

Instructor's Guide to Accompany
Digital Signal Processing: Fundamentals and Applications
Part 1: Chapter 2 – Chapter 8

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Jean Jiang

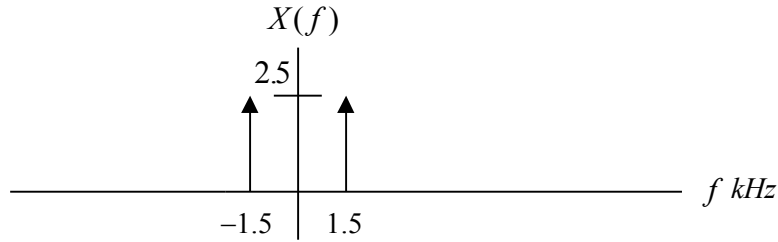
Chapter 2

2.1

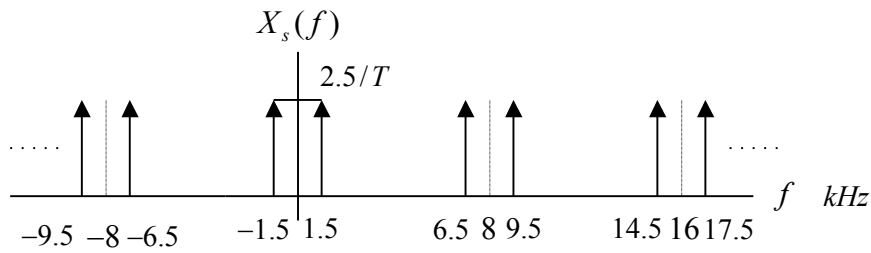
$$5 \cos(2\pi \times 1000t) = 5 \cdot \left(\frac{e^{j2\pi \times 1500t} + e^{-j2\pi \times 1000t}}{2} \right) = 2.5e^{j2\pi \times 1500t} + 2.5e^{-j2\pi \times 1500t}$$

$$c_1 = 2.5 \text{ and } c_{-1} = 2.5$$

a.



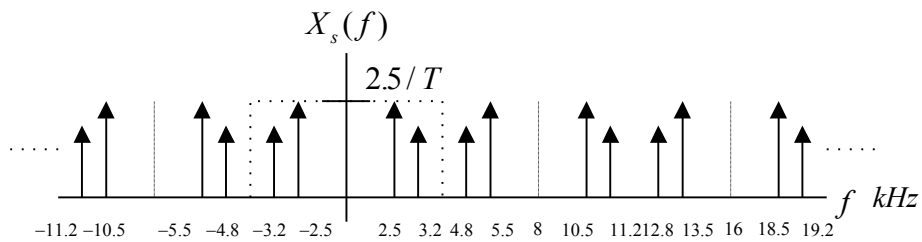
b.



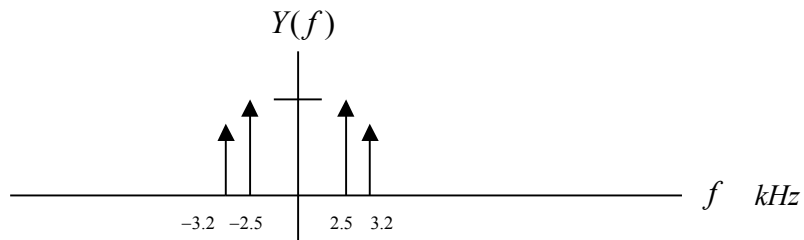
2.2

$$x(t) = e^{-j2\pi \cdot 3200t} + 2.5e^{-j2\pi \cdot 2500t} + 2.5e^{j2\pi \cdot 2500t} + e^{j2\pi \cdot 3200t}$$

a.

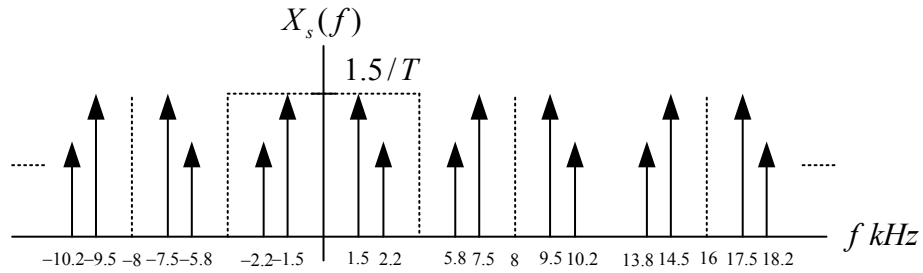


b.

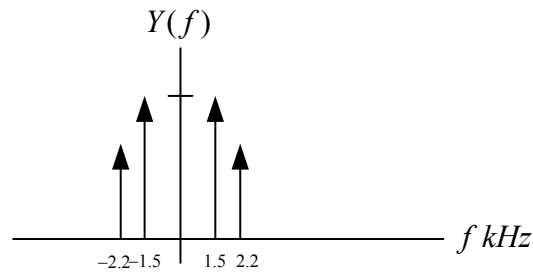


2.3 $x(t) = e^{-j2\pi \cdot 2200t} + 1.5e^{-j2\pi \cdot 1500t} + 1.5e^{j2\pi \cdot 1500t} + e^{j2\pi \cdot 2200t}$

a.

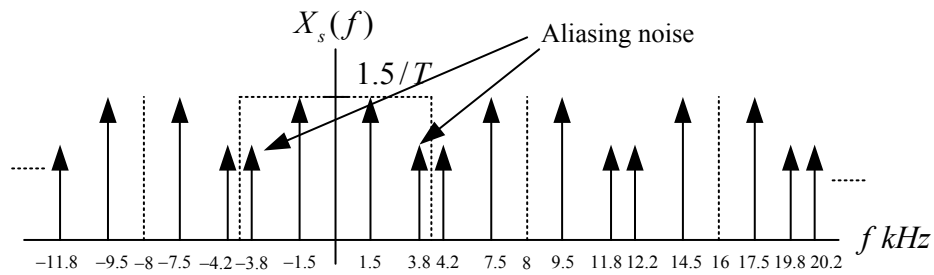


b.

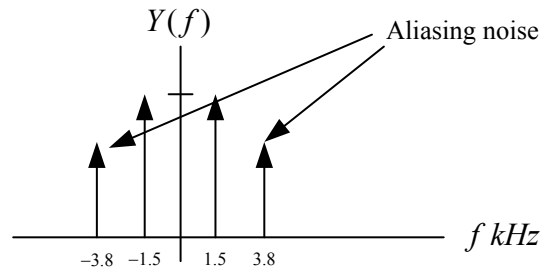


2.4 $x(t) = e^{-j2\pi \cdot 4200t} + 1.5e^{-j2\pi \cdot 1500t} + 1.5e^{j2\pi \cdot 1500t} + e^{j2\pi \cdot 4200t}$

a.



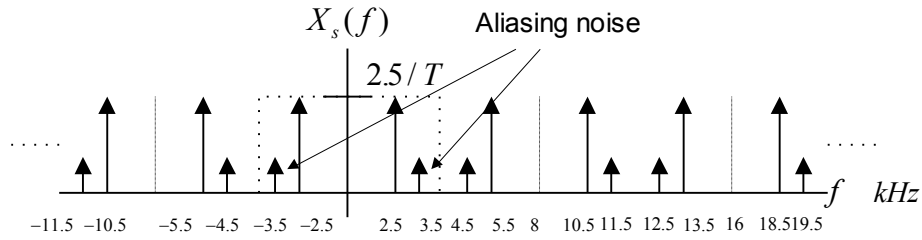
b.



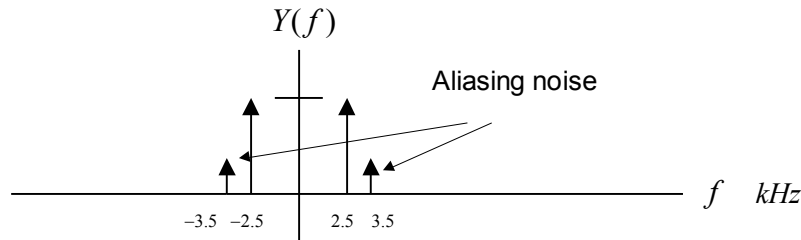
2.5

$$x(t) = e^{-j2\pi \cdot 4500t} + 2.5e^{-j2\pi \cdot 2500t} + 2.5e^{j2\pi \cdot 2500t} + e^{j2\pi \cdot 4500t}$$

a.



b.

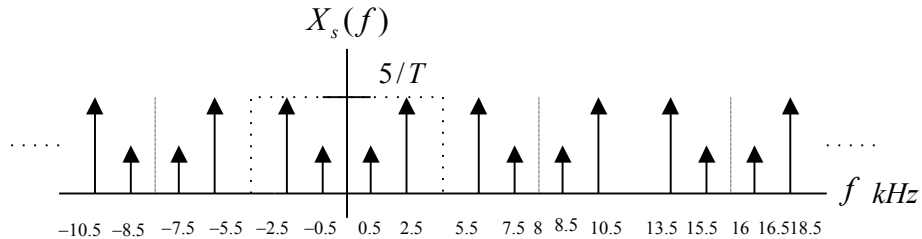


c. The aliasing frequency = 3.5 kHz

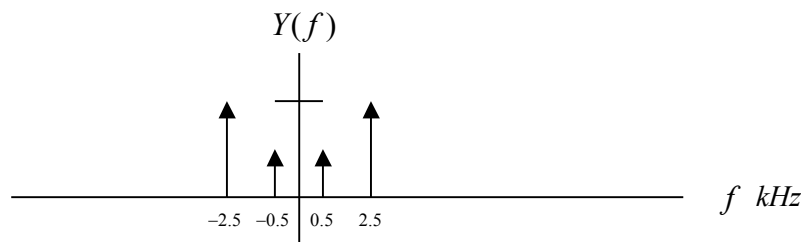
2.6

$$x(t) = \frac{-2.5}{j} e^{-j2\pi \cdot 4500t} + 5e^{-j2\pi \cdot 5500t} + 5e^{j2\pi \cdot 5500t} + \frac{2.5}{j} e^{j2\pi \cdot 7500t}$$

a.



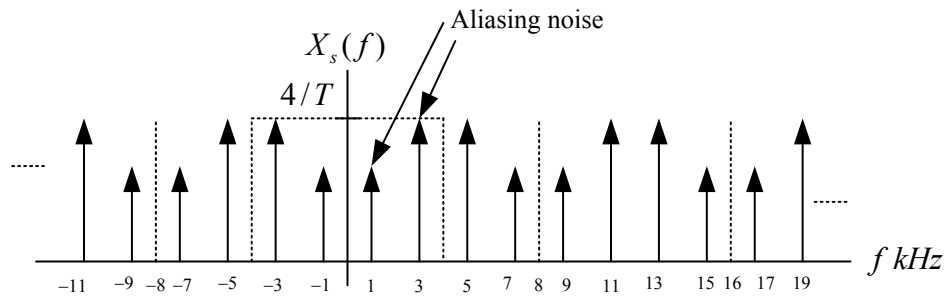
b.



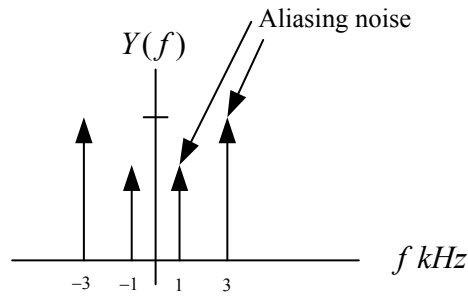
c. The aliasing frequencies: 0.5 kHz and 2.5 kHz.

$$2.7 \quad x(t) = \frac{-2.5}{j} e^{-j2\pi \cdot 7000t} + 4e^{-j2\pi \cdot 5000t} + 4e^{j2\pi \cdot 5000t} + \frac{2.5}{j} e^{j2\pi \cdot 7000t}$$

a.



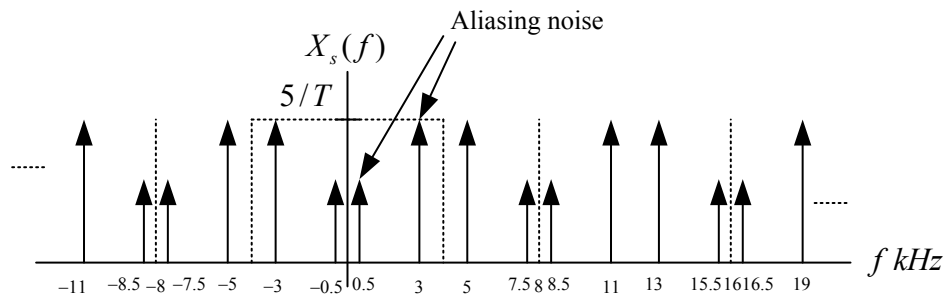
b.



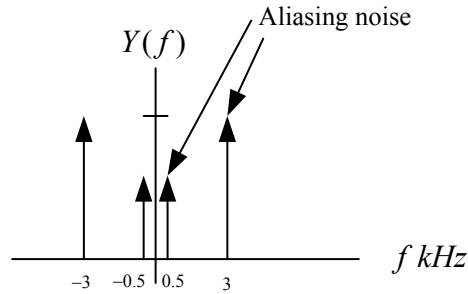
c. The aliasing frequencies: 1 kHz and 3 kHz.

$$2.8 \quad x(t) = \frac{-2.5}{j} e^{-j2\pi \cdot 7500t} + 5e^{-j2\pi \cdot 5000t} + 5e^{j2\pi \cdot 5000t} + \frac{2.5}{j} e^{j2\pi \cdot 7500t}$$

a.



b.



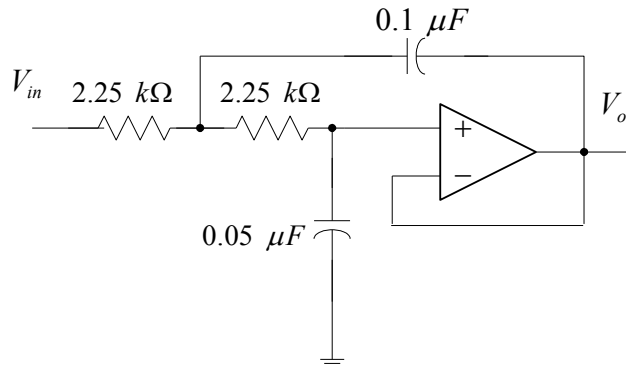
c. The aliasing frequencies: 0.5 kHz and 3 kHz.

2.9

Choose $C_2 = 0.1 \mu F$

$$R_1 = R_2 = \frac{1.4142}{C_2(2\pi f_c)} = \frac{1.4142}{0.1 \times 10^{-6} \times 2\pi \times 1000} = 2251 \Omega$$

$$C_1 = \frac{1}{R_1 R_2 (2\pi f_c)^2} = \frac{1}{2251 \times 2251 \times 0.1 \times 10^{-6} \times (2\pi \times 1000)^2} = 0.05 \mu F$$



2.10

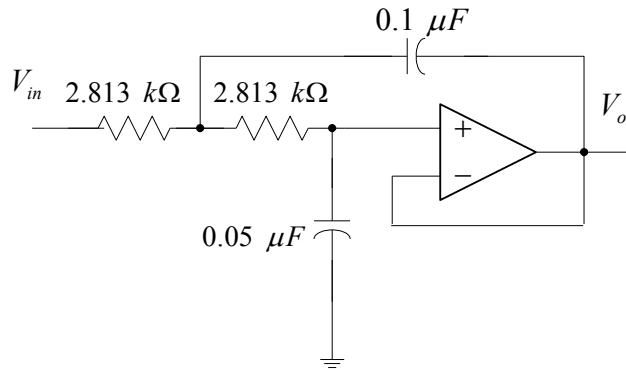
$$\% \text{ aliasing level} = \frac{\sqrt{1 + \left(\frac{f_a}{f_c}\right)^{2n}}}{\sqrt{1 + \left(\frac{f_s - f_a}{f_c}\right)^{2n}}} = \frac{\sqrt{1 + \left(\frac{500}{1000}\right)^4}}{\sqrt{1 + \left(\frac{4000 - 500}{1000}\right)^4}} = 8.39\%$$

2.11

Choose $C_2 = 0.1 \mu F$

$$R_1 = R_2 = \frac{1.4142}{C_2(2\pi f_c)} = \frac{1.4142}{0.1 \times 10^{-6} \times 2\pi \times 800} = 2813 \Omega$$

$$C_1 = \frac{1}{R_1 R_2 (2\pi f_c)^2} = \frac{1}{2813 \times 2813 \times 0.1 \times 10^{-6} \times (2\pi \times 800)^2} = 0.05 \mu F$$



2.12

$$\% \text{ aliasing level} = \frac{\sqrt{1 + \left(\frac{f_a}{f_c}\right)^{2n}}}{\sqrt{1 + \left(\frac{f_s - f_a}{f_c}\right)^{2n}}} = \frac{\sqrt{1 + \left(\frac{400}{800}\right)^4}}{\sqrt{1 + \left(\frac{4000 - 400}{800}\right)^4}} = 6.43\%$$

2.13

$$\text{a. } \% \text{ aliasing level} = \frac{\sqrt{1 + \left(\frac{f_a}{f_c}\right)^{2n}}}{\sqrt{1 + \left(\frac{f_s - f_a}{f_c}\right)^{2n}}} = \frac{\sqrt{1 + \left(\frac{3200}{3200}\right)^4}}{\sqrt{1 + \left(\frac{8000 - 3200}{3200}\right)^4}} = 57.44\%$$

$$\text{b. } \% \text{ aliasing level} = \frac{\sqrt{1 + \left(\frac{f_a}{f_c}\right)^{2n}}}{\sqrt{1 + \left(\frac{f_s - f_a}{f_c}\right)^{2n}}} = \frac{\sqrt{1 + \left(\frac{1000}{3200}\right)^4}}{\sqrt{1 + \left(\frac{8000 - 1000}{3200}\right)^4}} = 20.55\%$$

2.14

When $n = 7$

$$\% \text{ aliasing level} = \frac{\sqrt{1 + \left(\frac{f_a}{f_c}\right)^{2n}}}{\sqrt{1 + \left(\frac{f_s - f_a}{f_c}\right)^{2n}}} = \frac{\sqrt{1 + \left(\frac{3200}{3200}\right)^{2 \times 7}}}{\sqrt{1 + \left(\frac{8000 - 3200}{3200}\right)^{2 \times 7}}} = 8.26\%$$

The order of the anti-aliasing filter should be seven (7).

2.15

$$\text{a. } \% \text{ aliasing level} = \frac{\sqrt{1 + \left(\frac{f_a}{f_c}\right)^{2n}}}{\sqrt{1 + \left(\frac{f_s - f_a}{f_c}\right)^{2n}}} = \frac{\sqrt{1 + \left(\frac{3100}{3100}\right)^4}}{\sqrt{1 + \left(\frac{8000 - 3100}{3100}\right)^4}} = 52.55\%$$

$$\text{b. } \% \text{ aliasing level} = \frac{\sqrt{1 + \left(\frac{f_a}{f_c}\right)^{2n}}}{\sqrt{1 + \left(\frac{f_s - f_a}{f_c}\right)^{2n}}} = \frac{\sqrt{1 + \left(\frac{900}{3100}\right)^4}}{\sqrt{1 + \left(\frac{8000 - 900}{3100}\right)^4}} = 18.79\%$$

2.16

When $n = 6$

$$\% \text{ aliasing level} = \frac{\sqrt{1 + \left(\frac{f_a}{f_c}\right)^{2n}}}{\sqrt{1 + \left(\frac{f_s - f_a}{f_c}\right)^{2n}}} = \frac{\sqrt{1 + \left(\frac{3100}{3100}\right)^{2 \times 6}}}{\sqrt{1 + \left(\frac{8000 - 3100}{3100}\right)^{2 \times 6}}} = 9.05\%$$

The order of the anti-aliasing filter should be six (6).

2.17

$$\text{a. } fT = 3200 / 8000 = 0.4$$

$$\% \text{ distortion} = \left(1 - \frac{\sin(fT\pi)}{\pi fT}\right) \times 100\% = \left(1 - \frac{\sin(0.4\pi)}{0.4\pi}\right) \times 100\% = 24.32\%$$

$$\text{b. } fT = 1500 / 8000 = 0.1875$$

$$\% \text{ distortion} = \left(1 - \frac{\sin(fT\pi)}{\pi fT}\right) \times 100\% = \left(1 - \frac{\sin(0.1875\pi)}{0.1875\pi}\right) \times 100\% = 5.68\%$$

2.18

$$f = 4000 \text{ Hz}, fT = 4000 \times 1 / 20000 = 0.2, \text{ and } \text{gain} = \frac{\sin(0.2\pi)}{0.2\pi} = 0.9355 \text{ } (-0.58 \text{ dB})$$

$$f = 16000 \text{ Hz}, fT = 16000 \times 1 / 20000 = 0.8, \text{ and } \text{gain} = \frac{\sin(0.8\pi)}{0.8\pi} = 0.2339 \text{ } (-12.62 \text{ dB})$$

Maximum allowable gain variation from 0 to 4000 Hz = 2-0.58=1.42dB

40 -12.62 = 27.38 dB rejection at frequency 16000 Hz.

$$20 \log \left(1 + (4000 / f_c)^{2n} \right)^{1/2} \leq 1.42$$

$$20 \log \left(1 + (16000 / f_c)^{2n} \right)^{1/2} \geq 27.38$$

Then

$$n = \frac{1}{2} \log \left((10^{2.738} - 1) / (10^{0.142} - 1) \right) / \log(16000 / 4000) = 2.6158 \approx 3$$

$$f_c = \frac{4000}{(10^{0.142} - 1)^{1/6}} = 4686 \text{ Hz}$$

2.19

a. $fT = 3000 / 8000 = 0.375$

$$\% \text{ distortion} = \left(1 - \frac{\sin(fT\pi)}{\pi fT} \right) \times 100\% = \left(1 - \frac{\sin(0.375\pi)}{0.375\pi} \right) \times 100\% = 21.58\%$$

b. $fT = 1600 / 8000 = 0.2$

$$\% \text{ distortion} = \left(1 - \frac{\sin(fT\pi)}{\pi fT} \right) \times 100\% = \left(1 - \frac{\sin(0.2\pi)}{0.2\pi} \right) \times 100\% = 6.45\%$$

2.20

$$f = 4000 \text{ Hz}, fT = 4000 \times 1 / 22000 = 0.18182, \text{ and}$$

$$\text{gain} = \frac{\sin(0.18182\pi)}{0.18182\pi} = 0.9465 \text{ } (-0.48 \text{ dB})$$

$$f = 16000 \text{ Hz}, fT = 16000 \times 1 / 22000 = 0.72727, \text{ and}$$

$$\text{gain} = \frac{\sin(0.72727\pi)}{0.72727\pi} = 0.2104 \text{ } (-13.54 \text{ dB})$$

Maximum allowable gain variation from 0 to 4000 Hz = 2-0.48=1.52dB

40 -13.54 = 26.46 dB rejection at frequency 16000 Hz.

$$20 \log \left(1 + (4000 / f_c)^{2n} \right)^{1/2} \leq 1.52$$

$$20 \log \left(1 + (16000 / f_c)^{2n} \right)^{1/2} \geq 26.46$$

Then

$$n = \frac{1}{2} \log \left((10^{2.646} - 1) / (10^{0.152} - 1) \right) / \log(16000 / 4000) = 2.3138 \approx 3$$

$$f_c = \frac{4000}{(10^{0.152} - 1)^{1/6}} = 4624 \text{ Hz}$$

2.21

b1b0=01

2.22

$$V_0 = V_R \left(\frac{1}{2} b_1 + \frac{1}{4} b_0 \right) = 5 \times \left(\frac{1}{2} \times 0 + \frac{1}{4} \times 1 \right) = 1.25 \text{ Volts}$$

2.23

b1b0=10

2.24

$$\text{For b1b0=11, } V_0 = V_R \left(\frac{1}{2} b_1 + \frac{1}{4} b_0 \right) = 5 \times \left(\frac{1}{2} \times 1 + \frac{1}{4} \times 1 \right) = 3.75 \text{ Volts}$$

$$\text{For b1b0=10, } V_0 = V_R \left(\frac{1}{2} b_1 + \frac{1}{4} b_0 \right) = 5 \times \left(\frac{1}{2} \times 1 + \frac{1}{4} \times 0 \right) = 2.5 \text{ Volts}$$

2.25

$$\text{a. } L = 2^4 = 16 \text{ levels}$$

$$\text{b. } \Delta = \frac{x_{\max} - x_{\min}}{L} = \frac{5}{16} = 0.3125$$

$$\text{c. } x_q = 0 + 10 \times 0.3125 = 3.125$$

$$\text{d. } \frac{3.2}{0.3125} \Delta = 10.24 \Delta, i = \text{round} \left(\frac{x - x_{\min}}{\Delta} \right) = \text{round} \left(\frac{10.24 \Delta - 0}{\Delta} \right) = \text{round}(10.24) = 10$$

binary code = 1010

$$\text{e. } e_q = -0.075$$

2.26

$$\text{a. } L = 2^5 = 32 \text{ levels}$$

$$\text{b. } \Delta = \frac{x_{\max} - x_{\min}}{L} = \frac{4}{32} = 0.125$$

$$\text{c. } x_q = 0 + 10 \times 0.125 = 1.25$$

$$\text{d. } \frac{1.2}{0.125} \Delta = 9.6 \Delta, i = \text{round} \left(\frac{x - x_{\min}}{\Delta} \right) = \text{round} \left(\frac{9.6 \Delta - 0}{\Delta} \right) = \text{round}(9.6) = 10 \text{ binary}$$

code = 1010

$$\text{e. } e_q = 0.05$$

2.27

$$\text{a. } L = 2^3 = 8 \text{ levels}$$

$$\text{b. } \Delta = \frac{x_{\max} - x_{\min}}{L} = \frac{5}{8} = 0.625$$

c. $x_q = -2.5 + 2 \times 0.625 = -1.25$

d. $x = \frac{-1.2}{0.625} \Delta = -1.92\Delta,$

$$i = \text{round}\left(\frac{x - x_{\min}}{\Delta}\right) = \text{round}\left(\frac{-1.92\Delta - (-4\Delta)}{\Delta}\right) = \text{round}(2.08) = 2$$

binary code = 010

e. $e_q = -0.05$

2.28

a. $L = 2^8 = 256$ levels

b. $\Delta = \frac{x_{\max} - x_{\min}}{L} = \frac{5}{256} = 0.01953125$

c. $x_q = -2.5 + 205 \times 0.01953125 = 1.5039$

d. $x = \frac{1.5}{0.01953125} \Delta = 76.8\Delta,$

$$i = \text{round}\left(\frac{x - x_{\min}}{\Delta}\right) = \text{round}\left(\frac{76.8\Delta - (-128\Delta)}{\Delta}\right) = \text{round}(204.8) = 205$$

binary code = 11001101

e. $e_q = 0.0039$

2.29

a. $L = 2^6 = 64$ levels b. $\Delta = \frac{x_{\max} - x_{\min}}{L} = \frac{20}{64} = 0.3125$

c. $SNR_{dB} = 1.76 + 6.02 \times 6 = 37.88$ dB

2.30

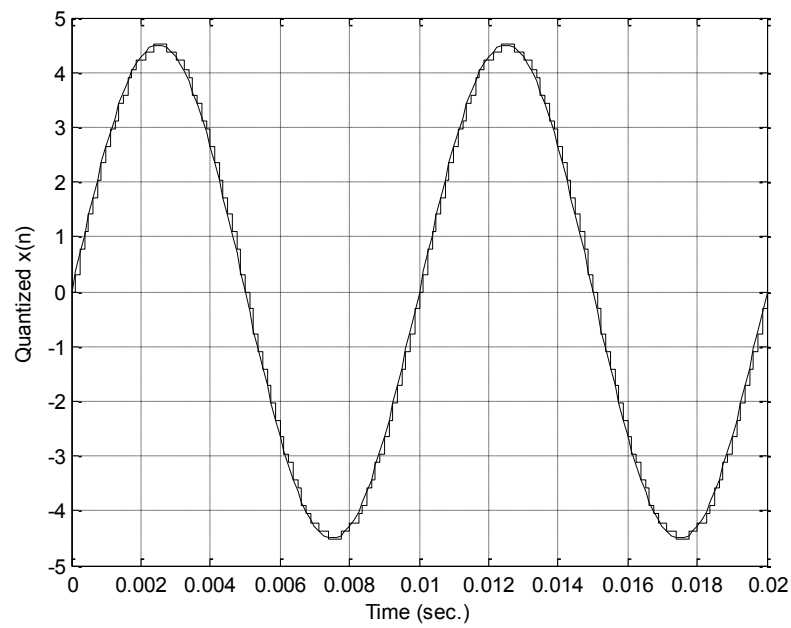
a. $L = 2^6 = 64$ levels b. $\Delta = \frac{x_{\max} - x_{\min}}{L} = \frac{5}{64} = 0.078125$

c. $SNR_{dB} = 4.77 + 20 \times \log(x_{rms} / |x|_{\max}) + 6.02 \times 6$
 $= 4.77 + 20 \times \log(0.25) + 36.12 = 28.85$ dB

2.31

a. See Program 2.1

b.

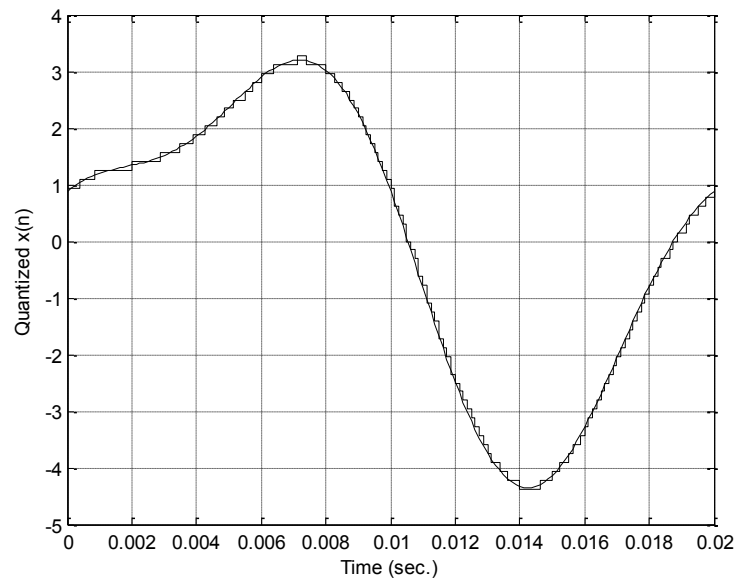


c. SNR = 37 dB

2.32

a. Use Program 2.1

b.

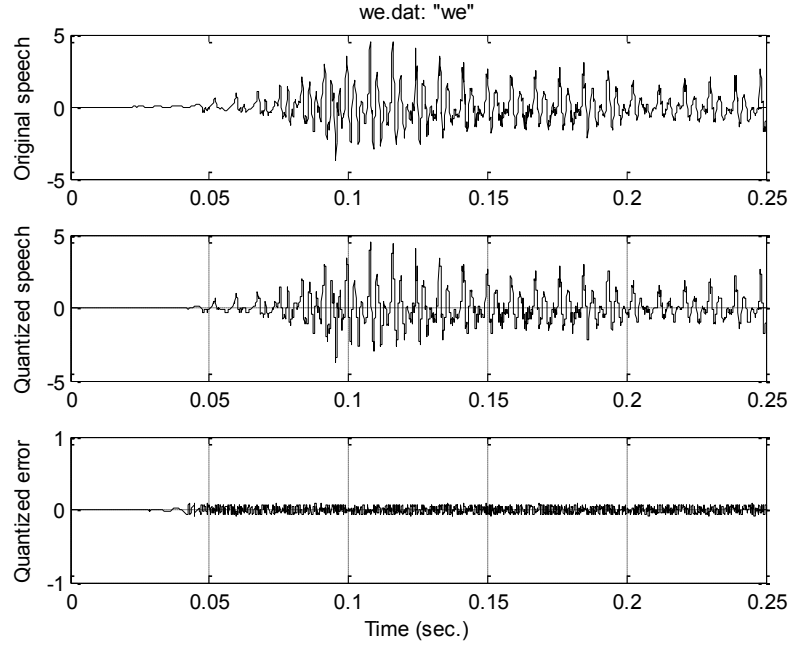


c. SNR = 35 dB

2.33

a. See Program 2.2

b.



c. SNR = 26.9 dB

2.36

$$(a) a_k = \frac{1}{T} \int_{-\infty}^{\infty} p(t) e^{-jk\omega_0 t} dt = \frac{1}{T} \int_{-\tau}^{\tau} \frac{0.5}{\tau} e^{-jk\omega_0 t} dt = \frac{1}{T} \frac{0.5}{\tau} \frac{e^{-jk\omega_0 t}}{-jk\omega_0} \bigg|_{-\tau}^{\tau} = \frac{1}{T} \frac{\sin(2\pi k\tau/T)}{2\pi k\tau/T},$$

$$\omega_0 = 2\pi/T = 2\pi f_s$$

$$p(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T} \frac{\sin(2\pi k\tau/T)}{2\pi k\tau/T} e^{-j2\pi k f_s t}$$

$$(b) X_s(f) = \frac{1}{T} \sum_{k=-\infty}^{\infty} \frac{\sin(2\pi k\tau/T)}{2\pi k\tau/T} X(f - f_s)$$

$$(c) Y_s(f) = X_s(f) = \frac{1}{T} \sum_{k=-\infty}^{\infty} \frac{\sin(2\pi k\tau/T)}{2\pi k\tau/T} X(f - f_s)$$

$$\text{For } -f_s/2 < f < f_s/2, Y_s(f) = X_s(f) = \frac{1}{T} X(f)$$

The output spectrum with sample and hold:

$$|Y_h(f)| = T \left| \frac{\sin(\pi fT)}{\pi fT} \right| |Y_s(f)|$$

The output spectrum with ideal recovery

$$|Y_{h-ideal}(f)| = T |Y_s(f)|$$

Distortion

$$Distortion = 1 - |Y_h(f)| / (|Y_{h-ideal}(f)|) = 1 - \left| \frac{\sin(\pi fT)}{\pi fT} \right|$$

2.37

$$(a) \quad a_k = \frac{1}{T} \int_{-\infty}^{\infty} p(t) e^{-jk\omega_0 t} dt = \frac{1}{T} \int_0^{\tau} \frac{1}{\tau} e^{-jk\omega_0 t} dt = \frac{1}{T} \left. \frac{e^{-jk\omega_0 t}}{-jk\omega_0} \right|_0^{\tau}$$

$$= \frac{1}{T} \frac{1 - e^{-jk\omega_0 \tau}}{jk\omega_0} = \frac{1}{T} e^{-jk\omega_0 \tau/2} \frac{e^{jk\omega_0 \tau/2} - e^{-jk\omega_0 \tau/2}}{jk\omega_0} = \frac{1}{T} e^{-jk\pi\tau/T} \frac{\sin(k\pi\tau/T)}{k\pi\tau/T}$$

$$\omega_0 = 2\pi/T = 2\pi f_s$$

$$p(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T} e^{-jk\pi\tau/T} \frac{\sin(k\pi\tau/T)}{k\pi\tau/T} e^{-j2\pi k f_s t}$$

$$(b) \quad X_s(f) = \frac{1}{T} \sum_{k=-\infty}^{\infty} e^{-jk\pi\tau/T} \frac{\sin(k\pi\tau/T)}{k\pi\tau/T} X(f - f_s)$$

$$(c) \quad Y_s(f) = X_s(f) = \frac{1}{T} \sum_{k=-\infty}^{\infty} e^{-jk\pi\tau/T} \frac{\sin(k\pi\tau/T)}{k\pi\tau/T} X(f - f_s)$$

$$\text{For } -f_c/2 < f < f_c/2, \quad Y_s(f) = X_s(f) = \frac{1}{T} X(f)$$

The output spectrum with sample and hold:

$$|Y_h(f)| = T \left| \frac{\sin(\pi f T)}{\pi f T} \right| |Y_s(f)|$$

The output spectrum with ideal recovery

$$|Y_{h-ideal}(f)| = T |Y_s(f)|$$

Distortion

$$Distortion = 1 - |Y_h(f)| / (|Y_{h-ideal}(f)|) = 1 - \left| \frac{\sin(\pi f T)}{\pi f T} \right|$$

2.38

$$(a) \quad a_k = \frac{1}{T} \int_{-\infty}^{\infty} p(t) e^{-jk\omega_0 t} dt = \frac{1}{T} \frac{[\sin(2\pi k\tau/T)]^2}{(2\pi k\tau/T)^2}, \quad \omega_0 = 2\pi/T = 2\pi f_s$$

$$p(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T} \frac{[\sin(2\pi k\tau/T)]^2}{(2\pi k\tau/T)^2} e^{-j2\pi k f_s t}$$

$$(b) \quad X_s(f) = \frac{1}{T} \sum_{k=-\infty}^{\infty} \frac{[\sin(2\pi k\tau/T)]^2}{(2\pi k\tau/T)^2} X(f - k f_s)$$

$$(c) \quad Y_s(f) = X_s(f) = \frac{1}{T} \sum_{k=-\infty}^{\infty} \frac{[\sin(2\pi k\tau/T)]^2}{(2\pi k\tau/T)^2} X(f - k f_s)$$

$$\text{For } -f_s/2 < f < f_s/2, \quad Y_s(f) = X_s(f) = \frac{1}{T} X(f)$$

The output spectrum with sample and hold:

$$|Y_h(f)| = T \left| \frac{\sin(\pi fT)}{\pi fT} \right| |Y_s(f)|$$

The output spectrum with ideal recovery

$$|Y_{h-ideal}(f)| = T |Y_s(f)|$$

Distortion

$$Distortion = 1 - |Y_h(f)| / (|Y_{h-ideal}(f)|) = 1 - \left| \frac{\sin(\pi fT)}{\pi fT} \right|$$

2.39

$$(a) AL = 1 / \sqrt{(1 + \varepsilon^2 C^2 [(f_s - f_a) / f_c])} / 1 / \sqrt{(1 + \varepsilon^2 C^2 (f_a / f_c))} = \frac{\sqrt{[1 + \varepsilon^2 C^2 (f_a / f_c)]}}{\sqrt{(1 + \varepsilon^2 C^2 [(f_s - f_a) / f_c])}}$$

$$(b) f_c = 3.4 \text{ kHz}, f_a = 1 \text{ kHz}, f_s = 8 \text{ kHz}, n = 4,$$

$$\varepsilon = 1 \text{ dB} = -20 \times \log_{10}(1 / \sqrt{1 + \varepsilon^2}) \Rightarrow \varepsilon^2 = 0.2589, f_a / f_c = 1 / 3.4 = 0.2941$$

$$(f_s - f_a) / f_c = 7 / 3.4 = 2.0588$$

$$C_4(0.2941) = \cos[4 \cos^{-1}(0.2941)] = 0.3679,$$

$$C_4(2.0588) = \cosh[4 \cosh^{-1}(2.0588)] = 110.8205$$

$$AL = \frac{\sqrt{[1 + 0.2589 \times 0.3679^2]}}{\sqrt{(1 + 0.2589 \times 110.8205^2)}} = 0.018$$

2.40

$$\Delta = 2 \sum_{i=1}^N A_i / 2^m, P = \sum_{i=1}^N A_i^2 / 2, N = \frac{\Delta^2}{12} = \frac{\left[2 \sum_{i=1}^N A_i / 2^m \right]^2}{12}, SNR = \frac{12 \sum_{i=1}^N A_i^2 / 2}{\left[2 \sum_{i=1}^N A_i / 2^m \right]^2}$$

2.41

$$\Delta = 2 A_1 A_2 / 2^m, N = \frac{\Delta^2}{12} = \frac{[2 A_1 A_2 / 2^m]^2}{12}$$

$$\text{Since } x(t) = A_1 A_2 \cos[(\omega_1 + \omega_2)t + \phi_1 + \phi_2] / 2 + A_1 A_2 \cos[(\omega_1 - \omega_2)t + \phi_1 - \phi_2] / 2.$$

$$P = (A_1 A_2 / 2)^2 / 2 + (A_1 A_2 / 2)^2 / 2 = (A_1 A_2)^2 / 4, SNR = \frac{3(A_1 A_2)^2}{[2 A_1 A_2 / 2^m]^2}$$

2.42

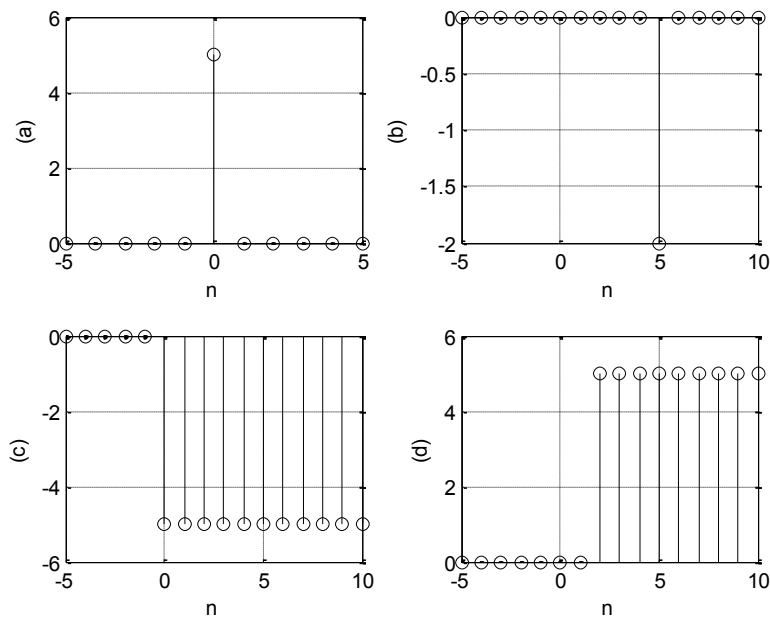
$$N = \int_{-\Delta}^0 e_q^2 \frac{1}{\Delta} de_q = \frac{1}{3\Delta} e_q^3 \Big|_{-\Delta}^0 = \frac{1}{3} \Delta^2, SNR = \frac{P}{N} = \frac{3 \sum_{i=1}^N A_i^2 / 2}{\left[2 \sum_{i=1}^N A_i / 2^m \right]^2}$$

2.43

$$N = \int_0^{\Delta} e_q^2 \frac{1}{\Delta} de_q = \frac{1}{3\Delta} e_q^3 \Big|_0^{\Delta} = \frac{1}{3} \Delta^2, \quad SNR = \frac{3(A_1 A_2)^2 / 4}{[2A_1 A_2 / 2^m]^2}$$

Chapter 3

3.1



3.2

a.

n	0	1	2	3	4	5	6	7
$x(n)$	1.000	0.5000	0.2500	0.1250	0.0625	0.0313	0.0156	0.0078

b.

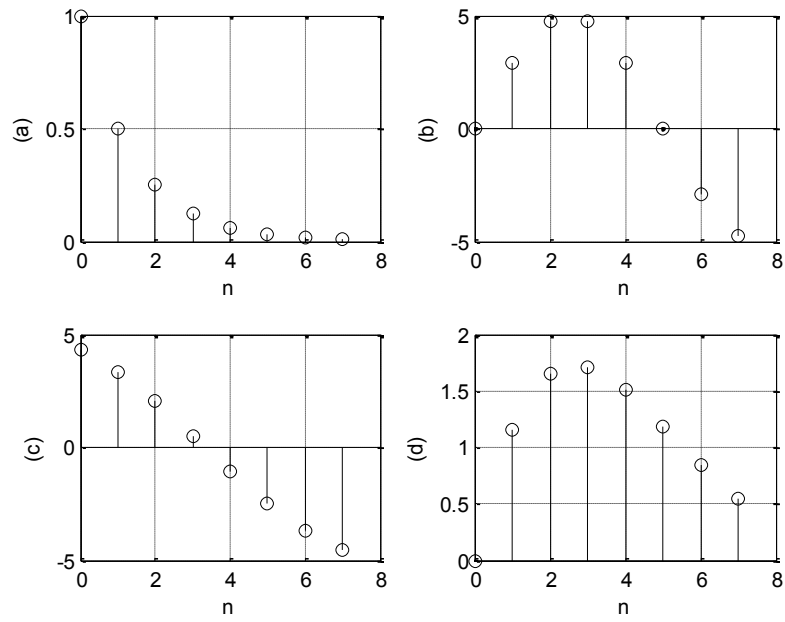
n	0	1	2	3	4	5	6	7
$x(n)$	0.0000	2.9389	4.7553	4.7553	2.9389	0.0000	-2.9389	-4.7553

c.

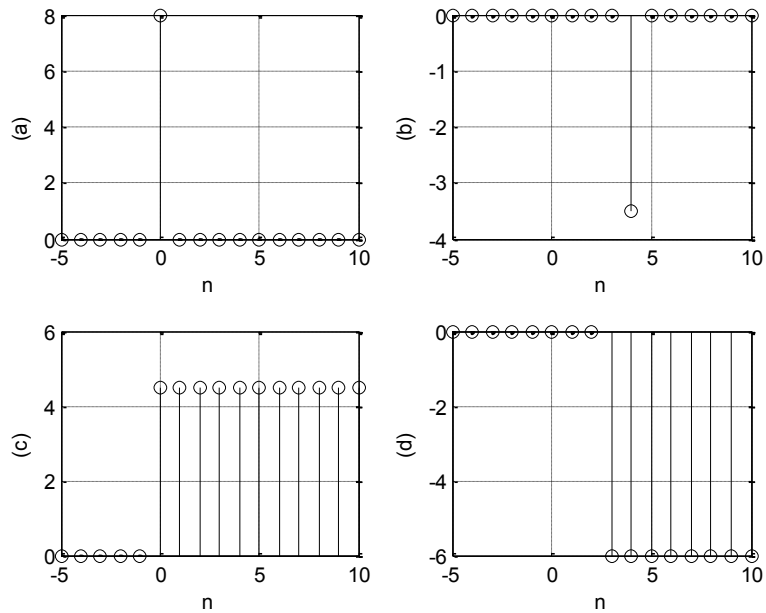
n	0	1	2	3	4	5	6	7
$x(n)$	4.3301	3.3457	2.0377	0.5226	-1.0396	-2.5000	-3.7157	-4.5677

d.

n	0	1	2	3	4	5	6	7
$x(n)$	0.0000	1.1588	1.6531	1.7065	1.5064	1.1865	0.8463	0.5400

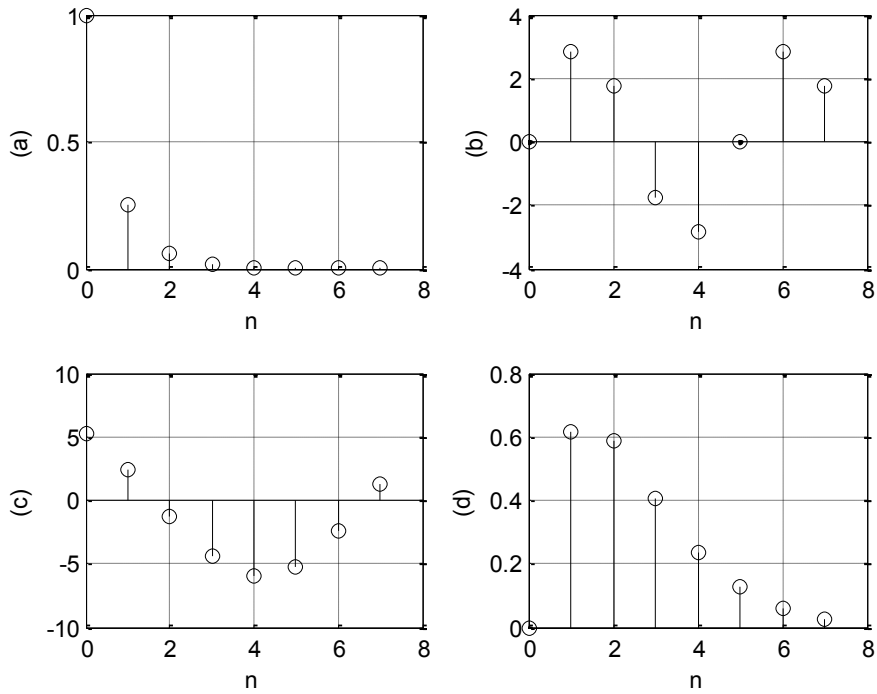


3.3

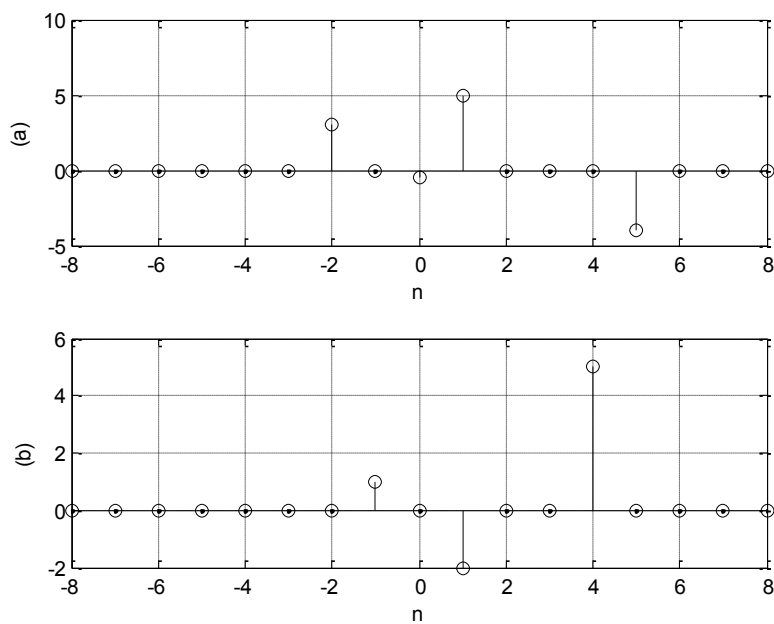


3.4

$x_a = [1.0000 \ 0.2500 \ 0.0625 \ 0.0156 \ 0.0039 \ 0.0010 \ 0.0002 \ 0.0001]$
 $x_b = [0 \ 2.8532 \ 1.7634 \ -1.7634 \ -2.8532 \ -0.0000 \ 2.8532 \ 1.7634]$
 $x_c = [5.1962 \ 2.4404 \ -1.2475 \ -4.4589 \ -5.9671 \ -5.1962 \ -2.4404 \ 1.2475]$
 $x_d = [0 \ 0.6180 \ 0.5878 \ 0.4045 \ 0.2378 \ 0.1250 \ 0.0594 \ 0.0253]$



3.5

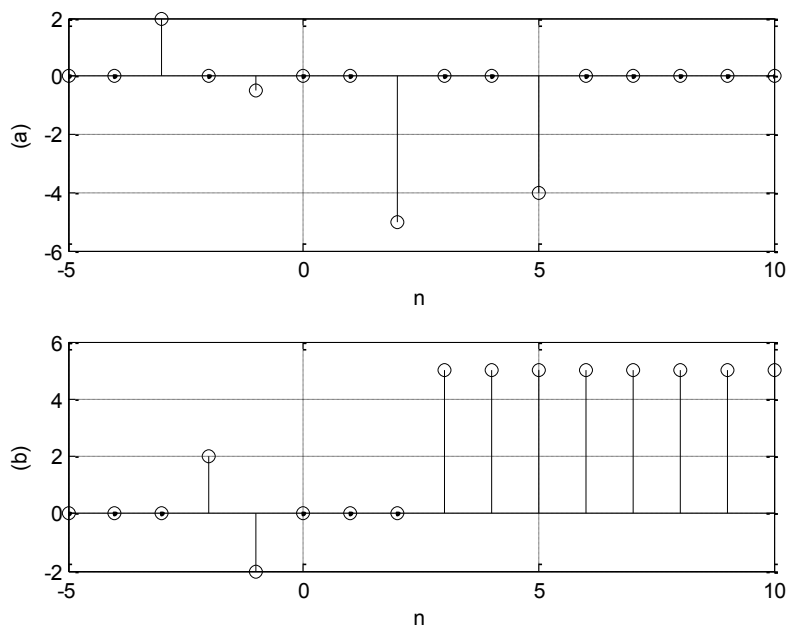


3.6

a. $x(n) = 3\delta(n) + \delta(n-1) + 2\delta(n-2) + \delta(n-3) + \delta(n-5)$

b. $x(n) = \delta(n-1) - \delta(n-2) + \delta(n-4) - \delta(n-5)$

3.7



3.8

a. $x(n) = \delta(n+1) + 0.5\delta(n-1) + 3\delta(n) + 2.5\delta(n-2) + \delta(n-3) + 0.5\delta(n-4) + \delta(n-5)$

b. $x(n) = -0.5\delta(n+1) + 0.5\delta(n) + \delta(n-1) - 0.5\delta(n-2) + \delta(n-4) - \delta(n-5)$

3.9

- a. $x(n) = e^{-0.5n}u(n) = (0.6065)^n u(n)$
- b. $x(n) = 5\sin(0.2\pi n)u(n)$
- c. $x(n) = 10\cos(0.4\pi n + \pi/6)u(n)$
- d. $x(n) = 10e^{-n}\sin(0.15\pi n)u(n) = 10(0.3679)^n \sin(0.15\pi n)u(n)$

3.10

- a. Let $y_1(n) = 5x_1(n) + 2x_1^2(n)$, $y_2(n) = 5x_2(n) + 2x_2^2(n)$

$$y_1(n) + y_2(n) = 5x_1(n) + 2x_1^2(n) + 5x_2(n) + 2x_2^2(n)$$

$$\text{For } x(n) = x_1(n) + x_2(n)$$

$$\begin{aligned} y(n) &= 5x(n) + 2x^2(n) = 5(x_1(n) + x_2(n)) + 2(x_1(n) + x_2(n))^2 \\ &= 5x_1(n) + 5x_2(n) + 2x_1^2(n) + 2x_2^2(n) + 4x_1(n)x_2(n) \end{aligned}$$

Since $y_1(n) + y_2(n) \neq y(n)$, the system is a nonlinear system.

- b. Let $y_1(n) = x_1(n-1) + 4x_1(n)$, $y_2(n) = x_2(n-1) + 4x_2(n)$

$$y_1(n) + y_2(n) = x_1(n-1) + x_2(n-1) + 4x_1(n) + 4x_2(n)$$

$$\text{For } x(n) = x_1(n) + x_2(n)$$

$$\begin{aligned} y(n) &= x(n-1) + 4x(n) = (x_1(n-1) + x_2(n-1)) + 4(x_1(n) + x_2(n)) \\ &= x_1(n-1) + x_2(n-1) + 4x_1(n) + 4x_2(n) \end{aligned}$$

Since $y_1(n) + y_2(n) = y(n)$, the system is a linear system.

- c. Let $y_1(n) = 4x_1^3(n) - 2x_1(n)$, $y_2(n) = 4x_2^3(n) - 2x_2(n)$

$$y_1(n) + y_2(n) = 4x_1^3(n) - 2x_1(n) + 4x_2^3(n) - 2x_2(n)$$

$$\text{For } x(n) = x_1(n) + x_2(n)$$

$$\begin{aligned} y(n) &= 4x^3(n) - 2x(n) = 4(x_1(n) + x_2(n))^3 - 2(x_1(n) + x_2(n)) \\ &= 4x_1^3(n) + 8x_1^2(n)x_2(n) + 8x_1(n)x_2^2(n) + 4x_2^3(n) - 2x_1(n) - 2x_2(n) \end{aligned}$$

Since $y_1(n) + y_2(n) \neq y(n)$, the system is a nonlinear system.

3.11

- a. $x(n) = e^{-0.5n}u(n) = (0.6065)^n u(n)$
- b. $x(n) = 4\sin(0.3\pi n)u(n)$
- c. $x(n) = 7.5\cos(0.1\pi n + \pi/3)u(n)$
- d. $x(n) = 20e^{-n}\sin(0.3\pi n)u(n) = 20(0.3679)^n \sin(0.3\pi n)u(n)$

3.12

- a. Let $y_1(n) = 4x_1(n) + 8x_1^3(n)$, $y_2(n) = 4x_2(n) + 8x_2^3(n)$

$$y_1(n) + y_2(n) = 4x_1(n) + 8x_1^3(n) + 4x_2(n) + 8x_2^3(n)$$

$$\text{For } x(n) = x_1(n) + x_2(n)$$

$$y(n) = 4x(n) + 8x^3(n) = 4(x_1(n) + x_2(n)) + 8(x_1(n) + x_2(n))^3$$

Since $y_1(n) + y_2(n) \neq y(n)$, the system is a nonlinear system.

b. Let $y_1(n) = x_1(n-3) + 3x_1(n)$, $y_2(n) = x_2(n-3) + 3x_2(n)$

$$y_1(n) + y_2(n) = x_1(n-3) + x_2(n-3) + 3x_1(n) + 3x_2(n)$$

For $x(n) = x_1(n) + x_2(n)$

$$\begin{aligned} y(n) &= x(n-3) + 3x(n) = (x_1(n-3) + x_2(n-3)) + 3(x_1(n) + x_2(n)) \\ &= x_1(n-1) + x_2(n-1) + 3x_1(n) + 3x_2(n) \end{aligned}$$

Since $y_1(n) + y_2(n) = y(n)$, the system is a linear system.

c. Let $y_1(n) = 5x_1^2(n-1) - 3x_1(n)$, $y_2(n) = 5x_2^2(n-1) - 3x_2(n)$

$$y_1(n) + y_2(n) = 5x_1^2(n-1) - 3x_1(n) + 5x_2^2(n-1) - 3x_2(n)$$

For $x(n) = x_1(n) + x_2(n)$

$$\begin{aligned} y(n) &= 5x^2(n-1) + 3x(n) = 5(x_1(n-1) + x_2(n-1))^2 - 3(x_1(n) + x_2(n)) \\ &= 5x_1^2(n-1) + 10x_1(n-1)x_2(n-1) + 5x_2^2(n-1) - 3x_1(n) - 3x_2(n) \end{aligned}$$

Since $y_1(n) + y_2(n) \neq y(n)$, the system is a nonlinear system.

3.13

a. For $x_1(n) = x(n-n_0)$, $y_1(n) = -5x_1(n-10) = -5x(n-10-n_0)$

Since $y(n-n_0) = -5x((n-n_0)-10) = -5x(n-10-n_0) = y_1(n)$

The system is time invariant.

b. For $x_2(n) = x(n-n_0)$ so that $x_2(n^2) = x(n^2-n_0)$, $y_2(n) = 4x_2(n^2) = 4x_2(n^2-n_0)$

Since shifting $y(n-n_0) = 4x((n-n_0)^2) = 4x(n^2-2nn_0+n_0^2) \neq y_2(n)$

The system is time invariant.

3.14

a. Since the output is depending on the current input and past inputs, the system is causal.

b. Since the output is depending on the future input $x(n+4)$, the system is a non-causal system.

3.15

a. causal system, since the system output depends on the current input and past inputs.

b. noncausal system, since the system output depends on a future input.

c. causal system, since the equation can be rewritten as

$$y(n) = -2x(n-3) + 2y(n-2)$$

The system output is depending on the past input and the past output which is in turn based on the past outputs.

3.16

a. $h(n) = 0.5\delta(n) - 0.5\delta(n-2)$

b. $h(n) = (0.75)^n$; $n \geq 0$

c. $h(n) = 1.25\delta(n) - 1.25(-0.8)^n; n \geq 0$

3.17

a. causal system, since the system output depends on the current input and past input and output.

b. noncausal system, since the system output depends on a future input $x(n+2)$.

c. causal system, since the equation can be rewritten as

$$y(n) = -0.2x(n-1) + 2y(n-2)$$

The system output is depending on the past input and the past output which is in turn based on the past outputs.

3.18

a. $h(n) = 0.2\delta(n) - 0.3\delta(n-2)$

b. $h(n) = 0.5(0.5)^n; n \geq 0$

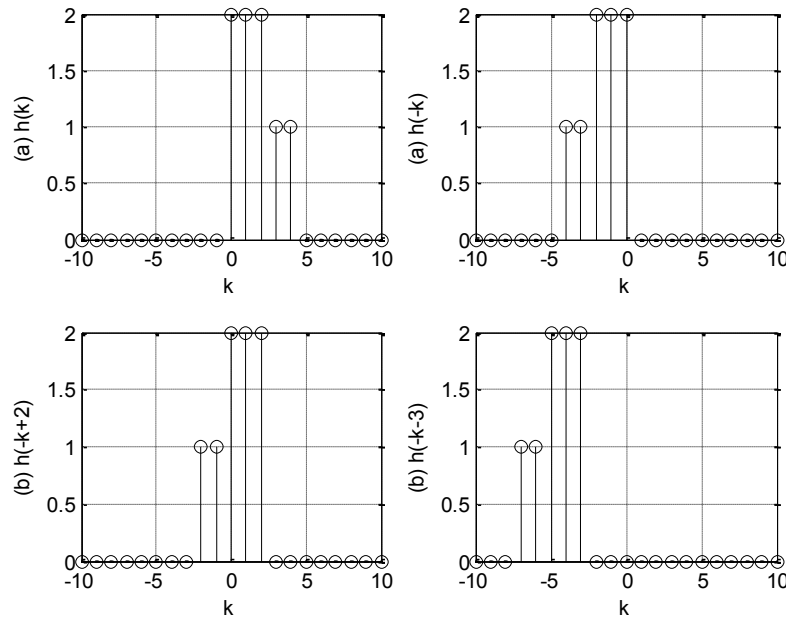
c. $h(n) = -(5/3)\delta(n) + (5/3)(-0.6)^n; n \geq 0$

3.19

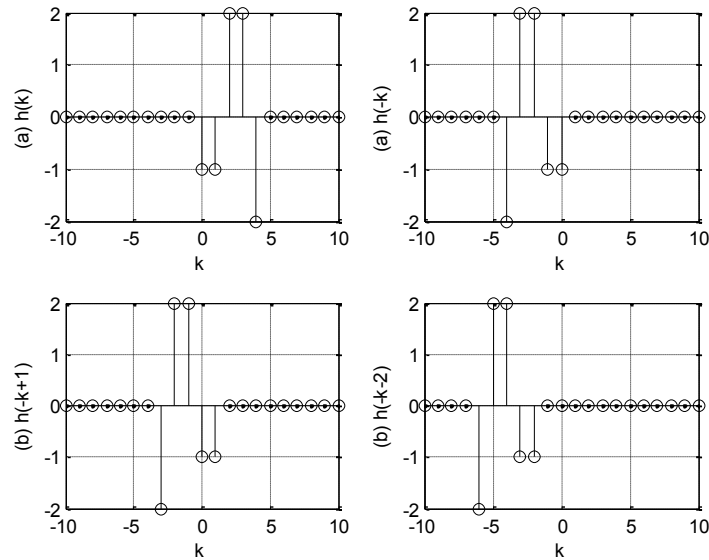
a. $h(n) = 5\delta(n-10)$

b. $h(n) = \delta(n) + 0.5\delta(n-1)$

3.20



3.21



3.22

$y(0) = 4$, $y(1) = 6$, $y(2) = 8$, $y(3) = 6$, $y(4) = 5$, $y(5) = 2$, $y(6) = 1$,
 $y(n) = 0$ for $n \geq 7$

3.23

$y(0) = -4$, $y(1) = -2$, $y(2) = 0$, $y(3) = 6$, $y(4) = 3$, $y(5) = -2$, $y(6) = -1$,
 $y(n) = 0$ for $n \geq 7$

3.24

$y(0) = 0$, $y(1) = 1$, $y(2) = 2$, $y(3) = 1$, $y(4) = 0$,
 $y(n) = 0$ for $n \geq 4$

3.25

Since $h(n) = 0.5\delta(n) + 100\delta(n-2) - 20\delta(n-10)$

and $S = 0.5 + 100 + 20 = 120.5 = \text{finite number}$, the system is stable.

3.26

a. $h(n) = 2.5\delta(n-5)$

b. $h(n) = 2\delta(n) + 1.2\delta(n-1)$

3.27

Since $h(n) = 5\delta(n) + 30\delta(n-3) - 10\delta(n-20)$

and $S = 5 + 30 + 10 = 45 = \text{finite number}$, the system is stable.

3.28

a. $h(n) = (0.75)^n u(n)$, $S = \sum_{k=0}^{\infty} (0.75)^k = 1/(1-0.75) = 4$ = finite, the system is stable.

b. $h(n) = (2)^n u(n)$, $S = \sum_{k=0}^{\infty} (2)^k = 1 + 2 + 2^2 + \dots$ = infinite, the system is unstable.

3.29

a. $h(n) = (-1.5)^n u(n)$, $S = \sum_{k=0}^{\infty} (-1.5)^k = \lim_{k \rightarrow \infty} \frac{1 - (-1.5)^{k+1}}{1 - (-1.5)} = \infty$ = infinite, the system is unstable.

b. $h(n) = (-0.5)^n u(n)$, $S = \sum_{k=0}^{\infty} (-0.5)^k = 1/(1 - (-0.5)) = 2/3$ = finite, the system is stable.

3.30

(a)

$$y(n) = x(-n+3)$$

(1)

Let $x_1(n) = \text{input}$, $y_1(n) = x_1(-n+3)$;

Let $x_2(n) = \text{input}$, $y_2(n) = x_2(-n+3)$;

Let $x_3(n) = \alpha x_1(n) + \beta x_2(n)$, $y_3(n) = x_3(-n+3) = \alpha x_1(-n+3) + \beta x_2(-n+3)$;

Check $\alpha y_1(n) + \beta y_2(n) = \alpha x_1(-n+3) + \beta x_2(-n+3) = y_3(n)$

The system is linear.

(2)

Let $x_1(n) = \text{input}$, $y_1(n) = x_1(-n+3)$;

Let $x_2(n) = x_1(n-n_0)$, $y_2(n) = x_2(-n+3) = x_1(-n+3-n_0)$;

Check $y_1(n-n_0) = x_1[-(n-n_0)+3] = x_1(-n+n_0+3) \neq y_2(n)$;

The system is time varying.

(3)

Since $y(-1) = x(1+3) = x(4)$ depending on the future input, the system is non-causal

(4)

Let $|x(n)| = M < \infty$, then $|y(n)| = |x(-n+3)| < M$

The system is BIBO stable

Linear, time varying, non-causal, stable

(b)

$$y(n) = x(n-1) + 0.5y(n-2);$$

Impulse response:

$$h(n) = 0, n < 1 \quad h(n) = 0, n < 1, \quad h(1) = 1, \quad h(2) = 0, \quad h(3) = 0.5,$$

$$|h(n)| \leq 1$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

(1)

Let $x_1(n) = \text{input}$, $y_1(n) = \sum_{k=-\infty}^{\infty} h(k)x_1(n-k)$

Let $x_2(n) = \text{input}$, $y_2(n) = \sum_{k=-\infty}^{\infty} h(k)x_2(n-k)$;

Let $x_3(n) = \alpha x_1(n) + \beta x_2(n)$,

$$y_3(n) = \sum_{k=-\infty}^{\infty} h(k)[\alpha x_1(n-k) + \beta x_2(n-k)] = y_3(n) = \alpha \sum_{k=-\infty}^{\infty} h(k)x_1(n-k) + \beta \sum_{k=-\infty}^{\infty} h(k)x_2(n-k)$$

Check $\alpha y_1(n) + \beta y_2(n) = \alpha \sum_{k=-\infty}^{\infty} h(k)x_1(n-k) + \beta \sum_{k=-\infty}^{\infty} h(k)x_2(n-k) = y_3(n)$;

The system is linear.

(2)

Let $x_1(n) = \text{input}$, $y_1(n) = \sum_{k=-\infty}^{\infty} h(k)x_1(n-k)$;

Let $x_2(n) = x_1(n-n_0)$, $y_2(n) = \sum_{k=-\infty}^{\infty} h(k)x_2(n-k) = \sum_{k=-\infty}^{\infty} h(k)x_1(n-n_0-k)$;

Check $y_1(n-n_0) = \sum_{k=-\infty}^{\infty} h(k)x_1(n-n_0-k) = y_2(n)$;

The system is time invariant.

(3)

Since $y(n)$ is depending on past input and past output, the system is causal

(4)

Let $|x(n)| = M < \infty$, then

$$|y(n)| < \left| \sum_{k=-\infty}^{\infty} h(k)x(n-k) \right| < M \sum_{k=-\infty}^{\infty} |h(k)| = M [1 + 0.5 + 0.5^2 \dots] = M \frac{1}{1-0.5} = 2M < \infty$$

The system is BIBO stable

Linear time invariant, causal, stable

(c)

$$y(n) = nx(n-1) + x(n)$$

(1)

Let $x_1(n) = \text{input}$, $y_1(n) = nx_1(n-1) + x_1(n)$;

Let $x_2(n) = \text{input}$, $y_2(n) = nx_2(n-1) + x_2(n)$;

Let $x_3(n) = \alpha x_1(n) + \beta x_2(n)$,

$$y_3(n) = nx_3(n-1) + x_3(n) = n\alpha x_1(n-1) + n\beta x_2(n-1) + \alpha x_1(n) + \beta x_2(n)$$
 ;

Check $\alpha y_1(n) + \beta y_2(n) = \alpha nx_1(n-1) + \alpha x_1(n) + \beta nx_2(n-1) + \beta x_2(n) = y_3(n)$

The system is linear.

(2)

Let $x_1(n) = \text{input}$, $y_1(n) = nx_1(n-1) + x(n)$;Let $x_2(n) = x_1(n-n_0)$, $y_2(n) = nx_2(n-1) + x_2(n) = nx_1(n-1-n_0) + x_1(n-n_0)$;Check $y_1(n-n_0) = (n-n_0)x_1(n-1-n_0) + x_1(n-n_0) \neq y_2(n)$;

The system is time varying.

(3)

Since $y(n)$ is depending on the current input and past input, the system is causal.

(4)

Let $|x(n)| = M < \infty$, then $|y(n)| < |nx(n-1)| + |x(n)| < (|n|+1)M \rightarrow \infty$

The system is not BIBO stable

Linear, time varying, causal, unstable

(d)

$$y(n) = |x(n)|$$

(1)

Let $x_1(n) = \text{input}$, $y_1(n) = |x_1(n)|$;Let $x_2(n) = \text{input}$, $y_2(n) = |x_2(n)|$;Let $x_3(n) = \alpha x_1(n) + \beta x_2(n)$, $y_3(n) = |x_3(n)| = |\alpha x_1(n) + \beta x_2(n)| < \alpha |x_1(n)| + \beta |x_2(n)|$;Check $\alpha y_1(n) + \beta y_2(n) = \alpha |x_1(n)| + \beta |x_2(n)| \neq y_3(n)$

The system is nonlinear.

(2)

Let $x_1(n) = \text{input}$, $y_1(n) = |x_1(n)|$;Let $x_2(n) = x_1(n-n_0)$, $y_2(n) = |x_2(n)| = |x_1(n-n_0)|$;Check $y_1(n-n_0) = |x_1(n-n_0)| = y_2(n)$;

The system is time invariant.

(3)

Since $y(n)$ depending on the current input, the system is causal

(4)

Let $|x(n)| = M < \infty$, then $|y(n)| = |x(n)| = M$

The system is BIBO stable

Nonlinear, time invariant, causal, stable

3.31

(a)

$$y(n) = \text{sign}[x(n)], \text{ where } \text{sign}(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

(1)

Let $x_1(n) = \text{input}$, $y_1(n) = \text{sign}[x_1(n)]$;

Let $x_2(n) = \text{input}$, $y_2(n) = \text{sign}[x_2(n)]$;

Let $x_3(n) = \alpha x_1(n) + \beta x_2(n)$, $y_3(n) = \text{sign}[x_3(n)] = \text{sign}[\alpha x_1(n) + \beta x_2(n)]$;

Check $\alpha y_1(n) + \beta y_2(n) = \alpha \times \text{sign}[x_1(n)] + \beta \times \text{sign}[x_2(n)] \neq y_3(n)$

The system is nonlinear.

(2)

Let $x_1(n) = \text{input}$, $y_1(n) = \text{sign}[x_1(n)]$;

Let $x_2(n) = x_1(n - n_0)$, $y_2(n) = \text{sign}[x_2(n)] = \text{sign}[x_1(n - n_0)]$;

Check $y_1(n - n_0) = \text{sign}[x_1(n - n_0)] = y_2(n)$;

The system is time invariant

(3)

Since $y(n)$ depending on the current input, the system is causal

(4)

Let $|x(n)| = M < \infty$, then $|y(n)| = \text{sign}[|x(n)|] \leq 1$

The system is BIBO stable

Nonlinear, time invariant, causal, stable

(b)

$y(n) = \text{truncate}[x(n)]$

(1)

Let $x_1(n) = \text{input}$, $y_1(n) = \text{truncate}[x_1(n)]$;

Let $x_2(n) = \text{input}$, $y_2(n) = \text{truncate}[x_2(n)]$;

Let $x_3(n) = \alpha x_1(n) + \beta x_2(n)$, $y_3(n) = \text{truncate}[x_3(n)] = \text{truncate}[\alpha x_1(n) + \beta x_2(n)]$;

Check $\alpha y_1(n) + \beta y_2(n) = \alpha \times \text{truncate}[x_1(n)] + \beta \times \text{truncate}[x_2(n)] \neq y_3(n)$

The system is nonlinear.

(2)

Let $x_1(n) = \text{input}$, $y_1(n) = \text{truncate}[x_1(n)]$;

Let $x_2(n) = x_1(n - n_0)$, $y_2(n) = \text{truncate}[x_2(n)] = \text{truncate}[x_1(n - n_0)]$;

Check $y_1(n - n_0) = \text{truncate}[x_1(n - n_0)] = y_2(n)$;

The system is time invariant.

(3)

Since $y(n)$ depending on the current input, the system is causal

(4)

Let $|x(n)| = M < \infty$, then $|y(n)| = |\text{truncate}[x(n)]| < M$

The system is BIBO stable

Nonlinear, time invariant, causal , stable

(c)

$y(n) = \text{round}[x(n)]$

(1)

Let $x_1(n) = \text{input}$, $y_1(n) = \text{round}[x_1(n)]$;

Let $x_2(n) = \text{input}$, $y_2(n) = \text{round}[x_2(n)]$;

Let $x_3(n) = \alpha x_1(n) + \beta x_2(n)$, $y_3(n) = \text{round}[x_3(n)] = \text{round}[\alpha x_1(n) + \beta x_2(n)]$;

Check $\alpha y_1(n) + \beta y_2(n) = \alpha \times \text{round}[x_1(n)] + \beta \times \text{round}[x_2(n)] \neq y_3(n)$

The system is nonlinear.

(2)

Let $x_1(n) = \text{input}$, $y_1(n) = \text{round}[x_1(n)]$;

Let $x_2(n) = x_1(n - n_0)$, $y_2(n) = \text{round}[x_2(n)] = \text{round}[x_1(n - n_0)]$;

Check $y_1(n - n_0) = \text{round}[x_1(n - n_0)] = y_2(n)$;

The system is time invariant

(3)

Since $y(n)$ depending on the current input, the system is causal

(4)

Let $|x(n)| = M < \infty$, then $|y(n)| = |\text{round}[x(n)]| < M + 0.5$

The system is BIBO stable

Nonlinear, time invariant, causal, stable

3.32

(a)

$y(n) = x(n)x(n-1)$

(1)

Let $x_1(n) = \text{input}$, $y_1(n) = x_1(n)x_1(n-1)$;

Let $x_2(n) = \text{input}$, $y_2(n) = x_2(n)x_2(n-1)$;

Let $x_3(n) = \alpha x_1(n) + \beta x_2(n)$,

$y_3(n) = x_3(n)x_3(n-1) = [\alpha x_1(n) + \beta x_2(n)][\alpha x_1(n-1) + \beta x_2(n-1)]$;

Check $\alpha y_1(n) + \beta y_2(n) = \alpha x_1(n)x_1(n-1) + \beta x_2(n)x_2(n-1) \neq y_3(n)$

The system is nonlinear.

(2)

Let $x_1(n) = \text{input}$, $y_1(n) = x_1(n)x_1(n-1)$;

Let $x_2(n) = x_1(n - n_0)$, $y_2(n) = x_2(n)x_2(n-1) = x_1(n - n_0)x_1(n - 1 - n_0)$;

Check $y_1(n - n_0) = x_1(n - n_0)x_1(n - n_0 - 1) = y_2(n)$;

The system is time invariant.

(3)

Since $y(n)$ depending on the current input and past input, the system is causal

(4)

Let $|x(n)| = M < \infty$, then $|y(n)| = |x(n)x(n-1)| = |x(n)||x(n-1)| < M^2 < \infty$

The system is BIBO stable

Nonlinear, time invariant, causal stable

(b)

$$y(n) = x(n) - 0.2x(n-1)y(n-2)$$

(1)

Let $x_1(n) = \text{input}$, $y_1(n) = x_1(n) - 0.2x_1(n-1)y_1(n-2)$;Let $x_2(n) = \text{input}$, $y_2(n) = x_2(n) - 0.2x_2(n-1)y_2(n-2)$;Let $x_3(n) = \alpha x_1(n) + \beta x_2(n)$,

$$y_3(n) = x_3(n) - 0.2x_3(n-1)y_3(n-2) = \alpha x_1(n) + \beta x_2(n) - 0.2[\alpha x_1(n-1) + \beta x_2(n-1)]y_3(n-2)$$

Check

$$\begin{aligned} \alpha y_1(n) + \beta y_2(n) &= \alpha x_1(n) - \alpha 0.2x_1(n-1)y_1(n-2) + \beta x_2(n) - \beta 0.2x_2(n-1)y_2(n-2) \\ &= \alpha x_1(n) + \beta x_2(n) - \alpha 0.2x_1(n-1)y_1(n-2) - \beta 0.2x_2(n-1)y_2(n-2) \neq y_3(n) \end{aligned}$$

The system is nonlinear.

