Squeezing of a quantum flux in a double rf-SQUID system

Kohji Takashima[1], **Shigemasa Matsuo**[1], **Toshiyuki Fujii**[1], **Noriyuki Hatakenaka**[1], **Susumu Kurihara**[2], **and Anton Zeilinger**[3]

[1]Graduate School of Integrated Arts and Sciences, Hiroshima University, Higashi-Hiroshima, 739-8521, Japan

[2]Department of Physics, Waseda University, Tokyo, 169-8555, Japan

[3]Institute of Experimental Physics, University of Vienna, Boltzmanngasse 5, 1090 Vienna, Austria

Abstract. We investigate the nonstationary boundary effect for a quantum flux in a double rf-SQUID system. In a superconducting ring interrupted by a dc-SQUID (so-called double rf-SQUID), the Josephson potential can be controlled by the magnetic flux through the dc-SQUID ring. This system is equivalent to an anharmonic oscillator with a time-dependent frequency. A rapid change of the magnetic flux in the dc-SQUID leads to the nonadiabatic mixing of the quantum states for a quantum flux in a double rf-SQUID. Therefore, this becomes a circuit analogue of the dynamical Casimir effect in quantum field theory. We perform numerical calculations for the quantum state evolution of the quantum flux within a harmonic approximation, taking account of the nonadiabatic effect. We found that the resulting state distribution has a super-Poissonian character that reflects flux squeezing caused by the Bogoliubov transformation between eigenstates at different times.

1. Introduction

The nonadiabatic effect has sometimes played a crucial role as regards our understanding of experimental results in a wide range of systems. A noteworthy example in connection with quantum field theory is the dynamical Casimir effect that predicts photon production out of the vacuum as the result of a nonadiabatic effect caused by quickly moving boundaries [1]. This effect has predominantly been studied in the electromagnetic field in a cavity with movable boundaries. Unfortunately, although there has been experimental verification of the *static* Casimir effect [2], there has been none of its dynamical counterpart. The main problems are the difficulties involved in making a mirror oscillate at high frequencies, and in detecting a microwave photon with high efficiency. One approach to imitating such a dynamical effect has been described by Dodonov et al. using a Josephson junction [3, 4, 5] based on parametric processes about two decades ago. Along the same lines, we proposed an alternative scheme for creating nonstationary situations using a double rf-SQUID, which we regarded as superconducting artificial atoms [6]. In this paper, we numerically investigate quantum-state evolution of a quantum flux in a double rf-SQUID system resulting from nonadiabatic changes in the Josephson critical current.
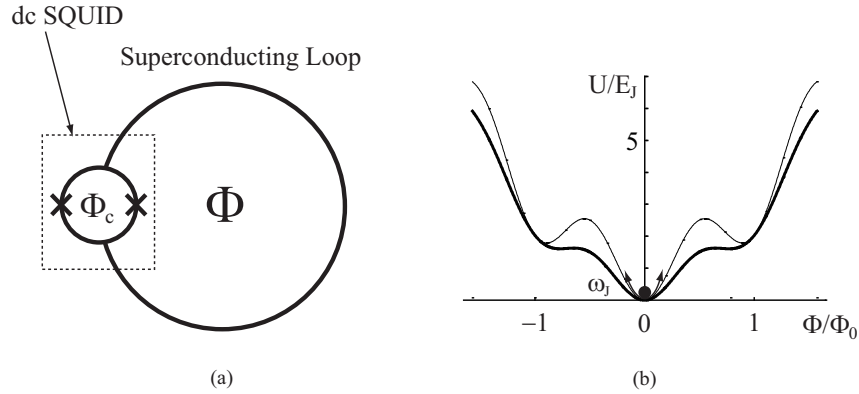
Figure 1. Schematics of a double rf-SQUID and its potential energy. (a) a double rf-SQUID (b) the potential energy of the double rf-SQUID at different Φ_c values: the thin solid line shows $\Phi_c = 0$ and the thick solid line shows $\Phi_c = \Phi_0/2\pi$.

2. Double rf-SQUID as superconducting artificial atom

Let us consider the system known as a double rf-SQUID shown in Fig. 1 (a), which consists of a superconducting loop with the inductance L interrupted by a dc-SQUID, i.e., two Josephson junctions. This device behaves as a normal rf-SQUID with *tunable* Josephson critical current. An external magnetic flux Φ_c applied to the dc-SQUID controls the current, equivalently, the Josephson plasma frequency ω_J. Figure 1 (b) shows the potential profile as a function of Φ_{ex} with different control flux values Φ_c, and clearly shows the Φ_c-dependent Josephson plasma frequency.

