

## Greenberger-Horne-Zeilinger-state analyzer

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Using just polarizing beam splitters and half-wave plates, we present a practical experimental scheme, by which one can conveniently identify two of the three-particle Greenberger-Horne-Zeilinger (GHZ) states. The scheme can easily be generalized to the  $N$ -particle case. The GHZ-state analyzer is essential for multiparticle generalizations of quantum dense coding, quantum teleportation, and entanglement swapping. [S1050-2947(98)06102-2]

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Maximally entangled states of three or more particles, so-called Greenberger-Horne-Zeilinger (GHZ) states [1], have been fascinating quantum systems to reveal the nonlocality of the quantum world. Most recently, the young quantum information theory [2] has shown that quantum entangled states can be exploited for information transmission and information processing, for example, in quantum dense coding [3,4] and in quantum teleportation [5,6]. It is well known that the preparation and measurement of Bell states is essential for quantum dense coding and quantum teleportation. The extension to the GHZ case will thus allow us to generalize the two to the multiparticle case. Also, in a generalization of entanglement swapping [7,8] a GHZ-state analyzer can actually be used to prepare multiparticle GHZ states out of entangled pairs.

Theoretically, using only single-quantum-bit (qubit) operations and the controlled-NOT gates [9] one can construct a suitable quantum network to produce and identify any of the maximally entangled states for any number of particles [10]. However, until now such quantum networks have not yet been built in the laboratory. On the experimental side, Kwiat *et al.* [11] have reported a high-intensity source of polarization-entangled photon pairs, by which one can easily prepare any of the four Bell states. Also, a general and realizable procedure for producing three-particle entanglements out of entangled pairs has been given recently [12]. Furthermore, while no complete Bell state measurement procedure exists, we can already experimentally identify two of the four Bell states [13].

In this paper, we present a universal scheme and practically realizable procedures for identifying two of the  $N$ -particle GHZ states based on the concept of quantum erasure. The basic elements of the experimental setup are just polarizing beam splitters (PBS) and half-wave plates (HWP). Meanwhile, we show that the scheme can also provide a conditional GHZ-state source by which one can experimentally demonstrate GHZ correlations [1] and entangled entanglement [14].

Before starting to discuss the GHZ-state analyzer, let us first give a modified version of the Bell-state analyzer, which is similar to but different from the former one shown in Fig. 1 of Ref. [4]. Consider the arrangement of Fig. 1. Two identical photons enter the Bell-state analyzer from modes  $A$  and  $B$ , respectively. Suppose they are in the most general polarization-superposition state

$$|\psi_{in}\rangle = \alpha|H_A\rangle|H_B\rangle + \beta|H_A\rangle|V_B\rangle + \gamma|V_A\rangle|H_B\rangle + \delta|V_A\rangle|V_B\rangle. \quad (1)$$

Because the PBS transmits only the horizontal polarization component and reflects the vertical component, after passing through  $PBS_{AB}$  the incident state will evolve into

$$|\psi_{in}\rangle \rightarrow \alpha|H_{A2}\rangle|H_{B1}\rangle + \beta|H_{A2}\rangle|V_{B2}\rangle + \gamma|V_{A1}\rangle|H_{B1}\rangle + \delta|V_{A1}\rangle|V_{B2}\rangle, \quad (2)$$

where the subscript  $ij$  ( $i=A,B, j=1,2$ ) denotes the transformation from input mode  $i$  to output mode  $j$ . Suppose that these two photons  $A$  and  $B$  arrive at the polarizing beam splitter  $PBS_{AB}$  simultaneously, and therefore their spatial wave functions overlap each other. Then, according to the indistinguishability of identical particles, we can directly denote  $H_{A2}$  as  $H_2$ ,  $V_{A1}$  as  $V_1$ ,  $H_{B1}$  as  $H_1$ , and  $V_{B2}$  as  $V_2$ . Thus, Eq. (2) reads

$$\alpha|H_1\rangle|H_2\rangle + \beta|H_2\rangle|V_2\rangle + \gamma|V_1\rangle|H_1\rangle + \delta|V_1\rangle|V_2\rangle. \quad (3)$$

