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How to create and detect *N*-dimensional entangled photons with an active phase hologram

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The experimental realization of multidimensional quantum states may lead to unexplored and interesting physics, as well as advanced quantum communication protocols. The orbital angular momentum of photons is a well suitable discrete degree of freedom for implementing high-dimensional quantum systems. The standard method to generate and manipulate such photon modes is to use bulk and fixed optics. Here the authors demonstrate the utilization of a spatial light modulator to manipulate the orbital angular momentum of entangled photons generated in spontaneous parametric downconversion. They show that their setup allows them to realize photonic entanglement of up to 21 dimensions, which in principle can be extended to even larger dimensions. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2752728]

The ability to manipulate the light modes of single photons is a crucial and necessary tool for any realization of high-dimensional quantum optics experiments based on the orbital angular momentum of photons.[1–5] In addition, many other schemes of quantum optics experiments such as quantum teleportation,[6–8] photonic quantum computing,[9–11] and higher-order photonic entanglement[12,13] directly rely on photon interference, which requires to overlap photon beams with high quality mode matching. The most common method is to use bulk optical components such as lenses, mirrors, and microfabricated holograms in these setups. However, as such optical components are static, each of them can only perform a predefined manipulation of the photon beam. Programmable diffractive optics can dramatically extend the capabilities of optical configurations by allowing the generation of arbitrary and flexible light patterns in real time.[14–16]

Our work makes original use of a spatial light modulator (SLM) as an active transformation in a quantum optics experiment for the manipulation of photons coming from spontaneous parametric downconversion, which have significantly less temporal and spatial coherence than photons from a laser. The SLM manipulates one photon of a photon pair entangled in its orbital angular momentum (Laguerre-Gaussian functions) of the light mode and hence replaces the fixed phase-singularity holograms used in preceding experiments.[1–3,17–20] We further demonstrate the possibility of generating 21-dimensional quantum states.

The SLM is an array of pixels (in our case 1024×768 with a total size of $19.5 \times 14.6\ \text{mm}^2$) acting as individually tunable retardation wave plates which can imprint a spatial phase modulation on a light beam. The SLM is made of nematic liquid crystal pixels whose birefringence is controlled by an external electric field. Our SLM is used in reflection mode, has a refresh rate of 75 Hz, and its diffraction efficiency into the first order, which corresponds to the desired output, is approximately 60%. For technical reasons we were limited by the nonideal modulation depth of the SLM, which has a maximal phase shift of 1.8π for our wavelength of 702 nm,[21] a reduced filling ratio of the pixels of 90% and a reflectivity of 75%.

In order to generate the patterns to be applied onto the SLM we implemented an algorithm on a computer in a MATLAB environment. The function generating the output value for each pixel on the SLM is

$$\text{SLM\_Pixel}\left(\frac{x \cdot 1024}{x_{\max} - x_{\min}}, \frac{y \cdot 768}{y_{\max} - y_{\min}}\right) = \text{mod}[(\text{angle}(\text{LG}(x - x_0, y - y_0, l, z, w_0, z_r)) \quad (1)$$

$$+ \frac{\pi \cdot 10^3}{\lambda} \cdot f_{\text{SLM}} \cdot (\text{ast} \cdot (x - x_{l0})^2 + (y - y_{l0})^2) \quad (2)$$

$$+ x \cdot k_x + y \cdot k_y), 2\pi] \cdot \frac{256}{2\pi}, \quad (3)$$

where λ is the wavelength of the photons impinging on the SLM. This function produces a normalized 8 bits gray value

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(0–255) picture over a grid corresponding to the pixel positions of the SLM. The standard range in meters for the values of x and y are $\{0.0098,-0.0098\}$ and $\{0.0073,-0.0073\}$, respectively, matching the size of the SLM.

Expression (1) of the function SLM_Pixel calculates the complex amplitude $\mathrm{LG}_{0,l}=\mathrm{LG}(x-x_0,y-y_0,l,z,w_0,z_r)$ of the desired Laguerre-Gaussian mode with the following parameters: z is the propagation direction of the light field, w_0 is the beam waist, z_r is the Rayleigh length, and x_0 and y_0 are the distance of the phase singularity from the origin. The function "angle" is a MATLAB function which returns the complex phase angle of the evaluated expression.

The index $l\in[\ldots,-1,0,+1,+2,\ldots]$ is the azimuthal mode index, with $2\pi l$ being the change in phase of a closed path around the propagation axis. This phase change gives the Laguerre-Gaussian modes a helical wave front, where the angular momentum per photon can only take integer multiples of $\hbar$.[22] A phase hologram with $l\neq 0$, imprints a phase $e^{-il\theta}$ on the incoming beam, which results in a phase singularity. Such a hologram with one singularity $(l=\pm 1)$ acts as a ladder operation on the reflected light mode, effectively raising the l index by 1 (hologram with $l=+1$) or lowering it by 1 (hologram with $l=-1$).