(2)

Let $x_1(n) = \text{input}$, $y_1(n) = x_1(n) - 0.2x_1(n-1)y_1(n-2)$;Let $x_2(n) = x_1(n-n_0)$, $y_2(n) = x_1(n-n_0) - 0.2x_1(n-n_0-1)y_2(n-2)$, Eqn (1)Check $y_1(n-n_0) = x_1(n-n_0) - 0.2x_1(n-n_0-1)y_1(n-n_0-2)$, Eqn (2)If $y_2(n) = y_1(n-n_0)$, Eqn 1 and Eqn 2 are the same.

The system is time invariant.

(3)

Since $y(n)$ depending on the current input, past input and past output, the system is causal

(4)

Let $|x(n)| = M < \infty$, then

$$|y(n)| = |x(n) - 0.2x(n-1)y(n-2)| < |x(n)| + 0.2|x(n-1)||y(n-2)| = M + 0.2M|y(n-2)|$$

$$|y(0)| < M + 0.2M|y(-2)| = M$$

$$|y(1)| < M + 0.2M|y(-1)| = M$$

$$|y(2)| < M + 0.2M|y(0)| = M + 0.2M^2$$

$$|y(3)| < M + 0.2M|y(1)| = M + 0.2M^2$$

$$|y(4)| < M + 0.2M|y(2)| = M + 0.2M(M + 0.2M) = M + 0.2M^2 + 0.2^2M^3$$

$$|y(5)| < M + 0.2M|y(3)| = M + 0.2M(M + 0.2M) = M + 0.2M^2 + 0.2^2M^3$$

$$|y(6)| < M + 0.2M|y(4)| = M + 0.2M[M + 0.2M^2 + 0.2^2M^3] = M + 0.2M^2 + 0.2^2M^3 + 0.2^3M^4]$$

.....

The system is not BIBO stable!

But when $|x(n)| < M = 1 < \infty$, $|y(n)| < 1 + 0.2 + 0.2^2 + \dots = 1/(1-0.2) = 1.25$ for $n \rightarrow \infty$,

then the system is stable.

Nonlinear, time invariant causal, not BIBO stable

3.33

$$(a) \quad r=1, \quad \sum_{k=M}^N r^k = \sum_{k=M}^N 1^k = \sum_{k=M}^N 1 = N - M + 1$$

$$r \neq 1, \quad \sum_{k=M}^N r^k = \sum_{n=0}^{N-M} r^{n+M} = r^M \sum_{n=0}^{N-M} r^n = r^M \frac{1-r^{N-M+1}}{1-r} = \frac{r^M - r^{N+1}}{1-r}$$

$$(b) \quad \text{when } |r| < 1, \quad M=0, \quad N \rightarrow \infty, \quad \text{from (a)} \quad \sum_{k=0}^{\infty} r^k = \frac{r^M - r^{N+1}}{1-r} = \frac{1}{1-r}$$

3.34

$$(a) \quad y(n) = ay(n-1) + x(n) = ay(n-1) + \delta(n)$$

$$n=0, \quad y(0) = ay(-1) + \delta(0) = a \times 0 + 1 = 1 = h(0)$$

$$n=1, \quad y(1) = ay(0) + \delta(1) = a \times 1 + 0 = a = h(1)$$

$$n=2, \quad y(2) = ay(1) + \delta(2) = a \times a + 0 = a^2 = h(2)$$

....

$$h(n) = a^n u(n)$$

(b)

$$\text{Try: } y(n) = a^2 y(n-2) + x(n) = a^2 y(n-2) + \delta(n)$$

$$n=0, \quad y(0) = a^2 y(-2) + \delta(0) = a^2 \times 0 + 1 = 1 = h(0)$$

$$n=1, \quad y(1) = a^2 y(-1) + \delta(1) = a^2 \times 0 + 0 = 0 = h(1)$$

$$n=2, \quad y(2) = a^2 y(0) + \delta(2) = a^2 \times 1 + 0 = a^2 = h(2)$$

$$n=3, \quad y(3) = a^2 y(1) + \delta(3) = a^2 \times 0 + 0 = 0 = h(3)$$

$$n=4, \quad y(4) = a^2 y(2) + \delta(4) = a^2 \times a^2 + 0 = a^4 = h(4)$$

....

We can predict $y(n) - a^2 y(n-2) = x(n)$.

3.35

$$y(n) = \frac{n}{n+1} y(n-1) + \frac{1}{n+1} \delta(n)$$

$$n=0, \quad y(0) = 0y(-1) + 1\delta(0) = 1 = h(0)$$

$$n=1, \quad y(1) = (1/2)y(0) + (1/2)\delta(1) = 1/2 = h(1)$$

$$n=2, \quad y(2) = (2/3)y(1) + (1/3)\delta(2) = (2/3) \times (1/2) = 1/3 = h(2)$$

$$n=3, \quad y(3) = (3/4)y(2) + (1/4)\delta(3) = (3/4) \times (1/3) = 1/4 = h(3)$$

....

We can predict $h(n) = 1/(n+1)$

3.36

(a)

$$y(n) = x(n) * h(n) = 0, \quad n < 0$$

$$y(n) = x(n) * h(n) = \sum_{k=0}^n a^k = \frac{1-a^{n+1}}{1-a}, \quad n \geq 0$$

(b)

$$n < 0, y(n) = h(n) * x(-n) = \sum_{k=-\infty}^{-|n|} a^{-k} = \sum_{m=0}^{\infty} a^{m-|n|} = a^{-|n|} \frac{1}{1-a} = \frac{a^n}{1-a}$$

$$n \geq 0, y(n) = h(n) * x(-n) = \sum_{k=-\infty}^0 a^{-k} = \sum_{m=0}^{\infty} a^m = \frac{1}{1-a}$$

3.37

$$\text{Let } m = n - k, y(n) = \sum_{k=-\infty}^n x(k)h(n-k) = \sum_{m=-\infty}^0 x(n-m)h(m) = \sum_{k=0}^{\infty} h(k)x(n-k)$$

3.38

$$x(k) = 0 \text{ for } k < 0, y(n) = \sum_{k=0}^{\infty} x(k)h(n-k),$$

$$\text{Let } m = n - k, y(n) = \sum_{m=-\infty}^{\infty} x(n-m)h(m) = \sum_{k=-\infty}^n x(n-k)h(k)$$

$$\text{Since } h(k) = 0 \text{ for } k < 0, \text{ then } y(n) = \sum_{k=0}^n h(k)x(n-k)$$

$$\text{Let } m = n - k, y(n) = \sum_{m=n}^0 x(m)h(n-m) = \sum_{k=0}^n x(k)h(n-k)$$

Chapter 4

4.1

$$X(0) = 1, X(1) = 2 - j, X(2) = -1, X(3) = 2 + j$$

4.2

$$X(0) = 10, X(1) = 2 - 2j, X(2) = 2, X(3) = 2 + 2j$$

4.3

$$X = [0.2000 \quad 0.4000 - 0.2000i \quad -0.2000 \quad 0.4000 + 0.2000i]$$

4.4

$$X = [2.0000 \quad 0.4000 - 0.4000i \quad 0.4000 \quad 0.4000 + 0.4000i]$$

4.5

$$\text{From 4.2: } X(0) = 10, X(1) = 2 - 2j, X(2) = 2, X(3) = 2 + 2j$$

$$x(0) = 4, x(1) = 3, x(2) = 2, x(3) = 1$$

4.6

$$X(0) = 10, X(1) = 3.5 - 4.3301j, X(2) = 2.5 - 0.8660j, X(3) = 2,$$

$$X(4) = 2.5 + 0.8660j, X(5) = 3.5 + 4.3301j$$

4.7

$$x = [0.8000 \quad 0.6000 \quad 0.4000 \quad 0.2000]$$

4.8

$$X = [2.00 \quad 0.70 - 0.866i \quad 0.50 - 0.1732i \quad 0.40 \quad 0.50 + 0.1732i \quad 0.70 + 0.866i]$$

4.9

From 4.4: $X(0) = 10$, $X(1) = 3.5 - 4.3301j$, $X(2) = 2.5 - 0.8660j$, $X(3) = 2$,
 $X(4) = 2.5 + 0.8660j$, $X(5) = 3.5 + 4.3301j$
 $\bar{x}(0) = 4$, $\bar{x}(4) = 0$

4.10

$$\Delta f = 2.5 \text{ Hz and } f_{\max} = 10 \text{ kHz}$$

4.11

$$\bar{x}(0) = 0.8, \bar{x}(4) = 0$$

4.12

$$N = 4000, \Delta f = 4 \text{ Hz}, f_{\max} = f_s / 2 = 8 \text{ kHz}$$

4.13

$$N = 4096, \Delta f = 0.488 \text{ Hz}$$

4.14

$$X(0) = 6, X(1) = 2 + 2j, X(2) = 6, X(3) = 2 - 2j$$

f Hz	0	25	50	75
A_k	1.5	0.707	1.5	0.707
P_k	2.25	0.5	2.25	0.5
ϕ_k degree	0	45	0	-45

4.15

a. $w = [0.0800 \quad 0.2532 \quad 0.6424 \quad 0.9544 \quad 0.9544 \quad 0.6424 \quad 0.2532 \quad 0.0800]$

b. $w = [0 \quad 0.1883 \quad 0.6113 \quad 0.9505 \quad 0.9505 \quad 0.6113 \quad 0.1883 \quad 0]$

4.16

a. $xw = [0 \quad 0.4000 \quad 0 \quad -0.8000 \quad 0 \quad 0]$

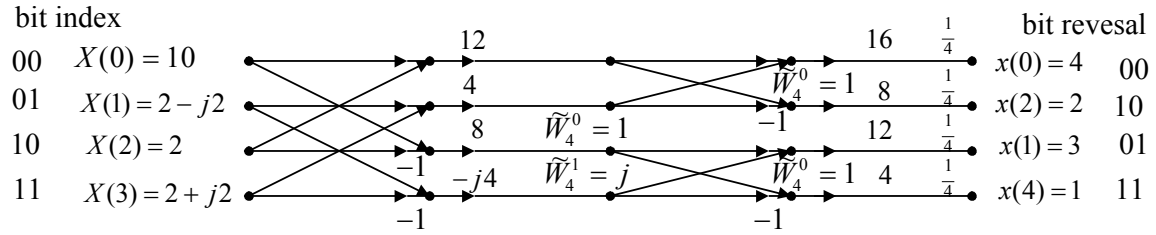
b. $xw = [0 \quad 0.3979 \quad 0 \quad -0.9121 \quad 0 \quad 0.0800]$

c. $xw = [0 \quad 0.3455 \quad 0 \quad -0.9045 \quad 0 \quad 0]$

4.17

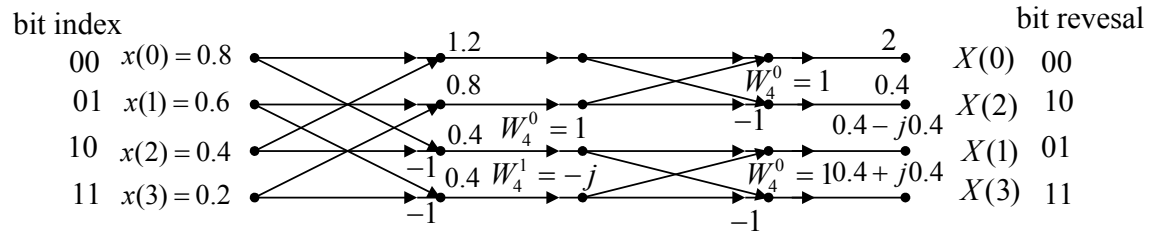
a.

$$w = [0.0800 \quad 0.1876 \quad 0.4601 \quad 0.7700 \quad 0.9723 \quad 0.9723 \quad 0.7700 \quad 0.4601 \quad 0.1876 \quad 0.0800]$$



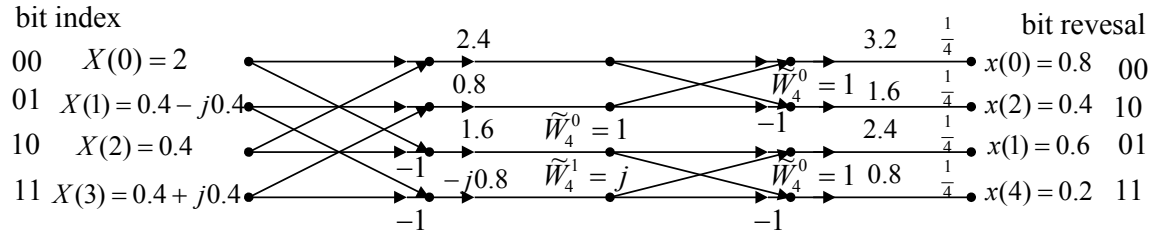
$x(0) = 4, x(1) = 3, x(2) = 2, x(3) = 1$, 4 complex multiplications

4.23



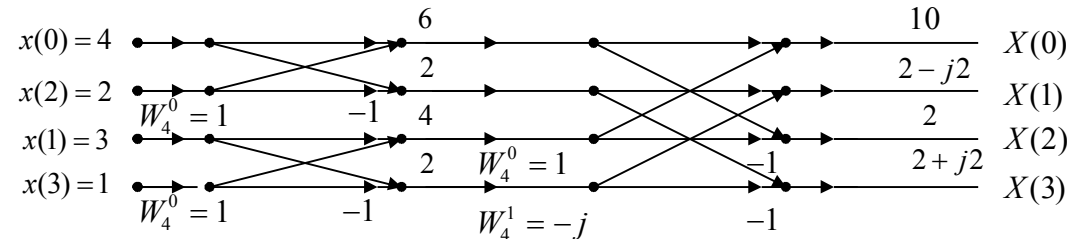
$X(0) = 2, X(1) = 0.4 - 0.4j, X(2) = 0.4, X(3) = 0.4 + 0.4j$, 4 complex multiplications

4.24



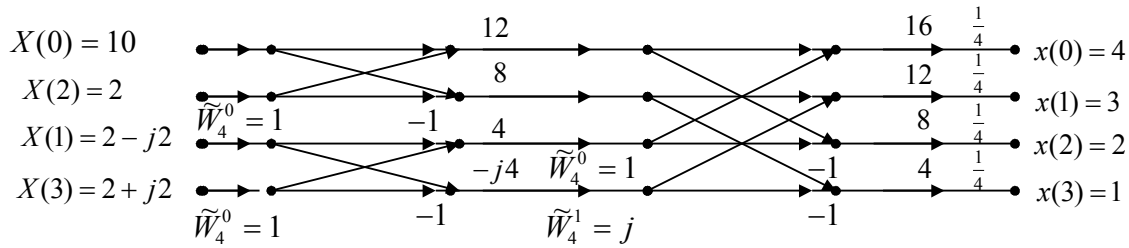
$x(0) = 0.8, x(1) = 0.6, x(2) = 0.4, x(3) = 0.2$, 4 complex multiplications

4.25



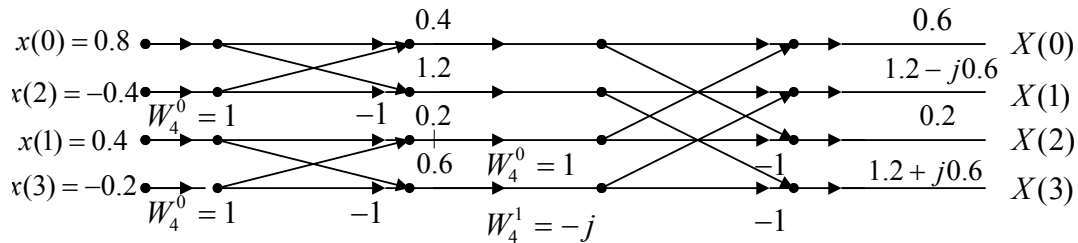
$X(0) = 10, X(1) = 2 - 2j, X(2) = 2, X(3) = 2 + 2j$, 4 complex multiplications

4.26



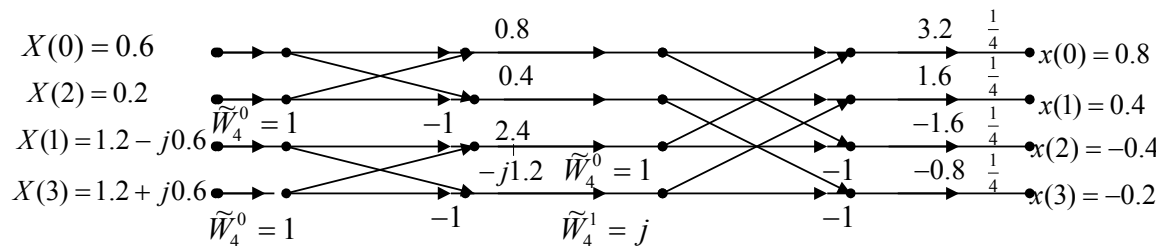
$x(0) = 4$, $x(1) = 3$, $x(2) = 2$, $x(3) = 1$, 4 complex multiplications

4.27



$X(0) = 0.6$, $X(1) = 1.2 - j0.6$, $X(2) = 0.2$, $X(3) = 1.2 + j0.6$, 4 complex multiplications

4.28



$x(0) = 0.8$, $x(1) = 0.4$, $x(2) = -0.4$, $x(3) = -0.2$, 4 complex multiplications

4.29

a.

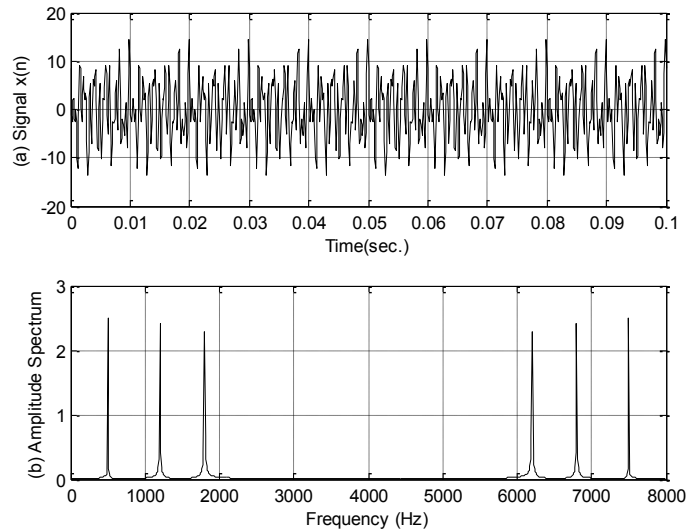
```

% generate the sine wave sequence
fs=8000; T=1/fs;           % Sampling rate and sampling period
t=0:T:0.1;
x1=5*cos(2*pi*500*t);
x2=5*cos(2*pi*1200*t+0.25*pi);
x3=5*cos(2*pi*1800*t+0.5*pi);
x=x1+x2+x3;
% apply the FFT algorithm
N=length(x);
index_t=[0:1:N-1];

```

```
f=[0:1:N-1]*fs/N;    %Map frequency bin to frequency (Hz)
Ak=abs(fft(x))/N;     %Calculate amplitude spectrum
subplot(2,1,1);plot(t,x);grid
xlabel('Time(sec.)');ylabel('(a) Signal x(n)');
subplot(2,1,2);plot(f,Ak);grid
xlabel('Frequency (Hz)');ylabel('(b) Amplitude Spectrum');
```

b.



4.30

a.

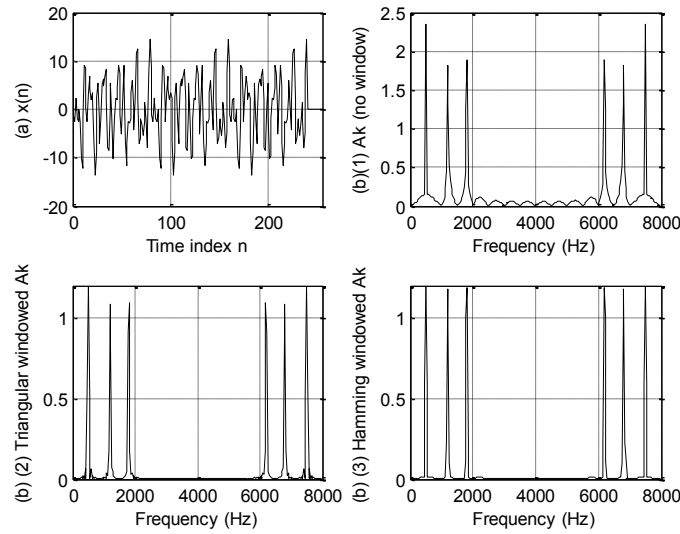
```
close all;clear all
% generate the sine wave sequence
fs=8000; T=1/fs;          % Sampling rate and sampling period
N=240;
t=0:T:(N-1)*T;
x1=5*cos(2*pi*500*t);
x2=5*cos(2*pi*1200*t+0.25*pi);
x3=5*cos(2*pi*1800*t+0.5*pi);
x=x1+x2+x3;
% apply the FFT algorithm with zero padding
x=[x,zeros(1,16)];
N=length(x);
index_t=[0:1:N-1];
f=[0:1:N-1]*fs/N;    %Map frequency bin to frequency (Hz)
xf=abs(fft(x))/N;    %Calculate amplitude spectrum
%using Bartlett window
x_b=x.*bartlett(N);    %Apply triangular window function
xf_b=abs(fft(x_b))/N; %Calculate amplitude spectrum
subplot(2,2,1);plot(index_t,x);grid
xlabel('Time index n'); ylabel('(a) x(n)');axis([0 255 -20 20]);
subplot(2,2,2);plot(f,xf);grid;axis([0 8000 0 2.5]);
xlabel('Frequency (Hz)'); ylabel('(b) (1) Ak (no window)');
subplot(2,2,3);plot(f,xf_b);grid; axis([0 8000 0 1.2]);
xlabel('Frequency (Hz)'); ylabel('(b) (2) Triangular windowed Ak');
%using Hamming window
```

```

x_hm=x.*hamming(N);    %Apply Hamming window function
xf_hm=abs(fft(x_hm))/N; %Calculate amplitude spectrum
subplot(2,2,4); plot(f,xf_hm);grid;axis([0 fs 0 1.2]);
xlabel('Frequency (Hz)'); ylabel('(b) (3) Hamming windowed Ak');

```

b.



c. Hamming window has least spectral leakage.

4.33

$$(a) W_N^N = e^{-j(2\pi/N)N} = e^{-j2\pi} = 1 \quad (b) W_N^{N/2} = e^{-j(2\pi/N)(N/2)} = e^{-j\pi} = -1$$

$$(c) W_N^{N/4} = e^{-j(2\pi/N)(N/4)} = e^{-j2\pi/4} = -j$$

$$(d) W_N^{-m} = e^{-j(2\pi/N)(-m)} = e^{j2\pi m/N} \times 1 = e^{j2\pi m/N} \times e^{-j2\pi N/N} = e^{-j2\pi(N-m)/N} = W_N^{N-m}$$

4.34

$$\text{Since } X(k) = \sum_{n=0}^{N-1} x(n)W_N^{nk},$$

$$X(N-k) = \sum_{n=0}^{N-1} x(n)W_N^{n(N-k)} = \sum_{n=0}^{N-1} x(n)W_N^{nN}W_N^{-nk} = \sum_{n=0}^{N-1} x(n)W_N^{-nk} = \left[\sum_{n=0}^{N-1} x(n)W_N^{nk} \right]^* = X^*(k)$$

4.35

$$x(n) = \delta(n-m)$$

$$X(k) = \sum_{n=0}^{N-1} x(n)W_N^{nk} = \sum_{n=0}^{N-1} \delta(n-m)W_N^{nk} = W_N^{km}$$

4.36

$$x(n) = 1$$

$$X(k) = \sum_{n=0}^{N-1} x(n)W_N^{nk} = \sum_{n=0}^{N-1} W_N^{nk} = \sum_{n=0}^{N-1} e^{(-j2\pi k/N)n} = \begin{cases} \sum_{n=0}^{N-1} 1 = N & k = 0 \\ \frac{1 - e^{(-j2\pi k/N)N}}{1 - e^{-j2\pi k/N}} = 0 & k \neq 0 \end{cases}$$

4.37

$$X(k) = \sum_{n=0}^{N-1} x(n)W_N^{nk} = \sum_{n=0, \text{even}}^{N-1} W_N^{nk} = \sum_{n=0, \text{even}}^{N-1} e^{(-j2\pi k/N)n}$$

$$\text{If } k = 0, X(k) = \sum_{n=0, \text{even}}^{N-1} e^{(-j2\pi k/N)n} = \sum_{n=0, \text{even}}^{N-1} 1 = N/2$$

$$\text{If } k = N/2, X(k) = \sum_{n=0, \text{even}}^{N-1} e^{(-j2\pi k/N)n} = \sum_{n=0, \text{even}}^{N-1} e^{-j\pi n} = \sum_{n=0, \text{even}}^{N-1} 1 = N/2$$

Else

$$X(k) = \sum_{m=0}^{N/2-1} e^{(-j4\pi k/N)m} = \frac{1 - e^{(-j4\pi k/N)(N/2)}}{1 - e^{(-j4\pi k/N)}} = 0$$

4.38

$$x(n) = 1/2 - (1/4)e^{j(2\pi n/N)} - (1/4)e^{-j(2\pi n/N)}$$

$$X(k) = \sum_{n=0}^{N-1} x(n)W_N^{nk} = (1/2) \sum_{n=0}^{N-1} e^{(-j2\pi k/N)n} - (1/4) \sum_{n=0}^{N-1} e^{(-j2\pi(k-1)/N)n} - (1/4) \sum_{n=0}^{N-1} e^{(-j2\pi(k+1)/N)n}$$

Use results in 4.36

$$k = 0, X(k) = (1/2) \sum_{n=0}^{N-1} e^{(-j2\pi k/N)n} - 0 - 0 = N/2$$

$$k = 1, X(k) = 0 - (1/4) \sum_{n=0}^{N-1} e^{(-j2\pi(k-1)/N)n} - 0 = -N/4$$

$$k = -1, X(k) = 0 - (1/4) \times 0 - (1/4) \sum_{n=0}^{N-1} e^{(-j2\pi(k+1)/N)n} = -N/4$$

Elsewhere

$$X(k) = 0$$

4.39

$$x(n) = n$$

Note :

$$S = 0 + 1 + 2 + 3 + \dots = N(N-1)/2;$$

$$S = 0 + r + 2r^2 + 3r^3 + \dots \quad |)r^{N-1} = \frac{r - r^N}{(1-r)^2} - \frac{(N-1)r^N}{1-r}$$

$$k = 0, X(k) = \sum_{n=0}^{N-1} x(n)W_N^{nk} = \sum_{n=0}^{N-1} ne^{(-j2\pi k/N)n} = \sum_{n=0}^{N-1} n = N(N-1)/2$$

$$k \neq 0, X(k) = \sum_{n=0}^{N-1} n(W_N^k)^n = \frac{W_N^k - W_N^{kN}}{(1 - W_N^k)^2} - \frac{(N-1)W_N^{kN}}{(1 - W_N^k)} = \frac{W_N^k - 1}{(1 - W_N^k)^2} - \frac{(N-1)}{(1 - W_N^k)} = \frac{-N}{(1 - W_N^k)}$$

4.40

$$\cos\left[\frac{\pi k(2n+1)}{2N}\right] \cos\left[\frac{\pi m(2n+1)}{2N}\right] = \frac{e^{j\pi k(2n+1)/(2N)} + e^{-j\pi k(2n+1)/(2N)}}{2} \times \frac{e^{j\pi m(2n+1)/(2N)} + e^{-j\pi m(2n+1)/(2N)}}{2}$$

$$= (1/4)e^{j\pi(k+m)(2n+1)/(2N)} + (1/4)e^{j\pi(k-m)(2n+1)/(2N)} + (1/4)e^{-j\pi(k-m)(2n+1)/(2N)} + (1/4)e^{-j\pi(k+m)(2n+1)/(2N)}$$

$$k = m = 0, \frac{1}{N} \sum_{n=0}^{N-1} \cos\left[\frac{\pi k(2n+1)}{2N}\right] \cos\left[\frac{\pi m(2n+1)}{2N}\right] = \frac{1}{N} \sum_{n=0}^{N-1} (1/4) + (1/4) + (1/4) + (1/4) = 1$$

 $k = m \neq 0$

$$\frac{1}{N} \sum_{n=0}^{N-1} \cos\left[\frac{\pi k(2n+1)}{2N}\right] \cos\left[\frac{\pi m(2n+1)}{2N}\right]$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} (1/4)e^{j2\pi k(2n+1)/(2N)} + \frac{1}{N} \sum_{n=0}^{N-1} 1/4 + \frac{1}{N} \sum_{n=0}^{N-1} 1/4 + \frac{1}{N} \sum_{n=0}^{N-1} e^{-j2\pi k(2n+1)/(2N)}$$

$$= \frac{1}{N} e^{j\pi k/N} \sum_{n=0}^{N-1} (1/4)e^{j2\pi kn/N} + 1/4 + 1/4 + \frac{1}{N} e^{-j\pi k/N} \sum_{n=0}^{N-1} e^{-j2\pi kn/N} = 1/2$$

Else

$$A = \frac{1}{N} \sum_{n=0}^{N-1} (1/4)e^{j\pi(k+m)(2n+1)/(2N)} = \frac{e^{j\pi(k+m)/(2N)}}{4N} \sum_{n=0}^{N-1} e^{j\pi(k+m)n/N} = \frac{e^{j\pi(k+m)/(2N)}}{4N} \frac{1 - e^{j\pi(k+m)N/N}}{1 - e^{j\pi(k+m)/N}}$$

$$B = \frac{1}{N} \sum_{n=0}^{N-1} (1/4)e^{j\pi(k-m)(2n+1)/(2N)} = \frac{e^{j\pi(k-m)/(2N)}}{4N} \sum_{n=0}^{N-1} e^{j\pi(k-m)n/N} = \frac{e^{j\pi(k-m)/(2N)}}{4N} \frac{1 - e^{j\pi(k-m)N/N}}{1 - e^{j\pi(k-m)/N}}$$

$$C = \frac{1}{N} \sum_{n=0}^{N-1} (1/4)e^{-j\pi(k-m)(2n+1)/(2N)} = \frac{e^{-j\pi(k-m)/(2N)}}{4N} \sum_{n=0}^{N-1} e^{-j\pi(k-m)n/N}$$

$$= \frac{e^{-j\pi(k-m)/(2N)}}{4N} \frac{1 - e^{-j\pi(k-m)N/N}}{1 - e^{-j\pi(k-m)/N}} = \frac{e^{-j\pi(k-m)/(2N)}}{4Ne^{-j\pi(k-m)/N}} \frac{1 - e^{-j\pi(k-m)}}{e^{j\pi(k-m)/N} - 1} = -\frac{e^{j\pi(k-m)/(2N)}}{4N} \frac{1 - e^{-j\pi(k-m)}}{1 - e^{j\pi(k-m)/N}}$$

$$D = \frac{1}{N} \sum_{n=0}^{N-1} (1/4)e^{-j\pi(k+m)(2n+1)/(2N)} = \frac{e^{-j\pi(k+m)/(2N)}}{4N} \sum_{n=0}^{N-1} e^{-j\pi(k+m)n/N}$$

$$= \frac{e^{-j\pi(k+m)/(2N)}}{4N} \frac{1 - e^{-j\pi(k+m)N/N}}{1 - e^{-j\pi(k+m)/N}} = \frac{e^{-j\pi(k+m)/(2N)}}{4Ne^{-j\pi(k+m)/N}} \frac{1 - e^{-j\pi(k+m)}}{e^{j\pi(k+m)/N} - 1} = -\frac{e^{j\pi(k+m)/(2N)}}{4N} \frac{1 - e^{-j\pi(k+m)}}{1 - e^{j\pi(k+m)/N}}$$

$$A + D = \frac{e^{j\pi(k+m)/(2N)}}{4N} \frac{1 - e^{j\pi(k-m)} - 1 + e^{-j\pi(k-m)}}{1 - e^{j\pi(k+m)/N}} = \frac{e^{j\pi(k+m)/(2N)}}{4N} \frac{-2j \sin[\pi(k-m)]}{1 - e^{j\pi(k+m)/N}} = 0$$

$$B + C = \frac{e^{j\pi(k-m)/(2N)}}{4N} \frac{1 - e^{j\pi(k-m)} - 1 + e^{-j\pi(k-m)}}{1 - e^{j\pi(k-m)/N}} = -\frac{e^{j\pi(k-m)/(2N)}}{4N} \frac{2j \sin[2\pi(k-m)]}{1 - e^{j\pi(k-m)/N}} = 0$$

Thus $A + B + C + D = 0$

4.41

Assuming the following is true,

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} a(k) X_{DCT}(k) \cos \left[\frac{\pi k(2n+1)}{2N} \right]$$

we can verify the $X_{DCT}(k)$.

$$\begin{aligned} \sum_{n=0}^{N-1} 2x(n) \cos \left[\frac{\pi m(2n+1)}{2N} \right] &= \sum_{n=0}^{N-1} 2 \frac{1}{N} \sum_{k=0}^{N-1} a(k) X_{DCT}(k) \cos \left[\frac{\pi k(2n+1)}{2N} \right] \left[\frac{\pi m(2n+1)}{2N} \right] \\ \sum_{n=0}^{N-1} 2x(n) \cos \left[\frac{\pi k(2m+1)}{2N} \right] &= \sum_{k=0}^{N-1} 2a(k) X_{DCT}(k) \frac{1}{N} \sum_{n=0}^{N-1} \cos \left[\frac{\pi k(2n+1)}{2N} \right] \left[\frac{\pi m(2n+1)}{2N} \right] \end{aligned}$$

Using results in Problem 4.41

If $k = m = 0$

$$\sum_{k=0}^{N-1} 2a(k) X_{DCT}(k) \frac{1}{N} \sum_{n=0}^{N-1} \cos \left[\frac{\pi k(2n+1)}{2N} \right] \left[\frac{\pi m(2n+1)}{2N} \right] = 2a(0) X_{DCT}(0) \times 1 = X_{DCT}(0)$$

That is,

$$\sum_{n=0}^{N-1} 2x(n) \cos \left[\frac{\pi m(2n+1)}{2N} \right] = X_{DCT}(0)$$

If $k = m \neq 0$

$$\sum_{k=0}^{N-1} 2a(k) X_{DCT}(k) \frac{1}{N} \sum_{n=0}^{N-1} \cos \left[\frac{\pi k(2n+1)}{2N} \right] \left[\frac{\pi m(2n+1)}{2N} \right] = 2a(m) X_{DCT}(m) \times \frac{1}{2} = X_{DCT}(m)$$

Thus, we prove

$$\sum_{n=0}^{N-1} 2x(n) \cos \left[\frac{\pi k(2m+1)}{2N} \right] = X_{DCT}(m)$$

for given

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} a(k) X_{DCT}(k) \cos \left[\frac{\pi k(2n+1)}{2N} \right]$$

4.42

$$X_{DHT}(k) = \frac{1}{N} \sum_{n=0}^{N-1} x(n) \left[\cos \left(\frac{2\pi nk}{N} \right) + \cos \left(\frac{2\pi nk}{N} \right) \right]$$

$$\begin{aligned} &\frac{1}{N} \sum_{k=0}^{N-1} X_{DHT}(k) \left[\cos \left(\frac{2\pi mk}{N} \right) + \sin \left(\frac{2\pi mk}{N} \right) \right] \\ &= \frac{1}{N} \sum_{k=0}^{N-1} \sum_{n=0}^{N-1} x(n) \left[\cos \left(\frac{2\pi nk}{N} \right) + \sin \left(\frac{2\pi nk}{N} \right) \right] \left[\cos \left(\frac{2\pi mk}{N} \right) + \sin \left(\frac{2\pi mk}{N} \right) \right] \end{aligned}$$

Changing the summation order leads to

$$\begin{aligned}
& \frac{1}{N} \sum_{k=0}^{N-1} X_{DHT}(k) \left[\cos\left(\frac{2\pi mk}{N}\right) + \sin\left(\frac{2\pi mk}{N}\right) \right] \\
&= \frac{1}{N} \sum_{n=0}^{N-1} x(n) \sum_{k=0}^{N-1} \left[\cos\left(\frac{2\pi nk}{N}\right) + \sin\left(\frac{2\pi nk}{N}\right) \right] \left[\cos\left(\frac{2\pi mk}{N}\right) + \sin\left(\frac{2\pi mk}{N}\right) \right] \\
& \frac{1}{N} \sum_{k=0}^{N-1} X_{DHT}(k) \left[\cos\left(\frac{2\pi mk}{N}\right) + \sin\left(\frac{2\pi mk}{N}\right) \right] \\
&= \frac{1}{N} \sum_{n=0}^{N-1} x(n) \sum_{k=0}^{N-1} \left[\cos\left(\frac{2\pi nk}{N}\right) \cos\left(\frac{2\pi mk}{N}\right) + \sin\left(\frac{2\pi nk}{N}\right) \cos\left(\frac{2\pi mk}{N}\right) \right. \\
& \quad \left. + \cos\left(\frac{2\pi nk}{N}\right) \sin\left(\frac{2\pi mk}{N}\right) + \sin\left(\frac{2\pi nk}{N}\right) \sin\left(\frac{2\pi mk}{N}\right) \right] \\
& \frac{1}{N} \sum_{k=0}^{N-1} X_{DHT}(k) \left[\cos\left(\frac{2\pi mk}{N}\right) + \sin\left(\frac{2\pi mk}{N}\right) \right] \\
&= \frac{1}{N} \sum_{n=0}^{N-1} \frac{1}{2} x(n) \sum_{k=0}^{N-1} \left[\cos\left(\frac{2\pi(n+m)k}{N}\right) + \cos\left(\frac{2\pi(n-m)k}{N}\right) + \sin\left(\frac{2\pi(n+m)k}{N}\right) + \sin\left(\frac{2\pi(n-m)k}{N}\right) \right. \\
& \quad \left. + \sin\left(\frac{2\pi(n+m)k}{N}\right) - \sin\left(\frac{2\pi(n-m)k}{N}\right) - \cos\left(\frac{2\pi(n+m)k}{N}\right) + \cos\left(\frac{2\pi(n-m)k}{N}\right) \right] \\
& \frac{1}{N} \sum_{k=0}^{N-1} X_{DHT}(k) \left[\cos\left(\frac{2\pi mk}{N}\right) + \sin\left(\frac{2\pi mk}{N}\right) \right] \\
&= \frac{1}{N} \sum_{n=0}^{N-1} x(n) \sum_{k=0}^{N-1} \left[\cos\left(\frac{2\pi(n-m)k}{N}\right) + \sin\left(\frac{2\pi(n+m)k}{N}\right) \right] \\
& \frac{1}{N} \sum_{k=0}^{N-1} X_{DHT}(k) \left[\cos\left(\frac{2\pi mk}{N}\right) + \sin\left(\frac{2\pi mk}{N}\right) \right] \\
&= \frac{1}{N} \sum_{n=0}^{N-1} x(n) \frac{1}{2} \sum_{k=0}^{N-1} \left[e^{j\frac{2\pi(n-m)k}{N}} + e^{-j\frac{2\pi(n-m)k}{N}} + \frac{1}{j} e^{j\frac{2\pi(n+m)k}{N}} + \frac{1}{j} e^{-j\frac{2\pi(n+m)k}{N}} \right]
\end{aligned}$$

When $m = n$, the right summation exists

$$\frac{1}{N} \sum_{k=0}^{N-1} X_{DHT}(k) \left[\cos\left(\frac{2\pi mk}{N}\right) + \sin\left(\frac{2\pi mk}{N}\right) \right] = \frac{1}{N} x(m) \frac{1}{2} \sum_{k=1}^{N-1} (1+1) = \frac{1}{N} x(m) \frac{1}{2} 2N = x(m)$$

That is,

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_{DHT}(k) \left[\cos\left(\frac{2\pi nk}{N}\right) + \sin\left(\frac{2\pi nk}{N}\right) \right]$$

4.42

```

function Xk= ifftdinf(x)
% IFFT using decimation-in-frequency method
XX=fftdinf2(x);
k=bitrev(1:length(XX));

```

```

        Xk=XX(k);
        Xk=Xk/length(Xk);
end
function Xk = fftdinf2(x)
% IFFT using decimation-in-frequency method
M=ceil(log2(length(x)));
x=[x zeros(1,2^M-length(x))]; %padding zeros to have a length of power
of 2
N=length(x);
if (N==1)
    Xk=x;
else
    a=x(1:N/2)+x(N/2+1:N);
    b=x(1:N/2)-x(N/2+1:N);
    Xk=[fftdinf2(a) fftdinf2(b.*exp(2*pi*j*[0:1:N/2-1]/N))];
end
end
function k = bitrev(x)
% bit reversal in terms of the integer
%   x=1:1:2^M
N=length(x);
if N==1
    k=x;
else
    k=[bitrev(x(1:2:N)) bitrev(x(2:2:N))];
    N=N/2;
end
end
end
4.42

```

```

function Xk = ifftdint(x)
% IFFT using decimation-in-time method, no need for bit reversal
algorithm
M=ceil(log2(length(x)));
x=[x zeros(1,2^M-length(x))]; %padding zeros to have a length of
power of 2
N=length(x);
if (N==1)
    Xk=x;
else
    G=fftdint(x(1:2:N));
    H=fftdint(x(2:2:N)).*exp(2*pi*j*[0:1:N/2-1]/N);
    Xk=[G+H G-H];
end
Xk=Xk/length(Xk);
end

```

Chapter 5

5.1

a. $X(z) = \frac{4z}{z-1}$, b. $X(z) = \frac{z}{z+0.7}$

$$c. X(z) = \frac{4z}{z - e^{-2}} = \frac{4z}{z - 0.1353},$$

$$d. X(z) = \frac{4z[z - 0.8 \times \cos(0.1\pi)]}{z^2 - [2 \times 0.8z \cos(0.1\pi)] + 0.8^2} = \frac{4z(z - 0.7608)}{z^2 - 1.5217z + 0.64}$$

$$e. X(z) = \frac{4e^{-3} \sin(0.1\pi)z}{z^2 - 2e^{-3}z \cos(0.1\pi) + e^{-6}} = \frac{0.06154z}{z^2 - 0.0947z + 0.00248}$$

5.2

$$a. X(z) = \frac{z}{z-1} + \frac{z}{z-0.5}$$

$$b. X(z) = \frac{z^{-4}z[z - e^{-3} \cos(0.1\pi)]}{z^2 - [2e^{-3} \cos(0.1\pi)]z + e^{-6}} = \frac{z^{-3}(z - 0.0474)}{z^2 - 0.0948z + 0.0025}$$

5.3

$$a. X(z) = \frac{3z^{-3}}{z-1}, \quad b. X(z) = \frac{2z}{z+0.5}$$

$$c. X(z) = \frac{5z^{-2}}{z - e^{-2}}, \quad d. X(z) = \frac{6z[z - 0.6 \cos(0.2\pi)]}{z^2 - 1.2 \cos(0.2\pi)z + 0.6^2} = \frac{6z[z - 0.4854]}{z^2 - 0.9708z + 0.36}$$

$$e. X(z) = \frac{4e^{-3} \sin(0.2\pi)}{z^2 - 2e^{-3} \cos(0.2\pi)z + e^{-6}} = \frac{0.1171}{z^2 - 0.0806z + 0.0025}$$

5.4

$$a. X(z) = \frac{-z}{z-1} + \frac{z}{z-0.75}$$

$$b. x X(z) = \frac{e^{-2} \sin(0.3\pi)z}{z^2 - 2e^{-2} \cos(0.3\pi)z + e^{-2 \times 2}} z^{-3} = \frac{e^{-2} \sin(0.3\pi)z^{-2}}{z^2 - 2e^{-2} \cos(0.3\pi)z + e^{-4}}$$

5.5

$$a. X(z) = 15z^{-3} - 6z^{-5}$$

$$b. x(n) = 15\delta(n-3) - 6\delta(n-5)$$

5.6

$$a. x(n) = 4\delta(n) - 10u(n) - (-0.5)^n u(n) \quad b. x(n) = -5u(n) + 10nu(n) + 2.5n(0.8)^n u(n)$$

$$c. x(n) = 1.25 \sin(126.87^\circ n) u(n)$$

$$d. x(n) = 4u(n-5) + (n-2)u(n-2) + \delta(n-8) + (0.5)^{n-6} u(n-6)$$

5.7

$$a. X(z) = X_1(z)X_2(z) = (-2 + 5z^{-2})(4z^{-4}) = -8z^{-4} + 20z^{-6}$$

$$b. x(n) = -8\delta(n-2) + 20\delta(n-6)$$

5.8

$$a. x(n) = 5\delta(n) - 7(-1)^n u(n) - 3(0.5)^n u(n)$$

- b. $x(n) = -3(0.5)^n u(n) + 8(0.8)^n u(n) + 2.5n(0.8)^n u(n)$
 c. $x(n) = 4.2433 \sin(135^\circ n) u(n)$
 d. $x(n) = 5u(n-6) + (n-3)u(n-3) + \delta(n-10) + (0.75)^{n-4} u(n-4)$

5.9

a. $X(z) = -25 + \frac{5z}{z-0.4} + \frac{20z}{z+0.1}$, $x(n) = -25\delta(n) + 5(0.4)^n u(n) + 20(-0.1)^n u(n)$

b. $X(z) = \frac{1.6667z}{z-0.2} - \frac{1.6667z}{z+0.4}$, $x(n) = 1.6667(0.2)^n u(n) - 1.6667(-0.4)^n u(n)$

c. $X(z) = \frac{1.3514z}{z+0.2} + \frac{Az}{z-P} + \frac{A^*z}{z-P^*}$

where $P = 0.5 + 0.5j = 0.707 \angle 45^\circ$, and $A = 1.1625 \angle -125.54^\circ$

$x(n) = 1.3514(-0.2)^n u(n) + 2.325(0.707)^n \cos(45^\circ \times n - 125.54^\circ)$

d. $X(z) = \frac{4.4z}{z-0.6} + \frac{-0.4z}{z-0.1} + \frac{-1.2z}{(z-0.1)^2}$,

$x(n) = 4.4(0.6)^n u(n) - 0.4(0.1)^n u(n) - 12n(0.1)^n u(n)$

5.10

$Y(z) = \frac{-4.3333z}{z-0.5} + \frac{5.3333z}{z-0.8}$, $y(n) = -4.3333(0.5)^n u(n) + 5.3333(0.8)^n u(n)$

5.11

a. $X(z) = \frac{50}{3} - \frac{50z}{z-0.2} + \frac{(100/3)z}{z-0.2}$

$x(n) = \frac{50}{3} \delta(n) - 50(-0.2)^n u(n) + \frac{100}{3}(-0.3)^n u(n)$

b. $X(z) = \frac{-1.25z}{z+0.3} + \frac{1.25z}{z-0.5}$, $x(n) = -1.25(-0.3)^n u(n) + 1.25(0.5)^n u(n)$

c. $X(z) = \frac{16z}{z-0.75} + \frac{Az}{z-P} + \frac{A^*z}{z-P^*}$

where $P = 0.5 + j0.5 = 0.7071 \angle 45^\circ$, and $A = -8 + j4 = 8.9443 \angle 153.43^\circ$

$x(n) = 16(0.75)^n u(n) + 17.8886(0.7071)^n \cos(45^\circ \times n + 153.43^\circ)$

d. $X(z) = \frac{-2.4z}{z+0.8} + \frac{2.4z}{z-0.2} + \frac{-0.4z}{(z-0.2)^2}$,

$x(n) = -2.4(-0.8)^n u(n) + 2.4(0.2)^n u(n) - 2n(0.2)^n u(n)$

5.12

$Y(z) = \frac{3.8z^2 + 0.06z}{(z-0.3)(z+0.2)} = \frac{2.4}{z-0.3} + \frac{1.4}{z+0.2}$

$y(n) = 2.4(0.3)^n u(n) + 1.4(-0.2)^n u(n)$

5.13

$$Y(z) = \frac{9.84z}{z-0.2} + \frac{-29.46z}{z-0.3} + \frac{20z}{z-0.4}$$

$$y(n) = 9.84(0.2)^n u(n) - 29.46(0.3)^n u(n) + 20(0.4)^n u(n)$$

5.14

$$\text{a. } Y(z) = \frac{-4z}{z-0.2} + \frac{5z}{z-0.5}, \quad y(n) = -4(0.2)^n u(n) + 5(0.5)^n u(n)$$

$$\text{b. } Y(z) = \frac{5z}{z-1} + \frac{-5z}{z-0.5} + \frac{z}{z-0.2},$$

$$y(n) = 5u(n) - 5(0.5)^n u(n) + (0.2)^n u(n)$$

5.15

$$Y(z) = \frac{1.12z^3 + 0.28z^2 + 0.08z}{(z^2 - 0.6z + 0.08)(z-0.5)} = \frac{16.6667z}{z-0.6} + \frac{-18.5600z}{z-0.4} + \frac{3.0133z}{z-0.2}$$

$$y(n) = 16.6667(0.5)^n u(n) - 18.5600(0.4)^n u(n) + 3.0133(0.2)^n u(n)$$

5.16

$$\text{a. } Y(z) = \frac{z^2 + z}{z^2 - 0.6z + 0.25} = \frac{Az}{z-P} + \frac{A^*z}{z-P^*}$$

$$\text{where } P = 0.3 + j0.4 = 0.5 \angle 53.13^\circ, \text{ and } A = 0.5 - j1.625 = 1.7002 \angle -72.90^\circ$$

$$y(n) = 3.4004(0.5)^n \cos(53.13^\circ \times n - 72.90^\circ)$$

$$\text{b. } Y(z) = \frac{z^3 + z^2}{(z-1)(z^2 - 0.6z + 0.25)} = \frac{3.0768z}{z-1} + \frac{Az}{z-P} + \frac{A^*z}{z-P^*}$$

$$\text{where } P = 0.3 + j0.4 = 0.5 \angle 53.13^\circ, \text{ and } A = -1.0385 - j0.1827 = 1.0544 \angle -170.22^\circ$$

$$y(n) = 3.0768u(n) + 2.1088(0.5)^n \cos(53.13^\circ \times n - 170.22^\circ)$$

5.17

$$\text{a. } Y(z) = \frac{Az}{z-P} + \frac{A^*z}{z-P^*}, \quad P = 0.2 + 0.5j = 0.5385 \angle 68.20^\circ, \quad A = 0.8602 \angle -54.46^\circ$$

$$y(n) = 1.7204(0.5382)^n \cos(n \times 68.20^\circ - 54.46^\circ)$$

$$\text{b. } Y(z) = \frac{1.6854z}{z-1} + \frac{Az}{z-P} + \frac{A^*z}{z-P^*},$$

$$\text{where } P = 0.2 + 0.5j = 0.5385 \angle 68.20^\circ, \quad A = 0.4910 \angle -136.25^\circ$$

$$y(n) = 1.6845u(n) + 0.982(0.5382)^n \cos(n \times 68.20^\circ - 136.25^\circ)$$

5.18

$$\text{a. } Y(z) = \frac{z^2 + 0.3z}{z^2 - 0.2z + 0.17} = \frac{Az}{z-P} + \frac{A^*z}{z-P^*}$$

where $P = 0.1 + j0.4 = 0.4123\angle 75.96^\circ$, and $A = 0.5 - j0.5 = 0.7071\angle -45.00^\circ$

$$y(n) = 1.4142(0.4123)^n \cos(75.96^\circ \times n - 45.00^\circ)$$

$$\text{b. } Y(z) = \frac{z^3 + 0.3z^2}{(z-1)(z^2 - 0.2z + 0.17)} = \frac{1.3402z}{z-1} + \frac{Az}{z-P} + \frac{A^*z}{z-P^*}$$

where $P = 0.3 + j0.4 = 0.5\angle 53.13^\circ$, and $A = -0.1701 - j0.2423 = 0.2960\angle -125.07^\circ$

$$y(n) = 1.3402u(n) + 0.5920(0.5)^n \cos(53.13^\circ \times n - 125.07^\circ)$$

5.19

(a)

$$x(0) = \lim_{z \rightarrow \infty} X(z) = \lim_{z \rightarrow \infty} \frac{1}{z^2 + 0.5z + 0.06} = 0$$

$$x(\infty) = \lim_{z \rightarrow 1} (z-1)X(z) = \lim_{z \rightarrow 1} \frac{z-1}{z^2 + 0.5z + 0.06} = 0$$

(b)

$$x(0) = \lim_{z \rightarrow \infty} X(z) = \lim_{z \rightarrow \infty} \frac{z}{(z+0.3)(z-0.5)} = 0$$

$$x(\infty) = \lim_{z \rightarrow 1} (z-1)X(z) = \lim_{z \rightarrow 1} \frac{(z-1)z}{(z+0.3)(z-0.5)} = 0$$

(d)

$$x(0) = \lim_{z \rightarrow \infty} X(z) = \lim_{z \rightarrow \infty} \frac{2z(z-0.4)}{(z-0.2)^2(z+0.8)} = 0$$

$$x(\infty) = \lim_{z \rightarrow 1} (z-1)X(z) = \lim_{z \rightarrow 1} \frac{2(z-1)z(z-0.4)}{(z-0.2)^2(z+0.8)} = 0$$

5.20

(a)

$$\begin{array}{r} z^{-2} - 0.5z^{-3} + 0.19z^{-4} \\ z^2 + 0.5z + 0.06 \overline{) 1} \\ \underline{1 + 0.5z^{-1} + 0.06z^{-2}} \\ -0.5z^{-1} - 0.06z^{-2} \\ \underline{-0.5z^{-1} - 0.25z^{-2} - 0.03z^{-3}} \\ 0.19z^{-2} + 0.03z^{-3} \\ \underline{0.19z^{-2} + 0.095z^{-3} + 0.0114z^{-4}} \end{array}$$

This leads to

$$X(z) = 0 + 0z^{-1} + z^{-2} - 0.5z^{-3} + 0.19z^{-4} + \dots$$

$$x(0) = x(1) = 0, x(2) = 1, x(3) = -0.5, x(4) = 0.19$$

(b)

$$\begin{array}{r}
 z^{-1} + 0.2z^{-2} + 0.19z^{-3} + 0.068z^{-4} \\
 z^2 - 0.2z - 0.15 \overline{) \phantom{z^2 - 0.2z - 0.15z^{-1}}} \\
 z \\
 z - 0.2 - 0.15z^{-1} \\
 \text{-----} \\
 0.2 + 0.15z^{-1} \\
 0.2 - 0.04z^{-1} - 0.03z^{-2} \\
 \text{-----} \\
 0.19z^{-1} + 0.03z^{-2} \\
 0.19z^{-1} - 0.038z^{-2} - 0.0285z^{-3} \\
 \text{-----} \\
 0.068z^{-2} + 0.0285z^{-3} \\
 0.068z^{-2} - 0.0136z^{-3} - 0.0102z^{-4}
 \end{array}$$

This leads to

$$X(z) = 0 + z^{-1} + 0.2z^{-2} + 0.19z^{-3} + 0.068z^{-4} + \dots$$

$$x(0) = 0, x(1) = 1, x(2) = 0.2, x(3) = 0.19, x(4) = 0.068$$

5.21

(a)

$$X(z)z^{n-1} = \frac{z^{n-1}}{(z^2 + 0.5z + 0.06)} = \frac{z^{n-1}}{(z + 0.2)(z + 0.3)}$$

For $n = 0$,

$$x(0) = R_{z=0} + R_{z=-0.2} + R_{z=-0.3} = \left. \frac{1}{(z + 0.2)(z + 0.3)} \right|_{z=0} + \left. \frac{1}{z(z + 0.3)} \right|_{z=-0.2} + \left. \frac{1}{z(z + 0.2)} \right|_{z=-0.3} = 0$$

For $n > 0$

$$x(n) = (R_{z=-0.2} + R_{z=-0.3})u(n-1)$$

$$= \left[\left. \frac{z^{n-1}}{(z + 0.3)} \right|_{z=-0.2} + \left. \frac{z^{n-1}}{(z + 0.2)} \right|_{z=-0.3} \right] u(n-1) = [10(-0.2)^{n-1} - 10(-0.3)^{n-1}] u(n-1)$$

(b)

$$X(z)z^{n-1} = \frac{z \times z^{n-1}}{(z + 0.3)(z - 0.5)} = \frac{z^n}{(z + 0.3)(z - 0.5)}$$

For $n \geq 0$

$$\begin{aligned}
 x(n) &= (R_{z=-0.3} + R_{z=0.5})u(n) \\
 &= \left[\frac{z^n}{(z-0.5)} \Big|_{z=-0.3} + \frac{z^n}{(z+0.3)} \Big|_{z=0.5} \right] u(n) = [-1.25(-0.3)^n - 1.25(0.5)^n]u(n)
 \end{aligned}$$

(d)

$$X(z)z^{n-1} = \frac{2z(z-0.4) \times z^{n-1}}{(z-0.2)^2(z+0.8)} = \frac{2(z-0.4)z^n}{(z-0.2)^2(z+0.8)}$$

For $n \geq 0$

$$\begin{aligned}
 x(n) &= (R_{z=-0.8} + R_{z=0.2})u(n) \\
 &= \left[\frac{2(z-0.4)z^n}{(z-0.2)^2} \Big|_{z=-0.8} + \frac{d}{dz} \left[\frac{2(z-0.4)z^n}{(z+0.8)} \right] \Big|_{z=0.2} \right] u(n)
 \end{aligned}$$

$$\begin{aligned}
 x(n) &= \left[-2.4(-0.8)^n + \frac{[2(n+1)z^n - 0.8nz^{n-1}](z+0.8) - (2z^{n+1} - 0.8z^n)}{(z+0.8)^2} \Big|_{z=0.2} \right] u(n) \\
 &= [-2.4(-0.8)^n - 2n(0.2)^n + 2.4(0.2)^n]u(n)
 \end{aligned}$$

5.22

$$Y(z) = \sum_{n=0}^{\infty} e^{-an} x(n) z^{-n} = \sum_{n=0}^{\infty} x(n) (e^a z)^{-n}$$

$$\text{Since } X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}, \text{ thus } Y(z) = X(ze^a)$$

5.23

$$\begin{aligned}
 Y(z) &= \sum_{n=0}^{\infty} nx(n)z^{-n}, \quad X(z) = \sum_{n=0}^{\infty} x(n)z^{-n}, \\
 \frac{dX(z)}{dz} &= -\sum_{n=0}^{\infty} nx(n)z^{-n-1} = -\frac{1}{z} \sum_{n=0}^{\infty} nx(n)z^{-n} = -\frac{Y(z)}{z}
 \end{aligned}$$

$$Y(z) = -z \frac{dX(z)}{dz}$$

5.24

(a)

$$Y(z) - [y(-1) + z^{-1}Y(z)] + [y(-2) + y(-1)z^{-1} + z^{-2}Y(z)] = 0$$

$$Y(z) = \frac{1 - z^{-1}}{1 - z^{-1} + z^2} = \frac{z(z-1)}{z^2 - z + 1} = \frac{z(z-1)}{(z - e^{j\pi/3})(z - e^{-j\pi/3})}$$

(b)

$$Y(z)z^{n-1} = \frac{z(z-1)z^{n-1}}{(z - e^{j\pi/3})(z - e^{-j\pi/3})} = \frac{(z-1)z^n}{(z - e^{j\pi/3})(z - e^{-j\pi/3})}$$

$$\begin{aligned}
y(n) &= \left. \frac{(z-1)z^n}{(z-e^{-j\pi/3})} \right|_{z=e^{j\pi/3}} + \left. \frac{(z-1)z^n}{(z-e^{j\pi/3})} \right|_{z=e^{-j\pi/3}} \\
&= \frac{(e^{j\pi/3}-1)e^{j\pi n/3}}{(e^{j\pi/3}-e^{-j\pi/3})} + \frac{(e^{-j\pi/3}-1)e^{-j\pi n/3}}{(e^{-j\pi/3}-e^{j\pi/3})} = \frac{(e^{j\pi/3}-1)e^{j\pi n/3}}{2j\sin(\pi/3)} - \frac{(e^{-j\pi/3}-1)e^{-j\pi n/3}}{2j\sin(\pi/3)} \\
&= \frac{e^{j\pi(n+1)/3} - e^{j\pi n/3} - e^{-j\pi(n+1)/3} + e^{-j\pi n/3}}{2j\sin(\pi/3)} = \frac{\sin[(n+1)\pi] - \sin(n\pi)}{\sin(\pi/3)}
\end{aligned}$$

5.25

$$\begin{aligned}
Y(z) &= 2\cos(\Omega_0)[y(-1) + z^{-1}Y(z)] - [y(-2) + y(-1)z^{-1} + z^{-2}Y(z)] \\
Y(z) &= 2\cos(\Omega_0)[- \sin(\Omega_0) + z^{-1}Y(z)] - [- \sin(2\Omega_0) - \sin(\Omega_0)z^{-1} + z^{-2}Y(z)] \\
Y(z) &= \frac{-2\cos(\Omega_0)\sin(\Omega_0) + \sin(2\Omega_0) + \sin(\Omega_0)z^{-1}}{1 - 2\cos(\Omega_0)z^{-1} + z^{-2}} = \frac{z\sin(\Omega_0)z^{-1}}{z^2 - 2z\cos(\Omega_0)z^{-1} + 1}
\end{aligned}$$

Thus, $y(n) = \sin(n\Omega_0)u(n)$

5.26

For $n \geq 1$, using long division leads to

$$\begin{array}{r}
\phantom{1 + a_1 z^{-1} + a_2 z^{-2}} \overline{b_0 + (b_1 - b_0 a_1) z^{-1}} \\
1 + a_1 z^{-1} + a_2 z^{-2} \overline{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} \dots} \\
\phantom{1 + a_1 z^{-1} + a_2 z^{-2}} b_0 + b_0 a_1 z^{-1} + b_0 a_2 z^{-2} \dots \\
\phantom{1 + a_1 z^{-1} + a_2 z^{-2}} \text{-----} \\
\phantom{1 + a_1 z^{-1} + a_2 z^{-2}} (b_1 - b_0 a_1) z^{-1} + (b_2 - b_0 a_2) z^{-2} + \dots \\
\phantom{1 + a_1 z^{-1} + a_2 z^{-2}} (b_1 - b_0 a_1) z^{-1} + (b_1 - b_0 a_1) a_1 z^{-2} + (b_1 - b_0 a_1) a_2 z^{-3} \dots \\
\phantom{1 + a_1 z^{-1} + a_2 z^{-2}} \text{-----} \\
\phantom{1 + a_1 z^{-1} + a_2 z^{-2}} (b_2 - x(0)a_2 - x(1)a_1) z^{-2} + \dots
\end{array}$$

We see

$$x(0) = b_0, x(1) = b_1 - x(0)a_1, x(2) = b_2 - [x(0)a_2 + x(1)a_1]$$

By prediction

$$x(n) = b_n - [x(0)a_n + x(1)a_{n-1} + x(2)a_{n-2} + \dots + x(n-1)a_1]$$

That is

$$x(n) = b_n - \sum_{k=1}^n x(n-k)a_k$$

5.27

$$X(z) = \sum_{n=0}^{\infty} x(n)z^{-n}, \quad X(z^{-1}) = \sum_{m=0}^{\infty} x(m)z^m$$

$$X(z)X(z^{-1}) = \sum_{n=0}^{\infty} x(n)z^{-n} \sum_{m=0}^{\infty} x(m)z^m = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} x(n)x(m)z^{m-n}$$

Applying line integral (Equation 5.21)

$$\frac{1}{2\pi j} \oint$$

Since

$$\frac{1}{2\pi j} \oint \begin{cases} n = m \\ \text{otherwise} \end{cases}$$

Then

$$\oint \dots^{-1} dz = \sum_{n=0}^{\infty} x^2(n)$$

5.28

$$z^{-1} X(z) X(z^{-1}) \Big|_{z=e^{j\Omega}} = e^{-j\Omega} X(e^{j\Omega}) X(e^{-j\Omega}) = e^{-j\Omega} |X(e^{j\Omega})|^2 \text{ and } dz = je^{j\Omega} d\Omega$$

$$\sum_{n=0}^{\infty} x^2(n) = \frac{1}{2\pi j} \oint$$

5.29

$$X(z) = \frac{z(a-1)}{(z-a)(z-1)}, \text{ and } X(z)z^{n-1} = \frac{(a-1)z^n}{(z-a)(z-1)}$$

$$\text{For } n < 0, x(n) = -\text{res}(X(z)z^{n-1}) \Big|_{z=1} = -\frac{(a-1)1^n}{(1-a)} = 1$$

$$\text{For } n \geq 0, x(n) = \text{res}(X(z)z^{n-1}) \Big|_{z=a} = \frac{(a-1)a^n}{(a-1)} = a^n$$

5.30

$$X(z) = \frac{(a-b)}{(1-az^{-1})(z-b)} = \frac{(a-b)z}{(z-a)(z-b)}, \text{ and } X(z)z^{n-1} = \frac{(a-b)z^n}{(z-a)(z-b)}$$

$$\text{For } n < 0, x(n) = -\text{res}(X(z)z^{n-1}) \Big|_{z=b} = -\frac{(a-b)b^n}{(b-a)} = b^n$$

$$\text{For } n \geq 0, x(n) = \text{res}(X(z)z^{n-1}) \Big|_{z=a} = \frac{(a-b)a^n}{(a-b)} = a^n$$

Chapter 6

6.1

- $y(0) = 0.5, y(1) = 0.25, y(2) = 0.125, y(3) = 0.0625, y(4) = 0.03125$
- $y(0) = 1, y(1) = 0, y(2) = 0.25, y(3) = 0, y(4) = 0.0625$

6.2

- $y = [0.1000 \quad 0.5600 \quad 0.5860 \quad 0.4766 \quad 0.3485]$

b. $y = [0 \quad 0.5000 \quad 0.5500 \quad 0.4550 \quad 0.3355]$

6.3

a. $y(0) = -2$, $y(1) = 2.3750$, $y(2) = -1.0312$, $y(3) = 0.7266$, $y(4) = -0.2910$

b. $y(0) = 0$, $y(1) = 1$, $y(2) = -0.2500$, $y(3) = 0.3152$, $y(4) = -0.0781$

6.4

a. $H(z) = 0.5 + 0.5z^{-1}$, b. $y(n) = 2\delta(n) + 2\delta(n-1)$, c. $y(n) = -5\delta(n) + 10u(n)$

6.5

a. $H(z) = \frac{1}{1+0.5z^{-1}}$, b. $y(n) = (-0.5)^n u(n)$, c. $y(n) = 0.6667u(n) + 0.3333(-0.5)^n u(n)$

6.6

$$H(z) = \frac{1 - 0.25z^{-2}}{1 + 1.1z^{-1} + 0.28z^{-2}}, B(z) = 1 - 0.25z^{-2}, A(z) = 1 + 1.1z^{-1} + 0.28z^{-2}$$

6.7

$$H(z) = \frac{0.5 + 0.5z^{-1}}{1 + 0.6z^{-2}}, B(z) = 0.5 + 0.5z^{-1}, A(z) = 1 + 0.6z^{-2}$$

6.8

$$H(z) = \frac{0.25z^{-2}}{1 - 0.5z^{-1} + 0.2z^{-2}}, B(z) = 0.25z^{-2}, A(z) = 1 - 0.5z^{-1} + 0.2z^{-2}$$

6.9

$$H(z) = 1 - 0.3z^{-1} + 0.28z^{-2}, A(z) = 1, B(z) = 1 - 0.3z^{-1} + 0.28z^{-2}$$

6.10

a. $y(n) = 0.5x(n) + 0.5x(n-1)$

b. $y(n) = 0.5x(n) + 0.3y(n-1)$

6.11

a. $y(n) = 0.1x(n) + 0.2x(n-1) + 0.3x(n-2)$

b. $y(n) = 0.5x(n) - 0.5x(n-2) + 0.3y(n-1) - 0.8y(n-2)$

6.12

a. $y(n) = x(n) - 0.25x(n-2) - 1.1y(n-1) - 0.18y(n-2)$

b. $y(n) = x(n-1) - 0.1x(n-2) + 0.3x(n-3)$

6.13

a. $H(z) = \frac{(z+1)(z+1)}{(z+2)(z+3)}$

$$\text{b. } H(z) = \frac{(z+0.4)(z-0.4)}{(z+0.2)(z+0.5)}$$

$$\text{c. } H(z) = \frac{(z+2-j)(z++j)}{z(z+1-j2.2361)(z+1+j2.2361)}$$

6.14

$$\text{a. Impulse response: } H(z) = \frac{13.3333}{z} - \frac{3.3333}{z+0.75},$$

$$y(n) = 13.3333u(n) - 3.3333(-0.75)^n u(n)$$

$$\text{Step response: } H(z) = \frac{11.4286}{z-1} - \frac{1.4286}{z+0.75}, \quad y(n) = 11.4286u(n) - 1.4286(-0.75)^n u(n)$$

$$\text{c. } H(z) = \frac{12.5}{z-1} - \frac{2.5}{z+0.75}, \quad y(n) = 12.5u(n) - 2.5(-0.75)^n u(n)$$

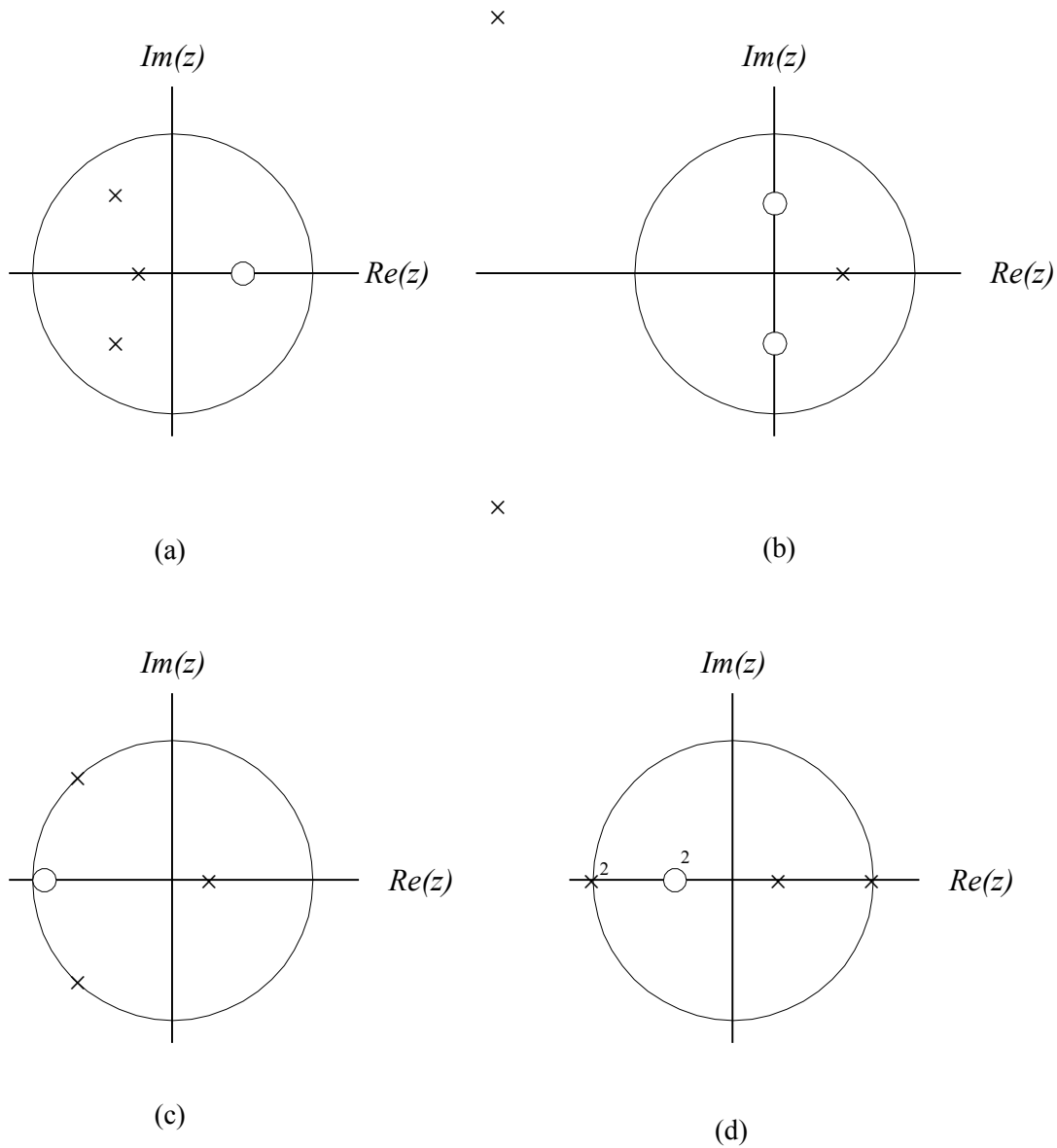
6.15

a. zero: $z = 0.5$, poles: $z = -0.25$ ($|z| = 0.25$), $z = -0.5 \pm 0.7416j$ ($|z| = 0.8944$), stable

b. zeros: $z = \pm 0.5j$, poles: $z = 0.5$ ($|z| = 0.5$), $z = -2 \pm 1.7321j$ ($|z| = 2.6458$), unstable

c. zero: $z = -0.95$, poles: $z = 0.2$ ($|z| = 0.2$), $z = -0.7071 \pm 0.7071j$ ($|z| = 1$), marginally stable

d. zeros: $z = -0.5$, $z = -0.5$, poles: $z = 1$ ($|z| = 1$), $z = -1$, $z = -1$ ($|z| = 1$), $z = 0.36$ ($|z| = 0.36$), unstable



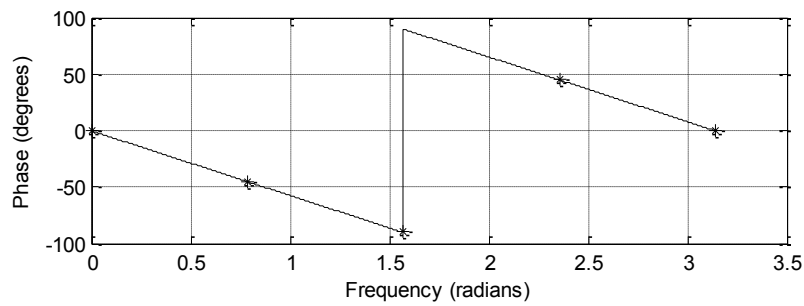
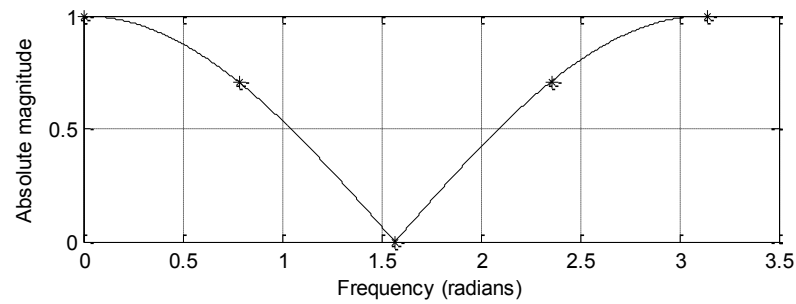
6.16

a. $H(z) = 0.5 + 0.5z^{-2}$, $H(e^{j\Omega}) = 0.5 + 0.5e^{-j2\Omega}$

$$|H(e^{j\Omega})| = 0.5\sqrt{(1 + \cos 2\Omega)^2 + (\sin 2\Omega)^2}, \quad \angle H(e^{j\Omega}) = \tan^{-1}\left(\frac{-\sin 2\Omega}{1 + \cos 2\Omega}\right)$$

b.

Ω (radians)	$f = \frac{\Omega}{2\pi} f_s$ (Hz)	$ H(e^{j\Omega}) $	$ H(e^{j\Omega}) _{dB}$	$\angle H(e^{j\Omega})$
0	0	1.000	0 dB	0.00°
0.25π	1000	0.707	-3.0102 dB	-45.00°
0.50π	2000	0.0	$-\infty$ dB	-90.00°
0.75π	3000	0.707	-3.0102 dB	45.00°
1.00π	4000	1.000	0 dB	0.00°



c. Bandstop filter

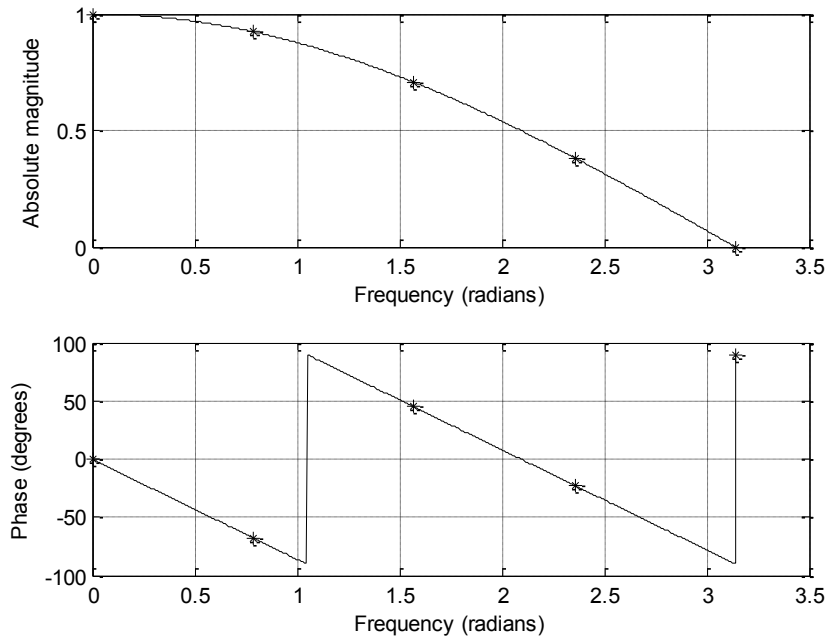
6.17

a. $H(z) = 0.5z^{-1} + 0.5z^{-2}$, $H(e^{j\Omega}) = 0.5e^{-j\Omega} + 0.5e^{-j2\Omega}$

$$|H(e^{j\Omega})| = 0.5\sqrt{(1 + \cos \Omega)^2 + (\sin \Omega)^2}, \quad \angle H(e^{j\Omega}) = \tan^{-1} \left(\frac{-\sin \Omega - \sin 2\Omega}{\cos \Omega + \cos 2\Omega} \right)$$

b.