Now let us consider time-dependent Φ_c situations. In such cases, the Josephson plasma frequency becomes time-dependent. Thus, the system can be regarded as a time-dependent anharmonic oscillator. With semiclassical Josephson junctions, namely, where the charging energy of the junction E_c is less than the Josephson coupling energy E_J, the system is further approximated by a harmonic oscillator. Therefore, the Hamiltonian that we consider hereafter is described as

$$H(t) = \frac{p^2}{2m} + \frac{1}{2}m\omega_J(t)^2 x^2 \tag{1}$$

where p and x are canonical variables, namely the momentum and position corresponding to the charge Q accumulated across the junction and Φ in the rf-SQUID, respectively. m is the flux mass, which is proportional to the junction capacitance.

3. Nonadiabatic quantum-state evolution

Here we investigate the quantum-mechanical evolution of a quantum flux in a double rf-SQUID potential with time-dependent frequencies. Let us assume that the solution of the Schrödinger equation can be approximated with the stationary eigenfunctions of the instantaneous Hamiltonian, so that a particular eigenfunction at one time goes over continuously into the corresponding eigenfunction at a later time. In other words, if the equation

$$H(t)u_n(t) = E_n(t)u_n(t) \tag{2}$$

can be solved at each instant of time, we expect that a system in a discrete nondegenerate state $u_m(0)$ with energy $E_m(0)$ at $t = 0$ is likely to be in the state $u_m(t)$ with energy $E_m(t)$ at time t, provided that $H(t)$ changes very slowly with time. In a nonadiabatic case, other states appear

in the expansion of ψ in terms of the u's. The wave function ψ satisfies the time-dependent Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = H(t)\psi. \tag{3}$$

We proceed by expanding ψ in terms of the u's in the following way:

$$\psi = \sum_m a_m(t) u_m(t) \exp\left[\frac{-i}{\hbar} \int_0^t E_m(t')dt'\right] \tag{4}$$

where we assume that the u_m are orthonormal, discrete, and nondegenerate. Substitution into Eq. (3) gives

$$\sum_m \left[\dot{a}_m u_m + a_m \frac{\partial u_m}{\partial t}\right] \exp\left[\frac{-i}{\hbar} \int_0^t E_m(t')dt'\right] = 0 \tag{5}$$

where use is made of Eq. (2). We multiply through on the left by u_n^* and integrate over all space to obtain [7]

$$\dot{a}_n(t) = -\sum_m a_m(t)\langle\psi_n(t)|\dot{\psi}_m(t)\rangle \exp\left[\frac{-i}{\hbar} \int_0^t (E_m(t') - E_n(t'))dt'\right] \tag{6}$$

In the case of time-dependent harmonic oscillator, the overlap term $\langle\psi_n(t)|\dot{\psi}_m(t)\rangle$ can be described as

$$
\begin{aligned}
\langle\psi_n(t)|\dot{\psi}_m(t)\rangle &= \frac{1}{E_m(t) - E_n(t)}\langle\psi_n(t)|\frac{\partial H(t)}{\partial t}|\psi_m(t)\rangle \\
&= \frac{1}{2}\frac{\dot{\omega}_J}{\omega_J}\frac{1}{m-n}\langle n|(a^\dagger + a)^2|m\rangle
\end{aligned}
\tag{7}
$$

where $a^\dagger(a)$ is a creation (annihilation) operator for the harmonic oscillator. This shows that the nonadiabatic element is proportional to $\dot{\omega}_J/\omega_J$. Thus, this term mixes the quantum states with different quantum numbers during the temporal evolution. In other words, the system stays in its initial state when the frequency changes are slower than the oscillation period, i.e., the inverse of the Josephson plasma frequency. Next we perform numerical simulations to investigate the quantum-mechanical evolution of the quantum flux states in the nonadiabatic dynamical regime.

4. Numerical simulations

The time-dependent frequency that we use in our numerical simulations is shown in Fig. 2 (a). It is given by $\omega_J(t) = \omega_J(\infty) + (\omega_J(0) - \omega_J(\infty))(1 - \tanh c(t-a))/2$ with c being a nonadiabatic parameter, and can be realized by using the interaction with a moving fluxon. This will be discussed in detail elsewhere. The nonadiabatic parameter is set at $c/\omega_J(0) = 10$. This means that the system should experience the nonadiabatic effect.

