Quantum Nonlocality Obtained from Local States by Entanglement Purification

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We have applied an entanglement purification protocol to produce a single entangled pair of photons capable of violating a Clauser-Horne-Shimony-Holt Bell inequality from two pairs that individually could not. The initial poorly entangled photons were created by a controllable decoherence that introduced complex errors. All of the states were reconstructed using quantum state tomography which allowed for a quantitative description of the improvement of the state after purification.

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Distributed entanglement is a prerequisite for quantum communication and quantum computation. Unfortunately, entanglement is very fragile and can prove difficult to maintain particularly between separated parties and over extended time scales. This places stringent limits on the range over which quantum communication can take place without some sort of restoration. In this experimental work, we create two-photon pairs with low entanglement and purity through controllable decoherence. This noise is very complex and is not engineered with a specific, but limited, purification scheme in mind [1,2]. After this decoherence, each pair cannot violate a Clauser-Horne-Shimony-Holt (CHSH) inequality [3] or any other known inequality [4] and we refer to the states as "local." We apply an entanglement purification protocol to these initial pairs and through quantum state tomography show quantitative improvement in the qualities of the purified pair. This is confirmed by the final pair violating a CHSH-Bell inequality proving that it cannot be described by a local realistic model and can serve as the crucial entanglement resource for a number of quantum communication protocols, such as quantum communication complexity [5].

Entanglement can be created via local interactions between particles. However, once the particles are no longer in direct contact, or connected by a quantum channel, their entanglement cannot be increased on average [6]. There are, however, methods for increasing the entanglement or state purity of a subset of the entangled particles [7,8]. In the general entanglement purification scheme proposed by Bennett *et al.* [9], Alice and Bob share two pairs of low-quality entangled particles such that each has one particle from each pair. They perform local entangling operations on their respective particles, and then each makes a local measurement on one of the particles. By comparing their measurement outcomes and performing local transformations based on those results, they retain a single shared pair of particles which can be of both higher entanglement and higher purity. After many rounds of this procedure, Alice and Bob can share a state asymptotically close to a maximally entangled pure state such as one of the Bell states $|\phi^{\pm}\rangle = (1/\sqrt{2})(|H\rangle|H\rangle \pm |V\rangle|V\rangle)$ and $|\psi^{\pm}\rangle = (1/\sqrt{2}) \times (|H\rangle|V\rangle \pm |V\rangle|H\rangle)$. While this scheme requires technically difficult controlled-NOT (CNOT) operations [10], it was shown that a commonly used linear optical element, the polarizing beam splitter (PBS), could be used in its place if one is satisfied with probabilistic operation [11].

The entanglement purification experiment of Pan *et al.* [12] served as a proof of principle, but the qualities of the input states were reduced in an artificial way. Input states were created via local polarization rotations on highly pure and entangled photon pairs. By averaging over two different polarization rotations, the authors argued they had an effectively mixed state. However, local polarization rotations cannot change either the purity or entanglement of the photon pairs. Furthermore, these rotations only introduce bit-flip errors and create mixtures of only two of the four Bell states. Mixtures of two Bell states always display nonlocal behavior (i.e., violate a Bell inequality), except for the case of an exactly equal mixture [13], which cannot be purified.

Here, we create effectively mixed polarization states by entangling the polarization degree of freedom from highly entangled photon pairs with their arrival times [1,8,14]; these time shifts are far too small to be measured by our detectors and constitute for all practical purposes irreversible interaction with the environment. Entanglement with this unobserved degree of freedom reduces the quantum coherences in the polarization states required for high entanglement and purity. Our decoherence creates states with incoherent contributions from all of the Bell states by introducing complex phase, bit-flip, and correlated errors. Using quantum state tomography [15], we reconstruct the full two-photon polarization states of our initial pairs and purified pair. The CHSH-Bell inequality [3] shows a conflict between local realism and measured polarization correlations when the Bell parameter, S (a function of those correlations), is larger than 2. We introduce sufficient decoherence to make our initial states incapable of violating such an inequality for any analyzer settings. We then quantitatively measure the improvement in the entangle-

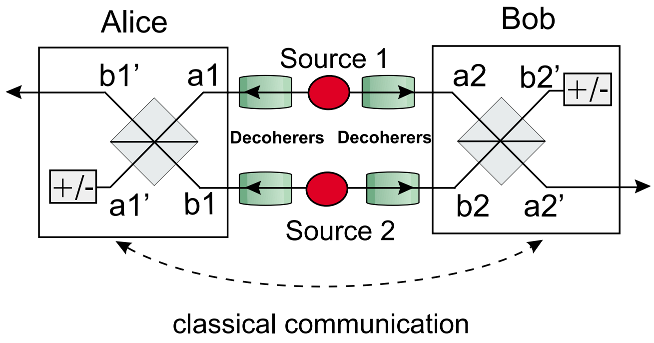

FIG. 1 (color). Schematic for entanglement purification. A source produces two pairs of entangled photons, one photon from each pair travels to Alice and the others to Bob. In each quantum channel, decoherence degrades the entanglement and purity of the quantum states. Each party mixes their photons through a PBS oriented in the H/V basis and measures the outputs a1$'$ and b2$'$ in the $|\pm\rangle = 1/\sqrt{2}(|H\rangle \pm |V\rangle)$ basis. Given that the four photons each take a different PBS output and both Alice and Bob find the same measurement outcome (in our experiment $|+\rangle$), the remaining photon pair in modes a2$'$ and b1$'$ has been purified.

