pulse position denotes the time of rise or fall of the pulse amplitude
(C) Modulation index μ is kept ≥ 1, to avoid distortion
(D) Facsimile (FAX) scans the contents of the document to create electronic signals.

61. For telecommunication through optical fibres, which of the following statements is not true?
(A) Optical fibres may have homogeneous core with a suitable cladding
(B) Optical fibres can be of graded refractive index
(C) Optical fibres are subject to electromagnetic interference from outside
(D) Optical fibres have extremely low transmission loss

62. In amplitude modulation, the bandwidth is
(A) Twice the audio signal frequency
(B) Thrice the audio signal frequency
(C) Thrice the carrier wave frequency
(D) Twice the carrier wave frequency

63. In an FM system a 7 kHz signal modulates 108 MHz carrier so that frequency deviation is 50 kHz. The carrier swing is
(A) 7.143 (B) 8
(C) 0.71 (D) 350

64. Maximum usable frequency (MUF) in F-region layer is x, when the critical frequency is 60 MHz and the angle of incidence is $70°$. Then x is
(A) 150 MHz (B) 170 MHz
(C) 175 MHz (D) 190 MHz

65. The antenna current of an AM transmitter is 8 A when only carrier is sent but increases to 8.96 A when the carrier is sinusoidally modulated. The percentage modulation is
(A) 50% (B) 60%
(C) 65% (D) 71%

66. A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of capacity 250 picofarad in parallel with a load resistance $100\text{ k}\Omega$. Find the maximum modulated frequency which could be detected by it
(A) 10.62 MHz (B) 10.62 kHz
(C) 5.31 MHz (D) 5.31 kHz

67. A 1000 kHz carrier wave is modulated by an audio signal of frequency range $100\text{-}5000\text{ Hz}$. Then the width of the channel in kHz is
(A) 10 (B) 20
(C) 30 (D) 40

68. The audio signal used to modulate $60\sin\left(2\pi \times 10^6 t\right)$ is $15\sin(300\pi t)$. The depth of modulation is
(A) 50% (B) 40%
(C) 25% (D) 15%

69. For good demodulation of AM signal of carrier frequency f, the value of RC should be
(A) $RC = \frac{1}{f}$ (B) $RC < \frac{1}{f}$
(C) $RC \geq \frac{1}{f}$ (D) $RC \gg \frac{1}{f}$

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1. **[Online January 2019]**
In a communication system operating at wavelength 800 nm, only one percent of source frequency is available as signal bandwidth. The number of channels accommodated for transmitting TV signals of band width 6 MHz are (take velocity of light $c = 3 \times 10^8\text{ ms}^{-1}$, $h = 6.6 \times 10^{-34}\text{ Js}$)
(A) 3.75×10^6 (B) 3.86×10^6
(C) 6.25×10^5 (D) 4.87×10^5

2. **[Online January 2019]**
A TV transmission tower has a height of 140 m and the height of the receiving antenna is 40 m. What is the maximum distance upto which signals can be

broadcasted from this tower in LOS (Line of Sight) mode? (Given: radius of earth $= 6.4\times10^6$ m)

(A) 65 km (B) 80 km
(C) 40 km (D) 48 km

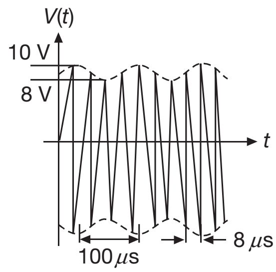
3. [Online January 2019]
The modulation frequency of an AM radio station is 250 kHz, which is 10% of the carrier wave. If another AM station approaches you for license what broadcast frequency will you allot?

(A) 2750 kHz (B) 2900 kHz
(C) 2000 kHz (D) 2250 kHz

4. [Online January 2019]
An amplitude modulated signal is given by $V(t)=10\left[1+0.3\cos\left(2.2\times10^4 t\right)\right]\sin\left(5.5\times10^5 t\right)$. Here t is in seconds. The sideband frequencies (in kHz) are, [Given $\pi=\frac{22}{7}$]

(A) 1785 and 1715 (B) 178.5 and 171.5
(C) 89.25 and 85.75 (D) 892.5 and 857.5

5. [Online January 2019]
An amplitude modulated signal is plotted below:



Which one of the following best describes the above signal?

(A) $\left(9+\sin\left(2\pi\times10^4 t\right)\right)\sin\left(2.5\pi\times10^5 t\right)$ V

(B) $\left(9+\sin\left(4\pi\times10^4 t\right)\right)\sin\left(5\pi\times10^5 t\right)$ V

(C) $\left(1+9\sin\left(2\pi\times10^4 t\right)\right)\sin\left(2.5\pi\times10^5 t\right)$ V

(D) $\left(9+\sin\left(2.5\pi\times10^5 t\right)\right)\sin\left(2\pi\times10^4 t\right)$ V

6. [Online January 2019]
A 100 V carrier wave is made to vary between 160 V and 40 V by a modulating signal. What is the modulation index?

(A) 0.5 (B) 0.4
(C) 0.6 (D) 0.3

7. [Online January 2019]
To double the covering range of a TV transmission tower, its height should be multiplied by

(A) $\sqrt{2}$ (B) 2
(C) $\frac{1}{\sqrt{2}}$ (D) 4

8. [Online April 2019]
The wavelength of the carrier waves in a modern optical fiber communication network is close to

(A) 600 nm (B) 900 nm
(C) 1500 nm (D) 2400 nm

9. [Online April 2019]
In a line of sight radio communication, a distance of about 50 km is kept between the transmitting and receiving antennas. If the height of the receiving antenna is 70 m, then the minimum height of the transmitting antenna should be (Radius of the Earth $= 6.4\times10^6$ m)

(A) 20 m (B) 51 m
(C) 32 m (D) 40 m

10. [Online April 2019]
A signal $A\cos(\omega t)$ is transmitted using $V_0\sin(\omega_0 t)$ as carrier wave. The correct amplitude modulated (AM) signal is

(A) $V_0\sin\omega_0 t+\frac{A}{2}\sin(\omega_0-\omega)t+\frac{A}{2}\sin(\omega_0+\omega)t$

(B) $(V_0+A)\cos\omega t\sin\omega_0 t$

(C) $V_0\sin\omega_0 t+A\cos\omega t$

(D) $V_0\sin\left[\omega_0(1+0.01\sin\omega t)t\right]$

11. [Online April 2019]
The physical sizes of the transmitter and receiver antenna in a communication system are

(A) Inversely proportional to modulation frequency
(B) Inversely proportional to carrier frequency
(C) Proportional to carrier frequency
(D) Independent of both carrier and modulation frequency

12. [Online April 2019]
A message signal of frequency 100 MHz and peak voltage 100 V is used to execute amplitude modulation on a carrier wave of frequency 300 GHz and peak voltage 400 V. The modulation index and difference between the two side band frequencies are

(A) 4; 2×10^8 Hz (B) 4; 1×10^8 Hz
(C) 0.25; 1×10^8 Hz (D) 0.25; 2×10^8 Hz

13. [Online April 2019]
Given below in the left column are different modes of communication using the kinds of waves given in the right column.

A. Optical Fibre communication	P. Ultrasound
B. Radar	Q. Infrared Light
C. Sonar	R. Microwaves
D. Mobile Phones	S. Radio Waves

From the options given below, find the most appropriate match between entries in the left and the right column.

(A) A-Q, B-S, C-P, D-R
(B) A-Q, B-S, C-R, D-P
(C) A-S, B-Q, C-R, D-P
(D) A-R, B-P, C-S, D-Q

14. [Online April 2019]
In an amplitude modulator circuit, the carrier wave is given by, $C(t)=4\sin(20000\pi t)$ while modulating signal is given by, $m(t)=2\sin(2000\pi t)$. The values of modulation index and lower side band frequency are

(A) 0.5 and 10 kHz (B) 0.3 and 9 kHz
(C) 0.4 and 10 kHz (D) 0.5 and 9 kHz

15. [2018]
A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz?

(A) 2×10^3 (B) 2×10^4
(C) 2×10^5 (D) 2×10^6

16. [Online 2018]
The number of amplitude modulated broadcast stations that can be accommodated in a 300 kHz band width for the highest modulating frequency 15 kHz will be

(A) 20 (B) 10
(C) 8 (D) 15

17. [Online 2018]
The carrier frequency of a transmitter is provided by a tank circuit of a coil of inductance 49 μH and a capacitance of 2.5 nF. It is modulated by an audio signal of 12 kHz. The frequency range occupied by the side bands is

(A) 18 kHz – 30 kHz
(B) 13482 kHz – 13494 kHz
(C) 63 kHz – 75 kHz
(D) 442 kHz – 466 kHz

18. [Online 2018]
A carrier wave of peak voltage 14 V is used for transmitting a message signal. The peak voltage of modulating signal given to achieve a modulation index of 80% will be

(A) 22.4 V (B) 11.2 V
(C) 7 V (D) 28 V

19. [2017]
In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m. The bandwidth $(\Delta\omega_m)$ of the signal is such that $\Delta\omega_m \ll \omega_c$. Which of the following frequencies is not contained in the modulated wave?

(A) ω_m (B) ω_c
(C) $\omega_m+\omega_c$ (D) $\omega_c-\omega_m$

20. [Online 2017]
A signal of frequency 20 kHz and peak voltage of 5 V is used to modulate a carrier wave of frequency 1.2 MHz and peak voltage 25 V. Choose the correct statement.

(A) Modulation index $=5$, side frequency bands are at 1400 kHz and 1000 kHz
(B) Modulation index $=0.2$, side frequency bands are at 1220 kHz and 1180 kHz
(C) Modulation index $=0.8$, side frequency bands are at 1180 kHz and 1220 kHz
(D) Modulation index $=5$, side frequency bands are at 21.2 kHz and 18.8 kHz

21. [Online 2017]
A signal is to be transmitted through a wave of wavelength λ, using a linear antenna. The length l of the antenna and effective power radiated P_{eff} will be given respectively as (K is a constant of proportionality)

(A) $\frac{\lambda}{5}$, $P_{\text{eff}}=K\left(\frac{l}{\lambda}\right)^{\frac{1}{2}}$ (B) λ, $P_{\text{eff}}=K\left(\frac{l}{\lambda}\right)^2$
(C) $\frac{\lambda}{16}$, $P_{\text{eff}}=K\left(\frac{l}{\lambda}\right)^3$ (D) $\frac{\lambda}{8}$, $P_{\text{eff}}=K\left(\frac{l}{\lambda}\right)$

22. [2016]
Choose the correct statement

(A) In amplitude modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.
(B) In amplitude modulation the frequency of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(C) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(D) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the frequency of the audio signal.

23. [Online 2016]

An audio signal consists of two distinct sounds: One a human speech signal in the frequency band of $200\ \text{Hz}$ to $2700\ \text{Hz}$, while the other is a high frequency music signal in the frequency band of $10200\ \text{Hz}$ to $15200\ \text{Hz}$. The ratio of the AM signal bandwidth required to send both the signals together to the AM signal bandwidth required to send just the human speech is

(A) 2 (B) 5

(C) 6 (D) 3

24. [Online 2016]

A modulated signal $C_m(t)$ has the form $C_m(t) = 30\sin(300\pi t) + 10(\cos 200\pi t - \cos 400\pi t)$.
The carrier frequency f_c, the modulating frequency (message frequency) f_m, and the modulation index μ are respectively given by

(A) $f_c = 200\ \text{Hz};\ f_m = 50\ \text{Hz};\ \mu = \dfrac{1}{2}$

(B) $f_c = 150\ \text{Hz};\ f_m = 50\ \text{Hz};\ \mu = \dfrac{2}{3}$

(C) $f_c = 150\ \text{Hz};\ f_m = 30\ \text{Hz};\ \mu = \dfrac{1}{3}$

(D) $f_c = 200\ \text{Hz};\ f_m = 30\ \text{Hz};\ \mu = \dfrac{1}{2}$

25. [2015]

A signal of $5\ \text{kHz}$ frequency is amplitude modulated on a carrier wave of frequency $2\ \text{MHz}$. The frequencies of the resultant signal is/are

(A) $2005\ \text{kHz}$, $2000\ \text{kHz}$ and $1995\ \text{kHz}$

(B) $2000\ \text{kHz}$ and $1995\ \text{kHz}$

(C) $2\ \text{MHz}$ only

(D) $2005\ \text{kHz}$ and $1995\ \text{kHz}$

26. [2012]

A radar has a power of $1\ \text{kW}$ and is operating at a frequency of $10\ \text{GHz}$. It is located on a mountain top of height $500\ \text{m}$. The maximum distance upto which it can detect object located on the surface of the earth (Radius of earth $= 6.4 \times 10^6\ \text{m}$) is

(A) 16 km (B) 40 km

(C) 64 km (D) 80 km

27. [2011]

This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1: Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.

Statement-2: The state of ionosphere varies from hour to hour, day to day and season to season.

(A) Statement-1 is true, Statement-2 is false.

(B) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.

(C) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1.

(D) Statement-1 is false, Statement-2 is true.

ANSWER KEYS—PRACTICE EXERCISES

Single Correct Choice Type Questions

1. B	**2.** D	**3.** A	**4.** B	**5.** C	**6.** C	**7.** D	**8.** A	**9.** A	**10.** D
11. C	**12.** C	**13.** D	**14.** B	**15.** B	**16.** B	**17.** B	**18.** A	**19.** C	**20.** A
21. A	**22.** A	**23.** A	**24.** C	**25.** A	**26.** B	**27.** C	**28.** A	**29.** B	**30.** D
31. A	**32.** A	**33.** D	**34.** B	**35.** B	**36.** D	**37.** D	**38.** C	**39.** A	**40.** B
41. A	**42.** A	**43.** A	**44.** D	**45.** B	**46.** C	**47.** B	**48.** D	**49.** B	**50.** B
51. A	**52.** B	**53.** D	**54.** A	**55.** B	**56.** C	**57.** D	**58.** C	**59.** A	**60.** C
61. C	**62.** A	**63.** A	**64.** C	**65.** D	**66.** B	**67.** A	**68.** C	**69.** D	

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1. C	**2.** A	**3.** C	**4.** C	**5.** A	**6.** C	**7.** D	**8.** C	**9.** C	**10.** A
11. B	**12.** D	**13.** A	**14.** D	**15.** C	**16.** B	**17.** D	**18.** B	**19.** A	**20.** B
21. B	**22.** A	**23.** C	**24.** B	**25.** A	**26.** D	**27.** B			

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HINTS AND EXPLANATIONS

CHAPTER 1: DUAL NATURE OF RADIATION AND MATTER

Test Your Concepts-I (Based on Photon Properties)

1. For completely absorbing surface,

$$P_{\text{rad}} = \frac{I}{C} = \frac{1.4 \times 10^3}{3 \times 10^8} = 4.7 \times 10^{-6} \text{ Nm}^{-2}$$

2. $f = \frac{v_1}{2\pi r_1} = \frac{c}{\lambda}$

$$\Rightarrow \quad \lambda = \frac{2\pi c r_1}{v_1} = \frac{(2\pi)(3\times 10^8)(0.529\times 10^{-10})(10^{10})}{(2.2\times 10^6)} \text{ Å}$$

$$\Rightarrow \quad \lambda = 453 \text{ Å}$$

3. For an electron, de-Broglie wavelength is given by,

$$\lambda = \sqrt{\frac{150}{V}} = \sqrt{\frac{150}{25}} = \sqrt{6}$$

$$\Rightarrow \quad \lambda \approx 2.5 \text{ Å}$$

4. Power of transmitter $= 10 \text{ kW} = 10^4 \text{ W}$

So, total energy emitted per second is 10^4 J

Energy of one photon,

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{500} \text{ J}$$

Number of photons emitted per second is

$$n = \frac{\text{Total energy emitted per second}}{\text{Energy of one photon}}$$

$$\Rightarrow \quad n = \frac{10^4 \times 500}{6.63 \times 10^{-34} \times 3 \times 10^8} = 2.51 \times 10^{31}$$

5. Energy of one photon is

$$E = \frac{hc}{\lambda}$$

$$\Rightarrow \quad E = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{5600 \times 10^{-10}} = 3.5 \times 10^{-19} \text{ J}$$

Since a 100 W bulb supplies 100 J of energy per second. So, energy released per second as visible photons is

$$E' = \frac{100 \times 5}{100} = 5 \text{ J}$$

$\therefore$ Number of photons emitted per second as visible light is

$$\frac{E'}{E} = \frac{5}{3.5 \times 10^{-19}} = 1.43 \times 10^{19}$$

6. Energy of each photon is

$$E = h\nu = 6.62 \times 10^{-34} \times 10^{12} = 6.62 \times 10^{-22} \text{ J}$$

Number of photons present in 6.62 J of radiation energy is calculated by using

$$E = N(h\nu)$$

$$\Rightarrow \quad N = \frac{E}{h\nu} = \frac{6.62}{6.62 \times 10^{-22}} = 10^{22}$$

7. (a) Energy of each photon,

$$E = h\nu = 6.63 \times 10^{-34} \times 6 \times 10^{14} = 3.98 \times 10^{-19} \text{ J}$$

(b) If N is the number of photons emitted per second by the source, then

Power transmitted in the beam is

$$P = N \text{ (energy of each photon)}$$

$$\Rightarrow \quad P = NE$$

$$\Rightarrow \quad N = \frac{P}{E} = \frac{2 \times 10^{-3} \text{ W}}{3.98 \times 10^{-19} \text{ J}}$$

$$\Rightarrow \quad N = 5 \times 10^{15} \text{ photons per second.}$$

8. The total energy of an electron with rest mass m_0 and momentum p is

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

where c is the speed of light in free space. Law of Conservation of Energy requires,

$$\begin{pmatrix}\text{Rest mass}\\ \text{of electron}\end{pmatrix} + \begin{pmatrix}\text{Energy of}\\ \text{photon}\end{pmatrix} = \begin{pmatrix}\text{Total energy of}\\ \text{moving electron}\end{pmatrix}$$

$$\Rightarrow \quad m_0 c^2 + h\nu = \sqrt{m_0^2 c^4 + p^2 c^2} \qquad \ldots(1)$$

Law of Conservation of Momentum requires.

$$\begin{pmatrix}\text{Initial}\\ \text{momentum}\\ \text{of electron}\end{pmatrix} + \begin{pmatrix}\text{Momentum}\\ \text{of photon}\end{pmatrix} = \begin{pmatrix}\text{Final}\\ \text{momentum}\\ \text{of electron}\end{pmatrix}$$

$$\Rightarrow \quad 0 + \frac{h}{\lambda} = p$$

$$\Rightarrow \quad \frac{h\nu}{c} = p \qquad \left\{\because \lambda = \frac{c}{\nu}\right\}$$

$$\Rightarrow \quad h\nu = pc \qquad \ldots(2)$$

Squaring both sides of equation (1), we get

$$m_0^2 c^4 + h^2 \nu^2 + 2 m_0 c^2 h\nu = m_0^2 c^4 + p^2 c^2$$

$\Rightarrow \quad 2(m_0c^2)(h\nu)=0$ {using (2)}

This is impossible. In a frame, where the initial electron is moving with uniform velocity, the same conclusion must hold because if a process is forbidden in one inertial frame, it is also forbidden in another inertial frame.

9. KE of an electron is

$$E_K = 50\text{ eV} = 1.6\times10^{-19}\times5\times10^{4}\text{ J} = 8\times10^{-15}\text{ J}$$

de-Broglie wavelength of electrons is

$$\lambda = \frac{h}{\sqrt{2mE_K}}$$

$$\Rightarrow \lambda = \frac{6.63\times10^{-34}}{\sqrt{2\times9.11\times10^{-31}\times8\times10^{-15}}}\text{ m}$$

$$\Rightarrow \lambda = \frac{6.63\times10^{-11}}{12.07}\text{ m} = 5.5\times10^{-12}\text{ m}$$

For yellow light, wavelength $\lambda = 5.9\times10^{-7}$ m

As resolving power is inversely proportional to wavelength, so the resolving power of an electron microscope is about 10^5 times greater than that of an optical microscope.

10. The kinetic energy of a particle of mass m can be expressed in terms of its momentum p as follows

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}\cdot\frac{(mv)^2}{m} = \frac{p^2}{2m}$$

As de-Broglie wavelength, $\lambda = \frac{h}{p}$

$$\Rightarrow \frac{\lambda_e}{\lambda_P} = \frac{p_P}{p_e} = \sqrt{\frac{2m_P\cdot KE}{2m_e\cdot KE}} = \sqrt{\frac{m_P}{m_e}}$$

As $m_e < m_P$, therefore, $\lambda_e > \lambda_P$

Thus, the electron has greater de-Broglie wavelength.

11. Kinetic energy of electron is

$$E_e = \frac{1}{2}mv^2 = \frac{1}{2}\cdot\frac{(mv)^2}{m} = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} \quad \left\{\because p = \frac{h}{\lambda}\right\}$$

Energy of a photon is totally kinetic and is given by

$$E_P = h\nu = \frac{hc}{\lambda}$$

$$\Rightarrow \frac{E_P}{E_e} = \frac{hc}{\lambda}\cdot\frac{2m\lambda^2}{h^2} = \frac{2mc\lambda}{h}$$

$$\Rightarrow \frac{E_P}{E_e} = \frac{m_P}{m_e}$$

Thus, the KE of a photon is greater than that of an electron of same wavelength.

12. The total relativistic energy of a particle is

$$E = \sqrt{m_0c^4 + p^2c^2}$$

As wavelength λ is same for both electron and proton,

Since, momentum, $p = \frac{h}{\lambda}$ is same for both particles and hence p^2c^2 is same for both.

But rest mass m_0 of a proton is greater than that of an electron, therefore, the energy of a proton is more than that of an electron of same wavelength.

13. Force on space vehicle is

$$F = \frac{P}{c} = \frac{100}{3\times10^8} = 3.33\times10^{-7}\text{ N}$$

Acceleration is $a = \frac{F}{m} = \frac{3.33\times10^{-7}}{50}$

$$\Rightarrow a = 6.66\times10^{-9}\text{ ms}^{-2}$$

14. Momentum of photon having wavelength λ is

$$p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}}{5200\times10^{-10}}\text{ kgms}^{-1}$$

Momentum of electron moving with a velocity v is

$$p_e = mv = 9.1\times10^{-31}v$$

Since the photon and the electron have same momentum

$$\Rightarrow 9.1\times10^{-31}v = \frac{6.626\times10^{-34}}{5200\times10^{-10}}$$

$$\Rightarrow v = 1400\text{ ms}^{-1}$$

15. (a) The energy of each photon (in eV) is

$$E = \frac{hc}{\lambda} = \frac{(4.14\times10^{-15}\text{ eVs})\times(3\times10^8\text{ ms}^{-1})}{496\text{ nm}}$$

$$\Rightarrow E = \frac{1240\text{ eVnm}}{496\text{ nm}} = 2.5\text{ eV}$$

$$\Rightarrow E = 4\times10^{-19}\text{ J}$$

In one second, 10 J of energy passes through any cross section of the beam. Thus, the number of photons crossing a cross section per second is

$$n = \frac{10}{4\times10^{-19}} = 2.5\times10^{19}\text{ photons/sec}$$

This is also the number of photons falling on the surface per second and being absorbed.

(b) The linear momentum of each photon is

$$p = \frac{h}{\lambda} = \frac{h\nu}{c}$$

The total change in momentum of all the photons falling on the surface is $\Delta p = N\left(\frac{h\nu}{c}\right) = (n\Delta t)\left(\frac{h\nu}{c}\right)$

As the photons are completely absorbed by the surface, this much momentum is transferred to the surface per second. The rate of change of the momentum of the surface, i.e., the force on it is

$$F = \frac{\Delta p}{\Delta t} = n\left(\frac{h\nu}{c}\right) = \frac{10}{3\times 10^8}$$

$$\Rightarrow \quad F = 3.33\times 10^{-8}\ \text{N}$$

16. The energy of each photon $= \dfrac{200\ \text{Js}^{-1}}{4\times 10^{20}\ \text{s}^{-1}} = 5\times 10^{-19}\ \text{J}$

Wavelength $= \lambda = \dfrac{hc}{E}$

$$\Rightarrow \quad \lambda = \frac{(6.63\times 10^{-34}\ \text{Js})\times(3\times 10^8\ \text{ms}^{-1})}{(5\times 10^{-19}\ \text{J})}$$

$$\Rightarrow \quad \lambda = 4.0\times 10^{-7}\ \text{m} = 400\ \text{nm}$$

17. The energy of each photon is

$$E = \frac{hc}{\lambda}$$

$$\Rightarrow \quad E = \frac{(6.63\times 10^{-34}\ \text{Js})\times(3\times 10^8\ \text{ms}^{-1})}{663\times 10^{-9}\ \text{m}}$$

$$\Rightarrow \quad E = 3.14\times 10^{-19}\ \text{J}$$

The energy of the laser emitted per second is 5×10^{-3} J. Thus, the number of photons emitted per second is

$$n = \frac{P}{E}$$

$$\Rightarrow \quad n = \frac{5\times 10^{-3}\ \text{J}}{3.14\times 10^{-19}\ \text{J}} = 1.6\times 10^{16}\ \text{photons/sec}$$

18. The linear momentum of the photon is

$$p = \frac{h}{\lambda} = \frac{6.63\times 10^{-34}\ \text{Js}}{122\times 10^{-9}\ \text{m}} = 5.43\times 10^{-27}\ \text{kgms}^{-1}$$

As the photon is absorbed and the atom stops, the total final momentum is zero.
From conservation of linear momentum, the initial momentum must be zero. The atom should move opposite to the direction of motion of the photon and they should have the same magnitudes of linear momentum.

$$\Rightarrow \quad (1.67\times 10^{-27}\ \text{kg})v = 5.43\times 10^{-27}\ \text{kgms}^{-1}$$

$$\Rightarrow \quad v = \frac{5.43\times 10^{-27}}{1.67\times 10^{-27}}\text{ms}^{-1} = 3.25\ \text{ms}^{-1}$$

Test Your Concepts-II
(Based on Photoelectric Effect)

1. Since,

$$E = h\nu = (4.316\times 10^{-15}\ \text{eV-sec})(1.5\times 10^{15}\ \text{sec}^{-1})$$

$$\Rightarrow \quad E \approx 6.5\ \text{eV}$$

$$\Rightarrow \quad K_{\max} = E - W = (6.5 - 3.7)\ \text{eV} = 2.8\ \text{eV}$$

2. The energy of incident photon in eV is given by

$$E = \frac{12400}{2480}\ \text{eV} = 5\text{eV}$$

$$\Rightarrow \quad E = (5)(1.6\times 10^{-19})\ \text{J} = 8\times 10^{-19}\ \text{J}$$

The number of photons emitted per second by the source is

$$n = \frac{P_{\text{source}}}{E_{\text{each photon}}} = \frac{40}{8\times 10^{-19}} = 5\times 10^{19}\ \text{photons/s}$$

The number of photons incident on the metal surface per second per unit area is

$$\phi_N = \frac{n}{A} = \frac{n}{4\pi r^2} = \frac{5\times 10^{19}}{4\times 3.14\times (2)^2} = 0.995\times 10^{18}$$

$$\Rightarrow \quad \phi_N \approx 1\times 10^{18}\ \text{s}^{-1}\text{m}^{-2}$$

The kinetic energy of the fastest electron is given by

$$K_{\max} = E - \phi_0$$

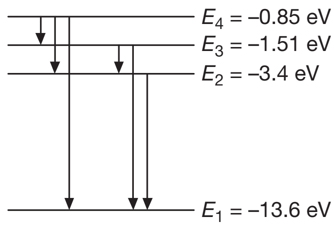
$$\Rightarrow \quad K_{\max} = 5\ \text{eV} - 3.7\ \text{eV} = 1.3\ \text{eV}$$

The threshold wavelength for the given metal surface for a work function of 3.7 eV is given by

$$\lambda_{\text{th}} = \frac{12400}{3.7}\ \text{Å} = 3351\ \text{Å}$$

3. $\dfrac{K_1}{K_2} = 5$

$$\Rightarrow \quad \frac{\Delta E_1 - W}{\Delta E_2 - W} = 5 \qquad \ldots(1)$$



Here, $\Delta E_1 = E_4 - E_1 = 12.75$ eV and

$$\Delta E_2 = E_3 - E_1 = 12.09 \text{ eV}$$

Substituting in equation (1) and solving we get

$$W = 11.93 \text{ eV}$$

4. $K_{max} = E - W$

$$\Rightarrow \quad 3 \text{ eV} = \frac{12375}{\lambda} - \frac{12375}{5000}$$

$$\Rightarrow \quad \lambda = 2260 \text{ Å}$$

5. Since, $E(\text{in eV}) = \dfrac{12375}{\lambda(\text{in Å})}$

$$\Rightarrow \quad E_1 = \frac{12375}{5000} = 2.475 \text{ eV}$$

$$E_2 = \frac{12375}{6000} = 2.06 \text{ eV and}$$

$$E_3 = \frac{12375}{7000} = 1.77 \text{ eV}$$

Since, W is 1.9 eV, photons of energy E_1 and E_2 can only emit photoelectrons. Charge emitted per second is

$$\frac{q}{t} = (1.6\times10^{-19})\left(\frac{1}{3}\right)\frac{(10^{-3})(10^{-4})}{\pi\times(10^{-3})^2\times2.475\times1.6\times10^{-19}} + (1.6\times10^{-19})\left(\frac{1}{3}\right)\frac{(10^{-3})(10^{-4})}{\pi\times(10^{-3})^2\times2.06\times1.6\times10^{-19}}$$

$$\Rightarrow \quad \frac{q}{t} = 9.28\times10^{-3} \text{ C} = 9.28 \text{ mC}$$

6. $E_1 = \dfrac{12375}{4000} = 3.1$ eV

$$E_2 = \frac{12375}{4800} = 2.57 \text{ eV}$$

$$E_3 = \frac{12375}{6000} = 2.06 \text{ eV and}$$

$$E_4 = \frac{12375}{7000} = 1.77 \text{ eV}$$

Therefore, light of wavelengths 4000 Å, 4800 Å and 6000 Å can only emit photoelectrons.

So, number of photoelectrons emitted per second is

$$n = \frac{I_1A_1}{E_1} + \frac{I_2A_2}{E_2} + \frac{I_3A_3}{E_3}$$

$$\Rightarrow \quad n = IA\left(\frac{E_1E_2 + E_2E_3 + E_1E_3}{E_1E_2E_3}\right)$$

$$\Rightarrow \quad n = \frac{(1.5\times10^{-3})(10^{-4})}{1.6\times10^{-19}} \times \left[\frac{3.1\times2.57 + 2.57\times2.06 + 3.1\times2.06}{3.1\times2.57\times2.06}\right]$$

$$\Rightarrow \quad n = 1.12\times10^{12}$$

7. (a) $h\nu_0 = W = 3.75\times1.6\times10^{-19}$ J

$$\Rightarrow \quad \nu_0 = \frac{3.75\times1.6\times10^{-19}}{6.6\times10^{-34}} \approx 10^{15} \text{ Hz}$$

(b) $E = \dfrac{12375}{1980}$ eV $= 6.25$ eV

$$\Rightarrow \quad K_{max} = 2.5 \text{ eV}$$

(c) $K_{max} = 2$ eV

8. $K_{max} = E - W = \dfrac{hc}{\lambda} - \dfrac{hc}{\lambda_0}$

$$0.18\times1.6\times10^{-19} = \frac{h(3\times10^8)}{5461\times10^{-10}} - \frac{h(3\times10^8)}{\lambda_0} \quad \ldots(1)$$

Further, $eV_0 = E - W = \dfrac{hc}{\lambda} - \dfrac{hc}{\lambda_0}$

$$\Rightarrow \quad 4.6\times1.6\times10^{-19} = \frac{h(3\times10^8)}{1849\times10^{-10}} - \frac{h(3\times10^8)}{\lambda_0} \quad \ldots(2)$$

Solving equations (1) and (2), for h and λ_0, we get

$$h = 6.6\times10^{-34} \text{ Js}$$

and $\lambda_0 = 5990.25$ Å

9. From Einstein's Photo-Electric Equation, we have

$$\frac{1}{2}mv_1^2 = \frac{hc}{\lambda_1} - W \quad \ldots(1)$$

$$\frac{1}{2}mv_2^2 = \frac{hc}{\lambda_2} - W \quad \ldots(2)$$

Dividing equation (1) with equation (2), and using the given fact that $v_1 = 2v_2$, we get

$$4 = \frac{\dfrac{hc}{\lambda_1} - W}{\dfrac{hc}{\lambda_2} - W}$$

$$\Rightarrow \quad 3W = 4\left(\frac{hc}{\lambda_2}\right) - \left(\frac{hc}{\lambda_1}\right) = \frac{4\times12375}{5400} - \frac{12375}{3500} = 5.63 \text{ eV}$$

$$\Rightarrow \quad W = 1.9 \text{ eV}$$

10. From Einstein's Photo-Electric Equation, we have

$$0.6 = \frac{12375}{4950} - W \quad \ldots(1)$$

$$1.1 = \frac{12375}{\lambda} - W \quad \ldots(2)$$

Solving above two equations, we get

$$W = 1.9 \text{ eV} \text{ and } \lambda = 4125 \text{ Å}$$

No change is observed.

11. Since, $W = 2.3$ eV and $\lambda = 2800$ Å

$$E(\text{in eV}) = \frac{12375}{\lambda(\text{in Å})} = \frac{12375}{2800} = 4.4 \text{ eV}$$

(a) From Einstein's Photo-Electric Equation, we have

$$K_{max} = E - W$$

$$\Rightarrow K_{max} = (4.4 - 2.3) \text{ eV}$$

$$\Rightarrow K_{max} = 2.1 \text{ eV}$$

(b) Since, $K_{max} = eV_0$

$$\Rightarrow 2.1 \text{ eV} = eV_0$$

$$\Rightarrow V_0 = 2.1 \text{ V}$$

12. Energy of incident photon in eV is

$$E = \frac{12375}{1500} \text{ eV} = 8.25 \text{ eV}$$

According to Einstein's photo electric equation, we have

$$E = \phi_0 + K_{max}$$

$$\Rightarrow \phi_0 = E - K_{max}$$

$$\Rightarrow \phi_0 = (8.25 - 3) \text{ eV} = 5.25 \text{ eV}$$

Threshold wavelength for the metal surface corresponding to work function 5.25 eV is given by

$$\lambda_{th} = \frac{12375}{5.25} \text{ Å} = 2357 \text{ Å}$$

Stopping potential for the ejected photoelectrons is given by

$$V_S = \frac{K_{max}}{e} = \frac{3 \text{ eV}}{e} = 3 \text{ V}$$

13. (a) The work function of tungsten cathode is

$$\phi_0 = \frac{hc}{\lambda_{th}}$$

$$\Rightarrow \phi_0 = \frac{12375}{2300} \text{ eV} \approx 5.4 \text{ eV}$$

(b) The energy, in eV, of incident photons is

$$E = \frac{hc}{\lambda}$$

$$\Rightarrow E = \frac{12375}{1800} \text{ eV} = 6.9 \text{ eV}$$

The maximum kinetic energy of ejected electrons can be given as

$$K_{max} = E - \phi_0$$

$$\Rightarrow KE_{max} = (6.9 - 5.4) \text{ eV} = 1.5 \text{ eV}$$

14. The energy (in eV) of the incident photons is

$$E = \frac{12375}{2250} \text{ eV} = 5.5 \text{ eV}$$

Since. the stopping potential for ejected electrons is 1.5 V, so the maximum kinetic energy of ejected photoelectrons is

$$K_{max} = eV_s = 1.5 \text{ eV}$$

Applying Einstein's photo electric equation, we get

$$\Rightarrow \phi_0 = E - K_{max} = (5.5 - 1.5) \text{ eV} = 4 \text{ eV}$$

Energy (in eV) for photons of high intensity light of wavelength 6875 Å is

$$E' = \frac{12375}{6875} \text{ eV} = 1.8 \text{ eV} < \phi_0$$

Since, $E' < \phi_0$, so the photocell will not work even if irradiated by this high intensity light.

15. (a) Given that the threshold wavelength of a metal is $\lambda_{th} = 2750$ Å. So, work function (ϕ_0) of metal is

$$\phi_0 = \frac{hc}{\lambda_{th}}$$

$$\Rightarrow \phi_0 = \frac{12430}{2750} \text{ eV} = 4.5 \text{ eV}$$

(b) The energy (E) of incident photon of wavelength 1800 Å on metal in eV is

$$E = \frac{12430}{1800} \text{ eV} = 6.9 \text{ eV}$$

So, maximum kinetic energy (K_{max}) of the ejected electrons is

$$K_{max} = E - \phi_0$$

$$\Rightarrow K_{max} = (6.9 - 4.5) \text{ eV} = 2.4 \text{ eV}$$

(c) If the maximum speed of ejected electrons is v_{max}, then

$$\frac{1}{2} m v_{max}^2 = 2.4 \text{ eV}$$

$$\Rightarrow v_{max} = \sqrt{\frac{2 \times 2.4 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} = 9.2 \times 10^5 \text{ ms}^{-1}$$

16. According to Einstein's photo-electric equation

$$K_{max} = \frac{1}{2}mv_{max}^2 = h\nu - \phi_0$$

$\Rightarrow \quad K_{max} = 4.9 - 4.5 = 0.4 \text{ eV}$

If p be the momentum of each ejected photo electron, then

$$p = \sqrt{2mK_{max}}$$

Also, we know that change in momentum of a body is equal to impulse. Hence the entire momentum of electron is gained when it is ejected out from the metal surface, so impulse (J) on the surface is given by

$$J = \Delta p = \sqrt{2mK_{max}}$$

Substituting the values, we get maximum impulse

$$J = \sqrt{2 \times 9.1 \times 10^{-31} \times 0.4 \times 1.6 \times 10^{-19}}$$

$\Rightarrow \quad J = 3.45 \times 10^{-25} \text{ kgms}^{-1}$

Single Correct Choice Type Questions

1. $E = nh\nu$

$\Rightarrow \quad \frac{E}{t} = P = \left(\frac{n}{t}\right)h\nu$

Hence, the correct answer is (C).

3. $5\,eV_0 = \frac{hc}{\lambda} - W \quad \ldots(1)$

$\Rightarrow \quad eV_0 = \frac{hc}{3\lambda} - W \quad \ldots(2)$

Solving equation (1) & (2), we get

$$\frac{hc}{\lambda} - W = \frac{5hc}{3\lambda} - 5W$$

$\Rightarrow \quad 4W = \frac{2hc}{3\lambda}$

$\Rightarrow \quad W = \frac{hc}{6\lambda}$

Hence, the correct answer is (A).

4. $\lambda = \frac{h}{\sqrt{2meV}}$

$$\lambda' = \frac{h}{\sqrt{2MeV}}$$

$\Rightarrow \quad \lambda' = \lambda\sqrt{\frac{m}{M}}$

Hence, the correct answer is (B).

8. $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$

$\Rightarrow \quad \lambda = kE^{-\frac{1}{2}}$

$\Rightarrow \quad \log_e \lambda = \log k - \frac{1}{2}\log E$

Hence, the correct answer is (A).

9. $E = mc^2$

$\Rightarrow \quad E = m_0c^2\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$

Since $\frac{v}{c} \ll 1$

$\Rightarrow \quad E \simeq m_0c^2\left(1 + \frac{v^2}{2c^2}\right)$

$\Rightarrow \quad E \simeq m_0c^2 + \frac{1}{2}m_0v^2$

Hence, the correct answer is (C).

10. Since both have same de Broglie wavelength, hence both must have equal value of momentum.

Since, $E = \frac{p^2}{2m}$

$\Rightarrow \quad \frac{E_e}{E_p} = \frac{m_p}{m_e} = 1840 \simeq 2000 \quad$ {nearest possible approximation to answer}

Hence, the correct answer is (C).

11. $E^2 = p^2c^2 + m_0^2c^4$

$\Rightarrow \quad E^2 = p^2c^2 + E_0^2 \quad$ {where $E_0 = m_0c^2$}

$\Rightarrow \quad p = \frac{E}{c}\sqrt{1 - \left(\frac{E_0}{E}\right)^2} \simeq \frac{E}{c} \quad (\text{for } E \gg E_0)$

$\Rightarrow \quad \lambda = \frac{h}{p} \simeq \frac{hc}{E}$

For a photon

$$E = h\nu = \frac{hc}{\lambda_\gamma}$$

$\Rightarrow \quad \lambda_\gamma = \frac{hc}{E} \simeq \lambda$

Hence, the correct answer is (A).

12. Energy of each lump before collision is

$$E = mc^2 = \frac{m_0c^2}{\sqrt{1 - \left(\frac{4}{5}\right)^2}} = \frac{5}{3}m_0c^2$$

The energy of composite lump after collision will be Mc^2.

By energy conservation principle we get

Total initial energy = Total final energy

$$\Rightarrow \quad \frac{5}{3}m_0c^2 + \frac{5}{3}m_0c^2 = Mc^2$$

$$\Rightarrow \quad M = \frac{10}{3}m_0$$

Hence, the correct answer is (A).

13. $r = \dfrac{mv}{qB}$

$$\Rightarrow \quad r = \frac{2\times 10^7}{1.76\times 10^{11}\times 2\times 10^{-2}}$$

$$\Rightarrow \quad r = 0.0055 \text{ m}$$

$$\Rightarrow \quad D = 2r = 0.011 \text{ m}$$

$$\Rightarrow \quad D = 1.1 \text{ cm}$$

Hence, the correct answer is (C).

14. $E_3 - E_2 = 13.6\left[\dfrac{1}{2^2} - \dfrac{1}{3^2}\right]$

$$\Rightarrow \quad \Delta E = \frac{13.6\times 5}{36} = 1.89 \text{ eV}$$

Photoelectrons with K_{max} are moving on circular path, so

$$r = \frac{mv}{qB}$$

$$\Rightarrow \quad mv = qBr$$

$$\Rightarrow \quad p = qBr = 1.6\times 10^{-19}\times\frac{1}{3200}\times 10^{-3}$$

$$\Rightarrow \quad p = \frac{1}{2}\times 10^{-24} = 5\times 10^{-25} \text{ kgms}^{-1}$$

Energy of photoelectron is $K_{max} = \dfrac{p^2}{2m}$

$$\Rightarrow \quad K_{max} = \frac{25\times 10^{-50}}{2\times 9.1\times 10^{-31}\times 1.6\times 10^{-19}} \text{ eV}$$

$$\Rightarrow \quad K_{max} = 0.86 \text{ eV}$$

Now using Einstein equation, we get

$$h\nu = \phi + K_{max}$$

$$\Rightarrow \quad 1.89 = 0.56 + \phi$$

$$\Rightarrow \quad \phi = 1.03 \text{ eV}$$

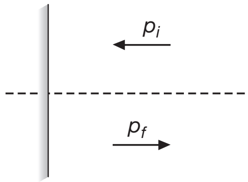
Hence, the correct answer is (A).

15. $F = \dfrac{I}{c}(\text{Effective Area})$

$$\Rightarrow \quad F = \frac{I}{c}(\pi R^2) = \frac{\pi R^2 I}{c}$$

Hence, the correct answer is (A).

16.



Let $p = \dfrac{E}{c}$

$$\Rightarrow \quad \Delta p = \frac{E}{c} - \left(-\frac{E}{c}\right) = \frac{2E}{c}$$

Hence, the correct answer is (B).

17. $\lambda = \dfrac{h}{\sqrt{2meV}}$

So, when V becomes 4 V, λ becomes $\dfrac{\lambda}{2}$

Hence, the correct answer is (C).

18. Energy of falling light

$$E = \frac{hc}{\lambda} = \frac{(4.14\times 10^{-15})(3\times 10^8)}{3\times 10^{-7}}$$

$$\Rightarrow \quad E = 4.14 \text{ eV}$$

So, electrons are emitted with a kinetic energy = 4.14 – 1 = 3.14 eV

$$\Rightarrow \quad \frac{1}{2}mv^2 = 3.14\times 1.6\times 10^{-19}$$

$$\Rightarrow \quad v = \sqrt{\frac{2\times 3.14\times 1.6\times 10^{-19}}{9.1\times 10^{-31}}}$$

$$\Rightarrow \quad v = 1.2\times 10^6 \text{ ms}^{-1}$$

Hence, the correct answer is (D).

19. $\lambda = \dfrac{h}{mv}$

$$\Rightarrow \quad \lambda = \frac{h}{m\sqrt{\frac{2qV}{m}}} \qquad \left\{\because \frac{1}{2}mv^2 = \text{eV}\right\}$$

$$\Rightarrow \quad \lambda = \frac{h}{\sqrt{2mqV}}$$

$$\Rightarrow \quad \frac{\lambda_P}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_P q_P}} = 2\sqrt{2} \qquad \{\because m_\alpha = 4m_P,\ q_\alpha = 2q_P\}$$

Hence, the correct answer is (C).

20. $R \propto \dfrac{1}{\lambda}$ and $\lambda = \dfrac{h}{\sqrt{2mqV}} \propto \dfrac{1}{\sqrt{V}}$

$$\Rightarrow \quad R \propto \sqrt{V}$$

$$\Rightarrow \quad \frac{R_2}{R_1} = \sqrt{\frac{80\ kV}{20\ kV}} = 2$$

$\Rightarrow \quad R_2 = 2R_1 = 2R$

Hence, the correct answer is (B).

21. $A = \dfrac{N_1A_1 + N_2A_2}{N_1 + N_2}$

$\Rightarrow \quad 10.81 = \dfrac{N_1(10) + N_2(11)}{N_1 + N_2}$

Solving, we get

$$\frac{N_1}{N_2} = \frac{0.19}{0.81} = \frac{19}{81}$$

Hence, the correct answer is (A).

23. Since plate is in air, so gravitational force will act on this

$F_{\text{gravitational}} = mg$ {downward}

$\Rightarrow \quad F_{\text{gravitational}} = 10 \times 10^{-3} \times 10$

$\Rightarrow \quad F_{\text{gravitational}} = 10^{-1}$ N

for equilibrium force exerted by light beam should be equal to $F_{\text{gravitational}}$

$F_{\text{photon}} = F_{\text{gravitational}}$

If power of light beam be P, the photon force is

$F_{\text{photon}} = \dfrac{P}{c}$

$\Rightarrow \quad \dfrac{P}{c} = 10^{-1}$

$\Rightarrow \quad P = 3.0 \times 10^8 \times 10^{-1}$

$\Rightarrow \quad P = 3 \times 10^7$ W

Hence, the correct answer is (B).

24. For a photon $\lambda = \dfrac{h}{p} = \dfrac{hc}{E}$

For a particle of mass m moving with a velocity v de Broglie relationship is given by

$$\lambda = \frac{h}{mv}$$

Hence, the correct answer is (B).

25. $E = n\left(\dfrac{hc}{\lambda}\right)$

Also $\dfrac{1}{2}\varepsilon_0 E_0^2 = \dfrac{\text{Energy}}{\text{Volume}}$

$\Rightarrow \quad \text{Energy} = \left(\dfrac{1}{2}\varepsilon_0 E_0^2\right)(\text{Volume of cavity})$

$\Rightarrow \quad \text{Energy} = \left(\dfrac{1}{2}\varepsilon_0 E_0^2\right)(\text{Area}(c\Delta t))$

$\Rightarrow \quad n\dfrac{hc}{\lambda} = \dfrac{1}{2}\varepsilon_0 E_0^2 (Ac\Delta t)$

$\dfrac{n}{A\Delta t} = \dfrac{(\text{Number of photons striking the desk})}{(\text{Area})(\text{time})}$

$\Rightarrow \quad \dfrac{n}{A\Delta t} = N = \dfrac{\lambda}{2}\dfrac{\varepsilon_0 E_0^2}{h}$

Hence, the correct answer is (C).

27. $Kx_0 = \dfrac{I}{C}(\pi R^2)$

$\Rightarrow \quad x_0 = \dfrac{I}{KC}(\pi R^2)$

Hence, the correct answer is (B).

31. According to Einstein's Photoelectric Equation

$E_K = h\nu_0 - \phi$

$\Rightarrow \quad 0 = h\nu_0 - \phi$

$\Rightarrow \quad \phi = h\nu_0 = \dfrac{hc}{\lambda_0}$

$\Rightarrow \quad 4 \times 1.6 \times 10^{-19} = \dfrac{(6.626 \times 10^{-34})(3 \times 10^8)}{\lambda_0}$

$\Rightarrow \quad \lambda_0 \simeq 3100$ Å

Hence, the correct answer is (C).

35. Stopping potential of 1.36 V implies $E_K = 1.36$ eV.

Since $h\nu = \phi_0 + E_K$

$\Rightarrow \quad \dfrac{(4.14 \times 10^{-15})(3 \times 10^8)}{5000 \times 10^{-10}} = \phi_0 + 1.36$

$\Rightarrow \quad 2.46 = \phi_0 + 1.36$

$\Rightarrow \quad \phi_0 = 1.1$ eV

Hence, the correct answer is (C).

36. Stopping potential is independent of intensity and depends upon frequency.

Hence, the correct answer is (A).

37. Saturation current $\propto$ (Intensity)

$\Rightarrow$ Saturation current $= 4(0.4\ \mu A)$

$\Rightarrow$ Saturation current $= 1.6\ \mu A$

Hence, the correct answer is (C).

38. $\dfrac{1}{2}mv^2 = \dfrac{hc}{\lambda} - W$ ...(1)

$\dfrac{1}{2}mv'^2 = \dfrac{hc}{3\lambda/4} - W$ ...(2)

Dividing, $\left(\dfrac{v'}{v}\right)^2 = \dfrac{\dfrac{4hc}{3\lambda} - W}{\dfrac{hc}{\lambda} - W}$

$$v' > v\sqrt{\frac{4}{3}}$$

Hence, the correct answer is (D).

39. $\frac{1}{2}mv^2 = k_B T$

$$v = \sqrt{\frac{2k_B T}{m}}$$

$$\Rightarrow \quad \lambda = \frac{h}{\sqrt{2mk_B T}}$$

Hence, the correct answer is (B).

40. $\frac{1}{2}m_p v^2 = eV$

$$\Rightarrow \quad v = \sqrt{\frac{2eV}{m_p}}$$

Since, $\lambda = \frac{h}{m_p v}$

$$\Rightarrow \quad \lambda = \frac{h}{\sqrt{2m_p eV}}$$

$$\Rightarrow \quad \lambda = \frac{0.287}{\sqrt{V}} \text{ Å} \left(\begin{array}{c}\text{is de-Broglie wavelength}\\ \text{for a proton accelerated}\\ \text{through a potential } V\end{array}\right)$$

$$\Rightarrow \quad \lambda = \frac{12.27}{\sqrt{V}} \text{ Å} \left(\begin{array}{c}\text{is de-Broglie wavelength}\\ \text{for an electron accelerated}\\ \text{through a potential } V\end{array}\right)$$

Hence, the correct answer is (B).

42. Number of photons per second entering human eye are

$$n = \frac{P\lambda}{hc}$$

$$\Rightarrow \quad n = \frac{10^{-10} \times 660 \times 10^{-9}}{6.63 \times 10^{-34} \times 3 \times 10^8} \times 10^{-4}$$

$$\Rightarrow \quad n = 3.31 \times 10^4$$

Hence, the correct answer is (C).

45. The net force on the plate is due to incidence of photons + due to emission of electrons. The number of photons incidents per second on the plate = number of electrons emitted per second

$$\Rightarrow \quad n = \frac{IS}{h\nu}$$

The momentum of photon is $\frac{h}{\lambda}$ and that of electron is $\sqrt{2m(h\nu - \phi)}$ where m is the mass of the electron. Hence the net force exerted on the metal plate is

$$F = \frac{IS}{h\nu}\left(\frac{h}{\lambda} + \sqrt{2m(h\nu - \phi)}\right)$$

Hence, the correct answer is (B).

51. $P = \frac{nhc}{\lambda t}$

$$\Rightarrow \quad 1.7 \times 10^{-18} = \frac{n \times 6.6 \times 10^{-34} \times 3 \times 10^8}{6000 \times 10^{-10} \times 1}$$

$$\Rightarrow \quad n = 5.15 \approx 5$$

Hence, the correct answer is (B).

52. $\frac{(e/m_P)}{(2e/4m_P)} = 2$

Hence, the correct answer is (D).

53. According to Plank's Quantisation Law.

$$E = nh\nu = n\left(\frac{hc}{\lambda}\right)$$

$$\Rightarrow \quad \frac{E}{t} = \left(\frac{n}{t}\right)\left(\frac{hc}{\lambda}\right)$$

$$\Rightarrow \quad 10^{-7} = \left(\frac{n}{t}\right)\frac{(6.626 \times 10^{-34})(3 \times 10^8)}{5000 \times 10^{-10}}$$

$$\Rightarrow \quad \frac{n}{t} = 2.5 \times 10^{11}$$

Hence, the correct answer is (B).

55. $\frac{hc}{\lambda} = \phi_0 + 2.5$

$$\frac{hc}{(\lambda/2)} = \phi_0 + E'_K$$

$$E'_K - 2.5 = \frac{hc}{\lambda}$$

$$E'_K = 2 \cdot 5 + 4 \cdot 14$$

$$E'_K = 6.64 \text{ eV}$$

So, stopping potential is $6.64 \text{ V} > 5 \text{ V}$

Hence, the correct answer is (D).

56. $\lambda = \frac{h}{mv}$

If E is kinetic energy of electron, then

$$E = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2E}{m}}$$

$$\Rightarrow \quad \lambda = \frac{h}{\sqrt{2mE}}$$

Hence, the correct answer is (B).

57. Energy of falling photon < Threshold energy. So, no photoelectric effect takes place.
Hence, the correct answer is (C).

58. Since $\lambda = \frac{h}{p}$

For identical p's, λ are identical
Hence, the correct answer is (D).

59. $h\nu = E_K + \phi$

$$\Rightarrow \quad h\nu - \phi = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2(h\nu - \phi)}{m}}$$

$$\Rightarrow \quad v = \sqrt{\frac{2(hc - \lambda\phi)}{m\lambda}}$$

Hence, the correct answer is (C).

61. Stopping potential is 4 V. So, maximum K.E. is 4 eV.
Hence, the correct answer is (B).

62. Photoelectric current ∝ Intensity

$$\Rightarrow \quad i \propto I$$

$$\Rightarrow \quad i = kI$$

Hence, the correct answer is (C).

63. $\frac{1}{2}mv^2 = eV$

$$\Rightarrow \quad v = \sqrt{\frac{2\,eV}{m}}$$

Hence, the correct answer is (B).

64. Since $\Delta E \Delta t \geq h$

$$\Rightarrow \quad \Delta E(10^{-8}) = 6.6 \times 10^{-34}$$

$$\Rightarrow \quad \Delta E = 6.6 \times 10^{-26}\text{ J}$$

Hence, the correct answer is (A).

66. $I = \frac{Nh\nu}{A\Delta t} = \frac{N\left(\frac{hc}{\lambda}\right)}{\frac{1\text{ mm}}{c} \times 1\text{ mm}^2}$

$$\Rightarrow \quad N = \frac{100 \times 10^{-9} \times 2640 \times 10^{-10}}{(3 \times 10^8)^2 \times 6.63 \times 10^{-34}}$$

$$\Rightarrow \quad N \simeq 442\text{ photons mm}^{-3}$$

Hence, the correct answer is (C).

67. $F = \frac{I}{c}(\text{Effective Area})$

$$\Rightarrow \quad F = \frac{I}{c}(\pi R^2) = \frac{\pi R^2 I}{c}$$

Hence, the correct answer is (A).

68. Initial de-Broglie wavelength is

$$\lambda_1 = \frac{h}{mv_0}$$

At time t, we have

$$\vec{v} = v_0\hat{j} + \left(\frac{qE}{m}\right)t\hat{i}$$

$$\Rightarrow \quad |\vec{v}| = \sqrt{v_0^2 + \frac{q^2E^2t^2}{m^2}}$$

$$\Rightarrow \quad \lambda_2 = \frac{h}{m|\vec{v}|} = \frac{h}{\sqrt{m^2v_0^2 + q^2E^2t^2}}$$

Since $\lambda_2 = \frac{\lambda_1}{2}$

$$\Rightarrow \quad \sqrt{m^2v_0^2 + q^2E^2t^2} = 2mv_0$$

$$\Rightarrow \quad 3m^2v_0^2 = q^2E^2t^2$$

$$\Rightarrow \quad t = \frac{\sqrt{3}mv_0}{qE}$$

Hence, the correct answer is (C).

69. Speed of first electron may increase or decrease, depending on the direction of electric field. However in the case of electron entering the magnetic field speed remains constant. Since, from de-Broglie relation

$$\lambda = \frac{h}{mv}$$

$$\Rightarrow \quad \lambda_1 > \lambda_2 \text{ OR } \lambda_2 > \lambda_1$$

are both the possibilities.
Hence, the correct answer is (D).

70. $\frac{h}{\lambda} = m_e v$

$$\Rightarrow \quad \lambda = 3.64\text{ nm}$$

Hence, the correct answer is (C).

71. $K_1 = \frac{hc}{\lambda_1} - W$ ...(1)

$K_2 = \frac{hc}{\lambda_2} - W$ ...(2)

Since, $\lambda_1 = 2\lambda_2$, so (1) becomes

$$K_1 = \frac{hc}{2\lambda_2} - W = \frac{1}{2}\left(\frac{hc}{\lambda_2}\right) - W$$

$$\Rightarrow \quad K_1 = \frac{1}{2}(K_2 + W) - W$$

$$\Rightarrow \quad K_1 = \frac{K_2}{2} - \frac{W}{2}$$

$$\Rightarrow \quad K_1 < \frac{K_2}{2}$$

Hence, the correct answer is (C).

72. Total energy radiated per minute from sun is

$$E_{\text{radiated}} = \sigma\left(4\pi R_{se}^2\right)$$

Energy radiated annually is given by

$$E_{\text{total}} = 24 \times 60 \times 365 \times E_{\text{radiated}}$$

$$\Rightarrow \quad \text{Annual loss of mass} = \Delta m = \frac{E_{\text{total}}}{c^2} = 1.38 \times 10^{17} \text{ kg}$$

Hence, the correct answer is (C).

73. Since $v = \alpha c$ $\{\text{where } \alpha \ll 1\}$

the electron is moving at non-relativistic speed.

By Law of Conservation of Momentum

$$\frac{h}{\lambda} + \frac{h}{\lambda'} = mv$$

$$\Rightarrow \quad \frac{hc}{\lambda} + \frac{hc}{\lambda'} = mcv = \frac{mv^2}{\alpha} \qquad \ldots(1)$$

By Law of Conservation of Energy

$$\frac{1}{2}mv^2 = \frac{hc}{\lambda} - \frac{hc}{\lambda'} \qquad \ldots(2)$$

Adding (1) & (2), we get

$$2\frac{hc}{\lambda} = \frac{1}{2}mv^2 + \frac{1}{\alpha}\left(mv^2\right)$$

$$\Rightarrow \quad \frac{hc}{\lambda} = E_\gamma = \frac{mv^2}{4} + \frac{mv^2}{2\alpha}$$

Since, $\alpha \ll 1$

$$\Rightarrow \quad \frac{1}{\alpha} \gg 1$$

$$\Rightarrow \quad \frac{mv^2}{4} \ll \frac{mv^2}{2\alpha}$$

$$\Rightarrow \quad E_\gamma \simeq \frac{1}{\alpha}\left(\frac{mv^2}{2}\right)$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \alpha E_\gamma$$

Hence, the correct answer is (A).

74. Photons are exchange particles for electromagnetic interactions.

Gravitons are exchange particles for gravitational interactions.

Mesons are exchange particles for nuclear interactions.

Whereas protons are not any sort of exchange particles.

Hence, the correct answer is (C).

75. The maximum K.E. of ejected photoelectron is

$$(KE)_{\max} = h\nu - \phi_0$$

If the frequency of photon is doubled, maximum kinetic energy of photon electron becomes

$$(\text{KE})'_{\max} = 2h\nu - \phi_0$$

$$\Rightarrow \quad \frac{(\text{KE})'_{\max}}{(\text{KE})_{\max}} = \frac{2\left(h\nu - \frac{\phi_0}{2}\right)}{h\nu - \phi_0} > 2$$

$$\text{Photo current} \propto \frac{\text{intensity of beam}}{h\nu}$$

If intensity and frequency both are doubled, the photo-current remains same.

Hence, the correct answer is (C).

76. Relativistic relative velocity of approach is given by

$$\vec{v}_r = \frac{\vec{v}_1 - \vec{v}_2}{\left(1 - \frac{\vec{v}_1 \cdot \vec{v}_2}{c^2}\right)}$$

$$\Rightarrow \quad v_r = \frac{c - (-c)}{1 - \frac{(c)(-c)}{c^2}}$$

$$\Rightarrow \quad v_r = \frac{2c}{1 + \frac{c^2}{c^2}}$$

$$\Rightarrow \quad v_r = c$$

Never expect the relative velocity to grow beyond the speed of light.

Hence, the correct answer is (B).

77. By Law of Conservation of Momentum, we have

$$0 = mv_1 + 2m(-v_2)$$

$$\Rightarrow \quad mv_1 = 2mv_2 \qquad \ldots(1)$$

Now, according to de-Broglie relation, we have

$$\lambda = \frac{h}{p}$$

$$\Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \frac{p_2}{p_1} = 1$$

Hence, the correct answer is (C).

78. The vector is perpendicular to initial velocity vector at $t = \frac{u}{g\sin\theta}$ and at this instant its speed is

$$v = u\cot\theta$$

Since, $\lambda = \frac{h}{mv}$

$$\Rightarrow \quad \lambda = \frac{h}{mu\cot\theta}$$

$$\Rightarrow \quad \lambda = \left(\frac{h}{mu}\right)\tan\theta$$

Hence, the correct answer is (C).

79. $F_{ex} = \frac{P}{c} = \frac{IA}{c}$

where $I = 1.4 \times 10^3 \text{ Wm}^{-2}$

$\Rightarrow \quad A = 4\pi r^2 = 4 \times 3.14 \times (2)^2 \text{ m}^2$

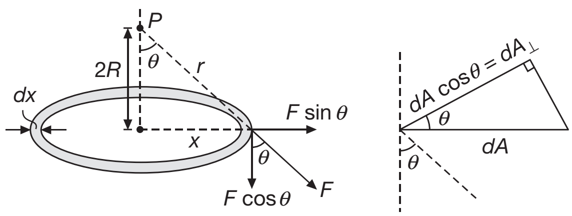
$\Rightarrow \quad F_{ex} = 2.35 \times 10^{-4} \text{ N}$

Hence, the correct answer is (A).

80. Number of photons striking per second are

$$n = \frac{IA}{h\nu}$$

where the area A is the area perpendicular to the direction of intensity or direction of energy flow.
Consider a ring of radius x and width dx on the disc.
Intensity I on the ring due to source is $I = \frac{P}{4\pi r^2}$



Since, $dA_{\perp} = dA\cos\theta$, where $dA = 2\pi x dx$

$\Rightarrow \quad dA_{\perp} = (2\pi x dx)\cos\theta$

A photon will exert force F as shown. Only the $F\cos\theta$ component will remain whereas $F\sin\theta$ will cancel out as we integrate for the ring, where

$$F = \frac{nh}{\lambda}$$

Since only $F\cos\theta$ component of force remains, so

$$dF = \left(\frac{IdA_{\perp}}{h\nu}\right)\left(\frac{h}{\lambda}\cos\theta\right)$$

$$\Rightarrow \quad \int_0^F dF = \int_0^R \left[\frac{P}{4\pi(4R^2 + x^2)}\right] \times \frac{(2\pi dx\cos\theta)}{\left(\frac{hc}{\lambda}\right)} \times \frac{h}{\lambda}\cos\theta$$

$$\left\{\because \text{As } r = \sqrt{4R^2 + x^2}\right\}$$

Solving we get $F = \frac{P}{20c}$

Hence, the correct answer is (D).

81. $\text{KE} = h\nu + \phi \qquad \ldots(1)$

$2\text{KE} = h\nu' + \phi \qquad \ldots(2)$

$\Rightarrow \quad 2(h\nu + \phi) = h\nu' + \phi$

$\Rightarrow \quad \nu' = 2\nu + \frac{\phi}{h}$

$\Rightarrow \quad \nu' > 2\nu$

Hence, the correct answer is (C).

82. $\lambda = \frac{h}{mv}$ where,

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

As $\quad v \to c$

$\quad m \to \infty$

$\Rightarrow \quad \lambda \to 0$

Hence, the correct answer is (C).

83. $E_K = h\nu - \phi_0$

$\Rightarrow \quad (E_K)_1 = 1 - 0.5 = 0.5 \text{ eV}$

Similarly $(E_K)_2 = 2.5 - 0.5 = 2 \text{ eV}$

$\Rightarrow \quad \frac{(E_K)_1}{(E_K)_2} = \frac{1}{4}$

$\Rightarrow \quad \frac{v_1^2}{v_2^2} = \frac{1}{4}$

$\Rightarrow \quad \frac{v_1}{v_2} = \frac{1}{2}$

Hence, the correct answer is (C).

84. Force $= \frac{2IA}{C}$

$\Rightarrow \quad Fx = \left(\frac{1}{2}Kx^2\right) \times 3$

$\Rightarrow \quad \left(\frac{2IA}{C}\right)x = \frac{3}{2}Kx^2$

$\Rightarrow \quad x = \frac{4}{3}\left(\frac{IA}{KC}\right)$

Hence, the correct answer is (D).

85. $\text{K.E.} = E_K = h\nu - h\nu_0 = h\nu - \phi_0$

$\Rightarrow \quad E_K = \frac{hc}{\lambda} - \phi_0$

$\Rightarrow \quad E_K = \frac{(4.14 \times 10^{-15})(3 \times 10^8)}{2 \times 10^{-7}} - 5.01$

$\Rightarrow \quad E_K = 6.21 - 5.01$

$\Rightarrow \quad E_K = 1.2 \text{ eV}$

So, stopping potential required is 1.2 V

Hence, the correct answer is (A).

86. For two particles to have same de-Broglie wavelength

$$\sqrt{mqV} = \text{constant}$$

$$\Rightarrow \quad m_P q_P V = m_\alpha q_\alpha V'$$

$$\Rightarrow \quad V' = \frac{m_P}{m_\alpha}\frac{q_P}{q_\alpha} V$$

$$\Rightarrow \quad V' = \frac{V}{8}$$

Hence, the correct answer is (D).

87. Under the given condition, energy of photon is made half the work function of the metal. Hence photo-emission shall stop altogether.
Hence, the correct answer is (A).

88. Energy of each photon,

$$E = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^{8}}{300\times10^{-9}}$$

$$E = 6.6\times10^{-19}\text{ J}$$

Power of source

$$P = IA = 1.0\times1.0\times10^{-4} = 10^{-4}\text{ watt}$$

So, number of photons per sec is

$$\frac{N}{t} = \frac{P}{E} = \frac{10^{-4}}{6.6\times10^{-19}}$$

Number of electrons emitted is

$$N' = \frac{1}{100}\times\frac{10^{-4}}{6.6\times10^{-19}}$$

$$\Rightarrow \quad N' = 1.51\times10^{12}\text{ per second}$$

Hence, the correct answer is (C).

89. Transverse deflection is

$$y = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{x}{v}\right)^2 = \frac{\frac{1}{2}qEx^2}{mv^2}$$

$$\Rightarrow \quad y = \frac{\frac{1}{2}qEx^2}{2\cdot\left(\frac{1}{2}mv^2\right)} = \frac{1}{4}\frac{qEx^2}{E_K}$$

For same electric field E, kinetic energy E_K and length x, $y \propto q$, q is smaller for proton, so y is smaller for proton. So proton's trajectory will be less curved than α-particle's trajectory.
Hence, the correct answer is (A).

91. $E_K = h\nu - h\nu_0$

$E_K \equiv y$, $\nu \equiv x$, $h\nu_0 = \text{constant}(k)$

$\Rightarrow \quad y = hx - k$ (Equation of a straight line)

Hence, the correct answer is (D).

93. $\lambda = \dfrac{h}{mv}$

$$\Rightarrow \quad \lambda = \frac{6.626\times10^{-34}}{(10^{-31})(10^{5})}$$

$$\Rightarrow \quad \lambda = 6.63\times10^{-8}\text{ m}$$

Hence, the correct answer is (A).

95. $E_K = h\nu - h\nu_0$
is the equation of a straight line with slope h.
Hence, the correct answer is (D).

96. $E_K = h\nu - h\nu_0$
Hence, the correct answer is (A).

97. The time rate of change of momentum equals force.

$$\Rightarrow \quad F = \frac{\Delta p}{\Delta t}$$

Force per unit area is pressure

$$\Rightarrow \quad P = \frac{1}{A}\frac{\Delta p}{\Delta t}$$

But for a photon $E = pc$, where
p = momentum of photon
E = energy of photon

$$\Delta E = c\Delta p$$

$$\Rightarrow \quad P = \frac{1}{Ac}\frac{\Delta E}{\Delta t} = \frac{1}{c}\left(\frac{\Delta E}{A\Delta t}\right) = \frac{1}{c}(I)$$

Because irradiance I is defined as energy per unit area per unit time.
Hence, the correct answer is (C).

98. $\lambda_e = \lambda_p$

$$\Rightarrow \quad \frac{h}{mv} = \frac{hc}{E}$$

$$\Rightarrow \quad \frac{p}{E} = \frac{1}{c}$$

Hence, the correct answer is (B).

99. Since, $K_{\max} = h\nu - h\nu_0$

$$\Rightarrow \quad K_{\max} = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

$$\Rightarrow \quad \frac{1}{\lambda_0} = \frac{1}{\lambda} - \frac{K_{\max}}{hc}$$

This kinetic energy of ejected electron is converted to electrostatic potential energy, $\Delta U = eEd$, as electrons come to rest while moving in the direction of electric field. Therefore, $K_{\max} = eEd$

CHAPTER 1

and $\lambda_0 = \left(\frac{1}{\lambda} - \frac{eEd}{hc}\right)^{-1}$

Hence, the correct answer is (B).

100. $E = \frac{hc}{\lambda}$

Number of photons emitted is

$$\frac{Pt}{\left(\frac{hc}{\lambda}\right)} = n_0$$

$$\Rightarrow \quad n_0 = \frac{P\lambda t}{hc}$$

Since the radiation is spherically symmetric, so total number of photons entering the sensor is n_0 times the ratio of aperture area to the area of a sphere of radius ℓ.

$$\Rightarrow \quad N = n_0 \frac{\pi(2d)^2}{4\pi\ell^2} = \frac{P\lambda t}{hc}\frac{d^2}{\ell^2}$$

Hence, the correct answer is (A).

101. $\frac{hc}{\lambda} = \phi_0 + 3V_0$ ...(1)

$\frac{hc}{2\lambda} = \phi_0 + V_0$ ...(2)

Subtracting

$$2V_0 = \frac{hc}{2\lambda}$$

$$\Rightarrow \quad V_0 = \frac{hc}{4\lambda}$$

$$\Rightarrow \quad \phi_0 = \frac{hc}{4\lambda}$$

$$\Rightarrow \quad \frac{hc}{\lambda_0} = \frac{hc}{4\lambda}$$

$$\Rightarrow \quad \lambda_0 = 4\lambda$$

Hence, the correct answer is (C).

102. $p = \frac{h\nu}{c}$ (for a photon)

$$\Rightarrow \quad 3.3\times10^{-29} = \frac{6.6\times10^{-34}\nu}{3\times10^8}$$

$$\Rightarrow \quad \nu = 1.5\times10^{13} \text{ Hz}$$

Hence, the correct answer is (D).

103. Decreasing the λ of incident photon means energy of incident light is increased, so E_k increases and hence stopping potential increases.

Hence, the correct answer is (D).

104. $E_K = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$

$$E_K = \frac{4.14\times10^{-15}\times3\times10^8}{10^{-7}}\left(\frac{1}{5} - \frac{1}{6}\right)$$

$$\Rightarrow \quad E_K = 0.414 \text{ eV}$$

Wherever necessary we take $h = 6.626\times10^{-34}$ Js

$$\Rightarrow \quad h = 4.14\times10^{-15} \text{ eVs}$$

Hence, the correct answer is (B).

105. $\lambda = \frac{h}{mv}$

$$\Rightarrow \quad \frac{d\lambda}{dt} = \frac{h}{m}\frac{d}{dt}(v^{-1}) = \frac{h}{m}(-1)v^{-2}\frac{dv}{dt}$$

$$\Rightarrow \quad \frac{d\lambda}{dt} = -\frac{h}{mv^2}\cdot a$$

Also $v = u + at$

$$\Rightarrow \quad v = at \qquad \{\because u = 0\}$$

$$\Rightarrow \quad \frac{d\lambda}{dt} = -\frac{h}{ma^2t^2}\cdot a = -\frac{h}{mat^2}$$

Also $ma = eE$

$$\Rightarrow \quad \frac{d\lambda}{dt} = -\frac{h}{eEt^2}$$

Hence, the correct answer is (A).

106. $\lambda = \frac{h}{mv}$

Since electron is the lightest particle of these four. So λ_e is maximum.

Hence, the correct answer is (A).

107. We have

$$\lambda_1 = 4100 \text{ Å}$$

$$\lambda_2 = 4960 \text{ Å}$$

$$\lambda_3 = 6200 \text{ Å}$$

$$\Rightarrow \quad E_1 = \frac{12400}{410} = 3 \text{ eV}$$

$$\Rightarrow \quad E_2 = \frac{12400}{4960} = 2.5 \text{ eV}$$

$$\Rightarrow \quad E_3 = \frac{12400}{6200} = 2 \text{ eV} < \phi_0$$

Hence only λ_1 and λ_2 can cause photoemission. Number of photons of wavelength λ_1 incident on the sodium surface in 1 sec is

$$n_1 = \frac{\frac{P}{3}}{E_1} = \frac{\frac{IA\cos\theta}{3}}{E_1} = \frac{\frac{144}{3}\times 10^{-4}\times\frac{1}{2}}{E_1} = \frac{2.4\times 10^{-3}}{E_1}$$

Similarly, $n_2 = \frac{P/3}{E_2} = \frac{2.4\times 10^{-3}}{E_2}$

So, total number of photoelectrons emitted in 1 sec is

$$n = n_1 + n_2$$

Photoelectric current is

$$I_P = (n_1 + n_2)e = 2.4\times 10^{-3}\left(\frac{1}{E_1} + \frac{1}{E_2}\right)e$$

$$\Rightarrow \quad I_P = 2.4\times 10^{-3}\left(\frac{1}{3} + \frac{1}{2.5}\right)A = \frac{44}{25}\text{ mA} = 1.76\text{ mA}$$

Hence, the correct answer is (A).

108. $I \propto \frac{1}{d^2}$

when d becomes $\frac{d}{4}$, I becomes $16I$

Hence, the correct answer is (D).

110. $h\nu = h\nu_0 + eV_0$

$$\Rightarrow \quad eV_0 = h\nu - h\nu_0$$

$$\Rightarrow \quad V_0 = \frac{h}{e}\nu - \frac{h}{e}\nu_0$$

which is again equation of a straight line with slope $\left(\frac{h}{e}\right)$.

Hence, the correct answer is (B).

112. 5% of 100 W is 5 Js^{-1}

$$\Rightarrow \quad 5 = \frac{n}{t}\left(\frac{hc}{\lambda}\right)$$

$$\Rightarrow \quad \frac{n}{t} = \frac{5\lambda}{hc}$$

$$\Rightarrow \quad \frac{n}{t} = \frac{5(5.6\times 10^{-7})}{6.626\times 10^{-34}\times 3\times 10^8}$$

$$\Rightarrow \quad \frac{n}{t} = 1.4\times 10^{19}$$

Hence, the correct answer is (A).

113. Since $\Delta x\Delta p \sim h$

$$\Rightarrow \quad (2\times 10^{-14})\Delta p = 6.6\times 10^{-34}$$

$$\Rightarrow \quad \Delta p = 3.3\times 10^{-20}\text{ kgms}^{-1}$$

Hence, the correct answer is (B).

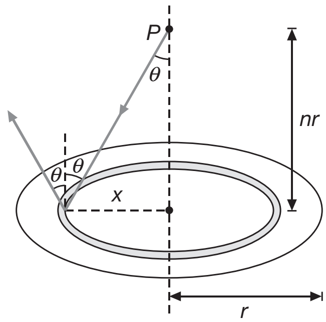
114. Consider a ring of radius x and width dx. Power incident on the ring

$$dP = \frac{P}{4\pi\left[(nr)^2 + x^2\right]}(2\pi x dx)\cos\theta$$

$$\Rightarrow \quad dP = \frac{(Pxdx)nr}{2\left[(nr)^2 + x^2\right]^{\frac{3}{2}}}$$

No. of photons falling (per unit time) on this area is

$$\frac{\lambda dP}{hc}$$



Momentum imparted due to one photon is

$$\Delta P = 2\left(\frac{h}{\lambda}\right)\cos\theta$$

So, force on the ring dF (in the downward direction) is

$$dF = \frac{\lambda dP}{hc}\left(\frac{2h}{\lambda}\cos\theta\right) = \frac{2dP\cos\theta}{c}$$

$$\Rightarrow \quad F = \int \frac{2Pxdxnr}{2\left[(nr)^2 + x^2\right]^{\frac{3}{2}}}\frac{\cos\theta}{c} = \frac{Pn^2r^2}{c}\int_0^r \frac{xdx}{(n^2r^2 + x^2)^2}$$

$$\Rightarrow \quad F = \frac{Pn^2r^2}{2c}\left[\frac{1}{n^2r^2(n^2+1)}\right] = \frac{P}{2c(n^2+1)}$$

Hence, the correct answer is (C).

115. $\nu_0 = 5\times 10^{14}$ Hz

Since, $c = \nu_0\lambda$

$$\Rightarrow \quad \lambda = \frac{3\times 10^8}{5\times 10^{14}}$$

$$\Rightarrow \quad \lambda = 6000\text{ Å}$$

Hence, the correct answer is (B).

116. Kinetic energy of recoil is

$$E_K = h\nu - h\nu'$$

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$\Rightarrow \quad E_K = 12.375 - 9.375$

$\Rightarrow \quad E_K = 3 \text{ eV}$

Hence, the correct answer is (A).

117. $\lambda = \dfrac{h}{\sqrt{2meV}}$

Putting values of h, m, e, we get

$$\Rightarrow \quad \lambda = \sqrt{\frac{150}{V}} \text{ Å}$$

$$\Rightarrow \quad \lambda = \frac{12.27}{\sqrt{V}} \text{ Å}$$

Hence, the correct answer is (A).

118. $\dfrac{\lambda_e}{\lambda_P} = \sqrt{\dfrac{m_P q_P}{m_e q_e}} = \sqrt{\dfrac{m_P}{m_e}}$ $\qquad \{\because q_e = q_P\}$

Hence, the correct answer is (D).

119. $\Delta x = \dfrac{2}{100}$ m

Since

$$\Delta x \Delta p = h$$

$$\Rightarrow \quad \Delta p = \frac{h}{\Delta x} = \frac{6.6 \times 10^{-34}}{(2/100)}$$

$$\Rightarrow \quad \Delta p = 3.3 \times 10^{-32} \text{ kgms}^{-1}$$

Hence, the correct answer is (A).

120. $\lambda = \dfrac{h}{\sqrt{2mE}}$

$\Rightarrow \quad mE = \text{constant}$

$m_e < m_P < m_\alpha$

$\Rightarrow \quad E_1 > E_3 > E_2$

Hence, the correct answer is (A).

