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EXPERIMENTAL PROPOSAL OF SWITCHED "DELAYED-CHOICE" FOR ENTANGLEMENT SWAPPING

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Entanglement swapping is a fascinating generalization of quantum teleportation, where the entanglement between pairs is interchanged leading to entanglement between particles that never interacted. Extending the idea of Peres, we propose an experimental setup where the choice which quantum systems are finally entangled is made at a time after they have been registered and do not even exist anymore. Our proposed setup can be used in Third-Man Quantum Cryptography where a third party can control whether Alice and Bob are able to communicate secretly, but he does know their secret key.

Keywords: Quantum information; quantum state teleportation; entanglement; entanglement swapping; qubit; photon; quantum optics.

1. Introduction

Entangled systems display one of the most interesting features of quantum mechanics — they are impossible to describe by the states of its (local) constituents

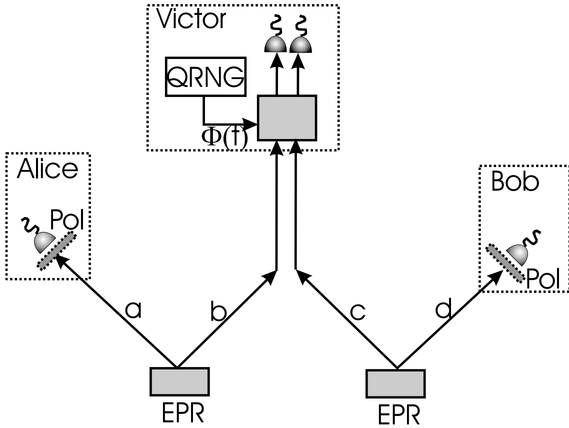

Fig. 1. Scheme of entanglement swapping with "delayed-choice." Two pairs of entangled particles $a - b$ and $b - c$ are produced by two Einstein–Podolsky–Rosen (EPR) sources. One particle from each of the pairs is sent to two separate observers Alice and Bob; for example, particle a is sent to Alice and particle d to Bob. The other particles b and c from each pair are sent to Victor, who may choose whether particles a and d become entangled although they have never interacted in the past, or whether particles a and d remain completely separated as in the initial state.

alone regardless of their spatial separation. It is still sometimes believed that to obtain entangled states, quantum systems *necessarily* need to interact (dynamically) with one another, either directly, or indirectly via other particles.

Yet, an alternative possibility to obtain entanglement between independent systems is to make use of the projection of the state of two particles onto an entangled state. The procedure is known as "entanglement swapping" and was suggested in 1993[1] and experimentally demonstrated for the first time in 1998[2] and later in 2002[4] with a proof of its truly quantum nature. In the experiment two pairs of entangled photons a, b and c, d are produced and one photon from each of the pairs is sent to two separated observers, say photon a is sent to Alice and photon d to Bob, as schematically shown in Fig. 1. The other photons, b from the first pair and c from the second pair, are sent to the third observer, Victor. He subjects photons b and c to a Bell-state measurement, by which photons a and d become automatically entangled. Note that the finally entangled photons a and d neither need to come from a common source nor to have interacted in the past.

2. "Delayed-Choice" in Entanglement Swapping

A seemingly paradoxical situation arises, if — as suggested by Peres[3] — "entanglement is produced *a posteriori*, after the entangled particles have been measured and may even no longer exist." In such a situation, particles a and d are detected before the Bell-state measurement has been performed. This seems paradoxical because Victor's measurement projects photons a and d into an entangled state *after they have been registered.* Furthermore, Victor is even free to choose the kind of measurement he wants to perform on photons b and c. Instead of a Bell-measurement he could also measure the polarization of these photons individually which would result in a well defined polarization for photons a and d, i.e. a separable product state. Whether Alice's and Bob's earlier results indicate entanglement of photons a and d therefore depends on Victor's later measurement. Recently, the delayed-choice entanglement swapping in the spirit of the proposal by Peres[3] was experimentally demonstrated using polarization-entangled photons.[4] In this experiment, the Bell-state measurement was realized by overlapping two photons on a balanced (50:50) beamsplitter and analyzing their distribution in the output ports. Specifically, for a projection on the spin-singlet state $|\psi^-\rangle = 1/\sqrt{(2)}(|H\rangle|V\rangle - |V\rangle|H\rangle)_{bc}$ the two photons exit in different ports of the beamsplitter. By including two 10 m optical fibre delays for both outputs of the Bell-state measurements, photons b and c hit the detectors delayed by about 50 ns with respect to detection of photons a and d. Importantly, it was shown that the observed fidelity of the entangled state of photons a and d matches the fidelity in the nondelayed case within experimental errors. This indicates that the relative temporal order of Alice's and Bob's events on the one hand, and Victor's events on the other is irrelevant.