ment and the purity of our purified state; our final state violates a Bell inequality by more than 2σ.

At the heart of this entanglement purification method is the PBS (Fig. 1). A PBS transmits horizontally polarized (H) light and reflects vertically polarized (V) light. If two photons enter the PBS from two different inputs, then the photons are sent to different outputs if and only if they are both H or both V. For this reason, the action of the PBS and postselection of photons in the two different outputs constitutes a determination that the parity of the photons is even [11,16]. The CNOT operation on a control (C) and target (T) photon polarization performs: $|H\rangle_C|H\rangle_T \rightarrow |H\rangle_C|H\rangle_T$, $|H\rangle_C|V\rangle_T \rightarrow |H\rangle_C|V\rangle_T$, $|V\rangle_C|H\rangle_T \rightarrow |V\rangle_C|V\rangle_T$, $|V\rangle_C|V\rangle_T \rightarrow |V\rangle_C|H\rangle_T$. These operations are related since, if the target photon is measured in the state H after a CNOT, then the input photons had even parity. However, the PBS-based scheme is limited to 50% of the efficiency of the CNOT scheme, since it only measures cases with even parity. Furthermore, the linear-optics parity check generates the requisite entanglement only when the photons overlap coherently at the PBS.

In our protocol (Fig. 1), Alice and Bob each have one photon from each entangled pair. They, locally, perform parity checks and measure the polarization of one of their PBS outcomes (modes a1$'$ and b2$'$) in the linear basis $|\pm\rangle = 1/\sqrt{2}(|H\rangle \pm |V\rangle)$. Alice and Bob keep the photons in the other modes (a2$'$ and b1$'$), only in those cases when they both obtain the $|+\rangle$ outcome. If each pair of entangled photons has a minimum fidelity $F > 0.5$ with a Bell state, and if both parity checks succeed, then those remaining photons can be both of higher purity and of higher entanglement [11]. The interested reader is referred to [9] for a mathematical analysis of purification using CNOT operations and to [11,17] for a detailed theoretical treatment of the linear-optics implementation.

In our experiment, we create two pairs of entangled photons using type-II parametric down-conversion [18]. An ultraviolet (UV) pulse passes twice through a β-barium borate (BBO) crystal, which emits highly entangled photons both into the forward pair of modes a2 and b2 and into the backward pair of modes a1 and b1 (Fig. 2). To counter the effects of birefringence in the down-conversion BBO crystal, one normally rotates the polarization of each photon by 90° with half-wave plates (HWP) and passes them through a second set of BBO crystals. Failure to do so results in degradation of the pair's entanglement from unwanted correlations between the polariza-

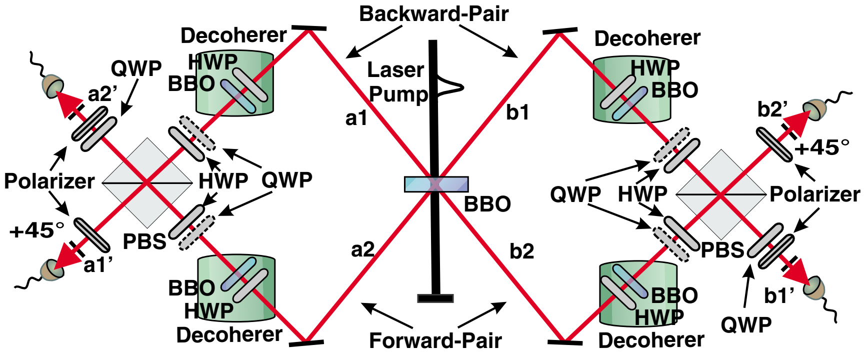

FIG. 2 (color). The experimental setup for purifying mixed entangled states. Entangled photon pairs are created when an ultraviolet laser pulse makes two passes through a β-barium borate (BBO) crystal. Rotating the half-wave plate (HWP) in front of each compensating BBO crystal by less than 90° creates a tunable degree of decoherence (Decoherer). All four HWPs are left in to rotate 45° to enhance the efficiency of purification under our experimental noise conditions. Modes from the two different pairs (a1 and a2, and b1 and b2) are combined at the two PBSs which perform parity checks. In the output modes a1$'$ and b2$'$, projective measurements are made onto the state $|+\rangle$ using single-mode fiber-coupled single-photon counting detectors behind 3-nm bandwidth interference filters. Tomographic and Bell inequality measurements are performed using the HWPs and quarter-wave plates (QWPs) in these output modes and two more fiber-coupled detectors also behind interference filters.

tion and the time of arrival of the photons. By rotating by an angle less than 90°, we introduce controllable effective decoherence in the polarization states of our photon pairs.