In addition, expression (2) of the function SLM_Pixel allows the spatial light modulator to act as a Fresnel lens. The parameter f_{SLM} is the focal length of the lens term in millimeters. Additionally, x_{l0} and y_{l0} are the distance of the lens term from the origin in x and y directions, respectively. The parameter ast is a weighting factor between the x and the y components of the lens term, which is used to compensate for any astigmatism. In the ideal case ast=1.

Furthermore, expression (3) of the function SLM_Pixel corresponds to a beam deflection, like a tilted mirror, where the parameters k_x and k_y define an inclined plane and are used to change the diffraction direction of the light beam, with $k_x\approx 2\pi\theta_x/\lambda$ in the approximation of a small diffraction angle θ_x (and analogous for k_y).

Effectively the SLM represents three adjustable diffractive optical elements: a phase singularity, a tunable lens and a tunable mirror, which are readily usable in experiments for achieving mode matching and beam pointing of a light beam.

The experimental versatility of the SLM is demonstrated with the manipulation of entangled photon pairs created by focusing an Ar^+ laser with a wavelength of 351 nm and a power of about 95 mW into a β-barium-borate crystal (BBO) with type-I mode matching (see Fig. 1 for the experimental setup). Due to spontaneous parametric downconversion (SPDC), pairs of photons entangled in their orbital angular momentum are produced,[1,23,24] called the signal and the idler beam, both at a central wavelength of 702 nm and a bandwidth of 2 nm full width at half maximum. The photons in the signal beam pass a lens L_1 ($f_1=250$ mm) to guarantee that they effectively couple into the final fibers. In contrast, the idler beam is widened using lenses L_2 ($f_2=-30$ mm) and L_3 ($f_3=100$ mm) to optimally illuminate the SLM. With another lens L_4 ($f_3=750$ mm) the beam is focused again after being reflected from the SLM. In order to discriminate between the +1, −1, and 0 modes we use a probabilistic mode analyzer (for a detailed description see Ref. 1 and 2).

In order to achieve a high pair coupling efficiency of both the photons from the SPDC into the optical fibers, which is required for maximal coincidence rates as well as

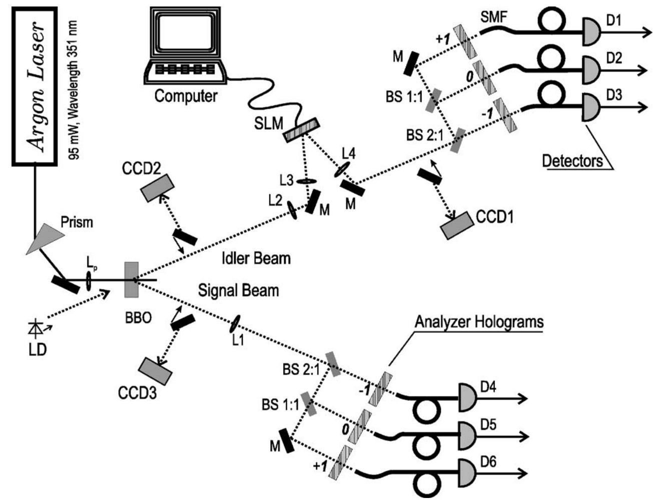

FIG. 1. Experimental setup to demonstrate the manipulation of entangled photons with a spatial light modulator. The optically nonlinear crystal (BBO) is pumped with an ultraviolet Ar^+ laser, which generates pairs of photons, entangled in their orbital angular momentum. In the idler beam the photons are controlled with a spatial light modulator and both signal and idler beam are analyzed with fixed phase holograms. The phase hologram of the spatial light modulator is actively controlled with a computer.

perfect correlation in the orbital angular momentum, the signal and idler beams must have well matched spatial mode functions. A striking advantage of the SLM over fixed optics is the fact that now the idler beam can be easily matched to the mode of the signal beam via tuning the lens term on the SLM. The dependence of the spot size of the idler beam at the crystal on the focal length f_{SLM} of the SLM is measured by sending a laser beam in reverse through the idler path, and measuring the mode diameter with a charge coupled device (CCD) camera. The focal length required to achieve the same spot size for the signal and idler beams is found to be $f_{\mathrm{SLM}}=940$ mm.

Since the beam is reflected not perfectly orthogonal from the SLM, the lens term applied to the SLM shows some astigmatism. Hence the resulting mode of the reflected light beam will no longer be a pure first order LG mode but a general superposition of Hermite-Gaussian modes. This is compensated by tuning the parameter ast which puts an astigmatism on the lens term, which also allows to perform experiments of photons involving Hermite-Gaussian modes.[25] In our setup the ideal value to obtain clean LG modes was ast=1.029 throughout the experiment [see Fig. 2(a)].