In this article we propose an experiment which would take this paradoxical situation to its extreme by making the choice of which of the particles are entangled only after the particles have been registered. This is because Victor may choose the type of measurement for photons b and c, which projects them either into a maximally entangled state, or leaves them separated, even after photons a and d have been detected. In general, Victor can even choose to what degree the particles are entangled. Therefore, depending on the states into which photons b and c are projected in Victor's later measurement, Alice's and Bob's earlier results correspond to either entangled or separable states of photons a and d. Furthermore, Victor could project his photons b and c onto an arbitrarily entangled state of photons b and c so that even the degree to which photons a and d are entangled could be defined with "delayed-choice." This degree is actually specified by the amount of information about the state of photon b (or equivalently c), which is gained in Victor's measurement of photons b and c. The information-theoretic description given in Ref. 5 shows that there exists a quantitative *complementarity relation* for the degree of entanglement of photons a and d and the amount of information about the state of photon b or c. In our setup, one single parameter, Φ, determines the information on photon b or c, and thus the complementary degree of entanglement for photons a and d. In addition, the information on the correlations of a and d is undefined even after Alice's and Bob's measurement, as long as the setting and outcome of Victor's measurement is known.

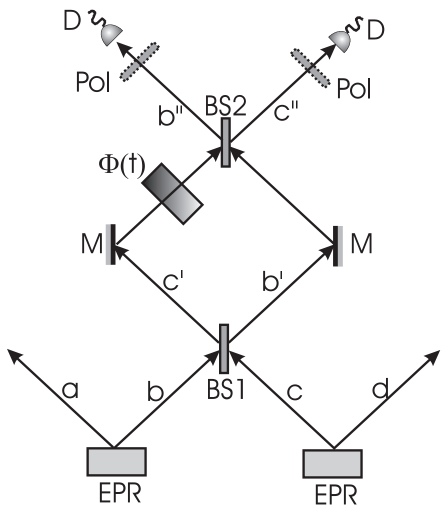

Fig. 2. Proposed setup of "delayed-choice" entanglement swapping. The two pairs of entangled photons $a-b$ and $c-d$ in the $|\Psi^-\rangle$ state are produced by Einstein–Podolsky–Rosen (EPR) sources. The photons b and c from each pair are both sent through an interferometer, which contains a phase modulator $\Phi(t)$. Depending on the setting of this phase, the photons b and c will either be fully independent of each other for $\Phi(t) = 0$, i.e. entangled with their original partners, or the photons b and c will become entangled for $\Phi(t) = \pi/2$ in the $|\Psi^-\rangle$ state, meaning that photons a and d also become entangled. The choice of the phase may be done just in the last moment before photons b and c pass through the interferometer.

3. The Proposal in Detail

We now explicitly describe our experimental proposal which would allow us to realize an entanglement swapping experiment where the measurement on photons b and c is actively chosen, e.g. by a random process. This will provide a very special "delayed-choice" configuration, where the decision on whether photons a and d will be entangled, is actively made after they were detected. The experimental setup is shown in Fig. 2. The joint measurement on photons b and c is realized with an interferometer, where the phase between the two arms b' and c' is tuned with a phase modulator $\Phi(t)$. This interferometer actually represents a beam splitter with tunable reflectivity. We will show in the following that this configuration works as an entangling Bell-state measurement for $\Phi(t) = \pi/2$, but completely keeps the two input photons separated for $\Phi(t) = 0$.

We use the formalism of second quantization, where $a^\dagger, b^\dagger, c^\dagger, d^\dagger$ are the creation operators for modes a, b, c, d. Hence, the state generated by the EPR source is an entangled state of the form

$$|\Psi^-\rangle_{ab} = \frac{1}{\sqrt{2}}\big[|H\rangle_a|V\rangle_b - |V\rangle_a|H\rangle_b\big] \equiv \frac{1}{\sqrt{2}}\big(a_H^\dagger b_V^\dagger - a_V^\dagger b_H^\dagger\big)|0\rangle_a|0\rangle_b, \qquad (1)$$

where H and V are the horizontal and vertical polarization modes of the photons, and $|0\rangle_{a,b}$ is the vacuum state of the corresponding modes. Therefore, the full state of the system will be

$$\frac{1}{2}\left(a_H^\dagger b_V^\dagger - a_V^\dagger b_H^\dagger\right)\left(c_H^\dagger d_V^\dagger - c_V^\dagger d_H^\dagger\right)|0\rangle_a|0\rangle_b|0\rangle_c|0\rangle_d$$
$$\rightarrow a_H^\dagger d_V^\dagger b_V^\dagger c_H^\dagger - a_H^\dagger d_H^\dagger b_V^\dagger c_V^\dagger - a_V^\dagger d_V^\dagger b_H^\dagger c_H^\dagger + a_V^\dagger d_H^\dagger b_H^\dagger c_V^\dagger, \tag{2}$$

where, for simplicity, in the second line we have neglected normalization as well as $|0\rangle_{a,b}$ as we will do for the equations from now on.

