

Quantum discord as resource for remote state preparation

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The existence of better-than-classical quantum information processing (QIP) models which consume very little or no entanglement suggests that separable or weakly entangled states could be extremely useful tools for information processing as they are much easier to prepare and control even in dissipative environments. It has been proposed that a resource of advantage is the generation of quantum discord, a measure of non-classical correlations that includes entanglement as a subset. Here we show that quantum discord is the necessary resource for quantum remote state preparation. We explicitly show that the geometric measure of quantum discord is related to the fidelity of this task, which provides its operational meaning. Our results are experimentally demonstrated using photonic quantum systems. Moreover, we demonstrate that separable states with non-zero quantum discord can outperform entangled states. Therefore, the role of quantum discord might provide fundamental insights for resource-efficient QIP.

Quantum computation and quantum communication are believed to allow for information processing with an efficiency that cannot be achieved by any classical device. It is usually assumed that a key resource for this enhanced performance is quantum entanglement¹. Quantum entanglement is widely recognized as one of the key resources for the advantages of QIP, including universal quantum computation², reduction of communication complexity^{3,4} or secret key distribution⁵. The creation and manipulation of entanglement, however, is a very demanding task, as it requires extremely precise quantum control and isolation from the environment. Thus, current experimental achievements are limited to rather small-scale entangled systems^{6–8}.

Whereas quantum computing with pure states requires the presence of quantum entanglement for exponential enhancements⁹, there is no proof that entanglement is needed for providing computational speed-up for mixed state quantum computing or enhanced QIP in general. The investigation of QIP protocols that allow for significant enhancements in the efficiency of data processing by using only separable states is of high interest. Obviously, such states have the benefit of being easier to prepare and more robust against losses and experimental imperfections. In fact, there are quantum computational models based on mixed, separable states, most notably the so-called deterministic quantum computation with one qubit¹⁰, that are believed to be classically intractable^{10,11}. Recently, they have been demonstrated experimentally^{12–14}. The quantum advantage of these models has been associated with the generation of so-called quantum discord^{15–17}, a measure of ‘non-classicality’ that captures quantum entanglement as a subset.

The role of quantum discord in QIP has been studied in the past^{18–23}. Quantum discord has been proposed as the

resource that can provide the enhancement for the computation in the deterministic quantum computation with a one qubit model^{24,25}, but its relation to the computational speed-up remains ambiguous^{26,27}. A relation to quantum communication was studied and has been shown to exist, for example in local broadcasting²² and quantum state merging^{19,20}. A more general approach has been adopted by Madhok and Datta²¹ showing the relation to all bipartite, unidirectional, memoryless quantum communication protocols, called the mother of all protocols.

Here we identify quantum discord as the necessary resource for quantum remote (pure) state preparation^{28,29} (RSP). This protocol is a variant of quantum state teleportation³⁰ in which the sender (Alice) knows the quantum state to be communicated to the receiver (Bob). We explicitly show that a quantum state retrieving non-trivial RSP fidelity is necessarily discordant. Our result is generic and holds for any two-qubit state, and does not rely on a particular measure of quantum discord. The experimental implementation was performed on a quantum optical platform using polarization-correlated single photons. We find that for a broad class of states the fidelity of RSP is directly given by the geometric measure of quantum discord. This provides an operational meaning to this measure of ‘quantumness’ of correlations in quantum information. Remarkably, states with no entanglement, yet non-zero geometric quantum discord, can outperform entangled states in accomplishing RSP.

Theory

Two systems are correlated if together they contain more information than taken separately. This intuitive definition is formally captured by (quantum) mutual information³¹ $I(A:B) = H(A) + H(B) - H(A,B)$, where A and B are random

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variables. In classical probability theory $H(\cdot)$ stands for the Shannon entropy $H(p) = -\sum_i p_i \log p_i$, where $p = (p_1, p_2, \dots)$ is the probability distribution vector, whereas in the quantum case it denotes the von Neumann entropy $H(\rho) = -\text{Tr} \rho \log \rho$ of a density matrix ρ . Classically, we can use the Bayes rule and find an equivalent expression for the mutual information $I(A : B) = H(A) - H(A|B)$, where $H(A|B)$ is the Shannon entropy of A conditioned on the measurement outcome of B . For quantum systems, these two expressions are inequivalent and their difference defines a non-negative quantity, the so-called quantum discord^{15,17}.