121. Least detectable intensity for the eye is

$$I = (5 \times 10^4)\left(\frac{hc}{\lambda}\right)$$

$$\Rightarrow \quad I = (5 \times 10)\left(\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{5 \times 10^{-7}}\right)$$

$$\Rightarrow \quad I = 1.99 \times 10^{-19} \text{ Wm}^{-2}$$

Thus, eye is more sensitive power detector.
Hence, the correct answer is (B).

122. $E = \dfrac{hc}{\lambda} - \phi \qquad \ldots(1)$

$2E = \dfrac{hc}{\lambda'} - \phi \qquad \ldots(2)$

$$\Rightarrow \quad 2E - E = hc\left[\frac{1}{\lambda'} - \frac{1}{\lambda}\right] = E$$

$$\Rightarrow \quad E + \frac{hc}{\lambda} = \frac{hc}{\lambda'}$$

$$\Rightarrow \quad \frac{E\lambda + hc}{\lambda} = \frac{hc}{\lambda'}$$

$$\Rightarrow \quad \lambda' = \left(\frac{hc\lambda}{E\lambda + hc}\right)$$

Hence, the correct answer is (B).

123. In order for scattering to occur, the wavelength of the waves must be of the same order of magnitude or smaller than the size of the object being observed. Hence the largest possible wavelength we can use in the present problem is $\lambda_{max} = 2.5$ Å. Hence minimum energy is

$$E_{min} = \frac{hc}{\lambda_{max}}$$

$$\Rightarrow \quad E_{min} = \frac{12.40 \times 10^3}{2.5 \text{ Å}} \text{eVÅ}$$

$$\Rightarrow \quad E_{min} = 4.96 \times 10^3 \text{ eV}$$

$$\Rightarrow \quad E_{min} = 5 \text{ keV}$$

Hence, the correct answer is (A).

124. $mc^2 = m_0 L$

$$(1)(3 \times 10^8)^2 = m_0 (80 \times 4200)$$

$$\Rightarrow \quad m_0 = 2.67 \times 10^{11} \text{ kg}$$

Hence, the correct answer is (A).

125. Density of star of mass M_0, radius R_0 is

$$\rho = \frac{M_0}{\frac{4}{3}\pi R_0^3}$$

On contracting the new mass becomes

$$M = \frac{4}{3}\pi R^3 \rho = M_0\left(\frac{R}{R_0}\right)^3$$

Loss in mass due to contraction $= -\Delta M$

$\Rightarrow \quad \text{Loss} = M_0 - M$

$\Rightarrow \quad$ Energy radiated $= c^2 \Delta M$

Hence, the correct answer is (D).

126. $F = \dfrac{\Delta p}{\Delta t} = \dfrac{n}{\Delta t}\left[\dfrac{h}{\lambda} - \left(-\dfrac{h}{\lambda}\right)\right] = N\left(\dfrac{2h}{\lambda}\right)$

{∵ Photons rebound with the same initial value of momentum}

So, total number of photons striking the totally reflecting screen is

$$N = \frac{F}{\left(\frac{2h}{\lambda}\right)} = \frac{F\lambda}{2h}$$

$\Rightarrow \quad N = 5 \times 10^{26}$

Hence, the correct answer is (D).

127. $\sqrt{2mE} = \frac{h}{\lambda}$

$$\Rightarrow \quad E = \frac{h^2}{2m\lambda^2} = \frac{(6.6 \times 10^{-34})^2}{2 \times 20 \times 1.66 \times 10^{-27} \times (6.6 \times 10^{-10})^2}$$

$$\Rightarrow \quad E = \frac{10^{-48+27}}{40 \times 1.66} = \frac{1}{4 \times 1.66} \times 10^{-22} \text{ J}$$

$\Rightarrow \quad E = 1.5 \times 10^{-23}$ J

Hence, the correct answer is (A).

128. $\frac{2nh}{\lambda} = 1$

$\Rightarrow \quad n = \frac{\lambda}{2h}$

Hence, the correct answer is (C).

129. Since, $\frac{hc}{\lambda} = \frac{1}{2}mv^2 + \phi$

$\Rightarrow \quad \phi = \frac{hc}{\lambda} - \frac{1}{2}mv^2$

$\Rightarrow \quad \phi = \frac{1240}{400} - 1.68 = 1.41$ eV

Hence, the correct answer is (B).

130. Both are independent of each other.

Hence, the correct answer is (A).

131. For a photon

$E = pc = h\nu$

$\Rightarrow \quad p = \frac{h\nu}{c}$

Hence, the correct answer is (B).

133. Potential of the sphere at any time is

$$V(t) = \frac{Q_0 + Qt}{4\pi\varepsilon_0 R} = V + \frac{\eta\lambda Pet}{4\pi\varepsilon_0 Rhc}$$

Here we have used, $Qt = \frac{P\lambda}{hc}\eta et$

Hence, the correct answer is (B).

135. $S_e = \frac{e}{m_e}$

$S_p = \frac{e}{m_p}$

$S_\alpha = \frac{2e}{4m_p} = \frac{e}{2m_p} = \frac{1}{2}S_p$

S_e is maximum, then comes S_p and then S_α

Hence, the correct answer is (A).

136. $I \propto \frac{1}{d^2}$

On doubling the distance the intensity becomes one fourth i.e. only one fourth of photons now strike the target in comparison to the previous number. Since photoelectric effect is a one photon-one electron phenomena, so only one-fourth photoelectrons are emitted out of the target hence reducing the current to one fourth the previous value.

Hence, the correct answer is (D).

137. The radiation pressure depend on the intensity of light used and not on its wavelength and frequency. Also, the radiation pressure depends on the nature of the surface on which light is falling. Hence (B).

Hence, the correct answer is (B).

138. Let threshold frequency be ν_0.

$\nu = 1.5\,\nu_0$

When the new frequency is halved

$\nu' = 0.75\nu_0 < \nu_0$

So, no photoelectric effect takes place.

Hence, the correct answer is (D).

139. $\frac{\lambda_P}{\lambda_\alpha} = \frac{m_\alpha v_\alpha}{m_P v_P}$

$\Rightarrow \quad \frac{1}{2} = 4\frac{v_\alpha}{v_P}$

$\Rightarrow \quad \frac{v_P}{v_\alpha} = 8$

Hence, the correct answer is (D).

140. $y = \frac{1}{2}at^2 = \frac{1}{2}\frac{eE}{m}\frac{l^2}{v^2}$

For same (nearly) circular path y is same

$\Rightarrow \quad \frac{E}{v^2} = \text{constant}$

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Here $E = x$

$$\Rightarrow \frac{x}{v^2} = \frac{x'}{(2v)^2}$$

$$\Rightarrow x' = 4x$$

Hence, the correct answer is (C).

142. Given that $\frac{h}{p} = \frac{hc}{E} = \frac{h}{mv}$

$$\Rightarrow E = mvc$$

$$\Rightarrow \frac{E_e}{E_p} = \frac{\frac{1}{2}mv^2}{mvc} = \frac{v}{2c}$$

Hence, the correct answer is (B).

143. $\lambda_{max} = 7500$ Å

So, $E = \frac{hc}{\lambda}$

$$\Rightarrow E = \frac{(4.14 \times 10^{-15} \text{ eVs})(3 \times 10^8 \text{ ms}^{-1})}{7500 \times 10^{-10} \text{ m}}$$

$$\Rightarrow E = 1.6 \text{ eV}$$

Hence, the correct answer is (B).

145. $eV_s = KE_{max} = h\nu - W$

$$V_s = \frac{hc}{e\lambda} - \frac{W}{e} \qquad \{\because W = \text{work function}\}$$

Hence, the correct answer is (B).

146. Change in momentum due to photon $= \frac{h}{\lambda}$

$F =$ rate of change of momentum

$$\Rightarrow F = n\frac{h}{\lambda} = ma$$

$$\Rightarrow a = \frac{nh}{\lambda m}$$

Hence, the correct answer is (B).

147. $E = nh\nu$

$$\Rightarrow \frac{E}{t} = \left(\frac{n}{t}\right)h\nu$$

$$\Rightarrow 10000 = \frac{n}{t}(6.6 \times 10^{-34})(880 \times 1000)$$

$$\Rightarrow \frac{n}{t} = 1.71 \times 10^{31}$$

Hence, the correct answer is (A).

148. Maximum microwave frequency = 3×10^{11} Hz

Since, $p = \frac{E}{c} = \frac{h\nu}{c}$

$$\Rightarrow p_{max} = \frac{(6.626 \times 10^{-34})(3 \times 10^{11})}{(3 \times 10^8)}$$

$$\Rightarrow p_{max} = 6.626 \times 10^{-31} \text{ kgms}^{-1}$$

Hence, the correct answer is (C).

149. $\lambda = \frac{h}{mv} = \frac{6.62 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^7}$

$$\Rightarrow \lambda = 0.24 \times 10^{-10} \text{ m}$$

Hence, the correct answer is (C).

150. $(m - m_0)c^2 = 2m_0c^2$

$$\Rightarrow m = 3m_0$$

Hence, the correct answer is (C).

152. Stopping potential = 1 V

$$\Rightarrow E_K = 1 \text{ eV}$$

$$\Rightarrow \frac{1}{2}mv^2 = 1.6 \times 10^{-19} \text{ J}$$

Hence, the correct answer is (C).

153. The energy of incident photons is given by

$$h\nu = eV_s + \phi_0 = 2 + 5 = 7 \text{ eV}$$

(V_s is stopping potential and ϕ_0 is work function)

$$\Rightarrow \text{Saturation current} = 10^{-5} \text{ A} = \frac{\eta P}{h\nu}e = \frac{10^{-5}P}{7 \times e}e$$

$$\Rightarrow P = 7 \text{ W}$$

Hence, the correct answer is (C).

154. $E_1 = \frac{1240}{550} = 2.25$ eV

$$E_2 = \frac{12400}{450} = 2.75 \text{ eV}$$

$$E_3 = \frac{1240}{350} = 3.54 \text{ eV}$$

E_1 cannot emit photoelectrons from q and r plates.
E_2 cannot emit photoelectrons from r.
Further, work function of p is least and it can emit photoelectrons from all three wavelengths. Hence magnitude of its stopping potential and saturation current both will be maximum.
Hence, the correct answer is (A).

155. $E = \frac{1}{2}k_B T$

at $T = 300$ K

$$\Rightarrow \quad E = \frac{1}{2}(1.38 \times 10^{-23})(300)$$

$$\Rightarrow \quad E = \frac{1}{2}\frac{(1.38 \times 10^{-23})(300)\text{eV}}{1.6 \times 10^{-19}}$$

$$\Rightarrow \quad E \simeq 0.01 \text{ eV}$$

Hence, the correct answer is (D).

156. $v = \sqrt{\frac{2qV}{m}}$

Hence, the correct answer is (D).

Multiple Correct Choice Type Questions

1. $\frac{hc}{\lambda} = \phi_0 + K_{\max}$

$$\Rightarrow \quad \frac{hc}{4000} = (1.9 + 1) \text{ eV} = 2.9 \text{ eV}$$

$$\Rightarrow \quad hc = (4000)(2.9) = 11600 \text{ eVÅ}$$

Now, for $\lambda = 500$ nm $= 5000$ Å, we have

$$\frac{11600}{5000} = 1.9 + K_{\max}$$

$$\Rightarrow \quad 2.32 = 1.9 + K_{\max}$$

$$\Rightarrow \quad K_{\max} = 0.42 \text{ eV}$$

The longest wavelength which will eject photoelectrons is given by

$$\lambda_{\max} = \frac{hc}{\phi_0} = \frac{11600}{1.9} = 6105 \text{ Å}$$

$$\Rightarrow \quad \lambda_{\max} \approx 6100 \text{ Å}$$

Hence, (A) and (C) are correct.

2. Since $P = \frac{I}{c} = 10^4 \text{ Nm}^{-2}$

$$\Rightarrow \quad P = \frac{F}{A} = \frac{1}{A}\frac{\Delta p}{\Delta t}$$

$$\Rightarrow \quad \Delta p = P \; A\Delta t = 10^{-5} \text{ kgms}^{-1}$$

Hence, (B) and (D) are correct.

3. The total quantity of charge carried by one pulse of current is

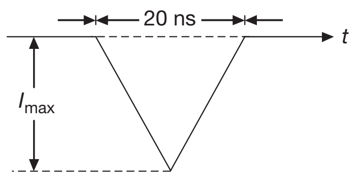
$$Q = \int I dt$$

which is the area of the triangle in figure. Thus

$$Q = \frac{1}{2}(20 \times 10^{-9})(80 \times 10^{-6}) = 8 \times 10^{-13} \text{ C}$$

and the number of electrons carried by one pulse is

$$n = \frac{Q}{e}$$



$$\Rightarrow \quad n = \frac{8 \times 10^{-13}}{1.6 \times 10^{-19}} = 5 \times 10^6$$

Then the number of photoelectrons emitted per light pulse is

$$n' = \frac{n}{10^6} = 5$$

and hence the number of photons in one light pulse is

$$N = \frac{n'}{0.1} = 50$$

Hence, (A), (B) and (C) are correct.

9. According to Heisenberg's Uncertainty Principle the product of

(a) uncertainty in position and uncertainty in momentum cannot be greater than $\frac{h}{4\pi}$.

(b) uncertainty in energy and uncertainty in time cannot be greater than $\frac{h}{4\pi}$.

(c) uncertainty in angular position and uncertainty in angular momentum cannot be greater than $\frac{h}{4\pi}$.

(d) uncertainty in generalised coordinate and generalised momentum cannot be greater than $\frac{h}{4\pi}$.

Hence, (A), (B), (C) and (D) are correct.

11. Wavelength of UV radiation is less than 5200 Å whereas wavelength for IR radiation is greater than 5200 Å. Hence photoelectric effect will be shown by UV radiation irrespective of its intensity.
Hence, (A) and (B) are correct.

14. For a photon

$$pc = h\nu$$

$\Rightarrow \quad p = \dfrac{hv}{c}$

$\Rightarrow \quad p = 8.8 \times 10^{-28}\ \text{kgms}^{-2}$

$\Rightarrow \quad p = 1.65 \times 10^{-6}\ \text{MeV/c}$

Hence, (B) and (C) are correct.

16. (A) $f = \dfrac{c}{\lambda} = \dfrac{3\times10^{8}}{600\times10^{-9}} = 5\times10^{-14}$

(B) $N = \dfrac{p}{hf}$

(C) $KE_{max} = \dfrac{1240}{600} - 1.07 = 1$

(D) KE_{max} depends upon frequency of incident photons and not distance of source.

Hence, (A), (B) and (C) are correct.

17. Since, saturation current i is proportional to the number of photoelectrons ejected per second (n) from the metal surface. Further, we know that (n) is proportional to $\dfrac{I}{hv}$. So, when frequency and intensity (I) both are doubled, then saturation photocurrent remains almost the same.
Also, we know that

$$\frac{1}{2}mv^2 = eV_S = h(v - v_0)$$

When the frequency is doubled, then

$$h(2v - v_0) > h(v - v_0)$$

Hence, $\dfrac{1}{2}mv^2$ and eV_S become more than double.

Hence, (A) and (D) are correct.

19. $\lambda_{red} > \lambda_{violet}$

VIBGYOR pattern shows that VIBG all have λ less than that of yellow colour and hence can initiate photoelectric effect irrespective of intensity.
Hence, (B) and (D) are correct.

20. $K_{max} = 4\times10^{-19}\ \text{J} = \dfrac{4\times10^{-19}}{1.6\times10^{-19}} = 2.5\ \text{eV}$

$\Rightarrow$ Stopping potential $= 2.5$ eV

Since, $K_{max} = hc\left(\dfrac{1}{\lambda} - \dfrac{1}{\lambda_0}\right)$

$\Rightarrow \quad 2.5 = \dfrac{1240}{300} - \dfrac{1240}{\lambda_0}$

$\Rightarrow \quad \lambda_0 = 759\ \text{nm}$

Hence, (A) and (B) are correct.

22. $K_{max} = 1.5$ eV

$$\lambda = \frac{h}{\sqrt{2mK}}$$

$\Rightarrow \quad \lambda = 1\ \text{nm}$

E of incident photon $= \dfrac{1241}{496} \simeq 2.5$ eV

$\Rightarrow \quad \phi = 2.5 - 1.5 = 1\ \text{eV}$

Since, $qvB = \dfrac{mv^2}{r}$

$\Rightarrow \quad B = \dfrac{mv}{qr} = \sqrt{\dfrac{2mK_{max}}{qr}}$

$\Rightarrow \quad B = \dfrac{\sqrt{2\times9.1\times10^{-31}\times e\times1.5}}{e\times1}$

$\Rightarrow \quad B = \sqrt{\dfrac{3\times9.1\times10^{-31}}{1.6\times10^{-19}}}$

$\Rightarrow \quad B = 4.13\times10^{-6}\ \text{T} \cong 4\ \mu\text{T}$

X-rays are of order of 0.1 nm $\simeq 12.4$ KeV

Hence, (A) and (C) are correct.

24. Energy of photon incident

$$hv = \frac{12400}{4000}\ \text{eV}$$

$\Rightarrow \quad hv = 3.1\ \text{eV}$

Since, $hv <$ work function of all metals
Hence no electron will come out

If $\lambda = 200$ nm

then $hv = \dfrac{hc}{\lambda} = \dfrac{12400}{2000} = 6.2$ eV

Since, 6.2 eV > work function of all metals, hence photoelectron will be emitted.
Hence, (B) and (D) are correct.

25. More current means more number of photoelectrons, so better is the photosensitive material. Also, more stopping potential means less energy is used in work function.
Hence, (A) and (D) are correct.

26. Work function is the intercept on K-axis i.e. 2 eV
Hence, the correct answer is (C).

27. $2 = 4.14 \times 10^{-15} \nu$

$\Rightarrow \quad \nu = 4.8 \times 10^{14}$ Hz

Hence, the correct answer is (A).

29. Photoelectric effect and Compton effect are explained on particle nature of light i.e. light is considered to be made up of a stream of photons.
Hence, (A) and (B) are correct.

Reasoning Based Questions

6. $\lambda = \frac{h}{p}$, Same for both

Hence, the correct answer is (D).

8. Kinetic energy; $E = \frac{1}{2}mv^2 = qV$

$\Rightarrow \quad v \propto \sqrt{\frac{q}{m}}$ because V is constant

$\Rightarrow \quad v_p : v_d : v_\alpha = \sqrt{\frac{q_p}{m_p}} : \sqrt{\frac{q_d}{m_d}} : \sqrt{\frac{q_\alpha}{m_\alpha}}$

$\Rightarrow \quad v_p : v_d : v_\alpha = \sqrt{\frac{e}{m}} : \sqrt{\frac{e}{2m}} : \sqrt{\frac{2e}{4m}}$

$\Rightarrow \quad v_p : v_d : v_\alpha = 1 : \frac{1}{\sqrt{2}} : \frac{1}{\sqrt{2}} = \sqrt{2} : 1 : 1$

Hence, the correct answer is (D).

13. Energy of photoelectron emitted is different because after absorbing the photon electrons within metals collide with other atom before being ejected out of metal.
Hence, the correct answer is (A).

14. $\frac{\lambda_e}{\lambda_{ph}} = \sqrt{\frac{E}{2m_e c^2}} = \sqrt{\frac{10^6 eV}{1MeV}} = 1$

Hence, the correct answer is (A).

15. $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

$\Rightarrow \quad 2m_0 = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

$\Rightarrow \quad v = \frac{\sqrt{3}}{2}c$

Hence, the correct answer is (A).

16. $K_{\max} = h\nu = \phi$

KE of emitted photoelectrons varies from zero to $K_{\max}$
Hence, the correct answer is (D).

17. $\lambda = \frac{12.27}{\sqrt{V}}$ Å

Hence, the correct answer is (D).

Linked Comprehension Type Questions

1. Since, $K_{\max} = \frac{hc}{\lambda} - \frac{hc}{\lambda_{th}}$

$$\Rightarrow \quad \frac{1}{2}mv^2_{\max_1} = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_{th}} \quad \ldots(1)$$

$$\frac{1}{2}mv^2_{\max_2} = \frac{hc}{\lambda_2} - \frac{hc}{\lambda_{th}} \quad \ldots(2)$$

Dividing (2) by (1), we get

$$\left(\frac{v_{\max_2}}{v_{\max_1}}\right)^2 = \frac{\frac{hc}{\lambda_2} - \frac{hc}{\lambda_{th}}}{\frac{hc}{\lambda_1} - \frac{hc}{\lambda_{th}}}$$

$$\Rightarrow \quad (2)^2 = \frac{\frac{1}{1500} - \frac{1}{\lambda_{th}}}{\frac{1}{3000} - \frac{1}{\lambda_{th}}}$$

$$\Rightarrow \quad \frac{4}{3000} - \frac{4}{\lambda_{th}} = \frac{1}{1500} - \frac{1}{\lambda_{th}}$$

$$\Rightarrow \quad \frac{4}{3000} - \frac{1 \times 2}{1500 \times 2} = \frac{3}{\lambda_{th}}$$

$$\Rightarrow \quad \frac{2}{3000} = \frac{3}{\lambda_{th}}$$

$$\Rightarrow \quad \lambda_{\text{th}} = 4500 \text{ Å}$$

Hence, the correct answer is (C).

2. $I_s \propto P\lambda$

$$\Rightarrow \quad \frac{I_{s_2}}{I_{s_1}} = \frac{P_2\lambda_2}{P_1\lambda_1}$$

$$\Rightarrow \quad \frac{I_{s_2}}{20} = \frac{5 \times 1500}{1 \times 3000}$$

$$\Rightarrow \quad I_{s_2} = 50\ \mu\text{A}$$

Hence, the correct answer is (A).

3. For CASE-1, we have

$$n = \frac{P\lambda}{hc}$$

Since, $n_e = \frac{I_s}{e}$

Efficiency of photoelectron generation per incident photon is

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$$\eta = \left(\frac{n_e}{n} \times 100\%\right) = \frac{I_s \times hc}{e \times P\lambda} \times 100$$

$$\Rightarrow \quad \eta = \frac{20 \times 10^{-6} \times 6.6 \times 10^{-34} \times 3 \times 10^{8} \times 100}{1.6 \times 10^{-19} \times 1 \times 10^{-3} \times 3000 \times 10^{-10}}$$

$$\Rightarrow \quad \eta = \left(\frac{20}{160}\right)\left(\frac{66}{10}\right) = \frac{66}{8} = 8.25\%$$

Hence, the correct answer is (B).

4. $\dfrac{mv^2}{r} = qvB$

$$\Rightarrow \quad \sqrt{2mE_K} = qBr$$

$$\Rightarrow \quad E_K = \frac{(qBr)^2}{2m}$$

$$\Rightarrow \quad E_K = 0.86 \text{ eV}$$

Hence, the correct answer is (C).

5. $E_3 - E_2 = 13.6\left(\dfrac{1}{2^2} - \dfrac{1}{3^2}\right)$

$$\Rightarrow \quad \Delta E = 1.89 \text{ eV}$$

Since, $\phi = E - K_{max}$

$$\Rightarrow \quad \phi = 1.89 - 0.86 = 1.03 \text{ eV}$$

Hence, the correct answer is (B).

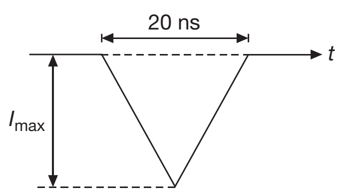
6. $\lambda = \dfrac{hc}{\Delta E}$

$$\Rightarrow \quad \lambda \simeq 6565 \text{ Å}$$

Hence, the correct answer is (A).

7. The total quantity of charge carried by one pulse of current is

$$Q = \int I dt$$



Since Q is the area under the current time graph, so the area of the triangle in Figure equals the charge flowing Q, given by

$$Q = \frac{1}{2} \times 20 \times 10^{-9} \times 80 \times 10^{-6} = 8 \times 10^{-13} \text{ C}$$

and the number of electrons carried by one pulse is

$$N = \frac{Q}{e} = \frac{8 \times 10^{-13}}{1.6 \times 10^{-19}} = 5 \times 10^{6}$$

The number of photoelectrons emitted per light pulse is

$$n' = \frac{N}{10^6} = 5$$

and hence the number of photons N' in one light pulse is

$$N' = \frac{N}{0.1} = 50$$

Hence, the correct answer is (C).

8. The probability that all the photons of a light pulse will go undetected is

$$\left(\frac{90}{100}\right)^N = (0.9)^{50} = 5.15 \times 10^{-3} = 0.52\%$$

N is the number of photons in one light pulse

Hence, the correct answer is (C).

9. In each pulse there is a finite probability for a certain number of photons not being detected. Thus, number of photons detected will vary from pulse to pulse. So, the maximum current I_{max} will fluctuate about a mean value. The greater the number of photons in a pulse, the smaller will be the fluctuation.

Hence, the correct answer is (A).

10. $\lambda \propto \dfrac{1}{\sqrt{mq}}$

Hence, the correct answer is (B).

11. $\lambda_e = \dfrac{12.27}{\sqrt{V}}$ Å

Hence, the correct answer is (A).

12. Since, $\lambda = \dfrac{h}{p}$

Further, $p_{electron} = p_{alpha}$

$\Rightarrow$ Wavelength will be same

Hence, the correct answer is (C).

13. Energy of photon, $E = \dfrac{hc}{\lambda}$

$$\Rightarrow \quad \frac{hc}{\lambda} = \frac{12400}{3100} = 4 \text{ eV}$$

For photo-electric effect, $E > \phi_0$

Since, 4 eV is greater than 2.5 eV and 3.5 eV, so electrons will be emitted from A and B.

Hence, the correct answer is (B).

14. The wavelength of light, which can emit electrons from D will also be able to emit electrons from A, B and C. So, for D

$$\lambda_0 = \frac{hc}{\phi_0} = \frac{12400}{5.5} = 2255 \text{ Å}$$

Hence, the correct answer is (D).

15. $$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-3}}{(66.3\times10^{-3})(5)}$$

$\Rightarrow \quad \lambda = 0.02$ m

Hence, the correct answer is (B).

16. Fringe width β is given by

$$\beta = \frac{\lambda D}{d} = \frac{12\times0.02}{0.6}$$

$\Rightarrow \quad \beta = 0.4$ m

Hence, the correct answer is (D).

17. de-Broglie wavelength of electron is

$$\lambda = \frac{6.63\times10^{-3}}{9.1\times10^{-31}\times10^{7}}$$

$\Rightarrow \quad \lambda \approx 10^{21}$ m

Fringe width $\beta = \frac{\lambda D}{d} \approx \left(\frac{12}{0.6}\right)\times10^{21} \approx 10^{22}$ m

Fringe width is so large that it is not possible to observe it.

Hence, the correct answer is (D).

18. The intensity of electromagnetic radiation is

$$I = \frac{\text{Energy}}{A\times t} = \frac{10\times10^{3}}{10^{-3}\times10^{-4}\times10^{-9}}$$

$\Rightarrow \quad I = 10^{20}$ Wm^{-2}

Also, we know that for an electromagnetic wave, the intensity I is given by

$$I = \left(\frac{1}{2}\varepsilon_0 E^2\right)c = 10^{20}$$

where E is the rms value of the electric field.

$$\Rightarrow \quad E = \sqrt{\frac{2I}{\varepsilon_0 c}} = \sqrt{\frac{2\times10^{20}}{8.85\times10^{-12}\times3\times10^{8}}}$$

$\Rightarrow \quad E = 1.94\times10^{11}$ Vm^{-1}

Peak value of electric field is

$$E_0 = \sqrt{2}E = 2.75\times10^{11} \text{ Vm}^{-1}$$

Hence, the correct answer is (A).

19. Radiation pressure is

$$P_R = \frac{I}{c} = \frac{10^{20}}{3\times10^{8}} = 3.33\times10^{11} \text{ Nm}^{-2}$$

Hence, the correct answer is (A).

20. As all the absorbed energy is convert into the kinetic energy of thermal motion of hydrogen atoms, so we have

$$KE = \frac{3}{2}NK_BT$$

where N is the number of hydrogen atoms involved and K_B is the Boltzmann's constant.
Considering the hydrogen atoms as an ideal gas, we have,

$$PV = NK_BT$$

So, the pressure is given by

$$P = \frac{NK_BT}{V} = \frac{2}{3}\left(\frac{KE}{V}\right)$$

$$P = \frac{2}{3}(KE)\left(\frac{4}{3}\pi R^3\right)^{-1}$$

Since the radius R of the sphere is related to the area A of the focal spot, so we have

$$\pi R^2 = A$$

$$\Rightarrow \quad P = \frac{KE}{2\pi}\left(\frac{\pi}{A}\right)^{\frac{3}{2}}$$

Since $KE = 10^4$ J, $A = 10^{-7}$ m^2, we get

$P = 2.8\times10^{14}$ pascal

$\Rightarrow \quad P = 280\times10^{12}$ pascal

$\Rightarrow \quad P = 280$ Tera Pascal

Hence, the correct answer is (A).

21. $$\lambda = \frac{h}{\sqrt{2mE}} = \sqrt{\frac{150}{1.5}} = 10 \text{ Å} = 100 \text{ nm}$$

Hence, the correct answer is (B).

22. $$qvB = \frac{mv^2}{r}$$

$$\Rightarrow \quad B = \frac{mv}{qr} = 4.1\times10^{-6} \text{ T}$$

Hence, the correct answer is (A).

23. Photon energy $= \frac{hc}{\lambda} = \frac{12375}{4960} = 2.5$ eV

Since, $V_s = 1.5$ V

So, work function is given by

$$\phi = 2.5 - 1.5 = 1 \text{ eV}$$

$$\Rightarrow \quad \lambda_{\min} = \frac{12375}{1} = 12375 \text{ Å} = 1237 \text{ nm}$$

$$\Rightarrow \quad \lambda_{\min} \cong 1250 \text{ nm}$$

Hence, the correct answer is (A).

24. $E = \dfrac{hc}{\lambda}$

$$\Rightarrow \quad \lambda = \frac{hc}{E} = \frac{12400}{2.5} = 4960 \text{ Å}$$

$$\Rightarrow \quad \lambda \approx 5000 \text{ Å}$$

Hence, the correct answer is (A).

25. Kinetic energy of the emitted photoelectron is given by applying Einstein's photoelectric equation, according to which

$$E = \phi_0 + K_{\max}$$

$$\Rightarrow \quad K_{\max} = (2.5 - 2) \text{ eV} = 0.5 \text{ eV}$$

$$\Rightarrow \quad K_{\max} = 0.5 \text{ eV} = (0.5)\left(1.6 \times 10^{-19}\right) \text{ joule}$$

$$\Rightarrow \quad K_{\max} = 8 \times 10^{-20} \text{ J}$$

de-Broglie wavelength of photoelectron is given by

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK_{\max}}} = \frac{6.626 \times 10^{-34}}{\sqrt{2\left(9 \times 10^{-31}\right)\left(8 \times 10^{-20}\right)}}$$

$$\Rightarrow \quad \lambda \approx 1.75 \text{ nm}$$

Hence, the correct answer is (B).

26. $p = \dfrac{I}{C} = \dfrac{3 \times 10^4}{3 \times 10^8} = 10^{-4} \text{ Pa}$

Hence, the correct answer is (B).

27. $\Delta p = PA\Delta t = 10^{-4} \times 10^{-4} \times 10 = 10^{-7} \text{ kgms}^{-1}$

Hence, the correct answer is (C).

28. $\dfrac{Nh}{\lambda} = \Delta p$

$$\Rightarrow \quad N = \frac{10^{-7} \times 663 \times 10^{-9}}{6.63 \times 10^{-34}} = 10^{20}$$

Hence, the correct answer is (B).

29. Since, $K_{\max} = h\nu - W$

If $K_{\max}$ and W both are in eV, but $h\nu$ is in joule, then we have

$$K_{\max} = \frac{h\nu}{e} - W$$

So, for both the frequency values, we have

$$0.5 = \frac{h\left(8 \times 10^{14}\right)}{e} - W \quad \ldots(1)$$

$$2 = \frac{h\left(12 \times 10^{14}\right)}{e} - W \quad \ldots(2)$$

Subtracting equation (1) from (2), we get

$$1.5 = \frac{h\left(4 \times 10^{14}\right)}{e}$$

$$\Rightarrow \quad h = \frac{1.5 \times 1.6 \times 10^{-19}}{4 \times 10^{14}}$$

$$\Rightarrow \quad h = 6 \times 10^{-34} \text{ Js}$$

Hence, the correct answer is (A).

30. From equation (1), we get

$$0.5 = \frac{\left(6 \times 10^{-34}\right)\left(8 \times 10^{14}\right)}{1.6 \times 10^{-19}} - W$$

$$\Rightarrow \quad 0.5 = 3 - W$$

$$\Rightarrow \quad W = 3 - 0.5 = 2.5 \text{ eV}$$

Hence, the correct answer is (C).

31. $\lambda = \dfrac{h}{\sqrt{2mK}} = \dfrac{12.27}{\sqrt{V}} \text{ Å}$

$$\Rightarrow \quad \lambda = \frac{12.27}{\sqrt{0.5}} \text{ Å} = 12.27\sqrt{2} \text{ Å}$$

$$\Rightarrow \quad \lambda = 17.35 \text{ Å}$$

Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

1. A → (p, r)
B → (p, r)
C → (q)
D → (s)

Consider two equations

$$eV_s = \frac{1}{2}mv_{\max}^2 = h\nu - \phi_0 \quad \ldots(1)$$

Number of photoelectron ejected per second (n) is proportional to

$$n \propto \frac{I}{h\nu} \quad \ldots(2)$$

(A) As frequency is increased keeping intensity constant, stopping potential $|V_s|$ will increase, so kinetic energy $\frac{1}{2}m\left(V_{\max}^2\right)$ also increases and saturation current will decrease.

(B) As frequency is increased and intensity is decreased, $|V_s|$ will increase $\frac{1}{2}m(v_{max}^2)$ will increase and saturation current will decrease.

(C) If work function is increased photo emission may stop.

(D) If intensity is increased and frequency is constant, saturation current will increase.

6. $A \to (p)$
$B \to (r)$
$C \to (s)$
$D \to (q)$

de-Broglie wavelength of electron in X-Ray tube

$$\lambda = \frac{h}{\sqrt{2mKE}} = \frac{h}{\sqrt{2meV}}$$

$$\Rightarrow \quad \lambda = \frac{1.227\times10^{-9}}{\sqrt{V}} = \frac{12.27}{\sqrt{V}} \text{ Å}$$

(A) Accelerating potential

$$V \simeq 10^4 \text{ eV in X-ray tube}$$

$$\lambda \simeq \frac{1.227}{10^2}\times10^{-9} = 1.227\times10^{-11} \text{ m}$$

$$\lambda \simeq 0.1 \text{ Å}$$

(B) Wavelength associated with X-rays

$$\lambda = \frac{12.4\times10^{-7}}{10^4} \simeq 12.4\times10^{-11} \text{ m}$$

$$\lambda = 1.2\times10^{-10} \text{ m}$$

$$\lambda \simeq 1 \text{ Å}$$

(C) de-Broglie wavelength of most energetic photoelectron

$$\lambda = \frac{1.227\times10^{-9}}{\sqrt{V}} = 1.227\times10^{-9} = 12.2\times10^{-10} \text{ m}$$

$$\lambda \simeq 10 \text{ Å}$$

(D) $E = \frac{hc}{\lambda}$

$$\Rightarrow \quad \lambda = \frac{hc}{E} = \frac{hc}{eV} = \frac{12.4\times10^{-7}\text{m}}{V}$$

$$V = 2.5 \text{ V}$$

$$E = \frac{12.4}{2.5}\times10^{-7} = 5000 \text{ Å}$$

Integer/Numerical Answer Type Questions

1. Since, $n = \frac{N}{t} = \frac{P}{E} = \frac{P\lambda}{hc}$

$$\Rightarrow \quad n = \frac{(1.7\times10^{-8})(6000\times10^{-10})}{(6.626\times10^{-34})(3\times10^8)}$$

$$\Rightarrow \quad n = 5.1\times10^{10} \text{ photons per sec}$$

$$\Rightarrow \quad n \approx 5\times10^{10}$$

$$\Rightarrow \quad \alpha = 5 \text{ and } \beta = 10$$

$$\Rightarrow \quad \frac{\beta}{\alpha} = 2$$

2. The target area is $S_1 = \pi(10^{-9})^2 = \pi\times10^{-18} \text{ m}^2$.

The area of a 5 metre sphere centred on the light source is, $S_2 = 4\pi(5)^2 = 100\ \pi\text{m}^2$.

Thus, if the light source radiates uniformly in all directions the rate P at which energy falls on the target is given by,

$$P = (10^{-3} \text{ watt})\left(\frac{S_1}{S_2}\right) = (10^{-3})\left(\frac{\pi\times10^{-18}}{100\times\pi}\right)$$

$$\Rightarrow \quad P = 10^{-23} \text{ Js}^{-1}$$

Assuming that all power is absorbed, the required time is,

$$t = \left(\frac{5 \text{ eV}}{10^{-23} \text{ Js}^{-1}}\right)\left(\frac{1.6\times10^{-19} \text{ J}}{1 \text{ eV}}\right) \approx 20 \text{ hr}$$

3. Rate of energy received from the sun

$$Q = 2 \text{ cal cm}^{-2} \text{ min}^{-1} = 2\times4.2$$

$$\Rightarrow \quad Q = 8.4 \text{ J cm}^{-2} \text{ min}^{-1}$$

Energy of a photon received from the sun is

$$E = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^8}{6600\times10^{-10}} = 3\times10^{-19} \text{ J}$$

If n is the number of photons reaching the earth per cm^2 per minute, then their total energy will be $(3\times10^{-19}n)$ joule.

$$\Rightarrow \quad (3\times10^{-19})n = 8.4$$

$$\Rightarrow \quad n = \frac{8.4}{3\times10^{-19}} = 28\times10^{18} \text{ photons per minute per cm}^2$$

$$\Rightarrow \quad x = 28$$

4. Mass of an electron is

$$m = 9.11\times10^{-31}\text{ kg and } T = 27+273 = 300\text{ K}$$

∴ de-Broglie wavelength of electrons is

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

$$\lambda = \frac{6.63\times10^{-34}}{\sqrt{3\times9.11\times10^{-31}\times1.38\times10^{-23}\times300}}\text{ m}$$

$$\Rightarrow \lambda = \frac{6.63\times10^{-8}}{\sqrt{3\times9.11\times1.38\times3}} = \frac{6.63\times10^{-8}}{10.64}$$

$$\Rightarrow \lambda = 6.2\times10^{-9}\text{ m}$$

Mean separation between two electrons in a metal is

$$r = 2\times10^{-10}\text{ m}$$

$$\Rightarrow \frac{\lambda}{r} = \frac{6.2\times10^{-9}}{2\times10^{-10}} = 31$$

5. The maximum kinetic energy of the emitted electron is

$$K_{max} = \frac{1}{2}mv_{max}^2 = \frac{hc}{\lambda} - W_0$$

$$\Rightarrow K_{max} = \frac{6.62\times10^{-34}\times3\times10^8}{180\times10^{-9}} - 2\times1.6\times10^{-19}$$

$$\Rightarrow K_{max} = 7.8\times10^{-19}\text{ J}$$

$$\Rightarrow v_{max} = \sqrt{\frac{K_{max}}{m}} = \sqrt{\frac{2\times7.8\times10^{-19}}{9.1\times10^{-31}}} = 1.3\times10^6\text{ ms}^{-1}$$

The magnetic field provides the centripetal force to the electron, so

$$Bev_{max} = \frac{mv_{max}^2}{r}$$

$$\Rightarrow r = \frac{mv_{max}}{Be} = \frac{9.1\times10^{-31}\times1.3\times10^6}{5\times10^{-5}\times1.6\times10^{-19}} = 0.148\text{ m}$$

$$\Rightarrow r = 148\text{ mm}$$

6. $F = \frac{IA_{effective}}{c} = \frac{I}{c}(\pi R^2)$

$$\Rightarrow F = \frac{(10^{-2})(3.14)\left(\frac{10}{100}\right)^2}{3\times10^8} = 1.046\times10^{-12}\text{ N}$$

$$\Rightarrow F \approx 1\times10^{-12}\text{ N} = 1\text{ pN}$$

7. The Einstein's photoelectric equation for the first case can be written as

$$\frac{hc}{\lambda} = W + K_1 \quad \ldots(1)$$

When illuminated with light of wavelength 2λ,

$$\frac{hc}{2\lambda} = W + K_2 \quad \ldots(2)$$

Subtracting (2) from (1), we get

$$\frac{hc}{\lambda} - \frac{hc}{2\lambda} = K_1 - K_2$$

$$\Rightarrow \lambda = \frac{hc}{2(K_1-K_2)} = \frac{(4\times10^{-15})(3\times10^8)}{2(30-10)}$$

$$\Rightarrow \lambda = 300\times10^{-10}\text{ m} = 300\text{ Å}$$

If λ_{max} is the maximum wavelength of the photons with which photoelectrons can be emitted, then

$$\frac{hc}{\lambda_{max}} = W_0 = \frac{hc}{\lambda} - K_1 = \frac{(4\times10^{-15})(3\times10^8)}{300\times10^{-10}} - 30\text{ eV}$$

$$\Rightarrow E = 40 - 30 = 10\text{ eV}$$

$$\Rightarrow \lambda_{max} = \frac{hc}{10\text{ eV}} = \frac{(4\times10^{-15})(3\times10^8)}{10}$$

$$\Rightarrow \lambda_{max} = 1200\times10^{-10}\text{ m}$$

$$\Rightarrow \lambda_{max} = 1200\text{ Å}$$

8. Since $K_{max} = h\nu - h\nu_0$

$$\Rightarrow \frac{1}{2}\text{m}(8\times10^6)^2 = h(5\nu_0 - \nu_0) \quad \ldots(1)$$

For the second case, we have

$$KE = \frac{1}{2}mv^2 = h(2\nu_0 - \nu_0) \quad \ldots(2)$$

$$\Rightarrow \frac{(8\times10^6)^2}{v^2} = 4$$

$$\Rightarrow v = 4\times10^6\text{ ms}^{-1}$$

9. Applying Einstein's photoelectric equation, we get

$$eV_0 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) \quad \ldots(1)$$

$$\frac{eV_0}{6} = hc\left(\frac{1}{3\lambda} - \frac{1}{\lambda_0}\right) \quad \ldots(2)$$

$$\Rightarrow \frac{hc}{6}\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) = hc\left(\frac{1}{3\lambda} - \frac{1}{\lambda_0}\right)$$

$$\Rightarrow \frac{1}{6\lambda} - \frac{1}{6\lambda_0} = \frac{1}{3\lambda} - \frac{1}{\lambda_0}$$

$$\Rightarrow \frac{1}{\lambda_0} - \frac{1}{6\lambda_0} = \frac{1}{3\lambda} - \frac{1}{6\lambda}$$

$$\Rightarrow \frac{5}{6\lambda_0} = \frac{1}{6\lambda}$$

$$\Rightarrow \lambda_0 = 5\lambda$$

$$\Rightarrow n = 5$$

10. $F = (\text{Area})\frac{I}{c}$ here effective Area $= \pi R^2$

$$\Rightarrow F = (\pi R^2)\frac{I}{c}$$

$$\Rightarrow F = \frac{22}{7} \times (21 \times 10^{-2})^2 \times \frac{1}{110} \times \frac{1}{3 \times 10^8}$$

$$\Rightarrow F = \frac{(22)(21 \times 10^{-2})(21 \times 10^{-2})}{7 \times 110 \times 3 \times 10^8}$$

$$\Rightarrow F = 42 \times 10^{-13} \text{ N}$$

$$\Rightarrow x = 42$$

11. Since, $eV_1 = \frac{hc}{\lambda_1} - \phi$ and

$$eV_2 = \frac{hc}{\lambda_2} - \phi$$

Subtracting, we get

$$e(V_2 - V_1) = hc\left(\frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2}\right)$$

$$\Rightarrow V_2 - V_1 = \frac{hc}{e}\left(\frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2}\right)$$

$$\Rightarrow V_2 - V_1 = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19}} \times \frac{100}{6 \times 10^{-5}}$$

$$\Rightarrow V_2 - V_1 = \frac{66}{32} \times 10^{-34+8+2+19+5}$$

$$\Rightarrow V_2 - V_1 = \frac{33}{16} \approx 2 \text{ V}$$

12. Since, $eV_s = h\nu - W$

$$\Rightarrow 5 \text{ eV} = h\nu - 2 \text{ eV}$$

$$\Rightarrow h\nu = 7 \text{ eV}$$

So, total number of photons incident is

$$n = \frac{P}{h\nu}$$

Since, $\eta = 10^{-3}\%$

$$\Rightarrow n_{\text{emitted}} = n \times \frac{10^{-3}}{100}$$

$$\Rightarrow i = \frac{q}{t} = \left(\frac{n_{\text{emitted}}}{t}\right)e$$

where, from the graph

$$i = 10 \times 10^{-6} \text{ A}$$

$$\Rightarrow 10 \times 10^{-6} = \frac{P}{h\nu} \times \frac{10^{-3}}{100} \times 1.6 \times 10^{-19}$$

$$\Rightarrow 10 \times 10^{-6} = \frac{P \times 10^{-3} \times 1.6 \times 10^{-19}}{7 \times 1.6 \times 10^{-19} \times 100}$$

$$\Rightarrow P = 7 \text{ W}$$

13. According to Einstein's Photo-Electric Equation, we have

$$K_{\max} = eV_s = h\nu - W$$

$$\Rightarrow eV_s = 12 \text{ eV} - 4 \text{ eV}$$

$$\Rightarrow eV_s = 8 \text{ eV}$$

$$\Rightarrow V_s = 8 \text{ V}$$

14. $$F = \left[\frac{\left(\frac{P}{4\pi a^2}\right)(\pi R^2)}{h\nu}\right]\left[(0.7)\frac{2h\nu}{c} + (0.3)\frac{h\nu}{c}\right]$$

$$\Rightarrow F = \frac{P}{4}\left(\frac{R}{a}\right)^2\left(\frac{1.7}{c}\right) = \left(\frac{12}{4}\right)\left(\frac{3}{39}\right)^2\left(\frac{1.7}{3 \times 10^8}\right)$$

$$\Rightarrow F = \left(\frac{3}{169}\right)\left(\frac{1.7}{3 \times 10^8}\right) \approx 1 \times 10^{-10} \text{ N}$$

$$\Rightarrow x = 1 \text{ and } y = 10$$

$$\Rightarrow \frac{y}{x} = 10$$

15. Magnetic force experienced by a charged particle in a magnetic field is given by

Since, $r = \frac{mv}{qB}$

CHAPTER 1

The de Broglie wavelength is given by

$$\lambda = \frac{h}{mv} = \frac{h}{qBr} \qquad \left\{\because r = \frac{mv}{qB}\right\}$$

$$\Rightarrow \frac{\lambda_p}{\lambda_\alpha} = \frac{q_\alpha r_\alpha}{q_p r_p}$$

Given, $\frac{r_\alpha}{r_p} = 1$ and $\frac{q_\alpha}{q_p} = 2$

$$\Rightarrow \frac{\lambda_p}{\lambda_\alpha} = 2$$

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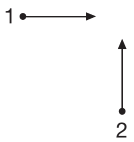
1. 

$$p_1 = \frac{h}{\lambda_1}$$

$$p_2 = \frac{h}{\lambda_2}$$

$$\Rightarrow p_f = \sqrt{p_1^2 + p_2^2}$$

$$\Rightarrow \frac{h}{\lambda} = \sqrt{\frac{h^2}{\lambda_1^2} + \frac{h^2}{\lambda_2^2}}$$

$$\Rightarrow \frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$$

Hence, the correct answer is (D).

2. $\lambda_A = \frac{h}{mV_A}$

Applying Conservation of Linear Momentum, we get

$$mV_A = \left(\frac{m}{2}\right)V - \left(\frac{m}{2}\right)\left(\frac{V}{2}\right) = \frac{mV}{4}$$

$$\Rightarrow \lambda_A = \frac{4h}{mV}$$

$$\Rightarrow V_A = \frac{V}{4}$$

$$\lambda_B = \frac{h}{\left(\frac{m}{2}\right)V} = \frac{2h}{mV} = \frac{\lambda_A}{2}$$

$$\lambda_C = \frac{h}{\left(\frac{m}{2}\right)\left(\frac{V}{2}\right)} = \frac{4h}{mV} = \lambda_A$$

Hence, the correct answer is (C).

3. $\lambda = 5 \times 10^{-7}$ m $= 5000$ Å

$$\Rightarrow E = \frac{12375}{5000} = 2.475 \text{ eV} \approx 2.48 \text{ eV}$$

Since, $K_{max} = E - \phi_0$

$$\Rightarrow K_{max} = 2.48 - 2 = 0.48 \text{ eV}$$

$$\Rightarrow V_s = 0.48 \text{ V}$$

Hence, the correct answer is (A).

4. Since $F_n = \frac{I}{c}(1 + r)$

$$\Rightarrow F_n = \frac{I}{c}(1 + 0.25) = \frac{(1.25)(50)}{3 \times 10^8} \approx 20 \times 10^{-8} \text{ N}$$

Hence, the correct answer is (A).

5. $p_1 = \frac{h}{\lambda_x}$ and $p_2 = \frac{h}{\lambda_y}$

Since particles are moving in opposite direction, so

$$p = p_1 - p_2 = h\left(\frac{1}{\lambda_x} - \frac{1}{\lambda_y}\right)$$

Also, the collision is perfectly inelastic, so

$$p_{\text{final}} = \sum p_{\text{initial}}$$

$$\Rightarrow \frac{h}{\lambda} = h\left(\frac{1}{\lambda_x} - \frac{1}{\lambda_y}\right)$$

$$\Rightarrow \frac{1}{\lambda} = \frac{|\lambda_y - \lambda_x|}{\lambda_x \lambda_y}$$

$$\Rightarrow \lambda = \frac{\lambda_x \lambda_y}{|\lambda_x - \lambda_y|}$$

Hence, the correct answer is (A).

6. Wavelength of incident wave $(\lambda) = 260$ nm

Cut off (threshold) wavelength $(\lambda_0) = 380$ nm

Since, $K_{max} = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$

$$\Rightarrow K_{max} = 1237\left(\frac{1}{260} - \frac{1}{380}\right)$$

$$\Rightarrow K_{max} = \frac{1237 \times 120}{380 \times 260} = 1.5 \text{ eV}$$

Hence, the correct answer is (D).

7. $E = \dfrac{hc}{\lambda}$

Let number of photons per second be n

$$\Rightarrow \quad n\left(\frac{hc}{\lambda}\right) = 2 \text{ mW} = 2\times10^{-3} \text{ W}$$

$$\Rightarrow \quad n = \frac{2\times\lambda}{hc} = \frac{(2\times10^{-3})(5000\times10^{-10})}{(6.6\times10^{-34})(3\times10^{8})}$$

$$\Rightarrow \quad n = 5\times10^{15} \text{ photons/sec}$$

Hence, the correct answer is (D).

8. $p = \dfrac{E}{c} = \dfrac{IAt}{c}$ $\qquad \left\{\because I = \dfrac{E}{At}\right\}$

$$\Rightarrow \quad p = \frac{(25\times25)\times40\times60}{3\times10^{8}} = 5\times10^{-3} \text{ Ns}$$

Hence, the correct answer is (C).

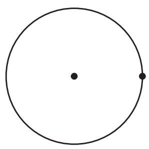
9. $\phi_0 = \dfrac{hc}{\lambda} = h\nu_0$

$$\Rightarrow \quad \phi_0 = h(4\times10^{14} \text{ Hz}) = 1.654 \text{ eV}$$

$$\Rightarrow \quad \phi_0 \approx 1.66 \text{ eV}$$

Hence, the correct answer is (D).

10. For $n = 3$



$$2\pi r = n\lambda$$

$$\Rightarrow \quad 2\pi r = 3\times\lambda$$

$$\Rightarrow \quad \lambda = \frac{2\pi\times4.65}{3} \text{ Å} = 9.7 \text{ Å}$$

Hence, the correct answer is (C).

11. $\dfrac{1}{2}m(2v)^2 = \dfrac{hc}{350} - \phi_0$

and $\dfrac{1}{2}mv^2 = \dfrac{hc}{540} - \phi_0$

$$\Rightarrow \quad 4\left(\frac{hc}{540}\right) - \left(\frac{hc}{350}\right) = 3\phi_0$$

$$\Rightarrow \quad 9.12 - 3.54 = 3\phi_0$$

$$\Rightarrow \quad \phi_0 \simeq 1.8 \text{ eV}$$

Hence, the correct answer is (A).

12. Maximum Angular Frequency is

$$\omega_{max} = 6.28\times10^{7}\times3\times10^{8} \text{ rads}^{-1}$$

$$\Rightarrow \quad f_{max} = 3\times10^{15} \text{ Hz}$$

$$E_{max} = hf_{max} = \frac{6.6\times10^{-34}\times3\times10^{15}}{1.6\times10^{-19}} \text{ eV}$$

$$= 12.375 \text{ eV} = 12.38 \text{ eV}$$

$$\Rightarrow \quad KE_{max} = 12.38 - 4.7 \simeq 7.7 \text{ eV}$$

Hence, the correct answer is (B).

13. Here $\lambda \simeq 7.5\times10^{-12}$ m

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE_k}} \qquad \left\{\because mv = \sqrt{2mE_k}\right\}$$

$$\Rightarrow \quad E_k = \frac{h^2}{2m\lambda^2}$$

$$\Rightarrow \quad E_k = \frac{(6.6\times10^{-34})^2 \text{ eV}}{2\times9.1\times10^{-31}\times(7.5\times10^{-12})^2\times1.6\times10^{-19}}$$

$$\Rightarrow \quad E_k = 25 \text{ keV}$$

Hence, the correct answer is (D).

14. $I = \left(\dfrac{Nhc}{\lambda}\right)\dfrac{1}{(\Delta t)(\Delta A)}$

$$\Rightarrow \quad \left(\frac{N}{\Delta t}\right) = \frac{16\times10^{-3}\times1\times10^{-4}\times\lambda}{hc} (\text{per sec})$$

Since, $\dfrac{hc}{\lambda} = 10$ eV

So total incident photons per second

$$\Rightarrow \quad \frac{N}{\Delta t} = \frac{16\times10^{-7}}{10 \text{ eV}} = 9.98\times10^{11}$$

Number of emitted electrons per sec is $n = \dfrac{10}{100}\left(\dfrac{N}{\Delta t}\right)$

$$\Rightarrow \quad n = 9.98\times10^{10}$$

$$\Rightarrow \quad n \approx 10^{11}$$

Maximum kinetic energy $= 10 \text{ eV} - 5 \text{ eV} = 5$ eV

Hence, the correct answer is (B).

15. $\lambda_{photon} = \dfrac{3\times10^{8}}{6\times10^{14}} = 0.5\times10^{-6}$ m

Since, $\lambda_e = \dfrac{\lambda_{photon}}{10^3} = 0.5\times10^{-9}$ m

$$\Rightarrow \quad \frac{h}{mv} = 0.5\times10^{-9}$$

$$\Rightarrow \quad v = \frac{6.63\times10^{-34}}{(9.1\times10^{-31})(0.5\times10^{-9})}$$

$$\Rightarrow \quad v = 1.45\times10^{6} \text{ ms}^{-1}$$

Hence, the correct answer is (B).

16. Since $h\nu = \phi_0 + eV_0$

$$\Rightarrow \quad \frac{hc}{\lambda_1} = \phi_0 + eV_1$$

$$\Rightarrow \quad \frac{hc}{\lambda_2} = \phi_0 + eV_2$$

$\Rightarrow \quad 1240\left[\frac{1}{300}-\frac{1}{400}\right]=e(V_1-V_2)$

$\Rightarrow \quad (V_1-V_2)\approx 1.0$ V

Hence, the correct answer is (A).

17. $\lambda=\frac{h}{p}=\frac{h}{\sqrt{2mqv}}$

$$\Rightarrow \quad \frac{\lambda_A}{\lambda_B}=\frac{\left(\frac{h}{2mq\times 50}\right)}{\left(\frac{h}{\sqrt{2\times 4m\times q\times 2500}}\right)}=\sqrt{200}=14.14$$

Hence, the correct answer is (B).

18. According to Einsteins Photoelectric Equation, we have

$h\nu=\phi_0+e\left(\frac{V_0}{2}\right)$

$\Rightarrow \quad 2h\nu=2\phi_0+eV_0$

Also, $\frac{h\nu}{2}=\phi+eV_0$

$\Rightarrow \quad \frac{3h\nu}{2}=\phi$

$\Rightarrow \quad \nu_0=\frac{3\nu}{2}$

Hence, the correct answer is (A).

19. Energy lost by electron is

$\Delta E=5.6-0.7=4.9$ eV

$\Rightarrow \quad \frac{hc}{\lambda_{\min}}=4.9$

$\Rightarrow \quad \lambda_{\min}=\frac{12375}{4.9}\approx 2500$ Å $=250$ nm

Hence, the correct answer is (C).

20. Momentum of two electrons are $\frac{h}{\lambda_1}\hat{i}$ and $\frac{h}{\lambda_2}\hat{j}$.

Velocity of centre of mass $\vec{V}_{CM}=\frac{h}{2m\lambda_1}\hat{i}+\frac{h}{2m\lambda_2}\hat{j}$

Velocity of first electron about centre of mass is

$\vec{V}_{CM}=\frac{h}{2m\lambda_1}\hat{i}-\frac{h}{2m\lambda_2}\hat{j}$

$$\Rightarrow \quad \lambda_{CM}=\frac{h}{\sqrt{\frac{h^2}{4\lambda_1^2}+\frac{h^2}{4\lambda_2^2}}}=\frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}$$

Hence, the correct answer is (A).

21. $\lambda_p=\frac{h}{p_p}=\frac{h}{m_pv_p}$ and $\lambda_\alpha=\frac{h}{m_\alpha v_\alpha}$

As, $\lambda_p=\lambda_\alpha$

$\Rightarrow \quad \frac{h}{m_pv_p}=\frac{h}{m_\alpha v_\alpha}$

$\Rightarrow \quad \frac{v_p}{v_\alpha}=\frac{m_\alpha}{m_p}=\frac{4m_p}{m_p}=\frac{4}{1}$

Hence, the correct answer is (D).

22. As de-Broglie wavelength is given by

$\lambda=\frac{h}{p}$

$\Rightarrow \quad \frac{\lambda_B}{\lambda_G}=\frac{p_G}{p_B}=\frac{mv_G}{mv_B}$

Since, $v\propto\frac{Z}{n}$

$\Rightarrow \quad \frac{\lambda_B}{\lambda_G}=\frac{n_B}{n_G}=\frac{3}{1}$

$\Rightarrow \quad \lambda_B=3\lambda_G$

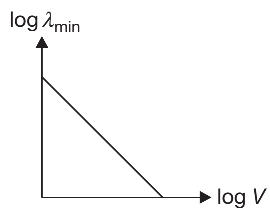
Hence, the correct answer is (B).

23. In X-ray tube

$\lambda_{\min}=\frac{hc}{eV}$

$\Rightarrow \quad \log(\lambda_{\min})=\log\left(\frac{hc}{e}\right)-\log V$

Slope is negative and intercept on y-axis is positive



Hence, the correct answer is (A).

24. $v_1=\frac{(m_1-m_2)v}{m_1+m_2}+0$

Since, $m_1=m$ and $m_2=\frac{m}{2}$

$\Rightarrow \quad v_1=\frac{v}{3}$

$\Rightarrow \quad p_1=m\left(\frac{v}{3}\right)$

and $v_2 = \dfrac{2m_1 v}{m_1 + m_2} + 0$

$\Rightarrow \quad v_2 = \dfrac{4v}{3}$

$\Rightarrow \quad p_2 = \dfrac{m}{2}\left(\dfrac{4v}{3}\right) = \dfrac{2mv}{3}$

Since $\lambda = \dfrac{h}{p}$

$\Rightarrow \quad \dfrac{\lambda_A}{\lambda_B} = \dfrac{p_2}{p_1} = 2:1$

Hence, the correct answer is (B).

25. $E_1 = \dfrac{1}{2}mv^2 = hn - \phi$

If incident frequency is increased to $3n$

$$E_2 = \frac{1}{2}mv'^2 = h(3n) - \phi$$

$\Rightarrow \quad \dfrac{1}{2}mv'^2 = 3(hn - \phi) + 2\phi$

$\Rightarrow \quad \dfrac{1}{2}mv'^2 = 3 \times \left(\dfrac{1}{2}mv^2\right) + 2\phi$

$\Rightarrow \quad v'^2 = 3v^2 + \dfrac{4\phi}{m}$

$\Rightarrow \quad v' > v\sqrt{3}$

Hence, the correct answer is (A).

26. Here, $\lambda = 660 \text{ nm} = 660 \times 10^{-9}$ m

$t = 60 \text{ ms} = 60 \times 10^{-3}$ s, $P = 0.5 \text{ kW} = 500$ W

$h = 6.62 \times 10^{-34}$ Js, $n = ?$

Since, $P = \dfrac{E}{t} = \dfrac{nhc}{\lambda t}$

$\Rightarrow \quad n = \dfrac{P\lambda t}{hc}$

$\Rightarrow \quad n = \dfrac{500 \times 660 \times 10^{-9} \times 60 \times 10^{-3}}{6.62 \times 10^{-34} \times 3 \times 10^8}$

$\Rightarrow \quad n \approx 10^{20}$

Hence, the correct answer is (D).

27. $\dfrac{1}{2}mv^2 = \dfrac{hc}{\lambda} - \phi_0 \quad \ldots(1)$

$$\frac{1}{2}mv'^2 = \frac{4hc}{3\lambda} - \phi_0$$

$\Rightarrow \quad \dfrac{1}{2}mv'^2 = \dfrac{4hc}{3\lambda} - \dfrac{4\phi_0}{3} + \dfrac{\phi_0}{3}$

$\Rightarrow \quad \dfrac{1}{2}mv'^2 = \dfrac{4}{3}\left(\dfrac{hc}{\lambda} - \phi_0\right) + \dfrac{\phi_0}{3}$

$\Rightarrow \quad \dfrac{1}{2}mv'^2 = \dfrac{4}{3}\left(\dfrac{1}{2}mv^2\right) + \dfrac{\phi_0}{3}$

Since $\dfrac{\phi_0}{3} > 0$

$\Rightarrow \quad \dfrac{1}{2}mv'^2 > \dfrac{4}{3}\left(\dfrac{1}{2}mv^2\right)$

$\Rightarrow \quad v' > v\left(\dfrac{4}{3}\right)^{\frac{1}{2}}$

Hence, the correct answer is (B).

28. Let the threshold wavelength for sphere be λ_0. According to Einstein's Photoelectric Equation, we have

$$eV_s = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

So, $eV = \dfrac{hc}{\lambda_1} - \dfrac{hc}{\lambda_0} \quad \ldots(1)$

$3eV = \dfrac{hc}{\lambda_2} - \dfrac{hc}{\lambda_0} \quad \ldots(2)$

$eV' = \dfrac{hc}{\lambda_3} - \dfrac{hc}{\lambda_0} \quad \ldots(3)$

From equations (1) and (2), we get

$$\frac{2hc}{\lambda_0} = \frac{3hc}{\lambda_1} - \frac{hc}{\lambda_2}$$

$\Rightarrow \quad \dfrac{hc}{\lambda_0} = \dfrac{3hc}{2\lambda_1} - \dfrac{hc}{2\lambda_2}$

Substituting in equation (3), we get

$$eV' = \frac{hc}{\lambda_3} - \frac{3hc}{2\lambda_1} + \frac{hc}{2\lambda_2}$$

$\Rightarrow \quad V' = \dfrac{hc}{e}\left(\dfrac{1}{\lambda_3} - \dfrac{3}{2\lambda_1} + \dfrac{1}{2\lambda_2}\right)$

Hence, the correct answer is (D).

29. According to Einstein's Photoelectric Equation, maximum energy of photoelectrons

$$(KE)_{\max} = h\nu - \phi_0$$

$\Rightarrow \quad (KE)_{\max} = \dfrac{hc}{\lambda} - \phi_0$

For first case, $K = \dfrac{hc}{\lambda} - \phi_0 \quad \ldots(1)$

For second case, $3K = \dfrac{2hc}{\lambda} - \phi_0 \quad \ldots(2)$

CHAPTER 1

From equations (1) and (2), we get

$$3\left(\frac{hc}{\lambda}-\phi_0\right)=\frac{2hc}{\lambda}-\phi_0$$

$$\Rightarrow \quad 2\phi_0=\frac{3hc}{\lambda}-\frac{2hc}{\lambda}=\frac{hc}{\lambda}$$

$$\Rightarrow \quad \phi_0=\frac{hc}{2\lambda}$$

Hence, the correct answer is (A).

30. Franck-Hertz Experiment-Discrete energy levels of atom. Photo-electric experiment – Particle nature of light Davisson-Germer Experiment – Wave nature of electron.
Hence, the correct answer is (A).

31. Momentum, $p=\sqrt{2mE}$ and $E=eV$

So, de-Broglie wavelength of the electron is given by,

$$\lambda=\frac{h}{p}=\frac{h}{\sqrt{2mE}}=\frac{h}{\sqrt{2meV}}$$

$$\Rightarrow \quad \lambda=\frac{6.6\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times50}}$$

$$\Rightarrow \quad \lambda=1.7\times10^{-10}\text{ m}=1.7\text{ Å}$$

Hence, the correct answer is (C).

32. de-Broglie wavelength of electron, $\lambda=\dfrac{h}{mv}$

Also $mvr=\dfrac{nh}{2\pi}$

$$\Rightarrow \quad \lambda=\frac{2\pi r}{n}$$

Since $r\propto n^2$

$$\Rightarrow \quad \lambda\propto n$$

For $n=4$, $\lambda_4=4\lambda_1$ i.e., the de-Broglie wavelength is four times that of ground state.

Hence, the correct answer is (B).

33. When wavelength exceeds a certain wavelength, photoelectric effect ceases to exist.
Hence, the correct answer is (D).

34. Davisson-Germer experiment showed that electron beams can undergo diffraction when passed through atomic crystals. This shows the wave nature of electrons as waves can exhibit interference and diffraction.
Hence, the correct answer is (B).

35. The maximum kinetic energy of the electron

$$K_{\max}=h\nu-h\nu_0$$

Here, ν_0 is threshold frequency.

The stopping potential is $eV_0=K_{\max}=h\nu-h\nu_0$

Therefore, if ν is doubled $K_{\max}$ and V_0 is not doubled.

Hence, the correct answer is (D).

36. Here, power of a source, $P=4\text{ kW}=4\times10^3\text{ W}$

Number of photons emitted per second, $N=10^{20}$

Energy of photons, $E=h\nu=\dfrac{hc}{\lambda}$

$$\Rightarrow \quad E=\frac{P}{N}$$

$$\Rightarrow \quad \frac{hc}{\lambda}=\frac{P}{N}$$

$$\Rightarrow \quad \lambda=\frac{Nhc}{P}=\frac{10^{20}\times6.63\times10^{-34}\times3\times10^{8}}{4\times10^{3}}$$

$$\Rightarrow \quad \lambda=4.972\times10^{-9}\text{ m}=49.72\text{ Å}$$

This lies in the X-ray region.

Hence, the correct answer is (B).

37. According to Einstein's Photoelectric Equation

$$K_{\max}=h\nu-\phi_0$$

where, $\nu=$ frequency of incident light

$\phi_0=$ Work function of the metal

Since $K_{\max}=eV_0$

$$V_0=\frac{h\nu}{e}-\frac{\phi_0}{e}$$

Since $\nu_{X\text{-rays}}>\nu_{\text{Ultraviolet}}$

Therefore, both $K_{\max}$ and V_0 increase when ultraviolet light is replaced by X-rays.

Statement-2 is False.
Hence, the correct answer is (A).

38. The wavelength of light illuminating the photoelectric surface is 400 nm.

$$\Rightarrow \quad h\nu=\frac{1240\text{ eVnm}}{400\text{ nm}}=3.1\text{ eV}$$

Maximum kinetic energy, K of the electrons is 1.68 eV

Since $h\nu=\phi_0+K$

$$\Rightarrow \quad \phi_0=h\nu-K$$

$$\Rightarrow \quad \phi_0=3.1-1.68\text{ eV}=1.42\text{ eV}$$

Hence, the correct answer is (B).

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Single Correct Choice Type Problems

1. According to photoelectric effect equation

$$KE_{max} = \frac{hc}{\lambda} - \phi_0$$

$$\frac{p^2}{2m} = \frac{hc}{\lambda} - \phi_0 \qquad \left\{\because KE = \frac{p^2}{2m}\right\}$$

$$\frac{\left(\frac{h}{\lambda_d}\right)^2}{2m} = \frac{hc}{\lambda} - \phi_0 \qquad \left\{\because p = \frac{h}{\lambda}\right\}$$

Assuming small changes, differentiating both sides,

$$\frac{h^2}{2m}\left(-\frac{2d\lambda_d}{\lambda_d^3}\right) = -\frac{hc}{\lambda^2}d\lambda, \ \frac{d\lambda_d}{d\lambda} \propto \frac{\lambda_d^3}{\lambda^2}$$

Hence, the correct answer is (D).

2. Since, $eV = \frac{hc}{\lambda} - \phi$

$$\Rightarrow \quad e(V_1 - V_2) = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2} = \frac{hc(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2}$$

$$\Rightarrow \quad h = \frac{e(V_1 - V_2)\lambda_1\lambda_2}{c(\lambda_2 - \lambda_1)}$$

$$\Rightarrow \quad h = \frac{1.6\times1\times0.3\times0.4\times10^{-12}\times10^{-19}}{3\times10^8\times0.1\times10^{-6}} = 6.4\times10^{-34}$$

Hence, the correct answer is (B).

3. Energy corresponding to 248 nm wavelength is

$$E_1 = \frac{1240}{248}\ \text{eV} = 5\ \text{eV}$$

Energy corresponding to 310 nm wavelength is

$$E_2 = \frac{1240}{310}\ \text{eV} = 4\ \text{eV}$$

$$\frac{KE_1}{KE_2} = \frac{u_1^2}{u_2^2} = \frac{4}{1} = \frac{5\ \text{eV} - W}{4\ \text{eV} - W}$$

$\Rightarrow \quad 16 - 4W = 5 - W$

$\Rightarrow \quad 11 = 3W$

$$\Rightarrow \quad W = \frac{11}{3} = 3.67\ \text{eV} \cong 3.7\ \text{eV}$$

Hence, the correct answer is (A).

4. $E_1 = \frac{1240}{550} = 2.25$ eV

$$E_2 = \frac{1240}{450} = 2.75\ \text{eV}$$

$$E_3 = \frac{1240}{350} = 3.54\ \text{eV}$$

E_1 cannot emit photoelectrons from q and r plates.

E_2 can not emit photoelectrons from r.

Further, work function of p is least and it can emit photoelectrons from all three wavelengths. Hence magnitude of its stopping potential and saturation current both will be maximum.

Hence, the correct answer is (A).

5. Momentum of striking electrons

$$p = \frac{h}{\lambda}$$

So k Kinetic energy of striking electrons is

$$K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

This is also, maximum energy of X-ray photons.

Therefore, $\frac{hc}{\lambda_0} = \frac{h^2}{2m\lambda^2}$

$$\Rightarrow \quad \lambda_0 = \frac{2m\lambda^2 c}{h}$$

Hence, the correct answer is (A).

6. As velocity (or momentum) of electron is increased, the wavelength $\left(\lambda = \frac{h}{p}\right)$ will decrease.

Hence, fringe width will decrease $(\beta \propto \lambda)$.

Hence, the correct answer is (C).

7. Saturation current is proportional to intensity while stopping potential increases with increase in frequency.

Hence, $f_a = f_b$ while $I_a < I_b$

Hence, the correct answer is (A).

8. $$\frac{\lambda_1}{\lambda_2} = \frac{\frac{h}{\sqrt{2mE}}}{\frac{hc}{E}}$$

$$\Rightarrow \quad \frac{\lambda_1}{\lambda_2} \propto E^{\frac{1}{2}}$$

Hence, the correct answer is (B).

9. By Law of Conservation of Momentum

$$\begin{pmatrix}\text{Momentum}\\ \text{before decay}\end{pmatrix} = \begin{pmatrix}\text{Momentum}\\ \text{after decay}\end{pmatrix}$$

CHAPTER 1

$\Rightarrow \quad 0 = p_1 + p_2$

$\Rightarrow \quad p_1 = -p_2$

(–) sign indicates recoil

$\Rightarrow \quad |p_1| = |p_2|$

$$\frac{\lambda_1}{\lambda_2} = \frac{|p_2|}{|p_1|} = 1$$

Hence, the correct answer is (C).

10. $\lambda(\text{in Å}) = \dfrac{12400}{E(\text{in eV})}$

$\Rightarrow \quad \lambda = \dfrac{12400}{4} = 3100\ \text{Å} = 310\ \text{nm}$

Hence, the correct answer is (C).

11. Stopping Potential $V_s = \dfrac{K_{\max}}{e}$

$\Rightarrow \quad V_s = 4\ \text{V}$

Hence, the correct answer is (B).

Multiple Correct Choice Type Problems

1. Maximum KE of electron just after ejection is

$$K_i = \frac{hc}{\lambda_{ph}} - \phi$$

Maximum KE of electron on reaching anode is

$$K_f = \left(\frac{hc}{\lambda_{ph}} - \phi\right) + eV$$

For $V >> \dfrac{\phi}{e}$

$\Rightarrow \quad K_f \approx eV$

$\Rightarrow \quad \lambda_{\text{electron}} = \dfrac{h}{\sqrt{2m(eV)}} \propto \dfrac{1}{\sqrt{V}}$

So, (A) is correct

If ϕ and λ_{ph} increase, then K_f decreases

$\Rightarrow \quad \lambda_{\text{electron}}$ increases

So, (B) is incorrect

$$\lambda_{\text{electron}} = \frac{h}{\sqrt{2m(K_f)}} = \lambda_e$$

$\Rightarrow \quad \dfrac{d\lambda_e}{dt} \neq \dfrac{d\lambda_{ph}}{dt}$

λ_e is independent of d

Hence, the correct answer is (A).

2. $eV_0 = \dfrac{hc}{\lambda} - W$

$$V_0 = \left(\frac{hc}{e}\right)\left(\frac{1}{\lambda}\right) - \frac{W}{e}$$

V_0 versus $\dfrac{1}{\lambda}$ graph is in the form $y = mx - c$

Therefore option (C) is correct.

Clearly, V_0 versus λ graph is not a straight line but V_0 decreases with increase in λ and V_0 becomes zero when $\dfrac{hc}{\lambda} = W$.

i.e. $\lambda = \lambda_0$ (Threshold wavelength)

Hence, (A) and (C) are correct.

3. From the relation,

$$eV = \frac{hc}{\lambda} - \phi \text{ or } V = \left(\frac{hc}{e}\right)\left(\frac{1}{\lambda}\right) - \frac{\phi}{e}$$

This is equation of straight line.

Slope is $\tan\theta = \dfrac{hc}{e}$

$$\phi_1 : \phi_2 : \phi_3 = \frac{hc}{\lambda_{01}} : \frac{hc}{\lambda_{02}} : \frac{hc}{\lambda_{03}} = \frac{1}{\lambda_{01}} : \frac{1}{\lambda_{02}} : \frac{1}{\lambda_{03}} = 1:2:4$$

$\dfrac{1}{\lambda_{01}} = 0.001\ \text{nm}^{-1} \quad \Rightarrow \quad \lambda_{01} = 10000\ \text{Å}$

$\dfrac{1}{\lambda_{02}} = 0.002\ \text{nm}^{-1} \quad \Rightarrow \quad \lambda_{02} = 5000\ \text{Å}$

$\dfrac{1}{\lambda_{03}} = 0.004\ \text{nm}^{-1} \quad \Rightarrow \quad \lambda_{03} = 2500\ \text{Å}$

Violet colour has wavelength $4000\ \text{Å}$.

So violet colour can eject photoelectrons from Metal -1 and Metal -2.

Hence, (A) and (C) are correct.

4. $h\nu = K.E.(T) + \text{Work function }(W)$

$\Rightarrow \quad h\nu = T + W$

$\Rightarrow \quad 4.25\ eV = T_A + W_A$ (for Metal A)

$\Rightarrow \quad 4.70\ eV = T_B + W_B$ (for Metal B)

Since $T_B = (T_A - 1.5)\ eV$

Also $\lambda = \dfrac{h}{p}$

$\Rightarrow \quad \lambda = \dfrac{h}{\sqrt{2mT}} \qquad \left\{\because \dfrac{p^2}{2m} = T = K.E.\right\}$

$\Rightarrow \quad \dfrac{\lambda_A}{\lambda_B} = \sqrt{\dfrac{T_B}{T_A}}$

Since $\lambda_A = \frac{1}{2}\lambda_B$

$\Rightarrow \quad T_A = 4T_B$

$\Rightarrow \quad T_B = T_A - 1.50$ gives

$\Rightarrow \quad T_B = 4T_B - 1.5$

$\Rightarrow \quad T_B = 0.5 \text{ eV}$

$\Rightarrow \quad T_A = 2 \text{ eV}$

$\Rightarrow \quad W_A = 2.25 \text{ eV}$

$\Rightarrow \quad W_B = 4.20 \text{ eV}$

Hence, (A), (B) and (C) are correct.

5. Cut off voltage is independent of intensity and hence remains the same. Since distance becomes 3 times, so I becomes $\frac{I}{9}$. Hence photocurrent also decreases by this factor i.e. becomes $\frac{18}{9} = 2 \text{ mA}$

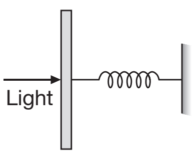
Hence, (B) and (D) are correct.

7. Infrared light has wavelength greater than 5200 Å and UV has $\lambda < 5200$ Å. So UV lamp will be able to get the photoelectrons emitted by a surface irrespective of intensity.

Hence, (C) and (D) are correct.

Integer/Numerical Answer Type Questions

1.



Momentum transferred to the mirror is

$$\Delta p = \frac{2Nh}{\lambda}$$

Given that $\Omega = \frac{10^{24}h}{4\pi m}$

So, the speed, acquired (V_0) by the mirror is given by applying conservation of linear momentum i.e.,

$$MV_0 = \frac{2Nh}{\lambda}$$

$$\Rightarrow \quad V_0 = \frac{2Nh}{\lambda M}$$

Since the system is also executing SHM, so

$$V_0 = A\Omega$$

where A is amplitude of SHM i.e. $A = 1\ \mu\text{m}$

$$\Rightarrow \quad \frac{2Nh}{\lambda M} = A\Omega$$

$$\Rightarrow \quad \frac{2Nh}{\lambda M} = A\left(\frac{10^{24}h}{4\pi M}\right)$$

$$\Rightarrow \quad N = \frac{10^{18} \times 8\pi \times 10^{-6}}{4\pi \times 2} = 1 \times 10^{12}$$

$$\Rightarrow \quad x = 1$$

2. $P = 200 \text{ Js}^{-1}$

No. of photons per second

$$(N) = \frac{200}{(6.25 \times 1.6 \times 10^{-19})} = 2 \times 10^{20}$$

$$\Delta p = \sqrt{2m(KE)} = \sqrt{2m(\text{eV})}$$

$$\Delta p = \sqrt{2 \times 9 \times 10^{-31} \times 1.6 \times 10^{-19} \times 500} = 12 \times 10^{-24}$$

$$\Rightarrow \quad F = \Delta p \times N = 12 \times 10^{-24} \times 2 \times 10^{20} = 24 \times 10^{-4} \text{ N}$$

3. $V = \frac{hf}{e} - \frac{\phi}{e}$

Slope $= \frac{h}{e}$

It is same for both, so

Ratio $= 1$

4. $\frac{hc}{\lambda} - \phi = ev_0$

$$\Rightarrow \quad v_0 = \frac{ne}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \quad \frac{1240}{200} \text{ eV} - 4.7 \text{ eV} = \left(\frac{xne}{4\pi\varepsilon_0 r}\right) \text{eV}$$

$$\Rightarrow \quad 6.2 - 4.7 = \frac{9 \times 19^9 \times n \times 1.6 \times 10^{-19}}{10^{-2}}$$

$$\Rightarrow \quad n = \frac{1.5 \times 10^{-2}}{9 \times 1.6 \times 10^{-10}} \approx 10^8$$

5. $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2qVm}}$

$$\Rightarrow \quad \lambda \propto \frac{1}{\sqrt{qm}}$$

$$\Rightarrow \quad \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{q_\alpha}{q_p} \cdot \frac{m_\alpha}{m_p}} = \sqrt{\frac{(2)(4)}{(1)(1)}} = 2.828$$

The nearest integer is 3.