Ω (radians)	$f = \frac{\Omega}{2\pi} f_s$ (Hz)	$ H(e^{j\Omega}) $	$ H(e^{j\Omega}) _{dB}$	$\angle H(e^{j\Omega})$
0	0	1.000	0 dB	0.00°
0.25π	1000	0.924	-0.6877 dB	-67.50°
0.50π	2000	0.707	-3.0102 dB	45.00°
0.75π	3000	0.383	-8.3432 dB	-22.50°
1.00π	4000	0.000	$-\infty$ dB	90.00°



c. Lowpass

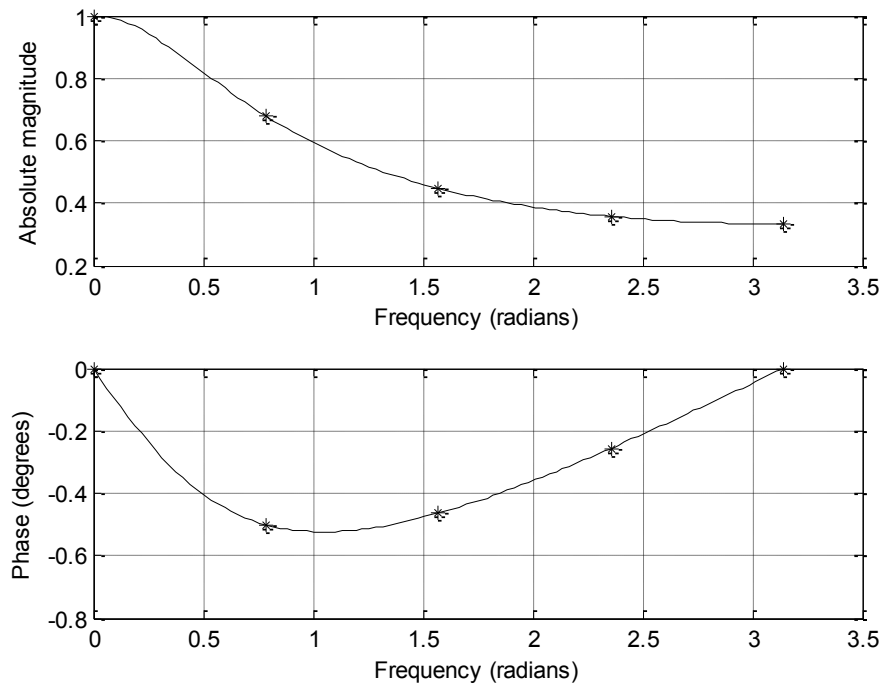
6.18

a. $H(z) = \frac{0.5}{1 - 0.5z^{-1}}$, $H(e^{j\Omega}) = \frac{0.5}{1 - 0.5e^{-j\Omega}}$

$$|H(e^{j\Omega})| = \frac{0.5}{\sqrt{(1 - 0.5\cos(\Omega))^2 + (0.5\sin(\Omega))^2}}, \quad \angle H(e^{j\Omega}) = -\tan^{-1}\left(\frac{0.5\sin(\Omega)}{1 - \cos(\Omega)}\right)$$

b.

Ω (radians)	$f = \frac{\Omega}{2\pi} f_s$ (Hz)	$ H(e^{j\Omega}) $	$ H(e^{j\Omega}) _{dB}$	$\angle H(e^{j\Omega})$
0	0	1.000	0 dB	0.00°
0.25π	1000	0.6786	-3.3677 dB	-28.68°
0.50π	2000	0.4472	-6.9897 dB	-26.57°
0.75π	3000	0.3574	-8.9367 dB	-14.64°
1.00π	4000	0.3333	-9.5424 dB	0.00°



c. Lowpass

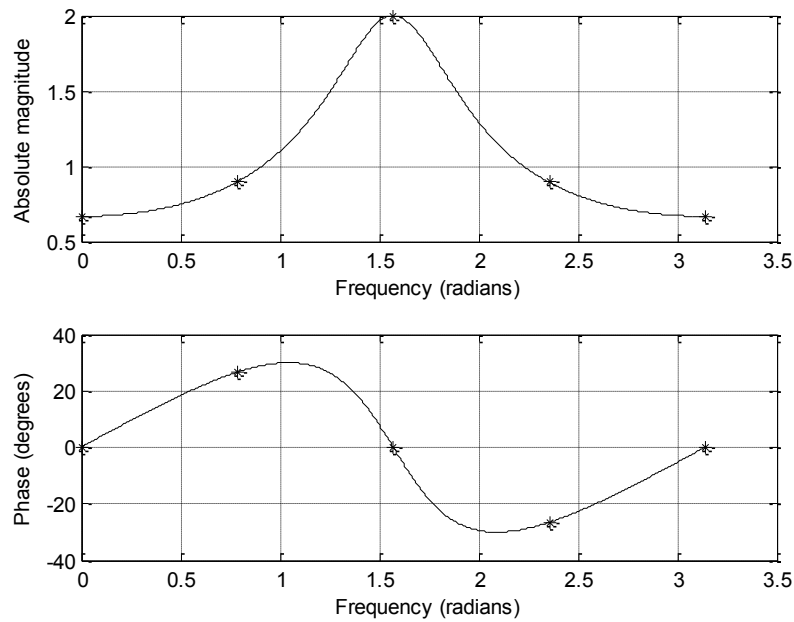
6.19

a. $H(z) = \frac{1}{1+0.5z^{-2}}$, $H(e^{j\Omega}) = \frac{1}{1+0.5e^{-j2\Omega}}$

$$|H(e^{j\Omega})| = \frac{1}{\sqrt{(1+0.5\cos 2\Omega)^2 + (0.5\sin 2\Omega)^2}}, \quad \angle H(e^{j\Omega}) = -\tan^{-1}\left(\frac{-0.5\sin 2\Omega}{1+0.5\cos 2\Omega}\right)$$

b.

Ω (radians)	$f = \frac{\Omega}{2\pi} f_s$ (Hz)	$ H(e^{j\Omega}) $	$ H(e^{j\Omega}) _{dB}$	$\angle H(e^{j\Omega})$
0	0	0.6667	0 dB	0.00°
0.25π	1000	0.8944	-3.0102 dB	26.57°
0.50π	2000	2.000	$-\infty$ dB	0.00°
0.75π	3000	0.8944	-3.0102 dB	-26.57°
1.00π	4000	0.6667	0 dB	0.00°

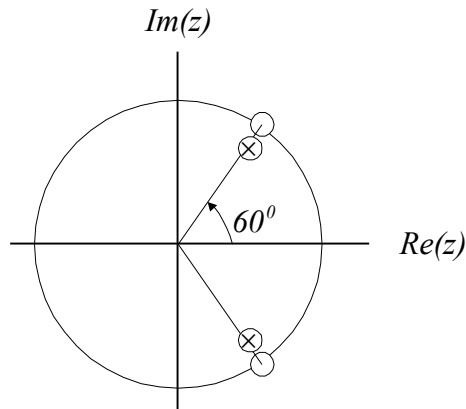


c. Bandpass filter

6.20

a.
$$H(z) = \frac{1 - z^{-1} + z^{-2}}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

b. Zeros: $0.5000 + 0.8660i$, $0.5000 - 0.8660i$,
Poles: $0.4000 + 0.6928i$, $0.4000 - 0.6928i$



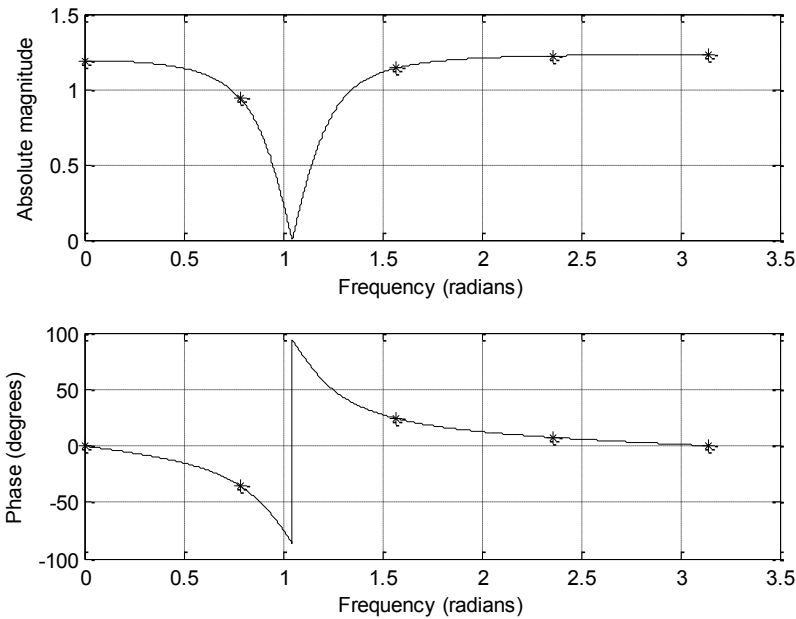
c. Stable since poles are inside the unit cycle.

d. and e.

$$|H(e^{j\Omega})| = \frac{\sqrt{(1 - \cos \Omega + \cos 2\Omega)^2 + (\sin \Omega + \sin 2\Omega)^2}}{\sqrt{(1 - 0.8 \cos \Omega + 0.64 \cos 2\Omega)^2 + (0.8 \sin \Omega + 0.64 \sin 2\Omega)^2}}$$

$$\angle H(e^{j\Omega}) = \tan^{-1} \left(\frac{-\sin \Omega - \sin 2\Omega}{1 - \cos \Omega + \cos 2\Omega} \right) - \tan^{-1} \left(\frac{-0.8 \sin \Omega - 0.64 \sin 2\Omega}{1 - 0.8 \cos \Omega + 0.64 \cos 2\Omega} \right)$$

Ω (radians)	$f = \frac{\Omega}{2\pi} f_s$ (Hz)	$ H(e^{j\Omega}) $	$ H(e^{j\Omega}) _{dB}$	$\angle H(e^{j\Omega})$
0	0	1.1905	1.5144 dB	0.00^0
0.25π	1000	0.9401	-0.5369 dB	-35.29^0
0.50π	2000	1.1399	1.1373 dB	24.23^0
0.75π	3000	1.2217	1.1792 dB	7.40^0
1.00π	4000	1.2295	1.7946 dB	0.00^0



6.21

(1) a. $H(z) = 0.5 + 0.5z^{-1}$, $H(e^{j\Omega}) = 0.5 + 0.5e^{-j\Omega}$

$$|H(e^{j\Omega})| = \sqrt{(0.5 + 0.5\cos\Omega)^2 + (0.5\sin\Omega)^2}, \quad \angle H(e^{j\Omega}) = \tan^{-1}\left(\frac{-0.5\sin\Omega}{0.5 + 0.5\cos\Omega}\right)$$

b. See table and plot

c. Lowpass filter

(2) a. $H(z) = 0.5 - 0.5z^{-1}$, $H(e^{j\Omega}) = 0.5 - 0.5e^{-j\Omega}$

$$|H(e^{j\Omega})| = \sqrt{(0.5 - 0.5\cos\Omega)^2 + (0.5\sin\Omega)^2}, \quad \angle H(e^{j\Omega}) = \tan^{-1}\left(\frac{0.5\sin\Omega}{0.5 - 0.5\cos\Omega}\right)$$

b. See table and plot

c. Highpass filter

(3) a. $H(z) = 0.5 + 0.5z^{-2}$, $H(e^{j\Omega}) = 0.5 + 0.5e^{-j2\Omega}$

$$|H(e^{j\Omega})| = \sqrt{(0.5 + 0.5\cos 2\Omega)^2 + (0.5\sin 2\Omega)^2}, \quad \angle H(e^{j\Omega}) = \tan^{-1}\left(\frac{-0.5\sin 2\Omega}{0.5 + 0.5\cos 2\Omega}\right)$$

b. See table and plot

c. Bandstop filter

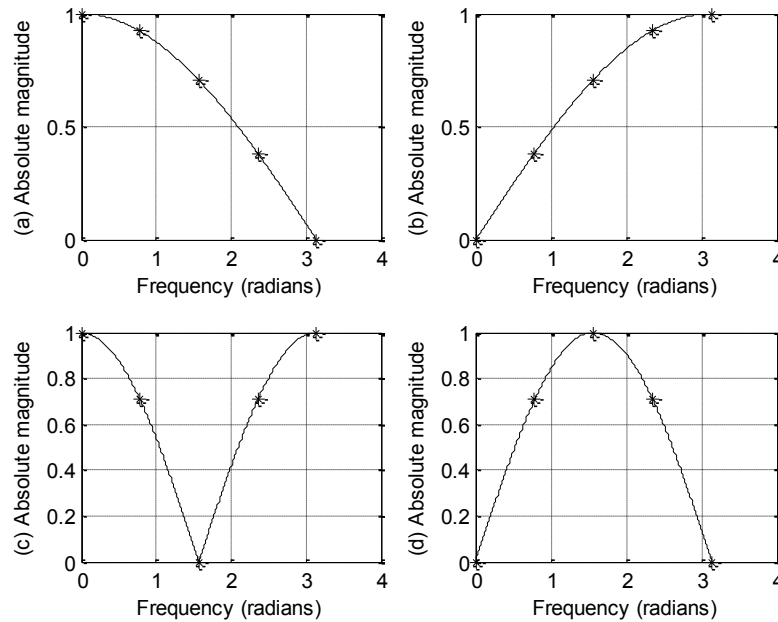
(4) a. $H(z) = 0.5 - 0.5z^{-2}$, $H(e^{j\Omega}) = 0.5 - 0.5e^{-j2\Omega}$

$$|H(e^{j\Omega})| = \sqrt{(0.5 - 0.5\cos 2\Omega)^2 + (0.5\sin 2\Omega)^2}, \quad \angle H(e^{j\Omega}) = \tan^{-1}\left(\frac{0.5\sin 2\Omega}{0.5 - 0.5\cos 2\Omega}\right)$$

b. See table and plot

c. Bandpass filter

Ω (radians)	(a) $ H(e^{j\Omega}) $	(b) $ H(e^{j\Omega}) $	(c) $ H(e^{j\Omega}) $	(d) $ H(e^{j\Omega}) $
0	1.0000	0.0000	1.0000	0.0000
0.25π	0.9239	0.3827	0.7071	0.7071
0.50π	0.7071	0.7071	0.0000	1.0000
0.75π	0.3827	0.9239	0.7071	0.7071
1.00π	0.0000	1.0000	1.0000	0.0000



6.22

a. $H(z) = \frac{0.5}{1 - 0.2z^{-1}}$

b. $Y(z) = \frac{-0.3333z}{z - 0.2} + \frac{0.8333z}{z - 0.5}$, $y(n) = -0.3333(0.2)^n u(n) + 0.8333(0.5)^n u(n)$

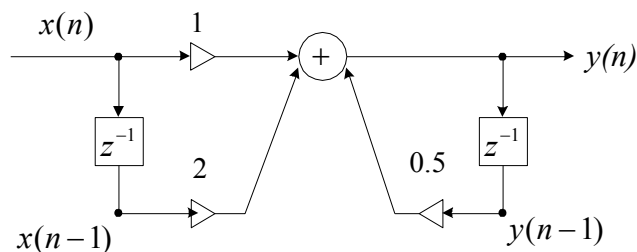
6.23

a. $H(z) = \frac{0.5}{1 + 0.7z^{-1} + 0.1z^{-2}}$

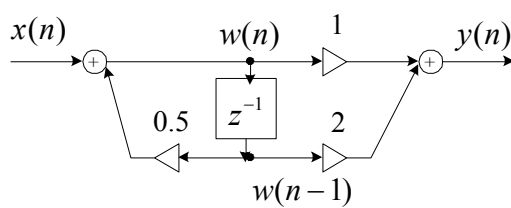
b. $y(n) = 0.5556u(n) - 0.111(-0.2)^n u(n) + 0.5556(-0.5)^n u(n)$

6.24

a. $y(n] = x(n) + 2x(n-1) + 0.5y(n-1)$

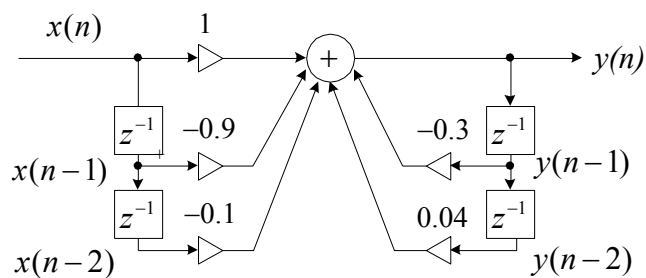


b. $w(n) = x(n) + 0.5w(n-1)$
 $y(n) = w(n) + 2w(n-1)$

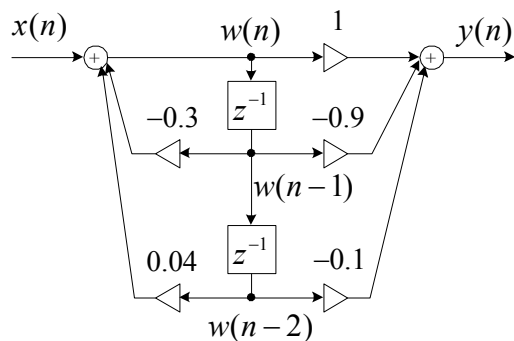


6.25

a. $y(n) = x(n) - 0.9x(n-1) - 0.1x(n-2) - 0.3y(n-1) + 0.04y(n-2)$



b. $w(n) = x(n) - 0.3w(n-1) + 0.04w(n-2)$
 $y(n) = w(n) - 0.9w(n-1) - 0.1w(n-2)$



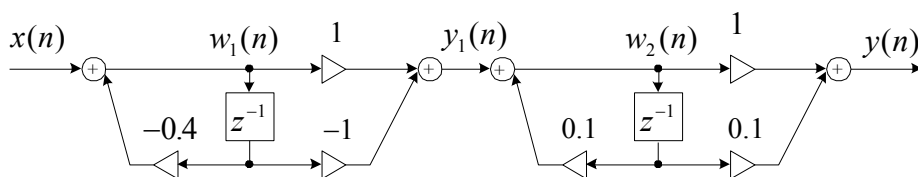
$$c. H(z) = \frac{(z-1)(z+0.1)}{(z+0.4)(z-0.1)} = \frac{1-z^{-1}}{1+0.4z^{-1}} \times \frac{1-0.1z^{-1}}{1-0.1z^{-1}}$$

$$w_1(n) = x(n) - 0.4w_1(n-1)$$

$$y_1(n) = w_1(n) - w_1(n-1)$$

$$w_2(n) = y_1(n) + 0.1w_2(n-1)$$

$$y(n) = w_2(n) + 0.1w_2(n-1)$$



$$d. H(z) = 2.5 + \frac{2.1z}{z+0.4} - \frac{3.6z}{z-0.1} = 2.5 + \frac{2.1}{1+0.4z^{-1}} + \frac{-3.6}{1-0.1z^{-1}}$$

$$y_1(n) = 2.5x(n)$$

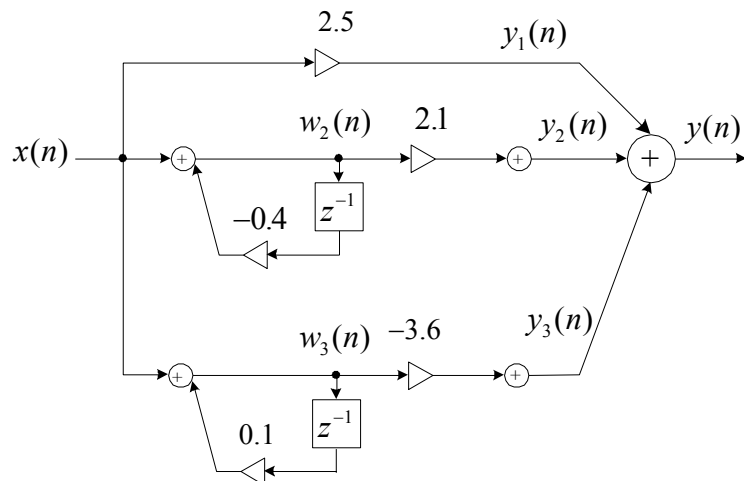
$$w_2(n) = x(n) - 0.4w_2(n-1)$$

$$y_2(n) = 2.1w_2(n)$$

$$w_3(n) = x(n) + 0.1w_3(n-1)$$

$$y_3(n) = -3.6w_3(n)$$

$$y(n) = y_1(n) + y_2(n) + y_3(n)$$



6.26

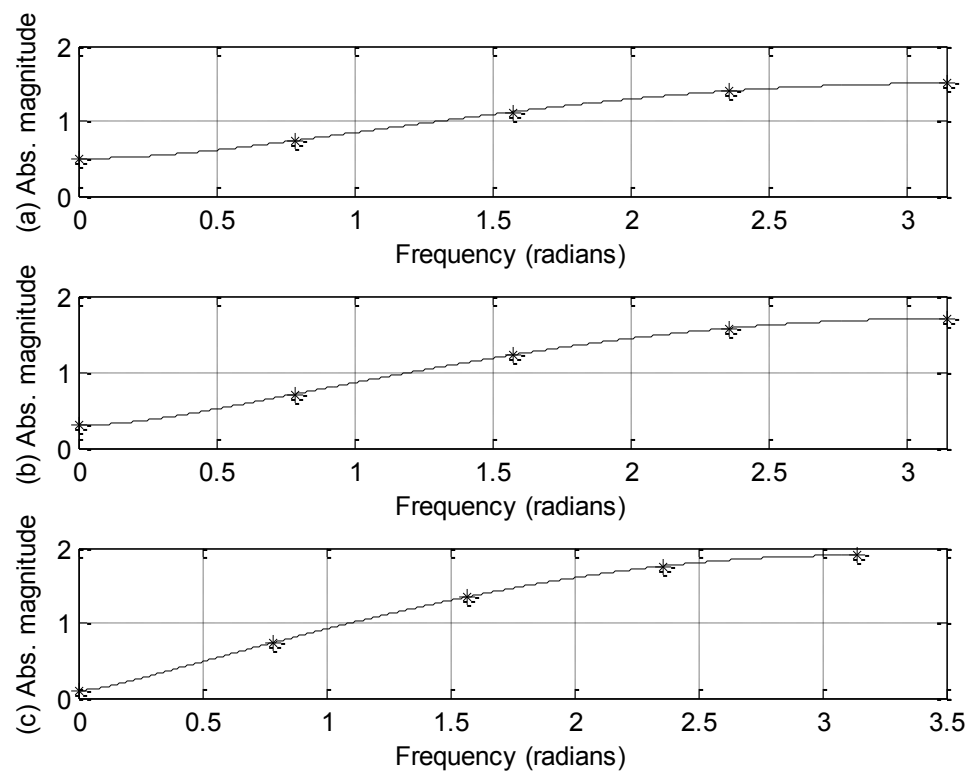
a. $y(n) = x(n) - 0.5x(n-1)$

$y(n) = x(n) - 0.7x(n-1)$

$y(n) = x(n) - 0.9x(n-1)$

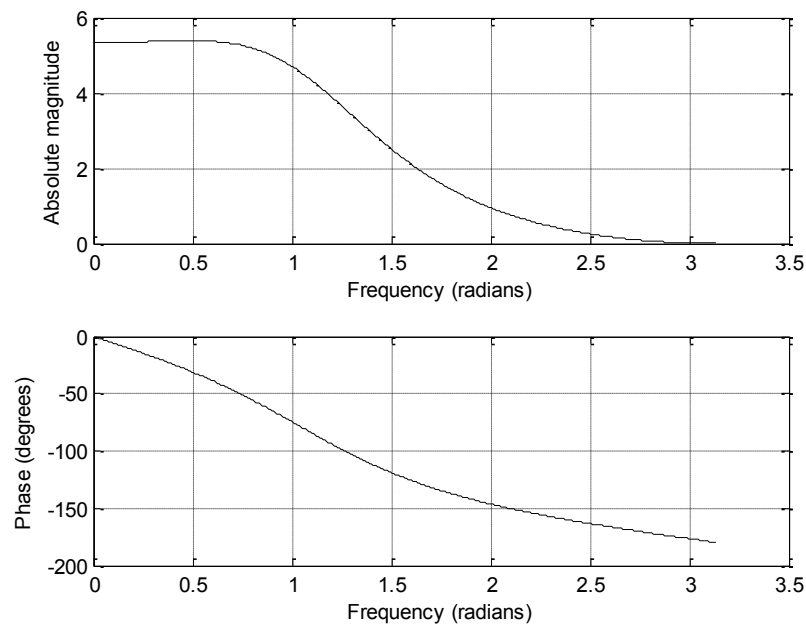
b. From frequency responses, the filter (c) emphasizes high frequencies most.

Ω (radians)	(a) $ H(e^{j\Omega}) $	(b) $ H(e^{j\Omega}) $	(c) $ H(e^{j\Omega}) $
0	0.5000	0.3000	0.1000
0.25π	0.6368	0.7071	0.7329
0.50π	1.1189	1.2207	1.3454
0.75π	1.3990	1.5748	1.7558
1.00π	1.5000	1.7000	1.9000



6.27

```
>> [H W]=freqz([1 2 1],[1 -0.5 0.25],512);
>> subplot(2,1,1);plot(W,abs(H),'k');grid;
>> xlabel('Frequency (radians)');ylabel('Absolute magnitude');
>> subplot(2,1,2);plot(W,angle(H)*180/pi,'k');grid;
>> xlabel('Frequency (radians)');ylabel('Phase (degrees)');
```



6.28

a.

```
>> x=[1 0.5 0.25 0.125 0.0625];
>> Xi=[-1 0]; Yi=[1 2];
>> Zi=filtic([0 1],[1 0.75 0.125],Yi,Xi);
>> y=filter([0 1],[1 0.75 0.125],x,Zi)
y = -2.0000  2.3750 -1.0313  0.7266 -0.2910
```

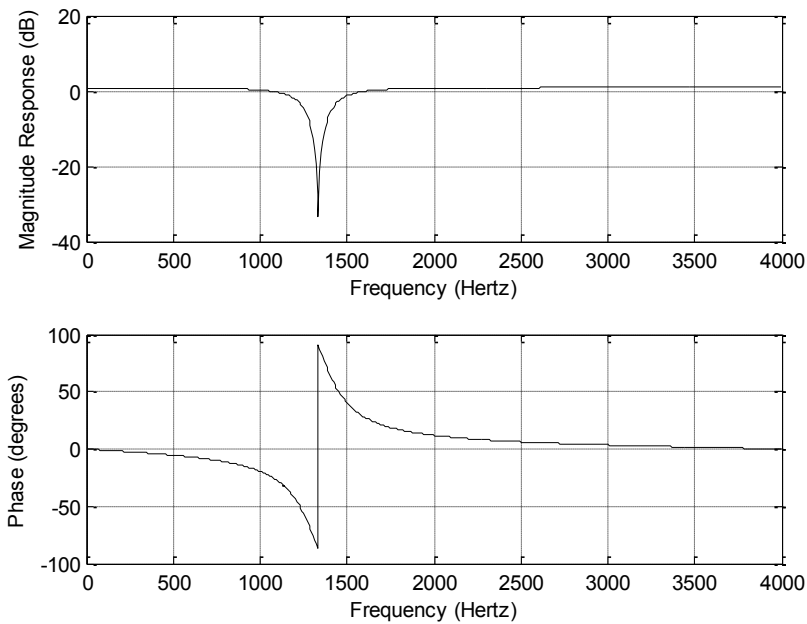
b.

```
>> x=[1 0.5 0.25 0.125 0.0625];
>> y=filter([0 1],[1 0.75 0.125],x)
y = 0  1.0000 -0.2500  0.3125 -0.0781
```

6.29

a.

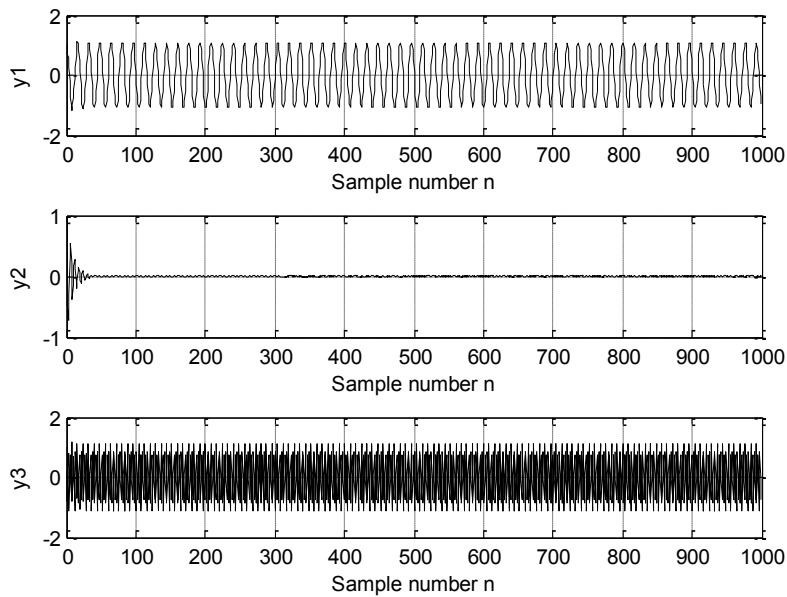
```
>> freqz([1 -1 1],[1 -0.9 0.81],512,8000)
```



b. Notch filter

c. $y(n) = x(n) - x(n-1) + x(n-2) + 0.9y(n-1) - 0.81y(n-2)$

d.



```

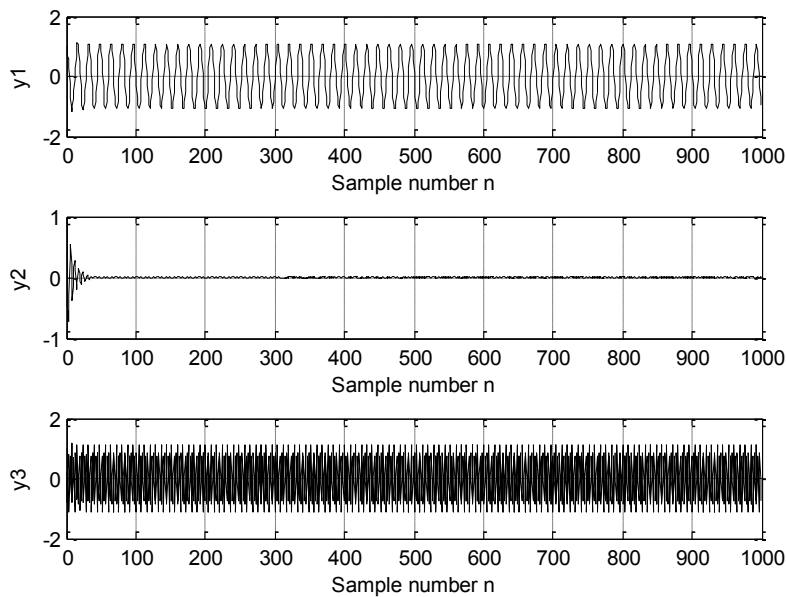
>>n=0:1:999;
>>x1=cos(pi*1000*n/8000);x1=[0 0 x1];
>>x2=cos((8/3)*pi*1000*n/8000); x2=[0 0 x2];
>>x3=cos(6*pi*1000*n/8000); x3=[0 0 x3];
>>w1=zeros(1,1002); w2=w1; w3=w1;y1=w1;y2=w2;y3=w3;
>>for nn=3:1:1002

```

```

w1(nn)=x1(nn)+0.9*w1(nn-1)-0.81*w1(nn-2);
y1(nn)=w1(nn)-w1(nn-1)+w1(nn-2);
w2(nn)=x2(nn)+0.9*w2(nn-1)-0.81*w2(nn-2);
y2(nn)=w2(nn)-w2(nn-1)+w2(nn-2);
w3(nn)=x3(nn)+0.9*w3(nn-1)-0.81*w3(nn-2);
y3(nn)=w3(nn)-w3(nn-1)+w3(nn-2);
>>end
>>y1=y1(3:1002);y2=y2(3:1002);y3=y3(3:1002);
>> subplot(3,1,1),plot(n,y1,'k');grid; xlabel('Sample number n');ylabel('y1');
>> subplot(3,1,2),plot(n,y2,'k');grid; xlabel('Sample number n');ylabel('y2');
>> subplot(3,1,3),plot(n,y3,'k');grid; xlabel('Sample number n'); ylabel('y3');

```



e.

```

>> n=0:1:999;
>> x1=cos(pi*1000*n/8000);
>> x2=cos((8/3)*pi*1000*n/8000);
>> x3=cos(6*pi*1000*n/8000);
>> y1=filter([1 -1 1],[1 -0.9 0.81],x1);
>> y2=filter([1 -1 1],[1 -0.9 0.81],x2);
>> y3=filter([1 -1 1],[1 -0.9 0.81],x3);
>> subplot(3,1,1),plot(n,y1,'k');grid; xlabel('Sample number n');ylabel('y1');
>> subplot(3,1,2),plot(n,y2,'k');grid; xlabel('Sample number n');ylabel('y2');
>> subplot(3,1,3),plot(n,y3,'k');grid; xlabel('Sample number n'); ylabel('y3');
See figure in (d)

```

6.30

```

>>n=0:1:999;
>>x1=cos(pi*1000*n/8000);x1=[0 0 x1];

```

```

>>x2=cos((8/3)*pi*1000*n/8000); x2=[0 0 x2];
>>x3=cos(6*pi*1000*n/8000); x3=[0 0 x3];
>>y1=zeros(1,1002); y2=y1; y3=y1;
>>for nn=3:1:1002
    y1(nn)=x1(nn)-x1(nn-1)+x1(nn-2)+0.9*y1(nn-1)-0.81*y1(nn-2);
    y2(nn)=x2(nn)-x2(nn-1)+x2(nn-2)+0.9*y2(nn-1)-0.81*y2(nn-2);
    y3(nn)=x3(nn)-x3(nn-1)+x3(nn-2)+0.9*y3(nn-1)-0.81*y3(nn-2);
>>end
>>y1=y1(3:1002);y2=y2(3:1002);y3=y3(3:1002);
>> subplot(3,1,1),plot(n,y1,'k');grid; xlabel('Sample number n');ylabel('y1');
>> subplot(3,1,2),plot(n,y2,'k');grid; xlabel('Sample number n');ylabel('y2');
>> subplot(3,1,3),plot(n,y3,'k');grid; xlabel('Sample number n'); ylabel('y3');

```

See plots in Problem 6.29

6.33

$$X(z) = z^{-N} (a_N z^N + a_{N-1} z^{N-1} + \dots + a_0 + \dots + a_N z^{-N})$$

$$X(re^{j\theta}) = (re^{j\theta})^{-N} [a_N (re^{j\theta})^N + a_{N-1} (re^{j\theta})^{N-1} + \dots + a_0 + \dots + a_N (re^{j\theta})^{-N}]$$

$$= (r^N e^{-jN\theta}) [a_N r^N e^{jN\theta} + a_{N-1} r^{N-1} e^{j(N-1)\theta} + \dots + a_0 + \dots + a_N r^{-N} e^{-jN\theta}] = 0$$

Assume $z = r^{-1} e^{-j\theta}$ is also a zero of $X(z)$ we can verify

$$X(re^{j\theta}) = (r^{-1} e^{-j\theta})^{-N} [a_N (r^{-1} e^{-j\theta})^N + a_{N-1} (r^{-1} e^{-j\theta})^{N-1} + \dots + a_0 + \dots + a_N (r^{-1} e^{-j\theta})^{-N}]$$

$$= (r^{-N} e^{jN\theta}) [a_N r^{-N} e^{-jN\theta} + a_{N-1} r^{-(N-1)} e^{-j(N-1)\theta} + \dots + a_0 + \dots + a_N r^N e^{jN\theta}] = 0$$

6.34

(a)

$$H(z) = 1 - 0.5z^{-1}$$

(b)

$$H(z) = \frac{1 - 0.7z^{-1} + 0.1z^{-2}}{1 - 0.2z^{-1}} = \frac{(1 - 0.5z^{-1})(1 - 0.2z^{-1})}{1 - 0.2z^{-1}} = 1 - 0.5z^{-1}$$

They are equivalent.

6.35

(a)

$$X(z) = \frac{z}{z - 0.5} \text{ and } Y(z) = \frac{z}{(z - 0.5)(z - 0.25)}$$

$$H(z) = Y(z) / X(z) = \frac{1}{z - 0.25}$$

(b)

$$\frac{Y(z)}{z} = \frac{1}{(z - 0.5)(z - 0.25)} = \frac{4}{(z - 0.5)} - \frac{4}{(z - 0.25)}$$

$$Y(z) = \frac{4z}{(z-0.5)} - \frac{4z}{(z-0.25)}, \quad y(n) = 4(0.5)^n u(n) - 4(0.25)^n u(n)$$

6.36

(a)

$$H(z) = b_0 + b_1 z^{-1} + b_1 z^{-2}, \quad H(e^{j\Omega}) = b_0 + e^{-1.5j\Omega} b_1 (e^{j0.5\Omega} + e^{-j0.5\Omega}) = b_0 + 2b_1 e^{-1.5j\Omega} \cos 0.5\Omega$$

$$\text{For } z = e^{j0}, \quad b_0 + 2b_1 = 1$$

$$\text{For } z = e^{j2\pi/3}, \quad b_0 - b_1 = 0$$

$$b_0 = 1/3, \quad b_1 = 1/3$$

(b)

$$H(e^{j\Omega}) = b_0 + b_1 e^{-j\Omega} + b_1 e^{-j2\Omega} = 1/3 + (2/3)e^{-j1.5\Omega} \cos(0.5\Omega)$$

Ω (radians)	$ H(e^{j\Omega}) $	$\angle H(e^{j\Omega})$
0	1.000	0.00°
0.25π	0.8047	-45.00°
0.50π	0.3333	-90.00°
$2\pi/3$	0.0000	-143.67°
0.75π	0.1381	45.00°
1.00π	0.3333	0.00°

6.37

$$H(e^{j\Omega}) = 1 - r e^{-j(\Omega-\theta)} = 1 - r \cos(\Omega-\theta) + jr \sin(\Omega-\theta)$$

(a)

$$\begin{aligned} |H(e^{j\Omega})| &= \sqrt{[1 - r \cos(\Omega-\theta)]^2 + [r \sin(\Omega-\theta)]^2} \\ &= \sqrt{1 - 2r \cos(\Omega-\theta) + r^2 \cos^2(\Omega-\theta) + r^2 \sin^2(\Omega-\theta)} = \sqrt{1 - 2r \cos(\Omega-\theta) + r^2} \end{aligned}$$

(b)

$$\angle H(e^{j\Omega}) = \tan^{-1} \left[\frac{r \sin(\Omega-\theta)}{1 - r \cos(\Omega-\theta)} \right]$$

(c)

$$r=0.8, \theta=\pi/3$$

Ω (radians)	$ H(e^{j\Omega}) $
0	0.9165
0.25π	0.3071
0.50π	0.5043
0.75π	1.1072
1.00π	1.5620

6.38

$$H(e^{j\Omega}) = \frac{1}{1 - re^{-j(\Omega - \theta)}} = \frac{1}{1 - r \cos(\Omega - \theta) + jr \sin(\Omega - \theta)}$$

(a)

$$|H(e^{j\Omega})| = 1 / \sqrt{[1 - r \cos(\Omega - \theta)]^2 + [r \sin(\Omega - \theta)]^2}$$

$$= 1 / \sqrt{1 - 2r \cos(\Omega - \theta) + r^2 \cos^2(\Omega - \theta) + r^2 \sin^2(\Omega - \theta)} = 1 / \sqrt{1 - 2r \cos(\Omega - \theta) + r^2}$$

(b)

$$\angle H(e^{j\Omega}) = -\tan^{-1} \left[\frac{r \sin(\Omega - \theta)}{1 - r \cos(\Omega - \theta)} \right]$$

(c)

r=0.8, theta=pi/3,

Ω (radians)	$ H(e^{j\Omega}) $
0	1.0911
0.25π	3.2527
0.50π	1.9828
0.75π	0.9032
1.00π	0.6402

6.39

(a)

$$H(z) = \frac{1-a}{1-az^{-1}} = \frac{(1-a)z}{z-a}, \quad H(e^{j\Omega}) = \frac{1-a}{1-ae^{-j\Omega}}, \quad |H(e^{j\Omega})| = \frac{1-a}{\sqrt{1-2a \cos(\Omega) + a^2}}$$

Lowpass filter

$$\text{Dc gain, } \Omega = 0, |H(e^{j0})| = \frac{1-a}{\sqrt{1-2a \cos(0) + a^2}} = 1$$

3-dB bandwidth

$$|H(e^{j\Omega})| = \frac{(1-a)}{\sqrt{1-2a \cos(\Omega) + a^2}} = \frac{1}{\sqrt{2}} \Rightarrow \Omega_{-3dB} = \cos^{-1} \left[\frac{1+a^2-2(1-a)^2}{2a} \right]$$

(b)

$$H(z) = \frac{1-a}{2} \frac{1+z^{-1}}{1-az^{-1}}, \quad H(e^{j\Omega}) = \frac{1-a}{2} \frac{1+e^{-j\Omega}}{1-ae^{-j\Omega}}, \quad |H(e^{j\Omega})| = \frac{1-a}{2} \frac{\sqrt{2+2 \cos \Omega}}{\sqrt{1-2a \cos(\Omega) + a^2}}$$

$$\text{Dc gain, } \Omega = 0, |H(e^{j0})| = \frac{1-a}{2} \frac{\sqrt{2+2 \cos 0}}{\sqrt{1-2a \cos(0) + a^2}} = 1$$

3-dB bandwidth

$$|H(e^{j\Omega})| = \frac{1-a}{2} \frac{\sqrt{2+2 \cos \Omega}}{\sqrt{1-2a \cos(\Omega) + a^2}} = \frac{1}{\sqrt{2}} \Rightarrow \Omega_{-3dB} = \cos^{-1} \left[\frac{2a}{1+a^2} \right]$$

(b) is better lowpass filter since parameter a can easily be used.

6.40

$$Y(z) = e^{j\Omega_0} z^{-1} Y(z) + 1, \quad Y(z) = \frac{1}{1 - e^{j\Omega_0} z^{-1}} = \frac{z}{z - e^{j\Omega_0}}$$

$$y(n) = (e^{j\Omega_0})^n u(n) = \cos n\Omega_0 u(n) + j \sin n\Omega_0 u(n)$$

6.41

(a)

$$H(z) = \frac{1 - z^{-1}}{1 - az^{-1}}, \quad H(e^{j\Omega}) = \frac{1 - e^{-j\Omega}}{1 - ae^{-j\Omega}}$$

$$\text{Passband gain } \Omega = \pi, \quad H(e^{j\pi}) = \frac{1 - e^{-j\pi}}{1 - ae^{-j\pi}} = \frac{2}{1 + a}$$

The lowpass filter can be

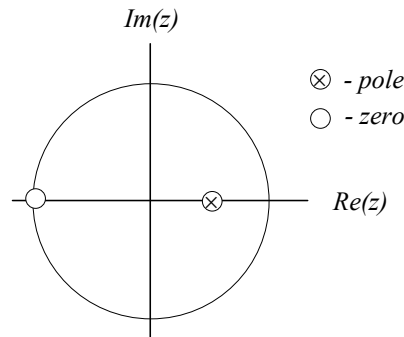
$$H_L(e^{j\Omega}) = \frac{2}{1 + a} - \frac{1 - e^{-j\Omega}}{1 - ae^{-j\Omega}},$$

$$\text{that is, } H_L(z) = \frac{2}{1 + a} - \frac{1 - z^{-1}}{1 - az^{-1}} = \frac{1 - a}{1 + a} \frac{1 + z^{-1}}{1 - az^{-1}}$$

$$y(n) = \frac{1 - a}{1 + a} [x(n) + x(n-1)] + ay(n-1)$$

(b)

Pole: $z = a$, zero: $z = -1$



(c)

Choose $a=0.5$

$$H_L(z) = \frac{1}{3} \frac{1 + z^{-1}}{1 - 0.5z^{-1}}, \quad H_L(e^{j\Omega}) = \frac{1}{3} \frac{1 + e^{-j\Omega}}{1 - 0.5e^{-j\Omega}}$$

Ω (radians)	$ H(e^{j\Omega}) $	$\angle H(e^{j\Omega})$
0	1.3333	0°
0.25π	0.8359	-51.18°
0.50π	0.4216	-71.57°
0.75π	0.1824	-82.14°
1.00π	0.0000	-90°

6.42

$$H(z) = 1 + z^{-M}, \text{ and } H(e^{j\Omega}) = 1 + e^{-jM\Omega} = 1 + \cos(M\Omega) - j\sin(M\Omega)$$

(a)

$$|H(e^{j\Omega})| = \sqrt{[1 + \cos(M\Omega)]^2 + [\sin(M\Omega)]^2} = \sqrt{2 + 2\cos(M\Omega)}$$

$$\angle H(e^{j\Omega}) = \tan^{-1} \left[\frac{-\sin(M\Omega)}{1 + \cos(M\Omega)} \right]$$

$$(b) \text{ set } H(e^{j\Omega}) = 1 + e^{-jM\Omega} = 0 \Rightarrow e^{-jM\Omega} = -1 = e^{-j(2k+1)\pi}, k = 0, 1, 2, \dots$$

$$\Omega = (2k+1)\pi / M, k = 0, 1, 2, \dots$$

6.43

$$H(e^{j\Omega}) = \frac{1 - e^{-j(\Omega-\theta)}}{1 - re^{-j(\Omega-\theta)}} = \frac{1 - \cos(\Omega-\theta) + j\sin(\Omega-\theta)}{1 - r\cos(\Omega-\theta) + jr\sin(\Omega-\theta)}$$

(a)

$$\begin{aligned} |H(e^{j\Omega})| &= \sqrt{[1 - \cos(\Omega-\theta)]^2 + [\sin(\Omega-\theta)]^2} / \sqrt{[1 - r\cos(\Omega-\theta)]^2 + [r\sin(\Omega-\theta)]^2} \\ &= \sqrt{1 - 2\cos(\Omega-\theta) + \cos^2(\Omega-\theta) + \sin^2(\Omega-\theta)} / \sqrt{1 - 2r\cos(\Omega-\theta) + r^2\cos^2(\Omega-\theta) + r^2\sin^2(\Omega-\theta)} \\ &= \sqrt{2 - 2\cos(\Omega-\theta)} / \sqrt{1 - 2r\cos(\Omega-\theta) + r^2} \end{aligned}$$

(b)

$$\angle H(e^{j\Omega}) = \tan^{-1} \left[\frac{\sin(\Omega-\theta)}{1 - \cos(\Omega-\theta)} \right] - \tan^{-1} \left[\frac{r\sin(\Omega-\theta)}{1 - r\cos(\Omega-\theta)} \right]$$

(c)

$$r=0.8, \theta=\pi/4,$$

Ω (radians)	$ H(e^{j\Omega}) $
0	1.0732
0.25π	0.0000
0.50π	1.0732
0.75π	1.1043
1.00π	1.1099

6.44

(a)

$$H(e^{j\Omega}) = \frac{(1 - e^{j\theta}z^{-1})(1 - e^{-j\theta}z^{-1})}{(1 - re^{j\theta}z^{-1})(1 - re^{-j\theta}z^{-1})} = \frac{1 - 2\cos\theta z^{-1} + z^{-2}}{1 - 2r\cos\theta z^{-1} + r^2 z^{-2}}$$

(b) Use results in 4.43

$$|H(e^{j\Omega})| = \frac{\sqrt{2 - 2\cos(\Omega-\theta)}}{\sqrt{1 - 2r\cos(\Omega-\theta) + r^2}} \times \frac{\sqrt{2 - 2\cos(\Omega+\theta)}}{\sqrt{1 - 2r\cos(\Omega+\theta) + r^2}}$$

(c)

 $r=0.8$, $\theta=\pi/4$,

Ω (radians)	$ H(e^{j\Omega}) $
0	1.1517
0.25π	0.0000
0.50π	1.1912
0.75π	1.2270
1.00π	1.2320

6.45

(a)

$$y(n) = \sum_{k=0}^{\infty} a^k x(n-kD), \quad Y(z) = \sum_{k=0}^{\infty} a^k X(z) z^{-kD}, \quad H(z) = \sum_{k=0}^{\infty} (az^{-D})^k = \frac{1}{1-az^{-D}}$$

(b)

$$Y(z) = \left(\frac{1}{a} - a \right) \sum_{k=0}^{\infty} a^k X(z) z^{-kD} - \frac{1}{a} X(z),$$

$$H(z) = \left(\frac{1}{a} - a \right) \sum_{k=0}^{\infty} a^k z^{-kD} - \frac{1}{a} = \left(\frac{1}{a} - a \right) \frac{1}{1-az^{-D}} - \frac{1}{a} = \frac{(1/a) - a - (1/a)(1-az^{-D})}{1-az^{-D}} = \frac{z^{-D} - a}{1-az^{-D}}$$

(c)

$$H(e^{j\Omega}) = \frac{e^{-jD\Omega} - a}{1 - ae^{-jD\Omega}},$$

$$\begin{aligned} |H(e^{j\Omega})| &= \frac{\sqrt{[\cos(D\Omega) - a]^2 + [\sin(D\Omega)]^2}}{\sqrt{[1 - a \cos(D\Omega)]^2 + [a \sin(D\Omega)]^2}} = \frac{\sqrt{\cos^2(D\Omega) - 2a \cos(D\Omega) + a^2 + \sin^2(D\Omega)}}{\sqrt{1 - 2a \cos(D\Omega) + a^2 \cos^2(D\Omega) + a^2 \sin^2(D\Omega)}} \\ &= \frac{\sqrt{1 - 2a \cos(D\Omega) + a^2}}{\sqrt{1 - 2a \cos(D\Omega) + a^2}} = 1 \end{aligned}$$

Chapter 7

7.1

a. $H(z) = 0.2941 + 0.3750z^{-1} + 0.2941z^{-2}$

$$y(n) = 0.2941x(n) + 0.3750x(n-1) + 0.2941x(n-2)$$

$$H(e^{j\Omega}) = e^{-j\Omega} (0.3750 + 0.5882 \cos(\Omega))$$

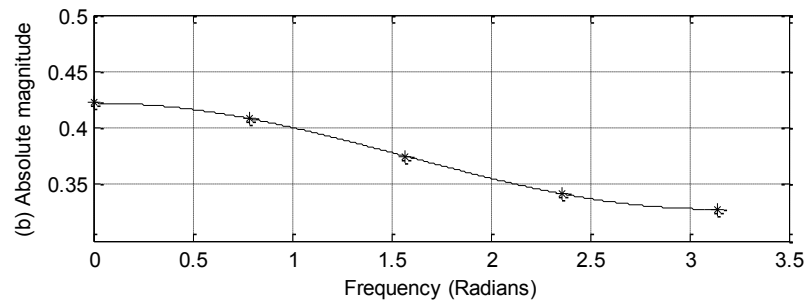
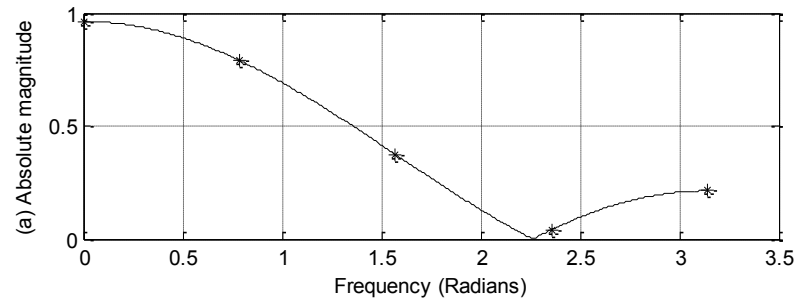
b. $H(z) = 0.0235 + 0.3750z^{-1} + 0.0235z^{-2}$

$$y(n) = 0.0235x(n) + 0.3750x(n-1) + 0.0235x(n-2)$$

$$H(e^{j\Omega}) = e^{-j\Omega} (0.3750 + 0.0470 \cos(\Omega))$$

Ω (radians)	(a) $ H(e^{j\Omega}) $	(b) $ H(e^{j\Omega}) $
0	0.9623	0.4220

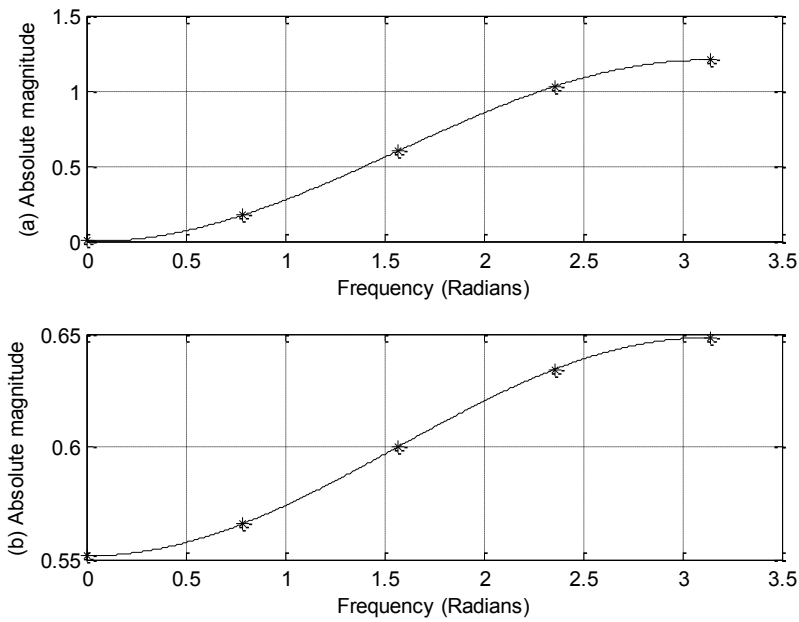
0.25π	0.7909	0.4082
0.50π	0.3750	0.3750
0.75π	0.0409	0.3418
1.00π	0.2132	0.3280



7.2

- a. $H(z) = -0.3027 + 0.6000z^{-1} - 0.3027z^{-2}$
 $y(n) = -0.3027x(n) + 0.6000x(n-1) - 0.3027x(n-2)$
 $H(e^{j\Omega}) = e^{-j\Omega} (0.6000 - 0.6054\cos(\Omega))$
- b. $H(z) = -0.0242 + 0.6000z^{-1} - 0.0242z^{-2}$
 $y(n) = -0.0242x(n) + 0.6000x(n-1) - 0.0242x(n-2)$
 $H(e^{j\Omega}) = e^{-j\Omega} (0.6000 - 0.0484\cos(\Omega))$

Ω (radians)	(a) $ H(e^{j\Omega}) $	(b) $ H(e^{j\Omega}) $
0	0.0054	0.5520
0.25π	0.1719	0.5661
0.50π	0.6000	0.6000
0.75π	1.0281	0.6339
1.00π	1.2054	0.6480



7.3

a. $H(z) = 0.1514 + 0.1871z^{-1} + 0.2000z^{-2} + 0.1871z^{-3} + 0.1514z^{-4}$

$$y(n) = 0.1514x(n) + 0.1871x(n-1) + 0.2x(n-2) + 0.1871x(n-3) + 0.1514x(n-4)$$

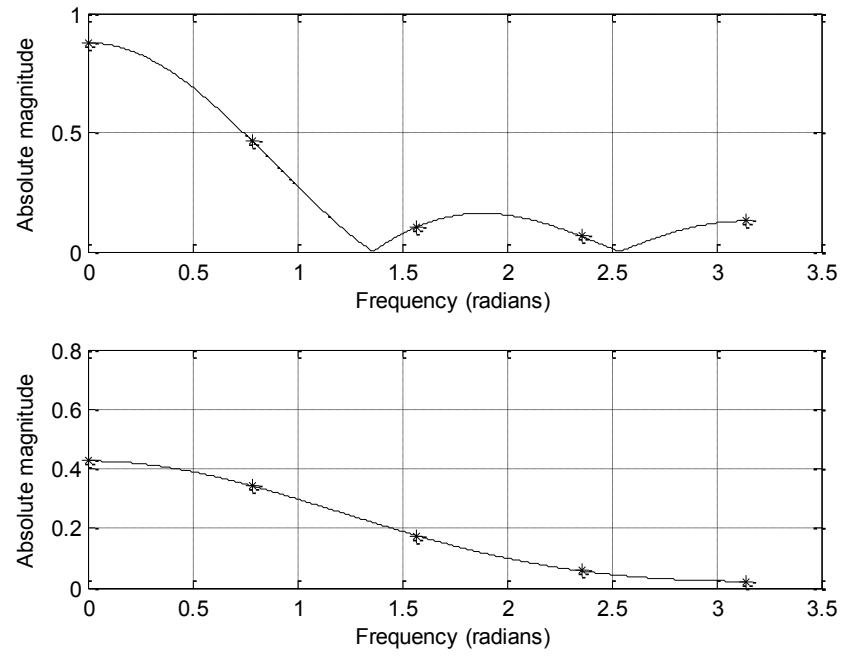
$$H(e^{j\Omega}) = e^{-j2\Omega} (0.2000 + 0.3562 \cos(\Omega) + 0.3028 \cos(2\Omega))$$

b. $H(z) = 0.0121 + 0.1010z^{-1} + 0.2000z^{-2} + 0.1010z^{-3} + 0.0121z^{-4}$

$$y(n) = 0.0121x(n) + 0.1010x(n-1) + 0.2x(n-2) + 0.1010x(n-3) + 0.0121x(n-4)$$

$$H(e^{j\Omega}) = e^{-j2\Omega} (0.2000 + 0.2020 \cos(\Omega) + 0.0242 \cos(2\Omega))$$

Ω (radians)	(a) $ H(e^{j\Omega}) $	(b) $ H(e^{j\Omega}) $
0	0.8770	0.4262
0.25π	0.4646	0.3428
0.50π	0.1028	0.1758
0.75π	0.0646	0.0572
1.00π	0.1286	0.0222



7.4

a. $H(z) = -0.0000 - 0.3183z^{-1} + 0.5000z^{-2} - 0.3183z^{-3} - 0.0000z^{-4}$

$$y(n) = -0.3183x(n-1) + 0.5x(n-2) - 0.3183x(n-3)$$

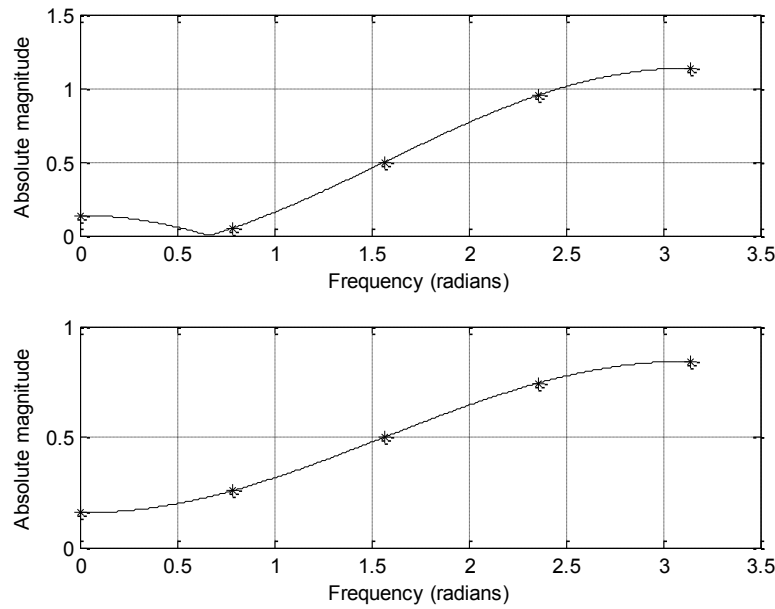
$$H(e^{j\Omega}) = e^{-j2\Omega} (0.5000 - 0.6366\cos(\Omega))$$

b. $H(z) = -0.0 - 0.1719z^{-1} + 0.5000z^{-2} - 0.1719z^{-3} - 0.0z^{-4}$

$$y(n) = -0.1719x(n-1) + 0.5x(n-2) - 0.1719x(n-3)$$

$$H(e^{j\Omega}) = e^{-j2\Omega} (0.5000 - 0.3438\cos(\Omega))$$

Ω (radians)	(a) $ H(e^{j\Omega}) $	(b) $ H(e^{j\Omega}) $
0	0.1366	0.1562
0.25π	0.0499	0.2569
0.50π	0.5000	0.5000
0.75π	0.9501	0.7431
1.00π	1.1366	0.8438



7.5

a. $H(z) = -0.0444 + 0.0117z^{-1} + 0.0500z^{-2} + 0.0117z^{-3} - 0.0444z^{-4}$

$y(n) = -0.0444x(n) + 0.0117x(n-1) + 0.0500x(n-2) + 0.0117x(n-3) - 0.0444x(n-4)$

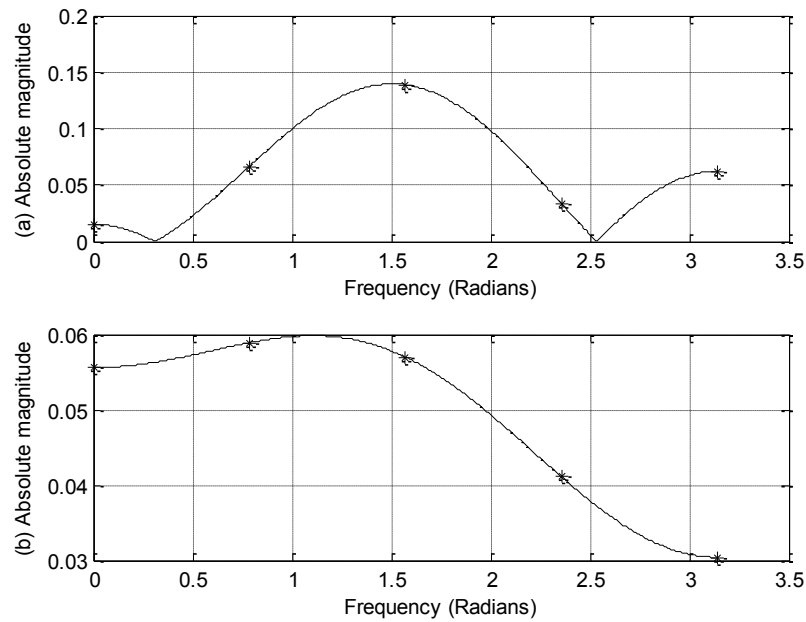
$H(e^{j\Omega}) = e^{-j2\Omega} (0.0500 + 0.0234\cos\Omega - 0.0888\cos 2\Omega)$

b. $H(z) = -0.0035 + 0.0063z^{-1} + 0.0500z^{-2} + 0.0063z^{-3} - 0.0035z^{-4}$

$y(n) = -0.0035x(n) + 0.0063x(n-1) + 0.0500x(n-2) + 0.0063x(n-3) - 0.0035x(n-4)$

$H(e^{j\Omega}) = e^{-j2\Omega} (0.0500 + 0.0126\cos\Omega - 0.0070\cos 2\Omega)$

Ω (radians)	(a) $ H(e^{j\Omega}) $	(b) $ H(e^{j\Omega}) $
0	0.0154	0.0556
0.25π	0.0665	0.0589
0.50π	0.1388	0.0570
0.75π	0.0335	0.0411
1.00π	0.0622	0.2132



7.6

a. $H(z) = 0.0444 - 0.0117z^{-1} + 0.9500z^{-2} - 0.0117z^{-3} + 0.0444z^{-4}$

$y(n) = 0.0444x(n) - 0.0117x(n-1) + 0.9500x(n-2) - 0.0117x(n-3) + 0.0444x(n-4)$

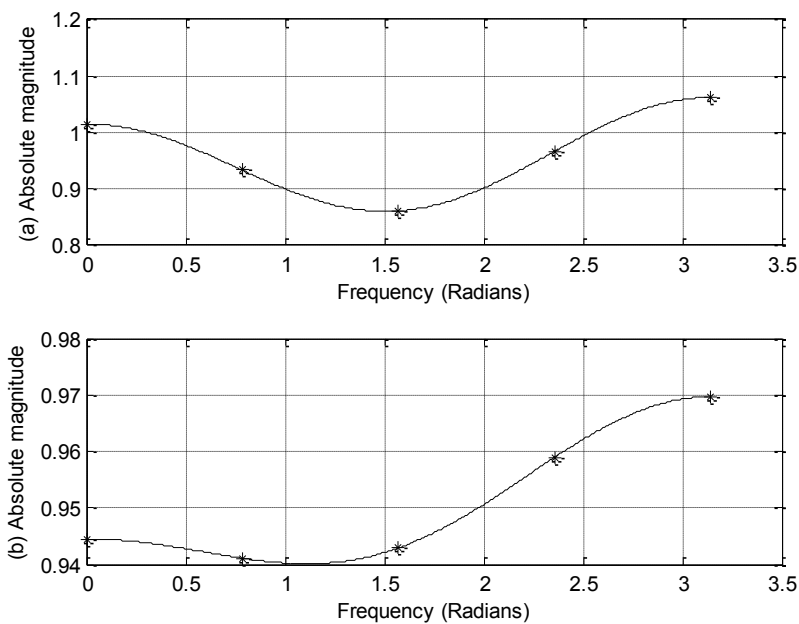
$H(e^{j\Omega}) = e^{-j2\Omega} (0.9500 - 0.0234 \cos \Omega + 0.0888 \cos 2\Omega)$

b. $H(z) = 0.0035 - 0.0063z^{-1} + 0.9500z^{-2} - 0.0063z^{-3} + 0.0035z^{-4}$

$y(n) = 0.0035x(n) - 0.0063x(n-1) + 0.9500x(n-2) - 0.0063x(n-3) + 0.0035x(n-4)$

$H(e^{j\Omega}) = e^{-j2\Omega} (0.9500 - 0.0126 \cos \Omega + 0.0070 \cos 2\Omega)$

Ω (radians)	(a) $ H(e^{j\Omega}) $	(b) $ H(e^{j\Omega}) $
0	1.0154	0.9444
0.25π	0.9335	0.9411
0.50π	0.8612	0.9430
0.75π	0.9665	0.9589
1.00π	1.0622	0.9696



7.7

a. Hanning window, b. filter length = 63, c. cut-off frequency = 1000 Hz

7.8

a. Blackman window, b. filter length = 89, c. cut-off frequency = 1750 Hz

7.9

a. Hamming window

b. filter length = 45

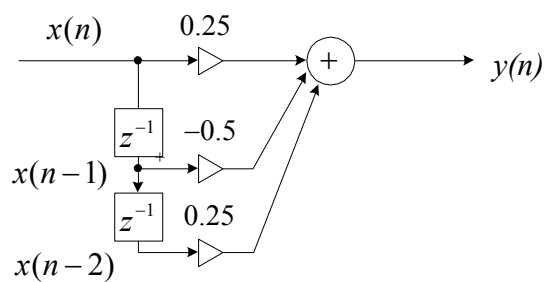
c. lower cut-off frequency = 1500 Hz, upper cut-off frequency = 2300 Hz

7.10

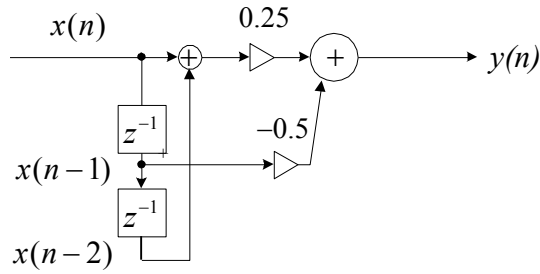
a. Blackman window, b. filter length = 111

c. lower cut-off frequency = 1400 Hz, upper cut-off frequency = 2200 Hz

7.11

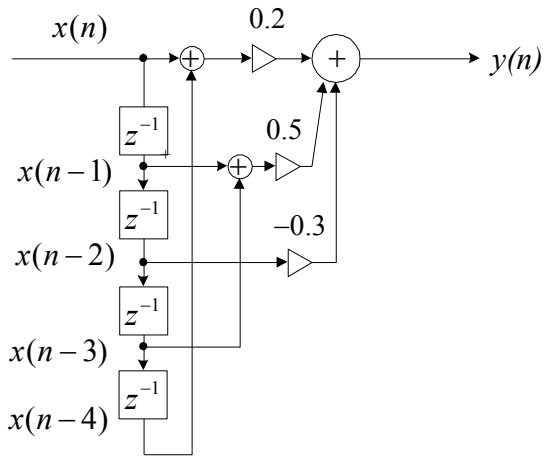
a. $y(n) = 0.25x(n) - 0.5x(n-1) + 0.25x(n-2)$ 

b. $y(n] = 0.25[x(n) + x(n-2)] - 0.5x(n-1)$



7.12

$$y(n) = 0.2(x(n) + x(n-4)) + 0.5(x(n-1) + x(n-3)) - 0.3x(n-2)$$



7.13

$$N = 3, \Omega_c = 3\pi/10, \Omega_0 = 0, H_0 = 1, \Omega_1 = 2\pi/3, H_1 = 0$$

$$H(z) = 0.3333 + 0.3333z^{-1} + 0.3333z^{-2}$$

7.14

$$N = 3, \Omega_c = \pi/2, \Omega_0 = 0, H_0 = 0, \Omega_1 = 2\pi/3, H_1 = 1$$

$$H(z) = -0.3333 + 0.6667z^{-1} - 0.3333z^{-2}$$

7.15

$$N = 5, \Omega_0 = 0, H_0 = 1, \Omega_1 = 2\pi/5, H_1 = 1, \Omega_2 = 4\pi/5, H_2 = 0$$

$$H(z) = -0.1236 + 0.3236z^{-1} + 0.6z^{-2} + 0.3236z^{-3} - 0.1236z^{-4}$$

7.16

$$N = 5, \Omega_0 = 0, H_0 = 0, \Omega_1 = 2\pi/5, H_1 = 0, \Omega_2 = 4\pi/5, H_2 = 1$$

$$H(z) = 0.1236 - 0.3236z^{-1} + 0.4z^{-2} - 0.3236z^{-3} + 0.1236z^{-4}$$

7.17

$$N = 7, \Omega_0 = 0, H_0 = 0, \Omega_1 = 2\pi/7, H_1 = 0,$$

$$\Omega_2 = 4\pi/7, H_2 = 1, \Omega_3 = 6\pi/7, H_3 = 0$$

$$H(z) = 0.1718 - 0.2574z^{-1} - 0.0636z^{-2} + 0.2857z^{-3} - 0.0636z^{-4} - 0.2574z^{-5} + 0.1781z^{-6}$$

7.18

$$N = 7, \Omega_0 = 0, H_0 = 1, \Omega_1 = 2\pi/7, H_1 = 1,$$

$$\Omega_2 = 4\pi/7, H_2 = 0, \Omega_3 = 6\pi/7, H_3 = 1$$

$$H(z) = -0.1718 + 0.2574z^{-1} + 0.0636z^{-2} + 0.7143z^{-3} + 0.0636z^{-4} + 0.2574z^{-5} - 0.1781z^{-6}$$

$$7.19 \quad \delta_p = 10^{(1/20)} - 1 = 0.1221, \delta_s = 10^{(-40/20)} = 0.01$$

$$\frac{\delta_p}{\delta_s} = \frac{0.1221}{0.01} \approx \frac{12}{1} = \frac{W_s}{W_p}$$

$$W_p = 1, W_s = 12$$

$$7.20 \quad \delta_p = 10^{(1/20)} - 1 = 0.1221, \delta_s = 10^{(-30/20)} = 0.0316$$

$$\frac{\delta_p}{\delta_s} = \frac{0.1221}{0.0316} \approx \frac{39}{10} = \frac{W_s}{W_p}$$

$$W_p = 10, W_s = 39$$

$$7.21 \quad \delta_p = 10^{(1/20)} - 1 = 0.1221, \delta_s = 10^{(-60/20)} = 0.001$$

$$\frac{\delta_p}{\delta_s} = \frac{0.1221}{0.001} \approx \frac{122}{1} = \frac{W_s}{W_p}$$

$$W_p = 1, W_s = 122$$

$$7.22 \quad \delta_p = 10^{(1/20)} - 1 = 0.1221, \delta_s = 10^{(-25/20)} = 0.0562$$

$$\frac{\delta_p}{\delta_s} = \frac{0.1221}{0.0562} \approx \frac{22}{10} = \frac{W_s}{W_p}$$

$$W_p = 10, W_s = 22$$

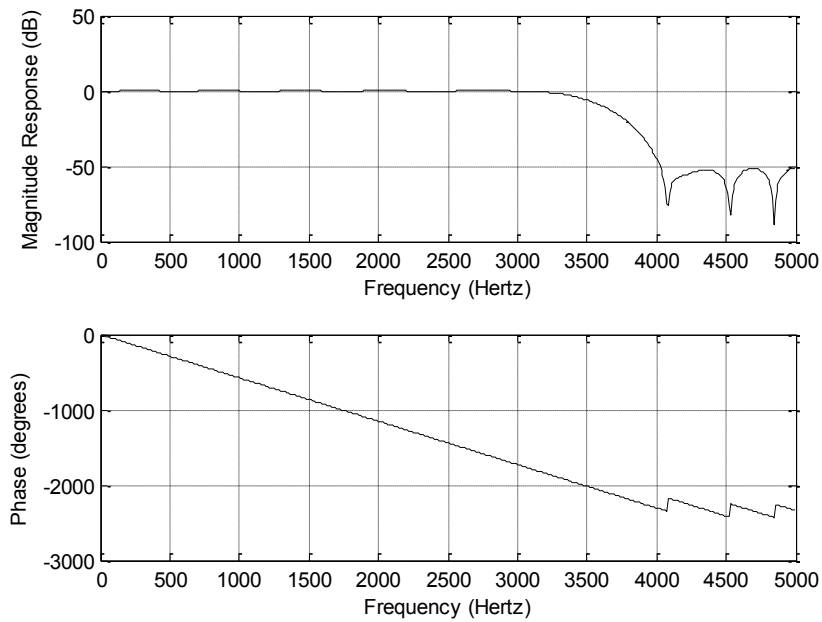
7.23

Hamming window, filter length= 33, lower cut-off frequency =3500 Hz

```
>> B=firwd(33,1,3500*2*pi/10000,0,4);
```

```
B = -0.0009  0.0019 -0.0015 -0.0012  0.0054 -0.0067  0.0000  0.0129 -0.0204
      0.0088  0.0223 -0.0506  0.0405  0.0302 -0.1461  0.2552  0.7000  0.2552
      -0.1461  0.0302  0.0405 -0.0506  0.0223  0.0088 -0.0204  0.0129  0.0000
      -0.0067  0.0054 -0.0012 -0.0015  0.0019 -0.0009
```

```
>> freqz(B,1,512,10000)
```



7.24

Hamming window, filter length=53,

lower cut-off frequency=1250 Hz, upper cut-off frequency=2250 Hz

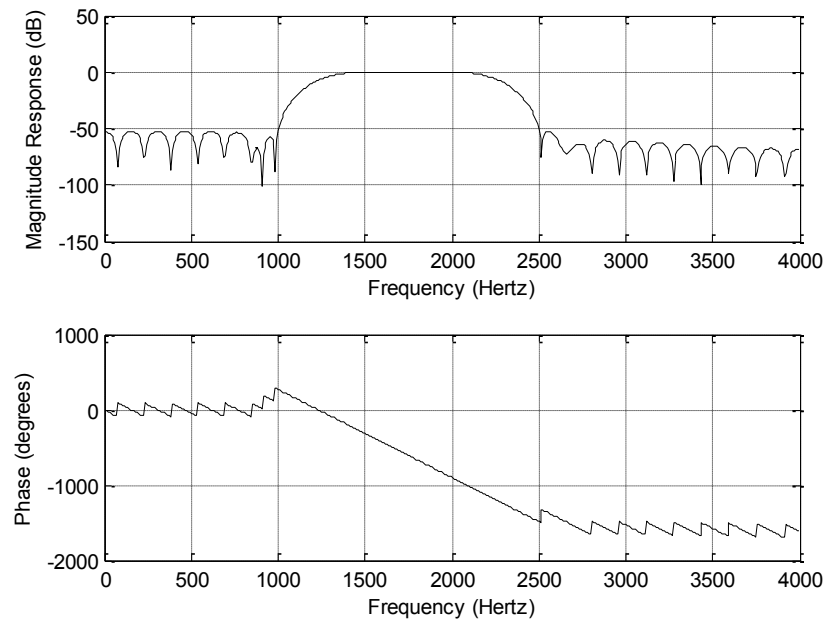
>> B=firwd(53,3,1250*2*pi/8000,2250*2*pi/8000,4);

```

B = 0.0005  0.0008  0  0.0011  0.0010 -0.0038 -0.0044  0.0040  0.0064
    -0.0009  0.0000  0.0014 -0.0144 -0.0136  0.0223  0.0289 -0.0121 -0.0200
    0 -0.0288 -0.0254  0.0898  0.1066 -0.1057 -0.2052  0.0474  0.2500
    0.0474 -0.2052 -0.1057  0.1066  0.0898 -0.0254 -0.0288  0 -0.0200
    -0.0121  0.0289  0.0223 -0.0136 -0.0144  0.0014  0.0000 -0.0009  0.0064
    0.0040 -0.0044 -0.0038  0.0010  0.0011  0  0.0008  0.0005

```

>> freqz(B,1,512,8000)