Whenever quantum discord vanishes the systems are classically correlated. A very simple expression for zero-discord states was proposed³², which will be illustrated in the following example. Consider the case of a two-qubit state shared by Alice and Bob, where the Hilbert space of each qubit is spanned by the orthogonal states $|0\rangle$ and $|1\rangle$. A general zero-discord state χ can be written as

$$\chi = p_1|0\rangle\langle 0| \otimes \rho_1 + p_2|1\rangle\langle 1| \otimes \rho_2 \quad (1)$$

where $p_1 + p_2 = 1$ and $\rho_{1,2}$ are arbitrary states of the second qubit. Intuitively, this can be understood as a joint system containing one classical bit (cbit) and one quantum bit (qubit), 1 cbit $\times$ 1 qubit. One of the systems can be identified as a cbit, because it is always in one of the perfectly distinguishable states $|0\rangle$ and $|1\rangle$. All other states possess some genuine quantum correlations.

Consider the representation of a two-qubit state ρ in terms of local Pauli matrices $\{\sigma_1, \sigma_2, \sigma_3\}$,

$$\rho = \frac{1}{4} \left(\mathbb{1} \otimes \mathbb{1} + \sum_{k=1}^3 a_k \sigma_k \otimes \mathbb{1} + \sum_{l=1}^3 b_l \mathbb{1} \otimes \sigma_l + \sum_{k,l=1}^3 E_{kl} \sigma_k \otimes \sigma_l \right)$$

where $E_{kl} = \text{Tr}(\sigma_k \otimes \sigma_l \rho)$ are the elements of correlation tensor E . The vector $\mathbf{a} = (a_1, a_2, a_3)$ with components $a_k = \text{Tr}(\sigma_k \rho_A)$ is the Bloch vector of the reduced density operator ρ_A of Alice and similarly $\mathbf{b}$ for Bob. One can always choose a local reference frame on Alice's and Bob's sides such that the correlation tensor becomes a diagonal matrix in the Schmidt canonical form³³ $E = \text{diag}[E_1, E_2, E_3]$, where E_i^2 are the eigenvalues of $E^T E$. A zero discord state has the Schmidt form $E = \text{diag}[E_1, 0, 0]$, which corresponds to correlation in one basis only.

There are many ways to quantify and verify quantum correlations^{34,35}. The advantage of the recently introduced geometric measure of quantum discord ($\mathcal{D}$) is that it can be evaluated explicitly and leads to an analytical closed form in many interesting cases^{26,36}. It is defined as the normalized trace distance to the set of classical states

$$\mathcal{D}^2(\rho) \equiv 2 \min_{\chi} \|\rho - \chi\|^2 \equiv 2 \min_{\chi} \text{Tr}(\rho - \chi)^2$$

where the minimum is taken over the set of zero-discord states χ , that is states of the form of equation (1). For states with maximally mixed marginals ($\mathbf{a} = \mathbf{b} = 0$) the geometric discord takes the following simple form²⁶

$$\mathcal{D}^2(\rho) = \frac{1}{2} (E_2^2 + E_3^2) \quad (2)$$

where $E_{2,3}^2$ are the two lowest eigenvalues of $E^T E$. In fact, the same expression holds for a much larger class of states (see Supplementary Information for details). We will show that equation (2) captures the quality of RSP, thereby providing an operational meaning for geometric discord.