CHAPTER 2: ATOMIC PHYSICS

Test Your Concepts-I
(Based on Atomic Structure and Properties)

1. (a) $E_3 - E_1 = \frac{(-13.6)(3)^2}{(3)^2} - \left[\frac{(-13.6)(3)^2}{(1)^2}\right] = 108.8 \text{ eV}$

$$\Rightarrow \quad \lambda = \frac{12375}{108.8} \text{ Å} = 113.74 \text{ Å}$$

(b) Number of lines in emission spectrum is

$$N = \frac{n(n-1)}{2}$$

$$\Rightarrow \quad N = \frac{(3)(3-1)}{2} = 3$$

2. Since, $E_n = -\frac{(13.6)Z^2}{n^2}$ eV

So, ionisation energy, $n = 1$, for $Li^{++}(Z = 3)$ is

$$IE = \frac{(13.6)(3)^2}{(1)^2} \text{ eV}$$

$$\Rightarrow \quad IE = 122.4 \text{ eV}$$

3. Energy of electron in ground state of hydrogen atom is -13.6 eV. Earlier it had a kinetic energy of 2 eV

Therefore, energy of photon released during formation of hydrogen atom,

$$\Delta E = 2 - (-13.6) = 15.6 \text{ eV}$$

$$\Rightarrow \quad \lambda = \frac{12375}{\Delta E} = \frac{12375}{15.6} = 793.3 \text{ Å}$$

4. For a deflection of $180°$, the α-particle must be approaching the nucleus head on. Since, we know that

$$r_0 = \frac{2}{4\pi\varepsilon_0}\left(\frac{2Ze^2}{m_\alpha v_\alpha^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{2Ze^2}{K_\alpha}\right)$$

where, $K_\alpha = 12.5 \text{ MeV} = 12.5 \times 1.6 \times 10^{-13}$ J, $r_0 = ?$

$$\Rightarrow \quad r_0 = \frac{2(9\times 10^9)(79)(1.6\times 10^{-19})^2}{12.5\times 1.6\times 10^{-13}} = 1.82\times 10^{-14} \text{ m}$$

5. Since the frequency of revolution of an electron in n^{th} orbit is

$$f_n = \frac{\omega_n}{2\pi}$$

$$\Rightarrow \quad f_n = \frac{v_n}{2\pi r_n}$$

$$\Rightarrow \quad f_n = \frac{2.18\times 10^6\left(\frac{Z}{n}\right)}{2\times 3.14\times 0.529\times 10^{-10}\left(\frac{n^2}{Z}\right)} \text{s}^{-1}$$

Thus, number of revolutions completed in 10^{-8} second in $n = 2$ state are

$$N = f_n \times 10^{-8}$$

$$\Rightarrow \quad N = \frac{2.18\times 10^6 \times 10^{-8}}{2\times 3.14\times 0.529\times 10^{-10}} \times \frac{Z^2}{n^3}$$

$$\Rightarrow \quad N = 8.2\times 10^6 \text{ revolutions}$$

6. The frequency of revolution of electron in n^{th} orbit is

$$\omega_n = \frac{v_n}{2\pi r_n}$$

ω_n is the number of revolutions made by electron in 1 second. For $n = 1$ orbit of hydrogen atom, we have

$$\omega_1 = \frac{2.18\times 10^6}{2\times 3.14\times 0.529\times 10^{-10}} \text{ rev/sec}$$

$$\Rightarrow \quad \omega_1 = 6.56\times 10^{15} \text{ rev/sec}$$

7. Since $U = -k\log_e r$

In the given situation, the centripetal force on electron in n^{th} orbit is given by

$$|F| = \left|-\frac{dU}{dr}\right| = \frac{k}{r_n}$$

If in n^{th} orbit speed of electron is v_n then, we have

$$\frac{mv_n^2}{r_n} = \frac{k}{r_n}$$

$$\Rightarrow \quad mv_n^2 = k \qquad \ldots(1)$$

According to Bohr's Quantization Rule, we have

$$mv_n r_n = \frac{nh}{2\pi} \qquad \ldots(2)$$

From (1) and (2), we get

$$r_n = \frac{nh}{2\pi\sqrt{mk}}$$

Energy of electron in n^{th} level is

$$E_n = KE_n + PE_n$$

$$\Rightarrow \quad E_n = \frac{1}{2}mv_n^2 - k\log_e r$$

$$\Rightarrow \quad E_n = \frac{k}{2} - k\log_e r$$

$$\Rightarrow \quad E_n = \frac{k}{2} - k\log_e\left(\frac{nh}{2\pi\sqrt{mk}}\right)$$

$$\Rightarrow \quad E_n = \frac{k}{2}\left[1 - \log_e\left(\frac{n^2h^2}{4\pi^2 mk}\right)\right]$$

8. (a) As the atoms finally emit radiation of only 3 different photon energies final excited state corresponds to $n = 3$.

So, the initial excited state corresponds to $n = 2$

$$\Rightarrow \quad Z^2(13.6)\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{12375}{1654}$$

$$\Rightarrow \quad Z = 2$$

Therefore, it is helium atom.

(b) Ionization energy is

$$IE = Z^2(13.6 \text{ eV}) = (2)^2(13.6 \text{ eV})$$

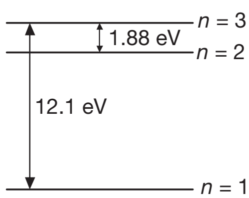
$$\Rightarrow \quad IE = 54.4 \text{ eV}$$

(c) $N = \dfrac{n(n-1)}{2} = 6$

So, $E = E_4 - E_2 = (13.6)(4)\left(\dfrac{1}{4} - \dfrac{1}{16}\right) = 10.2$ eV

9. Energy of photon corresponding to $\lambda = 6563$ Å is

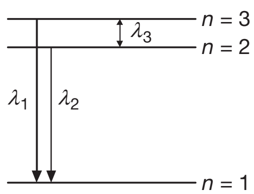
$$\Delta E = \frac{12375}{6563} \text{ eV} = 1.88 \text{ eV}$$



This is the difference in energy between $n = 3$ and $n = 2$. Hence the single electron in the hydrogen atom should excite at least upto $n = 3$ and for this the minimum energy of the striking electron should be 12.1 eV.

10. Given $E_3 = 0$

$$\lambda_1 = 460 \text{ Å}$$



$$\Rightarrow \quad E_3 - E_1 = \frac{12375}{460} = 26.9 \text{ eV}$$

Since, $E_3 = 0$

$$\Rightarrow \quad 0 - E_1 = 26.9 \text{ eV}$$

$$\Rightarrow \quad E_1 = -26.9 \text{ eV}$$

Further, $\lambda_3 = 1035$ Å

$$\Rightarrow \quad E_3 - E_2 = \frac{12375}{1035} \text{ Å} = 12 \text{ eV}$$

$$\Rightarrow \quad E_2 = -12 \text{ eV}$$

11. When hydrogen atom is excited, then we have

$$eV = E_0\left(\frac{1}{1} - \frac{1}{n^2}\right) \quad \ldots(1)$$

When ion is excited, then

$$eV = E_0 Z^2\left(\frac{1}{2^2} - \frac{1}{n_1^2}\right) \quad \ldots(2)$$

Wavelength of emitted light is

$$\frac{hc}{\lambda_1} = E_0\left(\frac{1}{1} - \frac{1}{n^2}\right) \quad \ldots(3)$$

$$\frac{hc}{\lambda_2} = E_0 Z^2\left(\frac{1}{1} - \frac{1}{n_1^2}\right) \quad \ldots(4)$$

Further it is given that

$$\frac{\lambda_1}{\lambda_2} = \frac{5}{1} \quad \ldots(5)$$

Solving the above equations, we get

$Z = 2$, $n = 2$, $n_1 = 4$ and $V = 10.2$ V

Energy of emitted photon by the hydrogen atom is

$$\Delta E = E_2 - E_1 = 10.2 \text{ eV}$$

and by the ion is

$$\Delta E' = E_4 - E_1 = (13.6)(2)^2\left(1 - \frac{1}{16}\right) = 51 \text{ eV}$$

12. Energy of photon of the first line of Lyman series is

$$E = E_2 - E_1 = (13.6)(2)^2\left(1 - \frac{1}{4}\right) = 40.8 \text{ eV}$$

Energy required to ionize the hydrogen atom is 13.6 eV. Therefore, kinetic energy of electron emitted from the hydrogen atom is

$$K = (40.8 - 13.6) \text{ eV} = 27.2 \text{ eV}$$

$$\Rightarrow \quad K = 4.352 \times 10^{-18} \text{ J}$$

Since, $K = \dfrac{1}{2}mv^2 = 4.352 \times 10^{-18}$

$$\Rightarrow \quad v=\sqrt{\frac{2\times 4.352\times 10^{-18}}{9.1\times 10^{-31}}}$$

$$\Rightarrow \quad v=3.1\times 10^{6}\ \text{ms}^{-1}$$

13. (a) From figure, we observe that

$$\begin{pmatrix}\text{Ionisation}\\ \text{Potential}\end{pmatrix}=15.6\ \text{eV}$$

(b) $\lambda_{\min}=\dfrac{12375}{5.3}=2335\ \text{Å}$

(c) $\Delta E_{31}=-3.08-(-15.6)=12.52\ \text{eV}$

Therefore, excitation potential for state $n=3$ is 12.52 V

(d) $\dfrac{1}{\lambda_{31}}=\dfrac{\Delta E_{31}}{12375}\ \text{Å}^{-1}=\dfrac{12.52}{12375}\ \text{Å}^{-1}$

$$\Rightarrow \quad \frac{1}{\lambda_{31}}\approx 1.01\times 10^{7}\ \text{m}^{-1}$$

(e) (i) $E_2-E_1=10.3\ \text{eV}>6\ \text{eV}$

Hence, the striking electron cannot excite the hypothetical atoms. So, the electron will keep its energy with itself.

$$\Rightarrow \quad K_{\min}=6\ \text{eV}$$

(ii) $E_2-E_1=10.3\ \text{eV}<11\ \text{eV}$

So, the electron can excite the atom.

$$\Rightarrow \quad K_{\min}=(11-10.3)\ \text{eV}=0.7\ \text{eV}$$

14. (a) $z=3$ for Li^{+2}. Further we know that $r_n=\dfrac{n^2}{z}a_0$

Substituting, $n=3$, $z=3$ and $a_0=0.529$ Å, we get r_3 for Li^{+2}

$$r_3=\frac{(3)^2}{(3)}(0.529)\ \text{Å}=1.587\ \text{Å}$$

(b) $z=2$ for He^{+}. Also, we know that

$$v_n=\frac{z}{n}v_1$$

Substituting $n=4$, $z=2$ and $v_1=2.19\times 10^{6}\ \text{ms}^{-1}$, we get for He^{+},

$$v_4=\left(\frac{2}{4}\right)(2.19\times 10^{6})\ \text{ms}^{-1}$$

$$\Rightarrow \quad v_4=1.095\times 10^{6}\ \text{ms}^{-1}$$

15. Magnetic moment $\mu=NiA=\left(\dfrac{e}{T}\right)(\pi r^2)$

$$\Rightarrow \quad \mu=\left(\frac{e}{2\pi r/v}\right)(\pi r^2)=\frac{evr}{2} \quad \ldots(1)$$

We know that $mvr=\dfrac{nh}{2\pi}$ …(2)

Solving equations (1) and (2)

$$\mu=\frac{neh}{4\pi m}$$

Magnetic induction, $B=\dfrac{\mu_0 i}{2r}=\dfrac{\mu_0 e}{2rT}$

$$\Rightarrow \quad B=\frac{\mu_0 ev}{(2r)(2\pi r)}=\frac{\mu_0 ev}{4\pi r^2} \quad \ldots(3)$$

From Newton's Second Law, we have

$$\frac{e^2}{4\pi\varepsilon_0 r^2}=\frac{mv^2}{r}$$

$$\Rightarrow \quad v^2=\frac{e^2}{4\pi\varepsilon_0 mr} \quad \ldots(4)$$

Solving these equations, we get

$$B=\frac{\mu_0\pi m^2e^7}{8\varepsilon_0 h^5 n^5}$$

16. The force at a distance r is,

$$F=-\frac{dU}{dr}=-2ar$$

Suppose r be the radius of nth orbit. Then the necessary centripetal force is provided by the above force. Thus,

$$\frac{mv^2}{r}=2ar \quad \ldots(1)$$

Further, the quantization of angular momentum gives,

$$mvr=\frac{nh}{2\pi} \quad \ldots(2)$$

Solving equations (1) and (2) for r, we get

$$r=\left(\frac{n^2h^2}{8am\pi^2}\right)^{1/4}$$

17. The time period T of an electron in a Bohr orbit of principal quantum number n is

$$T=\frac{n^3h^3}{4\pi^2k^2Z^2e^4m}$$

$$\Rightarrow \quad T\propto n^3$$

$$\Rightarrow \quad \frac{T_1}{T_2}=\frac{n_1^3}{n_2^3}$$

Since $T_1=8T_2$

$$\Rightarrow \quad \left(\frac{n_1}{n_2}\right)^3=8$$

$$\Rightarrow \quad n_1=2n_2$$

Thus, the possible values of n_1 and n_2 are

$n_1 = 2$, $n_2 = 1$

$n_1 = 4$, $n_2 = 2$

$n_1 = 6$, $n_2 = 3$ and so on …..

18. Since, $E_1 = -13.60$ eV

Also, we know that

$$TE = -KE = \frac{PE}{2}$$

$\Rightarrow \quad K_1 = -E_1 = 13.60$ eV

$\Rightarrow \quad U_1 = 2E_1 = -27.20$ eV

Further, $E_n = -\frac{13.6Z^2}{n^2}$

$\Rightarrow \quad E_2 = \frac{E_1}{(2)^2} = -3.40$ eV

$\Rightarrow \quad K_2 = 3.40$ eV and

$\Rightarrow \quad U_2 = -6.80$ eV

Now $U_1 = 0$, i.e., potential energy has been increased by 27.20 eV. So, we will increase U and E in all energy states by 27.20 eV while kinetic energy will remain unchanged. So, we have

Orbit	*K*(eV)	*U*(eV)	*E*(eV)
First	13.60	0	13.60
Second	3.40	20.40	23.80

19. Wavelengths corresponding to minimum wavelength (λ_{min}) or maximum energy will emit photoelectrons having maximum kinetic energy.

(λ_{min}) belonging to Balmer series and Lying in the given range (450 nm to 750 nm) corresponds to transition from $(n = 4$ to $n = 2)$. Here,

$$E_4 = -\frac{13.6}{(4)^2} = -0.85 \text{ eV}$$

and $E_2 = -\frac{13.6}{(2)^2} = -3.4$ eV

$\Rightarrow \quad \Delta E = E_4 - E_2 = 2.55$ eV

So, K_{max} = Energy of photon – work function

$\Rightarrow \quad K_{max} = 2.55 - 2 = 0.55$ eV

20. For $0 \le x \le 1$, $PE = E_0$

$\Rightarrow \quad (\text{Kinetic energy } K_1) = (\text{Total energy}) - (PE)$

$\Rightarrow \quad K_1 = 2E_0 - E_0 = E_0$

$$\Rightarrow \quad \lambda_1 = \frac{h}{\sqrt{2mE_0}} \qquad \ldots(1)$$

For $x > 1$, $PE = 0$

$\Rightarrow$ Kinetic energy K_2 = Total energy $= 2E_0$

$$\Rightarrow \quad \lambda_2 = \frac{h}{\sqrt{2m(E_0)}} \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\frac{\lambda_1}{\lambda_2} = \sqrt{2}$$

21. (a) Kinetic energy of electron in the orbits of hydrogen and hydrogen like atoms = |Total energy|

So, Kinetic energy = 3.4 eV

(b) The de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2Km}}$$

where K is the kinetic energy of an electron

Substituting the values, we get

$$\lambda = \frac{(6.6 \times 10^{-34} \text{ Js})}{\sqrt{2(3.4 \times 1.6 \times 10^{-19} \text{ J})(9.1 \times 10^{-31} \text{ kg})}}$$

$\Rightarrow \quad \lambda = 6.63 \times 10^{-10}$ m

$\Rightarrow \quad \lambda = 6.63$ Å

22. The energy lost by the electron in exciting the hydrogen atom from ground state (–13.6 eV) to first excited state (–3.4 eV) equal to $\Delta E_{12} = 10.2$ eV

$\Rightarrow \quad \Delta E = 16.36 \times 10^{-19}$ J

Now, the initial energy of electron is 20 eV $= 32 \times 10^{-19}$ J. Hence the kinetic energy of the scattered electron is

$$E_K = 32 \times 10^{-19} \text{ J} - 16.36 \times 10^{-19} \text{ J}$$

$\Rightarrow \quad E_K = 15.64 \times 10^{-19}$ J

The velocity v of the scattered electron is given by

$$\frac{1}{2}mv^2 = E_K$$

$$\Rightarrow \quad v = \left(\frac{2E_K}{m}\right)^{\frac{1}{2}} = \left(\frac{2 \times 15.64 \times 10^{-19}}{9.11 \times 10^{-31}}\right)^{\frac{1}{2}}$$

$\Rightarrow \quad v = 1.86 \times 10^6$ ms^{-1}

23. The maximum wavelength will correspond to the minimum energy transition of an electron. For ground state of hydrogen atom, the minimum energy transition is for $n = 1$ to $n = 2$, for which energy released will be

$$\Delta E_{12} = E_2 - E_1$$

$\Rightarrow$ $\Delta E_{12} = (-3.4 \text{ eV}) - (-13.6 \text{ eV})$

$\Rightarrow$ $\Delta E_{12} = 10.2 \text{ eV}$

Thus 10.2 eV energy is absorbed in the form of a photon. So, if λ be the wavelength of photon, then

$$\lambda = \frac{12400}{10.2} \text{ Å}$$

$\Rightarrow$ $\lambda = 1216 \text{ Å}$

For next smaller wavelength, the possibility is for an electron transition from $n = 1$ to $n = 3$, for which the absorbed energy photon required is

$$\Delta E_{13} = E_3 - E_1$$

$\Rightarrow$ $\Delta E_{13} = (-1.5 \text{ kV}) - (-13.6 \text{ eV})$

$\Rightarrow$ $\Delta E_{13} = 12.09 \text{ eV}$

If λ' be its wavelength, then we have

$$\lambda' = \frac{12400}{12.09} \text{ Å}$$

$\Rightarrow$ $\lambda' = 1026 \text{ Å}$

24. Minimum wavelength is corresponding to transition from $\infty \to 1$ hence

$$\lambda_{\min} = \frac{12431}{13.6} \text{ Å}$$

and maximum wavelength is for transition $2 \to 1$

$\Rightarrow$ $\lambda_{\max} = \frac{12431}{10.2} \text{ Å}$

$\Rightarrow$ $\frac{\lambda_{\min}}{\lambda_{\max}} = \frac{3}{4}$

25. The ionization energy for a hydrogenic atom can be given as energy required to excite electron from $n_1 = 1$ to $n_2 \to \infty$ given as

$$E = Rch\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$\Rightarrow$ $E = Rch\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$ joule

$\Rightarrow$ $E = (Rch)$ joule

If motion of nucleus is considered, then the value of Rydberg constant can be given by

$$R' = \frac{e^4}{8\varepsilon_0^2 ch^3}\left(\frac{mM_H}{m + M_H}\right)$$

So, the ionization energy of hydrogen and deuterium atom can be given by

$$E_H = \frac{e^4}{8\varepsilon_0^2 h^2}\left(\frac{mM_H}{m + M_H}\right), \text{ where } M_H = 1840 \text{ m}$$

and $E_D = \frac{e^4}{8\varepsilon_0^2 h^2}\left(\frac{mM_D}{m + M_D}\right)$, where $M_D = 3680$ m

The difference between the two energies E_D and E_H is

$$\Delta E = E_D - E_H$$

$\Rightarrow$ $\Delta E = \frac{e^4}{8\varepsilon_0^2 h^2}\left[\frac{mM_D}{m + M_D} - \frac{mM_H}{m + M_H}\right]$

$\Rightarrow$ $\Delta E = \frac{e^4}{8\varepsilon_0^2 h^2}\left(\frac{3680}{3681} - \frac{1840}{1841}\right)$

$\Rightarrow$ $\Delta E = 5.88 \times 10^{22} \text{ J}$

$\Rightarrow$ $\Delta E = 6.68 \times 10^{-3} \text{ eV}$

Test Your Concepts-II (Based on X-rays and Properties)

1. For $\lambda_c = 1 \text{ Å}$, we have

$$V = \frac{12400}{1} = 12.4 \text{ kV}$$

Photon energy, $E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{10^{-10}}$

$\Rightarrow$ $E \approx 2 \times 10^{-15} \text{ J}$

2. (a) Short wavelength is given as

$$\lambda_{\min} = 0.45 \text{ Å}$$

Maximum photo energy is given as

$$E_{\max} = h\nu_{\max} = \frac{hc}{\lambda_{\min}}$$

$\Rightarrow$ $E_{\max} = \frac{12431}{0.45} = 27624.44 \text{ eV}$

$\Rightarrow$ $E_{\max} = 27.624 \text{ keV}$

(b) The minimum accelerating voltage for electrons is

$$\frac{27.6 \text{ keV}}{e} = 27.6 \text{ kV}$$

i.e. of the order of 30 kV

3. If the short series limit of the Balmer series is corresponding to transition $n = \infty$ to $n = 2$ which is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = \frac{R}{4}$$

$\Rightarrow$ $R = \frac{4}{\lambda} = \frac{4}{3644} (\text{Å})^{-1}$

The shortest wavelength corresponds to $n \to \infty$ to $n = 1$.

Therefore λ_c is given as

$$\frac{1}{\lambda_c} = R(Z-1)^2\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right]$$

$$\Rightarrow \quad (Z-1)^2 = \frac{1}{\lambda_c R} = \frac{1}{1\text{ Å} \times \frac{4}{3644}\ (\text{Å})^{-1}}$$

$$\Rightarrow \quad (Z-1)^2 = \frac{3644}{4} = 911$$

$$\Rightarrow \quad Z - 1 = 30.2$$

$$\Rightarrow \quad Z = 31.2 \simeq 31$$

Thus, the atomic number of the element is 31 which is gallium.

4. Photons travel at speed of light, so time taken by both photons is

$$t = \frac{d}{c} = \frac{3\times 10^3}{3\times 10^8} = 10^{-5}\text{ s} = 10\ \mu\text{s}$$

5. Using the equation, $\sqrt{f} = a(Z-b)$ $(b = 1)$

Since, $b = 1$, so we get

$$\frac{f_{La}}{f_{Cu}} = \left(\frac{Z_{La}-1}{Z_{Cu}-1}\right)^2$$

$$\Rightarrow \quad f_{La} = f_{Cu}\left(\frac{Z_{La}-1}{Z_{Cu}-1}\right)^2$$

$$\Rightarrow \quad f_{La} = 1.88\times 10^{18}\left(\frac{57-1}{29-1}\right)^2$$

$$\Rightarrow \quad f_{La} = 7.52\times 10^{18}\text{ Hz}$$

6. Since, $\lambda = \frac{12375}{V}$, so we have

$$\lambda_1 - \lambda_2 = \frac{12375}{V_1} - \frac{12375}{V_2} = 12375\left(\frac{1}{V} - \frac{1}{1.5V}\right)$$

where $\lambda_1 - \lambda_2 = 26\text{ pm} = 0.26\text{ Å}$

$$\Rightarrow \quad 0.26 = (12375)\left(\frac{1}{3V}\right)$$

$$\Rightarrow \quad V = 15865\text{ volt}$$

7. Cutoff wavelength $\lambda_{\min}$ is given by,

$$\lambda_{\min}(\text{in Å}) = \frac{12375}{V(\text{in volt})} = \frac{12375}{40\times 10^3} = 0.31\text{ Å}$$

8. When an electron of charge e is accelerated through a potential difference V_0, it acquires energy eV_0. If m be the mass of the electron and $v_{\max}$ the maximum speed of electron, then

$$\frac{1}{2}mv_{\max}^2 = eV_0$$

$$\Rightarrow \quad v_{\max} = \sqrt{\frac{2eV_0}{m}}$$

Substituting the given values, we get

$$v_{\max} = \sqrt{\frac{2\times(1.6\times 10^{-19})\times 20000}{9\times 10^{-31}}}$$

$$\Rightarrow \quad v_{\max} = 8.4\times 10^7\text{ ms}^{-1}$$

9. Since tungsten is a multielectron atom, so due to the shielding of the nuclear charge by the negative charge of the inner core electrons, each electron is subject to an effective nuclear charge Z_{eff}, which is different for different shells. For an electron in the K shell $(b = 1)$ thus effective nuclear charge is given as

$$Z_{\text{eff}} = (Z - b) = Z - 1$$

Since an electron jumps from M shell $(n = 3)$ to K shell $(n = 1)$, the radiated emission is called K_β X-ray and from Mosley's law the wavelength emitted of K_β X-ray is given as

$$\frac{1}{\lambda_{K_\beta}} = R(Z-1)^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$

$$\Rightarrow \quad \frac{1}{\lambda_{K_\beta}} = 10967800\times(74-1)^2\left(\frac{8}{9}\right)$$

$$\Rightarrow \quad \lambda_{K_\beta} = 0.192\text{ Å}$$

10. The binding energy for K shell in eV is

$$E_k = \frac{hc}{\lambda_k} = \frac{12400}{0.2}\text{ eV} = 62\text{ keV}$$

The energy of the incident photon in eV is

$$E = \frac{hc}{\lambda} = \frac{12400}{0.15} \approx 83\text{ keV}$$

Therefore, the maximum energy of the photoelectrons emitted from the K shell is

$$E_{\max} = E - E_k = (83 - 62)\text{ keV} = 21\text{ keV}$$

11. Given that

$$2\left\{\frac{12375\times 10^{-10}}{34.3\times 10^3}\right\} = \frac{1}{1.09\times 10^7 (Z-1)^2\left(1 - \frac{1}{4}\right)}$$

$$\Rightarrow \quad Z = 42$$

12. Energy of K_2 line $= 100 - 24 = 76$ keV

$$\Rightarrow \quad \lambda_{K_\alpha} = \frac{12375}{76\times 10^3}\text{ Å}$$

$$\Rightarrow \quad \lambda_{K_\alpha} = 0.163\text{ Å}$$

Single Correct Choice Type Questions

1. $mvr = (2n)\frac{h}{2\pi}$

$\Rightarrow \quad v = \frac{2n\hbar}{mr}$

$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{2n\hbar}{mr}\right)^2 = \frac{(2n\hbar)^2}{2mr^2}$

Since, $\frac{Ze^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}$

$\Rightarrow \quad \frac{Ze^2}{4\pi\varepsilon_0 r^2} = \frac{(2n\hbar)^2}{mr^2(r)}$

$\Rightarrow \quad r = \frac{(2n\hbar)^2 4\pi\varepsilon_0}{mZe^2}$

Now, total energy of the atom is

$$E = KE + PE = -\frac{Ze^2}{8\pi\varepsilon_0 r} = -\frac{Z^2e^4m}{8\pi\varepsilon_0(2n\hbar)^2 4\pi\varepsilon_0}$$

$\Rightarrow \quad E = -\frac{Z^2e^4m}{32\varepsilon_0^2 n^2 h^2}$

For the actual hydrogen atom, the binding energy is

$$E_0 = -\frac{me^4Z^2}{8n^2h^2\varepsilon_0^2} = -13.6\left(\frac{Z^2}{n^2}\right) \text{ eV}$$

So, for the hypothetical hydrogen atom, we have

$$E = -\frac{Z^2e^4m}{32\varepsilon_0^2 n^2 h^2} = \frac{E_0}{4} = -3.4\left(\frac{Z^2}{n^2}\right) \text{ eV}$$

The longest wavelength is obtained when the electron makes a transition from $n_i = 2$ to $n_f = 1$

$\Rightarrow \quad \frac{hc}{\lambda} = \frac{12400}{\lambda} = 3.4\left(1 - \frac{1}{4}\right) = 2.55 \text{ eV}$

$$\lambda = \frac{12400}{2.55} = 4863 \text{ Å} \approx 486 \text{ nm}$$

Hence, the correct answer is (B).

2. First excitation energy is given by

$$\Delta E = Rch\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = Rch \times \frac{3}{4}$$

$\Rightarrow \quad \frac{3}{4}Rch = V_0$

$\Rightarrow \quad Rch = \frac{4V_0}{3}$

Hence, the correct answer is (C).

3. $B_n = \frac{\mu_0 I_n}{2r_n}$

$\Rightarrow \quad B_n \propto \frac{I_n}{r_n} \propto \frac{f_n}{r_n}$

$\Rightarrow \quad B_n \propto \frac{\left(\frac{v_n}{r_n}\right)}{r_n} \propto \frac{v_n}{(r_n)^2}$

$\Rightarrow \quad B_n \propto \frac{\left(\frac{z}{n}\right)}{\left(\frac{n^2}{z}\right)^2} \propto \frac{z^3}{n^5}$

Hence, the correct answer is (D).

5. $mvr = \frac{nh}{2\pi}$

$\Rightarrow \quad \frac{h}{mv} = \frac{(2\pi r)}{n}$

where, $\frac{h}{mv}$ = de-Broglie wavelength

Hence, the correct answer is (A).

6. $\Delta E = 13.6\left(\frac{1}{1^2} - \frac{1}{6^2}\right) = 13.22 \text{ eV}$

Since, $p = \frac{\Delta E}{c}$

$\Rightarrow \quad v = \frac{\Delta E}{mc} = \frac{13.22 \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27} \times 3 \times 10^8} = 4.2 \text{ ms}^{-1}$

Hence, the correct answer is (B).

7. Since, $E_{K_\alpha} < E_{K_\beta}$

$\Rightarrow \quad \lambda_{K_\alpha} > \lambda_{K_\beta}$

Hence, the correct answer is (C).

8. Momentum of electron in different states

$$p_n = \frac{h}{\lambda_n}, \quad p_g = \frac{h}{\lambda_g}$$

Kinetic energy, $K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$

Total energy in an orbit of hydrogen atom,

$$E = -K = -\frac{h^2}{2m\lambda^2}$$

$$E_n - E_g = \frac{h^2}{2m}\left(\frac{1}{\lambda_g^2} - \frac{1}{\lambda_n^2}\right)$$

$$\frac{h^2}{2m}\left(\frac{\lambda_n^2-\lambda_g^2}{\lambda_g^2\lambda_n^2}\right)=\frac{hc}{\Lambda_n}$$

$$\Rightarrow \quad \Lambda_n=\frac{2mc}{h}\left(\frac{\lambda_g^2\lambda_n^2}{\lambda_n^2-\lambda_g^2}\right)$$

$$\Rightarrow \quad \Lambda_n=\frac{2mc\lambda_g^2}{h}\left[1-\frac{\lambda_g^2}{\lambda_n^2}\right]^{-1}=\frac{2mc\lambda_g^2}{h}\left[1+\frac{\lambda_g^2}{\lambda_n^2}\right]$$

$$\{\because \lambda_g \ll \lambda_n\}$$

$$\Rightarrow \quad \Lambda_n=\frac{2mc\lambda_g^2}{h}+\left(\frac{2mc\lambda_g^4}{h}\right)\frac{1}{\lambda_n^2}=A+\frac{B}{\lambda_n^2}$$

where A and B are $A=\frac{2mc\lambda_g^2}{h}$, $B=\frac{2mc\lambda_g^4}{h}$

Hence, the correct answer is (A).

9. Since, $B=\frac{\mu_0 i}{2r}\propto\frac{i}{r}$

Magnetic field at the centre of hydrogen atom i.e. at nucleus, is found by calculating the current due to electron, given by

$$i=\frac{e}{T}=ef$$

Since, we know that $r\propto\frac{n^2}{Z}$ and $f\propto\frac{Z^2}{n^3}$

Since, $B\propto\frac{i}{r}\propto\frac{f}{r}$

$$\Rightarrow \quad B\propto\frac{\left(\frac{Z^2}{n^3}\right)}{\left(\frac{n^2}{Z}\right)}\propto\frac{Z^3}{n^5}$$

Hence, the correct answer is (B).

10. $t=\frac{2r}{v}=\frac{2\times10^{-15}\text{ m}}{3\times10^7\text{ ms}^{-1}}=\frac{2}{3}\times10^{-22}\text{ s}$

Hence, the correct answer is (B).

11. Since, $\Delta\lambda=\lambda_{K_\alpha}-\lambda_{\min}$ ...(1)

When V is made half, $\lambda_{\min}$ becomes two times, however λ_{K_α} remains the same.

$$\Rightarrow \quad \Delta\lambda'=\lambda_{K_\alpha}-2\lambda_{\min}$$

From (1), we have

$$\lambda_{\min}=\lambda_{K_\alpha}-\Delta\lambda$$

$$\Rightarrow \quad \Delta\lambda'=\lambda_{K_\alpha}-2\lambda_{K_\alpha}+2\Delta\lambda$$

$$\Rightarrow \quad \Delta\lambda'=2\Delta\lambda-\lambda_{K_\alpha}$$

$$\Rightarrow \quad \Delta\lambda'<2\Delta\lambda$$

Hence, the correct answer is (D).

12. Loss in $KE=[(-3.4)-(-13.6)]$ eV

$\Rightarrow$ Loss in KE = 10.2 eV

$$\Rightarrow \quad \frac{1}{4}m_H v^2=10.2\times1.6\times10^{-19}$$

$$\Rightarrow \quad v=6.25\times10^4\text{ ms}^{-1}$$

Hence, the correct answer is (A).

13. Assuming that ionization occurs as a result of a completely inelastic collision, we can write

$$mv_0=(m+m_H)V$$

where m is the mass of incident particle, m_H the mass of hydrogen atom, v_0 the initial velocity of incident particle and V the final common velocity of the particle after collision. Prior to collision, the KE of the incident particle was

$$E_0=\frac{mv_0^2}{2}$$

The total kinetic energy after collision

$$E=\frac{(m+m_H)V^2}{2}=\frac{m^2v_0^2}{2(m+m_H)}$$

The decrease in kinetic energy must be equal to ionization energy, so we have

$$E_i=E_0-E=\left(\frac{m_H}{m+m_H}\right)E_0$$

$$\Rightarrow \quad \frac{E_i}{E_0}=\frac{m_H}{m+m_H}$$

i.e., the greater the mass m, the smaller the fraction of initial kinetic energy that will be used for ionization.
Hence, the correct answer is (B).

14. KE of α-particle is $K_\alpha=qV_0=(2e)V_0$

PE of α-particle is $U_\alpha=\frac{1}{4\pi\varepsilon_0}\frac{(Ze)(2e)}{r_0}$

By Law of Conservation of Energy, $U_\alpha=K_\alpha$

$$2eV_0=\frac{2Ze^2}{4\pi\varepsilon_0 r_0}$$

$$\Rightarrow \quad r_0=\frac{Ze}{4\pi\varepsilon_0 V_0}$$

$$\Rightarrow \quad r_0=\frac{(9\times10^9)(1.6\times10^{-19})(Z)}{V_0}$$

CHAPTER 2

$$\Rightarrow \quad r_0 = 14.4 \times 10^{-10}\left(\frac{Z}{V_0}\right)m$$

$$\Rightarrow \quad r_0 = 14.4\left(\frac{Z}{V_0}\right)\text{Å}$$

Hence, the correct answer is (A).

15. According to Ritz Combination Principle

$$\nu_{m\to n} = \nu_{m\to i} + \nu_{i\to n} \qquad \{\text{for } m < i < n\}$$

e.g., $\nu_{4\to 1} = \nu_{4\to 3} + \nu_{3\to 1}$ or

$$\nu_{4\to 1} = \nu_{4\to 2} + \nu_{2\to 1}$$

Hence, the correct answer is (A).

16. $R = n^2 R_0$

$$\Rightarrow \quad \log R = \log R_0 + 2\log n$$

$$\Rightarrow \quad \log\left(\frac{R}{R_0}\right) = 2\log n$$

$$\Rightarrow \quad y = mx$$

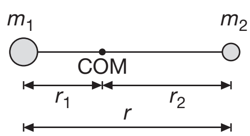
Hence, the correct answer is (D).

17. A diatomic molecule consists of two atoms of masses m_1 and m_2 at a distance r apart. Let r_1 and r_2 be the distances of the atoms from the centre of mass.

The moment of inertia of this molecule about an axis passing through its centre of mass and perpendicular to a line joining the atoms is

$$I = m_1 r_1^2 + m_2 r_2^2$$

As $m_1 r_1 = m_2 r_2$



$$\Rightarrow \quad r_1 = \frac{m_2}{m_1} r_2$$

$$\Rightarrow \quad r_1 + r_2 = r$$

$$\Rightarrow \quad r_1 = \frac{m_2}{m_1}(r - r_1)$$

On rearranging, we get $r_1 = \dfrac{m_2 r}{m_1 + m_2}$

Similarly, $r_2 = \dfrac{m_1 r}{m_1 + m_2}$

Therefore, the moment of inertia can be written as

$$I = m_1\left(\frac{m_2 r}{m_1 + m_2}\right)^2 + m_2\left(\frac{m_1 r}{m_1 + m_2}\right)^2 = \frac{m_1 m_2}{m_1 + m_2} r^2 \qquad \ldots(1)$$

According to Bohr's quantisation condition

$$L = \frac{nh}{2\pi} = n\hbar \qquad \left\{\because \hbar = \frac{h}{2\pi}\right\}$$

$$\Rightarrow \quad L^2 = \frac{n^2 h^2}{4\pi^2} = n^2\hbar^2 \qquad \ldots(2)$$

Rotational energy, $E = \dfrac{L^2}{2I}$

$$\Rightarrow \quad E = \frac{n^2}{8\pi^2 I} = \frac{n^2\hbar^2}{2I} \qquad \{\text{using (2)}\}$$

$$\Rightarrow \quad E = \frac{n^2 h^2 (m_1 + m_2)}{8\pi^2 (m_1 m_2) r^2} = \frac{n^2\hbar^2}{2}\frac{(m_1 + m_2)}{(m_1 m_2) r^2} \qquad \{\text{using (1)}\}$$

$$\Rightarrow \quad E = \frac{n^2\hbar^2 (m_1 + m_2)}{2 m_1 m_2 r^2}$$

Hence, the correct answer is (C).

18. Since $R' = \dfrac{\mu R}{m} = \dfrac{MR}{M + m}$

$$\Rightarrow \quad \frac{1}{\lambda} = R'\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\Rightarrow \quad \frac{1}{\lambda} = \frac{MR}{M + m}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Hence, the correct answer is (D).

19. $\dfrac{1}{\lambda} = R_H\left[\dfrac{1}{n^2} - \dfrac{1}{(n+1)^2}\right]$

$$\Rightarrow \quad \frac{1}{\lambda} = R_H\left[\frac{(2n+1)}{n^2(n+1)^2}\right]$$

$$\Rightarrow \quad \frac{1}{\lambda} \simeq R_H \frac{2n}{n^4}$$

$\{\because$ for large n, $2n + 1 \simeq 2n \quad n^2(n+1)^2 \simeq n^4\}$

$$\Rightarrow \quad \frac{1}{\lambda} \simeq R_H \frac{2}{n^3}$$

$$\Rightarrow \quad \nu = \frac{c}{\lambda} = \frac{2R_H c}{n^3}$$

Hence, the correct answer is (A).

20. $\dfrac{1}{\lambda} = RZ^2\left(\dfrac{1}{n_2^2} - \dfrac{1}{n_1^2}\right)$

$$\Rightarrow \quad \frac{1}{\lambda_{\text{Lyman}}} = RZ^2\left(\frac{1}{1} - \frac{1}{4}\right) = \frac{3}{4}RZ^2$$

$$\Rightarrow \quad \frac{1}{\lambda_{\text{Balmer}}} = RZ^2\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5}{36}RZ^2$$

$$\Rightarrow \quad \frac{\lambda_{\text{Lyman}}}{\lambda_{\text{Balmer}}} = \left(\frac{5}{36}\right)\left(\frac{4}{3}\right) = \frac{5}{27}$$

Hence, the correct answer is (C).

22. $n = 3$ ———————— $2E$

$n = 2$ ———————— $\frac{4E}{3}$

$n = 1$ ———————— E

For $3 \to 1$, $\frac{hc}{\lambda} = 2E - E = E$

For $2 \to 1$, $\frac{4E}{3} - E = \frac{E}{3} = \frac{hc}{\lambda'}$

$\Rightarrow \quad \lambda' = 3\lambda$

Hence, the correct answer is (D).

24. $m_\alpha v_\alpha = m_D v_D$

$\Rightarrow \quad v_D = \left(\frac{m_\alpha}{m_D}\right) v_\alpha = \left(\frac{4}{A-4}\right) v$

Hence, the correct answer is (C).

25. $\lambda = 2\pi r$

where $r =$ Radius of first orbit $= 0.53$ Å

$\Rightarrow \quad \lambda = 2\pi(0.53)$ Å $= 3.33$ Å

Hence, the correct answer is (D).

26. For UV radiation, $n_f = 1$

$\lambda = 1025$ Å

$\Rightarrow \quad \frac{1}{\lambda} = R(1)^2\left(\frac{1}{1^2} - \frac{1}{n_i^2}\right)$

$\Rightarrow \quad 1 - \frac{1}{n_i^2} = \frac{1}{\lambda R}$

$\sqrt{1 - \frac{1}{\lambda R}} = \frac{1}{n_i}$

$\Rightarrow \quad n_i = \frac{1}{\sqrt{1 - \frac{1}{\lambda R}}} = \frac{1}{\sqrt{1 - 0.89}} \approx 3$

Hence, the correct answer is (C).

27. Since, $r_n \propto n^2$

Given, $r_{n+1} - r_n = r_{n-1}$

$\Rightarrow \quad (n+1)^2 - n^2 = (n-1)^2$

$\Rightarrow \quad n = 4$

Hence, the correct answer is (D).

28. $\frac{nh}{2\pi} = \frac{2h}{\pi}$

$\Rightarrow \quad n = 4$

In fourth orbit, $KE = -TE = \frac{13.6}{(4)^2}$ eV $= 0.85$ eV

Hence, the correct answer is (C).

29. Since $F = -\frac{dU}{dr}$

$\Rightarrow \quad F = kr = \frac{mv^2}{r}$

$\Rightarrow \quad v^2 = \frac{k}{m} r^2$

$\Rightarrow \quad v = \sqrt{\frac{k}{m}} r$ …(1)

Also, $mvr = \frac{nh}{2\pi}$ …(2)

Solving (1) and (2), we get

$m\left(\sqrt{\frac{k}{m}} r\right) r = \frac{nh}{2\pi}$

$\Rightarrow \quad r \propto \sqrt{n}$

Since, $E = PE + KE$

$\Rightarrow \quad E = \frac{1}{2} kr^2 + \frac{1}{2} m\left(\sqrt{\frac{k}{m}} r\right)^2$

$\Rightarrow \quad E \propto r^2$

$\Rightarrow \quad E \propto n$

Hence, the correct answer is (B).

30. Since, $\frac{3h}{2\pi} = n\left(\frac{h}{2\pi}\right)$

$\Rightarrow \quad n = 3$

$\Rightarrow \quad K_n = \frac{K_1}{(3)^2} = \frac{13.6}{9} = 1.51$ eV

Hence, the correct answer is (A).

31. $f = \frac{c}{\lambda} = \frac{3 \times 10^8}{0.0709 \times 10^{-9}} = 4.23 \times 10^{18}$ Hz

From Moseley's law

$Z - 1 = \sqrt{\frac{4v}{3cR}} \approx 41$

$\Rightarrow \quad Z = 42$

This Z corresponds to Molybdenum.

Hence, the correct answer is (D).

32. Let E_K, E_L, E_M, E_N be the binding energies of K, L, M and N shell. Let E_P be energy of incident photon, then

$E_P - E_K = 24$ keV …(1)

$E_P - E_L = 100$ keV …(2)

$E_P - E_M = 110$ keV …(3)

$\Rightarrow \quad E(K_\alpha) = E_K - E_L = 100 - 24 = 76$ keV

Hence, the correct answer is (B).

33. $\lambda = \dfrac{hc}{eV_0}$

$\Rightarrow \quad \lambda = \dfrac{6.626\times10^{-24}\times3\times10^{8}}{\left(20\times10^{3}\right)\left(1.6\times10^{-19}\right)}$

$\Rightarrow \quad \lambda = 0.62\ \text{Å}$

Hence, the correct answer is (B).

34. Atomic number and melting point both should be high.
Hence, the correct answer is (D).

36. Since, $\lambda \propto \dfrac{1}{(Z-1)^2}$

$\Rightarrow \quad \dfrac{\lambda_1}{\lambda_2} = \left(\dfrac{Z_2-1}{Z_1-1}\right)^2 = \dfrac{1}{4}$

$\Rightarrow \quad 2(Z_2-1) = (Z_1-1)$

$\Rightarrow \quad Z_2 = 1 + \dfrac{Z_1-1}{2} = 1 + \dfrac{11-1}{2} = 6$

Hence, the correct answer is (A).

38. The frequency of emitted photon making a transition from n to $(n-1)$ level is

$f \propto \left(\dfrac{1}{(n-1)^2} - \dfrac{1}{n^2}\right)$

$\Rightarrow \quad f \propto \left[\dfrac{n^2-(n-1)^2}{n(n-1)^2}\right]$

$\Rightarrow \quad f \propto \dfrac{(2n-1)}{n^2(n-1)^2}$

For $n \gg 1$, we have $n-1 \approx n$, $2n-1 \approx 2n$

$\Rightarrow \quad f \propto \dfrac{2n}{n^4}$

$\Rightarrow \quad f \propto \dfrac{1}{n^3}$

Hence, the correct answer is (A).

39. For $0 < E_K < 10.2\ \text{eV}$, the collision of electron (having kinetic energy E_K) with a H-atom in its ground state will be elastic.
Hence, the correct answer is (A).

40. The binding energy is numerically equal to the kinetic energy of the electron, so

$\dfrac{1}{2}mv^2 = E_1$ ...(1)

Since, $mvr = \dfrac{nh}{2\pi}$ ...(2)

Dividing Equation (1) by Equation (2), we get

$\dfrac{v}{2\pi r} = \dfrac{2E_1}{nh}$

$\Rightarrow \quad f = \dfrac{2E_1}{nh}$ $\left\{\because \dfrac{v}{2\pi r} = \dfrac{\omega}{2\pi} = f\right\}$

Hence, the correct answer is (D).

41. Number of orbits i.e. number of revolutions i.e. frequency (f) is proportional to $\dfrac{Z^2}{n^3}$

Since $T_0 \propto \dfrac{n^3}{Z^2}$

$\Rightarrow \quad T' = 8T_0$

$\Rightarrow \quad N = \dfrac{\text{Life Time of Excited state}}{\text{Time for one Revolution}}$

$\Rightarrow \quad N = \dfrac{10^{-8}}{8T_0}$

Hence, the correct answer is (C).

42. $\dfrac{\text{Magnetic moment}}{\text{Angular momentum}} = \dfrac{e}{2m}$

$\Rightarrow$ Magnetic moment $\propto$ Angular Momentum

$\Rightarrow \quad M \propto n$ $\left\{\because L = n\dfrac{h}{2\pi}\right\}$

Hence, the correct answer is (B).

43. Since $v \propto \dfrac{1}{n}$ and $r \propto n^2$

$\Rightarrow \quad v \propto \dfrac{1}{\sqrt{r}}$

Since, $L = mvr$

$\Rightarrow \quad L \propto vr$

$\Rightarrow \quad L \propto \left(\dfrac{1}{\sqrt{r}}\right)r$

$\Rightarrow \quad L \propto \sqrt{r}$

Hence, the correct answer is (C).

44. Cut-off wavelength depends on the applied voltage not on the atomic number of the target. Characteristic wavelengths depend on the atomic number of target.
Hence, the correct answer is (B).

45. Bohr has assumed stationary orbits in which there is no gain or loss of energy and angular momentum in any orbit does not change.
Hence, the correct answer is (C).

46. $L = \sqrt{n(n+1)}\dfrac{h}{2\pi}$

$\Rightarrow \quad L = \sqrt{56}\left(\dfrac{h}{2\pi}\right)$

Hence, the correct answer is (D).

47. $a = \frac{v^2}{r} = \frac{Z^2}{n^2} \times \frac{Z}{n^2}$

$\Rightarrow \quad a \propto Z^3$

$\Rightarrow \quad \frac{a_{He^+}}{a_H} = \frac{Z^3_{He^+}}{Z^3_H} = \frac{2^3}{1^3} = \frac{8}{1}$

Hence, the correct answer is (B).

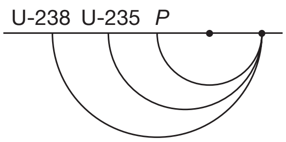
48. $R_n = n^2 R_0$

$\Rightarrow \quad R_2 = 4(0.528 \text{ Å})$

$\Rightarrow \quad R_2 = 2.112 \text{ Å}$

Hence, the correct answer is (B).

49.



$$r_{235} = \frac{mv}{qB} = \frac{(235m_p)v}{eB} = 235 \times 10 = 2350 \text{ m}$$

$$r_{238} = \frac{(238m_p)v}{eB} = 2380 \text{ mm}$$

The separation between the ions of U^{235} and U^{238} is

$\Delta r = 2(r_{238} - r_{235})$

$\Rightarrow \quad \Delta r = 2 \times (2380 - 2350)$

$\Rightarrow \quad \Delta r = 60 \text{ mm}$

Hence, the correct answer is (A).

50. Speed of electron in the nth orbit of H like atom is

$$v_n = \frac{1}{137}\left(\frac{cZ}{n}\right)$$

Hence, the correct answer is (C).

51. Since $v_n = \left(\frac{e^2}{2h\varepsilon_0}\right)\frac{Z}{n}$

$\Rightarrow \quad v = \left(\frac{e^2}{2h\varepsilon_0}\right)\left(\frac{2}{4}\right)$

$\Rightarrow \quad v = \frac{e^2}{4h\varepsilon_0}$

Hence, the correct answer is (B).

52. Since $\Delta p = \frac{h}{\lambda}$

$\Rightarrow \quad m_H v = hR_H\left(\frac{1}{1} - \frac{1}{25}\right)$

$\Rightarrow \quad v = \frac{hR_H}{m_H}\left(\frac{24}{25}\right)$

$\Rightarrow \quad v = 4 \text{ ms}^{-1}$

Hence, the correct answer is (C).

53. $r \propto n^2$

$\Rightarrow \quad \frac{5.3 \times 10^{-11}}{21.2 \times 10^{-11}} = \frac{1}{n^2}$

$\Rightarrow \quad n^2 = 4$

$\Rightarrow \quad n = 2$

Hence, the correct answer is (B).

54. (i) $a = \frac{v^2}{r} \propto \frac{Z^2}{n^2} \times \frac{Z}{n^2}$

$\Rightarrow \quad a \propto \frac{Z^2}{n^4}$

(ii) $L = mvr = \frac{nh}{2\pi}$

$\Rightarrow \quad L \propto n$

(iii) $KE = \frac{Ze^2}{8\pi\varepsilon_0 r}$

$KE \propto \frac{1}{r}$

$\Rightarrow \quad KE \propto \frac{1}{n^2}$

Hence, the correct answer is (C).

55. $R_H\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = R_H\left(\frac{9}{n_1^2} - \frac{9}{n_2^2}\right)$

Substituting $n_1 = 6$, $n_2 = 12$ makes both sides equal

Hence, the correct answer is (C).

56. Ionization energy, $E_0 \propto m$

$\Rightarrow \quad \frac{(E_0)_2}{(E_0)_1} = \frac{m_2}{m_1} = \frac{207\text{me}}{me}$

$\Rightarrow \quad (E_0)_2 = 207(E_0)_1$

$\Rightarrow \quad (E_0)_2 = 207 \times 13.6 \text{ eV}$

$\Rightarrow \quad (E_0)_2 = 2.82 \text{ keV}$

Hence, the correct answer is (C).

57. $a = \frac{v^2}{r}$

$\Rightarrow \quad a \propto \frac{(z)^2}{\left(\frac{1}{z}\right)} \qquad \{\text{for } n = 1\}$

$\Rightarrow \quad a \propto z^3$

$$\Rightarrow \quad \frac{a_1}{a_2} = \left(\frac{2}{1}\right)^3 = 8$$

Hence, the correct answer is (B).

58. $r_n = n^2 r_1$

$\Rightarrow \quad \log r_n = \log(n^2) + \log r_1$

$\Rightarrow \quad \log r_n - \log r_1 = 2\log n$

$$\Rightarrow \quad \log\left(\frac{r_n}{r_1}\right) = 2\log n$$

$\Rightarrow \quad y = kx$

So, $\log\left(\frac{r_n}{r_1}\right)$ vs $\log n$ is a straight line passing through origin, with slope $m = 2$

Hence, the correct answer is (A).

59. Maximum wavelength of Lyman series will correspond to the transition of electron from $n = 2$ to $n = 1$ and maximum wavelength of Paschen series will correspond to $n = 4$ to $n = 3$.

$$\Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \frac{\left(\frac{1}{9} - \frac{1}{16}\right)}{\left(\frac{1}{1} - \frac{1}{4}\right)} = \frac{7}{108}$$

Hence, the correct answer is (D).

60. Rydberg constant $R \propto m$

For positronium atom, reduced mass is $\mu = \frac{mM}{m+M}$.

Here, $m = M =$ mass of positron = mass of electron.

$$\Rightarrow \quad \mu = \frac{m}{2}$$

$$\frac{R'}{R} = \frac{\mu}{m} = \frac{1}{2}$$

$$\Rightarrow \quad R' = \frac{R}{2}$$

Hence, the correct answer is (C).

61. $v_n = \alpha\left(\frac{cZ}{n}\right)$ where $\alpha = \frac{e^2}{2h\varepsilon_0 c}$

is the fine structure constant $\left(\alpha \simeq \frac{1}{137}\right)$

$$v_{He^+} = \alpha\left(\frac{c(2)}{2}\right) = \alpha c$$

and $v_H = \alpha\frac{c(1)}{(1)} = \alpha c = v_{He^+}$

Hence, the correct answer is (B).

62. Since $r \propto \frac{n^2}{Z}$

$$\Rightarrow \quad \frac{a}{a_0} = \frac{(3)^2}{(2)^2}$$

$$\Rightarrow \quad a = \frac{9}{4}a_0$$

Hence, the correct answer is (B).

63. Momentum of striking electrons

$$p = \frac{h}{\lambda}$$

Kinetic energy of striking electrons

$$K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

This is also, maximum energy of X-rays photons.

$$\frac{hc}{\lambda_0} = \frac{h^2}{2m\lambda^2}$$

$$\Rightarrow \quad \lambda_0 = \frac{2m\lambda^2 c}{h}$$

Hence, the correct answer is (A).

64. $$\frac{1}{\lambda} = RZ^2\left(\frac{1}{3^2} - \frac{1}{\infty}\right) = R(1)^2\left(\frac{1}{1^2} - \frac{1}{\infty}\right)$$

$\Rightarrow \quad Z = 3$

Hence, the correct answer is (B).

65. Total power drawn by Coolidge tube is VI.

$\Rightarrow \quad P_{total} = VI = 200$ W

Since only 0.5% is carried by X-rays, so

$$P = \left(\frac{0.5}{100}\right)P_{total}$$

$$\Rightarrow \quad P = \frac{1}{200}(200)\text{W}$$

$\Rightarrow \quad P = 1$ W

Hence, the correct answer is (B).

66. $$evB = \frac{mv^2}{R}$$

$$\Rightarrow \quad v = \frac{eBR}{m}$$

Maximum kinetic energy of photoelectron is

$$K_{max} = \frac{1}{2}mv^2 = \frac{e^2B^2R^2}{2m} = 2.97 \times 10^{-15}\text{ J}$$

$\Rightarrow \quad K_{max} = 2.97 \times 10^{-15}$ J = 18.6 keV

$\Rightarrow \quad (BE)_{Kshell} = (24.8 - 18.6)$ keV

$\Rightarrow \quad (BE)_{\text{Kshell}} = 6.2 \text{ keV}$

Hence, the correct answer is (A).

67. $f_n = \dfrac{v_n}{2\pi r_n} = \dfrac{\dfrac{e^2}{2h\varepsilon_0 c}\dfrac{cZ}{n}}{2\pi\dfrac{n^2h^2\varepsilon_0}{\pi me^2Z}}$

$\Rightarrow \quad f_n = \left(\dfrac{me^4Z^2}{4\varepsilon_0^2h^3}\right)\dfrac{1}{n^3}$

Hence, the correct answer is (B).

68. Frequency, $f = \left(\dfrac{E_0}{h}\right)\left(\dfrac{2}{n^3}\right)$

Since, $E_0 = 13.6$ eV and $n = 2$

$\Rightarrow \quad f = 0.823 \times 10^{15}$ rev/sec

Number of revolutions $N = f \times \Delta t = 8.23 \times 10^6$ rev

Hence, the correct answer is (A).

70. $\dfrac{1}{\lambda} = RZ^2\left(\dfrac{1}{n_2^2} - \dfrac{1}{n_1^2}\right)$

$\dfrac{1}{912} = R(1)\left(\dfrac{1}{1} - \dfrac{1}{\infty}\right)$ and $\dfrac{1}{\lambda} = R(1)\left(\dfrac{1}{4} - \dfrac{1}{\infty}\right)$

$\Rightarrow \quad \dfrac{\lambda}{912} = 4$

$\Rightarrow \quad \lambda = 912 \times 4 = 3648$ Å

Hence, the correct answer is (A).

71. $\dfrac{hc}{\lambda_1} + \dfrac{hc}{\lambda_2} = RchZ^2\left(\dfrac{1}{1^2} - \dfrac{1}{n^2}\right)$

Substituting $\lambda_1 = 1026.7$ Å, $\lambda_2 = 304$ Å and $Z = 2$, we get

$n = 6$

Hence, the correct answer is (B).

72. $\dfrac{1}{\lambda} = R_H\left(\dfrac{1}{4} - \dfrac{1}{16}\right)$

Since $c = \nu\lambda$

$\Rightarrow \quad \nu = \dfrac{c}{\lambda}$

$\Rightarrow \quad \nu = 3 \times 10^8 \times 10^7 \times \left(\dfrac{3}{16}\right)$

$\Rightarrow \quad \nu = \dfrac{9}{16} \times 10^{15}$ Hz

Hence, the correct answer is (C).

73. $J_n = mvr = \dfrac{nh}{2\pi}$

$\Rightarrow \quad J_n \propto n$

Since, $E_n \propto \dfrac{1}{n^2}$

$\Rightarrow \quad E_n \propto \dfrac{1}{J_n^2}$

Hence, the correct answer is (D).

74. $E = R_\infty hc\left(1 - \dfrac{1}{25}\right)$

Momentum of photon emitted is

$p = \dfrac{E}{c} = R_\infty h\left(\dfrac{24}{25}\right)$

Recoil momentum of H-atom will also be p.

$\Rightarrow \quad mv = p$

$\Rightarrow \quad v = \dfrac{p}{m} = \dfrac{(1.097 \times 10^7)(6.626 \times 10^{-34})24}{(25)(1.67 \times 10^{-27})}$

$\Rightarrow \quad v = 4.178 \text{ ms}^{-1}$

Hence, the correct answer is (C).

76. Since $E = -\dfrac{13.6}{n^2}$ eV

$\Rightarrow \quad E_1 = -13.6$ eV

$\Rightarrow \quad E_2 = -3.4$ eV

$\Rightarrow \quad E_3 = -1.50$ eV

$\Rightarrow \quad E_4 = -0.85$ eV

From above we can see that

$E_3 - E_1 = 12.1$ eV

i.e. the electron must be making a transition from $n = 3$ to $n = 1$ level.

$\Rightarrow \quad \Delta L = (3-1)\dfrac{h}{2\pi} = \dfrac{h}{\pi}$

$\Rightarrow \quad \Delta L = \dfrac{6.626 \times 10^{-34}}{3.14}$

$\Rightarrow \quad \Delta L = 2.11 \times 10^{-34}$ Js

Hence, the correct answer is (B).

77. $PE = 2(TE)$

$\Rightarrow \quad PE = 2 \times (-13.6) = -27.2$ eV

Hence, the correct answer is (A).

78. Change in angular momentum

$$\Delta L = (n_f - n_i)\frac{h}{2\pi}$$

Since velocity of electron is

$$v \propto \frac{1}{n}$$

Hence linear momentum changes and difference in energy between energy levels is released as electromagnetic energy.
Hence, the correct answer is (D).

79. For the fifth electron to act as a dopant it must lie in the valence shell of P.
So $n = 3$. (TRICKY HINT !)

$$Z = 15 \qquad \text{\{for Phosphorus\}}$$

Since $r_n = \dfrac{n^2h^2\varepsilon_0}{\pi me^2Z} = \dfrac{n^2}{Z}r_0$

where $r_0 = 0.529$ Å $= 52.9$ pm

$$\Rightarrow \quad r_3 = \frac{9}{15}(52.9 \text{ pm}) \qquad \ldots(1)$$

When phosphorus acts as dopant in silicon, the expression for Bohr radius is

$$r_n' = \frac{n^2h^2\varepsilon}{\pi me^2Z} = kr_n$$

$$\Rightarrow \quad r_n' = 12r_n \qquad \{\because \varepsilon = k\varepsilon_0\}$$

$$\Rightarrow \quad r_3' = 12r_3 = 12\left[\frac{9}{15}(52.9)\text{pm}\right]$$

$$\Rightarrow \quad r_3' = 380.88 \text{ pm}$$

Hence, the correct answer is (A).

80. The energy difference between the energy levels in an atom remains fixed. Hence wavelength remains fixed and is given by

$$\lambda = \frac{hc}{\Delta E} = \frac{hc}{E_f - E_i}$$

Increasing number of atoms would increase the intensity absorbed.
Hence, the correct answer is (A).

81. Angular momentum, $L = n\left(\dfrac{h}{2\pi}\right) = mvr$

Magnetic moment, $\mu = iA = \left(\dfrac{e}{2\pi r}v\right)(\pi r^2)$

$$\Rightarrow \quad \mu = \frac{e(vr)}{2}$$

$$\Rightarrow \quad \mu = \frac{e}{2}\left(\frac{L}{m}\right)$$

$$\Rightarrow \quad \frac{\mu}{L} = \frac{e}{2m}$$

Hence, the correct answer is (AB).

82. Since, $T \propto n^3$ and $r \propto n^2$

$$\Rightarrow \quad T \propto r^{\frac{3}{2}}$$

Hence, the correct answer is (B).

83. $mvr = n\dfrac{h}{2\pi} = 2\left(\dfrac{h}{2\pi}\right)$

$$\Rightarrow \quad mvr = \frac{h}{\pi}$$

So, de-Broglie wavelength is

$$\lambda = \frac{h}{mv} = \pi r = (3.14)(2.116 \text{ Å}) = 6.64 \text{ Å}$$

Hence, the correct answer is (C).

84. λ is maximum

$\Rightarrow \quad E_{\min}$ i.e. transition from $3 \to 2$.

$$\frac{1}{\lambda_{\max}} = R(1)^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\Rightarrow \quad \frac{1}{\lambda_{\max}} = R\left(\frac{5}{36}\right)$$

$$\lambda_{\max} = \frac{36}{5R} = \frac{36}{5} \times 912 \text{ Å} = 6560 \text{ Å} = 656 \text{ nm}$$

Hence, the correct answer is (C).

85. Since, $r = \dfrac{\varepsilon_0 n^2h^2}{e^2\pi m}$

$$L = n\left(\frac{h}{2\pi}\right)$$

$$\Rightarrow \quad nh = 2\pi L$$

$$\Rightarrow \quad r = \frac{\varepsilon_0(2\pi L)^2}{e^2\pi m}$$

$$\Rightarrow \quad Lr^{-\frac{1}{2}} = \text{constant}$$

Hence, the correct answer is (D).

86. $\Delta E = 13.6\left(1 - \dfrac{1}{9}\right) = 12.1$ eV

Hence, the correct answer is (B).

87. Since K.E. = –T.E.
$\Rightarrow$ K.E. = +13.6 eV
Hence, the correct answer is (B).

88. The energy E_n of an electron in n^{th} orbit of positronium is given by

$$E_n = -\frac{\mu e^4}{8n^2h^2\varepsilon_0^2},$$

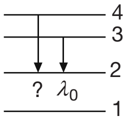
where $\mu_e = \frac{mM}{m+M} = \frac{m}{2} = \frac{m_e}{2}$ $\qquad \{\because M = m_e\}$

$$\Rightarrow \quad E_n = -\left(\frac{me^4}{8\varepsilon_0^2h^2}\right)\frac{1}{2n^2}$$

$$\Rightarrow \quad E_n = -\frac{13.6}{2n^2}\text{ eV} = -\frac{6.8}{n^2}\text{ eV}$$

Hence, the correct answer is (D).

90.



$$\frac{1}{\lambda_0} = R\left(\frac{1}{4}-\frac{1}{9}\right)$$

$$\lambda_0 = \frac{36}{5R} \qquad \ldots(1)$$

$$\frac{1}{\lambda} = R\left(\frac{1}{4}-\frac{1}{16}\right)$$

$$\lambda = \frac{16}{3R} \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{\lambda}{\lambda_0} = \frac{16/3R}{36/5R} = \frac{20}{27}$$

$$\Rightarrow \quad \lambda = \frac{20}{27}\lambda_0$$

Hence, the correct answer is (B).

91. According to Ritz Combination Principle

$$E_{C\to A} = E_{C\to B} + E_{B\to A}$$

$$\Rightarrow \quad \frac{hc}{\lambda_3} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$$

$$\Rightarrow \quad \lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}$$

Hence, the correct answer is (B).

92. For 12.1 eV energy, the electron is excited to third orbit, so total lines $N = \frac{n(n-1)}{2}$

$$\Rightarrow \quad N = \frac{3\times 2}{2} = 3\text{ lines}$$

Hence, the correct answer is (C).

93. For each principal quantum number n, number of electrons permitted equals the number of elements corresponding to the quantum number.

$$\Rightarrow \quad \begin{pmatrix}\text{Total Number}\\ \text{of Elements}\end{pmatrix} = \sum 2n^2 = \frac{n(n+1)(2n+1)}{3}$$

Hence, the correct answer is (D).

94. Since $E = \frac{\Delta V}{\Delta r}$

$$\Rightarrow \quad \Delta V = E\Delta r$$

$$\Rightarrow \quad 10.5 = (1.5\times 10^6)\Delta r$$

$$\Rightarrow \quad \Delta r = \frac{10.5}{1.5\times 10^6} = 7\times 10^{-6}\text{ m}$$

$$\Rightarrow \quad \Delta r = 7\ \mu\text{m}$$

Hence, the correct answer is (A).

95. $I = I_0 e^{-\mu x}$

Where I_0 is intensity at $x = 0$,

I is intensity at a distance x and

$\Rightarrow$ μ is absorption coefficient; $[\mu] = L^{-1}$

μ is maximum for lead as lead has maximum ability to absorb radiations in a minimum distance.

Hence, the correct answer is (B).

96. $K_\alpha = 7.7\text{ MeV}$

$$r_0 = \frac{(79e)(2e)}{4\pi\varepsilon_0 K_\alpha} = \frac{9\times 10^9\times 79\times 2\times e^2}{7.7\times 10^6\times e}$$

$$\Rightarrow \quad r_0 = \frac{9\times 79\times 2\times 10^9\times 1.6\times 10^{-19}}{7.7\times 10^6}$$

$$\Rightarrow \quad r_0 = \frac{9\times 79\times 2\times 1.6}{7.7}\times 10^{-16}\text{ m} = 3\times 10^{-14}\text{ m}$$

Hence, the correct answer is (B).

97. Since $v = \frac{e^2}{2h\varepsilon_0}\left(\frac{Z}{n}\right)$

For fourth orbit of Be^{+++}, $n = 4$, $Z = 4$

$$\Rightarrow \quad v = \frac{e^2}{2h\varepsilon_0}$$

Hence, the correct answer is (D).

98. $\lambda = \frac{hc}{E}$

$$\Rightarrow \quad \lambda = \frac{(6.626\times 10^{-34})(3\times 10^8)}{1.6\times 10^{-16}\ E(\text{in keV})}$$

$$\Rightarrow \quad \lambda = \frac{1.242\times 10^{-9}}{E(\text{in keV})}\text{ m}$$

$$\Rightarrow \quad \lambda = \frac{12.42}{E(\text{in keV})}\text{ Å}$$

Hence, the correct answer is (B).

99. Since $B \propto \frac{i}{r}$ $i \propto \frac{f}{r}$

$$\Rightarrow \quad B \propto \frac{Z^3}{n^5}$$

$$\Rightarrow \quad B' = \frac{B}{(2)^5} = \frac{B}{32}$$

Hence, the correct answer is (C).

100. $\tau = \left|\frac{\Delta L}{\Delta t}\right| = \left|\frac{L_f - L_i}{\Delta t}\right|$

$$\Rightarrow \quad \tau = \left|\frac{2\left(\frac{h}{2\pi}\right) - 3\left(\frac{h}{2\pi}\right)}{10^{-8}}\right|$$

$$\Rightarrow \quad \tau = 10^8\left(\frac{h}{2\pi}\right) = \frac{(10^8)(6.63 \times 10^{-34})}{2 \times 3.14}$$

$$\Rightarrow \quad \tau \approx 10^{-26} \text{ Nm}$$

Hence, the correct answer is (B).

101. Let the electron be initially in the n^{th} orbit, then the max energy is liberated for transition from $n \to 1$ and the minimum energy for transition between $n \to (n-1)$. If E_1 be the energy of the electron in the first orbit, then we have

$$E_1 - \frac{E_1}{n^2} = 52.224 \text{ eV} \quad \ldots(1)$$

$$\frac{E_1}{(n-1)^2} - \frac{E_1}{n^2} = 1.224 \text{ eV} \quad \ldots(2)$$

Solving (1) and (2), we get

$$E_1 = -54.4 \text{ eV}$$

Since, $E_1 = \frac{-13.6Z^2}{1^2}$

$$\Rightarrow \quad Z = 2$$

Hence, the correct answer is (C).

102. $E = -\frac{13.6}{n^2}$ eV

$$\Rightarrow \quad E_2 = -3.4 \text{ eV}$$

Hence, the correct answer is (D).

103. Energy of emitted photon is

$$E = \frac{hc}{\lambda} = 2.5 \text{ eV}$$

The excitation energy is the energy required to excite the atom to a level just above the ground state. Therefore, energy of the level is

$$E = -13.6 + 10.2 = -3.4 \text{ eV}$$

Since, photon arises from transition between energy levels such that

$$E_i - E_f = h\nu = 2.5 \text{ eV}$$

$$\Rightarrow \quad E_i = 2.5 + E_f$$

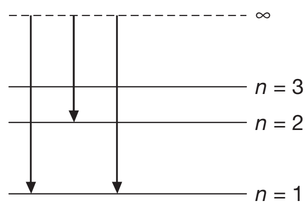
$$\Rightarrow \quad E_i = 2.54 + E = 2.54 - 3.4 \text{ eV} = -0.9 \text{ eV}$$

Hence, the correct answer is (D).

104. Series limit (i.e. shortest wavelength) of Lyman implies transition from $\infty \to 1$

First line (or longest wavelength) of Lyman series implies transition from $2 \to 1$

Series limit of Balmer implies transition from $\infty \to 2$



According to Ritz Combination Principle

$$E_{\infty \to 1} = E_{\infty \to 2} + E_{2 \to 1}$$

$$\Rightarrow \quad h\nu_1 = h\nu_3 + h\nu_2$$

$$\Rightarrow \quad \nu_1 - \nu_2 = \nu_3$$

Hence, the correct answer is (A).

105. $\Delta E = E_2 - E_1 = 10.2 \text{ eV} = -3.4 \text{ eV} + 13.6 \text{ eV}$

$$\Rightarrow \quad n_2 = 2 \text{ and } n_1 = 1$$

$$\Rightarrow \quad \Delta L = \frac{2h}{2\pi} - \frac{h}{2\pi} = \frac{h}{2\pi} = \frac{6.63 \times 10^{-34}}{2 \times 3.14} \text{ Js}$$

$$\Rightarrow \quad \Delta L = 1.05 \times 10^{-34} \text{ Js}$$

Hence, the correct answer is (A).

106. $E_n \propto \frac{1}{n^2}$ and $r_n \propto n^2$

So, $E_n r_n$ is independent of n

$$\Rightarrow \quad E_1 r_1 = (13.6 \text{ eV})(0.53 \text{ Å})$$

$$\Rightarrow \quad E_1 r_1 = 7.2 \text{ eVÅ}$$

$$\Rightarrow \quad E_1 r_1 = \text{constant}$$

Hence, the correct answer is (C).

107. For Balmer series, $n_f = 2$ and $n_i = n(>2)$

Since $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

$$\Rightarrow \quad \frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{n^2}\right)$$

$\Rightarrow \quad \lambda = \dfrac{4n^2}{R(n^2-4)}$

$\Rightarrow \quad k = \dfrac{4}{R}$

Hence, the correct answer is (D).

108. Since we assume the potential energy to be zero in the ground state. So,

Total Energy $= U + K =$ Kinetic Energy $\{\because U = 0\}$

$\Rightarrow$ Total Energy $= 13.6$ eV (in ground state)

If the potential energy is not assigned a zero value, then total energy is -13.6 eV.

So, we conclude that making potential energy zero increases the value of total energy by $13.6-(-13.6) = 27.2$ eV.

Now actual energy in second orbit $= -3.4$ eV

Hence new value is $(-3.4+27.2)$ eV $= 23.8$ eV

Hence, the correct answer is (C).

109. Loss in $KE = \dfrac{1}{2}\left[\dfrac{(m_H)(m_H)}{m_H + m_H}\right]v^2$

$\Rightarrow \quad -\Delta K = \dfrac{1}{4}m_H v^2$

This loss in kinetic energy must be equal to the energy required to take the electron from ground state to infinity i.e. 13.6 eV

$\Rightarrow \quad \dfrac{1}{4}m_H v^2 = 13.6 \times 1.6 \times 10^{-19}$

$\Rightarrow \quad v = 7.2 \times 10^4 \text{ ms}^{-1}$

Hence, the correct answer is (A).

110. Since, $E = -\dfrac{13.6}{n^2}$

$\Rightarrow \quad -3.4 = -\dfrac{13.6}{n^2}$

$\Rightarrow \quad n = 2$

Since, $\lambda = \dfrac{h}{mv}$

Velocity of electron in second orbit is

$$v_2 = \frac{c}{2(137)} = \frac{c}{274}$$

$\Rightarrow \quad \lambda = \dfrac{6.626 \times 10^{-34}}{(9.1 \times 10^{-31})\left(\dfrac{3 \times 10^8}{274}\right)}$

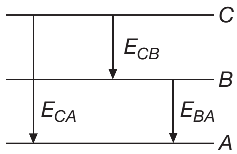
$\Rightarrow \quad \lambda \sim 6.6 \times 10^{-10}$ m

Further Kinetic Energy $= -($Total Energy$)$

$\Rightarrow \quad E = -(-3.4 \text{ eV}) = 3.4$ eV

Hence, the correct answer is (B).

111. Since $E_{C\to A} = E_{C\to B} + E_{B\to A}$



$\Rightarrow \quad \dfrac{hc}{\lambda_3} = \dfrac{hc}{\lambda_1} + \dfrac{hc}{\lambda_2}$

$\Rightarrow \quad \dfrac{1}{\lambda_3} = \dfrac{\lambda_1 + \lambda_2}{\lambda_1\lambda_2}$

$\Rightarrow \quad \lambda_3 = \dfrac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$

Hence, the correct answer is (D).

112. Shortest wavelength of Brackett series corresponds to the transition of electron between $n_1 = 4$ and $n_2 \to \infty$ and the shortest wavelength of Balmer series corresponds to the transition of electron between $n_1 = 2$ and $n_2 \to \infty$. So,

$$(Z^2)\left(\frac{13.6}{16}\right) = \left(\frac{13.6}{4}\right)$$

$\Rightarrow \quad Z^2 = 4$ or $Z = 2$

Hence, the correct answer is (A).

113. $\Delta L = \dfrac{nh}{2\pi} - \dfrac{(n-1)h}{2\pi} = \dfrac{h}{2\pi}$

Hence, the correct answer is (C).

114. The longest wavelength in a series is obtained when a transition takes place between the lowest consecutive levels. Here transition must take place from $n = 2$ to $n = 1$

Hence, the correct answer is (A).

115. K_α wavelength is smaller for target having larger Z. Cut-off wavelength is smaller for greater V.

Hence, the correct answer is (C).

116. $M = iA = \left(\dfrac{ev}{2\pi r}\right)\pi r^2 = \dfrac{evr}{2}$

Since, $v \propto \dfrac{1}{n}$ and $r \propto n^2$

$\Rightarrow \quad M \propto n$

Hence, the correct answer is (A).

117. Maximum angular speed will be in its ground state, so, we have

$$\omega_{max} = \frac{v_1}{r_1} = \frac{2.2 \times 10^6 \text{ ms}^{-1}}{0.529 \times 10^{-10} \text{ m}}$$

$\Rightarrow \quad \omega_{max} = 4.1 \times 10^{16} \text{ rads}^{-1}$

Hence, the correct answer is (B).

118. Since $v = \left(\dfrac{e^2}{2h\varepsilon_0}\right)\dfrac{Z}{n}$

$$\frac{v}{c} = \left(\frac{e^2}{2h\varepsilon_0 c}\right)\left(\frac{2}{1}\right) = \frac{e^2}{h\varepsilon_0 c}$$

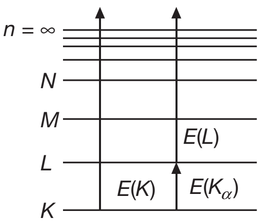
Hence, the correct answer is (C).

119. $\dfrac{hc}{\lambda} = Rhc\left(1 - \dfrac{1}{n^2}\right)$

$$\Rightarrow \quad n = \sqrt{\frac{\lambda R}{\lambda R - 1}}$$

Hence, the correct answer is (B).

120.



$$E(K) = \frac{hc}{\lambda_K} = \frac{12.4}{0.107} = 115.9 \text{ keV}$$

$$E(K_\alpha) = E(K) - E(L) = \frac{hc}{\lambda_\alpha} = 98.4 \text{ keV}$$

$$E_L = E(K) - E(K_\alpha) = 115.4 - 98.4$$

$$E_L = 17.5 \text{ keV}$$

$$\lambda_L = \frac{hc}{E_L} = \frac{12.4 \text{ keVÅ}}{17.5 \text{ keV}} = 0.709 \text{ Å}$$

Hence, the correct answer is (A).