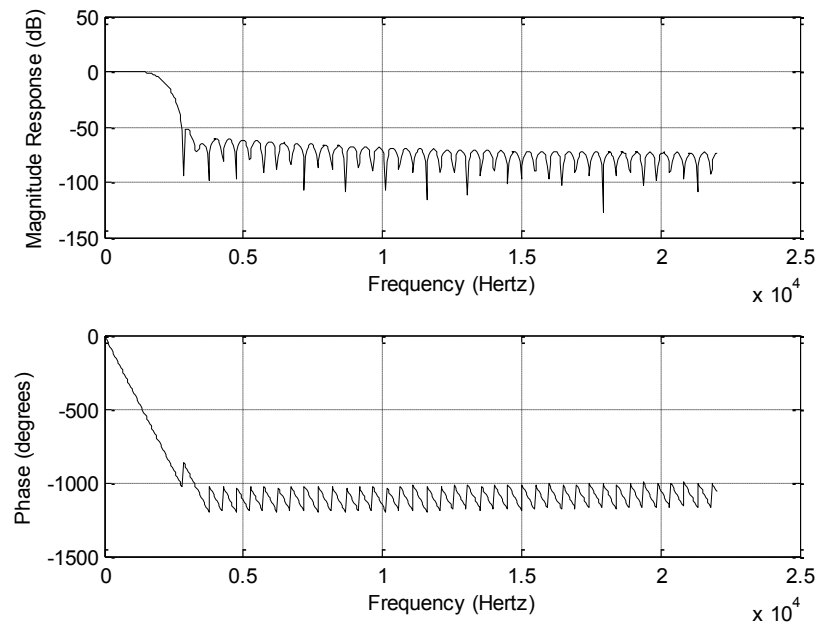


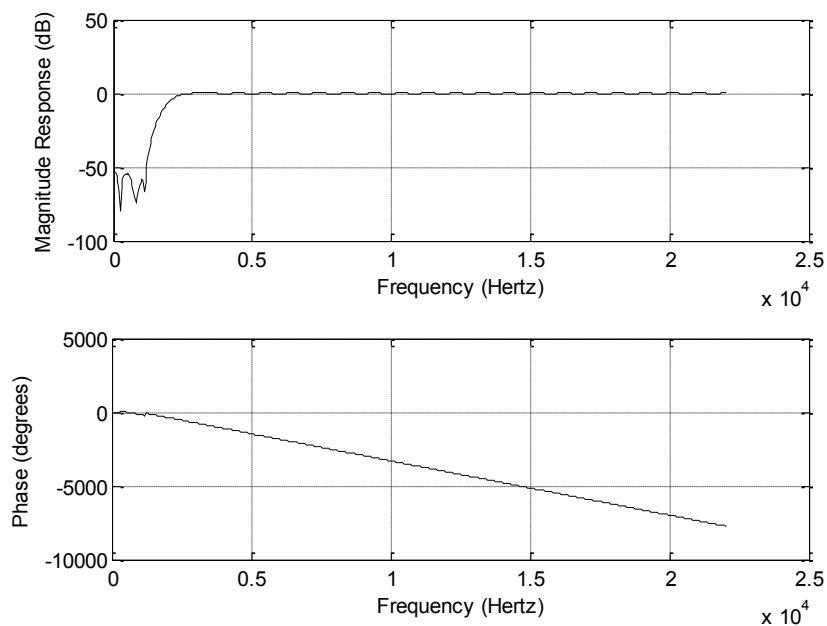
7.25

Lowpass filter: Hamming window, filter length=91, cut-off frequency =2000 Hz

High filter: Hamming window, filter length=91, cut-off frequency =2000 Hz

```
>> BL=firwd(91,1,2000*2*pi/44100,0,4);
>> freqz(BL,1,512,44100)
>> BH=firwd(91,2,0,2000*2*pi/44100,4);
>> freqz(BH,1,512,44100)
```

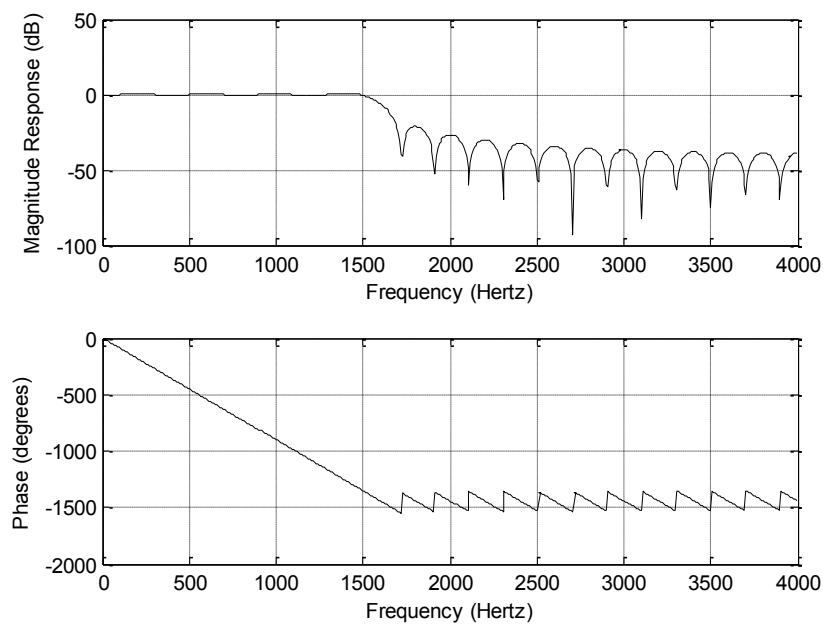




7.26

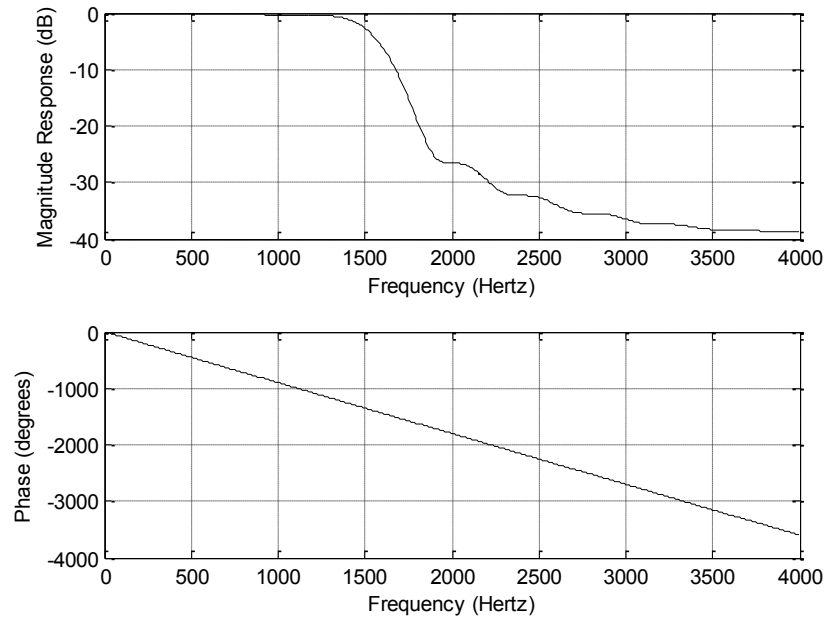
a.

```
>> B1=firwd(41,1,1600*2*pi/8000,0,1);
B = -0.0000 -0.0159 -0.0104  0.0110  0.0189 -0.0000 -0.0216 -0.0144  0.0156
    0.0275 -0.0000 -0.0336 -0.0234  0.0267  0.0505 -0.0000 -0.0757 -0.0624
    0.0935  0.3027  0.4000  0.3027  0.0935 -0.0624 -0.0757 -0.0000  0.0505
    0.0267 -0.0234 -0.0336 -0.0000  0.0275  0.0156 -0.0144 -0.0216 -0.0000
    0.0189  0.0110 -0.0104 -0.0159 -0.0000
>> freqz(B1,1,512,8000)
```



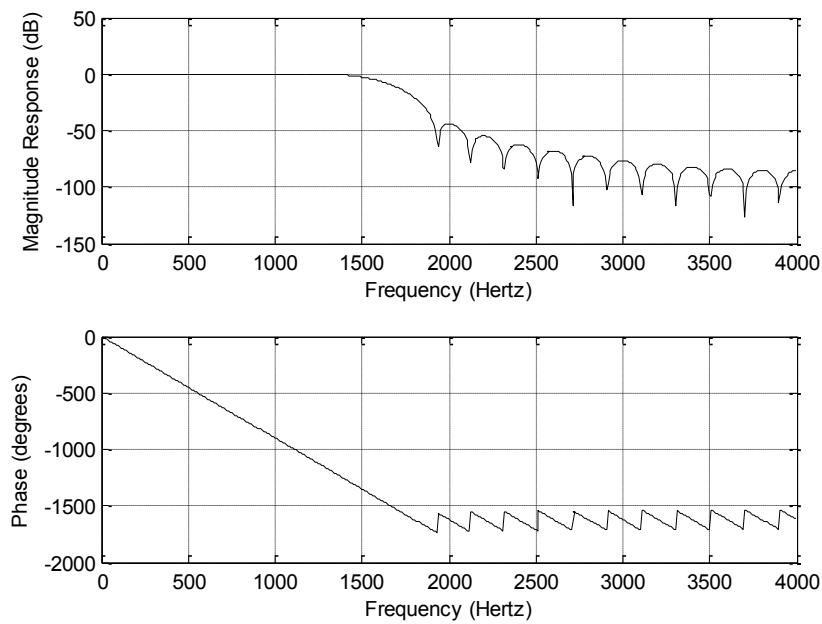
b.

```
>> B2=firwd(41,1,1600*2*pi/8000,0,2);
B = 0 -0.0008 -0.0010 0.0017 0.0038 -0.0000 -0.0065 -0.0050 0.0062
    0.0124 -0.0000 -0.0185 -0.0140 0.0174 0.0353 -0.0000 -0.0605 -0.0530
    0.0842 0.2876 0.4000 0.2876 0.0842 -0.0530 -0.0605 -0.0000 0.0353
    0.0174 -0.0140 -0.0185 -0.0000 0.0124 0.0062 -0.0050 -0.0065 -0.0000
    0.0038 0.0017 -0.0010 -0.0008 0
>> freqz(B2,1,512,8000)
```



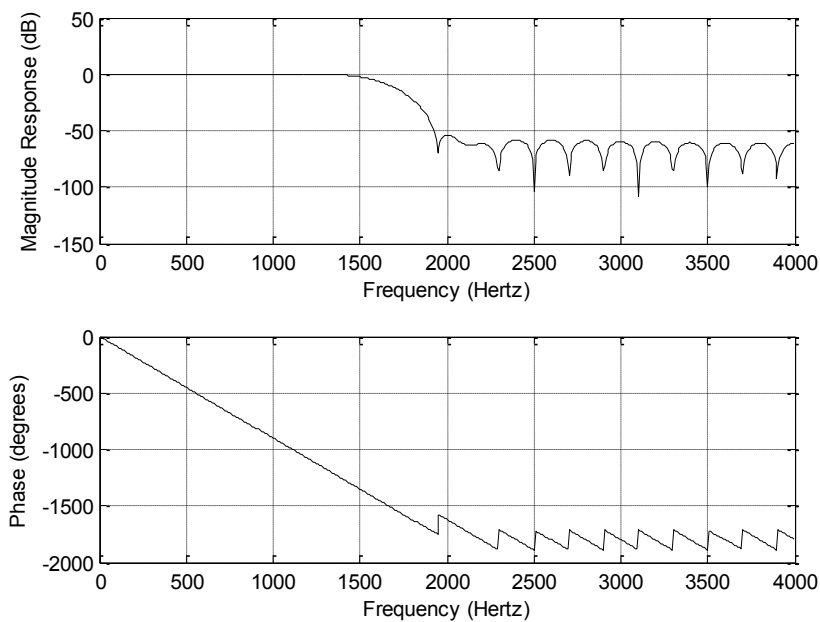
c.

```
>> B3=firwd(41,1,1600*2*pi/8000,0,3);
B = 0 -0.0001 -0.0003 0.0006 0.0018 -0.0000 -0.0045 -0.0039 0.0054
    0.0116 -0.0000 -0.0194 -0.0153 0.0194 0.0401 -0.0000 -0.0685 -0.0590
    0.0913 0.3009 0.4000 0.3009 0.0913 -0.0590 -0.0685 -0.0000 0.0401
    0.0194 -0.0153 -0.0194 -0.0000 0.0116 0.0054 -0.0039 -0.0045 -0.0000
    0.0018 0.0006 -0.0003 -0.0001 0
>> freqz(B3,1,512,8000)
```



d.

```
>> B4=firwd(41,1,1600*2*pi/8000,0,4);
B = -0.0000 -0.0014 -0.0011 0.0014 0.0032 -0.0000 -0.0058 -0.0048 0.0062
     0.0129 -0.0000 -0.0206 -0.0160 0.0200 0.0409 -0.0000 -0.0690 -0.0592
     0.0914 0.3010 0.4000 0.3010 0.0914 -0.0592 -0.0690 -0.0000 0.0409
     0.0200 -0.0160 -0.0206 -0.0000 0.0129 0.0062 -0.0048 -0.0058 -0.0000
     0.0032 0.0014 -0.0011 -0.0014 -0.0000
>> freqz(B4,1,512,8000)
```



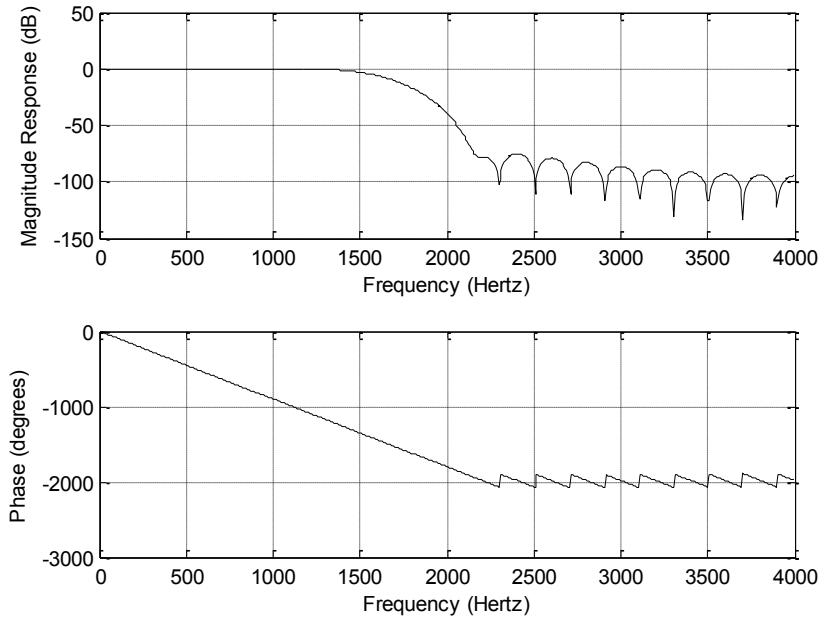
e.

```
>> B5=firwd(41,1,1600*2*pi/8000,0,5);
```

```

B = 0.0000 -0.0000 -0.0001 0.0002 0.0008 -0.0000 -0.0022 -0.0021 0.0031
    0.0073 -0.0000 -0.0142 -0.0119 0.0160 0.0348 -0.0000 -0.0643 -0.0569
    0.0898 0.2997 0.4000 0.2997 0.0898 -0.0569 -0.0643 -0.0000 0.0348
    0.0160 -0.0119 -0.0142 -0.0000 0.0073 0.0031 -0.0021 -0.0022 -0.0000
    0.0008 0.0002 -0.0001 -0.0000 0.0000
>> freqz(B5,1,512,8000)

```



7.27

a.

```

>> B1=firwd(21,1,1000*2*pi/8000,0,4);
B = 0.0025 0.0026 -0.0000 -0.0087 -0.0211 -0.0243 0.0000 0.0608 0.1452
    0.2200 0.2500 0.2200 0.1452 0.0608 0.0000 -0.0243 -0.0211 -0.0087
    -0.0000 0.0026 0.0025
>> [h1 f]=freqz(B1,1,512,8000);

```

b.

```

>> B2=firwd(31,1,1000*2*pi/8000,0,4);
B = -0.0012 -0.0020 -0.0021 0.0000 0.0048 0.0099 0.0099 -0.0000 -0.0189
    -0.0362 -0.0347 0.0000 0.0684 0.1528 0.2228 0.2500 0.2228 0.1528
    0.0684 0.0000 -0.0347 -0.0362 -0.0189 -0.0000 0.0099 0.0099 0.0048
    0.0000 -0.0021 -0.0020 -0.0012
>> [h2 f]=freqz(B2,1,512,8000);

```

c.

```

>> B3=firwd(41,1,1000*2*pi/8000,0,4);
B = 0.0000 0.0010 0.0018 0.0017 -0.0000 -0.0032 -0.0061 -0.0057 0.0000
    0.0096 0.0172 0.0153 -0.0000 -0.0241 -0.0430 -0.0390 0.0000 0.0713
    0.1556 0.2238 0.2500 0.2238 0.1556 0.0713 0.0000 -0.0390 -0.0430

```

```

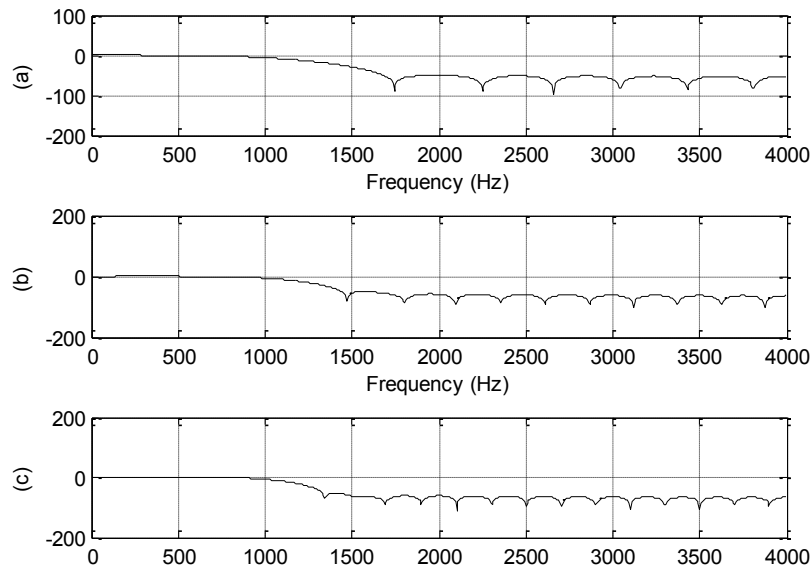
-0.0241 -0.0000 0.0153 0.0172 0.0096 0.0000 -0.0057 -0.0061 -0.0032
-0.0000 0.0017 0.0018 0.0010 0.0000
>> [h3 f]=freqz(B3,1,512,8000);

```

```

>> subplot(3,1,1),plot(f,20*log10(abs(h1)));grid
>> subplot(3,1,2),plot(f,20*log10(abs(h2)));grid
>> subplot(3,1,3),plot(f,20*log10(abs(h3)));grid

```



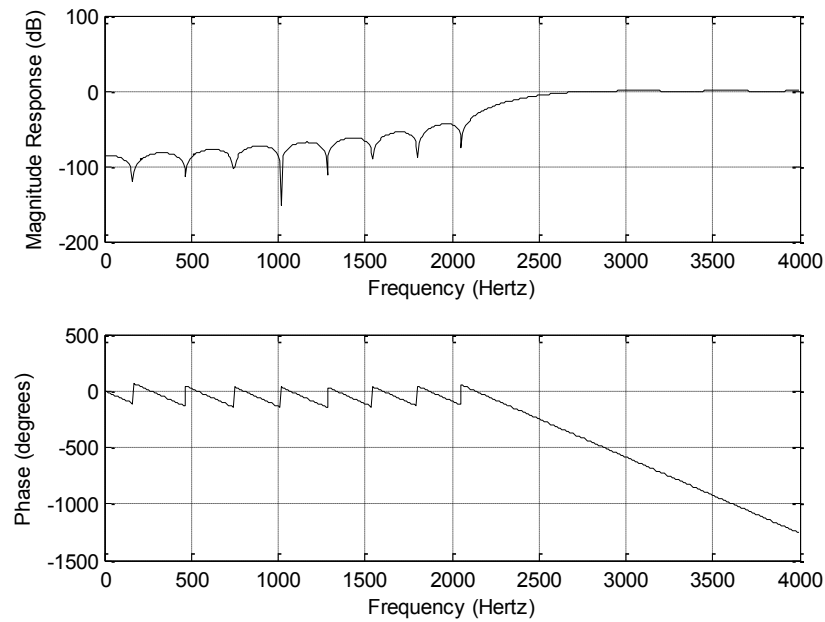
7.28

a.

```

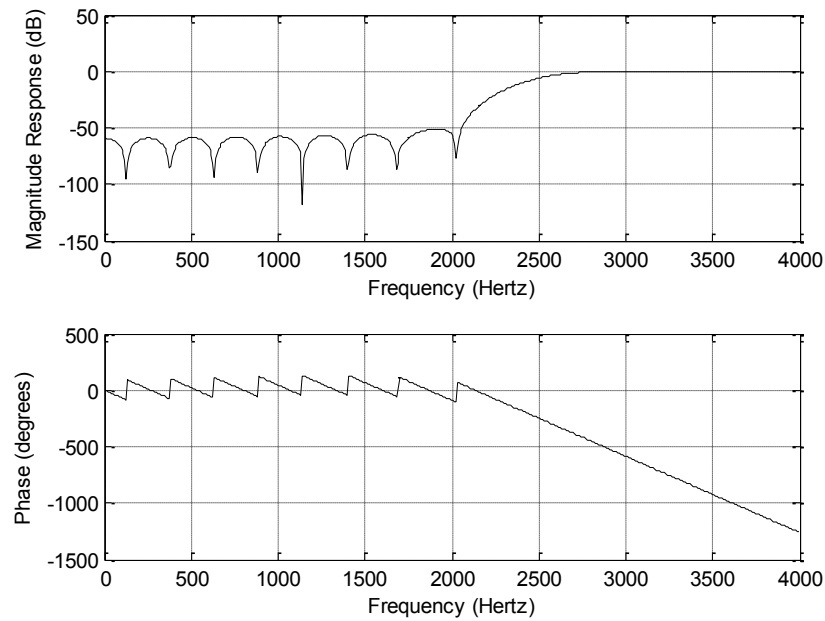
>> B1=firwd(31,2,0,2500*2*pi/8000,3);
B =
    0 -0.0002 -0.0004 0.0025 -0.0018 -0.0056 0.0113 -0.0000 -0.0232
    0.0246 0.0183 -0.0664 0.0367 0.1077 -0.2909 0.3750 -0.2909 0.1077
    0.0367 -0.0664 0.0183 0.0246 -0.0232 -0.0000 0.0113 -0.0056 -0.0018
    0.0025 -0.0004 -0.0002 0
>> freqz(B1,1,512,8000)

```



b.

```
>> B2=firwd(31,2,0,2500*2*pi/8000,4);
B = 0.0016 -0.0014 -0.0011 0.0045 -0.0026 -0.0070 0.0130 -0.0000 -0.0247
    0.0256 0.0188 -0.0675 0.0370 0.1081 -0.2911 0.3750 -0.2911 0.1081
    0.0370 -0.0675 0.0188 0.0256 -0.0247 -0.0000 0.0130 -0.0070 -0.0026
    0.0045 -0.0011 -0.0014 0.0016
>> freqz(B2,1,512,8000)
```



c.

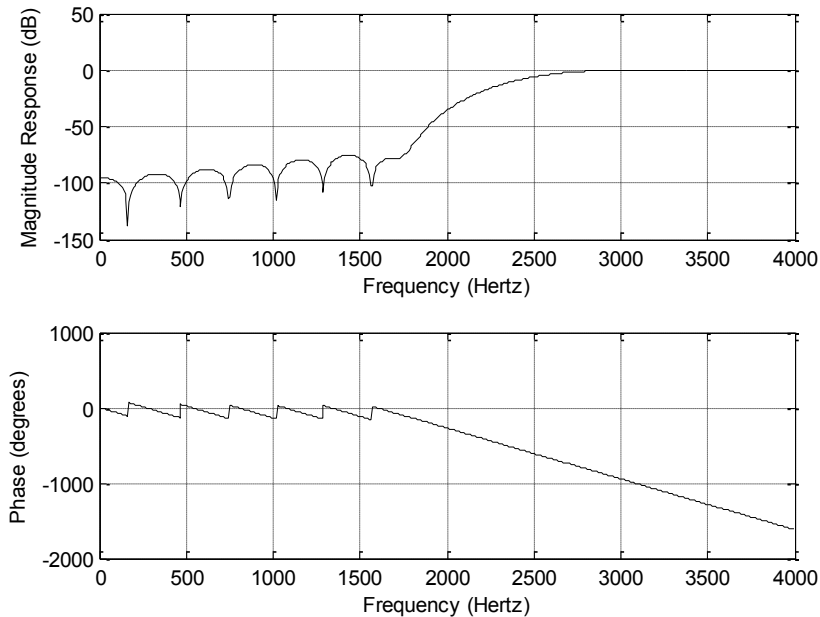
```
>> B3=firwd(31,2,0,2500*2*pi/8000,5);
```

```

B = -0.0000 -0.0001 -0.0002 0.0011 -0.0009 -0.0029 0.0066 -0.0000 -0.0166
     0.0191 0.0153 -0.0594 0.0345 0.1047 -0.2888 0.3750 -0.2888 0.1047
     0.0345 -0.0594 0.0153 0.0191 -0.0166 -0.0000 0.0066 -0.0029 -0.0009
     0.0011 -0.0002 -0.0001 -0.0000

```

```
>> freqz(B3,1,512,8000)
```



7.29

a.

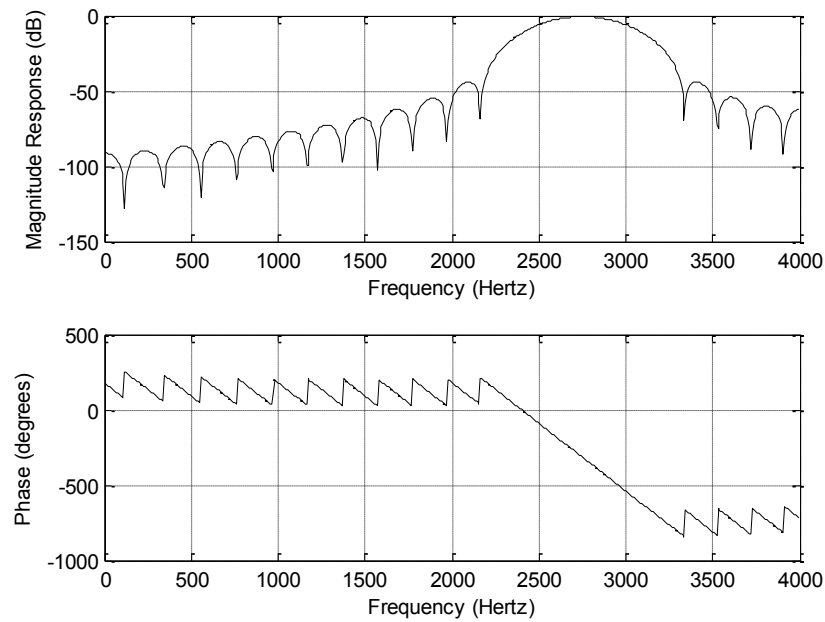
```
>> B1=firwd(41,3,2500*2*pi/8000,3000*2*pi/8000,3);
```

```

B = 0 0.0001 -0.0001 -0.0002 -0.0000 0.0007 0.0014 -0.0073 0.0092
     0.0040 -0.0272 0.0334 -0.0000 -0.0539 0.0719 -0.0176 -0.0720 0.1093
    -0.0455 -0.0686 0.1250 -0.0686 -0.0455 0.1093 -0.0720 -0.0176 0.0719
    -0.0539 -0.0000 0.0334 -0.0272 0.0040 0.0092 -0.0073 0.0014 0.0007
    -0.0000 -0.0002 -0.0001 0.0001 0

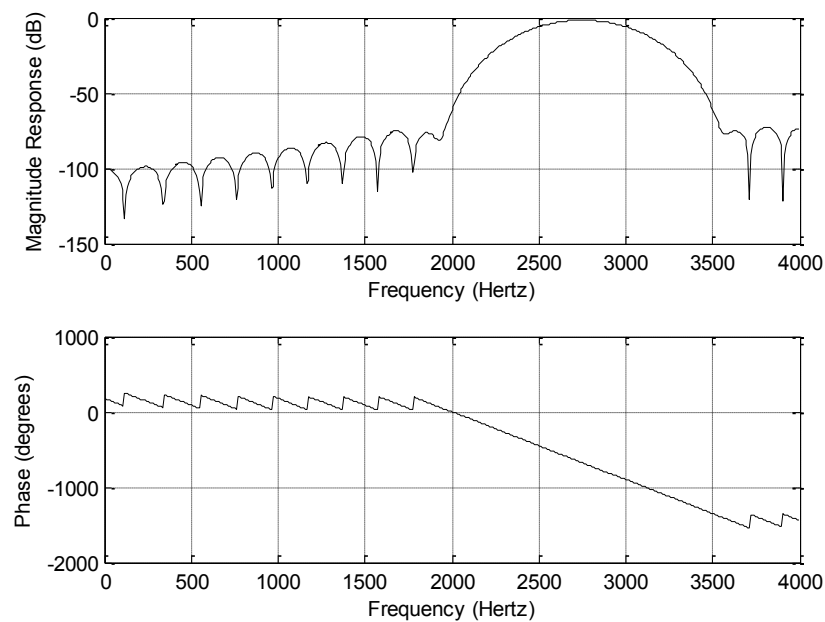
```

```
>> freqz(B1,1,512,8000)
```



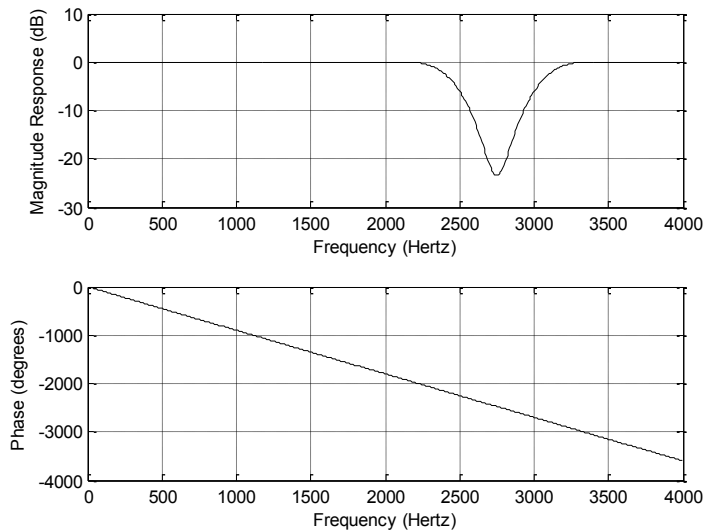
b.

```
>> B2=firwd(41,3,2500*2*pi/8000,3000*2*pi/8000,5);
B = 0.0000 0.0000 -0.0000 -0.0001 -0.0000 0.0003 0.0007 -0.0039 0.0053
     0.0025 -0.0185 0.0244 -0.0000 -0.0445 0.0624 -0.0160 -0.0676 0.1055
    -0.0448 -0.0683 0.1250 -0.0683 -0.0448 0.1055 -0.0676 -0.0160 0.0624
    -0.0445 -0.0000 0.0244 -0.0185 0.0025 0.0053 -0.0039 0.0007 0.0003
    -0.0000 -0.0001 -0.0000 0.0000 0.0000
>> freqz(B2,1,512,8000)
```



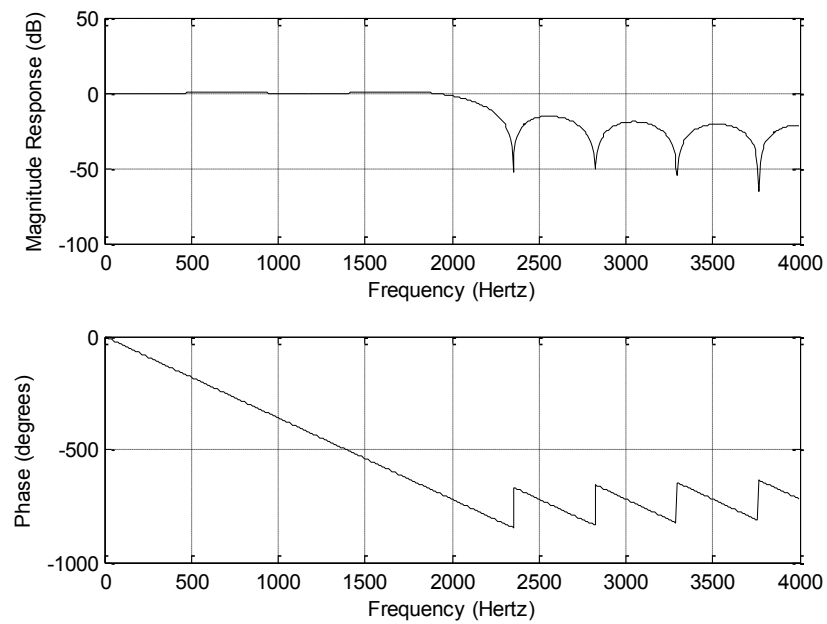
7.30

```
>> B=firwd(41,4,2500*2*pi/8000,3000*2*pi/8000,4);
B = 0.0013 -0.0016 0.0005 0.0005 0.0000 -0.0010 -0.0018 0.0088 -0.0106
    -0.0044 0.0293 -0.0353 0.0000 0.0555 -0.0734 0.0179 0.0726 -0.1098
    0.0456 0.0686 0.8750 0.0686 0.0456 -0.1098 0.0726 0.0179 -0.0734
    0.0555 0.0000 -0.0353 0.0293 -0.0044 -0.0106 0.0088 -0.0018 -0.0010
    0.0000 0.0005 0.0005 -0.0016 0.0013
>> freqz1(B,1,512,8000)
```



7.31

```
>> B=firfs(17,[1 1 1 1 1 0 0 0 0])
B = 0.0398 -0.0488 -0.0346 0.0660 0.0315 -0.1075 -0.0299 0.3188 0.5294
    0.3188 -0.0299 -0.1075 0.0315 0.0660 -0.0346 -0.0488 0.0398
>> freqz(B,1,512,8000)
```

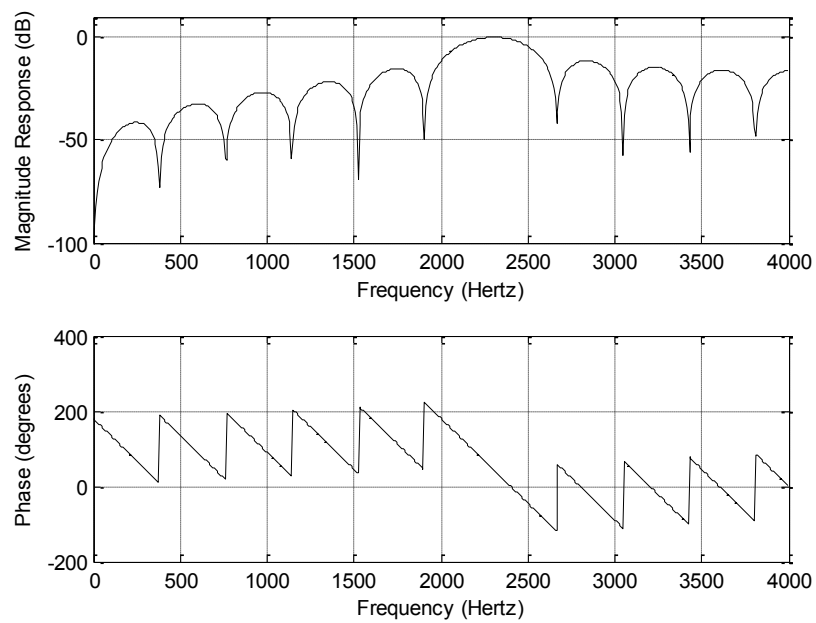


7.32

```
>> B=firfs(21,[0 0 0 0 0 0 1 0 0 0 0])
```

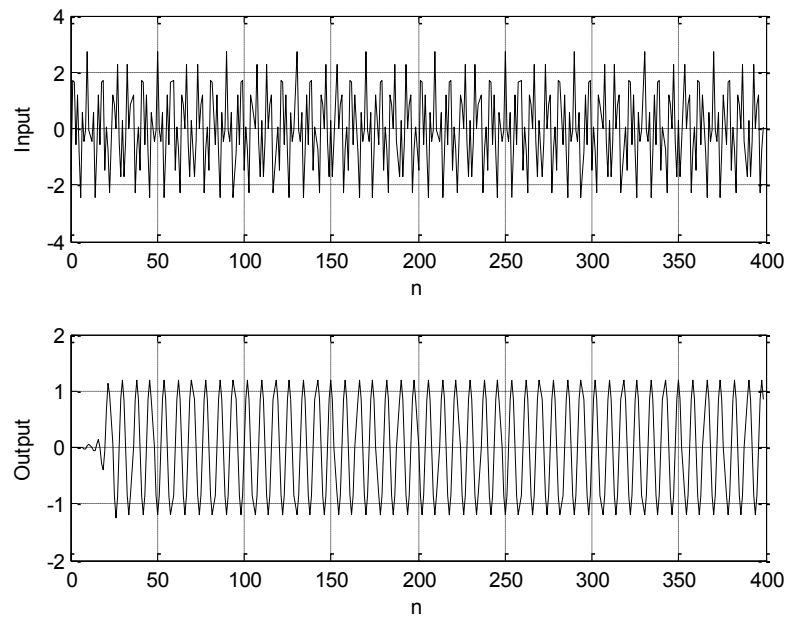
```
B = 0.0594 -0.0858 -0.0212 0.0952 -0.0212 -0.0858 0.0594 0.0594 -0.0858  
-0.0212 0.0952 -0.0212 -0.0858 0.0594 0.0594 -0.0858 -0.0212 0.0952  
-0.0212 -0.0858 0.0594
```

```
>> freqz(B,1,512,8000)
```



7.33

```
>> B=firwd(41,1,1600*2*pi/8000,0,4);
>> x=1.2*sin(2*pi*1000*[0:1:399]/8000)-1.5*cos(2*pi*2800*[0:1:399]/8000);
>> y=filter(B,1,x);
>> n=0:1:399;
>> x=1.2*sin(2*pi*1000*n/8000)-1.5*cos(2*pi*2800*n/8000);
>> y=filter(B,1,x);
>> subplot(2,1,1),plot(n,x);xlabel('n');ylabel('Input');grid
>> subplot(2,1,2),plot(n,y);xlabel('n');ylabel('Output');grid
```

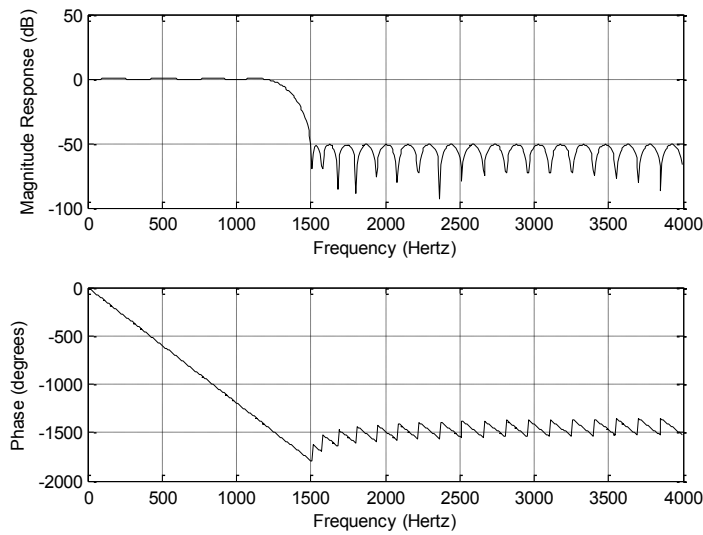


7.34

$$\delta_p = 10^{\left(\frac{1}{20}\right)} - 1 = 0.1220, \quad \delta_s = 10^{\left(\frac{-40}{20}\right)} = 0.01, \quad W_s = 12 \text{ and } W_p = 1$$

```
>> fs=8000;
>> f=[0 0.3 0.375 1];
>> m=[1 1 0 0];
>> W=[1 12];
>> b=remez(53,f,m,W);
>> freqz(b,1,512,fs)
```

```
B=[ 0.0020 -0.0003 -0.0043 -0.0086 -0.0089 -0.0032 0.0049 0.0083 0.0029
-0.0073 -0.0121 -0.0048 0.0098 0.0173 0.0075 -0.0134 -0.0252 -0.0120
0.0190 0.0387 0.0204 -0.0300 -0.0684 -0.0420 0.0656 0.2114 0.3156
0.3156 0.2114 0.0656 -0.0420 -0.0684 -0.0300 0.0204 0.0387 0.0190
-0.0120 -0.0252 -0.0134 0.0075 0.0173 0.0098 -0.0048 -0.0121 -0.0073
0.0029 0.0083 0.0049 -0.0032 -0.0089 -0.0086 -0.0043 -0.0003 0.0020]
```



7.35

$$\delta_p = 10^{\left(\frac{1}{20}\right)} - 1 = 0.1220, \quad \delta_s = 10^{\left(\frac{-40}{20}\right)} = 0.01, \quad W_s = 12 \text{ and } W_p = 1$$

```

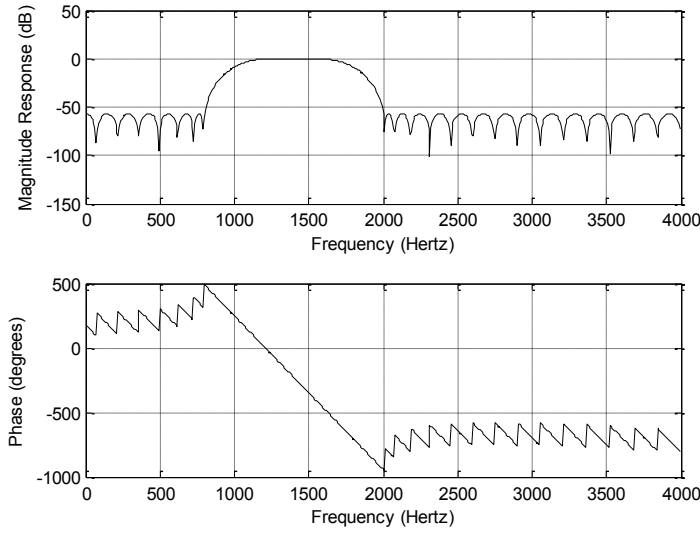
>> fs=8000;
>> f=[0 0.2 0.3 0.4 0.5 1];
>> m=[0 0 1 1 0 0];
>> W=[12 1 12];
>> b=remez(53,f,m,W);
>> freqz1(b,1,512,fs)

```

```

B = [ -0.0004 -0.0013 -0.0000 0.0003 -0.0006 -0.0002 0.0041 0.0068 -
0.0010
-0.0140 -0.0140 0.0049 0.0207 0.0126 -0.0057 -0.0066 0.0027 -0.0108
-0.0393 -0.0232 0.0545 0.1041 0.0301 -0.1138 -0.1533 -0.0139 0.1565
0.1565 -0.0139 -0.1533 -0.1138 0.0301 0.1041 0.0545 -0.0232 -0.0393
-0.0108 0.0027 -0.0066 -0.0057 0.0126 0.0207 0.0049 -0.0140 -0.0140
-0.0010 0.0068 0.0041 -0.0002 -0.0006 0.0003 -0.0000 -0.0013 -0.0004

```



7.38

$$\begin{aligned}
 h(n) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\Omega}) e^{j\Omega n} d\Omega = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} j\Omega e^{j\Omega n} d\Omega \\
 &= \frac{1}{2\pi} \left[\frac{\Omega}{n} e^{j\Omega n} - \frac{1}{jn^2} e^{j\Omega n} \right]_{-\pi/2}^{\pi/2} = \frac{\cos(n\pi/2)}{2n} - \frac{\sin(n\pi/2)}{n^2\pi}
 \end{aligned}$$

7.39

$$\begin{aligned}
 h(n) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\Omega}) e^{j\Omega n} d\Omega = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos \Omega / 2 e^{j\Omega n} d\Omega = \frac{1}{4\pi} \int_{-\pi}^{\pi} [e^{j\Omega(n+1/2)} + e^{j\Omega(n-1/2)}] d\Omega \\
 &= \frac{1}{4\pi} \left[\frac{e^{j\Omega(n+1/2)}}{j(n+1/2)} + \frac{e^{j\Omega(n-1/2)}}{j(n-1/2)} \right]_{-\pi}^{\pi} = \frac{\sin[(n-1/2)\pi/2]}{2\pi(n+1/2)} + \frac{\sin[(n-1/2)\pi/2]}{2\pi(n-1/2)}
 \end{aligned}$$

7.40

$$\begin{aligned}
 h(n) &= \frac{1}{N} \left[H_0 + 2 \sum_{k=1}^M H_k \cos \left(\frac{2\pi k(n-M)}{2M+1} \right) \right], \text{ for } 0 \leq n < M \\
 \Omega_k &= 2\pi k / (2M+1), \quad H_k = |j\Omega_k| = \left(\frac{2\pi k}{2M+1} \right) \text{ and } H_0 = 0 \\
 h(n) &= \frac{1}{N} \left[2 \sum_{k=1}^M \left(\frac{2\pi k}{2M+1} \right) \cos \left(\frac{2\pi k(n-M)}{2M+1} \right) \right], \text{ for } 0 \leq n < M
 \end{aligned}$$

7.41

$$\begin{aligned}
 h(n) &= \frac{1}{N} \left[H_0 + 2 \sum_{k=1}^M H_k \cos \left(\frac{2\pi k(n-M)}{2M+1} \right) \right], \text{ for } 0 \leq n < M \\
 \Omega_k &= 2\pi k / (2M+1) \text{ and } H_k = \cos \Omega_k / 2 = \cos[\pi k / (2M+1)], \quad H_0 = 1 \\
 h(n) &= \frac{1}{N} \left[1 + 2 \sum_{k=1}^M \cos \left(\frac{\pi k}{2M+1} \right) \cos \left(\frac{2\pi k(n-M)}{2M+1} \right) \right], \text{ for } 0 \leq n < M
 \end{aligned}$$

7.42

$$\begin{aligned}
 h(n) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\Omega}) e^{j\Omega n} d\Omega = \frac{1}{2\pi} \int_{-\pi}^{\pi} (j\Omega)^2 e^{j\Omega n} d\Omega = -\frac{1}{2\pi} \int_{-\pi}^{\pi} \Omega^2 d\Omega \\
 &= -\frac{1}{2\pi} \left[\frac{\Omega^2}{jn} e^{j\Omega n} - \frac{2\Omega}{(jn)^2} e^{j\Omega n} + \frac{2}{(jn)^3} e^{j\Omega n} \right]_{-\pi}^{\pi} = -\frac{\pi \sin n\Omega}{n} - \frac{2 \cos n\Omega}{n^2} + \frac{2 \sin n\Omega}{n^2 \pi}
 \end{aligned}$$

7.43

$$\begin{aligned}
 h(n) &= \frac{1}{N} \left[H_0 + 2 \sum_{k=1}^M H_k \cos \left(\frac{2\pi k(n-M)}{2M+1} \right) \right], \text{ for } 0 \leq n < M \\
 \Omega_k &= 2\pi k / (2M+1), \quad H_k = |j\Omega_k|^2 = \left(\frac{2\pi k}{2M+1} \right)^2 \text{ and } H_0 = 0 \\
 h(n) &= \frac{1}{N} \left[2 \sum_{k=1}^M \left(\frac{2\pi k}{2M+1} \right)^2 \cos \left(\frac{2\pi k(n-M)}{2M+1} \right) \right], \text{ for } 0 \leq n < M
 \end{aligned}$$

4.44

$$\begin{aligned}
 h(n) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\Omega}) e^{j\Omega n} d\Omega = \frac{1}{2\pi} \int_{-\pi/2}^0 j e^{j\Omega n} d\Omega - \frac{1}{2\pi} \int_0^{\pi/2} j e^{j\Omega n} d\Omega \\
 &= \frac{1}{2\pi} \left[\frac{1}{n} e^{j\Omega n} \right]_{-\pi/2}^0 - \frac{1}{2\pi} \left[\frac{1}{n} e^{j\Omega n} \right]_0^{\pi/2} = \frac{1 - \cos(n\pi/2)}{n\pi} = \frac{2 \sin^2(n\pi/4)}{n\pi}
 \end{aligned}$$

Chapter 8

8.1

$$\begin{aligned}
 H(z) &= \frac{0.3333 + 0.3333z^{-1}}{1 - 0.3333z^{-1}} \\
 y(n) &= 0.3333x(n) + 0.3333x(n-1) + 0.3333y(n-1)
 \end{aligned}$$

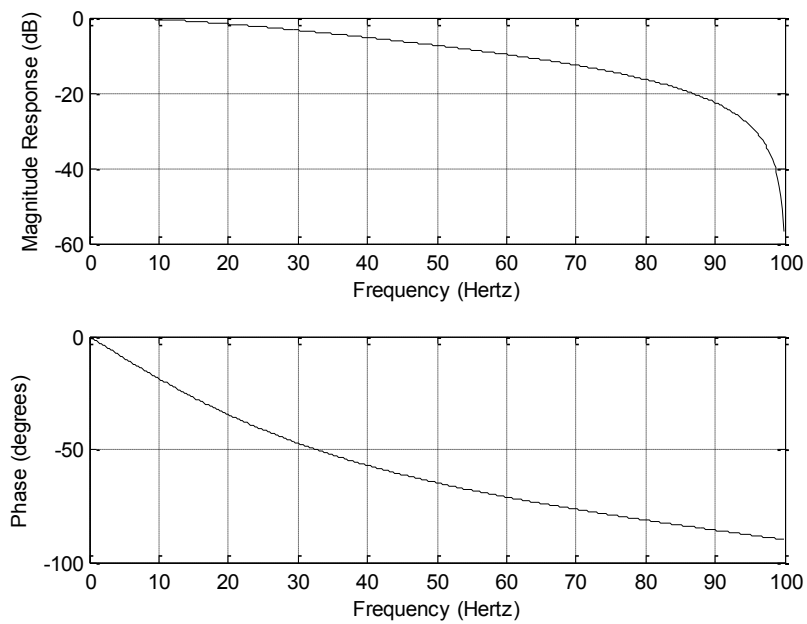
8.2

a.

$$\begin{aligned}
 H(z) &= \frac{0.3375 - 0.3375z^{-1}}{1 - 0.3249z^{-1}} \\
 y(n) &= 0.3375x(n) + 0.3375x(n-1) + 0.3249y(n-1)
 \end{aligned}$$

b.

$$\text{freqz}([0.3375 \ 0.3375], [1 \ -0.3249], 512, 200)$$



8.3

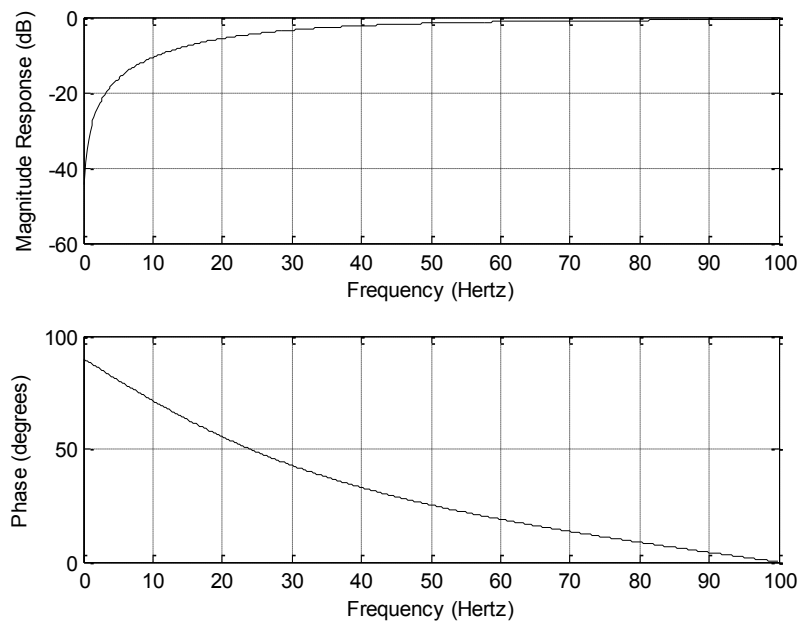
a.

$$H(z) = \frac{0.6625 - 0.6625z^{-1}}{1 - 0.3249z^{-1}}$$

$$y(n] = 0.6225x(n] - 0.6225x(n-1] + 0.3249y(n-1]$$

b.

```
>> freqz([0.6225 -0.6225],[1 -0.3594],512,200)
```



8.4

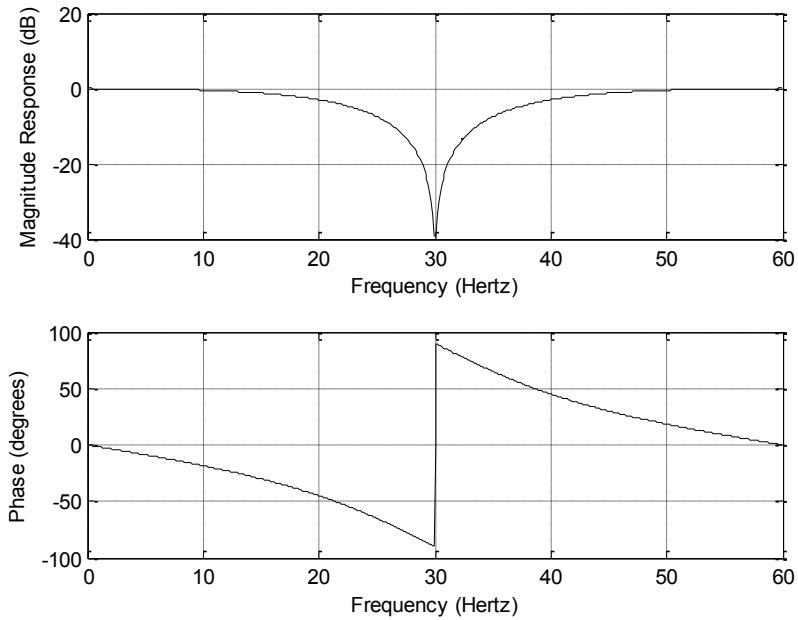
a.

$$H(z) = \frac{0.6340 - 0.6340z^{-2}}{1 + 0.2679z^{-2}} \quad \text{b.}$$

$$y(n] = 0.6340x(n) - 0.6340x(n - 2) - 0.2679y(n - 2)$$

b.

```
>> freqz([0.6340 0 0.6340],[1 0 0.2679],512,120)
```



8.5

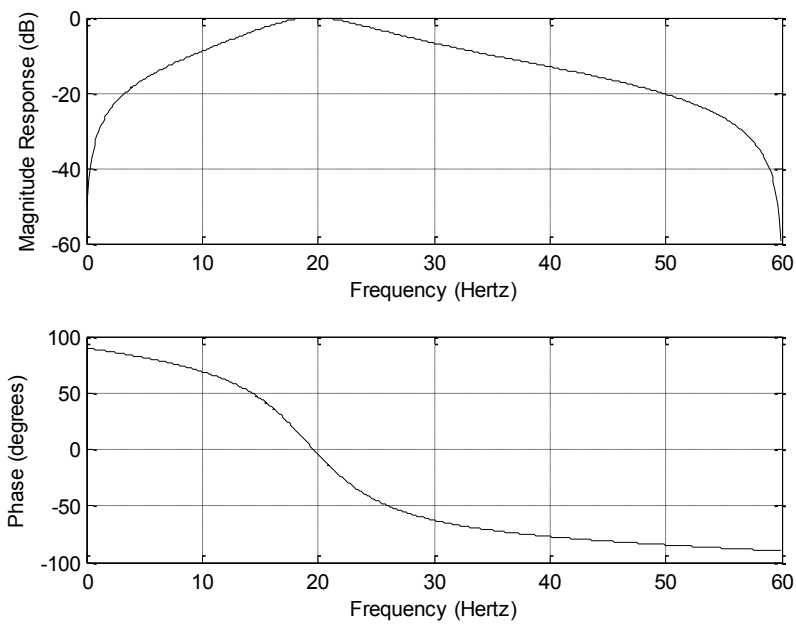
a.

$$H(z) = \frac{0.2113 - 0.2113z^{-2}}{1 - 0.8165z^{-1} + 0.5774z^{-2}}$$

$$y(n] = 0.2113x(n) - 0.2113x(n - 2) + 0.8165y(n - 1) - 0.5774y(n - 2)$$

b.

```
>> freqz([0.2113 0 -0.2113],[1 -0.8165 0.5774],512,120)
```



8.6

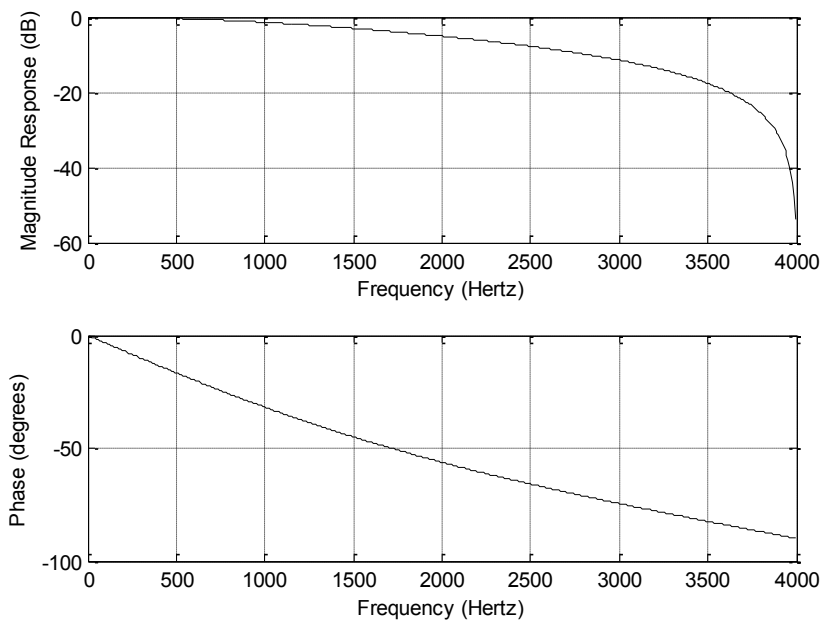
a.

$$H(z) = \frac{0.4005 + 0.4005z^{-1}}{1 - 0.1989z^{-1}}$$

$$y(n] = 0.4005x(n] + 0.4005x(n-1] + 0.1989y(n-1]$$

b.

>> freqz([0.4005 0.4005],[1 -0.1989],512,8000)



8.7

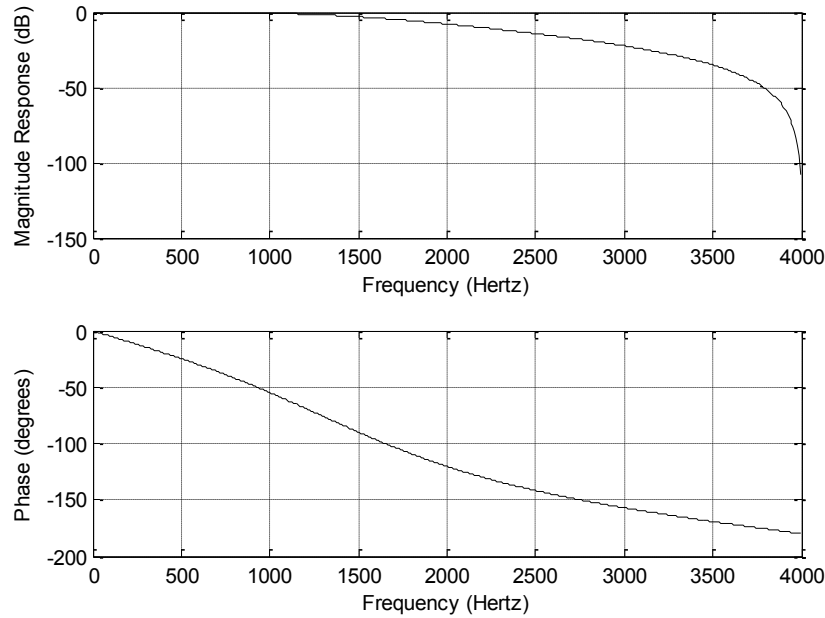
a.

$$H(z) = \frac{0.1867 + 0.3734z^{-1} + 0.1867z^{-2}}{1 - 0.4629z^{-1} + 0.2097z^{-2}}$$

$$y(n] = 0.1867x(n) + 0.3734x(n-1) + 0.1867x(n-2) + 0.4629y(n-1) - 0.2097y(n-2)$$

b.

```
>> freqz([0.1867 0.3734 0.1867],[1 -0.4629 0.2097],512,8000)
```



8.8

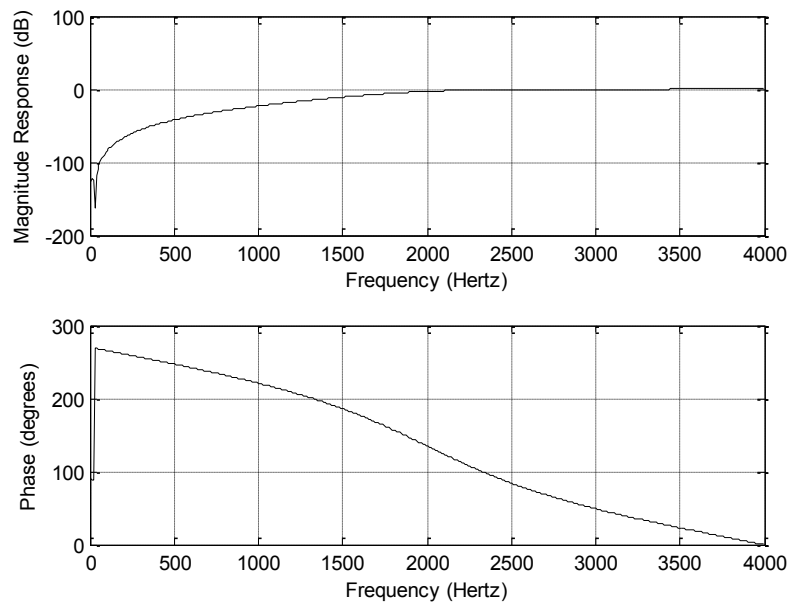
a.

$$H(z) = \frac{0.1667 - 0.5000z^{-1} + 0.5000z^{-2} - 0.1667z^{-3}}{1 + 0.3333z^{-2}}$$

$$y(n] = 0.1667x(n) - 0.5x(n-1) + 0.5x(n-2) - 0.1667x(n-3) - 0.3333y(n-2)$$

b.

```
>> freqz([0.1667 -0.5 0.5 -0.1667],[1 0 0.3333 0],512,8000)
```



8.9

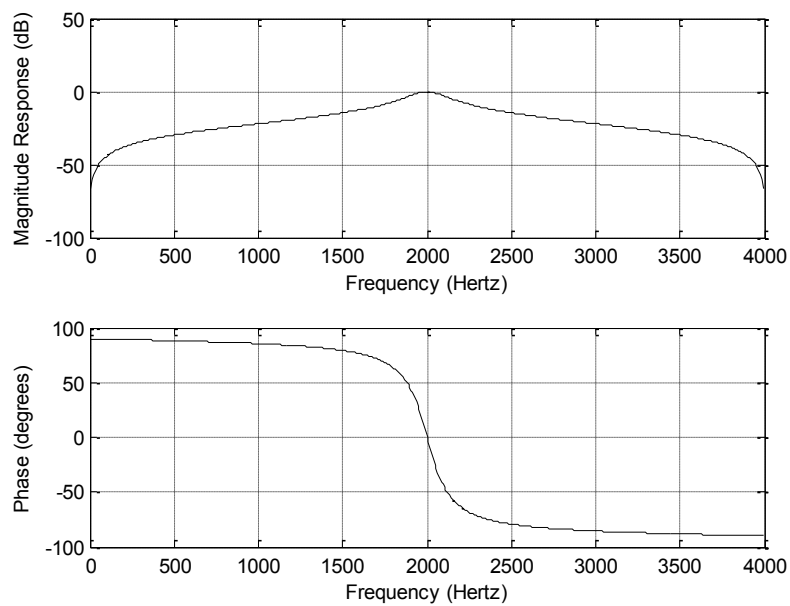
a.

$$H(z) = \frac{0.0730 - 0.0730z^{-2}}{1 + 0.8541z^{-2}}$$

$$y(n] = 0.0730x(n) - 0.0730x(n-2) - 0.8541y(n-2)$$

b.

```
>> freqz([0.0730 0 -0.0730],[1 0 0.8541],512,8000)
```



8.10

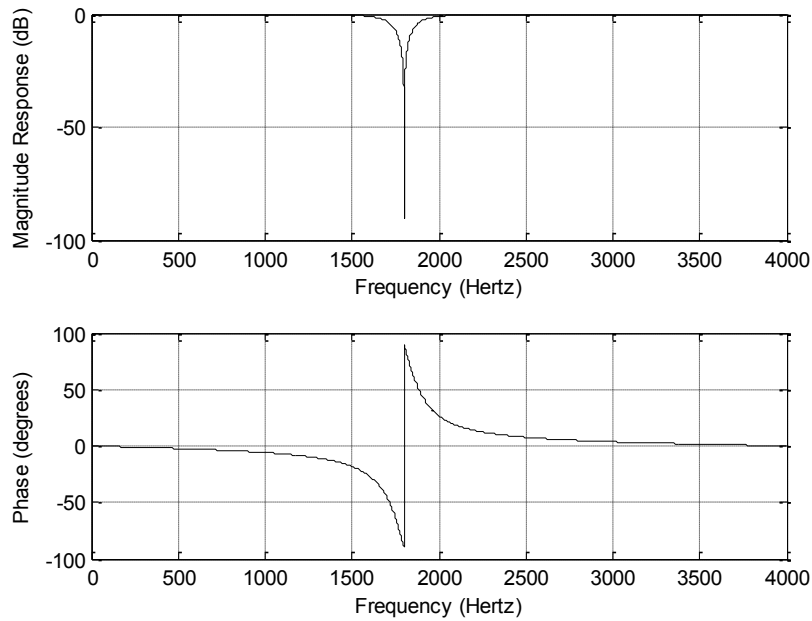
a.

$$H(z) = \frac{0.9266 - 0.2899z^{-1} + 0.9266z^{-2}}{1 - 0.2899 + 0.8532z^{-2}}$$

$$y(n) = 0.9266x(n) - 0.2899x(n-1) + 0.9266x(n-2) \\ + 0.2899y(n-1) - 0.8532y(n-2)$$

b.

```
>> freqz([0.9266 -0.2899 0.9266],[1 -0.2899 0.8532],8000,8000)
```



8.11

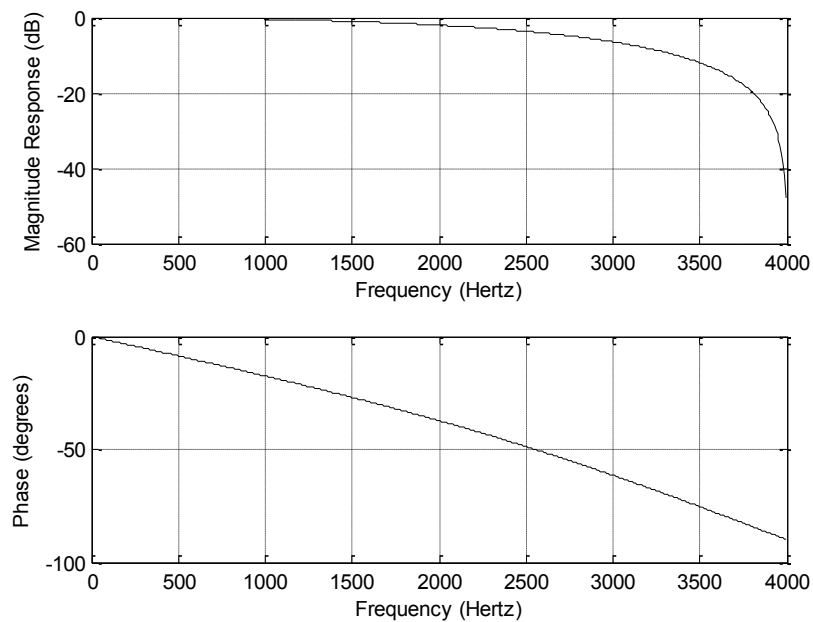
a.

$$H(z) = \frac{0.5677 + 0.5677z^{-1}}{1 + 0.1354z^{-1}}$$

$$y(n) = 0.5677x(n) + 0.5677x(n-1) - 0.1354y(n-1)$$

b.

```
>> freqz([0.5667 0.5667],[1 0.1354],512,8000)
```



8.12

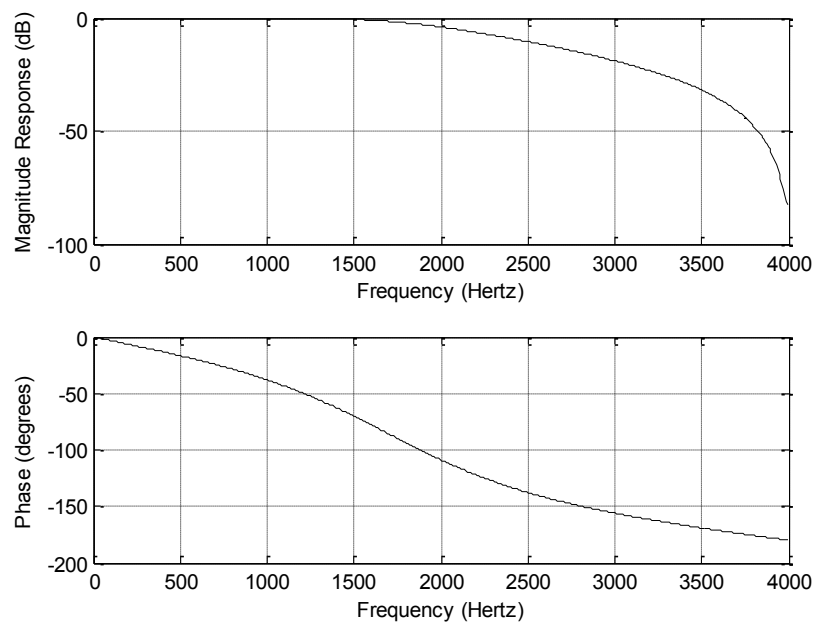
a.

$$H(z) = \frac{0.2430 + 0.4861z^{-1} + 0.2430z^{-2}}{1 - 0.2457 + 0.2755z^{-2}}$$

$$y(n) = 0.2430x(n) + 0.4861x(n-1) + 0.2430x(n-2) \\ + 0.2457y(n-1) - 0.2755y(n-2)$$

b.

```
>> freqz([0.2430 0.4861 0.2430],[1 -0.2457 0.2755],512,8000)
```



8.13

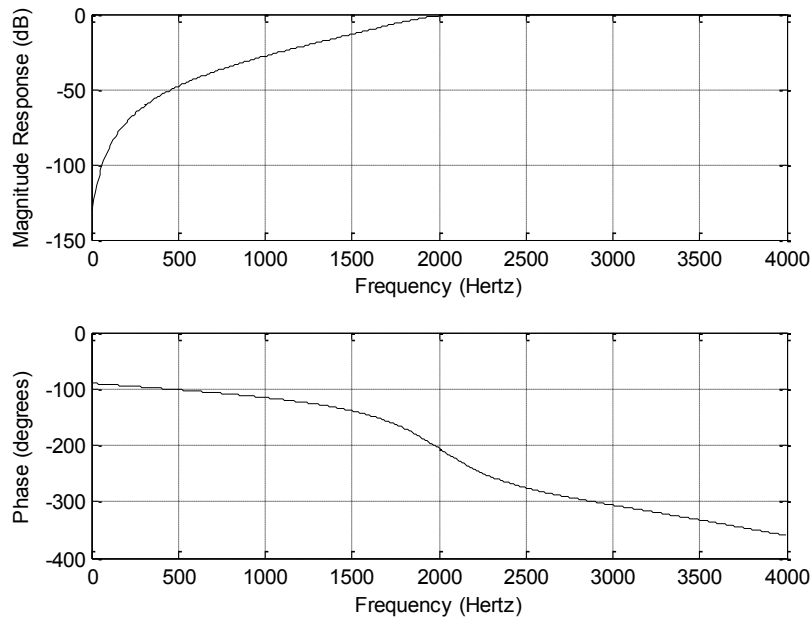
a.

$$H(z) = \frac{0.1321 - 0.3964z^{-1} + 0.3964z^{-2} - 0.1321z^{-3}}{1 + 0.3432z^{-1} + 0.6044z^{-2} + 0.2041z^{-3}}$$

$$y(n) = 0.1321x(n) - 0.3964x(n-1) + 0.3964x(n-2) - 0.1321x(n-3) \\ - 0.3432y(n-1) - 0.6044y(n-2) - 0.2041y(n-3)$$

b.

```
>> freqz([0.1321 -0.3964 0.3964 -0.1321],[1 0.3432 0.6044 0.2041],512,8000)
```



8.14

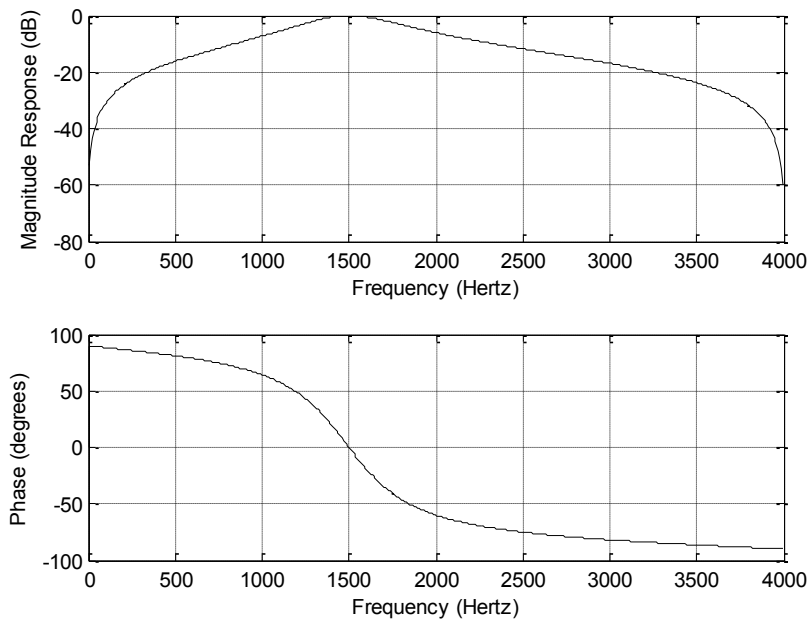
a.

$$H(z) = \frac{0.1815 - 0.1815z^{-2}}{1 - 0.6265z^{-1} + 0.6370z^{-2}}$$

$$y(n) = 0.1815x(n) - 0.1815x(n-2) + 0.6265y(n-1) - 0.6370y(n-2)$$

b.

```
>> freqz([0.1815 0 -0.1815],[1 -0.6265 0.6370],512,8000)
```



8.15

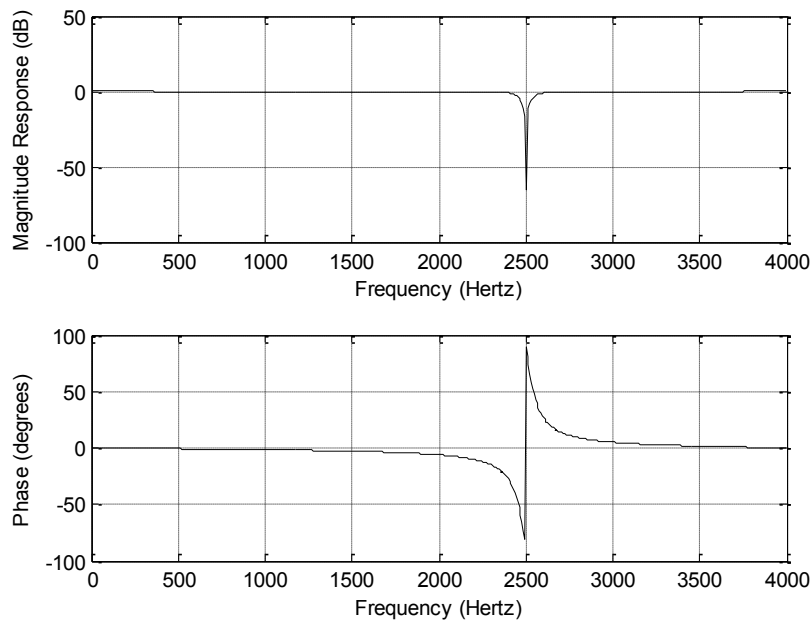
a.

$$H(z) = \frac{0.9609 + 0.7354z^{-1} + 0.9609z^{-2}}{1 + 0.7354z^{-1} + 0.9217z^{-2}}$$

$$y(n] = 0.9609x(n) + 0.7354x(n-1) + 0.9609x(n-2) \\ - 0.7354y(n-1) - 0.9217y(n-2)$$

b.

```
>> freqz([0.9609 0.7354 0.9609],[1 0.7354 0.9217],512,8000)
```



8.16

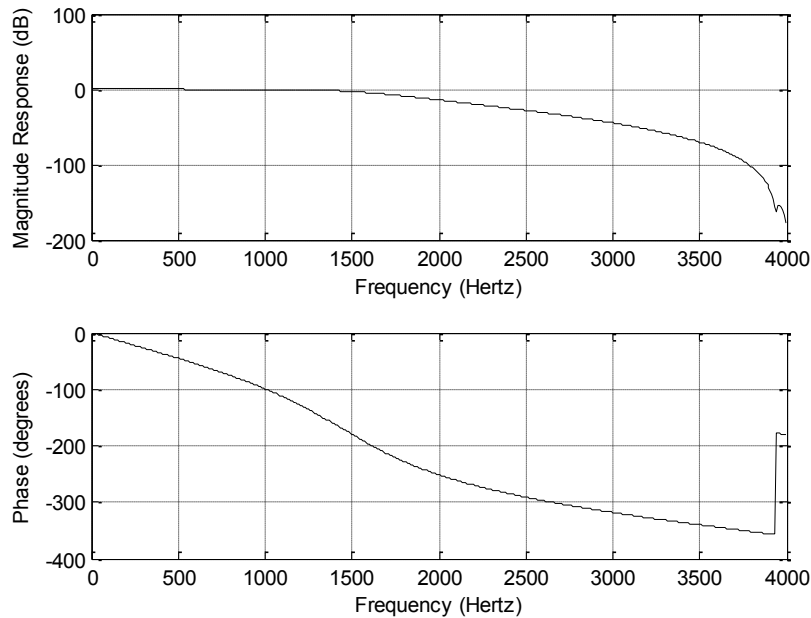
a.

