Experimental Violation of a Cluster State Bell Inequality

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Cluster states are a new type of multiqubit entangled states with entanglement properties exceptionally well suited for quantum computation. In the present work, we experimentally demonstrate that correlations in a four-qubit linear cluster state cannot be described by local realism. This exploration is based on a recently derived Bell-type inequality [V. Scarani *et al.*, Phys. Rev. A **71**, 042325 (2005)] which is tailored, by using a combination of three- and four-particle correlations, to be maximally violated by cluster states but not violated at all by GHZ states. We observe a cluster-state Bell parameter of 2.59 ± 0.08, which is more than 7σ larger than the threshold of 2 imposed by local realism.

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Multiparticle entanglement is a complex and relatively unexplored landscape. For two qubits, there exists only one class of entanglement [1], for three qubits there are two classes of genuine three-particle entangled states [2,3], and for four qubits at least nine different classes of entanglement have been identified [4]. Recently, a great deal of attention has been devoted to a class of multiparticle entangled states called cluster states. This attention is largely due to the application of cluster states in Raussendorf and Briegel's "one-way" model for universal quantum computation [5]. In that model, one can drive a quantum computation entirely through single-qubit measurements and feed forward instead of unitary evolution. In addition to being a practical alternative to the standard model for quantum computing, it has also called into question the requirements for quantum computing and the relationship between measurement and dynamics [6]. One-way quantum computation based on cluster states demonstrating one-qubit gates, two-qubit gates, and a quantum search algorithm was recently realized experimentally [7].

Aside from their fascinating use for quantum computing, cluster states are a novel kind of multiparticle entangled states with fundamentally new and different properties. They share some properties with multiparticle extensions of both Greenberger-Horne-Zeilinger (GHZ) states $|\mathrm{GHZ}\rangle = 1/\sqrt{2}(|000\rangle_{123} + |111\rangle_{123})$ [8–10] and W states $|\mathrm{W}\rangle = 1/\sqrt{3}(|100\rangle_{123} + |010\rangle_{123} + |001\rangle_{123})$ [2,11,12]. Each single-qubit constituent of a cluster state is completely mixed, characteristic of GHZ states. Also, any two of the four cluster qubits can be projected into a Bell state by choosing an appropriate basis, similar to a GHZ state, but cluster states also share their *persistency of entanglement* [13] with the W states. Recent theoretical investigations of the "nonlocality" of these cluster states have constructed new types of Bell inequalities and even GHZ-type arguments to refute local realism with the specific correlations of cluster states in mind [14,15].

Bell's inequalities are specifically designed to put quantum physics to the test against local realistic models. For two-qubit entangled states, the CHSH-Bell inequality [16,17] is perhaps the best-known example. The inequality is constructed from two-qubit spin or, in our case, polarization correlation functions. Similarly, the Mermin inequality [18], testing local realism in three-qubit entangled states, is made entirely of three-qubit correlations. Its generalization is based entirely on N-qubit correlations [19]. In general, the choice of these correlations determines the optimality of a Bell inequality, i.e., whether entanglement is detected by a maximal violation of the inequality. For example, for the specific case of three qubits the inclusion of lower-order correlations can lead to an optimal Bell inequality for a W state, which could not detect GHZ entanglement [20]. This ambiguity and selectivity of which type of entangled state produces a maximal Bell violation makes the connection between entanglement and Bell's inequality tenuous especially in multiparticle states [21]. Nevertheless, it nicely highlights the fundamentally different ways in which the GHZ and W states manifest violations of local realism. Since the number of distinct classes of entanglement grows rapidly with the number of qubits, one might expect to find other Bell inequalities optimal for different states. Specifically, for a Bell inequality optimal for cluster states, lower-order correlations will be of importance, since cluster states can be generated by nearest-neighbor Ising interaction [5].