121. According to Moseley's law, we have

$$\sqrt{\nu} \propto Z - 1$$

$$\Rightarrow \quad \nu \propto (Z-1)^2$$

$$\Rightarrow \quad \frac{\lambda_C}{\lambda_M} = \frac{(Z_M - 1)^2}{(Z_C - 1)^2}$$

$$\Rightarrow \quad \lambda_C = (0.71)\left(\frac{41}{28}\right)^2 = 1.52 \text{ Å}$$

Hence, the correct answer is (C).

122. When one electron is removed, the remaining atom is hydrogen like atom whose energy in first orbit is

$$E_1 = -(2)^2(13.6 \text{ eV}) \qquad (Z = 2)$$

$$\Rightarrow \quad E_1 = -54.4 \text{ eV}$$

Therefore, to remove the second electron an additional energy of 54.4 eV is required. Thus, to remove both the electrons $(24.6 + 54.4)$ eV $= 79$ eV energy is required.

Hence, the correct answer is (D).

123. Using $\dfrac{1}{\lambda} = R(Z-1)^2\left[\dfrac{1}{n_2^2} - \dfrac{1}{n_1^2}\right]$

For K_α radiation, $n_1 = 2$ and $n_2 = 1$

For metal A, we have

$$\frac{1875R}{4} = R(Z_1 - 1)^2\left(\frac{3}{4}\right)$$

$$\Rightarrow \quad Z_1 = 26$$

For metal B, we have

$$675R = R(Z_2 - 1)^2\left(\frac{3}{4}\right)$$

$$\Rightarrow \quad Z_2 = 30$$

Therefore, 4 elements lie between A and B.

Hence, the correct answer is (D).

124. $|E_K| = \dfrac{hc}{\lambda_K} = \dfrac{12.4 \text{ KeV Å}}{0.15 \text{ Å}} = 82.7 \text{ keV}$

The energy of incident photon

$$E = \frac{hc}{\lambda} = \frac{12.4}{0.1} = 124 \text{ keV}$$

The maximum kinetic energy is

$$K_{\max} = E - |E_K| = 41.3 \text{ keV} \approx 41 \text{ keV}$$

Hence, the correct answer is (A).

125. Number of possible emission lines are $\dfrac{n(n-1)}{2}$ when electron jumps from nth state to ground state. In this question this value is

$$N = \frac{(n-1)(n-2)}{2}$$

$$\Rightarrow \quad 10 = \frac{(n-1)(n-2)}{2}$$

Solving this, we get

$$n = 6$$

Hence, the correct answer is (A).

126. Frequency corresponding to wavelength of 0.180 nm is

$$\nu = \frac{c}{\lambda} = 1.67 \times 10^{18} \text{ Hz}$$

From Moseley's law, we have

$$\nu = \frac{3}{4}Rc(Z-1)^2$$

$$\Rightarrow \quad Z - 1 = \sqrt{\frac{4\nu}{3cR}} \approx 26$$

$\Rightarrow \quad Z = 27$

Hence element is cobalt.

Hence, the correct answer is (B).

127. First excited state is $n = 2$ and second excited state is $n = 3$.

Also,

$$E_n \propto \frac{1}{n^2}$$

$$\Rightarrow \frac{E_2}{E_3} = \frac{9}{4}$$

Hence, the correct answer is (D).

128. Since $\frac{1}{\lambda} = R\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$

$$\Rightarrow \frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R}{36}$$

$$\Rightarrow \frac{1}{\lambda_2} = R\left(\frac{1}{1} - \frac{1}{4}\right) = R\left(\frac{3}{4}\right) = \frac{3}{4}R$$

$$\Rightarrow \frac{\lambda_2}{\lambda_1} = \frac{5/36}{3/4} = \frac{5}{36} \times \frac{4}{3} = \frac{5}{27}$$

$$\Rightarrow \lambda_2 = \frac{5}{27}\lambda_1$$

$$\Rightarrow \lambda_2 = \frac{5}{27} \times 6563 \text{ Å} = 1215.4 \text{ Å}$$

Hence, the correct answer is (A).

129. $\frac{1}{\lambda_B} = R_H\left(\frac{1}{4} - \frac{1}{9}\right)$

$$\frac{\lambda_L}{\lambda_B} = \frac{\frac{5}{36}}{\frac{3}{4}} = \frac{5}{27}$$

Hence, the correct answer is (B).

130. Considering rotation of nucleus about common centre of mass of nucleus and electron, we get different values of wavelength depending on mass of nucleus.

Hence, the correct answer is (C).

132. For Balmer series

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$$

When $n_i \to \infty$, $\frac{1}{\lambda_{\min}} = \frac{R}{4}$

$$\Rightarrow \lambda_{\min} = \frac{4}{R}$$

When $n_i = 3$, $\frac{1}{\lambda_{\max}} = R\left(\frac{5}{36}\right)$

$$\Rightarrow \lambda_{\max} = \frac{36}{5R}$$

$$\Rightarrow \frac{\lambda_{\min}}{\lambda_{\max}} = \frac{(4/R)}{(36/5R)} = \frac{20}{36}$$

$$\Rightarrow \frac{\lambda_{\min}}{\lambda_{\max}} = \frac{5}{9}$$

Hence, the correct answer is (A).

133. $\frac{1}{\lambda} = Z^2 R_\infty\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

For K_α line, $n_1 = 1$ and $n_2 = 2$

$$\Rightarrow \frac{1}{\lambda} = Z^2 R_\infty\left(\frac{3}{4}\right)$$

$$\Rightarrow Z = \sqrt{\frac{4}{3\lambda R_\infty}}$$

$$\Rightarrow Z = 39.9 \simeq 40$$

Hence, the correct answer is (B).

134. $R = R_0 A^{\frac{1}{3}}$

$$\Rightarrow \frac{R}{R_0} = A^{\frac{1}{3}}$$

$$\Rightarrow \log_e\left(\frac{R}{R_0}\right) = \frac{1}{3}\log_e A$$

$$\Rightarrow y = \frac{1}{3}x$$

A straight line passing through origin with slope $\frac{1}{3}$

Hence, the correct answer is (D).

135. Since in Rutherford experiment

$$N \propto \frac{1}{\sin^4\left(\frac{\phi}{2}\right)}$$

$$\Rightarrow \frac{N_{90}}{N_{60}} = \frac{\sin^4(30)}{\sin^4(45)}$$

$$\Rightarrow \frac{N_{90}}{N_{60}} = \frac{\frac{1}{16}}{\frac{1}{4}}$$

$$\Rightarrow N_{60} = 100$$

Hence, the correct answer is (A).

136. Energy of n^{th} orbit in H-atom is same as the energy of $3n^{th}$ state in Li^{++}.

So, $3 \rightarrow 1$ transition in H-atom would give same energy as the $3(3) \rightarrow 3(1) = 9 \rightarrow 3$ transition in Li^{++}.

Hence, the correct answer is (D).

137. Total Energy $= -(K.E.) = -13.6$ eV

Hence, the correct answer is (C).

138. $$\frac{\lambda_1}{\lambda_2} = \frac{\left(\frac{1}{9} - \frac{1}{16}\right)}{\left(\frac{1}{1} - \frac{1}{4}\right)} = \frac{7}{108}$$

Hence, the correct answer is (D).

139. $R_n \propto n^2$

$\Rightarrow \quad A_n \propto n^4$

Hence

$$\frac{A_2}{A_1} = \frac{n_2^4}{n_1^4} = \frac{16}{1}$$

Hence, the correct answer is (D).

140. λ_{min} corresponds to the maximum frequency, which occurs when all the electron's kinetic energy goes to photon.

$$\lambda_{min} = \frac{hc}{eV_0}$$

$$\Rightarrow \quad \lambda_{min} = \frac{1.24 \times 10^{-6}}{10^4} = 1.24 \times 10^{-10} \text{ m}$$

Hence, the correct answer is (A).

141. $$\frac{1}{\lambda} = R_H\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\Rightarrow \quad \lambda = \frac{36}{5R_H} = 6563 \text{ Å}$$

Hence, the correct answer is (A).

142. λ_{K_α} will not change.

Hence, the correct answer is (B).

143. For an elastic collision to take place there must be no loss in the energy of electron. The hydrogen atom will absorb energy from the colliding electron only if it can go from ground state to first excited state i.e. from $n = 1$ to $n = 2$ state. For this Hydrogen atom must absorb energy

$$E_2 - E_1 = -3.4 - (-13.6) = 10.2 \text{ eV}.$$

So, if the electron possesses energy less than 10.2 eV it would never loose it and hence collision would be elastic.

Hence, the correct answer is (B).

144. When number of electrons increases, then number of photons emitted also increases, thereby increasing the intensity. Energy, wavelength and frequency of photon are related to potential difference.

Hence, the correct answer is (A).

145. Continuous X-rays are produced by the deceleration of electrons whose K.E. depends on the potential difference. Discrete X-rays are produced when an electron makes transition between energy levels which have fixed values and hence does not depend on potential difference.

Hence, the correct answer is (B).

147. $$\frac{mv^2}{r} = \frac{3q^2}{4\pi\varepsilon_0 r^2}$$

$$\Rightarrow \quad mvr = \frac{3q^2}{4\pi\varepsilon_0 v}$$

According to Bohr's Quantisation rule, we have

$$mvr = \frac{nh}{2\pi}$$

For $n = 1$, we get

$$\frac{h}{2\pi} = \frac{3q^2}{4\pi\varepsilon v}$$

$$\Rightarrow \quad v = \frac{3q^2}{2\varepsilon_0 h}$$

Hence, the correct answer is (A).

148. $$L_1 = (1)\frac{h}{2\pi} \qquad \ldots(1)$$

{Using Bohr's Quantisation Rule}

In the first excited state of Li

$$L_2 = (2)\frac{h}{2\pi} \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{L_2}{L_1} = 2$$

Hence, the correct answer is (C).

149. For $2 \rightarrow 1$ transition energy emitted is $\Delta E = (10.2)Z^2$ eV, which is maximum.

Hence, the correct answer is (C).

150. $$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$n_f = 2$, $n_i = 3$ and $R = 1.097 \times 10^7$ m^{-1}

$\Rightarrow \quad \lambda = 656$ nm

Hence, the correct answer is (A).

151. $\frac{e^2}{2h\varepsilon_0 c}$ = Fine structure constant $(\alpha) = \frac{1}{137}$

and $[\alpha] = M^0L^0T^0$

Hence, the correct answer is (C).

152. $\frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$

For Balmer Series, $n_f = 2$ and $n_i = 3, 4, 5....$

$$\frac{1}{\lambda} = 1.097\times10^7\left[\frac{1}{4} - \frac{1}{9}\right]$$

$$\Rightarrow \frac{1}{\lambda} = 1.097\times10^7\left[\frac{5}{36}\right]$$

$$\Rightarrow \lambda = \frac{36}{5}\times\frac{10^{-7}}{1.097}\text{ m}$$

$$\Rightarrow \lambda = 6.56\times10^{-7}\text{ m}$$

$$\Rightarrow \lambda = 656\times10^{-9}\text{ m}$$

This wavelength falls in visible range.
Hence, the correct answer is (C).

153. The series in U-V region is Lymen series. Longest wavelength corresponds to minimum energy which occurs in transition from $n = 2$ to $n = 1$.

$$\Rightarrow 122 = \frac{1/R}{\left(\frac{1}{1^2} - \frac{1}{2^2}\right)} \quad ...(1)$$

The smallest wavelength in the infrared region corresponds to maximum energy of Paschen series.

$$\Rightarrow \lambda = \frac{1/R}{\left(\frac{1}{3^2} - \frac{1}{\infty}\right)} \quad ...(2)$$

Solving equations (1) and (2), we get

$$\lambda = 823.5\text{ nm}$$

Hence, the correct answer is (B).

154. $V = \frac{12400}{0.1} = 124000\text{ V} = 124\text{ kV}$

Hence, the correct answer is (C).

155. $E = 13.6Z^2 = 13.6\times(11)^2\text{ eV} = 13.6\times121\text{ eV}$

$\Rightarrow E = 1645.6\text{ eV} \approx 1.65\text{ keV}$

Hence, the correct answer is (D).

156. $\lambda_n = \frac{(2\pi r)}{n}$

$\Rightarrow \lambda_n \propto n \quad \left\{\because r \propto n^2\right\}$

and $J_n \propto n \quad \left\{\because J_n = \frac{nh}{2\pi}\right\}$

$\Rightarrow \lambda_n \propto J_n$

Hence, the correct answer is (A).

157. $\Delta E = E_i - E_f$

$$\Rightarrow \Delta E = mZ^2E_0\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$\Rightarrow \Delta E = (207)(82)^2\times13.6\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$\Rightarrow \Delta E = (19\text{ MeV})\times\frac{3}{4} = 14\text{ MeV}$$

Hence, the correct answer is (B).

158. After absorbing a photon of energy 12.1 eV electron jumps from ground state $(n = 1)$ to second excited state $(n = 3)$. Therefore, change in angular momentum is

$$\Delta L = L_3 - L_1$$

$$\Rightarrow \Delta L = 3\left(\frac{h}{2\pi}\right) - \frac{h}{2\pi} = \frac{h}{\pi}$$

$$\Rightarrow \Delta L = \frac{6.6\times10^{-34}}{3.14}\text{ Js} = 2.11\times10^{-34}\text{ Js}$$

Hence, the correct answer is (C).

159. A rough estimate of size of the nucleus is given by the distance of closest approach r_0 of an α-particle incident head-on on a nucleus of charge Ze.

Since $r_0 = \frac{2Ze^2}{4\pi\varepsilon_0 K_\alpha}$, where $K_\alpha = 6$ MeV

$$\Rightarrow r_0 = \frac{2\times100\times(1.6\times10^{-19})^2\times9\times10^9}{6\times10^6\times1.6\times10^{-19}}$$

$$\Rightarrow r = 5\times10^{-14}\text{ m}$$

Hence, the correct answer is (B).

160. $r_{n+1} - r_n = r_{n-1}$

Since $r \propto n^2$

$\Rightarrow (n+1)^2 - n^2 = (n-1)^2$

$\Rightarrow n^2 + 2n + 1 - n^2 = n^2 - 2n + 1$

$\Rightarrow n^2 - 4n = 0$

$\Rightarrow n(n-4) = 0$

Since $n \neq 0$

$\Rightarrow n = 4$

Hence, the correct answer is (D).

161. $\frac{E_{2n} - E_n}{E_{4n} - E_{2n}} = \frac{\frac{E_1}{4n^2} - \frac{E_1}{n^2}}{\frac{E_1}{16n^2} - \frac{E_1}{4n^2}} = 4 = \text{constant}$

Hence, the correct answer is (D).

CHAPTER 2

162. Since $N \propto \dfrac{1}{\sin^4\left(\dfrac{\theta}{2}\right)}$

$$\Rightarrow \frac{N_{120^\circ}}{N_{60^\circ}} = \frac{\sin^4(30^\circ)}{\sin^4(60^\circ)} = \frac{\frac{1}{16}}{\frac{9}{16}}$$

$$\Rightarrow N_{120^\circ} = \left(\frac{1}{9}\right)N_{60^\circ} = \frac{9\times 10^6}{9} = 10^6$$

Hence, the correct answer is (C).

163. Since, $\Delta E = K\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right) = \dfrac{hc}{\lambda}$

In the given diagram, emission from $n = 4 \to n = 2$ would give photon of maximum energy (shortest wavelength) and $n = 4 \to n = 3$ transition would give photon of minimum energy (longest wavelength).
Hence, the correct answer is (C).

164. Since $E_0 = -\dfrac{me^4Z^2}{8\varepsilon_0^2h^2} = -13.6\left(\dfrac{Z^2}{n^2}\right)$ eV

When $m \to 2m$

$$\Rightarrow E_0 \to 2E_0 = -27.2 \text{ eV}$$

Also, $a_0 = \dfrac{h^2\varepsilon_0}{\pi me^2}$

When $m \to 2m$

$$\Rightarrow a_0 \to \frac{a_0}{2} = 0.27 \text{ Å}$$

Hence, the correct answer is (A).

165. Energy of photon is

$$E = 13.6\left(1 - \frac{1}{25}\right) \text{ eV}$$

$$\Rightarrow E = 13 \text{ eV}$$

Since, momentum is conserved, so we have

$$\frac{E}{c} = mv$$

$$\Rightarrow v = \frac{E}{mc}$$

$$\Rightarrow v = \frac{13\times 1.6\times 10^{-19}}{1.67\times 10^{-27}\times 3\times 10^8} = 4 \text{ ms}^{-1}$$

Hence, the correct answer is (C).

166. $F = \dfrac{mv^2}{r} = \dfrac{Ze^2}{4\pi\varepsilon_0 r^2}$

Since $v \propto \dfrac{1}{n}$ and $r \propto n^2$

$$\Rightarrow F \propto \frac{1}{n^4}$$

Hence, the correct answer is (D).

167. Because radius of nth orbit is equal to $r_n = n^2 r_0$ where $r_0 \simeq 0.529$ Å

$$\Rightarrow \frac{0.0001}{2}\times 10^{-3} = \left(0.5\times 10^{-10}\right)n^2$$

$$\Rightarrow n^2 = 1000$$

$$\Rightarrow n \simeq 31$$

Hence, the correct answer is (D).

168. $\lambda_{max} = 2d\sin(90^\circ) = 2d$

Since, $d = 10^{-7}$ cm $= 10^{-9}$ m $= 10$ Å

$$\Rightarrow \lambda_{max} = 2d = 20 \text{ Å}$$

Hence, the correct answer is (D).

170. The short limit of the Balmer series is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = \frac{R}{4}$$

$$\Rightarrow R = \frac{4}{\lambda} = \left(\frac{4}{3646}\right)\times 10^{10} \text{ m}^{-1}$$

Further the wavelengths of the K_α series are given by the relation

$$\frac{1}{\lambda} = R(Z-1)^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

The maximum wave number corresponds to $n \to \infty$ and therefore, we must have

$$\frac{1}{\lambda} = R(Z-1)^2$$

$$\Rightarrow (Z-1)^2 = \frac{1}{R\lambda} = \frac{3646\times 10^{-10}}{4\times 1\times 10^{-10}} = 911.5$$

$$\Rightarrow (Z-1) = \sqrt{911.5} \cong 30.2$$

$$\Rightarrow Z = 31.2 \cong 31$$

So, the atomic number of the element concerned is 31. The element having atomic number $Z = 31$ is Gallium.
Hence, the correct answer is (B).

171. For incident electron, we have

$$\lambda_1 = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2meV}}$$

For shortest wavelength of X-rays, we have

$$\lambda_2 = \frac{hc}{eV}$$

$$\Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \frac{1}{c}\sqrt{\frac{eV}{2m}}$$

Substituting the values, we get

$$\frac{\lambda_1}{\lambda_2} = 1$$

Hence, the correct answer is (D).

Multiple Correct Choice Type Questions

1. Since $L = \frac{nh}{2\pi} = \frac{3h}{2\pi}$

$\Rightarrow \quad n = 3$

Since, $r_n = a_0 \frac{n^2}{Z}$

$\Rightarrow \quad 4.5a_0 = a_0 \frac{n^2}{Z}$

$\Rightarrow \quad Z = 2$

Possible transitions are $3 \to 2$, $3 \to 1$ and $2 \to 1$

For $3 \to 2$, we have

$$\frac{1}{\lambda} = R(2)^2\left(\frac{1}{4} - \frac{1}{9}\right) = 4R\left(\frac{9-4}{36}\right) = \frac{5R}{9}$$

$\Rightarrow \quad \lambda = \frac{9}{5R}$

For $3 \to 1$, we have

$$\frac{1}{\lambda} = R(2)^2\left(1 - \frac{1}{9}\right) = 4R\left(\frac{8}{9}\right) = \frac{32R}{9}$$

$\Rightarrow \quad \lambda = \frac{9}{32R}$

For $2 \to 1$, we have

$$\frac{1}{\lambda} = R(2)^2\left(1 - \frac{1}{4}\right) = 4R\left(\frac{3}{4}\right) = 3R$$

$\Rightarrow \quad \lambda = \frac{1}{3R}$

Hence, (A) and (C) are correct.

2. $r = \frac{mv}{qB} = \frac{\sqrt{2mE}}{qB}$

$\Rightarrow \quad r \propto \frac{\sqrt{m}}{q}$

$\Rightarrow \quad$ Deflection $\propto \frac{q}{\sqrt{m}}$

Hence, (A) and (C) are correct.

3. Let electron jump from $n_1 \to n_2$.

So $\Delta L = (n_2 - n_1)\frac{h}{2\pi}$

(According to Bohr's Quantisation Rule)

Since, n_1 and n_2 are integers $(n_1, n_2 > 1)$, so $n_2 - n_1$ is also an integral value and hence ΔL must be an integral multiple of $\frac{h}{2\pi}$.

Hence, (B) and (C) are correct.

4. $U \propto \frac{1}{r} \propto \frac{1}{n^2}$

$K \propto \frac{1}{r} \propto \frac{1}{n^2}$

$v \propto \frac{1}{n}$ and

$L \propto n$

Hence, (B) and (C) are correct.

5. In Bohr's model of Hydrogen atom

$R \propto n^2$

$V \propto \frac{1}{n}$

$T \propto n^3$ and $E \propto \frac{1}{n^2}$

$\Rightarrow \quad VR \propto n$

$\Rightarrow \quad TE \propto n$

$\Rightarrow \quad \frac{T}{R} \propto n$

$\Rightarrow \quad \frac{V}{E} \propto n$

Hence, (A), (C) and (D) are correct.

6. Characteristic X-rays depend upon the atomic number Z, so

$\lambda_1 = \lambda_2 = \lambda_3$

OR $\quad \lambda_2 = \sqrt{\lambda_1 \lambda_3}$

Hence, (A) and (D) are correct.

7. Since, $|F| = \frac{dU}{dr} = \frac{Ke^2}{r^4}$ ...(1)

$\Rightarrow \quad \frac{Ke^2}{r^4} = \frac{mv^2}{r}$ ...(2)

According to Bohr's Quantisation Rule, we have

$$mvr = \frac{nh}{2\pi} \quad \ldots(3)$$

From (2) and (3), we get

$$r = \left(\frac{4\pi^2 e^2 K}{h^2}\right)\frac{m}{n^2} = K_1\left(\frac{m}{n^2}\right) \quad \ldots(4)$$

Since total energy E is half the potential energy, so we have

$$E = -\frac{Ke^2}{6r^3} = -\frac{Ke^2}{6\left(\frac{K_1 m}{n^2}\right)^3} = -\frac{Ke^2 n^6}{6K_1^3 m^3}$$

So, total energy is $E \propto n^6$ and $E \propto m^{-3}$

Hence, (A) and (B) are correct.

8. Since, $\frac{hc}{\lambda} = \Delta E$

$$\Rightarrow \quad \lambda = \frac{hc}{\Delta E}$$

$$\lambda_{\min} = \frac{hc}{\Delta E_{\max}} \text{ and } \lambda_{\max} = \frac{hc}{\Delta E_{\min}}$$

Emission will take place when transition takes place from $n=3$ to $n=1$

$$\Rightarrow \quad \lambda_{\min} = \frac{12400}{10(\text{eV})} = 1240\ \text{Å} = 1.24\times10^{-7}\ \text{m}$$

Absorption will take place when transition takes place from $n=3$ to $n=4$.

$$\Rightarrow \quad \lambda_{\max} = \frac{12400}{2(\text{eV})} = 6200\ \text{Å} = 6.2\times10^{-7}\ \text{m}$$

Lowest frequency photon that can ionise the atom will have to remove the electron from $n=3$ state, so we have $E = 15$ eV

$$\Rightarrow \quad 15\times1.6\times10^{-19} = 6.63\times10^{-31}\nu$$

$$\Rightarrow \quad \nu = 3.62\times10^{15}\ \text{Hz}$$

The total number of ways of de-excitation of atom to ground state is 2.

Hence, (A), (B) and (C) are correct.

9. Since reduced mass is

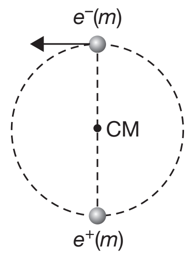
$$\mu = \frac{m_1 m_2}{m_1+m_2} = \frac{m}{2}$$

Now, energy is directly proportional to mass, so

$$E_0 \propto m$$

$$\Rightarrow \quad E \propto \mu \propto \frac{m}{2}$$

$$\Rightarrow \quad \frac{E}{E_0} = \frac{1}{2}$$



Since, $E_0 = R_0 hc$

So, Rydberg constant is

$$R_0 = \frac{E_0}{hc} \propto m$$

$$\Rightarrow \quad R \propto \mu$$

$$\Rightarrow \quad \frac{R}{R_0} = \frac{\mu}{m} = \frac{1}{2}$$

Radius of orbit is

$$r = \frac{n^2 h^2 \varepsilon_0}{\pi m e^2 Z}$$

So, for $n=1$, $Z=1$, we have

$$r_0 = \frac{h^2\varepsilon_0}{\pi m e^2} \propto \frac{1}{m}$$

$$\Rightarrow \quad r \propto \frac{1}{\mu}$$

$$\Rightarrow \quad \frac{r}{r_0} = \frac{m}{\mu} = 2$$

Velocity of electron in first orbit is

$$v = \frac{e^2}{2h\varepsilon_0}$$

which is independent of mass, hence for positronium atom and hydrogen atom velocity of electron is same in both cases.

Hence, (A), (B), (C) and (D) are correct.

10. Under normal conditions, the total energy (E), kinetic energy (K) and potential energy (U) are

$$\left.\begin{aligned} E_1 &= -13.6\ \text{eV} \\ K_1 &= 13.6\ \text{eV} \\ U_1 &= -27.2\ \text{eV} \end{aligned}\right] \text{ for ground state}$$

$$\left.\begin{aligned} E_2 &= -3.4\ \text{eV} \\ K_2 &= 3.4\ \text{eV} \\ U_2 &= -6.8\ \text{eV} \end{aligned}\right] \text{ for first excited state}$$

If PE in ground state is taken to be zero, then KE remains unchanged, however new PE and TE are increased by 27.2 eV.

So, for ground state, we have

$$E_1 = (-13.6+27.2)\ \text{eV} = 13.6\ \text{eV}$$

$$K_1 = 13.6\ \text{eV (same)}$$

$$U_1 = (-27.2+27.2)\ \text{eV} = 0\ \text{eV}$$

For first excited state, we have

$$E_2 = (-3.4+27.2)\ \text{eV} = 23.8\ \text{eV}$$

$K_2 = 3.4$ eV (same)

$U_2 = (-6.8 + 27.2)$ eV $= 20.4$ eV

Hence, (A), (B), (C) and (D) are correct.

11. Ground state is $n = 1$

So, first excited state is $n = 2$

$$KE = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{2r} \quad \{\text{for } Z = 1\}$$

$$\Rightarrow \quad KE = \frac{14.4 \times 10^{-10}}{2r} \text{ eV}$$

Since, $r = 0.53n^2$ Å

$$\Rightarrow \quad (KE)_1 = \frac{14.4 \times 10^{-10}}{2 \times 0.53 \times 10^{-10}} \text{ eV} = 13.58 \text{ eV}$$

$$\Rightarrow \quad (KE)_2 = \frac{14.4 \times 10^{-10}}{2 \times 0.53 \times 10^{-10} \times 4} \text{ eV} = 3.39 \text{ eV}$$

$\Rightarrow$ KE decreases by 10.2 eV

Now $PE = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} = -\frac{14.4 \times 10^{-10}}{r}$ eV

$$\Rightarrow \quad (PE)_1 = -\frac{14.4 \times 10^{-10}}{0.53 \times 10^{-10}} \text{ eV} = -27.1 \text{ eV}$$

$$\Rightarrow \quad (PE)_2 = -\frac{14.4 \times 10^{-10}}{0.53 \times 10^{-10} \times 4} = -6.79 \text{ eV}$$

$\Rightarrow$ PE increases by 20.4 eV

Now Angular momentum is $L = mvr = \frac{nh}{2\pi}$

$$\Rightarrow \quad L_2 - L_1 = \frac{h}{2\pi} = \frac{6.6 \times 10^{-34}}{6.28} = 1.05 \times 10^{-34} \text{ Js}$$

Hence, (B), (C) and (D) are correct.

13. Since, $v = CRZ^2\left(\frac{1}{n_2^{\,2}} - \frac{1}{n_1^{\,2}}\right)$

$$\Rightarrow \quad v_{3\to 2} = (3 \times 10^8)(1.1 \times 10^7)(1)\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\Rightarrow \quad v_{3\to 2} = (3.3 \times 10^{15})\left(\frac{5}{36}\right)$$

$$\Rightarrow \quad v_{3\to 2} = \frac{16.5}{36} \times 10^{15} = 4.6 \times 10^{14} \text{ Hz}$$

Similarly, for transition from 3 to 1, we have

$$v_{3\to 1} = (3 \times 10^8)(1.1 \times 10^7)\left(1 - \frac{1}{9}\right)$$

$$\Rightarrow \quad v_{3\to 1} = 2.9 \times 10^{15} \text{ Hz}$$

Please note that (D) is incorrect as it says "must be" because from second excited state i.e., $n = 3$, photons are emitted for electron transitions from $3 \to 2$, $2 \to 1$, $3 \to 1$.

Hence, (A) and (C) are correct.

14. $E \propto \frac{1}{r^2} \propto \frac{1}{n^2}$ …(1)

$P \propto v \propto \frac{1}{n}$ …(2)

$r \propto n^2$ …(3)

$\Rightarrow \quad PEr \propto \frac{1}{n}$

$\Rightarrow \quad \frac{P}{E} \propto n$

$\Rightarrow \quad Er \propto n^{\circ}$

$\Rightarrow \quad Pr \propto n$

Hence, (A), (B), (C) and (D) are correct.

15. Actually, in the ground state, we have

$KE = +13.6$ eV

$PE = -27.2$ eV and

$TE = -13.6$ eV

In the first excited state, we have

$KE = +3.4$ eV, $PE = -6.8$ eV, $TE = -3.4$ eV

Now PE and TE both will be increased by

$13.6 - (-27.2) = 40.8$ eV

KE remains unchanged being independent of reference.

Hence, (A), (B) and (D) are correct.

16. Since, $r \propto \frac{1}{m}$

and $E_n \propto m$

For an orbit, $mvr = \frac{nh}{2\pi}$ is a constant, because $n =$ constant

$\Rightarrow \quad v \propto m$

Hence, (B) and (D) are correct.

17. From $n = 3$ to $n = 2$

$$\frac{1}{\lambda} \propto \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Since, $E = \frac{hc}{\lambda}$

$$\Rightarrow \quad E \propto \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For a photon of energy E, momentum p is

$$p=\frac{E}{v}\text{, where } v \text{ is the speed of light.}$$

$$\Rightarrow \quad \frac{\lambda_1}{\lambda_2}=\frac{\frac{1}{1^2}-\frac{1}{2^2}}{\frac{1}{2^2}-\frac{1}{3^2}}=\frac{\frac{3}{4}}{\frac{5}{36}}$$

$$\Rightarrow \quad \frac{\lambda_1}{\lambda_2}=a=\frac{27}{5}$$

Further, $\dfrac{E_1}{E_2}=\dfrac{\frac{hc}{\lambda_1}}{\frac{hc}{\lambda_2}}=\dfrac{\lambda_2}{\lambda_1}=\dfrac{5}{27}$

$$\Rightarrow \quad c=\frac{5}{27}=\frac{1}{a}$$

So, $b=\dfrac{p_1}{p_2}=\dfrac{E_1}{E_2}=c=\dfrac{5}{27}$

Hence, (A), (C) and (D) are correct.

18. $\Delta E=204=13.6\,Z^2\left(\frac{1}{1}-\frac{1}{4n^2}\right)$

$$\Rightarrow \quad 40.8=13.6\,Z^2\left(\frac{1}{n^2}-\frac{1}{4n^2}\right)$$

Satisfied for, $Z=4$ and $n=2$

Hence, (B) and (D) are correct.

19. $v\propto\dfrac{1}{n}$ $\quad\left\{\because v=\left(\dfrac{e^2}{2h\varepsilon_0}\right)\dfrac{Z}{n}\right\}$

$$E\propto\frac{1}{n^2}$$

$$r\propto n^2$$

$\Rightarrow \quad \dfrac{E}{v^2}$, Er, v^2r are independent of n.

Hence, (B), (C) and (D) are correct.

20. Intensity of incident light may increase by decreasing distance of source without increasing frequency.
Hence, (B) and (C) are correct.

21. Since, $E_n=-(13.6)\left(\dfrac{Z^2}{n^2}\right)$ eV

$$E_1=-13.6\text{ eV for } Z=1 \text{ and } n=1$$

Similarly, for $Z=n$, we have

$$E=E_1=-13.6\text{ eV}$$

i.e., for second orbit of He^+ and for third orbit of Li^{++}, we have energy equal to -13.6 eV

Hence, (A) and (D) are correct.

22. Let the transition in He^+ ion be from level $a\to b$ and that in H atom be from $p\to q$. Then $\lambda_1=\lambda_2$ gives

$$4\left(\frac{1}{b^2}-\frac{1}{a^2}\right)=\left(\frac{1}{q^2}-\frac{1}{p^2}\right)$$

$$\Rightarrow \quad \frac{4}{b^2}-\frac{4}{a^2}=\frac{1}{q^2}-\frac{1}{p^2}$$

This equation will be satisfied when

$$a=2p \text{ and } b=2q$$

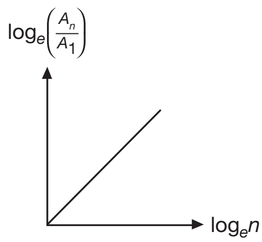
So, a and b are even integers greater than 1, satisfied by (A) and (D).
Hence, (A) and (D) are correct.

23. $E=E_0\dfrac{Z^2}{n^2}$, $r=a_0\dfrac{n^2}{Z}$, $v=v_0\dfrac{Z}{n}$

where E_0 is energy of electron in ground state, a_0 is radius of ground state orbit and v_0 is velocity of electron in ground state.
Hence, (C) and (D) are correct.

24. $r=a_0\dfrac{Z^2}{n^2}$

$$\Rightarrow \quad A=\pi r^2=\pi\frac{a_0{}^2n^4}{Z^2}$$



$$\Rightarrow \quad A_n=A_1n^4$$

$$\Rightarrow \quad \log_e A_n=\log A_1+\log n^4$$

$$\Rightarrow \quad \log_e A_n-\log_e A_1=4\log_e n$$

$$\Rightarrow \quad \log_e\left(\frac{A_n}{A_1}\right)=4\log_e N$$

So, the graph will be a straight line passing through origin having a slope of four units at all the points.
Hence, (A), (B) and (D) are correct.

25. $E_n=\dfrac{-13.6Z^2}{n^2}$ eV

Since, $E_1=-54.4$ eV

$$\Rightarrow \quad -\frac{13.6Z^2}{(1)^2}=-54.4$$

$\Rightarrow \quad Z^2 = 4$

$\Rightarrow \quad Z = 2$

Also, 40.8 eV is the difference between two energy levels $n = 2$ and $n = 1$.

Also, the electron cannot fall from the ground state and hence cannot emit photon. So,

$$-E = K \text{ and } E = \frac{U}{2}$$

Hence, (B), (C) and (D) are correct.

28. Since, $p = mv$ where $v \propto \frac{1}{n}$

$$\Rightarrow \quad p \propto \frac{1}{n}$$

Since, $KE \propto \frac{1}{r}$ and $r \propto n^2$

$$\Rightarrow \quad KE \propto \frac{1}{n^2}$$

Further $L = \frac{nh}{2\pi}$

$$\Rightarrow \quad L \propto n$$

Hence, (A), (C) and (D) are correct.

29. Any transition in the Balmer series must end up at $n = 2$. This must be followed by the transition from $n = 2$ to $n = 1$ by emitting a photon of energy 10.2 eV. This 10.2 eV photon corresponds to a wavelength of about 122 nm, which belongs to the Lyman series.
Hence, (B) and (C) are correct.

30. According to Ritz Combination Principle, we have

$$E_3 = E_1 + E_2$$

$$\Rightarrow \quad hv_3 = hv_1 + hv_2$$

$$\Rightarrow \quad v_3 = v_1 + v_2$$

$$\Rightarrow \quad \frac{c}{\lambda_3} = \frac{c}{\lambda_1} + \frac{c}{\lambda_2} \qquad \left\{\because v = \frac{c}{\lambda}\right\}$$

$$\Rightarrow \quad \lambda_3 = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

Hence, (B) and (C) are correct.

Reasoning Based Questions

1. Cut-off wavelength depends on the accelerating voltage, not the characteristic wavelengths. Further, approximately 2% kinetic energy of the electrons is utilised in producing X-ray. Rest 98% is lost in heat.
Hence, the correct answer is (B).

2. According to classical electromagnetic theory, an accelerated charge continuously emits radiation. As electrons revolving in circular paths are constantly experiencing centripetal acceleration, hence, they will be losing their energy continuously and the orbital radius will go on decreasing and form spirals and finally the electron will fall on the nucleus.
Hence, the correct answer is (D).

4. $\frac{1}{\lambda} \propto R \propto \mu$, where μ is the reduced mass of the system.

$$\Rightarrow \quad \mu_D > \mu_H$$

$$\Rightarrow \quad R_D > R_H$$

$$\Rightarrow \quad \lambda_D < \lambda_H$$

In the centre of mass frame both the nucleus and the electron revolve about the common axis passing through the centre of mass.
Hence, the correct answer is (D).

5. Since $\lambda = \frac{hc}{\Delta E}$, so the wavelength will be inversely proportional to the energy difference between the levels. The energy difference is more when the transition takes place from $n \to \infty$ to $n = 2$ than when the transition takes place from $n = 2$ to $n = 1$.
Hence, the correct answer is (C).

7. Both Statement-1 & Statement-2 are correct & Statement-2 is the correct explanation of Statement-1.
Hence, the correct answer is (A).

8. Both Statement-1 and Statement-2 are true and Statement-2 is the correct explanation of Statement-1.
Hence, the correct answer is (A).

9. Total energy is negative because electron is bound to the atom due to coulomb attraction and in the bound system energy is negative.
Hence, the correct answer is (B).

10. At outermost orbit total energy is zero and electron is free from the influence of the nucleus of the atom.
Hence, the correct answer is (B).

11. Speed of electron in H like atom is $v_{H\text{ Like}} = \left(\frac{e^2}{2h\varepsilon_0}\right)\frac{Z}{n}$
Hence, the correct answer is (C).

12. No. of lines in emission spectrum is $N = \frac{n(n-1)}{2}$

$$N = \frac{4(4-1)}{2} = 6$$

and this depends on number of energy levels available for transition.
Hence, the correct answer is (C).

13. Lyman series - its energy is in the ultra violet region
Balmer series - its energy is in visible region
Hence, the correct answer is (C).

15. Statement–1 is false, the penetration power depends upon accelerating potential.
Statement–2 is true, increasing current increases the temperature of filament causing it to emit more electrons.
Hence, the correct answer is (D).

17. $hf_1 = 0 - E_2$ and

$hf_2 = 0 - E_3$

$\Rightarrow \quad h(f_1 - f_2) = E_3 - E_2$

So, Statement-1 and Statement-2 both are correct and Statement-2 correctly explains Statement-1.
Hence, the correct answer is (A).

Linked Comprehension Type Questions

1. $U = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r}$, where $q_1 = +2e$, $q_2 = 82e$

$$\Rightarrow \quad U = \frac{(9\times10^9)(2e)(82e)}{6.5\times10^{-14}} = 5.82\times10^{-13}\ \text{J}$$

$\Rightarrow \quad U = 3.63$ MeV

Hence, the correct answer is (C).

2. Applying Law of Conservation of Energy, we get

$K_1 + U_1 = K_2 + U_2$

$\Rightarrow \quad K_1 + 0 = 0 + 3.63$ MeV

$\Rightarrow \quad K_1 = 3.63$ MeV

Hence, the correct answer is (C).

3. Total energy of the hydrogen like atom (atomic number Z) in the n^{th} Bohr orbit is

$$E_n = -\left(\frac{me^4}{8\varepsilon_0^2 h^2}\right)\frac{Z^2}{n^2} = -(13.6)\frac{Z^2}{n^2}\ \text{eV}$$

$$\Rightarrow \quad E_2 = -\frac{13.6}{4}Z^2 = -3.4Z^2\ \text{eV}$$

and $E_3 = -\dfrac{13.6}{9}Z^2 = -1.5Z^2$ eV

Since $E_3 - E_2 = 68$ eV

$\Rightarrow \quad E_3 - E_2 = -1.5Z^2 - (-3.4Z^2)$

$\Rightarrow \quad 1.9Z^2 = 68$

$\Rightarrow \quad Z = 6$

Hence, the correct answer is (D).

4. Total energy in the first Bohr orbit is

$E_1 = -13.6 \times Z^2$ eV

$\Rightarrow \quad E_1 = -13.6 \times 36$ eV $= -489.6$ eV

Since $KE = -(TE)$

So, KE of electron in the first Bohr orbit is

$KE = -$Total energy $= +489.6$ eV

Hence, the correct answer is (C).

5. Energy required to eject the electron from the first Bohr orbit E_1 to infinity $(E_\infty \to 0)$ is

$\Delta E = E_\infty - E_1 = 489.6$ eV

$$\Rightarrow \quad \lambda = \frac{12375}{489.6}$$

$\Rightarrow \quad \lambda = 25.28$ Å

Hence, the correct answer is (B).

6. $eV = 3\times10^{-15}$

$$\Rightarrow \quad V = \frac{3\times10^{-15}}{1.6\times10^{-19}} = 1.875\times10^4\ \text{V}$$

Hence, the correct answer is (A).

7. $\Delta E = 3\times10^{-15} - 0.3\times10^{-15}$ J

$\Rightarrow \quad \Delta E = 2.7\times10^{-15}$ J

Hence, the correct answer is (C).

8. The difference of the energy will be gained by the emitted electron as kinetic energy, so

$\text{KE} = 2.7\times10^{-15} - 3\times10^{-17}$

$\Rightarrow \quad \text{KE} = 2.67\times10^{-15}$ J

Hence, the correct answer is (D).

9. If n_E is the number of photons with energy E, then the distribution of n_E is given by

$$P(n_E) \approx \frac{I(E)}{E} = E^4 e^{-\frac{E}{k_B T}}$$

The most probable energy E_m of photons satisfies the equation

$$\left.\frac{dP}{dE}\right|_{E=E_m} = \left(4E^3 - \frac{E^4}{k_B T}\right)e^{-\frac{E}{k_B T}} = 0$$

$\Rightarrow \quad E_m = 4KT$

Hence, the correct answer is (D).

10. The Balmer lines of hydrogen are emitted when electrons transit from energy levels of $n > 3$ to that of $n = 2$. Thus, the maximum energy of the Balmer line

photons is when the electron makes transition from $n_i \to \infty$ to $n_f = 2$. So, we have

$$\Delta E_{max} = 0 - (-3.4) = 3.4 \text{ eV}$$

Hence, the correct answer is (B).

11. Given that the human eye is most sensitive to sunlight, the visible light is the most probable frequency band of the light emitted by the sun, and the visible light corresponds to the frequency range of the Balmer lines, the surface temperature of the Sun is given by

$$E_m = 4k_B T = 3.4 \text{ eV}$$

$$\Rightarrow \quad T = \frac{3.4}{4k_B} \text{ eV} = 1.06 \times 10^4 \text{ K}$$

Hence, the correct answer is (A).

12. $$\sqrt{\frac{m^{-3} \times C^2 \times Nm^{-2}}{Kg \times C^2}} = \sqrt{s^{-2}} \qquad \{N = \text{kg ms}^{-2}\}$$

Hence, the correct answer is (C).

13. $$\lambda = \frac{2\pi C}{\omega}$$

$$\Rightarrow \quad \lambda = \frac{2\pi C}{\sqrt{\frac{Ne^2}{m\varepsilon_0}}} \approx 600 \text{ nm}$$

Hence, the correct answer is (B).

14. The correct answer is (C).

15. The correct answer is (A).

16. The correct answer is (C).

17. The correct answer is (C).

Combined solution of 14, 15, 16 & 17

When hydrogen atom is excited, then we have

$$eV = E_0\left(\frac{1}{1} - \frac{1}{n^2}\right) \qquad \ldots(1)$$

When ion is excited, then

$$eV = E_0 Z^2\left(\frac{1}{2^2} - \frac{1}{n_1^2}\right) \qquad \ldots(2)$$

Wavelength of emitted light is

$$\frac{hc}{\lambda_1} = E_0\left(\frac{1}{1} - \frac{1}{n^2}\right) \qquad \ldots(3)$$

$$\frac{hc}{\lambda_2} = E_0 Z^2\left(\frac{1}{1} - \frac{1}{n_1^2}\right) \qquad \ldots(4)$$

Further it is given that

$$\frac{\lambda_1}{\lambda_2} = \frac{5}{1} \qquad \ldots(5)$$

Solving the above equations, we get

$$Z = 2, \ n = 2, \ n_1 = 4 \text{ and } V = 10.2 \text{ V}$$

Energy of emitted photon by the hydrogen atom is

$$\Delta E = E_2 - E_1 = 10.2 \text{ eV}$$

and by the ion is

$$\Delta E' = E_4 - E_1 = (13.6)(2)^2\left(1 - \frac{1}{16}\right) = 51 \text{ eV}$$

Hence, the correct answer is (C).

18. $P = (1\%) \times 40 \times 10^3 \times 10 \times 10^{-3}$

$\Rightarrow \quad P = 4 \text{ W}$

Hence, the correct answer is (C).

19. Total power is $P = VI = 40 \times 10^3 \times 10 \times 10^{-3}$

$\Rightarrow \quad P_{total} = 400 \text{ W}$

So, heat produced per second is

$$P_{total} - P_{emitted} = 400 - 4 = 396 \text{ W}$$

Hence, the correct answer is (B).

20. $$\lambda_{min} = \frac{12400}{40 \times 1000} = 0.3 \text{ Å}$$

Hence, the correct answer is (A).

21. $$K_{max} = \frac{12400}{855} - 13.6 = 0.9 \text{ eV}$$

Hence, the correct answer is (A).

22. H_α in Balmer series corresponds to $n = 3$ to $n = 2$

$$\Rightarrow \quad E_3 - E_1 = 13.6\left(1 - \frac{1}{9}\right) = 12.09 \text{ eV}$$

Hence, the correct answer is (A).

23. Minimum wavelength for $n = 3$ to $n = 1$

$$\lambda = \frac{12400}{12.09} \approx 1026 \text{ Å} \approx 103 \text{ nm}$$

Hence, the correct answer is (B).

24. $$F = -\frac{dU}{dr}$$

$$\Rightarrow \quad F = \frac{4k}{r^5}$$

$$\Rightarrow \quad \frac{mv^2}{r} = \frac{4k}{r^5}$$

$$\Rightarrow \quad mv^2 = \frac{4k}{r^4} \qquad \ldots(1)$$

Now, according to Bohr's Quantisation rule, we have

$$v = \frac{nh}{2\pi mr} \qquad \ldots(2)$$

$$\Rightarrow \quad m\left(\frac{n^2h^2}{4\pi^2m^2r^2}\right)=\frac{4k}{r^4}$$

$$\Rightarrow \quad \frac{n^2h^2}{4\pi^2m}=\frac{4k}{r^2}$$

$$\Rightarrow \quad r^2=\frac{16\pi^2mk}{n^2h^2}$$

$$\Rightarrow \quad r=\frac{4\pi}{nh}\sqrt{mk} \qquad \ldots(3)$$

Hence, the correct answer is (C).

25. Substituting (3) in (2), we get

$$v=\frac{nh}{2\pi m\left(\frac{4\pi}{nh}\sqrt{mk}\right)}$$

$$\Rightarrow \quad v=\frac{n^2h^2}{8\pi^2m\sqrt{mk}}$$

Hence, the correct answer is (D).

26. $KE=\frac{1}{2}mv^2=\frac{1}{2}m\left(\frac{n^4h^4}{64\pi^4m^3k}\right)$

$$\Rightarrow \quad KE=\frac{n^4h^4}{128\pi^4m^2k}$$

Hence, the correct answer is (B).

27. $PE=U=-\frac{k}{r^4}$

$$\Rightarrow \quad U=-\frac{k}{\left(\frac{16\pi^2mk}{n^2h^2}\right)^2}=-\frac{kn^4h^4}{256\pi^4m^2k^2}$$

$$\Rightarrow \quad U=-\frac{n^4h^4}{256\pi^4m^2k}$$

Hence, the correct answer is (D).

28. $TE=KE+PE$

$$\Rightarrow \quad TE=\frac{n^4h^4}{128\pi^4m^2k}-\frac{n^4h^4}{256\pi^4m^2k}$$

$$\Rightarrow \quad TE=\frac{n^4h^4}{256\pi^4m^2k}$$

Hence, the correct answer is (C).

29. The energy transitions for the given wavelengths are

$$\Delta E_1=\frac{12375}{\lambda_1}=\frac{12375}{1085}=11.40 \text{ eV}$$

$$\Delta E_2=\frac{12375}{\lambda_2}=\frac{12375}{304}=40.70 \text{ eV}$$

Total energy emitted $\Delta E=\Delta E_1+\Delta E_2=52.1$ eV

Hence, the correct answer is (C).

30. Since, $\Delta E=13.6Z^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$ eV

where, ΔE is the energy emitted

$$\Rightarrow \quad 52.1=13.6\times 2^2\left(\frac{1}{1^2}-\frac{1}{n^2}\right)$$

$$\Rightarrow \quad n=5$$

Hence, the correct answer is (D).

31. The energy of electron after collision is

$$\Delta E=100-52.1=47.9 \text{ eV}$$

Hence, the correct answer is (C).

32. Since, $\frac{1}{\lambda}=RZ^2\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$

$$\Rightarrow \quad \frac{hc}{\lambda}=(Rhc)Z^2\left(\frac{1}{4}-\frac{1}{9}\right)$$

where $Rhc=13.6$ eV

Also, given that $\frac{hc}{\lambda}=47.2$ eV

$$\Rightarrow \quad 47.2=13.6Z^2\left(\frac{5}{36}\right)$$

$$\Rightarrow \quad Z^2\approx 25$$

$$\Rightarrow \quad Z=5$$

Hence, the correct answer is (A).

33. Since, $r_n=(0.53)\frac{n^2}{Z}$ Å

$$\Rightarrow \quad r_1=\frac{0.53}{5} \text{ Å}$$

$$\Rightarrow \quad r_1=0.106 \text{ Å}$$

Hence, the correct answer is (B).

34. Since, $E_2=\frac{E_1}{4}$

$$\Rightarrow \quad E_1=4E_2=4(-144)=-576 \text{ eV}$$

So, ionization energy $=576$ eV

Hence, the correct answer is (B).

35. $E_1=-576$ eV

Hence, the correct answer is (D).

36. Since, $E_n = -\frac{E_1 z^2}{n^2}$

So, graph of E_n vs $\frac{1}{n^2}$ is a straight line passing through origin with negative slope. The distance between successive points is non-uniform.

Hence, the correct answer is (A).

37. $\frac{mv^2}{r} = evB$ ...(1)

$\Rightarrow \frac{v}{r} = \frac{eB}{m}$ ...(2)

According to Bohr Quantisation rule,

$mvr = \frac{nh}{2\pi}$

$\Rightarrow vr = \frac{nh}{2\pi m}$ ...(3)

From (2) & (3), we get

$r = \sqrt{\frac{nh}{2\pi Be}}$ ...(4)

Hence, the correct answer is (B).

38. Kinetic energy, $K = \frac{1}{2}mv^2$

$\Rightarrow K = \frac{1}{2}m\left(\frac{n^2h^2}{m^2r^2}\right)$

$\Rightarrow K = \frac{1}{2}nh\left(\frac{Be}{m}\right)$

Hence, the correct answer is (C).

39. Since, $T = \frac{2\pi r}{v}$

$\Rightarrow i = \frac{e}{T} = \frac{e^2B}{2\pi m}$

Since, $\text{Area} = \pi r^2 = \pi\left(\frac{nh}{Be}\right)$ $\left\{\because r^2 = \frac{nh}{Be}\right\}$

$\Rightarrow M = i\text{Å} = \left(\frac{e^2B}{2\pi m}\right)\left(\frac{\pi nh}{Be}\right) = \frac{neh}{2m}$

Since, $PE = U = MB\sin(90°)$

$\Rightarrow U = \frac{nhBe}{2m}$

Hence, the correct answer is (A).

40. $\phi = \pi r^2 B$

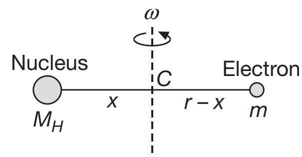
$\Rightarrow \phi = \pi\left(\frac{nh}{Be}\right)B = \frac{\pi nh}{e}$

Hence, the correct answer is (B).

41. Centre of mass of the atom

Hence, the correct answer is (C).

42. Let x be the distance of the nucleus from the common centre of mass C. The distance of the electron will be $(r - x)$.



$x = \frac{mr}{M_H + m}$ and $r - x = \frac{M_H r}{M_H + m}$

$\Rightarrow L = \left[m(r-x)^2 + M_H X^2\right]\omega$

$\Rightarrow L = \left(\frac{M_H m}{M_H + m}\right)r^2\omega = \mu r^2 \omega$

where $\mu = \frac{M_H m}{M_H + m}$ is called the reduced mass of the electron revolving around the heavy nucleus having finite mass.

Hence, the correct answer is (D).

43. Theoretically, the energy of the electron in the first orbit of the atom whose nucleus is assumed to be infinitely heavy is $E_0 = -\frac{me^4}{8h^2\varepsilon_0^2} = -13.6 \text{ eV}$. However, practically the nucleus is heavy and has finite mass. So, the energy of the electron in the first orbit of the atom is $E = -\frac{\mu e^4}{8h^2\varepsilon_0^2}$, where $\mu = \frac{M_H m}{M_H + m}$ is the reduced mass of the electron and $\mu < m$. So, in the corrected model or the practical model of hydrogen atom, $E > E_0$ and hence the new ground state energy of electron will be more than that found with Bohr's theoretical model.

Hence, the correct answer is (A).

44. $\Delta E = 13.6\, Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\Rightarrow 47.2 = 13.6\, Z^2\left(\frac{1}{4} - \frac{1}{9}\right)$

$\Rightarrow Z = 5$

Hence, the correct answer is (C).

45. Total energy $TE = 13.6(5)^2 = 340 \text{ eV}$

Since, $TE = KE$

$\Rightarrow KE = 340 \text{ V}$

Hence, the correct answer is (C).

46. $\frac{hc}{\lambda}=\frac{13.6(Z)^2}{n^2}$

$\Rightarrow \lambda=\frac{hc\,n^2}{(13.6\ eV)Z^2}$

$\Rightarrow \lambda=\frac{6.63\times10^{-34}\times3\times10^{8}\times9}{13.6\times25\times1.6\times10^{-19}}$

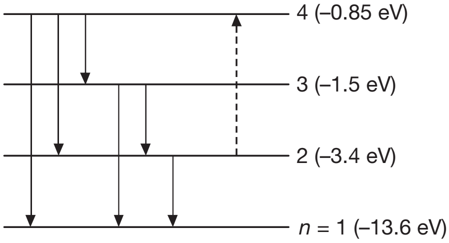
$\Rightarrow \lambda=329$ Å

Hence, the correct answer is (A).

47. $E_2-E_4=-2.5\text{ eV}<2.7\text{ eV}$

So, the electron will be making a transition to $n=4$.

$\Rightarrow n=2$



Hence, the correct answer is (B).

48. $E_n=-\frac{13.6}{n^2}Z^2$

$\Rightarrow E_B=E_2=-3.4\ Z^2$

and $E_C=E_4=-0.85\ Z^2$

Now, $E_C-E_B=2.7$ eV

$\Rightarrow Z=1$

The ionisation energy is $IE=13.6$ eV

Hence, the correct answer is (A).

49. $E_{\text{max}}=E_4-E_1=12.75$ eV

and $E_{\text{min}}=E_4-E_3=0.66$ eV

Hence, the correct answer is (A).

50. $100(E_n-E_1)=\left(\frac{4800}{49}\right)Rch$

$\Rightarrow Rch\left(\frac{1}{1^2}-\frac{1}{n_i^2}\right)=\left(\frac{48}{49}\right)Rch$

$\Rightarrow \frac{1}{n_i^2}=\frac{1}{49}$

$\Rightarrow n_i=7$

So, the atoms are in the sixth excited state and hence $n=6$.

Hence, the correct answer is (B).

51. The maximum energy of the emitted photon is $\left(\frac{48}{49}\right)Rch$

Hence, the correct answer is (C).

52. $N=\frac{n(n-1)}{2}=\frac{7(7-1)}{2}=21$

So, maximum number of photons emitted is 100 N = 2100

Hence, the correct answer is (D).

54. $E_4-E_2=\frac{hc}{\lambda}$

$\Rightarrow -\frac{13.6\times(1)^2}{16}-\frac{(-13.6)\times(1)^2}{4}=\frac{hc}{\lambda}$

$\Rightarrow -0.85+3.4=2.55\text{ eV}=\frac{hc}{\lambda}$

$\Rightarrow \lambda=\frac{12400}{2.55}=4862\text{ Å}=486.2\text{ nm}$

Hence, the correct answer is (C).

55. $\Delta E=\frac{hc}{\lambda}=\frac{12400}{1025}=12.09$

Hence, the correct answer is (D).

56. Since, $KE=\frac{Ze^2}{8\pi\varepsilon_0 r}$

As n increases, r increases, so kinetic energy decreases.

Hence, the correct answer is (B).

57. $E=-13.6\quad E_n=-13.6\left(\frac{Z^2}{n^2}\right)$ eV

$\Rightarrow E_n=-13.6\left(\frac{2^2}{3^2}\right)\text{ eV}=-\frac{54.4}{9}\text{ eV}\approx 6\text{ eV}$

Hence, the correct answer is (A).

58. $E=3.4Z^2$

$\Rightarrow E=3.4\times1^2=3.4$ eV

Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

1. A → (q)
B → (r)
C → (s)
D → (p)

$$r\propto\frac{n^2}{Z},\ v\propto\frac{Z}{n}$$

Since, $i \propto \frac{v}{r}$

$$\Rightarrow \quad i \propto \frac{Z^2}{n^3}$$

Since, $B \propto \frac{i}{r}$

$$\Rightarrow \quad B \propto \frac{Z^3}{n^5}$$

2. A → (r)
B → (p)
C → (s)
D → (t)

$$E = -13.6\left(\frac{Z^2}{n^2}\right) \text{ eV}$$

(A) For second excited state $(n = 3)$ of hydrogen atom $(Z = 1)$, we have

$$E = -13.6\frac{(1)^2}{(3)^2} = -1.51 \text{ eV}$$

(B) For fourth state $(n = 4)$ of $He^+ (Z = 2)$, we have

$$E = -13.6\left(\frac{2}{4}\right)^2 = -3.4 \text{ eV}$$

(C) For first excited state $(n = 2)$ of $Li^{++} (Z = 3)$, we have

$$U = 2E = -27.2\left(\frac{Z^2}{n^2}\right)$$

$$\Rightarrow \quad U = -27.2\left(\frac{9}{4}\right) = -61.2 \text{ eV}$$

(D) For second excited state $(n = 3)$ of $Li^{++} (Z = 3)$, we have

$$K = -E = 13.6\left(\frac{Z^2}{n^2}\right) \text{ eV}$$

$$\Rightarrow \quad K = 13.6\left(\frac{3}{3}\right)^2 = 13.6 \text{ eV}$$

4. A → (r)
B → (s)
C → (p)
D → (q, s)

For H-like atom (atomic number Z), the energy of electron in n^{th} orbit is $E_n = -(13.6)\frac{Z^2}{n^2}$ eV

(A) For $Li^{++} (Z = 3)$, so we have

$$E = -13.6 \times 9 = -122.4 \text{ eV}$$

So, ionisation energy is 122.4 eV

(B) For $Be^{+++} (Z = 4)$ and $n = 2$, we have

$$E = -\frac{13.6(4)^2}{(2)^2} = -54.4 \text{ eV}$$

$$\Rightarrow \quad |E| = 54.4 \text{ eV}$$

(C) For $B^{++++} (Z = 5)$ and $n = 1$, we have

$$E = -\frac{(13.6)(5)^2}{(1)^2} = -340 \text{ eV}$$

So, ionisation Energy is 340 eV

(D) For $He^+ (Z = 1)$, the assembling energy is equal to ionisation energy i.e.

$$|E_n| = +13.6\left(\frac{Z^2}{n^2}\right) \text{ eV}$$

$$\Rightarrow \quad |E_n| = 13.6\frac{(2)^2}{(1)^2} = 54.4 \text{ eV}$$

For $He^+ (Z = 1)$, in the third excited state i.e. $n = 4$, we have

$$|E_n| = 13.6\left(\frac{2}{4}\right)^2$$

$$\Rightarrow \quad |E_n| = \frac{13.6}{4} \text{ eV} = 3.4 \text{ eV}$$

9. A → (p, s)
B → (u)
C → (r)
D → (t)

$$L = mvr = \frac{nh}{2\pi}$$

For $n = 1$, $L = \frac{h}{2\pi}$

For $n = 4$, $L = \frac{2h}{\pi}$

$$E_n = -13.6\left(\frac{Z^2}{n^2}\right) \text{ eV}$$

$$\Rightarrow \quad E_n = -13.6(2)^2 = -54.4 \text{ eV}$$

$\Rightarrow \quad U_n = 2E_n = -108.8 \text{ eV}$

Since, $K_n = -E_n = (13.6)\dfrac{Z^2}{n^2}$

$\Rightarrow \quad K_n = (13.6)\left(\dfrac{2}{2}\right)^2$

$\Rightarrow \quad K_n = 13.6 \text{ eV}$

11. A $\rightarrow$ (s)
B $\rightarrow$ (r)
C $\rightarrow$ (q)
D $\rightarrow$ (p)
For a given atomic number (Z), the energy and hence frequency of K-series is more than the L-series. In one series also, β-line has more energy or frequency compared to that of α-line.

12. A $\rightarrow$ (p, s)
B $\rightarrow$ (q, s)
C $\rightarrow$ (q, s)
D $\rightarrow$ (s)

$E \propto \dfrac{1}{n^2}$

$\Rightarrow \quad p \propto \dfrac{1}{n}$

$\Rightarrow \quad r \propto n^2$

$\Rightarrow \quad \dfrac{E}{p} \propto \dfrac{1}{n}$ and $Epr \propto \dfrac{1}{n}$

$\Rightarrow \quad \dfrac{r}{p} \propto n^3$ and $Er = \text{constant}$

13. A $\rightarrow$ (p, r, t)
B $\rightarrow$ (q, s)
C $\rightarrow$ (q, s)
D $\rightarrow$ (p, r, t)

$L \propto n$, $r \propto \dfrac{n^2}{Z}$, $v \propto \dfrac{Z}{n}$

$\Rightarrow \quad f \propto \dfrac{v}{r} \propto \dfrac{Z^2}{n^3}$

Since, $i \propto \dfrac{v}{r}$

$\Rightarrow \quad i \propto \dfrac{Z^2}{n^3}$

Also, $M = iA$

$\Rightarrow \quad M = i(\pi r^2)$

$\Rightarrow \quad M \propto \left(\dfrac{Z^2}{n^3}\right)\left(\dfrac{n^4}{Z^2}\right)$

$\Rightarrow \quad M \propto n$

15. A $\rightarrow$ (s)
B $\rightarrow$ (t)
C $\rightarrow$ (p)
D $\rightarrow$ (q)
E $\rightarrow$ (r)

Since, $\Delta E = \dfrac{hc}{\lambda}$

For $(n+1) \rightarrow n$ transition, $\lambda = \lambda_{\text{max}}$

$\Rightarrow \quad \lambda_{\text{max}} = \dfrac{n^2(n+1)^2}{(2n+1)R}$

For $\infty \rightarrow n$ transition, $\lambda = \lambda_{\text{min}}$

$\Rightarrow \quad \lambda_{\text{min}} = \dfrac{n^2}{R}$

For Lyman series, $n = 1$

$\Rightarrow \quad \lambda_{\text{max}} = \dfrac{4}{3R}$ and $\lambda_{\text{min}} = \dfrac{1}{R}$

Balmer series, $n = 2$

$\Rightarrow \quad \lambda_{\text{max}} = \dfrac{36}{5R}$ and $\lambda_{\text{min}} = \dfrac{4}{R}$

Paschen series, $n = 3$

$\Rightarrow \quad \lambda_{\text{max}} = \dfrac{144}{7R}$ and $\lambda_{\text{min}} = \dfrac{9}{R}$

Brackett series, $n = 4$

$\Rightarrow \quad \lambda_{\text{max}} = \dfrac{400}{9R}$ and $\lambda_{\text{min}} = \dfrac{16}{R}$

Pfund series, $n = 5$

$\Rightarrow \quad \lambda_{\text{max}} = \dfrac{900}{11R}$ and $\lambda_{\text{min}} = \dfrac{25}{R}$

16. A $\rightarrow$ (p, q, t)
B $\rightarrow$ (r)
C $\rightarrow$ (s)
D $\rightarrow$ (t)

$$v \propto \frac{1}{n},\ KE \propto \frac{1}{n^2},\ L \propto n,\ \omega = \frac{v}{r} \text{ and } i \propto \frac{v}{r}$$

Since, $r \propto n^2$ and $v \propto \frac{1}{n}$

$$\Rightarrow \quad \omega \propto \frac{1}{n^3} \text{ and } i \propto \frac{1}{n^3}$$

18. A → (p)
B → (p)
C → (q)
D → (s)

$$f \propto \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \quad f = k\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right), \text{ where } k \text{ is a constant}$$

For f_1, we have $n_1 = 1$ and $n_2 \to \infty$

$$\Rightarrow \quad f_1 = k$$

For f_2, we have $n_1 = 1$ and $n_2 = 2$

$$\Rightarrow \quad f_2 = \frac{3k}{4}$$

For f_3, we have $n_1 = 2$ and $n_2 \to \infty$

$$\Rightarrow \quad f_3 = \frac{k}{4}$$

$$\Rightarrow \quad f_1 - f_2 = f_3$$

19. A → (p)
B → (s)
C → (q)
D → (s)

$$\frac{1}{\lambda} \propto \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \quad \frac{\lambda_2}{\lambda_1} = \frac{\left(1/n_1^2 - 1/n_2^2\right)_i}{\left(1/n_1^2 - 1/n_2^2\right)_f}$$

$$\Rightarrow \quad \lambda_2 = \frac{(1/4 - 1/16)\lambda}{\left(1/n_1^2 - 1/n_2^2\right)_f}$$

$$\Rightarrow \quad \lambda_2 = \left(\frac{3}{16}\lambda\right)\left[\frac{1}{\left(1/n_1^2 - 1/n_2^2\right)_f}\right]$$

(A) For first line of Balmer series,

$$n_1 = 2,\ n_2 = 3$$

$$\Rightarrow \quad \lambda_2 = \left(\frac{27}{20}\right)\lambda$$

(B) For third line of Balmer series,

$$n_1 = 2,\ n_2 = 5$$

$$\Rightarrow \quad \lambda_2 = \left(\frac{25}{28}\right)\lambda$$

(C) For first line of Lyman series,

$$n_1 = 1,\ n_2 = 2,$$

$$\Rightarrow \quad \lambda_2 = \frac{\lambda}{4}$$

(D) For second line of Lyman series,

$$n_1 = 2,\ n_2 = 3$$

$$\Rightarrow \quad \lambda_2 = \left(\frac{27}{128}\right)\lambda$$

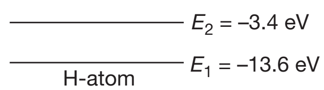
20. A → (r)
B → (p)
C → (s)
D → (q)

$$E_n = -13.6\left(\frac{Z^2}{n^2}\right) \text{ eV}$$

21. A → (s)
B → (r)
C → (s)
D → (p)

For He^+ atom, $Z = 2$ and $E_n = -13.6\left(\frac{Z^2}{n^2}\right)$ eV

$$\Rightarrow \quad E \propto \frac{Z^2}{n^2}$$



Ionization energy from first excited state of H-atom is given to be

$$E = |E_2| = 3.4 \text{ eV}$$

(A) For He^+ atom $(Z=2)$, so

$$|E_1| = (13.6\text{ eV})(2)^2 = 16(3.4\text{ eV}) = 16E$$

(B) $U_2 = 2E_2 = 2(-13.6)\dfrac{(2)^2}{(2)^2}$

$\Rightarrow \quad U_2 = -8(3.4\text{ eV}) = -8E$

(C) $K_1 = |E_1| = 16E$

(D) $|E_2| = (13.6)\dfrac{(2)^2}{(2)^2} = 4(3.4\text{ eV}) = 4E$

Integer/Numerical Answer Type Questions

1. Acceleration of the revolving electron is $a = \dfrac{v^2}{r}$. Since, we know that $v \propto \dfrac{Z}{n}$ and $r \propto \dfrac{n^2}{Z}$. So, for the same orbit of both the atoms, we have

$$a \propto \frac{Z^2}{(1/Z)}$$

$\Rightarrow \quad a \propto Z^3$

$\Rightarrow \quad \dfrac{a_1}{a_2} = \left(\dfrac{Z_1}{Z_2}\right)^3 = \left(\dfrac{2}{1}\right)^3 = 8$

2. After removing the first electron it will become He^+ ion. The ionization energy of single electron in He^+ ion $(Z=2)$ is

$$IE = 13.6(Z^2) = 54.4\text{ eV}$$

Therefore, total energy required to remove both the electrons is given by

$$E = (24.6 + 54.4)\text{ eV} = 79\text{ eV}$$

3. $v = \sqrt{\dfrac{2eV_0}{m}} = \sqrt{\dfrac{2(1.6\times10^{-19})(18000)}{9\times10^{-31}}}$

$\Rightarrow \quad v = 8\times10^7\text{ ms}^{-1}$

$\Rightarrow \quad x = 8$

4. $E_n - E_1 = \dfrac{12375}{1085} + \dfrac{12375}{304}$

$\Rightarrow \quad (13.6)(Z^2)\left(1 - \dfrac{1}{n^2}\right) = 52.1\text{ eV}$

$\Rightarrow \quad 1 - \dfrac{1}{n^2} = 0.96 \qquad \{\because Z = 2\}$

$\Rightarrow \quad n \approx 5$

5. Since, we know that inside a material of coefficient of absorption μ, the intensity of X-rays decays exponentially with distance x as $I = I_0 e^{-\mu x}$. So, we have

$$\frac{I_0}{2} = I_0 e^{-\mu x}$$

$\Rightarrow \quad \mu x = \ell n2$

$\Rightarrow \quad x = \dfrac{\ell n2}{\mu} = \dfrac{0.693}{50} = 0.01386\text{ cm}$

$\Rightarrow \quad x = 138.6\ \mu\text{m} \approx 139\ \mu\text{m}$

6. Since, $\dfrac{1}{\lambda} = R(Z-1)^2\left(\dfrac{1}{1^2} - \dfrac{1}{2^2}\right)$

$\Rightarrow \quad \dfrac{1}{0.76\times10^{-10}} = (1.09\times10^7)(Z-1)^2\left(\dfrac{3}{4}\right)$

$\Rightarrow \quad Z - 1 \approx 40$

$\Rightarrow \quad Z \approx 41$

7. $E = e\Delta V = \dfrac{hc}{\lambda}$

$\Rightarrow \quad E = \dfrac{hc}{\lambda} = \dfrac{12400}{3.1} = 4000\text{ eV}$

$\Rightarrow \quad \Delta V = \dfrac{E}{e} = \dfrac{4000\text{ eV}}{e} = 4000\text{ V} = 4\text{ kV}$

8. Let m_1 and m_2 be the mass of α-particle and hydrogen atom.

By Law of Conservation of Momentum, we get

$$m_1 u_1 = (m_1 + m_2)v$$

where u_1 is the initial velocity of the incident α-particle and v is the final common velocity (or velocity of centre of mass) of the particles.

By Law of Conservation of Energy, we get

$$\frac{1}{2}m_1 v_1^2 = \frac{1}{2}(m_1 + m_2)v^2 + \Delta E_0$$

where ΔE_0 is the Ionization Energy

The loss in KE of the α-particles must be gained by the atom as ionisation energy, so we have

$$\frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)v_1^2 = \Delta E_0$$

$$\Rightarrow \quad K_1 = \frac{1}{2}m_1v_1^2 = \left(\frac{m_1+m_2}{m_2}\right)\Delta E_0$$

$$\Rightarrow \quad K_1 = \left(1+\frac{m_1}{m_2}\right)\Delta E_0$$

$$\Rightarrow \quad K_1 = \left(1+\frac{4}{1}\right)(13.6)\text{ eV}$$

$$\Rightarrow \quad K_1 = 68\text{ eV}$$

9. $P = VI$

Total power drawn by tube is $P = VI = 200$ W

As 0.5% of the energy is carried by the electrons, so power possessed by the X-ray beam is

$$P' = \left(\frac{0.5}{100}\right)(200) = 1\text{ W}$$

10. $U = -1.7$ eV

$$\Rightarrow \quad E = \frac{U}{2} = -0.85\text{ eV} = \frac{-13.6}{n^2}$$

$$\Rightarrow \quad n = 4$$

Ejected photoelectron will have minimum de-Broglie wavelength corresponding to transition from $n = 4$ to $n = 1$, so we have

$$\Delta E = E_4 - E_1 = -0.85 - (-13.6) = 12.75\text{ eV}$$

Using Einstein's Photo-Electric Equation, we get

$$K_{max} = \Delta E - W = 10.45\text{ eV}$$

$$\Rightarrow \quad \lambda = \sqrt{\frac{150}{10.45}}\text{ Å} \qquad \text{\{for an electron\}}$$

$$\Rightarrow \quad \lambda = 3.8\text{ Å} \approx 4\text{ Å}$$

11. Shortest wavelength of Brackett is obtained when transition takes place from $n = 4$ to $n \to \infty$ and is obtained when transition takes place from $n = 2$ to $n \to \infty$.

$$\Rightarrow \quad (13.6)Z^2\left(\frac{1}{16}-\frac{1}{\infty}\right) = 13.6\left(\frac{1}{4}-\frac{1}{\infty}\right)$$

$$\Rightarrow \quad Z = 2$$

12. (a) Since, $N = \dfrac{n(n-1)}{2} = 6$

$$\Rightarrow \quad n = 4$$

i.e., if $n_1 = n$, then $n_2 = n - 4$

$$\Rightarrow \quad \frac{(13.6)Z^2}{n^2} = 0.544 \qquad \ldots(1)$$

$$\Rightarrow \quad \frac{(13.6)Z^2}{(n-4)^2} = 0.85 \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$Z = 4 \text{ and } n = 20$$

(b) $\lambda_{min} = \dfrac{12375}{-0.544-(0.85)} = 40441$ Å

13. Given, $\lambda_1 = 4102$ Å, $\lambda_2 = 4861$ Å

For Balmer Series, we have

$$\Rightarrow \quad \frac{1}{\lambda_1} = R\left(\frac{1}{2^2}-\frac{1}{n_1^2}\right)$$

Substituting values of λ_1 and R, we get

$$n_1 = 6$$

Similarly, for Balmer Series, we have

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2}-\frac{1}{n_2^2}\right)$$

Substituting values of λ_2 and R, we get

$$n_2 = 4$$

Now difference of wave numbers of above two lines is

$$\frac{1}{\lambda_1}-\frac{1}{\lambda_2} = \frac{1}{\lambda} = R$$

$$\Rightarrow \quad \frac{1}{\lambda_1}-\frac{1}{\lambda_2} = \frac{1}{\lambda} = R\left(\frac{1}{n_2^2}-\frac{1}{n_1^2}\right)$$

Transition $n_2 \to n_1$ or $6 \to 4$ corresponds to second line of Brackett series, whose wavelength is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{4^2}-\frac{1}{6^2}\right)$$

Substituting the values, we get

$$\lambda = 26206\text{ Å}$$

14. $\dfrac{hc}{\lambda_1}+\dfrac{hc}{\lambda_2} = RchZ^2\left(\dfrac{1}{1^2}-\dfrac{1}{n^2}\right)$

$$\Rightarrow \quad n \approx 6$$

15. From the given conditions

$$E_n - E_2 = (10.2+17)\text{ eV} = 27.2\text{ eV} \qquad \ldots(1)$$

and $E_n - E_3 = (4.25+5.95)\text{ eV} = 10.2\text{ eV} \qquad \ldots(2)$

CHAPTER 2

Equation (1) – (2) gives

$$E_3 - E_2 = 17 \text{ eV}$$

$$\Rightarrow \quad Z^2(13.6)\left(\frac{1}{4}-\frac{1}{9}\right)=17$$

$$\Rightarrow \quad Z^2(13.6)\left(\frac{5}{36}\right)=17$$

$$\Rightarrow \quad Z^2 = 9$$

$$\Rightarrow \quad Z = 3$$

From equation (1), we get

$$Z^2(13.6)\left(\frac{1}{4}-\frac{1}{n^2}\right)=27.2$$

$$\Rightarrow \quad (3)^2(13.6)\left(\frac{1}{4}-\frac{1}{n^2}\right)=27.2$$

$$\Rightarrow \quad \frac{1}{4}-\frac{1}{n^2}=0.222$$

$$\Rightarrow \quad \frac{1}{n^2}=0.0278$$

$$\Rightarrow \quad n^2 = 36$$

$$\Rightarrow \quad n = 6$$

16. The energy required to remove both the electrons from the atom is

$$E = 25.6 + 13.6(2)^2\left(\frac{1}{1^2}-\frac{1}{\infty}\right)$$

$$\Rightarrow \quad E = 25.6 + 54.4 = 80 \text{ eV}$$

$$\Rightarrow \quad N = 8$$

17. (a) 1 rydberg $= 2.2\times10^{-18}$ J $=$ Rhc

Ionisation energy is given as 4 rydberg, so

$$IE = 8.8\times10^{-18}\text{ J} = \frac{8.8\times10^{-18}}{1.6\times10^{-19}} = 55 \text{ eV}$$

$\Rightarrow$ Energy in first orbit is $E_1 = -55$ eV

Energy of radiation emitted when electron jumps from first excited state $(n=2)$ to ground state $(n=1)$ is

$$E_{21} = \frac{E_1}{(2)^2} - E_1 = -\frac{3}{4}E_1 = 41.25 \text{ eV}$$

So, wavelength of photon emitted in this transition is

$$\lambda = \frac{12375}{41.25} = 300 \text{ Å}$$

(b) Let Z be the atomic number of given element. Then

$$E_1 = (-13.6)(Z^2)$$

$$\Rightarrow \quad -55 = (-13.6)(Z^2)$$

$$\Rightarrow \quad Z \approx 2$$

(c) Since, $r = \frac{a_0}{Z}$, so the radius of first orbit of this atom is

$$r_1 = \frac{a_0}{2}$$

$$\Rightarrow \quad * = 2$$

18. According to Moseley's Law $\frac{1}{\lambda} \propto (Z-1)^2$

$$\Rightarrow \quad \frac{\lambda_2}{193} = \frac{(26-1)^2}{(29-1)^2} = \left(\frac{25}{28}\right)^2$$

$$\Rightarrow \quad \lambda_2 = 193\left(\frac{25}{28}\right)^2 \approx 154 \text{ pm}$$

19. $E_n = -\frac{(13.6)z^2}{n^2}$ eV

Substituting $z = 3$, we get

$$E_n = -\frac{122.4}{n^2} \text{ eV}$$

$$\Rightarrow \quad E_1 = -\frac{122.4}{(1)^2} = -122.4 \text{ eV}$$

and $E_3 = -\frac{122.4}{(3)^2} = -13.6$ eV

$$\Rightarrow \quad \Delta E = E_3 - E_1 = 108.8 \text{ eV}$$

The corresponding wavelength is

$$\lambda = \frac{12375}{\Delta E(\text{in eV})}\text{ Å} = \frac{12375}{108.8}\text{ Å} = 113.74 \text{ Å}$$

$$\Rightarrow \quad \lambda \approx 114 \text{ Å}$$

20. Total number of electrons striking the target per second

$$n = \frac{10\times10^{-3}}{1.6\times10^{-19}} = 6.25\times10^{16}$$

Kinetic energy of an electron is

$$K = 40\times10^3 \text{ eV} = 40\times10^3\times1.6\times10^{-19} = 6.4\times10^{-15} \text{ J}$$

$\therefore$ Total energy of electrons striking the target (per second) is

$$E = 6.25\times10^{16}\times6.4\times10^{-15} = 400 \text{ J}$$

(a) Total power emitted as X-rays is 1% of 400 W. So

$$P = 4\text{ W}$$

(b) Heat produced per second is

$$H = (400 - 4)\text{ Js}^{-1} = 396\text{ Js}^{-1}$$

21. $E_L - E_K = \dfrac{hc}{\lambda}$

$$\Rightarrow \lambda = \frac{hc}{E_L - E_K} = \frac{12375}{\left(\frac{15.525}{1000}\right)}$$

$$\Rightarrow \lambda = 7.97 \times 10^{-11}\text{ m} \approx 8 \times 10^{-11}\text{ m}$$

$$\Rightarrow x = 8$$

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1. Energy released by hydrogen atom

$$\Delta E_1 = 13.6 \times \left(\frac{1}{1} - \frac{1}{4}\right) = \frac{3}{4} \times 13.6\text{ eV} = 10.2\text{ eV}$$

Also, energy absorbed by He^+ ion in transition $n = 2 \to n = 4$ is

$$\Delta E_2 = 13.6 \times 4 \times \left(\frac{1}{4} - \frac{1}{16}\right) = 10.2\text{ eV}$$

So, possible transition is $n = 2 \to n = 4$

Hence, the correct answer is (A).