We perform numerical calculations at the above time-dependent frequency using Eqs. (6) and (7). The quantum states of a quantum flux evolve as shown in Fig. 2 (b). The system initially occupies its ground state. The occupation probability of the ground state suddenly decreases followed by the frequency change as shown in Fig. 2 (a). On the other hand, the occupation probabilities for the other states increase with the amplitude of each nonadiabatic element. The distribution of the occupation probabilities is shown in Fig. 2 (c). This distribution $|a_n|^2$ can be fitted by a super-Poissonian distribution $P(n)$ expressed as

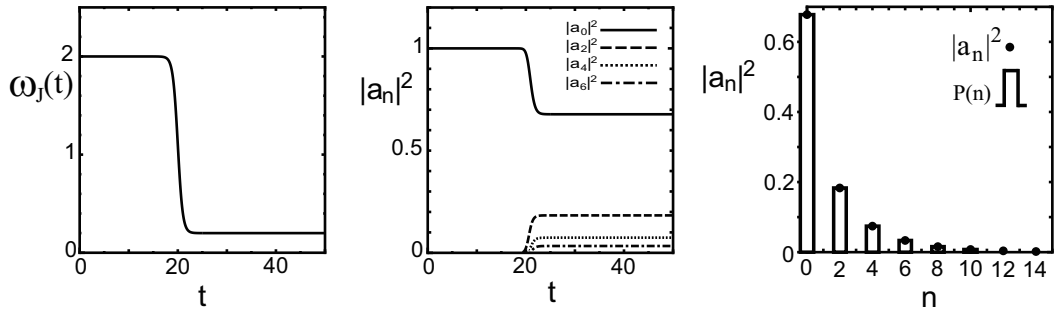$$P(n) = \frac{\nu^n}{2^n n! \mu^{n+1}} H_n(0)^2 \tag{8}$$

Figure 2. Numerical simulations: (a) the time-dependent Josephson plasma frequency $\omega_J(t)$ ($\omega_J(0)/\omega_J(\infty) = 10$) (b) the occupation probability for each quantum state as a function of time: the solid line shows $|a_0|^2$, the dashed line shows $|a_2|^2$, the dotted line shows $|a_4|^2$, and the dot-dashed line shows $|a_6|^2$, (c) the occupation probability distribution. Solid points $|a_n|^2$ refer to numerical calculations, histograms $P(n)$ to theory Eq. (8)

where $\nu = \sinh(r)$ and $\mu = \cosh(r)$ with r being a squeezing parameter. $H_n(x)$ is the Hermite polynominal. This indicates that the produced states are squeezed vacuums since squeezed vacuums always exhibits super-Poissonian statistics [8]. In fact, the system Hamiltonian involves the off-diagonal element at a finite time, i.e., the Hamiltonian can be rewritten by

$$H(t) = \hbar\omega_J(0)\left(a^\dagger a + \frac{1}{2}\right) + \hbar\omega_J(0)\left[\left(\frac{\omega_J(t)}{\omega_J(0)}\right)^2 - 1\right]\left[\left(a^\dagger a + \frac{1}{2}\right) + \left(a^2 + a^{\dagger 2}\right)\right] \qquad (9)$$

This Hamiltonian should be diagonalized using the Unitary operator $V = \exp[\hbar\omega_J(t)(a^{\dagger 2} - a^2)/2]$, so that the Hamiltonian at $t = t$ is described by $\hbar\omega_J(0)(b^\dagger b + 1/2)$ with $b^\dagger(b)$ being a creation (annihilation) operator at $t = t$. Therefore, a squeezed vacuum might be produced as a result of the Bogoliubov transformation between eigenstates at different times [9].

5. Summary
We have investigated the nonadiabatic effect of a quantum flux in a double rf-SQUID system. The system is equivalent to an anharmonic oscillator with a time-dependent frequency. We have performed numerical calculations to investigate the quantum-state evolution of the quantum flux within a harmonic approximation, taking account of the nonadiabatic effect. We found that the resulting state distribution has a super-Poissonian character that reflects flux squeezing caused by the Bogoliubov transformation between eigenstates at different times.

[1] Moore G T 1970 *J. Math. Phys.* **11** 2679
[2] Casimir H B G 1948 *Proc. K. Ned. Akad. Wet.* **51** 793
[3] Dodonov V V, Man'ko V I and Man'ko O V 1989 *J. Sov. Laser Res.* **10** 413.
[4] Man'ko V I 1991 *J. Sov. Laser Res.* **12** 383.
[5] Dodonov V V 2001 *Adv. Chem. Phys.* **119** 309.
[6] Takashima K, Hatakenaka N, Kurihara S and Zeilinger A 2008 *J. Phys. A:Math. Theor.* **41** 146036
[7] Schiff L I 1968 *Quantum Mechanics* (New York: McGraw-Hill Inc.)
[8] Walls D and Milburn G J 1994 *Quantum Optics* (Berlin: Springer)
[9] Hartly J G and Ray J R 1982 *Phys. Rev. D* **25** 382