Next, we compute how the general modes $b_\alpha^\dagger, c_\beta^\dagger$, where $\alpha, \beta \in H, V$, evolve through the interferometer. First, the modes b and c will evolve through beam splitter 1 as follows:

$$b_\alpha^\dagger c_\beta^\dagger \overset{\text{BS1}}{\longrightarrow} \left(b_\alpha'^\dagger + ic_\alpha'^\dagger\right)\left(c_\beta'^\dagger + ib_\beta'^\dagger\right). \tag{3}$$

Next, mode c picks up the phase $\Phi(t)$:

$$\overset{\Phi(t)}{\longrightarrow} \left(b_\alpha'^\dagger + i\mathrm{e}^{i\Phi(t)}c_\alpha'^\dagger\right)\left(\mathrm{e}^{i\Phi(t)}c_\beta'^\dagger + ib_\beta'^\dagger\right). \tag{4}$$

Now, we let the two beams b', c' interfere in BS2, which leads to

$$\overset{\text{BS2}}{\longrightarrow} \left[b_\alpha''^\dagger + ic_\alpha''^\dagger + i\mathrm{e}^{i\Phi(t)}\left(c_\alpha''^\dagger + ib_\alpha''^\dagger\right)\right]\left[i\mathrm{e}^{i\Phi(t)}\left(c_\beta''^\dagger + ib_\beta''^\dagger\right) + i\left(b_\beta''^\dagger + ic_\beta''^\dagger\right)\right] \tag{5}$$
$$= -b_\alpha''^\dagger b_\beta''^\dagger \sin\Theta\cos\Theta - b_\alpha''^\dagger c_\beta''^\dagger \sin^2\Theta + c_\alpha''^\dagger b_\beta''^\dagger \cos^2\Theta + c_\alpha''^\dagger c_\beta''^\dagger \sin\Theta\cos\Theta, \tag{6}$$

where in the last line we neglected the overall phase term, and substituted the phase by $\Phi(t)/2 \equiv \Theta$ and all normalization factors for simplicity. If we now only consider the case where one photon is found in either of the modes b'', c'', we obtain

$$\rightarrow -b_\alpha''^\dagger c_\beta''^\dagger \sin^2\Theta + c_\alpha''^\dagger b_\beta''^\dagger \cos^2\Theta \tag{7}$$

for the general input $b_\alpha^\dagger, c_\beta^\dagger$.

Next, we can transform each of the four terms for photons b, c in the initial state (2) to the corresponding outputs:

$$b_V^\dagger c_H^\dagger \rightarrow -b_V''^\dagger c_H''^\dagger \sin^2\Theta + c_V''^\dagger b_H''^\dagger \cos^2\Theta, \tag{8}$$
$$b_V^\dagger c_V^\dagger \rightarrow b_V''^\dagger c_V''^\dagger(\cos^2\Theta - \sin^2\Theta) = b_V''^\dagger c_V''^\dagger \cos 2\Theta, \tag{9}$$
$$b_H^\dagger c_H^\dagger \rightarrow b_H''^\dagger c_H''^\dagger \cos 2\Theta, \tag{10}$$
$$b_H^\dagger c_V^\dagger \rightarrow -b_H''^\dagger c_V''^\dagger \sin^2\Theta + c_H''^\dagger b_V''^\dagger \cos^2\Theta. \tag{11}$$

Taking these transformations into account, the whole system will be transformed into the final state

$$\begin{aligned}\rightarrow\ & a_H^\dagger d_V^\dagger[-b_V''^\dagger c_H''^\dagger \sin^2\Theta + c_V''^\dagger b_H''^\dagger \cos^2\Theta] \\ & -a_H^\dagger d_H^\dagger[b_V''^\dagger c_V''^\dagger \cos 2\Theta] \\ & -a_V^\dagger d_V^\dagger[b_H''^\dagger c_H''^\dagger \cos 2\Theta] \\ & +a_V^\dagger d_H^\dagger[-b_H''^\dagger c_V''^\dagger \sin^2\Theta + c_H''^\dagger b_V''^\dagger \cos^2\Theta].\end{aligned} \tag{12}$$