2. $$\frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$$

$$\Rightarrow \frac{5\lambda_1}{36} = \frac{12\lambda_2}{4 \times 16}$$

$$\Rightarrow \lambda_2 = \frac{5 \times 660 \times 64}{36 \times 12} = 489\text{ nm}$$

Hence, the correct answer is (B).

3. $$E_n = -\left(\frac{13.6Z^2}{n^2}\right)\text{ eV}$$

For He^+ ion ($Z = 2$) in first excited state $(n = 2)$, we get

$$E_n = -13.6\text{ eV}$$

$\Rightarrow$ Ionisation energy is 13.6 eV

Hence, the correct answer is (A).

4. $$\Delta E = \frac{hc}{\lambda}$$

$$\Rightarrow \Delta E = (13.4)(3)^2\left(1 - \frac{1}{16}\right)\text{ eV}$$

$$\Rightarrow \lambda = \frac{1242 \times 16}{(13.4) \times (9)(15)}\text{ nm} \approx 10.8\text{ nm}$$

Hence, the correct answer is (D).

5. $$\Rightarrow \Delta E_n = -\frac{13.6Z^2}{n^2}$$

Let it start from n_1 to n_2 and from n_2 to ground.

Then $$13.6 \times 4\left|1 - \frac{1}{n_2^2}\right| = \frac{hc}{30.4\text{ nm}}$$

$$\Rightarrow 1 - \frac{1}{n_2^2} = 0.7498$$

$$\Rightarrow 0.25 = \frac{1}{n_2^2}$$

$$\Rightarrow n_2 = 2$$

$$13.6 \times 4\left(\frac{1}{4} - \frac{1}{n_1^2}\right) = \frac{hc}{108.5 \times 10^{-9}}$$

$$\Rightarrow n_1 \approx 5$$

Hence, the correct answer is (A).

6. $$\frac{1}{\lambda_1} = R\left(\frac{1}{9} - \frac{1}{16}\right) = R\frac{7}{144} \quad \ldots(1)$$

$$\frac{1}{\lambda_2} = R\left[\frac{1}{4} - \frac{1}{9}\right] = R\frac{5}{36} \quad \ldots(2)$$

$$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{5 \times 144}{36 \times 7} = \frac{20}{7}$$

Hence, the correct answer is (D).

7. $$\Delta E = \frac{hc}{\lambda}$$

$$\Rightarrow \Delta E = \frac{12500}{980} = 12.76\text{ eV}$$

$$\Rightarrow E_n - E_1 = 12.76$$

$$\Rightarrow E_n = E_1 + 12.76$$

$$\Rightarrow E_n = -13.6 + 12.76$$

Since $E_n = -\dfrac{13.6}{n^2}$ eV

$$\Rightarrow E_n = -0.84\text{ eV} = \frac{-13.6}{n^2}\text{ eV}$$

CHAPTER 2

$\Rightarrow \quad n = 4$

$\Rightarrow \quad r_n = 16a_0$

Hence, the correct answer is (D).

8. $\dfrac{1}{\lambda} = RZ^2\left[\dfrac{1}{4} - \dfrac{1}{9}\right] = \dfrac{5RZ^2}{36}$

$$\frac{1}{\lambda'} = RZ^2\left[\frac{1}{4} - \frac{1}{16}\right] = \frac{3RZ^2}{16}$$

$$\lambda' = \frac{16}{3RZ^2} \text{ and } \lambda = \frac{36}{5RZ^2}$$

$$\Rightarrow \quad \frac{\lambda'}{\lambda} = \frac{16 \times 5}{3 \times 36} = \frac{20}{27}$$

$$\Rightarrow \quad \lambda' = \frac{20}{27}\lambda$$

Hence, the correct answer is (C).

9. Since $F = -\dfrac{dU}{dr}$

$$\Rightarrow \quad F = kr = \frac{mv^2}{r}$$

$$\Rightarrow \quad v^2 = \frac{k}{m}r^2$$

$$\Rightarrow \quad v = \sqrt{\frac{k}{m}}r \qquad \ldots(1)$$

Also, $mvr = \dfrac{nh}{2\pi}$ $\qquad \ldots(2)$

Solving (1) and (2), we get

$$m\left(\sqrt{\frac{k}{m}}r\right)r = \frac{nh}{2\pi}$$

$$\Rightarrow \quad r \propto \sqrt{n}$$

Since, $E = PE + KE$

$$\Rightarrow \quad E = \frac{1}{2}kr^2 + \frac{1}{2}m\left(\sqrt{\frac{k}{m}}r\right)^2$$

$$\Rightarrow \quad E \propto r^2$$

$$\Rightarrow \quad E \propto n$$

Hence, the correct answer is (B).

10. Momentum of electron in different states

$$p_n = \frac{h}{\lambda_n}, \; p_g = \frac{h}{\lambda_g}$$

Kinetic energy, $K = \dfrac{p^2}{2m} = \dfrac{h^2}{2m\lambda^2}$

Total energy in an orbit of hydrogen atom, $E = -K = -\dfrac{h^2}{2m\lambda^2}$

$$E_n - E_g = \frac{h^2}{2m}\left(\frac{1}{\lambda_g^2} - \frac{1}{\lambda_n^2}\right)$$

$$\frac{h^2}{2m}\left(\frac{\lambda_n^2 - \lambda_g^2}{\lambda_g^2\lambda_n^2}\right) = \frac{hc}{\Lambda_n}$$

$$\Rightarrow \quad \Lambda_n = \frac{2mc}{h}\left(\frac{\lambda_g^2\lambda_n^2}{\lambda_n^2 - \lambda_g^2}\right)$$

$$\Rightarrow \quad \Lambda_n = \frac{2mc\lambda_g^2}{h}\left[1 - \frac{\lambda_g^2}{\lambda_n^2}\right]^{-1} = \frac{2mc\lambda_g^2}{h}\left[1 + \frac{\lambda_g^2}{\lambda_n^2}\right]$$

$$\{\because \lambda_g \ll \lambda_n\}$$

$$\Rightarrow \quad \Lambda_n = \frac{2mc\lambda_g^2}{h} + \left(\frac{2mc\lambda_g^4}{h}\right)\frac{1}{\lambda_n^2} = A + \frac{B}{\lambda_n^2}$$

where A and B are $A = \dfrac{2mc\lambda_g^2}{h}$, $B = \dfrac{2mc\lambda_g^4}{h}$

Hence, the correct answer is (A).

11. Frequency of emitted photon in a hydrogen atom is given by $\nu = Rc\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

For Lyman series, series limit condition is given by

$$n_2 = \infty, \; n_1 = 1.$$

$$\Rightarrow \quad \nu_L = Rc\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = Rc \qquad \ldots(1)$$

For Pfund series, series limit condition is given by,

$$n_2 = \infty, \; n_1 = 5$$

$$\Rightarrow \quad \nu_P = Rc\left(\frac{1}{5^2} - \frac{1}{\infty^2}\right) = \frac{Rc}{25} \qquad \ldots(2)$$

From equation (1) and (2), $\nu_P = \dfrac{\nu_L}{25}$

Hence, the correct answer is (D).

12. Energy required to remove an electron from singly ionized helium atom $= 54.4$ eV.

Energy required to remove the electron from helium atom be x eV

Given $54.4 \text{ eV} = 2.2x$

$\Rightarrow \quad x = 24.73$ eV

Total energy required to ionize helium atom is

$$E = 54.4 + 24.73 = 79.13 \text{ eV}$$

Hence, the correct answer is (B).

14. As photon energy, $E = \frac{hc}{\lambda}$

$\Rightarrow \quad \frac{\lambda_N}{\lambda_A} = \frac{E_A}{E_N}$

where E_A and E_N are energies of photons from atom and nucleus respectively. E_N is of the order of MeV and E_a in few eV.

So $\frac{\lambda_N}{\lambda_A} = 10^{-6}$

Hence, the correct answer is (B).

15. Since $\lambda = \frac{hc}{|\Delta E|}$, so from energy level diagram, we have

$$\lambda_1 = \frac{hc}{E}$$

$$\lambda_2 = \frac{hc}{\left(\frac{E}{3}\right)}$$

$$\Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \frac{1}{3}$$

Hence, the correct answer is (D).

16. Magnetic field at the centre, $B_n = \frac{\mu_0 I}{2r_n}$

For a hydrogen atom, radius of n^{th} orbit is given by

$$r_n = \left(\frac{n^2}{m}\right)\left(\frac{h}{2\pi}\right)^2 \frac{4\pi\varepsilon_0}{e^2}$$

$$\Rightarrow \quad r_n \propto n^2$$

$$I = \frac{e}{T} = \frac{e}{\frac{2\pi r_n}{v_n}} = \frac{ev_n}{2\pi r_n}$$

Also, $v_n \propto n^{-1}$

$\Rightarrow \quad I \propto n^{-3}$ Hence, $B_n \propto n^{-5}$

Hence, the correct answer is (D).

17. For first orbit of hydrogen atom $(n = 1)$,

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \qquad \ldots(1)$$

$$mvr = \frac{h}{2\pi} \qquad \ldots(2)$$

Squaring equation (2), we get $m^2v^2r^2 = \frac{h^2}{4\pi^2}$

Dividing both sides by r^3, we get

$$\frac{m^2v^2}{r} = \frac{h^2}{4\pi^2 r^3}$$

$$\Rightarrow \quad \frac{v^2}{r} = \frac{h^2}{4\pi^2 r^3 m^2}$$

This is required acceleration of the electron.

Hence, the correct answer is (B).

18. Energy of emitted photon

$$E = \left[\frac{1}{1^2} - \frac{1}{2^2}\right] \times 13.6 \text{ eV} = \frac{3}{4} \times 13.6 \text{ eV}$$

Energy required to completely remove the electron from n^{th} excited state of doubly ionized lithium,

$$E' = \frac{13.6 Z^2}{n^2}\text{eV} = \frac{13.6 \times 9}{n^2}\text{eV}$$

Since $E \geq E'$

$$\Rightarrow \quad \frac{3}{4} \times 13.6 \geq \frac{13.6 \times 9}{n^2}$$

$$\Rightarrow \quad n^2 \geq 3 \times 4$$

$$\Rightarrow \quad n \geq \sqrt{12} = 3.5$$

$\Rightarrow$ Least quantum number for the excited state = 4.

Hence, the correct answer is (B).

19. $KE \propto \frac{z^2}{n^2}$

So, as n decreases, KE decreases, PE decreases and TE decreases.

Hence, the correct answer is (A).

20. $$mvR = \frac{nh}{2\pi} \qquad \ldots(1)$$

and $qvB = \frac{mv^2}{R}$; $qB = \frac{mv}{R}$ ...(2)

From equations (1) and (2), we get $qB\left(\frac{nh}{2\pi mv}\right) = mv$

$$\frac{1}{2}mv^2 = \frac{1}{4\pi m} nhqB$$

$$\Rightarrow \quad E = n\left(\frac{hqB}{4\pi m}\right)$$

Hence, the correct answer is (B).

21. Since, $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

$$\Rightarrow \quad \lambda \propto \frac{1}{Z^2}$$

$$\Rightarrow \quad \lambda Z^2 = \text{constant}$$

$$\Rightarrow \quad \lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$$

Hence, the correct answer is (C).

22. $f \propto \left(\frac{1}{(n-1)^2} - \frac{1}{(n)^2}\right)$

$$\Rightarrow \quad f \propto \frac{n^2-(n-1)^2}{n^2(n-1)^2}$$

$$\Rightarrow \quad f \propto \frac{n^2-n^2-1+2n}{n^2(n-1)^2}$$

$$\Rightarrow \quad f \propto \frac{2n-1}{n^2(n-1)^2}$$

Since, $n \gg 1$

$$\Rightarrow \quad f \propto \frac{1}{n^3}$$

Hence, the correct answer is (D).

23. Number of spectral lines in the emission spectra,

$$N = \frac{n(n-1)}{2}$$

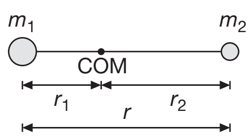
Here, $n = 4$

$$\Rightarrow \quad N = \frac{4(4-1)}{2} = 6$$

Hence, the correct answer is (C).

24. A diatomic molecule consists of two atoms of masses m_1 and m_2 at a distance r apart. Let r_1 and r_2 be the distances of the atoms from the centre of mass.
The moment of inertia of this molecule about an axis passing through its centre of mass and perpendicular to a line joining the atoms is

$$I = m_1 r_1^2 + m_2 r_2^2$$



As $m_1 r_1 = m_2 r_2$

$$\Rightarrow \quad r_1 = \frac{m_2}{m_1} r_2$$

$$\Rightarrow \quad r_1 + r_2 = r$$

$$\Rightarrow \quad r_1 = \frac{m_2}{m_1}(r - r_1)$$

On rearranging, we get $r_1 = \frac{m_2 r}{m_1 + m_2}$

Similarly, $r_2 = \frac{m_1 r}{m_1 + m_2}$

Therefore, the moment of inertia can be written as

$$I = m_1\left(\frac{m_2 r}{m_1+m_2}\right)^2 + m_2\left(\frac{m_1 r}{m_1+m_2}\right)^2 = \frac{m_1 m_2}{m_1+m_2} r^2 \quad \ldots(1)$$

According to Bohr's quantisation condition

$$L = \frac{nh}{2\pi} = n\hbar \qquad \left\{\because \hbar = \frac{h}{2\pi}\right\}$$

$$\Rightarrow \quad L^2 = \frac{n^2h^2}{4\pi^2} = n^2\hbar^2 \quad \ldots(2)$$

Rotational energy, $E = \frac{L^2}{2I}$

$$\Rightarrow \quad E = \frac{n^2}{8\pi^2 I} = \frac{n^2\hbar^2}{2I} \qquad \{\text{using (2)}\}$$

$$\Rightarrow \quad E = \frac{n^2h^2(m_1+m_2)}{8\pi^2(m_1m_2)r^2} = \frac{n^2\hbar^2}{2}\frac{(m_1+m_2)}{(m_1m_2)r^2} \qquad \{\text{using (1)}\}$$

$$\Rightarrow \quad E = \frac{n^2\hbar^2(m_1+m_2)}{2m_1m_2r^2}$$

Hence, the correct answer is (C).

25. Using, $E_n = \frac{13.6Z^2}{n^2}\text{eV}$

Here, $Z = 3$ (For Li^{++})

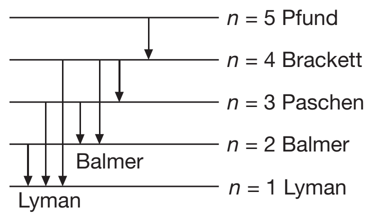
$$\Rightarrow \quad E_1 = -\frac{13.6(3)^2}{(1)^2}\text{eV}$$

$$\Rightarrow \quad E_1 = -122.4 \text{ eV and } E_3 = \frac{-13.6\times(3)^2}{(3)^2} = -13.6 \text{ eV}$$

$$\Delta E = E_3 - E_1 = -13.6 + 122.4 = 108.8 \text{ eV}$$

Hence, the correct answer is (C).

26.



Transition $4 \rightarrow 3$ is in Paschen series. This is not in the ultraviolet region but this is in infrared region.
Transition $5 \rightarrow 4$ will also be in infrared region (Brackett).
Hence, the correct answer is (D).

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Single Correct Choice Type Problems

1. K_α transition takes place from $n_1 = 2$ to $n_2 = 1$

$$\Rightarrow \frac{1}{\lambda} = R(Z-b)^2\left[\frac{1}{(1)^2} - \frac{1}{(2)^2}\right]$$

For K-series, $b = 1$

$$\Rightarrow \frac{1}{\lambda} \propto (Z-1)^2$$

$$\Rightarrow \frac{\lambda_{Cu}}{\lambda_{Mo}} = \frac{(z_{Mo}-1)^2}{(z_{Cu}-1)^2} = \frac{(42-1)^2}{(29-1)^2}$$

$$\Rightarrow \frac{\lambda_{Cu}}{\lambda_{Mo}} = \frac{41\times 41}{28\times 28} = \frac{1681}{784} = 2.144$$

Hence, the correct answer is (B).

2. $$\frac{1}{\lambda_1} = R\left[\frac{1}{4} - \frac{1}{9}\right]$$

$$\frac{1}{\lambda_2} = 4R\left[\frac{1}{4} - \frac{1}{16}\right]$$

$$\Rightarrow \frac{\lambda_2}{\lambda_1} = \frac{5}{27}$$

$$\Rightarrow \lambda_2 = 1215\ \text{Å}$$

Hence, the correct answer is (A).

3. Cut-off wavelength depends on the applied voltage not on the atomic number of the target. Characteristic wavelengths depend on the atomic number of target.
Hence, the correct answer is (B).

4. The series in U-V region is Lyman series. Longest wavelength corresponds to minimum energy which occurs in transition from $n = 2$ to $n = 1$.

$$\Rightarrow 122 = \frac{\frac{1}{R}}{\left(\frac{1}{1^2} - \frac{1}{2^2}\right)} \quad \ldots(1)$$

The smallest wavelength in the infrared region corresponds to maximum energy of Paschen series.

$$\Rightarrow \lambda = \frac{\frac{1}{R}}{\left(\frac{1}{3^2} - \frac{1}{\infty}\right)} \quad \ldots(2)$$

Solving Equations (1) and (2), we get

$\lambda = 823.5$ nm

Hence, the correct answer is (B).

5. $$\frac{1}{\lambda} \propto (Z-1)^2$$

$$\Rightarrow \frac{\lambda_1}{\lambda_2} = \left(\frac{Z_2 - 1}{Z_1 - 1}\right)^2$$

$$\Rightarrow \frac{1}{4} = \left(\frac{Z_2 - 1}{11 - 1}\right)^2$$

Solving this, we get, $Z_2 = 6$

Hence, the correct answer is (A).

6. The first photon will excite the hydrogen atom (in ground state) in first excited state (as $E_2 - E_1 = 10.2$ eV). Hence, during de-excitation a photon of 10.2 eV will be released. The second photon of energy 15 eV can ionise the atom.
Hence the balance energy i.e., $(15 - 13.6)$ eV $= 1.4$ eV is retained by the electron.
Therefore, by the second photon an electron of energy 1.4 eV will be released.
Hence, the correct answer is (C).

7. $$U = eV = eV_0 \log_e\left(\frac{r}{r_0}\right)$$

$$|F| = \left|-\frac{dU}{dr}\right| = \frac{eV_o}{r}$$

This force will provide the necessary centripetal force.

Hence, $$\frac{mv^2}{r} = \frac{eV_o}{r}$$

$$\Rightarrow v = \sqrt{\frac{eV_o}{m}} \quad \ldots(1)$$

Since by Bohr's Quantisation rule, we have

$$mvr = \frac{nh}{2\pi} \quad \ldots(2)$$

Dividing equation (2) by (1), we get

$$mr = \left(\frac{nh}{2\pi}\right)\sqrt{\frac{m}{eV_o}}$$

$$\Rightarrow r_n \propto n$$

Hence, the correct answer is (A).

8. Second excited state implies $n = 3$

$$\Rightarrow L_{\text{H}} = n\left(\frac{h}{2\pi}\right) = 3\left(\frac{h}{2\pi}\right)$$

$$\Rightarrow L_{\text{Li}} = n\left(\frac{h}{2\pi}\right) = 3\left(\frac{h}{2\pi}\right)$$

$$|E_H| = \frac{Z^2}{n^2}(13.6)\ \text{eV}$$

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$\Rightarrow \quad |E_H| = \frac{1^2}{9}(13.6)\text{ eV}$

$\Rightarrow \quad |E_{Li}| = \frac{3^2}{3^2}(13.6)\text{ eV}$

$\Rightarrow \quad |E_{Li}| = 13.6\text{ eV}$

$\Rightarrow \quad |E_{Li}| = 9|E_H|$

$\Rightarrow \quad |E_{Li}| > |E_H|$

Hence, the correct answer is (B).

9. $I = \frac{q}{t} = \frac{ne}{t}$

$\Rightarrow \quad 3.2\times10^{-3} = \left(\frac{n}{t}\right)(1.6\times10^{-19})$

$\Rightarrow \quad \left(\frac{n}{t}\right) = 2\times10^{16}$

Hence, the correct answer is (A).

10. λ_C decreases with increase in accelerating voltage in accordance with the expression given by

$$\lambda_C = \frac{hc}{eV}$$

Wavelength for K_α line is not affected as it is due to the electronic transition between $n=2$ and $n=1$ in the target element. Hence $(\lambda_K - \lambda_C)$ increase with increase in the accelerating voltage.

Hence, the correct answer is (A).

11. Energy of infrared radiation is less than the energy of ultraviolet radiation. In options (A), (B) and (C), energy released will be more, while in option (D) only, energy released will be less.
Hence, the correct answer is (D).

12. $v_n \propto \frac{1}{n}$

$\Rightarrow \quad KE \propto \frac{1}{n^2}$ (with positive sign)

Since potential energy is given by $U_n = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r_n}$

$\Rightarrow \quad U_n$ is negative and $U_n \propto \frac{1}{r_n} \propto \frac{1}{n^2}$

$\{\text{because } r_n \propto n^2\}$

Similarly, total energy $E_n \propto \frac{1}{n^2}$ (with negative sign)

Therefore, when an electron jumps from some excited state to the ground state, value of n will decrease. Therefore, kinetic energy will increase (with positive sign), potential energy and total energy will also increase but with negative sign.
Thus, finally kinetic energy will increase, while potential and total energies will decrease.
Also, for hydrogen and hydrogen-like atoms, we have

$$E_n = -13.6\frac{Z^2}{n^2}\text{eV}$$

$$U_n = 2E_n = -27.2\frac{Z^2}{n^2}\text{ eV}$$

and $K_n = |E_n| = 13.6\frac{Z^2}{n^2}$ eV

From these three relations we can see that as n decreases, K_n will increase but E_n and U_n will decrease. When an electron comes closer to the nucleus, the electrostatic force (which provides the necessary centripetal force) increases or speed (or KE) of the electron increases
Hence, the correct answer is (A).

13. $E_n = -\frac{me^4}{8n^2h^2\varepsilon_0^2}$, m = mass of electron

$\Rightarrow \quad E_n = -\frac{m_{\text{hypothetical}}\,e^4}{8n^2h^2\varepsilon_0^2}$

$\Rightarrow \quad E_n = -2\left(\frac{Rhc}{n^2}\right)$

The longest wavelength (or minimum energy) photon corresponds to transition between adjacent states.
i.e. $n=3$ to $n=2$

$\Rightarrow \quad \frac{hc}{\lambda_{\max}} = E_3 - E_2 = -2Rhc\left(\frac{1}{9}-\frac{1}{4}\right)$

$\Rightarrow \quad \frac{hc}{\lambda_{\max}} = 2Rhc\left(\frac{5}{36}\right)$

$\Rightarrow \quad \lambda_{\max} = \frac{18}{5R}$

Hence, the correct answer is (C).

14. $\lambda_0 = \frac{hc}{E} = 1.55\times10^{-11}$ m

$\Rightarrow \quad \lambda_0 = 0.155$ Å

is the minimum wavelength of continuous X-rays which carry energy equivalent to energy of incident electrons.
As this energy of incident radiation is more than that of K-shell electrons, the characteristic X-rays appear as peaks on the continuous spectrum.
Hence, the correct answer is (D).

15. $\dfrac{\Delta\lambda}{\lambda} = \mp\dfrac{v}{c}$

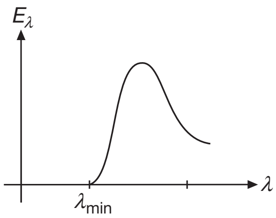
(–) sign to be used for approaching
(+) sign to be used for receding

$\Rightarrow \quad \dfrac{\Delta\lambda}{\lambda} = \dfrac{706-656}{656} = +\dfrac{v}{c}$

$\Rightarrow \quad v \approx 2\times10^7 \text{ ms}^{-1}$

Hence, the correct answer is (B).

16. The continuous X-ray spectrum is shown in figure.



All wavelengths $\lambda > \lambda_{min}$ are found, where

$$\lambda_{min} = \frac{12375}{V\,(\text{in volt})}\,\text{Å}$$

Here, V is the applied voltage.

Hence, the correct answer is (B).

17. $E_n = \dfrac{Z^2}{n^2}(13.6 \text{ eV})$

$\Rightarrow \quad E_n = 9(13.6 \text{ eV})$

$\Rightarrow \quad E_n = 122.4 \text{ eV}$

Hence, the correct answer is (D).

18. $\lambda_{K_\alpha} = 0.021 \text{ nm} = 0.21 \text{ Å}$

Since, λ_{K_α} corresponds to the transition of an electron from $L-$shell to $K-$shell, therefore,

$$E_L - E_K = (\text{in eV}) = \frac{12375}{\lambda(\text{in Å})} = \frac{12375}{0.21} \approx 58928 \text{ eV}$$

$\Rightarrow \quad \Delta E \approx 59 \text{ keV}$

Hence, the correct answer is (C).

20. For quantum number n we have $2n^2$ electrons.

For $n=1$ we have 2 elements
For $n=2$ we have 8 elements
For $n=3$ we have 18 elements
For $n=4$ we have 32 elements
So, total number of elements is

$$2+8+18+32=60$$

Hence, the correct answer is (C).

21. Shortest wavelength or cut-off wavelength depend only upon the voltage applied to the Coolidge tube.

Hence, the correct answer is (B).

Multiple Correct Choice Type Problems

1. $\dfrac{hc}{\lambda_a} = (E_4 - E_1)$ and $\dfrac{hc}{\lambda_e} = (E_4 - E_m)$

$$\Rightarrow \quad \frac{\lambda_a}{\lambda_e} = \frac{(E_4 - E_m)}{E_4 - E_1} = \frac{1}{5} = \frac{\frac{1}{m^2} - \frac{1}{16}}{\frac{15}{16}}$$

$$\Rightarrow \quad \frac{15}{16\times5} = \frac{1}{m^2} - \frac{1}{16}$$

$$\Rightarrow \quad \frac{1}{m^2} = \frac{1}{4}$$

$\Rightarrow \quad m = 2$

$$\Delta p_a = \frac{(E_4 - E_1)}{c} \text{ and } \Delta p_e = \frac{(E_4 - E_m)}{c}$$

$$\Rightarrow \quad \frac{\Delta p_a}{\Delta p_e} = \frac{(E_4 - E_1)}{(E_4 - E_m)} = \frac{15\times16}{16\times3} = 5$$

$$\frac{hc}{\lambda_e} = (13.6 \text{ eV})\times\frac{3}{16}$$

$$\Rightarrow \quad \frac{1242\times16}{3\times13.6}\text{ nm} = \lambda_e$$

$\Rightarrow \quad \lambda_e \approx 487 \text{ nm}$

Hence, (C) and (D) are correct.

2. Since, $r_n = \left(\dfrac{n^2}{Z}\right)r_0$

$$\Rightarrow \quad \frac{\Delta r_n}{r_n} = 2\left(\frac{\Delta n}{n}\right)$$

Since, $\Delta n = 1$ (for two consecutive orbits)

$$\Rightarrow \quad \frac{\Delta r_n}{r_n} \propto \frac{1}{n}$$

Since $L = \dfrac{nh}{2\pi}$

$$\Rightarrow \quad \frac{\Delta L}{L} = \frac{\Delta n}{n} = \frac{1}{n}$$

$$\Rightarrow \quad \frac{\Delta L}{L} \propto \frac{1}{n}$$

Hence, (A), (B) and (D) are correct.

3. Since $L = \dfrac{nh}{2\pi} = \dfrac{3h}{2\pi}$

$\Rightarrow \quad n = 3$

CHAPTER 2

Since, $r_n = \left(\frac{n^2}{Z}\right)a_0$

$\Rightarrow \quad 4.5a_0 = \left(\frac{n^2}{Z}\right)a_0$

$\Rightarrow \quad Z = 2$

Possible transitions are $3 \to 2$, $3 \to 1$ and $2 \to 1$

For $3 \to 2$, we have

$$\frac{1}{\lambda} = R(2)^2\left(\frac{1}{4} - \frac{1}{9}\right) = 4R\left(\frac{9-4}{36}\right) = \frac{5R}{9}$$

$\Rightarrow \quad \lambda = \frac{9}{5R}$

For $3 \to 1$, we have

$$\frac{1}{\lambda} = R(2)^2\left(1 - \frac{1}{9}\right) = 4R\left(\frac{8}{9}\right) = \frac{32R}{9}$$

$\Rightarrow \quad \lambda = \frac{9}{32R}$

For $2 \to 1$, we have

$$\frac{1}{\lambda} = R(2)^2\left(1 - \frac{1}{4}\right) = 4R\left(\frac{3}{4}\right) = 3R$$

$\Rightarrow \quad \lambda = \frac{1}{3R}$

Hence, (A) and (C) are correct.

4. $T \propto n^3$

$\Rightarrow \quad \frac{T_1}{T_2} = 8$

Satisfied by both (A) and (D)

Hence, (A) and (D) are correct.

5. $\lambda_m(\text{in Å}) = \frac{12375}{V(\text{in volts})}$

With increase in V, λ_m will decrease. With decrease in λ_m energy of emitted photons will increase. And hence intensity will increase even if number of photons emitted per second are constant. Because intensity is basically energy per unit area per unit time.

Hence, (A) and (D) are correct.

6. $r_n \propto \frac{n^2}{Z}$ and $|PE| = 2(KE)$

Hence, (A) and (D) are correct.

Reasoning Based Questions

1. Cut-off wavelength depends on the accelerating voltage, not the characteristic wavelengths. Further, approximately 2% kinetic energy of the electrons is utilised in producing X-rays. Rest 98% is lost in heat.

Hence, the correct answer is (B).

Comprehension Type Questions

1. $L = I\omega = \frac{nh}{2\pi}$

$\Rightarrow \quad \omega = \frac{nh}{2\pi I}$

$\Rightarrow \quad K = \frac{1}{2}I\omega^2 = \frac{1}{2}I\left(\frac{nh}{2\pi I}\right)^2 = \frac{n^2h^2}{8\pi^2 I}$

Hence, the correct answer is (D).

2. $hv = K_2 - K_1 = \frac{3h^2}{8\pi^2 I}$

$\Rightarrow \quad I = \frac{3h}{8\pi^2 f} = \frac{3 \times 2\pi \times 10^{-34} \times \pi}{8 \times \pi^2 \times 4 \times 10^{11}}$

$\Rightarrow \quad I = 1.87 \times 10^{-46}\ \text{kgm}^2$

Hence, the correct answer is (B).

3. $I = \mu r^2$ (where, $\mu =$ reduced mass)

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{48}{7}\ \text{amu} = 11.43 \times 10^{-27}\ \text{kg}$$

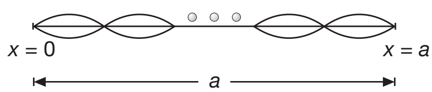
Substituting in $I = \mu r^2$ we get,

$$r = \sqrt{\frac{I}{\mu}} = \sqrt{\frac{1.87 \times 10^{-46}}{11.43 \times 10^{-27}}}$$

$\Rightarrow \quad r = 1.28 \times 10^{-10}\ \text{m}$

Hence, the correct answer is (C).

4. $a = \frac{n\lambda}{2}$



$\Rightarrow \quad \lambda = \frac{2a}{n} = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$ ...(1)

$\Rightarrow \quad \sqrt{E} \propto \frac{1}{a}$

$\Rightarrow \quad E \propto \frac{1}{a^2}$

Hence, the correct answer is (A).

5. From equation (1), we get

$$E = \frac{n^2h^2}{8a^2m}$$

In ground state $n = 1$

$$\Rightarrow \quad E_1 = \frac{h^2}{8ma^2}$$

Substituting the values, we get

$$E_1 = 8 \text{ meV}$$

Hence, the correct answer is (B).

6. Since, we have already calculated that

$$\lambda = \frac{2a}{n} = \frac{h}{p} = \frac{h}{\sqrt{2mE}} \quad \ldots(1)$$

From equation (1), we get

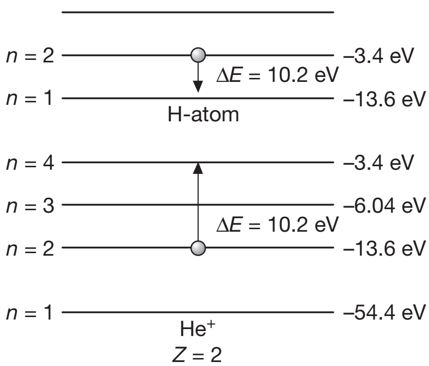
$$p \propto n$$

$$\Rightarrow \quad mv \propto n$$

$$\Rightarrow \quad v \propto n$$

Hence, the correct answer is (D).

7. Energy given by H-atoms in transition from $n = 2$ to $n = 1$ is equal to energy taken by He^+ atom in transition from $n = 2$ to $n = 4$.



Hence, the correct answer is (C).

8. Visible light lies in the range, $\lambda_1 = 4000$ Å to $\lambda_2 = 7000$ Å. Energy of photons corresponding to these wavelengths (in eV) would be

$$E_1 = \frac{12375}{4000} = 3.09 \text{ eV}, \quad E_2 = \frac{12375}{7000} = 1.77 \text{ eV}$$

From energy level diagram of He^+ atom we can see that in transition from $n = 4$ to $n = 3$ energy of photon released will lie between E_1 and E_2.

$$\Delta E_{43} = -3.4 - (-6.04) = 2.64 \text{ eV}$$

Wavelength of photon corresponding to this energy,

$$\lambda = \frac{12375}{2.64} \text{ Å} = 4687.5 \text{ Å}$$

$$\lambda = 4.68 \times 10^{-7} \text{ m}$$

Hence, the correct answer is (C).

9. Kinetic energy $K \propto Z^2$

$$\Rightarrow \quad \frac{K_H}{K_{He^+}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Hence, the correct answer is (A).

Integer/Numerical Answer Type Questions

1. $$13.6\left(\frac{1}{1} - \frac{1}{4}\right)Z^2 = 74.8 + 13.6 \times \left(\frac{1}{4} - \frac{1}{9}\right)Z^2$$

$$\Rightarrow \quad 13.6Z^2\left(\frac{3}{4} - \frac{5}{36}\right) = 74.8$$

$$\Rightarrow \quad Z^2 = 9$$

$$\Rightarrow \quad Z = 3$$

2. $$\frac{120e \times e}{4\pi\varepsilon_0 (10 \times 10^{-15})} = \frac{p^2}{2m} = \frac{h^2}{\lambda^2 \times 2m}$$

$$\Rightarrow \quad \lambda^2 = \frac{h^2}{2m} \times \frac{4\pi\varepsilon_0 \times 10^{-14}}{120e^2}$$

$$\Rightarrow \quad \lambda = \frac{h}{e} \times \sqrt{\frac{4\pi\varepsilon_0 \times 10^{-4}}{2 \times 120 \times m}}$$

$$\Rightarrow \quad \lambda = 4.2 \times 10^{-15} \times \sqrt{\frac{10^{-14} \times 3}{9 \times 10^9 \times 2 \times 120 \times 5 \times 10^{-27}}}$$

$$\Rightarrow \quad \lambda = 7 \times 10^{-15} \text{ m}$$

$$\Rightarrow \quad \lambda = 7 \text{ fm}$$

3. Potential energy of hydrogen atom $(Z = 1)$ in n^{th} orbit is

$$PE = U = -\frac{27.2}{n^2} \text{ eV}$$

$$\Rightarrow \quad \frac{U_f}{U_i} = \frac{-\frac{27.2}{n_f^2}}{-\frac{27.2}{n_i^2}} = \frac{1}{6.25}$$

$$\Rightarrow \quad 6.25 = \frac{n_f^2}{n_i^2}$$

$$\Rightarrow \quad \frac{n_f}{n_i} = 2.5 = \frac{5}{2}$$

Hence the answer is 5.

4. Energy available $E = \frac{hc}{\lambda} = \frac{1.237 \times 10^{-6} \text{ eVm}}{970 \times 10^{-10} \text{ m}} = 12.75 \text{ eV}$

This energy corresponds to 4^{th} energy level of hydrogen atom.

Hence, the number of lines present in the emission spectrum is

$$N = {}^4C_2 = \frac{4(4-1)}{2} = 6$$

5. Kinetic energy of ejected electron is equal to the energy of incident photon minus the energy required to ionize the electron from n^{th} orbit (all in eV)

$$\Rightarrow \quad 10.4=\frac{1242}{90}-\left|E_n\right|$$

Since, $E_n=-13.6\left(\frac{Z^2}{n^2}\right)$ eV, so for the nth orbit of hydrogen atom, we have,

$$E_n=-\left(\frac{13.6}{n^2}\right) \text{eV}$$

$$\Rightarrow \quad 10.4=\frac{1242}{90}-\frac{13.6}{n^2}$$

Solving this equation, we get

$$n=2$$

6. Angular momentum $=n\left(\frac{h}{2\pi}\right)=3\left(\frac{h}{2\pi}\right)$

$$\Rightarrow \quad n=3$$

Now, $r_n=\left(\frac{n^2}{Z}\right)a_0$

$$\Rightarrow \quad r_3=\frac{(3)^2}{3}\left(a_0\right)=3a_0$$

Now, $mv_3r_3=3\left(\frac{h}{2\pi}\right)$

$$\Rightarrow \quad mv_3\left(3a_0\right)=3\left(\frac{h}{2\pi}\right)$$

$$\Rightarrow \quad \frac{h}{mv_3}=2\pi a_0$$

$$\Rightarrow \quad \frac{h}{p_3}=2\pi a_0 \qquad \{\because p_3=mv\}$$

$$\Rightarrow \quad \lambda_3=2\pi a_0 \qquad \left\{\because \lambda=\frac{h}{p_3}\right\}$$

$\Rightarrow$ Answer is 2.

7. $$\frac{120e\times e}{4\pi\varepsilon_0\left(10\times10^{-15}\right)}=\frac{p^2}{2m}=\frac{h^2}{\lambda^2\times 2m}$$

$$\Rightarrow \quad \lambda^2=\frac{h^2}{2m}\times\frac{4\pi\varepsilon_0\times10^{-14}}{120e^2}$$

$$\Rightarrow \quad \lambda=\frac{h}{e}\times\sqrt{\frac{4\pi\varepsilon_0\times10^{-4}}{2\times120\times m}}$$

$$\Rightarrow \quad \lambda=4.2\times10^{-15}\times\sqrt{\frac{10^{-14}\times3}{9\times10^{9}\times2\times120\times5\times10^{-27}}}$$

$$\Rightarrow \quad \lambda=7\times10^{-15}\text{ m}$$

$$\Rightarrow \quad \lambda=7\text{ fm}$$

Test Your Concepts-I (Based on Nucleus Properties and Binding Energy)

1. Since, we know that $R = R_0 A^{1/3}$

$$\Rightarrow \quad R = R_0 A^{1/3} = (1.1 \text{ fm})(70)^{1/3}$$

$$\Rightarrow \quad R = (1.1 \text{ fm})(4.12) = 4.53 \text{ fm}$$

2. The radius of a ^{12}C nucleus is $R = R_0 A^{1/3}$

$$\Rightarrow \quad R = (1.1 \text{ fm})(12)^{1/3} = 2.52 \text{ fm}$$

The separation between the centres of the nuclei is $2R = 5.04$ fm. The potential energy of the pair is $U = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$.

$$\Rightarrow \quad U = (9 \times 10^9) \frac{(6 \times 1.6 \times 10^{-19})^2}{5.04 \times 10^{-15}}$$

$$\Rightarrow \quad U = 1.64 \times 10^{-12} \text{ J} = 10.2 \text{ MeV}$$

3. Average atomic mass A is given by

$$A = \frac{p_1 A_1 + p_2 A_2}{p_1 + p_2}$$

$$\Rightarrow \quad A = \frac{(7.5)(6.0152) + (92.5)(7.016004)}{100}$$

$$\Rightarrow \quad A = \frac{45.39 + 648.98}{100} = 6.94 \text{ amu}$$

4. The surface area S of a sphere is given by

$$S = 4\pi R^2$$

Since, $R = R_0 A^{\frac{1}{3}}$

$$\Rightarrow \quad S = 4\pi R^2 \propto A^{2/3}$$

$$\Rightarrow \quad \frac{S_1}{S_2} = \left(\frac{A_1}{A_2}\right)^{\frac{2}{3}}$$

5. The radius of $^{12}_{6}C$ nucleus is given by

$$R = R_0 A^{\frac{1}{3}}$$

$$\Rightarrow \quad R = 1.2 \times 10^{-15} \times (12)^{\frac{1}{3}}$$

$$\Rightarrow \quad R = 2.75 \times 10^{-15} \text{ m}$$

The atomic mass of $^{12}_{6}\text{C}$ is 12 amu. Neglecting the masses and binding energies of the six electrons, the nuclear density is given by

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} = \frac{12 \times 1.66 \times 10^{-27}}{\left(\frac{4}{3}\pi\right)(2.7 \times 10^{-15})^3}$$

$$\Rightarrow \quad \rho = 2.4 \times 10^{17} \text{ kgm}^{-3}$$

6. $$\Delta m = \frac{E}{c^2} = \frac{4.2 \times 10^3}{(3 \times 10^8)^2} \text{ kg}$$

$$\Rightarrow \quad \Delta m = 4.7 \times 10^{-14} \text{ kg}$$

7. The $^{35}_{17}\text{Cl}$ nucleus has 17 protons and 18 neutrons. Therefore, the mass M of constituent nucleons of $^{35}_{17}\text{Cl}$ is

$$M = 17 m_p + 18 m_n$$

$$\Rightarrow \quad M = 17(1.007825) + 18(1.008665)$$

$$\Rightarrow \quad M = 35.289 \text{ amu}$$

Now, mass defect for the nucleus is

$$\Delta m = \frac{298 \text{ MeV}}{9312 \text{ MeV/amu}} = 0.3200 \text{ amu}$$

Since, mass defect Δm is

$$\Delta m = (17 m_p + 18 m_n) - m({}_{17}\text{Cl}^{35}) = M - m({}_{17}\text{Cl}^{35})$$

So, atomic mass of $^{35}_{17}\text{Cl}$ is approximately equal to $m({}_{17}\text{Cl}^{35})$, given by

$$m({}_{17}\text{Cl}^{35}) = M - \Delta m = (35.289 - 0.3200) \text{ amu}$$

$$\Rightarrow \quad m({}_{17}\text{Cl}^{35}) = 34.969 \text{ amu}$$

8. The alpha particle contains 2 protons and 2 neutrons. The binding energy is

$$BE = (2 \times 1.007826 \text{ u} + 2 \times 1.008665 \text{ u} - 4.00260 \text{ u})c^2$$

$$\Rightarrow \quad BE = (0.03038 \text{ u})c^2$$

$$\Rightarrow \quad BE = 0.03038 \times 931 \text{ MeV} = 28.3 \text{ MeV}$$

9. The binding energy of $^{8}_{4}\text{Be}$ is determined by the equation

$$BE({}^{8}_{4}\text{Be}) = (\Delta m)c^2$$

where, $\Delta m = \left[4 m_n + 4 m({}^{1}_{1}\text{H}) - m({}^{8}_{4}\text{Be})\right]$

$\Rightarrow \quad \Delta m = \left(4(1.008665) + 4(1.007825)\right) - 8.005305$

$\Rightarrow \quad \Delta m = 0.06065$ amu

$\Rightarrow \quad BE\left({}^{8}\text{Be}\right) = (0.06065)(931.5) = 56.5$ MeV

Now, let us calculate the binding energy of the decay of ${}^{8}_{4}\text{Be}$ in two α-particles through the reaction

$${}^{8}\text{Be} \rightarrow 2\alpha + \Delta E$$

where, $\Delta E = (\Delta m)c^2$ and

$$\Delta m = 2m_{\alpha} - m\left({}^{8}_{4}\text{Be}\right)$$

$\Rightarrow \quad \Delta m = 2(4.002603) - 8.005305 = -0.000099$ amu

$\Rightarrow \quad \Delta E = (-0.000099)(931.5)$ MeV $= -0.092$ MeV

Since, binding energy is negative for this reaction, hence ${}^{8}_{4}\text{Be}$ is unstable against decay to two alpha particles.

10. Mass defect Δm is given by

$$\Delta m = 3(1.007825) + 4(1.008665) - 7.016005$$

$\Rightarrow \quad \Delta m = 0.04213$ amu

$\Rightarrow \quad BE = (0.04213)(931.5)$ MeV $= 39.231$ MeV

Binding energy per nucleon is

$$\frac{BE}{A} = \frac{39.231}{7} = 5.604 \text{ MeV}$$

Test Your Concepts-II (Based on Radioactivity)

1. In 1 second 90% of the nuclei have remained undecayed, so in another 1 second 90% of 90 i.e., 81 nuclei will remain undecayed.

2. Since, $N = \frac{10^{-3}}{210} \times 6.02 \times 10^{23} = 2.87 \times 10^{18}$

During one mean life period 63.8% nuclei are decayed. Hence, energy released is

$$E = 0.638 \times 2.87 \times 10^{18} \times 5.3 \times 1.6 \times 10^{-13} \text{ J}$$

$\Rightarrow \quad E = 1.55 \times 10^{6}$ J

3. $N = \frac{2}{238} \times 6.02 \times 10^{23} = 5.06 \times 10^{21}$

Since, $R = \lambda N$

$\Rightarrow \quad \lambda = \frac{R}{N} = \frac{2.5 \times 10^{4}}{5.06 \times 10^{21}} = 4.94 \times 10^{-18} \text{ s}^{-1}$

$\Rightarrow \quad t_{1/2} = \frac{0.693}{\lambda} = 1.4 \times 10^{17}$ s

4. $\frac{R_1}{R_2} = \frac{\lambda_1 N}{\lambda_2 N} = \frac{1.2}{98.8}$

$\Rightarrow \quad \lambda_2 = 82.33\lambda_1 \quad \ldots(1)$

Further, $\lambda = \frac{0.693}{21.8} \text{ year}^{-1} = 0.0318 \text{ year}^{-1} \quad \ldots(2)$

Also, $\left(-\frac{dN}{dt}\right) = \left(-\frac{dN_1}{dt}\right) + \left(-\frac{dN_2}{dt}\right)$

$\Rightarrow \quad \lambda N = \lambda_1 N + \lambda_2 N$

$\Rightarrow \quad \lambda = \lambda_1 + \lambda_2 \quad \ldots(3)$

Solving equations (1), (2) and (3), we get

$$\lambda_1 = 3.81 \times 10^{-4} \text{ year}^{-1} \text{ and } \lambda_2 = 3.14 \times 10^{-2} \text{ year}^{-1}$$

5. (a) $R = R_0 e^{-\lambda t}$

$\Rightarrow \quad 2700 = 4750 e^{-5\lambda}$

$\Rightarrow \quad \lambda = 0.113 \text{ min}^{-1}$

(b) $t_{1/2} = \frac{0.693}{\lambda} = 6.132$ min

6. $R_0 = \lambda N$

$\Rightarrow \quad R_0 = \frac{0.693}{14.3 \times 3600 \times 24} \times 6.02 \times 10^{23}$ per sec

$\Rightarrow \quad R_0 = 3.37 \times 10^{17}$ per sec

After 70 hours activity

$$R = R_0 e^{-\lambda t}$$

$\Rightarrow \quad R = \left(3.37 \times 10^{17}\right) e^{-\left(\frac{0.693}{14.3} \times 24\right)(70)} = 2.92 \times 10^{17}$ per sec

In fruits, the activity was observed $1\,\mu$Ci or 3.7×10^{4} per sec.

Therefore, percentage P of activity transmitted from root to the fruit is

$$\begin{pmatrix} \text{Percentage of} \\ \text{Activity Transmitted} \end{pmatrix} = \frac{3.7 \times 10^{4}}{2.92 \times 10^{17}} \times 100$$

$\Rightarrow \quad P = 1.26 \times 10^{-11}\,\%$

7. (a) $N = \frac{1}{109} \times 6.02 \times 10^{23}$

$\Rightarrow \quad R = \lambda N = \frac{0.693}{2.7 \times 10^{7}} \times \frac{1}{109} \times 6.02 \times 10^{23}$

$\Rightarrow \quad R = 1.42 \times 10^{14}$ per year

(b) After 2 year, $R = R_0 e^{-\lambda t}$

$\Rightarrow \quad R = \left(1.42 \times 10^{14}\right) e^{\left[-0.693/2.7 \times 10^{7}\right](2)}$

$\Rightarrow \quad R = 1.41 \times 10^{14}$ per year

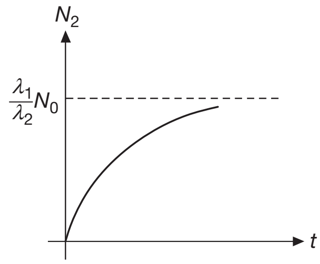
(c) After 2 half lives activity reduces to 25% of the original value, so we have

$$t = 2t_{1/2} = 5.4 \times 10^{7} \text{ years}$$

8. Since, $N_2 = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$

When $\lambda_1 \gg \lambda_2$ we have, $N_2 \approx N_0 e^{-\lambda_2 t}$

Physically this means that parent nuclei practically instantly transform into daughter nuclei, which then decay According to the Law of Radioactive Decay with a certain decay constant.



$$N_2 \approx \frac{\lambda_1}{\lambda_2} N_0 e^{-\lambda_1 t}$$

When $\lambda_1 \ll \lambda_2$, then $N_2 \approx \frac{\lambda_1}{\lambda_2} N_0 e^{-\lambda_1 t}$

i.e., N_2 versus t graph in this case is as shown in the figure.

9. Probability of a nucleus to decay in time t is

$$\frac{N}{N_0} = 1 - e^{-\lambda t} = 1 - e^{-(1/10)(5)} = 0.39$$

10. (a) $R = R_0 e^{-\lambda t}$

$$\Rightarrow \quad 7.3 = 9.3 e^{-\lambda(4500-2500)}$$

$$\Rightarrow \quad \lambda = 1.21 \times 10^{-4} \text{ year}^{-1}$$

$$\Rightarrow \quad t_{1/2} = \frac{0.693}{\lambda} = 5724 \text{ year}$$

(b) Further applying, $R = R_0 e^{-\lambda t}$

$$\Rightarrow \quad R_0 = \text{R}e^{\lambda t} = (7.3)e^{(1.21\times 10^{-4})(4500)}$$

$$\Rightarrow \quad R_0 = 12.58 \text{ dismin}^{-1}\text{g}^{-1}$$

11. $N = \frac{R}{\lambda} = \frac{5\times 10^{-3} \times 3.7 \times 10^{10}}{\left(\frac{0.693}{138\times 24 \times 3600}\right)} = 3.2 \times 10^{15}$

$$\Rightarrow \quad m = \left(\frac{3.2\times 10^{15}}{6.02 \times 10^{23}}\right)(210)g$$

$$\Rightarrow \quad m = 1.12 \times 10^{-6} \text{ g}$$

12. Let x the desired ratio, then, mass of Co^{58} in 1 g be x

$$\Rightarrow \quad N = \frac{x}{58} \times 6.02 \times 10^{23}$$

Given, $2.2\times 10^{12} = \lambda N = \frac{0.693}{71.3\times 24\times 3600} \times \frac{x}{58} \times 6.02 \times 10^{23}$

$$\Rightarrow \quad x = 1.88 \times 10^{-3}$$

13. (a) 13.0 g of ${}^{13}_{7}N$ will contain 6.023×10^{23} nuclei (Avogadro's number).

The number of nuclei N_0 that we have initially in 1.49×10^{-6} g of ${}^{13}_{7}N$, is

$$N_0 = \left(6.023 \times 10^{23}\right)\left(\frac{1.49\times 10^{-6}\text{ g}}{13.0\text{ g}}\right),$$

$$\Rightarrow \quad N_0 = 6.90 \times 10^{16} \text{ nuclei.}$$

(b) Since, we have $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{600\text{ s}} = 1.16\times 10^{-3}\text{ s}^{-1}$

So, activity at $t = 0$ is

$$A_0 = \lambda N_0 = \left(1.16\times 10^{-3}\text{ s}^{-1}\right)\left(6.90 \times 10^{16}\right)$$

$$\Rightarrow \quad A_0 = 8.00 \times 10^{13} \text{ decay/s}$$

(c) 60 minutes is 6 half-lives, so we have

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$$

$$\Rightarrow \quad A = \frac{A_0}{64} = \frac{8.00\times 10^{13}}{64} = 1.25 \times 10^{12}\text{ s}^{-1}$$

14. The number of parent nuclei decaying in a short time interval t to $t + dt$ is $\lambda_p N_p dt$. This is also the number of daughter nuclei decaying during the same time interval is $\lambda_d N_d dt$. The number of the daughter nuclei will be constant if

$$\lambda_p N_p dt = \lambda_d N_d dt$$

$$\Rightarrow \quad \lambda_p N_p = \lambda_d N_d$$

15. In one half-life the number of active nuclei reduces to half the original number. Thus, in two half lives the number is reduced to $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$ of the original number.

The number of remaining active nuclei is given by

$$N = \frac{N_0}{4} = \frac{6\times 10^{18}}{4} = 1.5 \times 10^{18}.$$

16. Since the number of ^{206}Pb atoms equal the number of ^{238}U atoms, half of the original ^{238}U atoms have decayed. It takes one half-life to decay half of the active nuclei. Thus, the sample is 4.5×10^9 years old.

17. 63.5 g of copper has 6×10^{23} atoms. Thus, the number of atoms in 1 μg of Cu is

$$N = \frac{6 \times 10^{23} \times 1\ \mu g}{63.5\ g} = 9.45 \times 10^{15}$$

The activity is $A = \lambda N$

$$A = \left(1.516 \times 10^{-5}\ s^{-1}\right) \times \left(9.45 \times 10^{15}\right)$$

$$\Rightarrow \quad A = 1.43 \times 10^{11} \text{ disintegrations/s}$$

$$\Rightarrow \quad A = \frac{1.43 \times 10^{11}}{3.7 \times 10^{10}} \text{ Ci} = 3.86 \text{ Ci}$$

18. 40 hours $\Rightarrow$ 2 half lives

Since $A = \dfrac{A_0}{2^n}$

$$\Rightarrow \quad A = \frac{A_0}{(2)^2} = \frac{A_0}{4}$$

$$\Rightarrow \quad \frac{A}{A_0} = \frac{1}{4}$$

So, one fourth of the original activity will remain after 40 hours.

19. Let N_0 be the initial amount of uranium in the rock and let N be the present amount of uranium after 1.5×10^9 years.

So, the amount of lead present in the rock is $\left(N_0 - N\right)$

So, the required ratio is $\dfrac{N_0 - N}{N} = \dfrac{1 - e^{-\lambda t}}{e^{-\lambda t}}$

Since, $T_{\frac{1}{2}} = \dfrac{\ln 2}{\lambda}$

$$\Rightarrow \quad \lambda = \frac{\ln 2}{T_{\frac{1}{2}}}$$

$$\Rightarrow \quad e^{-\lambda t} = e^{-\left(\frac{\log_e 2}{T_{\frac{1}{2}}}\right)t} = e^{-\log(2)\frac{t}{T_{\frac{1}{2}}}} = 2^{-\frac{t}{T_{\frac{1}{2}}}}$$

$$\Rightarrow \quad \frac{N_0 - N}{N} = \frac{1 - e^{-\left(\frac{\ln 2}{T_{\frac{1}{2}}}\right)t}}{e^{-\left(\frac{\ln 2}{T_{\frac{1}{2}}}\right)t}} = \frac{1 - 2^{-\left(\frac{t}{T_{\frac{1}{2}}}\right)}}{2^{-\left(\frac{t}{T_{\frac{1}{2}}}\right)}} = \frac{1 - 2^{-\frac{1}{3}}}{2^{-\frac{1}{3}}}$$

$$\Rightarrow \quad \frac{N_0 - N}{N_0} = 2^{\frac{1}{3}} - 1 = 0.259$$

20. At maximum, the rate of production of nuclei is equal to the rate of decay

$$\Rightarrow \quad r_{\text{production}} = r_{\text{decay}}$$

$$\Rightarrow \quad 10^{21} = \frac{1}{30} N$$

$$\Rightarrow \quad N = 30 \times 10^{21}$$

Test Your Concepts-III
(Based on Nuclear Reactions, Alpha, Beta, Gamma Decay, Fission and Fusion)

1. A reaction is possible (spontaneously) if the binding energy of products is larger than that of $^{238}_{92}U$.

(a) Total binding energy of $^{238}_{92}U = 238 \times 7.57 = 1801.66$ MeV and binding energy of $^{206}_{82}Pb = 206 \times 7.83 = 1612.981$ MeV the binding energy of products is less than that of $^{238}_{92}U$. Note that the protons and neutrons are free and do not have any binding energy. Hence, this reaction is not possible spontaneously. It can take place only if $1801.66 - 1612.98 = 188.68$ MeV of energy is supplied from outside.

(b) In the second case binding energy of products is larger than the binding energy of the parent nucleus. Hence, reaction is possible spontaneously.

2. Since, the masses are given in atomic mass units, so, we shall first calculate the mass difference between reactants and products in the same units and then multiply the mass difference by $931.5\ \text{MeVu}^{-1}$. Thus, we have

$$Q = \left(14.00307 + 4.00260 - 1.00783 - 16.99913\right)(931.5)$$

$$\Rightarrow \quad Q = -1.20 \text{ MeV}$$

Q value is negative, so the reaction is endothermic.

Hence, the minimum kinetic energy of α-particle to initiate this reaction is given by

$$K_{\min} = |Q|\left(\frac{m_\alpha}{m_N} + 1\right)$$

$$\Rightarrow \quad K_{\min} = (1.20)\left(\frac{4.00260}{14.00307} + 1\right)$$

$$\Rightarrow \quad K_{\min} = 1.54 \text{ MeV}$$

3. Total energy released is

$$\Delta E = \frac{1}{235} \times 6.02 \times 10^{26} \times 185 \times 1.6 \times 10^{-13} \text{ J}$$

$$\Rightarrow \quad \Delta E = 7.58 \times 10^{13} \text{ J}$$

Power $P = 100 \times 10^6 = 10^8 \text{ Js}^{-1}$

Therefore, time

$$t = \frac{7.58 \times 10^{13}}{10^8} \text{sec} = 8.78 \text{ day}$$

4. ${}_0n^1 \rightarrow {}_1\text{H}^1 + {}_{-1}e^0$

Mass defect $\Delta m = m_n - (m_p + m_e)$

$$\Rightarrow \quad \Delta m = (1.6747 \times 10^{-27}) - (1.6725 \times 10^{-27} + 9 \times 10^{-31})$$

$$\Rightarrow \quad \Delta m = 0.0013 \times 10^{-27} \text{kg}$$

Energy released $Q = (\Delta m)c^2$

$$\Rightarrow \quad Q = (0.0013 \times 10^{-27}) \times (3 \times 10^8)^2 = 1.17 \times 10^{-13} \text{ J}$$

$$\Rightarrow \quad Q = \frac{1.17 \times 10^{-13}}{1.6 \times 10^{-19}} = 0.73 \times 10^6 \text{ eV} = 0.73 \text{ MeV}$$

5. By Law of Conservation of Linear Momentum, we have

$$M_\alpha v_\alpha = mv$$

$$\Rightarrow \quad v = \frac{M_\alpha v_\alpha}{M}$$

Kinetic energy of $T\ell$ atom is

$$(KE)_{T\ell} = \frac{1}{2}Mv^2 = \frac{1}{2}M\left(\frac{M_\alpha^2 v_\alpha^2}{M^2}\right) = \left(\frac{1}{2}M_\alpha v_\alpha^2\right)\left(\frac{M_\alpha}{M}\right)$$

$$\Rightarrow \quad (KE)_{T\ell} = (\text{KE})_\alpha \left(\frac{M_\alpha}{M}\right)$$

$$\Rightarrow \quad (KE)_{T\ell} = 6.082 \times \frac{4}{208} \text{ MeV}$$

$$\Rightarrow \quad (KE)_{T\ell} = 0.1308 \text{ MeV}$$

6. (a) Since the neutron and boron are both initially at rest, so the total momentum both before the reaction and after the reaction is zero. So, we get

$$M_{\text{Li}} v_{\text{Li}} = M_{\text{He}} v_{\text{He}}$$

Please understand that, here we can use the Classical approach rather than the Relativistic approach. Since $v_{He} = 9.30 \times 10^6 \text{ ms}^{-1}$, which is very small compared to the speed of light $c = 3 \times 10^8 \text{ ms}^{-1}$, so we can take kinetic energy as $KE \approx \frac{1}{2}mv^2$. The error in the calculations due to this Classical approach will be negligible or will even be less because $M_{\text{Li}} > M_{\text{He}}$. So, we have kinetic energy of Li as

$$K_{\text{Li}} = \frac{1}{2}M_{\text{Li}} v_{\text{Li}}^2 = \frac{1}{2}M_{\text{Li}}\left(\frac{M_{\text{He}} v_{\text{He}}}{M_{\text{Li}}}\right)$$

$$\Rightarrow \quad K_{\text{Li}} = \frac{M_{\text{He}}^2 v_{\text{He}}^2}{2M_{\text{Li}}}$$

Substituting the values, we get

$$K_{\text{Li}} = \frac{(4.0026)^2 (1.66 \times 10^{-27})^2 (9.30 \times 10^6)^2}{2(7.0160)(1.66 \times 10^{-27})}$$

$$\Rightarrow \quad K_{\text{Li}} = 1.64 \times 10^{-13} \text{ J} = 1.02 \text{ MeV}$$

(b) Now, $Q = K_{\text{Li}} + K_{\text{He}}$

where $K_{\text{He}} = \frac{1}{2}M_{\text{He}} v_{\text{He}}^2$

$$\Rightarrow \quad K_{\text{He}} = \frac{1}{2}(4.0026)(1.66 \times 10^{-27})(9.30 \times 10^6)^2$$

$$\Rightarrow \quad K_{\text{He}} = 2.87 \times 10^{-13} \text{ J} = 1.80 \text{ MeV}$$

$$\Rightarrow \quad Q = 1.02 \text{ MeV} + 1.80 \text{ MeV} = 2.82 \text{ MeV}$$

7. Please observe that here, atomic masses are given (not the nuclear masses), but still we can use them for calculating the mass defect because mass of electrons get cancelled both sides. Thus,

Mass defect $\Delta m = (22.9945 - 22.9898) = 0.0047u$

$$\Rightarrow \quad Q = (0.0047u)(931.5 \text{ MeVu}^{-1})$$

$$\Rightarrow \quad Q = 4.4 \text{ MeV}$$

Hence, the energy of beta particles can range from 0 to 4.4 MeV.

8. $Q = \left| m_c' - (m_B' + m_p) \right| c^2$

where m_c' and m_B' are the nuclear mass of ${}^{11}\text{C}$ and ${}^{11}\text{B}$, so

$$m_B = m_B' + 5m_e$$

$$m_c = m_c' + 6m_e$$

$$\Rightarrow \quad Q = \left[(m_c - 6m_e) - (m_B - 5m_e + m_p)\right]c^2$$

Since $m_{\text{electron}} = m_{\text{positron}}$

$$\Rightarrow \quad Q = \left[m_c - m_B - 2m_p\right]c^2$$

$$\Rightarrow \quad Q = (11.011434 - 11.009305 - 2 \times 0.0005486)931$$

$$\Rightarrow \quad Q = 0.961 \text{ MeV}$$

The disintegration energy is equal to the maximum energy of the emitted photon.

9. $\Delta m = 2(\text{mass of } {}_1\text{H}^2) - (\text{mass of } {}_2\text{He}^4) = 0.0256 \text{ u}$

$$\Rightarrow \quad \Delta E = 0.0256 \times 931.5 \text{ MeV} = 23.85 \text{ MeV}$$

Total energy required per day is

$$E = 200 \times 10^6 \times 24 \times 3600 \text{ J} = 1.728 \times 10^{13} \text{ J}$$

Let m be the mass of deuterium required. Then energy required for reactor is

$$E' = \left(\frac{25}{100}\right)\left(\frac{m/2}{2}\right)(6.02\times10^{23})(23.85\times1.6\times10^{-13})$$

This should be equal to 1.728×10^{13} J

$$\Rightarrow \quad m = \frac{4\times100\times1.728\times10^{13}}{25\times6.02\times10^{23}\times23.85\times1.6\times10^{-13}}\text{ g}$$

$$\Rightarrow \quad m = 120.35 \text{ g}$$

10. (a) Total number of atoms in 1 kg of U^{238} is

$$N = \frac{1}{238}\times6.02\times10^{26} = 2.53\times10^{24}$$

$\Rightarrow$ Total energy released,

$$\Delta E = (200\times2.53\times10^{24})\text{ MeV}$$

$$\Rightarrow \quad \Delta E = 8.09\times10^{13}\text{ J}$$

(b) $m = \dfrac{8.09\times10^{13}}{30\times10^{3}}\text{ g} = 2.7\times10^{9}\text{ g}$

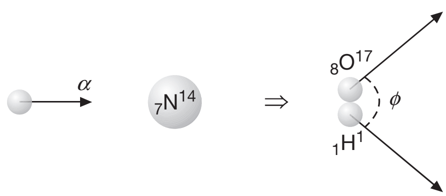
$$\Rightarrow \quad m = 2.7\times10^{6}\text{ kg}$$

11. (a) $Q\text{-value} = (\Delta m)931.5 \text{ MeV} = -1.18 \text{ MeV}$

(b) By Law of Conservation of Linear Momentum, we have

$$P_\alpha^2 = P_0^2 + P_H^2 + 2P_0P_H\cos\phi$$

$$\Rightarrow \quad 2m_\alpha K_\alpha = 2m_0K_0 + 2m_HK_H + 2\sqrt{(2m_0K_0)(2m_HK_H)}\cos\phi \quad \ldots(1)$$



By Law of Conservation of Energy, we have

$$K_0 = Q + K_\alpha - K_H = (-1.18 + 7.7 - 5.5)\text{ MeV}$$

$$\Rightarrow \quad K_0 = 1.02 \text{ MeV}$$

$$\Rightarrow \quad \cos\phi = -\left(\frac{m_0K_0 - m_\alpha K_\alpha - m_HK_H}{2\sqrt{m_\alpha m_H K_\alpha K_H}}\right) = 0.73$$

$$\Rightarrow \quad \phi \approx 43°18'$$

12. The corresponding decay equations can be

(i) ${}^{230}_{92}\text{U} \to n + {}^{229}_{92}\text{U}$ and

(ii) ${}^{230}_{92}\text{U} \to p + {}^{229}_{91}\text{Pa}$

In first equation the energy released is

$$Q = (230.033927 - 229.033496 - 1.008665)(931.5)$$

$$\Rightarrow \quad Q = -7.7 \text{ MeV}$$

Since $Q < 0$, so a spontaneous neutron decay is not possible.

Similarly, for the second reaction, the energy released is

$$Q = (230.033927 - 229.032089 - 1.007825)(931.5)$$

$$\Rightarrow \quad Q = -5.6 \text{ MeV}$$

Since $Q < 0$, so a spontaneous proton decay is also not possible.

13. Since, $\Delta E = (8m_p + 8m_n - m_o)\times931.5$ MeV

$$\Rightarrow \quad \Delta E = 127.6 \text{ MeV}$$

14. The endothermic reaction can be written as

$${}^1_1\text{H} + {}^{13}_6\text{C} \to {}^{13}_7\text{N} + {}^1_0 n + Q$$

where $Q = (m_{\text{final}} - m_{\text{initial}})c^2$

$$\Rightarrow \quad Q = (m_{\text{final}} - m_{\text{initial}})\times931.5\text{ MeV}$$

Now, $m_{\text{final}} = 13.0067384 + 1.0086654 = 14.0154038$ u

$m_{\text{initial}} = 13.0033554 + 1.0078254 = 14.0111808$ u

$$\Rightarrow \quad \Delta m = 0.004223 \text{ u}$$

$$\Rightarrow \quad Q = (0.004223)(931.5) \approx 4.00 \text{ MeV}$$

The minimum amount of energy that a bombarding particle must possess in order to initiate the reaction is

Threshold energy $E_{\text{th}} = Q\left(\dfrac{m_1}{m_2} + 1\right)$

where m_1 is mass of the projectile, m_2 is mass of target

$$\Rightarrow \quad E_{\text{th}} = 4.00\left(\frac{1.0078254}{13.0033554} + 1\right) = 4.31 \text{ MeV}$$

15. $P = 1\text{ MW} = 10^6\text{ W} = 10^6\text{ Js}^{-1}$

$t = 1\text{ day} = 24\times60\times60 = 86400$ s

So, energy released in one day is $E = Pt = 86400\times10^6$ J

Since energy released per fusion is given to be 20 MeV

$$\Rightarrow \quad (\Delta E)_{\text{per fusion}} = 20\times1.6\times10^{-13} = 3.2\times10^{-12}\text{ J}$$

Mass of ${}_1H^2$ consumed in one fusion $({}_1H^2 + {}_1H^2)$ is $4u$

$$\Rightarrow \quad (\Delta m)_{\text{per fusion}} = 4\times1.66\times10^{-27}\text{ kg} = 6.64\times10^{-27}\text{ kg}$$

$$\Rightarrow \quad (\Delta m)_{1\text{ day}} = \frac{6.64\times10^{-27}}{3.2\times10^{-12}}\times86400\times10^6 = 1.79\times10^{-4}\text{ kg}$$

Single Correct Choice Type Questions

1. When sample is 1000 m above ground i.e. it has fallen by 2000 m. Since $s = \frac{1}{2}gt^2$

$$\Rightarrow \quad 2000 = \frac{1}{2}(10)t^2$$

$$\Rightarrow \quad t = 20 \text{ s}$$

$$\Rightarrow \quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{\frac{20}{10}} = \frac{1}{4}$$

$$\Rightarrow \quad N = \frac{1000}{4} = 250$$

Hence, the correct answer is (B).

2. Let N be the number of nuclei at any time t. Then

$$\frac{dN}{dt} = 200 - \lambda N$$

$$\Rightarrow \quad \int_0^N \frac{dN}{200 - \lambda N} = \int_0^t dt$$

$$\Rightarrow \quad N = \frac{200}{\lambda}\left(1 - e^{-\lambda t}\right)$$

Given that $N = 100$ and $\lambda = 1 \text{ s}^{-1}$

$$\Rightarrow \quad 100 = 200\left(1 - e^{-t}\right)$$

$$\Rightarrow \quad e^{-t} = \left(\frac{1}{2}\right)$$

$$\Rightarrow \quad t = \log_e(2) \text{ s}$$

Hence, the correct answer is (B).

3. Since, $A_1 = \lambda N_0 e^{-\lambda t_1}$

$$\Rightarrow \quad t_1 = \frac{1}{\lambda}\log_e\left(\frac{\lambda N_0}{A_1}\right) \qquad \ldots(1)$$

Since, $A_2 = (\lambda)(2N_0)e^{-\lambda t_2}$

$$\Rightarrow \quad t_2 = \frac{1}{\lambda}\log_e\left(\frac{2\lambda N_0}{A_2}\right) \qquad \ldots(2)$$

$$\Rightarrow \quad t_1 - t_2 = \frac{1}{\lambda}\log_e\left(\frac{A_2}{2A_1}\right)$$

$$\Rightarrow \quad t_1 - t_2 = \frac{T}{\log_e(2)}\log_e\left(\frac{A_2}{2A_1}\right)$$

Hence, the correct answer is (C).

4. Using $10.81 = 10x + 11(1 - x)$

$$\Rightarrow \quad 10.81 = 11 - x$$

$$\Rightarrow \quad x = 0.19$$

$$\Rightarrow \quad 1 - x = 0.81$$

$$\Rightarrow \quad \frac{x}{1-x} = \frac{19}{81}$$

Hence, the correct answer is (A).

6. Since, $N = N_0 e^{-\lambda t}$

$$\Rightarrow \quad \frac{2N_0}{3} = N_0 e^{-\lambda t_0}$$

$$\Rightarrow \quad e^{-\lambda t_0} = \frac{3}{2} \qquad \ldots(1)$$

Taking log both sides, we get

$$\lambda t_0 = \log_e\left(\frac{3}{2}\right)$$

$$\Rightarrow \quad \lambda = \frac{1}{t_0}\log_e\left(\frac{3}{2}\right)$$

Also $\frac{N_0}{3} = N_0 e^{-\lambda t_1}$

$$\Rightarrow \quad \lambda t_1 = \log_e(3)$$

$$\Rightarrow \quad t_1 = \frac{1}{\lambda}\log_e(3) = \frac{t_0 \log_e(3)}{\log_e\left(\frac{3}{2}\right)}$$

Hence, the correct answer is (C).

7. $t = 1 \text{ week} = (7)(24 \text{ hr})$

Since $T_{1/2} = 12 \text{ hr}$

$$\Rightarrow \quad n = \frac{t}{T_{1/2}} = 14 \text{ and } A_0 = 1 \text{ Ci}$$

$$\Rightarrow \quad A = \frac{A_0}{2^n} = \frac{1}{2^{14}}Ci = \frac{1}{16384}$$

$$\Rightarrow \quad A \approx 60\ \mu\text{Ci}$$

Hence, the correct answer is (C).

8. $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$

Probabilities of getting α and β-particles are same. So, rate of disintegration are equal.

$$\Rightarrow \quad \lambda_A N_A = \lambda_B N_B$$

$$\Rightarrow \quad \frac{N_A}{N_B} = \frac{\lambda_B}{\lambda_A} = 2$$

Hence, the correct answer is (C).

9. $\dfrac{m}{m_0} = \dfrac{N}{N_0} = \left(\dfrac{1}{2}\right)^{\frac{540}{270}} = \dfrac{1}{4}$

$\Rightarrow \quad m = \dfrac{m_0}{4} = \dfrac{100 \text{ mg}}{4} = 25 \text{ mg}$

Hence, the correct answer is (C).

10. $N = N_0 e^{-\lambda t}$

$$t_{\frac{1}{2}} = \frac{0.693}{\lambda}$$

$\Rightarrow \quad \lambda = \dfrac{0.693}{3.8}$

Since $\dfrac{N}{N_0} = \dfrac{1}{10}$

$\Rightarrow \quad \dfrac{1}{10} = e^{-\frac{0.693}{3.8}t}$

$\Rightarrow \quad \log 10 = \dfrac{0.693}{3.8}t$

$\Rightarrow \quad 2.3 = \dfrac{0.693}{3.8}t$

$\therefore \quad t = 12.62$ days

Hence, the correct answer is (B).

11. $\dfrac{N}{N_0} = \left(\dfrac{1}{2}\right)^n = \left(\dfrac{1}{2}\right)^{\frac{1}{2}} = \dfrac{1}{\sqrt{2}}$

Hence, the correct answer is (C).

12. Mass of product should be less than the mass of reactant because energy is released in a fission reaction.

Hence, the correct answer is (A).

14. Decay process is a probable process and hence nothing can be said for a small sample.

Hence, the correct answer is (D).

15.

	X	$\longrightarrow$	Y
$t = 0$	N_0		0
$t = t$	N		$(N_0 - N)$

$\Rightarrow \quad N_Y = N_0 - N$

Since, $N = N_0 e^{-\lambda t}$

$\Rightarrow \quad N_Y = N_0\left(1 - e^{-\lambda t}\right)$

Rate of formation of Y is

$$\frac{dN_y}{dt} = N_0 \lambda e^{-\lambda t}$$

The above curve is decreasing with time.

Hence, the correct answer is (C).

17. The decay constant λ is the reciprocal of the mean life τ. For a simultaneous decay process, we have

$$\lambda = \lambda_\alpha + \lambda_\beta, \text{ where } \lambda = \frac{1}{\tau}$$

Since, $\lambda_\alpha = \dfrac{1}{1620}$ per year and $\lambda_\beta = \dfrac{1}{405}$ per year

So, total decay constant is

$$\lambda = \lambda_\alpha + \lambda_\beta$$

$\Rightarrow \quad \lambda = \dfrac{1}{1620} + \dfrac{1}{405} = \dfrac{1}{324}$ per year

Since, $N = N_0 e^{-\lambda t}$

When $\dfrac{3}{4}$th part of the sample has disintegrated, then the sample left undecayed is

$$N = \frac{N_0}{4}$$

$\Rightarrow \quad \dfrac{N_0}{4} = N_0 e^{-\lambda t}$

$\Rightarrow \quad e^{\lambda t} = 4$

Taking logarithm of both sides, we get

$$\lambda t = \log_e 4$$

$\Rightarrow \quad t = \dfrac{1}{\lambda}\log_e (2)^2 = \dfrac{2}{\lambda}\log_e 2$

$\Rightarrow \quad t = 2 \times 324 \times 0.693 = 449$ year

Hence, the correct answer is (B).

18. λ remains uncharged.

Hence, the correct answer is (A).

19. Three half-lives of A is equivalent to six half-lives of B. Hence, we have

$$N_A\left(\frac{1}{2}\right)^3 = N_B\left(\frac{1}{2}\right)^6$$

$\Rightarrow \quad \dfrac{N_A}{N_B} = \dfrac{1}{8}$

Hence, the correct answer is (D).