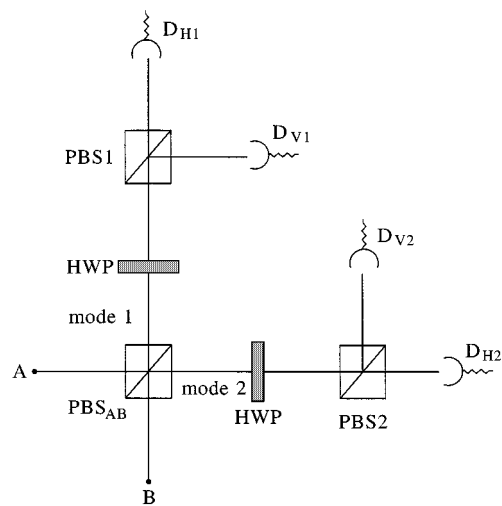


FIG. 1. A Bell-state analyzer. Two identical photons enter the Bell-state analyzer from input ports  $A$  and  $B$ .  $PBS_{AB}$ ,  $PBS_1$ , and  $PBS_2$  are three polarizing beam splitters, which transmit the horizontal polarization component and reflect the vertical component.  $D_{H1}$ ,  $D_{V1}$ ,  $D_{H2}$ , and  $D_{V2}$  are four trigger detectors.

Note that for the terms  $|H_1\rangle|H_2\rangle$  and  $|V_1\rangle|V_2\rangle$  the two photons are in different output ports, and while for the terms  $|H_2\rangle|V_2\rangle$  and  $|V_1\rangle|H_1\rangle$  both photons are in the same output port. Therefore, we can identify states  $|\psi_d\rangle = \alpha|H_1\rangle|H_2\rangle + \delta|V_1\rangle|V_2\rangle$  and  $|\psi_s\rangle = \beta|H_2\rangle|V_2\rangle + \gamma|H_1\rangle|V_1\rangle$  using the coincidence between detectors in mode 1 and in mode 2. Obviously, in terms of the Bell states,  $\Psi_d$  can be rewritten as

$$|\psi_d\rangle = \frac{1}{\sqrt{2}}(\alpha + \delta)\phi^+ + \frac{1}{\sqrt{2}}(\alpha - \delta)\Phi^-, \quad (4)$$

where

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|H_1\rangle|H_2\rangle + |V_1\rangle|V_2\rangle),$$

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}(|H_1\rangle|H_2\rangle - |V_1\rangle|V_2\rangle).$$

Thus, in order to finish the Bell-state measurement we now only need to identify states  $|\phi^+\rangle$  and  $|\phi^-\rangle$ , that is, we have to determine the relative phase between terms of  $|H_1\rangle|H_2\rangle$  and  $|V_1\rangle|V_2\rangle$ . Let the angle between the HWP axis and the horizontal direction be  $22.5^\circ$  such that it corresponds to a  $45^\circ$  rotation of the polarization. Therefore, for a photon passing the HWP, its polarization state will undergo the following unitary transformation:

$$|H_i\rangle \rightarrow \frac{1}{\sqrt{2}}(|H_i\rangle + |V_i\rangle),$$

$$|V_i\rangle \rightarrow \frac{1}{\sqrt{2}}(|H_i\rangle - |V_i\rangle),$$

where  $i=1,2$ . Finally,  $|\phi^+\rangle$  and  $|\phi^-\rangle$  will thus be transformed into

$$|\phi^+\rangle \rightarrow |\phi^+\rangle = \frac{1}{\sqrt{2}}(|H_1\rangle|H_2\rangle + |V_1\rangle|V_2\rangle), \quad (5)$$

$$|\phi^-\rangle \rightarrow |\psi^+\rangle = \frac{1}{\sqrt{2}}(|H_1\rangle|V_2\rangle + |V_1\rangle|H_2\rangle). \quad (6)$$