A recent theoretical work has found a GHZ-type argument for cluster states [14]. As in the original GHZ article [8], the new work showed that there exists a combination of observables whose expectation values cannot be consistent with a set of local realistic properties. However, in contrast to GHZ states, cluster states can even fulfill a GHZ argument using combinations of three- and four-qubit correlations [22]. This leads to the development of a Bell inequality that can be maximally violated by cluster states but cannot be violated at all by GHZ states. In this experimental work, we use four-qubit cluster states encoded into the polarization state of photons to test that Bell inequality.

A linear cluster state arises when a line of qubits, each in the $|+\rangle$ state, where $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$, experience nearest-neighbor CPhase operations, i.e., $|j\rangle|k\rangle \rightarrow (-1)^{jk}|j\rangle|k\rangle$, $j, k \in \{0, 1\}$ [5]. Linear cluster states of two and three qubits are equivalent under local unitary transformations, or "locally equivalent," to Bell states and GHZ states, respectively [5]. In contrast, the four-qubit cluster is not locally equivalent to either the four-qubit GHZ or W states. In the present work, we use photon polarization to encode qubits with horizontal (vertical) polarization corresponding to $|0\rangle$ ($|1\rangle$). Our target cluster state is of the form,

$$|\phi_4\rangle = \frac{1}{2}(|HHHH\rangle_{1234} + |HHVV\rangle_{1234} + |VVHH\rangle_{1234} - |VVVV\rangle_{1234}), \tag{1}$$

where the subscripts 1, 2, 3, and 4 label different photons in separated spatial modes. This state is locally equivalent, under a Hadamard transformation $H = \frac{1}{\sqrt{2}}(\sigma_X + \sigma_Z)$ on the first and last qubit, to the four-qubit linear cluster state

$$|\phi_4'\rangle = \frac{1}{2}(|0+0+\rangle_{1234} + |0-1-\rangle_{1234} + |1-0+\rangle_{1234} + |1+1-\rangle_{1234}), \tag{2}$$

where $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$ represents the complementary linear polarization. The linear cluster state, $|\phi_4'\rangle$, has a set of 15 nontrivial stabilizer operators, S_i', each made up of products of four Pauli operators such that $S_i'|\phi_4'\rangle = \pm|\phi_4'\rangle$ [14]. Since each of the Pauli operators, σ_X, σ_Y, σ_Z, and σ_0, has eigenvalues of ± 1 (σ_0 is the identity), each such stabilizer operators represents a property of the state that is fulfilled *with certainty*, i.e., an element of physical reality [23]. Following the reasoning of GHZ one can then find sets of 4 of these stabilizers, e.g., $\sigma_Z\sigma_Y\sigma_Y\sigma_Z$, $\sigma_Z\sigma_Y\sigma_X\sigma_Y$, $\sigma_0\sigma_Z\sigma_X\sigma_Z$, $\sigma_0\sigma_Z\sigma_Y\sigma_Y$ with expectation values $+1$, -1, $+1$, and $+1$, which are inconsistent with local realism. In addition, these stabilizers can be used to construct a Bell inequality. Since $|\phi_4'\rangle$ and $|\phi_4\rangle$ are equivalent only up to local transformations, the stabilizer operators required for the GHZ argument need to be interconverted. Obviously, the GHZ argument and Bell inequality remain intact. Making use of the relations $H\sigma_X = \sigma_Z H$, $H\sigma_Z = \sigma_X H$, $H\sigma_Y = -\sigma_Y H$, and $H\sigma_0 = \sigma_0 H$, we can convert the four operators, S_i', to a new set, S_i, to $\sigma_X\sigma_Y\sigma_Y\sigma_X$, $\sigma_X\sigma_Y\sigma_X\sigma_Y$, $\sigma_0\sigma_Z\sigma_X\sigma_X$, and $\sigma_0\sigma_Z\sigma_Y\sigma_Y$, where the expectation values for $|\phi_4\rangle$ are $+1$, $+1$, $+1$, and -1, respectively. These stabilizers can now be used to construct the Bell inequality optimized for our cluster state $|\phi_4\rangle$. The Bell parameter, S_C, is given by

$$S_C = |\sigma_X\sigma_Y\sigma_Y\sigma_X + \sigma_X\sigma_Y\sigma_X\sigma_Y| + |\sigma_0\sigma_Z\sigma_X\sigma_X - \sigma_0\sigma_Z\sigma_Y\sigma_Y|. \tag{3}$$