$$H(z) = \frac{0.0380 + 0.1519z^{-1} + 0.2278z^{-2} + 0.1519z^{-3} + 0.0380z^{-4}}{1 - 0.9784z^{-1} + 0.7901z^{-2} - 0.2419z^{-3} + 0.0377z^{-4}}$$

$$y(n) = 0.0380x(n) + 0.1519x(n-1) + 0.2278x(n-2) + 0.1519x(n-3) + 0.0380x(n-4) \\ + 0.9784y(n-1) - 0.7901y(n-2) + 0.2419y(n-3) - 0.0377y(n-4)$$

b.

```
>> freqz([0.0380 0.1519 0.2278 0.1519 0.0380],[1 -0.9784 0.7901 -0.2419
0.0377],512,8000)
```



8.17

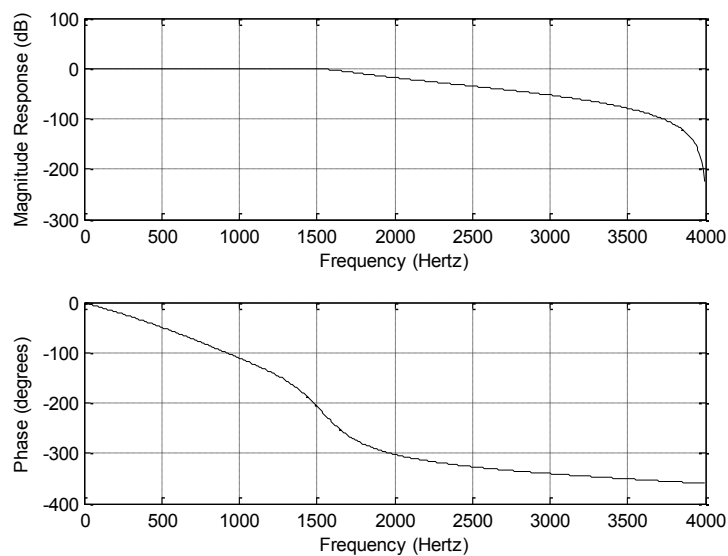
a.

$$H(z) = \frac{0.0242 + 0.0968z^{-1} + 0.1452z^{-2} + 0.0968z^{-3} + 0.0242z^{-4}}{1 - 1.5895z^{-1} + 1.6690z^{-2} - 0.9190z^{-3} + 0.2497z^{-4}}$$

$$y(n) = 0.0242x(n) + 0.0968x(n-1) + 0.1452x(n-2) + 0.0968x(n-3) + 0.0242x(n-4) \\ + 1.5895y(n-1) - 1.6690y(n-2) + 0.9190y(n-3) - 0.2497y(n-4)$$

b.

```
>> freqz([0.0242 0.0968 0.1452 0.0968 0.0242],[1 -1.5895 1.6690 -0.9190
0.2497],512,8000)
```



8.18

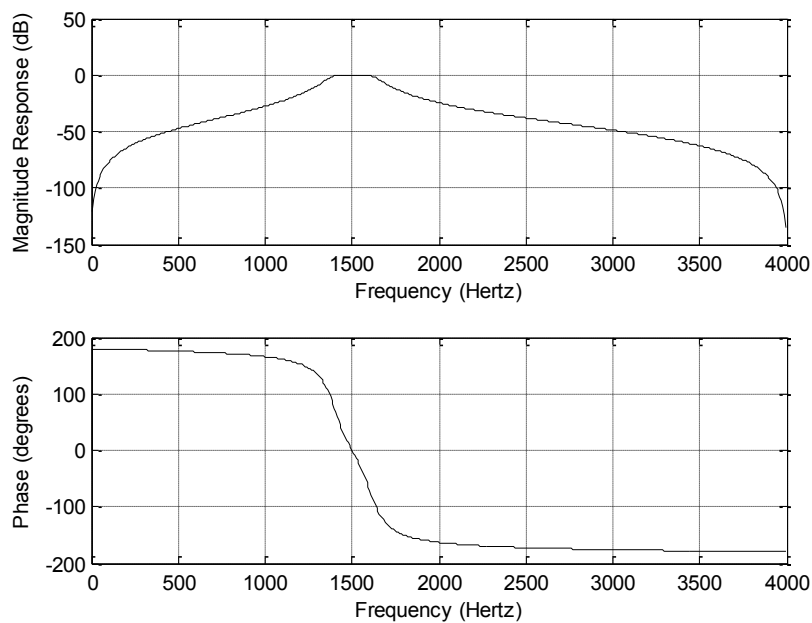
a.

$$H(z) = \frac{0.00767 - 0.01534z^{-2} + 0.00767z^{-4}}{1 - 1.4428z^{-1} + 2.2935z^{-2} - 1.2918z^{-3} + 0.8027z^{-4}}$$

$$y(n) = 0.00767x(n) - 0.01534x(n-2) + 0.00767x(n-4) \\ + 1.4428y(n-1) - 2.2935y(n-2) + 1.2918y(n-3) - 0.8027y(n-4)$$

b.

```
>> freqz([0.00767 0 -0.01534 0 0.00767],[1 -1.4428 2.2935 -1.2918 0.8027],512,8000)
```



8.19

a.

$$H(z) = \frac{1}{1 - 0.3679z^{-1}}$$

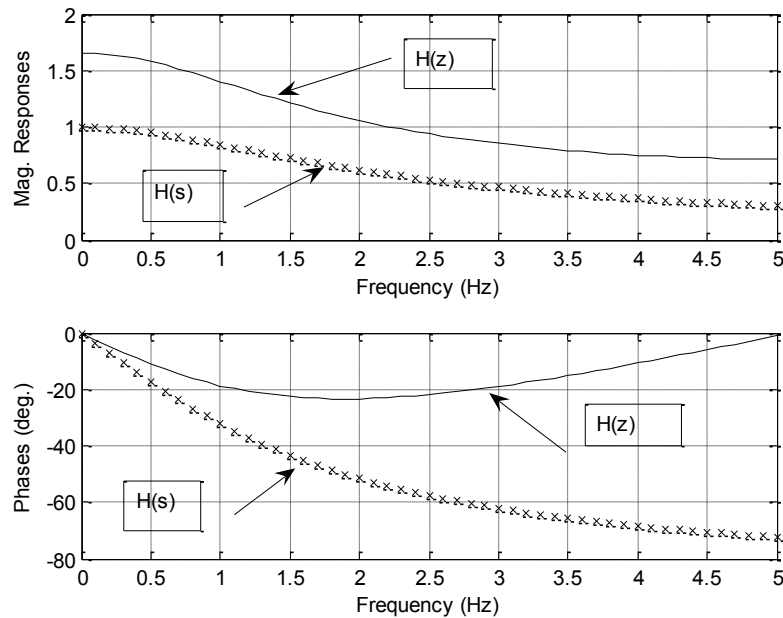
$$y(n) = x(n) + 0.3679y(n-1)$$

b. and c.

```

f=0:0.1:5;T=0.1           % frequency range and sampling interval
w=2*pi*f;                 % frequency range in radians/sec
hs=freqs([10], [1 10],w); % analog magnitude frequency response
phis=180*angle(hs)/pi;
% for the z-transfer function H(z)
hz=freqz([1],[1 -0.3967],length(w)); % digital magnitude frequency response
phiz=180*angle(hz)/pi;
%plot magnitude and phase responses.
subplot(2,1,1), plot(f,abs(hs),'kx',f, abs(hz),'k-'),grid; axis([0 5 0 2]);
xlabel('Frequency (Hz)'); ylabel('Mag. Responses')
subplot(2,1,2), plot(f,phis,'kx',f, phiz,'k-'); grid;
xlabel('Frequency (Hz)'); ylabel('Phases (deg.)');

```



8.20

a.

$$H(z) = \frac{0.0086z^{-1}}{1 - 1.7326z^{-1} + 0.7408z^{-2}}$$

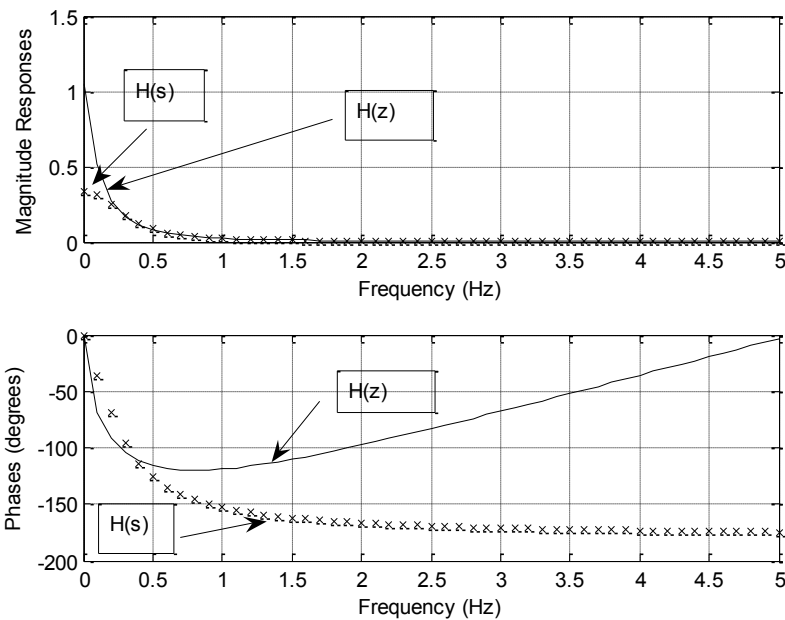
$$y(n) = 0.0086x(n-1) + 1.7326y(n-1) + 0.7408y(n-2)$$

b. and c.

```

f=0:0.1:5;T=0.1; % initialize analog frequency range in Hz and sampling interval
w=2*pi*f;        % convert the frequency range to radians/second
hs=freqs([1], [1 3 3],w); % calculate analog filter frequency responses
phis=180*angle(hs)/pi;
% for the z-transfer function H(z)
% calculate digital filter frequency responses
hz=freqz([0.0086],[1 -1.7326 0.7408],length(w));
phiz=180*angle(hz)/pi;
% plot magnitude and phase responses
subplot(2,1,1), plot(f,abs(hs),'kx',f, abs(hz),'k-'),grid;
xlabel('Frequency (Hz)'); ylabel('Magnitude Responses')
subplot(2,1,2), plot(f,phis,'kx',f, phiz,'k-'); grid;
xlabel('Frequency (Hz)'); ylabel('Phases (degrees)')

```



8.21

a.

$$H(z) = \frac{0.1 - 0.09781z^{-1}}{1 - 1.6293z^{-1} + 0.6703z^{-2}}$$

$$y(n) = 0.1x(n) - 0.0978x(n-1) + 1.6293y(n-1) - 0.6703y(n-2)$$

b. and c.

```

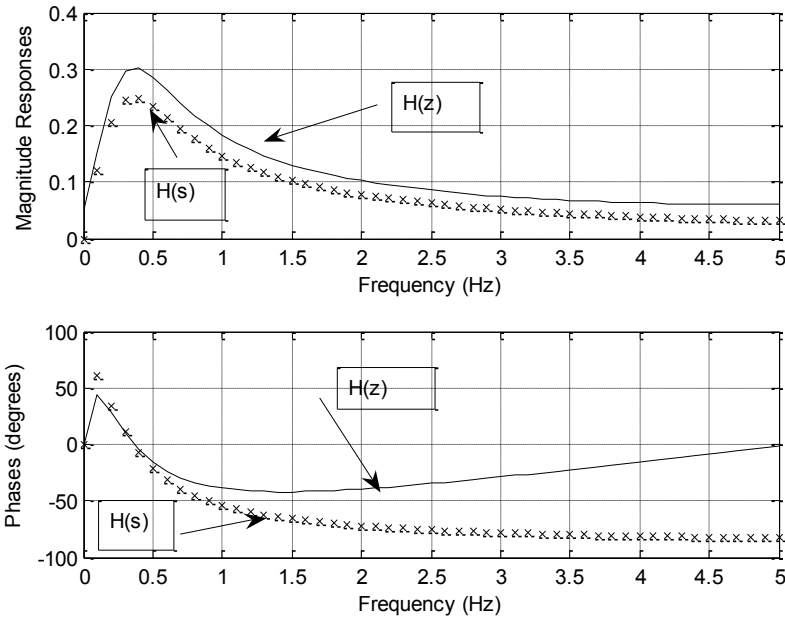
f=0:0.1:5;T=0.1; % initialize analog frequency range in Hz and sampling interval
w=2*pi*f;        % convert the frequency range to radians/second
hs=freqs([1 0], [1 4 5],w); % calculate analog filter frequency responses
phis=180*angle(hs)/pi;
% for the z-transfer function H(z)
% calculate digital filter frequency responses

```

```

hz=freqz([0.1 -0.09781],[1 -1.6293 0.6703],length(w));
phiz=180*angle(hz)/pi;
% plot magnitude and phase responses
subplot(2,1,1), plot(f,abs(hs),'kx',f, abs(hz),'k-'),grid;
xlabel('Frequency (Hz)'); ylabel('Magnitude Responses')
subplot(2,1,2), plot(f,phis,'kx',f, phiz,'k-'); grid;
xlabel('Frequency (Hz)'); ylabel('Phases (degrees)')

```



8.22

$$H(z) = \frac{0.0385 - 0.0385z^{-2}}{1 + 0.9230z^{-2}}$$

$$y(n) = 0.0385x(n) - 0.0385x(n-2) - 0.9230y(n-2)$$

8.23

$$H(z) = \frac{0.9320 - 1.3180z^{-1} + 0.9320z^{-2}}{1 - 1.3032z^{-1} + 0.8492z^{-2}}$$

$$y(n) = 0.9320x(n) - 1.3180x(n-1) + 0.9320x(n-2) + 1.3032y(n-1) - 0.8492y(n-2)$$

8.24

$$H(z) = \frac{0.0785 + 0.0785z^{-1}}{1 - 0.8429z^{-1}}$$

$$y(n) = 0.0785x(n) + 0.0785x(n-1) - 0.8429y(n-1)$$

8.25

$$H(z) = \frac{0.9215 + 0.9215z^{-1}}{1 + 0.8429z^{-1}}$$

$$y(n] = 0.9215x(n) + 0.9215x(n-1) - 0.8429y(n-1)$$

8.26

$$H(z) = \frac{0.0589 - 0.0589z^{-1}}{1 + 0.8822z^{-1}}$$

$$y(n] = 0.0589x(n) - 0.0589x(n-1) - 0.8822y(n-1)$$

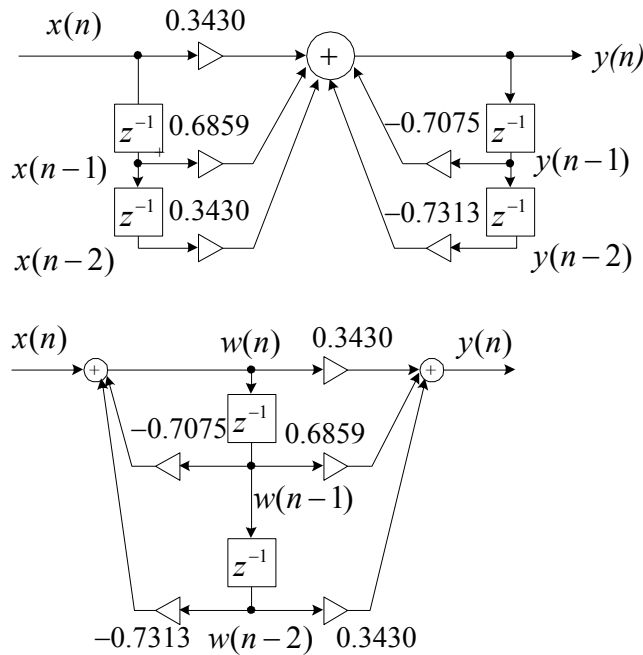
8.27

$$H(z) = \frac{0.9607 - 0.9607z^{-1}}{1 - 0.9215z^{-1}}$$

$$y(n] = 0.9607x(n) - 0.9607x(n-1) + 0.9215y(n-1)$$

8.28

a.



b.

Direct-form I:

$$y(n] = 0.3430x(n) + 0.6859x(n-1) + 0.3430x(n-2) - 0.7075y(n-1) - 0.7131y(n-2)$$

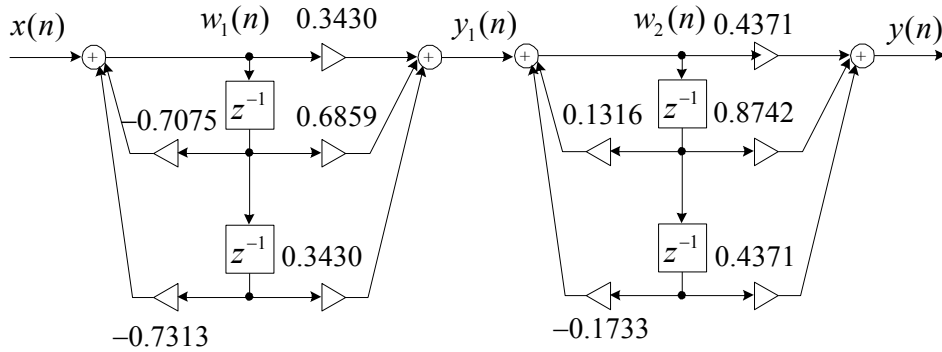
Direct-form II:

$$w(n] = x(n] - 0.7075w(n-1) - 0.7313w(n-2)$$

$$y_1(n] = 0.3430w(n) + 0.6859w(n-1) + 0.3430w(n-2)$$

8.29

a.

b. for section 1: $w_1(n) = x(n) - 0.7075w_1(n-1) - 0.7313w_1(n-2)$

$$y_1(n) = 0.3430w_1(n) + 0.6859w_1(n-1) + 0.3430w_1(n-2)$$

for section 2: $w_2(n) = y_1(n) + 0.1316w_2(n-1) - 0.1733w_2(n-2)$

$$y_2(n) = 0.4371w_2(n) + 0.8742w_2(n-1) + 0.4371w_2(n-2)$$

8.30

$$\Omega_0 = 2\pi \times 200/1000 = 0.4\pi, \quad H(z) = \frac{0.9511z^{-1}}{1.0000 - 0.6180z^{-1} + z^{-2}}$$

$$y(n) = 0.9511x(n-1) + 0.618y(n-1) - y(n-2)$$

8.31

$$\Omega_0 = 2\pi \times 250/8000 = 0.0625\pi, \quad H(z) = \frac{0.1951z^{-1}}{1.0000 - 1.9616z^{-1} + z^{-2}}$$

$$y(n) = 0.1951x(n-1) + 1.9616y(n-1) - y(n-2)$$

$$8.32 \quad a. \quad H_{852}(z) = \frac{0.6203z^{-1}}{1 - 1.5687z^{-1} + z^{-2}}, \quad H_{1477}(z) = \frac{0.9168z^{-1}}{1 - 0.7986z^{-1} + z^{-2}}$$

$$b. \quad y_{852}(n) = 0.6203x(n-1) + 1.5678y_{852}(n-1) - y_{852}(n-2)$$

$$y_{1477}(n) = 0.9168x(n-1) + 0.7986y_{1477}(n-1) - y_{1477}(n-2)$$

$$y_9(n) = y_{1477}(n) + y_{852}(n)$$

$$8.33 \quad a. \quad H_{697}(z) = \frac{0.5205z^{-1}}{1 - 1.7077z^{-1} + z^{-2}}, \quad H_{1477}(z) = \frac{0.9168z^{-1}}{1 - 0.7986z^{-1} + z^{-2}}$$

$$b. \quad y_{697}(n) = 0.5205x(n-1) + 1.7077y_{697}(n-1) - y_{697}(n-2)$$

$$y_{1477}(n) = 0.9168x(n-1) + 0.7986y_{1477}(n-1) - y_{1477}(n-2)$$

$$y_3(n) = y_{1477}(n) + y_{697}(n)$$

$$8.34 \quad X(0) = 2, |X(0)|^2 = 4, A_0 = 0.5 \text{ (single side)}$$

$$X(1) = 1 - j3, |X(1)|^2 = 10, A_1 = 1.5811 \text{ (single side)}$$

$$8.35 \quad X(2) = 0, |X(2)|^2 = 0, A_2 = 0.0 \text{ (single side)}$$

$$X(3) = 1 + j3, |X(3)|^2 = 10, A_3 = 1.5811 \text{ (single side)}$$

$$8.36 \quad A_0 = 2.5, A_2 = 0.5$$

$$8.37 \quad A_1 = 0.7071, A_3 = 0.7071$$

8.38

Chebyshev notch filter: order =2

$$H(z) = \frac{0.9915 - 1.9042z^{-1} + 0.9915z^{-2}}{1.0000 - 1.9042z^{-1} + 0.9830z^{-2}}$$

$$y(n) = 0.9915x(n) - 1.9042x(n-1) + 0.9915x(n-2)$$

$$+ 1.9042y(n-1) - 0.9830y(n-2)$$

```
fs=8000;T=1/fs;
w0=2*pi*360; wa0=(2/T)*tan(w0*T/2);
wL=2*pi*330; waL=(2/T)*tan(wL*T/2);
wH=2*pi*390; waH=(2/T)*tan(wH*T/2);
waaL=wa0*wa0/waH;BW1=waH-waaL
waaH=wa0*wa0/waL;BW2=waaH-waL
[B,A]=lp2bs(2.8628,[1 2.8628],wa0,BW2);
[b,a]=bilinear(B,A,fs)
freqz(b,a,8000,8000);
```

8.39 Chebyshev notch filter 1: order =2

$$H(z) = \frac{0.9915 - 1.9042z^{-1} + 0.9915z^{-2}}{1.0000 - 1.9042z^{-1} + 0.9830z^{-2}}$$

$$y(n) = 0.9915x(n) - 1.9042x(n-1) + 0.9915x(n-2)$$

$$+ 1.9042y(n-1) - 0.9830y(n-2)$$

See Problem 8.38.

Chebyshev notch filter 2: order =2

$$H(z) = \frac{0.9917 - 1.3117z^{-1} + 0.9917z^{-2}}{1.0000 - 1.3117z^{-1} + 0.9835z^{-2}}$$

$$y(n) = 0.9917x(n) - 1.3117x(n-1) + 0.9917x(n-2)$$

$$+ 1.3117y(n-1) - 0.9835y(n-2)$$

```

fs=8000;T=1/fs;
w0=2*pi*1080; wa0=(2/T)*tan(w0*T/2);
wL=2*pi*1050; waL=(2/T)*tan(wL*T/2);
wH=2*pi*1110; waH=(2/T)*tan(wH*T/2);
waaL=wa0*wa0/waH;BW1=waH-waaL
waaH=wa0*wa0/waL;BW2=waaH-waL
[B,A]=lp2bs(2.8628,[1 2.8628],wa0,BW2);
[b,a]=bilinear(B,A,fs)
freqz(b,a,8000,8000);

```

8.40

$$\omega_{dp} = 2\pi(3000) = 6000\pi \quad \omega_{ds} = 2\pi(4000) = 8000\pi$$

$$\omega_{ap} = \frac{2}{T} \tan\left(\frac{\omega_{dp}T}{2}\right) = 20000 \times \tan\left(\frac{6000\pi/10000}{2}\right) = 27528 \text{ radians/se.}$$

$$\omega_{as} = \frac{2}{T} \tan\left(\frac{\omega_{dp}T}{2}\right) = 20000 \times \tan\left(\frac{8000\pi/10000}{2}\right) = 61554 \text{ radians/sec}$$

$$v_s = \omega_{as} / \omega_{ap} = 61554 / 27528 = 2.24 \text{ and } A_s = 25 \text{ dB}$$

$$\epsilon^2 = 10^{0.1 \times 3} - 1 = 1$$

$$n = \frac{\log_{10}(10^{0.1 \times 25} - 1)}{2 \cdot \log_{10}(2.24)} = 3.5669 \text{ Choose } n = 4$$

Butterworth filter order = n=4

$$H(z) = \frac{0.1672 + 0.6687z^{-1} + 1.0031z^{-2} + 0.6687z^{-3} + 0.1672z^{-4}}{1.0000 + 0.7821z^{-1} + 0.6800z^{-2} + 0.1827z^{-3} + 0.0301z^{-4}}$$

```

fs=10000;T=1/fs;
wd=2*pi*3000; wa=(2/T)*tan(wd*T/2);
[B,A]=lp2lp(1,[1 2.6131 3.4142 2.6131 1],wa);
[b,a]=bilinear(B,A,fs)
freqz(b,a,512,fs);

```

8.41

$$\omega_{dp} = 2\pi f = 2\pi(3000) = 6000\pi \text{ radians/second,}$$

$$\omega_{ds} = 2\pi f = 2\pi(4000) = 8000\pi \text{ radians/second, and } T = 1/f_s = 1/8000 \text{ sec.}$$

$$\omega_{ap} = \frac{2}{T} \tan\left(\frac{\omega_{dp}T}{2}\right) = 20000 \times \tan\left(\frac{6000\pi/10000}{2}\right) = 27528 \text{ radians/se.}$$

$$\omega_{as} = \frac{2}{T} \tan\left(\frac{\omega_{dp}T}{2}\right) = 20000 \times \tan\left(\frac{8000\pi/10000}{2}\right) = 61554 \text{ radians/sec}$$

$$v_s = \omega_{as} / \omega_{ap} = 61554 / 27528 = 2.24 \text{ and } A_s = 35 \text{ dB}$$

$$\varepsilon^2 = 10^{0.1 \times 1} - 1 = 0.2589$$

$$n \geq \frac{\cosh^{-1} \left[\left(\frac{10^{0.1 \times 35} - 1}{0.2589} \right)^{0.5} \right]}{\cosh^{-1}(2.24)} = 3.7342$$

Chebyshev filter order = 4;

$$H(z) = \frac{0.1103 + 0.4412z^{-1} + 0.6618z^{-2} + 0.4412z^{-3} + 0.1103z^{-4}}{1.0000 + 0.1509z^{-1} + 0.8041z^{-2} - 0.1619z^{-3} + 0.1872z^{-4}}$$

fs=10000;T=1/fs;

wd=2*pi*3000; wa=(2/T)*tan(wd*T/2);

[B,A]=lp2lp(0.2456,[1 0.9528 1.4539 0.7426 0.2756],wa);

[b,a]=bilinear(B,A,fs)

freqz(b,a,512,fs);

$$8.42 \quad r = 1 - (500/8000) \times \pi = 0.8037 \text{ and } \theta = (1750/8000) \times 360^\circ = 78.75^\circ$$

$$K = \frac{(1 - 0.8037) \sqrt{1 - 2 \times 0.8037 \cos 157.5^\circ + 0.8037^2}}{2 |\sin 78.75^\circ|} = 0.1771$$

>> r=1-500*pi/8000;

>> theta=1750*2*pi/8000;

>> K=(1-r)*sqrt(1-2*r*cos(2*theta)+r*r)/(2*abs(sin(theta)));

r = 0.8037

K = 0.1771

-2*r*cos(theta)

ans = -0.3136

$$H(z) = \frac{0.1771 - 0.1771z^{-2}}{1 - 0.3136z^{-1} + 0.6459z^{-2}}$$

8.43

$$\omega_{dpL} = 2\pi(1500) = 3000\pi \quad \omega_{dpH} = 2\pi(2000) = 4000\pi$$

$$\omega_{dsL} = 2\pi(1000) = 2000\pi \quad \omega_{dsH} = 2\pi(2500) = 5000\pi$$

$$\omega_{apL} = 16000 \times \tan\left(\frac{3000\pi/8000}{2}\right) = 10691 \quad \omega_{apH} = 16000 \times \tan\left(\frac{4000\pi/8000}{2}\right) = 16000$$

$$\text{Fixing } \omega_{asL} = 16000 \times \tan\left(\frac{2000\pi/8000}{2}\right) = 6627$$

$$\omega_{asH} = \omega_{apL} \omega_{apH} / \omega_{asL} = 25812 \quad W_s = 19185$$

Fixing $\omega_{asH} = 16000 \times \tan\left(\frac{5000\pi/8000}{2}\right) = 23946$,

$$\omega_{asL} = \omega_{apL} \omega_{apH} / \omega_{asH} = 7143, W_s = 16803$$

We choose a smaller bandwidth for aggressive design:

$$\omega_{asH} = 23946, \omega_{asL} = \omega_{apL} \omega_{apH} / \omega_{asH} = 7143$$

$$v_s = (\omega_{asH} - \omega_{asL}) / (\omega_{apH} - \omega_{apL}) = (23946 - 7143) / (16000 - 10691) = 3.17 \text{ and } A_s = 20 \text{ dB}$$

$$\varepsilon^2 = 10^{0.1 \times 3} - 1 = 1$$

$$n = \frac{\log_{10}(10^{0.1 \times 20} - 1)}{2 \cdot \log_{10}(3.17)} = 1.9914 \text{ Choose } n = 2$$

filter order $2n = 4$

$$H(z) = \frac{0.0300 - 0.0599z^{-2} + 0.0300z^{-4}}{1.0000 - 0.6871z^{-1} + 1.5741z^{-2} - 0.5176z^{-3} + 0.5741z^{-4}}$$

```
fs=8000;T=1/fs;
wL=2*pi*1500; waL=(2/T)*tan(wL*T/2);
wH=2*pi*2000; waH=(2/T)*tan(wH*T/2);
wa0=sqrt(waL*waH); BW=waH-waL
[B,A]=lp2bp(1,[1 1.4142 1],wa0,BW);
[b,a]=bilinear(B,A,fs)
freqz(b,a,512,8000);
```

8.44

a.

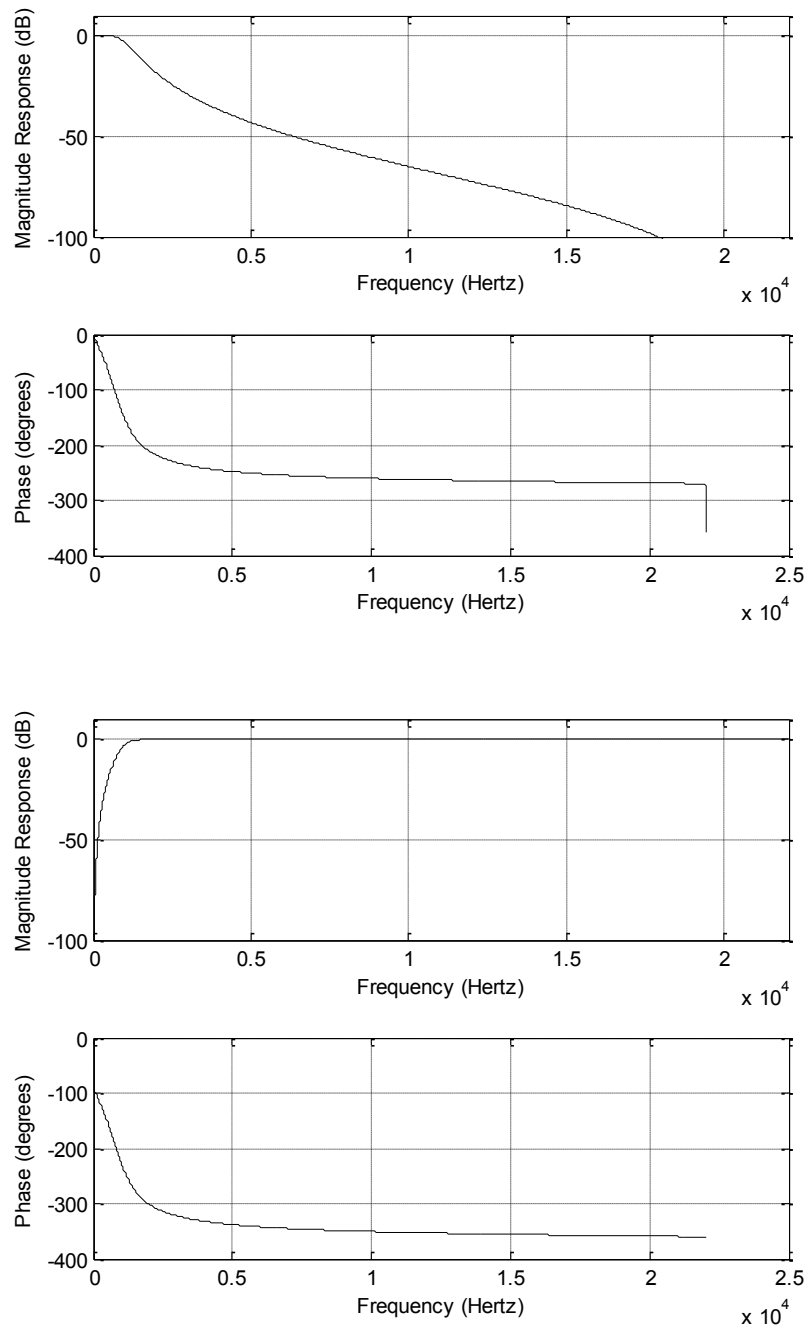
$$H_{LP}(z) = \frac{0.0003151 + 0.0009452z^{-1} + 0.0009452z^{-2} + 0.0003151z^{-3}}{1.0000 - 2.7153z^{-1} + 2.4697z^{-2} - 0.7519z^{-3}}$$

$$y_{LP}(n) = 0.0003151x(n) + 0.0009452x(n-1) + 0.0009452x(n-2) + 0.0003151x(n-3) \\ 2.7153y_{LP}(n-1) - 2.4697y_{LP}(n-2) + 0.7519y_{LP}(n-3)$$

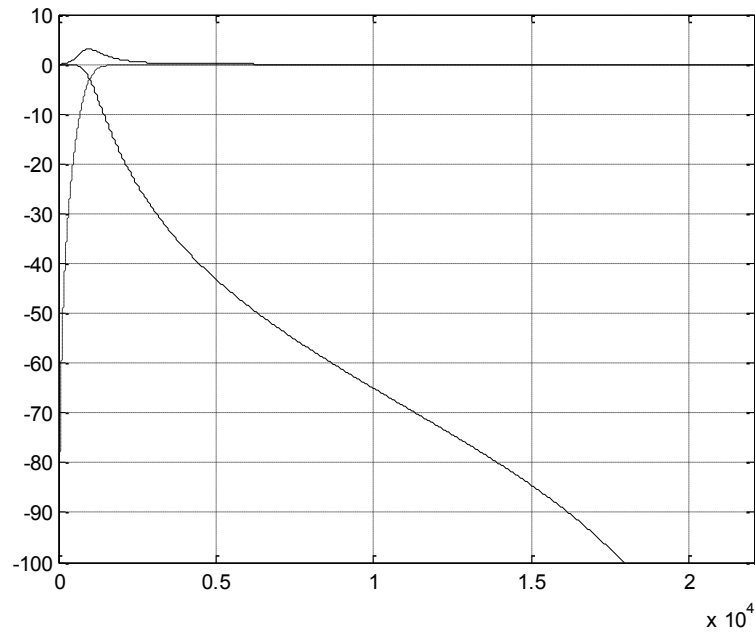
$$H_{HP}(z) = \frac{0.8671 - 2.6013z^{-1} + 2.6013z^{-2} - 0.8671z^{-3}}{1.0000 - 2.7153z^{-1} + 2.4697z^{-2} - 0.7519z^{-3}}$$

$$y_{HP}(n) = 0.8671x(n) - 2.6013x(n-1) + 2.6013x(n-2) - 0.8671x(n-3) \\ 2.7153y_{HP}(n-1) - 2.4697y_{HP}(n-2) + 0.7519y_{HP}(n-3)$$

b.



c.



```

fs=44100;T=1/fs;
wd=2*pi*1000; wa=(2/T)*tan(wd*T/2);
[B,A]=lp2lp(1,[ 1 2 2 1],wa);
[bL,aL]=bilinear(B,A,fs)
freqz(bL,aL,512,fs);
[hL,ff]=freqz(bL,aL,512,fs);
figure
[B,A]=lp2hp(1,[ 1 2 2 1],wa);
[bH,aH]=bilinear(B,A,fs)
freqz(bH,aH,512,fs);
[hH,ff]=freqz(bH,aH,512,fs);
figure
H=abs(hL)+abs(hH);
plot(ff,20*log10(abs(hL)),ff,20*log10(abs(hH)),'-.', ff,20*log10(H));

```

8.45

a.

$$H(z) = \frac{0.5878z^{-1}}{1 - 1.6180z^{-1} + z^2}$$

b.

$$y(n) = 0.5878x(n-1) + 1.6180y(n-1) - y(n-2)$$

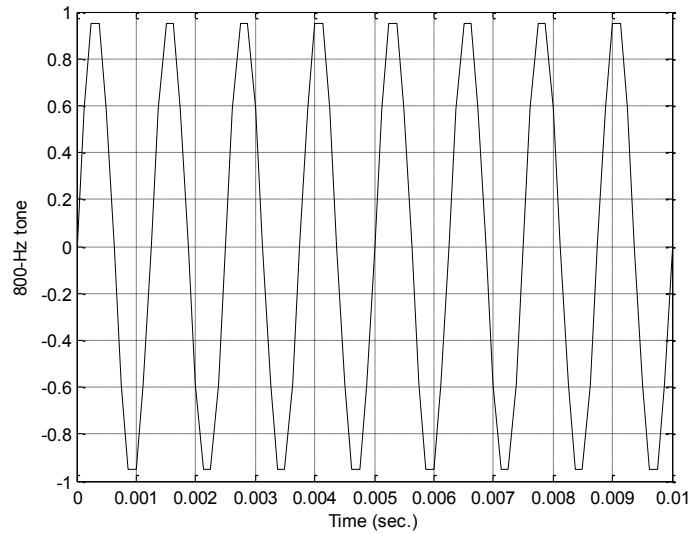
c.

```

>> fs=8000; T=1/fs;
>> t=0:T:0.01;

```

```
>> x=zeros(1,length(t)); x(1)=1;
>> y=filter([0 0.5878],[1 -1.6180 1],x);
>> plot(t,y);grid;xlabel('Time (sec.)');ylabel('800-Hz tone');
```



8.46

a.

$$H_1(z) = \frac{0.5686z^{-1}}{1 - 1.6453z^{-1} + z^2} \quad H_2(z) = \frac{0.2593z^{-1}}{1 - 1.9316z^{-1} + z^2}$$

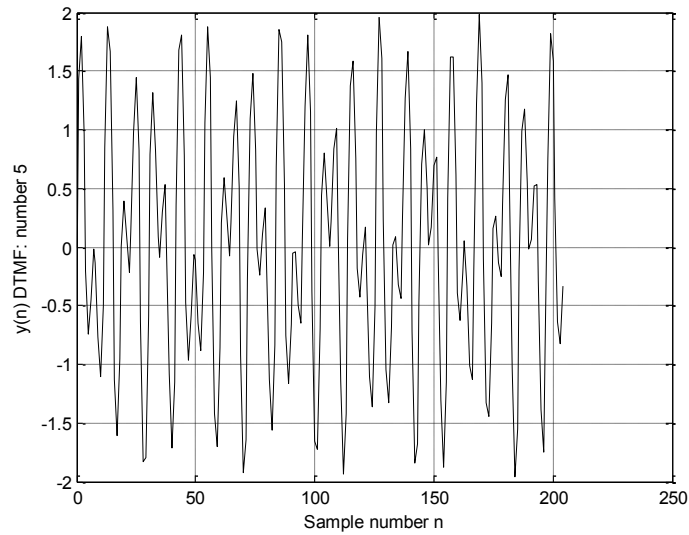
b.

$$y_1(n) = 0.5686x(n-1) + 1.6453y_1(n-1) - y_1(n-2)$$

$$y_2(n) = 0.2593x(n-1) + 1.9316y_2(n-1) - y_2(n-2)$$

c.

```
fs=8000;T=1/fs;
x=zeros(1,205);x(1)=1;
y1=filter([0 sin(2*pi*770/fs)],[1 -2*cos(2*pi*770/fs) 1],x);
y2=filter([0 sin(2*pi*1336/fs)],[1 -2*cos(2*pi*1336/fs) 1],x);
y=y1+y2;
plot(N,y,'k');grid
ylabel('y(n) DTMF: number 5');xlabel('Sample number n')
```



8.47

$$x(4) = 0$$

for $n = 0, 1, 2, 3, 4$

$$v_0(n) = 2v_0(n-1) - v_0(n-2) + x(n)$$

$$y_0(n) = v_0(n) - v_0(n-1) \quad \text{with } v_0(-2) = 0, v_0(-1) = 0$$

$$X(0) = y_0(4)$$

$$|X(0)|^2 = v_0^2(4) + v_0^2(3) - 2v_0(4)v_0(3)$$

a. $X(0) = 1$; b. $|X(0)|^2 = 1$; c. $A_0 = 0.25$

for $n = 0, 1, 2, 3, 4$

$$v_1(n) = -v_1(n-2) + x(n)$$

$$y_1(n) = v_1(n) + jv_1(n-1) \quad \text{with } v_1(-2) = 0, v_1(-1) = 0$$

$$X(1) = y_1(4)$$

$$|X(1)|^2 = v_1^2(4) + v_1^2(3)$$

a. $X(1) = 1 - j2$; b. $|X(1)|^2 = 5$; c. $A_1 = 1.12$ (single side)

8.48

$$x(4) = 0$$

for $n = 0, 1, 2, 3, 4$

$$v_2(n) = -2v_2(n-1) - v_2(n-2) + x(n)$$

$$y_2(n) = v_2(n) + v_2(n-1) \quad \text{with } v_2(-2) = 0, v_2(-1) = 0$$

$$X(2) = y_2(4)$$

$$|X(2)|^2 = v_2^2(4) + v_2^2(3) + 2v_2(4)v_2(3)$$

a. $X(2) = 1$; b. $|X(2)|^2 = 1$; c. $A_2 = 0.25$

for $n = 0, 1, 2, 3, 4$

$$v_3(n) = -v_3(n-2) + x(n)$$

$$y_3(n) = v_3(n) - jv_3(n-1) \text{ with } v_3(-2) = 0, v_3(-1) = 0$$

$$X(3) = y_3(4)$$

$$|X(3)|^2 = v_3^2(4) + v_3^2(3)$$

a. $X(3) = 1 + j2$; b. $|X(3)|^2 = 5$; c. $A_3 = 0.559$

8.49

$$k_L = \frac{770}{8000} \times 205 \approx 20, \text{ and } k_H = \frac{1336}{8000} \times 205 \approx 34$$

a.

$$H_{20}(z) = \frac{1}{1 - 1.6359z^{-1} + z^{-2}} \text{ and } H_{34}(z) = \frac{1}{1 - 1.0088z^{-1} + z^{-2}}$$

b.

$$v_{20}(n) = 1.6359v_{20}(n-1) - v_{20}(n-2) + x(n) \text{ with } x(205) = 0, \text{ for } n = 0, 1, \dots$$

$$v_{34}(n) = 1.0088v_{34}(n-1) - v_{34}(n-2) + x(n) \text{ with } x(205) = 0, \text{ for } n = 0, 1, \dots$$

c.

$$|X(20)|^2 = (v_{20}(205))^2 + (v_{20}(204))^2 - 1.6359(v_{20}(205)) \times (v_{20}(204))$$

$$A_{20} = \frac{2\sqrt{|X(20)|^2}}{205}$$

and

$$|X(34)|^2 = (v_{34}(205))^2 + (v_{34}(204))^2 - 1.0088(v_{34}(205)) \times (v_{34}(204))$$

$$A_{34} = \frac{2\sqrt{|X(34)|^2}}{205}$$

d.

```

fs=8000;T=1/fs;
x=zeros(1,205);x(1)=1;
y1=filter([0 sin(2*pi*770/fs)],[1 -2*cos(2*pi*770/fs) 1],x);
y2=filter([0 sin(2*pi*1336/fs)],[1 -2*cos(2*pi*1336/fs) 1],x);
y=y1+y2;
xDTMF=[y 0];
v20=filter(1,[1 -2*cos(2*pi*20/205) 1],xDTMF);
v34=filter(1,[1 -2*cos(2*pi*34/205) 1],xDTMF);
X20=sqrt(v20(206)^2+v20(205)^2-2*cos(2*pi*20/205)*v20(206)*v20(205));
X34=sqrt(v34(206)^2+v34(205)^2-2*cos(2*pi*34/205)*v34(206)*v34(205));
A20=2*X20/205
A34=2*X34/205

```

$$A_{20} = 0.8818$$

$$A_{34} = 0.9147$$

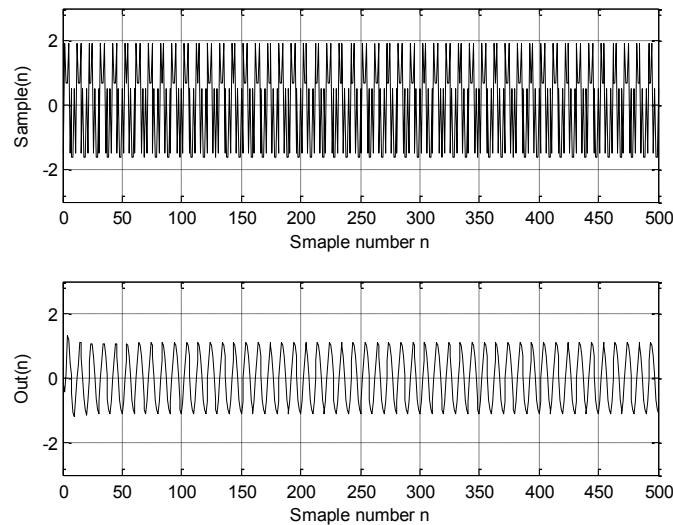
8.50

a.

```

n=0:N-1;
sample=1.2*sin(2*pi*1000*n/10000)-1.5*cos(2*pi*4000*n/10000);
%direct-form I implementation
x=[0 0 0 0 0]; %input buffer [x(n) x(n-1) ..]
y=[0 0 0 0 0]; %output buffer [y(n) y(n-1) ...]
b=[0.1103 0.4412 0.6618 0.4412 0.1603]; %Numerator coefficients [b0 b1 ...]
a=[1 0.1509 0.9041 -0.1619 0.1872]; %Denominator coefficients [1 a1 ...]
KKb=length(b); KKa=length(a);
for n=1:1:length(sample) % loop processing
    for k=KKb:-1:2 % shift input by one sample
        x(k)=x(k-1);
    end
    x(1)=sample(n); % get new sample
    for k=KKa:-1:2 % shift input by one sample
        y(k)=y(k-1);
    end
    y(1)=0; % perform IIR filtering
    for k=1:1:KKb
        y(1)=y(1)+b(k)*x(k);
    end
    for k=2:1:KKa
        y(1)=y(1)-a(k)*y(k);
    end
    out(n)=y(1); %send filtered sample to the output array
end
subplot(2,1,1);plot(sample);grid;axis([0 500 -3 3]);
xlabel('Sample number n');ylabel('Sample(n)');
subplot(2,1,2);plot(out);grid;axis([0 500 -3 3]);
xlabel('Sample number n');ylabel('Out(n)');

```



b.

```

-----
n=0:N-1;
sample=1.2*sin(2*pi*1000*n/10000)-1.5*cos(2*pi*4000*n/10000);
%direct-form II implementation
w=[0 0 0 0 0]; %filter states [w(n) w(n-1) ..]
b=[0.1103 0.4412 0.6618 0.4412 0.1603]; %Numerator coefficients [b0 b1 ...]
a=[1 0.1509 0.9041 -0.1619 0.1872]; %Denominator coefficients [1 a1 ...]
KKb=length(b); KKa=length(a); KKw=length(w);
for n=1:1:length(sample) % loop processing
    for k=KKw:-1:2 % shift input by one sample
        w(k)=w(k-1);
    end
    w(1)=sample(n);
    for k=2:1:KKa
        w(1)=w(1)-a(k)*w(k); %IIR filtering
    end
    sum=0;
    for k=1:1:KKb
        sum=sum+b(k)*w(k); %FIR filtering
    end
    out(n)=sum; %send filtered sample to the output array
end
subplot(2,1,1);plot(sample);grid;axis([0 500 -3 3]);
xlabel('Sample number n');ylabel('Sample(n)');
subplot(2,1,2);plot(out);grid;axis([0 500 -3 3]);
xlabel('Sample number n');ylabel('Out(n)');
-----

```

Plots are the same as ones in (b).

$$a = a' \approx b = b' \quad r_2 \approx r_4$$

$$1-r \approx r_3 \quad BW_{3dB}(rad) = 2(1-r)$$

$$2\pi BW_{3dB}(Hz)/f_s = 2(1-r) \quad \theta = \Omega_0 = 2\pi f_0/f_s$$

$$r = 1 - \left(\frac{BW_{3dB}(Hz)}{f_s} \right) \pi \quad 0 \leq r < 1$$

$$\theta = \Omega_0 = \frac{f_0}{f_s} \times 360^\circ$$

$$\frac{|H(e^{j\theta_{3dB}})|}{|H(e^{j0})|} = \frac{a}{r_1} \frac{1-\alpha}{2} = \frac{1}{\sqrt{2}}$$

$$1-\alpha \approx 2 \quad -1 < \alpha \ll -0.9$$

$$\frac{a}{r_1} \approx \frac{1}{\sqrt{2}}$$

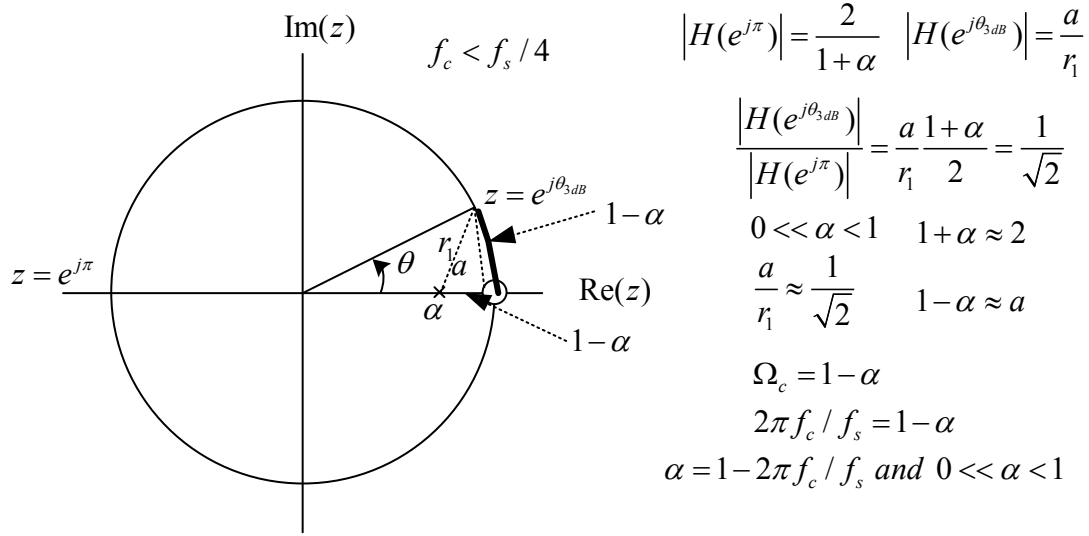
$$a = 1 + \alpha$$

$$\Omega_c = \pi - a = \pi - 1 - \alpha$$

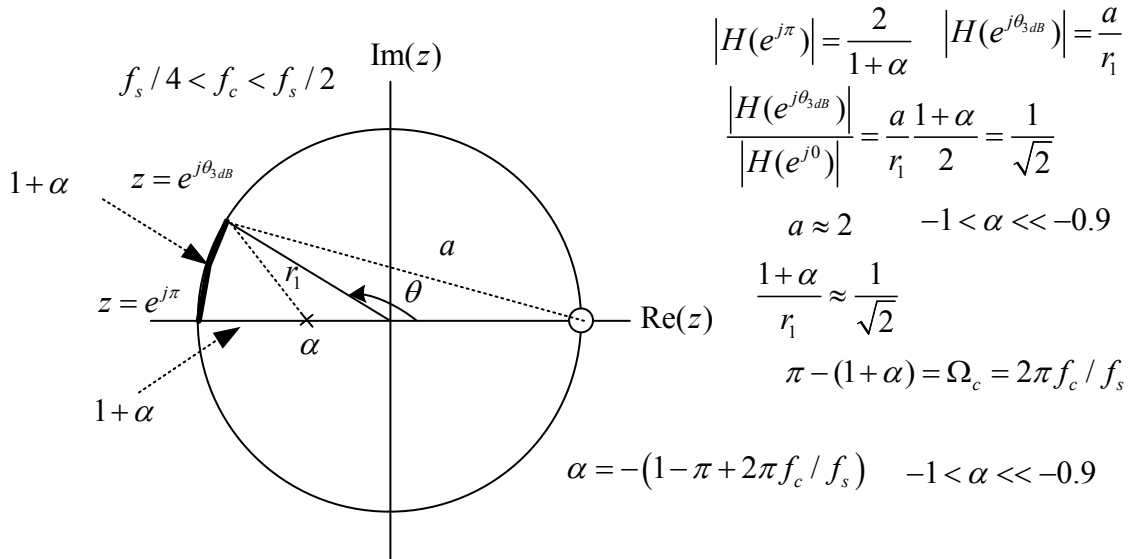
$$2\pi f_c / f_s = \pi - 1 - \alpha$$

$$\alpha = -(1 - \pi + 2\pi f_c / f_s) \quad -1 < \alpha \ll -0.9$$

8.56



8.57



8.58

$$h(n) = -a_1 h(n-1) - a_2 h(n-2) - \dots - a_N h(n-N) + b_0 \delta(n) + b_1 \delta(n-1) + \dots + b_M \delta(n-M)$$

(a)

$$\text{When } 0 \leq n \leq M, \quad b_0 \delta(n) + b_1 \delta(n-1) + \dots + b_M \delta(n-M) = b_n$$

Thus

$$h(n) = -a_1 h(n-1) - a_2 h(n-2) - \dots - a_N h(n-N) + b_n$$

(b)

$$\text{When } n > M, \quad b_0 \delta(n) + b_1 \delta(n-1) + \dots + b_M \delta(n-M) = 0$$

Thus

$$h(n) = -a_1 h(n-1) - a_2 h(n-2) - \dots - a_N h(n-N)$$

(c)

$$\text{For } M < n \leq M+N$$

$$h_d(M+1) = -a_1 h_d(M) - a_2 h_d(M-1) - \dots - a_N h_d(M+1-N)$$

$$h_d(M+2) = -a_1 h_d(M+1) - a_2 h_d(M) - \dots - a_N h_d(M+2-N)$$

...

$$h_d(M+N) = -a_1 h_d(M+N-1) - a_2 h_d(M+N-2) - \dots - a_N h_d(M)$$

This leads to the matrix form 1

For $0 \leq n \leq M$

$$h_d(M+1) = -a_1 h_d(M) - a_2 h_d(M-1) - \dots - a_N h_d(M+1-N)$$

$$h_d(M+2) = -a_1 h_d(M+1) - a_2 h_d(M) - \dots - a_N h_d(M+2-N)$$

...

$$h_d(M+N) = -a_1 h_d(M+N-1) - a_2 h_d(M+N-2) - \dots - a_N h_d(M)$$

This leads to the matrix form 1

$$h_d(0) = b_0$$

$$h_d(1) = -a_1 h_d(0) + b_1$$

...

$$h_d(M) = -a_1 h_d(M-1) - a_2 h_d(M-2) - \dots - a_M h_d(0) + b_M$$

This leads to the second matrix

8.59

$$M=5, N=5, h_d(n) = \frac{1}{n+1} u(n)$$

$$A = [-1/6 \ -1/5 \ -1/4 \ -1/3 \ -1/2;$$

$$-1/7 \ -1/6 \ -1/5 \ -1/4 \ -1/3;$$

$$-1/8 \ -1/7 \ -1/6 \ -1/5 \ -1/4;$$

$$-1/9 \ -1/8 \ -1/7 \ -1/6 \ -1/5;$$

$$-1/10 \ -1/9 \ -1/8 \ -1/7 \ -1/6];$$

$$B = [1/7 \ 1/8 \ 1/9 \ 1/10 \ 1/11]';$$

$$>> a = \text{inv}(A) * B$$

a =

-2.7273

2.7273

-1.2121

0.2273

-0.0130

$$>> C = [1 \ 0 \ 0 \ 0 \ 0;$$

$$1/2 \ 1 \ 0 \ 0 \ 0;$$

$$1/3 \ 1/2 \ 1 \ 0 \ 0;$$

$$1/4 \ 1/3 \ 1/2 \ 1 \ 0;$$

$$1/5 \ 1/4 \ 1/3 \ 1/2 \ 1];$$

$$>> b = C * a$$

b =

-2.7273
1.3636
-0.7576
-0.1515
-0.1670

8.60

(a)

$$E_1 = \sum_{n=M+1}^{\infty} \left[h_d(n) + \sum_{k=1}^N a_k h_d(n-k) \right]^2$$

Taking derivate of E_1 to a_m and setting the result to zero leads:

$$\frac{\partial E_1}{\partial a_m} = \sum_{n=M+1}^{\infty} 2 \left[h_d(n) + \sum_{k=1}^N a_k h_d(n-k) \right] h_d(n-m) = 0$$

$$\sum_{k=1}^N a_k \sum_{n=M+1}^{\infty} h_d(n-k) h_d(n-m) = - \sum_{n=M+1}^{\infty} h_d(n) h_d(n-m), \quad m = 1, \dots, N$$

(b)

$$E_2 = \sum_{n=0}^{\infty} \left[h_d(n) - \sum_{k=0}^M b_k v(n-k) \right]^2$$

Taking derivate of E_2 to b_m and setting the result to zero leads:

$$\frac{\partial E_2}{\partial b_m} = \sum_{n=0}^{\infty} 2 \left[h_d(n) - \sum_{k=0}^M b_k v(n-k) \right] v(n-m) = 0$$

$$\sum_{k=0}^M b_k \sum_{n=0}^{\infty} v(n-k) v(n-m) = \sum_{n=0}^{\infty} h_d(n) v(n-m), \quad m = 0, 1, \dots, M$$

8.61

$$M = N = 1, h_d(n) = 2(0.5)^n u(n)$$

$$a_1 \sum_{n=2}^{\infty} h_d(n-1) h_d(n-1) = - \sum_{n=2}^{\infty} h_d(n) h_d(n-1)$$

$$a_1 = \frac{- \sum_{n=2}^{\infty} h_d(n) h_d(n-1)}{\sum_{n=2}^{\infty} h_d(n-1) h_d(n-1)} = \frac{- \sum_{n=2}^{\infty} 2(1/2)^n 2(1/2)^{n-1}}{\sum_{n=2}^{\infty} 2(1/2)^{n-1} 2(1/2)^{n-1}} = \frac{-8 \sum_{n=2}^{\infty} (1/2)^n (1/2)^n}{16 \sum_{n=2}^{\infty} (1/2)^n (1/2)^n} = -0.5$$

$$H_1(z) = \frac{1}{1-0.5z^{-1}} = \frac{z}{z-0.5}, \quad h_1(n) = v(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$b_0 \sum_{n=0}^{\infty} v(n)v(n-m) + b_1 \sum_{n=0}^{\infty} v(n-1)v(n-m) = \sum_{n=0}^{\infty} h_d(n)v(n-m)$$

Then

$$b_0 \sum_{n=0}^{\infty} v(n)v(n) + b_1 \sum_{n=0}^{\infty} v(n-1)v(n) = \sum_{n=0}^{\infty} h_d(n)v(n)$$

$$b_0 \sum_{n=0}^{\infty} v(n)v(n-1) + b_1 \sum_{n=0}^{\infty} v(n-1)v(n-1) = \sum_{n=0}^{\infty} h_d(n)v(n-1)$$

$$\sum_{n=0}^{\infty} v(n)v(n) = \sum_{n=0}^{\infty} (1/2)^n (1/2)^n = \sum_{n=0}^{\infty} (1/4)^n = 4/3$$

$$\sum_{n=0}^{\infty} v(n-1)v(n-1) = \sum_{n=0}^{\infty} (1/2)^{n-1} u(n-1) (1/2)^{n-1} u(n-1) = 4 \sum_{n=1}^{\infty} (1/4)^n = 4/3$$

$$\sum_{n=0}^{\infty} v(n)v(n-1) = \sum_{n=0}^{\infty} (1/2)^n u(n) (1/2)^{n-1} u(n-1) = 2 \sum_{n=1}^{\infty} (1/4)^n = 2/3$$

$$\sum_{n=0}^{\infty} h_d(n)v(n) = \sum_{n=0}^{\infty} 2(1/2)^n (1/2)^n = 8/3$$

$$\sum_{n=0}^{\infty} h_d(n)v(n-1) = \sum_{n=0}^{\infty} 2(1/2)^n u(n) (1/2)^{n-1} u(n-1) = 4 \sum_{n=1}^{\infty} (1/4)^n = 4/3$$

$$\begin{bmatrix} 4/3 & 2/3 \\ 2/3 & 4/3 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \begin{bmatrix} 8/3 \\ 4/3 \end{bmatrix}, \quad \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

Finally

$$H(z) = \frac{2}{1 + 0.5z^{-1}}$$

Instructor's Guide to Accompany
Digital Signal Processing: Fundamentals and Applications
Part 2: Chapter 9 – Chapter 14

Lizhe Tan
Jean Jiang

Chapter 9

9.1

$$\frac{dJ}{dw} = -40 + 10 \times 2w = 0, \quad w^* = 2$$

$$J_{\min} = J|_{w=w^*} = 50 - 40w + 10w^2|_{w=1} = 50 - 40 \times 2 + 10 \times 2^2 = 10$$

9.2

$$\frac{dJ}{dw} = 20 + 10 \times 2w = 0, \quad w^* = -1$$

$$J_{\min} = J|_{w=w^*} = 15 + 20w + 10w^2|_{w=1} = 15 + 20 \times (-1) + 10 \times (-1)^2 = 5$$

9.3

$$\frac{dJ}{dw} = 20 + 2 \times 2w = 0, \quad w^* = -5$$

$$J_{\min} = J|_{w=w^*} = 100 + 20w + 2w^2|_{w=1} = 100 + 20 \times (-5) + 2 \times (-5)^2 = 50$$

9.4

$$\frac{dJ}{dw} = -30 + 2 \times 15w = 0, \quad w^* = 1$$

$$J_{\min} = J|_{w=w^*} = 10 - 30w + 15w^2|_{w=1} = 10 - 30 \times 1 + 15 \times 1^2 = -5$$

9.5

$$\frac{dJ}{dw} = -40 + 20w$$

$$\mu \frac{dJ}{dw} = 0.04 \times (-40 + 20w_0)|_{w_0=0} = -1.6, \quad w_1 = w_0 - \mu \frac{dJ}{dw} = 0 - (-1.6) = 1.6$$

$$\mu \frac{dJ}{dw} = 0.04 \times (-40 + 20w_1)|_{w_1=1.6} = -0.32, \quad w_2 = w_1 - \mu \frac{dJ}{dw} = 1.6 - (-0.32) = 1.92$$

$$\mu \frac{dJ}{dw} = 0.04 \times (-40 + 20w_2)|_{w_2=1.92} = -0.064, \quad w_3 = w_2 - \mu \frac{dJ}{dw} = 1.92 - (-0.064) = 1.984$$

$$J_{\min} \approx 40 - 20w + 10w^2|_{w=0.992} = 50 - 40 \times 1.984 + 10 \times 1.984^2 = 10.0026$$

$$w^* \approx w_2 = 1.984, \text{ and } J_{\min} = 10.0026$$

9.6

$$\frac{dJ}{dw} = 20 + 20w$$

$$\mu \frac{dJ}{dw} = 0.04 \times (20 + 20w_0)|_{w_0=0} = 0.8, \quad w_1 = w_0 - \mu \frac{dJ}{dw} = 0 - (0.8) = -0.8$$

$$\mu \frac{dJ}{dw} = 0.04 \times (20 + 20w_1)|_{w_1=-0.8} = 0.16, \quad w_2 = w_1 - \mu \frac{dJ}{dw} = -0.8 - (0.16) = -0.96$$

$$\mu \frac{dJ}{dw} = 0.04 \times (20 + 20w_2) \big|_{w_2=-0.96} = 0.032 \quad w_3 = w_2 - \mu \frac{dJ}{dw} = -0.96 - (0.032) = -0.992$$

$$J_{\min} \approx 40 - 20w + 10w^2 \big|_{w=-0.992} = 15 + 20 \times (-0.992) + 10 \times (-0.992)^2 = 5.00064$$

$$w^* \approx w_3 = -0.992, \text{ and } J_{\min} = 5.00064$$

9.7

$$\frac{dJ}{dw} = 20 + 4w$$

$$\mu \frac{dJ}{dw} = 0.2 \times (20 + 4w_0) \big|_{w_0=-4} = 0.8, \quad w_1 = w_0 - \mu \frac{dJ}{dw} = -4 - (0.08) = -4.8$$

$$\mu \frac{dJ}{dw} = 0.2 \times (20 + 4w_1) \big|_{w_1=-4.8} = 0.16 \quad w_2 = w_1 - \mu \frac{dJ}{dw} = -4.8 - (0.16) = -4.96$$

$$\mu \frac{dJ}{dw} = 0.2 \times (20 + 4w_2) \big|_{w_2=-4.96} = 0.032 \quad w_3 = w_2 - \mu \frac{dJ}{dw} = -4.96 - (0.032) = -4.992$$

$$J_{\min} \approx 100 + 20w + 2w^2 \big|_{w=-4.992} = 100 + 20 \times (-4.992) + 2 \times (-4.992)^2 = 5.0001$$

$$w^* \approx w_3 = -4.992, \text{ and } J_{\min} = 5.0001$$

9.8

$$\frac{dJ}{dw} = -30 + 30w$$

$$\mu \frac{dJ}{dw} = 0.02 \times (-30 + 30w_0) \big|_{w_0=2} = 0.6, \quad w_1 = w_0 - \mu \frac{dJ}{dw} = 2 - (0.6) = 1.4$$

$$\mu \frac{dJ}{dw} = 0.02 \times (-30 + 30w_1) \big|_{w_1=1.4} = 0.24 \quad w_2 = w_1 - \mu \frac{dJ}{dw} = 1.4 - (0.24) = 1.16$$

$$\mu \frac{dJ}{dw} = 0.02 \times (-30 + 30w_2) \big|_{w_2=1.16} = 0.096 \quad w_3 = w_2 - \mu \frac{dJ}{dw} = 1.16 - (0.096) = 1.064$$

$$J_{\min} \approx 10 - 30w + 15w^2 \big|_{w=1.064} = 10 - 30 \times (1.064) + 15 \times (1.064)^2 = -4.93856$$

$$w^* \approx w_3 = 1.064, \text{ and } J_{\min} = -4.93856$$

9.9

a.

$$y(n) = w_0 x(n) + w_1 x(n-1)$$

$$e(n) = d(n) - y(n)$$

$$w_0 = w_0 + 0.2 \times e(n) x(n)$$

$$w_1 = w_1 + 0.2 \times e(n) x(n-1)$$

b.

$$\text{for } n = 0: y(0) = 0, e(0) = 3, w_0 = 1.8, w_1 = 1$$

$$\text{for } n = 1: y(1) = 1.2, e(1) = -3.2, w_0 = 2.44, w_1 = -0.92$$

$$\text{for } n = 2: y(2) = 5.8, e(2) = -4.8, w_0 = 0.52, w_1 = 0.04$$

c.

$$W = \begin{bmatrix} 0 & 0 \end{bmatrix}^T, \quad X(n) = \begin{bmatrix} x(n) & x(n-1) \end{bmatrix}^T$$

$$Q = \delta \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\alpha = d(n) - X^T(n)W$$

$$k = \frac{QX(n)}{\lambda + X^T(n)QX(n)}$$

$$W = W + k\alpha$$

$$Q = [Q - kX^T(n)Q] / \lambda$$

$$y(n) = W^T X(n)$$

$$e(n) = d(n) - y(n)$$

d.

```
close all; clear all; clc
W=[0 0]'; X=[0 0]';
x=[3 -1 2]; d=[3 -2 1]; y=[];
lamda=0.96; delta=0.5; Q=eye(2)*delta;
for n=1:3
    X=[x(n); X(1:1)];
    alpha=d(n)-X'*W
    k=Q*X/(lamda+X'*Q*X)
    W=W+k*alpha
    Q=(Q-k*X'*Q)/lamda
    y(n)=W'*X
    e(n)=d(n)-y(n)
end
```

n=0:

alpha = 3

k = 0.2747

0

W = 0.8242

0

Q = 0.0916 0

0 0.5208

y = 2.4725

e = 0.5275

n=1:

alpha = -1.1758

k = -0.0160

0.2723

W = 0.8429

-0.3201

Q = 0.0939 0.0260

0.0260 0.0994

$$y = 2.4725 \quad -1.8033$$

$$e = 0.5275 \quad -0.1967$$

n2:

$$\alpha = -1.0060$$

$$k = 0.1215$$

$$-0.0357$$

$$W = 0.7207$$

$$-0.2842$$

$$Q = 0.0773 \quad 0.0331$$

$$0.0331 \quad 0.1018$$

$$y = 2.4725 \quad -1.8033 \quad 1.7256$$

$$e = 0.5275 \quad -0.1967 \quad -0.7256$$

9.10

(a)

$$d(n) = 0.25x(n) + 0.25x(n-1) + 0.5d(n-1)$$

$$y(n) = w_0x(n) + w_1x(n-1) + w_2x(n-2) + w_3x(n-3) + w_4x(n-4)$$

$$e(n) = d(n) - y(n)$$

for $k = 0, \dots$

$$w_k = w_k + 2\mu e(n)x(n-i)$$

or

$$w_0 = w_0 + 2\mu e(n)x(n)$$

$$w_1 = w_1 + 2\mu e(n)x(n-1)$$

$$w_2 = w_2 + 2\mu e(n)x(n-2)$$

$$w_3 = w_3 + 2\mu e(n)x(n-3)$$

$$w_4 = w_4 + 2\mu e(n)x(n-4)$$

(b)

$$d(n) = 0.25x(n) + 0.25x(n-1) + 0.5d(n-1)$$

$$W = [0 \quad 0 \quad 0 \quad 0 \quad 0]^T, \quad X(n) = [x(n) \quad x(n-1) \quad x(n-2) \quad x(n-3) \quad x(n-4)]^T$$

$$Q = \delta I_{5 \times 5}$$

$$\alpha = d(n) - X^T(n)W$$

$$k = \frac{QX(n)}{\lambda + X^T(n)QX(n)}$$

$$W = W + k\alpha$$

$$Q = [Q - kX^T(n)Q] / \lambda$$

$$y(n) = W^T X(n)$$

$$e(n) = d(n) - y(n)$$

9.11

a.

$$y(n) = w_0 x(n) + w_1 x(n-1) + w_2 x(n-2)$$

$$e(n) = d(n) - y(n)$$

$$w_0 = w_0 + 0.2 \times e(n)x(n)$$

$$w_1 = w_1 + 0.2 \times e(n)x(n-1)$$

$$w_2 = w_2 + 0.2 \times e(n)x(n-2)$$

b.

$$\text{for } n = 0: y(0) = 0, e(0) = 3, w_0 = 1.8, w_1 = 0, w_2 = 0$$

$$\text{for } n = 1: y(1) = 1.2, e(1) = -0.2, w_0 = 1.84, w_1 = -0.12, w_2 = 0.0$$

$$\text{for } n = 2: y(2) = 3.8, e(2) = -2.8, w_0 = 0.72, w_1 = 0.44, w_2 = -1.68$$

c.

$$W = [0 \ 0 \ 0]^T, X(n) = [x(n) \ x(n-1) \ x(n-2)]^T$$

$$Q = \delta \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\alpha = d(n) - X^T(n)W$$

$$k = \frac{QX(n)}{\lambda + X^T(n)QX(n)}$$

$$W = W + k\alpha$$

$$Q = [Q - kX^T(n)Q] / \lambda$$

$$y(n) = W^T X(n)$$

$$e(n) = d(n) - y(n)$$

d.

```
close all; clear all; clc
W=[0 0 0]'; X=[0 0 0]';
x=[3 -1 2]; d=[3 -2 1]; y=[];
lamda=0.96; delta=0.5; Q=eye(3)*delta;
for n=1:3
    X=[x(n); X(1:2)];
    alpha=d(n)-X'*W
    k=Q*X/(lamda+X'*Q*X)
    W=W+k*alpha
    Q=(Q-k*X'*Q)/lamda
    y(n)=W'*X
    e(n)=d(n)-y(n)
end
```

n=0:

alpha = 3

k = 0.2747

0

0

W = 0.8242

0

0

$$Q = \begin{bmatrix} 0.0916 & 0 & 0 \\ 0 & 0.5208 & 0 \\ 0 & 0 & 0.5208 \end{bmatrix}$$

$$y = 2.4725$$

$$e = 0.5275$$

n=1:

$$\alpha = -1.1758$$

$$k = -0.0160$$

$$0.2723$$

$$0$$

$$W = 0.8429$$

$$-0.3201$$

$$0$$

$$Q = \begin{bmatrix} 0.0939 & 0.0260 & 0 \\ 0.0260 & 0.0994 & 0 \\ 0 & 0 & 0.5425 \end{bmatrix}$$

$$y = 2.4725 \quad -1.8033$$

$$e = 0.5275 \quad -0.1967$$

n=2:

$$\alpha = -1.0060$$

$$k = 0.0260$$

$$-0.0076$$

$$0.2619$$

$$W = 0.8167$$

$$-0.3124$$

$$-0.2635$$

$$Q = \begin{bmatrix} 0.0934 & 0.0283 & -0.0441 \\ 0.0283 & 0.1032 & 0.0130 \\ -0.0441 & 0.0130 & 0.1211 \end{bmatrix}$$

$$y = 2.4725 \quad -1.8033 \quad 1.1554$$

$$e = 0.5275 \quad -0.1967 \quad -0.1554$$

9.12

(a)

$$d(n) = 0.2x(n) + 0.3x(n-1) + 0.2x(n-2)$$

$$y(n) = w_0x(n) + w_1x(n-1) + w_2x(n-2) + w_3x(n-3) + w_4x(n-4)$$

$$e(n) = d(n) - y(n)$$

for $k = 0, \dots$

$$w_k = w_k + 2\mu e(n)x(n-k)$$

or

$$w_0 = w_0 + 2\mu e(n)x(n)$$

$$w_1 = w_1 + 2\mu e(n)x(n-1)$$

$$w_2 = w_2 + 2\mu e(n)x(n-2)$$

$$w_3 = w_3 + 2\mu e(n)x(n-3)$$

$$w_4 = w_4 + 2\mu e(n)x(n-4)$$

(b)

$$d(n) = 0.2x(n) + 0.3x(n-1) + 0.2x(n-2)$$

$$W = [0 \ 0 \ 0 \ 0 \ 0]^T, \quad X(n) = [x(n) \ x(n-1) \ x(n-2) \ x(n-3) \ x(n-4)]^T$$

$$Q = \delta I_{5 \times 5}$$

$$\alpha = d(n) - X^T(n)W$$

$$k = \frac{QX(n)}{\lambda + X^T(n)QX(n)}$$

$$W = W + k\alpha$$

$$Q = [Q - kX^T(n)Q] / \lambda$$

$$y(n) = W^T X(n)$$

$$e(n) = d(n) - y(n)$$

9.13

$$a. \quad n(n) = 0.5 \cdot x(n-5)$$

$$b. \quad xx(n) = 5000 \cdot \delta(n), \quad yy(n) = 0.7071xx(n-1) + 1.4141yy(n-1) - yy(n-2)$$

$$c. \quad d(n) = yy(n) - n(n)$$

$$d. \quad \text{for } k = 0, \dots, \quad w_k = 0$$

$$y(n) = \sum_{k=0}^{24} w_k x(n-k)$$

$$e(n) = d(n) - y(n)$$

$$\text{for } k = 0, \dots$$

$$w_k = w_k + 2\mu e(n)x(n-k)$$

(e)

$$W = [0 \ 0 \ \dots \ 0 \ 0]_{24 \times 1}^T, \quad X(n) = [x(n) \ x(n-1) \ x(n-2) \ \dots \ x(n-23)]^T$$

$$Q = \delta I_{24 \times 24}$$

$$\alpha = d(n) - X^T(n)W$$

$$k = \frac{QX(n)}{\lambda + X^T(n)QX(n)}$$

$$W = W + k\alpha$$

$$Q = [Q - kX^T(n)Q] / \lambda$$

$$y(n) = W^T X(n)$$

$$e(n) = d(n) - y(n)$$

9.14

a. $w_0 = w_1 = 0, \mu = 0.1$

$$y(n) = w_0 x(n) + w_1 x(n-1)$$

$$e(n) = d(n) - y(n)$$

$$w_0 = w_0 + 0.2e(n)x(n)$$

$$w_1 = w_1 + 0.2e(n)x(n-1)$$

b. for $n = 0$: $y(0) = 0, e(0) = 0, w_0 = 0, w_1 = 0$

for $n = 1$: $y(1) = 0, e(1) = 2, w_0 = 0.4, w_1 = 0.4$

for $n = 2$: $y(2) = 0, e(2) = -1, w_0 = 0.6, w_1 = 0.2$

c.