20. Since $E = (\Delta m)c^2 = (1)(3\times10^8)^2$

$\Rightarrow \quad E = 9\times10^{16}$ J

$\Rightarrow \quad E = \dfrac{9\times10^{16}}{1.6\times10^{-13}}$ MeV

$\Rightarrow \quad E = \dfrac{9}{1.6}\times10^{29}$ MeV

$\Rightarrow \quad E = 5.625\times10^{29}$ MeV

Hence, the correct answer is (D).

21. Positron emission means β^+ decay and electron capture is a reverse β decay process.
Hence, the correct answer is (C).

22. Fraction of nuclei which remain undecayed is

$$f = \frac{N}{N_0} = \frac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$$

$$\Rightarrow \quad f = e^{-\left(\frac{\log_e(2)}{T}\right)\left(\frac{T}{2}\right)} = e^{-\frac{1}{2}\log_e(2)}$$

$$\Rightarrow \quad f = \frac{1}{e^{\log_e\sqrt{2}}} = \frac{1}{\sqrt{2}}$$

Hence, the correct answer is (A).

23. The number of undecayed nuclei in sample at time t is

$$N = N_0 e^{-\lambda t}$$

So, number of decayed nuclei of sample is

$$N_0 - N = N_0(1-e^{-\lambda t})$$

$$\Rightarrow \quad 10^5 = N_0(1-e^{-36\lambda}) \qquad \ldots(1)$$

$$\Rightarrow \quad 1.11\times10^5 = N_0(1-e^{-108\lambda}) \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{1-e^{-108\lambda}}{1-e^{-36\lambda}} = 1.11 = \frac{10}{9}$$

If $e^{-36\lambda} = x$ (say), then

$$\frac{1-x^3}{1-x} = \frac{10}{9}$$

$$\Rightarrow \quad \frac{(1-x)(1+x^2+x)}{(1-x)} = \frac{10}{9}$$

$$\Rightarrow \quad 9+9x^2+9x = 10$$

$$\Rightarrow \quad 9x^2+9x-1 = 0$$

$$\Rightarrow \quad x = \frac{-9\pm\sqrt{117}}{18} = \frac{-9+10.81}{18}$$

$$\Rightarrow \quad x = \frac{1.81}{18} \approx 0.1$$

$$\Rightarrow \quad e^{-36\lambda} = 0.1$$

$$\Rightarrow \quad e^{36\lambda} = 10$$

$$\Rightarrow \quad \lambda = \frac{\ell n(10)}{36}\,\text{s}^{-1}$$

$$\Rightarrow \quad T_{1/2} = \frac{\ell n2}{\lambda} = \frac{36\ell n2}{\ell n10} \approx 10.8 \text{ s}$$

Hence, the correct answer is (B).

24. For radioactive equilibrium, we have

$$\lambda_1 N_1 = \lambda_2 N_2$$

$$\Rightarrow \quad \frac{N_1}{T_1} = \frac{N_2}{T_2}$$

$$\Rightarrow \quad T_2 = \frac{N_2}{N_1}\times T_1$$

$$\Rightarrow \quad T_2 = 1.8\times10^4\times2.5\times10^5$$

$$\Rightarrow \quad T_2 = 4.5\times10^9 \text{ year}$$

Hence, the correct answer is (A).

25. ${}^{27}_{12}\text{Mg} \rightarrow {}^{27}_{13}\text{Al} + e^- + \overline{v}$

This is example of beta decay in which isotope ${}^{27}_{12}\text{Mg}$ is converted to an isotope of aluminium ${}^{27}_{13}\text{Al}$.
Hence, the correct answer is (C).

26. **The correct answer is (C).**

27. **Method-I**

$$\frac{N_1}{N_0} = 90\% = 0.9 = \left(\frac{1}{2}\right)^{\frac{5}{T}}$$

$$\frac{N_2}{N_0} = \left(\frac{1}{2}\right)^{\frac{20}{T}} = (0.9)^4 = 0.6561 \equiv 65.61\%$$

So, percentage decayed is $(100-65.61) \approx 34.4\%$

Method-II

$$\underset{\text{(Initial undecayed)}}{100\%} \xrightarrow{5 \text{ yrs}} 90\% \xrightarrow{5 \text{ yrs}} 81\% \xrightarrow{5 \text{ yrs}}$$

$$72.9\% \xrightarrow{5 \text{ yrs}} \underset{\text{Final undecayed}}{65.61\%}$$

So, percentage decayed is $(100-65.61) \approx 34.4\%$

Hence, the correct answer is (D).

29. Energy released ΔE is

$$\Delta E = (\text{Final BE}) - (\text{Initial BE})$$

$$\Rightarrow \quad \Delta E = 110\times8.2+90\times8.2-200\times7.4$$

$\Rightarrow \quad \Delta E = 160 \text{ MeV}$

Hence, the correct answer is (D).

30. $A = 232 - 4 = 228$

From conservation of momentum

$$p_\alpha = p_Y$$

$$\Rightarrow \quad \sqrt{2m_\alpha K_\alpha} = \sqrt{2m_Y K_Y}$$

$$\Rightarrow \quad \frac{K_\alpha}{K_Y} = \frac{m_Y}{m_\alpha} = \frac{228}{4}$$

$$\Rightarrow \quad K_\alpha = \left(\frac{228}{228+4}\right)K_{\text{Total}}$$

$$\Rightarrow \quad K_\alpha = \left(\frac{228}{232}\right)K_{\text{Total}}$$

Hence, the correct answer is (B).

31. $E = mc^2$

$$\Rightarrow \quad E = m_0c^2\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

Since $\frac{v}{c} \ll 1$

$$\Rightarrow \quad E \simeq m_0c^2\left(1 + \frac{v^2}{2c^2}\right)$$

$$\Rightarrow \quad E \simeq m_0c^2 + \frac{1}{2}m_0v^2$$

Hence, the correct answer is (C).

32. Since we know that $\tau = \frac{1}{\lambda}$

$$\Rightarrow \quad A = A_0 e^{-\frac{t}{\tau}}$$

$$\Rightarrow \quad y = xe^{-\frac{(t_2 - t_1)}{\tau}}$$

Number of atoms disintegrated during $t_2 - t_1$ is

$$\Delta N = (x - y)\tau$$

Hence, the correct answer is (D).

33. Efficiency of 50% means that we are getting only 100 MeV of energy by the fission of one uranium nucleus.

Number of nuclei per second is

$$\frac{N}{t} = \frac{\text{Energy required per second}}{\text{Energy obtained in one fission}}$$

$$\Rightarrow \quad \frac{N}{t} = \frac{16 \times 10^6}{100 \times 1.6 \times 10^{-13}} = 10^{18}$$

Hence, the correct answer is (D).

34. Let R_0 be the initial activity. Then

$$R_1 = R_0 e^{-\lambda t_1} \text{ and } R_2 = R_0 e^{-\lambda t_2}$$

$$\Rightarrow \quad \frac{R_2}{R_1} = e^{\lambda(t_1 - t_2)}$$

$$\Rightarrow \quad R_2 = R_1 e^{\lambda(t_1 - t_2)}$$

Hence, the correct answer is (D).

35. $R_1 = \lambda N_1$

$$\Rightarrow \quad N_1 = \frac{R_1}{\lambda}$$

and $R_2 = \lambda N_2$

$$\Rightarrow \quad N_2 = \frac{R_2}{\lambda}$$

So, number of atoms decayed is

$$N = N_1 - N_2$$

$$\Rightarrow \quad N = \left(\frac{R_1 - R_2}{\lambda}\right)$$

Hence, the correct answer is (D).

36. Since $N = \frac{N_0}{2^n}$, where $n = \frac{t}{T_{1/2}}$

For A, $N_A = \frac{N_0}{2^{\frac{80}{20}}} = \frac{N_0}{2^4}$

For B, $N_B = \frac{N_0}{2^{\frac{80}{40}}} = \frac{N_0}{2^2}$

$$\Rightarrow \quad \frac{N_A}{N_B} = \frac{1}{4}$$

Hence, the correct answer is (C).

37. ${}_{92}\text{U}^{238} \to {}_{82}\text{Pb}^{206} + n_1\left({}_2\alpha^4\right) + n_2\left({}_{-1}e^0\right)$

$\Rightarrow \quad 4n_1 + 0 \times n_2 + 206 = 238$

$\Rightarrow \quad n_1 = 8$

Also, $92 = 82 + 2n_1 - n_2$

$\Rightarrow \quad 10 = 2 \times 8 - n_2$

$\Rightarrow \quad n_2 = 6$

Hence, the correct answer is (C).

38. Speed of the neutrons in the beam is

$$\frac{1}{2}mv^2 = E_K = 5 \text{ eV}$$

$$\Rightarrow \quad v = \sqrt{\frac{2(5) \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27}}}$$

$$\Rightarrow \quad v = 31 \text{ kms}^{-1}$$

During a time of $T_{1/2} = 12.8$ min, half the neutrons may have decayed from the beam. The distance travelled by the undecayed during this time is

$$x = vt = (31 \text{ kms}^{-1})(12.8 \text{ min})(60 \text{ s/min})$$

$$\Rightarrow \quad x = 23800 \text{ km}$$

Hence, the correct answer is (B).

39. For, successive disintegration, we have

$$A \xrightarrow[\lambda_1]{} B \xrightarrow[\lambda_2]{} C$$

So, rate of decay of B is

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B$$

In equilibrium, $\frac{dN_B}{dt} = 0$ i.e. rate of production of B equals the rate of disintegration of B.

Hence, the correct answer is (A).

41. Count rate for 100 cc volume is c. If some volume is discarded and left out volume is V, then new count rate should be $\left(\frac{c}{100}\right)V$. After 3 half life of remaining liquid, count rate is $\frac{c}{10}$.

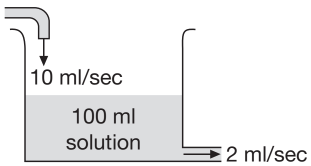
$$\Rightarrow \quad \frac{c}{100} = \frac{cV}{100}\left(\frac{1}{2}\right)^3$$

$$\Rightarrow \quad V = 80 \text{ cc}$$

Hence, the correct answer is (D).

42. Rate of disintegration does not change with temperature, pressure or humidity.
Hence, the correct answer is (D).

43.



The volume of water added in time t is $8t$ and the volume of liquid ejected in t seconds is $2t$. Hence, the volume of liquid in beaker at any instant of time t is

$$V = 100 + 8t$$

Number of active atoms being taken out is

$$-dN = \frac{N}{V}(2dt)$$

$$\Rightarrow \quad -\frac{dN}{dt} = \frac{2N}{V} = \frac{2N}{100 + 8t}$$

Multiplying both sides with disintegration constant λ, we get

$$-\lambda dN = \lambda N\left(\frac{2dt}{V}\right) \qquad \ldots(1)$$

Since, $\lambda dN = dA$ and $\lambda N = A$, so equation (1) becomes

$$-dA = A\left(\frac{2dt}{V}\right)$$

where A is activity of the solution.

Since the time taken for 10 ml solution to come out is 5 s

$$\Rightarrow \quad \int_{A_0}^{A} \frac{dA}{A} = \int_0^5 \frac{-2t}{100 + 8t} dt$$

$$\Rightarrow \quad A = A_0\left(\frac{5}{7}\right)^{\frac{1}{4}}$$

So, required activity of the ejected solution is

$$A - A_0 = A_0\left[1 - \left(\frac{5}{7}\right)^{\frac{1}{4}}\right]$$

Hence, the correct answer is (C).

44. $N = \frac{N_0}{(2)^{t/T_{1/2}}}$

$$\Rightarrow \quad (2)^{t/T_{1/2}} = \frac{N_0}{N}$$

$$\Rightarrow \quad (2)^{t/T_{1/2}} = \frac{8}{1}$$

$$\Rightarrow \quad t = 3T_{1/2} = 4.2 \times 10^9 \text{ year}$$

Hence, the correct answer is (C).

45. Given $T_{1/2} = 30$ min

At $t = 2\,hr$, $A = 5 \text{ sec}^{-1}$ and $A_0 = ?$

Since, $n = \frac{t}{T_{1/2}} = \frac{2 \times 80}{30} = 4$

Since, $\frac{A}{A_0} = \left(\frac{1}{2}\right)^n$

$$\Rightarrow \quad A_0 = 2^n A = 2^4 \times 5$$

$$\Rightarrow \quad A_0 = 16 \times 5 = 80 \text{ sec}^{-1}$$

Hence, the correct answer is (B).

46. $\frac{7}{8}$ decays $\Rightarrow$ $\frac{1}{8}$ left undecayed

$\Rightarrow$ since $\frac{1}{8} = \left(\frac{1}{2}\right)^n$

$\Rightarrow \quad x = 3$

$\Rightarrow \quad \dfrac{t}{T_{1/2}} = 3$

Since $t = 12$ day

$\Rightarrow \quad T_{1/2} = 4$ day

So, fraction undecayed after 24 days is

$$\frac{N'}{N_0} = \left(\frac{1}{2}\right)^{\frac{24}{4}} = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$$

Hence, the correct answer is (C).

47. ${}_0n^1 \to {}_1p^1 + {}_{-1}e^0 + \bar{\nu}$

The electron comes out with a spectrum of energies. The energy released is shared between electron and neutrino.

Hence, the correct answer is (D).

49. $A_2 = A_1\left(\dfrac{90}{100}\right)^{\frac{20}{5}}$

$\Rightarrow \quad \dfrac{A_2}{A_1} \times 100\% = \left(\dfrac{9}{10}\right)^4 100\% = 65.61\%$

Hence, the correct answer is (B).

51. When the rate production is equal to the rate of disintegration then the number of nuclei is maximum.

$\Rightarrow \quad \lambda N = A$

$\Rightarrow \quad \dfrac{\ln 2}{T} N = A$

$\Rightarrow \quad N = \dfrac{AT}{\ln 2} = \text{maximum}$

Hence, the correct answer is (D).

52. $\dfrac{dN_2}{dt} = \lambda N_1 = 2\lambda N_2$

For N_2 to be maximum, we have

$$\frac{dN_2}{dt} = 0$$

$\Rightarrow \quad \lambda N_1 = 2\lambda N_2$

$\Rightarrow \quad \dfrac{N_1}{N_2} = 2$

Hence, the correct answer is (B).

54. ${}_ZX^A \to {}_{Z-2}Y^{A-4} + {}_2\alpha^4$

Hence, the correct answer is (C).

55. Charge number and mass number for γ-ray is zero.

Hence, the correct answer is (C).

56. In the mixture, initially we have

$$N = N_1 + N_2$$

At any time t, we have

$$N(t) = N_1(t) + N_2(t)$$

$\Rightarrow \quad N(t) = N_1 e^{-\lambda_1 t} + N_2 e^{-\lambda_2 t} \quad \ldots(1)$

The decay rate $-\dfrac{dN}{dt}$ at time t is obtained by differentiating equation (1)

$\Rightarrow \quad -\dfrac{dN}{dt} = N_1\lambda_1 e^{-\lambda_1 t} + N_2\lambda_2 e^{-\lambda_2 t}$

Hence, the correct answer is (C).

58. $R_0 = 15 \times 200 = 3000$ decay/min from 200 g carbon.

Using, $R = R_0\left(\dfrac{1}{2}\right)^n$

$\Rightarrow \quad 375 = 3000\left(\dfrac{1}{2}\right)^n$

$\Rightarrow \quad \dfrac{1}{8} = \left(\dfrac{1}{2}\right)^n$

$\Rightarrow \quad n =$ number of half lives $= 3$

$\Rightarrow \quad t = 5730 \times 3 = 17190$ yr

Hence, the correct answer is (C).

59. Since $\dfrac{A}{A_0} = \dfrac{3}{5} = \left(\dfrac{1}{2}\right)^{\frac{t}{T}}$

$\Rightarrow \quad \ell n\left(\dfrac{3}{5}\right) = \dfrac{t}{T}\ell n\left(\dfrac{1}{2}\right)$

$\Rightarrow \quad t = \dfrac{T(\ell n5 - \ell n3)}{\ell n2} \approx 4000$ year

Also, we can think that, $\dfrac{3}{5} = 60\%$ and 50% decay takes 5570 year.

So 40% decay takes a little less number of year.

Hence, the correct answer is (D).

60. Since, $\frac{1}{4}=\frac{1}{2^n}$

$\Rightarrow\ n=2$

$\Rightarrow\ t=nT_{1/2}=n\left(\frac{T_1\times T_2}{T_1+T_2}\right)=2\left(\frac{1620\times 810}{1620+810}\right)$

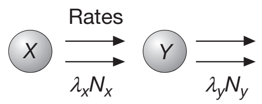
$\Rightarrow\ t=1080$ yr

Hence, the correct answer is (A).

61. Number of radionuclei become constant, when

$X=\lambda N$

$\Rightarrow\ N=\frac{x}{\lambda}$



$\Rightarrow\ N=\frac{X}{\frac{\log_e(2)}{Y}}=\frac{XY}{\log_e(2)}$

Hence, the correct answer is (C).

62. $T=\frac{4\times 12}{16}=3$ year

$\frac{N}{N_0}=\left(\frac{1}{2}\right)^{\frac{12}{3}}=\frac{1}{16}=6.25\%$

Hence, the correct answer is (A).

63. Since decay is to be regarded as a statistical spontaneous process, hence α decay can be regarded as an Adiabatic process.

Hence, the correct answer is (C).

64. $R=R_0A^{1/3}$

$\Rightarrow\ \log R=\log R_0+\frac{1}{3}\log A$

$\Rightarrow\ \log\left(\frac{R}{R_0}\right)=\frac{1}{3}\log A$

Hence, the correct answer is (D).

65. Total initial binding energy is 236×7.6

$\Rightarrow\ U_i=1793.6$ MeV

Total final binding energy is $2(117)(8.5)$

$\Rightarrow\ U_f=1989$ MeV

$\Rightarrow\ \Delta U=195.4$ MeV

Hence, the correct answer is (B).

66. $t=0,\ N_0=8\times 10^4$

$N=1\times 10^4$

at $t=?$

Since, $T_{1/2}=3$ year

and $\frac{N}{N_0}=\frac{1\times 10^4}{8\times 10^4}=\frac{1}{8}=\frac{1}{2^n}$

$\Rightarrow\ n=3$

Since $t=nT_{1/2}$

$\Rightarrow\ t=3\times 3=9$ year

Hence, the correct answer is (A).

67. $R=R_0e^{-\lambda t}$

$\Rightarrow\ \lambda=\frac{1}{t}\log_e\left(\frac{R_0}{R}\right)=\frac{2.3}{t}\log_{10}\left(\frac{R_0}{R}\right)$

$\therefore$ Decay constant

$\lambda=\frac{1}{5}\times 2.3\log_{10}\frac{9750}{975}=\frac{2.3}{5}=0.461\ \text{min}^{-1}$

Hence, the correct answer is (C).

68. $N=N_0e^{-\lambda t}$

$\Rightarrow\ N=N_0e^{-\frac{\ell n2}{T_{1/2}}t}$, where $t=6$ months and $T_{1/2}=1$ year

$\Rightarrow\ N=N_0e^{-\frac{\ell n2}{2}}=N_0e^{-\ell n\sqrt{2}}$

$\Rightarrow\ N=\frac{N_0}{\sqrt{2}}$

So, amount decayed is $(N_0-N)=N_0\left(1-\frac{1}{\sqrt{2}}\right)$

$\Rightarrow\ (N_0-N)=N_0(1-0.707)=0.293N_0$

$\Rightarrow\ \frac{N_0-N}{N_0}\times 100\%=29.3\%$

So, percentage amount decayed is 29.3%

Hence, the correct answer is (A).

70. $R=\frac{dN}{dt}=\lambda N$

$\Rightarrow\ \frac{R}{N}=\lambda=$ Decay Constant

Hence, the correct answer is (D).

71. $\dfrac{N}{N_0}=\left(\dfrac{1}{2}\right)^4$

$\Rightarrow \quad \dfrac{N}{N_0}=\dfrac{1}{16}$

Hence, the correct answer is (D).

72. Decay probability per second i.e. $\left|\dfrac{dN}{N}\dfrac{1}{dt}\right|$ is just the decay constant (λ), because $\dfrac{dN}{dt}=-\lambda N$

$\Rightarrow \quad \left|\dfrac{dN}{N}\dfrac{1}{dt}\right|=\lambda$

$\Rightarrow \quad \lambda=\dfrac{0.693}{t_{1/2}}=\dfrac{0.693}{2.7 \text{ days}}$

$\Rightarrow \quad \lambda=\dfrac{0.693}{2.7\times 24\times 60\times 60}$

$\Rightarrow \quad \lambda=2.97\times 10^{-6}\ \text{s}^{-1}$

Hence, the correct answer is (B).

73. Since nuclear density is independent of mass of nucleus, hence all possess equal density.

Hence, the correct answer is (C).

74. Probability that a nucleus will not decay is given by

$$p=\frac{N}{N_0}=\left(\frac{1}{2}\right)^n=\left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

When n is the number of half lives lapsed, so $n=4$

$\Rightarrow \quad p=\dfrac{N}{N_0}=\left(\dfrac{1}{2}\right)^4=\dfrac{1}{16}$

Probability that a nucleus will decay is $q=1-p$

$\Rightarrow \quad q=1-p=1-\dfrac{1}{16}=\dfrac{15}{16}$

Hence, the correct answer is (C).

76. For A, $N_A=\dfrac{N_{0A}}{(2)^{\frac{16}{4}}}=\dfrac{N_{0A}}{16}=\dfrac{10^{-2}}{16}$ kg

$\Rightarrow \quad N_A=0.625$ mg

For B, $N_B=\dfrac{N_{0B}}{(2)^{\frac{16}{8}}}=\dfrac{N_{0B}}{4}=\dfrac{10^{-2}}{4}$ kg

$\Rightarrow \quad N_B=2.5$ mg

Hence, the correct answer is (B).

77. If R_0 be the initial activity of the sample, then $R_1=R_0e^{\lambda t_1}$ and $R_2=R_0e^{-\lambda t_2}$

where $\lambda=\dfrac{1}{T}$ $\quad\left\{\because \text{Mean life } =T=\dfrac{1}{\lambda}\right\}$

$\Rightarrow \quad \dfrac{R_2}{R_1}=\dfrac{e^{-\lambda t_2}}{e^{-\lambda t_1}}=e^{\lambda(t_1-t_2)}$

$\Rightarrow \quad R_2=R_1\exp\left(\dfrac{t_1-t_2}{T}\right)$

Hence, the correct answer is (C).

78. For x, $A_x=A_0\left(\dfrac{1}{2}\right)^{\frac{48}{24}}=\dfrac{A_0}{4}$

For y, $A_y=A_0\left(\dfrac{1}{2}\right)^{\frac{48}{16}}=\dfrac{A_0}{8}$

When mixed, total activity is

$$A=A_x+A_y=\frac{A_0}{4}+\frac{A_0}{8}=\frac{3A_0}{8}$$

Hence, the correct answer is (D).

79. Rate of decay of sample 1, $R_1=\lambda_1N_1e^{-\lambda_1 t}$ and for sample 2, $R_2=\lambda_2N_2e^{-\lambda_2 t}$.

When they are mixed, we have $R=R_1R_2$

$\Rightarrow \quad R=\lambda_1N_1e^{-\lambda_1 t}\lambda_2N_2e^{-\lambda_2 t}$

$\Rightarrow \quad R=\lambda_1N_1\lambda_2N_2e^{-(\lambda_1+\lambda_2)t}$

Hence, the correct answer is (D).

80. Speed of light and γ-rays is c while $v_\beta<c$

Hence, the correct answer is (C).

81. $\left(T_{1/2}\right)_R=\dfrac{\log_e(2)}{4.5\times 10^{-3}}\ \text{yr}^{-1}$

$\left(T_{1/2}\right)_S=\dfrac{\log_e(2)}{3\times 10^{-3}}\ \text{yr}^{-1}$

$\Rightarrow \quad \dfrac{\left(T_{1/2}\right)_R}{\left(T_{1/2}\right)_S}=\dfrac{2}{3}$

$\Rightarrow \quad 3\left(T_{1/2}\right)_R=2\left(T_{1/2}\right)_S$

So, after three half life of R, we see that two half life of S have lapsed.

$$N_R=(N_0)_R\left(\frac{1}{2}\right)^3$$

$$N_S=(N_0)_S\left(\frac{1}{2}\right)^2$$

$$\Rightarrow \quad \frac{N_R}{N_S} = \left[\frac{(N_0)_R}{(N_0)_S}\right]\left(\frac{1}{2}\right)$$

Since $\frac{(N_0)_R}{(N_0)_S} = 2$

$$\Rightarrow \quad \frac{N_R}{N_S} = 2\left(\frac{1}{2}\right) = 1$$

Hence, the correct answer is (C).

82. $240 \to 120 \to 60 \to 30$

Since, 3 half lives = 60 minute

$\Rightarrow$ 1 half life = 20 minute

Hence, the correct answer is (A).

83. Activity $R = \lambda N$

Number of nuclei (N) per mole are equal for both the substances, so we have

$$R \propto \lambda$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \frac{\lambda_1}{\lambda_2} = \frac{4}{3}$$

Hence, the correct answer is (C).

84. $$\frac{2}{3} = \left(\frac{1}{2}\right)^{\frac{t_1}{T_{1/2}}} \quad \ldots(1)$$

$$\frac{1}{3} = \left(\frac{1}{2}\right)^{\frac{t_2}{T_{1/2}}} \quad \ldots(2)$$

Divide (1) by (2) and rearrange, we get

$$2 = 2^{\frac{(t_2 - t_1)}{T_{1/2}}}$$

$$\Rightarrow \quad \frac{t_2 - t_1}{T_{1/2}} = 1$$

$$\Rightarrow \quad \Delta t = T_{1/2} = 20 \text{ min}$$

Hence, the correct answer is (B).

85. Net rate of formation of Y at any time t is

$$\frac{dN_y}{dt} = \lambda_x N_x - \lambda_y N_y$$

N_y is maximum when $\frac{dN_y}{dt} = 0$

$$\Rightarrow \quad \lambda_x N_x = \lambda_y N_y$$

Hence, the correct answer is (D).

86. $\lambda = \lambda_1 + \lambda_2$

$$\Rightarrow \quad \frac{\ln 2}{T} = \frac{\ln 2}{T_1} + \frac{\ln 2}{T_2} \qquad (T = \text{Half life})$$

$$\Rightarrow \quad T = \frac{T_1 T_2}{T_1 + T_2} = 20 \text{ yr}$$

$\frac{1}{4}$th sample remains after 2 half lives or 40 yr

Hence, the correct answer is (C).

88. By Conservation of Linear Momentum, we have

$$p_D = p_\alpha$$

$$\Rightarrow \quad \sqrt{2m_D K_D} = \sqrt{2m_\alpha K_\alpha}$$

$$\Rightarrow \quad K_D = \frac{m_\alpha}{m_D} K_\alpha = \frac{4}{214} \times 6.7 \text{ MeV} = 0.125 \text{ MeV}$$

Hence, the correct answer is (A).

89. Activity of $S_1 = \frac{1}{2}$ (activity of S_2)

$$\Rightarrow \quad \lambda_1 N_1 = \frac{1}{2}(\lambda_2 N_2)$$

$$\Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \frac{N_2}{2N_1}$$

$$\Rightarrow \quad \frac{T_1}{T_2} = \frac{2N_1}{N_2} \qquad \left\{T = \text{half life} = \frac{\log_e 2}{\lambda}\right\}$$

Given $N_1 = 2N_2$

$$\Rightarrow \quad \frac{T_1}{T_2} = 4$$

Hence, the correct answer is (A).

90. For A, $m_1 = m_0\left(\frac{1}{2}\right)^{\frac{6}{1}} = \frac{m_0}{64}$

For B, $m_2 = m_0\left(\frac{1}{2}\right)^{\frac{6}{2}} = \frac{m_0}{8}$

$$\Rightarrow \quad \frac{A_1}{A_2} = \frac{\lambda_1 N_1}{\lambda_2 N_2} = \left(\frac{T_2}{T_1}\right)\left(\frac{m_1}{m_2}\right)$$

$$\Rightarrow \quad \frac{A_1}{A_2} = \frac{2}{1} \times \frac{m_0/64}{m_0/8} = \frac{16}{64} = \frac{1}{4}$$

Hence, the correct answer is (A).

91. Rest mass of parent nucleus should be greater than the rest mass of daughter nuclei.

Hence, the correct answer is (A).

92. $A_0 = \lambda N_0 = \lambda\left(\frac{m}{M}\right)N_A$

Since $A = A_0 e^{-\lambda t}$

$\Rightarrow \quad A=\left(\frac{\lambda m N_A}{M}\right)e^{-\lambda t}$

Hence, the correct answer is (C).

93. Total energy radiated per minute from sun is

$$E_{\text{radiated}}=\sigma\left(4\pi R_{se}^2\right)$$

Energy radiated annually is given by

$$E_{\text{total}}=24\times 60\times 365\times E_{\text{radiated}}$$

$\Rightarrow \quad$ Annual loss of mass $=\Delta m=\frac{E_{\text{total}}}{c^2}=1.38\times 10^{17}$ kg

Hence, the correct answer is (C).

94. $\frac{N_0}{N}=25\%=\frac{1}{4}=\frac{1}{2^n}$

$\Rightarrow \quad n=$ Number of half lifes lapsed is 2

$\Rightarrow \quad t=nT_{1/2}$

$\Rightarrow \quad T_{1/2}=\frac{t}{n}=\frac{16}{2}=8$ day

Hence, the correct answer is (B).

95. Combining two given equations, we have

$$3\,{}_1H^2={}_2He^4+p+n$$

Now, $\quad \Delta m=3\times 2.014-4.001-1.007-1.008$

$\Rightarrow \quad \Delta m=0.026u$

Energy released by 3 deutrons is

$$\Delta E=0.026\times 931.5\times 1.6\times 10^{-13}\text{ J}$$

$\Rightarrow \quad \Delta E=3.9\times 10^{-12}$ J

$\Rightarrow \quad \left(10^{16}\times t\right)=\left(\frac{10^{40}}{3}\right)\left(3.9\times 10^{-12}\right)$

Solving we get,

$$t\approx 1.3\times 10^{12}\text{ s}$$

Hence, the correct answer is (C).

96. $\frac{m}{m_0}=\frac{1\text{ g}}{16\text{ g}}=\left(\frac{1}{2}\right)^4=\left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$

$\Rightarrow \quad t=4T_{1/2}=4\times 140\text{ day}=560\text{ day}$

Hence, the correct answer is (B).

97. $A\longrightarrow B\longrightarrow C$

Let b be the number of nuclei of B, then

$$\frac{dB}{dt}=x-\lambda b$$

$\Rightarrow \quad y=x-\lambda b$

$\Rightarrow \quad \lambda b=x-y$

Activity of B is $A_B=-\lambda b=y-x$

Hence, the correct answer is (B).

98. $\tau=\frac{\tau_\alpha\tau_\beta}{\tau_\alpha+\tau_\beta}=\frac{1620\times 520}{1620+520}=394$ year

$\Rightarrow \quad \frac{1}{\lambda}=394$

Time of decay, $t=\frac{2.303}{\lambda}\log_{10}\left(\frac{N_0}{N}\right)$

$\Rightarrow \quad t=2.303(394)\left(\log_{10}(4)\right)$

$\Rightarrow \quad t=540$ year

Hence, the correct answer is (A).

99. $f=\frac{\Delta m}{A}=\frac{M-A}{A}$

Hence, the correct answer is (D).

100. Since $N=N_0e^{-\lambda t}$

$\Rightarrow \quad P=\frac{N}{N_0}=e^{-\lambda t}$

Hence, the correct answer is (A).

102. Since $\frac{N}{N_0}=\frac{1}{64}=\frac{1}{2^n}$

$\Rightarrow \quad n=\frac{t}{T_{1/2}}=6$

$\Rightarrow \quad t=nT_{1/2}$

$\Rightarrow \quad T_{1/2}=\frac{t}{n}=\frac{60}{6}=10$ sec

Hence, the correct answer is (B).

103. $\frac{N}{N_0}=\left(\frac{1}{2}\right)^n=\left(\frac{1}{2}\right)^{\frac{1}{2}}=\frac{1}{\sqrt{2}}$

Hence, the correct answer is (C).

104. When 75% decays, 25% is left undecayed. This required a time $t=2T_{1/2}$, where $T_{1/2}=\frac{\ell n2}{\lambda}$ and also, $\lambda=\frac{1}{T}$

$\Rightarrow \quad t=2\left(\frac{\ell n2}{\lambda}\right)=2T\ell n(2)$

Hence, the correct answer is (D).

105. $E=mc^2$

$\Rightarrow \quad E=\left(10^{-6}\right)\left(9\times 10^{16}\right)$

$\Rightarrow \quad E=9\times 10^{10}$ J

Hence, the correct answer is (A).

106. $\frac{A}{A_0} = \frac{128}{1024} = \frac{1}{8} = \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{\frac{3\text{ min}}{T_{1/2}}}$

$\Rightarrow \quad T_{1/2} = 1\text{ min}$

$\Rightarrow \quad A' = A_0\left(\frac{1}{2}\right)^{\frac{5}{1}} = \frac{A_0}{32} = \frac{1024}{32} = 32$

Hence, the correct answer is (D).

107. Since $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$

where $n = \frac{\text{Time lapsed}}{\text{Half life}}$

According to the problem $n = \frac{\frac{T_{1/2}}{2}}{T_{1/2}} = \frac{1}{2}$

$\Rightarrow \quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^{1/2} = \frac{1}{\sqrt{2}}$

Hence, the correct answer is (C).

108. Energy released is $\Delta E = (BE)_{\text{final}} - (BE)_{\text{initial}}$

$\Rightarrow \quad \Delta E = (7.0 - 1.1)4$

$\Rightarrow \quad \Delta E = 23.6\text{ MeV}$

Hence, the correct answer is (B).

110. $\frac{\lambda_1}{\lambda_2} = \frac{\lambda_1 N}{\lambda_2 N}$

$\Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \frac{\text{decay rate of } \alpha \text{ decay}}{\text{decay rate of } \beta \text{ decay}}$

$\Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \frac{\text{probability of } \alpha \text{ decay}}{\text{probability of } \beta \text{ decay}}$

$\Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \frac{\frac{75}{100}}{\frac{25}{100}} = 3$

Hence, the correct answer is (A).

111. ${}_yA^x \rightarrow {}_{y-2}B^{x-4} + {}_2\text{He}^4$

${}_{y-2}B^{x-4} \rightarrow {}_yC^{x-4} + 2\,{}_{-1}e^0$

Hence, the correct answer is (C).

112. $\Delta A = 235 - 219 = 16$

$\Rightarrow$ Number of α-particles emitted $= \frac{16}{4} = 4$

Hence, the correct answer is (A).

113. $\frac{N_0}{16} = N_0\left(\frac{1}{2}\right)^n$

$\Rightarrow \quad n = 4$

So, $3t$ times is equivalent to four half lives. Hence one half life is equal to $\frac{3t}{4}$.

The given time $\frac{11}{2}t - t = \frac{9}{2}t = 6\left(\frac{3t}{4}\right)$ is equivalent to 6 half lives

$\Rightarrow \quad N = N_0\left(\frac{1}{2}\right)^6 = \frac{N_0}{64}$

Hence, the correct answer is (B).

114. $A = A_0 e^{-\lambda t}$

$\Rightarrow \quad 100 = 800e^{-\lambda(6\times 60)}$

$\Rightarrow \quad e^{-360\lambda} = \frac{1}{8}$

$\Rightarrow \quad -360\lambda = \ell n\left(\frac{1}{8}\right) = -\ell n 8$

$\Rightarrow \quad \lambda = \frac{\ell n(2^3)}{360} = \frac{\ell n 2}{120}$

$\Rightarrow \quad T_{\frac{1}{2}} = \frac{\ell n 2}{\lambda} = \frac{\ell n 2}{\left(\frac{\ell n 2}{120}\right)} = 120\text{ min}$

$\Rightarrow \quad T_{\frac{1}{2}} = 2\text{ hrs}$

Hence, the correct answer is (B).

115. Charge number decreases by 4, but actually it must decrease by 8 (due to emission of 4α-particles). Hence 4β particles must have also been emitted.

Hence, the correct answer is (A).

116. ${}_{86}A^{222} \longrightarrow {}_{84}B^{210}$

$\Rightarrow \quad n_\alpha = \frac{A - A'}{4} = \frac{222 - 210}{4} = 3$

and $n_\beta = 2n_\alpha - Z + Z'$

$\Rightarrow \quad n_\beta = 2\times 3 - 86 + 84$

$\Rightarrow \quad n_\beta = 4$

Hence, the correct answer is (B).

117. $N = N_0\left(\frac{1}{2}\right)^{\frac{15}{5}} = \frac{N_0}{8}$

So, amount of substance undecayed is

$\left(N_0 - \frac{N_0}{8}\right) = \frac{7N_0}{8}$

Hence, the correct answer is (C).

118. $\frac{dN}{dt} = t^2 - \lambda N$

For $\frac{dN}{dt}$ to be minimum, we have $\frac{d^2N}{dt^2} = 0$

$$\Rightarrow \left.\frac{d^2N}{dt^2}\right|_{t=t_0} = 2t - \lambda\frac{dN}{dt} = 2t_0 - \lambda\left(t_0^2 - \lambda N\right) = 0$$

$$\Rightarrow N = \frac{2t_0 - \lambda t_0^2}{\lambda^2}$$

Hence, the correct answer is (A).

119. Probability of decay is $p = 1 - \frac{N}{N_0} = 1 - \left(\frac{1}{2}\right)^n$

where $n = \frac{\text{Time lapsed}}{T_{1/2}} = \frac{10}{5} = 2$

$$\Rightarrow p = 1 - \frac{1}{2^2} = \frac{3}{4} = 75\%$$

Hence, the correct answer is (B).

120. $A_P = A_0 e^{-\lambda t_1}$ ...(1)

$A_Q = A_0 e^{-\lambda t_2}$ ...(2)

From (1), $\lambda_{t_1} = \ln\left(\frac{A_0}{A_P}\right)$

$$\Rightarrow t_1 = \frac{1}{\lambda}\ln\left(\frac{A_0}{A_P}\right) = T\ln\left(\frac{A_0}{A_P}\right)$$

Similarly, from (2), $t_2 = T\ln\left(\frac{A_0}{A_Q}\right)$

$$\Rightarrow t_1 - t_2 = T\ln\left(\frac{A_Q}{A_P}\right)$$

Hence, the correct answer is (B).

121. $\frac{4}{3}\pi R^3 \propto A$

$$\Rightarrow R \propto A^{\frac{1}{3}}$$

Hence, the correct answer is (B).

122. After $t = 9$ year

$$\frac{A_0}{3} = A_0(2)^{-\frac{9}{T}}$$

$$\Rightarrow T = 9\frac{\log(2)}{\log(3)}$$

Nine years further i.e. at $t = 18$ year, we have

$$A = A_0(2)^{-\frac{18}{T}}$$

$$\Rightarrow 18\left[\frac{\log(2)}{\log\left(\frac{A_0}{A}\right)}\right] = 9\left[\frac{\log(2)}{\log(3)}\right]$$

$$\Rightarrow \left(\frac{A_0}{A}\right) = (9)$$

$$\Rightarrow A = \frac{A_0}{9}$$

Hence, the correct answer is (C).

123. Power of Sun is

$$P_S = 1.4 \times 10^3 \times 4\pi\left(1.5 \times 10^{11}\right)^2$$

energy released by Sun per day is

$$E_{\text{sun/day}} = P_S \times 86400$$

Mass lost by sun per day is

$$\Delta m = \frac{E_{\text{sun/day}}}{c^2}$$

$$\Rightarrow \Delta m = \frac{1.4 \times 10^3 \times 4 \times 3.14 \times \left(1.5 \times 10^{11}\right)^2 \times 86400}{\left(3 \times 10^8\right)^2}$$

$$\Rightarrow m = 8.79 \times 10^{14} \text{ kg}$$

Hence, the correct answer is (D).

124. The nuclear force of interaction between any pair of nucleons is identical i.e. force between two neutrons (F_2) equals the force between neutron and proton (F_3). However, between two protons net force is equal to the resultant of nuclear force between them (attractive in nature) and electrostatic force between them (repulsive in nature). Hence F_1 is a value lesser than F_2 and F_3. So $F_1 < F_2 = F_3$.

Hence, the correct answer is (C).

125. $N = N_0 e^{-\lambda t}$

$$\Rightarrow D = N_0 - N = N_0\left(1 - e^{-\lambda t}\right)$$

Since $R = R_0 e^{-\lambda t}$

$$\Rightarrow \frac{R}{N} = \frac{R_0 e^{-\lambda t}}{N_0 e^{-\lambda t}} = \frac{R_0}{N_0} = \lambda = \text{constant}$$

Hence, the correct answer is (D).

126.

	A	B
$T_{1/2}$	1 year	2 year
M_0	10 g	1 g
$t = ?$	M	M

For A, $M = \frac{10}{2^{t/1}}$ ...(1)

For B, $M = \frac{1}{2^{t/2}}$ ...(2)

Equating (1) and (2), we get

$$\frac{10}{2^t} = \frac{1}{2^{t/2}}$$

$$\Rightarrow \quad 2^{t/2} = 10$$

Taking log both sides, we get

$$\log_{10}(2^{t/2}) = \log_{10} 10$$

$$\Rightarrow \quad \frac{t}{2}\log_{10}(2) = \log_{10}(10) = 1$$

$$\Rightarrow \quad t = \frac{2}{0.3010} = 6.62 \text{ year}$$

Hence, the correct answer is (A).

128. From given graph we have

$$\ell n A = -\left(\frac{2.5}{25}\right)t + 2.5$$

$$\Rightarrow \quad A = e^{\left(-\frac{t}{10}+2.5\right)}$$

$$\Rightarrow \quad A = e^{2.5}e^{-0.1t}$$

$$\Rightarrow \quad A = 12e^{-0.1t}$$

Hence, the correct answer is (D).

129. $\frac{dN}{dt} = n - \lambda N$

Because the population N is simultaneously increasing at rate n and decreasing due to decay at rate λN.

$$\Rightarrow \quad \int_{N_0}^{N} \frac{dN}{n - \lambda N} = \int_0^t dt$$

$$\Rightarrow \quad \frac{1}{\lambda}\ell n\left(\frac{n - \lambda N_0}{n - \lambda N}\right) = t$$

$$\Rightarrow \quad N = \frac{n}{\lambda} + \left(N_0 - \frac{n}{\lambda}\right)e^{-\lambda t}$$

OBJECTIVE TRICK

At $t = 0$, $N = N_0$ which is satisfied by **(C)** only.

Hence, the correct answer is (C).

130. Since $\frac{N}{N_0} = \frac{1}{2^n}$, where $n = \frac{t}{T_{1/2}} = \frac{1}{2}$

$$\Rightarrow \quad \frac{N}{10000} = \frac{1}{(2)^{\frac{10}{20}}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad N = 7070$$

Hence, the correct answer is (A).

131. From momentum conservation,

$$p_{\text{photon}} = p_{\text{nucleus}}$$

$$\Rightarrow \quad \frac{E}{c} = \sqrt{2mK}$$

$$\Rightarrow \quad K = \frac{E^2}{2mc^2}$$

$$\Rightarrow \quad K = \frac{(7 \times 1.6 \times 10^{-13})^2}{2(24)(1.67 \times 10^{-27})(3 \times 10^8)^2}(1.6 \times 10^{-16})\text{keV}$$

$$\Rightarrow \quad K = 1.1 \text{ keV}$$

Hence, the correct answer is (B).

132. Mass of product should be less than the mass of reactant because energy is released in a fission reaction.
Hence, the correct answer is (A).

133. $R_1 = R_2$

$$\Rightarrow \quad R_{01}e^{-\lambda_1 t} = R_{02}e^{-\lambda_2 t}$$

$$\Rightarrow \quad \lambda_1 N_0 e^{-\lambda_1 t} = \lambda_2 N_0 e^{-\lambda_2 t}$$

where, $\lambda_1 = \frac{\ln 2}{t_1} = \frac{0.693}{t_1}$

and $\lambda_2 = \frac{0.693}{t_2}$

Substituting these values of λ_1 and λ_2 in Equation (1), we get

$$t = \frac{t_1 t_2}{0.693(t_2 - t_1)}\ln\left(\frac{t_2}{t_1}\right)$$

Hence, the correct answer is (A).

134. $R_1 = \lambda N_1$ and $R_2 = \lambda N_2$ (λ is same for a given sample)
As $N_2 < N_1$
Number of atoms disintegrated in time $(t_2 - t_1)$ is

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$$N_1-N_2=\frac{R_1}{\lambda}-\frac{R_2}{\lambda}=\left(\frac{R_1-R_2}{\lambda}\right)$$

$$N_1-N_2=\frac{(R_1-R_2)T}{0.693}\propto(R_1-R_2)T$$

Hence, the correct answer is (D).

138. Expected mass of Cu must be less than that of zinc. So it is unstable and radioactive, decaying to Zn through β decay.

Hence, the correct answer is (D).

139. f_1 is the fraction decayed in one half life $T_{1/2}$ and f_2 is the fraction decayed in one mean life (T).

$$T_{1/2}=\frac{0.693}{\lambda}=0.693T$$

$\Rightarrow \quad T>T_{1/2}$

Further, since the fraction decayed is $(1-e^{-\lambda t})$

So, $f_1=1-e^{-\lambda T_{1/2}}=(1-e^{-0.693})=\frac{1}{2}$ and

$$f_2=1-e^{-\lambda T}=1-e^{-1}=0.632$$

$\Rightarrow \quad f_2>f_1$

However, if we had been asked about the fraction of sample undecayed, then we must give our answer in the light of formula $\frac{N}{N_0}=e^{-\lambda t}$ where N is number of undecayed nuclei in sample at time t.

Hence, the correct answer is (B).

142. Q-value = Final binding energy

Initial binding energy is

$$(BE)_{\text{initial}}=E_2N_2+E_3N_3-E_1N_1$$

Hence, the correct answer is (B).

143. $t=\frac{0.693}{\lambda}$

$T=\frac{1}{\lambda}$

$\Rightarrow \quad t=0.693\text{ T}$

$\Rightarrow \quad T>t$

Hence, the correct answer is (C).

145. $80\% \xrightarrow{T} 40\% \xrightarrow{T} 20\%$

$\Rightarrow \quad t=2T=40$ minutes

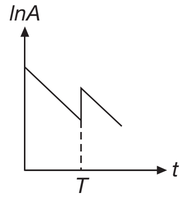
Hence, the correct answer is (A).

146. $A=A_0e^{-\lambda t}$

Taking log both sides, we get

$$\ell nA=\ell nA_0-\lambda t$$

Hence, the correct answer is (B).



147. Activity of a radioactive substance $R=\lambda N$

$\Rightarrow \quad \lambda=\frac{R}{N}$

Since, $R=N_2$ particles per second

and $N=N_1$

$\Rightarrow \quad \lambda=\frac{N_2}{N_1}$

Hence, the correct answer is (A).

148. ${}_{92}U^{238}\xrightarrow{\alpha}{}_{90}X^{234}\xrightarrow{\beta^-}{}_{91}Y^{234}$

$\Rightarrow \quad Z=91,\ A=234$

Hence, the correct answer is (D).

149. Radioactivity is a probable phenomenon that is not influenced by physical conditions like temperature, pressure, humidity, electronic configuration etc.

Hence, the correct answer is (D).

150. Since, $\tau=\frac{\tau_1\tau_2}{\tau_1+\tau_2}=\frac{1620\times 660}{2280}=469$

$\Rightarrow \quad T_{1/2}=\frac{0.693}{\lambda}=0.693\tau$

$\Rightarrow \quad N=\frac{N_0}{4}=\frac{N_0}{2^{t/T_{1/2}}}$

$\Rightarrow \quad t=2T_{1/2}=2(0.693)(469)$

$\Rightarrow \quad t=650$ year

Hence, the correct answer is (C).

151. During fusion, binding energy of daughter nucleus is always greater than the total binding energy of the parent nuclei. The difference of binding energies is the energy released. Hence,

$$Q=E_2-2E_1$$

Hence, the correct answer is (C).

152. Train moving with uniform speed implies that $F_{ext}=0$

$\Rightarrow \quad x' = x = (v_{234} + v_\alpha)t$

Hence, the correct answer is (C).

153. Maximum number of nuclei will be present when

Rate of decay = Rate of formation

$\Rightarrow \quad \lambda N = \alpha$

$\Rightarrow \quad N = \dfrac{\alpha}{\lambda}$

Hence, the correct answer is (A).

154. At time $t = 0$, A gets converted to B which has a higher activity. Therefore, activity increases.

Hence, the correct answer is (B).

155. $P(\text{survival}) = \dfrac{N(t)}{N_0} = \dfrac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$

For, $t = \dfrac{1}{\lambda}$, we have

$P(\text{survival}) = \dfrac{1}{e}$

Hence, the correct answer is (B).

156. Energy released is given by

$$\Delta E = \begin{pmatrix}\text{Total Binding} \\ \text{Energy of } {}_2He^4\end{pmatrix} - 2\begin{pmatrix}\text{Total Binding} \\ \text{Energy of } {}_1H^2\end{pmatrix}$$

$\Rightarrow \quad \Delta E = (4)(7) - 2(1.1)(2) = 23.6 \text{ MeV}$

Hence, the correct answer is (D).

157. Let $\lambda_A = \lambda$ and $\lambda_B = 2\lambda$

Initially rate of disintegration of A is λN_0 and that of B is $(2\lambda)N_0$.

After one half-life of A, rate of disintegration of A will becomes $\dfrac{\lambda N_0}{2}$ and that of B would also be $\dfrac{\lambda N_0}{2}$, because

$$(\text{Half-life of B}) = \frac{1}{2}(\text{Half-life of A})$$

So, after one half-life of A or two half-lives of B, we have

$$\left(-\frac{dN}{dt}\right)_A = \left(-\frac{dN}{dt}\right)_B$$

$\Rightarrow \quad n = 1$

Hence, the correct answer is (D).

158. On the last day we have 100% decay i.e. on the ninth day 50% decay must be there or 50% must be left.

Hence, the correct answer is (D).

160. In a beta decay process, mass number cannot increase.

Hence, the correct answer is (C).

164. The mean free path in vacuum is the distance travelled by a neutron in its lifetime i.e., from generation to decay which is about 10^3 s. Since, its energy i.e. 5 eV is much less than its rest energy 940 MeV so, non-relativistic approximation can be used and hence its velocity is given by using the formula for kinetic energy i.e.

$$E_K = \frac{1}{2}mv^2$$

$\Rightarrow \quad v = \sqrt{\dfrac{2E_K}{m}} = c\sqrt{\dfrac{2E_K}{mc^2}} = \sqrt{\dfrac{2 \times 5 \times 10^{-6}}{940}} \times 3 \times 10^8$

$\Rightarrow \quad v = 10^4 \text{ ms}^{-1}$

$\Rightarrow \quad x = vt = 10^4 \text{ km}$

Hence, the correct answer is (D).

165. $N = N_0 e^{-\lambda t}$

For $t = T_{\text{av}} = \dfrac{1}{\lambda}$, we have

$N = \dfrac{N_0}{e}$

Hence, the correct answer is (B).

167. $\dfrac{m}{t} = 4 \times 10^9 \text{ kgs}^{-1}$

$E = mc^2$

$\Rightarrow \quad \dfrac{E}{t} = \left(\dfrac{m}{t}\right)c^2$

$\Rightarrow \quad \dfrac{E}{t} = 4 \times 10^9 \times 9 \times 10^{16}$

$\Rightarrow \quad \dfrac{E}{t} = 3.6 \times 10^{26} \text{ Js}^{-1}$

$\Rightarrow \quad \dfrac{E}{t} = 3.6 \times 10^{26} \text{ W}$.

Hence, the correct answer is (A).

168. Activity $= \left|\dfrac{dN}{dt}\right| = \lambda N = \lambda\left(\dfrac{m}{M}\right)L$

Hence, the correct answer is (B).

169. After time t, sample left undecayed is 90% i.e. $0.9N_0$
After time $2t$, sample left undecayed is

$$N = N_0(0.9)^2$$

$$\Rightarrow \quad N = 0.81N_0$$

So, 81% of initial value is left
Hence percentage of the initial sample decayed is $(100-81) = 19\%$

Hence, the correct answer is (B).

170. $$92 \xrightarrow{\alpha} 90 \xrightarrow{2\beta^-} 92 \xrightarrow{5\alpha} 82 \xrightarrow{2\beta^-} 84 \xrightarrow{\alpha}$$

$$82 \xrightarrow{2\beta^+} 80 \xrightarrow{\alpha} 78$$

Hence, the correct answer is (C).

171. Since $\dfrac{A}{A_0} = \dfrac{1}{128} = \dfrac{1}{2^n}$

$$\Rightarrow \quad n = 7$$

Since $t = nT_{1/2}$

$$\Rightarrow \quad t = 7 \times 1 = 7 \text{ hr}$$

Hence, the correct answer is (B).

173. $${}_1H^2 + {}_1H^2 \longrightarrow {}_2He^4 + Q$$

$$\Rightarrow \quad \Delta m = m\left({}_2He^4\right) - 2m\left({}_1H^2\right)$$

$$\Rightarrow \quad \Delta m = 4.0024 - 2(2.0141)$$

$$\Rightarrow \quad \Delta m = -0.0258 \text{ u}$$

Since, $Q = c^2\Delta m$

$$\Rightarrow \quad Q = (0.0258)(931.5) \text{ MeV}$$

$$\Rightarrow \quad Q \simeq 24 \text{ MeV}$$

Hence, the correct answer is (C).

174. Since, $\dfrac{dN}{dt} = \lambda N$

$$\Rightarrow \quad \lambda dt = \frac{dN}{N}$$

Hence, the correct answer is (A).

175. Let N_0 be the number of atoms of X at time $t = 0$. Then at $t = 4$ h (two half lives), we have

$$N_x = \frac{N_0}{4} \text{ and } N_y = \frac{3N_0}{4}$$

$$\Rightarrow \quad \frac{N_x}{N_y} = \frac{1}{3}$$

and at $t = 6h$ (three half-lives), we have

$$N_x = \frac{N_0}{8} \text{ and } N_y = \frac{7N_0}{8}$$

$$\Rightarrow \quad \frac{N_x}{N_y} = \frac{1}{7}$$

The given ratio $\dfrac{1}{4}$ lies between $\dfrac{1}{3}$ and $\dfrac{1}{7}$

Therefore, t lies between 4 h and 6 h

Hence, the correct answer is (C).

176. $$\frac{N}{N_0} = \frac{1}{8} = \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{\frac{6}{T}}$$

$$\Rightarrow \quad T = \frac{6}{3} = 2 \text{ days}$$

$$\frac{N'}{N_0} = \left(\frac{1}{2}\right)^{\frac{10}{2}} = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

$$\Rightarrow \quad \text{Fraction Decayed } = 1 - \frac{N'}{N_0} = 1 - \frac{1}{32} = \frac{31}{32}$$

Hence, the correct answer is (C).

177. $${}^A_Z X \xrightarrow{\alpha} {}^{A-4}_{Z-2}Y \xrightarrow{2\beta^-} {}^{A-4}_{Z}X$$

Hence, the correct answer is (B).

178. In K-capture process, the nucleus captures the K shell electron. So,

$${}^A_Z X + {}_{-1}e^0 \rightarrow {}^A_{Z-1}Y, \text{ where } {}_{-1}X^0 \text{ is } {}_{-1}e^0$$

Hence, the correct answer is (A).

179. A and B can be isotopes if number of β-decays is two times the number of α-decays.

Hence, the correct answer is (B).

180. $N = N_0 e^{-\lambda t}$

Number of nuclei decayed $= (N_0 - N)$

Percentage of initial nuclei decayed is

$$\%\text{age} = \frac{N_0 - N}{N_0} \times 100 = \left(1 - \frac{N}{N_0}\right) \times 100$$

$$\%\text{age} = \left(1 - e^{-\lambda t}\right) \times 100 = (1 - 0.37) \times 100$$

$$\%\text{age} = 63\%$$

Hence, the correct answer is (C).

181. In $2s$ only 90% nuclei are left behind. So, in the next $2s$ 90% of 900 i.e., 810 nuclei will be left.

Hence, the correct answer is (D).

182. Let number of α-decays be x and number of β-decays be y. Then

$$92 - 2x + y = 85$$

$$\Rightarrow \quad 2x - y = 7 \qquad \ldots(1)$$

and $238 - 4x = 210$

$$\Rightarrow \quad x = 7$$

Substituting this value in equation (1), we get

$$y = 7$$

Hence, the correct answer is (C).

Multiple Correct Choice Type Questions

1. We have 6.25% left undecayed

$$\Rightarrow \quad \frac{N}{N_0} = \frac{6.25}{100} = \frac{1}{16}$$

$$\Rightarrow \quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \frac{1}{16}$$

$$\Rightarrow \quad n = \frac{t}{T_{1/2}} = 4$$

$$\Rightarrow \quad T_{1/2} = \frac{t}{4}$$

$$\Rightarrow \quad T_{1/2} = \frac{4}{4} = 1\text{ hr}$$

Since $T_{1/2} = \frac{\ln 2}{\lambda}$ and mean life is $T_m = \frac{1}{\lambda}$

After further 4 hours, the amount left over would be

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^8 = 0.39\%$$

Hence, (A), (B), (C) and (D) are correct.

2. By Law of Conservation of energy, we get

$$m_x c^2 + K_x = m_Y c^2 + K_Y + m_y c^2 + K_y$$

$$Q = \left(m_x c^2 - m_Y c^2 - m_y c^2\right) = K_y + K_Y - K_X$$

Since, X is at rest, so the energy released Q is

$$Q = \left(m_x c^2 - m_Y c^2 - m_y c^2\right) = K_Y + K_y$$

The Q value i.e., energy released is the rest energy of X minus rest energy of Y and y.
When binding energy of products is more than binding energy of parent nucleus, then energy is released.

Hence, (B) and (C) are correct.

4. $\frac{N}{N_0}$ is the fraction of nuclei that will not decay

$\left(1 - \frac{N}{N_0}\right)$ is the fraction of nuclei that will decay

$$\Rightarrow \quad 1 - \frac{N}{N_0} = 1 - e^{-\lambda t} = \left(\begin{array}{l}\text{Probability that a}\\ \text{nucleus will decay}\end{array}\right)$$

Also, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$, where $n = \frac{t}{T_{1/2}}$

$$\Rightarrow \quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

$$\Rightarrow \quad 1 - \frac{N}{N_0} = 1 - \frac{1}{16} = \frac{15}{16}$$

Probability that a nucleus will decay is

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

So, fraction of nuclei that will remain after two half lives is $\frac{1}{4}$ or 25%

Hence, (A), (B) and (D) are correct.

6. For ${}^{20}_{10}\text{Ne}$ nucleus to exist

$$M_1 < 10\, m_p + 10\, m_n$$

$$M_1 < 10\left(m_p + m_n\right)$$

Further since ${}^{40}_{20}\text{Ca}$ has 20 more nucleons and thus it requires more energy to hold all of them together and hence $M_1 \neq 2M_2$ instead $M_1 > 2M_2$ because some additional mass defect must occur to provide an additional B.E. to ${}^{40}_{20}\text{Ca}$ nucleus.

Hence, (C) and (D) are correct.

7. $R = R_0 A^{\frac{1}{3}}$

$$\Rightarrow \quad R \propto A^{\frac{1}{3}}$$

Hence, (B) and (C) are correct.

9. $N = N_0 e^{-\lambda t}$

$N =$ Number of undecayed nuclei in the sample at time t.

Total number of undecayed nuclei equals $\left(N_0 - N\right)$

$$\Rightarrow \quad \left(N_0 - N\right) = N_0\left(1 - e^{-\lambda t}\right)$$

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which is growing exponentially with time.

Activity $R=-\lambda N=\frac{dN}{dt}$

Hence, (A), (B) and (D) are correct.

11. **(B)** $R \propto \sqrt[3]{A}$

$\Rightarrow$ Volume $\propto A$

$\Rightarrow$ Mass $\propto A$

$\Rightarrow$ Density is independent of mass number A

(D) $v \propto \frac{1}{n}$

$r \propto n^2$

$\Rightarrow \frac{v^2}{r} \propto \frac{1}{n^4}$

Hence, (B) and (C) are correct.

13. α **decay:** ${}_{Z}^{A}X \rightarrow {}_{Z-2}^{A-4}\gamma + {}_{2}^{4}\text{He}$

β^+ **decay:** ${}_{Z}^{A}X \rightarrow {}_{Z-1}^{A}\gamma + \beta^+ + v$

β^- **decay:** ${}_{Z}^{A}X \rightarrow {}_{Z+1}^{A}\gamma + \beta^- + \bar{v}$

In γ decay process, only the quantum state of nucleons change

Hence, (A) and (B) are correct.

15. Due to the emission of an α-particle, atomic number decreases by 2 and due to the emission of two beta particles atomic number increases by 2. Hence net atomic number remains unchanged.

Hence, (A), (B) and (C) are correct.

16. Since Bohr's radius is given by

$$r=\frac{n^2h^2\varepsilon_0}{\pi me^2Z}$$

$$\Rightarrow r \propto \frac{1}{m}$$

$$\Rightarrow \frac{r}{r_0}=\frac{m_e}{m_\mu}$$

$$\Rightarrow r=\frac{r_0}{212}=\frac{0.53\text{ Å}}{212}=250\text{ fm}$$

Since, $F \propto m$

$$\Rightarrow E=E_0\left(\frac{m_\mu}{m_e}\right)$$

$$\Rightarrow E=(13.6\text{ eV})(212)$$

$$\Rightarrow E=2883\text{ eV}$$

Angular momentum is given as

$$L=mvr=\frac{nh}{2\pi}$$

For ground state $n=1$, so we have

$$L=\frac{h}{2\pi}$$

Hence, (A), (B) and (C) are correct.

18. $y=\lambda x=\left(\frac{\ln 2}{T}\right)\cdot x$

$$\Rightarrow \frac{x}{y}=\frac{1}{\lambda}=\text{constant}$$

$$\Rightarrow \frac{x}{y}=\frac{T}{\ln 2}$$

$$\Rightarrow \frac{x}{y}>T \qquad \{\text{as } \ln 2=0.693\}$$

Further, $xy=x(\lambda x)=\lambda x^2$

After one half life, x remains half. Hence, x^2 remains $\frac{1}{4}$th .

Hence, (A), (B) and (D) are correct.

19. ${}_{7}^{14}\text{N}+{}_{0}^{1}n \longrightarrow {}_{3}^{7}\text{Li}$

Difference in mass number $=8$

Difference in charge number $=4$

(A) ${}_{2}^{14}\text{N}+{}_{0}^{1}n \longrightarrow {}_{3}^{7}\text{Li}+4\,{}_{1}^{1}\text{H}+4\,{}_{0}^{1}n$

This reaction is balanced hence (A) is correct.

(B) ${}_{7}^{14}\text{N}+{}_{0}^{1}n \longrightarrow {}_{3}^{7}\text{Li}+5\,{}_{1}^{1}\text{H}+1\,{}_{-1}^{0}e$

This reaction is not balanced

(C) ${}_{7}^{14}\text{N}+{}_{0}^{1}n \longrightarrow 2\,{}_{2}^{4}\text{He}+2\gamma+{}_{3}^{7}\text{Li}$

This reaction is balanced

(D) ${}_{7}^{14}\text{N}+{}_{0}^{1}n \longrightarrow {}_{3}^{7}\text{Li}+{}_{2}^{4}\text{He}+4\,{}_{1}^{1}\text{H}+2\,{}_{-1}^{0}\beta$

This reaction is balanced.

Hence, (A), (C) and (D) are correct.

21. $Q=\left[M(\text{Po}^{210})-M(\text{Pb}^{206})-M(\alpha)\right]931\text{ MeV}$ and $\lambda=\frac{0.693}{T_{1/2}}$

Hence, (A), (B) and (C) are correct.

23. In a beta decay process, a neutron decays to a proton

$${}_0^1n \rightarrow p^+ + e^-$$

Spin of p^+, e^- and ${}_0^1n$ is $\frac{1}{2}$

Therefore spin (R.H.S.) is either 0 or 1 whereas spin (L.H.S.) is $\frac{1}{2}$

Since, Spin (R.H.S.) $\neq$ Spin (L.H.S.)

Hence spin angular momentum is not conserved.

However, total energy, mass number and charge is conserved in the process.

Hence, (A), (B) and (C) are correct.

25. In ground frame

Kinetic energy, $K_x = |-Q|\left[1+\frac{m_{\text{projectile}}}{m_{\text{target}}}\right]$

$\Rightarrow \quad K_x > Q$

In centre of mass frame

Kinetic energy, $K_x = Q$

In nuclear reactions, linear momentum is conserved.

Hence, (A), (B) and (C) are correct.

26. In fusion two or more lighter nuclei combine to make a comparatively heavier nucleus.

In fission, a heavy nucleus breaks into two or more comparatively lighter nuclei.

Further, energy will be released in a nuclear process if total binding energy increases.

Hence, (B) and (D) are correct.

27. At $t = 4T$

Number of half lives of first is $n_1 = 4$ and number of half lives of second is $n_2 = 2$

Since $N = N_0\left(\frac{1}{2}\right)^n$

$$\Rightarrow \quad x = \frac{N_1}{N_2} = \frac{N_0\left(\frac{1}{2}\right)^4}{N_0\left(\frac{1}{2}\right)^2} = \frac{1}{4}$$

Since $R = R_0\left(\frac{1}{2}\right)^n = \lambda N_0\left(\frac{1}{2}\right)^n$

$$\Rightarrow \quad y = \frac{R_1}{R_2} = \frac{\lambda_1 N_0\left(\frac{1}{2}\right)^4}{\lambda_2 N_0\left(\frac{1}{2}\right)^2}$$

$$\Rightarrow \quad y = \frac{\lambda_1}{4\lambda_2} = \frac{T_2}{4T_1}$$

$$\Rightarrow \quad y = \frac{2T}{4T} = \frac{1}{2}$$

Hence, (B) and (C) are correct.

Reasoning Based Questions

1. $(1\text{ amu})c^2 = 931.48$ MeV

$\Rightarrow \quad 1\text{ amu} = 931.5\ \text{MeV}/c^2$

Hence, the correct answer is (D).

3. By the emission of one α-particle atomic number decreases by 2 and mass number by 4. But by the emission of one β-particle, atomic number increases by 1 and mass number remains unchanged.

Hence, the correct answer is (B).

5. α-particles are heaviest. Hence, their ionising power is maximum.

Hence, the correct answer is (B).

6. In moving from lower energy state to higher energy state, electromagnetic waves are absorbed.

Hence, the correct answer is (C).

8. Antineutrino is also produced during β-decay.

Hence, the correct answer is (D).

10. After 200 days the number of un-decayed nuclei in the sample will be $\frac{1}{4}$ the initial number of un-decayed nuclei in the sample initially.

Hence, the correct answer is (D).

12. In binding energy per nucleon versus mass number graph, the binding energy per nucleon of daughter nuclei should increase (for release of energy) or, the daughter nuclei should lie towards the peak of the graph.

Hence, (A) and (B) are correct.

15. A very large amount of energy is involved in any nuclear process, which cannot be increased or decreased by pressure or temperature.

Hence, the correct answer is (B).

17. Total binding energy per nucleon is more important for stability.

Hence, the correct answer is (D).

19. Some lighter nuclei are also radioactive.

Hence, the correct answer is (D).

Linked Comprehension Type Questions

1. $$\frac{dN_X}{dt} = \alpha - \lambda N_X$$

$$\Rightarrow \quad N_X = \frac{1}{\lambda}\left[\alpha - (\alpha - \lambda N_0)e^{-\lambda t}\right]$$

At $t = T_{1/2} = \dfrac{\log_e 2}{\lambda}$, we have

$$N_x = \frac{1}{\lambda}\left(\alpha - (\alpha - \lambda N_0)e^{-\lambda\left(\frac{\log_e 2}{\lambda}\right)}\right)$$

$$\Rightarrow \quad N_x = \frac{1}{\lambda}\left[\alpha - (\alpha - \lambda N_0)e^{-\log_e 2}\right]$$

Since, $e^{\log_e x} = x$

$$\Rightarrow \quad e^{-\log_e 2} = e^{\log_e\left(\frac{1}{2}\right)} = \frac{1}{2}$$

$$\Rightarrow \quad N_x = \frac{1}{\lambda}\left[\alpha - \left(\frac{\alpha - \lambda N_0}{2}\right)\right]$$

$$\Rightarrow \quad N_x = \frac{\alpha + \lambda N_0}{2\lambda}$$

Hence, the correct answer is (D).

2. Further, due to the decay of X to the stable nucleus Y, we have

$$\frac{dN_Y}{dt} = \lambda N_X$$

Since, $N_X = \dfrac{1}{\lambda}\left[\alpha - (\alpha - \lambda N_0)e^{-\lambda t}\right]$

Substituting the value of N_x and solving, we get

$$\Rightarrow \quad N_Y = \alpha t + \left(\frac{\alpha - \lambda N_0}{\lambda}\right)e^{-\lambda t} - \left(\frac{\alpha - \lambda N_0}{\lambda}\right)$$

Hence, the correct answer is (B).

3. Since, $N_Y = \alpha t + \left(\dfrac{\alpha - \lambda N_0}{\lambda}\right)e^{-\lambda t} - \left(\dfrac{\alpha - \lambda N_0}{\lambda}\right)$

Substituting the value of $T_{1/2} = \dfrac{\log_e(2)}{\lambda}$ in this equation, we get

$$N_Y = \frac{\alpha}{\lambda}\log_e(2) + \left(\frac{\alpha - \lambda N_0}{2\lambda}\right) - \left(\frac{\alpha - \lambda N_0}{\lambda}\right)$$

$$\Rightarrow \quad N_Y = \frac{\alpha}{\lambda}\log_e(2) - \frac{1}{2}\left(\frac{\alpha - \lambda N_0}{\lambda}\right)$$

Hence, the correct answer is (C).

4. $$m\left({}_1^2\text{H}\right) + m\left({}_2^4\text{He}\right) = 2.014102 + 4.002603$$

$$\Rightarrow \quad m\left({}_1^2\text{H}\right) + m\left({}_2^4\text{He}\right) = 6.016705 \text{ u}$$

Since, $m\left({}_3^6\text{Li}\right) = 6.015123$ u

$$\Rightarrow \quad m_1 + m_2 > M$$

So, (A) is incorrect.

$$m\left({}_1^1\text{H}\right) + m\left({}_{83}^{209}\text{Bi}\right) = 1.007825 + 208.980388$$

$$m\left({}_1^1\text{H}\right) + m\left({}_{83}^{209}\text{Bi}\right) = 209.988213 \text{ u}$$

Since, $m\left({}_{84}^{210}\text{Po}\right) = 209.982876$ u

$$\Rightarrow \quad m_1 + m_2 > M$$

So, B is incorrect

$$m\left({}_1^2\text{H}\right) + m\left({}_2^4\text{He}\right) = 2.014102 + 4.002603$$

$$\Rightarrow \quad m\left({}_1^2\text{H}\right) + m\left({}_2^4\text{He}\right) = 6.016705 \text{ u}$$

Since, ${}_3^6\text{Li} = 6.015123$ u

$$\Rightarrow \quad (m_3 + m_4) > M'$$

So, (C) is correct and hence deuteron and alpha particle can go complete fusion.

$$m\left({}_{30}^{70}\text{Zn}\right) + m\left({}_{34}^{82}\text{Se}\right) = 69.925325 + 81.916709$$

$$\Rightarrow \quad m\left({}_{30}^{70}\text{Zn}\right) + m\left({}_{34}^{82}\text{Se}\right) = 151.842034 \text{ u}$$

Since, ${}_{64}^{152}\text{Gd} = 151.919803$ u

$$\Rightarrow \quad m_3 + m_4 < M'$$

So, (D) is incorrect.

Hence, the correct answer is (C).

5. $${}_{84}^{210}\text{Po} = {}_{82}^{206}\text{Pb} + {}_2^4\text{He} + \Delta E$$

$$m\left({}_{82}^{206}\text{Pb}\right) = 205.974455 \text{ u}$$

$$m\left({}_2^4\text{He}\right) = 4.002603 \text{ u}$$

$$\Rightarrow \quad m\left({}_{82}^{206}\text{Pb}\right) + m\left({}_2^4\text{He}\right) = 209.977058 \text{ u}$$

Now, $\Delta m = 209.977058 - 209.982876$

$\Rightarrow \Delta m = 0.005818$ u

$\Rightarrow Q = \Delta E = 0.005818 \times 931.5$

$\Rightarrow Q = 5.419467$ MeV $= 5419.467$ keV

$\Rightarrow Q = 5419.5$ keV

By Law of Conservation of Momentum, we have

$$0 = p_\alpha - p_{\text{lead}}$$

$\Rightarrow p_\alpha = p_{\text{lead}}$

$\Rightarrow \sqrt{2m_\alpha E_\alpha} = \sqrt{2m_{Pb}E_{Pb}}$

$\Rightarrow 4E_\alpha = 206E_{Pb}$

$\Rightarrow E_\alpha = \dfrac{103}{2}E_{Pb}$

Now, since $E_\alpha = \left(\dfrac{m_{Pb}}{m_{Pb} + m_\alpha}\right)Q$

$\Rightarrow E_\alpha = \left(\dfrac{206}{206+4}\right)Q$

$\Rightarrow E_\alpha = \dfrac{103}{105}(5.422) = 5319$ MeV

Hence, the correct answer is (A).

6. Since, $\dfrac{dN}{dt} = P - \lambda N$

$\Rightarrow \dfrac{dN}{P - \lambda N} = dt$, where $\lambda = \dfrac{\log_e 2}{T}$

$\Rightarrow N = \dfrac{PT}{\log_e 2}\left(1 - e^{-\frac{t\log_e 2}{T}}\right)$

Hence, the correct answer is (A).

7. Since, $A = \lambda N = P\left(1 - e^{-\frac{t\log_e 2}{T}}\right)$

$\Rightarrow$ Rate of energy release is $AE_0 = PE_0\left(1 - e^{-\frac{t\log_e 2}{T}}\right)$

Hence, the correct answer is (B).

8. Energy released upto time t is

$$E_{\text{released}} = (Pt - N)E_0$$

Hence, the correct answer is (A).

9. $Z_1 - (2)(2) + (3)(1) = Z_2 - (2)(1) + (5)(1) = Z_c$

$\Rightarrow Z_1 - Z_2 = 4$

$A_1 - (4)(2) = A_2 - (1)(4) = A_c$

$\Rightarrow A_1 - A_2 = 4$

Hence, the correct answer is (B).

10. For A

$$4N_0 \xrightarrow{1\text{ min}} 2N_0 \xrightarrow{1\text{ min}} N_0 \xrightarrow{1\text{ min}} \frac{N_0}{2} \xrightarrow{1\text{ min}} \frac{N_0}{4}$$

For B

$$N_0 \xrightarrow{2\text{ min}} \frac{N_0}{2} \xrightarrow{2\text{ min}} \frac{N_0}{4}$$

After 4 minute, we have

$$N_A = N_B = \frac{N_0}{4}$$

$\Rightarrow N_C = (4N_C + N_0) - \left(\dfrac{N_0}{4} + \dfrac{N_0}{4}\right) = \dfrac{9N_0}{2}$

Hence, the correct answer is (C).

11. Given $R_A = R_B$

$\Rightarrow \lambda_A N_A = \lambda_B N_B$

$\Rightarrow \left(\dfrac{\log_e 2}{T_A}\right)\left(4N_0 e^{-\lambda_A t_0}\right) = N_0\left(\dfrac{\log_e 2}{T_B}\right)e^{-\lambda_B t_0}$

$\Rightarrow t_0 = 6$ min

Hence, the correct answer is (B).

12. If $x + X \longrightarrow Y + y$

For the above nuclear reaction, threshold energy is given as

$$E_{th} = -Q\left(1 + \frac{m_x}{m_X}\right)$$

The Q-value of reaction is given by

$$Q = (1.007825 + 3.016049 - 2\times 2.014102)\times 931.5$$

$\Rightarrow Q = -4.033$ MeV

When protons are incident on ${}_1^3\text{H}$, then

$$x = {}_1^1\text{H} \text{ and } X = {}_1^3\text{H}$$

$$K_{th} = 4.033 \text{ MeV}\left(1 + \frac{1.007825}{3.016049}\right)$$

$\Rightarrow K_{th} = 5.381$ MeV

Hence, the correct answer is (A).

13. When ${}_1^3H$ is incident on protons

$$x = {}_1^3H \text{ and } X = {}_1^1H$$

$\Rightarrow K_{th} = (4.033 \text{ MeV})\left(1 + \dfrac{3.016049}{1.007825}\right)$

$\Rightarrow K_{th} = 16.10$ MeV

Hence, the correct answer is (C).

14. From above calculations we observe that less energy is required for a nuclear reaction, which a light particle is incident on a heavy target than if a heavy particle is incident on a light target.
Hence, the correct answer is (C).

16. B.E. per nucleon for intermediate nucleus is more than lighter or heavier nuclei.
Hence, the correct answer is (C).

17. $V_{Nu} = \frac{4}{3}\pi r_{Nu}^3$

Since, $r_{Nu} = r_0 A^{\frac{1}{3}}$

$\Rightarrow \quad V_{Nu} = \left(\frac{4}{3}\pi r_0^3\right)A$

$\Rightarrow \quad V_{Nu} \propto A$

Hence, the correct answer is (D).

18. Mass defect in the reaction is

$$\Delta m = (2(2.014102) - 3.016049 - 1.007825)\text{ u}$$

$\Rightarrow \quad \Delta m = 0.0043\text{ u}$

So, energy released in the reaction is

$$\Delta E = (\Delta m)(931.5)\text{ MeV}$$

$\Rightarrow \quad \Delta E = (0.0043)(931.5)\text{ MeV}$

$\Rightarrow \quad \Delta E \approx 4\text{ MeV}$

Hence, the correct answer is (C).

19. Let N number of fusion reactions be required for the purpose. Then

$$N(4 \times 1.6 \times 10^{-13}) = 10^3 \times 3600$$

$\Rightarrow \quad N = 5.625 \times 10^{18}$

Hence, the correct answer is (B).

20. In one fusion reaction two ${}_1^2\text{H}$ nuclei are used. Hence total number of ${}_1^2\text{H}$ nuclei are

$$N' = 2N = 1.125 \times 10^{19}$$

So, mass m required in kg is

$$m = \left(\frac{1.125 \times 10^{19}}{6.02 \times 10^{26}}\right)(2)\text{ kg}$$

$\Rightarrow \quad m = 3.7 \times 10^{-8}\text{ kg}$

Hence, the correct answer is (D).

21. $Q = (N_2E_2 + N_3E_3) - N_1E_1$

Hence, the correct answer is (B).

24. Since, $1\text{ kW} \equiv 3.1 \times 10^{13}\ \frac{\text{fission}}{\text{sec}}$

$\Rightarrow \quad 1.6 \times 10^3\text{ kW} \equiv 1.6 \times 10^3 \times 3.1 \times 10^{13}$

$= 5 \times 10^{16}$ fission per second

Hence, the correct answer is (D).

26. The reaction for the second stage is given by

$${}_1T^3 + {}_1D^2 \longrightarrow {}_2\text{He}^4 + {}_0n^1 + E_2$$

$\Rightarrow \quad \Delta m = (3.016049 + 2.014102) - (4.002603 + 1.008665)$

$\Rightarrow \quad \Delta m = 0.01888\text{ u}$

$\Rightarrow \quad E_2 = 0.01888 \times 931 = 17.587\text{ MeV}$

Hence, the correct answer is (B).