Down-conversion is a probabilistic source of entangled photons and thus can produce two-photon pairs in the same pair of modes (e.g., a2 and b2) with about the same probability as two pairs into four different modes. It has been shown that, if the phases of the forward double pair and backward double pair are held stable, then the double pair emission can enhance the efficiency of purification over the case with no double pair emission [17]. We, therefore, kept the path lengths strictly controlled and enclosed the setup to minimize air fluctuations for the duration of the measurements.

The density matrices of the forward- and backward-emitted pairs were reconstructed via a "maximum likelihood" method [19,20] from our experimental data as shown in Figs. 3(a) and 3(b), respectively. The HWP in each decoherer rotated the photon polarizations by 50° for the forward pair and 62° for the backward pair, instead of the ideal 90°. These rotation angles were chosen to reduce the maximum possible Bell parameter to $S < 2$. The angles differ most likely due to experimental asymmetries in the pump characteristics and to spatial filtering. The two initial density matrices are similar, but have interesting differences. Both input pairs contain large diagonal elements in the HH and VV positions with nonmaximal negative coherences so both states are primarily $|\phi^-\rangle$. The extra diagonal terms and coherences in the density matrices, more prominent for the forward pair, indicate that there are also $|\psi^\pm\rangle$ components in our states. The smaller than maximal coherences between HH and VV, which are more prominent for the backward pair, show that our states are highly mixed. Using the method of Horodecki *et al.* [13] on our initial states, we find that the maximum CHSH-Bell parameter one could possibly measure is $S_{\max} = 1.89 \pm 0.02$ for the forward pair and $S_{\max} = 1.90 \pm 0.014$ for the backward pair. The errors on quantities extracted from the density matrices were calculated via a Monte Carlo procedure. Both of our initial states are more than 5σ below the nonlocality border of $S = 2$ and cannot serve as the crucial entanglement resource for many quantum communication protocols. Our initial states were also checked against the 3322 and 3422 inequalities of Collins and Gisin [4], which are complementary to the CHSH inequality, and found that these inequalities could also not be violated.

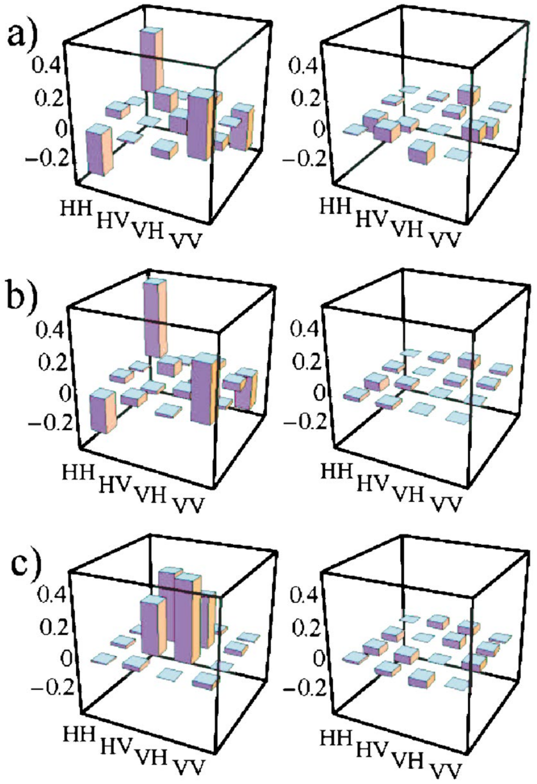

FIG. 3 (color). Tomographic reconstruction of the density matrices before and after purification. The real (left-hand side) and imaginary (right-hand side) parts of the density matrices of (a) the initial forward pair, (b) the initial backward pair, and (c) the purified pair. The maximum possible Bell parameter, $S_{\max}$, from the CHSH-Bell inequality for any set of measurements is 1.89 ± 0.02 for the initial forward pair and 1.90 ± 0.014 for the initial backward pair. For the purified state we measured the Bell parameter with the polarizer settings −22.5° and 22.5° in mode a3 and 0° and 45° in mode b4 to be $S_{\text{meas}} = 2.29 \pm 0.13 > 2$—a 2.2-$\sigma$ Bell inequality violation. The final purified state cannot be described by any local realistic theory.