The above analysis shows that we can readily identify two of the four incident Bell states. Specifically, if we observe a coincidence either between detectors  $D_{H1}$  and  $D_{H2}$  or  $D_{V1}$  and  $D_{V2}$ , then the incident state was  $(1/\sqrt{2})(|H_A\rangle|H_B\rangle + |V_A\rangle|V_B\rangle)$ . On the other hand if we observe coincidences between detectors  $D_{H1}$  and  $D_{V2}$  or  $D_{V1}$  and  $D_{H2}$ , then the incident state was  $(1/\sqrt{2})(|H_A\rangle|H_B\rangle - |V_A\rangle|V_B\rangle)$ . The other two incident Bell states will lead to no coincidence between detectors in mode 1 and in mode 2. Such states are signified by some kind of superposition of  $|H_A\rangle|V_B\rangle$  and  $|V_A\rangle|H_B\rangle$ . This concludes our demonstration that we can identify two of the four Bell states using the coincidence between modes 1 and 2.

The reason we discuss the modified version of the Bell-state analyzer is that the above scheme can directly be generalized to the  $N$ -particle case. Making use of its basic idea,

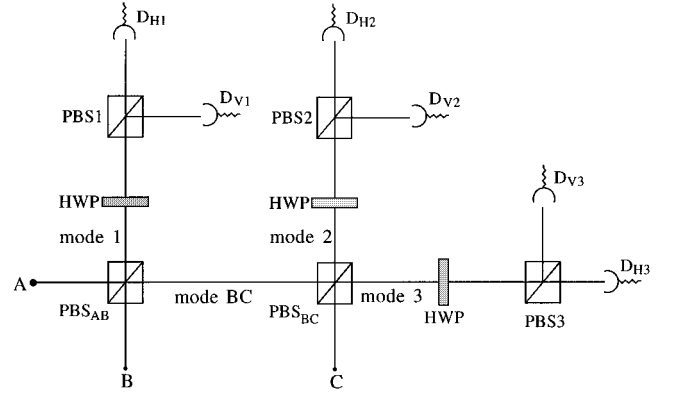


FIG. 2. A GHZ-state analyzer. Three photons incident one each in modes  $A$ ,  $B$ , and  $C$  will give rise to distinct threefold coincidences if they are in the GHZ states  $|\Phi^+\rangle$  or  $|\Phi^-\rangle$ . All the notations are the same as in Fig. 1.

one can easily construct a type of GHZ-state analyzer by which one can immediately identify two of the  $2^N$  maximally entangled GHZ states.

For example, in the case of three identical photons, the eight maximally entangled GHZ states are given by

$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle|H\rangle \pm |V\rangle|V\rangle|V\rangle),$$

$$|\Psi_1^\pm\rangle = \frac{1}{\sqrt{2}}(|V\rangle|H\rangle|H\rangle \pm |H\rangle|V\rangle|V\rangle),$$

$$|\Psi_2^\pm\rangle = \frac{1}{\sqrt{2}}(|H\rangle|V\rangle|H\rangle \pm |V\rangle|H\rangle|V\rangle),$$

$$|\Psi_3^\pm\rangle = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle|V\rangle \pm |V\rangle|V\rangle|H\rangle),$$

where  $|\Psi_i^\pm\rangle$  designates that GHZ state where the polarization of photon  $i$  is different from the other two. Consider now the setup of Fig. 2 and suppose that three photons enter the GHZ analyzer, each one from modes  $A$ ,  $B$ , and  $C$ , respectively. A suitable arrangement can be realized such that the photon coming from mode  $A$  and the one coming from mode  $B$  overlap at  $\text{PBS}_{AB}$  and thus they are correspondingly transformed into mode 1 and into mode  $BC$ . Let us further suppose that the photons from mode  $BC$  and mode  $C$  overlap each other at  $\text{PBS}_{BC}$ . Thus, following the above demonstration for the case of the Bell-state analyzer, it is easy to find that the eight GHZ states above will correspondingly evolve into

$$\frac{1}{\sqrt{2}}(|H_1\rangle|H_2\rangle|H_3\rangle \pm |V_1\rangle|V_2\rangle|V_3\rangle), \quad (7)$$

$$\frac{1}{\sqrt{2}}(|H_1\rangle|H_3\rangle|V_3\rangle \pm |V_1\rangle|V_2\rangle|H_2\rangle), \quad (8)$$