4. How to Make a Choice on the Entangling Operation

There are two special cases for $\Phi(t)$ we want to consider. The first is $\Phi(t) = 0 = \Theta$, which will let the final state become

$$\xrightarrow{\Phi(t)=0} a_H^\dagger d_V^\dagger b_V''^\dagger c_H''^\dagger - a_H^\dagger d_H^\dagger b_V''^\dagger c_V''^\dagger - a_V^\dagger d_V^\dagger b_H''^\dagger c_H''^\dagger + a_V^\dagger d_H^\dagger b_H''^\dagger c_V''^\dagger \tag{13}$$

$$= \left(a_H^\dagger b_V''^\dagger - a_V^\dagger b_H''^\dagger\right)\left(c_H''^\dagger d_V^\dagger - c_V''^\dagger d_H^\dagger\right), \tag{14}$$

where the second line is just the initial state. This will mean that if we set a phase shift of 0, then photons b'', c'' will not interfere. As a consequence the photon pairs a, b and c, d will remain in their maximally entangled states, and there will be no correlations between the photons a, d and b, c.

However, if we consider the phase $\Phi(t) = \pi/2 \Rightarrow \Theta = \pi/4$, we will obtain the final state

$$\xrightarrow{\Phi(t)=\pi/2} a_H^\dagger d_V^\dagger \left[-b_V''^\dagger c_H''^\dagger + c_V''^\dagger b_H''^\dagger\right] + a_V^\dagger d_H^\dagger \left[-b_H''^\dagger c_V''^\dagger + c_H''^\dagger b_V''^\dagger\right] \tag{15}$$

$$= \left(a_H^\dagger d_V^\dagger - a_V^\dagger d_H^\dagger\right)\left(b_H''^\dagger c_V''^\dagger - b_V''^\dagger c_H''^\dagger\right). \tag{16}$$

Now, the final state describes maximal entanglement between the photons a, d and b'', c'', and no correlations between the photons in initially entangled a, b'' and c'', d pairs. Therefore, with a phase $\Phi = \pi/2$, the system performs the full entanglement swapping procedure. The entanglement between photons a and d can be tested, e.g. by violation of the Clauser–Horne–Shimony–Holt (CHSH) inequality.[7,8]

In the general case of $0 \leq \Phi \leq \pi/2$, one can observe a "complementarity relation" between the degree of entanglement of the pairs a, b'' and c'', d, on the one hand, and the degrees of entanglement of the pairs a, c'' and b'', d, on the other, as introduced in Ref. 5. In Ref. 6, it was found that no two pairs of qubits of a three qubit system can violate the CHSH inequality simultaneously. This is because if two systems are highly entangled, they cannot be entangled highly to other systems. Thus, if in our case photon pairs a, b'' and c'', d violate the CHSH inequality, the pairs a, c'' and b'', d cannot and *vice versa*. This can also be explicitly shown for our state expression (12) of the four photons.

5. Third-Man Quantum Cryptography

Our set-up can be used for a specific quantum cryptography protocol that involves a third party and can be called Third-Man cryptography. The Third Man — in our example Victor — can decide at the very last instant long after Alice and Bob perform the polarization measurements on photons a and d respectively, whether he wants (a) to set $\Phi = \pi/2$ and thus observe photons b'' and c'' in a Bell-state, or (b) to set $\Phi = 0$ and then measure the polarization of photons b'' and c'' separately.

If the Third Man chooses (a) then only if he publicly announces at which time he observers one photon in either of the two modes b'' and c'' (i.e. at which time a Bell-state analysis was performed) are Alice and Bob able to establish the secret key by applying the Ekert scheme.[9] If, on the other hand, the Third Man chooses

(b) then, instead of a key between Alice and Bob, he can establish a secret key with either of them or both. Therefore, he can control whether Alice and Bob can communicate secretly or he himself can communicate with them secretly. It is important to note that in the case (a) the Third Man has no access whatsoever to the key and thus on the content of the communication between Alice and Bob. To ensure the secrecy of their communication Alice and Bob can check for a subset of their data, whether their photons are still entangled or not.

6. Conclusions

We propose a "delay-choice" experimental setup allowing which quantum systems are finally entangled to be decided after they have been registered. The experiment is based solely on linear optical elements. The set-up can be used for implementing Third-Man Quantum Cryptography in which a third party can control (or testify) whether two parties can communicate secretly, yet it can be excluded that he has knowledge of the secret keys.

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