The PBS acts in a preferred basis, the H/V basis. This makes purification more efficient at purifying Bell states with different H/V correlations (i.e., $|\psi^\pm\rangle$ states from $|\phi^\pm\rangle$ noise) than it is from purifying Bell states with the same H/V correlations (i.e.,$|\psi^\pm\rangle$ states from $|\psi^\pm\rangle$ noise). One can convert between different Bell states through local polarization rotations without changing the entanglement or purity. With colored noise, these local rotations can increase the efficiency of purification. We perform polarization rotations using the four HWP which convert $|\phi^-\rangle$ to $|\psi^+\rangle$ and also permute the noise contributions. This allows higher degree of purification and converts our output state to $|\psi^+\rangle$.

Figure 3(c) shows the density matrix of our final state after the purification protocol. Whereas the initial state measurements were based on direct two-photon coincidence counts, the final state measurements were based on fourfold coincidence counts—two counts signal that Alice and Bob both measure the $|+\rangle$ outcome and the second two are used for tomography of the purified state. Fourfold coincidences were accumulated for 6000 s per point and were normalized by the square of the coincidences between the two fixed measurement detectors (modes a1′ and b2′) to account for changes in the laser power. The final state contains large diagonal elements in the HV and VH positions and has large positive coherences between them—this is the signature of $|\psi^+\rangle$. The larger coherences and the lack of significant terms in other diagonal density

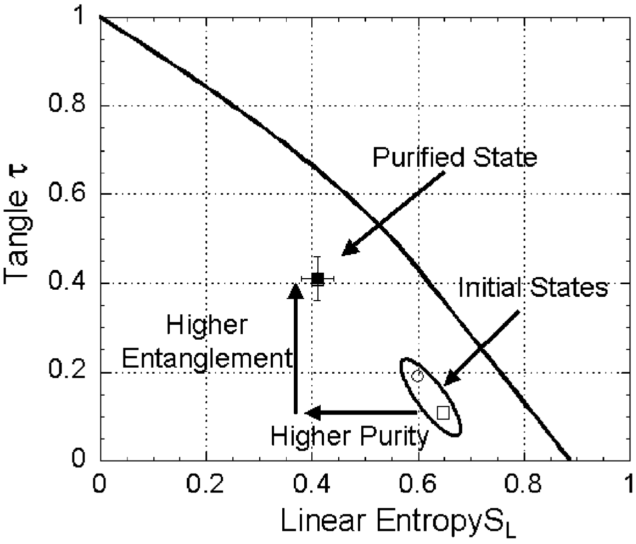

FIG. 4. Quantifying the state's improvement through purification. Quantum state purity and entanglement can be characterized by the linear entropy of the quantum state and by its tangle. The initial states (open circle and open square) are of high linear entropy and low tangle; i.e., they are highly mixed and have low entanglement. After purification, the new state (solid square) is of higher purity and higher entanglement. For reference, all physical states lie below the solid line [22].

matrix elements indicate qualitatively that our state has both improved purity and entanglement characteristics. We express the improvement quantitatively by computing the maximum Bell parameter one could measure from this state for optimal measurement settings, $S_{\max} = 2.28 \pm 0.06$. From our tomographic reconstruction, we conclude that our final state can violate a Bell inequality by 4σ. Using this same final state, we explicitly measured the Bell parameter for the measurement settings of $-22.5°$, $22.5°$ for Alice (mode a2$'$) and $0°$, $45°$ for Bob (mode b1$'$) to be $S_{\text{meas}} = 2.29 \pm 0.13$—a 2.2σ violation. For the same settings, we predict the Bell parameter from the density matrix (DM) and find $S_{\text{DM}} = 2.17 \pm 0.07$, in good agreement with our measured results.

For bipartite states, the entanglement can be quantified by the tangle [21], while the purity can be quantified by the linear entropy $S_L = \frac{4}{3}(1 - \text{Tr}[\rho^2])$. To show that the entanglement and purity of our state are improved through the purification process, we plot the input and final states in Fig. 4 [22]. The initial states have very low tangle (low entanglement) and high linear entropy (low purity). Both the tangle and entropy of our final state are significantly improved by the purification process.

Decoherence is one of the most difficult obstacles facing experimental quantum information processing and communication. In this work, two-photon pairs were decohered by complex noise that introduced both phase and bit-flip errors. We have used a linear-optics-based protocol to purify the states, and quantitatively measured the resulting improvement in their quantum properties. Our initial states were so noisy that their polarization correlations could not violate a CHSH inequality and would thus be useless for many quantum communication tasks. In contrast, the nonlocality of our final state was explicitly demonstrated by violation of a Bell inequality. The newly created nonlocal correlations constituted an entanglement resource for quantum communication protocols. Counteracting the negative effects of decoherence is essential to the success of quantum information.

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