$$\frac{1}{\sqrt{2}}(|H_2\rangle|V_2\rangle|H_3\rangle \pm |H_1\rangle|V_1\rangle|V_3\rangle), \quad (9)$$

$$\frac{1}{\sqrt{2}}(|H_1\rangle|V_1\rangle|H_2\rangle \pm |V_2\rangle|H_3\rangle|V_3\rangle) \quad (10)$$

immediately after these three photons passed through  $\text{PBS}_{AB}$  and  $\text{PBS}_{BC}$ , and before they enter the half-wave plates. Here, e.g.,  $H_i$  denotes a photon with polarization  $H$  in output mode  $i$ .

From Eqs. (8)–(11), it is evident that one can observe threefold coincidences between modes 1, 2, and 3 only for the state of Eq. (8). For the other states, there are always two particles in the same mode. We can thus distinguish the two states  $(1/\sqrt{2})(|H\rangle|H\rangle|H\rangle \pm |V\rangle|V\rangle|V\rangle)$  from the other six GHZ states. Furthermore, after the states of Eq. (8) pass through the HWP, we finally obtain

$$\begin{aligned} & \frac{1}{\sqrt{2}}(|H_1\rangle|H_2\rangle|H_3\rangle + |V_1\rangle|V_2\rangle|V_3\rangle) \\ & \rightarrow \frac{1}{2}(|H_1\rangle|H_2\rangle|H_3\rangle + |H_1\rangle|V_2\rangle|V_3\rangle \\ & \quad + |V_1\rangle|H_2\rangle|V_3\rangle + |V_1\rangle|V_2\rangle|H_3\rangle), \end{aligned} \quad (11)$$

$$\begin{aligned} & \frac{1}{\sqrt{2}}(|H_1\rangle|H_2\rangle|H_3\rangle - |V_1\rangle|V_2\rangle|V_3\rangle) \\ & \rightarrow \frac{1}{2}(|H_1\rangle|H_2\rangle|V_3\rangle + |H_1\rangle|V_2\rangle|H_3\rangle \\ & \quad + |V_1\rangle|H_2\rangle|H_3\rangle + |V_1\rangle|V_2\rangle|V_3\rangle). \end{aligned} \quad (12)$$

Thus, using threefold coincidences we can readily identify the relative phase between states  $|H\rangle|H\rangle|H\rangle$  and  $|V\rangle|V\rangle|V\rangle$ . This is because only the initial state  $|\Phi^+\rangle$  leads to coincidences between detectors  $D_{H1}$ ,  $D_{H2}$ , and  $D_{H3}$  (or  $H_1V_2V_3$ ,  $V_1H_2V_3$ ,  $V_1V_2H_3$ ). On the other hand, only the state  $|\Phi^-\rangle$  leads to coincidences between detectors  $D_{H1}$ ,  $D_{H2}$ , and  $D_{V3}$  (or  $H_1V_2H_3$ ,  $V_1H_2H_3$ ,  $V_1V_2V_3$ ). Or, in conclusion, states  $|\Phi^+\rangle$  and  $|\Phi^-\rangle$  are identified by coincidences between all three output modes 1, 2, and 3. They can be distinguished because behind the half-wave plates  $|\Phi^+\rangle$  results in one or three horizontally, and zero or two vertically polarized photons, while  $|\Phi^-\rangle$  results in zero or two horizontally polarized photons and one or three vertically polarized ones.

Our GHZ-state analyzer has many possible applications, for example, the three photons entering via the modes  $A$ ,  $B$ , and  $C$ , respectively could each come from one entangled pair. Then projection of these three photons using the GHZ-state analyzer onto the GHZ state  $\Phi^+$  or  $\Phi^-$  implies that the other three photons emerging from each pair will be prepared in a GHZ state. It is clear that our scheme can readily be generalized to analyze entangled states consisting of more than three photons by just adding more polarizing beam splitters and half-wave plates. Also, identification of analogs