The assumptions of locality and realism put a limit on the strength of the correlations such that $S_C \leq 2$. However, since the four terms in the Bell inequality are stabilizers of the cluster state, with the last term having opposite sign, the cluster state can violate this bound up to the algebraic limit of this expression, i.e., $S_C = 4$. It is a curious fact that the GHZ state, which is often said to be a maximally entangled multiparticle state, cannot violate this inequality. Notice that the four properties for the cluster state include not only four-particle correlations as in the original GHZ argument, but also three-particle correlations. Those terms involving a measurement of σ_0 of photon 1 completely ignore the state of polarization of that photon. Recall that in a GHZ state this tracing out of one of the qubits leaves the remaining state completely mixed with only classical correlations between qubits. This is not the case in the cluster state as its persistency of entanglement allows for some particles to be ignored before all entanglement is lost. Thus different classes of multiparticle entanglement can exhibit stronger violations of local realism depending on the nature of the correlations in the Bell inequality.

To create the cluster state, we use a method first demonstrated in Ref. [7]. For the experiment, we generate polarization-entangled photon pairs using type-II parametric down-conversion [24]. A UV-laser pulse with a central wavelength of 395 nm and a pulse duration of 200 fs makes two passes through a β-barium borate (BBO) crystal which emits entangled photons into the forward pair of modes a and b and into the backward pair of modes c and d (Fig. 1). Transversal and longitudinal walk-off effects are erased by compensating crystals, which exist of a half wave plate (HWP) implementing a 90° rotation and an additional BBO crystal. These compensators are placed in each of the four modes. Final HWPs, one in mode a and another in mode c, and the tilt of the compensation crystals allow the generation of any of the four Bell states. The forward pair of modes a and b are coherently superimposed with the backward pair of modes c and d at the two polarizing beam splitters (PBS) by adjusting the position of the delay mirror for the UV pump. The preparation of the cluster state relies on all of the lowest-order processes which result in the simultaneous emission of four photons.

Recall that the PBS is a device which transmits horizontally polarized light and reflects vertically polarized light. If two photons enter a PBS from opposite input ports, they will only emerge separately if their polarizations are the same in the H/V basis. If two photons enter a PBS from the same input port, they only emerge separately if they are oppositely polarized in the H/V basis. In the present case, the source was aligned to produce the Bell state $|\phi^-\rangle$ into modes a and b and $|\phi^+\rangle$ into modes c and d. If one pair of photons is emitted into modes a and b and another into c and d, then, after the two PBSs, the four-photon state $|HHHH\rangle_{1234} - |VVVV\rangle_{1234}$ is left, provided the photons emerge into four different output modes. Emission of two pairs of photons in a single direction occurs with approximately equal probability, and contributes two more terms to the final state $-|HHVV\rangle_{1234}$ coming from the first pass and $|VVHH\rangle_{1234}$ from the second. Provided that all of