$$W = [0 \ 0]^T, X(n) = [x(n) \ x(n-1)]^T$$

$$Q = \delta \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\alpha = d(n) - X^T(n)W$$

$$k = \frac{QX(n)}{\lambda + X^T(n)QX(n)}$$

$$W = W + k\alpha$$

$$Q = [Q - kX^T(n)Q] / \lambda$$

$$y(n) = W^T X(n)$$

$$e(n) = d(n) - y(n)$$

d.

```
close all; clear all; clc
W=[0 0]'; X=[0 0]';
x=[1 1 -1 2]; d=[0 2 -1 1]; y=[];
lamda=0.96; delta=0.5; Q=eye(2,2)*delta;
for n=1:3
    X=[x(n); X(1:1)];
    alpha=d(n)-X'*W
    k=Q*X/(lamda+X'*Q*X)
    W=W+k*alpha
    Q=(Q-k*X'*Q)/lamda
    y(n)=W'*X
    e(n)=d(n)-y(n)
end
```

n=0:

$$\alpha = 0$$

$$k = 0.3425$$

$$0$$

$$W = 0$$

$$0$$

$$Q = \begin{bmatrix} 0.3425 & 0 \\ 0 & 0.5208 \end{bmatrix}$$

$$0 \quad 0.5208$$

```

y = 0
e = 0
n=1:
alpha = 2
k = 0.1878
    0.2857
W = 0.3757
    0.5713
Q = 0.2897 -0.1019
    -0.1019 0.3876
y = 0 0.9470
e = 0 1.0530
n=2:
alpha = -1.1957
k = -0.2127
    0.2659
W = 0.6300
    0.2534
Q = 0.2150 0.0023
    0.0023 0.2682
y = 0 0.9470 -0.3766
e = 0 1.0530 -0.6234

```

9.15

- a. $w_0 = w_1 = w_2 = 0$, $\mu = 0.1$
 $y(n) = w_0 x(n) + w_1 x(n-1) + w_2 x(n-2)$
 $e(n) = d(n) - y(n)$
 $w_0 = w_0 + 0.2e(n)x(n)$
 $w_1 = w_1 + 0.2e(n)x(n-1)$
 $w_2 = w_2 + 0.2e(n)x(n-2)$
- b. for $n = 0$: $y(0) = 0$, $e(0) = 0$, $w_0 = 0$, $w_1 = 0$, $w_2 = 0$
for $n = 1$: $y(1) = 0$, $e(1) = 2$, $w_0 = 0.4$, $w_1 = 0.4$, $w_2 = 0$
for $n = 2$: $y(2) = 0$, $e(2) = -1$, $w_0 = 0.6$, $w_1 = 0.2$, $w_2 = -0.2$

c.

$$W = [0 \ 0 \ 0]^T, \quad X(n) = [x(n) \ x(n-1) \ x(n-2)]^T$$

$$Q = \delta \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\alpha = d(n) - X^T(n)W$$

$$k = \frac{QX(n)}{\lambda + X^T(n)QX(n)}$$

$$W = W + k\alpha$$

$$Q = [Q - kX^T(n)Q] / \lambda$$

$$y(n) = W^T X(n)$$

$$e(n) = d(n) - y(n)$$

d.

```
close all; clear all; clc
W=[0 0 0]'; X=[0 0 0]';
x=[1 1 -1 2]; d=[0 2 -1 1]; y=[];
lamda=0.96; delta=0.5; Q=eye(3)*delta;
for n=1:3
    X=[x(n); X(1:2)];
    alpha=d(n)-X'*W;
    k=Q*X/(lamda+X'*Q*X);
    W=W+k*alpha
    Q=(Q-k*X'*Q)/lamda;
    y(n)=W'*X
    e(n)=d(n)-y(n)
end
```

n=0:

W = 0

0

0

y = 0

e = 0

n=1:

W = 0.3757

0.5713

0

y = 0 0.9470

e = 0 1.0530

n=2:

W = 0.5721

0.3258

-0.2721

y = 0 0.9470 -0.5185

e = 0 1.0530 -0.4815

9.16

a. $w_0 = w_1 = 0$, $\mu = 0.1$

$$x(n) = d(n-2)$$

$$y(n) = w_0 x(n) + w_1 x(n-1)$$

$$e(n) = d(n) - y(n)$$

$$w_0 = w_0 + 0.2e(n)x(n)$$

$$w_1 = w_1 + 0.2e(n)x(n-1)$$

- b. for $n = 0$: $x(0) = 0, y(0) = 0, e(0) = -1, w_0 = 0, w_1 = 0$
 for $n = 1$: $x(1) = 0, y(1) = 0, e(1) = 1, w_0 = 0, w_1 = 0$
 for $n = 2$: $x(2) = -1, y(2) = 0, e(2) = -1, w_0 = 0.2, w_1 = 0$
 for $n = 3$: $x(3) = 1, y(3) = 0.2, e(3) = 0.8, w_0 = 0.36, w_1 = -0.16$
 for $n = 4$: $x(4) = -1, y(4) = -0.52, e(4) = -0.48, w_0 = 0.456, w_1 = -0.256$

c.

$$W = [0 \ 0]^T, \quad X(n) = [x(n-2) \ x(n-3)]^T$$

$$Q = \delta \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\alpha = d(n) - X^T(n)W$$

$$k = \frac{QX(n)}{\lambda + X^T(n)QX(n)}$$

$$W = W + k\alpha$$

$$Q = [Q - kX^T(n)Q] / \lambda$$

$$y(n) = W^T X(n)$$

$$e(n) = d(n) - y(n)$$

d.

```
close all; clear all; clc
W=[0 0]'; X=[0 0]';
d=[-1 1 -1 1 -1 1 -1]; x=[ 0 0 -1 1 -1 1 -1]; % x(n)=d(n-2)
y=[];
lamda=0.96; delta=0.5; Q=eye(2)*delta;
for n=1:5
    X=[x(n); X(1:1)];
    alpha=d(n)-X'*W;
    k=Q*X/(lamda+X'*Q*X);
    W=W+k*alpha;
    Q=(Q-k*X'*Q)/lamda;
    y(n)=W'*X;
    e(n)=d(n)-y(n);
end
```

$$y = \quad 0 \quad 0 \quad -0.3611 \quad 0.6748 \quad -0.7849$$

9.17

- a. $w_0 = w_1 = 0, \mu = 0.1$
 $x(n) = d(n-3)$
 $y(n) = w_0 x(n) + w_1 x(n-1)$
 $e(n) = d(n) - y(n)$
 $w_0 = w_0 + 0.2e(n)x(n)$
 $w_1 = w_1 + 0.2e(n)x(n-1)$
- b. for $n = 0$: $x(0) = 0, y(0) = 0, e(0) = -1, w_0 = 0, w_1 = 0$
 for $n = 1$: $x(1) = 0, y(1) = 0, e(1) = 1, w_0 = 0, w_1 = 0$

for $n = 2$: $x(2) = 0$, $y(2) = 0$, $e(2) = -1$, $w_0 = 0$, $w_1 = 0$

for $n = 3$: $x(3) = -1$, $y(3) = 0$, $e(3) = 1$, $w_0 = -0.2$, $w_1 = 0$

for $n = 4$: $x(4) = 1$, $y(4) = -0.2$, $e(4) = -0.8$, $w_0 = -0.36$, $w_1 = 0.16$

c.

$$W = [0 \ 0 \ 0]^T, \quad X(n) = [x(n) \ x(n-1) \ x(n-2)]^T$$

$$Q = \delta \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\alpha = d(n) - X^T(n)W$$

$$k = \frac{QX(n)}{\lambda + X^T(n)QX(n)}$$

$$W = W + k\alpha$$

$$Q = [Q - kX^T(n)Q] / \lambda$$

$$y(n) = W^T X(n)$$

$$e(n) = d(n) - y(n)$$

d.

```
close all; clear all; clc
W=[0 0 0]'; X=[0 0 0]';
d=[-1 1 -1 1 -1 1 -1]; x=[ 0 0 0 -1 1 -1 1 -1]; % x(n)=d(n-3)
y=[];
lamda=0.96; delta=0.5; Q=eye(3)*delta;
for n=1:5
    X=[x(n);X(1:2)];
    alpha=d(n)-X'*W;
    k=Q*X/(lamda+X'*Q*X);
    W=W+k*alpha;
    Q=(Q-k*X'*Q)/lamda;
    y(n)=W'*X;
    e(n)=d(n)-y(n);
end
```

$$y = 0 \quad 0 \quad 0 \quad 0.3705 \quad -0.6851$$

9.18

a. 30 coefficients

b. for $k = 0, \dots$, $w_k = 0$

$$y(n) = \sum_{k=0}^{29} w_k x(n-k-\Delta)$$

$$e(n) = d(n) - y(n)$$

for $i = 0, \dots$

$$w_k = w_k + 2\mu e(n)x(n-k)$$

c.

$$W = [0]_{1 \times 30}^T, \quad X(n) = [x(n-\Delta) \quad \cdots \quad 29-\Delta]^T$$

$$Q = \delta I_{30 \times 30}$$

$$\alpha = d(n) - X^T(n)W$$

$$k = \frac{QX(n)}{\lambda + X^T(n)QX(n)}$$

$$W = W + k\alpha$$

$$Q = [Q - kX^T(n)Q] / \lambda$$

$$y(n) = W^T X(n)$$

$$e(n) = d(n) - y(n)$$

9.19

(a)

10 coefficients

(b)

$$\text{for } k = 0, \dots, \quad w_k = 0$$

$$y(n) = \sum_{i=0}^9 w(i)x(n-i-\Delta)$$

$$e(n) = d(n) - y(n)$$

$$\text{for } i = 0, \dots$$

$$w(i) = w(i) + 2\mu e(n)x(n-i)$$

(c)

$$W = [0]_{1 \times 10}^T, \quad X(n) = [x(n-\Delta) \quad \cdots \quad 9-\Delta]^T$$

$$Q = \delta I_{30 \times 30}$$

$$\alpha = d(n) - X^T(n)W$$

$$k = \frac{QX(n)}{\lambda + X^T(n)QX(n)}$$

$$W = W + k\alpha$$

$$Q = [Q - kX^T(n)Q] / \lambda$$

$$y(n) = W^T X(n)$$

$$e(n) = d(n) - y(n)$$

9.20

$$\text{for } k = 0, \dots, \quad w(i) = 0$$

$$y(n) = \sum_{k=0}^{19} w_k x(n-k)$$

$$e(n) = d(n) - y(n)$$

$$\text{for } k = 0, \dots$$

$$w_k = w_k + 2\mu e(n)x(n-k)$$

9.21

$$y_A(n) = \sum_{k=0}^3 w_k x_B(n-k)$$

$$e_A(n) = d_A(n) - y_A(n)$$

for $k = 0, \dots$

$$w_k = w_k + 2\mu e_A(n) x_B(n-k)$$

9.22

a. See Section 10.4.3, b. See Section 10.4.3

$$c. \quad y_A(n) = \sum_{k=0}^{N-1} w_k x_B(n-k)$$

$$e_A(n) = d_A(n) - y_A(n)$$

for $k = 0, \dots$

$$w_k = w_k + 2\mu e_A(n) x_B(n-k)$$

9.23

close all; clear all

w1(1)=0;w2(1)=0;mu=0.001;

for k=1:500

J(k)=100+100*w1(k)^2+4*w2(k)^2-100*w1(k)-8*w2(k)+10*w1(k)*w2(k);

dj1(k+1)=200*w1(k)-100+10*w2(k);

dj2(k+1)=8*w2(k)-8+10*w1(k);

w1(k+1)=w1(k)-mu*dj1(k);

w2(k+1)=w2(k)-mu*dj2(k);

end

subplot(3,1,1);plot(w1,'k');grid;

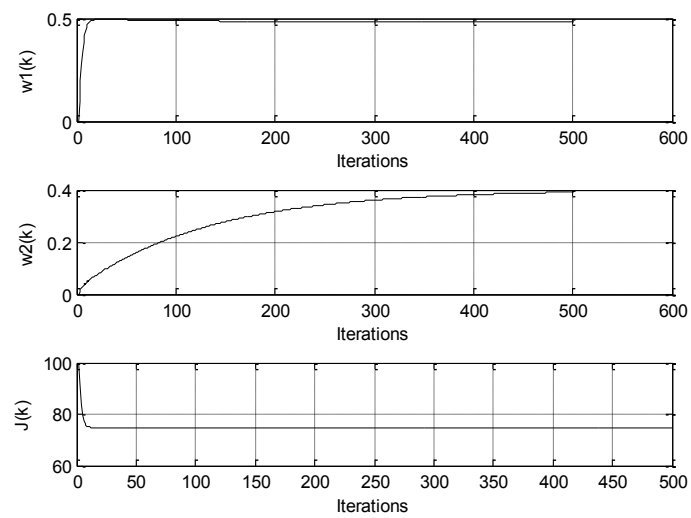
xlabel('Iterations');ylabel('w1(k)');

subplot(3,1,2);plot(w2,'k');grid

xlabel('Iterations');ylabel('w2(k)');

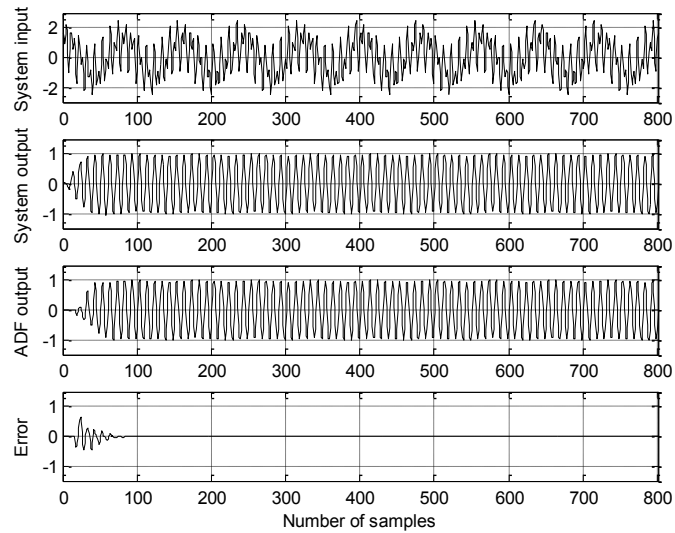
subplot(3,1,3);plot(J,'k');grid

xlabel('Iterations');ylabel('J(k)');

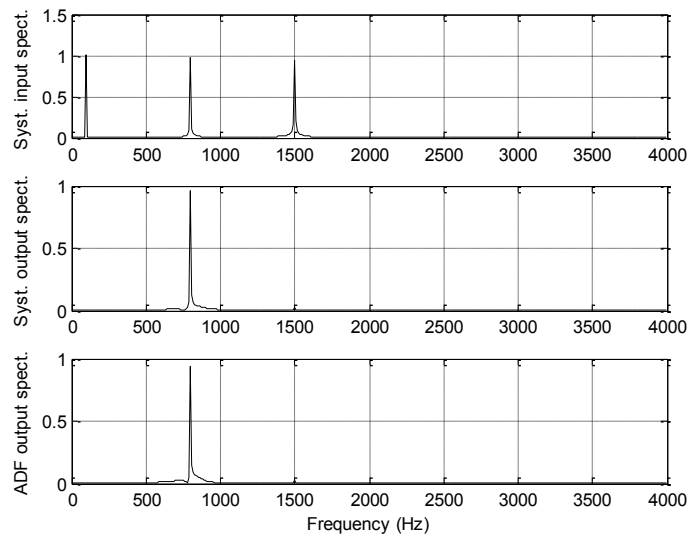


9.24 (a) and (b) Modify Program 9.2 in Section 9.3.2.

c.



d.



e.

```

close all; clear all
%Create the unknown system
fs=8000; T=1/fs; % Sampling rate
and sampling period
% Bandpass filter design
%for the assumed unknown system using the bilinear transformation
%(BLT) method (see Chapter 8)
wd1=700*2*pi; wd2=900*2*pi;
wa1=(2/T)*tan(wd1*T/2); wa2=(2/T)*tan(wd2*T/2);
BW=wa2-wa1;
w0=sqrt(wa2*wa1);
[B,A]=lp2bp([1],[1 1.4141 1],w0,BW);
[b,a]=bilinear(B,A,fs);
freqz(b,a,512,fs); axis([0 fs/2 -80 1]); % Frequency response plots
figure
t=0:T:0.1; % Generate the time vector
x=cos(2*pi*100*t)+sin(2*pi*800*t)+cos(2*pi*1500*t+pi/4);
d=filter(b,a,x); % Produce unknown system output
W=zeros(1,21)'; y=zeros(1,length(t)); % Initialize the coefficients and
output
e=y; % Initialize the
error vector
lamda=0.96; delta=0.5;Q=eye(21)*delta;
X=zeros(1,21)';
for n=1:length(x);
    X=[x(n); X(1:20)];
    alpha=d(n)-X'*W;
    k=Q*X/(lamda+X'*Q*X);
    W=W+k*alpha;
    Q=(Q-k*X'*Q)/lamda;
    y(n)=W'*X;
    e(n)=d(n)-y(n);
end

% Calculate the single-sided amplitude spectrum for the input
X=2*abs(fft(x))/length(x);X(1)=X(1)/2;

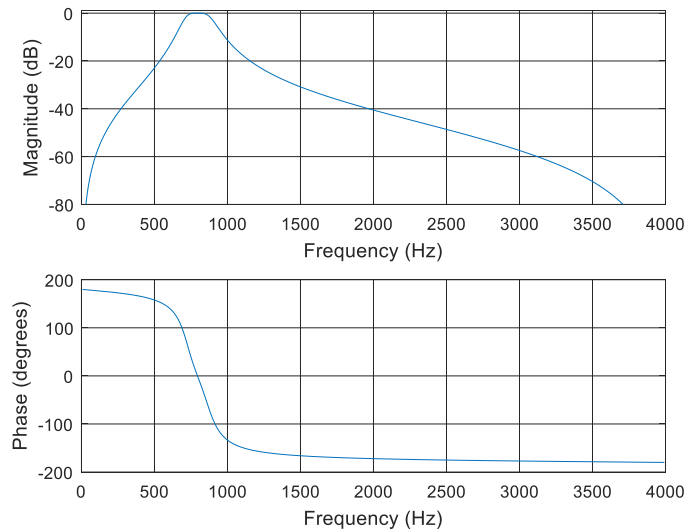
```

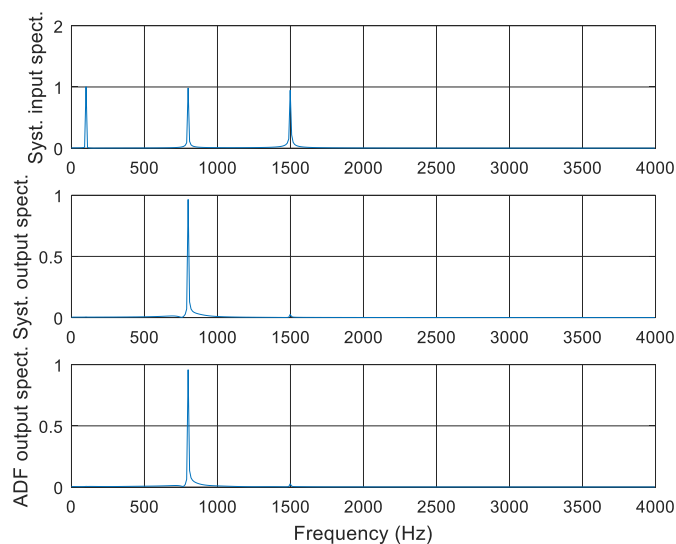
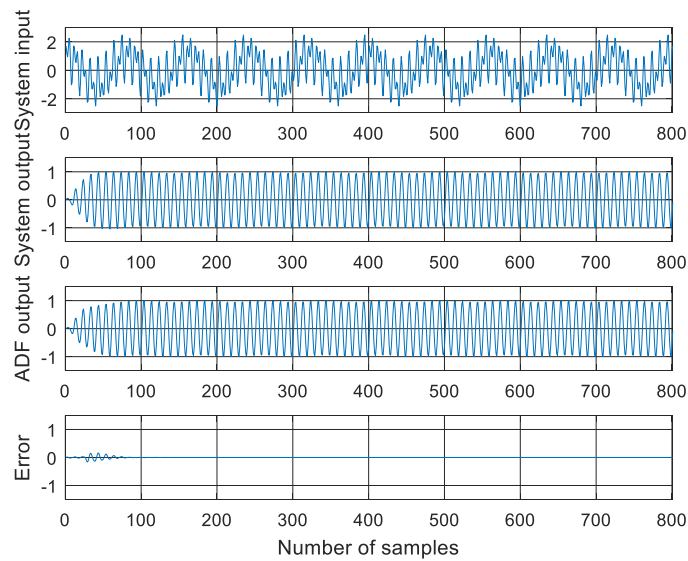
```

% Calculate the single-sided amplitude spectrum for the unknown system
output
D=2*abs(fft(d))/length(d);D(1)=D(1)/2;
% Calculate the single-sided amplitude spectrum for the adaptive filter
output
Y=2*abs(fft(y))/length(y);Y(1)=Y(1)/2;
% Map the frequency index to its frequency in Hz
f=[0:1:length(x)/2]*fs/length(x);

% Plot signals and spectra
subplot(4,1,1), plot(x);grid; axis([0 length(x) -3 3]);
ylabel('System input');
subplot(4,1,2), plot(d);grid; axis([0 length(x) -1.5 1.5]);
ylabel('System output');
subplot(4,1,3), plot(y);grid; axis([0 length(y) -1.5 1.5]);
ylabel('ADF output')
subplot(4,1,4), plot(e);grid; axis([0 length(e) -1.5 1.5]);
ylabel('Error'); xlabel('Number of samples')
figure
subplot(3,1,1), plot(f,X(1:length(f)));grid; ylabel('Syst. input
spect.')
subplot(3,1,2), plot(f,D(1:length(f)));grid; ylabel('Syst. output
spect.')
subplot(3,1,3), plot(f,Y(1:length(f)));grid
ylabel('ADF output spect. '); xlabel('Frequency (Hz)');

```





9.25

a.

```

close all; clear all
fs=8000;T=1/fs;
t=0:T:0.1;
x=randn(1,length(t));
n=filter([ 0 0 0 0 0 0 0 0 0.8 ],1,x);
d=sin(2*pi*300*t)+n;          % generate signal plus noise
mu=0.01;w=zeros(1,16);y=zeros(1,length(t));
e=y;
% adaptive filtering using LMS algorithm

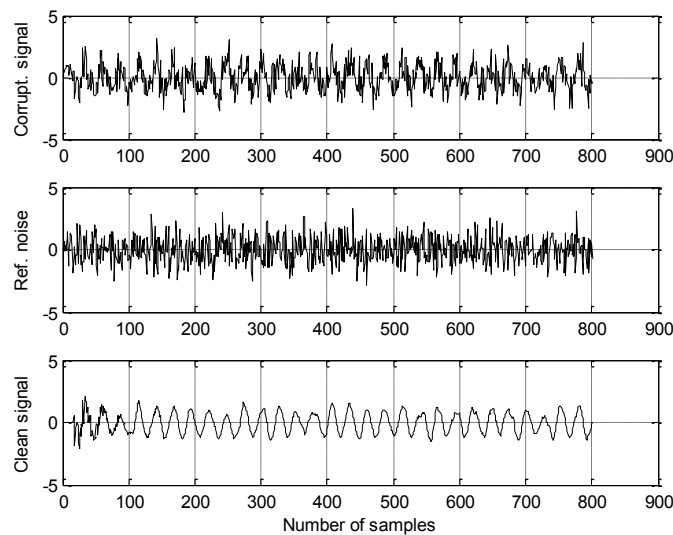
```

```

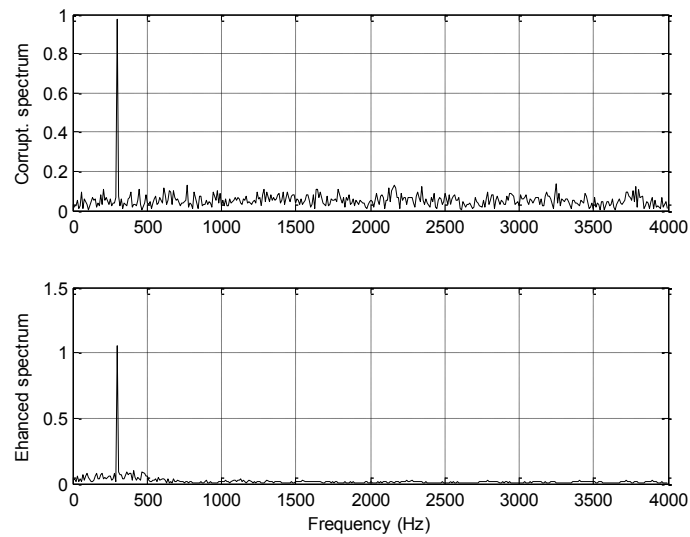
for m=17:1:length(t)-1
    sum=0;
    for i=1:1:16
        sum=sum+w(i)*x(m-i);
    end
    y(m)=sum;
    e(m)=d(m)-y(m);
    for i=1:1:16
        w(i)=w(i)+2*mu*e(m)*x(m-i);
    end
end
D=2*abs(fft(d))/length(d);D(1)=D(1)/2;
f=[0:1:length(d)/2]*8000/length(d);
E=2*abs(fft(e))/length(e);E(1)=E(1)/2;
subplot(3,1,1),plot(d,'k');grid; ylabel('Corrupt. signal')
subplot(3,1,2),plot(x,'k');grid;ylabel('Ref. noise');
subplot(3,1,3),plot(e,'k');grid; ylabel('Clean signal');
xlabel('Number of samples');
figure
subplot(2,1,1),plot(f,D(1:length(f)),'k');grid; ylabel('Corrupt. spectrum')
subplot(2,1,2),plot(f,E(1:length(f)),'k');grid
ylabel('Enhanced spectrum'); xlabel('Frequency (Hz)');
-----

```

b.



c.



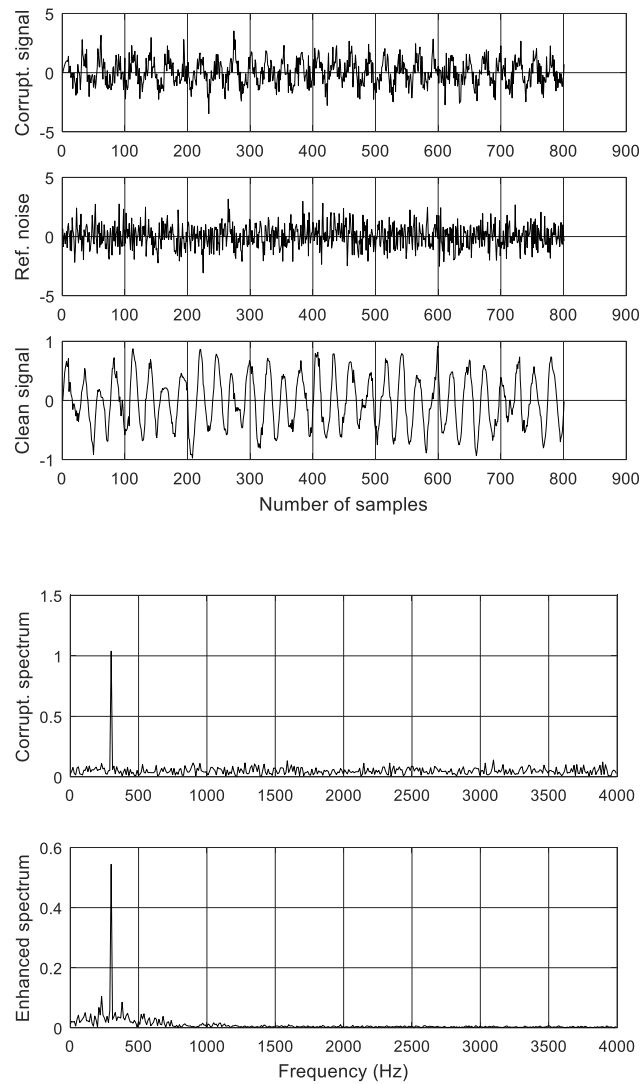
d

```

close all; clear all
fs=8000; T=1/fs;
t=0:T:0.1;
x=randn(1,length(t));
n=filter([ 0 0 0 0 0 0 0 0 0 0.8 ],1,x);
d=sin(2*pi*300*t)+n; % generate signal plus noise
W=zeros(1,16)'; y=zeros(1,length(t)); % Initialize the coefficients and
output
e=y; % Initialize the
error vector
lamda=0.96; delta=0.5; Q=eye(16)*delta;
X=zeros(1,16)';
% RLS algorithm
for n=1:length(x);
    X=[x(n); X(1:15)];
    alpha=d(n)-X'*W;
    k=Q*X/(lamda+X'*Q*X);
    W=W+k*alpha;
    Q=(Q-k*X'*Q)/lamda;
    y(n)=W'*X;
    e(n)=d(n)-y(n);
end

D=2*abs(fft(d))/length(d); D(1)=D(1)/2;
f=[0:1:length(d)/2]*8000/length(d);
E=2*abs(fft(e))/length(e); E(1)=E(1)/2;
subplot(3,1,1), plot(d, 'k'); grid; ylabel('Corrupt. signal')
subplot(3,1,2), plot(x, 'k'); grid; ylabel('Ref. noise');
subplot(3,1,3), plot(e, 'k'); grid; ylabel('Clean signal');
xlabel('Number of samples');
figure
subplot(2,1,1), plot(f, D(1:length(f)), 'k'); grid; ylabel('Corrupt.
spectrum')
subplot(2,1,2), plot(f, E(1:length(f)), 'k'); grid
ylabel('Enhanced spectrum'); xlabel('Frequency (Hz)');

```



9.26

a.

```

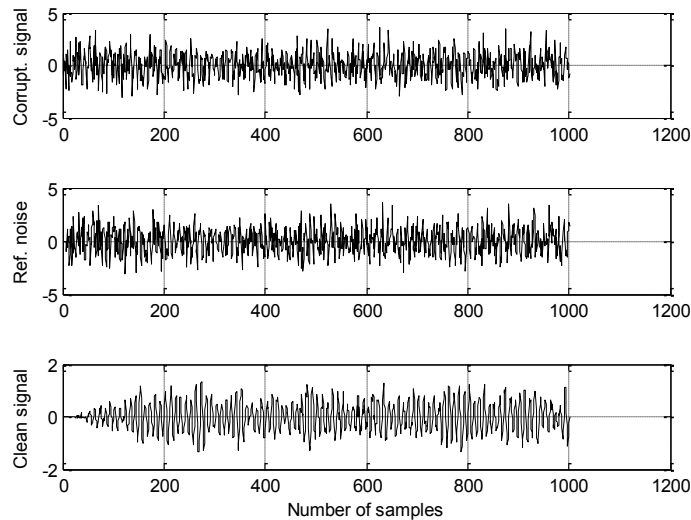
close all; clear all;clc
fs=1000;T=1/fs;
t=0:T:1;
n=randn(1,length(t));
d=sin(2*pi*100*t)+n;
x=filter([ 0 0 0 0 0 0 1 ],1,d);
mu=0.001;w=zeros(1,16);y=zeros(1,length(t));
e=y;
% adaptive filtering using LMS algorithm
for m=17:1:length(t)-1
    sum=0;
    for i=1:1:16

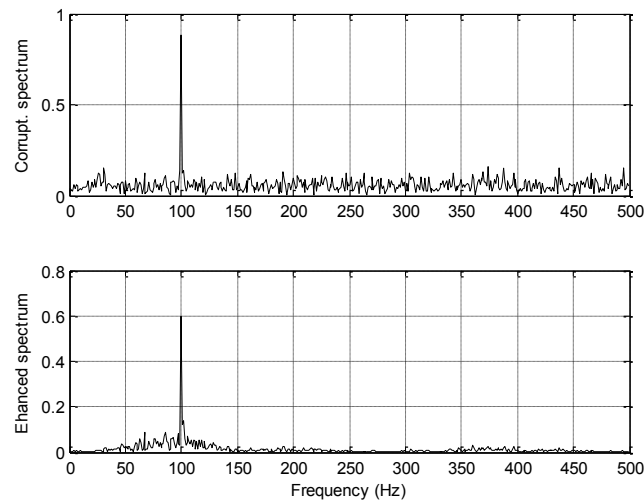
```

```

    sum=sum+w(i)*x(m-i);
end
y(m)=sum;
e(m)=d(m)-y(m);
for i=1:1:16
    w(i)=w(i)+2*mu*e(m)*x(m-i);
end
end
D=2*abs(fft(d))/length(d);D(1)=D(1)/2;
f=[0:1:length(d)/2]*1000/length(d);
Y=2*abs(fft(y))/length(y);Y(1)=Y(1)/2;
subplot(3,1,1),plot(d,'k');grid; ylabel('Corrupt. signal')
subplot(3,1,2),plot(x,'k');grid;ylabel('Ref. noise');
subplot(3,1,3),plot(y,'k');grid; ylabel('Clean signal');
xlabel('Number of samples');
figure
subplot(2,1,1),plot(f,D(1:length(f)),'k');grid; ylabel('Corrupt. spectrum')
subplot(2,1,2),plot(f,Y(1:length(f)),'k');grid
ylabel('Enhanced spectrum'); xlabel('Frequency (Hz)');

```

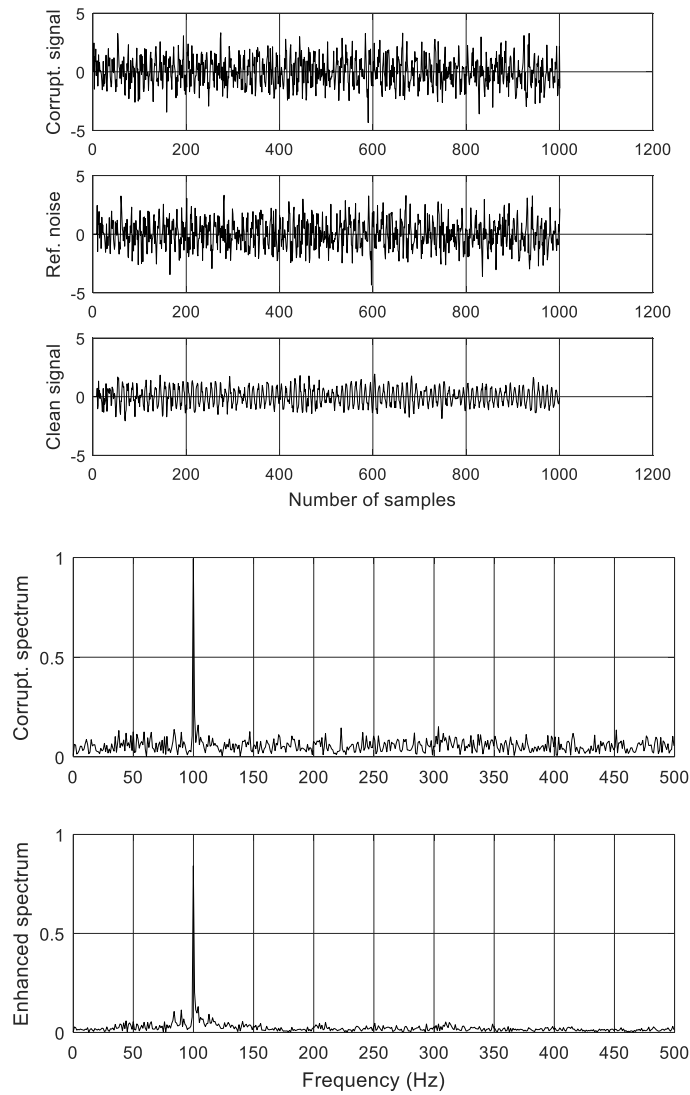




b. Δ is around 8 samples.

c.

```
close all; clear all; clc
fs=1000; T=1/fs;
t=0:T:1;
n=randn(1,length(t));
d=sin(2*pi*100*t)+n;
x=filter([ 0 0 0 0 0 0 0 1 ],1,d);
W=zeros(1,16)'; y=zeros(1,length(t)); % Initialize the coefficients and
output
e=y; % Initialize the
error vector
lamda=0.995; delta=0.5; Q=eye(16)*delta;
X=zeros(1,16)';
% RLS algorithm
for n=1:length(x);
    X=[x(n); X(1:15)];
    alpha=d(n)-X'*W;
    k=Q*X/(lamda+X'*Q*X);
    W=W+k*alpha;
    Q=(Q-k*X'*Q)/lamda;
    y(n)=W'*X;
    e(n)=d(n)-y(n);
end
D=2*abs(fft(d))/length(d); D(1)=D(1)/2;
f=[0:1:length(d)/2]*1000/length(d);
Y=2*abs(fft(y))/length(y); Y(1)=Y(1)/2;
subplot(3,1,1), plot(d, 'k'); grid; ylabel('Corrupt. signal')
subplot(3,1,2), plot(x, 'k'); grid; ylabel('Ref. noise');
subplot(3,1,3), plot(y, 'k'); grid; ylabel('Clean signal');
xlabel('Number of samples');
figure
subplot(2,1,1), plot(f, D(1:length(f)), 'k'); grid; ylabel('Corrupt.
spectrum')
subplot(2,1,2), plot(f, Y(1:length(f)), 'k'); grid
ylabel('Enhanced spectrum'); xlabel('Frequency (Hz)');
```



9.30

(a)

$$E\{\sin(n\Omega)\} = 0$$

(b)

$$E\{e^{jn\Omega}\} = E\{\cos(n\Omega)\} + jE\{\sin(n\Omega)\} = 0 + j0 = 0$$

(c)

$$E\{(A \sin n\Omega)^2\} = (A^2/2)E\{1 - \cos(2n\Omega)\} = A^2/2$$

(d)

$$E\{(A \cos n\Omega)^2\} = (A^2/2)E\{1 + \cos(2n\Omega)\} = A^2/2$$

(e)

$$E\{(A \sin n\Omega)(A \cos n\Omega)\} = (A^2/2)E\{\sin(2n\Omega)\} = 0$$

(f)

$$E\{[A \cos(n - n_0)\Omega]^2\} = (A^2/2)E\{1 + \cos[2(n - n_0)\Omega]\} = A^2/2$$

(g)

$$E\{[A \sin(n - n_0)\Omega][A \cos(n\Omega)]\} = (A^2/2)E\{\sin(2n - n_0)\Omega + \sin(-n_0\Omega)\} = -(A^2/2)\sin(n_0\Omega)$$

9.31

$$W^* = R^{-1}P$$

$$r_{11} = E\{d^2(n - n_1)\} = E\{A^2 \sin^2(n - n_1)\Omega\} = A^2/2$$

$$r_{22} = E\{d^2(n - n_2)\} = E\{A^2 \sin^2(n - n_2)\Omega\} = A^2/2$$

$$r_{33} = E\{d^2(n - n_3)\} = E\{A^2 \sin^2(n - n_3)\Omega\} = A^2/2$$

$$r_{12} = r_{21} = E\{d(n - n_1)d(n - n_2)\} = E\{A^2 \sin(n - n_1)\sin(n - n_2)\Omega\} = (A^2/2)\cos(n_2 - n_1)$$

$$r_{13} = r_{31} = E\{d(n - n_1)d(n - n_3)\} = E\{A^2 \sin(n - n_1)\sin(n - n_3)\Omega\} = (A^2/2)\cos(n_3 - n_1)$$

$$r_{23} = r_{32} = E\{d(n - n_2)d(n - n_3)\} = E\{A^2 \sin(n - n_2)\sin(n - n_3)\Omega\} = (A^2/2)\cos(n_3 - n_1)$$

$$p_1 = E\{d(n)d(n - n_1)\} = E\{A^2 \sin(n)\sin(n - n_1)\Omega\} = (A^2/2)\cos(n_1)$$

$$p_2 = E\{d(n)d(n - n_2)\} = E\{A^2 \sin(n)\sin(n - n_2)\Omega\} = (A^2/2)\cos(n_2)$$

$$p_3 = E\{d(n)d(n - n_3)\} = E\{A^2 \sin(n)\sin(n - n_3)\Omega\} = (A^2/2)\cos(n_3)$$

$$W^* = \begin{bmatrix} A^2/2 & (A^2/2)\cos(n_2 - n_1)\Omega & (A^2/2)\cos(n_3 - n_1)\Omega \\ (A^2/2)\cos(n_2 - n_1)\Omega & A^2/2 & (A^2/2)\cos(n_3 - n_2)\Omega \\ (A^2/2)\cos(n_3 - n_1)\Omega & (A^2/2)\cos(n_3 - n_2)\Omega & A^2/2 \end{bmatrix}^{-1} \begin{bmatrix} (A^2/2)\cos(n_1\Omega) \\ (A^2/2)\cos(n_2\Omega) \\ (A^2/2)\cos(n_3\Omega) \end{bmatrix}$$

$$W^* = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 1 & \cos(n_2 - n_1)\Omega & \cos(n_3 - n_1)\Omega \\ \cos(n_2 - n_1)\Omega & 1 & \cos(n_3 - n_2)\Omega \\ \cos(n_3 - n_1)\Omega & \cos(n_3 - n_2)\Omega & 1 \end{bmatrix}^{-1} \begin{bmatrix} \cos(n_1\Omega) \\ \cos(n_2\Omega) \\ \cos(n_3\Omega) \end{bmatrix}$$

9.32

$$J = 16 + 2w_0^2 + w_1^2 - 2w_0 + 2w_1$$

$$\frac{\partial J}{\partial w_0} = 4w_0 - 2 = 0, \quad \frac{\partial J}{\partial w_1} = 2w_1 + 2 = 0$$

$$w_0 = 1/2, \quad w_1 = -1$$

$$J_{\min} = 16 + 2w_0^2 + w_1^2 - 2w_0 + 2w_1 = 16 + 2 \times (0.5)^2 + (-1)^2 - 2 \times (1/2) + 2 \times (-1) = 14.5$$

9.33

$$J = 16 + 2w_0^2 + w_1^2 - 2w_0 + 2w_1 - 2w_0w_1$$

$$\frac{\partial J}{\partial w_0} = 4w_0 - 2 - 2w_1 = 0, \quad \frac{\partial J}{\partial w_1} = 2w_1 + 2 - 2w_0 = 0$$

$$w_0 = 0, \quad w_1 = -1$$

$$J_{\min} = 16 + 2w_0^2 + w_1^2 - 2w_0 + 2w_1 - 2w_0w_1 = 16 + 2 \times (0)^2 + (-1)^2 - 2 \times (0) + 2 \times (-1) - 2 \times 0 \times (-1) = 15$$

9.34

(a)

$$J = 16 + 2w_0^2 + w_1^2 - 2w_0 + 2w_1$$

$$\frac{\partial J}{\partial w_0(n)} = 4w_0(n) - 2, \quad \frac{\partial J}{\partial w_1(n)} = 2w_1(n) + 2$$

The steepest descent algorithm:

$$w_0(n) = 0, \quad w_1(n) = 0$$

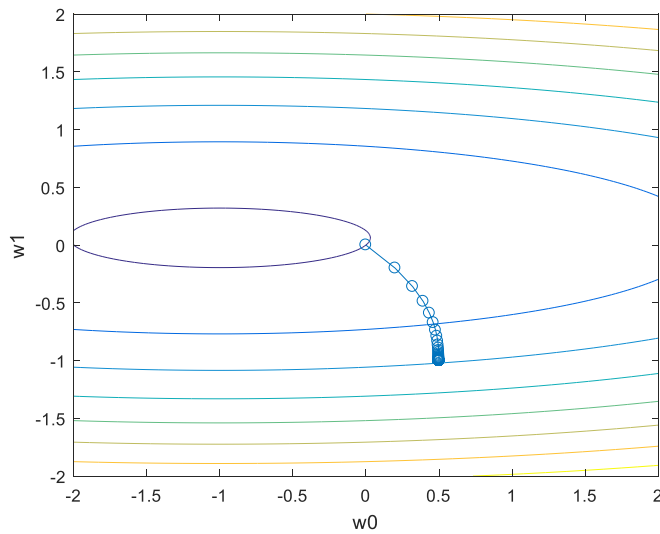
$$w_0(n+1) = w_0(n) - \mu[4w_0(n) - 2]$$

$$w_1(n+1) = w_1(n) - \mu[2w_1(n) + 2]$$

$$J(n+1) = 16 + 2w_0^2(n+1) + w_1^2(n+1) - 2w_0(n+1) + 2w_1(n+1)$$

(b)

```
close all; clear all; clc;
w0(1)=1;w1(1)=0;mu=0.1;
J=zeros(1,500);
w0=J;w1=J;
for k=2:500
    w0(k)=w0(k-1)-mu*(4*w0(k-1)-2+2*w1(k));
    w1(k)=w1(k-1)-mu*(2*w1(k-1)+2);
    %J(k)=16+2*w0(k)^2+w1(k)^2-2*w0(k)+2*w1(k);
end
x=-2:0.01:2;
y=-2:0.01:2;
for i=1:length(x)
    for j=1:length(y)
        z(i,j)=16*x(i)^2+y(j)^2-2*x(i)+2*y(j);
    end
end
contour(x,y,z);xlabel('w0'),ylabel('w1');hold
plot(w0,w1,'o-');
```



9.35

(a)

$$J = 16 + 2w_0^2 + w_1^2 - 2w_0 + 2w_1 - 2w_0w_1$$

$$\frac{\partial J}{\partial w_0(n)} = 4w_0(n) - 2 - 2w_1(n), \quad \frac{\partial J}{\partial w_1(n)} = 2w_1(n) + 2 - 2w_0(n) = 0$$

The steepest descent algorithm:

$$w_0(0) = -1, w_1(0) = 0$$

$$w_0(n+1) = w_0(n) - \mu[4w_0(n) - 2 - 2w_1(n)]$$

$$w_1(n+1) = w_1(n) - \mu[2w_1(n) + 2 - 2w_0(n)]$$

$$J(n+1) = 16 + 2w_0^2(n) + w_1^2(n+1) - 2w_0(n+1) + 2w_1(n+1) - 2w_0(n+1)w_1(n+1)$$

(b) `close all; clear all; clc;`

`J=zeros(1,500);`

`w0=J;w1=J;`

`w0(1)=1;w1(1)=0;mu=0.1;`

`for k=2:500`

`w0(k)=w0(k-1)-mu*(4*w0(k-1)-2-2*w1(k-1));`

`w1(k)=w1(k-1)-mu*(2*w1(k-1)+2-w0(k-1));`

`%J(k)=16+2*w0(k)^2+w1(k)^2-2*w0(k)+2*w1(k);`

`end`

`x=-2:0.01:2;`

`y=-2:0.01:2;`

`for i=1:length(x)`

`for j=1:length(y)`

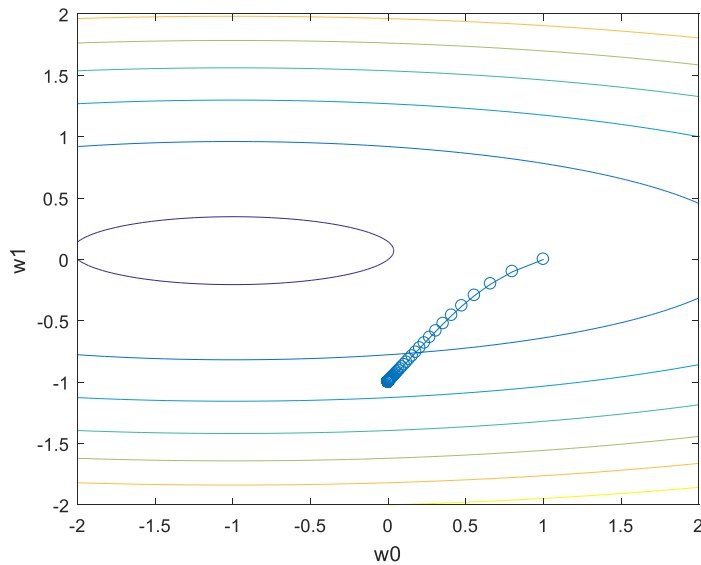
`z(i,j)=16*x(i)^2+y(j)^2-2*x(i)+2*y(j)-2*x(i)*y(i);`

`end`

`end`

`contour(x,y,z);xlabel('w0'),ylabel('w1');hold`

`plot(w0,w1,'o-');`



9.36

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = - \begin{bmatrix} A^2/2 & (A^2/2)\cos(n_2 - n_1)\Omega \\ (A^2/2)\cos(n_2 - n_1)\Omega & A^2/2 \end{bmatrix}^{-1} \begin{bmatrix} (A^2/2)\cos(n_1\Omega) \\ (A^2/2)\cos(n_2\Omega) \end{bmatrix}$$

$$\begin{aligned}
\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} &= - \begin{bmatrix} 1 & \cos(n_2 - n_1)\Omega \\ \cos(n_2 - n_1)\Omega & 1 \end{bmatrix}^{-1} \begin{bmatrix} \cos(n_1\Omega) \\ \cos(n_2\Omega) \end{bmatrix} \\
&= - \begin{bmatrix} 1 & -\cos(n_2 - n_1)\Omega \\ -\cos(n_2 - n_1)\Omega & 1 \end{bmatrix} \begin{bmatrix} \cos(n_1\Omega) \\ \cos(n_2\Omega) \end{bmatrix} / \sin^2(n_2 - n_1)\Omega \\
&= - \begin{bmatrix} \cos(n_1\Omega) - \cos(n_2 - n_1)\Omega \cos(n_2\Omega) \\ -\cos(n_1\Omega) \cos(n_2 - n_1)\Omega + \cos(n_2\Omega) \end{bmatrix} / \sin^2(n_2 - n_1)\Omega \\
\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} &= - \begin{bmatrix} \cos(n_1\Omega) - \cos(n_2\Omega) \cos(n_1\Omega) \cos(n_2\Omega) - \sin(n_2\Omega) \sin(n_1\Omega) \cos(n_2\Omega) \\ -\cos(n_1\Omega) \cos(n_2\Omega) \cos(n_1)\Omega - \sin(n_1\Omega) \sin(n_2\Omega) \cos(n_1)\Omega + \cos(n_1\Omega) \end{bmatrix} / \sin^2(n_2 - n_1)\Omega \\
\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} &= - \begin{bmatrix} \cos(n_1\Omega) \sin^2(n_2\Omega) - \sin(n_2\Omega) \sin(n_1\Omega) \cos(n_2\Omega) \\ \sin^2(n_1\Omega) \cos(n_2\Omega) - \sin(n_1\Omega) \sin(n_2\Omega) \cos(n_1)\Omega \end{bmatrix} / \sin^2(n_2 - n_1)\Omega \\
\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} &= - \begin{bmatrix} \sin(n_2\Omega) \sin(n_2 - n_1)\Omega \\ -\sin(n_1\Omega) \sin(n_2 - n_1)\Omega \end{bmatrix} / \sin^2(n_2 - n_1)\Omega = - \begin{bmatrix} \frac{\sin(n_2\Omega)}{\sin(n_2 - n_1)\Omega} \\ -\frac{\sin(n_1\Omega)}{\sin(n_2 - n_1)\Omega} \end{bmatrix}
\end{aligned}$$

for $(n_2 - n_1)\Omega \neq k\pi, k = 0, \pm 1, \pm 2, \dots$

9.37

$$\begin{aligned}
r(\Delta) &= E \left\{ [\sqrt{2} \sin(n\pi/8) + \cos(n\pi/16)] [\sqrt{2} \sin((n - \Delta)\pi/8) + \cos((n - \Delta)\pi/16)] \right\} \\
&= \cos(\Delta\pi/8) + \cos(\Delta\pi/16) / 2 \\
R &=
\end{aligned}$$

1.5000	1.4143	1.1690	0.7984
1.4143	1.5000	1.4143	1.1690
1.1690	1.4143	1.5000	1.4143
0.7984	1.1690	1.4143	1.5000

P =

1.4143
1.1690
0.7984
0.3536

a = -inv(R)*P

a =

-3.8093
5.6245
-3.8093
1.0000

>> W = -a;

>> Jmin = r(1) - W'*P

Jmin = 1.6174e-12

9.38

$$A = \begin{bmatrix} 2 & -1/2 \\ 0 & 1/2 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, D = 1$$

$$A^{-1} = B - BC(D + C^T BC)^{-1} C^T B$$

$$\text{Left equation} = A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{\text{adj} \begin{bmatrix} 2 & -1/2 \\ 0 & 1/2 \end{bmatrix}}{\det \begin{bmatrix} 2 & -1/2 \\ 0 & 1/2 \end{bmatrix}} = \frac{\begin{bmatrix} 1/2 & 1/2 \\ 0 & 2 \end{bmatrix}}{1} = \begin{bmatrix} 1/2 & 1/2 \\ 0 & 2 \end{bmatrix}$$

$$\text{Right side equation} = B - BC(D + C^T BC)^{-1} C^T B$$

$$C^T BC = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1$$

$$(D + C^T BC)^{-1} = (1 + 1)^{-1} = 1/2$$

$$\begin{aligned} B - BC(D + C^T BC)^{-1} C^T B &= \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 1/2 & 1/2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 1/2 & 1/2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ 0 & 2 \end{bmatrix} \end{aligned}$$

9.39

$$e_f(n) = x(n) - \hat{x}(n) = x(n) + a_0 + a_1 x(n - n_1) + a_2 (n - n_2)$$

Taking derivative

$$E \left\{ \frac{\partial e_f^2(n)}{\partial a_p} \right\} = 0$$

and setting the results to zero leads

$$\begin{bmatrix} 1 & m_x & m_x \\ m_x & r(0) & r(n_2 - n_1) \\ m_x & r(n_2 - n_1) & r(0) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = - \begin{bmatrix} m_x \\ r(n_1) \\ r(n_2) \end{bmatrix}$$

$$J_{\min} = r(0) - a^T P$$

For $x(n) = A \sin^2(n\Omega)$, we have

$$m_x = E \{ A \sin^2(n\Omega) \} = A/2; \quad r(0) = E \{ A^2 \sin^4(n\Omega) \} = 3A^2/8$$

$$r(n_1) = E \{ A^2 \sin^2(n\Omega) \sin^2(n - n_1)\Omega \} = \frac{A^2}{8} (1 + 2 \cos^2 n_1 \Omega)$$

$$r(n_1) = E \{ A^2 \sin^2(n\Omega) \sin^2(n - n_2)\Omega \} = \frac{A^2}{8} (1 + 2 \cos^2 n_2 \Omega)$$

$$r(n_2 - n_1) = E \{ A^2 \sin^2(n - n_2)\Omega \sin^2(n - n_1)\Omega \} = \frac{A^2}{8} (1 + 2 \cos^2(n_2 - n_1)\Omega)$$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = - \begin{bmatrix} 1 & m_x & m_x \\ m_x & r(0) & r(n_2 - n_1) \\ m_x & r(n_2 - n_1) & r(0) \end{bmatrix}^{-1} \begin{bmatrix} m_x \\ r(n_1) \\ r(n_2) \end{bmatrix}$$

For $A=1$, $n_1=1$, $n_2=2$, $\omega=\pi/8$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = - \begin{bmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 0.375 & 0.125(1+2\cos^2 \pi/8) \\ 0.5 & 0.125(1+2\cos^2 \pi/8) & 0.375 \end{bmatrix}^{-1} \begin{bmatrix} 0.5 \\ 0.125(1+2\cos^2 \pi/8) \\ 0.125(1+2\cos^2 \pi/4) \end{bmatrix}$$

$R =$

$$\begin{bmatrix} 1.0000 & 0.5000 & 0.5000 \\ 0.5000 & 0.3750 & 0.3384 \\ 0.5000 & 0.3384 & 0.3750 \end{bmatrix}$$

$P =$

$$\begin{bmatrix} 0.5000 \\ 0.3384 \\ 0.2500 \end{bmatrix}$$

$\gg a = \text{inv}(R) * P$

$a =$

$$\begin{bmatrix} -0.2929 \\ -1.4142 \\ 1.0000 \end{bmatrix}$$

$\gg W = -a;$

$\gg J_{\min} = 3/8 - W' * P$

$J_{\min} =$

$$1.6653e-16$$

10.1

a. $\Delta = \frac{5}{2^3 - 1} = 0.714 \text{ volts}$

b. for $x = 1.6 = 2.24\Delta$ volts, binary code=110, $x_q = 1.428$ volts, and $e_q = -0.172$ volts
for $x = -0.2 = -0.28\Delta$ volts, binary code=000, $x_q = 0$ volts, and $e_q = 0.2$ volts

10.2

a. $\Delta = \frac{8}{2^3 - 1} = 1.143 \text{ volts}$

b. for $x = -2.6 = -2.275\Delta$ volts, binary code=010, $x_q = 2.286$ volts, and
 $e_q = -0.314$ volts

for $x = 0.1 = 0.0875\Delta$ volts, binary code=100, $x_q = 0$ volts, and $e_q = -0.1$ volts

10.3

a. $\Delta = \frac{10}{2^3 - 1} = 1.4286 \text{ volts}$

b. for $x = -2.6 = -1.82\Delta$ volts, binary code=010, $x_q = -2.8572$ volts, and $e_q = -0.2572$ volts

for $x = 3.5 = 2.4500\Delta$ volts, binary code=110, $x_q = 2.8572$ volts, and $e_q = -0.6482$ volts

10.4

a. $\Delta = \frac{20}{2^3 - 1} = 2.8571$ volts

b. for $x = -5 = -1.75\Delta$ volts, binary code=010, $x_q = -5.7142$ volts, and $e_q = -0.7142$ volts

for $x = 0 = 0\Delta$ volts, binary code=100, $x_q = 0$ volts, and $e_q = 0$ volts

for $x = 7.2 = 2.52\Delta$ volts, binary code=111, $x_q = 8.5713$ volts, and $e_q = 1.3713$ volts

10.5

for $x = 1.6$ volts, $y = \text{sign}(1.6) \frac{\ln\left(1 + 255 \frac{|1.6|}{2.5}\right)}{\ln(1 + 255)} = 0.920$

$$\Delta = \frac{2}{2^3 - 1} = 0.286, y = \frac{0.920}{0.286} = 3.216\Delta, \text{ binary code}=111$$

$$y_q = 3\Delta = 0.858 \quad x_q = 2.5 \times \text{sign}(0.858) \frac{(1 + 255)^{|0.858|} - 1}{255} = 1.132$$

$$e_q = x_q - x = 1.132 - (1.6) = -0.468 \text{ volts}$$

for $x = -0.2$, we get

$$y = \text{sign}(-0.2) \frac{\ln\left(1 + 255 \frac{|-0.2|}{2.5}\right)}{\ln(1 + 255)} = -0.552$$

$$y = -\frac{0.552}{0.286} \Delta = -1.93\Delta \text{ and binary code}=010$$

$$y_q = -2\Delta = -0.572, x_q = 2.5 \times \text{sign}(-0.572) \frac{(1 + 255)^{|-0.572|} - 1}{255} = -0.224 \text{ volts}$$

$$e_q = -0.224 - (-0.2) = -0.024 \text{ volts}$$

10.6

for $x = -2.6$ volts, $y = \text{sign}(-2.6) \frac{\ln\left(1 + 255 \frac{|-2.6|}{4}\right)}{\ln(1 + 255)} = -0.923$

$$\Delta = \frac{2}{2^3 - 1} = 0.286, y = \frac{-0.923}{0.286} = -3.226\Delta, \text{ binary code}=011$$

$$y_q = -3\Delta = -0.858, \quad x_q = 4 \times \text{sign}(-0.858) \frac{(1+255)^{|-0.858|} - 1}{255} = -1.842$$

$$e_q = x_q - x = -1.842 - (-2.6) = 0.758 \text{ volts}$$

for $x = 0.1$, we get

$$y = \text{sign}(0.2) \frac{\ln\left(1 + 255 \frac{|0.1|}{4}\right)}{\ln(1+255)} = 0.360$$

$$y = \frac{0.360}{0.286} \Delta = 1.26\Delta \text{ and binary code} = 101$$

$$y_q = \Delta = 0.286, \quad x_q = 4 \times \text{sign}(0.286) \frac{(1+255)^{|0.286|} - 1}{255} = 0.061 \text{ volts}$$

$$e_q = 0.061 - 0.1 = -0.039 \text{ volts}$$

10.7

$$\text{for } x = -2.6 \text{ volts, } y = \text{sign}(-2.6) \frac{\ln\left(1 + 255 \frac{|2.6|}{5}\right)}{\ln(1+255)} = -0.8827$$

$$\Delta = \frac{2}{2^3 - 1} = 0.286, \quad y = \frac{-0.920}{0.286} = -3.086\Delta, \text{ binary code} = 011$$

$$y_q = -3\Delta = -0.858, \quad x_q = 5 \times \text{sign}(-0.858) \frac{(1+255)^{|-0.858|} - 1}{255} = -2.2644$$

$$e_q = x_q - x = -2.2644 - (-2.6) = 0.3356 \text{ volts}$$

for $x = 3.5$, we get

$$y = \text{sign}(3.5) \frac{\ln\left(1 + 255 \frac{|3.5|}{5}\right)}{\ln(1+255)} = 0.9360$$

$$y = \frac{0.9360}{0.286} \Delta = 3.27\Delta \text{ and binary code} = 111$$

$$y_q = 3\Delta = 0.858, \quad x_q = 5 \times \text{sign}(0.858) \frac{(1+255)^{|0.858|} - 1}{255} = 2.2644 \text{ volts}$$

$$e_q = 2.2644 - 3.5 = -1.2356 \text{ volts}$$

10.8

$$\text{for } x = -5 \text{ volts, } y = \text{sign}(-5) \frac{\ln\left(1 + 255 \frac{|-5|}{10}\right)}{\ln(1 + 255)} = -0.8757$$

$$\Delta = \frac{2}{2^3 - 1} = 0.286, y = \frac{-0.8757}{0.286} = -3.0619\Delta, \text{ binary code} = 011$$

$$y_q = -3\Delta = -0.858, x_q = 10 \times \text{sign}(-0.858) \frac{(1 + 255)^{|-0.858|} - 1}{255} = -4.5288$$

$$e_q = x_q - x = -4.5288 - (-5) = 0.4712 \text{ volts}$$

for $x = 0$, we get

$$y = \text{sign}(0) \frac{\ln\left(1 + 255 \frac{|0|}{10}\right)}{\ln(1 + 255)} = 0.0$$

$$y = \frac{0.0}{0.286} \Delta = 0.0\Delta \text{ and binary code} = 100$$

$$y_q = 0\Delta = 0.0, x_q = 10 \times \text{sign}(0.0) \frac{(1 + 255)^{|0.0|} - 1}{255} = 0.0 \text{ volts}$$

$$e_q = 0.0 - 0.0 = 0.0 \text{ volts}$$

for $x = 7.2$, we get

$$y = \text{sign}(7.2) \frac{\ln\left(1 + 255 \frac{|7.2|}{10}\right)}{\ln(1 + 255)} = 0.9410$$

$$y = \frac{0.9410}{0.286} \Delta = 3.29\Delta \text{ and binary code} = 111$$

$$y_q = 3\Delta = 0.858, x_q = 10 \times \text{sign}(0.0) \frac{(1 + 255)^{|0.858|} - 1}{255} = 4.5288 \text{ volts}$$

$$e_q = 4.5288 - 7.2 = -2.6712 \text{ volts}$$

10.9

a. 00010101 b. 11100111

10.10

a. 000000000111 b. 101100110000

10.11

a. 01100101 b. 10001101

10.12

a. 000000111011 b. 100000000101

10.13

$$\tilde{x}(0) = -1 = 0, d(0) = x(0) - \tilde{x}(0)$$

$$d_q(0) = Q[d(0)] = -5, \text{ binary code} = 010, \hat{x}(0) = \tilde{x}(0) + d_q(0) = -5$$

$$\tilde{x}(1) = -5, d(1) = x(1) - \tilde{x}(1) = 0 - (-5) = 5$$

$$d_q(1) = Q[d(1)] = 2, \text{ binary code} = 001, \hat{x}(1) = \tilde{x}(1) + d_q(1) = -5 + 2 = -3$$

$$\tilde{x}(2) = -3, d(2) = x(2) - \tilde{x}(2) = 0 - (-3) = 3$$

$$d_q(2) = Q[d(2)] = 5, \text{ binary code} = 010, \hat{x}(2) = \tilde{x}(2) + d_q(2) = -3 + 5 = 2$$

10.14

for binary code = 110

$$\tilde{x}(0) = 1 = 0, d_q(0) = 5, \hat{x}(0) = \tilde{x}(0) + d_q(0) = 0 + 5 = 5$$

for binary code = 100

$$\tilde{x}(1) = 5, d_q(1) = 0, \hat{x}(1) = \tilde{x}(1) + d_q(1) = 5 + 0 = 5$$

for binary code = 110

$$\tilde{x}(2) = 5, d_q(2) = 2, \hat{x}(2) = \tilde{x}(2) + d_q(2) = 5 + 2 = 7$$

10.15

$$\tilde{x}(0) = -1 = 0, d(0) = x(0) - \tilde{x}(0)$$

$$d_q(0) = Q[d(0)] = 5, \text{ binary code} = 110, \hat{x}(0) = \tilde{x}(0) + d_q(0) = 5$$

$$\tilde{x}(1) = 5, d(1) = x(1) - \tilde{x}(1) = 0 - 5 = -5$$

$$d_q(1) = Q[d(1)] = 2, \text{ binary code} = 101, \hat{x}(1) = \tilde{x}(1) + d_q(1) = 5 + 2 = 7$$

$$\tilde{x}(2) = 7, d(2) = x(2) - \tilde{x}(2) = 0 - 7 = -7$$

$$d_q(2) = Q[d(2)] = 5, \text{ binary code} = 110, \hat{x}(2) = \tilde{x}(2) + d_q(2) = 7 + 5 = 12$$

10.16

for binary code = 010

$$\tilde{x}(0) = 1 = 0, d_q(0) = -5, \hat{x}(0) = \tilde{x}(0) + d_q(0) = 0 - 5 = -5$$

for binary code = 000

$$\tilde{x}(1) = -5, d_q(1) = 0, \hat{x}(1) = \tilde{x}(1) + d_q(1) = -5 + 0 = -5$$

for binary code = 001

$$\tilde{x}(2) = -5, d_q(2) = -2, \hat{x}(2) = \tilde{x}(2) + d_q(2) = -5 - 2 = -7$$

10.17

a. 1:1, b. 2:1, c. 4:1

10.18

a. 128 kbps, b. 64 kbps, c. 32 kbps

10.19

a. 1:1, b. 2:1, c. 4:1

10.20

a. 529.2 kbps, b. 352.8 kbps, c. 176.4 kbps

10.21

a. 12 channels, b. 24 channels, c. 48 channels

10.22

$$X_{DCT}(0) = 54, X_{DCT}(1) = 0.5412, X_{DCT}(2) = -4, X_{DCT}(3) = -1.3066$$

10.23

$$X_{DCT}(0) = 38.8909, X_{DCT}(1) = -3.5355$$

10.24

$$X_{DCT}(0) = 41.7193, X_{DCT}(2) = -4.2376, X_{DCT}(4) = 0.7071, X_{DCT}(6) = -2.8373$$

10.25

$$X_{DCT}(1) = 33.9730, X_{DCT}(3) = -10.4308, X_{DCT}(5) = 1.2001, X_{DCT}(7) = -1.6102$$

10.26

a. The inverse DCT: 10.0845 6.3973 13.6027 -2.0845

>> idct([14 6 -6 8])

ans = 10.0845 6.3973 13.6027 -2.0845

b. Quantized DCT coefficients: 16, 8 -8, 8

Recovered inverse DCT: 11.3910 8.9385 15.0615 -3.3910

>> idct([16 8 -8 8])

ans = 11.3910 8.9385 15.0615 -3.3910

c. Quantization error: 1.3066 2.5412 1.4588 -1.3066

10.27

a. The inverse DCT: 11.4546 5.3128 -1.3128 6.5454

>> idct([11 5 7 -3])

ans = 11.4546 5.3128 -1.3128 6.5454

b. Quantized DCT coefficients: 8, 8 -8, 0

Recovered inverse DCT: 5.2263 10.1648 5.8352 -5.2263

>> idct([8 8 -8 0])

ans = 5.2263 10.1648 5.8352 -5.2263

c. Quantization error: -6.2284 4.8519 7.1481 -11.7716

10.28

a.

$$\begin{aligned} f\left(n + \frac{N}{2}\right) h\left(n + \frac{N}{2}\right) &= \sin\left(\frac{\pi}{N}(n + N/2 + 0.5)\right) \sin\left(\frac{\pi}{N}(n + N/2 + 0.5)\right) \\ (1) \quad &= \sin\left(\frac{\pi}{N}(n + 0.5) + \frac{\pi}{2}\right) \sin\left(\frac{\pi}{N}(n + 0.5) + \frac{\pi}{2}\right) = \cos^2\left(\frac{\pi}{N}(n + 0.5)\right) \end{aligned}$$

and

$$f(n)h(n) = \sin\left(\frac{\pi}{N}(n+0.5)\right)\sin\left(\frac{\pi}{N}(n+0.5)\right) = \sin^2\left(\frac{\pi}{N}(n+0.5)\right)$$

Hence

$$\begin{aligned} f\left(n + \frac{N}{2}\right)h\left(n + \frac{N}{2}\right) + f(n)h(n) &= \cos^2\left(\frac{\pi}{N}(n+0.5)\right) + \sin^2\left(\frac{\pi}{N}(n+0.5)\right) = 1 \\ (2) \quad f\left(n + \frac{N}{2}\right)h(N-n-1) &= \sin\left(\frac{\pi}{N}(n+N/2+0.5)\right)\sin\left(\frac{\pi}{N}(N-n-1+0.5)\right) \\ &= \sin\left(\frac{\pi}{N}(n+0.5) + \frac{\pi}{2}\right)\sin\left(\pi - \frac{\pi}{N}(n+0.5)\right) = \cos\left(\frac{\pi}{N}(n+0.5)\right)\sin\left(\frac{\pi}{N}(n+0.5)\right) \\ f(n)h(N/2-n-1) &= \sin\left(\frac{\pi}{N}(n+0.5)\right)\sin\left(\frac{\pi}{N}(N/2-n-1+0.5)\right) \\ &= \sin\left(\frac{\pi}{N}(n+0.5)\right)\sin\left(\frac{\pi}{2} - \frac{\pi}{N}(n+0.5)\right) = \sin\left(\frac{\pi}{N}(n+0.5)\right)\cos\left(\frac{\pi}{N}(n+0.5)\right) \end{aligned}$$

Hence

$$\begin{aligned} f\left(n + \frac{N}{2}\right)h(N-n-1) &= \sin\left(\frac{\pi}{N}(n+N/2+0.5)\right)\sin\left(\frac{\pi}{N}(N-n-1+0.5)\right) - f(n)h(N/2-n-1) \\ &= \cos\left(\frac{\pi}{N}(n+0.5)\right)\sin\left(\frac{\pi}{N}(n+0.5)\right) - \sin\left(\frac{\pi}{N}(n+0.5)\right)\cos\left(\frac{\pi}{N}(n+0.5)\right) = 0 \end{aligned}$$

b.

$$\begin{aligned} (-1)^{\frac{N}{2}+1} X_{MDCT}(N-1-k) &= (-1)^{\frac{N}{2}+1} 2 \sum_{n=0}^{N-1} x(n)h(n) \cos\left[\frac{2\pi}{N}(n+0.5+N/4)(N-1-k+0.5)\right] \\ &= (-1)^{\frac{N}{2}+1} 2 \sum_{n=0}^{N-1} x(n)h(n) \cos\left[\frac{2\pi}{N}(n+0.5+N/4)(N-k-0.5)\right] \\ &= (-1)^{\frac{N}{2}+1} 2 \sum_{n=0}^{N-1} x(n)h(n) \cos\left[2\pi n + \pi + \frac{N\pi}{2} + \frac{2\pi}{N}(n+0.5+N/4)(k+0.5)\right] \\ &= (-1)^{\frac{N}{2}+1} 2 \sum_{n=0}^{N-1} x(n)h(n)(-1) \cos\left[\frac{N\pi}{2} + \frac{2\pi}{N}(n+0.5+N/4)(k+0.5)\right] \end{aligned}$$

for $N = \text{even number}$

$$\begin{aligned} (-1)^{\frac{N}{2}+1} X_{MDCT}(N-1-k) &= (-1)^{\frac{N}{2}+1} 2 \sum_{n=0}^{N-1} x(n)h(n)(-1)(-1)^{\frac{N}{2}} \cos\left[\frac{2\pi}{N}(n+0.5+N/4)(k+0.5)\right] \\ &= (-1)^{N+2} 2 \sum_{n=0}^{N-1} x(n)h(n) \cos\left[\frac{2\pi}{N}(n+0.5+N/4)(k+0.5)\right] \\ &= 2 \sum_{n=0}^{N-1} x(n)h(n) \cos\left[\frac{2\pi}{N}(n+0.5+N/4)(k+0.5)\right] = X_{MDCT}(k) \end{aligned}$$

a.

```

First block data:    1 2 3 4
Second block data:   3 4 5 4
Third block data:    5 4 3 2
>> wmdct([1 2 3 4])
ans = -9.0711 -0.5858
>> wmdct([3 4 5 4])
ans = -13.3137 -0.0000
>> wmdct([5 4 3 2])
ans = -7.8995 0.5858

```

b.

```

>> x1=wimdct([-9.0711 -0.5858])
x1 = -0.5607 1.3536 3.9749 1.6465
>> x2=wimdct([-13.3137 -0.0000])
x2 = -0.9749 2.3536 5.6820 2.3536
>> x3=wimdct([-7.8995 0.5858])
x3 = -0.6820 1.6465 3.2678 1.3536
>> [x1 0 0 0 0]+[0 0 x2 0 0]+[0 0 0 0 x3]
-0.5607 1.3536 3.0000 4.0000 5.0000 4.0000 3.2678 1.3536

```

The recovered first two subblocks have values as 3, 4, 5, 4 which are consistent with the input data.

10.30

a.

```

First block data:    1 2 3 4 5 4
Second block data:   4 5 4 3 2 1
Third block data:    3 2 1 2 3 4
>> wmdct([1 2 3 4 5 4])
ans = -18.1953 -1.7932 -1.2247
>> wmdct([4 5 4 3 2 1])
ans = -11.1242 2.8284 -0.1895
>> wmdct([3 2 1 2 3 4])
ans = -9.5713 -3.8637 0.7071

```

b.

```

>> x1=wimdct([-18.1953 -1.7932 -1.2247])
x1 = -0.6830 0.0000 2.5490 4.7321 5.0000 1.2680
>> x2=wimdct([-11.1242 2.8284 -0.1895])
x2 = -0.7320 0.0000 2.7320 3.0490 2.0000 0.8170
>> x3=wimdct([-9.5713 -3.8637 0.7071])
x3 = -0.0490 0.0000 0.1830 2.8660 3.0000 0.7679
>> [x1 0 0 0 0 0]+[0 0 0 x2 0 0]+[0 0 0 0 0 x3]
-0.6830 0.0000 2.5490 4.0000 5.0000 4.0000 3.0000 2.0000 1.0000 2.8660 3.0000 0.7679

```

The recovered first two subblocks have values as 4, 5, 4, 3, 2, 1 which are consistent with the input data.

10.31

- a. (1) unacceptable, (2) intelligent, (3) good
 b. (1) unacceptable, (2) good, (3) excellent

10.32 Excellent

10.33

For (a), (b), (c), and (d), the decoded speech quality using the M-DCT coding is better.

