

A NOVEL PROTOCOL-AUTHENTICATION ALGORITHM RULING OUT A MAN-IN-THE MIDDLE ATTACK IN QUANTUM CRYPTOGRAPHY

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In this work we review the security vulnerability of quantum cryptography with respect to “man-in-the-middle attacks” and the standard authentication methods applied to counteract these attacks. We further propose a modified authentication algorithm which features higher efficiency with respect to consumption of mutual secret bits.

Keywords: Quantum cryptography; QKD; authentication.

1. Introduction

Quantum Key Distribution (QKD) or “quantum cryptography” is a quantum mechanics based cryptographic primitive which, in principle, holds the potential of absolutely secure communication that cannot be compromised by any eavesdropping technique. The strength of the QKD primitive is the unconditionally secure simultaneous generation of two identical bit streams at two distinct locations which subsequently could be used as a key in symmetric (unconditionally or computationally secure) encryption schemes. However, it is well known that QKD requires a public channel with trusted integrity as otherwise a potential adversary (Eve) can easily mount a “man-in-the-middle attack.” In case the eavesdropper can manipulate messages on the public channel there is no way to guarantee that in the course of a QKD protocol the two legitimate communication parties (Alice and Bob) are really exchanging the messages they are sending to each other.

Eve can simply cut the quantum channel and subsequently communicate over both the quantum and the public channels with Bob as if she would be Alice and with Alice as if she would be Bob. Eventually, she would thus share two independent keys with the two legitimate parties and gain full control of all the subsequently transmitted encrypted information without being noticed at all. The described type of attack can be counteracted by authenticating the QKD protocol messages transmitted over the public channel. Basically, public key authentication methods and symmetric key authentication methods can be used (see Ref. 1 for a discussion of the relative merits and drawbacks of these methods). It is, however, straightforward to notice that unconditionally secure key generation by means of QKD is only feasible if it is combined with methods providing unconditionally secure authentication. Standard public key methods are automatically ruled out if one were to stick to this requirement as the latter are only computationally secure and potentially subject to cryptanalysis by means of quantum computers. Therefore, already in Ref. 2 it was proposed to use unconditionally secure symmetric message authentication methods as, for example, developed in Ref. 3 to ensure the integrity of the public channel. The main idea of the application of these methods in QKD is to intertwine the transcript of the public channel communication with an independent secret, which the two legitimate parties share, and on this basis provide a mechanism for authenticating this communication. Alice and Bob need therefore an initial secret key, which they use only once. Subsequently, in each QKD session they repeatedly renew the mutual secret by reserving part of the newly generated key. This key is to be used for channel authentication purposes in the next session. This paradigm has been elaborated in subsequent publications.^{4,5} It should be noted that while thus the unconditional security of QKD is retained, it is basically degraded from a secret key generation scheme in the strict sense to a secret key growing technique.

In what follows we restrict our discussion to symmetric key message authentication methods and, similar to Wegman and Carter,³ base our approach on strongly universal₂ functions. In Sec. 2 we discuss a general method for producing message authentication tags using only a moderate amount of the secret key. In Sec. 3 we briefly discuss the details of the authentication algorithm in relation to the QKD protocol. We also present a modular integrated software library implementing full scale QKD-protocols including public channel authentication used in the framework of a recent quantum cryptographic experiment.⁶

2. Message Authentication Primitive

A broad class of unconditionally secure symmetric key message authentication approaches follow the method described in Ref. 3. This method is independent of the context in which authentication is applied, and therefore we refer to it in what follows as the authentication primitive. Before discussing the primitive itself, we briefly review the foundations of message authentication by means of a family of strongly universal₂ functions. Let H_A be such a family of functions which