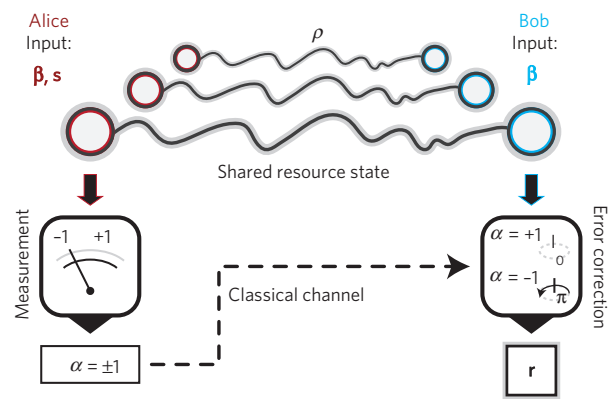


Figure 1 | RSP. Alice and Bob share a quantum state ρ . She is supposed to prepare a state s in the plane orthogonal to the direction β (that is announced to Bob as well). After performing the measurement along the direction α she sends her result $\alpha = \pm 1$ to Bob via a classical channel. Conditioned on the value of α , Bob applies a correction to his qubit to obtain the state r .

For the implementation of the RSP protocol Alice and Bob share a quantum state ρ , which can possess various correlations, for example classical correlation, entanglement and quantum discord. In the case of RSP, the task of preparing a specific state at Bob's location can be accomplished with fewer resources than in the case of quantum teleportation³⁰. Preparing an arbitrary unknown state via teleportation requires communicating two cbits, while sharing a maximally entangled state. However, if Alice only wants to remotely prepare a quantum state $|\psi\rangle$ on the equatorial plane of the Bloch sphere, for example $|\psi\rangle = (|0\rangle + e^{i\phi}|1\rangle)/\sqrt{2}$, one cbit of communication is sufficient. The protocol that achieves this uses a maximally entangled state as a shared resource and a single cbit, which is Alice's measurement outcome along the direction of the state she wants to prepare. Depending on the value of the received cbit Bob needs to correct his qubit by a π rotation about the z axis to generate the state envisaged by Alice.

In the ideal case a shared maximally entangled state enables Alice to deterministically prepare any state in the equatorial plane of Bob's Bloch sphere. In general, when a mixed state is used as quantum resource for the RSP protocol, then Bob obtains a quantum state with a reduced fidelity³⁷. For the investigation of the underlying resource for the RSP protocol, photon pairs with different polarization correlations are generated and shared by Alice and Bob. Alice uses this shared quantum state ρ to remotely prepare a state s in the plane orthogonal to the direction β on Bob's side. She initializes the state preparation on Bob's side by performing a local measurement along the direction α . The measurement outcome $\alpha = \pm 1$ is then sent to Bob as one cbit of information. For $\alpha = -1$ Bob applies a π rotation about β to his system, whereas no correction is required for $\alpha = 1$. The resulting state on Bob's side is denoted by r (see Fig. 1). To evaluate the efficiency of the protocol we first define a payoff-function $\mathcal{P} \equiv (\mathbf{r} \cdot \mathbf{s})^2$ (The pay-off function is directly related to the state fidelity. The fidelity F_s between states s and r is given by $F_s = (1 + \mathbf{r} \cdot \mathbf{s})/2$, hence $\mathcal{P} = (2F_s - 1)^2$), which is calculated for each run. Alice can optimize the payoff for a given s and β by her choice of the local measurement direction α . The fidelity of the protocol is calculated as an averaged payoff and minimized over all possible choices of the direction β , which captures the worst case scenario. The payoff $\mathcal{P}$ is non-zero if and only if Alice is able to prepare a state effectively different from a completely mixed state. In particular $\mathcal{P} = 1$, when using a maximally entangled state as shared resource, while $\mathcal{P} = 0$ for a totally mixed state.