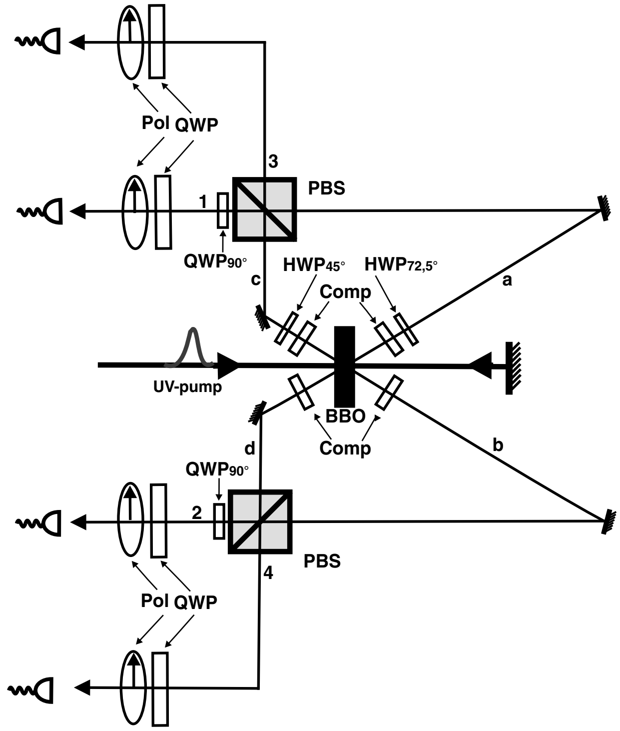

FIG. 1. Experimental setup for the generation of four-photon cluster states. An ultraviolet laser pulse passes twice through a nonlinear crystal which is aligned to produce polarization-entangled photon pairs on both the first and second pass. Compensators (Comp) are placed in the modes *a*, *b*, *c*, and *d* to compensate birefringent effects and half wave plates (HWP) in mode *a*, *c* to manipulate the emitted entangled pairs. In each output mode quarter-wave plates (QWP) and polarizers (Pol) are placed to project onto any desired state. Including the possibility of double-pair emission and the action of the polarizing beam splitters (PBS), the four components of the cluster state are prepared. The incorrect phase on the $HHVV$ amplitude can easily be changed by using the HWP in mode *a*. Using these processes and multiphoton coincidence postselection, the four-photon cluster state $|\phi_4\rangle$ is generated in modes 1–4.

these processes are indistinguishable and their relative phases are fixed, the final state is a coherent superposition of all four terms. The requisite π-phase shift on the $-|HHVV\rangle_{1234}$ term to $+|HHVV\rangle_{1234}$ was implemented using the HWP in mode a. A HWP rotation by an angle, θ, modifies the amplitude of this term according to the relation $-\cos 2\theta |HHVV\rangle_{1234}$; thus a rotation of larger than 45° adds the required phase shift. Note that this rotation also changes the amplitudes of the $|HHHH\rangle_{1234}$ and $|VVVV\rangle_{1234}$ terms by a factor of $\cos\theta$. Single-mode fibre-coupled photon counters were used in modes 1–4 to detect the photons. Controlling the coincidence counting rates from the forward and backward pairs give the extra degrees of freedom to balance the four amplitudes in the state.

The required expectation values, comprising products of Pauli operators, were reconstructed from sets of multiparticle polarization correlation measurements. Each of the 48 measurements was performed for 600 s using combinations of quarter-wave plates (QWPs) and linear polarizers in each of the 4 output modes (1–4). The Pauli operators $\sigma_{X,Y,Z}$ were measured by projective polarization measurements, $|H/V\rangle$ for the σ_Z operator, $|+/-\rangle = \frac{1}{\sqrt{2}}(|H\rangle \pm |V\rangle)$ for the σ_X operator, and $|R/L\rangle = \frac{1}{\sqrt{2}}(|H\rangle \pm i|V\rangle)$ for the σ_Y operator. For the linear polarization measurements the QWP was set parallel to the orientation of the polarizer, whereas for the circular polarization measurements, the QWP was fixed to +45° while the polarizer was horizontally or vertically oriented. In order to extract the expectation value, 16 (four-particle correlations) or 8 (three-particle correlations) measurements are required.

Experimental imperfections, including partial distinguishability in the four relevant four-photon emission processes and phase instabilities, lead to imperfect correlations which give some coincidence counts even when theory predicts none. For the three-particle correlations, we removed the polarizer from mode 1. However, since the state preparation method was reliant upon postselection, fourfold coincidences were still collected. Those measurements made without the polarizer show an increase in the coincidence rate as well as an imbalance most likely due to changes in the sensitive single-mode spatial filtering.