A further, important parameter is the position of the displacement of the hologram in the x direction x_0 and in the y direction y_0 with respect to the beam center. Tuning this parameter can be used to obtain superpositions of different LG modes, as was done in order to violate a Bell-type inequality[2] and to obtain a secret key in a quantum cryptography scheme.[19] Figure 2(b) shows the resulting mode for different values of x_0 and y_0.

The most important aspect of our experiment is to show that entangled photons from downconversion can easily be manipulated with the SLM device.[26] We therefore place a CCD camera with a long-pass filter (cutoff at 800 nm) in the idler beam and apply transformations between LG modes with different l indices on the SLM. As the majority of the downconversion photons are produced in the $\mathrm{LG}_{0,0}$ state, the intensity profiles on the CCD correspond to the l index of the

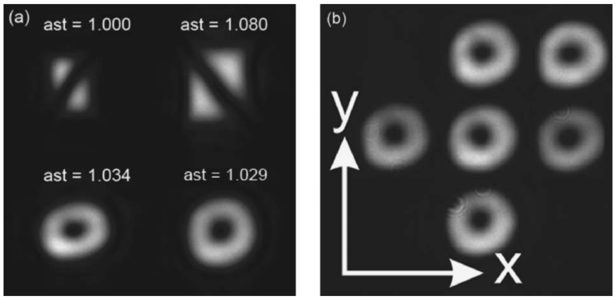

FIG. 2. Images (a) show how the astigmatism of the lens term is compensated with the parameter ast. For ast=1.029 an $LG_{0,1}$ is obtained (on the lower right), whereas for all other values general superpositions of Hermite-Gaussian modes are produced. (b) Pictures of the resulting mode for different values of x_{l0} and y_{l0}, which determine the position of the lens term on the SLM. The camera is kept at the same position throughout the measurements.

transformation on the SLM. Figure 3(a) shows images of the idler beam recorded for various values of l ($f_{SLM}=131$ mm and $k_y=2\times10^4$ rad/m). The mode with the highest index shown is $l=10$, which can in principle be used for quantum communication and information protocols with dimension of up to 21. The transformations and therefore the accessible Hilbert spaces for quantum optics experiments are in principle not limited.

As a test of the SLM we analyze the perfect correlations of the entangled photon pairs in their orbital angular momentum. The signal beam is transformed with conventional quartz holograms and the idler beam with the SLM scheme. These correlations are the crucial ingredients for any quantum information processing scheme such as quantum cryptography. The expected correlations for a maximally entangled qutrit (a trinary quantum system; $l=-1,0,+1$) state for four different couplers on the signal and idler side are shown in Fig. 3(b). Various phase holograms calculated for the desired transformations were displayed on the SLM and the experimentally obtained coincidences for the different couplers are also shown in Fig. 3(b). The entangled character of the produced pairs can be clearly seen and the ability of the spatial light modulator to manipulate the pairs on the single photon level is proven. The reason for the reduced visibility of the observed correlations in the trinary quantum systems is mainly due to residual mismatching of the two SPDC modes, as well as the nonperfect initial correlation of the photon pairs produced in the downconversion process.

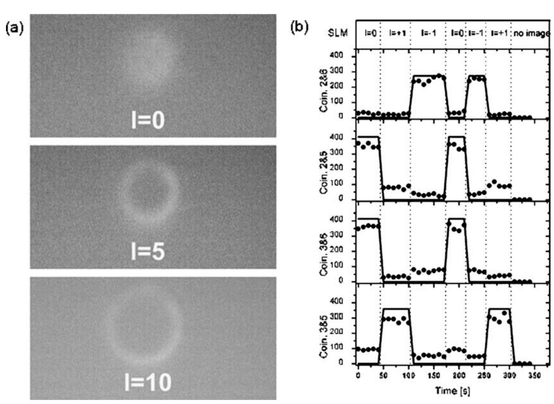

FIG. 3. (a) Pictures of the downconverted light, transformed into higher-order modes by corresponding phase holograms on the SLM. (b) Coincidence counts of the downconverted beams per 10 s. With the SLM a transformation is performed and the correlations are observed via probabilistic mode analyzers. The individual settings are coupler 2, $l=0$; coupler 3, $l=-1$; coupler 5, $l=0$; and coupler 6, $l=+1$. The solid lines show the expected coincidences for different settings of the transformation hologram applied to the spatial light modulator (SLM). Only photon pairs with a total angular momentum $l_{total}=0$ are expected to show correlations due to the entangled nature of the generated two-photon state.

We have shown a scheme of manipulating entangled photons produced by spontaneous parametric downconversion based on a spatial light modulator. This scheme allows high flexibility for generating arbitrary superpositions of orbital angular momentum states of photons by tuning the position and the amplitude of phase singularity. Transformations of the l index of Laguerre-Gaussian modes up to 10 have been demonstrated, which paves the way to experiments with 21-dimensional quantum systems. The SLM scheme provides a fast and unique way of managing spatial modes of single photons in quantum optics experiments.

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[26]Our SLM-device is a LC-R 2500 from HoloEye Photonics AG, Germany.