27. Total energy released $E = E_1 + E_2 = 4.033 + 17.587$

$\Rightarrow \quad E = 21.62\text{ MeV}$

So, energy released/deuteron is $\frac{21.62}{3} = 7.207\text{ MeV}$

Hence, the correct answer is (D).

28. $\left(\begin{array}{c}\text{\% of Rest Mass of}\\ \text{deuterium released}\end{array}\right) = \frac{7.207 \times 100}{2.014102 \times 931.4} = 0.384\%$

Hence, the correct answer is (C).

29. $T_M = \frac{1}{\lambda}$ and $T_H = \frac{0.693}{\lambda}$

$\Rightarrow \quad T_M > T_H$

Hence, the correct answer is (A).

30. $A = \frac{dN}{dt} = \lambda N$

$\Rightarrow \quad n = \lambda N = \frac{0.693}{T}\text{ N}$

$\Rightarrow \quad T = \frac{0.693}{n}$

Hence, the correct answer is (C).

31. $R_1 = R_0 e^{-\lambda t}$ and $R_2 = R_0 e^{-\lambda_2 t}$

$\Rightarrow \quad \frac{R_1}{R_2} = e^{\lambda(t_2 - t_1)}$

$\Rightarrow \quad R_2 = R_1 e^{-\lambda(t_2 - t_1)}$

Hence, the correct answer is (C).

32. $N_0 = \frac{mN_A}{M}$

Hence, the correct answer is (C).

33. The number of undecayed nuclei after time t is $N = N_0 e^{-\lambda t}$

So, the number of decayed nuclei is

$$N' = N_0 - N = N_0\left(1 - e^{-\lambda t}\right)$$

Hence, the correct answer is (C).

34. Activity $= A = -\dfrac{dN}{dt}$

$$\Rightarrow \quad A = A_0 e^{-\lambda t} = \lambda N_0 e^{-\lambda t}$$

Hence, the correct answer is (A).

36. From conservation of mechanical energy, we have

$$U_i + K_i = U_f + K_f$$

$$\Rightarrow \quad 0 + 2(1.5\text{ kT}) = \frac{1}{4\pi\varepsilon_0}\frac{(e)(e)}{d} + 0$$

Substituting the values, we get

$$T = 1.4 \times 10^9 \text{ K}$$

Hence, the correct answer is (A).

37. As given in the paragraph, a reactor is termed successful, if

$$nt_0 > 5 \times 10^{14} \text{ s cm}^{-3}$$

Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

2. A → (p)
B → (p)
C → (p)
D → (s)

When the daughter nuclei lie towards peak of this graph then energy is released. Hence the binding energy per nucleon or total binding energy in the nuclear process increases.

3. A → (p)
B → (q)
C → (r)
D → (q)

$$\gamma \rightarrow e^- + e^+$$

For pair production, we have

$$E = 2m_e c^2 = 2 \times 0.51 \text{ MeV} = 1.02 \text{ MeV}$$

Inverse photoelectric effect is X-ray production and energy involved in it is of order of tens of KeV

For de-excitation of Be^{+4} from first excited state, we have

$$E = \frac{Z^2}{n^2} \times 13.6 = \frac{4^2}{Z^2} \times 13.6$$

$$\Rightarrow \quad E \approx 54.4 \text{ eV} = 55 \text{ eV}$$

For K_α X-ray photon of molybdenum, we have

$$E(K_\alpha) = h\nu = \frac{h \times 3}{4} cR(Z-1)^2$$

$$\Rightarrow \quad E(K_\alpha) = \frac{3}{4}(hcR)(Z-1)^2 = \frac{3}{4}E_0(Z-1)^2$$

$$\Rightarrow \quad E(K_\alpha) = \frac{3}{4} \times 13.6 \times (42-1)^2 = 17.146 \times 10^3 \text{ eV}$$

$$\Rightarrow \quad E(K_\alpha) \approx 17 \text{ KeV}$$

5. A → (p, r)
B → (q, r, s)
C → (q, r)
D → (p, r)

For α-decay, we have

$$^A_zX \longrightarrow {}^{A-4}_{z-2}Y + {}^4_2\text{He}$$

$$Q = (KE)_Y + (KE)_\alpha$$

Since, $(KE)_Y \ll (KE)_\alpha$, we have

$$Q = (KE)_\alpha$$

So, kinetic energy of all the emitted α- particles is equal to Q i.e., mono-energetic α- particles are emitted.
Angular momentum is conserved in α- decay

For β-Decay, we have

a neutron decaying to a proton, so

$$^A_zX \longrightarrow {}^A_{z+1}Y + \bar{\nu} + e^-$$

$$Q = (KE)_Y + (KE)_e + E_{\bar{\nu}}$$

Since, $(KE)_Y \ll (KE)_e$

$$\Rightarrow \quad Q \approx (KE)_e + E_{\bar{\nu}}$$

Also note that,

(a) $E_{\bar{\nu}}$ is the energy of anti-neutrino and $E_{\bar{\nu}}$ takes on values from zero to maximum. Hence poly-energetic particles are emitted. i.e., poly-energetic antineutrinos are emitted.

(b) Due to emission of antineutrinos spin angular momentum is conserved.

CHAPTER 3

(c) Neutron can decay in free space i.e., outside nucleus

For Positron emission, we have

a proton decaying to a neutron, so

$$ {}_Z^AX \longrightarrow {}_{Z-1}^{A}Y + \nu + e^+ $$

Same explanation as above in case of Beta decay can be applied here too.

Proton cannot decay in free space i.e., outside nucleus because rest mass of proton is less than that of neutron.

For Electron capture, we have

$$ {}_Z^AX + e^- \longrightarrow {}_{Z-1}^{A}Y + \nu $$

$$ Q = (KE)_Y + E_\nu $$

Since, $(KE)_Y \ll E_\nu$

$$ \Rightarrow \quad Q \approx E_\nu $$

All the neutrinos emitted are of equal energies and their energies are approximately equal to Q. That is mono-energetic neutrinos are emitted. So, for electron capture we have that

(a) angular momentum is conserved.

(b) it cannot take place outside nucleus i.e., in free space.

7. A $\rightarrow$ (q)
B $\rightarrow$ (s)
C $\rightarrow$ (p)
D $\rightarrow$ (s)

Let N_0 be the total initial number of nuclei of A, B and C. So, $N_0 = N_A + N_B + N_C$. Now, when A decreases, then C increases, so (A + B) will continuously decrease, because C is formed only from A and B. So, A is continuously decreasing and hence $(C+B)$ will continuously increase.

9. A $\rightarrow$ (s)
B $\rightarrow$ (p, r)
C $\rightarrow$ (s)
D $\rightarrow$ (q, r)

(a) $Z' = (Z-2)+1 = Z-1$ and

$A' = A-4$

(b) $Z' = 2(Z-2)+1 = Z-3$ and

$A' = A-(2)(4) = A-8$

(c) $Z' = (Z-2)+(2)(1) = Z$ and

$A' = A-4$

(d) $Z' = 2(Z-2)+(2)(2) = Z-2$ and

$A' = A-2\times 4 = A-8$

12. A $\rightarrow$ (q)
B $\rightarrow$ (p)
C $\rightarrow$ (s)
D $\rightarrow$ (r)

In alpha decay, charge number decreases by 2 and mass number decreases by 4.

In β^+- decay, charge number decreases by 1 and mass number remains same.

In proton emission, charge no, decreases by 1 and mass no, decreases by 1.

13. A $\rightarrow$ (s)
B $\rightarrow$ (p)
C $\rightarrow$ (r)
D $\rightarrow$ (s)

Since, $\left(-\dfrac{dN}{dt}\right) = \lambda N$

$$ \Rightarrow \quad y = \lambda x $$

$$ \Rightarrow \quad \lambda = \frac{y}{x} $$

$$ \Rightarrow \quad T_{1/2} = \frac{\ln 2}{\lambda} = (\ln 2)\left(\frac{x}{y}\right) $$

Also, $R = R_0 e^{-\lambda t}$

$$ \Rightarrow \quad R = y e^{-\lambda\left(\frac{1}{\lambda}\right)} = \frac{y}{e} $$

$$ \Rightarrow \quad R = \frac{y}{e} = \lambda N $$

$$ \Rightarrow \quad N = \frac{y}{e\lambda} = \frac{y}{e\left(\frac{y}{x}\right)} = \frac{x}{e} $$

Integer/Numerical Answer Type Questions

1. $A = \left|\dfrac{dN}{dt}\right| = \lambda N_0$

$$ \Rightarrow \quad A = \frac{\ell n 2}{T_{\frac{1}{2}}} N_0 = \left(\frac{0.693}{T_{\frac{1}{2}}}\right) N_0 $$

$$ \Rightarrow \quad A = \frac{0.693\times 0.5\times 10^{-3}\times 6.02\times 10^{23}}{3.8\times(24\times 3600)\times 222\times(3.7\times 10^{10})} $$

$\Rightarrow \quad A = 77.35$ Ci

$\Rightarrow \quad A \approx 77$ Ci

2. $N = N_0\left(\frac{1}{2}\right)^n$

$$20 = 80\left(\frac{1}{2}\right)^n$$

$$\Rightarrow \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^2$$

$\Rightarrow \quad n = 2$

$\Rightarrow \quad t = nT_{\frac{1}{2}} = 2 \times 4 = 8$ minutes

3. Let rate of production be R

$$\Rightarrow \frac{dN}{dt} = R - \lambda N$$

$$\Rightarrow \frac{dN}{dt} + \lambda N = R$$

Multiplying both sides by $e^{\lambda t}$, we get

$$e^{\lambda t}\frac{dN}{dt} + \lambda N e^{\lambda t} = \text{R}e^{\lambda t}$$

$$\Rightarrow \frac{d\left(Ne^{\lambda t}\right)}{dt} = \text{R}e^{\lambda t}$$

$$\Rightarrow d\left(Ne^{\lambda t}\right) = \text{R}e^{\lambda t}\,dt$$

Integrating, we get

$$Ne^{\lambda t} = \frac{\text{R}e^{\lambda t}}{\lambda} + C$$

where C is a constant of integration

Now, at $t = 0$, $N = 0$

$$\Rightarrow C = -\frac{R}{\lambda}$$

$$\Rightarrow N = \frac{R}{\lambda}\left(1 - e^{-\lambda t}\right)$$

At equilibrium, we have $N = \frac{R}{\lambda}$, for $t \to \infty$

For 50% of the equilibrium quantity, we have $N = \frac{R}{2\lambda}$

$$\Rightarrow \frac{R}{2\lambda} = \frac{R}{\lambda}\left(1 - e^{-\lambda t}\right)$$

$$\Rightarrow e^{-\lambda t} = \frac{1}{2}$$

$$\Rightarrow t = \frac{\ell n 2}{\lambda} = T_{\frac{1}{2}} = 100 \text{ years}$$

4. From our knowledge of Collision Theory, the fraction of kinetic energy lost by neutron is

$$\frac{\Delta K}{K_i} = \frac{4m_1 m_2}{\left(m_1 + m_2\right)^2} = \frac{4(1)(2)}{(1+2)^2} = \frac{8}{9}$$

where K_i is the initial kinetic energy of neutron and ΔK is the energy loss.

After first collision $\Delta K_1 = \frac{8}{9}K_0$

After second collision $\Delta K_2 = \frac{8}{9}K_1$ and so on

So, total energy loss is

$$\Delta K = \Delta K_1 + \Delta K_2 + \ldots + \Delta K_n = \frac{8}{9}\left(K_0 + K_1 + \ldots. + K_{n-1}\right)$$

where, $K_1 = K_0 - \Delta K_1 = \frac{K_0}{9}$

$$K_2 = \frac{K_1}{9} = \left(\frac{1}{9}\right)^2 K_0$$

$$\Rightarrow K_{n-1} = \left(\frac{1}{9}\right)^{n-1} K_0$$

$$\Rightarrow \Delta K = \frac{8}{9}K_0\left[1 + \frac{1}{9} + \ldots\left(\frac{1}{9}\right)^{n-1}\right]$$

$$\Rightarrow \Delta K = \frac{8}{9}K_0\left(\frac{1 - \frac{1}{9^n}}{1 - \frac{1}{9}}\right) = 1 - \frac{1}{9^n}$$

Since, $K_0 = 10^6$ eV and $\Delta K = \left(10^6 - 0.025\right)$ eV

$$\Rightarrow \frac{1}{9^n} = \frac{K_0 - \Delta K}{K_0} = \frac{0.025}{10^6}$$

$\Rightarrow \quad 9^n = 4 \times 10^7$

Taking log both sides, we get

$n = 8$

5. Given, $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$

Since the probabilities of getting α and β particles are equal. So, rate of disintegration is equal for both.

$$\Rightarrow \quad \lambda_A N_A = \lambda_B N_B$$

$$\Rightarrow \quad \frac{N_A}{N_B} = \frac{\lambda_B}{\lambda_A} = 2$$

6. At time t, let say there are N atoms of ^{7}Be (radioactive). Then net rate of formation of ^{7}Be nuclei at this instant is,

$$\frac{dN}{dt} = \frac{10^{-4}}{1.6\times 10^{-19}\times 1000} - \lambda N$$

$$\Rightarrow \quad \frac{dN}{dt} = 6.25\times 10^{11} - \lambda N$$

$$\Rightarrow \quad \int_0^{N_0} \frac{dN}{6.25\times 10^{11} - \lambda N} = \int_0^{3600} dt$$

where N_0 are the number of nuclei at $t = 1\text{ hr}$ or 3600 second.

$$\Rightarrow \quad -\frac{1}{\lambda}\log_e\left(\frac{6.25\times 10^{11} - \lambda N_0}{6.25\times 10^{11}}\right) = 3600$$

Since, $\lambda N_0 =$ Activity of ^{7}Be at $t = 1\text{ hr} = 1.8\times 10^8$ disintegrations/second

$$\Rightarrow \quad -\frac{1}{\lambda}\log_e\left(\frac{6.25\times 10^{11} - 1.8\times 10^8}{6.25\times 10^{11}}\right) = 3600$$

$$\Rightarrow \quad \lambda = 8\times 10^{-8}\text{ sec}^{-1}$$

Therefore, half life

$$t_{1/2} = \frac{0.693}{8\times 10^{-8}} = 8.66\times 10^6\text{ sec} = 100.26\text{ days}$$

$$\Rightarrow \quad t_{1/2} \cong 100\text{ days}$$

7. In 2 sec only 90% of nuclei are left. Thus, in next 2 seconds 90% of 900 i.e. 810 nuclei will be left.

8. Since, $R = R_0\left(\frac{1}{2}\right)^n$

where, n is the number of half lives.

Given, $R = \frac{R_0}{16}$

$$\Rightarrow \quad \frac{R_0}{16} = R_0\left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad n = 4$$

Four half lives are equivalent to 8 s. Hence, 2 s is equal to one half life. So in one half life activity will remain half of 1600 Bq i.e., 800 Bq

9. $_6C^{13} \longrightarrow {}_6C^{12} + {}_0n^1$

Energy required is equal to difference in binding energy of parent nucleus and daughter nucleus.

10. For a simultaneous decay process, we have

$$\frac{1}{T} = \frac{1}{T_\alpha} + \frac{1}{T_\beta}$$

$$\Rightarrow \quad T = \frac{T_\alpha T_\beta}{T_\alpha + T_\beta} = 324\text{ years}$$

Since $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$, where $n = \frac{t}{T_{\frac{1}{2}}}$

According to problem, we have

$$\frac{N}{N_0} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{t}{T_{\frac{1}{2}}} = 2$$

$$\Rightarrow \quad t = 2T_{\frac{1}{2}}$$

$$\Rightarrow \quad t = 2(0.693T) \approx 449\text{ yr}$$

11. The activity of a sample is

$$A = \lambda N = \left(\frac{0.693}{T_{\frac{1}{2}}}\right)N$$

$$\Rightarrow \quad N = \frac{AT_{\frac{1}{2}}}{0.693} = \frac{6\times 10^6\times 30.2\times 3.16\times 10^7}{0.693}$$

$$\Rightarrow \quad N = 3.1\times 10^{26}$$

One mole of ^{137}Cs has a mass 137 g and one mole possesses 6.02×10^{23} nuclei. So, the mass of ^{137}Cs released was

$$m = \left(\frac{137}{6.02\times 10^{23}}\right)(3.1\times 10^{26}) = 70\times 10^3\text{ g}$$

$$\Rightarrow \quad m = 70\text{ kg}$$

$$\Rightarrow \quad x = 7$$

12. $\frac{N}{N_0 - N} = \frac{3}{1}$

$$\Rightarrow \quad \frac{N}{N_0} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{N}{N_0} = \frac{3}{4} = e^{-\lambda t} \qquad \left\{\because \frac{N}{N_0} = e^{-\lambda t}\right\}$$

$$\Rightarrow \quad \lambda t = \log_e\left(\frac{4}{3}\right)$$

$$\Rightarrow \quad t = \left(\frac{\log_e\left(\frac{4}{3}\right)}{0.693}\right)T_{\frac{1}{2}}$$

$$\Rightarrow \quad t = 4.5\times10^9\left[\frac{0.1249}{0.3010}\right]$$

$$\Rightarrow \quad t = 1.8678\times10^9 \text{ years} \approx 2\times10^9 \text{ years}$$

13. At time $t = t$

$$N_1 = N_0 e^{-\lambda_1 t}$$

and $N_2 = \dfrac{N_0\lambda_1}{\lambda_2-\lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$

$$\Rightarrow \quad N_3 = N_0 - N_1 - N_2$$

$$\Rightarrow \quad N_3 = N_0\left[1 - e^{-\lambda_1 t} - \frac{\lambda_1}{\lambda_2-\lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)\right]$$

$$\Rightarrow \quad \frac{N_3}{N_0} = 1 - e^{-\lambda_1 t} - \frac{\lambda_1}{\lambda_2-\lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

Since, $\lambda_1 = \dfrac{0.693}{30} = 0.0231 \text{ min}^{-1}$

$$\lambda_2 = \frac{0.693}{45} = 0.0154 \text{ min}^{-1}$$

and $t = 60$ minutes

$$\Rightarrow \quad \frac{N_3}{N_0} = 1 - e^{-0.0231\times60} - \frac{0.0231}{0.0154-0.0231}\left(e^{-0.0231\times60} - e^{-0.0154\times60}\right)$$

$$\Rightarrow \quad \frac{N_3}{N_0} = 1 - 0.25 + 3(0.25-0.4) = 0.31$$

$$\Rightarrow \quad \%\text{age} = \frac{N_3}{N_0}\times100\% = 31\%$$

14. $\lambda = \dfrac{h}{p}$ for α-particle

Kinetic energy of α-particle is

$K_\alpha = \dfrac{p^2}{2m_\alpha}$ and kinetic energy of nucleus is

$$K_{\text{nucleus}} = \frac{p^2}{2m_n}$$

$$\Rightarrow \quad E = \frac{p^2}{2}\left(\frac{1}{m_\alpha} + \frac{1}{m_n}\right) = 6.25 \text{ MeV}$$

So, mass of parent nucleus is $M = m_n + m_\alpha + \dfrac{E}{c^2}$

$$\Rightarrow \quad M = 227.62 \text{ amu}$$

15. Charge on capacitor is $Q = Q_0 e^{-\frac{t}{RC}}$ and activity of sample is $A = A_0 e^{-\lambda t}$

$$\Rightarrow \quad \frac{Q}{A} = \frac{Q_0 e^{-\frac{t}{RC}}}{A_0 e^{-\lambda t}} = \frac{Q_0}{A_0} e^{\left(\lambda - \frac{1}{RC}\right)t}$$

For $\dfrac{Q}{A}$ to be independent of time, we have

$$\lambda - \frac{1}{RC} = 0$$

$$\Rightarrow \quad \lambda = \frac{1}{RC}$$

$$\Rightarrow \quad R = \frac{1}{C\lambda} = \frac{T_m}{C} = \frac{20\times10^{-3}}{100\times10^{-6}} = 200\ \Omega$$

16. Cross-sectional area of the torus is

$$A = \pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{4} \text{ m}^2$$

Circumference of torus is

$$\ell = 2\pi\left(\frac{3}{2}\right) = 3\pi \text{ metre}$$

So, volume of the torus is

$$V = A\ell = \left(\frac{\pi}{4}\right)(3\pi) = \left(\frac{3\pi^2}{4}\right) \text{ m}^3$$

$$\Rightarrow \quad V = \left(\frac{3\pi^2}{4}\right) \text{ m}^3 = 7.5 \text{ m}^3$$

Pressure of the gas is

$$P = 10^{-5}\times13.6\times10^3\times10 = 1.36 \text{ Nm}^{-2}$$

According to ideal gas equation, we have

$$PV = Nk_B T_0$$

where $T_0 = 27 + 273 = 300$ K

$$\Rightarrow \quad Nk_B = \frac{PV}{T_0} = \frac{(1.36)(7.5)}{300} = 0.034$$

The energy obtained from the discharge is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(1200\times10^{-6}\right)\left(4\times10^4\right)^2$$

$$\Rightarrow \quad U = 9.6\times10^5 \text{ J}$$

This energy is transferred to the plasma in the form of kinetic energy. So, we have

$$U = KE = 9.6\times10^5 \text{ J}$$

Since we know that, at temperature T, the average kinetic energy associated with the gas molecule is

$$E = \frac{3}{2}Nk_B T$$

So, the kinetic energy gained by the plasma molecules is $E_K = \left(\dfrac{10}{100}\right)U = 9.6\times10^4$ J

Further it is given that, each deuterium molecule produces two ions and two electrons, hence the total energy of the plasma at temperature T is given by

CHAPTER 3

$$E_{\text{Total}} = 4\left(\frac{3}{2}Nk_BT\right) = 6Nk_BT = 9.6\times10^4$$

$$T = 4706\times10^2 \text{ K}.$$

17. Let $\lambda_A = \lambda$ and $\lambda_B = 2\lambda$

Initially rate of disintegration of A is λN_0 and that of B is $(2\lambda)N_0$.

After one half life of A, rate of disintegration of A will becomes $\frac{\lambda N_0}{2}$ and that of B would also be $\frac{\lambda N_0}{2}$.

So, after one half life of A or two half life of B.

$$\left(-\frac{dN}{dt}\right)_A = \left(-\frac{dN}{dt}\right)_B$$

$\Rightarrow$ $n = 1$

18. $BE = (\Delta m)c^2$

$\Rightarrow$ $BE = 0.0302\times930$

$\Rightarrow$ $BE = 28.086$

$\Rightarrow$ $\frac{BE}{4} = \frac{28.086}{4} \approx 7$ MeV

19. 3 half lives of A is equivalent to 6 half lives of B.

$\Rightarrow$ $N_A\left(\frac{1}{2}\right)^3 = N_B\left(\frac{1}{2}\right)^6$

$\Rightarrow$ $\frac{N_B}{N_A} = 8$

20. Since, $\Delta E = (m_n - m_p)\times 931.5 \text{ MeV} = 1.4 \text{ MeV}$

Also, $\Delta E = E_1 + E_2$ ...(1)

where $E_1 = m_0c^2 + K_1 =$ Energy of electron

$E_2 =$ Energy of antineutrino (having zero rest mass)

By conservation of momentum, we have

$$p_1 = -p_2$$

$\Rightarrow$ $p_1^2 = p_2^2$

Since $E = \sqrt{m_0^2c^4 + p^2c^2}$

$\Rightarrow$ $p = \frac{1}{c}\sqrt{E^2 - m_0^2c^4}$

$\Rightarrow$ $\frac{1}{c}\sqrt{E_1^2 - m_0^2c^4} = \frac{1}{c}\sqrt{E_2^2 - 0}$

$\Rightarrow$ $E_1^2 = m_0^2c^4 = E_2^2$ Using (1), we get

$$E_1^2 - m_0^2c^4 = (\Delta E - E_1)^2$$

$\Rightarrow$ $E_1 = \frac{(\Delta E)^2 + m_0^2c^4}{2\Delta E} = 0.7937$ MeV

$\Rightarrow$ $E_2 = 1.4 - 0.7937 = 0.6068$ MeV

$\Rightarrow$ $E_2 \approx 6\times10^5$ eV

21. Let ${}^A_ZX \longrightarrow {}^{A}_{Z+1}Y + e^- + \bar{\nu}$

$\Rightarrow$ ${}^{23}_{10}\text{Ne} \longrightarrow {}^{23}_{11}\text{Na} + e^- + \bar{\nu}$

$$Q = \left[m\left({}^{23}\text{Ne}\right) - m\left({}^{23}\text{Na}\right)\right]\times 931.5 \text{ MeV}$$

$\Rightarrow$ $Q = 4.375 \text{ MeV} = 4.4 \text{ MeV}$

$\Rightarrow$ $Q \approx 4$ MeV

Since, $Q = (KE)_Y + (KE)_e + E(\bar{\nu})$

As, $(KE)_Y$ is very very small, so

$$Q \approx (KE)_e + E(\bar{\nu})$$

When $(KE)_e$ is maximum, then $E(\bar{\nu})$ is negligible

$\Rightarrow$ $(KE)_e \approx Q = 4$ MeV

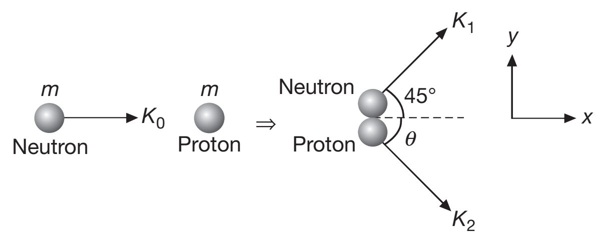
22. ${}_{92}U^{238} \longrightarrow {}_2He^4 + {}_{90}Th^{234} + Q$

$$\Delta m = 238.05081 - (4.00260 + 234.04363)$$

$\Rightarrow$ $\Delta m = 0.00458$ u

$\Rightarrow$ $Q = \Delta m \times 931.5 \text{ MeV} = 4.26$ MeV

23. Mass of neutron $\approx$ mass of proton $= m$



Applying Law of Conservation of Linear Momentum along y-direction, we get

$$\sqrt{2mK_1}\sin45° = \sqrt{2mK_2}\sin\theta \quad ...(1)$$

along x-direction, we get

$$\sqrt{2mK_0} - \sqrt{2mK_1}\cos45° = \sqrt{2mK_2}\cos\theta \quad ...(2)$$

Squaring and adding equations (1) and (2), we get

$$K_2 = K_1 + K_0 - \sqrt{2K_0K_1} \quad ...(3)$$

By Law of Conservation of Energy, we get

$$K_2 = K_0 - K_1 \quad ...(4)$$

Solving equations (3) and (4), we get

$$K_1 = \frac{K_0}{2}$$

$\Rightarrow$ $\Delta K = K_0 - K_1 = \frac{K_0}{2}$

i.e., after each collision energy remains half. Therefore, after n collisions, we have

$$K_n = K_0\left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad 0.23 = (4.6\times10^6)\left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad 2^n = \frac{4.6\times10^6}{0.23}$$

Taking log both sides, we get

$n \approx 24$

24.

	A^{236}	:	A^{234}
Nuclei	$4N_0$	:	N_0
Half life	30	:	60

Activity is $A = \lambda N = \lambda N_0 e^{-\lambda t}$

$$\lambda_1(4N_0)e^{-\frac{0.693}{30}t} = \lambda_2 N_0 e^{-\frac{0.693}{60}t}$$

$$\Rightarrow \quad \frac{0.693}{30}(4N_0)e^{-\frac{0.693}{30}t} = \frac{0.693}{60}(N_0)e^{-\frac{0.693}{60}t}$$

$$\Rightarrow \quad 8 = e^{0.693t\left(\frac{1}{30}-\frac{1}{60}\right)}$$

$$\Rightarrow \quad 8 = e^{+\frac{0.693}{60}t}$$

Taking natural log both sides, we get

$$3\times0.693 = \frac{0.693t}{60}$$

$$\Rightarrow \quad t = 180 \text{ min}$$

25. When a substance decays by α and β emission simultaneously, the average disintegration constant λ_{av} is given by

$$\lambda_{av} = \lambda_\alpha + \lambda_\beta$$

where λ_α = disintegration constant for α-emission only

λ_β = disintegration constant for β-emission only

Mean life is given by $T_m = \dfrac{1}{\lambda_{av}}$

$$\Rightarrow \quad \lambda_{av} = \lambda_\alpha + \lambda_\beta$$

$$\Rightarrow \quad \frac{1}{T_m} = \frac{1}{T_\alpha} + \frac{1}{T_\beta} = \frac{1}{1200} + \frac{1}{600}$$

$$\Rightarrow \quad T_m = 400 \text{ yr}$$

Since, $t = \dfrac{1}{\lambda}\log_e\left(\dfrac{N_0}{N_t}\right) = T_m \log_e\left(\dfrac{N_0}{N_t}\right)$

$$\Rightarrow \quad t = 400\log_e\left(\frac{100}{25}\right) = 400\log_e(4)$$

$$\Rightarrow \quad t = 400\times1.4 = 560 \text{ year}$$

26. 235 kg contains 6.023×10^{26} atoms

So, 1.5 kg contains 0.384×10^{25} atoms

Energy released is $\Delta E = 0.384\times10^{25}\times200$ MeV

$$\Rightarrow \quad \Delta E = 7.7\times10^{26} \text{ MeV}$$

$$\Rightarrow \quad \Delta E = 12.32\times10^{13} \text{ J}$$

$$\Rightarrow \quad \Delta E \approx 1.23\times10^{14} \text{ J}$$

27. Let $\lambda_A = \lambda$ and $\lambda_B = 2\lambda$

Initially rate of disintegration of A is λN_0 and that of B is $(2\lambda)N_0$.

After one half life of A, rate of disintegration of A will becomes $\dfrac{\lambda N_0}{2}$ and that of B would also be $\dfrac{\lambda N_0}{2}$.

So, after one half life of A or two half life of B.

$$\left(-\frac{dN}{dt}\right)_A = \left(-\frac{dN}{dt}\right)_B$$

$$\Rightarrow \quad n = 1$$

28.

$$\underset{6.5}{{}^{200}X} \longrightarrow \underset{7}{{}^{80}Y} + \underset{8}{{}^{120}Z}$$

So, energy released is

$$\Delta E = 80\times7 + 120\times8 - 200\times6.5$$

$$\Rightarrow \quad \Delta E = 220 \text{ MeV} = 2200\times10^5 \text{ eV}$$

29. Energy per fission

$$E_0 = 200\times10^6\times1.6\times10^{-19} \text{ J}$$

$$\Rightarrow \quad E_0 = 3.2\times10^{-11} \text{ J}$$

$$P = 4 \text{ MW} = 4\times10^6 \text{ Js}^{-1}$$

In time $t(=1 \text{ year})$, the energy delivered is $E = Pt$

Number of fission required is $N = \dfrac{Pt}{E_0}$

$$\Rightarrow \quad N = \frac{4\times10^6\times365\times24\times3600}{3.2\times10^{-11}}$$

$$\Rightarrow \quad N = 3.942\times10^{24}$$

So, mass of uranium required is

$$m = \frac{3.942\times10^{24}\times235}{6.02\times10^{23}} \text{ g}$$

$$\Rightarrow \quad m = 1538.8 \text{ g}$$

$$\Rightarrow \quad m \approx 1539 \text{ g}$$

30. $N_1 = N_0 e^{-\lambda(10)}$

Let age of sample be x, them

$$N_2 = N_0 e^{-\lambda x}$$

$$\Rightarrow \frac{N_2}{N_1} = \frac{4}{100} = e^{\lambda(10-x)}$$

$$e^{\lambda(x-10)} = \frac{100}{4}$$

$$\Rightarrow \lambda(x-10) = \ell n(100) - \ell n 4 = 2\ell n(10) - 2\ell n(2)$$

$$\Rightarrow \lambda(x-10) = 2(2.3) - 2(0.693)$$

$$\Rightarrow \lambda(x-10) = 3.22$$

Since $\lambda = \frac{0.693}{12.5}\ \text{yr}^{-1}$

$$\Rightarrow x - 10 = \left(\frac{12.5}{0.693}\right)(3.22) = 58.08$$

$$\Rightarrow x \approx 68 \text{ years}$$

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1. Densities of nucleus happens to be constant, irrespective of mass number.

Hence, the correct answer is (B).

2. Number of nuclei present at any time t

$$N = N_0 e^{-\lambda t}$$

Given that $\frac{N_A}{N_B} = \frac{1}{e}$

$$\Rightarrow \frac{N_A}{N_B} = e^{(\lambda_B - \lambda_A)t} = \frac{1}{e}$$

$$\Rightarrow (\lambda_A - \lambda_B)t = 1$$

$$\Rightarrow t = \frac{1}{-\lambda + 10\lambda} = \frac{1}{9\lambda}$$

Hence, the correct answer is (C).

3. At time t, $N_x = N_0 e^{-5\lambda t}$

At time t, $N_y = N_0 e^{-\lambda t}$

Since $\frac{N_x}{N_y} = \frac{1}{e^2} = e^{-4\lambda t}$

$$\Rightarrow 4\lambda t = 2$$

$$\Rightarrow t = \frac{2}{4\lambda} = \left(\frac{1}{2\lambda}\right)$$

Hence, the correct answer is (A).

4. $N_A = \frac{N_0}{(2)^{\frac{60}{10}}} = \frac{N_0}{64}$

$$N_B = \frac{N_0}{(2)^{\frac{60}{20}}} = \frac{N_0}{8}$$

So, number of decayed nuclei in A is $N'_A = N_0 - N_A$ and number of decayed nuclei in B is $N'_B = N_0 - N_B$

So, required ratio is $\frac{N'_A}{N'_B} = \frac{N_0 - \frac{N_0}{64}}{N_0 - \frac{N_0}{8}}$

$$\Rightarrow \frac{N'_A}{N'_B} = \frac{63 \times 8}{64 \times 7} = \frac{9}{8}$$

Hence, the correct answer is (A).

5. $10 \text{ mCi} = \lambda_A N_A(t)$ ...(1)

$20 \text{ mCi} = \lambda_B N_B(t)$ ...(2)

Since $N_A(t) = 2N_B(t)$

$$\Rightarrow \frac{1}{2} = \frac{\lambda_A N_A(t)}{\lambda_B N_B(t)}$$

$$\Rightarrow \frac{1}{2} = \left(\frac{\lambda_A}{\lambda_B}\right)2$$

$$\Rightarrow \lambda_B = 4\lambda_A$$

$$\Rightarrow (t_B)_{\frac{1}{2}} = \frac{(t_A)_{\frac{1}{2}}}{4}$$

$$\Rightarrow (t_A)_{\frac{1}{2}} = 4(t_B)_{\frac{1}{2}}$$

Hence, the correct answer is (B).

6. $R_A = R_0 e^{-\lambda_A t}$

$$R_B = R_0 e^{-\lambda_B t}$$

Since $\frac{R_B}{R_A} = e^{-3t}$

$$\Rightarrow \frac{R_B}{R_A} = e^{-(\lambda_B - \lambda_A)t} = e^{-3t}$$

$$\Rightarrow \lambda_B - \lambda_A = 3$$

$$\Rightarrow \frac{\ln 2}{T_2} - \frac{\ln 2}{\ln 2} = 3$$

$$\Rightarrow T_2 = \frac{\ln 2}{4}$$

Hence, the correct answer is (C).

7. $(2)^N = \left(\frac{1600}{100}\right)$

$N = 4$

$4t = 8$

$t = 2$ s

Count Rate $= \frac{1600}{(2)^3} = 200$

Hence, the correct answer is (D).

8. Amount of energy released is

$$\Delta E = 2(\text{BE of He}^4) + (\text{BE of C}^{12}) - (\text{BE of Ne}^{20})$$

$$\Rightarrow \quad \Delta E = (2)(4)(7.07) + (12)(7.86) - (20)(8.03)$$

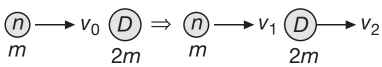
$$\Rightarrow \quad \Delta E = -9.72 \approx -9.7 \text{ MeV}$$

So, 9.7 MeV of energy will be supplied.

Hence, the correct answer is (D).

9. Let initial speed of neutron is v_0 and kinetic energy is K.

For 1st collision



Using momentum conservation principle,

$$mv_0 = mv_1 + 2mv_2$$

$$\Rightarrow \quad v_1 + 2v_2 = v_0 \qquad \ldots(1)$$

As, $e = 1$, so

$$v_2 - v_1 = v_0 \qquad \ldots(2)$$

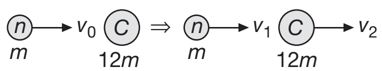
From equations (1) and (2), we get

$$v_2 = \frac{2v_0}{3}, \; v_1 = -\frac{v_0}{3}$$

Fractional loss of energy is $p_d = \dfrac{\frac{1}{2}mv_0^2 - \frac{1}{2}m\left(-\frac{v_0}{3}\right)^2}{\frac{1}{2}mv_0^2}$

$$\Rightarrow \quad p_d = \frac{8}{9} \approx 0.89$$

For 2nd collision



Using momentum conservation principle,

$$mv_0 = mv_1 + 12mv_2$$

$$\Rightarrow \quad v_1 + 12v_2 = v_0 \qquad \ldots(1)$$

As $e = 1$;

$$\Rightarrow \quad v_2 - v_1 = v_0 \qquad \ldots(2)$$

From equations (1) and (2), we get

$$v_2 = \frac{2v_0}{13}; \; v_1 = \frac{-11v_0}{13}$$

Now fraction loss of energy

$$P_c = \frac{\frac{1}{2}mv_0^2 - \frac{1}{2}m\left(-\frac{11v_0}{13}\right)^2}{\frac{1}{2}mv_0^2} = \frac{48}{169} \approx 0.28$$

Hence, the correct answer is (A).

10. Let total volume of blood is V

Initial activity $A_0 = 0.8\ \mu$Ci

Its activity at time t, $A = A_0 e^{-\lambda t}$

Activity of solution of volume x is

$$A_1 = \left(\frac{A}{V}\right)x = x\left(\frac{A_0}{V}\right)e^{-\lambda t}$$

$$\Rightarrow \quad V = x\left(\frac{A_0}{A_1}\right)e^{-\lambda t}$$

$$\Rightarrow \quad V = (1 \text{ cm}^3)\left(\frac{8\times 10^{-7} \times 3.7 \times 10^{10}}{\frac{300}{60}}\right)(0.84)$$

$$\Rightarrow \quad V = 4.97 \times 10^3 \text{ cm}^3 = 4.97 \text{ litres} \approx 5 \text{ litres}$$

Hence, the correct answer is (C).

11. As, $\frac{v_1}{v_2} = \frac{8}{27}$; $\frac{r_1}{r_2} = ?$

Using law of conservation of linear momentum,

$$0 = m_1v_1 - m_2v_2$$

(As both are moving in opposite directions.)

$$\Rightarrow \quad \frac{m_1}{m_2} = \frac{v_2}{v_1} = \frac{27}{8}$$

$$\Rightarrow \quad \frac{\rho\left(\frac{4}{3}\pi r_1^3\right)}{\rho\left(\frac{4}{3}\pi r_2^3\right)} = \frac{27}{8}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{3}{2}$$

Hence, the correct answer is (A).

12. As per question, $N_1 = 2N_2$

Also $A_1 = 5\ \mu$Ci, $A_2 = 10\ \mu$Ci

As, $A = \lambda N = \dfrac{\ln 2}{T_{\frac{1}{2}}}N$

CHAPTER 3

$$\Rightarrow \quad \frac{A_1}{A_2}=\frac{\left(T_{\frac{1}{2}}\right)_2}{\left(T_{\frac{1}{2}}\right)_1}\times\frac{N_1}{N_2}$$

$$\frac{\left(T_{\frac{1}{2}}\right)_1}{\left(T_{\frac{1}{2}}\right)_2}=\frac{N_1}{N_2}\times\frac{A_2}{A_1}=2\times 2=4$$

Hence, the correct answer is (C).

13. $\dfrac{N_B}{N_A}=\dfrac{N_0-N_0e^{-\lambda t}}{N_0e^{-\lambda t}}=0.3$

$\Rightarrow \quad e^{\lambda t}=1.3$

$\Rightarrow \quad \lambda t=\ln 1.3$

$\Rightarrow \quad \left(\dfrac{\ln 2}{T}\right)t=\ln(1.3)$

$\Rightarrow \quad t=T\cdot\dfrac{\ln(1.3)}{\ln 2}$

$\Rightarrow \quad t=T\dfrac{\log(1.3)}{\log 2}$

Hence, the correct answer is (B).

14. ${}_1H^2+{}_1H^2\rightarrow {}_2He^4$

Energy released is $\Delta E=4\left(B.E.\left({}_1^2H\right)\right)-4\left(B.E.\left({}_2^4He\right)\right)$

$\Rightarrow \quad \Delta E=4\times 7-4\times 1.1=23.6$ MeV

Hence, the correct answer is (D).

15. Here, $P=10^9$ W, $c=3\times 10^8$ ms^{-1}, $\dfrac{\Delta m}{\Delta t}=?$

We know, $P=\dfrac{E}{\Delta t}=\dfrac{\Delta mc^2}{\Delta t}$

$\Rightarrow \quad \dfrac{\Delta m}{\Delta t}=\dfrac{P}{c^2}=\dfrac{10^9}{(3\times 10^8)^2}=\dfrac{10^{-7}}{9}$ kgs^{-1}

$\Rightarrow \quad \dfrac{\Delta m}{\Delta t}=\dfrac{10^{-7}}{9}\times 1000\times 3600$ gh$^{-1}=4\times 10^{-2}$ gh^{-1}

Hence, the correct answer is (A).

16. $t=80$ min $=4T_A=2T_B$

So, number of nuclei of A decayed $=N_0-\dfrac{N_0}{2^4}=\dfrac{15N_0}{16}$

So, number of nuclei of B decayed $=N_0-\dfrac{N_0}{2^2}=\dfrac{3N_0}{4}$

$\Rightarrow$ Required ratio $=\dfrac{5}{4}$

Hence, the correct answer is (A).

17. Using conservation of linear momentum,

$$\left(\begin{matrix}\text{Total momentum}\\ \text{before collision}\end{matrix}\right)=\left(\begin{matrix}\text{Total momentum}\\ \text{after collision}\end{matrix}\right)$$

$\Rightarrow \quad mv=(m+m)v'$

$\Rightarrow \quad v'=\dfrac{v}{2}$

Loss in kinetic energy during the process,

$$\Delta K=\frac{1}{2}mv^2-\frac{1}{2}(2m)\left(\frac{v}{2}\right)^2=\frac{1}{4}mv^2$$

For minimum kinetic energy of neutron, lost kinetic energy should be used by the electron to jump from first orbit to second orbit.

$\Rightarrow \quad \dfrac{1}{4}mv^2=(13.6-3.4)\text{eV}=10.2$ eV

$\Rightarrow \quad \dfrac{1}{2}mv^2=20.4$ eV $=K.E.$ of neutron for inelastic collision.

Hence, the correct answer is (A).

18. Half life $=15$ hrs $=\dfrac{0.693}{\lambda}$

$\Rightarrow \quad \lambda=0.0462$ hr^{-1}

$N_0=\dfrac{1}{24}$ moles of Na, $t=7.5$ hrs

Number of β-particles disintegrated, $N_\beta=N_0\left(1-e^{-\lambda t}\right)$

$\Rightarrow \quad N_\beta=\left(\dfrac{1}{24}\text{moles}\right)\left(1-e^{-(0.0462\times 7.5)}\right)$

$\Rightarrow \quad N_\beta=\left(\dfrac{1}{24}\text{moles}\right)\left(1-e^{-0.35}\right)$

$\Rightarrow \quad N_\beta=0.0122$ moles $=0.0122\times 6.023\times 10^{23}$

$\Rightarrow \quad N_\beta=7.4\times 10^{21}$

Hence, the correct answer is (B).

19. Since, $r=\dfrac{mv}{qB}=\dfrac{mv}{eB}$

$\Rightarrow \quad KE=\dfrac{1}{2}mv^2=\dfrac{e^2B^2r^2}{2m}$

is the KE of the ejected photoelectrons. Now, according to Law of Photo-electricity, we have

$$hv=hv_0+\frac{1}{2}mv^2$$

$\Rightarrow \quad hv=\phi_0+\dfrac{1}{2}mv^2$ ...(1)

Now $\frac{1}{2}mv^2 = \frac{e^2B^2r^2}{2m} = 0.8$ eV

$$hv = 13.6\left(\frac{1}{4}-\frac{1}{9}\right) = \frac{68}{36} = 1.9 \text{ eV}$$

So, from (1), we get

$$1.9 = \phi_0 + 0.8$$

$\Rightarrow \quad \phi_0 = 1.1$ eV

Hence, the correct answer is (B).

20. Mass defect, $\Delta m = m_p + m_e - m_n$

$\Rightarrow \quad \Delta m = \left(1.6725\times10^{-27} + 9\times10^{-31} - 1.6725\times10^{-27}\right)$ kg

$\Rightarrow \quad \Delta m = 9\times10^{-31}$ kg

Energy released is $\Delta E = \Delta mc^2 = 9\times10^{-31}\times\left(3\times10^8\right)^2$ J

$\Rightarrow \quad \Delta E = \frac{9\times10^{-31}\times9\times10^{16}}{1.6\times10^{-13}}$ MeV $= 0.51$ MeV

Hence, the correct answer is (D).

21. Number of undecayed atoms after time t_2,

$$\frac{N_0}{3} = N_0e^{-\lambda t_2} \quad \ldots(1)$$

Number of undecayed atoms after time t_1,

$$\frac{2}{3}N_0 = N_0e^{-\lambda t_1}$$

Dividing (2) by (1), we get $2 = e^{\lambda(t_2-t_1)}$

$\Rightarrow \quad \ln 2 = \lambda(t_2 - t_1)$

$\Rightarrow \quad (t_2 - t_1) = \frac{\ln 2}{\lambda}$

As per question, $t_{\frac{1}{2}}$ = half life time = 20 min

$\Rightarrow \quad t_2 - t_1 = 20$ min $\quad \left\{\because t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}\right\}$

Hence, the correct answer is (C).

22. When a radioactive nucleus emits an alpha particle, its mass number decreases by 4 while the atomic number decreases by 2.
When a radioactive nucleus, emits a β^+ particle (or positron (e^+)) its mass number remains unchanged while the atomic number decreases by 1.

$$\Rightarrow \quad {}_Z^AX \xrightarrow{3\alpha} {}_{Z-6}^{A-12}Y \xrightarrow{2e^+} {}_{Z-8}^{A-12}W$$

In the final nucleus,

Number of protons, $N_p = Z - 8$

Number of neutrons, $N_n = A - 12 - (Z - 8) = A - Z - 4$

$\Rightarrow \quad \frac{N_n}{N_p} = \frac{A-Z-4}{Z-8}$

Hence, the correct answer is (C).

23. Mass defect, $\Delta M = \left[(M+\Delta m) - \left(\frac{M}{2}+\frac{M}{2}\right)\right] = \Delta m$

Energy released, $Q = \Delta Mc^2 = \Delta mc^2 \quad \ldots(1)$

According to law of conservation of momentum, we get

$$(M+\Delta m)\times 0 = \frac{M}{2}\times v_1 - \frac{M}{2}\times v_2$$

$\Rightarrow \quad v_1 = v_2$

Also, $Q = \frac{1}{2}\left(\frac{M}{2}\right)v_1^2 + \frac{1}{2}\left(\frac{M}{2}\right)v_2^2 - \frac{1}{2}(M+\Delta m)\times(0)^2$

$\Rightarrow \quad Q = \frac{M}{2}v_1^2 \quad \{\because v_1 = v_2\} \quad \ldots(2)$

Equating equations (1) and (2), we get $\left(\frac{M}{2}\right)v_1^2 = \Delta mc^2$

$$v_1^2 = \frac{2\Delta mc^2}{M}$$

$\Rightarrow \quad v_1 = c\sqrt{\frac{2\Delta m}{M}}$

Hence, the correct answer is (C).

24. After decay, the daughter nuclei will be more stable, hence binding energy per nucleon of daughter nuclei is more than that of their parent nucleus.
Hence, $E_2 > E_1$.

Hence, the correct answer is (D).

25. When two nucleons combine to form a third one, and energy is released, one has fusion reaction. If a single nucleus splits into two, one has fission. The possibility of fusion is more for light elements and fission takes place for heavy elements. Out of the choices given for fusion, only A and B are light elements and D and E are heavy elements. Therefore $A + B \to C + \varepsilon$ is correct. In the possibility of fission is only for F and not C. Therefore $F \to D + E + \varepsilon$ is the correct choice.

Hence, the correct answer is (A).

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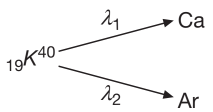
Single Correct Choice Type Problems

1. Since, $\frac{dN}{dt} = -\lambda_1 N - \lambda_2 N$

$\Rightarrow \frac{dN}{dt} = -(\lambda_1 + \lambda_2)dt$

$\Rightarrow N = N_0 e^{-(\lambda_1+\lambda_2)t}$



Since $N = N_0 - 99\%$ of N_0 i.e., $N = 0.01N_0$, we have

$$t = \frac{\ln(100)}{\lambda_1 + \lambda_2} = \frac{2.3 \times 2}{5 \times 10^{-10}}$$

$\Rightarrow t = 9.2 \times 10^9$ year

Hence, the correct answer is (B).

2. Required activity $= \frac{\text{Initial activity}}{64} = \frac{\text{Initial activity}}{2^6}$

Time required = 6 half lives

$\Rightarrow t = 6 \times 18$ days

$\Rightarrow t = 108$ days

Hence, the correct answer is (C).

3. For ${}^{15}_{8}\text{O}$, $E_0 = \frac{3}{5} \times \frac{8 \times 7}{R} \times \frac{e^2}{4\pi\varepsilon_0} = \frac{3}{5} \times \frac{8 \times 7}{R} \times 1.44$ MeV

For ${}^{15}_{7}\text{N}$, $E_N = \frac{3}{5} \times \frac{7 \times 6}{R} \times \frac{e^2}{4\pi\varepsilon_0} = \frac{3}{5} \times \frac{7 \times 6}{R} \times 1.44$ MeV

$\Rightarrow |E_0 - E_N| = \frac{3}{5} \times \frac{1.44}{R} \times 7(2)$ ...(1)

Mass defect of N atom is

$\Delta m_N = 8 \times 1.008665 + 7 \times 1.007825 - 15.000109$

$\Rightarrow \Delta m_N = 0.1239864\ u$

$\Rightarrow$ Binding energy is $B_N = 0.1239864 \times 931.5$ MeV

Mass defect of O atom is

$\Delta m_0 = 7 \times 1.008665 + 8 \times 1.007825 - 15.003065$

$\Rightarrow \Delta m_0 = 0.12019044\ u$

$\Rightarrow$ Binding energy $B_0 = 0.12019044 \times 931.5$ MeV

So $|B_0 - B_N| = 0.0037960 \times 931.5$ MeV ...(2)

Equating (1) and (2), we get

$R = 3.42$ fm

Hence, the correct answer is (C).

4. From conservation laws of mass number and atomic number, we can say that $x = n$, $y = n$

$\left(x = {}^{1}_{0}n,\ y = {}^{1}_{0}n\right)$

$\Rightarrow$ Only (A) and (D) options may be correct.

From conservation of momentum, $|P_{xe}| = |P_{sr}|$

From $K = \frac{P^2}{2m}$

$\Rightarrow K \propto \frac{1}{m}$

$\frac{K_{sr}}{K_{xe}} = \frac{m_{xe}}{m_{sr}}$

$\Rightarrow K_{sr} = 129$ MeV, $K_{xe} = 86$ MeV

Note: There is no need of finding total energy released in the process.

Hence, the correct answer is (A).

5. Activity of $S_1 = \frac{1}{2}$ (activity of S_2)

$\Rightarrow \lambda_1 N_1 = \frac{1}{2}(\lambda_2 N_2)$

$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{N_2}{2N_1}$

$\Rightarrow \frac{T_1}{T_2} = \frac{2N_1}{N_2}$ $\left\{T = \text{half life} = \frac{\log_e 2}{\lambda}\right\}$

Given $N_1 = 2N_2$

$\Rightarrow \frac{T_1}{T_2} = 4$

Hence, the correct answer is (A).

6. Rest mass of parent nucleus should be greater than the rest mass of daughter nuclei. Therefore, option (A) will be correct.

Hence, the correct answer is (A).

7. After two half lives $\frac{1}{4}$th fraction of nuclei will remain undecayed. Or, $\frac{3}{4}$th fraction will decay. Hence, the probability that a nucleus decays in two half lives is $\frac{3}{4}$.

Hence, the correct answer is (B).

8. $4\left({}_2\text{He}^4\right) = {}_8\text{O}^{16}$

Mass defect, $\Delta m = (4(4.0026) - 15.9994) = 0.011$ amu

Energy released per oxygen nuclei is

$\Delta E = (0.011)(931.48)\text{MeV} = 10.24$ MeV

Hence, the correct answer is (C).

9. Activity reduces from 6000 dps to 3000 dps in 140 days. It implies that half-life of the radioactive sample is 140 days. In 280 days (or two half-lives) activity will remain $\frac{1}{4}$th of the initial activity. Hence, the initial activity of the sample is

$$4 \times 6000 \text{ dps} = 24000 \text{ dps}$$

Hence, the correct answer is (D).

10. Given that $K_1 + K_2 = 55$ MeV ...(1)

From Conservation of Linear Momentum,

$$P_1 = P_2$$

$$\Rightarrow \sqrt{2K_1(216m)} = \sqrt{2K_2(4m)}$$

as $P = \sqrt{2Km}$

$$\Rightarrow K_2 = 54K_1 \quad ...(2)$$

Solving equations (1) and (2), we get $K_2 = KE$ of α-particle $= 5.4$ MeV.

Hence, the correct answer is (B).

11. Nuclear density is constant hence, mass $\propto$ volume or $m \propto V$.

Hence, the correct answer is (A).

12. $(r_m) = \left(\frac{m^2}{z}\right)(0.53 \text{ Å}) = (n \times 0.53) \text{ Å}$

$$\Rightarrow \frac{m^2}{z} = n$$

$m = 5$ for ${}_{100}\text{Fm}^{257}$ (the outermost shell) and $z = 100$

$$\Rightarrow n = \frac{(5)^2}{100} = \frac{1}{4}$$

Hence, the correct answer is (D).

13. During γ-decay atomic number (Z) and mass number (A) does not change. So, the correct option is (C) because in all other options either Z, A or both is/are changing.

Hence, the correct answer is (C).

14. $\frac{1}{16} = \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^{\left(\frac{\text{Time lapsed}}{T_{1/2}}\right)}$

$$\Rightarrow \frac{\text{Time lapsed}}{T_{1/2}} = 4$$

$$\Rightarrow \text{Time lapsed} = 400\ \mu s$$

Hence, the correct answer is (A).

15. The total number of atoms can neither remain constant (as in option A) nor can ever increase (as in options B and C). They will continuously decreases with time. Therefore, (D) is the appropriate option.

Hence, the correct answer is (D).

16. During β-decay, a neutron is transformed into a proton and an electron. This is why atomic number ($Z =$ number of protons) increases by one and mass number ($A =$ number of protons + neutrons) remains unchanged during beta decay.

Hence, the correct answer is (C).

17. $N_1 = N_0 e^{-10\lambda t}$

$$N_2 = N_0 e^{-\lambda t}$$

$$\Rightarrow \frac{N_1}{N_2} = \frac{1}{e} = e^{(-10\lambda + \lambda)t}$$

$$\Rightarrow 9\lambda t = 1$$

$$\Rightarrow t = \frac{1}{9\lambda}$$

Hence, the correct answer is (D).

18. $\frac{0.693}{\lambda_X} = \frac{1}{\lambda_Y}$

$$\Rightarrow \lambda_Y > \lambda_X$$

Since, $\left(-\frac{dN}{dt}\right)_X = \lambda_X N$

And $\left(-\frac{dN}{dt}\right)_Y = \lambda_Y N$

$$\Rightarrow \left(-\frac{dN}{dt}\right)_Y > \left(-\frac{dN}{dt}\right)_X$$

Decay rate of Y > Decay rate of X

Hence, the correct answer is (C).

19. Both the beta rays and the cathode rays are made up of electrons. So, only option (A) is correct.

(B) Gamma rays are electromagnetic waves.

(C) Alpha particles are doubly ionized helium atoms and

(D) Protons and neutrons have approximately the same mass.

Therefore, (B), (C) and (D) are wrong options.

Hence, the correct answer is (A).

20. ${}^{22}_{10}\text{Ne} \longrightarrow 2\,{}^{4}_{2}\text{He} + {}^{A}_{Z}X$

$$\Rightarrow A = 14$$

$$\Rightarrow Z = 6$$

i.e. ${}^{14}_{6}\text{C}$ is the unknown nucleus.

Hence, the correct answer is (B).

21. $\rho = \dfrac{A(1.67\times10^{-27})}{\frac{4}{3}\pi R^3}$

Since, $R = R_0 A^{\frac{1}{3}}$ where $R_0 = 1.1$ fm

$\Rightarrow \quad \rho = \dfrac{A(1.67\times10^{-27})}{\frac{4}{3}\pi R_0^3 A}$

$\Rightarrow \quad \rho = 2.9\times10^{17}\ \text{kgm}^{-3}$

Hence, the correct answer is (B).

22. If it was A

$Y \to 2Z$

Reactant: $R = 60\times8.5 = 510$ MeV

Product: $P = 2\times30\times5 = 300$ MeV

$\Delta E = -210$ MeV

ENDOTHERMIC

If it was B

$W \to X + Z$

$R = 120\times7.5 = 900$ MeV

$P = 90\times8 + 30\times5 = 870$ MeV

$\Delta E = -30$ MeV

ENDOTHERMIC

If it was C

$W \to 2Y$

$R = 120\times7.5 = 900$ MeV

$P = 2\times60\times8.5 = 1020$ MeV

$\Delta E = 120$ MeV

EXOTHERMIC

If it was D

$X \to Y + Z$

$R = 90\times8.0 = 720$ MeV

$P = 60\times8.5 + 30\times5.0 = 660$ MeV

$\Delta E = -60$ MeV

ENDOTHERMIC

Hence, the correct answer is (C).

23. Number of nuclei decreases exponentially

$N = N_o e^{-\lambda t}$ and

Rate of decay $\left(-\dfrac{dN}{dt}\right) = \lambda N$

Therefore, decay process lasts upto $t \to \infty$ Therefore, a given nucleus may decay at any time after $t = 0$.

Hence, the correct answer is (D).

24. In beta decay, atomic number is increased by 1 whereas the mass number remains the same. Therefore, following equation can be possible

$${}^{64}_{29}\text{Cu} \to {}^{64}_{30}\text{Zn} + {}_{-1}e^{o}$$

Hence, the correct answer is (D).

25. Penetrating power is maximum for $\gamma-$rays, then of $\beta-$particles and then $\alpha-$particles because basically it depends on the velocity. However, ionization power is in reverse order.

Hence, the correct answer is (A).

26. Heavy water is used as moderators in nuclear reactors to slow down the neutrons.

Hence, the correct answer is (B).

27. ${}_1\text{H}^2 + {}_1\text{H}^2 \longrightarrow {}_1\text{H}^3 + p$

${}_1\text{H}^2 + {}_1\text{H}^3 \longrightarrow {}_2\text{He}^4 + n$

$\Rightarrow \quad 3\,{}_1\text{H}^2 \longrightarrow {}_2\text{He}^4 + p + n$

$\Delta m = m({}_2\text{He}^4) + m(p) + m(n) - 3m({}_1\text{H}^2)$

$\Rightarrow \quad \Delta m = [4.001 + 1.007 + 1.008 - 3(2.014)]\,u$

$\Rightarrow \quad \Delta m = -0.026\,u$

$\Rightarrow \quad |\Delta E| = c^2|\Delta m|$

$\Rightarrow \quad \Delta E = (9\times10^{16})(0.026\times1.67\times10^{-27})$

$\Rightarrow \quad \Delta E = (931.5)(0.026)$ MeV

$\Rightarrow \quad \Delta E = 3.87\times10^{-12}$ J

As each reaction involves 3 deutrons, so total number of reactions involved in the process $= \dfrac{10^{40}}{3}$. If each reaction produces an energy ΔE, then

$$E_{\text{total}} = \frac{10^{40}}{3}\Delta E = 1.29\times10^{28}\ \text{J}$$

$$E_{\text{total}} = Pt$$

Time of exhaustion of the star

$$t = \frac{1.29\times10^{28}}{10^{16}}$$

$\Rightarrow \quad t = 1.29\times10^{12}$ s

Hence, the correct answer is (C).

29. From $R = R_o\left(\dfrac{1}{2}\right)^n$

We have, $1 = 64\left(\dfrac{1}{2}\right)^n$

or $n = 6$ = number of half-lives.

$\therefore \quad t = n \times t_{1/2} = 6 \times 2 = 12$ h

Hence, the correct answer is (B).

32. Following nuclear reaction takes place

$${}_0n^1 \rightarrow {}_1\mathrm{H}^1 + {}_{-1}e^o + \bar{v}$$

$\bar{v}$ is antineutrino.

Hence, the correct answer is (C).

33. β and $\gamma-$ decay take place from a radioactive nucleus.
Hence, options (A) and (B) are wrong. During fusion process two or more lighter nuclei combine to form a heavy nucleus.
Hence, the correct answer is (C).

34. Beta particles are fast moving electrons which are emitted by the nucleus.
Hence, the correct answer is (C).

35. Using $N = N_0 e^{-\lambda t}$ where $\lambda = \frac{\log_e 2}{t_{1/2}} = \frac{\log_e(2)}{3.8}$

$$\Rightarrow \quad \frac{N_0}{20} = N_0 e^{-\frac{\log_e(2)}{3.8}t}$$

Solving this equation with the help of given data we find $t = 16.5$ days.

Hence, the correct answer is (B).

Multiple Correct Choice Type Problems

1. $${}_zX^A \xrightarrow{\alpha\text{-Decay}} {}_{Z-2}Y^{A-4}$$

$${}_zX^A \xrightarrow{\beta^-\text{-Decay}} {}_{Z+1}X_1^A$$

$${}_{90}\mathrm{Th}^{232} \longrightarrow {}_{82}\mathrm{Pb}^{212}$$

Number of α-particles emitted $= \frac{232-212}{4} = 5$

Since Z decreases by $(90-82) = 8$ only

Hence number of β^- decay $= 2$

Hence, (A) and (C) are correct.

2. In fusion two or more lighter nuclei combine to make a comparatively heavier nucleus.
In fission, a heavy nucleus breaks into two or more comparatively lighter nuclei.
Further, energy will be released in a nuclear process if total binding energy increases.
Hence, (B) and (D) are correct.

3. Due to mass defect (which is finally responsible for the binding energy of the nucleus), mass of a nucleus is always less than the sum of masses of its constituent particles.

${}^{20}_{10}\mathrm{Ne}$ is made up of 10 protons plus 10 neutrons.

Therefore, mass of ${}^{20}_{10}\mathrm{Ne}$ nucleus,

$$M_1 < 10(m_p + m_n)$$

Also, heavier the nucleus, more is the mass defect.

Thus, $20(m_n + m_p) - M_2 > 10(m_p + m_n) - M_1$

$$\Rightarrow \quad 10(m_p + m_n) > M_2 - M_1$$

$$\Rightarrow \quad M_2 < M_1 + 10(m_p + m_n)$$

Now since, $M_1 < 10(m_p + m_n)$

$$\Rightarrow \quad M_2 < 2M_1$$

Hence, (C) and (D) are correct.

4. In nuclear fusion two or more lighter nuclei are combined to form a relatively heavy nucleus and thus, releasing the energy.
Hence, (A) and (D) are correct.

5. Cut off voltage is independent of intensity and hence remains the same. Since distance becomes 3 times, so I becomes $\frac{I}{9}$. Hence photocurrent also decreases by this factor i.e. becomes $\frac{18}{9} = 2$ mA

Hence, (B) and (D) are correct.

6. In case of ${}_1\mathrm{H}^1$, mass number and atomic number are equal and in case of ${}_1\mathrm{H}^2$, mass number is greater than its atomic number.
Hence, (C) and (D) are correct.

7. In fusion reaction, two or more lighter nuclei combine to form a comparatively heavier nucleus.
Hence, (B) and (C) are correct.

Comprehension Type Questions

1. $$r = \frac{1-a}{1+a} \qquad \ldots(1)$$

$$\Rightarrow \quad \ln r = \ln(1-a) - \ln(1+a)$$

$$\Rightarrow \quad \frac{\Delta r}{r} = \frac{\Delta a}{1-a} + \frac{\Delta a}{1+a}$$

$$\Rightarrow \quad \frac{\Delta r}{r} = \frac{2\Delta a}{1-a^2}$$

Substituting value of r from equation (1), we get

$$(1+a)\frac{\Delta r}{(1-a)}=\frac{2(\Delta a)}{(1-a^2)}$$

$$\Rightarrow \quad \Delta r=\frac{2\Delta a}{(1+a)^2}$$

Hence, the correct answer is (B).

2. Let number of nuclei decayed be N

$$N=N_0(1-e^{-\lambda t})$$

$$\lambda t=\ln\left(\frac{N_0}{N_0-N}\right)$$

$$\lambda t=\ln N_0-\ln(N_0-N)$$

$$(\Delta\lambda)t=\frac{dN}{(N_0-N)}$$

$$(\Delta\lambda)=\frac{40}{(3000-1000)}=0.02\ \text{s}^{-1}$$

Hence, the correct answer is (C).

3. $m({}^2_1\text{H})+m({}^4_2\text{He})=2.014102+4.002603$

$$\Rightarrow \quad m({}^2_1\text{H})+m({}^4_2\text{He})=6.016705\ \text{u}$$

Since, $m({}^6_3\text{Li})=6.015123\ \text{u}$

$$\Rightarrow \quad m_1+m_2>M$$

So, (A) is incorrect.

$$m({}^1_1\text{H})+m({}^{209}_{83}\text{Bi})=1.007825+208.980388$$

$$m({}^1_1\text{H})+m({}^{209}_{83}\text{Bi})=209.988213\ \text{u}$$

Since, $m({}^{210}_{84}\text{Po})=209.982876\ \text{u}$

$$\Rightarrow \quad m_1+m_2>M$$

So, B is incorrect

$$m({}^2_1\text{H})+m({}^4_2\text{He})=2.014102+4.002603$$

$$\Rightarrow \quad m({}^2_1\text{H})+m({}^4_2\text{He})=6.016705\ \text{u}$$

Since, ${}^6_3\text{Li}=6.015123\ \text{u}$

$$\Rightarrow \quad (m_3+m_4)>M'$$

So, (C) is correct and hence deuteron and alpha particle can go complete fusion.

$$m({}^{70}_{30}\text{Zn})+m({}^{82}_{34}\text{Se})=69.925325+81.916709$$

$$\Rightarrow \quad m({}^{70}_{30}\text{Zn})+m({}^{82}_{34}\text{Se})=151.842034\ \text{u}$$

Since, ${}^{152}_{64}\text{Gd}=151.919803\ \text{u}$

$$\Rightarrow \quad m_3+m_4<M'$$

So, (D) is incorrect.

Hence, the correct answer is (C).

4. ${}^{210}_{84}\text{Po}={}^{206}_{82}\text{Pb}+{}^4_2\text{He}+\Delta E$

$$m({}^{206}_{82}\text{Pb})=205.974455\ \text{u}$$

$$m({}^4_2\text{He})=4.002603\ \text{u}$$

$$\Rightarrow \quad m({}^{206}_{82}\text{Pb})+m({}^4_2\text{He})=209.977058\ \text{u}$$

Now, $\Delta m=209.977058-209.982876$

$$\Rightarrow \quad \Delta m=0.005818\ \text{u}$$

$$\Rightarrow \quad Q=\Delta E=0.005818\times 931.5$$

$$\Rightarrow \quad Q=5.419467\ \text{MeV}=5419.467\ \text{keV}$$

$$\Rightarrow \quad Q=5419.5\ \text{keV}$$

By Law of Conservation of Momentum, we have

$$0=p_\alpha-p_{\text{lead}}$$

$$\Rightarrow \quad p_\alpha=p_{\text{lead}}$$

$$\Rightarrow \quad \sqrt{2m_\alpha E_\alpha}=\sqrt{2m_{Pb}E_{Pb}}$$

$$\Rightarrow \quad 4E_\alpha=206E_{Pb}$$

$$\Rightarrow \quad E_\alpha=\frac{103}{2}E_{Pb}$$

Now, since $E_\alpha=\left(\frac{m_{Pb}}{m_{Pb}+m_\alpha}\right)Q$

$$\Rightarrow \quad E_\alpha=\left(\frac{206}{206+4}\right)Q$$

$$\Rightarrow \quad E_\alpha=\frac{103}{105}(5.422)=5319\ \text{MeV}$$

Hence, the correct answer is (A).

5. Maximum energy of the antineutrino will be nearly 0.8×10^6 eV

Hence, the correct answer is (C).

6. Minimum kinetic energy of electron can be zero or greater than zero. But maximum kinetic energy will be less than 0.8×10^6 eV.

Hence, the correct answer is (D).

8. From conservation of mechanical energy, we have

$$U_i+K_i=U_f+K_f$$

$$\Rightarrow \quad 0+2(1.5\ \text{kT})=\frac{1}{4\pi\varepsilon_0}\frac{(e)(e)}{d}+0$$

Substituting the values, we get

$$T=1.4\times10^9\ \text{K}$$

Hence, the correct answer is (A).

9. As given in the paragraph, a reactor is termed successful, if

$$nt_0 > 5\times10^{14}\ \text{s cm}^{-3}$$

Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

3. A $\to$ (q)
B $\to$ (p)
C $\to$ (s)
D $\to$ (r)

(p) In α-decay mass number decrease by 4 and atomic number decreases by 2.

(q) In β^+-decay mass number remains unchanged while atomic number decreases by 1.

(r) In fission, parent nucleus breaks into all most two equally fragments.

(s) In proton emission both mass number and atomic number decreases by 1.

Integer/Numerical Answer Type Questions

1. $\Delta m = (226.005 - 222 - 4) = 0.005$ amu

$\Rightarrow\quad Q = \Delta mc^2$

$\Rightarrow\quad Q = 931.5\times0.005 = 4.655$ MeV

Since momentum is conserved, kinetic energy is in inverse ratio of masses.

$$K_T = 4.44 + K_{Rn}$$

$$\Rightarrow\quad K_{Rn} = \frac{4.44\times4}{222} = 0.08\ \text{MeV}$$

$$E_{\gamma\text{-photon}} = 4.655 - 4.520$$

$$\Rightarrow\quad E_{\gamma\text{-photon}} = 0.135\ \text{MeV} = 135\ \text{keV}$$

2. $\text{I}^{131} \xrightarrow[T_{\frac{1}{2}}=8\text{ Days}]{} \text{Xe}^{131} + \beta$

$A_0 = 2.4\times10^5\ \text{Bq} = \lambda N_0$

Let the volume is V,

$t = 0 \qquad A_0 = \lambda N_0$

$t = 11.5\ \text{h} \qquad A = \lambda N$

$$115 = \lambda\left(\frac{N}{V}\times2.5\right)$$

$$\Rightarrow\quad 115 = \frac{\lambda}{V}\times2.5\times\left(N_0e^{-\lambda t}\right)$$

$$\Rightarrow\quad 115 = \frac{(N_0\lambda)}{V}\times(2.5)\times e^{-\frac{\ln 2}{8\text{ day}}(11.5\text{ h})}$$

$$\Rightarrow\quad 115 = \frac{(2.4\times10^5)}{V}\times(2.5)\times e^{-\frac{1}{24}}$$

$$\Rightarrow\quad V = \frac{2.4\times10^5}{115}\times2.5\left[1-\frac{1}{24}\right]$$

$$\Rightarrow\quad V = \frac{2.4\times10^5}{115}\times2.5\left[\frac{23}{24}\right]$$

$$\Rightarrow\quad V = \frac{10^5\times23\times25}{115\times10^2} = 5\times10^3\ \text{ml} = 5\ \text{L}$$

3. $\text{Q value} = \left[12.014u - \left(12u + 4.041\dfrac{\text{MeV}}{c^2}\right)\right]c^2$

$$\Rightarrow\quad Q = (0.014u\times931.5)\text{MeV} - (4.041)\text{MeV} = 9\ \text{MeV}$$

Hence, β particle will have a maximum KE of 9 MeV

4. Let initial numbers are N_1 and N_2.

$$\frac{\lambda_1}{\lambda_2} = \frac{\tau_2}{\tau_1} = \frac{2\tau}{\tau} = 2 = \frac{T_2}{T_1} \qquad (T = \text{Half life})$$

$$A = \frac{-dN}{dt} = \lambda N$$

Initial activity is same

$$\Rightarrow\quad \lambda_1N_1 = \lambda_2N_2$$

Activity at time t,

$$A = \lambda N = \lambda N_0e^{-\lambda t}$$

$$A_1 = \lambda_1N_1e^{-\lambda_1 t}$$

$$\Rightarrow\quad R_1 - = \frac{dA_1}{dt} = \lambda_1^2N_1e^{-\lambda_1 t}$$

Similarly, $R_2 = \lambda_2^2N_2e^{-\lambda_2 t}$

After $t = 2\tau$

$$\lambda_1 t = \frac{1}{\tau_1}(t) = \frac{1}{\tau}(2\tau) = 2$$

$$\lambda_2 t = \frac{1}{\tau_2}(t) = 1 = \frac{1}{2\tau}(2\tau) = 1$$

$$\frac{R_P}{R_Q} = \frac{\lambda_1^2N_1e^{-\lambda_1 t}}{\lambda_2^2N_2e^{-\lambda_2 t}}$$

$$\Rightarrow\quad \frac{R_P}{R_Q} = \frac{\lambda_1}{\lambda_1}\left(\frac{e^{-2}}{e^{-1}}\right) = \frac{2}{e}$$

5. Let initial power available from the plant is P_0. After time $t = nT$ or n half lives, this will become $\left(\frac{1}{2}\right)^n P_0$. Now, it is given that, $\left(\frac{1}{2}\right)^n P_0 = 12.5\%$ of $P_0 = (0.125)P_0$

Solving this equation we get, $n = 3$

6. Since, $N = N_0 e^{-\lambda t}$

$$\Rightarrow \quad \frac{N}{N_0} = e^{-\lambda t}$$

$$\Rightarrow \quad \frac{N}{N_0} = e^{-\frac{\log_e 2}{1386} \times 80}$$

$$\Rightarrow \quad \frac{N}{N_0} = e^{-\frac{0.693 \times 80}{1386}}$$

$$\Rightarrow \quad \frac{N}{N_0} = e^{-0.04}$$

$$\Rightarrow \quad \frac{N}{N_0} = \left(\frac{1}{e}\right)^{0.04}$$

Fraction of nuclei decayed is

$$1 - \frac{N}{N_0} = 1 - \left(\frac{1}{e}\right)^{0.04} = 0.04 = 4\%$$

7. $\frac{dN}{dt} = -\lambda N = -10^{10}$

$$\Rightarrow \quad N = 10^{10} \times \left(\frac{1}{\lambda}\right)$$

$$\Rightarrow \quad m_{\text{total}} = N \times m_1$$

$$\Rightarrow \quad m_{\text{total}} = 10^{10} \times 10^{9} \times 10^{-25} \times 10^{6} \text{ mg}$$

$$\Rightarrow \quad m_{\text{total}} = 1 \text{ mg}$$

8. $\left|\frac{dN}{dt}\right| = |\text{Activity of radioactive substance}|$

$$= \lambda N = \lambda N_0 e^{-\lambda t}$$

Taking log both sides

$$\ln\left|\frac{dN}{dt}\right| = \ln(\lambda N_0) - \lambda t$$

Hence, $\ln\left|\frac{dN}{dt}\right|$ versus t graph is a straight line with slope $-\lambda$.

From the graph we can see that,

$$\lambda = \frac{1}{2} = 0.5 \text{ yr}^{-1}$$

Now applying the equation,

$$N = N_0 e^{-\lambda t} = N_0 e^{-0.5 \times 4.16}$$

$$\Rightarrow \quad N = N_0 e^{-2.08} = 0.125 N_0 = \frac{N_0}{8}$$

i.e., nuclei decrease by a factor of 8.

Hence, the answer is 8.

CHAPTER 4: SEMICONDUCTOR DEVICES AND APPLICATIONS

Single Correct Choice Type Questions

1. The emitter is most heavily doped.
Hence, the correct answer is (A).

2. Collector current,

$$I_C = 10 \text{ mA}$$

$$\alpha = \frac{90}{100} = 0.9$$

$$\alpha = \frac{I_C}{I_E}$$

$\Rightarrow$ Emitter current, $I_E = \frac{I_C}{\alpha}$

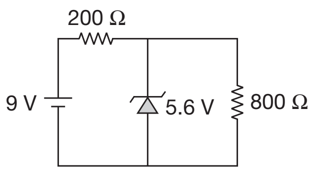
$$I_E = \frac{10}{0.9} = 11 \text{ mA}$$

Base current, $I_B = I_E - I_C$

$$I_B = 11 - 10 = 1 \text{ mA}$$

Hence, the correct answer is (B).

3.



$$I_{800\ \Omega} = \frac{5.6}{800} \text{ A} = 7 \text{ mA}$$

$$I_{200\ \Omega} = \frac{9 - 5.6}{200} = 17 \text{ mA}$$

$$\Rightarrow \quad I_Z = 17 - 7 = 10 \text{ mA}$$

Hence, the correct answer is (C).

5. Wave nature of electron and covalent bonds are correlated.
Hence, the correct answer is (A).

7. Copper is conductor and germanium is semiconductor. When cooled, the resistance of copper strip decreases and that of germanium increases.
Hence, the correct answer is (C).

8. Since $I = e^{\frac{1000V}{T}} - 1$

$$\Rightarrow \quad I + 1 = e^{\frac{1000V}{T}}$$

$$\Rightarrow \quad \log_e(I+1) = \frac{1000 \text{ V}}{T}$$

$$\Rightarrow \quad d[\log_e(I+1)] = d\left(\frac{1000 \text{ V}}{T}\right)$$

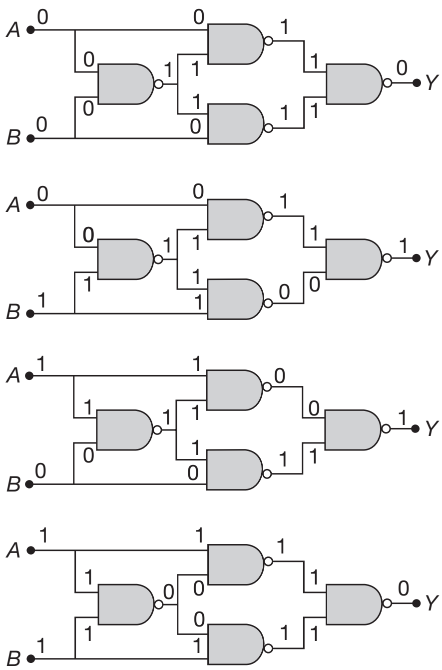
$$\Rightarrow \quad \frac{dI}{I+1} = \left(\frac{1000}{T}\right) dV$$

$$\Rightarrow \quad \frac{dI}{(5+1)mA} = \left(\frac{1000}{300}\right)(0.01)$$

$$\Rightarrow \quad dI = 0.2 \text{ mA}$$

Hence, the correct answer is (A).

9.



Hence, the correct answer is (D).

11. In oscillator, feedback is positive gain with positive feedback

$$A_f = \frac{A}{1 - \beta A} > 1$$

Hence, the correct answer is (B).