maps the set of all messages A , typically the set of binary strings of length m , to the set of all authentication tags B , typically the set of binary strings of length $n < m$. One can then authenticate a message by sending an authentication tag in addition to the message itself over the communication channel. An adversary willing to manipulate the original message must also be able to produce the proper tag for the manipulated message. The authentication system is unbreakable with probability p when B is chosen to have at least $1/p$ elements.³ The term “unbreakable with probability p ” is used in the following sense: if a message from $l \in A$ yields a tag $t \in B$ through a randomly chosen function $f \in H_A$, $t = f(l)$, and if an eavesdropper knows m and t but not f , she has only a probability lower than p to find the proper tag t' of a different message l' , with respect to f , $t' = f(l')$. The legitimate parties share a secret key, which is used as an index in the function space H_A . In this sense the secret sharing is symmetric. The secret can be used only once. The problem with this basic approach is that most of the well known families of strongly universal₂ functions are typically larger than the space of all messages. Therefore, the key needed to authenticate a message is longer than the message itself. This is a particular problem in quantum cryptography, where the key growth factor directly depends on the portion of generated key, reserved for a subsequent authentication. While it is necessary to minimize the length of the messages to be authenticated as discussed in Sec. 3, it is also strongly desirable to restrict the space of applied hash functions, reduce the secure key consumption for authentication purposes and thus get efficient authentication methods. At the expense of increasing the “security parameter” p to $2p$, Wegman and Carter propose a method for building a relatively restricted family of almost strongly universal₂ hash functions,³ which uses a basic class of strongly universal₂ hash functions into intermediate spaces as a kernel. Wegman and Carter choose a specific multiplicative family of hash functions (denoted as H_1 in Ref. 7), to map strings of length $2s$ to those of length s , where

$$s = n + \log_2 \log_2 m. \quad (1)$$

Note that by definition the cardinality of this class, being a function of s , only grows slowly with m . The original message l is then divided into substrings of a defined length $2s$ and a randomly chosen hash function from the mentioned class is applied to the substrings. The set of resulting tags is then concatenated to produce an intermediate message. The latter is then once again subdivided into substrings of the length $2s$ and a new hash function from the described family is applied to each string. This process is applied until only one tag remains. The lower order n bits are taken for the final authentication tag t . One can show³ that this method defines an almost strongly universal₂ family of functions from A onto B . Wegman and Carter also prove that the key length needed to index this family is

$$k = 4s \log_2 m. \quad (2)$$

This method constitutes a general primitive for symmetric key authentication. The definition f of the almost strongly universal₂ class of hash functions is independent of the underlying kernel class of intermediate strongly universal₂ functions and any such class can be used. The authentication of the public channel in QKD discussed so far in the literature (see, for example, Refs. 2, 5 and 8) are almost exclusively based on the discussed primitive developed in Ref. 3, including the choice of the basic intermediate class of strongly universal₂ ($2s$ to s) hash functions. It is obvious that this method is suitable for authenticating long messages. As an example, for authentication tags which are 64 bits long, the message length exceeds the key length if the former is longer than 3138 bits. For messages longer than 20000 bits the message length exceeds the key lengths already by a factor of four. However, in certain settings, and in particular in the QKD case, it is highly relevant to have an efficient authentication primitive also for short messages. To this end, we propose a new primitive, which includes a two-step procedure. First of all, one maps the initial message l from A to Z , where Z is the set of all binary strings of length r ($m > r > n$), by means of a single publicly known hash function f_0 so that $z = f_0(l)$. The second step is a direct application of the basic approach as discussed above. One sends m over the communication channel alongside $t = f(z)$, where f is a randomly chosen secret strongly universal₂ hash function from H_Z mapping Z onto B . We first discuss the security of this primitive and then assess the amount of secret key needed for its implementation. The security of the primitive is given by the probability p of an adversary to produce a proper authentication tag for a modified message (cf. the discussion above). Obviously

$$p = p_1 + p_2, \quad p_2 = 1/|B|. \quad (3)$$

Here, p_2 is the probability for the eavesdropper of breaking the strongly universal₂ family H_Z (see Ref. 3) while p_1 is the probability that the initial message and the modified message yield the same tag q under the chosen fixed hash function f_0 :

$$p_1 = \max_l (\Pr\{f_0(l) = f_0(l') | l \neq l'\}). \quad (4)$$

Clearly all messages $A_0 = f_0^{-1}(z)$ yield the same authentication tag z and thus

$$p_1 = \max_l (\Pr\{l' \in \bar{A}_0 = \{A_0 \setminus l\}\}). \quad (5)$$

In the case when A_0 is independent of the choice of l and all l are equally probable (the distribution of meaningful messages in the space of all bit strings is uniform) then

$$p_1 = (|A|/|Z| - 1)/|A| < 1/|Z| \quad \text{for all values of } |A|, \quad p < 1/|B| + 1/|Z|. \quad (6)$$