10.34

(a)

$$y = \text{sign}(x) \lim_{\mu \rightarrow 0} \frac{\ln \left(1 + \mu \frac{|x|}{|x|_{\max}} \right)}{\ln(1 + \mu)} = \text{sign}(x) \lim_{\mu \rightarrow 0} \frac{\frac{|x|}{|x|_{\max}} / \left(1 + \mu \frac{|x|}{|x|_{\max}} \right)}{1/(1 + \mu)} = \text{sign}(x) \frac{|x|}{|x|_{\max}} = \frac{x}{|x|_{\max}}$$

(b)

$$\begin{aligned} y &= \text{sign}(x) \lim_{\mu \rightarrow \infty} \frac{\ln \left(1 + \mu \frac{|x|}{|x|_{\max}} \right)}{\ln(1 + \mu)} = \text{sign}(x) \lim_{\mu \rightarrow \infty} \frac{\frac{|x|}{|x|_{\max}} / \left(1 + \mu \frac{|x|}{|x|_{\max}} \right)}{1/(1 + \mu)} = \text{sign}(x) \lim_{\mu \rightarrow \infty} \frac{\frac{|x|}{|x|_{\max}} (1 + \mu)}{\left(1 + \mu \frac{|x|}{|x|_{\max}} \right)} \\ &= \text{sign}(x) \lim_{\mu \rightarrow \infty} \frac{\frac{|x|}{|x|_{\max}} (1 + \mu)}{\left(1 + \mu \frac{|x|}{|x|_{\max}} \right)} = \text{sign}(x) \frac{\frac{|x|}{|x|_{\max}}}{\frac{|x|}{|x|_{\max}}} = \text{sign}(x) \end{aligned}$$

10.35

$$y_q = \text{sign}(x) \frac{\ln \left(1 + \mu \frac{|x_q|}{|x|_{\max}} \right)}{\ln(1 + \mu)}, \quad \ln(1 + \mu)^{y_q / \text{sign}(x)} = \ln \left(1 + \mu \frac{|x_q|}{|x|_{\max}} \right)$$

$$(1 + \mu)^{y_q / \text{sign}(x)} = 1 + \mu \frac{|x_q|}{|x|_{\max}}, \quad |x_q| = |x|_{\max} \frac{(1 + \mu)^{|y_q|} - 1}{\mu}$$

$$x_q = |x|_{\max} \text{sign}(y_q) \frac{(1 + \mu)^{|y_q|} - 1}{\mu}$$

10.36

The easiest way is to use the Jury test to guarantee

$$Q(z) = z^2 - a_1 z - a_2 = 0 \text{ has roots inside the unit circle in the } z\text{-plane.}$$

It requires that

$$Q(1) > 0,$$

$$(-1)^2 Q(-1) > 0$$

$$| -a_2 | < 1$$

$$Q(1) = 1 - a_1 - a_2 > 0 \Rightarrow a_1 < 1 - a_2$$

$$(-1)^2 Q(-1) = 1 + a_1 - a_2 > 0 \Rightarrow -(1 - a_2) < a_1$$

$$|a_2| < 1$$

That is, $|a_1| < 1 - a_2$ and $|a_2| < 0.75$

10.37

From Problem 4.40:

$$\frac{1}{N} \sum_{n=0}^{N-1} \cos \left[\frac{\pi k(2n+1)}{2N} \right] \cos \left[\frac{\pi m(2n+1)}{2N} \right] = \begin{cases} 1 & k = m = 0 \\ 1/2 & k = m \neq 0 \\ 0 & k \neq m \end{cases}$$

Given

$$x(n) = \sqrt{\frac{2}{N}} \sum_{k=0}^{N-1} C(k) X_{DCT}(k) \cos \left[\frac{(2n+1)k\pi}{2N} \right]$$

Show that

$$X_{DCT}(k) = \sqrt{\frac{2}{N}} C(k) \sum_{n=0}^{N-1} x(n) \cos \left[\frac{(2n+1)k\pi}{2N} \right]$$

Now

$$\sqrt{\frac{2}{N}} C(m) \sum_{n=0}^{N-1} x(n) \cos \left[\frac{(2n+1)m\pi}{2N} \right] = \sqrt{\frac{2}{N}} C(m) \sum_{n=0}^{N-1} \sqrt{\frac{2}{N}} \sum_{k=0}^{N-1} C(k) X_{DCT}(k) \cos \left[\frac{(2n+1)k\pi}{2N} \right] \cos \left[\frac{(2n+1)m\pi}{2N} \right]$$

$$\sqrt{\frac{2}{N}} C(m) \sum_{n=0}^{N-1} x(n) \cos \left[\frac{(2n+1)m\pi}{2N} \right] = \frac{2}{N} C(m) \sum_{k=0}^{N-1} C(k) X_{DCT}(k) \sum_{n=0}^{N-1} \cos \left[\frac{(2n+1)k\pi}{2N} \right] \cos \left[\frac{(2n+1)m\pi}{2N} \right]$$

For $m = k = 0$

$$\frac{2}{N} C(m) \sum_{k=0}^{N-1} C(k) X_{DCT}(k) \sum_{n=0}^{N-1} \cos \left[\frac{(2n+1)k\pi}{2N} \right] \cos \left[\frac{(2n+1)m\pi}{2N} \right] = 2C^2(0) X_{DCT}(0) \times 1 = X_{DCT}(0)$$

For $m = k \neq 0$

$$\frac{2}{N} C(m) \sum_{k=0}^{N-1} C(k) X_{DCT}(k) \sum_{n=0}^{N-1} \cos \left[\frac{(2n+1)k\pi}{2N} \right] \cos \left[\frac{(2n+1)m\pi}{2N} \right] = 2C^2(m) X_{DCT}(m) \times (1/2) = X_{DCT}(m)$$

This concludes

$$\sqrt{\frac{2}{N}} C(m) \sum_{n=0}^{N-1} x(n) \cos \left[\frac{(2n+1)m\pi}{2N} \right] = X_{DCT}(m)$$

10.38

$$S(k) = \sum_{n=0}^{2N-1} s(n) W_{2N}^{kn} = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/(2N)} + \sum_{n=N}^{2N-1} x(2N-n-1) e^{-j2\pi kn/(2N)}$$

Let $m = 2N - n - 1$

$$\begin{aligned}
S(k) &= \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/(2N)} + \sum_{m=N-1}^0 x(m)e^{-j2\pi k(2N-m-1)/(2N)} = \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/(2N)} + \sum_{n=0}^{N-1} x(n)e^{j2\pi kn/(2N)} e^{-j2\pi k(2N-1)/(2N)} \\
&= W_{2N}^{-k/2} \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/(2N)} W_{2N}^{k/2} + \sum_{n=0}^{N-1} x(n)e^{j2\pi kn/(2N)} e^{-j2\pi k(2N)/(2N)} e^{j2\pi k/(2N)} \\
&= W_{2N}^{-k/2} \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/(2N)} e^{-j\pi k/(2N)} + W_{2N}^{-k/2} \sum_{n=0}^{N-1} x(n)e^{j2\pi kn/(2N)} e^{j\pi k/(2N)} \\
&= W_{2N}^{-k/2} \left[\sum_{n=0}^{N-1} x(n)e^{-j\pi k(2n+1)/(2N)} + \sum_{n=0}^{N-1} x(n)e^{j\pi k(2n+1)/(2N)} \right] = W_{2N}^{-k/2} 2 \sum_{n=0}^{N-1} x(n) \cos\left(\frac{\pi(2n+1)k}{2N}\right)
\end{aligned}$$

10.39

(a)

$$\begin{aligned}
S(k) &= W_{2N}^{-k/2} 2 \sum_{n=0}^{N-1} x(n) \cos\left(\frac{\pi(2n+1)k}{2N}\right) \\
S(N) &= W_{2N}^{-N/2} 2 \sum_{n=0}^{N-1} x(n) \cos\left(\frac{\pi(2n+1)N}{2N}\right) = e^{j\pi/2} 2 \sum_{n=0}^{N-1} x(n) \cos(\pi(n+1/2)) = 0,
\end{aligned}$$

since $\cos(\pi(n+1/2)) = 0$

(b) Based on DFT definition

$$\begin{aligned}
s(n) &= \frac{1}{2N} \sum_{k=0}^{2N-1} S(k) W_{2N}^{-kn} = \frac{1}{2N} \left(\sum_{k=0}^{N-1} S(k) W_{2N}^{-kn} + \sum_{k=N}^{2N-1} S(k) W_{2N}^{-kn} \right) \\
&= \frac{1}{2N} \left(S(0) + \sum_{k=1}^{N-1} S(k) W_{2N}^{-kn} + S(N) W_{2N}^{-kN} + \sum_{k=N+1}^{2N-1} S(k) W_{2N}^{-kn} \right)
\end{aligned}$$

Since $S(N) = 0$

$$s(n) = \frac{1}{2N} \left(S(0) + \sum_{k=1}^{N-1} S(k) W_{2N}^{-kn} + \sum_{k=N+1}^{2N-1} S(k) W_{2N}^{-kn} \right)$$

Now let $j = 2N - k$ in the second sum

$$\begin{aligned}
s(n) &= \frac{1}{2N} \left(S(0) + \sum_{k=1}^{N-1} S(k) W_{2N}^{-kn} + \sum_{j=N-1}^1 S(2N-j) W_{2N}^{-(2N-j)n} \right) \\
&= \frac{1}{2N} \left(S(0) + \sum_{k=1}^{N-1} S(k) W_{2N}^{-kn} + \sum_{k=1}^{N-1} S(2N-k) W_{2N}^{-2N} W_{2N}^{kn} \right)
\end{aligned}$$

Since

 $W_{2N}^{-2N} = 1$, then

$$s(n) = \frac{1}{2N} \left(S(0) + \sum_{k=1}^{N-1} S(k) W_{2N}^{-kn} + \sum_{k=1}^{N-1} S(2N-k) W_{2N}^{kn} \right)$$

From Problem

$$\begin{aligned}
S(2N-k) &= W_{2N}^{-(2N-k)/2} 2 \sum_{n=0}^{N-1} x(n) \cos\left(\frac{\pi(2n+1)(2N-k)}{2N}\right) \\
&= W_{2N}^{-N} W_{2N}^{k/2} 2 \sum_{n=0}^{N-1} x(n) \cos\left(\pi(2n+1) - \frac{\pi(2n+1)k}{2N}\right) = (-1) W_{2N}^{k/2} 2 \sum_{n=0}^{N-1} x(n) (-1) \cos\left(\frac{\pi(2n+1)k}{2N}\right) \\
&= W_{2N}^{k/2} 2 \sum_{n=0}^{N-1} x(n) \cos\left(\frac{\pi(2n+1)k}{2N}\right) = S^*(k)
\end{aligned}$$

$$s(n) = \frac{1}{2N} \left(S(0) + \sum_{k=1}^{N-1} S(k)W_{2N}^{-kn} + \left(\sum_{k=1}^{N-1} S(k)W_{2N}^{-kn} \right)^* \right)$$

and $S(0) = 2 \sum_{n=0}^{N-1} x(n) = V(0) = \text{real}$

$$s(n) = \frac{1}{N} \operatorname{Re} \left(\frac{S(0)}{2} + \sum_{k=1}^{N-1} S(k)W_{2N}^{-kn} \right)$$

10.40

$$S(0) = 2 \sum_{n=0}^{N-1} x(n) = V(0) = \text{real} \text{ and } V(k) = 2 \sum_{n=0}^{N-1} x(n) \cos \left[\frac{\pi(2n+1)k}{2N} \right] = \text{real}$$

For $0 \leq n < N$

$$\sum_{k=1}^{N-1} S(k)W_{2N}^{-kn} = \sum_{k=1}^{N-1} W_{2N}^{-k/2} V(k)W_{2N}^{-kn} = \sum_{k=1}^{N-1} V(k)W_{2N}^{-k(2n+1)/2} = \sum_{k=1}^{N-1} V(k)e^{j\pi k(2n+1)/(2N)}$$

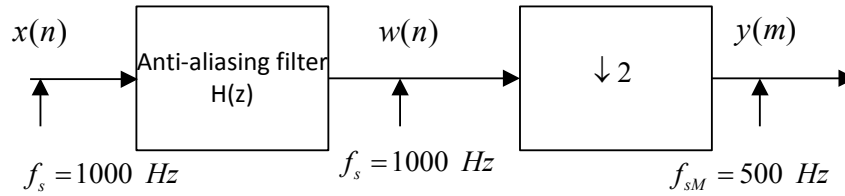
From 10.38

$$s(n) = \frac{1}{N} \left\{ \frac{V(0)}{2} + \sum_{k=1}^{N-1} V(k) \cos \left[\frac{\pi k(2n+1)}{2N} \right] \right\}$$

Chapter 11

11.1

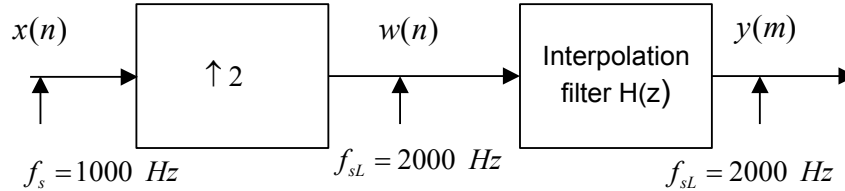
a.



b. Blackman window, $N = 37$, $f_c = 175$ Hz

11.2

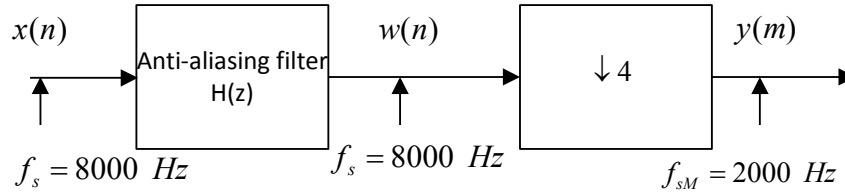
a.



b. Hamming window, $N = 19$, $f_c = 325$ Hz

11.3

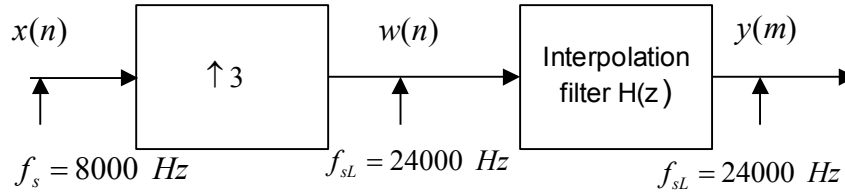
a.



b. Hamming window, $N = 133$, $f_c = 900$ Hz

11.4

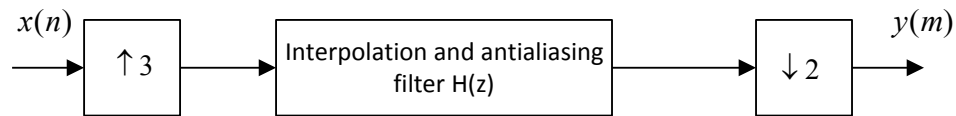
a.



b. Hamming window, $N = 133$, $f_c = 3700$ Hz

11.5

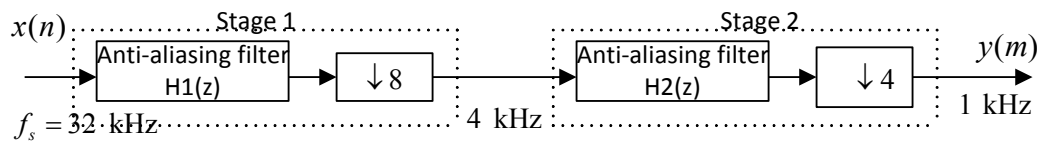
a.



b. Choose interpolation filter $H(z)$ and anti-aliasing requirement is automatically satisfied. Hamming window, $N = 25$, $f_c = 1200$ Hz

11.6

a.



b. the sampling rate after stage is 4 kHz and the sampling rate after the second stage is 1 kHz. $M_1 = 8$, and $M_2 = 4$

c.

Filter specification for $H_1(z)$:

Passband frequency range: 0 – 250 Hz

Passband ripples: $= 0.05/2 = 0.025$ (δ_p dB $= 20 \log_{10}(1 + \delta_p) = 0.212$ dB)

Stop band frequency range: 2000 Hz – 16000 Hz

Stop band attenuation $= 0.005$, δ_s dB $= 20 \times \log_{10}(\delta_s) = -46$ dB

Filter type: FIR type, Hamming window

$$f_{stop} = \frac{f_s}{M_1} - \frac{f_s}{2 \times M} = \frac{32000}{8} - \frac{32000}{2 \times 32} = 3500 \text{ Hz}$$

$$\Delta f = \frac{f_{stop} - f_{pass}}{f_s} = \frac{3500 - 250}{32000} \quad N = \frac{3.3}{\Delta f} = 32.49, \text{ choose } N = 33$$

$$f_c = \frac{f_{pass} + f_{stop}}{2} = \frac{3500 + 250}{2} = 1875 \text{ Hz}$$

d.

Filter specification for $H_2(z)$:

Passband frequency range: 0 – 250 Hz

Passband ripples: $=0.05/2=0.025$ (0.212 dB)

Stop band frequency range: 500 Hz – 2000 Hz

Stop band attenuation = 0.005, δ_s dB = -46 dB

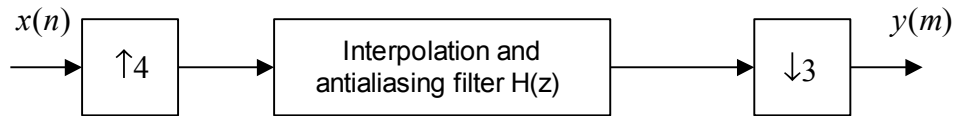
Filter type: FIR type, Hamming window

$$\Delta f = \frac{f_{stop} - f_{pass}}{f_{sM1}} = \frac{500 - 250}{4000}, \quad N = \frac{3.3}{\Delta f} = 52.8, \text{ choose } N = 53,$$

$$f_c = \frac{f_{pass} + f_{stop}}{2} = \frac{500 + 250}{2} = 375 \text{ Hz}$$

11.7

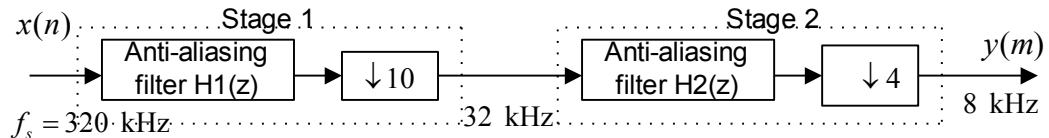
a.



b. Choose interpolation filter $H(z)$ and anti-aliasing requirement is automatically satisfied. Hamming window, $N = 133$, $f_c = 2700$ Hz

11.8

a.



b. the sampling rate after stage is 32 kHz and the sampling rate after the second stage is 8 kHz. $M_1 = 10$, and $M_2 = 4$

c.

Filter specification for $H_1(z)$:

Passband frequency range: 0 – 3400 Hz

Passband ripples: $=0.05/2=0.025$ (δ_p dB = $20 \log_{10}(1 + \delta_p) = 0.212$ dB)

Stop band frequency range: 28000 Hz – 160000 Hz

Stop band attenuation = 0.005, δ_s dB = $20 \times \log_{10}(\delta_s) = -46$ dB

Filter type: FIR type, Hamming window

$$f_{stop} = \frac{f_s}{M_1} - \frac{f_s}{2 \times M} = \frac{320000}{10} - \frac{320000}{2 \times 40} = 28000 \text{ Hz}$$

$$\Delta f = \frac{f_{stop} - f_{pass}}{f_s} = \frac{28000 - 3400}{320000} \quad N = \frac{3.3}{\Delta f} = 42.9, \text{ choose } N = 43$$

$$f_c = \frac{f_{pass} + f_{stop}}{2} = \frac{28000 + 3400}{2} = 15700 \text{ Hz}$$

d.

Filter specification for $H_2(z)$:

Passband frequency range: 0 – 3400 Hz

Passband ripples: $=0.05/2=0.025$ (0.212 dB)

Stop band frequency range: 4000 Hz – 16000 Hz

Stop band attenuation = 0.005, $\delta_s \text{ dB} = -46 \text{ dB}$

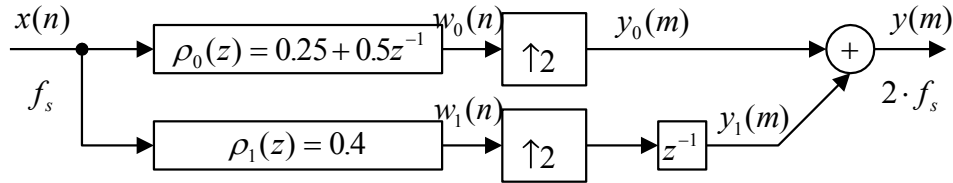
Filter type: FIR type, Hamming window

$$\Delta f = \frac{f_{stop} - f_{pass}}{f_{sM1}} = \frac{4000 - 3400}{32000}, \quad N = \frac{3.3}{\Delta f} = 176, \text{ choose } N = 177,$$

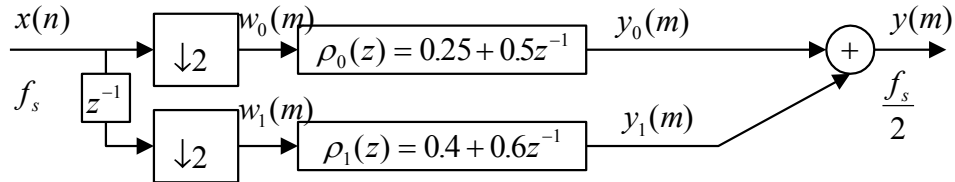
$$f_c = \frac{f_{pass} + f_{stop}}{2} = \frac{4000 + 3400}{2} = 3700 \text{ Hz}$$

11.9

a.

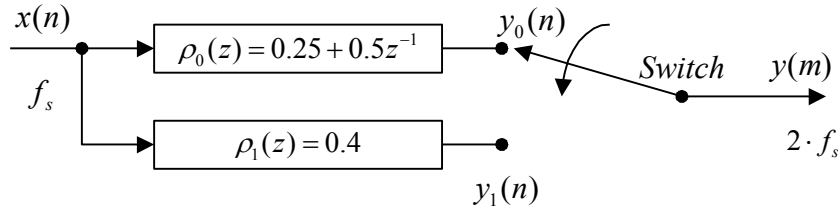


b.

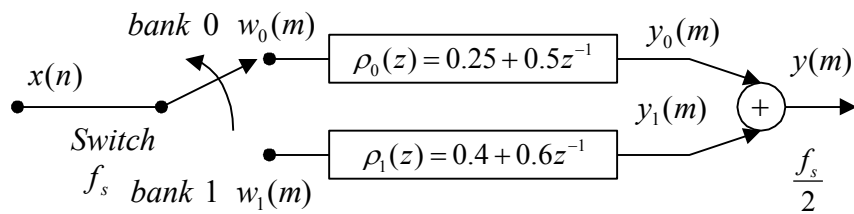


11.10

a.

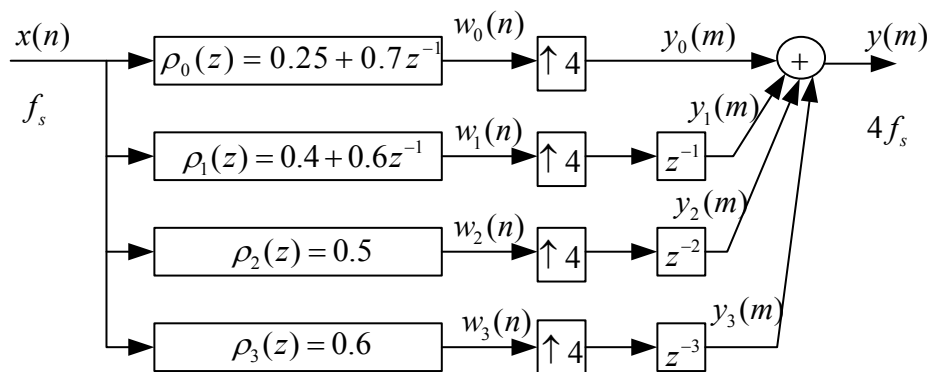


b.

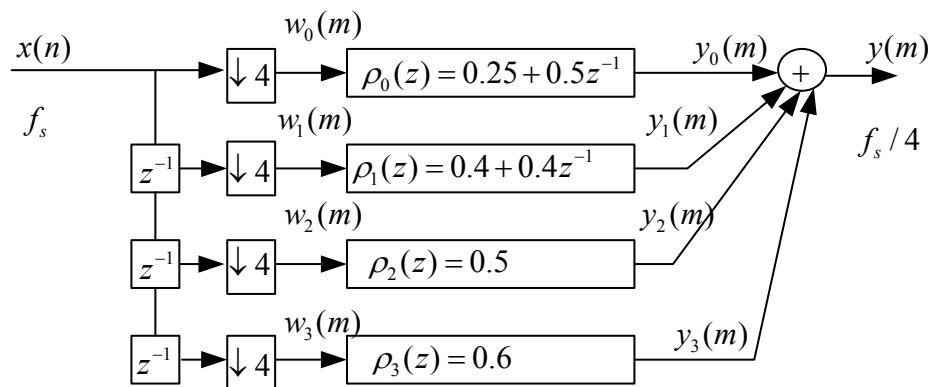


11.11

a.

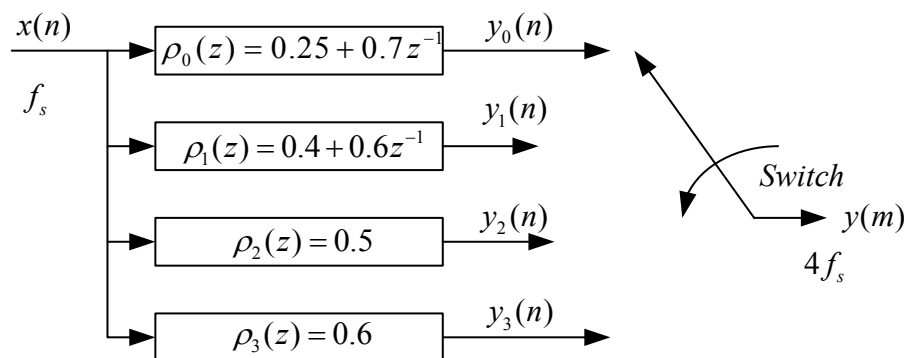


b.

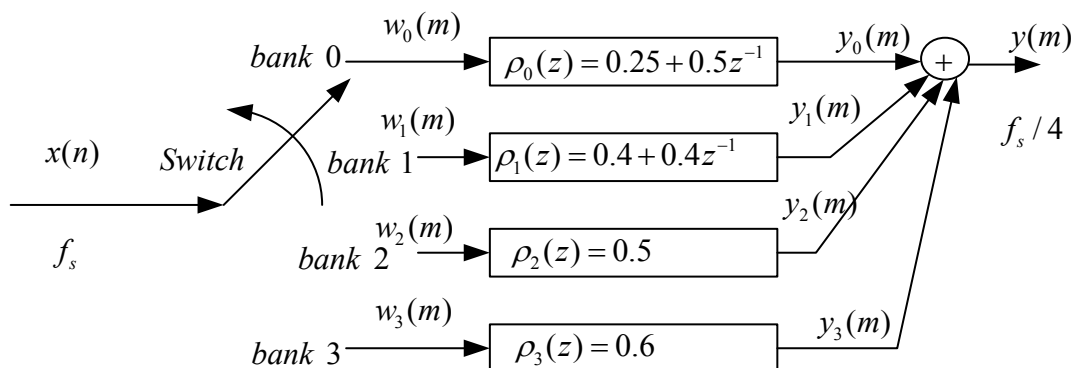


11.12

a.



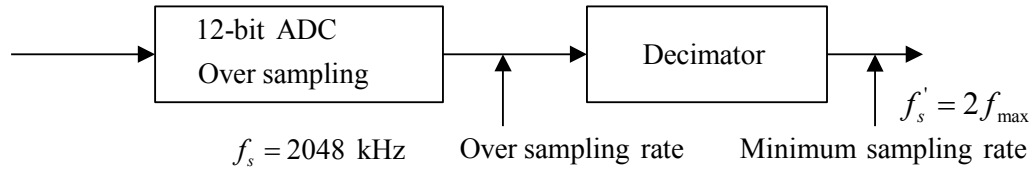
b.



11.13

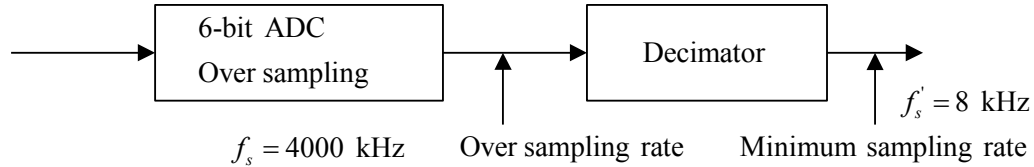
a. $f_s = 2f_{\max} 2^{2(n-m)} = 2 \times 4 \times 2^{2 \times (16-12)} = 2048 \text{ kHz}$

b.



11.14

a.



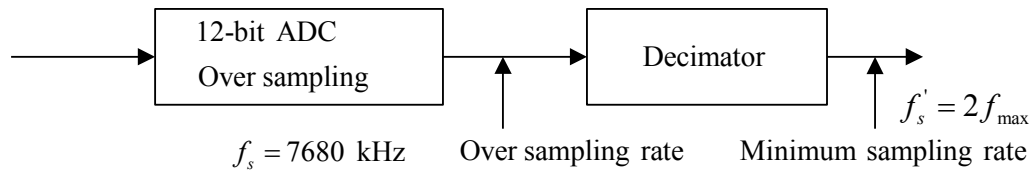
b.

$$n = m + 0.5 \times \log_2 \left(\frac{f_s}{2f_{\max}} \right) = 6 + 0.5 \times \log_2 \left(\frac{4000 \text{ kHz}}{2 \times 4 \text{ kHz}} \right) = 10.48 \approx 10 \text{ bits}$$

11.15

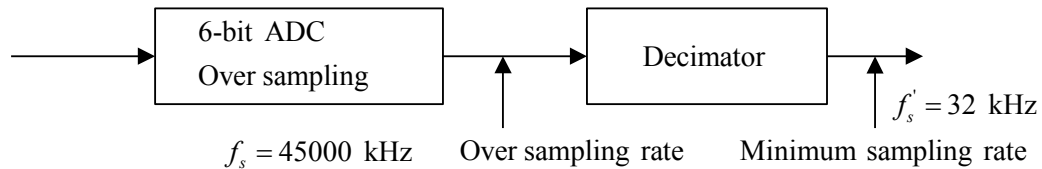
a. $f_s = 2f_{\max} 2^{2(n-m)} = 2 \times 15 \times 2^{2 \times (16-12)} = 7680 \text{ kHz}$

b.



11.16

a.

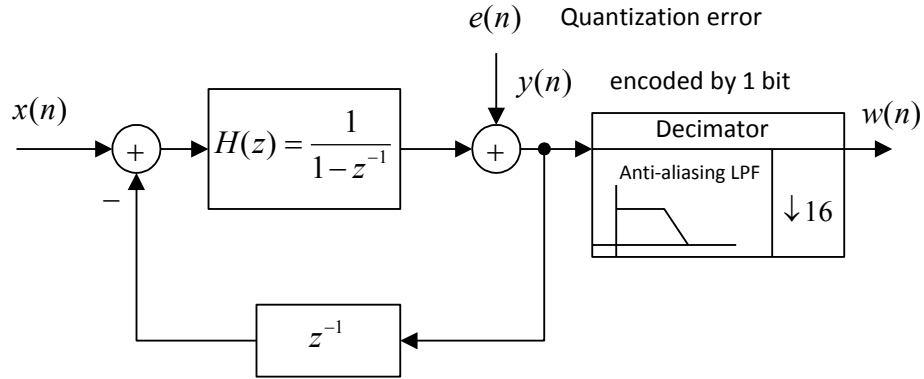


b.

$$n = m + 0.5 \times \log_2 \left(\frac{f_s}{2f_{\max}} \right) = 6 + 0.5 \times \log_2 \left(\frac{45000 \text{ kHz}}{2 \times 15 \text{ kHz}} \right) = 11.28 \approx 11 \text{ bits}$$

11.17

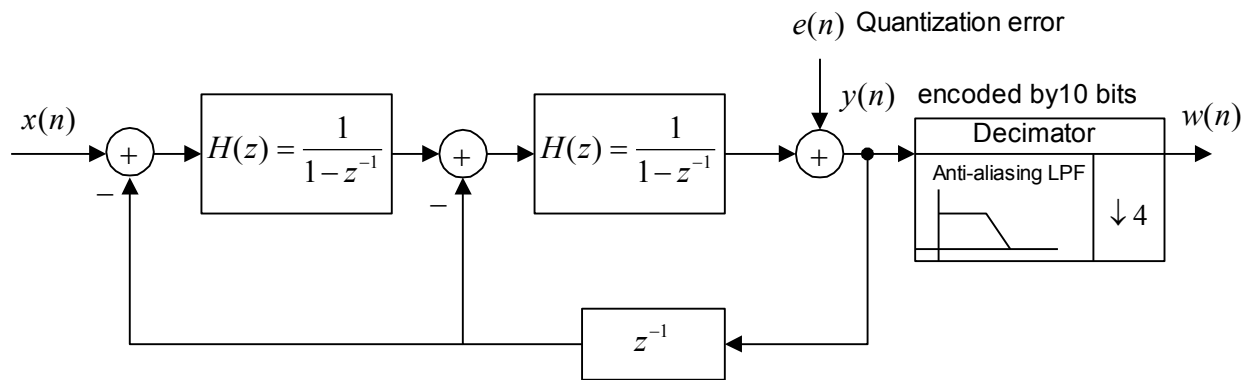
a.



b. $n = 1 + 1.5 \times \log_2 \left(\frac{128}{2 \times 4} \right) - 0.86 \approx 6$ bits

11.18

a.

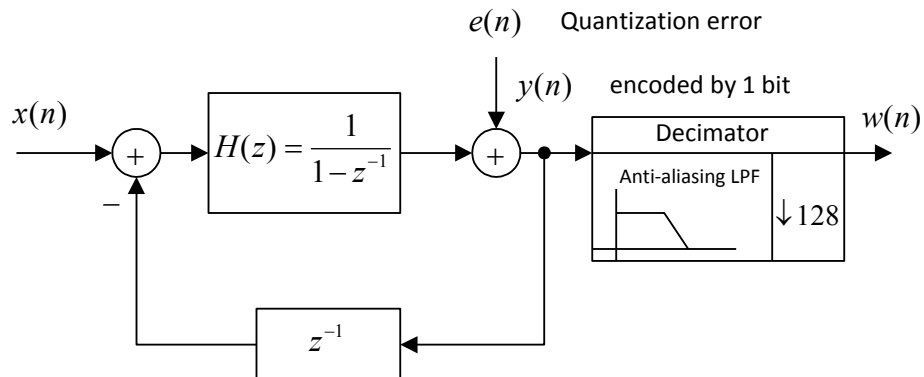


b.

$$n = m + 2.5 \times \log_2 \left(\frac{f_s}{2f_{\max}} \right) - 2.14 = 10 + 2.5 \times \log_2 \left(\frac{160}{2 \times 20} \right) - 2.14 = 12.86 \approx 13 \text{ bits}$$

11.19

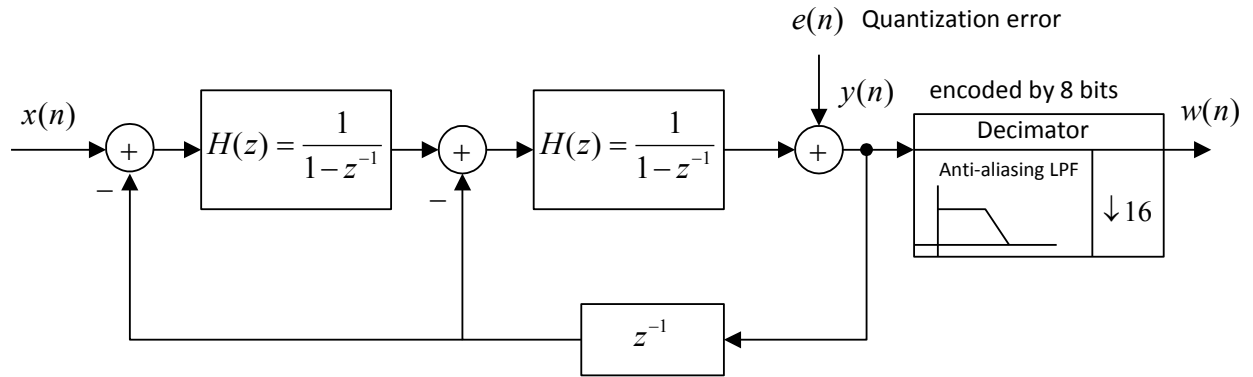
a.



b. $n = 1 + 1.5 \times \log_2 \left(\frac{128}{2 \times 0.5} \right) - 0.86 \approx 10$ bits

11.20

a.

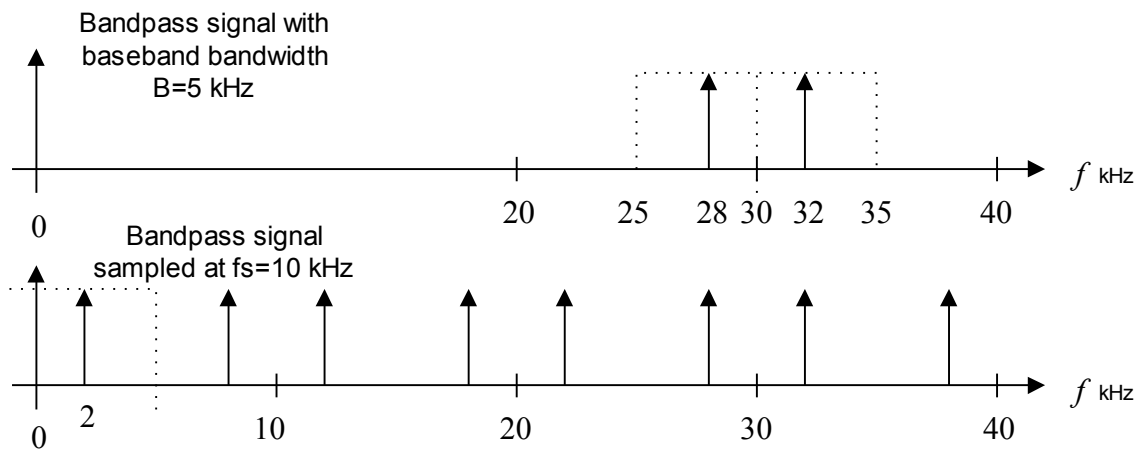


b.

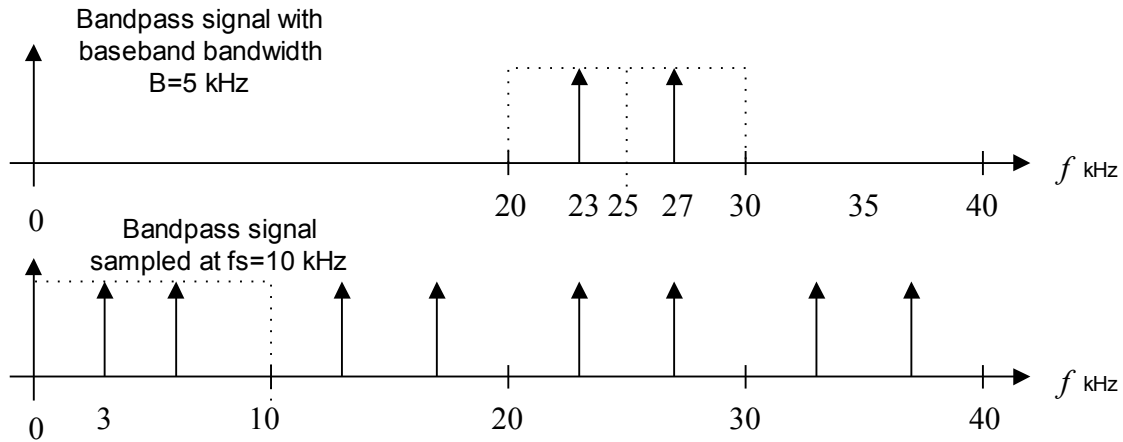
$$n = m + 2.5 \times \log_2 \left(\frac{f_s}{2f_{\max}} \right) - 2.14 = 8 + 2.5 \times \log_2 \left(\frac{16}{2 \times 0.5} \right) - 2.14 = 15.86 \approx 15 \text{ bits}$$

11.21

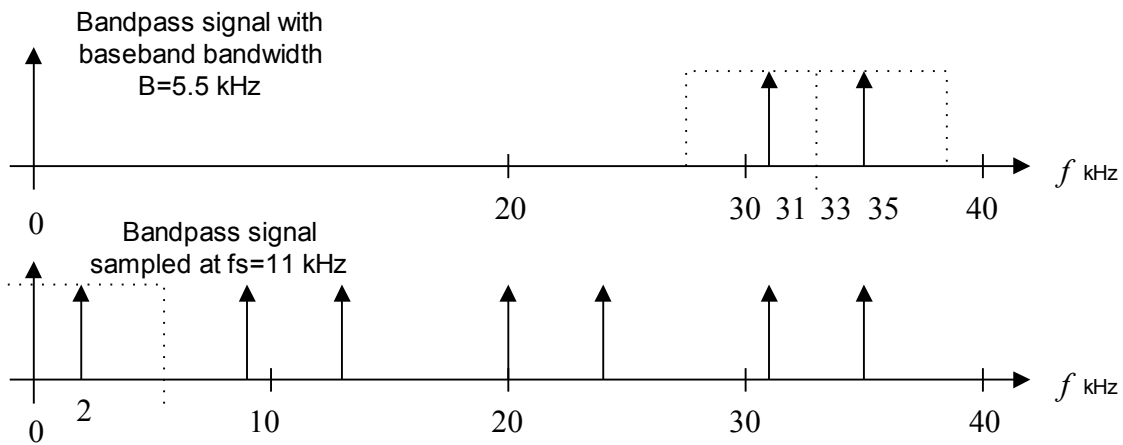
a. $f_c/B = 6$ is an even number, which is the case 1, we select $f_s = 10$ kHz



b. Since $f_c / B = 5$ is an odd number, we select $f_s = 10$ kHz

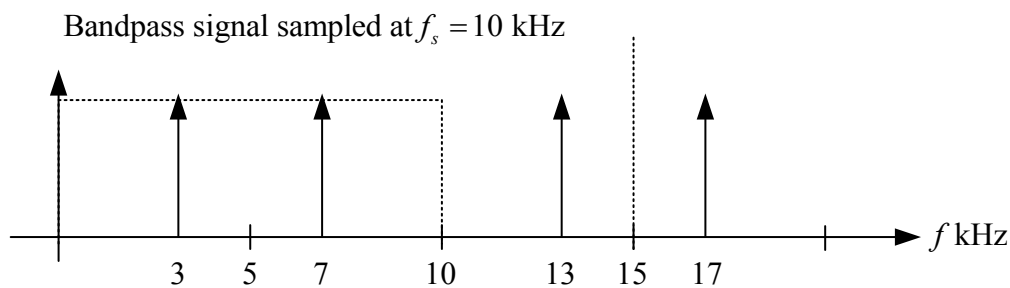
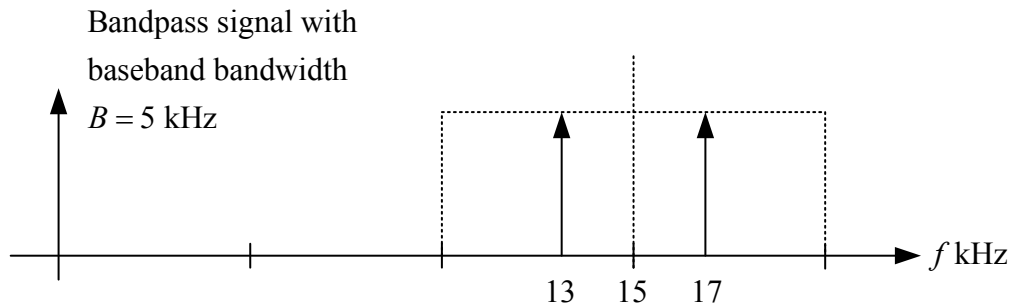


c. Now, $f_c / B = 6.6$ which is a non integer. We extend the band width $\bar{B} = 5.5$ kHz, so $f_c / \bar{B} = 6$ and $f_s = 2\bar{B} = 11$ kHz.

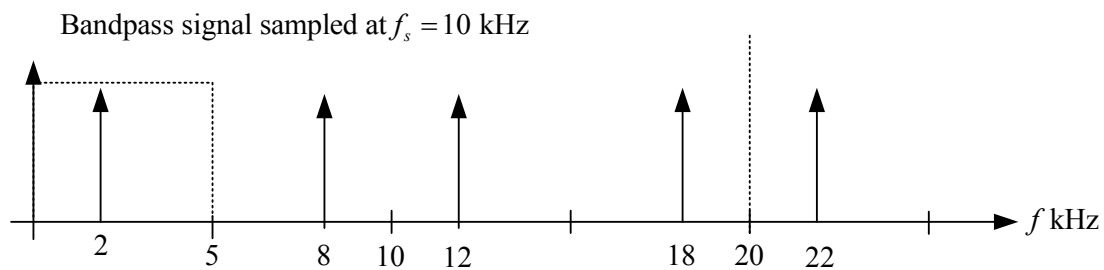
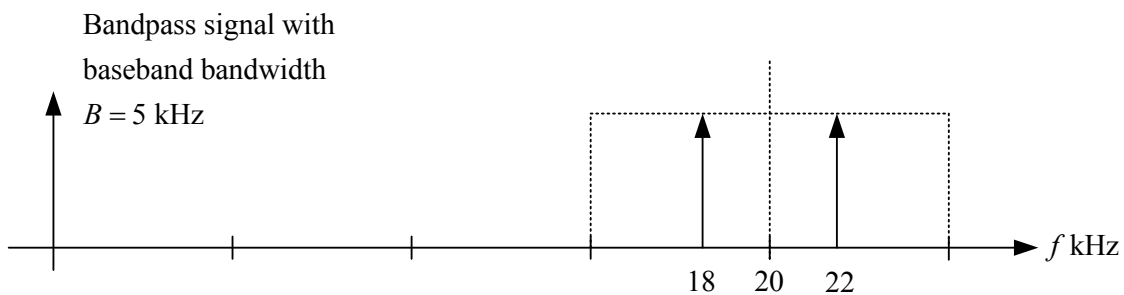


11. 22

a. Since $f_c / B = 3$ is an odd number, we select $f_s = 10$ kHz

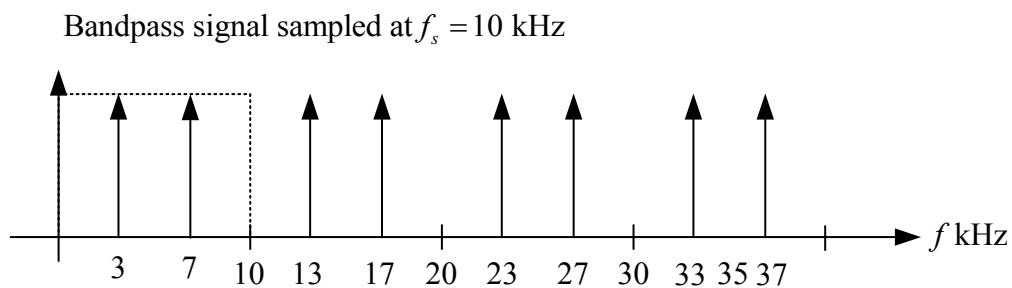
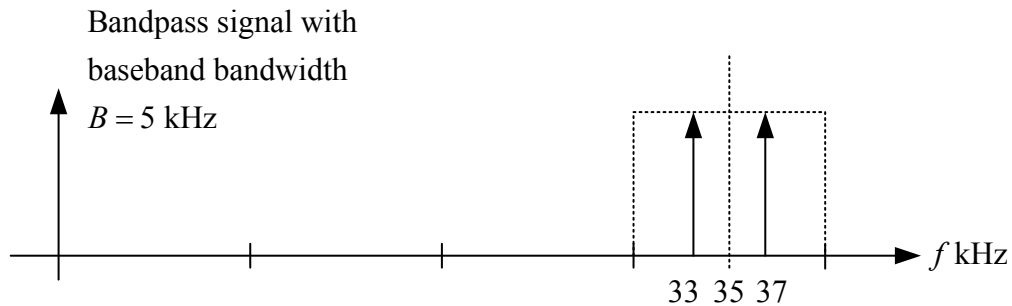


b. Since $f_c / B = 4$ is an even number, we select $f_s = 10$ kHz

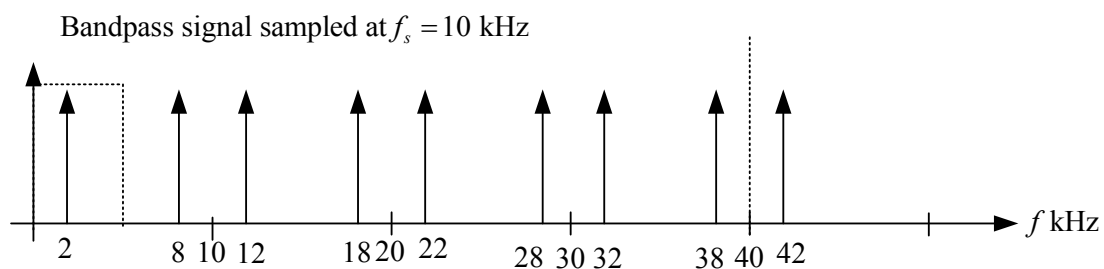
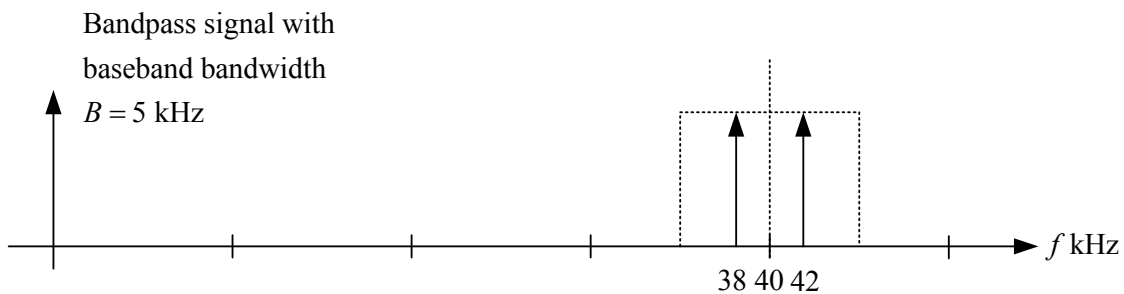


11.23

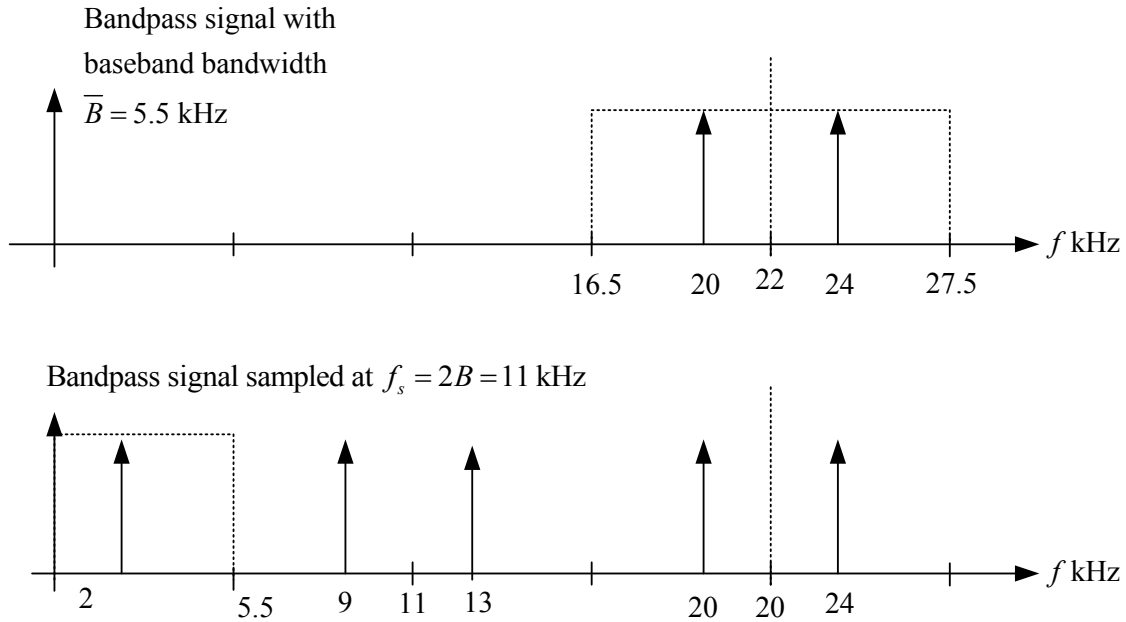
a. Since $f_c / B = 7$ is an odd number, we select $f_s = 10$ kHz



b. Since $f_c / B = 8$ is an even number, we select $f_s = 10$ kHz



c. Now, $f_c / B = 22 / 5 = 4.4$ which is a non integer. We extend the bandwidth $\bar{B} = 5.5$ kHz, so $f_c / \bar{B} = 4$ and $f_s = 2\bar{B} = 11$ kHz.

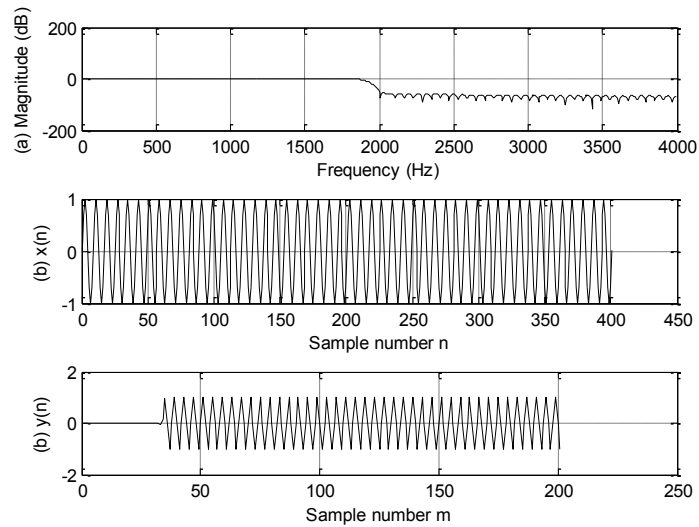


11.24

```

fs=8000; T=1/fs;
t=0:T:0.05;
x=sin(2*pi*1000*t);
b=firwd(133,1,2*pi*1900/fs,0,4);
[h,f]=freqz(b,1,512,fs);
figure(2)
w=filter(b,1,x);
y=w(1:2:length(w));
subplot(3,1,1);plot(f,20*log10(abs(h)));grid
xlabel('Frequency (Hz)');ylabel('(a) Magnitude (dB)');
subplot(3,1,2);plot(x);grid
xlabel('Sample number n');ylabel('(b) x(n)');
subplot(3,1,3);plot(y);grid
xlabel('Sample number m');ylabel('(b) y(n)');

```

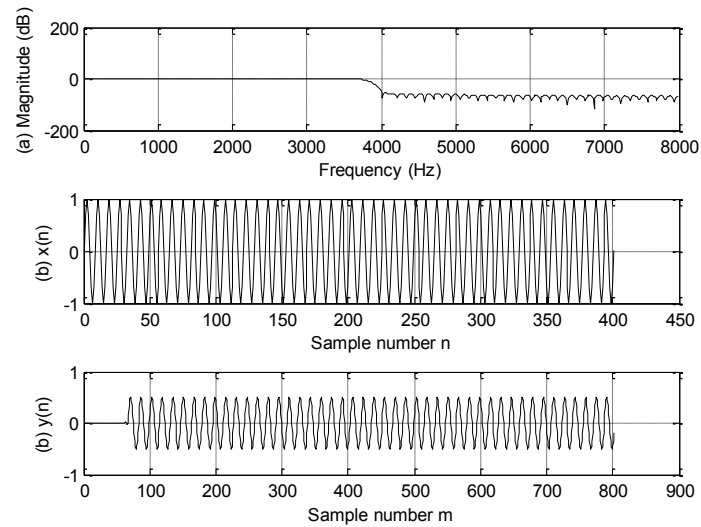


11.25

```

fs=8000; T=1/fs;fsL=16000;
t=0:T:0.05;
x=sin(2*pi*1000*t);
b=firwd(133,1,2*pi*3800/fsL,0,4);
[h,f]=freqz(b,1,512,fsL);
figure(2)
w=zeros(1,2*length(x));
w(1:2:length(w))=x;
y=filter(b,1,w);
subplot(3,1,1);plot(f,20*log10(abs(h)));grid
xlabel('Frequency (Hz)');ylabel('(a) Magnitude (dB)');
subplot(3,1,2);plot(x);grid
xlabel('Sample number n');ylabel('(b) x(n)');
subplot(3,1,3);plot(y);grid
xlabel('Sample number m');ylabel('(b) y(n)');

```

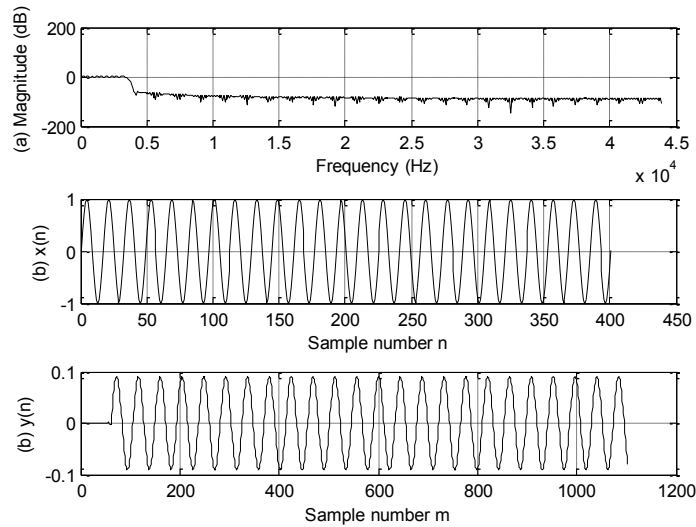


11.26

```

-----
fs=8000; T=1/fs; fsL=88000; fsM=22000;
t=0:T:0.05;
x=sin(2*pi*500*t);
b=firwd(485,1,2*pi*3600/fsL,0,4);
[h,f]=freqz(b,1,512,fsL);
figure(2)
w=zeros(1,11*length(x));
w(1:11:length(w))=x;
y1=filter(b,1,w);
y=y1(1:4:length(y1));
subplot(3,1,1);plot(f,20*log10(abs(h)));grid
xlabel('Frequency (Hz)');ylabel('(a) Magnitude (dB)');
subplot(3,1,2);plot(x);grid
xlabel('Sample number n');ylabel('(b) x(n)');
subplot(3,1,3);plot(y);grid
xlabel('Sample number m');ylabel('(b) y(n)');
-----

```



11.27

```

fs=8000; T=1/fs;
t=0:T:0.05;
x=sin(2*pi*1000*t);
b=firwd(133,1,2*pi*1900/fs,0,4);
[h,f]=freqz(b,1,512,fs);
%polyphase decimator
p0=b(1:2:length(b)); p1=b(2:2:length(b));
x_delay=filter([0 1],1,x);
w0=x(1:2:length(x)); w1=x_delay(1:2:length(x_delay));
y0=filter(p0,1,w0); y1=filter(p1,1,w1);
y=y0+y1;

```

11.28

```

fs=8000; T=1/fs; fsL=16000;
t=0:T:0.05;
x=sin(2*pi*1000*t);
b=firwd(133,1,2*pi*3800/fsL,0,4);
[h,f]=freqz(b,1,512,fsL);
%polyphase interpolator
p0=b(1:2:length(b)); p1=b(2:2:length(b));
w0=filter(p0,1,x); w1=filter(p1,1,x);
y0=zeros(1,2*length(x)); y1=y0;
y0(1:2:length(y0))=w0; y1(1:2:length(y1))=w1;
y1=filter([0 1],1,y1);
y=y0+y1;

```

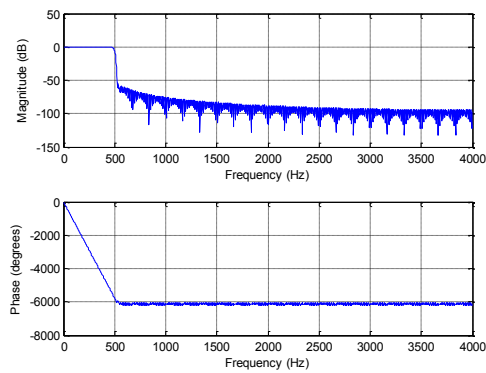
11.29

a. and b. for L=8

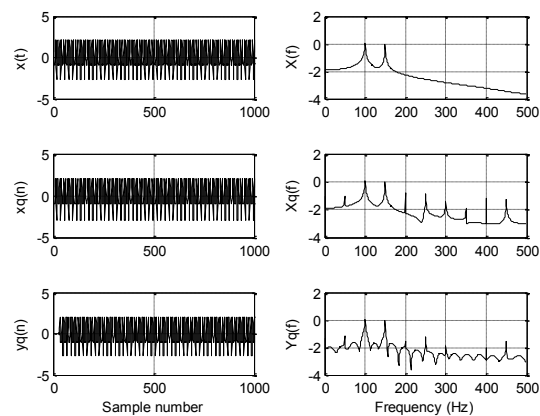
```

clear all; close all, clc
ntotal=1024;
n=0:ntotal; % Number of samples
L=8; % over saampling factor
nL=0:ntotal*L; % Number of samples for over sampling
numb=3; % Number of bits
A=2^(numb-1)-1; %Peak value
f1=100; C1=1.8; f2=150; C2=1; %Frequencies and amplitudes
fmax=500; fs=1000; T=1/fs; % Maximum frequency, sampling rate, sampling
period
fsL=L*fs; TL=1/fsL; %Over sampling rate, and over sampling period
% Sampling at fs=1000 Hz
x=C1*sin(2*pi*f1*n*T)+C2*sin(2*pi*f2*T*n+pi/4);
xq=round(x); %Qunrized signal at the minimum sampling rate
NN=length(n);
f=[0:ntotal-1]*fs/NN;
M=32*L; nd=M/L; %Number of delay in samples due to anti-aliasing
filtering
B=firwd(2*M+1, 1, 2*pi*fmax/fsL, 0, 4); % anti-aliasing filter design
(ensure 5% normalized transition BW)
figure(1); freqz(B, 1, 1000, fsL)
% Oversampling
xx=C1*sin(2*pi*f1*nL*TL)+C2*sin(2*pi*f2*nL*TL+pi/4);
xxq=round(xx); % Quantized signal
% down sampling
y=filter(B, 1, xxq); %Anti-aliasing filtering
yd=y(1:L:length(y)); % down sample
figure (2)
subplot(3,2,1); plot(n, x, 'k'); grid; axis([0 1000 -5 5]); ylabel('x(t)')
Ak=2*abs(fft(x))/NN; Ak(1)=Ak(1)/2
subplot(3,2,2); plot(f(1:NN/2), log10(Ak(1:NN/2)), 'k'); grid; ylabel('X(f)')
); axis([0 500 -4 2])
subplot(3,2,3); plot(n, xq, 'k'); grid; axis([0 1000 -5 5]); ylabel('xq(n)');
Ak=2*abs(fft(xq))/NN; Ak(1)=Ak(1)/2
subplot(3,2,4); plot(f(1:NN/2), log10(Ak(1:NN/2)), 'k'); grid; ylabel('Xq(f)')
); axis([0 500 -4 2])
subplot(3,2,5); plot(n, yd, 'k'); grid; axis([0 1000 -5 5]); ylabel('yq(n)');
xlabel('Sample number');
Ak=2*abs(fft(yd))/NN; Ak(1)=Ak(1)/2
subplot(3,2,6); plot(f(1:NN/2), log10(Ak(1:NN/2)), 'k'); grid; ylabel('Yq(f)')
); axis([0 500 -4 2])
xlabel('Frequency (Hz)');
figure (3)
plot(n(1:50), x(1:50), 'k', 'LineWidth', 2); hold % plot of first 50
samples
stairs(n(1:50), xq(1:50), 'b');
stairs(n(1:50), yd(1+nd:50+nd), 'r', 'LineWidth', 2); grid
axis([0 50 -5 5]); xlabel('Sample number'); ylabel('Amplitudes')
snr(x, xq);
snr(x(1:ntotal-nd), yd(1+nd:ntotal));

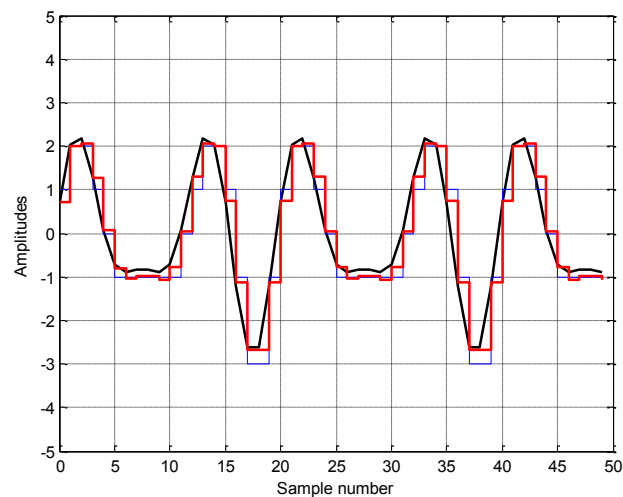
```



Anti-aliasing filter



Signal and spectra continuous case, quantization with regular sampling, and quantization with oversampling $L=8$



Detailed comparisons, continuous (black), regular (blue, SQNR=16 dB), oversampling (red, SQNR=24 dB)

c. SQNR > 23 dB for $L \geq 8$

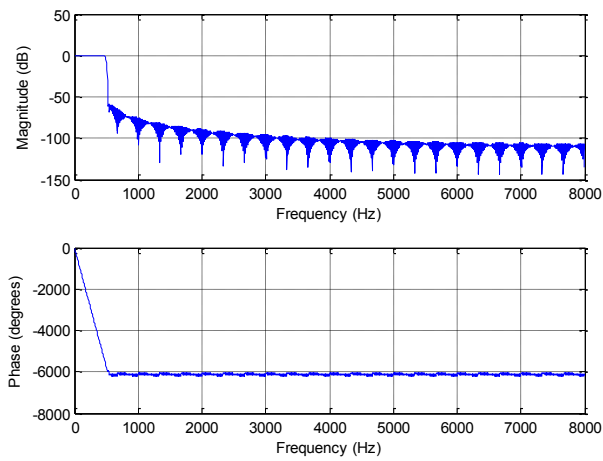
11.30

a. and b. (use the model from Figure 12.30)

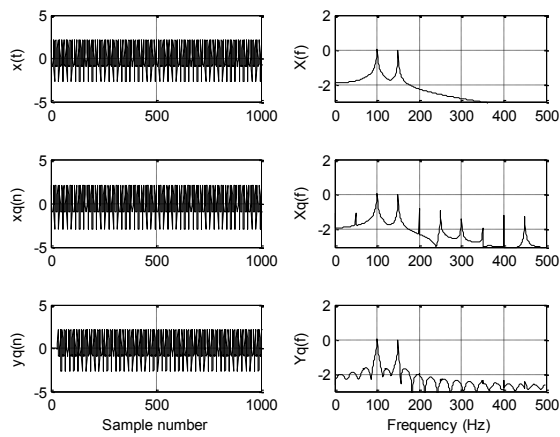
```

clear all; close all; clc
ntotal=1024; %Number of samples
n=0:ntotal;
L=16; %Oversampling factor
nL=0:ntotal*L; numb=3; A=2^(numb-1)-1; %Peak value
f1=100; C1=1.8; f2=150; C2=1; %Frequencies and amplitudes
fmax=500; fs=1000; T=1/fs % Sampling rate and sampling period
fsL=L*fs; TL=1/fsL; % Oversampling rate and sampling period
% Sampling at fs-1000 Hz
x=C1*sin(2*pi*f1*n*T)+C2*sin(2*pi*f2*T*n+pi/4);
xq=round(x); %Quantization
NN=length(n);
M=32*L; nd=M/L; %Delay in terms of samples for anti-filtering
B=firwd(2*M+1,1,2*pi*fmax/fsL,0,4); % Deisgn of anti-aliasing filter
figure(1)
freqz(B,1,1000,fsL);
% oversampling
xx=C1*sin(2*pi*f1*nL*TL)+C2*sin(2*pi*f2*nL*TL+pi/4);
% the first-order SDM processing
yq=zeros(1,ntotal*L+1); %Initializign the buffer
y=yq;
for i=1:ntotal*L
    y(i+1)=(xx(i+1)-yq(i))+y(i);
    yq(i+1)=round(y(i+1));
end
xxq=yq(1:ntotal*L+1); %Signal Quantization
% down sampling
y=filter(B,1,xxq);
yd=y(1:L:length(y));
f=[0:ntotal-1]*fs/NN;
figure(2)
subplot(3,2,1); plot(n,x,'k'); grid; axis([0 1000 -5 5]); ylabel('x(t)');
Ak=2*abs(fft(x))/NN; Ak(1)=Ak(1)/2;
subplot(3,2,2); plot(f(1:NN/2),log10(Ak(1:NN/2)),'k'); grid;
axis([0 500 -3 2]); ylabel('X(f)');
subplot(3,2,3); plot(n,xq,'k'); grid; axis([0 1000 -5 5]); ylabel('xq(n)');
Ak=2*abs(fft(xq))/NN; Ak(1)=Ak(1)/2;
subplot(3,2,4); plot(f(1:NN/2),log10(Ak(1:NN/2)),'k'); grid
axis([0 500 -3 2]); ylabel('Xq(f)');
subplot(3,2,5); plot(n,yd,'k'); grid; axis([0 1000 -5 5]); ylabel('yq(n)');
xlabel('Sample number');
Ak=2*abs(fft(yd))/NN; Ak(1)=Ak(1)/2;
subplot(3,2,6); plot(f(1:NN/2),log10(Ak(1:NN/2)),'k'); grid
axis([0 500 -3 2]); ylabel('Yq(f)'); xlabel('Frequency (Hz)');
figure(3)
plot(n(1:50),x(1:50),'k','LineWidth',2); hold
stairs(n(1:50),xq(1:50),'b');
stairs(n(1:50),yd(1+nd:50+nd),'r','LineWidth',2);
axis([0 50 -5 5]); grid; xlabel('Sample number'); ylabel('Amplitudes');
snr(x,xq);
snr(x(1:ntotal-nd),yd(1+nd:ntotal));

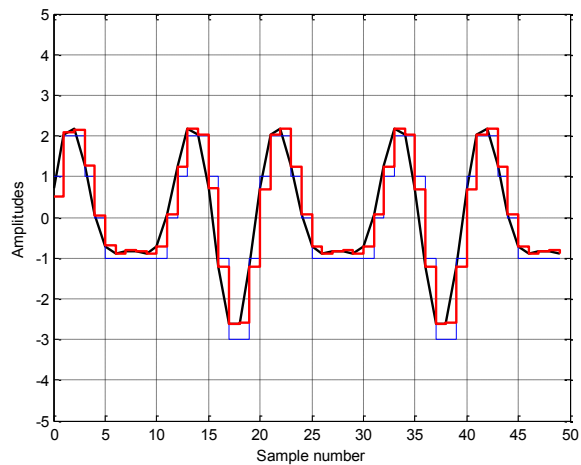
```



Anti-aliasing filter



Continuous, regular sampling, oversampling SDM with $L=16$;



Continuous (black), regular (blue, SQNR=16 dB), oversampling SDM with $L=16$ (red, SQNR=43.6 dB)

c. (use the model from Figure 12.32)

```

clear all; close all; clc
ntotal=1024; %Number of samples
n=0:ntotal;
L=16; %Oversampling factor
nL=0:ntotal*L; numb=3; A=2^(numb-1)-1; %Peak value
f1=100; C1=1.8; f2=150; C2=1; %Frequencies and amplitudes
fmax=500; fs=1000; T=1/fs % Sampling rte and sampling period
fsL=L*fs; TL=1/fsL; % Oversampling rate and sampling period
% Sampling at fs-1000 Hz
x=C1*sin(2*pi*f1*n*T)+C2*sin(2*pi*f2*T*n+pi/4);
xq=round(x); %Quantization
NN=length(n);
M=32*L; nd=M/L; %Delay in terms of samples for anti-filtering
B=firwd(2*M+1,1,2*pi*fmax/fsL,0,4); % Deisgn of anti-aliasing filter
figure(1)
freqz(B,1,1000,fsL);
% oversampling
xx=C1*sin(2*pi*f1*nL*TL)+C2*sin(2*pi*f2*nL*TL+pi/4);
% the second-order SDM processing
yq=zeros(1,ntotal*L+1+1); %Initilizign the buffer
y=yq; y1=y;
for i=1:ntotal*L
    y1(i+1)=(xx(i+1)-yq(i))+y1(i); %first-stage SDM
    y(i+1)=(y1(i+1)-yq(i))+y(i); %second-stage SDM
    yq(i+1)=round(y(i+1));
end
xxq=yq(1:ntotal*L+1); %Signal Quantization
% down sampling
y=filter(B,1,xxq);
yd=y(1:L:length(y));
f=[0:ntotal-1]*fs/NN;
figure(2)
subplot(3,2,1); plot(n,x,'k'); grid; axis([0 1000 -5 5]); ylabel('x(t)');
Ak=2*abs(fft(x))/NN; Ak(1)=Ak(1)/2;
subplot(3,2,2); plot(f(1:NN/2),log10(Ak(1:NN/2)),'k'); grid;
axis([0 500 -3 2]); ylabel('X(f)');
subplot(3,2,3); plot(n,xq,'k'); grid; axis([0 1000 -5 5]); ylabel('xq(n)');
Ak=2*abs(fft(xq))/NN; Ak(1)=Ak(1)/2;
subplot(3,2,4); plot(f(1:NN/2),log10(Ak(1:NN/2)),'k'); grid
axis([0 500 -3 2]); ylabel('Xq(f)');
subplot(3,2,5); plot(n,yd,'k'); grid; axis([0 1000 -5 5]); ylabel('yq(n)');
xlabel('Sample number');
Ak=2*abs(fft(yd))/NN; Ak(1)=Ak(1)/2;
subplot(3,2,6); plot(f(1:NN/2),log10(Ak(1:NN/2)),'k'); grid
axis([0 500 -3 2]); ylabel('Yq(f)'); xlabel('Frequency (Hz)');
figure(3)
plot(n(1:50),x(1:50),'k','LineWidth',2); hold
stairs(n(1:50),xq(1:50),'b');
stairs(n(1:50),yd(1+nd:50+nd),'r','LineWidth',2);
axis([0 50 -5 5]); grid; xlabel('Sample number'); ylabel('Amplitudes');
snr(x,xq);
snr(x(1:ntotal-nd),yd(1+nd:ntotal));

```

SQNR =47.9 dB is better than SQNR=43.6 in (b)

11.31



Verification for causal system (non-causal system can be verified similarly)

(a)

$$y(m) = v(m) * h(m) = \sum_{k=-\infty}^{\infty} h(k)v(m-k) = \sum_{k=0}^{\infty} h(k)v(m-k), \quad v(m) = x(mM)$$

$$y(0) = \sum_{k=0}^{\infty} h(k)v(-k) = h(0)v(0) + h(1)v(-1) + \dots = h(0)x(0 \times M) + h(1)x(-1 \times M) + \dots$$

$$y(1) = \sum_{k=0}^{\infty} h(k)v(1-k) = h(0)v(1) + h(1)v(0) + \dots = h(0)x(1 \times M) + h(1)x(0 \times M) + \dots$$

$$y(2) = \sum_{k=0}^{\infty} h(k)v(2-k) = h(0)v(2) + h(1)v(1) + h(2)v(0) + \dots = h(0)x(2M) + h(1)x(M) + h(2)x(0M) \dots$$

(b)

$$\text{Let } \bar{H}(z) = H(z^M), \text{ that is, } \bar{h}(n) = \begin{cases} h(n/M) & \text{if } n/M = \text{integer} \\ 0 & \text{else} \end{cases}$$

$$\text{and } y(m) = w(mM) \text{ and } w(n) = \sum_{k=0}^{\infty} \bar{h}(k)x(n-k)$$

$$y(0) = w(0) = \sum_{k=0}^{\infty} \bar{h}(k)x(0-k) = \bar{h}(0)x(0) + \bar{h}(1)x(-1) + \dots$$

$$+ \bar{h}(M)x(-M) + \bar{h}(M+1)x(-M-1) + \dots = h(0)x(0) + h(1)x(-M) + \dots$$

$$y(1) = w(M) = \sum_{k=0}^{\infty} \bar{h}(k)x(M-k) = \bar{h}(0)x(M) + \bar{h}(1)x(M-1) + \dots$$

$$+ \bar{h}(M)x(M-M) + \bar{h}(M+1)x(M-M-1) + \dots = h(0)x(M) + h(1)x(0) + \dots$$

$$y(2) = w(2M) = \sum_{k=0}^{\infty} \bar{h}(k)x(2M-k) = \bar{h}(0)x(2M) + \bar{h}(1)x(2M-1) + \dots$$

$$+ \bar{h}(M)x(2M-M) + \bar{h}(M+1)x(2M-M-1) + \dots$$

$$+ \bar{h}(2M)x(2M-2M) + \bar{h}(2M+1)x(-1) + \dots = h(0)x(2M) + h(1)x(M) + h(2)x(0) \dots$$

We can conclude (a)=(b).