12. Variation of number of charge carriers with temperature is responsible for variation of resistance in a metal and a semiconductor.

Hence, the correct answer is (B).

13. $i = \dfrac{V_F - V_B}{R}$

$\Rightarrow \quad R = \dfrac{V_F - V_B}{i}$

$i = \dfrac{5-0.7}{10^{-3}} = 4.3 \times 10^3 \ \Omega = 4.3 \text{ k}\Omega$

Hence, the correct answer is (C).

16. Electric field is zero in the middle of the depletion layer of a reverse biased p-n junction.

Hence, the correct answer is (A).

17. Output of upper OR gate $= W + X$

Output of lower OR gate $= W + Y$

Net output $F = (W+X)(W+Y)$

$= WW + WY + XW + XY$

$= W + WY + XW + XY$ (since $W.W = W$)

$= W(1+Y) + XW + XY$ (since $1+Y = 1$)

$= W.1 + XW + XY$

$= W(1+X) + XY = W + XY$

Hence, the correct answer is (C).

18. $\beta = \dfrac{\alpha}{1-\alpha} = \dfrac{0.9}{1-0.9} = 9$

Hence, the correct answer is (D).

19. $\dfrac{1}{\alpha} - \dfrac{1}{\beta} = \dfrac{1}{(\Delta I_C/\Delta I_E)} - \dfrac{1}{(\Delta I_C/\Delta I_B)}$

$\dfrac{1}{\alpha} - \dfrac{1}{\beta} = \dfrac{\Delta I_E - \Delta I_B}{\Delta I_C} = \dfrac{\Delta I_C}{\Delta I_C} = 1$

Hence, the correct answer is (A).

21. Electrons of n-type emitter move from emitter to base and then base to collector when npn transistor is used as an amplifier

Hence, the correct answer is (A).

25. In common emitter configuration, current gain is

$$A_i = \frac{-(h_{fe})}{1+(h_{oe})(R_L)} = \frac{-50}{1+(25\times 10^{-6})\times(1\times 10^3)}$$

$$\Rightarrow \quad A_i = -\frac{50}{1+0.025} = \frac{-50}{1.025} = -48.78$$

Hence, the correct answer is (D).

27. Copper is a conductor.

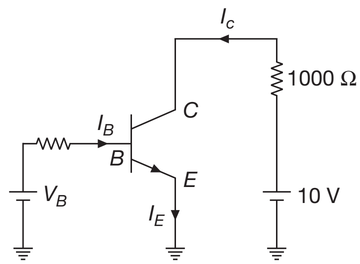
Germanium is a semiconductor.

When cooled, the resistance of copper decreases and that of germanium increases.

Hence, the correct answer is (C).

30. For saturation, $V_{CC} - I_C \times R_C = 0$

$\Rightarrow \quad I_C = \dfrac{V_{CC}}{R_C} = \dfrac{10}{10^3} = 10^{-2} \text{ A}$



Since, $\beta = \dfrac{I_C}{I_B} = 250$

$\Rightarrow \quad I_B = \dfrac{I_C}{250} = \dfrac{10^{-2}}{250} = 40 \ \mu\text{A}$

Hence, the correct answer is (D).

32. Pauli's exclusion principle explains band structure of solids.

Hence, the correct answer is (B).

33. $\alpha = \dfrac{I_C}{I_E} = \dfrac{I_E - I_B}{I_E}$

$\Rightarrow \quad 0.95 = \dfrac{10 - I_B}{10}$

$\Rightarrow \quad I_B = 0.5 \text{ mA}$

Hence, the correct answer is (B).

34. In circuit 1, I diode is forward biased and II diode is reverse biased; so $R_1 \neq R_2$ hence $V_1 \neq V_2$. In circuit 2, both diodes are forward biased $R_1 = R_2$; so $V_1 = V_2$. In circuit 3, both diodes are reverse biased so $R_1 = R_2$ and hence $V_1 = V_2$

Hence, the correct answer is (B).

36. The energy band gap is maximum in insulators.

Hence, the correct answer is (C).

38. When p-n junction diode is forward biased, both the depletion region and barrier height are reduced.

Hence, the correct answer is (C).

39. $\sigma = n_d e \mu_e$

$$\Rightarrow \quad n_d = \frac{\sigma}{e\mu_e} = \frac{1}{\rho e \mu_e}$$

$$n_d = \frac{1}{0.1 \times 1.6 \times 10^{-19} \times 0.05}$$

$$n_d = 1.25 \times 10^{21}$$

Hence, the correct answer is (C).

41. In a common base amplifier, the phase difference between the input signal and output voltage is zero.
Hence, the correct answer is (A).

42. $$i = \frac{V_F - V_B}{R} = \frac{2 - 0.7}{20 + 180} = \frac{1.3}{200}$$

$$i = 6.5 \times 10^{-3} \text{ A} = 6.5 \text{ mA}$$

Hence, the correct answer is (C).

43. $$A_v = \beta \cdot \frac{R_2}{R_1} = 100 \times \frac{10 \times 10^3}{1 \times 10^3} = 1000$$

Hence, the correct answer is (B).

44. Band gap ΔE_g corresponds to the energy of photon of $\lambda = 2480$ nm

$$\Rightarrow \quad \Delta E_g = \frac{hc}{\lambda} \text{ J} = \frac{hc}{\lambda e} \text{ eV}$$

$$\Rightarrow \quad \Delta E_g = \frac{(6.63 \times 10^{-34}) \times (3 \times 10^8)}{(2480 \times 10^{-9}) \times (1.6 \times 10^{-19})} \text{ eV}$$

$$\Rightarrow \quad \Delta E_g = 0.5 \text{ eV}$$

Hence, the correct answer is (A).

45. Under forward bias, the width of depletion layer decreases.
Hence, the correct answer is (B).

46. Output of OR gate $= A + B$

Output of NAND gate $= \overline{A \cdot B}$

$\Rightarrow$ Output Y from AND gate $= (A + B) \cdot \overline{AB}$

$$= (A + B) \cdot (\overline{A} + \overline{B}) = A\overline{A} + B\overline{B} + A\overline{B} + B\overline{A}$$

$$= 0 + 0 + A\overline{B} + B\overline{A}$$

This is Boolean expression for XOR gate.

Hence, the correct answer is (B).

47. $$\beta = \frac{I_C}{I_B} = 200$$

$$R_i = \frac{10 \times 10^{-3}}{15 \times 10^{-6}} = 0.67 \text{ k}\Omega$$

So, voltage gain is $A_v = \beta\left(\frac{R_o}{R_i}\right) = 300$

Hence, the correct answer is (D).

48. Semiconductor devices are suitable for low voltages.

Hence, the correct answer is (C).

49. $$i = \frac{\Delta V}{R} = \frac{3 - 1}{100} = 2 \times 10^{-2} \text{ A} = 20 \text{ mA}$$

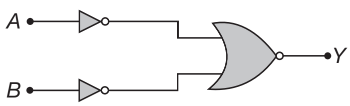
Hence, the correct answer is (C).

50. Frequency of full wave rectifier is

$$f = 2 \times \text{input frequency} = 2 \times 50 = 100 \text{ Hz}$$

Hence, the correct answer is (A).

51.



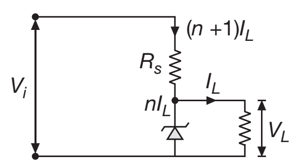
By de Morgan's theorem, $(\overline{\overline{A} + \overline{B}}) = A \cdot B$

A	B	$\overline{A}$	$\overline{B}$	$\overline{A} + \overline{B}$	$\overline{\overline{A} + \overline{B}}$	$A \cdot B$
1	1	0	0	0	1	1
0	0	1	1	1	0	0
0	1	1	0	1	0	0
1	0	0	1	1	0	0

This is the same as AND Gate of A and B.

Hence, the correct answer is (A).

53.



Voltage drop across Zener diode is V_L, so voltage drop across R_S is,

$$V_{RS} = V_i - V_L = (n + 1) I_L R_S$$

$\Rightarrow \quad R_S = \dfrac{V_i - V_L}{(n+1)I_L}$

Hence, the correct answer is (C).

54. $\dfrac{\text{Reverse resistance}}{\text{Forward resistance}} > 10000$

Hence, the correct answer is (D).

55. Such a solid is formed by covalent bonding.

Hence, the correct answer is (C).

57. $I_C = I_E - I_B = 4 \text{ mA} - 40\ \mu\text{A} = (4 - 0.04) = 3.96 \text{ mA}$

$$\alpha = \frac{I_C}{I_E} = \frac{3.96}{4} = 0.99$$

Hence, the correct answer is (A).

59. For conductor, ρ increases as temperature rises. For semiconductor, ρ decreases as temperature rises.

Hence, the correct answer is (C).

60. Drift velocity $v_d = \dfrac{I}{neA}$

$$\Rightarrow \quad \frac{(v_d)_{\text{electron}}}{(v_d)_{\text{hole}}} = \left(\frac{I_e}{I_h}\right)\left(\frac{n_h}{n_e}\right) = \frac{7}{4} \times \frac{5}{7} = \frac{5}{4}$$

Hence, the correct answer is (D).

61. $A_f = \dfrac{A}{1 + \beta A}$

$$\Rightarrow \quad 25 = \frac{150}{1 + \beta \times 150}$$

$$\Rightarrow \quad \beta = \frac{1}{30}$$

Hence, the correct answer is (A).

62. Junction diode is reverse biased, so current is zero.

Hence, the correct answer is (A).

65. $\beta = \dfrac{I_C}{I_B} = \dfrac{I_C}{I_E - I_C}$

$$\Rightarrow \quad \beta = \frac{5.488}{5.60 - 5.488} = \frac{5.488}{0.112} = 49$$

Hence, the correct answer is (B).

66. $R = \dfrac{\Delta V}{\Delta i} = \dfrac{2.3 - 0.3}{10 \times 10^{-3}}$

$$R = 0.2 \times 10^3\ \Omega = 0.2\ \text{k}\Omega$$

Hence, the correct answer is (B).

68. $Y = \overline{\overline{A} \cdot \overline{B}} = A + B$ (Using De-Morgan's Law)

$\Rightarrow$ OR gate

Hence, the correct answer is (A).

69. Semiconductors, like Si, Ge, act as insulators at low temperature.

Hence, the correct answer is (C).

71. E_c and E_v decrease but E_g increases if the lattice constant of the semiconductor is decreased.

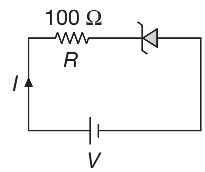
Hence, the correct answer is (D).

73. Potential drop across Zener diode

$$V_Z = V - IR = V - 100I$$

Since, power, $P = V_Z I_Z$

$$\Rightarrow \quad P = (V - 100I)I$$



Given that, $P = 1$ W

$$\Rightarrow \quad (V - 100I)I = 1$$

$$\Rightarrow \quad 100I^2 - VI + 1 = 0$$

For I to be real, $V^2 - 4 \times 100 \times 1 \geq 0$

$$\Rightarrow \quad V \geq 20 \text{ V}$$

Hence, the correct answer is (B).

74. A bridge rectifier uses four identical junction diodes.

Hence, the correct answer is (D).

76. $\alpha = \dfrac{\beta}{\beta + 1} = \dfrac{100}{100 + 1} = 0.99$

Hence, the correct answer is (C).

79. $P_{\text{gain}} = \beta^2 \left(\dfrac{R_{\text{out}}}{R_{\text{in}}}\right)$ and $I_{\text{gain}} = \beta$

$$\Rightarrow \quad 10^6 = \beta^2 \left(\frac{10000}{100}\right)$$

$$\Rightarrow \quad \beta = 100$$

Hence, the correct answer is (C).

80. Figure (A) represent a reverse biased diode.

Hence, the correct answer is (A).

81. $\sigma = e(n_e\,\mu_e + n_h\,\mu_h)$

In an intrinsic semiconductor

$n_e = n_h = n_i$

$\therefore\quad \sigma = n_i e(\mu_e + \mu_h)$

$$\Rightarrow\quad n_i = \frac{\sigma}{e(\mu_e+\mu_h)} = \frac{2.13}{1.6\times10^{-19}(0.38+0.18)}$$

$$\Rightarrow\quad n_i = \frac{2.13\times10^{19}}{1.6\times0.56} = 2.37\times10^{19}\ \text{m}^{-3}$$

Hence, the correct answer is (A).

82. $I_B = I_E - I_C = 1\text{ mA} - 0.95\text{ mA} = 0.05\text{ mA}$

Hence, the correct answer is (B).

84. Input current $= \frac{V_i}{R_i} = \frac{0.01}{10^3}\text{A} = 10^{-5}\text{ A}$

$\Rightarrow$ Output collector current

$i_C = \beta i_B = 50\times10^{-5}\text{ A}$

$i_C = 500\ \mu\text{A}$

Hence, the correct answer is (B).

87. In forward biasing current is due to majority charge carriers which diffuse from higher concentration region to lower concentration region.

In reverse biasing the current is due to minority charge carriers which drift due to applied reverse potential difference.

Hence, the correct answer is (B).

88. $A_v = \beta\times\frac{R_2}{R_1} = 60\times\frac{24}{3} = 480$

Hence, the correct answer is (D).

90. Since diode D_1 is reverse biased, therefore it will act like an open circuit.

Effective resistance of the circuit is

$R = 4 + 2 = 6\ \Omega$

Current in the circuit is $I = \frac{E}{R} = \frac{12}{6} = 2\text{ A}$

Hence, the correct answer is (C).

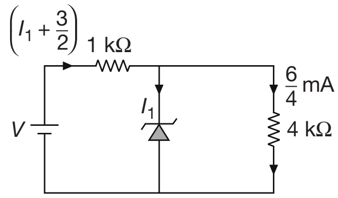
91. Peak value $E_0 = E_{rms}\sqrt{2} = 100\sqrt{2}$

Also $E = \sqrt{E_{\text{diode}}^2 + E_C^2} = E_C$, since $R_F = 0$

$\Rightarrow\quad (E_C)_{\text{maximum}} = E_0 = 100\sqrt{2}$ volt

Hence, the correct answer is (B).

94.



$$I_1 = \left(8 - 6 - \frac{3}{2}\right) = \frac{1}{2} = 0.5\text{ mA}$$

$$I_2 = \left(16 - 6 - \frac{3}{2}\right) = 8.5\text{ mA}$$

Hence, the correct answer is (B).

95. Depletion layer contains only fixed positive and negative ions; positive ions on N-side and negative ions on P-side.

Hence, the correct answer is (D).

98. The current will flow through R_L when diode is forward biased.

Hence, the correct answer is (A).

99. In first half cycle junction diode 2 is forward biased and 1 reverse biased; while in next half cycle junction diode 1 is forward biased and 2 is reverse biased. So, contributions from diode 1 are B, D.

Hence, the correct answer is (C).

102. $Y = \overline{A\cdot(A+B)} = \overline{A} + \overline{(A+B)}$

$\Rightarrow\quad Y = \overline{A} + \overline{A}\cdot\overline{B} = \overline{A}(1+\overline{B})$

$\Rightarrow\quad Y = \overline{A}$

Hence, the correct answer is (D).

104. $i = \frac{\Delta V}{R} = \frac{V_F - V_B}{R} = \frac{3-0.7}{100}$

$i = 2.3\times10^{-2}\text{ A} = 23\text{ mA}$

Hence, the correct answer is (D).

106. $\beta = \frac{I_c}{I_b} = \frac{5\times10^{-3}}{100\times10^{-6}} = 50$

Voltage gain $A_v = \beta\left(\frac{R_0}{R_i}\right) = 5\times10^4$

Power gain $A_p = \beta$ (voltage gain)

$\Rightarrow\quad A_p = 250\times10^4 = 2.5\times10^6$

Hence, the correct answer is (A).

107. C, Si and Ge have the same lattice structure and their valence electrons are 4. For C, these electrons are in the second orbit, for Si it is third and for germanium it is the fourth orbit. In solid state, higher the orbit, greater the possibility of overlapping of energy bands. Ionization energies are also less therefore Ge has more conductivity compared to Si. Both are semiconductors and carbon is an insulator.

Hence, the correct answer is (A).

108. Ge conducts at 0.3 V and silicon at 0.7 V. Both Ge and Si diodes are connected in parallel. When current begins to flow, the potential difference remains at 0.3 V, so no current flows through Si-diode.

$\therefore$ Potential difference across

$$R_L = 12 - 0.3 = 11.7 \text{ V}$$

$\therefore$ Potential of $Y = 11.7$ V

Hence, the correct answer is (D).

109. When polarity of Ge diode is reversed, Ge diode is reverse biased, now for conduction potential difference across Si becomes 0.7 V.
So potential difference across

$$R_L = 12 - 0.7 = 11.3 \text{ V}$$

$\therefore$ Potential of $Y = 11.3$ V

Hence, the correct answer is (C).

111. I_Z is maximum when input voltage is 16 V.

$$I_S = \frac{10}{2\times 10^3} = 5 \text{ mA}$$

$$I_L = \frac{6}{4\times 10^3} = 1.5 \text{ mA}$$

$$\Rightarrow \quad I_{Z(\max)} = I_S - I_L = 3.5 \text{ mA}$$

Hence, the correct answer is (D).

112. Output of upper AND gate $= A\bar{B}$

Output of lower AND gate $= B\bar{A}$

Output $Y = A\bar{B} + B\bar{A}$
This is Boolean expression for XOR gate.

Hence, the correct answer is (B).

114. Output of OR gate is 0 when all inputs are 0 and output is 1 when atleast one of the input is 1. Observing output x It is 0 when all inputs are 0 and it is 1 when atleast one of the input is 1. So, OR gate.

Hence, the correct answer is (D).

116. $\Delta E = \dfrac{hc}{\lambda}$

$$\Rightarrow \quad \lambda = \frac{hc}{\Delta E} = \frac{12375}{(\Delta E \text{ in eV})} \text{ Å}$$

$$\lambda = \frac{12375}{1.14} \text{ Å} = 10855 \text{ Å}$$

Hence, the correct answer is (A).

118. $I = neA\mu E$

$$I = \frac{neA\mu V}{L}$$

$$\frac{V}{I} = \frac{L}{neA\mu} = \rho\frac{L}{A}$$

$$\Rightarrow \quad \rho = \frac{1}{ne\mu} = \frac{1}{10^{19}\times 1.6\times 10^{-19}\times 1.6} = \frac{1}{2.56}$$

$$\Rightarrow \quad \rho = 0.4 \ \Omega\text{m}$$

Hence, the correct answer is (C).

119. $I_C = 19$ mA

and $\alpha = \dfrac{I_C}{I_E}$

$$\Rightarrow \quad I_E = \frac{I_C}{\alpha}$$

$$\Rightarrow \quad I_E = \frac{19}{0.95} \text{ mA} = 20 \text{ mA}$$

$$\Rightarrow \quad I_B = I_E - I_C = 20 - 19 = 1 \text{ mA}$$

Hence, the correct answer is (A).

121. $I_b = \dfrac{120-50}{5\times 10^3} = \dfrac{70}{5}\times 10^{-3}$

$$\Rightarrow \quad I_b = 14 \text{ mA and } I_L = \frac{50}{10\times 10^3} = 5 \text{ mA}$$

$$\Rightarrow \quad I_Z = 14 - 5 = 9 \text{ mA}$$

Hence, the correct answer is (B).

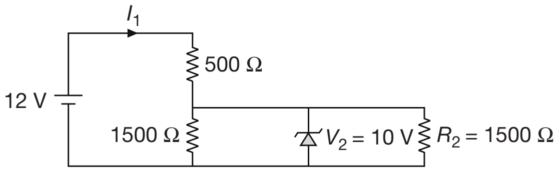
124. $A_v = \beta\dfrac{R_2}{R_1} = 10\times\dfrac{100 \ k\Omega}{20 \ k\Omega} = 50$

$\therefore$ Power gain $= \beta A_v = 10\times 50 = 500$

Hence, the correct answer is (C).

125. As is pentavalent impurity and In is trivalent impurity; so junction, X-Y is reverse biased n-p junction.
Hence, the correct answer is (D).

126.



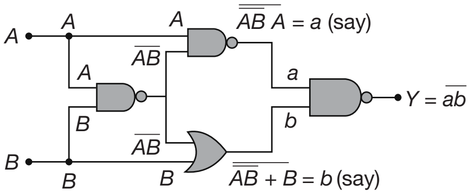
$$\left(V_{R_2}\right)_{max} = \frac{12 \times 750}{1250}$$

$$\left(V_{R_2}\right)_{max} < V_Z$$

So, current through Zener diode is zero.

Hence, the correct answer is (B).

129.



$$a = \overline{\overline{AB}\,A}$$

$$\Rightarrow \quad a = \overline{A} + \overline{\overline{AB}}$$

$$\Rightarrow \quad a = \overline{A} + AB$$

Now, $b = \overline{AB} + B$

$$\Rightarrow \quad b = \overline{A} + \overline{B} + B$$

$$\Rightarrow \quad b = \overline{A} + \left(\overline{B} + B\right)$$

$$\Rightarrow \quad b = \overline{A} + 1 = 1$$

Now $Y = \overline{ab}$

where $a = \overline{A} + AB$ and $b = 1$

$$\Rightarrow \quad Y = \overline{\overline{A} + AB}$$

$$\Rightarrow \quad Y = \overline{\overline{A}}\,\overline{AB} = A\left(\overline{A} + \overline{B}\right)$$

$$\Rightarrow \quad Y = A\overline{A} + A\overline{B}$$

Since $A\overline{A} = 0$

$$\Rightarrow \quad Y = A\overline{B}$$

Hence, the correct answer is (B).

133. $V_{CC} - I_C R_C = 0$

$$\Rightarrow \quad I_C = \frac{V_{CC}}{R_C} = \frac{5}{1000} \text{ A}$$

$$V_{BB} - I_B R_B = V_{BE} \qquad \ldots(1)$$

Also, $\beta = \dfrac{I_C}{I_B} = 200$

$$\Rightarrow \quad I_C = 200 I_B$$

$$\Rightarrow \quad I_B = \frac{I_C}{200} = 25\ \mu\text{A}$$

So, from equation (1), we get

$$V_{BB} - 25 \times 10^{-6} \times 100 \times 10^{3} = 1$$

$$\Rightarrow \quad V_{BB} = 3.5 \text{ V}$$

Hence, the correct answer is (A).

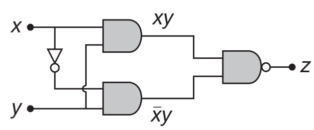
134. $\Delta E = \dfrac{hc}{\lambda} = \dfrac{12375}{\lambda \text{ in Å}} \text{ eV} = \dfrac{12375}{24800} = 0.5$

Hence, the correct answer is (C).

138. It is *npn* transistor with *R* as collector. If it is connected to base, it will be in forward bias.

Hence, the correct answer is (A).

141.



Output, $z = \overline{(xy)(\overline{x}y)} = \overline{0y} = \overline{0} = 1$

$$\{\because\ x\overline{x} = 0 \text{ and } yy = y\}$$

Whatever be the inputs to the given digital circuit, output will always be one.

Hence, the correct answer is (A).

143. Current in R_1 is $I_1 = \dfrac{5}{500} = 10 \times 10^{-3}$ A = 10 mA

Current in R_2 is $I_2 = \dfrac{10}{1500}$ A $= \dfrac{20}{3}$ mA

Current through Zener diode is

$$I_z = I_1 - I_2 = \left(10 - \frac{20}{3}\right) \text{ mA} = \frac{10}{3} \text{ mA} \approx 3.3 \text{ mA}$$

Hence, the correct answer is (B).

144. It is OR gate. When either of them conducts, the gate conducts.

Hence, the correct answer is (A).

CHAPTER 4

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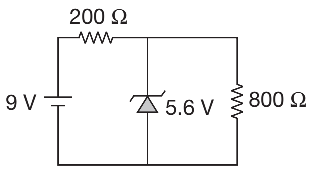
1.



$$I_{800\ \Omega} = \frac{5.6}{800}\ \text{A} = 7\ \text{mA}$$

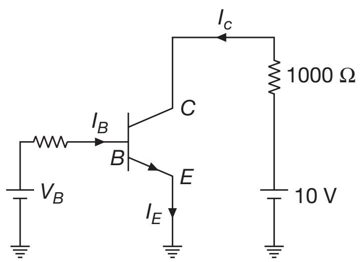
$$I_{200\ \Omega} = \frac{9-5.6}{200} = 17\ \text{mA}$$

$\Rightarrow \quad I_Z = 17 - 7 = 10\ \text{mA}$

Hence, the correct answer is (C).

2. For saturation, $V_{CC} - I_C \times R_C = 0$

$$\Rightarrow \quad I_C = \frac{V_{CC}}{R_C} = \frac{10}{10^3} = 10^{-2}\ \text{A}$$



Since, $\beta = \dfrac{I_C}{I_B} = 250$

$$\Rightarrow \quad I_B = \frac{I_C}{250} = \frac{10^{-2}}{250} = 40\ \mu\text{A}$$

Hence, the correct answer is (D).

3. $\beta = \dfrac{I_C}{I_B} = 200$

$$R_i = \frac{10 \times 10^{-3}}{15 \times 10^{-6}} = 0.67\ \text{k}\Omega$$

So, voltage gain is $A_v = \beta\left(\dfrac{R_o}{R_i}\right) = 300$

Hence, the correct answer is (D).

4. $Y = \overline{\overline{A}\cdot\overline{B}} = A + B$ (Using De-Morgan's Law)

$\Rightarrow$ OR gate

Hence, the correct answer is (A).

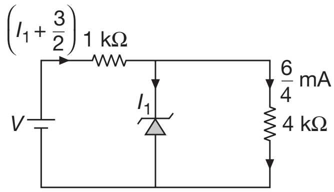
5. $P_{\text{gain}} = \beta^2\left(\dfrac{R_{\text{out}}}{R_{\text{in}}}\right)$ and $I_{\text{gain}} = \beta$

$$\Rightarrow \quad 10^6 = \beta^2\left(\frac{10000}{100}\right)$$

$\Rightarrow \quad \beta = 100$

Hence, the correct answer is (C).

6.



$$I_1 = \left(8 - 6 - \frac{3}{2}\right) = \frac{1}{2} = 0.5\ \text{mA}$$

$$I_2 = \left(16 - 6 - \frac{3}{2}\right) = 8.5\ \text{mA}$$

Hence, the correct answer is (B).

7. $Y = \overline{A\cdot(A+B)} = \overline{A} + \overline{(A+B)}$

$\Rightarrow \quad Y = \overline{A} + \overline{A}\cdot\overline{B} = \overline{A}(1+\overline{B})$

$\Rightarrow \quad Y = \overline{A}$

Hence, the correct answer is (D).

8. $\beta = \dfrac{I_c}{I_b} = \dfrac{5 \times 10^{-3}}{100 \times 10^{-6}} = 50$

Voltage gain $A_v = \beta\left(\dfrac{R_0}{R_i}\right) = 5 \times 10^4$

Power gain $A_p = \beta$ (voltage gain)

$\Rightarrow \quad A_p = 250 \times 10^4 = 2.5 \times 10^6$

Hence, the correct answer is (A).

9. I_Z is maximum when input voltage is 16 V.

$$I_S = \frac{10}{2 \times 10^3} = 5\ \text{mA}$$

$$I_L = \frac{6}{4 \times 10^3} = 1.5\ \text{mA}$$

$\Rightarrow \quad I_{Z(\max)} = I_S - I_L = 3.5\ \text{mA}$

Hence, the correct answer is (D).

10. $I = neA\mu E$

$$I = \frac{neA\mu V}{L}$$

$$\frac{V}{I} = \frac{L}{neA\mu} = \rho\frac{L}{A}$$

$$\Rightarrow \quad \rho = \frac{1}{ne\mu} = \frac{1}{10^{19} \times 1.6 \times 10^{-19} \times 1.6} = \frac{1}{2.56}$$

$$\Rightarrow \quad \rho = 0.4 \ \Omega\text{m}$$

Hence, the correct answer is (C).

11. Voltage drop across diode will change from 0.3 to 0.7 V. Value of V_0 changes by 0.4 V.

Hence, the correct answer is (B).

12. $A = \left[\overline{(\bar{X}+\bar{Y}) + \overline{\overline{X\bar{Y}}}}\right]$

$$\Rightarrow \quad A = \overline{\bar{\bar{X}} + Y + \bar{\bar{X}} + Y}$$

$$\Rightarrow \quad A = \left(\overline{\bar{\bar{X}} + Y}\right)$$

Output is 1 when $X = 1$, $Y = 0$

Hence, the correct answer is (C).

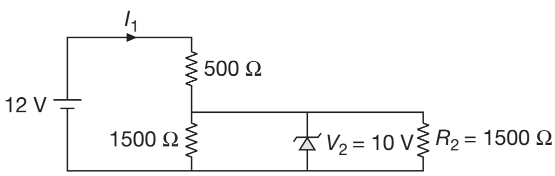
13. $I_b = \frac{120-50}{5 \times 10^3} = \frac{70}{5} \times 10^{-3}$

$$\Rightarrow \quad I_b = 14 \text{ mA and } I_L = \frac{50}{10 \times 10^3} = 5 \text{ mA}$$

$$\Rightarrow \quad I_Z = 14 - 5 = 9 \text{ mA}$$

Hence, the correct answer is (B).

14.



$$(V_{R_2})_{max} = \frac{12 \times 750}{1250}$$

$$(V_{R_2})_{max} < V_Z$$

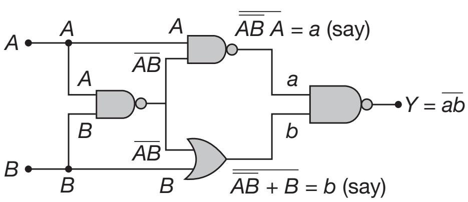
So, current through Zener diode is zero.

Hence, the correct answer is (B).

15. $I = \frac{6}{50+150+100} = \frac{6}{300} \text{ A} = 0.02 \text{ A}$

Hence, the correct answer is (B).

16.



$$a = \overline{\overline{AB}\,A}$$

$$\Rightarrow \quad a = \bar{A} + \overline{\overline{AB}}$$

$$\Rightarrow \quad a = \bar{A} + AB$$

Now, $b = \overline{AB} + B$

$$\Rightarrow \quad b = \bar{A} + \bar{B} + B$$

$$\Rightarrow \quad b = \bar{A} + (\bar{B} + B)$$

$$\Rightarrow \quad b = \bar{A} + 1 = 1$$

Now $Y = \overline{ab}$

where $a = \bar{A} + AB$ and $b = 1$

$$\Rightarrow \quad Y = \overline{\bar{A} + AB}$$

$$\Rightarrow \quad Y = \bar{\bar{A}}\,\overline{AB} = A(\bar{A} + \bar{B})$$

$$\Rightarrow \quad Y = A\bar{A} + A\bar{B}$$

Since $A\bar{A} = 0$

$$\Rightarrow \quad Y = A\bar{B}$$

Hence, the correct answer is (B).

17. $V_{CC} - I_C R_C = 0$

$$\Rightarrow \quad I_C = \frac{V_{CC}}{R_C} = \frac{5}{1000} \text{ A}$$

$$V_{BB} - I_B R_B = V_{BE} \qquad \ldots(1)$$

Also, $\beta = \frac{I_C}{I_B} = 200$

$$\Rightarrow \quad I_C = 200 I_B$$

$$\Rightarrow \quad I_B = \frac{I_C}{200} = 25 \ \mu\text{A}$$

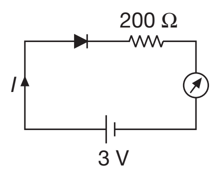
So, from equation (1), we get

$$V_{BB} - 25 \times 10^{-6} \times 100 \times 10^3 = 1$$

$$\Rightarrow \quad V_{BB} = 3.5 \text{ V}$$

Hence, the correct answer is (A).

18. Current in the circuit,



$$I = \frac{V - V_{diode}}{R}$$

$$\Rightarrow \quad I = \frac{3 - 0.7}{200} = \frac{2.3}{200} \text{ A}$$

$$\Rightarrow \quad I = \frac{2300}{200} \text{ mA} = 11.5 \text{ mA}$$

Hence, the correct answer is (C).

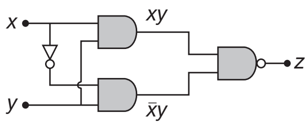
19. Current gain, $\beta = \dfrac{\Delta I_C}{\Delta I_B}$

Voltage gain, $A_V = \dfrac{\Delta V_{CE}}{R_{BE}\Delta I_B} = \beta \dfrac{R_L}{R_{BE}}$

Power gain, $A_P = \beta A_V = \beta^2 \dfrac{R_L}{R_{BE}}$

Hence, the correct answer is (B).

20.



Output, $z = \overline{(xy)(\bar{x}y)} = \overline{0y} = \bar{0} = 1$

$\{\because x\bar{x} = 0 \text{ and } yy = y\}$

Whatever be the inputs to the given digital circuit, output will always be one.
Hence, the correct answer is (A).

21. Current in R_1 is $I_1 = \dfrac{5}{500} = 10 \times 10^{-3}\text{ A} = 10\text{ mA}$

Current in R_2 is $I_2 = \dfrac{10}{1500}\text{ A} = \dfrac{20}{3}\text{ mA}$

Current through Zener diode is

$$I_z = I_1 - I_2 = \left(10 - \frac{20}{3}\right)\text{mA} = \frac{10}{3}\text{ mA} \approx 3.3\text{ mA}$$

Hence, the correct answer is (B).

22. In common emitter configuration for *n-p-n* transistor, phase difference between output and input voltage is $180°$.

Hence, the correct answer is (D).

23. Forward bias resistance is

$$R_{FB} = \frac{\Delta V}{\Delta I_{\text{for}}} = \frac{0.8 - 0.7}{(20 - 10)\times 10^{-3}} = \frac{0.1}{10 \times 10^{-3}} = 10$$

Reverse bias resistance, $R_{RB} = \dfrac{10}{1 \times 10^{-6}} = 10^7$ then, the ratio of forward to reverse bias resistance

$$\frac{R_{FB}}{R_{RB}} = \frac{10}{10^7} = 10^{-6}$$

Hence, the correct answer is (D).

24. Given that $n_e = 5 \times 10^{18}\text{ m}^{-3}$, $n_h = 5 \times 10^{19}\text{ m}^{-3}$,

$\mu_e = 2\text{ m}^2\text{V}^{-1}\text{s}^{-1}$ and $\mu_h = 0.01\text{ m}^2\text{V}^{-1}\text{s}^{-1}$

So, conductivity is

$$\sigma = e(n_e\mu_e + n_h\mu_h)$$

Substituting values, we get

$$\sigma = 1.6 \times 10^{-19}(5 \times 10^{18} \times 2 + 5 \times 10^{19} \times 0.01)$$

$$\Rightarrow \sigma = 1.6 \times 10^{-19}(10^{19} + 0.05 \times 10^{19}) = 1.68\ (\Omega\text{m})^{-1}$$

Hence, the correct answer is (B).

25. Since, $\beta = 69$, $I_e = 7$ mA, $I_c = ?$

$$\Rightarrow \alpha = \frac{\beta}{1+\beta} = \frac{69}{70}$$

Also, $\alpha = \dfrac{I_c}{I_e}$

$$\Rightarrow \frac{69}{70} = \frac{I_c}{7}$$

$$\Rightarrow I_c = \frac{69}{70} \times 7 = 6.9\text{ mA}$$

Hence, the correct answer is (B).

26. Output of OR gate is 0 when all inputs are 0 and output is 1 when atleast one of the input is 1. Observing output x It is 0 when all inputs are 0 and it is 1 when atleast one of the input is 1. So, OR gate.

(A) Zener diode, Solar cell, Simple diode, Light dependent resistance
(B) Simple diode, Zener diode, Solar cell, Light dependent resistance
(C) Zener diode, Simple diode, Light dependent resistance, Solar cell
(D) Solar cell, Light dependent resistance, Zener diode, Simple diode

Hence, the correct answer is (D).

28. $\alpha = \dfrac{I_C}{I_e}$, $\beta = \dfrac{I_C}{I_b}$

$$I_e = I_b + I_c$$

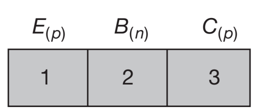
$$\Rightarrow \frac{I_e}{I_c} = \frac{I_b}{I_c} + 1$$

$$\Rightarrow \frac{1}{\alpha} = \frac{1}{\beta} + 1$$

$$\alpha = \frac{\beta}{1+\beta}$$

Hence, the correct answer is (A, C).

29. p-n-p transistor

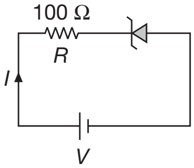


Positive at terminal 2 and negative at terminal 1 implies p-n junction is reverse biased and hence offers high resistance.

Hence, the correct answer is (B).

30. Potential drop across Zener diode

$$V_Z = V - IR = V - 100I$$



Since, power, $P = V_Z I_Z$

$$\Rightarrow \quad P = (V - 100I)I$$

Given that, $P = 1$ W

$$\Rightarrow \quad (V - 100I)I = 1$$

$$\Rightarrow \quad 100I^2 - VI + 1 = 0$$

For I to be real, $V^2 - 4 \times 100 \times 1 \geq 0$

$$\Rightarrow \quad V \geq 20 \text{ V}$$

Hence, the correct answer is (B).

31. The given truth table represents OR gate.
Hence, the correct answer is (A).

32. $R = \dfrac{\text{Output resistance } (r_o)}{\text{Input resistance } (r_i)} \equiv 1 - 10$

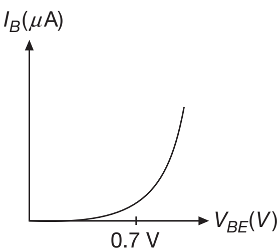
Hence, the correct answer is (B).

33. Output obtained is $Y = (a + b)c$

So, $Y = 1$, when $c = 1$ and $a = 0$, $b = 1$ or $a = 1$, $b = 0$

Hence, the correct answer is (C).

34. For common emitter configuration, the input characteristic



$$r_i = \frac{\Delta V_{BE}}{\Delta I_B}$$

$$\Rightarrow \quad \frac{1}{r_i} = \frac{dI_B}{dV_{BE}} = \text{Slope of } (I_B - V_{BE}) \text{ curve}$$

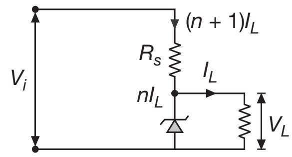
Slope of the input characteristic is almost constant upto knee voltage (0.7 V). Then it increases sharply. Hence option (C) is the correct choice.

Hence, the correct answer is (C).

35. Electron concentration in n-region is more as compared to that in p-region. So, electrons diffuse from n-side to p-side.

Hence, the correct answer is (C).

36.



Voltage drop across Zener diode is V_L, so voltage drop across R_S is,

$$V_{RS} = V_i - V_L = (n+1)I_L R_S$$

$$\Rightarrow \quad R_S = \frac{V_i - V_L}{(n+1)I_L}$$

Hence, the correct answer is (C).

37. When positive terminal of battery is connected to A, current passes through diode D_1.

$$\Rightarrow \quad \text{Current supplied} = \frac{2 \text{ V}}{5\ \Omega} = 0.4 \text{ A}$$

When positive terminal is connected to B current passes through D_2.

$$\Rightarrow \quad \text{Current supplied} = \frac{2 \text{ V}}{10\ \Omega} = 0.2 \text{ A}$$

Hence, the correct answer is (B).

38. Since $I = e^{\frac{1000V}{T}} - 1$

$$\Rightarrow \quad I + 1 = e^{\frac{1000V}{T}}$$

$$\Rightarrow \quad \log_e(I+1) = \frac{1000 \text{ V}}{T}$$

$$\Rightarrow \quad d[\log_e(I+1)] = d\left(\frac{1000 \text{ V}}{T}\right)$$

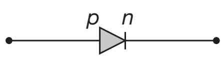
$$\Rightarrow \quad \frac{dI}{I+1} = \left(\frac{1000}{T}\right)dV$$

$$\Rightarrow \quad \frac{dI}{(5+1)mA} = \left(\frac{1000}{300}\right)(0.01)$$

$$\Rightarrow \quad dI = 0.2 \text{ mA}$$

Hence, the correct answer is (B).

39.



In forward bias, the p-side of diode is at higher potential with respect to the potential of n-side.

Hence, the correct answer is (B).

CHAPTER 4

40. When wavelength exceeds a certain wavelength, then photoelectric effect ceases to exist.
Hence, the correct answer is (D).

41. $\tau = RC = 100\times10^3 \times 250\times10^{-12}$ s

$\Rightarrow \quad \tau = 2.5\times10^7 \times 10^{-12}$ s

$\Rightarrow \quad \tau = 2.5\times10^{-5}$ s

The higher frequency which can be detected with tolerable distortion is

$$f = \frac{1}{2\pi mRC} = \frac{1}{2\pi\times0.6\times2.5\times10^{-5}}\ \text{Hz}$$

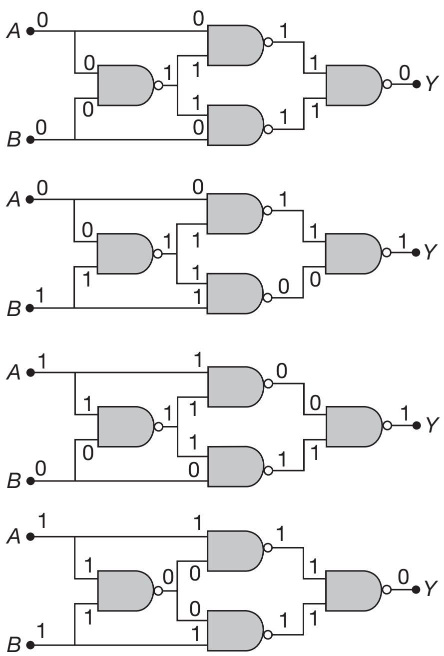
$$\Rightarrow \quad f = \frac{100\times10^4}{25\times1.2\pi}\ \text{Hz}$$

$$\Rightarrow \quad f = \frac{4}{1.2\pi}\times10^{-4}\ \text{Hz}$$

$$\Rightarrow \quad f = 10.61\ \text{KHz}$$

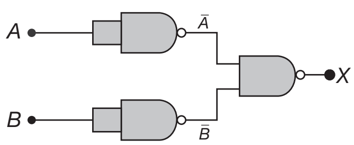
This condition is obtained by applying the condition that rate of decay of capacitor voltage must be equal or less than the rate of decay modulated signal voltage for proper detection of modulated signal.
Hence, the correct answer is (B).

42.



Hence, the correct answer is (D).

43.



The Boolean expression of the given circuit is

$$X = \overline{\bar{A}\cdot\bar{B}}$$

$$= \bar{\bar{A}} + \bar{\bar{B}} \qquad \text{(Using De Morgan's theorem)}$$

$$= A + B \qquad \text{(Using Boolean identity)}$$

This is same as the Boolean expression of OR gate.

Alternative method

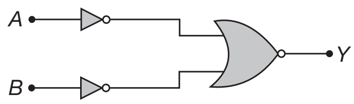
The truth table of the given circuit is as shown in the table

A	B	$\bar{A}$	$\bar{B}$	$\bar{A}\bar{B}$	$X = \overline{\bar{A}\bar{B}}$
0	0	1	1	1	0
0	1	1	0	0	1
1	0	0	1	0	1
1	1	0	0	0	1

This is same as that of OR gate.

Hence, the correct answer is (B).

44.



By de Morgan's theorem, $\left(\overline{\bar{A}+\bar{B}}\right) = A\cdot B$

A	B	$\bar{A}$	$\bar{B}$	$\bar{A}+\bar{B}$	$\overline{\bar{A}+\bar{B}}$	$A\cdot B$
1	1	0	0	0	1	1
0	0	1	1	1	0	0
0	1	1	0	1	0	0
1	0	0	1	1	0	0

This is the same as AND Gate of A and B.

Hence, the correct answer is (A).

45. (A) is original wave, (B) is a full-wave rectified, (C) is the correct choice. The negative waves are cut off when the diode is connected in reverse bias, (D) is not the diagram for alternating current.

Hence, the correct answer is (C).

CHAPTER 5: COMMUNICATION SYSTEMS

Single Correct Choice Type Questions

1. In point-to-point communication mode, communication takes place over a link between a single transmitter and a receiver. Telephony is an example of such a mode of communication.
Hence, the correct answer is (B).

2. Radio waves can be transmitted from one place to another as ground wave or sky wave or space wave propagation.
Hence, the correct answer is (D).

3. VHF (Very High Frequency) band having frequency range 30 MHz to 300 MHz is typically used for TV and RADAR transmission.
Hence, the correct answer is (A).

4. It mixes weak signals with carrier signals.
Hence, the correct answer is (B).

5. An antenna is a metallic structure used to radiate or receive EM waves.
Hence, the correct answer is (C).

6. In optical fibre, light travels inside it, due to total internal reflection.
Hence, the correct answer is (C).

7. Remote sensing is the technique to collect information about an object in respect of its size, colour, nature, location, temperature etc. without physically touching it. There are some areas or locations which are inaccessible. So, to explore these areas or locations, a technique known as remote sensing is used, Remote sensing is done through a satellite.
Hence, the correct answer is (D).

8. Duplex or full duplex refers to the simultaneous transmission of data in two directions. A mobile phone is a full duplex device because both people can talk at once and hear each at the same time. Walky-talky is a half duplex device because only one person can talk at a time.
Hence, the correct answer is (A).

9. Modulation index, $\mu = \dfrac{A_m}{A_c}$

$$\Rightarrow \quad \mu = \frac{0.5V}{10V} = 0.05$$

The side bands frequencies are $f_{SB} = f_c \pm f_m$

$$\Rightarrow \quad f_{SB} = (1 \pm 0.010)\text{ MHz}$$

Hence, the correct answer is (A).

10. Optical source frequency $f = \dfrac{c}{\lambda}$

$$\Rightarrow \quad f = \frac{3\times 10^8\text{ ms}^{-1}}{1200\times 10^{-9}\text{ m}} = 2.5\times 10^{14}\text{ Hz}$$

Bandwidth of channel (2% of the source frequency), so

$$\Delta f = 5\times 10^{12}\text{ Hz}$$

Number of channels is

$$N = \frac{\text{Total bandwidth}}{\text{Bandwidth needed per channel}}$$

$$\Rightarrow \quad N = \frac{5\times 10^{12}\text{ Hz}}{5\times 10^{6}\text{ Hz}} = 10^6 = 1\text{ million}$$

Hence, the correct answer is (D).

11. Power radiated by the antenna is proportional to $\left(\dfrac{\ell}{\lambda}\right)^2$. When both the length of the antenna ℓ and wavelength of the signal λ are doubled, the power radiated by the antenna remains constant.
Hence, the correct answer is (C).

12. Modem performs the functions of both the modulator and the demodulator. Modem acts as a modulator in the transmitting mode and it acts as a demodulator in the receiving mode.
Hence, the correct answer is (C).

13. When $m_a > 1$, then carrier is said to be over modulated.
Hence, the correct answer is (D).

14. Here,

$$C_m(t) = 30\sin(300\pi t) + 10(\cos(200\pi t) - \cos(400\pi t))$$

Compare this equation with standard equation of amplitude modulated wave i.e.,

$$C_m(t) = A_c\sin\omega_c t - \frac{\mu A_c}{2}\cos(\omega_c + \omega_m)t + \frac{\mu A_c}{2}\cos(\omega_c - \omega_m)t$$

So, we get

$$A_c = 30\text{ V},\ \omega_c = 300\pi$$

$$\Rightarrow \quad 2\pi f_c = 300\pi$$

$$\Rightarrow \quad f_c = 150\text{ Hz}$$

Since $\omega_c - \omega_m = 200\pi$

$\Rightarrow \quad f_c - f_m = 100 \text{ Hz}$

$\Rightarrow \quad f_m = 150 - 100 = 50 \text{ Hz}$

Also, $\dfrac{\mu A_c}{2} = 10$ and $A_c = 30$

$\Rightarrow \quad \mu = \dfrac{10}{15} = \dfrac{2}{3}$

Hence, the correct answer is (B).

15. $P_t = P_c\left(1+\dfrac{m^2}{2}\right) = 9\left(1+\dfrac{(0.4)^2}{2}\right)$

$\Rightarrow \quad P_t = 9\left(1+\dfrac{0.16}{2}\right) \qquad \{\because m = 40\% = 0.4\}$

$\Rightarrow \quad P_t = 9(1.08) = 9.72 \text{ kW}$

Hence, the correct answer is (B).

16. $d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$\Rightarrow \quad 40\times1000 = \sqrt{2\times6.4\times10^6\times h} + \sqrt{2\times6.4\times10^6\times45}$

$\Rightarrow \quad 40\times10^3 = \sqrt{2\times6.4\times10^6\times h} + 24\times10^3$

$\Rightarrow \quad h = \dfrac{[\{40-24\}10^3]^2}{2\times6.4\times10^6} = 20 \text{ m}$

Hence, the correct answer is (B).

17. $P_t = P_c\left(1+\dfrac{m_a^2}{2}\right)$, where $m_a = 1$

$\Rightarrow \quad 1800 = P_c\left(1+\dfrac{(1)^2}{2}\right)$

$\Rightarrow \quad P_c = 1200 \text{ W}$

Hence, the correct answer is (B).

18. Since, $f_c \approx 9(N_{\max})^{\frac{1}{2}}$

$\Rightarrow \quad f_c \approx 2 \text{ MHz}$

Hence, the correct answer is (A).

19. The sky waves are the radio waves of frequency between 2 MHz to 30 MHz. These waves can propagate through atmosphere and are reflected back the ionosphere of earth's atmosphere.

Hence, the correct answer is (C).

20. Here, $f_c = 1.5 \text{ MHz} = 1500 \text{ kHz}$, $f_m = 10 \text{ kHz}$

$\Rightarrow$ Low side band frequency is

$f_{LSB} = f_c - f_m = 1500 \text{ kHz} - 10 \text{ kHz} = 1490 \text{ kHz}$

Upper side band frequency is

$f_{USB} = f_c + f_m = 1500 \text{ kHz} + 10 \text{ kHz} = 1510 \text{ kHz}$

Hence, the correct answer is (A).

21. Some important wireless communication frequency bands

Name of Service	Frequency bands	Remarks
Standard *AM* broadcast	540-1600 kHz	
FM broadcast	88-108 MHz	
Television	54-72 MHz	VHF (very high frequencies)
	76-88 MHz	(VHF) TV
	174-216 MHz	UHF (ultra high frequencies)
	420-890 MHz	(UHF) TV
Cellular Mobile Radio	896-901 MHz	Mobile to base station
	840-935 MHz	Base station to mobile
Satellite communication	5.925-6.425 GHz	Uplink
	3.7-4.2 GHz	Downlink

Hence, the correct answer is (A).

22. Since, modulation index, $\mu = 0.5$

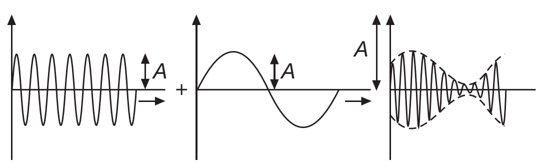
Amplitude of the carrier wave is A_c

Amplitude of the side band is $A_{SB} = \mu\dfrac{A_c}{2}$

So, their ratio is $\dfrac{A_C}{A_{SB}} = \dfrac{2}{\mu} = \dfrac{2}{0.5} = \dfrac{4}{1}$

Hence, the correct answer is (A).

23. When signal amplitude is equal to the carrier amplitude, the amplitude of carrier wave varies between $2A$ and zero.



$m_a = \dfrac{\text{Amplitude of charge carrier}}{\text{Amplitude of normal carrier}} = \dfrac{2A - A}{A}\times100 = 100\%$

Hence, the correct answer is (A).

24. $\omega_U = (2.2\times10^4 + 5.5\times10^5)$ rads^{-1} and

$\omega_L = (5.5\times10^5 - 2.2\times10^4)$ rads^{-1}

$\Rightarrow \omega_U = (2.2+55)\times10^4 = 57.2\times10^4$ rads^{-1}

$\Rightarrow \omega_L = 52.8\times10^4$ rads^{-1}

$\Rightarrow f_U = \frac{572}{2\pi}$ kHz $\simeq$ 91 kHz

$\Rightarrow f_L = \frac{528}{2\pi}$ kHz $\simeq$ 84 kHz

Hence, the correct answer is (C).

25. Optical source frequency

$$f = \frac{c}{\lambda} = \frac{3\times10^8}{(800\times10^{-9})} = 3.8\times10^{14} \text{ Hz}$$

Bandwidth of channel (1% of above) $= 3.8\times10^{12}$ Hz

Number of channels is

$$N = \frac{\text{Total bandwidth of channel}}{\text{Bandwidth needed per channel}}$$

$$\Rightarrow N = \frac{3.8\times10^{12}}{8\times10^3} \approx 4.8\times10^8$$

Hence, the correct answer is (A).

26. The frequency of AM channel is 1020 kHz whereas for the FM it is 89.5 MHz (given). For higher frequencies (MHz), space wave communication is needed. Very tall towers are used as antennas.
Hence, the correct answer is (B).

27. $P_{sb} = P_c\left(\frac{m_a}{2}\right)^2 = P_c\frac{(0.5)^2}{4} = 0.0625P_c$

Also $P = P_c\left(1+\frac{m_a^2}{2}\right) = P_c\left(1+\frac{(0.5)^2}{2}\right) = 1.125P_c$

$$\Rightarrow \%\text{saving} = \frac{(1.125P_c - 0.0625P_c)}{1.125P_c}\times100 = 94.4\%$$

Hence, the correct answer is (C).

28. Modulation is a process of superposing a low frequency audio signals (called modulating signal) on a high frequency radio wave called carrier wave.
Hence, the correct answer is (A).

29. The formula for modulating index is given by

$$m_f = \frac{\Delta f}{f_m} = \frac{\text{Frequency variation}}{\text{Modulating frequency}} = \frac{10\times10^3}{2\times10^3} = 5$$

Hence, the correct answer is (B).

30. Carrier frequency > audio frequency
Hence, the correct answer is (D).

31. $C_m = (A_C + A_m\sin\omega_m t)\sin(\omega_C t)$

From the graph and

$A_C + A_m = 10$

$A_C - A_m = 8$

$\Rightarrow A_C = 9$ V , $A_m = 1$ V

Since, $\omega_m = \frac{2\pi}{100\times10^{-6}} = 2\pi\times10^4$ s^{-1} and

$\omega_C = \frac{2\pi}{8\times10^{-6}} = 2.5\pi\times10^5$ s^{-1}

$\Rightarrow C_m = (9+\sin 2\pi\times10^4 t)\sin(2.5\pi\times10^5 t)$ V

Hence, the correct answer is (A).

32. The critical frequency of a sky wave for reflection from a layer of atmosphere is given by $f_c = 9(N_{max})^{\frac{1}{2}}$

$\Rightarrow 10\times10^6 = 9(N_{max})^{\frac{1}{2}}$

$$\Rightarrow N_{max} = \left(\frac{10\times10^6}{9}\right)^2 \simeq 1.2\times10^{12} \text{ m}^{-3}$$

Hence, the correct answer is (A).

33. Maximum amplitude, $A_{max} = A_c + A_m$ ...(1)

Minimum amplitude, $A_{min} = A_c - A_m$ ...(2)

Solving (1) and (2), we get

$A_c = \frac{A_{max}+A_{min}}{2}$ and

$A_m = \frac{A_{max}-A_{min}}{2}$

Modulation index μ is defined as

$$\mu = \frac{A_m}{A_c} = \frac{A_{max}-A_{min}}{A_{max}+A_{min}}$$

$$\Rightarrow \mu = \frac{25\ V - 5\ V}{25\ V + 5\ V} = \frac{20}{30} = \frac{2}{3}$$

Hence, the correct answer is (D).

34. $\mu(\%) = \frac{E_{max}-E_{min}}{E_{max}+E_{min}}\times100$

$\Rightarrow \mu(\%) = \frac{12-4}{12+4}\times100 = 50\%$

Hence, the correct answer is (B).

35. In frequency modulation the frequency of the modulated wave is the linear function of the amplitude of the modulating wave.
Hence, the correct answer is (B).

36. Given that, $\frac{\mu_1-\mu_2}{\mu_1}=\frac{0.88}{100}$

$\Rightarrow \quad \frac{\mu_2}{\mu_1}=0.9912$

$\Rightarrow \quad$ Critical angle $\theta_c=\sin^{-1}\left(\frac{\mu_2}{\mu_1}\right)$

$\Rightarrow \quad \theta_c=\sin^{-1}(0.9912)=82°24'$

Hence, the correct answer is (D).

37. Here, $V_{\max}=\frac{24}{2}=12\text{ mV}$ and $V_{\min}=\frac{8}{2}=4\text{ mV}$

Now, $m=\frac{V_{\max}-V_{\min}}{V_{\max}+V_{\min}}=\frac{12-4}{12+4}=\frac{8}{16}=\frac{1}{2}=0.5=50\%$

Hence, the correct answer is (D).

38. Velocity of electromagnetic waves in free space and wavelength are

$$v=3\times10^8\text{ ms}^{-1}\text{ and }\lambda=150\text{ m}$$

The frequency f of radio waves is given by

$$f=\frac{v}{\lambda}=\frac{3\times10^8}{150}=2\times10^6\text{ Hz}=2\text{ MHz}$$

Hence, the correct answer is (C).

39. Bandwidth Δf is equal to twice the frequency of modulating signals, so

$$\Delta f=2f_m=2\times4000\text{ Hz}=8\text{ kHz}$$

Hence, the correct answer is (A).

40. For demodulation $\frac{1}{f_c}\ll RC$

$$\frac{1}{f_c}=\frac{1}{100\times10^3}=10^{-5}\text{ s}$$

$$RC=10^3\times10\times10^{-12}\text{ s}=10^{-8}\text{ s}$$

We see that $\frac{1}{f_c}$ here is not less than RC as required by the above condition. Hence, this is not good.

Hence, the correct answer is (B).

41. If n is the number of bits per sample, then number of quantisation levels $=2^n$

Since the number of quantisation levels is 16

$\Rightarrow \quad 2^n=16$

$\Rightarrow \quad n=4$

$$\Rightarrow \quad \begin{pmatrix}\text{Bit}\\\text{Rate}\end{pmatrix}=\begin{pmatrix}\text{Sampling}\\\text{Rate}\end{pmatrix}\times\begin{pmatrix}\text{Number of}\\\text{bits per sample}\end{pmatrix}$$

$\Rightarrow \quad$ Bit Rate $=8000\times4=32{,}000$ bits/s

Hence, the correct answer is (A).

42. A maximum frequency deviation of 75 kHz is permitted for commercial FM broadcast stations in the 88 to 108 MHz VHF band.

Hence, the correct answer is (A).

43. Frequency modulation requires much wider channel (7 to 15 times) as compared to AM.

Hence, the correct answer is (A).

44. Problems which are faced while transmitting audio signals directly are

(i) These signals are relatively of short range.

(ii) If every body started transmitting these low frequency signals directly, mutual interference will render all of them ineffective.

(iii) Size of antenna required for their efficient radiation would be larger, i.e., about 75 km

Hence, the correct answer is (D).

45. Area covered by T.V. signals is

$$A=\pi d^2=\pi\left(\sqrt{2hR}\right)^2$$

$\Rightarrow \quad A=2\pi hR=2\times3.14\times100\times6.4\times10^6=128\pi\times10^8$

$\Rightarrow \quad A=1.28\pi\times10^3\text{ km}^2$

Hence, the correct answer is (B).

46. For demodulation

$$\frac{1}{f_C}\ll RC$$

where f_C is the frequency of the carrier signal

Here, $R=1\text{ k}\Omega$, $C=1\ \mu\text{F}$

$\Rightarrow \quad RC=1\times10^3\ \Omega\times1\times10^{-6}\text{ F}=10^{-3}\text{ s}$

In OPTION (C)

$$\frac{1}{f_C}=\frac{1}{10\times10^3}=0.1\times10^{-3}\text{ s}$$

Since, only in OPTION (C), $\frac{1}{f_c}\ll RC$, hence (C) is correct.

Hence, the correct answer is (C).

47. $n_{eff}=n_0\sqrt{1-\left(\frac{81\text{ N}}{f^2}\right)}=1\sqrt{1-\frac{81\times(400\times10^6)}{(55\times10^6)^2}}\approx1$

Also $n_{eff}=\frac{\sin i}{\sin r}$

$\Rightarrow \quad \sin r=\sin i$

$\Rightarrow \quad r=i=45°$

Hence, the correct answer is (B).

48. A very small part of light energy is lost from an optical fibre due to absorption or due to light leaving the fibre as a result of scattering of light sideways by impurities in the glass fibre.
Hence, the correct answer is (D).

49. Maximum Range of the radar is given by

$$R_{max} = \left(\frac{P_t A^2 S}{4\pi\lambda^2 P_{min}}\right)^{\frac{1}{4}}$$

where P_t is the, peak value of transmitted power, A is capture area of the receiving antenna, S is radar cross-sectional area, λ is wavelength of RADAR wave, P_{min} is minimum receivable power of the receiver.
Hence, the correct answer is (B).

50. $m_f = \frac{\Delta f}{f_m} = \frac{2250}{500} = 4.5$

$\Rightarrow$ New deviation $= 2(m_f f_m) = 2\times 4.5\times 6 = 54$ kHz

Hence, the correct answer is (B).

51. Let $x(t) = A_c \sin\omega_c t$ represents carrier wave $y(t) = A_m \sin\omega_m t$ represents the modulating signal. The modulated signal $x_m(t)$ can be written as

$$x_m(t) = (A_c + A_m \sin\omega_m t)\sin\omega_c t$$

$$\Rightarrow x_m(t) = A_c\left(1 + \frac{A_m}{A_c}\sin\omega_m t\right)\sin\omega_c t$$

$$\Rightarrow x_m(t) = A_c\sin(\omega_c t) + \mu A_c \sin(\omega_m t)\sin(\omega_c t)$$

where $\mu = \frac{A_m}{A_c}$ is the modulation index.

Since, $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

$$\Rightarrow x_m(t) = A_c\sin\omega_c t + \frac{\mu A_c}{2}[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t]$$

$$\Rightarrow x_m(t) = A_c\sin(\omega_c t) + \frac{\mu A_c}{2}\cos(\omega_c - \omega_m)t - \frac{\mu A_c}{2}\cos(\omega_c + \omega_m)t$$

Amplitude modulated wave contains the frequencies ω_c, $(\omega_c - \omega_m)$ and $(\omega_c + \omega_m)$. So, the frequency ω_m is not contained in the amplitude modulated wave.
Hence, the correct answer is (A).

52. The distance of coverage of a transmitting antenna is

$$d_T = \sqrt{2Rh_T}$$

where R is the radius of the earth and h_T is the height of the transmitting antenna

$$h_T = \frac{d_T^2}{2R} = \frac{(12.8\times 10^3 \text{ m})^2}{2\times 6400\times 10^3 \text{ m}} = 12.8 \text{ m}$$

Hence, the correct answer is (B).

53. $f_{upper} = f_c + f_s$

$\Rightarrow$ $f_{upper} = (2510 + 12)$ kHz

$\Rightarrow$ $f_{upper} = 2522$ kHz

$f_{lower} = f_c - f_s = 2498$ kHz

Hence, the correct answer is (D).

54. $d = \sqrt{2hR}$

$\Rightarrow$ $d \propto h^{\frac{1}{2}}$

Hence, the correct answer is (A).

55. Acceptance angle of the core is

$$\theta = \sin^{-1}\sqrt{n_1^2 - n_2^2}$$

Hence, the correct answer is (B).

56. $v = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times 3.14\sqrt{10\times 10^{-6}\times 1\times 10^{-9}}} = 1592$ kHz

Hence, the correct answer is (C).

57. Few advantages of optical fibres are that the number of signals carried by optical fibres is much more than that carried by the Cu wire or radio waves. Optical fibres are practically free from electromagnetic interference and problem of cross talks whereas ordinary cables and microwave links suffer a lot from it.
Hence, the correct answer is (D).

58. In *PWM* (pulse width modulation), which is also called *PDM* (Pulse duration modulation), width of pulses vary with modulating signal
Hence, the correct answer is (C).

59. $d = \sqrt{2hR} = \sqrt{2\times 500\times 6.4\times 10^6}$ m

$\Rightarrow$ $d = 80{,}000$ m $= 80$ km

Hence, the correct answer is (A).

60. Modulation index μ is kept ≤ 1 to avoid distortion
Hence, the correct answer is (C).

61. Optical fibres are not subjected to electromagnetic interference from outside.
Hence, the correct answer is (C).

62. In amplitude modulation, bandwidth is

$$\Delta f = f_{USB} - f_{LSB}$$

$$\Rightarrow \quad \Delta f = (f_c + f_m) - (f_c - f_m) = 2f_m$$

So, bandwidth is twice the frequency of modulating signal frequency.

Hence, the correct answer is (A).

63. Carrier swing $= \dfrac{\text{Frequency deviation}}{\text{Modulating frequency}} = \dfrac{50}{7} = 7.143$

Hence, the correct answer is (A).

64. $MUF = \dfrac{f_c}{\cos\theta} = \dfrac{60}{\cos 70^\circ} = 175 \text{ MHz}$

Hence, the correct answer is (C).

65. We know that $\left(\dfrac{I_t}{I_c}\right)^2 = 1 + \dfrac{m^2}{2}$

where, $I_t = 8.96\ A$ and $I_c = 8\ A$

$$\Rightarrow \quad \left(\frac{8.96}{8}\right)^2 = 1 + \frac{m^2}{2}$$

$$\Rightarrow \quad 1.254 = 1 + \frac{m^2}{2}$$

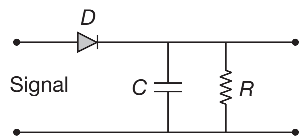
$$\Rightarrow \quad \frac{m^2}{2} = 0.254$$

$$\Rightarrow \quad m^2 = 0.508$$

$$\Rightarrow \quad m = 0.71 = 71\%$$

Hence, the correct answer is (D).

66.



The higher frequency which can be detected with tolerable distortion is

$$f = \frac{1}{2\pi\mu RC} = \frac{1}{2\pi \times 0.6 \times 100 \times 10^3 \times 250 \times 10^{-12}} \text{ Hz}$$

$$\Rightarrow \quad f = \frac{10^6}{2 \times 3.14 \times 0.6 \times 25} = 10.61 \text{ kHz}$$

This condition is obtained by applying the condition that rate of decay of capacitor voltage must be equal or less than the rate of decay modulated signal voltage for proper detection of modulated signal.

Hence, the correct answer is (B).

67. Band width $\Delta f = 2 \times$ frequency of modulating

$$\Rightarrow \quad \Delta f = 2 \times 5000 \text{ Hz} = 10 \text{ kH}_Z$$

Hence, the correct answer is (A).

68. $m_a = \dfrac{E_m}{E_c} = \dfrac{15}{60} \times 100 = 25\%$

Hence, the correct answer is (C).

69. For good demodulation,

$$\frac{1}{f} \ll RC$$

$$\Rightarrow \quad RC \gg \frac{1}{f}$$

Hence, the correct answer is (D).

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1. The frequency of the wave is

$$f = \frac{3 \times 10^8}{800 \times 10^{-9}} = \frac{3 \times 10^{15}}{8} \text{ Hz}$$

Signal bandwidth is 1%, so

$$\Delta f = \left(\frac{3 \times 10^{15}}{8}\right)\left(\frac{1}{100}\right) = \frac{3}{8} \times 10^{13} \text{ Hz}$$

Number of channels is

$$N = \frac{\Delta f}{f} = \frac{3 \times 10^{13}}{8 \times 6 \times 10^6} = 6.25 \times 10^5$$

Hence, the correct answer is (C).

2. Maximum distance upto which signals can be broadcasted is given by

$$d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

$$\Rightarrow \quad d = 42.33 + 22.6$$

$$\Rightarrow \quad d \approx 65 \text{ km}$$

Hence, the correct answer is (A).

3. $f_c = 2500 \text{ kHz}$, $f_m = 250 \text{ kHz}$

$$f_c + f_m = 2750 \text{ kHz}$$

$$f_c - f_m = 2250 \text{ kHz}$$

For accepted frequency, two bandwidths do not overlap

$$f_2 = f_c + 2f_m = 3000 \text{ kHz}$$

and $f_c - 2f_m = 2000$ kHz

Hence, the correct answer is (C).

4. $\omega_U = \left(2.2\times10^4 + 5.5\times10^5\right)$ rads^{-1} and

$$\omega_L = \left(5.5\times10^5 - 2.2\times10^4\right) \text{ rads}^{-1}$$

$$\Rightarrow \omega_U = (2.2+55)\times10^4 = 57.2\times10^4 \text{ rads}^{-1}$$

$$\Rightarrow \omega_L = 52.8\times10^4 \text{ rads}^{-1}$$

$$\Rightarrow f_U = \frac{572}{2\pi} \text{ kHz} \simeq 91 \text{ kHz}$$

$$\Rightarrow f_L = \frac{528}{2\pi} \text{ kHz} \simeq 84 \text{ kHz}$$

Hence, the correct answer is (C).

5. $C_m = \left(A_C + A_m \sin\omega_m t\right)\sin\left(\omega_C t\right)$

From the graph and

$$A_C + A_m = 10$$

$$A_C - A_m = 8$$

$$\Rightarrow A_C = 9 \text{ V},\ A_m = 1 \text{ V}$$

Since, $\omega_m = \dfrac{2\pi}{100\times10^{-6}} = 2\pi\times10^4 \text{ s}^{-1}$ and

$$\omega_C = \frac{2\pi}{8\times10^{-6}} = 2.5\pi\times10^5 \text{ s}^{-1}$$

$$\Rightarrow C_m = \left(9+\sin 2\pi\times10^4 t\right)\sin\left(2.5\pi\times10^5 t\right) \text{ V}$$

Hence, the correct answer is (A).

6. $A_c + A_m = 160$ and $A_c - A_m = 40$

$$\Rightarrow A_c = 100 \text{ V}$$

$$\Rightarrow A_m = 60 \text{ V}$$

Modulation index $\mu = \dfrac{A_m}{A_c} = \dfrac{3}{5} = 0.6$

Hence, the correct answer is (C).

7. Since, we know that

$$d = \sqrt{2hR}$$

For d' to be $2d$, we have

$$\frac{2d}{d} = \frac{\sqrt{2h'R}}{\sqrt{2hR}}$$

$$\Rightarrow 4h = h'$$

Hence, the correct answer is (D).

8. Wavelength of carrier waves in modern optical fiber communication is most widely used near about 1500 nm .
Hence, the correct answer is (C).

9. Since, it is given that

$$\sqrt{2\times70\times R_E} + \sqrt{2\times h_R\times R_E} = 50\times10^3$$

Substituting $R_E = 6.4\times10^6$ m and solving we get

$$h_R = 32 \text{ m}$$

Hence, the correct answer is (C).

10. $A_m = \left(V_0 + A\cos\omega t\right)\sin\omega_0 t$

$$\Rightarrow A_m = V_0\sin\left(\omega_0 t\right) + \frac{A}{2}\left[\sin\left(\omega_0-\omega\right)t + \sin\left(\omega_0+\omega\right)t\right]$$

Hence, the correct answer is (A).

11. Size of antenna depends on wavelength of carrier wave.
Hence, the correct answer is (B).

12. Range of frequency is from $\left(f_c - f_m\right)$ to $\left(f_c + f_m\right)$
So, band width is given by

$$\Delta f = 2f_m = 2\times100\times10^6 \text{ Hz}$$

$$\Rightarrow \Delta f = 2\times10^8 \text{ Hz}$$

and modulation index is given by

$$\mu = \frac{A_m}{A_C} = \frac{100}{400} = 0.25$$

Hence, the correct answer is (D).

13. Optical fibre communication - Infrared Light
Radar – Radio Waves
Sonar – Ultrasound
Mobile Phones – Microwaves
Hence, the correct answer is (A).

14. Modulation index is

$$\mu = \frac{A_m}{A_c} = \frac{2}{4} = 0.5$$

From the equations given in the question, we have

$$f_c = \frac{\omega_c}{2\pi} = \frac{20000\pi}{2\pi} = 10000 \text{ Hz and}$$

$$f_m = \frac{\omega_m}{2\pi} = \frac{2000\pi}{2\pi} = 1000 \text{ Hz}$$

So, lower side band frequency is

$$f_{LSB} = (10000 - 1000) \text{ Hz}$$

$$\Rightarrow f_{LSB} = 9 \text{ kHz}$$

Hence, the correct answer is (D).

15. Frequency of carrier wave is

$$f = 10\times10^9 \text{ Hz}$$

Total available bandwidth is 10% of f, so we have

$$\Delta f = 10\% \text{ of } 10\times10^9 \text{ Hz}$$

$$\Rightarrow \quad \Delta f = 10^9 \text{ Hz}$$

Bandwidth for each telephonic channel is $5 \text{ kHz} = 5\times10^3 \text{ Hz}$, so number of channels is given by

$$N = \frac{\text{Total Bandwidth of Channels}}{\text{Bandwidth Needed per Channel}}$$

$$\Rightarrow \quad N = \frac{10^9}{5\times10^3} = 2\times10^5$$

Hence, the correct answer is (C).

16. If modulating frequency is $f_m = 15 \text{ kHz}$ then bandwidth of one channel is $\Delta f = 2f_m = 30 \text{ kHz}$. So, the number of channels that can be accommodated is

$$N = \frac{300 \text{ kHz}}{30 \text{ kHz}} = 10$$

Hence, the correct answer is (B).

17. Carrier frequency, $f_c = \dfrac{1}{2\pi\sqrt{LC}}$

$$f_c = \frac{1}{6.28\times\sqrt{49\times10^{-6}\times2.5\times10^{-9}}}$$

$$\Rightarrow \quad f_c = \frac{10^8}{6.28\times7\times5} \text{ Hz} = 454 \text{ kHz}$$

Side bands frequency range is from $(f_c - f_m)$ to $(f_c + f_m)$ i.e. from $(454-12) \text{ Hz} = 442 \text{ Hz}$ to $(454+12) \text{ Hz} = 466 \text{ Hz}$.

Hence, the correct answer is (D).

18. Modulation index is given by

$$\mu = \frac{A_m}{A_c}$$

$$\Rightarrow \quad A_m = \mu A_c$$

Since, $\mu = 80\% = 0.8$

$$\Rightarrow \quad A_m = 0.8\times14 = 11.2 \text{ V}$$

Hence, the correct answer is (B).

19. Let $x(t) = A_c \sin\omega_c t$ represents carrier wave $y(t) = A_m \sin\omega_m t$ represents the modulating signal. The modulated signal $x_m(t)$ can be written as

$$x_m(t) = (A_c + A_m \sin\omega_m t)\sin\omega_c t$$

$$\Rightarrow \quad x_m(t) = A_c\left(1 + \frac{A_m}{A_c}\sin\omega_m t\right)\sin\omega_c t$$

$$\Rightarrow \quad x_m(t) = A_c \sin(\omega_c t) + \mu A_c \sin(\omega_m t)\sin(\omega_c t)$$

where $\mu = \dfrac{A_m}{A_c}$ is the modulation index.

Since, $2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$$\Rightarrow \quad x_m(t) = A_c \sin\omega_c t + \frac{\mu A_c}{2}\left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right]$$

$$\Rightarrow \quad x_m(t) = A_c \sin(\omega_c t) + \frac{\mu A_c}{2}\cos(\omega_c - \omega_m)t - \frac{\mu A_c}{2}\cos(\omega_c + \omega_m)t$$

Amplitude modulated wave contains the frequencies ω_c, $(\omega_c - \omega_m)$ and $(\omega_c + \omega_m)$. So, the frequency ω_m is not contained in the amplitude modulated wave.

Hence, the correct answer is (A).

20. Modulation index, $\mu = \dfrac{A_m}{A_c} = \dfrac{5}{25} = 0.2$

Frequency of carrier wave,

$$f_c = 1.2\times10^3 \text{ kHz} = 1200 \text{ kHz},$$

Frequency of modulate wave $f_m = 20 \text{ kHz}$

$$f_{LSB} = f_c - f_m = 1200 - 20 = 1180 \text{ kHz}$$

$$f_{USB} = f_c + f_m = 1200 + 20 = 1220 \text{ kHz}$$

Hence, the correct answer is (B).

21. For transmitting a signal, the size of antenna should be comparable to the wavelength of the signal (λ). A linear antenna of length (ℓ) radiates power which is proportional to $\left(\dfrac{\ell}{\lambda}\right)^2$ i.e. $P_{eff} = K\left(\dfrac{\ell}{\lambda}\right)^2$.

Hence, the correct answer is (B).

22. Carrier wave : $y_c = A_c \sin(\omega_c t)$

Message signal : $y_m = A_m \sin(\omega_m t)$

Amplitude of the modulated carrier wave is given by

$$y = (A_c + A_m \sin\omega_m t)\sin\omega_c t$$

Hence, the correct answer is (A).

23. Frequency band of human speech ranges from 200 Hz to 2700 Hz.

Band width for human speech is

$$2700 - 200 = 2500 \text{ Hz}$$

Frequency band of high frequency music is from 10200 Hz to 15200 Hz.

Band width for high frequency music required to just send the human speech is

$$15200 - 200 = 15000 \text{ Hz}$$

So, the required ratio is

$$\frac{\text{Bandwidth for both signals}}{\text{Bandwidth for human speech}} = \frac{15000}{2500} = 6$$

Hence, the correct answer is (C).

24. Here,

$C_m(t) = 30\sin(300\pi t) + 10(\cos(200\pi t) - \cos(400\pi t))$

Compare this equation with standard equation of amplitude modulated wave i.e.,

$$C_m(t) = A_c \sin\omega_c t - \frac{\mu A_c}{2}\cos(\omega_c + \omega_m)t + \frac{\mu A_c}{2}\cos(\omega_c - \omega_m)t$$

So, we get

$A_c = 30$ V , $\omega_c = 300\pi$

$\Rightarrow \quad 2\pi f_c = 300\pi$

$\Rightarrow \quad f_c = 150$ Hz

Since $\omega_c - \omega_m = 200\pi$

$\Rightarrow \quad f_c - f_m = 100$ Hz

$\Rightarrow \quad f_m = 150 - 100 = 50$ Hz

Also, $\frac{\mu A_c}{2} = 10$ and $A_c = 30$

$\Rightarrow \quad \mu = \frac{10}{15} = \frac{2}{3}$

Hence, the correct answer is (B).

25. Given, $f_m = 5$ kHz , $f_c = 2$ MHz $= 2000$ kHz

The frequencies of the resultant signal are $(f_c - f_m)$, f_c and $(f_c + f_m)$. Hence

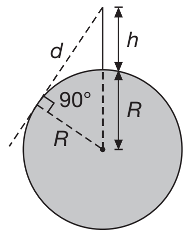
$f_c - f_m = (2000 - 5)$ kHz $= 1995$ kHz

$f_c = 2000$ kHz and

$f_c + f_m = (2000 + 5)$ kHz $= 2005$ kHz

Hence, the correct answer is (A).

26. If maximum distance on earth where object can be detected is d, then



$(h + R)^2 = d^2 + R^2$

$\Rightarrow \quad d^2 = h^2 + 2Rh$

Since $h \ll R$

$\Rightarrow \quad d = \sqrt{2Rh}$

$\Rightarrow \quad d = \sqrt{2 \times 6.4 \times 10^6 \times 500} = 8 \times 10^4$ m $= 80$ km

Hence, the correct answer is (D).

CHAPTER 5