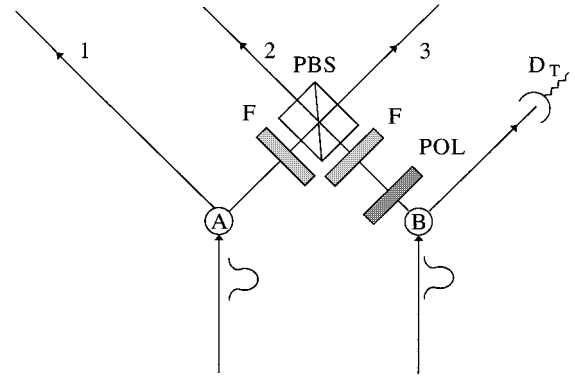


FIG. 3. A three-particle polarization-entanglement source. The photon sources,  $A$  and  $B$ , pumped by short pulses, each one emits a photon pair. Source  $A$  emits a pair of photons in the superposition  $HH+VV$ , and one of the two photons emitted by source  $B$  in the state  $(1/\sqrt{2})(H+V)$  by inserting a one-channel polarizer  $\text{POL}$ .  $F$  is narrow filter,  $D_T$  is a trigger detector. Then, one photon coming from source  $A$  and the one coming from source  $B$  overlap at the PBS.

of our scheme for GHZ-state analysis of atoms or of mode entangled states instead of polarization entangled ones is straightforward.

Here, it is worth noting that an extension of the above scheme would provide us with a conditional GHZ-state source by which we can conveniently observe three-particle correlations and also demonstrate entangled entanglement. Consider two sources  $A$  and  $B$  (see Fig. 3) each one emitting a photon pair. Consider for simplicity that the state emitted by source  $A$  is the maximally entangled state  $(1/\sqrt{2})(|H\rangle|H\rangle + |V\rangle|V\rangle)$ , and let us assume that one of the two photons emitted by source  $B$  is projected into the state  $(1/\sqrt{2})(|H\rangle + |V\rangle)$  by inserting a simple one-channel polarizer. The other photon from source  $B$  then serves as a trigger in order to reduce unwanted backgrounds. Then, using very analogous arguments as above we find that the state of the three particles immediately after passage through the polarizing beam splitter PBS will be the superposition

$$\begin{aligned} & \frac{1}{2}(|H_1\rangle|H_2\rangle|H_3\rangle + |V_1\rangle|V_2\rangle|V_3\rangle + |H_1\rangle|H_3\rangle|V_3\rangle \\ & \quad + |V_1\rangle|V_2\rangle|H_2\rangle). \end{aligned} \quad (13)$$

Again, only for the superposition  $|H_1\rangle|H_2\rangle|H_3\rangle + |V_1\rangle|V_2\rangle|V_3\rangle$  we would observe threefold coincidence. Therefore, we then know these three particles are in the superposition  $|H_1\rangle|H_2\rangle|H_3\rangle + |V_1\rangle|V_2\rangle|V_3\rangle$  as soon as we observe threefold coincidence. Note that here the GHZ state is not directly prepared but we know that the three particles are in a GHZ state under the condition that one particle each is detected in each of the outgoing beams 1, 2, 3. This is a much weaker condition than any postselection procedure that might be based on properties of the particles. In an experiment our case will not be distinguishable from the real situation occurring anyway because of finite detector efficiencies. That is, from a practical point of view, even if one definitely prepares a full GHZ state one only will observe three or fourfold coincidences in a fraction of time anyway. Thus, we conclude that in the scheme of Fig. 3 one will be

able to experimentally demonstrate all features of a GHZ state including entangled entanglement.

Finally we would like to note that in all these schemes we have used the principle of quantum erasure in a way that behind our GHZ-state analyzer at least some of the photons registered cannot be identified anymore as to which source they came from. This implies very specific experimental schemes, because the particles might have been created at different times. One theoretical possibility is to apply the principle of ultracoincidence [7]. This means that the photons must be registered within a time short compared to their coherence time. For practical reasons, i.e., the unavailability of sufficiently fast detectors, the scheme cannot be realized at

present. Yet, alternatively, one can create the particles within a time interval small compared to their coherence times. This in practice implies the use of pulsed sources and of filters behind them, which introduce coherence times larger than the pulse length [15]. Such a scheme has been successfully used in the first experimental demonstration of quantum teleportation [6].

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