Careful analysis shows that the described RSP-fidelity ($\mathcal{F}$) is given by the following expression (see Supplementary Information for details)

$$\mathcal{F} \equiv \min_{\beta} \langle \mathcal{P}_{\text{opt}} \rangle = \frac{1}{2} (E_2^2 + E_3^2) \quad (3)$$

where E_2^2 and E_3^2 are the two lowest eigenvalues of $E^T E$. As noted above, this quantity captures the suitability of a certain resource state for RSP. From equation (3) it follows that $\mathcal{F} > 0$ if and only if the correlation tensor has at least two non-zero eigenvalues. Recall that a zero-discord state has a correlation tensor of the form $E = \text{diag}[E_1, 0, 0]$. In other words, $\mathcal{F} = 0$ for all zero-discord states (states of the form of equation (1)). Therefore non-zero fidelity necessarily implies the presence of quantum discord. This shows that quantum discord is a necessary resource for RSP. We further observe that for a broad class of states the geometric measure of quantum discord has the form of equation (2) and therefore coincides with the RSP-fidelity (see Supplementary Information for details). In this case geometric discord is provided with an operational meaning.

We now consider the Werner states³⁸, ρ_W :

$$\rho_W = \lambda |\psi^-\rangle \langle \psi^-| + \frac{(1-\lambda)}{4} \mathbb{1}_4 \quad (4)$$

which have isotropic correlations equal to the weighting parameter λ , that is $E = -\lambda \mathbb{1}$. Here $\mathbb{1}_4 = \text{diag}[1, 1, 1, 1]$ denotes an equal mixture of the four Bell states $|\psi^\pm\rangle = (|10\rangle \pm |01\rangle)/\sqrt{2}$ and $|\phi^\pm\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$. For $1/3 < \lambda \leq 1$ the Werner states are entangled. By choosing $\lambda = 1/3$ we obtain a separable state, which we denote by $\tilde{\rho}_W$. However, it has non-zero discord, which, according to the relation we established earlier, awards the state with a limited suitability for RSP quantified by $\mathcal{D}^2 = \mathcal{F} = \lambda^2 = 1/9$.

Useful resource states for RSP can easily be missed when looking only for entanglement. In fact, the situation is more delicate: not only does entanglement overlook the capability of certain states for RSP, its verdict can actually be misleading. This deficit can be explicitly illustrated by introducing another type of state, ρ_B :

$$\rho_B = \frac{1-k}{4} |\psi^+\rangle \langle \psi^+| + \frac{1+3k}{4} |\psi^-\rangle \langle \psi^-| + \frac{1-2t-k}{4} |00\rangle \langle 00| + \frac{1+2t-k}{4} |11\rangle \langle 11| \quad (5)$$

where k and t are real parameters specifying the mixture of Bell states and computational basis states. Such a state has a Bloch representation with local vectors $\mathbf{a} = \mathbf{b} = t\mathbf{e}_z$ and correlation tensor $E = -k\mathbb{1}$ (isotropic correlations). We set $k = 1/5$, $t = 2/5$ to obtain a state with non-zero entanglement, as measured by a concurrence³⁹ of $\mathcal{C} = 1/5$, and denote it by $\tilde{\rho}_B$. It is tempting to assume that this state might be better suited for RSP than any separable state. However, the RSP-fidelity of $\mathcal{F} = \mathcal{D}^2 = k^2 = 1/25$, compared to the value of $\mathcal{F} = 1/9$ for the state $\tilde{\rho}_W$, shows that this is not the case. This underlines that entanglement does not qualify as a distinctive resource for RSP. This possibility was also anticipated in the work of Chaves and de Melo⁴⁰. They show unexpected behaviour that the computation fidelity is higher when the entanglement is more fragile against disturbances (less entanglement corresponds to higher fidelity). Furthermore, they show the similar tendency for the original measure of quantum discord¹⁵. Here we demonstrate that geometric discord can assess the situation correctly.