The four extracted correlations are shown in Fig. 2. We obtained positive expectation values of 0.61 ± 0.05,

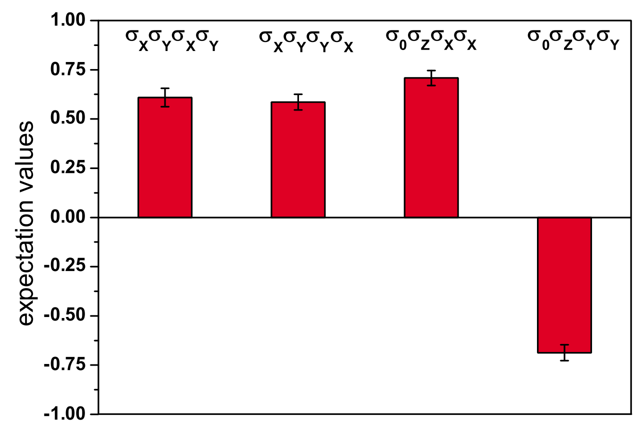

FIG. 2 (color online). Experimentally extracted polarization correlations. The cluster-state Bell inequality requires four different polarization correlations. These are extracted from a complete set of 48 fourfold coincidence measurements. The four-photon correlations, $\sigma_X\sigma_Y\sigma_X\sigma_Y$ and $\sigma_X\sigma_Y\sigma_Y\sigma_X$, are combinations of 16 coincidence rates, whereas the three-photon correlations, $\sigma_0\sigma_Z\sigma_X\sigma_X$ and $\sigma_0\sigma_Z\sigma_Y\sigma_Y$, are combinations of 8 coincidence rates. Each measurement run was recorded for 600 s. A quarter-wave plate and linear polarizer were used for each polarization projection. The polarizer could be completely removed for those cases where σ_0 was measured. The values for the correlations $\sigma_X\sigma_Y\sigma_X\sigma_Y$, $\sigma_X\sigma_Y\sigma_Y\sigma_X$, $\sigma_0\sigma_Z\sigma_X\sigma_X$, and $\sigma_0\sigma_Z\sigma_Y\sigma_Y$ are $(+0.61 \pm 0.05)$, $(+0.59 \pm 0.04)$, $(+0.71 \pm 0.04)$, and (-0.69 ± 0.04), respectively. Substituting these into the Bell inequality in Eq. (3) yields a Bell parameter, $S_C = 2.59 \pm 0.08$, which violates the local realism threshold by more than 7σ.

0.59 ± 0.04, and 0.71 ± 0.04 for the measurements $\sigma_X\sigma_Y\sigma_X\sigma_Y$, $\sigma_X\sigma_Y\sigma_Y\sigma_X$, and $\sigma_0\sigma_Z\sigma_X\sigma_X$, respectively, and the negative value -0.69 ± 0.04 for $\sigma_0\sigma_Z\sigma_Y\sigma_Y$. Adding these four correlations together according to the Bell inequality from Eq. (3) results in $S_C = 2.59 \pm 0.08$, where the uncertainty is due to Poisson counting statistics. The threshold for a local realistic modeling of these correlations is $S_C \leq 2$, which our experiment violates by 7σ.

The remarkable entanglement properties of cluster states can be readily used for the alternative "one-way" model of quantum computing [5], as was recently demonstrated experimentally [7]. Different from the two well-known classes of multiparticle entanglement, GHZ and W type, the properties of cluster states, such as their robustness against decoherence and their persistency of entanglement, make them practical for experimental study and interesting for quantum foundations. In this experiment, we have addressed a question of more fundamental rather than practical interest, namely, how the novel family of cluster states can be used to demonstrate the nonlocal facets of quantum physics. We investigated a new kind of Bell inequality based on a GHZ argument for cluster states. The inequality detects cluster-state entanglement optimally, while GHZ states would not violate the inequality. Our experimentally produced cluster violates the inequality by more than 7σ. Our result demonstrates how specifically tailored Bell inequalities (e.g., by using specific correlations of the state) can become a useful tool to tackle the interesting questions between multiparticle entanglement and quantum nonlocality.

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