In addition to the two basic assumptions in the derivation of expression Eq. (6) one should note that we implicitly assume that the message l' is random and fixed, i.e. the eavesdropper cannot choose the manipulation message at will. While this cannot be taken for granted in general, in the case of man-in-the-middle attacks

in quantum cryptography l and l' are definitely randomly and independently fixed beside the scope of influence of the adversary. (In the case of a man-in-the-middle attack l and l' are protocol extracts from the communication between Alice and Eve and Eve and Bob, respectively, whereby these are generated through physical random processes and Eve has no opportunity at all to change either l or l' .) The assumption that A_0 is independent from l can be guaranteed by any suitably chosen hash function that constitutes a homomorphism of A onto Z . Finally, the assumption of a uniform distribution of all possible messages depends on the choice of the protocol extracts and cannot always be granted. However, one can initially perform a uniform randomizing operation, e.g. by means of XORing the message l with a completely random bit string of the same dimension. The latter can be obtained by means of a deterministic pseudorandom generator whereby a number of secret bits from the joint secret are used for the seed. The application of other appropriate uniform randomizers, possibly integrated in the definition of f_0 , is also feasible.

We should now point out that the secret key needed in this approach is exactly the number of bits needed to index the family H_Z . Obviously, if r (the dimension of Z) is chosen to be moderate and an appropriate restricted strongly universal₂ class is selected, then the amount of secret key required can also be reduced. To estimate this amount exactly one needs to specify the function family applied. We choose the set of affine transformations:

$$H_Z(\cdot) = \{f = (\Phi, \beta) | \Phi - \text{all } (n \times r) \text{ binary Toeplitz matrices}; \\ \beta - \text{all } (n \times 1) \text{ binary vectors}\}, \quad f(z) = \Phi z + \beta \text{ mod}(2). \quad (7)$$

This function family^{9,10} is strongly universal₂ as shown in Ref. 10 and is indexed by $r + 2 \times n - 1$ parameters. For $r = 256$ and $n = 64$, obviously the message length exceeds the required secret key already for strings longer than 384 bits. In contrast to the primitive used by Wegman and Carter, this amount is constant by definition and does not increase with m .

3. Protocol Implementation

In Sec. 2 we have only given a general description of a new primitive suitable for authentication in QKD settings, leaving the question of the exact protocol extracts to be authenticated completely aside. It is beyond the scope of the present paper to address this topic in detail. This issue has however been thoroughly discussed in Refs. 4 and 5. Certainly, an authentication of the full protocol transcript is one (inefficient) extreme possibility. In Ref. 5 it is shown that authenticating the sifting phase discussion and the results of the error correction phase is sufficient. In this paper it is also suggested that (the relevant parts of) the transcript are not authenticated at once but rather the bit strings to be authenticated are separately processed as soon as they are generated in the respective QKD protocol phases.

A particular advantage of this approach with respect to the cryptographic primitive proposed in Sec. 2 above, is that all the bit strings to be authenticated are randomly and uniformly distributed in the space of all possible strings of corresponding dimension. Thus, if this protocol is employed, an initial randomization is not required for a secure application the primitive described above.

We have implemented an authentication algorithm based on the primitive presented in this paper, whereby, provisionally, SHA is used as the initial hash function. This algorithm is a part of a constantly developed modular software-set up, which is integrated in the framework of an embedded general purpose QKD hardware-software prototype dedicated to data acquisition and subsequent QKD-protocol processing and data encryption (currently AES and One Time Pad are implemented). A public demonstration of the functionality of this QKD prototype together with an optic segment implementing entangled-photon key generation took place recently in the form of a “Q-Banking” transaction, which was carried out between two buildings in Vienna, Austria — the Rathaus (city hall) and the seat of Bank Austria Creditanstalt.⁶ The current version of the software set-up is designed in the form of a C library “QKD III” which allows application by choice of alternative quantum-acquisition protocols, error correction, privacy amplification and authentication algorithms and can alternatively be compiled for usage in PC or embedded environments. The QKD protocol implemented, in contrast to an earlier version used in the “Q-Banking” experiment, follows the approach suggested in Ref. 5. This protocol is prone by design to a potential loophole in this earlier version. The latter is discussed in detail in Ref. 11.

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