Experiment and results

We have implemented an experimental test of the RSP protocol using polarization-encoded photonic qubits (Fig. 2). We generate

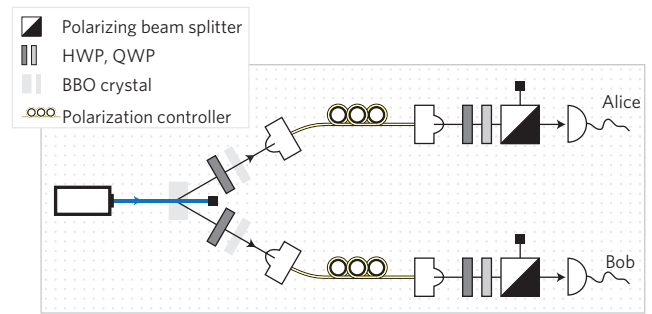


Figure 2 | Experimental set-up to realize the RSP protocol. A β -barium borate (BBO) crystal is pumped with a laser beam (394.5 nm, 76 MHz) such that entangled photon pairs are created via type-II spontaneous parametric down-conversion at a wavelength of 789 nm. The photons are spectrally and spatially filtered by 3 nm narrow-bandwidth filters and by coupling into single-mode fibres, respectively. All four Bell states $|\phi^\pm\rangle$, $|\psi^\pm\rangle$ can be obtained by rotating polarizations and introducing phase shifts. A combination of quarter-wave plate (QWP), half-wave plate (HWP) and polarizing beam splitter is used for the characterization of the prepared quantum states.

Table 1 | Characterization of the experimentally created states $\tilde{\rho}_W$ and $\tilde{\rho}_B$.

	State $\tilde{\rho}_W$	State $\tilde{\rho}_B$
State fidelity	0.998	0.993
Purity	0.33	0.36
Concurrence	0.00	0.12
Geometric discord	0.097	0.036
RSP-fidelity	0.098	0.036

State fidelity, purity and concurrence have been extracted from the density matrices, while the geometric discord has been calculated according to ref. 26 and the RSP-fidelity using the procedure given in the Supplementary Information. The errors computed by simulating Poissonian counting statistics are below 10^{-3} .

all four Bell states $|\phi^\pm\rangle$, $|\psi^\pm\rangle$ and the product states $|00\rangle$, $|11\rangle$ to have access to all the states given in equations (4) and (5). For the RSP protocol Alice remotely prepares 58 states uniformly distributed on Bob's Bloch sphere for each of the two resource states $\tilde{\rho}_W$ and $\tilde{\rho}_B$.

To evaluate the data for the intended resource states we combine the corresponding coincidence counts from different states constituting the mixtures. The relative weights in the mixture are accounted for by appropriate measurement durations^{41,42}. This approach results in state fidelities higher than 0.99, where the statistical errors, constituting a lower limit, are below 10^{-4} for $\tilde{\rho}_W$ and $\tilde{\rho}_B$.

The characterization of $\tilde{\rho}_W$ shows that this state is indeed separable, as indicated by a vanishing concurrence, and has a higher value of geometric discord than the entangled state $\tilde{\rho}_B$ (see Table 1). The remotely prepared states and the respective payoffs $\mathcal{P}$ are presented in Fig. 3. We find the separation $\Delta\mathcal{P} = \mathcal{P}_{\tilde{\rho}_W} - \mathcal{P}_{\tilde{\rho}_B}$ between two corresponding values to be larger than $\Delta\mathcal{P} = 0.0434 \pm 0.0007$ for all prepared states, which confirms the better performance of the separable state $\tilde{\rho}_W$ by 62 standard deviations. Although the prepared resource states are of high state fidelity, smallest experimental imperfections, in the form of a slight rotation of the Bloch sphere axis, lead to fluctuations in the data (see Fig. 3a). This effect leads to periodic oscillations instead of the expected constant behaviour of values.