11.32



Verification for causal system (non-causal system can be verified similarly)

(a)

$$v(n) = \sum_{k=0}^{\infty} h(k)x(n-k), \quad y(m) = \begin{cases} v(m/L) & \text{if } m/L = \text{integer} \\ 0 & \text{else} \end{cases}$$

$$y(0) = v(0) = \sum_{k=0}^{\infty} h(k)x(n-k) = h(0)x(0) + h(1)x(-1) + \dots$$

$$y(L) = v(1) = \sum_{k=0}^{\infty} h(k)x(1-k) = h(0)x(1) + h(1)x(0) + \dots$$

$$y(2L) = v(2) = \sum_{k=0}^{\infty} h(k)x(2-k) = h(0)x(2) + h(1)x(1) + h(2)x(0) + \dots$$

(b)

Let $\bar{H}(z) = H(z^L)$, that is, $\bar{h}(m) = \begin{cases} h(m/L) & \text{if } m/L = \text{integer} \\ 0 & \text{else} \end{cases}$

$$w(m) = \begin{cases} x(m/L) & \text{if } m/L = \text{integer} \\ 0 & \text{else} \end{cases}$$

$$y(m) = \sum_{k=0}^{\infty} \bar{h}(k)w(m-k)$$

$$y(0) = \sum_{k=0}^{\infty} \bar{h}(k)w(0-k) = \bar{h}(0)w(-0) + \bar{h}(1)w(-1) + \dots + \bar{h}(L)w(-L) + \dots$$

$$= h(0)x(0) + h(1)x(-1) + \dots$$

$$y(1) = \sum_{k=0}^{\infty} \bar{h}(k)w(1-k) = \bar{h}(0)w(1-0) + \bar{h}(1)w(1-1) + \dots + \bar{h}(L)w(1-L) + \dots = 0$$

....

$$y(L) = \sum_{k=0}^{\infty} \bar{h}(k)w(L-k) = \bar{h}(0)w(L-0) + \bar{h}(1)w(L-1) + \dots + \bar{h}(L)w(L-L) + \dots$$

$$+ \bar{h}(L)w(0) + \dots = h(0)x(1) + h(1)x(0)$$

$$y(L+1) = \sum_{k=0}^{\infty} \bar{h}(k)w(L+1-k) = \bar{h}(0)w(L+1-0) + \bar{h}(1)w(L+1-1) + \dots + \bar{h}(L)w(L+1-L) + \dots$$

$$+ \bar{h}(L)w(1) + \dots = 0$$

...

$$y(2L) = \sum_{k=0}^{\infty} \bar{h}(k)w(2L-k) = \bar{h}(0)w(2L-0) + \bar{h}(1)w(2L-1) + \dots + \bar{h}(L-1)w(2L-L-1) + \dots$$

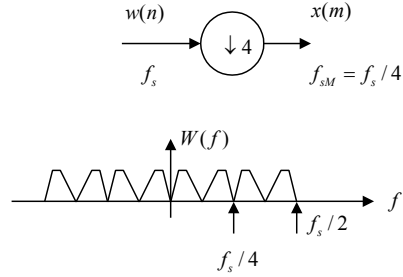
$$+ \bar{h}(L)w(2L-L) + \bar{h}(L+1)w(2L-L-1) + \dots + \bar{h}(2L)w(2L-2L) = h(0)x(2) + h(1)x(1) + h(2)x(0) + \dots$$

By comparison: (a)=(b)

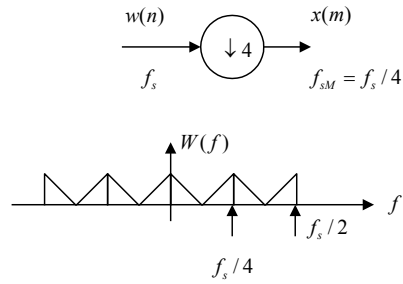
Chapter 12

12.1

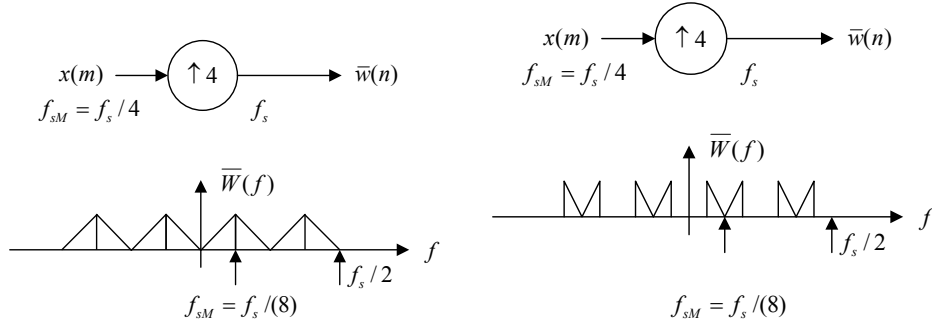
a. $B = f_{sM} / 2 = f_s / (2M)$ $f_c = 2(f_s / (2M)) = 2B$, $f_c / B = 2 = \text{even}$



b. $B = f_{sM} / 2 = f_s / (2M)$ $f_c = f_s / (2M) = B$, $f_c / B = 1 = \text{odd}$



12.2

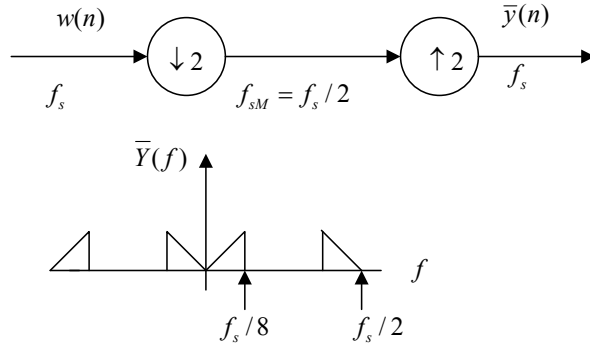


12.3

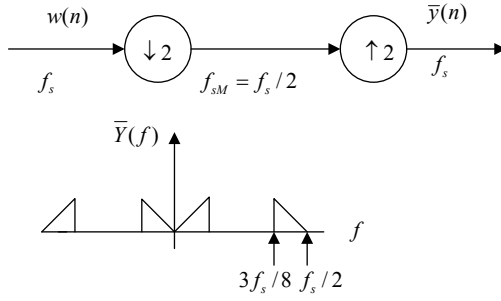
From Equation (13.7), $\bar{Y}(z) = \frac{1}{2}(W(z) + W(e^{-j\pi}z))$,

$\bar{Y}(e^{j\Omega}) = \frac{1}{2}(W(e^{j\Omega}) + W(e^{j(\Omega-\pi)}))$, $W(e^{j(\Omega-\pi)})$ is the shifted version of $W(e^{j\Omega})$ by $f_s / 2$.

a.



b.

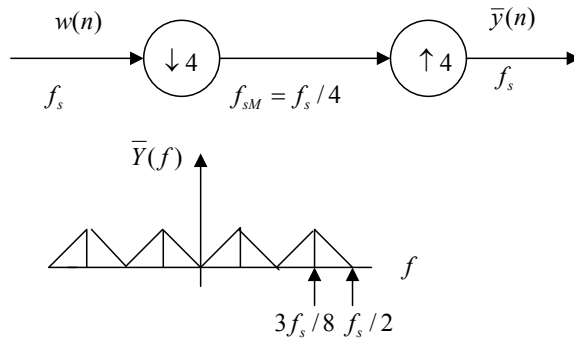


12.4

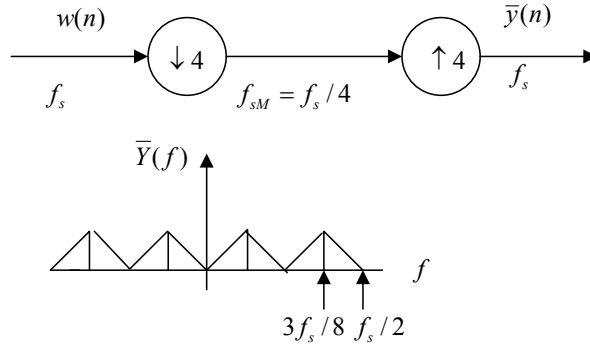
From Equation (13.7), $\bar{Y}(z) = \frac{1}{4}(W(z) + W(e^{-j\pi/2}z) + W(e^{-j\pi}z) + W(e^{-j3\pi/2}z))$,

$$\bar{Y}(e^{j\Omega}) = \frac{1}{2}(W(e^{j\Omega}) + W(e^{j(\Omega-\pi/2)}) + W(e^{j(\Omega-\pi)}) + W(e^{j(\Omega-3\pi/2)}))$$

a.



b.



12.5

Since $H_0(z) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}z^{-1}$, $N = 2$

$$H_1(z) = z^{-(N-1)}H_0(-z^{-1}) = z^{-(2-1)}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}(-z^{-1})^{-1}\right) = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}z^{-1}$$

$$G_0(z) = -H_1(-z) = -\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}(-z)^{-1}\right) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}z^{-1}$$

$$G_1(z) = H_0(-z) = \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}(-z)^{-1}\right) = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}z^{-1}$$

12.6

a. $h_0(k) = [0.707 \ 0.707]$, $\rho(2n) = \sum_{k=0}^{N-1} h_0(k)h_0(k+2n)$

$$n = 0$$

$$\rho(0) = \sum_{k=0}^{2-1} h_0(k)h_0(k) = h_0(0)h_0(0) + h_0(1)h_0(1) = 0.707 \times 0.707 + 0.707 \times 0.707 = 1$$

$$n = 1$$

$$\rho(1) = \sum_{k=0}^{2-1} h_0(k)h_0(k+2) = h_0(0)h_0(2) + h_0(1)h_0(3) = 0.707 \times 0 + 0.707 \times 0 = 0$$

Similarly, for $n = 2, 3, \dots$, $\rho(n) = 0$

$$n = -1$$

$$\rho(-1) = \sum_{k=0}^{2-1} h_0(k)h_0(k-2) = h_0(0)h_0(-2) + h_0(1)h_0(-3) = 0.707 \times 0 + 0.707 \times 0 = 0$$

Similarly, for $n = -2, -3, \dots$, $\rho(n) = 0$.

b.

$$H_0(z) = 0.707 + 0.707z^{-1}$$

$$R(z) = H_0(z)H_0(1/z) = (0.707 + 0.707z^{-1})(0.707 + 0.707z) = 0.5z^{-1} + 1 + 0.5z^{-1}$$

$$R(-z) = -0.5z^{-1} + 1 - 0.5z^{-1}$$

$$R(z) + R(-z) = 2$$

12.7

$$H_0(-z^{-1}) = 0.483 - 0.837z + 0.224z^2 + 0.129z^3$$

$$H_1(z) = z^{-(4-1)}H_0(-z^{-1}) = 0.129 + 0.224z^{-1} - 0.837z^{-2} + 0.483z^{-3}$$

$$G_0(z) = -H_1(-z) = -0.129 + 0.224z^{-1} + 0.837z^{-2} + 0.483z^{-3}$$

$$G_1(z) = H_0(-z) = 0.483 - 0.837z^{-1} + 0.224z^{-2} + 0.129z^{-3}$$

12.8

$$\rho(n) = \sum_{k=0}^3 h_0(k)h_0(k+n) = h_0(0)h_0(n) + h_0(1)h_0(1+n) + h_0(2)h_0(2+n) + h_0(3)h_0(3+n)$$

$$\rho(0) = h_0(0)h_0(0) + h_0(1)h_0(1) + h_0(2)h_0(2) + h_0(3)h_0(3)$$

$$= 0.483^2 + 0.837^2 + 0.224^2 + (-0.129)^2 = 1.00$$

$$\rho(2) = h_0(0)h_0(2) + h_0(1)h_0(3) + h_0(2)h_0(4) + h_0(3)h_0(5)$$

$$= 0.483 \times 0.224 + 0.837 \times (-0.129) + 0.224 \times 0 + (-0.129) \times 0 = 0.000219 \approx 0$$

$$\rho(-2) = h_0(0)h_0(-2) + h_0(1)h_0(-1) + h_0(2)h_0(0) + h_0(3)h_0(1)$$

$$= 0.483 \times 0 + 0.837 \times 0 + 0.224 \times 0.483 + (-0.129) \times 0.837 \approx 0$$

$$\rho(4) = h_0(0)h_0(4) + h_0(1)h_0(5) + h_0(2)h_0(6) + h_0(3)h_0(7) = 0$$

$$\rho(-4) = h_0(0)h_0(-4) + h_0(1)h_0(-3) + h_0(2)h_0(-2) + h_0(3)h_0(-1) = 0$$

Similarly, $\rho(\pm 6) = \rho(\pm 8) = \dots$

$$H_0(z) = 0.483 + 0.837z^{-1} + 0.224z^{-2} - 0.129z^{-3},$$

$$H_0(z^{-1}) = 0.483 + 0.837z + 0.224z^2 - 0.129z^3$$

$$R(z) = H_0(z)H_0(z^{-1})$$

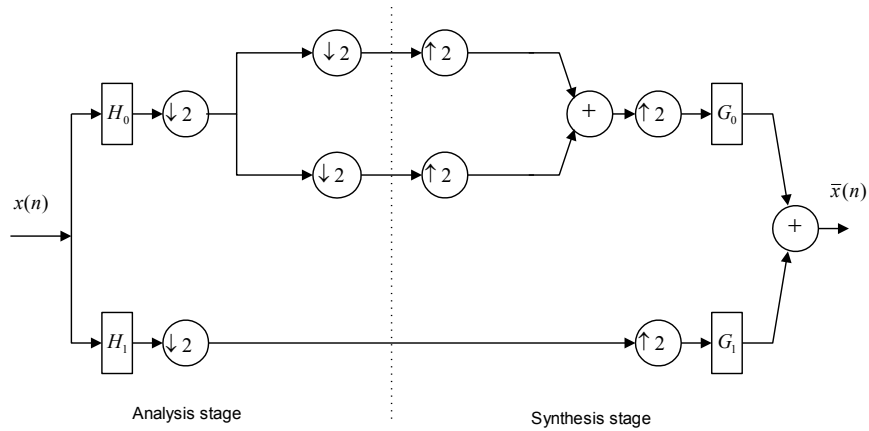
$$= (0.483 + 0.837z^{-1} + 0.224z^{-2} - 0.129z^{-3})(0.483 + 0.837z + 0.224z^2 - 0.129z^3)$$

$$R(z) = -0.0623z^{-3} + 0.5629z^{-1} + 1 + 0.5269z - 0.0623z^3$$

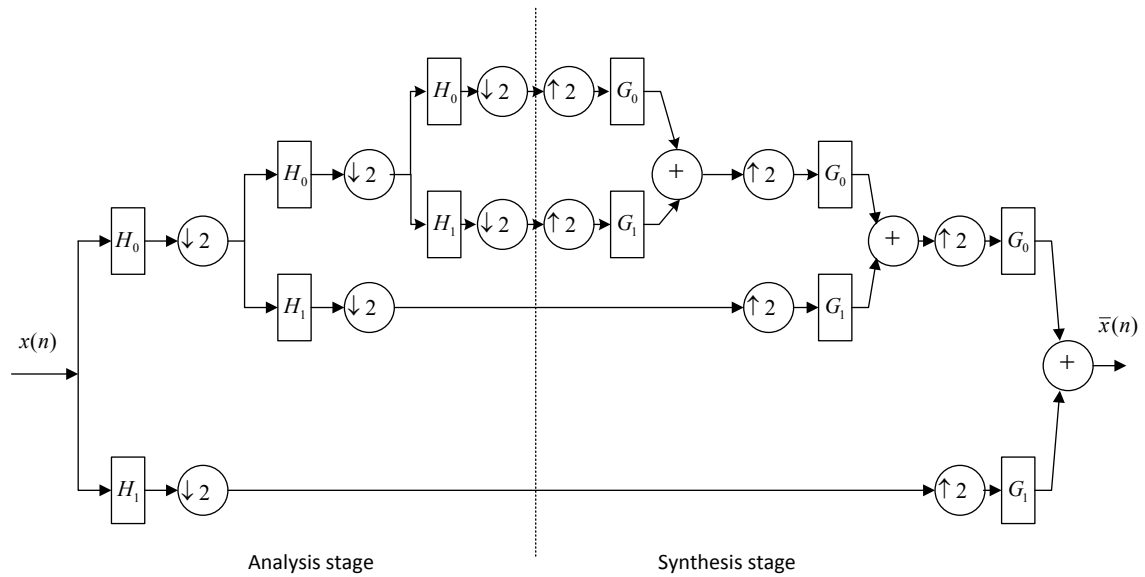
$$R(-z) = 0.0623z^{-3} - 0.5629z^{-1} + 1 - 0.5269z + 0.0623z^3$$

$$\text{Then } R(z) + R(-z) = 2$$

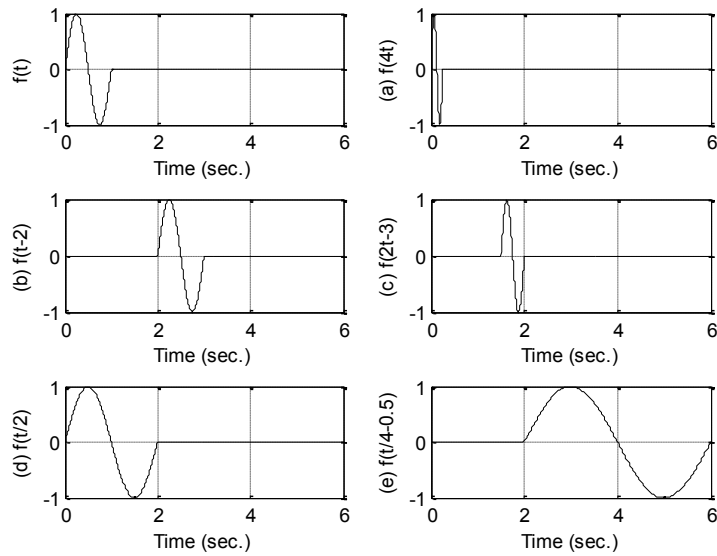
12.9



12.10



12.11

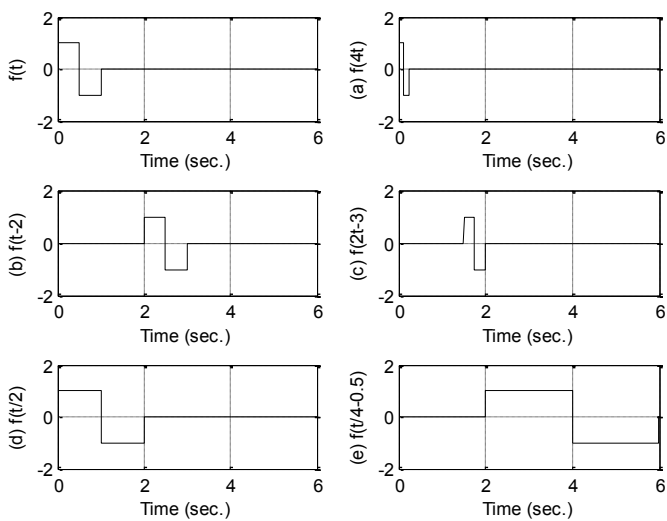


12.12

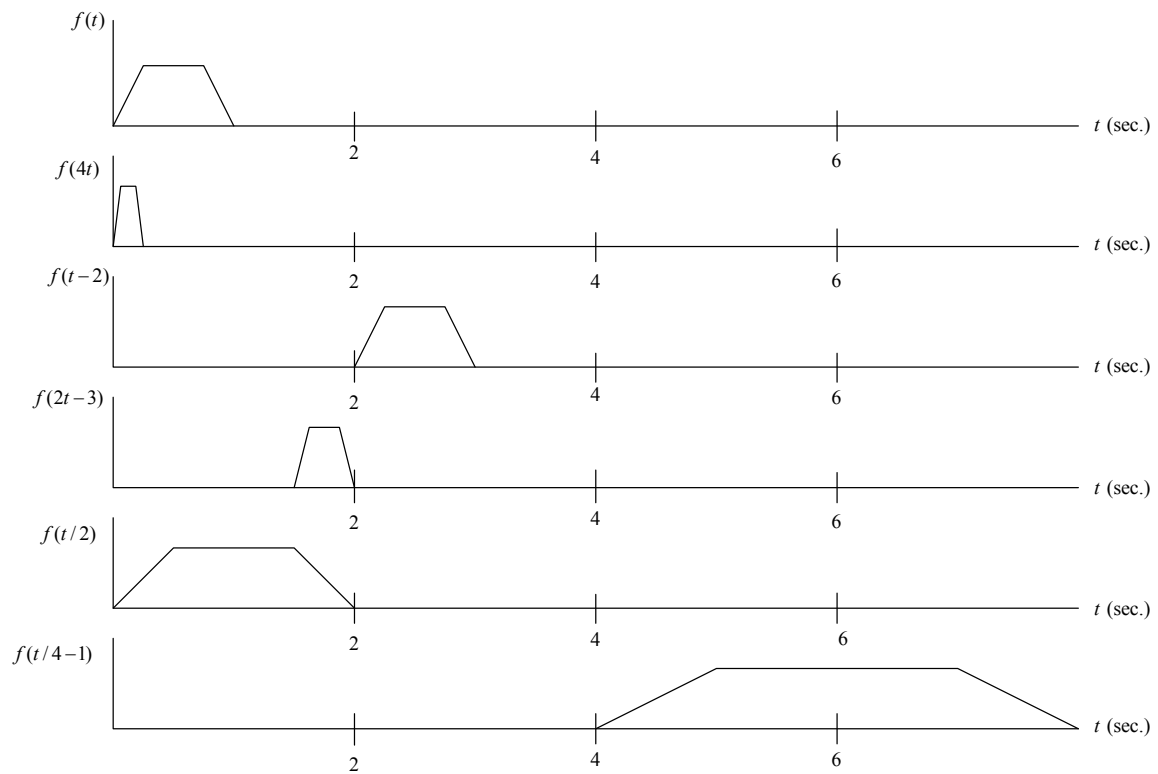
For Figure 13.52 (b), $a = 0.25$, $b = 1.5$

For Figure 13.52 (c), $a = 0.125$, $b = 2$

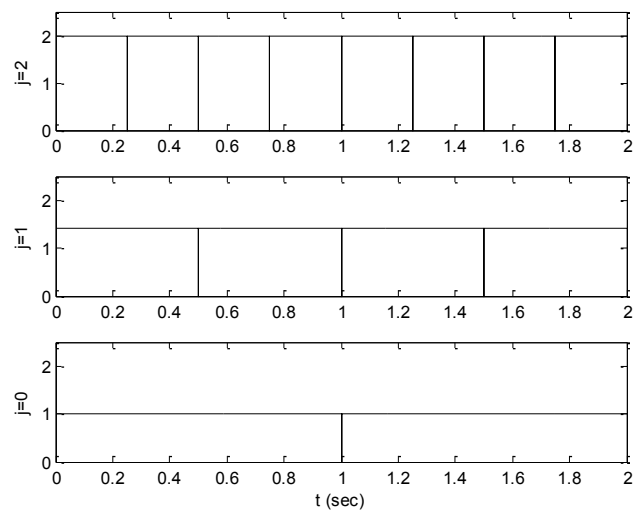
12.13



12.14



12.15



12.16

Repeat Figure 13.7 for one more second.

12.17

a. $f(t) = \sum_{k=-\infty}^{\infty} c_1(k) 2^{1/2} \phi(2t-k)$

$$c_1(k) = \langle f(t) \phi_{1k}(t) \rangle = \int f(t) 2^{1/2} \phi(2t-k) dt$$

$$c_1(0) = \langle f(t) \phi_{10}(t) \rangle = \int_0^{1/2} f(t) 2^{1/2} \phi(2t) dt = 2 \times 2^{1/2}$$

$$c_1(1) = \langle f(t) \phi_{11}(t) \rangle = \int_{1/2}^1 f(t) 2^{1/2} \phi(2t-1) dt = -2^{1/2}$$

$$f(t) = c_1(0) 2^{1/2} \phi(2t) + c_1(1) 2^{1/2} \phi(2t-1) = 4\phi(2t) - 2\phi(2t-1)$$

b. $f(t) = \sum_{k=-\infty}^{\infty} c_0(k) \phi_{0k}(t) + \sum_{k=-\infty}^{\infty} d_0(k) \psi_{0k}(t)$

$$c_0(0) = \langle f(t) \phi_{00}(t) \rangle = \int_0^1 f(t) \phi(t) dt = 1$$

$$d_0(0) = \langle f(t) \psi_{00}(t) \rangle = \int_0^1 f(t) \psi(t) dt = 3$$

$$f(t) = c_0(0) \phi(t) + d_0(0) \psi(t) = \phi(t) + 3\psi(t)$$

12.18

a. $f(t) = \sum_{k=-\infty}^{\infty} c_2(k) 2\phi(4t-k)$

$$c_2(k) = \langle f(t) \phi_{2k}(t) \rangle = \int f(t) 2\phi(4t-k) dt$$

$$c_2(0) = \langle f(t) \phi_{20}(t) \rangle = \int_0^{1/4} f(t) 2\phi(4t) dt = \int_0^{1/4} 8dt = 2$$

$$c_2(1) = \langle f(t) \phi_{21}(t) \rangle = \int f(t) 2\phi(4t-1) dt = \int_{1/4}^{2/4} (-2) 2dt = -1$$

$$c_2(2) = \langle f(t) \phi_{22}(t) \rangle = \int f(t) 2\phi(4t-2) dt = \int_{2/4}^{3/4} 2 \times 2dt = 1$$

$$c_2(3) = \langle f(t) \phi_{23}(t) \rangle = \int f(t) 2\phi(4t-3) dt = \int_{3/4}^1 (-1) 2dt = -0.5$$

$$f(t) = c_2(0) 2\phi(4t) + c_2(1) 2\phi(4t-1) + c_2(2) 2\phi(4t-2) + c_2(3) 2\phi(4t-3)$$

$$= 4\phi(4t) - 2\phi(4t-1) + 2\phi(4t-2) - 1\phi(4t-3)$$

b. From Equation (13.46), $f(t) = \sum_{k=-\infty}^{\infty} c_1(k) 2^{1/2} \phi(2t-k) + \sum_{k=-\infty}^{\infty} d_1(k) 2^{1/2} \psi(2t-k)$

$$c_1(0) = \langle f(t) \phi_{10}(t) \rangle = \int f(t) 2^{1/2} \phi(2t) dt = \int_0^{1/4} 4 \times 2^{1/2} dt + \int_{1/4}^{1/2} (-2) \times 2^{1/2} dt = 0.5 \times 2^{1/2}$$

$$c_1(1) = \langle f(t) \phi_{11}(t) \rangle = \int f(t) 2^{1/2} \phi(2t-1) dt = \int_{1/2}^{3/4} 2 \times 2^{1/2} dt + \int_{1/4}^{1/2} (-1) \times 2^{1/2} dt = 0.25 \times 2^{1/2}$$

$$d_1(0) = \langle f(t) \psi_{10}(t) \rangle = \int f(t) 2^{1/2} \psi(2t) dt = \int_0^{1/4} 4 \times 2^{1/2} dt + \int_{1/4}^{1/2} (-2) \times 2^{1/2} (-1) dt = 1.5 \times 2^{1/2}$$

$$d_1(1) = \langle f(t) \psi_{11}(t) \rangle = \int f(t) 2^{1/2} \psi(2t-1) dt = \int_{1/2}^{3/4} 2 \times 2^{1/2} dt + \int_{1/4}^{1/2} (-1) \times 2^{1/2} (-1) dt = 0.75 \times 2^{1/2}$$

$$f(t) = c_1(0) 2^{1/2} \phi(2t) + c_1(1) 2^{1/2} \phi(2t-1) + d_1(0) 2^{1/2} \psi(2t) + d_1(1) 2^{1/2} \psi(2t-1)$$

$$= \phi(2t) + 0.5\phi(2t-1) + 3\psi(2t) + 1.5\psi(2t-1)$$

c. From Equation (13.47),

$$\begin{aligned}
 f(t) &= \sum_{k=-\infty}^{\infty} c_0(k)\phi(t-k) + \sum_{j=0}^1 \sum_{k=-\infty}^{\infty} d_j(k)2^{j/2}\psi(2^j t - k) \\
 &= c_0(0)\phi(t) + d_0(0)\psi(t) + d_1(0)2^{1/2}\psi(2t) + d_1(1)2^{1/2}\psi(2t-1) \\
 c_0(0) &= \langle f(t)\phi_{00}(t) \rangle = \int f(t)\phi(t)dt = 0.75 \\
 d_0(0) &= \langle f(t)\psi_{00}(t) \rangle = \int f(t)\psi(t)dt = 0.25 \\
 d_1(0) &= \langle f(t)\psi_{10}(t) \rangle = \int f(t)2^{1/2}\psi(2t)dt = \int_0^{1/4} 4 \times 2^{1/2}dt + \int_{1/4}^{1/2} (-2) \times 2^{1/2}(-1)dt = 1.5 \times 2^{1/2} \\
 d_1(1) &= \langle f(t)\psi_{11}(t) \rangle = \int f(t)2^{1/2}\psi(2t-1)dt = \int_{1/2}^{3/4} 2 \times 2^{1/2}dt + \int_{1/4}^{1/2} (-1) \times 2^{1/2}(-1)dt = 0.75 \times 2^{1/2} \\
 f(t) &= 0.75\phi(t) + 0.25\psi(t) + 3\psi(2t) + 1.5\psi(2t-1)
 \end{aligned}$$

12.19

$$\begin{aligned}
 \text{a. } f(t) &= \sum_{k=-\infty}^{\infty} c_1(k)2^{1/2}\phi(2t-k) \\
 c_1(k) &= \langle f(t)\phi_{1k}(t) \rangle = \int f(t)2^{1/2}\phi(2t-k)dt \\
 c_1(0) &= \langle f(t)\phi_{10}(t) \rangle = \int_0^{1/2} \sin(2\pi t)2^{1/2}\phi(2t)dt = (1/\pi) \times 2^{1/2} \\
 c_1(1) &= \langle f(t)\phi_{11}(t) \rangle = \int_{1/2}^1 \sin(2\pi t)2^{1/2}\phi(2t-1)dt = (-1/\pi)2^{1/2} \\
 f(t) &= c_1(0)2^{1/2}\phi(2t) + c_1(1)2^{1/2}\phi(2t-1) = (2/\pi)\phi(2t) - (2/\pi)\phi(2t-1) \\
 \text{b. } f(t) &= \sum_{k=-\infty}^{\infty} c_0(k)\phi_{0k}(t) + \sum_{k=-\infty}^{\infty} d_0(k)\psi_{0k}(t) \\
 c_0(0) &= \langle f(t)\phi_{00}(t) \rangle = \int_0^1 \sin(2\pi t)\phi(t)dt = 0 \\
 d_0(0) &= \langle f(t)\psi_{00}(t) \rangle = \int_0^1 \sin(2\pi t)\psi(t)dt = 2/\pi \\
 f(t) &= c_0(0)\phi(t) + d_0(0)\psi(t) = (2/\pi)\psi(t)
 \end{aligned}$$

12.20

$$\begin{aligned}
 \text{a. } f(t) &= \sum_{k=-\infty}^{\infty} c_1(k)2^{1/2}\phi(2t-k) \\
 c_1(k) &= \langle f(t)\phi_{1k}(t) \rangle = \int f(t)2^{1/2}\phi(2t-k)dt \\
 c_1(0) &= \langle f(t)\phi_{10}(t) \rangle = \int_0^{1/2} e^{-5t}2^{1/2}\phi(2t)dt = \frac{1}{5}(1-e^{-2.5}) \times 2^{1/2} \\
 c_1(1) &= \langle f(t)\phi_{11}(t) \rangle = \int_{1/2}^1 e^{-5t}2^{1/2}\phi(2t-1)dt = \frac{1}{5}(e^{-2.5}-e^{-5})2^{1/2} \\
 f(t) &= c_1(0)2^{1/2}\phi(2t) + c_1(1)2^{1/2}\phi(2t-1) = (2/5)(1-e^{-2.5})\phi(2t) + (2/5)(e^{-2.5}-e^{-5})\phi(2t-1) \\
 \text{b. } f(t) &= \sum_{k=-\infty}^{\infty} c_0(k)\phi_{0k}(t) + \sum_{k=-\infty}^{\infty} d_0(k)\psi_{0k}(t)
 \end{aligned}$$

$$c_0(0) = \langle f(t)\phi_{00}(t) \rangle = \int_0^1 e^{-5t} \phi(t) dt = (1/5)(1 - e^{-5})$$

$$d_0(0) = \langle f(t)\psi_{00}(t) \rangle = \int_0^1 e^{-5t} \psi(t) dt = \int_0^{1/2} e^{-5t} dt + \int_{1/2}^1 e^{-5t} (-1) dt = (1/5)(1 - 2e^{-2.5} + e^{-5})$$

$$f(t) = c_0(0)\phi(t) + d_0(0)\psi(t) = (1/5)(1 - e^{-5})\phi(t) + (1/5)(1 - 2e^{-2.5} + e^{-5})\psi(t)$$

12.21

$$h_0(k) = [0.707 \ 0.707] \text{ and } h_1(k) = [0.707 \ -0.707]$$

$$\begin{aligned} \text{a. } \sum_{k=-\infty}^{\infty} \sqrt{2}h_0(k)\phi(4t-k) &= \sqrt{2}h_0(0)\phi(4t) + \sqrt{2}h_0(1)\phi(4t-1) \\ &= \sqrt{2} \times 0.707\phi(4t) + \sqrt{2} \times 0.707\phi(4t-1) = \phi(4t) + \phi(4t-1) = \phi(2t) \end{aligned}$$

$$\begin{aligned} \text{b. } \sum_{k=-\infty}^{\infty} \sqrt{2}h_1(k)\phi(4t-k) &= \sqrt{2}h_1(0)\phi(4t) + \sqrt{2}h_1(1)\phi(4t-1) \\ &= \sqrt{2} \times 0.707\phi(4t) + \sqrt{2}(-0.707)\phi(4t-1) = \phi(4t) - \phi(4t-1) = \psi(2t) \end{aligned}$$

12.22

$$h_0(k) = [0.483 \ 0.837 \ 0.224 \ -0.129]$$

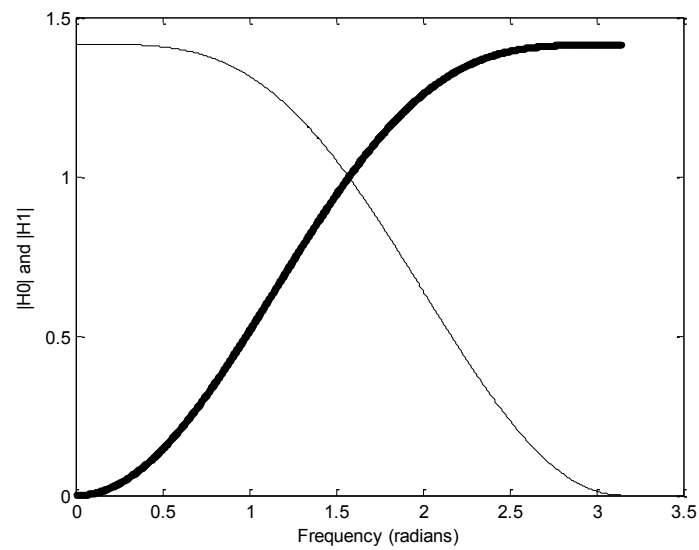
$$h_1(k) = (-1)^k h_0(4-1-k), \quad h_1(0) = (-1)^0 h_0(4-1-0) = h_1(3) = -0.129$$

$$h_1(1) = (-1)^1 h_0(4-1-1) = -h_1(2) = -0.224, \quad h_1(2) = (-1)^2 h_0(4-1-2) = h_1(1) = 0.837$$

$$h_1(3) = (-1)^3 h_0(4-1-3) = -h_1(0) = -0.483$$

$$h_1(k) = [-0.129 \ -0.224 \ 0.837 \ -0.483]$$

```
>> h0=[0.483 0.837 0.224 -0.129]
>> h1=[-0.129 -0.224 0.837 -0.483 ]
>> [H0,w]=freqz(h0,1,1000);
>> [H1,w]=freqz(h1,1,1000);
>> plot(w,abs(H0),'k',w,abs(H1),'k');
>> xlabel('Frequency (radians)');ylabel('|H0| and |H1|')
```



12.23

$$w(k) = [5.5000 \quad 0.5000 \quad 7.0711 \quad 2.1213]$$

12.24

$$w(k) = [3.5355 \quad 5.6569 \quad -0.5000 \quad 0.5000 \quad 7.0711 \quad 0.7071 \quad 0.7071 \quad -1.4142]$$

12.25

$$c(k) = [2.2929 \quad 3.7071 \quad 2.4142 \quad -0.4142]$$

12.26

$$c(k) = [1.6213 \quad 1.6213 \quad 2.6213 \quad 2.6213 \quad 1.7071 \quad 1.7071 \quad -0.2929 \quad -0.2929]$$

12.27

$$c(k) = [2.1213 \quad 3.5355 \quad 2.8284 \quad 0]$$

12.28

a. $W = [100 \ 20 \ 16 \ -5 \ -3 \ 4 \ 2 \ -6]$

b. $W = [100 \ 20 \ 16 \ -5]$

c. $W = [100 \ 20]$

d. $W = [100]$

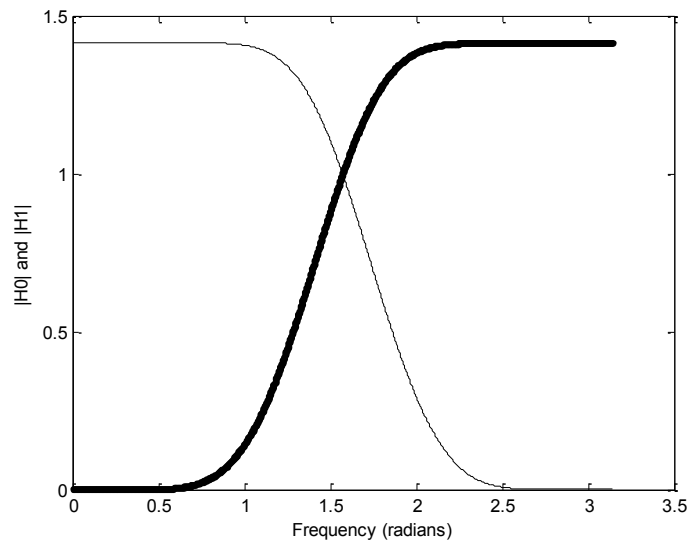
12.29

MATLAB program

```

-----
clc, clear all; close all;
%Problem 13.29
h0=[0.054415842243144    0.312871590914520    0.675630736297712    ...
     0.585354683654425   -0.015829105256675   -0.284015542962009   ...
     0.000472484573805    0.128747426620538   -0.017369301001845   ...
    -0.044088253930837    0.013981027917411    0.008746094047413   ...
    -0.004870352993456   -0.000391740373377    0.000675449406451   ...
    -0.000117476784125];
for k=0:15
    h1(k+1)=(-1)^(k)*h0(16-k);
end
disp('Output p(2n)=>');
p=xcorr(h0,h0);
p(2:2:length(p))
disp('output R(z)=>');
p
disp('Output R(-z)=');
pp=xcorr(h1,h1)
disp('Output R(z)+R(-z)=');
p+pp
[H0,w]=freqz(h0,1,1000);
[H1,w]=freqz(h1,1,1000);
plot(w,abs(H0),'k',w,abs(H1),'k. ');
xlabel('Frequency (radians)');ylabel('|H0| and |H1|')
-----

```



12.30

See answers in problems 12.23-12.27.

12.31

MATLAB program

```

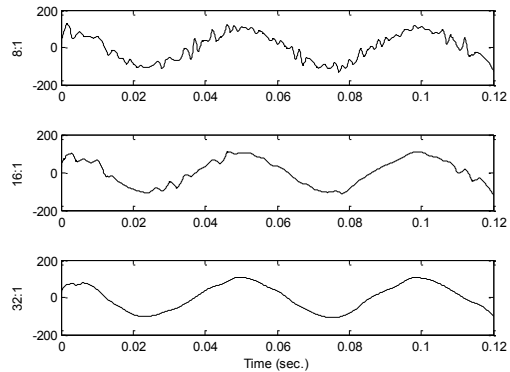
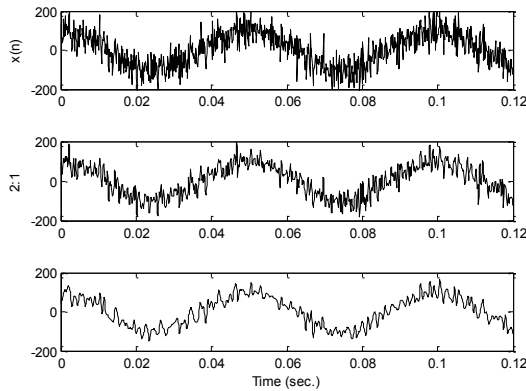
close all; clear all; clc
t=0:1:1023;t=t/8000;
x=100*cos(20*2*pi*t)+50*randn(1,1024);
h0=[0.230377813308896    0.714846570552915    0.630880767929859 ...
    -0.027983769416859   -0.187034811719092    0.030841381835561 ....
    0.032883011666885   -0.010597401785069];
N=1024; nofseg=1
rec_sig=[]; rec_sig2t1=[]; rec_sig4t1=[]; rec_sig8t1=[];
rec_sig16t1=[];
rec_sig32t1=[];
for i=1:nofseg
    sp=x((i-1)*1024+1:i*1024);
    w=dwt(h0,sp,10);
% Quantization
    wmax=round(max(abs(w)));
    wcode=round(2^15*w/wmax); % 16-bit code for storage
    w=wcode*wmax/2^15; % Recovered wavelet coefficients
    w(513:1024)=zeros(1,512); % 2:1 compression ratio
    sig_rec2t1=idwt(h0,w,10);
    rec_sig2t1=[rec_sig2t1 sig_rec2t1'];
    w(257:1024)=0; % 4:1 compression ratio
    sig_rec4t1=idwt(h0,w,10);
    rec_sig4t1=[rec_sig4t1 sig_rec4t1'];
    w(129:1024)=0; % 8:1 compression ratio
    sig_rec8t1=idwt(h0,w,10);
    rec_sig8t1=[rec_sig8t1 sig_rec8t1'];
    w(65:1024)=0; % 16:1 compression ratio
    sig_rec16t1=idwt(h0,w,10);
    rec_sig16t1=[rec_sig16t1 sig_rec16t1'];
    w(33:1024)=0; % 32:1 compression ratio
    sig_rec32t1=idwt(h0,w,10);
    rec_sig32t1=[rec_sig32t1 sig_rec32t1'];
end
subplot(3,1,1),plot(t,x,'k'); axis([0 0.12 -200 200]);ylabel('x(n)');
subplot(3,1,2),plot(t,rec_sig2t1,'k'); axis([0 0.12 -200
200]);ylabel('2:1');
subplot(3,1,3),plot(t,rec_sig4t1,'k'); axis([0 0.12 -200
200]);ylabel(4:1);
xlabel('Time (sec.)')
NN=min(length(x),length(rec_sig2t1)); axis([0 0.12 -200 200]);
figure (2)
subplot(3,1,1),plot(t,rec_sig8t1,'k'); axis([0 0.12 -200
200]);ylabel('8:1');
subplot(3,1,2),plot(t,rec_sig16t1,'k'); axis([0 0.12 -200
200]);ylabel('16:1');
subplot(3,1,3),plot(t,rec_sig32t1,'k'); axis([0 0.12 -200
200]);ylabel('32:1');
xlabel('Time (sec.)')
NN=min(length(x),length(rec_sig2t1)); axis([0 0.12 -200 200]);
err=rec_sig2t1(1:NN)-x(1:NN);
SNR=sum(x.*x)/sum(err.*err);
disp('Reconstruction SNR dB (2:1)=>');

```

```

SNR=10*log10(SNR)
err=rec_sig4t1(1:NN)-x(1:NN);
SNR=sum(x.*x)/sum(err.*err);
disp('Reconstruction SNR dB (4:1)=>');
SNR=10*log10(SNR)
err=rec_sig8t1(1:NN)-x(1:NN);
SNR=sum(x.*x)/sum(err.*err);
disp('Reconstruction SNR dB (8:1)=>');
SNR=10*log10(SNR)
err=rec_sig16t1(1:NN)-x(1:NN);
SNR=sum(x.*x)/sum(err.*err);
disp('Reconstruction SNR dB (16:1)=>');
SNR=10*log10(SNR)
err=rec_sig32t1(1:NN)-x(1:NN);
SNR=sum(x.*x)/sum(err.*err);
disp('Reconstruction SNR dB (32:1)=>');
SNR=10*log10(SNR)

```



Reconstruction SNR dB (2:1)=>
SNR =8.0710

Reconstruction SNR dB (4:1)=>
SNR = 6.1910

Reconstruction SNR dB (8:1)=>

SNR =5.4304
Reconstruction SNR dB (16:1)=>

SNR = 5.0725
Reconstruction SNR dB (32:1)=>

SNR = 4.7970

Chapter 13

13.1

a. 76.8 Kbytes; b. 921.6 Kbytes; c. 1920.768 Kbytes.

13.2

a. $2^{16} = 65536$ b. $2^{24} = 16777216 = 16.777 \times 10^6$

13.3

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.274 & -0.322 \\ 0.212 & -0.523 & 0.311 \end{bmatrix} \begin{bmatrix} 200 \\ 10 \\ 100 \end{bmatrix} = \begin{bmatrix} 142 \\ 54 \\ 11 \end{bmatrix}$$

$Y = 142, I = 54, Q = 11$

13.4

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1.000 & 0.956 & 0.621 \\ 1.000 & -0.272 & -0.647 \\ 1.000 & -1.106 & 1.703 \end{bmatrix} \begin{bmatrix} 141 \\ 46 \\ 5 \end{bmatrix} = \begin{bmatrix} 188 \\ 125 \\ 99 \end{bmatrix}$$

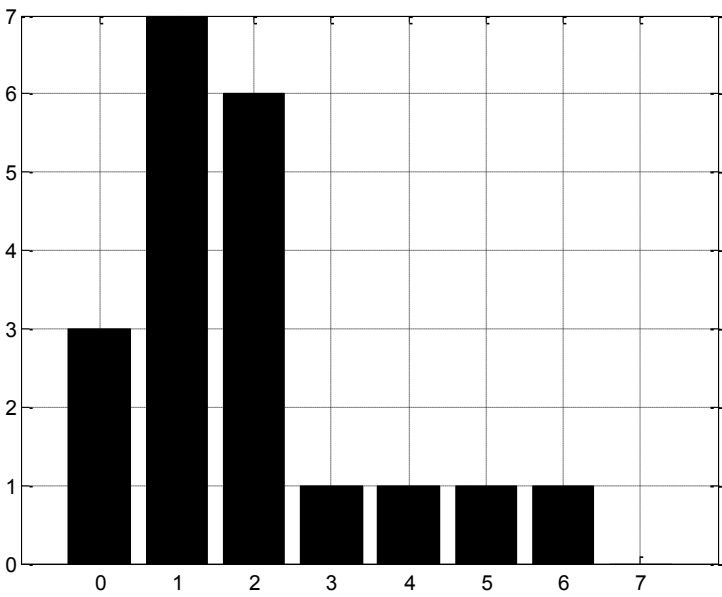
$R = 188, G = 125, B = 99$

13.5

$$Y = 0.299 \times \begin{bmatrix} 100 & 50 \\ 100 & 150 \end{bmatrix} + 0.587 \times \begin{bmatrix} 20 & 40 \\ 10 & 30 \end{bmatrix} + 0.114 \times \begin{bmatrix} 100 & 50 \\ 200 & 150 \end{bmatrix} = \begin{bmatrix} 53 & 44 \\ 59 & 50 \end{bmatrix}$$

13.6

Pixel $p(m,n)$ level	Number of pixels
0	3
1	7
2	6
3	1
4	1
5	1
6	1
7	0

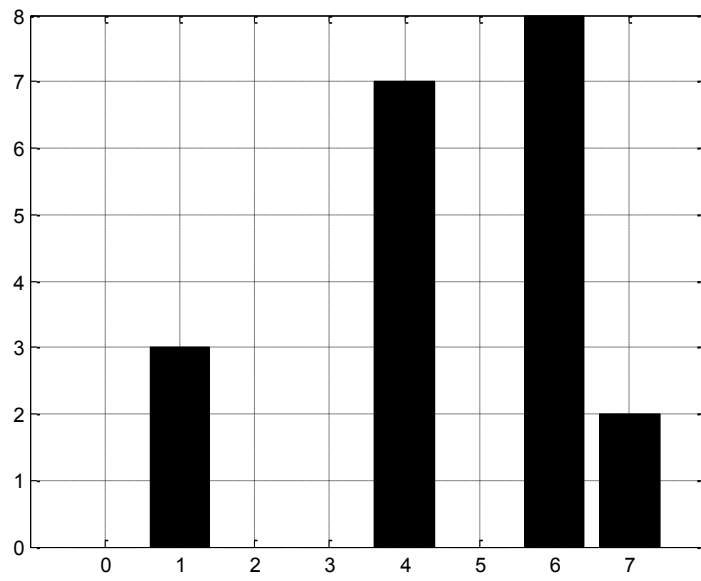


13.7

Pixel $p(m,n)$ level	Number of pixels	Number of pixels $\leq p(m,n)$	Equalized pixel level
0	3	3	1
1	7	10	4
2	6	16	6
3	1	17	6
4	1	18	6
5	1	19	7
6	1	20	7
7	0	20	7

Equalized image $\begin{bmatrix} 1 & 4 & 6 & 6 & 1 \\ 6 & 4 & 4 & 6 & 4 \\ 4 & 4 & 6 & 6 & 6 \\ 1 & 6 & 7 & 7 & 4 \end{bmatrix}$

Pixel $p(m,n)$ level	Number of pixels
0	0
1	3
2	0
3	0
4	7
5	0
6	8
7	2



13.8

Full range: $H = 4$, $L = 2$, Top = $2^3 - 1 = 7$, Bottom=0

Shift up: $H = 4$, $L = 2$, Top = 7, Bottom=3

Shift-down: $H = 4$, $L = 2$, Top = 3, Bottom=0

Pixel $p(m,n)$ level	Full range	Range [3 7]	Range [0-3]
2	0	3	0
3	4	5	2
4	7	7	3

$$\text{Full range: } \begin{bmatrix} 0 & 7 & 7 & 0 \\ 0 & 4 & 4 & 4 \\ 7 & 7 & 7 & 0 \\ 4 & 0 & 4 & 7 \end{bmatrix}, \text{ Shift up: } \begin{bmatrix} 3 & 7 & 7 & 3 \\ 3 & 5 & 5 & 5 \\ 7 & 7 & 7 & 3 \\ 5 & 3 & 5 & 7 \end{bmatrix}, \text{ Shift down: } \begin{bmatrix} 0 & 3 & 3 & 0 \\ 0 & 2 & 2 & 2 \\ 3 & 3 & 3 & 0 \\ 2 & 0 & 2 & 3 \end{bmatrix}$$

13.9

Padding zeros for boundaries:

93	116	109	108	0
92	107	103	108	0
84	107	86	107	0
87	113	106	99	0
0	0	0	0	0

$$4 \times 4 \text{ Enhanced image: } \begin{bmatrix} 102 & 109 & 104 & 51 \\ 98 & 101 & 101 & 54 \\ 98 & 103 & 100 & 51 \\ 50 & 55 & 51 & 25 \end{bmatrix} \quad 4 \times 4 \text{ original image: } \begin{bmatrix} 100 & 100 & 100 & 100 \\ 100 & 100 & 100 & 100 \\ 100 & 100 & 100 & 100 \\ 100 & 100 & 100 & 100 \end{bmatrix}$$

13.10

padding zeros for boundaries:

0	0	0	0	0	0
0	100	255	100	100	0
0	0	255	255	100	0
0	100	0	100	0	0
0	100	255	100	100	0
0	0	0	0	0	0

$$4 \times 4 \text{ filtered image: } \begin{bmatrix} 0 & 100 & 100 & 0 \\ 0 & 100 & 100 & 100 \\ 0 & 100 & 100 & 100 \\ 0 & 100 & 0 & 0 \end{bmatrix} \quad 4 \times 4 \text{ original image: } \begin{bmatrix} 100 & 100 & 100 & 100 \\ 100 & 100 & 100 & 100 \\ 100 & 100 & 100 & 100 \\ 100 & 100 & 100 & 100 \end{bmatrix}$$

13.11

a.

$$\text{Vertical Sobel detector : } \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \text{ Processed image } \begin{bmatrix} 225 & 125 & 130 & 33 \\ 249 & 119 & 136 & 6 \\ 249 & 119 & 136 & 6 \\ 255 & 125 & 130 & 0 \\ 255 & 128 & 128 & 30 \end{bmatrix}$$

$$\text{b. Laplacian edge detector : } \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{ Processed image: } \begin{bmatrix} 0 & 106 & 106 & 0 \\ 106 & 255 & 255 & 106 \\ 106 & 255 & 255 & 106 \\ 117 & 223 & 223 & 117 \\ 0 & 117 & 117 & 0 \end{bmatrix}$$

13.12 The green color is dominant in the area pointed by the arrow; the blue color is dominant in the background

13.13 The blue color is dominant in the area pointed by the arrow; the red color is dominant in the background.

13.14

$$X(1,2) = 90.00 - j51.96 \text{ and } A(1,2) = 11.5470$$

13.15

$$X(u,v) = \begin{bmatrix} 460 & -40 \\ -240 & -140 \end{bmatrix} \text{ and } A(u,v) = \begin{bmatrix} 115 & 10 \\ 60 & 35 \end{bmatrix}$$

13.16

$$\text{Forward DCT: } F(u,v) = \begin{bmatrix} 230 & -20 \\ -120 & -70 \end{bmatrix}$$

13.17

$$\text{Inverse DCT: } p(i,j) = \begin{bmatrix} 110 & 100 \\ 100 & 90 \end{bmatrix}$$

13.18

$$d(n) = DC(n) - DC(n-1)$$

DPCM sequence: 400, -10, -40, 10, 10

DPCM sequence using the Huffman table:

(9, 110010000), (4, 0101), (6, 010111), (4, 1010), (4, 1010)

13.19

- a. (0,-2) (3,4), (2,-3), (0,7), (4, -2), (0,0)
 b. (0000, 0010, 01), (0011, 0011, 100), (0010, 0010, 00),
 (0000, 0011, 111), (0100, 0010, 01), (0000, 0000)

13.20

w = 230.0000 -20.0000;
 -120.0000 -70.0000

13.21

f = 110.0000 100.0000;
 100.0000 90.0000

13.22

W= 210.0000 130.0000 -30.0000 -20.0000
 185.0000 155.0000 15.0000 -15.0000
 40.0000 10.0000 -20.0000 0
 45.0000 25.0000 -25.0000 35.0000

13.23

f = 115.0000 145.0000 25.0000 45.0000
 105.0000 135.0000 5.0000 25.0000
 30.0000 20.0000 7.5000 27.5000
 10.0000 -0.0000 -7.5000 12.5000

13.24

The trace jumping from the end of horizontal scan line to the beginning of the next horizontal line scan in the same frame is called the horizontal retrace. The trace from the end of the bottom line to the beginning of the top line in the next frame is called the vertical retrace. The vertical retrace takes longer time. See Section 13.9.1

13.25

The interlaced scanning contains the odd field and even field per frame. The purpose of using interlaced scanning is to transmit a full frame quickly to reduce flicker. See Section 14.9.1

13.26

NTSC bandwidth = 6 MHz, luminance Y bandwidth = 4.2 MHz, I bandwidth= 1.6 MHz, and Q bandwidth = 0.6 MHz. See section 14.9.1

13.27

Frequency modulated (FM) using a peak frequency deviation of 25 kHz. Assuming the audio baseband bandwidth is 15 kHz, the stereo FM audio requires a transmission bandwidth of 80 kHz with an audio carrier located at 4.5 MHz relative to the picture carrier. FM (frequency modulation). See Section 14.9.1

13.28

$$\begin{aligned} \text{Composite} \times 2 \sin(2\pi f_{sc} t) &= Y \times 2 \sin(2\pi f_{sc} t) + I \cos(2\pi f_{sc} t) \times 2 \sin(2\pi f_{sc} t) + Q \times 2 \sin^2(2\pi f_{sc} t) \\ &= Y \times 2 \sin(2\pi f_{sc} t) + I \sin(2 \times 2\pi f_{sc} t) + Q - Q \cos(2 \times 2\pi f_{sc} t) \end{aligned}$$

Then apply lowpass filtering

13.29

The back porch of the blanking contains the color sub-carrier burst for the color demodulation. The color burst carrier is centered at 3.58 MHz above the picture carrier and has the 8 cycles. See Section 13.9.1

13.30

See Table 13.9.1.

13.31

The scan line rate 525 lines per frame x 30 frames per second = 15.75 kHz

The vertical synchronizing pulse rate (used with equalization pulses to provide timing) = 31.5 kHz. See Section 13.9.1

13.32

The method (c) since the chroma is down sampled both horizontally and vertically by a factor of 2. See Section 13.9.2

13.33

The progressive scanning traces a whole picture which is called the frame via row-wise, the interlaced scanning retraces the odd field and even field in each frame alternatively. CIF uses the progressive scan. See Section 13.9.2

13.34

b. takes more computations since finding the motion vectors requires using the sequential search algorithm to obtain the best matched microblock in the reference frame. See Section 13.10

13.35

$$\frac{80 \times 80}{16 \times 16} (16^2 \times 32^2 \times 3) = 19.661 \times 10^6 \text{ operations}$$

13.36

Refer to Program 13.1.

13.37

Refer to Program 13.2.

The bigger the SIGMA, the more blurred the image is.

The bigger the kernel size, the more blurred the image is

13.38 Refer to Program 13.2.

The bigger the kernel size of the median filter, the blurred the image is

13.39

Refer to Program 13.2.

13.40

Refer to Program 13.3

13.41

```

close all; clear all; clc
% This test program is a part of simulation for Grayscale JPEG image
% compression.
% image size: 240x320
X=imread('cruiseorg','TIFF');           %note the image size= 240x320
Y=rgb2gray(X);                          %Y =grayscale image
size(Y)
Q=[ 16 11 10 16 24 40 51 61;           %quantization quality factor
    12 12 14 19 26 58 60 55;
    14 13 16 24 40 57 69 56;
    14 17 22 29 51 87 80 62;
    18 22 37 56 68 109 103 77;
    24 35 55 64 81 104 113 92;
    49 64 78 87 103 121 120 101;
    72 92 95 98 112 100 103 99];
factor=1;
Q=factor*Q;
count=0; % number of image blocks; total number of blocks =30x40=1200
for i=1:30
    for j=1:40
        count=count+1;
        P(1:8,1:8,count)=Y((i-1)*8+1:i*8,(j-1)*8+1:j*8);
    end
end
for m=1:1200
    F(:,:,m)=round(dct2(P(:,:,m))); % 2-D DCT transform
    FF(:,:,m)=round(F(:,:,m) ./Q); % quantization of DCT coefficients
end
%Coding and lossless compression. (not included)
%decoding
for m=1:1200
    FC(:,:,m)=FF(:,:,m) .*Q; % recover 2-D DCT coefficients
    PP(:,:,m)=round(idct2(FC(:,:,m))); % recover the 8x8 image image
    err(:,:,m)=single(PP(:,:,m))-single(P(:,:,m)); % quantization vector
end

```

```

count=0;
for i=1:30
    for j=1:40
        count=count+1;
        YY((i-1)*8+1:i*8,(j-1)*8+1:j*8)=PP(1:8,1:8,count); % recover image
    end
end
YY=uint8(YY); % level adjustment
figure
subplot(1,2,1),imshow(Y); % original image
title('Original image');
subplot(1,2,2),imshow(YY) % JPEG image
title('JPEG compressed image');

```

I. Excellent image, II. Begin to see the block effects.

III. Image degradation with the block effects become severe. The image quality is unacceptable.

13.42

(a), (b), and (c)

```

close all; clear all; clc
X=imread('cruiseorg','TIFF');
Y=rgb2gray(X);
h0=[ 0.230377813308896    0.714846570552915    0.630880767929859 ...
     -0.027983769416859   -0.187034811719092    0.030841381835561 ...
      0.032883011666885   -0.010597401785069];
M= length(h0);
h1(1:2:M-1) = h0(M:-2:2);h1(2:2:M) = -h0(M-1:-2:1);%Obtain QMF highpass
filter
[m n]=size(Y);
%level-1 transform
[m n]=size(Y);
for i=1:m
    W1(i,:)=dwt(h0,double(Y(i,:)),1)';
end
for i=1:n
    W1(:,i)=dwt(h0,W1(:,i),1); % Wavelet coefficients at level-1
end
% finished
%level-2 transform
Y1=W1(1:m/2,1:n/2); %Obtain LL subband
[m n]=size(Y1);
for i=1:m
    W2(i,:)=dwt(h0,Y1(i,:),1)';
end
for i=1:n
    W2(:,i)=dwt(h0,W2(:,i),1);
end
% finished

```

```

W22=W1;
W22(1:m,1:n)=W2; % wavelet coefficients at level-2 transform
wmax=max(max(abs(W22)));
% 8-bit Quantization
W22=round(W22*2^7/wmax);
W22=double(W22)*wmax/2^7;
figure(1), imshow(uint8(W22));
% Reconstruction with various methods
[m, n]=size(W22);
method=input('Reconstuction methods: 1, 2, 3, 4, 5=>');
% Method I: use all the coefficnets
if method ==1
end
% Method II: only use LL2
if method ==2
    WW=zeros(m,n);
    WW(1:m/4,1:n/4)=W22(1:m/4,1:n/4);
    W22=WW;%Discard HL2,LH2, HH2, HL1, LH1, HH1 subbands
end
% Method III: Use LL2, HL2, LH2, and HH2
if method ==3
    WW=zeros(m,n);
    WW(1:m/2,1:n/2)=W22(1:m/2,1:n/2);
    W22=WW;
end
% Method IV: LL2, HL2, LH2
if method ==4
    WW=zeros(m,n);
    WW(1:m/2,1:n/2)=W22(1:m/2,1:n/2);
    W22=WW;
    WW22(m/4+1:m/2:n/4+1:n/2)=0; % set HH2 to be zeros
end
% Method V: LL2, HL2, LH2, HH2, LH1, HL1
if method == 5
    W22(m/2+1:m,n/2+1:n)=0;
end
% decoding from Level-2 transform
[m,n]=size(W22);
Wd2=W22(1:m/2,1:n/2);

%level-2
[m n]=size(Wd2);
for i=1:n
    Wd1(:,i)=idwt(h0,double(Wd2(:,i)),1);
end
for i=1:m
    Wd1(i,:)=idwt(h0,double(Wd1(i,:))',1);
end
%level-1
[m, n]=size(W22);Yd11=W22;
Yd11(1:m/2,1:n/2)=Wd1;
for i=1:n
    Yd(:,i)=idwt(h0,Yd11(:,i),1);
end
for i=1:m
    Yd(i,:)=idwt(h0,double(Yd(i,:))',1);
end

```

```
% finished
figure (2), imshow(Y);
Y11=uint8(Yd);
figure (3), imshow(Y11);
```

d.

Quality in decreasing order: I, V, III, IV, II

Chapter 14

14.1

$$0.2560123 \times 2^{15} = 8389_{10} = 0010000011000101$$

$$0.2560123 \text{ (decimal)} = 0.010000011000101 \text{ (Q-15)}$$

14.2

$$-0.2160123 \times 2^{15} = -7078_{10} = 1110010001011010$$

$$-0.2160123 \text{ (decimal)} = 1.110010001011010 \text{ (Q-15)}$$

14.3

$$-0.3567921 \times 2^{15} = -11691_{10} = 1101001001010101$$

$$-0.3567921 \text{ (decimal)} = 1.101001001010101 \text{ (Q-15)}$$

14.4

$$0.4798762 \times 2^{15} = 15725_{10} = 0011110101101101$$

$$0.4798762 \text{ (decimal)} = 0.011110101101101 \text{ (Q-15)}$$

14.5

$$0.101010001011110 = 0.6591186$$

$$1.010101110100010 \text{ (Q-15)} = -0.6591186$$

14.6

$$0.001000111101110 \text{ (Q-15)} = 0.1400756$$

14.7

$$0.110101000100010 \text{ (Q-15)} = 0.8291626$$

14.8

$$0.110111011010001 = 0.3657532$$

$$1.101000100101111 \text{ (Q-15)} = -0.3657532$$

14.9

$$1.101010111000001 + 0.010001111011010 \\ = 1.111100110011011$$

14.10

$$0.001010101000001 + 0.010101111010010 \\ = 0.100000100010011$$

14.11

$$1.001010101000001 + 1.010101111010010 \\ = 0.100000100010011 \text{ (overflow, since adding two negative numbers results in a positive number)}$$

14.12

$$0.001010101000001 + 1.010101111010010 \\ = 1.100000100010011$$

14.13

a. $0.1101235 = 0.880988 \times 2^{-3}$, $-3 = 1101$ (exponent bits)
 $0.880988 \times 2^{11} = 1804_{10} = 011100001100$ (Q-11)

Cascading the exponent bits and the mantissa bits yields

$$1101 \ 011100001100$$

b. $-10.430527 = -0.651908 \times 2^4$, $4 = 0100$ (exponent bits)
 $-0.651908 \times 2^{11} = -1335_{10} = 101011001001$ (Q-11)

Cascading the exponent bits and the mantissa bits yields

$$0100 \ 101011001001$$

14.14

a. $2.5568921 = 0.6392230 \times 2^2$, $2 = 0010$ (exponent bits)
 $0.6392230 \times 2^{11} = 1309_{10} = 010100011101$ (Q-11)

Cascading the exponent bits and the mantissa bits yields

$$0010 \ 010100011101$$

b. $-0.678903 = -0.678903 \times 2^0$, $0 = 0000$ (exponent bits)
 $-0.678903 \times 2^{11} = -1390_{10} = 101010010010$ (Q-11)

Cascading the exponent bits and the mantissa bits yields

$$0000 \ 101010010010$$

c. $0.0 = 0.0 \times 2^0$, $0 = 0000$ (exponent bits)
 $0.0 \times 2^{11} = 0_{10} = 000000000000$ (Q-11)

Cascading the exponent bits and the mantissa bits yields

$$0000 \ 000000000000$$

d. $-1.0 = -0.5 \times 2^1$, $0 = 0001$ (exponent bits)
 $-0.5 \times 2^{11} = -1024_{10} = 110000000000$ (Q-11)

Cascading the exponent bits and the mantissa bits yields

$$0001 \ 110000000000$$

14.15

$$1101\ 011100011011\ (\text{floating}) = 0.8881835 \times 2^{-3}\ (\text{decimal})$$

$$0100\ 101111100101\ (\text{floating}) = -0.5131835 \times 2^4\ (\text{decimal})$$

$$0.8881835 \times 2^{-3}\ (\text{decimal}) = 0.0069389 \times 2^4\ (\text{decimal}) = 0100\ 000000001110\ (\text{floating})$$

$$0100\ 101111100101\ (\text{floating}) + 0100\ 000000001110\ (\text{floating})$$

$$= 0100\ 10111110010\ (\text{floating}) = -8.1016\ (\text{decimal})$$

14.16

$$0111\ 110100011011\ (\text{floating}) = -0.3618164 \times 2^7\ (\text{decimal})$$

$$0101\ 001000100101\ (\text{floating}) = 0.2680664 \times 2^5\ (\text{decimal})$$

$$0.2680664 \times 2^5\ (\text{decimal}) = 0.0670166 \times 2^7\ (\text{decimal}) = 0111\ 000010001001\ (\text{floating})$$

$$0111\ 110100011101\ (\text{floating}) + 0111\ 000010001001\ (\text{floating})$$

$$= 0111\ 110110100100\ (\text{floating}) = -37.75\ (\text{decimal})$$

14.17

$$0001\ 0000000010011\ (\text{floating}) = 0.00927734 \times 2^1\ (\text{decimal})$$

$$0100\ 0010000000101\ (\text{floating}) = 0.25244141 \times 2^4\ (\text{decimal})$$

$$0.00927734 \times 2^1\ (\text{decimal}) = 0.00115967 \times 2^4\ (\text{decimal}) = 0100\ 000000000010\ (\text{floating})$$

$$0100\ 000000000010\ (\text{floating}) + 0100\ 0010000000101\ (\text{floating})$$

$$= 0100\ 001000000111\ (\text{floating}) = 4.0546875\ (\text{decimal})$$

14.18

$$(-1)^1 \times 1.025 \times 2^{160-127} = -8.8047 \times 10^9$$

14.19

$$(-1)^0 \times 1.625 \times 2^{164-127} = 2.2334 \times 10^{11}$$

14.20

$$(-1)^0 \times 1.625 \times 2^{1536-1023} = 4.3575 \times 10^{154}$$

14.21

$$(-1)^0 \times 1.325 \times 2^{1536-1023} = 3.5531 \times 10^{154}$$

14.22

$$B = 1$$

$$S = |h(0)| + |h(1)| + |h(2)| = 0.2 + 0.6 + 0.2 = 1$$

$$x_s(n) = x(n), \quad y_s(n) = -0.2x_s(n) + 0.6x_s(n-1) + 0.2x_s(n-2), \quad y(n) = y_s(n)$$

14.23

$$H(z) = \frac{0.6z}{z-0.3}, \quad h(n) = 0.6(0.3)^n u(n), \quad S = 0.6(1 + 0.3 + 0.3^2 + \dots) \quad \frac{1}{1-0.3} = \frac{6}{7}$$

Select: $S = 1, B = 1, C = 1$

Direct form I:

$$x_s(n) = x(n), \quad y_s(n) = 0.6x_s(n) + 0.3y_f(n-1), \quad y_f(n) = y_s(n), \quad y(n) = y_s(n)$$

14.24

$$A(z) = \frac{1}{1-0.3z^{-1}} = \frac{z}{z-0.3}, \quad h(n) = (0.3)^n u(n), \quad S = (1 + 0.3 + 0.3^2 + \dots) \quad \frac{1}{1-0.3} = \frac{10}{7}$$

Select: $S = 2, A = 1, B = 1$

Direct form II:

$$x_s(n) = x(n)/2, \quad w_s(n) = x_s(n) + 0.3w(n-1), \quad w(n) = w_s(n)$$

$$y_s(n) = 0.6w(n), \quad y(n) = (B \times S)y_s(n) = 2y_s(n)$$

14.25

 $B = 2$

$$S = \frac{1}{2}(|h(0)| + |h(1)| + |h(2)|) = \frac{1}{2}(0.36 + 1.6 + 0.36) = 1.16, \text{ choose } S = 2$$

$$x_s(n) = \frac{x(n)}{2}, \quad y_s(n) = -0.18x_s(n) + 0.8x_s(n-1) + 0.18x_s(n-2), \quad y(n) = 4y_s(n)$$

14.26

 $C = 2$

$$H(z) = \frac{1.35}{1-0.3z^{-1}} = \frac{1.35z}{z-0.3} \quad h(n) = 1.35 \times (0.3)^n u(n)$$

$$S = 0.5 \times (1.35(0.3)^0 + 1.35(0.3)^1 + 1.35(0.3)^2 + \dots) \quad \frac{1.35 \times 1}{1-0.3} = 0.9643$$

Choose $S = 1$

$$x_s(n) = x(n), \quad y_s(n) = 0.675x_s(n) + 0.15y_f(n-1), \quad y_f(n) = 2y_s(n), \quad y(n) = y_f(n)$$

14.27

 $A = 1, B = 2$

$$A(z) = \frac{1}{1-0.3z^{-1}} = \frac{z}{z-0.3} \quad h(n) = (0.3)^n u(n)$$

$$S = 0.5 \times ((0.3)^0 + (0.3)^1 + (0.3)^2 + \dots) \quad \frac{1}{1-0.3} = \frac{5}{7}$$

Choose $S = 1$

Direct form II:

$$x_s(n) = x(n), \quad w_s(n) = x_s(n) + 0.3w(n-1), \quad w(n) = w_s(n)$$

$$y_s(n) = 0.675w(n), \quad y(n) = (B \times S)y_s(n) = 2y_s(n)$$

14.28

$$H(z) = \frac{0.72 + 1.42z^{-1} + 0.72z^{-2}}{1 + 1.35z^{-1} + 0.5z^{-2}}$$

Using MATLAB function leads

>> h=impz([0.72 1.42 0.72],[1 1.35 0.5]);

>> sf=sum(abs(h))

sf = 1.7876

Choose $S = 2$.Choosing $C = 2$ to scale down the filter coefficients by half.

$$x_s(n) = x(n)/2,$$

$$y_s(n) = 0.36x_s(n) + 0.71x_s(n-1) + 0.36x_s(n-2) - 0.675y_f(n-1) - 0.25y_f(n-2),$$

$$y_f(n) = 2y_s(n), \quad y(n) = 2y_f(n)$$

14.29

$$A(z) = \frac{1}{1 + 1.35z^{-1} + 0.5z^{-2}}$$

Using MATLAB function leads

>> h=impz(1,[1 1.35 0.5]);

>> sf=sum(abs(h))

sf = 7.0504

Choose $S = 8$.Choosing $A = 2$ to scale down the denominator coefficients by half.

Since the second adder output after scaling is

$$y_s(n) = \frac{0.72}{B}w(n) + \frac{1.42}{B}w(n-1) + \frac{0.72}{B}w(n-2)$$

$$B = 4$$

$$x_s(n) = x(n)/8, \quad w_s(n) = 0.5x_s(n) - 0.675w(n-1) - 0.25w(n-2), \quad w(n) = 2w_s(n)$$

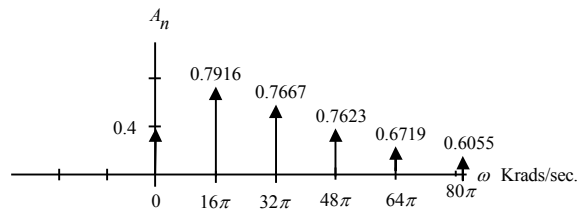
$$y_s(n) = 0.18w(n) + 0.355w(n-1) + 0.18w(n-2), \quad y(n) = 32y_s(n)$$

Appendix B

B.1

a. $A_0 = 0.4, A_1 = 0.7916, A_2 = 0.7667, A_3 = 0.7263, A_4 = 0.6719, A_5 = 0.6055$

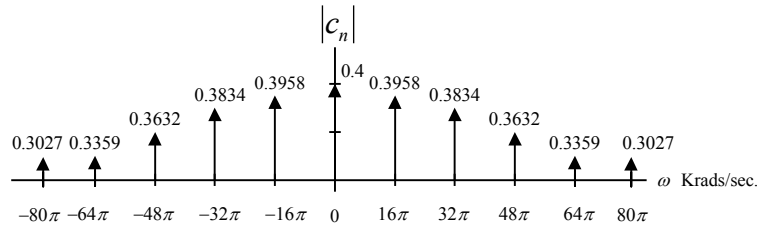
$$\omega_0 = 2\pi \times 8000 = 16000\pi$$



b.

$$|c_0| = 0.4, \quad |c_1| = |c_{-1}| = 0.3958,$$

$$|c_2| = |c_{-2}| = 0.3834, |c_3| = |c_{-3}| = 0.3632, |c_4| = |c_{-4}| = 0.3359, |c_5| = |c_{-5}| = 0.3027$$



B.2

a.

$$x(t) = \frac{4A}{\pi} \left(\cos \omega_0 t - \frac{1}{3} \cos 3\omega_0 t + \frac{1}{5} \cos 5\omega_0 t - \frac{1}{7} \cos 7\omega_0 t + \dots \right)$$

$$A = 10, T_0 = 1 \text{ ms}, f_0 = 1000 \text{ Hz}, \omega_0 = 2000\pi \text{ radians/second}$$

$$x(t) = \frac{40}{\pi} \cos 2000\pi t - \frac{40}{3\pi} \cos 3000\pi t + \frac{40}{5\pi} \cos 10000\pi t - \frac{40}{7\pi} \cos 14000\pi t + \dots$$

$$x(t) = 12.73 \cos 2000\pi t + 4.24 \cos(3000\pi t + 180^\circ) \\ + 2.55 \cos 10000\pi t + 1.82 \cos(14000\pi t + 180^\circ) + \dots$$

b.

$$f_3 = 3000 \text{ Hz}, A_3 = 4.24 \text{ volts}$$

B.3

a.

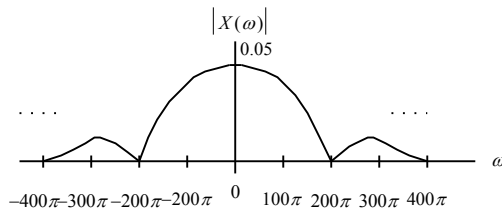
$$x(t) = 2 + 3.7420 \times \cos(2000\pi t) + 3.0273 \times \cos(4000\pi t) \\ + 2.0182 \times \cos(6000\pi t) + 0.9355 \times \cos(8000\pi t) + \dots$$

b.

$$f_2 = 2000 \text{ Hz}, A_2 = 3.0273$$

B.4

$$X(f) = 0.05 \left(\frac{\sin 0.01\pi f}{0.01\pi f} \right)$$



B.5

$$X(f) = 5 \left(\frac{\sin \pi f}{\pi f} \right)^2$$

B.6

$$X(f) = 5 \left(\frac{\sin \pi f}{\pi f} \right)$$

B.7

a. $X(s) = 10$, b. $X(s) = -100/s^2$, c. $X(s) = \frac{10}{s+2}$

d. $X(s) = \frac{2e^{-5s}}{s}$, e. $X(s) = \frac{10s}{s^2+9}$, f. $X(s) = \frac{14.14+7.07s}{s^2+9}$

g. $X(s) = \frac{3(s+2)}{(s+2)^2+9}$, h. $X(s) = \frac{12000}{s^6}$

B.8

a. $x(t) = 10e^{-2t}u(t)$, b. $x(t) = 100e^{-2t}u(t) - 100e^{-3t}u(t)$

c. $x(t) = -66.67e^{-2t}u(t) + 166.67e^{-5t}u(t)$, d. $x(t) = 5e^{-2t} \sin(5t)u(t)$

B.9

a. $X(s) = \frac{7.5}{s(s+1.5)}$, b. $x(t) = 5u(t) - 5e^{-1.5t}u(t)$

B.10

a. $X(s) = \frac{10}{s(s^2+3s+2)}$

b. $X(s) = \frac{10}{s(s+1)(s+2)} = \frac{5}{s} + \frac{-10}{s+1} + \frac{5}{s+2}$, $x(t) = 5u(t) - 10e^{-t}u(t) + 5e^{-2t}u(t)$

B.11

a. zero: $s = 3$, poles: $s = -2$, $s = -2$, stable

b. zeros: $s = 0$, $s = \pm 2.236j$, poles: $s = \pm 3j$, $s = -1 \pm 1.732j$, marginally stable

c. zeros: $s = \pm j$, $s = -1$, poles: $s = 0$, $s = -3$, $s = -4$, $s = -8$, $s = 1$, unstable

B.12

a. $h(t) = 5e^{-5t}u(t)$ b. $Y(s) = \frac{5}{s(s+5)}$ c. $y(t) = u(t) - e^{-5t}u(t)$

B.13

a. $H(j\omega) = \frac{1}{\frac{j\omega}{5} + 1}$, b. $A(\omega) = \frac{1}{\sqrt{1 + \left(\frac{\omega}{5}\right)^2}}$ $\beta(\omega) = \angle -\tan\left(\frac{\omega}{5}\right)$

c. $Y(j2) = 4.6424 \angle -21.80^\circ$ that is, $y_{ss}(t) = 4.6424 \sin(2t - 21.80^\circ)u(t)$

B.14

$$x(t) = u(t), \quad h(t) = 5e^{-5t}u(t)$$

$$y(t) = h(t) * u(t) = \int_0^{\infty} h(\tau)u(t-\tau)d\tau$$

$$= \int_0^t 5e^{-5\tau}d\tau = -e^{-5\tau} \Big|_0^t = -e^{-5t} - (-e^{-5 \times 0})$$

$$= 1 - e^{-5t}$$