We showed that non-zero quantum discord is the necessary resource for RSP. This is demonstrated by using a variety of

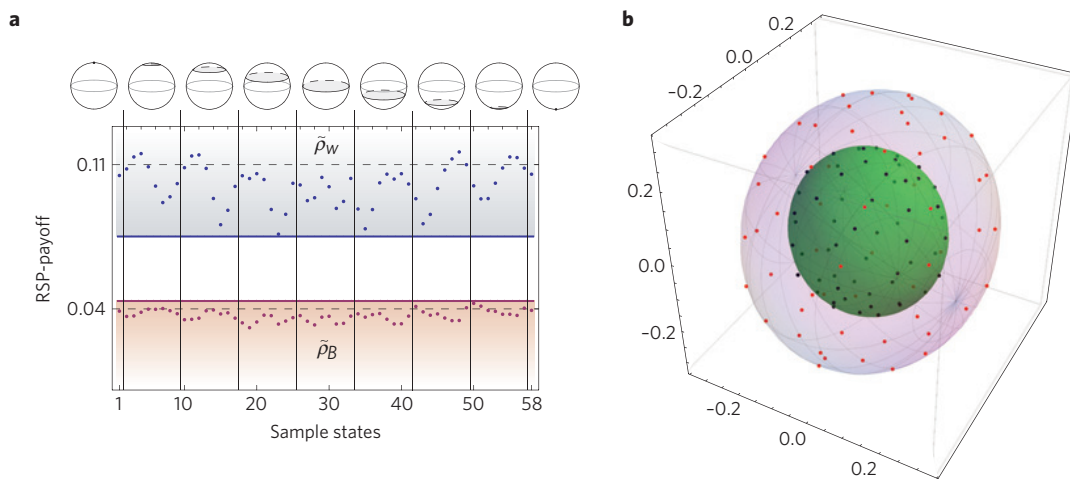


Figure 3 | Experimentally achieved RSP-payoff $\mathcal{P}$ for 58 distinct states on Bob's Bloch sphere. **a, Shown are the respective values for the two resource states $\tilde{\rho}_W$ (red) and $\tilde{\rho}_B$ (blue). The dashed lines represent the theoretical expectations. There is a clear separation between the two resource states, which indicates that the separable state $\tilde{\rho}_W$ is a better resource for RSP than the entangled state $\tilde{\rho}_B$. The errors from Poissonian noise are below 6×10^{-4} , which is smaller than the point size. Oscillations in the measured data result from imperfections in the optical setup and demonstrate the sensitivity of the measurements even for mixed quantum states. **b**, Each state is shown on the Bloch sphere. The outer red points (purple sphere) correspond to $\tilde{\rho}_W$ and the inner black points (green sphere) are the same quantum states using $\tilde{\rho}_B$ as a resource.**

polarization-correlated photon pairs. Furthermore, we show that the geometric measure of quantum discord is directly linked to the fidelity of RSP for a broad class of states, providing an operational interpretation for this measure. Our demonstration that separable states can achieve higher fidelities in RSP than entangled states underlines that quantum discord quantifies the non-classical correlations required for the task, not entanglement. This insight might be of importance for future quantum-enhanced applications that rely on resources different from quantum entanglement.

Received 12 February 2012; accepted 22 June 2012;
published online 5 August 2012

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Acknowledgements

We acknowledge support from the European Commission, Q-ESSENCE (No 248095), ERC Advanced Senior Grant (QIT4QAD), and the ERA-Net CHIST-ERA project QUASAR, the John Templeton Foundation, Austrian Science Fund (FWF): (SFB-FOCUS) and (Y585-N20) and the doctoral programme CoQuS, and the Air Force Office of Scientific Research, Air Force Material Command, United States Air Force,

under grant number FA8655-11-1-3004. The work is supported by the National Research Foundation and Ministry of Education in Singapore.

Author contributions

Y.O.L., X.M. and M.R. designed and carried out the experiment, analysed data and wrote the manuscript. B.D., T.P. and V.V. provided the theoretical analysis, analysed data and wrote the manuscript. S.B. designed the experiment, discussed the results and edited the manuscript. S.K. programmed the software. A.Z. supervised the project and edited the manuscript. C.B. supervised the project, provided theoretical analysis and wrote the manuscript. P.W. supervised the project, designed the experiment and wrote the manuscript.

Additional information

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Competing financial interests

The authors declare no competing financial interests.